REVIEW OF THE THEORY OF BOUND THREE-NUCLEON SYSTEMS: EXPERIMENTAL AND THEORETICAL ANALYSIS OF THE CHARGE FORM FACTORS OF ^3H AND ^3He

By

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TITLE: Review of the Theory of Bound Three-Nucleon Systems: Experimental and Theoretical Analysis of the Charge Form Factors of ^3H and ^3He

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SCOPE AND CONTENTS:

A review of theoretical analyses and predictions of the ^3He and ^3H bound-state properties is presented. The predictions of various theoretical models, with the choice of different nucleon-nucleon potentials and methods for solving the Schroedinger equation for the chosen potentials, are compared with one another. For given nucleon-nucleon forces (provided the potential has a soft-core), the diagonalisation method seems to be preferable to other methods. Among the nucleon-nucleon potentials, the Riihimaki potential is favourable compared to other realistic N-N forces.

The experimental data on ^3H and ^3He form factors together with Schiff's theoretical analysis of the ^3H and

^{3}He charge form factors is brought so that to study phenomenologically the wave function of a bound three-nucleon ($T = 1/2$, $J = 1/2$) system.

ACKNOWLEDGEMENTS

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INTRODUCTION

Study of three-nucleon form factors is beneficial from the point of understanding nuclear forces. Just like binding energies, the form factors are quantities, that can be measured to a sufficient degree of accuracy. Experiments on electron elastic scattering, done at the Stanford Linear Accelerator with 4-momentum transfer squared as high as 20 fm^{-2}, can tell about inner structure of the scattering nucleus. [It specifically applies to ^3He nucleus.]

The el.m. form factors of ^3H and ^3He can provide information about the ground state wave function of a three-nucleon bound system. L. I. Schiff gave an extensive theoretical analysis of ^3H and ^3He form factors [1]. It is known that in the ground state both ^3H and ^3He have J = 1/2. If nuclear forces are charge independent, ^3H and ^3He can be treated as belonging to an isospin doublet (T = 1/2) three-nucleon system. [In a model calculation of three scalar nucleons (i.e., three spin zero nucleons), admixture of T = 3/2 component in ^3He wave function was found to be of the order 0.01 - 0.001%.] [2] There can be three possible $^2S_{1/2}$ states of three-nucleon system: 1) fully symmetric in space coordinates of all three nucleons (S state), 2) antisymmetric in interchanging spatial coordinates (S_a), 3) mixed symmetry (S') state (which means that the state is either symmetric or antisymmetric under permutation of two nucleons out of three). Also, there can be three $^2P_{1/2}$ states (L = 1, S = 1/2), $^4P_{1/2}$ (L = 1, S = 3/2) state, and three $^4D_{1/2}$ (L = 2, S = 3/2) states, present in the wave function. Spin part consists of three types of functions. Let S_z be +1/2.

$$\left.\begin{aligned} X_1 &= 6^{-1/2}[(++-) + (+-+) - 2(-++)] \\ X_2 &= 2^{-1/2}[(++-) - (+-+)] \end{aligned}\right\} \quad S = 1/2$$

$$X_3 = 3^{-1/2}[(++-) + (+-+) + (-++)] \qquad S = 3/2 \quad .$$

A + (or −) in, say, the second position of a paranthesis means that nucleon 2 has spin up (down). Isospin part of three-nucleon wave function consists of

$$\eta_1 = 6^{-1/2}[(++-) + (+-+) - 2(-++)]$$

$$\eta_2 = 2^{-1/2}[(++-) - (+-+)] \quad , \quad \text{for } {}^3\text{He} \ (T_z = 1/2)$$

$$\eta_1 = 6^{-1/2}[(--+) + (-+-) - 2(+--)]$$

$$\eta_2 = 2^{-1/2}[(--+) - (-+-)] \quad , \quad \text{for } {}^3\text{H} \ (T_z = -1/2) \ .$$

The wave function, consisting of all possible products of spatial, spin and isospin components, has to be totally antisymmetrised, since the system consists of fermions. (Given explicitly in Chapters II and III).

The charge and magnetic moment density operators, if no distortion or mutual interference of nucleons is considered, are

$$\rho_C(\bar{r};\bar{r}_1,\bar{r}_2,\bar{r}_3) = \sum_{i=1}^{3} [\tfrac{1}{2}(1+\tau_{iz}) f^p_{ch}(\bar{r}-\bar{r}_i) + \tfrac{1}{2}(1-\tau_{iz}) f^n_{ch}(\bar{r}-\bar{r}_i)] \quad (1)$$

$$\rho_M(\bar{r};\bar{r}_1,\bar{r}_2,\bar{r}_3) = \sum_{i} [\tfrac{1}{2}\, \sigma_{iz}(1+\tau_{iz})\mu_p f^p_{mag}(\bar{r}-\bar{r}_i) + \tfrac{1}{2}\, \sigma_{iz}(1-\tau_{iz})\mu_n f^n_{mag}(\bar{r}-\bar{r}_i)] \quad . \quad (2)$$

σ's and τ's are unit amplitude Pauli matrices; μ's are nucleon magnetic moments. f's can be regarded as spatial distribution functions of charge and magnetic moments, i.e., inverse Fourier transforms of $F^p_{ch}(\bar{q})$, $F^n_{ch}(\bar{q})$, $F^p_{mag}(\bar{q})$, $F^n_{mag}(\bar{q})$.

The form factors of ^3H and ^3He are calculated (under the stated assumptions) as

$$ZF_{ch}(\bar{q}) = \frac{1}{\int d\bar{r}_1 d\bar{r}_2 d\bar{r}_3 \psi^* \psi} \cdot \int d\bar{r}\; e^{i\bar{q}\bar{r}} \int d\bar{r}_1 d\bar{r}_2 d\bar{r}_3$$

$$\times\ \psi^* \rho_C(\bar{r};\bar{r}_1,\bar{r}_2,\bar{r}_3)\psi \quad , \tag{3}$$

$$\mu F_{mag}(\bar{q}) = \frac{1}{\int d\bar{r}_1 d\bar{r}_2 d\bar{r}_3 \psi^* \psi} \cdot \int d\bar{r}\; e^{i\bar{q}\bar{r}} \int d\bar{r}_1 d\bar{r}_2 d\bar{r}_2$$

$$\times\ \psi^* \rho_M(\bar{r};\bar{r}_1,\bar{r}_2,\bar{r}_3)\psi \quad , \tag{4}$$

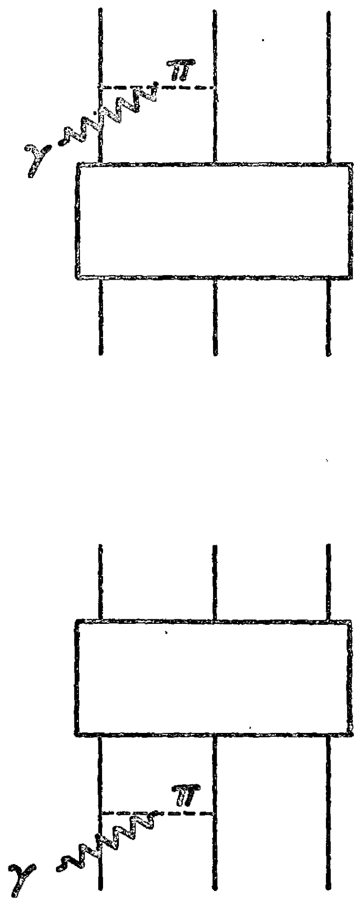
where $Z = 1$, $\mu = 2.975$ for ^3H and $Z = 2$, $\mu = -2.125$ for ^3He. The magnetic moments μ, μ_p, μ_n are in magnetons. From the equalities $F^p_{ch}(0) = 1$, $F^n_{ch}(0) = 0$, $F^p_{mag}(0) = F^n_{mag}(0) = 1$, it can be deduced that

$$F^{^3H}_{ch}(0) = F^{^3He}_{ch}(0) = 1 \quad .$$

Also,

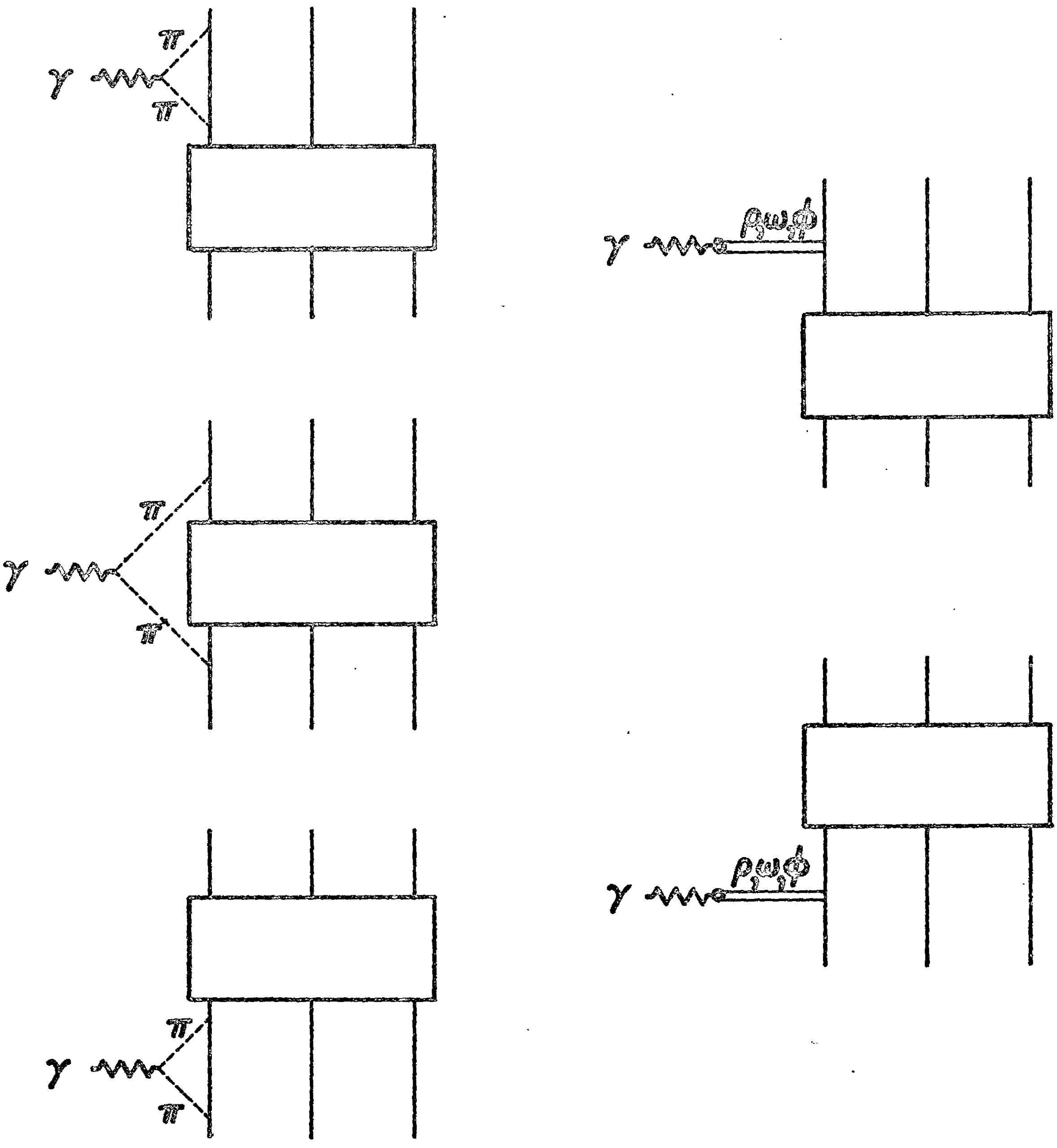
$$F^{^3H}_{mag}(0) = F^{^3He}_{mag}(0) = 1 \quad .$$

The spatial components in ψ are the same for ^3H and ^3He. Mutual interference of nucleons is found to make negligible contribution to the charge density operator, whereas contribution to magnetic moments and magnetic form factors is significant. Among various mechanisms of interference effects, the interaction of external el.m. field with a charged pion exchanged by two bound nucleons is dominant. Coupling of the photon with two exchanged pions and coupling with ρ, ω, ϕ mesons is an effect, smaller by an order of magnitude [2]. There are few qualitative calculations done on the distortion of nucleons in bound systems. It is believed that contribution from them is less than that from interference effects. Essentially, distortion of a nucleon is (Nπ) resonance. Diagrams of interference effects and distortion are given in Figs. 1(a,b) and 2 respectively. A. M. Green and T. H. Schucan [36] did calculation of magnetic moments of ^3H and ^3He considering the contribution from Δ(1236) resonance, and found that at most a 1/4% correction to the isoscalar moment and about 2% correction to the isovector moment will result, whereas one pion exchange contribution to the magnetic moments is of the order of 5%. Hence, to a reasonably good approximation, the nucleon form factors in Eq. (3) are taken to be those of free nucleons in order to calculate the ^3H and ^3He charge form factors, and the information about the ^3H and ^3He charge form factors can be regarded as a first step in studying the wave function of a three-nucleon bound system.



Coupling of external photon with charged pion exchanged by two nucleons.

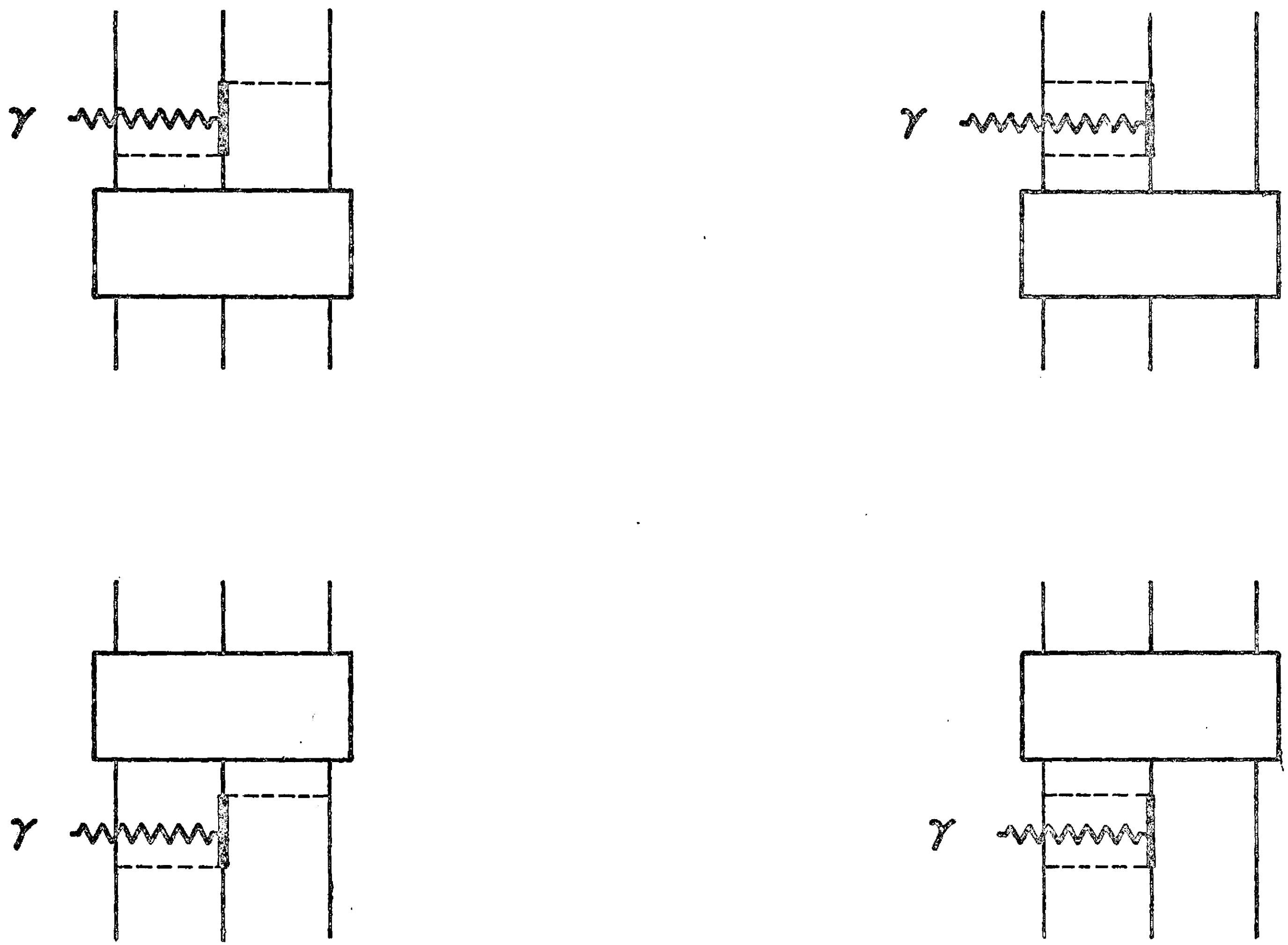
Fig. 1(a)



Coupling of external photon with short range pions and ρ,ω,φ mesons.

Fig. 1(b)

Fig. 2



Coupling of external photon with the (Nπ) resonance

CHAPTER I

EXPERIMENTAL DATA ON FORM FACTORS OF ^{3}H AND ^{3}He

The elastic scattering cross section of an electron by a spin 1/2 particle with the assumption of one photon exchange, is described according to the Rosenbluth equation ($\hbar = c = 1$). See, e.g., Ref. 3.

$$\frac{d\sigma}{d\Omega} = \sigma_{NS} \left\{ \frac{F_{ch}^2(q^2) + \frac{q^2}{4M^2}(1+\kappa)^2 F_{mag}^2(q^2)}{1 + \frac{q^2}{4M^2}} + 2\,\frac{q^2}{4M^2}(1+\kappa)^2 F_{mag}^2(q^2) \tan^2\frac{\theta}{2} \right\} ,$$

where

$$\sigma_{NS} = \frac{e^4}{4E_0^2}\,\frac{\cos^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}} \cdot \frac{1}{1 + \frac{2E_0}{M}\sin^2\frac{\theta}{2}}$$

is the cross section for a point-particle scatterer. The expression in curly brackets depends on the electromagnetic structure of the scatterer.

E_0 is the incident energy of electron in lab system,

q is the 4-momentum transfer,

M is the mass of the scattering nucleus,

$F_{ch}(q^2)$ is the charge form factor with unit normalisation for a charged particle: $F_{ch}(0) = 1$,

$F_{mag}(q^2)$ is the magnetic form factor with unit normalisation: $F_{mag}(0) = 1$,

κ is the anomalous magnetic moment expressed in magnetons.

Collard et al. [3] analysed the cross section data for ^3H and ^3He for q^2 up to 8 fm^{-2}. Validity of the one-photon-exchange assumption was checked by plotting $d\sigma/d\Omega:\sigma_{NS}$ versus $\tan^2 \theta/2$ for a given q^2, and seeing whether it is a straight line as the Rosenbluth equation indicates. From the Rosenbluth plots for every q^2, the absolute values of charge and magnetic form factors of ^3H and ^3He were extracted. Data on the ^3H and ^3He form factors are given in Table I. The charge form factors of ^3H and ^3He (absolute) are drawn in Fig. 3.

McCarthy et al. [4] have analysed data for q^2 up to 20 fm^{-2} in order to find the ^3He charge form factor. First order correction in the Born approximation due to Coulomb distortion is taken into account by using, instead of q, a corrected 4-momentum transfer. Absolute value of the Fourier transform of a preliminary charge distribution, compatible with the phase shift results of a relativistic electron, scattered by the charge distribution, was compared to the experimental (absolute) charge form factor. Thus, an expression for the charge distribution with appropriately fitted parameters was adopted.

Table I Experimental Data on ^3H and ^3He Form Factors (Ref. 3)

TABLE I

$q^2 (fm^{-2})$	$F_{ch}(^3H)$	$F_{mag}(^3H)$	$F_{ch}(^3He)$	$F_{mag}(^3He)$
1.0	0.622 ± 0.007	0.653 ± 0.022	0.567 ± 0.004	0.676 ± 0.075
1.5	0.503 ± 0.007	0.475 ± 0.015	0.431 ± 0.004	0.479 ± 0.046
2.0	0.387 ± 0.007	0.379 ± 0.012	0.329 ± 0.004	0.385 ± 0.031
2.5	0.312 ± 0.006	0.312 ± 0.008	0.258 ± 0.003	0.291 ± 0.020
3.0	0.267 ± 0.005	0.242 ± 0.006	0.209 ± 0.002	0.203 ± 0.014
3.5	0.215 ± 0.004	0.199 ± 0.005	0.1614 ± 0.0017	0.167 ± 0.010
4.0	0.175 ± 0.004	0.167 ± 0.004	0.1326 ± 0.0015	0.128 ± 0.009
4.5	0.187 ± 0.003	0.139 ± 0.003	0.1013 ± 0.0010	0.118 ± 0.005
5.0	0.118 ± 0.004	0.109 ± 0.005	0.0813 ± 0.0012	0.093 ± 0.008
6.0	0.0758 ± 0.0041	0.0792 ± 0.0032	0.0548 ± 0.0015	0.0566 ± 0.0056
8.0	0.0295 ± 0.0039	0.0416 ± 0.0018	0.0173 ± 0.0010	0.0318 ± 0.0026

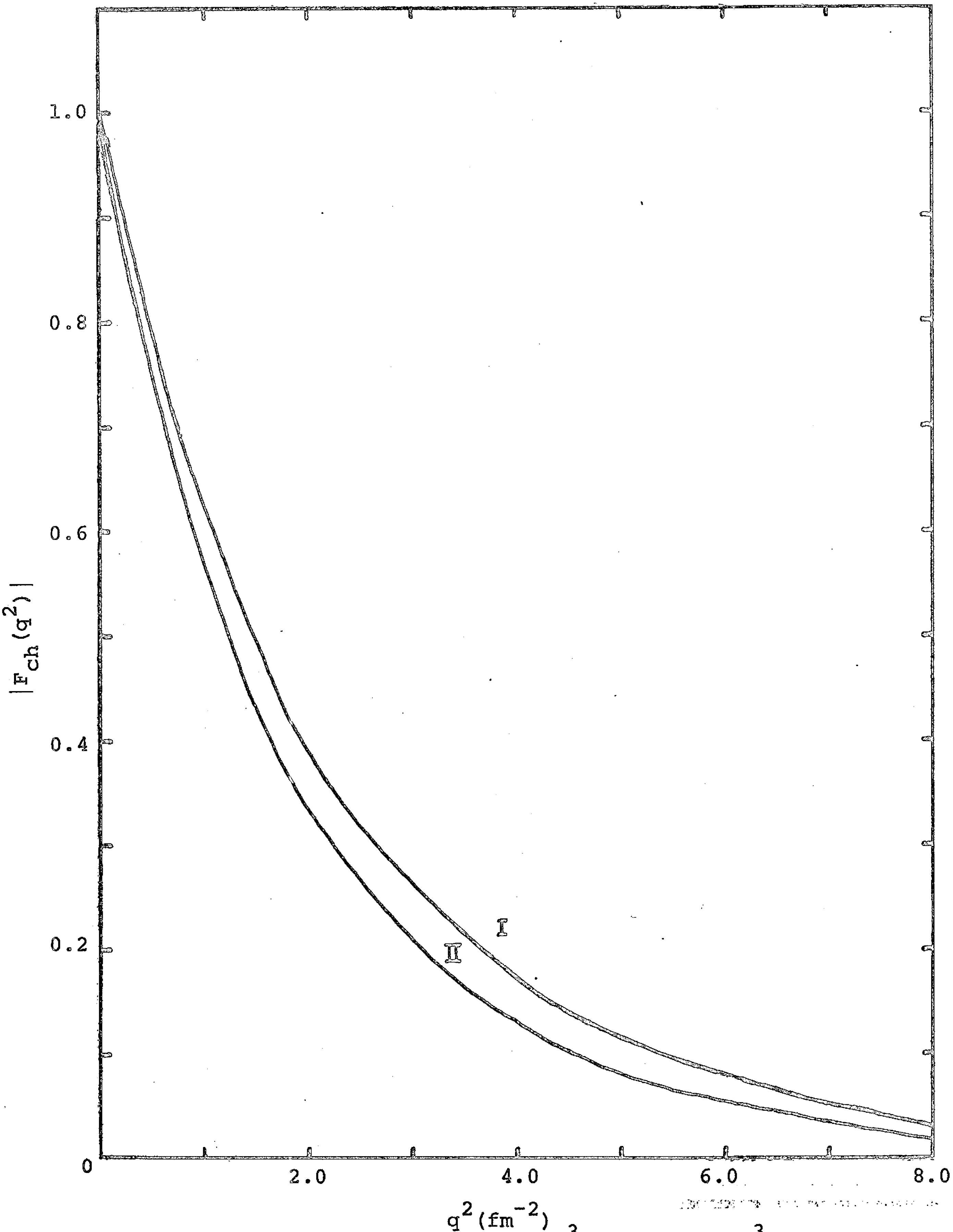


Fig. 3 Experimental data on $|F_{ch}^{^3H}(q^2)|$ and $|F_{ch}^{^3He}(q^2)|$ are fitted by the curves I and II respectively (Ref. 3).

$$\rho(r) = Z\left[\frac{\exp(-\frac{r^2}{4a^2})}{8\pi^{3/2}\, a^3} - \frac{b^2}{2\pi^{3/2}} \cdot \frac{16c^2-r^2}{16c^7} \exp(-\frac{r^2}{4c^2})\right] \quad , \tag{1.1}$$

in the Born approximation,

$$F_B(q^2) = e^{-a^2q^2} - b^2q^2e^{-c^2q^2} \quad , \tag{1.2}$$

which is the 3-d Fourier transform of $1/Z\ \rho(r)$; $q^2 = \bar{q}^2$ for elastic scattering. The form factor $F_B(q^2)$ can fit well the experimental data on

$$|F_{ch}^{^3He}(q^2)|$$

up to 8 fm^{-2}. So as to produce the diffraction minimum, appearing at $q^2 = 11.6\ fm^{-2}$, a correction $\Delta\rho(r)$ is added to $\rho(r)$.

$$\Delta\rho(r) = \frac{Zdpq_0^2}{2\pi^{3/2}}\left[\frac{\sin q_0r}{q_0r} + \frac{p^2}{2q_0^2}\cos q_0r\right]\exp(-\tfrac{1}{4}p^2r^2) \quad , \tag{1.3}$$

three-dimensional Fourier transform of $\Delta\rho(r)$ is

$$\Delta F_B(q^2) = d\ \exp[-(\frac{q-q_0}{p})^2] \quad . \tag{1.4}$$

Best fit was obtained for the values of parameters

$a = 0.675 \pm 0.008$ fm

$b = 0.366 \pm 0.025$ fm

$c = 0.836 \pm 0.032$ fm

$d = (-6.78 \pm 0.83) \times 10^{-3}$

$q_0 = 3.98 \pm 0.09$ fm^{-1}

$p = 0.90 \pm 0.16$ fm^{-1}.

The root mean square charge radius is $r_{rms} = 1.88$ fm. (See Fig. 4).

In Ref. 5 it is argued that near the minimum, two photon exchange correction in the elastic cross section is about 30 - 40%. Corrections from vector-meson dominance (i.e., the electron exchanges a photon by coupling through ρ-meson with the ^3He nucleus) were shown by E. Lehman [37)] to be unable to account simultaneously for the form factor dip at 11.6 fm^{-2}, and the behaviour at high momentum transfer. But these corrections are quite small, by themselves.

According to the theory given by L. I. Schiff, charge form factors of ^3H and ^3He depend linearly on F^p_{ch} and F^n_{ch}. Experimental data on F^p_{ch} are obtained from elastic electron-proton scattering [38)]. One photon exchange is a good approximation for q^2 up to 22 fm^{-2}. (See Table II). Neutron form factors are derived from elastic electron-deuteron scattering [6)]. The nucleon form factors were earlier fitted by Kirson [7)], and de Vries [8)] by so-called three-pole fit, which is a three-pole approximation to the dispersion theory of nucleon form factors.

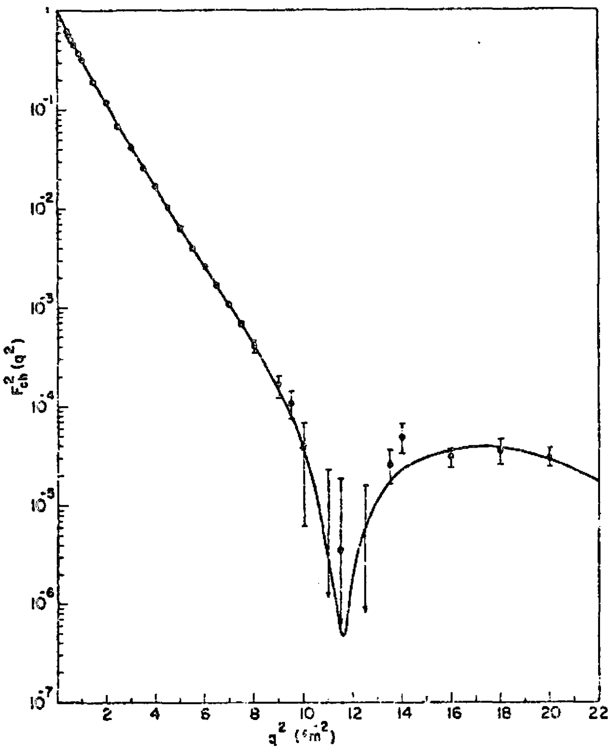


Fig. 4 The ^{3}He charge form factor squared is fitted by parametric functions [Eqs. (1.2) and (1.4)] with parameters

$a = 0.675 \pm 0.008$ fm,

$b = 0.366 \pm 0.025$ fm,

$c = 0.836 \pm 0.032$ fm,

$d = (-6.78 \pm 0.83) \times 10^{-3}$,

$q_0 = 3.98 \pm 0.09$ fm^{-1},

$p = 0.90 \pm 0.16$ fm^{-1}.

The charge root mean square radius for the charge distribution [Eqs. (1) and (3)] is equal to the experimentally found number.

Table II Experimental and Theoretical Values of the Proton Form Factors

TABLE II

$q^2 (fm^{-2})$	Experimental Values [37]		Three-pole Fit [7,8]	
	F^p_{ch}	F^p_{mag}	F^p_{ch}	F^p_{mag}
4.00	0.689 ± 0.019	0.623 ± 0.018	0.658	0.644
4.6	0.615 ± 0.015	0.611 ± 0.010	0.624	0.610
5.0	0.599 ± 0.026	0.618 ± 0.021	0.603	0.588
6.0	0.577 ± 0.019	0.533 ± 0.014	0.554	0.540
7.0	0.521 ± 0.021	0.490 ± 0.010	0.511	0.499
7.5	0.504 ± 0.022	0.472 ± 0.011	0.492	0.480
8.0	0.453 ± 0.020	0.466 ± 0.009	0.474	0.462
9.0	0.422 ± 0.027	0.437 ± 0.011	0.440	0.430
10.0	0.424 ± 0.017	0.400 ± 0.007	0.410	0.402
11.0	0.398 ± 0.025	0.379 ± 0.009	0.383	0.376
12.0	0.363 ± 0.020	0.355 ± 0.007	0.359	0.354
13.0	0.349 ± 0.040	0.327 ± 0.015	0.337	0.333
14.0	0.315 ± 0.028	0.316 ± 0.008	0.317	0.314
15.0	0.304 ± 0.053	0.297 ± 0.015	0.299	0.297
16.0	0.271 ± 0.024	0.282 ± 0.006	0.283	0.282
17.0	0.234 ± 0.041	0.277 ± 0.008	0.267	0.268
18.0	0.274 ± 0.026	0.250 ± 0.005	0.253	0.254
19.0	0.254 ± 0.039	0.245 ± 0.008	0.240	0.242
20.0	0.187 ± 0.073	0.237 ± 0.010	0.228	0.231
22.0	0.166 ± 0.075	0.224 ± 0.007	0.207	0.211
26.0		0.178 ± 0.005	0.171	0.179
		0.160 ± 0.006	0.157	0.166
		0.145 ± 0.006	0.144	0.154

Behaviour of the isoscalar form factors $G_{ES} = \frac{1}{2} (F^p_{ch} + F^n_{ch})$ and $G_{MS} = \frac{1}{2} [(1+\kappa_p)F^p_{mag} + \kappa_n F^n_{mag}]$ is supposedly dominated by intermediate states of two pions, coupled to ω and ϕ mesons. Isovector form factors $G_{EV} = \frac{1}{2} (F^p_{ch} - F^n_{ch})$ and $G_{MV} = \frac{1}{2} [(1+\kappa_p)F^p_{mag} - \kappa_n F^n_{mag}]$ are dominated by effects of the ρ-meson.

$$G_{ES} = 0.5 \left\{\frac{s_{e1}}{1 + \frac{q^2}{15.7}} + \frac{s_{e2}}{1 + \frac{q^2}{26.7}} + 1 - s_{e1} - s_{e2}\right\} , \qquad (1.5)$$

$$G_{MS} = 0.44 \left\{\frac{s_{m1}}{1 + \frac{q^2}{15.7}} + \frac{s_{m2}}{1 + \frac{q^2}{26.7}} + 1 - s_{m1} - s_{m2}\right\} , \qquad (1.6)$$

$$G_{EV} = 0.5 \left\{\frac{v_{e1}}{1 + \frac{q^2}{8.19}} + 1 - v_{e1}\right\} , \qquad (1.7)$$

$$G_{MV} = 2.353 \left\{\frac{v_{m1}}{1 + \frac{q^2}{8.19}} + 1 - v_{m1}\right\} . \qquad (1.8)$$

Fitted parameters are $s_{e1} = 2.50$, $s_{e2} = -1.60$, $s_{m1} = 3.33$, $s_{m2} = -2.77$, $v_{e1} = 1.16$, $v_{m1} = 1.11$. (See Fig. 5).

Theoretical expression for the elastic electron-deuteron scattering cross section involves deuteron wave function, which is dependent on the choice of internucleon potential. S. Galster et al. [6] used parametric function

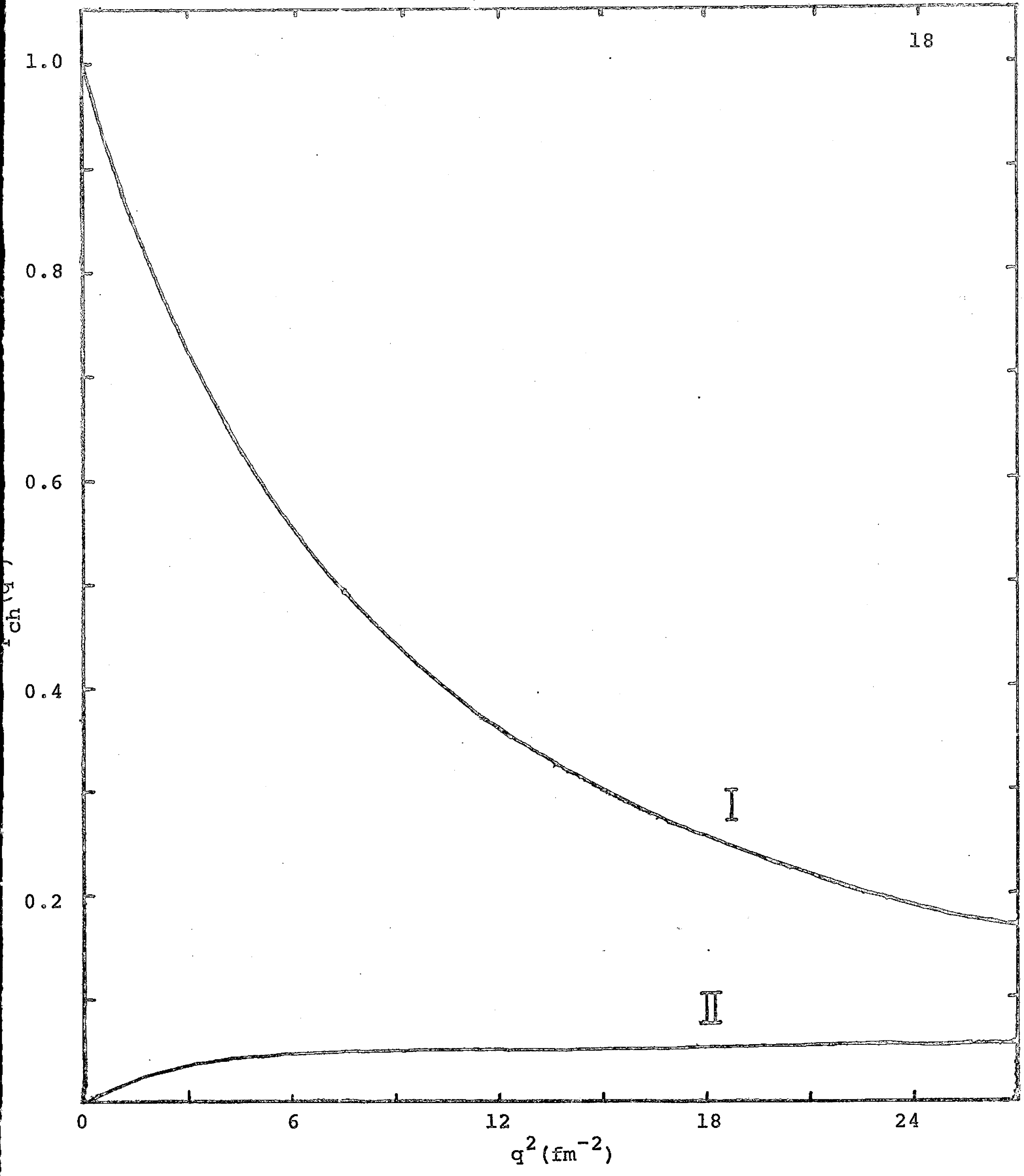


Fig. 5 Curves I and II, corresponding to $F^p_{ch}(q^2)$ and $F^n_{ch}(q^2)$ respectively, are obtained from the Eqs. (1.5) and (1.7).

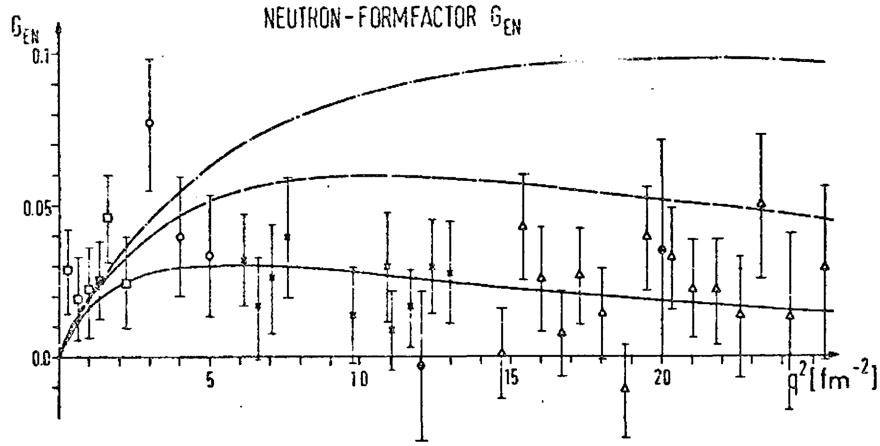
$$F^{n}_{ch} = -\mu_n \frac{\frac{q^2}{4M^2}}{1 + p\frac{q^2}{4M^2}} F^{p}_{ch} \qquad (1.9)$$

to fit the parameter p, using different deuteron wave functions. For the McGee [23], Lomon-Feshbach [24] and Hamada-Johnston [25] wave functions, the corresponding values of p were found equal to 19.7, 5.6 and 10.7. The dependence of F^{n}_{ch} on the choice of deuteron wave function is shown in Figs. 6(a,b,c).

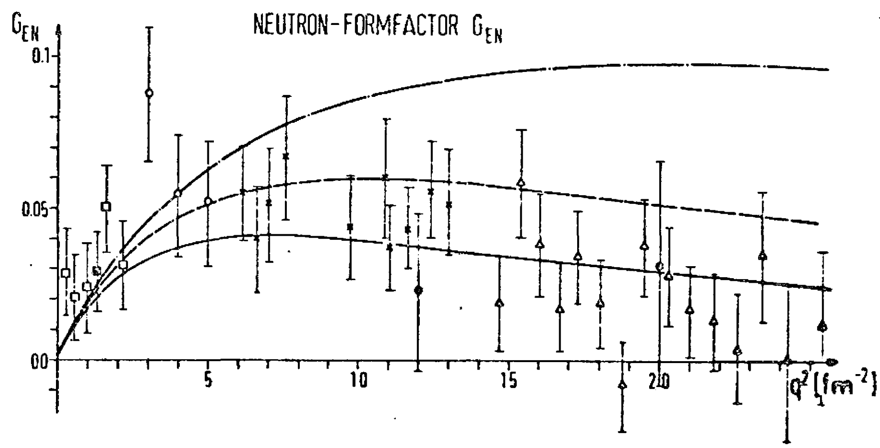
Fig. 6(a) The solid curve is obtained with the fitting parameter p equal to 19.7, by using the McGee deuteron wave function. The dot-dashed curve corresponding to $F^n_{ch} = -\mu_n (q^2/4M^2) F^p_{ch}$ and the dashed one corresponding to $F^n_{ch} = -\mu_n (q^2/4M^2)/[1 + (q^2/M^2)] F^p_{ch}$ are drawn for comparison in the three Figs. 6(a,b,c).

Fig. 6(b) The solid curve is obtained with the fitting parameter p equal to 10.7, by using the Hamada-Johnston wave function.

Fig. 6(c) The solid curve is obtained by using the Lomon-Feshbach wave function with p = 5.6.


NEUTRON-FORMFACTOR G_{EN}
G_{EN}
0.1
0.05
0.0
5
10
15
20
q^2 [fm^{-2}]

(a)


NEUTRON-FORMFACTOR G_{EN}
G_{EN}
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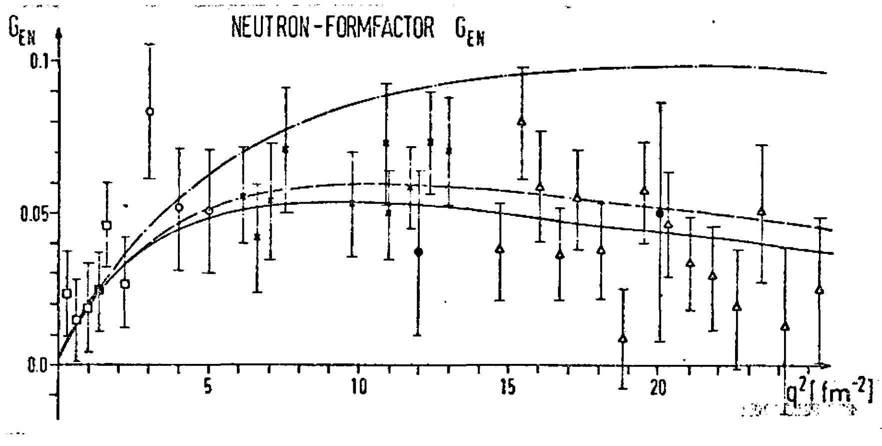
0.05
0.0
5
10
15
20
q^2 [fm^{-2}]

(b)


NEUTRON-FORMFACTOR G_{EN}
G_{EN}
0.1
0.05
0.0
5
10
15
20
q^2 [fm^{-2}]

(c)

CHAPTER II

DETERMINATION OF THREE-NUCLEON WAVE FUNCTION AND CALCULATIONS WITH IT

The three-body problem for realistic two-nucleon forces has been tackled in three different ways. Variational calculations with forces like Hamada-Johnston have been done by L. M. Delves et al.[9] The problem has been solved by diagonalising the hamiltonian for soft-core potentials: for Reid soft-core[22] - by A. D. Jackson et al.[10,11,12,26], and for Riihimaki potential[19] by P. Nunberg et al.[13]. For Reid soft-core potential, the Faddeev equations have been solved in momentum space by Malfliet and Tjon[14], and Harper, Kim and Tubis[15]. Harms et al.[16,17,29] have used unitary pole approximation method in solving the Faddeev equations. The Faddeev equations have been solved in configuration space for Reid and Sprung, de Tourreil forces (Ref. 30).

A. Variational Calculations

A two-nucleon potential is called realistic in the sense that it describes properties of two-nucleon systems fairly well. In a trinucleon system, interactions are considered between nucleons 1 and 2, 2 and 3, 3 and 1. Variational procedure implies choosing a trial function. Any function of three particles F(1,2,3) can be decomposed into three mutually orthogonal functions.

$F = F_s + F_m + F_a$; F_s is completely symmetric in interchanging any two particles. F_a is completely antisymmetric, $F_m = F_{m_1} + F_{m_2}$ - is a sum of two functions:
1) symmetric under, for instance, $1 \leftrightarrow 2$ permutation F_{m_1};
2) antisymmetric under $1 \leftrightarrow 2$ permutation F_{m_2}. Trial function is the sum of antisymmetric products of radial and angular-spin-isospin parts [2).

$$\psi = \sum_k f_k(r_{12},r_{23},r_{31})Y_k(\alpha\beta\gamma,\zeta_1\zeta_2\zeta_3,\eta_1\eta_2\eta_3)$$

$$\equiv f_sY_a + f_aY_s + f_{m1}Y_{m2} - f_{m2}Y_{m1} \quad . \tag{2.1}$$

The angular part for given orbital angular momentum L of the system is a combination of the Euler angle functions $D^L_{\mu M}(\alpha\beta\gamma)$, μ, M = -L, ..., L. The spin-isospin part, in general, has states of S = 1/2, 3/2 and T = 1/2, 3/2. $(\zeta_1\zeta_2\zeta_3)$ describes the spin and $(\eta_1\eta_2\eta_3)$ - the isospin parts.

In Ref. 9, the trial function of the bound state was taken as

$$\psi_Q = \sum_{i,k} f^{(Q)}_{i,k}(r_{12},r_{23},r_{31})Y_{i,k}(\alpha\beta\gamma,\zeta_1\zeta_2\zeta_3,\eta_1\eta_2\eta_3) \quad , \tag{2.2}$$

Q is an integer parameter, indicating the degree of trial function.

1,2,3 are i-index values for L=0 states (i.e., S, S_a, S'),

4,5,6,7 are i-index values for L=1 states,

8,9,10 are i-index values for L=2 "old" states,

11,12,13,14 are i-index values for L=2 "new" states.

k is the index for different symmetries.

Radial components $f^{(Q)}_{i,k}$ are products of a suitable set of one-dimensional functions $\{\phi_\ell(r), \ell = 1,2,3,\ldots\}$.

$$f^{(Q)}_{i,k} = \sum_{\substack{\ell,m,n \\ \ell+m+n\leq Q}} a_{i\ell mn}\, P_k\{\phi_\ell(r_{12})\phi_m(r_{23})\phi_n(r_{31})\} \quad , \tag{2.3}$$

P_k's are symmetrising operators. The P_k's are written out in equations 2.6(a-d).

$$\phi_\ell(r) = \{1 - e^{-\delta(r-c)}\}\,\frac{e^{-\gamma r}}{r^p}$$

$$\times \; \{1 + \sum_{k=0}^{\ell-1} a_{k\ell} \exp[-2k\gamma(r-c)]\} \quad , \tag{2.4}$$

c is core radius. $p = 0.5$. Inside the core, $\phi_\ell(r) = 0$. The factor in curly brackets is to account for boundary condition: $\phi_\ell(c) = 0$. γ, δ and the $a_{k\ell}$ are nonlinear variational parameters. $\phi_\ell(r)$ is taken in the form similar to deuteron wave function.

$$\delta = 4.0 \quad , \quad \gamma = 0.25 \quad \text{for all L=0 states,}$$

$$\delta = 4.0 \quad , \quad \gamma = 0.2 \quad \text{for all L=2 states.}$$

P states contribute only 0.08 MeV for H-J potential.

The expectation value of hamiltonian over the trial function with unit normalisation is minimized by Ritz variational principle. In that way probabilities of different components are estimated. $a_{k\ell}$ in the functions $\phi_\ell(r)$ are not independent variational parameters, but are chosen so that as many as possible of the sequence of symmetric S- and D-states, are orthogonal. The coefficients $a_{i\ell mn}$ are variational parameters.

The energy of ^3H is obtained by extrapolating the curve E(Q) as $E_\infty + AQ^{-r}$. E(Q) is the expectation value of hamiltonian, calculated for ψ_Q, Q can be 1, Q can be 2, up to a finite Q_0. A and r are appropriately taken to fit the curve E(Q) up to Q_0. E_∞, the energy for H-J potential, was found to be -6.5 ± 0.2 MeV. The trial functions converge to some wave function with increasing Q. In Ref. 9, the trial function corresponding to Q = 7 was taken as the trinucleon wave function. For ^3He Delves and Hennell did variational calculations including both the H-J potential and the Coulomb interaction between two point protons, and got $E(^3He) = -5.95 \pm 0.2$, neglecting the T = 3/2 admixture. In

Table III are given properties of the ^3H and ^3He ground state wave functions for H-J potential. Also, see Fig. 7.

Adding a phenomenological three-body force $V_3(\alpha) = - V_3 \exp[-\alpha(r_{12} + r_{23} + r_{31})]$ so as to adjust the binding energy of ^3H to the experimental number 8.418 MeV, moves the diffraction minimum of ^3H charge form factor to 14 fm^{-2}. Probabilities of different components are not altered much. The experimental value of $E(^3He)$ is -7.664 MeV. $\langle r_{ch}\rangle(^3H) = 1.70$ fm, $\langle r_{ch}\rangle(^3He) = 1.87$ fm are the values, determined by Collard et al. [3)] experimentally.

Table III Properties of ^3H and ^3He for H-J Potential ; in case of ^3He Coulomb Forces are included as well

TABLE III

Property	^3H	^3He
Energy (MeV)	−6.5 ± 0.2	−5.95 ± 0.2
$<r_{ch}>$(fm)	1.85 ± 0.02	1.90
r_{mass}(fm)	2.00 ± 0.01	2.11
P(S) (%)	89.2	89.4
P(S_a) (%)	2×10^{-5}	0.0
P(S') (%)	1.8	1.93
P(P) (%)	0.03	0.0
P(D) (%)	9.0	8.63
Diffraction minimum of $\|F_{ch}\|$ (fm^{-2})	13.4 ± 0.3	12.5 ± 0.3

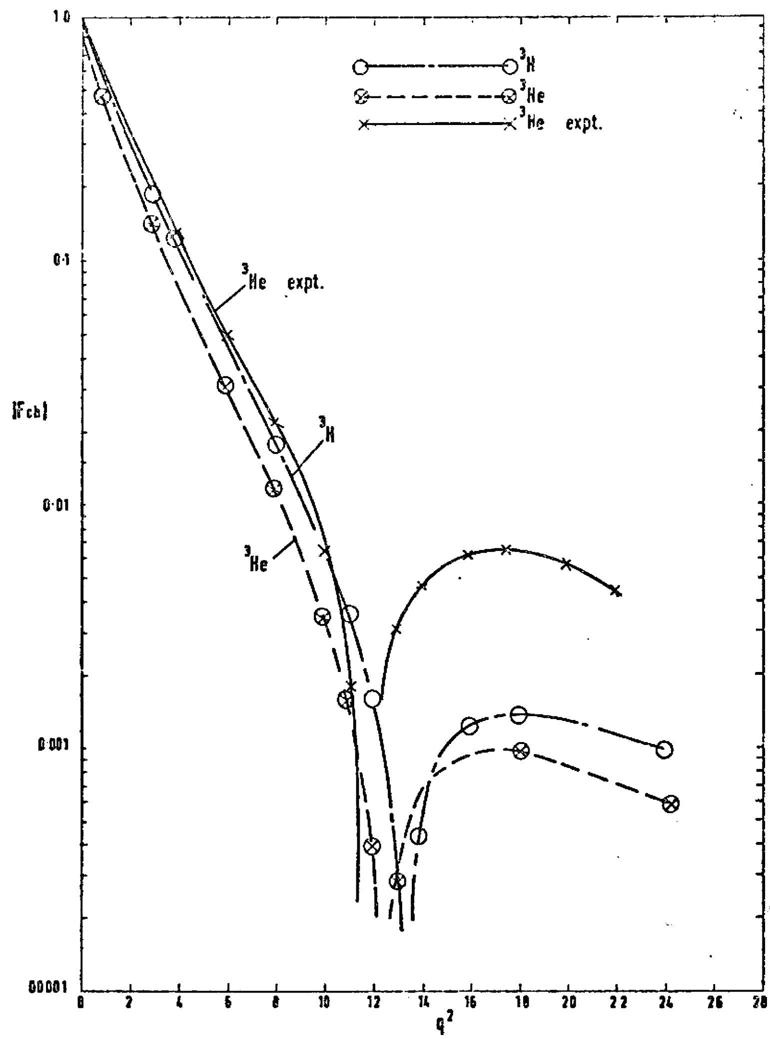


Fig. 7 The absolute charge form factors of ^{3}H and ^{3}He for Hamada-Johnston potential (adding the Coulomb repulsion between point protons in ^{3}He) are compared to the experimental curve of $|F_{ch}^{^3He}(q^2)|$ (taken from Ref. 9).

B. Diagonalisation of the Hamiltonian Matrix in the Basis of Harmonic Oscillator Eigenfunctions

The harmonic oscillator hamiltonian of three equal mass nucleons (harmonic oscillator forces of frequency $\omega/\sqrt{3}$ are acting between two nucleons) in terms of the Jacobi coordinates is

$$H_{ho} = -\frac{\hbar^2}{2m}(\bar{\nabla}_\zeta^2 + \bar{\nabla}_\eta^2) + \frac{m\omega^2}{2}(\bar{\zeta}^2 + \bar{\eta}^2) \quad ;$$

$$\bar{\zeta} = 6^{-1/2}(\bar{x}_1 + \bar{x}_2 - 2\bar{x}_3) \quad ,$$

$$\bar{\eta} = 2^{-1/2}(\bar{x}_1 - \bar{x}_2) \quad . \tag{2.5}$$

Eigenvalues of H_{ho} are $\hbar\omega(\rho+3)$, $\rho = 2n_\zeta+\ell_\zeta+2n_\eta+\ell_\eta$ is the total number of oscillator quanta. Angular momenta $\bar{\ell}_\zeta$ and $\bar{\ell}_\eta$ couple to orbital angular momentum $\vec{L}$ of the system. n_ζ and n_η are radial quantum numbers. The eigenfunctions of H_{ho}, $\bar{\ell}_\zeta^2$ and $\bar{\ell}_\eta^2$: $\langle\bar{\zeta},\bar{\eta}|n_\zeta\ell_\zeta,n_\eta\ell_\eta,L\rangle$, form spatial parts in the total wave function.

We define operators

$$P_s = \frac{1}{6}[1 + (1,2) + (1,3) + 2,3) + (1,2,3) + (1,3,2)] \quad , \tag{2.6a}$$

where (1,2) is $1 \leftrightarrow 2$ permutation, (1,3,2) means $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$;

$$P_a = \frac{1}{6}\,[1 - (1,2) - (1,3) - (2,3) + (1,2,3) + 1,3,2)]\ ; \quad (2.6b)$$

$$P_{m1} = \frac{1}{6}\,[21 - (1,2,3) - (1,3,2)][1 + (1,2)]\ , \quad (2.6c)$$

$$P_{m2} = \frac{1}{6}\,[21 - (1,2,3) - (1,3,2)][1 - (1,2)]\ , \quad (2.6d)$$

and note that

$P_s\langle\bar{\zeta},\bar{\eta}|n_\zeta\ell_\zeta,n_\eta\ell_\eta,L\rangle$ is spatially symmetric , (2.7a)

$P_a\langle\bar{\zeta},\bar{\eta}|n_\zeta\ell_\zeta,n_\eta\ell_\eta,L\rangle$ is spatially antisymmetric , (2.7b)

$P_{m1}\langle\bar{\zeta},\bar{\eta}|n_\zeta\ell_\zeta,n_\eta\ell_\eta,L\rangle$ is symmetric under $1\leftrightarrow 2$ permutation , (2.7c)

$P_{m2}\langle\bar{\zeta},\bar{\eta}|n_\zeta\ell_\zeta,n_\eta\ell_\eta,L\rangle$ is antisymmetric under $1\leftrightarrow 2$ permutation . (2.7d)

In the totally antisymmetric wave function, one couples $P_k[\langle\bar{\zeta},\bar{\eta}|n_\zeta\ell_\zeta,n_\eta\ell_\eta,L\rangle]$ with the spin-isospin function of adjoint symmetry: L should couple with S to J = 1/2, J_z = +1/2, or J = 1/2, J_z = -1/2. The eigenstates of H_{ho}, antisymmetrised according to the way as discussed above, form the basis for diagonalisation of the hamiltonian

$$H = H_{ho} + [V(1,2) + V(2,3) + V(1,3) - \frac{1}{2}\,m\omega^2(\bar{\zeta}^2 + \bar{\eta}^2)]\ . \quad (2.8)$$

ω' serves as a parameter, chosen so as to obtain maximum binding energy.

Jackson, Landé and Sauer [11] used Reid soft-core potential including all partial waves $\ell \leq 2$. The basis for diagonalisation consisted of 34 quanta of symmetric S states [ρ = 34, and the number of states is 273], 12 quanta of mixed symmetry S' states [27 states], and 20 quanta of D states [220 states]. The oscillator length $b = (\hbar/m\omega)^{1/2}$ was treated as a nonlinear variational parameter and the value of b = 0.85 fm was obtained. Complete basis should consist of an infinite number of quanta in all the L = 0, L = 2 states. L = 1 states have negligible probability, and are not taken in the basis.

Calculated triton ground state properties (Ref. 11)

ρ(S) L=0	ρ(D) L=2	ρ(S') L=0	b(fm)	B=-E(MeV)	P_D(%)	$P_{S'}$(%)	V(MeV)	r(fm)
26	18	10	0.80	5.712	9.05	0.39	-58.007	1.588
30	18	10	0.80	5.858	8.91	0.38	-57.005	1.616
26	22	10	0.80	5.867	9.07	0.40	-57.684	1.600
26	18	14	0.80	5.798	9.22	0.62	-58.369	1.590
26	18	2	0.80	5.341	8.39	0.00	-56.065	1.596
34	20	12	0.80	6.091	8.92	0.49	-56.316	1.647
34	20	12	0.85	6.265	8.89	0.53	-55.352	1.695
34	20	12	0.85	6.298	8.92	0.52	-56.109	1.669
(34	20	12)	0.85	6.057	8.52	0.43	-54.884	1.680

The last row contains results for potential, restricted to even partial waves only (ℓ is even).

The calculated root mean square radius of 1.669 fm yields the root mean square charge radius of 1.85 fm when effects of finite proton size are included. Collard et al.[3] found $\langle r_{ch}\rangle({}^3H) = 1.70$ fm.

Difference $\Delta[\rho(S)] = E(\infty,\rho(D),\rho(S'))-E(\rho(S),\rho(D),\rho(S'))$ of energies for infinite and finite number of quanta in S state is fitted to the type of wave $\kappa\rho(S)^{-p}$. $E(\infty,\rho(D),\rho(S'))$ need not be known; $E(\rho(S),\rho(D),\rho(S'))$ for different $\rho(S)$ can tell what κ and p should be. Then $E(\infty,\rho(D),\rho(S'))$ is found. Similarly, $E(\rho(S),\infty,\rho(S'))$ and $E(\rho(S),\rho(D),\infty))$ are evaluated.

ρ(S)	ρ(D)	ρ(S')	B=-E(MeV)	Δ(MeV)
∞	18	10	5.999	0.286
26	∞	10	6.009	0.298
26	18	∞	5.917	0.206

Then the actual energy E is found as

$$E \equiv E(\infty,\infty,\infty) = E(\rho(S),\rho(D),\rho(S')) + \Delta[\rho(S)] + \Delta[\rho(D)] + \Delta[\rho(S')] \quad . \tag{2.9}$$

Binding energy was found to be 6.50 MeV.

With the trinucleon wave function thus obtained in the basis of 484 harmonic oscillator states, Yang and Jackson [12)] found the minimum in ^{3}He absolute charge form factor to occur at $q^2 \simeq 12.9\ fm^{-2}$. Coulomb forces were not taken into account in finding the ^{3}He wave function. See Fig. 8.

Hadjimichael and Jackson [26)] calculated binding energy and charge form factor of triton for phase equivalent potentials of the Reid soft-core (only $^1S_0$ and $^3S_1-^3D_1$ partial waves of the N-N interaction were included). For Reid potential itself, binding energy was found equal to 6.25 MeV; dip in $|F_{ch}^{^3H}(q^2)|$ occurs at $q^2 = 13.5\ fm^{-2}$. Position of the dip in $|F_{ch}^{^3H}(q^2)|$ varies for different phase equivalent potentials, which give different binding.

If B_T = 4.753 MeV, dip in $|F_{ch}^{^3H}|$ is at $q^2 = 13.7\ fm^{-2}$,

5.34 12.4 ,

6.03 12.7 .

For Riihimaki potential (in Ref. 13) all partial waves in the N-N interaction with $\ell \leq 4$ were taken into account. Maximum number of oscillator quanta was taken equal to $\rho = 24$, and the total number of basis states equal to 348. The curve for triton binding energy $B_T(\rho)$ is extrapolated as $B_T^\infty - A\rho^{-B}$. $\langle r^2\rangle^{1/2}(^3H)$ and $\langle r^2\rangle^{1/2}(^3He)$ were also extrapolated by an exponential formula. Following results were obtained.

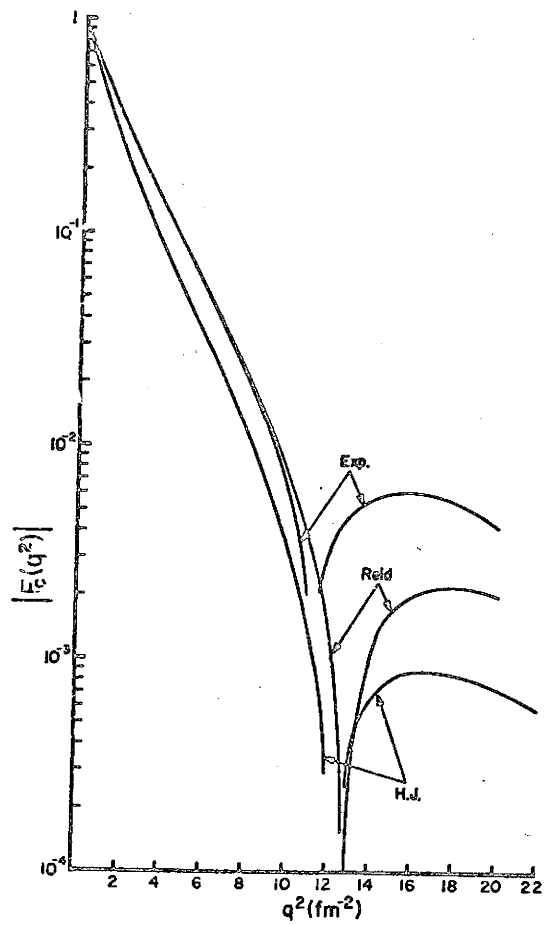


Fig. 8 The absolute charge form factor of ^3He for Reid soft-core potential compared to the experimental curve $|F_{ch}^{^3He}(q^2)|$ and the curve obtained by Delves and Hennell (Ref. 9) (taken from Ref. 12).

$$\left.\begin{aligned} B_T &= 6.28 \text{ MeV} , \\ \langle r^2\rangle^{1/2}(^3H) &= 1.77 \text{ fm} , \\ \langle r^2\rangle^{1/2}(^3He) &= 2.10 \text{ fm} , \\ P_D &= 4.6\% . \end{aligned}\right\} \quad \rho = 24 .$$

Extrapolated values are

$$B_T^{\infty} = 7.0 \pm 0.3 \text{ MeV} ,$$

$$\langle r^2\rangle^{1/2}(^3H) = 1.83 \text{ fm} ,$$

$$\langle r^2\rangle^{1/2}(^3He) = 2.14 \text{ fm} ,$$

$$P_D = 4.6\% .$$

C. Solution of the Faddeev Equations

The nonrelativistic 3-particle scattering matrix T is equal to

$$T(s) = V - V(H-s)^{-1} V \quad , \tag{2.10}$$

where hamiltonian H is the sum of the kinetic energy operator (in the c.m. frame) H_0 and potential energy operator $V = \sum_{i=1}^{3} V_i$, V_i acts between particles $j,k \neq i$. s is the total energy of three-nucleon system.

$$G_0(s) = (H_0 - s)^{-1} \tag{2.11}$$

is Green's function for free nucleons. As Faddeev shows, the T-matrix can be decomposed into three parts

$$T = T^{(1)} + T^{(2)} + T^{(3)} \quad . \tag{2.12}$$

$$T^{(i)}(s) = T_i(s) - \sum_{j \neq i} T_i(s) G_0(s) T^{(j)}(s) \quad ,$$

i = 1,2,3, are a set of three Faddeev equations; T_i's are off-shell two-body T matrices, satisfying the Lippmann-Schwinger equations.

$$T_i(s) = V_i - V_i G_0(s) T_i(s) \quad . \tag{2.13}$$

In the transformation of momentum variables (Refs. 14,20)

$$\bar{p} = \frac{m_2\bar{k}_1 - m_1\bar{k}_2}{[2m_1m_2(m_1+m_2)]^{1/2}} \quad , \tag{2.14}$$

$$\bar{q} = \frac{m_3(\bar{k}_1+\bar{k}_2)-(m_1+m_2)\bar{k}_3}{[2m_3(m_1+m_2)(m_1+m_2+m_3)]^{1/2}} \quad , \tag{2.15}$$

$$H_0 = \bar{p}^2 + \bar{q}^2 \quad . \tag{2.16}$$

Eigenstate of H_0, which is simultaneously eigenstate of three-nucleon orbital angular momentum, spin and total angular momentum, is multiplied by isospin functions of definite (τ,τ_z).

$$|p,q,\alpha\rangle \equiv |[pq(L\ell)\mathcal{L},(Ss)\mathcal{S}]JJ_z,(Tt)\tau\tau_z\rangle \quad , \tag{2.17}$$

and the product is antisymmetrised with respect to interchanging any two nucleons. The bound state wave function is a sum over antisymmetric basis vectors $|p,q,\alpha\rangle_A$.

$$|\psi_B\rangle = \underset{\alpha}{S} \langle pq\alpha|\psi_B\rangle|pq\alpha\rangle_A \quad . \tag{2.18}$$

$$(H - E_B)|\psi_B\rangle = 0 \tag{2.19a}$$

$$(H_0 - \bar{p}^2 - \bar{q}^2)|p,q,\alpha\rangle_A = 0 \quad . \tag{2.19b}$$

For the energy $s = E_B$, $\langle pq\alpha|T(s)|\psi\rangle_A$ has a bound-state pole, and residue at the pole gives us the value of the component $\langle pq\alpha|\psi_B\rangle$. Equations for $\langle pq\alpha|T^{(i)}(s)|\psi\rangle_A$ $(i = 1,2,3)$ are inhomogeneous integral equations of the type

$$F(s) = F_0(s) - \kappa(s)F(s) \quad . \tag{2.20}$$

If the kernel $\kappa(s)$ is multiplied by complex variable λ, one has a different equation

$$F(s,\lambda) = F_0(s) - \lambda\kappa(s)F(s,\lambda) \quad . \tag{2.21}$$

Kernel $\lambda\kappa(s)$ is of Hilbert-Schmidt type. So, $F(s,\lambda)$ is a meromorphic function of λ with poles at characteristic values $\lambda_\alpha(s)$ for given s. That is, $F(s,\lambda)$ can be written as (see Ref. 14)

$$F(s,\lambda) = \sum_\alpha \frac{B_\alpha(s)}{\lambda_\alpha(s)-\lambda} + R(s,\lambda) \quad . \tag{2.22}$$

$F(s,\lambda)$ is expanded in power series of λ [Neuman Series].

$$F(s,\lambda) = F_0(s) - \lambda\kappa(s)F_0(s) + \lambda^2\kappa^2(s)F_0(s) \ldots$$

$$= \sum_{n=0}^{\infty} (-1)^n \lambda^n \kappa^n(s) F_0(s) \equiv \sum_{n=0}^{\infty} \lambda^n F_n(s) \quad . \tag{2.23}$$

Since $R(s,\lambda)$ is an entire function,

$$R(s,\lambda) = \sum_{n=0}^{\infty} \lambda^n R_n(s) \quad . \tag{2.24}$$

So,

$$F_n(s) = \sum_{\alpha} \frac{B_\alpha(s)}{[\lambda_\alpha(s)]^{n+1}} + R_n(s) \quad . \tag{2.25}$$

$$\lim_{n\to\infty} \frac{F_{n+1}(s)}{F_n(s)} = \frac{1}{\lambda_0(s)} \quad , \tag{2.26}$$

$\lambda_0(s)$ is the smallest characteristic value where $F(s,\lambda)$ has a pole.

Since the original equation corresponds to $\lambda = 1$, the pole λ_0 should be equal to one for the energy $s = E_B$. This way of finding the pole is known as the ratio method (or iteration method).

Padé approximants method is following [39]. N^{th} Padé approximant to $F(s,\lambda)$ is

$$F_{[N,N]}(s,\lambda) = \sum_{n=0}^{N} An\lambda^n / (1 + \sum_{n=1}^{N} Bn\lambda^n) \quad ; \tag{2.27}$$

this is compared to $\sum_{n=0}^{2N+1} \lambda^n F_n(s)$, and the coefficients A_n and B_n found. The pole of $F_{[N,N]}(s,\lambda)|_{\lambda=1}$ gives the binding energy $-E_B$. Harper, Kim and Tubis [20] considered set of 8 different α, whereas Malfliet and Tjon considered only the set of first 3 α.

The basis states [39)] are

Component α	$(L\ell)\mathcal{L}$	$(Ss)\,\mathcal{S}$	(Tt) $\tau=1/2$, $\tau_z=+1/2$ for ^3He, $=-1/2$ for ^3H
1	(00)0	(0 1/2)1/2	(1 1/2)
2	(00)0	(1 1/2)1/2	(0 1/2)
3	(20)2	(1 1/2)3/2	(0 1/2)
4	(02)2	(1 1/2)3/2	(0 1/2)
5	(22)2	(1 1/2)3/2	(0 1/2)
6	(22)1	(1 1/2)1/2	(0 1/2)
7	(22)1	(1 1/2)3/2	(0 1/2)
8	(22)0	(1 1/2)1/2	(0 1/2)

The Lippmann-Schwinger equations [β stands for angular, spin and isospin dependence] are

$$t_\beta(p,p'',z) = V_\beta(p,p'') - 4\pi \int_0^\infty dp' p'^2 \frac{V_\beta(p,p')}{p'^2 - z} t_\beta(p',p'';z) \qquad (2.28a)$$

$$-2\pi^2 t_\beta(p,p;p^2) = \frac{1}{p} \sin\delta_\beta(p) e^{i\delta_\beta(p)} , \qquad (2.28b)$$

z is the energy of two-nucleon system and δ_β is the phase shift.

In Ref. 21, Harper, Kim and Tubis have solved the Faddeev equations for the Reid soft-core potential, effective only in the two-nucleon $^1S_0$ and $^3S_1$-$^3D_1$ states. Yang and Jackson [12] used $^1D_2$, $^3D_2$ and $^3D_3$ Reid interactions also, which do not have significant effect on the final results. Only the first five basis states are retained. The probabilities are P(S) = 89.7%, P(S') = 1.68%, P(D) = 8.56%. Malfliet and Tjon [14] using the same forces found P(S) = 89.9%, P(S') = 1.8%, P(D) = 8.1%. The difference in P(D) is due to the discarding of 4 and 5 components by Malfliet and Tjon. Authors of Refs. 14 and 21 get the same energy E_B = -6.4 MeV, when components 4 and 5 are left aside. If 4 and 5 components are included, E_B = -6.7 MeV [21]. General expression for the ^{3}He form factor in momentum variables is (r is symmetry index of spin-isospin function [21])

$$2F_{ch}^{^3He}(Q^2) = \int_0^\infty p^2 dp \int_0^\infty q^2 dq \sum_{L,\ell} \sum_{\mathcal{L},\mathcal{S}} \sum_{r,r'} \sum_{\lambda=0}^{\ell} (-1)^\lambda \left(\frac{Q}{\sqrt{3}}\right)^\lambda q^{\ell-\lambda}$$

$$\times \frac{\ell!}{(\ell-\lambda)!\lambda!} \frac{1}{2} \int_{-1}^{1} dz \frac{P_\lambda(z)}{q_1{}^\lambda} \langle \psi_B^{^3He} | pq_1 (L\ell)\mathcal{L} W_{\mathcal{S}}^{r'} JJ_z \rangle$$

$$\times \langle W_{\mathcal{S}}^{r'} | \sum_{i=1}^{3} [f_{ch}^p(Q^2)\tfrac{1}{2}(1+\tau_{iz}) + f_{ch}^n(Q^2)\tfrac{1}{2}(1-\tau_{iz})] | W_{\mathcal{S}}^r \rangle$$

$$\times \langle pq(L\ell)\mathcal{L} W_{\mathcal{S}}^r JJ_z | \psi_B^{^3He} \rangle ,$$

$$q_1 = \left(q^2 + \frac{Q^2}{3} - \frac{2q}{\sqrt{3}} Qz\right)^{1/2} , \quad J = J_z = \tau = \tau_z = \frac{1}{2} . \tag{2.29}$$

Diffraction minimum in $|F_{ch}^{^3He}(Q^2)|$ by taking all eight components in $|\psi_B\rangle$ was found to occur at $Q^2 \approx 15.5\ fm^{-2}$. See Fig. 9. The slope of $|F_{ch}^{^3He}(Q^2)|$ at $Q^2 = 0$ gives charge radius 1.96 fm. [Experimental number is 1.88 ± 0.05 fm.]

Using only the $\mathcal{L} = 0$ components in trinucleon wave function, derived for the ($^1S_0$ and $^3S_1$-$^3D_1$) Reid soft-core potential, Tjon et al. [27] calculated charge form factors of ^3H and ^3He as

$$2F_{ch}^{^3He} = (2F_{ch}^{p} + F_{ch}^{n})F_1 - \frac{2}{3}(F_{ch}^{p} - F_{ch}^{n})F_2 \quad , \tag{2.30a}$$

$$F_{ch}^{^3H} = (2F_{ch}^{n} + F_{ch}^{p})F_1 + \frac{2}{3}(F_{ch}^{p} - F_{ch}^{n})F_2 \quad , \tag{2.30b}$$

where

$$F_1(\bar{q}) = \int d^3P\ d^3Q\ U(P,Q)U(P,|\bar{Q} - \frac{\bar{q}}{(3M)^{1/2}}|) \quad , \tag{2.31a}$$

$$F_2(\bar{q}) = -3 \int d^3P\ d^3Q\ U(P,Q)V_1(P,|\bar{Q} - \frac{\bar{q}}{(3M)^{1/2}}|) \quad . \tag{2.31b}$$

$$\bar{P} = \frac{1}{2} M^{-1/2} (\bar{k}_2 - \bar{k}_3) \quad , \quad \bar{Q} = \frac{1}{2}(3M)^{-1/2}(\bar{k}_2 + \bar{k}_3 - 2\bar{k}_1) \quad .$$

Functions U and V_1 correspond to components 1 and 2. For this case

$$E(^3H) = -6.8\ MeV \quad ,$$

$$R_{ch}(^3He) = 2.05\ fm \quad ,$$

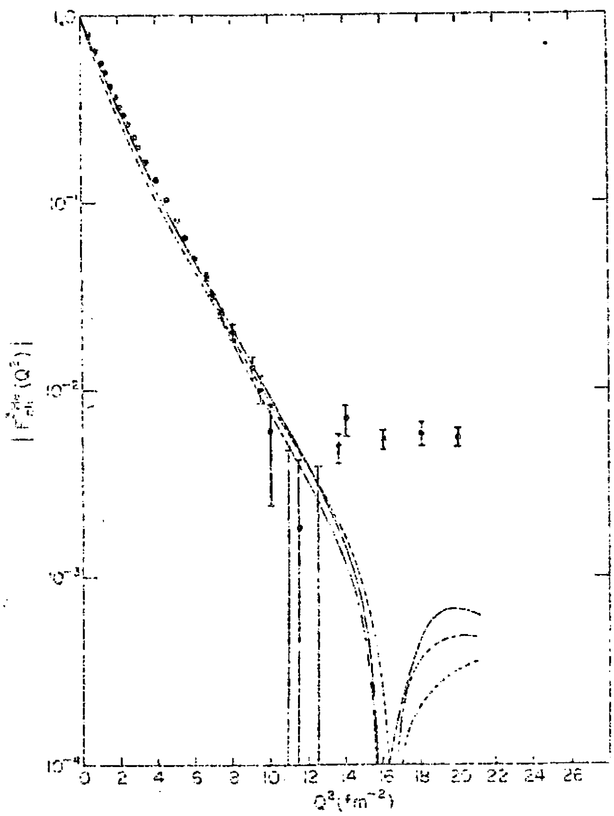


Fig. 9 The solid curve of $|F_{ch}^{^3He}(Q^2)|$ for Reid soft-core potential (with even partial waves) was obtained by using all components of the total wave function, dashed curve - by using S and S' states in the total wave function only, and dot-dashed curve was obtained by using (L = 0, ℓ = 0) $\mathcal{L}$ = 0 in the S state and S' state in the total wave function (taken from Ref. 21).

$$R_{ch}(^3H) = 1.80 \text{ fm} \quad .$$

Experimental numbers are

$$R_{ch}(^3He) = 1.88 \pm 0.05 \text{ fm} \quad ,$$

$$R_{ch}(^3H) = 1.70 \pm 0.05 \text{ fm} \quad .$$

The minimum in $|F_{ch}^{^3He}(q^2)|$ is at $q^2 = 17.0 \text{ fm}^{-2}$. (See Fig. 10.)

For calculation of processes that are not strongly dependent upon the short range behaviour of the wave function, the unitary pole approximation wave function is reliable [29].

The unitary pole approximation is a method for solving the Lippmann-Schwinger equations. For two spinless bosons interacting through S-wave potential only, there is one L-S equation (taken as an example).

$$T(s_0) = V + VG_0(s_0)T(s_0) \quad . \tag{2.32}$$

Consider the homogeneous L-S equation.

$$T(s_0) = VG_0(s_0)T(s_0) \quad , \quad G_0(s_0) = (s_0-H_0)^{-1} \quad . \tag{2.33a}$$

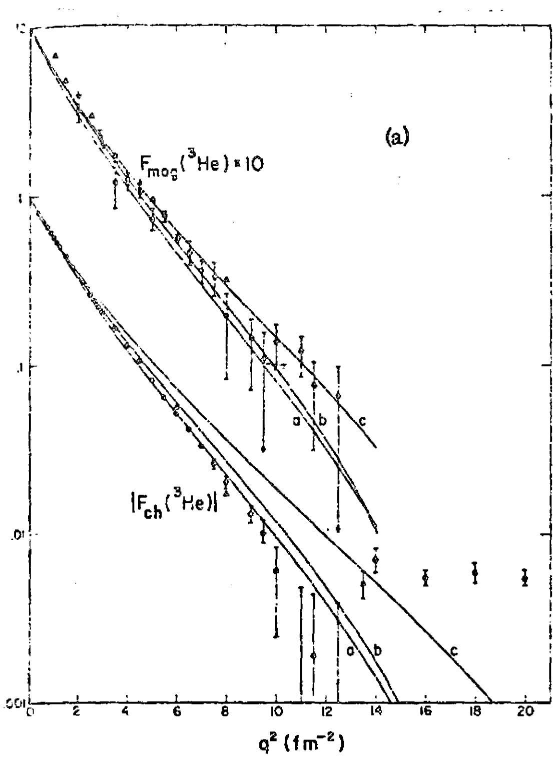


Fig. 10 The curve (a) is obtained for Reid soft-core potential (taken from Ref. 27).

Or,

$$|\psi_n(s_0)\rangle = \lambda_n(s_0)VG_0(s_0)|\psi_n(s_0)\rangle \quad . \tag{2.33b}$$

Fixing s_0 at a negative value -B [B is the two-particle binding energy], s_0-dependence is dropped from $|\psi_n\rangle$ and λ_n.

$$|\psi_n\rangle = \lambda_n VG_0(-B)|\psi_n\rangle \quad . \tag{2.34a}$$

The adjoint equation is

$$\langle\chi_n| = \mu_n\langle\chi_n|VG_0(-B) \quad , \tag{2.34b}$$

with $\lambda_n = \mu_n$ being real;

$$|\chi_n\rangle = G_0(-B)|\psi_n\rangle \quad . \tag{2.34c}$$

The orthonormality condition is

$$\langle\psi_n|G_0(-B)|\psi_m\rangle = -\delta_{nm} \quad . \tag{2.35}$$

The $|\psi_n\rangle$'s and λ_n's are such that $V = \sum_{m=1}^{N} - \frac{|\psi_m\rangle\langle\psi_m|}{\lambda_m}$ (with N possibly infinite). Then the homogeneous equation without s_0-dependence is satisfied. Truncating it to finite N means instead of the correct potential, an apporoximation of a finite rank separable potential is made. Thus the

Lippmann-Schwinger equations can be solved with an arbitrary small degree of approximation.

$$T(s) = V + VG_0(s)T(s)$$

$$= \sum_m - \frac{|\psi_m><\psi_m|}{\lambda_m} [1 + G_0(s)T(s)] \quad . \tag{2.36}$$

The unitary pole expansion matrix T_{UPE}-matrix is

$$T_{UPE}(s) = \sum_{m,n=1}^{N} |\psi_m>\Delta_{m,n}(s)<\psi_n| \quad , \tag{2.37a}$$

$$- [\Delta(s)]^{-1}_{mn} = \lambda_m \delta_{mn} + <\psi_m|G_0(s)|\psi_n> \quad . \tag{2.37b}$$

In each term of V, $|\psi_n>$'s correspond to form factors in a separable potential for different λ_n, and a particular energy s_0. If the two nucleons are known to be bound by an energy B, s_0 is taken to be -B. For antibound nucleons, $s_0 = 0$, and for unbound nucleons any convenient s_0 is taken. $T_{UPE}(s)$ is explicitly solved through $\Delta(s)$, and substituted into the Faddeev equations [16)]. T_{UPE} gives correct deuteron and low energy two-body scattering wave functions. Matrix elements of two-nucleon T_i-matrices in the three-nucleon space $(\bar{p},\bar{q})$ are

$$<\bar{p}\bar{q}|T_i(E)|\bar{p}'\bar{q}'> = \delta(\bar{p}-\bar{p}')<\bar{q}(t_{jk}(E - \frac{3}{4} p^2)|\bar{q}'> \quad , \tag{2.38}$$

t_{jk} is the T_{UPE}-matrix.

The unitary pole approximation to Reid (singlet and triplet) soft-core potential was performed by E. Harms et al.[16] Only 1 and 2 components in the wave function were retained. Results agree with those of Tjon, Gibson and O'Connell[27].

$$R_{ch}(^3He) = 1.97 \text{ fm} \quad ,$$

$$R_{ch}(^3H) = 1.76 \text{ fm} \quad ,$$

and q^2 of the dip in the ^3He absolute charge form factor is 17.0 fm^{-2}. See Fig. 11.

Faddeev equations were first solved for separable Yamaguchi-type potentials analytically by A. N. Mitra[34].

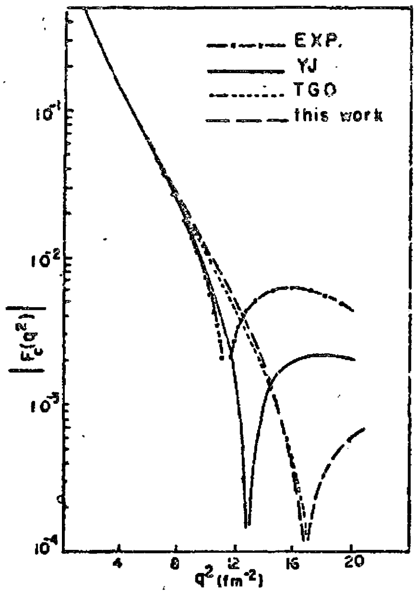


Fig. 11 Absolute charge form factor of ^3He. The long-dashed curve is obtained by using unitary-pole approximation method. The short-dashed curve is from the work of Tjon, Gibson and O'Connell (Ref. 27).

CHAPTER III

THREE-NUCLEON CHARGE FORM FACTORS WITH PHENOMENOLOGICAL WAVE FUNCTIONS

Construction of phenomenological three-nucleon wave functions from the symmetry properties has been shown by Gibson and Schiff [31] (also see Ref. 32). Spin functions for $SS_z = (1/2,1/2)$ are χ_1 and χ_2. (3/2,1/2) spin function is χ_s.

$$\chi_1 = 6^{-1/2}[(++-) + (+-+) - 2(-++)] \quad , \tag{3.1a}$$

$$\chi_2 = 2^{-1/2}[(++-) - (+-+)] \quad , \tag{3.1b}$$

$$\chi_s = 3^{-1/2}[(++-) + (+-+) + (-++)] \quad . \tag{3.1c}$$

Similar functions η_1, η_2 and η_s in isospin space are written for ^3He nucleus. For ^3H, there will be 'minus' instead of 'plus' and vice versa. Different combinations of spin-isospin functions can be formed from χ_1,χ_2 and η_1,η_2.

$$\phi_1 = \chi_2\eta_2 - \chi_1\eta_1 \quad , \tag{3.2a}$$

$$\phi_2 = \chi_2\eta_1 + \chi_1\eta_2 \quad , \tag{3.2b}$$

$$\phi_s = \chi_2\eta_2 + \chi_1\eta_1 \quad , \tag{3.2c}$$

$$\phi_a = \chi_2\eta_1 - \chi_1\eta_2 \quad . \tag{3.2d}$$

Jacobi coordinates of three nucleons can be

$$\bar{\xi} = 6^{-1/2}(\bar{x}_2 + \bar{x}_3 - 2\bar{x}_1) \tag{3.3a}$$

$$\bar{\eta} = 2^{-1/2}(\bar{x}_2 - \bar{x}_3) \tag{3.3b}$$

$$\bar{R} = 3^{-1/2}(x_1 + x_2 + x_3) \quad . \tag{3.3c}$$

Under the permutations P_{12}, P_{23}, P_{13} of two nucleons, the quantities χ_1, η_1, ϕ_1 and $\bar{\xi}$ behave in an analogous way.

$$P_{23}\phi_1 = \phi_1 \quad , \tag{3.4a}$$

$$P_{12}\phi_1 = \frac{1}{2}\ (\sqrt{3}\phi_2 - \phi_1) \quad , \tag{3.4b}$$

$$P_{13}\phi_1 = -\frac{1}{2}\ (\sqrt{3}\phi_2 + \phi_1) \quad . \tag{3.4c}$$

So do quantities χ_2, η_2, ϕ_2 and $\bar{\eta}$.

$$P_{23}\phi_2 = -\phi_2 \quad , \quad P_{12}\phi_2 = \frac{1}{2}(\phi_2 + \sqrt{3}\phi_1)$$

$$P_{13}\phi_2 = \frac{1}{2}(\phi_2 - \sqrt{3}\phi_1) \quad .$$

χ_s, η_s, ϕ_s and $\bar{R}$ are completely symmetric.

One defines

$$S_1 = \bar{\eta}^2 - \bar{\xi}^2 \tag{3.5a}$$

$$S_2 = 2\bar{\eta}\bar{\xi} \tag{3.5b}$$

$$S_s = \bar{\eta}^2 + \bar{\xi}^2 \tag{3.5c}$$

which are elementary scalar functions in spatial coordinates, behaving like ϕ_1, ϕ_2 and ϕ_s respectively under permutations P_{12}, P_{23}, P_{13}. $S_a = 0$.

The spin function χ_1 is also equal to

$$12^{-1/2}\,\bar{\sigma}_1(\bar{\sigma}_2 - \bar{\sigma}_3)\chi_2 \equiv 12^{-1/2}\,\bar{\sigma}_1\bar{\sigma}_{23}\chi_2 \quad . \tag{3.6}$$

The spatially symmetric component of the trinucleon wave function can be taken as

$$\psi_1 = \phi_a f_1(S_s) \quad , \tag{3.7}$$

f_1 is a spatially symmetric function. Second component is

$$\psi_2 = (\phi_2 S_1 - \phi_1 S_2) f_2(S_s) \quad . \tag{3.8}$$

T = 3/2 admixture can be introduced as

$$\psi_{(3/2)} = (S_2\chi_1 - S_1\chi_2)\eta_s f_{(3/2)}(S_s) \quad . \tag{3.8'}$$

Due to its small contribution, it is neglected.

P-state even parity components of the wave function contain a space vector of even parity $\bar{\xi} \times \bar{\eta}$, multiplied by vector functions of spin.

$$\bar{\Pi}_1 = 12^{-1/2}[\bar{\sigma}_{23} + i(\bar{\sigma}_1 \times \bar{\sigma}_{23})]\chi_2 \quad , \tag{3.9a}$$

$$\bar{\Pi}_2 = \bar{\sigma}_1\chi_2 \quad , \tag{3.9b}$$

$$\bar{\Pi}_s = [\bar{\sigma}_{23} - \frac{1}{2}\, i(\bar{\sigma}_1 \times \bar{\sigma}_{23})]\chi_2 \quad . \tag{3.9c}$$

$\bar{\Pi}_s$ is S = 3/2 function.

$$\psi_3 = (\bar{\Pi}_2\eta_2 + \bar{\Pi}_1\eta_1)(\bar{\xi} \times \bar{\eta})f_3(S_s) \tag{3.10}$$

$$\psi_4 = [(\bar{\Pi}_2S_1 + \bar{\Pi}_1S_2)\eta_2 + (\bar{\Pi}_2S_2 - \bar{\Pi}_1S_1)\eta_1]$$

$$\times (\bar{\xi} \times \bar{\eta})f_4(S_s) \tag{3.11}$$

are two S = 1/2, L = 1 functions.

$$\psi_s = (S_2\eta_2 + S_1\eta_1)\overline{\Pi}_s(\bar{\xi} \times \bar{\eta}) f_s(S_s) \tag{3.12}$$

is of quartet spin.

Symmetric quartet spin states (S = 3/2) are the following [symmetric only under spin permutations].

$$D_1 = [(\bar{\sigma}_1\bar{\eta})(\bar{\sigma}_{23}\bar{\eta}) - (\bar{\sigma}_1\bar{\xi})(\bar{\sigma}_{23}\bar{\xi}) - \tfrac{1}{3}(\bar{\sigma}_1\bar{\sigma}_{23})(\bar{\eta}^2 - \bar{\xi}^2)]\chi_2 \quad , \tag{3.13}$$

$$D_2 = [(\bar{\sigma}_1\bar{\eta})(\bar{\sigma}_{23}\bar{\xi}) + (\bar{\sigma}_1\bar{\xi})(\bar{\sigma}_{23}\bar{\eta}) - \tfrac{2}{3}(\bar{\sigma}_1\bar{\sigma}_{23})\bar{\eta}\bar{\xi}]\chi_2 \quad , \tag{3.14}$$

$$D_s = [(\bar{\sigma}_1\bar{\eta})(\bar{\sigma}_{23}\bar{\eta}) + (\bar{\sigma}_1\bar{\xi})(\bar{\sigma}_{23}\bar{\xi}) - \tfrac{1}{3}(\bar{\sigma}_1\bar{\sigma}_{23})(\bar{\xi}^2 + \bar{\eta}^2)]\chi_2 \quad , \tag{3.15}$$

which transforms under both spatial a spin permutations like $\bar{\xi}$, $\bar{\eta}$ and $\bar{R}$ respectively.

The 3 components L = 2, S = 3/2 in the total wave function are

$$\psi_6 = D_s(S_2\eta_1 - S_1\eta_2) f_6(S_s) \quad , \tag{3.16}$$

$$\psi_7 = (D_2\eta_1 - D_1\eta_2)f_7(S_s) \quad , \tag{3.17}$$

$$\psi_8 = [(D_2S_1 + D_1S_2)\eta_1 - (D_2S_2 - D_1S_1)\eta_2]f_8(S_s) \quad . \tag{3.18}$$

ψ_6 and ψ_7 are not orthogonal, and therefore, instead of ψ_6 the following linear combination of ψ_6 and ψ_7 is taken.

$$\psi_6 = [(5D_2S_2 - 2D_2S_s)\eta_1 - (5D_sS_1 - 2D_1S_s)\eta_2]f_6(S_s) \quad . \tag{3.16}$$

$f_1, \ldots, f_8$ are invariant functions of $S_s = \overline{\xi}^2 + \overline{\eta}^2$ (under all permutations P_{12}, P_{23}, P_{13}). Choosing the components to be homogeneous in S_1, S_2 and S_s of the same order, the set of 8 components, orthogonal to one another, is following.

$$\psi_1 = (\chi_2\eta_1 - \chi_1\eta_2)(S_1^2 + S_2^2)f_1(S_s)$$

$$\psi_2 = [(\chi_2(S_2^2 - S_1^2) + 2\chi_1S_1S_2)\eta_1$$

$$+ (\chi_1(S_2^2 - S_1^2) - 2\chi_2S_1S_2)\eta_2]f_2(S_s)$$

$$\psi_3 = (\overline{\Pi}_1\eta_1 + \overline{\Pi}_2\eta_2)(\overline{\xi} \times \overline{\eta})S_sf_3(S_s)$$

$$\psi_4 = [(\overline{\Pi}_2S_2 - \overline{\Pi}_1S_1)\eta_1 + (\overline{\Pi}_2S_1 + \overline{\Pi}_1S_2)\eta_2](\overline{\xi} \times \overline{\eta})f_4(S_s)$$

$$\psi_5 = (S_1\eta_1 + S_2\eta_2)\overline{\Pi}_s(\overline{\xi} \times \overline{\eta})f_5(S_s)$$

$$\psi_6 = [(5D_sS_2 - 2D_2S_s)\eta_1 - (5D_sS_1 - 2D_1S_s)\eta_2]f_6(S_s)$$

$$\psi_7 = (D_2S_s\eta_1 - D_1S_s\eta_2)f_7(S_s)$$

$$\psi_8 = [(D_2S_1 + D_1S_2)\eta_1 - (D_2S_2 - D_1S_1)\eta_2]f_8(S_s) \quad .$$

$$\psi = \sum_i \psi_i = u_2\eta_1 - u_1\eta_2 \quad ; \tag{3.19}$$

u_1 and u_2 are spatial-spin functions. The wave function ψ has the following well-defined quantum numbers: $J = 1/2$, $S_z = 1/2$, $T = 1/2$ and $T_z = +1/2$ for ^3He, and $-1/2$ for ^3H. Then

$$2\iint d\overline{\xi}d\overline{\eta}(u_1^*u_1 + u_2^*u_2) \times F_{ch}^{^3He}(\overline{q}) = \frac{3}{2}[F_{ch}^p(\overline{q}) + F_{ch}^n(\overline{q})]$$
$$\times \iint e^{-i\sqrt{2/3}\,\overline{q}\overline{\xi}}(u_2^*u_2 + u_1^*u_1)d\overline{\xi}d\overline{\eta} + \frac{3}{2}[F_{ch}^p(\overline{q})$$
$$- F_{ch}^n(\overline{q})] \times \iint e^{-i\sqrt{2/3}\,\overline{q}\overline{\xi}}(-\frac{1}{3}u_2^*u_2 + u_1^*u_1)d\overline{\xi}d\overline{\eta} \quad . \tag{3.20}$$

For ^3H, F_{ch}^p and F_{ch}^n should be interchanged.

$$\iint d\bar{\xi}d\bar{\eta}(u_1^*u_1 + u_2^*u_2) \times F^{^3H}_{ch}(\bar{q}) = \frac{3}{2}[F^p_{ch}(\bar{q}) + F^n_{ch}(\bar{q})]$$

$$\times \iint e^{-i\sqrt{2/3}\ \bar{q}\bar{\xi}}(u_2^*u_2 + u_1^*u_1)d\bar{\xi}d\bar{\eta} - \frac{3}{2}[F^p_{ch}(\bar{q})$$

$$- F^n_{ch}(\bar{q})] \times \iint e^{-i\sqrt{2/3}\ \bar{q}\bar{\xi}}(-\frac{1}{3}\ u_2^*u_2 + u_1^*u_1)d\bar{\xi}d\bar{\eta} \quad . \tag{3.21}$$

Nonzero scalar products involving different spin functions are

$$\chi_1^*\chi_1 = \chi_2^*\chi_2 = 1 \qquad \chi_1^*\Pi_1 = \chi_2^*\Pi_2 = (0,0,1) \quad .$$

$$[\Pi_1(\bar{\xi} \times \bar{\eta})]^*\Pi_1(\bar{\xi} \times \bar{\eta}) = [\Pi_2(\bar{\xi} \times \bar{\eta})]^*\Pi_2(\bar{\xi} \times \bar{\eta})$$

$$= \frac{1}{4}\ (S_s^2 - S_1^2 - S_2^2) \quad .$$

$$[\Pi_s(\bar{\xi} \times \bar{\eta})]^*\Pi_s(\bar{\xi} \times \bar{\eta}) = \frac{3}{2}\ (S_s^2 - S_1^2 - S_2^2)$$

$$D_1^*D_1 = 2(S_s^2 + \frac{1}{3}S_1^2 - S_2^2) \qquad D_1^*D_2 = 4(\frac{2}{3}S_1S_2 + iS_s(\bar{\eta} \times \bar{\xi})z)$$

$$D_2^*D_2 = 2(S_s^2 - S_1^2 + \frac{1}{3}S_2^2) \qquad D_1^*D_s = 4(\frac{2}{3}S_1S_s + iS_2(\bar{\eta} \times \bar{\xi})z)$$

$$D_s^*D_s = 2(\frac{1}{3}S_s^2 + S_1^2 + S_2^2) \qquad D_2^*D_s = \frac{8}{3}S_2S_s \quad .$$

It is convenient to use the so-called Irving transformation

$$|\bar{\xi}| = \rho\cos\theta \qquad , \qquad |\bar{\eta}| = \rho\sin\theta \quad .$$

$$\bar{\xi} = |\bar{\xi}| \; (\sin\theta_\xi\cos\phi_\xi, \; \sin\theta_\xi\sin\phi_\xi, \; \cos\theta_\xi)$$

$$\bar{\eta} = |\bar{\eta}| \; (\sin\theta_\eta\cos\phi_\eta, \; \sin\theta_\eta\sin\phi_\eta, \; \cos\theta_\eta) \quad .$$

Then

$$S_1 = -\rho^2\cos2\theta \quad ,$$

$$S_2 = \rho^2\sin2\theta\cos\theta_{\xi\eta} \quad ,$$

$$S_s = \rho^2 \quad ,$$

and

$$\int d\bar{\xi}d\bar{\eta} = \frac{1}{8}\int_0^\infty d\rho\rho^5 \int d\Omega_\xi d\Omega_\eta \int_0^{\frac{\pi}{2}} d\theta(1 - \cos4\theta) \quad .$$

With this transformation, normalisation integrals become

$$\int d\bar{\xi}d\bar{\eta} \; |\psi_1|^2 = \frac{2}{3}\,\pi^3 \int d\rho\rho^{13}f_1^2(\rho) \tag{3.22}$$

$$\int d\bar{\xi}d\bar{\eta} \; |\psi_2|^2 = \frac{2}{3}\,\pi^3 \int d\rho\rho^{13}f_2^2(\rho) \tag{3.23}$$

$$\int d\bar{\xi}d\bar{\eta} \; |\psi_3|^2 = \frac{1}{4}\,\pi^3 \int d\rho\rho^{13}f_3^2(\rho) \tag{3.24}$$

$$\int d\bar{\xi}d\bar{\eta} \; |\psi_4|^2 = \frac{1}{12}\pi^3 \int d\rho\rho^{13}f_4^2(\rho) \tag{3.25}$$

$$\int d\bar{\xi}d\bar{\eta}\ |\psi_5|^2 = \frac{1}{4}\pi^3 \int d\rho\rho^{13}f_5^2(\rho) \tag{3.26}$$

$$\int d\bar{\xi}d\bar{\eta}\ |\psi_6|^2 = \frac{35}{3}\pi^3 \int d\rho\rho^{13}f_6^2(\rho) \tag{3.27}$$

$$\int d\bar{\xi}d\bar{\eta}\ |\psi_7|^2 = \frac{10}{3}\pi^3 \int d\rho\rho^{13}f_7^2(\rho) \tag{3.28}$$

$$\int d\bar{\xi}d\bar{\eta}\ |\psi_8|^2 = \frac{14}{3}\pi^3 \int d\rho\rho^{13}f_8^2(\rho) \quad . \tag{3.29}$$

The expression for $F_{ch}^{^3He}$ is

$$2\int d\bar{\xi}d\bar{\eta}(u_2^*u_2 + u_1^*u_1) \times F_{ch}^{^3He}(\bar{q})$$

$$= 2\pi^3\int d\rho\rho^{13}\{\tfrac{2}{3}(f_1^2+f_2^2)+\tfrac{1}{4}f_3^2+\tfrac{1}{12}f_4^2+\tfrac{1}{4}f_5^2+\tfrac{35}{3}f_6^2+\tfrac{10}{3}f_7^2+\tfrac{14}{9}f_8^2\}\times F_{ch}^{^3He}(\bar{q})$$

$$= \tfrac{3}{2}(F_{ch}^{p}+F_{ch}^{n})\int d\bar{\xi}d\bar{\eta}e^{-i\sqrt{2/3}\ \bar{q}\bar{\xi}}\{2(S_1^2+S_2^2)^2(f_1^2+f_2^2)+\tfrac{1}{2}S_s^2(S_s^2-S_1^2-S_2^2)f_3^2$$

$$+\tfrac{1}{2}(S_1^2+S_2^2)(S_s^2-S_1^2-S_2^2)f_4^2+\tfrac{3}{2}(S_1^2+S_2^2)(S_s^2-S_1^2-S_2^2)f_5^2+[16S_s^4+50(S_1^2+S_2^2)^2$$

$$-42S_s^2(S_1^2+S_2^2)]f_6^2+4S_s^2[S_s^2-\tfrac{1}{3}(S_1^2+S_2^2)]f_7^2+4(S_1^2+S_2^2)[S_s^2-\tfrac{1}{3}(S_1^2+S_2^2)]f_8^2$$

$$+16S_s^2(2S_1^2+2S_2^2-S_s^2)f_6f_7+8S_1S_s\cdot 2(\tfrac{4}{3}S_2^2-S_1^2)f_6f_8+16S_1S_s(-\tfrac{1}{3}S_1^2+S_2^2)f_7f_8\}$$

$$+(F_{ch}^{p}-F_{ch}^{n})\int d\bar{\xi}d\bar{\eta}e^{-i\sqrt{2/3}\ \bar{q}\bar{\xi}}\{(S_1^2+S_2^2)^2(f_1^2+f_2^2)+\tfrac{1}{4}S_s^2(S_s^2-S_1^2-S_2^2)f_3^2$$

$$+\tfrac{1}{4}(S_1^2+S_2^2)(S_s^2-S_1^2-S_2^2)f_4^2+2(S_1^4-S_2^4)(f_1^*f_2+f_2^*f_1)+\tfrac{1}{2}S_1S_s(S_s^2-S_1^2-S_2^2)$$

$$\times(f_3^*f_4+f_4^*f_3)+\tfrac{3}{4}(3S_2^2-S_1^2)(S_s^2-S_1^2-S_2^2)f_5^2+[8S_s^4+25(3S_1^2-S_2^2)(S_1^2+S_2^2)$$

$$-47S_s^2(S_1^2+S_2^2)+52S_2^2S_s^2]f_6^2+2S_s^2(S_1^2+S_s^2-\tfrac{5}{3}S_2^2)f_7^2+2[S_s^2(S_1^2+S_2^2)-\tfrac{26}{3}S_1^2S_2^2$$

$$+S_1^4+S_2^4]f_8^2+8S_s^2(4S_1^2-S_s^2)f_6f_7+8S_1S_s(2S_s^2-5S_1^2+\tfrac{2}{3}S_2^2)f_6f_8$$

$$+8S_1S_s(\tfrac{4}{3}S_2^2-S_s^2)f_7f_8\} \quad . \tag{3.30}$$

Converting to Irving coordinates, this becomes $[\bar{\kappa} = \sqrt{2/3}\ \bar{q}]$.

$$2\int d\rho\rho^{13}\{\tfrac{2}{3}(f_1^2+f_2^2)+\tfrac{1}{4}f_3^2+\tfrac{1}{12}f_4^2+\tfrac{1}{4}f_5^2+\tfrac{35}{3}f_6^2+\tfrac{10}{3}f_7^2+\tfrac{14}{9}f_8^2\}\times F_{ch}^{^3He}(\bar{q})$$

$$= [2F_{ch}^p(\bar{q})+F_{ch}^n(\bar{q})]\int d\rho\rho^5\int d\theta(1-\cos 4\theta)\frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta}[8(S_1^4+\tfrac{8}{3}\eta^2\xi^2 S_1^2$$

$$+\tfrac{16}{5}\eta^4\xi^4)(f_1^2+f_2^2)+\tfrac{1}{2}\rho^4(\rho^4-S_1^2-\tfrac{8}{3}\eta^2\xi^2)f_3^2+2(S_1^2\rho^4-S_1^4-\tfrac{8}{3}\eta^2\xi^2 S_1^2$$

$$+\tfrac{4}{3}\eta^2\xi^2\rho^4-\tfrac{16}{5}\eta^4\xi^4)f_4^2]+[F_{ch}^p(\bar{q})-F_{ch}^n(\bar{q})]\int d\rho\rho^5\int d\theta(1-\cos 4\theta)$$

$$\times\frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta}[8(S_1^4-\tfrac{16}{5}\xi^4\eta^4)(f_1^*f_2+f_2^*f_1)+2S_1S_s(\rho^4-S_1^2-\tfrac{2}{3}\xi^2\eta^2)$$

$$\times(f_3^*f_4+f_4^*f_3)\}+9(F_{ch}^p+F_{ch}^n)\int d\rho\rho^5\int d\theta(1-\cos 4\theta)\frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta}$$

$$\times(\rho^4 S_1^2-S_1^4-\tfrac{8}{3}\eta^2\xi^2 S_1^2-\tfrac{16}{5}\eta^4\xi^4)f_5^2+9(F_{ch}^p-F_{ch}^n)\int d\rho\rho^5\int d\theta(1-\cos 4\theta)$$

$$\times\frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta}(\tfrac{4}{3}\rho^4\eta^2\xi^2-\tfrac{8}{9}S_1^2\eta^2\xi^2-\tfrac{16}{5}\eta^4\xi^4)f_5^2+12(F_{ch}^p+F_{ch}^n)$$

$$\times\int d\rho\rho^5\int d\theta(1-\cos 4\theta)\frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta}\{[8\rho^8+25S_1^4+\tfrac{200}{3}S_1^2\xi^2\eta^2+80\xi^4\eta^4$$

$$-21\rho^4(S_1^2+\tfrac{4}{3}\xi^2\eta^2)]f_6^2+2\rho^4[\rho^4-\tfrac{1}{3}(S_1^2+\tfrac{4}{3}\xi^2\eta^2)]f_7^2+2[\rho^4(S_1^2+\tfrac{4}{3}\xi^2\eta^2)$$

$$-\tfrac{1}{3}(S_1^4+\tfrac{8}{3}S_1^2\xi^2\eta^2+\tfrac{16}{5}\xi^4\eta^4)]f_8^2+8\rho^4(2S_1^2+\tfrac{8}{3}\xi^2\eta^2-\rho^4)f_6f_7+\tfrac{8}{3}\rho^2 S_1(4\xi^2\eta^2$$

$$-S_1^2)f_7f_8+8\rho^2 S_1(\tfrac{16}{9}\xi^2\eta^2-S_1^2)f_6f_8\}+4(F_{ch}^p-F_{ch}^n)\int d\rho\rho^5\int d\theta(1-\cos 4\theta)$$

(continued over)

$$\times \frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta}\{[8\rho^8+25(3S_1^4+\frac{8}{3}S_1^2\xi^2\eta^2-\frac{16}{5}\xi^4\eta^4)-47\rho^4S_1^2+\frac{20}{3}\rho^4\xi^2\eta^2]f_6^2$$

$$+2\rho^4(S_1^2-\frac{20}{9}\xi^2\eta^2+\rho^4)f_7^2+2[\rho^4(S_1^2+\frac{4}{3}\xi^2\eta^2)-\frac{26.4}{9}S_1^2\xi^2\eta^2+S_1^4+\frac{16}{5}\xi^4\eta^4]f_8^2$$

$$+8\rho^4(4S_1^2-\rho^4)f_6f_7+8S_1\rho^2(\frac{16}{9}\xi^2\eta^2-\rho^4)f_7f_8+8S_1\rho^2(2\rho^4-5S_1^2+\frac{8}{9}\xi^2\eta^2)f_6f_8\}. \tag{3.31}$$

The following formula is used

$$\int_0^{\frac{\pi}{2}} d\theta\ \sin(\kappa\rho\cos\theta)\cos n\theta = \frac{1}{2}\pi(-1)^{\frac{n-1}{2}} J_n(\kappa\rho) \quad . \tag{3.32}$$

and

$$J_{n-1}(z) + J_{n+1}(z) = \frac{2n}{z} J_n(z) \quad . \tag{3.33}$$

Coefficient of $\underline{\underline{f_1f_2}}$ in the integration over ρ is

$$2(F^p_{ch} - F^n_{ch})\rho^5 \int d\theta(1 - \cos4\theta)\ \frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta} \cdot 8(S_1^4 - \frac{16}{5}\xi^4\eta^4)$$

$$= \frac{16}{\kappa}(F^p_{ch}-F^n_{ch})\frac{\rho^{12}}{4}\int d\theta\ \sin(\kappa\rho\cos\theta)(2\cos5\theta-2\cos7\theta+\frac{2}{5}\cos9\theta-\frac{2}{5}\cos11\theta)$$

$$= \frac{8}{\kappa}(F^p_{ch}-F^n_{ch})(J_5+J_7+\frac{1}{5}J_9+\frac{1}{5}J_{11})\rho^{12} = \underline{\frac{32}{\kappa^2}(F^p_{ch}-F^n_{ch})(3J_6+J_{10})\rho^{11}}.$$

$\underline{\underline{f_1^2 \ \& \ f_2^2}}$

$$(2F^p_{ch}+F^n_{ch})\rho^5 \int d\theta(1-\cos 4\theta)\frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta} \cdot 8(S_1^4+\frac{8}{3}\xi^2\eta^2S_1^2+\frac{16}{5}\xi^4\eta^4)$$

$$= \frac{8}{\kappa}(2F^p_{ch}+F^n_{ch})\rho^{12}\int d\theta\sin(\kappa\rho\cos\theta)(\frac{2}{3}\cos\theta-\frac{2}{3}\cos 3\theta+\frac{1}{3}\cos 5\theta-\frac{1}{3}\cos 7\theta$$

$$+\frac{1}{15}\cos 9\theta-\frac{1}{15}\cos 11\theta)$$

$$= \frac{16}{3\kappa}(2F^p_{ch}+F^n_{ch})(J_1+J_3+\frac{1}{2}J_5+\frac{1}{2}J_7+\frac{1}{10}J_9+\frac{1}{10}J_{11})$$

$$= \frac{32}{\kappa^2}(2F^p_{ch}+F^n_{ch})(J_2+3J_6+J_{10})\rho^{11} \quad .$$

$\underline{\underline{f_3^2}}$

$$(2F^p_{ch} + F^n_{ch})\rho^5 \int d\theta(1 - \cos 4\theta) \frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta} \cdot \frac{1}{2}\rho^4(\rho^4-S_1^2-\frac{8}{3}\xi^2\eta^2)$$

$$= \frac{2F^p_{ch}+F^n_{ch}}{2\kappa}\rho^{12}\int d\theta \sin(\kappa\rho\cos\theta)(\frac{1}{2}\cos\theta-\frac{1}{2}\cos 3\theta-\frac{1}{6}\cos 5\theta+\frac{1}{6}\cos 7\theta)$$

$$= \frac{2F^p_{ch}+F^n_{ch}}{\kappa^2}(J_2 - J_6)\rho^{11} \quad .$$

$\underline{\underline{f_3 f_4}}$

$$(F^p_{ch} - F^n_{ch})\rho^5 \int d\theta (1 - \cos 4\theta) \frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta} \cdot 2S_1\rho^2(\rho^4 - S_1^2 - \tfrac{2}{3}\xi^2\eta^2)$$

$$= \frac{20}{3} \frac{F^p_{ch} - F^n_{ch}}{\kappa^2} (J_4 - J_8)\rho^{11} \quad .$$

$\underline{\underline{f_4^2}}$

$$(2F^p_{ch} + F^n_{ch})\rho^5 \int d\theta (1-\cos 4\theta) \frac{\sin(\kappa\rho\cos\theta)}{\kappa\rho\cos\theta} \cdot 2(S_1^2\rho^4 - S_1^4 - \tfrac{8}{3}\xi^2\eta^2 S_1^2$$

$$+ \tfrac{4}{3}\xi^2\eta^2\rho^4 - \tfrac{16}{5}\xi^4\eta^4)$$

$$= \frac{8(2F^p_{ch} + F^n_{ch})}{3\kappa} \cdot \frac{1}{4}(J_1 + J_3 - \tfrac{1}{5}J_9 - \tfrac{1}{5}J_{11})\rho^{12}$$

$$= \frac{8(2F^p_{ch} + F^n_{ch})}{3\kappa^2} (J_2 - J_{10})\rho^{11} \quad .$$

$\underline{\underline{f_5^2}}$

$$- \frac{2(F^p_{ch} + 5F^n_{ch})}{\kappa^2} (J_2 - 3J_6 + 2J_{10})\rho^{11} \quad .$$

$\underline{\underline{f_6^2}}$

$$\frac{32.35}{3\kappa^2}(2F^p_{ch}+F^n_{ch})\rho^{11}J_2+\frac{48}{\kappa^2}(13F^p_{ch}-5F^n_{ch})\rho^{11}J_6+\frac{25.16}{3\kappa^2}(7F^p_{ch}-F^n_{ch})\rho^{11}J_{10} .$$

$\underline{\underline{f_6 f_7}}$

$$\frac{24.64}{\kappa^2}\rho^{11} F^p_{ch} J_6 .$$

$\underline{\underline{f_8^2}}$

$$\frac{7.64}{9\kappa^2}(2F^p_{ch}+F^n_{ch})\rho^{11}J_2+\frac{2.64}{3\kappa^2}(2F^p_{ch}+F^n_{ch})\rho^{11}J_6+\frac{2.8}{9\kappa^2}(25F^p_{ch}-67F^n_{ch})\rho^{11}J_{10} .$$

$\underline{\underline{f_6 f_8}}$

$$-\frac{4.8.8}{9\kappa^2}[(11F^p_{ch}+4F^n_{ch})\rho^{11}J_4+(43F^p_{ch}+4F^n_{ch})\rho^{11}J_8] .$$

$\underline{\underline{f_7^2}}$

$$\frac{32.2}{3\kappa^2}\rho^{11}[5(2F^p_{ch}+F^n_{ch})J_2-(2F^p_{ch}-5F^n_{ch})J_8] .$$

$\underline{\underline{f_7 f_8}}$

$$-\frac{16^2}{9\kappa^2}\rho^{11}[7(F^p_{ch}-F^n_{ch})J_4+(11F^p_{ch}+7F^n_{ch})J_8] .$$

$$h_i(\rho) = \rho^6 f_i(\rho) \qquad \bar{\kappa} = \sqrt{2/3}\,\bar{q} .$$

$$F^{^3He}_{ch}(\bar{q})\cdot\int_0^\infty d\rho\times\rho\{\tfrac{2}{3}(h_1^2+h_2^2)+\tfrac{1}{4}h_3^2+\tfrac{1}{12}h_4^2+\tfrac{1}{4}h_5^2+\tfrac{35}{3}h_6^2+\tfrac{10}{3}h_7^2+\tfrac{14}{9}h_8^2\}$$

$$= \frac{1}{2\kappa^2}\int_0^\infty d\rho\frac{1}{\rho}\{32(2F^p_{ch}(\bar{q})+F^n_{ch}(\bar{q})(2J_2(\kappa\rho)+3J_6(\kappa\rho)+J_{10}(\kappa\rho))$$

$$\times (h_1^2+h_2^2)+32(F^p_{ch}-F^n_{ch})(3J_6+J_{10})h_1h_2+(2F^p_{ch}+F^n_{ch})(J_2-J_6)h_3^2$$

$$+\frac{8}{3}(2F^p_{ch}+F^n_{ch})(J_2-J_{10})h_4^2+\frac{20}{3}(F^p_{ch}-F^n_{ch})(J_4-J_8)h_3h_4-2(F^p_{ch}+5F^n_{ch})$$

$$\times (J_2-3J_6+2J_{10})h_5^2+[\frac{32.35}{3}(2F^p_{ch}+F^n_{ch})J_2+48(13F^p_{ch}-5F^n_{ch})J_6$$

$$+\frac{25.16}{3}(7F^p_{ch}-F^n_{ch})J_{10}]h_6^2+24\times 64F^p_{ch}(\bar{q})h_6h_7+\frac{64}{3}[5(2F^p_{ch}+F^n_{ch})J_2(\kappa\rho)$$

$$-(2F^p_{ch}-5F^n_{ch})J_6(\kappa\rho)]h_7^2-(\frac{16}{3})^2[7(F^p_{ch}-F^n_{ch})J_4+(11F^p_{ch}+7F^n_{ch})J_8]h_7h_8$$

$$+[\frac{448}{9}(2F^p_{ch}+F^n_{ch})J_2+\frac{128}{3}(2F^p_{ch}+F^n_{ch})J_6+\frac{16}{9}(25F^p_{ch}-67F^n_{ch})J_{10}]h_8^2$$

$$-(\frac{16}{3})^2[(11F^p_{ch}+4F^n_{ch})J_4+(43F^p_{ch}+4F^n_{ch})J_8]h_6h_8\} . \qquad (3.34)$$

Function h can be taken in the form $c\rho^{b}e^{-a\rho}$ (b = 2 for the Irving function). In practice, it is necessary to investigate the importance of each h_i some way. Since the magnetic moments of ^3H and ^3He appear not to depend on the probability of P states, the terms containing h_3, h_4 and h_5 may be ignored. The parametric functions h_i can be fitted to the experimental numbers on $F_{ch}^{^3He}(\bar{q})$, $F_{ch}^{p}(\bar{q})$, $F_{ch}^{n}(\bar{q})$. In Ref. 35, the author has given expressions fitting the experimental data on ^3H and ^3He charge form factors. The factor $e^{-a\rho}$ defines behaviour of the wave function near the outer edge of the nuclear potential. Parameter a is roughly of the same magnitude for different h_i. For short distances, $h_i(\rho)$ approaches $\rho = 0$ as some power of ρ: ρ^{b_i}. L. I. Schiff considered three types of phenomenological wave functions: exponential, Gaussian and Irving functions, and compared the physical properties of ^3H and ^3He calculated with the three different wave functions. Irving function was found to be a physically plausible phenomenological wave function, that is reasonably easy to work with.

SUMMARY

Results of the variational calculations do not give unambiguous information about true nature of the trinucleon wave function for a given hamiltonian. It is, in principle, possible for different types of trial functions to converge to different wave functions and give the same binding energy.

Method of diagonalising the hamiltonian is applicable for soft-core potentials only. If the basis is taken sufficiently large, diagonalisation method gives a reliable wave function. Diagonalisation of the hamiltonian for Reid soft-core potential in the basis of 484 harmonic oscillator states (discarding the P states) yielded a binding energy of the three-nucleon bound system equal to 6.30 MeV. The extrapolated value is 6.5 MeV. With this wave function, minimum in the ^{3}He absolute charge form factor was found to be at $q^2 = 12.9$ fm^{-2}. Probability of the D states equals to 7.8% (Ref. 11). Nunberg et al.[13] diagonalised the hamiltonian for Riihimaki potential in the basis of 348 harmonic oscillator states. Binding energy in the truncated basis was found equal to 6.28 MeV, and the probability of D states - 4.6%. In finding the extrapolated value of the binding energy (7.0 MeV), P states were not completely neglected. From rough estimates of the isoscalar magnetic moment it is concluded by Nunberg et al. that P(D) $\lesssim$ 4%, and therefore, Riihimaki potential is more favoured with respect to other realistic potentials. Form factor calculations for the Riihimaki potential have not been performed. Extrapolating procedures are different in the two diagonalisation calculations.

For solving Faddeev's equations, the wave function is expanded over a basic set of states [See Chap. II]. Each basic state is characterised by $(L\ell)\mathcal{L}(Ss)\mathcal{S}(Tt)\tau\tau_z$. $\mathcal{L} = 0$ should couple with $\mathcal{S} = 1/2$, since the total angular momentum is J = 1/2. L and ℓ should both be equal to each other, i.e., $L = \ell = n$, where n = 0, 1, 2, 3, ..., so that $\mathcal{L}$ be zero. Harper, Kim and Tubis, and others consider only n = 0, 1, 2. Similar remarks can be made for $\mathcal{L} = 1$, $\mathcal{L} = 2$ basic states, used in expanding the wave function. This can be a reason of disagreement in the results of Jackson et al. and Harper et al. and others for the same potential, i.e., Reid soft-core. Therefore, among different calculations on the three-nucleon bound system for given two-nucleon forces, diagonalisation method seems to be suitable, provided the basis is quite large.

The summary of the calculations by different methods for particular realistic nucleon-nucleon forces is given in Table IV.

Table IV Summary of the Calculations, presented in the Review

TABLE IV

Ref.	Potential	Triton binding energy (MeV)	$\langle r^2 \rangle_{ch}^{1/2}(^3H)$ (fm)	$\langle r^2 \rangle_{ch}^{1/2}(^3He)$ (fm)	$q^2 (fm^{-2})$ of the minimum in $\lvert F_{ch}^{^3He}(q^2) \rvert$
9	Hamada-Johnston (Coulomb forces are included in case of ^3He)	6.5 ± 0.2	1.85 ± 0.02	1.90	12.5 ± 0.3
13	Riihimaki	7.0	1.83	2.14	
11	Reid soft-core	6.5	1.87	2.09	
12					12.9
26		6.25			
14		6.5			
16			1.76	1.97	17.0
21		6.7		1.96	15.5
27		6.8 ± 0.5	1.80	2.05	17.0
17		7.58			
	Experimental	8.48	1.70 ± 0.05	1.88 ± 0.05	11.6

From the table it can be seen that all the calculations give a binding energy 1-2 MeV less than the experimental binding energy. The mentioned calculations assume only two-nucleon forces to be acting in the three-nucleon system, and the discrepancy between theory and experiment suggests that three-body forces may play a significant role in obtaining the correct binding energy. If the experimental data on ^3He are considered as well, and compared with those on ^3H, one can get information about the charge dependence of the nucleon-nucleon forces.

The wave function of (T = 1/2, J = 1/2) three-nucleon bound system can be investigated from the form factor data.

CORRECTION (on page 54)

In constructing phenomenological wave function, it seems likely to choose

$$\psi_1 = (\chi_2\eta_1 - \chi_1\eta_2)S_s^2 f_1(S_s) \quad .$$

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