ESSAYS ON THE ECONOMIC IMPACT OF INTANGIBLE CAPITAL AND INVESTMENT

ESSAYS ON THE ECONOMIC IMPACT OF INTANGIBLE CAPITAL AND INVESTMENT

By

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Abstract

This thesis investigates the role of intangible capital and intangible investment (the intangibles) in explaining modern economic activity. It presents an in depth analysis of the context in which the intangibles are studied in the economic literature, and modifies existing theoretical real business cycle (RBC) models to account for the presence of the intangibles. The newly developed models are further used to address previously documented issues such as the Canadian productivity puzzle and the quantity anomaly.

Chapter 1 provides a detailed explanation of the concept of the intangibles in the economic literature. It also highlights the importance of accounting for the intangibles during economic analysis and presents a detailed analysis of how they are measured and modeled in practice. The main findings indicate that the intangibles have contributed positively to economic growth and productivity. The need for improvements in the measurement and modeling of the intangibles is also identified. Specifically, there is a need to improve the estimates of the depreciation rates and price deflators that are used in the measurement of intangible assets; and a need for proper

model specification testing to validate the inclusion of the intangibles when modeling economic activity.

Chapter 2 explores the role of the intangibles in explaining business cycles in a small open economy. The benchmark two-sector model developed in this chapter is tailored to the Canadian economy and allows for the examination of the relationship between intangible investment and the trade balance, which has not been attempted to date in the RBC literature. Overall, this chapter finds that technological change in the production of intangible investment plays an important role in explaining labour productivity and business cycles in a small open economy. Simulations based on the benchmark two-sector model highlight the circumstances under which the trade-balance to business sector output ratio tends to be procyclical. The extended model is further used to make predictions about the Canadian productivity puzzle, where the main findings reinforce the need to re-evaluate the traditional measure of productivity in business cycle models.

Chapter 3 is motivated by the rising levels of intangible investment in the U.S. and Europe. These investments have been expensed in the national accounts rather than capitalized (unmeasured investment) and this practice has resulted in the traditional measures of investment, productivity and output underestimating their true levels. In order to investigate the economic impact of this practice in an international setting, the standard two-country business cycle model is extended to include such intangibles. The main results imply that the traditional measures of output and labour productivity differences across countries are understated when intangible investment is not properly accounted for. The modeling of intangible investment also improves the fit of the model based upon recent data on international business cycles. This is most evident in the international correlation of investment, which the standard model predicts to be low (0.13) and the extended model correctly predicts to be high (0.66) as seen in the data (0.74).

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Introduction

Recent evidence shows that the increases in output and labour productivity over the past two decades in the U.S., Canada and many European countries have been partially driven by inputs that are not physical in nature such as advertising and software. The traditional theory of production stipulates that the production of output requires two inputs: physical capital (structures and equipment) and labour. In recent times where intangible inputs play an increasingly larger role in producing output, the traditional theory of production does not provide as good a fit as it once did (when intangible inputs were not as relevant as they are today). This lesser fit arises because the theoretical foundations of the traditional theory of production do little to address the role of intangible inputs. The traditional theory must therefore be modified to improve the fit of production models with modern economic activity.

This thesis investigates the role of aggregate intangible capital and intangible investment (the intangibles) in explaining aggegate economic activity such as labour productivity and business cycles. The intangibles constitute intangible economic activity that have been around for a large amount of time but have not received sufficient attention in the literature and have historically not been properly accounted for during economic analysis. Most recently, with the contributions of researchers such as Corrado, Hulten, and Sichel (2005, 2009) (for the US) and Baldwin, Gu, and Macdonald (2012) (for Canada) the intangibles have gained some traction in the economic literature and the 3 chapters of this thesis contribute to the advancement of knowledge on the economic impact of intangible capital and intangible investment.

Like its physical counterpart, intangible capital has many components such as software, design, organizational capital, research and development (R&D) and advertising. While there has been some effort to account for the role of some components of intangible capital such as organizational capital (Chang, Gomes, and Schorfheide (2002) and Cooper and Johri (2002)) and R&D (Wälde (1999, 2002)), there has been relatively little effort devoted towards the study of aggregate intangible capital. This is analogous to accounting for only structures or equipment instead of both when counting the stock of physical capital which distorts the true quantity of physical capital. The challenge going forward is to come up with ways to effectively account for the intangibles during economic analysis.

There are two main sources of motivation for this thesis. The first source is based on the recently constructed data by Corrado, Haskel, Jona-Lasinio, and Iommi (2012) which shows that between 1995 and 2010 there were rising levels of intangible economic activity. The average annual growth of both intangible capital and intangible investment were approximately three times higher than physical capital and physical investment, respectively. The rising levels of intangibles were further linked to gains in labour productivity and output.

The second source of motivation is based on the finding of McGrattan and Prescott (2010) which demonstrates that modeling the intangibles leads to closer agreement between the predictions of the traditional closed economy business cycle model (that does not feature the intangibles) and the observed U.S. data.[1] These findings suggest that the intangibles play an important role in explaining and modeling labour productivity and business cycles. They also provide sufficient motivation to seek further insight about the state of the intangibles in the economic literature and how they affect business cycles in a small open economy (SOE) and in an international setting.

Business cycles and aggregate intangible capital and investment have historically been studied separately and the key contribution of this research is to integrate both strands of the economic literature by describing the evolution of the intangibles and investigating their role in explaining business cycles.

Chapter 1 describes the state of the intangibles in the economic literature and discusses the range of economic topics they have been linked to, such as firm valuation, productivity and business cycles. It also highlights key issues related to the proper measurement of intangible assets and highlights the difficulties that arise when modeling intangible assets. The benefits and disadvantages of the two most popular methods of incorporating the intangibles into economic analysis (the one-sector and

[1] The closed economy model is one of three types of business cycle models that are currently used in practice. The other two are the small open economy model (Mendoza (1991)) and the international business cycle model (Backus, Kehoe, and Kydland (1993)).

two-sector models) are also discussed. The lack of empirically backed estimates for the depreciation rates of some components of intangible capital and the lack of proper model specificaton testing are identified as the main shortcomings in the economic literature for the measurement and modeling of intangible assets. Some of the modeling assumptions such as the rivalrous nature of inputs are also discussed to provide some context for the modeling choices made in Chapters 2 and 3.

Chapter 2 uses the Canadian economy as a case study to investigate the role of the intangibles in explaining business cycles in a SOE. The novel contribution of this chapter lies in the inclusion of intangible investment in the SOE real business cycle (RBC) framework which has not been done to date. The SOE RBC model that was first introduced by Mendoza (1991) is modified to include intangible capital and intangible investment, and simulations based on the new model are used to generate moments which are compared to the moments generated from recent data. An impulse response analysis of the newly developed model is further used to highlight the model's internal propagation mechanism. Similar to the findings of McGrattan and Prescott (2010) for the U.S. economy, the main findings here are that the intangibles play a crucial role in explaning Canadian labour productivity and business cycles, and also contribute to the understanding of the dynamics of debt (trade-balance) in a SOE which has not been previously reported in the literature.

Chapter 3 investigates the role of the intangibles in explaining international business cycles. Using the U.S. and an aggregate of 19 European countries as a case study, the main finding is that introducing the intangibles into the standard Backus et al. (1993) two-country two-sector international business cycle model yields significant improvements in the performance of the model. Relative to the Backus et al. (1993) model, the predictions of the extended model provide a better fit to the recent data. The newly developed model features two sectors (business and intangible) and three goods (final output, intermediate business good and intangible investment) where, similar to McGrattan and Prescott (2010), each sector has its own level of total factor productivity (TFP). It is important to note that this chapter was developed independently of the work of Baldi and Bodmer (2016) which shares a similar title with this Chapter. Their work utilizes a different set of modeling assumptions that are more restrictive than the set of assumptions featured in my analysis. For example, they assume that the intangible sector and business sector share the same level of TFP whereas I assume different levels of TFP across sectors, among others.

Even though Chapters 2 and 3 make unique contributions to the literature, due to the shared theme of business cycle analysis, there is some overlap in the methodology and theoretical underpinnings of both chapters. Both chapters feature the use of dynamic stochastic general equilibrium (DSGE) modeling, simulation techniques and impulse response analysis. Both chapters also rely on the theoretical foundations of incorporating the intangibles into business cycle analysis that were established by McGrattan and Prescott (2010).

As highlighted in Chapter 1, it is sometimes difficult to model the intangibles because there is no established range for some of the key parameters required to properly model the intangibles. This difficulty is evident in Chapter 2 where Bayesian estimation - a data driven method that chooses model parameters which maximize the likelihood of observing the data for a given model - is used to derive model parameter estimates, and Chapter 3 where ordinary least squares (OLS) regression analysis is used to derive some parameter estimates. In Chapter 3, the uncertainty surrounding the use of newly derived estimates - for which there are no directly comparable estimates in the existing literature - is abated by examining the sensitivity of the main results to small changes in key parameters.

The main results of this thesis can be classified into two categories: the data contribution and the modeling contribution to the literature. The data contribution is present in Chapters 1 and 3, and the modeling contribution is present in Chapters 1, 2 and 3.

The data contibution arises in Chapter 1 with the documentation of the issues that affect the proper measurement of the intangibles (such as the lack of adequate depreciation rates and price deflators). In Chapter 3, the data contribution is in the form of an update to the previously reported stylized facts on business cycles which indicates that there is now a higher level of correlation in international business cycles. In addition to this update, the business cycle properties of intangible investment, which include the positive correlation between intangible investment and output within countries and the positive correlation in intangible investment across countries, are also documented. Both properties of intangible investment have not been previously reported in the literature.

The modeling contribution arises in Chapter 1 where the need for proper model specification testing required to verify the validity of assumed functional forms when modeling the intangibles is higlighted, and in Chapters 2 and 3 with the documentation of the predictions of the SOE RBC and international real business cycle (IRBC) models, respectively. In discussing the predictions of the newly developed models, the predictions of the models are compared with the predictions of the traditional SOE and IRBC models. This comparison makes clear that modeling the intangibles leads to significant improvements.

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Chapter 1

Much Ado About the Intangibles

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1.1 Introduction

The landmark contributions of Corrado, Hulten, and Sichel (2005, 2009) reignited the revolution for the proper study of intangible capital, intangible investment (henceforth the intangibles) and their impact on economic growth and productivity. This technological revolution calls for the inclusion of a new input (intangible capital) and output (intangible investment) in the measurement and modeling of modern economic activity.

This chapter presents a detailed analysis of the status of the intangibles in the economic literature. Several key aspects of the intangibles are discussed with special emphasis being placed on issues related to the measurement and modeling of the intangibles. Overall, although there is room for improvement in how the intangibles are measured and modeled in practice, the general consensus in the literature is that the intangibles have contributed positively to economic growth and productivity. As demonstrated by the wide range of estimates for the intangibles, the exact magnitude of this positive contribution remains unknown as it depends on exactly how one measures intangibles. The study of the intangibles is especially important because they have been linked to higher output and productivity growth rates which are essential to the economic prosperity of any nation.[1]

In the existing literature, there are two key issues that have limited our ability to properly account for the intangibles and their contributions to the economy. The first key issue is the lack of data which arises as a result of the lack of a standardized system of measurement for intangible capital (data issue). The second issue arises due to the difficulty of modeling intangible economic activity.

The lack of data persists because the intangibles have historically not contributed to economic activity in the way that they do today. For instance, based on recent estimates from the US national accounts, the volume of intangible capital relative to total capital in the US economy has not always been large (5.8% in 1970 vs. 10.8% in 2014) and this has created a situation where the intangibles were easily neglected. In absolute terms, according to data from the US national accounts, the volume of intangible capital increased from about $627 Billion in 1970 to $3.38 Trillion in 2014. These data show that significant growth has occurred over time, however, these

[1]See Corrado et al. (2005, 2009) and Corrado, Haskel, Jona-Lasinio, and Iommi (2012).

figures have not always been available and they are currently included in the national accounts as a result of recent efforts to properly account for the intangibles.

Furthermore, while there are some guidelines for the measurement of some components of the intangibles such as research and development (R&D) and advertising, there is currently no generally accepted guideline for the measurement of aggregate intangible capital and investment. As a result, there have been a wide range of estimates for the level of intangibles reported by different researchers. This inconsistency motivated the contributions of Corrado et al. (2012) who constructed estimates for the intangibles that are more robust and that allow for the inclusion of aggregate intangibles during economic analysis.

The second key hinderence that is addressed in this chapter is the difficulty of modeling intangible economic activity. This issue arises because the set of assumptions that led to the creation of the traditional theory of production with two inputs (capital and labour) does not allow for the effective study of the role of intangibles. It is therefore important to address the suitability of recently proposed models that include intangible capital and intangible investment.

It is also important to note that the measures of aggregate intangible capital in the national accounts of many countries are incomplete. For instance, the data from the US national accounts only represents a fraction of aggregate intangible capital as reported by Corrado et al. (2012) (55% in 2010). Hence, the measure from the national accounts is viewed as a proxy for aggregate intangible capital and investment. A recurring theme in this chapter is the comparison of the Corrado et al. (2012) (CHJI) estimates of aggregate intangible capital with the estimates from the US national accounts (OECD).

The remainder of this chapter is structured as follows; Section 1.2 outlines the definition of the intangibles and highlights the various categories and components of intangible capital. Section 1.3 describes the evolution of intangible capital in the economic literature and highlights the important role it plays in explaining economic activity. Sections 1.4 and 1.5 outline some of the common measurement and modeling practices in the economic literature and address some of the challenges related to the proper measurement and modeling of the intangibles. Section 1.6 provides concluding remarks and suggestions for the direction of future research.

1.2 Intangible Capital and Intangible Investment

Defining intangible capital and intangible investment requires a cautious approach that accurately captures the essence of intangible economic activity. The simple definition of intangible capital is: an asset that is not physical in nature.[2] However, because the definition of an asset that is intangible varies from firm to firm, this definition is very broad and results in a desire for a more precise explanation of the concept. Since the definition of the word intangible is straightforward, a more precise description relies on the definition of the word capital. Hunter, Webster, and Wyatt

[2]Intangible investment refers to expenditure on assets that are not physical in nature.

(2005) define capital as assets retained by firms that lead to the generation of profits and this definition of capital forms the basis of how intangible capital is viewed in this thesis.

Popular examples of intangible capital include R&D, patents, marketing and advertising, and intellectual property products (IPP). In the literature, there is some inconsistency about the definition of intangible capital and IPP: some such as Joia (2000) view them as synonymous while others such as Cummins (2005), Hunter et al. (2005) and Van Ark, Hao, Corrado, and Hulten (2009) view IPP as a subset of intangible capital. This issue appears to stem from the evolution of the intangibles. They were initially considered as synonymous because IPP such as R&D and software were part of the first set of intangibles to be identified and included in the national income accounts. However, over time, more intangible assets have emerged which are not easily classified as IPP (e.g. training and organizational structure), so more categories have been created to accommodate the different forms of intangible capital.

Following the comprehensive estimate of aggregate intangible capital and investment constructed by Corrado et al. (2012), there are nine types of intangible capital classified into three main categories: computerized information, innovative property and economic competencies.[3] Table 1.1 contains a list of the categories and types of intangible capital used by Corrado et al. (2012).

[3]Hunter et al. (2005) assume four groups of intangible capital: human, intellectual, organizational and customer.

1.3 Evolution of the Intangibles

The traditional theory on the production of output mostly assumes that production relies on two inputs: physical capital (plant and equipment) and labour. This assumption has served economists well for a long time as it accurately represented the dynamics of firm production in an era where increasing production required buying more machines or hiring more workers. In recent times however, the increase in output and productivity in the global economy has been increasingly reliant on investments that are not physical in nature. As a result, the importance of a new input to sustained economic growth has gradually emerged. Figure 1.1 compares the evolution of tangible capital from 1995-2010 to two measures of intangible capital for the US economy.[4] The first measure of intangible capital is the series labeled "OECD" which was sourced from the OECD national income accounts database and the second measure labeled "CHJI" is the series constructed by Corrado et al. (2012). Similarly, Figure 1.2 compares the evolution of tangible investment from 1995-2010 to two measures of intangible investment where the labels and sources of data are as described above. It is clear that both measures of intangible capital (investment) have grown at a faster rate than their tangible counterparts.

These findings are reinforced by the annual growth rates reported in Table 1.2. Considering the rates derived from the OECD data, the average annual growth rates

[4] See the Appendix for more details about the data.

of intangible capital (3.8%) and intangible investment (4.8%) are approximately two times higher than the growth rates of tangible capital (2.1%) and tangible investment (2.5%). This discrepancy is even larger when the average annual growth rates of intangible capital (6.6%) and intangible investment (7.5%) derived from the Corrado et al. (2012) estimates are considered.

There is also some evidence to support the notion that intangible capital and investments play an important role in explaining macroeconomic and microeconomic firm-level productivity and growth. The majority of this evidence is based on different components of intangible capital as opposed to aggregate intangible capital, which is more difficult to measure.

In the macroeconomic setting, the works of Corrado et al. (2012) and Niebel, O'Mahony, and Saam (2013) are examples of recent contributions that highlight the importance of accounting for the intangibles in the macroeconomic setting. Corrado et al. (2012) find that intangible capital accounts for about 25% of the productivity growth experienced in the US and in many large European countries. They also find that the contribution of each category of intangible capital to productivity growth varies by country. For example, in the US, Finland and Germany, innovative property ranks as the top contributor to productivity growth, whereas in the UK, Netherlands and France, economic competencies contribute the most to productivity growth. Computerized information also contributes significantly to productivity growth as it is ranked as the top contributor for Italy and the second highest contributor for the

UK, Netherlands and France.

At the sectoral level, Niebel et al. (2013) find that the intangibles are a key driver of sectoral productivity growth in many European countries; especially in the manufacturing (goods producing) and financial intermediation (service) sectors. The intangibles have also been linked to explainig business cycles. McGrattan and Prescott (2010) find that the intangibles play an important role in resolving the US labor productivity puzzle which arises because the standard business cycle model (without intangible capital) produces counterintuitive predictions for US labour productivity. By introducing the intangibles into the standard business cycle model setup, they find that the predictions of the model are more aligned with the data.

Although the majority of evidence in support of the intangibles contributing to productivity growth is for developed economies, as demonstrated by Chen, Niebel, and Saam (2015), the intangibles also play an important role in explaing economic activity in developing countries. Chen et al. (2015) find that the intangibles are more productive in information, communications and technology (ICT) intensive sectors of the Indian economy.

In the microeconomic setting, Hirschey and Weygandt (1985) and Van Ark et al. (2009) document how under-reported intangible capital explains some of the difference between stock market valuations and financial statement valuations, thereby demonstrating how accounting for the intangibles yields important insights that contribute to the understanding of modern economic activity.[5]

[5]Other examples of micro level studies that highlight the significance of the intangibles include

The scope of the significance of the intangibles is not restricted to economic growth and productivity. The intangibles have also been linked to other areas of economic significance such as labour markets (Connolly, Hirsch, and Hirschey (1986)) and the formulation of business strategy (Joia (2000)). Connolly et al. (1986) established a relationship between unionization and intangible capital where unions are able to extract a portion of the returns to firm-specific intangible capital such as R&D, which in turn has a negative effect on intangible investment.

In summary, it is evident that the intangibles have grown at a higher rate than the tangibles and have contributed to an improved understanding of economic growth, productivity and other areas of economic significance. The emergence of the intangibles however, does not mean that all economic puzzles related to growth and productivity are resolved, and the use of intangibles to explain economic phenomena has not always yielded positive results. For example, Megna and Mueller (1991) investigated the role of intangible capital in explaining the difference in profit rates across firms and industries, and their results indicate that, even though conventional accounting methods fail to account for intangible capital stocks, the difference in profit rates remains even after intangibles are accounted for.

the contributions of Hunter et al. (2005) on the international practices in the measurement of firm level intangible capital and Cummins (2005) on the valuation of intangible capital.

1.4 Measuring the Intangibles

Thus far, the preceding sections have discussed the evolution of the intangibles and their contribution towards a better understanding of modern economic activity. This section discusses key approaches to measuring the intangibles. A review of the existing literature reveals that the proper measurement of the intangibles requires the harmonious union of economic theory and accounting principles and standards, which unfortunately does not yet exist in practice. Researchers such as Hunter et al. (2005) have noted that there is still much confusion surrounding the nature and contribution of various forms of intangible capital and intangible manifestations of human activity.

This point is compounded by the fact highlighted by Van Ark et al. (2009) that there is no consensus on the unit of measurement for knowledge. Accounting practices differ across firms, industries and countries, and there is no standardized methodology for the measurement of the intangibles.[6] The measurement of the intangibles is therefore a very important aspect of establishing their presence as a driving force for economic growth. After all, how can the magnitude of the contribution of an asset to economic growth and productivity be established if the asset itself cannot be measured?

More recently, the annual estimates of intangible capital and investment constructed by Corrado et al. (2012) are regarded as the most comprehensive measures

[6] Hunter et al. (2005) note that accounting standards mandate the declaration of intangibles only when an exchange transaction has occurred where a firm has acquired external items or entities as is commonly reported as the value of licenses and trademarks when companies are acquired.

available. This annual dataset covers the 16-year period 1995 to 2010.[7] Niebel et al. (2013) provide a sectoral breakdown of the Corrado et al. (2012) dataset, which has also been used for sectoral productivity and growth analysis as seen in the work of Crass, Licht, and Peters (2015). In Equation (1.1), we see how Corrado et al. (2009) modify the traditional national accounting identity (excluding government spending) to include intangible capital and intangible investment.[8]

$$P^y Y = P^c C + P^i I_I + P^k I_T = WN + R^i K_I + R^k K_T \tag{1.1}$$

Here, Y is aggregate output, C consumption, I_I intangible investment, I_T tangible investment and P represents the price indices with the appropriate superscript to reflect the type of economic activity. The inputs used in production are N for labour, K_I for intangible capital and K_T for tangible capital with W, R^i and R^k as their rental rates respectively.

There is some debate on the classification of some components of the intangibles as expenses rather than capital, but this is not to be confused with the measurement issue being addressed here.[9] Formally, according to Hunter et al. (2005) there are two types of classification errors: the first is a "Type 1" error which occurs when assets are classified as expenses and the second is a "Type 2" error which occurs when expenses are classified as investment. They further provide a critique that current accounting

[7]There is currently no quarterly data on the intangibles.

[8]Van Ark et al. (2009) note that the current national account formulation treats some expenditure on I_I as an intermediate input in the production of C and I_T.

[9]See Bontempi and Mairesse (2008) and Corrado et al. (2012) for more details on the debate.

standards seek to minimize Type 2 errors and do not focus enough on Type 1 errors and this is an adjustment that needs to be made to create more accurate measures of the intangibles. The classification debate is related to the measurement issue discussed here in the sense that a Type 1 or Type 2 error biases the total value of the intangibles. Here however, the focus is on how the components are measured and not how they are classified.

The measurement of the components of intangible capital occurs directly or indirectly. The direct approach involves measuring the intangibles on a cost basis at the time expenditure occurs, while the indirect approach involves measuring the intangibles as a residual difference between two values.[10] During measurement, some components of the intangibles are further classified into an own-account component, which refers to intangible capital that is produced and consumed in-house, and a purchased component, which refers to intangible capital that is purchased. Software is an example that has an own-account component, which is measured as the cost of labour and other intermediate inputs required to produce the software, and a purchased component, which is measured using the purchase price of pre-packaged software.

The direct cost-based approach is seen in the work of Hand and Lev (2003) who construct estimates of intangible capital with the intent to establish the acceptable range of values. They considered expenditures on advertising and software, and the cost of inputs that contribute to the production of intangibles as direct measures of

[10]Hunter et al. (2005) recommend a cost approach that classifies investment as tangible or intangible based on the intention of management at the time the cost is incurred.

intangible capital.[11]

The indirect approach to measuring the intangibles is generally more complicated than the direct approach. For example, the equity market approach measures intangible capital as the residual difference between the market value and book value of a company. Although this indirect measure yields estimates of intangible capital which are highly correlated with other direct cost-based measures, it is only valid under the assumption that investors incorporate the value of intangibles into the market value of firms. This would imply that investors have access to information on the value of intangibles which is not typically the case. Hence this approach is not widely adopted. Cummins (2005) investigated a new indirect approach to the valuation of intangible capital where organizational capital is measured as the difference between installed information technology (IT) versus uninstalled IT. Using software as an example, uninstalled IT is defined as database software and installed IT is defined as database software loaded with data which is more valuable than the software alone.[12] Despite the more complicated nature of the indirect approach, both direct and indirect approaches are necessary for capturing the wide range of intangibles listed in Table 1.1.

[11] Labour costs are measured as the income share of occupations classified as creative such as engineers and scientist.

[12] The theory here is that organizational capital explains why some firms with the same IT as other firms are able to charge a higher premium.

1.4.1 Data Sources

As discussed in Section 1.1, most measures of the intangibles, including the measures reported in the national income accounts of many countries, are incomplete and simply proxy for the true value of aggregate intangible capital. In addition, the data provided by reputable statistical agencies such as the BEA, OECD, Eurostat and EU-KLEMS also fall under the proxy category. In order to derive aggregate estimates, researchers have had to piece together aggregate intangible capital by combining various proxy measures from different sources. For example, Van Ark et al. (2009) and Corrado et al. (2012) combined data from reputable sources such as national accounts, reports of statistical agencies, and trade associations, with data from unconventional sources such as *Screen Digest*, a magazine that provides data on the production cost of movies for 59 countries.

1.4.2 Price Deflators

Another key issue that prevents the proper measurement of the intangibles is the lack of prices and consequently price deflators for many of its components. Price deflators are required to convert nominal expenditure on intangible assets into real values. As shown in Equation (1.1) accounting for the intangibles requires data for the price level for intangible investment (P^i) and the rate of return for intangible capital (R^i), so consequentially, a lack of prices and price deflators pose a major obstacle to the proper accounting of the intangibles. While there are price deflators for R&D and software, none exist for components such as brand equity and human capital. This issue exists because the intangibles have not always been measured and, as such, it was not necessary to keep track of the prices of the various components. Corrado et al. (2009) and Van Ark et al. (2009) use the overall output price as a place holder for the price of intangibles (that is, $P^y = P^i$ in Equation (1.1)) until such a time as a more satisfactory method exists and this has become standard practice. This is an area that requires further research to derive more applicable prices for various components of intangible capital.

1.4.3 Depreciation Rates

The perpetual inventory model (PIM) is the most widely used method to account for intangible capital and intangible investment. As shown in (1.2), this method requires measures for the depreciation rates (δ^i) of various components of intangible capital.[13]

$$K^i_{I_{(t+1)}} = I^i_{I_{(t)}} + (1 - \delta^i) K^i_{I_{(t)}} \tag{1.2}$$

Here i=[1,...,N] indexes the various components of intangible capital and their depreciation rates. The difficulty arises in the assignment of depreciation rates for various components of intangible capital for which empirically backed estimates are not readily available. This is the main challenge of using the PIM methodology and it

[13]Depreciation rates are generally assigned based on the expected service life of the asset.

is evident in the literature where researchers have assumed a wide range of estimates for depreciation rates. For example, McGrattan and Prescott (2010) assume a rate of 0% for aggregate intangible capital while Corrado et al. (2009) assume different rates (20% to 60%) for the various components of intangible capital.

In theory, it is quite difficult to assign depreciation rates because of the constantly evolving nature of some forms of intangible capital. For example, when comparing printed advertising to TV advertising, it becomes clear that it is rather difficult to assign a depreciation rate to the aggregate advertising component of intangible capital as both forms of advertising cannot be expected to depreciate at the same rate. Similarly, there is also a wide range of estimates for the depreciation rate of R&D in different industries. Li (2011) reports 12% for the pharmaceutical industry and 38% for IT hardware while Mead (2007) assumes an aggregate depreciation rate for R&D of 15%. These examples highlight the difficulty encountered when applying PIM to the measurement of intangible capital.

In summary, there is a wide range of approaches to measuring the components of intangible capital. Some components such as R&D and advertising are measured on a cost basis while others are derived using other measures such as Cummins (2005) who derived organizational capital as the difference between installed IT and uninstalled IT. In addition, as there is no consensus on the classification of some intangible resources as intangible investment or expense, this chapter identifies the recommendation of Hunter et al. (2005) that intangible resources should be classified based on the intention of management at the time costs are incurred as a well-rounded approach due to the variation in the definition of intangibles across firms and industries.[14]

Also, while there are many reasons to measure intangible capital, according to Van der Meer-Kooistra and Zijlstra (2001) who focus on the importance of measuring intellectual capital in a knowledge-based economy, there are also some disadvantages to measuring and reporting intangibles. The extensive literature review of Marr, Gray, and Neely (2003) revealed that measuring the intangibles assists with the following: strategy formulation, strategy execution, diversification and expansion decisions, and compensation and shareholder communication. Examples of the disadvantages highlighted by Van der Meer-Kooistra and Zijlstra (2001) include: accounting costs, loss of confidentiality and the creation of excessive expectations due to the perceptions of progress implied by investment activities.

1.5 Modeling the Intangibles

Given the emergence of the role of the intangibles, it is important to investigate how they are typically accounted for during economic analysis. Here, this is achieved by comparing two model specifications that feature intangible capital and intangible investment. The most popular model specification in the macroeconomic literature that accounts for the intangibles converts the standard two-input one-sector pro-

[14] Corrado et al. (2012) recommend that expenditure on intangible resources should be capitalized based on the expected service life of the resource. The resource is regarded as capital if its service life exceeds a year and as an expense if less than a year.

duction model into a three-input two-sector model that accounts for business sector output and intangible investment as a function of inputs. There is also a one-sector approach which simply introduces an additional input (intangible capital) into the traditional one-sector model and uses a measure of output that includes intangible investment. Others have also introduced one or more components of intangible capital into the production setup. For example, De and Dutta (2007) include human capital, organizational capital and brand capital.

1.5.1 A Two-Sector Model

In the two-sector setup, the first sector produces business output using tangible capital, intangible capital and labour as in Equation (1.3) and the second sector produces intangible investment using the same inputs as in Equation (1.4). This setup requires assumptions about the functional forms of the production functions and the rivalrous use of inputs across sectors to make the model applicable to the data. Corrado et al. (2012) and McGrattan and Prescott (2010) assume that the total stock of intangible capital (K_I) is simultaneously used to produce intangible investment and output. That is, K_I is used in a non-rivalrous way, whereas the stock of capital (K_T) and labour (N) are split across sectors as in equations (1.5) and (1.6).

$$Y_{b_t} = g(A_t^1, K_{T_t}^1, K_{I_t}, N_t^1) \quad (1.3) \qquad I_{I_t} = j(A_t^2, K_{T_t}^2, K_{I_t}, N_t^2) \quad (1.4)$$

$$K_{T_t} = K_{T_t}^1 + K_{T_t}^2 \quad (1.5) \qquad N_t = N_t^1 + N_t^2, \quad (1.6)$$

where Y_b refers to business sector output and I_I refers to the intangible sector output.

Aggregate output in this setup is defined as $Y = Y_b + qI_I$ where q is the relative price of intangible capital (P^i/P^y from Equation (1.1)) which is difficult to measure. A^1 and A^2 represent the level of productivity in the business and intangible sectors respectively. K_T^1 (K_T^2) and N^1 (N^2) represent the portion of tangible capital and labour allocated to the business (intangible) sector.

For the model specification, the most popular production specification is the Cobb-Douglas production technology which assumes a complementary relationship between inputs. Both Corrado et al. (2012) and McGrattan and Prescott (2010) assume equations (1.3) and (1.4) above to be of the Cobb-Douglas form shown in equations (1.7) and (1.8).

$$Y_{b_t} = A_t^1 (K_{T_t}^1)^{\alpha_1} (K_{I_t})^{\epsilon_1} (N_t^1)^{(1-\alpha_1-\epsilon_1)} \tag{1.7}$$

$$I_{I_t} = A_t^2 (K_{T_t}^2)^{\alpha_2} (K_{I_t})^{\epsilon_2} (N_t^2)^{(1-\alpha_2-\epsilon_2)} \tag{1.8}$$

Here, $0 < \alpha_1, \alpha_2, \epsilon_1, \epsilon_2 < 1$. As expected, there are several issues with the set of assumptions required to make the two-sector setup a plausible representation of modern economic activity. The three key assumptions that are worth highlighting here are the assumed functional form, the rivalrous or non-rivalrous use of inputs and the assumption of different productivity levels across sectors.

Ramsey and Alexander (1982) warn that econometricians must learn to test their models before attempting to make inference; and Griffin, Montgomery, and Rister

(1987) note that functional form selection is sometimes overlooked in applied research in production analysis.[15] Both critiques apply to the modeling of the intangibles as most researchers fail to test their model specification.

The lack of testing generally occurs because there are some challenges that restrict our ability to test the validity of any assumed model specification. For example, a proper investigation of the Cobb-Douglas specification for the two-sector model requires data on the output of both sectors (Y_b and I_I), the aggregate inputs (K_T, K_I and N) and the split of inputs across sectors (K_T^1, K_T^2, N^1 and N^2), however, all the required data is not available. While there are data on the aggregates required, there are currently no data on the split of inputs across sectors, it is therefore difficult to test for correct specification of Equations (1.7) and (1.8).

In addition to the unavailability of data on the split of input across sectors, it is also difficult to obtain measures of sector-specific productivity (A^1 and A^2) to include in the model in order to proceed with the testing exercise. It is therefore not surprising that there is no mention of model specification testing in the literature.[16]

[15] Griffin et al. (1987) identified 20 functional forms and provided guidelines for selection based on underlying assumptions and properties such as differentiability, separability and asymptotic convergence.

[16] While there was some work in the early days on the appropriate model specification for a two-input production model, it appears that this is an area of research that most researchers avoid working on.

1.5.2 A One-Sector Model

Relative to the two-sector model, the one-sector model is easier to work with as it does not require data on the split of inputs. Equations (1.9) to (1.11) show three variations of a Cobb-Douglas model specification for a one-sector model.

$$Y_t = A_t K_{T_t}^{\alpha} K_{I_t}^{\epsilon} N_t^{(1-\alpha-\epsilon)} \tag{1.9}$$

$$Y_t = A_t \left[K_{T_t} + \gamma K_{I_t}\right]^{\alpha} N_t^{\epsilon} \tag{1.10}$$

$$Y_t = A_t \left[K_{T_t}^{-\rho} + K_{I_t}^{-\rho}\right]^{-\nu/\rho} N_t^{\epsilon} \tag{1.11}$$

Here, the measure of aggregate ouput is Y and it is equivalent to aggregate output from the two-sector model ($Y = Y_b + qI_I$), A is the level of productivity and K_T, K_I and N refer to the aggregate stock of physical capital, intangible capital and labour respectively. These functional forms are taken from Bontempi and Mairesse (2008). They use the one-sector approach and consider panel data for Italian manufacturing firms in order to compare the results from the multiplicative, additive and constant elasticity of substitution (CES) Cobb-Douglas model specifications shown in Equations (1.9) to (1.11). The model parameters were then estimated using ordinary least squares (OLS). For example, Equation (1.9) was estimated as (1.12) (where variables in lower case are in logs) with the following parameter estimates: α=0.131, ϵ= 0.026

and (1-α-ϵ) = 0.843.[17]

$$y_t = a_t + k_{T_t}\alpha + k_{I_t}\epsilon + n_t(1 - \alpha - \epsilon) \tag{1.12}$$

The major drawback of testing for model specification in this setup lies in the measure of productivity (A) required to make testing econometrically sound. In the macroeconomic literature, A is usually estimated as a residual using data on Y, K_T, K_I, and N (rearranging Equation (1.12)), hence this measure of A cannot be included during macroeconomic analysis.[18] The lack of model specification testing for models that include intangible capital is not especially surprising, given that there are few recent papers that focus on testing for proper model specification in the traditional two-input two-sector production model ($Y = f(K_T, N)$). This appears to be an area avoided by most researchers, and is quite puzzling given the constant evolution of economic activity that requires consistent testing to ensure the validity of assumptions. With the introduction of the intangibles into production analysis, it is now a more complex task to test for model specification. Most researchers simply test for the statistical significance of their estimates (t-test) and do not focus on the tests for model specification like the Ramsey RESET test. This situation needs to change if we are to establish a sound theoretical foundation for modeling the intangibles.

It is worth noting that the use of data driven methods such as Bayesian estimation

[17] These parameter estimates correspond to the estimates derived under the assumption of constant returns to scale. Relaxing this assumption does not change the estimated parameters significantly.

[18] For microeconomic analysis, it is possible to include the aggregate measure of A, hence testing should be more feasible in such a setup.

to derive model parameter estimates is quite common in the macroeconomic business cycle literature. These methods assign parameter values that maximize the likelihood of observing the data for a given model. These methods however do not address the testing for model specification raised here. There is also a consensus in the literature that the inclusion of intangible capital in the production function reduces the explanatory power of labour as an input. Van Ark et al. (2009) and Koh, Santaeulalia-Llopis, and Zheng (2014) report a decline in the labour share after accounting for intangible capital.[19] The implication of this for a Cobb-Douglas model specification is a reduction in the labour share $(1 - \alpha - \epsilon)$ due to the inclusion of ϵ which has now become the standard practice in the literature when assigning parameter values in the Cobb-Douglas setup.

In summary, any of the numerous functional forms commonly assumed during production analysis can be modified to include intangible capital. The Cobb-Douglas specification appears to be the most widely assumed specification in the intangible capital literature. This assumption however, has not been thoroughly tested and verified, and the inference from such analysis should be viewed as suspect. Despite this important critique, the contributions to the literature thus far constitute a good starting point for the modeling and analysis of the intangibles.

[19]Koh et al. (2014) find a reduction in the U.S. labour share from 0.68 in 1947 to 0.60 in 2013. The labour share is defined as (1 - (K_y / Y)) where K_y is total capital income and Y is total output.

1.6 Conclusion

This chapter presents an overview of the state of intangible capital and intangible investment in the economic literature. To describe the state of the intangibles, a wide range of issues related to the measurement and modeling of intangible resources are discussed. The current measures of intangible capital are plagued by issues such as: the inconsistency in the classification of some components of intangible capital (capital vs. expenses) and the lack of empirically backed estimates for depreciation rates and price deflators. Given these data issues, the measures of intangible capital that exist today still constitute a very important step toward quantifying the role of intangibles during economic analysis. The modeling issues highlighted in this chapter are centered on the need for proper model specification testing to validate the introduction of the intangibles into the traditional production framework that formerly only included tangible capital and labour. This is seen as a necessary step towards ensuring the credibility of the inference that is derived from models that include intangible capital and intangible investment. The suggested direction for future research is to focus on the construction of more accurate price deflators and depreciation rates for the components of intangible capital and the rigorous testing of model specifications when modeling the intangibles.

The remaining chapters of this thesis feature the application of the two-sector model described in Section 1.5.1 to the study of business cycles. The model is extended to accomodate the study of business cycles in the small open economy setting of Chapter 2, and the two-country international business cycle setting of Chapter 3.

1.7 Appendix

This chapter makes reference to two datasets; OECD and Corrado et al. (2012). Annual intellectual property products and GDP data (2009 real dollars) for the US economy from 1995 to 2010 was sourced from the OECD. The IPP data includes R&D, software and artistic originals. The comprehensive annual data on intangible capital and investment for the U.S. was constructed by Corrado et al. (2012) and sourced from INTAN-invest. These data cover the time period 1995 to 2010 and were reported in nominal U.S. dollars and converted to 2009 real currency using the software deflator provided by Corrado et al. (2012). “CHJI” is used to refer to this dataset.

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Table 1.1: Categories of Intangible Capital

Asset type	National Accounts
Computerized information	
1. Software	Yes
2. Databases	No
Innovative property	
3. Mineral exploration	Yes
4. R&D (Scientific)	Yes
5. Entertainment and artistic originals	Yes
6. New products/systems in financial services	No
7. Design and other new product/systems	No
Economic competencies	
8. Brand equity	
a. Advertising	No
b. Market research	No
9. Firm-specific resources	
a. Employer-provided training	No
b. Organizational structure	No

This table, copied from Corrado et al. (2012), contains the categories and components of intangible capital. The national accounts column captures whether a specific component is currently included in the US national accounts.

Table 1.2: Descriptive Statistics (Annual data, 1995-2010)

	Mean Growth (%)	Minimum (%)	Maximum (%)
Output	2.5	-2.8	4.6
Capital (Agg)	2.3	1	3.4
Intangible Capital (CHJI)	6.6	-7.7	13
Intangible Capital (OECD)	3.8	2.6	5.3
Tangible Capital (OECD)	2.1	1	3.2
Investment (Agg)	3.1	-12.3	8.7
Intangible Investment (CHJI)	7.5	-0.4	11.6
Intangible Investment (OECD)	4.8	1	9.5
Tangible Investment (OECD)	2.5	-17.3	8.7

This table contains the descriptive statistics of the growth rates of various variables for the US economy.

Table 1.3: Depreciation Rates for Intangible Capital

Asset type	Depreciation Rate
Computerized information	
1. Software	.315
2. Databases	.315
Innovative property	
3. Mineral exploration	.075
4. R&D (Scientific)	.15
5. Entertainment and artistic originals	.20
6. New products/systems in financial services	.20
7. Design and other new product/systems	.20
Economic competencies	
8. Brand equity	
a. Advertising	.550
b. Market research	.550
9. Firm-specific resources	
a. Employer-provided training	.40
b. Organizational structure	.40

This table, copied from Corrado et al. (2012), contains the depreciation rates for the various components of intangible capital.

Figure 1.1: Stock of Tangible and Intangible Assets in the U.S.
(Annual, 1995=100, 1995-2010)

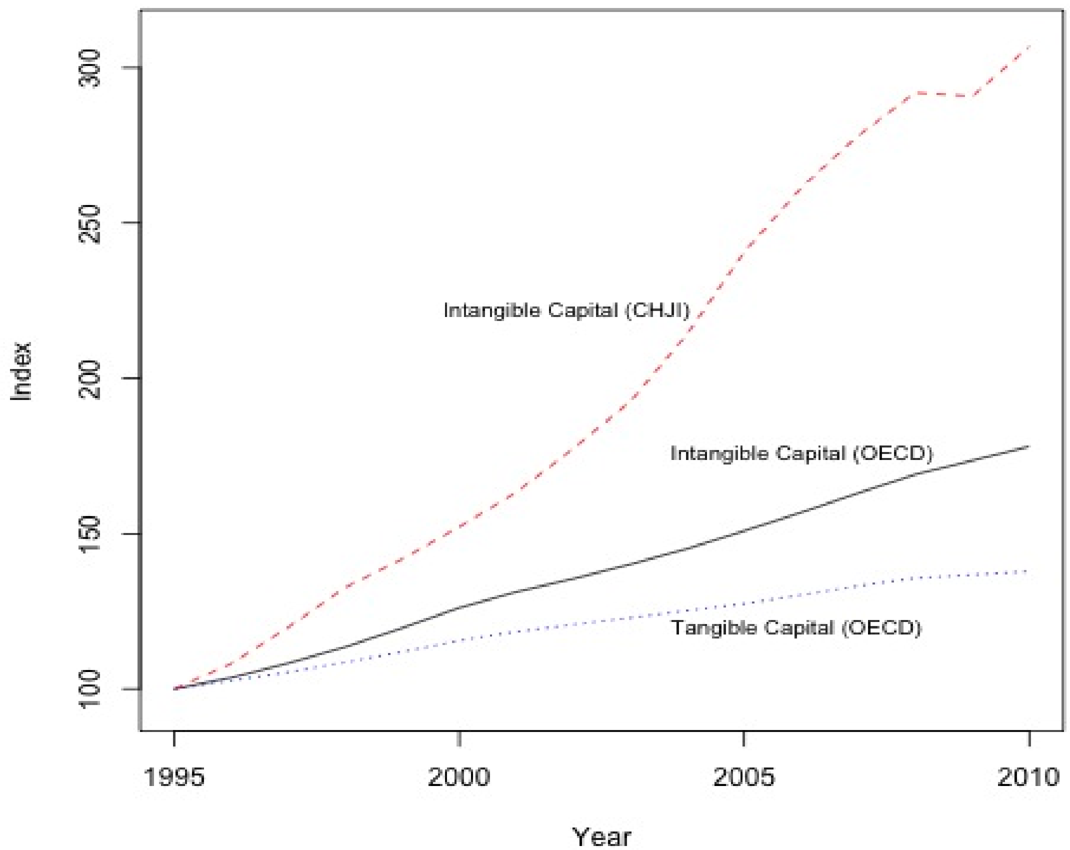


Figure 1.2: Investment in Tangible and Intangible Assets in the U.S. (Annual, 1995=100, 1995-2010)

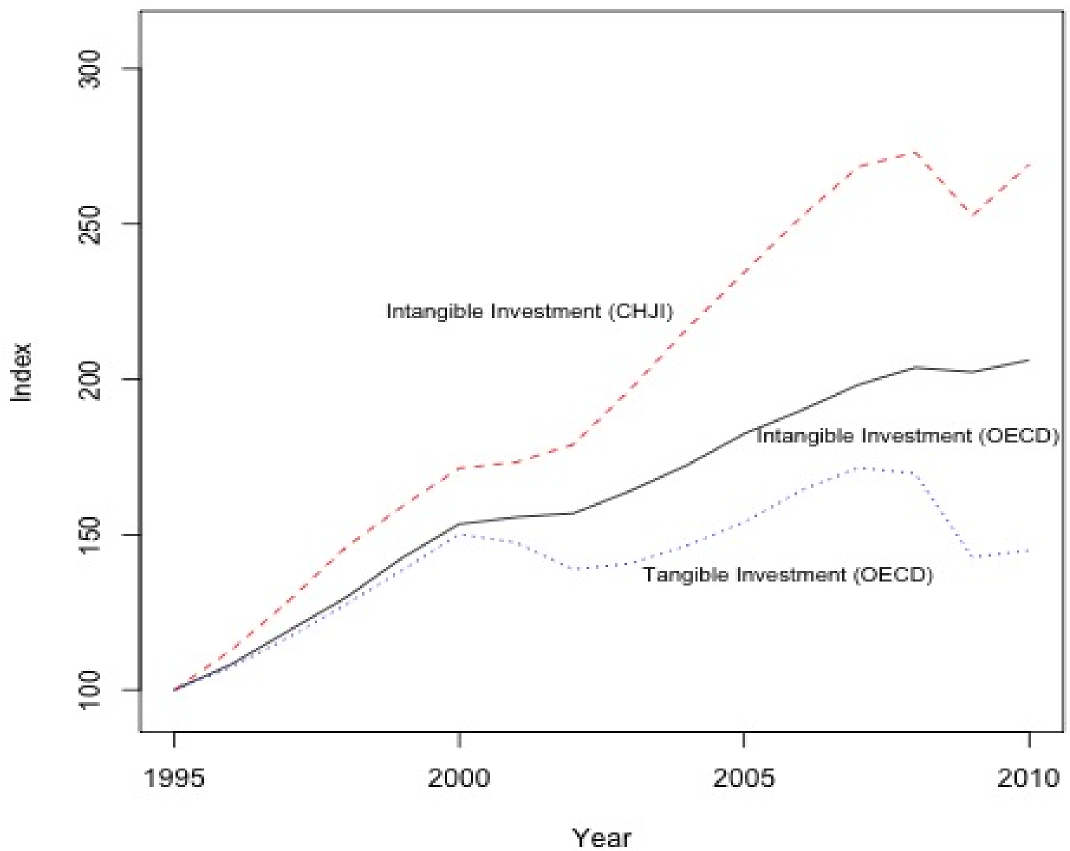
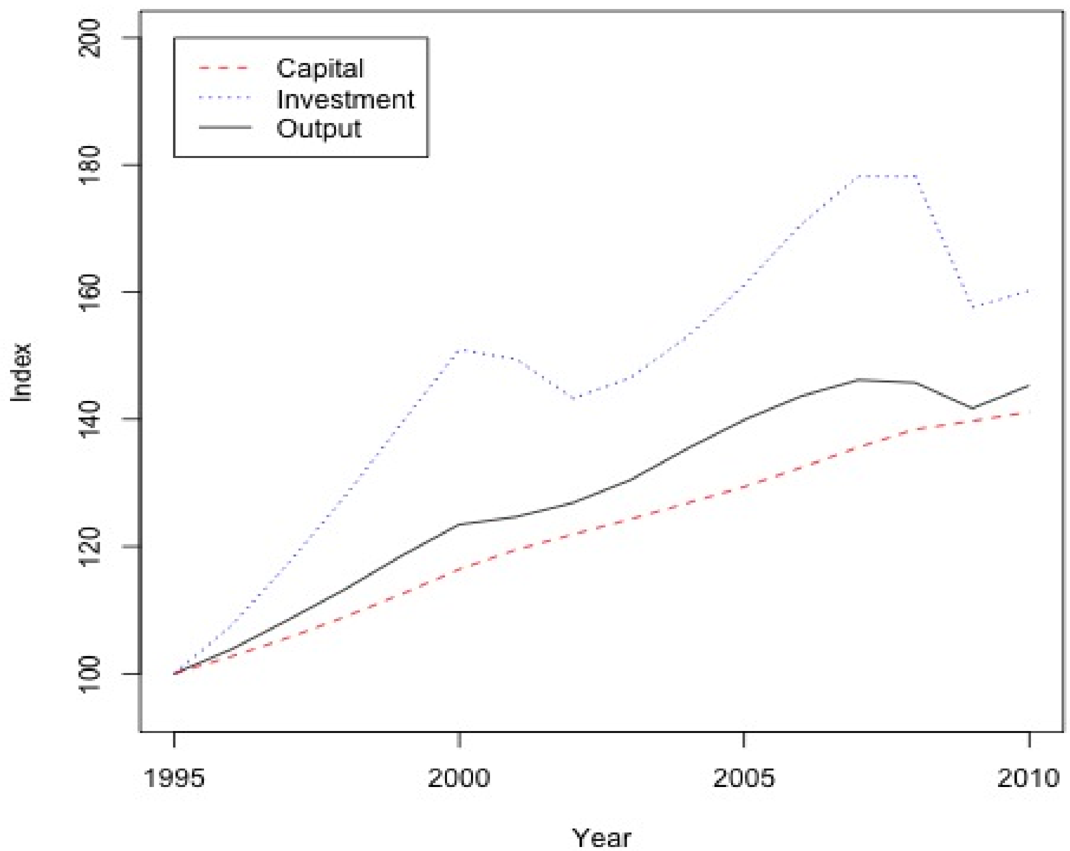


Figure 1.3: Aggregate Output, Capital and Investment in the U.S. (Annual, 1995=100, 1995-2010)



Chapter 2

Intangible Investment in a Small Open Economy

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2.1 Introduction

Since the seminal work of Kydland and Prescott (1982) on real business cycle (RBC) analysis (the neoclassical RBC model), the general philosophy of RBC theories and models that rely on them has been that aggregate fluctuations in the economy can be explained with the use of stochastic disturbances in a dynamic stochastic general equilibrium (DSGE) model. Stochastic disturbances are modeled as shocks to total factor productivity (TFP) that have an impact on the decision making process of agents in the model economy. If a model is to deliver results and make predictions that are in agreement with the data, it is important that the measure of TFP in the model reflects the economic climate it seeks to explain.

The puzzling US boom, where the basic RBC model predicts a recession in the US economy in the early 90's when in fact there was a boom, is a classic example of a puzzling prediction that can arise if the measure of TFP in a model does not accurately reflect the economic climate. As McGrattan and Prescott (2010) note, this puzzling prediction is due in part to unmeasured economic activity that is not captured by the measure of TFP and productivity in the standard model. Thus, the performance of RBC models is heavily influenced by the economic climate they seek to explain and there is a need for additional research to improve the performance of models to accurately reflect the ever evolving economic climate they seek to explain.

McGrattan and Prescott (2010) present a solution to the puzzling US boom by introducing intangible capital and intangible investment (modeled as a production process that uses non-neutral technology) into the basic business cycle model. In doing so, they transformed the standard one-sector model into a two-sector model that allows for additional dynamics. This enabled them to create a new model that generates the observed boom in the US economy in the 90's. Their analysis also revealed that standard accounting measures underestimate the boom in business sector productivity that occurred in the 90's.

This Chapter uses the extended theory of McGrattan and Prescott (2010) to explore the impact of including investment in intangible capital in a small open economy

(SOE) RBC model tailored to the Canadian economy. Due to the high degree of interconnectedness between the US and Canadian economies, it is expected that intangible capital plays as important a role in explaining business cycles in Canada as it does in the US.

Intangible capital in this Chapter is defined as a business input that is required to produce both output (final goods) and new intangible capital as in the two-sector model discussed in section 1.5.1. The production of new intangible capital is referred to as "investment in intangible capital". As mentioned earlier, examples of intangible capital include: advertising, marketing, patent purchases, research and development (R&D), organizational capital and the creation of new brands. Many papers feature some type of intangible capital, however, there are only a few that feature investment in intangible capital. For example, in contrast to how intangible capital is modeled here, Johri, Letendre, and Luo (2011) model organizational capital as a production input that accumulates as a by product of regular production activity and requires no designated inputs.

There is indeed some evidence to support the claim that investments in intangible capital play a crucial role in the performance of the Canadian economy. For example, the increased reliance on internet-based technologies to facilitate economic growth constitutes compelling evidence of the significance of intangible capital for economic growth. A good measure of this effect can be found in the reported growth in internet-based advertising revenue. According to the Interactive Advertising Bureau of Canada, Internet advertising revenue increased from $364 million in 2004 to $3.5 billion in 2013. Similarly, there has been some significant growth in R&D in Canada. The number of researchers in R&D (per million people) increased from 3058 researchers in 1996 to 4562 researchers in 2011. As shown in Figure 2.1, there is an upward trend in the number of patent and trademark applications by Canadian residents which is a measure of the output from R&D.[1] This evidence shows the growing significance of intangible investments and their contribution to the Canadian economy. It is therefore important to account for the impact of this type of intangible investment activity on the aggregate economy in the RBC framework.

This Chapter constitutes a new theoretical contribution to the SOE RBC literature, because it is the first to feature investment in intangible capital in the way suggested by McGrattan and Prescott (2010) in a SOE RBC model. As such, it allows for a new type of analysis for a SOE that was not previously possible. In particular, the benchmark two-sector model developed in this Chapter allows for the examination of the relationship between intangible investment, business sector productivity and output in a SOE. It also allows for the exploration of the link between trade balance and investment in intangible capital, which has not been done until now in the RBC literature. Futhermore, this Chapter is the first to provide a detailed explanation of the mechanism behind the performance of a model that features investment in intangible capital in the manner proposed by McGrattan and Prescott

[1] Data on researchers in R&D, patents and trademarks was sourced from the Science and Technology section of the World Bank development indicators for Canada.

(2010).

The key difference between this Chapter and the work of McGrattan and Prescott (2010) lies in the modelling of the additional debt dynamic that is present in an open economy but not in a closed economy model. In particular, the closed economy model of McGrattan and Prescott (2010) is an extension of the basic RBC model of Kydland and Prescott (1982) – without debt - whereas this Chapter extends the SOE model of Schmitt-Grohé and Uribe (2003) - with debt - to include investment in intangible capital.[2] Borrowing and lending is an additional dynamic for a SOE that is not present in the closed economy model of McGrattan and Prescott (2010). Modeling the debt dynamics allows for the exploration of the interaction between intangible capital and the trade balance in a SOE.

To test the implications of the newly developed model with investment in intangible capital and non-neutral technology, the framework is used to examine the Canadian productivity puzzle that has been well documented in recent years. This puzzle arises due to the low annual growth of business sector productivity in Canada (0.86% from 2000-2013) despite the implementation of policies that are designed to increase productivity.[3], [4] Since McGrattan and Prescott (2010) find that intangible investment plays a crucial role in explaining US business sector productivity, the proposed extended model is well suited for investigating the Canadian productivity

[2]The Schmitt-Grohé and Uribe (2003) model is an extension of Mendoza (1991). Hereinafter referred to as SGU (2003).

[3]According to Drummond, Ryan, and Veall (2013), the annual growth rate of 0.86% is low by historical and international standards.

[4]An example of a policy is the Canada Research Chair Program that involved spending over $2 billion to attract and retain high quality researchers in various academic disciplines.

puzzle.

The key mechanism behind the performance of the benchmark model lies in the reallocation of resources across sectors in response to sector specific shocks. This mechanism is highlighted by a discussion of the impulse response functions generated by the model. The overall performance of the benchmark model is measured by its ability to replicate key stylized facts about the Canadian economy that the standard SOE RBC model is able to mimic. In other words, the simulated properties of the benchmark model are compared to the simulated properties of the basic SOE model and Canadian data.

In general, this Chapter finds that intangible capital plays an important role in explaining productivity and economic growth in a SOE. The variance decomposition confirms that TFP shocks in the intangible capital sector play a vital role in explaining variations in business sector output. The impulse response functions (IRF) show that positive shocks to the intangible capital sector eventually translate to a boom in the business sector and vice versa. Furthermore, the benchmark model reveals a different (procyclical) behaviour of trade balance in response to a positive TFP shock in the intangible capital sector, which is in contrast to the countercyclical behavior in response to a positive business sector TFP shock. Lastly, similar to the finding of McGrattan and Prescott (2010), this Chapter finds that the absence of an intangible capital sector leads to a negative business sector productivity bias that arises as a result of mismeasurement. This implies that measurement error could be partially responsible for the observed low business sector productivity in the Canadian data.

The structure of the remainder of this Chapter is as follows: Sections 2.2 and 2.3 contain the literature review and description of the data and methodology used in this study. Section 2.4 features the description of the two models considered and a discussion of key modeling assumptions. Section 2.5 presents the results, a comparison of both models, and a discussion of the impulse response functions to explain the mechanism behind the performance of the benchmark model. Section 2.6 contains concluding remarks and the suggested direction for future research.

2.2 Literature Review

The importance of various forms of intangible capital has been well documented since the late 90's. Using models that combine R&D and capital accumulation, Bental and Peled (1996), Matsuyama (1999, 2001), Wälde (1999, 2002) show that intentional R&D by profit maximizing firms are a source of long run growth and short run fluctuations. While it is possible to come up with cost estimates for these types of investment activities that use up valuable business inputs (capital and labour), their exact benefits are difficult to measure and this creates the problem of unmeasured economic activity that distorts the true picture of the economy.

In recent years, there have been numerous inquiries into the role of investment in intangible capital in a SOE. However, none of these inquiries utilize the RBC framework, instead they mostly utilize applied econometrics techniques. Baldwin, Gu, Lafrance, and Macdonald (2009) conclude that investments in intangible capital have become increasingly important to sustained economic growth. Baldwin, Gu, and Macdonald (2012) estimate that the ratio of intangible investment to tangible investment in Canada increased from 0.23 in 1976 to 0.66 in 2008. This implies that the size of unmeasured economic activity is increasing and highlights the need to properly account for the role that they play in the economy.

An important question that naturally arises in a business cycle setting is whether or not intangible capital is procyclical. Fatas (2000) argues that R&D is procyclical for the US between 1961 and 1996. Cincera, Cozza, Tübke, and Voigt (2012), using evidence from the great recession of 2009, suggest that credit constraints play an important role that reinforces procyclical R&D adjustments as firms tend to cut down their R&D and innovation activities in a crisis, but increase them after the crisis. López-García, Montero, and Moral-Benito (2012), use panel data on Spanish firms to test the opportunity cost theory, similar to Cincera et al. (2012), they find that on aggregate, credit constraints in bad economic times counteract the effects of the opportunity cost theory and lead to the procyclical behavior of R&D.[5] However, they find that patent purchases are acyclical, on-the-job-training expenditure is countercyclical and both are not affected by credit constraints.

There appears to be no clear consensus on the cyclical nature of all types of intangible capital. This Chapter finds intangible capital to be procyclical and investment

[5] The opportunity cost theory dictates that R&D and productivity improving activities should increase in recessions since their opportunity costs are relatively lower in comparison to good economic times.

in intangible capital to be countercyclical.[6] In order to obtain a measure of the direct impact of modelling investment in intangible capital in an otherwise basic SOE model, credit constraints, taxes and other financial frictions are not modelled during this analysis.

2.3 Description of Data and Methodology

This Chapter uses the DSGE approach to model investment in intangible capital in a SOE. Two models are considered: the first is a basic SOE model a la Mendoza (1991) - without intangible capital - and the second is the benchmark model of this Chapter which is an extension of McGrattan and Prescott (2010) to the case of a SOE.[7] To assess the contribution of investment in intangible capital, the results from the benchmark model will be compared with the results from the basic model.[8] In the case of the benchmark model, there is no established range for some of the model parameters, hence Bayesian estimation is used to achieve more robust parameter estimates that maximize the likelihood of observing the data used to motivate this study. The Bayesian estimation and stochastic simulations presented in this Chapter were carried out using Dynare.

The annual data used in this study was sourced from Statistics Canada and covers

[6]Investigating a version of the benchmark model with credit constraints could provide insight on the cyclicality of intangible capital. This task is intentionally omitted in this version of the Chapter.

[7]The benchmark model features intangible capital and non neutral technology in the creation of new intangible capital.

[8]Results here refer to the properties and impulse response functions generated from both models.

the period from 1981 to 2013. Canadian data on output (GDP), consumption, investment, hours, capital, trade balance-to-output ratio and current account-to-output ratio is used. All data are in constant 2007 dollars except for hours, trade balance to business output ratio (tb/y_b) and current account to business output ratio (ca/y_b). Appendix 2.7.1 contains the description of variables. As is standard practice in the literature, to generate the statistics used to characterize the empirical regularities of the Canadian economy, all data except the ratios are in per capita form (using population older than 14), logged and detrended using log-quadratic detrending. [9] Table 2.2 shows the properties of the Canadian data that were used for this analysis.

McGrattan and Prescott (2010) do not report a table of summary statistics for their benchmark model for the US economy, hence there is no direct point of comparison for the benchmark model presented in this Chapter. This Chapter is also the first of its kind to present the properties for a SOE model that implements investment in intangible capital in the way proposed by McGrattan and Prescott (2010).

2.4 Model and Parameterization

This Chapter makes reference to two types of capital (physical and intangible), to ensure consistency, the term "capital" will refer to physical capital and intangible capital will refer to intangible capital. It helps to think of the basic model in this Chapter as a one-sector model where there is a single output (final good) that is

[9] Where the residuals from the regressions of the log of each variable on a time trend and its square i.e. log $Y = \beta_0 + \beta_1 t + \beta_2\ t^2$ are used to calculate the properties.

produced using two inputs (capital and labour). This output is used for consumption, debt repayment and investment in capital. The benchmark model is a two-sector model where the first sector produces final goods using three inputs (capital, labour and intangible capital) that is used for consumption, debt repayment and investment in capital, and the second sector produces new intangible capital using the same three inputs. The sole purpose of the output from the second sector is to replenish the stock of intangible capital, hence, output from that sector is referred to as "investment in intangible capital".

2.4.1 Basic and Benchmark Small Open Economy Models

In the basic model, given an initial stock of capital $k_{T,0}$ and debt d_0, households maximize the expected present value of lifetime utility subject to a series of intertemporal budget constraints. Each period the household chooses consumption c_t, hours h_t, future capital stock $k_{T,t+1}$ and debt d_{t+1}. Capital and debt (portfolio) adjustment costs are included to reduce the volatility of investment in capital $I_{T,t}$ and to close the model.[10]

Households maximize:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t - \frac{h_t^{\omega}}{\omega} \right)^{(1-\gamma)} - 1}{1-\gamma} \right]$$

[10] According to SGU(2003) the debt adjustment cost is required to induce stationarity and ensure the independence of steady state from initial conditions. The model uses a small adjustment cost to achieve this objective.

Subject to:

$$c_t + i_{Tt} = r_{Tt}k_{Tt} + w_t h_t + d_{t+1} - d_t(1 + r_w) - (\frac{\psi}{2})(d_{t+1} - \bar{d})^2 - (\frac{\phi}{2})(k_{T,t+1} - k_{Tt})^2 \tag{2.1}$$

$$k_{T,t+1} = i_{Tt} + (1 - \delta_T)k_{Tt} \tag{2.2}$$

where all variables are written in per capita terms. $r_{T,t}$ is the rental rate of capital, w_t is the wage rate of labour, r_w is the interest cost of debt that is exogenously determined because this is a SOE. β is the discount rate of households and δ_T is the depreciation rate of capital. ϕ and ψ are the adjustment cost parameters for capital and debt respectively.

Firms are assumed to operate in a perfectly competitive environment and choose the optimal amount of capital $k_{T,t}$ and labour h_t that maximize profit to use in production.

Firms maximize:

$$\pi_t = y_t - r_{Tt}k_{Tt} - w_t h_t$$

subject to:

$$y_t = A_t(k_{Tt})^{\alpha_1}(h_t)^{(1-\alpha_1)} \tag{2.3}$$

The second model considered here is the benchmark model of this Chapter. The household problem is similar to the one-sector model with the additional choice of next period intangible capital $k_{I,t+1}$. Given an initial stock of capital $k_{T,0}$, intangible capital $k_{I,0}$ and debt d_0, households maximize the expected present value of lifetime utility subject to a series of intertemporal budget constraints by choosing consumption c_t, hours h_t, future capital stock $k_{T,t+1}$, intangible capital stock $k_{I,t+1}$ and debt d_{t+1}. Capital and debt adjustment costs are also included here for the same reasons as above.

Households maximize:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t - \frac{h_t^{\omega}}{\omega} \right)^{(1-\gamma)} - 1}{1-\gamma} \right]$$

Subject to:

$$\begin{aligned} c_t + i_{Tt} + q_t i_{It} = & r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_t + d_{t+1} - d_t(1+r_w) - (\frac{\psi}{2})(d_{t+1} - \bar{d})^2 \qquad (2.4) \\ & - (\frac{\phi}{2})(k_{T,t+1} - k_{Tt})^2 \end{aligned}$$

$$k_{T,t+1} = i_{Tt} + (1-\delta_T) k_{Tt} \qquad (2.5)$$

$$k_{I,t+1} = i_{It} + (1-\delta_I) k_{It} \qquad (2.6)$$

$r_{I,t}$ is the rental rate of intangible capital and q_t is the relative price of intangible capital in consumption terms. Other variables are as described above. Firms operate in a perfectly competitive environment and choose the optimal amount of capital, intangible capital and labour to use in the production of final goods and new intangible capital.

It is assumed that hours and capital are split across sectors whereas intangible capital is used simultaneously. Therefore, in addition to choosing the optimal total amount of inputs, firms must decide upon the allocation of capital and labour to the final goods sector (k_T^1 & h^1) and the intangible investment sector (k_T^2 & h^2).

The firm maximizes

$$\pi_t = y_t - r_{Tt}k_{Tt} - r_{It}k_{It} - w_t h_t$$

subject to:

$$y_t = y_{bt} + q_t i_{It} \tag{2.7}$$

$$y_{bt} = A_t^1 (k_{Tt}^1)^{\alpha_1} (k_{It})^{\epsilon_1} (h_t^1)^{(1-\alpha_1-\epsilon_1)} \tag{2.8}$$

$$i_{It} = A_t^2 (k_{Tt}^2)^{\alpha_2} (k_{It})^{\epsilon_2} (h_t^2)^{(1-\alpha_2-\epsilon_2)} \tag{2.9}$$

$$k_{Tt} = k_{Tt}^1 + k_{Tt}^2 \tag{2.10}$$

$$h_t = h_t^1 + h_t^2 \tag{2.11}$$

where

$$ln(A_t^1) = \rho_1 ln(A_{t-1}^1) + e_{1t} \tag{2.12}$$

$$ln(A_t^2) = \rho_2 ln(A_{t-1}^2) + e_{2t} \tag{2.13}$$

$0 < \rho_i < 1$ and $e_{i,t}$ is iid with $E(e_{i,t})=0$ and $var(e_{i,t})=\sigma^2_{ei}$ for i=1,2. The equilibrium conditions of the model are presented in Appendix 2.7.2.

2.4.2 Model Parameters

To solve both models, values are assigned to model parameters using four methods: i) some parameters are calibrated to ensure that the model matches key properties for the Canadian data, ii) some parameters are assigned based on actual data and the structure implied by the model, iii) some parameters are drawn from the relevant existing empirical literature and iv) for the benchmark model, Bayesian estimation is used to determine the values of some parameters.

The values of the parameters used in the basic model are shown in Table 2.3. For this model, some parameter values are borrowed from SGU (2003). Even though the basic model here is the same as the model with debt adjustment costs from SGU (2003), there is a need to re-calibrate and assign some parameters for one main reason: the data set used to generate the empirical regularities of the Canadian economy in this Chapter is different from the data set considered by SGU (2003). This Chapter presents statistics based on 1981-2013 annual data, whereas SGU (2003) use the same data as Mendoza (1991); 1946-1985 data. As a result, there are differences across the statistics generated from both datasets that require the re-calibration of parameters instead of adopting the exact parameter values used in SGU (2003). In what follows, α, γ, ω, δ_T, and r_w are borrowed from SGU (2003), whereas ϕ, ψ, ρ_a and σ_e are calibrated. The parameter ϕ is set to match the relative volatility of investment and output.

The values of the parameters used in the benchmark model are shown in Table 2.4. For this model, the parameters from the benchmark model of McGrattan and Prescott (2010) are used as a guide. However, since the parameter values assigned in their model are based on US estimates, there is a need to re-estimate these parameters using Canadian data. There is also need to assign values to parameters not present in their model such as ψ and ϕ which determine the adjustment costs of debt and capital respectively. The parameters r_w, γ and δ_T are assigned the same values as in the basic model. The capital adjustment cost parameter ϕ is selected to match the relative volatility of investment in capital to output.

The values of ω, α_1, α_2, ϵ_1, ϵ_2, ρ_1, ρ_2, σ_{e1}, σ_{e2} and δ_I are derived via Bayesian estimation where parameter values from McGrattan and Prescott (2010) are used as the mean of the prior distribution for some of the parameters and standard distribution shapes such as Beta, Normal and Inverse gamma that are commonly used in the literature are used as the prior distribution for the estimated parameters. The prior and posterior distribution of the estimated parameters for the benchmark model are provided in Table 2.7.

Because the model variables are in log form and there is no growth in the model, the observables used for the Bayesian estimation are the deameaned Canadian GDP (*dy*) and productivity (*dprod*) growth rate data.[11] Figure 2.7 shows the historical

[11]Where GDP growth is calculated as $ln(GDP_t) - ln(GDP_{t-1})$. Productivity is measured as

and smoothed paths for the observables and we can see that both lines are a perfect fit for both observables, implying that the dynamics of the model are important for this analysis. With Bayesian estimation, the number of observables included in the analysis is constrained by the number of shocks in the model. In this case, there are two shocks in the benchmark model, hence only two observables are used.

In general, the values derived for all parameters in the benchmark model fall within the range of commonly used values in the literature and are listed in Table 2.4. In the case of ω, the estimate is 1.43 and the implied Frisch elasticity of labour supply is 2.3, which is within the range of 2 to 4 that is common for macroeconomic analysis as noted by Peterman (2014).

Both models feature two types of adjustment costs: debt adjustment cost and capital adjustment cost. The formulation used for the adjustment costs are of the simplest form that have the desired properties of reducing volatility and inducing stationarity.

2.5 Results

2.5.1 Variance decomposition

Table 2.8 shows the variance decomposition for the benchmark model. It captures the importance of including intangible investment in an otherwise basic RBC model. It is

$prod_t = y_{bt}/h_t$ where y_b is business sector output and h is total hours worked across all sectors. Productivity growth ($dprod$) is measured as $prod_t - prod_{t-1}$

evident that A^2 shocks explain a significant amount of variation in the observables used for this analysis. They account for 13% and 45% of the variation in the observables GDP growth (dy) and productivity growth ($dprod$), respectively. Furthermore, shocks to intangible investments explain a significant amount of variation in other variables in the model economy. They account for about 28% to 47% of the variation in the major aggregate variables - except investment in physical capital (i_T [9%]) - including: capital, consumption, hours, trade balance, observable productivity (y_b/h) and business sector output. These findings are the first estimates of the explanatory power of shocks to intangible investments in a SOE RBC framework and imply that the benchmark model has some explanatory power.

To check the robustness of these results, other shocks are introduced into the benchmark model to see if A^2 shocks maintain their explanatory power. Equations (2.14) to (2.16) show how key model equations change with the addition of a preference shock b_t, an investment specific technology (IST) shock υ_t and a marginal efficiency of investment (MEI) shock μ_t. Equations (2.17) to (2.19) describe the processes for the new shocks.

$$E_t \sum_{t=0}^{\infty} b_t \left[\beta^t \left[\frac{\left(c_t - \frac{h_t^{\omega}}{\omega} \right)^{(1-\gamma)} - 1}{1-\gamma} \right] \right] \tag{2.14}$$

$$k_{T,t+1} = \mu_t i_{Tt} + (1 - \delta_T) k_{Tt} \tag{2.15}$$

$$c_t + \frac{i_{Tt}}{\upsilon_t} + q_t i_{It} = r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_t + d_{t+1} - d_t(1 + r_w) - (\frac{\psi}{2})(d_{t+1} - \bar{d})^2 - (\frac{\phi}{2})(k_{T,t+1} - k_{Tt})^2 \quad (2.16)$$

$$ln(b_t) = \rho_b ln(b_{t-1}) + e_{b_t} \quad (2.17)$$

$$ln(\mu_t) = \rho_\mu ln(\mu_{t-1}) + e_{\mu_t} \quad (2.18)$$

$$ln(\upsilon_t) = \rho_\upsilon ln(\upsilon_{t-1}) + e_{\upsilon_t} \quad (2.19)$$

Table 2.9 contains the variance decomposition for the version of the benchmark model including MEI, IST and preference shocks which are quite popular in the RBC literature. The results show that A^2 shocks continue to have some explanatory power even after accounting for other shocks. When simultaneously accounting for all shocks, A^2 shocks explain a significant but slightly lower amount of variation in the observables; 9% for GDP growth and 42% for productivity growth. A^2 shocks continue to explain a high amount of variation in the key macroeconomic variables in the model excluding investment in capital. The percentage variation explained ranges from 6% for trade balance to 40% for observable productivity. Preference shocks account for very little variation in the model economy and thus have no real impact on the results in this analysis. Overall, the results imply that shocks to intangible investment play a significant role in explaining aggregate economic activity in a SOE.

2.5.2 Table of Summary Statistics

The statistics generated from the Canadian data are provided in Table 2.2 and the properties of the basic and benchmark models are shown in Tables 2.5 and 2.6 respectively. In this section, the properties of both models are compared with the statistics from the data. During the calibration of both models, the relative volatility of i_T and tb were targeted to match the data. However, the autocorrelation of GDP was targeted in the basic model, but not the benchmark model. It is therefore a positive result that the benchmark model produces an autocorrelation for GDP of 0.79 which is close to 0.75 from the data. There are not many differences in the autocorrelation of variables and ranking of relative volatilities between both models.[12] The major difference lies in the predicted correlation of variables to output. The introduction of intangible capital and investment is able to break the perfect correlation between output, hours and productivity that is observed in the basic model. This result is more in-line with the observed correlations in the data.

The benchmark model also improves the predicted correlation between output and the trade balance to business sector output ratio. The basic model predicts a negative correlation of -0.10 whereas the benchmark model predicts a mild postive correlation of 0.15 that is closer to the 0.34 correlation in the data. The IRF discussion below

[12]Similar to the basic model, the benchmark model is unable to deliver the ranking of relative volatility observed in the data. Both models imply that the relative volatility of consumption to output is higher than the relative volatility of hours to output whereas the reverse is the case in the data. This is a well documented shortcoming of most SOE RBC models.

provides some insight as to why the correlation between tb/yb and GDP changes sign when A^2 shocks are added. Overall, the comparison of the properties of both models suggests that the benchmark model provides a better fit to the data than the basic model.

2.5.3 Impulse Response Functions

The discussion in this section is centered around the dynamics of the benchmark model. This is so because the performance of the basic model has been well documented in many studies such as SGU (2003). Figure 2.2 contains the impulse response functions for the basic model and Figures 2.3 and 2.4 show the IRF for the benchmark model for an A^1 and A^2 shock respectively. In all plots, the variables are measured in percentage deviations from steady state values.

Comparing an A^1 shock in the benchmark model (Figure 2.3) to an A shock in the basic model (Figure 2.2) it becomes evident that all aggregate variables: k_T, h, i_T, d, y_b and tb/y_b respond in a similar manner across models. This implies that, in the case of an A^1 shock, the introduction of investment in intangible capital does not alter the dynamics of the model at the aggregate level. There are still sector-specific reallocations of inputs which are discussed below, but at the aggregate level, a business sector A^1 shock in the benchmark model is equivalent to an A shock in the one-sector basic model since the sole sector is the business sector. This is not the case when there is an A^2 shock, as shown in Figure 2.4. In general, with the introduction of an additional sector and new variables, there are certainly additional dynamics at play that we now discuss.

Positive Productivity Shock in the Business Sector

The initial outcome of a positive e_{A^1} shock is an increase in business sector output (y_b) and a decrease in investment in intangible capital (i_I). The mechanism of change is as follows: following the positive technology shock in the final goods sector, the relative price of intangible capital (an input in both sectors) increases because it is now relatively scarce. In an attempt to capitalize on the shock, producers demand more labour (h) and allocate relatively more capital and labour to the more productive final goods sector. That is, k_T^1 and h^1 increase while k_T^2 and h^2 decrease. There is no initial adjustment to k_I as its stock is fixed and is not split between sectors. The resulting increase in y_b is used to fund higher consumption and increase investment in capital i_T.

Producers also realize that intangible capital is a valuable input to production in both sectors and its stock needs to be built up in order to fully capitalize on the shock. As a result, after the initial reallocation of inputs to the more productive sector, there is a gradual increase in the inputs (capital and labour) allocated to the intangible capital sector. This leads to a gradual rise in i_I and k_I from their lower levels in the period in which the shock occurred. Hence, both y_b and i_I are above their steady state values shortly after the shock.

The impact of the positive A^1 shock on debt d is two-fold. Initially, there is an increase in debt because additional resources are required to simultaneously increase investment in capital i_T and consumption c. However, due to household's desire to smooth consumption, there is a subsequent reduction in debt. i.e. households begin to lend out resources to ensure higher levels of future consumption when repayment occurs.

Positive Productivity Shock in the Intangible Sector

Figure 2.4 shows the IRF for a positive A^2 shock. Due to the high degree of persistence of A^2, the IRF for the aggregate variables appear hump-shaped. The initial impact of this shock is a decrease in y_b and increase in i_I. The mechanism of change is as follows: following the A^2 shock, there is a flow of resources to the relatively more productive intangible investment sector. k_T^2 and h^2 rise and this increases i_I which builds up the stock of intangible capital k_I. In the business sector, k_T^1 and h^1 decrease and this causes the decline in y_b. Because output from the intangible investment sector cannot be directly consumed or invested in physical capital, the reallocation effect does not last very long. Producers quickly reallocate resources to the business sector to increase y_b and facilitate the observed increase in consumption c and capital investment i_T. This builds up the stock of k_T which is further used to increase output in both sectors.

The impact of the A^2 shock on labour demand in this case is different from the impact of the A^1 shock. Due to the relatively smaller size of the intangible investment sector, the increase in labour initially required to effectively capitalize on the shock in the intangible capital sector is relatively small. Hence, there is only a small increase in labour demand in the period in which the shock hits. In the periods following the shock, there is a further increase in labour demand to facilitate the increase in output in the relatively larger business sector that allows the economy build up the physical capital stock and increase consumption.

The behavior of debt following the A^2 shock is different from the response to an A^1 shock. There is initially a gradual increase in borrowing that leads to progressively higher levels of debt followed by periods of gradual reduction in debt towards pre-shock levels. This is so because the A^2 shock only leads to an increase in i_I in the initial period and additional resources are required to increase business sector output and achieve higher levels of capital and consumption in subsequent periods. This leads the economy to borrow resources to increase investment in capital and sustain higher levels of consumption in the periods following the shock.

In both cases (A^1 and A^2), after the initial reallocation of resources to the relatively more productive sector where the positive shock occurred (which causes an increase in output in that sector and decrease in output in the other sector), there is a subsequent reallocation of resources to the sector where the shock did not occur and increase in output of that sector. This effectively explains how a shock to one sector eventually translates to a boom in the other sector even after the initial negative impact. On aggregate, there is an increase in total output ($y = y_b + qi_I$) in both cases which highlights the importance of accounting for unmeasured economic activity. If all that is observed is business sector output, in the event of an A^2 shock, the observed impact on the economy would be negative whereas the actual total impact would be positive. This relationship is crucial for any discussion on productivity.

The overall impact of the A^2 shock on the final goods sector is significantly less than the impact of the A^1 shock on the intangible investment sector. This is so because of the relative sizes of the sectors (the final goods sector is significantly larger than the intangible investment sector).

2.5.4 Note on Trade Balance

As stated earlier, the benchmark model of this Chapter allows for the exploration of the link between trade balance and investment in intangible capital. Figure 2.5 shows the response of the trade balance to business sector output ratio, tb/y_b, to positive A^1 and A^2 shocks. This measure of the trade balance is used instead of the trade balance to aggregate output ratio tb/y because there is currently no data for y as there is for yb. Furthermore, tb/y_b in the benchmark model is equivalent to tb/y in the basic one-sector model, which has been studied extensively.

The response of tb/y_b to a positive A^1 shock is countercyclical. This is similar to the response of tb/y when there is an A shock in the basic model and the intuition behind the response is the same. In response to the positive A^1 shock, business sector output and debt increase. The increase in debt is accompanied by a decrease in the trade balance and these two effects combine to cause the observed reduction in tb/y_b.

As the shock wears off and y_b reverts to its pre-shock levels, consumers reduce debt in an attempt to smooth consumption, which leads to an increase in tb/y_b as y_b falls. This characterizes the countercyclical behaviour of tb/y_b in response to a positive A^1 shock.

This response is in contrast to the procyclical behaviour of tb/y_b when there is an A^2 shock. In response to the A^2 shock, the reallocation of resources to the intangible investment sector causes yb to decrease. This, combined with the increase in debt that leads to a reduction in the trade balance, causes the initial decrease in tb/y_b. As more resources are allocated to the business sector to achieve the desired increase in consumption and investment i_T, y_b begins to grow. This, combined with the gradual increase and subsequent decrease in debt, causes the observed increase in tb/y_b after the initial decrease.

The percentage variation in the trade balance explained by A^2 shocks shown in Table 2.8 is 40%. This, combined with the procyclical response of the trade balance to output ratio in response to A^2 shocks, and the finding of Baldwin et al. (2012) that the ratio of intangible investments to tangible investments is on the rise, imply that the increasingly important role of intangible investments are a potential explanation for the increasingly positive correlation between the trade balance to business output ratio (tb/y_b) and business output (y_b) that is observed in the Canadian data. Mendoza (1991) reports a correlation of -0.13 in 1945 - 1989 annual Canadian data whereas this study finds a correlation of 0.34 in 1981 - 2013 data.

2.5.5 Model Prediction for Canadian Productivity Puzzle

In this section, different measures of productivity from the benchmark model are compared and a productivity bias is found to be induced by unmeasured investment. The accurate measure of business sector productivity in this setting requires that the inputs used in the production of final goods and the inputs used in the production of new intangible capital be separated. According to the two-sector benchmark model presented in this Chapter, the true measure of business sector productivity is y_b/h^1 where h^1 is the total labour allocated to the production of final goods. This differs from the standard measure of business sector productivity y_b/h (where $h = h^1 + h^2$) that is often used in practice due to the availability of data.[13] Figure 2.6 contains the plots for the IRF of the three measures of productivity examined here: y_b/h^1, the true measure of business sector productivity, y_b/h, the measure of business sector productivity currently used in practice and y/h, the measure of aggregate labour productivity.

When there is an A^1 shock, all measures of productivity are predicted to rise. However, in the event of an A^2 shock, the prediction for business sector productivity depends on the measure considered. The second plot in Figure 2.6 shows that using the true measure of business sector productivity y_b/h^1 leads to the conclusion that productivity in that sector rises as a result of the A^2 shock. However, if the conven-

[13] According to the theory of the two-sector benchmark model, y_b/h^1 captures the true productivity of the business sector as it does not assign the hours used in the production of new intangible capital h^2 to the final goods sector.

tional measure (y_b/h) is used instead, it leads to the conclusion that the initial impact of a positive A^2 shock is a reduction in business sector productivity. This result has significant implications for the Canadian productivity puzzle.

If this is truly an era of significant investments in intangible capital and the Canadian economy is partly driven by shocks to the intangible investment sector, then according to the two-sector benchmark model studied here, using the conventional measure of productivity negates the impact of A^2 shocks and creates a productivity bias that leads to the unfavourable conclusion that business sector productivity is low when, in fact, there is an increase in the true measure of productivity. It can also be seen from the true measure of aggregate productivity in this model y/h in the third plot of Figure 2.6, that aggregate productivity increases in response to both A^1 and A^2 shocks. This further highlights the consequences of drawing conclusions about productivity based on an incomplete measure. A bias is found to be induced by a mismeasurement in either hours (y_b/h^1 Vs. y_b/h) or output (y_b/h Vs. y/h). The above discussion implies that the Canadian productivity puzzle could partially be caused by the measurement error associated with the increasingly important role of unmeasured intangible investments in the Canadian economy.

2.6 Conclusion

The main findings of this research imply that modelling intangible investment in the way suggested by McGrattan and Prescott (2010) (with non-neutral technology) is a valuable tool for explaining business cycles in a SOE. The benchmark model is capable of replicating some of the stylized facts of the Canadian economy and introducing additional dynamics (i.e. the reallocation of inputs across sectors in response to shocks) that help explain the evolving Canadian economy. The model is also able to highlight the circumstances under which the trade-balance to business sector output ratio tends to be procyclical and shows how shock spillovers occur even when technologies and their disturbances are uncorrelated across sectors.

Furthermore, the predictions of the model for the Canadian productivity puzzle highlights the need to re-evaluate the traditional measure of productivity, which is a key driver of the performance of business cycle models. The three measures of productivity considered above show that using the traditional measure leads to a misrepresentation of the true state of business sector productivity. This finding is similar to the results of McGrattan and Prescott (2010) for the US economy. The challenge going forward is to find ways to accurately capture the unmeasured investment that is at the root of the negative business sector productivity bias. It is also important to investigate the relationship between intangible investment and other frictions such as credit constraints and taxes that are not present in this Chapter.

This Chapter provides some insight about the contributions of intangible capital and intangible investment in explaining business cycles in a SOE. The next Chapter features an in depth analysis of the contributions of intangible capital and intangible investments in explaining international business cycles.

2.7 Appendix

2.7.1 Data Appendix

The data used for this study is of annual frequency and covers the period 1981-2013. The data were downloaded from CANSIM and are in 2007 constant prices unless otherwise specified. The data on patents and trademarks were sourced from The World Bank DataBank.

Table 2.1: Data Sources

Variable	Description	Table number
Population	Total number of canadians aged 15 and over	051-0001
Output	Gross domestic product at market prices	380-0106
Consumption	Household final consumption expenditure	380-0106
Capital	Geometric end-yer net stock of total non-residential capital	031-0005
Investment	Business gross fixed capital formation + Business investment in inventory	380-0106
Hours	Total actual hours, all jobs (full-time and part-time)	282-0028
Exports	Exports of goods and services	380-0106
Imports	Imports of goods and services	380-0106
Current account	Total current account balance(Annual dollars)	376-0101
Patents	Patent applications, residents	[IP.PAT.RESD]
Trademarks	Trademark applications, direct resident	[IP.TMK.RESD]

2.7.2 Equilibrium Conditions

Benchmark Model

Consumption

$$\lambda_t = \left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{(-\gamma)} \tag{2.20}$$

Optimal debt level

$$\lambda_t(1 - \psi(d_{t+1} - \bar{d})) = \beta\lambda_{t+1}(1 + r_w) \tag{2.21}$$

Optimal labour supply

$$h_t^{\omega-1} = \left(\frac{(1 - \alpha_1 - \epsilon_1)y_{bt}}{h_t^1}\right) \tag{2.22}$$

Optimal allocation of labour

$$\frac{(1 - \alpha_2 - \epsilon_2)q_t i_{It}}{h_t^2} = \frac{(1 - \alpha_1 - \epsilon_1)y_{bt}}{h_t^1} \tag{2.23}$$

Optimal level of capital

$$\lambda_t(1 + \phi(k_{T,t+1} - k_{Tt})) = \beta\lambda_{t+1}\left(\frac{\alpha_1 y_{b,t+1}}{k_{T,t+1}^1} + (1 - \delta_T) + \phi(k_{T,t+2} - k_{T,t+1})\right) \tag{2.24}$$

Optimal allocation of capital

$$\frac{\alpha_2 q_t i_{It}}{k_{Tt}^2} = \frac{\alpha_1 y_{bt}}{k_{Tt}^1} \tag{2.25}$$

Optimal level of intangible capital

$$q_t \lambda_t = \beta \lambda_{t+1} \left(\frac{\epsilon_1 y_{b,t+1} + \epsilon_2 q_{t+1} i_{I,t+1}}{k_{I,t+1}} + q_{t+1}(1 - \delta_I) \right) \tag{2.26}$$

Productivity processes

$$ln(A_t^1) = \rho_1 ln(A_{t-1}^1) + e_{1t} \tag{2.27}$$

$$ln(A_t^2) = \rho_2 ln(A_{t-1}^2) + e_{2t} \tag{2.28}$$

where $0 < \rho_i < 1$ and $e_{i,t}$ is iid with $E(e_{i,t})$=0 and $var(e_{i,t})$=σ_{ei}^2 for i=1,2

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Table 2.2: Properties of the Canadian Data (1981-2013)

	Standard Dev	SD/SD(Y)	Autocorrelation	Correlation(GDP)
GDP	3.33	1.00	0.75	1.00
Consumption	2.32	0.70	0.74	0.79
Investment	10.78	3.24	0.50	0.86
Capital	1.57	0.47	0.82	-0.14
Hours	2.72	0.82	0.66	0.83
Productivity	1.52	0.46	0.84	0.68
TB/Y ratio	2.05	0.61	0.89	0.34
CA/Y ratio	2.73	0.82	0.65	0.31

Table 2.3: Basic Model Parameters

Parameter	Value	Source
β	0.96	$1/(1+r_w)$
ω	1.455	Schmitt-Grohe and Uribe (2003)
γ	2	Schmitt-Grohe and Uribe (2003)
α	0.32	Mendoza (1991)
ϕ	0.03	Calibrated to match $\sigma_{i_T}/\sigma_{y_b}$=3.3
ψ	0.0016	Calibrated to match $\sigma_{tb/y_b}/\sigma_{y_b}$=0.64
ρ_a	0.54	Calibrated to match ρ_{y_b}=0.75
σ_e	0.02	Set equal to the value from the benchmark model
r_w	0.04	Schmitt-Grohe and Uribe (2003)
$\bar{d}$	1.487	Calibrated to match tb/y_b=0.04
δ_T	0.1	Schmitt-Grohe and Uribe (2003)

Table 2.4: Benchmark Model Parameters

Parameter	Value	Source
β	0.96	$1/(1+r_w)$
ω	1.43	Bayesian estimation
γ	2	Schmitt-Grohe and Uribe (2003)
α_1	0.27	Bayesian estimation
α_2	0.25	Bayesian estimation
ϵ_1	0.05	Bayesian estimation
ϵ_2	0.07	Bayesian estimation
ϕ	0.035	Calibrated to match $\sigma_{i_T}/\sigma_{y_b}$=3.2
ψ	0.0068	Calibrated to match $\sigma_{tb/y_b}/\sigma_{y_b}$=0.61
ρ_1	0.56	Bayesian estimation
ρ_2	0.87	Bayesian estimation
σ_{e1}	0.02	Bayesian estimation
σ_{e2}	0.08	Bayesian estimation
r_w	0.04	Schmitt-Grohe and Uribe (2003)
$\bar{d}$	.8	Calibrated to match tb/y_b=0.042
δ_T	0.1	Schmitt-Grohe and Uribe (2003)
δ_I	0.095	Bayesian estimation

Table 2.5: Properties of the Basic Model

	SD/SD(Y)	Autocorrelation	Correlation(GDP)
GDP	1.00	0.74	1.00
Consumption	0.88	0.84	0.90
Investment	3.20	0.16	0.60
Capital	0.60	0.85	0.95
Hours	0.69	0.74	1.00
Productivity	0.46	0.74	1.00
TB/Y	0.61	0.36	-0.10
CA/Y	0.55	0.28	0.01

Table 2.6: Properties of the Benchmark Model

	SD/SD(YB)	Autocorrelation	Correlation(GDP)
GDP(YB)	1.00	0.79	1.00
Consumption	0.80	0.88	0.92
Investment	3.16	0.12	0.48
Capital	0.74	0.90	0.91
Hours	0.66	0.83	0.97
Productivity	0.46	0.69	0.92
TB/YB	0.61	0.36	0.15
CA/YB	0.59	0.36	0.22
Int. Capital	2.84	0.98	0.44
Int. Investment	6.73	0.65	-0.09

Table 2.7: Prior and Posterior Distribution of Estimated Parameters

	Prior Distribution			Posterior Distribution			
	Distr.	Mean	st Dev.	Mode	Mean	5%	95%
α_1	Beta	0.26	0.02	0.268	0.269	0.237	0.300
α_2	Beta	0.26	0.02	0.246	0.248	0.214	0.279
ϵ_1	Beta	0.067	0.02	0.054	0.057	0.031	0.082
ϵ_2	Beta	0.067	0.02	0.063	0.073	0.042	0.100
δ_I	Beta	0.1	0.03	0.091	0.096	0.050	0.137
ρ_1	Beta	0.5	0.05	0.571	0.563	0.481	0.643
ρ_2	Beta	0.9	0.05	0.891	0.886	0.815	0.970
ω	Normal	1.5	0.05	1.428	1.433	1.354	1.514
σ_{e_1}	Invgamma	0.1	2	0.021	0.022	0.016	0.027
σ_{e_2}	Invgamma	0.1	2	0.063	0.076	0.043	0.116

Table 2.8: Variance Decomposition: Benchmark Model

Variables	A^1	A^2
dy	87.00	13.00
$dprod$	55.00	45.00
k_T^1	55.00	45.00
h^1	67.00	33.00
y_b	68.00	32.00
k_T^2	7.00	93.00
h^2	4.00	96.00
i_I	3.00	97.00
c	72.00	28.00
d	71.00	29.00
q	2.00	98.00
i_T	91.00	9.00
k_T	58.00	42.00
k_I	1.00	99.00
y	68.00	32.00
h	68.00	32.00
y_b/h	53.00	47.00
y_b/h^1	68.00	32.00
y/h	68.00	32.00
tb/y_b	60.00	40.00
ca/y_b	58.00	42.00

Table 2.9: Variance Decomposition: A^2 Robustness Check

	A^1	A^2	MEI	PREF	IST
y_b	50.00	20.00	14.00	1.00	14.00
c	49.00	20.00	14.00	3.00	14.00
i_T	8.00	2.00	31.00	0.00	59.00
k_T	22.00	9.00	33.00	2.00	33.00
h	50.00	18.00	15.00	1.00	15.00
y_b/h	40.00	40.00	10.00	1.00	10.00
tb/y_b	5.00	6.00	32.00	1.00	57.00
dy	84.00	9.00	3.00	0.00	3.00
$dprod$	55.00	42.00	1.00	0.00	1.00

Figure 2.1: Patent and Trademark Applications

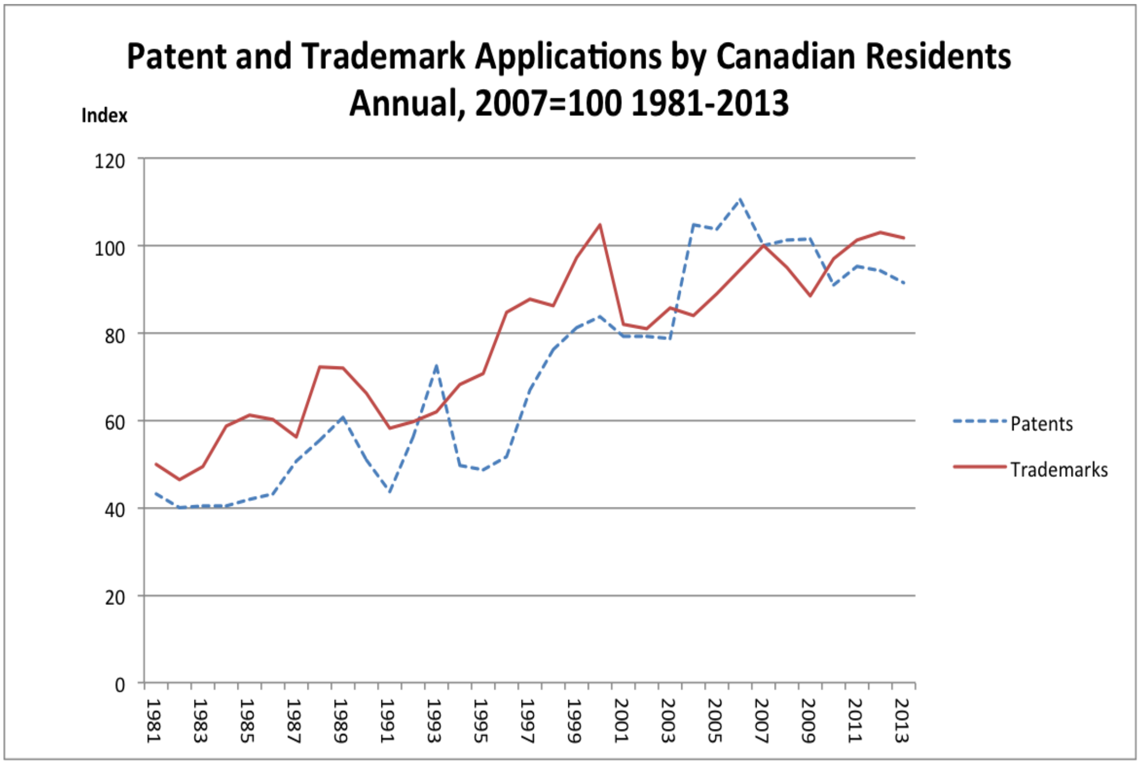


Figure 2.2: Response to Positive TFP Shock (A)- Basic Model

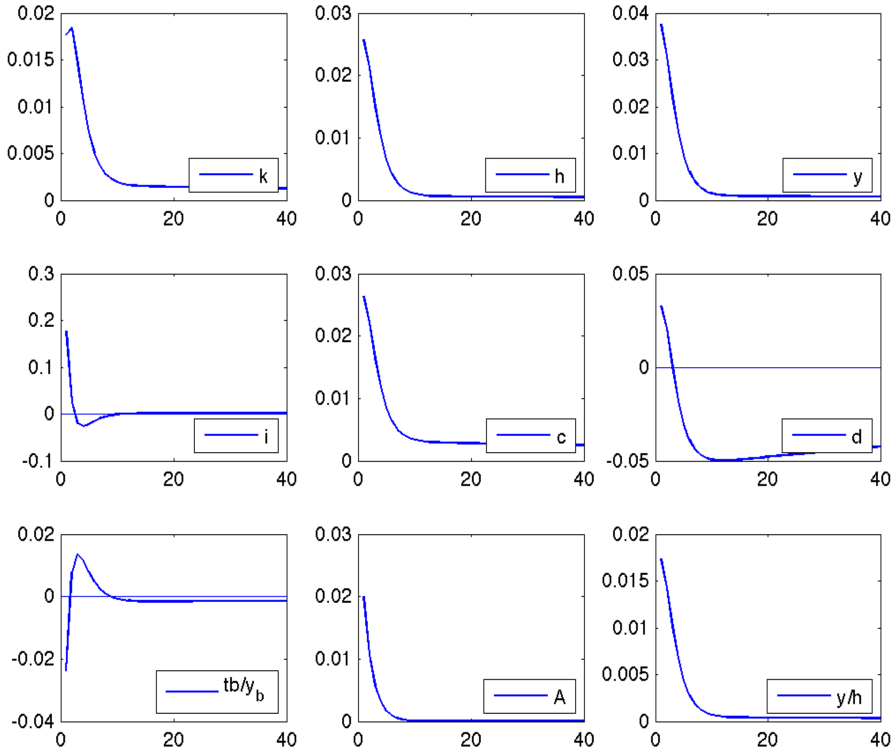


Figure 2.3: Impulse Response Functions: 1 s.d. Shock to Business Sector TFP (A^1)

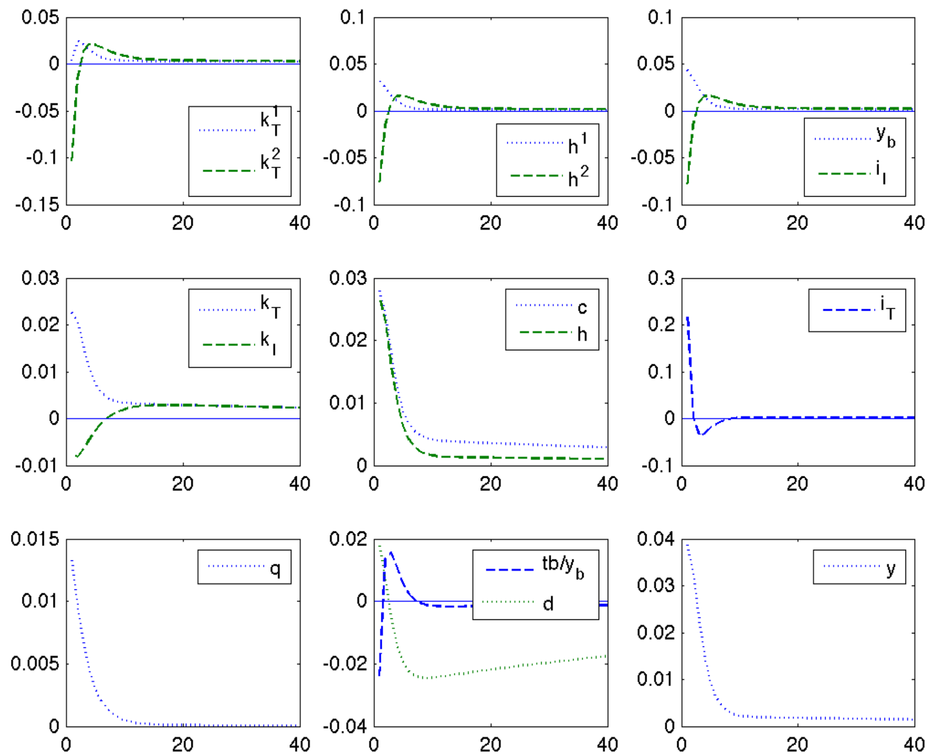


Figure 2.4: Impulse Response Functions: 1 s.d. Shock to Intangible Sector TFP (A^2)

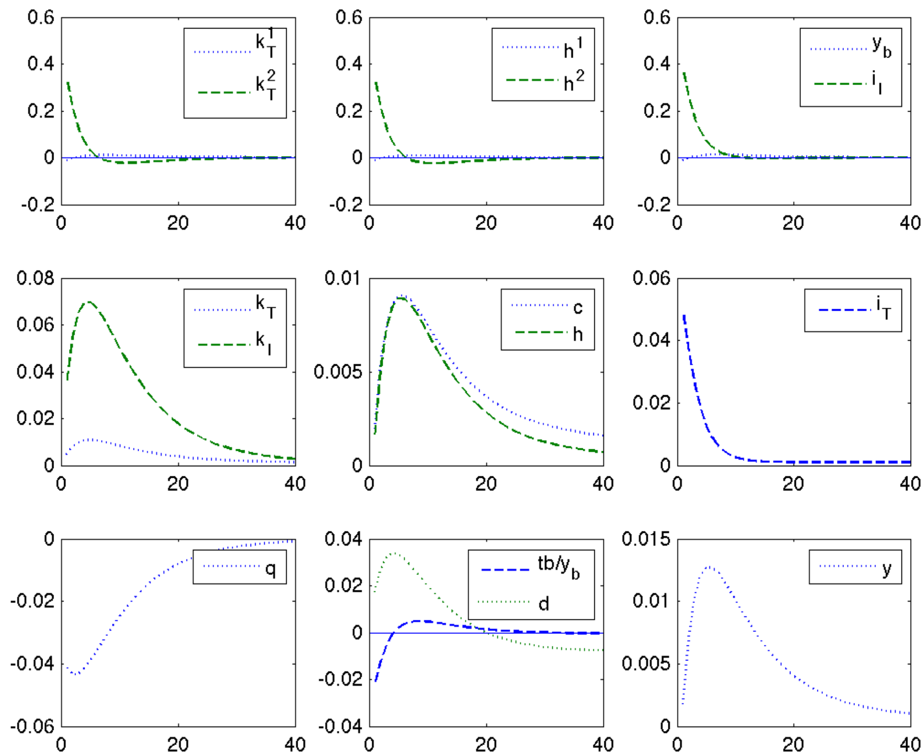


Figure 2.5: Response of TB/Y_b to A^1 and A^2 Shocks

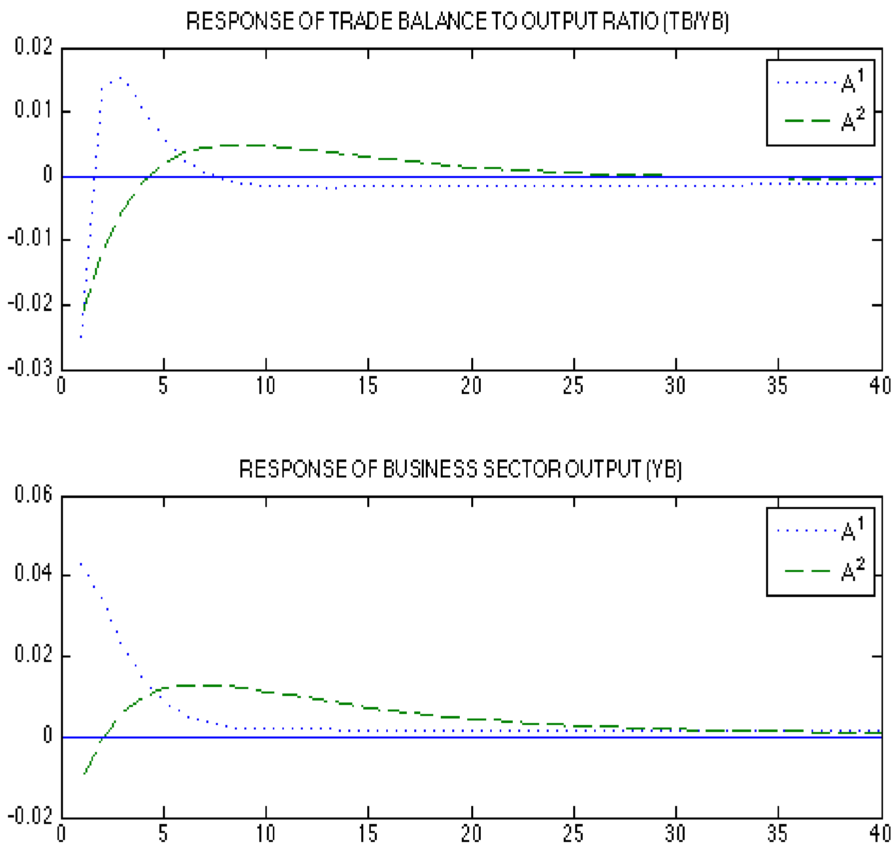


Figure 2.6: Response of Productivity to A^1 and A^2 Shocks

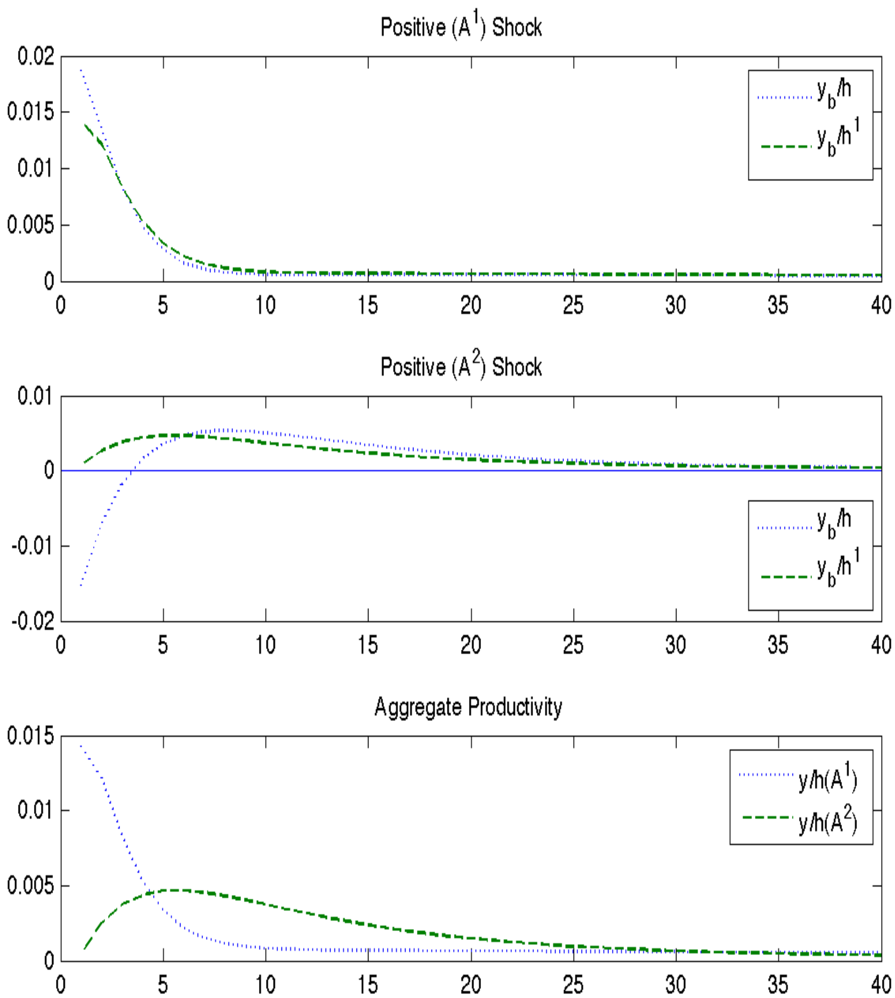
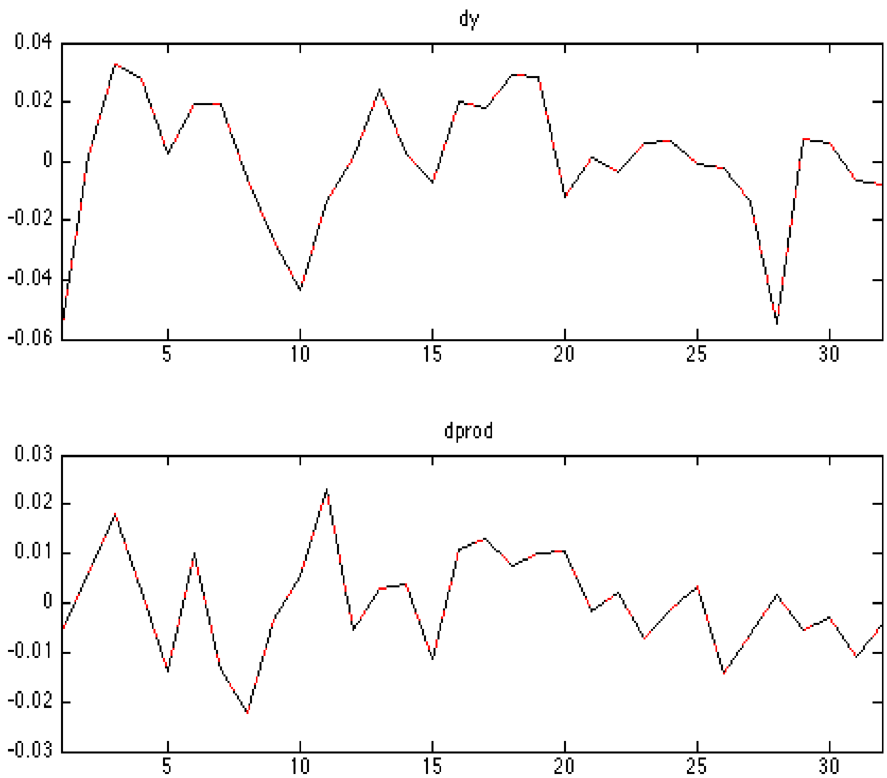


Figure 2.7: Historical and Smoothed Variables (Observables)



Chapter 3

Intangible Investment and International Business Cycles: The Rise of the Intangibles

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3.1 Introduction

Recently constructed data by Corrado, Haskel, Jona-Lasinio, and Iommi (2012) show that there has been a large average annual increase in the volume of intangible investment in the U.S. (6.6%) and Europe (5.3%) between 1995 and 2010. These growth rates are approximately 3 times higher than the average annual growth of GDP in the U.S. (2.4%) and Europe (1.6%) over the same time period. In relation to business cycles, the data reveals two key facts that have not been documented in the international real business cycle (IRBC) literature to date. The first fact is that there is a positive correlation between GDP and intangible investment within the U.S. and many European economies. The second fact is the presence of a positive international correlation in intangible investment between U.S. and Europe at the business cycle frequency. By construction, the standard IRBC models of Backus, Kehoe, and Kydland (1992, 1993) henceforth referred to as "BKK" and their offshoots are unable to produce these two facts.

The main contribution of this Chapter is to propose an extended IRBC model that includes investment in intangible capital. The main difference in the structure of the proposed model and the standard model is the introduction of a new input (intangible capital) and a new sector producing intangible investment. As recommended by McGrattan and Prescott (2010), the new sector features non-neutral technological change in the production of intangible investment. The Chapter also presents a detailed comparison of the dynamics of the proposed model and the standard IRBC model, where non-neutral technological change and the allocation of resources across sectors play a crucial role in explaining the dynamics of the proposed model.

A key finding is that the model with intangible investment (IRBCii) produces realistic movements in intangible investment. More specifically, the model produces a positive correlation between GDP and intangible investment and a positive cross-country correlation for intangible investment. Another key finding and contribution is that the IRBCii model produces significantly larger cross-country correlations in the main aggregates than does the standard model. On this important dimension, the IRBCii model matches the 1995-2014 data much better than the standard model.

The latter finding is especially important in light of two facts. Firstly, as documented in many IRBC papers such as Heathcote and Perri (2002), Baxter and Farr (2005) and Johri, Letendre, and Luo (2011), the standard model struggles to produce international correlations that are as high as the levels observed in the data. Secondly, as documented in this Chapter, the level of international correlations in the recent data are higher than in the data from earlier periods. These facts imply that there is now an even bigger mismatch between the predictions of the standard model and the data and further motivates the need to bridge the gap which the extended model is able to do.

This Chapter also contributes to the literature on the mismeasurement of productivity that arises when intangible investments are not properly accounted for. As noted by Corrado et al. (2012) and discussed in Chapter 1, this issue arises because a significant portion of intangible investments have been expensed rather than capitalized in the national income accounts of many countries. As a result, the reported measures of output and productivity underestimate their true measures which account for all intangible investments. McGrattan and Prescott (2010) show how this underestimation arises in the context of a closed economy RBC model. They find that modeling intangible investment as a production process with non-neutral technology improves the mapping between the model variables and the data which leads to an improvement in the model's measure of output and productivity.

In a more recent paper, McGrattan and Prescott (2014) call for a thorough investigation of their proposed theory on intangible capital and investments in the RBC literature. Since their proposed extension improved the performance of the standard closed economy model, will it improve the performance of the standard IRBC model? This is the context in which it is considered as an extension here. Using the IRBCii model that features non-neutral technology in the production of intangible investment, this Chapter finds that there is an underestimation in the measure of output and productivity differences across countries when intangible investments are not properly accounted for. The impulse response analysis shows that, following a productivity shock in the intangible investment sector of the home country, the traditional measure of labour productivity incorrectly implies that labour is more productive abroad than at home. This further demonstrates the importance of correctly accounting for intangible investment.

The remainder of this Chapter is structured as follows: Section 3.2 discusses the importance of intangible investment and the obstacles to analyzing its economic impact. Section 3.3 describes the data and stylized facts on international business cycles. Section 3.4 outlines the extended model and discusses the assignment of model parameters. Section 3.5 presents the results which explain the propagation mechanism of the model with intangible investment via a discussion of the impulse response functions, cross-country correlations and other relevant statistics. Section 3.6 presents the model's sensitivity to key parameters. Section 3.7 provides concluding remarks and suggested direction for future research.

3.2 Evidence of Intangible Investment Activity

This Chapter makes reference to two types of capital: tangible and intangible. To ensure consistency, the term "capital" will refer to physical capital such as machinery and equipment and the term "intangible capital" will refer to intangible capital. Similarly, investment and intangible investment will refer to tangible and intangible investment, respectively.

The major obstacle to a more thorough analysis of the economic impact of intangible capital and investments is the availability of data. There is currently no comprehensive data for intangible capital stocks and flows at the quarterly frequency.[1] However, there are two datasets worth considering. The first is the comprehensive annual data on intangible capital stock and investment that was recently constructed by Corrado et al. (2012) for the U.S. and select European countries for the period (1995-2010). The second dataset is the revised version of the U.S. national accounts (1980-2014) that was published by the Bureau of Economic Analysis (BEA). This data is of quarterly frequency but it is not comprehensive.[2]

[1] The majority of the IRBC literature is based on analysis at the quarterly frequency.

[2] There is also quarterly data for some European countries from the OECD, but these data are

This first dataset includes data on 10 types of intangible capital listed in Appendix 3.8.1. In this data, there is a positive correlation in intangible investments across countries (0.64 for the U.S. and EU), which is a relationship that the standard IRBC model cannot produce because there is no intangible investment in the model.[3]

In the second dataset, estimates of GDP were adjusted to include R&D, software and artistic originals as investments.[4] However, because these components are only a small fraction of the many types of intangible capital, this dataset is not the comprehensive dataset that is required in order to perform a thorough analysis. It is considered here to provide a second estimate of the volatility of intangible investment due to its relatively larger sample size in comparison to the first dataset.[5]

Figure 3.1 shows the share of tangible and intangible assets as a percentage of GDP for a group of developed economies as reported by the Organization for Economic Co-operation and Development (OECD). The figure reveals that intangible assets constitute a significant portion of total assets across the board, especially in Canada, U.S., Japan, Finland and Sweden. Figure 3.2 shows the change in intangible investment and GDP over time for the U.S. and EU aggregate.[6] The plot reveals that intangible investments have grown at a significant rate over time across countries. It is therefore important to account for this type of economic activity in the cross-country business cycle framework that has so far only featured investment in tangible capital.

also not comprehensive. They only account for a small fraction of intangible capital and investment.

[3]EU here refers to an aggregate of 9 European countries listed in the Appendix 3.8.1. Henceforth referred to as EU9. The data used to calculate the correlation is HP-filtered annual data.

[4]See McCulla, Holdren, and Smith (2013) for details on the revised U.S. national accounts.

[5]This dataset is considered for a robustness check on the volatility of intangible investment.

[6]The plot of intangible investment is based on the Corrado et al. (2012) annual dataset.

3.3 Data and Stylized Facts

Quarterly data on output, consumption, employment, investment, population and net exports for the U.S. and a Euro area aggregate of 19 countries constructed by the OECD (henceforth referred to as EU19) are used to calculate the summary statistics presented in this study.[7] All data are per capita and in constant 2010 U.S. Dollars and Euros, except as otherwise noted. As is standard practice in the literature, the data used to generate the summary statistics are logged (the net export to output ratio is not logged) and HP-filtered using Lambda=1600.[8] The quarterly data covers the period 1995:1 to 2014:4 and was sourced from the OECD's quarterly national accounts and the International Monetary Fund (IMF) International Financial Statistics (IFS) database.[9]

As stated earlier, this Chapter also makes use of annual data on intangible investment provided by Corrado et al. (2012) and R&D data from the BEA to characterize the properties of intangible investment. These data are also logged and HP-filtered using Lambda=6.25.[10] The Corrado et al. (2012) dataset is in constant 2010 U.S. Dollars and Euros. This data is used to calculate the volatility of intangible investment, its correlation with GDP and the cross-country correlation of intangible investment

[7]Note that the EU19 aggregate is different from the EU9 aggregate that is used for the analysis of intangible investment. The sample size for the analysis of intangible investment is smaller due to limited availability of data.

[8]The optimal smoothing parameter for quarterly data recommended by Ravn and Uhlig (2002).

[9]A detailed description of the data series and adjustments are present in the data appendix.

[10]The optimal smoothing parameter for annual data recommended by Ravn and Uhlig (2002).

between the U.S. and EU9 described in the data appendix. The BEA data is in constant 2010 U.S. dollars and it is used to provide a second estimate of the volatility of intangible investment. This statistic is calculated to form a basis of comparison for the volatility of intangible investment that was calculated from the relatively small sample size of 16 years using the Corrado et al. (2012) dataset.

The first column of Table 3.3 presents the summary statistics from the data referenced in this Chapter. The data properties reported are the average values from the U.S. and EU19 data except for terms of trade (TOT) which is for the U.S. only. As expected, consumption, hours and net export are less volatile than output, while investment and TOT are more volatile than output. The correlation of consumption, investment, hours and TOT with output are positive and the correlation of net exports with output is negative, as usual. The cross-country correlations of variables are all positive as expected and are discussed in greater detail in Section 3.3.1.

There are three new statistics reported which have not been documented in the IRBC literature to date. The volatility of intangible investment relative to output, which is greater than one, the correlation of intangible investment with output, which is positive and the international correlation of intangible investment, which is also positive. These are the key statistics that the IRBCii model is constructed to investigate.

3.3.1 Stylized Facts on International Business Cycles

Given that this Chapter uses recent data to generate the international business cycle statistics that are used to assess the performance of the IRBCii model, it is important to discuss the fit of these statistics with the stylized facts on international business cycles.

Ambler, Cardia, and Zimmermann (2004) test the robustness of the stylized facts on international business cycles that were established by Backus et al. (1993). Using a sample of 20 industrialized countries, Ambler et al. (2004) focused on the cross-country correlations of major aggregates - output, consumption, investment, employment and productivity. They show that the only robust stylized facts are that the cross-country correlations are positive, tend to be not very high and are of a similar order of magnitude. This is a result that most IRBC models fail to produce especially in relation to the international correlation of investment.

Comparing the statistics generated from the data used for this study with the statistics from the data used by BKK (1993) and Ambler et al. (2004) as shown in Table 3.1, a new result that redefines the quantity anomaly emerges.[11] The cross-country correlation of consumption is higher than output in the new data but lower than output in the old data. However, rather than focusing on the observed ranking of international correlations and declaring a new type of anomaly, this study follows

[11]The quantity anomaly refers to the inability of the standard IRBC model to generate a cross-country correlation in output that is higher than consumption which was found in the 1970:1-1990:2 data used by BKK (1993).

Ambler et al. (2004) and places more emphasis on the observed levels of correlations and the relative order of magnitude of the correlations.

The main finding in the recent data is thus that the international correlations are generally higher than previously reported and continue to be of a similar order of magnitude, which is consistent with the findings of Ambler et al. (2004). This is a result that the IRBCii model is able to produce. This Chapter also contributes to the IRBC stylized facts by documenting a positive international correlation in intangible investment.

3.4 Model with Intangible Investment

The proposed model extends the two-country BKK (1993) model to include intangible capital and investment in intangible capital that is produced using non-neutral technology.[12] In order to study the impact of modeling intangible capital and investment in this manner, the properties of the newly developed model are compared with the statistics from recent data. The impulse response functions generated by the model are also discussed to highlight the internal propagation mechanism of the model.

In the standard BKK (1993) model, there are two sectors: the intermediate good sector and final good sector. The proposed extension introduces an additional input to production (intangible capital), and a new sector that produces intangible investment into the standard model. This implies that in each country, there are now three

[12]The use of non-neutral technology is recommended by McGrattan and Prescott (2010).

immobile domestic inputs (capital, labour and intangible capital) and three goods produced in two sectors. Given the presence of a new good in the extended model, it helps to redefine the sectors as business and intangible in reference to the type of economic activity instead of the stage of production (intermediate and final). Accordingly, there are two goods in the business sector (one tradeable intermediate good and one non-tradeable final good) and one final good in the intangible sector that is not tradeable.

The assumption of symmetry between countries is preserved from the BKK (1993) setup. This assumption requires that both countries have the same number of identical households, intermediate good producers, and final good producers and that parameters are the same across both countries. This assumption of symmetry allows for the study of a representative agent within each country. In the discussion that follows, m is used to index countries, where $m \in \{h, f\}$.

3.4.1 Households

Households maximize the expected present value of their lifetime utility subject to a series of budget constraints. Households are also assumed to exhibit Greenwood, Hercowitz, and Huffman (1988) preferences - henceforth referred to as GHH preferences - where the households labour supply decision is not sensitive to changes in wealth (no income effect). Raffo (2008) recommends the use of GHH preferences in the IRBC setting to bring a model's dynamics of cross-country correlations and net exports closer to the observed dynamics in the data. Under GHH preferences, the utility function for the representative agent in country m takes on the following functional form:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_{m_t} - \frac{n_{m_t}^{\omega}}{\omega} \right)^{(1-\gamma)} - 1}{1-\gamma} \right], \qquad 0 < \beta < 1,\ \gamma > 1,\ \omega > 0$$

where c_{m_t} denotes consumption and n_{m_t} denotes labour supply. The household's flow budget constraint is:

$$c_{m_t} + i_{Tm_t} + q_{m_t} i_{Im_t} = r_{Tm_t} k_{Tm_t} + r_{Im_t} k_{Im_t} + w_{m_t} n_{m_t} \tag{3.1}$$

Each period, the household chooses consumption c_{m_t}, investment in capital i_{Tm_t}, intangible investment i_{Im_t} and how much labour to supply n_{m_t}. q_{m_t} is the relative price of intangible capital, r_{Tm_t} is the rental rate of capital, r_{Im_t} is the rental rate of intangible capital and w_{m_t} is the wage rate.

3.4.2 Intermediate Business Good Producers

The production of the intermediate business good in this setup combines capital ($k_{Tm_t}^1$), labour ($n_{m_t}^1$) and intangible capital (k_{Im_t}) as inputs to production assuming Cobb-Douglas technology with constant returns to scale.[13] The output of the intermediate business good is sold to both domestic and foreign final business good producers,

[13] The superscript 1 refers to the inputs used in the business sector.

who use it as an input to production. The representative intermediate good producer in each country maximizes its profits (3.2) subject to (3.3).

$$\pi_{m_t} = p^y_{m_t} y^b_{m_t} - r_{Tm_t} k^1_{Tm_t} - r_{Im_t} k_{Im_t} - w_{m_t} n^1_{m_t} \tag{3.2}$$

$$y^b_{m_t} = A^1_{m_t} (k^1_{Tm_t})^{\alpha_1} (k_{Im_t})^{\epsilon_1} (n^1_{m_t})^{(1-\alpha_1-\epsilon_1)} \tag{3.3}$$

The resource constraints for the intermediate business good in the home and foreign country are as follows:

$$y^b_{h_t} = a_{h_t} + a_{f_t} \tag{3.4}$$

$$y^b_{f_t} = b_{h_t} + b_{f_t} \tag{3.5}$$

The variables $k^1_{Tm_t}$ and $n^1_{m_t}$ are the share of capital and labour used in the production of the intermediate business good in each country. a_{h_t} (b_{h_t}) is the share of the home (foreign) intermediate business good sold to final business good producers in the home country and a_{f_t} (b_{f_t}) is the share of the home (foreign) intermediate business good sold to final business good producers in the foreign country. $p^y_{m_t}$ is the relative price of the intermediate business good in country m (relative to the numeraire final business good).

3.4.3 Final Business Good Producers

In both countries, the final business good is produced using a combination of domestic and foreign intermediate inputs. The production process is modeled using the Armington aggregator and producers choose a_{m_t} and b_{m_t} to maximize profit.

$$\pi^b_{h_t} = G(a_{h_t}, b_{h_t}) - p_{a_t} a_{h_t} - p_{b_t} b_{h_t} \tag{3.6}$$

$$\pi^b_{f_t} = G(b_{f_t}, a_{f_t}) - p^*_{a_t} a_{f_t} - p^*_{b_t} b_{f_t} \tag{3.7}$$

where

$$G(a_{h_t}, b_{h_t}) = ((\omega_y a_{h_t})^{\eta_y} + ((1-\omega_y) b_{h_t})^{\eta_y})^{1/\eta_y} \tag{3.8}$$

$$G(b_{f_t}, a_{f_t}) = ((\omega_y b_{f_t})^{\eta_y} + ((1-\omega_y) a_{f_t})^{\eta_y})^{1/\eta_y} \tag{3.9}$$

$$0 < \omega_y < 1, \ \eta_y > 0 \tag{3.10}$$

and where p_{a_t} and p_{b_t} are the relative prices of the home and foreign intermediate business goods in terms of the home numeraire good (final business output) whose price has been normalized to 1.[14] Other variables are as described in the intermediate business good producers section. $p^*_{a_t}$ and $p^*_{b_t}$ are the relative prices of the home and foreign intermediate business goods in terms of the foreign numeraire good.

[14] $p_{a_t} = p^y_{h_t}$ and $p^*_{b_t} = p^y_{f_t}$

The final business good is used to finance consumption and investment in physical capital and this is captured by the following resource constraints:

$$G(a_{h_t}, b_{h_t}) = c_{h_t} + i_{Th_t} \tag{3.11}$$

$$G(b_{f_t}, a_{f_t}) = c_{f_t} + i_{Tf_t} \tag{3.12}$$

3.4.4 Intangible Investment Producers

Intangible investment is produced in a manner similar to the intermediate business good. It is modeled using the Cobb-Douglas form with constant returns to scale that requires the combination of capital, labour and intangible capital. The sole purpose of intangible investment is to replenish the stock of intangible capital. The producers choose $k^2_{Tm_t}$, $n^2_{m_t}$ and k_{Im_t} to maximize profit (3.13) subject to (3.14).[15]

$$\pi^I_{m_t} = q_{m_t} i_{Im_t} - r_{Tm_t} k^2_{Tm_t} - r_{Im_t} k_{Im_t} - w_{m_t} n^2_{m_t} \tag{3.13}$$

$$i_{Im_t} = A^2_{m_t} (k^2_{Tm_t})^{\alpha_2} (k_{Im_t})^{\epsilon_2} (n^2_{m_t})^{(1-\alpha_2-\epsilon_2)} \tag{3.14}$$

where $k^2_{Tm_t}$ and $n^2_{m_t}$ are the share of capital and labour used in the production of intangible investment in each country. q_{m_t} is the relative price of intangible investment.

It is important to note the usage of the three main domestic inputs (capital,

[15]The superscript 2 refers to the inputs used in the intangible sector.

labour and intangible capital). In both countries, the stock of capital and labour are split between the production of the intermediate business good ($y^b_{m_t}$) and intangible investment(i_{Im_t}). This specification results in the addition of input allocation constraints (3.15) and (3.16) to the system of equations. Meanwhile, the stock of intangible capital is used simultaneously across both production processes in a non-rivalrous way - that is, its stock is not split across sectors.

$$k_{Tm_t} = k^1_{Tm_t} + k^2_{Tm_t} \tag{3.15}$$

$$n_{m_t} = n^1_{m_t} + n^2_{m_t} \tag{3.16}$$

The stock of capital and intangible capital are assumed to evolve as follows:

$$k_{Tm_{t+1}} = i_{Tm_t} + (1-\delta_T)k_{Tm_t} - \frac{\phi(K_{Tm_{t+1}} - K_{Tm_t})^2}{2} \tag{3.17}$$

$$k_{Im_{t+1}} = i_{Im_t} + (1-\delta_I)k_{Im_t} \tag{3.18}$$

Capital adjustment costs are included in the capital accumulation equation to reduce the volatility of investment and bring it in-line with the level observed in the data. The evolution of total factor productivity (TFP) in each sector is assumed to follow a stationary AR(1) process with no technology spillover across countries:

$$ln(A^1_{m_t}) = \rho^1_A ln(A^1_{m_{t-1}}) + e^1_{m_t} \tag{3.19}$$

$$ln(A^2_{m_t}) = \rho^2_A ln(A^2_{m_{t-1}}) + e^2_{m_t} \tag{3.20}$$

where $0 < \rho^i_A < 1$ and $e^i_{m_t}$ are innovations with $E(e^i_{m_t})$=0 and $var(e^i_{m_t})$=σ^2_e for i=1,2 and $m \in \{h, f\}$. Parameters ρ^1_A and ρ^2_A control the degree of persistence of TFP in each sector and do not vary across countries.

The shocks in both sectors within each country are further assumed to be related as follows:

$$e^1_{h_t} = u^{12}_{h_t} + u^1_{h_t} \tag{3.21}$$

$$e^2_{h_t} = u^{12}_{h_t} + u^2_{h_t} \tag{3.22}$$

$$e^1_{f_t} = u^{12}_{f_t} + u^1_{f_t} \tag{3.23}$$

$$e^2_{f_t} = u^{12}_{f_t} + u^2_{f_t} \tag{3.24}$$

where $u^{12}_{m_t}$, $u^1_{m_t}$ and $u^2_{m_t}$ are innovations with zero mean and constant variance σ^2_u. $u^{12}_{m_t}$ is the assumed common shock that causes co-movement in the business and intangible sector TFP within each country. $u^1_{m_t}$ and $u^2_{m_t}$ are idiosyncratic shocks that affect the level of productivity in the business and intangible sectors respectively. The idea of a common and idiosyncratic shock component is motivated by the observation that the detrended annual GDP and intangible investment series (shown in Figures 3.3 and 3.4) evolve in a similar manner. This suggests that there are similar forces driving them. The cross-country relationships between the shock components are discussed in Section 3.4.5. It might be helpful to think of the common shock as a federal policy initiative that leads to an improvement in the general economic climate within a country, which creates comovement in the TFPs, and to think of the idiosyncratic shocks as targeted policies that are intended to create sector specific improvements. The impact of the idiosyncratic shocks is discussed in Section 3.5. To solve the model, the social planner approach is adopted and the first order conditions are presented in Appendix 3.8.2.

To allow for the comparison of the model's predictions and the data, we need to define new variables. The terms of trade is defined as the relative price of the home country's imports in terms of its exports $TOT{=}(p_{b_t}/p_{a_t})$. The model's measure of GDP is $Y_m{=}(p^y_{m_t}\ y^b_{m_t} + q_{m_t}\ i_{Im_t})$. It accurately accounts for all the economic activity (tangible and intangible). However, this measure of GDP is not directly comparable to the measure of GDP in the data due to the absence of a significant portion of intangible investment in the data. Hence, for the remainder of this Chapter, the measure of GDP from the model that is comparable to the data is used, namely, $GDP_m{=}p^y_{m_t}\ y^b_{m_t}$ which is the output from the business sector. The net export to GDP ratio for the home country is defined as $NX = (a_{f_t} - (TOT)b_{h_t})/y^b_{h_t}$. This variable allows us to quantify the impact of various shocks on trade.

3.4.5 Parameterization

In order to solve the model, it is necessary to assign values to the parameters in the model. As stated earlier, there are three agents in the model economy and the discussion that follows is based on the parameters that need to be assigned to each agent. For the households, the parameters to be assigned are those present in the utility function, β, ω and γ. ω is the elasticity of labour supply and is assigned a value of 1.455 which is the same value used in Schmitt-Grohé and Uribe (2003); γ governs the risk aversion and the intertemporal elasticity of substitution and its value is set to 2, which is the standard value used in the literature. For the value of β, the households discount factor, it is standard practice to set it to a number that is close to 1. Here, it is set to 0.99 which is the same value used by BKK (1993).

The intermediate business good producers and intangible investment producers combine capital, labour and intangible capital using a Cobb-Douglas production function with constant returns to scale technology that requires values for α_1 and α_2 (the capital shares), ϵ_1 and ϵ_2 (the intangible capital shares) and (1-α_1 - ϵ_1) and (1- α_2 - ϵ_2) (the labour shares). Because of the relatively new way that intangible capital is modeled in this Chapter, there are not many estimates for the values of ϵ_1 and ϵ_2 in the literature. Following McGrattan and Prescott (2010) α_1 and α_2 are set to 0.26 while ϵ_1 and ϵ_2 are set to 0.067.[16] There are also not many estimates of the depreci-

[16]Koh, Santaeulalia-Llopis, and Zheng (2014) find that there has been a reduction in the U.S. labour share from 0.68 in 1947 to 0.60 in 2013 caused by an increase in the stock of intangible capital. The use of a lower labour share (0.60 vs 0.673) in favor of a higher intangible capital share

ation rate of intangible capital. There is however, a wide range of estimates for the depreciation rate of R&D. Li (2011) reports estimates of the depreciation rate of R&D that range between 12% for the pharmaceutical industry and 38% for IT hardware while Mead (2007) assumes an aggregate R&D depreciation rate of 15%. Similar to Mead (2007) this Chapter assumes an annual depreciation rate of 15% which implies a quarterly depreciation rate of $\delta_I = 3.75\%$.

The final business good producers combine domestic and foreign intermediate inputs using the Armington aggregator that requires values for ω_y, η_y and Im_y. η_y controls the elasticity of substitution between domestic and imported foreign intermediate inputs. In this case, η_y is set to 0.33 to achieve an elasticity of substitution (σ= -1/(η_y -1)) of 1.5. There is no real consensus in the literature on the value of this elasticity. For example, Raffo (2008) assumes an elasticity of 1.5 while Benigno and Thoenissen (2008) assume 2 and 0.5 for different sectors. This study examines the model's sensitivity to the value of η_y by considering a different value in the sensitivity analysis section below. Im_y is the assumed import ratio - the ratio of imported intermediate business good b_h (a_f) to the total domestic intermediate business good Y_h^b (Y_f^b) - that is required to solve for the steady state values of several variables in the model.[17] Im_y is set to 0.15 as in BKK (1993). ω_y is known as the "home bias" parameter and is generally assigned a value that is derived from the model's equilibrium conditions as in BKK (1993). Equation (3.25) shows how ω_y is calculated and

(0.14 vs 0.067) does not affect the main results of this Chapter.

[17]See BKK (1993) for details.

it depends on the values assigned to Im_y and η_y. 0.7616 is the implied value of ω_y based on the values assigned to Im_y and η_y.

$$\omega_y = \frac{((1 - Im_y)/Im_y)^{(1-\eta_y)}}{(1 + ((1 - Im_y)/Im_y)^{(1-\eta_y)})} \tag{3.25}$$

Exogenous Shock Processes

In the home (foreign) country, there are two productivity processes: one for the business sector A_h^1 (A_f^1) and the other for the intangible sector A_h^2 (A_f^2). These processes require persistence parameters (ρ_A^1 and ρ_A^2), standard deviations (σ_{e^1}, σ_{e^2}) and correlations for the disturbances. Since the disturbances are assumed to have a common and idiosyncratic component, additional parameters are required, namely, σ_{u^1}, σ_{u^2} and $\sigma_{u^{12}}$ for the standard deviations of each component and ρ_{u^1}=corr(u_h^1, u_f^1), ρ_{u^2}=corr(u_h^2, u_f^2) and $\rho_{u^{12}}$=corr(u_h^{12}, u_f^{12}) for the correlation of shocks across countries.

According to the theory of the model with intangible investment, proper estimation of the TFP series for both sectors in both countries requires estimating for $m \in \{h, f\}$:

$$ln(A_{m_t}^1) = ln(y_{m_t}^b) - \alpha_1 ln(k_{Tm_t}^1) - \epsilon_1 ln(k_{Im_t}) - (1 - \alpha_1 - \epsilon_1) ln(n_{m_t}^1) \tag{3.26}$$

$$ln(A_{m_t}^2) = ln(i_{Im_t}) - \alpha_2 ln(k_{Tm_t}^2) - \epsilon_1 ln(k_{Im_t}) - (1 - \alpha_1 - \epsilon_1) ln(n_{m_t}^2) \tag{3.27}$$

However, this estimation requires data on GDP, intangible investment, total hours and the stock of capital and intangible capital for the U.S. and EU19 or EU9 which is not available at the quarterly frequency. Hence, in order to assign parameters, this Chapter follows BKK (1992) and defines the aggregate TFP series as:

$$ln(TFP_{h_t}) = ln(GDP_{h_t}) - (1 - \alpha_1 - \epsilon_1) ln(n_{h_t}) \tag{3.28}$$

$$ln(TFP_{f_t}) = ln(GDP_{f_t}) - (1 - \alpha_1 - \epsilon_1) ln(n_{f_t}) \tag{3.29}$$

Using this method, we generate one TFP series for each country that is detrended and used to estimate the parameters governing the AR(1) process in each country shown in (3.30) and (3.31) with the U.S. as the home country.[18] Also following BKK (1992), this Chapter assumes symmetry across countries. Hence, the average value of the parameter estimates from each country is used as the single value in the symmetric set-up. This Chapter further assumes that the TFP series in both sectors evolve in a similar manner. Hence, the average estimates are applied to both business sector and intangible sector TFP.

$$ln(TFP_{h_t}) = 0.69 ln(TFP_{h_{t-1}}) + \hat{e}_{h_t} \tag{3.30}$$

$$ln(TFP_{f_t}) = 0.85 ln(TFP_{f_{t-1}}) + \hat{e}_{f_t} \tag{3.31}$$

[18] 2002:2 - 2014:1 data on GDP and employment for the U.S. and EU19 is used to calculate the TFP series.

where $\sigma_{\hat{e}_h}$=0.0049 , $\sigma_{\hat{e}_f}$=0.0055 and corr($\hat{e}_h$,$\hat{e}_f$)= 0.57.

Therefore, ρ_A^1=ρ_A^2= 0.77, σ_e=0.0052 and corr(e_h^1,e_f^1) = corr(e_h^2,e_f^2) = ρ_e = 0.57. The shock components are parameterized in a way that preserves the properties of the disturbances (e_h^1, e_h^2, e_f^1 and e_f^2) and the relationship between them. Setting $\rho_{u^1} = \rho_{u^2} = \rho_{u^{12}} = 0.57$ and $\sigma_{u^1} = \sigma_{u^2} = \sigma_{u^{12}} = 0.003677$ ensures that $\rho_e = 0.57$ and σ_e=0.0052.[19] The main results are not sensitive to moderate changes in the parameters assigned to the shock components as long as $\sigma_e = 0.0052$ and $\rho_e = 0.57$ are preserved. Table 3.2 contains the values that are assigned to all parameters in the model. The model's sensitivity to key parameters is tested in Section 3.6.

3.5 Results

In this section, we compare the dynamics of the model with intangible investment to those of the standard model. The standard model referred to here retains all the features of the model described in Section 3.4 except that it excludes intangible capital, intangible investment and the intangible sector.

3.5.1 Business Cycle Statistics

The last section of Table 3.3 displays the international correlations.[20] It is evident that the IRBCii model outperforms the standard model in its ability to produce positive

[19] The presence of a common shock creates a correlation of 0.5 between the level of productivity in the business sector and intangible sector.

[20] The properties reported for the standard model provide a better fit with the data than the properties of the original BKK (1993) model.

cross-country correlations that are of a similar order of magnitude especially in the international correlation of investment.[21] In the model with intangible investment, the level of international correlation in investment and intangible investment are slightly lower than the levels observed in the data.

It is important to note that the data properties of intangible investment are based on annual data whereas the properties of the model are reported at the quarterly frequency. This implies that the properties of intangible investment generated by the model are not directly comparable to the statistics from the data, while the properties of the other variables reported are directly comparable.

In the extended model, the predicted level of international correlation for both types of investment are also lower than the predicted international correlation for consumption, output and hours. While the IRBCii model's performance is not a perfect fit with the data, it represents a considerable improvement from the standard model. Hence, the fact that the extended model is able to generate high positive cross-country correlations (especially in investment, which is one of the major shortcomings of the standard IRBC model) is a significant achievement. Furthermore, the properties reported for the extended model constitute the first estimates of the properties of intangible investment derived from an IRBC model.

[21]Where parameters common to both models are assigned the same values. For the production share of the inputs, under the constant returns to scale assumption, there will always be a mismatch in the assumed production shares assigned across both models because there are only two inputs in the standard model whereas there are three inputs in the extended model.

International Correlation of Investment

A relevant question to ask at this point is why the model with intangible investment is able to deliver a level of international correlation in investment that is higher than the standard model predicts. To answer this, it helps to set the international correlation of all shocks in both models to zero. This allows us to see the baseline predictions of both models and highlight the main mechanism responsible for the higher level of correlation in the model with intangible investment.

When the international correlations of the shocks are set to zero, the standard model generates a negative international correlation in investment of -0.46 while the extended model generates a low positive correlation of 0.14. The reason the correlation is higher in the extended model is because of the additional decisions that are present. In the standard model, the main decision in response to a shock is the consumption - investment decision, whereas in the extended model, there are additional decisions, namely, the allocation of capital and labour across sectors as well as the intangible investment decision. The capital and labour allocation decisions are the main drivers of the positive international correlation of investment.

Negative International Correlation of Investment (Standard Model)

In the setup of the standard IRBC model, there is no intangible capital or intangible investment. There are only two goods: the intermediate business good y_h^b (that is traded) and the final business good $G(a_h, b_h)$ (that is not traded). In the standard model, because the stock of capital is predetermined, after a positive shock in the intermediate business good sector (A^1) of the home country, the only way agents in the home country can capitalize on the shock in the period in which it occurs is to increase labour supply and increase investment to build up the stock of capital. The combination of the increase in productivity and labour input causes y_h^b to increase, which in turn lowers its relative price because it is now relatively more abundant. As a result, the final business producers in both countries demand more of the home intermediate business good a. To increase $G(-)$ they also demand more of the foreign intermediate good b to combine with the relatively more abundant good a, but its supply is restricted. This is so because there is no change in the level of productivity in the foreign intermediate business sector. Hence, the incentive for the foreign agents to increase labour supply to produce more intermediate business goods is not as high as in the home country.

The cheaper price of good a creates a small positive income effect in the foreign country which causes the foreign households to increase consumption. With the use of GHH preferences, there is no decrease in labour supply, instead, there is a small increase in labour supply. This occurs in order to increase the production of the foreign intermediate business good which the foreign intermediate producers export more of to the home country because the price of good b is now relatively higher. This causes a decrease in the allocation of b to the foreign final good producers which in turn causes a decrease in the production of final business good in the foreign country, even though there is an increase in the supply of good a. This, combined with the increase in consumption, leads to a decrease in investment in the foreign country. In the home country, both investment and consumption increase because the home final good producers have access to more of good a and b which leads to a higher level of the home final business good. These opposite responses in investment cause the observed negative international correlation of investment in the standard model.

Positive International Correlation of Investment (IRBCii Model)

In the IRBCii model, the stock of capital and intangible capital are predetermined, but the allocation of capital and labour across sectors is not. Therefore, in addition to increasing total labour supply and investment, agents in the home country can also allocate more capital and labour to the more productive intermediate business sector in the period in which the shock occurs. This additional mechanism allows for a larger increase in y_h^b in comparison to the standard model. This in turn causes the relative price of the home intermediate good (p_a) to drop even further than it does in the standard model. In response to the lower price, there is an increase in demand of the home intermediate business good by the final business good producers in both countries (a_h and a_f increase).

In order to produce efficiently, the assumed Armington aggregator production function requires an optimal mix of inputs. This causes the home final business good producers to increase their demand for b_h which puts upward pressure on p^b. Similarly, the increase in the demand of a_f by foreign final business good producers in this case is such that they also find it optimal to increase their demand for b_f. This provides the additional incentive for foreign agents to also allocate more capital and labour to the business sector (even though there is no change in the level of foreign productivity) and increase y_f^b, which is split between home and foreign final business good producers. This leads to an increase in the output of the final business good in both countries, which leads to the observed increase in consumption and investment in both countries. This effectively explains why the model with intangible investment is able to generate a positive correlation in investment even when there is no correlation in the shocks to both countries. The impact of the described mechanism becomes magnified when the shocks are correlated.

Additionally, the dynamics in the benchmark model contain an extra incentive to increase the stock of capital: agents want to take full advantage of the high productivity level and therefore decide to increase intangible investment. This incentive is described in more detail in Section 3.5.3.

3.5.2 Other Statistics

So far, the fit of the model in terms of cross-country correlation has been discussed. In this section, the other statistics that determine the overall fit of the model are discussed. These are the volatilities of the major macro aggregates relative to business sector output (GDP) and their correlations with GDP.

The first section of Table 3.3 shows that the IRBCii model provides a good fit for the standard deviation of all variables except TOT and NX. The model understates the volatility of TOT and NX partly because of the high degree of substitution in intermediate business inputs that is assumed. The model's sensitivity to η_y is examined in Section 3.6. Even though the model generates the correct ranking of volatilities, the magnitude of the volatility of i_I relative to GDP is overstated. This finding motivated the test of the model's sensitivity to σ_{e^2} which determines the volatility of i_I in Section 3.6.

The second section of Table 3.3 illustrates that the model is able to generate the observed high level of correlation between the major macro aggregates and GDP. However, the correlations of NX and TOT with GDP are overstated and understated, respectively. These are both issues that carry over from the standard model and are well documented in the literature. The level of correlation of NX is significantly more negative in the current data than BKK (1993) reported (-0.51 vs -0.37) and the model is unable to match this new level of correlation. The model also produces a low positive level of contemporaneous correlation between GDP and i_I which is below the level of correlation in the data (0.21 vs 0.86).[22]

Overall, the model with intangible investment is able to make correct prediction about the positive correlation between GDP and i_I and the high positive cross-country correlation of intangible investment. These are two newly documented statistics that were generated from the Corrado et al. (2012) dataset which the standard IRBC model is unable to generate.

[22]The data figure 0.86 is based on annual data.

3.5.3 Impulse Response Functions

The discussion in this section highlights the mechanism by which the IRBCii model generates the observed correlations within and across countries. Figures 3.5 to 3.11 show the response of the major macro aggregates to an increase in productivity in the business and intangible sectors in the home country. The increase in productivity considered here is caused by a one standard deviation increase in the idiosyncratic components in each sector (u_h^1 and u_h^2). The impact of the common shock is not studied here because the common shock causes similar movements across sectors and the main objective of this section is to highlight the differences in the response to sector specific changes in productivity. In each plot, the variables are measured in percentage deviation from steady state values and the dotted line represents the response of the variable in the home country while the dashed line represents the response of the same variable in the foreign country.

Overall, even though there are differences in the magnitudes of response across countries, it is clear that in the case of an A^1 shock (a shock to the intermediate business sector), all variables respond in a similar manner across countries. This result is partly due to the cross-country correlation (0.57) in the shocks to business sector productivity. However, when there is an A^2 shock (a shock to the intangible sector), the result is different and is discussed in detail below.

Positive Shock to Productivity in the Business Sector

Figures 3.5 and 3.6 show the response of the variables in the IRBCii model economy to a one standard deviation increase in the idiosyncratic shock to the business sector (u_h^1) in the home country. The initial impact of the shock is an increase in the level of productivity and output of the intermediate business good producers in both countries.[23] In order to fully capitalize on the shock and achieve the observed increase in consumption and investment, both countries need to increase the output of the non-tradeable final business good that is used to fulfill the consumption and investment needs of households.

As a result, both home and foreign final good producers increase their demand for both domestic and foreign intermediate business inputs. The increase in demand is higher in the home country due to the relatively higher level of productivity. The higher level of productivity in the home country also implies that the home intermediate business good is relatively more abundant, or that the foreign intermediate good is relatively scarce. This is captured by the increase in TOT, which when combined with the changes in imports and exports leads to a decrease in NX.

In both countries, to achieve the observed increase in the intermediate business good output, labour demand (n) increases and resources are reallocated from the intangible sector to the more productive business sector. Hence, $k_{T_t}^1$ and n_t^1 increase while $k_{T_t}^2$ and n_t^2 decrease. This reduction of inputs to the intangible sector leads

[23] Impact in the foreign country occurs via the assumed correlation in shocks.

to the observed decrease in the level of intangible investment in both countries. The reallocation of inputs to the business sector does not last very long. In both countries, after the initial reallocation when the shock occurs, there is a subsequent reallocation of inputs to the intangible sector. This happens because agents understand that in order to fully capitalize on the shock and ensure higher levels of intermediate business goods as the shock wears off in future periods, the stock of intangible capital has to be built up as well. This happens at a faster rate in the home country because that is where the shock occurred and producers have the opportunity to capitalize earlier. The end result is an increase in i_{Ih} and i_{If} in future periods after the initial decrease.

It is also important to note that even though the initial impact of the shock is an increase in business sector output and decrease in intangible investment in both countries, the combined impact is an increase in the model's measure of GDP in both countries Y_h and Y_f.

Positive Shock to Productivity in the Intangible Sector

The response of the model economy to an A^2 shock (Figures 3.7 and 3.8) is quite different from the response to an A^1 shock. In this case, the shock occurs to the level of productivity in the sector producing intangible investment, which is not traded or directly consumed. The change occurs in the home country and the foreign country via the assumed correlation between A_h^2 and A_f^2. It is important to note that the responses in this case are generally smaller due to the relatively smaller size of the intangible investment sector and the relatively small share of intangible capital in the assumed Cobb-Douglas production functions.

In response to the A^2 shock, there is a reallocation of inputs to the relatively more productive intangible sector. k_T^2 and n^2 increase while k_T^1 and n^1 decrease. This causes the observed increase in intangible investment i_I and decrease in the output of the intermediate business good y^b. The decrease in y_h^b and y_f^b imply that there is now a lower level of intermediate business input that can be utilized in the production of the final business goods in both countries. Hence, $G(a_{h_t}, b_{h_t})$ and $G(b_{f_t}, a_{f_t})$ decrease, the constraint in equation (3.11) tightens and there are now less resources to be shared between consumption and investment. In the period the shock occurs, in an attempt to smooth consumption (in anticipation of the future increase in y^b), agents in the home country reduce consumption by less than investment and increase them as the shock wears off. The lower level of consumption lasts for only 2 periods in the home country.

Because the reduction in y_h^b is larger than the reduction in y_f^b, it is now relatively scarce and there is a reduction in TOT. Conventional wisdom dictates that a reduction in TOT is accompanied by an increase in NX, however this is not the case here. The decrease in TOT puts upward pressure on NX, but the larger decrease in y_h^b relative to y_f^b implies that there are now relatively less goods for the home country to export in comparison to the foreign country. Hence, exports decrease by more than imports. This quantity effect dominates the price effect and the net impact is a reduction in NX. The decrease in home TOT means there is an increase in foreign TOT

which causes the foreign final goods producers to decrease their demand for the home intermediate business good. This leads to a reduction in $G(b_{f_t}, a_{f_t})$ that causes the observed initial decrease in both C_f and I_{Tf}.

It is also important to highlight the impact of the shock on the data and true model measures of output in the home and foreign countries. GDP_h and GDP_f are the data measure and Y_h and Y_f are the true model measure. In both countries, there is a contraction in the business sector and this is accurately captured by the data measure of output which represents the measure of output that is used in practice due to the lack of data on intangible investment. However, when intangible investments are taken into account, the conclusion is different. Even though there is a contraction in the business sector in the home country, because of the increase in output in the intangible sector, the model's measure of total output (Y_h) increases. In the foreign country, the opposite occurs. The increase in the intangible sector output is not sufficient to make up for the contraction in the business sector and the net impact is a very slight reduction in the model's measure of total output (Y_f).

Note on Business Sector Labour Productivity

In this section, three measures of labour productivity from the model with intangible investment are used to investigate the bias that is induced from using inaccurate measures of business sector labour productivity. The measures are also used to examine the international co-movement of labour productivity. The first measure that is considered is the traditional data measure of business sector labour productivity (GDP/n) for which the model equivalent is $prod = p^y y^b/(n^1+n^2)$.[24] This is the measure that is often used in practice, and it is inaccurate because it attributes hours from the intangible sector (n^2) to the business sector. The second measure considered is the true measure of labour productivity in the business sector $prodb = p^y\ y^b/n^1$ and the last measure studied is the true measure of aggregate labour productivity $Y/n = (p^y\ y^b + q\ i_I)/(n^1+n^2)$. The impulse response plots in Figures 3.9 and 3.10 illustrate the pitfalls of using the wrong measure of productivity within each country.

As Figure 3.9 shows, after a positive A^1 shock, all measures of productivity in both countries are predicted to rise. The true measures of aggregate and business sector labour productivity respond in a similar manner because of the relatively small size of the intangible sector. The traditional measure of labour productivity overstates the increase in labour productivity in both countries because the increase in GDP is larger than the increase in Y. Overall, the risks associated with using the traditional measure are low as all measures of labour productivity increase and the use of the traditional measure slightly overstates the increase in labour productivity.

When there is an A^2 shock, the risks are much higher. As shown in Figure 3.10 all measures of labour productivity in the foreign country are predicted to decrease and the traditional measure overstates the decrease in comparison to the other true measures. In the home country, the traditional measure implies a decrease in labour productivity of around $-13\text{x}10^{-4}$ whereas the true measures shows an increase of

[24] This measure is used due to lack of data on intangible investment.

around 9x10^{-5}. This mismatch occurs because the business output decreases and intangible investment increases. Hence when intangible investment is not accounted for, there is a negative bias in the measure of labour productivity.

The model's predictions for business sector labour productivity within the home country are generally in agreement with the findings of McGrattan and Prescott (2010) and the facts reported by Corrado et al. (2012); Corrado, Hulten, and Sichel (2009). They reported that the growth rate of output per worker increases when intangible investments are properly accounted for, and the addition of intangible investment to the standard IRBC model introduces dynamics that help explain the increase in labour productivity when intangibles are properly accounted for.

International Differences in Output and Productivity

The measure of the difference in the level of output between countries is also affected by the presence of unmeasured investment. Figure 3.11 shows what happens to the measure of the difference in output across countries when the incomplete measure of output $(GDP_h - GDP_f)$ is used versus when the total measure that takes intangible investment into account is used $(Y_h - Y_f)$. It also shows the difference in the difference of the level of labour productivity across countries based on the measure of labour productivity considered $(GDP_h/n_h - GDP_f/n_f)$ vs $(Y_h/n_h - Y_f/n_f)$.

In the event of a positive shock to business sector productivity, the model predicts that using the incomplete measure of output to measure the difference in output across countries leads to a 0.002% overstatement in the percentage difference of output across countries in the period the shock occurs. There is also an overstatement of the difference in labour productivity of 0.012%. These errors are relatively small and do not cause much cause for concern in the short-run. However, when there is a positive shock to the intangible sector productivity, the margin of error is larger. The model predicts that the use of the incomplete measure leads to a 0.026% understatement in the percentage difference of output across countries in the period the shock occurs, and a 0.06% understatement in the measure of labour productivity difference between countries.

If we interpret the fact that we are in an era of increasing intangible investments as a series of positive shocks to the intangible investment sector, the predictions of the model lead to the conclusion that the use of the traditional measures of output and labour productivity instead of the true measures that include intangible investment results in the underestimation of the difference in output and productivity across countries.

3.6 Sensitivity Analysis

This section studies the sensitivity of the main results of the IRBCii model to different parameter values. The model's sensitivity to two parameters is tested and the results are presented in Table 3.4. The first parameter tested is η_y, the parameter that controls the elasticity of substitution between domestic and foreign intermediate inputs. This experiment is titled "low elasticity" and the value of η_y is reduced from

0.33 to -1 which generates an elasticity of 0.5 that is on the lower end of the values documented in the literature. For this experiment, the adjustment cost parameter is also changed to keep the relative volatility of investment equal to 3.1 (ϕ=0.75).

The lower degree of substitution leads to an increase in the home bias term ω_y which implies that each country now relies less on the intermediate goods of each other. The reduction in elasticity also leads to an increase in the volatilities of TOT and NX that constitute a better fit with the data. However, there is also an increase in the level of international correlation of hours and business output that makes them higher than the levels in the data. The lower elasticity does not lead to an improvement in the fit of the model with the recent data on international business cycles.

Another parameter of interest is the standard deviation of A^2 shocks σ_{e^2}. It was initially assumed to be equal to σ_{e^1} in the initial calibration and here σ_{u^2} is set to 0.0018 (about half the size of σ_{u^1}) to reduce the volatility of σ_{e^2}. This experiment is of interest because intangible investment is less volatile than investment in the data, but the model with intangible investment generates a higher level of volatility in intangible investment than investment. When σ_{e^2} is reduced, the main impact is a decrease in the volatility of I_I and an increase in the correlation between GDP and I_I from 0.21 to 0.38.

Overall, as depicted in Table 3.4, the main results from the IRBCii model are robust to a wide range of parameter values. In all cases, the relative standard deviation of intangible investment is greater than 1, the correlation of I_I and GDP is positive and the international correlation of intangible investment is positive. The main parameter that impacts the results is the level of correlation between the idiosyncratic business sector shock across countries, however, this value was assigned based on estimates from the data and not to generate any particular result. The only result that was targeted in the initial calibration is the relative volatility of investment at 3.1. It is therefore reassuring that the extended model is able to generate a level of correlation that is high and much closer to the levels observed in the data than the standard model predicts for a wide range of parameter values.

3.7 Conclusion

The extended model of this Chapter introduces a new input (intangible capital) and output (intangible investment) into the setup of the standard two-country business cycle model. This modification provides a better mapping between the two-country business cycle model and the recent data, which exhibits rising levels of intangible investment. The IRBCii model produces positive cross-country correlations in output, consumption, hours, tangible investment and intangible investment that are a better fit with the recent data than the predictions of the standard model.

The two key features of the extended model that enable it to outperform the standard model are the use of non-neutral technology in the production of intangible investment and the split of the stock of capital and labour across the business and intangible sectors. With the use of non-neutral technology, the evolution of intangible investment is not completely tied to the level of productivity in the business sector. This creates an environment where intangible investment can grow at a faster rate than business sector output which is in line with the data for U.S. and Europe. The split of the stock of capital and labour across sectors creates an allocation decision which plays a crucial role in the extended model's ability to generate a high level of international correlation in investment.

The IRBCii model predicts that the measures of output and labour productivity differences between countries are underestimated when technological change in the production of intangible investment is not fully accounted for. Hence, the main implication of this research is that intangible investments should be accounted for when conducting cross-country comparison analysis.

The main mechanism produces a positive correlation between business sector output and intangible investment, which is a novel result in the IRBC literature. The magnitude of the predicted positive correlation (at the quartely frequency) is roughly 25% of the level observed in the annual data. Hence, the accuracy of the prediction remains an open question until there is an equivalent measure that is based on quarterly data to form a basis of comparison. It is important to realize that this study abstracts from other typical shocks and frictions (like preference shocks and taxes) which can potentially account for the unexplained gap. Incorporating these shocks and frictions is however beyond the scope of this Chapter and is left for future research.

3.8 Appendix

3.8.1 Data Appendix

This study makes use of both quarterly and annual data for the U.S. and an aggregate of European countries. The EU19 consists of the following 19 European countries: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovak Republic, Slovenia and Spain.

The data was acquired from three main sources: Organization for Economic Cooperation and Development (OECD) quarterly national accounts, International Monetary Fund (IMF) International Financial Statistics (IFS) database and INTAN-invest. The quarterly (1995:1 - 2014:4) GDP, investment, consumption, net export and population data for the U.S. and EU19 was acquired from the OECD in nominal U.S. dollars/Euros and deflated using the OECD's quarterly deflator series.[25] The quarterly (2001:4 - 2014:3) employment data for the EU19 was acquired from the IMF and required the addition of Lithuania's employment data to the IMF's EU18 employment dataset to match the OECD's EU19 data.[26]

The population data for the EU19 and the employment data for the U.S. and EU19 were seasonally adjusted assuming an additive seasonal component. The seasonal

[25] All data except population for the EU19 were acquired in a seasonally adjusted format.

[26] EU18 data is available from 1999:1 - 2014:3, but Lithuania's data is only available from 2001:4 - 2014:3.

adjustment resulted in a reduced final sample of (1995:3 - 2014:2) for all variables except employment which is now from (2002:2 - 2014:2). The employment sample size for the EU19 restricted the final sample that was used to calculate the solow residuals to (2002:2 - 2014:2).

The terms of trade (TOT) defined as the ratio of the price of imports to the price of exports for the U.S. was constructed using the import and export quarterly (1995:3 - 2014:2) price indices acquired from the IMF IFS database.

For the analysis of intangible investment, intellectual property products and GDP data (2010 real dollars) for the U.S. from 1995:1 to 2014:4 was sourced from the BEA's revised national accounts. This data was used to calculate the relative volatility of IPP (2.26) that served as a robustness check for the relative volatility of intangible investment that was calculated from the annual US and EU9 data. The IPP data includes R&D, software and artistic originals. The comprehensive annual data on intangible capital and investment for the U.S. and aggregate of 9 European countries (Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Slovenia and Spain) was constructed by Corrado et al. (2012) and sourced from INTAN-invest. The data covers the time period 1995 to 2010 and was reported in nominal U.S. dollars/Euros and converted to 2010 real currency using the software deflator provided by Corrado et al. (2012). Because a set of the components of intangible investment are already included in the current measure of GDP in the national accounts (Software, Entertainment, Literary and Artistic Originals, and Mineral Explorations) these sums are subtracted from GDP and included in the measure of intangible investment not included (R&D, Design, New Financial Products, Advertising, Market Research, Training and Organisational Capital) to create a more accurate measure of aggregate intangible investment that is used during the analysis.

3.8.2 Equilibrium Conditions

Home Country First Order Conditions

Consumption

$$\lambda_{1h_t} = \left(c_{h_t} - \frac{n_{h_t}^{\omega}}{\omega}\right)^{(-\gamma)} \tag{3.32}$$

Optimal labour supply

$$n_{h_t}^{\omega-1} = \left(\frac{(1-\alpha_1-\epsilon_1)p_{h_t}^{y}y_{h_t}^{b}}{n_{h_t}^{1}}\right) \tag{3.33}$$

Optimal allocation of labour

$$\frac{(1-\alpha_2-\epsilon_2)q_t i_{Ih_t}}{n_{h_t}^{2}} = \frac{(1-\alpha_1-\epsilon_1)p_{h_t}^{y}y_{h_t}^{b}}{n_{h_t}^{1}} \tag{3.34}$$

Optimal level of capital

$$\lambda_{1h_t}(1+\phi(k_{Th_{t+1}}-k_{Th_t})) = E_t\beta\lambda_{1h_{t+1}}\left(\frac{\alpha_1 p_{h_t}^{y}y_{h_{t+1}}^{b}}{k_{Th_{t+1}}^{1}} + (1-\delta_T) + \phi(k_{Th_{t+2}}-k_{Th_{t+1}})\right) \tag{3.35}$$

Optimal allocation of capital

$$\frac{\alpha_2 q_t i_{Ih_t}}{k_{Th_t}^{2}} = \frac{\alpha_1 p_{h_t}^{y}y_{h_t}^{b}}{k_{Th_t}^{1}} \tag{3.36}$$

Optimal level of intangible capital

$$q_t\lambda_{1h_t} = E_t\beta\lambda_{1h_{t+1}}\left(\frac{\epsilon_1 p^y_{h_{t+1}} y^b_{h_{t+1}} + \epsilon_2 q_{t+1} i_{Ih_{t+1}}}{k_{Ih_{t+1}}} + q_{t+1}(1-\delta_I)\right) \tag{3.37}$$

Optimal allocation of a_{h_t}

$$G_1(a_{h_t}, b_{h_t}) = p^y_h \tag{3.38}$$

Optimal allocation of b_{h_t}

$$G_2(a_{h_t}, b_{h_t}) = p^y_f \tag{3.39}$$

where

$$G_1(a_{h_t}, b_{h_t}) = ((\omega_y a_{h_t})^{\eta_y} + ((1-\omega_y)b_{h_t})^{\eta_y})^{(1-\eta_y/\eta_y)}\omega_y a_{h_t}^{(\eta_y-1)} \tag{3.40}$$

$$G_2(a_{h_t}, b_{h_t}) = ((\omega_y a_{h_t})^{\eta_y} + ((1-\omega_y)b_{h_t})^{\eta_y})^{(1-\eta_y/\eta_y)}(1-\omega_y) b_{h_t}^{(\eta_y-1)} \tag{3.41}$$

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Table 3.1: Historical Comparison of Cross-Country Correlations

	Data (1995:3 - 2014:2)	Ambler et al. (2004) (1960:1-2000:4)	Backus et al. (1992) (1970:1-1990:2)
Corr(GDP_h,GDP_f)	0.70	0.48	0.66
Corr(C_h,C_f)	0.80	0.47	0.51
Corr(N_h,N_f)	0.71	0.65	0.33
Corr(I_{Th},I_{Tf})	0.74	0.49	0.53
Corr(I_{Ih},I_{If})*	0.64	-	-

* This moment is calculated based on annual data.

Table 3.2: IRBCii Model Parameters

	Parameter	Value
Utility	β	0.99
	ω	1.455
	γ	2
Technology	$\alpha_1 = \alpha_2$	0.26
	$\epsilon_1 = \epsilon_2$	0.067
	ω_y	0.7616
	η_y	0.33
Productivity	$\rho_1 = \rho_2$	0.77
	$\sigma_{e^1} = \sigma_{e^2}$	0.0052
	$\text{corr}(e_h^1, e_f^1)$	0.57
	$\text{corr}(e_h^2, e_f^2)$	0.57
Innovations	$\sigma_{u^1} = \sigma_{u^2} = \sigma_{u^{12}}$	0.003677
	$\rho_{u^1} = \rho_{u^2} = \rho_{u^{12}}$	0.57
Others	Im_y	0.15
	ϕ	1.4
	δ_T	0.025
	δ_I	0.0375

Table 3.3: IRBCii Model vs Standard Model

	Data	IRBCii	Standard
RelSD(C)	0.80	0.68	0.62
RelSD(I_T)	3.07	3.11	2.46
RelSD(N)	0.76	0.69	0.69
RelSD(TOT)	2.06	0.41	0.18
RelSD(NX)	0.36	0.04	0.17
RelSD(I_I)	> 1	3.58	-
Corr(GDP, C)	0.92	0.99	1.00
Corr(GDP, I_T)	0.94	0.98	0.97
Corr(GDP, N)	0.78	0.98	1.00
Corr(GDP, TOT)	0.56	0.36	0.28
Corr(GDP, NX)	-0.51	-0.24	-0.44
Corr(GDP, I_I)	> 0	0.21	-
Corr(GDP_h, GDP_f)	0.70	0.73	0.58
Corr(C_h, C_f)	0.80	0.73	0.60
Corr(N_h, N_f)	0.71	0.70	0.58
Corr(I_{Th}, I_{Tf})	0.74	0.66	0.13
Corr(I_{Ih}, I_{If})	> 0	0.52	-

RelSD(x) is the standard deviation of (x) relative to GDP (SD(x)/SD(GDP)). Corr(x, y) is the correlation between x and y. With the exception of the international correlations and the statistics related to I_I and TOT, the numbers in the "Data" column are the averages of the statistics from the U.S. and EU19 quarterly data (1995:3 - 2014:2). Annual data (1995-2010) for the US and EU9 is used to calculate the average statistics for RelSD(I_I)= 2.54 (2.26^{BEA}), Corr(GDP, I_I) = 0.86 and to calculate Corr(I_{Ih}, I_{If}) = 0.64. U.S. is the home country and EU19 is the foreign country. The superscript BEA refers to the moment that was generated using quarterly data (1995:1 - 2014:4) on U.S. intellectual property products from the Bureau of Economic Analysis.

Table 3.4: Sensitivity Analysis

	IRBCii	η_y = -1	σ_{u^2} = 0.0018
RelSD(C)	0.68	0.72	0.68
RelSD(I_T)	3.11	3.10	3.05
RelSD(N)	0.69	0.71	0.69
RelSD(TOT)	0.41	0.79	0.40
RelSD(NX)	0.04	0.13	0.04
RelSD(I_I)	3.58	3.84	2.70
Corr(GDP, C)	0.99	0.99	0.99
Corr(GDP, I_T)	0.98	0.96	0.99
Corr(GDP, N)	0.98	0.98	0.99
Corr(GDP, TOT)	0.36	0.24	0.36
Corr(GDP, NX)	-0.24	-0.26	-0.26
Corr(GDP, I_I)	0.21	0.35	0.38
Corr(GDP_h, GDP_f)	0.73	0.87	0.73
Corr(C_h, C_f)	0.73	0.78	0.73
Corr(N_h, N_f)	0.70	0.81	0.70
Corr(I_{Th}, I_{Tf})	0.66	0.62	0.65
Corr(I_{Ih}, I_{If})	0.52	0.46	0.49

Figure 3.1: Investment in Tangible and Intangible Assets as a Share of GDP (2006)

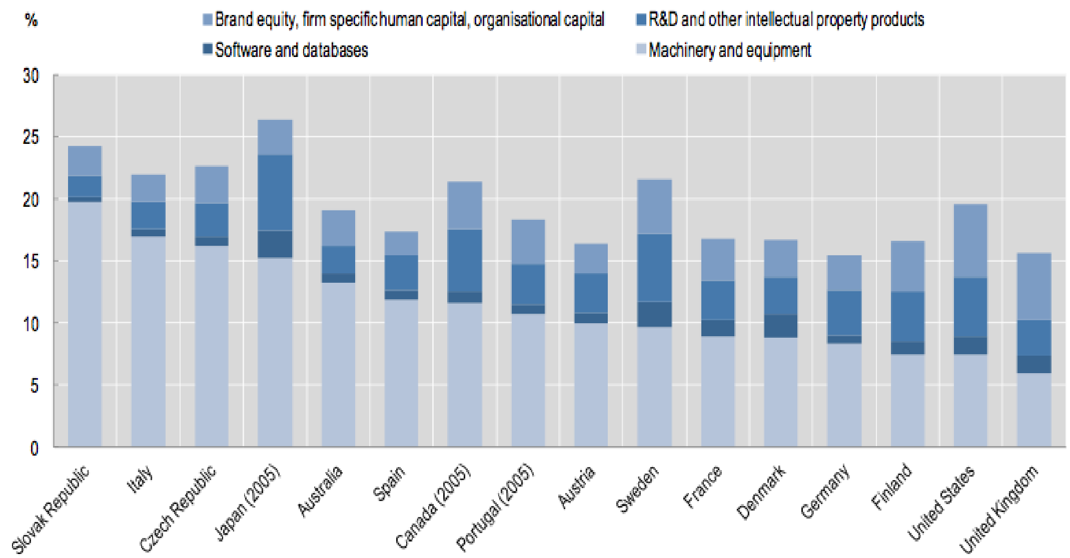


Figure 3.2: GDP and Intangible Investment in the U.S. and EU
(Annual, 1995=100, 1995-2010)

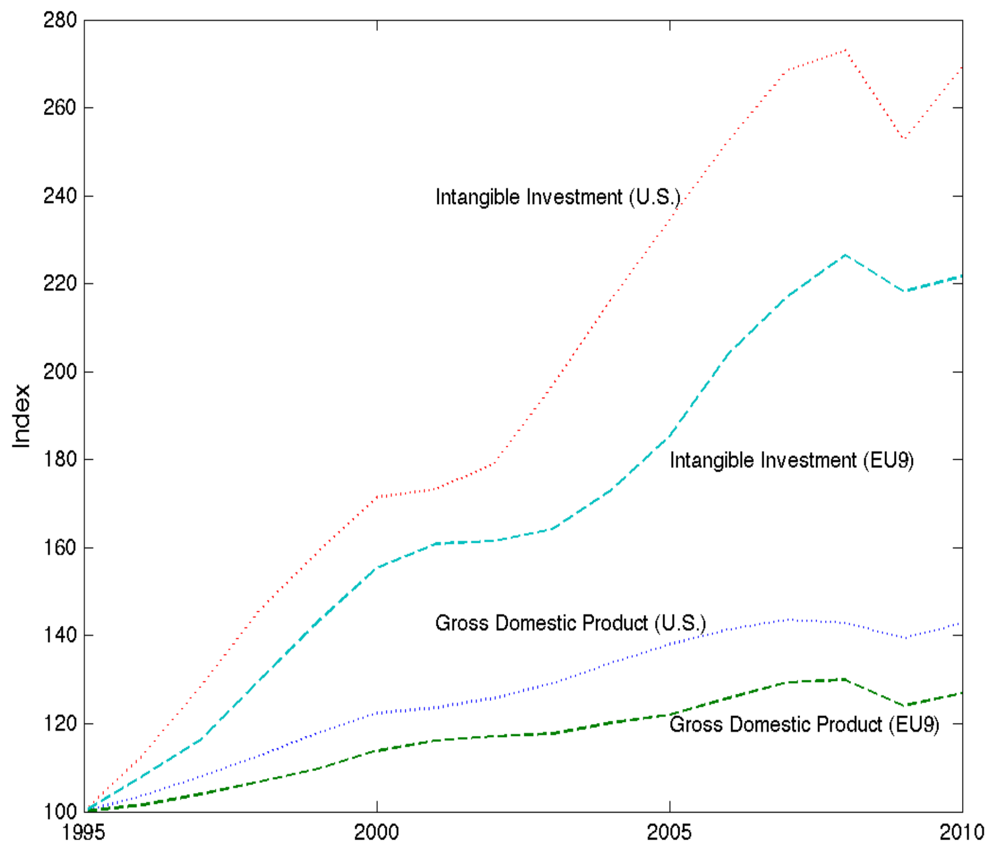


Figure 3.3: GDP and Intangible Investment in the U.S.
(Detrended Annual Data, 1995-2010)

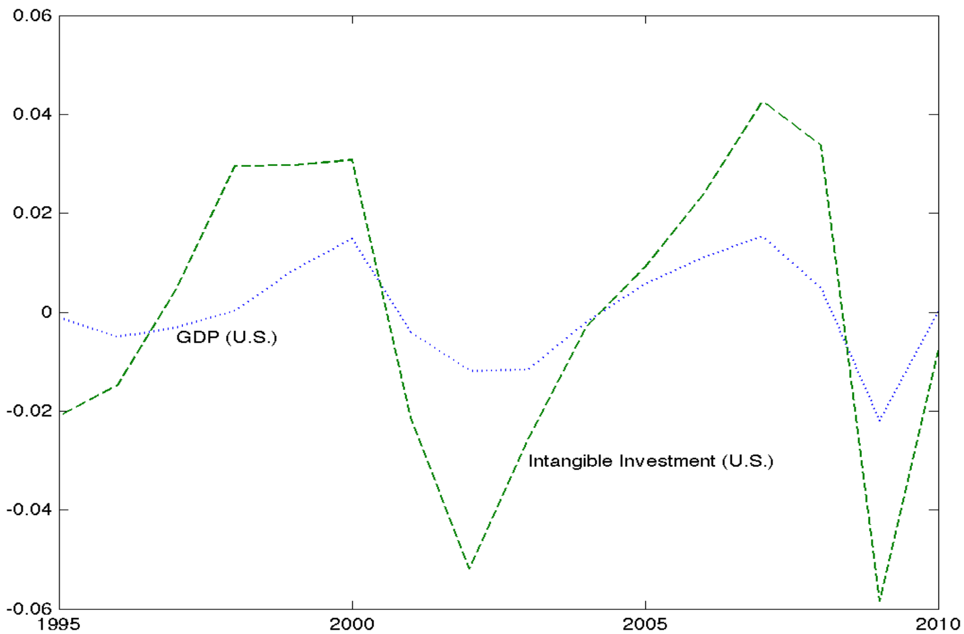


Figure 3.4: GDP and Intangible Investment in the EU9
(Detrended Annual Data, 1995-2010)

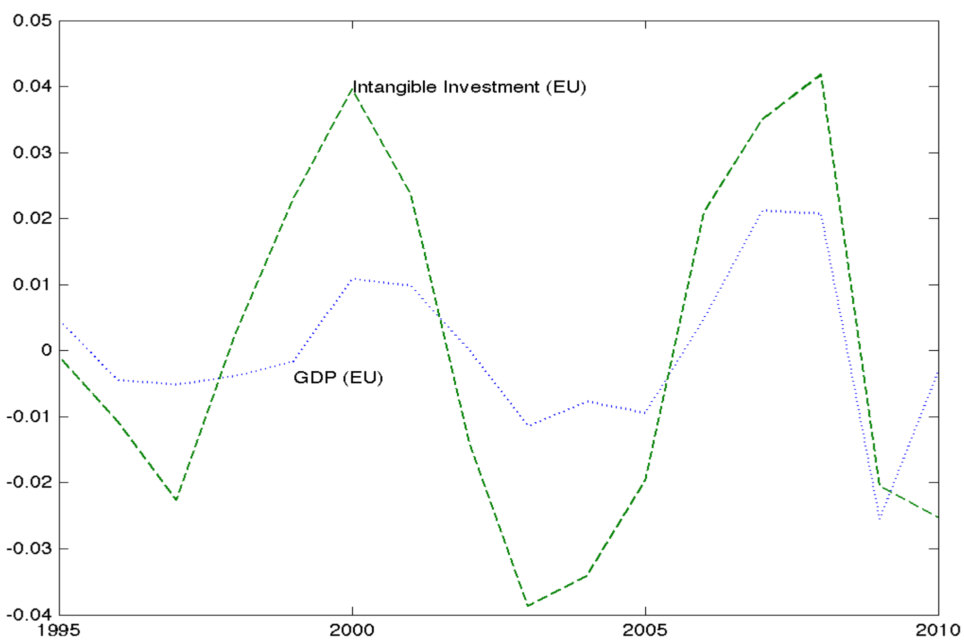


Figure 3.5: Positive A^1 shock (1 s.d) (IRBCii Model)

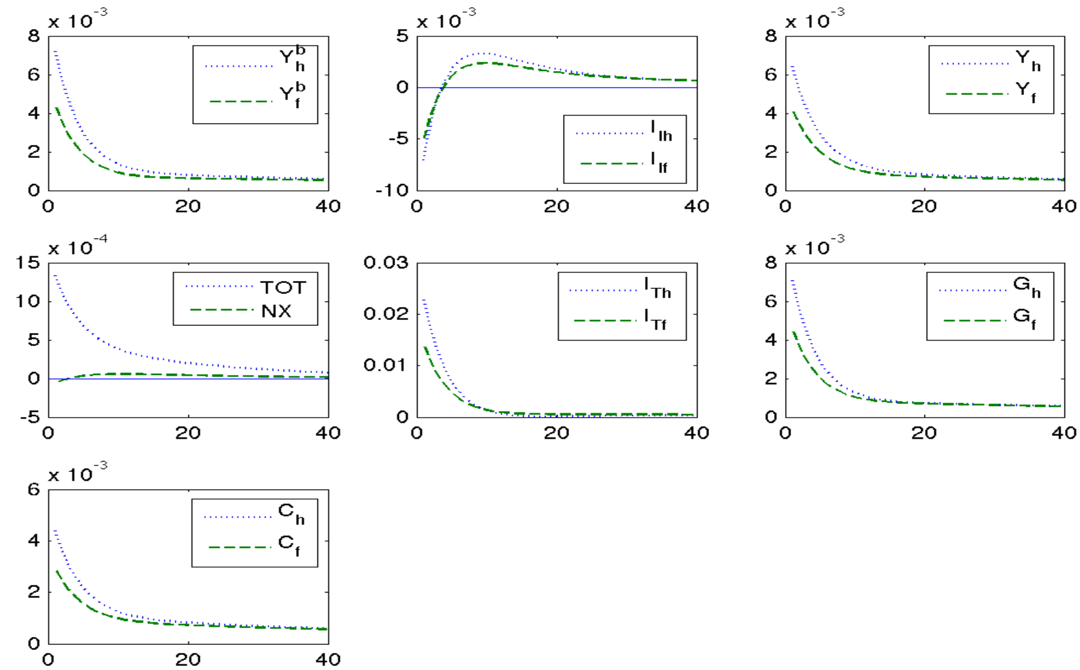


Figure 3.6: Resource Allocation: Positive A^1 shock (1 s.d) (IRBCii Model)

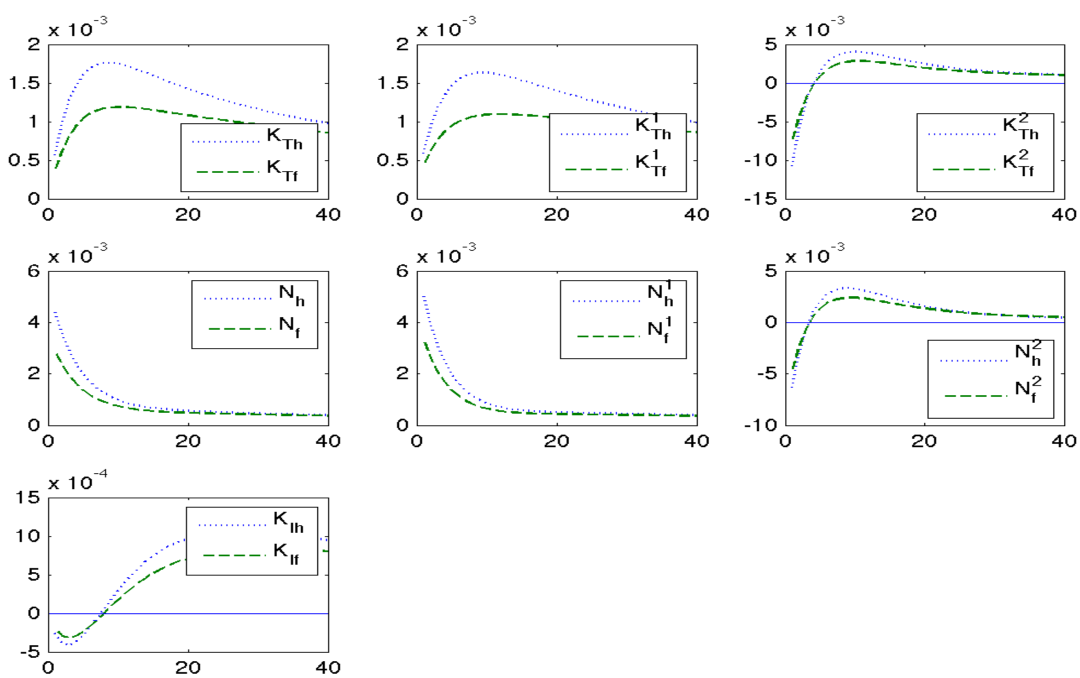


Figure 3.7: Positive A^2 shock (1 s.d) (IRBCii Model)

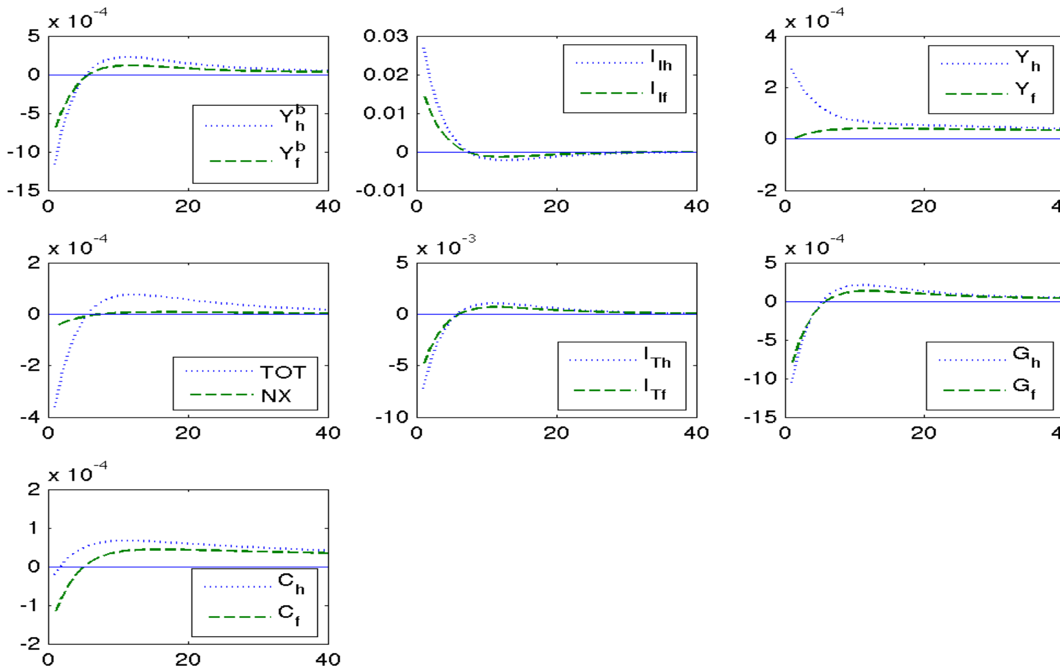


Figure 3.8: Resource Allocation: Positive A^2 shock (1 s.d) (IRBCii Model)

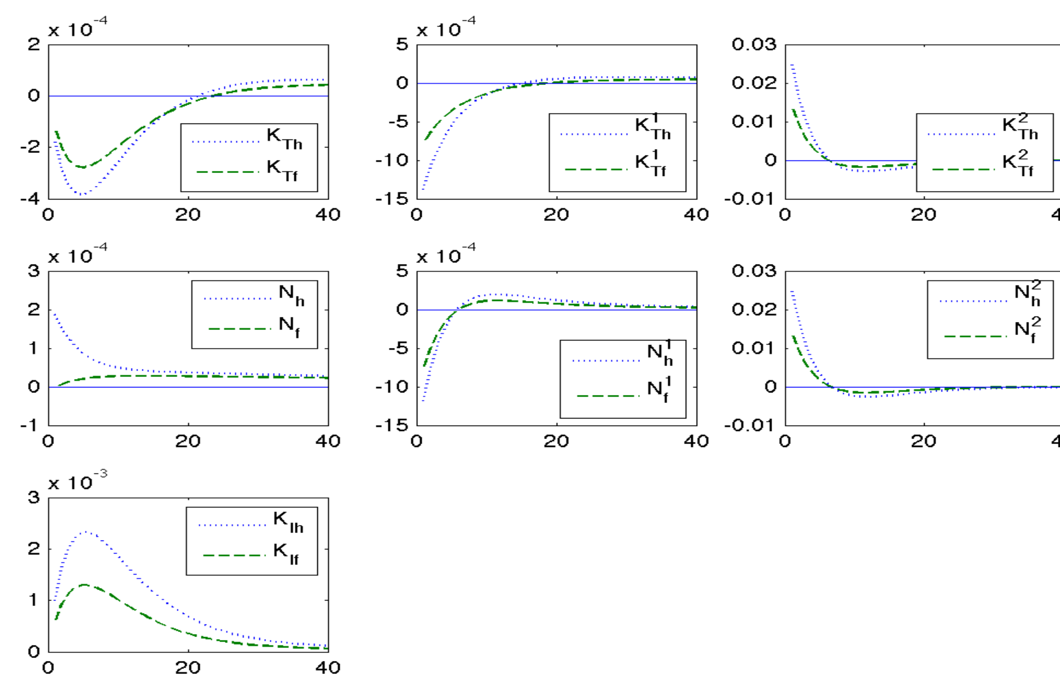


Figure 3.9: Measuring Labour Productivity: Positive A^1 shock (1 s.d) (IRBCii Model)

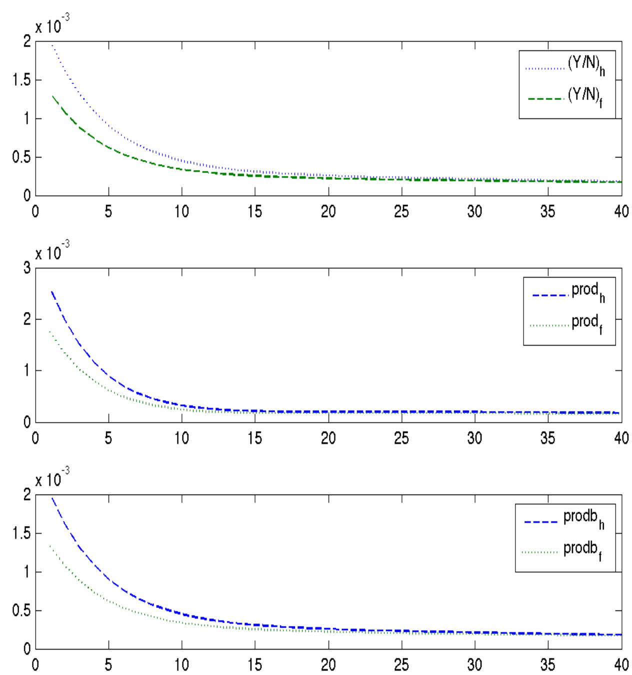


Figure 3.10: Measuring Labour Productivity: Positive A^2 shock (1 s.d) (IRBCii Model)

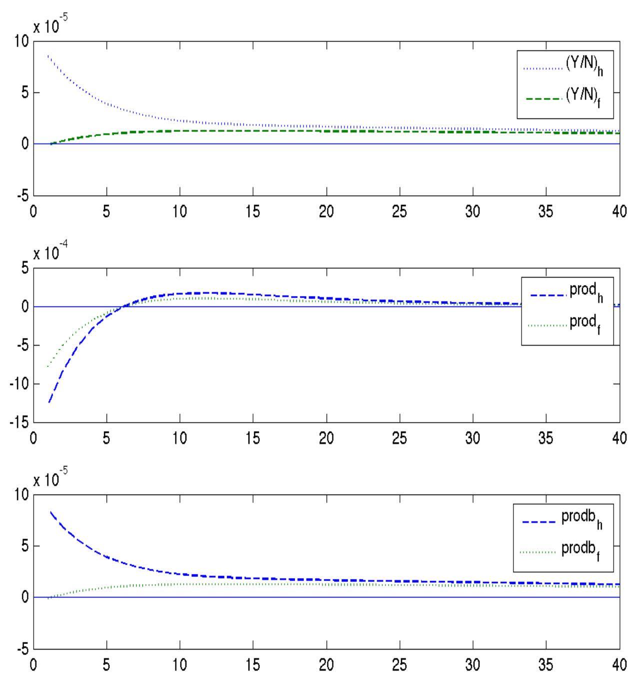
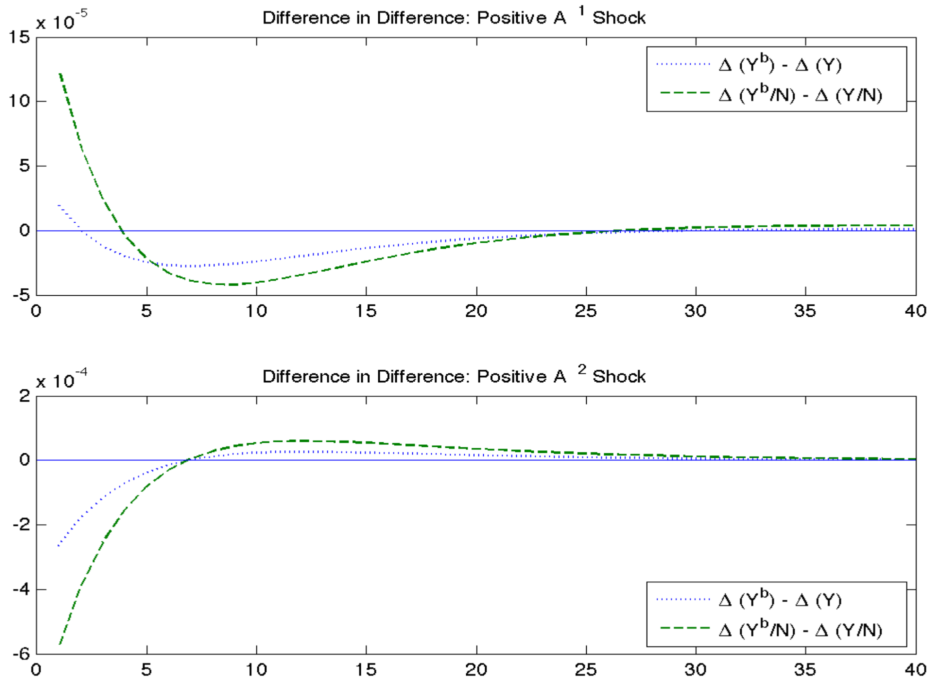


Figure 3.11: Output and Labour Productivity (IRBCii Model)



$\Delta(Y^b)$ is the difference in output across countries using Y^b as the measure of output in both countries. $\Delta(Y)$ is the difference in output across countries using Y as the measure of output in both countries. $\Delta(Y^b/N)$ is the difference in labour productivity across countries using Y^b/N as the measure of labour productivity in both countries. $\Delta(Y/N)$ is the difference in labour productivity across countries using Y/N as the measure of labour productivity in both countries.

Conclusion

With the data and analytical contributions of Corrado, Haskel, Jona-Lasinio, and Iommi (2012) and Corrado, Hulten, and Sichel (2005, 2009) the stage has been set for incorporating the intangibles into economic analysis. We now have the opportunity to conduct empirically-backed research that will improve the credibility of inference from a new class of models that feature intangible capital and intangible investment. In the business cycle setting, the contributions of McGrattan and Prescott (2010) set the stage for incorporating the intangibles into business cycle analysis by providing the theoretical foundations required to model them.

In Chapter 1, the obstacles that have prevented the proper measurement and modeling of the intangibles are presented. These challenges were encountered during the preparation of Chapters 2 and 3, and it is by addressing these issues that the inference from analysis involving the intangibles will become more credible. Some of the issues such as the lack of model specification testing are relatively easy to address, while others such as the lack of adequate price deflators require a longer time frame to address as we simply need to start keeping track of the prices for the various components of intangible capital going forward, especially because they were not documented in the past and a heavy reliance on guess-work is required to create historical price deflators.

The variance decomposition analysis in Chapter 2 shows that shocks to the level of productivity in the intangible sector explain a significant amount of variation in the main macroeconomic aggregates including labour productivity. This finding holds even after the traditional business sector productivity shocks are accounted for. This result is in line with the findings of Niebel, O'Mahony, and Saam (2013) who report that the intangibles play a key role in explaining sectoral labour productivity growth for many European countries. In addition, the impulse response analysis reveals that the intangibles play an important role in explaining business cycles in a small open economy.

Chapter 3 is an example of the type of progress that can be achieved when there is data on the intangibles (provided by Corrado et al. (2012)) to combine with the theoretical contributions of Backus, Kehoe, and Kydland (1992) and McGrattan and Prescott (2010). We now understand that total factor productivity shocks in the intangible sector play an important role in explaining international business cycles and we can now account for economic variables that, by construction, the traditional two-country model cannot accomodate.

There are some key issues that were identified but not addressed in this thesis. In relation to the measurement of the intangibles described in Chapter 1, the classification of some components of intangible capital as expenditure rather than capital was identified as a key issue that is left for future research. For the business cycle analysis in Chapters 2 and 3, the task of incorporating other shocks and frictions not present in this thesis is also left for future research.

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