

DESIGN AND ANALYSIS OF FREQUENCY MODULATED
FIBER-OPTIC COMMUNICATION SYSTEM

DESIGN AND ANALYSIS OF FREQUENCY MODULATED FIBER-
OPTIC COMMUNICATION SYSTEM

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Abstract

Despite the fact that frequency modulation (FM) was firstly applied to radio signaling 80 years ago (1936, by Edwin Howard Armstrong), it has never been deployed in fiber-optic communication systems. In this thesis, a novel frequency modulated fiber-optic communication system with optical discriminator is proposed. The noise configuration and anti-dispersion property of the FM system are investigated through an analytical model that has been derived and validated with numerical simulations. The performance of the proposed FM system is compared with an amplitude modulated (AM) fiber-optic communication system, owing to the fact that the widely used modulation formats, intensity modulation and quadrature amplitude modulation (QAM), can be extracted as a model of the basic AM format. Depending on the property of the filter, two types of frequency discriminators are discussed: the leading edge filter (LEF) and the tail edge filter (TEF). Since the amplified spontaneous emission (ASE) noise is averagely distributed without any frequency dependence, the noise characteristics are not affected by the choice of the frequency discriminator. However, when it comes to the dispersion

impairment, the difference between two frequency discriminators is dramatic because the distortion induced by dispersion strongly hinges on the operated frequency.

The results show that, with the presence of noise, the proposed FM scheme can lead to one or two orders of magnitude enhancement in the system's output signal-to-noise ratio (SNR) as compared to that of the conventional AM scheme. Also, with the presence of dispersion, it is proved that the span of the FM system can reliably reach 110km with bit rate up to 10Gbit/s, surpassing the AM system with a maximum signal reach of 70km. A real application, with the presence of both noise and dispersion, demonstrates the overall superiority of the FM system's performance over that of the AM system. The obtained results suggest a promising future for the FM technique in fiber-optic communication.

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Last but not least, I am also grateful to all my family members, friends, and lab mates in Photonics CAD Lab for their patience, support and unwavering encouragement.

Abbreviations

| | |
|------|--------------------------------|
| AM | Amplitude modulation |
| ASE | Amplified spontaneous emission |
| BER | Bit error rate |
| DCD | Duty cycle distortion |
| FM | Frequency modulation |
| FP | Fabry-Perot |
| FSR | Free spectral range |
| FTTH | Fiber-to-the-home |
| FTTx | Fiber-to-the-x |
| GVD | Group velocity dispersion |

| | |
|------|-------------------------------------|
| ISI | Inter symbol interference |
| LEF | Leading edge filter |
| MA | Mask area of an eye diagram |
| MZI | Mach-Zehnder interferometer |
| NF | Noise figure |
| NFI | FM over AM noise figure improvement |
| PON | Passive optical network |
| PRBS | Pseudo-random bit sequence |
| PSD | Power spectral density |
| PDF | Probability density function |
| QAM | Quadrature amplitude modulation |
| RIN | Relative intensity noise |
| RMS | Root mean square |
| SNR | Signal-to-noise ratio |
| TEF | Tail edge filter |

UI Unit interval

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Chapter 1

Introduction

1.1 Background

It is well known that the explosion of optical network capacity stimulates the growth of internet-based communications services as an underpinning of the innovative digital age nowadays [1]. The expansion of the fiber-optic communication technology largely depends on an accompanied price drop per bit of data transmission. Four critical factors are widely recognized as being responsible for the major cost reduction: increased bit rate within a single wavelength slot, increased number of wavelengths a single fiber can carry, increasing flexible optical wavelength routing, and increased optical transmission reach [2].

Especially, in datacom networks or access networks, there is a serious problem when very few clients share the cost of the network while a certain level of system performance should be guaranteed [3]. Hence, one of the major topics today is how to save the cost

within the optical communication network by realizing one or more of the abovementioned factors.

On the other hand, it is also important to specify the physical limitations of the practical fiber-optic communication system. If any of those limitations are resolved or at least weakened, an obvious progress would be made in the area. More specifically, in a fiber-optic communication system, limitations can be divided into two categories: linear and nonlinear. Linear impairments include fiber loss, chromatic dispersion and polarization mode dispersion (PMD). Optical power loss due to attenuation can be easily compensated by an optical amplifier, which will induce amplified spontaneous emission (ASE) noise. Dispersion is another major linear limitation for the optical communication system. Since in this work, the data rate is not extremely high and the transmission length is fairly low, PMD can be ignored. Nonlinear degrading effects induced by fiber nonlinearity are self-phase modulation (SPM), cross phase modulation (XPM), four-wave mixing (FWM), stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) [4]. However, in this thesis it is possible to ignore the adverse effect of nonlinearity as the focus lies on short-haul optic communication systems for the following reasons.

Datacom and telecom composes the modern communication application networks, where the former refers to the transmission network that allows computers to exchange data throughout a certain region such as a campus or a company, while the latter spans countries or bridges continents and can be categorized into core networks, switch networks and access networks. The rise in short-haul networks marks a clear shift since

2010 when datacom took up 40% of the market, and 40% in access networks, whereas only 20% was left in the backbone long-haul network - core network. In 2014, the deployment of the worldwide FTTx (Fiber-to-the-x), which is regarded as the ultimate last-mile component of a broadband architecture, passed 100 million connections and more than 100 cities are connected with gigabit-capable broadband networks mostly with FTTH (Fiber to the home, a subset of FTTX) technologies [5]. The rapid growth of these “last mile” short-haul technologies continues today, and have led to the proliferation of applications such as video streaming, the cloud computing, and the “Internet of Things”.

1.2 Motivation

While the majority of the researchers are focusing on quadrature amplitude modulation (QAM), polarization division multiplexing (PDM) or spacing division multiplexing (SDM), the potential of frequency modulation (FM) technology in the optical communication domain is usually out of scope.

On the basis of the Shannon-Hartley theorem $C = B \log(1 + S/N)$ [6], On theoretically the upper bound of net bit rate C that can be communicated with a specified bandwidth at an arbitrarily low error rate is decisive by the signal-to-noise ratio (SNR, S/N) over the bandwidth B . Therefore, wide band frequency modulation is capable of reaching high SNR as long as enough transmission bandwidth is guaranteed, providing an edge on the noise characteristics of fiber communication systems. It is feasible because the bandwidth of the fiber is ultra-wide so that usually in a fiber-optic communication system, only a small portion of the bandwidth around low loss band is utilized in short-

haul networks. In this way, the potential of the noise depression characteristics of the FM format impelled us to do further research on the FM scheme.

In addition, the potential of better dispersion performance of the FM scheme is also a crucial factor in determining the topic of my thesis. The main impairment of dispersion for the AM format is that in the time domain, broadening of a single bit pulse leads to overlapping with the neighboring symbols, which is inter symbol interference (ISI) [4]. Thus in the recovery circuit, the mistaken of bit “0” (or “1”) by bit “1” (“0”) will ensue. On the contrary, theoretically frequency modulation is capable of converting the factor of ISI - phase dispersion in the time domain, into amplitude distortion, largely depressing the extension of the single bit in the time domain. And the amplitude distortion can be easily reduced or even removed with the help of digital signal processing technology. In this way, the FM technique may satisfy the needs of reducing or eliminating the ISI phenomenon and at the same bit rate it may reach longer transmission distance comparing to the AM system.

The main hurdle of the FM technique is that the transmitting message is loaded on the frequency part of the optical signal, while the amplitude must be kept constant. When the FM signal reaches the end of the system, a proper receiver is needed in order to reconstruct the message signal accurately. The traditional approach requires extracting the full optical field information at the receiver - which, in turn, means that the receiver systems have to evolve from direct detection of the optical pulse intensity to other kinds of complicated and costly detections [2]. However, with the recent advent of the optical differentiator, it becomes possible to use the costless and robust direct detection technique

at the receiver to extract the message signal once the FM optical signal is converted to AM-FM signal by the optical differentiator. This optical differentiator should be highly linear over tens of GHz required, in order to avoid distorting the received message signal.

In this way, it is likely that the FM technique is applicable and can get the best of both worlds: better noise characteristics and improved anti-dispersion property. The growing demand in high-performance optical communication systems is the impelling factor that motivates us to discover whether the frequency modulation format in optical communication systems can bring any benefits over the traditional amplitude modulation format.

1.3 Thesis Organization

The remainder of the thesis is organized as follows:

In Chapter 2, the design outline of the FM communication system and the critical technique is provided, including details of the FM transmitter, the optical channel as well as the receiver with an explicit explanation for the optical differentiator.

Chapter 3 introduces the anti-noise performance of the FM system with the proper analytical model. An analytical model for the calculation of signal-to-noise ratio (SNR) for both FM and AM systems is obtained based on analog transmission channel. A parameter NFI (FM over AM noise figure improvement) is induced to intuitively show the difference of the noise figure between FM and AM systems. It can be proved that the anti-noise characteristics of the FM system significantly outweighs that of the AM system as long as it is governed by the wide band frequency modulation, at a cost of

enlarged bandwidth as the ratio of bandwidth of FM system is in scale of its frequency deviation. Numerical simulations are conducted to exemplify that the above argument is valid.

In Chapter 4, the effect of dispersion is studied as in short-haul access networks. Dispersive compensation methods are not widely used for their high cost and length dependency. After comparison between the two kinds of slope filters, the leading edge filter (LEF) and the tail edge filter (TEF), we decided to apply TEF as the optical discriminator in the FM system because it offers higher transmission distance and less pulse broadening for a single pulse. The simulation results show that the transmission distance limitation of the FM system can be much longer compared to the conventional AM system since the performance of the FM system is less sensitive to fiber dispersion.

A practical application of the proposed FM communication system is provided and analyzed within Chapter 5, showing the application prospect of the FM system. A 10GBps, 70km short-span access network is realized with the application of Corning SMF-28 optical fiber (ITU-T Recommendation G.652.D compliant optical fiber). Again, an AM system is used for comparison to exhibit the privilege of the novel FM system. The simulation results show that the BER of the FM system is below 10^{-4} , whereas the BER of the AM system is in the order of 10^{-1} , which means FM gives better performance than the AM system.

Conclusions and the future work are given in Chapter 6.

1.4 Contribution of The Thesis

Aiming at the challenges discussed above, the main contributions of this thesis are as follows:

Compared with the AM optical communication system, the optical FM communication system is less investigated. In this thesis, a novel frequency modulated optical communication system with optical discriminator is proposed. We mainly focused on the performance of the proposed optical FM communication system with the presence of noise and dispersion separately, and the evaluation of the novel FM scheme is based on the comparison with the AM optical communication system. To investigate the performances of the proposed FM communication system and the referenced AM communication system, analytical models and numerical simulations go hand in hand. The effect of each limitation on FM and AM systems is analyzed separately, by deriving the analytical model and validated by numerical simulations.

As for the optical frequency discriminator (slope filter), it is divided into two categories: the leading edge filter (LEF) and the tail edge filter (TEF), according to the slope of the magnitude response of the filter. As they have roughly the same anti-noise property, the tail edge filter is selected to apply in the FM communication system due to its more advantageous performance in the presence of fiber dispersion.

Chapter 2

Optical FM System Configuration

In this Chapter, we present the configuration of the optical FM communication system by analyzing the design process for it.

Basically, as is shown in Figure 2-1 [7], a typical fiber-optic communication system consists of an optical transmitter to load the message signal on the carrier optical signal to be launched, the fiber-optic channel to span the undersea or terrestrial distance that is needed and an optical receiver to extract the message signal from the received optical signal [8].

We would present the detail of transmitter, optical channel, and receiver of the FM system respectively in this chapter.

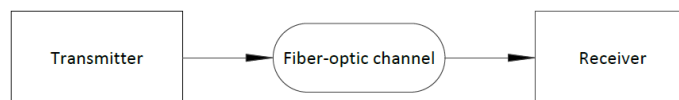


Figure 2-1 A typical fiber-optic communication system

2.1 Transmitter

The role of an optical transmitter is to convert the electrical signal into optical form and to launch the resulting optical signal into the optical fiber [4]. The transmitter in this FM system should be capable of loading the electrical message signal on the frequency of the carrier wave while the amplitude of the carrier wave maintains constant, which is the critical technological difficulty of this device. We would like to refer to the transmitter as frequency modulator in the rest of the thesis.

In frequency modulation [9], the instantaneous angular frequency of the carrier varies linearly with the message signal $f(t)$.

$$\omega_i(t) = \omega_c + k_f f(t), \quad (2.1)$$

where ω_c is the carrier angular frequency ($\omega_c = 2\pi f_c$), k_f indicates the frequency sensitivity of the modulator, or the frequency deviation, which represents the maximum shift away from the carrier frequency in one direction assuming the message signal is limited to the range (-1,1).

Hence, assuming the initial phase is φ_0 , the instantaneous phase becomes

$$\varphi_i(t) = \int_0^t \omega_i(\tau) d\tau + \varphi_0 = \omega_c t + k_f \int_0^t f(\tau) d\tau + \varphi_0. \quad (2.2)$$

The mathematical form of the frequency modulated optical signal is interpreted as

$$E_0(t) = A \cos[\omega_c t + k_f \int f(t) dt + \varphi_0], \quad (2.3)$$

or
$$E_0(t) = A \exp[j(\omega_c t + k_f \int f(t) dt + \varphi_0)]. \quad (2.4)$$

Constant A represents the envelope and the optical signal $E_0(t)$ is a non-linear function of the message signal $f(t)$.

If we denote the peak message amplitude as $|f(t)|_{\max}$, then the instantaneous angular frequency will always satisfy that [6]

$$\omega_c - k_f |f(t)|_{\max} < \omega_i(t) < \omega_c + k_f |f(t)|_{\max} . \quad (2.5)$$

Let's define that the maximum frequency deviation to be $\Delta\omega = k_f |f(t)|_{\max}$ and the bandwidth of the message signal is $2\omega_m$, then

$$m_f = \Delta\omega / \omega_m = k_f |f(t)|_{\max} / \omega_m \quad (2.6)$$

is named as deviation ratio (modulation index).

According to Carson's rule of thumb, the bandwidth of an FM signal can be approximately expressed as the following:

$$W = 2(\Delta\omega + 2\omega_m) = 2k_f |f(t)|_{\max} + 4\omega_m, \quad (2.7)$$

and the relationship can be summarized as: For narrow band FM, $m_f \ll 1$, $W = 4\omega_m$; for wide band FM, $m_f \gg 1$, $W = 2\Delta\omega = 2k_f |f(t)|_{\max}$.

When it comes to the message pulse shape, the raised-cosine pulse is commonly used in communications because of its wide shape in the time domain and compactness in the frequency spectrum. Pulse generator are used to generate raised cosine pulses to form the message signal in which bit "1" (bit"0") is represented by a pulse with amplitude 1 (0). The NRZ pulse is applied throughout this thesis because it is proved to have a narrower bandwidth than RZ pulse. Thus the message signal can be represented as

$$f(t) = \sum_m a_m h(t - m\tau), \quad (2.8)$$

a_m is 1 or 0 depending on the launched bit sequence, and $h(t)$ is the raised cosine pulse shape.

$$h(t) = \frac{E}{2} \left[1 + \cos\left(\frac{\pi t}{\tau}\right) \right], 0 \leq t \leq \tau. \quad (2.9)$$

E is the magnitude of the raised cosine pulse, and in this thesis we set it to be 1. τ draws boundary of the time t .

For example, when the pseudo bit sequence is [0,0,1,1,0,1,0,1], the message signal is plotted in Figure 2-2 and the envelope of the output FM optical signal after FM laser is shown in Figure 2-3, supposing the peak launch power is 0dBm (1mW).

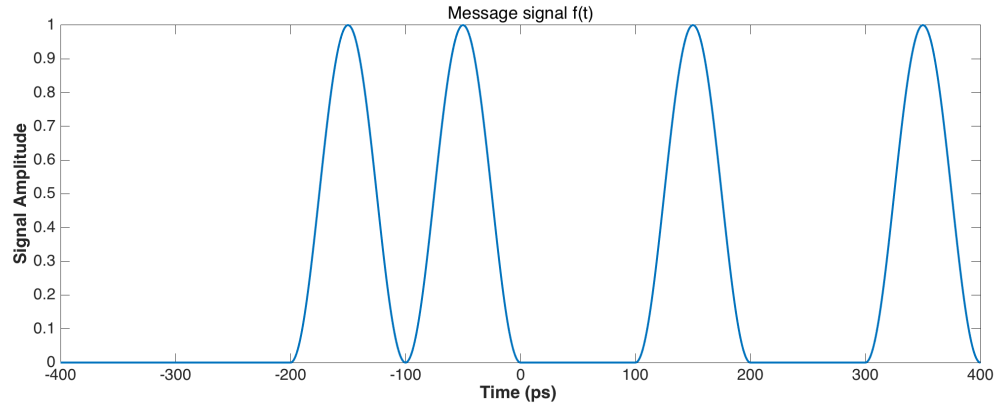


Figure 2-2 Message signal $f(t)$ when the pseudo bit sequence is [0,0,1,1,0,1,0,1].

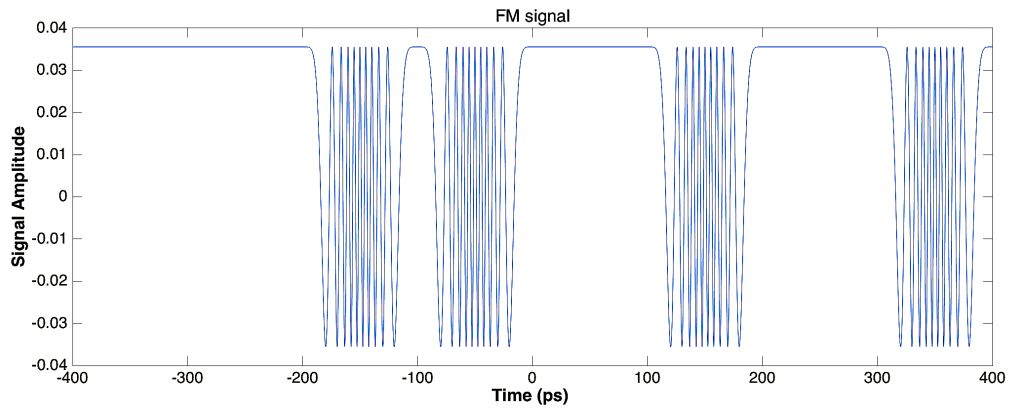


Figure 2-3 Envelope of FM signal when the pseudo bit sequence is [0,0,1,1,0,1,0,1] and the peak launch power is 0dBm.

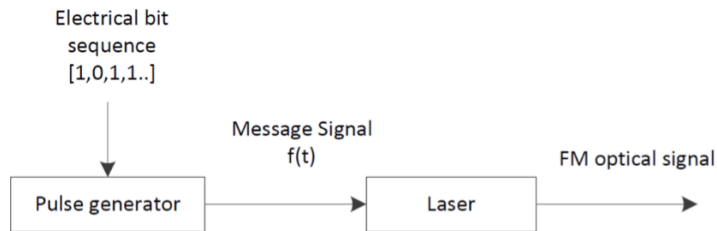


Figure 2-4 Internal Modulation

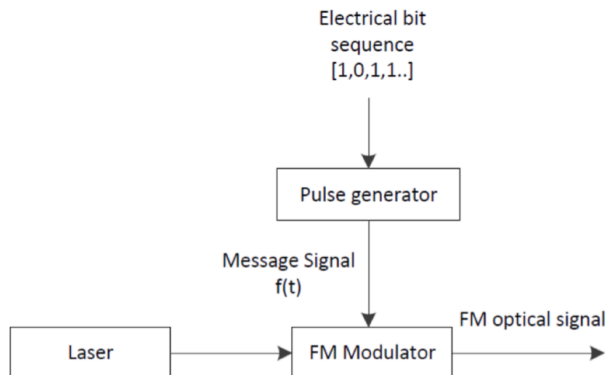


Figure 2-5 External Modulation

Figure 2-4 and Figure 2-5 introduces two modulation techniques: internal modulation (or direct modulation) and external modulation [10]. Either can be used in our optical FM communication configuration.

Figure 2-6 introduce a more practical way of frequency modulation [11], which is obtaining the integration of the message signal and loading it to the phase part of the optical carrier wave, utilizing the phase modulator.

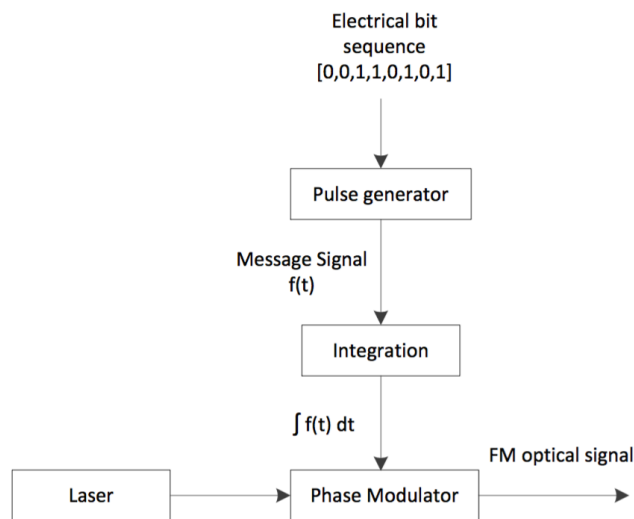


Figure 2-6 Indirect Frequency Modulation

In this thesis, we utilize wide band FM so that the parameters should satisfy that $\omega_c \gg k_f |f(t)|_{\max} \gg \omega_m$. This can also be written as $f_c \gg K_f |f(t)|_{\max} \gg f_m$ supposing the frequency deviation is $k_f = 2\pi K_f$. To be specific, in this thesis, we apply the 1550nm frequency modulated laser ($f_c = 193THz, \omega_c = 2\pi f_c$), the range of the frequency deviation for frequency modulation is (100GHz,300GHz) and the message is based on the raised

cosine pulse shape with 10Gbps bit rate so that $f_m = 20GHz, \omega_m = 2\pi f_m$, making sure that $f_c \gg K_f |f(t)|_{\max} \gg f_m$ is perfectly satisfied.

2.2 Optical Channel

The optical channel of the FM system is basically the optical fiber link spanned with the amplifiers periodically. Some related detail about the optical channel is stated as follows.

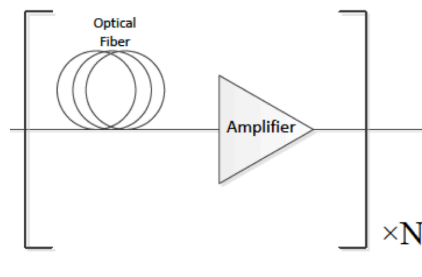


Figure 2-7 Optical fiber link with amplifiers spanned in it with equal distance

The detailed scheme of the optical fiber link is shown in Figure 2-7 [7]. The transmission fiber guides the optical signal to propagate over the distance, while the signal power degrades concomitantly with the presence of fiber loss. Demonstrated in Figure 2-8, the amplifier right after the fiber would compensate for the fiber loss in order to maintain the signal power above a certain level that is sufficient for the subsequent receiver to detect. However, ASE noise would be induced once the amplifier optical gain is high enough. In addition, the optical signal would also experience dispersion throughout the fiber link and this also leads to the degradation of the signal quality. The former structure would repeat itself for N times, which is decided by the gain of the amplifiers and the overall reach of the transmission system. Apart from the loss, optical

fiber would also induce dispersion, and the total transfer function of the fiber is $H_f(\Omega, z) = \exp[-\alpha z - i(\beta_1\Omega + \frac{1}{2}\beta_2\Omega^2)z]$. In this thesis we utilize a single-mode fiber that can only support one guided mode [7], which is regarded as a typical fiber of optical communication systems.

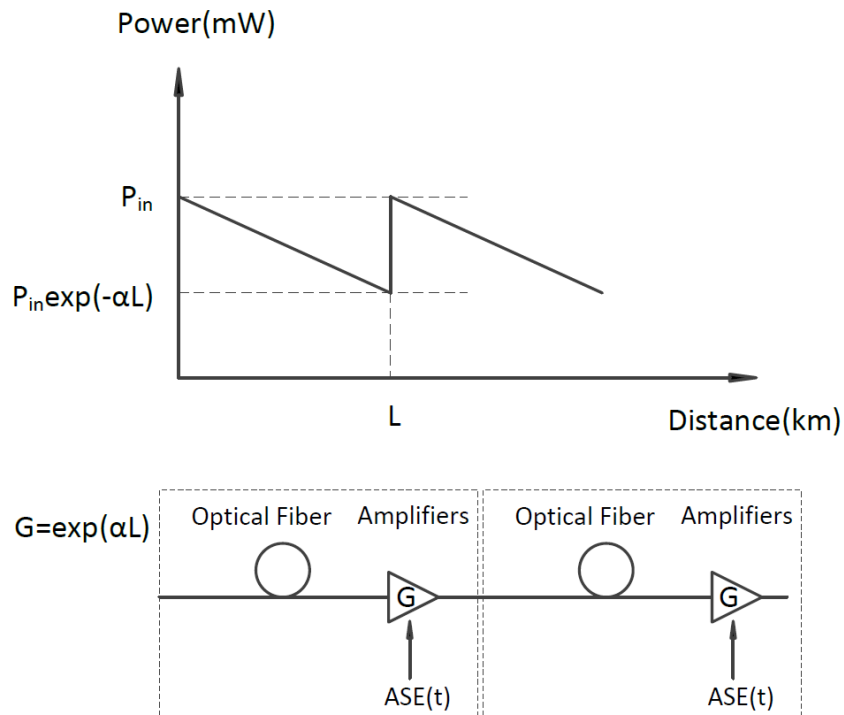


Figure 2-8 Power analysis of optical fiber link

After the transmission link, an optical band pass filter may be useful to reduce the noise before the optical signal is launched into the subsequent devices.

2.3 Receiver

A typical direct detection receiver is composed of three parts: front end, linear channel and data recovery, and the detail has been depicted in Figure 2-9 [12]. In this thesis, we simplify the model by ignoring the pre-amplifier at the front end, the high-gain amplifier in linear channel as well as the data recovery system, since we ignore the noise caused by the receiver and only focus on the shape of the analog signals. In addition, in order to detect frequency-modulated signal, an optical discriminator is essential.

The receiver used in this thesis consists of an optical differentiator, a photo detector followed by an electrical low pass filter as shown in Figure 2-10. The optical differentiator plays a role in converting FM signals into AM signals, the detail of which will be discussed in the sub sections. The photo-detector serves as a linear optical to electrical converter and the output current should be directly proportional to the input optical power [13]. Taking cost and performance into account, a photo diode would be a suitable choice for the photo-detector [14]. The electrical low pass filter plays a role of reserving the message signal and filtering out the unwanted noise outside the frequency band of the message signal [15]

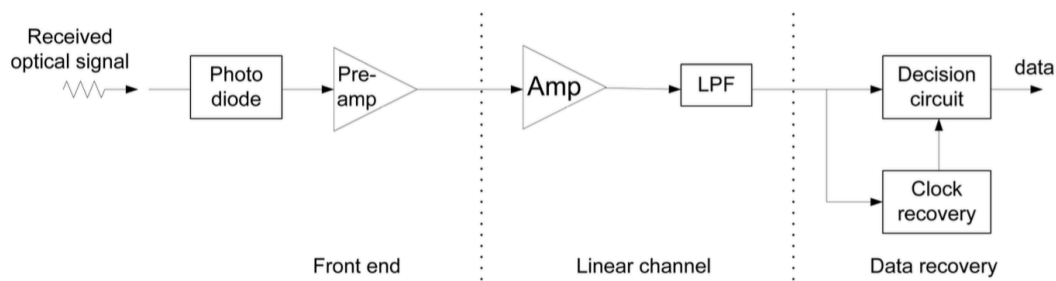


Figure 2-9 Block diagram of a direct detection receiver

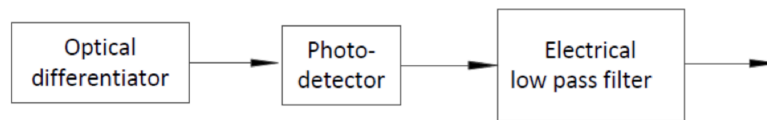


Figure 2-10 Block diagram of the receiver used in this thesis

2.3.1. Optical Differentiator (Slope Filter)

Previously, one of the major technology hurdles of FM systems is the lack of suitable photo-detectors that can directly extract and demodulate a message from the frequency component. But using optical frequency discriminators to convert FM to AM becomes a possible solution because the converted signal can be detected with the conventional receiver.

Hypothetically, the transfer function of the discriminator is ideally linear, which corresponds to a differential operation in the time domain according to the property of the Fourier transform, i.e.

$$\alpha\omega \leftrightarrow -i\alpha \frac{d}{dt}. \quad (2.10)$$

This also explains the reason that the optical discriminator is also named as slope filter, or differentiator.

Therefore, suppose the input of the differentiator is the frequency-modulated signal:

$$E_{in}(t) = A \exp[i(\omega_c t + k_f \int f(t) dt + \varphi_i)], \quad (2.11)$$

the output signal would be expressed as

$$E_{out}(t) = A\alpha(\omega_c + k_f f(t)) \exp[i(\omega_c t + k_f \int f(t) dt + \varphi_i)]. \quad (2.12)$$

Strictly speaking, it is an AM-FM signal, but we would simplify it to be AM signal owing to the fact that the message signal $f(t)$ can be directly extracted from the amplitude of the output signal with the help of the direct photo-detector.

The optical differentiator can implement the differentiation of ultrafast optical signals in the optical domain directly without electronic processing [16]. It needs to be highly linear to avoid introducing spurious distortion and higher harmonics within the operation band of about several hundred GHz required by an FM signal [17]. Numerous techniques have been proposed to implement optical temporal differentiation, including the use of a phase shifted fiber bragg grating [18], the silicon-on-insulator micro-ring optical filter [19] [20], as well as a conventional F-P [21] or M-Z interferometer [22].

In order to guarantee the transmission system operates normally, the slope filter must satisfy the following conditions: 1) The linear region of the slope filter should be large enough that it can cover the entire bandwidth of the frequency modulated signal, which means it is expected to have a bandwidth of 100-300GHz, decided by the frequency

modulation index (k_f). Along with the free spectral range (FSR) of the amplitude transfer function, the shape of it is also a decisive factor to the linear region. 2) The Notch of the optical differentiator that denotes the lowest value of the amplitude transfer function should be low enough to ensure that the message from the input FM signal can be extracted accurately. In this thesis we would choose the differentiator with the notch below -10dB.

We tried two categories of frequency differentiators, leading edge filter (LEF) and tail edge filter (TEF). The simulation models are physically practical, which means apart from the pure ideal linear term, we would also consider the high order nonlinear terms. In both model, the closer the frequency approaches the carrier frequency, the more linear the transfer function would be. That's why we model two differentiators separately, rather than using the left and right part of one single differentiator.

After the discussion in Chapter 4, we will find out that the performance of FM systems would be significantly improved with TEF as compared to LEF with the presence of dispersion.

2.3.2. Leading Edge Filter (Low Frequency Suppression)

The slope of the magnitude response of the leading edge filter is a positive value, which means it suppresses the low frequency band of the input signal, while as the frequency increases, the amplitude of the output signal also increases. Since it is a passive device, the value of the magnitude transfer function is between 0 and 1.

LEF is based on the technique of tunable fractional order photonic differentiator using an electrically tuned silicon-on-insulator Mach-Zehnder interferometer to realize the FM to AM conversion.

The transfer function of the LEF can be realized by a single MZI and it can be expressed as [23]

$$H_f(f) = \frac{1}{2}[1 + \beta \cdot \exp[i(2\pi\tau f + \Phi_0)]]. \quad (2.13)$$

Assuming β is determined by the power splitting ratio of the two arms of the MZI, the value of it hinges on the voltage applied on one arm of the MZI. Here τ and Φ_0 are the relative time delay and phase difference between two arms, respectively. Critical coupling happens when $\beta = 1$. In the following simulation, we set $\beta = 0.99$, $\tau = 1ps$, and $\Phi_0 = \pi$.

Figure 2-11 (a) and (b) depict the magnitude response and phase response separately. It is a causal system and totally satisfies our prerequisite because the linear region of the magnitude transfer function is about 400GHz (the device can differentiate a pulse with a bandwidth >300GHz), while the notch of the optical differentiator is about -23dB (<-10dB), and therefore low enough for the proposed FM system.

Conversion from FM to AM can be achieved by positioning the spectrum of the output signal of fiber channel on the positive slope of LEF as is shown in Figure 2-13 (a).

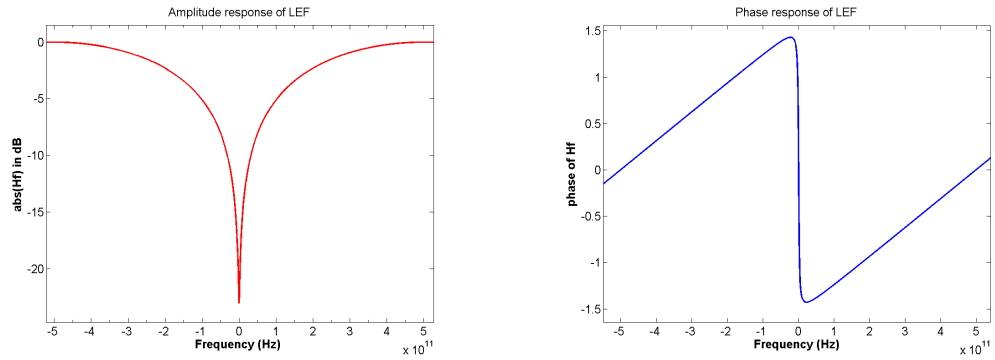


Figure 2-11 (a) LEF Magnitude response in dB (b) LEF Phase response

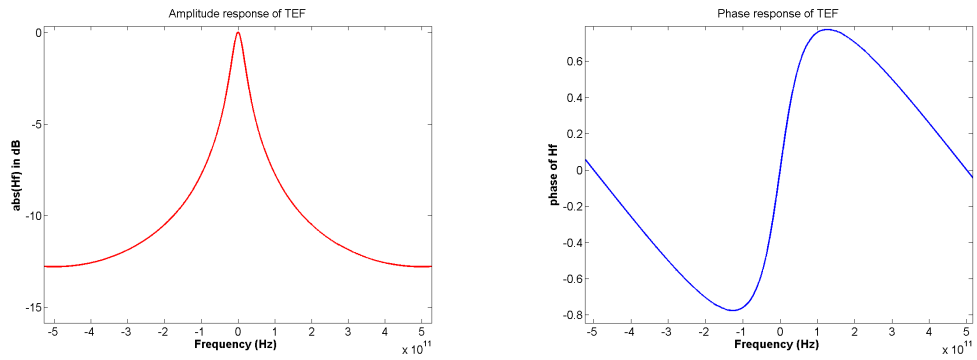
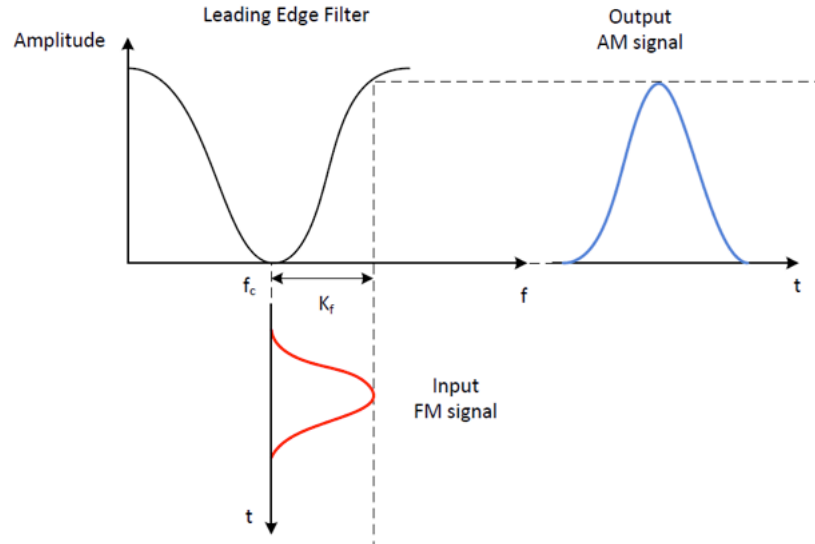


Figure 2-12 (a) TEF Magnitude response in dB (b)TEF Phase response

(a)



(b)

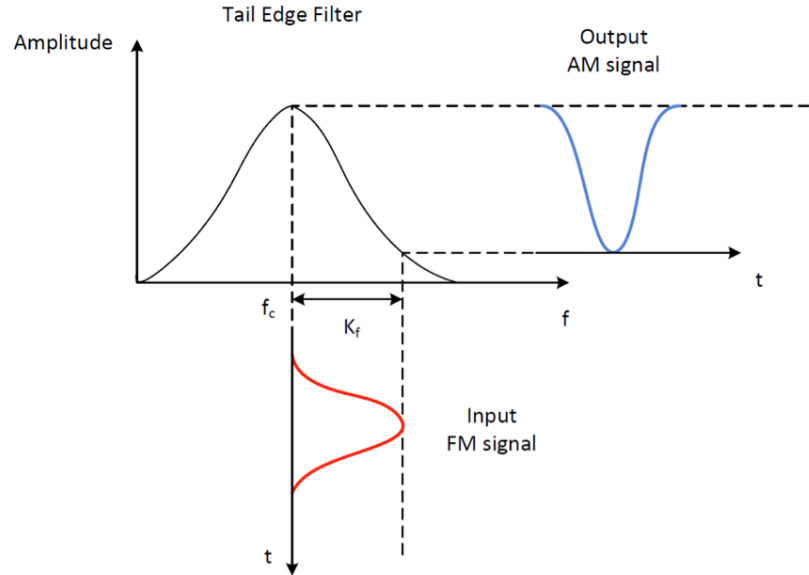


Figure 2-13(a) Conversion of FM to AM with LEF (b) Conversion of FM to AM with

TEF

2.3.3. Tail Edge Filter (High Frequency Suppression)

The slope of the magnitude response of the tail edge filter is a negative value. Absolutely opposite to the mechanism of the LEF, the TEF encourages low frequency band of the input signal while restraining the higher frequency band. It is also a passive device, thus the value of the magnitude transfer function is between 0 and 1.

The transfer function of the TEF is given by

$$H_f(f) = \frac{1}{10[1 + \beta \cdot \exp[i(2\pi\tau f + \Phi_0)]]} \quad (2.14)$$

The value of β is tunable. τ and Φ_0 are the relative time delay and phase difference between the two arms, respectively. Critical coupling happens when $\beta=1$ [16]. In the following simulation we set $\beta=0.9$ $\tau=1ps$ $\Phi_0=\pi$.

Figure 2-12 (a) and (b) depict the magnitude transfer function and phase transfer function, respectively. It is a causal system and completely satisfies our prerequisite because the linear region of the magnitude transfer function is about 300GHz while the notch is about -13dB (<-10dB).

Conversion from FM to AM can be achieved by positioning the output signal spectrum of the fiber channel on the negative slope of TEF as is shown in Figure 2-13 (b). An inverted AM signal is generated at the output.

Chapter 3

Noise Characteristics

In this chapter, ASE noise impairment is studied and an analytical model for the calculation of SNR is obtained based on both single channel fiber-optic FM and AM systems. It is worth mentioning that the two categories of frequency discriminator, LEF and TEF, would make no difference when noise impairment is discussed because the Gaussian white noise is averagely distributed without any frequency preference. Both can be used in the FM system when we focus on noise impairment.

Analytically it is derived that the anti-noise performance of FM system is much better than the AM system as long as the frequency deviation is sufficiently large. Then the numerical experiment is conducted, proving that the analytical results are in agreement with the numerical results.

3.1 ASE Noise

ASE noise is often assumed as a zero mean stationary Gaussian white noise. In fact, it is not a white noise but a “colored” noise because the frequency band of ASE noise cannot contain equal amounts of all frequencies from negative infinity to positive infinity. However, as the power spectral density of ASE noise is nearly constant over the transmission bandwidth, it is sufficient to regard ASE as a Gaussian white noise in our discussion [7]. Some related definitions are listed below.

A random variable takes its values from the outputs of a random experiment. For independent identically distributed random variables,

$$z = x_1 + x_2 + \dots + x_n, \quad (3.1)$$

and z becomes convergent to Gaussian distribution as n goes to infinity. A random process is a time-varying function that assigns the outcome of a random experiment to each time instant. For a fixed time t , the random process becomes a random variable, where the mean value can be calculated as

$$\mu_x(t) = E[X(t)] = \int_{-\infty}^{\infty} xf_x(t) dx. \quad (3.2)$$

Here $f_x(t)$ denotes the probability density function of the variable x . In general, the mean is a function of time t .

A random process is wide-sense stationary if the following two conditions are satisfied [24]:

1. Its mean does not depend on time,

$$\mu_X(t) = \mu_X. \quad (3.3)$$

2. Its autocorrelation function only depends on time difference

$$R_X(t, t + \tau) = R_X(\tau). \quad (3.4)$$

In communications, noise is often modeled as a stationary Gaussian random process for the above reasons [25].

The Gaussian distribution is given by [26]

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} \text{ for } -\infty < x < \infty \quad (3.5)$$

$$F_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^x e^{-\frac{(y-m)^2}{2\sigma^2}} dy \quad (3.6)$$

where the mean value of X is m , the RMS value is σ .

For white noise, the power spectrum density (PSD) is constant over all frequencies, i.e.

$$S_N(f) = \frac{N}{2}, -\infty < f < \infty. \quad (3.7)$$

The term “white” is analogous to white light, which contains equal amounts of all frequencies. It is only defined for stationary noise. The infinite bandwidth is a purely theoretic assumption and cannot be reached in reality. As for Gaussian white noise, the probability density function (PDF) at any time instant is Gaussian, and the power spectrum density (PSD) is constant.

Similar to other transmission systems, optical signals suffer from power loss as they propagate through the fiber. Despite the fact that the loss is relatively low compared to other transmission systems, i.e. about 0.2dB/km around 1550nm wavelength, it will

degrade the performance of the optical signal dramatically when the system reach is in long distance. Optical amplifiers are often used in optical fiber transmission links in order to compensate for the fiber power loss to ensure the signal-to-noise ratio (SNR) is beyond an acceptable limit. When the amplifier optical gain is high enough, the amplified spontaneous emission (ASE) becomes the principal source of noise. In this thesis, only ASE noise is taken into account, whereas other sources of noise such as double Rayleigh backscatter, thermal noise and shot noise of the receiver are ignored when we consider the noise impairment between the two modulation formats since they are much smaller than ASE noise in multi-span fiber-optic system.

The ASE noise is assumed as a zero mean stationary Gaussian white noise, with power spectral density (PSD) given by [4]

$$\rho_{ASE} = n_{sp}(G-1)h\nu, \quad (3.8)$$

where n_{sp} is the spontaneous noise factor, G is the amplifier gain, h is the Plank's constant, ν is the optical carrier frequency.

In the following simulations we set $n_{sp} = 1.5$, $\nu = 193THz$ and the bandwidth B of the ASE noise spectrum is the same as sampling rate. The amplifier gain G can exactly compensate the power loss in the previous fiber link so that it relates to the length of one fiber span [27]. The ASE noise is generated using a random number generator that follows the zero mean Gaussian white distribution with variance equal to $\rho_{ASE}B$.

3.2 Noise Characteristics of the FM system

3.2.1. System Configuration

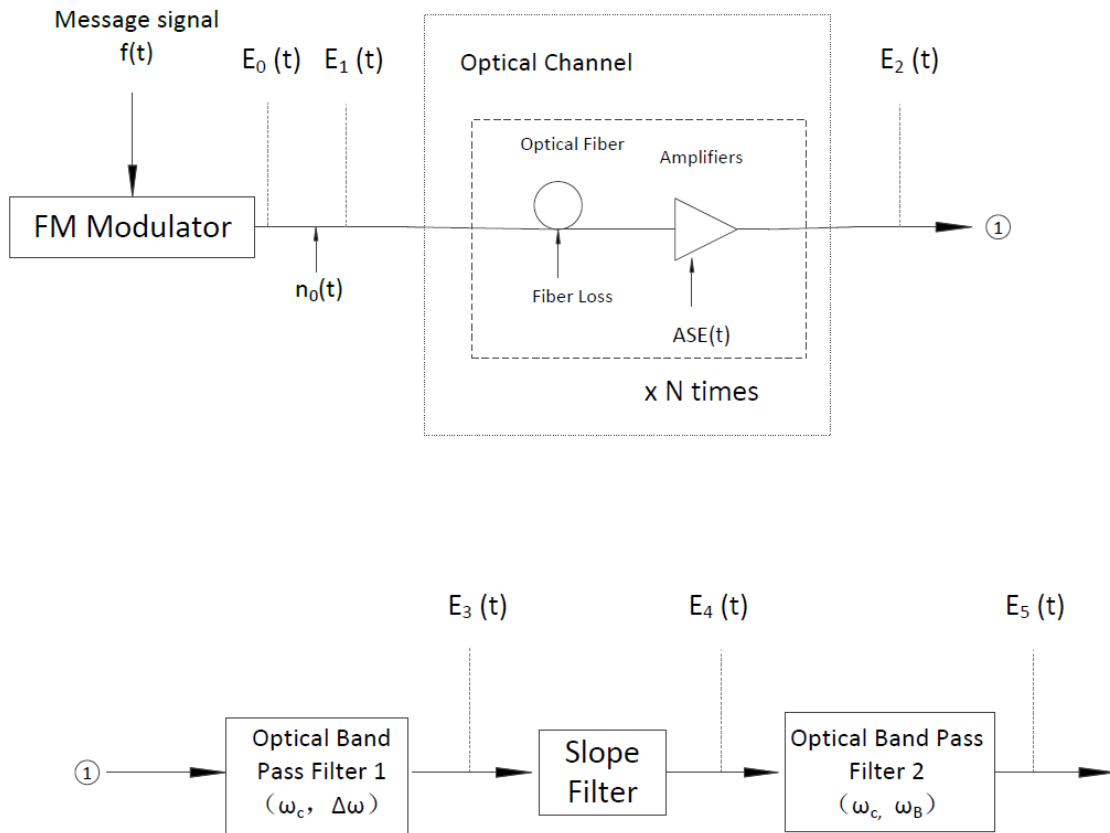


Figure 3-1 Schematic model of FM communication system when noise characteristics is focused.

Figure 3-1 shows the schematic model of the basic optical frequency modulated fiber communication scheme that is used when we discover the anti-noise property of the FM system. The unit of a span of fiber with loss followed by an amplifier is repeated for N

times in the optical channel. The central frequency and the bandwidth of the optical band pass filter 1 are denoted as ω_c and $\Delta\omega$, respectively; the central frequency and the bandwidth of the optical band pass filter 2 are denoted as ω_c and ω_B , respectively.

The system is composed of the transmitter - FM modulator, the optical fiber transmission link with several equally spaced amplifiers, and the optical band pass filter. As for the noise impairment, we only consider the ASE noise by ignoring the shot noise and thermal noise which would be induced by the photo-detector because both the AM and FM system will encounter the exact same receiver. To compare the AM system and FM system there would be meaningless if those noise factors are involved. Hence, the receiver component is out of our scope within this chapter. In other words, in this chapter we only consider how much the modulation scheme and the optical devices in the fiber path affect the noise property of the optical transmission system.

In this FM fiber-optic communication scheme, the FM modulator will create a carrier optical wave and load the message signal $f(t)$ to its frequency part. For the convenience of the following discussion, we will call the interface at the output of the FM modulator to be Port 0, corresponding to the output optical signal $E_0(t)$. Then there will be some additive background noise, and the interface is named Port 1 with the optical signal called $E_1(t)$. Here we assume them to be Gaussian white noise.

After that the optical signal will become the incident light into the fiber transmission path. It is necessary to consider the fiber loss and compensate for it when the optical fiber link is relatively long. Otherwise, the optical signal would be too weak to be detected

reliably [4]. For here we use several equally spaced optical amplifiers to compensate for the loss. The advantage of the optical amplifier is that it can amplify the power of the optical signal directly without requiring conversion of the signal to the electric domain. However, it would induce the ASE noise and will worsen the impact of fiber dispersion and nonlinearity effect because over multiple amplifiers the signal degradation continues to accumulate. The interface after the optical link is named as Port 2 with the total optical signal called $E_2(t)$.

After that, the optical wave goes through a wide band pass filter which will filter out the noise outside the FM signal frequency band and we denote the interface after the optical band pass filter to be Port 3 with the total optical signal here named $E_3(t)$.

The optical wave is then incident to the slope filter, which can also be called frequency discriminator, and it is used to transfer the FM signal to AM-FM signal. We denote the interface after the optical band pass filter to be Port 4 with the total optical signal here named $E_4(t)$.

Finally the output optical wave from the fiber path will be injected into an optical band pass filter, whose bandwidth is equal to the modulated optical signal, $\omega_b = \omega_m$. So it will filter out the noise outside the pass band while the signal part will not be affected. We denote the interface after the optical band pass filter to be Port 5 with the total optical signal here named $E_5(t)$.

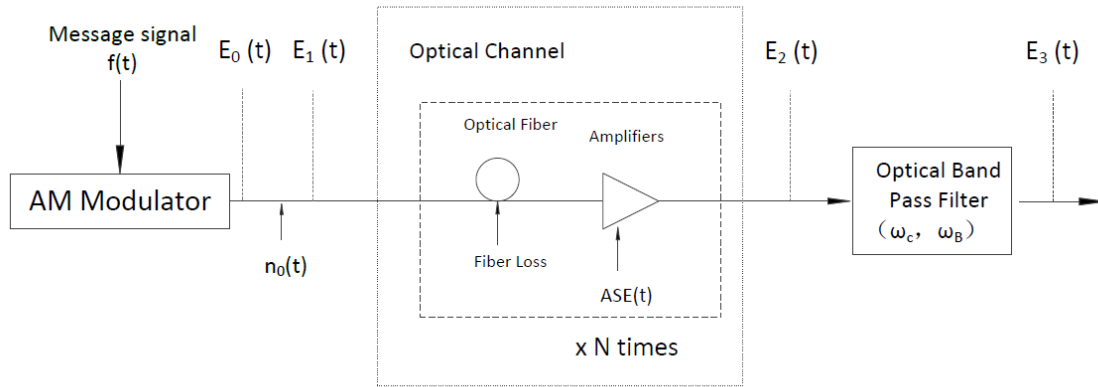


Figure 3-2 Schematic model of AM communication system when noise characteristics is focused

Figure 3-2 shows the schematic model of the basic amplitude modulated fiber communication scheme that is used as a comparison group when we discover the anti-noise property of the AM system. The unit of a span of fiber with loss followed by an amplifier is repeated for N times in the optical channel. The central frequency and the bandwidth of the optical band pass filter are denoted as ω_c and ω_B respectively.

The system is composed of the transmitter - AM modulator, the optical fiber transmission link with several equally spaced amplifiers, and the optical band pass filter. The AM modulator plays a role in creating a carrier optical wave and loading the message signal $f(t)$ to its amplitude. For the convenience of the following discussion, we will call the interface at output the AM modulator to be Port 0, corresponding to the output optical signal $E_0(t)$. Then there will be some additive background Gaussian white noise, and the interface is named Port 1 with the optical signal called $E_1(t)$.

Next the optical signal will become the incident light into the fiber transmission path. The transmission fiber link is exactly the same as in the FM scheme with equally spaced amplifiers in the fiber channel. The interface location after the optical link is named as Port 2 with the total optical signal called $E_2(t)$.

Then the output optical wave from the fiber path will be injected in to an optical band pass filter, whose bandwidth is equal to the modulated optical signal, $\omega_B = \omega_m$. So it will filter out the noise outside the pass band while the signal part will not be affected. We denote the interface after the optical band pass filter to be Port 3 and the total optical signal $E_3(t)$.

3.2.2. Analytical Expression

The analytical expression of each signal in the FM system scheme is given below.

At Port 0, the frequency modulated optical signal $E_0(t)$ is a non-linear function of the message signal $f(t)$ and it can be written as

$$E_0(t) = A \cos[\omega_c t + k_f \int f(t) dt + \varphi_0], \quad (3.9)$$

assuming the initial phase is φ_0 , the magnitude of the carrier wave is constant A, ω_c is the carrier angular frequency. After some background Gaussian white noise $n_0(t)$ was added, the optical signal at Port 1 would be

$$E_1(t) = A \cos[\omega_c t + k_f \int f(t) dt + \varphi_0] + n_0(t). \quad (3.10)$$

Then the optical signal will be injected into an optical fiber path. To compensate for the fiber loss, several amplifiers located with a fixed distance would be added. When we

ignore the effect of fiber dispersion and only consider the Gaussian white ASE noise induced by the m amplifiers, the total signal at Port 2 will be:

$$E_2(t) = A \cos[\omega_c t + k_f \int f(t) dt + \varphi_0] + n_0(t) + mn_{ASE}(t), \quad (3.11)$$

where $n_{ASE}(t)$ denotes the ASE noise, it is independent with $n_0(t)$ and m is the number of the amplifiers in the fiber path. According to the additive property of Gaussian white noise, the total noise $n(t)$ is also Gaussian white noise:

$$n(t) = n_0(t) + mn_{ASE}(t). \quad (3.12)$$

The PSD of the total noise is denoted as $N/2$, i.e.,

$$S_n(\omega) = \frac{N}{2}. \quad (3.13)$$

The signal part of the optical signal at Part 2 is the same as the output signal of the FM transmitter:

$$E_{2s} = A \cos[\omega_c + k_f \int f(t) dt + \varphi_0] \quad (3.14)$$

Thus, for the whole system, the input signal power can be calculated as [9]

$$S_i = S_{2s} = \frac{1}{2} A^2. \quad (3.15)$$

If the limited bandwidth of the optical amplifiers placed periodically along the fiber link is ω_f , the input noise power is yielded as

$$N_i = N_3 = \frac{1}{\pi} \int_{\omega_c - \omega_f}^{\omega_c + \omega_f} S_n(\omega) d\omega = N \frac{\omega_f}{\pi}. \quad (3.16)$$

The band pass filter removes any signals outside the bandwidth of $\omega_c \pm \Delta\omega$ and for here $\Delta\omega$ is the single side bandwidth of the FM signal. The ideal transfer function of BPF is

$$H(\omega) = \begin{cases} 1, & |\omega - \omega_c| \leq \Delta\omega \\ 0, & |\omega - \omega_c| \geq \Delta\omega \end{cases} \quad (3.17)$$

and in the time domain it can be expressed as

$$h(t) = F^{-1}[H(\omega)] = \frac{\Delta\omega}{\pi} e^{j\omega_c t} \text{sinc}(\Delta\omega t). \quad (3.18)$$

The optical signal at Port 3 must in the form

$$\begin{aligned} E_3(t) &= E_2(t) * h(t) \\ &= E_{2s}(t) * h(t) + E_{2n}(t) * h(t) \\ &= E_{3s}(t) + E_{3n}(t), \end{aligned} \quad (3.19)$$

which the numerical calculations would be based on. However, in the analytical approach, because we have to give all the accurate formulations, simplification is required or the expression will be too unnecessarily complicated. Because the bandwidth of the BPF is the same as the bandwidth of the signal part of $E_2(t)$, the sinc function can be simplified as the Dirac Delta function. Hence, the signal part will maintain the same when this simplification is done.

$$\int_{-\infty}^{+\infty} h(t) dt = \int_{-\infty}^{+\infty} \frac{\Delta\omega}{\pi} e^{j\omega_c t} \sin c(\Delta\omega t) dt = H(\omega_c) = 1, \quad (3.20)$$

and

$$h(t) \approx \delta(t). \quad (3.21)$$

However, because the bandwidth of Gaussian white noise is significantly larger than the bandwidth of the BPF, BPF affects the noise part severely. For this reason, the above simplification cannot be realized. This is the expression of a narrow band Gaussian noise:

$$\begin{aligned} E_{3n}(t) &= E_{2n}(t) * h(t) \\ &= n_i(t) \\ &\approx n_c(t) \cos \omega_c t + n_s(t) \sin \omega_c t. \end{aligned} \quad (3.22)$$

The optical expression at port 3 is

$$E_3(t) = E_{3s}(t) + n_i(t), \quad (3.23)$$

$$E_{3s} = A \cos[\omega_c t + k_f \int f(t) dt + \varphi_0]. \quad (3.24)$$

After that, the frequency modulated optical signal is then fed into an optical frequency discriminator, which is usually an optical filter with a transfer function having a linear slope within the used bandwidth [28]. This device helps to realize the FM-AM conversion. As for the derivation for port 4, we would like to split the calculation into two parts: Part A is to calculate the pure signal only without any noise disturbance; Part B only focuses on the analytical expression of the noise part.

Part A.

Take the complex form of $E_{3s}(t)$ to derive the output signal expression of the FM discriminator:

$$\tilde{E}_{3s}(t) = A \exp[j(\omega_c t + k_f \int f(t) dt + \varphi_0)]. \quad (3.25)$$

It is known that a linear frequency discriminator for LEF has a frequency response given by

$$H_1(\omega) = \alpha(\omega - \omega_0), \quad (3.26)$$

α denotes the slope angle of the frequency response and ω_0 is the angular frequency for $H_1(\omega) = 0$. It can be easily proved that if we apply TEF here, the same result will be derived, as Gaussian white noise is evenly distributed on the frequency band with no frequency dependence.

Suppose that $\tilde{E}_{3s}(\omega)$ is the Fourier transform of $\tilde{E}_{3s}(t)$. For a linear time-invariant system the output signal in the frequency domain is a multiplication of the Fourier transform of the input signal and the frequency response of the system.

Thus, when the frequency-modulated light is launched to the linear frequency discriminator, the output signal in the frequency domain is given by

$$\tilde{E}_{4s}(\omega) = \tilde{E}_{3s}(\omega)H_1(\omega) = \alpha\omega\tilde{E}_{3s}(\omega) - \alpha\omega_0\tilde{E}_{3s}(\omega) \quad (3.27)$$

where $\tilde{E}_{4s}(\omega)$ represents the output signal in the frequency domain. Applying the inverse Fourier transform to (3.20), we obtain the output signal in the time domain

$$\begin{aligned} \tilde{E}_{4s}(t) &= -j\alpha \frac{d}{dt} \tilde{E}_{3s}(t) - \alpha\omega_0 \tilde{E}_{3s}(t) \\ &= A[\alpha(\omega_c - \omega_0) + \alpha k_f f(t)] \exp[j(\omega_c t + k_f \int f(t) dt + \varphi_0)]. \end{aligned} \quad (3.28)$$

In our design it is easy to set $\omega_c = \omega_0$, so that

$$\tilde{E}_{4s}(t) = A\alpha k_f f(t) \exp[j(\omega_c t + k_f \int f(t) dt + \varphi_0)], \quad (3.29)$$

$$E_{4s}(t) = \text{Re}\{\tilde{E}_{4s}(t)\} = A\alpha k_f f(t) \cos[\omega_c t + k_f \int f(t) dt + \varphi_0]. \quad (3.30)$$

Since $\omega_c \gg k_f f(t)$, when we calculate the signal power, we can approximate as below,

$$E_{4s}(t) \approx A\alpha k_f f(t) \cos[\omega_c t]. \quad (3.31)$$

In this way, for the whole system, the output signal power can be calculated as

$$S_o = S_4 = \frac{1}{2} A^2 \alpha^2 k_f^2 \overline{f(t)^2}. \quad (3.32)$$

Part B.

For high SNR, the noise output is approximately independent of the message signal, so we can compute the noise power at Port 4 without message signal $f(t)$, and accept that the result holds for the case with noise too [6].

$$E_3(t) = A \cos[\omega_c t + k_f \int f(t) dt + \varphi_0] + n_c(t) \cos(\omega_c t) + n_s(t) \sin(\omega_c t) \quad (3.33)$$

Suppose $f(t) = 0, \varphi_0 = 0$ is the case, and only carrier light and noise remains, the above expression becomes

$$E_{3n}(t) = A \cos(\omega_c t) + n_c(t) \cos(\omega_c t) + n_s(t) \sin(\omega_c t), \quad (3.34)$$

where $K(t)$ denotes the instantaneous amplitude noise and $\varphi(t)$ the instantaneous phase noise.

$$K(t) = \sqrt{(A + n_c(t))^2 + n_s(t)^2}, \quad (3.35)$$

and

$$\varphi(t) = -\tan^{-1}\left(\frac{n_s(t)}{A + n_c(t)}\right) \approx -\tan^{-1}\left(\frac{n_s(t)}{A}\right) \approx -\frac{n_s(t)}{A}. \quad (3.36)$$

Take the complex form of $E_{3n}(t)$ to be $\tilde{E}_{3n}(t)$, and use the property of the linear time-invariant system to obtain the output noise signal $\tilde{E}_{4n}(\omega)$ and $\tilde{E}_{4n}(t)$ in the frequency domain and time domain respectively.

$$\tilde{E}_{3n}(t) = \sqrt{(A + n_c(t))^2 + n_s(t)^2} \exp[j(\omega_c t - \frac{n_s(t)}{A})] \approx A \exp[j(\omega_c t - \frac{n_s(t)}{A})] \quad (3.37)$$

and

$$\tilde{E}_{4n}(\omega) = \tilde{E}_{3n}(\omega) H_1(\omega) = \alpha \omega \tilde{E}_{3n}(\omega) - \alpha \omega_0 \tilde{E}_{3n}(\omega). \quad (3.38)$$

According to the differentiation property of the Fourier's law, the noise signal expression in the time domain can be written as

$$\begin{aligned} E_{4n}(t) &= -j\alpha \frac{d}{dt} \tilde{E}_{3n}(t) - \alpha \omega_0 \tilde{E}_{3n}(t) \\ &= [A\alpha(\omega_c - \omega_0) - \alpha \frac{dn_s(t)}{dt}] \exp[j(\omega_c t - \frac{n_s(t)}{A})]. \end{aligned} \quad (3.39)$$

For simplicity, in our design we set $\omega_c = \omega_0$, so the noise can be expressed as

$$E_{4n}(t) = -\alpha \frac{dn_s(t)}{dt} \exp[j(\omega_c t - \frac{n_s(t)}{A})] \quad (3.40)$$

$$E_{4n}(t) = \text{Re}\{\tilde{E}_{4n}(t)\} = -\alpha \frac{dn_s(t)}{dt} \cos(\omega_c t - \frac{n_s(t)}{A}). \quad (3.41)$$

The power spectral density of the output noise can be derived by

$$\begin{aligned}
 S_{n_0}(\omega) &= \frac{\alpha^2}{2} S_{n_s}(\omega) = \frac{\alpha^2 \omega^2}{2} S_{n_s}(\omega) \\
 &= \frac{\alpha^2 \omega^2}{2} [S_n(\omega + \omega_c) + S_n(\omega - \omega_c)].
 \end{aligned}
 \tag{3.42}$$

After the FM discriminator, there is a narrow band pass filter, which is used to filter out the useless signal and only obtain the message signal.

$$S_{n_0}(\omega) = \begin{cases} \frac{\alpha^2 \omega^2}{2} [S_n(\omega + \omega_c) + S_n(\omega - \omega_c)], & |\omega| \leq \omega_m \\ 0, & |\omega| \geq \omega_m \end{cases}
 \tag{3.43}$$

Because the power spectral density of the added Gaussian white noise, we get

$$S_{n_0}(\omega) = \begin{cases} \frac{\alpha^2 \omega^2}{2} N, & |\omega| \leq \omega_m \\ 0, & |\omega| \geq \omega_m \end{cases}
 \tag{3.44}$$

Therefore, the output noise power is

$$\begin{aligned}
 N_o &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{n_0}(\omega) d\omega = \frac{1}{2\pi} \int_{-\omega_m}^{+\omega_m} \frac{\alpha^2 \omega^2}{2} N d\omega \\
 &= \frac{\alpha^2 N \omega_m^3}{6\pi}.
 \end{aligned}
 \tag{3.45}$$

From the previous analysis we can obtain the input SNR (Signal-to-Noise ratio), output SNR and the ratio of the two:

$$(SNR)_{FMinput} = \left(\frac{S_i}{N_i}\right)_{FM} = \frac{\pi A^2}{2N\omega_f}, \quad (3.46)$$

$$(SNR)_{FMoutput} = \left(\frac{S_o}{N_o}\right)_{FM} = \frac{3\pi A^2 k_f^2 \overline{f(t)^2}}{N\omega_m^3}, \quad (3.47)$$

and

$$\frac{S_o/N_o}{S_i/N_i} = \frac{6k_f^2 \overline{f(t)^2} \omega_f}{\omega_m^3}. \quad (3.48)$$

The ratio of output SNR and input SNR for the FM system is influenced by the message power, the frequency deviation k_f and the bandwidth of the FM signal ω_f . This is one of the significant results of this research.

As for the compared group, the AM system, the analytical expression of SNR is given below.

At Port 0, the amplitude modulated optical signal $E_0(t)$ is a linear function of the message signal $f(t)$ and it can be written as

$$E_0(t) = A \cdot f(t) \cdot \cos(\omega_c t + \varphi_0) \quad (3.49)$$

Assuming the initial phase is φ_0 , the magnitude of the carrier wave is constant A , ω_c is the carrier angular frequency.

For the whole AM system, the input signal power is

$$S_i = \frac{1}{2} A^2 \cdot \overline{f(t)^2}. \quad (3.50)$$

Assume that the limited bandwidth of the optical amplifiers placed periodically along the fiber link is ω_f , the input noise power becomes

$$N_i = \frac{1}{\pi} \int_{\omega_c - \omega_f}^{\omega_c + \omega_f} S_n(\omega) d\omega = N \frac{\omega_f}{\pi}. \quad (3.51)$$

Following the same procedure as in the previous section, the output signal power can be expressed as

$$S_o = \frac{1}{2} A^2 \cdot \overline{f(t)^2}, \quad (3.52)$$

and the output noise power is

$$N_o = \frac{1}{\pi} \int_{\omega_c - \omega_m}^{\omega_c + \omega_m} \frac{N}{2} d\omega = N \frac{\omega_m}{\pi}. \quad (3.53)$$

The input SNR and output SNR of the AM system can be written as

$$(SNR)_{AMinput} = \left(\frac{S_i}{N_i} \right)_{AM} = \frac{\pi A^2 \overline{f(t)^2}}{2N\omega_f}, \quad (3.54)$$

and

$$(SNR)_{AMoutput} = \left(\frac{S_o}{N_o} \right)_{AM} = \frac{\pi A^2 \cdot \overline{f(t)^2}}{2N\omega_m}. \quad (3.55)$$

In the subsequent system, the output amplitude-modulated optical signal for both scheme will be received by an photo-detector, which can be modeled as envelope detection, and the output electrical signal would enter the data recovery system in which

the launched digital signal would be restore with error probability. To be specific, the output current of the photo detector equals [29]

$$I(t) = \begin{cases} \sqrt{[A_m + n_c(t)]^2 + n_s^2(t)}, & \text{for "1"} \\ \sqrt{n_c^2(t) + n_s^2(t)}, & \text{for "0"} \end{cases} \quad (3.56)$$

The upper (lower) one corresponds to the circumstances when bit “1” (bit “0”) is transmitted. It is sampled and then to be decided according to the given threshold current b , so the received symbol is determined to be 1 or 0. It is prescribed that when $I > b$, the decision is received “1”; when $I \leq b$, the decision is received “0”. And for here suppose the responsivity of the photo detector is R, the maximum electrical current value A_m can be denoted as

$$A_m = RA^2 / 2. \quad (3.57)$$

Thus the probability density function (PDF) can be written as

$$p(I) = \begin{cases} p_1(I) = \frac{I}{\sigma_n^2} I_0\left(\frac{A_m I}{\sigma_n^2}\right) e^{-(I^2 + A_m^2)/2\sigma_n^2}, & \text{for "1"} \\ p_0(I) = \frac{I}{\sigma_n^2} e^{-I^2/2\sigma_n^2}, & \text{for "0"} \end{cases}, \quad (3.58)$$

where σ_n^2 is the variance of the $n(t)$, equals to N ; and $I_0(*)$ is the first class zero order modified Bessel function. The optimum threshold current b is the intersection of the two probability density curves $p_1(b) = p_0(b)$. Thus the optimum threshold b is found to be $A_m / 2$, which is in electrical domain. In this circumstance, the total error probability P_e can be expressed as [29]

$$\begin{aligned}
 P_e &= P(1)P(0/1) + P(0)P(1/0) \\
 &= P(1)[1 - Q(\sqrt{2(SNR)}, b_0)] + P(0)e^{-b_0^2/2}
 \end{aligned} \tag{3.59}$$

Assuming the SNR is the received signal-to-noise ratio, b_0 is the normalized threshold, the probability of transmitting bit “1” is $P(1)$ while the probability of transmitting bit “0” is $P(0)$. Q is the Marcum Q function and the normalized optimum threshold is $\sqrt{SNR/2}$. In this model suppose the probability of bit “1” equals to that of bit “0”

$$P(1) = P(0) = \frac{1}{2}. \tag{3.60}$$

Therefore, the total error probability can be written as

$$P_e = \frac{1}{2}[1 - Q(\sqrt{2(SNR)}, b_0)] + \frac{1}{2}e^{-b_0^2/2}. \tag{3.61}$$

From this equation we get that the larger the SNR is, the lower the total error probability becomes, and hence the performance of the total transmission system is better. In other words, the transmission system with higher SNR can be guaranteed to have an outstanding performance.

3.3 Comparison on Noise Characteristics

We have derived the mathematical expressions for input SNR of the FM system (3.46), output SNR of the FM system (3.47), input SNR of the AM system (3.54), and output SNR of the AM system (3.55) in the above section.

To make a fair comparison between the two systems, we should guarantee that the message signal $f(t)$, the baseband signal bandwidth ω_m and the fiber bandwidth ω_f should all be the same in the AM system and FM system.

NF (Noise Figure) is the ratio of input SNR to the output SNR of a communication system. A parameter NFI (FM over AM Noise Figure Improvement) is defined to evaluate the extent to which the FM system outperforms the AM system in terms of noise impairment:

$$NFI = \frac{(SNR)_{FMoutput} / (SNR)_{FMinput}}{(SNR)_{AMoutput} / (SNR)_{AMinput}} = \frac{NF_{AM}}{NF_{FM}} = 6 \left(\frac{k_f}{\omega_m} \right)^2 \overline{f(t)^2}, \quad (3.62)$$

in which the message signal $f(t)$ is a dimensionless parameter, k_f is the frequency deviation with the dimension rad/s, same as that of baseband bandwidth ω_m . NFI is a dimensionless parameter. All of the formulas related are tabulated in Table 3-1.

Table 3-1 Summary of the FM over AM system noise figure improvement

| | FM | AM |
|--------------------|--------------------------------|---|
| Input Signal Power | $S_i = \frac{1}{2} A^2$ | $S_i = \frac{1}{2} A^2 \cdot \overline{f(t)^2}$ |
| Input Noise Power | $N_i = N \frac{\omega_f}{\pi}$ | $N_i = N \frac{\omega_f}{\pi}$ |

| | | |
|---|--|---|
| Input SNR | $\frac{S_i}{N_i} = \frac{\pi A^2}{2N\omega_f}$ | $\frac{S_i}{N_i} = \frac{\pi A^2 \overline{f(t)^2}}{2N\omega_f}$ |
| Output Signal Power | $S_o = \frac{1}{2} A^2 \alpha^2 k_f^2 \overline{f(t)^2}$ | $S_o = \frac{1}{2} A^2 \cdot \overline{f(t)^2}$ |
| Output Noise Power | $N_o = \frac{\alpha^2 N \omega_m^3}{6\pi}$ | $N_o = N \frac{\omega_m}{\pi}$ |
| Output SNR | $\frac{S_o}{N_o} = \frac{3\pi A^2 k_f^2 \overline{f(t)^2}}{N \omega_m^3}$ | $\frac{S_o}{N_o} = \frac{\pi A^2 \cdot \overline{f(t)^2}}{2N \omega_m}$ |
| Ratio (Reciprocal of Noise Figure) | $\frac{S_o/N_o}{S_i/N_i} = \frac{6k_f^2 \overline{f(t)^2} \omega_f}{\omega_m^3}$ | $\frac{S_o/N_o}{S_i/N_i} = \frac{\omega_f}{\omega_m}$ |
| $NFI = \frac{(SNR)_{FMoutput} / (SNR)_{FMinput}}{(SNR)_{AMoutput} / (SNR)_{AMinput}} = \frac{NF_{AM}}{NF_{FM}} = 6 \left(\frac{k_f}{\omega_m} \right)^2 \overline{f(t)^2}$ | | |

When the average power of the message signal $\overline{f(t)^2}$ is not too small, NFI would be much larger than 1 as long as the wide band optical frequency modulation ($f_c > K_f > f_m, K_f = k_f / 2\pi$) is satisfied, and this corresponds to the circumstances that the FM system gives better anti-noise performance than the AM system. Theoretically, in this circumstance, it is also true that the FM signal will take up the bandwidth ($K_f |f(t)|_{\max}$), which is much more than AM signal (f_m). Actually, this conclusion shows a trade-off between the anti-noise property and the transmission bandwidth. In

practical use, if the transmission bandwidth is sufficient, we can apply the FM modulation format in order to get better anti-noise performance.

This conclusion could be invalid only when the noise goes beyond the threshold - in which case, the pre-emphasizing at the transmitter and de-emphasizing at the receiver can be implemented to ensure that the FM performance will always surpass the AM's.

3.4 Modeling and Simulation

We carried out the numerical simulations of the FM fiber-optic system and the AM fiber-optic system in order to test the validity of the derived analytical result. The span of transmission fiber is followed by an amplifier that fully compensates the loss of the fiber but induce the ASE noise at the same.

The signal power can be easily obtained by going through the system with no ASE noise added. To calculate the input noise power and output noise power numerically, the numerical simulations were performed 100 times with different noise signal each time, then we computed the variance at each time node throughout those 100 times, and obtained the time average power by summing the variance up and divided by the total number of the time nodes.

3.4.1. Simulation Model

A pseudo-random bit sequence (PRBS) is generated at the beginning of the simulation. The message signal $f(t)$ is the bit sequence multiplied by raised cosine pulses, in which bit “1” (bit“0”) is represented by a pulse with amplitude 1 (0). And the modulator plays a role in loading the message signal to the frequency part of the carrier wave, forming the

optical signal $E_0(t)$. After adding the additive background noise, the optical signal $E_1(t)$ is obtained. Next the optical signal will become the incident light into the fiber transmission path. It is necessary to consider the fiber loss and note that the amplifiers are located equidistantly throughout the fiber. Utilizing the split-step method, for each span, the input signal's Fourier transform is multiplied by the transfer function of the fiber with the loss term only, and then the inverse Fourier transform is applied to achieve the weakened signal in the time domain. The output of the span is equal to the weakened signal multiplied by the amplifier's gain value accompanied by the ASE noise. The ASE noise is generated using a random number generator that follows the zero mean Gaussian white distribution with variance equal to $\rho_{ASE}B$. The signal quality degradation would keep on accumulating as it passes through the optical channel. If the number of span is denoted as N , the above procedure will repeat N times and the output signal at Port 2, $E_2(t)$ is achieved. The wide band pass filter in this simulation is defined as the transfer function in the frequency domain. Based on the convolution theorems of the Fourier transform, input signal would experience the operation as follows to obtain the output signal of the band pass filter $E_3(t)$: 1) Take the Fourier transform to obtain the input spectrum. 2) Multiply the input spectrum by the transfer function of the device, which in this case is the band pass filter. 3) Take the inverse Fourier transform to obtain the output signal. The above operation is known as convolution theorems [30]. The proceeding frequency differentiator is also given in the form of transfer function, and the output signal $E_4(t)$ is obtained with convolution theorems.

Finally the output optical wave from the fiber path will be injected into an optical band pass filter whose bandwidth is as narrow as that of the modulated optical signal, $\omega_B = \omega_m$. The optical signal $E_s(t)$ can be anticipated with the help of convolution theorems.

3.4.2. Parameter Setting

The following parameters were used for the simulation: A pseudo-random bit sequence (PRBS) of length $2^8 - 1$, a bit interval of 100ps, bit rate of $B = 10\text{Gbit} / \text{s}$, and an operating wavelength of 1550nm (carrier frequency of 194THz). NRZ format is applied and the message signal $f(t)$ is based on the raised cosine pulses. Thus the half bandwidth of the baseband signal is about $\omega_m = 10\text{GHz}$. We apply the G.652.D single mode fiber with the fiber loss $\alpha = 0.2\text{dB} / \text{km}$. When the ASE noise is additive Gaussian white noise and the only impairment is noise, the result at the optical band pass filter of adding the noise at the end of each fiber span for multi-span system is equivalent to adding it only once as long as the total length of the transmission link is the same. Thus here we only assume the system is a 5-span system. As for the amplifiers, the spontaneous emission factor or population- inversion factor is $n_{sp} = 1.5$, and the gain is $G = e^{\alpha L}$ which will exactly compensate the fiber loss of the last span. The frequency deviation range K_f for the FM signal is tuned between 100GHz to 300GHz. The optical receiver filter applies 6-th order butter worth band pass filter and the bandwidth is equal to the base band signal, $\omega_B = \omega_m$.

As previously mentioned, other impairments including laser phase noise, fiber dispersion and polarization effects are ignored in this chapter.

3.4.3. Results and Discussion

The SNR ratio NFI estimated analytically is found to be in good agreement with numerical simulations. Thus in the future we can easily get the result from analytical expressions, which would significantly reduce the computational time to estimate the system performance in the presence of noise impairment.

From the analytical equation $NFI = 6\left(\frac{k_f}{\omega_m}\right)^2 \overline{f(t)^2}$, we can conclude that the value of

NFI should be only determined by 3 parameters: the message signal $f(t)$, the baseband bandwidth ω_m and the frequency deviation k_f (or for convenience we use K_f , with $K_f = k_f / 2\pi$).

The message signal $f(t)$ is composed of several raised cosine pulses with the amplitude 1 (0) present bit “1” (bit “0”). Since the width of the pulse is equal to the bit interval (100ps), if the number of “1” bits and the number of “0” bits is equal, the average power of the message signal $\overline{f(t)^2}$ is about 0.1875 (dimensionless). Thus $\overline{f(t)^2}$ cannot be tuned as a parameter because the value of it will change within a small range only when the number of “1” bits and “0” bits are not the same or when the bit rate is changed which is out of our scope.

The baseband bandwidth ω_m is also a fixed parameter once the bit rate and the message pulse shape is settled, so it is not able to be tuned as a parameter in order to change the value of NFI.

The frequency deviation K_f , however, is a parameter that can be tuned in order to achieve different values of NFI. Thus in the following we would do more research on the relationship between these two parameters numerically in order to testify the analytical result derived above.

In addition, from the analytical derivation we can conclude that NFI has no relationship with the input noise power as well as average input signal power of the optical FM system and AM system. And it is easy to understand that the input noise power is linearly related to the transmission fiber length according to the property of the amplifier.

Thus, apart from frequency deviation, we will also plot the relationship between NFI and the transmission fiber length as well as the input signal power.

3.4.2.1 NFI vs. Frequency deviation

Figure 3-3 shows the analytical and numerical NFI value as a function of the frequency deviation K_f when the average input launch power for both systems is set to be 0dBm, the transmission length is 100km, and the probability of bit “1” is equal to that of bit “0”. To calculate NFI numerically, we proceed as follows. First we generate a random message signal $f(t)$ that includes $2^8 - 1$ bits and launch it into AM system, then the

$\overline{f(t)^2}$ stays invariant when we consider the variable K_f . At each interface of the system, we calculate the signal power and the noise power separately. The signal power can be easily obtained by going through the system with no ASE noise added. To calculate the input noise power and output noise power numerically, the numerical simulations were performed 100 times with a different noise signal each time. Then we computed the variance at each time node within those 100 times, and obtained the time average power by summing the variance and dividing by the total number of time nodes. Thus we can calculate the $(SNR)_{AMoutput} / (SNR)_{AMinput}$ for the AM system. Second we launched the same message signal into the FM system and tuned K_f from 100GHz to 300GHz, computing $(SNR)_{FMoutput} / (SNR)_{FMinput}$ numerically the same way as described before. Then we obtain the numerical value of NFI by dividing the resulting value of $(SNR)_{FMoutput} / (SNR)_{FMinput}$ for each K_f by the value of $(SNR)_{AMoutput} / (SNR)_{AMinput}$.

The discrepancy between the analytical model and the numerical model is less than 15%, which means that the analytical expression for NFI is validated using numerical simulations when the message signal power $\overline{f(t)^2}$ and the signal bandwidth is fixed. The larger the frequency deviation K_f is, the better the FM system becomes than the AM system. In addition, according to the analytical model, the NFI scales as K_f^2 , which is in good agreement with the numerical results. The discrepancy is mainly due to the inaccuracy of the model of the slope filter in the numerical simulation, in order to

guarantee every device model to be physically practical rather than purely ideal as is the case in the analytical model.

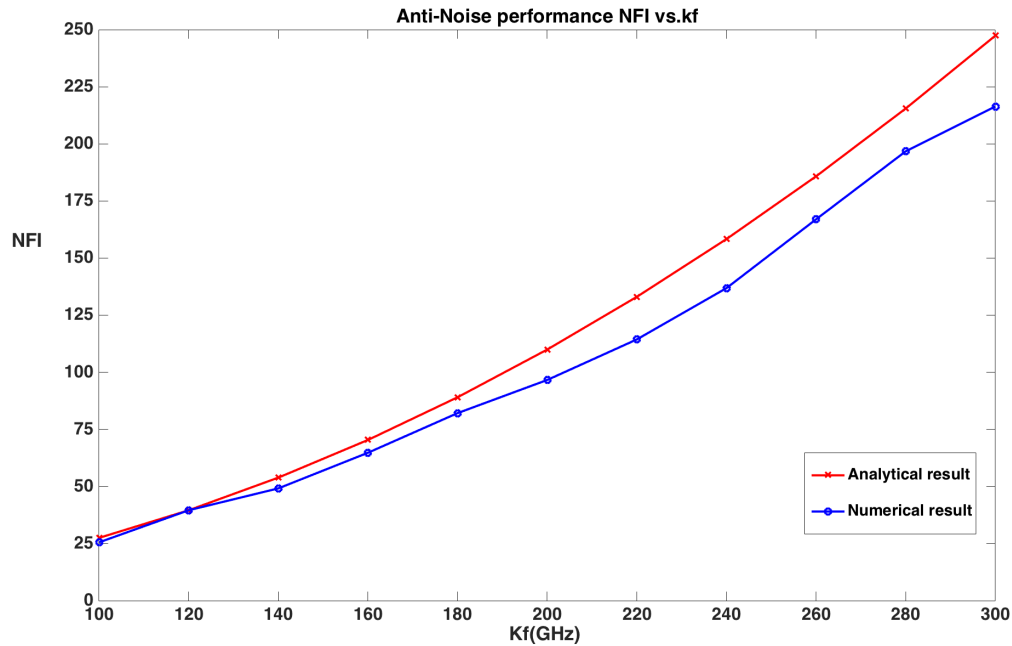


Figure 3-3 Analytical and numerical NFI value vs. frequency deviation for average input power 0dBm, transmission length 100km.

3.4.2.2 NFI vs. Transmission fiber length

From Figure 3-4 it suffices to conclude that with the increase of the transmission length, the power of the ASE noise induced by the amplifiers would be more drastic, but the input SNR and the output SNR would be degraded at the same time so that the noise figure of the optical system would maintain the same. This means that

$(SNR)_{F_{Output}} / (SNR)_{F_{Input}}$ will not change with the transmission length, and neither does

$(SNR)_{AMoutput} / (SNR)_{AMinput}$, and therefore the value of NFI should maintain stable analytically.

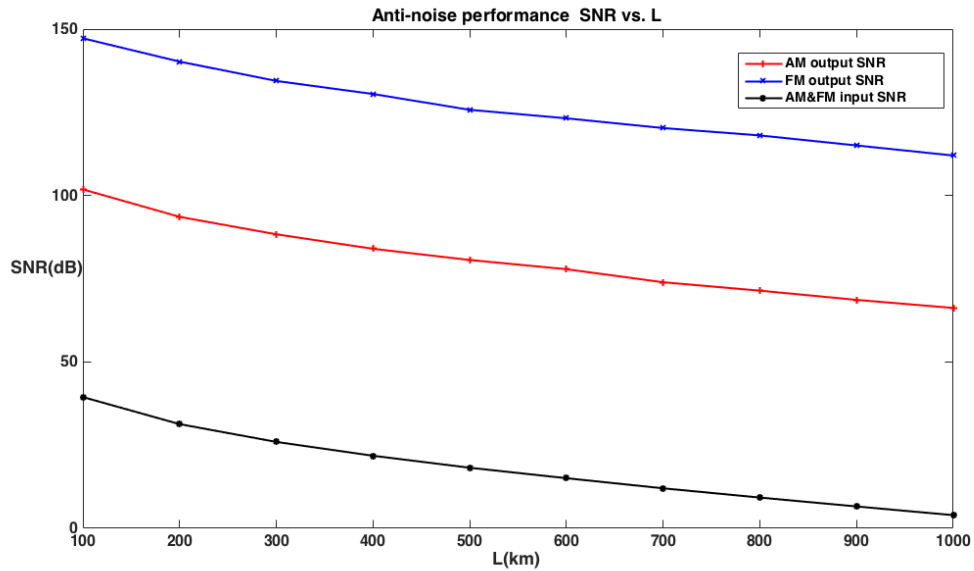


Figure 3-4 Signal-to-Noise ratio SNR vs. transmission length L for AM and FM system when average input power 0 dBm, frequency deviation for FM system is 200GHz.

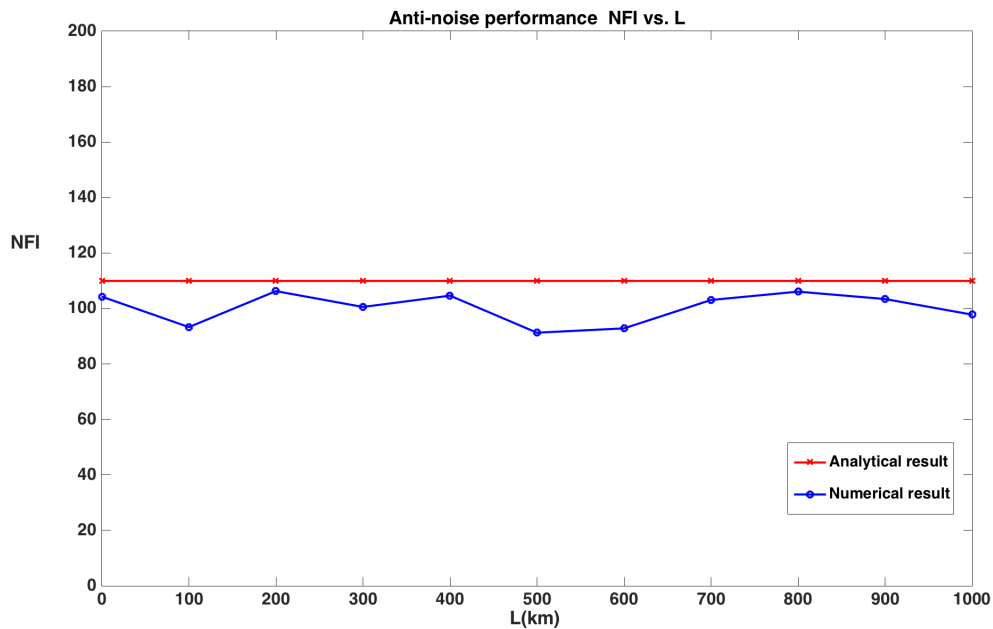


Figure 3-5 Analytical and numerical NFI value vs. transmission length L for average input power 0 dBm, frequency deviation for FM system is 200GHz.

Figure 3-5 depicts the analytical and numerical value of NFI vs. the transmission fiber length when $k_f=200\text{GHz}$, the average input launch power for both systems is set to be 0dBm, and the probability of bit “1” is equal to that of bit “0”. It turns out that the value of NFI will not change with the increase of the transmission fiber length L, which testify the analytical result that the NFI value C is irrelevant to transmission fiber length L or input noise power as long as both systems have the same transmission distance when we calculate the NFI. The discrepancy between the analytical model and the numerical model is less than 10%. This deviation is caused by simulation limitations. It can be assured that if the simulation is reiterated infinitely, the numerical result will ultimately converge to the analytical result.

3.4.2.3 NFI vs. Input signal power

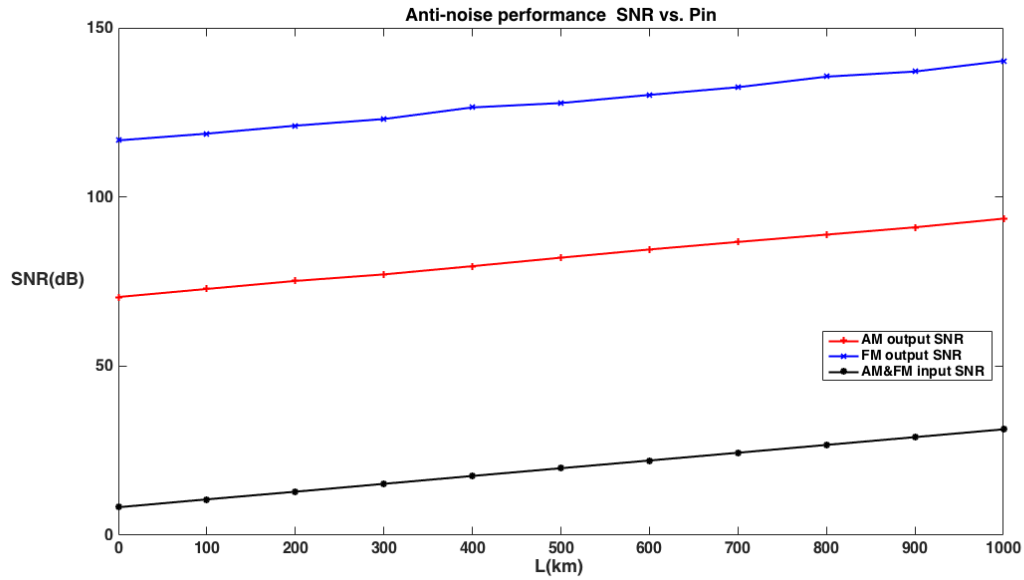


Figure 3-6 Signal-to-Noise ratio SNR vs. transmission length L for AM and FM system when transmission length 100km, frequency deviation for FM system

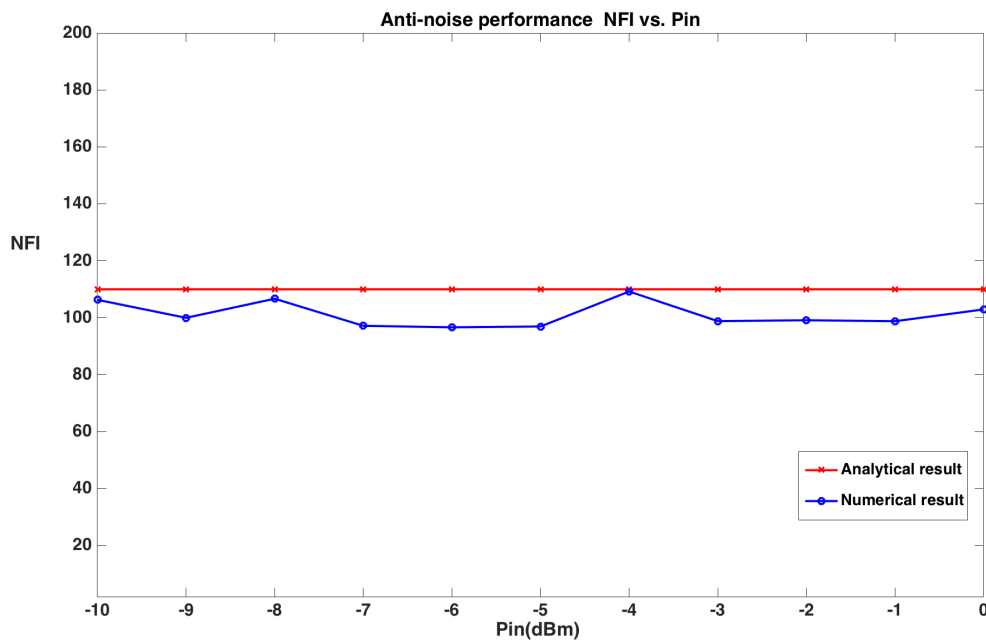


Figure 3-7 Analytical and numerical NFI value vs. average input power P_{in} for transmission length 100km, frequency deviation for FM system 200GHz.

As a matter of fact, with the increase of the input signal power, the input SNR and the output SNR would be increased simultaneously but the noise figure would stay constant, as is depicted in Figure 3-6. To be specific, $(SNR)_{FMoutput} / (SNR)_{FMinput}$ will not change with the input signal power, and neither does $(SNR)_{AMoutput} / (SNR)_{AMinput}$. Figure 3-7 depicts the analytical and numerical NFI value vs. the input signal power when $k_f=200\text{GHz}$, the transmission fiber length for both systems is set to be 100km, and the probability of bit “1” is equal to that of bit “0”. As can be concluded from the figure, the value of NFI stays constant when the input signal power changes, which means that NFI is not related to the input signal power and this also proves the validity of the analytical results. The discrepancy between the analytical model and the numerical model is less than 10%, which is also caused by simulation limitation. It can be assured that if the simulation is reiterated infinitely, the numerical result will ultimately converge to the analytical result.

Chapter 4

Dispersion Analysis

In this chapter, dispersion tolerance is studied for the FM system because it is considered as one of the critical obstacles for short-haul access networks in which dispersive compensation methods such as dispersion compensation fiber (DCF) or adaptive equalizer are not widely used for its high cost and length dependency. Zero dispersion is also undesired due to the high nonlinearities that would appear at 1550nm. The main impairment of the dispersion is that the time-domain broadening of the pulses leads to the overlapping of neighboring symbols—inter symbol interference (ISI) [4]. It is worth mentioning that in this chapter amplifiers are not induced to compensate for the power loss, and noise impairment is out of scope. The numerical method is used to evaluate the optical FM system performance degraded only by dispersion. The simulation results show that the transmission distance limitation of an FM system can be much longer compared

to a conventional AM system since the performance of the FM system is less sensitive to fiber dispersion.

4.1 Introduction to Dispersion

As is mentioned before, in order to avoid the intermodal dispersion, which only exists in MMF (multi-mode fibers), we apply SMF (single-mode fibers) in our optical communication systems. However, the intra-modal dispersion [a phenomenon also referred to as group-velocity dispersion (GVD)] still exists in SMF and will also cause pulse broadening of the input signal pulse in AM schemes, affecting the transmission quality of the fiber. Specifically, intra-modal dispersion describes the phenomenon in which different frequency components of the input signal pulse undergo different amounts of delay and arrive at different times at the receiver, leading to pulse broadening [4].

If the incident light of the SMF is from a single frequency laser operating at the frequency ω , the optical field distribution ψ in the fiber can be written as [7]

$$\psi(x, y, z, t) = \Phi(x, y, \omega)W(\omega)\exp[i(\omega t - \beta(\omega)z)], \quad (4.1)$$

where Φ represents the transverse field distribution and W denotes the mode weight factor, both of which could vary with frequency ω . z is the distance along the fiber and t is time.

If the field envelope of the incident optical signal to the SMF fiber is a pulse, which means the optical signal takes up a narrow bandwidth rather than an ideal Delta function in the frequency domain, the total optical field can be written as

$$\psi(x, y, z, t) = \Phi(x, y) \int_{-\infty}^{\infty} W(\omega) \exp[i(\omega t - \beta(\omega)z)] d\omega. \quad (4.2)$$

It can be understood as a superposition of each frequency component with a mode weight factor.

The impact of the fiber is characterized by the propagation constant $\beta(\omega)$.

$$\beta(\omega) = \beta_r(\omega) - i\alpha(\omega) / 2, \quad (4.3)$$

in which $\beta_r(\omega)$ and $\alpha(\omega)$ are related to the phase term and amplitude term, respectively.

In this chapter we ignore the loss ($\alpha(\omega)=0$) and nonlinearity and only consider the intra-model dispersion. Because the frequency band of the input optical field is around the carrier frequency ω_c , using Taylor series, we can calculate the following approximation:

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_c) + \frac{1}{2}\beta_2(\omega - \omega_c)^2 + \dots \quad (4.4)$$

where

$$\beta_0 = \beta_r(\omega_c), \quad (4.5)$$

$$\beta_1 = \left. \frac{d\beta_r}{d\omega} \right|_{\omega=\omega_c} = \frac{1}{v_g}, \quad (4.6)$$

and

$$\beta_2 = \left. \frac{d^2\beta_r}{d\omega^2} \right|_{\omega=\omega_c}. \quad (4.7)$$

Usually we can only consider the first three terms by ignoring the high order term. β_1 is the inverse group velocity and β_2 is the second order dispersion coefficient, which is

also known as the group velocity dispersion parameter (ps^2/km), while D is the dispersion parameter and it is expressed in units of $ps/(nm.km)$ [30]

$$D = -\frac{2\pi c}{\lambda^2} \beta_2. \quad (4.8)$$

Suppose the frequency deviation is Ω and let $A(\omega)$ to be a function directly of it, i.e.,

$$\Omega = \omega - \omega_c \quad (4.9)$$

$$W(\omega) = W(\omega_c + \Omega) = S(\Omega), \quad (4.10)$$

and the optical field in the fiber becomes

$$\begin{aligned} \psi(x, y, z, t) &= \Phi(x, y) \int_{-\infty}^{\infty} A(\omega) \exp[i(\omega t - \beta(\omega)z)] d\omega \quad (4.11) \\ &= \Phi(x, y) \cdot \exp[i(\omega_c t - \beta_0 z)] \cdot \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t - (\beta_1 \Omega + \frac{1}{2} \beta_2 \Omega^2)z)] d\Omega. \end{aligned}$$

Simplify the above equations by assuming

$$F(z, t) = \exp[i(\omega_c t - \beta_0 z)] \cdot \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t - (\beta_1 \Omega + \frac{1}{2} \beta_2 \Omega^2)z)] d\Omega, \quad (4.12)$$

and

$$s(z, t) = \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t - (\beta_1 \Omega + \frac{1}{2} \beta_2 \Omega^2)z)] d\Omega. \quad (4.13)$$

Thus the total field can be expressed as

$$\begin{aligned} \psi(x, y, z, t) &= \Phi(x, y) F(t, z) \quad (4.14) \\ &= \Phi(x, y) \cdot \exp[i(\omega_c t - \beta_0 z)] \cdot s(t, z). \end{aligned}$$

It contains 3 parts, the transverse field distribution $\Phi(x, y)$, the carrier wave with a distance relate phase delay $\exp[i(\omega_c t - \beta_0 z)]$ and the optical field envelope $s(t, z)$. When $z=0$, which corresponds to the incident signal

$$s(0, t) = \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t)] d\Omega, \quad (4.15)$$

from which we see that $s(0, t)$ and $S(\Omega)$ is a Fourier transform pair. The transfer function of the fiber can be defined as

$$H_f(\Omega, z) = \exp[-i(\beta_1 \Omega + \frac{1}{2} \beta_2 \Omega^2) z]. \quad (4.16)$$

So if $s(z, t)$ and $S_{out}(\Omega)$ are Fourier transform pairs, we can obtain that

$$s(z, t) = \int_{-\infty}^{\infty} S(\Omega) H_f(\Omega, z) \exp[i(\Omega t)] d\Omega, \quad (4.17)$$

and

$$S_o(\Omega) = S(\Omega) H_f(\Omega, z). \quad (4.18)$$

The system can be imagined as a linear system with the transfer function $H_f(\Omega, z)$, and the optical signal propagation in a single-mode fiber can be summarized as

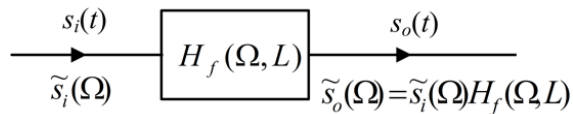


Figure 4-1 Optical signal propagation model in a single mode fiber

The above result can also be derived from the Nonlinear Schrodinger Equation originally from the governor of all electromagnetic phenomenon---Maxwell's equation [31].

For convenience we neglect the phase $\beta_0 z$ and group delay $\beta_1 z$ because both of them would only produce a phase delay of the carrier signal plus a time delay of the modulation signal with no influence on the distortion or degradation of the signal. Therefore, we obtain

$$F(z,t) = \exp[i(\omega_c t)] \cdot \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t - (\frac{1}{2}\beta_2 \Omega^2)z)] d\Omega. \quad (4.19)$$

Normally when we consider the output of a modulated single mode laser by a message signal as the incident light of the SMF, we usually express the combination of the carrier wave and the field envelope, ignoring the transverse field. So the input optical field can be written as

$$E_{in}(t) = F(0,t) = \exp[i(\omega_c t)] \cdot \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t)] d\Omega. \quad (4.20)$$

To better imply that this is a modulated optical signal, we usually write it in the following form:

$$E_{in}(t) = A_{in}(t) \exp[i(\omega_c t + \varphi_{in}(t))]. \quad (4.21)$$

If it is the AM signal, the amplitude part $A_{in}(t)$ will be time variant and the phase part $\varphi_{in}(t)$ will be time invariant; if it is the PM or FM signal, the amplitude $A_{in}(t)$ will stay constant while the phase part $\varphi_{in}(t)$ would change with time. And the field envelope of the input signal has 2 expressions:

$$E_{in}(t) = \exp[i(\omega_c t)] s(0,t), \quad (4.22)$$

and

$$s(0,t) = \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t)] d\Omega = A_{in}(t) \exp[i(\varphi_{in}(t))]. \quad (4.23)$$

Following the previous results, the output signal can be expressed as

$$\begin{aligned} s(z,t) &= \int_{-\infty}^{\infty} S(\Omega) H_f(\Omega, z) \exp[i(\Omega t)] d\Omega \\ &= \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t - i(\frac{1}{2} \beta_2 \Omega^2) z)] d\Omega \\ &= \exp[i(\frac{1}{2} \beta_2 z \frac{d^2}{dt^2})] A_{in}(t) \exp[i(\varphi_{in}(t))], \end{aligned} \quad (4.24)$$

and

$$\begin{aligned} E_{out}(t) &= \exp[i(\omega_c t)] s(z,t) \\ &= \exp[i(\omega_c t)] \int_{-\infty}^{\infty} S(\Omega) H_f(\Omega, z) \exp[i(\Omega t)] d\Omega \\ &= \exp[i(\omega_c t)] \int_{-\infty}^{\infty} S(\Omega) \exp[i(\Omega t - i(\frac{1}{2} \beta_2 \Omega^2) z)] d\Omega \\ &= \exp[i(\omega_c t)] \exp[i(\frac{1}{2} \beta_2 z \frac{d^2}{dt^2})] \{A_{in}(t) \exp[i(\varphi_{in}(t))]\}. \end{aligned} \quad (4.25)$$

The operator $\exp[i(\frac{1}{2} \beta_2 z \frac{d^2}{dt^2})]$ will be operated on the $A_{in}(t) \exp[i(\varphi_{in}(t))]$, which is $s(0,t)$. It can also be concluded in the form

$$E_{out}(t) = \exp[i(\omega_c t)] s(z,t) = A_{out}(t) \exp[i(\omega_c t + \varphi_{out}(t))]. \quad (4.26)$$

According to reference [32], with the small signal analysis, the difference between the output and input signal is small enough

$$\Delta E(t) = |E_{out}(t) - E_{in}(t)| \ll |E_{in}(t)|. \quad (4.27)$$

Therefore, the following relationship is obtained:

$$A_{out}(t)^2 = A_{in}(t)^2 + 2 \operatorname{Re} \{A_{in}(t) \exp[-i(\varphi_{in}(t))] (\exp[i(\frac{1}{2} \beta_2 z \frac{d^2}{dt^2})] - 1) [A_{in}(t) \exp[i(\varphi_{in}(t))]]\} \quad (4.28)$$

$$\varphi_{out}(t) = \varphi_{in}(t) + \text{Im}\left(\frac{(\exp[i(\frac{1}{2}\beta_2 z \frac{d^2}{dt^2})] - 1)A_{in}(t)\exp[i(\varphi_{in}(t))]}{A_{in}(t)\exp[i(\varphi_{in}(t))]}.\right) \quad (4.29)$$

From the above we can conclude that the group velocity dispersion (GVD) parameter β_2 has the opposite sign with dispersion parameter D . The fiber exhibits normal dispersion when $D < 0$ whereas anomalous dispersion occurs when $D > 0$ [33]. In this thesis we apply a G.652.D single mode fiber with $D = 15 \text{ ps} / (\text{nm} \cdot \text{km})$, exhibiting anomalous dispersion. In the anomalous dispersion regime, high frequency components of the signal travel faster than low frequency components. Thus the frequency components near the leading edge arrive early and frequency components near the tailing edge arrive late. This explains why the pulse is broadened at the fiber output for a single pulse. On the contrary, in the normal dispersion regime, the situation is the opposite [7].

4.2 Dispersion of the FM System

4.2.1. System Configuration

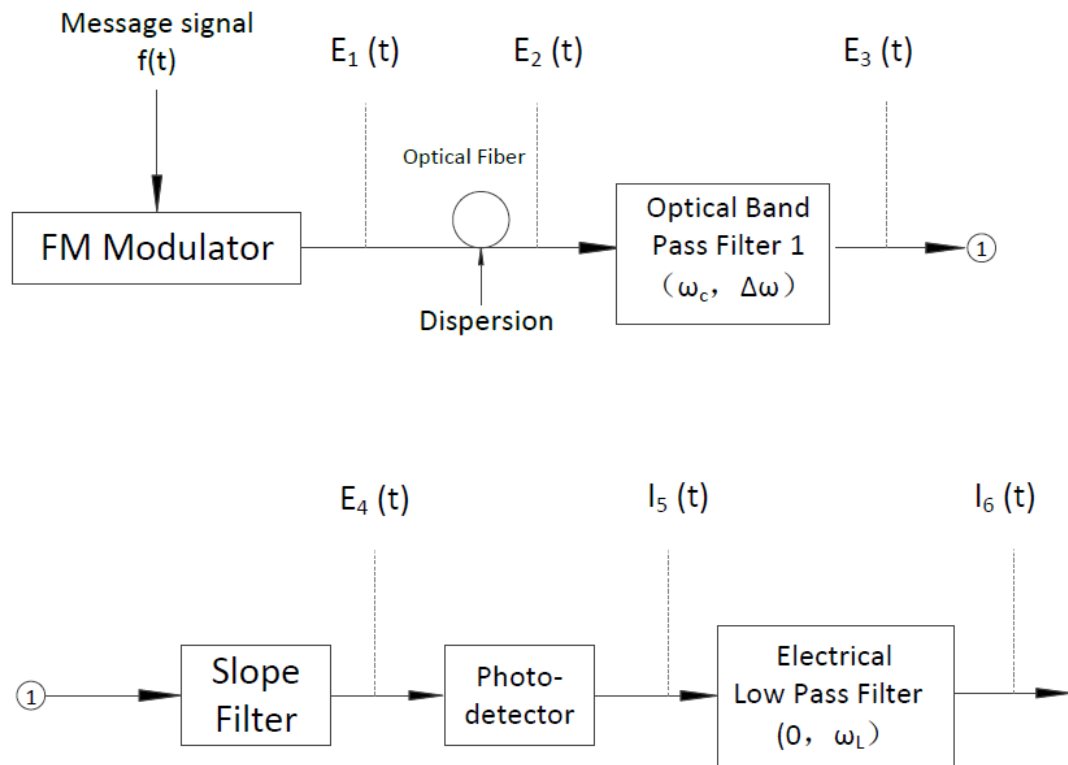


Figure 4-2 Schematic model of FM communication system when dispersion is focused

The FM system for the discovery of dispersion impairment is shown in Figure 4-2. It consists of a FM laser, the dispersive fiber ignoring fiber loss and without amplifiers, an optical filter with the 3dB bandwidth equal to the bandwidth of the frequency modulated signal, a differential discriminator, the photo detector assumed as ideal square-law detector and an electrical low pass filter with the 3dB bandwidth between 5GHz-10GHz

for the bit rate 10Gbps. The central frequency and the bandwidth of the optical band pass filter 1 are denoted as ω_c and $\Delta\omega$, respectively; the central frequency and the bandwidth of the electrical low pass filter 2 is 0 and ω_L , respectively.

It is similar to the previous FM scheme although for this chapter we induced the photo detector and the electrical low pass filter to replace the narrow band optical filter as a detection system. The reason for this change is because it is necessary to draw the eye diagram in this chapter for the purpose of analyzing the quality of the received analog signal. The photo-detector serves as a linear optical-to-electrical (O/E) converter and the output current of it should be directly proportional to the input optical power.

The AM system serves as a compare group and the detail of the structure is analyzed as follows.

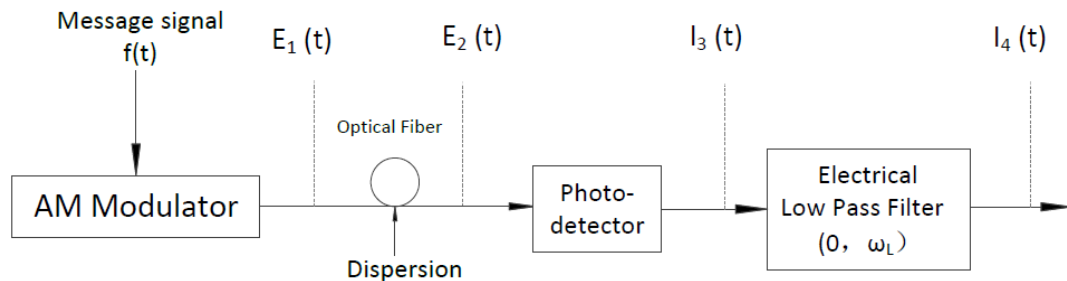


Figure 4-3 Schematic model of AM communication system when dispersion is focused

The structure of the AM system for the discovery of dispersion impairment is shown in Figure 4-3. It consists of an AM laser, the dispersive fiber without fiber loss and amplifier, a photo detector assumed as ideal square-law detector and an electrical low

pass filter with the 3dB bandwidth between 5GHz-10GHz for the bit rate 10Gbps. The central frequency and the bandwidth of the Electrical Band Pass Filter 2 is 0 and ω_L , respectively.

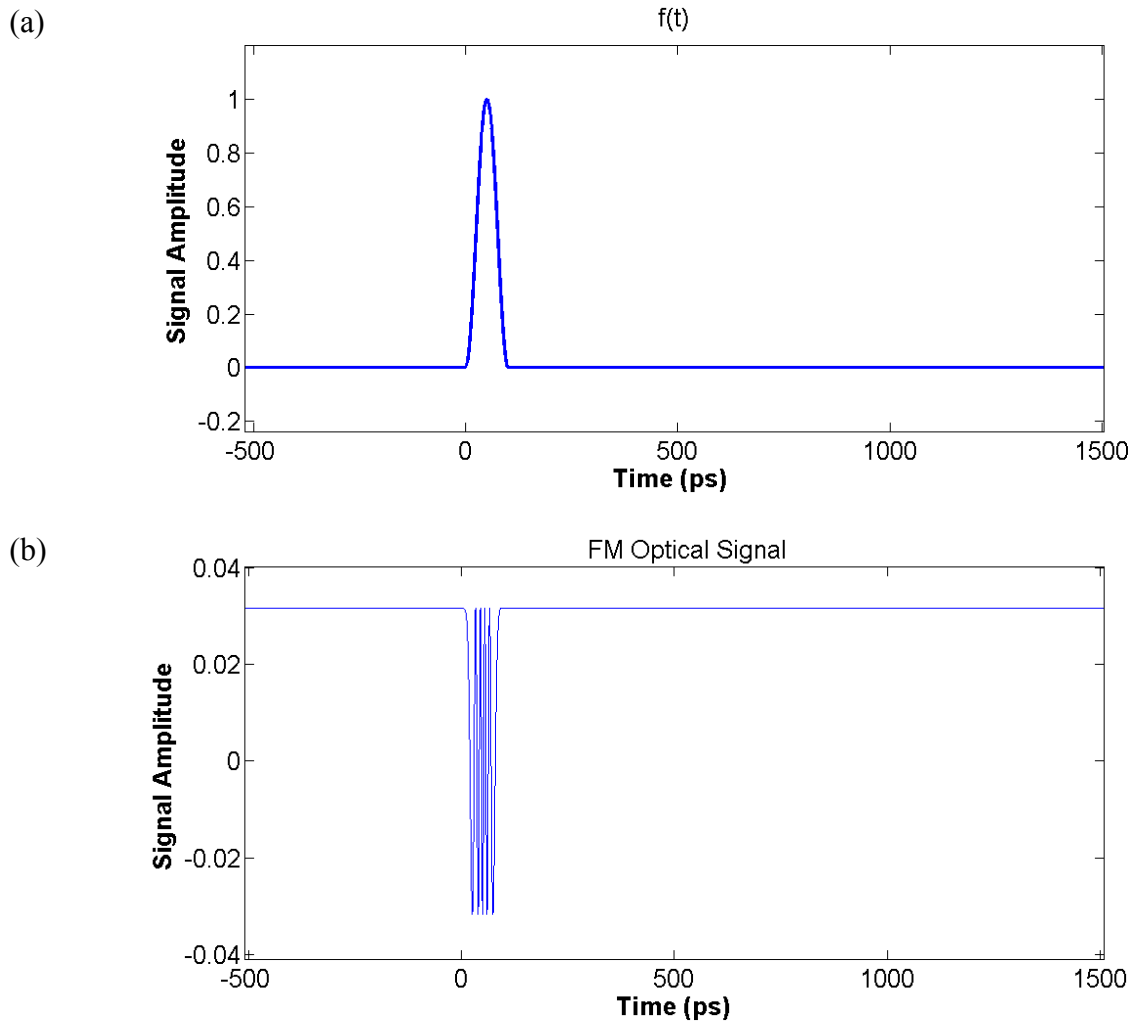
4.2.2. Selection of Optical Differentiator

Whether LEF or TEF is chosen as the frequency differentiator in the FM system makes a significant difference when we focus on the dispersion impairment, because the distortion induced by dispersion strongly hinges on the value of the frequency, while the frequency preference is the main difference between the two differentiators.

In order to analyze the anti-dispersion property of the FM system with LEF and TEF separately, we listed the transmission process of the message signal with a one bit single pulse, made comparison of the pulse broadening extent, and finally chose the one that is more suitable for the FM system to mitigate the effect of dispersion.

Previously we have mentioned that in this case for anomalous dispersion fiber ($D>0$), the high frequency components travel faster than low frequency components and the difference in the time domain will increase with the growth of transmission length. It can be drawn from the result of the simulation that the phase term $\frac{d\phi(t)}{dt}$ runs faster than the message signal $f(t)$, indicating that the higher the frequency component the more severe the broadening effect. Consequently, TEF outperforms the LEF because it preserves the low frequency components with less broadening effect to represent the input signal rather than the high frequency components.

To show the process of the pulse broadening in a simplified manner, a single pulse with the bit interval of 100ps is injected to the system as the message signal $f(t)$ and the transmission length is 50km with the dispersion parameter to be $D = 15 \text{ ps} / (\text{nm} \cdot \text{km})$.



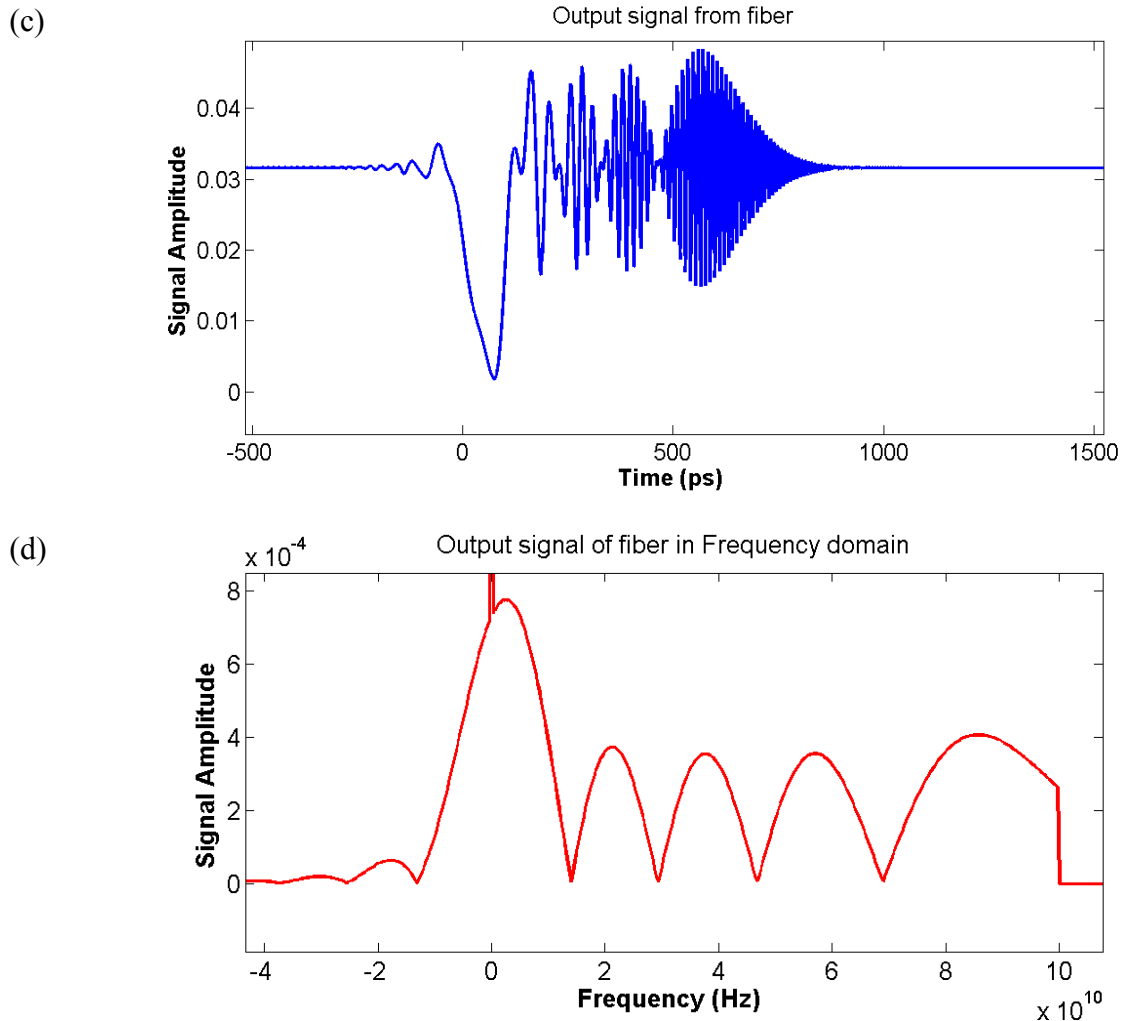


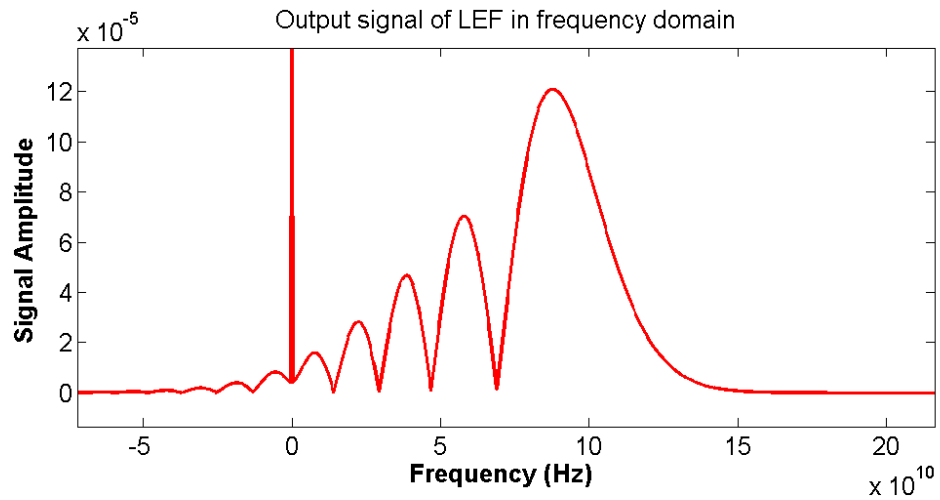
Figure 4-4 (a) Message signal $f(t)$ (b) FM optical signal $E_1(t)$ (c) The output signal from anomalous dispersive fiber $E_2(t)$ (d) Frequency domain of output signal from fiber

Figure 4-4 depicts the message signal (a), the input FM optical signal of fiber (b), the output signal from fiber in the time domain (c) and frequency domain (d), respectively. (c) is the consequence of the fact that low frequency components travel slower than high frequency components in anomalous dispersive fibers. (d) shows the frequency domain of

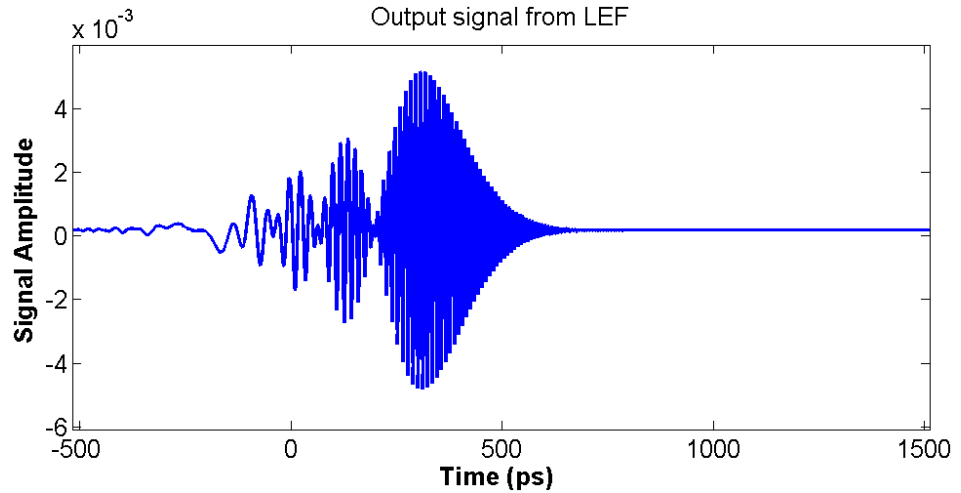
the output signal. The frequency 0 here denotes the carrier frequency so that its amplitude is higher than rest of the frequency components.

A. If we apply LEF as the optical differentiator, the detail of the following signals are as below:

(a)



(b)



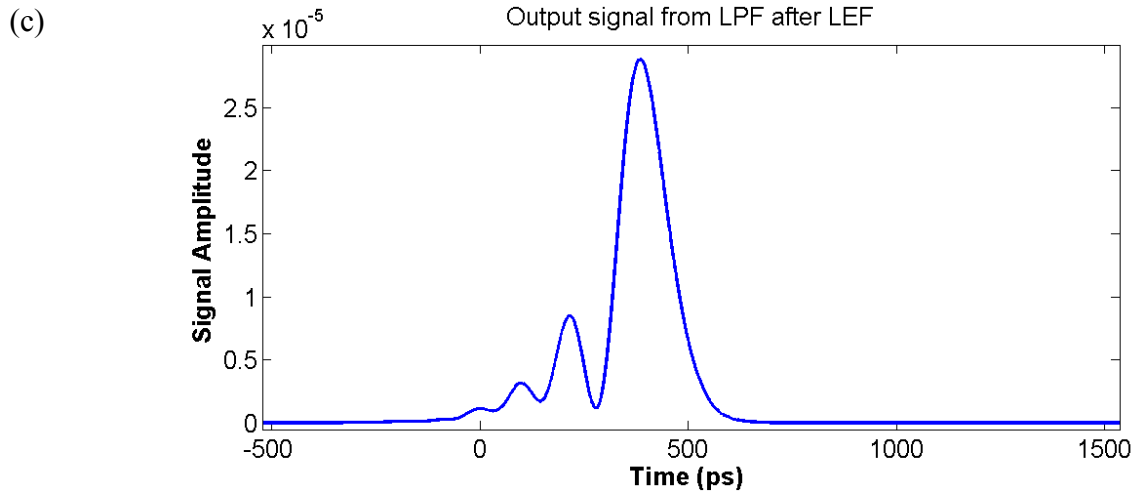


Figure 4-5 (a) Frequency domain of output signal from LEF (b) Time domain of output signal from LEF (c) The output signal from electrical low pass filter $I_6(t)$

If the differentiator is a LEF (Leading Edge Filter), the output signal in the frequency domain (a) and in the time domain (b) as well as the final output signal from the electrical low pass filter of receiver (c) is shown in Figure 4-5 respectively. From (a) it is clear that the LEF suppress the low frequency component. (b) and (c) depict the shape and envelope of the high frequency component of the fiber output, respectively.

B. If we apply TEF as the optical differentiator, the details of the following signals are as follows:

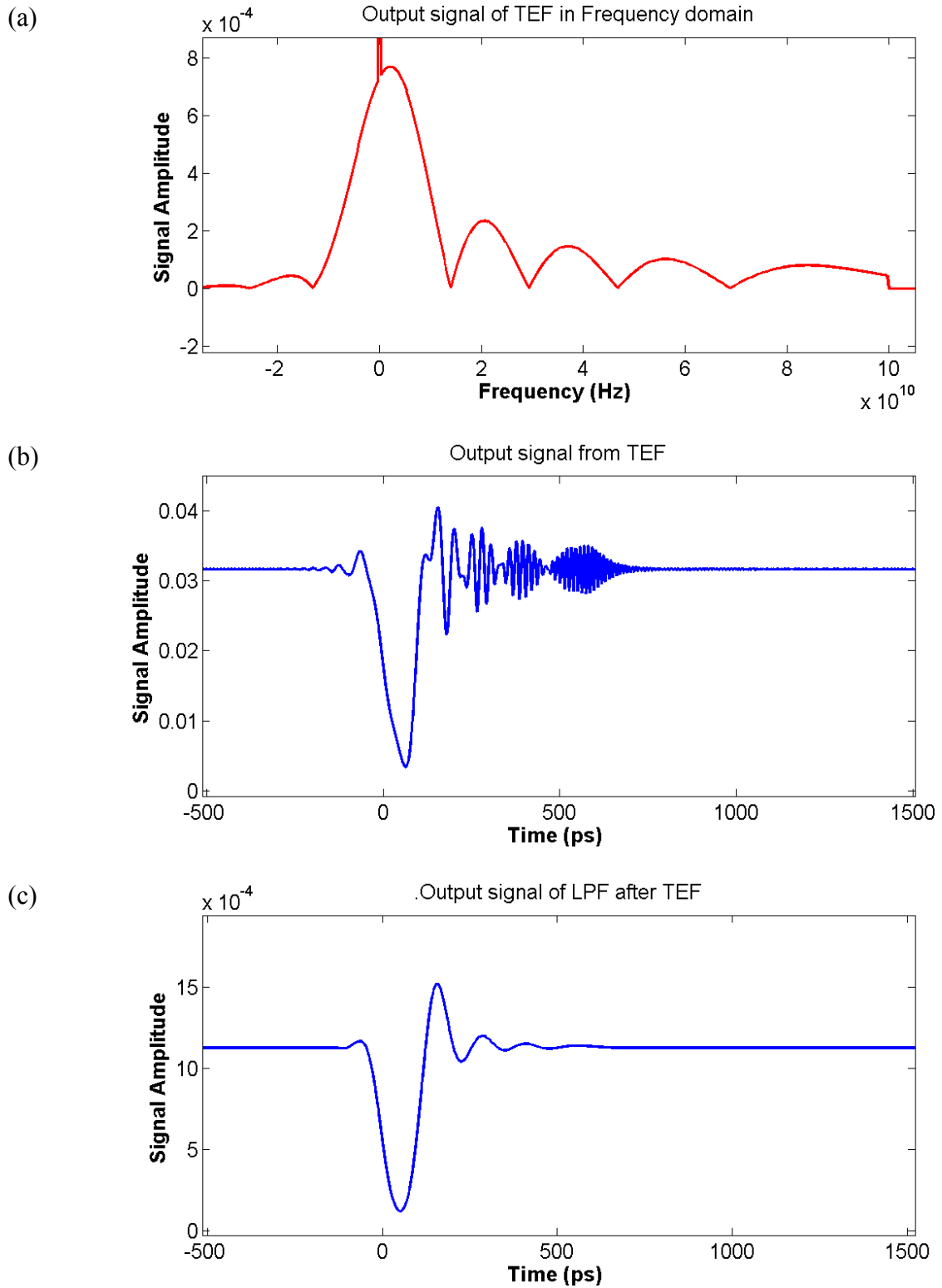


Figure 4-6 (a) Frequency domain of output signal from TEF, (b) Time domain of output signal from TEF, (c) The output signal from electrical low pass filter $I_6(t)$

If the differentiator is a TEF (Tail Edge Filter), the output signal in the frequency domain (a) and in the time domain (b) as well as the final output signal from the electrical low pass filter of receiver (c) is shown in Figure 4-6, respectively. This time the TEF suppresses the high frequency component as shown in (a). And it suffices to say that (b) and (c) shows the shape and envelope of the low frequency component of the fiber output.

The width of the input message pulse is equal to 100ps, which is the inversion of 10 Gb/s bit rate. After 50km, the output pulse will be broadened to about 600ps if we apply the LEF as the frequency discriminator in the FM system, while the width of the output pulse will be less than 300 ps if we apply TEF in the optical transmission system. This implies that the ghost pulses are significantly suppressed in the case of TEF as compared to LEF.

In conclusion, TEF is superior to LEF, offering higher transmission distance and narrower broadening effect for a single pulse. Thus we would apply TEF in our FM scheme.

4.2.3. Analytical Expression

Same as previously discussed, at Port 1, the frequency modulated optical signal $E_1(t)$ is a non-linear function of the message signal $f(t)$ and it can be expressed as

$$E_1(t) = A \exp[i(\omega_c t + k_f \int f(t) dt + \varphi_i)], \quad (4.30)$$

assuming the initial phase is φ_i , the magnitude of the carrier wave is constant A, and ω_c is the carrier angular frequency. According to (4.21), when it is injected into the dispersive fiber, the expression can also be written as

$$E_1(t) = A_{in}(t) \exp[i(\omega_c t + \varphi_{in}(t))]. \quad (4.31)$$

Since the corresponding term should be equal, we have

$$A_{in}(t) = A, \quad (4.32)$$

and

$$\varphi_{in}(t) = k_f \int f(t) dt + \varphi_i. \quad (4.33)$$

Following the rule as derived above, after it passes through the dispersive fiber link the optical signal at Port 2 should be expressed in the form of

$$E_2(t) = A_{out}(t) \exp[i(\omega_c t + \varphi_{out}(t))], \quad (4.34)$$

where the amplitude term $A_{out}(t)$ and phase term $\varphi_{out}(t)$ would be specified as follows:

$$\begin{aligned}
A_{out}^2(t) &= A_{in}^2(t) + 2\text{Re}[A_{in}(t)e^{-j\varphi_m(t)}(e^{\frac{j}{2}\beta_2 L \frac{d^2}{dt^2}} - 1)A_{in}(t)e^{j\varphi_m(t)}] \\
&= A^2 + 2\text{Re}[A^2 e^{-j\varphi_m(t)}(e^{\frac{j}{2}\beta_2 L \frac{d^2}{dt^2}} - 1)e^{j\varphi_m(t)}] \\
&= A^2 + 2\text{Re}[A^2 e^{-j(k_f \int f(t) dt + \varphi_i)}(e^{\frac{j}{2}\beta_2 L \frac{d^2}{dt^2}} - 1)e^{j(k_f \int f(t) dt + \varphi_i)}] \\
&= \begin{cases} A^2, \beta_2 L = 0 \\ 2A^2 \text{Re}[e^{-j(k_f \int f(t) dt + \varphi_i)} e^{\frac{j}{2}\beta_2 L \frac{d^2}{dt^2}} e^{j(k_f \int f(t) dt + \varphi_i)}], \beta_2 L \neq 0 \end{cases} \\
&= \begin{cases} A^2, \beta_2 L = 0 \\ 2A^2 \text{Re}[K(t)], \beta_2 L \neq 0 \end{cases},
\end{aligned} \tag{4.35}$$

$$\begin{aligned}
\varphi_{out}(t) &= \varphi_{in}(t) + \text{Im}\left[\frac{(e^{\frac{j}{2}\beta_2 L \frac{d^2}{dt^2}} - 1)e^{j\varphi_m(t)}}{e^{j\varphi_m(t)}}\right] \\
&= (k_f \int f(t) dt + \varphi_i) + \text{Im}\left[\frac{(e^{\frac{j}{2}\beta_2 L \frac{d^2}{dt^2}} - 1)e^{j(k_f \int f(t) dt + \varphi_i)}}{e^{j(k_f \int f(t) dt + \varphi_i)}}\right].
\end{aligned} \tag{4.36}$$

For better understanding, it can also be written as

$$E_2(t) = A_{out}(t) \exp[i(\omega_c t + k_f \int f(t) dt + \varphi(t))], \tag{4.37}$$

in which the time variant phase term induced by the dispersion is given by

$$\begin{aligned}
\varphi(t) &= \varphi_i + \text{Im}\left[\frac{(e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2}} - 1)e^{j(k_f \int f(t) dt + \varphi_i)}}{e^{j(k_f \int f(t) dt + \varphi_i)}}\right] \\
&= \begin{cases} \varphi_i, \beta_2 L = 0 \\ \varphi_i + \text{Im}[e^{-j(k_f \int f(t) dt + \varphi_i)} (e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2}} - 1)e^{j(k_f \int f(t) dt + \varphi_i)}], \beta_2 L \neq 0 \end{cases} \\
&= \begin{cases} \varphi_i, \beta_2 L = 0 \\ \varphi_i + \text{Im}[K(t)], \beta_2 L \neq 0 \end{cases} .
\end{aligned} \tag{4.38}$$

We can introduce $K(t)$ as the substitution of the following expression:

$$K(t) = e^{-j(k_f \int f(t) dt + \varphi_i)} e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2}} e^{j(k_f \int f(t) dt + \varphi_i)} . \tag{4.39}$$

Next, the signal part would not change after passing through the optical wide band pass filter

$$E_3(t) = A_{out}(t) \exp[i(\omega_c t + k_f \int f(t) dt + \varphi(t))]. \tag{4.40}$$

A. If the slope filter is a LEF

$$H_1(\omega) = \alpha(\omega - \omega_0), \tag{4.41}$$

the following expression will be valid

$$\tilde{E}_4(\omega) = \tilde{E}_3(\omega)H_1(\omega) = \alpha\omega \tilde{E}_3(\omega) - \alpha\omega_0 \tilde{E}_3(\omega). \tag{4.42}$$

$\tilde{E}_3(\omega)$ and $\tilde{E}_4(\omega)$ are the input and output signal of the LEF in the frequency domain, respectively. Applying the inverse Fourier transform, we obtain the output signal in the time domain, by setting $\omega_0 = \omega_c$:

$$\begin{aligned}
E_4(t) &= -j\alpha \frac{d}{dt} E_3(t) - \alpha\omega_0 E_3(t) & (4.43) \\
&= -j\alpha \left(\frac{dA_{out}(t)}{dt} \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))] \right. \\
&\quad \left. + jA_{out}(t)(\omega_c + k_f f(t) + \frac{d\varphi(t)}{dt}) \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))] - \alpha\omega_0 E_3(t) \right) \\
&= \left[-j\alpha \frac{dA_{out}(t)}{dt} \frac{1}{A_{out}(t)} + \alpha(\omega_c - \omega_0 + k_f f(t) + \frac{d\varphi(t)}{dt}) \right] E_3(t) \\
&= \left[-j\alpha \frac{dA_{out}(t)}{dt} + A_{out}(t) \alpha(\omega_c - \omega_0 + k_f f(t) + \frac{d\varphi(t)}{dt}) \right] \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))] \\
&= \left[-j\alpha \frac{dA_{out}(t)}{dt} + A_{out}(t) \alpha(k_f f(t) + \frac{d\varphi(t)}{dt}) \right] \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))].
\end{aligned}$$

B. If the slope filter is a TEF,

$$H_1(\omega) = 1 - \alpha(\omega - \omega_0) \quad (4.44)$$

the following expression will be valid

$$\tilde{E}_4(\omega) = \tilde{E}_3(\omega) H_1(\omega) = \tilde{E}_3(\omega) - \alpha\omega \tilde{E}_3(\omega) + \alpha\omega_0 \tilde{E}_3(\omega) \quad (4.45)$$

$\tilde{E}_3(\omega)$ and $\tilde{E}_4(\omega)$ are the input and output signal of the TEF in the frequency domain, respectively. Applying the inverse Fourier transform to (4.45), we obtain the output signal in the time domain,

$$\begin{aligned}
E_4(t) &= E_3(t) + j\alpha \frac{d}{dt} E_3(t) + \alpha \omega_0 E_3(t) & (4.46) \\
&= E_3(t) + j\alpha \left(\frac{dA_{out}(t)}{dt} \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))] \right. \\
&\quad \left. + jA_{out}(t)(\omega_c + k_f f(t) + \frac{d\varphi(t)}{dt}) \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))] + \alpha \omega_0 E_3(t) \right) \\
&= E_3(t) + [j\alpha \frac{dA_{out}(t)}{dt} \frac{1}{A_{out}(t)} - \alpha(\omega_c - \omega_0 + k_f f(t) + \frac{d\varphi(t)}{dt})] E_3(t) \\
&= [j\alpha \frac{dA_{out}(t)}{dt} + A_{out}(t)(1 - \alpha(\omega_c - \omega_0 + k_f f(t) + \frac{d\varphi(t)}{dt}))] \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))] \\
&= [j\alpha \frac{dA_{out}(t)}{dt} + A_{out}(t)(1 - \alpha(k_f f(t) + \frac{d\varphi(t)}{dt}))] \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))],
\end{aligned}$$

setting $\omega_0 = \omega_c$. Simplify the equation by writing it in the form of

$$E_4(t) = A_4(t) \exp[i(\omega_c t + \varphi_4(t))]. \quad (4.47)$$

The output of the slope filter is collected by a photo detector which transmits the optical signal into the current signal. The output current from the photo detector with responsivity R would become

$$I_5(t) = R |E_4(t)|^2 = R |A_4(t)|^2. \quad (4.48)$$

When there is no dispersion impairment, $GVD=0$ or $L=0$,

$$\begin{aligned}
I_5(t) &= R |E_4(t)|^2 = R |A_4(t)|^2 & (4.49) \\
&= \begin{cases} 2A^2 R \alpha^2 k_f^2 f(t)^2, \beta_2 L = 0 [\text{for LEF}] \\ 2A^2 R \alpha^2 k_f^2 f(t)^2 - 4A^2 R \alpha k_f f(t) + 2A^2 R, \beta_2 L = 0 [\text{for TEF}]. \end{cases}
\end{aligned}$$

On the contrary, if the dispersion does exist, the expression would become

$$\begin{aligned}
I_5(t) &= R |A_4(t)|^2 \tag{4.50} \\
&= \left\{ \begin{aligned}
&f(t)^2 2A^2 R \alpha^2 k_f^2 \operatorname{Re}[K(t)] + f(t) 4A^2 R \alpha^2 k_f \operatorname{Re}[K(t)] \frac{d \operatorname{Im}[K(t)]}{dt} \\
&+ 2A^2 R \alpha^2 \operatorname{Re}[K(t)] \left(\frac{d \operatorname{Im}[K(t)]}{dt} \right)^2 + A^2 R \alpha^2 \frac{d \operatorname{Re}[K(t)]}{2 \operatorname{Re}[K(t)]}, \beta_2 L \neq 0 [\text{for LEF}] \\
&f(t)^2 2A^2 R \alpha^2 k_f^2 \operatorname{Re}[K(t)] + f(t) 4A^2 R \alpha k_f \operatorname{Re}[K(t)] \left(\alpha \frac{d \operatorname{Im}[K(t)]}{dt} - 1 \right) \\
&+ 2A^2 R \operatorname{Re}[K(t)] \left\{ 1 - 2\alpha \frac{d \operatorname{Im}[K(t)]}{dt} + \alpha^2 \left(\frac{d \operatorname{Im}[K(t)]}{dt} \right)^2 \right\} \\
&+ A^2 R \alpha^2 \frac{d \operatorname{Re}[K(t)]}{2 \operatorname{Re}[K(t)]}, \beta_2 L \neq 0 [\text{for TEF}]
\end{aligned} \right.
\end{aligned}$$

If an optical amplitude limiter is inset to the front part of the slope filter device, the input signal, the following relationships will be valid,

$$A_{out}(t) = A_o \tag{4.51}$$

$$E_3(t) = A_o \exp[i(\omega_c t + k_f \int f(t) dt + \varphi(t))]. \tag{4.52}$$

$$E_4(t) = \begin{cases} [A_o \alpha(k_f f(t) + \frac{d\varphi(t)}{dt})] \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))], [\text{for LEF}] \\ [A_o (1 - \alpha(k_f f(t) + \frac{d\varphi(t)}{dt}))] \exp[j(\omega_c t + k_f \int f(t) dt + \varphi(t))], [\text{for TEF}] \end{cases} \tag{4.53}$$

$$I_5(t) = R |E_4(t)|^2 = \begin{cases} R [A_o \alpha(k_f f(t) + \frac{d\varphi(t)}{dt})]^2, [\text{for LEF}] \\ R [A_o (1 - \alpha(k_f f(t) + \frac{d\varphi(t)}{dt}))]^2, [\text{for TEF}] \end{cases} \tag{4.54}$$

The output of the photo detector is directed to a low pass filter. The low pass filter will change the current signal by filtering out the frequency component out of the pass band. The output current signal can be expressed as

$$I_6 = I_5(t) * h_2(t), \quad (4.55)$$

where $h_2(t)$ is the impulse response of the low pass filter, and in this chapter we apply the 6th order butter worth low pass filter with the cut off frequency 8.5GHz.

As for the AM system, the analytical expression for each interface can be derived as follows.

At Port 1, the amplitude modulated optical signal $E_0(t)$ is a linear function of the message signal $f(t)$ and it can be written as

$$E_1(t) = A \cdot f(t) \exp[i(\omega_c t + \varphi_i)], \quad (4.56)$$

where the initial phase is φ_i , the magnitude of the carrier wave is A, ω_c is the carrier angular frequency. It is injected into the dispersive fiber, so it can also be written as

$$E_1(t) = A_{in}(t) \exp[i(\omega_c t + \varphi_{in}(t))]. \quad (4.57)$$

The corresponding term should be equal, so

$$A_{in}(t) = A \cdot f(t), \quad (4.58)$$

$$\varphi_{in}(t) = \varphi_i. \quad (4.59)$$

Following the rule derived above, after it goes through the dispersive fiber link the optical signal at Port 2 would be

$$E_2(t) = A_{out}(t) \exp[i(\omega_c t + \varphi_{out}(t))]. \quad (4.60)$$

The amplitude term $A_{out}(t)$ and phase term $\varphi_{out}(t)$ in the above equation can be specified as follows:

$$\begin{aligned} A_{out}^2(t) &= A_{in}^2(t) + 2 \operatorname{Re}[A_{in}(t) e^{-j\varphi_{in}(t)} (e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2}} - 1) A_{in}(t) e^{j\varphi_{in}(t)}] \\ &= A^2 f(t)^2 + 2 \operatorname{Re}[A f(t) e^{-j\varphi_i} (e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2}} - 1) A f(t) e^{j\varphi_i}] \\ &= A^2 f(t)^2 + 2 \operatorname{Re}[A^2 f(t) (e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2}} - 1) f(t)] \\ &= \begin{cases} A^2 f(t)^2, \beta_2 L = 0 \\ -A^2 f(t)^2 + 2A^2 f(t) \operatorname{Re}[e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2} f(t)}], \beta_2 L \neq 0 \end{cases}, \end{aligned} \quad (4.61)$$

$$\begin{aligned} \varphi_{out}(t) &= \varphi_{in}(t) + \operatorname{Im}\left[\frac{(e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2}} - 1) e^{j\varphi_{in}(t)}}{e^{j\varphi_{in}(t)}}\right] \\ &= \varphi_i + \operatorname{Im}\left[\frac{(e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2}} - 1) e^{j\varphi_i}}{e^{j\varphi_i}}\right] \\ &= \varphi_i + \operatorname{Im}\left[\frac{1 - e^{j\varphi_i}}{e^{j\varphi_i}}\right]. \end{aligned} \quad (4.62)$$

The second term of $A_{out}^2(t)$ describes the extent of the pulse broadening effect quantitatively.

The output of the slope filter is collected by a photo detector(PD). The output current from the PD with responsivity R would become

$$\begin{aligned}
I_3(t) &= R |E_2(t)|^2 = RA_{out}(t)^2 & (4.63) \\
&= \begin{cases} A^2 Rf(t)^2, \beta_2 L = 0 \\ -A^2 Rf(t)^2 + 2A^2 Rf(t) \operatorname{Re}[e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2} f(t)}], \beta_2 L \neq 0 \end{cases}
\end{aligned}$$

It is proportional to the square of the message signal $f(t)^2$ and $A^2 R$ is a constant once the system is fixed. Dispersion impairment can be expressed as the distortion term $2A^2 Rf(t) \operatorname{Re}[e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2} f(t)}]$, which is related to the magnitude A corresponding to the peak launch power A^2 , the responsivity of PD R , value of GVD β_2 , the transmission distance L and also the message signal $f(t)$. There would be no dispersive distortion if we set GVD or the transmission length equal to zero, and in these circumstances the output current would be proportional to $f(t)^2$ so that the message signal can be well extracted from it.

The low pass filter will change the current signal by filtering out the frequency component from the pass band, so the output current signal can be expressed as

$$I_4 = I_3(t) * h_2(t), \quad (4.64)$$

where $h_2(t)$ is the impulse response of the low pass filter. Same as FM system, we also apply the 6th order butter worth low pass filter with the cut off frequency 8.5GHz. Ideally the low pass filter would not affect the input current since the function of LPF is to filter out the noise disturbance.

4.3 Comparison on Dispersion Performance

In AM systems, the output current signal can be expressed as

$$\begin{aligned}
 I_3(t) &= R |E_2(t)|^2 = RA_{out}(t)^2 \\
 &= \begin{cases} A^2 R f(t)^2, \beta_2 L = 0 \\ -A^2 R f(t)^2 + 2A^2 R f(t) \operatorname{Re}[e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2} f(t)}], \beta_2 L \neq 0 \end{cases}
 \end{aligned} \tag{4.65}$$

On the contrary, in FM systems, the output current signal is given by:

- a. When there is no dispersion impairment, $GVD=0$ or $L=0$,

$$\begin{aligned}
 I_5(t) &= R |E_4(t)|^2 = R |A_4(t)|^2 \\
 &= \begin{cases} f(t)^2 A^2 R \alpha^2 k_f^2, \beta_2 L = 0 [\text{for LEF}] \\ f(t)^2 A^2 R \alpha^2 k_f^2 - f(t) 2A^2 R \alpha k_f + A^2 R, \beta_2 L = 0 [\text{for TEF}] \end{cases}
 \end{aligned} \tag{4.66}$$

- b. If the dispersion does exist, the expression would be

$$\begin{aligned}
 I_5(t) &= R |E_4(t)|^2 = \begin{cases} R[A_0 \alpha(k_f f(t) + \frac{d\varphi(t)}{dt})]^2, \beta_2 L \neq 0 [\text{for LEF}] \\ R[A_0(1 - \alpha(k_f f(t) + \frac{d\varphi(t)}{dt}))]^2, \beta_2 L \neq 0 [\text{for TEF}] \end{cases} \\
 &= \begin{cases} R[A_0 \alpha(k_f f(t) + \frac{d}{dt} \operatorname{Im}[e^{-j(k_f \int f(t) dt + \varphi_1)} (e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2} f(t)} - 1) e^{j(k_f \int f(t) dt + \varphi_1)}))]^2, \beta_2 L \neq 0 [\text{for LEF}] \\ R[A_0(1 - \alpha(k_f f(t) + \frac{d}{dt} \operatorname{Im}[e^{-j(k_f \int f(t) dt + \varphi_1)} (e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2} f(t)} - 1) e^{j(k_f \int f(t) dt + \varphi_1)}))]^2, \beta_2 L \neq 0 [\text{for TEF}] \end{cases}
 \end{aligned} \tag{4.67}$$

From the complex mathematical derivation, AM system would come across with a pulse

broadening which can be represented by $2A^2 R f(t) \operatorname{Re}[e^{j\frac{1}{2}\beta_2 L \frac{d^2}{dt^2} f(t)}]$, with the message signal

tangled in it. As for the case in the FM system, fiber dispersion would cause an amplitude

distortion that can be specified as $A_0 \alpha \frac{d\varphi(t)}{dt}$. In other words, FM system converts the

phase distortion horizontally into the amplitude distortion vertically, which may largely depress the dispersion caused extension of a single pulse. This provides a significant edge for FM system because the broadening effect in the time domain, i.e. horizontally distortion, is a fatal degradation in the transmission system, whereas the vertically amplitude distortion can be easily handled with the help of digital signal processing technology. It is hard to quantify the extent of the degradation of input signal for each system, which makes the numerical results more valuable.

Since we have already drawn to the conclusion that TEF outperforms LEF in FM system, in the following contents the term “FM system” means the frequency modulated optical communication system with TEF frequency differentiator.

4.4 Modeling and Simulation

Due to the unavailability of an analytical solution of the nonlinear Schrodinger equation, abundant numerical algorithms can be applied to solve this nonlinear partial differential equation, such as the inverse scattering method, the finite difference methods, and the pseudo spectral methods. The one that employs the fast Fourier transform (FFT) algorithm to increase the computational speed has been extensively used, that is the split-step Fourier scheme (SSFS). The main idea of the SSFS is to separate the linear and nonlinear operations of the nonlinear Schrodinger equation in order to calculate the effect of them one at a time [34]. In this thesis, only the dispersion part is considered while ignoring the nonlinear effect of the optical fiber.

Utilizing the numerical method mentioned above, we conduct numerical simulations to calculate the maximum S in the presence of dispersion impairments for FM and AM systems.

4.4.1. Definition of Mask Area S of an Eye Diagram

The electrical signal after the low pass filter has a filtering effect, hence the eye diagram of the system is degraded with time jitter and noise effect. Typically an eye mask captures all of the output signal performance including rise and fall times, time jitter, duty cycle distortion (DCD) and noise. Thus we intend to use a single parameter representing the characteristics of the eye mask to replace all of those parameters.

International standard for different products category has different eye mask requirement. In passive optical network (PON) we usually follow the requirement established by IEEE 802.3ah [35], ITU-T F9.842 [36] or standard IEC 61280-2-2 [37]. The input waveform must remain outside the mask regions (fail regions) in order to comply with the industry standard. Any acquired data point that falls inside a fail region appears in red and is logged as a mask test failure [38]. Usually the test can be divided into two steps: 1) Use a template (mask) that consists of pass/fail region to verify that a displayed eye diagram complies with an industry-standard waveform shape within a certain amount of time or hit rate. 2) After it passes the first test, exaggerate the mask to create the margin and measure the maximum margin the signal can pass with a limited number of mask test failures [39].

In this work, we define a mask area (MA) parameter S , which is the area of the maximum allowable hexagon mask of the eye diagram, to represent the characteristics of

the eye diagram, allowing a trade-off among all of the parameters mentioned above. It is highly efficient since it quantifies both time and amplitude parameters in one measurement. In addition, it is also a combination of the traditional mask and margin test. After viewing IEEE 802.3ah standard and standard IEC 61280-2-2, we decided to apply the “eye mask auto margin” technique to compare the performance of two systems, and the hit ratio is set to be lower than 1/20000 hits per sample.

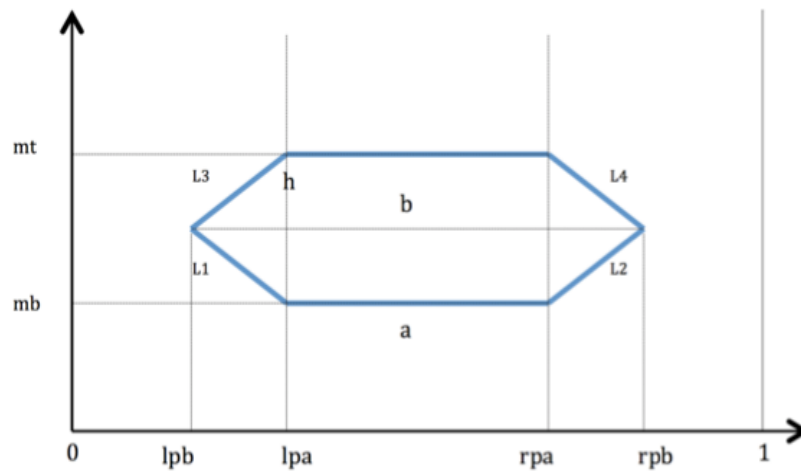


Figure 4-7 Definition of mask area S

Figure 4-7 explains the definition of an eye mask presented in 1 unit interval (UI). For example, if the final mask shows that $lpb = 0.253$ and $lpa = 0.348$, then we can calculate that $a = 2(0.5 - lpa) = 0.304$ and $b = 2(0.5 - lpb) = 0.53$, which means width **a** takes up 30.4% UI while width **b** takes up 53%UI. And **mt** means the mask top whereas the **mb** means the bottom of the mask.

The area of the mask MA can be denoted as

$$S = (mt - mb)(rpa - lpb). \quad (4.68)$$

For each eye diagram produced after the electrical receiver, we can search for an optimized eye mask that can best fit within the eye diagram and then calculate the area of that S area. It not only takes in to account the degree of opening of the eye diagram, but also the tolerance of the horizontal time jitter. With the same input peak launch power, the larger S corresponds to better performance of the optical communication system. The dimension of S is $mA \cdot ps$.

The reasons why we are not using BER (Bit Error Rate) to evaluate the performance of the system in this thesis are listed in the following aspects: In this chapter only the dispersion impairment is concerned and all of the noise is shut off, thus no random variable that can only be described by a probability distribution exists. Once the dispersion parameter and the transmission length are fixed, the eye diagram would be specified and that would not be largely dependent on the number of bit stream in the message signal or the number of tests. For example, the system transmission distance is 70km and the dispersion parameter $D = 15ps / (nm \cdot km)$, if the message signal can be accurately extracted (BER=0) from the receiver when the number of bits is 2^8-1 it should also be accurately extracted (BER=0) when the number of bits enlarged to 2^9-1 , $2^{10}-1$ or even $2^{12}-1$ regardless of the number of tests performed.

4.4.2. Simulation Model

The detail of modeling of the same component in this section is identical to that of Chapter 3, whereas some difference exists for the different components.

A pseudo-random bit sequence (PRBS) is generated at the beginning of the simulation. The message signal $f(t)$ is the bit sequence multiplied by raised cosine pulses, in which bit “1” (bit“0”) is represented by a pulse with amplitude 1 (0). And the modulator plays a role in loading the message signal to the frequency part of the carrier wave, forming the optical signal $E_1(t)$. This optical signal becomes the incident light into the dispersive fiber transmission path. Utilizing the split-step method, for each span, the input signal’s Fourier transform is multiplied by the transfer function of the dispersive fiber, and then the inverse Fourier transform is applied to achieve the output optical signal in the time domain $E_2(t)$. The wide band pass filter and optical differentiator in this numerical method are defined as the transfer function in the frequency domain. Thus the optical signal $E_3(t)$ and $E_4(t)$ can be obtained with the help of the convolution theorem. Next, $E_4(t)$ is collected by a photo-detector, which is assumed as an ideal square-law detector and the output electrical signal is denoted as $I_5(t)$. The electrical signal $E_5(t)$ at the output of the low pass filter can also be anticipated using the convolution theorem.

4.4.3. Parameter Setting

The following parameters were used for the simulation: A pseudo-random bit sequence (PRBS) of length $2^8 - 1$, a bit interval of 100ps, a bit rate of $B = 10\text{Gbit} / s$, and an operating wavelength of 1550nm. NRZ format is applied and the message signal $f(t)$ is based on the raised cosine pulses. The peak launch power is 0dBm. The frequency deviation range K_f for the FM signal is 100GHz. We apply the Corning SMF-28 single

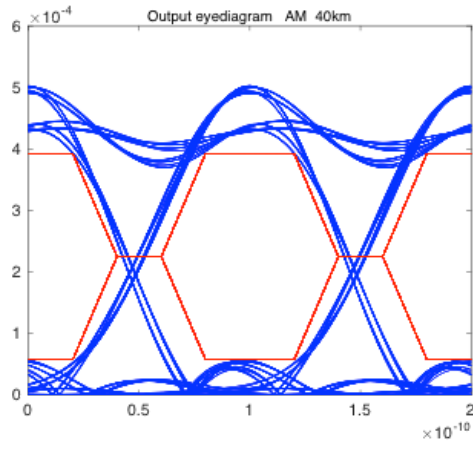
mode fiber with the dispersion coefficient D to be the value of $15 \text{ ps} / (\text{nm} \cdot \text{km})$, and the fiber loss is ignored in this chapter. The responsivity of photo detector is $R=0.9 \text{ A/W}$. The electrical receiver filter applies 6-th order butter worth low pass filter with the 3dB cut off bandwidth at 8.5GHz, which means $\omega_B = 0.85 \times \omega_m$. Laser phase noise, polarization effects, and receiver imperfections are also ignored in this chapter.

All results have been obtained considering the parameters above and unless otherwise stated, these parameters are used throughout this chapter.

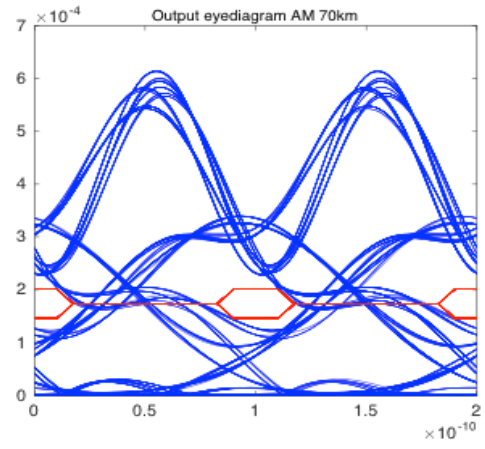
4.4.4. Results and Discussion

There are two factors that determine the relationship between the performance of the system and the length of the fiber: the fiber dispersion and the modulation format.

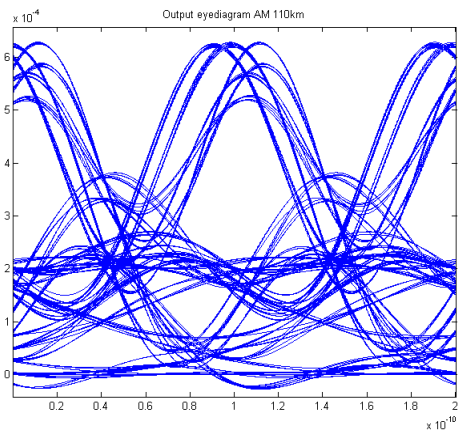
The eye diagram of the demodulated signal from both FM and AM systems with the transmission distance at 40km, 70km and 110km are shown in Figure 4-8 when the peak launch power for both systems are 0dBm. As can be noticed, the dispersion causes system performance degradation with the increase of transmission distance. When the distance is 40km, the AM system and the FM system have roughly the same performance. When the distance is extended to 70km, FM has higher dispersion tolerance than AM system and therefore it gives better performance. In the case when distance is 110km, only the FM system can survive while the AM output signal is completely distorted such that we cannot extract the message signal from it.



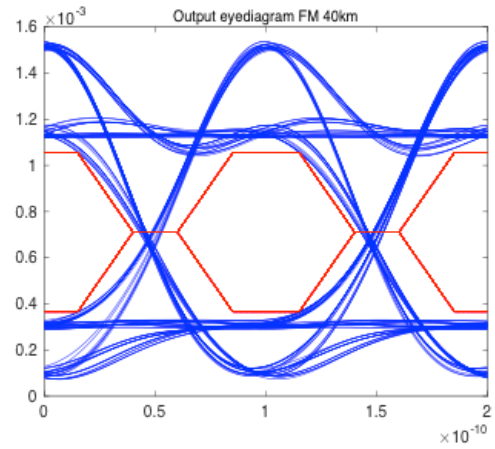
(a)



(b)



(c)



(d)

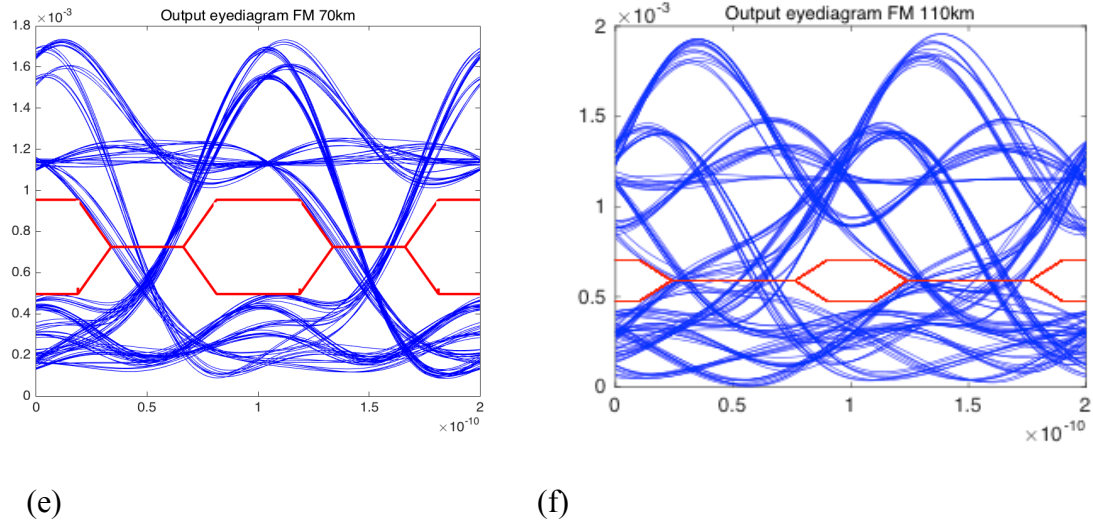


Figure 4-8 Eye diagram and maximum allowable hexagon mask for AM system with the transmission fiber length (a) 40km, (b) 70km, (c) 110km and FM system with the transmission fiber length (d) 40km, (e) 70km and (f) 110km, with peak launch power 0dBm.

To express the result in a more explicit way, Figure 4-9 is plotted, showing the MA value S versus the fiber transmission distance L for two cases, AM and FM scheme, when the peak launch power is 0dBm. When the transmission distance is shorter than 40km, the MA value S for both systems is stable and the message signal can be clearly extracted from the output current. However, when the transmission distance is longer, the MA value S for the FM system outperforms the AM system drastically. When the transmission distance is around 70 km, AM performance becomes severely degraded and the message cannot be well extracted from it, while FM performance is still good enough to transmit the message signal correctly. If we consider a required S of $2mA \cdot ps$, the

maximum distances for reliable data transmission is limited to 70km for AM systems, which can be extended to more than 110 km using an FM scheme instead.

In conclusion, FM has higher dispersion tolerance than AM systems and therefore it gives better performance in terms of fiber dispersion.

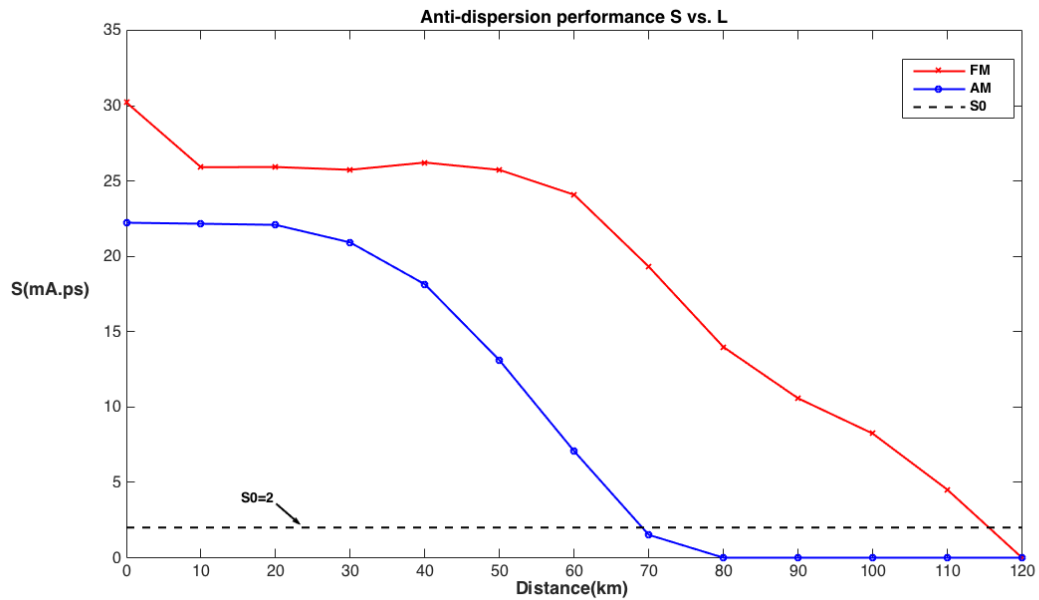


Figure 4-9 System performance MA value S versus fiber transmission distance when the peak launch power is 0dBm.

Chapter 5

System Simulation

In this chapter, a practical application with both noise and dispersion impairments are considered, showing the application prospect of the FM fiber-optic communication system. All the parameters are from the real-world short-span networks in order to exemplify that the novel FM system shows overall superiority to the traditional AM system as long as the wide band frequency modulation is satisfied. Numerical simulations of both systems illustrate that the FM system is able to effectively mitigate the signal distortions caused by both noise and dispersion.

5.1 System Configuration

Typically, the structure of FM and AM systems is described by the combination of Chapter 3 and Chapter 4. To be specific, here the amplifiers are spanned within the transmission link impaired by loss and dispersion, and the receiver component is complete. All the denotation can be inherited from chapter 4.

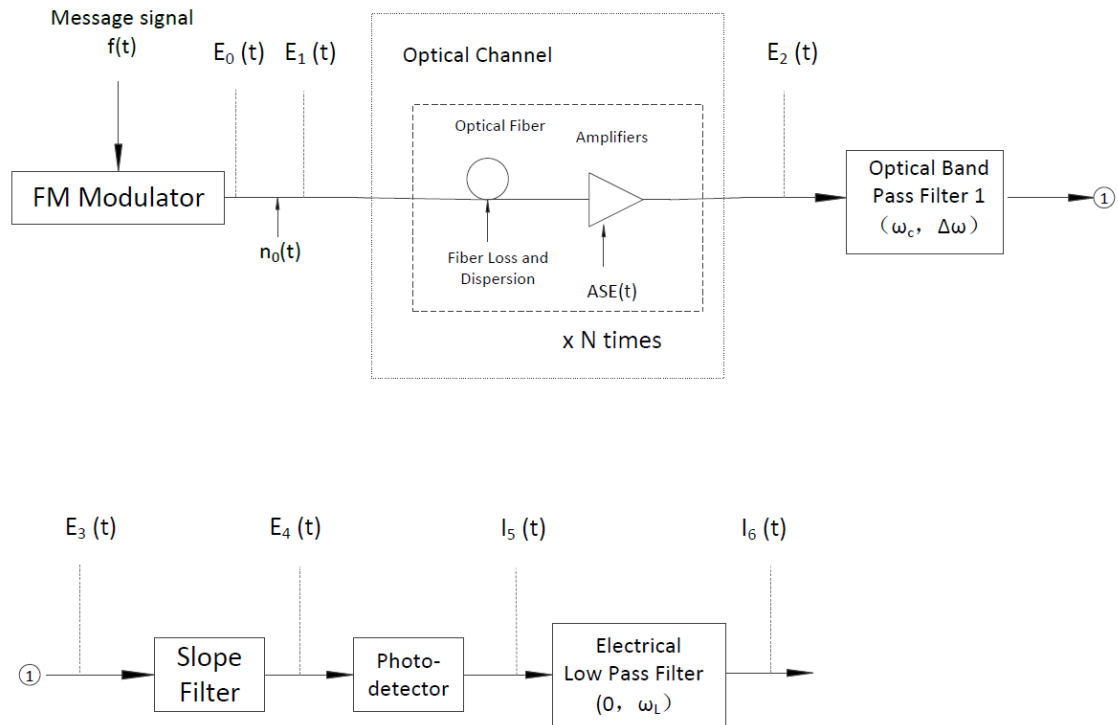


Figure 5-1 Schematic model of FM communication system for practical application

Figure 5-1 shows the schematic model of the basic frequency modulated fiber communication scheme that is used for the practical access network case we are discussing. The transmission system consists of a FM modulator, optical fibers, amplifiers, slope filter, photo detector and electrical low pass filter. The unit of a span of fiber with loss and dispersion followed by an amplifier is repeated for N times in the optical channel. The central frequency and the bandwidth of the optical band pass filter 1 are denoted as ω_c and $\Delta\omega$, respectively; the central frequency and the bandwidth of the electrical band pass filter 2 is 0 and ω_L , respectively.

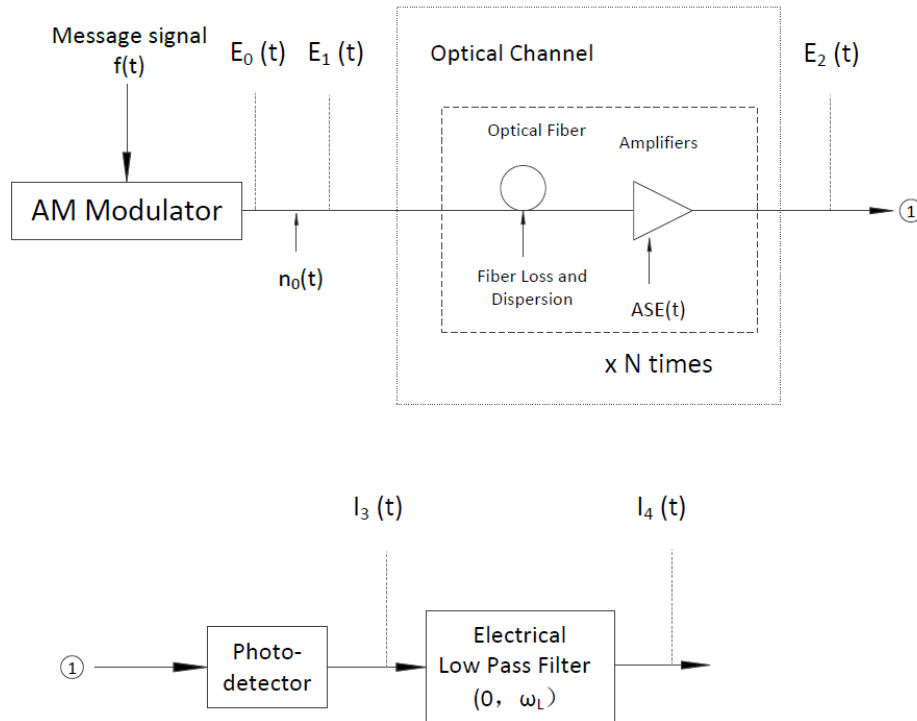


Figure 5-2 Schematic model of AM communication system for practical application

Figure 5-2 shows the schematic model of the basic amplitude modulated fiber communication scheme that is used as a reference when the practical access network is considered. The AM system consists of an AM modulator, optical fibers, amplifiers, photo detector and electrical low pass filter. The unit of a span of fiber with loss and dispersion followed by an amplifier is repeated for N times in the optical channel. The central frequency and the bandwidth of the electrical band pass filter 2 is 0 and ω_L , respectively.

5.2 Simulation Model

A pseudo-random bit sequence (PRBS) is generated at the beginning of the simulation. The message signal $f(t)$ is the bit sequence multiplied by raised cosine pulses, in which bit “1” (bit“0”) is represented by a pulse with amplitude 1 (0). And the modulator plays a role in loading the message signal to the frequency part of the carrier wave, forming the optical signal $E_0(t)$. After adding the background Gaussian noise, the optical signal $E_1(t)$ is obtained. Next the optical signal will become the incident light into the fiber transmission path. It is necessary to consider the fiber dispersion and ASE noise at the same time in the real application. Split-step method is applied to derive the output signal from the fiber channel. For each span, the input signal’s Fourier transform is multiplied by the transfer function of the fiber with the loss term and dispersion term, and then the inverse Fourier transform is applied to obtain a weakened signal in the time domain. The output of the span is equal to the weakened signal multiplied by the amplifier’s gain value, accompanied by the ASE noise. Both the background noise and ASE noise are generated using a random number generator that follows the zero mean Gaussian white distribution. The signal quality degradation would continue accumulating as it passes through the optical channel. If the number of span is denoted as N , the above procedure will repeat N times and the output signal at Port 2, $E_2(t)$ is achieved. The wide band pass filter and optical differentiator are defined as the transfer function. Thus the optical signal $E_3(t)$ and $E_4(t)$ can be obtained based on the convolution theorem. Thereby, an FM signal is

converted to an AM-FM signal $E_4(t)$. Next, $E_4(t)$ is collected by the photo-detector, which is assumed as an ideal square-law detector and the output electrical signal is denoted as $I_5(t)$. The electrical signal $E_5(t)$ at the output of the low pass filter can also be anticipated by the convolution theorem because the transfer function is defined in the frequency domain.

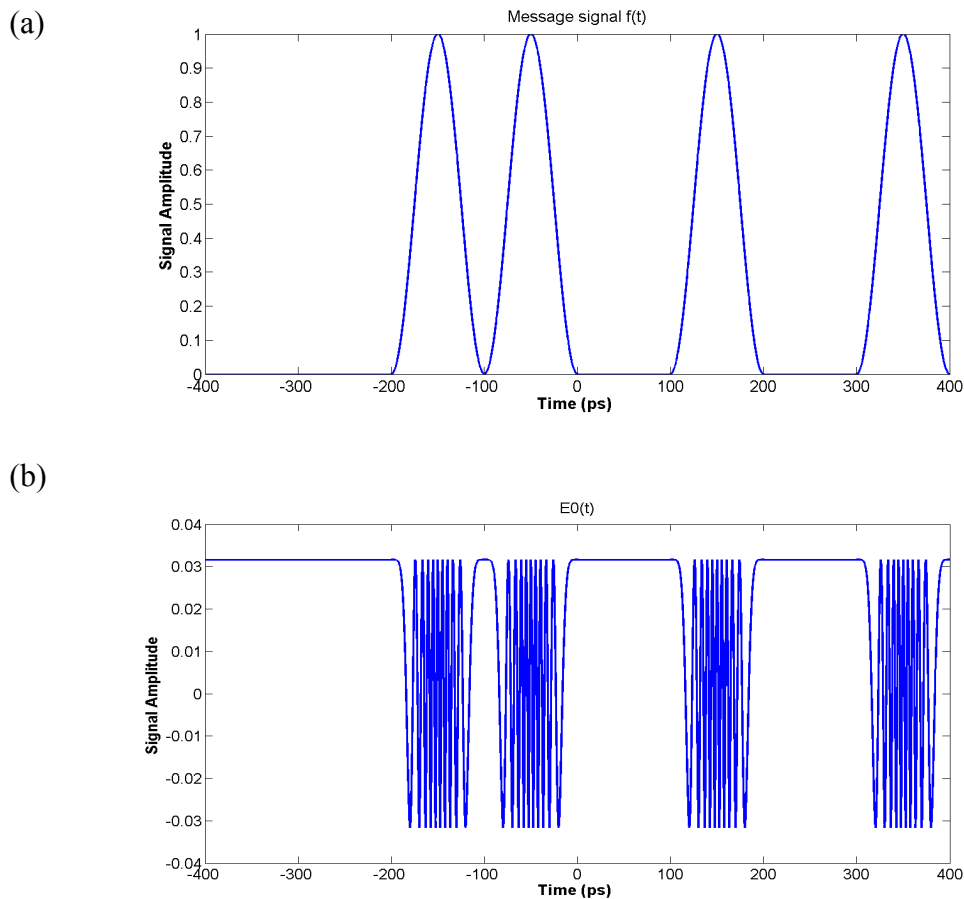
5.3 Parameter setting

The following parameters were used in this real application case: A pseudo-random bit sequence (PRBS) of length $2^8 - 1$, a bit interval of 100ps, a bit rate of $B = 10\text{Gbit} / \text{s}$, and an operating wavelength of 1550nm. NRZ format is applied and the message signal $f(t)$ is based on the raised cosine pulses. Thus the half bandwidth of the baseband signal is about $\omega_m = 10\text{GHz}$. The peak launch power is 0dBm. The Corning SMF-28 single mode fiber (G.652.D) is used as the optical fiber in this practical application, with attenuation coefficient 0.2dB/km and dispersion coefficient 15 ps/(nm.km) at wavelength 1550nm. We only assume the transmission system is of 5-span with the total distance to be 70km. As for the amplifiers, the spontaneous emission factor is $n_{sp} = 1.5$, and the gain of the amplifier is $G = e^{\alpha L}$ which will exactly compensate the fiber loss. The frequency deviation range K_f for the FM signal is set to be 200GHz. The responsivity of photo detector is $R=0.9\text{A/W}$. The electrical low pass filter applies 6-th order butter worth band pass filter with the 3dB cut off bandwidth at 8.5GHz, which means $\omega_L = 0.85 \times \omega_m$.

5.4 Simulation Results and Discussion

5.4.1. Transmission signal at each interface

To better exhibit the signal shape, only the first 8 bits are plotted in this section, and the message bit sequence is [0,0,1,1,0,1,0,1]. The envelope of the transmission signal at each interface for the optical FM communication system is shown in Figure 5-3 and Figure 5-4, with both noise and dispersion considered. The transmission distance is $L=70\text{km}$, fiber loss is 0.2dB/km , and the dispersion coefficient is $15\text{ ps}/(\text{nm}\cdot\text{km})$.



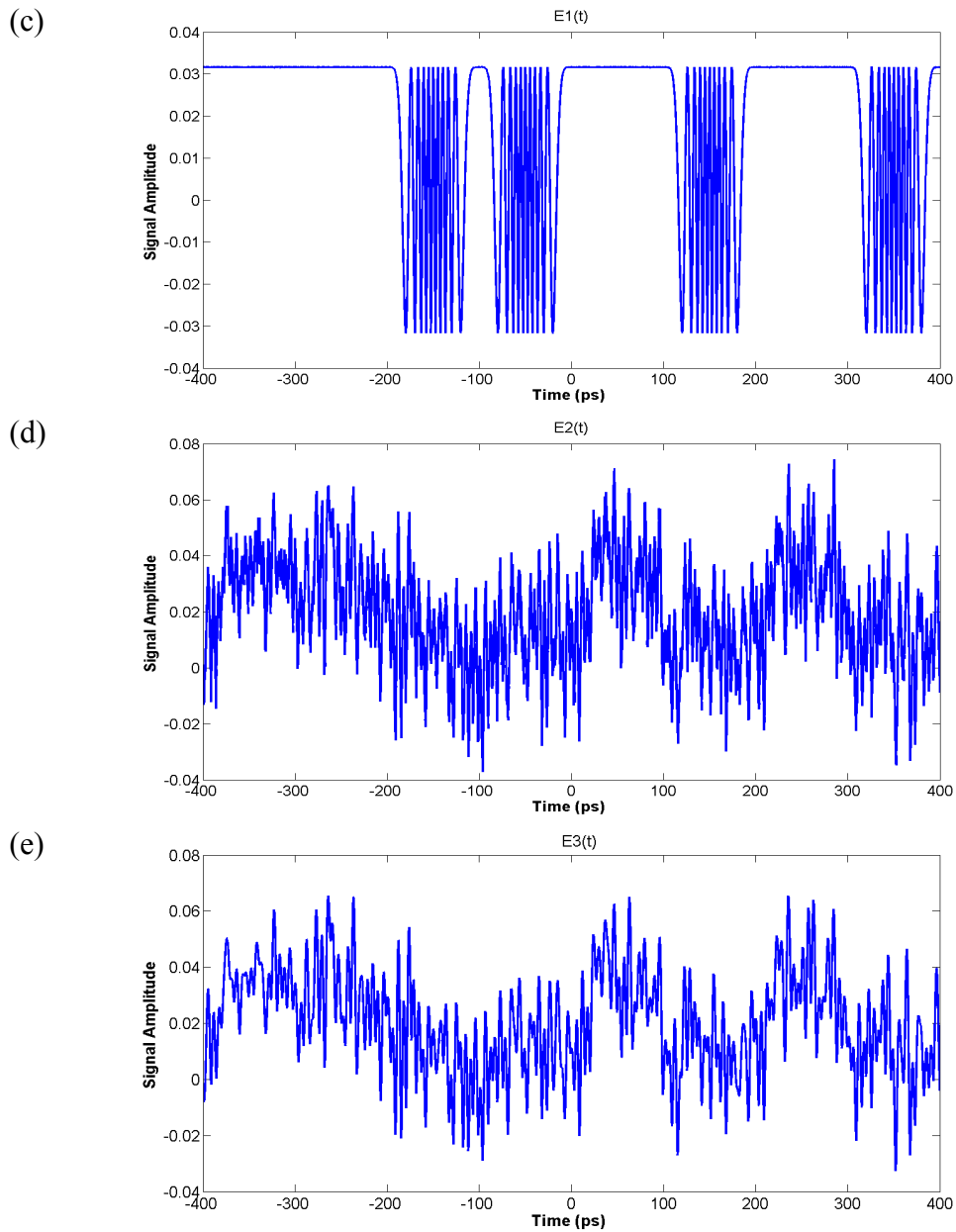
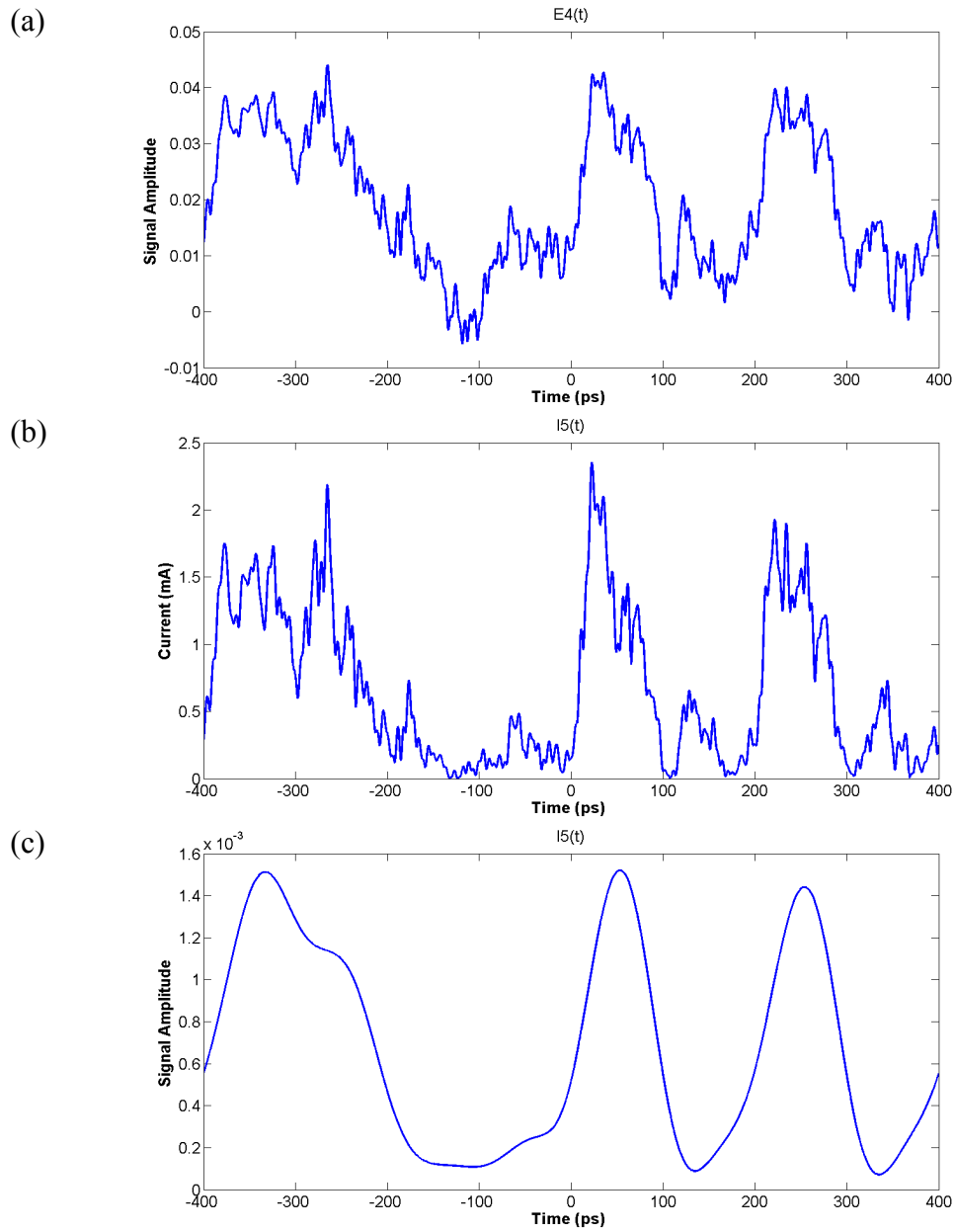


Figure 5-3 (a) Message signal $f(t)$ (b) FM optical signal $E_0(t)$ (c) FM optical signal after background noise is added $E_1(t)$ (d) The output signal from noisy and dispersive fiber channel $E_2(t)$ (e) The output signal from Optical band pass filter $E_3(t)$



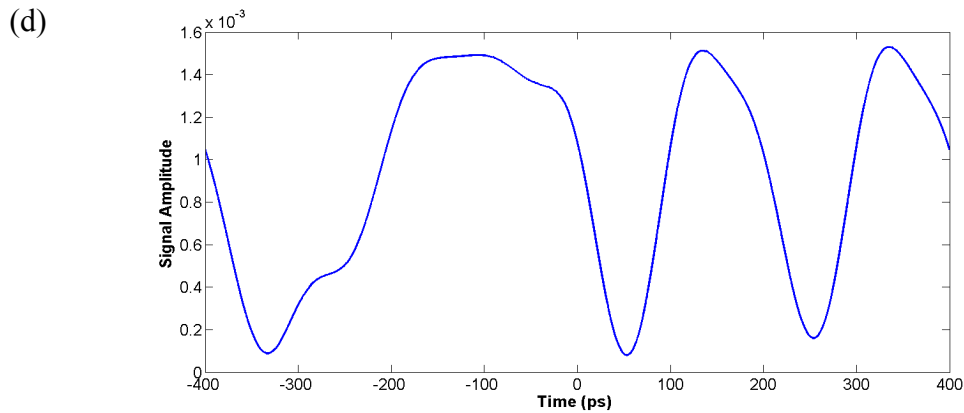


Figure 5-4 (a) The output signal from TEF $E_4(t)$ (b) The output signal from photo-detector $I_5(t)$ (c) The output signal from Electrical low pass filter $I_6(t)$ (d) Reversion of $I_6(t)$

From the plot of reversion of $I_6(t)$, it can be clearly distinguished that the launched message bit sequence is [0,0,1,1,0,1,0,1], which means that the span of the proposed FM system could reliably reach 70km.

The envelope of the transmission signal at each interface for the AM optical communication system is shown in Figure 5-5 and Figure 5-6, with both noise and dispersion considered. The message bit sequence is [0,0,1,1,0,1,0,1], transmission distance is $L=70\text{km}$, fiber loss is 0.2dB/km , dispersion coefficient is $15\text{ ps}/(\text{nm.km})$.

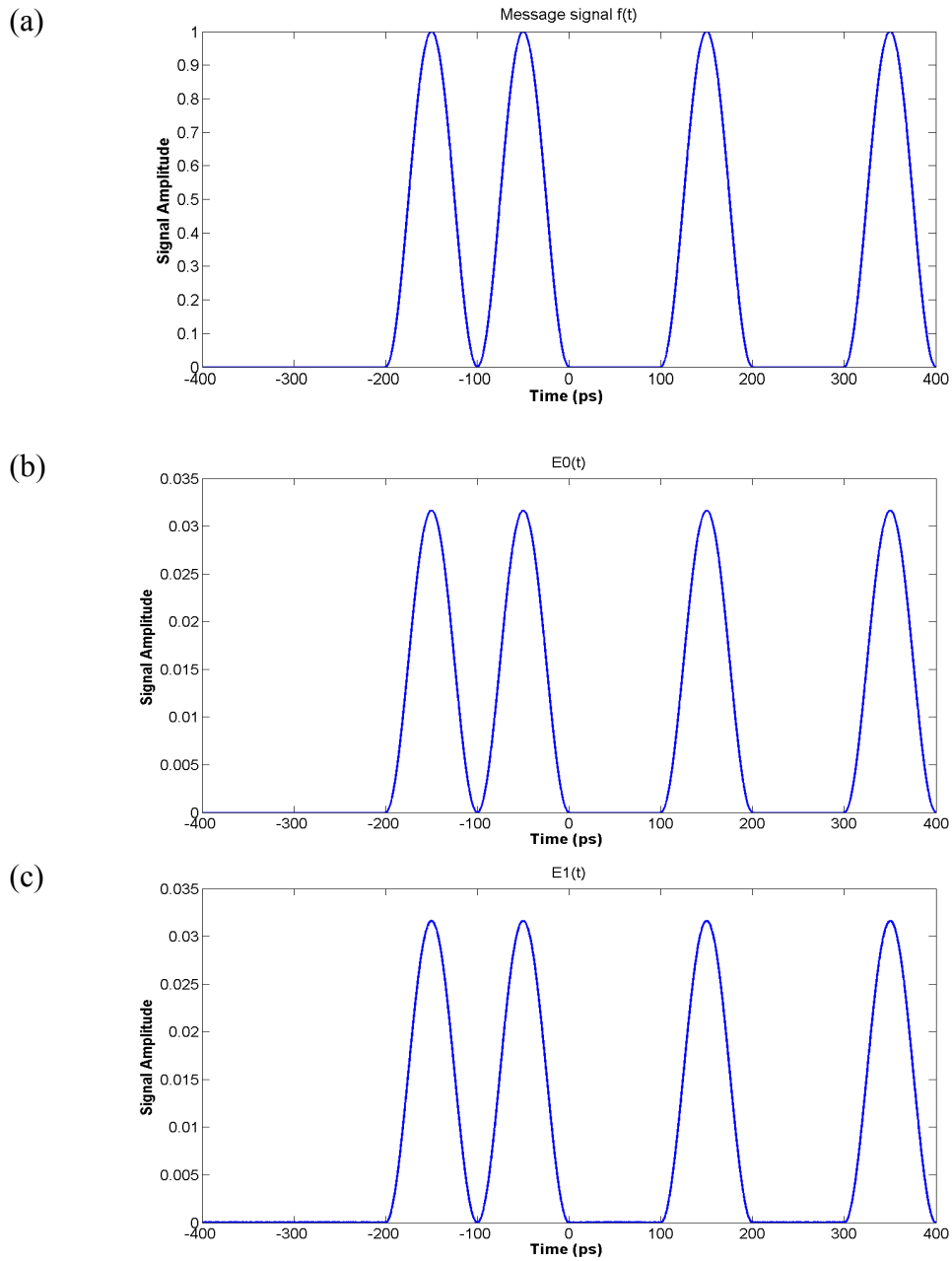


Figure 5-5 (a) Message signal $f(t)$ (b) AM optical signal $E_0(t)$ (c) AM optical signal after background noise is added $E_1(t)$

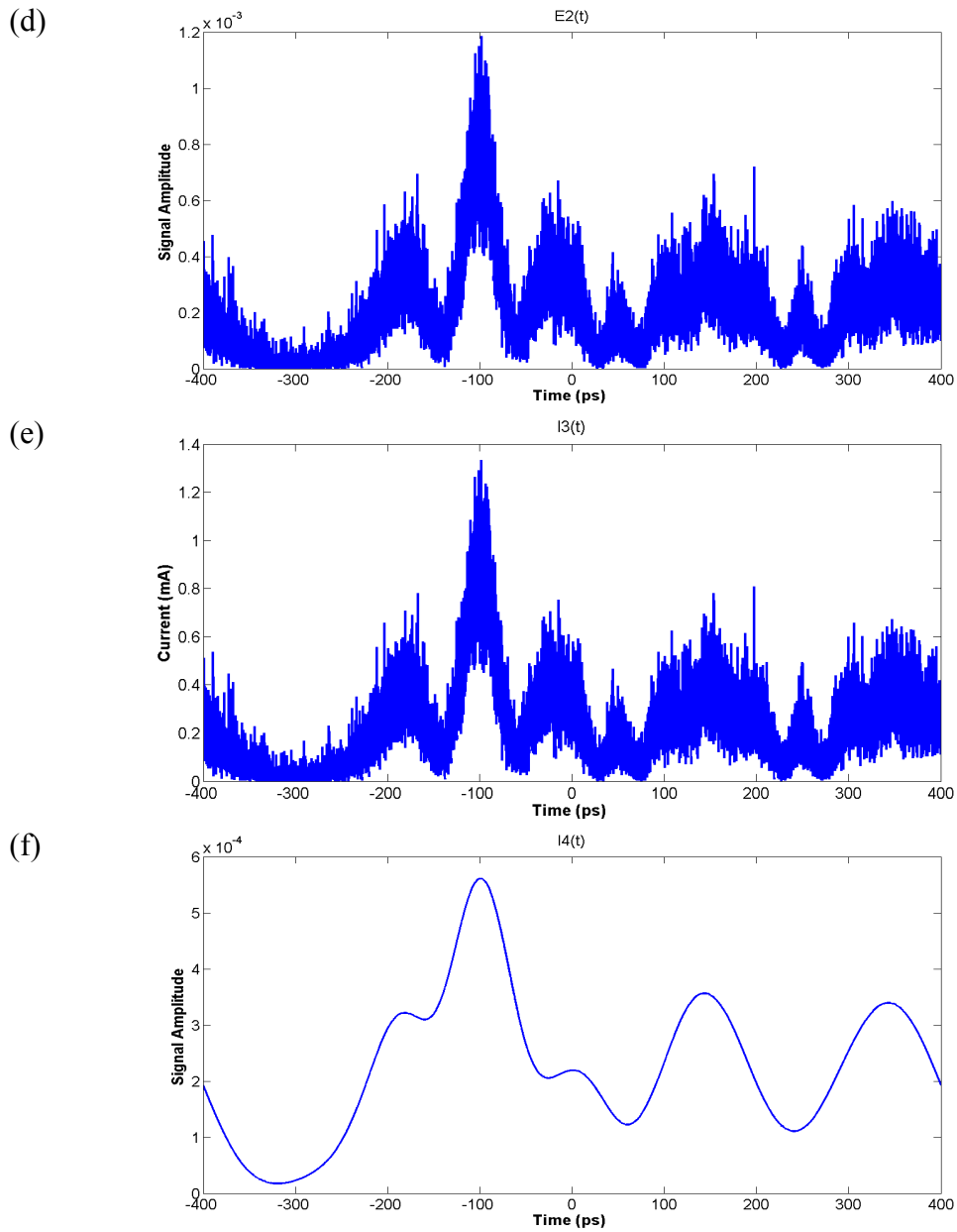


Figure 5-6 (a) The output signal from noisy and dispersive fiber channel $E_2(t)$ (b) The output signal from photo-detector $I_3(t)$ (c) The output signal from Electrical low pass filter $I_4(t)$

The launched message bit sequence [0,0,1,1,0,1,0,1] can be barely distinguished from the output signal $I_4(t)$, which means that the span of AM system could hardly reach 70km.

5.4.2. System Performance

In order to obtain a robust eye diagram, the length of the bit sequence should be long enough. The result for a pseudo-random bit sequence (PRBS) of length $2^8 - 1$ when both noise and dispersion impairments are considered is shown in Figure 5-7 and Figure 5-8. They can be compared with Figure 4-8 (b) and (e), which is the case when only dispersion is considered for the 70km transmission at 10Gb/s on a single mode fiber. The MA value of S for the AM system drops from $1.52mA \cdot ps$ to $1.18mA \cdot ps$ with the presence of noise impairment, whereas it increases from $24.04mA \cdot ps$ to $32.11mA \cdot ps$ for the FM system. With the uncertainty induced by the ASE noise, BER can also be used to evaluate the performance of the system if we induce the data recovery component at the end of the receiver [40]. And the simulation result implies that the BER for the FM system is below 10^{-4} , while the BER for the AM system is as high as 10^{-1} . In conclusion, all of the above results demonstrate that the optical FM communication system we proposed has higher noise and dispersion tolerance than the traditional AM optical communication system. In other words, the FM system gives better performance than AM systems in this practical application, in the case when the peak launch power is 0dBm with the speed of 10Gb/s over a distance of 70km on the Corning SMF-28 single mode fiber.

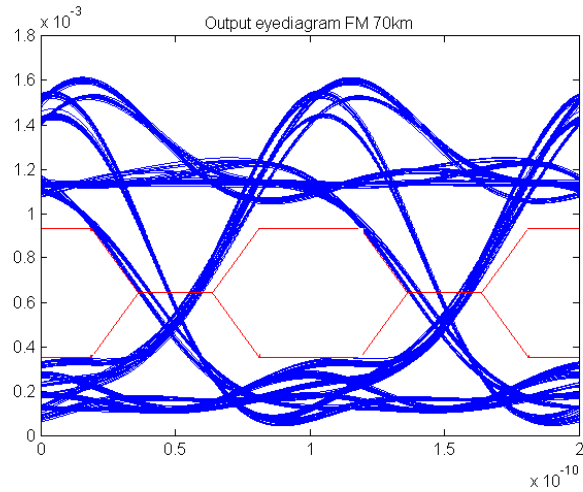


Figure 5-7 Eye diagram for a 10Gbit/s FM system with transmission distance 70km and peak launch power 0dBm, $S = 32.1 mA \cdot ps$

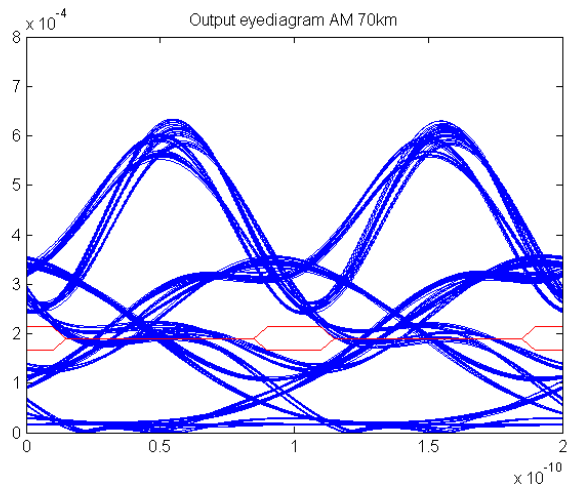


Figure 5-8 Eye diagram for a 10Gbit/s AM system with transmission distance 70km and peak launch power 0dBm, $1.18 mA \cdot ps$

Chapter 6

Conclusion

This thesis proposed a novel frequency modulated fiber-optic communication system with optical discriminator. We mainly investigated about the performance of frequency modulated optical communication systems with the presence of noise impairment and dispersion impairment separately, and the amplitude modulated optical communication system has been utilized throughout this thesis to evaluate the merits of the FM system.

Chapter 2 provides a design outline of the FM system from the aspect of the composition of optical communication systems. And it also explains the specific principle and features of the two kinds of optical differentiators — LEF and TEF. Chapter 3 introduces the anti-noise performance of the FM system with a proper analytical model. A parameter NFI is induced to intuitively show that the FM system can lead to about two orders of improvement in noise figure as compared to the AM system as long as it obeys wide band frequency modulation in the impairment of ASE noise, at a cost of enlarged bandwidth as the ratio of bandwidth of FM system is in scale of its frequency deviation.

Numerical simulations are conducted to exemplify that the above argument is valid. Chapter 4 studies the FM system performance with dispersion impairment. First, we decide to apply TEF as the optical differentiator after a series of comparisons. Second, and crucially, we present the relationship between the transmission distance and the system performance by introducing a parameter S to evaluate the opening of the eye diagram. Theoretical modeling and numerical simulation have been performed and the result addressed that the span of the FM system can faithfully reach 110 km with bit rate up to 10Gbit/s, showing overall superiority over the AM system with a maximum signal reach of 70 km. Finally in Chapter 5, a practical application of FM fiber-optic communication systems has been investigated with both noise and dispersion considered. In the case when the peak launch power is 0dBm with the speed of 10Gb/s over a distance of 70km on the Corning SMF-28 single mode fiber, the BER of FM system is below 10^{-4} while the BER for AM system is in the scale of 10^{-1} . In conclusion, FM systems show an overall superiority to AM systems.

Several topics are related and remained for further study as listed in the following.

Firstly, all of the numerical simulations in this thesis are at a speed of 10Gbps, while increased bit rate within a single wavelength slot can enhance the system efficiency and reduce the cost at the same time. Thus, the extension to a higher bit rate is promising.

Secondly, as discussed in the previous chapters, apart from ASE noise and chromatic dispersion, many other types of impairments would also decrease the quality of the optical communication system based on a single channel fiber, such as mode partition noise. It would be a breakthrough if it can be proved that FM systems have less sensitivity

for mode partition noise because that would dramatically decrease the requirement for the single mode laser, reducing the cost of the whole system.

In addition, the impact of fiber impairment on FM systems can be compared with other modulation schemes in the future.

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