VARIATIONS OF AXIAL MIXING
IN AN AERATION TANK

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IN AN AERATION TANK

By

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SCOPE AND CONTENTS:

The axially dispersed plug flow model was used to study the variations of axial mixing in a laboratory model aeration tank, with variations of temperature, air flow rate, and water flow rate. The effect of geometry was also considered briefly by comparing two different width to depth ratios. Other theoretical models used to describe an aeration tank were examined by comparing the predicted theoretical tracer out-put with the experimental out-put.

Full scale field tests were made on an aeration tank and the results compared to the laboratory model.

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CHAPTER 1

INTRODUCTION

The Function of an Aeration Tank

An aeration tank is the continuous-flow reactor section of the activated sludge process. Microorganisms, in the presence of dissolved oxygen, grow primarily at the expense of colloidal and dissolved organic matter in the waste. Subsequently the microorganisms are separated from the waste usually by sedimentation and returned to the aeration tank in sufficient quantity to insure steady state operations. The desired product of the process is a waste of lower organic content.

Air is usually released by a series of diffusers near the bottom of one side of the aeration tank resulting in a spiral bulk flow of the contents. This bulk rotation of the fluid in a rectangular tank combined with the mixing effects of rising air bubbles, results in a high degree of turbulent mixing. The possibility of short-circuiting by length-wise bulk flow is usually eliminated by making the tank substantially longer than it is wide. An aeration tank with very little longitudinal mixing would provide a higher degree of treatment than a highly mixed tank. However, in order to reduce the effects of shock loading, a considerable amount of longitudinal mixing is desired.

The primary function of an aeration tank is to provide sufficient time, and a suitable environment for the assimilation of organics. Initially assimilation occurs by a rapid absorption or adsorption process. After this initial contact phenomena between the waste and microorganisms, the biochemical reactions predominate, and the rate of organic removal decreases. Hence, the mean aeration period is not as critical as the time for a significant quantity of influent to reach the effluent.

Purpose of the Investigation

This study attempts to determine what effects variations of air-flow, mean residence time, temperature, and tank geometry have upon the rate of longitudinal mixing, and thus upon the expected reduction of organics in a substrate. Knowing these effects the optimization of the activated sludge process may be possible.

Because of the difficulty in evaluation of longitudinal mixing for reactors close to ideal mixing, a technique was developed to allow the rapid and accurate analysis of experimental tracer response curves. The applicability of the various theoretical models proposed for use with aeration tanks was evaluated through a comparison of theoretical and experimental response curves.

Two full scale aeration tank tests were made in order to study the possibility of geometric scale-up of model to prototype.

CHAPTER 2

LITERATURE REVIEW

In their 1914 publication Ardern and Lockett (1914) launched the extensive use of the activated sludge aeration tank for stabilization of organics. Eighty ounce batch units, aerated for various lengths of time were used to substantiate the validity of aeration. A proper appreciation for the dynamics of the process was indicated by their immediate advancement to the test of a continuous flow system.

The first large activated sludge plants in operation in North America, as reported by Greeley (1945), were at Houston (1916), Milwaukee (1920), Pasadena (1924), and Indianapolis (1926). Hurd (1929) in discussing the operation of the newly developed "spiral circulation" at Indianapolis, indicated a concern that "short-circuiting" was possible. He stated that the air diffusers were:

> "Set in a series of 'V'-shaped formations to break up uniformity and prevent any possibility of short-circuiting."

In 1934, for the same plant Calvert and Bloodgood (1934) reported that return activated sludge added 238 feet from the influent end "soon worked back in reducing concentration nearly to the influent end." This phenomenon was explained by the "mixing and dividing action" of the air rising from the "checker-board" diffuser arrangement. It was felt that the smoothing of the influent B.O.D. variations in the effluent as reported by Calvert (1932) could be traced to this "mixing and dividing action".

In 1932, at the Salinas, California, activated sludge plant, Hasseltine (1932) reported that he believed at peak flows the sewage entering at the three-eighths point backed up considerably into the sludge reaeration section. It was observed during current-meter velocity determinations that relatively little motion could be detected in the center core of the aerators. In an effort to increase the effective volume of the tank by eliminating the stagnant core and thereby reducing the short-circuiting ("the actual aeration period was 50 to 80 percent of the theoretical period"), some baffles were inclined at 45° to the horizontal above the air diffusers. Reportedly, a 15 percent increase in treatment was realized and up to 25 percent was anticipated when all the aerators were converted.

Kessener (1934) first used an oxygen absorption test to indicate "dead" areas of low oxygen content and as an "indication of the circulation and mixing of the tank contents". Later (1935) he performed the first quantitative analysis of short-circuiting. The model tested was the Kessener type brush-aeration tank consisting of three 48 cubic meter cells separated by baffles with under water openings. A large quantity of salt was added at the entrance of the first cell and the resulting concentrations observed throughout the three cells. Significant concentrations of salt appeared in the effluent from the last cell in less than one-tenth of the theoretical detention time. He made the important differentiation between "absolute short-circuiting" (circulating or axial bulk-flow of sewage) and "relative short-circuiting" (mixing of tank contents). These "separate" mechanisms are today known as hydraulic short-circuiting and turbulent dispersion. By comparing the time of passage of concentration peak heights with the theoretical detention time, he concluded that no "absolute short-circuiting" was occurring, but that mixing was an important parameter. He suggested that mixing was an advantage because of the changing characteristics of the sewage.

The following year Kehr (1936) reported on the experimental tests made to determine the detention periods of aeration tanks, prior to the opening of the experimental activated sludge plant at Cincinnati in 1932. The experimental model was 15 feet by 6 feet and 3.75 feet deep with a water-tight baffle extended down the middle. The tank could be used as an "around the end" or "direct flow through" type. Aeration was provided by "a single row of diffuser tubes in the center of each side". Chloride (concentration from a salt solution) was used as a tracer for the step function approach (as against the unit pulse of Kessener). Kehr presented the basic equations for concentration in one or more tanks with perfect mixing and tracer step-up function.

For one tank:

$$\frac{C}{C_o} = 1 - e^{-t/\bar{t}} \qquad (1)$$

where: C = concentration at time t,

C_o = concentration of the tracer input, and

$\bar{t}$ = theoretical detention time.

A plot of the experimental points demonstrated that this model tank was very close to perfect mixing. By modifying the tank to the "around the end" type a close correlation was obtained to two perfectly mixed tanks in series.

In the reorganization of Kessener's data, Kehr illustrated that the tracer pulse function also demonstrated a general correspondence of Kessener's model aeration tank to the ideal mixing tank.

In applying the perfectly mixed tank concept to the reduction of organic matter Kehr utilized the concept of the monomolecular reaction:

$$\frac{dS}{dt} = - kS \tag{2}$$

where: S = decomposable organic matter, and

k = reaction constant,

to develop the basic equation for the conversion of organic matter by a first order (monomolecular) reaction with respect to the substrate concentration:

$$\frac{S}{S_o} = \frac{1}{1 + k\bar{t}} \tag{3}$$

Kehr concluded that for small aeration tanks, perfect mixing could be assumed, and even with large tanks the mixing phenomena was a major factor.

In 1939 Gould (1939) published details of a modification of the Tallmans Island Activated Sludge Plant which subsequently became known as step aeration. Sewage was added at multiple points throughout the aeration tank in an attempt to make more effective use of the air supply and tank volume. McKee and Fair (1942) later published a paper which essentially agreed with that of Gould. The basic concepts involved were to allow flexibility in the degree of sludge reaeration, the distribution of shock loads over the entire tank, and the use of less air and more sewage for a given tank. Gould also claimed that a longer solids detention was possible (sludge age increase) and that a more even distribution of oxygen utilization would be realized. The Water Pollution Control Federation (1959) suggested that tapered aeration and high sludge return rates also help to minimize the difference in oxygen demand between the influent and the effluent tank sections.

Step aeration, tapered aeration, and high sludge return rates all assume that essentially plug flow exists, or that there is a negligible amount of longitudinal mixing or short-circuiting between baffles. This point was emphasized by McKee and Fair (1942). They indicated that a "sufficient number of baffles must be inserted to keep longitudinal mixing within bounds no matter what the method of operation". Recognizing the high degree of mixing between baffles they asserted that the continuous even distribution of the influent along the tank was not necessary, and that discharge into successive compartments, formed by transverse baffles, would accomplish the desired distribution. McKee and Fair also suggested that under conventional aeration tank conditions the mechanism of longitudinal mixing may be performing the equivalent of short-circuiting.

One of the most complete studies made to date on longitudinal mixing in aeration tanks was published by Thomas and McKee (1944). They reviewed the previous work by Kessener and Kehr and rather than employ the assumption of perfect mixing, they developed a method to describe the actual degree of mixing attained. No significant difference was observed when water was used as the test media instead of a sewage-sludge mixture. The tracer was salt and its presence was determined by both conductivity measurements and titration techniques. Diffused air turbulence and mechanical paddle turbulence essentially yielded the same results, however, mechanical agitation was used because of the greater degree of control possible.

A turbulence expression (b^2) was derived by dimensional analysis and evaluated experimentally:

$$b^2 = \frac{180\ \ell^2}{\pi^2\ t_{90}}\ (\text{ft.}^2/\text{hr.}) \qquad (4)$$

where: ℓ = Length of the tank, and

t_{90} = Time to reach 90 percent ultimate concentration at exit with no flow through the tank.

The differential equation of longitudinal mixing was obtained, by taking a materials balance for a cross-section of the tank:

$$b^2 \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} - \frac{\partial C}{\partial t} = 0 \qquad (5)$$

where: V = Mean Displacement Velocity,

C = Concentration, and

$\frac{\partial C}{\partial x}$ = Concentration Gradient.

The solution of this equation for a tracer pulse input to a closed vessel was given for the concentration C at time t, and the distance x along the vessel:

$$\frac{C}{C_o} = 2 \sum_{n=1}^{\infty} \frac{a_n \ell (B \sin a_n x + a_n \cos a_n x)}{(a_n^2 + B^2)\ell + 2B} \exp\,[Bx - k_n^2 t] \qquad (6)$$

where: C_o = Weight of Salt added, divided by the Tank Volume,

ℓ = L = Tank Length,

$B = V/2b^2$,

$a_n = a_1, a_2, a_3, \ldots,$ defined by $\tan a_n \ell = \frac{2Ba_n}{a_n^2 - B^2}$, and

$k_n^2 = b^2(a_n^2 + B^2)$.

By making the following substitutions:

$$a_n = \mu_n/\ell, \text{ where } \mu_n = \cot^{-1}[(\frac{\mu_n}{U} - \frac{U}{\mu_n})/2]$$

$$b^2 = D,$$

$$V = u,$$

$$B = U/L,$$

$$t/\bar{t} = t/(L/V) = \phi \text{ (dimensionless time)},$$

the following is obtained:

$$D\frac{\partial^2 C}{\partial x^2} - u\frac{\partial C}{\partial x} - \frac{\partial C}{\partial t} = 0 \tag{7}$$

and

$$\frac{C}{C_o} = 2\sum_{n=1}^{\infty} \frac{\mu_n(U \sin \mu_n + \mu_n \cos \mu_n)}{(U^2 + 2U + \mu_n^2)} \exp [U - \frac{(U^2 + \mu_n^2)}{2U} \phi]. \tag{8}$$

This is the solution given by Miyachi (1953), for a pulse input for a dispersion bed with fixed-end boundaries.

Using their solution Thomas and McKee were able to fit the theoretical curve to the experimental tracer pulse outputs by varying the values of $V\ell/2b^2$. It was pointed out that the assumption of perfect mixing made by Kehr (1936) was not justified in all cases. In an aeration tank under certain conditions it would be possible to have $V\ell/2b^2 = 1.0$, with a corresponding peak exit concentration at 0.4 detention times. Under these conditions the perfect mixing assumption would not be valid.

The tank-in-series solution assuming perfect mixing between baffles was also used:

$$\frac{C}{C_o} = \frac{n^n}{(n-1)!} \frac{t}{t}^{\,n-1} \exp[-nt/\bar{t}] \tag{9}$$

where: n = the number of tanks.

The experimental results from an aerated turbulent tank showed a good correlation with the theoretical. As the number of chambers increased, the degree of longitudinal mixing decreased. The perfect mixed, tank-in-series assumption was also used later by Camp (1953) and Archibald (1949) for flocculation and sedimentation studies.

By increasing air flow rates Thomas and McKee found that the tank tended to greater mixing, and tapered aeration resulted in a "relative decrease in short-circuiting" and greater effective dispersion. The results of multiple-point dosing was found to reduce the effect of baffling and increase the effective mixing.

Since this excellent work by Thomas and McKee, essentially no work was done for almost 20 years to determine the hydraulic characteristics of aeration tanks.

For most laboratory work on aeration tanks, complete mixing has been assumed. (See Smith (1953), Thomas (1953), Stack and Conway (1959) and Busch (1962).) The widespread use of small package plants, with long detention times and thus essentially a completely mixed activated sludge, prompted the study of the mechanism and benefits involved. McKinney (1960) reported that complete mixing allows protection from shock loads, is unaffected by sludge hydraulic loads, and provides a maximum utilization of air.

A perusal of the current texts and design manuals shows a great deal of confusion. The need for substantial mixing to reduce shock loads is recognized by all, but the degree of mixing, the use of baffles, the tank dimensions, and the methods of introduction of the sewage to the aeration tank, are all considered from an empirical standpoint. Apparently the tracer response of a full scale tank has never been obtained, and information concerning backmixing has usually been obtained indirectly (Calvert and Bloodgood (1934), Hasseltine (1932), Rohlich and Sawyer (1943)). The concept that the aeration tank has plug flow is implied in many cases, and design on the basis of detention time is still accepted, though not recommended, by the Water Pollution Control Federation (1959).

The methods of investigating vessel flow-through characteristics have been improving rapidly. The use of radio-active tracers, Archibald (1949), Thomas and Archibald (1952), and others, has provided a continuous and more accurate method than the previous method of dosing with salt. Later, fluorescent tracers were used (Turner Associates (no date), Carpenter (1960), Pritchard and Carpenter (1960)), and resulted in the same accuracy as radioactive tracers, but eliminated the health hazards and many inconveniences. The fluorescent tracer combined with a continuous flow analyzer and recorder has been used in the study of dispersion and mixing in rivers (Krenkel and Orlob (1962), Patterson (1963)).

Feuerstein and Selleck (1963) reported a complete study on the technique of fluorometry for dispersion measurements, and compared the behaviour of fluorescent tracers, Rhodamine B, Pontacyl Brilliant Pink B, and Fluorescein in waters of various qualities. Fluorescein showed a high rate of photo-chemical decay and natural background levels. Rhodamine B showed a higher rate of absorption than Pontacyl Brilliant Pink B, when suspended particles were present. For high quality waters, however, both of these two tracers were essentially equivalent. The cost of Rhodamine B is less and hence is preferred for tracer work using tap water in laboratory scale models.

There have been extensive studies made mainly in chemical engineering journals over the past 10 years concerning the flow characteristics, and dispersion in chemical reactors. Danckwerts (1953), Levenspiel (1957, 1962b), and many others in Chemical Engineering Science have developed an extensive literature in this field, based upon the dispersed plug flow model.

The foundation for this approach is the assumption of the dispersion model using an equation similar to Fick's second law for molecular diffusion. The basic equation is:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \tag{10}$$

where D is the axial dispersion coefficient for random fluctuations in the axial direction.

Taking a basic differential material balance for any cross-section as was done by Thomas and McKee (1944), the differential equation describing the concentration for an n^{th} order reaction is obtained:

$$D \frac{\partial^2 C}{\partial x^2} - u \frac{\partial C}{\partial x} - kC^n = 0 \tag{11}$$

For non-first order reactions this equation can only be solved numerically (Fan and Bailie (1960), Levenspiel and Bischoff (1959,1961)). However, for a first-order reaction, Wehner and Wilhelm (1956) have solved this equation analytically. For all entrance and exit conditions, the fraction of the reactant unconverted is:

$$\frac{C}{C_o} = \frac{4a \exp[\frac{1}{2} \frac{uL}{D}]}{(1 + a)^2 \exp[\frac{a}{2} \frac{uL}{D}] - (1 - a)^2 \exp[- \frac{a}{2} \frac{uL}{D}]} \qquad (12)$$

where: $a = \sqrt{1 + 4\ k\bar{t}\ (D/uL)}$

With the limits $D = 0$, and $D = \infty$, this equation simplifies to the two basic equations of plug flow and complete mixing for first-order reactions:

$$\frac{C}{C_o} = e^{-k\bar{t}}\ ,\quad D = 0 \quad \text{(plug flow)} \qquad (13)$$

where: $\bar{t}$ = theoretical detention time, and

$$\frac{C}{C_o} = \frac{1}{1 + k\bar{t}}\ ,\quad D = \infty \quad \text{(completely mixed)}\ . \qquad (14)$$

To match the experimental and the theoretical curves the variance (σ^2) is usually used as a measurement of spread about the mean. By Laplace transforms for various boundary conditions, a relationship is developed between σ^2 and D/uL. For a closed tank ($D = 0$ at inlet and exit), and for a constant D throughout the vessel, with a dimensionless plot of concentration (C/C_o) and time ($t/\bar{t}$), Van der Laan (1957) has shown:

$$\sigma^2 = 2\left(\frac{D}{uL}\right) - 2\left(\frac{D}{uL}\right)^2 \left(1 - \exp[- \frac{uL}{D}]\right) \qquad (15)$$

Dawkins (1963) suggested that these results could be applied to the activated sludge biological system following a first-order reaction. He stated that provided the rate constant, and the tracer pattern are known, then the effluent concentration can be predicted for the reactor. As an illustration of the importance of dispersion for a certain plug flow reactor the B.O.D. reduction would be fifty-fold, while with back mixing the reduction would be only five-fold. Obviously the degree of mixing markedly affects the degree of treatment that can be expected from an activated sludge tank.

Many mixing models besides the dispersion model have been proposed to describe an aeration tank. Originally plug flow was assumed, however, the other extreme, ideal mixing was later found to provide a better description of the system. Pipes et al (1964a, 1965) have suggested additional models which are the tank-in-series (a number of perfectly mixed, equal-volume tanks in series) and the mixed model (various combinations of complete mixing, plug flow, short-circuiting, stagnant zones, and recycle). The former model is a special case of the latter model. A small laboratory model was used to approximate the tank-in-series response to a step-down function. For short detention times an eight cell unit coincided with the expected response for the tank-in-series configuration. No correlation of the tank-in-series model with an actual aeration tank was attempted.

Levenspiel (1962a) when comparing the tank-in-series with the dispersion model concluded that there was no clear cut relationship between the two models. For small values of D/uL (or a large number of equal tanks) both models become identical, but for larger deviations from plug flow the tracer response curves differed by an increasing amount. Pipes et al

(1964b) discussed the two models and felt that the satisfaction to be gained in distinguishing between the models was not worth the labour involved in testing, in spite of the fact that they recognized the predictions of the two models differed considerably. The tank-in-series method was felt superior to the axial dispersion method because non-linear equations (non-first order reactions) could be more easily formulated.

Both the tank-in-series, and the dispersion model have been considered as unsuitable for flow patterns which deviate a great deal from plug flow. Levenspiel (1962b) stated that these models cannot satisfactorily be used:

> "when the gross flow pattern of fluid deviates greatly from plug flow because of channeling or recirculation of fluid, eddies in odd corners, etc."

Milbury (1964) felt that the dispersion model was not satisfactory for an aeration tank:

> "because the elementary process on which this model is based is viewed to be statistical in nature, therefore usually taking place in relatively small regions of the reaction vessel."

The mixed model is convenient to use for first order reactions because the order of location of the various components of the system does not matter. For non-linear systems as indicated by Dawkins (1963), the principle of superposition does not apply. In this case the residence time distribution and the rate constant are not sufficient to predict the reaction conversion. Cholette *et al* (1960) discuss this problem of fitting models for non-linear systems.

An overall review of the literature to date presents a confused picture as to the effects of design and operation on the degree of treatment possible with the activated sludge technique. The use of arbitrary design values has developed over the years and these may entail a great waste of money through unnecessarily conservative or improper design. A better understanding of the mechanism of removal and the flow characteristics is essential for a more rational design of activated sludge aeration tanks.

CHAPTER 3

EQUIPMENT AND TECHNIQUES

Laboratory Model

The rectangular laboratory model aeration tank consisted of a 5.0 feet long test section with a maximum depth of 2.5 feet and a maximum width of 3.0 feet. A false back was constructed for the tank so that the width of the tank could be altered. The front of the test section was constructed of one inch thick acrylic plastic. Uniform diffused air was provided along the entire length of the tank by diffuser tubes. These were recessed into the base of the tank along one edge so that their crown was level with the tank bottom. The tank water depth was indicated by a water column. Because of rapid radial diffusion and relatively large transverse velocities, inlet and outlet conditions had relatively little effect. Short weirs at the rear of the tank were used for both entrance and exit. The tank and associated equipment are shown in Fig. 1, and represented schematically in Fig. 2.

Air flow measurements were made on an Al-800 gas meter manufactured by the American Meter Company. This positive displacement type meter was rated for air flows up to 20 p.s.i. and 650 cu. ft./hr. Air flow rates were determined using an even number of revolutions on the five cubic foot dial for periods of at least five minutes. The pressure at the meter was read and the volume flow rate of air was determined for existing atmospheric pressure at the start of the test. Two pressure compensating control valves were used in series to eliminate the line fluctuations. A

FIGURE 1 EXPERIMENTAL EQUIPMENT

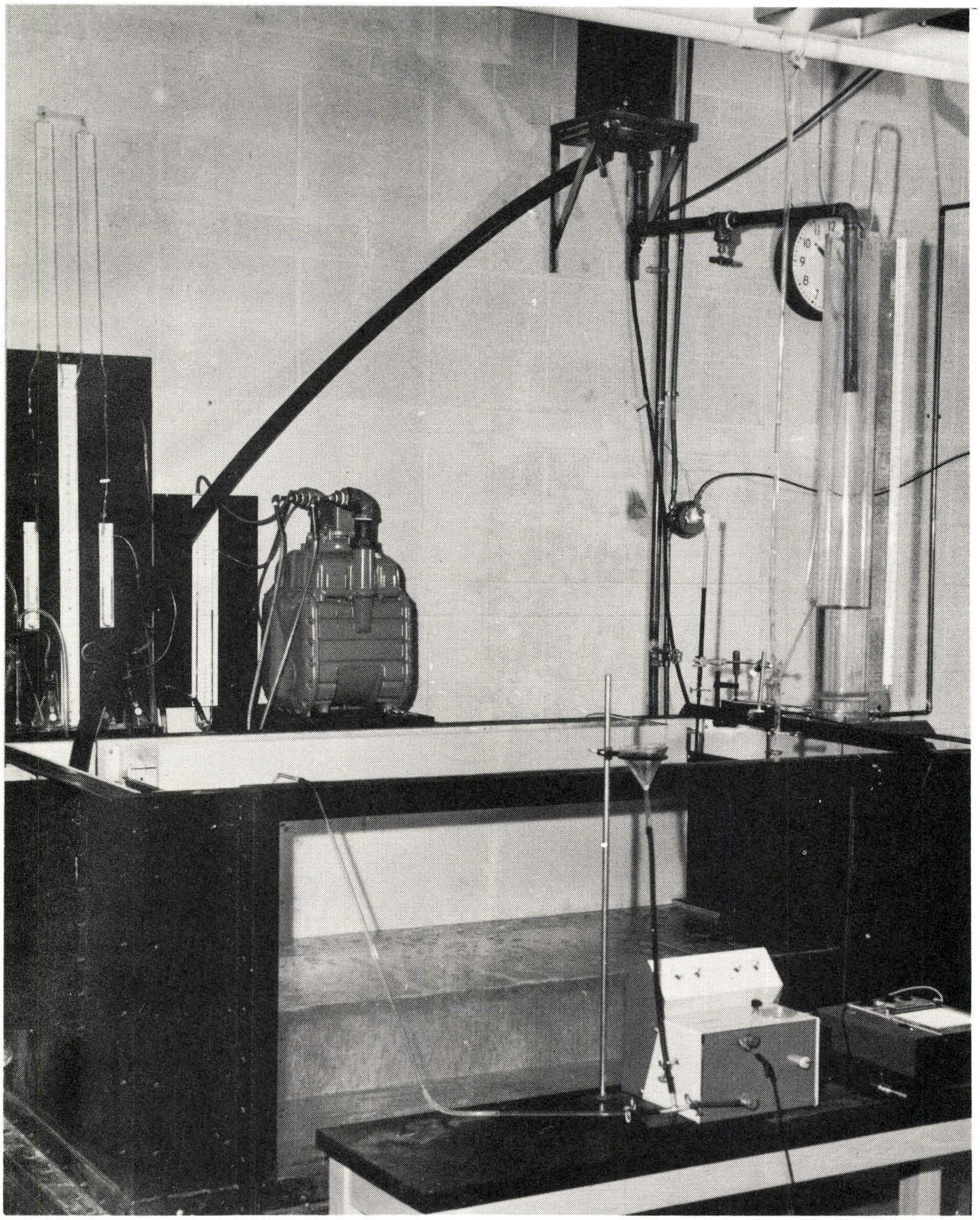
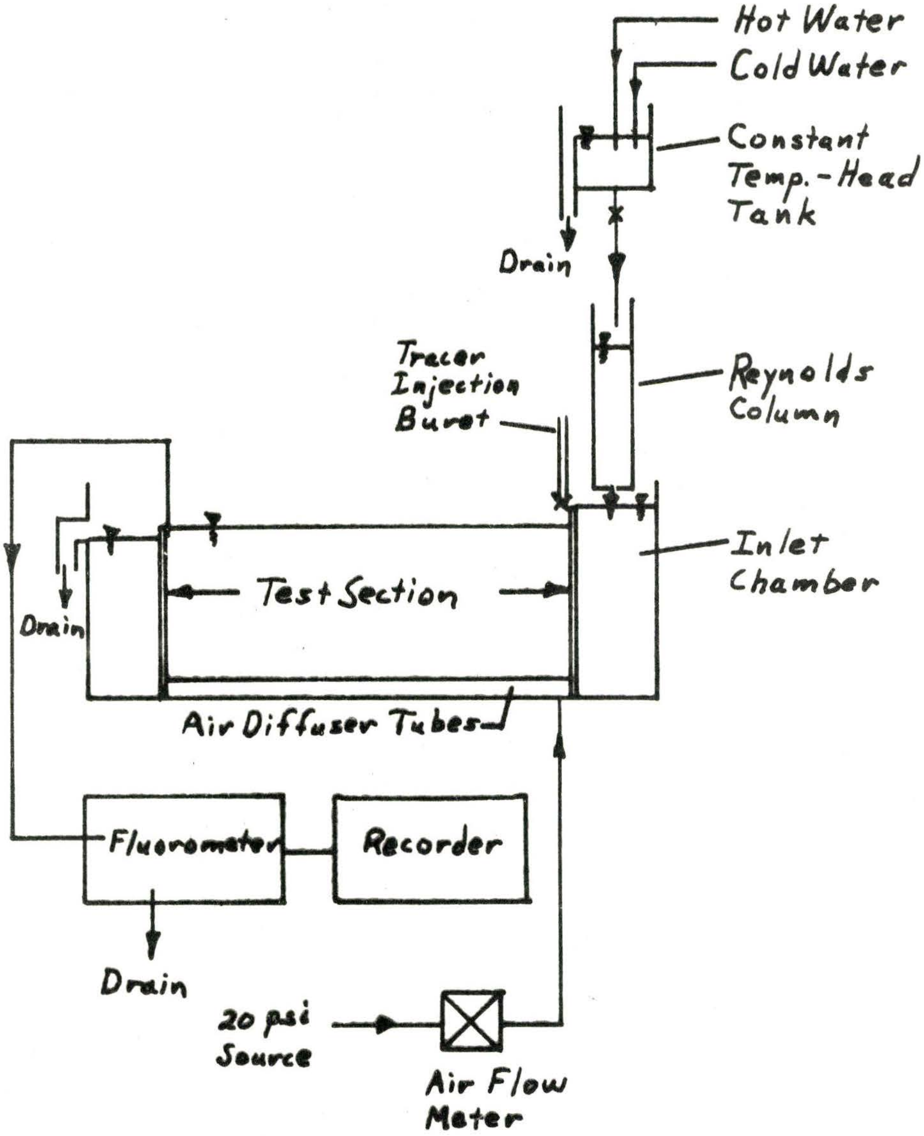


FIGURE 2 SCHEMATIC OF EXPERIMENTAL EQUIPMENT



pressure differential reading over an orifice was used to indicate whether any drift had occurred in the air flow during the test.

The water flow measurements were made by a six inch diameter acrylic plastic Reynolds Column with a maximum water head of 4.5 feet. Six different diameters were used for the brass orifice plates in order to make possible any flow from 0.18 US gpm to 18.0 US gpm. The orifice plates were calibrated for a given height of water above a given plate by measuring the time required to fill a 20 lt. bottle. The resulting calibration curves are shown in Appendix "A". The flow rate was kept constant during tests by using a constant head tank with a continuous overflow.

The water temperature was maintained at a constant 20.0°C (68.0°F) throughout all the tests except those specifically studying temperature effects. The temperature was controlled by a T-275 Valve Top Thermostat valve manufactured by Johnson Controls Limited on the hot water line with the control probe injected into the pipe leaving the constant head tank.

The air diffuser tubes were standard grade 60 carborundum cylinders with an outside diameter of three inches and an average pore diameter of 0.16 inches. From measurements made by Sueishi (1964) using a high speed camera, the mean air bubble diameter was 3.22 mm., and the mean rise velocity of the bubble with respect to the fluid was 18.5 cm./sec. The diameter was calculated for a sphere with the same volume as the bubble which was assumed as ellipsoidal. In order to maintain a constant diffusion for the test section, the air diffusers were jetted with air or water streams to respectively increase or decrease the air flow in a given section. This process was continued until the air bubble density was judged to be constant as viewed through the acrylic plastic front.

Tracer flow-through studies were accomplished using a 1.0 percent stock solution of Rhodamine Lissamine B-200 (colour index - 45100) obtained from Canadian Industries Limited. This stock solution was added at the entrance weir at the rate of about 0.03 US gpm over a period of five to ten seconds. Half way through this tracer injection an event marker button connected to the recorder was depressed to indicate the start of the test. It was possible to represent the ideal pulse function by the above method because the short duration of injection and the injection rate were negligible in terms of the tank detention time and the flow-through rate. The volume of the tracer injected was determined for most of the tests in order to calculate the percentage recovery.

When a part of the tracer added had reached the effluent weir, a continuous sampling tube syphoned part of the effluent through the fluorometer. The rate of syphoning was 0.06 US gpm and there was an associated lag of about 30 seconds for the sample to pass through the system and for the fluorometer to respond.

The fluorometer used was a G. K. Turner and Associates Model 111 with a connected Bausch and Lomb V.O.M.-5 strip chart recorder, the fluorometer was calibrated at 20°C using the primary filters 1-60, and 58 with a secondary filter 23A. The fluorometer had changed its calibration during the tests, and for tests made after number 40.0 the calibration curves II were used. The calibration curves are shown in Appendix "B".

All of the tests were made with the maximum tank size except for one series which was made at a depth of only 13.45 inches, and a width of 27.27 inches. These dimensions were chosen to yield a geometric scale 1:13.2 for model:prototype. The prototype will be described in the following section. Initial runs for these dimensions showed that hydraulic short-circuiting was a significant factor. Large vortices developed along the rear of the tank and, in order to minimize this bulk flow, five vertical baffles, 4.0 inches wide extending over the tank depth, were arbitrarily placed so as to minimize the hydraulic short-circuiting. These baffles were effective in minimizing the short-circuiting. However, it was assumed that they would not appreciably change the turbulent dispersion of the tank.

In order to examine the characteristics of the flow patterns about the axis of the tank, a separate tank was used which was designed as a one foot thick cross-section through the test tank. A one inch thick acrylic plastic front on the cross-section tank made it possible to effectively "look down" the axis of the tank. Silver-coated saran "Christmas tinsel" was cut into short lengths and immersed into the cross-section tank. These short silver bits followed the flow streamlines and also reflected light. Time exposures were made of the cross-section tank under various air flow rates. The pictures were taken from behind a black partition in order to eliminate reflections from the acrylic plastic.

A continuous flow laboratory aeration tank was operated for several months to study the effects of microorganisms on Rhodamine B. After the tracer was introduced to the system the effluent was collected for about five detention times. When the microorganisms were removed from the effluent high percent recoveries were realized. The amount of Rhodamine B absorbed by or on the microorganisms appeared to be negligible. Hence the testing of a full scale operating aeration tank was attempted.

Field Prototype Tests

Two field tests were made on an activated sludge aeration tank which was 66 feet long, 15 feet deep, and 30 feet wide. It was attempted to operate with all conditions the same as those of the laboratory aeration tank. However, it was difficult to control all the variables of the system because of the scale of the tests.

To minimize fluctuations of the inlet flow, extra water when needed was obtained from a nearby fire-hydrant. In addition, the sewage lift-station pump was set to start and stop as frequently as practical. An inlet baffle was installed in order to lessen the chances of significant hydraulic short-circuiting. The sewage inlet temperature was 19.0°C for the first test and 19.5°C for the second test. The sewage flow into the aeration tank was calculated at frequent intervals using a rectangular weir in conjunction with the Francis formula.

The air diffusers were of the "Sparger" type. The air flow was adjusted by eye to yield a constant air flow along the tank length. The air flow meter had been installed to measure the total air flow to two parallel tanks. Only one of these was being used for the test and it was assumed that each tank received one half the total air flow.

A ten litre jar was filled with the stock Rhodamine B solution and emptied into the influent at the start of the test. Any recirculation of the Rhodamine B to the aeration tank was eliminated by using only tap water for the foam spray controls, eliminating the wasting of final settling tank solids to the primary settling tank, and stopping the return of solids to the aeration tank.

Samples of the effluent were taken frequently during the first hour of the test and then at longer and longer intervals as the test progressed. The samples were centrifuged for about five minutes to remove the solids and then the fluorescence was determined using the discrete sampling door with the fluorometer.

Suspended solids were determined during the first test as outlined by Standard Methods (1960). The solids reduced from 1400 mg./lt. at the start to 240 mg./lt. after 16 hours. The 240 mg./lt. represented the approximate level at which the solids would be maintained in the aeration tank without any return sludge.

Control of the many variables possible in the prototype tests was fairly easily obtained except for surging of the influent when the lift station pump started. Some irregularities of the tank flow from the air pipes installed inside the tank, and fillets could not be eliminated. However, the full scale prototype test appeared to be reasonably well-represented by the laboratory model of 1/13th geometric scale.

CHAPTER 4

DATA ANALYSIS

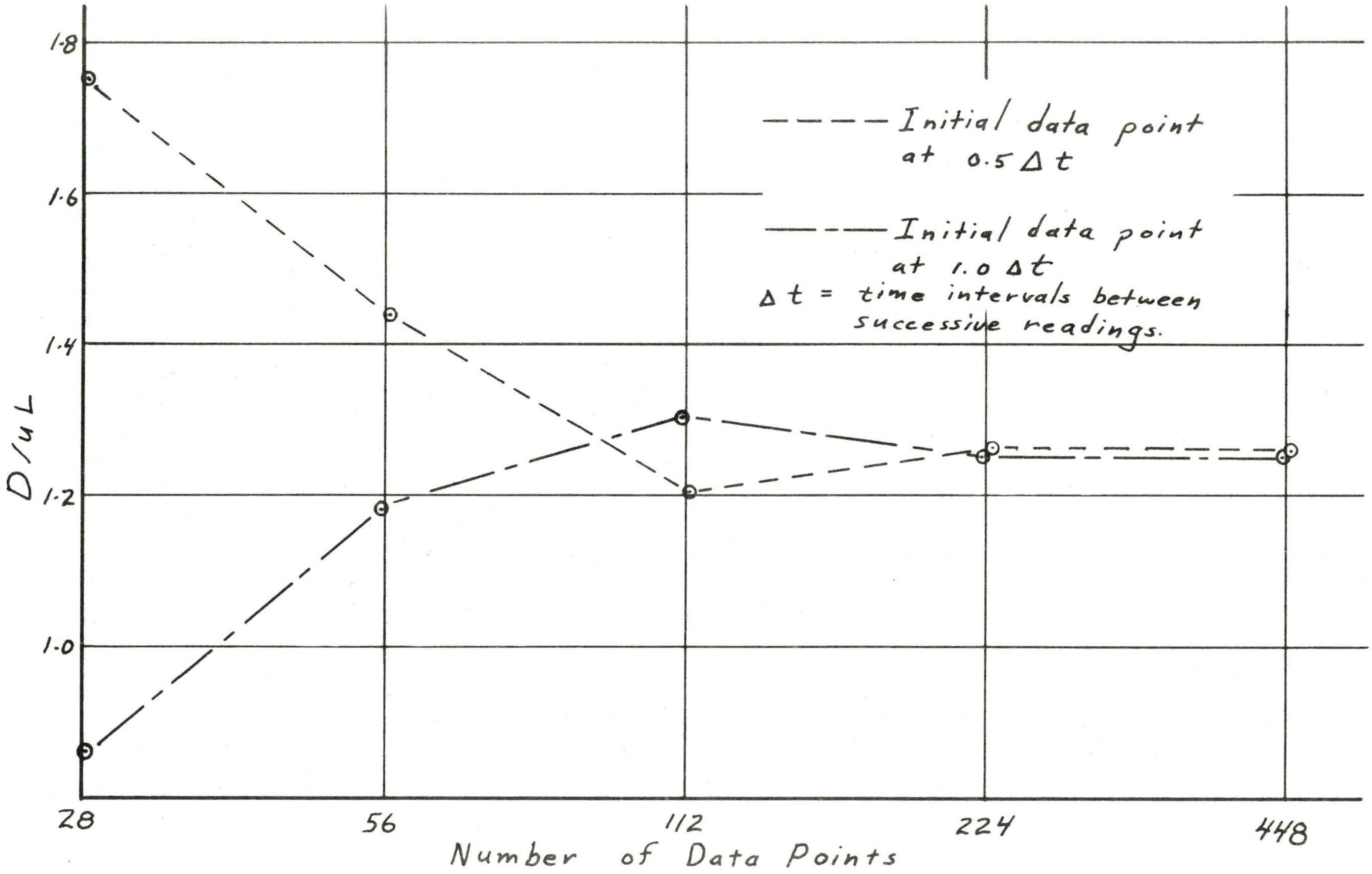
During laboratory tests continuous recordings of the effluent concentration were obtained from a recorder strip chart. The resulting curves were transfered to I.B.M. data cards taking readings at equal intervals. A computer programme was developed using the calibration curves of Appendix "B" to convert the fluorometer output readings to concentrations, and to calculate the percent recovery and the mean residence time. The variance of the curve was calculated as outlined by Levenspiel (1962b), and the associated value of D/uL was obtained from equation 15.

The programmes used for this and other calculations are reproduced in Appendix "C". The data, the results of the analyses, and the output curves which have been used as illustrations are presented in Appendix "D".

The magnitude of the calculated value of D/uL varied with the number of points selected to characterize the response curve (Fig. 3). The value of D/uL was essentially a constant for 200 or more data points when the initial point was taken at either one-half or one time interval. At least 200 data points were used to characterize all response curves.

The experimental response curves were reduced to dimensionless plots of C/C_o and $t/\bar{t}$ so that the experimental curves could be compared

FIGURE 3 EFFECT OF DATA ON D/uL - VARIANCE TECHNIQUE



to the theoretical curves as defined by equation 8 using the value of D/uL obtained by the variance technique. Figure 4 indicates a poor fit of the experimental and the theoretical curves was obtained using the variance technique. A study was made of the theoretical curve in an effort to explain the poor fit of experimental and theoretical curves. Calculations were made of the mean residence time and D/uL values for different cut off points on the theoretical response curve. The results for two different D/uL values are presented in Fig. 5. To obtain 80 percent of the actual value of D/uL by the standard variance technique, accurate readings must be obtained to 8.1 detention times or values of $C_o/4000$ for highly dispersed flow (D/uL = 6.0). In contrast, for a relatively low degree of dispersion (D/uL = 0.85) accurate readings to 4.9 detention times or $C_o/300$ are sufficient to yield 80 percent of the actual D/uL. Readings of $C_o/300$ are obtainable using common laboratory equipment, but readings of $C_o/4000$ are not usually possible. Levenspiel (1962b) only considers values of D/uL much less than 0.85 and thus the variance technique is adequate at this level. As an aeration tank usually is designed with a high degree of dispersion, it was not possible to use the standard variance technique effectively.

Alternately a trial and error method of graphical fitting could be used to obtain the value of D/uL which best represents the experimental curve. As this method could be time consuming and laborious, a different technique was sought.

The most prominent characteristic of a response curve is the peak. The time of peak exit concentration ($t_p/\bar{t}$) was correlated to D/uL for the theoretical dispersion model, and produced a good fit of experimental

FIGURE 4 FIT OF DISPERSION CURVE TO EXPERIMENTAL CURVE - VARIANCE TECHNIQUE

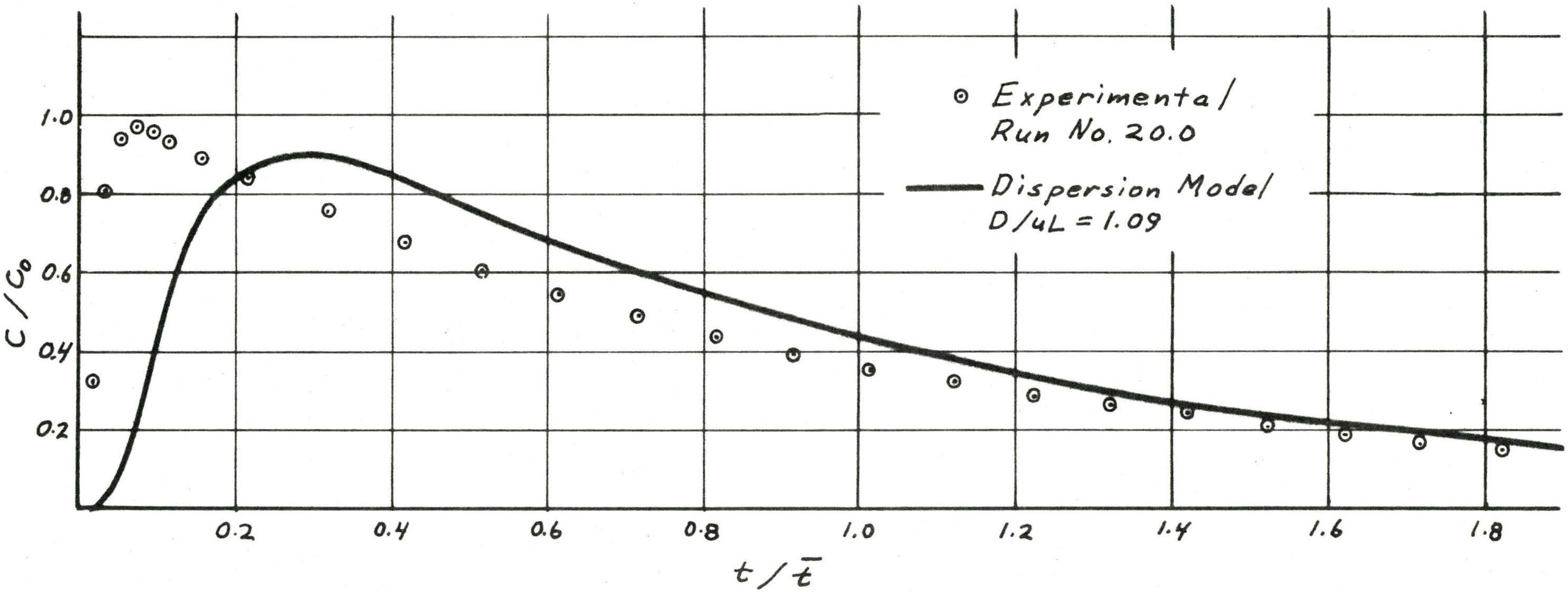
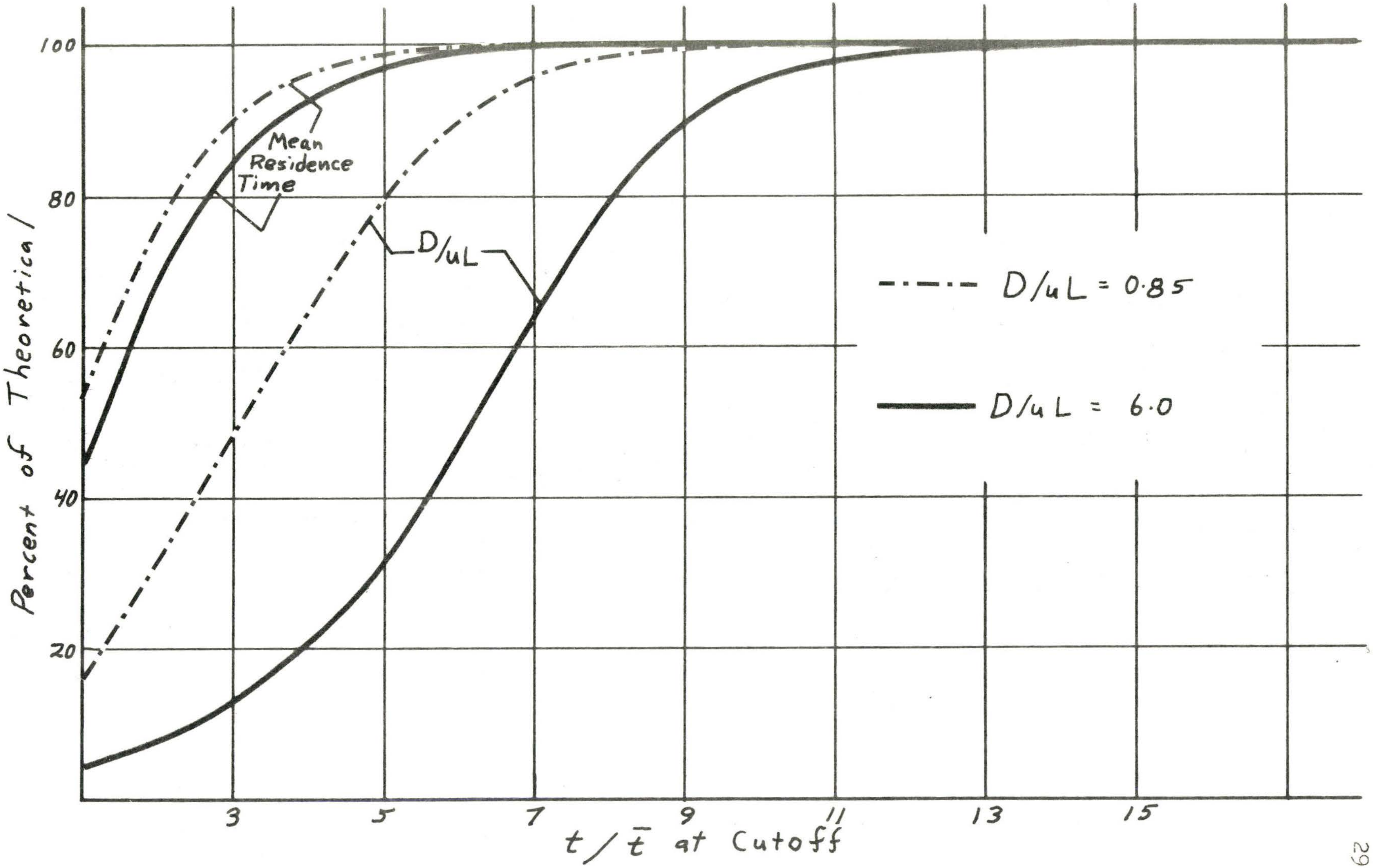


FIGURE 5 EFFECT OF CUTOFF POINT FOR THEORETICAL DISPERSION MODEL



data to the theoretical response curve. This method was suggested by work of Levenspiel and Smith (1957). The time of peak exit concentration can be obtained from equation 8 by setting the differential of C/Co equal to zero and solving for the $t_p/\bar{t}$ value associated with each D/uL. The resulting plot is presented in Fig. 6 and Fig. 7. These plots may be approximated by the empirical relationships:

$$D/uL = 0.2\ (t_p/\bar{t})^{-1.34} \qquad (16)$$

where: $0.03 < (t_p/\bar{t}) < 0.3$, and

$$D/uL = 4.027\ (10)^{-2.09\ (t_p/\bar{t})} \qquad (17)$$

where: $0.3 < (t_p/\bar{t}) < 0.8$.

The typical fit of theoretical and experimental response curves using the peak time technique, is illustrated in Fig. 8.

For run number 20.0 the peak time technique yielded D/uL = 6.0. The variance technique used for this run with a cutoff at 2.87 detention times yielded D/uL = 0.703, or 11.7 percent of that predicted by the peak time technique. From Fig. 5 for D/uL = 6.0 and the same cutoff point for the theoretical model, the variance technique predicted D/uL = 12 percent of the actual D/uL. A similar analysis of run 15.0 yielded D/uL = 0.85 by the peak time technique, while the variance technique with a cutoff at 3.04 detention times yielded D/uL = 0.378, or 44.5 percent of that predicted by the peak time technique. From Fig. 5, for D/uL = 0.85, and the same cutoff point for the theoretical model, the variance technique predicted D/uL = 48 percent of the actual D/uL.

FIGURE 6 DISPERSION PARAMETER D/uL VERSUS TIME OF PEAK EXIT CONCENTRATION

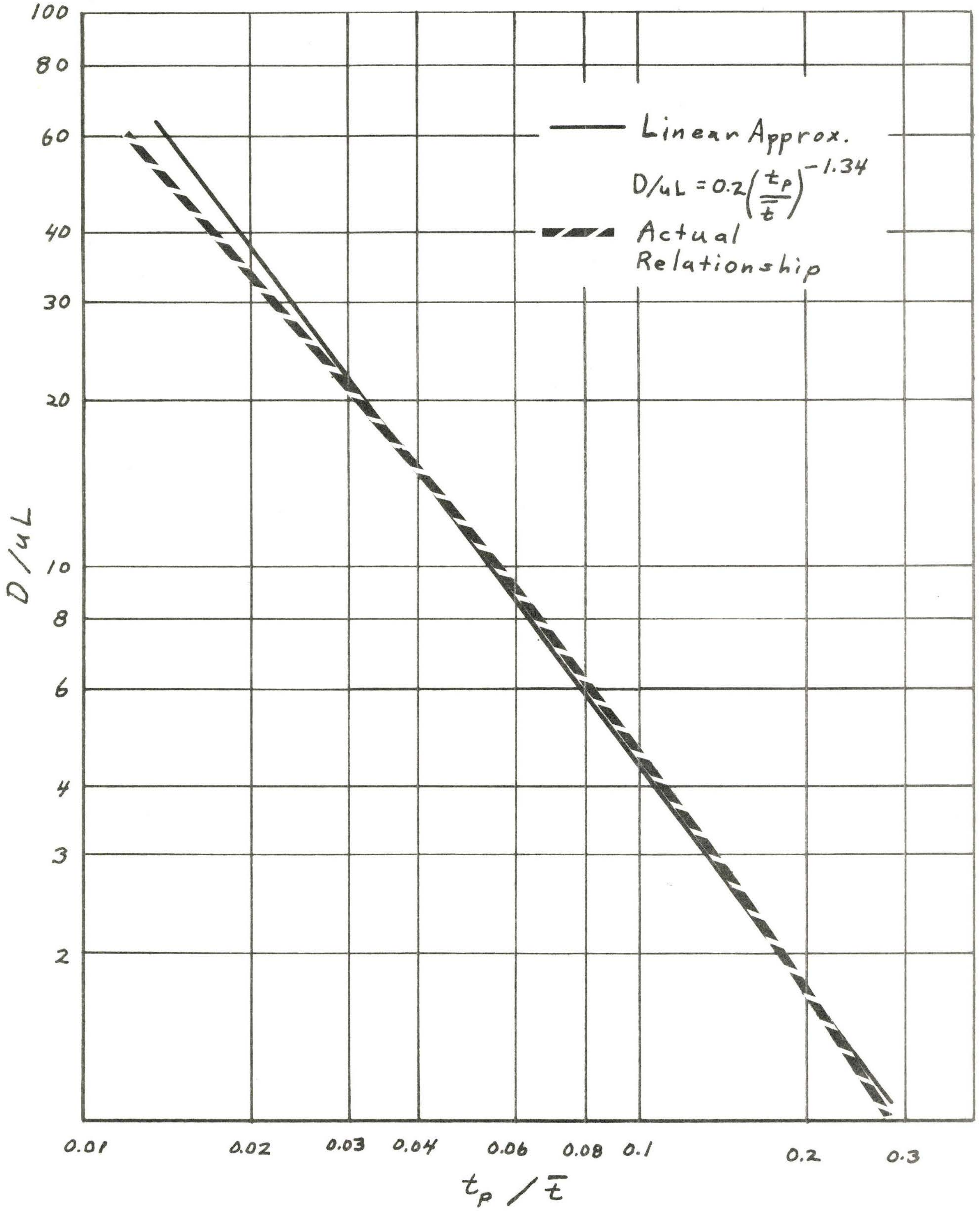


FIGURE 7 DISPERSION PARAMETER D/UL VERSUS TIME OF PEAK EXIT CONCENTRATION

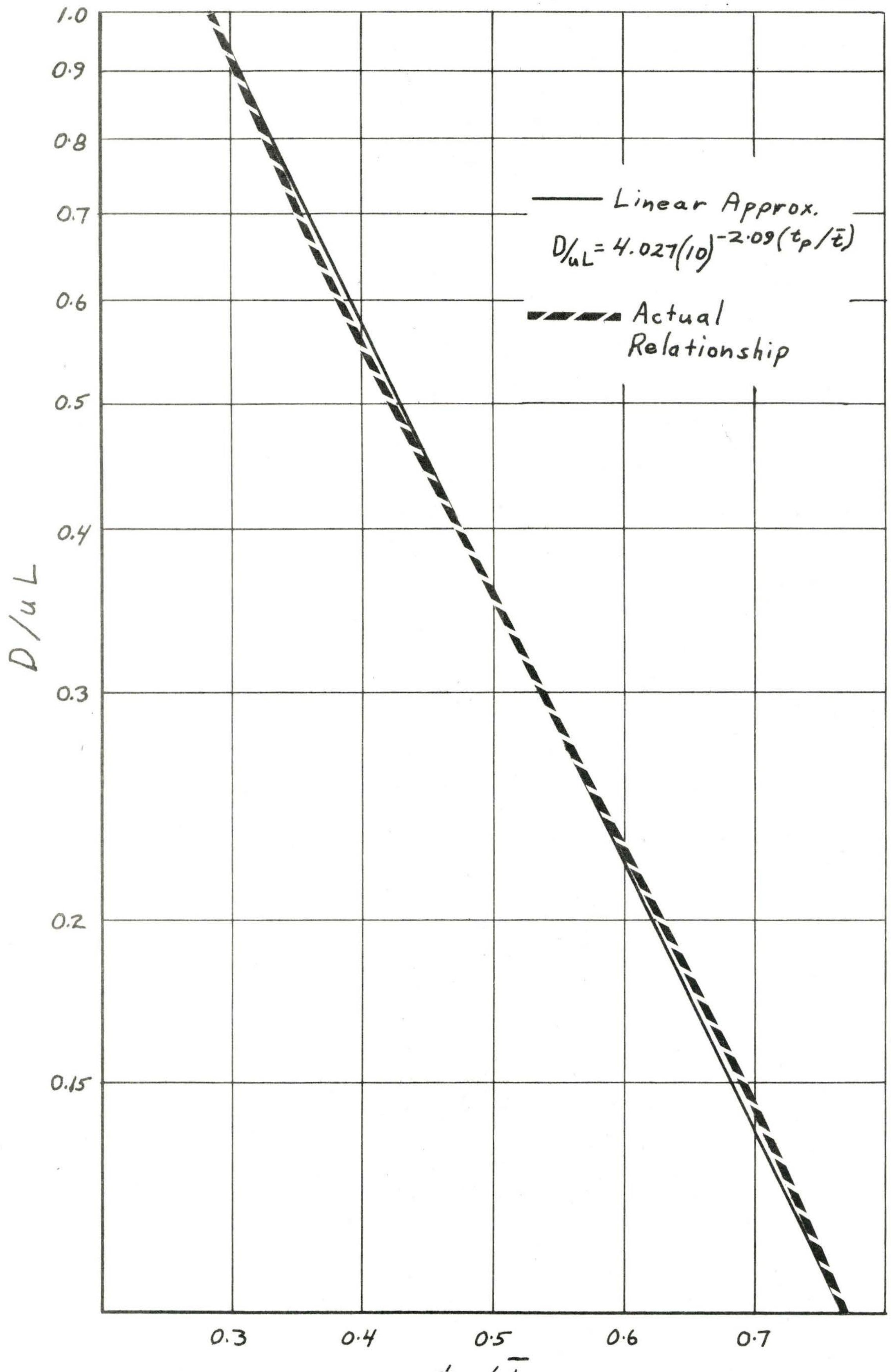
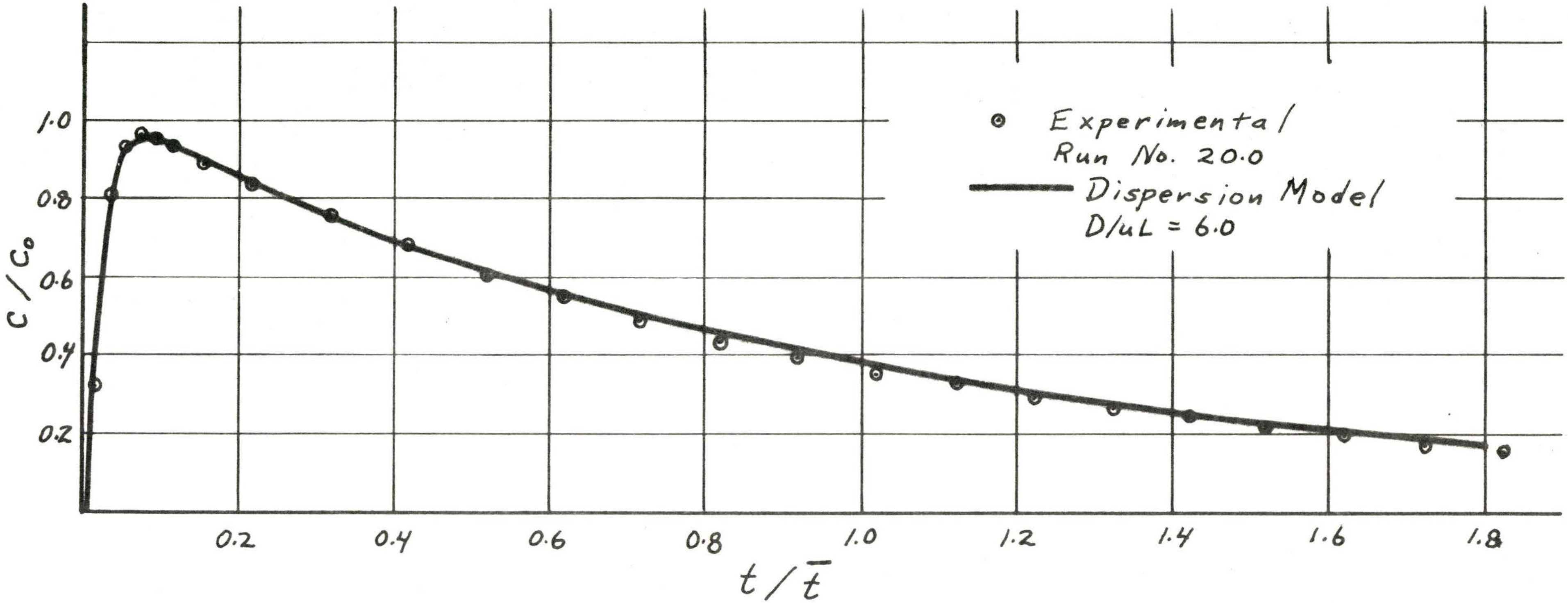


FIGURE 8 FIT OF DISPERSION CURVE TO EXPERIMENTAL CURVE - PEAK TIME TECHNIQUE



These results indicate that the standard variance technique could not be used satisfactorily to analyze an aeration tank response curve if the curve is truncated prematurely. The peak time technique as illustrated in Fig. 8 does provide an adequate fit of the theoretical to the experimental response curve.

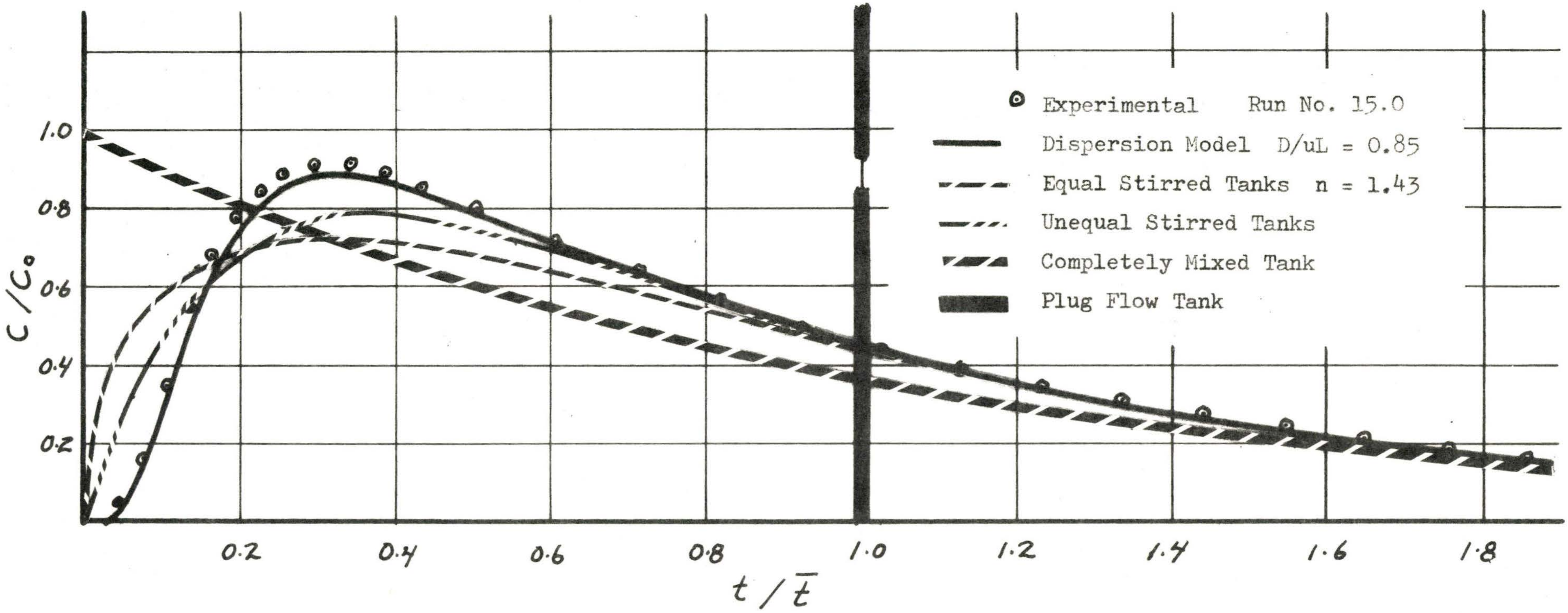
CHAPTER 5

COMPARISON OF MODELS

Commonly mathematical models have been used by all branches of science to aid in understanding problems. Initially a simple model may prove adequate, but as more information is gathered, the model may have to be discarded in favour of a more sophisticated version. In sanitary engineering, the plug flow model was originally considered adequate to describe the activated sludge aeration tank. This model gave way to the complete mixing model which provided a more accurate description of the aeration tank. In recent years the limitations of the complete mixing model resulted in a search for another model that would provide a better description of an aeration tank.

The effectiveness of various mixing models to describe the experimental response curve from an aeration tank is illustrated in Fig. 9. The two extremes of plug flow and complete backmixing obviously are not adequate. Some other models which have been suggested for reactor design in the literature, are also illustrated. The dispersion model would appear to provide a better representation of the response curve than the equal, or unequal tanks-in-series models when the variance of the curve is used as the criteria of comparison.

FIGURE 9 FIT OF THEORETICAL MODEL CURVES TO EXPERIMENTAL CURVES
D/uL= 0.85



The out-put curve for the equal perfect mixed tanks-in-series model was obtained from equation 9, and the relationship given by Levenspeil (1962 b);

$$\sigma^2 = \frac{1}{n} \tag{18}$$

For a D/uL of 0.850 and an associated variance of 0.701 the number of equal tanks in series is 1.43. This paradox of 1.43 "equal" tanks is not theoretically justified because the derivation of equation 9 assumes that n is an integer. The response curve can be estimated by using the gamma function to estimate the non-integer factorial.

$$\Gamma(n) = (n - 1)! \tag{19}$$

The theoretical response curve of the equal tanks-in-series solution is shown in Fig. 9 and Fig. 10. This model cannot satisfactorily represent the experimental curves because there is no lag time until a significant amount of tracer reaches the exit. The Water Pollution Control Federation (1959) correctly indicates that:

> "most of the sewage pollutants are adsorbed
> by the sludge floc in 15 to 45 minutes when
> the sewage and sludge are mixed together."

Because the equal tanks-in-series solution predicts an immediate rapid rise of the exit concentration for less than two tanks, the important lag time associated with the spiral flow aeration tank is not taken into account.

In an effort to obtain better agreement between the theoretical and experimental curves, and to eliminate the paradox of an unequal number of equal tanks, a mathematical model was developed on the basis of two unequal, completely mixed tanks in series. Kandiner (1948) has derived an equation for the tracer response to a step-up function for n unequal ideally mixed tanks in series,

$$C_n/C_o = 1 - \sum_{i=1}^{n} \frac{\exp[-t/\bar{t}_i]}{\prod_{\substack{j=1\\ j\neq i}}^{j=n} (1 - \bar{t}_j/\bar{t}_i)} \tag{20}$$

where: C_n = concentration in the n^{th} tank,

$\bar{t}_i$ = detention time of the i^{th} tank, and

t = time .

To transform a step-up function to the pulse function the derivative of equation 20 is taken. The resulting equation for two tanks in series is

$$\frac{C}{C_o} = \frac{\exp[-\theta/\bar{\theta}_1] - \exp[-\theta/(1-\bar{\theta}_1)]}{2\bar{\theta}_1 - 1} \tag{21}$$

where: θ = t/mean detention time of the system,

$\bar{\theta}_1 = \bar{t}_1$/mean detention time of the system,

$\bar{\theta}_1 + \bar{\theta}_2 = 1.0$, and

$\bar{\theta}_2 = \bar{t}_2$/mean detention time of the system.

By the use of equation 21, and the basic definition of variance it can be shown that:

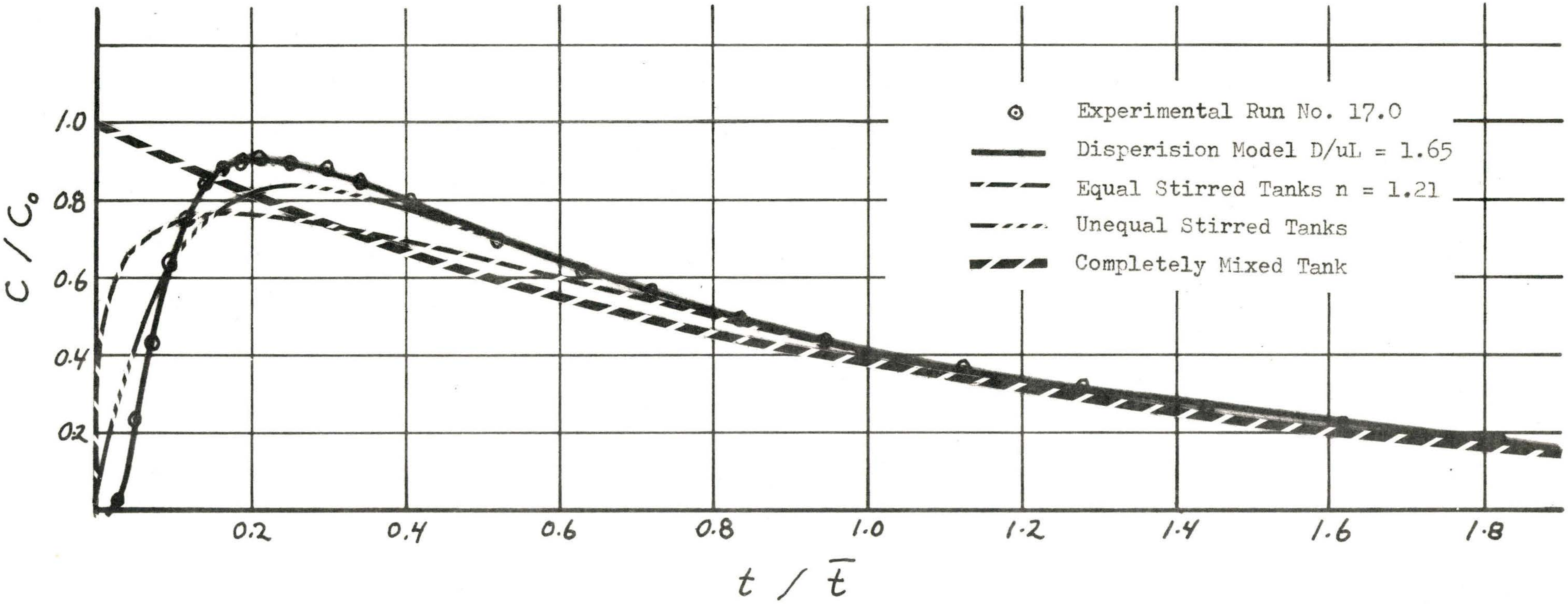
$$\sigma^2 = \frac{1 - 2\,\bar{\theta}_1 + 2\,\bar{\theta}_1^{\,3} - 2(1-\bar{\theta}_1)^3}{2\,\bar{\theta}_1 - 1} \qquad (22)$$

For a dimensionless curve with a known variance, $\bar{\theta}_1$ can be found from equation 22. The associated tracer response curve can then be obtained from equation 21. Plots of the unequal tanks-in-series solution were compared with the experimental response curves, as in Fig. 9 and Fig. 10. The unequal tanks-in-series solution is a better representation of the experimental than the equal tanks-in-series solution, but does not present as good a description of the actual response curve as the dispersion model.

After one detention time both of the above models essentially predict the same response curve as the dispersion model. This result would be expected because the basis of comparing the two models was the variance. The most important contribution to the variance of a curve is from the "tail" section, and hence for equal variances, the "tail" sections should be almost equivalent. However, the initial response of a curve is important because the response curve represents the exit age distribution. The fit of a theoretical model response curve to an experimental model response curve after one detention time is not a sufficient justification for the use of a particular theoretical model.

Both the equal and the unequal tanks-in-series solutions have one characteristic in common. The curvature of the response curves for less than two equal tanks commence at zero time with a negative curvature, in contrast to the dispersion model which represents the actual tank with an initial positive curvature followed by an inflection point and a negative curvature.

FIGURE 10 FIT OF THEORETICAL MODEL CURVES TO EXPERIMENTAL CURVES - D/uL = 1.65



Another method of comparing the validity of theoretical models is by considering the performance of the system as a reactor. From Levenspiel (1962 b) a plot of 1/n versus D/uL shows that if percent conversion is used as the criteria, the higher the conversion that is required the greater the value of n for a given D/uL value. As n becomes larger, the similarity in the shape of the two curves increases, but the shape remains significantly different.

The criticisms of the tank-in-series solutions could be overcome by using a mixed model. A plug flow section and a tank in series section could be combined to more closely approximate the actual response curves. As Levenspeil (1962 b) pointed out, this generalized model has the disadvantage that as the number of parameters increase, the model may have very little correspondence with actual conditions, and

> "an unrealistic many-parameter model may closely fit all present data after the fact, but may be quite unrealistic for prediction in new untried situations."

The dispersion model is a one parameter model and closely represents the tracer response of an actual aeration tank under a great variety of conditions. Hence the dispersion model appears to be an adequate model to describe spiral flow aeration tanks, and is used to interpret the data obtained in this study.

CHAPTER 6

EXPERIMENTAL RESULTS

Preliminary Studies

Runs number 27.0, 28.0, and 29.0 were made on three different days in order to study the reproducibility of the D/uL derived by the peak time technique. It was not possible to measure any differences among these runs for the time to peak exit concentration and it was concluded that this technique yielded reproducible results.

The effect of the temperature upon D/uL was studied over the range of 12 to 30°C (54 to 86°F). This range covers the usual limits of temperature variation for the activated sludge process. Fig. 11 illustrates that no significant difference of D/uL was observed for either short, or long detention times over the temperature range studied.

Effects of Detention Time

The effects of detention time on D/uL for a constant air flow were studied. At an air flow (Q_A) of 0.865 cu. ft./min., a variation of detention time from 0.28 hours to 15.0 hours resulted in an increase of D/uL from 0.825 to 43.0 as illustrated in Fig. 12. D/uL was directly proportional to the detention time, and the associated value of D can be considered

FIGURE 11 EFFECTS OF TEMPERATURE ON D/uL

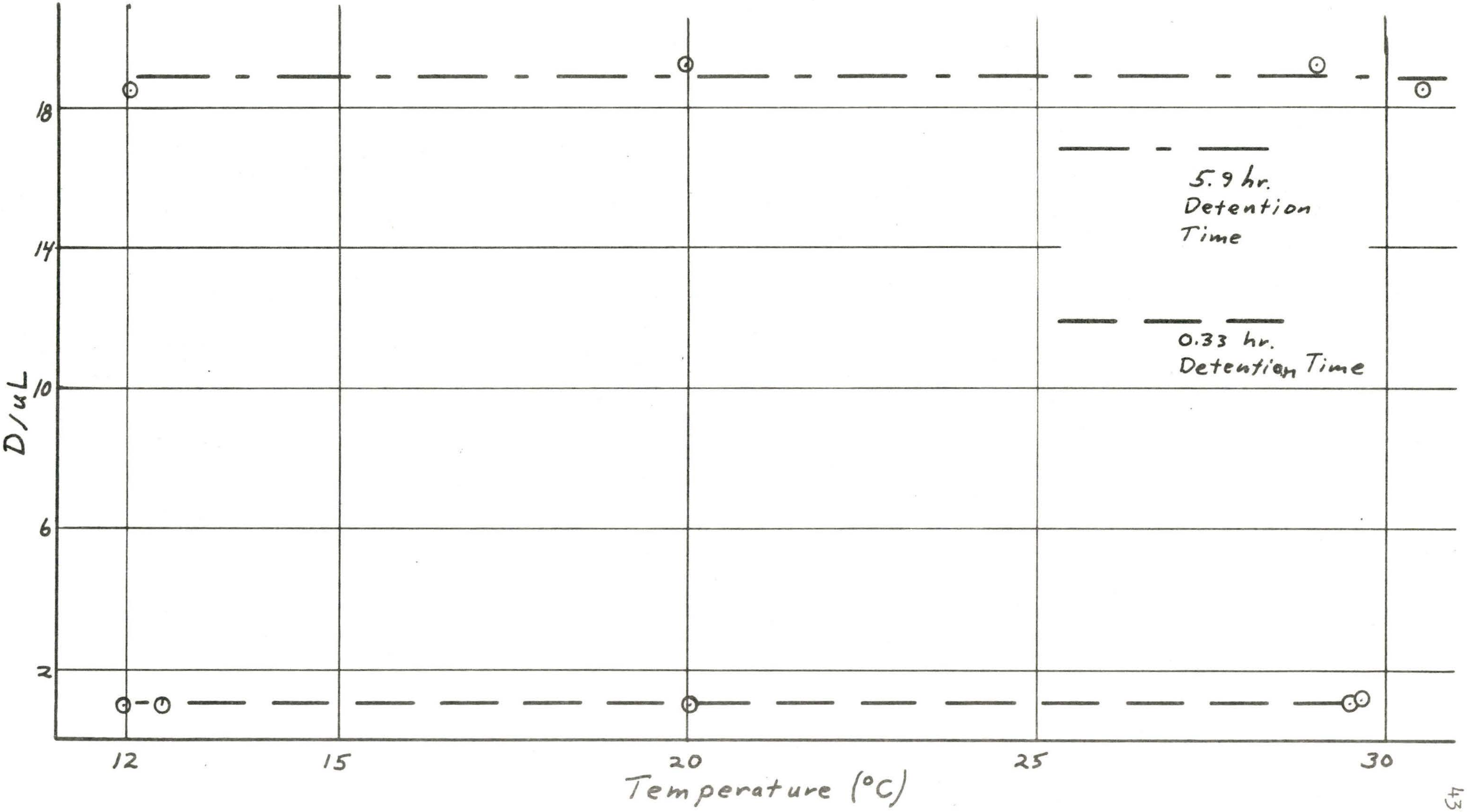
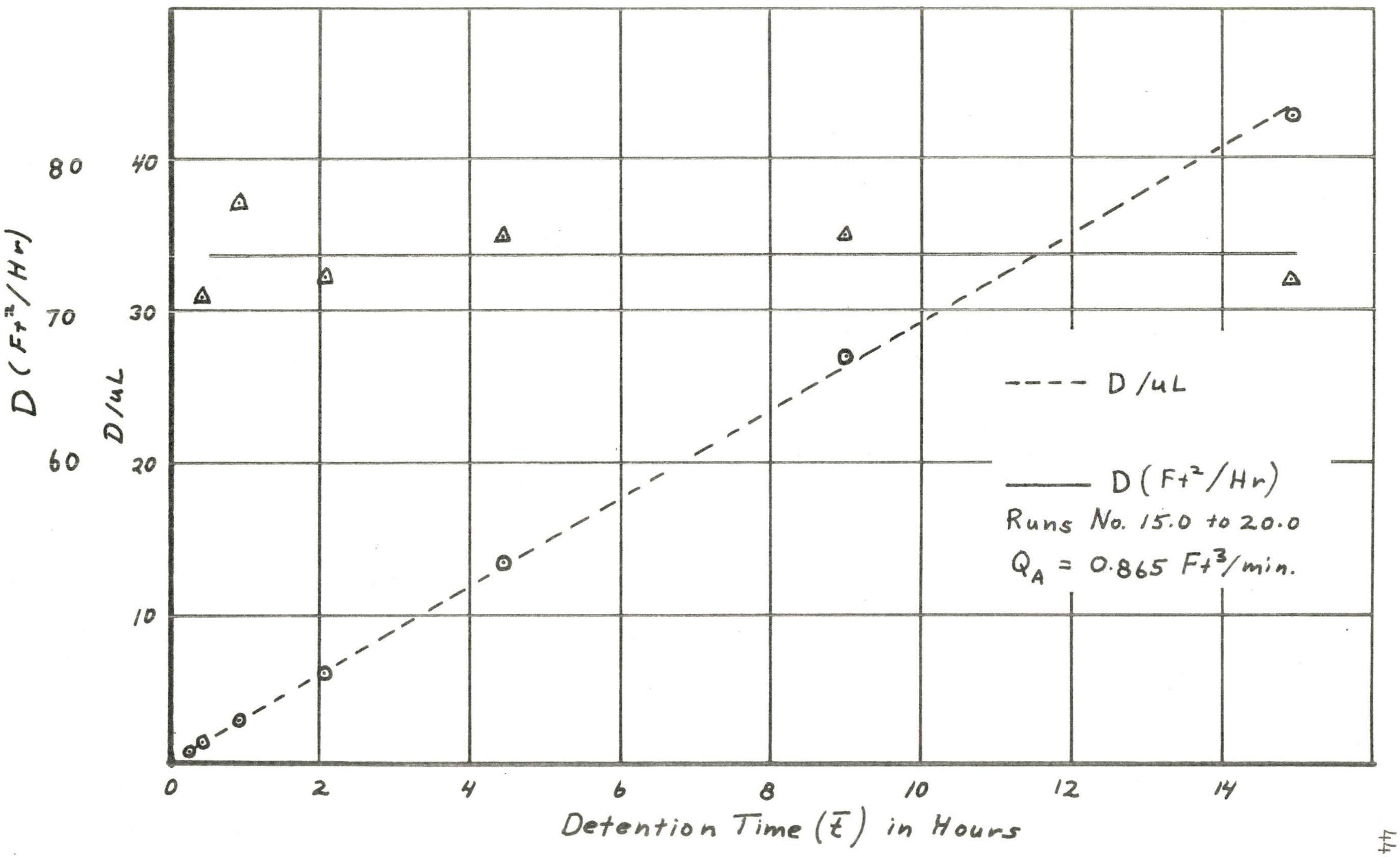


FIGURE 12 EFFECT OF DETENTION TIME ON D/uL AND D



as constant over this range. This indicates that the hydraulic effects of different detention times or axial bulk flow velocity do not alter the mixing characteristics measured by D. As the detention time of a particular tank increases, the axial bulk flow velocity decreases and the tank approaches the mixing conditions characteristic of complete mixing or $D/uL = \infty$.

Effects of Air Flow

The common criteria of 1.0 to 2.0 cu. ft. of air per gallon of domestic waste, suggested a study to determine the effect of the addition of a constant volume of air per gallon of influent. The results, presented in Fig. 13, indicate that D/uL is not directly proportional to the mean residence time for a constant airflow per gallon of influent. It can be shown by a logarithmic plot of the data contained in Fig. 13 that D/uL is directly proportional to the detention time taken to a constant exponent.

The effect of air flow variations upon D/uL for different detention times is presented in Fig. 14. Interpolated points from Fig. 13 are also indicated. The effect of air flow on D/uL decreases as the air flow increases. As with Fig. 13, these plots may be linearized by logarithmic plotting.

Using the fact that D is a constant for any detention time at a given air flow, Figs. 12, 13 and 14 can all be combined into one plot of D versus air flow as in Fig. 15. This plot can be linearized by a logarithmic plot of the data (Fig. 16). Using the least squares method of fitting a straight line it can be shown that:

FIGURE 13 EFFECT OF DETENTION TIME ON D/uL FOR CONSTANT AIR FLOWS

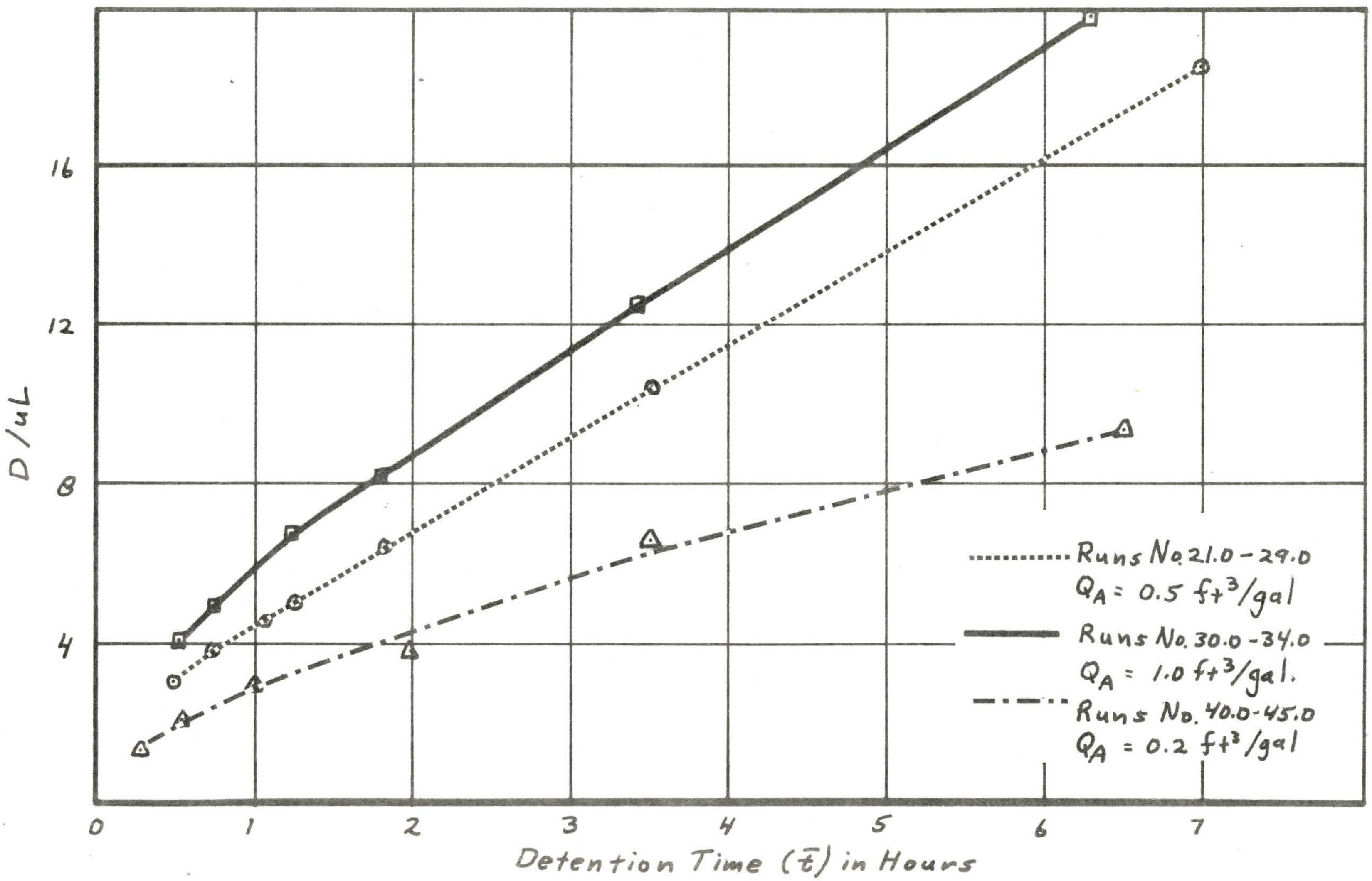


FIGURE 14 EFFECT OF AIR FLOW ON D/uL FOR CONSTANT DETENTION TIMES

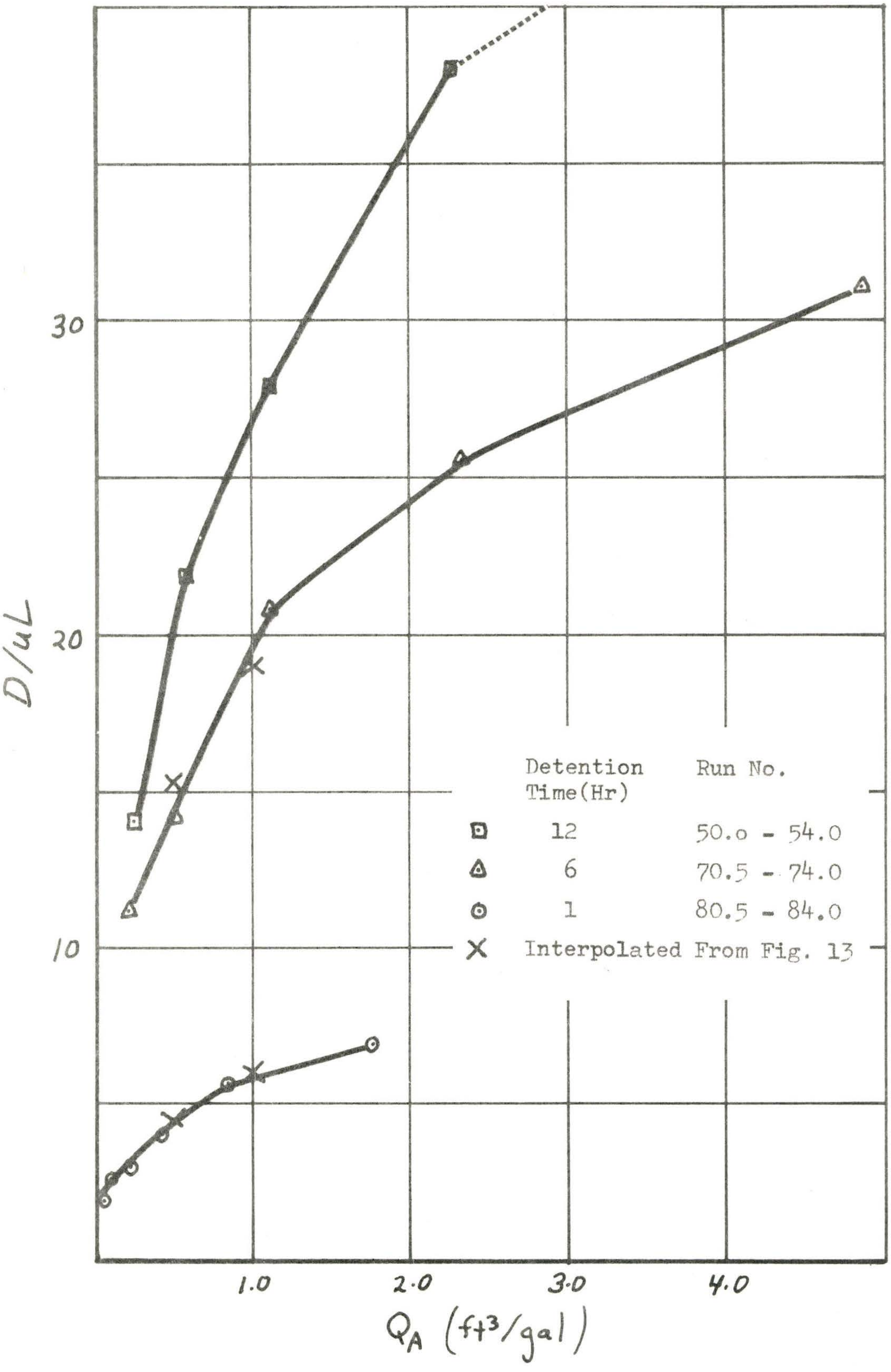


FIGURE 15 EFFECT OF AIR FLOW ON THE DISPERSION COEFFICIENT D

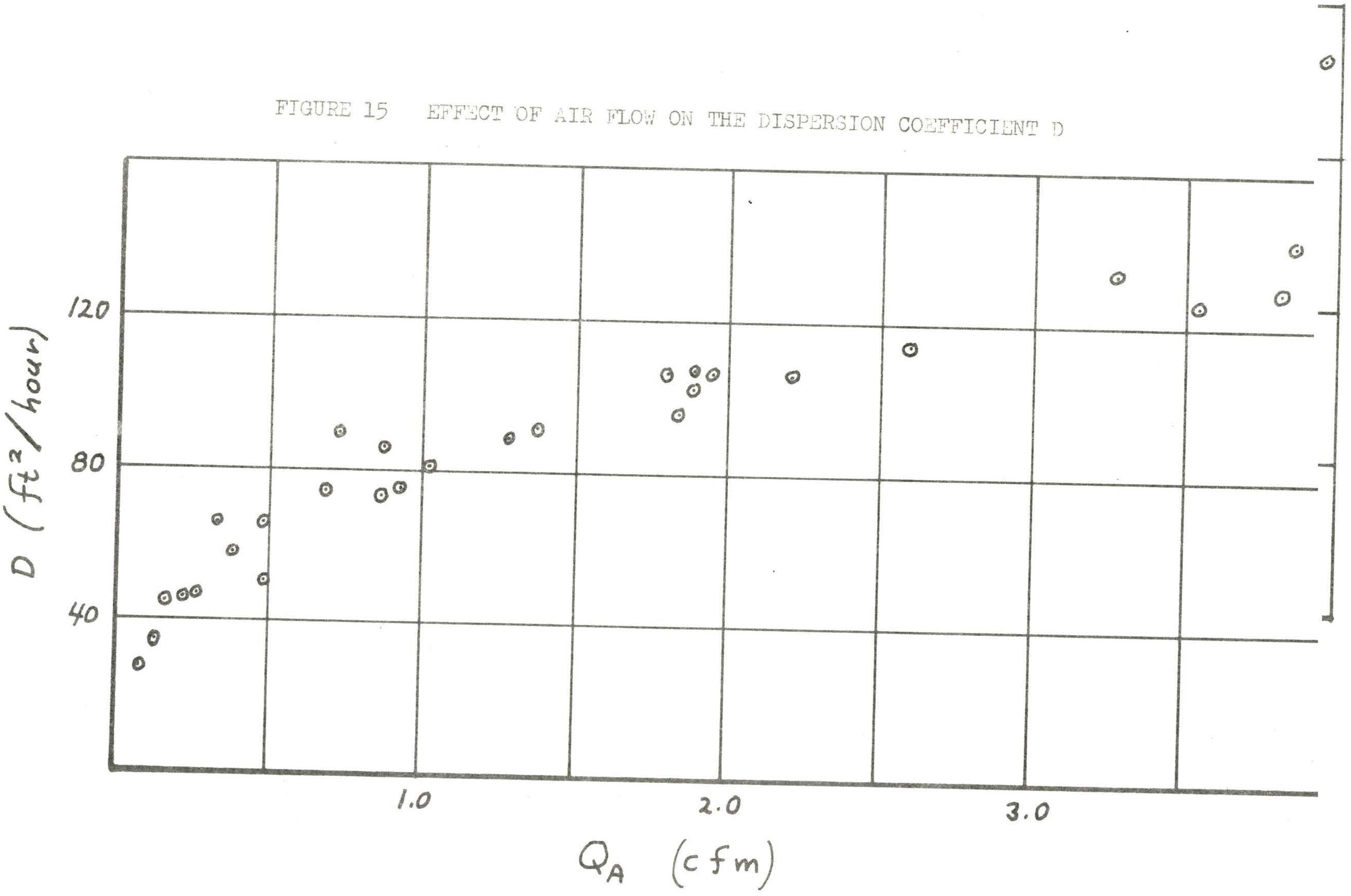
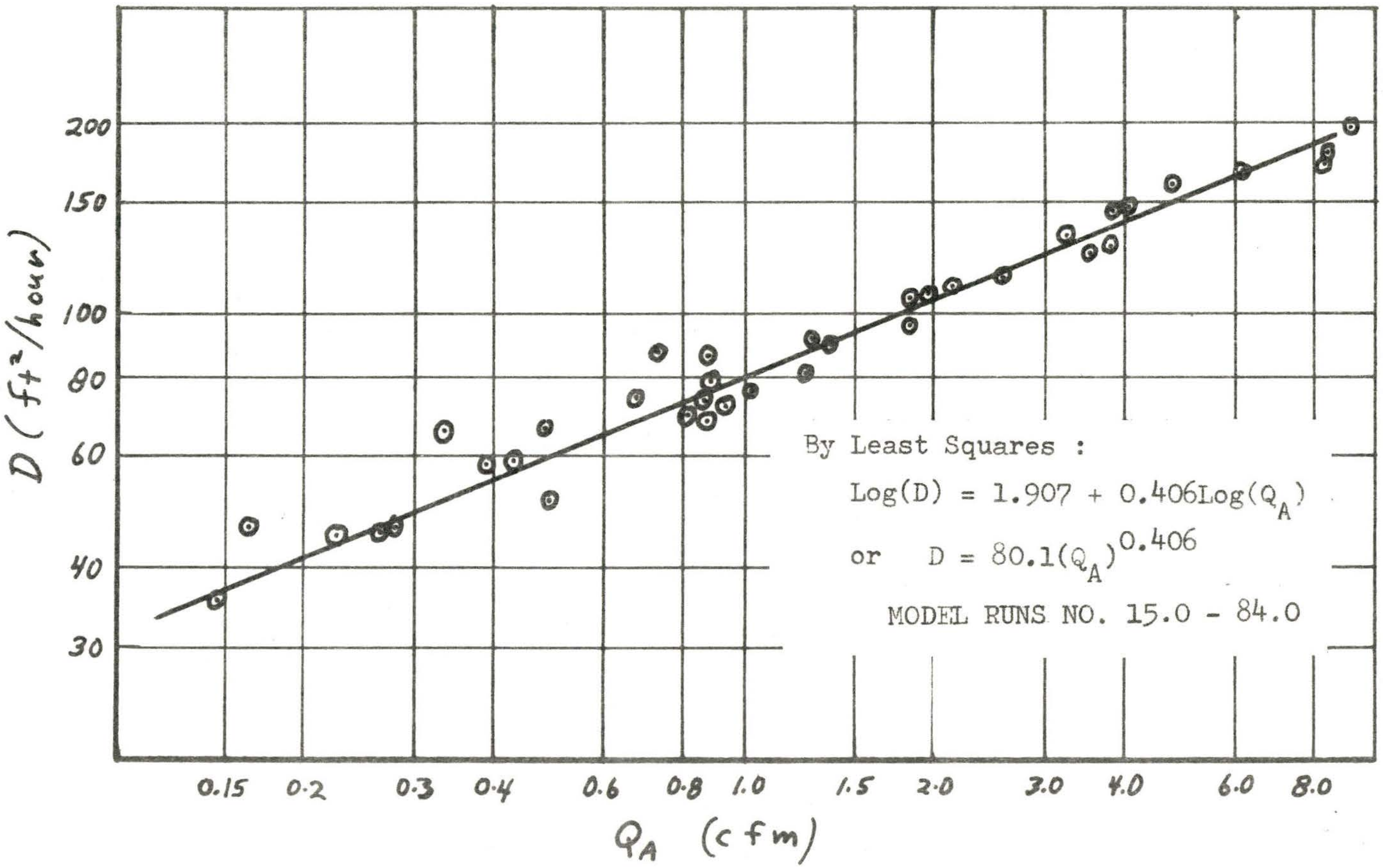


FIGURE 16 EFFECT OF AIR FLOW ON THE DISPERSION COEFFICIENT - LOGARITHMIC



By Least Squares :

$\text{Log}(D) = 1.907 + 0.406\text{Log}(Q_A)$

or $D = 80.1(Q_A)^{0.406}$

MODEL RUNS NO. 15.0 - 84.0

$$\log D = 1.9067 + 0.406 \log (Q_A) \tag{23}$$

or that:

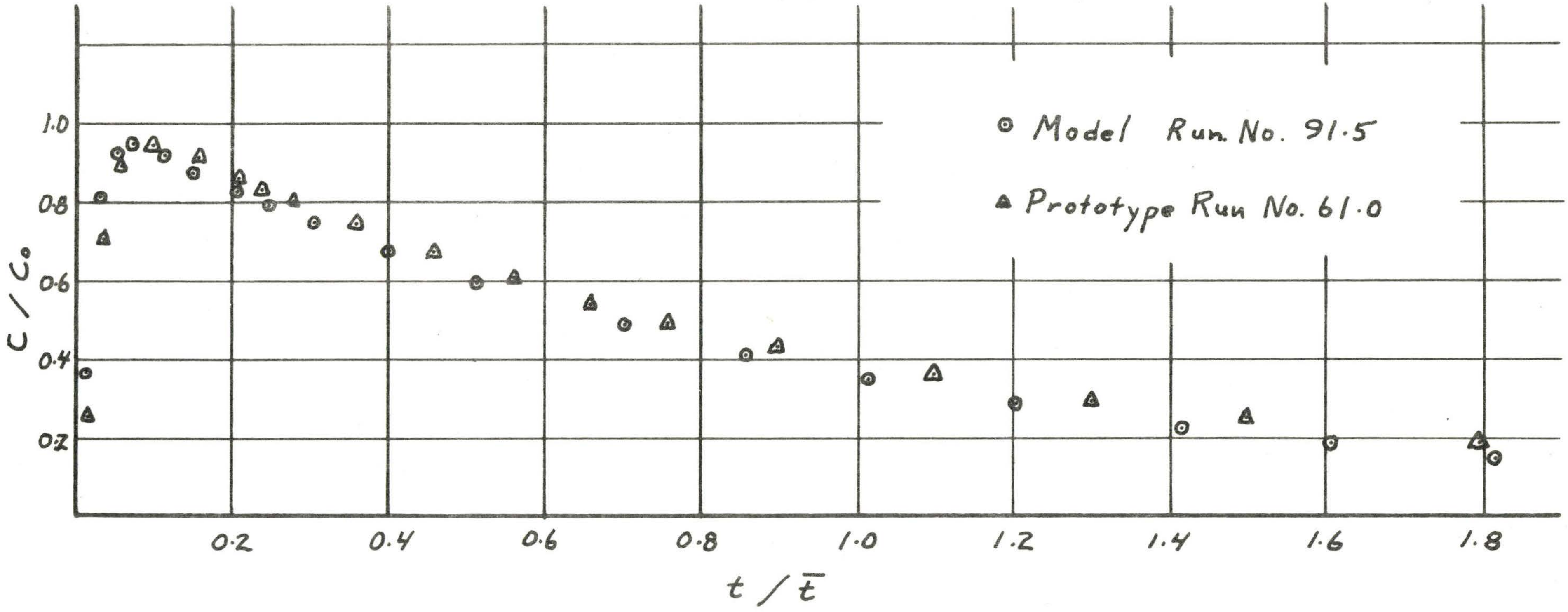
$$D = 80.1 (Q_A)^{0.406} \tag{24}$$

Model-Prototype Study

The response curves of the field prototype study were analyzed in the same manner as the laboratory response curves. Both the prototype response curves gave a peak value of C/Co greater than 1.0, and recoveries of approximately 130%. The reason for these excessive values could be errors in quantity dosed, interaction between the organic substrate and the fluorescence, or the lack of adequate temperature control with the discrete sample door on the fluorometer. To compare the model and prototype response curves, the peak values of C/Co were adjusted to equal those of the laboratory model by altering the value of the input dosage. The resulting percent recoveries were 86 and 100 percent. Fig. 17 shows the response curve comparison of model and prototype for the same air flow (cu.ft./min.-ft.^{3} tank) dimension ratio, and detention time. The laboratory model response curve duplicated the field prototype curve.

The D/uL predicted by the model was 7.2 for test 91.5. The D/uL of the prototype was 5.2 for test 61.0, and 7.5 for test 60.0. The detention time of both tests 91.5 and 61.0 was 4.4 hours. The prototype test 60.0 had a detention time of 5.2 hours, and this different detention time resulted in a D/uL not directly comparable to the D/uL of a shorter detention time.

FIGURE 17 COMPARISON OF MODEL AND PROTOTYPE RESPONSE CURVES



D/uL can be expressed as $D\bar{t}/L^2$. For a given tank (L = const.) and air flow (D = const.) the corresponding D/uL for test 60.0 for 4.4 hours detention time would be 6.4. For the one air flow rate used in the prototype test, a close agreement between model and prototype values of D/uL is indicated in Fig. 18. The average prototype D/uL value was 19 percent less than the model D/uL value.

Model Geometry Studies

The most important aspects of the study were to substantiate the assumption of the dispersion model, and to study the relationships between a model and a prototype. Consequently only tentative relationships concerning the effects of geometry can be reported as only two different tank geometries were studied.

The effect of changing the tank length is compensated by the "L" term in D/uL. The only effect of geometry on the mixing is thus by the width and depth values. Fig. 19 indicates that when the width to depth ratio was increased, the mixing level decreased for a constant air flow per volume of tank. Because both the absolute depth and the width to depth ratio were varied simultaneously, no definite relationship to either parameter could be obtained. However, the tank geometry definitely does affect the mixing level.

Radial Mixing Study

For tests 99.0, 99.3 and 99.4, sampling was at the centre of the cross section one foot from the exit. The times of peak exit concentration for these tests were compared to those of tests 99.1, 99.2 and 99.5 where sampling was from a point three inches below the water surface. There

FIGURE 18 EFFECT OF AIR FLOW ON D/uL FOR MODEL AND PROTOTYPE

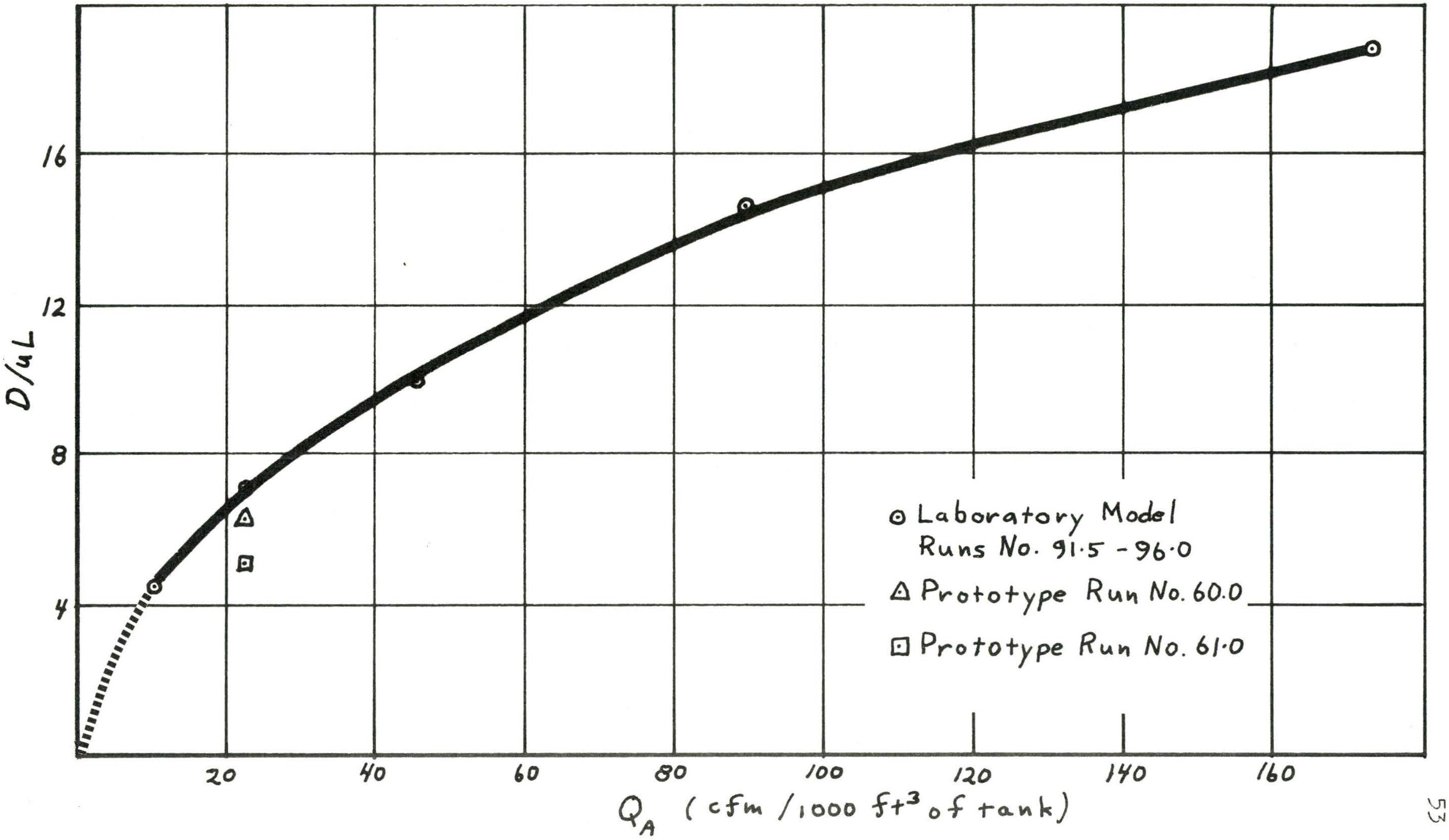
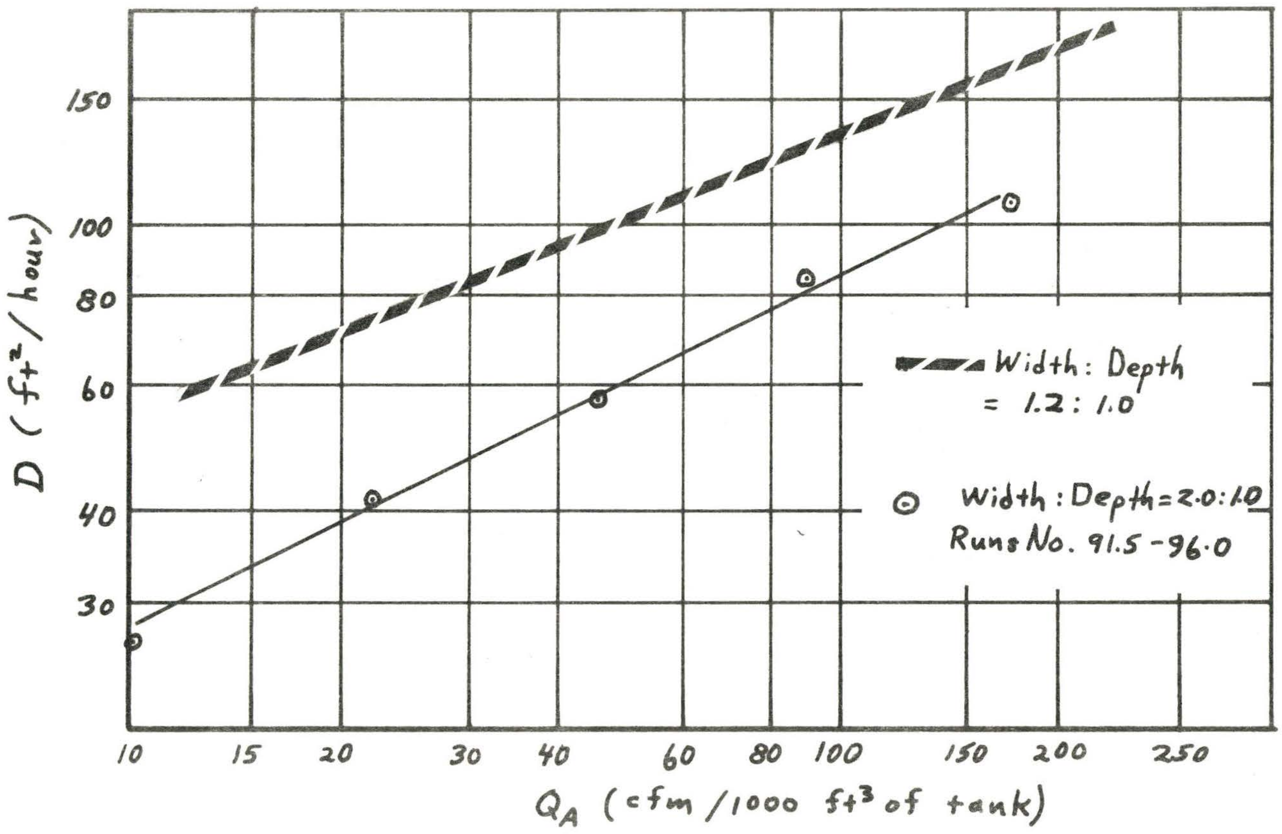


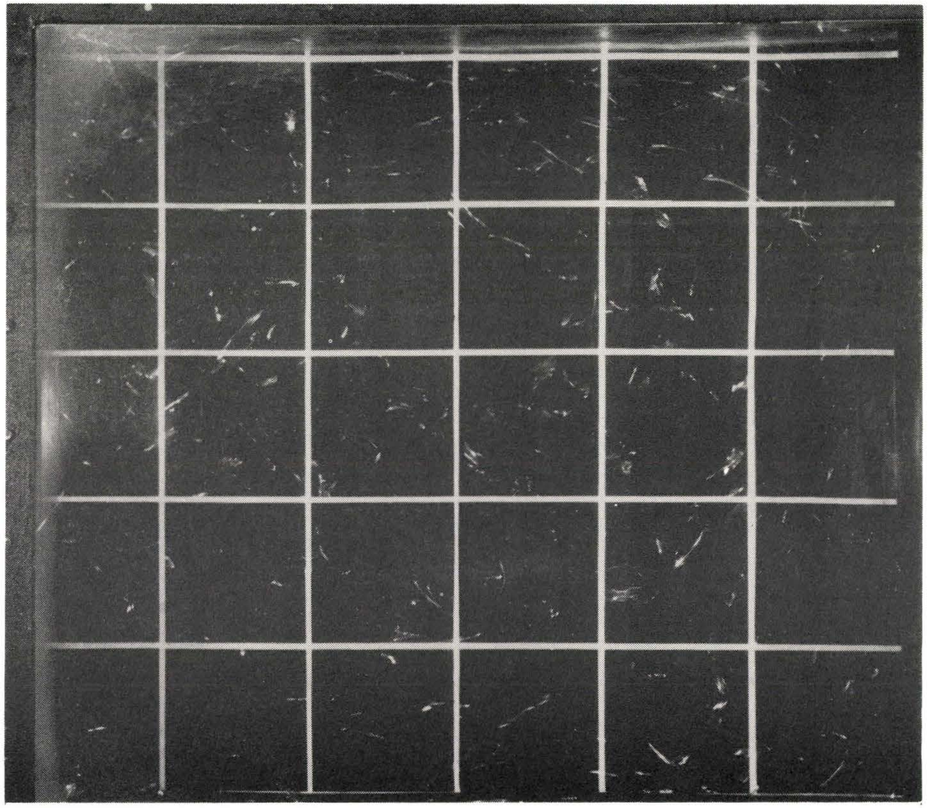
FIGURE 19 EFFECT OF AIR FLOW ON D FOR MODEL GEOMETRY CHANGE



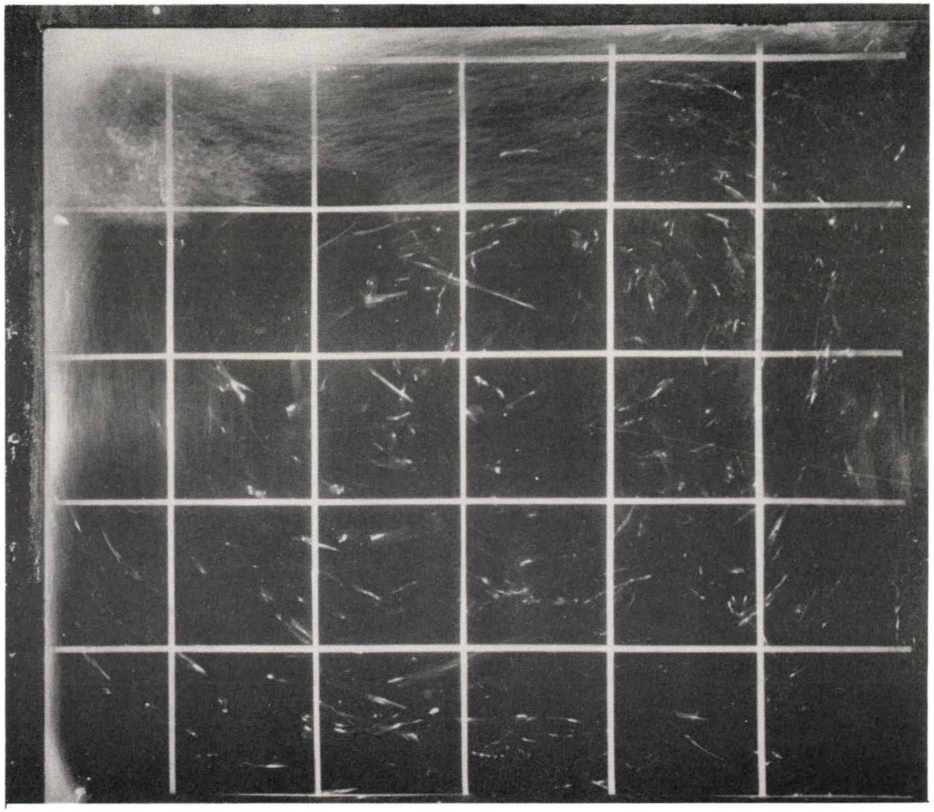
was no difference in the mean time to peak for the two different sampling points. This result indicated that no difference between the longitudinal dispersion in the centre core and the periphery of the tank could be detected.

No "stagnant" core could be detected, probably because of the degree of radial mixing. The degree of radial mixing is illustrated in Fig. 20 with pictures of the cross-section tank. The water streamlines as traced by the reflections from the saran tinsel follow a general trend indicating the bulk roll of the tank contents. At almost all points the streamlines were observed to fluctuate by as much as 40 degrees. This random fluctuation of streamline direction accounts for the high degree of radial mixing, and appears to explain why no "stagnant" core could be detected.

FIGURE 20 RADIAL FLOW PATTERNS IN THE CROSS SECTION TANK



Run No. 1.0, Exposure of 1.0 sec. at f 16, Q_A = 0.377 cfm, Dimensions: Length 1.0 ft., Width 3.0 ft., Depth 2.5 ft.



Run No. 3.0, Exposure of ½ sec. at f 11, Q_A = 1.62 cfm, Dimensions: Length 1.0 ft., Width 3.0 ft., Depth 2.5 ft.

CHAPTER 7

CONCLUSIONS, DISCUSSION AND RECOMMENDATIONS

The studies presented in the previous chapters indicate that the dispersed plug flow model adequately describes the longitudinal mixing conditions of a spiral flow aeration tank. Although the literature indicates that this model cannot satisfactorily characterize flow patterns which deviate greatly from plug flow, the experimental data verifies the use of this model for highly mixed spiral flow aeration tanks. The axial dispersion coefficient D ($ft.^2/hr.$) can be combined with the tank length L (ft.) and the longitudinal bulk flow velocity u (ft./hr.) to yield the dimensionless parameter D/uL which characterizes the theoretical tracer response curve. Using the dispersion model the following conclusions can be made from tests performed on a laboratory model 5.0 ft. long, 3.0 ft. wide, and 2.5 ft. deep.

1. The theoretical response curve can be easily fitted to an experimental response curve by finding the time of peak exit concentration which can be correlated to the theoretical D/uL value. The comparison of the theoretical and experimental curves by their variances is not feasible for the mixing levels common in aeration tanks.
2. For a constant air flow and detention time D/uL can be considered a constant over the range of 12 to 30 degrees Centigrade.

3. The axial dispersion coefficient (D) can be considered a constant over a range of 0.3 to 15.0 hours detention time for a constant air flow ($ft.^3$/min.).
4. The axial dispersion coefficient (D) may be related to the air flow rate (Q_A, $ft.^3$/min.) for the tank used in this study by

$$D = 80.1 \, (Q_A)^{0.406}$$

5. Correlations of the dimensionless parameter D/uL with the detention time or air flow ($ft.^3$/gal. of influent) are possible. All of these variables contain the detention time of the system and thus are not as significant as the correlation of D and Q_A above.
6. Apparently radial diffusion was rapid as no differences were detected in the time to peak exit tracer concentration for the centre core and the periphery. This result indicated that the tank core was not "stagnant".

By comparing the full scale prototype tank to the 1/13th geometric scale laboratory model, and by comparing two different models with different geometries, the following conclusions can be made:

7. A scaled-down laboratory model provided good approximation of the dimensionless curve obtained from a full scale aeration tank with the same detention time and air flow (cfm/cu. ft. of tank). The laboratory model D/uL was 7.2, and the prototype yielded D/uL equal to 5.2 and 6.4 for the same detention time.

8. The effect of changing the tank length is compensated by the "L" term in D/uL. In the two tests run at a different absolute depth and width to depth ratio, no definite relationship to either parameter could be obtained. However, the tank geometry definitely does affect the mixing level.

Further studies are needed to fully establish the correlation of tank geometry and mixing levels. The dispersion model should also be used to re-evaluate the effects of the many variations of the conventional activated sludge tank. Step aeration, tapered aeration, tanks-in-series or baffles, and possibly even solids concentration could be evaluated for the effects on mixing using the dispersion model.

Because of the statistical nature of the dispersion model, it should be possible to study the effects on mixing levels by the air bubble size, the geometric position of air injection, type of injector, and bubble detention period.

Finally the verification of the dispersion model utilizing a biological or chemical reactor would establish its use as an operational and design criteria. The accurate prediction of reduction in substrate for a known first order reaction would verify the residence time distribution. If the prediction could be made for a second order reaction, then it could be concluded that the dispersion model is the correct detailed flow model of this reactor.

REFERENCES

1. Archibald, J. M., "Radioactive Tracers in Flow Tests", Proceedings of the Boston Society of Civil Engineers, 37, 49, (1949).

2. Ardern, E., and Lockett, W. T., "Experiments on the Oxidation of Sewage Without the Aid of Filters", Journal of the Society of Chemical Industries, London, 33, 523, (1914).

3. Busch, A. W., "Activated Sludge Kinetics and Effluent Quality", Proc. A.S.C.E., Journal of Sanitary Engineering Division, 88, SA6, 1, (1962).

4. Calvert, C. K., and Bloodgood, D. E., "Operation Experiments at the Indianapolis Activated Sludge Plant", Sewage Works Journal, 6, 1068, (1934).

5. Calvert, C. K., "The Hourly Variations of Indianapolis Sewage", Sewage Works Journal, 4, 815, (1932).

6. Camp, T. R., "Flocculation and Flocculation Basins", Trans. A.S.C.E., 120, 1, (1955).

7. Carpenter, J. H., "Tracer for Circulation and Mixing in Natural Waters", Public Works Magazine, 91, 110, (1960).

8. Chollette, A., Blanchet, J., and Cloutier, L., "Performance of Flow Reactors at Various Levels of Mixing", Canadian Journal of Chemical Engineering, 38, 1 (1960).

9. Danckwerts, P. V., "Continuous Flow Systems - Distribution of Residence Times", Chemical Engineering Science, 2, 1 (1953).

10. Dawkins, G. S., "Mixing Patterns and Residence Time Predictions", 18th Industrial Waste Conference, Purdue University, 562, (1963).

11. Fan, L. T., and Bailie, R. C., "Axial Diffusion in Isothermal Tubular Flow Reactors", Chemical Engineering Science, 13, 63, (1960).

12. Feuerstein, D. L., and Selleck, R. E., "Fluorescent Tracers for Dispersion Measurements", Proc. A.S.C.E., Journal of Sanitary Engineering Division, 89, SA4, 2, (1963).

13. Gould, R. H., "Tallmans Island Works Opens for World's Fair", Municipal Sanitation, 10, 185, (1939).

14. Greeley, S. A., "The Development of the Activated Sludge Method of Sewage Treatment", Sewage Works Journal, 17, 1135, (1945).

15. Hasseltine, T. R., "The Activated Sludge Processes at Salinas, California, with Particular Reference to Causes and Control of Sludge Bulking", Sewage Works Journal, 4, 461, (1932).

16. Hurd, C. H., "Distinctive Characteristics of the Indianapolis Sewage Treatment Plant", Sewage Works Journal, 1, 578, (1929).

17. Kehr, R. W., "Detention of Liquids Being Mixed in Continuous Flow Tanks", Sewage Works Journal, 8, 915, (1936).

18. Kandiner, H. J., "Sampling Lag and Purging Time in Mixing Vessels in Series", Chem. Eng. Progress, 44, 383, (1948).

19. Kessener, H. J. N. H., and Ribbius, F. J., "Comparison of the Aeration Systems for the Activated Sludge Process", Sewage Works Journal, 6, 423, (1934).

20. Kessener, H. J. N. H., "Annual Report, Government Institute for Purification of Waste Waters, 1933", Sewage Works Journal, 7, 135, (1935).

21. Krenkel, P. A., and Orlob, G. T., "Turbulent Diffusion and the Reaeration Coefficient", Proc. A.S.C.E., Journal of the Sanitary Engineering Division, 88, SA2, 53, (1962).

22. Levenspiel, O., "Comparison of the Tank-in-series and Dispersion Models for Non-ideal Flow of Fluid", Chemical Engineering Science, 17, 576, (1962a).

23. Levenspiel, O., Chemical Reaction Engineering, Wiley, New York (1962b).

24. Levenspiel, O., and Bischoff, "Backmixing in the Design of Chemical Reactors", Industrial Engineering Chemistry, 51, 1431 (1959); 53, 313, (1961).

25. Levenspiel, O., and Smith, W. K., "Notes on the Diffusion-type Model for Longitudinal Mixing of Fluids in Flow", Chemical Engineering Science, 6, 227, (1957).

26. McKee, J. E., and Fair, G. M., "Load Distribution in the Activated Sludge Process", Sewage Works Journal, 14, 121, (1942).

27. McKinney, R. E., "Complete Mixing Activated Sludge", Water and Sewage Works, 107, 69, (1960).

28. Milbury, W. F., "A Development and Evaluation of a Theoretical Model Describing the Effects of Hydraulic Regime in Continuous Microbial Systems", Northwestern University, Ph.D. Thesis (1964).

29. Miyauchi, T., "Residence Time Curves", Chemical Engineering (Japan), 17, 382, (1953).

30. Patterson, C. C., "The Dispersion of Radionuclides in Open Channel Flow", The University of Texas, Ph.D. Thesis, Dissertation Abstracts, 24, 5321, (1964).

31. Pipes, W. O., et al, "A Laboratory Study of Mixing Conditions in Small Aeration Vessels", 19th Ind. Waste Conf., Purdue University, 927, (1964a).

32. Pipes, W. O., et al, "A Comparison of the Axial Dispersion and Volume Apportionment Methods for Describing Mixing Conditions in an Aeration Tank", 19th Ind. Waste Conf., Purdue University, 940, (1964b).

33. Pipes, W. O., et al, "Compartmentalization of Aeration Tanks", Proc. A.S.C.E, Journal of the Sanitary Engineering Division, 91, SA3, 45, (1965).

34. Pritchard, D. W., and Carpenter, J. H., "Measurements of Turbulent Diffusion in Estuarine and Inshore Waters", International Assoc. of Sc. Hydrology, 20, 37, (1960).

35. Rohlich, G. A., and Sawyer, C. N., "Studies on the Activated Sludge Process at Two Rivers, Wisconsin; Part II, Stage Addition of Activated Sludge", Sewage Works Engineering, New York, 14, 234, (1943).

36. Smith, D. B., "Measurements of Respiratory Activity of Activated Sludge", Sewage and Industrial Wastes, 25, 767, (1953).

37. Stack, V. T., and Conway, R. A., "Design Data for Completely Mixed Activated Sludge Treatment", Sewage and Industrial Wastes, 31, 1181, (1959).

38. Standard Methods for the Examination of Water and Waste Water, American Public Health Association, 11th Ed., New York, (1960).

39. Sueishi, T., Unpublished Laboratory Data, McMaster University (1964).

40. Thomas, H. A., and McKee, J. E., "Longitudinal Mixing in Aeration Tanks", Sewage Works Journal, 16, 42, (1944).

41. Thomas, H. A., and Archibald, J. M., "Longitudinal Mixing Measured by Radioactive Tracers", Trans. A.S.C.E., 117, 839, (1952).

42. Thomas, H. A., "Measurements of the Respiratory Activity of Activated Sludge", Sewage and Industrial Wastes, 25, 776, (1953).

43. Turner Associates, G. K., "Manual of Fluorometric Clinical Procedures", (no date).

44. Van der Laan, E. T., "Notes on Diffusion-type Model for Longitudinal Mixing in Flow", Chemical Engineering Science, 7, 187, (1958).

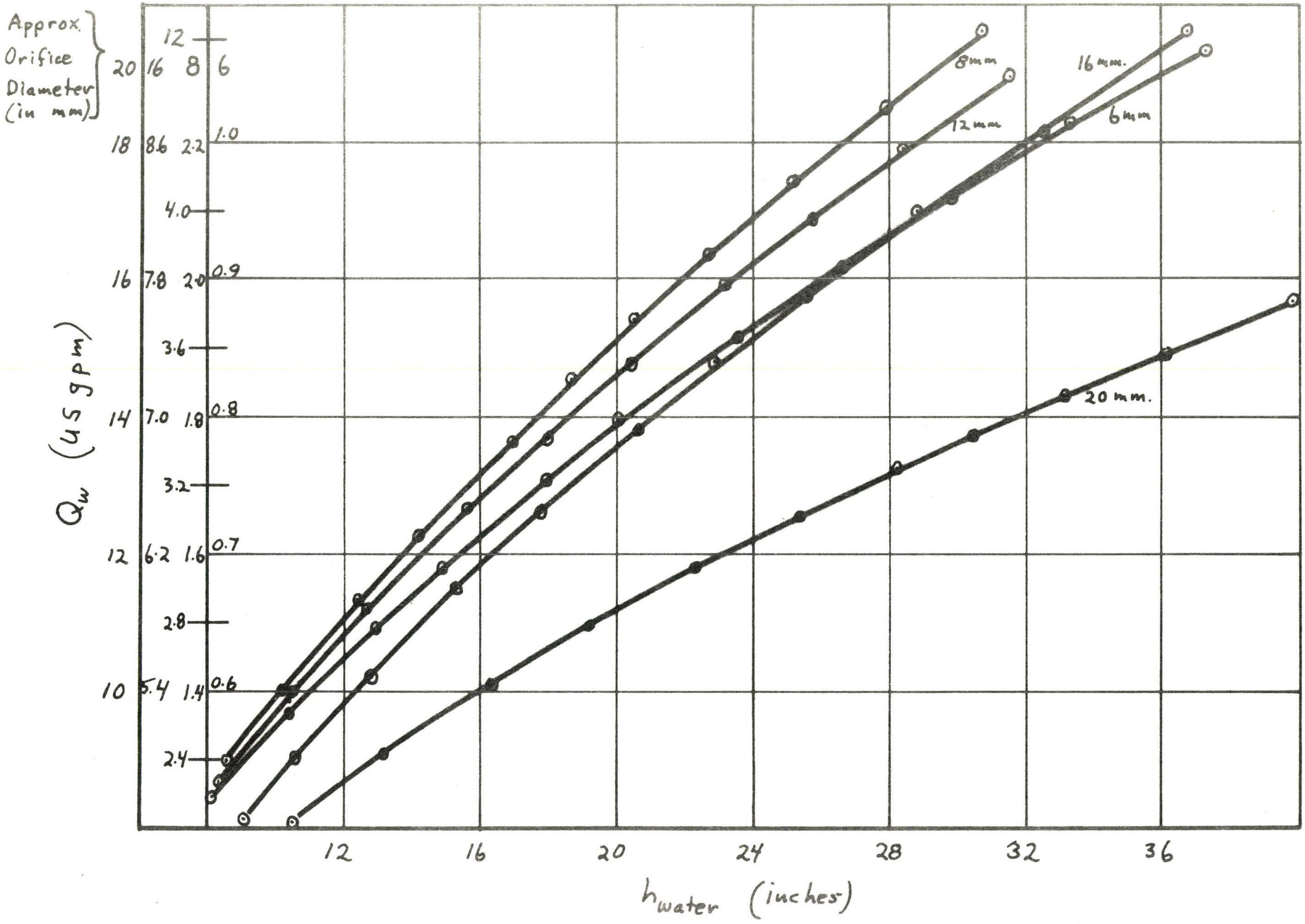
45. Water Pollution Control Federation, Manual of Practice No. 8, Sewage Treatment Plant Design, (1959).

46. Wehner, J. F., and Wilhelm, R. H., "Boundary Conditions of Flow Reactor", Chemical Engineering Science, 6, 89, (1956).

APPENDIX "A"

Reynolds Column Calibration Curves

FIGURE 21 REYNOLDS COLUMN CALIBRATION CURVES



APPENDIX "B"

Fluorometer Calibration Curves

FIGURE 22 FLUOROMETER CALIBRATION CURVES - LOW VALUES

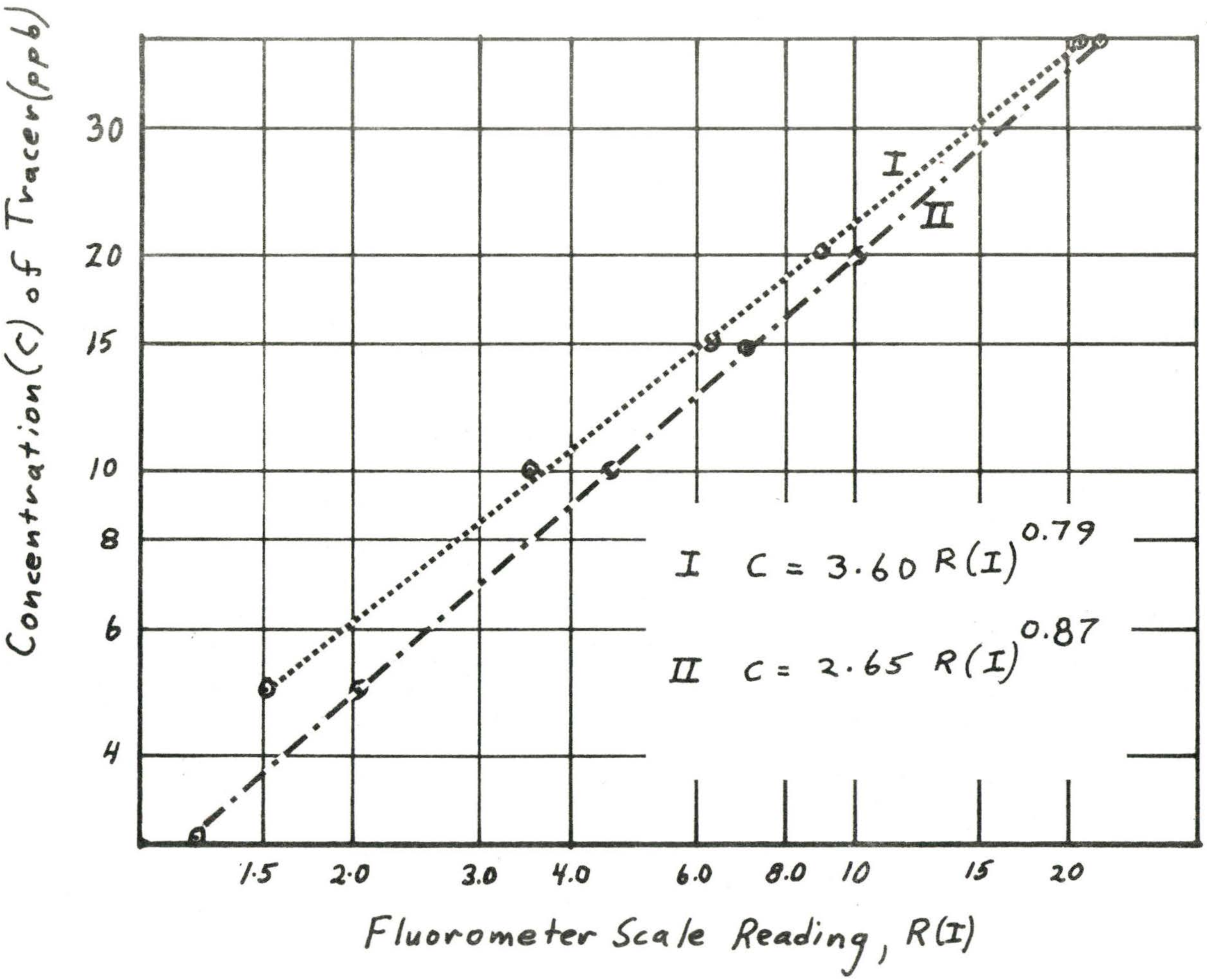
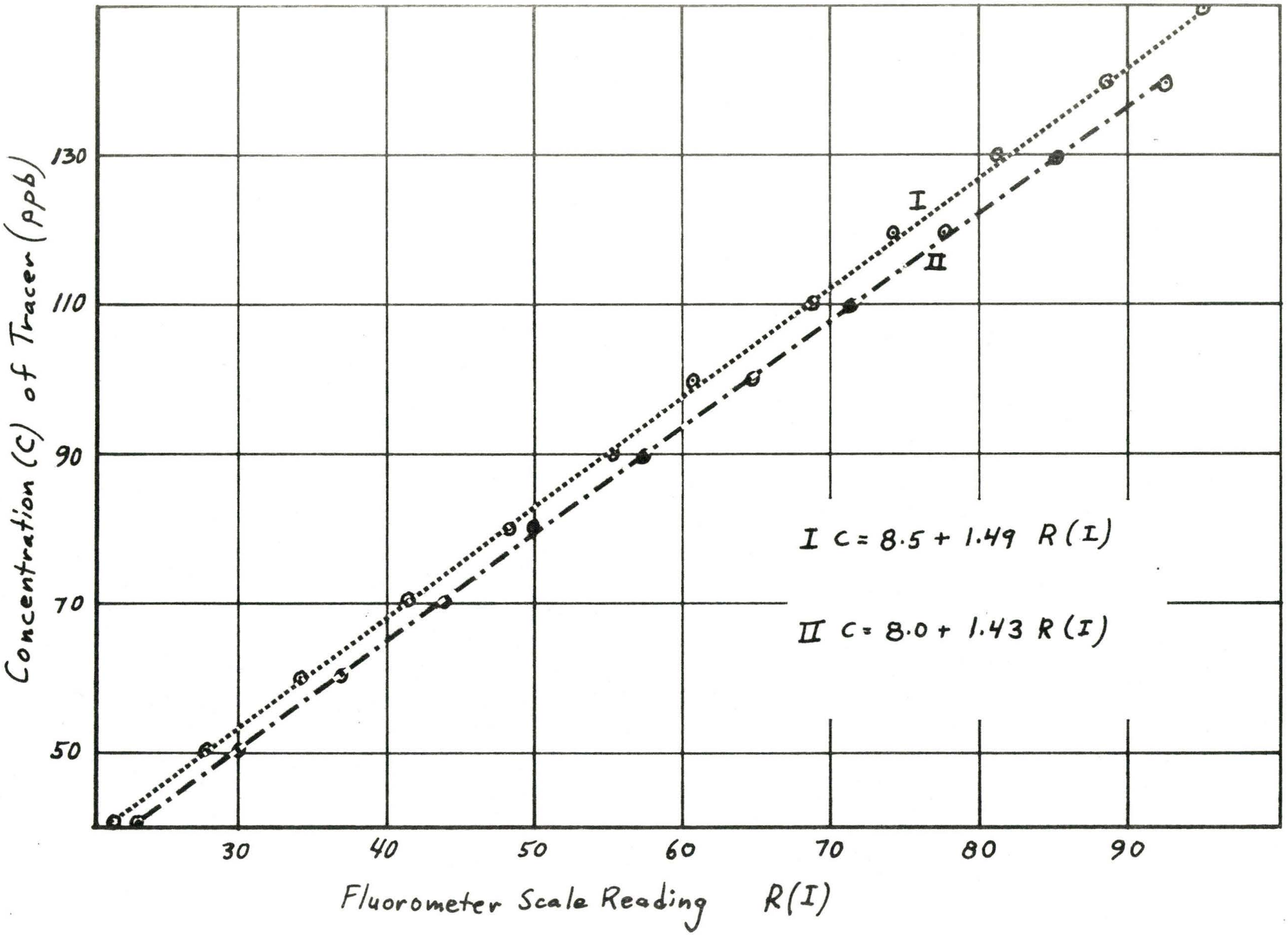


FIGURE 23 FLUOROMETER CALIBRATION CURVES - HIGH VALUES



APPENDIX "C"

Computer Programmes Used for Analysis

TABLE I

Nomenclature Used for Computer Programmes

Programme Symbol	Meaning or Equivalent	Programme Symbol	Meaning or Equivalent
A	a	PF	Function
C	C	PR	Percent Removal
C(I)	C	PT	t_p
CO	Co	PTO	$t_p/\bar{t}$
D	Depth	PTP	Peak Time D/uL
DC(I)	C	PTO2	Peak Time σ^2
DET	Theoretical Det. Time	PU	Percent Substrate Remaining
DT	Time Inc.	QA	Q_A
FO	Function	QW	Q_W
FR	Function	R(I)	Fluorometer Reading or μ_i
I, J, K	Counters		
		RK	k
N	Number of Data Pts.	RN	Run No.
NCC	Fluorometer Cal. Curve No.	RT	Mean Tracer Det. Time
O	$t/\bar{t}$	SC	C/C_o
O1	θ_1	SC(I)	C/C_o
O2	θ_2, or σ^2	SUMC	$\sum C$
P	D/uL	SUMTC	$\sum tC$
PCR	Percent of Tracer Recovery	SUMT2C	$\sum t^2C$

TABLE I (Continued)

Programme Symbol	Meaning or Equivalent	Programme Symbol	Meaning or Equivalent
T	t	V	σ^2
TI	Time Inc.- Dimensionless	VI	Tracer Vol Dosed
TL	Tank Length	W	Tank Width
TT(I)	$t/\bar{t}$		
U	u		

TABLE II

Experimental Response Curve Analysis

```
      DISPERSION CURVE ANALYSIS
      DIMENSION R(1000),SC(1000),DC(1000),TT(1000)
1000  READ 20,RN,QA,QW,TL,W,D
      READ 21,DT,VI,N,NCC,PT
      READ 22 ,(R(I),I=1,N)
   20 FORMAT (6F10.5)
   21 FORMAT (2F10.5,2I10,F10.5)
   22 FORMAT (16F5.1)
      PRINT 30
   30 FORMAT(1H1, 131HRUN    AIR      WATER     L      W       D
     1RESIDENCE TIME   PERCENT  VARIANCE   D/UL     U   PERCENT     PEAK
     2 TIME METHOD         /
     3 131H NO.    FLOW     FLOW                            THEOR.    ME
     4AN     RECOVERY                         REMOVAL    O    D/UL VARIA
     5NCE            /
     6113H         (CFM)    (GPM)   (FT)   (FT)    (FT)      (HR)     (H
     7R)                             (FT/HR) (K=1.5)              )
      D=D/12.0
      DET=(TL*W*D)/(QW*8.0208)
   40 I=1
      C=0.0
      T=0.0
      SUMTC=0.0
      SUMC=0.0
      SUMT2C=0.0
      IF(NCC.EQ.1) GO TO 46
   41 IF(R(I))60,42,42
   42 IF(R(I).LE.21.0)  GO TO 44
      IF(R(I).LE.100.0)  GO TO 43
      C=26.5+5.91133*(R(I)-100.0)
      GO TO 50
   43 C=8.0+1.43077*R(I)
      GO TO 50
   44 C=2.65*R(I)**0.871047
      GO TO 50
   46 IF(R(I)) 60,47,47
   47 IF(R(I).LE.21.0) GO TO 48
      C=8.5+1.48993*R(I)
      GO TO 50
   48 C=3.6*R(I)**0.790706
   50 DC(I)=C
      T=T+DT
      SUMC=SUMC+C
      SUMTC=SUMTC+T*C
      SUMT2C=SUMT2C+(T**2)*C
      I=I+1
      IF(NCC.EQ.1) GO TO 46
      GO TO 41
```

TABLE II (Continued)

```
 60 J=I-1
    PCR=(QW*0.03785*SUMC*DT)/VI
    RT=SUMTC/(SUMC*60.0)
    U=TL/RT
    O2=((SUMT2C/SUMC)-(SUMTC/SUMC )**2)/(RT*RT*3600.0)
    P=0.0
 71 CONTINUE
    P=P+0.001
    PF=O2-2.0*P+2.0*P*P*(1.0-EXP(-1.0/P))
    IF (PF) 72,72,71
 72 RK=1.5
    A=SQRT(1.0+4.0*RK*RT*P)
    PU=(400.0*A*EXP(1.0/(2.0*P)))/((((1.0+A)**2)*EXP(A/(2.0*P)))-(((1.
   10-A)**2)*EXP(-A/(2.0*P))))
    PR=100.0-PU
    PTO=PT/(60.0*DET)
    PTP=0.2*PTO**(-1.3355)
    PTO2=2.0*PTP-2.0*PTP**2*(1.0-EXP(-1.0/PTP))
    PRINT 75,RN,QA,QW,TL, W,D,DET,RT,PCR,O2,P, U ,PR,PTO,PTP,PTO2
 75 FORMAT(1H0,F5.1,F8.3,F8.3,F7.2,F7.2,F9.4,2X,2(F8.4,2X),F7.2,1X,F8.
   14,2X,F7.3,    F8.2,F6.2,3F8.4    )
    TI=DT/(DET*60.0)
    PRINT 100,TI
100 FORMAT(1H0,//30X,33HDIMENSIONLESS DISPERSION CURVE AT ,F8.5,10H IN
   1TERVALS )
    C0=(VI/(TL*W*D*28316.0))*10.0**7
    TT(1)=TI-0.5/(DET*60.0)
    DO 110 I=1,J
    SC(I)=DC(I)/C0
110 TT(I+1)=TT(I)+TI
    PRINT 115
115 FORMAT(1H0,8(16H C/C0    TIME      ))
    PRINT 120,(SC(I),TT(I),I=1,J )
120 FORMAT(1H0,8(F6.3,F7.4,3X))
    GO TO 1000
    END
```

TABLE III

Unequal Tanks In Series Solution

```
   UNEQUAL TANKS IN SERIES SOLUTION
   DIMENSION C(100)
 1 READ 2,V
 2 FORMAT(F10.5)
   O1=0.0
10 O1=O1+0.001
   FO=  V-((  1.0-2.0*O1+2.0*O1**3-2.0*(1.0-O1)**3)/(2.0*O1-1.0))
   IF(FO) 10,10,15
15 O1=O1-0.00001
   FO=  V-((  1.0-2.0*O1+2.0*O1**3-2.0*(1.0-O1)**3)/(2.0*O1-1.0))
   IF(FO) 20,15,15
20 O2=O1
   O1=1.0-O1
   PRINT 30,V,O1,O2
30 FORMAT(1H1,9HVARIANCE= ,F8.5,10X,3HO1= ,F8.5,10X,3HO2=, F8.5,  ///
  1     40H  DIMENSIONLESS CURVE AT 0.01 INTERVALS      ,80X,5HTIME   )
   T=0.0
   DT=0.01
35 DO 40 I=1,10
   T=T+DT
40 C(I) =  EXP(-T/O1)/(O1*(1.0-O2/O1))+ EXP(-T/O2)/(O2*(1.0-O1/O2))
   PRINT 50,(C(I),I=1,10),T
50 FORMAT(1H0,10F10.5,15X,F10.4 )
   IF(T.LE.5.0) GO TO 35
   GO TO 1
   END
```

TABLE IV

Theoretical Dispersion Curve Analysis for a Pulse Input

```
   THEORETICAL DISPERSION CURVE ANALYSIS - PULSE
   DIMENSION         R(1000),SC(100),C(100)
 1 READ 2,P,O2
 2 FORMAT(2F10.5)
   IF(O2.LE.0.0001) GO TO 5
   P=0.0
 3 P=P+0.001
   PF=O2-2.0*P+2.0*P*P*(1.0-EXP(-1.0/P))
   IF(PF) 4,3,3
 4 P=P-0.000001
   PF=O2-2.0*P+2.0*P*P*(1.0-EXP(-1.0/P))
   IF(PF) 4,6,6
 5 O2=2.0*P-2.0*P*P*(1.0-EXP(-1.0/P))
 6 PRINT 7,P,O2
 7 FORMAT(1H1,6H D/UL=,F9.4,10X,38HDIMENSIONLESS CURVE AT 0.01 INTERV
  1ALS       ,10X,3HO2=,F10.5,30X,6H TIME   )
   I=1
   U=0.5/P
   R(I)=1.4
   O=0.0
10 R(I)=R(I)-0.001
   FR=COS(R(I))/SIN(R(I))-R(I)*P+0.25/(R(I)*P)
   IF(FR) 10,10,20
20 R(I)=R(I)+0.00001
   FR=COS(R(I))/SIN(R(I))-R(I)*P+0.25/(R(I)*P)
   IF(FR) 25,20,20
25 R(I)=R(I)-0.0000001
   FR=COS(R(I))/SIN(R(I))-R(I)*P+0.25/(R(I)*P)
   IF(FR) 25,28,28
28 I=I+1
   R(I)=R(I-1)+3.1417
   IF(I.LE.49 )  GO TO 10
   PRINT 29 ,(R(I),I=1,50 )
29 FORMAT(1H ,10F10.5)
30 DO 45 K=1,10
   O=O+0.01
   SC(K)=0.0
   DO 40 I=1,50
   C(I)=2.*R(I)*(U*SIN(R(I))+R(I)*COS(R(I)))*EXP(U-((U**2+R(I)**2)/(2
  1.0*U))*O)/(U**2+2.0*U+R(I)**2)
   SC(K)=SC(K)+C(I)
40 IF(ABS(C(I)+C(I-1)).LE.0.000001)  GO TO 45
45 CONTINUE
   PRINT 50,(SC(K),K=1,10),O
50 FORMAT(1H0,10F10.4,15X,F10.4)
   IF(O.LE.5.0) GO TO 30
   GO TO 1
   END
```

TABLE V

Peak Time Analysis for Theoretical Dispersion Curve

```
    THEORETICAL DISPERSION CURVE ANALYSIS PEAK TIME
    DIMENSION        R(1000),       C(100)
    PRINT 3
  3 FORMAT(1H1,33H     D/UL     PEAK TIME      -C/CO'    )
100 READ 1,P
  1 FORMAT( F10.5)
  2 I=1
    U=0.5/P
    R(I)=3.0
    O=0.1
 10 R(I)=R(I)-0.001
    FR=COS(R(I))/SIN(R(I))-R(I)*P   +0.25/(R(I)*P)
    IF(FR) 10,10,20
 20 R(I)=R(I)+0.00001
    FR=COS(R(I))/SIN(R(I))-R(I)*P   +0.25/(R(I)*P)
    IF(FR) 25,20,20
 25 R(I)=R(I)-0.0000001
    FR=COS(R(I))/SIN(R(I))-R(I)*P   +0.25/(R(I)*P)
    IF(FR) 25,28,28
 28 I=I+1
    R(I)=R(I-1)+3.1417
    IF(I.LE.10 )  GO TO 10
 30 O=O+0.01
    SC=0.0
    DO 40 I=1,10
    C(I)=  -R(I)*(U*SIN(R(I))+R(I)*COS(R(I)))*EXP(U-((U**2+R(I)**2)/(2
   1.0*U))*O)/(U**2+2.0*U+R(I)**2)*(U**2+R(I)**2)/U
    SC=SC+C(I)
 40 IF(ABS(C(I)+C(I-1)).LE.0.000001)  GO TO 41
 41 IF(SC) 50,30,30
 50 O=O-0.0001
    SC=0.0
    DO 60 I=1,10
    C(I)=  -R(I)*(U*SIN(R(I))+R(I)*COS(R(I)))*EXP(U-((U**2+R(I)**2)/(2
   1.0*U))*O)/(U**2+2.0*U+R(I)**2)*(U**2+R(I)**2)/U
    SC=SC+C(I)
 60 IF(ABS(C(I)+C(I-1)).LE.0.000001)  GO TO 61
 61 IF(SC) 50,50,62
 62 O=O+0.000001
    SC=0.0
    DO 65 I=1,10
    C(I)=  -R(I)*(U*SIN(R(I))+R(I)*COS(R(I)))*EXP(U-((U**2+R(I)**2)/(2
   1.0*U))*O)/(U**2+2.0*U+R(I)**2)*(U**2+R(I)**2)/U
    SC=SC+C(I)
 65 IF(ABS(C(I)+C(I-1)).LE.0.000001)  GO TO 66
 66 IF(SC) 67,67,62
 67 PRINT 70,P,O,SC
 70 FORMAT(1H0,3F11.5)
    GO TO 100
    END
```

APPENDIX "D"

Some Response Curves, Data, and Results of Analyses

TABLE VI

Some Experimental Response Curves

```
15.0          .865      16.8         5.0         3.0        30.25
  .25       14.86             288           1    5.39
                   00    20    45   100   200   270   370   450   510   580   625   665   695
730   750   770   780   787   792   792   794   792   784   781   773   761   750   742   739
722   720   695   688   676   663   652   642   630   620   613   600   590   580   570   560
550   542   530   520   510   500   490   475   468   461   450   440   430   420   413   407
400   395   380   373   368   361   353   345   342   332   322   318   310   303   300   294
288   278   272   271   263   256   254   250   243   240   235   231   223   221   216   212
208   205   200   193   190   190   185   175   172   170   170   164   158   155   151   149
144   141   140   135   130   128   125   121   119   117   115   112   108   104   102   100
 98    95    92    89    85    82    80    80    79    78    76    75    75    74    71    70
 70    70    67    65    62    61    58    58    57    56    56    55    55    55    55    54
 54    52    52    51    50    50    50    50    48    48    44    44    42    41    41    41
 40    40    37    35    33    33    32    31    31    31    30    30    30    30    29    29
 27    25    25    25    25    25    24    23    23    23    23    23    22    22    21    21
 21    21    20    20    19    19    19    18    18    17    15    12    12    12    12    12
 12    12    12    12    12    12    12    12    12    12    12    12    11    11    11    11
 11    11    11    10    10    10    10    10    10    10    10    10    10    07    07    05
 05    05    05    05    05    04    04    04    02    02    02    02    02    02    01    01
 01    01   -10
```

TABLE VI (Continued)

17.0 .865 10.1 5.0 3.0 30.25
.625 13.96 192 1 5.7

00	10	150	320	500	600	680	710	730	735	735	725	720	715	695	685
670	650	635	615	600	580	570	550	530	520	505	495	480	470	455	450
435	420	410	400	388	375	365	355	345	340	325	315	310	300	290	285
275	265	260	255	248	240	235	228	222	218	208	200	195	190	182	178
172	168	162	154	150	145	142	138	132	130	128	125	122	115	112	108
105	102	100	98	95	90	90	85	80	80	80	78	72	72	70	68
68	65	62	62	60	55	52	52	50	50	48	45	45	42	40	40
40	38	38	32	32	31	30	30	30	28	28	28	25	25	25	22
22	22	22	21	20	20	20	20	20	20	19	18	18	17	15	13
13	13	12	11	11	11	11	11	11	11	10	10	10	10	10	10
10	10	9	9	9	9	9	9	9	8	8	8	7	7	7	7
7	7	7	7	7	5	5	5	2	2	2	2	2	2	2	-10

20.0 .865 2.27 5.0 3.0 30.25
2.5 14.72 208 1 9.93

240	690	810	833	825	805	785	765	755	735	715	695	685	665	650	635
625	605	595	580	570	555	540	530	520	505	495	480	470	460	450	435
425	410	400	393	382	372	365	358	350	340	330	325	318	310	302	295
290	282	272	270	260	255	250	243	238	230	228	222	215	210	205	200
195	190	182	180	175	170	168	162	160	155	150	145	145	140	135	132
130	128	125	120	115	110	105	105	105	102	98	98	92	85	82	80
80	78	78	72	71	71	70	70	68	65	62	58	55	53	53	53
52	51	50	50	47	47	45	42	40	40	40	38	38	37	35	32
31	31	30	30	30	29	28	28	26	25	22	22	22	22	21	21
20	20	19	18	18	15	15	15	15	12	12	12	11	11	11	10
10	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8
8	8	6	6	5	5	5	5	4	3	3	3	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	-10

TABLE VI (Continued)

```
61.0        660.0       850.0       66.0        30.0        180.00
 5.0      10300.0             224           2   23.5
170   550   710   755   755   745   740   730   715   690   675   660   650   635   625   610
600   585   570   560   545   530   520   505   493   480   473   465   452   440   428   415
408   400   388   375   370   365   360   355   348   340   335   330   320   310   303   295
290   285   280   275   270   265   260   255   250   245   240   235   229   223   217   210
205   200   195   190   187   185   183   180   177   175   173   170   166   162   158   155
152   150   148   145   141   137   133   130   128   125   123   120   118   115   113   110
108   106   104   102   100    98    96    95    93    91    90    88    86    84    82    80
 78    76    74    72    70    68    66    65    64    63    62    60    59    58    57    55
 54    53    52    51    50    49    48    47    46    45    44    43    42    41    40    40
 39    38    38    37    36    35    34    34    33    32    31    30    29    28    28    27
 26    25    24    24    23    22    21    20    19    19    18    18    17    17    16    16
 15    15    14    14    13    13    12    12    11    11    10    10     9     9     8     8
  8     7     7     7     6     6     6     5     5     5     4     4     4     3     3     3
  2     2     2     2     2     1     1     1     1     1     1     1     1    -1

 91.5          .282        .365        5.0          2.2725      13.45
 5.0          4.57                224           2   17.9
269   670   770   778   771   757   739   721   707   693   677   659   648   635   621   607
597   583   567   559   543   533   516   505   494   481   471   458   450   440   431   423
413   406   394   387   379   359   357   353   343   336   325   319   310   302   298   293
288   278   272   267   260   251   248   243   238   232   228   220   212   207   206   201
199   193   188   180   176   172   169   165   162   157   154   146   144   140   134   131
131   127   127   127   124   119   117   116   114   110   110   108   106   102   102   099
097   092   089   086   082   081   078   078   078   077   075   072   072   069   067   064
061   059   057   057   057   057   057   057   057   057   056   056   054   052   050   049
047   046   042   042   042   042   042   042   042   042   042   039   039   039   039   039
036   035   035   033   030   027   027   027   027   025   023   023   023   022   021   020
020   019   019   019   019   019   019   019   019   019   019   015   015   015   015   015
015   015   015   010   010   010   010   010   010   010   010   010   010   010   010   007
007   007   007   007   007   007   007   007   007   007   004   004   004   004   004   004
004   004   002   002   002   002   002   002   002   002   002   002   002  -001
```

TABLE VII

Experimental Data

Run No.	Air Flow (cfm)	Water Flow (US gpm)	Temp. °C	Tank Length (ft)	Tank Width (ft)	Tank Depth (ft)	Theoretical Det. Time (hr)
8.0	0.86	14.1	12.0	5.0	3.0	2.52	0.334
9.0	0.86	14.1	12.5	5.0	3.0	2.52	0.334
11.0	0.86	0.80	29.0	5.0	3.0	2.52	5.89
11.5	0.86	0.80	30.5	5.0	3.0	2.52	5.89
12.0	0.86	0.80	12.0	5.0	3.0	2.52	5.89
13.0	0.86	14.1	29.5	5.0	3.0	2.52	0.334
13.5	0.86	14.1	20.0	5.0	3.0	2.52	0.334
14.0	0.86	14.1	29.5	5.0	3.0	2.52	0.334
14.5	0.86	0.80	20.0	5.0	3.0	2.52	5.89
15.0	0.865	16.8	20.0	5.0	3.0	2.52	0.281
16.0	0.865	1.06	20.0	5.0	3.0	2.51	4.44
17.0	0.865	10.1	20.0	5.0	3.0	2.52	0.467
18.0	0.865	0.525	20.0	5.0	3.0	2.52	8.98
18.5	0.865	0.314	20.0	5.0	3.0	2.52	14.9
19.0	0.865	5.04	20.0	5.0	3.0	2.52	0.935
20.0	0.865	2.27	20.0	5.0	3.0	2.52	2.08
21.0	4.82	9.64	20.0	5.0	3.0	2.52	0.489
22.0	3.26	6.50	20.0	5.0	3.0	2.53	0.728
23.0	2.20	4.35	20.0	5.0	3.0	2.52	1.08
24.0	1.28	2.57	20.0	5.0	3.0	2.50	1.82
25.0	0.674	1.34	20.0	5.0	3.0	2.52	3.51
26.0	0.332	0.667	20.0	5.0	3.0	2.50	7.01
27.0	1.88	3.75	20.0	5.0	3.0	2.52	1.26
28.0	1.88	3.75	20.0	5.0	3.0	2.52	1.26
29.0	1.88	3.75	20.0	5.0	3.0	2.52	1.26
30.0	2.59	2.58	20.0	5.0	3.0	2.52	1.82
31.0	9.05	9.05	20.0	5.0	3.0	2.50	0.517
32.1	1.37	1.37	20.0	5.0	3.0	2.50	3.42
33.0	6.22	6.22	20.0	5.0	3.0	2.49	0.749
34.0	3.86	3.86	20.0	5.0	3.0	2.49	1.21
35.0	0.725	0.725	20.0	5.0	3.0	2.50	6.45
40.0	0.489	2.43	20.0	5.0	3.0	2.51	1.93
41.0	0.267	1.34	20.0	5.0	3.0	2.50	3.50
42.0	3.52	17.6	20.0	5.0	3.0	2.52	0.268
43.0	1.87	9.35	20.0	5.0	3.0	2.51	0.503
44.0	0.938	4.70	20.0	5.0	3.0	2.50	0.998
45.0	0.143	0.715	20.0	5.0	3.0	2.49	6.54
50.0	8.33	4.7	20.0	5.0	3.0	2.49	0.99
51.0	0.263	4.7	20.0	5.0	3.0	2.52	1.00
52.0	1.02	4.7	20.0	5.0	3.0	2.52	1.00

TABLE VII (Continued)

Run No.	Air Flow (cfm)	Water Flow (US gpm)	Temp. °C	Tank Length (ft)	Tank Width (ft)	Tank Depth (ft)	Theoretical Det. Time (hr)	
52.1	0.482	4.7	20.0	5.0	3.0	2.52	1.00	
53.0	1.94	4.7	20.0	5.0	3.0	2.52	1.00	
54.0	3.95	4.7	20.0	5.0	3.0	2.52	1.00	
60.0	660	714	19.0	66.0	30.0	15.0	5.19	*
61.0	660	850	19.5	66.0	30.0	15.0	4.36	*
70.5	0.873	0.78	20.0	5.0	3.0	2.50	6.00	
71.0	0.391	0.78	20.0	5.0	3.0	2.50	6.00	
72.0	1.82	0.78	20.0	5.0	3.0	2.50	6.00	
73.0	3.81	0.78	20.0	5.0	3.0	2.50	6.00	
74.0	0.161	0.78	20.0	5.0	3.0	2.50	6.00	
80.5	0.091	0.39	20.0	5.0	3.0	2.50	12.0	
81.0	0.224	0.39	20.0	5.0	3.0	2.50	12.0	
82.0	0.432	0.39	20.0	5.0	3.0	2.50	12.0	
83.0	0.880	0.39	20.0	5.0	3.0	2.50	12.0	
84.0	1.84	0.39	20.0	5.0	3.0	2.50	12.0	
91.5	0.282	0.365	20.0	5.0	2.27	1.12	4.35	
93.0	0.128	0.365	20.0	5.0	2.27	1.12	4.35	
94.0	0.579	0.365	20.0	5.0	2.27	1.12	4.35	
95.0	1.14	0.365	20.0	5.0	2.27	1.12	4.35	
96.0	2.21	0.365	20.0	5.0	2.27	1.12	4.35	
99.0	0.86	0.78	20.0	5.0	3.0	2.50	6.00	C
99.1	0.86	0.78	20.0	5.0	3.0	2.50	6.00	P
99.2	0.86	0.78	20.0	5.0	3.0	2.50	6.00	P
99.3	0.85	0.78	20.0	5.0	3.0	2.50	6.00	C
99.4	0.85	0.78	20.0	5.0	3.0	2.50	6.00	C
99.5	0.85	0.78	20.0	5.0	3.0	2.50	6.00	P

* Prototype Test

C Sampling tube at centre of section

P Sampling tube at periphery of section

TABLE VIII

Results of Analyses of Experimental Data

Run No.	Mean Tracer Det. Time (hr)	Percent Recovery	Curve Variance (σ^2)	Associated D/uL	Peak Time ($t_p/\bar{t}$)	Associated	
						D/uL	D (ft^2/hr)
8.0	-	-	-	-	0.287	0.990	-
9.0	-	-	-	-	0.284	1.00	-
11.0	-	-	-	-	0.0318	19.2	-
11.5	-	-	-	-	0.0324	18.5	-
12.0	-	-	-	-	0.0324	18.5	-
13.0	-	-	-	-	0.270	1.07	-
13.5	-	-	-	-	0.280	1.02	-
14.0	-	-	-	-	0.268	1.09	-
14.5	-	-	-	-	0.0318	19.2	-
15.0	0.284	102.1	0.571	0.507	0.320	0.85	73.5
16.0	4.07	95.0	0.789	1.32	0.0425	13.3	74.9
17.0	0.463	100.5	0.671	0.746	0.204	1.32	70.7
18.0	7.76	91.5	0.773	1.21	0.0239	27.0	75.1
18.5	-	-	-	-	0.0157	43.0	72.0
19.0	0.863	98.9	0.716	0.912	0.140	2.88	77.0
20.0	1.87	95.2	0.753	1.09	0.0797	6.00	72.2
21.0	0.490	101.7	0.782	1.27	0.133	3.15	161
22.0	0.722	102.5	0.807	1.47	0.113	3.90	134
23.0	1.05	100.0	0.812	1.52	0.0953	4.60	106
24.0	1.76	101.2	0.805	1.45	0.0747	6.50	89.5
25.0	3.51	101.5	0.855	2.04	0.0518	10.5	74.9
26.0	6.69	96.9	0.881	2.55	0.0325	18.5	66.0
27.0	1.17	99.1	0.813	1.53	0.0914	5.10	102
28.0	1.17	99.5	0.804	1.44	0.0914	5.10	102
29.0	1.15	98.4	0.788	1.31	0.0914	5.10	102
30.0	1.80	103.8	0.864	2.20	0.0620	8.30	114
31.0	0.457	97.3	0.727	0.959	0.108	4.10	198
32.1	3.33	103.1	0.856	2.06	0.0455	12.4	90.7
33.0	0.640	95.0	0.693	0.821	0.0919	5.00	167
34.0	0.988	92.2	0.687	0.802	0.0718	6.90	142
35.0	5.91	96.7	0.875	2.41	0.0272	23.0	89.4
40.0	1.86	100.7	0.794	1.36	0.118	3.90	50.5
41.0	3.20	97.5	0.761	1.14	0.0744	6.60	47.0
42.0	0.253	97.0	0.601	0.567	0.236	1.34	125
43.0	0.478	98.2	0.713	0.899	0.172	2.15	107
44.0	0.908	94.3	0.712	0.895	0.136	3.00	75.3
45.0	6.03	97.6	0.877	2.45	0.0566	9.40	35.9
50.0	0.930	99.3	0.816	1.55	0.0709	7.00	176
51.0	0.931	95.4	0.669	0.742	0.188	1.85	46.2
52.0	0.980	98.3	0.812	1.52	0.129	3.22	80.5

TABLE VIII (Continued)

Run No.	Mean Tracer Det. Time (hr)	Percent Recovery	Curve Variance (σ^2)	Associated D/uL	Peak Time ($t_p/\bar{t}$)	Associated D/uL	Associated D (ft^2/hr)	
52.1	0.953	99.1	0.786	1.30	0.149	2.66	66.4	
53.0	0.964	97.7	0.777	1.24	0.105	4.30	106	
54.0	0.951	95.4	0.779	1.25	0.0824	5.80	145	
60.0	4.38	85.6	0.778	1.24	0.0675	7.50	6310	*
61.0	4.16	100.0	0.735	0.995	0.0899	5.15	5150	*
70.5	-	-	-	-	0.0297	20.7	86.5	
71.0	-	-	-	-	0.0403	14.0	58.4	
72.0	-	-	-	-	0.0250	25.5	106	
73.0	-	-	-	-	0.0208	31.0	129	
74.0	-	-	-	-	0.0494	11.1	46.3	
80.5	-	-	-	-	0.0430	13.3	27.7	
81.0	-	-	-	-	0.0285	21.7	45.2	
82.0	-	-	-	-	0.0232	28.0	58.4	
83.0	-	-	-	-	0.0176	38.0	79.1	
84.0	-	-	-	-	0.0149	46.0	95.9	
91.5	3.99	94.1	0.780	1.26	0.0686	7.2	42.0	
93.0	-	-	-	-	0.0967	4.60	26.4	
94.0	-	-	-	-	0.0532	10.0	57.4	
95.0	-	-	-	-	0.0394	14.7	84.4	
96.0	-	-	-	-	0.0321	18.8	108	
99.0	-	-	-	-	0.0342	-	-	C
99.1	-	-	-	-	0.0338	-	-	P
99.2	-	-	-	-	0.0368	-	-	P
99.3	-	-	-	-	0.0361	-	-	C
99.4	-	-	-	-	0.0353	-	-	C
99.5	-	-	-	-	0.0353	-	-	P

*	Prototype Test
C	Sampling tube at center of section
P	Sampling tube at periphery of section

APPENDIX "E"

Nomenclature

TABLE IX

Nomenclature

$a = \sqrt{1 + 4k\,\bar{t}\,(D/uL)}$

$a_n = a_1, a_2, a_3, \ldots$ defined by $\tan a_n \ell = \dfrac{2Ba_n}{a_n^2 - B^2}$

$B = V/2b^2 = U/L$ (hr/ft)

$b^2 = 180\,\ell^2/\pi^2 t_{90}$, a turbulence expression (ft^2/hr)

C = exit concentration of tank at time t (mg/ℓt)

C_o = concentration of input for step functions, or weight of tracer added divided by volume of tank for pulse functions (mg/ℓt)

D = axial dispersion coefficient (ft^2/hr)

k = reaction constant (hr^{-1})

$k_n^2 = b^2(a_n^2 + B^2)$

L, ℓ = tank length (ft)

n = number of tanks in series

Q_A = air flow (ft^3/min, or ft^3/min x ft^3 of tank)

S = concentration of substrate at exit (mg/ℓt)

S_o = concentration of substrate feed to tank (mg/ℓt)

t = time (hr)

$\bar{t}$ = theoretical detention time or mean residence time for entire tank volume used (hr)

$\bar{t}_i$ = theoretical detention time or mean residence time of i^{th} tank for entire volume used (hr)

t_p = time of peak exit concentration (hr)

t_{90} = time to reach 90 percent ultimate tracer concentration at tank exit with no flow (hr)

$U = uL/2D$

TABLE IX (Continued)

u, V = mean displacement velocity along tank length (ft/hr)

x = distance along tank (ft)

Greek Symbols

Θ = time divided by the theoretical detention time, or mean residence time for entire system volume used (hr)

$\bar{\Theta}_1, \bar{\Theta}_2 = \bar{t}_1$ or $\bar{t}_2$ divided by the theoretical detention time, or the mean residence time for the entire system volume used.

$\mu_n = \mu_1, \mu_2, \mu_3, \ldots, \mu_n = \cot^{-1}[\frac{1}{2}(\frac{\mu_n}{U} - \frac{U}{\mu_n})]$

σ^2 = variance or second moment of a curve about its mean (dimensionless in $t/\bar{t}$ units

σ_t^2 = variance or second moment of a curve about its mean (hr^2)

$$= \bar{t}^2\sigma^2 = \int_0^\infty (t - \bar{t})^2\, C\, dt \Big/ \int_0^\infty C\, dt$$

$$= \frac{\sum t^2 C}{\sum C} - \left(\frac{\sum tC}{\sum C}\right)^2 \quad \text{for equal time intervals}$$