SPACE FRAMES - STATIC ANALYSIS

VIBRATION ANALYSIS AND DESIGN OPTIMISATION STUDIES OF SPACE FRAMES

I - STATIC ANALYSIS

BY

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SCOPE AND CONTENT:

An oblique four bar structural model with fixed member ends, being the most general building block for space frames, is analysed for establishing its influence coefficients, using the Finite Element Matrix Method.

Experimental techniques for measurement of the flexibility influence coefficients of the model are described.

Experimental results have been compared against analytical ones.

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ABSTRACT

This research programme has the general objective of establishing analytical techniques for analysis of indeterminate spatical frames and shells under dynamic loading, and the design optimisation of these structures under the constraints of dynamic loading. Although techniques developed should have wide applicability, emphasis will be placed, for experimental and illustrative purposes, on structural configurations common to machine structures.

Recent success of the finite element matrix method and progress in the field of nonlinear optimisation provide a rational basis for the synthesis of space frames, with emphasis on configurations common to mechanical engineering structures.

For the initial stage of the project a highly redundant oblique four bar space frame was selected to investigate into the nature of problems involved in the optimisation of generalised space frames subject to dynamic constraints.

The present work relates to the static analysis of the frame including a theoretical analysis based on the finite element approach and experimental determination of the influence coefficients.

The average percentage error in actual measurement varies approximately from four percent for larger flexibility influence coefficients to about ten percent for smaller ones.

Related studies will examine the dynamic analysis of the structure, and the optimisation problem.

INTRODUCTION

This research programme has the general objective of establishing analytical techniques for analysis of indeterminate spatial frames and shells under dynamic loading, and the design optimisation of these structures under the constraints of dynamic loading. Although techniques developed should have wide applicability, emphasis will be placed, for experimental and illustrative purposes, on structural configurations common to machine structures.

This present work relates to the first stage of this programme in which the problem is explored by examining a simple discrete element space frame with generalized characteristics. More specifically, this thesis is concerned with the static analysis of the structure, including a theoretical and experimental determination of influence coefficients and stress-load transfer matrices. Related studies will examine the dynamic analysis of the structure, and the optimisation problem. The following discussion reviews the overall problem.

Design systhesis essentially is an evolutionary spiral process involving a complex feed back interrelating the fields of creativity, past experience and tools of analysis. The role of the designer is to optimise the value of a synthesis on the basis of some criteria through a balanced exploitation of the evergrowing information from all the three fields. The basic techniques and the criteria of evaluation themselves need refinement from time to time in the light of achievements in the foregoing areas.

The process has been marked with a rather slow progress in the field of mechanical engineering structures, mainly due to their inherent nature. These have not received the intensive investigation that civil and aerospace engineering configurations have. Analysis of mechanical engineering structures has perhaps lagged behind because they are much more difficult to categorise than in the other fields where a few highly typical configurations can be recognised, modelled and studied in a concentrated way. In addition, the analytical tools available until lately have had their own limitations.

These methods can be broadly classified into two divisions,[1,2] -

(1) Methods based on exact solution of the differential equations describing the structure.

Apart from the difficulties in setting up and solving the equations subject often to awkward boundary conditions, in case of complex structures the basic assumptions proved too restrictive for accurate solution.

(2) Approximate methods involving mathematical approximations can be subclassified into -

(a) Those based on finite difference procedures. These are unsatisfactory in their formulation of boundary conditions and convergence characteristics, and

(b) Those which approximate the stress or displacement distribution by a series of analytical expressions and hence are unsuited for complex structures.

The classical analytical tools are thus incapable of providing an integrated approach even for structures of moderate complexity. Hence it is not surprising that the practical design of mechanical engineering structures has relied more on past practical experience supported by rough analytical checks wherever possible, rather than on the analytical tools.

The need for a tool well suited to complex configurations was most acute in the aircraft industry where the designer had to work within extremely narrow margins of practical expediency[3]. The extensive efforts over years by numerous and often isolated workers culminated in the finite element approach which is a major break-through from the past.

Based on structural as against mathematical approximation, the method essentially seeks to idealise the structure into an assembly of a finite number of discrete elements connected at a finite number of points, and then proceeds to solve for the system response on an exact mathematical basis. It is the finite connectivity which permits a complex continuous structure to be analysed by a system of algebraic equations and forms the basis of the technique. Although earlier work was restricted to the field of aeronautical engineering, recently results of applications to nonaeronautical problems[4,5,6] and extensions to three dimensional discrete elements[7] have been reported.

It is realised that, although the finite element technique is still developing, it provides a unified approach to the analysis of any type of structural assembly, from any field and with any combination of one, two or three dimensional elements of different characteristics[4]. It thus provides a reliable analytical tool which is a prerequisite for design systhesis.

A rather limited amount of work appears to have been done on the general problem of elastic vibration of structures and the problem of optimisation under vibrational constraints, although techniques for calculating the natural modes and frequencies of lumped mass spatial structures are fairly well established for essentially beam like aircraft structures, and to a lesser extent the rectangular frames of civil engineering. The significance of rotary inertia in spatial frames does not appear to have been studied. Archer[8,9] has provided two useful new papers in this field and has related it to the finite element stiffness matrix technique. Hurty[10] has developed a method for analysing complex structural systems that can be divided into interconnected components.

The concept of optimum design has registered a drastic change since the advent of high speed digital computers. Earlier, the magnitude of computation involved acted as a deterent and a feasible solution was accepted in lieu of the optimum. With computers to handle the arithmetic, systematic design synthesis has become a reality.

Very many general techniques of optimisation appear in the literature that might be applied to structural optimisation. Most promising are the Direct Search Method first suggested by Hooke and Jeeves and further developed by Flood and Leon[11], the Method of Successive Linear Approximation due to Griffith and Stewart[12], and the Random Method of Dickinson[13].

Minimisation of weight, weight stiffness ratio, cost, volume for a homogeneous structure, etc. have been suggested as criteria for optimisation of structures, but minimisation of weight appears to have been accepted as the most satisfactory one even though the minimum weight design is not always the minimum cost design.

The optimisation of a statically determinate truss subjected to single loading is a problem in analysis rather than synthsis. For strength design, member cross sections are proportioned to develop maximum allowable stress for the required failure mode. For optimum stiffness design based on minimisation of weight per unit stiffness, stiffness being defined as the reciprocal of strain energy, the members should carry stresses proportional to the square root of the product of the modulus of elasticity and specific weight. The constant of proportionality is based on stiffness requirements[14].

For a given determinate truss under multiple load condition the problem essentially remains the same. All the member cross sections carry the maximum allowable stress, based on strength or stiffness design, at least under one load condition. The optimum design has come to be recognised as a fully stressed design.

In the case of indeterminate trusses, for a given configuration, applied loading and allowable stress, the cross sectional area of the members and hence the weight of the structure are functions of forces in the redundant members. Sved[15] has shown analytically that under single load conditions the minimum weight structure is always determinate.

Using the Lagrange multiplier technique, L. C. Schmidt[16] has shown that under alternative loads numerous fully stressed designs of an indeterminate truss exist. Due to the prohibitive nature of computations involved in arriving at the minimum weight he has suggested two complementary relaxation methods to arrive at a fully stressed design.

The beginning of the present decade marked a radical departure in the approach to structural optimisation. It came to be accepted as a problem in mathematical programming with Schmit[17] as the pioneer. Utilising the joint force and displacement formulation of structural analysis as first proposed by Klein[18], he has optimised a fixed configuration three bar truss subject to three alternate loads. He treated it as a problem in nonlinear programming by adopting a modified steepest descent method designated as the method of alternate steps. On encountering an inequality constraint, which must be convex, the search moves along a constant weight plane in the feasible region until the constraint is again contacted. It then steps back halfway, and then continues to move along the steepest slope. On the basis of numerical results he concludes that in terms of design parameter space the minimum weight design need not be a fully stressed design lying at the apex of constraint hyper-planes.

Subsequently[19,20] in collaberation with Mallett and Kicher he extended the above to the problem of selecting a suitable configuration and material for the three bar truss. Various optimum designs were compiled by changing the material or configuration, one at a time in discrete steps. The best of all these design was chosen.

Dorn et al[21] have proposed a linear programming method which selects the optimum combination of configuration and member cross section from a wide classes of admissible trusses defined by a given number of admissible joints connected in all possible ways by linear members. The optimisation is based on a modified simplex method capable of handling large number of equations. The results provide an interesting study in the behaviour of optima due to change in load and the height-span ratio of the truss. The configuration remains the same for the load for a certain change in height-span ratio α, and then alters, as α continues to change. Thus a continuous spectrum is provided from which the value of α giving the absolute minimum weight truss and the configuration itself could be selected.

Best[22] has optimised a contilever box beam by the steepest descent method. It has one unique feature. The partical derivatives of stress and deflections with respect to the design parameters are calculated by the finite difference approximation using the stiffness matrix, which must be inverted to obtain the deflections. To avoid the time consuming process of inversion at every step he adopts an interative scheme to obtain the deflections. Only the incremental stiffness matrix for a given change in design parameter is calculated which, in conjunction with the previously inverted stiffness matrix, rapidly converges to the required displacements on iterations. This feature is said to substantially reduce the calculation time. Constraints on stresses and deflections are handled by a version of the reduced gradient method. His solution is a maximum stress solution, and thus forced to be on a boundary.

The presentation of the structural synthesis as an unconstrained minimisation problem by Schmit and Fox[23] is unique. It is based on the method of solving linear simultaneous equations by minimising the sum of squares of the residuals to zero. This expression is set up for the equality constraints defining the stresses. To this is added penalty terms for violated inequality constraints, which are all simple upper and lower bounds. The actual quantity to be optimised, the weight, is treated as an inequality constraint, requiring that the weight be less than an arbitrarily defined draw down weight. The problem is now an unconstrained optimisation problem solved by a gradient method. It is repeated using progressively lower draw-down weights until the optimisation function cannot be made zero. This indicates that the draw-down weight is lower than the inherent minimum weight. The method thus actually requires a series of optimisations. It does not seem too applicable to complex problems; as the constraints must be expressed explicitly in order to set up the residuals. The implicit matrix form of equality constraints are ruled out.

Razani[24] has proposed an unconventional approach using an iterative technique in which areas are changed by successive increments from an initial feasible solution so that each member is fully stressed in at least one of the several possible load conditions. This gives a feasible solution forced to be on a boundary. The true minimum may not be on a boundary if the stress is indeterminate.

The gradient projection technique has been successfully adopted by Brown and Alfredo[25] to optimise a portal frame and a two storey single bay frame. The search begins at a feasible starting point until constraints are encountered. At this point the constraint hypersurfaces are approximated by hyperplanes and gradient of the objective function is projected on the line of intersection of these planes. After a move along the indicated direction a correction is indicated due to the nonlinearity of the constraint hypersurfaces. The authors have proposed the use of only one design parameter for a member as variable while the rest of the parameters for the same member are expressed as functions of the selected one. As moment of inertia of the members has a predominant effect on the behaviour of the structure, other parameters are expressed as functions of moment of inertia. Inspite of this simplification the procedure seems too invloved for complex structures.

Young and Christiansen[14] have provided the first known optimal structural design technique using vibrational constraints using an iterative technique. Adjustment of the member area to achieve a fully stressed design simultaneously with the required resonant frequency characteristic is the main feature. An application to pin jointed space truss is included.

THE PHYSICAL MODEL

For the first stage of the project it was decided to examine a simple but highly redundant space frame with generalised characteristics. An oblique four bar frame without symmetry and with fixed member ends illustrated in Figure 1 was selected. The obliquity of the bars ensured assymmetry of static and dynamic response. The four bars are welded at the base to a half inch thick aluminum alloy plate at a 24 inch square spacing. The top ends of the bars are brought close together and welded to another half an inch thick aluminum plate at a square spacing of 2.5 inches. It is extremely important for the accuracy of the results that deflection of the bars at the ends fixed to the base plate are small as compared to the relative displacement between points on the structure due to applied load. To ensure this the base plate is bolted firmly around each leg of the structure between 1.25 inches thick steel plates. The point at which the external loads are applied is located at the centre of the top plate.

THEORETICAL ANALYSIS

The decision was made to first examine the problem using the finite element - matrix approach. Static analysis of structures by this method is well established. During the earlier stages of development the literature was in the form of a number of short individual publications marked by variety of notations and seemingly different approaches to the same basic technique, often applied to specific situations. The most oft quoted work as the first comprehensive presentation of the subject is by Argyris and Kelsey[26]. Results of a more recent survey of the finite element method with a view to unifying the techniques and classifying the approaches to derivation of element properties has been presented by Gallagher et al[27]. The very recent book by Zienkiewicz[30] is particularly comprehensive and useful.

The finite element analysis of a general structure consists of three distinct phases[4] -

(1) Structural idealisation wherein the original structure is represented as an assembly of discrete elements,

(2) Evaluation of element properties,

(3) Analysis of the discretised structure.

Phase 1 introduces into the anlaysis the first of the approximations. Judgement is required to provide a discretisation capable of reasonably accurate results but in general is not a difficult problem.

The success of the method and its extension to new areas depend almost entirely on the second phase. It is therefore not surprising that the derivation of element properties of various shapes has received such wide attention. Two levels of approximations are involved in the development of these properties - assumptions about the essential element behaviour and secondly representation of distributed stress and displacements in terms of nodal forces and displacements[27].

The third phase like any other method of structural analysis seeks to satisfy the conditions of equilibrium, compatibility and force displacement relationship, simultaneously. Depending on whether compatibility or equilibrium equations are utilised first, the methods can be classified into the equilibrium or displacement method and the compatibility or force method. The procedural details of these methods are readily available in a number of excellent recent publications [28,29,30].

The Mathematical Model:

The analytical procedure uses two mathematical models - static and dynamic. The later is usually an extension of the static model. The accuracy of analytical dynamic response depends on the number of nodes selected for formulation of the mass matrix. The amount of experimental work involved in determination of the influence coefficients, however, increases in proportion to the number of nodes. As a compromise each member was discretised into three equal lenghts.

For the static model the top plate was treated as rigid due to its relatively small size and comparatively large thickness. It was idealised into four rigid bars each joining the centre of the top plate to the point of intersection of a member axis with the plate.

The combination of the rigid element and the corresponding flexible member element was treated as an integral finite element, and the element property was formulated as such.

The static mathematical model thus consisted of twelve discrete elements. The arrangement of the elements in the physical model is shown in Figure 8 .

Analysis:

The analysis was subject to the usual limitations of small displacements and linearity of stress-strain and force-displacement relationships. The weight of the structural elements was neglected as being small compared to their load capacity. In evaluating the element characteristic matrix the deflection due to shear was neglected

The displacement method was adopted as it is simple and straightforward to programme.

The basic assumptions used in this method are -

(1) Boundary displacements of adjacent elements are mutually compatible,

(2) Stresses in the elements due to the boundary displacements are equilibrated by a set of forces at the element boundary in the direction of the displacements, and

(3) Element forces are related to the corresponding element displacements by an element stiffness matrix expressed by

the matrix equation.

$$\{P_i\} = [k_i]\{\delta_i\} \tag{1}$$

where $\{P_i\}$ is the element boundary forces vector,

$[k_i]$ is the element stiffness matrix,

$\{\delta_i\}$ is the element boundary displacement vector,

i refers to the element number

For all the elements treated as unconnected Equation (1) can be written as $\{P\} = [k] \{\delta\}$ (2)

where $\{P\} = \begin{Bmatrix} P_i \\ P_j \\ \vdots \\ P_n \end{Bmatrix}$ and n is the total number of elements in the structure

Also

$$[k] = \begin{bmatrix} k_i & & & \\ & k_j & & \\ & & \ddots & \\ & & & k_n \end{bmatrix}$$

and

$$\{\delta\} = \begin{Bmatrix} \delta_i \\ \delta_j \\ \vdots \\ \delta_n \end{Bmatrix}$$

When elements are connected together to form a structure Equations (1) can combined into a single relationship.

$$\{F\} = [K] \{u\} \tag{3}$$

where $\{F\}$ is the structural joint forces vector

$[K]$ is the structural assembly stiffness matrix and $\{u\}$ is the structural joint displacement vector.

Element boundaries have the same displacements as the structural joints to which they are connected if both are referred to a common coordinate system. This can be expressed by

$$\{\delta\} = [\beta]\{u\} \qquad (4)$$

where $[\beta]$ is the displacement transformation matrix. It consists of zero and one as elements. If the displacements are referred to different coordinate systems the relation (4) still holds but elements of matrix $[\beta]$ contain elements of the coordinate transformation matrix. The work done in loading the structure by the applied loads $\{F\}$ must be equal to the internal energy. Therefore from Equation (2) and (3)

$$\frac{1}{2}\{u^T\}\{F\} = \frac{1}{2}\{\delta^T\}\{P\}$$

$$= \frac{1}{2}\{\delta^T\}[k]\{\delta\}$$

$$= \frac{1}{2}\{\delta^T\}[k][\beta]\{u\} \qquad (5)$$

But

$$\{\delta^T\} = \{u^T\}[\beta^T] \qquad (6)$$

Therefore

$$\frac{1}{2}\{u^T\}\{F\} = \frac{1}{2}\{u^T\}[\beta^T][k][\beta]\{u\}$$

or

$$\{F\} = [\beta^T][k][\beta]\{u\} \qquad (7)$$

Comparing Equations (3) and (7) we see that

$$[K] = [\beta]^T [k] [\beta] \qquad (8)$$

$[\beta]$ is a sparsely populated matrix. The matrix product indicated by Equation (8), therefore, can be a very time consuming step particularly for large structures. However, the structural stiffness matrix $[K]$ can be generated directly by assembling the element stiffness matrices already transformed into the structural coordinate system. This method has come to be known as the Direct Stiffness Method[3,33].

The assumption of rigidity of the plate and the selection of the displacement method precluded the choice of more than one node on the top plate. If more than one node is selected the flexibility matrix will have dependent displacement rows. Hence its inverse, the stiffness matrix, does not exist[28]. Imposition of a one node condition dictated the use of integral elements made up of flexible and rigid components. The combination shown in Figure (8) was achieved by the transformation of the stiffness matrix of the flexible component BC by the equilibrium matrix of the rigid component CD according to the expressions given below[31,32].

$$\begin{aligned} [K_{11}]_{BD} &= [K_{11}]_{BC} \\ [K_{12}]_{BD} &= [K_{12}]_{BC} [H_{CD}^{-1}]^T \\ [K_{22}]_{BD} &= [H_{CD}^{-1}] [K_{22}]_{BC} [H_{CD}^{-1}]^T \end{aligned} \qquad (9)$$

where

BD is the composite element,

$[H_{CD}]$ is the equilibrium matrix of CD

K_{11}, K_{12} etc. are 6 x 6 submatrices of the element stiffnesses in system coordinates.

The following computer programmes were used in the analysis.

Subroutine TRANF:

With position coordinates of member ends and components of a unit vector along the member Y axis as the input, the subroutine calculates the coordinate transformation matrix and checks it for orthogonality. If this condition is not satisfied execution is terminated after printing out an error message.

Subrountine STIFCO:

Area of cross section and moment of inertia are read in along with the elastic modulii. Depending upon the value of an index, the output is a stiffness matrix in member coordinates or in system coordinates, or is the stiffness matrix of a composite element.

Subroutine TRANQL:

This subroutine transforms the stiffness matrix by the equilibrium matrix to provide the stiffness matrix of the composite element. In case of shortage of storage space it can be used to provide the 12 x 12 stiffness matrix of a free element from the 6 x 6 stiffness matrix of the same element when fixed at one end.

Subroutine ASSEMBL:

Once the stiffness matrix of all the elements has been worked out the subroutine can assemble the structural stiffness matrix.

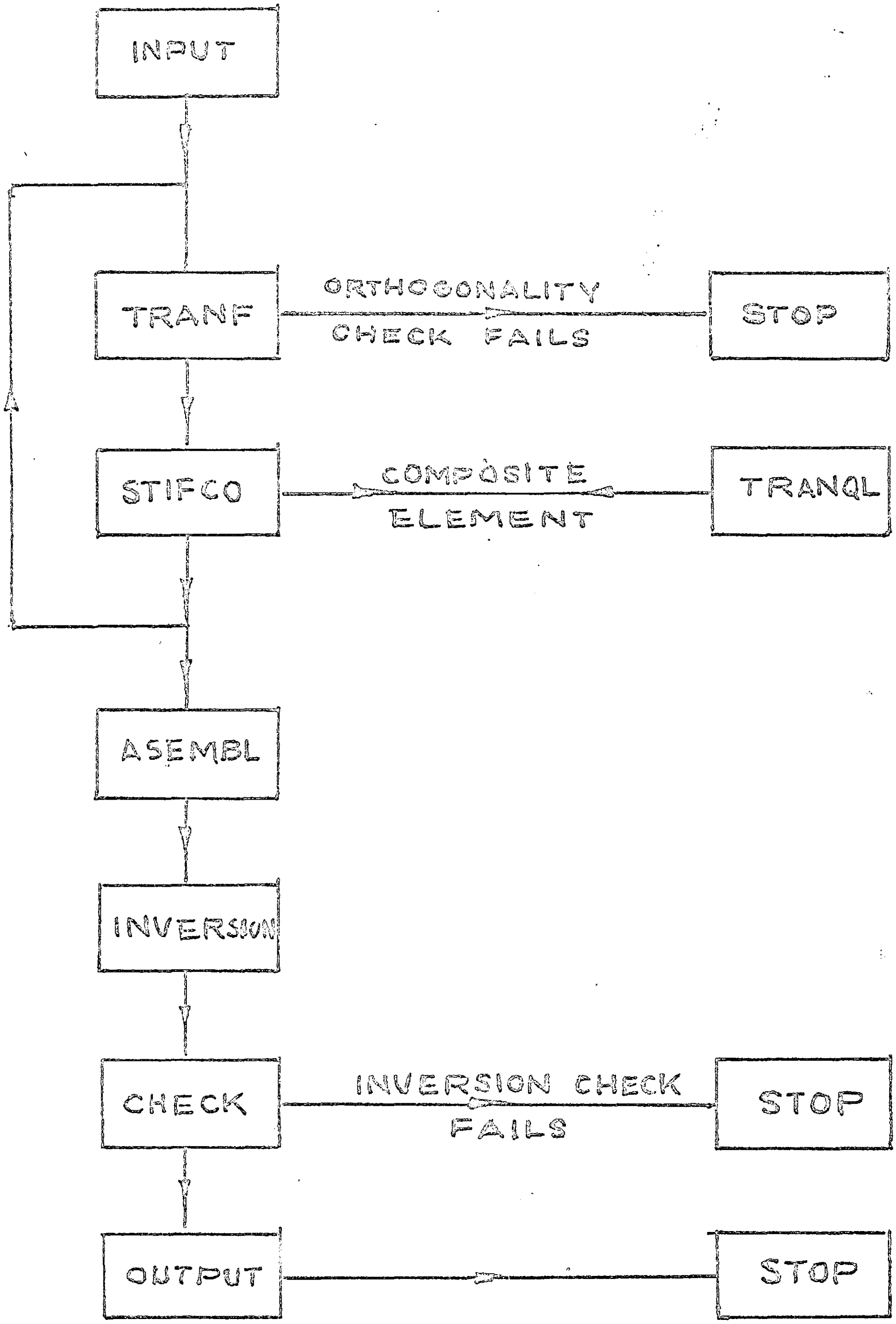
Main Programme:

The main programme utilises the subroutines to obtain the structural stiffness matrix, inverts it to supply the flexibility matrix, and checks the validity of inversion

The subroutine approach provides the more flexible and more general programmes.

The scheme of computations is indicated as follows on the next page.

FLOW CHART


INPUT
TRANF
ORTHOGONALITY CHECK FAILS
STOP
STIFCO
COMPOSITE ELEMENT
TRANQL
ASEMBL
INVERSION
CHECK
INVERSION CHECK FAILS
STOP
OUTPUT
STOP

For optimisation analysis a relationship between external loads and resultant forces on member ends was required. The external loads were to act at the apex of the structure. Hence only one node at the top plate was required. The structure was therefore idealised into four flexible members integral with a short rigid element contributed by the top plate. Further the lower ends of all the four members are fixed to the foundation, hence the structural assembly matrix was a superimposition of submatrices for each composite element. The structural stiffness matrix can be expressed as

$$[K] = \sum_{i=1}^{4} [H_i^{-1}][T_i][k_{22}^i][T_i][H_i^{-1}]^T \qquad (10)$$

where $[k_{22}]$ is the element stiffness submatrix in member coordinates 6 x 6,

$[T]$ is the transformation matrix to system coordinate, 6 x 6,

$[H]$ is the equilibrium matrix, 6 x 6

i is the member number

If the displacements at the node are given by $\{u\}$ and external load by $\{F\}$

then $$\underset{6\times1}{\{u\}} = \underset{6\times6}{[K^{-1}]}\ \underset{6\times1}{\{F\}} \qquad (11)$$

The member end displacements $\{d\}$ are given by

$$\{d\} = [\beta]\{u\} \qquad (12)$$

where $[\beta]$ is a transformation matrix defined by expression (4) .

For the present structure it is a column vector of four 6 x 6 indentity matrices. Substituting (11)in(12) gives

$$\underset{24\times 1}{\{d\}} = \underset{24\times 6}{[\beta]}\underset{6\times 6}{[K^{-1}]}\underset{6\times 1}{\{F\}} \tag{13}$$

Due to the special nature of $[\beta]$, for the present case we can write

$$\underset{6\times 1}{\{d_i\}} = \underset{6\times 6}{[K^{-1}]}\underset{6\times 1}{\{F\}} \tag{14}$$

where $\{d_i\}$is the end displacement of the i^{th} member in system coordinates. Transforming these displacements to the ends of the flexible component

$$\underset{6\times 1}{\{d_{if}\}} = \underset{6\times 6}{[H_i^T]^{-1}}\underset{6\times 1}{\{d_i\}} \tag{15}$$

where $\{d_{if}\}$ represents the displacement at the flexible end of the i^{th} member, connected to the rigid part. Transforming $\{d_{if}\}$ to member coordinates to give $\{d_{if}\}_m$ we have

$$\underset{6\times 1}{\{d_{if}\}_m} = \underset{6\times 6}{[T_i]}\underset{6\times 1}{\{d_{if}\}} \tag{16}$$

Substituting (14) and (15) into (16) we get

$$\underset{6\times 1}{\{d_{if}\}_m} = \underset{6\times 6}{[T_i]}\underset{6\times 6}{[H_i^T]^{-1}}\underset{6\times 6}{[K]^{-1}}\underset{6\times 1}{\{F\}} \tag{17}$$

Forces $\{P_i\}_m$ at the free flexible end of each element are given by

$$\{P_i\}_m = [k_{22}^i]\{d_{if}\}_m \tag{18}$$

Substituting (9) in (10)

$$\{P_i\}_m = [K_{22}^i][T_i][H_i^T]^{-1}[K]^{-1}\{F\} \tag{19}$$
$$\scriptstyle 6\times1 \qquad\qquad 6\times6 \quad 6\times6 \quad 6\times6 \quad 6\times6 \quad 6\times1$$

The stresses in the member can be obtained by transforming $\{P_i\}_m$ by a general transformation matrix $[R]$. Let $\{S_i\}$ be a column vector of axial stress S_{xi} and transverse shear stresses S_{xyi} and S_{xzi}, where xyz refer to member coordinates.

Then
$$\underset{3\times1}{\{S_i\}} = \underset{3\times6}{[R]}\,\underset{6\times1}{\{P_i\}_m} \tag{20}$$

Substituting (19) into (20) gives

$$\underset{3\times1}{\{S_i\}} = \underset{3\times6}{[R]}\,\underset{6\times6}{[K_{22}^i]}\,\underset{6\times6}{[T_i]}\,\underset{6\times6}{[H_i^T]^{-1}}\,\underset{6\times6}{[K^{-1}]}\,\underset{6\times1}{\{F\}} \tag{21}$$

The exact nature of elements in $[R]$ will depend on the member cross-section. For a hollow circular cross-section it is given by the following matrix.

$$\begin{bmatrix} \frac{1}{A} & -\frac{L\cdot R_o \cos\theta}{I_z} & -\frac{L\cdot R_o \sin\theta}{I_y} & 0 & \frac{R_o \sin\theta}{I_y} & -\frac{R_o \cos\theta}{I_z} \\ 0 & -\frac{MA_z \sin\theta}{I_z\cdot t} & \frac{MA_y \sin\theta}{I_y\cdot t} & -\frac{R_o \sin\theta}{I_x} & 0 & 0 \\ 0 & \frac{MA_z\cdot \cos\theta}{I_z\cdot t} & -\frac{MA_y \cos\theta}{I_y\cdot t} & \frac{R_o \cos\theta}{I_x} & 0 & 0 \end{bmatrix} \tag{22}$$

where the significance and sign convention for the terms in matrix R are shown in Figures 11 and 11a.

EXPERIMENTAL ANALYSIS

Both static and dynamic analysis may require the stiffness or the flexibility matrix of the entire structure, depending upon the approach selected for subsequent analysis. Experimental compilation of the stiffness matrix would require measurement of forces or moments at all the coordinates, for a given displacement at one of them, while the displacements at all the rest are of zero magnitude. It is obvious that direct measurement of the stiffness coefficients would be extremely inconvenient. Hence the only alternative is the determination of the flexibility coefficients.

The experimental procedure generally would be to apply a known force or a moment at a given coordinate i and measure the corresponding linear or angular displacements at coordinate j. The magnitude of the displacement at j per unit force or moment at i gives the coefficient ij of the flexibility matrix. Inversion of the flexibility matrix thus obtained can, if desired, provide the stiffness matrix. Instrumentation was, therefore, designed to measure flexibilities.

The loading technique was to transmit sufficient force at a node to cause displacements of convenient magnitude without appreciably stiffening the structure. The requirement was fulfilled by two parts of a split collar, held together by countersunk screws, clamped on to the member over a 3/4 inch length, at the point of load application.

A small gap between the split parts ensured a friction grip between the member and the collar which had an outer spherical diameter of 2 inches. A split spherical shell having 2 inches inner and 3 inches outer spherical diameters, was locked on to the collar so that its axis was in any position within 30° of the member axis. It carried two pins screwed at diametrically opposite locations on the outer surface. Load was applied to the pins by the ends of a fine steel wire which supports a pulley in a vertical plane. A load supported by the pulley gives rise to identical tensions equal to half the applied load, in each section of the steel wire leaving the pulley vertically. These tensions were transmitted through an intermediate system of pulleys, supported by an independent tubular framework, to the pins in the same or opposite sense according as a force or a moment was being applied. Theoretically, the arrangement ensured that the two equal tensions would give rise, at the node, to a resultant force or a pure moment passing through the inaccessible member axis. The cross hairs of a levelling instrument were used as a reference to align the sections of the steel wire in the required horizontal or vertical directions.

The load application to the pulley was achieved by means of a pneumatic cylinder working off 100 psi air, through a spring balance calibrated at 0.2 lbs per division up to 100 lbs. Alternatively the load could be applied directly by steel weights supported on a pan held by the pulley.

The objective of the measurment of displacement was to isolate the required component, angular or linear, for measurement while suppressing all the other components. The points at which displacements were desired were inaccessible. The situation is represented in Figure 12. The fixed point O is lying on a member axis when the structure is unloaded. An orthogonal right handed coordinate system ijk parallel to the structural coordinate system is eatablished with origin at O G is a point on the member axis coinciding with O before displacement. Linear and angular displacements of G are defined respectively by vectors $\bar{L}$ and $\bar{A}$ with subscripts indicating their components along corresponding coordinate direction. The components are positive along positive coordinates. As point G is inaccessible, measurements are to be made at P fixed with respect to ijk . GH is a rigid link.

A plane ABCD always passing through H can be oriented perpendicular to the desired coordinate axis. H coincides with P before displacement. The displacement of H from P to Q is given by $\bar{d}$ where

$$\bar{d} = \bar{L} + \bar{A} \times \bar{R} \tag{23}$$

where $\bar{R}\ (R_i, R_j, R_k)$ is the position vector of P and $\bar{A}$ is small.

In terms of scaler components

$$d_i = L_i + A_j \cdot R_k - A_k \cdot R_j$$
$$d_j = L_j + A_k \cdot R_i - A_i \cdot R_k \quad (24)$$
$$d_k = L_k + A_i \cdot R_j - A_j \cdot R_i$$

Suppose L_i is to be measured. The surface ABCD is oriented perpendicularly to the i axis at P and locked to H. ABCD has a rigid body motion with GH but it can be considered first to move parallel to itself from P to Q with displacement d and then at Q take the required orientation A. The displacement transducer axis is fixed and passes through P along the i axis. The moving element of the transducer derives its displacement from each phase of movement of the surface ABCD, referred to above. The recorded displacement d_{ri} can therefore be expressed as the sum

$$\begin{aligned} d_{ri} &= \underbrace{d_i}_{\text{PHASE I}} + \underbrace{A_k \cdot d_j - A_j \cdot d_k}_{\text{PHASE II}} \\ &= L_i + A_j R_k - A_k R_j \\ &\quad + A_k \left(L_j + A_k \cdot R_i - A_i \cdot R_k \right) \\ &\quad + A_j \left(L_k + A_i \cdot R_j - A_j \cdot R_i \right) \end{aligned} \quad (25)$$

It is noted that displacements due to phase II are of higher order and can be neglected, generally

$$d_{ri} = L_i + A_j R_k - A_k \cdot R_j \quad (26)$$

It is obvious from Equation (26)that if the transducer axis coincides with the i axis R_j and R_k will be zero and thus the effects of all the other components are suppressed.

This may not always be possible. Care must be taken to maintain the instrument either in the ij plane or the ik plane so that only one error term is effective. Further, if surface ABCD is not axactly perpendicular to the i axis, phase I will contribute two more error terms $+A_{k0} \cdot d_j$ and $-A_{j0} \cdot d_k$, where A_{j0} and A_{k0} are the components in initial angular displacement of the perpendicular to ABCD from i.

By cyclic permutation of the indices, the expressions for the other two coordinate directions can be obtained. The problem of resolution of angular displacements will be discussed along with the method of angular measurement.

To achieve the requirements for linear measurements as discussed above two clamps were designed as shown in Figure 6. The inner clamp grips the member along a 1/2 inch length while providing an outer spherical surface having a 2 inch diameter over which the outer clamp can be fixed in any position. The outer clamp carries a ball-and-socket joint which in turn carries an optically flat first surface mirror, off which the linear displacements were measured. The mirror thus corresponds to plane ABCD. The ball-and-socket joint permitted the morror surface to be aligned perpendicular to the required coordinate, while the outer clamp allowed the transducer axis to coincide with the coordinate axis along which measurements were desired. The mirror was aligned perpendicular to the coordinate axis by ensuring simultaneously that

(1) Reference lines on the base of the structure and parallel to the coordinate axis were in line with their image in the mirror, and

(2) A vertical plumb line was parallel to its own image in the mirror.

The mirror provided an optically flat surface and was used to align the transducer axis with its image, thus putting the transducer perpendicular to the mirror and parallel to the desired coordinate axis.

A capacitance type proximity transducer coupled through an oscillator and reactance convertor to a cathode ray oscilloscope was adopted as the linear measuring device. The transducer consists of a fixed electrode. Any flat conducting surface parallel to the fixed electrode can act as the moving electrode. Normally, the moving electrode is fastened to the component whose displacement is to be measured. In the present application the structure had both linear as well as angular displacements whereas the proximity transducer is designed to work when electrodes remain parallel while moving towards or away from each other. When angular displacements are present too high results are registered.

To eliminate this defect the moving electrode was mounted on the transducer itself. The spindle of the moving electrode was supported jointly by two 2-1/2 inch x 1/2 inch x 5/1000 inch thick stainless steel strips, parallel to each other, which forced the electrode surfaces to remain parallel during relative motion. This is illustrated in Figures 6 and 7 . The spindle on the moving electrode was kept in positive contact with the mirror during displacement by the steel strips, which exerted a force on the structure of the order of 0.02 lbs or less as against 0.2 to 0.4 lbs exerted by an average dial indicator with a least count of 0.0001 inches. The errors introduced due to friction were thus minimised.

The transducer system is based on frequency modulation of a carrier wave. The capacitance of the electrodes is in parallel with another fixed capacitance. The combination forms a series resonant circuit with an inductance. The change in distance between the electrodes, due to loading of the structure caused a change in reactance in the resonant circuit which is used to change the frequency of the signal delivered by the oscillator. The signal is amplified and detected to provide a proportional D.C. voltage which was metered on the oscilloscope. Unloading the structure restores the initial gap between electrodes. The transducer can then be calibrated by the integral micrometer producing a deflection on the oscilloscope of the same order as that obtained due to the load. The calibration enabled the displacement to be evaluated. The least count of the micrometer was 0.01 mm which could be further subdivided by the oscilloscope. A initial gap of 0.5 to 1.5 mm between the electrodes was used.

The resolution of angular displacement into its components was achieved by a simple optical device. As the magnitude of rotations involved was small, vectorial resolution was possible. The optically flat first surface mirror used for linear measurements was also used to view the image of an illuminated transparent grid of 1/40 inch squares through a levelling telescope equipped with optical axis and base spirit levels together with vertical and horizontal cross hairs. The grid and the telescope were so placed that the line of sight was within 5° of the perpendicular to the mirror at the point of incidence.

The cross hairs were made to coincide with a pair of lines at right angles to each other on the grid. Rotation of the mirror due to loading of the structure had three components - one about an axis perpendicular to the mirror and two about axes in its plane. The first had no effect on the grid image as viewed through the telescope. The other two caused a displacement of the grid image with respect to the cross hairs. The vertical and horizontal displacement of the grid was directly proportional to twice the rotation of the mirror about the horizontal and vertical axes respectively.

Thus viewing the mirror along the i axis provides rotations along j and k axes and that along j axis those about i and k axes. Readings with these two settings provided all the three rotations and values about z provided a check.

An additional optically flat first surface mirror, mounted close to the objective of the telescope, could be used to view the grid reflected by both the mirrors, thereby increasing the optical arm. The arrangement is shown in Figures 4 & 5 . The telescope was further equipped with a lateral displacement type optical vernier , with a least count of 0.001 inches which could be used to further subdivide the 1/40 inch grid. The set up could accurately provide measurements with a least count of 0.1×10^{-4} radians up to a distance of 200 inches between the grid and the telescope along the line of sight.

The linear displacement of the mirror which shifted the point of incidence along the line of sight affected the readings as represented in Figure 10 , which is self explanatory. The error ϵ in the reading recorded for rotation of the mirror about an axis in its own plane but perpendicular to the plane containing the line of sight is given by

$$\epsilon \approx \pm 2 \cdot d_i \cdot \sin\alpha \qquad (27)$$

ignoring the change in the included angle α due to mirror rotation. We can determine d_i from the equation (26). The correct reading without the linear displacement of the mirror is

$$d_k = 2 \cdot LL \cdot A_k \tag{28}$$

Therefore the percentage error $\simeq \pm \dfrac{2 \cdot d_i \cdot \sin\alpha}{2 \cdot LL \cdot A_k} \times 100$

where LL is the distance between the mirror and the grid

In the worst case d_i and $LL \cdot A_k$ are of the same order. Hence the percentage error is determined by 100 sin α approximately.

It is thus obvious that by increasing the distance between the mirror and grid and decreasing the angle between the line of sight and the perpendicular to the mirror, the percentage error can be decreased. In the event that errors are still large, the reading can be repeated after shifting the mirror, mounted on the telescope, through 90°.

RESULTS AND CONCLUSIONS

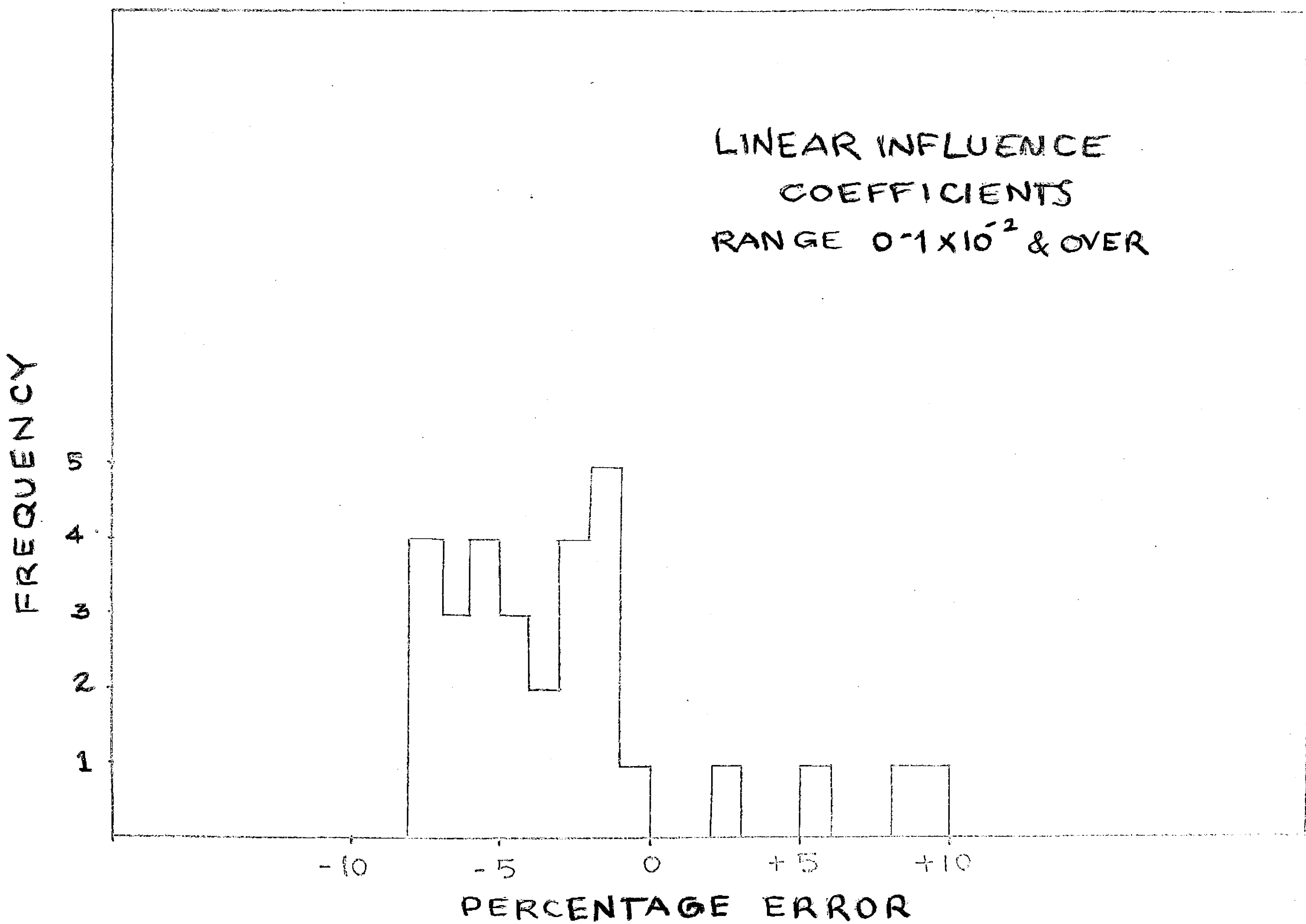
The analytical influence coefficients, their experimental values and the percentage error are listed in the Appendices II-IV in the form of a lower triangular matrix. No attempts have been made to measure the coefficients if the theoretical influence coefficients were below the absolute value of 0.25×10^{-5} for angular ones and 0.5×10^{-5} for the linear ones.

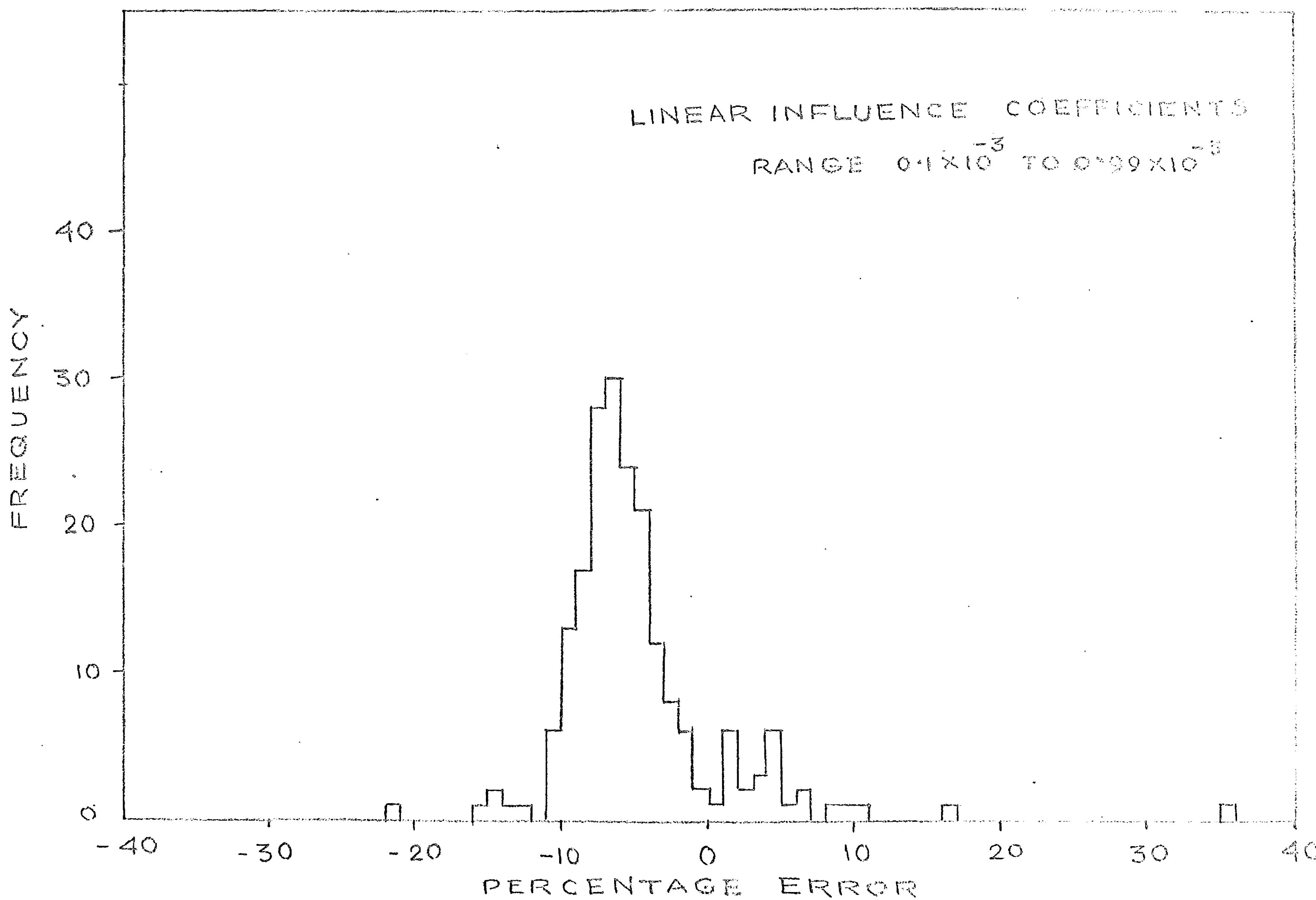
The coefficients are classified into linear and angular measurements. Each of these are further subdivided into three groups each according to the order of absolute magnitude of the analytical coefficients. The results have been presented in the form of histograms for each of the above groups.

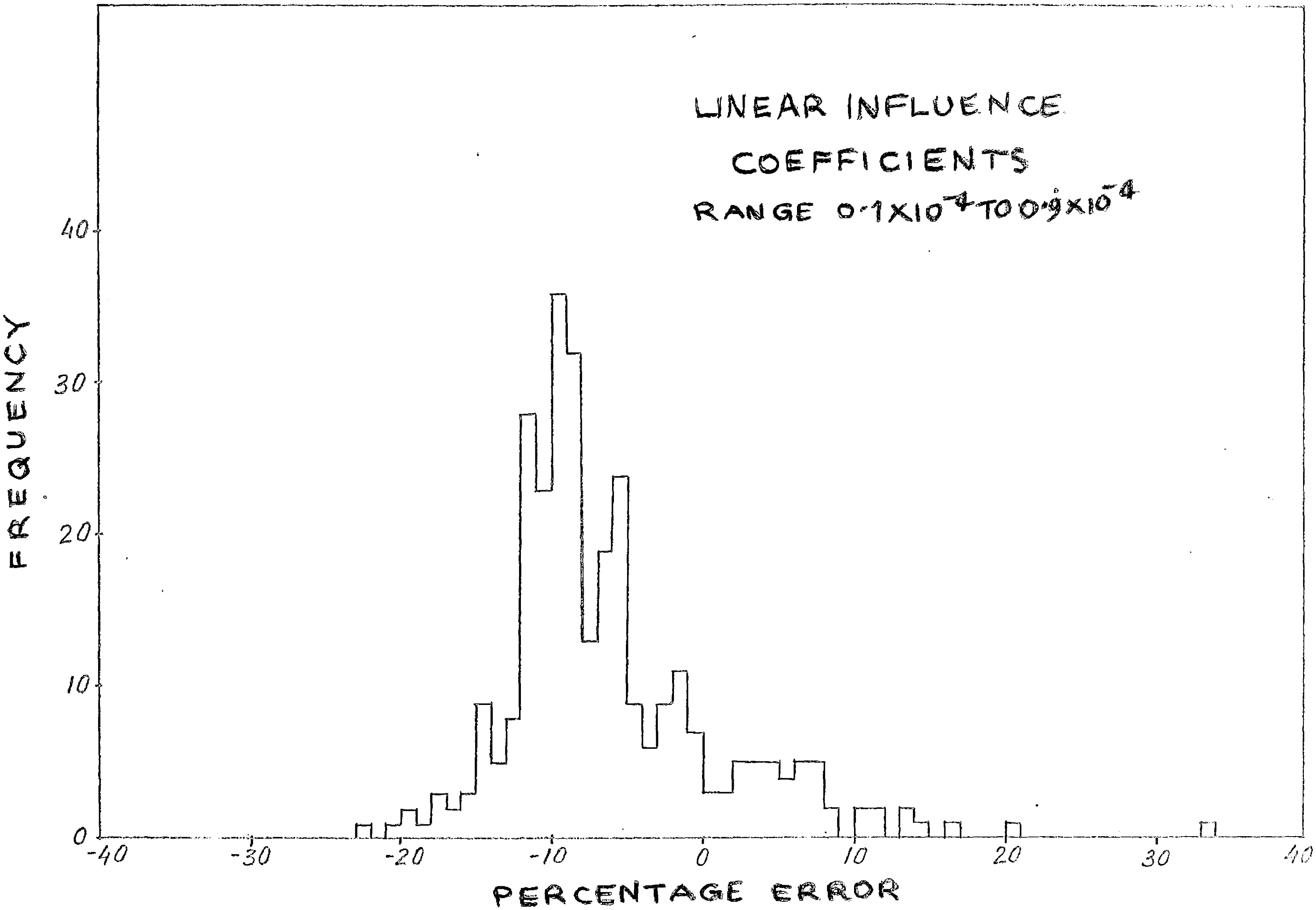
Although some of the results are higher, in general the experimental values are lower than the theoretical ones. The largest single reason is apparently the error in load application. For larger values the scatter in the readings is small. The scatter in error for smaller values of influence coefficients would indicate that limits of instrumentation have been reached.

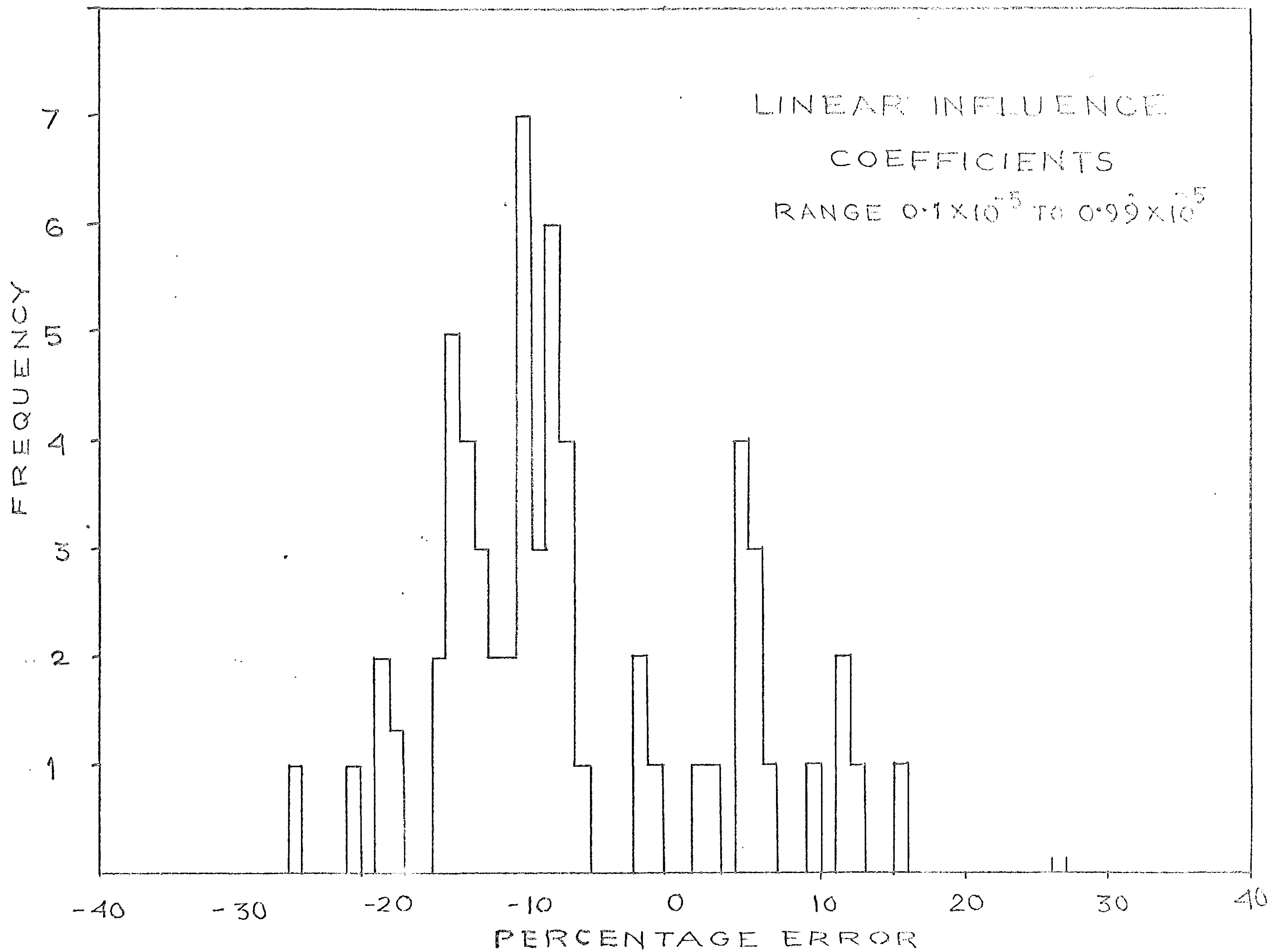
The resultant error is dependent upon a number of other independent factors like error in location of the nodes, deviation of the instrument axis from the coordinate direction, errors in taking observations, other personal errors - to name a few. Effect of each of these factors vary about some average value in a random fashion during experimentation. Hence it is natural to expect a more or less normal distribution for the resultant error which is a function of all the independent factors. The tendency is brought out clearly by the histograms.

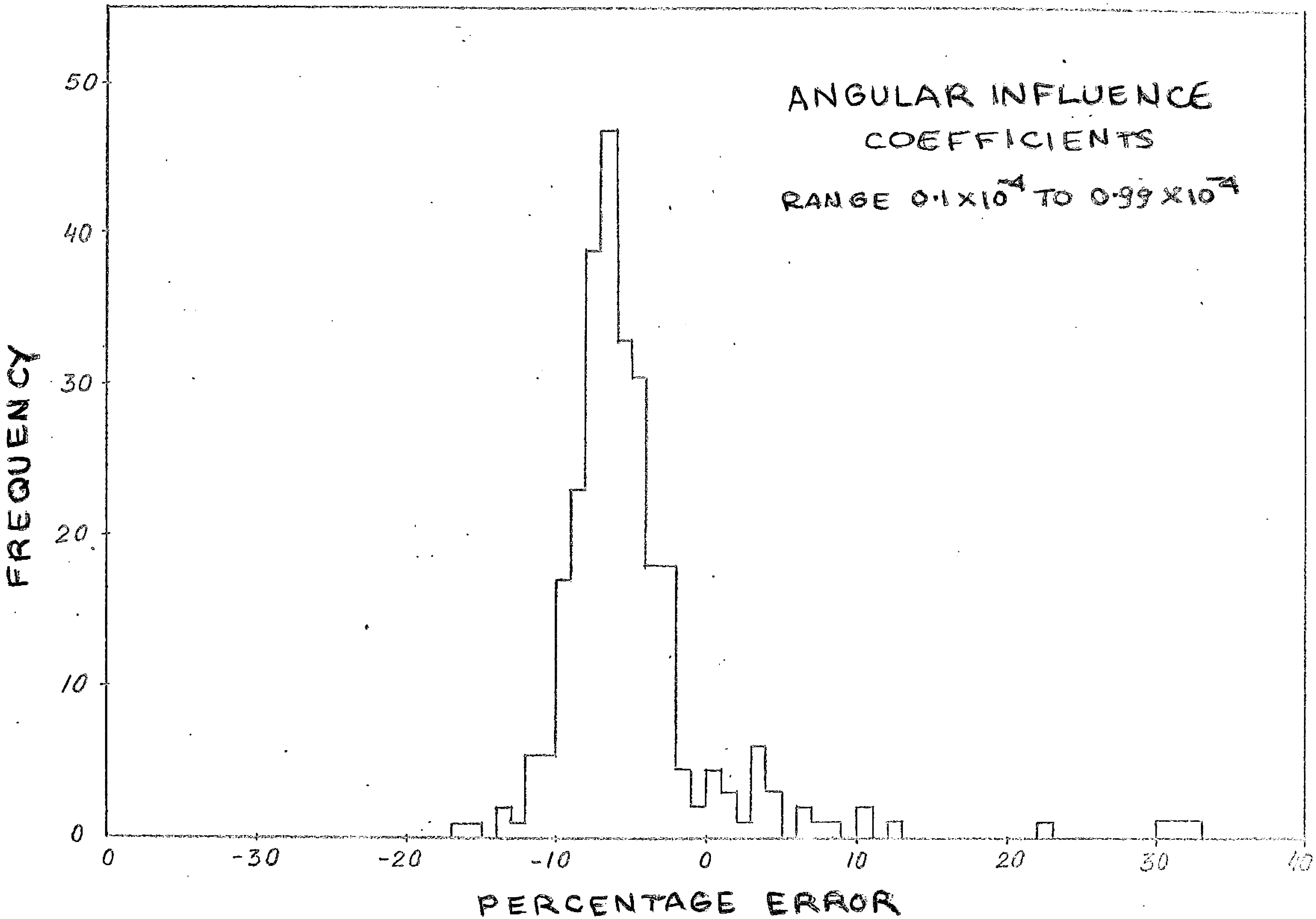
Considering the order of magnitudes of the displacements invloved, it can be concluded that a reasonably accurate technique for measurement of the flexibility influence coefficients, both linear and angular, for a generalised space frame has been established. Proximity transducer has been successfully adopted for the measurement of linear displacements in a component having linear displacement coupled with rotation. Large frictional and application error inherrent in the use of a dial indicator for measurement of linear displacements have been eliminated.


LINEAR INFLUENCE
COEFFICIENTS
RANGE 0·1 X 10⁻² & OVER
FREQUENCY
5
4
3
2
1
-10
-5
0
+5
+10
PERCENTAGE ERROR

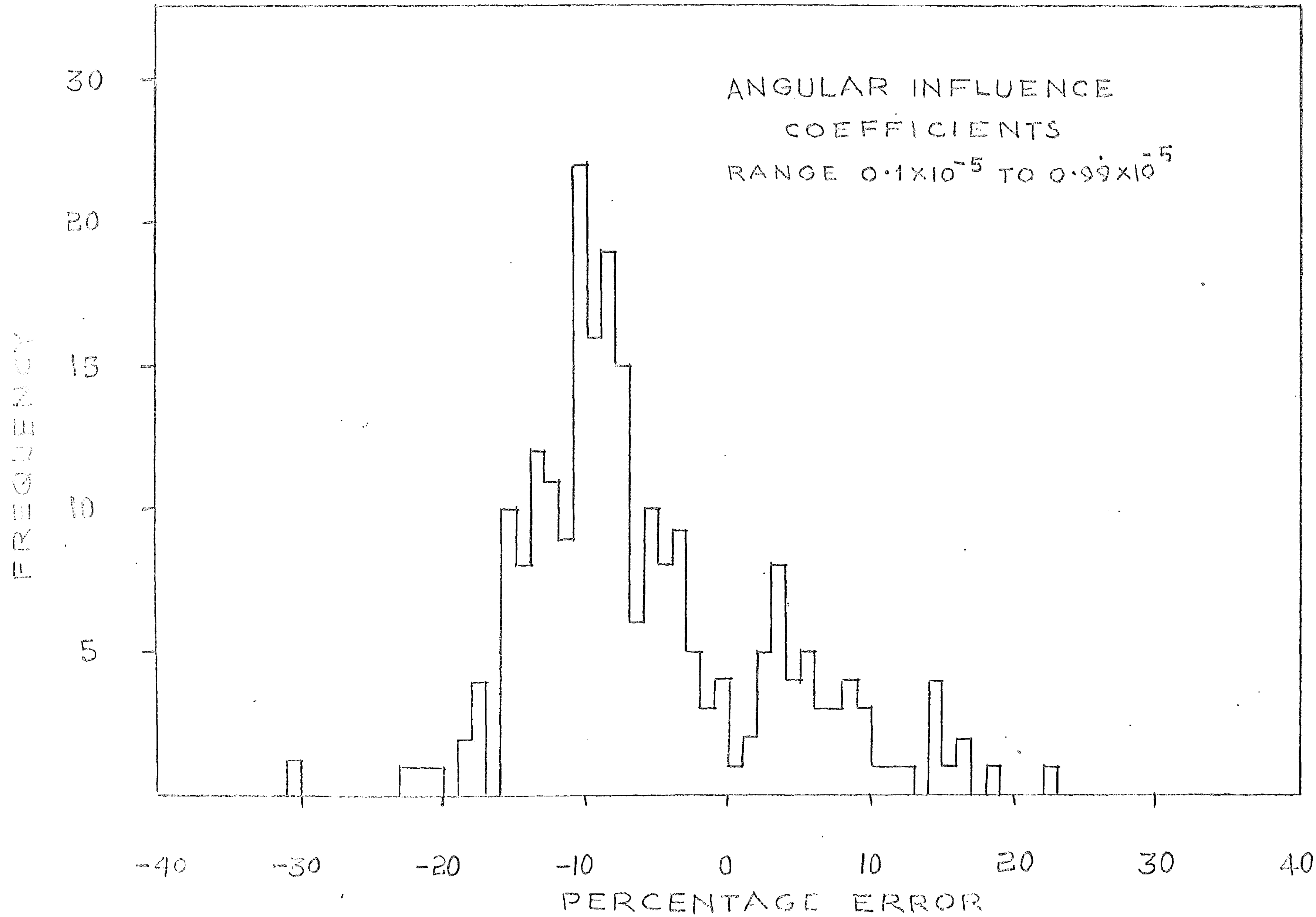

LINEAR INFLUENCE COEFFICIENTS
RANGE 0·1×10^-3 TO 0·99×10^-3
FREQUENCY
0
10
20
30
40
-40
-30
-20
-10
0
10
20
30
40
PERCENTAGE ERROR


LINEAR INFLUENCE
COEFFICIENTS
RANGE 0·1X10⁻⁴ TO 0·9X10⁻⁴
FREQUENCY
PERCENTAGE ERROR
40
30
20
10
0
-40
-30
-20
-10
0
10
20
30
40


LINEAR INFLUENCE
COEFFICIENTS
RANGE 0·1×10-5 TO 0·99×10-5
FREQUENCY
1
2
3
4
5
6
7
-40
-30
-20
-10
0
10
20
30
40
PERCENTAGE ERROR



ANGULAR INFLUENCE COEFFICIENTS
RANGE 0·1 x 10⁻⁴ TO 0·99 x 10⁻⁴
FREQUENCY
50
40
30
20
10
0
0
-30
-20
-10
0
10
20
30
40
PERCENTAGE ERROR



ANGULAR INFLUENCE
COEFFICIENTS
RANGE 0·1×10^{-5} TO 0·99×10^{-5}
FREQUENCY
30
20
15
10
5
-40
-30
-20
-10
0
10
20
30
40
PERCENTAGE ERROR

FIGURES

Figure 1 Overall picture of set-up

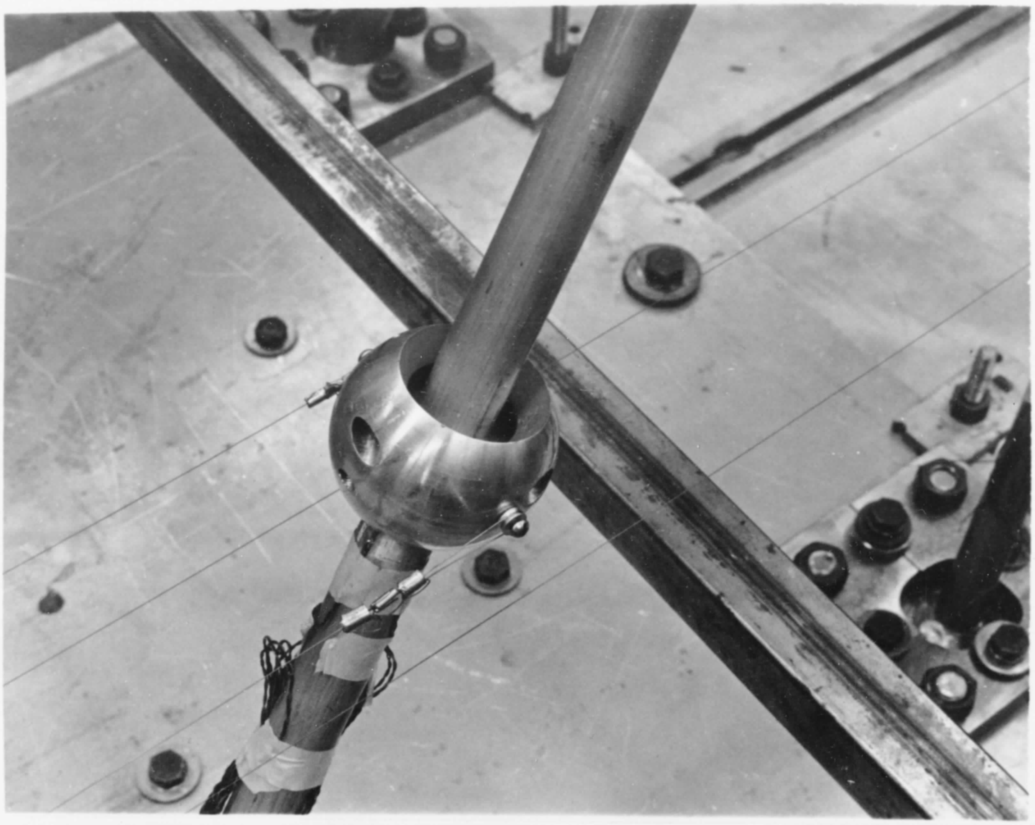
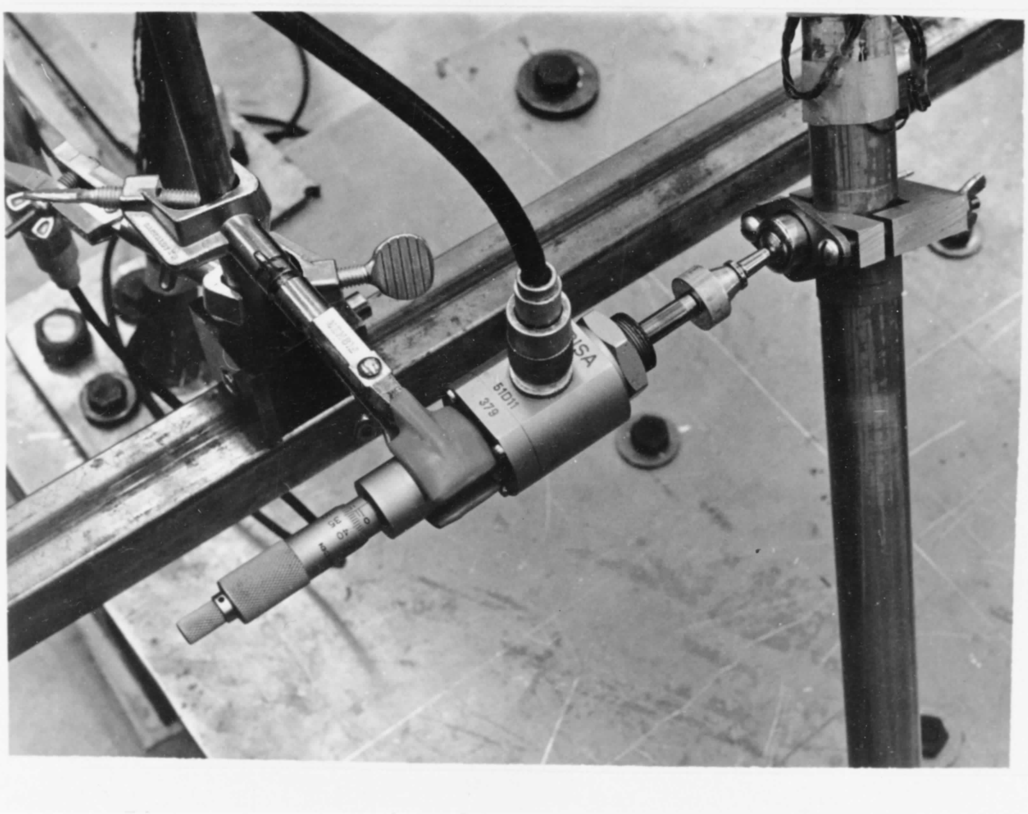


Figure 3 Detail of ball joint for loading frame



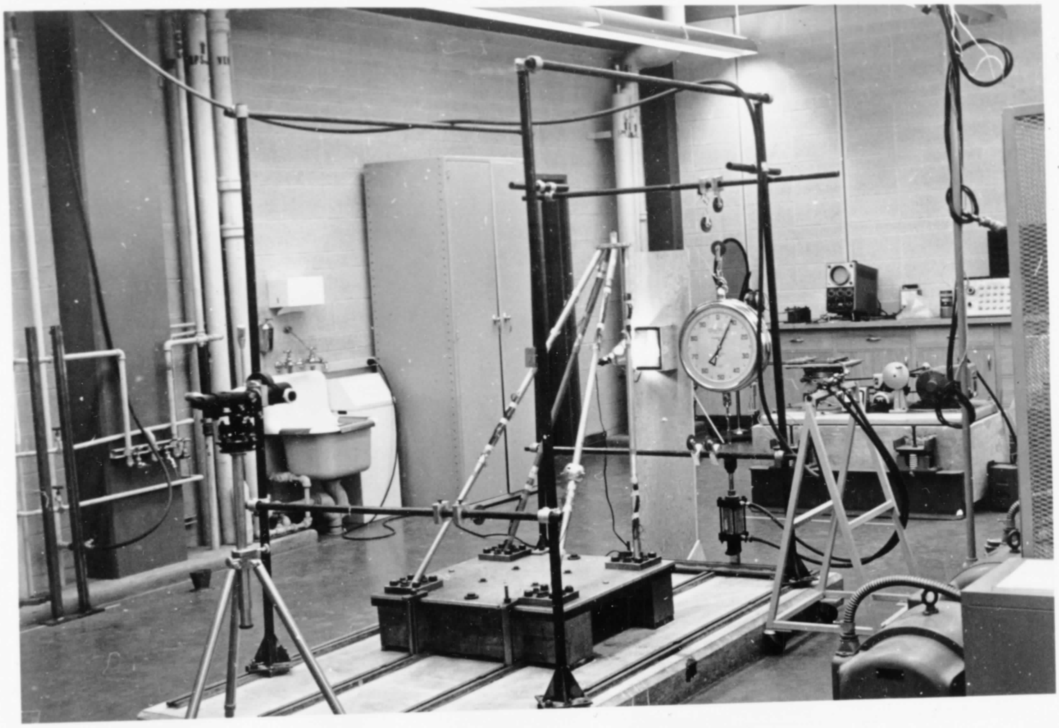


Figure 4   Overall picture of angular measurement arrangement

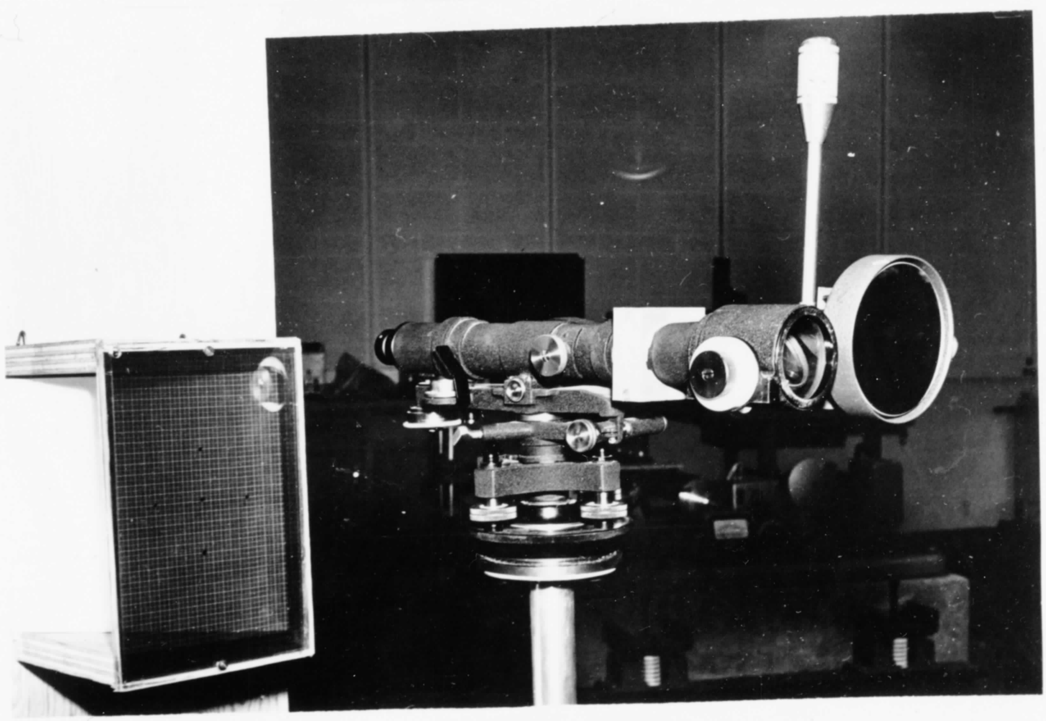


Figure 5   Detail of telescope

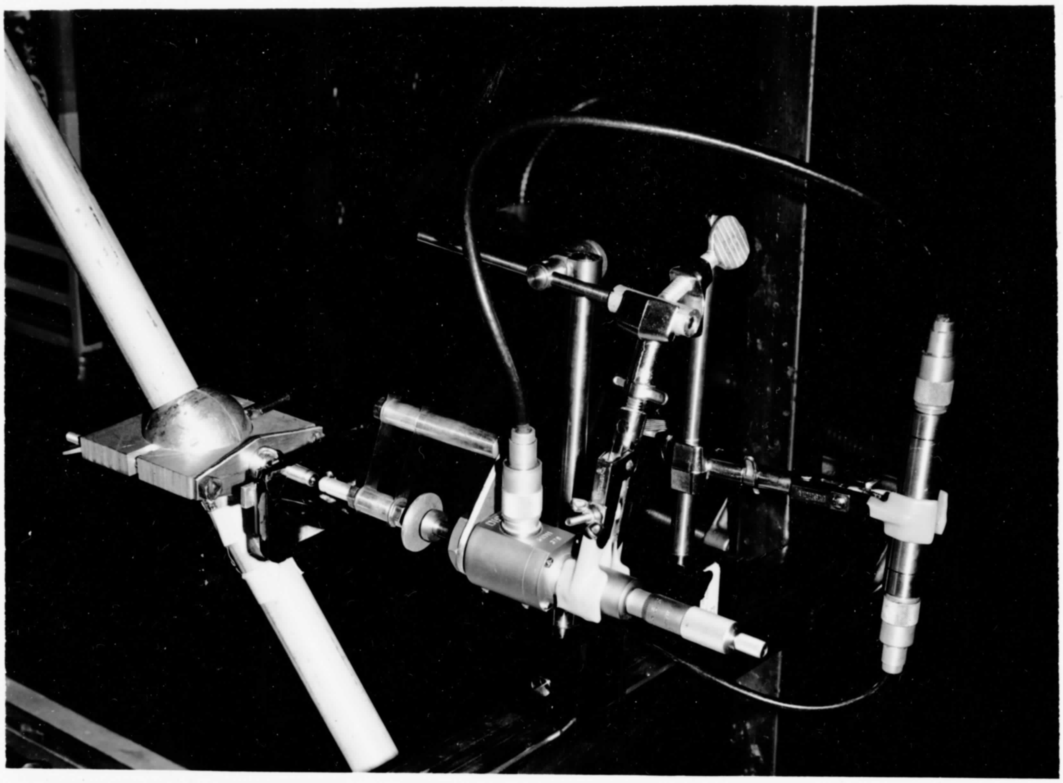


Figure 6. Installation of Device to Eliminate Rotation Error in Linear Displacement Measurement

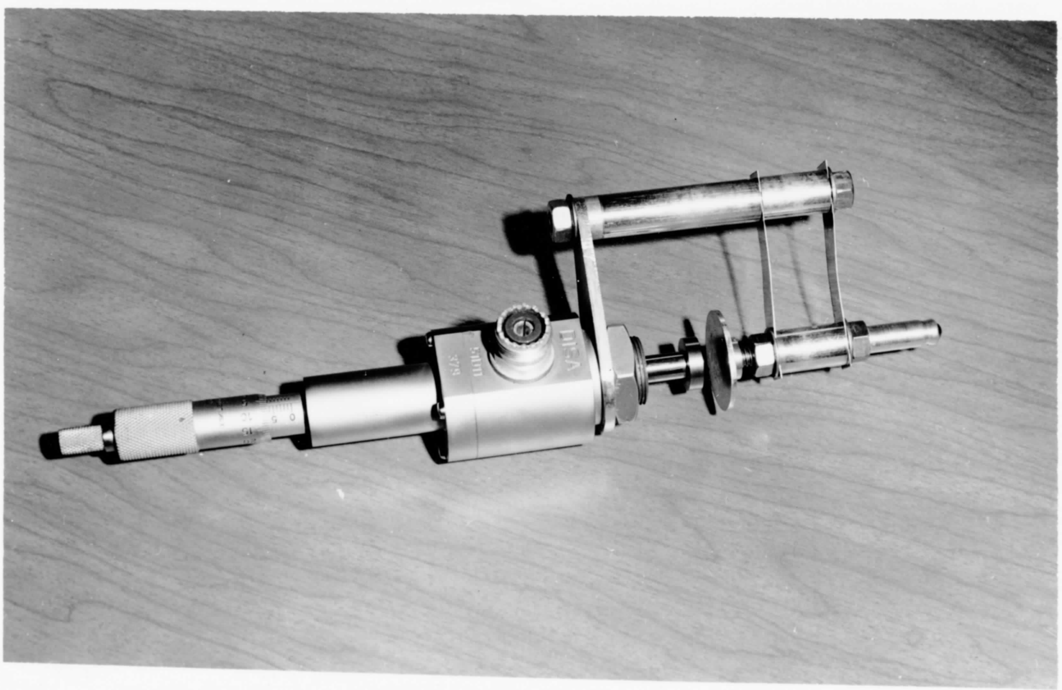


Figure 7. Device to Eliminate Rotation Error in Linear Displacement Measurements

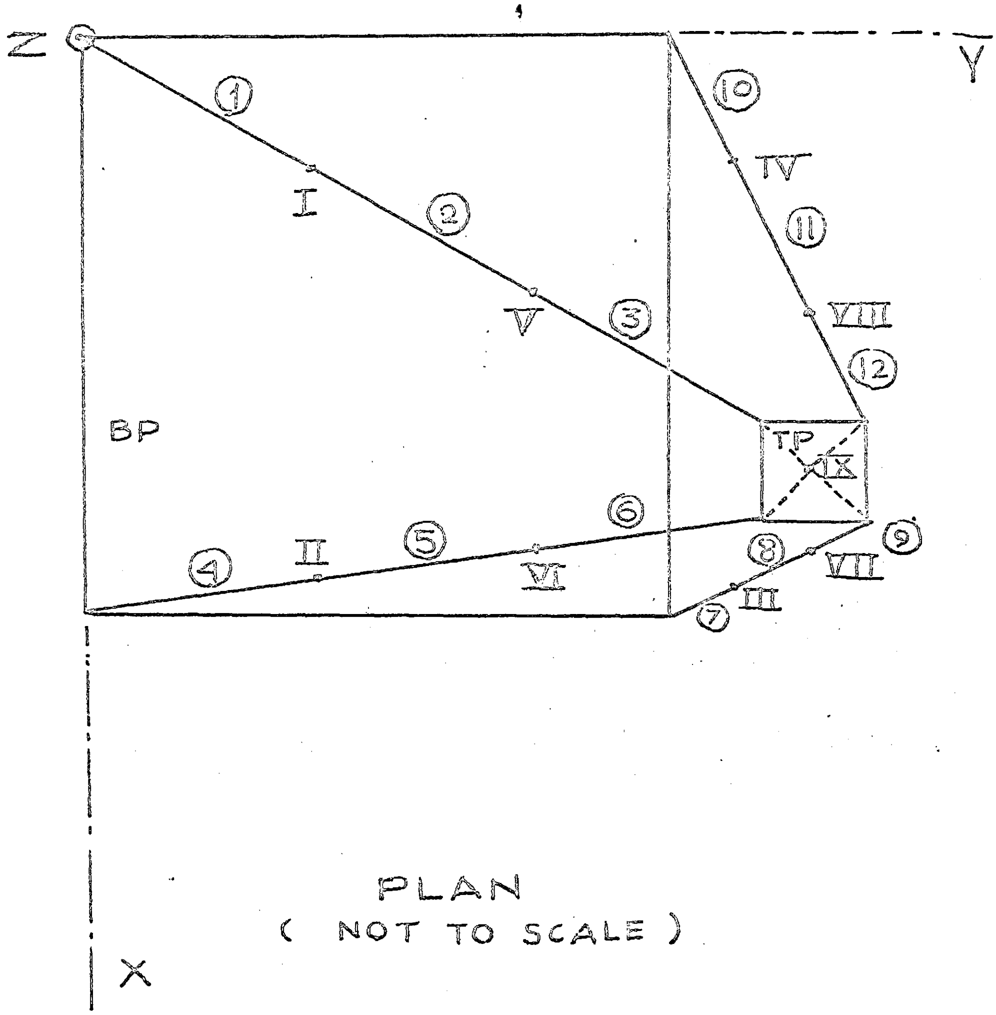


FIGURE 8 DISCRETISED MATHEMATICAL MODEL

I ETC	...	NODE POINTS
① ETC	...	DISCRETE ELEMENTS
TP	...	TOP PLATE
BP	...	BOTTOM PLATE
XYZ	...	SYSTEM COORDINATE AXES

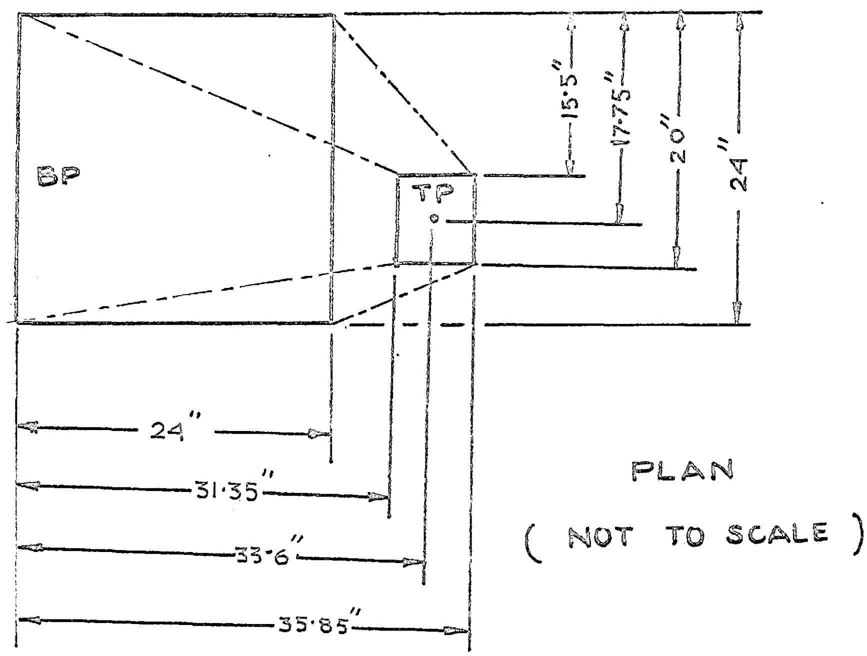
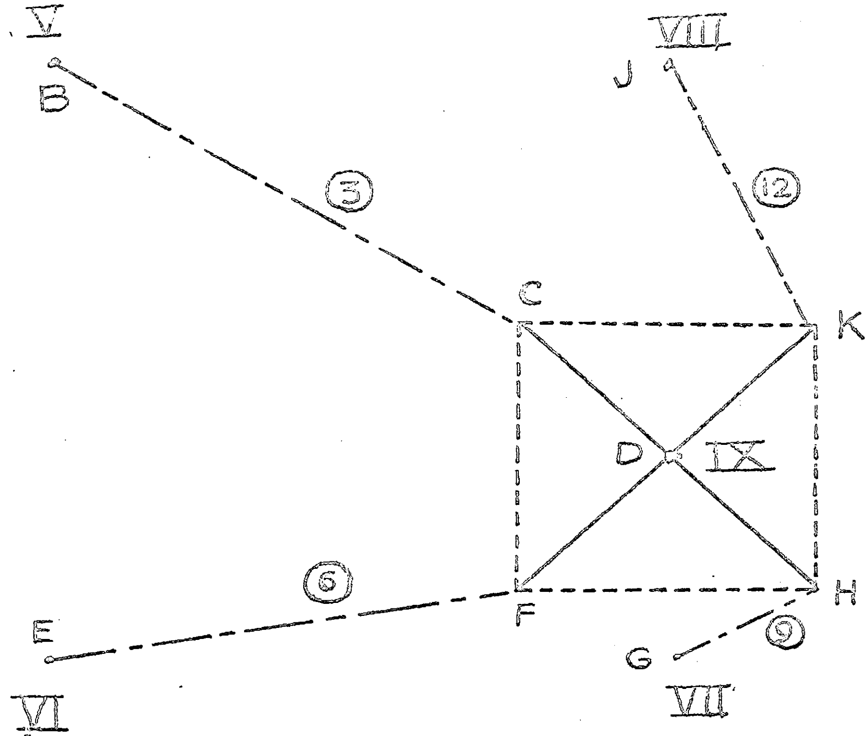


FIGURE 8a
MODEL DIMENSIONS AND MATERIAL PROPERTIES

OUTER DIA. OF ELEMENTS	...	1·05"
INNER DIA. OF ELEMENTS	...	0·824"
AREA OF CROSS SECTION OF ELEMENTS	...	0·326 SQ.IN.
MOMENT OF INERTIA OF ELEMENT CROSS SECTION	...	0·03635 $IN.^4$
POLAR MOMENT OF INERTIA	...	0·07272 $IN.^4$
MODULUS OF ELASTICITY	...	10,300,000 $LBS/IN.^2$
MODULUS OF RIGIDITY	...	3,850,000 $LBS/IN.^2$



PART PLAN
(NOT TO SCALE)

FIGURE 9 TOP PLATE AND THE COMPOSITE ELEMENT

BD, ED ETC	...	COMPOSITE ELEMENTS
------	...	TOP PLATE BOUNDARY
———	...	RIGID ELEMENT
— - —	...	FLEXIBLE ELEMENT
VI ETC	...	NODE POINTS
⑨ ETC	...	DISCRETE ELEMENTS

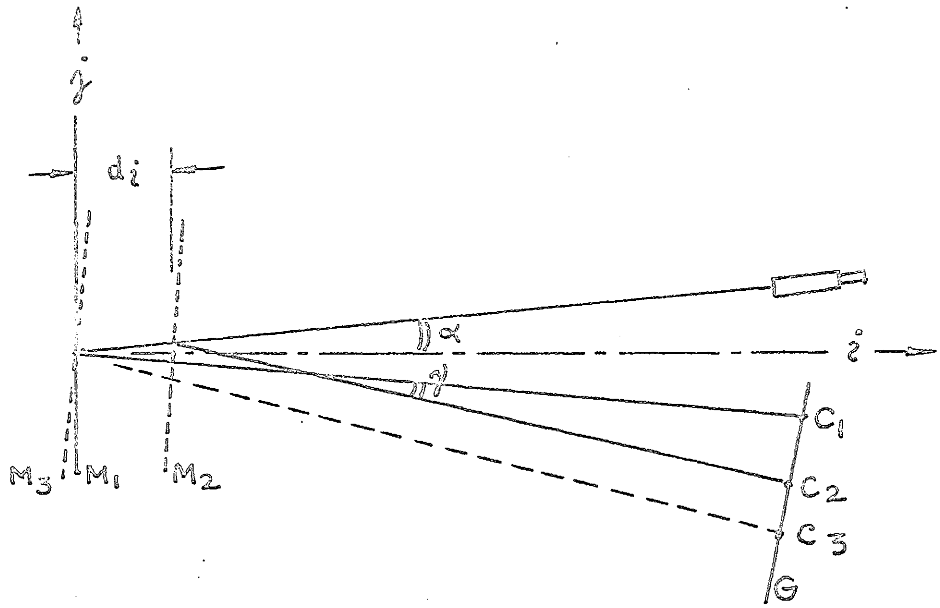


FIGURE 10 ERROR IN ANGULAR MEASUREMENT

T	...	TELESCOPE
M	...	MIRROR
M_1	...	INITIAL POSITION
M_2	...	DISPLACED POSITION
M_3	...	DISPLACED ANGULAR POSITION WITHOUT LINEAR DISPLACEMENT
C_1, ETC	...	CROSSHAIR INTERSECTION ON GRID CORRESPONDING TO M_1 ETC
$C_1 C_2$	...	RECORDED LINEAR GRID DISPLACEMENT
$C_2 C_3$	...	ERROR DUE TO THE LINEAR DISPLACEMENT OF THE MIRROR
$C_1 C_3$	...	CORRECT DISPLACEMENT
d_i	...	LINEAR MIRROR DISPLACEMENT ALONG i AXIS, EQUALS L_i

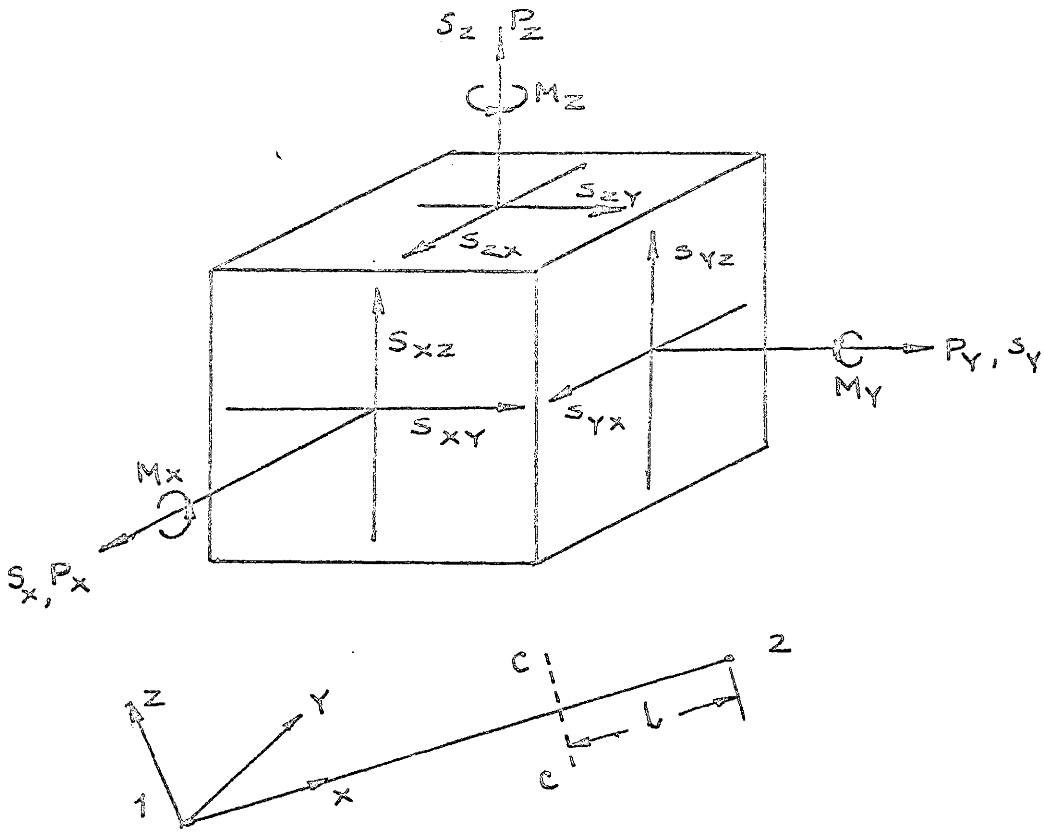


FIGURE 11 SIGN CONVENTIONS

Px ETC	...	FORCES AT MEMBER END 2 IN X ETC DIRECTION
Mx ETC	...	CORRESPONDING MOMENTS AT MEMBER END 2
Sx ETC	...	DIRECT STRESSES ALONG COORDINATE AXES
Sxy ETC	...	TRANSVERSE STRESSES
xyz	...	MEMBER COORDINATE AXES
1 2	...	MEMBER AXIS

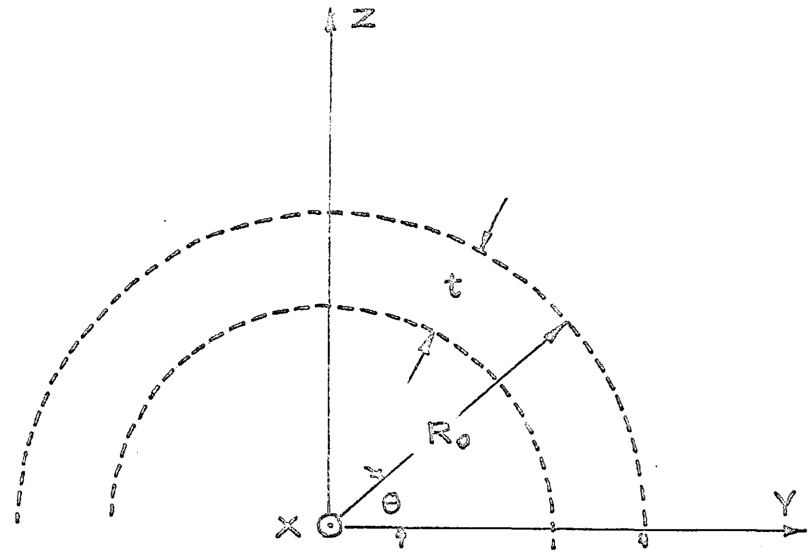


FIGURE 11a
TERMS IN MATRIX R

XYZ	...	MEMBER AXES
t	...	THICKNESS OF THE HOLLOW CYLINDRICAL ELEMENT
R_o	...	OUTER RADIUS
θ	...	ANGLE BETWEEN THE POSITION VECTOR OF ANY POINT AND Y AXIS
MA_Y	...	MOMENT OF AREA BETWEEN THE RADIUS VECTOR AND AXIS Y ABOUT THE Z AXIS

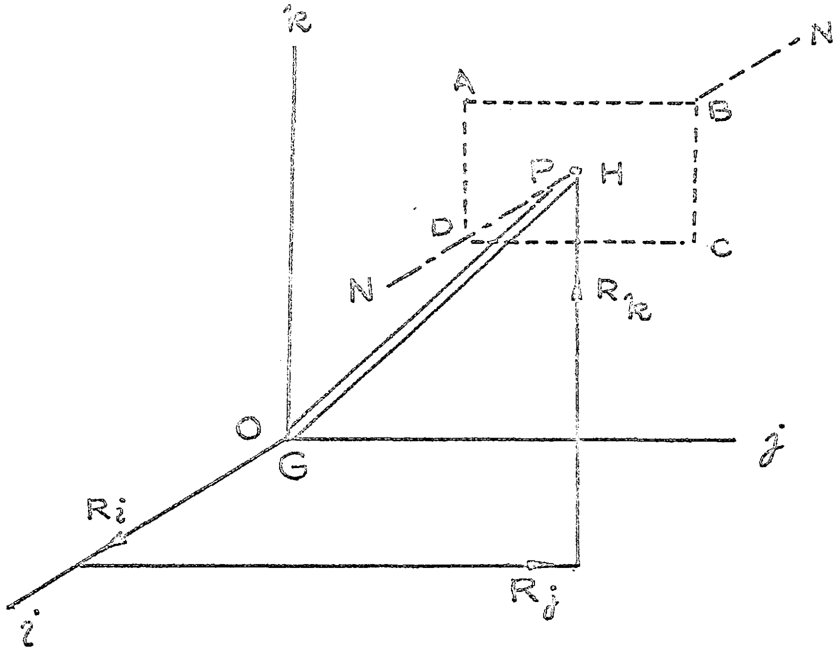


FIGURE 12 RESOLUTION OF LINEAR DISPLACEMENT

oijk	...	RIGHT HANDED COORDINATE SYSTEM, FIXED IN SPACE
GH	...	RIGID LINK FIXED TO THE NODE POINT & EQUAL TO $\overline{R}$
ABCDH	...	FLAT SURFACE, CAN BE LOCKED TO GH
R_i, R_j, R_k	...	COMPONENTS OF POSITION VECTOR $\overline{R}$ OF H W.R.T. G
P	...	A POINT FIXED IN SPACE, COINCIDENT WITH H BEFORE IT'S DISPLACEMENT
NN	...	AXIS PARALLEL TO i AXIS.

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APPENDIX I

COMPUTER PROGRAMMES

FORTRAN PARAMETERS

NELEM	...	NO. OF ELEMENTS
NJOIN	...	NO. OF JOINTS
MI	...	JOINT NO. TO WHICH THE FIRST END OF THE ELEMENT IS CONNECTED
MJ	...	JOINT NO. TO WHICH SECOND END OF THE ELEMENT IS CONNECTED
SAM	...	STRUCTURAL ASSEMBLY MATRIX
SAMT	...	TEMPORARY STORAGE FOR SAM FOR INVERSION
STIFF	...	ELEMENT STIFFNESS MATRIX
TRANS	...	ELEMENT COORDINATE TRAVSFORMATION MATRIX
XP	...	COORDINATES OF THE FIRST END OF THE ELEMENT
XQ	...	COORDINATES OF THE SECOND END OF THE ELEMENT
YM	...	COORDINATES OF THE ELEMENT Y AXIS
XL	...	RELATIVE COORDINATES OF END TWO WITH RESPECT TO END ONE OF AN ELEMENT
ALONG	...	ELEMENT LENGTH
AREA	...	AREA OF CROSSSECTION OF A MEMBER
PINERT	...	MOMENT OF INRTIA ABOUT THE ELEMENT X AXIS
YINERT	...	MOMENT OF INRTIA ABOUT THE ELEMENT Y AXIS
ZINERT	...	MOMENT OF INRTIA ABOUT THE ELEMENT Z AXIS
EMOD	...	MODULUS OF ELASTICITY
GMOD	...	MODULUS OF RIGIDITY
NO	...	TRANSFER OF CONTROL PARAMETER
IND	...	ADDITIONAL TRANSFER OF CONTROL PARAMETER
INDEX	...	CONTROL PARAMETER FOR COORDINATE TEANSFORMATION OF THE ELEMENT STIFFNESS MATRIX
NI	...	TEMPORARY STORAGE REQUIRED BY THE INVERSION SUBROUTINE

```
$JOB            003102S.K.TIWARI
$IBJOB          DECK
$IBFTC MOD
C
      COMMON STIFF(20,12,12), TRANS(50,3,3), XP(3), XQ(3),
     1   YM(3), ALONG(50), EMOD, GMOD, NO, IND, SAM( 54,54 ),
     2   SAMT( 54,54 ), MI( 20 ), MJ( 20 ), N1( 54 ), MEM
C
      DIMENSION AA( 54 )
C
C     READ THE NO. OF ELEMENTS AND NO. OF JOINTS.
      READ (5,100) NELEM, NJOIN
C
C     READ THE NO. OF JOINT TO WHICH FIRST END OF ELEMENT
C     IS CONNECTED
      READ (5,100) ( MI(I), I = 1, NELEM )
C
C     READ THE NO. OF JOINT TO WHICH SECOND END OF ELEMENT
C     IS CONNECTED
      READ (5,100) ( MJ(I), I = 1, NELEM )
C
C     READ THE ELASTIC CONSTANTS.
      READ (5,101) EMOD, GMOD
C
C     CALCULATE THE COORDINATE TRANSFORMATION AND THE
C     STIFFNESS MATRIX FOR ALL THE ELEMENTS
C
      DO 10 MEM = 1, NELEM
C     CALCULATE THE COORDINATE TRANSFORMATION MATRIX.
      CALL TRANF(MEM)
C
C     CALCULATE THE ELEMENT STIFFNESS MATRIX IN
C     SYSTEM COORDINATES
      CALL STIFCO(MEM, INDEX )
   10 CONTINUE
C
C     START ASSEMBLY.
      CALL ASEMBL
C
C     INVERT THE ASSEMBLED STIFFNESS MATRIX FOR
C     THE STRUCTURE
C
      CALL INVMAT ( SAMT, NM, NM, 1.0E-08, IERR, N1 )
```

```
C
C     CHECK THE ACCURACY OF THE INVERSION
C
      DO 13 I = 1, NM
      DO 11 J = 1, NM
   11 AA( J ) = SAMT( J,I )
      DO 13 J = 1, NM
      X = 0.0
      DO 12 K = 1, NM
   12 X = X + SAM( J,K ) * AA( K )
      SAMT( J,I ) = X
   13 CONTINUE
      DO 15 I = 1, NM
      X = 0.0
      DO 14 J = 1, NM
   14 X = X + SAMT( I,J )
      IF ( ABS( SAMT( I,I )-1.0 ) .LT. 0.1E-03 .AND.
     1   ABS( X) .LT. 0.1E-03 ) GO TO 12
      WRITE ( 6, 110 ) I, SAMT( I,I ), X
   15 CONTINUE
C
C     OUTPUT
      WRITE ( 6, 115 )
      DO 13 I = 1, NM
      WRITE ( 6, 120 ) ( SAMT( I,J ), J = 1, I )
      WRITE ( 6, 200 )
   13 CONTINUE
      STOP
  100 FORMAT ( 16I5 )
  101 FORMAT ( 4F20.8 )
  110 FORMAT ( 1X, 22HINVERSION UNSUCCESSFUL ,//,
     1   1X, 15HROW NUMBER     =   ,I3, //, 9HDIAGONAL,
     2   12HELEMENT  =   ,E15.8, //, 1X, 10HSUM OF NON ,
     3   19HDIAGONAL TERMS  =    , E15.8, // )
  115 FORMAT ( 1X, 22HTHE FLEXIBILITY MATRIX , // )
  120 FORMAT ( 1X, 6E20.5 )
  200 FORMAT ( // )
      END
```

```
$IBFTC TRAN
C
      SUBROUTINE TRANF(I)
C
      COMMON STIFF(20,12,12), TRANS(50,3,3), XP(3), XQ(3),
     1   YM(3), ALONG(50), EMOD, GMOD, NO, IND, SAM( 54,54 ),
     2   SAMT( 54,54 ), MI( 20 ), MJ( 20 ), N1( 54 ), MEM
C
      READ (5,100) (XP(IL), IL=1,3), (XQ(J), J=1,3),
     1    ( YM( K ), K=1,3 ), NO, IND
      IF ( NO .EQ. 7 ) GO TO 22
C
C     CALCULATION OF THE FIRST ROW OF THE MATRIX.
      T = 0.0
      DO 10 J = 1,3
      XQ(J) = XQ(J) - XP(J)
      TL(J) = TL(J) + XQ(J)
      IF ( IND .EQ. 1 ) GO TO 10
      T = T + XQ(J)**2
   10 CONTINUE
      IF ( IND .EQ. 1 ) RETURN
      ALONG(I) = SQRT(T)
      DO 11 J = 1,3
   11 TRANS(I,1,J) = XQ(J) / ALONG(I)
      IF ( XQ(3).EQ.0.0 ) GO TO 20
C
C     CALCULATION OF THE SECOND ROW OF THE MATRIX.
      T = 0.0
      DO 12 J = 1,3
   12 T = T + YM(J)**2
      T = SQRT(T)
      DO 13 J = 1,3
   13 TRANS(I,2,J) = YM(J) / T
C
C     CALCULATION OF THE THIRD ROW OF THE MATRIX.
      XP(1) = - XQ(3) * YM(2) + XQ(2) * YM(3)
      XP(2) = - XQ(1) * YM(3) + XQ(3) * YM(1)
      XP(3) = - XQ(2) * YM(1) + XQ(1) * YM(2)
      T = 0.0
      DO 14 J = 1,3
   14 T = T + XP(J)**2
      T = SQRT(T)
      DO 15 J = 1,3
   15 TRANS(I,3,J) = XP(J) / T
C
```

```
C     CHECK IF THE SUM OF SQUARES OF ELEMENTS IN THE SAME
C     ROW OR COLUMN IS EQUAL TO UNITY
      DO 19 L = 1,2
      DO 19 K = 1,3
      T = 0.0
      DO 18 J = 1,3
      GO TO ( 16,17 ), L
   16 T = T + TRANS(I,K,J)**2
      GO TO 18
   17 T = T + TRANS(I,J,K)**2
   18 CONTINUE
      IF ( ABS( T-1.0 ).GT.1.0E-03 ) GO TO 21
   19 CONTINUE
      RETURN
C
C     SECOND AND THIRD ROW OF THE MEMBER IN THE XY PLANE.
   20 TRANS(I,2,1) = - TRANS(I,1,2)
      TRANS(I,2,2) = TRANS(I,1,1)
      TRANS(I,2,3) = 0.0
      TRANS(I,3,1) = 0.0
      TRANS(I,3,2) = 0.0
      TRANS(I,3,3) = 1.0
      RETURN
C
C     THE CHECK HAS FAILED, STOP.
   21 WRITE(6,200) I
      WRITE ( 6,300 ) (( TRANS( I, IL, IM ), IM=1,3 ),
     1   IL=1,3 )
      STOP
   22 DO 23 II = 1, 3
      DO 23 JJ = 1, 3
   23 TRANS ( I, II, JJ ) = TRANS ( I-1, II, JJ )
      RETURN
C
  100 FORMAT ( 8F10.0/ F10.0, 2I10 )
  200 FORMAT ( //, 1X, 31HCHECK THE COORDINATES OF MEMBER,
     1   7H NUMBER, I3 // )
  300 FORMAT ( 3F20.5 )
      END
```

```
$IBFTC STIFF
      SUBROUTINE STIFCO (MEM )
C
      COMMON STIFF(20,12,12), TRANS(50,3,3), XP(3), XQ(3),
     1   YM(3), ALONG(50), EMOD, GMOD, NO, IND, SAM( 54,54 ),
     2   SAMT( 54,54 ), MI( 20 ), MJ( 20 ), N1( 54 ), MEM
C
      IF ( NO .EQ. 7 ) GO TO 22
C     READ IN THE MEMBER INCIDENCES.
      READ (5,1) AREA, PINERT, YINERT, ZINERT, INDEX
C     LET ALL THE ELEMENTS BELOW THE MAIN DIAGONAL BE ZERO.
      DO 50 I = 2,12
      IM = I - 1
      DO 50 J = 1,IM
   50 STIFF(MEM,I,J) = 0.0
C
C     CALCULATE THE VALUES OF THE ELEMENTS.
      TEMP = EMOD / ALONG(MEM)
      STIFF(MEM,1,1) = AREA * TEMP
      STIFF(MEM,11,5) = 2. * TEMP * YINERT
      STIFF(MEM,12,6) = 2. * TEMP * ZINERT
      STIFF(MEM,5,5) = 2. * STIFF(MEM,11,5)
      STIFF(MEM,6,6) = 2. * STIFF(MEM,12,6)
      STIFF(MEM,5,3) = - 3. * STIFF(MEM,11,5) / ALONG(MEM)
      STIFF(MEM,6,2) = 3. * STIFF(MEM,12,6) / ALONG(MEM)
      STIFF(MEM,8,6) = - STIFF(MEM,6,2)
      STIFF(MEM,12,8) = STIFF(MEM,8,6)
      STIFF(MEM,9,5) = - STIFF(MEM,5,3)
      STIFF(MEM,11,9) = STIFF(MEM,9,5)
      STIFF(MEM,11,3) = STIFF(MEM,5,3)
      STIFF(MEM,12,2) = STIFF(MEM,6,2)
      STIFF(MEM,2,2) = 2. * STIFF(MEM,6,2) / ALONG(MEM)
      STIFF(MEM,3,3) = 2. * STIFF(MEM,9,5) / ALONG(MEM)
      STIFF(MEM,4,4) = GMOD * PINERT / ALONG(MEM)
      STIFF(MEM,10,4) = - STIFF(MEM,4,4)
C
C     FILL IN THE VALUES OF ELEMENTS THAT REPEAT.
      DO 151 I = 1,6
  151 STIFF(MEM,I+6,I+6) = STIFF(MEM,I,I)
      DO 52 I = 1,3
   52 STIFF(MEM,I+6,I) = - STIFF(MEM,I,I)
C
C     IF K-BAR MATRIX IS NOT REQUIRED, RETURN
C     TO MAIN PROGRAMME
      IF ( INDEX.EQ.0 ) GO TO 70
C
C     TRANSFORM THE THREE BY THREE SUBMATRICES
C     WITH DIAGONAL ELEMENTS
      M = 0
      N = 0
      DO 59 L = 1,3
```

```
C
C     STORE THE DIAGONAL ELEMENTS TEMPORARILY.
      DO 54 I = 1,3
      MI = M + I
      NI = N + I
   54 ST(I) = STIFF(MEM,MI,NI)
C
C     TRANSFORMATION PROPER.
      DO 56 I = 1,3
      MI = M + I
      DO 56 J = 1,I
      TEMP = 0.0
      DO 55 K = 1, 3
   55 TEMP = TEMP + ST(K) * TRANS(MEM,K,J) * TRANS(MEM,K,I)
      NJ = N + J
      STIFF(MEM,MI,NJ) = TEMP
   56 CONTINUE
      GO TO ( 57,58,59 ), L
   57 M = 3
      N = 3
      GO TO 59
   58 M = 9
   59 CONTINUE
      DO 61 L = 1,2
      IL = L * L
      LL = IL + 2
      DO 61 I = IL,LL
      DO 61 J = IL,I
      STIFF(MEM,I+6,J+6) = STIFF(MEM,I,J)
      GO TO ( 60,61 ), L
   60 STIFF(MEM,I+6,J) = - STIFF(MEM,I,J)
   61 CONTINUE
      M = 6
      N = 0
      DO 64 L = 1,2
      DO 62 I = 1,2
      MI = M + I
      NI = N + I
      II = I + 1
      DO 62 J = II,3
      MJ = M + J
      NJ = N + J
   62 STIFF(MEM,MI,NJ) = STIFF(MEM,MJ,NI)
      GO TO ( 63,64 ), L
   63 M = M + 3
   64 N = N + 3
C
C     TRANSFORMATION OF THE SUBMATRICES WITH
C     TWO NONDIAGONAL ELEMENTS
      M = 3
      N = 0
      DO 68 L = 1,2
```

```
C
C     STORE THE TWO ELEMENTS TEMPORARILY.
      DO 65 I = 2,3
      MI = M + I
      NN = N + 5 - I
   65 ST(I) = STIFF(MEM,MI,NN)
C
C     TRANSFORMATION PROPER.
      DO 67 I = 1,3
      NI = N + I
      MI = M + I
      DO 67 J = 1,I
      TEMP = 0.0
      TEM = 0.0
      DO 66 K = 2, 3
      KK = 5 - K
      TEMP = TEMP + ST(K) * TRANS(MEM,KK,J) * TRANS(MEM,K,I)
      IF ( I.EQ.J ) GO TO 66
      TEM = TEM + ST(KK) * TRANS(MEM,KK,J) * TRANS(MEM,K,I)
   66 CONTINUE
      MJ = M + J
      NJ = N + J
      STIFF(MEM,MI,NJ) = TEMP
      IF ( I.EQ.J ) GO TO 67
      STIFF(MEM,MJ,NI) = TEM
   67 CONTINUE
      M = M + 3
      N = N + 3
   68 CONTINUE
      DO 69 I = 4,6
      DO 69 J = 1,3
      STIFF(MEM,I+6,J) = STIFF(MEM,I,J)
      STIFF(MEM,I+6, J+6) = - STIFF(MEM,I,J)
   69 CONTINUE
   70 DO 111 JL = 1,11
      JLP = JL + 1
      DO 111 KL = JLP,12
  111 STIFF(MEM,JL,KL) = STIFF(MEM,KL,JL)
      IF ( INDEX .EQ. 2 ) CALL TRANQL
      RETURN
   22 DO 23 II = 1, 12
      DO 23 JJ = 1, 12
   23 STIFF ( MEM, II, JJ ) = STIFF ( MEM-1, II, JJ )
      RETURN
    1 FORMAT ( 4F10.0, 4I10 )
  100 FORMAT ( 8F10.8 )
  601 FORMAT ( 6E20.5 )
  602 FORMAT ( / )
      END
```

```
$IBFTC TRANQ
      SUBROUTINE TRANQL
C
      COMMON STIFF(20,12,12), TRANS(50,3,3), XP(3), XQ(3),
     1   YM(3), ALONG(50), EMOD, GMOD, NO, IND, SAM( 54,54 ),
     2   SAMT( 54,54 ), MI( 20 ), MJ( 20 ), N1( 54 ), MEM
C
  112 READ ( 5,100 ) ( XL(I), I = 1,3 )
      DO 20 L = 1,2
      PP = -1.0
      IP = ( L-1 ) * 6 + 1
      IQ = L * 6
      DO 20 I = IP, IQ
      F = PP
      DO 20 J = 1,3
      F = - F
      JJ = 6 - J
      SUM = 0.0
      DO 19 K = 1,3
      IF ( K .EQ. J ) GO TO 19
      KJ = JJ - K
      SUM = SUM + STIFF ( MEM, I, K+6 ) * XL( KJ ) * F
      F = - F
   19 CONTINUE
      STIFF ( MEM, I, J+9 ) = STIFF ( MEM, I, J+9 ) + SUM
   20 CONTINUE
      PP = - 1.0
      DO 10 I = 1, 6
      F = PP
      DO 10 J = 1, 3
      F = - F
      JJ = 6 - J
      SUM = 0.0
      DO 9 K = 1, 3
      IF ( K .EQ. J ) GO TO 9
      KJ = JJ - K
      SUM = SUM + STIFF ( MEM, K+6, I+6 ) * XL ( KJ ) * F
      F = - F
    9 CONTINUE
      STIFF ( MEM, J+9, I+6 ) = STIFF ( MEM, J+9, I+6 ) + SUM
   10 CONTINUE
      DO 21 I = 1, 6
      DO 21 J = 1, 6
   21 STIFF ( MEM, J+6, I ) = STIFF ( MEM, I, J+6 )
      RETURN
C
      END
```

```
$IBFTC ASMBL
      SUBROUTINE ASEMBL
C
      COMMON STIFF(20,12,12), TRANS(50,3,3), XP(3), XQ(3),
     1   YM(3), ALONG(50), EMOD, GMOD, NO, IND, SAM( 54,54 ),
     2   SAMT( 54,54 ), MI( 20 ), MJ( 20 ), N1( 54 ), MEM
C
C     INITIALISE THE STRUCTURAL ASSEMBLY MATRIX.
      NM = 6 * NJOIN
      DO 549 I = 1, NM
      DO 549 J = 1, I
  549 SAM(I,J) = 0.0
C
C     START ASSEMBLY.
C
      DO 552 MEM = 1, NELEM
C
C     CALCULATE LOCATION OF ELEMENT STIFFNESS
C     SUBMATRICES IN SAM
   53 M = ( MI(MEM) - 1 ) * 6
      N = ( MJ(MEM) - 1 ) * 6
C
C     TRANSFER OF ELEMENT STIFFNESS SUBMATRICES TO SAM.
C
      DO 552 K = 1, 6
      MK = M + K
      NK = N + K
C
C     TRANSFER SUBMATRIX K22.
      DO 550 L = 1, K
      NL = N + L
  550 SAM( NK,NL ) = SAM( NK,NL ) + STIFF( MEM,K+6,L+6 )
C
C     IF FIRST END OF THE ELEMENT IS FIXED,
C     TAKE UP THE NEXT ELEMENT
      IF ( MI(MEM) .EQ. 0 ) GO TO 552
C
C     TRANSFER OF SUBMATRIX K11.
      DO 551 LI = 1, K
      ML = M + LI
  551 SAM( MK,ML ) = SAM( MK,ML ) + STIFF( MEM,K,LI )
C
C     TRANSFER OF SUBMATRIX K12.
      DO 570 LJ = 1, 6
      ML = M + LJ
  570 SAM( NK,ML ) = SAM( NK,ML ) + STIFF( MEM,K+6,LJ )
  552 CONTINUE
      RETURN
      END
```

```
$ENTRY
C
C
NELEM AND NJOIN
C
   12    9
C
M ( I )
C
          1    5         2    6         3    7         4    8
C
M ( J )
C
   1    5    9    2    6    9    3    7    9    4    8    9
C
EMOD AND GMOD
C
10300000.0          3850000.0
C
THE DATA INCLUDED BETWEEN STARRED LINES IS REPEATED IN
THE DO LOOP NUMBER TEN OF THE MAIN PROGRAMME.
FOR INDEX EQUAL TO TWO SUBROUTINE TRANQL IS CALLED.
IN TRANQL RELATIVE COORDINATES OF END TWO OF THE RIGID
ELEMENT WITH RESPECT TO END ONE ARE READ AS XL.
WHEN NO IS EQUAL TO SEVEN TRANS AND STIFF FOR THE ELEMENT
UNDER CONSIDERATION IS EQUAL TO THAT OF THE PREVIOUS ELEMENT.
A LINE WITHOUT A C IN THE FIRST GOLUMN IS A BLANK DATA CARD
************************************************************
XP ( IL ), XQ ( J ), YM ( K ), NO AND IND
C
0.0       0.0       0.0       5.5       10.7833   15.92
10.7833   -5.5      0.0
C
AREA, PINERT, YINERT, ZINERT AND INDEX
C
0.326     0.07272   0.03635   0.03635             1
************************************************************
C
NO IS SEVEN
C

                    7
************************************************************
```

```
0.0         0.0         0.0         5.5         10.7833    51.92
10.7833     -5.5        0.0
0.326       0.07272     0.03635     0.03635                 2
C
XL ( I ) IS READ
C
1.25        1.25
************************************************************
24.0        0.0         0.0         22.6666     10.7833    15.92
10.7833     1.3333      0.0
0.326       0.07272     0.03635     0.03635                 1
************************************************************

                   7
************************************************************
24.0        0.0         0.0         22.6666     10.7833    15.92
10.7833     1.3333      0.0
0.326       0.07272     0.03635     0.03635                 2
-1.25       1.25
************************************************************
24.0        24.0        0.0         22.6666     27.6166    15.92
3.6166      1.3333      0.0
0.326       0.07272     0.03635     0.03635                 1
************************************************************

                   7
************************************************************
24.0        24.0        0.0         22.6666     27.6166    15.92
3.6166      1.3333      0.0
0.326       0.07272     0.03635     0.03635                 2
-1.25       -1.25
************************************************************
0.0         24.0        0.0         5.5         27.6166    15.92
3.6166      -5.5        0.0
0.326       0.07272     0.03635     0.03635                 1
************************************************************

                   7
************************************************************
0.0         24.0        0.0         5.5         27.6166    15.92
3.6166      -5.5        0.0
0.326       0.07272     0.03635     0.03635                 2
1.25        -1.25
************************************************************
$IBSYS
```

APPENDIX II

ANALYTICAL FLEXIBILITY MATRIX

ANALYTICAL FLEXIBILITY MATRIX

NODE I

0.22353E-02		
-0.38419E-03	0.17158E-02	
-0.50982E-03	-0.10250E-02	0.87498E-03
0.52497E-05	-0.84155E-04	0.55071E-04
0.16867E-04		
0.87321E-04	-0.67434E-05	-0.25575E-04
0.62465E-05	0.25206E-04	
-0.48015E-04	0.22279E-04	0.14547E-05
0.84578E-05	0.16159E-04	0.36504E-04

NODE II

0.28443E-03	-0.71992E-04	-0.48792E-04
0.52565E-05	0.24328E-04	-0.53470E-05
0.21643E-02		
0.40040E-04	0.13726E-03	-0.10559E-03
-0.96968E-05	0.11245E-04	0.10849E-04
0.11292E-03	0.14933E-02	
-0.38088E-05	-0.97864E-04	0.67748E-04
0.69385E-05	-0.56190E-05	-0.77229E-05
0.10389E-03	-0.99766E-03	0.68904E-03
-0.35714E-05	-0.12756E-04	0.97807E-05
0.82843E-06	-0.11749E-05	-0.12200E-05
-0.35655E-05	-0.80063E-04	0.53827E-04
0.13549E-04		
0.24806E-04	-0.12604E-04	0.16329E-07
0.14993E-05	0.29457E-05	0.75074E-06
0.85512E-04	0.65685E-05	0.26908E-05
-0.20649E-05	0.25347E-04	
-0.46632E-05	-0.81911E-05	0.70892E-05
0.14529E-05	0.81885E-06	0.18862E-05
-0.45781E-04	-0.11850E-05	-0.29708E-05
-0.25308E-05	0.16932E-04	0.37116E-04

NODE III

0.16855E-03	0.11403E-04	-0.64746E-04
-0.66001E-05	0.57199E-05	-0.16020E-04
0.16850E-03	-0.19019E-04	0.25919E-04
0.32525E-05	0.49706E-05	-0.17151E-04
0.13514E-02		

0.40041E-04	0.13711E-03	-0.10561E-03
-0.96866E-05	0.11244E-04	0.10842E-04
0.39968E-04	0.19417E-03	-0.12660E-03
-0.19338E-04	0.65657E-05	0.38941E-05
0.18260E-05	0.12969E-02	

0.39869E-05	-0.30872E-04	0.18831E-04
0.17064E-05	-0.21782E-05	-0.37990E-05
0.40205E-05	-0.46383E-04	0.31806E-04
0.47705E-05	-0.11815E-05	-0.23089E-05
0.11158E-03	-0.29440E-03	0.80963E-04

-0.48676E-05	-0.12277E-04	0.98605E-05
0.77608E-06	-0.13097E-05	-0.12331E-05
-0.48587E-05	-0.19687E-04	0.12754E-04
0.19768E-05	-0.87371E-06	-0.58457E-06
0.27956E-05	-0.71255E-04	0.16534E-04
0.11935E-04		

0.19049E-04	-0.21980E-05	-0.49778E-05
-0.16497E-06	0.11156E-05	-0.11176E-05
0.19045E-04	-0.30128E-06	0.16977E-05
0.11995E-06	0.11036E-05	-0.11377E-05
0.68654E-04	-0.29556E-06	0.57191E-05
-0.47451E-06	0.12761E-04	

0.10829E-04	-0.14581E-04	0.60467E-05
0.21227E-05	0.24250E-05	0.20107E-05
0.10832E-04	0.69302E-05	-0.37052E-05
-0.87937E-06	0.26953E-05	0.24116E-05
-0.23116E-04	0.26004E-05	-0.25220E-05
-0.33003E-05	0.59863E-05	0.41001E-04

NODE IV

0.16861E-03	0.11346E-04	-0.64696E-04
-0.65923E-05	0.57250E-05	-0.16021E-04
0.16856E-03	-0.19082E-04	0.25937E-04
0.32586E-05	0.49765E-05	-0.17152E-04
0.19979E-03	-0.19010E-04	0.20177E-04
0.28036E-05	0.17069E-04	-0.11396E-04
0.14114E-02		

-0.71917E-04	0.21772E-03	-0.12103E-03
-0.21148E-04	-0.67409E-05	0.51975E-06
-0.72009E-04	0.13711E-03	-0.97875E-04
-0.12744E-04	-0.12603E-04	-0.81853E-05
0.11337E-04	0.13698E-03	-0.30779E-04
-0.12260E-04	-0.22031E-05	-0.14572E-04
-0.78884E-04	0.15069E-02	

-0.40701E-04	-0.54075E-04	0.50545E-04
0.71851E-05	-0.34013E-06	0.53893E-05
-0.40688E-04	-0.25315E-04	0.13309E-04
0.18329E-05	0.12634E-05	0.77636E-05
-0.70483E-04	-0.25232E-04	0.62732E-06
0.19072E-05	-0.52632E-05	0.72455E-05
-0.46728E-03	-0.31499E-03	0.23773E-03

0.99959E-05	-0.22188E-04	0.11390E-04
0.24092E-05	0.14471E-05	0.68231E-06
0.10006E-04	-0.79714E-05	0.61336E-05
0.65469E-06	0.19949E-05	0.14968E-05
-0.46920E-05	-0.79568E-05	0.14928E-05
0.57987E-06	0.97898E-07	0.24548E-05
-0.46834E-05	-0.76426E-04	0.19091E-04
0.15983E-04		

0.18479E-04	0.62911E-06	-0.66993E-05
-0.43410E-06	0.10771E-05	-0.10914E-05
0.18474E-04	0.17836E-05	0.25218E-06
-0.76806E-07	0.98537E-06	-0.12291E-05
0.17261E-04	0.17868E-05	0.92184E-06
-0.14110E-06	0.15970E-05	-0.44646E-06
0.71525E-04	0.62228E-06	-0.24762E-04
0.18885E-05	0.13203E-04	

0.84192E-05	-0.22318E-05	-0.14470E-05
0.95095E-06	0.22642E-05	0.21235E-05
0.84204E-05	0.16005E-04	-0.10039E-04
-0.17355E-05	0.21847E-05	0.20069E-05
-0.10423E-04	0.15995E-04	-0.45188E-05
-0.18195E-05	-0.35110E-06	0.29629E-05
-0.22751E-04	0.16518E-04	0.40953E-05
0.10351E-04	0.56388E-05	0.39553E-04

NODE V

0.19844E-02	-0.37663E-03	-0.42690E-03
0.11813E-04	0.13370E-03	-0.65522E-04
0.64309E-03	0.89565E-04	-0.92774E-05
-0.80310E-05	0.56042E-04	-0.10653E-04
0.38261E-03	0.89655E-04	0.82863E-05
-0.10920E-04	0.43200E-04	0.24263E-04
0.38281E-03	-0.16191E-03	-0.91502E-04
0.22496E-04	0.41893E-04	0.18845E-04
0.34075E-02		
-0.37708E-03	0.15215E-02	-0.39407E-03
-0.12651E-03	-0.15252E-04	0.28362E-04
-0.16255E-03	0.31026E-03	-0.21992E-03
-0.28810E-04	-0.28421E-04	-0.18442E-04
0.25357E-04	0.30974E-03	-0.70558E-04
-0.27794E-04	-0.49959E-05	-0.32840E-04
0.25135E-04	0.49134E-03	-0.12305E-03
-0.50132E-04	0.14025E-05	-0.50412E-05
-0.67808E-03	0.26102E-02	
-0.42779E-03	-0.89556E-03	0.75631E-03
0.81375E-04	-0.35811E-04	0.33398E-05
-0.11063E-03	-0.23866E-03	0.15246E-03
0.22103E-04	-0.11950E-07	0.16032E-04
-0.14694E-03	-0.23859E-03	0.43525E-04
0.22326E-04	-0.11312E-04	0.13685E-04
-0.14680E-03	-0.27367E-03	0.11467E-03
0.25814E-04	-0.15202E-04	-0.31962E-05
-0.71084E-03	-0.15213E-02	0.12709E-02
0.64302E-05	0.67589E-04	-0.48208E-04
-0.21743E-05	0.58702E-05	0.77283E-05
0.64421E-05	0.22920E-05	-0.11577E-05
-0.35763E-06	0.16825E-05	0.15524E-05
-0.74596E-05	0.23124E-05	-0.10335E-05
-0.41248E-06	-0.20878E-06	0.22992E-05
-0.74469E-05	-0.11124E-04	0.52639E-05
0.17141E-05	-0.33276E-06	0.17747E-05
0.14476E-04	0.37599E-04	-0.30880E-04
0.16097E-04		

-0.61355E-04	-0.10314E-04	0.28257E-04
0.61977E-05	0.59310E-05	0.14996E-04
0.17383E-04	0.14486E-04	-0.84373E-05
-0.16252E-05	0.33149E-05	0.23695E-05
-0.82800E-05	0.14486E-04	-0.41578E-05
-0.17558E-05	0.83719E-07	0.39291E-05
-0.82712E-05	-0.10314E-04	0.53816E-05
0.22575E-05	0.31956E-08	0.35894E-05
-0.23607E-04	-0.23322E-04	0.24100E-04
0.77285E-05	0.26939E-04	

0.65226E-04	-0.39088E-04	0.38295E-05
0.89066E-05	0.16349E-04	0.16480E-04
0.11896E-04	0.19628E-04	-0.12148E-04
-0.22347E-05	0.37014E-05	0.37444E-05
-0.20665E-04	0.19611E-04	-0.61963E-05
-0.23724E-05	-0.82255E-06	0.53175E-05
-0.20667E-04	-0.11875E-04	0.98187E-05
0.28684E-05	-0.91076E-06	0.49269E-05
0.69232E-04	-0.48543E-04	0.87370E-05
0.11110E-04	0.21391E-04	0.42935E-04

NODE VI

0.64323E-03	-0.16171E-03	-0.10972E-03
0.11806E-04	0.54963E-04	-0.12189E-04
0.19355E-02	0.13983E-03	0.64833E-04
-0.80369E-05	0.13193E-03	-0.62057E-04
0.38265E-03	0.89728E-04	0.82527E-05
-0.10929E-04	0.43204E-04	0.24260E-04
0.38285E-03	-0.16182E-03	-0.91514E-04
0.22485E-04	0.41898E-04	0.18844E-04
0.14568E-02	-0.36550E-03	-0.24977E-03
0.14464E-04	0.39387E-04	0.26562E-04
0.33368E-02		

0.90726E-04	0.31009E-03	-0.23741E-03
-0.21909E-04	0.25358E-04	0.24435E-04
0.14066E-03	0.13317E-02	-0.88410E-03
-0.11958E-03	0.14862E-04	0.24415E-05
-0.42556E-04	0.43831E-03	-0.10579E-03
-0.44499E-04	-0.64685E-06	0.15643E-04
-0.42769E-04	0.30960E-03	-0.58055E-04
-0.18076E-04	0.40708E-05	0.36061E-04
0.20313E-03	0.70263E-03	-0.53817E-03
0.49254E-05	0.32689E-04	0.44240E-04
0.27634E-03	0.22919E-02	

-0.85998E-05	-0.22130E-03	0.15224E-03
0.15689E-04	-0.12652E-04	-0.17425E-04
0.65518E-04	-0.88551E-03	0.60769E-03
0.80112E-04	0.93914E-06	-0.67295E-05
0.58722E-04	-0.28623E-03	0.72453E-04
0.28877E-04	0.38538E-05	-0.84004E-05
0.58824E-04	-0.22120E-03	0.30799E-04
0.13919E-04	0.57772E-06	-0.22654E-04
-0.20521E-04	-0.49885E-03	0.34373E-03
-0.24144E-05	-0.19004E-04	-0.27441E-04
0.87169E-04	-0.15170E-02	0.10396E-02

-0.23909E-05	-0.27432E-05	0.25026E-05
0.46689E-07	-0.66068E-06	-0.63475E-06
-0.23797E-05	0.68522E-04	-0.46792E-04
-0.48247E-05	-0.16578E-05	-0.21007E-05
0.23929E-05	-0.73380E-05	0.20305E-05
0.78819E-06	0.76133E-07	-0.70282E-06
0.24039E-05	-0.27206E-05	-0.88310E-07
-0.40457E-07	0.24532E-07	-0.91171E-06
-0.53680E-05	-0.64320E-05	0.58483E-05
-0.34605E-06	-0.98501E-06	-0.13455E-05
-0.53790E-05	0.43854E-04	-0.30514E-04
0.11970E-04		

0.17862E-04	-0.16175E-04	0.48834E-05
0.23411E-05	0.32221E-05	0.21653E-05
-0.58028E-04	0.98100E-05	-0.11568E-04
-0.22914E-05	0.67440E-05	0.15808E-04
-0.90292E-05	0.98080E-05	-0.31612E-05
-0.13199E-05	0.71760E-07	0.41994E-05
-0.90197E-05	-0.16176E-04	0.69851E-05
0.28052E-05	-0.88513E-07	0.35099E-05
0.40465E-04	-0.36491E-04	0.10934E-04
0.26503E-05	0.47716E-05	0.61654E-05
-0.20240E-04	0.22193E-04	-0.16854E-04
-0.22061E-05	0.27265E-04	

0.12577E-04	-0.20593E-04	0.94639E-05
0.32164E-05	0.35872E-05	0.34740E-05
0.63983E-04	0.19034E-04	-0.73965E-05
-0.32148E-05	0.17208E-04	0.17865E-04
-0.21797E-04	0.12664E-04	-0.47063E-05
-0.17238E-05	-0.84259E-06	0.57184E-05
-0.21798E-04	-0.20580E-04	0.12193E-04
0.36829E-05	-0.10485E-05	0.48103E-05
0.28099E-04	-0.46382E-04	0.21430E-04
0.36198E-05	0.61874E-05	0.85884E-05
0.69215E-04	0.33687E-04	-0.16746E-04
-0.31420E-05	0.22482E-04	0.44088E-04

NODE VII

0.38270E-03	0.25677E-04	-0.14560E-03
-0.14840E-04	0.13128E-04	-0.36183E-04
0.38255E-03	-0.43041E-04	0.57621E-04
0.72998E-05	0.11455E-04	-0.38735E-04
0.12446E-02	-0.28528E-04	0.10744E-03
0.62906E-05	0.10323E-03	-0.40362E-04
0.45305E-03	0.25462E-04	-0.15843E-03
-0.10547E-04	0.39168E-04	-0.23520E-04
0.87114E-03	0.56615E-04	-0.33130E-03
-0.16774E-04	-0.18307E-04	-0.46640E-04
0.87129E-03	-0.96004E-04	0.13086E-03
0.53664E-05	-0.19980E-04	-0.49192E-04
0.21810E-02		

0.90721E-04	0.31023E-03	-0.23739E-03
-0.21919E-04	0.25358E-04	0.24442E-04
0.90457E-04	0.43915E-03	-0.28486E-03
-0.43707E-04	0.14864E-04	0.88031E-05
-0.28326E-04	0.11968E-02	-0.27655E-03
-0.10901E-03	-0.65262E-06	0.10250E-04
-0.42840E-04	0.30973E-03	-0.58136E-04
-0.18090E-04	0.40674E-05	0.36071E-04
0.20303E-03	0.70313E-03	-0.53823E-03
0.49049E-05	0.32689E-04	0.44256E-04
0.20329E-03	0.99360E-03	-0.64594E-03
-0.16882E-04	0.22195E-04	0.28617E-04
-0.75443E-04	0.20972E-02	

0.93767E-05	-0.69686E-04	0.42259E-04
0.38537E-05	-0.48669E-05	-0.85717E-05
0.94653E-05	-0.10473E-03	0.71288E-04
0.10751E-04	-0.26299E-05	-0.52198E-05
0.10870E-03	-0.27522E-03	0.76420E-04
0.25517E-04	0.85984E-05	-0.57193E-05
0.45926E-04	-0.69453E-04	0.15349E-05
0.33826E-05	0.21206E-05	-0.10189E-04
0.20055E-04	-0.15962E-03	0.97785E-04
-0.22859E-05	-0.93061E-05	-0.13981E-04
0.19967E-04	-0.23907E-03	0.16272E-03
0.46107E-05	-0.70691E-05	-0.10629E-04
0.19323E-03	-0.48735E-03	0.13648E-03

-0.36872E-05	-0.22645E-05	0.25827E-05
-0.56365E-08	-0.79552E-06	-0.64793E-06
-0.36731E-05	-0.76984E-05	0.46574E-05
0.79073E-06	-0.69054E-06	-0.48510E-06
0.19360E-05	0.56825E-04	-0.12576E-04
-0.38692E-05	-0.42994E-06	-0.27347E-05
0.19490E-05	-0.22365E-05	-0.13921E-07
-0.11526E-06	-0.39771E-07	-0.99576E-06
-0.82576E-05	-0.54169E-05	0.60720E-05
-0.40090E-06	-0.11156E-05	-0.14833E-05
-0.82717E-05	-0.17664E-04	0.10773E-04
0.39545E-06	-0.10107E-05	-0.13204E-05
0.43573E-05	0.33894E-04	-0.69936E-05
0.10351E-04		

0.12106E-04	-0.57693E-05	-0.11075E-06
0.67678E-06	0.13919E-05	0.29695E-06
0.12099E-04	0.29402E-05	-0.11207E-05
-0.33045E-06	0.14728E-05	0.41298E-06
-0.61428E-04	0.29467E-05	-0.59831E-05
-0.83230E-06	-0.30934E-05	0.64348E-05
0.30727E-05	-0.57759E-05	0.45850E-06
0.90822E-06	0.52313E-06	0.97411E-06
0.27622E-04	-0.13066E-04	-0.36531E-06
0.75902E-06	0.15404E-05	0.16415E-05
0.27629E-04	0.66845E-05	-0.24988E-05
-0.24820E-06	0.16213E-05	0.17576E-05
-0.44282E-04	0.66778E-05	-0.55634E-05
-0.83845E-06	0.11878E-04	

0.28069E-04	-0.26983E-04	0.84215E-05
0.38862E-05	0.51934E-05	0.35985E-05
0.28074E-04	0.15709E-04	-0.81308E-05
-0.18941E-05	0.56459E-05	0.42697E-05
-0.13915E-05	0.21093E-04	-0.49293E-05
-0.40505E-05	0.52257E-05	0.25079E-04
-0.16042E-04	-0.26967E-04	0.11675E-04
0.46409E-05	-0.26586E-06	0.57663E-05
0.63016E-04	-0.60780E-04	0.19082E-04
0.43666E-05	0.77470E-05	0.10162E-04
0.63010E-04	0.35448E-04	-0.18417E-04
-0.14136E-05	0.81995E-05	0.10833E-04
-0.24448E-04	0.39786E-04	-0.11128E-04
-0.39590E-05	0.88349E-05	0.50871E-04

NODE VIII

0.38264E-03	0.25735E-04	-0.14565E-03
-0.14848E-04	0.13123E-04	-0.36181E-04
0.38249E-03	-0.42980E-04	0.57604E-04
0.72938E-05	0.11449E-04	-0.38734E-04
0.45280E-03	-0.42769E-04	0.44439E-04
0.62828E-05	0.38738E-04	-0.25711E-04
0.12859E-02	-0.36515E-04	-0.43159E-03
-0.10555E-04	0.10699E-03	-0.38928E-04
0.87094E-03	0.56836E-04	-0.33145E-03
-0.16787E-04	-0.18316E-04	-0.46638E-04
0.87108E-03	-0.95795E-04	0.13076E-03
0.53554E-05	-0.19990E-04	-0.49192E-04
0.10291E-02	-0.96007E-04	0.10160E-03
0.43444E-05	0.72988E-05	-0.36168E-04
0.22405E-02		

-0.16224E-03	0.49213E-03	-0.27224E-03
-0.47785E-04	-0.15254E-04	0.11562E-05
-0.16253E-03	0.31040E-03	-0.21991E-03
-0.28822E-04	-0.28422E-04	-0.18448E-04
0.25423E-04	0.30988E-03	-0.70651E-04
-0.27811E-04	-0.49908E-05	-0.32849E-04
-0.36831E-04	0.13779E-02	-0.30267E-03
-0.11798E-03	0.14091E-05	0.18387E-04
-0.36568E-03	0.11129E-02	-0.61739E-03
-0.25419E-04	-0.23323E-04	-0.26790E-04
-0.36538E-03	0.70311E-03	-0.49894E-03
-0.64547E-05	-0.36490E-04	-0.46395E-04
0.56829E-04	0.70363E-03	-0.15985E-03
-0.54448E-05	-0.13060E-04	-0.60795E-04
-0.33114E-04	0.24029E-02	

-0.92913E-04	-0.12208E-03	0.11357E-03
0.16192E-04	-0.85561E-06	0.12182E-04
-0.92842E-04	-0.57184E-04	0.30127E-04
0.41547E-05	0.27404E-05	0.17530E-04
-0.15997E-03	-0.56985E-04	0.19086E-05
0.43288E-05	-0.11983E-04	0.16341E-04
-0.43283E-03	-0.30140E-03	0.22221E-03
0.30671E-04	-0.37104E-04	0.92473E-05
-0.20988E-03	-0.27795E-03	0.25833E-03
0.11902E-04	0.11988E-04	0.22161E-04
-0.20995E-03	-0.13163E-03	0.69913E-04
-0.13551E-06	0.15584E-04	0.27508E-04
-0.36069E-03	-0.13182E-03	0.44610E-05
0.38703E-07	0.86101E-06	0.26319E-04
-0.75779E-03	-0.53916E-03	0.39269E-03

0.11176E-04	-0.12176E-04	0.41125E-05
0.16275E-05	0.19613E-05	0.12675E-05
0.11192E-04	0.40173E-05	-0.19626E-05
-0.53136E-06	0.21781E-05	0.15963E-05
-0.55516E-05	0.40422E-05	-0.12471E-05
-0.60868E-06	0.54082E-07	0.26313E-05
-0.55380E-05	0.55683E-04	-0.10565E-04
-0.57685E-06	0.16431E-05	0.96487E-05
0.25158E-04	-0.27754E-04	0.95604E-05
0.17256E-05	0.28977E-05	0.37576E-05
0.25143E-04	0.87588E-05	-0.41845E-05
-0.43320E-06	0.31145E-05	0.40863E-05
-0.12480E-04	0.87338E-05	-0.27570E-05
-0.51052E-06	0.99046E-06	0.51214E-05
-0.12494E-04	0.26472E-04	-0.13539E-05
0.15386E-04		

0.11535E-04	-0.29422E-05	-0.18322E-05
0.40765E-06	0.13534E-05	0.32320E-06
0.11529E-04	0.50251E-05	-0.25663E-05
-0.52721E-06	0.13546E-05	0.32149E-06
0.32615E-05	0.50292E-05	-0.10578E-05
-0.58728E-06	0.56517E-06	0.10577E-05
-0.64556E-04	-0.29503E-05	0.23137E-04
0.23521E-05	-0.34363E-05	0.59604E-05
0.26316E-04	-0.66677E-05	-0.42552E-05
0.63504E-06	0.14599E-05	0.15533E-05
0.26322E-04	0.11402E-04	-0.57750E-05
-0.29980E-06	0.14611E-05	0.15516E-05
0.77324E-05	0.11398E-04	-0.23187E-05
-0.35989E-06	0.67164E-06	0.22878E-05
-0.46535E-04	-0.66594E-05	0.17918E-04
0.29262E-05	0.12236E-04	

0.25659E-04	-0.14634E-04	0.92776E-06
0.27145E-05	0.50325E-05	0.37113E-05
0.25663E-04	0.24784E-04	-0.14465E-04
-0.27502E-05	0.51353E-05	0.38650E-05
-0.15069E-04	0.24764E-04	-0.69161E-05
-0.29588E-05	-0.56026E-07	0.62697E-05
0.33795E-06	-0.38055E-04	0.85247E-05
0.11011E-04	0.48158E-05	0.22601E-04
0.57598E-04	-0.32981E-04	0.22012E-05
0.38422E-05	0.74072E-05	0.97710E-05
0.57595E-04	0.55866E-04	-0.32671E-04
-0.16225E-05	0.75100E-05	0.99248E-05
-0.33977E-04	0.55885E-04	-0.15598E-04
-0.18311E-05	0.23187E-05	0.12329E-04
-0.21650E-04	-0.51738E-04	0.19226E-04
0.13665E-04	0.81941E-05	0.48416E-04

NODE IX

0.27011E-03	-0.33431E-04	-0.63653E-04
-0.78179E-06	0.18004E-04	-0.12693E-04
0.26985E-03	0.10571E-04	0.92165E-05
-0.25379E-06	0.17904E-04	-0.12986E-04
0.22302E-03	0.10911E-04	0.99338E-05
-0.11516E-05	0.21925E-04	-0.73427E-06
0.22339E-03	-0.33784E-04	-0.62159E-04
0.29472E-05	0.21607E-04	-0.15144E-05
0.62278E-03	-0.77664E-04	-0.14851E-03
-0.61983E-06	0.64962E-05	-0.57223E-05
0.62304E-03	0.25883E-04	0.22203E-04
-0.91769E-07	0.63964E-05	-0.60156E-05
0.51768E-03	0.25541E-04	0.25023E-04
-0.98965E-06	0.10417E-04	0.62364E-05
0.51730E-03	-0.77311E-04	-0.14647E-03
0.31091E-05	0.10099E-04	0.54563E-05
0.35000E-03		

-0.17444E-04	0.20813E-03	-0.12651E-03
-0.18032E-04	0.24679E-05	0.64631E-05
-0.17963E-04	0.19528E-03	-0.12528E-03
-0.18769E-04	-0.33354E-05	-0.23929E-05
-0.43969E-05	0.19416E-03	-0.49555E-04
-0.19032E-04	-0.14465E-05	-0.42667E-05
-0.48305E-05	0.20710E-03	-0.50621E-04
-0.17942E-04	0.14690E-05	0.78487E-05
-0.40762E-04	0.47965E-03	-0.29393E-03
-0.64174E-05	0.22928E-05	0.44744E-05
-0.40245E-04	0.45042E-03	-0.29147E-03
-0.71541E-05	-0.35105E-05	-0.43816E-05
-0.10390E-04	0.45153E-03	-0.11360E-03
-0.74174E-05	-0.16216E-05	-0.62555E-05
-0.99575E-05	0.48068E-03	-0.11624E-03
-0.63272E-05	0.12940E-05	0.58599E-05
-0.13434E-04	0.28865E-03	

-0.27082E-04	-0.88599E-04	0.66972E-04
0.82385E-05	-0.34810E-05	-0.12554E-05
-0.26954E-04	-0.88552E-04	0.56808E-04
0.84997E-05	-0.80424E-06	0.27678E-05
-0.26953E-04	-0.88247E-04	0.22193E-04
0.86772E-05	-0.21789E-05	0.20152E-05
-0.26684E-04	-0.88245E-04	0.32185E-04
0.76537E-05	-0.34955E-05	-0.33348E-05
-0.62342E-04	-0.20402E-03	0.15493E-03
0.29311E-05	-0.18866E-05	-0.12687E-05
-0.62469E-04	-0.20435E-03	0.13135E-03
0.31921E-05	0.79006E-06	0.27545E-05
-0.62180E-04	-0.20465E-03	0.50089E-04
0.33698E-05	-0.58456E-06	0.20019E-05
-0.62448E-04	-0.20437E-03	0.73842E-04
0.23463E-05	-0.19013E-05	-0.33481E-05
-0.38324E-04	-0.12383E-03	0.64025E-04

0.35411E-05	0.30037E-04	-0.21833E-04
-0.23451E-05	0.15425E-05	0.17556E-05
0.35567E-05	0.35966E-04	-0.24289E-04
-0.35582E-05	0.54953E-06	0.29841E-06
-0.25787E-05	0.35997E-04	-0.82195E-05
-0.35657E-05	-0.13145E-06	0.52962E-06
-0.25638E-05	0.30070E-04	-0.57634E-05
-0.20854E-05	0.30400E-06	0.24713E-05
0.79886E-05	0.67132E-04	-0.48761E-04
0.34740E-07	0.19206E-05	0.26674E-05
0.79727E-05	0.80504E-04	-0.54311E-04
-0.11782E-05	0.92770E-06	0.12102E-05
-0.58001E-05	0.80472E-04	-0.18419E-04
-0.11858E-05	0.24672E-06	0.14414E-05
-0.58152E-05	0.67099E-04	-0.12870E-04
0.29448E-06	0.68218E-06	0.33831E-05
0.48593E-06	0.34843E-04	-0.15922E-04
0.71396E-05		

-0.20831E-04	-0.10714E-04	0.14601E-04
0.25253E-05	0.82905E-06	0.42437E-05
-0.20837E-04	0.97244E-05	-0.84554E-05
-0.13512E-05	0.11077E-05	0.46519E-05
-0.41999E-04	0.97270E-05	-0.59390E-05
-0.13385E-05	-0.30956E-05	0.45124E-05
-0.41988E-04	-0.10713E-04	0.17165E-04
0.24310E-05	-0.32217E-05	0.39756E-05
-0.46732E-04	-0.24211E-04	0.32839E-04
0.29034E-05	0.43701E-05	0.73922E-05
-0.46726E-04	0.21994E-04	-0.19058E-04
-0.97300E-06	0.46488E-05	0.78004E-05
-0.94306E-04	0.21991E-04	-0.13318E-04
-0.96036E-06	0.44553E-06	0.76609E-05
-0.94317E-04	-0.24206E-04	0.38522E-04
0.28091E-05	0.31944E-06	0.71241E-05
-0.34524E-04	-0.52527E-06	0.47829E-05
0.11345E-05	0.10623E-04	

0.51720E-04	-0.37206E-04	0.71244E-05
0.52905E-05	0.83051E-05	0.47633E-05
0.51728E-04	0.26335E-04	-0.13277E-04
-0.30441E-05	0.88518E-05	0.55744E-05
-0.13937E-04	0.26309E-04	-0.71921E-05
-0.34178E-05	0.88521E-06	0.99204E-05
-0.13937E-04	-0.37185E-04	0.13288E-04
0.65584E-05	0.54180E-06	0.84103E-05
0.11626E-03	-0.83819E-04	0.16192E-04
0.62023E-05	0.11454E-04	0.14532E-04
0.11625E-03	0.59415E-04	-0.30050E-04
-0.21323E-05	0.12000E-04	0.15343E-04
-0.31371E-04	0.59440E-04	-0.16227E-04
-0.25060E-05	0.40337E-05	0.19689E-04
-0.31372E-04	-0.83840E-04	0.29936E-04
0.74702E-05	0.36903E-05	0.18179E-04
0.20912E-04	-0.59663E-05	-0.39836E-07
0.27355E-05	0.94455E-05	0.29307E-04

APPENDIX III

EXPERIMENTAL FLEXIBILITY MATRIX

EXPERIMENTAL FLEXIBILITY MATRIX

NODE I

0.21250E-02		
-0.34450E-03	0.18090E-02	
-0.46320E-03	-0.96800E-03	0.82630E-03
0.60920E-05	-0.81220E-04	0.50870E-04
0.15830E-04		
0.90310E-04	-0.56710E-05	-0.23560E-04
0.54310E-05	0.24710E-04	
-0.44240E-04	0.21230E-04	0.14680E-05
0.80800E-05	0.15730E-04	0.35150E-04

NODE II

0.26410E-03	-0.61290E-04	-0.47990E-04
0.54790E-05	0.22910E-04	-0.45760E-05
0.23630E-02		
0.35810E-04	0.13910E-03	-0.95110E-04
-0.10270E-04	0.11020E-04	0.10860E-04
0.17520E-03	0.14720E-02	
-0.	-0.85800E-04	0.60930E-04
0.75670E-05	-0.57510E-05	-0.70410E-05
0.16340E-03	-0.97010E-03	0.70010E-03
-0.38600E-05	-0.11820E-04	0.83390E-05
-0.	-0.	-0.12560E-05
-0.33580E-05	-0.84000E-04	0.50510E-04
0.14550E-04		
0.23020E-04	-0.11920E-04	-0.
-0.	0.29930E-05	-0.
0.82100E-04	0.59520E-05	-0.
-0.23690E-05	0.33100E-04	
-0.43700E-05	-0.86630E-05	0.72370E-05
0.15040E-05	-0.	0.17650E-05
-0.42280E-04	-0.	-0.24320E-05
-0.	0.15800E-04	0.35390E-04

NODE III

0.15980E-03	0.10470E-04	-0.60520E-04
-0.58880E-05	0.49970E-05	-0.14760E-04
0.15570E-03	-0.17230E-04	0.22930E-04
-0.	0.41930E-05	-0.15490E-04
0.12880E-02		

0.34800E-04	0.12950E-03	-0.10090E-03
-0.98620E-05	0.10450E-04	0.10290E-04
0.39620E-04	0.17800E-03	-0.11630E-03
-0.20110E-04	0.68610E-05	0.37870E-05
-0.	0.12030E-02	

-0.	-0.28130E-04	0.17090E-04
-0.	-0.	-0.35120E-05
-0.	-0.41580E-04	0.29070E-04
0.40010E-05	-0.	-0.
0.10110E-03	-0.28800E-03	0.80440E-04

-0.42390E-05	-0.11230E-04	0.81410E-05
-0.	-0.	-0.
-0.49810E-05	-0.18280E-04	0.11610E-04
-0.	-0.	-0.
0.22210E-05	-0.67920E-04	0.15480E-04
0.10490E-04		

0.17840E-04	-0.	-0.47240E-05
-0.	-0.	-0.
0.18080E-04	-0.	-0.
-0.	-0.	-0.
0.65620E-04	-0.	0.40110E-05
-0.	0.12900E-04	

0.10100E-04	-0.14220E-04	0.58760E-05
-0.	-0.	-0.
0.10350E-04	0.59600E-05	-0.39620E-05
-0.	-0.	-0.
-0.22040E-04	-0.	-0.
-0.30010E-05	0.89140E-05	0.50190E-04

NODE IV

0.15940E-03	0.97500E-05	-0.59900E-04
-0.55690E-05	0.49810E-05	-0.15140E-04
0.16120E-03	-0.19650E-04	0.24180E-04
-0.	-0.	-0.15210E-04
0.18520E-03	-0.17430E-04	0.28910E-04
-0.	0.15610E-04	-0.11920E-04
0.13920E-02		

-0.63840E-04	0.20150E-03	-0.12900E-03
-0.20700E-04	-0.59200E-05	-0.
-0.65910E-04	0.13200E-03	-0.20820E-04
-0.12010E-04	-0.12500E-04	-0.86180E-05
0.11750E-04	0.12200E-03	-0.30310E-04
-0.12270E-04	-0.24630E-05	-0.14310E-04
-0.72510E-04	0.14140E-02	

-0.41920E-04	-0.47910E-04	0.49910E-04
0.57030E-05	-0.	0.49280E-05
-0.37720E-04	-0.24120E-04	0.11810E-04
-0.	-0.	0.60130E-05
-0.70110E-04	-0.25010E-04	-0.
-0.	-0.43900E-05	0.64540E-05
-0.43730E-03	-0.29120E-03	0.22270E-03

0.87650E-05	-0.20880E-04	0.10920E-04
-0.	-0.	-0.
0.91280E-05	-0.78950E-05	0.54090E-05
-0.	-0.	-0.
-0.43140E-05	-0.69240E-05	-0.
-0.	-0.	-0.
-0.40590E-05	-0.77390E-04	0.17700E-04
0.15020E-04		

0.16820E-04	-0.	-0.59890E-05
-0.	-0.	-0.
0.16760E-04	-0.	-0.
-0.	-0.	-0.
0.16510E-04	-0.	-0.
-0.	-0.	-0.
0.68150E-04	-0.	-0.22350E-04
-0.	0.12130E-04	

0.71680E-05	-0.	-0.
-0.	-0.	-0.
0.76930E-05	0.14830E-04	-0.91250E-05
-0.	-0.	-0.
-0.10090E-04	0.15210E-04	-0.40400E-05
-0.	-0.	0.26980E-05
-0.21540E-04	0.18280E-04	0.36390E-05
0.91860E-05	0.53100E-05	0.37840E-04

NODE V

0.19300E-02	-0.34250E-03	-0.40010E-03
0.10740E-04	0.12210E-03	-0.68440E-04
0.61110E-03	0.89200E-04	-0.12970E-04
-0.63930E-05	0.49750E-04	-0.98920E-05
0.35890E-03	0.85670E-04	0.76910E-05
-0.99410E-05	0.38910E-04	0.22310E-04
0.36130E-03	-0.14820E-03	-0.85130E-04
0.21150E-04	0.43830E-04	0.17110E-04
0.33590E-02		

-0.35930E-03	0.15650E-02	-0.86210E-03
-0.13510E-03	-0.13460E-04	0.26940E-04
-0.14500E-03	0.34410E-03	-0.21020E-03
-0.24210E-04	-0.25310E-04	-0.17320E-04
0.22540E-04	0.31920E-03	-0.67930E-04
-0.23290E-04	-0.	-0.30020E-04
0.26510E-04	0.43920E-03	-0.11590E-03
-0.47900E-04	-0.	-0.
-0.63200E-03	0.25900E-02	

-0.39210E-03	-0.94610E-03	0.70720E-03
0.76780E-04	-0.31320E-04	-0.
-0.10320E-03	-0.24180E-03	0.14110E-03
0.20720E-04	-0.	0.15770E-04
-0.13610E-03	-0.22220E-03	0.39910E-04
0.20190E-04	-0.10130E-04	0.12090E-04
-0.14420E-03	-0.25160E-03	0.10390E-03
0.22900E-04	-0.12250E-04	-0.
-0.67910E-03	-0.14530E-02	0.12510E-02

0.54910E-05	0.62290E-04	-0.41820E-04
-0.	0.53900E-05	0.72100E-05
0.70610E-05	-0.	-0.
-0.	-0.	-0.
-0.67420E-05	-0.	-0.
-0.	-0.	-0.
-0.66750E-05	-0.12090E-04	0.48050E-05
-0.	-0.	-0.
0.13510E-04	0.34210E-04	-0.32100E-04
0.14510E-04		

-0.55310E-04	-0.10480E-04	0.25080E-04
0.73550E-05	0.60900E-05	0.14340E-04
0.18510E-04	0.13760E-04	-0.74050E-05
-0.	-0.	-0.
-0.72690E-05	0.13810E-04	-0.37420E-05
-0.	-0.	0.42130E-05
-0.71150E-05	-0.95880E-05	0.49700E-05
-0.	-0.	0.35080E-05
-0.24410E-04	-0.21750E-04	0.22730E-04
0.74590E-05	0.25310E-04	

0.59210E-04	-0.35960E-04	0.33040E-05
0.80820E-05	0.15770E-04	0.15800E-04
0.11110E-04	0.19230E-04	-0.11790E-04
-0.	0.31140E-05	0.33890E-05
-0.19090E-04	0.17730E-04	-0.52410E-05
-0.	-0.	0.46110E-05
-0.19290E-04	-0.11620E-04	0.86380E-05
0.24910E-05	-0.	0.45190E-05
0.65430E-04	-0.45520E-04	0.99890E-05
0.10430E-04	0.20030E-04	0.40110E-04

NODE VI

0.61320E-03	-0.15520E-03	-0.99780E-04
0.11600E-04	0.61170E-04	-0.11090E-04
0.21060E-02	0.15110E-03	0.70220E-04
-0.85940E-05	0.12420E-03	-0.57030E-04
0.35450E-03	0.80920E-04	0.10420E-04
-0.11190E-04	0.38560E-04	0.26000E-04
0.37170E-03	-0.16520E-03	-0.85540E-04
0.19950E-04	0.39840E-04	0.18110E-04
0.14350E-02	-0.35170E-03	-0.24610E-03
0.13160E-04	0.35670E-04	0.24970E-04
0.33000E-02		

0.85260E-04	0.29610E-03	-0.23320E-03
-0.23610E-04	0.22340E-04	0.23760E-04
0.16320E-03	0.12920E-02	-0.84510E-03
-0.10730E-03	0.12500E-04	-0.
-0.38990E-04	0.41330E-03	-0.99630E-04
-0.39890E-04	-0.	0.12920E-04
-0.37920E-04	0.30570E-03	-0.61040E-04
-0.16520E-04	-0.	0.30850E-04
0.19340E-03	0.68110E-03	-0.50020E-03
-0.	0.30120E-04	0.41830E-04
0.26140E-03	0.22010E-02	

-0.77880E-05	-0.21950E-03	0.15660E-03
0.14420E-04	-0.10020E-04	-0.15680E-04
0.79220E-04	-0.22910E-03	0.35950E-03
0.75470E-04	-0.	-0.75110E-05
0.53690E-04	-0.28990E-03	0.67590E-04
0.26410E-04	-0.	-0.78350E-05
0.53020E-04	-0.21420E-03	0.33520E-04
0.11420E-04	-0.	-0.22010E-04
-0.18120E-04	-0.45160E-03	0.33350E-03
-0.	-0.17830E-04	-0.25310E-04
0.79130E-04	-0.14730E-02	-0.

-0.26410E-05	-0.24120E-05	-0.
-0.	-0.	-0.
-0.27010E-05	0.63750E-04	-0.43620E-04
-0.59270E-05	-0.	-0.24340E-05
-0.	-0.78640E-05	-0.
-0.	-0.	-0.
-0.	-0.26950E-05	-0.
-0.	-0.	-0.
-0.55550E-05	-0.58310E-05	0.59930E-05
-0.	-0.	-0.
-0.59030E-05	0.41120E-04	-0.28230E-04
0.11120E-04		

0.15490E-04	-0.16870E-04	0.44800E-05
-0.	0.29800E-05	-0.
-0.56630E-04	0.80970E-05	-0.10780E-04
-0.	0.56650E-05	0.14950E-04
-0.75750E-05	0.10690E-04	0.33210E-05
-0.	-0.	0.38790E-05
-0.80320E-05	-0.15770E-04	0.62820E-05
0.25120E-05	-0.	0.33210E-05
0.37250E-04	-0.33450E-04	0.10130E-04
-0.	0.42750E-05	0.60320E-05
-0.18730E-04	0.21150E-04	-0.15720E-04
-0.	0.24910E-04	

0.16710E-04	-0.21250E-04	0.87350E-05
0.29520E-05	0.35250E-05	0.36400E-05
0.62090E-04	0.19190E-04	-0.64280E-05
-0.28950E-05	0.16670E-04	0.17170E-04
-0.20510E-04	0.11340E-04	-0.43170E-05
-0.	-0.	0.51320E-05
-0.20030E-04	-0.20330E-04	0.11630E-04
0.34950E-05	-0.	0.43640E-05
0.25360E-04	-0.43540E-04	0.20120E-04
0.33240E-05	0.58030E-05	0.47050E-05
0.65210E-04	0.31820E-04	-0.15590E-04
-0.33450E-05	0.21220E-04	0.41110E-04

NODE VII

0.35160E-03	0.22820E-04	-0.12350E-03
-0.11820E-04	0.11530E-04	-0.32170E-04
0.30100E-03	-0.38920E-04	0.61520E-04
0.82180E-05	0.99920E-05	-0.35220E-04
0.11480E-02	-0.25730E-04	0.98450E-04
0.56190E-05	0.97620E-04	-0.42070E-04
0.40930E-03	0.25000E-04	-0.14630E-03
-0.12030E-04	0.35720E-04	-0.22130E-04
0.81110E-03	0.53370E-04	-0.31080E-03
-0.15010E-04	-0.16130E-04	-0.43350E-04
0.79980E-03	-0.91120E-04	0.12720E-03
-0.	-0.22130E-04	-0.49950E-04
0.21030E-02		

0.81320E-04	0.29050E-03	-0.23150E-03
-0.20110E-04	0.23120E-04	0.24930E-04
0.85440E-04	0.41830E-03	-0.26670E-03
-0.42630E-04	0.13450E-04	0.76030E-05
-0.31450E-04	0.11130E-02	-0.25670E-03
-0.10610E-03	-0.	0.11930E-04
-0.38220E-04	0.28170E-03	-0.51190E-04
-0.15530E-04	-0.	0.37210E-04
0.20950E-03	0.70530E-03	-0.53020E-03
-0.	0.33240E-04	0.42060E-04
0.19430E-03	0.94720E-03	-0.61370E-03
-0.17950E-04	0.23650E-04	0.23880E-04
-0.62020E-04	0.19630E-02	

0.78930E-05	-0.59900E-04	0.39750E-04
-0.	-0.	-0.98730E-05
0.85120E-05	-0.99210E-04	0.67590E-04
0.11370E-04	-0.	-0.
0.10390E-03	-0.27030E-03	0.75910E-04
0.23920E-04	0.90120E-05	-0.
0.43250E-04	-0.63500E-04	-0.
-0.	-0.	-0.10930E-04
0.18130E-04	-0.14950E-03	0.13030E-03
-0.	-0.91120E-05	-0.12190E-04
0.22590E-04	-0.22530E-03	0.15130E-03
-0.	-0.61450E-05	-0.99220E-05
0.18690E-03	-0.47560E-03	0.12740E-03

-0.42150E-05	-0.	-0.
-0.	-0.	-0.
-0.42150E-05	-0.85710E-05	0.42510E-05
-0.	-0.	-0.
-0.	0.53740E-04	-0.11620E-04
-0.30110E-05	-0.	-0.25180E-05
-0.	-0.20620E-05	-0.
-0.	-0.	-0.
-0.79610E-05	-0.56990E-05	0.54970E-05
-0.	-0.	-0.
-0.76740E-05	-0.16450E-04	0.99730E-05
-0.	-0.	-0.
0.40320E-05	0.32170E-04	-0.73920E-05
0.10150E-04		
0.11520E-04	-0.59680E-05	-0.
-0.	-0.	-0.
0.11380E-04	0.25510E-05	-0.
-0.	-0.	-0.
-0.57750E-04	0.33980E-05	-0.62500E-05
-0.	-0.29700E-05	0.64200E-05
0.31770E-05	-0.54420E-05	-0.
-0.	-0.	-0.
0.25270E-04	-0.12290E-04	-0.
-0.	-0.	-0.
0.27350E-04	0.60320E-05	-0.
-0.	-0.	-0.
-0.47030E-04	0.63950E-05	-0.
-0.	0.11310E-04	
0.26000E-04	-0.26090E-04	0.86100E-05
0.32270E-05	0.50100E-05	0.33940E-05
0.25890E-04	0.14240E-04	-0.74520E-05
-0.	0.50400E-05	0.39720E-05
-0.	0.19700E-04	-0.43740E-05
-0.35190E-05	0.43970E-05	0.23070E-04
-0.15110E-04	-0.25990E-04	0.10680E-04
0.39920E-05	-0.	0.53570E-05
0.62610E-04	-0.58390E-04	0.18230E-04
0.45370E-05	0.71490E-05	0.97310E-05
0.58830E-04	0.32620E-04	-0.18010E-04
-0.	0.88520E-05	0.10120E-04
-0.23210E-04	0.35650E-04	-0.10690E-04
-0.43050E-05	0.87400E-05	0.48870E-04

MODE VIII

0.36010E-03	0.22120E-04	-0.12750E-03
-0.13820E-04	0.11790E-04	-0.34820E-04
0.35540E-03	-0.40130E-04	0.49790E-04
0.58870E-05	0.10320E-04	-0.36750E-04
0.43290E-03	-0.37920E-04	0.42050E-04
0.52970E-05	0.35090E-04	-0.24780E-04
0.12010E-02	-0.31790E-04	-0.40490E-03
-0.12030E-04	0.98130E-04	-0.31150E-04
0.80150E-03	0.52030E-04	-0.30520E-03
-0.12950E-04	-0.16350E-04	-0.48390E-04
0.83290E-03	-0.87900E-04	0.12090E-03
-0.	-0.22080E-04	-0.43600E-04
0.95240E-03	-0.94210E-04	0.23590E-04
-0.	0.61350E-05	-0.38920E-04
0.21170E-02		

-0.15130E-03	0.45750E-03	-0.24740E-03
-0.45930E-04	-0.14010E-04	-0.
-0.14920E-03	0.29140E-03	-0.20620E-03
-0.26010E-04	-0.24370E-04	-0.19840E-04
0.22990E-04	0.28120E-03	-0.65150E-04
-0.27000E-04	-0.	-0.29890E-04
-0.32510E-04	0.13090E-02	-0.28210E-03
-0.11930E-03	-0.	0.19540E-04
-0.34580E-03	0.10560E-02	-0.57110E-03
-0.23130E-04	-0.20100E-04	-0.25730E-04
-0.39890E-03	0.66670E-03	-0.46620E-03
-0.10450E-04	-0.36800E-04	-0.44300E-04
0.48300E-04	0.68100E-03	-0.15130E-03
-0.40010E-05	-0.12210E-04	-0.55490E-04
-0.30140E-04	0.22740E-02	

-0.84510E-04	-0.11250E-03	0.10400E-03
-0.	-0.	0.11290E-04
-0.88140E-04	-0.54120E-04	0.27100E-04
-0.	-0.	0.15820E-04
-0.15510E-03	-0.58900E-04	-0.
-0.	-0.11330E-04	0.14540E-04
-0.39650E-03	-0.28120E-03	0.20490E-03
0.28120E-04	-0.34620E-04	0.82030E-05
-0.18150E-03	-0.25210E-03	0.22110E-03
0.10950E-04	0.10820E-04	0.20320E-04
-0.19820E-03	-0.13820E-03	0.60240E-04
-0.	0.14120E-04	0.28200E-04
-0.31010E-03	-0.13620E-03	-0.
-0.	-0.	0.24730E-04
-0.73290E-03	-0.50120E-03	0.36030E-03

0.10410E-04	-0.11420E-04	0.34980E-05
-0.	0.16900E-05	0.11340E-05
0.10680E-04	0.34420E-05	-0.
-0.	-0.	-0.
-0.57750E-05	0.35180E-05	-0.
-0.	-0.	-0.
-0.50010E-05	0.56220E-04	-0.96250E-05
-0.	-0.	0.92710E-05
0.24350E-04	-0.25970E-04	0.98930E-05
-0.	0.25010E-05	0.39450E-05
0.25320E-04	0.80140E-05	-0.37520E-05
-0.	0.26600E-05	0.36020E-05
-0.11630E-04	0.79250E-05	-0.
-0.	-0.	0.45920E-05
-0.11920E-04	0.25170E-04	-0.
0.13990E-04		

0.10640E-04	-0.33030E-05	-0.15520E-05
-0.	-0.	-0.
0.10510E-04	0.43520E-05	-0.
-0.	-0.	-0.
0.37190E-05	0.43810E-05	-0.
-0.	-0.	-0.
-0.61620E-04	-0.24620E-05	0.21750E-04
-0.	-0.31780E-05	0.57630E-05
0.24440E-04	-0.67950E-05	-0.38920E-05
-0.	-0.	-0.
0.25210E-04	0.10650E-04	-0.51090E-05
-0.	-0.	-0.
0.69140E-05	0.10830E-04	-0.
-0.	-0.	-0.
-0.43750E-04	-0.59830E-05	0.16350E-04
0.34130E-05	0.11560E-04	

0.24130E-04	-0.14270E-04	-0.
0.28940E-05	0.48300E-05	0.34000E-05
0.22890E-04	0.23060E-04	-0.13310E-04
-0.24110E-05	0.48900E-05	0.35800E-05
-0.14470E-04	0.21940E-04	-0.57920E-05
-0.24290E-05	-0.	0.55550E-05
-0.	-0.35380E-04	0.76110E-05
0.10230E-04	0.43820E-05	0.21810E-04
0.53870E-04	-0.32010E-04	-0.
0.34450E-05	0.67120E-05	0.89750E-05
0.53920E-04	0.52130E-04	-0.30090E-04
-0.	0.81220E-05	0.88520E-05
-0.31170E-04	0.51470E-04	-0.14120E-04
-0.	-0.	0.11320E-04
-0.20320E-04	-0.48220E-04	0.17270E-04
0.12350E-04	0.77210E-05	0.46190E-04

NODE IX

0.24130E-03	-0.33010E-04	-0.57500E-04
-0.	0.15830E-04	-0.11250E-04
0.25110E-03	0.94320E-05	0.83570E-05
-0.	0.15900E-04	-0.11920E-04
0.21330E-03	0.96820E-05	0.91340E-05
-0.	0.20350E-04	-0.
0.20590E-03	-0.31740E-04	-0.59130E-04
-0.	0.20350E-04	-0.
0.59120E-03	-0.72250E-04	-0.20080E-03
-0.	0.58110E-05	-0.59520E-05
0.60350E-03	0.24130E-04	0.20080E-04
-0.	0.59210E-05	-0.63290E-05
0.52560E-03	0.22940E-04	0.22130E-04
-0.	0.98630E-05	0.55830E-05
0.50020E-03	-0.69850E-04	-0.13730E-03
-0.	0.10350E-04	0.48320E-05
0.31930E-03		

-0.15930E-04	0.19560E-03	-0.12610E-03
-0.16110E-04	-0.	0.63380E-05
-0.19210E-04	0.20310E-03	-0.11820E-03
-0.15990E-04	-0.	-0.
-0.	0.18350E-03	-0.52520E-04
-0.16890E-04	-0.	-0.
-0.	0.19670E-03	-0.46110E-04
-0.17520E-04	-0.	0.71390E-05
-0.36800E-04	0.45360E-03	-0.27030E-03
-0.71310E-05	-0.	-0.
-0.42210E-04	0.42130E-03	-0.27120E-03
-0.65330E-05	-0.	-0.
-0.90320E-05	0.41820E-03	-0.10830E-03
-0.68890E-05	-0.	-0.53350E-05
-0.85450E-05	0.44220E-03	-0.10790E-03
-0.57780E-05	-0.	-0.
-0.13190E-04	0.27030E-03	

-0.25930E-04	-0.81150E-04	0.59620E-04
0.71320E-05	-0.	-0.
-0.26320E-04	-0.79320E-04	0.56110E-04
0.76390E-05	-0.	-0.
-0.24170E-04	-0.85720E-04	0.20120E-04
0.74220E-05	-0.	-0.
-0.23920E-04	-0.84390E-04	0.29170E-04
0.70220E-05	-0.	-0.
-0.53330E-04	-0.18830E-03	0.14480E-03
-0.	-0.	-0.
-0.56480E-04	-0.19130E-03	0.12340E-03
-0.	-0.	-0.
-0.55690E-04	-0.19020E-03	0.45430E-04
-0.	-0.	-0.
-0.54920E-04	-0.19500E-03	0.66210E-04
-0.	-0.	-0.
-0.31320E-04	-0.11680E-03	0.56970E-04

0.30210E-05	0.30170E-04	-0.20320E-04
-0.	-0.	-0.
0.34150E-05	0.33980E-04	-0.22820E-04
-0.28950E-05	-0.	-0.
-0.	0.31800E-04	-0.75610E-05
-0.39120E-05	-0.	-0.
-0.	0.31010E-04	-0.55270E-05
-0.	-0.	-0.
0.62620E-05	0.64320E-04	-0.45130E-04
-0.	-0.	0.20170E-05
0.71130E-05	0.75370E-04	-0.51190E-04
-0.	-0.	-0.
-0.48820E-05	0.78200E-04	-0.16240E-04
-0.	-0.	-0.
-0.50780E-05	0.65520E-04	-0.11910E-04
-0.	-0.	0.29960E-05
-0.	0.32160E-04	-0.14520E-04
0.67130E-05		

-0.20640E-04	-0.99450E-05	0.13920E-04
-0.	-0.	0.41600E-05
-0.22990E-04	0.87820E-05	-0.75980E-05
-0.	-0.	0.45410E-05
-0.38960E-04	0.98150E-05	-0.57910E-05
-0.	-0.26250E-05	0.43220E-05
-0.34920E-04	-0.11200E-04	0.15450E-04
-0.	-0.33450E-05	0.36850E-05
-0.44480E-04	-0.22270E-04	0.31120E-04
-0.	0.39880E-05	0.73200E-05
-0.45720E-04	0.21130E-04	-0.21510E-04
-0.	0.46800E-05	0.74700E-05
-0.85320E-04	0.19930E-04	-0.13000E-04
-0.	-0.	0.73100E-05
-0.91130E-04	-0.22850E-04	0.37520E-04
0.31180E-05	-0.	0.74930E-05
-0.30820E-04	-0.	0.43200E-05
-0.	0.10850E-04	

0.48710E-04	-0.36310E-04	0.66510E-05
0.44920E-05	0.91220E-05	0.45000E-05
0.43500E-04	0.25030E-04	-0.13490E-04
-0.25090E-05	0.91430E-05	0.51500E-05
-0.12930E-04	0.23540E-04	-0.65660E-05
-0.30130E-05	-0.	0.91920E-05
-0.12780E-04	-0.35120E-04	0.12540E-04
0.58530E-05	-0.	0.79850E-05
0.11210E-03	-0.78130E-04	0.15190E-04
0.57420E-05	0.19830E-04	0.13250E-04
0.10790E-03	0.55200E-04	-0.29250E-04
-0.	0.10910E-04	0.15830E-04
-0.29130E-04	0.57130E-04	-0.14430E-04
-0.	0.33910E-05	0.18130E-04
-0.29430E-04	-0.80020E-04	0.27250E-04
0.67050E-05	0.38110E-05	0.16150E-04
0.19530E-04	-0.42050E-05	-0.
-0.	0.90050E-05	0.26900E-04

APPENDIX IV

PERCENTAGE ERROR MATRIX

PERCENTAGE ERROR IN EXPERIMENTAL FLEXIBILITY MATRIX

NODE I

-4.93					
-10.33	5.43				
-9.14	-5.56	-5.56			
-16.04	5.49	7.63	6.15		
3.42	-15.90	-7.88	-13.06	-1.97	
-7.86	-4.71	0.91	-4.47	-2.65	-3.71

NODE II

-7.15	-14.87	-1.62	4.23	-5.83	-14.42
9.18					
-10.56	1.34	-9.93	5.91	-2.00	0.10
55.15	-1.43				
	-12.33	-10.06	9.06	2.35	-8.83
57.28	-2.76	1.61			
8.08	-7.34	-14.74			2.95
-5.82	4.92	-6.16	7.39		
-7.20	-5.43			1.61	
-3.99	-10.91		14.73	30.59	
-6.29	5.76	2.08	3.52		-6.43
-7.65		-18.14		-6.69	-4.65

NODE III

-5.19	-8.18	-6.53	-10.79	-12.64	-7.87
-7.60	-9.41	-11.92		-15.64	-9.68
-4.69					

-13.09	-5.55	-4.46	1.81	-7.06	-5.09
-0.87	-8.33	-8.14	3.99	4.50	-2.75
	-7.24				

	-8.88	-9.25			-7.55
	-10.36	-8.60	-16.13		
-9.39	-2.17	-0.65			

-12.91	-8.53	-17.44			
2.52	-7.15	-8.97			
-20.55	-4.68	-6.37	-12.11		

-6.35		-5.10			
-5.07					
-4.42		-15.88		1.09	

-6.73	-2.48	-3.15			
-4.45	-14.00	6.93			
-4.65			-9.07	48.91	22.41

NODE IV

-5.46	-14.07	-7.41	-15.52	-13.00	-5.50
-4.37	2.98	-6.77			-11.32
-7.30	-8.31	43.28		-8.55	4.60
-1.37					

-11.23	-7.45	6.59	-2.12	-12.18	
-8.47	-3.73	-7.21	-5.76	-0.32	5.29
3.64	-10.94	-1.52	0.08	11.80	-1.80
-8.08	-6.16				

3.00	-11.40	-1.26	-20.63		-8.56
-7.29	-4.72	-11.26			-22.55
-0.53	-0.88			-16.59	-10.92
-6.42	-7.55	-6.32			

-12.31	-5.90	-4.13			
-8.77	-0.96	-11.81			
-8.06	-12.98				
-13.33	1.26	-7.29	-6.03		

-8.98		-10.60			
-9.28					
-4.35					
-4.72		-9.74		-8.13	

-14.86					
-8.64	-7.34	-9.10			
-3.19	-4.91	-10.60			-8.94
-5.32	10.67	-11.14	-11.25	-5.83	-4.33

NODE V

-2.74	-9.06	-6.28	-9.08	-8.68	4.45
-4.97	-0.41	39.80	-20.40	-11.23	-7.14
-6.20	-4.44	-7.18	-8.97	-9.93	-8.05
-5.62	-8.47	-6.96	-5.98	4.62	-9.21
-1.42					

-4.72	2.86	-3.58	6.79	-11.75	-5.01
-10.80	10.91	-4.42	-15.97	-10.95	-6.08
-11.11	3.05	-3.72	-16.20		-8.59
5.47	-10.61	-5.81	-4.45		
-6.80	-0.77				

-8.34	5.64	-6.49	-5.65	-12.54	
-6.72	1.32	-7.45	-6.26		-1.63
-7.38	-6.87	-8.31	-9.57	-10.45	-11.66
-1.77	-8.06	-9.39	-11.29	-19.42	
-4.47	-4.49	-2.26			

-14.61	-7.84	-13.25		-8.18	-6.71
9.61					
-9.62					
-10.37	8.68	-8.72			
-6.67	-9.01	3.95	-9.86		

-9.85	1.61	-11.24	18.67	2.68	-4.37
-6.48	5.01	12.23	100.00	100.00	100.00
-12.21	-4.67	-10.00			7.23
-13.98	-7.04	-7.65			-2.27
3.40	-6.74	-5.68	-3.49	-6.05	

-9.22	-8.00	-13.72	- 9.26	-3.54	-4.13
-6.61	-2.03	-2.95		-15.87	-9.49
-7.62	-9.59	-15.42			-13.29
-6.66	-2.15	-12.03	-13.16		-8.28
-5.49	-6.23	14.33	-6.12	-6.36	-6.58

NODE VI

-4.67	-4.03	-9.06	-1.74	11.29	-9.02
8.81	8.06	8.31	6.93	-5.86	-8.10
-7.36	-9.82	26.26	2.39	-10.75	7.17
-2.91	2.09	-6.53	-11.27	-4.91	-3.90
-1.50	-3.78	-1.47	-9.02	-9.44	-5.99
-1.10					

-6.02	-4.51	-1.77	7.76	-11.90	-2.76
16.02	-2.98	-4.41	-10.27	-15.89	
-8.38	-5.71	-5.82	-10.36		-17.41
-11.34	-1.26	5.14	-8.61		-14.45
-4.79	-3.06	-7.06		-7.86	-5.45
-5.41	-3.97				

-9.44	-0.81	2.86	-8.09	-20.80	-10.01
20.91	4.92	-40.84	-5.79		11.61
-8.57	1.28	-6.71	-8.54		-6.73
-9.87	-3.16	8.83	-17.95		-2.84
-11.70	-9.47	-2.98		-6.18	-7.77
-9.22	-2.90				

10.46	-12.07				
13.50	-6.96	-6.78	22.85		15.87
	7.17				
	-0.94				
3.48	-9.34	2.47			
9.74	-6.23	-7.49	-7.10		

-13.28	4.30	-8.26		-7.51	
-2.41	-17.46	-6.81		-16.00	-5.43
-16.11	8.99	-205.06			-7.63
-10.95	-2.51	-10.07	-10.45		-5.38
-7.95	-8.33	-7.35		-10.41	-2.16
-7.46	-4.70	-6.73		-8.64	

32.86	3.19	-7.70	-8.22	-1.73	4.78
-2.96	0.82	-13.09	-9.95	-3.13	-3.89
-5.90	-10.45	-8.27			-10.25
-8.11	-1.21	-4.62	-5.10		-9.28
-9.75	-6.13	-6.11	-8.17	-6.21	-45.22
-5.79	-5.54	-6.90	6.46	-5.61	-6.75

NODE VII

-8.13	-11.13	-15.18	-20.35	-12.17	-11.09
-21.32	-9.57	6.77	12.58	-12.77	-9.07
-7.76	-9.81	-8.37	-10.68	-5.43	4.23
-9.66	-1.81	-7.66	14.06	-8.80	-5.91
-6.89	-5.73	-6.19	-10.52	-11.89	-7.05
-8.21	-5.09	-2.80		10.76	1.54
-3.58					

-10.36	-6.36	-2.48	-8.25	-8.83	2.00
-5.55	-4.75	-6.38	-2.46	-9.51	-13.63
11.03	-7.00	-7.18	-2.67		16.39
-10.78	-9.05	-11.95	-14.15		3.16
3.19	0.31	-1.49		1.69	-4.96
-4.42	-4.67	-4.99	6.33	6.56	-16.55
-17.79	-6.40				

-15.82	-14.04	-5.94			15.18
-10.07	-5.27	-5.19	5.76		
-4.42	-1.79	-0.67	-6.26	4.81	
-5.83	-8.57				7.27
-9.60	-6.34	33.25		-2.09	-12.81
13.14	-5.76	-7.02		-13.07	-6.65
-3.28	-2.41	-6.65			

14.51					
14.75	8.74	-8.73			
	-5.43	-7.60	-22.18		-7.92
	-7.80				
-3.59	5.21	-9.47			
-7.23	-6.87	-7.43			
-7.47	-5.09	5.70	-1.94		

-4.84	3.44				
-5.94	-13.24				
-5.99	15.32	4.46		-3.99	-0.23
3.39	-5.78				
-8.51	-5.94				
-1.01	-9.76				
6.21	-4.23			-4.78	

-7.37	-3.31	2.24	-16.96	-3.53	-5.68
-7.78	-9.35	-8.35		-10.73	-6.97
	-6.60	-11.27	-13.12	-15.86	-8.01
-5.81	-3.62	-8.52	-13.98		-7.10
-0.64	-3.93	-4.46	3.90	-7.72	-4.24
-6.63	-7.98	-2.21		7.96	-6.58
-5.06	-10.40	-3.94	8.74	-1.07	-3.93

NODE VIII

-5.89	-14.05	-12.46	-6.92	-10.16	-3.76
-7.08	-6.63	-13.57	-19.29	-9.86	-5.12
-4.39	-11.34	-5.38	-15.69	-9.42	-3.62
-6.60	-12.94	-6.18	13.97	-8.28	-19.98
-7.97	-8.46	-7.92	-22.86	-10.73	3.76
-4.38	-8.24	-7.54		10.46	-11.37
-7.45	-1.87	-7.88		-15.95	7.61
-5.51					

-6.74	-7.04	-9.12	-3.88	-8.16	
-8.20	-6.12	-6.23	-9.76	-14.26	7.55
-9.57	-9.26	-7.79	-2.92		-9.01
-11.73	-5.00	-6.47	1.12		6.27
-5.44	-5.11	-7.50	-9.01	-13.82	-3.96
9.17	-5.18	-6.16	61.90	0.85	-4.82
-15.01	-3.22	-5.35	-26.52	-6.51	-8.73
-8.98	-5.36				

-9.04	-7.85	-8.43			-7.32
-5.06	-5.36	-10.05			-9.75
-3.04	3.36			-5.45	-11.02
-8.39	-6.70	-7.79	-8.32	-6.69	-11.29
-13.52	-9.30	-14.41	-8.00	-9.74	-8.31
-5.60	4.99	-13.84		-9.39	2.52
-14.03	3.32				-6.04
-3.28	-7.04	-8.25			

-6.85	-6.21	-14.94		-12.83	-10.53
-4.57	-14.32				
4.02	-12.97				
-9.70	0.96	-8.90			-3.91
-3.21	-6.43	3.48		-13.69	4.99
0.70	-8.50	-10.34		-14.59	-11.85
- 6.81	- 9.26				- 10.34
-4.59	-4.92		-9.07		

-7.76	12.26	-15.29			
-8.84	-13.39				
14.03	-12.89				
-4.55	-16.55	-5.99		-7.52	-3.31
-7.13	1.91	-8.54			
-4.22	-6.60	-11.53			
-10.58	-4.98				
-5.98	-10.16	-8.75	16.64	-5.52	

-5.96	-2.49		6.61	-4.02	-8.39
-10.81	-6.96	-7.98	-12.33	-4.78	-7.37
-3.98	-11.40	-16.25	-17.91		-11.40
	-7.03	-10.72	-7.09	-9.01	-3.50
-6.47	-2.94		-10.34	-9.39	-8.15
-6.38	-6.69	-7.90		8.15	-10.81
-8.26	-7.90	-9.48			-8.18
-6.14	-6.80	-10.17	-9.62	-5.77	-4.60

NODE IX

-10.67	-1.26	-9.67		-12.08	-11.37
-6.95	-10.77	-9.33		-11.19	-8.21
-4.36	-11.26	-8.05		-7.18	
-7.83	-6.05	-4.87		-5.82	
-5.07	-6.97	35.21		-10.55	4.01
-3.14	-6.77	-9.56		-7.43	5.21
1.53	-10.18	-11.56		-5.32	-10.48
-3.31	-9.65	-6.26		2.49	-11.44
-8.77					

-8.68	-6.02	-0.32	-10.66		-1.94
6.94	4.00	-5.57	-14.81		
	-5.49	5.98	-11.25		
	-5.02	-8.91	-2.35		-9.04
-9.72	-5.43	-8.04	11.12		
4.88	-6.47	-6.95	-8.68		
-13.07	-7.38	-4.67	-7.12		-14.72
-14.19	-8.01	-7.17	-8.68		
-1.82	-6.36				

-4.25	-8.41	-10.98	-13.43		
-2.35	-10.43	-1.23	-10.13		
-10.33	-2.86	-9.34	-14.47		
-10.36	-4.37	-9.37	-8.25		
-14.46	-7.71	-6.54			
-9.59	-6.39	-6.05			
-10.44	-7.06	-9.30			
-12.05	-4.58	-10.34			
-18.28	-5.68	-11.02			

-14.69	0.44	-6.88			
-3.98	-5.52	-6.05	-18.64		
	-11.66	-8.01	9.71		
	3.13	-3.93			
-21.61	-4.19	-7.45			5.61
-10.78	-6.38	-5.75			
-15.83	-2.71	-8.03			
-12.68	-2.35	-7.46			-11.44
	-7.70	-8.81	-5.98		

-0.92	-7.18	-5.35			-1.97
10.33	-9.69	-10.14			-2.38
-7.24	-9.38	-2.49		-15.20	-4.22
-16.83	4.50	-9.99		3.83	-7.31
-4.82	-8.02	-5.23		-8.74	-0.98
-2.15	-3.93	12.87		0.67	-4.24
-9.53	-9.37	-2.39			-4.58
-3.38	-5.60	-2.63	11.00		5.18
-10.73		-9.49		2.14	

-5.82	-2.41	-6.64	-15.09	-2.20	-5.53
-15.91	-4.96	-5.93	-17.58	3.29	-7.45
-7.23	-10.52	-8.71	-11.84		-7.34
-8.30	-5.55	-5.63	-10.76		-6.25
-3.58	-6.79	-6.19	-7.42	-5.45	-8.82
-7.18	-7.09	-2.66		-9.08	3.17
-7.14	-3.89	-11.07		-15.93	-7.92
-6.19	-4.56	-8.97	-10.24	3.27	-11.16
-6.61	-29.52			-4.66	-7.91