PRECISE MATHEMATICAL MODELS FOR ELECTROMAGNETIC AND PERMANENTIC MOTORS .

by

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C O N D E N S E

La présente étude traite de l'établissement d'un modèle mathématique, comprenant toutes les non-linéarités,d'un moteur à courant continu.

Il a été nécessaire de rechercher pour cela de nouvelles techniques de mesures afin d'obtenir les différents paramètres avec un maximum de précision. L'originalité de ces mesures tient dans le fait que la machine est étudiée dans des conditions dynamiques d'expérience.

A l'aide de ce modèle non-linéaire, il a été possible alors de montrer un éventail de comparaisons entre un moteur électromagnetique et un moteur à aimants permanents, dans le domaine des systèmes asservis.

Cette étude démontre l'efficacité de l'outil que fournit ce modèle.

R E M E R C I E M E N T S

Ce travail a été effectué au Laboratoire d'Electrotechnique de l'Université Mc MASTER à Hamilton, grâce à l'agrément du Directeur de ce département, le Docteur C.CAMPBELL.

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A B R E V I A T I O N S U S E D

ie	excitation current (field current)
E	no-load voltage
Ø	excitation field under one pole
Ω	speed in rd/s , N speed in rpm
K	voltage constant
U	output voltage (or input) under load
ε	armature reaction
I	dc load current (armature current)
a,b	parameters of losses
λm	motor torque
λr or λω	reactive torque
J	moment of inertia
i	armature current function of time(transient)
L	inductance at the output of the machine
PM.	permanent magnet
EM.	electromagnet

T A B L E   O F   C O N T E N T S -

ILLUSTRATIONS

GRAPHS:

INTRODUCTION

If a motor is used as an element of a system, it is in common practice to assume a simple linear model of it, that is to say taking the parameters as constants. But the study of the performances shows that this assumption is not sufficient and leads to the need for a more accurate model.

It becomes necessary to be able to determine an exact mathematical model in order to predict system performances.

The purpose of this work is to determine accurately the models of both the direct current electromagnetic motor and the permanentic motor, so that a comparison can be made of each type of machine as a control systems element.

CHAPTER 1

STUDY OF THE PERFORMANCES OF A D.C. MACHINE:

Given characteristics: machine A.S.E.A. n° 4 925 546

1.1 kW
220 v
6.3 A
2o50 rpm

In order to approach the performances of a D.C. machine, one must analyse the magnetic characteristics which are given by the no-load voltage curves of the machine running as a generator.

Then the voltage drops should be determined to obtain the load characteristics of the machine.

The mechanical parameters and the inductance of the machine are required for the purpose of transient response studies.

In the following experiments, the usual measurement techniques are mentioned, and if their results are not sufficient, which is almost every time the case, a search for a better approach is developped.

This will lead to the determination of the parameters of the machine, including the non-linearities which have to be taken into consideration.

I) STUDY OF THE NO-LOAD VOLTAGE:

1) DETERMINATION OF THE NO-LOAD VOLTAGE CURVES:

The no-load voltage $E(i_e)$ is measured directly at different speeds, while the machine is running as a generator, with separate excitation. But taking care of the hysteresis of the machine, the measurements are performed with raising and decreasing excitation currents. For further studies the curves for negative values of i_e have even been plotted.

The results are summarized in the curves of figure 1.

To be able to utilize those curves mathematically, it is tried to approach their equations by a polynomial. A computer program using a least mean square criterion, fits a polynomial equation to the data that have been fed in (see program P1).

Different degrees of polynomials have been tried, and by comparing the calculated values to the measured data, it has been chosen, as best approach, a degree of eight. The error resulting from this approximation does not exceed 0.1% of the original data. This is largely sufficient and less than measurements errors.

The equations to be utilized further on at 2050 rpm are the following:

increasing excitation at 2050 rpm: equ. 1

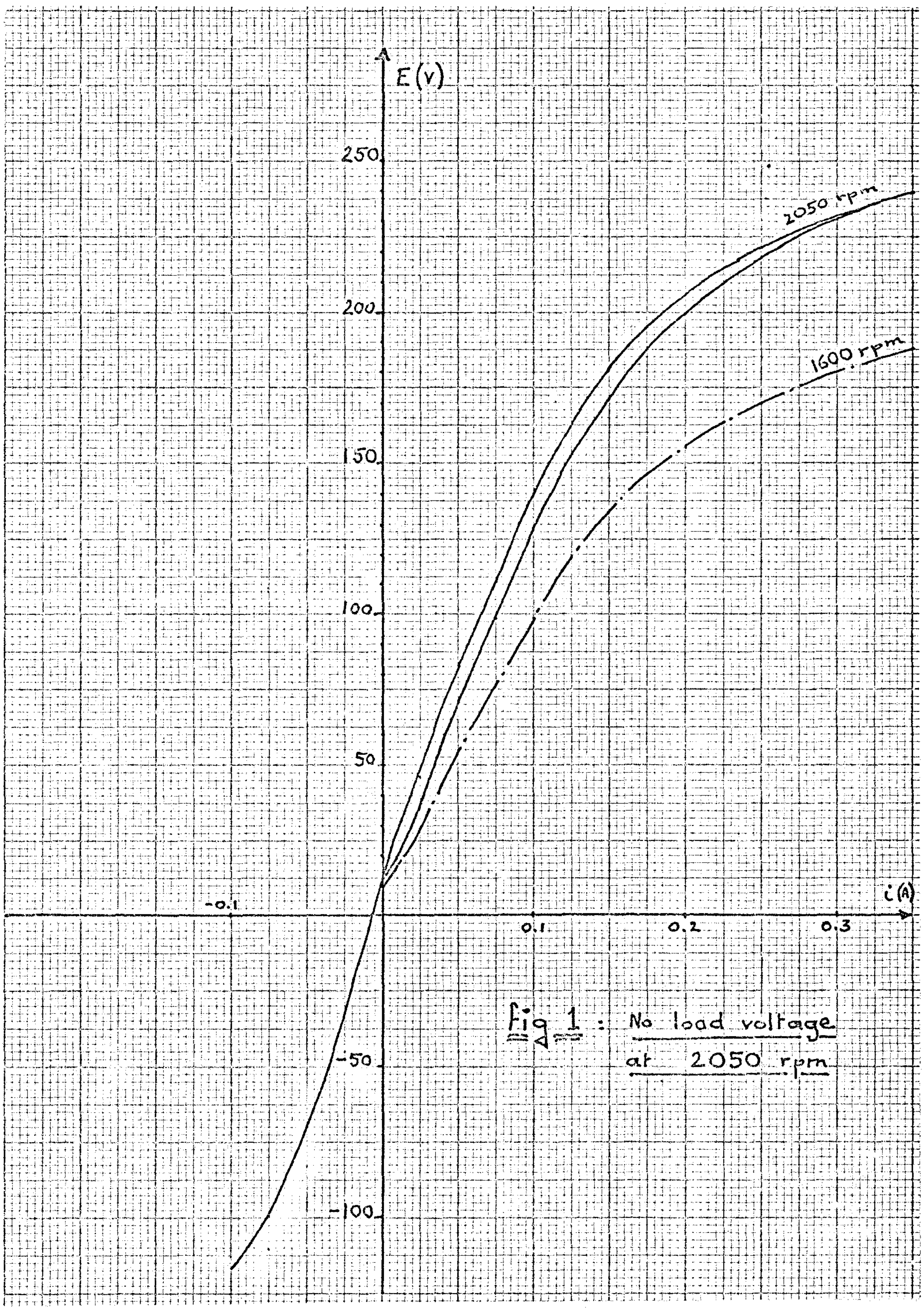


Fig 1 : No load voltage at 2050 rpm

P1

```
$IBFTC SURFIT
C     THIS SUBROUTINE CALLS DLESQ TO FIT TO A POLYNOMIAL OF DEGREE M=M1-1
C     A SET N OF DATAS.IT IS NECESSARY TO DECLARE IN DOUBLE PRECISION X(N)
C     Y(N),B(M1),A(M2) WHERE M2=(M+2)**2/2
C     THIS SUBROUTINE GIVES RESULTS IN THOSE FORMS
C   1 WRITES COEFFICIENTS B(I)
C   2 WRITES VALUES OF E(C) FROM CI-INITIAL VALUE IN STEPS OF VC,TO A MAX CM
C   3 PUNCHES ON CARDS B(I)
C   4 RETURNS
C     SEQUENCES.J1 LEADS TO THE FIRST PROCESS
C      P1 ENDS WITH J2
C      P2 ENDS WITH J3
C      P3 ENDS WITH J4
C
      SUBROUTINE FIT(X,Y,B,A,N,M1,M2,CI,VC,CM,J1,J2,J3,J4)
      DOUBLE PRECISION X(N),Y(N),B(M1),A(M2)
      DIMENSION Z(10)
C     WRITE TITLE
      DO 666 MMM=1,2
      READ(5,16)  (Z(I),I=1,10)
666   WRITE(6,17) (Z(I),I=1,10)
16    FORMAT(10A6)
17    FORMAT(20X,10A6)
C
      M=M1-1
      DO 100 I=1,N
100   READ(5,9)X(I),Y(I)
      CALL DLESQ(A,B,X,Y,M,N)
      GO TO(1,2,3,4),J1
C  WRITING B(I)
1     WRITE(6,11)
      DO 111 L=1,M1
      L1=L-1
111   WRITE(6,12)L1,B(L)
      WRITE(6,10)
      GO TO(1,2,3,4),J2
2     WRITE(6,14)
      C=CI
70    E=B(1)
      DO 112 I=2,M1
      L=I-1
112   E=E+B(I)*(C**L)
      WRITE(6,15)C,E
      C=C+VC
      IF(C.GT.CM) WRITE(6,10)
      IF(C.GT.CM) GO TO(1,2,3,4),J3
      GO TO 70
3     WRITE(7) B
      GO TO(1,2,3,4),J4
4         RETURN
9     FORMAT(2(F20.8))
10     FORMAT(1H1)
11     FORMAT(10X,35HEQUATION BY LEAST MEAN SQUARE ERROR/1H-,
     1  1X,16HCOEFFICIENTS OF,)
12     FORMAT(1H0,20X,3HX**,I2,4X,1PE10.3)
14     FORMAT(     10X,25HSAMPLING THE FITTED CURVE/1H0,
     1   10X,1HC,12X,5HV1(M)/1H0)
15     FORMAT(4X,F10.3,5X,F10.3)
         END
```

$$E1(i_e)=12.97+570\, i_e+25310\, i_e^2-4.27\times10^5\, i_e^3+3.89\times10^6\, i_e^4-2.15\times10^7\, i_e^5$$
$$+6.99\times10^7\, i_e^6-1.21\times10^8\, i_e^7+8.62\times10^7\, i_e^8$$

decreasing excitation at 2050 rpm (equ.2)

$$E2(ie)=13+1410\, ie+183.3\, i_e^2-1.49\times10^4\, i_e^3-2.53\times10^4\, ie^4+1.9\times10^5\, ie^5$$
$$+5.5\times10^5\, ie^6-3.24\times10^6\, ie^7+3.57\times10^6\, ie^8$$

2) DETERMINATION OF THE VOLTAGE CONSTANT:

If Nc is the total number of conductors

p is the number of pairs of poles

2a is the number of parallel paths

Ø is the utilizable flux under one pole

n is the rotation per second of the rotor

And ifthe poles of same polarity are assumed to be identical, each parallel path has Nc/4a sections in series and the e.m.f. induced in a section in between two neutral lines is:

(3) $$e_{average}=\frac{\Delta\emptyset}{\Delta t}$$ with $\Delta\emptyset=2\emptyset$ and $\Delta t=1/2pn$

the total e.m.f. will be

(4) $$E_{av}=\frac{Nc.2\emptyset.2pn}{4a}$$

If the speed is called Ω (in rd/s), (4) leads to

(5) $$E=\frac{p\ \Omega\ Nc\ \emptyset}{2\pi a}$$

The quantity $pNc/2\pi a=A$ is a constant of the machine hence (5) becomes :

(6) $E = A\,\Omega\,\phi$

If the excitation is maintained at a constant, the flux ϕ, which depends only upon this excitation and the construction of the machine, will remain constant and one can write :

(7) $E = K\,\Omega$ where $K = A\,\phi$

This very important constant of the machine can be measured utilizing curves of figure 1.

For an excitation of 0.255 A, corresponding to no-load voltage of 220 V(increasing excitation)

$$K = 1.02 \text{ V/rd/s}$$

II) VOLTAGE DROPS:

1) CLASSICAL METHODS:

The fundamental equation of a d.c. generator is :

(8) $U = E(ie) - RI - \varepsilon(I)$

where
- $E(ie)$ is the no-load voltage with excitation ie
- R is the total resistance at the output
- ε is the armature reaction
- I is the load current
- U is the output voltage under a load I

a) In a first step the voltage U(I) is measured directly and the previous knowledge of E(ie) gives the quantity E-U.

b)Then a load current is passed through the armature, rotor blocked, and voltage and current are registered. This should give a linear resistance. It can be done better if the rotor is rotated by hand, but the readings are very inaccurate because of the great dispertion due to the commutation.

The ideal would be to desaturate the machine completely, which is of course impossible in practice most of the time.

Hence the armature reaction is obtained out of 3 measurements. The practical results are very poor. At the best one can expect E with 0.5 V accuracy(with normal excitation and speed that gives an error $\Delta E/E = 1/400$ and likewise for U;hence $\Delta U/U = 1/400$). But E-U is very small(varying from zero to 20 V) and the error becomes:

$$\frac{\Delta(E-U)}{E-U} = \frac{\Delta E + \Delta U}{E - U} = \frac{1}{E-U}$$

and it will lie in a range of 5% to 100%, which is very poor, especially in the beginning of the curve where E-U is very small. If one now adds the errors due to R, it is found that the armature reaction really cannot be measured with those techniques, and in fact, cannot be used in further studies.

New techniques of measurements are needed to be investigated. In the following methods the errors on ε will be minimized.

2) DIRECT MEASUREMENT OF σ=E-U :

A very constant excitation is needed (with batteries for d.c.

machines).Hence this applies well to a permanent magnet machine.

Figure (2a) shows the device.The voltage U to be measured is applied to a potential divideor composed of r,fixed value,and X varying resistor,and Uo is a potential reference.At the point of equilibrium found by the galvanometer one can obviously write:

$$U = \frac{r+X}{r} \times Uo \qquad (9)$$

Now a measurement performed for I=O similarly gives:

$$E = \frac{r+Xo}{r} \times Uo \qquad (10)$$

and the quantity required becomes from (9) and (10):

$$\sigma = E-U = (Xo-X)\frac{Uo}{r} \qquad (11)$$

Uo and r are fixed values, therefore can be chosen with very good accuracy The readings of Xo and X are performed with a minimum error if one has matched r and X correctly, that is to say ,if X can be used in its total range of decades.

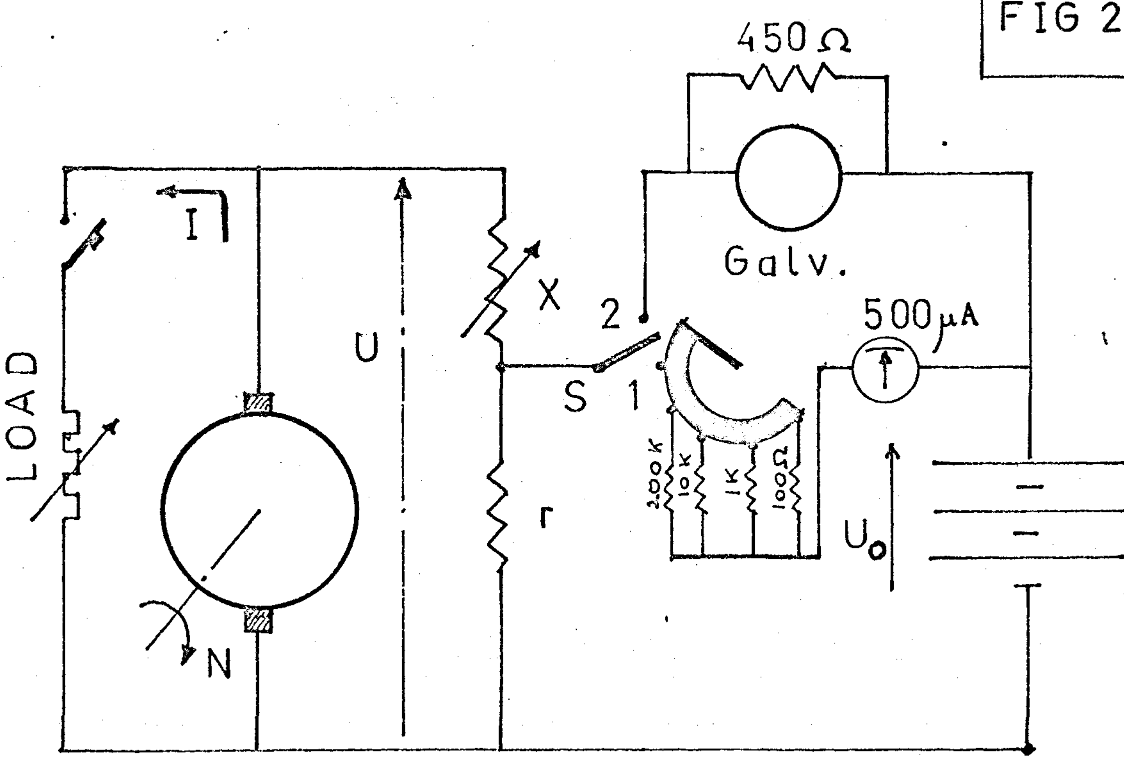
For the study of the E.M. machine the following values have been set up: Ωo=2050 rpm ie =0.255 A

Uo = 25 V r = 10 k

The following measurements read and compiled using (11) give:

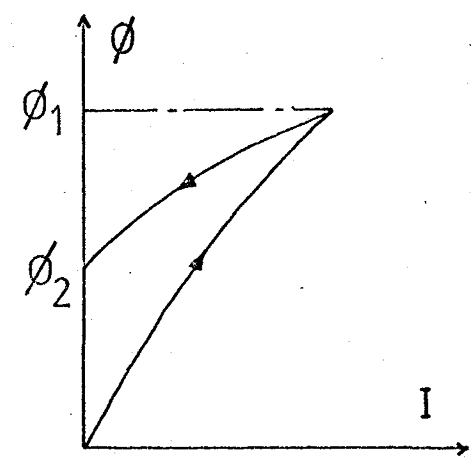
I(A)	0	0.214	0.420	0.640	0.850	1.05	1.26	1.47	1.67	1.98
X(KΩ)	78	77.6	77.3	77.2	77.0	76.8	76.7	76.5	76.4	76.3
Xo-X(KΩ)	0	.4	.7	.8	1.	1.2	1.3	1.5	1.6	1.7
σ(I) V	0	1.	1.75	2.	2.5	3.	3.25	3.75	4.	4.25

FIG 2

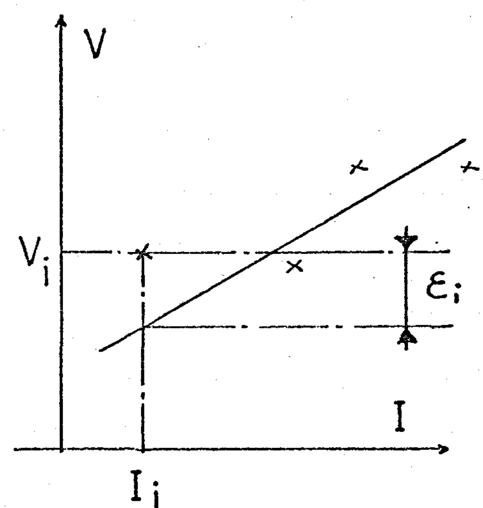


a) Measuring $\delta(I) = E(i_e) - U(I)$

Note: To find the equilibrium point the switch S1 has to be closed and short-circuiting the series resistors we approximate the point with a micro-amperemeter.Then switching to S2 the exact zero is found by the galvanometer.



b) REMANENCE



c)FIT A SET (Vi,Ii) TO A STRAIGHT LINE

I	2.45	3.05	3.5	3.9	4.52	5.	5.5	6.	6.5
X	75.9	75.2	74.5	74.05	73.1	72.4	71.5	70.5	69.
Xo-X	2.1	2.8	3.5	3.95	4.9	5.6	6.5	7.5	9.
σ(I)	5.25	7.	8.75	9.88	12.25	14.	16.25	18.75	22.5

The X resistors are used on the entire scale and one can encompass the point of equilibrium with the last decade.That gives an error on E-U of :

(12) $$\frac{\Delta(E-U)}{E-U} = \frac{\Delta(Xo-X)}{Xo-X} + \frac{\Delta Uo}{Uo} + \frac{\Delta r}{r}$$

The two last parts can be performed with:

$$\frac{\Delta Uo}{Uo} = \frac{1}{1000} \quad \text{and} \quad \frac{\Delta r}{r} = \frac{1}{500}$$

For Δ(Xo-X) / (Xo-X) a simple calculation would not give a real result because the sense of the error is known. Therefore Xo can be considered almost as a reference.The error on the first measurement hence is:

$$\frac{\Delta(Xo-X)}{Xo-X} = \frac{5}{380} = 1.3\%$$

The total error given by 12 will be for the first point 1.6% This is the worst because Xo-X is the smallest.For the last point it steps down to 0.5%.This range of 1.6% to 0.5% is far better than the results of the classical method by direct measurement.

Likewise we have performed a set of curves σ(I) with various excitations,from ie=0.255 A to ie=0 in steps of 0.025 A.The curves are plotted on figure 3.

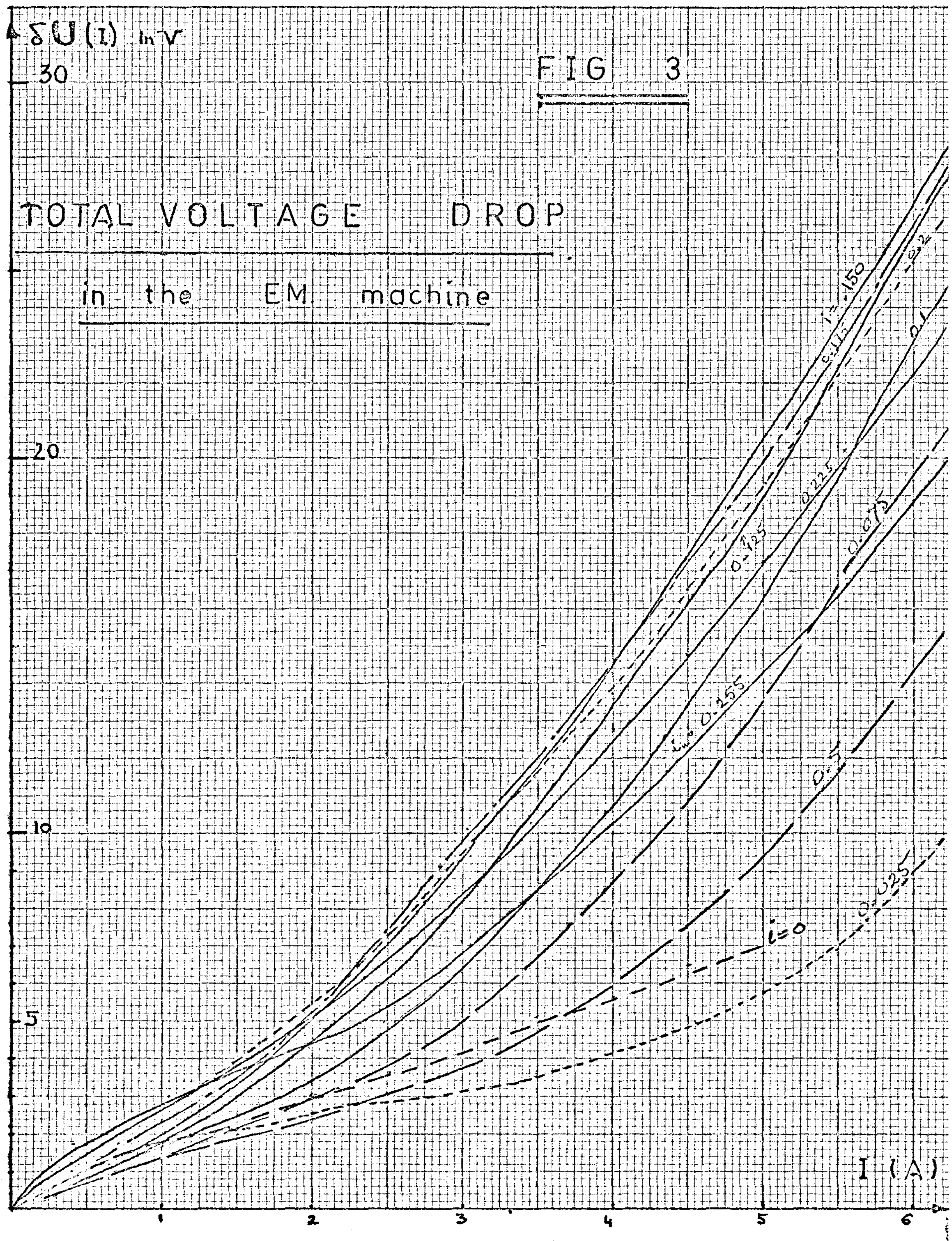


FIG 3

TOTAL VOLTAGE DROP

in the EM machine

3) DETERMINATION OF THE BRUSH VOLTAGE DROP:

This is basicly a dynamic mode, this means that factors such as speed and temperature must be considered.

Running the machine as a generator, a battery is inserted in series with the armature and in opposition with the voltage delivered by the generator.

First of all the poles are demagnetized by changing the sense of the excitation while decreasing the current. Then applying the battery voltage the current and voltage are read.

But this current will generate an armature reaction $\varepsilon(I)$, which will magnetize the poles. Hence if the battery is disconnected a remaining voltage can be read, which must be subtracted from the previous one. This subtraction will give the curve $\Delta U(I)$ at zero excitation and almost no remanence.

If this curve is recorded on figure 4 it can be seen that beginning from a certain amount of current, the curve, instead of remaining asymptotic, is beginning to increase more. This is caused by the armature reaction. As shown on figure 2b, $\varepsilon(I)$ will generate a flux $\phi 1$ and when the battery is disconnected, a remanent flux $\phi 2$ appears. The assumption has been made that $\phi 2$ is equal to $\phi 1$, which is fairly good if $\varepsilon(I)$ is low, but is not valid if I grows. It can be seen that for 2/3 of I nominal ε is small, and then increases.

The curve is taken up to the inflexion point and this data is fitted to the theoretical equation:

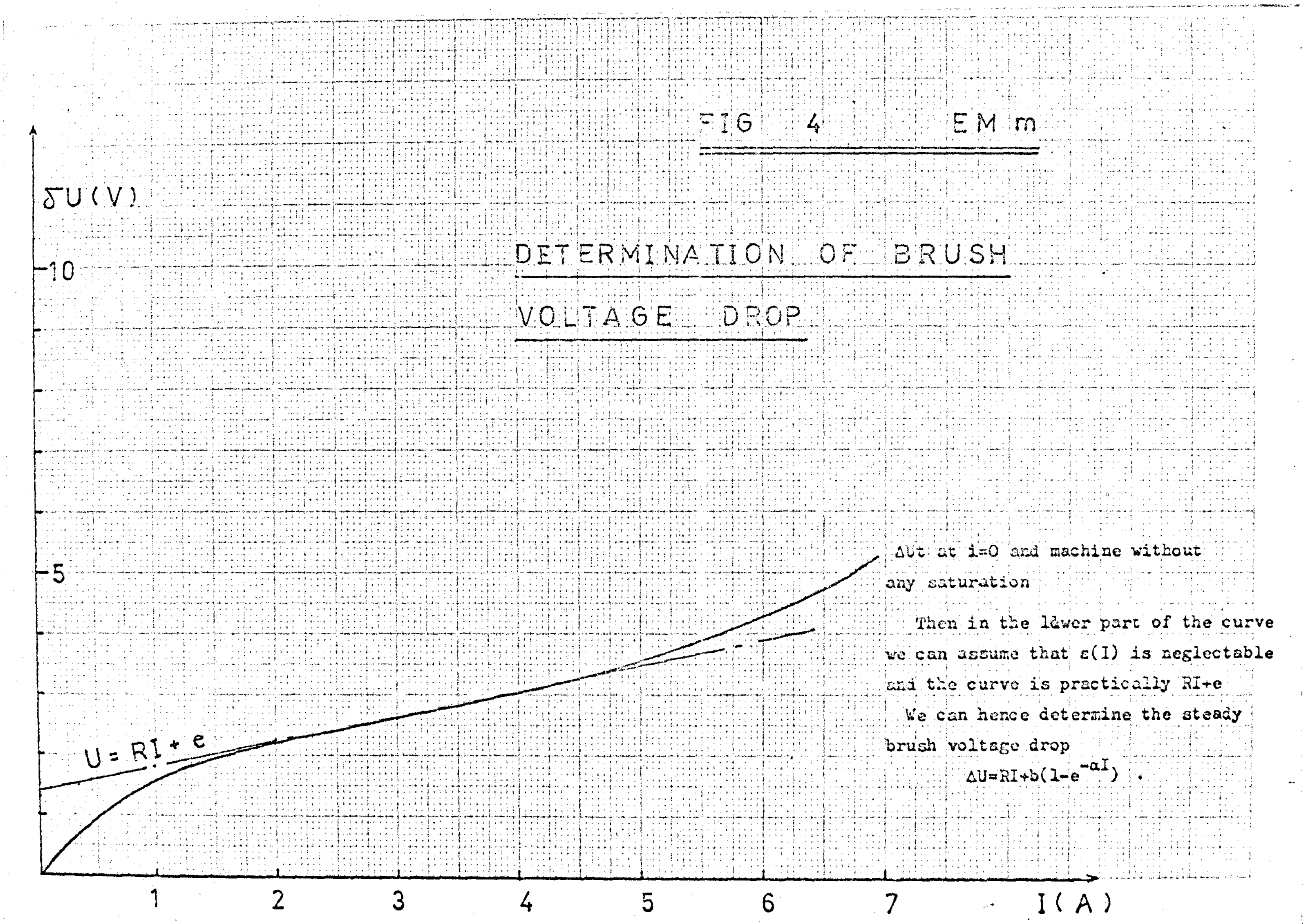


FIG 4 EMm
DETERMINATION OF BRUSH VOLTAGE DROP
δU (V)
10
5
U = RI + e
1
2
3
4
5
6
7
I (A)
ΔUt at i=0 and machine without any saturation
Then in the lower part of the curve we can assume that ε(I) is neglectable and the curve is practically RI+e
We can hence determine the steady brush voltage drop
$\Delta U = RI + b(1 - e^{-\alpha I})$.

(13) $$\Delta U = RI + b(1-e^{-\alpha I})$$

where
- ΔU is the total brush voltage drop
- R the linear part of the resistance
- $b(1-e^{-\alpha I})$ the non-linear part
- I is the load current

α) DETERMINATION OF THE ASYMPTOTE:

The least mean square error rule is applied as seen in Appendix A;

The linear part (around the inflexion) is fitted to the equation: $U = RI + b$

Because if I is large enough $e^{-\alpha I}$ will vanish. This gives the coefficients R and b .

β) DETERMINATION OF THE NON-LINEAR PART:

According to (13) every point of the curve must satisfy:

(14) $$e^{-\alpha I_k} = 1 - \frac{\Delta Uk - RIk}{b} \qquad \text{for all } k$$

or

(15) $$\alpha Ik = \text{Log} \frac{b}{b + RIk - \Delta Uk} \qquad \text{for all } k$$

b and R are known from the linear part, so every element $(Ik, \Delta Uk)$ will give :

(16) $$Kk = \text{Log} \frac{b}{b + RIk - \Delta Uk}$$

Equation (15) becomes:

(17) $$\alpha Ik = Kk$$

and one can again apply the results of appendix A to reach the least mean square error approach

(18) $$\alpha=\frac{\Sigma(Kk_{\cdot}Ik)}{\Sigma(Ik^2)}$$

The curve has been fitted with the maximum accuracy to the data points and the total brush voltage drop is obtained as follow:

(19) $$\Delta U = 0.43\ I + 1.32(1- e^{-1.29\ I})$$

It is impossible to discuss an exact error,but this approach certainly seems to be a better approach than the static mode.

4)ANALYSIS OF THE ARMATURE REACTION: MODIFIED ROSENBERG'S THEORY:

Let us consider a bipolar machine as a simple example.The poles give an excitation flux Ød(see figure 15a and a deeper analysis further in section IIa note 4).If the armature is flown through by a current I,it generates a quadrature flux which leads on the excitation flux in the senseof the movement.Hence N'S' is a neutral line,if the machine is not saturated and if we put the brushes on the neutral line no armature reaction in voltage meaning will appear.Because of the linearity of the non-saturation zone,we can superpose the two states. Vectors will serve to add the fluxes.In the leading tips of the shoes the sum of the two vectors will go beyond the saturation of the machine;however,in the trailing tips,the vectors being in opposition,the resulting vector will be of a lesser magnitude than that of the no-load condition.Distortion of the lines appear,but the resulting flux remains constant.

If saturation occurs the flux distortion will oversaturate the leading tips.There will be a decrease of flux.

Let us consider an assumption.On figure 5a,the operating point P representing the leading tips,is moving on the rising part of the curve f1,and the trailing tips,represented by N on the decreasing part f2.

M and N represent the magnetic state of the circuit(induction)

For a load I in the trailing tips there is an m.m.f. of nI-kI,and in the leading tips, nI+kI ,where k is a coefficient proportional to the physical dimension of the machine,the number of poles the windings etc...

Let x= nie and a=kI

In figure 5b we suppose the machine non-saturated.The area N'P'p'n' represents the flux which induces the e.m.f. In this case the area is given by:

$$A = \left(\frac{nN' + pP'}{2}\right) 2a = 2(mM)a$$

but mM= f1(x) so that

(20) $$A = 2af1(x)$$

In figure 5c saturation occurs.F3 is the auxilliary decreasing curve from M to N.Hence the area A' representing the flux will be given by the area (nNMPp)

(21) $$A' = \int_{x-a}^{x} f3(x)\,dx + \int_{x}^{x+a} f1(x)\,dx$$

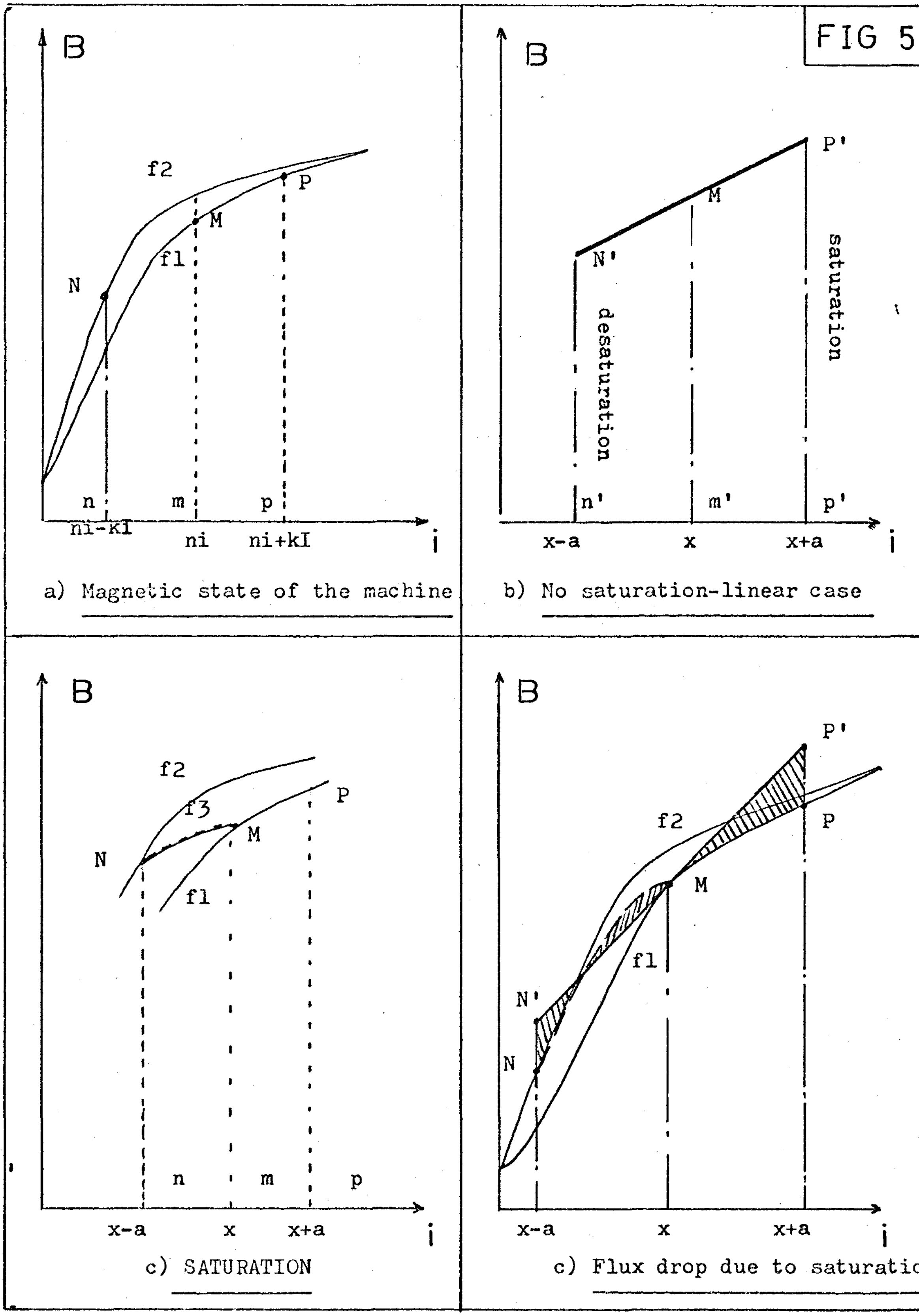


FIG 5

a) Magnetic state of the machine

b) No saturation-linear case

c) SATURATION

c) Flux drop due to saturation

The flux drop can be calculated by subtracting (21) from (20)

$$\text{(22)} \qquad \Delta A = 2\,a\,f1(x) - \left(\int_{x-a}^{x} f3(x)\,dx - \int_{x}^{x+a} f1(x)\,dx \right)$$

NOTE: Normally the partial characteristic f3(x) has to be taken into consideration.We do not know this curve,it depends on M and N.But one can see that f3(x) in its largest part follows very closely f2(x). Therefore a great error will not appear if f2 is considered instead of f3.Hence (22) becomes:

$$\text{(23)} \qquad \Delta A = 2\,a\,f1(x) - \int_{x-a}^{x} f2(x)\,dx + \int_{x}^{x+a} f1(x)\,dx$$

ΔA gives the diminution of excitation flux in function of the excitation current and load current.This represents the quadrature armature reaction in voltage.

RESULTS:

α)One can postulate that the armature flux density is of the same importance as the flux density of the excitation.Hence a is of the same order as x.We have tried with $i_{max}=0.255$ A and $0 < a \leq 0.1$

A computer program calculates the curves $\varepsilon(I,ie)$ with the modified Rosenberg's theory.The results are plotted on figure 6.

From figures 3 and 4 the curves of figure 7 can be drawn,representing the armature reaction with respect to the excitation,and figure 8,with respect to the load current.

β)The curves of figure 7 show that $\varepsilon(I)$ is the

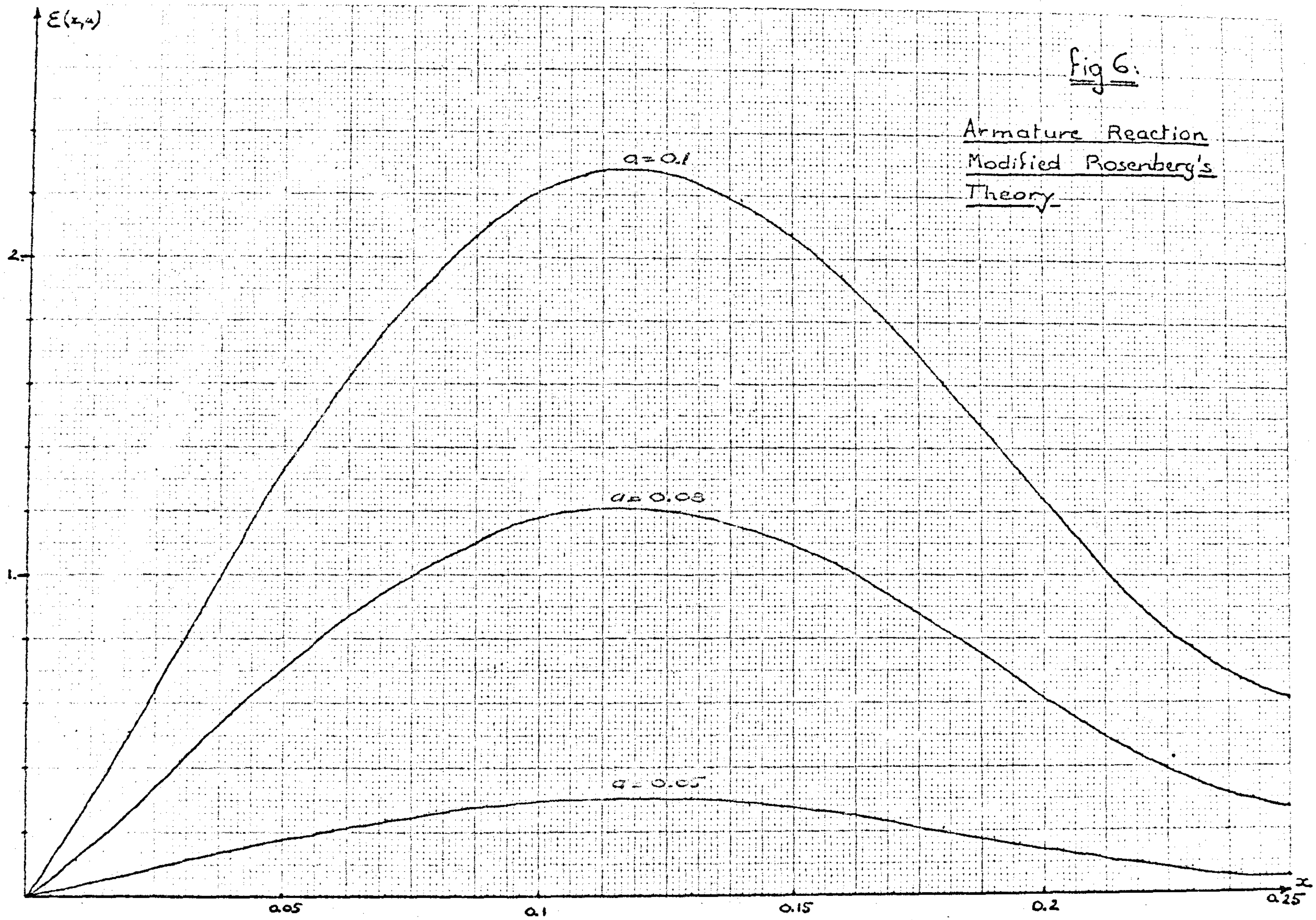


Fig 6.

Armature Reaction Modified Rosenberg's Theory.

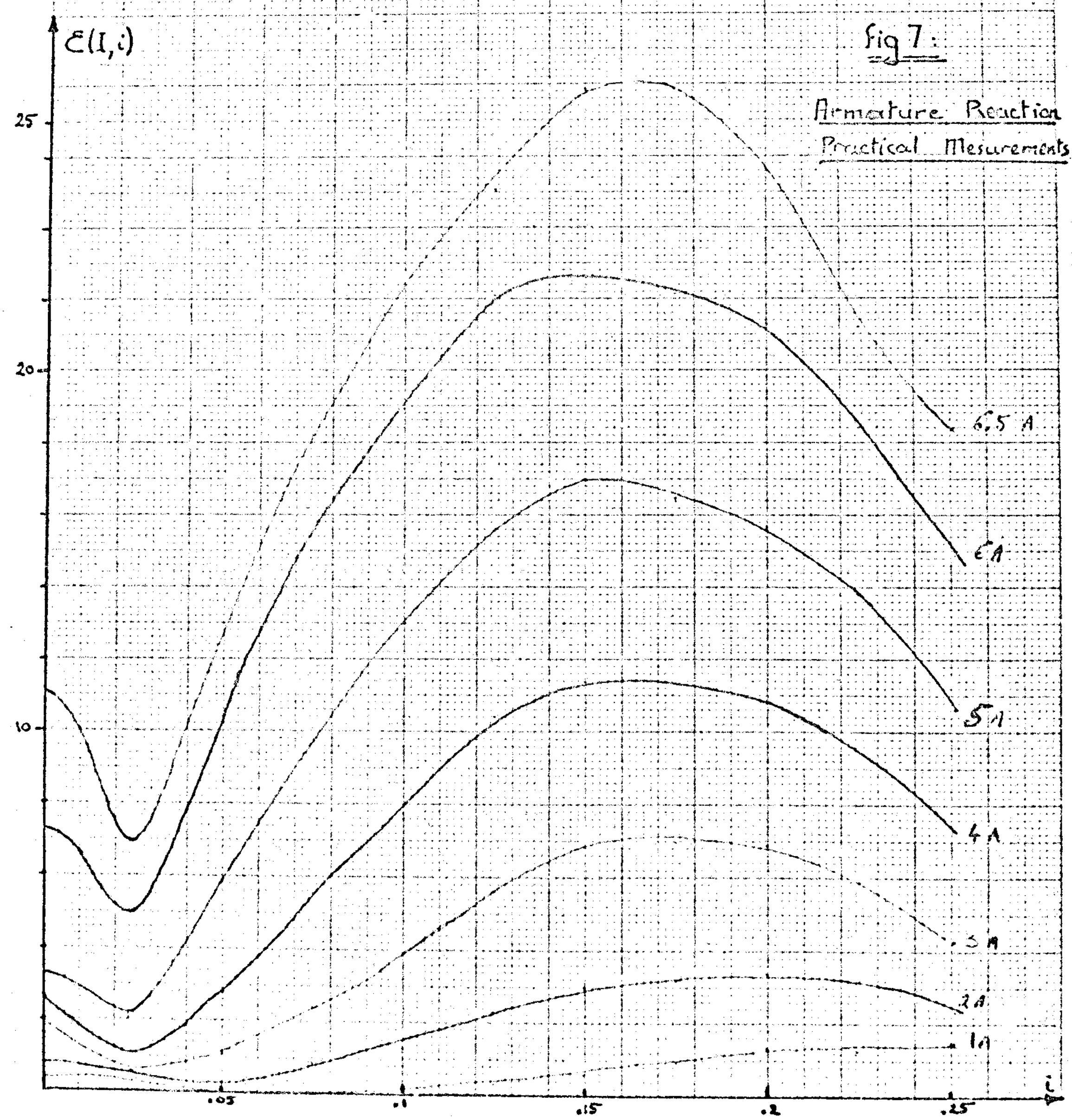
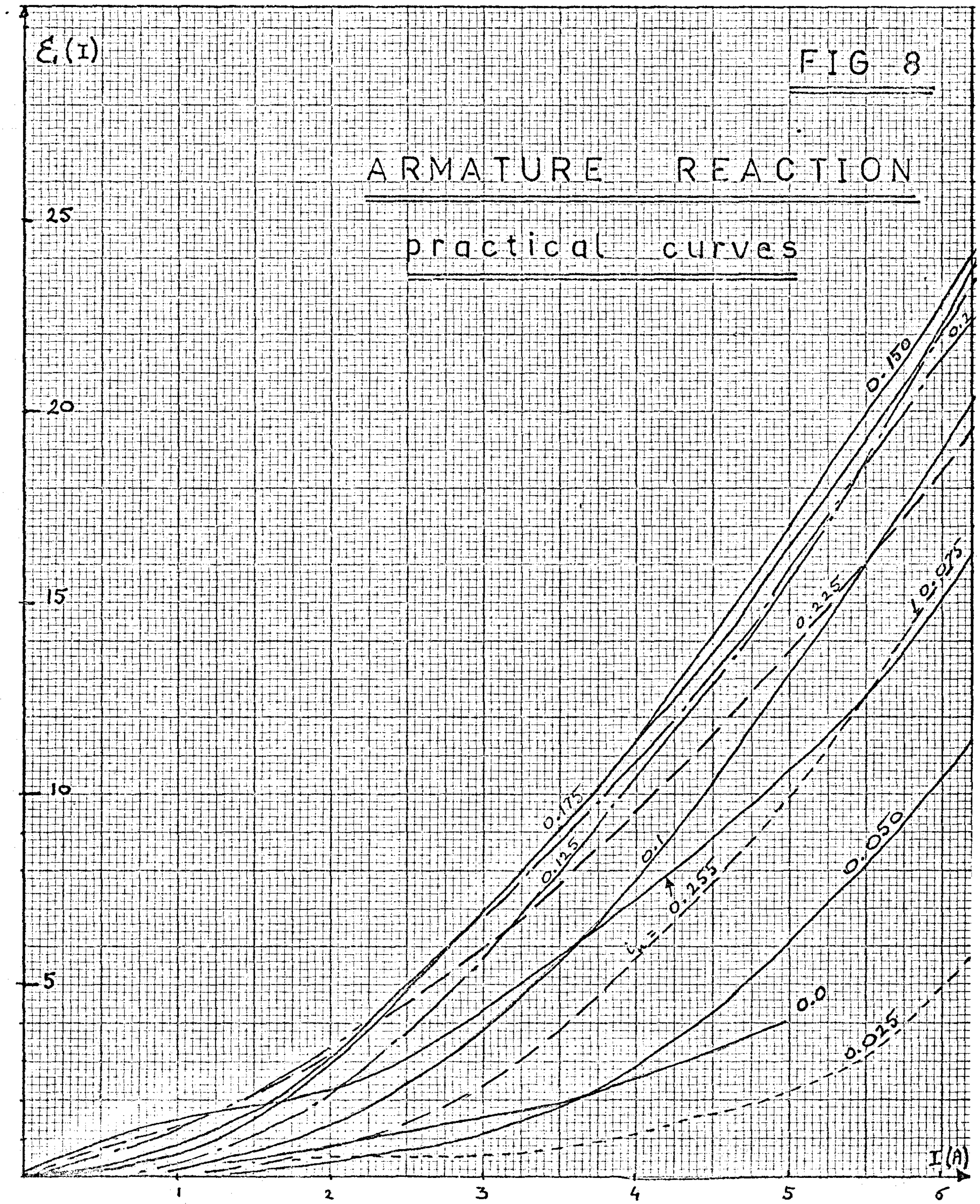


fig 7:

Armature Reaction
Practical Mesurements



$\mathcal{E}_1(I)$
FIG 8
ARMATURE REACTION
practical curves
25
20
15
10
5
0.150
0.1
0.225
0.075
0.175
0.125
0.1
i_n = 0.255
0.050
0.0
0.025
I(A)
1
2
3
4
5
6

maximum for the excitation lying in the range of the beginning of saturation and also for low values of excitation .Normally on the curves of figure 6,Rosenberg's theory gives the same maximum for ie in the range of saturation,but a zero value is found for zero excitation which contradicts the experimental curves.

The modification introduced in Rosenberg's construction shows that the quadrature flux is responsible of the voltage drop whatever the excitation flux is(even zero)

The comparison of the figures 7 and6 shows a large direct armature reaction when the excitation is low,because the experimental curves are shifted to the non-saturated part.But this reaction decreases with the growing saturation.(see theoretical demonstration further on).

III) MECHANICAL PARAMETERS:

1) LOSSES:

The different losses can be approximated in a machine as follows:

*Joule effect: If we know the equation of the voltage drop $\Delta U(I)$ (resistance of windings and brush voltage drop) the copper losses are: $Pj = \Delta U \times I$

One must include,for the efficiency calculation only,the excitation energy which is Uxie , U being the excitation voltage and ie the excitation current.

*Iron losses: In a first approximation one can assume that they do not depend upon the load, but only upon the flux and the speed of the machine.

The Eddy current losses are proportionnal to the square of the frequency, or to the speed .

The hysteresis losses are proportionnal to the speed.

*Mechanical losses:

Likewise the machanical losses depend only upon the speed of the machine

Friction losses are proportionnal to the speed(friction torque constant for bearings and friction of brushes on the bars)

The air friction (cooling) losses are proportionnal to the square of the speed.

— The following experiment can be done:

If the machine is taken under load condition, and runs as a motor, we can measure thepower given to the machine by:

Pg = UI in the armature plus Ulxie for the excitation windings.

According to the analysis standing above, one may write:

(24) $$UI = \Delta U.I + a.\Omega^2 + b.\Omega$$

a and b are losses constants to be determined, and Ω is the speed.

Performing the measurements at different speeds the following points can be plotted:

(25) $$\frac{U.I - \Delta U.I}{\Omega} = a.\Omega + b$$

The experiments show this curve to be a straight line in the range of points measured.From this the coefficients a and b are determined utilizing the least mean square error approach of appendix A the results are the following:

$$\begin{cases} a = 10^{-3} \text{ watt/(rad/s)}^2 \\ b = 0.35 \text{ watt/rd/s} \end{cases}$$

NOTE:The stray losses have not been taken in consideration.

2) EFFICIENCY:

The efficiency of the machine can be determined as follows:

as a generator: (26) $$n_g = \frac{U.I}{U.I + U'.ie + R.I^2 + Po}$$

where Po is the total mechanical and iron losses at a certain speed.

as a motor: (27) $$n_m = \frac{U.I - R.I^2 - Po}{U.I + U'.ie}$$

The efficiency for different speeds and voltages has been calculated.If the normal conditions are taken(2050 rpm,220 V),they are similar to the conditions (1700 rpm, 187 V).Figure 9 shows the efficiency ofthe machine as a generator and as a motor,and it can be observed that the efficiency is at its maximum under the normal conditions in full load.This machine has probably been built for power purpose.

3) RUNNING DOWN:

The motor is running at a speed No.The torque pro-

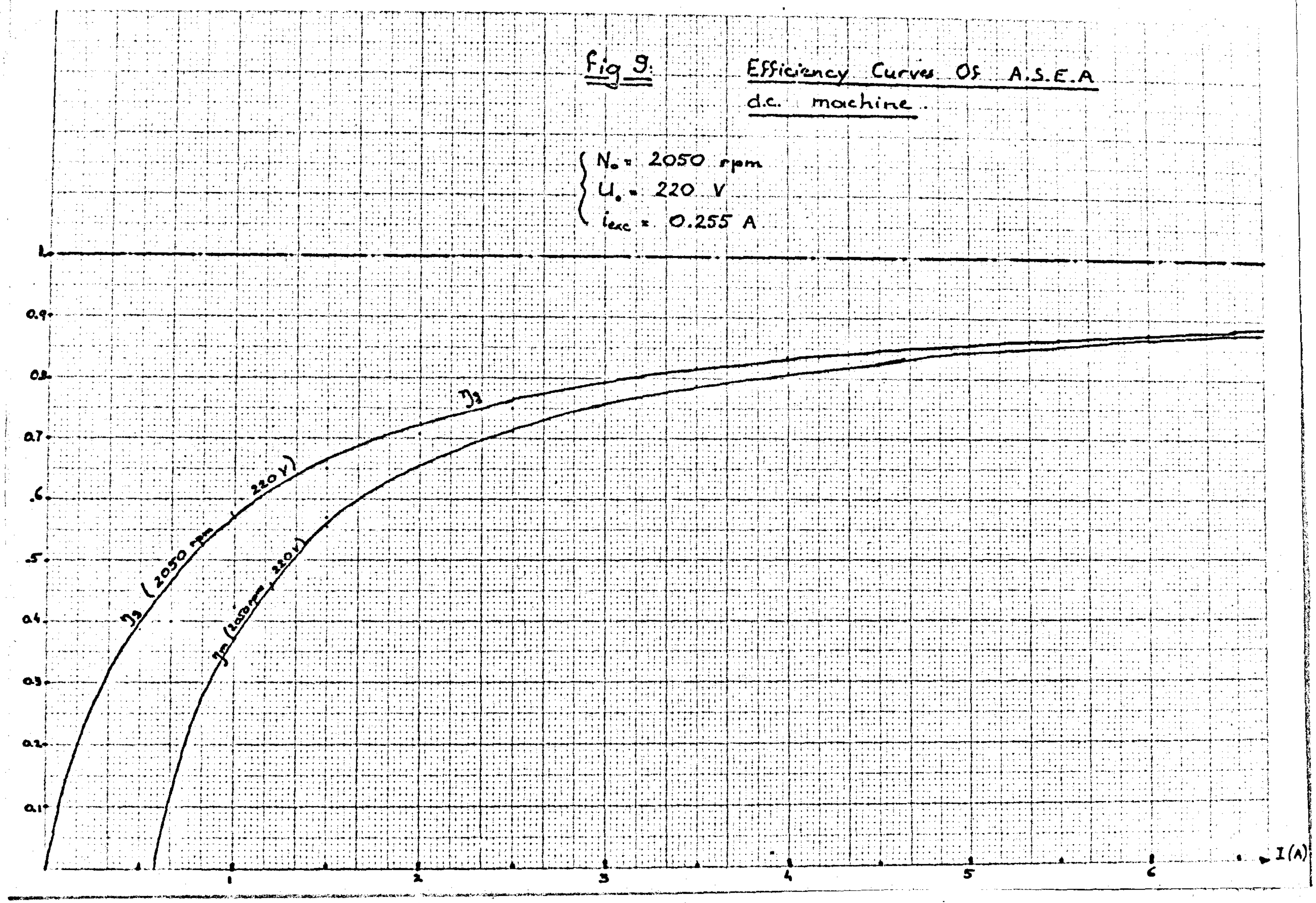


fig 9.
Efficiency Curves Of A.S.E.A d.c. machine.
N₀ = 2050 rpm
U₀ = 220 V
i_exc = 0.255 A
η_g
η_g (2050 rpm , 220 V)
η_m (2050 rpm , 220 V)
1
0.9
0.8
0.7
.6
.5
0.4
0.3
0.2
0.1
1
2
3
4
5
6
I(A)

duced is equal to the resistive torque and the inertial torque. The equation is stated as:

(28) $$\lambda m = \lambda r + J\frac{d\Omega}{dt}$$

But (29) $\lambda r.\Omega = a\Omega^2 + b\Omega$ so that from (28) and (29) one can write:

(30) $$\lambda m = a\Omega + b + J\frac{d\Omega}{dt}$$

The motor torque is suddenly interrupted. The motor will slow down. During this movement the equation to be solved is (30) in which $\lambda m = 0$ hence

(31) $$0 = a\,\Omega + b + J\frac{d\Omega}{dt}$$

Starting from Ωo and running down time being T, it is found

(32) $$(\Omega o + b/a)\exp(-aT/J) = b/a$$

Knowing a and b from the losses, the moment of inertia can be easily determined as :

(33) $$J = \frac{aT}{\text{Log}(1+\Omega o.a/b)}$$

4) RESULTS AND MACHANICAL TIME CONSTANT:

We have performed 4 measurements, starting from various speeds and the time T is registered on the scope. The average value of the moment of inertia is found to be :

$$J = 1.5 \times 10^{-2} \ \text{kg/m}^2$$

If the asymptotical value for the armature resistor is taken, which is 0.43 Ω and for the voltage constant K=1.02 v/rd/s, a mechanical time constant can be defined:

(34) $$tm = \frac{J.R}{k^2}$$ which gives tm =6.2 ms

IV) DETERMINATION OF THE INDUCTANCE OF THE MACHINE:

a) CLASSICAL STATIC MODE:

One needs this measurement for the starting of the motor.

The rotor is blocked and the establishement of the armature current from a step voltage input Uo is recorded.

The exponential rise of current is given by :

$$i = io(1- e^{-Rt/L})$$

where io = Uo/R, R and L being the resistor and inductance of the circuit.(We measure Uo and io for steady state and that gives R).

The half rise time T is read and that leads to:

(35) $$io/2 = io(1-e^{-RT/L})$$ and further on

(36) $$L = \frac{RT}{Log\ 2}$$

The only assumption is that Uo is a real step input and that the resistance and inductance remain constant.That is the usual method of measurement.But those assumptions are erroneous and another technique has to be found.

b) ADAPTED MEASUREMENT FOR INDUCTANCE, FUNCTION OF ARMATURE CURRENT AND SPEED:

The following methods can be applied for statical mode, but here the dynamique mode will be discussed, because it is the more difficult case.

α) FOR LOW CURRENTS:

The machine running as a generator at a speed No, the establishement of the current in a resistor is recorded. The assumption that No and Uo do not vary during the operation is made, hence the same rising curve of exponential shape is found, but now the time constant is a function of the load:

(37) $$i = io(1-e^{-\frac{R(i)}{L(i)}t})$$

On figure 10a this curve is drawn. The inductance for the current kio ($k<1$) is wanted. L and R are assumed not to vary considerably in the range (k1.io , k2.io) surrounding (kio). Let k=(k1+k2)/2 and let k1 and k2 be of the same order as k. If the rise time T between these two values is read, from (37) it comes:

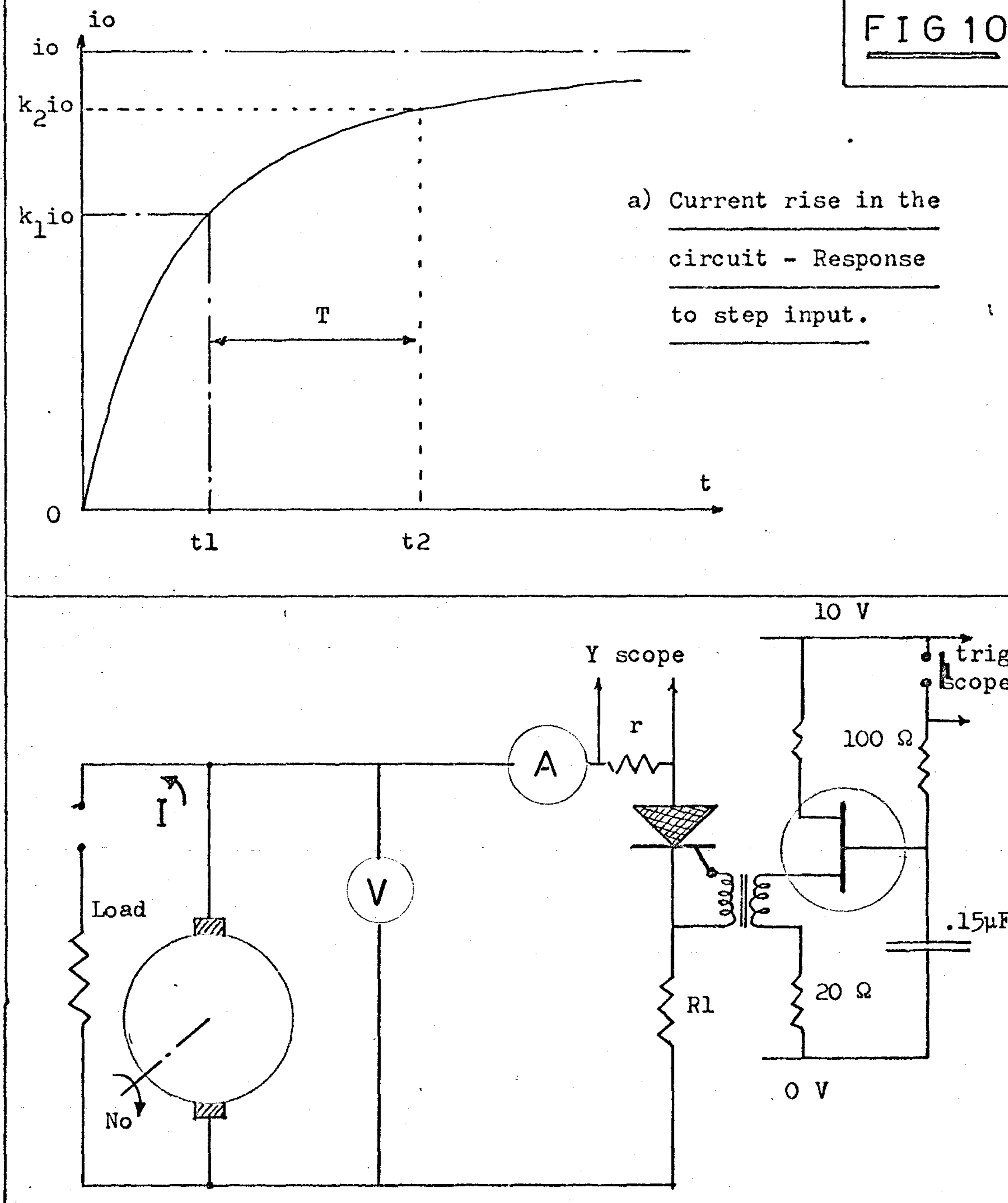
(38) $$k1.io= io(1-e^{-R.t1/L})$$ or

(38') $$Log(1-k1) = -R1.t1/L1$$

and similarily:

(38'') $$Log(1-k2) = -R2.t2/L2$$

FIG 10



a) Current rise in the circuit - Response to step input.

b) Low load current case:

The assumption L1=L2 and R1=R2 has been made provided k-k1 is very small. From (38') and (38'') it comes:

(39)
$$L_{k.io} = \frac{RT}{\mathrm{Log}\left(\frac{1-k1}{1-k2}\right)}$$

The figure 10b states the experiment, which is very simple. r has to be fitted so that the voltage drop rIo gives a sufficient deflection on the scope. 30 Volts, on caliber 5 V/cm is suggested. That gives an easy reading while attenuating the noise due to the commutation bars.

The readings of U, io and T give

(40)
$$R = R_{motor} + r + R1 = U/Io$$

and $L_{k.io}$ is given by (39)

But this method is only valid for small currents because if Io increases, U will vary and a transient response superposes itself which gives an erronuous reading (an overshoot even appears). The method has to be adapted in conscequence as follows.

β) HIGH CURRENTS:

The device of figure 11 is used.

The purpose is to start with a current I1 and increase it to I2, then record this current rise on the scope. If I2-I1 is small, then No and Uo will not vary and the assumption can be made that L does not vary a lot in this range and its value is assumed to be:

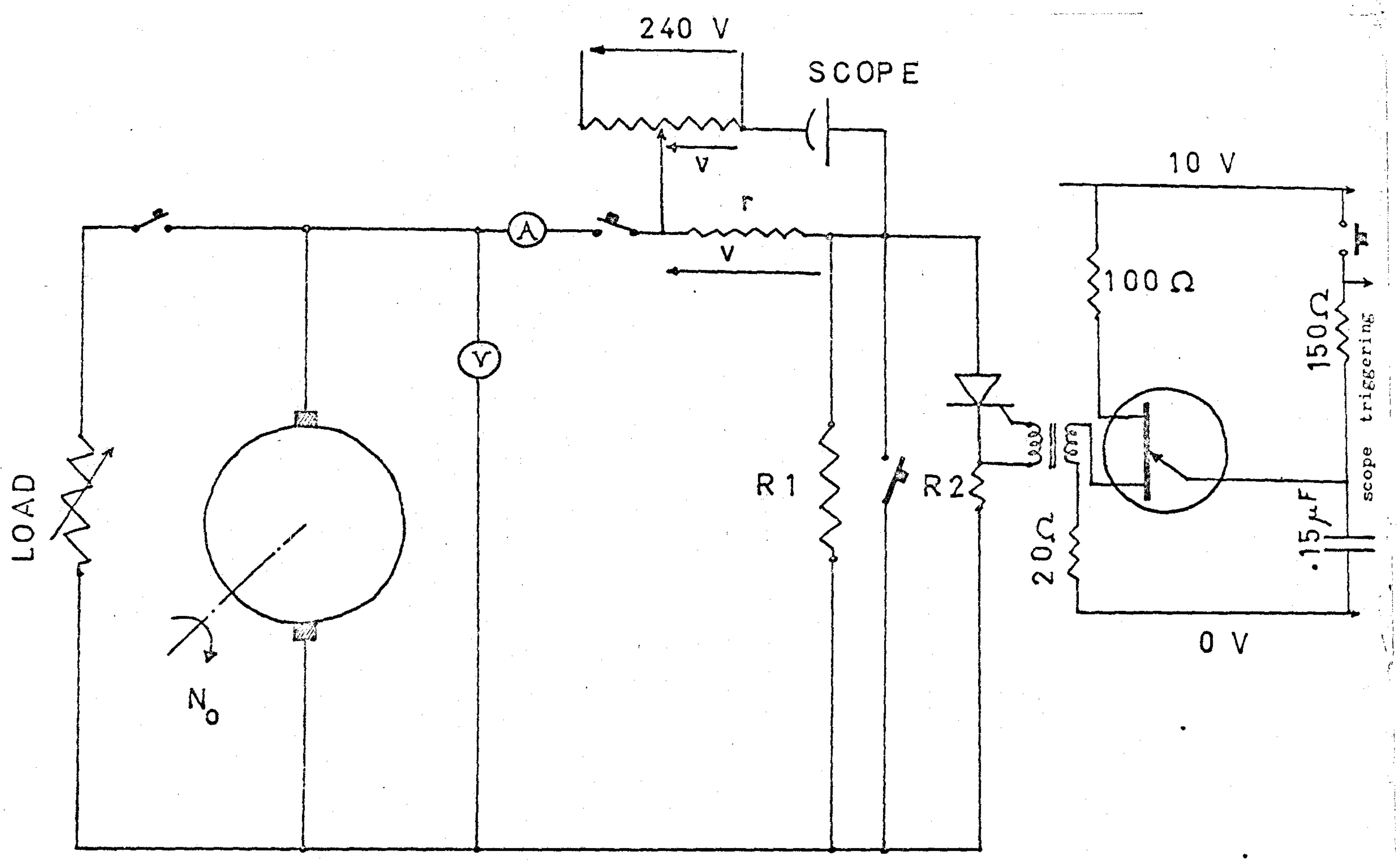


FIG 11 MEASUREMENT OF DYNAMIC INDUCTANCE

$L(\frac{I1+I2}{2})$.

The problems occuring are to suppress the noise in the readings.Therefore r is chosen so that $\Delta v=r(I2-I1)$ gives a sufficient voltage to be able to read this with the greatest caliber of the scope. Actually caliber 5 v/cm is sufficient.A large display on the scope is needed.Therefore,having r fixedone must put in opposition a d.c. voltage equal to v=(r.I1) to the input of the scope.

Nowone has to measure only a voltage increase of about 30v . R1 and R2 have to be adjusted to make the variation (I2-I1) needed. One tries to maintain this variation small. It has been noticed experimentally that a variation of less than I1/20 at least keeps U and No constant.

Let notice the very simple method and the small amount of equipment required.The opposition voltage needed is at most the voltage given by the generator.

The differential equation of the circuit at time $t > 0$ (t=0 is the step input corresponding to the variation of R) is

(41) $$U = Rf.i + L\frac{di}{dt}$$

where $Rf = R_{motor} + r + R_e = U/I2$ $\qquad (R_e = \frac{R1.R2}{R1+R2})$ (42)

If L is considered constant in the range (I1,I2) the solving of 41 gives:

(43) $$i = ke^{-\frac{Rf.t}{L}} + I2$$

for t=0 i=I1 hence k= I1-I2

(44) $$i = (I1-I2)\, e^{-\frac{Rf.t}{L}} + I2$$

Let measure the half rise time T corresponding to $I=(I1+I2)/2$. From (44) comes

$$(44)\quad \frac{I1+I2}{2} = (I1-I2)\,e^{-\frac{Rf.T}{L}} + I2$$

or

$$(45)\quad L=\frac{Rf.T}{Log2}$$

Figure 12 summarizes the different results obtained.

The curve Lo is the static inductance when the excitation current io=0.One can observe that it begins to grow with the load current I,then between 3 and 4 amperes the saturation occurs.

The curves L1 and L2,static and dynamic inductances when the excitation is normal,i=0.255 A,present the same features.When I is low L is decreasing very rapidly and over 4 A the saturation causes an asymptotically value which is about 1/3 of the no-load condition.

It can be seen now that considering L constant is very rough. The variation of L has multiple causes,but one of the mainones is the behaviour under commutation.During the commutation one section is always in short-circuit,so that the mutual inductance between the sections can vary a lot when I is growing.

In the static mode measurement a greater value of L is found when I is small because now there is no commutation,hence no parasite flux occuring which brings down the value of the mutual inductance.

γ) ELECTRICAL TIME CONSTANT:

In our case that does not represent a real magnitude because

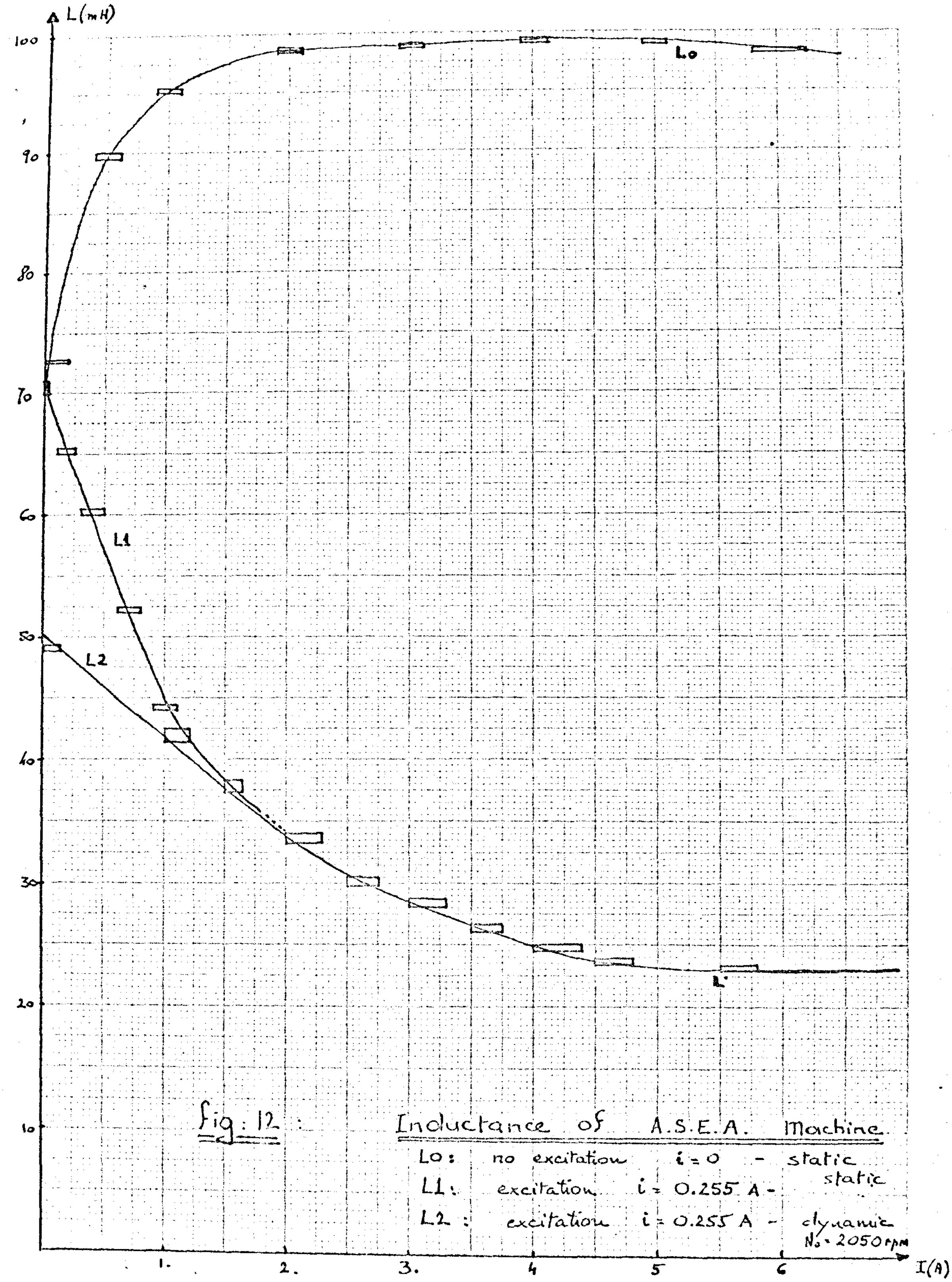


fig: 12 : Inductance of A.S.E.A. Machine it is not constant. Both L and R are functions of I, so that a comparison can be stated with the mechanical time constant at no-load:

$$te = \frac{Lo}{Ro} = 162 \text{ ms}$$

which is about 10 times greater than the mechanical time constant.

CHAPTER 2 :

APPROACH OF A MATHEMATICAL MODEL FOR CONTROL SYSTEM PURPOSES OF THE ELECTROMAGNET MOTOR:

I) GROSS' LINEAR APPROACH:

In this usual approach the problem has been linearized. Resistance and inductance are considered as constants and so is the flux.

If λm is the motor torque and λr the resistive torque other than the inertia torque, the torque equation of the motor is stated as

(46) $$\lambda m = \lambda r + J \frac{d\Omega}{dt}$$

The electrical equation then is

(47) $$U = E + Ri + L \frac{di}{dt}$$ where U is the applied voltage and E the no-load voltage.

Let consider now the case of a no-load condition. The resistive torque has been determined previously as

(48) $$\lambda r = a\Omega + b$$

and the equation (46) in this case turns out to be the equation (30)

(30) $$\lambda m = a\Omega + b + J \frac{d\Omega}{dt}$$

The motor torque can be expressed as:

(49) $$\lambda m = \frac{E_{load} \cdot i}{\Omega}$$

but out of (7)

(50) $$\lambda m = \frac{(K\Omega - \varepsilon) i}{\Omega} = Ki - \frac{\varepsilon i}{\Omega}$$

In the linear approach the armature reaction which is non linear is neglected, hence 30 becomes:

(51) $$Ki = a\Omega + b + J\frac{d\Omega}{dt}$$

and (47) becomes

(52) $$U = K\Omega + Ri + L\frac{di}{dt}$$

(51) and (52) is a set of two linear differential equations that one can solve by Laplace Transform in the case of a start from zero conditions at $t=0$ $\Omega o=0$, $io=0$ and a step voltage U is applied

hence

(53) $$\begin{cases} \Omega(a+Js) - Ki = -b/s \\ \Omega K + I(R+Ls) = U/s \end{cases}$$

The determinant of the system is

(54) $$\gamma(s) = JL\left(s^2 + \left(\frac{R}{L} + \frac{a}{J}\right)s + \left(\frac{K^2}{JL} + \frac{aR}{JL}\right) \right)$$

(54) can also be written as

(55) $$\gamma = JL\left((s+A)^2 + \omega \right) \quad \text{where} \quad \begin{cases} A = \frac{RJ+aL}{2JL} \\ \omega = \frac{K^2+aR}{JL} - \left(\frac{RJ+aL}{2JL}\right)^2 \end{cases}$$

The system now can be solved

(56) $$I=\frac{\alpha s+\beta}{s(\ (s+A)^2+\omega\ )} \quad \text{where} \quad \begin{cases} \alpha=U/L \\ \beta=\dfrac{bK+aU}{JL} \end{cases}$$

further on (56) becomes

(57) $$I= Io/s + \frac{-Io.s+\rho}{(s+A)^2+\omega} \quad \text{where} \quad \begin{cases} Io=\dfrac{\beta}{A^2+\omega} \\ \rho=\alpha-\dfrac{2A\beta}{A^2+\omega} \end{cases}$$

Similarily for the speed:

(58) $$\Omega=\frac{\Omega o}{s}+\frac{-\Omega o.s+\rho'}{(s+A)^2+\omega} \quad \text{with} \quad \begin{cases} \alpha'=-b/J \\ \beta'=\dfrac{-bR+KU}{JL} \\ A=\dfrac{RJ+aL}{2JL} \\ \omega=\dfrac{K^2+aR}{JL}-A^2 \end{cases}$$

and

$$\Omega o=\frac{\beta'}{A^2+\omega} \qquad \rho'=\alpha'-\frac{2A\beta}{A^2+\omega}$$

Solving in time domain:

<u>FIRST CASE</u> : if ω is positive

(59) $$i(t)=Io+A'\,e^{-At}\sin(\sqrt{\omega}t+\ell)$$

where

$$A'=\frac{1}{\omega}\sqrt{\omega Io^2+(\rho+AIo)^2}$$

$$tg\ \ell=\frac{-\sqrt{\omega}\,Io}{\rho+A.Io}$$

and

(60) $$\Omega(t)=\Omega o+A''\,e^{-At}\sin(\sqrt{\omega}t+\ell)$$

where $A'' = \frac{1}{\omega}\sqrt{\omega\Omega o^2 + (\rho + A\Omega o)^2}$

$tg\,\ell = \frac{-\Omega o\,\sqrt{\omega}}{\rho + A.Io}$

<u>SECOND CASE:</u> $\omega=0$

(61) $i(t) = Io + A1.t.\exp(-At)$

where $Io = \beta/A^2$ and $A1 = (-\alpha A + \beta)/(-A)$

and

(62) $\Omega(t) = \Omega o + \Omega 1.t.\exp(-At)$

where $\Omega o = \beta'/A^2$ and $\Omega 1 = (-\alpha' A + \beta')/(-A)$

<u>THIRD CASE:</u> ω is negative

(63) $i(t) = Io + I1.e^{-s1.t} + I2.e^{-s2.t}$

where $Io = \beta/(A^2+\omega)$ $I1 = \frac{\alpha s1 + \beta}{2\sqrt{-\omega}\,s1}$ $I2 = \frac{\alpha.s2 + \beta}{-2\sqrt{-\omega}\,s2}$

with $s1 = -A + \sqrt{-\omega}$ $s2 = -A - \sqrt{-\omega}$

and

(64) $\Omega(t) = \Omega o + \Omega 1.e^{-s1.t} + \Omega 2.e^{-s2.t}$

where $\Omega o = \beta'/(A^2+\omega)$ $\Omega 1 = \frac{\alpha'.s1 + \beta'}{2\sqrt{-\omega}\,s1}$ $\Omega 2 = \frac{\alpha'.s2 + \beta'}{-2\sqrt{-\omega}\,s2}$

A computer program P2 calculates by Laplace transform the equations of the starting of the motor. One has just to feed in the fix parameters of the machine: a,b,J,K,R,L and the conditions of the experiment as

Ro= starting resistor in series with the armature
U step voltage applied

SZ

P 2

```
 A4173.
 RUN(S)
 LGO.
 '        6400 END RECORD
       PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
 $JOB             003521 SZABADOS B        100   010   030
 $IBJOB           DECK
 $IBFTC MAIN
 C    TRANSFER FUNCTION BY LAPLACE TRANSFORM OF A MOTOR
 C      N= NUMBER OF DATA GIVEN
        CALL TRFUN(1)
        STOP
        END
 $IBFTC SO
        SUBROUTINE TRFUN(L)
        DIMENSION Z(10)
        DO 44 I=1,L
 C    TITLE
        WRITE(6,3)
        DO 98 M=1,2
        READ(5,96)Z
 98     WRITE(6,97)Z
 96     FORMAT(10A6)
 97     FORMAT(53X,10A6)
        READ(5,1)A1,B1,RL1,R1,RJ1,RK1,U
        WRITE(6,2)A1,B1,RL1,R1,RJ1,RK1,U
        ALFA=U/RL1
        BETA=(B1*RK1+A1*U)/(RJ1*RL1)
        A=(R1*RJ1+A1*RL1)/(2.0*RJ1*RL1)
        OMEG=(RK1**2+A1*R1)/(RJ1*RL1)-A**2
        ALFA1=-B1/RJ1
        BETA1=(-B1*R1+RK1*U)/(RJ1*RL1)
        IF(OMEG.LT.0.0) CALL OMNEG(ALFA,BETA,ALFA1,BETA1,A,OMEG)
        IF(OMEG.EQ.0.0) CALL OMNOT(ALFA,BETA,ALFA1,BETA1,A,OMEG)
        IF(OMEG.GT.0.0) CALL OMPOS(ALFA,BETA,ALFA1,BETA1,A,OMEG)
 44     CONTINUE
 1      FORMAT(7F10.3)
 2      FORMAT(    10X,18HTRANSFER  FUNCTION/1H0,20X,2HA=,  E10.3/1H0,
       1 20X,2HB=,  E10.3/1H0,20X,2HL=,  E10.3/1H0,20X,2HR=,  F10.3/1H0,
       220X,2HJ=,  E10.3   /1H0,20X,2HK=,  E10.3/1H0,20X,2HU=,F10.3/1H-)
 3      FORMAT(1H1)
        RETURN
          END
 $IBFTC S1
        SUBROUTINE OMNEG(ALFA,BETA,ALFA1,BETA1,A,OMEG)
        N=1
        OMEG2=-OMEG
        OMEGA=SQRT(OMEG2)
        S1=-A+OMEGA
        S2=-A-OMEGA
 22     CIO=BETA/(A**2-OMEG2)
        CI1=(ALFA*S1+BETA)/(+2.0*OMEGA*S1)
        CI2=(ALFA*S2+BETA)/(-2.0*OMEGA*S2)
        IF( N.EQ.1) WRITE(6,1) CIO,CI1,S1,CI2,S2
        IF( N.EQ.2) WRITE(6,2) CIO,CI1,S1,CI2,S2
        ALFA=ALFA1
        BETA=BETA1
        IF(N.EQ.2) RETURN
        N=N+1
```

```
      GO TO 22
1     FORMAT(1H-,10X,5HI(T)=,F10.4,1H+,  E10.3,5H*EXP(,E10.3,4H*T)+,
     1 E10.3,5H*EXP(,E10.3,4H *T)/1H-)
2     FORMAT(1H-,10X,5HN(T)=,F10.4,1H+,  E10.3,5H*EXP(,E10.3,4H*T)+,
     1 E10.3,5H*EXP(,F10.3,4H *T)/1H-)
          END
$IBFTC S2
      SUBROUTINE OMNOT(ALFA,BETA,ALFA1,BETA1,A,OMEG)
      N=1
22    CIO=BETA/(A**2)
      CI1=(-ALFA*A+BETA)/(-A)
      IF(N.EQ.1) WRITE(6,1)CIO,CI1,A
      IF(N.EQ.2) WRITE(6,2)CIO,CI1,A
      ALFA=ALFA1
      BETA=BETA1
      IF(N.EQ.2) RETURN
      N=N+1
      GO TO 22
1     FORMAT(1H-,10X,5HI(T)=,F10.4,1H+,F10.4,8H*T*EXP(-,  E10.3,3H*T)/1H
     1-)
2     FORMAT(1H-,10X,5HN(T)=,F10.4,1H+,F10.4,8H*T*EXP(-,  E10.3,3H*T)/1H
     1-)
          END
$IBFTC S3
      SUBROUTINE OMPOS(ALFA,BETA,ALFA1,BETA1,A,OMEG)
      N=1
22    CIO=BETA/(A**2+OMEG)
      GAMMA=ALFA-(2.0*A*BETA)/(A**2+OMEG)
      OMEGA=SQRT(OMEG)
      AP=SQRT(OMEG*CIO+(GAMMA+A*CIO)**2)/OMEGA
      C1=-OMEGA*CIO
      C2=GAMMA+A*CIO
      PHI=ATAN2(C1,C2)
      IF(N.EQ.1) WRITE(6,6) CIO,AP,A,OMEGA,PHI
      IF(N.EQ.2) WRITE(6,7) CIO,AP,A,OMEGA,PHI
      ALFA=ALFA1
      BETA=BETA1
      IF(N.EQ.2) RETURN
      N=N+1
      GO TO 22
6     FORMAT(10X,17HTRANSFER FUNCTION/1H-,5X,5HI(T)=,F10.4,
     11H+,  E10.3,6H*EXP(-,F10.4,8H*T)*SIN(,F10.5,3H*T+,F10.4,1H)/1H0)
7     FORMAT(10X,17HTRANSFER FUNCTION/1H-,5X,5HN(T)=,F10.4,
     11H+,  E10.3,6H*EXP(-,F10.4,8H*T)*SIN(,F10.5,3H*T+,F10.4,1H)/1H0)
          END
$ENTRY
'        6400 END RECORD
P.M. MOTOR STARTING WITH NO LOAD,AND R0=39.2 OHMS   /U=120 V
*************************************************************
0.0108    0.323     0.0024    40.4      0.019     1.13      120.
$IBSYS
'        6400 END FILE
```

II) ESTABLISHMENT OF THE MATHEMATICAL MODEL:

1)

Notation used: units in MKSA system

Ω	speed in rd/s
i	armature current
U	voltage applied
$R(i)$	dynamic resistance of the armature
$L(i)$	dynamic inductance
a and b	coefficients of losses
J	moment of inertia
E	no-load voltage
$\varepsilon(i)$	armature reaction
λm	motor torque

The electrical differential equation of the motor is written as

(65) $$U=E+Ri+L\frac{di}{dt}-\varepsilon$$

The motor torque equation can be stated from the conservation of energy

(66) $$\lambda m.\Omega=(E-\varepsilon)i$$

But E and ε are proportionnàl to Ω , hence

(67) $$\begin{cases} E=K\Omega & K \text{ is a voltage constant} \\ \varepsilon=K'.\Omega & K' \text{ depends upon } i \end{cases}$$

Hence the motor torque is given by:

(68) $$\lambda m=(K-K'(i)).i$$

and the differential equation (30) ,if the no-load condition is taken, becomes:

(69) $$(K-K'(i)).i = a\Omega + b + J\frac{d\Omega}{dt}$$

Finally the system to be solved is

(70) $$\begin{cases} U=K\Omega - K'(i).\Omega+R(i).i+L(i)\dfrac{di}{dt} \\ Ki-K'(i).i= a\Omega+b+J\dfrac{d\Omega}{dt} \end{cases}$$

If the very general case is taken, where U is a function of t that has to be known, and $\lambda\omega$ is the torque which loads the machine(its characteristic must be known against Ω) (70) is changed into

(70') $$\boxed{\begin{array}{l} U(t)=K\Omega-K'(i).\Omega+R(i)i+L(i)\dfrac{di}{dt} \\ Ki=K'(i).i+a\Omega+b+J\dfrac{d\Omega}{dt}+\lambda\omega(\Omega) \end{array}}$$

NOTE 1 : The constant resistor, if this is the case, is included in R(i).

NOTE 2: (70') cannot be valid for the starting of the motor. At t=0 if i=0 and Ω=0 the second equation of (70') becomes

(71) $$0= b+J\frac{d\Omega}{dt}+ \lambda\omega \qquad \text{hence}$$

(71') $$\frac{d\Omega}{dt} = -\frac{b+\lambda\omega(0)}{J}$$

But b, $\lambda\omega$, and J being positive, $d\Omega/dt$ is shown to be negative, which is impossible because Ω=0 cannot have a decelaration. The equations should be modified as:

(72) if $i \leqslant \frac{b+\lambda\omega(0)}{K}$

$$\boxed{U=Ri+L\frac{di}{dt} \qquad \text{and} \qquad \Omega=0}$$

if $i > \frac{b+\lambda\omega(0)}{K}$ (70') can be applied

This has a physical meaning. A certain current i has to be provided to overcome the inertial frictions before the motor can start.

NOTE 3: The armature reaction ε has been determined by experiment in chapter A. But the curves have only been obtained till the maximum current allowed, here 6.5 A

The secund equation of (65) permits to have K'(i). The curve $\varepsilon(I)$ at 2050 rpm and normal excitation ie=0.255 A is taken and one can obtain K' by dividing ε by 215 rd/s.

Utilizing the computer program P1 a polynomial has been fitted to the experimental curve. The result is given by the following polynomial:

$$\varepsilon=0.24+0.879I+0.9I^2+0.025I^3+7.68\times10^{-4}I^4+.....$$

But this is valid only if $i \leqslant 6.5$ A.

The study of the armature reaction has not been conducted in the area $i > 6.5$ A, and the experiment cannot indicate its behaviour. One can be sure that the shape of the curve which is rising very rapidly will not follow this tendency because of the saturation. If there would be no saturation for a certain high current the equation (65) would mean a non-sens, ε growing very quickly would reach E and $E-\varepsilon$ would become negative.

One cannot determine the beginning of the saturation of the armature,therefore,if one solve those equations of (70'),one must be careful that i does not reach those high values,otherwise the equation of K' is not valid.

Before undertaking the solving of those non-linear equations, it is suggested to approximate the current with Laplace transform,to see if it remains in the correct range.

2) COMPARISON OF THE GROSS' LINEAR APPROACH AND THE ACTUAL MODEL:

a) EXPERIMENTAL MEASUREMENT:

To be able to compare the results of Laplace transform and the mathematical model,an experiment has been set up,where the motor starts under no-load conditions.

As shown on figure 13a,there is a 30 Ω resistor in series with the armature to limit the current,and a 240 V step voltage is applied through an SCR device.The curve of the current rise is recorded and the shape of figure 13b is obtained.The bump around 150ms can be noticed.This is very probably due to the presence of SCR,which introduces a parasite capacity which causes the circuit composed by the inductance and resistance of the machine to ring lightly.When one observe the curve without SCR switch,this bump disapears,but the triggering of the scope and the record of the curve is very difficult to reach,due to the bounces of the mechanical contacts.

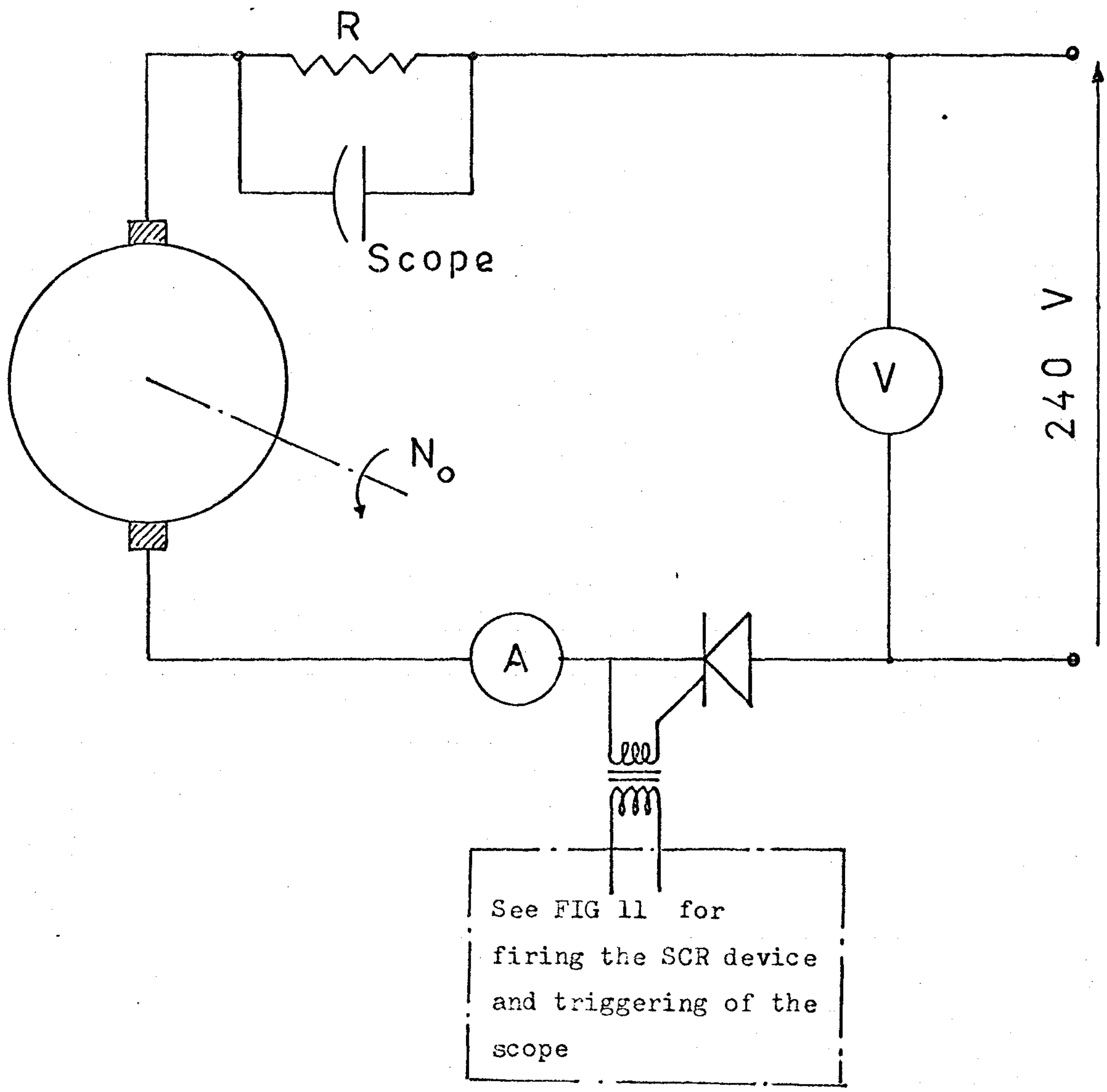


FIG 13 a Experimental device recording the starting current of an electromagnet motor

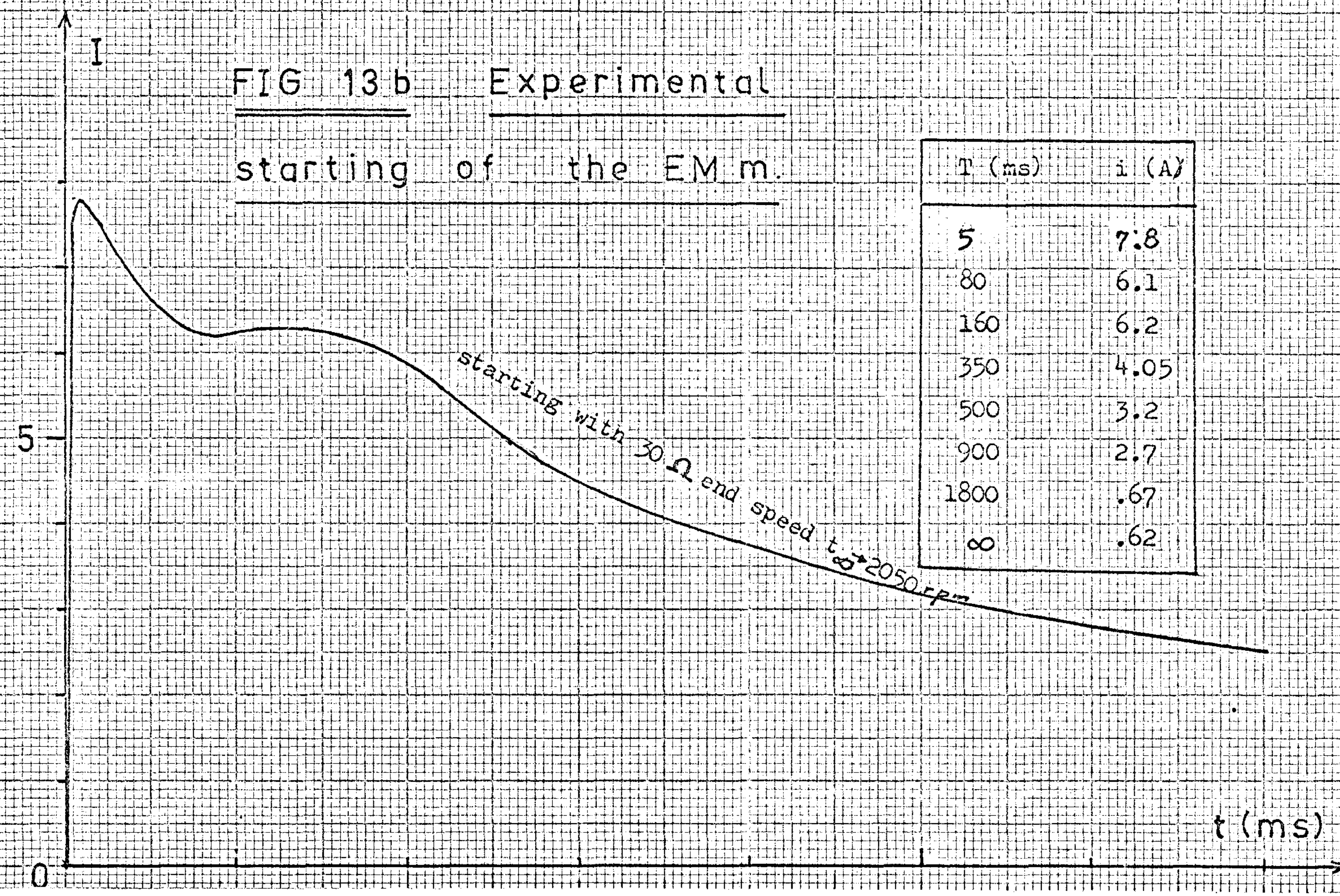


FIG 13b Experimental starting of the EM m.

T (ms)	i (A)
5	7.8
80	6.1
160	6.2
350	4.05
500	3.2
900	2.7
1800	.67
∞	.62

b) LAPLACE TRANSFORM RESOLUTION:

Utilizing the computer program P2 ,Ro is set 30 Ω and U=240V The result equations obtained for i(t) and Ω(t),current and speed, are the following:

$$\begin{cases} i(t)=.56+7.4\ \exp(-2.4xt)\ -7.97\ \exp(-433.2t) \\ \Omega(t)=218.6-220\ \exp(-2.4\ t)+1.25\ \exp(-433.2) \end{cases}$$

c)SOLVING THE NON-LINEAR DIFFERENTIAL EQUATION SYSTEM:

The parameters beeing defined as

$$\begin{cases} R=.43+1.32(1-\exp(-1.29xI))/I \\ K=1.02\quad V/rd/s \\ J=0.015\ kg/m^2 \\ a=10^{-3}\quad W/rd^2/s^2 \\ b=0.35\ W/rd/s \\ \varepsilon(I)=0.24+.879\ I+.09\ I^2+\ .025\ I^3+7.68x10^{-4}I^4+\ldots \\ L(I)=50-6.85\ I-0.736\ I^2+0.215\ I^3 \quad \text{(in mH)} \quad \text{if } i\ \ 4.65\ A \\ L=\ 23.7\ mH \quad \text{if } i\ \ 4.65\ A \end{cases}$$

Setting the torque λω to zero,equation (72) and (70') are solved by the computer program P3.

d) COMPARISON:

Figure 14 shows the plot of the three curves.L1 is the expe-

P3

```
$JOB            003521 SZABADOS B        100    010    030
$IBJOB          NOMAP,NODECK
$IBFTC MAIN
C   SOLVING A NON LINEAR DIFF EQUQTION SYSTEM
      DIMENSION Z(10),WORK(5),Y(2),DY(2),CUR(200),VIT(200)
      COMMON A,B,RJ,RK,SO,S1,S2,S3,S4,SF,U,RO
      EXTERNAL SUBNO,SUB2
      DO 11 I=1,2
      Y(I)=0.0
11    DY(I)=0.0
      WRITE(6,100)
C   READ TITLE ON CARDS, 4CARDS PRINTED ON HEADINGS
      DO 1 LL=1,4
      READ(5,101)  (Z(I),I=1,10)
1     WRITE(6,102) (Z(I),I=1,10)
C
C   READ DATA AND WRITE THEM
C
      READ(5,103)A,B,RJ,RK,U,RO
      READ(5,104)SO,S1,S2,S3,S4,SF
      WRITE(6,105)RO,U
C
C   SOLVING THE STARTING OF THE MOTOR
C
      N=1
      BK=B/RK
      T=0.0
      CALL DEQ(T,1.0E-06,1,Y(1),DY(1),WORK,SUBNO)
2     CALL DEQ
      IF(Y(1).GE.BK)   GO TO 3
      N=N+1
      IF(N.EQ.200) WRITE(6,108) Y(1)
      IF(N.EQ.200)STOP
      GO TO 2
3     WRITE(6,106) T
      CALL DEQSET
C
C SOLVING THE TWO EQUATIONS
C
      CALL DEQ(T,TSTEP,2   ,Y,DY,WORK,SUB2)
      DO 4 I=1,200
      TSTEP=1.0E-04
      IF(I.EQ.1) TSTEP=TSTEP-T
      CALL DEQ
      CUR(I)=Y(1)
      VIT(I)=Y(2)
      IF(VIT(I).LT.0.0 .OR. CUR(I).LT.0.0) STOP
4     WRITE(6,107) T,Y(1),Y(2)
      STOP
100   FORMAT(1H1)
101   FORMAT(10A6)
102   FORMAT(50X,10A6)
103   FORMAT(6F10.3)
104   FORMAT(6E10.3)
105   FORMAT(1H-,10X,3HRO=,F10.3,20X,2HU=,F10.1,1X/1H-)
106   FORMAT(10X,27HTHE MOTOR STARTS AT TIME T=,F10.3/1H-,
     117X,6HT(SEC),24X,6HI(AMP),22X,11HOMEGA(RD/S)/1H+)
107   FORMAT(15X,F8.5,21X,E12.5,18X,E12.5)
108   FORMAT(10X,32HINTEGRATION INTERVAL TOO SMAL,I=,F10.6)
```

```
      END
$IBFTC S1
      SUBROUTINE SUBNO(C,DC,T)
      COMMON A,B,RJ,RK,SO,S1,S2,S3,S4,SF,U,RO
      IF(C .GT.0.0) R=0.43+RO+1.32*(1.-EXP(-1.29*C))/C
      IF(C.GT.0.0)S=((S4*C+S3)*C+S2)*C+S1
      IF(C.EQ.0.0) DC=U/SO
      IF(C.GT.0.0) DC=(U-R*C)/S
      RETURN
      END
$IBFTC S2
      SUBROUTINE SUB2(Y,DY,T)
      DIMENSION Y(2),DY(2)
      COMMON A,B,RJ,RK,SO,S1,S2,S3,S4,SF,U,RO
      R=0.43+RO+1.32*(1.-EXP(-1.29*Y(1)))/Y(1)
      PK=(0.24+Y(1)*(0.879+Y(1)*(0.0902+Y(1)*(0.0254+Y(1)*7.675E-04))))/
     1 215.
      IF(Y(1).GT.6.0) PK=5.62*Y(1)-18.75
      IF(Y(1).LE.4.5) S=((S4*Y(1)+S3)*Y(1)+S2)*Y(1)+S1
      IF(Y(1).GT.4.5) S=SF
      DY(1)=(U-R*Y(1)-RK*Y(2)+PK*Y(2))/S
      DY(2)=(RK*Y(1)-A*Y(2)-B-PK*Y(1))/RJ
      RETURN
      END
$ENTRY
RESOLUTION BY DEQ OF NON LINEAR DIFF. EQUATIONS
SOLVING THE STARTING OF A.S.E.A. MOTOR
        NO LOAD  RO=30.    / U=240
****************************************************************
0.001      0.35       0.015      1.02       240.       30.
+7.000E-02+5.000E-02-6.850E-03-7.360E-04+2.150E-04+2.370E-02
$IBSYS
```

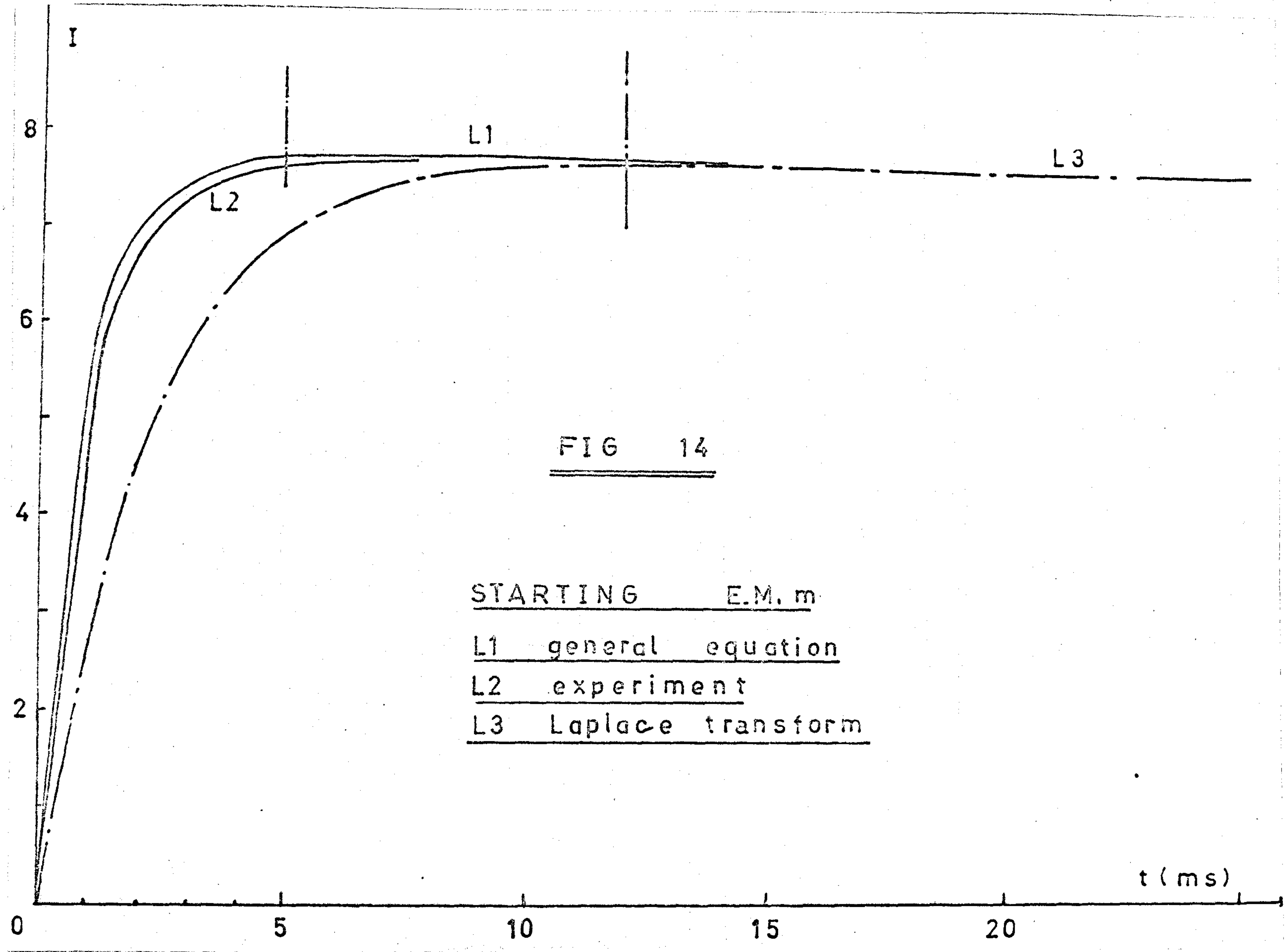

I
8
6
4
2
0
5
10
15
20
t (ms)
L1
L2
L3
STARTING E.M.m
L1 general equation
L2 experiment
L3 Laplace transform

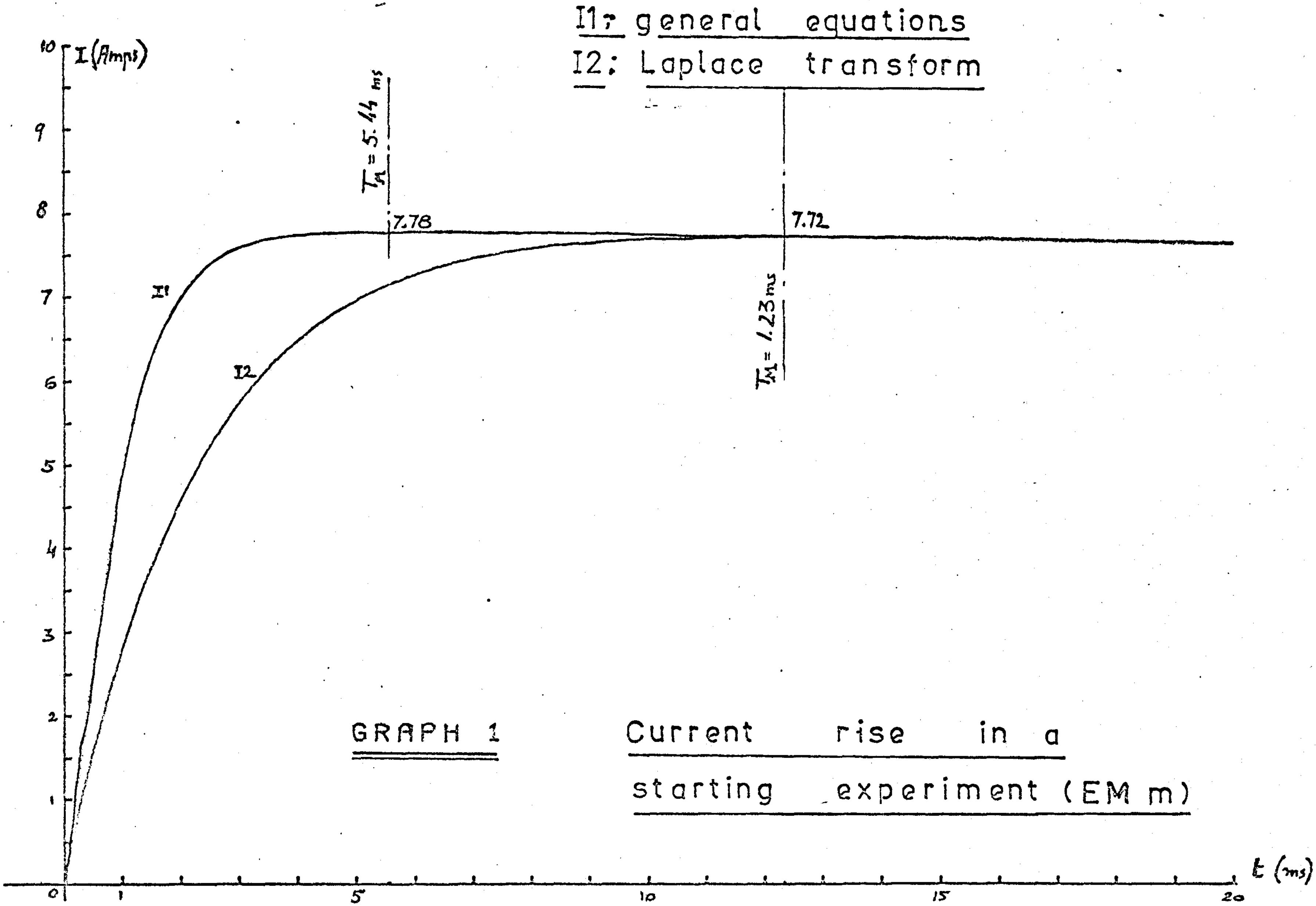
FIG 14

rimental curve, L2 is the result of the solving of the non-linear differential equations and L3 is the plot of Laplace transform. Very precise computer plots, on graph 1 and 2 compare the linear and non-linear approach.

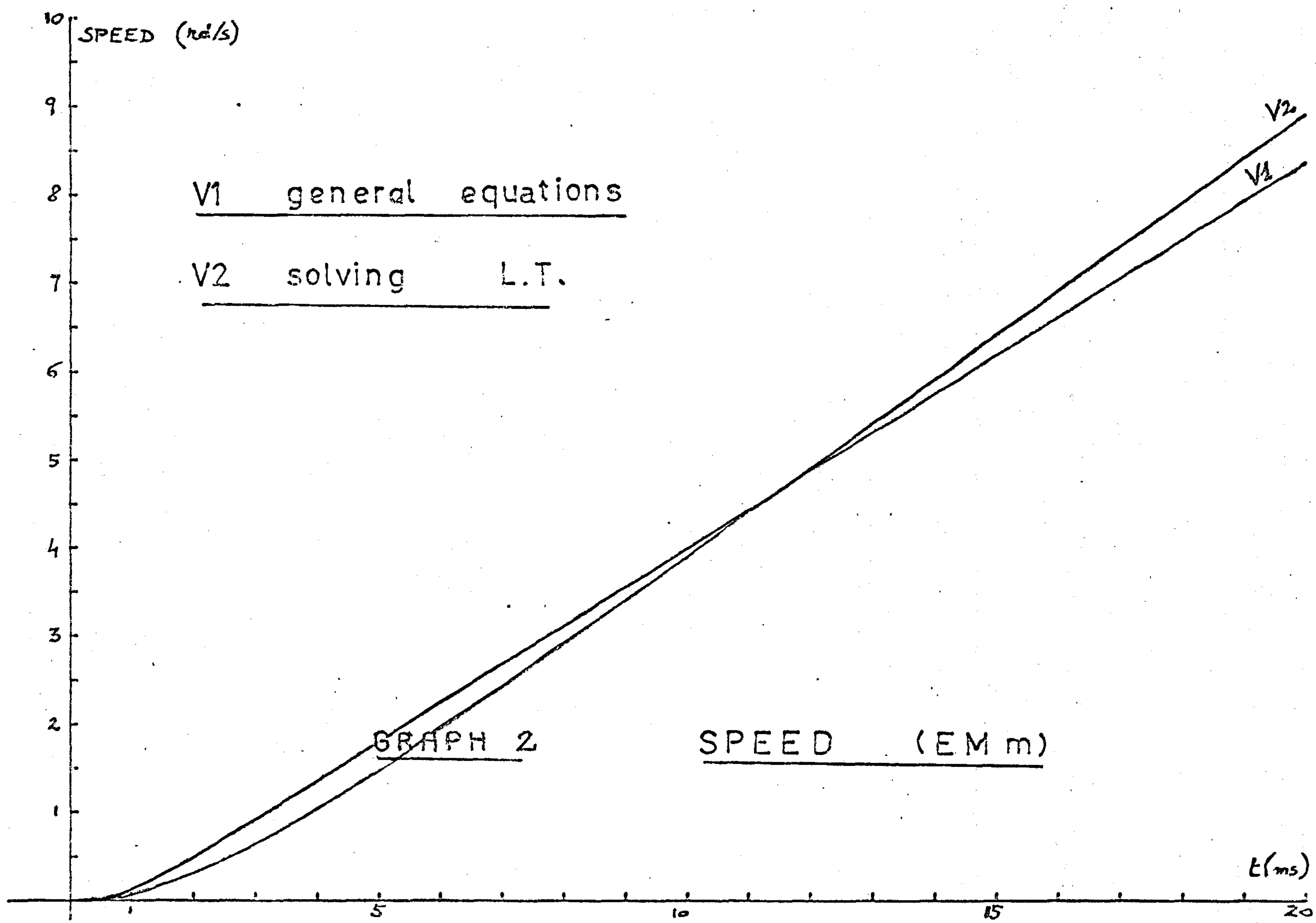
As it can be seen the modulus of the current is approached quite exactly by both methods. But as far as the rise time is concerned there is a very large difference. For example the maximum current is reached in 12 ms by Gross' linear approach and only in 5.45 ms by the mathematical model study. The experimental curve shows that this maximum lies within 5 and 6 ms, which enforces the exactitude of the model.

CONCLUSION

Hence there is no doubt that in power studies the linear approach can be used, but in control systems, where the time response is very important, a better model is required, and the set of non-linear differential equations as defined by (70') and (72) seems to fit this purpose adequately.



GRAPH 1 Current rise in a starting experiment (EM m)


SPEED (rd/s)
t(ms)
V1 general equations
V2 solving L.T.
V1
V2
GRAPH 2
SPEED (EM m)
0
5
10
15
20
1
2
3
4
5
6
7
8
9
10

CHAPTER 3 :

PERMANENTIC MOTOR PERFORMANCES:

P.M. motor: n° D2729dv

1 HP / 90 V dc
9.5 A / 650 rpm

The same study will be performed. But a special attention has been given to the justification of the armature reaction.

I) DETERMINATION OF THE VOLTAGE CONSTANT:

A strictly constant excitation flux is used here, and equation (7) can be fully applied, and K considered as a constant.

The plot of the curve $E(\Omega)$ gives by least mean square error approximation(Appendix A), a very good straight line. Only in very low speeds, which cannot give any accurate measurements, a very light non-linearity appears. The following results have been found:

$$K = 1.13 \text{ V/rd/s}$$

II) VOLTAGE DROPS:

a) RESISTANCE AND ARMATURE REACTION:

The experiments used for a dc motor cannot be applied here, unless the poles can be taken out, or demagnetized. In the PM machine it is impossible because the poles are stuck to the stator.We have no action on the excitation flux.

Let us do another set of experiments.The machine,running as a generator,delivers a voltage Ug and satisfies the equation

(73) $$Ug=E-RI-\varepsilon$$

where E is the no-load voltage and R the non-linear resistance,ε being the armature reaction.

If the machine is running now as a motor,one must have

(74) $$Um=E+RI-\varepsilon$$

where Um is the driving voltage

But now the motor has to rotate in the opposite sens to have the same armature reaction characteristics.

Using the measurements techniques seen previously,E-U can be determined at different loads,first in generator then in motor.This will give

(75) $$\begin{cases} \alpha_g = E - Ug \\ \alpha_m = E - Um \end{cases}$$

from (73),(74) and (75) let write

(76) $$\begin{cases} \alpha_g = RI + \varepsilon \\ \alpha_m = -RI + \varepsilon \end{cases}$$

and that leads to

(77) $$\begin{cases} \varepsilon = \dfrac{\alpha g + \alpha m}{2} \\ RI = \dfrac{\alpha_g - \alpha_m}{2} \end{cases}$$

The assumption made was that the armature reaction was the same for each experiment in motor and generator mode.

The construction of those curves(figure 17) shows that ε is always very small.It does not exceed the asymptotical value of 1 V. This fact emphasises the assumption made in NOTE 3,that the armature reaction saturates.In a dc. machine this saturation occurs over the maximum admissible current but with the PM motor it is found to lie in the range of low currents.

In the following studies the armature reaction can be neglected.This is not in contradiction with the theory as shown in the following note.

NOTE 4: ARMATURE REACTION: THEORETICAL JUSTIFICATION

For the purpose of simplification let take a bipolar dc. machine as shown on figure 15a.The poles induce an excitation flux following the direct axis of the machine.

The neutral axis being defined as in quadrature with the direct axis,let have the brushes on those.

The load current I will behave as shown on figure 15a.It will induce a flux in leading quadrature on the direct flux Ød.

ASSUMPTION:If no saturation occurs and the brushes axis is on the neutral axis there is no armature reaction.In fact,in a non-saturated machine,the linearity allows to add each componant of the flux to give the resulting flux.The excitation flux,under no-loa ,induces the no-load voltage E.The quadrature flux,completely symetrical cannot induce any flux within the neutral axis.Therefore the no-load condition is equal to the load condition and no armature reaction occurs .

FIG 15

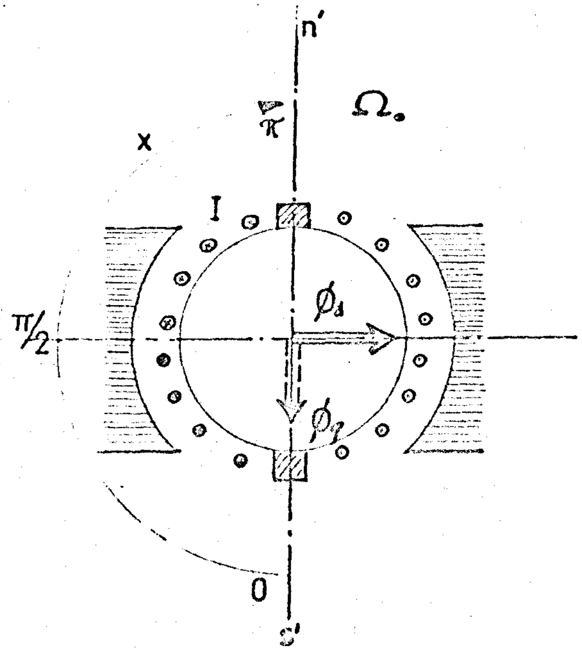


FIG 15 a

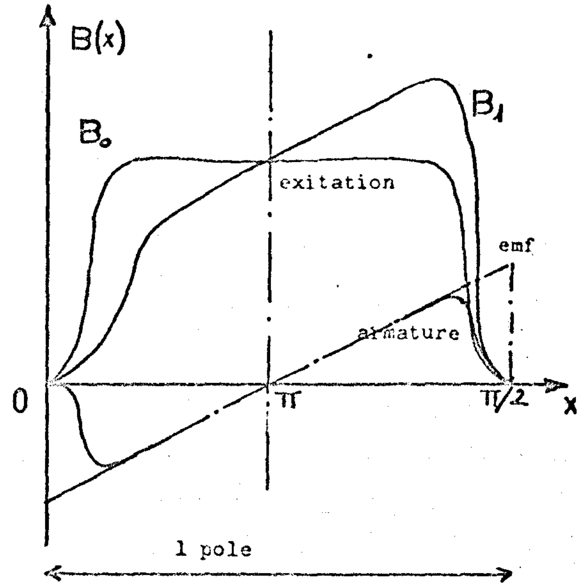


FIG 15 b

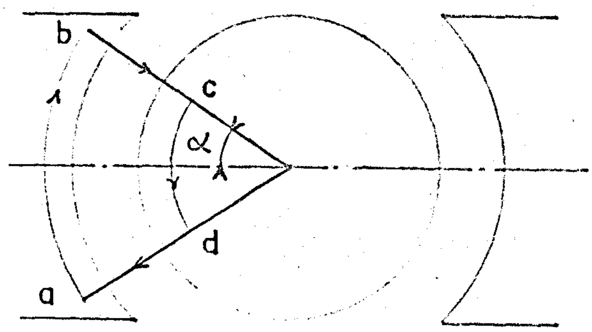
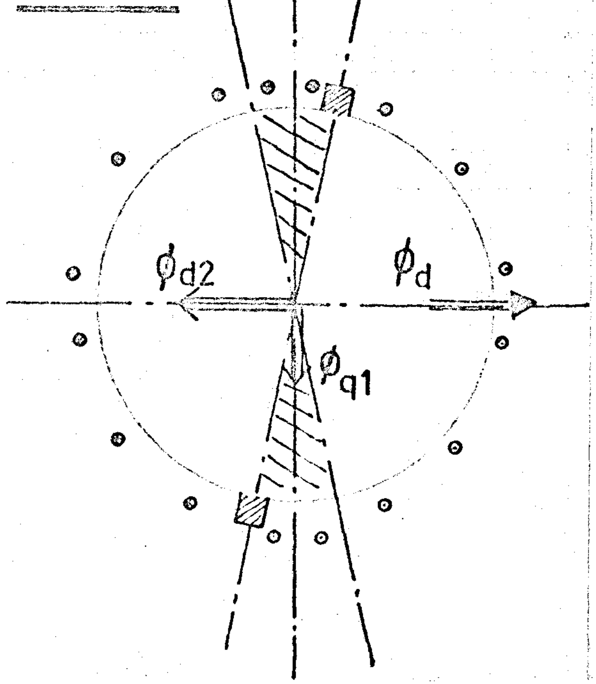


FIG 15 c

FIG 15 d



1) INFLUENCE OF THE SATURATION

On figure 15b the variation of the flux density with x(electrical angle of the machine)can be seen.Let consider now a portion α of the rotor as shown on figure 15c.The Ampere Law applied to the circulation(abcd),gives the e.m.f. induced in α to be proportional to the ampere turns lying in the angle α,hence to α itself(if we neglect the circulation along cd and ba which is reasonable because of the radial induction).

The variation of the e.m.f. is hence triangular with the zero minimum at $\pi/2$.But because of the tips,the variation $B(\alpha)$ (armature flux density) does not follow the e.m.f. curve at α=0 and π;the shape of the curve is given approximately on figure 15b.So that the resulting flux available to generate the e.m.f. under load is $B1(\alpha)$.

But it is possible to see that in the trailing tips the two flux densities are subtracted one from the other,there is no saturation But in the leading tips the two flux densities are added one to the other and because of the saturation the final result of this vector addition is less than the sum of the absolute values of each flux. Therefore the area under $B1(\alpha)$ is less than that of $Bo(\alpha)$.The total available flux is hence less than the unload condition.This constitutes the quadrature armature reaction.

2) SHIFTING OF THE BRUSH AXIS:

On figure 15d the armature can be separated into two regions.

In region (I) the current I determines the quadrature flux Øq1 as seen previously.In the cone(2α) it can be seen that the current I induces a flux Ød2 which lies along the direct axis and subtracts the excitation flux from Ød.This constitutes the direct armature reaction.

3) COMPARISON OF THE BEHAVIOUR OF THE ARMATURE REACTION IN AN E.M. AND A P.M. MACHINE:

*Usually the PM motor works in an area which is less saturated than the normal EM machine.Therefore the sum of the two fluxes in the leading tips saturate relatively later than in those of an EM machine.Hence the diminuation of flux is less in a PM motor and the quadrature armature reaction is less than in an EM machine.

*Under the effect of the commutation the brushes axis shifts because the current density in a brush is not symetrical.So that even if geometrical brushes axis is on the neutral axis,effectively a fictitious shift will be observed and a direct armature reaction will appear,decreasing the available flux.

-*In the case of an EM machine:

On figure 16a the behaviour of the different fluxes is analysed The componant along the axis qb perpendicular to the brushes axis(b) gives the flux which induces the utilizable e.m.f.

In an EM machine the quadrature axis has a high reluctance. Therefore the shift of the brushes axis does not affect the quadrature

FIG 16

FIG 16 a

flux analysis in an EMm

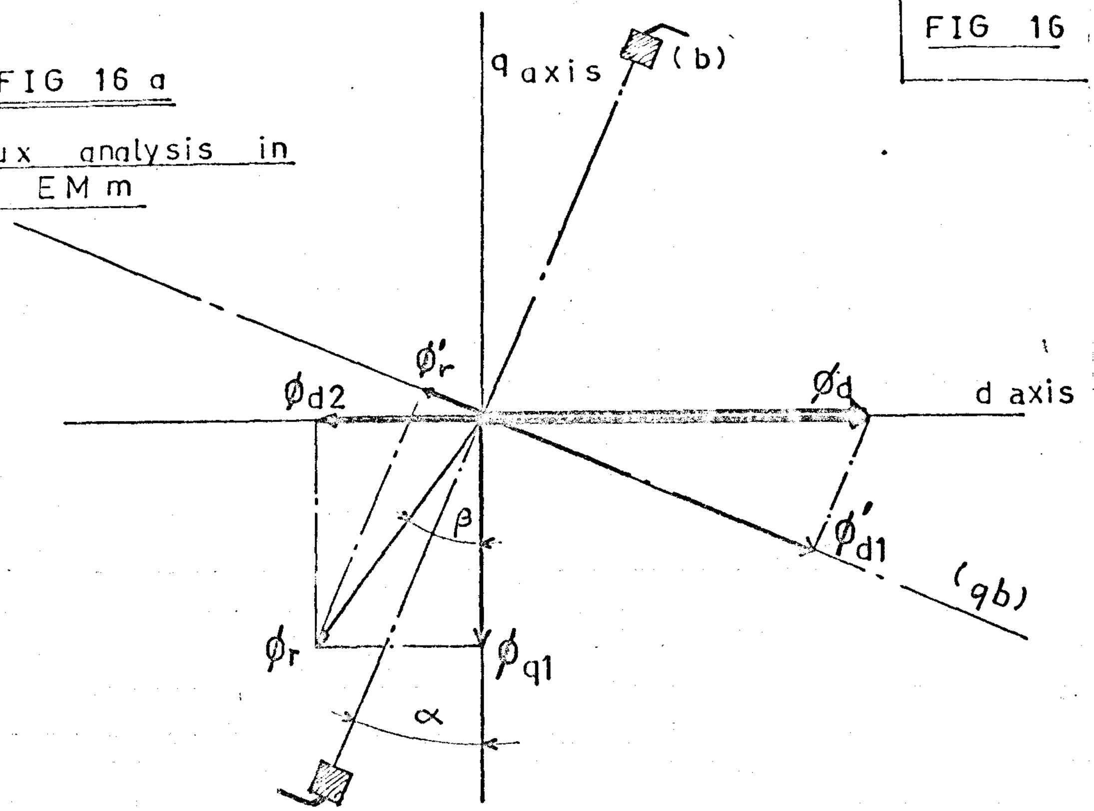
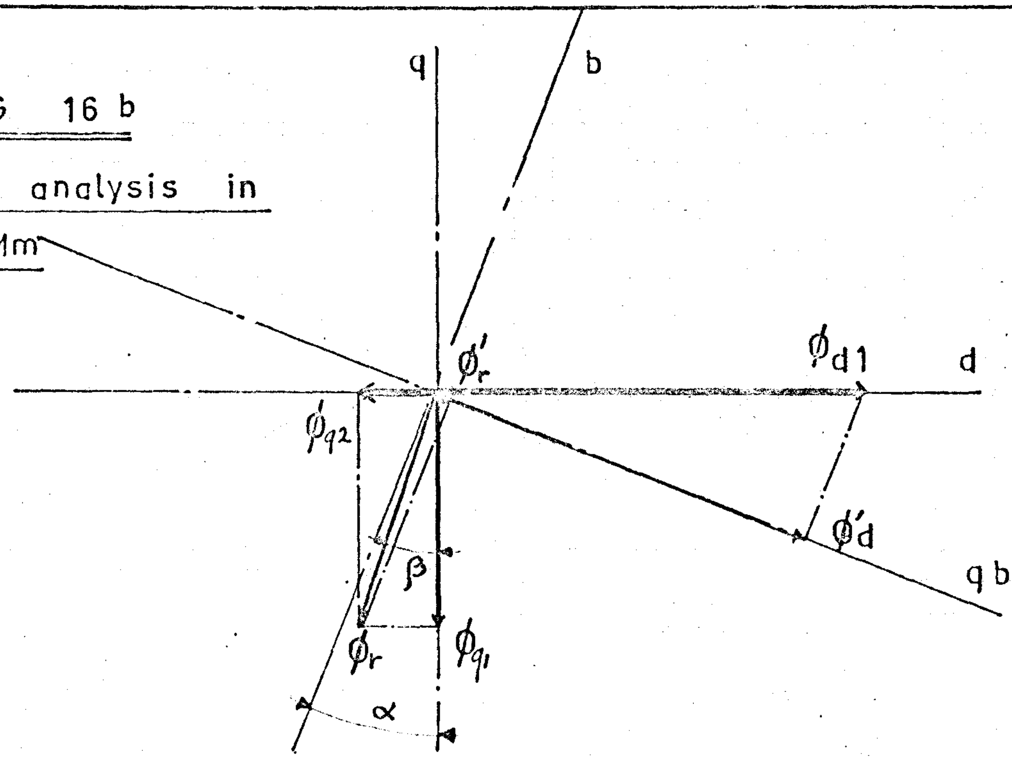


FIG 16 b

flux analysis in a PMm



flux $\phi q1$.But the direct axis has a low reluctance,therefore the flux resulting from the shift is relatively high as shown by $\phi d2$.

The resulting vector ϕr has rotated more than α and the component of ϕr on the axis (qb) is on the opposite side of the component of $\phi d1$,direct excitation flux,on this same axis.The resulting flux $|\phi' d1| - |\phi' r|$ is less than $\phi' d1$.The resulting direct armature reaction is positive and acts against the excitation. $\phi d2$ being proportional to the load current,this armature reaction grows with I and adds its effects to the quadrature armature reaction.

-*In a PM machine:

The direct axis has a high reluctance due to the low permeance of the magnetic materials of the poles,and the quadrature reluctance is relatively low.Therefore the shift will cause an increase in the quadrature flux and as the direct axis reluctance is high,the direct armature reaction $\phi d2$ is relatively small(figure 16b).

Hence the resulting flux ϕr has made a turn of an angle β less than α and the active component $\phi' r$ on (qb) adds to the active component $\phi' d1$ of $\phi d1$.

So that now the direct armature reaction adds its effects to the excitation and the quadrature subtracts.The result is of course ,a total armature reaction which is very low even negative for small intensities,hence small shifts.

But as the load increases the shifts increase and are stabilized at a certain moment,the brush axis will be at the trailing edge of the brush.But $\phi d2$ increases while $\phi q1$ increases less and β

becomes larger until becomming superior to α. The armature reaction increases but remains very small and stabilizes at a saturation point which is very low.

We have tried to justify in theory the result that the armature reaction in a PM. motor is small and negligible, which is in accordance with the experiment.

RESULTS

The plots of figure 17 have been fed into a computer program and the result gives the equation of the resistance

$$R=1.2+.68(1-\exp(-.277\ I))/I$$

III) MECHANICAL PARAMETERS:

1) LOSSES:

No energy has to be provided for the excitation. A test with the machine running as a motor will give the curve

$$W= a\,\Omega^2 + b\,\Omega$$

. As seen for the EM machine a and b are determined by the least mean square error and given by the result sheet as

$$\begin{cases} a = 10^{-2}\ \ W/(rd/s)^2 \\ b = 0.323\ \ W/rd/s \end{cases}$$

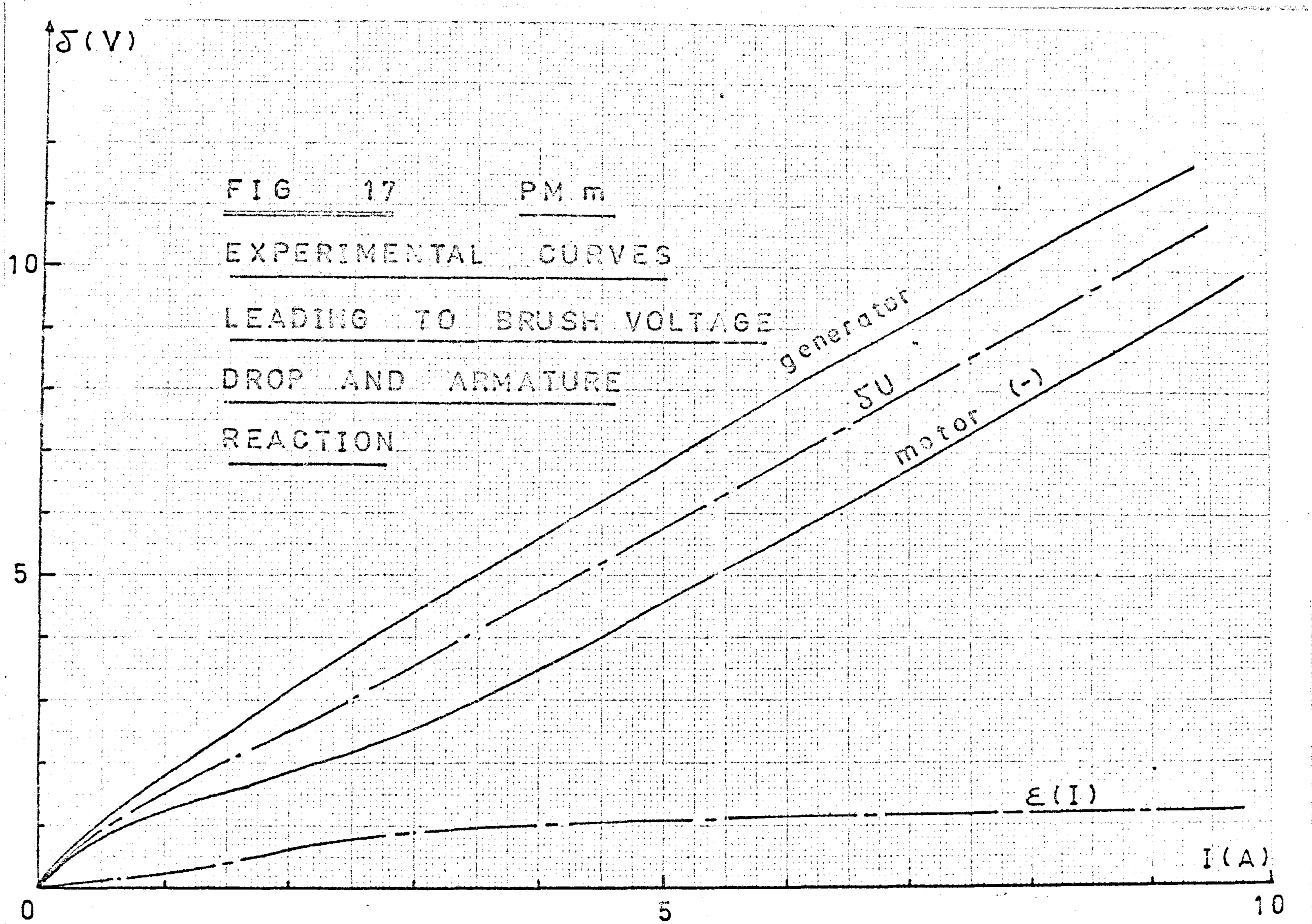


FIG 17 PM m

EXPERIMENTAL CURVES LEADING TO BRUSH VOLTAGE DROP AND ARMATURE REACTION

2) RUNNING DOWN:

Starting from various speeds, the predetermined parameters (a,b) are used to find the moment of inertia.The average is found to be

$$J = 0.0192 \quad kg/m^2$$

But we have tried to solve the set of non-linear equations by a computer program P4

(32)

$$\left(Ni + \frac{b}{a}\right) e^{-\frac{a}{J}Ti} = \frac{b}{a}$$

where Ni are starting speeds and Ti the corresponding running down times.The following values are calculated:

$$\begin{cases} a = .0105 \quad W/(rd/s)^2 \\ b = .323 \quad W/rd/s \\ J = 0.01905 \quad kg/m^2 \end{cases}$$

Compared to the values found in the above sections, one can due to the cross checquing,state with a remarkable precision that

$$\begin{cases} a = 10^{-2} \quad W/(rd/s)^2 \\ b = .323 \quad W/rd/s \\ J = 0.019 \quad kg/m^2 \end{cases}$$

3) EFFICIENCY OF THE PM MACHINE:

If Po is the total mechanical and iron losses at a certain

```
A4173,T100.
RUN(S)
LGO.
'        6400 END RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     SOLVING A NON LINEAR SYSTEM
      DIMENSION X(3),Y(3),Z(10)
      COMMON T(3),OMEG(3)
      EXTERNAL RMIN
      PIS=3.159265/30.
      WRITE(6,9)
      DO 4 M=1,2
      READ(5,96) (Z(I),I=1,10)
4     WRITE(6,97)(Z(I),I=1,10)
C    X(1)=J,  X(2)=A,  X(3)=B
C    WE READ THE INITIAL VALUES OF J,A,B, WHICH HAVE TO BE APPROXIMATED
      READ(5,10) X
      WRITE(6,11) X
      DO 2  I=1,3
      READ(5,12)OMEG(I),T(I)
2     OMEG(I)=OMEG(I)*PIS
      H=.1
      NB=10
      DO 3 I=1,4
      CALL GRAD(RMIN,3,X,H,NB,V,Y)
      WRITE(6,13) I,V
      WRITE(6,11) X
      ABSV=ABS(V)
      IF(ABSV.LT.0.01) STOP
3     NB=NB*5
      STOP
9     FORMAT(1H1,10X,27HSOLVING A NON LINEAR SYSTEM/1H-,10X,14HINITIAL V
     1ALUES)
10    FORMAT(3F20.5)
11    FORMAT(10X,2HJ=,F10.5/1H-,34X,2HA=,F10.5     ,34X,2HB=,F10.5/1H-)
12    FORMAT(2F20.6)
13    FORMAT(10X,12HFINAL VALUES,30X,5HSTEP ,I1/1H-,34X,2HV=,E15.3)
96    FORMAT(10A6)
97    FORMAT(30X,10A6)
      END
$IBFTC
      FUNCTION RMIN(X,Y)
      DIMENSION X(3),Y(3)
      COMMON  T(3),OMEG(3)
      R=0.0
      DO 1 I=1,3
      Y(I)=-X(3)/X(2)+(OMEG(I)+X(3)/X(2))*EXP(-T(I)*X(2)/X(1))
1     R=R+Y(I)**2
      RMIN=R
      RETURN
      END
'        6400 END RECORD
 A.S.E.A.   ALONE  WITHOUT ANY LOAD
************************************
0.0114              0.0005              0.195
  2100.             9.
 1600.             8.4
 1300.             6.9
'        6400 END FILE
```

17086

speed the efficiency in each mode is given by 26 and 27 where ie is set equal to zero:

(26')
$$\begin{cases} n_g = \dfrac{UI}{UI+RI^2+Po} \\ n_m = \dfrac{UI-RI^2-Po}{UI} \end{cases}$$

Figure 18 shows those curves which have been plotted for the normal speed of 650 rpm and the corresponding voltage applied 76.5 V.

4) MECHANICAL TIME CONSTANT:

As defined for the EM, a mechanical time constant can be calculated(taking R=1.2 Ω for very large currents)

$$t_m = \frac{J\ R}{K^2} = 18\ x10^{-3}\ s$$

$$t_m = 18\ ms$$

IV) ELECTRICAL TIME CONSTANT:

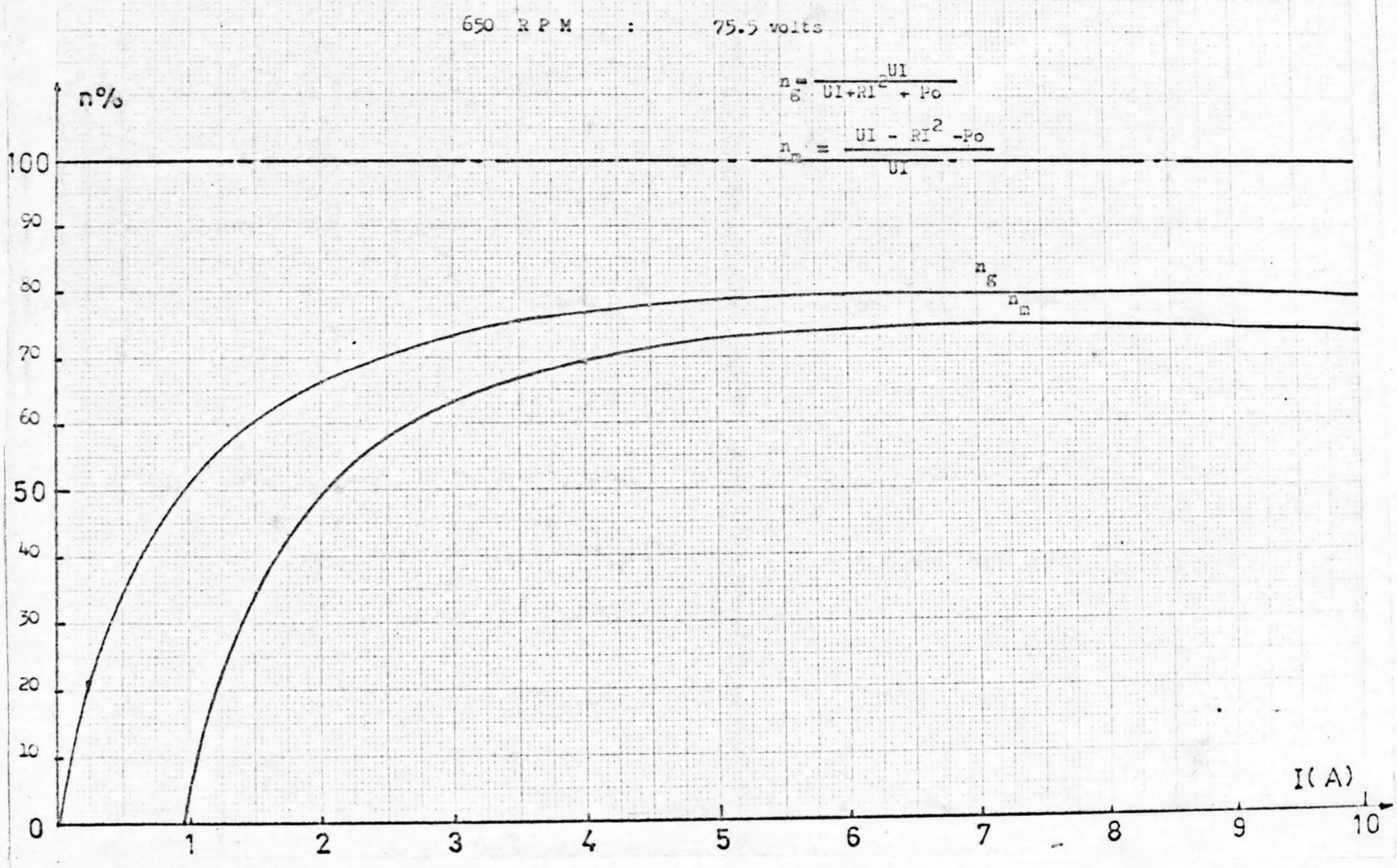
The global inductance L of the machine is then measured.

In an EM machine this inductance is the sum of the armature inductance La and the mutual inductance of the armature and poles which one is high due to the presence of iron.

Here a very low inductance can be expected because no winding is present on the stator hence there is no mutual inductance.

The experiment is performed as previously:

FIG 18 EFFICIENCY CURVES OF PM m



1) STATIC MODE:

On the blocked rotor a step voltage variation is applied,and the rise time of the current is recorded.

2) DYNAMIC MODE:

Great troubles appear when performing this measurement. First of all the commutation is very bad(there is no inter-pole)so the shape of the steady state current flowing through the armature is given on figure 19a.

We have tried the measurement in the region where I is constant,hence in a range of 0.5 ms.But the experiment shows that a range of 200 μs is only utilizable.The best readings are on caliber 20 μs/cm for the scope.If we go below this caliber the half rise time cannot be reached because of the commutation of the SCR which rings the circuit,as shown on figure 19b.The response is assumed to be

(78) $F(t) = \exp(-Rt/L) + f(t)$ where $f(t) = A\, e^{-\alpha t} \sin \omega t$ and α is large while ω shows a high frequency

f(t) being a damped sinewave(due to the capacitor of the SCR and the inductance of the circuit) disappears in a very short time of about 20 μs.

Therefore the half rise time reading must be over 50 μs. This gives a restriction on the resistor:

FIG 19

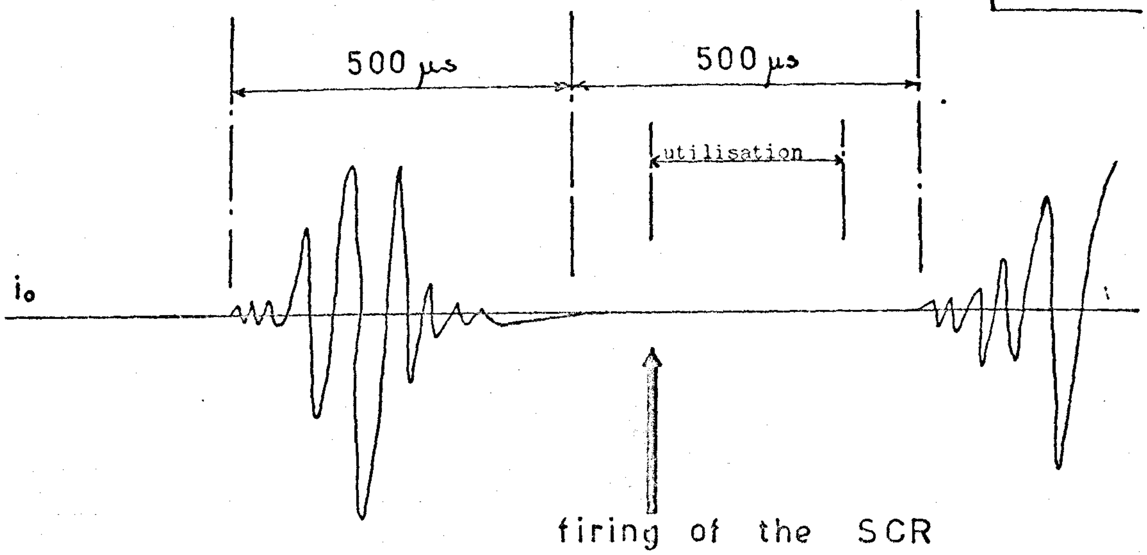


FIG 19a NOISE DUE TO COMMUTATION (PM m)

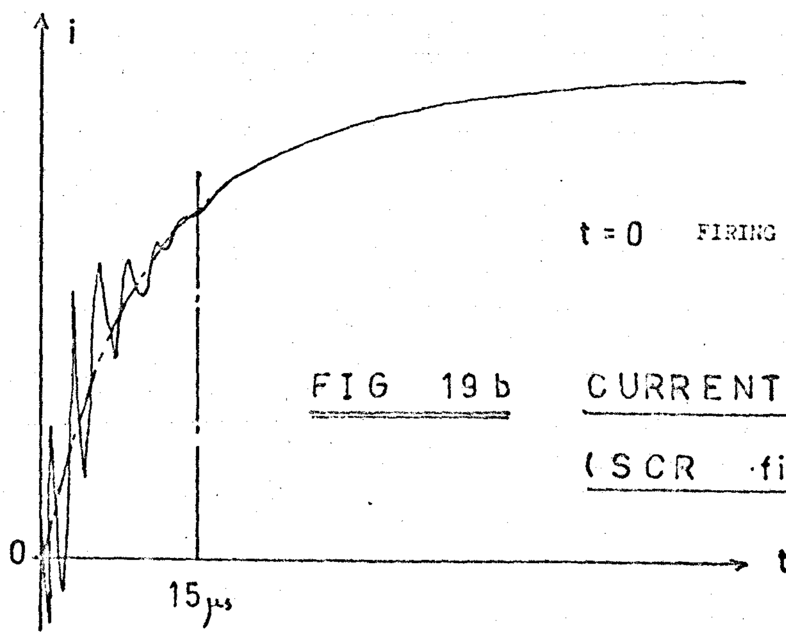


FIG 19b CURRENT RAISE (SCR firing)

$R < 25\ \Omega$

and therefore in full speed that implies a current $I > 3$ A.

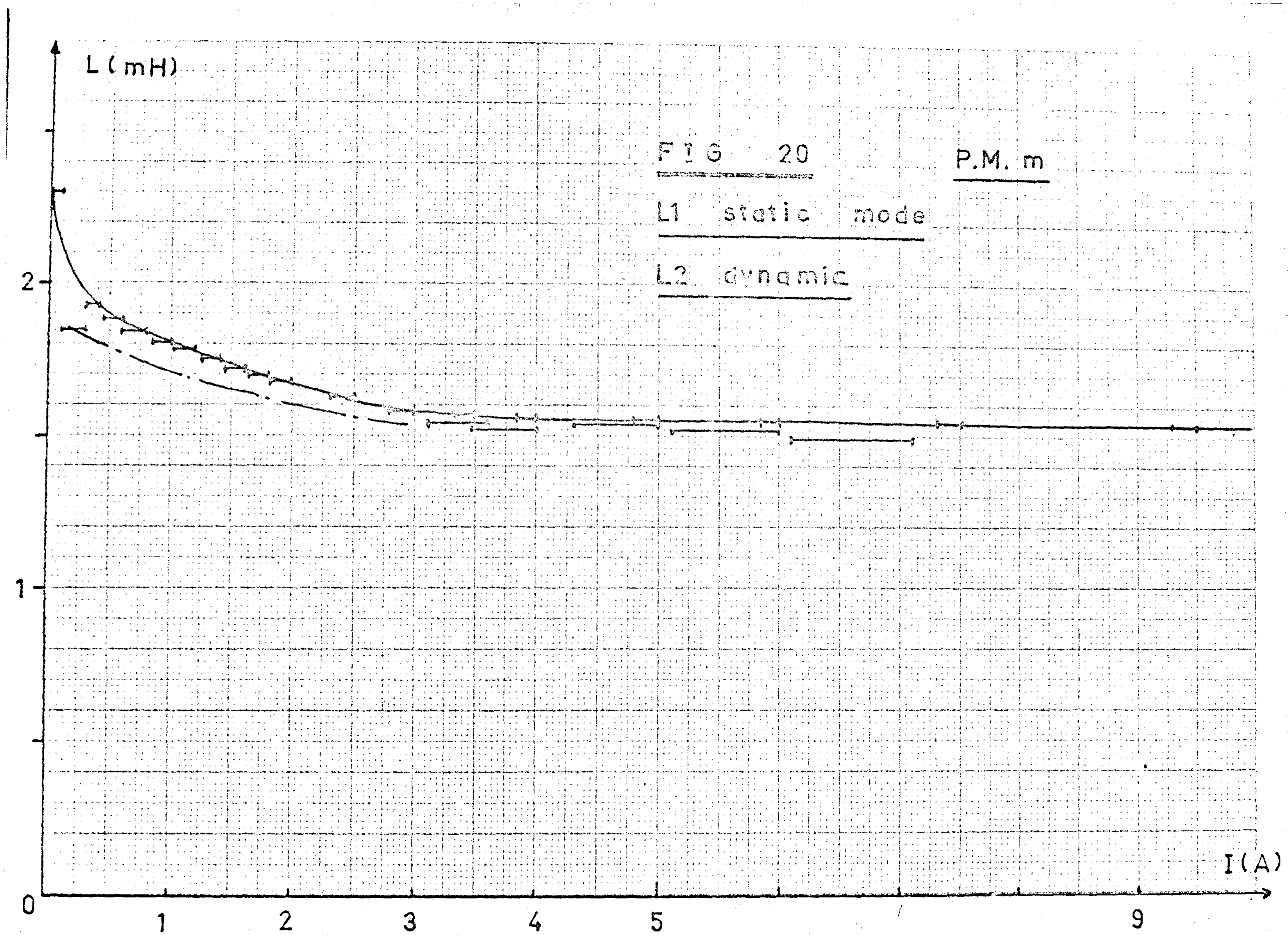
For this range one can measure the half rise time with the same accuracy as for immobility.But for low currents R is required very large and T lies in a range of 2 µs which is impossible to read correctly.Only a very rough approximation can be done.

CRITICISM :

Large jumps of intensity must occur,that means of course, that the assumptions of U and Ω constant are not true.

Experimentally the results are 1 or 2 volts drops on U and a maximum of 10 rpm on the speed.

Figure 20 shows the plots of the inductance in static and dynamic modes.A very similar behaviour of the inductance in the PM and the EM machine can be noticed.


L (mH)
FIG 20
P.M. m
L1 static mode
L2 dynamic
2
1
0
1
2
3
4
5
9
I (A)

CHAPTER 4:

MODEL OF THE PERMANENTIC MOTOR:

I) LINEAR APPROACH:

If one considersthe different parameters as constants the same equations as for the EM motor can be applied(see chapter B-I) The same computer program P2 is used to determine the following equations:

Starting with no-load, with a step input of U=75.6 V

$$\begin{cases} i(t) = .9 + 84 \exp(-65\ t) - 85 \exp(-436\ t) \\ \Omega(t) = 66 - 77.5 \exp(-65\ t) + 12 \exp(-436\ t) \end{cases}$$

II) MATHEMATICAL MODEL:

The equations (70') and (72) apply and the following model is defined:

U voltage applied

$R(I)_\Omega = 1.2 + .68(1 - \exp(-.277\ I))/I$

$\varepsilon(I) = 0$ | $J = 0.019$

$a = 10^{-3}$ | $K = 1.13$

$b = .323$ | $\lambda m = \lambda m(t)$

if $I \leqslant 3.8$ A

$$L(I)_{mH} = 2.35-1.26\,I+.98\,I^2-.39\,I^3+.076\,I^4-.007I^5+2.6x10^{-4}I^6$$

if $I > 3.8$ A

$$L = 1.54 \text{ mH}$$

The equations to be solved are :

(79)

if $i \leqslant \frac{b + \lambda m(0)}{K}$

$$\begin{cases} U(t)=R(i)i + L(i)\frac{di}{dt} \\ \Omega(t)=0 \end{cases}$$

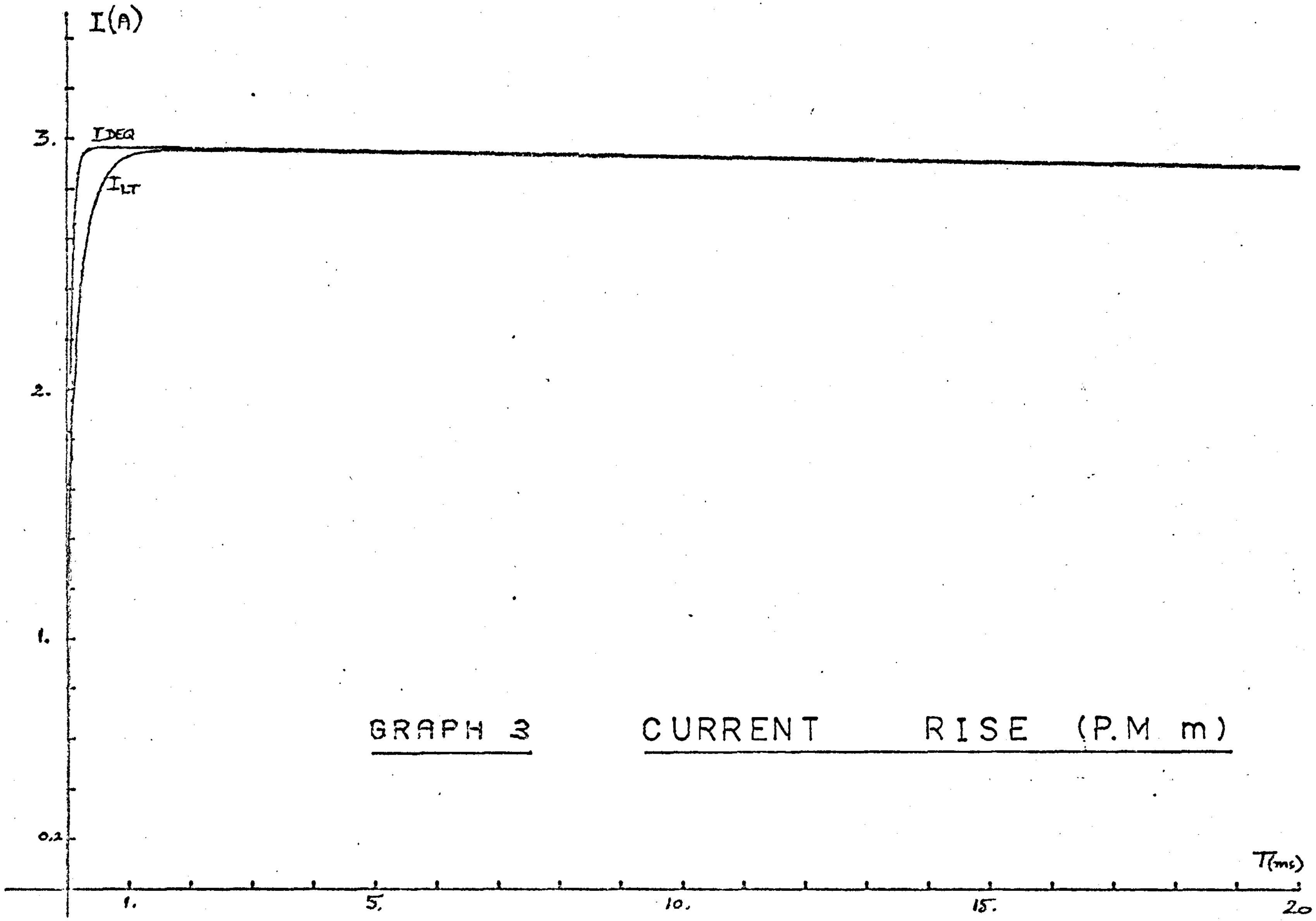
if $i > \frac{b + \lambda m(0)}{K}$

$$\begin{cases} U(t)=K\Omega+R(i)i+L(i)\frac{di}{dt} \\ Ki = a\Omega+b+J\frac{d\Omega}{dt} + \lambda m(t) \end{cases}$$

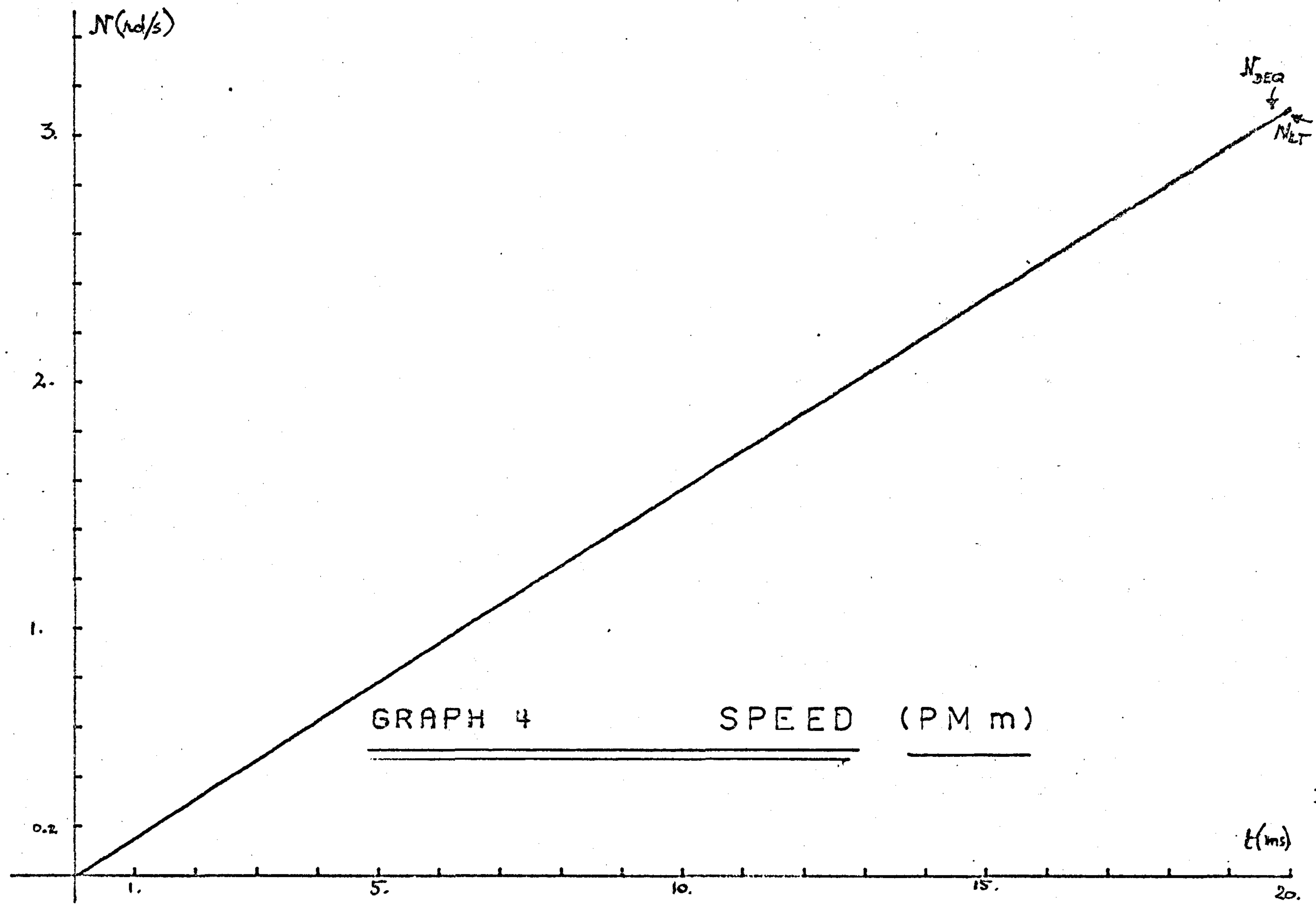
A similar experiment as with the EM motor has been done and it shows in the same way that the mathematical model of the differential equations is better ,especially in control system analysis.

The plots of graphs 3 and 4 are very interesting.They show the comparison of the linear approach and the solution by solving (72) and (70). For the PM motor the result on the speed is very acceptable with both methods.But as far as the current is concerned,a great diference in the methods appears(maximum reached at a time about three times more with Laplace transform than with the actual method)

Of course as the torque follows the current,one has to give up the linear approach method in precise control system design.



GRAPH 3 CURRENT RISE (P.M m)



GRAPH 4 SPEED (PM m)

CHAPTER 5:

COMPARISON OF THE TWO MACHINES:

To be able to compare two machines one has to define some criteria and a domain of comparison.Here two aspects are developed both oriented towards control system applications.

The principal object of this following study is not to try a very fair comparison,the two motors are not designed for the same purpose,but it is to show that the machines can be analysed and their behaviour predicted very precisely,using the non-linear mathematical model.

I) STARTING:

1) STARTING WITH NO-LOAD:

a) ELECTROMAGNET MOTOR:

A step voltage of 220 V applied directly without any current limiting resistor in the circuit would give the curve C1 of figure21a

Previously a normal excitation should be provided.By the same way the curve $\Omega 1$ of figure 21b shows the speed under the previous conditions.

It can be seen that the motor cannot accept those terrible

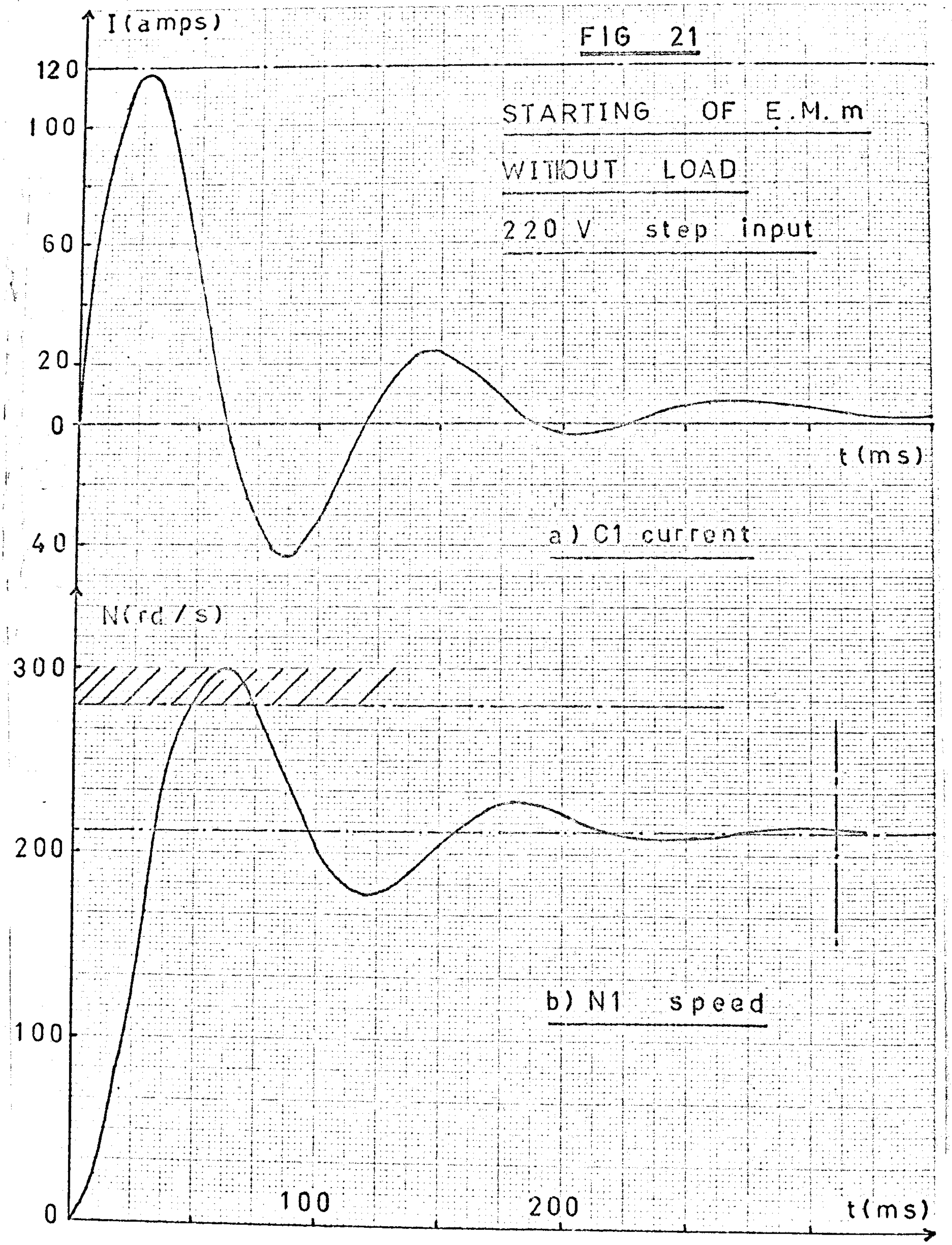


FIG 21
STARTING OF E.M. m
WITHOUT LOAD
220 V step input
I(amps)
120
100
60
20
0
40
t(ms)
a) C1 current
N(rd/s)
300
200
100
0
100
200
t(ms)
b) N1 speed

constraints neither in current nor in speed(the maximum speed allowed being 280 rd/s). A current limiting resistor is required for the starting.

b) PERMANENT MAGNET MOTOR:

On the same basis if a step voltage of 75.6 V is applied to the PM. motor, the curve of figure 22 shows the behaviour of the current.

The starting can be done easily without any inconvinien ces.

2) STARTING UNDER LOAD CONDITIONS:

It is very important and interesting to try to start the motor which has been loaded. The following conditions are chosen:

*A resistive constant torque of 3/4 of the maximum power is applied.

*A step voltage step voltage starts the motor.

Figure 23 shows the behaviour of the EM motor starting with full excitation and no current limiting resistor. The speed overshoots the maximum speed allowed during 25 ms which may damage the machine. Anyway the speed reaches the 1% steady state bandwidth in 310 ms only.

The current goes up to 120 A and remains during more than 50 ms above the maximum current allowed.

We may sometimes try those constraints, but it will damage

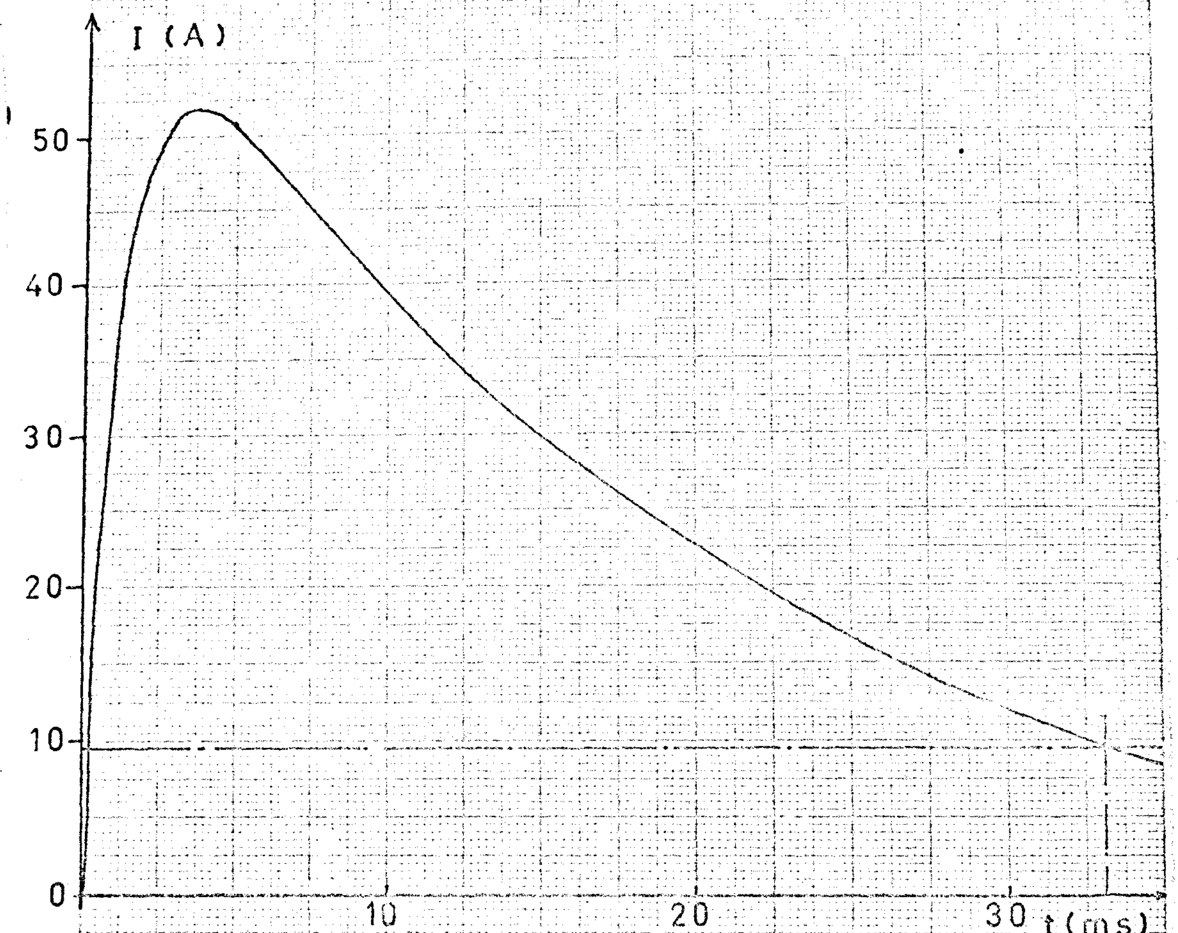
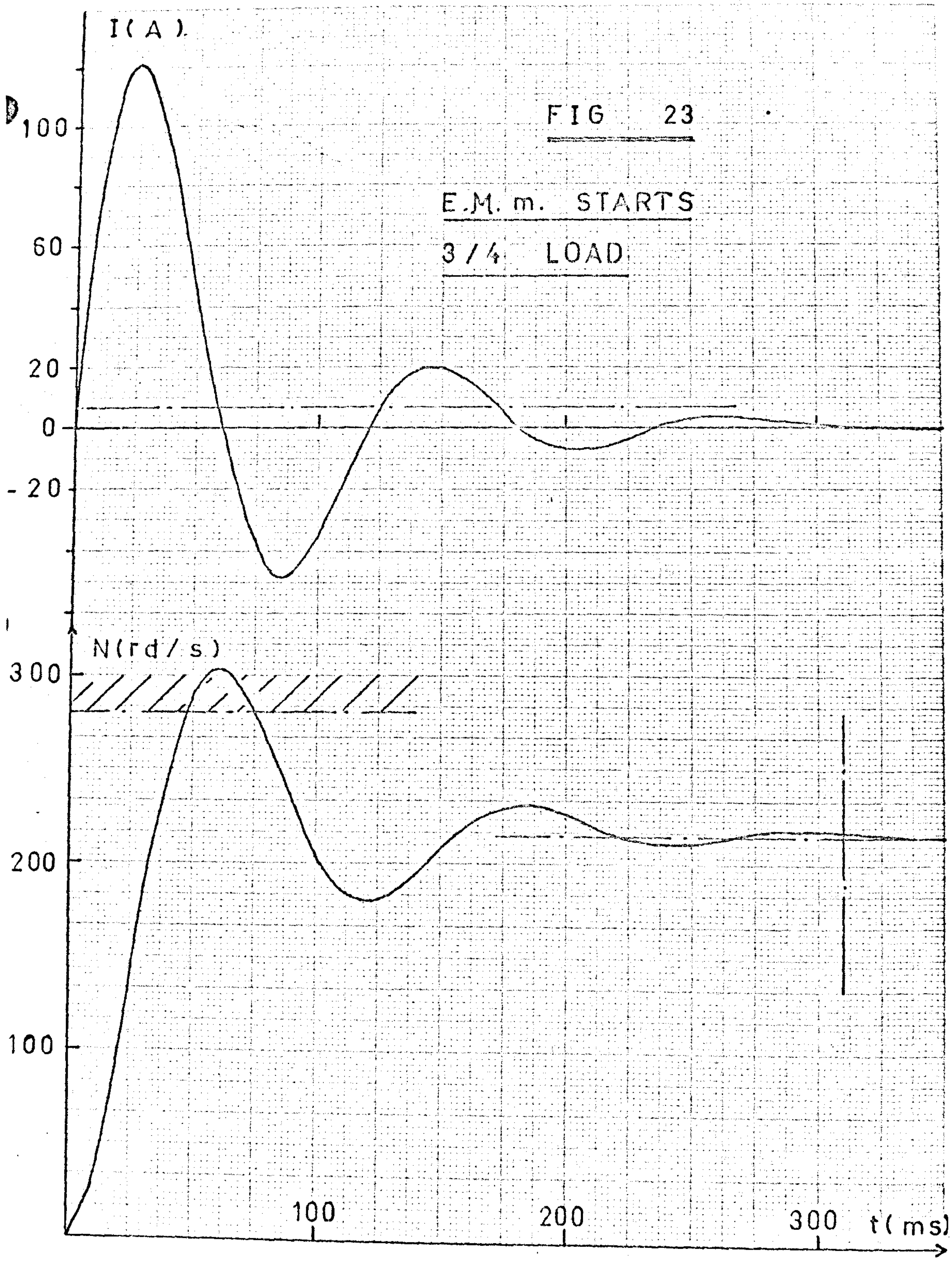


FIG 22

P.M. m. STARTING WITH NO LOAD

Speed reached in 81 ms
(1% bandwidth)


I(A)
100
60
20
0
-20
FIG 23
E.M. m. STARTS
3/4 LOAD
N(rd/s)
300
200
100
100
200
300
t(ms)

the machine and there is no way to use this motor for a control system which requires startings.

Figure 24 shows the behaviour of the PM motor.The current overshoot goes up to 55 A and in 55 ms stays below the maximum current allowed in the steady state.

The designer specifies that 210 A instantaneous current is allowed which is far from this constraint.But the best result is on the speed.No oscillation occurs and the steady speed is reached in the 1% bandwidth in 83 ms which is really a nice performance.

3) STOPPING UNDER LOAD CONDITIONS:

The running down equation (30) can be written as

(30) $$\lambda m = a\Omega + b + J\frac{d\Omega}{dt}$$

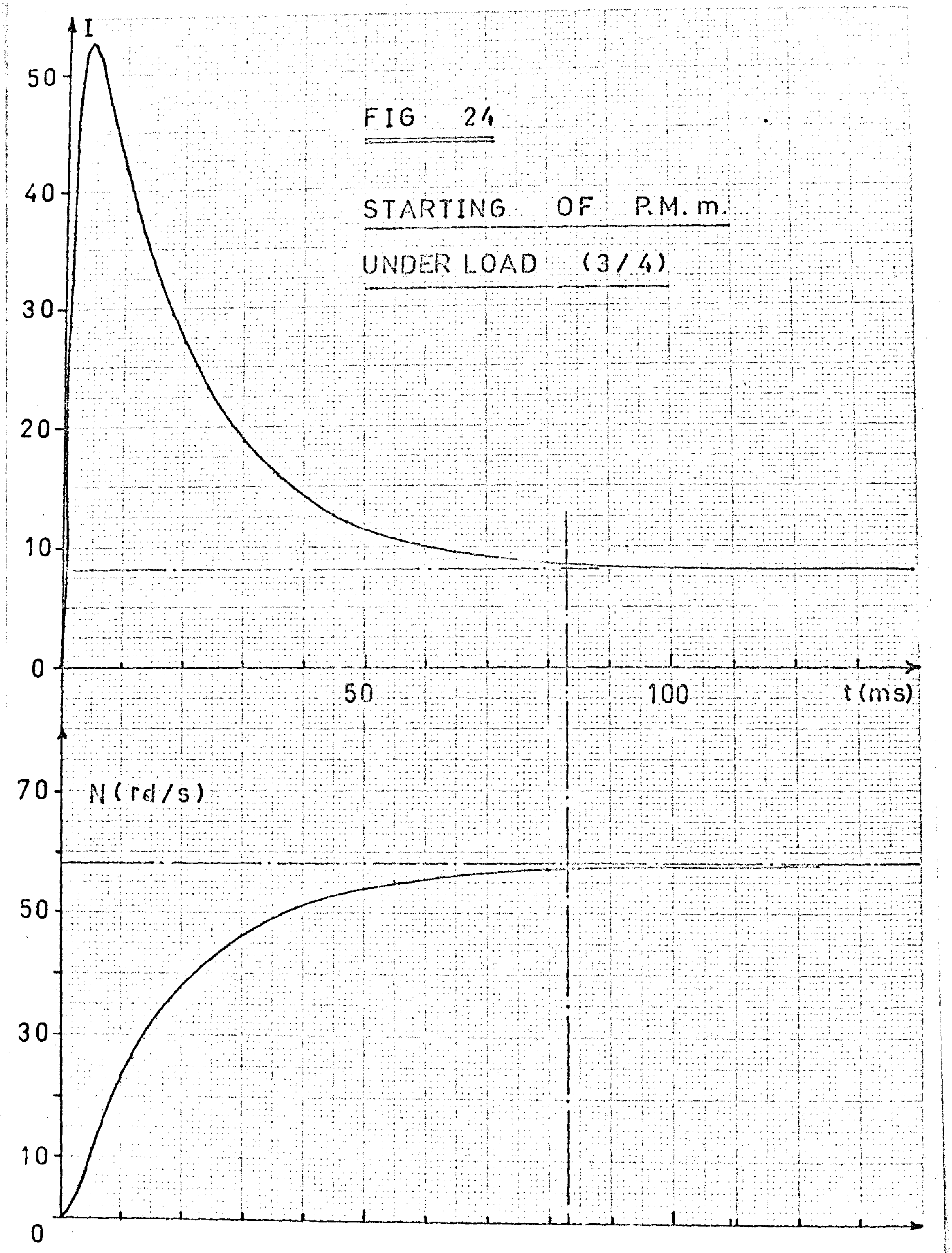
but under load conditions it becomes

(30') $$\lambda m = a\,\Omega + b + J\frac{d\Omega}{dt} + \lambda r$$ where λr is the resistive load torque

If one replace $B = b + \lambda r$ (supposing a constant torque) in equation (30'), one obtain the same equation as (30), the solving of which gives the running down time as

(33') $$T = \frac{J}{a}\,\mathrm{Log}\left(1 + \frac{\Omega o . a}{b + \lambda r}\right)$$

Applied to both motors the results are the following:


I
50
40
30
20
10
0
FIG 24
STARTING OF P.M. m.
UNDER LOAD (3/4)
50
100
t (ms)
70
N (rd/s)
50
30
10
0

EM motor ...... T1 = 0.75 s

PM. motor ..... T2 = 0.147 s

4) CONCLUSION:

If the fact that the EM motor would be damaged if starting occurs very often,is neglected,the stepping mode operation can be a criterion of comparison.

Figure 23 gives the starting time of 310 ms and the stopping time will be 750 ms. The complete operation requires 1.06 s of transitory response time.So that this EM motor cannot be used for steps less than 1 second.

In the same way the PM motor has a starting time of 83 ms and a stopping time of 147 ms which gives a transitory response time of 0.23 s,which is 5 times better than the EM motor.

In a first conclusion of this study,without any doubt the stepping mode control system requires PM motors.It is almost impossible to obtain the same results with an EM motor.

II) TORQUE DISTURBANCES:

Both motors are running at normal speed.We load them with a resistive constant torque corresponding to 3/4 of the maximum power. A disturbance in the torque occurs.The behaviour of each motor is studied.

a) STEADY STATE DETERMINATION:

α) Electromagnet machine:

The maximum power will be

$$P_{max} = 1.1 \text{ kW}$$

so that the 3/4 of this power is

$$P_{3/4} = 0.825 \text{ kW}$$

at a normal speed of 2050 rpm and a normal excitation that leads to a torque of

$$\lambda r = 3.84 \text{ J/rd/s}$$

β) Permanent magnet machine:

$$P_{max} = 0.736 \text{ kW}$$

$$P_{3/4} = 0.55 \text{ kW}$$

at normal speed of 650 rpm the torque will be

$$\lambda r = 8.1 \text{ J/rd/s}$$

NOTE: The torque developped in the PM motor is greater than in the EM motor, that could help the faster speed up.

γ) Determination of the voltage and the current at sready state:

The (70') become for steady state

(70'') $$\begin{cases} U = K\Omega - K'\Omega + Ri \\ Ki = K'i + a\Omega + b + \lambda r \end{cases}$$

Knowing the equations of $K'(i)$, from the second equation i can be determined so that:

$$i(K-K') = a\Omega o + b + \lambda r$$ where Ωo is the normal speed

Having io, the voltage to be applied is given by:

$$Uo = K\Omega o - K'(io)\Omega o + R(io)io$$

Applying this for both motors we have found:

EM	$\Omega o = 215$ rd/s	$\lambda r = 3.84$ J/rd/s
	$io = 4.372$ A	$Uo = 219.85$ V
PM	$\Omega o = 68$ rd/s	$\lambda r = 8.135$ J/rd/s
	$io = 8.086$ A	$Uo = 87.23$ V

b) DISTURBANCE:

Let us choose a torque disturbance of 10% of the initial value. The voltage remains constant, and so does the excitation of the EM motor.

At time t=0 the disturbance is applied and one has to solve the equations of (70') with the initialy predetermined conditions.

1) THE TORQUE DISTURBANCE IS A STEP INPUT:

Figure 25 shows the response of the PM motor . The speed reaches its steady value in 38 ms (within a bandwidth of 1‰) and without any oscillation. It can be seen that the speed has droped from 68 rd/s to 67.37 rd/s , that is to say a 9.5‰ drop.

Figure 26 shows the behaviour of the EM motor. Here the response is oscillatory. The 1‰ bandwidth of the steady state is reached in 50 ms only, but the speed drop represents only 1.3‰. That is to say 7 times less than the PM motor.

This comparison shows that the PM motor is most affected by a disturbance. As far as the power domain is concerned, constant speed maintenance without any feed back regulation, the EM motor shows its advantages.

2) TORQUE DISTURBANCE IS A RAMP INPUT:

A 5 ms raising time of the ramp is considered and then the torque remains constant (a step input never occurs, there is always a ramp rising part).

Figure 27 shows on curve C(5) the behaviour of the response if the ramp is stated as 20 ms. The drop of the speed is obviously the same, but the time is now 30 ms for a 5 ms ramp and 39 ms for a 20 ms ramp.

This result should not be taken as a cause for concern. The PM motor follows the torque variation very closely, so that when the torque has reached a speed which is close to the steady response

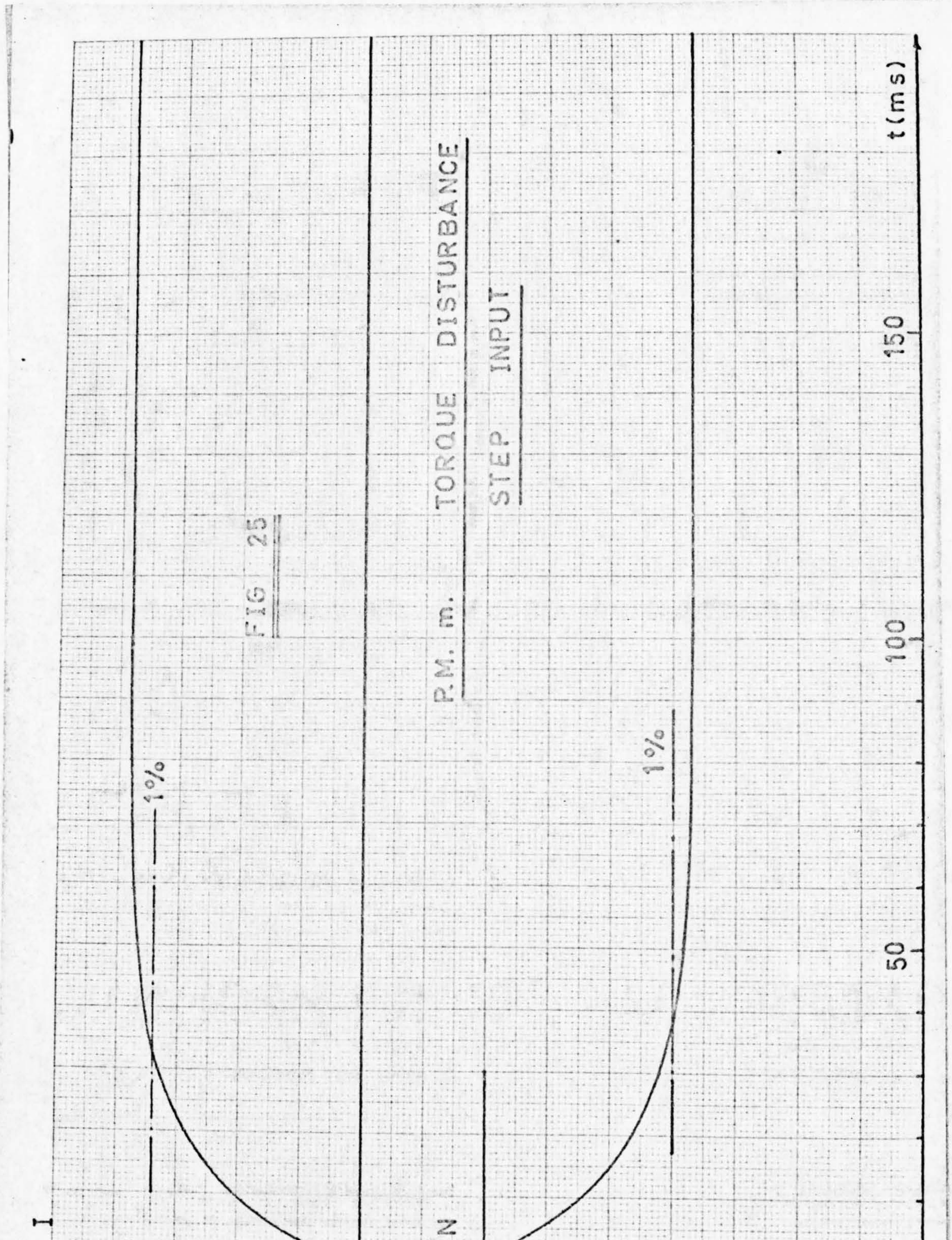


FIG 25

P.M. m. TORQUE DISTURBANCE

STEP INPUT

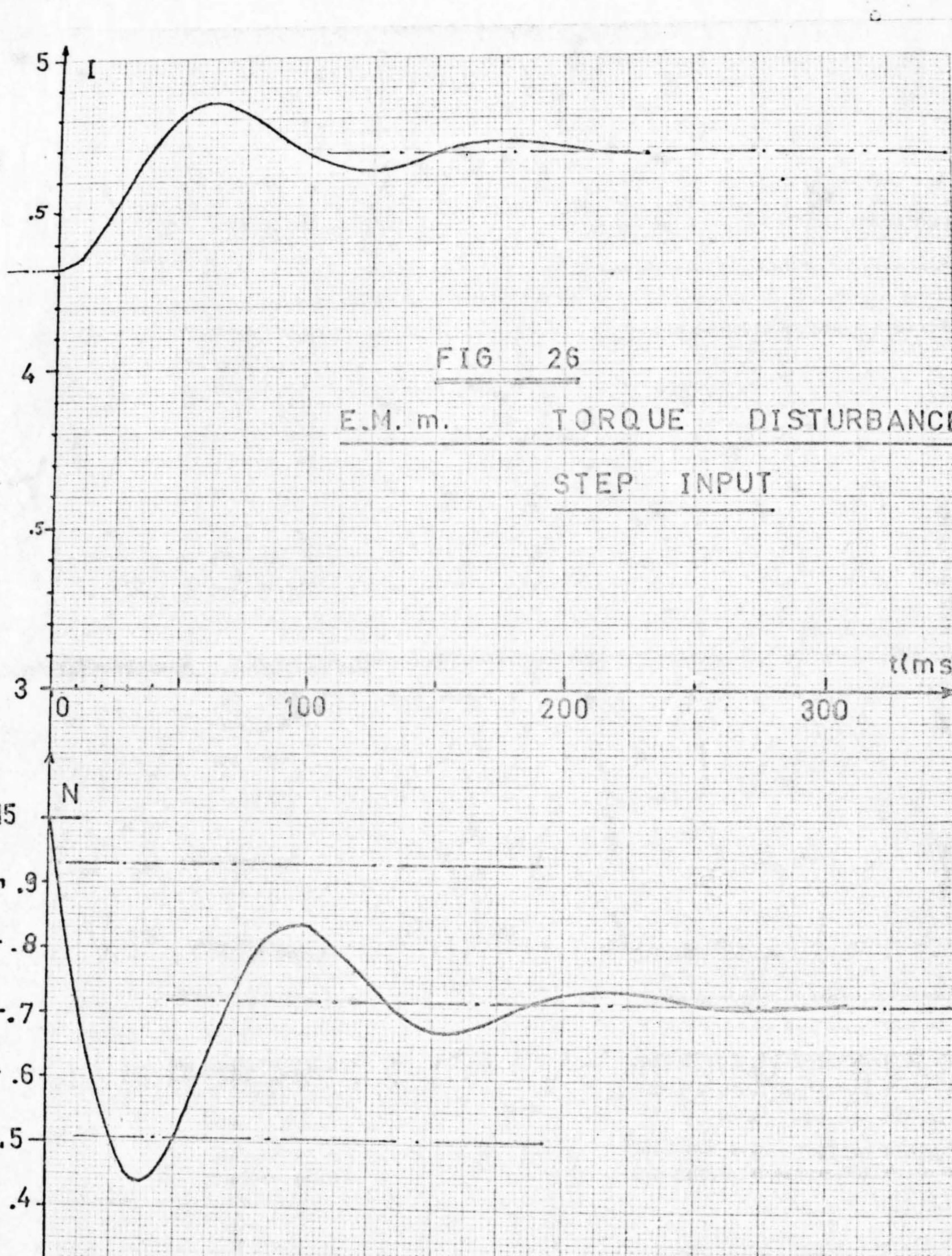


FIG 26

E.M. m. TORQUE DISTURBANCE

STEP INPUT

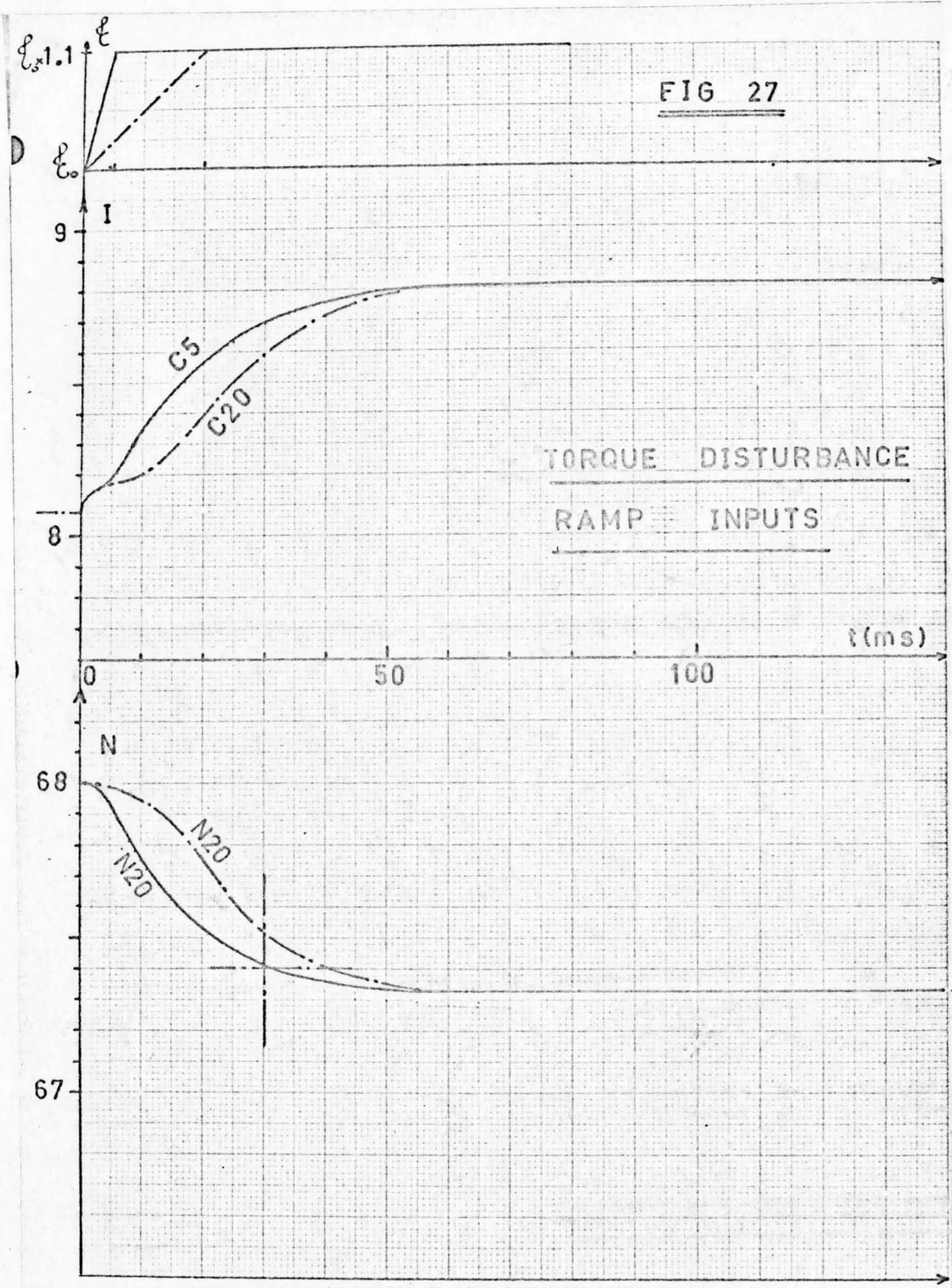


FIG 27
TORQUE DISTURBANCE
RAMP INPUTS
ξ_s 1.1
ξ
ξ_0
I
9
8
C5
C20
t(ms)
0
50
100
N
68
67
N20
N20

and the 1‰ bandwidth is very rapidly reached even if the steady state is reached later than for the step input.

On figure 28, C5 and C20 shows the same reactions. But now the 47 ms time for a 5 ms ramp input is explained by an overshoot which is less in this case than for a step input. Besides the 20 ms ramp gives a response of 54 ms. The EM motor has more difficulties to follow the input.

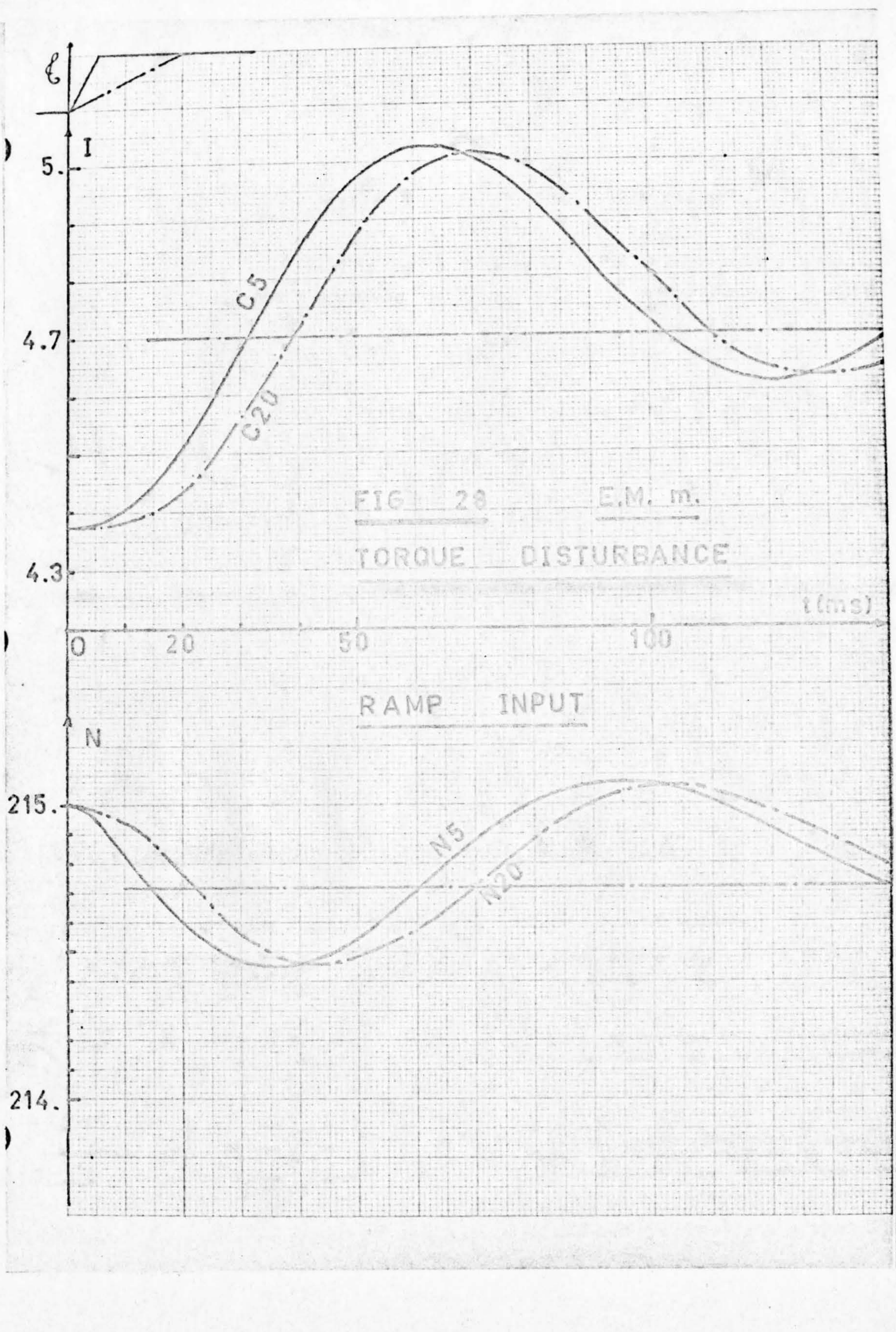
3) TORQUE INPUT IS A SINEWAVE:

A 50 ms period sinewave and an amplitude of 10% of the initial torque is chosen here.

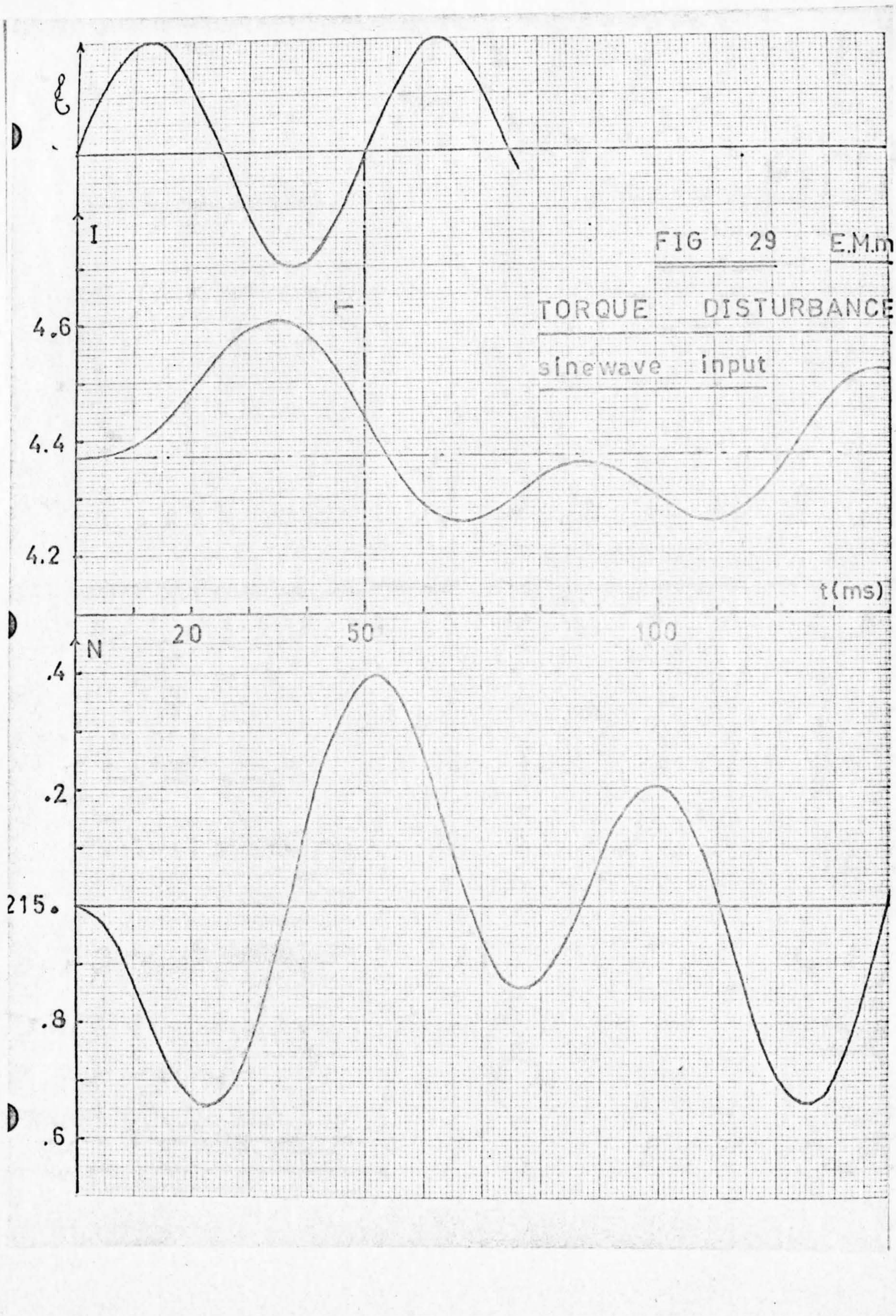
Figure 29 shows the response of the EM motor. A phase angle corresponding to 12 ms is observed. But as far as the amplitude is concerned the speed presents even harmonics in the response.

By comparison figure 30 shows the behaviour of the PM motor. Only a 7 ms phase angle is observed and after the first oscillation which has a very light overshoot, the following oscillations are steady.

The conclusion of the two last studies and especially the sinewave input can be stated as follow. The PM motor follows the disturbance very closely. It can represent a very good transducer. But the EM motor responds two slowly and therefore parasite frequencies are introduced, hence it cannot be utilized properly as a transducer.



ℓ
I
5.
4.7
4.3
C5
C20
FIG 28
E.M. m.
TORQUE DISTURBANCE
t(ms)
0
20
50
100
RAMP INPUT
N
215.
N5
N20
214.



ℰ
I
FIG 29 E.M.m
TORQUE DISTURBANCE
sinewave input
4.6
4.4
4.2
t(ms)
20
50
100
N
.4
.2
215.
.8
.6

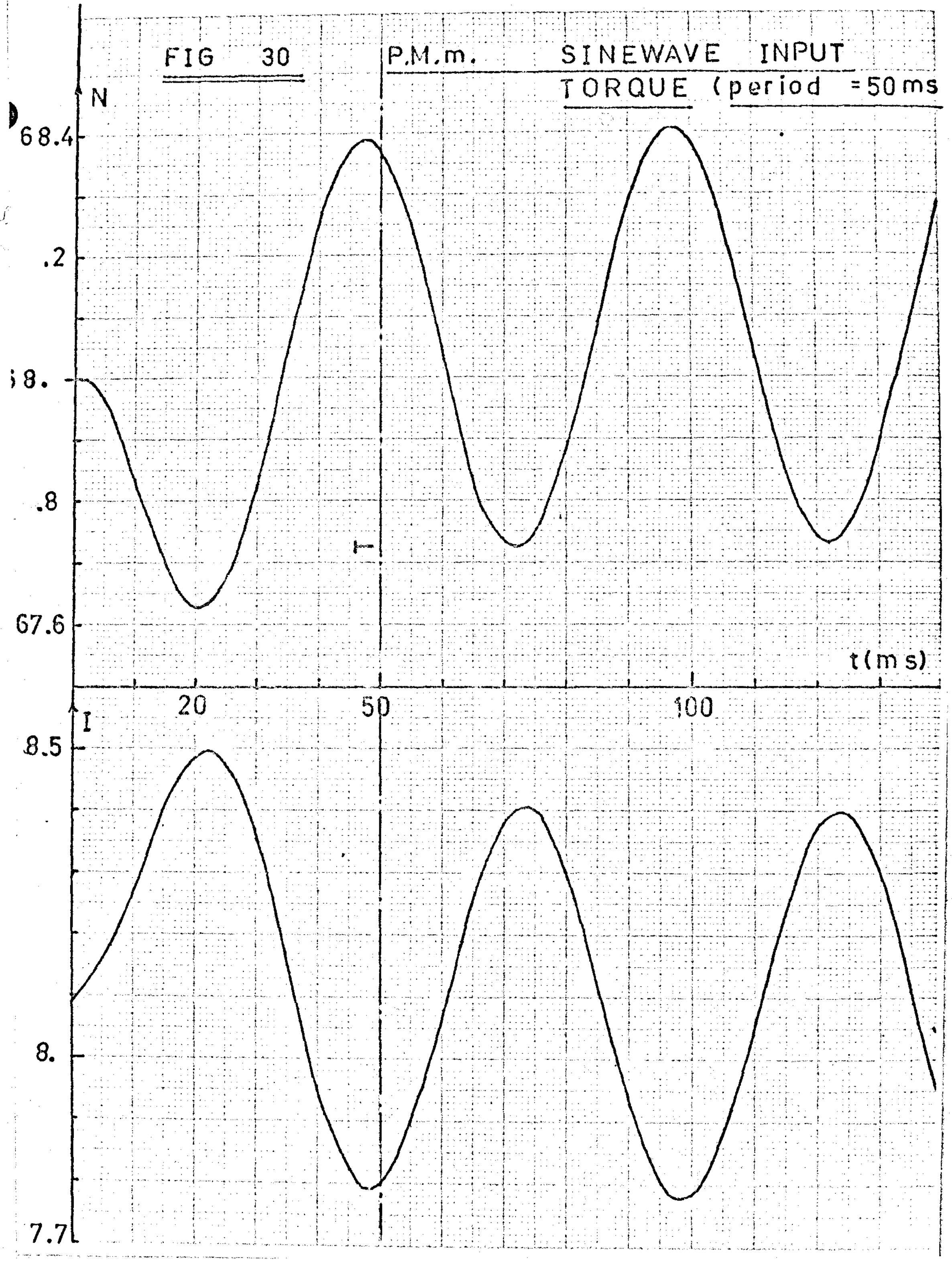


FIG 30
P.M.m. SINEWAVE INPUT
TORQUE (period =50ms
N
68.4
.2
8.
.8
67.6
t(ms)
20
50
100
I
8.5
8.
7.7

c) LIMIT CONSTRAINTS ON A MOTOR:

An interesting analysis can be made to study the limit constraints. A step input torque disturbance of magnitude $\Delta\lambda$ is applied to the motors running at 3/4 load and normal speed. But now after a time T the disturbance disappears and the torque is brought back to the initial position.

We have chosen some constraints limits. First a maximum overshoot of instantaneous current of 160 A is tolerated for the PM motor and an overshoot above 9.5 A during 10 ms can be endured by the motor without any harm.

For the EM motor we defined the constraints as 140 A maximum instantaneous current and a 10 ms overshoot above 6.5 A.

Figure 31 gives the constraint curve of the torque variation applied in function of the duration of the step.

Below 5 ms the curve is quite a straight line and tends to the point ($\Delta\lambda$=1000 , T= 1 ms). The comparison between those constraints is shown in figure 32. On the ordinate variations in percentage of the initial torque have been plotted.

It is remarkuable to see that below T=15 ms the two motors behave in exactly the same way, and above, the EM motor has a better performance. This can be easily explained. The constraints affects the first oscillation of the EM motor.So that for large torque variations PM and EM motors respond the same way as far as the constraints are concerned. But with lower torque variations, the EM motor having less

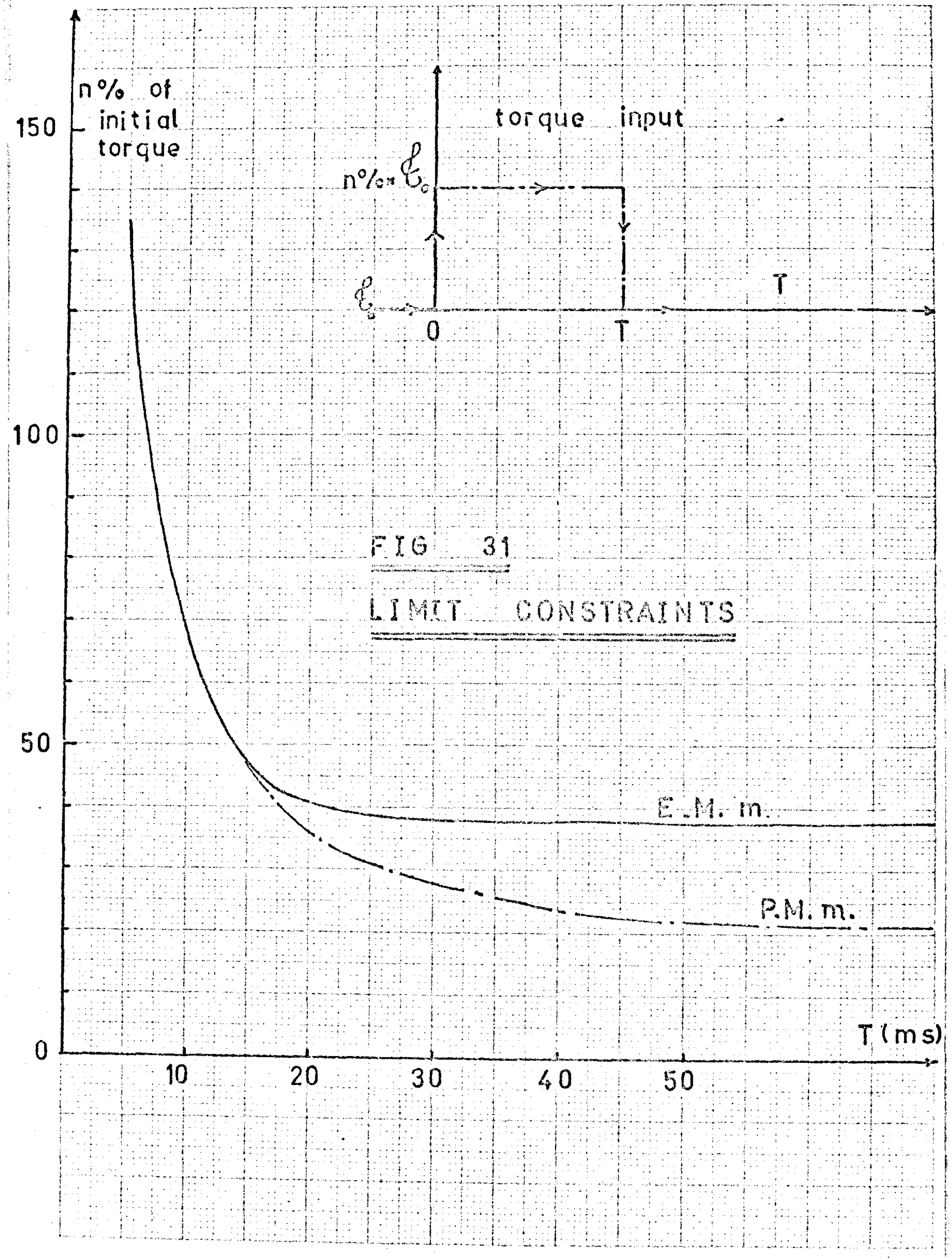


FIG 31

LIMIT CONSTRAINTS

speed-drops than the permanentic motor, the maximum constraints are less affected.

It has to be pointed out at this stage, that the results of the different comparisons, and especially the last one, are not absolute in themselves. It was wanted to show that the mathematical model is a sufficient tool to deal with a large number of various kind of problems.

CONCLUSION

After having analysed the usual equations of a dc. machine, using the simplifications normally introduced, it was found that the usual methods of determining the parameters of a dc machine do not lead to a powerful representation by mathematical model.

It was therefore necessary to search for new techniques of measurements which will take into consideration the dynamic characteristics of the parameters, and introduce the non-linearities in the results.

Most of those techniques have been found very handy, because they do not require a special instrumentation.

On the other hand, no assumptions were made on the size of the motors, and those techniques can be applied in a very large range of power.

The comparison of the linear normal model and the new precise non-linear mathematical representation, shows some important differences in the results. As far as the steady state is concerned, it was found that both models agree, hence the linear model can be handy, because simple. But if the transient response is involved, and that is the case in control systems, the difference in the responses given by each model is so large that the linear approach cannot be used. The experiment gives a credit to the new non-linear model which was found very precise in the range studied.

Having determined and checked the precise mathematical model of the electromagnetic and the permanentic motor, two examples of applications have been taken and analysed.

Those two examples, chosen in control system domain, show the power of the model defined and the variety of problems which can now be solved precisely. They even lead to a comparison of the behaviour of an electromagnetic and a permanentic motor.

In the case of the electromagnetic motor, a limitation occurs. The measurements of the armature reaction can be performed only in the range of admissible currents in the machine studied. But there is no way to predict and extrapolate the curve in the high current domain.

This weakness of the model has been pointed out in Note 3, and a deeper investigation would be helpful. This weakness disappears for the permanentic motor model after the theoretical justification of the predictability of the curve.

The precise model obtained for a dc machine could now permit an investigation of the influences of each parameter on the behaviour of the responses. This can be done on an analogue computer and may lead to very interesting results in the design of the motor itself or of the establishement of a required control system, avoiding a lot of experimental work.

APPENDIX A

The problem is to fit a set of n data recorded as (Vi,Ii) to the equation:

$$\text{(a)} \qquad V = pI + q$$

p and q have to be determined so that the errors εi , the spread of data around the straight line, have a zero mean value (see figure 2-c).

For each element one must have:

$$\text{(b)} \qquad Vi = pIi + q - \varepsilon i$$

If the matricial notation is introduced, where

$$(V) = \begin{bmatrix} v1 \\ v2 \\ \cdot \\ \cdot \\ vn \end{bmatrix} \qquad (X) = \begin{bmatrix} p \\ q \end{bmatrix} \qquad (I) = \begin{bmatrix} I1 & 1 \\ I2 & 1 \\ \cdot & \\ \cdot & \\ In & 1 \end{bmatrix} \qquad (\varepsilon) = \begin{bmatrix} \varepsilon 1 \\ \varepsilon 2 \\ \cdot \\ \cdot \\ \varepsilon n \end{bmatrix}$$

equation (b) becomes

$$\text{(b')} \qquad (V) = (I)(X) - (\varepsilon)$$

the mean square error is then minimized

$$\text{(c)} \qquad (\varepsilon) = (I)(X) - (V)$$

hence

$$\text{(d)} \qquad (\varepsilon)^2 = (\,(I)(X)-(V)\,)^* \,(\,(I)(X)-(V)\,)$$

developping this expression one can find:

$$\text{(e)} \qquad (\varepsilon)^2 = X^* I^* IX - V^* IX - X^* I^* V + V^* V$$

To have $(\varepsilon)^2$ minimum it has to satisfy:

(f) $$\frac{\partial(\varepsilon)^2}{\partial X} = 0$$

taking in consideration that: $(X) = \begin{bmatrix} p \\ q \end{bmatrix}$ $\quad (X)^* = \begin{bmatrix} p & q \end{bmatrix}$

and $$\frac{\partial X}{\partial p} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \frac{\partial X}{\partial q} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{leads to} \frac{\partial (X)}{\partial X} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \underline{1}$$

likewise: $$\frac{\partial X}{\partial p} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \frac{\partial X^*}{\partial q} = \begin{bmatrix} 0 & 1 \end{bmatrix} \text{ and } \frac{\partial (X)^*}{\partial X} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \underline{1}$$

so that the differentiation of e gives:

(g) $$\frac{\partial \varepsilon^2}{\partial X} = I^* I X + X^* I^* I - V^* I - I^* V = I^* \varepsilon + \varepsilon^* I$$

which is the sum of the matrix and its transposed. To be able to apply (f) one must have:

(h) $$(I)^*(\varepsilon) = (0)$$ because non of the matrix (I) or (ε) are not

The equation (h) leads to the set of equations using (c) :

(i) $$\begin{bmatrix} \sum_1^n Ii^2 & \sum_1^n Ii \\ \sum_1^n Ii & n \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} \sum_1^n Vi\ Ii \\ \sum_1^n Vi \end{bmatrix}$$

The parameters are given by:the solution of (i) :

(j) $$p = \frac{n\sum_1^n(ViIi) - \sum_1^n(Vi)\sum_1^n(Ii)}{n(\sum_1^n Ii^2) - (\sum_1^n Ii)^2} \qquad q = \frac{(\sum_1^n Vi)(\sum_1^n Ii^2) - (\sum_1^n ViIi)(\sum_1^n Ii)}{n(\sum_1^n Ii^2) - (\sum_1^n Ii)^2}$$

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