The Dynamical Evolution of Accreted Star Clusters in the Milky Way
The Dynamical Evolution of Accreted Star Clusters in the Milky Way

By

Meghan Catherine Miholics, B.Sc.

A Thesis
Submitted to the School of Graduate Studies in Partial Fulfilment of the Requirements for the Degree Master of Science

McMaster University
©Copyright by Meghan Miholics, August 2016
MASTER OF SCIENCE (2016) McMaster University
(Physics and Astronomy) Hamilton, Ontario

TITLE: The Dynamical Evolution of Accreted Star Clusters in the Milky Way

AUTHOR: Meghan Miholics, B.Sc. (McMaster University)

SUPERVISORS: Alison Sills

NUMBER OF PAGES: 9, 59
Abstract

We perform $N$-body simulations of star clusters in time-dependant galactic potentials. Since the Milky Way was built up through mergers with dwarf galaxies, its globular cluster population is made up of clusters formed both during the initial collapse of the Galaxy and in dwarf galaxies that were later accreted. Throughout a dwarf Milky Way merger, dwarf galaxy clusters are subject to a changing galactic potential. We investigate how this changing galactic potential affects the evolution of a cluster’s half-mass radius. In particular, we simulate clusters on circular orbits around a dwarf galaxy that either falls into the Milky Way or evaporates as it orbits the Milky Way. We find that the dynamical evolution of a star cluster is determined by whichever galaxy has the strongest tidal field at the position of the cluster. Thus, clusters entering the Milky Way undergo changes in size as the Milky Way tidal field becomes stronger and that of the dwarf diminishes. We find that ultimately accreted clusters become the same size as a cluster born in the Milky Way on the same orbit. The change in size for accreted clusters occurs on a short time-scale, comparable to 1-2 cluster half-mass relaxation times. Assuming their initial sizes are similar, clusters born in the Galaxy and those that are accreted cannot be separated based on their current size alone.
Acknowledgements

First and foremost I would like to thank my supervisors, Alison Sills, Jeremy Webb and Diederik Kruijssen, for guiding me through the journey that has been the last few years. Alison, your wonderful blend of compassion and toughness has taught me so much. Working for such a strong, capable woman has been truly inspiring. Jeremy, although you were still a student when we met, you never failed to give amazing advice. Thanks for listening to all the rants and celebrating all the victories. Diederik, your hospitality during my working visit in Heidelberg was more than I could have hoped for. Thank you for teaching me so much scientifically and for all the chats about my future path. I would also like to acknowledge the members of my Master’s committee, Bill Harris and James Wadsley, for being excellent teachers and always keeping me on my toes.

Now for the really sappy part...

To my wonderful parents, I would be nothing without your ongoing support and your reminders that I am always loved, unconditionally, no matter what I do in life. Mum, you are such an inspirational woman and have taught me so much about life. A big thank you to you and Jeff is owed for all the wonderful Sunday dinners (J-ello fights FTW) and life chats that we have shared. Dad, thank you for always taking an interest in my work and life and letting me talk to you until your ears fall off. I will always cherish our dinners out together. Thank you also to your wonderful partner Lisa, for providing yet another example of what a strong woman can achieve when she wants to.

I would also like to thank my brother Evan who has always encouraged me and has never failed to be there with a hug when I have needed it most. Thank you also to my cousin Jennifer, who has been like a sister to me. Our chats over tea got
me through some of the roughest times. Thank you as well to everyone else in my extended family, especially my beautiful Grandma and my Auntie Karen. I always want to make you proud.

Next, I need to thank my best friend. Pelinsu, do I even need to say it? Probably not, but I will anyway. Thank you for being there through everything and, in particular, constantly reminding me that I could do this. What would a girl be without her best friend?

Thank you to my officemates, past and present, Jessica, Aisha, Niloufar, Ian and Fraser, for so many laughs and beers at the Phoenix its hard to count. To the rest of the McMaster crew, in particular those senior grad students, I owe you one for constantly reminding me that other people are going through this and that its ok to cry sometimes. In particular, a special thanks is owed to Ben (I know we have a lot of them around here but I am sure you can guess which one!). We’ve only known each other for a short while but your support has been indispensable while completing this degree. Thanks for travelling around Europe with me too!

I would also like to warmly acknowledge all the people I met and worked with during my working visit at the ARI in Heidelberg this year. I really learned a lot from you.

Finally, to anyone that has ever offered me any words of encouragement, I thank you.

P.S. I am so sorry if I forgot you but I believe the last blanket statement has got me covered :p
Dedicated to my Mum and Dad
# Table of Contents

Descriptive Notes ii

Abstract iii

Acknowledgements iv

List of Figures ix

Chapter 1 Introduction 1

1.1 The Dynamical Evolution of Star Clusters 5

1.2 N-body Simulations 8

1.3 Summary 10

Chapter 2 The Dynamical Evolution of Accreted Star Clusters in the Milky Way 16

2.1 Introduction 16

2.2 Methods 21

2.2.1 The Combined Galactic Potential 21

2.2.2 Simulations and Parameters 24

2.3 Results 25

2.3.1 Dwarf Falls into Milky Way 26

2.3.2 Dwarf Evaporates 33

---

Chapter 3  Further Questions and Future Work  47

3.1  Additional Simulations  . . . . . . . . . . . . . . . . . . 47

3.1.1  Dwarf Falls: Effect of a Smaller Initial Size  . . . . . . . 47

3.1.2  Dwarf Evaporates: Effect of a Retrograde Cluster Orbit  51

3.2  Conclusions  . . . . . . . . . . . . . . . . . . . . . . . . 54

3.3  Future Work  . . . . . . . . . . . . . . . . . . . . . . . . 56
List of Figures

2.1 Accreted cluster evolution in the dwarf falls scenario . . . . . . . 27

2.2 Accreted cluster evolution in the dwarf falls scenario with a larger initial cluster size . . . . . . . . . . . . . . . . . . . . . . . . . . 30

2.3 Accreted cluster evolution in the dwarf falls scenario with a more massive dwarf galaxy . . . . . . . . . . . . . . . . . . . . . . . . . . 32

2.4 Accreted cluster evolution in the dwarf evaporates scenario . . . 34

3.1 Accreted cluster evolution in the dwarf falls scenario with a smaller initial size . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 49

3.2 Accreted cluster evolution in the dwarf evaporates scenario with a retrograde cluster orbit . . . . . . . . . . . . . . . . . . . . . . . . 52
Chapter 1

Introduction

Stars form together in clustered environments (Lada & Lada, 2003). A significant fraction, 30 - 35 %, of all stars that have formed in the Universe were born in bound stellar clusters with a wide range of masses (Kruijssen, 2012). Three main types of star clusters exist in the Universe, open clusters, young massive clusters and globular clusters. A summary of the properties of each type is given in Portegies Zwart et al. (2010) (see their Table 1). Open clusters are low mass systems (< $10^4 M_\odot$) which tend to form with low densities and are only marginally gravitationally bound. Consequently, open clusters dissolve quickly and have short lifetimes. They are preferentially found in the discs of galaxies, where stars and clusters form. The second subset of clusters are young massive clusters and as their name suggests they are massive (> $10^4 M_\odot$) and young (ages < 100 Myr). They are found in a wide variety of environments including the star forming regions of the Milky Way and in galaxy mergers (see Portegies Zwart et al. (2010) for a comprehensive review). The final subset of star clusters are known as globular clusters. Although the precise definition of a globular cluster is somewhat uncertain, they are generally old (ages > 10.0 Gyr) but could also include intermediate age clusters, massive (> $10^4 M_\odot$) and
live in the haloes of their host galaxies (away from the star forming component, see Kruijssen (2015) for an interesting discussion of the definition of a globular cluster). Globular clusters also tend to be very compact objects. A common measure of a globular cluster’s size is the radius containing half the light, which is known as the half-light radius. The globular clusters in the Milky Way, of which they are approximately 150, have a wide but sharply peaked distribution of half-light radii. The peak/average of this distribution occurs around $\approx 2.5$ pc (Harris, 1996). Globular clusters also have a peaked mass function with a roughly lognormal shape. The peak occurs around approximately $10^{5.3} M_\odot$ and is roughly the same in all galaxies studied (Harris, 1996; Brodie & Strader, 2006). A consequence of the high masses and small sizes of globular clusters is that they are extremely dense, some of the densest stellar systems in the Universe (Binney & Tremaine, 1987). Hence, globular clusters are much more gravitationally bound than open clusters and have lifetimes greater than the age of Universe.

In the present day Universe, globular clusters are found in all types of galaxies, from small dwarf galaxies, which typically only harbour a few clusters, to giant ellipticals that are home to thousands (Harris et al., 2013). The old ages of most globular clusters imply that they formed in the very early Universe (Krauss & Chaboyer, 2003; VandenBerg et al., 2013) during the first stages of galaxy formation. The ages of globular clusters combined with their ubiquity means that globular clusters are witnesses to all stages of galaxy formation and evolution, making them powerful probes of these processes. For example, globular clusters have a great potential to trace structure formation in the Universe, which is known to be hierarchical (White & Rees, 1978).
initial stages of massive galaxy formation at high redshift are characterized by many rapid mergers of smaller galaxies. As the Universe expanded and the density of galaxies became lower, the merger rate for galaxies decreased and the rapid assembly process came to an end. At present day, galaxies continue to grow both through the merging of smaller galaxies and continuous accretion of dark matter/gas and in situ star formation. A consequence of this hierarchical process for building galaxies is that the globular cluster population of a given galaxy is composed of globular clusters formed in the smaller constituent galaxies that merged to form it (Searle & Zinn, 1978; Brodie & Strader, 2006). The power of globular clusters lies in that they may bare traces of these processes either as a group or individually.

The Milky Way’s globular cluster population is an example of a globular cluster population with more than one constituent population. At present day, galaxies like the Milky Way continue to grow through mergers with dwarf galaxy satellites. Evidence for the process of dwarf galaxy accretion is abundant in the Local Group, where streams of former dwarf galaxies litter the haloes of both the Milky Way and Andromeda (see Grillmair & Carlin (2016) and Ferguson & Mackey (2016) for comprehensive reviews). The most prominent example of this accretion process in the Local Group is the Sagittarius dwarf galaxy which is currently being disrupted by the Milky Way and has tidal debris wrapping around the majority of the Galaxy (Ibata et al., 1994; Majewski et al., 2003). Consequently, in the Milky Way the globular cluster population is composed of two distinct groups, a group formed in the initial collapse of the Galaxy and another that was later accreted from dwarf galaxies that merged with the Milky Way. These groups are often referred to as in situ
and accreted respectively. It is somewhat difficult to make a clear distinction between these two groups since during the initial stages of galaxy formation (the rapid assembly process discussed above) many small satellites are accreted which may host globular clusters, blurring the distinction between in situ and accreted. However, for the purposes of this thesis we would like to make the distinction between those that were accreted very early in the lifetime of the galaxy and those that were not. Therefore, we refer to the globular clusters formed before or during the rapid assembly phase of the galaxy as in situ while everything that entered the galaxy after will be referred to as accreted. Direct observational evidence for the existence of these two groups in the Milky Way is abundant (for more details see Chapter 2). The existence of these sub-populations of globular clusters in the Milky Way provide an excellent opportunity to trace galaxy evolution but only if we can first understand how the properties of the globular clusters depend on the galaxy merger process.

One property of globular clusters that has yet to be investigated in the context of galaxy mergers is their internal dynamics. Globular clusters are extremely dynamic objects which can be heavily influenced by the galaxies in which they live. The galaxy provides an underlying gravitational potential which gives rise to a tidal field, a force field that results from the differential force felt at the cluster’s centre and at any other point in the cluster. The tidal field acts on the cluster as an outward forces that opposes the inward self gravity of the cluster. The field imposes a limit to the cluster’s extent, known as the Jacobi radius, beyond which a cluster star feels a greater force towards the galaxy then it does back towards the cluster and can be removed from the cluster (for more details on the effect of tides on star clusters see Chapter 4).
2). Globular clusters can also be affected by substructure in the galaxies that they live in. Encounters with such substructure inject energy into globular clusters via tidal shocks that result from the rapid variation of the tidal field over a short period of time. Examples of substructure include stellar/gaseous discs, giant molecular clouds (GMCs) and smaller dark matter haloes. Many globular clusters in the Milky Way have orbits that take them through the disc. The process of injecting energy into globular clusters by passing through the disc is known as disc shocking. Disc shocking increases the escape rate of stars from the cluster and speeds up dissolution (Webb et al., 2014). A dwarf galaxy globular cluster that is accreted by the Milky Way will feel a change in the gravitational potential (and hence, tidal field) it experiences before, during and after the merger. This change in gravitational potential is expected to alter the dynamical evolution of these clusters since the tidal radius will be constantly changing throughout this process. A key question to answer is how the structural properties, in particular the size, of accreted clusters will change throughout the merger. The investigation of this problem is the subject of this thesis.

1.1 The Dynamical Evolution of Star Clusters

The long term dynamical evolution of globular cluster systems has been studied extensively for many decades (von Hoerner, 1960). Globular clusters are some of the most interesting dynamical systems in our Universe since they have high mass densities in their central regions, typically on the order of $\rho \approx 10^3 - 10^4 M_\odot pc^{-3}$ (Peterson & King, 1975), several orders of magnitude
more than the mass density in the Solar neighbourhood (Binney & Tremaine, 1987). Since the density of stars inside globular clusters is so high, encounters between stars are frequent. Orbits of stars within the cluster are therefore constantly changing, leading to an ever evolving system.

The evolution of stellar orbits inside the cluster through gravitational interactions is the dominant mechanism driving the dynamical evolution of globular clusters. This process is known as two body relaxation. The time-scale over which these interactions significantly change the trajectories of the particles in the system (i.e. $|\Delta \vec{v}| \approx |\vec{v}|$) is called a relaxation time. The relaxation time is often evaluated within the half mass radius (radius containing half the mass) of the system. The half mass relaxation time, $t_{rh}$ is given by the following formula:

$$t_{rh} = 0.138 \frac{N^{1/2}r_m^{3/2}}{m^{1/2}G^{1/2}ln\Lambda}$$ (1.1)

(Spitzer, 1987) where $N$ is the number of stars in the system, $r_m$ is the half mass radius, $m$ is the mass of the particles in the system (or the average mass for systems with unequal masses) and $ln \Lambda$ is the Coulomb logarithm which goes as $\Lambda \approx N$. Hence, one can see that for a given number of stars, the smaller (or more dense) a stellar system is, the shorter the relaxation time. Systems with high densities such as globular clusters have short relaxation times and hence evolve quickly relative to lower density astrophysical systems like galaxies. Typical relaxation times for globular clusters are in the range $10^8 - 10^{10}$ yr with the large majority $< 10^{9.5}$ yr (Harris, 1996). Since most globular clusters have ages $\geq 10^{10}$ yr, they have undergone several relaxation times in their lifetime and are dynamically evolved systems.
Several processes result from relaxation inside globular clusters. These processes have been modelled extensively with a variety of methods, from analytical models to gravitational N-body simulations. Some of the pioneering work in this field is the work of Henon who produced simple idealized models for star clusters evolving in a tidal field and in isolation (Hénon, 1961, 1965, 1975). His models firmly established that star clusters evolve through a constant exchange of energy. Energy produced in the core of the cluster is continuously transported outward through two body interactions. Although Henon did not know the exact mechanism that would produce this energy, N-body simulations later showed that this energy comes primarily from hard binaries that give energy to other stars through three body interactions, transferring them to higher orbits (Giersz & Heggie, 1994). This exchange of energy causes the outer layers of the cluster to expand, while the core contracts. Gieles et al. (2011) put the ideas of Henon together to construct a complete model of compact star clusters evolving in a tidal field. In their model, the life cycle of a star cluster is composed of two stages. During the first stage, coined ‘expansion dominated’, the initially compact cluster expands towards its tidal limit (Jacobi radius) through two body relaxation. In this stage, the dynamical evolution of the cluster is dominated by the internal effects of two body relaxation rather than removal of stars by the tidal field. In the second stage, known as ‘evaporation dominated’, the rate at which stars escape the cluster tends to a constant value and the half mass radius shrinks as the cluster heads towards complete dissolution. In contrast to the first stage, in the ‘evaporation dominated’ stage the tidal field is playing a large part in the evolution of the cluster.
1.2 $N$-body Simulations

Analytical models of star clusters have been useful for establishing a baseline for the evolution of star clusters. Unfortunately, real star clusters possess a variety of characteristics that increase the complexity of these models and make them difficult to work with. Some examples of these complications are the presence of a stellar mass spectrum, stellar evolution, binary populations, cluster rotation, spherical asymmetry and tidal fields. Taking into account these complexities, it becomes necessary to use computer simulations to gain further insight into the evolution of star clusters.

NBODY6 (Aarseth, 1999, 2003; Nitadori & Aarseth, 2012) is an $N$-body simulation code that has been developed to simulate star clusters. The code represents each star as a particle that has a mass, position and velocity. The motion of each star through phase space is calculated according to the gravitational forces it experiences from the other stars. The main disadvantage of this approach is computational expense. Since calculating the force between each pair of stars is normally an $N^2$ calculation, simulating star clusters with an $N$ similar to globular clusters ($>10^5$) can be challenging. However, many efforts have been taken over the last five decades to develop algorithms that allow fast and accurate integration of the stars’ equations of motion, decreasing the computational expense of these simulations (Aarseth, 1999). Recently simulations of star clusters with $10^6$ bodies have been completed within a reasonable amount of time (Wang et al., 2016) using the state of the art version of NBODY6: NBODY6++GPU, which uses MPI parallelization as well as GPUs.
The current publicly available version of NBODY6, used for this thesis, can be run in parallel over several CPUs or with GPUs. The simulations presented in this thesis were performed with 1 GPU at a time (computations performed with 8 CPUs in parallel are about a factor of two slower). Approximately 5-7 days of wall clock time was needed to simulate 12.0 Gyr of star cluster evolution ($N = 5 \times 10^4$). The exact amount of time required to complete each simulation depends on the tidal field experienced by the cluster. If the tidal field is stronger stars will escape the cluster more quickly, decreasing the amount of stars left in the simulation and the remaining computation time. Additionally, most star clusters simulated in this thesis have $r_m > 3.0$ pc. Star clusters with smaller initial sizes will take longer to simulate since higher densities imply more close encounters between stars which are relatively computationally expensive.

NBODY6 can accommodate the aforementioned complexities of star cluster evolution that analytical models cannot. The initial stellar masses, positions and velocities can be set by the user allowing for the use of a realistic stellar initial mass function and cluster density profile as well as the implementation of other characteristics such as cluster rotation or asymmetry which is sometimes desirable. The code includes routines for stellar evolution such that a variety of stellar properties, most importantly mass, can be updated over time using formulae fitted to detailed stellar evolution models (Tout et al., 1997; Hurley et al., 2000). An initial binary population can also be implemented by the user and routines for calculating the effects of binary evolution and collisions (Tout et al., 1997) are included in the code.
One final advantage of using an $N$-body simulation code versus an analytical model that is essential to the completion of this work, is a more realistic treatment of the tidal field. Many analytical models impose a tidal field by placing a circle down around the cluster (with radius equal to the assumed Jacobi radius) and defining stars to have escaped the cluster after they cross that boundary. This assumption is an over simplification of tidal evaporation for several reasons. For example, under this assumption stars can escape in all directions from the cluster centre but in reality stars can only escape through the two points lying along the line connecting the galactic centre to the cluster centre (one on either side of the cluster), where the tidal force is exactly balanced by the force back towards the cluster (Renaud et al., 2011). These points are known as the Lagrange points and are further discussed in Chapter 2. Using an $N$-body simulation code, like NBODY6, allows for the implementation of an actual force field that acts on the cluster stars, by computing the difference between the galactic force at the cluster centre and at the position of the star. Using such a direct calculation also allows for the use of time dependent tidal fields, essential to this work. The implementation of tidal fields in an $N$-body code is discussed further in Chapter 2.

1.3 Summary

The aim of this thesis is to study how the dynamical evolution of globular clusters is effected by the changing tidal field of a galaxy merger. Specifically, we will investigate how the size of a globular cluster changes as it moves from a dwarf galaxy into the Milky Way during a merger. There are two main
physical processes that will effect the gravitational potential that an accreted cluster experiences. The first is that the Milky Way tidal field acting on the cluster will increase as the cluster falls closer to the centre of the Milky Way. The second is that the dwarf tidal field acting on the cluster will decrease as the Milky Way strips material from the dwarf itself, decreasing its mass.

To investigate the effects of these two processes, we designed two scenarios for the external gravitational potential experienced by the cluster. The first scenario, coined ‘dwarf falls’ has the cluster on a circular orbit around the point mass dwarf galaxy which gradually moves closer to the centre of the Milky Way, thereby investigating the effect of increasing the Milky Way tidal field. The second scenario, which we call ‘dwarf evaporates’, puts the cluster on a circular orbit around a point mass dwarf which in turn executes a circular orbit around the Milky Way. The mass of the dwarf is slowly decreased to zero after several billion years of evolution in this configuration, which allows for the investigation of the response to the dwarf tidal field decreasing.

We find several common results from all of our simulations. The first is that the cluster’s size is always determined by the galaxy with the strongest tidal field at the position of the cluster. For example, if the dwarf tidal field is strongest at the position of the cluster than the cluster will have the same size as a cluster that lives in the dwarf only. All of our scenarios are designed such that at some point the Milky Way’s tidal field becomes stronger than the dwarf’s (as would realistically be the case for any accreted cluster). We find that when the transition to the Milky Way’s tidal field occurs, the cluster’s half mass radius starts to adjust immediately, expanding or contracting accordingly. The accreted cluster’s size always becomes the same as an in situ
cluster on the same orbit in the Milky Way on a short time-scale (1-2 cluster half-mass relaxation times). Since we observe this outcome in simulations from both scenarios we can draw the conclusion that real clusters undergoing this type of galaxy merger will undergo the same changes in size when the two processes are happening simultaneously. Hence, we have shown that no size differences should exist between the accreted and in situ globular clusters in the Milky Way and therefore it is not possible to identify accreted clusters using only their size.
Bibliography

—. 2003, Gravitational N-Body Simulations, 430

Binney, J. & Tremaine, S. 1987, Galactic dynamics


Hénon, M. 1961, Annales d'Astrophysique, 24, 369
—. 1965, Annales d'Astrophysique, 28, 62


von Hoerner, S. 1960, ZAp, 50


Chapter 2

The Dynamical Evolution of Accreted Star Clusters in the Milky Way

2.1 Introduction

Globular clusters are giant groups of stars in bound spherical configurations, that reside in all types of galaxies. Most globular clusters formed in the early Universe, coeval with their parent galaxies (Krauss & Chaboyer, 2003). Hence, globular cluster populations trace the hierarchical formation of structure in the Universe, in which large galaxies are formed through the merging of smaller galaxy building blocks (White & Rees, 1978). Each galaxy’s globular cluster population will be made up of both clusters that were formed in the initial collapse of the galaxy as well as clusters that were formed elsewhere and were later accreted on to the galaxy through galaxy mergers (Searle & Zinn, 1978). In particular, the globular cluster population in the Milky Way is

\footnote{This chapter was originally published as Miholics M., Webb J.J., & Sills A., MNRAS, 456, 240.}
composed of clusters formed in the Milky Way and clusters that were formed in dwarf galaxies that have merged with the Milky Way in the past.

Observational evidence for two distinct populations of globular clusters in the Milky Way is abundant. Perhaps the most compelling evidence for this scenario comes from the dwarf galaxy Sagittarius which is merging with the Milky Way at present day (Ibata et al., 1994). Several of the Galaxy’s globular clusters (anywhere from 5 to 9) have been shown to be coherent with the Sagittarius stream in position and velocity space and thus can be associated with the original dwarf galaxy (e.g. Da Costa & Armandroff, 1995; Palma et al., 2002; Bellazzini et al., 2003; Law & Majewski, 2010). Additional evidence comes from examining the present day properties of Milky Way globular clusters. Zinn (1993) was the first to note that the Galaxy’s clusters are divided into two groups that have distinct horizontal branch morphologies, kinematics and spatial distributions from one another. More recent work has shown that certain subsets of Milky Way clusters have similar horizontal branch morphologies, core radii and relationships between age and metallicity as clusters found in Milky Way satellites such as the LMC, SMC, Fornax and Sagittarius (e.g Mackey & Gilmore, 2004; Marín-Franch et al., 2009; Forbes & Bridges, 2010; Leaman et al., 2013). These studies typically find about 25-35 per cent of clusters in the Milky Way have properties similar to clusters observed in dwarf galaxies, indicating that as many as 40-50 of the Galaxy’s clusters were accreted from dwarf galaxies. These estimates suggest that the Milky Way has undergone $\approx 6$-8 mergers with galaxies like the Sagittarius dwarf over its history. Evidence for accretion of clusters on to one galaxy from another exists in
many other systems such as Andromeda and in larger systems such as galaxy clusters (Collins et al., 2009; Mackey et al., 2014; D'Abrusco et al., 2015).

Although it is well established that the Milky Way is host to a population of accreted clusters, little work has been done so far to determine how the changing Galactic environment affects these clusters. The dynamical evolution of globular clusters in static potentials has been studied extensively and is significantly affected by the galaxy in which the cluster lives (e.g. Baumgardt & Makino, 2003; Webb et al., 2014a,b). Clusters are subject to two body relaxation as well as tidal forces from the host galaxy. Two body relaxation alters the orbits of stars through their dynamical interactions with one another and ultimately causes the outer layers of the cluster to expand. The difference between the force from the galaxy at the cluster’s centre and the force from the galaxy acting at some other position in the cluster creates tidal forces on the stars in the cluster. The points where the tidal forces from the galaxy exactly balance the force from the cluster are known as the Lagrange points. The first and second Lagrange points lie along the axis through the galaxy centre and the cluster. Stars in general escape the cluster through one of these two points. Hence, the tidal forces set a rough maximum size for the cluster as it expands (known as the Jacobi or tidal radius, $r_j$) which is the distance between the cluster centre and the first Lagrange point. Outside of this radius stars feel a stronger gravitational force from the galaxy than the cluster and are thus stripped away from the cluster. For clusters in spherically symmetric
potentials (or in axisymmetric potentials on circular orbits in the plane of symmetry), the Jacobi radius is given by:

\[ r_j = R_G \left( \frac{M_c}{3M_G} \right)^{1/3} \]  

(2.1)

where \( R_G \) is the radius of the circular orbit, \( M_c \) is the cluster’s mass and \( M_G \) is the mass of the galaxy enclosed by the cluster’s orbit. For a more detailed discussion of the above ideas, see Renaud et al. (2011).

The actual physical size of a cluster can be smaller or approximately equal to this radius. If a cluster occupies a large majority of the volume set by the tidal radius it is said to be tidally filling, if not it is underfilling. The ratio of the cluster’s half light radius (radius which contains half the light) to the Jacobi radius, \( r_h/r_j \), is used to measure the degree to which a cluster fills its tidal radius and is referred to as the tidal filling factor. It is also useful to compare \( r_j \) to the 95 or 99 percent Lagrange radii (radius that contains 95 or 99 per cent of the mass) to obtain an idea of how close the outer region of the cluster is to \( r_j \). A cluster’s tidal filling factor can have a great influence on its evolution since clusters that are more tidally filling are more susceptible to the tidal forces of the galaxy and will experience higher mass-loss rates than clusters that are underfilling.

The evolution of globular clusters has been studied in a variety of static galactic potentials (e.g. Baumgardt & Makino, 2003; Webb et al., 2014a,b) as well as some semi-static potentials where changes in the potential are made instantaneously (Madrid et al., 2014; Miholics et al., 2014). However, given that star clusters are embedded in ever-evolving environments, it is important to be able to study clusters in time-dependent potentials. Such a task has been
made possible with the recent inception of \texttt{nbody6tt}, an extension of the $N$-body code \texttt{nbody6} \cite{Renaud2011, Renaud2015a}. \texttt{nbody6tt} allows the user to evolve stellar clusters in arbitrary time dependent galactic potentials and has been used to study the evolution of clusters in major galaxy mergers \cite{Renaud2013} as well as clusters embedded in slowly growing dark matter haloes \cite{Renaud2015b}. Similar techniques have also been implemented by Rieder et al. \cite{Rieder2013} who studied clusters in the tidal field extracted from a cosmological dark matter simulation.

Recently, steps have been taken towards understanding the evolution of dwarf galaxy globular clusters that undergo a change in galactic potential. Bianchini et al. \cite{Bianchini2015} simulated clusters in the centre of a dwarf galaxy potential that is instantaneously or slowly removed. They found that clusters expand in response to the changing potential but never become as extended as they would if they evolved solely in isolation. In our previous work, Miholics et al. \cite{Miholics2014}, we studied dwarf galaxy clusters in a similar context. We simulated clusters undergoing an instantaneous change in galactic potential, from that of a dwarf galaxy to the Milky Way, to understand the ultimate evolution of a cluster that has been brought into the Milky Way by a merger with a dwarf. We found that a cluster’s size will adjust rapidly in response to the new galactic potential until it is the same size as a Milky Way cluster on that orbit. In our current work, we simulate clusters in idealized time dependent potentials representative of a dwarf Milky Way merger. This method allows us to study the evolution of a cluster throughout the whole merger process rather than just before and after. We start all of our simulations by placing the cluster on a circular orbit around a dwarf galaxy that does one of two
things: falls into the Milky Way or evaporates as it orbits around the Milky Way. We always keep the Milky Way potential fixed but explore the effects of varying the cluster’s size as well as the mass of the dwarf galaxy.

2.2 Methods

2.2.1 The Combined Galactic Potential

We simulate clusters with the \textsc{nbody6} (Aarseth, 1999, 2003; Nitadori & Aarseth, 2012). Traditionally, \textsc{nbody6} is able to evolve clusters under the gravitational influence of a single galaxy. However, to study the evolution of star clusters in a dwarf Milky Way merger, we need to simulate them under the combined influence of both the dwarf galaxy and Milky Way. To accomplish this goal, we utilize the extension \textsc{nbody6tt} (Renaud et al., 2011; Renaud & Gieles, 2015a) which allows for integration of star clusters in arbitrary galactic potentials. \textsc{nbody6tt} offers two possible methods for implementing the galactic potential. The option which we utilize here uses an expansion of the galactic potential which yields the force on a cluster star (as a function of position with respect to the cluster centre):

\[
\nabla \phi_G(r) = \nabla \phi_G(0) - T_t(r) \cdot r + O(r^2)
\]

where \( r \) is the position of the star in the cluster, \( \phi_G \) is the galactic potential and \( T_t \) is a tensor (referred to as the tidal tensor) given by the following:

\[
T_t^{ij}(r) = \left(- \frac{\partial^2 \phi_G}{\partial x^i \partial x^j}\right)_r
\]
To use this method, a series of tidal tensors, evaluated at the cluster’s position within the potential, must be supplied to the code at discrete timesteps. The position of the cluster within the potential as a function of time, i.e. the cluster’s orbit, is completely specified by the user. This method allows for the simple calculation of the tidal tensor from two galaxies since it is simply the summation of the tidal tensors for each individual galaxy (evaluated at the cluster’s position with respect to that galaxy). Note, however, that this summation is a vector summation. The effects of the two tidal field strengths are, in practice, not additive since the points where the first galaxy’s tidal field is the strongest (i.e. the Lagrange points, see Section 1) are not located at the same position around the cluster as the points where the second galaxy’s tidal field is the strongest.

To emulate the potential a cluster would feel during a dwarf-Milky Way merger, we simulate clusters in two main scenarios. In the first scenario, the cluster begins its evolution on a circular orbit around a point mass dwarf galaxy which falls into the Milky Way. The initial separation between the galaxies’ centres is always set to 50 kpc. Starting at 3 Gyr, the separation between the two galaxies is decreased at a constant rate (10 kpc/750 Myr) until the dwarf galaxy reaches a certain distance from the Milky Way centre and stops. We chose to decrease the separation at a constant rate in order to keep the comparison of the tidal forces from each galaxy simple. The rate was chosen such that the cluster falls into the Milky Way relatively slowly compared to its orbit around the dwarf. We keep the cluster on a circular orbit around the dwarf galaxy as the dwarf falls in. This method allows us to directly study how
the varying tidal field strength of the Milky Way affects the cluster’s evolution while maintaining a constant tidal field strength for the dwarf.

In the second scenario, the cluster is simulated on a circular orbit around a point mass dwarf galaxy that evaporates over time. Initially the cluster orbits around the dwarf galaxy which in turn executes a circular orbit around the Milky Way. After 3 Gyr of evolution in this combined system, we decrease the mass of the dwarf according to the following equation:

$$M_D(t) = M_o e^{-6(t-3.0)}$$

(2.4)

where $t$ is the time in Gyr and $M_o$ is the original mass. As the mass decreases, we also decrease the radius of the cluster’s orbit around the dwarf according to the following:

$$R(t) = R_o e^{-(t-3.0)}$$

(2.5)

where $t$ is again the time in Gyr and $R_o$ is the original radius. The functional form for $M_D(t)$ was chosen such that the mass of the dwarf is effectively zero with respect to the cluster’s mass after 3.0 Gyr. $R(t)$ was then chosen such that the $r_j$ in the dwarf potential only increases as a function of time. Ultimately, the cluster is left orbiting around the Milky Way on a circular orbit at the same distance that the dwarf galaxy was from the centre. Although we choose the orbit of the cluster, this method allows us to study how the diminishing tidal field strength of the dwarf affects the evolution of the cluster while keeping the average tidal field strength of the Milky Way constant.

The scenarios described above probe the two key processes in a dwarf Milky Way merger that will affect the cluster; the increase in tidal field strength of
the Milky Way as the dwarf falls into the galaxy and the decrease in the tidal field strength of the dwarf as it is stripped by the Milky Way. The relative contributions of these two processes will be determined by the realistic orbit of the cluster in the combined potential. However, combining the results from these two scenarios will allow us to obtain a full picture of a star cluster’s evolution in a dwarf Milky Way galaxy merger.

2.2.2 Simulations and Parameters

All clusters are simulated with $N = 50,000$ stars distributed according to a Plummer density profile (Plummer, 1911) with no primordial binaries. Velocities are assigned such that the cluster is initially in virial equilibrium. Initial masses are assigned using a Kroupa (2001) initial mass function with masses from 0.1 to $50M_\odot$ and an average mass of $0.6M_\odot$. The effects of stellar and binary evolution are implemented throughout the simulation as per the prescriptions in Tout et al. (1997); Hurley et al. (2000); Hurley (2008). To characterize the actual size of a cluster, we use the half-mass radius (radius that contains half the mass), $r_m$, since it is a proxy for the half light radius measured by observers. For most of our simulations, we set the initial $r_m$ equal to 3.2 pc, unless otherwise stated.

The clusters are simulated in the two scenarios described above which we will call for simplicity ‘dwarf falls’ and ‘dwarf evaporates’ for the dwarf falling into the Milky Way and the dwarf losing mass, respectively. In all simulations, the Milky Way is modelled as a point mass bulge, Miyamoto & Nagai (1975) disc and logarithmic halo, details of which can be found in Miholics et al.
For the base case in the dwarf falls scenario, we simulate a cluster on a $R = 4.0$ kpc circular orbit around a $10^9 M_\odot$ dwarf. The initial separation between the two galaxies is set to 50 kpc. We allow the dwarf to fall into the Milky Way until it reaches a distance of 15 kpc from the Milky Way’s centre. We also examine the effect of changing the initial half-mass radius by simulating a cluster in the same potential but with an initial $r_m = 4.0$ pc. Additionally, we investigate the effect of varying the dwarf’s tidal field strength by performing a simulation with the mass of the dwarf, $M_D = 10^{10} M_\odot$. In the dwarf evaporates scenario, we simulate the cluster around a $10^{10} M_\odot$ dwarf with a $R = 4.0$ kpc circular orbit and place the dwarf on a circular orbit around the centre of the Milky Way at a radius of $R = 20.0$ kpc. For all our simulations, we must also perform two comparison simulations in which the same cluster evolves in the dwarf potential only and the Milky Way potential only. The comparison clusters simulated in the Milky Way are always given a circular orbit at the same distance from the centre that the cluster will ultimately end up at.

2.3 Results

For clarity, in all figures, we plot quantities corresponding to the cluster as it evolves in the combined potential, dwarf + Milky Way, in black. Quantities that represent cluster properties in the dwarf potential only and Milky Way potential only will always appear in red and blue, respectively.
2.3.1 Dwarf Falls into Milky Way

In our first simulation, we evolve the cluster on a circular orbit around a point mass dwarf \( M_D = 10^9 M_\odot \) and \( R = 4.0 \text{kpc} \) which is located 50 kpc from the Milky Way centre. After 3 Gyr, we allow the distance between the dwarf and Milky Way to decrease until the separation between the two galaxies is 15 kpc. In Figure 2.1a, we plot the half-mass radius for a cluster in such a potential. For comparison, we also show clusters in the dwarf only and in the Milky Way only (orbiting on a circular orbit of \( R = 15 \text{kpc} \)). Additionally, to demonstrate how the cluster’s position in the Milky Way potential changes over time, we plot the distance between the cluster and the Milky Way centre (multiply by 4 to get the correct value in kpc). The periodic variation in this distance is due to the cluster’s circular orbit around the dwarf galaxy.

We can understand the cluster’s size evolution in the combined potential by considering Figure 2.1b, where we plot the Jacobi radius of the cluster in each of the individual galaxies as a function of time. We also plot in Figure 2.1b the 99 % Lagrange radius, \( r_{99} \), a measure for the overall size of the cluster. The calculation of the Jacobi radius in each galaxy is done by considering the cluster’s orbit through that potential and ignoring the effects of the other galaxy. Hence, to calculate \( r_j \) for the cluster in the dwarf, we simply use the expression given in Equation 2.1 for circular orbits. However, no analytic expression for \( r_j \) in the Milky Way exists for the particular orbit that we have used (effectively, a spiral inwards towards the Galactic Centre). To obtain a sensible estimate for \( r_j \) in this case, we use Equation 1 and evaluate it at the instantaneous position of the cluster in the Milky Way all along its orbit. The mass used in
Figure 2.1: Simulation of a cluster around a $10^9 M_\odot$ dwarf that falls into the Milky Way with an initial half-mass radius of $r_m = 3.2$ pc.

(a) Half mass radius in parsecs over time for the cluster in three potentials: dwarf only (red) at 4 kpc, Milky Way at 15 kpc (only) and in the combined potential of the dwarf falling into the Milky Way (black). The magenta line gives 0.25 x the distance between the cluster and the Milky Way centre in kiloparsecs.

(b) The Jacobi radius of the cluster in the dwarf (red) and in the Milky Way (blue) as defined by Equation 2.1. We also plot the cluster’s 99 per cent Lagrange radius, $r_{99}$ in black.
this equation is the mass enclosed by a circular orbit of radius equal to the position of the cluster in the Milky Way at any given time. Therefore, we see a decrease in $r_j$ for the Milky Way due to both the cluster’s decreasing mass and decreasing Galactocentric distance in the Milky Way. The decrease in $r_j$ for the dwarf corresponds to the decreasing mass of the cluster only since the mass of the dwarf and radius of the cluster’s orbit stays constant throughout the simulation. From this point in the text, the two values of Jacobi radius in the dwarf and Jacobi radius in the Milky Way will be abbreviated as $r_{jD}$ and $r_{jMW}$ respectively.

The evolution of the cluster’s half-mass radius in the combined potential is almost identical to its evolution in only the dwarf galaxy over the first several Gyr of the cluster’s lifetime. This similarity indicates that when the dwarf is far from the Milky Way centre, the dwarf tides dominate the cluster. This idea is reinforced by Figure 2.1b which shows that $r_{jD}$ is much smaller than $r_{jMW}$ during the first stage of the cluster’s life. However, as the dwarf falls into the Milky Way, the $r_{jMW}$ shrinks (the tides become stronger) and the Milky Way becomes the dominant galaxy in terms of tidal field strength. At about 5.5 Gyr (2.5 Gyr after the dwarf starts to fall into the Milky Way) the cluster’s half-mass radius suddenly starts to decrease. By examining Figure 1, we see that this sudden decrease corresponds to the time when $r_{jMW}$ has decreased to a value such that $r_{99}$ becomes roughly equal to it. At this point, the cluster completely fills its Jacobi radius in the Milky Way and becomes very susceptible to tidal stripping. Also at this time, periodic bumps in the half-mass radius emerge corresponding to the cluster passing closer to the Milky Way centre and then further away on its orbit around the dwarf galaxy.
A short time after the dwarf reaches its final position in the Milky Way, at a separation of 15 kpc, the cluster’s half-mass radius completely overlaps with the half-mass radius of the cluster that has evolved in the Milky Way only. This point of overlap corresponds to a physical time of about 8.0 Gyr. At the point in time when the cluster starts to respond to the Milky Way potential, the half-mass relaxation time is approximately 1.3 Gyr. Hence, the cluster takes about 1.9 relaxation times to fully adjust to its new potential.

### 2.3.1.1 Effect of Cluster Size

To investigate how the initial size of the cluster affects our results, we perform another simulation with the same potential and larger initial half-mass radius, \( r_m = 4.0 \) pc. The results of this simulation are plotted in Figure 2.2, showing the same information as Figure 2.1, discussed above. Comparing the evolution of the differently sized clusters, we see that they follow the same overall pattern: the cluster follows the same evolution it would have in the dwarf only early on, at some point the Milky Way tides begin to dominate and the cluster eventually has the same half-mass radius as the cluster in the Milky Way only simulation. However, the half-mass radius of the cluster with a larger initial size starts to decrease in response to the increasing tidal strength of the Milky Way before the the smaller cluster, at about 5.0 Gyr. Examining Figure 2.2b reveals that this effect is due to the overlap of \( r_{99} \) and \( r_{j}^{MW} \) occurring at different times since differences in initial half-mass radius correspond to differences in \( r_{99} \).
Figure 2.2: Simulation of a cluster around a $10^9 M_\odot$ dwarf that falls into the Milky Way with an initial half-mass radius of $r_m = 4.0$ pc.

(a) Half mass radius over time in the three potentials as well as 0.25 x the Galactocentric distance in the Milky Way. Colours are the same as in Figure 2.1a.

(b) The Jacobi radius in each potential and $r_{99}$ over time. Colours are the same as in Figure 2.1b.
The larger cluster fully adjusts by 7.25 Gyr or about 1.4 relaxation times after the cluster starts to adjust (the half-mass relaxation time at the cluster’s first response is 1.6 Gyr). Hence, in addition to the larger cluster starting to respond to the new potential earlier, it also takes a smaller amount of time (both in physical units and relaxation times) to complete its adjustment. Although bigger clusters have longer relaxation times they adjust more quickly because they are more vulnerable to the tidal field (their filling factors are larger). This result suggests that mass-loss due to tidal stripping plays a more dominant role in changing the cluster’s size than internal relaxation driven processes.

For clusters with smaller initial sizes, we expect the response to the Milky Way potential to begin at later times. However, it is conceivable that a cluster could be so small that, at the final position of the cluster in the Milky Way, \( r_j^{MW} \) could still be larger than \( r_{99} \). In this case, as long as \( r_j^{MW} \) is smaller than \( r_j^D \), the tides of the Milky Way will dominate and the cluster’s size will be similar to the evolution of a Milky Way only cluster. However, in this case, the adjustment will not be as rapid since the tidal field of the Milky Way will not be able to strip stars as efficiently.

### 2.3.1.2 Effect of a More Massive Dwarf

We also study the effect of changing the dwarf’s tidal field strength by performing a simulation with \( M_D = 10^{10} M_\odot \). In this simulation, we allow the dwarf galaxy to fall into the Milky Way farther, to a separation of just 10 kpc. These parameters were chosen to make the final tidal field strengths of both
Figure 2.3: Simulation of a cluster around a $10^{10}M_{\odot}$ dwarf that falls into the Milky Way.

(a) Half mass radius over time in the three potentials as well as 0.25 x the Galactocentric distance in the Milky Way. Colours are the same as in Figure 2.1a.

(b) The Jacobi radius in each potential and $r_{99}$ over time. Colours are the same as in Figure 2.1b.
galaxies roughly equal. This equality is demonstrated in Figure 2.3b, where we see that when the dwarf reaches its final position in the Milky Way, $r_j^D$ and the time averaged $r_j^{MW}$ are approximately equal. In Figure 2.3a we again show the half-mass radius of the system in the combined potential as well as in the dwarf and Milky Way alone. We see that the cluster seems largely unaffected by the Milky Way throughout its entire evolution, deviating only slightly from its evolution in the dwarf potential only. Essentially, the cluster behaves as though only one galaxy were present in setting its size. This effect occurs because the summation of the tidal forces from each galaxy is a vector summation. Since the cluster is orbiting around the dwarf galaxy, there will be phases of the cluster’s orbit where the acceleration a star experiences towards the Milky Way is increased due to the presence of the dwarf galaxy (e.g. when the dwarf is between the cluster and the Milky Way). In this case, the instantaneous $r_j$ will be less than $r_j^{MW} = r_j^D$. However, since this stage represents a small part of the cluster’s orbit around the dwarf the time averaged $r_j$ will be close to the value of $r_j$ obtained by considering only one of the galaxies.

2.3.2 Dwarf Evaporates

In our second scenario, we evolve a cluster on a circular orbit around a dwarf galaxy that in turn executes a circular orbit around the centre of the Milky Way. We use the same dwarf galaxy and orbit ($M_D = 10^{10} M_\odot$, $R = 4.0$ kpc) as in the third “dwarf falls” simulations presented above. The dwarf galaxy executes a circular orbit of radius $R = 20.0$ kpc around the centre of the Milky Way. After 3 Gyr of evolution in this system, we decrease the mass of
Figure 2.4: Simulation of a cluster orbiting around a $10^{10} M_\odot$ dwarf galaxy that evaporates as it orbits the Milky Way on a circular orbit of $R = 20.0$ kpc.

(a) Half mass radius over time in the three potentials as well as $0.25 \times$ the Galactocentric distance in the Milky Way. Colours are the same as in Figure 2.1a.

(b) Mass normalized size ($r_m/M^{(1/3)}$) of the cluster over time in the three potentials: dwarf only (red), Milky Way (blue), combined potential (black).
the dwarf and radius of the cluster's orbit in the dwarf as described in Section 2.2.1, eventually leaving the cluster on a circular orbit in the Milky Way at 20.0 kpc.

In Figure 2.4, we show the cluster’s half-mass radius over time in this potential. Initially, the cluster follows an evolution similar to its evolution in the dwarf galaxy only, showing that the dwarf’s tides dominate over the Milky Way tides in this configuration. When the mass of the dwarf begins to decrease, the tides weaken and we see an immediate increase in the cluster’s half-mass radius. By 6 Gyr, the dwarf’s mass is effectively zero and the cluster is orbiting in the Milky Way only. The half-mass radius of the cluster continues to expand and becomes very close to the half-mass radius of the cluster living in the Milky Way its whole life. To remove the effect of varying mass-loss rates in different potentials, in Figure 2.4b, we have shown the mass normalized
radius \( (r_m/M^{1/3}) \) of the cluster in the three potentials. After the dwarf evaporates we see that mass normalized radius overlaps completely with the mass normalized radius for the cluster in the Milky Way potential only. Hence, the remaining difference in the clusters’ half-mass radii in these two potentials can be attributed to a difference in cluster mass, which we have plotted in Figure 2.4c.

We see in Figure 2.4c, a large difference in the mass-loss rate of the cluster in the Milky Way in comparison to the combined potential over the first 3 Gyr of evolution. Consequently, when the dwarf begins to evaporate and the cluster starts to adjust, a large difference in the masses of the clusters in the two potentials exists. For the size of the cluster to completely adjust the mass of the cluster must as well. Hence, the mass-loss rate slows and the masses in the two potentials become comparable. However, the masses never completely overlap because the mass-loss rate of the cluster in the combined potential cannot slow enough to compensate for such a large difference in cluster masses. This effect occurs when the cluster moves into a tidal field that is weaker than the original tidal field. There is no such a effect when the cluster moves from a weak tidal field to a strong one, as in the “dwarf falls” case shown in Figure 2.1. In this case, an acceleration of the mass-loss rate is driving the evolution of the cluster, rather than a deceleration. The increased mass-loss rate acts quickly to bring the cluster’s mass down to what it would have been had it been living in the stronger tidal field its whole life.

In the simulation presented in Figure 2.4, we start with a dwarf galaxy that has a stronger tidal field at \( R = 4.0 \) kpc than the Milky Way. To examine
the effects of starting the cluster in a dwarf galaxy that has a weaker tidal field than the Milky Way, we performed additional simulations in which we decreased the mass of the dwarf and let it orbit at a distance closer to the centre of the Milky Way. Before the dwarf evaporates, it has little effect on the cluster because the Milky Way’s tidal field is stronger. The orbit of the cluster in the Milky Way (passing closer and farther away from the centre due to the orbit around the dwarf) does not have an effect on the cluster’s size. Even though the tidal field of the Milky Way varies quite a bit on such an orbit, the cluster appears to evolve according to the mean tidal field strength of the Milky Way along that path. The dwarf evaporating decreases its tidal field strength even more and so the cluster continues to have a half-mass radius as dictated by the Milky Way’s tidal field.

2.4 Summary and Discussion

In our simulations, we simulate a cluster that orbits around a dwarf galaxy which either falls into the Milky Way or evaporates and leaves the cluster orbiting in the Milky Way only. The size of a cluster is determined by whichever galaxy has the strongest tidal field strength. Hence, in the ‘dwarf falls’ simulations, when the separation between the dwarf cluster pair and Milky Way is such that the dwarf tidal field strength is stronger (has a smaller \( r_j \)), the cluster has the same size as it would have if it evolved solely in the dwarf. Conversely, when the dwarf is close enough to the Milky Way centre such that the Milky Way’s tides begin to dominate, the cluster size decreases and eventually becomes the same size as a cluster which spends its entire lifetime in the Milky Way.
Way. This adjustment occurs because the stronger tidal field of the Milky Way can strip stars closer to the centre of the cluster, keeping the cluster confined to a region where the tides from the dwarf are not strong enough to strip stars from the cluster. The transition between these two regimes occurs when $r_j^{MW}$ becomes comparable to $r_{99}$, i.e. the physical limit of the cluster. In the “dwarf evaporates” simulations, the cluster responds to the weakening tidal field of the dwarf galaxy immediately, deviating from the size it would have in the non-evaporating dwarf. As the dwarf tides go to zero, the Milky Way begins to dominate and the subsequent size evolution is determined by the Milky Way tides entirely, leaving the cluster with a similar size as a cluster that has evolved completely in the Milky Way only. Any remaining size difference can be attributed to slightly different masses since the mass-normalized size of the cluster in the combined potential and the Milky Way converge almost immediately after the dwarf has evaporated.

Taking all the results from the above simulations together, we can construct the full picture of a star cluster’s evolution inside a dwarf-galaxy-Milky-Way merger. The two scenarios used for the gravitational potential on the cluster, ‘dwarf falls’ and ‘dwarf evaporates’ represent the key processes affecting a cluster in this type of galaxy merger. In particular, in the merger these processes will happen simultaneously since as the separation between the two galaxies diminishes, the Milky Way will strip mass away from the dwarf. These processes have the same net effect, to increase the Milky Way’s tidal field strength with respect to that of the dwarf. The simulations we perform in this work show that a cluster will have a size determined by whichever tidal field is the strongest at any one point. Whenever the Milky Way’s tides take over, whether
its due to the dwarf evaporating, the cluster becoming close to the Milky Way centre or both, the cluster will respond quickly to the new potential and ultimately become the same size as a cluster that has evolved solely in the Milky Way.

The results of this work are consistent with our previous findings detailed in Miholics et al. (2014). In that paper, we used an instantaneous change in galactic potential, from dwarf galaxy to Milky Way. Using this method, we found the same result as in this work: clusters adjust quickly in response to the new Milky Way potential, such that their size appear as if they had always been orbiting in the Milky Way on that particular orbit. The main difference between this and our previous work are the time-scales over which adjustments occur. Naturally, since the potential experienced by the cluster changes gradually over time in the simulations presented here, the changes in the cluster’s half-mass radius take longer to occur. However, once the potential reaches its final configuration and the tides on the cluster no longer vary, the cluster generally adjusts quickly.

For the changes in potential modelled here, the size adjustment takes place within $\approx 1-2$ current cluster half-mass relaxation times. We found that clusters with a larger initial size take less time to adjust to their new potential than their smaller counterparts. Such differences in adjustment time exist because larger clusters are more susceptible to stripping by the tidal field. This effect occurs despite the longer relaxation times of larger clusters, suggesting that the new tidal filling factor of the cluster plays the largest role in determining the adjustment time-scale. Hence, even the most extended globular clusters
in the Milky Way with long relaxation times would have adjusted quickly (if they were accreted) as long as they were not too tidally underfilling. We also note that altering the relative field strengths (initial versus final), may slightly alter the time-scale over which adjustments occur.

Our work is also consistent with recent simulations performed by Bianchini et al. (2015). These simulations are similar to our ‘dwarf evaporates’ case but in their case after the dwarf evaporates the cluster is left in isolation, rather than in the Milky Way. They also found that the cluster would respond to the change in potential by expanding rapidly, achieving sizes similar to (but always slightly less than) the size of a completely isolated cluster. The results of these simulations in conjunction with our work show that clusters accreted from dwarf galaxies should not be larger than clusters born in the Milky Way on the same orbit. In particular, the extended nature of some observed clusters in the Milky Way (Harris, 1996) is not a consequence of being accreted from a dwarf galaxy and experiencing a change in tides. Instead, the large size of these clusters most likely reflects some difference in their initial conditions at birth such as a large initial size (Zonoozi et al., 2011, 2014). Bianchini et al. (2015) also investigate the effect of turning the dwarf off instantaneously versus smoothly over time and reach similar conclusions as we do when comparing our current work with our previous work, as discussed above.

Our results suggest that it should not be possible to determine the origin of a cluster in the Milky Way based on its size alone. In particular, the distributions of tidally filling and underfilling clusters in the Milky Way should not be affected by the presence of clusters accreted by dwarf galaxies (assuming that
initial cluster sizes born in dwarf galaxies are similar to those born in the Milky Way). This result is consistent with the findings of Baumgardt et al. (2010). Although, they found evidence for two distinct populations of clusters in the Milky Way, one filling and one underfilling, they found no correlation between these groups and the group of clusters that are expected to be accreted from dwarf galaxies based on other properties.

An interesting extension of the work presented here would be to study a cluster inside a dwarf that is on an elliptical orbit around the Milky Way. In this case, the galaxy with the dominant tidal field strength might change along the orbit. If these oscillations occurred on a long time-scale, our results suggest that the cluster’s size would also oscillate between its size in the dwarf galaxy only and the Milky Way only, quickly adjusting to whatever tidal field was the strongest. However, if these oscillations happened on a short time-scale then the cluster would probably exhibit a size as determined by the average tidal field strength of the two galaxies. It is worth noting that such a situation would probably be short lived since the dwarf would have to be quite close to the centre of the Milky Way for this to occur and the cluster would be stripped away from the dwarf quickly.

A possible next step in this work is to perform a galaxy simulation of a merger between a dwarf and the Milky Way and extract from it the potential to use in the $N$-body simulation. This method would allow one to study the orbits of clusters inside the combined potential of the two galaxies (such as the elliptical ones discussed in the previous paragraph) as well as help to quantify the relative timing of the two processes studied here (falling and evaporating).
Since our models probe the key processes important to cluster evolution in galaxy mergers, this method should yield similar results on the sizes of most accreted clusters. However, using a galaxy simulation would allow for the study of accreted clusters with more dramatic tidal fields than the ones studied here (such as strong encounters with the Milky Way’s disc, dark matter substructure or other clusters) and may lead to the discovery of important physical processes that affect some accreted clusters and not others.
Bibliography

—. 2003, Gravitational N-Body Simulations, 430


Chapter 3

Further Questions and Future Work

3.1 Additional Simulations

Two additional simulations are presented here which were not included in the original work to investigate some other important questions surrounding the dynamics of accreted clusters.

3.1.1 Dwarf Falls: Effect of a Smaller Initial Size

The dwarf falls simulations, presented in Chapter 2, investigate how a star cluster behaves if placed on a circular orbit around a dwarf galaxy which starts at a separation of $R = 50.0$ kpc from the centre of a Milky Way potential and travels to some closer distance ($R = 10.0, 15.0$ kpc). Our original investigation included simulations of star clusters with two different initial half mass radii, $r_m = 3.2, 4.0$ pc. We now present an additional simulation of a star cluster in the same ‘dwarf falls’ potential with an initial half mass radius of $r_m = 1.6$ pc. Investigating a wide range of initial sizes is desirable for this work since a wide range of present day half-mass radii are observed for the globular clusters in
the MW as well as in dwarf galaxies (which tend to be somewhat larger than those found in the MW (Georgiev et al., 2009)). Simulating a cluster with such a small initial size is particularly interesting since a considerable fraction of globular clusters are very compact and are expected to have formed with initial sizes $\lesssim 1$ pc (Baumgardt et al., 2010).

In Figure 3.1, we present the results of this simulation. An interesting consequence of the small initial size is that the evolution of the cluster’s half mass radius when simulated in the dwarf galaxy potential only (red) is almost identical to when simulated in the Milky Way potential only at $R = 15.0$ kpc (blue). These similarities are present despite the higher tidal field strength in the Milky Way. This behaviour is due to the cluster’s small initial tidal filling factor ($< 0.05$ in both galaxies). The dynamics of very underfilling clusters is driven primarily by internal dynamics rather than the external tidal field since only a few outer stars are affected by the tidal field.

At $\approx 4.0$ Gyr, the evolution of the cluster in the two galaxies starts to differ (the cluster in the Milky Way becomes slightly larger than the cluster in the dwarf) likely due to slightly different core collapse times. When these slight differences start to arise, we see that the cluster’s evolution in the combined potential (black) tracks the evolution in the Milky Way only potential. The cluster’s evolution in the combined potential is more similar to the Milky Way cluster’s evolution because, at this point, the cluster has reached a distance close enough to the Milky Way centre where $r_{jMW}^j < r_{jD}^j$, as is evident in Figure 3.1b. This result supports the findings discussed in Chapter 2 since we again see that the galaxy with the strongest tidal field determines the cluster’s evolution.
Figure 3.1: Simulation of a cluster around a $10^9 M_\odot$ dwarf that falls into the Milky Way with an initial half mass radius of $r_m = 1.6$ pc.

(a) Half mass radius over time in the three potentials as well as $0.25 \times$ the Galactocentric distance in the Milky Way. Colours are the same as in Figure 2.1a.

(b) The Jacobi radius in each potential and $r_{99}$ over time. Colours are the same as in Figure 2.1b.
After $\approx 6.0$ Gyr the dwarf galaxy cluster becomes larger than the Milky Way cluster. Both clusters are still expanding due to two body relaxation but the dwarf galaxy cluster expands more since it is less tidally limited. The cluster in the combined potential continues to evolve as a Milky Way cluster for the rest of the simulation since the Milky Way has the strongest tidal field at the position of the cluster.

Although this extra simulation supports the conclusions drawn from the other simulations, it raises an important point about tidally underfilling clusters. The cluster with the smaller initial size is tidally underfilling for a significant portion of its life, in contrast with the clusters presented previously. Since the cluster becomes tidally filling at some point in its life, we see a difference in the evolution of the cluster when evolved in the Milky Way versus the dwarf. However, one can envision real clusters that were tidally underfilling at birth and are still underfilling at present day. Indeed, the only reason why the cluster we simulate becomes tidally filling within a Hubble time is that it has a very short initial relaxation time ($\approx 100$ Myr) and the cluster expands rapidly due to two body relaxation. For more realistic globular cluster-like systems, $N$ is much higher and thus typical initial relaxation times should be longer, yielding a slower expansion rate than the one witnessed in this simulation. Interestingly, if an accreted cluster is underfilling throughout its whole lifetime it will not undergo a change in size when accreted into the Milky Way from a dwarf. Since approximately 2/3 of the Milky Way’s globular cluster population is estimated to be in the tidally underfilling expansion driven stage of their evolution (Gieles et al., 2011), it is likely that a significant fraction of accreted clusters never experienced a change in size at all. This point is subtle but im-
important for any future work regarding this topic. For example, although similar conclusions about the dynamics of tidally filling versus underfilling accreted clusters can be drawn, other aspects such as kinematics may differ. It remains a possibility that the kinematics of outer cluster stars may bear the imprints of a recent rapid change in size, thus providing a way to distinguish accreted clusters from in situ clusters. Although this approach would be useful for the subset of accreted clusters that are tidally filling, clusters that remain tidally underfilling would not bear the same signature, thus limiting the applicability of any such method.

3.1.2 Dwarf Evaporates: Effect of a Retrograde Cluster Orbit

One unexplored avenue is the evolution of an accreted cluster that is on a retrograde orbit inside the dwarf galaxy with respect to the orbit of the dwarf galaxy around the Milky Way. The ‘dwarf evaporates’ simulations presented in Chapter 2 are all done assuming that the cluster orbits around the dwarf in the same sense that the dwarf orbits the Milky Way. We perform a simulation that is otherwise identical to the one presented in Section 2.3.2 but with the dwarf orbiting the Milky Way in the opposite sense, to investigate the effect of the cluster executing a retrograde orbit.

In Figure 3.2, we show the half mass radius over time for the cluster on a retrograde orbit. Also shown are the usual comparison simulations, one in the Milky Way at $R = 20.0$ kpc and one in the dwarf galaxy only, as well as the half mass radius evolution for the same cluster on a prograde orbit (presented in Section 2.3.2). We see very little difference in the evolution of the cluster
on a retrograde orbit compared to a prograde orbit and the same conclusions can be drawn about the dynamical evolution. We again see that the galaxy with the strongest tidal field determines the evolution of the cluster and that ultimately the accreted cluster is similar in size to the Milky Way cluster. Any size difference between the accreted clusters and the Milky Way cluster can be attributed to mass differences, as discussed in Section 2.3.2. Any differences in the evolution of the two accreted clusters (prograde and retrograde) can be attributed to the stochastic nature of $N$-body simulations.
We can explain the behaviour of the retrograde orbit cluster by considering the effect of the dwarf's orbital motion on the cluster. Normally, when a cluster orbits around a galaxy, the side of the cluster that faces the centre of the galaxy constantly changes. From the perspective of a coordinate system centred on the cluster with the x-direction fixed along the line from the galaxy centre to the cluster centre, the cluster is rotating. This effective rotation causes a centrifugal force that acts on the cluster stars, increasing the effective tidal field strength of the galaxy (Renaud et al., 2011). When this effect is not taken into account, the expression one derives for the Jacobi radius is slightly different from the one in Equation 2.1:

\[ r_j = R_G \left( \frac{M_c}{2M_G} \right)^{1/3} \] (3.1)

(notice the 3 in the original equation becomes a 2). In our simulations, the cluster is orbiting around a dwarf galaxy as the dwarf orbits around the Milky Way. This configuration causes the side of the cluster that faces the centre of the Milky Way to stay nearly fixed, nearly eliminating the centrifugal force that normally arises from the cluster's orbital motion, and decreasing the tidal field strength of the Milky Way. The decrease in the tidal field strength can be found by taking the ratio between the two expressions for the Jacobi radius in Equations 2.1 and 3.1: \((2/3)^{1/3} = 0.87\). This decrease is independent of whether the cluster is on a retrograde or prograde orbit with respect to the dwarf’s orbit around the Milky Way. Hence, we see no difference in the evolution of accreted clusters that have prograde orbits and those with retrograde orbits.

The effect described above is nearly independent of our choice of parameters (i.e. the orbital distances, the shape of the cluster’s orbit in the dwarf)
since the side of the cluster that faces the Milky Way will stay nearly fixed over a wide range of these parameters. The side of the cluster that faces the Milky Way will change by at most $\theta = \tan^{-1}\left(\frac{\max(y_D)}{R_G}\right)$ where $y_D$ is the distance between the dwarf and the cluster in the direction perpendicular to the line connecting the dwarf and the Milky Way and $R_G$ is the radius of the dwarf’s orbit in the Milky Way. For the choice of parameters in the simulation, $\theta = \tan^{-1}(4.0/20.0) = 11.3^\circ$, implying that the centrifugal force has a minimal effect. The centrifugal force would become important again when $y_D$ is a significant fraction of $R_G$. However, at such a point the cluster is likely to be unbound from the dwarf and so even then a prograde/retrograde orbit would matter little to the evolution of the cluster.

3.2 Conclusions

Globular cluster populations are powerful tools for understanding the evolution of their host galaxies. In particular, since they are formed early in the Universe, they trace the hierarchical formation of structure; any galaxy will contain clusters formed in the galaxies that have merged to create it. However, before using star clusters as an observational tool for peering into a galaxy’s past, we must first use simulations to systematically understand how the properties of individual clusters and cluster populations depend on their host galaxy’s history. This thesis has examined how the properties of individual globular clusters, in particular their size, depend on their tidal history, or in other words how their host galaxy has changed. This work was done in the specific context of studying how dwarf galaxy globular clusters evolve if they
are accreted by the Milky Way in a dwarf-Milky Way merger. However, the main conclusions can be applied equally well to any other scenario where globular clusters undergo changes in their local tidal field. The main conclusions of this work are:

- The instantaneous evolution of a cluster’s half mass radius is determined by whichever galaxy (dwarf or Milky Way) has the strongest tidal field at the position of the cluster.

- A star cluster that has recently undergone a change in which galaxy’s tidal field is the strongest adjusts to the new tidal field by shrinking or expanding on a short time-scale ($\approx 1\text{-}2$ cluster half-mass relaxation times) such that its size is indistinguishable from a cluster that has always lived on that orbit.

- The distribution of half mass radii for a cluster population should be unaffected by the tidal history of the individual clusters. The distribution should be determined by the initial half mass radii of the clusters (which could have a dependence on the origin of the cluster) and the present day orbit of the cluster inside the galaxy only.

In the future, it would be constructive to extend the work presented in this thesis by performing a galaxy simulation of a dwarf Milky Way merger and use the tidal field obtained to simulate accreted clusters. The benefits of such an investigation are discussed in detail in Section 2.4.
3.3 Future Work

Although this work has shown that it is not possible to trace the accretion history of the Milky Way through the internal dynamics of star clusters, there are still plenty of opportunities to use star clusters as tracers of galactic evolution, especially if the population as a whole is considered. Over the past several months I have worked with Diederik Kruijssen at Heidelberg University to start the process of characterizing how the properties of star cluster populations are affected by their host galaxy environment. We have worked on the suite of galaxy simulations presented by Kruijssen et al. (2011, 2012) that tracks star cluster populations in isolated disc galaxies by modelling the formation and destruction of star clusters through tidal shocks. Through detailed analysis of these simulations, I studied how the properties of star cluster populations, especially their age distributions, depend on the galaxy properties, like gas density, velocity dispersion and encounter rate. The next step in this project would be to analyze the galaxy merger simulations in the same suite of galaxy simulations in a similar way. These results can then be compared to observations of cluster populations in real galaxies to determine their merger and star formation histories.

Another project that could be pursued in the future would again focus on the dynamical evolution of individual clusters. The total evolution of a star cluster is composed of two main parts. The first part occurs in the birth environment of the cluster, expected to be a gas-rich disc with high pressures and densities (Elmegreen & Efremov, 1997; Kruijssen, 2015). The second component occurs after the cluster has migrated away from its birth environment,
into the halo of its host galaxy. The second stage is characterized by a slow leaking of stars out of the cluster due to the tidal field of the galaxy while the first is much more violent due to interactions with giant molecular clouds (GMCs), other star clusters and merging galaxies. Many studies have focused on simulating the latter part of a star cluster’s life. This work has been essential for understanding the long term dynamical evolution of star clusters over the majority of their lifetimes. The initial conditions for these simulations come from assumptions about the properties of clusters after they have completed the first part of their evolution in the birth environment. These assumptions include a spherically symmetric mass distribution for the stars (usually a King or Plummer profile), virial equilibrium to assign velocities, a mass function given by the initial mass function, no mass segregation and an initial half mass radius comparable to the average globular cluster half mass radii observed today. Using these assumptions, although they are a good first approximation, makes comparisons to real observed clusters difficult. This issue could be remedied by performing N-body simulations of globular clusters in a fully realistic tidal field, from immediately after their birth to the present day with initial conditions given by the final products of star formation. These simulations could be done by extracting tidal histories for clusters formed in the Kruijssen et al. (2011, 2012) simulations or from a cosmological simulation with baryons (e.g. the EAGLE project, Schaye et al. (2015)), and using them as the tidal field in an N-body simulation. The results from the detailed analysis of the Kruijssen et al. (2011, 2012) simulations, including the galaxies mergers, would help to characterize what a typical tidal history for a globular star cluster is, as well as how much tidal histories can vary from cluster to
cluster. Such an approach would allow one to obtain the typical evolution for a globular cluster and how much we can expect that evolution to vary from cluster to cluster. This project would allow for better comparison of simulations to observations of real globular clusters.

The field of research focussed on the dynamics of globular clusters has come a long way over the last several decades. However, we are just now starting to tap into using globular clusters as tools to answer questions in other astronomical areas of research. Since globular clusters witness all the stages of galaxy evolution, they will be excellent tools for advancing the field of galaxy formation and evolution in the near future. This thesis has taken a critical step towards using globular clusters to probe galaxy formation and evolution by answering the question of how individual globular clusters behave in galaxy mergers. The current and future work described above will answer how populations of globular clusters as a whole evolve over cosmic time as a function of their environment. With these two sets of work acting as stepping stones, it seems reasonable that we will soon be able to use globular clusters to unlock the history of our entire Universe.
Bibliography


This page intentionally left blank.