

A COMPARISON OF FREEWAY  
FLOW-OCCUPANCY RELALATIONSHIP

BY

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## ABSTRACT

Traffic operations can be described by the relationship that occurs between flow and occupancy. This paper investigates the flow-occupancy relationship of an Ontario system and a Minnesota system to see if the same general relationship occurs in different locations. It is hoped that this investigation will help to further the work being done at McMaster in developing a new incident detection algorithm.

In comparing the two data sets, simple analytical procedures were employed to compare the full data set, the uncongested regime, and the calculated fitted lines for the uncongested data. Visual comparison was the basis for much of the analysis.

When the comparisons were conducted the relationships were indeed very similar, signifying that the flow-occupancy relationship is the same for different locations.

## ACKNOWLEDGEMENTS

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## 1. Introduction

Freeway traffic management systems are used in many cities for the purpose of improving traffic operations. As a part of their monitoring, these systems measure the rates of flow, occupancy, and, in some cases, speed. Flow refers to the number of vehicles per unit time passing a point on the roadway. Occupancy is the percentage of time a detector on the roadway is occupied by vehicles. The goal of the research paper is to see whether the same flow-occupancy relationship exists in different locations. To compare the flow-occupancy relationship two locations were chosen: an Ontario system in the Hamilton area, and a Minnesota system from the Minneapolis area. The results of the comparison can help to validate a new incident detection approach for freeway traffic management systems.

In order to explain the purpose more completely, it is first necessary to give some background information about freeway management systems. It is also necessary to understand what these systems do and how they can be used to their fullest extent.

The first attempt at freeway management in Ontario was undertaken by the Ontario Ministry of Transportation and Communications in July 1975 (Case & Williams). Two basic goals were established for this project: to operate the freeway system at a high flow rate and a reasonable level of service; and to minimize collisions on the freeway system. The goal of

minimizing collisions would be achieved by recognizing conditions likely to cause collisions and then providing adequate warnings, as well as by rapidly recognizing and responding to collisions (thereby reducing the risk of secondary collisions).

Case and Williams give a detailed description of the system located on the Queen Elizabeth Way between Oakville and Toronto. This particular system provided lane and station values of volume, occupancy, speed, and vehicle-length distribution. The system comprised several mainline detector stations with induction loop pairs, ramp metering on entrance ramps and closed-circuit television surveillance cameras operated from a control centre. The data were collected during weekday morning peak periods, and represented approximately two and one half hours collection each day. For each pair of induction loops, the occupancy at the downstream loop, volume of vehicles, and average speed were obtained for each lane in 5-minute intervals. The data were stored on magnetic tapes, and a complete log of daily weather conditions and incidents was available (Case & Williams, 1975).

The Ontario experience is not limited to the system described by Case & Williams. Freeway traffic management systems are being used in other areas. The Mississauga section of the Queen Elizabeth Way used in the Case and Williams study is still in operation and has since been expanded from 9



on the Burlington Skyway portion of the QEW. Other systems are underway for Highway 401 and the Gardiner Expressway.

Elsewhere, facilities are present in areas ranging from California and Washington to Minnesota and Long Island.

The basic aim of all of these systems is to improve traffic operations. This is accomplished through reliable automatic incident detection methods (an incident can be an accident, a breakdown, etc.). In North America the most commonly used logic for incident detection appears to be the California-type algorithm in which specified differences in traffic operations between two adjacent stations indicate the presence of an incident. In an ideal setting the logic would detect all incidents immediately on occurrence and would not produce false alarms when there are no incidents present (Persaud & Hall, 1988).

The current comparative approach is being challenged by a new algorithm - the McMaster algorithm. According to Persaud and Hall (1988), this new logic improves on the current logic in several aspects. This new logic is most efficient if all three variables of speed, flow and occupancy are available. Given a system which provides reliable values for these variables the McMaster logic improves chances of detecting incidents. The new logic also makes it possible to detect incidents by looking at data for a single station, providing an advantage when highway geometric conditions vary between successive detector stations. The proposed logic can be based

on median lane data only, thereby eliminating the need to use average lane occupancy which is subject to considerable random fluctuation because of the presence of trucks (Persaud & Hall, 1988).

The present research is important to the McMaster algorithm development. Since this algorithm uses flow-occupancy data it is necessary to show that flow-occupancy relationships are similar from one place to another. Should this be the case it would be possible to develop a generalizable incident detection logic. The two locations that have been chosen for comparison are the Burlington Skyway on the Queen Elizabeth Way in Ontario and a system in Minneapolis, Minnesota. Both of these systems provide the necessary flow and occupancy data. Speed is given in the Ontario data and will be used to analyze the Ontario portion of the data. Unfortunately, the Minnesota data do not offer speed and therefore will have to be analyzed somewhat differently.

The research involves extensive use of the data sets. The nature of the data makes it possible to view the entire freeway activity, that is both congested and uncongested behaviour. Several graphs are used to illustrate this fact. Similarities were sought in two respects: the general tendency of the uncongested data, and the appearance of the complete data set. Most of the focus was placed on the uncongested portion of the data sets.

The remaining text is organized as follows. First,

there is a short literature review which identifies relevant models of flow-occupancy. Second, a description of the data is provided and the analysis is discussed. Third, a section is devoted to the comparison of the results from the data analysis. Finally, a conclusion is offered.

## 2. Literature Review

Within the field of freeway management and incident detection there appear to be different views of how the relationship between the variables should be modelled. Although the focus of this research is the flow-occupancy relationship, most of the previous work has dealt with three variables: flow, occupancy, and speed. Any efforts made at identifying the relationships between these variables have usually investigated them two at a time, thus finding the relationship between speed and flow, between speed and density, and between flow and density. Many of the papers deal with flow-density relationships instead of flow-occupancy. This review will deal with the relationship between only those two variables of interest for this paper.

The relationship between flow rates and vehicular concentration on freeways has been discussed in several papers. Many studies on traffic flow use density (vehicles/km) as the measure of concentration. Another measure of concentration is occupancy. When occupancy is compared to density there is considerable scatter in the data for congested operations. For uncongested operations, the ratio of density to occupancy is relatively unvarying (Hall, 1986).

The relevant literature, listed chronologically, provides necessary background information for the research foundations of this paper.

Edie (1961), after observing a number of data sets, noted that empirical flow-density plots could be represented by two curves. One curve represents the uncongested state and the other curve represents the congested state. Within this model, Edie suggested a distinct discontinuity in the region of maximum flow and showed that the two curves fit the data better than a single curve.

Gazis, Herman, and Rothery (1961) mention Edie's observation of the apparent discontinuity of the flow near the peak of the flow versus concentration curve. In their paper, Gazis et. al. describe the discontinuity as reflecting what they call a possible bimodal character of the flow curve. Bimodal, in this case, would refer to the apparent two-regime representation ( $\wedge$ ). Perhaps it would have been better to refer to the "bimodal" character as two separate models, one for low concentrations and one for high concentrations. Regardless, their interpretation is that the initial portion of a realistic flow-concentration curve at low concentrations arises from the fact that there are very few vehicles and they do not interact. As the concentration increases the flow increases, but because of the increase of the vehicle interactions the flow reaches a maximum and then decreases with increasing concentration. With high concentration, large relative speeds become improbable so that the flow pattern may become ordered. Therefore, the flow curve may exhibit some kind of a "bimodal" character.

Athol (1965) used lane occupancy and compared it to volume and speed. Lane occupancy was chosen because of the practical advantages in representing the degree of concentration existing within a moving traffic stream. According to Athol as traffic breaks down and speed is reduced, the accuracy of speed determination diminishes, whereas the accuracy of occupancy determination increases. Athol's use of occupancy appears to be the first use of such a measure and therefore lends support for the use of occupancy and flow in this research effort.

Drake, Schofer, and May (1967) compared hypotheses describing the relationships between basic traffic stream characteristics by regressing speed versus density. The volume-density and speed-volume relationships were verified visually. The general criterion guiding the entire Drake et. al. research effort was the ability of the various functions to predict the entire range of flow characteristics. The tests used were designed to cover this range. The discontinuous hypotheses were examined independently for significant differences between the congested and uncongested regimes. The hypotheses were also tested for a slope significantly different than zero. On the basis of this investigation, Drake, Schofer, and May concluded that the Edie discontinuous hypothesis was best among the hypotheses tested. Drake, Schofer, and May also found that the Edie hypothesis yielded a comparatively low value for the standard error of estimate.

Hillegas (1974), in an investigation of flow-density discontinuity, favoured distinct discontinuous ranges of linear and non-linear behaviour. Through the use of time-series analysis, flow-density curves were generated with the data. The curves were broken into hourly segments such that they were divided according to traffic conditions, ie., free flow, congested, and transitional. The analysis yielded three results: a range of distinct linear behaviour, a nonlinear range of behaviour, and a combined linear and nonlinear behaviour. In order to distinguish between the three operational states, some density value  $k$  would be necessary to distinguish free flow from congested behaviour. Evaluation of the density criterion function involved finding the standard deviation of average density for both congested and uncongested data.

Using the general car-following equation developed by Gazis (1960) and others as a starting point, Easa (1983) selected two-regime models (congested and uncongested) for the traffic-flow data. Easa observed that two-regime representation would, in general, provide a better fit to the traffic-flow data than the single-regime representation. This would be particularly true if there is a wide range of flow disturbance near capacity. The reason given was that two-regime models account for the variability of the data in the intermediate ranges of operations through the use of auxiliary criteria. Single-regime models consider only basic criteria.

The traffic-flow criteria that Easa refers to as basic criteria are speed and density. The auxiliary criteria simply account for the variability of speed and density in the intermediate ranges of operations.

In another instance, Hall (1986) selected the flow-occupancy relationship for consideration over the speed-flow or speed-occupancy relationships because, in his analysis, it provided the clearest distinction between congested and uncongested regimes.

Occurrences of gaps in freeway speed-density and flow-density have led researchers to suggest that discontinuous functions are necessary to describe traffic behaviour. It was the contention of Hall, Allen & Gunter (1986) that gaps found in the data do not necessarily imply a discontinuous function. Instead, an inverted V shape (continuous, but not continuously differentiable) is suggested. The "gaps", or areas of sparse data, usually occur in high flow ranges, at speeds normally associated with near-capacity operations. They are typically located in the congested regime. Daily time traces were used to observe the nature of observations and of transitions between congested and uncongested regimes. These plots gave rise to the conclusion that there is additional support for the use of continuous relationships to describe the data obtained from freeway operations.

To try to understand "gaps", Hall (1987) used catastrophe theory as a means for understanding the behaviour



of freeway operations. "Catastrophe theory takes its name from the sudden, discrete changes that occur in one variable of interest, while other related variables are exhibiting smooth and continuous change." (Hall, 1987) The theory was developed by Thom (1975) and further refined by Zeeman (1977). Hall concluded that catastrophe theory, in particular the cusp catastrophe, replicated very well the functions derived from the data collected on the Queen Elizabeth Way in Ontario. A feasible explanation for the occurrence of jumps in the data is shown to be provided by simple linear transformations between traffic operations variables and catastrophe theory variables. Hall's paper does not offer a single unique solution but does provide new insight into the operation of freeways and new areas of research. In the paper, the reasoning used was primarily visual, rather than mathematical.

The review of the literature shows that there seems to be widespread belief that traffic stream flows can be represented by discontinuous functions. If the data can be fitted by two distinct curves there is justification for dealing simply with one side of the curve, that is the curve representing uncongested conditions. With respect to catastrophe theory, empirical findings show that uncongested operations occur fairly close to the 'edge' on the upper fold of the partially folded catastrophe surface. Therefore, it would be easy to deal with only the upper surface.

Following from the literature there appears to be

support for the choice of using a flow-occupancy relationship, as was used by Athol (1965) and Hall (1986). A further reason for the use of occupancy rather than density is the fact that it is the variable directly measured by the freeway management system. Also, freeway management systems from elsewhere do not collect information on speed.

With respect to analytical methods, the literature shows that regression analysis and curve fitting provide the best method. Drake, Schofer, and May (1967) do mention the fact that one should be leary of any results obtained as their statistical tests were not able to distinguish among the different hypotheses tested. It is possible that visual verification may have to be used.

Review of relevant literature has assisted the formulation of specific objectives for the research. First, the entire data set must be understood. In order to do this it will be necessary to graph the entire data set to see, visually, the full range of congested and uncongested data. Second, the uncongested data will have to be identified. The third objective entails fitting a line to the uncongested portion of the data so that the fitted lines obtained from the data sets can be compared.

### 3. Data and Analysis

This section provides a description of the data being used. Following this description, the data selection methods are discussed with emphasis placed on the method used to select and identify the uncongested data. The final part of this section is devoted to the analysis of the two data sets.

#### 3.1 Description

The Ontario system provided information from several surveillance stations on the southbound lanes of the Queen Elizabeth Way on the Burlington Skyway and of these, only three were chosen. The data, obtained in August 1987, came from the median lane for all three stations.

The data consisted of volume, occupancy and speed, based on 30 second intervals. For each interval two values were given for occupancy and flow, one value corresponding to the upstream detector and one value for the downstream detector. For this research the values from the downstream detector were used.

Quite often the values obtained for the upstream detector were different than those obtained at the downstream detector. An explanation that can be offered is based on the finite time interval. Because of the 30-second intervals a vehicle might pass over the upstream induction loop detector and before it reaches the downstream detector the 30 second interval has elapsed. The differences between the upstream and

downstream detectors were never more than 2 vehicles.

Speed was given in kilometres per hour (km/h). Speed was important for identifying the uncongested data for Ontario. The Minnesota data do not provide speed.

The Minnesota data were collected in October 1988 on Interstate 35 in Minneapolis. The Minnesota data set, like the Ontario data set, was extremely large and therefore only a few of the stations were dealt with so that the size could be reduced. Three stations in particular were focussed on. Station 53, 54 and 57, were located on the eastbound lanes of the I-35 just west of the Minneapolis central business district.

The Minnesota data provided occupancy and volume values based on 5-minute intervals instead of 30-second intervals as was used in Ontario. An error flag was also provided. The values for the error flag were either 1 or 0, where 1 signified that the data was suspect, and 0 signified that the data was acceptable. The suspect data were not used in this analysis. The values for volume had to be changed to hourly rates so that they would be comparable to the rates provided in the Ontario data. The Ontario data had also been converted to an hourly rate from the original 30-second intervals.

### 3.2 Identification of Uncongested Data

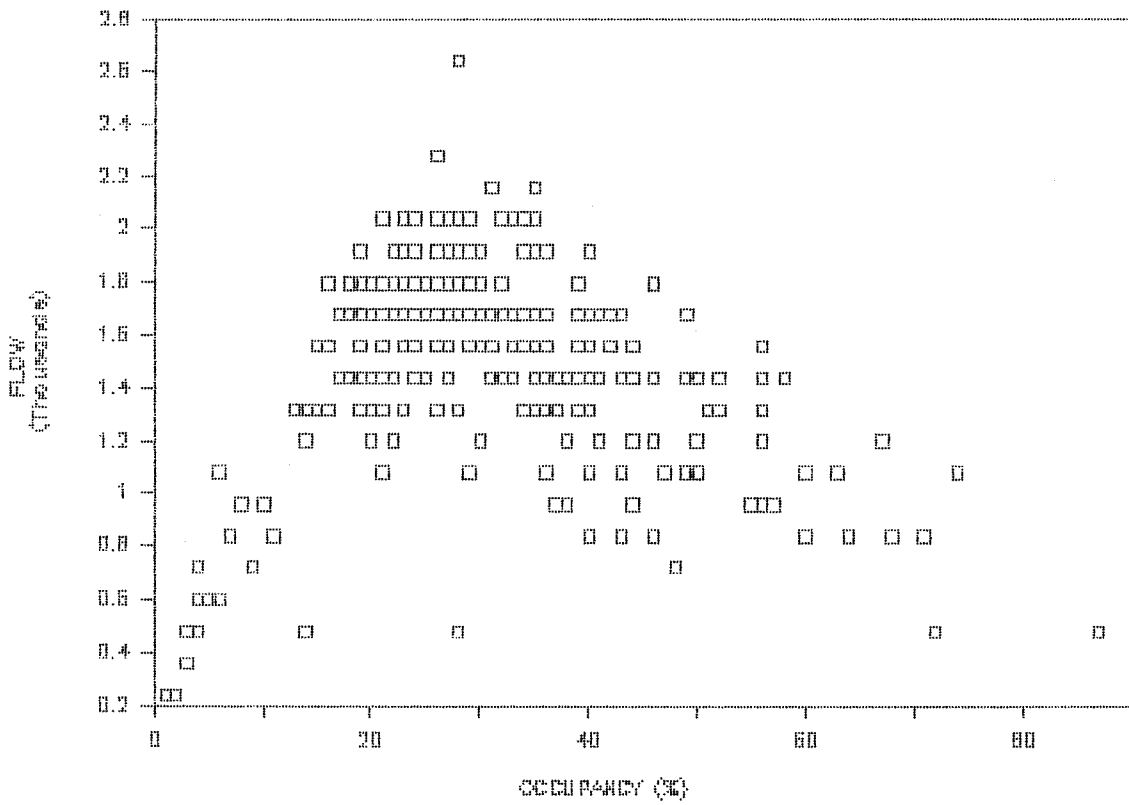
Based on the evidence presented in the Edie paper (1961) the entire data set was studied in two separate

portions, the uncongested and the congested. For this research most of the focus was placed on the uncongested portion of the data. The first step was to graph and understand the entire data sets. Graphing was required so that it was possible to see what was happening and where it was happening. From these graphs it was possible to identify areas of congested and uncongested behaviour.

Figure 1 shows the complete data set for station 5 on the Burlington Skyway. From this graph it is possible to see visually the reverse lambda shape ( $\wedge$ ) that Koshi et. al. obtained in their study. It is also possible to see how the entire set can be divided into two separate regimes. The uncongested traffic behaviour corresponds to lower values of occupancy while congested behaviour corresponds to higher values of occupancy. In the case of the Ontario data congested behaviour is also linked with lower speeds.

The uncongested behaviour for station 5 (figure 1) occurs at occupancies less than 27%. The data points in this area appear to be more ordered and capable of being fit by a straight line. The congested behaviour for station 5 is visible at occupancies greater than 27%. In this area the data points are more scattered and spread out. It does not appear that data in this area could be fit well by a straight line. The transition area occurs in figure 1 between 20 and 30% occupancy range. This transition area shows a tight clustering of the data points.

FIGURE 1: FULL DATA SET FOR STATION 5



Figures 2 and 3 show the other Ontario stations. Station 6 data looks very similar to station 5. Station 7 also looks similar but has a greater transition zone (15-30%).

The Minnesota stations shown in figures 4, 5, and 6, show very little transition data. For all three stations the uncongested behaviour occurred at lower occupancies than those observed for Ontario.

In order to fulfill the objective of fitting a line to the uncongested portion of the data it was first necessary to define what could be considered congested. Once critical values for both speed and occupancy were determined it was possible to identify the uncongested behaviour. Since Ontario provided speed this became a logical starting point in defining what was congested and uncongested behaviour. A cut-off point was necessary to indicate the point where traffic moved from an uncongested state to a congested state. Three values to be tested were chosen, 60km/h, 70 km/h, and 80km/h. This decision was made arbitrarily in the knowledge that the normal speed limit in Ontario on highways is 100 km/h.

Taking the cut-off value for speed and looking through the data it was possible to look at only those occupancy and flow rates that had corresponding speeds of greater than 60 km/h. Once this was accomplished, and any values corresponding to speeds of less than 60 km/h were rejected, the amount of the data was greatly reduced. When the remaining values were graphed some stray points appeared, as can be seen in figure 7,

FIGURE 2: FULL DATA SET FOR STATION 6

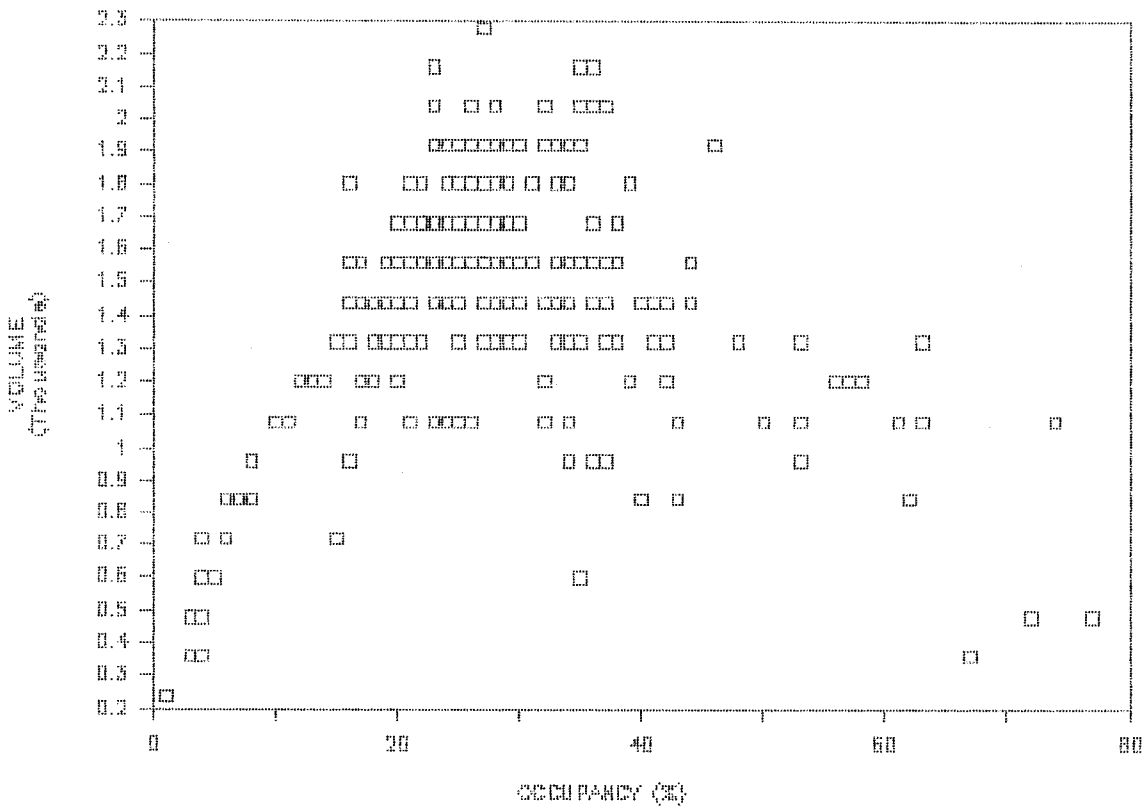




FIGURE 3: FULL DATA SET FOR STATION 7

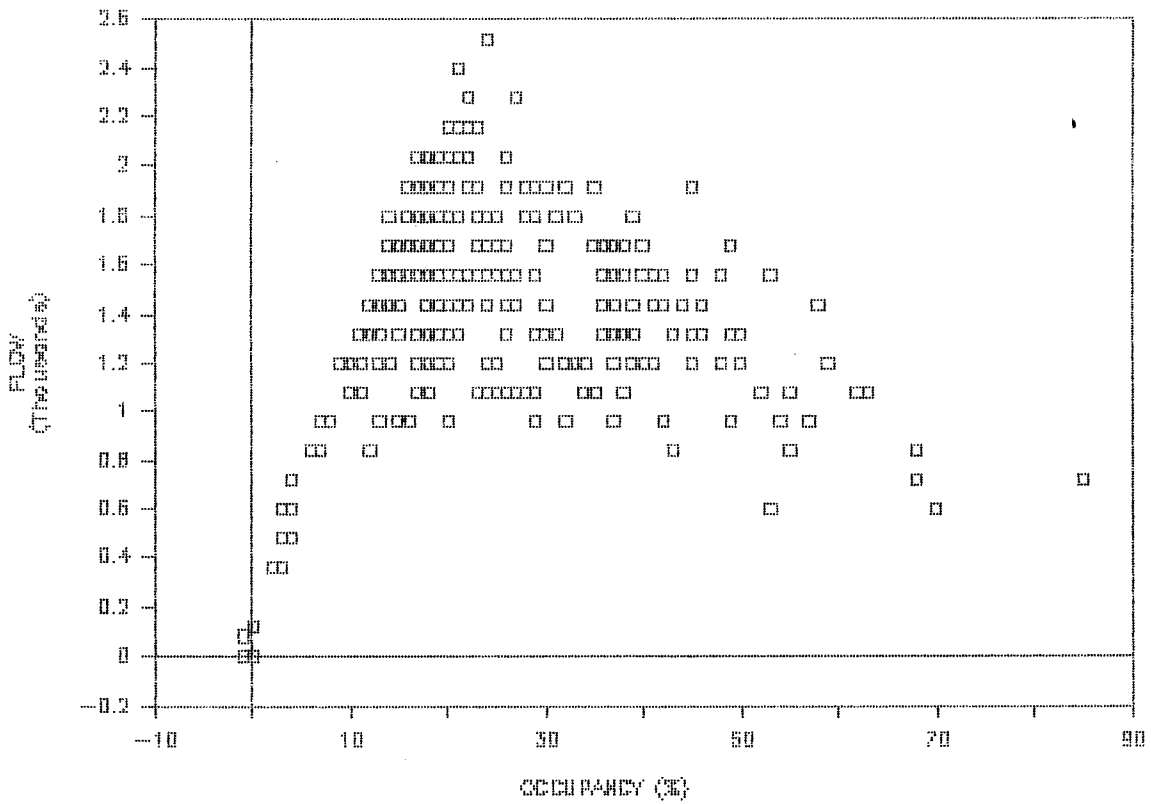


FIGURE 4: FULL DATA SET FOR MINNESOTA 305

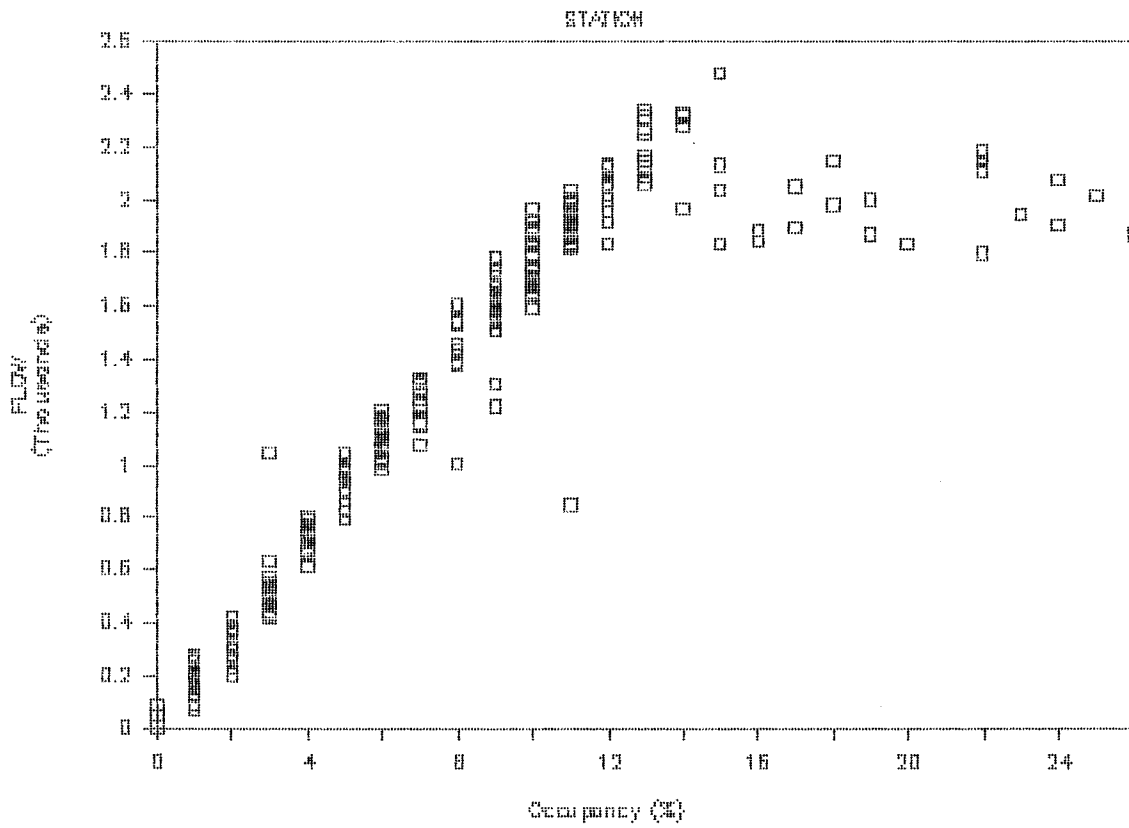
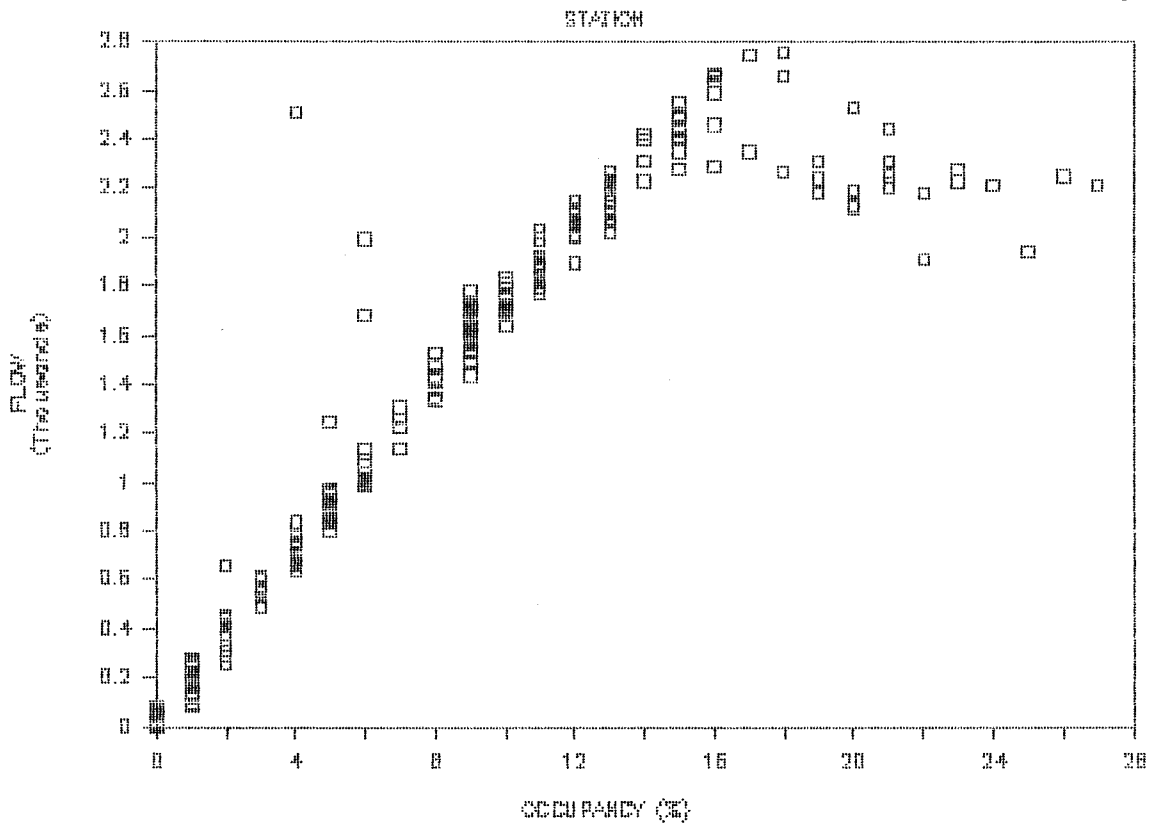


FIGURE 5: FULL DATA SET FOR MINNESOTA 308





that were not consistent with the rest of the data points. These values, in spite of having speeds above 60 km/h, correspond to very low flow rates and high occupancy rates. It was obvious that these few points were not representative of uncongested behaviour. Instead, further sorting of the data had to be done. In this instance the remaining data were sorted, except this time with respect to occupancy. This meant that a cut-off point also had to be established for occupancy so that the stray points could be eliminated. When the resultant data points were plotted they were representative, at this point, of the uncongested data. Figure 8 shows these plotted data points.

The same procedure was carried out for a cut-off point of 70 km/h (figure 9) and 80 km/h (figure 10). From a visual comparison of the three data sets it was clear that with the increased speed cut-off the uncongested data was more distinct, even before the use of an occupancy threshold. This implies that for the higher thresholds there were fewer points that were suspected of possibly being in the transition area. The idea of presenting only the uncongested data points becomes very important later when lines are fit to the uncongested data. If there are points present that in fact are not representative of uncongested behaviour the equations of the fitted lines will be affected, possibly preventing accurate comparison of the final data sets.

From visual inspection of the Ontario data (Fig. 8) it

FIGURE 7: DATA FOR ONTARIO STATION 5

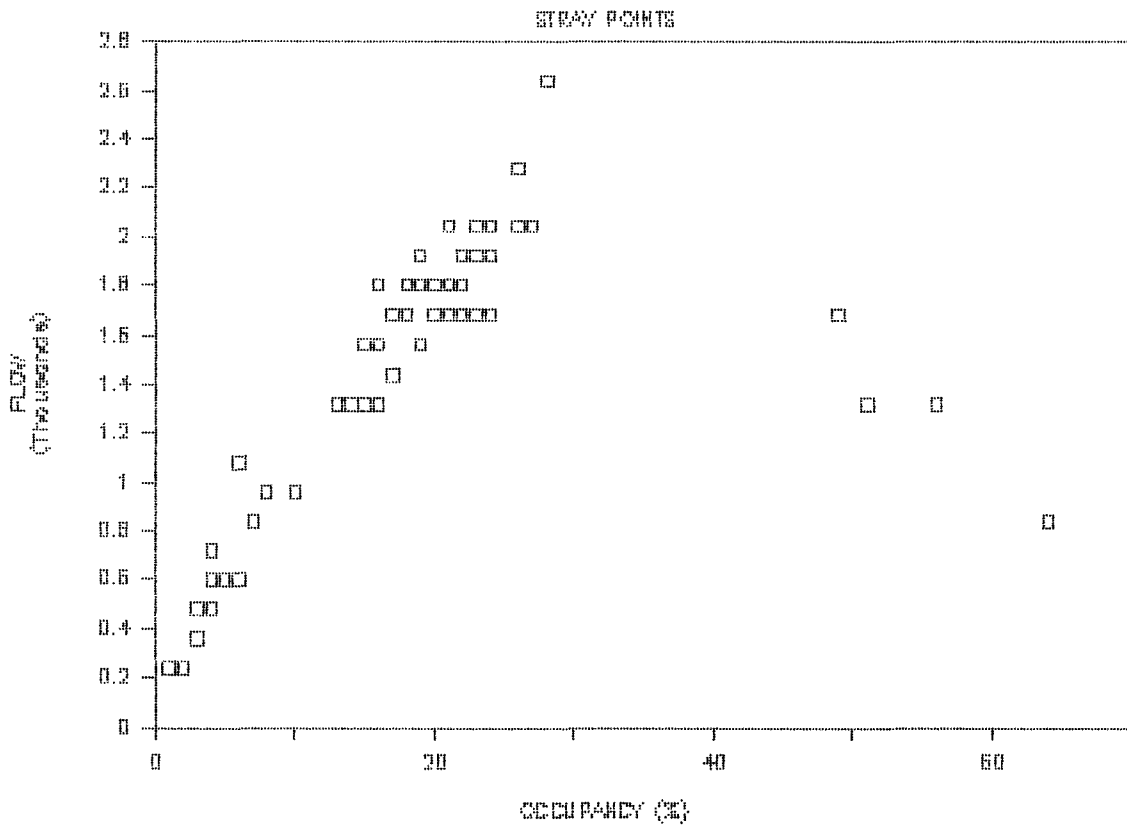


FIGURE 8: DATA FOR ONTARIO STATION 5  
NO STRAY POINTS 60km/h CUT-OFF

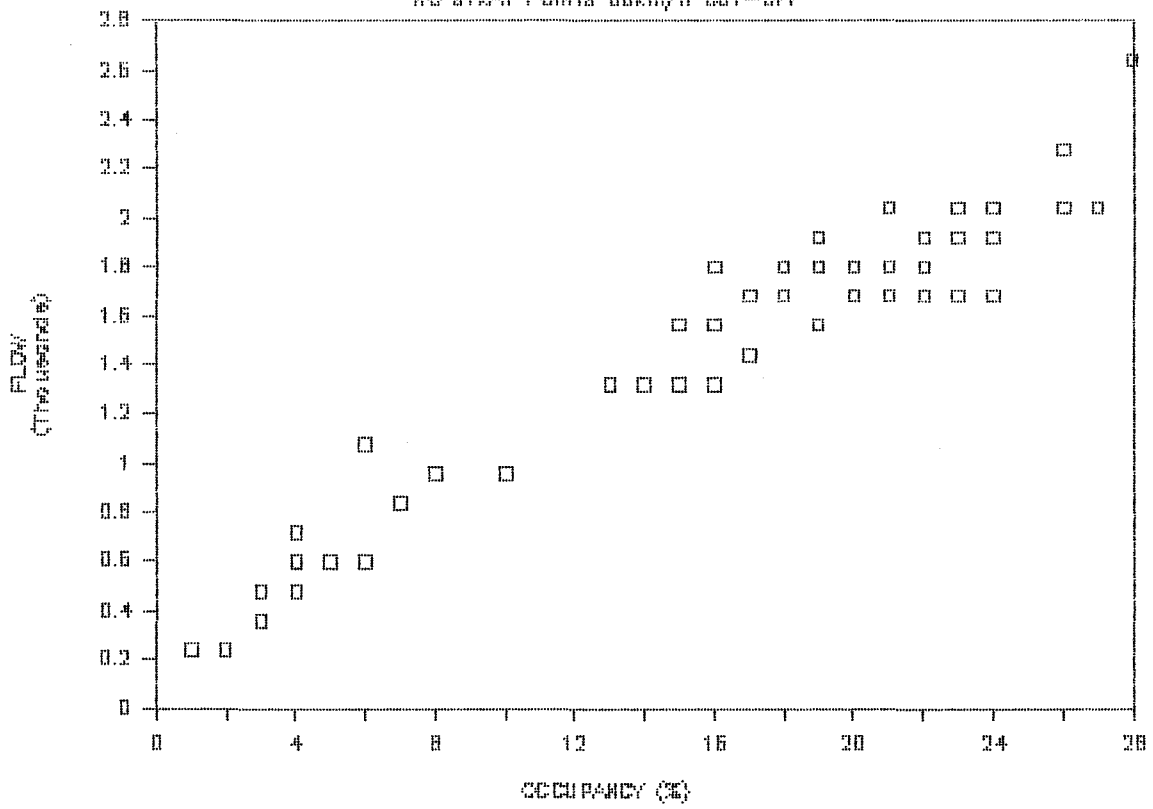
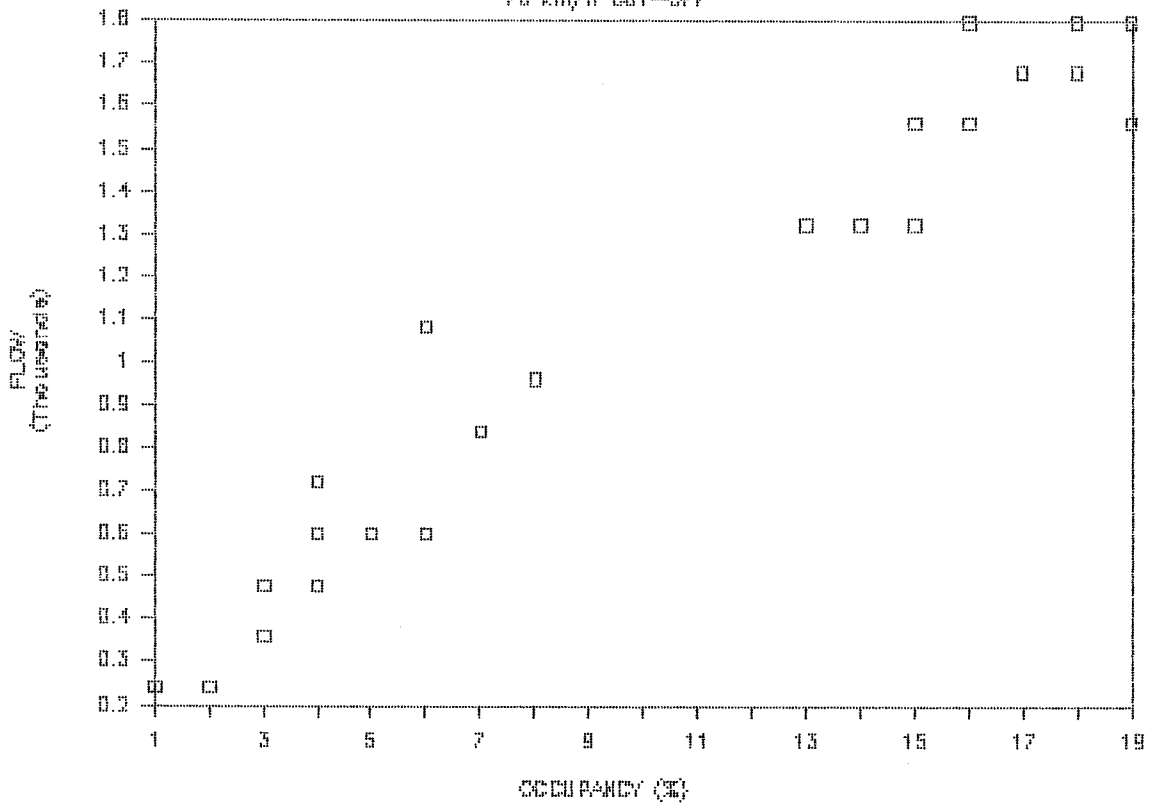


FIGURE 9: DATA FOR ONTARIO STATION 5  
70 km/h CUT-OFF





appeared that when 60km/h was used as the cut-off, the uncongested data were occurring at occupancies less than a value of approximately 27. When using 70km/h (figure 9) as the cut-off the uncongested data were occurring at occupancies less than a value of approximately 19. When using 80km/h (figure 10) as the cut-off the uncongested behaviour was occurring at occupancies of less than 16. These values, for all three cut-off points, correspond to station 5. There was some variation between both station 6 and 7 for the different cut-off points. It appeared, using visual inspection, that employing 80km/h as the cut-off point was the best choice in determining both uncongested behaviour and the resultant maximum observed occupancy. Because of the reduction in the size of the data set when using 80km/h as the cut-off it is not correct to call this maximum observed occupancy the critical occupancy. The critical occupancy is that point where the traffic moves from an uncongested state to a congested state. For each cut-off speed used there was a different maximum observed occupancy.

Turning to the Minnesota data, it was necessary to do some initial work within the Minnesota set. Since the Ontario data was obtained from the median lane at all stations studied, it was imperative that the median lane was identified and was used for Minnesota. Detector numbers 303 and 305 from station 53 were compared visually against one another. Using Hall and Gunter's (1986) comparison of the median lane, middle lane, and shoulder lane, a decision could be made as to which detector

represented the median lane.

Detector 305 (figure 11) illustrated characteristics of a median lane as described by Hall and Gunter (1986). The maximum flow rates are higher in the median lane than in the shoulder lane. Detector 305 shows flow rates of 2500 whereas detector 303 (figure 12) never reaches this maximum. Detector 303, typical of a shoulder lane affected by entrance ramp merging, has flow rates in the congested regime that are higher than in the uncongested regime. The explanation for this, given by Hall and Gunter, may be a consequence of decreased flows on the metered ramps, leading to increased main-line flows as the system becomes more congested.

With the proper identification of the median lane for all Minnesota stations the task of selecting the uncongested data was the next step. The data did not provide speed so any procedure undertaken had to be based only on occupancy. The maximum observed occupancy that was valid for Ontario was applied to Minnesota. This implies that values with occupancies of less than 16% were plotted. When this task was performed, visual inspection showed that the maximum observed occupancy found for Ontario was somewhat similar, however some further manipulation was necessary. The values eventually used was 15 after some stray points were excluded from the set. These stray points were the same as in the Ontario data set, i.e. the points associated with very high values of occupancy but very low values of volume. Again, these points are not

FIGURE 10: DATA FOR ONTARIO STATION 5

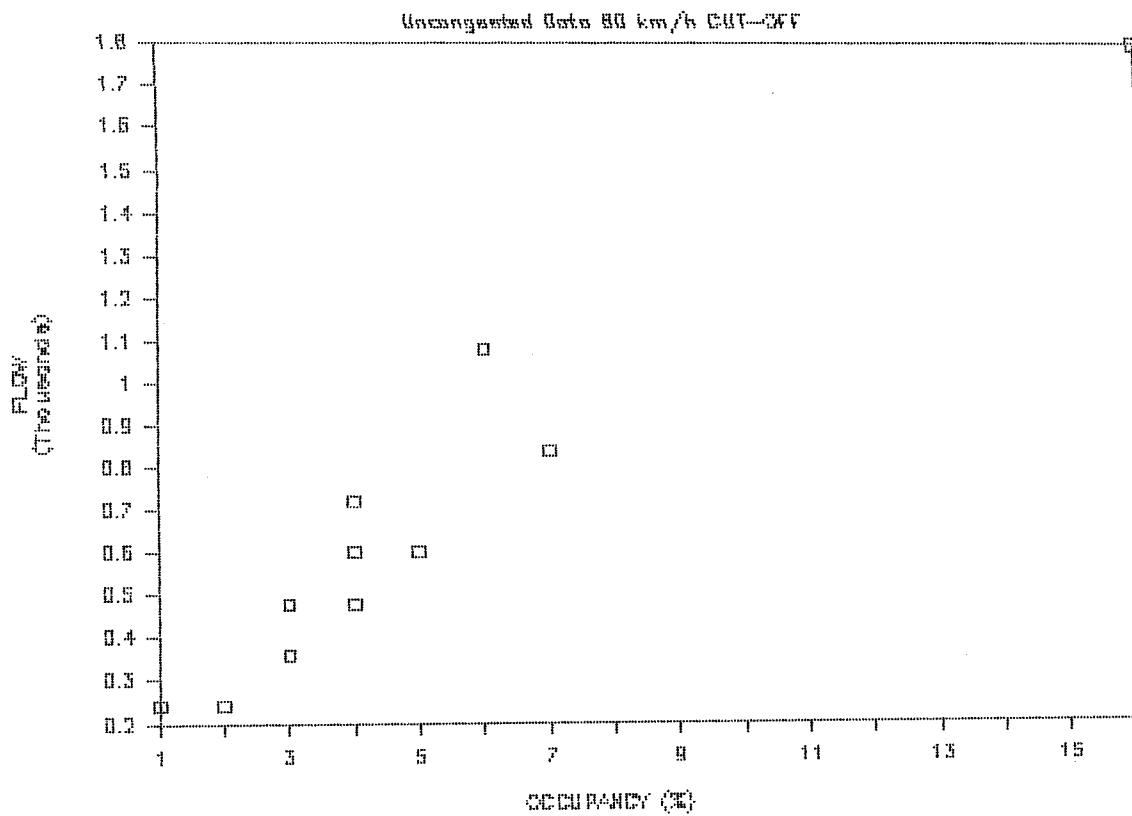
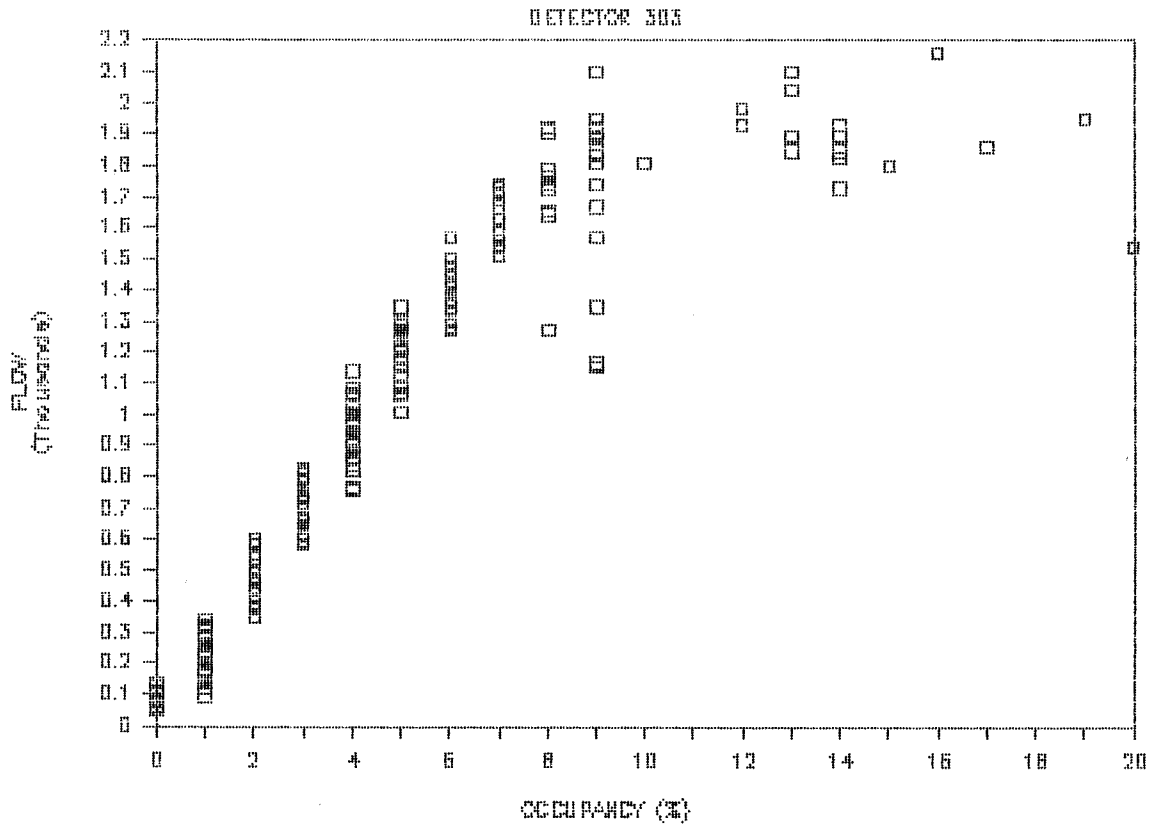




FIGURE 12: DATA FOR MINNESOTA STATION 53



representative of uncongested behaviour. The final result looked like figure 13 which corresponds to station 53.

Using the observed occupancy from Ontario, obtained with 80km/h as the cut-off, proved to be slightly too high for Minnesota. This occurrence provides further justification for not using 60km/h or 70km/h as the cut-off point for finding uncongested behaviour. It should also be noted that differences were expected when dealing with the Minnesota data. This stems from the fact the Minnesota data use 5-minute intervals. Five minute volumes and the associated averaged occupancies may not provide enough definition to clearly distinguish the complete transition from uncongested to congested behaviour. The five minute averages also makes it difficult to distinguish those data points in the transition area.

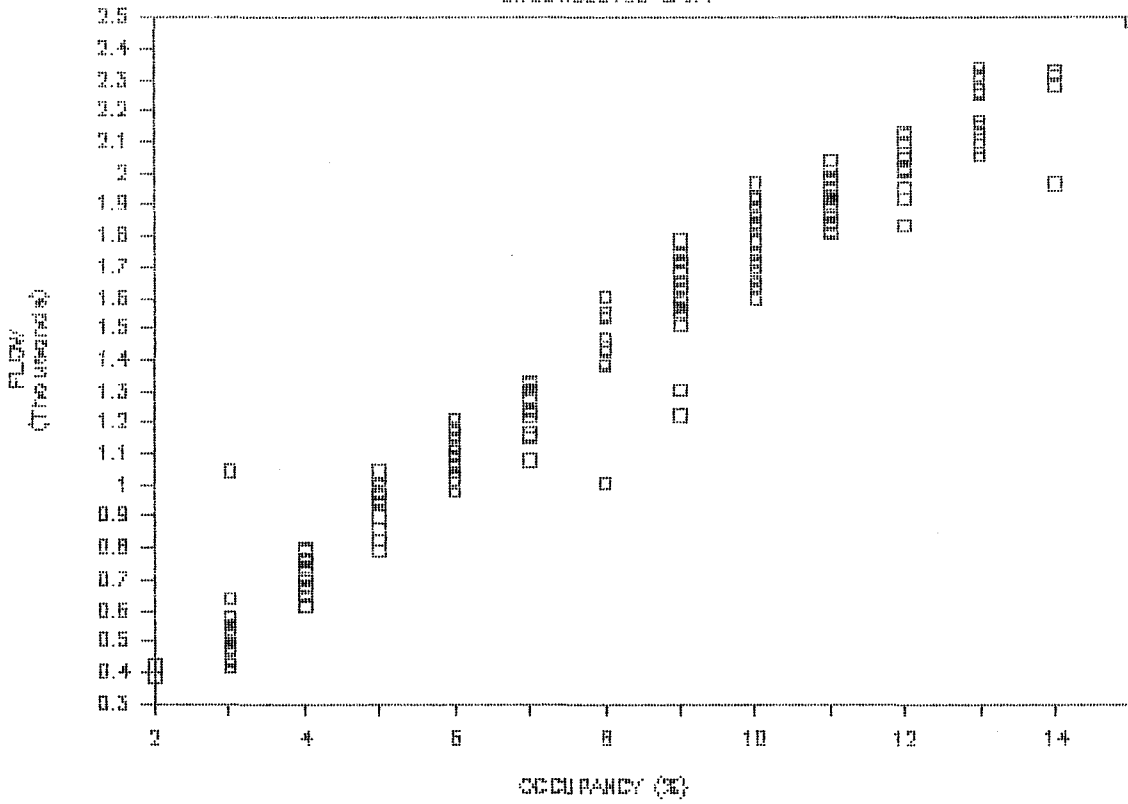
### 3.3 Analytical Procedures

After the uncongested data had been found at all stations for both data sets, the final analytical step was to conduct a line fitting procedure so that all lines could eventually be compared. The approach taken for the curve fitting was as follows. An initial functional form had to be chosen and then, using all available uncongested data, equations were fitted to each station separately.

After inspection two functional forms appeared to be plausible, these being a power function and a linear function.

FIGURE 13: MINNESOTA STATION 53

UNCONGESTED DATA



The linear function was quickly rejected because it does not necessarily pass through the origin, which is necessary for an equation representing flow and occupancy. Therefore, the function used was

$$\text{flow} = a' * \text{occupancy}^b$$

The model actually estimated was

$$\log \text{flow} = \log a' + b * \log \text{occupancy}$$

Letting

$$\log a' = a$$

$$\log \text{flow} = y$$

$$\log \text{occupancy} = x$$

this can be written as  $Y = a + bX$ . Formulae for linear regression could then be used in a spreadsheet to obtain values a and b.

The equations used to determine values of a and b were as follows:

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

n = number of observations

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

Once the lines were fit to the different data sets they could be compared. Figures 14 and 15 show the lines that were fit to station 5 and station 53 respectively. The lines that were fit to all the data sets appeared to provide a good fit.



FIGURE 14: STATION 5 FITTED LINE

80 km/h CUT-OFF

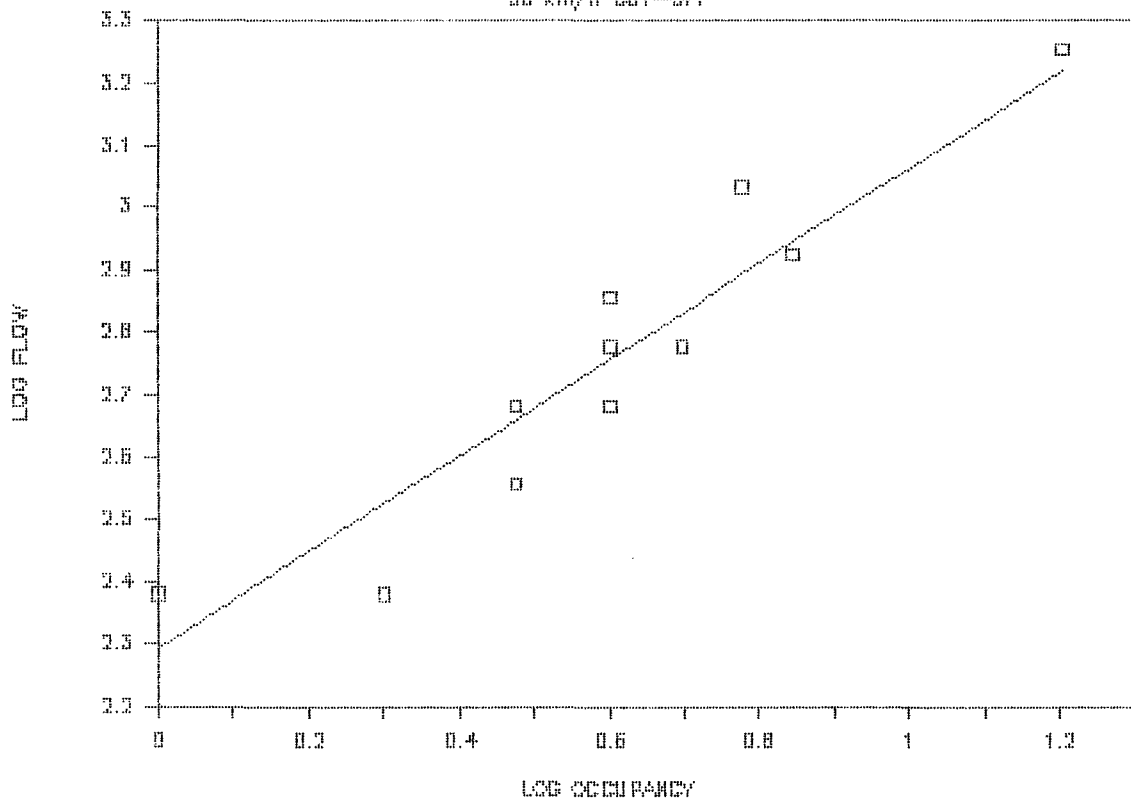
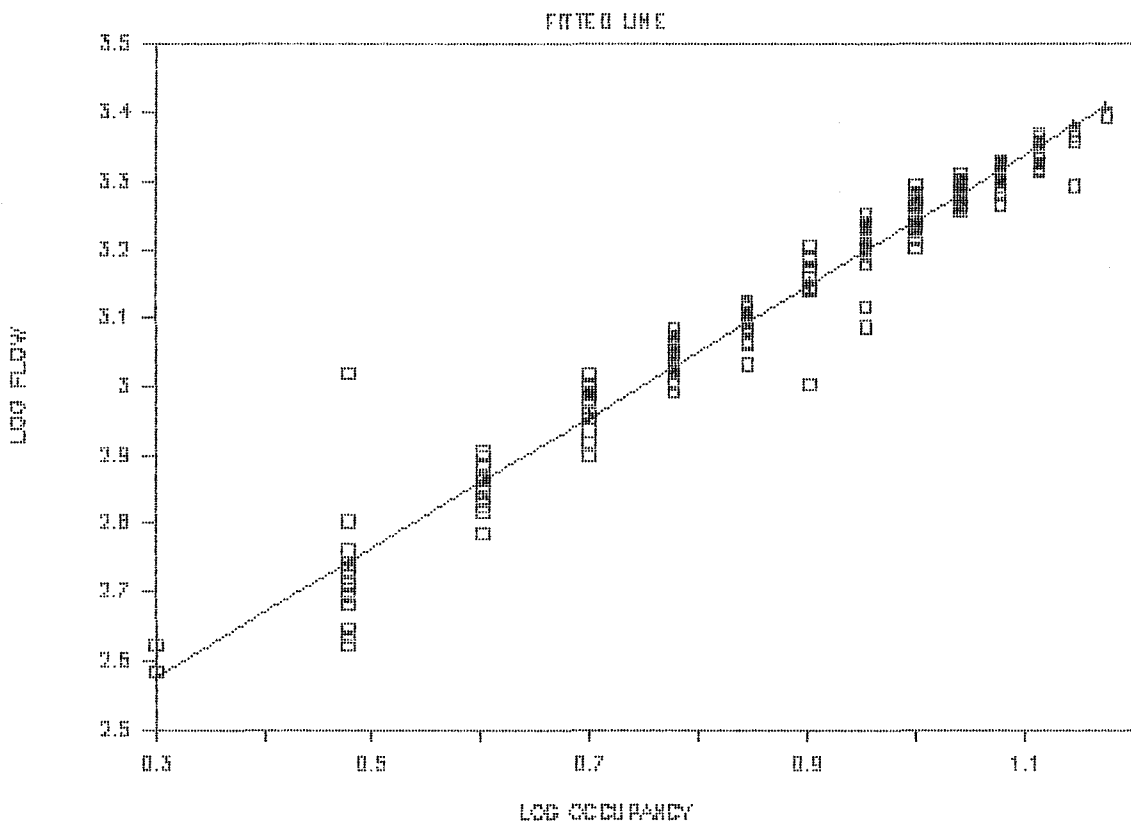


FIGURE 15: MINNESOTA STATION



#### 4. Data Comparison and Analysis

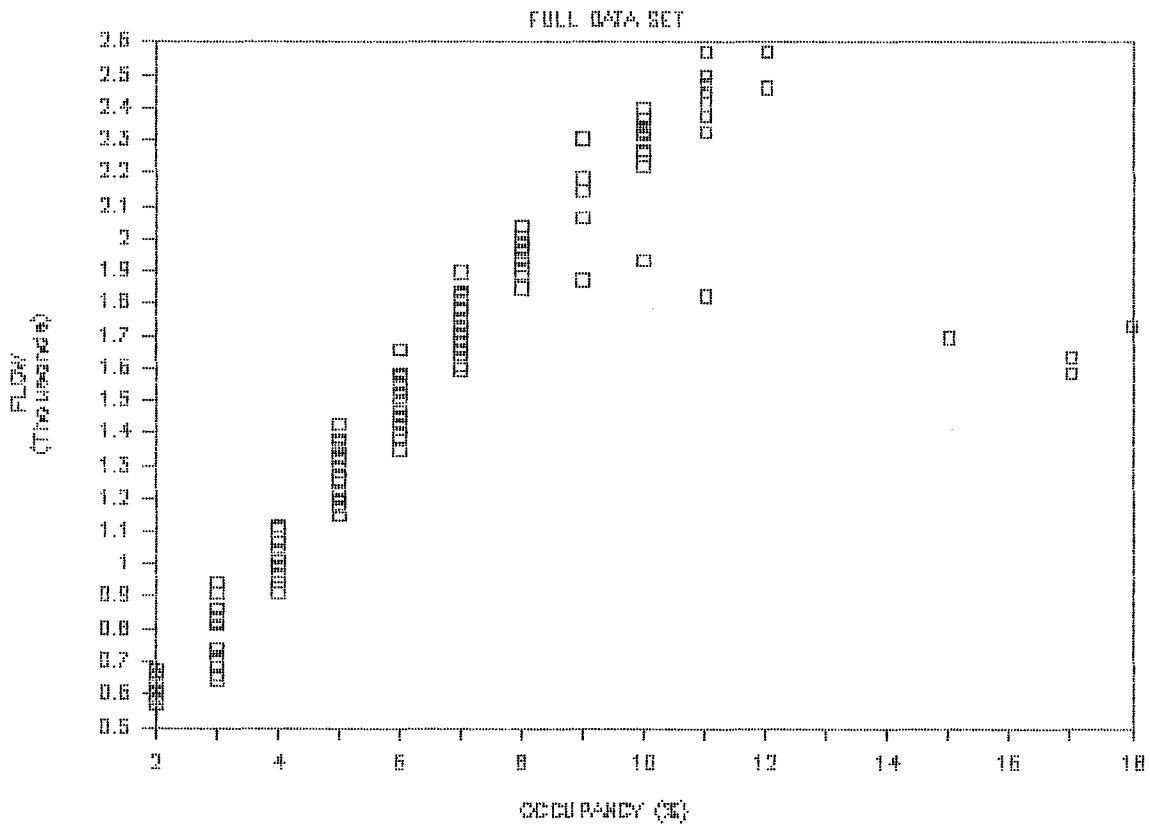
In this section several comparisons are made. The data sets are compared with respect to the entire data set, the uncongested portion of the data and finally, the fitted lines. Following this task it is possible to observe whether the flow-occupancy relationship on the Skyway Bridge in Ontario is similar to the relationship found in Minnesota.

##### 4.1 Comparison of the Full Data Set

Upon initial comparison of the data sets it is not totally obvious that the same general shape exists. The Ontario stations illustrate very well the reverse lambda shape observed by Koshi et. al. The Minnesota stations show very little congested data and therefore the reverse lambda shape is not as defined, but in spite of this is visible (figure 16). The Minnesota data also show very little transition data points, possibly resulting from the 5-minute intervals. When 5-minute intervals are used the data points are more aggregated whereas when using 30-second intervals the data points "jump" around a lot more. From careful visual inspection it is obvious that the maximum observed occupancies differ little between Ontario and Minnesota.

The congested regime exhibits somewhat more scatter. This behaviour occurs in the 25-70% occupancy range. Even though the congested regime is less likely to be fit by a straight line there is still a consistency that is observed in

FIGURE 16: DATA FOR MINNESOTA STATION



both Ontario and Minnesota data sets. The congested regime also tends to intersect the uncongested regime at flow rates somewhere less than the maximum flow rates.

#### 4.2 Comparison of the Uncongested Data

Once again a visual approach was the first step taken to compare the different data sets. From this visual inspection the uncongested data appeared to be replicated between Ontario and Minnesota. In the case of Ontario, as the higher cut-off values for speed were implemented the uncongested data became clearer. The uncongested branch is virtually linear and these points appear to scatter very little. Through this area, the relationship appears to be very well defined for both Ontario and Minnesota. Upon closer analysis of the graphs for the different stations the maximum observed occupancy is higher for the Ontario stations. Station 5 on the Skyway Bridge demonstrates maximum occupancy in the area of 16% (figure 10). Stations 6 (figure 17) and 7 (figure 18) show maximum observed occupancy in the areas of 16% and 14% respectively. Station 53 (figure 13) shows maximum observed occupancy at 15%, station 54 (figure 19) at approximately 15%, and station 57 (figure 20) at 12%. Ontario illustrates higher maximum observed occupancies.

The flow-occupancy relationship is well defined in the lower occupancy ranges, which is synonymous with the uncongested regime. The relationship is somewhat less defined

in the higher occupancy ranges. More transitional data points are available for the Ontario stations because of the 30-second interval data collection.

FIGURE 17: DATA FOR STATION 6

UNCONGESTED DATA, 80km/h CUT-OFF

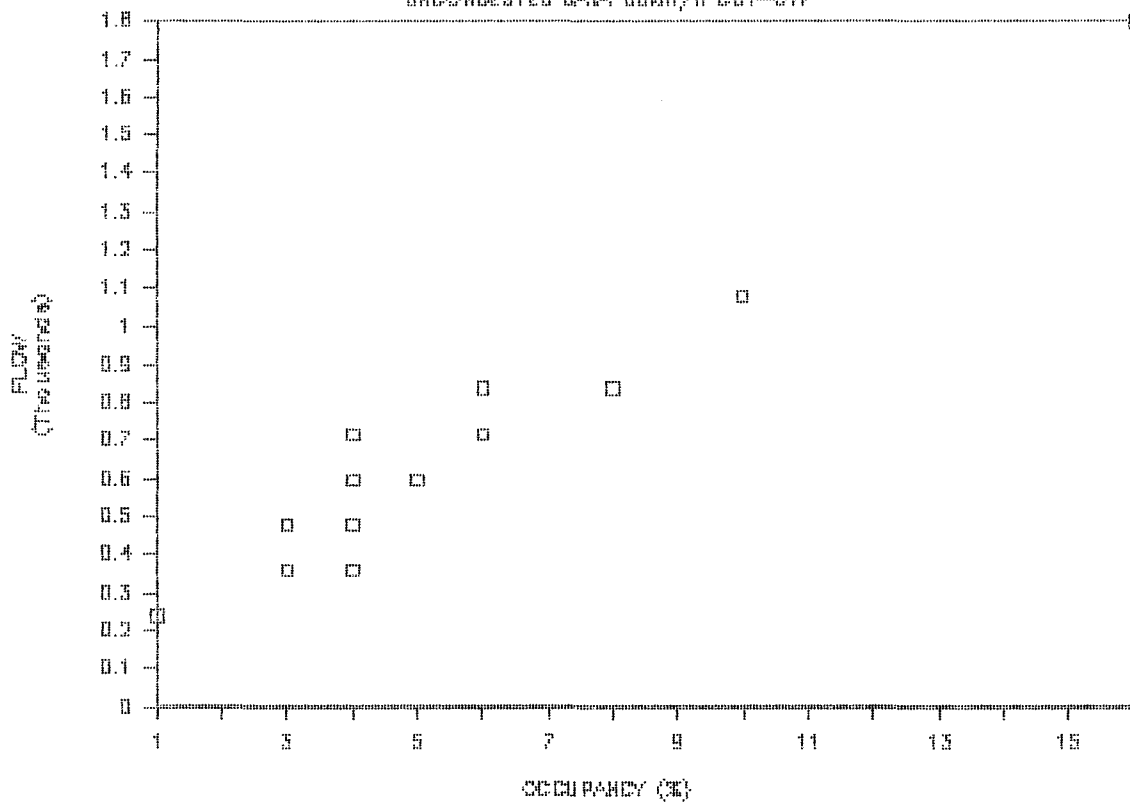


FIGURE 18: DATA FOR STATION 7

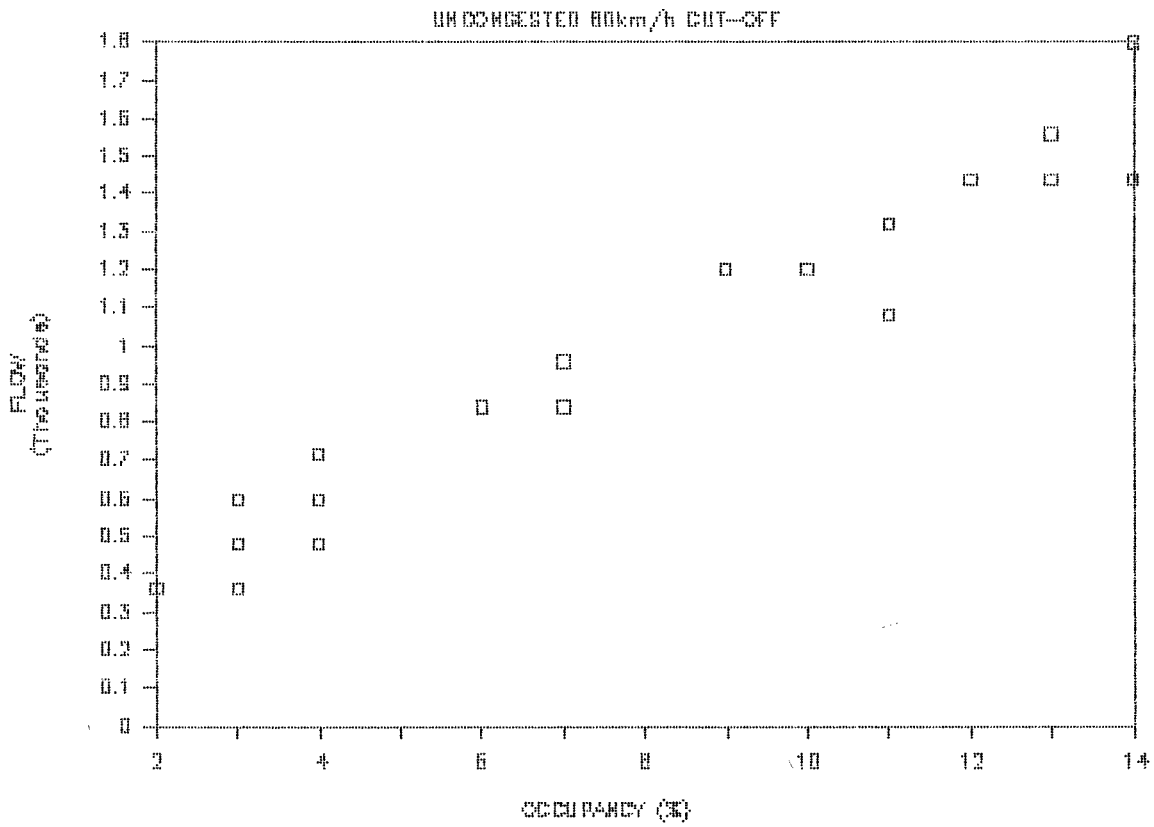




FIGURE 19: MINNESOTA STATION 54  
UNCONGESTED DATA

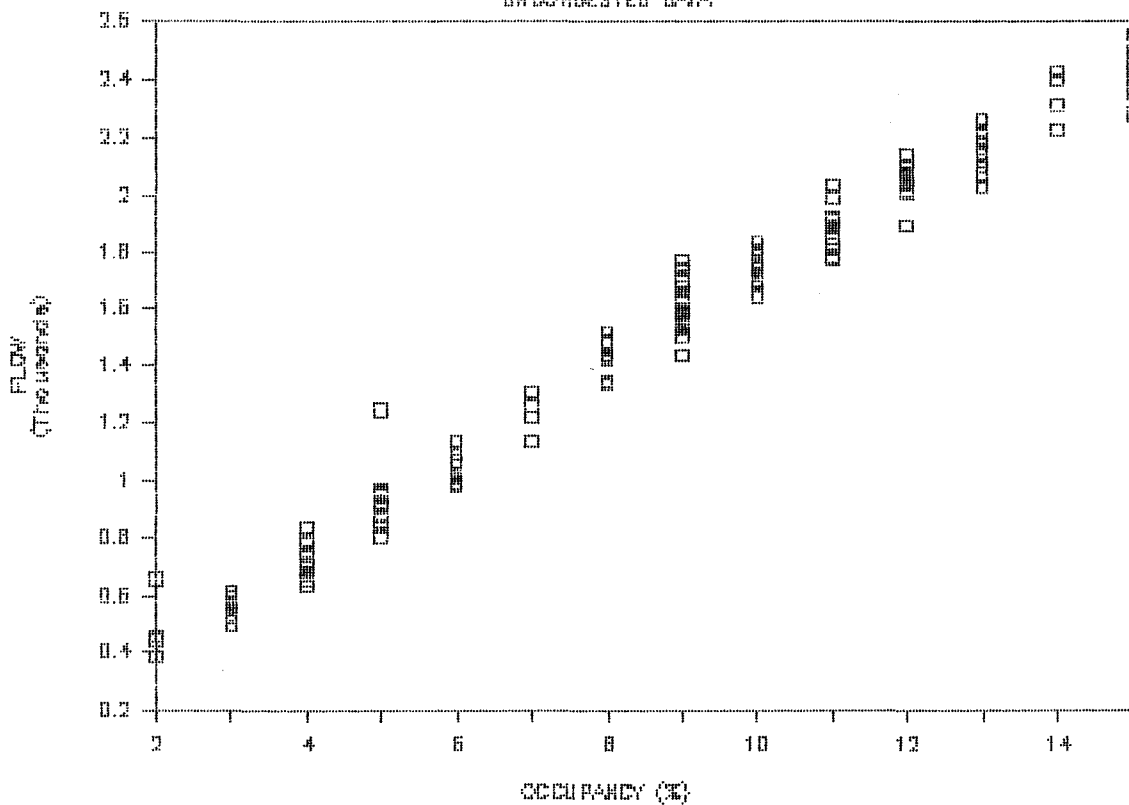
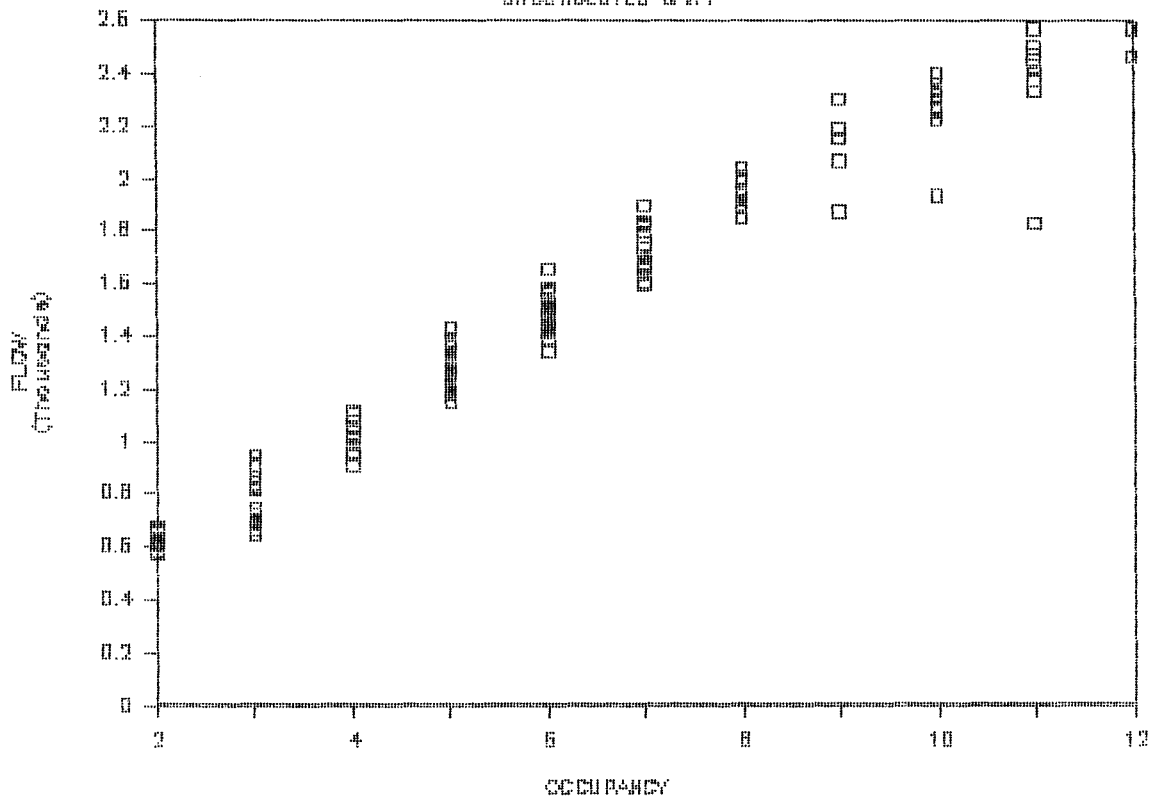


FIGURE 20: MINNESOTA STATION 57  
UNCONGESTED DATA



The proper identification of the uncongested regime was very important in the task of fitting lines to the uncongested behaviour. Improper identification would have led to fitted lines that were not precise enough to demonstrate any possible relationship that may exist between flow and occupancy. This point is of particular importance when comparing the uncongested data points at stations from different locations, i.e. different freeway management systems.

#### 4.3 Comparison of the Fitted Lines

The comparison of the fitted lines proves to be the most important part of this research paper. When calculated  $a$  and  $b$  values are substituted into the power function it is possible to see the relationship that exists between the different data sets. Since the maximum observed occupancy was lower for the Minnesota stations, after the  $a$  and  $b$  values were estimated it was necessary to look only at that portion of the Ontario data with an occupancy corresponding to the maximum observed occupancy for Minnesota.

The  $a$  and  $b$  values that were determined for each of the data sets are listed in table 1. The  $a$  values were all very similar for the different stations while the  $b$  values were somewhat different. This gave reason to believe that the relationship between flow and occupancy should be similar. To determine if this was, in fact, the case the values found for  $a$  and  $b$  were substituted into the equation representative

TABLE 1: The Estimated a and b Values

## Ontario Data

Station 5	a= 2.293678 b= 0.7692234
Station 6	a= 2.336026 b= 0.6733984
Station 7	a= 2.304485 b= 0.7845406

## Minnesota Data

Station 53	a= 2.290529 b= 0.9030362
Station 54	a= 2.33233 b= 0.9030362
Station 57	a= 2.503231 b= 0.8629011

of the relationship between flow and occupancy.

The relationship is identified by:

$$\text{Flow} = a * \text{occupancy}^b$$

The substitution method was carried out and the results are found in table 2. The results were also graphed and can be seen in figure 21.

As the occupancy increased there was a slight deviation between the Minnesota sets and the Ontario sets. A possible reason for the difference in slopes for the two data sets may be due to the vehicle detection hardware. The slope of the lines for the Minnesota stations are steeper than the stations for the Ontario stations. The maximum flow rates are also occurring with lower occupancies, while the maximum flow rates are comparable. The main differences in the occupancy values are due to the vehicle detection hardware. The magnetic detectors employed in the Minnesota system have a small diameter and therefore a very short effective detection zone. The QEW equipment consists of 6' X 6' induction loops that have a much larger detection area. Whereas the Minnesota detector may have a detection zone between 0.3m and 0.5m, the Ontario induction loop may cover about 3m.

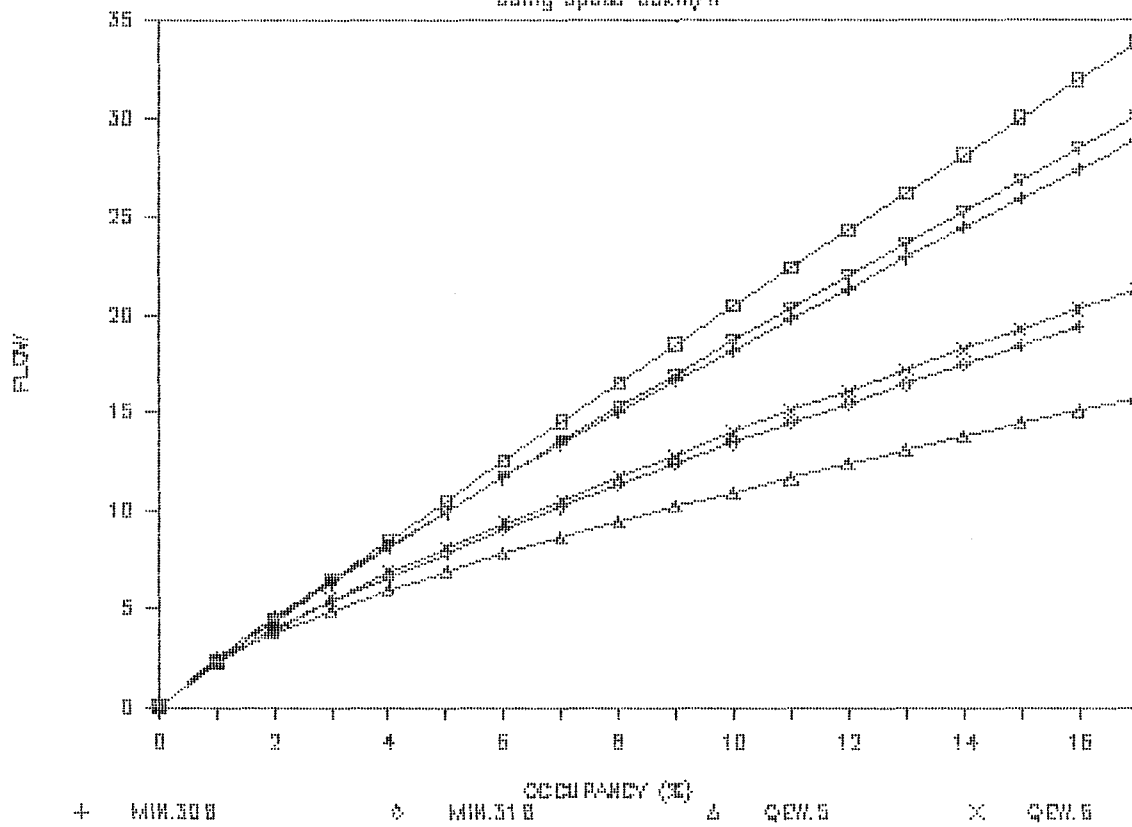
Table: 2 Results of the Substitution Method

$$\text{Flow} = a * \text{occupancy}^b$$

Flow						
Occ.	Min.305	Min.308	Min.318	QEW.5	QEW.6	QEW.7
0	0	0	0	0	0	0
1	2.29053	2.3323	2.5032	2.2946	2.3360	2.3045
2	4.42774	4.3615	4.5526	3.9110	3.7255	3.9696
3	6.51067	6.2899	6.4596	5.3424	4.8952	5.4563
4	8.55910	8.1559	8.2797	6.6656	5.9416	6.8378
5	10.58223	9.9767	10.0378	7.9138	6.9049	8.146
6	12.58553	11.7622	11.7480	9.1053	7.8069	9.3986
7	14.57239	13.5189	13.4194	10.2516	8.6609	10.6069
8	16.5453	15.2515	15.0583	11.3606	9.4758	11.7784
9	18.5061	16.9631	16.6692	12.4379	10.2580	12.9186
10	20.45622	18.6563	18.2557	13.4879	11.0123	14.0318
11	22.39676	20.3332	19.8203	14.5139	11.7422	15.1213
12	24.32864	21.9953	21.3660	15.5186	12.4508	16.1896
13	26.25263	23.6440	22.8939	16.5042	13.1403	17.2388
14	28.16936	25.2805	24.4058	17.4723	13.8127	18.2708
15	30.07936	26.9056	25.9029	18.4246	14.4696	19.2870
16	31.98312	28.5203	27.3863	19.3624	15.1123	20.2887

FIGURE 21: COMPARISON OF FITTED LINES

Using Speed 80km/h



## 5. Conclusions

In spite of the simple analytical procedures that were undertaken in this paper, a few conclusions seem warranted. The first conclusion should be obvious, the two data sets exhibit areas where the relationship is very similar and areas where it is dissimilar. Second, there is a good relationship between the uncongested regimes of the data sets. Third, the lines obtained from the substitution method (using the a and b values) were also similar even though the slopes were different for Ontario and Minnesota.

The full data set, encompassing congested, uncongested, and transitional behaviour, demonstrated well the flow-occupancy relationship that has been found in other research efforts. Although the data sets were not completely identical they both covered all the ranges of freeway activity. Even though the volume rates were converted to hourly rates for both Ontario and Minnesota the initial difference between the interval data collection had an effect on the overall shape of the flow-occupancy graphs. This fact also makes it very important to know the details about the systems when performing any comparisons, a lack of this knowledge may lead to false conclusions.

The fact that the different regimes can be seen clearly provides reinforcement for looking at the entire data set as illustrating discontinuous behaviour. In supporting the discontinuous relationship it is logical to focus on the



separate regimes as was done when comparing the uncongested regimes.

The second conclusion states that there is a good relationship between the uncongested regimes of the flow-occupancy relationship. Even though there was a difference in the maximum observed occupancy the maximum flow rates achieved were very close. In both data sets the congested regimes intersected the uncongested regimes at lower than the maximum flow rate. Upon visual inspection the uncongested regimes for all stations looked very similar. The uncongested regimes provide a good basis for conducting line fitting procedures.

The third conclusion states that there are areas where the fitted lines are very similar and areas where they are dissimilar. The difference in slope was explained by the difference in detection hardware between different systems. In spite of this, at very low occupancies the lines virtually superimpose on one another. Since low occupancies are associated with high speeds there is good reason to suggest that at optimal highway conditions the traffic behaviour is identical from one location to another.

Finally, based on these conclusions there is good reason to believe that the flow-occupancy relationship is similar between Ontario and Minnesota. The strong relationship between the uncongested regimes indicate that normal highway operations are the same from one location to the next. It appears that knowledge of this will aid in furthering the work

on a new incident detection algorithm.

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