TRANSIENT STABILITY OF THE WIEN BRIDGE OSCILLATOR

TRANSIENT STABILITY OF THE WIEN BRIDGE OSCILLATOR

by

RICHARD PRESCOTT SKILLEN, B. Eng.

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AUTHOR: Richard Prescott Skillen, B. Eng., (McMaster University)

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SCOPE AND CONTENTS: In many Resistance-Capacitance Oscillators the oscillation amplitude is controlled by the use of a temperature-dependent resistor incorporated in the negative feedback loop. The use of thermistors and tungsten lamps is discussed and an approximate analysis is presented for the behaviour of the tungsten lamp. The result is applied in an analysis of the familiar Wien Bridge Oscillator both for the presence of a linear circuit and a cubic nonlinearity. The linear analysis leads to a highly unstable transient response which is uncommon to most oscillators. The inclusion of the slight cubic nonlinearity, however, leads to a result which is in close agreement to the observed response.

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LIST OF PRINCIPAL SYMBOLS

Symbol	Meaning
s	complex frequency $\alpha + j\omega_m$ of modulation
j	complex operator
ω	frequency in radians per second equal to $2\pi f$
f	frequency in cycles per second
t	time in seconds
e	exponential operator
v	instantaneous voltage in volts
E_1	peak voltage in volts
i	instantaneous current in amps
P	radiated power in watts
P_o	steady state radiated power in watts
W	supplied power in watts to lamp
Wo	steady state supplied power in watts to lamp
R	lamp resistance in ohms
Ro	steady state lamp resistance in ohms
Ri	for i equal 1,2,3,...Resistances in ohms
C	capacitance in microfarads
θ	temperature of filament in degrees Kelvin
θ_o	steady state temperature of filament in ^oK.
τ_1	time constant of transient response in seconds
C_T	thermal heat capacity of lamp in watt-sec/degree
G_T	differential thermal conductance in watts/degree
τ	lamp bridge thermal time constant in sec.
m_o	constant - slope of log R versus log P for lamp
γ	temperature coefficient of resistance for tungsten

Symbol	Meaning
A	gain of operational amplifier
$\beta(\omega)$	feedback function of frequency-determining network
A'	closed loop gain of system
$L(s)$	Laplace transform of change in output voltage amplitude
δE_1	change of output voltage amplitude in peak volts
ΔE_1	peak value of δE_1 extrapolated back to time $t=0$
δR	small perturbation of lamp resistance = $dRf(t)$
ϵ	ratio of dR to R_o or relative change in lamp resistance
ω_o	steady state oscillator frequency in radian/sec.
f_o	steady state oscillator frequency in cycles/sec.
ω_m	transient modulation frequency in radian/sec.
f_m	transient modulation frequency in cycles/sec.
D_3	fractional third harmonic distortion of output voltage

I Introduction

In recent years, resistors which exhibit nonlinear electric properties by virtue of a change in resistance with applied power, have found many new uses. One application lies in exercising control over the amplitude of a sinusoidal waveform without introducing the customary nonlinearity associated with most simple nonlinear limiters. Although many devices exhibit this property to some extent, we will primarily concern ourselves with two; the incandescent lamp, and the solid state thermistor. Although the electrical characteristics are extremely different for the two, they have in common the key to their ability to control without distortion; their associated time constant due to thermal inertia.

Although the thermal properties of the incandescent lamp had been qualitatively understood for a considerable time, Meacham(1) was probably the first to analyse and employ its dynamic characteristics to stabilize the amplitude of a Meachan Oscillator in 1938. His work aroused considerable interest at that time in the nonlinear aspects of the lamps and Glynne(2) and others(3),(4),(5),(6) followed by studying the dynamic behaviour of the lamps.

By the middle of the century, thermistors were also being analysed and employed for this control problem. Bollman and Kreer(7) were two of the first to realize the potential value of these solid state devices and to set up the differential equation for their dynamic behaviour. Patchett(8) employed a directly heated type of thermistor in a bridge circuit and showed that it had a figure of merit, which he defined as the change in output voltage for a change in input voltage, of about fifty. Typically the figure of merit for the lamp bridge is less than unity. Thus the use of the thermistor would seem to indicate a more sensitive amplitude control for oscillator amplitude stabilization.

Theoretical work concerning the amplitude stability of harmonic oscillators had attracted considerable interest in the years prior to the thermal amplitude control. With its introduction some renewed interest was brought to bear on the subject.

Edson[9] examined the amplitude stability in a lamp stabilized oscillator by examining the changes in amplitude and phase of the modulation vector when a lightly modulated wave passed through the open loop feedback circuit. For the modulated wave to persist the open loop gain and phase shift of the input signal were required to be zero. An increase in gain would indicate instability or growth, while a decrease would indicate attenuation of the modulation. In his analysis the low frequency component is not considered and the feedback factor must be symmetrical about the centre frequency for a bandwidth of at least twice the highest modulating frequency used to insure that the output will be independent of phase modulation.

Gladwin examined the problem of amplitude stability in valve oscillators[11]. For the class of oscillators known as separable oscillators he examined the stability of the system from the viewpoint of its characteristic equation[12]. In his analysis the oscillator is initially oscillating in the steady state with a sinusoidal output of constant amplitude and frequency. The stability is examined by subjecting the output to a small variation by changing some parameter or introducing an input perturbation. This perturbation gives rise to oscillations at certain complex frequencies which are characteristic of the feedback loop. If these terms are then considered to modulate the output, the oscillator output will consist of an infinite number of waves of the form;

$$v_{out} = \sum_{n=-\infty}^{+\infty} x_n e^{(s+jn\omega_o)t}$$

where ω_o is the unperturbed oscillator frequency and $s = \alpha + j\omega_m$ is the complex frequency of the disturbance or modulation. If the oscillating circuit has a reasonably high quality factor the above series can be approximated, with a good degree of accuracy, to terms of the fundamental component and the limits of the series reduce to -1 to +1. We are thus left with three components of a small perturbation.

These components can be introduced into the network equations, and give rise to three linearized equations which when solved in a matrix analysis yield the matrix determinant equal to zero. This matrix determinant is referred to as the characteristic equation of the system's stability.

For stable oscillation all of the roots of the characteristic equation must have negative real parts to insure that the amplitude of any transient will decrease with time. Although the characteristic equation cannot always be factored for complex systems, the stability can be determined by the Routh-Hurwitz criterion for certain steady state conditions.

For oscillators where the feedback factor is symmetrical about the oscillator centre frequency for a range greater than twice the highest modulating frequency, the characteristic equation can be factored(12) to yield independent criteria for the amplitude and frequency stability.

The Wien Bridge Oscillator with a thermal amplitude compensator can exhibit a very poor transient response during tuning or switching of the oscillator frequency control. Several investigations have been aimed at predicting its peculiar oscillatory transient response. Cooper[10] analysed a lamp stabilized Wien Bridge type of oscillator assuming a linear operational amplifier and a simple time constant for the lamp bridge circuit. The result was applied to an amplitude modulated waveform and the value of the natural modulation frequency was obtained which was in close agreement with his experimental data. The analysis did not however, predict the amplitude of the transient modulation or the time constant to any satifactory degree. Oliver[14] pointed out that a slight nonlinearity of the operational amplifier could explain these discrepancies in his analysis of the problem.

The Wien Bridge oscillator was thus analysed for both a linear amplifier and an amplifier containing a small amount of cubic non-linearity using the general approach of Gladwin[12] [13]. The findings are compared with experimental results to check the approximations made in the analysis.

II SIMPLE FEEDBACK OSCILLATORS

(1) Conditions for Oscillation

The concept of feedback plays an important role in almost every branch of engineering and physics. Although the word feedback is in common use it is surprisingly difficult to find a precise definition. In many physical systems it is extremely hard to identify a feedback loop. From a general point of view however, we can iden tify feedback as a closed series of cause and effect relationships. When feedback is introduced intentionally for a desired purpose, its definition becomes considerably simpler and can usually be expressed in a mathematical form.

Figure (1) shows in block diagram form the circuit for a simple feedback oscillator

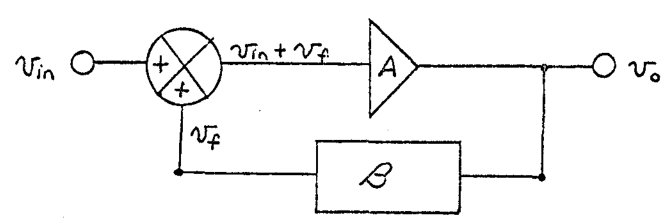


Figure (1)

Here A is defined as the gain of the operational amplifier and β is the so called feedback factor or feedback function. In figure (1) the closed loop or feedback loop is easily identified and the system is defined mathematically by the equations

$$v_o=(v_{in}+v_f)A$$

and $$v_f= v_o\beta$$

from which we can write the closed loop gain A' for the system as;

$$A' = \frac{v_o}{v_{in}} = \frac{A}{1-A\beta} \qquad \text{--(1)}$$

If we consider the amplifier gain A to be real and positive, then the feedback function β will be real and positive at some complex frequency s for the case of the oscillator. Here v_{in} would be zero while v_{out} is finite. For the case of a sinusoidal oscillator, equation(1) must have a pair of imaginary poles at $s = +j\omega_o$ and $s = -j\omega_o$ which means that the feedback network will contain at least two energy storage elements. If we write the feedback function as some function of the complex frequency $s = \alpha + j\omega$ we can equate the denominator of equation (1) to zero to obtain the conditions for oscillation as;

$$A\beta(s) = 1$$

For the steady state $s = j\omega$ and the above reduces to the familiar form for steady state oscillation often referred to as the Barkhausen criterion for oscillation or the characteristic equation,

$$A\beta(j\omega) = 1 \qquad \text{--(2)}$$

This concept implies unity loop gain as the criterion for oscillation, however unity loop gain at a single frequency is a necessary but not a sufficient condition for self-sustained oscillation. Clearly if the β network provides zero net phase shift at more than one frequency, the criteria for steady oscillation is further complicated.

For simple oscillators however, where equation (2) is sufficient, the complex equation will yield two independent criterion for oscillation.

$$I_m(\beta) = 0 \qquad \text{--(3A)}$$

which will determine the steady state frequency of oscillation and,

$$AR_e(\beta) = 1 \qquad \text{--(3B)}$$

which stipulates the necessary gain requirement for steady state oscillation to exist. Here $I_m(\beta)$ and $R_e(\beta)$ stand for the imaginary part of β and real part of β respectively.

(2) Selectivity

Any practical oscillator will need a value for the amplifier gain somewhat larger than that predicted by equation (3B). Consequently its output will increase until the amplifier limits the output resulting in distortion of the steady state output. Each of the harmonics present at the amplifier output will be affected similar to additional signals injected into the feedback loop and will be reduced or enhanced by the factor;

$$F(n\omega_o) = \left| \frac{1}{1-A\beta(n\omega_o)} \right| \qquad \text{--(4)}$$

where $\beta(n\omega_o)$ is the feedback factor evaluated at the various harmonic frequencies $(n\omega_o)$; $n = 2,3,4.....$ Equation (4) can be considered as a measure of the system's selectivity. For a properly designed oscillator the feedback should change from positive to negative in the frequency range ω_o to $2\omega_o$ ensuring that all harmonics will be small. For this condition equation (4) would be small provided $[A\beta(n\omega_o)] \gg 1$. To insure that the feedback at the frequency $n\omega_o$ is negative,the factor $A\beta(n\omega_o)$ must of course be negative.

(3) Phase - Frequency Stability

Oscillators are also susceptible to frequency changes caused by variation of circuit elements in other than the beta network. Since the natural frequency of oscillation is identical to the frequency of zero net phase shift around the closed loop, changes in the resistive loading or phase shift in the amplifier can alter the natural frequency from the condition of equation (3A). To ensure a constant frequency of oscillation, the beta network should thus exhibit as rapid a change of phase with frequency about ω_0 as possible. We can define a measure of phase-frequency stability for the oscillator as;

$$G = \left| \frac{\Delta \phi}{\Delta\omega / \omega_0} \right| \qquad \text{--(5)}$$

which is the change in phase $\Delta\phi$ for a relative change in frequency ω about ω_0 for the beta network. Equation (5) would increase for increasing frequency stability with respect to variations in amplifier or load which would effect the overall phase shift. For the limiting case where $G \rightarrow \infty$ the natural frequency of oscillation ω_0 would be entirely due to the phase-frequency chacteristic of the beta network.

III BETA NETWORKS FOR R-C OSCILLATORS

(1) R-C Oscillators

At frequencies below one kilocycle per second it becomes impractical to use L-C circuits for frequency determining networks due to their large size and associated low quality factors. The large size associated with these circuits can be greatly reduced by the use of resistor capacitor networks (R-C networks) at the expense of increasing the amplifier gain and the number of parameters which must be varied to alter the oscillator frequency. There are two basic groups of R-C networks which can be employed for oscillator use, the phase shift networks, and the null type networks. Examples of each type are treated in this chapter with regard to their general merits.

(2) The "R-C Oscillator Network"

One of the simplest types of beta networks required for a feedback oscillator is shown in figure (2).

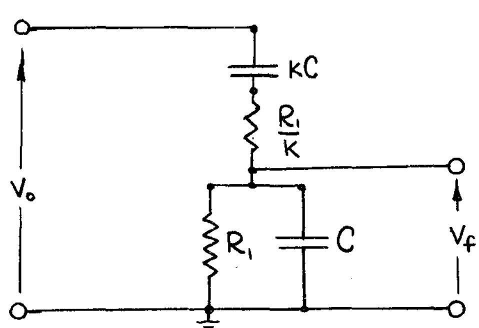


Figure (2)

Here k is a constant and C and R_1 are capacitors and resistors respectively.

This network is characterized by the feedback function;

$$\beta(s) = \frac{v_f}{v_o} = \frac{ksCR_1}{s^2C^2R_1^2+(k+2)sCR_1+1} \quad \text{—(6)}$$

which for the steady state where $s = j\omega$ reduces to;

$$\beta(j\omega) = \frac{k}{(k+2)} \quad \frac{1}{1+\frac{1}{(k+2)}\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right]} \qquad \text{--(7)}$$

where

$$\omega_o = \frac{1}{CR_1} \qquad \text{--(8)}$$

The imaginary portion of β will vanish for the frequency $\omega = \omega_o$ given by equation (8) at which frequency beta reduces to,

$$\beta(\omega_o) = k/(k+2)$$

The necessary amplifier gain for the circuit of figure (1) would be

$$A = \frac{(k+2)}{k}$$

for threshold of oscillation.

The transient response of this network can also be obtained from the roots of the equation $1 - A\beta(s)$ for this oscillator.

Writing the equation for $1 - A\beta(s) = 0$ we obtain

$$s^2 + s\omega_o(k+2-Ak) + \omega_o^2 = 0 \qquad \text{--(9)}$$

or

$$(s-s_x)(s-s_y) = 0$$

This equation has two roots s_x, s_y which for the particular case of $k = 1$ will describe the locus shown in figure (3).

$$s_x, s_y = -\frac{\omega_o(3-A)}{2} \pm \frac{\omega_o}{2}\sqrt{(A-1)(A-5)}$$

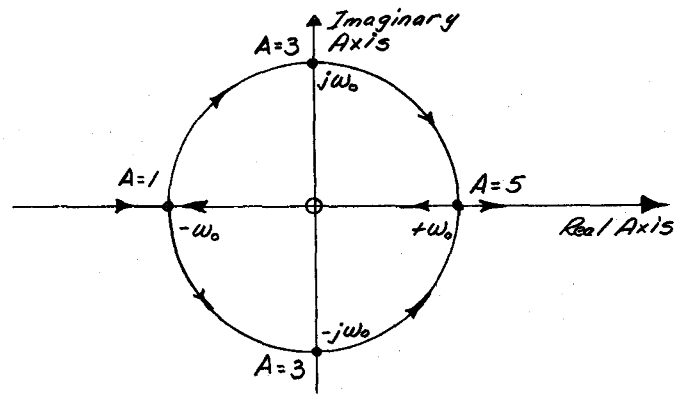


Figure (3)

We see that as the gain A increases from zero, the two roots will coalesce for A = 1 and then separate at $-\omega_0$ on the negative real axis as shown. When the roots reach $\pm j\omega_0$ on the imaginary axis corresponding to a gain of A = 3, we have the condition corresponding to the threshold of oscillation; the location of the imaginary roots giving the frequency of oscillation. In the region $1 < A < 5$ the transient response can be obtained from equation (9) and will be of the general form,

$$v(t) = Ke^{\alpha_3 t} \cos(\omega_1 t + \phi) \qquad \text{--(10)}$$

where $\omega_0 = \sqrt{\alpha_3^2 + \omega_1^2}$ and ϕ is the phase shift. The two roots are located at $\alpha_3 \pm j\omega_1$. For $1 < A < 3$, α_3 will be negative and the transient will damp out with the time constant given by equation (10). For $3 < A < 5$ the output will increase until the amplifier limits slightly making the average gain over one cycle equal to 3.

For $A > 5$ the output will no longer be oscillatory, but will approach the action of the multivibrator.

This simple beta circuit would thus seem to have a poor amplitude stability due to the dependence on A. The network also exhibits very poor selectivity. For example equation (4) would yield for the second harmonic where A = 3;

$$F(2\omega_0) = 2.237$$

This unfortunately means that any second harmonic term present in the output would be increased in amplitude rather than reduced. This beta network would thus not seem to be very useful for oscillator use.

(3) The "Phase Shift Network"

A second simple way of achieving the required beta network for an oscillator is the familiar "phase shift network" shown below in figure (4)

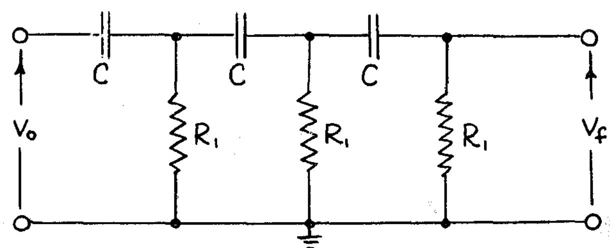


Figure (4)

This circuit is a simple R-C ladder type filter with a transfer characteristic .

$$\beta_1(s) = \frac{1}{1+\frac{6}{sCR_1}+5\left(\frac{1}{sCR_1}\right)^2+\left(\frac{1}{sCR_1}\right)^3} \quad \text{--(11)}$$

For the steady state s = $j\omega$ and the above reduces to;

$$\beta_1(j\omega) = \frac{1}{1+\frac{6}{j\omega CR_1}-\frac{5}{\omega^2C^2R_1^2}-\frac{1}{j\omega^3C^3R_1^3}} \quad \text{--(12)}$$

The ladder network could also contain more than three sections. A simple solution for a more complex ladder network can be found by the use of Pascal's triangle(16), but this increases the number of parameters which must be changed to alter the oscillator frequency.

From equation (12) we see that the odd power terms in ω contribute to the imaginary part of $\beta_1(\omega)$. Hence for the imaginary part to vanish

$$\frac{6}{\omega CR_1} - \frac{1}{\omega^3 C^3 R_1^3} = 0$$

or

$$\omega_o = \frac{1}{\sqrt{6}\, CR_1} \qquad \text{—(13)}$$

Solving for β_1 at $\omega = \omega_o$ yields,

$$\beta_1(\omega_o) = -\frac{1}{29} \qquad \text{—(14)}$$

which defines the threshold gain for the oscillator of figure (1) to be A = −29. Although the amplitude is dependent on gain variations for this oscillator, the selectivity is considerably better than for the previous oscillator of section III-2. From equation (4) for second harmonic terms

$$F_1(2\omega_o) = 0.368$$

and the second harmonic terms are actually reduced in the closed loop.

We conclude that this oscillator is superior to the first, although variation of the oscillator frequency requires that three components be tracked simultaneously rather than two for the first oscillator. This factor alone probably confines its use to a fixed frequency oscillator.

(4) The Wien Bridge Network

Although the oscillator of section III-(2) seemed at first glance to be extremely poor, we can convert the circuit into a type of bridge circuit such that we can exchange loop gain for added selectivity. A common circuit; the Wien Bridge, is shown in figure (5).

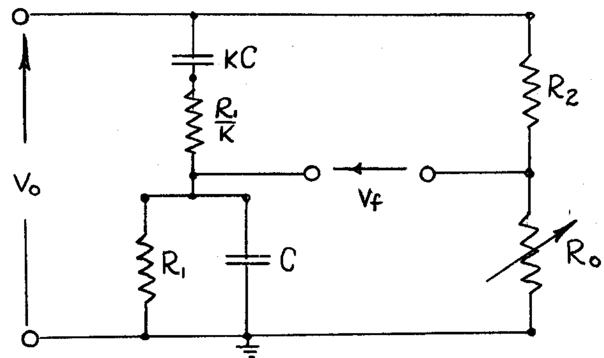


Figure (5)

Here the additional arm of the bridge is composed of ordinary carbon resistors R_2 and R_o.

The transfer characteristic β_2 of the above network is simply

$$\beta_2(j\omega) = \beta(j\omega) - \frac{R_o}{R_o + R_2} \qquad \text{--(15)}$$

where $\beta(j\omega)$ has been given by equation (7). For an oscillator employing this beta network we find for $\omega = \omega_o$, the condition for threshold of oscillation is;

$$\frac{k}{k+2} - \frac{R_o}{R_o + R_2} = \frac{1}{A} \qquad \text{--(16)}$$

For given values of the amplifier gain A, resistance R_2, and parameter k, equation (16) defines the unique value of R_o necessary for oscillation to exist. It is obvious that the higher the degree of balance of the bridge, the higher the gain must be for the operational amplifier.

The marked improvement of the selectivity of the bridge circuit over that of section III-(2) can be easily found as the oscillator of section III-(2) corresponds to the completely unbalanced bridge where $A = (k+2)/k$. The ratio of equation (4) for each oscillator is thus,

$$\frac{F_2(n\omega_o)}{F\,(n\omega_o)} = \frac{\dfrac{1}{A\left(\frac{k}{k+2} - \beta(\omega)\right)}}{\dfrac{1}{\frac{k+2}{k}\left(\frac{k}{k+2} - \beta(\omega)\right)}} = \frac{k+2}{kA} \qquad \text{—(17)}$$

where $F_2(n\omega_o)$ is for the Wien Bridge Oscillator. We see from equation (17) that the selectivity will be a maximum for the Wien Bridge Oscillator for a maximum value of amplifier gain A or high degree of bridge balance.

Up to this point the value of k has been left arbitrary. In the network of figure (5) however, its choice should be made to yield the maximum amplitude and frequency selectivity for the bridge circuit. From equation (16) we see that

$$\frac{R_o}{R_o+R_2} = \frac{k}{k+2} - \frac{1}{A} \qquad \text{—(18)}$$

which means that the bridge is never fully balanced for finite amplifier gain A. The bridge output voltage in the steady state can be found as;

$$v_f = v_o\left[\frac{k/(k+2)}{1 + j\frac{z}{(k+2)}}\right] - v_o\frac{k}{(k+2)} + \frac{v_o}{A}$$

where

$$z = \left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right]$$

For frequencies close to $\omega = \omega_o$, $z \ll 1$ and the equation for the output voltage can be approximated for $A \gg 1$ as;

$$\left|\frac{v_f}{v_o}\right|_{\omega \approx \omega_o} \simeq \frac{zk}{(k+2)^2} \qquad \text{—(19)}$$

This should be a maximum for maximum frequency sensitivity. Differentiating equation (19) with respect to k and setting the result equal to zero yields

$$z\left[\frac{(k+2)^2 - 2(k+2)k}{(k+2)^4}\right] = 0$$

from which we find that maximum frequency sensitivity occurs for,

$$k = 2 \qquad \text{—(20)}$$

This result would require that $R_o \approx R_2$ for a very high gain amplifier and this is also the value for R_o and k for maximum amplitude sensitivity for the bridge circuit.

(5) The "Twin T" Network

Practically any three terminal null network may be used for a frequency determining branch in a bridge type oscillator. A common type of null network which could be employed is shown in figure (6). This network is commonly referred to as the "Twin T" network.

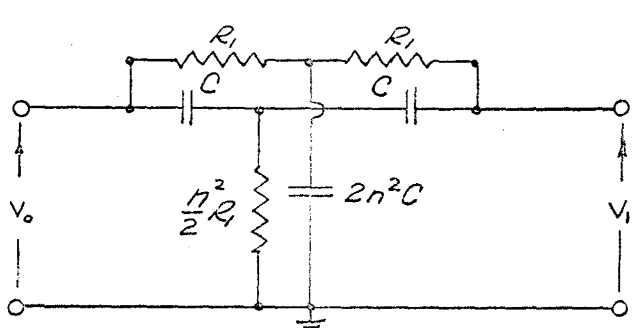


Figure (6)

The transfer function for the twin T network of figure (6) can be found as;

$$\beta_3(j\omega) = \frac{1}{1 - j\,\frac{2(n^2+1)}{n}\left[\frac{1}{\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}\right]} \qquad \text{--(21)}$$

where $\omega_o = \frac{1}{nCR_1}$

The network thus exhibits a reciprocal type of response to the network of section III-(2). As $\omega \to \omega_o$ the beta function goes towards zero and hence the name of "null network" is applicable. For use as an oscillator the network can be incorporated in a bridge circuit as shown in figure (7)

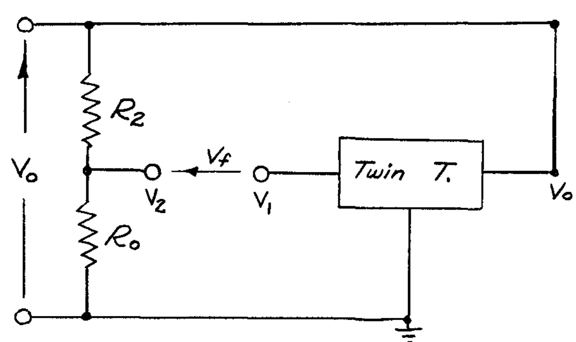


Figure (7)

The complete feedback function for the oscillator of figure (1) is then given by

$$\beta_4(j\omega) = \frac{R_o}{R_o+R_2} - \beta_3(j\omega) \qquad \text{--(22)}$$

For the resonant frequency $\omega = \omega_o$ we obtain the condition for threshold oscillation as;

$$\frac{R_o}{R_o+R_2} - \beta_3(\omega_o) = \frac{1}{A}$$

and since $\beta_3(\omega_o) = 0$ this reduces to

$$A = \frac{R_o+R_2}{R_o}$$

If the network is examined with regard to selectivity, we find as for the case of the Wien Bridge that selectivity is a maximum for A approaching infinity.

(6) Phase-Frequency Characteristic of Bridge Networks

It was pointed out in Chapter II that the phase-frequency characteristic of the beta network can be used as a measure of the frequency stability of the oscillator with respect to component phase shifts, in components other than the beta network itself. The Wien Bridge Oscillator can be examined with regard to equation (5). When examined the phase shift of the beta network can be found from the approximate value for phase shift close to resonance ϕ,

$$\beta_2 \simeq \frac{1}{A} - j\,\frac{k}{(k+2)^2}\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right] \qquad \text{—(24)}$$

$$\phi \simeq \tan^{-1} \frac{kA}{(k+2)^2}\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right] \qquad \text{—(25)}$$

Equation (5) can now be found close to resonance as approximately,

$$G = \omega_o \frac{d\phi}{d\omega} \simeq \frac{\frac{kA}{(k+2)^2}\left[1 + \frac{\omega_o^2}{\omega^2}\right]}{1 + \left[\frac{kA}{(k+2)^2}\right]^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2} \qquad \text{—(26)}$$

which at $\omega = \omega_o$ reduces to;

$$G(\omega_o) \simeq \frac{2kA}{(k+2)^2} \qquad \text{--(27)}$$

For the above to be a maximum the parameter k will have the value k = 2 which is in agreement with the section III-(4). For k = 2 the result reduces to,

$$G(\omega_o) = \frac{A}{4} \qquad \text{--(28)}$$

and the effect of a high gain amplifier is seen to make the oscillator almostentirely dependent on the beta network as the frequency determining element. This oscillator would thus seem to behave suitably if some means could be found with which to maintain the amplifier gain constant.

(7) The Nonlinear Bridge Compensator

Any compensation which could be carried out in the oscillator circuit for changes in amplifier gain would involve the use of a network with a nonlinear voltage characteristic. Compensation can be achieved if a passive nonlinear element is included in the feedback network at a point where it will change the magnitude of the feedback without affecting the steady state frequency. This is commonly called feedback limiting. For the bridge type oscillator, the bridge balance can be controlled by the thermal characteristics of a tungsten lamp or a thermistor.

As an example of the effectiveness of this type of amplitude compensation, we can analyse the oscillator circuit shown in figure(8). For the analysis we will assume that the thermal time constant of the lamps R is much greater than the period of the steady state frequency of oscillation.

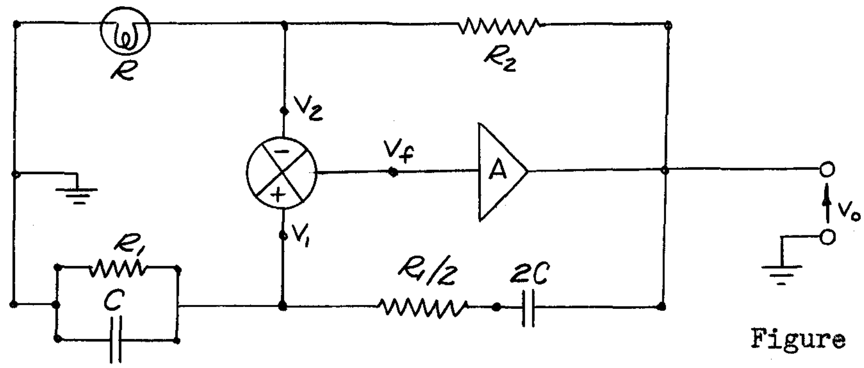


Figure (8)

Here we have introduced two lamps in series represented by R for the variable resistor R_o shown in figure (5). In the steady state the nonlinear characteristic of the lamps R can be represented to a first approximation by the experimentally determined expression,

$$R = 900 + 170E_L \qquad \text{---(29)}$$

where R will be in ohms and E_L is the R.M.S. lamp voltage in volts. For an oscillator output of v_o = 8.0 volts R.M.S. and an amplifier gain of A_1 = 400 we find for threshold of oscillation;

$$\frac{R}{R+R_2} = \frac{1}{2} - \frac{1}{400} = 4.975 \times 10^{-1} \qquad \text{---(30)}$$

which means that the portion of the output voltage across the lamp is,

$$E_L = 4.975 \times 10^{-1} \times 8.0 = 3.980 \text{ volts R.M.S.} \qquad \text{---(31)}$$

Substituting this value of E_L back into equation (29) to obtain the steady state lamp resistance yields,

$$R = 1577\,\Omega$$

and from equation (31) we obtain,

$$R_2 = 1591\,\Omega$$

which will be a constant for the oscillator.

If the amplifier gain changes over a period of time due to component deterioration to some new value $A_2 = 200$ we can calculate the change in R; ΔR and hence the new output condition to be respectively,

$$R + \Delta R = 1560\,\Omega$$

$$v_o + \Delta v_o = 7.76 \text{ volts R.M.S.} \qquad \text{---(32)}$$

Hence we see that for a 50% change in amplifier gain, the oscillator output is only changed by 3%. This method of amplitude limiting would thus seem to be quite effective in maintaining a constant steady state output voltage. In the next chapter we will examine the dynamic behavior of several thermal devices with a view towards predicting their transient effect on the oscillator.

IV TEMPERATURE DEPENDENT RESISTORS

(1) Incandescent Lamps

Although considerable literature on lamps has been written concerning their use in illumination, comparatively little work has ever been published with regard to their dynamic behaviour. An extensive bibliography found in Patchett's work[(3)] contains the majority of the early literature.

Patchett[(3)] examined a wide variety of incandescent lamps or filaments with regard to their suitability for bridge circuits which could be used as feedback limiters. His work mentioned many of the undesirable characteristics associated with lamps when used for control elements. Intermittent bridge misbalance was claimed to be one of their greatest drawbacks. He found that lamps were extremely susceptible to vibrations which in turn caused random variations in the bridge output voltage. This effect was especially pronounced for the coiled filament type of lamp. Presence of gas in the lamp envelope was also found to alter the characteristic of the lamp, as added conduction terms alter the heat transfer equations especially at certain temperatures where the gas ionizes.

Patchett[(3)] derived an approximate solution for the thermal response time of the vacuum lamp, but the result involved a knowledge of the physical mass of the filament which makes its use somewhat limited. The thermal response time was also developed by Glynne, but the temperature coefficient of resistance used in his formula referred to the lamp's operating temperature θ_o. This coefficient would thus vary over a range of about two to one in the normal operating region that he used.

(2) Lamp Dynamic Behaviour

In the steady state, the exact mechanism of the heat transfer is of little concern to us, but from experimental data it has been found that,

$$P = K_1 \theta^a \qquad \text{--(33)}$$

and

$$R = K_2 \theta^b \qquad \text{--(34)}$$

holds fairly well apart from a small region close to the ambient or room temperature. Here P is the power radiated in watts, R is the resistance of the lamp filament in ohms at some temperature θ in degrees Kelvin, and K_1,K_2,a and b are constants. Strictly speaking "a" varies somewhat with temperature, but equation (33) is a good approximation for the restricted temperature range of the compensator used. For an effective lamp,of the two lamps used in series in the experimental section,typical values for "a" and "b" would be a = 5.3 and b = 1.2 .

If we define W(t) as the instantaneous power supplied to the lamp and C_T as the thermal heat capacity of the filament, we can write the power balance equation for the lamp

$$W(t) = \frac{d}{dt}(C_T\theta) + P(t)$$

$$\simeq C_T \frac{d\theta}{dt} + P(t) \qquad \text{--(35)}$$

Here C_T has been assumed constant over the temperature range of operation.

The first term on the R.H.S. of equation (35) will be the stored energy per second and $P(t)$ will be the instantaneous power radiated by the filament. Combining equations (33) and (34) to elliminate temperature and substituting the result in equation (35) yields,

$$W(t) = C_T \frac{d}{dt}\left[\frac{R(t)}{K_2}\right]^{\frac{1}{b}} + K_1 \left[\frac{R(t)}{K_2}\right]^{\frac{a}{b}} \qquad \text{--(36)}$$

For the case of the feedback limiter, the lamp R will be in series with the fixed resistor R_2 as shown in figure (9)

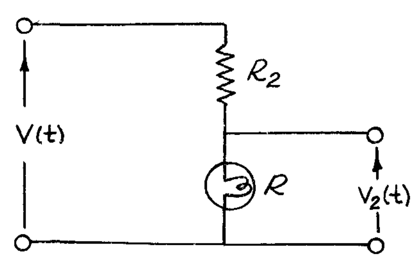


Figure (9)

The power supplied to the lamp would thus be;

$$W(t) = v^2(t) \frac{R(t)}{(R(t) + R_2)^2} \qquad \text{--(37)}$$

Combining equations (36) and (37) yields upon simplification,

$$\frac{d}{dt}\left[R(t)\right]^{\frac{1}{b}} + K_1 K_2^{\frac{1-a}{b}} \left[R(t)\right]^{\frac{a}{b}} = \frac{R(t)}{(R(t) + R_2)^2} \frac{v^2(t)}{C_T K_2^{-\frac{1}{b}}} \qquad \text{--(38)}$$

This is the approximate nonlinear differential equation for the dynamic behaviour of the lamp bridge arm shown in figure (9) and is not of any apparent standard form. A solution could be obtained by recourse to numerical or graphical methods, but the perturbation method will yield a solution about a steady state value in an analytical form.

Let us then consider the system of figure (9) for small changes about some specified steady state temperature θ_o and resistance R_o. The power supplied to the lamp has been given by equation (37) and if the steady state voltage varies as $v + \delta v$, then the temperature will vary as $\theta_o(1+x)$ where δv and x are small time varying parameters. From equation (34) we see that,

$$R = K_2\theta_o^b\,(1+x)^b$$

$$\simeq R_o\,(1+bx) \qquad \text{--(39)}$$

where $R_o = K_2\theta_o^b$ and $x \ll 1$

Similarily from equation (33) we obtain,

$$P \cong P_o\,(1+ax) \qquad \text{--(40)}$$

where $P_o = K_1\theta_o^a$ is the steady state radiated power by definition.

If we write the variation of the supplied power as $W_o(1+\delta)$ then equation (37) becomes,

$$W_o(1+\delta) = \frac{(v+\delta v)^2 Ro(1+bx)}{(R_o(1+bx) + R_2)^2}$$

$$\simeq W_o\left[1 + 2\frac{\delta v}{v} + bx - \frac{2R_o bx}{(R_o+R_2)}\right] \qquad \text{--(41)}$$

for $x \ll 1$

Defining $\rho = \frac{R_o-R_2}{R_o+R_2}$ to simplify the notation the above reduces to;

$$\delta = 2\frac{\delta v}{v} - b\rho x \qquad \text{--(42)}$$

From the power balance equation for small changes we find that;

$$\frac{C_T\theta_o}{W_o}\frac{dx}{dt} = 1+\delta - \frac{P_o}{W_o}(1+ax) \qquad \text{---(43)}$$

In the steady state the supplied power W_o is equal to the radiated power P_o and the above becomes,

$$\frac{\theta_o C_T}{W_o}\frac{dx}{dt} = \delta - ax$$

For the value of δ found in equation (42) the final result is;

$$\frac{\theta_o C_T}{W_o}\frac{dx}{dt} + ax = 2\frac{\delta v}{v} - \rho bx \qquad \text{---(44)}$$

which is a linear differential equation with a solution;

$$x = K_3 e^{-\frac{(a+b\rho)W_o}{C_T\theta_o}t} + \frac{2\delta v}{v(a+\rho b)} \qquad \text{---(45)}$$

Thus the system would appear to have a simple time constant given by equation (46).

$$\tau = \frac{C_T\theta_o}{(a+\rho b)W_o} \qquad \text{---(46)}$$

For the particular case of the Wien Bridge Oscillator employing a high gain amplifier $R_o \simeq R_2$ and thus $\rho \simeq 0$. For this case equation (46) reduces to the simple form;

$$\tau = \frac{C_T\theta_o}{aW_o} \qquad \text{---(46A)}$$

Although "a" and C_T are not strictly speaking constant, equation (46) or (46A) should yield τ to a good approximation for small values of the perturbation δv. The steady state temperature θ_o should also be kept well above the temperature of the lamp's surroundings.

A useful expression for variations of the lamp resistance in terms of power can be obtained by further manipulation of equations (33) and (34). If we take the logarithms of both equations and combine the two resultants to eliminate the temperature, we can obtain;

$$\log R_o = \frac{b}{a} \log P_o + \log K_2 - \frac{b}{a} \log K_1 \qquad \text{--(47)}$$

which is the equation of a straight line with slope

$$m_o = \frac{b}{a} \qquad \text{--(48)}$$

and y intercept

$$y_o = \log K_2 - \frac{b}{a} \log K_1 \qquad \text{--(49)}$$

The actual characteristic will of course deviate from this predicted result at low power inputs where the temperature or resistance of the filament would be dependent on room temperature. The actual characteristic would thus be log R equals a constant value for low values of applied power and then gradually experience a transition where the room temperature becomes decreasingly important until equation (47) is satisfied.

For useful power inputs in the region of operation, the slope of the characteristic should obey equation (47). For typical values of "a" and "b" for the lamps used in the experimental portion the slope will be;

$$m_o = \frac{b}{a} \quad \frac{1.2}{5.3} = 0.23$$

We have seen that the lamp bridge shown in figure (9) is characterized by a simple time constant τ. This time constant will set a lower frequency limit on the use of the bridge as a control element. If the frequency is reduced for the oscillator output we would eventually reach a point where the lamp resistance would follow the oscillator output and fluxuate at twice the frequency of the oscillator. This would produce an output which would be entirely unsuitable as a control signal.

(3) Low Frequency Response

To obtain an idea of the output from the lamp bridge at low frequencies, let us examine the circuit shown in figure (10).

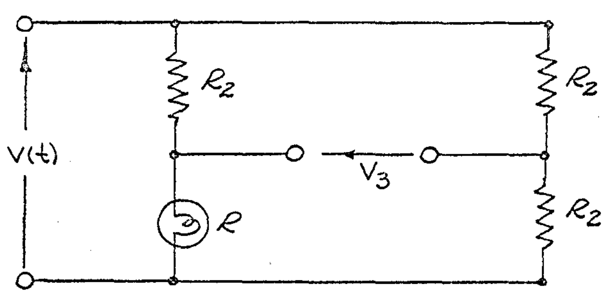


Figure (10)

Let the input to the bridge be a sinusoidal voltage $v = \sqrt{2}\, E \sin \omega_o t$ where E is the R.M.S. voltage and ω_o is the frequency in radians per second. At low frequencies as the lamp follows the sinusoid, let the resistance vary slightly about the mean value R_o so that

$$R = R_o(1+y) \qquad \text{—(50)}$$

where y is assumed to be a small time varying parameter. If the bridge arms are adjusted so that $R_2 = R_o$ then the bridge output will be;

$$v_3 = v\left[\frac{R}{R+R_2} - \frac{1}{2}\right] = v\left[\frac{R_o(1+y)}{R_o(1+y)+R_o}\right] - \frac{v}{2}$$

$$\simeq \frac{v}{4}y \qquad \text{--(51)}$$

The instantaneous current "i" through the lamp arm of the bridge is,

$$i = \frac{v}{R+R_o} = \frac{v}{R_o(2+y)} \qquad \text{--(52)}$$

and the instantaneous power supplied to the lamp is;

$$W = \frac{v^2R_o(1+y)}{\left[R_o(1+y)+R_o\right]^2} = \frac{v^2R_o(1+y)}{4R_o^2(1+\frac{y}{2})^2}$$

$$\simeq \frac{v^2}{4R_o} \qquad \text{--(53)}$$

where terms in y^2 have been neglected. Corresponding to the variation in resistance, let the temperature of the filament vary about some mean value θ_o such that,

$$\theta = \theta_o(1+x) \qquad \text{--(54)}$$

where x is a small time varying parameter. The actual radiated power P is in general some nonlinear function of this temperature say $P = f(\theta)$ and this can be expressed as a few terms of a Taylor Series for small nonlinearity as,

$$P = f(\theta_o) + \theta_o x f'(\theta_o) + \ldots\ldots\ldots\ldots\ldots$$

$$\simeq P_o + \theta_o x G_T \qquad \text{--(55)}$$

where G_T has been defined as the differential thermal conductance of the filament at a temperature θ_o ie ($G_T = f'(\theta_o)$). If we again write C_T as the thermal heat capacity of the filament, the power balance for the lamp arm of the bridge can be written as;

$$\frac{d(C_T\theta)}{dt} = W - P \qquad \text{--(56)}$$

Noting that the steady state radiated power is $P_o = E^2/4R_o$ we can evaluate equation (56) as;

$$C_T\theta_o\frac{dx}{dt} + G_T\theta_o x = -\frac{E^2}{4R_o}\cos 2\omega_o t \qquad \text{--(57)}$$

This is a simple linear differential equation with a solution;

$$C_T\theta_o x = \frac{E^2}{4R_o}\left[\frac{\omega_T}{4\omega_o^2+\omega_T^2}\cos 2\omega_o t + \frac{2\omega_o}{4\omega_o^2+\omega_T^2}\sin 2\omega_o t\right] \qquad \text{--(58)}$$

where $\omega_T = G_T/C_T$

If we define γ as the temperature coefficient of resistivity for variations about a termperature θ_o, we can write $y = \gamma\theta_o x$. Upon substitution in equation (51) along with the value of $\theta_o x$ from equation (58) we can obtain the bridge output voltage as;

$$v_3 = \frac{E^3\gamma}{16\sqrt{2}\,C_T R_o}\left[\frac{\omega_T}{4\omega_o^2+\omega_T^2}\left\{\sin\omega_o t - \sin 3\omega_o t\right\} - \frac{2\omega_o}{4\omega_o^2+\omega_T^2}\left\{\cos\omega_o t - \cos 3\omega_o t\right\}\right] \qquad \text{--(59)}$$

In the above form the result is somewhat cumbersome, however at practical low frequencies $2\omega_o$ is still much larger than ω_T for the tungsten lamps of the type used. For $2\omega_o \gg \omega_T$ equation (59) can be approximated further to yield;

$$v_3 \simeq \frac{E^3 \gamma}{32\sqrt{2}\, C_T R_o \omega_o} \left\{ \cos 3\omega_o t - \cos \omega_o t \right\} \qquad \text{---(60)}$$

This result means that at the bridge balance point where we would normally expect zero output voltage, we still have an output given by equation (60). This output consists of two components of equal amplitude and ninety degrees out of phase with the bridge input voltage. The first of these components is a third harmonic of v(t) while the second is at the same frequency as v(t). The components should decrease quite rapidly with frequency as is seen by equation (60). In the oscillator these components would be further reduced by the factor $\frac{1}{1-A\beta}$ in the feedback loop. As long as the feedback loop is quite selective the unwanted output from the bridge can be tolerated for even very low frequencies, however, the lamp bridge will cease to maintain the amplitude constant.

(4) Thermistors

Thermistors are basically electronic devices which utilize the change in resistivity of a semiconductor with a change in temperature or applied voltage. The devices can be either directly heated by the current flow through the semiconductor or indirectly heated by heaters depending on the type and application. The active portion of the devices is composed of complex metallic-oxide compounds using such typical oxides as manganese, nickel, copper, and cobalt.

The important factor in their application is the region of negative temperature coefficient of resistance; the resistance decreases approximately exponentially with the inverse of the absolute temperature.

Figure (11) shows a typical characteristic curve for a directly heated bead thermistor which would be suitable for the feedback compensator of the oscillator used in the experimental section.

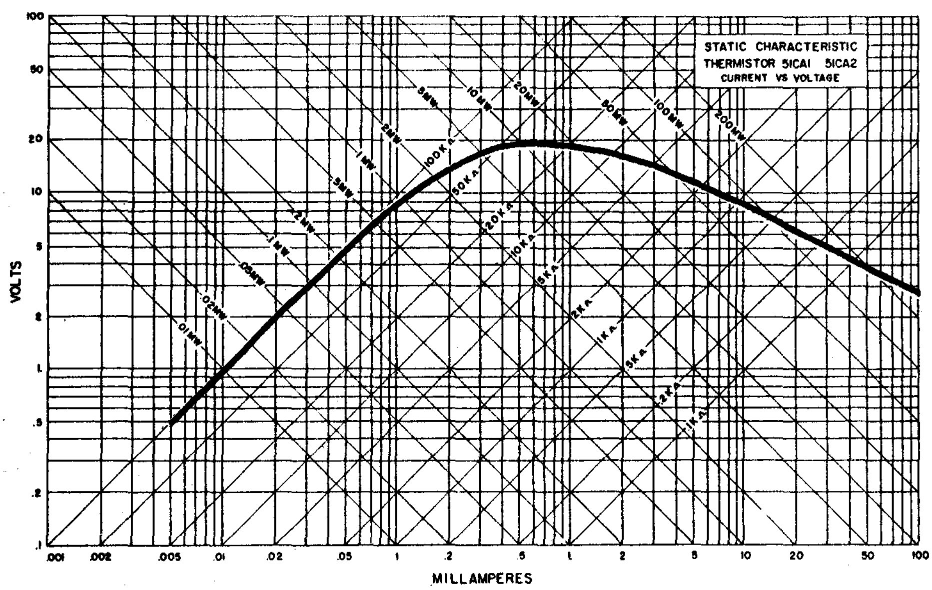


Figure (11)

Bollman and Kreer[7] have developed an expression for the dynamic behaviour of the thermistor in the form of a nonlinear differential equation. Although the equation is highly nonlinear it can be shown that the device can be characterized by a simple time constant to a good approximation.

The design of the thermistor bridge arm can be shown quite easily via figure (12).

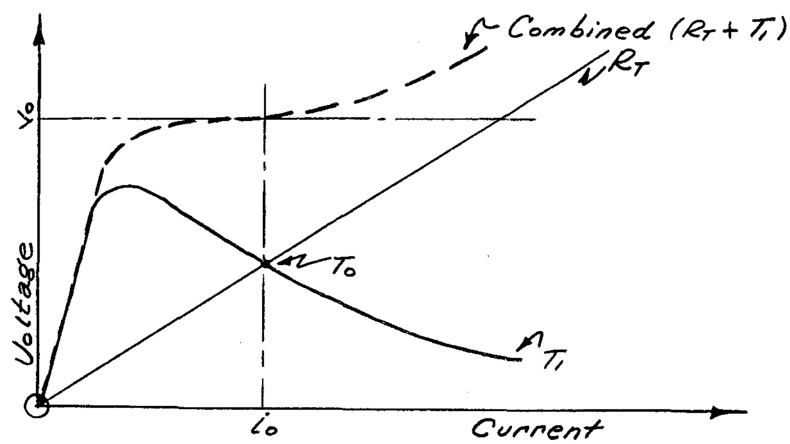


Figure (12)

Figure (12) shows three curves, the thermistor T_1, the series resistor R_T, and the combined characteristic of the two in series. Resistor R_T and thermistor T_1 could form one arm of a bridge circuit as shown in figure (13). Let T_o be the steady state resistance of T_1.

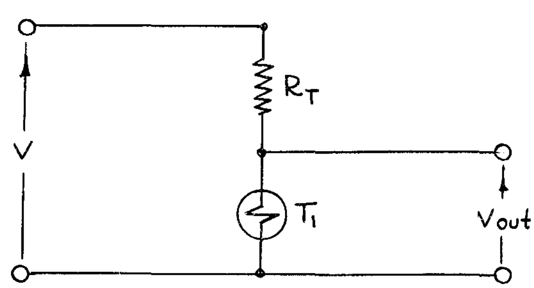


Figure (13)

For a proper choice of R_T and thermistor T_1, the steady state input voltage v_o will be such that $R_T = T_o$ for the bridge balance, and corresponding to this condition the combined characteristic $(T_1 + R_T)$ should have dv/di very close to zero. Under these conditions any change in the steady state voltage v from v_o would create a large change in current i and a large change in the output voltage v_{out} which would unbalance the bridge considerably. This would thus seem to be a very sensitive source of an error voltage if used as a feedback compensator.

For use in the oscillator of section III-(7) figure (8), the thermistor T_1 would take the place of the resistor R_2 and the resistor R_T would take the place of the lamps R. For additional sensitivity the lamps R could remain, replacing resistor R_T, although this would further complicate the selection of the lamps R and thermistor T_1.

One additional benefit which could be obtained from the use of a thermistor for the compensator is the independence of the device with respect to vibrations and their associated bridge misbalance.

V WIEN BRIDGE OSCILLATOR WITH AMPLITUDE COMPENSATOR

(1) Sensitivity of Wien Bridge Circuit

In Chapter III it was found that the optimum choice for the parameter k in figure (5) was k = 2 for maximum frequency sensitivity. For R_o, a passive linear resistor, this is also the criterion for maximum amplitude sensitivity for the bridge.

When a nonlinear resistor R is substituted for the resistor R_o as in figure (14), the situation is further complicated and the parameter k will have a new optimum value for maximum bridge amplitude sensitivity.

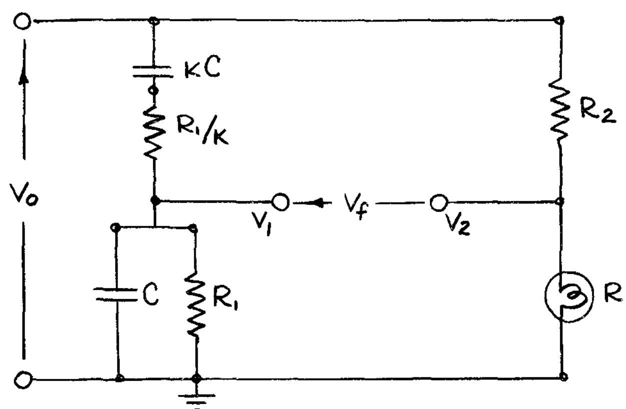


Figure (14)

If the voltage input to the bridge is given in the steady state by,

$$v_o = E_1 \cos \omega_o t \qquad \text{--(61)}$$

and the mean value of R is R_o at some mean temperature θ_o, we can examine the effect of small voltage amplitude changes on the bridge balance. Let the input to the bridge vary in amplitude as $(E_1+\delta E_1)\cos\omega_o t$ and the temperature of the lamp's filament vary as $(\theta_o+\delta\theta)$. If γ is defined as the temperature coefficient of resistivity for the filament, then the lamp resistance will vary as $R_o(1+\gamma\delta\theta)$.

The power radiated P can be found from equation (55) as,

$$P = P_o + \delta\theta G_T \qquad \text{--(62)}$$

where G_T is again the differential thermal conductance and P_o is the steady state radiated power.

The power supplied to the lamp W is,

$$W = \frac{R_o(1+\gamma\delta\theta)(E_1+\delta E_1)^2}{2\left(R_2+R_o(1+\gamma\delta\theta)\right)^2} \qquad \text{--(63)}$$

which for small perturbations δE_1 where $|\delta E_1| \ll 1$ and $|\delta\theta| \ll 1$ reduces to;

$$W \simeq W_o\left[1 + 2\frac{\delta E_1}{E_1} + \gamma\delta\theta\left(1 - \frac{2R_o}{R_o+R_2}\right)\right] \qquad \text{--(64)}$$

where $W_o = \dfrac{E_1 R_o}{(R_o+R_2)^2}$ is the steady state power supplied and hence radiated. ie $(P_o = W_o)$

On the average the power supplied and radiated must be equal. Thus we can equate equations (64) and (62) to obtain;

$$\left[1 + 2\frac{\delta E_1}{E_1} + \gamma\delta\theta\left(1 - \frac{2R_o}{R_o+R_2}\right)\right] = 1 + \frac{\delta\theta G_T}{P_o} \qquad \text{--(65)}$$

If we recall the constant m_o from section IV-(2) we can obtain

$$m_o = \frac{\log R}{\log P} = \frac{\dfrac{\delta R}{R_o}}{\dfrac{\delta P}{P_o}} = \frac{\dfrac{R_o\gamma\delta\theta}{R_o}}{\dfrac{\delta\theta G_T}{P_o}} = \frac{\gamma P_o}{G_T} \qquad \text{--(66)}$$

and note from equation (16) that for an oscillator employing a high gain amplifier,

$$\frac{R_o}{R_o+R_2} \simeq \frac{k}{k+2}$$

then we can reduce equation (65) to;

$$2\frac{\delta E_1}{E_1} \simeq \gamma\delta\theta\left[\frac{1}{m_o} - \left(1 - \frac{2k}{k+2}\right)\right] \qquad \text{---(67)}$$

The output voltage v_f of figure (14) can thus be written and approximated as;

$$v_f = (E_1+\delta E_1)\cos\omega_o t\left[\frac{k}{k+2} - \frac{R_o(1+\gamma\delta\theta)}{R_o(1+\gamma\delta\theta)+R_2}\right]$$

and

$$\delta v_f \simeq -2\delta E_1 \frac{2k}{(k+2)^2}\frac{1}{\left[\frac{1}{m_o} -1+\frac{2k}{(k+2)}\right]} \qquad \text{---(68)}$$

For maximum amplitude sensitivity $\left|\delta v_f/\delta E_1\right|$ should yield a maximum. For this condition for real values of k we find

$$k = 2\sqrt{\frac{1-m_o}{1+m_o}} \qquad \text{---(69)}$$

We note that the value of k for maximum sensitivity has been altered by the factor $\sqrt{\frac{1-m_o}{1+m_o}}$ from the case of the linear resistor. Since we have seen that the expected value of m_o is small for the tungsten lamp, a value of k = 2 will not greatly alter the amplitude sensitivity and yet it will allow the oscillator to be designed for maximum frequency sensitivity.

(2) Oscillator Stability

Stability for the oscillator is extremely important as the oscillator output should not experience any violent changes for finite changes in the circuit. We say that a linear circuit is stable when all transients decay in a finite time leaving a predominent steady state. It is thus essential to study a system's transient response when investigating the system stability.

For an oscillator, we must know the transient response to small perturbations about each of the possible "steady states". The "steady state" is where the output is a wave of constant amplitude and frequency. The equilibrium point or steady state point is then investigated by perturbing the system and examining the resultant transient. If the particular steady state is stable, the transient disturbance must decrease with time, and if unstable the transient will be enhanced. In some systems a large perturbation can cause a change in the steady state while a small perturbation will not. The time constant with which a disturbance decreases may also be of sufficient duration to be annoying. For these reasons some qualification must be imposed on the meaning of stability for a system depending on its application.

If we perturb an oscillator which is initially oscillating with a sinusoidal output, the perturbations will give rise to a modulation of the output[12] at some complex frequency $s = \alpha + j\omega_m$ which can be obtained by solving the network equations for the perturbation. The transient response can then be evaluated with regard to stability.

(3) Transient Response of Wien Bridge Oscillator

In this section we will examine the transient response of the Wien Bridge Oscillator for small perturbations of the lamp resistance R for the circuit shown in figure (15).

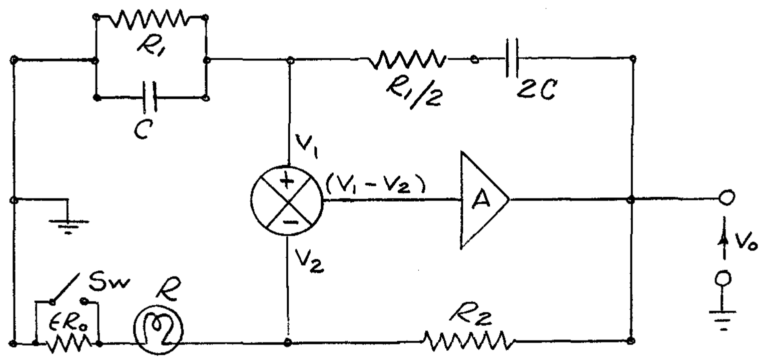


Figure (15)

The amplifier A is assumed to be linear with gain $A\underline{/0^\circ}$. The lamp R is perturbed by introducing or removing the resistor ϵR_o via the switch Sw.

During the steady state oscillation the switch Sw is closed and the output voltage v_o will be given by;

$$v_o = E_1 \cos(\omega_o t + \phi) \qquad \text{--(70)}$$

where $\omega_o = 1/CR_1$ is the steady state frequency. For oscillator frequencies above the lower frequency limit of the lamp bridge, the output amplitude and frequency are constant in the steady state and the lamp resistance will be a steady state value R_o. If the amplifier has a high gain such that $A \gg 1$, the resistor R_2 will also have the approximate value $R_2 \simeq R_o$.

If the resistance ϵR_o is introduced into the circuit at time t=0 via the switch Sw, the output must change correspondingly by a time varying parameter δv_o such that.

$$v_o + \delta v_o = (E_1 + \delta E_1)\cos(\omega_o t + \phi) \qquad \text{--(71)}$$

The change in the output voltage will be a function of the amplitude only as shown in equation (71) if the response characteristic of the feedback network is symmetrical about the oscillation frequency[11]. For any other condition the changes in amplitude would be accompanied by changes in the oscillation frequency. We are justified in making this approximation here at least for small perturbations.

Equation (71) can be written in the exponential form using the exponential identity for the cosine term to yield,

$$\delta v_o = \frac{E_1 d}{2}\left[e^{(s+j\omega_o)t+j\phi} + e^{(s-j\omega_o)t-j\phi}\right] \qquad \text{--(72)}$$

where the relative change in E_1 has been written as

$$\frac{\delta E_1}{E_1} = d e^{st} \qquad \text{--(73)}$$

where $S = \alpha + j\omega_m$ is the complex frequency of the modulation.

The change in the output is seen from equation (72) to be composed of two terms at the complex frequencies $S \pm j\omega_o$.

In the steady state the output v_1 from the frequency selective arm of the bridge of the circuit, figure (15) was,

$$v_1 = v_o \frac{1}{2 + \frac{j}{2}\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right]} \qquad \text{--(74)}$$

The change in v_1 for the change in v_o will thus be

$$\delta v_1 = \delta v_o \frac{1}{2 + \frac{j}{2}\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right]}$$

for each of the two complex frequencies of the output δv_o given in equation (72). The output from the frequency selective arm δv_1 is thus;

$$\delta v_1 = \frac{E_1 d}{2}\left[\frac{e^{(s+j\omega_o)t+j\phi}}{2+\frac{1}{2}\left(j+\frac{s}{\omega_o}+\frac{1}{j+s/\omega_o}\right)}+\frac{e^{(s-j\omega_o)t-j\phi}}{2+\frac{1}{2}\left(-j+\frac{s}{\omega_o}+\frac{1}{-j+s/\omega_o}\right)}\right]$$

Fortunately for the oscillator parameters involved we can make the assumption $|s| \ll |\omega_o|$ which greatly reduces this equation. For the above approximation we obtain;

$$\delta v_1 \simeq \frac{E_1 d}{4}\left[\frac{e^{(s+j\omega_o)t+j\phi}+e^{(s-j\omega_o)t-j\phi}}{\left(1+\frac{s}{2\omega_o}\right)}\right]$$

or

$$\delta v_1 \simeq \frac{\delta v_o}{\left(2+\frac{s}{\omega_o}\right)} \qquad \text{--(75)}$$

If the transfer function for changes in the envelope is defined as $F_1(s)$ for the frequency selective arm we can write;

$$F_1(s) = \frac{\delta V_1}{\delta E_1} = \frac{1}{\left(2+\frac{s}{\omega_o}\right)} \qquad \text{--(76)}$$

where δV_1 is the amplitude of δv_1 and δE_1 is the amplitude of δv_o. If the Laplace transforms of δV_1 and δE_1 are $L_1(s)$ and $L(s)$ respectively we can write,

$$L_1(s) = \frac{L(s)}{2+\frac{s}{\omega_o}} \qquad \text{--(77)}$$

In order to evaluate L(s) in terms of the circuit constants we must obtain a relationship for the amplitude sensitive arm of the bridge of figure (15) which consists of the lamps R and resistor R_2. In the steady state for the high gain amplifier, the lamp R had a resistance R_o and R_2 was approximately $R_2 \simeq R_o$. The power supplied and hence radiated in the steady state was thus;

$$W_o = P_o = \frac{E_1^2}{8R_o} \qquad \text{--(78)}$$

where E_1 is the peak voltage given in equation (70). Upon perturbation the lamp temperature will vary by a small amount,

$$\theta = \theta_o(1+x)$$

where x is the small time varying parameter. Similarily the lamp resistance will vary as;

$$R = R_o(1+y)$$

where y is a small time varying parameter. If the perturbation in R occurs from switching the resistance ϵR_o into the circuit at time t=0, we can write the time function for the change in R as;

$$\delta R = \epsilon R_o h(t)$$

where h(t) is the unit step function. Upon perturbation the power supplied to the lamp, W becomes;

$$W = W_o + \delta W = \frac{(E_1+\delta E_1)^2 \{R_o(1+y)\}}{2(R_2+\delta R+R_o(1+y))^2} \qquad \text{--(79)}$$

Assuming a small perturbation such that $\epsilon \ll 1$ this can be approximated by the binomial expansion as;

$$\delta W \simeq W_o\left[\frac{2\delta E_1}{E_1} - \epsilon h(t)\right] \qquad \text{--(80)}$$

The power radiated by the lamp was given in equation (55) as;

$$P = P_o + \theta_o x G_T$$

where G_T was the differential thermal conductance and P_o is the steady state radiated power. The variation in the radiated power is thus;

$$\delta P = x\theta_o G_T \qquad \text{—(81)}$$

The thermal heat capacity of the filament has been defined previously as C_T so that we can write the power balance equation for the lamp as;

$$\frac{d}{dt}\left[C_T\theta_o(1+x)\right] = \delta W - \delta P \qquad \text{—(82)}$$

which can be evaluated by substitution from equation (80) and (81) to obtain,

$$C_T\theta_o\frac{dx}{dt} + G_T\theta_o x = \frac{2P_o}{E_1}\,\delta E_1 - P_o\epsilon h(t) \qquad \text{—(83)}$$

If the Laplace transforms of $\theta_o x$ and δE_1 are $L_x(s)$ and $L(s)$ respectively the transform equation for (83) is

$$L_x(s)\{C_T s + G_T\} = \frac{2P_o}{E_1}\,L(s) - \frac{P_o\epsilon}{S} \qquad \text{—(84)}$$

The temperature coefficient of resistivity for the filament material has been defined as γ, and it was shown in equation (66) that

$$m_o = \frac{\gamma P_o}{G_T} \qquad \text{—(66)}$$

Substituting equation (66) in (84) and letting $\omega_T = G_T/C_T$ we obtain,

$$L_x(s) = \frac{\omega_T m_0}{\gamma(s+\omega_T)}\left\{\frac{2}{E_1} L(s) - \frac{\epsilon}{s}\right\} \qquad \text{--(85)}$$

The voltage output from the thermal arm of the bridge is v_2 in the steady state and will change in response to δv_0 to;

$$v_2 + \delta v_2 = \frac{(v_0+\delta v_0)\left[R_0(1+y) + \delta R\right]}{R_2 + R_0(1+y) + \delta R} \qquad \text{--(86)}$$

For small perturbations and noting that $R_2 \simeq R_0$ this can be approximated as;

$$\delta v_2 \simeq \frac{1}{2}\left[\delta v_0 + \frac{1}{2} v_0 (y + \epsilon h(t))\right] \qquad \text{--(87)}$$

Noting that $y = \gamma \theta_0 x$ by definition, and writing δV_2 as the change in the envelope of δv_2, we obtain,

$$\delta V_2 \simeq \frac{1}{2}\left[\delta E_1 + \frac{1}{2} E_1 (\gamma \theta_0 x + \epsilon h(t))\right]$$

from which we can write the transform equation if we write $L_2(s)$ as the transform of δV_2.

$$L_2(s) \simeq \frac{1}{2}\left[L(s) + \frac{E_1}{2}(\gamma L_x(s) + \frac{\epsilon}{s})\right] \qquad \text{--(88)}$$

In the above $L(s)$ is the transform of δE_1 and $L_x(s)$ the transform of $\theta_0 x$.

From the circuit of figure (15) we see that before perturbation,

$$v_0 = A(v_1 - v_2)$$

and after perturbation this would become

$$v_0 + \delta v_0 = A(v_1 + \delta v_1 - v_2 - \delta v_2)$$

from which the variation term is;

$$\delta v_o = A(\delta v_1 - \delta v_2)$$

Since $A \gg 1$ we see that $\delta v_1 \simeq \delta v_2$ or $\delta V_1 \simeq \delta V_2$. Thus the transforms will be approximately equal.

$$L_1(s) \simeq L_2(s) \qquad \text{--(89)}$$

Solving equations (89),(88),(85) and (77) for L(s) we obtain the transform of the output disturbance for the perturbation $\epsilon R_o h(t)$ as;

$$L(s) \simeq -\frac{E_1 \epsilon}{2s}\left[\frac{s^2 + s\{\omega_T(1-m_o) + 2\omega_o\} + 2\omega_o\omega_T(1-m_o)}{s^2 + s\omega_T(1+m_o) + 2\omega_o\omega_T m_o}\right] \qquad \text{--(90)}$$

Equation (90) is seen to have three poles which are located at the roots of the denominator. $s = 0$ and $s = s_1, s_2$ where,

$$s_1, s_2 = -\frac{(1+m_o)\omega_T}{2} \pm j\sqrt{2\omega_o\omega_T m_o - \frac{(1+m_o)^2\omega_T^2}{4}} \qquad \text{--(91)}$$

or abbreviating

$$s_1, s_2 = \alpha \pm j\omega_m$$

where $\alpha = -\dfrac{(1+m_o)\omega_T}{2}$

and $\omega_m = \sqrt{2\omega_o\omega_T m_o - \dfrac{(1+m_o)^2\omega_T^2}{4}}$ --(92)

The existance of a negative α means that the poles of L(s) will all lie in the left hand half of the s plane. This will ensure absolute stability in that the transient will die out in a finite time. Formal stability in terms of the pole locations is often benificial as a pair of poles which are located close to the $j\omega$ axis in the left half plane, correspond to a lightly damped sinusoid which may be inadmissible where the amplitude is sufficient to seriously alter the steady state output. From equation (92) we see that since m_o is small, α will be correspondingly small with an expected poor transient response. For actual "squegging" to exist for small perturbations, the constant m_o must have a value approaching $m_o = -1$ which of course is not possible for devices which exhibit a positive temperature coefficient of resistance.

The transient response can be found by taking the inverse transform of equation (90). Using the method of residues[15] at the three poles $s = 0, s_1$, and s_2, the time function can be obtained as;

$$\delta E_1 = \frac{-E_1 \epsilon e^{\alpha t}}{2\omega_m}\left[\left\{\left(\frac{1}{2} - m_o - \frac{1}{2m_o}\right)\omega_T + 2\omega_o\right\}\sin\omega_m t + \left(2 - \frac{1}{m_o}\right)\omega_m \cos\omega_m t\right] - \frac{E_1\epsilon(1-m_o)\,h(t)}{2m_o} \quad \text{---(93)}$$

In the analysis we have already assumed that $\omega_o \gg \omega_T$. Employing this restriction to equation (93) we obtain the approximation

$$\delta E_1 \simeq -\frac{E_1 \epsilon \omega_o}{\omega_m} e^{\alpha t} \sin\omega_m t - \frac{E_1 \epsilon}{2}\,\frac{(1-m_o)}{m_o}\,h(t) \quad \text{---(94)}$$

For normal oscillator frequencies the constant term of equation (94) can be ignored because of its relative magnitude. The variable part of equation (94) extrapolated back to t=0 would be

$$\left|\delta E_1\right|_{t=o} = \frac{E_1 \epsilon \omega_o}{\omega_m}$$

and would take the form of a damped sinusoid with the frequency ω_m, dying out with the time constant $\tau_1 = 1/\alpha$ where α and ω_m are given by equations (92).

The transfer function $F_2(s)$ for changes in the output envelope with relative changes in the lamp resistance $\delta R/R_0$ can be written from equation (90) as;

$$F_2(s) = \frac{L(s)}{L_3(s)} = -\frac{E_1}{2}\left[\frac{s^2 + s(\omega_T(1-m_o) + 2\omega_o) + 2\omega_o\omega_T(1-m_o)}{s^2 + s\omega_T(1+m_o) + 2\omega_o\omega_T m_o}\right] \qquad \text{--(95)}$$

where $L_3(s)$ is the Laplace transform of the relative change in R. The "enhancement factor" which is defined in most literature as the relative change in the output for relative changes of the input can be found for sinusoidal variations of the resistor R by writing $s = j\omega$ in (95) as;

$$\frac{\delta E_1/E_1}{\delta R/R_o} = -\frac{1}{2}\left[\frac{\omega^2 - j\omega\{\omega_T(1-m_o) + 2\omega_o\} - 2\omega_o\omega_T(1-m_o)}{\omega^2 - j\omega\{1 + m_o\}\omega_T - 2\omega_o\omega_T m_o}\right] \qquad \text{--(96)}$$

The maximum enhancement will occur for the real frequency corresponding to the poles of equation (96) which is;

$$\omega^2 \simeq 2m_o\omega_o\omega_T \simeq \omega_m^2$$

Upon substitution of this value in equation (96) we may write the approximate "enhancement factor" as

$$\left|\frac{\delta E_1/E_1}{\delta R/R_o}\right|_{\omega=\omega_m} \simeq \frac{\omega_o}{\omega_T(1+m_o)} \qquad \text{--(97)}$$

Actually we could expect that the true magnitude of the enhancement would be somewhat less than this value due to neglected dissipation in the capacitors and other components and the fact that the gain of the amplifier is not really infinite. The enhancement would however be expected to be unusually large at high oscillator frequencies. Although extremely linear amplifiers do cause the oscillator to behave poorly with respect to its transient response, the enhancement at high oscillator frequencies is always much better than is predicted by this theory. In fact, the transient response usually improves with increasing oscillator frequency ω_o rather than become worse as is predicted by the theory.

(4) Effect on Transient Response for Nonlinear Amplifier

It was noted in the preceeding section that transient disturbances die out much more rapidly than is predicted by the linear amplifier theory especially at higher oscillator frequencies. It is known that small amounts of nonlinearity will prevent the oscillation amplitude from building up indefinitely by effectively altering the gain and thus creating a new steady state operating point. The analysis is thus repeated including the effect of a small nonlinearity present in the operational amplifier.

In practice the highly stabilized amplifier used is characterized by a range which is relatively free from distortion followed by a critical level often referred to as the power point of the amplifier. After this critical level is reached the output is highly nonlinear and usually almost independent of input voltage increases. Below the power point however, the nonlinearity increases very slowly and can be closely approximated by a few terms of a power series such as;

$$Av_{in} = a_1 v_o + a_2 v_o^2 + a_3 v_o^3 + \cdots\cdots$$

where A is the amplifier gain, v_{in} is the amplifier input voltage, and v_o is the amplifier output voltage. The constants $a_1, a_2, a_3 \ldots\ldots$

will vary depending on the amount and type of nonlinearity. In the closed loop operation, the output voltage will contain distortion terms and will be of the form;

$$v_o = E_1 \cos(\omega_o t + \phi) + D_2E_1 \cos(2\omega_o t + \phi_1) + D_3E_1 \cos(3\omega_o t + \phi_2) + \dots\dots$$

where $D_2, D_3, \dots\dots$ are the fractional distortions at the various harmonic frequencies of the oscillator natural frequency ω_o. Even power nonlinearities in the amplifier will contribute a d.c. term and even harmonics to the output, while odd power nonlinearity will contribute a term at the fundamental frequency plus odd harmonics. Since we are concerned with terms which will limit the amplitude of the output, we will deal only with an odd power nonlinearity and in its simplest form — the cubic. Let us assume then that the amplifier can be represented by the approximation

$$Av_{in} \simeq v_o + bv_o^3 \qquad \text{--(98)}$$

and for the steady state the output voltage from the oscillator will contain a third harmonic term

$$v_o = E_1 \cos(\omega_o t + \phi) - D_3E_1 \cos(3\omega_o t + \phi_2) \qquad \text{--(99)}$$

where D_3 is the fractional distortion of the output voltage at the frequency $3\omega_o$. The oscillator natural frequency is altered somewhat by this additional term, but as long as $D_3 \ll 1$ the change will be insignificant. The approximate amplifier input can be evaluated as;

$$v_{in} \simeq \frac{E_1}{A}\frac{(1+3bE_1^2)}{4} \cos(\omega_o t + \phi) - \frac{D_3E_1}{A} \cos(3\omega_o t + \phi_2) + \frac{bE_1^3}{A4} \cos(3\omega_o t + 3\phi) \qquad \text{--(100)}$$

The Wien Bridge circuit must in effect adjust slightly for this new condition. We can account for this change by allowing k to vary slightly from previous value of k = 2, to a new value $k = 2(1 + 2\delta_1)$ where $2\delta_1 \ll 1$.

The steady state transfer function $T_1(j\omega)$ for the Wien Bridge for the new value of k is simply

$$T_1(j\omega) = \frac{\frac{1}{2}k}{\frac{1}{2}(k+2) + \frac{1}{2}j\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right]} - \frac{1}{2}$$

$$\simeq \frac{1}{2}\left[\frac{1 + \delta_1}{1 + \frac{(1-\delta_1)}{4}\, j(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega})} - 1\right] \qquad \text{--(101)}$$

For the term at the fundamental frequency ω_o this reduces to,

$$T_1(\omega_o) \simeq \frac{\delta_1}{2} \qquad \text{--(102)}$$

while for the term at the third harmonic frequency,

$$T_1(3\omega_o) \simeq -\frac{e^{j\alpha_2}}{\sqrt{13}} \qquad \text{--(103)}$$

where $\alpha_2 = \tan^{-1}(\frac{3}{2})$

Since the input to the bridge is given by equation (99) we can find a second value for v_{in} which is the bridge output,

$$v_{in} = \frac{1}{2}\delta_1 E_1 \cos(\omega_o t + \phi) + \frac{D_3 E_1}{\sqrt{13}} \cos(3\omega_o t + \phi_2 + \alpha_2) \qquad \text{--(104)}$$

Equating equations (104) and (100) to evaluate the unknown parameters we find;

(a) $$\frac{\delta_1 E_1}{2} = \frac{E_1}{A}(1 + \frac{3bE_1^2}{4})$$

or $$\delta_1 = \frac{2}{A}(1 + \frac{3bE_1^2}{4}) \qquad \text{--(105)}$$

$$\text{(b)}\quad \frac{D_3E_1}{\sqrt{13}}\cos(3\omega_o t + \phi_2 + \alpha_2) = -\frac{D_3E_1}{A}\cos(3\omega_o t + \phi_2) + \frac{bE_1^{\,3}}{4A}\cos(3\omega_o t + 3\phi)$$

For the case of the high gain amplifier where $A \gg 1$ the above can be approximated as

$$D_3 \simeq \frac{b\sqrt{13}\,E_1^{\,2}}{4A} \qquad \text{and } (\phi_2 + \alpha_2) \simeq 3\phi \qquad \text{--(106)}$$

which when combined with equation (105) yields;

$$\delta_1 \simeq \frac{6D_3}{\sqrt{13}} \qquad \text{--(106A)}$$

If the oscillator is again subjected to a perturbation in the lamp resistance as for the previous analysis, the oscillator output will again be modulated. The third harmonic term in the output will also be modulated in a similar manner.

If the output varies about the steady state by an amount δv_o the output will thus be;

$$v_o + \delta v_o = (E_1 + \delta E_1)\cos(\omega_o t + \phi) - (D_3 + \delta D_3)(E_1 + \delta E_1)\cos(3\omega_o t + \phi_2) \qquad \text{--(107)}$$

where δE_1 is again a function of time as defined in equation (73).

The amplifier input of figure (15) and hence the bridge output voltage, is given from equation (98) as;

$$v_{in} = \frac{v_o}{A} + \frac{b}{A}v_o^{\,3} \qquad \text{--(98)}$$

For the term involving the fundamental frequency ω_o in equation (107) this is approximately;

$$\left[v_{in} + \delta v_{in}\right]_{\omega=\omega_o} = \frac{E_1 + \delta E_1}{A}\left[1 + \frac{3b}{4}(E_1 + \delta E_1)^2\right]\cos(\omega_o t + \phi)$$

and hence

$$\delta V_{in}\Big|_{\omega=\omega_o} \simeq \frac{\delta E_1}{A}\left[1 + \frac{9b}{4} E_1^2\right] \qquad \text{--(108)}$$

where δV_{in} is the change in the envelope of δv_{in} defined as $\delta v_{in} = \delta V_{in} \cos(\omega_o t + \emptyset)$. Substituting the value of "b" from equation (106) in (108) yields;

$$\delta V_{in}\Big|_{\omega=\omega_o} \simeq \delta E_1 \frac{9D_3}{\sqrt{13}} \qquad \text{--(109)}$$

A second value for δV_{in} is seen from figure (15) to be

$$\delta V_{in} = \delta V_1 - \delta V_2 \qquad \text{--(110)}$$

The steady state transfer function $T(j\omega)$ for the frequency selective arm of the bridge which is needed to evaluate δV_1 is simply the first term of equation (101). Thus

$$T(j\omega) = \frac{\frac{1}{2}(1 + \delta_1)}{1 + \frac{(1-\delta_1)}{4}\left[\frac{j\omega}{\omega_o} + \frac{\omega_o}{j\omega}\right]} \qquad \text{--(111)}$$

Again for the complex frequencies $j\omega = (s \pm j\omega_o)$ associated with the oscillator output, this becomes

$$T(s + j\omega) + T(s - j\omega) = \frac{\frac{1}{2}(1 + \delta_1)}{1 + \frac{(1-\delta_1)}{4}\left[\frac{s + j\omega_o}{\omega_o} + \frac{\omega_o}{s + j\omega_o}\right]} + \frac{\frac{1}{2}(1 + \delta_1)}{1 + \frac{(1-\delta_1)}{4}\left[\frac{s - j\omega_o}{\omega_o} + \frac{\omega_o}{s - j\omega_o}\right]}$$

$$\simeq \frac{\frac{1}{2}(1+\delta_1)}{1 + \frac{(1-\delta_1)}{2}\frac{s}{\omega_o}} \qquad \text{--(112)}$$

where it has been assumed that $|s| \ll |\omega_o|$.

The transfer function $F_3(s)$ for changes in the envelope through the frequency selective arm is thus defined as,

$$F_3(s) = \frac{\delta V_1}{\delta E_1} = \frac{\frac{1}{2}(1 + \delta_1)}{1 + \frac{(1-\delta_1)}{2}\frac{s}{\omega_o}} \qquad \text{--(113)}$$

If we again define L(s) as the Laplace transform of δE_1, then $L_4(s)$, the Laplace transform of δV_1, is given by;

$$L_4(s) = L(s)\, F_3(s) \qquad \text{--(114)}$$

The output from the amplitude sensitive arm of the bridge is again defined by equation (88) and (85) as;

$$L_2(s) = \frac{1}{2}\left[L(s) + \frac{1}{2}E_1\left(\gamma L_x(s) + \frac{\epsilon}{s}\right)\right] \qquad \text{--(88)}$$

where;

$$L_x(s) = \frac{\omega_T m_o}{\gamma(s+\omega_T)}\left[\frac{2L(s)}{E_1} - \frac{\epsilon}{s}\right] \qquad \text{--(85)}$$

If we now define $L_5(s)$ as the Laplace Transform of δV_{in} we can write from equation (109) and (110);

$$L_5(s) = L(s)\frac{3}{2}\delta_1 = L_4(s) - L_2(s) \qquad \text{--(115)}$$

and we can solve equations (115),(85),(88),(114) and (113) for L(s) to yield,

$$L(s) = \left[\frac{\frac{E_1\epsilon}{2s}\left[1 - \frac{m_o\omega_T}{s+\omega_T}\right]}{\frac{1 + \delta_1}{1 + \frac{(1-\delta_1)s}{2\omega_o}} - 1 - 3\delta_1 - \frac{m_o\omega_T}{s+\omega_T}}\right] \qquad \text{--(116)}$$

Neglecting terms in δ_1^2 and higher this can be written as the ratio of two polynomials in s.

$$L(s) = \frac{-E_1\epsilon}{2s}\left[\frac{(1-3\delta_1)\{s^2+s[(1-m_o)\omega_T+2(1+\delta_1)\omega_o]+2(1+\delta_1)(1-m_o)\omega_o\omega_T\}}{s^2+s[(1+m_o(1-3\delta_1))\omega_T+4\omega_o\delta_1]+2\omega_o\omega_T\{m_o(1-2\delta_1)+2\delta_1\}}\right]$$

Since D_3 is small δ_1 is also small and will only have a significant influence on the coefficient of s in the denominator. Hence the above can be approximated as;

$$L(s) \simeq -\frac{E_1\epsilon}{2s}\left[\frac{s^2 + s\{(1-m_o)\omega_T + 2\omega_o\} + 2(1-m_o)\omega_o\omega_T}{s^2 + s\{(1+m_o)\omega_T + 4\omega_o\delta_1\} + 2m_o\omega_o\omega_T}\right] \qquad \text{--(117)}$$

where $\delta_1 \ll |\omega_o|$.

Equation (117) has three poles, $s=0, s=s_3$ and $s=s_4$ given by the roots of the denominator of equation (117). Thus;

$$s_3, s_4 = \alpha_1 \pm j\omega_{m1} \qquad \text{--(118)}$$

where α_1 and ω_{m1} are defined as;

$$\alpha_1 = -\frac{(1+m_o)\omega_T + 4\omega_o\delta_1}{2} \qquad \text{--(119)}$$

and

$$\omega_{m1}^2 = 2\omega_o\omega_T m_o - \left[\frac{(1+m_o)\omega_T + 4\omega_o\delta_1}{2}\right]^2 \qquad \text{--(120)}$$

For normal oscillator frequencies where $\omega_o \gg \omega_T$ this reduces to approximately the same value as for the linear amplifier.

$$\omega_{m1}^2 \simeq 2\omega_o\omega_T m_o = \omega_m^2 \qquad \text{--(121)}$$

We should note however, that for increased nonlinearity and hence increased δ_1, the second term of equation (120) would become more important and the modulation frequency would be decreased.

Since α_1 is negative for real values of the parameters m_o, ω_T, δ_1 and ω_o, the transient will decrease with time. Furthermore since $|\alpha_1|$ is increased by the term $2\omega_o\delta_1$, the time constant with which the transient decreases will be much shorter especially for the higher oscillator frequencies. The actual transient δE_1 can again be obtained from the inverse transform of equation (117) as;

$$\delta E_1 = -\frac{E_1\epsilon}{2}\frac{(1-m_o)}{m_o}h(t) - \frac{\epsilon E_1}{2\omega_{m1}} e^{\alpha_1 t}\left[(2\omega_o + \{\tfrac{1}{2} - m_o - \tfrac{1}{2m_o}\}\omega_T)\sin\omega_{m1}t + (2-\tfrac{1}{m_o})\omega_{m1}\cos\omega_{m1}t\right] \quad \text{--(122)}$$

which is identical to the result obtained for the linear amplifier except that α has been replaced with α_1 and ω_m has been replaced with ω_{m1}. The approximate transient for $\omega_o \gg \omega_T$ can again be written.

$$\delta E_1 \simeq -\frac{E_1\epsilon}{\omega_{m1}} e^{\alpha_1 t}\omega_o \sin\omega_{m1} t - \frac{E_1\epsilon}{2}\frac{(1-m_o)}{m_o}h(t) \quad \text{--(123)}$$

The time constant with which the transient dies out is the reciprocal of α_1 and is thus;

$$\tau_1 \simeq \frac{2}{(1+m_o)\omega_T + 4\omega_o\delta_1} \quad \text{--(124)}$$

The value of ΔE_1 which is the magnitude of δE_1 extrapolated back to t=0 will be unchanged from the case of the linear amplifier. However, since the time constant will be much shorter for the nonlinear amplifier, the associated first peak of the sinusoid will be much smaller than for the linear amplifier. In figure (16) the oscillator output response is shown for $\delta R = -\epsilon R_o h(t)$ which is the negative of the result obtained in equation (122) and was caused by closing the switch in figure (15) at time t=0.

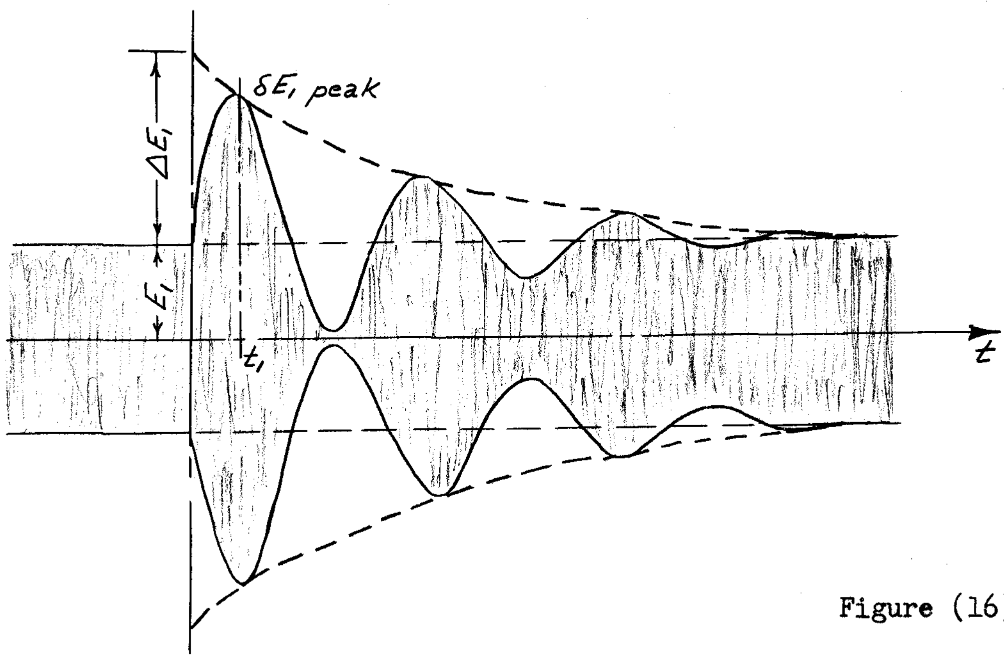


Figure (16)

where the variable part of δE_1 is given by;

$$\delta E_1 \simeq \frac{E_1 \epsilon}{\omega_{m1}} e^{\alpha_1 t} \omega_o \sin \omega_{m1} t \qquad \text{--(125)}$$

The variable part will thus reach a value δE_1 peak shown in figure(16) at some time $t = t_1$. For this condition we can solve for t_1 by differentiating equation (125) and setting the result equal zero to yield;

$$\frac{d}{dt}\left[\delta E_1\right] = 0 = \frac{E_1 \epsilon \omega_o}{\omega_{m1}}\left[\alpha_1 e^{\alpha_1 t_1} \sin \omega_{m1} t_1 + \omega_{m1} e^{\alpha_1 t_1} \cos \omega_{m1} t_1\right]$$

thus
$$t_1 = \frac{1}{\omega_{m1}} \tan^{-1}\left[\frac{\omega_{m1}}{-\alpha_1}\right] \qquad \text{--(126)}$$

The value for δE_1 peak can also be calculated for the value of t_1 as;

$$\delta E_1/_{peak} = \frac{E_1 \epsilon \omega_o}{\sqrt{2 m_o \omega_o \omega_T}} e^{\alpha_1 t_1} \qquad \text{--(127)}$$

where t_1 is given by equation (126).

Since the time constant will be larger for the linear case, the value of δE_1 peak will be greater than for the case of the nonlinear amplifier. In addition if the first peak is large, the nonlinearity will tend to limit the increasing amplitude peak. The most important result of the nonlinearity is that the time constant will decrease to an extremely low value at the higher oscillator frequencies as is shown by equation (124). For increased nonlinearity the frequency ω_{m1} would also become lower and the time t_1 given by equation (126) would increase while the time constant τ_1 decreased. The net result would also decrease the amplitude of the first peak.

The response of the system to sinsusoidal variations in the lamp resistance can be found in the same manner as for the linear amplifier. For a variation $\frac{\delta R}{R_0} = \epsilon_0 e^{j\omega t}$ we obtain;

$$\delta E_1 = -\frac{E_1 \epsilon_0 e^{j\omega t}}{2}\left[\frac{\omega^2 - j\omega\{(1-m_0)\omega_T + 2\omega_0\} - 2(1-m_0)\omega_0\omega_T}{\omega^2 - j\omega\{(1+m_0)\omega_T + 4\delta_1\omega_0\} - 2m_0\omega_0\omega_T}\right] \qquad \text{--(128)}$$

For a range of oscillator frequencies where $\omega_0 \gg \omega_T$ the enhancement will be a maximum at the frequency,

$$\omega^2 \simeq 2m_0\omega_0\omega_T$$

and the "enhancement factor" will thus be;

$$\left|\frac{\delta E_1/E_1}{\delta R/R_0}\right|_{\omega=\omega_m} \simeq \frac{\omega_0}{(1+m_0)\omega_T + 4\delta_1\omega_0} \qquad \text{--(129)}$$

The enhancement for the nonlinear amplifier case is thus improved by the factor $4\delta_1\omega_0$ in the denominator as can be seen by comparison of equations (129) and (97).

The analysis presented has been entirely for perturbations of the lamp resistance δR. The analysis can be extended to variations in any of the closed loop parameters through the use of the basic oscillator equation (16)

$$\frac{k}{k+2} - \frac{R_o}{R_o+R_2} = \frac{1}{A} \qquad \text{--(16)}$$

and the chain rule. For example if the transfer function for changes in output for relative changes in the lamp resistance is $F_2(s)$, then an expression for the output voltage envelope corresponding to a variation in the amplifier gain $\delta A/A$ would be

$$\delta E_1 \simeq \frac{\delta A}{A}(s)\frac{\partial R_o}{\partial A}\,\frac{A}{R_o}\,F_2(s)$$

$$= \frac{\delta A}{A}(s).\frac{4}{A}F_2(s)$$

The enhancement of variations of amplifier gain would thus be reduced by the factor $4/A$ which is in agreement with the general concept of the use of the bridge circuit. Since the value of δ_1 decreases proportional to A the enhancement of variations of amplifier gain should be independent of A for high frequencies and the condition that $A \gg 1$.

VI THE EXPERIMENTAL CIRCUIT

(1) The Basic Oscillator

In order to evaluate the theory presented in the preceeding sections, an oscillator was constructed from which the transients could be measured. To keep the transients as large as possible, the amplifier used was constructed as linear as was possible. The basic oscillator described in the theory can be drawn in a block diagram form such as that of figure (17)

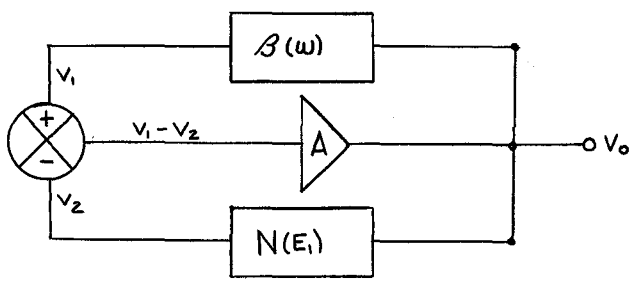


Figure (17)

The oscillator consists of the lamp Bridge Arm $N(E_1)$, the Frequency Selective Arm $\beta(\omega)$, and the Amplifier A which contains a small amount of third harmonic distortion. The summing amplifier shown can be a part of the amplifier A and has unity gain.

(2) The Frequency Selective Network

The frequency selective network shown in figure (18) was discussed in sections III and V.

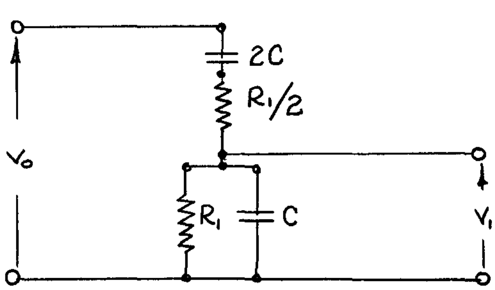


Figure (18)

The values of the components R_1 and C are to a large extent governed by the input and output impedances of the amplifier. The value of R_1 for the low frequency end of the tuning range must be much smaller than the input impedance of the amplifier. Similarily, at the high end of the tuning range, the value of C must be much larger than the input capacitance of the amplifier. The total impedance of the frequency selective network must also be high enough not to severely load the amplifier. In particular the network impedance should be much larger than the amplifier output impedance for the analysis given. These effects can be reduced by using cathode follower inputs and output for the amplifier, but should not be ignored in the design.

(3) The Thermal Bridge Arm

The thermal bridge arm contained two tungsten lamps as shown in figure (19).

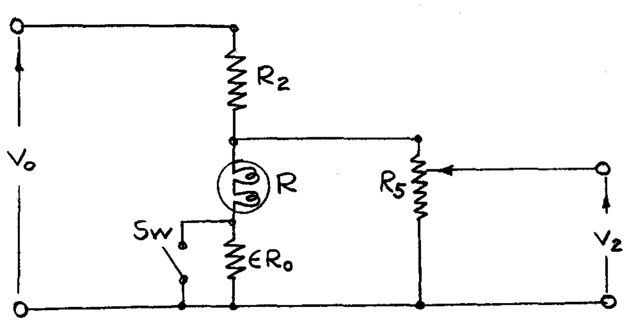


Figure (19)

By the inclusion of R_5, the resistor R_2 can be adjusted exactly to $R_2 = R_o$ and still maintain the slight bridge misbalance necessary for oscillation. The value of R_5 should however, be much larger than R_2 or R_o. The two lamps used for the variable resistance R should have sufficiently high resistance not to load the amplifier output and yet still have a characteristic which would enable them to be used above their transition region for low oscillator output voltages. The switch Sw and resistor ϵR_o were included as a source of the perturbation $\delta R = -\epsilon R_o h(t)$.

(4) The Linear Amplifier

The high gain amplifier must consist of at least two stages in order to achieve the zero net phase shift required by the analysis. As mentioned earlier the output impedance of the amplifier must be very low in order that the Wien Bridge circuit does not overload the amplifier. This can be easily accomplished by using a heavy duty cathode follower for the output stage. The high gain and zero net phase shift can be obtained from any simple two stage preamplifier. The frequency response of the entire amplifier should be flat from very low frequencies to frequencies well above those used in the tests. The net phase shift of the amplifier should be kept close to zero degrees although a small phase shift will not greatly alter the oscillation frequency. In order to minimize the harmonic distortion and decrease the amplifier output impedance, the amplifier employed approximately 20db of overall negative voltage feedback, and operation was confined to a region of about one quarter of its designed maximum output level.

(5) The Differential Amplifier

The summing amplifier used should be as free from common mode effect as possible. Several different types of differential amplifier circuits are discussed in the literature (17),(18). The circuit selected, and designed consisted of a double triode T_1 with a common cathode consisting of a triode T_2 and its cathode resistor R_K.

The combination of triode T_2 and resistor R_K provided an effective A.C. common cathode resistance of $r_{p2} + (1+\mu_2)R_K$ for the double triode T_1. Here r_{p2} is the plate resistance of T_2 and μ_2 is mhu of T_2. This provided a minumum common mode voltage and a maximum input impedance for the differential amplifier. A double triode should be used for the tube T_1 to minumize differences in characteristics between the two inputs. This amplifier was constructed as a part of the total high gain amplifier A.

(6) The Envelope Detector

Although this piece of equipment was not a part of the actual oscillator, it was used wherever the time constant permitted to detect the modulation frequency of the transient and amplify the envelope to a level suitable to drive a Beckman Counter. The block diagram of the detector is shown in figure (20).

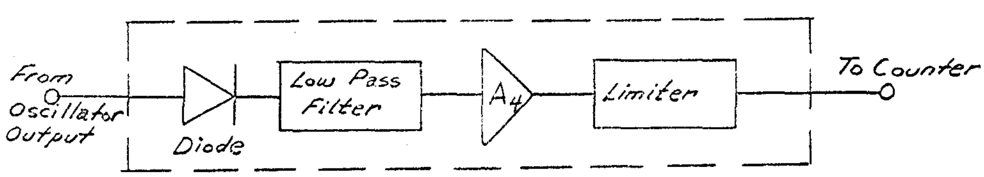


Figure (20)

The circuit used consisted of an indium bonded diode type 1N100 followed by a low pass non-tapered R-C filter which comprised the actual detector. The detected envelope was then amplified by the amplifier A_4 and clipped at approximately two volts to supply the desired amplitude for the counter. The low pass filter was a non-tapered four section R-C filter designed to eliminate any carrier frequency above 190 cps. The limiter consisted of a pair of reverse biased diodes which were adjusted to clip at two volts peak to peak.

VII EXPERIMENTAL OUTLINE

(1) General Outline

The experimental portion of the thesis was carried out to evaluate the various parameters and to verify the theoretical results obtained in the analysis. The oscillator circuit was designed and constructed to meet the requirements indicated in the previous chapter. The transient waveform of the oscillator was then measured for a perturbation of the lamp resistance at various oscillator frequencies and amplitudes.

The static characteristics of the lamp bridge used in the oscillator were measured and the experimental values were compared with those predicted by the theory. The time constant due to the thermal inertia of the lamp was measured and the result was used in calculating the expected transient response for the oscillator.

A study of the low frequency response of the lamp bridge was also made. The results obtained were used to check the result of equation (60) with regard to the unwanted output at the bridge balance point, and the variation of the unwanted output with bridge input voltage and frequency.

(2) Measurement of Static Lamp Characteristics

The static current-voltage characteristic was measured for the two Westinghouse ten watt 250 volt lamps connected in series. The voltage supply used was a John Fluke Stabilized Power Supply model 407 and the voltage was measured with a John Fluke Digital Voltmeter. A standard 0.5% laboratory d.c. milliameter was used to measure the current through the lamps. The power supplied to the lamps and hence radiated in the steady state and the resistance of the lamps were then calculated for the various lamp voltages. These results are tabulated in table (1) and are shown plotted in graphs number (1),(2), and (3).

The static current-voltage relationship is shown on graph number one for the two lamps connected in series. The characteristic is seen to be composed of a linear region, a transition region, and an almost straight line region. A graphical analysis of this characteristic in the region of oscillator operation yields an approximate curve given by,

$$E \simeq 0.884\ I^{1.64}$$

where E and I are the d.c. voltage and current supplied to the lamp in volts d.c. and m.a.d.c.

Graph number two is a plot of the lamp resistance R as a function of the lamp voltage E which can be approximated by the straight line relationship

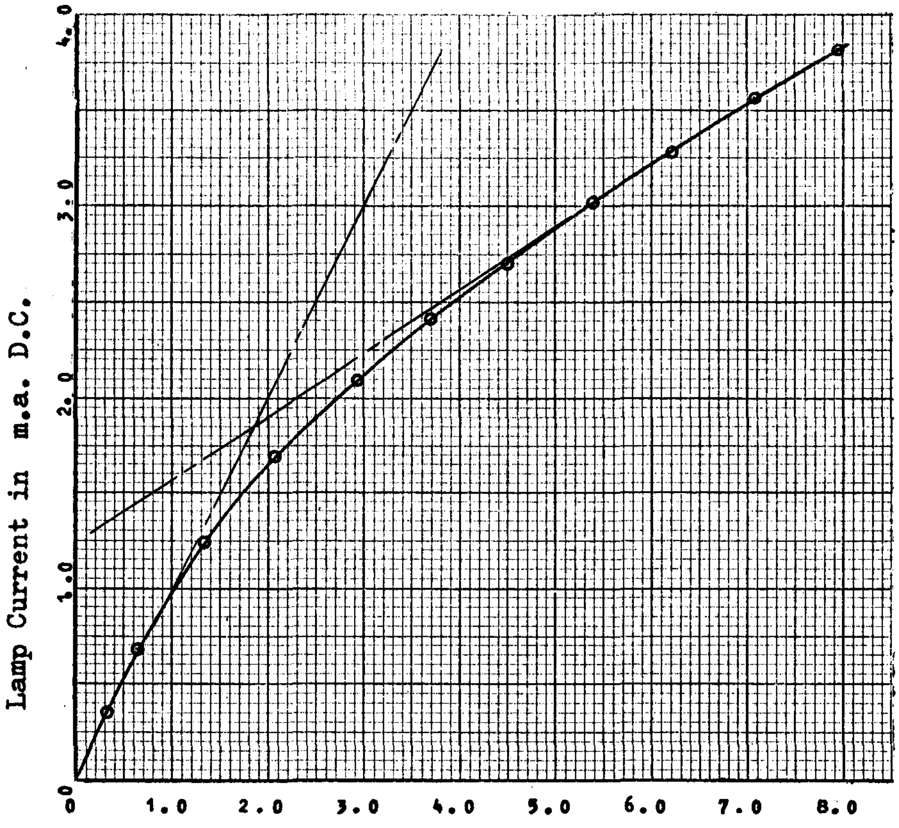
$$R \simeq 900 + 170E$$

in the region of oscillator operation.

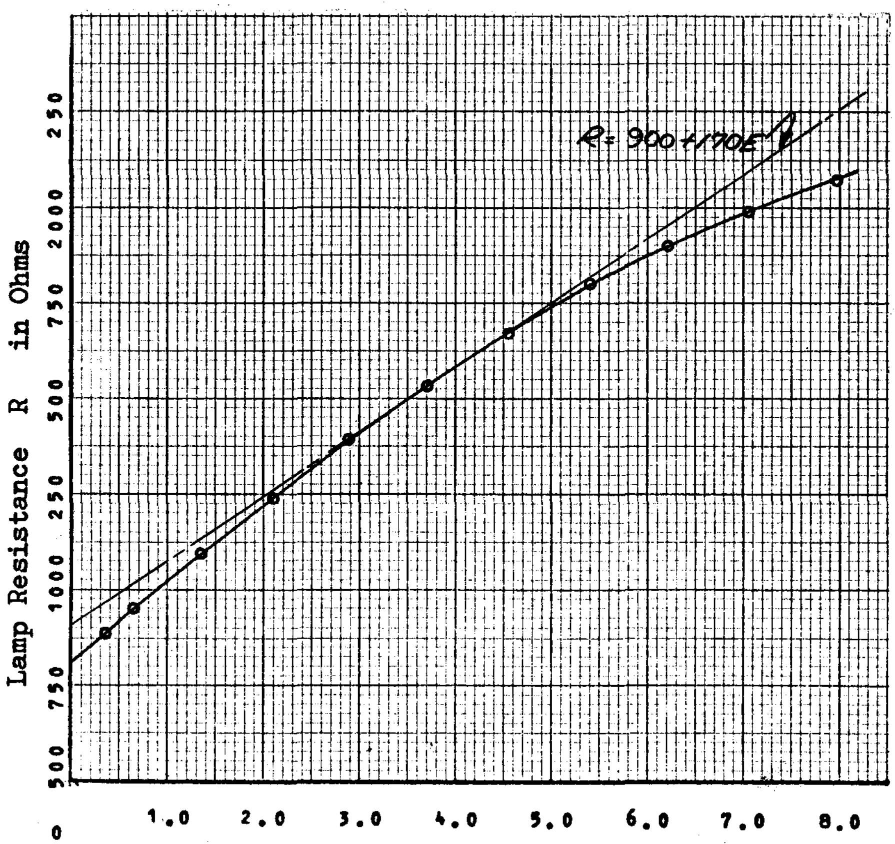
The third graph, graph (3), shows the relationship between the lamp resistance R and power supplied or radiated in the steady state. As mentioned in the theory we would expect a straight line relationship between log R and log W excepting the low power inputs where the supplied power has little effect on the lamp temperature. Graph (3) shows this straight line region quite clearly with a slope

$$m_o = 0.226$$

We also see from graph (3) that there is a linear region for very low power inputs which corresponds to the region in which the lamp temperature is completely governed by the ambient temperature. The



GRAPH 1



GRAPH 2

GRAPH 3

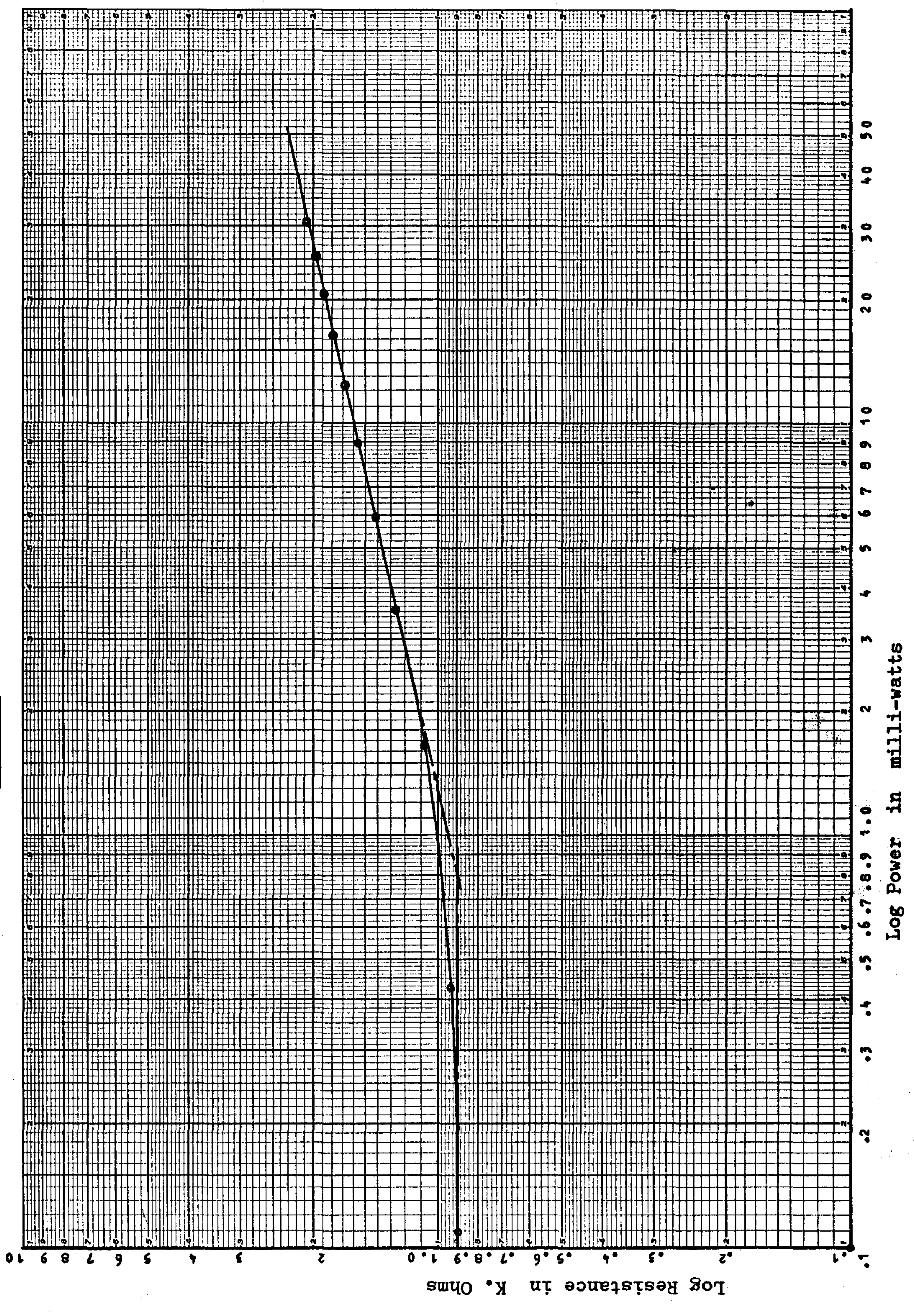

Log Power in milli-watts
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 2 3 4 5 6 7 8 9 10 20 30 40 50
Log Resistance in K. Ohms
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 2 3 4 5 6 7 8 9 10

TABLE (1)

Lamp Voltage in Volts d.c.	Lamp Current in m.a. d.c.	Lamp Resistance in Ohms	Power Supplied to Lamps in m. Watts
0.3110	0.3504	884.8	0.1086
0.6330	0.6768	935.3	0.4284
1.331	1.233	1079	1.642
2.088	1.682	1242	3.512
2.883	2.060	1400	5.939
3.699	2.399	1542	8.874
4.531	2.709	1673	12.27
5.372	3.002	1789	16.13
6.215	3.292	1888	20.46
7.069	3.561	1985	25.17
7.927	3.823	2074	30.30

curved region which follows is simply the transition region in which the effect of the room temperature plays a decreasing role with increasing power in establishing the filament temperature. Operation below a lamp voltage of two volts would be inadvisable due to these ambient temperature effects on the lamp performance.

It should be noted that since the lamps used were designed and manufactured for illumination purposes, variations between the static characteristics of two identical lamps often exceeds 10%. The characteristics shown apply to the two lamps used in the oscillator and should be accurate within one half of one percent, but will only be typical values for lamps of their general rating and species.

(3) Measurement of Thermal Time Constant

The thermal time constant of the lamps can be measured by connecting the lamp bridge to a constant voltage supply and subjecting the lamp bridge to a small change in operating voltage. The circuit used is shown in figure (21) and enabled the measurement of the variable term on the oscilloscope.

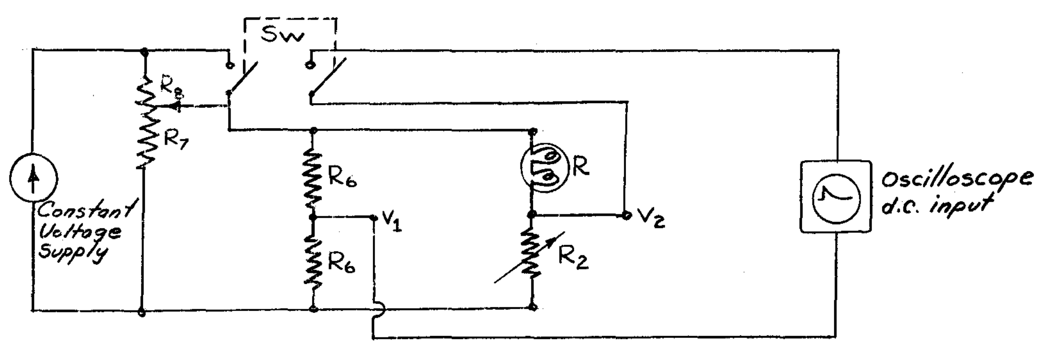


Figure (21)

At normal oscillator frequencies the lamp will behave in the same manner as it would to a direct current voltage. We can thus replace the alternating constant voltage supply for the regulated d.c.

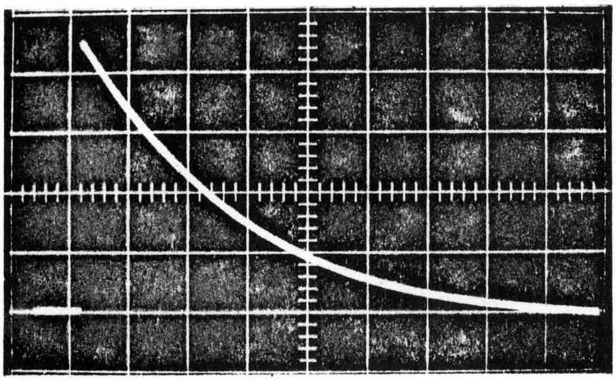
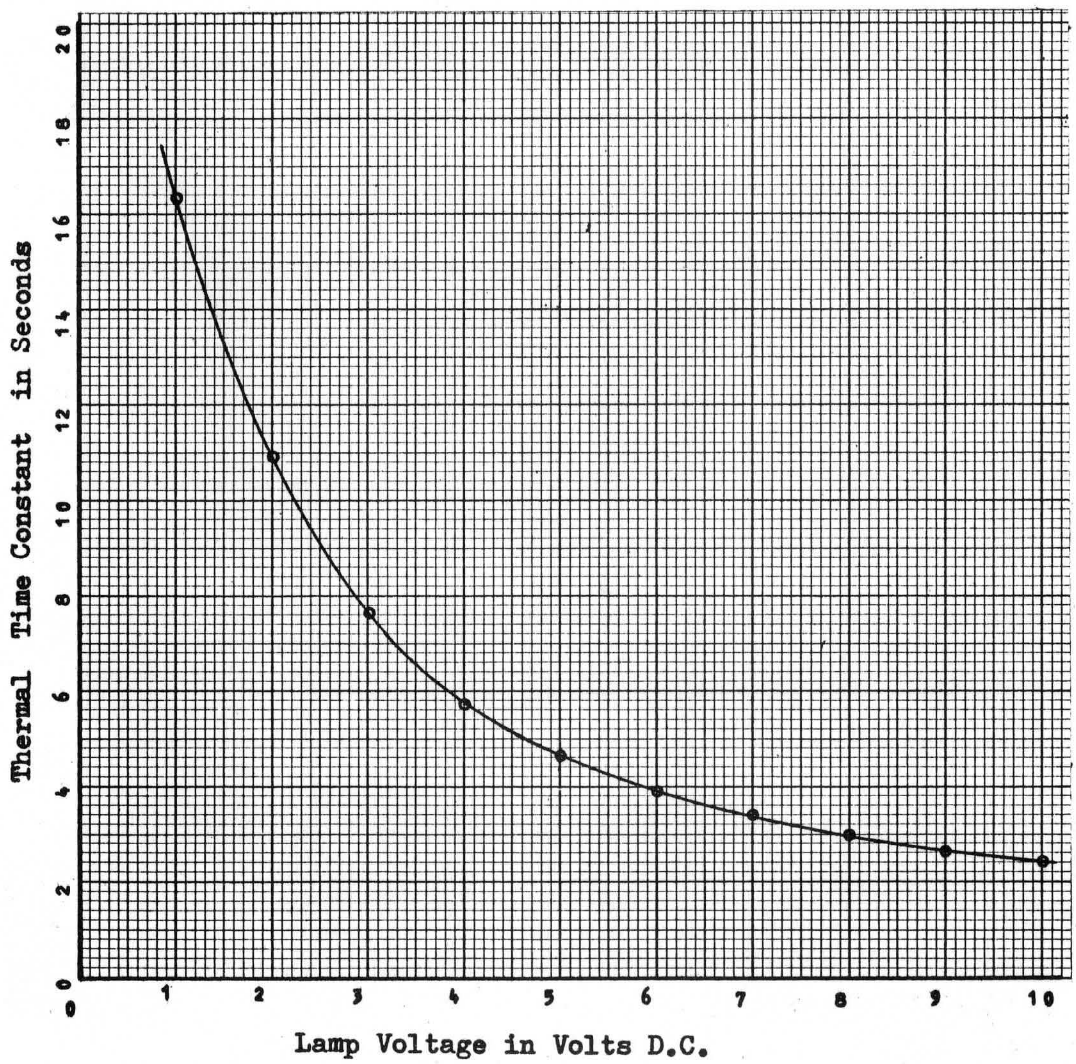


FIGURE 22
Horiz. Scale 1.0 Sec./ Div.
Lamp Voltage 10 Volts D.C.



GRAPH 4

power supply shown in figure (21). Resistors R_7 and R_8 are part of a low resistance potentiometer included to facilitate the change in voltage and to improve the regulation of the power supply. Resistor R_8 was adjusted for each bridge voltage to yield approximately a ten percent change in input voltage when switch s_w was closed. Resistors R_6 were a matched pair of resistors much higher than the mean lamp resistance and yet much smaller than the oscilloscope input impedence. A value of R_6 = 10K was used. The bridge was initially balanced with switch s_w closed which brings the oscilloscope into the circuit along with the increased voltage. For this condition the oscilloscope trace was along the zero voltage axis. The switch s_w was then opened and after the lamp had reached its new steady state, switch s_w was again closed and the transient noted.

If we let the input voltage to the bridge vary due to a step change δV_o to $V_o(1+\varepsilon)$ at time t=0 the resistance R will vary as $R_o(1+y)$ as the filament temperature varies as $\theta_o(1+x)$. It can be shown that the bridge output voltage $v_{out} = (v_1 - v_2)$ is given by;

$$v_{out} \simeq V_o(1+\varepsilon h(t))\left[\frac{1}{2} - \frac{R_o}{R_o + R_o(1+y)}\right]$$

where $y \simeq 2\varepsilon m_o\left[1 - e^{-\omega_T t}\right]h(t)$

which reduces to the form

$$v_{out} \simeq \frac{V_o\varepsilon m_o}{2}\left[1 - e^{-\omega_T t}\right](1+\varepsilon)h(t)$$

which is a constant term plus a variable term. When the variable term is separated and measured as in figure (21) we can find the thermal time constant for the lamp bridge $\tau = \frac{1}{\omega_T}$ by noting the amplitudes of the trace at two instants in time. If the amplitude of the trace is V_1 at time t_1 and V_2 at time t_2 then the time constant would be;

$$\tau = \frac{1}{\omega_T} = \frac{(t_1 - t_2)}{\ln \frac{V_2}{V_1}} \qquad \text{--(130)}$$

The time constant can thus be evaluated by obtaining the difference in time between where the trace crosses two successive or particular graticule lines on the oscilloscope face. The values should be measured close to the steady state where the lamp voltage is close to half the bridge input voltage.

The values for the time constant were found for ten lamp voltages given in table (2). These values are the average values of a successive number of trials. A d.c. amplifier not shown in figure (21) was constructed and used in conjunction with the oscilloscope to provide additional gain especially for low lamp voltages. The oscilloscope was used on d.c. input because of the time constant associated with the coupling capacitors of the oscilloscope on A.C. input.

The time constant due to the thermal inertia of the lamp is plotted as a function of lamp voltage E on graph number four. Figure (22) shows a typical oscillogram taken from the oscilloscope of figure (21) for a lamp voltage at balance of E = 10 volts D.C. The values are used later in predicting the expected transient for the oscillator.

(4) Low Frequency Lamp Response

The circuit of figure (23) was constructed using the lamps employed in the oscillator limiter and the bridge output was measured

TABLE (2)

Lamp Voltage in Volts dc.	Lamp Resistance in Ohms	Measured Time Constant In Seconds
1.00	1024	16.3
2.00	1223	10.9
3.00	1400	7.60
4.00	1575	5.70
5.00	1745	4.60
6.00	1873	3.90
7.00	2076	3.40
8.00	2070	3.00
9.00	2135	2.60
10.00	2200	2.40

at balance for low frequencies using a Muirhead Wave Analyser type D 729-B.

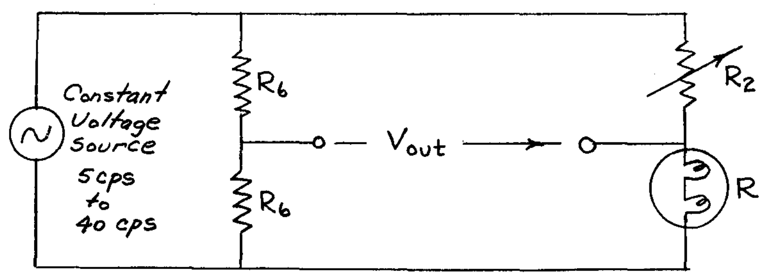


Figure (23)

The resistors R_6 were much larger than R and were a matched pair. Resistor R_2 was adjusted for each bridge input voltage to make the bridge output a minumum. The constant voltage source consisted of a Hewlett Packard model 200-CD Wide Range Oscillator operating at a low output and feeding a high power low output impedence amplifier. The output from the bridge was also fed through an amplifier to an oscilloscope. An oscillogram figure (24) was taken from the oscilloscope for a bridge driving frequency of five cycles per second. The output is seen from figure (24) to be composed of two frequency components of equal amplitude and opposite phase. The first is at the fundamental frequency of the bridge input voltage and the second is at the third harmonic; both are shifted by ninety degrees from the bridge input voltage.

A plot of one of these two frequency components in shown in graph (5) and the results are tabulated for the two in table (3). The input voltage was measured with a thermal milliameter and series resistor which had been calibrated to read in volts R.M.S.

The theoretical value for the bridge output voltage can be found from section IV-(3) equation (60) if we note that $\gamma = m_o G_T / P_o$ and $P_o = E^2/4R_o$ where $\omega_T = G_T/C_T$. The bridge output would thus be

$$v_{out} = \frac{E m_o \omega_T}{8\sqrt{2}\,\omega_o}\left[\cos 3\omega_o t - \cos\omega_o t\right]$$

The plots of graph (5) vary quite closely as the reciprocal of the driving frequency ω_o as is indicated by the theory. As a further check the equation can be evaluated at some bridge voltage say E=10 volts RMS, $m_o = 0.226$ and ω_T can be evaluated from table (2) at a lamp voltage of 5 volts RMS as $\omega_T = (1/4.6)\ \text{sec}^{-1}$. For these values the output is;

TABLE (3)

Driving Voltage Frequency in c.p.s. f_o	Driving Voltage Across Bridge V. R.M.S.	Amplitude of Fundamental Frequencies at Balance mv. R.M.S.	Amplitude of 3rd Harmonic at Balance mv. R.M.S.
5.0	10.0	8.90×10^{-1}	8.90×10^{-1}
10.0		4.24	4.25
15.0		2.76	2.76
20.0		2.10	2.10
30.0		1.60	1.60
40.0		1.10	1.10
5.0	8.00	5.60×10^{-1}	5.60×10^{-1}
10.0		2.70	2.70
15.0		1.75	1.76
20.0		1.25	1.27
30.0		1.00	1.00
5.0	6.00	3.25×10^{-1}	3.20×10^{-1}
10.0		1.52	1.52
15.0		0.90	0.90
20.0		0.70	0.70
30.0		0.60	0.60
5.0	5.00	2.30×10^{-1}	2.25×10^{-1}
10.0		1.00	1.00
15.0		0.64	0.65
20.0		0.45	0.45
30.0		0.37	0.37

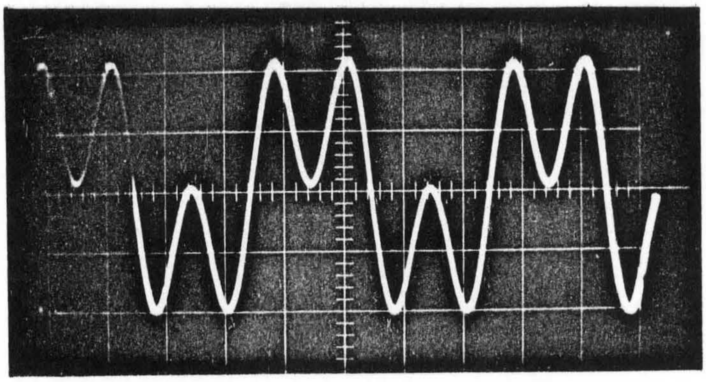
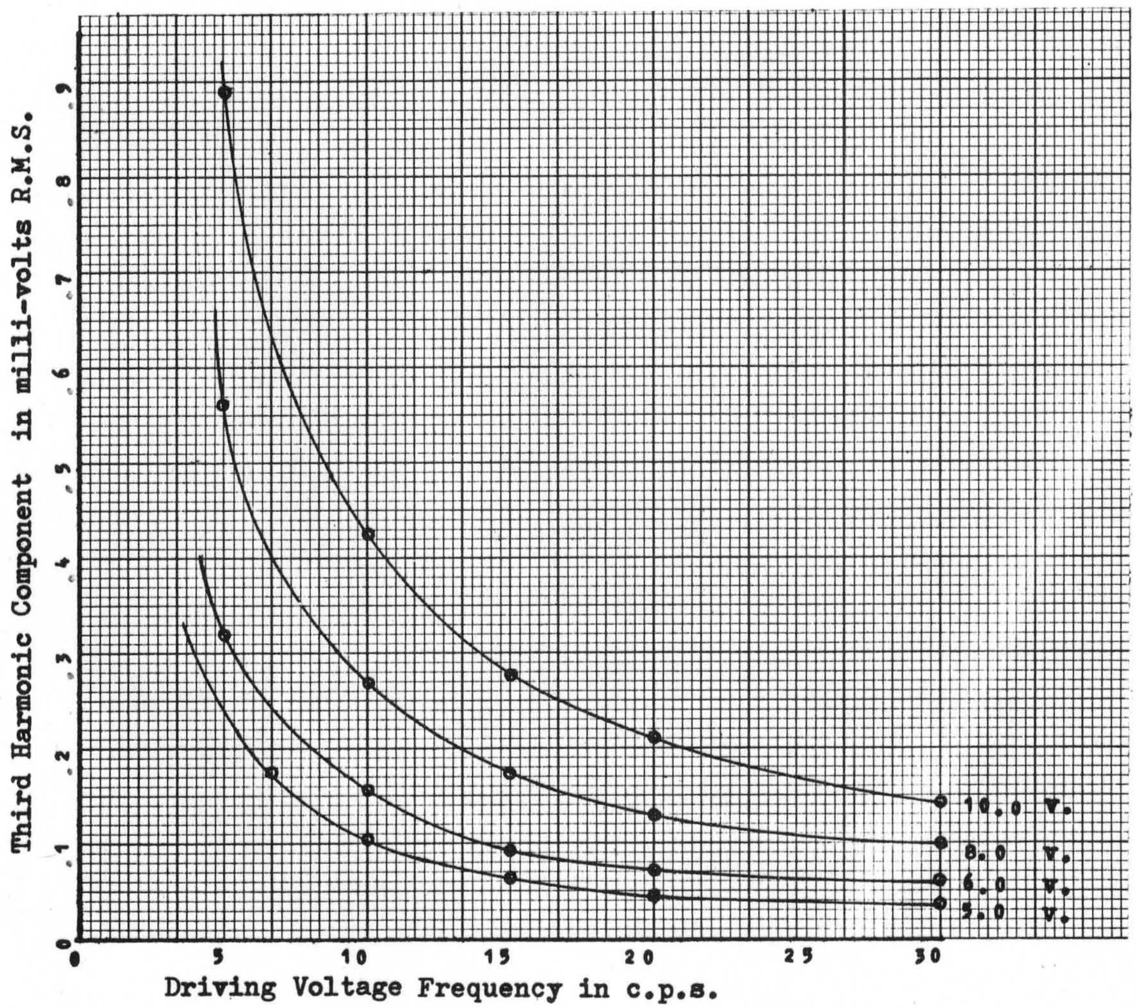


FIGURE 24

Unwanted Bridge Output at a frequency of 5 c.p.s.



GRAPH 5

v_{out} at 5 cps = 0.977 m. volts R.M.S.

v_{out} at 30 cps = 0.163 m. volts R.M.S.

which agrees quite closely to 0.89 m. volts and 0.16 m. volts measured and recorded in table (3).

The magnitude of the unwanted component should thus not be objectionable for oscillator frequencies of five cycles or higher for even extremely high gain amplifiers used in the Wien Bridge Oscillator circuit. Below five cycles per second the unwanted components increase quite rapidly and hence sets a lower limit on the usefulness of the thermal compensator as a control unit.

(5) <u>Transient Oscillator Response</u>

The oscillator circuit was constructed with the value of k in figure (14) chosen as k=2. This choice of k=2 allows the Wien Bridge circuit to operate at the condition of maximum frequency sensitivity. The capacitors 2C and C were of a fixed value mica type and were matched for the two to one ratio. These remained constant during all of the tests.

The resistors R_1 and $R_1/2$ were of the deposited carbon type and were mounted in shielded plug in units to allow changes in oscillator frequency. The values of all the components were adjusted to form balanced pairs. To allow a continual frequency variation, a dual potentiometer was mounted in a shielded plug in unit. Although the tracking was not particularly good on this device it was suitable to obtain the frequencies where hum enhancement occurred. The values of the circuit frequency determining elements are listed with the expected oscillator frequencies.

$C = 24140$ p.f.

Resistance Unit	Expected Frequency
$R_1^{\,1} = 28420$ ohms	232.0 c.p.s.
$R_1^{\,2} = 19120$ ohms	344.8 c.p.s.
$R_1^{\,3} = 15520$ ohms	424.8 c.p.s.
$R_1^{\,4} = 14180$ ohms	464.9 c.p.s.
$R_1^{\,5} = 10870$ ohms	606.5 c.p.s.
$R_1^{\,6} = 9447$ ohms	697.9 c.p.s.
$R_1^{\,7} = 7613$ ohms	866.0 c.p.s.

The dual potentiometer had a value of 0 - 50K, 0 - 25K ohms and would thus tune the above range of frequencies.

The transient was introduced by shorting the small resistance ϵR_o in series with the two lamps at a time $t = 0$. The result obtained was a damped oscillatory transient as shown in figure (25). These transients were produced and measured for various oscillator frequencies and oscillator output levels.

The observed transient waveform can be easily correlated with the theory with the aid of figure (26).

Here the various terms are defined by

E_1 = oscillator steady state peak amplitude in peak volts.

ΔE_1 = magnitude of transient extrapolated back to $t = 0$ in peak volts.

$\delta E_{1\,peak}$ = magnitude of first peak of transient in peak volts.

f_m = modulation frequency in c.p.s. = $\dfrac{1}{t_2 - t_1}$

τ_1 = time constant of transient wave form in sec.

From equation (93) these terms can be approximated for the linear theory as

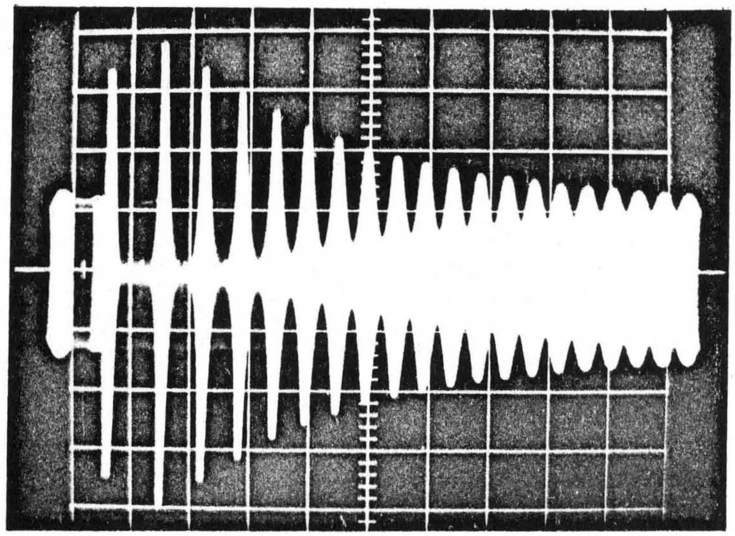


FIGURE 25

Oscillator Transient caused by a typical switching disturbance

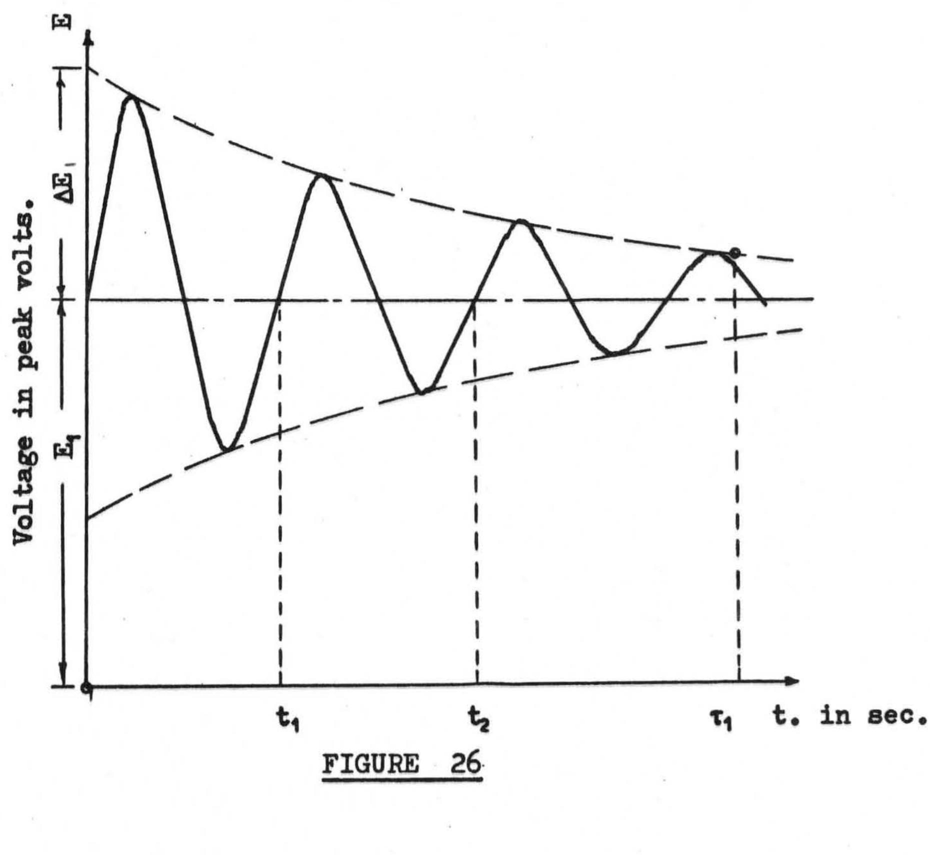


FIGURE 26

$$\delta E_1/_{peak} \simeq \Delta E_1 = \frac{E_1 \epsilon \omega_o}{\omega_m} \qquad \text{--(131)}$$

$$fm \simeq \frac{1}{2\pi}\sqrt{2m_o \omega_o \omega_T} \qquad \text{--(132)}$$

and

$$\tau_1 \simeq \frac{2}{(1+m_o)\omega_T} \qquad \text{--(133)}$$

Similarily where the nonlinearity of the amplifier is accounted for, the terms become,

$$\delta E_1/_{peak} \simeq \frac{E_1 \epsilon \omega_o}{\omega_m} e^{-\frac{1}{\tau_1 \omega_m}\tan^{-1}(\tau_1 \omega_m)} \qquad \text{--(134)}$$

$$f_m \simeq \frac{1}{2\pi}\sqrt{2m_o \omega_o \omega_T} \qquad \text{--(135)}$$

$$\tau_1 \simeq \frac{2}{(1+m_o)\omega_T + 4\omega_o \delta_1} \qquad \text{--(136)}$$

The presence of even a minute amount of cubic nonlinearity should have a profound effect on the transient time constant τ_1 as is seen in equation number (136) while the modulation frequency of the oscillator will remain practically unchanged for small nonlinearities such that D<.01 ie (1%).

The experimental values for the peak amplitude $\delta E_1/_{peak}$, the time constant τ_1, and the modulation frequency f_m were measured from the transient waveform displayed on a Tecktronic Oscilloscope model 536. The oscillator frequency and the modulation frequency were measured on a Beckman Eput and Timer model 7370 wherever the transient time constant would permit its use. The oscillation level was estabilished by the use of a thermal voltmeter which was constructed from a thermal milliameter and calibrated to read R.M.S. voltages. The results which are shown tabulated in table (4) are for a step function of resistance $\delta R = -2.018\ h(t)$ which was introduced by shorting the 2.018 ohm resistor with a switch at time $t = 0$. The value of R_2 which was a carbon potentiometer was adjusted for each oscillator voltage on a resistance bridge so that $R_2 = R_o$. The potentiometer R_5 of figure (19) was then adjusted to provide the small misbalance needed to obtain the exact oscillator output voltage. The time constant was evaluated from the time base of the oscilloscope which was checked at low sweep rates with a stop watch and higher sweep rates with the counter and a known frequency signal. The measured values for these readings are shown in graphs numbers (6), (7) and (8). In graph (6) the peak amplitude $\delta E_1/_{peak}$ is plotted as a function of the square root of the oscillator frequency. In graph (7) the modulation frequency is shown as a function of the square root of the oscillator frequency, and in graph (8) the transient time constant is shown as a function of the oscillator frequency.

The theoretical results for the oscillator with a linear high gain amplifier were next calculated from equations (131), (132), and (133) and are listed in table (5). The values for f_o are taken from the expected value of f_o for the constants used in each of the plug in frequency determining units. The values of ϵ and ω_T are for a resistance change of 2.018Ω and the time constant given on graph (4). The values for the steady state lamp resistances are given on graph (2) or table (2). The theoretical curves are

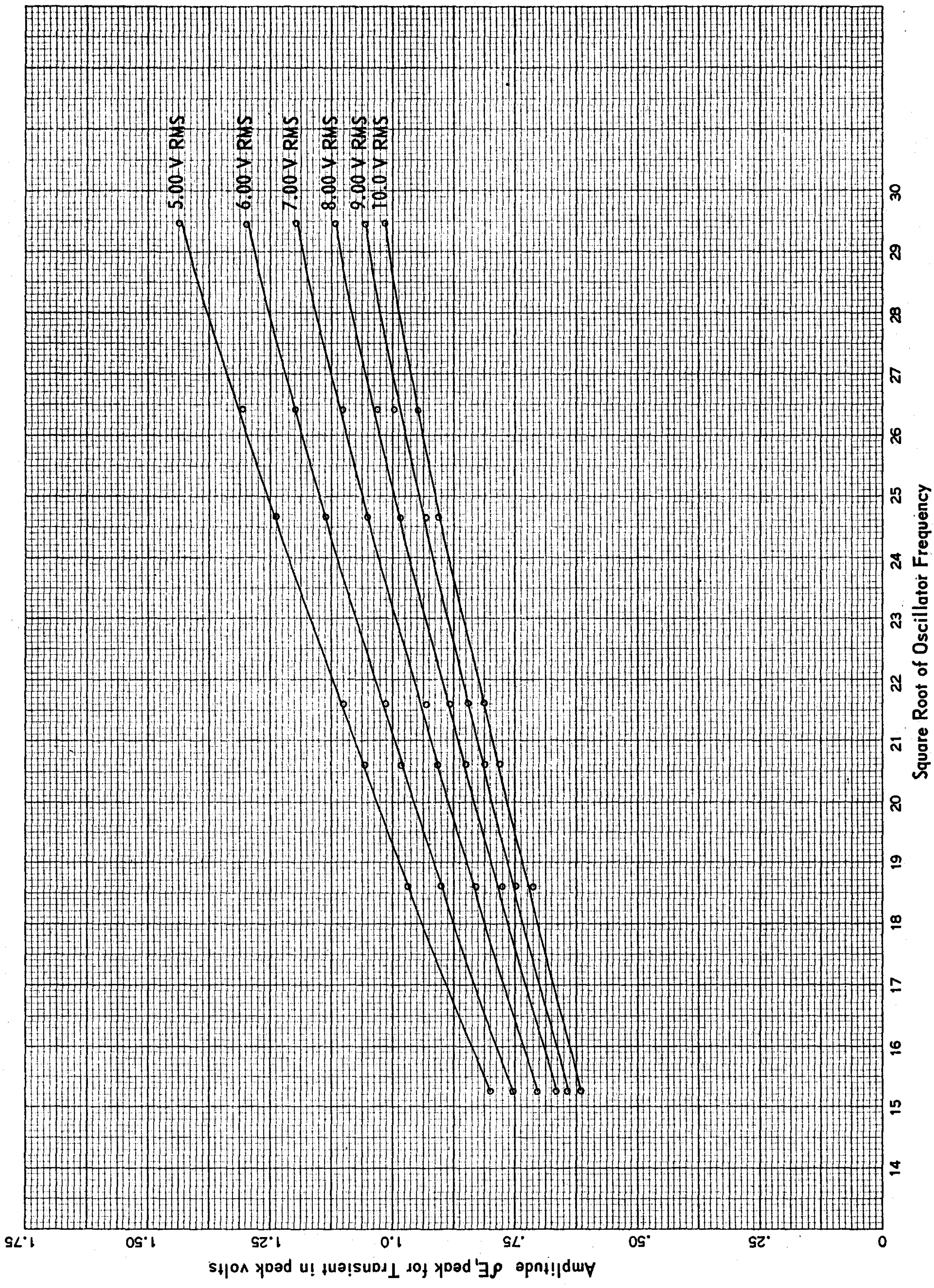
TABLE (4) EXPERIMENTAL RESULTS

E_1 v RMS	f_o in cps	δE_1/peak peak volts	f_m in cps	τ_1 in sec	ϵ X10^{-3}	dR in ohms	R_o in KΩ
5.00	237	0.83	1.32	5.00	1.570	2.018	1.285
	351	0.97	1.62	3.75			
	432	1.06	1.79	3.20			
	469	1.10	1.87	3.00			
	613	1.24	2.15	2.40			
	704	1.31	2.30	2.20			
	870	1.44	2.56	1.75			
6.00	237	0.76	1.45	3.75	1.435	2.018	1.405
	351	0.89	1.75	2.80			
	432	0.98	1.96	2.35			
	469	1.01	2.05	2.20			
	613	1.14	2.35	1.80			
	703	1.20	2.50	1.60			
	870	1.30	2.80	1.35			
7.00	237	0.71	1.54	2.80	1.344	2.018	1.500
	351	0.83	1.87	2.08			
	432	0.90	2.10	1.75			
	469	0.93	2.19	1.65			
	613	1.05	2.53	1.31			
	703	1.10	2.73	1.13			
	870	1.20	3.02	0.95			

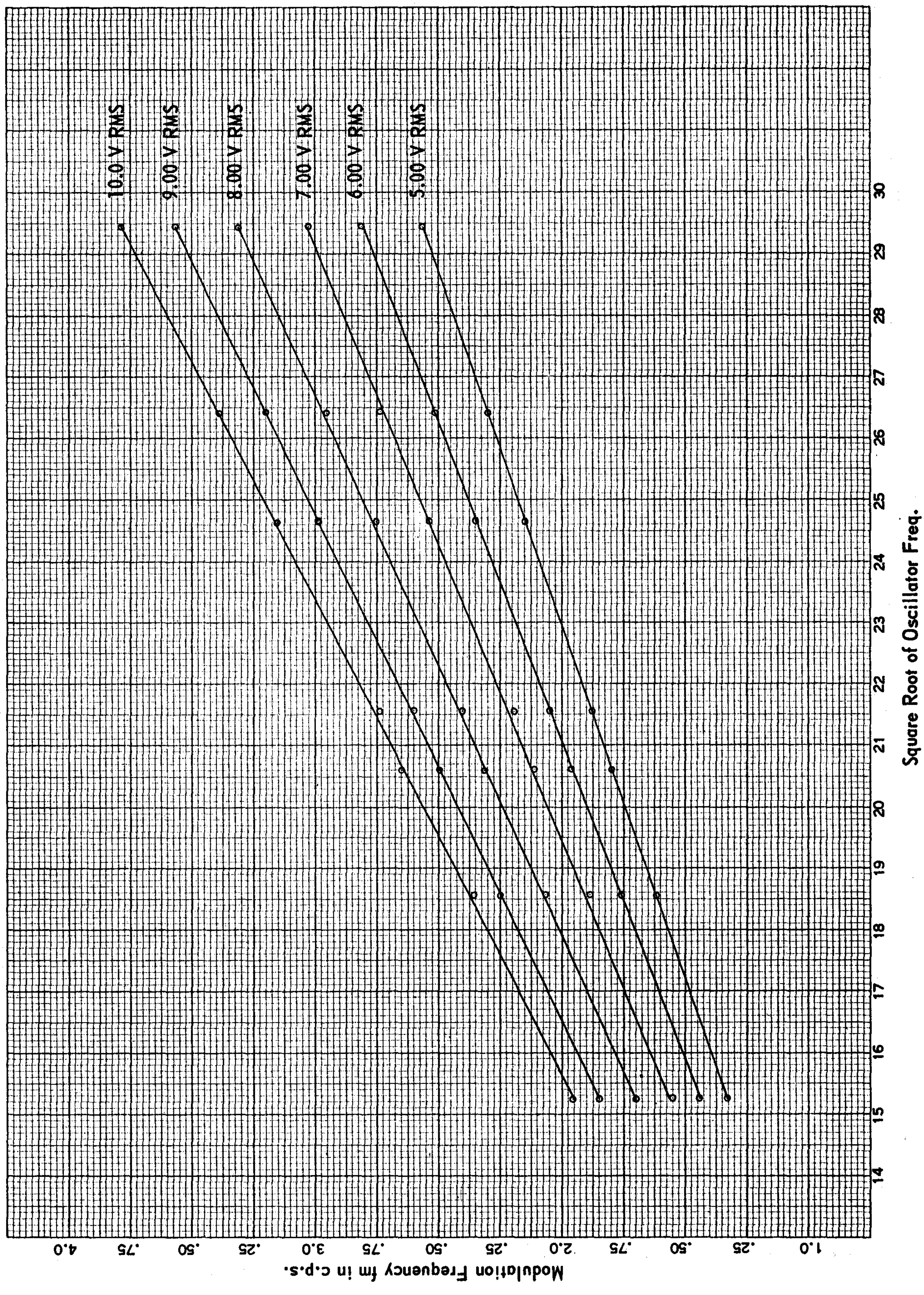
TABLE (4) EXPERIMENTAL RESULTS (CONTINUED)

E_1 v. RMS	f_o in cps	δE_1/peak peak volts	f_m in cps	τ_1 in sec	ϵ X10^{-3}	dR in ohms	R_o in KΩ
8.00	237	0.67	1.70	2.22	1.276	2.018	1.580
	351	0.77	2.07	1.60			
	432	0.85	2.32	1.30			
	469	0.88	2.40	1.20			
	613	0.98	2.75	0.90			
	703	1.03	2.95	0.80			
	870	1.12	3.30	0.65			
9.00	237	0.64	1.85	1.75	1.210	2.018	1.665
	351	0.74	2.25	1.25			
	432	0.81	2.50	1.05			
	469	0.84	2.60	1.00			
	613	0.93	3.00	0.75			
	703	1.00	3.20	0.63			
	870	1.06	3.56	0.50			
10.00	237	0.620	1.96	1.40	1.160	2.018	1.738
	351	0.71	2.35	1.00			
	432	0.79	2.65	0.85			
	469	0.81	2.73	0.80			
	613	0.91	3.15	0.60			
	703	0.95	3.39	0.50			
	870	1.02	3.78	0.37			

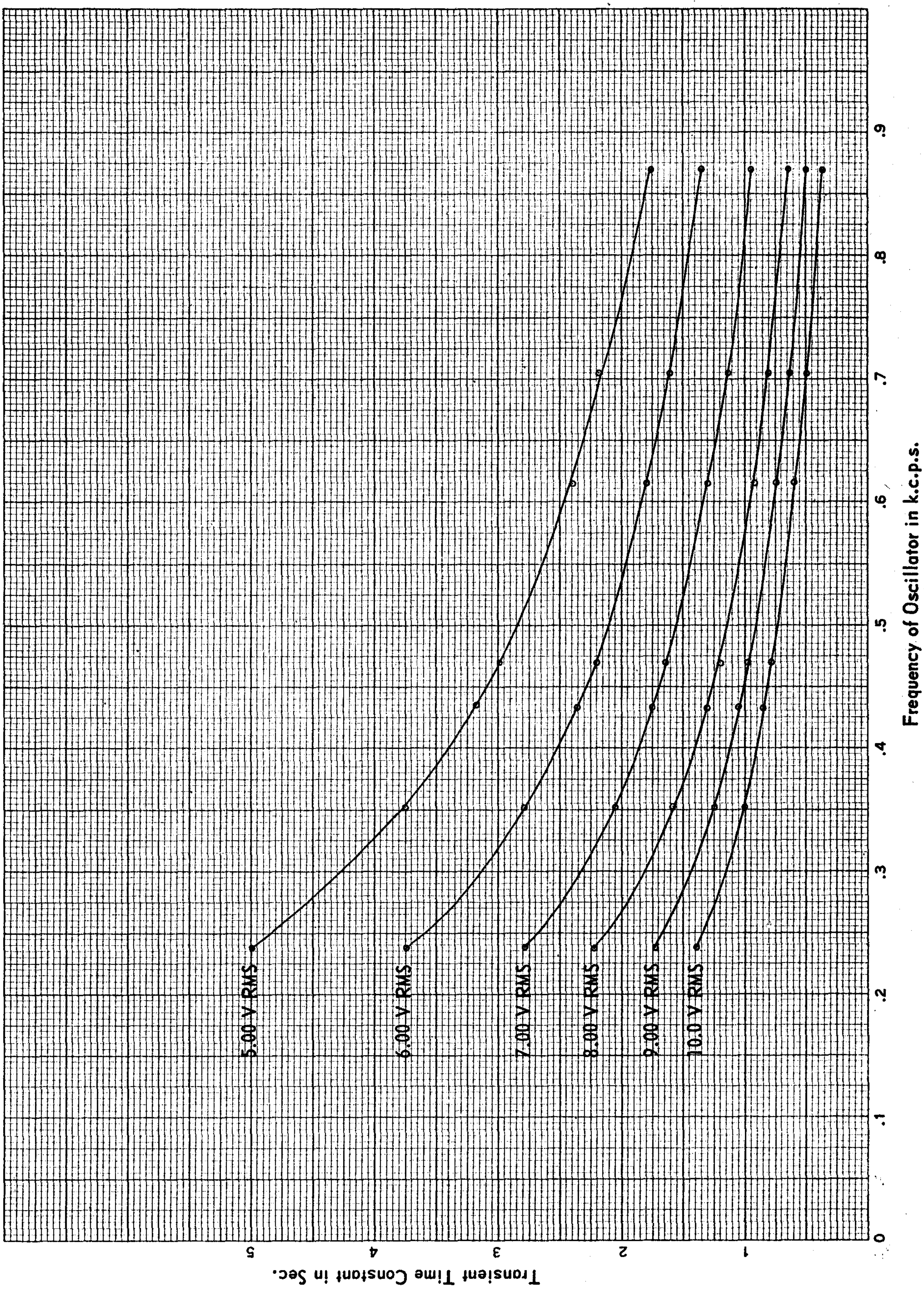
Graph (6)


5.00 V RMS
6.00 V RMS
7.00 V RMS
8.00 V RMS
9.00 V RMS
10.0 V RMS
Square Root of Oscillator Frequency
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
Amplitude δE, peak for Transient in peak volts
0
.25
.50
.75
1.0
1.25
1.50
1.75

Graph (7)


10.0 V RMS
9.00 V RMS
8.00 V RMS
7.00 V RMS
6.00 V RMS
5.00 V RMS
Square Root of Oscillator Freq.
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
Modulation Frequency fm in c.p.s.
1.0
.25
.50
.75
2.0
.25
.50
.75
3.0
.25
.50
.75
4.0

Graph (8)


Frequency of Oscillator in k.c.p.s.
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
Transient Time Constant in Sec.
1
2
3
4
5
5.00 V RMS
6.00 V RMS
7.00 V RMS
8.00 V RMS
9.00 V RMS
10.0 V RMS

plotted in graphs (9) and (10) for the modulation frequency f_m as a function of the square root of the oscillator frequency and the amplitude of the peak versus the square root of the oscillator frequency. Clearly the linear theory does not predict the decrease in the time constant for increasing oscillator frequency and the transient peak is larger than is measured. The linear theory does however, predict the modulation frequency with a reasonable degree of accuracy.

From graph (8) we see that the measured time constant is much shorter than is predicted by equation (133) and decreases quite rapidly with increasing oscillator frequency. The peak value of the transient apart from being smaller than is expected from the linear theory, decreases slightly from the straight line predicted for graph (10) for increasing oscillator frequency.

The small amount of cubic nonlinearity present in the operational amplifier of the Wien Bridge Oscillator was next measured for the steady state with the General Radio Wave Analyser. Readings for low voltage oscillator outputs were inconclusive but for a voltage output of 10.0 volts R.M.S. it was found that the third harmonic fractional distortion was approximately $D_3 \simeq 1.1 \times 10^{-4}$. Values for D_3 for lower voltages were calculated using this result and the relationship given by equation (106). The value for δ_1 can be found from equation (106A) and equations (134), (135) and (136) can be evaluated in the same manner as for the linear theory. Equation (134) may be further approximated for computations as;

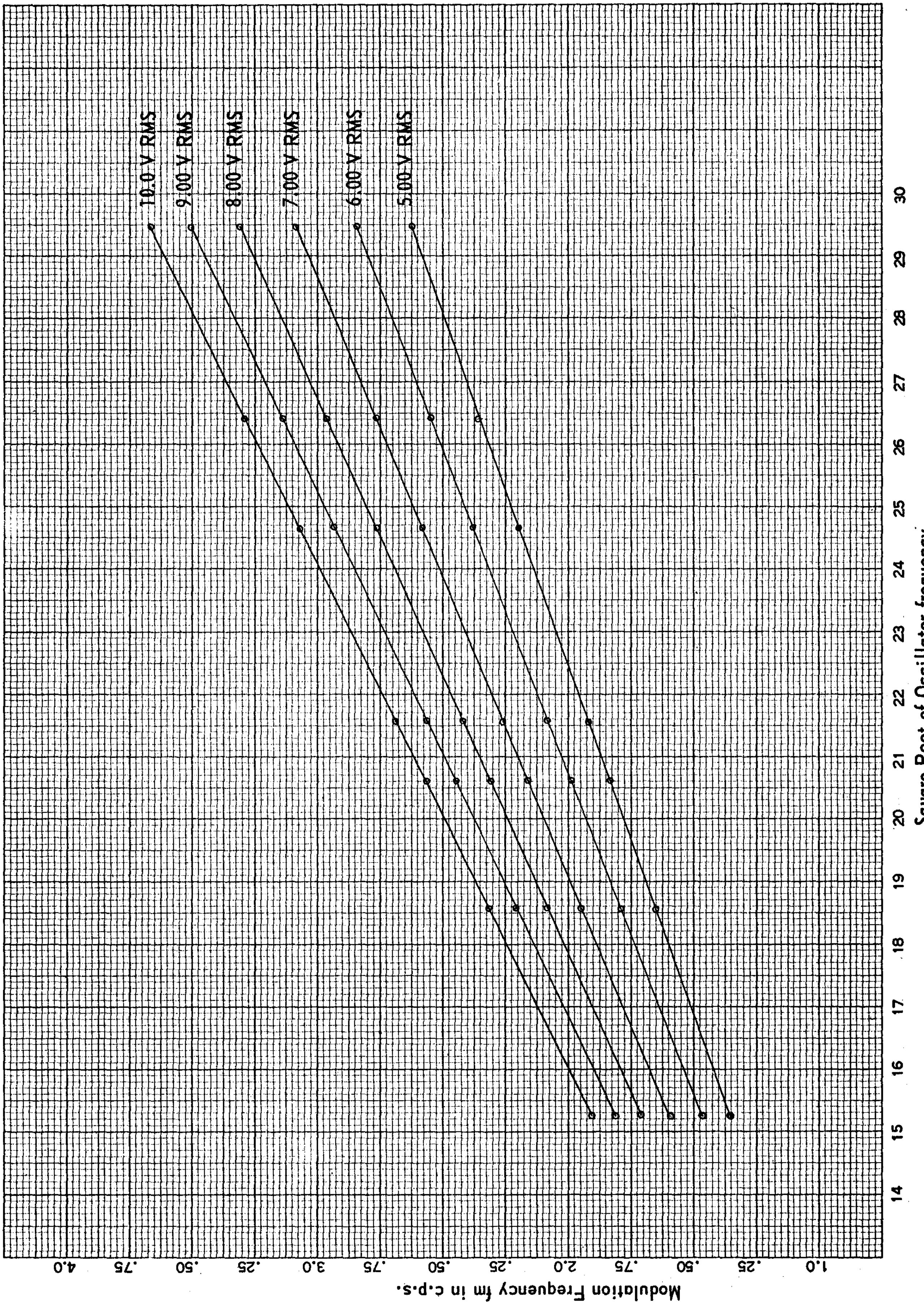
TABLE (5) LINEAR AMPLIFIER

E_1 v. RMS	f_o in cps	ΔE in peak volts	f_m in cps	τ_1 in sec	ω_T $X10^{-1}$	ϵ $X10^{-3}$
5.00	232.0	1.91	1.35	14.85	1.10	1.570
	344.8	2.32	1.65			
	424.8	2.57	1.83			
	464.9	2.79	1.91			
	606.5	3.08	2.19			
	697.9	3.30	2.35			
	866.0	3.68	2.62			
6.00	232.0	1.92	1.47	12.56	1.30	1.435
	344.8	2.34	1.79			
	424.8	2.60	1.99			
	464.9	2.72	2.08			
	606.5	3.11	2.38			
	697.9	3.33	2.55			
	866.0	3.71	2.84			
7.00	232.0	1.93	1.60	10.67	1.53	1.344
	344.8	2.37	1.95			
	424.8	2.62	2.16			
	464.9	2.73	2.26			
	606.5	3.12	2.58			
	697.9	3.35	2.77			
	866.0	3.74	3.08			

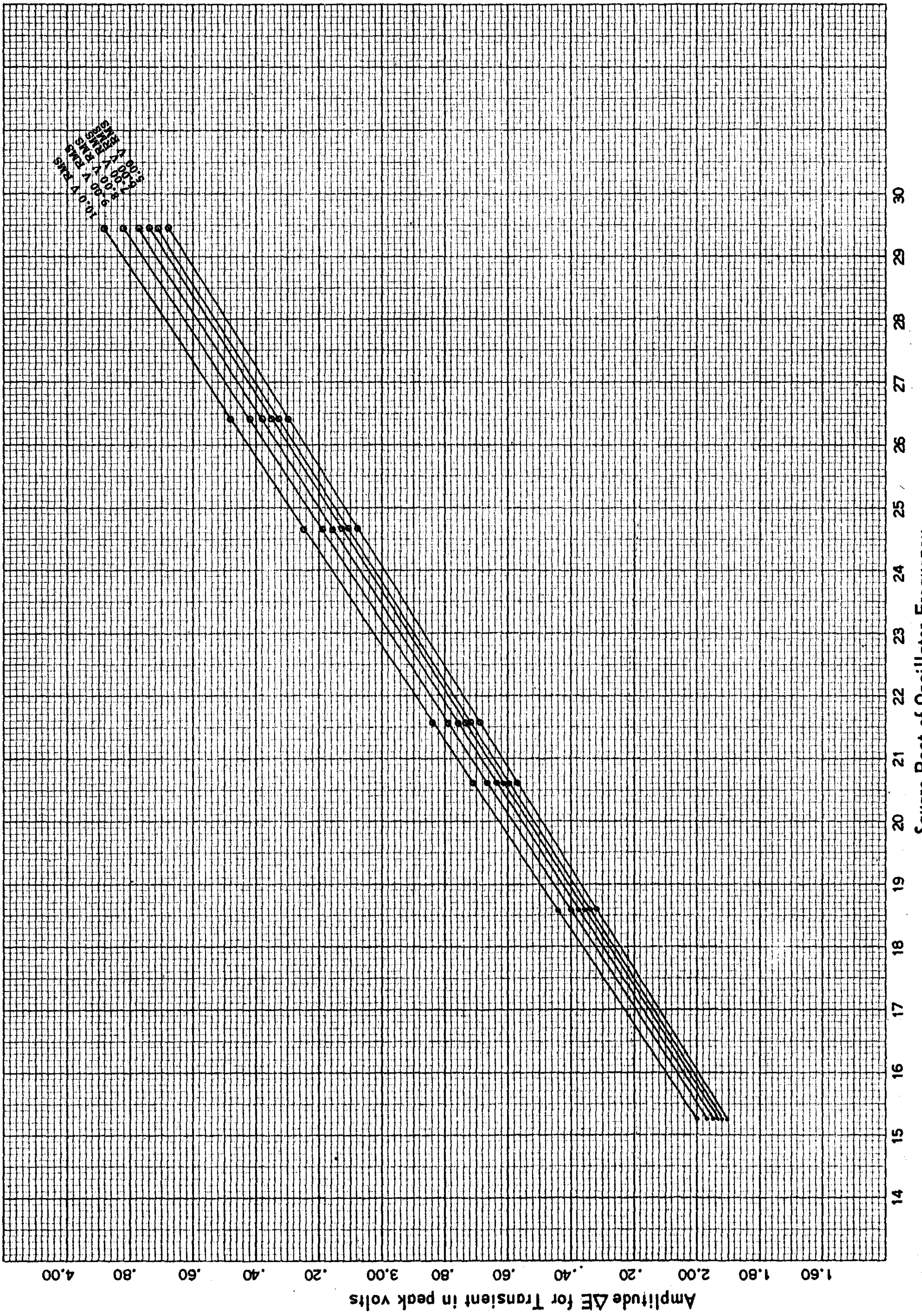
TABLE (5) LINEAR AMPLIFIER (CONTINUED)

E_1 v.RMS	f_o in cps	ΔE in peak volts	f_m in cps	τ_1 in sec	ω_T X10^{-1}	ϵ X10^{-3}
8.00	232.0	1.95	1.71	9.33	1.75	1.276
	344.8	2.37	2.08			
	424.8	2.64	2.31			
	464.9	2.76	2.41			
	606.5	3.16	2.76			
	697.9	3.38	2.96			
	866.0	3.77	3.30			
9.00	232.0	1.97	1.81	8.32	1.96	1.210
	344.8	2.40	2.21			
	424.8	2.67	2.45			
	464.9	2.79	2.56			
	606.5	3.19	2.93			
	697.9	3.42	3.14			
	866.0	3.82	3.50			
10.00	232.0	2.01	1.90	7.55	2.16	1.160
	344.8	2.44	2.31			
	424.8	2.71	2.57			
	464.9	2.84	2.68			
	606.5	3.25	3.07			
	697.9	3.48	3.29			
	866.0	3.88	3.67			

GRAPH (9)


10.0 V RMS
9.00 V RMS
8.00 V RMS
7.00 V RMS
6.00 V RMS
5.00 V RMS
Square Root of Oscillator frequency
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
Modulation Frequency fm in c.p.s.
1.0
.25
.50
.75
2.0
.25
.50
.75
3.0
.25
.50
.75
4.0

GRAPH (10)


10.0 V RMS
9.00 V RMS
8.00 V RMS
Square Root of Oscillator Frequency
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
Amplitude ΔE for Transient in peak volts
4.00
.80
.60
.40
.20
3.00
.80
.60
.40
.20
2.00
1.80
1.60

$$\delta E_{1\,\text{peak}} \simeq \frac{E_1 \epsilon \omega_0}{\sqrt{2 m_0 \omega_0 \omega_T}} e^{-\frac{1}{\tau_1 \omega_m}\left[\frac{\pi}{2} - \frac{1}{\tau_1 \omega_m}\right]}$$

for small nonlinearity such that $(\tau_1 \omega_m)^2 \gg 1$. The results are tabulated in table (6) and are plotted in graphs (11) and (12).

From graph (11) we see that the peak amplitude of the transient is somewhat larger than was measured, but it experiences the deviation from the straight line for increasing oscillator frequencies as is found in practice. The deviation of the experimental from the theoretical can perhaps be attributed to two things, first the switching time of the perturbation may not be infinitesimally small as assumed in the analysis, and second the larger peaks will be reduced by the increasing nonlinearity. This mechanism seems to be present in figure (25) for the large transient caused by switching frequencies. All of the increasing peaks are flattened while all of the decreasing peaks are lengthened by the cubic nonlinearity. If these two things were included the result would be a smaller value of δE_1 peak which would be in accordance with the experimental results.

From graph (12) the transient time constant τ_1 is seen to be markedly changed by the inclusion of the small cubic nonlinearity. In most practical problems this nonlinearity would most certainly be neglected in the first approximation and yet in this case it has a profound effect on the transient response. The experimental values of the time constant are seen to agree quite closely with equation (136) or graph (12).

(6) Hum Enhancement

The oscillator constructed was extremely poor with respect

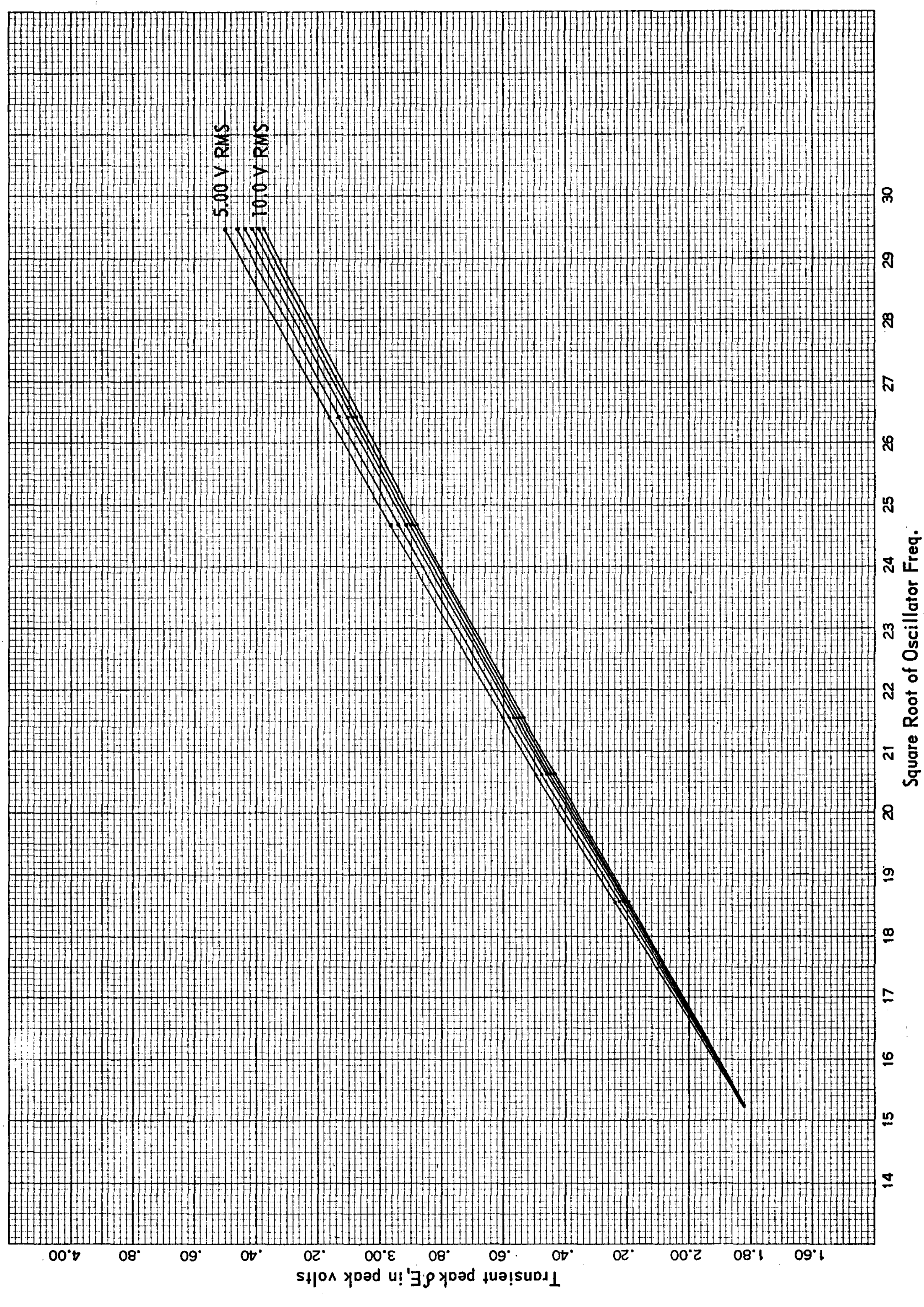
TABLE (6) CUBIC NONLINEARITY

E_1 v. RMS	f_o	δE_1/peak peak volts	f_m cps	τ_1 sec	ω_T $X10^{-1}$	ϵ $X10^{-3}$	D_3 $X10^{-5}$
5.00	232.0	1.83	1.35	4.98	1.10	1.570	2.75
	344.8	2.24	1.65	3.77			
	424.8	2.50	1.83	3.22			
	464.9	2.60	1.91	2.99			
	606.5	2.97	2.19	2.40			
	697.9	3.17	2.35	2.13			
	866.0	3.50	2.62	1.77			
6.00	232.0	1.83	1.47	3.68	1.30	1.435	3.96
	344.8	2.22	1.79	2.74			
	424.8	2.47	1.99	2.32			
	464.9	2.58	2.08	2.16			
	606.5	2.94	2.38	1.73			
	697.9	3.14	2.55	1.53			
	866.0	3.46	2.84	1.26			
7.00	232.0	1.83	1.60	2.81	1.53	1.344	5.40
	344.8	2.22	1.95	2.07			
	424.8	2.46	2.16	1.74			
	464.9	2.57	2.26	1.62			
	606.5	2.91	2.58	1.28			
	697.9	3.11	2.77	1.13			
	866.0	3.44	3.08	0.930			

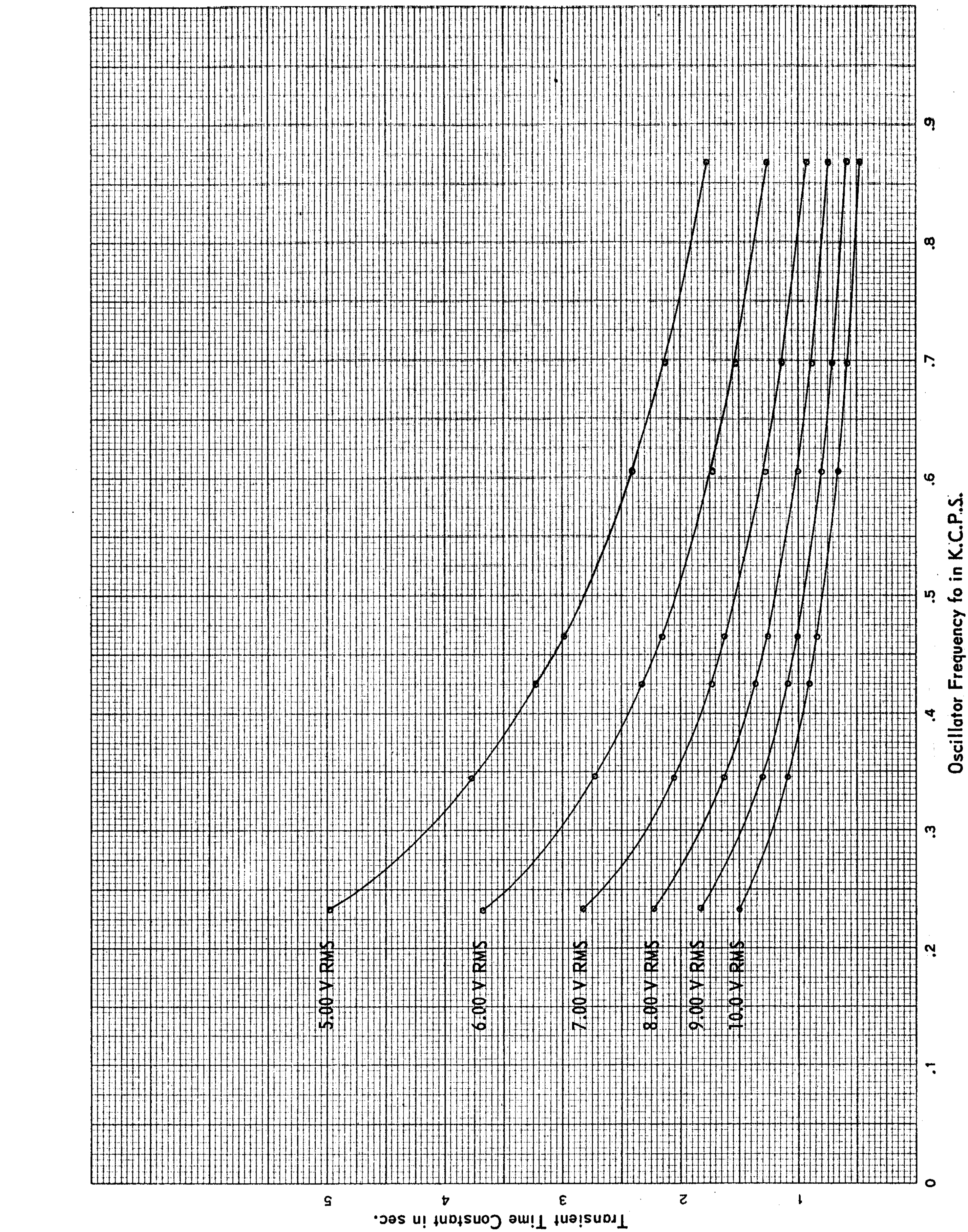
TABLE (6) CUBIC NONLINEARITY (CONTINUED)

E_1 v. RMS	f_o in cps	δE_1/peak peak volts	f_m in cps	τ_1 in sec	ω_T X10^{-1}	ϵ X10^{-3}	D_3 X10^{-5}
8.00	232.0	1.83	1.71	2.23	1.75	1.276	7.04
	344.8	2.21	2.08	1.63			
	424.8	2.45	2.31	1.36			
	464.9	2.55	2.41	1.26			
	606.5	2.90	2.76	1.00			
	697.9	309	2.96	0.883			
	866.0	3.41	3.30	0.723			
9.00	232.0	1.82	1.81	1.81	1.96	1.210	8.90
	344.8	2.21	2.21	1.32			
	424.8	2.44	2.45	1.10			
	464.9	2.54	2.56	1.01			
	606.5	2.89	2.93	0.800			
	697.9	3.08	3.14	0.705			
	866.0	3.40	3.50	0.578			
10.00	232.0	1.82	1.90	1.50	2.16	1.160	11.0
	344.8	2.20	2.31	1.08			
	424.8	2.43	2.57	0.902			
	464.9	2.54	2.68	0.832			
	606.5	2.88	3.07	0.655			
	697.9	3.07	3.29	0.575			
	866.0	3.38	3.67	0.470			

Graph (11)


5.00 V RMS
10.0 V RMS
Square Root of Oscillator Freq.
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
Transient peak δE, in peak volts
1.60
1.80
2.00
.20
.40
.60
.80
3.00
.20
.40
.60
.80
4.00

Graph (12)


5.00 V RMS
6.00 V RMS
7.00 V RMS
8.00 V RMS
9.00 V RMS
10.0 V RMS
Oscillator Frequency fo in K.C.P.S.
Transient Time Constant in sec.
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
2
3
4
5

to disturbances in the output. The chassis was well grounded and the lamp assembly was shock mounted and screened to minimize random variations from being originated from outside the oscillator. Even with these precautions random variations due to internal noise and power hum were present to a small extent for low oscillator output levels where the amplifier nonlinearity was a minumum.

Random noise and power hum could be regarded as a variation of the amplifier gain as the two are probably introduced in the amplifier through resistor and tube noise and A.C. power frequencies in the d.c. power supply. For the case of the amplifier used two harmonics of the power frequency were found to cause a continuous modulation where they provided one of the two components necessary in the output for continuous modulation. For the two frequencies 180 cps and 300 cps corresponding to the third and fifth harmonics of the power frequency, four oscillator frequencies were found where continuous modulation at the frequency f_m occurred. The oscillator frequencies were of course $f_o = (180 \pm f_m)$ and $f_o = (300 \pm f_m)$ where f_m is the expected modulation frequency at the oscillator frequency f_o.

It was shown in the theoretical section that sinusoidal variations of the amplifier gain would be reduced by the factor 4/A from the case of sinusoidal variations in the lamp resistance R or for that matter sinusoidal variations in the parameter k of the frequency selective arm. From equation (106) we have seen that the distortion is proportional to the reciprocal of the amplifier gain. Hence the enhancement of variations in the amplifier gain will be approximately independent of the gain A so long as $A \gg 1$.

The most sensitive portion of the oscillator to variations is of course the two feedback loops. Electrical radiation or mechanical vibration could cause a severe disturbance in the output. For this reason these components were particularily well shielded and shock mounted.

VIII CONCLUSION AND SUGGESTIONS

Although we have shown that the simple lamp bridge is an effective long time control for the oscillator amplitude with respect to variations in the amplifier gain, the thermal inertia of the lamp gives the system a poor transient response for any changes in the closed loop. In addition the oscillator's operating region is confined to frequencies above a lower limit indicated in section IV-(3). Below this lower limit, the lamp resistance will vary over the individual cycle and hence distort the output. The latter difficulty is inherent in this type of automatic level control, but the former can be improved by the presence of odd power non-linearity in the operational amplifier. The presence of this nonlinearity is extremely effective and unless the amplifier contains a third harmonic distortion greater than 100db down, it would be impossible to obtain a steady state output as noise and radiation would cause the output to be swamped by transients.

In general this type of automatic level control is inexpensive and quite simple to build. The time constant for the thermal bridge should be kept as short as possible consistent with the condition that it is long with respect to the period of the oscillation frequency. The transient response can be improved by increasing the value of the constant m_o which would mean choosing a different type of lamp or thermal device. For the thermistor of figure (11) a similar experimental value for m_o would be $m_o \simeq -2.56$ and if this device was used in its appropriate bridge, the transient output would be improved. From equations (134), (135), and (136) we see that for a larger m_o, the peak δE_1 is smaller, the frequency f_m is higher, and the time constant of the transient T_1 is shorter at the lower oscillator frequencies f_o where the transient was the most pronounced.

The oscillator transient response can also be improved by choosing a high gain amplifier with a high degree of linearity and inserting a pair of zener diodes back to back across the amplifier output as shown in figure (27).

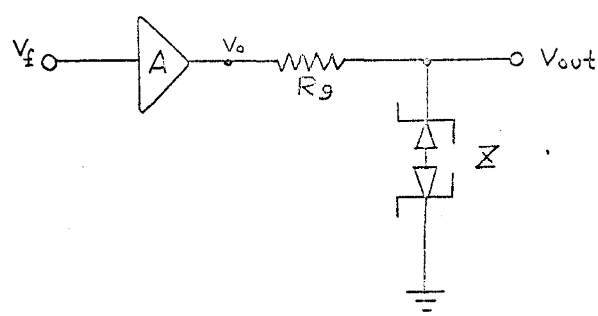


Figure (27)

The resistor R_9 is in series with the device and the amplifier A. The zener voltage is chosen equal to the peak voltage of the oscillator output in the steady state. The resistor R_9 should be a much higher resistance than the dynamic resistance of the zener above the peak voltage E_1, and at the same time must be much smaller than the feedback loop load. For any transient within the time response of the zener diode, the zener pair will limit the positive peaks creating components at the harmonic frequencies $3\omega_o$, $5\omega_o$, $7\omega_o$.... as well as harmonics and products of ω_m with ω_o. For small perturbations the amplitude of the fundamental frequency component ω_o will be reduced by the presence of harmonics in the transient state and the transient time constant and first peak δE_1 peak will be reduced in the same manner as section V-(4). The important thing is that for an ideal zener diode pair, the nonlinearity in the steady state will be determined by the nonlinearity of the amplifier.

A double diode combination was constructed from two standard diodes and batteries to provide the same type of limiter characteristic as the zener diodes. The batteries were adjusted to give the proper level of limiting and the oscillator output was observed to contain only a negligible amount of additional distortion in the steady state. The transient response however, was greatly reduced especially with regard to the transient time constant. With the diode limiter adjusted properly, a steady state harmonic distortion of 100db down should be allowable without serious transient disturbances.

A somewhat more subtle use of this method can also be accomplished by adjusting the amplifier to an operating point just below its power point. An amplifier such as the single ended push-pull circuit provides an almost linear characteristic up to the power point and would be suitable for this application. Both of these methods would allow operation with a minumum amount of steady state distortion and still reduce the transient response to an acceptable level for oscillator use.

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