A KINEMATIC STORM MODEL FOR
URBAN DRAINAGE STUDY

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by

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ABSTRACT

This study describes the development and use of a numerical storm model as a pre-processor for a detailed urban runoff model. The storm model simulates the spatial and temporal growth and decay of a system of storm cells as they move across an urban catchment system by generating hyetographs for each subcatchment.

Traditionally, design storms are developed from statistical analysis of point rainfall records that include all types of rainstorms. This methodology was considered appropriate for flood predictions based on the so-called rational formula. However, rain distributions resulting from point rainfall analysis are unlike any type of observed rainstorm. This synthetic temporal distribution is typically applied uniformly across the catchment and hydrographs are consequently unlike observed hydrographs.

The storm model presented in this study is based on synoptic observations of rain cells reported in weather radar literature. Statistics of the size and distribution of rain cells can be obtained from reported weather radar studies. Large static or slow-moving cells of uniform rainfall intensity are rare even in prolonged frontal events. Convective cells tend to be circular with a circular rainfall intensity pattern. Rain cells in frontal precipitation events tend to be elliptical, aligned sub-parallel to the front and moving sub-parallel to it. Rainfall is typically most intense near the leading edge of the cell. Fast moving storms produce very rapid point-intensity-duration changes.

A model with these features is developed. The model is applied to urban catchments of the City of Hamilton in Southern Ontario. The sensitivity of the time-to-peak and rate of rise of hydrographs and pollutographs indicate that storm cell kinematics are significant in peak runoff estimates and water pollutant loading estimates.

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CHAPTER 1

INTRODUCTION

1.1 Study Objective

The City of Hamilton contains diverse land use characteristics ranging from intense industrial activity to large expanses of open recreational parkland.

Hamilton Harbour receives stormwater runoff from virtually the entire city. Much of this runoff is polluted. The Ontario Ministry of the Environment is currently developing a numerical model to investigate the effects of urban stormwater discharge on the quality of the water in the harbour (22,27).

At McMaster University, Dr. James is supervising research to formulate a model of the Hamilton urban drainage system. This model will be used to estimate the following annual pollutant loadings to the harbour: suspended solids, BOD5, Nitrogen, Phosphate and Coliforms. James and Robinson (25) have recently examined interactive design using microprocessor communicating with large scale batch-oriented packages at remote mainframes. Mitri (34) developed a procedure to compute the overflow from a side weir diversion structure. El-Zawahry (10) wrote an algorithm for sediment deposition and resuspension, and applied it to the Chedoke Creek. Meanwhile, Shivalangaiah (24) is using the SWMM-STORAGE/TREATMENT Block to evaluate pollutional loads from stormwater, and Henry (23) is studying the feasible alternative solutions to the problem. Further publications on the project are also available (22,23,24,26,27). After suitable calibration, an urban drainage system model will be interfaced with the model of the receiving waters in order to evaluate the long-term effect of urban stormwater discharge on the harbour environment and to evaluate management alternatives (22,27).

The purpose of this study is to develop a model to generate hyetographs for each subcatchment and thus simulate the spatial and temporal growth and decay of a system of storm cells as they move across an urban catchment. This approach replaces the usual concept of static storms which uses a uniform precipitation across the entire catchment.

Computer simulation of urban runoff has become an integral part of hydrological analysis. Runoff models have attained an adequate level of sophistication and there needs to be more emphasis on developing adequate simulation techniques for precipitation, the input to runoff models. Rainfall models are not conventionally used in conjunction with runoff models. Yet accurate precipitation input to a runoff model seems to be essential to the simulation of storm hydrographs.

On the other hand, the spatial variability of actual rainfall may be damped out by means of the runoff process itself. There is not clear definition of what is meant by accurate and acceptable precipitation input.

Modern rainfall-runoff models, such as the Stormwater Management Model (SWMM) (Huber, 1975), utilize up to six rainfall hyetographs and discretize the catchment into (typically) 40-100 subcatchments. Slight modifications allow SWMM to accept 10-20 hyetographs distributed across the subcatchments. Thus, separate hyetographs allow simulation of a moving storm tracking across the catchment in any given direction, as well as growth and decay of the spatial size of the storm cell and the rainfall intensity distribution across the cells (20).

It is unlikely that storms will spontaneously occur, grow and die off while remaining stationary over a typically small urban basin. For smaller subcatchments, the assumption is that areal extent of the basin is small compared to spatial variation of storm rain. Translatory storms can be expected to produce very different runoff hydrographs and pollutant loadings than those produced by the usually accepted static design storm. Static design storms may introduce unjustified errors and thus lead to either costly drainaged works or damage from flooding, erosion and increasing pollution.

1.2 Urbanization

Due to demographic, economic and social reasons, the urban environment is growing rapidly and often in an uncontrolled manner.

An important component of urban development is the storm drainage system, the purpose of which is to collect and remove urban storm runoff in such a way as to minimize disturbances to the industrial, commercial and social activities of the urban environment. Recently, impacts on the natural environment have become an important consideration.

During the last 40 years, much work has been done to develop methods for urban storm runoff computations. These methods vary in their degree of complexity and mathematical sophistication. Urban drainage engineers are now more disposed to adopt and apply research work carried out at the academic level.

The various methods for urban storm runoff computation can be classified into two groups: stochastic methods and deterministic methods.

In stochastic methods, the variables involved are related statistically. Such methods are not yet widespread for the urban environment because measured periods of record are not long enough to permit development of reliable relations.

Deterministic methods are based on wholly predictable relations between the variables involved; such methods can be divided according to their approach, as follows:

The Empirical and Traditional Approach: Consisting of simple formulas, graphic relations, recommendations and rules-of-thumb, which are based on the experience and know-how of the engineers during the nineteenth century. The "Rational Method", so popular among urban drainage engineers, belongs in this category.

The Macroscopic Approach: Based on, for example, the unit hydrograph, which expresses the hydrologic behaviour of the urban environment in a concentrated form. Such aproaches tell us nothing of the constituent processes and their effects in the drainage system (9,40).

The Microscopic Approach: Based on the detailed analysis of the various hydraulic and hydrologic processes occurring in the urban environment during and after the storm. This is the approach used in this study (33).

1.3 Background - Design Storms as Model Input

This study is based on a paper written by James and Drake (20), that paper (20) succinctly outlined the scope and objectives of this study. Part of the following paragraphs have been abstracted directly from their paper.

Generally a prescribed return period is specified for a design project and then a design storm that has the same return period is selected. The project is designed not to fail when subjected to calculated flood produced by the design storm; it is assumed that the capacity of the design drainage system has a return period equal to that of the design storm (20).

The validity of this assumption of a linear relationship between some measure of the runoff hydrograph and a description of the rainfall hyetograph is of importance (47,48) and open to question. This is especially true where antecedent moisture conditions are significant and variable, and where surcharging is involved. Continuous modelling, though expensive, is seen to be helpful in this regard, but few continuous models account for the complex actions of the combined sewer overflow and diversion structures commonly encountered (20).

Design storms are usually either developed by a simple statistical analysis of point rainfall records that include rain of all types, or an historic storm is used (particularly for rare events). In the former case, the resulting temporal distribution of rain may be quite unlike the rainstorms occurring in nature (42) and thus may result in runoff peaks that can vary significantly from peak flows calculated statistically from long-term simulation using recorded rainfall and calibrated models. Synthetic storms attempt to aggregate intensity duration data from many storms of all types, thunderstorms and cyclonic rains, into a single and hence, impossible, storm event. Moreover, the general design hyetograph shape is based on data from many geographic regions, some involving orographic and other local effects. Most synthetic design storms are tantamount to an attempt to replace the several precipitation types and kinematics by a single simplistic rain hyetograph applied uniformly across a catchment supposedly appropriate to all shapes, sizes and kinds of catchments (20).

A number of studies have been carried out on the various published methods of synthesizing design storms from rainfall records (see, for example, Marsalek, (31); Arnell (1)) and these draw conclusions regarding the adequacy or otherwise of the various synthetic design storm techniques. None of the studies available to us has accounted for the storm kinematics or dynamics, notwithstanding the fact that storm movement may significantly affect the computed catchment response, especially pollutographs, or pollutant loadings to the receiving waters (20).

A promising path through the jungle has been proposed by Walesh, Lau and Liebman (46): hyetographs of major rainfall events are assembled from a long historic record and applied to a calibrated event model.

Rain intensities thus still relate to actual precipitation types. Unfortunately, once again the historic record is usually based on independent single point rainfall observations, incapable of accounting for storm dynamics. Dahlstrom (7) has suggested that the spatial variability of precipitation intensity can be taken into account by a complicated analysis of precipitation records from a limited number of adjacent stations for urban areas of large size. However, Dahlstrom evidently does not believe that current knowledge of meso-scale rainstorm characteristics would justify the development of a useful storm model for urban hydrology. Both Dahlstrom and Arnell appear to support Walesh's general approach to the use of many storms from the long-term rainfall record, but do not caution against overlooking storm kinematics (20).

An interesting research paper was done by Wilson, Valdas and Rodriguez-Iturbe in Puerto Rico (49), in which two mathematical models were used in the investigation; a deterministic runoff model based on the kinematic wave approximation and a non-stationary, time-varying, multidimensional rainfall generation model. They explored the influence of the spatial distribution of the rainfall input on the discharge by using one raingauge or 20 raingauges to record the synthetic storms.

Their model is appropriate for the frontal type of precipitation, when storms are not intense, not of short duration, and not localized in space.

A recent paper (Yen and Chow, (51)), although another attempt to apply a simple, approximate, spatially uniform, and static design hyetograph, does discuss the effect of the inherent storage and attentuation of a basin on the selection of the design hyetograph. These authors define a small basin as one sensitive to high-intensity rainfalls of short durations and sensitive to land use; inherent storage-channel characteristics do not suppress these sensitivities (20).

The problem is not new. Clark (6) has emphasized the case for including inherent stochasticity and error in the modelling procedure and has discussed this relationship to the observation network density. Van Nguyen et al. (44) advocate the use of a radar system rather than a dense network of raingauges. Both of these studies appear to hint at the need to model moving storms.

In summary, as stated by James and Drake (1980, (20)), the trend seems to be increasingly critical of the use of a simple, static, spatially-uniform design storm based on a single raingauge record or of a single spatially-averaged hyetograph using several adjacent raingauges and towards the use of a number of historic storms selected from the long-term record, or even continuous modelling using the entire long-term record. There is also increasing interest in better sampling of the rainfall inputs, but at present, the accent is on data analysis rather than modelling of dynamic storms. The use of storm models based on known synoptic characteristics of storms does not seem to have been considered a design alternative. Hydrometeorologists have perhaps been too cautious to advocate the use of storm models incorporating cell kinematics.

CHAPTER 2

PRECIPITATION TYPES SIGNIFICANT TO STORMWATER MODELLING

2.1 Precipitation Types

It is usual to identify three main types of precipitation, according to the mode of uplift of the air: convective, cyclonic and orographic (4).

2.1.1 Convective Precipitation

This is associated with towering cumulus and cumulonimbus clouds. There are three known subcategories:

1. Scattered convective cells develop through strong heating of the land surface in summer, the precipitation, including hail, is of the thunderstorm type.
2. Showers of rain, snow or soft hail pellets may form in cold, moist, unstable air passing over a warmer surface.
3. In tropical cyclones, cumulonimbus cells become organized about the vortex in spiralling bands (4).

2.1.2 Cyclonic Precipitation

Precipitation characteristics vary according to the type of low-pressure system and its stage of development, but the essential mechanism is ascent of air through horizontal convergence of airstreams in an area of low pressure (4). This type of precipitation is not studied here.

2.1.3 Orographic Precipitation

Orography, dependent on the alignment and size of the barrier, may: (a) trigger conditional or convective instability by giving an initial upward motion or by differential heating of the mountain slopes, (b) increase cyclonic precipitation by retarding the rate of movement of the depression system, or (c) cause convergence and uplift through the funnelling effects of valleys on airstreams (4). This type of precipitation is not dominant in Southern Ontario.

2.2 Thunderstorms

Thunderstorms are usually associated with rapid upward and downward movements of the air. They occur: (a) as rising cells of excessively heated moist air; (b) along a squall line in association with the triggering off of conditional instability by uplift over mountains or by excessive local convergence.

The life cycle of a storm cell lasts only one to two hours, and begins when a parcel of air is either warmer than the air surrounding it or is actively undercut by colder encroaching air. In both instances, the air begins to rise and the embryo thunder cell forms as an unstable updraught of warm air. As condensation begins to form cloud droplets, latent heat is released and the initial upward impetus of the air parcel is augmented by an expansion and a decrease in density until the whole mass becomes completely out of thermal equilbrium with the surrounding air. The constant release of latent heat continuously injects fresh supplies of heat energy, which accelerates the updraught and does not permit it to slacken. The rise of the air mass will continue as long as its temperature remains greater than that of the surrounding air (4).

2.3 Significant Precipitation in Hamilton

In the Hamilton area, as is stated by James and Drake (20), orographic uplift over the Niagara Escarpment rarely leads to rainfall, although it may generate clouds. Cyclonic precipitation, associated with frontal systems, has two components: a broad belt of relatively low intensity rainfall lying along the warm front, and a narrower belt of relatively high intensity rainfall lying along the following cold front. Convective precipitation in summer may be caused by differential local surface heating and generate small, intense rain cells, or the cold front rainband may in fact be composed of a linear set of convective cells associated with the air mass moving from Lake Erie over the land. Occasionally, very intense, widespread convective rainfall may be associated with the incursion of a tropical storm into the area (e.g. Hurricane Hazel in 1954).

Adiabatic cooling is the cause of condensation and rainfall, and vertical transport of humid air masses is a requirement. In convective precipitation, heated air at the ground expands and reduces in weight. The warm moisture-laden air becomes unstable and pronounced vertical currents are developed. Dynamic cooling then causes condensation and precipitation in the form of light showers, storms or thunderstorms of high intensity.

Thunderstorms begin as cumulus clouds characterized by strong updrafts that reach 25,000 feet. During development of the storm, additional moisture is provided by a considerable horizontal inflow of air. The storm enters a mature stage when the strong updrafts produce precipitation. Gusty surface winds move outward from the region of rainfall and heavy rainfall occurs for a period of 15 to 30 minutes. In the final dissipating stage of the storm, the downdrafts predominate and precipitation tails off and ends.

In the Hamilton urban catchment, the greatest rainfall rates are associated with convective precipitation. Although major structures may be designed on the basis of an exceptional recorded event, urban storm-water structures are designed on the basis of a composite synthetic storm, derived from raingauge data that is assumed to begin, peak and end simultaneously over the whole catchment. In fact, the greatest rates of runoff are usually associated with a linear set of convection cells containing individuals that are continually being generated and dissipated, and which moves across the area.

Thunderstorms experienced at a point on the ground comprise one or more such cells moving overhead in varying stages of development, the life cycle of which is usually completed in an hour or less. However, such storms tend to be self-propagating by the formation of new cells and in the Hamilton area, generally move from the south-west or south-east at speeds of 10-50 km/hr and in broken lines or bands up to 80 km in width. Severe storms may produce 5 cm of rain in less than half an hour, while slowly moving storms may appear to remain in one locality for an hour or more and produce a total point rainfall as great as 20 cm.

In summary, rainstorms travel in preferred directions; they do not spontaneously grow and die over one spot, as suggested by current practice in the analysis of point rainfall data. Cells have substantial speeds and intensity variations across areas typically appropriate to urban runoff studies (e.g. 5-5000 acres). A substantial body of information is available and the general characteristics of stormcells can be described.

It is preferable to specify the expected speed and direction of movement of cells, and even cell size and rainfall intensity distribution, rather than to assume no speed or direction, and excessively large cells with uniform rainfall intensities.

Finally, thunderstorms have different characteristics from cylconic events. Point rainfall data does not distinguish between rainfall types, and statistical analyses of rain data includes intensities from all types. Point rainfall data cannot generally provide information on storm cell kinematics (20).

CHAPTER 3

THE DESIGN STORM CONCEPT

A rainfall hyetograph is the required input for most runoff models. This input may be in the form of observed rainfall events, representative design storms, the widely used intensity duration frequency curves, or other statistical analysis of rainfall records.

3.1 Definition of Design Storm

The general idea of a design storm is to provide a means of estimating a discharge or runoff volume of specified recurrence interval for planning or design purposes. A recurrence interval is assigned to the design storm, a rainfall-runoff procedure is used to convert the rainfall to runoff, and the recurrence interval of the design storm is transferred to the resulting runoff discharge or volume.

There are two types of design storms:

a) Direct Use of Intensity-Duration-Frequency (IDF) Curves.

b) Development of a Synthetic Hyetograph.

3.1.1 Direct Use of Intensity-Duration-Frequency (IDF) Curves

The IDF relationships are used to obtain a uniform intensity for a given duration and recurrence interval; commonly used in applying the rational method of storm sewer design.

3.1.1.1 <u>The Rational Method</u>

The rational method for estimating peak flows is based on a simple rainfall-runoff relationship:

$$Q = CIA$$

where: Q = runoff (cfs)

C = runoff coefficient

I = intensity (in/hr)

A = area (acres)

In the metric system,

$$Q = 2.78\ CIA$$

where: I = rain intensity in mm/hr

A = area in hectares

Q = runoff in litre/sec (l/s)

When using the rational method, the following assumptions are made:

(a) The rainfall intensity is distributed uniformly over the entire watershed and is constant during the entire storm duration.

(b) The maximum runoff rate occurs when the rainfall lasts at least as long as the time of concentration.

(c) The time of concentration is the time required for the runoff from the most remote part of the watershed to reach the point under design.

3.1.2 Development of a Synthetic Hyetograph

Hyetographs are usually simply not available for all locations. Sometimes standard hyetographs are required for drainage system design. In such cases, a synthetic hyetograph may be derived from IDF relationships, historic storms, or by other means.

Examples of design storms of this type are: The Chicago hyetograph and its variations, and the quartile hyetograph developed by the Illinois State Water Survey (32).

3.1.2.1 Chicago Design Storms

The formulation of the Chicago synthetic hyetograph was presented over twenty years ago. The Chicago method has been rather widely incorporated in North American practice because it can be readily derived from available rainfall intensity-duration-frequency relationships and partly because of limited alternative approaches.

In an attempt to preserve correspondence with actual rainfall events, the Chicago methods takes into account the maximum rainfalls of individual durations, the average amount of rainfall antecedent to the peak intensity, and the relative timing of the peak intensity. The first step in applying the method is determination of the time antecedent to the peak intensity, expressed as a dimensionless ratio tr. tr=tp/T where tp is the elapsed time from the onset of rainfall to the peak intensity and T is the total storm duration. Values of tr are determined individually for a number of historical storms and their mean value is used for the design hyetograph.

The hydrograph intensities on either side of the peak are obtained from local intensity-duration-frequency relationships in the form

$$Iav = (A/Td^{**}b+c)$$

where Iav is the average maximum rainfall intensity over a duration Td, and the constants A, b, c are chosen to fit local data. Typically, one to six hours is selected as the total storm duration, T. The choice of T does not affect the magnitudes of the peak rainfall intensity or the dimensionless time to peak (32).

3.1.2.2 Illinois State Water Survey Design Storms

In this procedure, maximum hourly rainfall depths are taken from local data or from intensity-duration-frequency relationships for various return periods. For application elsewhere (out of Illinois) observed storms are first divided into a number of groups in accordance with the relative timing of the peak intensity. Distributions over time are next determined for the predominant group of storms and their median distribution is used for the design storm.

A median rainfall distribution is determined for a group of data and expressed as:

$$Rcp = f(Tcp)$$

where: Rcp is the cumulative percent of rainfall,

Tcp is the cumulative percent of storm time,

f is the empirical function (32).

3.2 Relevant Processes of Storm Runoff

The conversion of rainfall into runoff is affected by many factors, especially in the varied environment of urban communities. Their relative importance must be clearly recognized in the interpretation of storm runoff in relation to intense rainfalls.

Flows normally reach their crest at a given point on a stream or within a drainage scheme when runoff from rainfall begins to pour in from all parts of the tributary area. There are exceptions to this rule, but they are few. An important exception is a storm travelling upstream or sweeping across a catchment area so rapidly that runoff from distant points cannot reach the outlet until long after the central storm has moved on. This effect is rarely taken into consideration in North American practice; but it should be in some circumstances. Because rainfall decreases in overall average intensity with increasing duration, the shorter the response time over which the entire area is tributary to the point of concentration, the larger are the flows.

This so-called "time of concentration" is shortest for small, broad, steep areas with rapidly shedding surfaces. It is lengthened by dry soil, surface inequalities and indentations, vegetal cover, and storage in water courses, on flood plains, and in reservoirs. The volume of runoff is swelled by snow and ice melt, infiltration from bank storage, and release of water from impoundages either deliberately or accidentally.

Maximum discharges are widely thought to be obtained when storms move downstream at speeds that bring them to the point of discharge in about the time of concentration, making it possible for the runoff from the most intense rainfall to arrive at the point of discharge at nearly the same instant as the peak of the runoff flood wave. Actually, rapidly moving storms precipitate less water in a catchment (12).

3.3 Predictive Use of Models (29)

Hydrological problems arise from both the quantity and the quality of runoff. The textbook approach to runoff control requires determination of a flood frequency curve for the existing conditions and a second flood frequency curve for conditions after completion of a particular runoff control system. These curves can be converted into damage frequency curves by assuming a relation between peak flow and flood damage. The area under the curves then becomes average annual damage in the existing condition and average annual damage after improvements have been made. The difference between these two damage figures represents the benefits of the flood control project and can be compared with the costs of an economic evaluation. In many urban situations the damage is little more than nuisance and a decision is made rather arbitrarily to limit the probability of this nuisance to some acceptable level (29).

In dealing with pollution from urban storms, the magnitude and frequency of the pollutant loads should also be known. A determination should be made of acceptable levels of pollution frequency given some information on the magnitude of the polluting load, in the same manner as for floods.

Consequently, it is necessary to define the probability of peak flows, and, where storage is being considered, the volume characteristics of the streamflow as well. This is necessary for each of the design alternatives.

It is usual to select the maximum rain hyetograph each year and convert this to a flow hydrograph with a discrete event simulation model. But the most intense rainfall in a year does not necessarily produce the maximum peak or the maximum runoff volume.

A short high-intensity rainfall may produce a very large peak flow but with a low runoff volume. This flood might be severely reduced by available storage. On the other hand, modest rainfalls extending over many hours or even days, may produce a large volume which could fill a storage reservoir. The effect will of course depend on the amount of storage and its outlet capacities. The point is that the critical storm depends upon the catchment characteristics, not only on the climatic region (29).

At present, there are few suitable continuous simulation models available for design and it appears to be advisable to use a tried-and-tested model in a flexible manner. The development of FASTSWMM is an attempt to provide rapid and easy use of the stormwater management model (SWMM).

In hydrologic synthesis, meaningful input over a period T for which the simulation is to be run such that the output can be used objectively, is required. A design storm hyetograph and its associated frequency of occurrence are required. The basis of the methods of precipitation analysis as discussed in the Ontario Manual of Drainage Design are now reviewed (29). SWMM has been improved by extending the number of hyetographs from 6 to 10.

3.3.1 Selection of Frequency of Occurrence of the Design Event (29)

In stormwater calculations, it is often (erroneously) assumed that the frequency of occurrence of a rainfall event is identical to the frequency of occurrence of the resulting runoff. The selection of this design frequency for drainage projects is a compromise between periodic inconvenience and damages due to flooding and the cost of preventing this flooding through a larger storm sewer system.

The most common recurrence intervals used vary between five and ten years. Many minor drainage systems do not warrant a detailed analysis of the relationship between the cost of flood protection and flood damage and consequently design periods for such minor drainage components are specified in municipal stormwater drainage design criteria. Major drainage elements are also sometimes included in such criteria (29).

3.3.2 Rainfall Intensity-Duration Curves (29)

To obtain the rainfall intensity during an individual storm, or a shorter period, chart recorders are necessary. Rain intensity varies markedly with the time interval selected.

Traditionally, the design rainfall was completely defined by rainfall intensity-duration curves which were used in conjucntion with the rational method for calculating runoff (29).

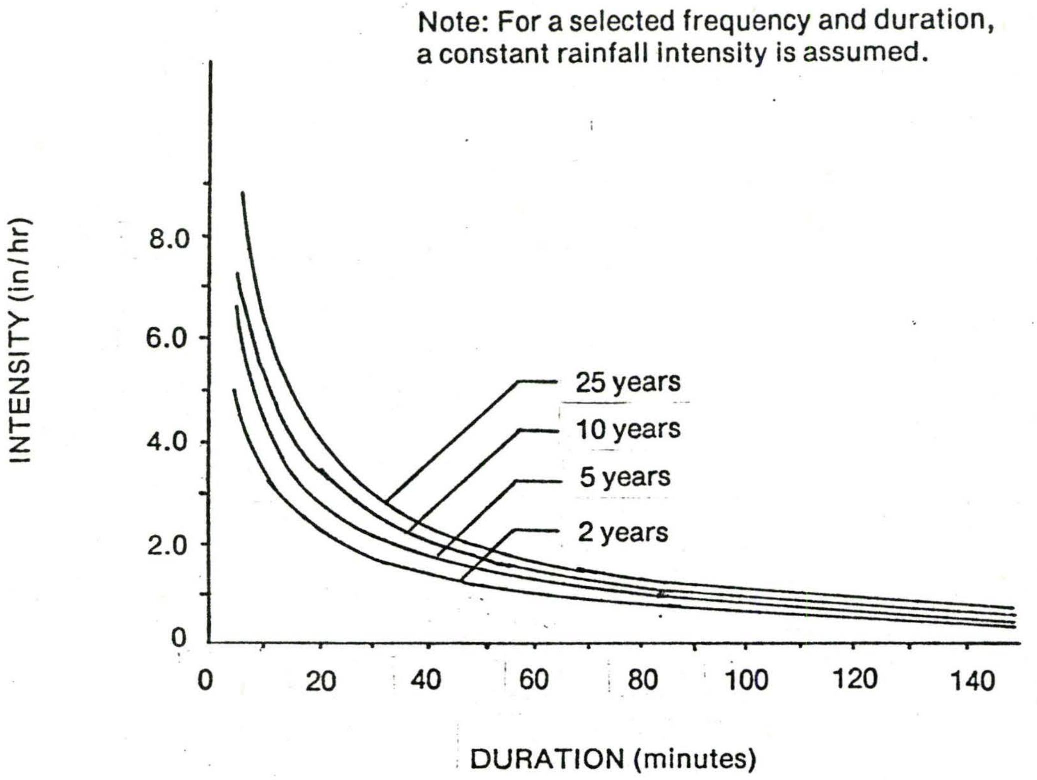


Figure 1: Intensity-Duration Rainfall Curves, source (29).

3.3.3 Synthetic Design Storm Hyetographs (29)

As it has been stated by James (29): The analysis of synthetic design storms is undergoing rapid development and may be subject to criticism. Synthetic storms may even be less well defended than an historical storm event.

When developing a design storm, the time distribution, the storm position and movement should be considered. The resulting time and spatially variable design storm pattern represent a statistical summary of historical precipitation records (29).

The following steps are typically taken:

1. derive a set of rainfall depth-duration-frquency curves
2. establish a temporal rainfall intensity distribution
3. establish a spatial distribution of rainfall intensity

The motion of the storm is not considered. Depth-duration-frequency curves may be readily derived from the intensity-duration-frequency curves. In the next step, an area-depth relationship is found for events of various frequencies.

This relationship is required only for catchments of larger areas than several sq. km., otherwise point precipitation is acceptable. The precipitation depth should be reduced somewhat for an increasing catchment area.

The temporal rainfall pattern during the storm is determined in the next step. Here one may either utilize some of the rainfall patterns reported in the literature, or preferably, derive such patterns from the precipitation data. Among the temporal rainfall patterns reported in the literature, the best known appear to be the Chicago Design, the U.K. Meteorological Office Design Storm, and distributions reported by Huff. The general applicability of these three distributions has not been studied and therefore it is preferable to derive a distribution by a statistical analysis on local data (29).

Finally, the spatial rainfall intensity distribution is determined by a three-dimensional statistical analysis of the problem - two dimensions refer to space and one is an occurrence component. The occurrence component represents the recorded rainfall depth for a certain event and duration at each gauge.

The development of a design storm may be substantially simplified for small catchments (area of the order of several sq. km.). In that case, the aerial distribution effects are negligible and the development of the design storm is reduced to defining the precipitation depth and its distribution in time.

A comparison of the Chicago and Hamburg design storm is shown in Figure 2; a large discrepancy between both storms is obvious (29).

3.3.4 Historical Design Storms

As it has been stated by James (29): Difficulties with the development of synthetic design storms, as well as uncertainties involved in these storms, led some designers to an alternative approach - adoption of an historical design storm. The selection of such a storm is done either directly or indirectly.

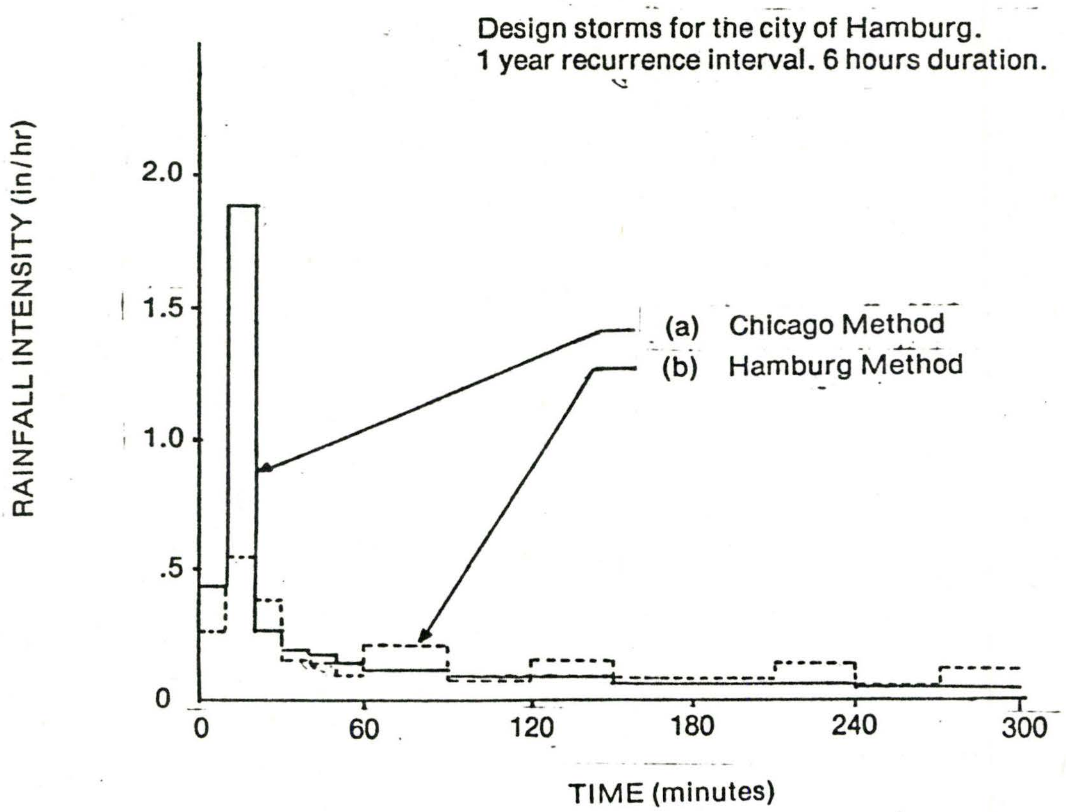


Figure 2: Comparison of Design Storms, source (29).

In the Direct Method an historical storm which is well documented and for which the drainage system behaviour is also somewhat documented (the extent of flooding and damages) is selected and used for future designs. The same event may also be used as a regional storm (eg. Hurrican Hazel in Ontario).

The frequency of occurrence of these events can be estimated. The approach described may be applicable to major drainage elements, whereas minor drainage elements are typically designed from more frequent events.

The indirect method is based on the frequency of occurrence of runoff events. A precipitation record is first translated by means of a continuous simulation model into a runoff record. A statistical analysis of the runoff record is then performed and the frequency of occurrence of various runoff flows is determined for a selected runoff flow. The corresponding storm can be identified and used as an historical design storm on catchments of similar size in the study area (29).

3.3.5 Risk-based Design (29)

Risk-based design of large hydraulic projects is well accepted in engineering practice. A simialr approach has been proposed for storm sewers by Tang and Yen (52). Such an approach is not yet common in engineering practice and consequently, only a general discussion of the underlying principles follows. Risk-based design may be particularly useful for drainage systems in which considerable flood damages can be expected.

The procedure considers uncertainties involved in runoff computations, such as, e.g. in the Rational Method or any other tecnique.

A design safety factor, SF, is defined as

$$SF = Qc - Qd$$

where Qc is the sewer pipe capacity, and Qd is the design flow. Using the Rational Method and rainfall intensity-duration-frequency curves, Tang and Yen established the risk vs. safety factor curves for a particular location.

After designing the entire sewer system, risks associated with the individual pipes can be evaluated from the risk vs. safety factor curves.

Should these risks be too high, the system is redesigned by either selecting a lower fequency storm, or by selecting the maximum acceptable risk and deriving the corresponding safety factor. Details can be found in the references.

Note that when the maximum acceptable risk is specified, the design event frequency may vary during the calculation. The reduction of a maximum acceptable risk for a particular pipe is equivalent to designing the sewer pipe for a less frequent storm and a safety factor of one (29).

3.3.6 Water Quality Oriented Design Storms (29)

The preceding paragraph on design storms dealt exclusively with runoff quantities. When water quality aspects are to be considered, a different analysis of the precipitation data may be required. The frequency of occurrence of the pollution load of a certain magnitude differs significantly from the frequency of the corresponding storm. The total pollution load produced by an event depends not only on the event itself, but also on the length of the antecedent dry weather period.

Hence the design storm approach is not used in quality-oriented design of drainage. Instead, a continuous simulation of runoff quality and the associated costs of quality control are usually used and provide a good basis for selecting a cost effective measure for runoff quality control.

Alternatively, single-event simulations may be performed for a series of typical storm events of return periods varying from several days to several years, the drainage system response to these events is determined and a cost effective runoff quality control measure is selected. In other cases the selection of the quality control measure is based on the desired degree of protection of the receiving waters.

While the quantity-based design calls for design events with the return period of the order of several years, the most cost-effective runoff quality measures may be obtained for design events with return periods of several weeks or months (29).

3.4 Shortcomings of the Design Storm Concept (14)

1. A fundamental assumption associated with the design storm concept as it is usually applied is that the recurrence interval of the design storm may be transferred to the discharge or volume produced with the storm. This equivalency of recurrence interval has not been confirmed.
2. Casual observations of rainfall-runoff data for watersheds reveal instances in which strikingly similar hyetographs produce markedly different hydrographs. Also very similar hydrographs can be shown to have been generated by markedly different hyetographs. This suggests the important role of factors such as

antecedent moisture conditions and the distribution of rainfall over the catchment.

3. Some of the methods used to construct design storms may involve incorrect use of IDF relationships.
4. Design storms do not usually yield all the probability information desired for planning and design studies. For example, whereas a design storm might provide a flood flow of specified recurrence interval, it is not suitable for developing a flow duration curve. The need for additional probability based hydrologic-hydraulic data is expected to become more pressing with increased work in the area of non-point source pollution.
5. If the design storm concept is generally less than adequate for determination of discharges of specified recurrence interval, then existing design storms are even less likely to be suitable for determination of volumes of specified recurrence interval. Design storms are typically constructed so as to generate peak discharges. Therefore, design storms usually do not include the long duration-high volume rainfall events likely to be important in determining required sizes of detention/retention facilities. Design storms are even less likely to be suitable for determining non-point source pollutant loads and concentrations in receiving waters.
6. The variability of rainfall may be an important factor in determining discharge and volume at the catchment outlet. This variability is typically ignored in applying design storms.

7. Finally, engineers and other professionals involved in planning and design may expose themselves to criticism and even liability through use of an "unproven technique" (14).

3.5 Positive Aspects of the Design Storm Concept (14)

The design storm concept enjoys widespread use. This is apparently in response to the following positive aspects of the concept:

1. Use of design storms requires minimal resources in terms of time and money.
2. Design storms appear to give conservative results, that is, high discharges and volume.
3. Application of the design storm approach is generally accepted in practice and one can argue for maintenance of consistency of methodology in a given jurisdiction or geographic area (14).

CHAPTER 4

STORM MODEL THOR

4.1 Introduction

From the earlier discussion, it is clear that rainstorms travel in preferred directions: they do not spontaneously grow and die over one spot, as suggested by current practice in the analysis of point rainfall data. Convective cells have substantial speeds and intensity variations across areas typically appropriate to urban runoff studies (eg. 5-5000 acres) (20).

A substantial body of information is available and the general characteristics of storm cells can be described. It is preferable to specify the expected speed and direction of movement of cells, and even cell size and rainfall intensity distribution rather than to assume no speed or direction, and hence excessively large cells with uniform rainfall intensitives (20).

Thunderstorms have different characteristics from cyclonic events. Point rainfall data does not distinguish between rainfall types, and statistical analyses of rain data includes intensities from all types. Point rainfall data cannot generally provide information on storm cell kinematics (20).

Radar studies of summer rain events resulting from moving clusters of sub-circular convective cells have shown that the cells are relatively short-lived (Austin (2)), that they tend to have an exponentially-decreasing intensity away from the cell centre (Konrad, (30)), and that their statistical properties can be matched to those of ground-based precipitation records (Drufaca, (8)). Gupta and Waymire (16) have proposed a stochastic model of rainfall from such clusters, but no a priori single-event design storm for the Hamilton area can be derived from it (20).

Studies of "line-convection" rainbands associated with extra-tropical cyclones have shown them to be longer lasting and their structure to be one of sets of extended elliptical cells oriented sub-parallel to the front with a component of motion along it (Hobbs and Biswas, (17); James and Browning, (19)). This pattern of rainfall, oriented across the drainage basin and moving down from head to mouth is apparently common in Hamilton, and forms the basis of the present model. The form of the model is an infinitely wide rainband in which the rainfall intensity decays exponentially away from the line of peak intensity at different rates ahead and behind it. This model has the further advantage of being similar to the Chicago design storm, already in common practice among Civil Engineering Hydrologists. Thus

$$P=PO*EXP(-K1(tp-t)) \qquad t<tp$$

$$P=PO*EXP-(K2(t-tp)) \qquad t>tp$$

where PO is the instantaneous point peak intensity tp is the time-of-peak at a point and t is time at a point. Statistical studies of rainfall rates before and after peak rates recorded by raingauges in several parts of North America indicate that k2 = 0.54 k1. However from statistical analysis of 36 storms in the Hamilton area (summer, 1980), K2 = 0.0818 + 0.299K1, the relationship adopted in the thunderstorm model THOR. The two parameters PO and K1 (or K2) could have been evaluated from intensity frequency duration curves published for the Hamilton area, by assuming that events of all durations for a given recurrence interval are embedded in one storm. However excellent correlations were found between total observed precipitation PTOT and PO, and also between PO and K1. The relations

$$K1 = 0.101 + 0.0025PO \text{ and } PO = 2.6 + 1.81 \text{ PTOT}$$

were adopted in THOR. Konrad (30) and other studies have suggested that convective cells have a similar peak intensity distance-decay exponent of about 0.5 km, also could be used in THOR. Good correlation were obtained between the ground-level storm wind speed WV and storm speed SV computed from analysis of 3 synchronized hyetographs (data from summer, 1980). The relationship adopted was SV=7.39 + 0.933WV.

Input data processed by THOR include three observed hyetographs of total precipitation, wind velocity and wind direction (21).

THOR will produce time-averaged and space-averaged hyetographs for any basic time step and subcatchment area, representing a moving storm tracking across the catchment in any given direction.

Figures 3 and 4 show the typical form of the hyetographs generated by the model.

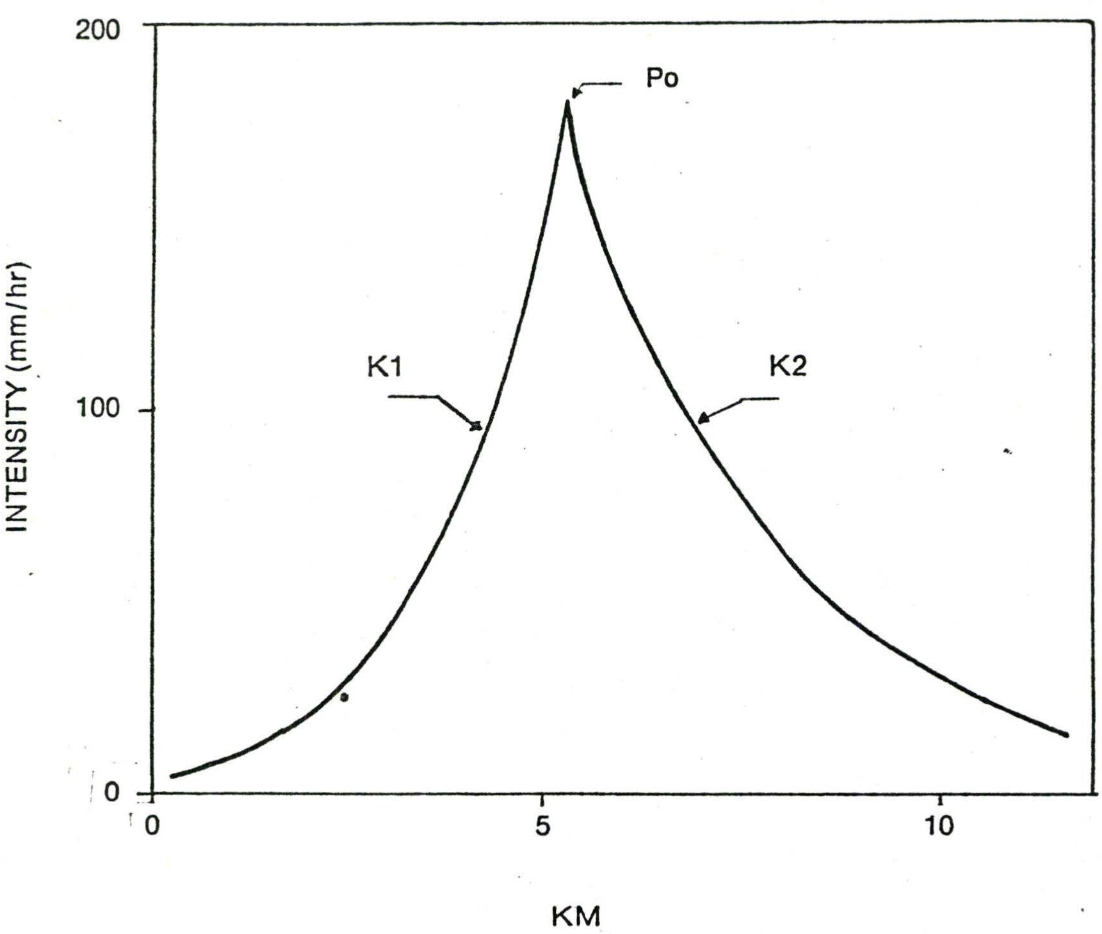


Figure 3: Cross-Section of Typical Line Convective Cells

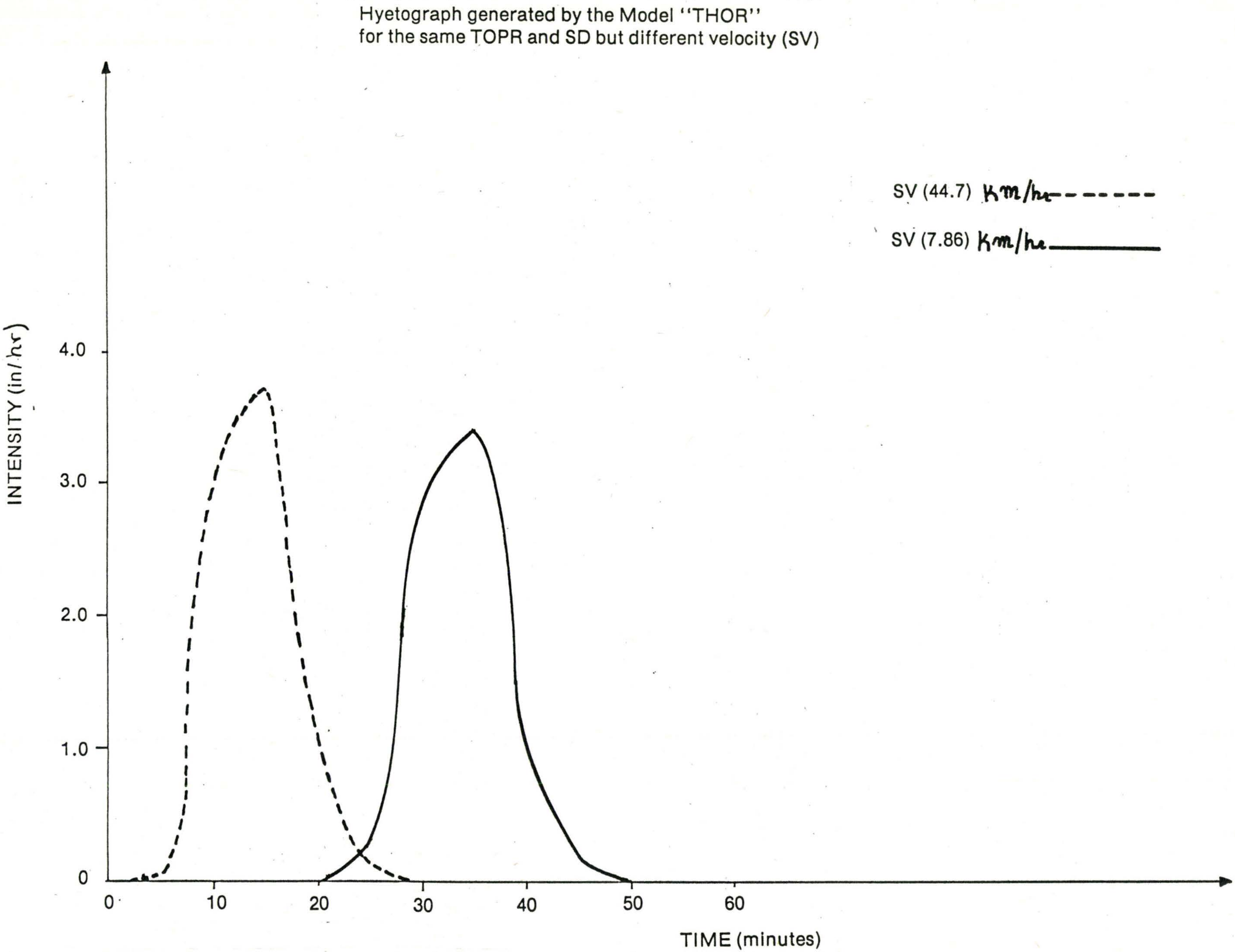


Figure 4: Hyetographs Generated by the Model THOR for the Same TOPR and SD but Different Velocity

4.2 Input Data

1. At least one raingauge is required inside the catchment. A total of 3 raingauges are required to derive the hyetograph motion.

2. At least one station is required in the catchment measuring wind speed (km/hr) and wind direction at ground level continuously. More than one station would naturally provide more accurate and realistic data.

3. Certain subcatchment data is required. Using a map, the size of the subcatchment in all directions, and the distance from the general coordinate system to the subcatchment is measured.

   In Figure 5,

   DX – is the size of the subcatchment in the direction of storm motion

   XO – is the distance to the subcatchment centroid from the general coordinate system

XO is measured from the line perpendicular to the direction (southwest) of the storm as can be seen from catchment no. 7 in Figure 5.

In the computer program THOR, 8 directions that the storm can move are used: N, NW, W, SW, S, SE, E, NE and therefore just four different measurements of DX are required (since the measurement SW is the same as that for NE, etc.). However, for XO, 8 different

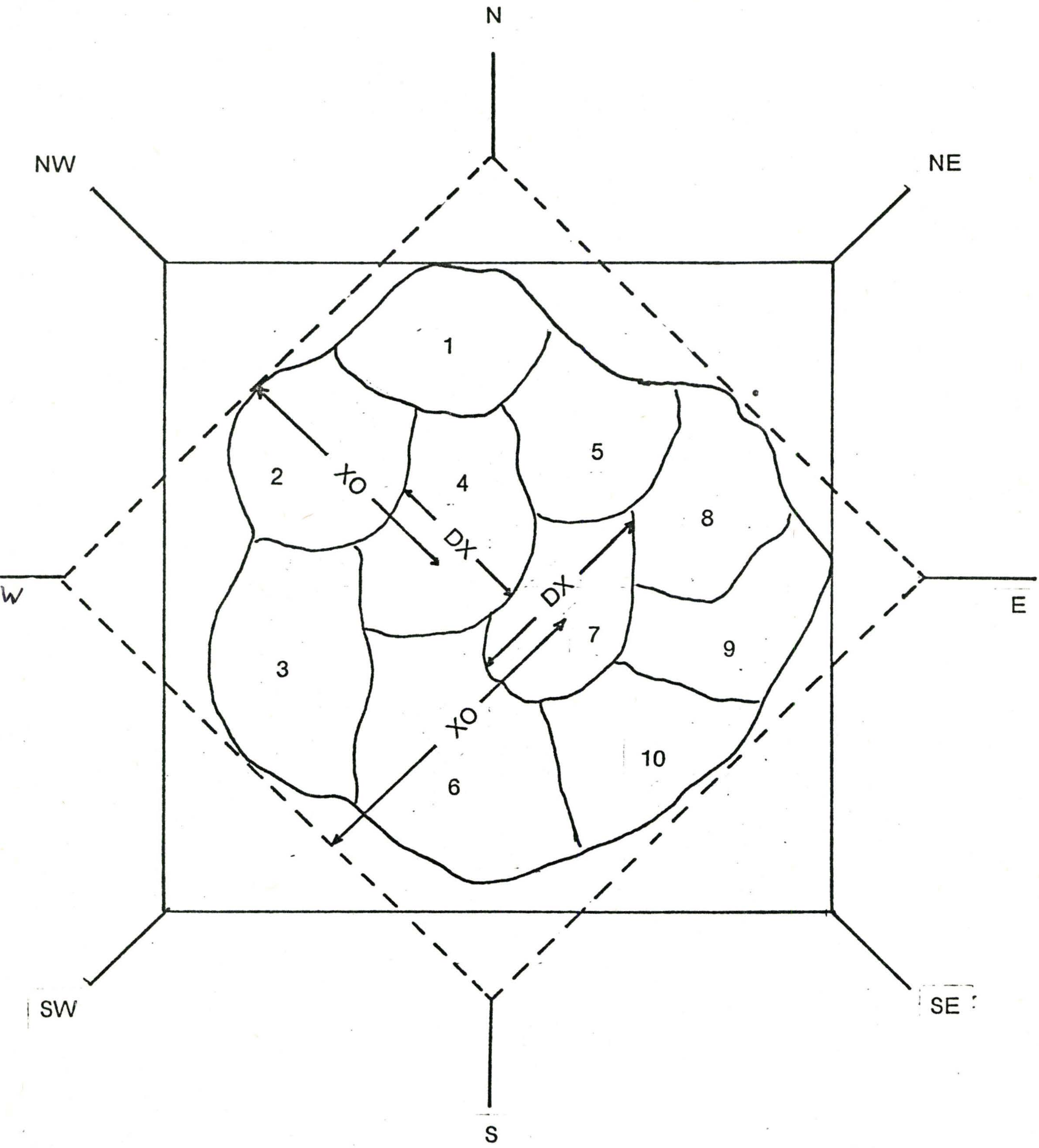


Figure 5: General Co-ordinate for DX and XO

measurements are required.

The program determines the correct size of (DX) and distance to (XO) the subcatchment according to the given storm direction.

4. THOR models thunderstorms with high intensities and short durations. If the duration is longer than 1 hour, the model stops and declares that the model is not appropriate for rainfall with long duration.

4.3 Data Supplied to the Model

Three basic parameters, easily obtained from field stations, are required:

1. TOPR - Total precipitation (mm)
2. WV - Wind speed (km/hr)
3. WD - Wind direction

4.4 Model Equations and Constraints

1. TOPR, WV, WD, - supplied by the user
2. PO = 2.6 + 1.81*TOPR
3. K1 = 0.101 + 0.0025 * PO
4. K2 = 0.0818 + 0.299 * K1
5. VS = 7.39 + 0.933 * VW
6. SD = 33 + 0.884 * WD

7. TR = ALOG (20)/(K1) TR between 0.05 Po and Po

8. TF = ALOG (20)/(K2) TF between 0.05 Po and Po

9. TS = TR + TF

10. DX,XO Supplied by the User

11. XOT = XO * 60/VS

12. DXT = DX * 60/VS

13. TP = XOT + TR

14. P = Po * EXP (-K1 * (TP-T))

Subroutines used in the Model include:

1. ICSICU to calculate point instantaneous precipitation for X1-X2.
2. DCSQDU to calculate the integration for t1 - t2.
3. ICSICU to calculate instantaneous average precipitation over a basin DX long.
4. DSCQDU to calculate average precipitation over basin DX long for duration DT.

THOR generates hyetographs for each subcatchment, thus simulating the motion of a system of spatially-limited storm cells as they move across an urban catchment.

4.5 Incorporating THOR as Part of SWMM

The SWMM is one of many urban runoff models used in Canada and the U.S.A. The RUNOFF block of SWMM requires rainfall data as input in order to calculate hydrographs and pollutographs. THOR has been incorporated as a part of SWMM in such a way that the user has the options to use the model to generate hyetographs or to supply hyetographs to the SWMM on the basis of a static storm. THOR has been incorporated as a part of the general procedure for executing the SWMM.

The hyetographs which are generated by the THOR model retained as a disc file and are passed on subsequently as input to the RUNOFF block of the Stormwater Management Model.

4.6 THOR Sensitivity

The idea behind this sensitivity analysis was to determine how the results produced by SWMM were affected by changing parameters in THOR such as wind velocity, wind direction and total precipitation.

Figure 6 shows the sensitivity of peak flow, peak BOD and peak S.S. to different wind velocities (0.1, 20, 40 km/hr) in two different directions: (a) 45 - direction of the storm against the drainage systems (up), (b) 225 - direction of the storm in the same direction as the drainage system (down).

Figure 7 shows the sensitivity of total flow, total BOD and total S.S. to different wind velocities and different wind directions.

Figure 8 illustrates the computed hydrograph at a wind velocity of 20 km/hr. From Figures 6 and 8 we see that computed results are sensitive, especially at a wind velocity of 20 km/hr and to direction especially 225, down the catchment.

The sensitivity of peak and total flows, S.S. and BOD to total precipitation and to stationary or moving storms was tested. For a stationary storm, the "Hamilton-Wentworth Design Storm" applied

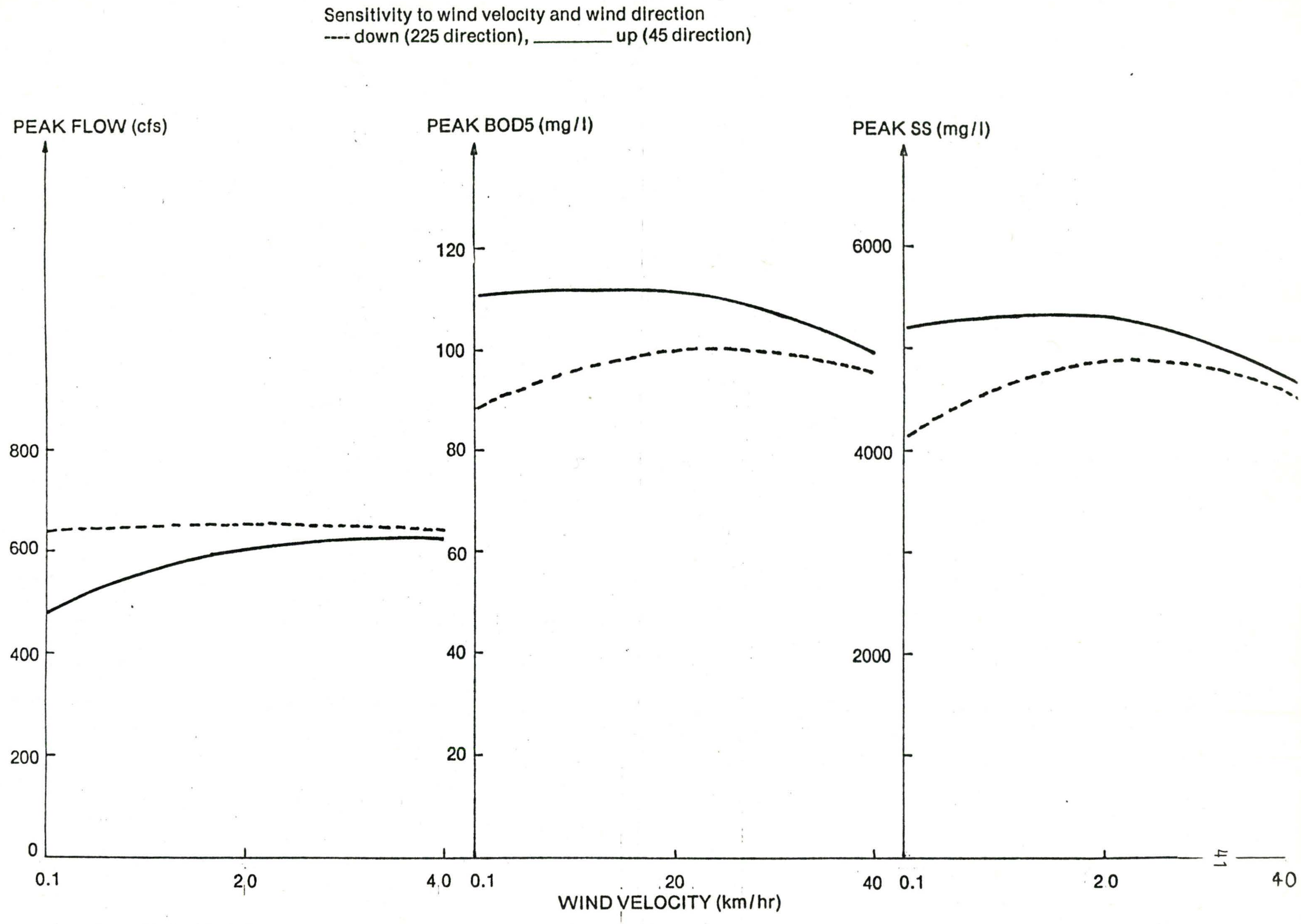


Figure 6: Sensitivity to Wind Velocity and Wind Direction

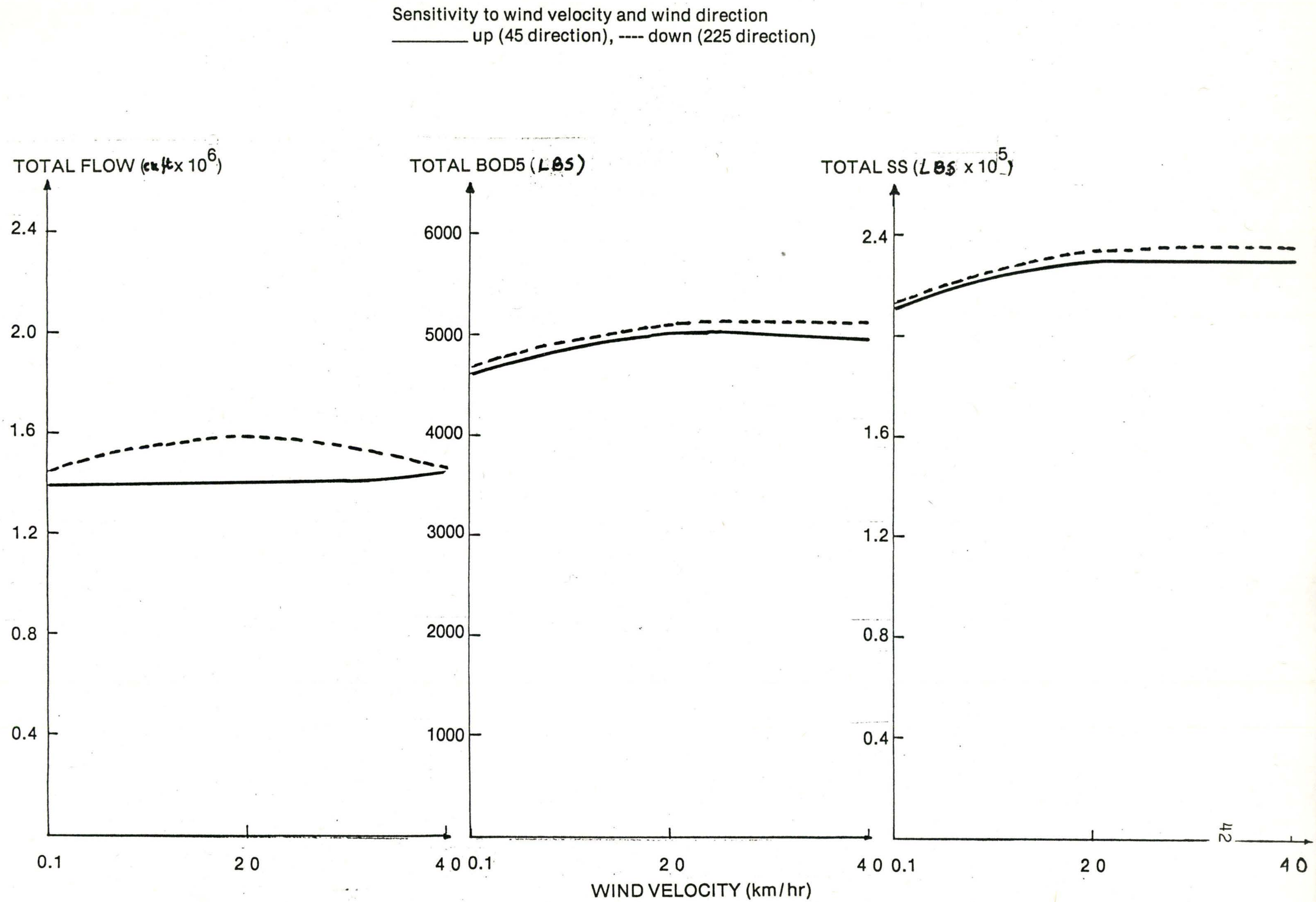


Figure 7: Sensitivity to Wind Velocity and Wind Direction

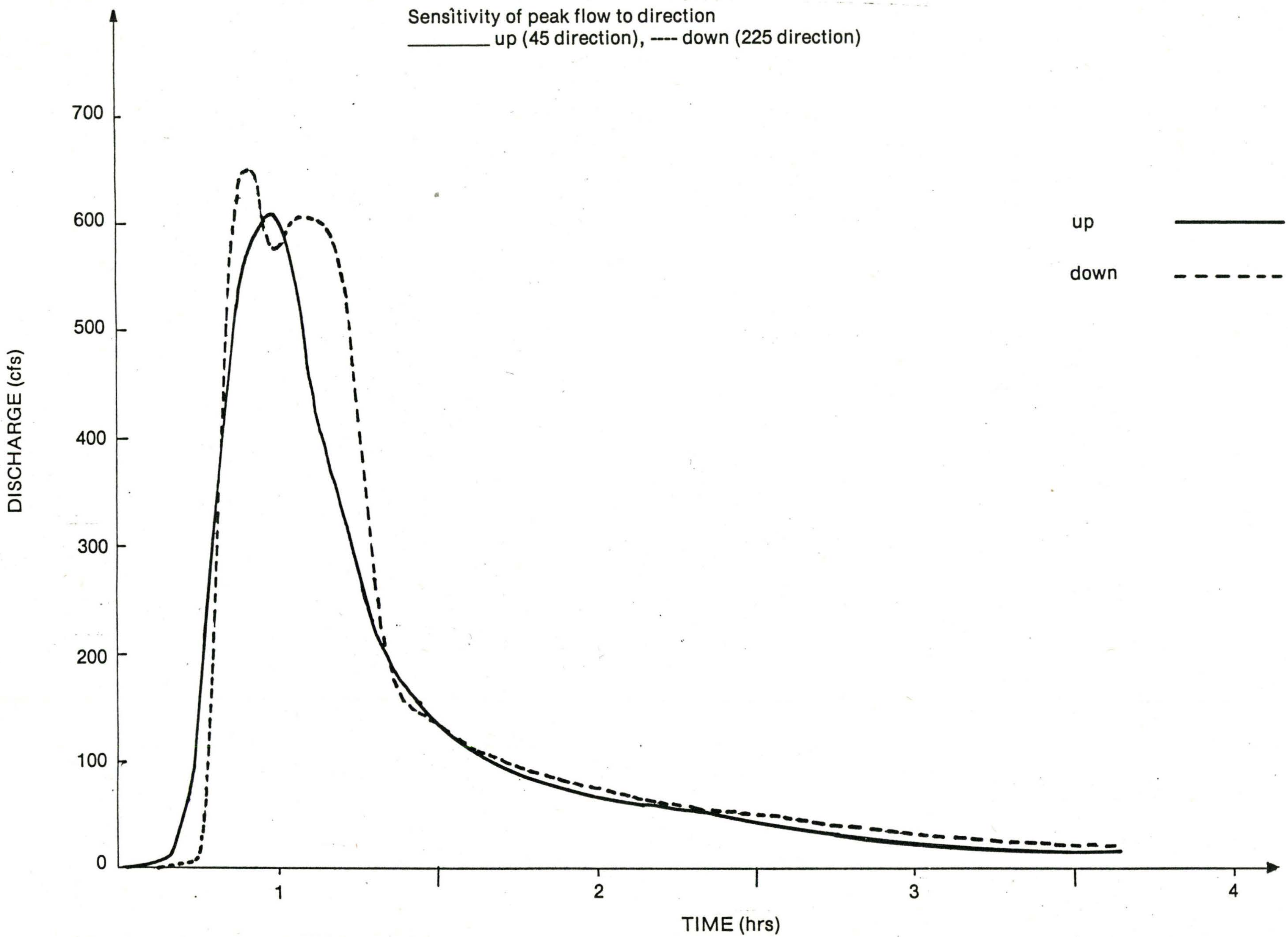


Figure 8: Sensitivity of Peak Flow to Direction

uniformly across the catchment (2.53 in.). For moving storms, total precipitation of 65, 130, 200 mm for 20-km/hr wind speed and 225-wind direction were used.

A total precipitation of 65 mm is produced by a stationary "Hamilton-Wentworth Design Storm". Figures 9 and 10 show the results.

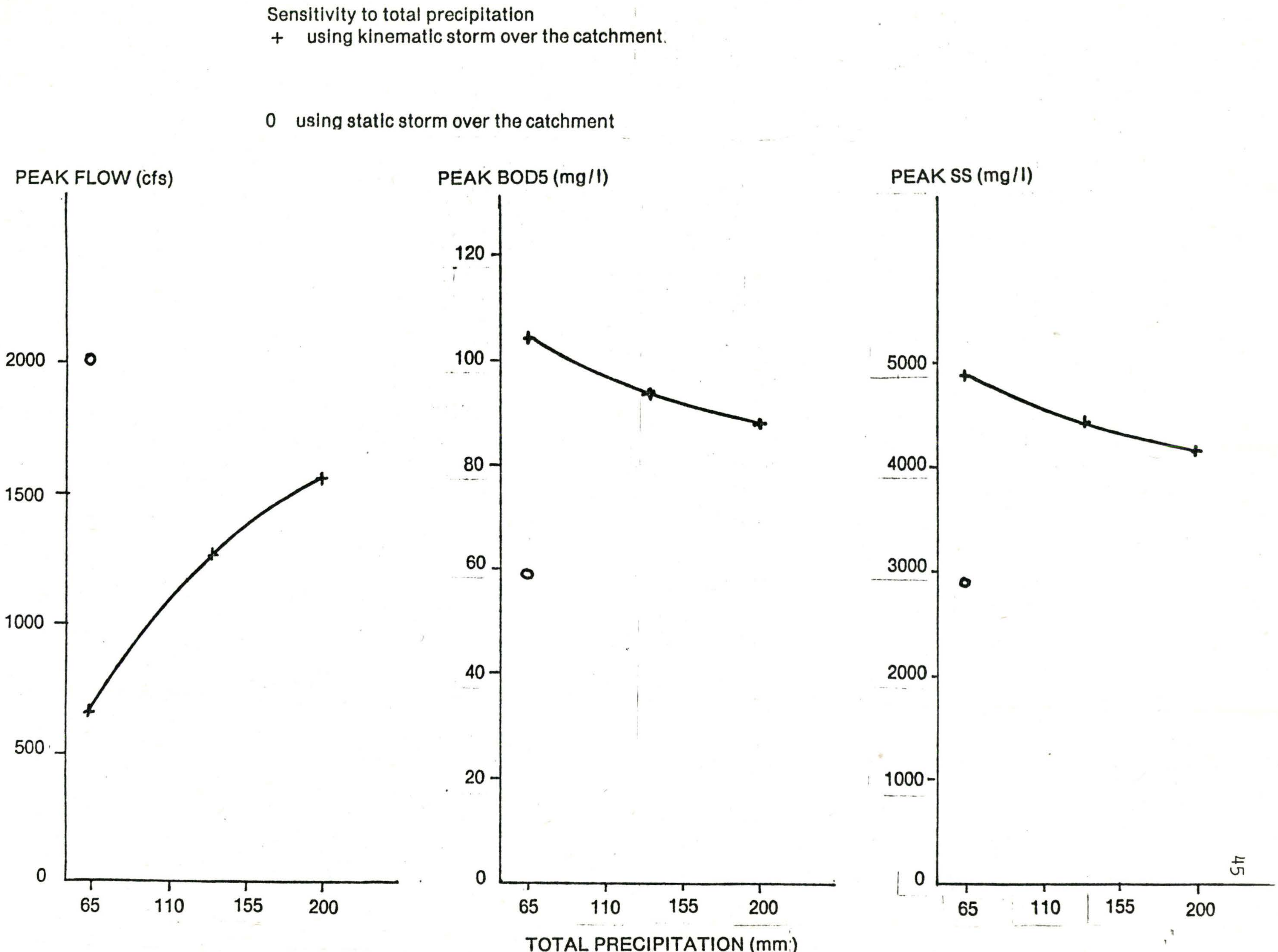


Figure 9: Sensitivity to Total Precipitation

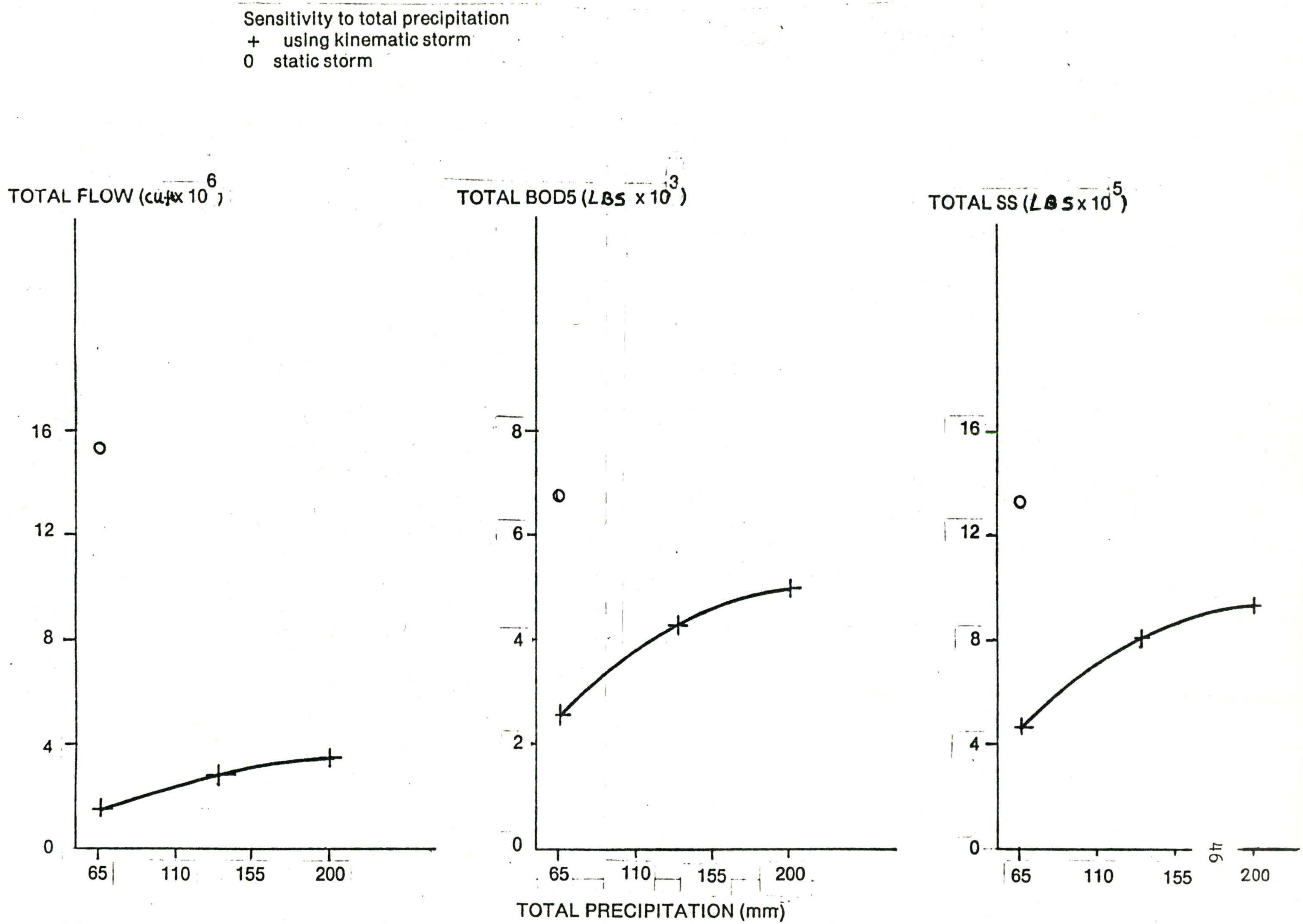


Figure 10: Sensitivity to Total Precipitation

CHAPTER 5

DATA COLLECTION AND ANALYSIS

5.1 Introduction

Beginning in early summer 1980 a network of streamflow and rain gauges was established in the City of Hamilton to provide the following information for all significant storms (22,27):

1. Precipitation, spatial and temporal distribution,
2. Runoff quantity,
3. Runoff quality.

The data was used to calibrate SWMM for the Hamilton urban drainage system.

The precipitation measurements provide information on thunderstorm kinematics in the City, which is complicated by a valley and escarpment.

The Chedoke Creek drainage basin was used to calibrate SWMM in this study. In this basin the "Hydro substation" streamflow gauge was installed to measure the flow in the drainage channel. Water quality samples were collected at the streamflow gauge site for each major storm in the period June-August. The criterion for initiating sampling is that the rainfall intensity must exceed 0.25 in. (6.35 mm) per hour.

Water quality samples were analyzed by the Ministry of the Environment laboratory in Toronto for suspended solids, BOD5, total-N and total PO4. This chapter describes the installation and calibration of the field equipment and the analysis of the data used to develop THOR (22, 27).

5.2 The Chedoke Creek Catchment

Hamilton is possibly the most heavily industrialized city in Canada. In addition to the two primary steel making companies in Canada, Hamilton supports a population exceeding 300,000. The intense industrial activity and highly urbanized nature of the city strain the receiving waters bordering the city; the Hamilton Harbour and Coote's Paradise. As a result of their recreational and educational benefits to the city, concern has grown regarding the pollutant loading to these receiving waters (22, 27).

Hamilton is naturally divided into two zones by the Niagara Escarpment. An elevation difference of about 350 feet exists between the old part of the city built around the Harbour and newer part of the city on the plateau above the escarpment. Below the escarpment the stormwater drainage system is essentially a combined system while above the escarpment separated storm and sanitary sewers dominate.

The Chedoke Creek catchment is located in the west end of the city. The catchment is typical of the Hamilton area but does not include any industry. Most of the area is single family residential and underdeveloped area or parks. The Chedoke Creek drains to Coote's Paradise.

5.2.1 Catchment Topography and Land Use

Ignoring the vicinity of the escarpment, the catchment is gently sloping (average slope = 2%) from the south towards the drainage outfall located in the northeast corner. The entire catchment area is zoned as single family residential or undeveloped area for parks.

5.2.2 Hydrologic Characteristics

The three main hydrologic characteristics of a watershed which influence the generation of surface runoff during storm events are: the degree of imperviousness, the depression storage, and the infiltration capacity. A brief description of these parameters is presented below.

5.2.2.1 Imperviousness -

The impervious areas consist of the roof area, driveways, parking lots, streets and sidewalks. The overall imperviousness is approximately 20%.

5.2.2.2 Depression Storage -

Field inspection of the impervious areas and study of available plans showed that the potential for depression storage in the parking lots, on the road pavements and on the flat roofs of the food stores and apartment buildings could be useful for stormwater management. The roofs of single-family units have conventional slopes inducing only an initial wetting loss.

5.2.2.3 Ground Infiltration Capacity -

Most of the upper Hamilton area above the Niagara Escarpment has a shallow clay cover over flat fissured limestone layers. Below the Escarpment the soils range from Ancaster silt loam to Oneida clay loam complex to Beverly slit loam.

A great part of the study area is located above the escarpment, and this geological formation is expected to influence the infiltration characteristics of the pervious surfaces. Other factors affecting the infiltration capacity are the type of soil, vegetation, slope and the maintenance of the pervious surface.

The actual ground infiltration capacity and its variation has not been monitored or measured for this area. The hydrologic characteristics of the study are further dealt with in the presentation and discussion of the observed and model results.

5.2.2.4 Street Maintenance -

The Department of Engineering of the Regional Municipality provided the following street sweeping frequencies in the test catchment:

single family residential	once per month
undeveloped and parks	once per month
commercial and industrial	once per month

5.2.2.5 Meterological Data -

The wind velocity and direction was obtained from the Atmospheric Environment Service (AES) gauge at Mount Hope Airport. In general the wind direction is southwesterly or southeasterly.

5.3 Field Instrumentation

The following paragraphs 5.3, 5.4, 5.5 were abstracted directly from a report written by James (27).

5.3.1 General Criteria

With the ultimate objective of field verification of the quantity and quality simulation subroutines for the Stormwater Management model, the key elements of the required field data acquisition system included:

- rain gauge
- streamflow rate
- sampling
- recording facilities (eg. chart recorder)

Selection criteria for instrumentation required for this study were: accuracy of sensing and recording, repeatability of records, reliable performance under adverse weather conditions, continuous monitoring, ability to cover as wide a range of values as possible from dry to wet weather conditions, and the vital necessity of providing synchronized data sets for each monitored event. The rainfall and flow data monitoring, recording system and sample collection are briefly described in the following sections (27).

5.3.2 Instrumentation Set-Up

A penumatic level sensor (bubbler gauge) was installed at the Hydro Substation gauging station. The pneumatic level sensor measures the pressure required to force a bubble of air or nitrogen from a tube against a head of water (27).

5.3.3 Rain Gauges

Three rain gauges were installed: at McMaster University, Mohawk College, and Garth Street Reservoir. In addition, the existing gauges at Hamilton Airport and the Royal Botanical Gardens were also used.

Rain gauges were also already in existence at the Mt. Albion Falls and Christie Dam Conservation areas. Photocopies of all rain events recorded at these sites were obtained.

Permission was obtained to install gauges at the Garth Street Reservoir, Mohawk College and McMaster University from the relevant authorities. Tipping bucket gauges were installed on a building roof at each location to protect against vandalism. The recorders, located inside the building, were connected to a 12 V power source and the tipping bucket gauge by 18-gauge speaker wire. A totalizing gauge was installed beside each tipping bucket gauge. The totals are used to obtain correction factors for the tipping bucket gauges.

Maintenance of the rain gauge at a particular location depend on the type of recorder being used (27).

5.3.3.1 <u>Types of Recorders (27)</u>

1. Honeywell Electronic 194 - This is a continuous strip chart recorder which uses an ink pen. The chart speed was set at 6 in./hr. The recorder is located in the Computational Hydraulics Group's Laboratory, Room 233, JHE Building at McMaster University. The chart drive operates from a 110 V power source while the pen is driven by a 6 V dry cell battery through the tipping bucket gauge. Timing marks are set manually several times during a workday. The inkwell requires refilling at the beginning and end of each weekend and at least once during the week.

2. Rustrak - This is a continuous chart recorder recording on pressure sensitive paper. Chart speed on these recorders was set at 5 in./hr. The chart drive operates from a 12 V power source and each chartroll lasts for just over 6 days. The power must be switched off when the rolls are being changed because of the sensitivity of the 2 amp fuse in the power source. Timing marks need to be made when changing rolls and should be made as soon as possible before and after each rain event. These types of recorders were located at Garth Reservoir and Mohawk College.

5.3.3.2 <u>General Difficulties with the Rain Gauges (27)</u> -

1. The chart speeds of 6 in./hr. (Honeywell) and 5 in./hr. (Rustrak) generate large amounts of paper (12 and 10 ft/day respectively). The paper for both recorders cannot be recycled. The pressure

sensitive paper cost $5.6 per roll and lasts about 6 days.

2. Doorbell buttons were incorporated into the Rustrak/tipping bucket circuit to facilitate setting timing marks.

3. Sporadic problems occurred with the ink and drive-belt of the Honeywell recorder resulting in a loss of data as well as additional maintenance.

4. A maintenance circuit by car of the rain and flow monitoring sites takes approximately four hours.

5. There is some error in the timing of the recorders. This error has been as great as +1/2 hour per day. To minimize errors, the Rustraks should be operated in the tear-off mode.

6. The resolution of the Honeywell and Rustrak recorders is one minute.

7. The Mohawk gauge can only be serviced during normal working hours from Monday to Friday.

5.3.4 Rainfall Rate

The rain gauges were of the tipping bucket type and the charts from the rain gauges were removed after each storm. Five minute rainfall intensities were calculated for all tipping bucket stations and one minute insensities were calculated for the Mohawk, McMaster and Garth Street Stations.

The five minute intensities were plotted aginst time, to obtain hyetographs for each station and for each individual storm (27).

5.3.5 Flow Rate

5.3.5.1 Construction and Installation of Field Site (27)

At the Hydro Substation site the weir was installed in a single trapezoidal channel with a bottom width of 12 ft.

At this location the Chedoke Creek channel flows into a single box culvert passing beneath an access ramp of the Chedoke Expressway. Using sandbags in a semicircular fashion to divert flow around the area of construction, angle-iron brackets and galvanized tin flashing were installed in the channel. The angle-iron brackets were made of painted 3/16 in. thick steel with pre-drilled bolt holes for fastening to the weir and to the channel bed. Two sets of 16 gauge galvanized iron flashing and angle-iron brackets were set up forming a slot into which the weir could fit. The flashing was used to compensate for the wood not being flush with the concrete floor of the channel. Caulking was applied along the flashing to prevent water seeping under the weir. The angle-iron brackets were bolted to the channel using 4 in. long, 1/2 in. diameter wedge-anchor redhead bolts obtained from R.G. Glover Ltd. Holes for the bolts were made using a power drill with a 1/2 in. bit. A 2750 watt gasoline generator provide the power to operate the drill. The weir was set in place and bolted to the angle-iron using 2.5 in.

long, 3/8 in. diameter bolts.

After the weir was in place, the sandbags were removed. The steel box for housing the bubbler-gauge recorder batteries and air cylinder was installed next.

The box was supported by steel brackets from a concrete wall on the north side of the channel. A piece of 2 in. flashing was bolted to the top of the weir and adjusted so that the top of the weir was approximately level across the width of the channel. A flexible plastic tube, attached to the air cylinder within the recorder box was housed in a copper sleeve, bolted to the side of the channel and terminated approximately 9 inches below the crest of the weir (27).

5.3.5.2 Weir Calibration (27) -

The height of the water over the weir (h) and the discharge (Q) were obtained from streamflow gaugings. Points were obtained over a range of flows. When as much data as possible had been collected, curves of the following approximate form were derived:

Q = A*h**b

where: A is constant coefficient and b is a constant exponent.

A theoretical equation was developed for the weir. A least-squares linear regression analysis of the gauging data was made using an HP 41-C calculator to solve for constants A and b. The correlation coefficient between both equations was computed.

No. of Gauging Points	Theoretical Equation	Field Equation	Correlation Coefficient
8	Q=36.8h**1.5	Q=74.5h**1.534	0.982

5.4 Runoff Quality Sampling

5.4.1 Sampling Equipment

Sampling equipment consisted of a pole, bottles, a flashlight, rain parka and rubber boots, adjustable wrench, clipboard, pen and polyethylene bag and sheet. Bottles were provided by the Ministry of the Environment and the remainder of the equipment was purchased from a variety of sources.

A sampling pole consisted of three, 5 foot lengths (1.32 m) of glavanized steel pipe. The pipes were of three diameters (1/2 in., 3/4 in., 1 in.) in order to permit them to fit within one another in a telescopic manner, making the pole easier to transport.

The sections, when extended, were held in place by a screw fitted through holes drilled in each pipe and held in place by a wingnut. Bottles were fixed to the pole using adjustable strapping welded on to the end of each pole (27).

5.4.2 Water Quality Sampling

A "rain alert" existed on days when weather forecasts predicted a significant amount of rain. These forecasts were obtained from the radio, the Hamilton Airport (679-6065) or the Atmospheric Environment Service in Toronto (676-3066 tape, 676-4567 detailed). The alert is based on forecasts that heavy rain is expected but the final decision to sample was based on local judgement.

For any rainstorm with an intensity exceeding 0.25 inches (6.35 mm) per hour sampling was attempted. We ensured that the sampling "kits" were complete and ready for use well in advance of the storms. These kits consisted of a rain parka, a clipboard, masking tape, a couple of pens, a flashlight, a polyethylene bag and sheet, a screw driver, a wrench and a sampling pole.

On the day of a "rain alert" watches were synchronized to a quartz clock in our laboratory, set to National Research Council time in Ottawa. Timing marks were made on the tipping bucket and flow gauge recorders, both before and after the storm whenever possible.

The following procedure was followed at the sampling site:

1) A timing mark was made on the recorder.

2) The sampling pole was assembled.

3) The data and bottle number was marked on a bottle and the box as follows:

Station	Box	Bottle
A	2	5

4) The bottle was attached to the pole.

5) The time of the sample was marked on the box.

6) The sample bottle was filled from the true left bank of the channel.

7) Chart readings were taken with each sample as soon as possible after the sampling bottle was filled.

8) The bottle was removed from the pole and placed back in the box in its proper place.

9) Steps 3 to 8 were repeated as frequently as possible on the rising limb of the hydrograph (about every three minutes), Less frequent sampling (5 to 10 minute intervals) were required on the recession limb.

10) Typically, a total of 24 to 30 samples were taken during a storm.

After the sampling was completed, all samples were returned to the laboratory at McMaster where submission sheets were filled out for each box of bottles and the samples were checked to make sure the data and sample numbers were properly recorded on each bottle label.

The samples were transported by car within a day or so to the Ministry of the Environment Laboratory in Toronto for analysis.

Sample analysis was carried out for the following parameters:

Solids - suspended solids (SS)

Oxygen Demand - biochemical oxygen demand (BOD)

Nutrients - (ammonia (NH_3) nitrates and nitrates (NO_3 + NO_2) = Total-N

- Phosphorous -(P)

Pollutographs, concentration (mg/l) versus time, were plotted for each pollutant (27).

5.4.3 General Difficulties

1. Weather forecasts should not be used to make asampling decision but simply to alert personnel to be prepared to make such a decision.

2. Transportation posed a serious problem for data collection. Several storms were not sampled because transportation was not available for personnel willing to sample. As well, loading bottle boxes into cars was time consuming. A van would be more suitable. A supply of boxes could be stored in the van. All personnel could be transported to and from the site more easily.

3. Kits, bottles and sampling assignments should be made ready as early as possible to ensure that as much of the rising limb as possible is sampled.

4. Where possible, recording equipment should be checked piror to a storm. Again transportation needs sometimes hindered

this. Equipment failure resulted in incomplete data for several storms.

5. Marking data on the flaps of the boxes during storms proved easier than trying to write data on forms. The boxes could be kept fairly dry beneath the polyethylene sheets. This data could easily be transferred from the boxes to the forms at the laboratory (27).

5.5 Raingauge Data - Interpretation

This study was done in the west end of the city of Hamilton. The rainfall-runoff model was run for that part of the Chedoke Creek Catchment within subcatchment No. 1 (see Figure 11). The raingauges closest to the area No. 1 were used:
Royal Botanical Gardens, McMaster University, Mohawk College, Garth Street Reservoir and Hamilton Airport.

Charts from the raingauges were removed after each storm unless the storm occurred outside normal working hours. The charts were corrected for timing errors.

One minute and five minute intensities were calculated using tip capacities of 0.01 inches/tip (0.254 mm/tip) at McMaster and Garth Street and 0.2 mm/tip at Mohawk, Hamilton Airport, and RBG.

Rain data was coded in SWMM format and archived on the computer. When coding, the following rules were observed:

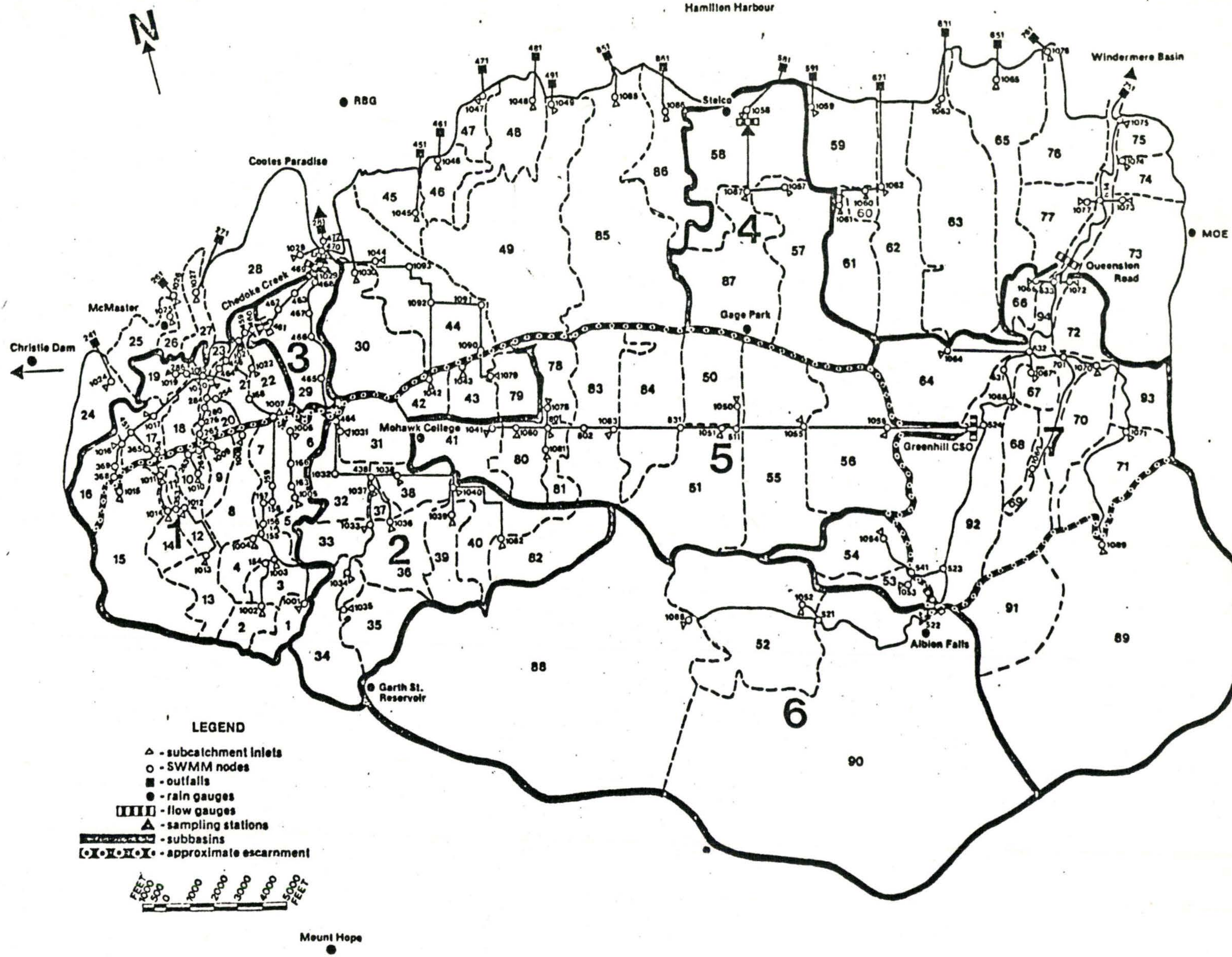


Figure 11: Subcatchment Discretization, source (22)

1. 10 intensities per line.

2. A zero was entered for any period when there was no rain.

3. For each storm the first card was used to identify the date, starting time, frequency, interval and location of the storm. For example,

Storm	Year	Month	Day
1	80	06	15

Time	Location	Dt (time interval)
0911	Mac	5 (minute)

4. The following codes were used to identify the different raingauge sites

RBG Royal Botanical Gardens

MAC McMaster University

MOH Mohawk College

GAR Garth Street Reservoir

HAP Hamilton Airport

Five minute rainfall intensities were calculated for all tipping bucket stations and one minute intensities were calculated for the Mohawk, McMaster and Garth Street stations.

The five minute intensities were plotted against time to obtain hyetographs for each station and for each individual storm (27).

5.5.1 General Suggestions

1. The maximum resolution that could be obtained for the Hamilton Airport and RBG charts was 5 minutes.

2. For a 1-minute frequency, 1 tip represents 0.6 in./hr. (15.2 mm/hr) for McMaster and Garth Street and 0.473 in./hr. (12 mm/hr) for Mohawk.

3. For a 5 minute frequency, 1 tip represents 0.12 in/hr (3.0 mm/hr) for McMaster and Garth and 0.095 in./hr. (2.4 mm/hr) for Mohawk, RBG and Hamilton Airport.

4. The Bendix Friez recorders at Christie Dam and Mount Albion Falls were not used in this study because: (a) the charts had a resolution of only 30 minutes, (b) they are located too far outside the catchment.

5. The Garth Street recorder was observed to experience timing errors of up to 30 minutes per day (27).

5.6 Correction Factor for Timing Errors

Initially there were two raingauges with timing errors: McMaster and Garth Street Reservoir. The error at McMaster was fixed, the paper moved at a velocity of 143 mm/hr instead of 151 mm/hr. Thus, the timing was 3 1/2 minutes per hour slow.

At Garth Street the error was random, not fixed. The paper was changed as soon as possible after each storm ended and a correction factor applied according to the velocity of the moving paper and the times marked on the paper before and after the storm.

5.7 Hyetograph Characteristics

It is usual to use data from several gauges to estimate the average precipitation for an area and to evaluate the reliability of any one gauge.

A single recording raingauge measures point rainfall only at the location. This may be a poor estimate of precipitation prevailing over the area as a whole.

Given the records of one or more recording raingauges within or reasonably near a given area, rainfall is found to vary in intensity (1) during the course or duration of individual storms (2) throughout the area covered by individual storms, and (3) from storm to storm. These variations establish respectively: (1) the time-intensity or intensity-duration relationship of individual storms, (2) the areal distribution of individual storms, and (3) the frequency of storms of given intensity and duration.

Precipitation data are used to estimate the areal variability of rain and for developing design storm characteristics for small watersheds. These design storms do not represent a real storm moving across an area. Thus, this procedure has many misconceptions associated with it.

In this study the rain model is an infinitely wide rainband across which the rainfall intensity decays exponentially at different rates ahead and behind, the peak intensity: i.e. the hyetographs have a short duration, high intensity and have an exponential form:

P=PO*EXP(-K1(TP-T))   T<TP

P=PO*EXP(-K2(T-TP))   T>TP

The relationship between the following components of the individual recorded hyetographs are studied:

PO - The maximum peak intensity (in./hr. or mm./hr.)

tp - The time to the maximum peak

TOPR - Total precipitation during the storm

K1,K2 - The exponential shape factors of the hyetographs

5.8 Hyetographs Data Analysis

For every storm during the period May - September 1980, the hyetographs from all 5 raingauges were plotted. For each raingauge PO, TOPR, TP, and K1, K2 were calculated.

5.8.1 Calculation of K1 and K2

In all cases, the dependent variable Y is plotted along the ordinate and the independent variable X along the abscissa. The mean relationship between X and the corresponding Y is referred to as the regression curve of Y on X,

The regression equation is of the form

$$Y=b0+b1x1+b2x2+.....+(bn)(Xn)$$

and in all cases the regression finds the least squares linear equation for predicting Y from n predictors X1, X2...Xn. b0, b1,...bn are called the regression coefficients, and were calculated using the "Minitab" general purpose statistical computer package.

K1 and K2 were calculated as follows:

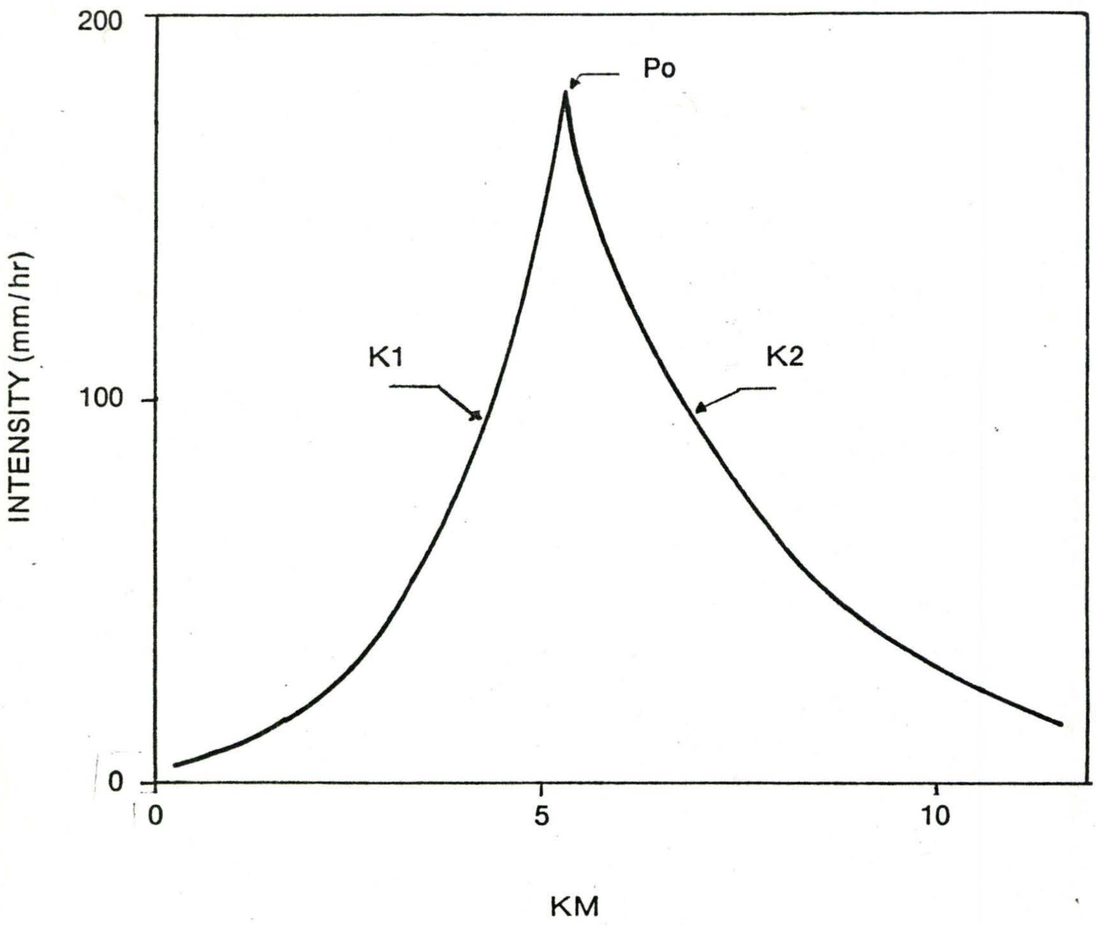
$$LOGE(P/Po)=LOGE(EXP(-K1(TP-T)))$$

$$LOG\ P-LOG\ Po=-K1(TP-T)$$

By assuming TP=0, the exponential shape factor of the hyetographs on both sides of Po is calculated:

$$LOG\ P = LOG\ Po + K1T$$

This equation represents a straight line and fits the regression equation. P,Po,t were selected from the hyetographs, and input to "Minitab" using the command "Regression" to set K1 and K2. Minitab also gives the regression equation and other statistics coefficients such as standard deviation, T-Ratio and R-Squared. An example follows:



ure 12: Cross-Section of Typical Line Convective Cell

```
--
? READ C1 C2
--
? 0 14.4
--
? 5 7.2
--
? 10 2.4
--
? LOGE C2 C3
--
? REGR C3 1 C1
THE REGRESSION EQUATION IS
Y = 2.73 - 0.179 X1
                                      ST. DEV.   T-RATIO =
          COLUMN  COEFFICIENT   OF COEF.   COEF/S.D.
            --       2.7348       .1511       18.10
X1        C1       -.1792       .0234       -7.65

THE ST. DEV. OF Y ABOUT REGRESSION LINE IS
S =    .1655
WITH ( 3- 2) = 1 DEGREES OF FREEDOM

R-SQUARED = 98.3 PERCENT
R-SQUARED = 96.6 PERCENT, ADJUSTED FOR D.F.

ANALYSIS OF VARIANCE

 DUE TO          DF          SS          MS=SS/DF

REGRESSION        1       1.60520       1.60520
RESIDUAL          1       0.02740       0.02740
TOTAL             2       1.63260
```

C1 - represents time t

C2 - represents the intensities P

C3 - is the natural log of P.

Therefore the regression equation is:

y = 2.73 - 0.179 X1

i.e. K1 = 0.179

5.8.2 Summary of Hyetographs Data:

Table 1 is a summary of 36 observed hyetographs. For all the hyetographs the total precipitation in mm. was calculated by hand, and the maximum peak intensity (mm/hr) was estimated by plotting.

Table 1

Summary of Hyetographs Data

	Po	K1	K2	TOPR
1	4.8	.0693	.0693	1.40
2	4.8	.2260	.0693	.80
3	4.8	.2260	.0693	1.00
4	48.0	.1560	.1960	9.08
5	7.20	.1950	.1950	1.80
6	31.20	.1100	.1390	8.55
7	21.40	.1960	.0200	3.55
8	30.40	.1570	.1230	8.37
9	75.00	.1460	.1170	17.37
10	48.00	.0900	.1000	14.60
11	56.00	.1540	.1870	11.63
12	60.00	.1560	.1480	14.80
13	52.00	.2130	.2160	10.33
14	21.80	.1980	.1270	6.10
15	9.80	.1000	.1000	2.45
16	14.40	.1220	.0960	3.80
17	30.50	.1630	.1260	9.17
18	40.40	.1910	.1160	10.73
19	29.00	.2580	.1730	6.57
20	27.80	.2230	.1590	6.55
21	115.00	.5400	.2380	24.75
22	125.00	.5500	.1940	19.91
23	62.00	.1300	.1270	9.66
24	76.00	.1560	.3200	11.25
25	7.20	.1630	.1630	2.00
26	12.20	.3200	.3200	1.53
27	28.80	.0850	.0570	12.20
28	30.00	.2000	.1600	8.33
29	34.00	.1830	.1340	8.80
30	24.00	.1270	.1410	9.32
31	14.40	.1020	.0600	5.40
32	24.40	.2400	.1000	6.15
33	19.20	.1600	.1080	5.60
34	4.80	.2260	.1360	2.05
35	15.40	.1090	.0471	7.92
36	14.40	.1450	.0855	3.37

5.9 Correlation and Regression

The correlation between the different hyetograph variables was examined. The correlation coefficient between data in any two columns was calculated by the usual (Pearson moment) correlation between the two columns. When a reasonable correlation was found the regression between these parameters was examined.

Table 2 represents the correlation between: Po, K1, K2, TOPR over 36 data items.

Table 2:

Correlation of Hyetograph Data

	P_o	K_1	K_2
K_1	0.573		
K_2	0.453	0.458	
TOPR	0.919	0.416	0.292

Comparison to the critical value of $r_{n,\alpha}$, of the linear correlation coefficient r shows:

n \ α	20%	10%	5%	2%	1%	0.2%
36	0.219	0.279	0.329	0.386	0.424	0.498

The values underlined show that the probability from an uncorrelated population is less than 1% or less than 0.2%. Thus apparent correlation is real and true for more than 99% or 99.8% of the cases. The reasons for the correlation are discussed later.

The regression between Po and K1:

K1 = 0.119 + 0.002 Po (r=0.573)

The regression between Po and TOPR:

Po = -4.25 + 4.8 TOPR (r=0.919)

The regression between K2 and K1:

K2 = 0.0818 + 0.299K1 (r=0.458)

The latter relationship was adopted in THOR. The statistical background to the analysis is presented in Appendix D-1.

5.10 The Storm Data

The storm data were deduced from three synchronized hyetographs from the three raingauges: McMaster University, Garth Street Reservoir and Mohawk College.

Data for 22 separate storms was collected. The following hyetograph characteristics were evaluated: total precipitation (TOPR); peak rainfall intensity (Po); time to peak intensity (tp); the shape constants for the hyetographs (K1); wind speed (VW) and direction (WD) were measured during the storm only at Mount Hope Airport.

5.10.1 TOPR - Total Precipitation (mm)

The computation of the average precipitation for the storm over a given area (catchment no. 1) may be calculated by the "Thiessen Polyson" method. In this method the precipitation at each station is weighted in porportion to the area the station is assumed to represent. A Thiessen network is constructed by connecting adjacent stations on a map by straight lines and erecting perpendicular bisectors to each connecting line.

The polygon formed by the perpendicular bisectors around a station encloses an area which is everywhere closer to that station than to any other station. To compute the average rainfall, the area represented by each station is expressed as a percentage of the total area. The average rainfall is the sum of the individual station amounts, each multiplied by the percentage of the area.

$$TOPR=PAVE=SUM(Ka*Pi)/A$$

where: Ka are constant subareas

A is total catchment area = SUM(Ka)

The calculation for total precipitation in mm has been done for 22 storms during the summer 1980.

5.10.2 Po - Maximum Peak Intensity (mm/hr)

Each storm included three observed hyetographs from which the average peak intensity for each storm in (mm/hr) was calculated.

5.10.3 TP - Time to Peak Intensity

Precipitation varies with time within each particular storm. The time to peak varies as well when the storm is moving across the catchment. The time to peak for the 3 observed hyetographs provides a basis for evaluation of the direction (SD) in which the storm is moving and the storm speed (SV).

5.10.4 SD - Storm Direction and SV - Storm Speed

The three raingauges: McMaster, Mohawk and Garth comprise a triangular network. When the distances between the raingauges are available it is simple to use "sine" or "cosine" rule to calculate the angles between the sides of the triangle (see Figure 13).

From the map (distance in km):

AB = 5.33, AC = 3.175, BC = 3.81

A = 44.90	B = 36.03	C = 99.06
COSA = 0.70	COSB = 0.087	COSC = 0.1579
SINA = 0.7059	SINB = 0.5882	SINC = 0.9875

By knowing the time to peak for the different raingauges we can estimate the direction of the storm. The storm direction SD and the distance between the raingauges define the storm velocity (SV).

Assume the storm moving in any direction α and from Station A (see Figure 13).

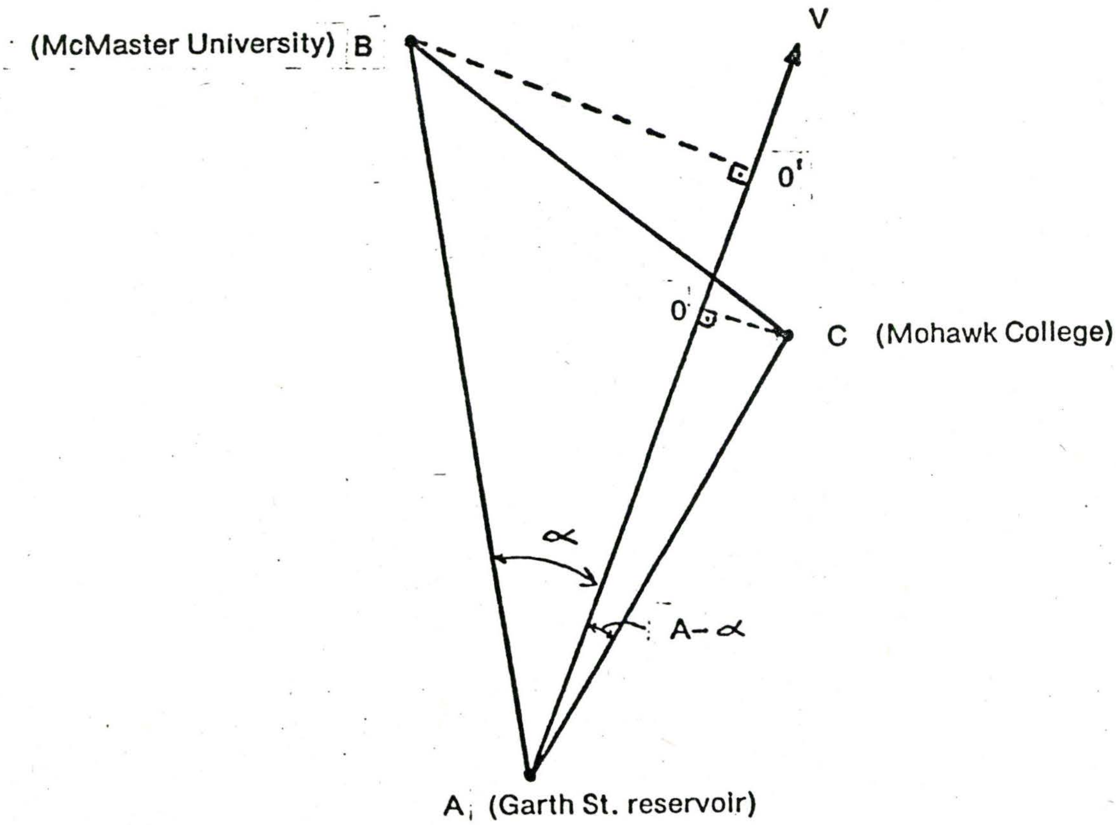


Figure 13: Storm Direction and Storm Velocity

TPab,TPac - is the difference in time between station A and B and A and C, or the time it takes for the storm to move from A to B or A to C.

AB,AC,BC - distances between the raingauge

V - the velocity of the storm

The following steps are required to calculate α and V (V=SV)

(1) $\cos\alpha = AO'/AB$

(2) $AO' = AB \cdot \cos\alpha$

(3) $tp_{AB} = AO'/V$

(4) $V = AO'/tp_{AB} = AB \cdot \cos\alpha/tp_{AB}$

(5) $\cos(A-\alpha) = AO/AC$

(6) $AO = AC \cdot \cos(A-\alpha)$

(7) $\cos(A-\alpha) = \cos A \cdot \cos\alpha + \sin A \cdot \sin\alpha$

(8) $tp_{AC} = AO/V$

(9) $V = AC \cdot \cos(A-\alpha)/tp_{AC}$

(10) $$\frac{AB \cos\alpha}{tp_{AB}} = \frac{AC \cdot \cos(A-\alpha)}{tp_{AC}}$$

Solving equation (10) by substituting distances AB and AC and time to peak tpAB, tpAC, permits evaluation of the exact direction, α in which the storm is moving. Using α for equations (4) and (9) and the distances AB, AC and tpAB, tpAC define the storm speed (SV).

Example: Storm No. 13 Date: 22-08-80

TP - time to peak intensity = (tp)

McMaster (B)	Mohawk (C)	Garth (A)
12:30	12:35	12:22

It is obvious that the storm is moving somewhere between AB toward C, we assume the storm moving in direction α from station A (Garth).

tp_{AB} = 8/60 (hr) tp_{AC} = 13/60 (hr)

AB = 5.33 (km) AC = 3.175 (km)

$A = 44^{\circ}.90^{\circ}$ cos A = 0.7083 sin A = 0.7059

$$tp_{AB} = \frac{AB \cos\alpha}{V} \rightarrow V = AB \cos\alpha / tp_{AB} = \frac{5.33 \cdot \cos\alpha}{(8/60)}$$

(a) $V = 39.9750 \cdot \cos\alpha$

$$tp_{AC} = \frac{AC \cdot \cos(A-\alpha)}{V} \rightarrow V = AC \cdot \cos(A-\alpha)/tp_{AC}$$

$$V = AC\,[\cos A \cdot \cos\alpha + \sin A \cdot \sin\alpha]/tp_{AC}$$

(b) $V = 3.175(0.7083 \cos\alpha + 0.7059 \sin\alpha)/(13/60)$

Solution of (a) and (b)

$$39.9750 \cos\alpha = 3.175(0.7083 \cos\alpha + 0.7059 \cdot \sin\alpha)/(13/60)$$

$$29.5957 \cos\alpha = 10.3442 \cdot \sin\alpha$$

$\sin\alpha/\cos\alpha = \tan\alpha = 2.8611$

$\alpha = 70.73$

$V = 39.975 \cdot \cos\alpha = 39.975 \cdot \cos(70.73)$

$V = 13.19$ (km/hr)

The storm is moving at an angle of 70.73 degrees from the side AB, at a velocity of 13.19 km/hr. For each of our 22 storms, angles between a station and a side are calculated. To establish a uniform coordinate system assume the following:

1. 0 = EAST
   90 = NORTH
   180 = WEST
   270 = SOUTH
   360 = 0 = EAST
2. The direction SD is the direction from where the storm is coming. Example: For SD = 225, the storm is from SW (south-west). In the previous example $\alpha = 70.73$ refers to the side AB.

In Figure 14, the dashed line represents the direction SD in which the storm is moving.

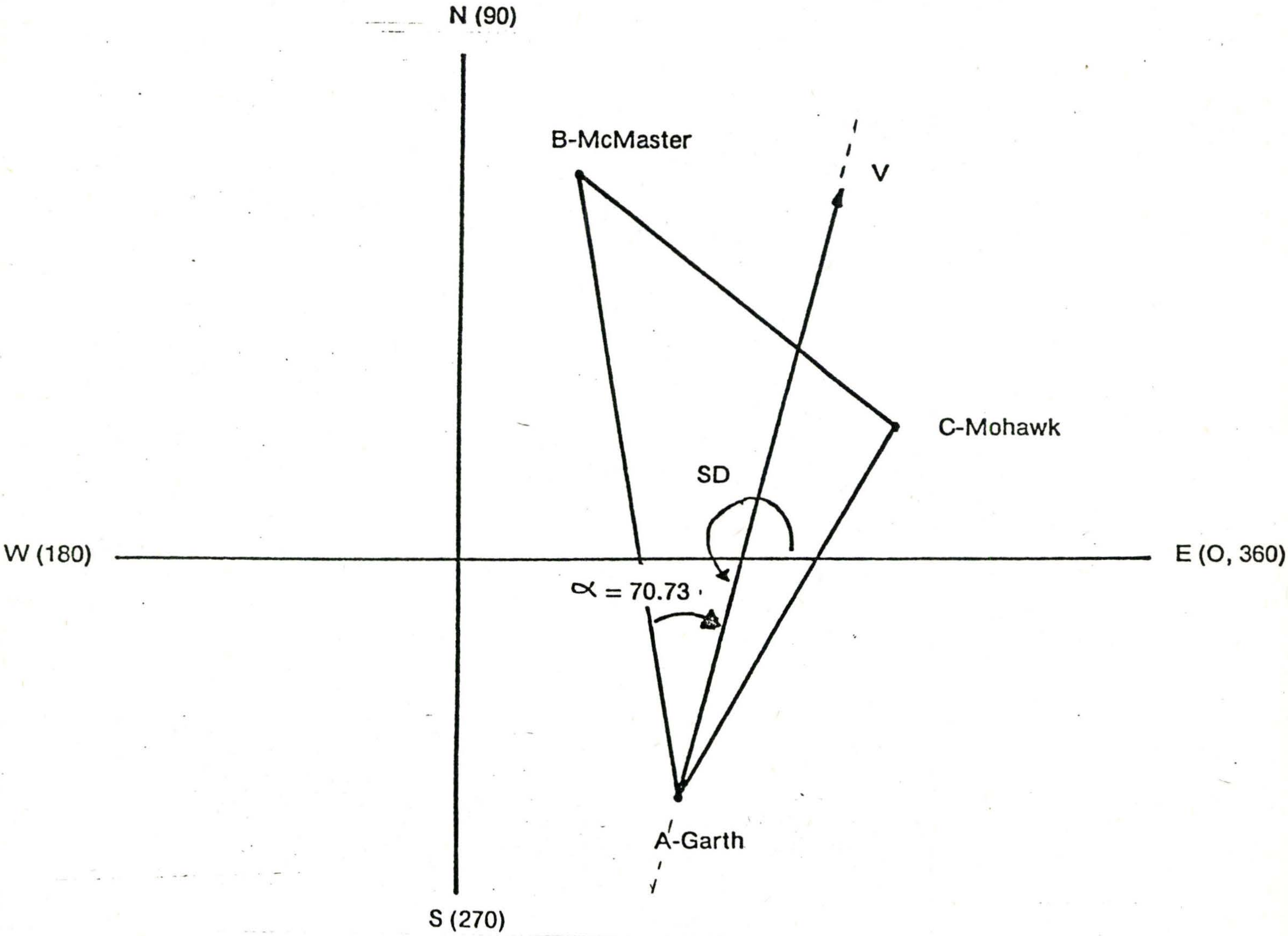


Figure 14: General Co-ordinate System

5.10.5 K1 - Shape of the Hyetographs

For each storm we have 3 different hyetographs and therefore 3 different computed shape factors for the hyetographs. The shape constants K1 used to represent the storm are calculated from the mean of the three.

5.10.6 WD - Wind Direction, WV - Wind Speed

During the summer 1980 additional data were obtained from the Atmospheric Environment Service (AES) meteorological stations at the Hamilton Airport at Mount Hope and at the Royal Botanical Gardens. These stations are located several miles outside the Chedoke Creek Basin. The continuous data measured at these stations provide the long-term records available for Hamilton. These stations provided rainfall data on a 5-minute time interval as well as measurements of wind speed and wind directions during the storm.

At the end of each month we collected a "Monthly Weather Summary" from the stations which included a wind summary (km/hr) for each day and for each hour as well as wind directions.

The observed wind speed (WV) and wind direction used in this study was the calculated mean between the two raingauges.

The wind speed (WV) measured in km/hr and the wind direction (WD) refer to the coordinate system used in this study.

5.10.7 Summary of Storm Data

Table No. 3 summarizes data for 22 separate storms. For each storm the following storm parameters were calculated:

TOPR - Total precipitation - computed using Thiessen polygons (mm)

SV - Storm velocity computed from analysis of 3 synchronized observed hyetographs

WV - wind velocity during the storm measured at RBG and HAP raingauges (km/hr)

Po - maximum peak intenstiy (mm/hr)

K1 - hyetographs shape factors

SD - storm direction computed from analysis of three observed synchronized hyetographs

WD - mean wind direction during the storm observed at RBG and HAP weather stations

Table No. 3

Summary of Storm Data

storm #	Po (mm/hr)	SV (km/hr)	WV (km/hr)	TOPR (mm)	K1	SD	WD
1	4.80	26.30	17.00	1.52	.125	225.0	210.0
2	3.60	43.54	33.00	1.80	.226	.5	30.0
3	10.30	20.86	9.00	24.27	.163	222.0	180.0
4	54.67	56.66	31.50	26.93	.133	137.0	135.0
5	42.00	10.14	15.00	13.94	.205	311.0	300.0
6	40.73	12.27	7.00	25.46	.100	345.0	325.0
7	15.40	10.50	10.00	8.58	.122	107.5	150.0
8	17.40	18.42	15.00	18.10	.122	92.0	60.0
9	29.60	28.34	19.00	12.14	.122	105.0	120.0
10	19.20	7.43	19.00	8.80	.163	335.0	300.0
11	32.10	13.19	10.50	19.35	.222	301.0	325.0
12	112.00	25.87	11.00	31.60	.545	268.0	330.0
13	63.30	46.40	20.00	10.87	.130	238.0	240.0
14	22.30	12.64	11.00	8.56	.156	275.0	315.0
15	29.40	12.98	13.00	20.35	.116	212.0	180.0
16	14.90	27.45	35.00	5.91	.228	297.5	330.0
17	27.80	21.64	24.00	26.57	.141	290.0	270.0
18	16.10	34.11	17.00	9.92	.157	120.0	0.5
19	4.27	25.89	6.00	2.91	.091	344.0	270.0
20	9.87	13.59	7.00	2.70	.070	108.0	120.0
21	3.60	15.86	15.00	2.05	.226	262.0	270.0
22	14.93	26.67	28.00	10.98	.099	337.0	300.0

5.10.8 Correlation and Regression for Storm Data

The correlation between the different storm parameters was examined. The correlation coefficient between the data was calculated by the usual (Pearson product moment) correlation between two sets of variables. When a reasonable correlation was found the regression between these parameters was examined. Table No. 4 presents the correlations between the various storm characteristics for 22 storms.

Table No. 4

Correlation Between Storm Parameters

	Po	SV	WV	TOPR	K_1	SD
SV	0.224					
WV	0.046	0.625 *				
TOPR	0.677 *	0.036	0.097			
K_1	0.637 *	0.05	0.045	0.325		
SD	0.169	0.380	0.184	0.169	0.107	
WD	0.305	0.386	0.135	0.177	0.290	0.918 *

Comparison with the critical value of the linear correlation coefficient (Appendix D-2) show:

$n \backslash \alpha$	20 %	10 %	5 %	2 %	1 %	0.2 %
22	0.284	0.360	0.423	0.492	0.537	0.622

The values flagged with an asterisk show a correlation that is true for more than 99.8% of the cases, and were incorporated in THOR:

1. The regression between Po and TOPR:

   Po = 2.60 + 1.81 TOPR

2. The regression between Po and K1:

   K1 = 0.101 + 0.0025.Po

3. The regression between SV and WV:

   SV = 7.39 + 0.933WV

4. The regression between SD and WD:

   SD = 33 + 0.884.WD

Clearly, at least 1 raingauge and windgauge is available within the catchment to measure TOPR, WV and WD, will permit generation of hyetographs as the storm moves across the area.

In general the correlations are found to be reasonable because our model does not take into account the aging of the cell nor the 2 dimensional plan form of the cell, it would be unreasonable to expect better correlations. The results, however, can be used to build a model based on expected relations. The variance appears to be large enough to warrant further study before a probabilistic interpretation can be made.

CHAPTER 6

APPLICATION OF THOR

6.1 Introduction

The Stormwater Management Model (SWMM), a computer program capable of simulating urban stormwater runoff and combined sewage overflows, was chosen for use in this study.

The main purpose of this part of the study was to use the RUNOFF block of the program to generate single event storms. Comparisons between observed and computed flow hydrographs and pollutographs were made for two different storm inputs.

1. A standard design storm: a hypothetical stationary storm based on AES raingauge data from the Hamilton Airport.
2. A moving storm based on the THOR program, using the three raingauges installed for summer, 1980.

The following paragraphs (6.2, 6.3.2, 6.4) were abstracted directly from reports written by James (22).

6.2 The Stormwater Management Model

6.2.1 Development of SWMM

The development of SWMM, a U.S. computer program, was a joint effort of Metcalf and Eddy, Inc., Water Resources Engineers, Inc., and the University of Florida. Since its inception in 1971, there have been numerous corrections and updates to the model. A major revision is due to be released in 1981 (22).

6.2.2 Basic Structure of SWMM (22)

The program requires rainfall, pollutant and basin characteristics as input and calculates stormwater quantity and quality.

These processes are performed by five main code segments or blocks: RUNOFF TRANSPORT, EXTENDED TRANSPORT, STORAGE/TREATMENT and RECEIVE. In this study, only the RUNOFF BLOCK was used.

RUNOFF BLOCK

Rainfall hyetographs are distributed evenly over each subcatchment within each time interval (typically one to five minutes). The drainage area is characterized by its size, degree of imperviousness, slope and several factors describing pollutant accumulation over the area. The available depression storage and infiltration potential are satisfied before runoff occurs. Overland flow is then considered, using the kinematic wave formula based on Manning's equation and continuity at each time interval.

Overland flow may be routed through small pipes and gutters in its travel to an inlet manhole for the TRANSPORT system. The rate of overland flow determines the amount of the available surface pollutants washed off.

Thus at the inlet, a temporal description of the flow and the pollutant mass washoff is available. These hydrographs and pollutographs are the output results from the RUNOFF Block (22).

6.2.3 FASTSWM (22)

To run the RUNOFF block in this study, the FASTSWM package was used. FASTSWM is a program package that makes it possible to run parts of SWMM from a terminal in a pseudo-conversational mode. FASTSWM was originally developed by James in Sweden and known as SWESWMM. FASTSWM (November 1979) is available on the CDC Multi-processor computers at Multiple Access, Toronto, CYBERNET, and the CYBER system at McMaster University.

The package is easily maintained by simply updating the basic SWMM modules in accordance with the latest modifications and corrections issued by the original authors. These basic modules have not been modified for FASTSWM.

The basic processes included in the package are:

1. Solicit job control data to identify or set up the necessary user files (user name, case number, new file).
2. Solicit and accept user-directed basic SWMM input in conversational free-format mode, usually from a remote terminal, if a new file.
3. Check the validity of the free format input data and return values of certain dependent parameters to the user terminal.
4. Copy and save the user's free format input in a special file defined by the user's initials and a case number, if a new file.
5. Convert from metric units to the Imperial units required by SWMM,

if necessary.

6. Copy and save the properly formatted data in Imperial units in a special SWMM input file also defined by the user's initials and case number.
7. Submit the blocks of SWMM in the order required.
8. Save the SWMM output file, identified by the user's initials and case number.
9. Return selected output from the SWMM output file to the user terminal and reconvert to metric data if required.
10. List the SWMM output file as directed by the user at the central site.
11. Re-submit the SWMM blocks as directed and return selected output, and repeat as required.

Steps 1-7 are handled by the pre-processor, steps 8-11 by the post-processor. The procedure has been carefully designed to minimize user errors and reduce total design turnaround times. The FASTSWM package considerably reduces the complexity of the SWMM job submission; users are able to focus on the hydrologic, hydraulic and water quality processes modelled. No knowledge of FORTRAN formats, or systems control language is required.

FASTSWM comprises three parts: PRESWM, SWMM and POSTSWM. A copy of the user's free format input file is saved as NNNDATX where NNN are the user's initials and X is the study case number. X may be any character. This file may be freely accessed and edited using the usual system editing facilities. Similarly the final output file NNNoutX, may also be accessed, edited and listed at the user's terminal (22).

SWMM is the main part of the program and is automatically submitted to a central processor unit as a remote batch job by the FASTSWM program.

PRESWMM works in an interactive mode and prepares the input for SWMM.

PRESWMM makes it possible for the user to enter his input in free format. It also converts units from metric to Imperial (FPS).

POSTSWM also works interactively. It returns selected results to the user and reconverts the data to metric units (22).

6.3 Input Data Required

6.3.1 Runoff Quantity Modelling

The necessary program input is specified in the SWMM user's manual. The required physical data were extracted from plans and contour maps supplied by the Hamilton-Wentworth Regional Engineering Department.

6.3.1.1 Watershed Discretization -

The relatively large size (2449 acres) of the catchment facilitated detailed modelling of various land uses, surface and pipe slopes. The study area was discretized into 10 subcatchments ranging in size from 143 acres to 506 acres. All imperviousness coefficients were computed individually accounting for different land uses, road patterns and lot depths as determined from the details in the plans and verified by field investigations. The input data included 31 gutters and pipes.

6.3.1.2 Catchment Input Data -

The program default values for infiltration rates and surface storage were modified for the initial simulation. Later these were changed during calibration. The surface flow resistance factors were also left at default values.

For the impervious areas, the model assumes a constant proportion of the area to be generating immediate runoff which, for all the watershed, was assigned the default value of 25%.

6.3.1.3 Input Rainfall Data -

Two different sets of data were supplied to the SWMM model:

1. Rainfall recorded by the Hamilton Airport raingauge, representing a stationary storm using one point rainfall station.
2. Rainfall or hyetographs supplied by THOR thus modelling a moving storm.

6.3.2 Runoff Quality Modelling (22)

During runoff-generating storm events, pollutants from impervious surfaces are washed into the sewer system, from the dust and dirt accumulated during dry weather ont the surface. The land uses and antecedent conditions, such as street sweeping practices and length of intervening dry weather periods, largely determine the magnitude of these loadings. Apart from this surface washoff, resuspension of pollutants accumulated in catch basins and deposited in the sewer system are the other two important sources of pollutant loadings during such storm events. In the SWMM program the modelling technique initially estimates the dust and dirt load based on the land use details and the antecedent dry weather and street cleaning data; then it expresses the modelled pollutants as fractions of this dust and dirt load, which are then combined with the runoff hydrograph to obtain pollutographs. The present version of SWMM can model the following eight pollution parameters:

Suspended Solids (SS)	Settleable Solids
BOD	Total Nitrogen (N)
COD	Phosphate (PO4)
Total Coliforms	Grease and Oil

The RUNOFF block is designed to simulate surface pollutant loads and catch basin loads and needs the following additional information:

- subcatchment data such as land use types, catch basin density, total curb length
- erosion data
- general quality data such as catch basin volume, catch basin BOD concentration, antecedent dry days and street cleaning data.

6.4 Sensitivity Analysis

Sensitivity analyses were carried out on both the quantity and quality parameters in the RUNOFF Block. The following summarizes the results.

6.4.1 RUNOFF-Block-Quantity-Summary (22)

In general, the quantity algorithms in the RUNOFF block are most sensitive to the percentage imperviousness of the subcatchment. An almost linear relationship exists between percentage imperviousness and both peak flow and volume. Percentage imperviousness for an area can be estimated quite accurately given up-to-date mapping (land use plans or aerial photography).

The SWMM model displays moderate sensitivity to the subcatchment width. A large number of techniques have been proposed for estimating the width but none have proven applicable in all situations. It is preferable to calibrate the model for an area using this parameter since it incoprorates inherent uncertainty (22).

The quantity model parameters may be listed in order of decreasing significance in terms of sensitivity:

- percentage imperviousness
- subcatchment width
- infiltration rates
- detention storage
- ground slope
- percentage of impervious area with zero detention storage (22)

6.4.2 RUNOFF Block-Quality-Summary (22)

The quality section of the RUNOFF block was found to be sensitive to changes in parameters affecting suspended or settleable solids.

Examination of the equations used in the model algorithms indicates that a variation in the amount of suspended or settleable solids washed off will have an effect, potentially overwhelming, on the amount of other constituent pollutants washed off.

The parameters affecting RUNOFF quality simulation and the degree to which the model is sensitive to them are summarized below:

6.4.2.1 High Sensitivity (22) -

1. number of dry days (DRYDAY)
2. street sweeping interval (CLFREQ)
3. street sweeping efficiency (REFF)
4. exponential coefficient in washoff equation (RCDEF), at low runoff rates
5. dust and dirt loadings (DDFACT)
6. insoluble fraction due to suspended solids (F2)
7. availability factors for suspended and settleable solids (NAVAIL)
8. total gutter length (GQ)

6.4.2.2 Medium Sensitivity (22) -

1. street sweeping availability factor (AVSWP)
2. insoluble fractions due to settleable solids (FA)
3. pollutant fractions (QFACT)

6.4.2.3 Low Sensitivity (22) -

1. catchbasin storage volume (CBVOL)
2. initial concentration of BOD5 in each catchbasin (CBFACT(4))
3. number of catchbasins (BA)

6.5 Comparison of Observed and Computed Results

6.5.1 General

Six independent storms observed at Hamilton Airport gauge were chosen for calibrating SWMM. The six hyetographs in this first series of runs thus represent a stationary storm, as commonly used in engineering practice. The output was compared to observed runoff hydrographs and pollutographs of suspended solids and BOD5. Then THOR was used to process five storms. The hyetographs produced by THOR represent moving, spatially limited storms. The SWMM model was calibrated for these as well.

The parameters that were used in calibrating the model were: width of subcatchment, percentage imperviousness, initial and final infiltration rates, infiltration decay constant, dust and dirt loadings, and suspended solids and BOD5 pollutant fractions.

The calibration procedure produced two sets of calibrated SWMM parameters: one set for observed stationary Hamilton Airport hyetographs, the second for the moving hyetographs generated by THOR.

After calibration the six storms were again input with the final set of SWMM calibration parameters to compare the observed peak flow and the total volume of runoff with the results computed by the SWMM model.

The resulting peaks and volumes were checked statistically by the R**2 test.

Three entirely different storms and responses, using the two methods and their sets of calibration parameters were then compared.

Significantly different results were obtained, using the AES data recorded at the airport and the stationary areally uniform model, on the one hand, and using THOR and simulating a spatially limited storm moving across the catchment, on the other hand.

Some storms observed at Hamilton Airport did not represent simultaneously observed rains in Hamilton. Figures 35-49 present observed and computed hydrographs and pollutographs for the Chedoke Creek Catchment for storms 8, 10, 14 not used in the original calibration.

6.5.2 RUNOFF Quantity Modelling

6.5.2.1 Discussion of Calibration Parameters -

The most sensitive hydrologic parameters warranting adjustment were: subcatchment width, percentage imperviousness and infiltration rates.

The percent imperviousness, which can normally be estimated with fair accuracy, was reviewed to ascertain the indirectly draining impervious area, often overlooked initially. The overland flow width is one of the prime calibration parameters directly influencing the subcatchment time and thereby the flow hydrograph.

The infiltration rates used in the initial simulation were:

Maximum infiltration: 3.0 in/hr

Minimum infiltration: 0.52 in/hr

Infiltration decay rate: 0.00115/sec

Manning's n, Impervious area: 0.013

Manning's n, Pervious area: 0.25

Detention Storage (inches), Impervious area: 0.062

Detention Storage (inches), Pervious area: 0.184

PCTZER, Percent impervious area with zero detention: 25%

6.5.2.2 Calibration Results

In general the parameters that changed were: width of the catchment, percentage impervious, minimum infiltration rate and decay rate. Details of parameter ajdustments are tabulated in Tables 5 and 6.

Table No. 5

Subcatchment Data for Stationary Storm

Subcatchment Data

Subcatchment No.	Width (Ft.)	Area (AC)	Percent Imperv.	Slope (Ft/Ft)	Decay Rate (1/Sec)
1	7060.	361.	25.00	.0500	.01000
2	6020.	244.	31.00	.0250	.01000
3	5520.	163.	35.00	.0125	.01000
4	4760.	235.	9.00	.0350	.01000
5	6340.	164.	26.00	.0225	.01000
6	4900.	143.	23.00	.0250	.01000
7	5560.	109.	12.00	.0300	.01000
8	5480.	506.	8.00	.0500	.01000
9	6180.	288.	27.00	.0300	.01000
10	5560.	236.	24.00	.0300	.01000

Table No. 6

Subcatchment Data for THOR Moving Storm

Subcatchment Data

Subcatchment No.	Width (Ft.)	Area (AC)	Percent Imperv.	Slope (Ft/Ft)	Decay Rate (1/Sec)
1	5130.	361.	20.00	.0300	.01000
2	4270.	244.	24.00	.0250	.01000
3	3450.	163.	28.00	.0125	.01000
4	3020.	235.	6.00	.0350	.01000
5	4720.	164.	22.00	.0225	.01000
6	3800.	143.	20.00	.0250	.01000
7	3220.	109.	9.00	.0300	.01000
8	4300.	506.	4.00	.0500	.01000
9	4870.	288.	23.00	.0300	.01000
10	4100.	236.	21.00	.0300	.01000

Resistance factor: Impervious and Pervious, surface storage impervious and pervious, infiltration rate: maximum and minimum were adopted as default data.

Figures 15 and 16 illustrate the calibration results for a stationary storm. The final results after calibration are summarized in Table 7.

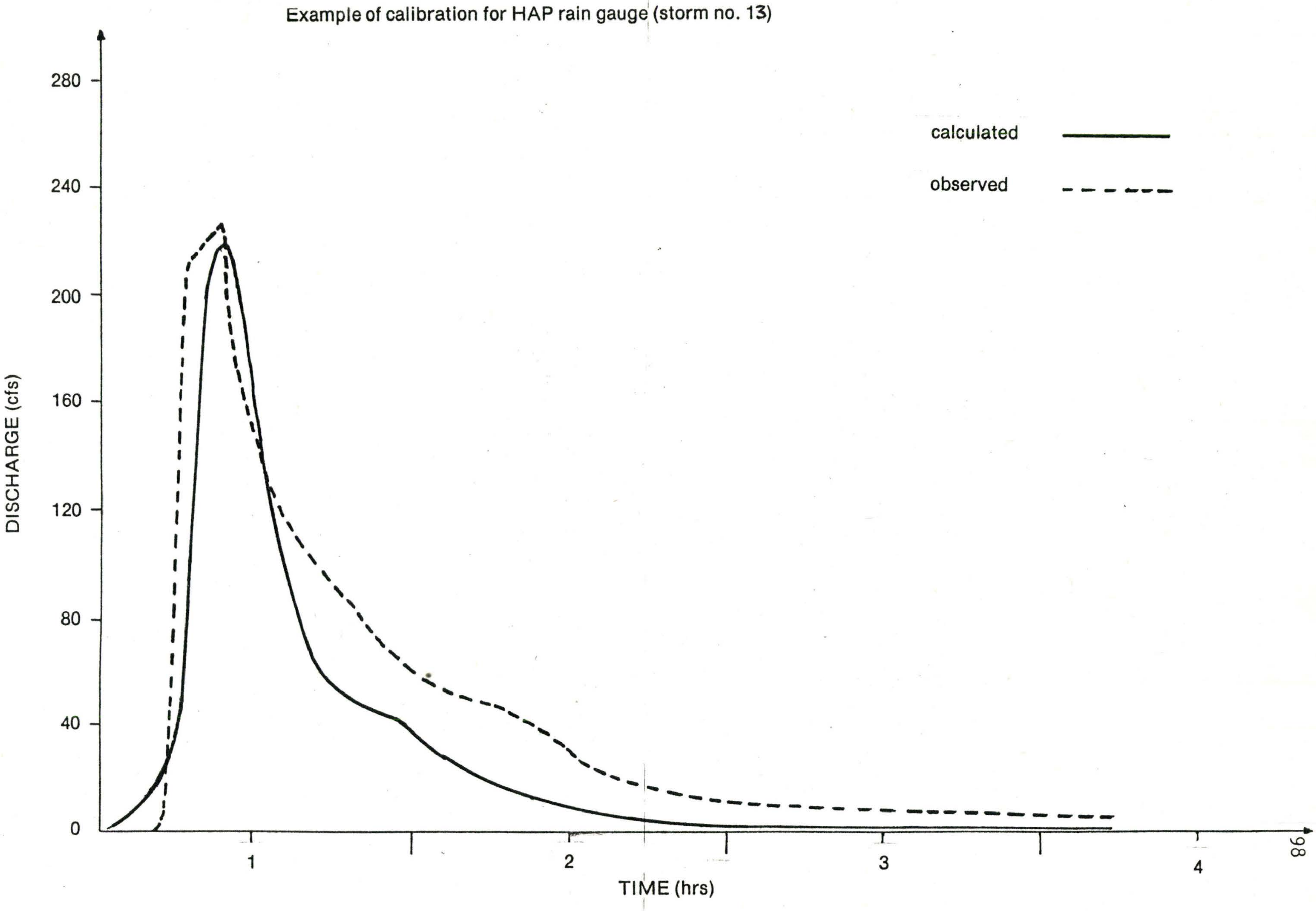


Figure 15: Example of Calibration for Hamilton-Airport Raingauge (Storm No. 13)

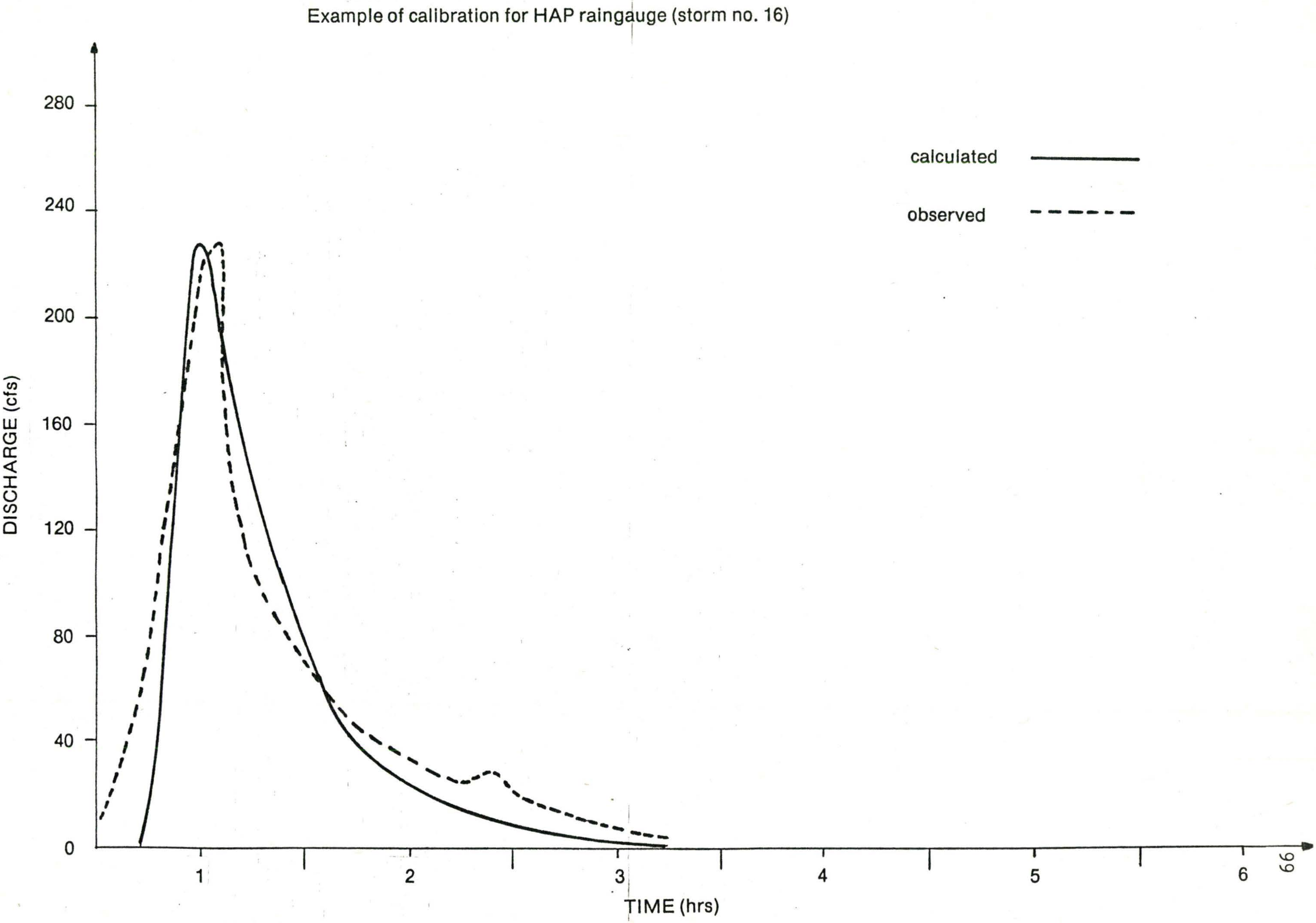


Figure 16: Example of Calibration for Hamilton-Airport Raingauge (Storm No. 16)

Table No. 7

Stationary Storm

Comparison Between Observed and Computed Flows After Calibration

Peak Flows (cfs)		Volume (ft3)	
Observed	SWMM	Observed	SWMM
83.5	63	755000	450000
93	56	230000	176000
228	215	720000	525000
228	224	525000	513000
29	44	198000	200000
183	221	1152000	1148000

Correlation coefficient for peak flow = 0.957

R**2 = 91.5%

Correlation coefficient for total volume = 0.937

R**2 = 87.7%

Figures 17 and 18 illustrate the calibration results for a moving storm. The final results for MOV (moving storm) after calibration are summarized in Table 8.

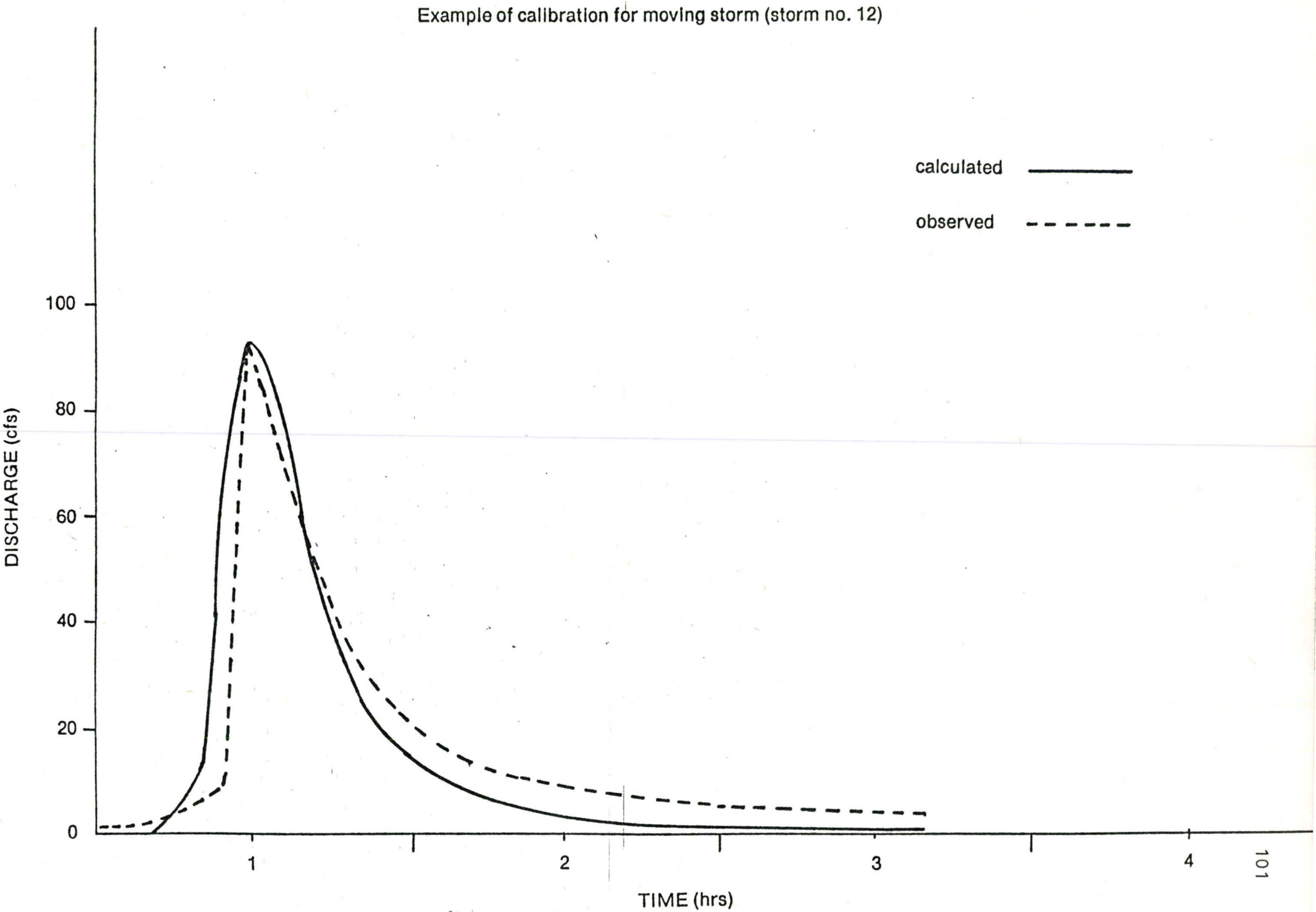


Figure 17: Example of Calibration for Moving Storm (Storm No. 12)

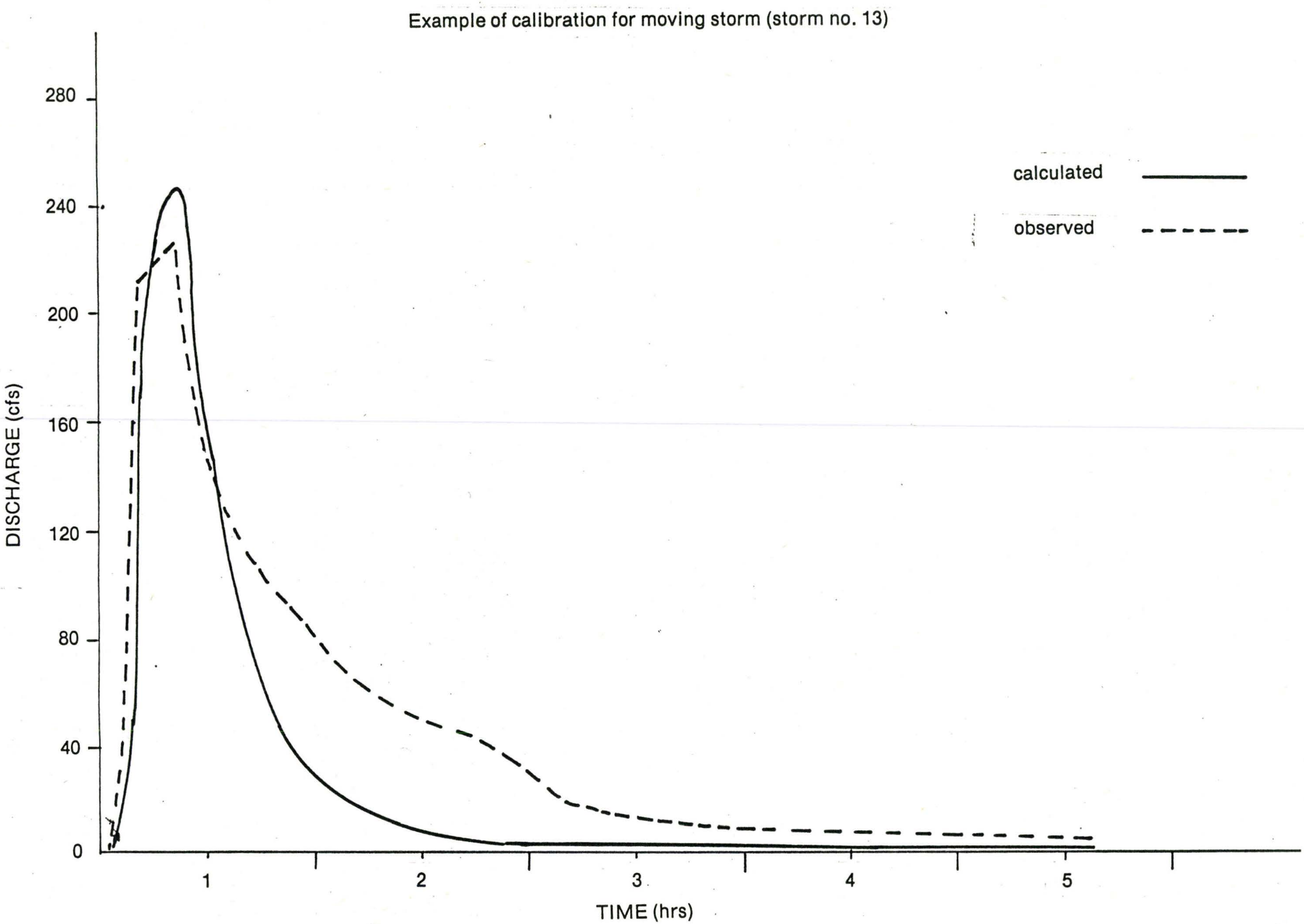


Figure 18: Example of Calibration for Moving Storm (Storm No. 13)

Table No. 8

Moving Storms:
Comparison Between Observed and Computed Flows After Calibration

Peak Flows (cfs)		Total Volume (ft3)	
Observed	SWMM	Observed	SWMM
228	243	720000	440550
338	428	1200000	740000
93	89	230000	218250
236	283	1000000	615000
29	20	198000	138000

Correlation coefficient for peak flow = 0.995

R**2 = 99%

Correlation coefficient for total volume = 0.995

R**2 = 99.1%

Note that not all the storms in Tables 7 and 8 are identical.

6.5.3 RUNOFF Quality Modelling

6.5.3.1 Discussion of Calibration Parameters -

The quality calibration was started after the quantity simulations were completed. The subcatchment data were the same for both the quantity and quality simulations.

The methodology adopted was basically the same as that for flow calibration described earlier.

The major pollution parameters needing adjustment were: dust and dirt loadings, suspended solids and BOD5, and pollutant fractions.

Data such as number of dry days were based on the observed values. The following values were used in the initial simulation:

dust and dirt accumulation:

(1)	for single family	2.1
(2)	for undeveloped/parkland	4.5

pollutant per gram of dust and dirt:

(1)	SUS.SOL:	single family residence	1000
(2)	BOD:	single family residence	5.0
		underdeveloped/parkland	5.0
(3)	N:	single family residence	0.48
		underdeveloped/parkland	0.05
(4)	PO4:	single family residence	0.05
		underdeveloped/parkland	0.01

parameters to account for insoluble fraction:

(a) fraction of settleable solids:

SET.SOL	SUS S	COLI	BOD	COD	N	PO4	GREASE
0.0	0.0	0.0	0.2	0.02	0.01	0.001	0

(b) fraction of suspended solids

SET.SOL	SUS S	COLI	BOD	COD	N	PO4	GREASE
0	0	0	0.05	0.05	0.045	0.0045	0

6.5.3.2 Calibration Results -

In general the parameters changed were: dust and dirt loadings, suspended solids and BOD and pollutant fractions. Details of parameter adjustments made are tabulated in Tables 9 and 10.

Table No. 9

Quality Data for Stationary Storm

.....QUALITY SIMULATION INCLUDED IN THIS RUN.....

INPUT PARAMETERS AS FOLLOWS

NUMBER OF CONSTITUENTS	8
NUMBER OF DRY DAYS	3.0
STD CATCHBASIN VOLUME	19.0 FT3
CATCHBASIN CONTENTS BOD	100.0 MG/L
POL. WASHOFF EQN. COEF.	4.60

USES AVAILABILITY FACTORS GIVEN IN DOCUMENTATION. (NAVAIL = 0).

STREET SWEEPING DATA

STREET SWEEPING EFFICIENCY .70

LAND USE	CLEANING INTERVAL (DAYS)	AVAILABILITY FACTOR
1 = SINGLE FAMILY RES.	30.0	.50
2 = MULTIPLE FAMILY RES.	30.0	.50
3 = COMMERCIAL	7.0	.50
4 = INDUSTRIAL	7.0	.50
5 = UNDEVELOPED/PARK	30.0	.50

DUST AND DIRT ACCUMULATION (DDFACT)

LAND USE	LB D^D/100 FT - CURB - DAY
1 = SINGLE FAMILY RES.	4.000
2 = MULTIPLE FAMILY RES.	13.800
3 = COMMERCIAL	19.800
4 = INDUSTRIAL	27.600
5 = UNDEVELOPED/PARKLAND	7.500

MG OR MPN POLLUTANT PER GRAM OF DUST AND DIRT (QFACT)

LAND USE	SET. SOL.	SUS. SOL.	BOD	N	PO4
1 = SINGLE FAMILY RES.	100.00	6500.00	1.50	.120	.020
2 = MULTIPLE FAMILY RES.	100.00	1000.00	7.20	.610	.150
3 = COMMERCIAL	100.00	1000.00	7.70	.410	.070
4 = INDUSTRIAL	100.00	1000.00	3.00	.430	.030
5 = UNDEVELOPED/PARKLAND	100.00	6500.00	1.50	.010	.020
CONCENTRATION IN EACH CATCHBASIN (CBFACT)	0.00	0.00	100.00	0.00	0.00

PARAMETERS TO ACCOUNT FOR INSOLUBLE FRACTIONS:

FRACTION OF SET. SOLIDS CONCENTRATION ADDED TO POLLUTANT (F1)	0.0000	0.0000	.0200	.0100	.0010
FRACTION OF SUS. SOLIDS CONCENTRATION ADDED TO POLLUTANT (F2)	0.0000	0.0000	.0200	.0050	.0020

Table No. 10

Quality Data for Moving Storm

```
.....QUALITY SIMULATION INCLUDED IN THIS RUN.....
INPUT PARAMETERS AS FOLLOWS
NUMBER OF CONSTITUENTS              8
NUMBER OF DRY DAYS                  3.0
STD CATCHBASIN VOLUME               19.0 FT3
CATCHBASIN CONTENTS BOD             100.0 MG/L
POL. WASHOFF EQN. COEF.             4.60
USES AVAILABILITY FACTORS GIVEN IN DOCUMENTATION. (NAVAIL = 0).
STREET SWEEPING DATA
STREET SWEEPING EFFICIENCY          .70
                                    CLEANING         AVAILABILITY
LAND USE                         INTERVAL (DAYS)       FACTOR
1 = SINGLE FAMILY RES.              30.0                .50
2 = MULTIPLE FAMILY RES.            30.0                .50
3 = COMMERCIAL                       7.0                .50
4 = INDUSTRIAL                       7.0                .50
5 = UNDEVELOPED/PARK                30.0                .50
DUST AND DIRT ACCUMULATION (DDFACT)
LAND USE                    LB D^D/100 FT - CURB - DAY
 1 = SINGLE FAMILY RES.            4.500
 2 = MULTIPLE FAMILY RES.         13.800
 3 = COMMERCIAL                   19.800
 4 = INDUSTRIAL                   27.600
 5 = UNDEVELOPED/PARKLAND          9.000
MG OR MPN POLLUTANT PER GRAM OF DUST AND DIRT (QFACT)
LAND USE                    SET. SOL.  SUS. SOL.  BOD     N     PO4
 1 = SINGLE FAMILY RES.      100.00     6800.00   5.80   .080   .010
 2 = MULTIPLE FAMILY RES.    100.00     1000.00   7.20   .610   .050
 3 = COMMERCIAL              100.00     1000.00   7.70   .410   .070
 4 = INDUSTRIAL              100.00     1000.00   3.00   .430   .030
 5 = UNDEVELOPED/PARKLAND    100.00     6800.00   5.60   .010   .010
CONCENTRATION IN EACH
    CATCHBASIN (CBFACT)        0.00        0.00 100.00   0.00   0.00
PARAMETERS TO ACCOUNT FOR INSOLUBLE FRACTIONS:
FRACTION OF SET. SOLIDS
  CONCENTRATION ADDED
  TO POLLUTANT (F1)         0.0000     0.0000     .0200   .0100 .0010
FRACTION OF SUS. SOLIDS
  CONCENTRATION ADDED
  TO POLLUTANT (F2)         0.0000     0.0000     .0200   .0050 .0020
```

Figures 19 to 26 illustrate the final results of the calibration for S.S., BOD, PO4 and Total Nitrogen for stationary storms. The figures include storms 12 and 13. Figures 27 to 34 illustrate the final calibration for S.S., BOD, PO4 and Total Nitrogen for moving storms. The figures include storms 12 and 13.

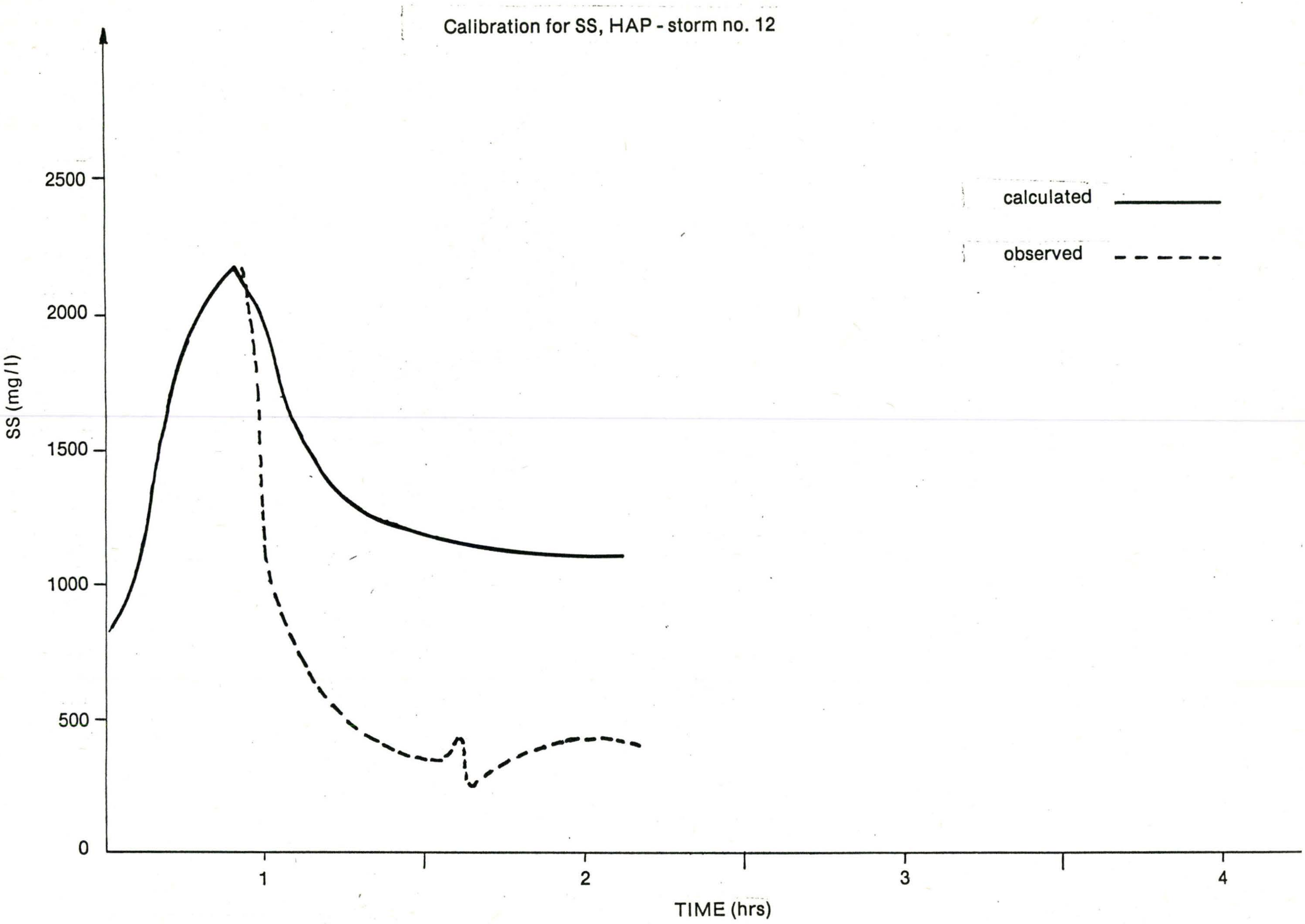


Figure 19: Calibration for S.S., HAP - Storm No. 12

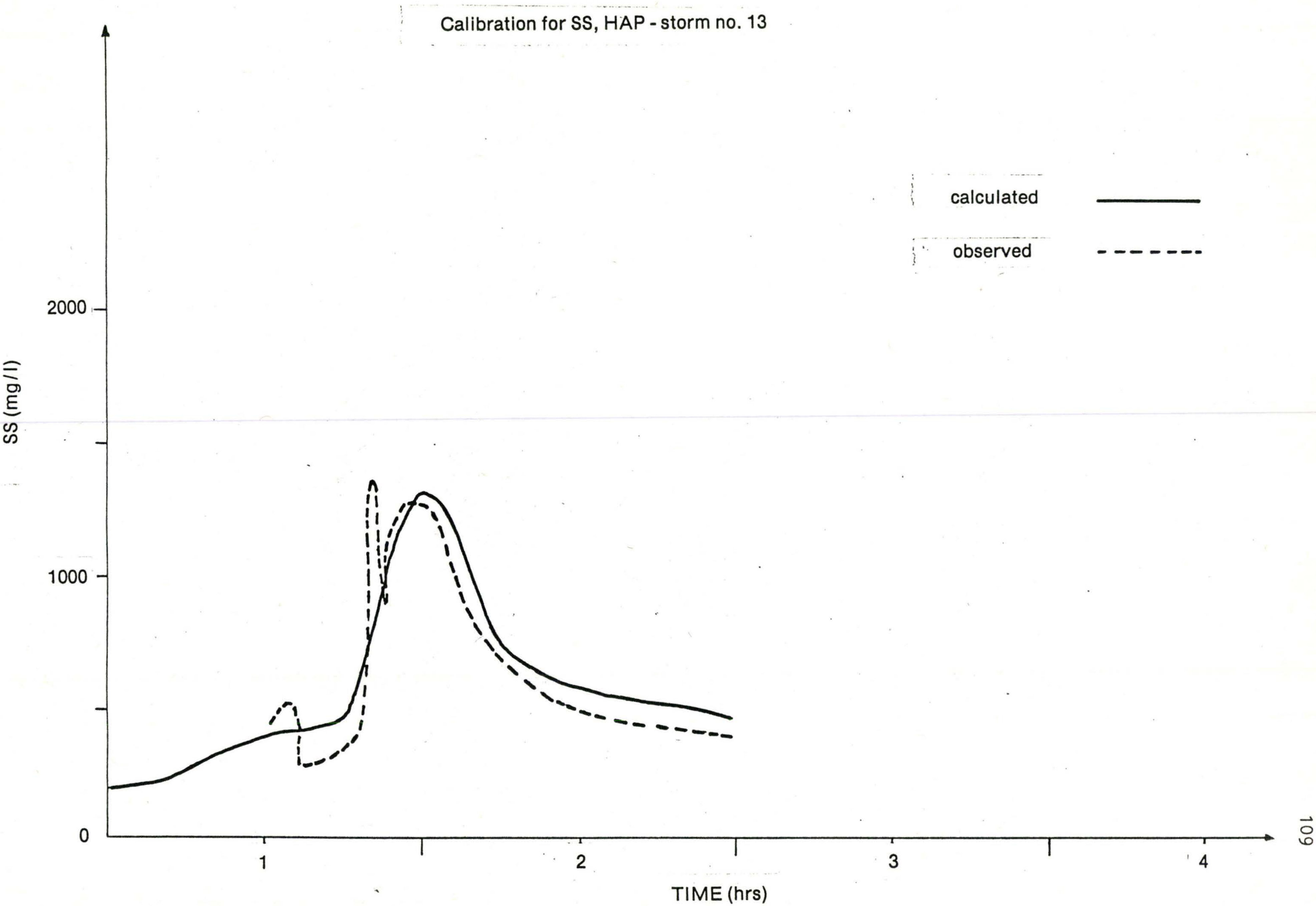


Figure 20: Calibration for S.S., HAP - Storm No. 13

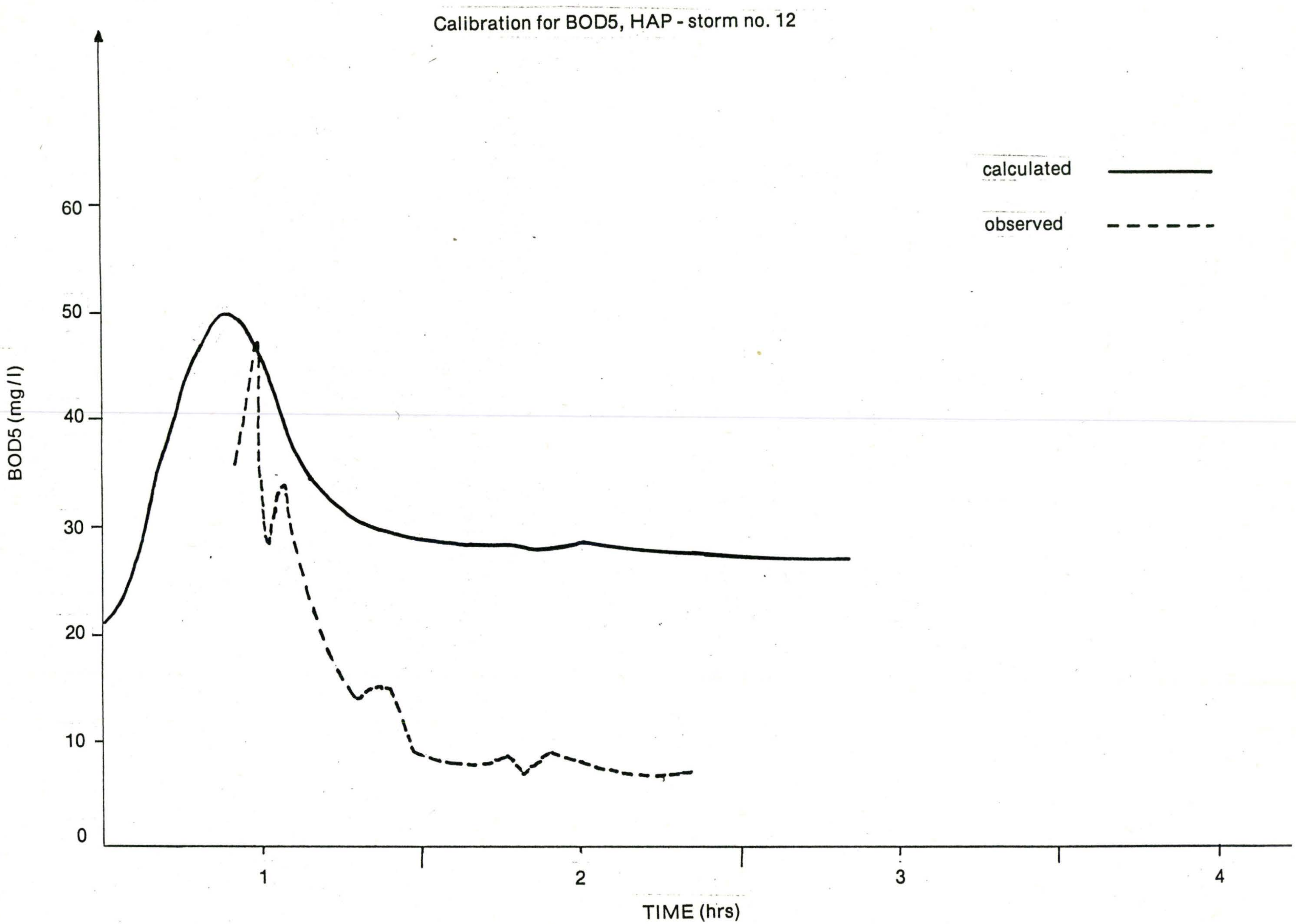


Figure 21: Calibration for BOD, HAP - Storm No. 12

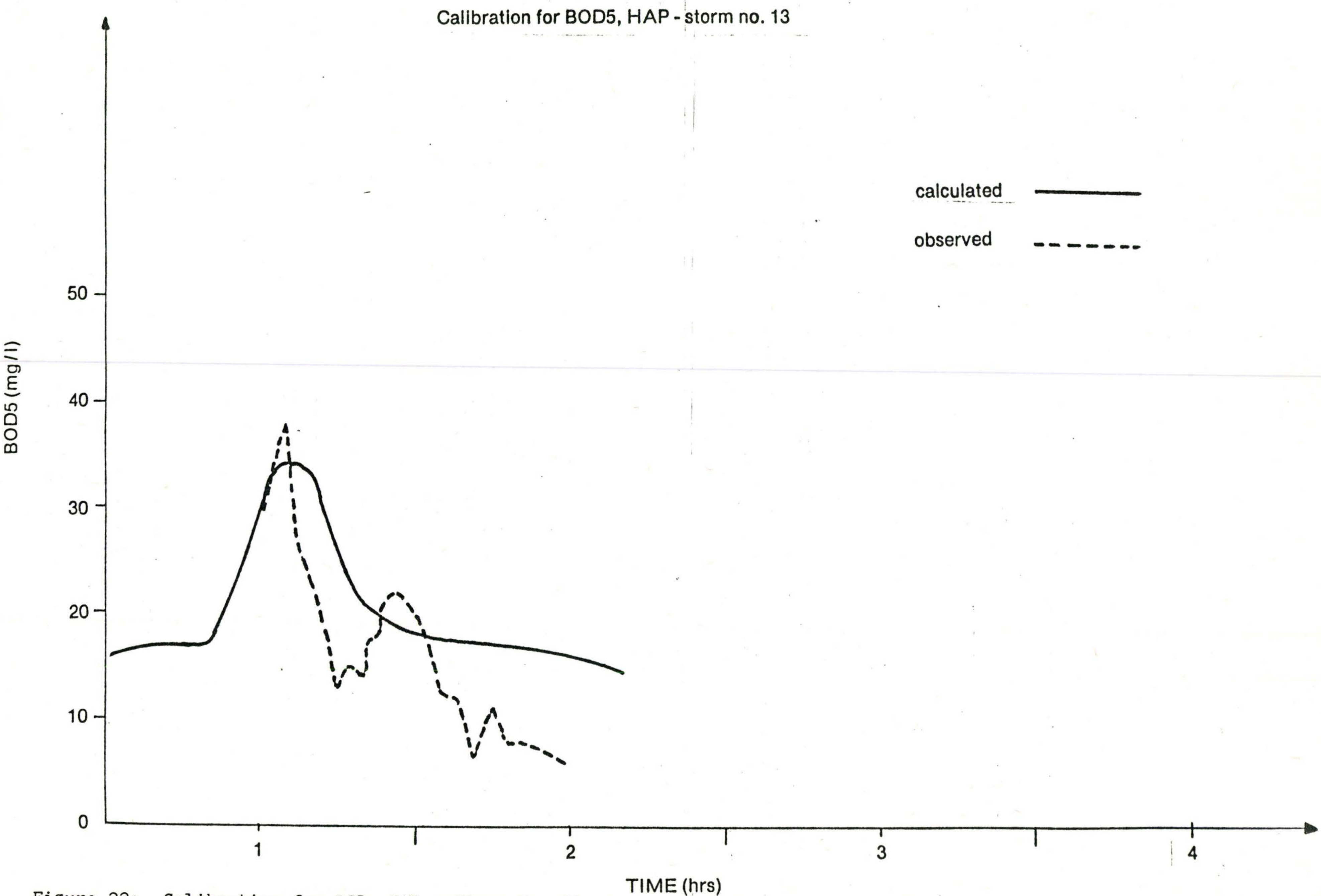


Figure 22: Calibration for BOD, HAP - Storm No. 13

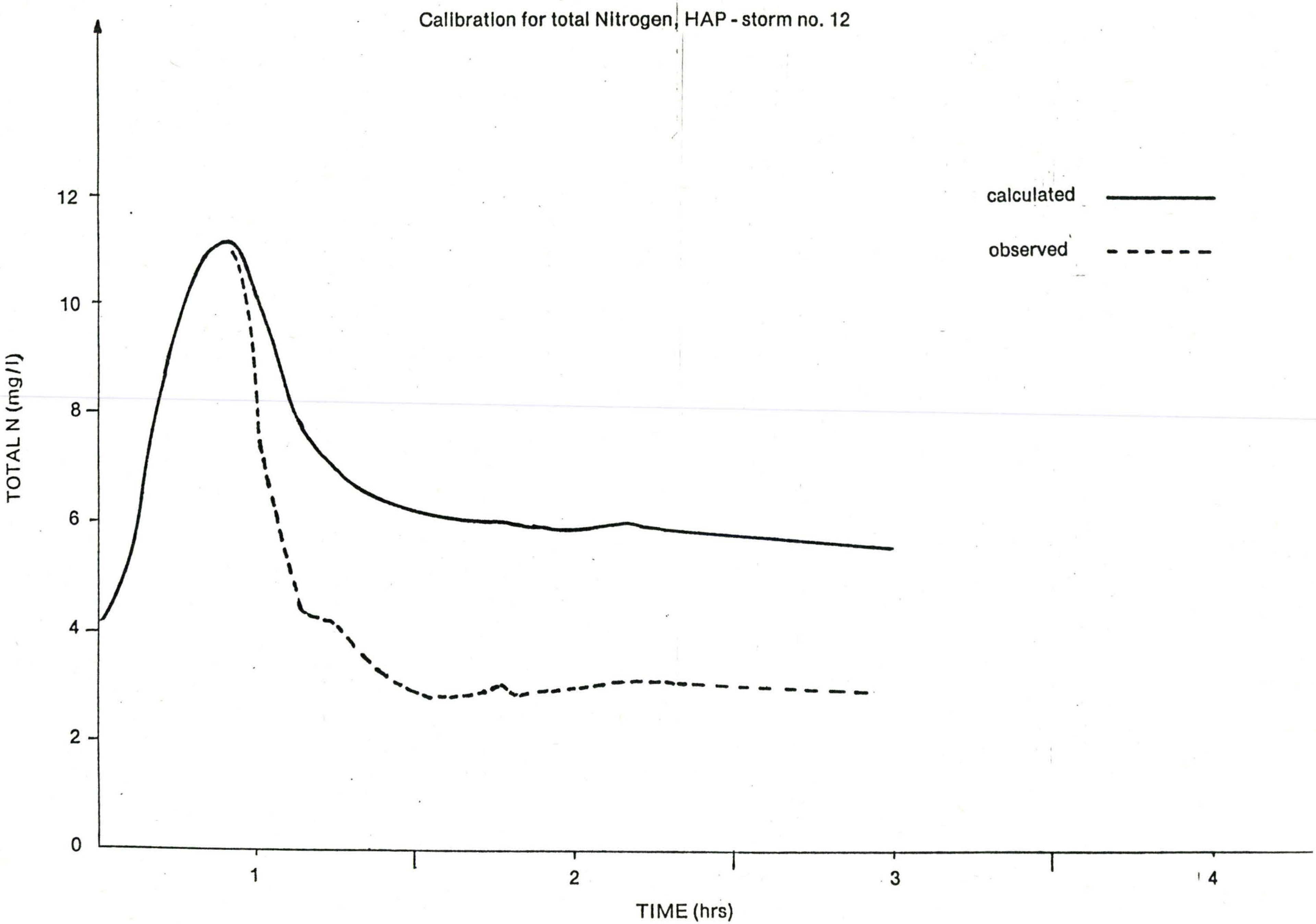


Figure 23: Calibration for Total Nitrogen, HAP - Storm No. 12

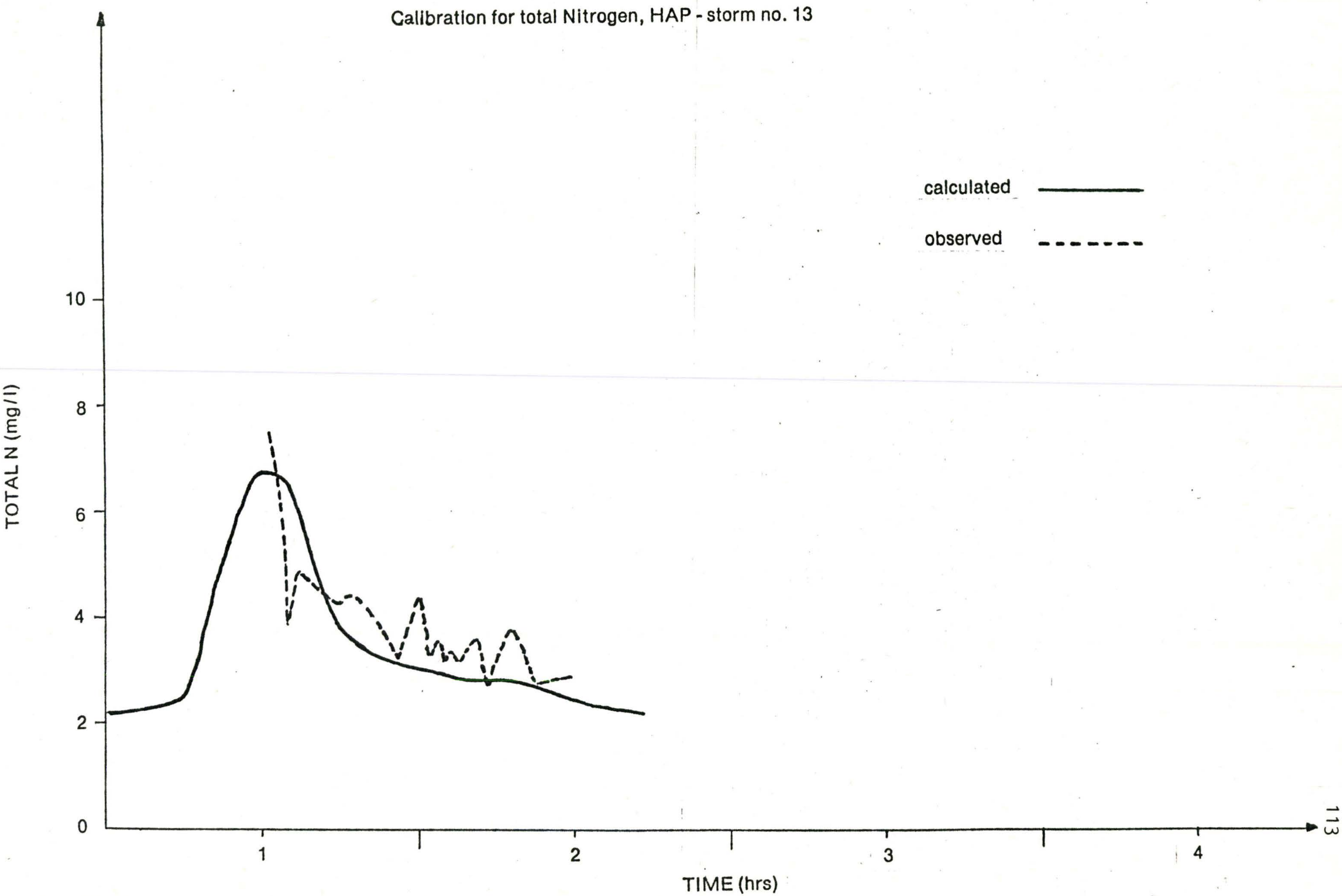


Figure 24: Calibration for Total Nitrogen, HAP - Storm No. 13

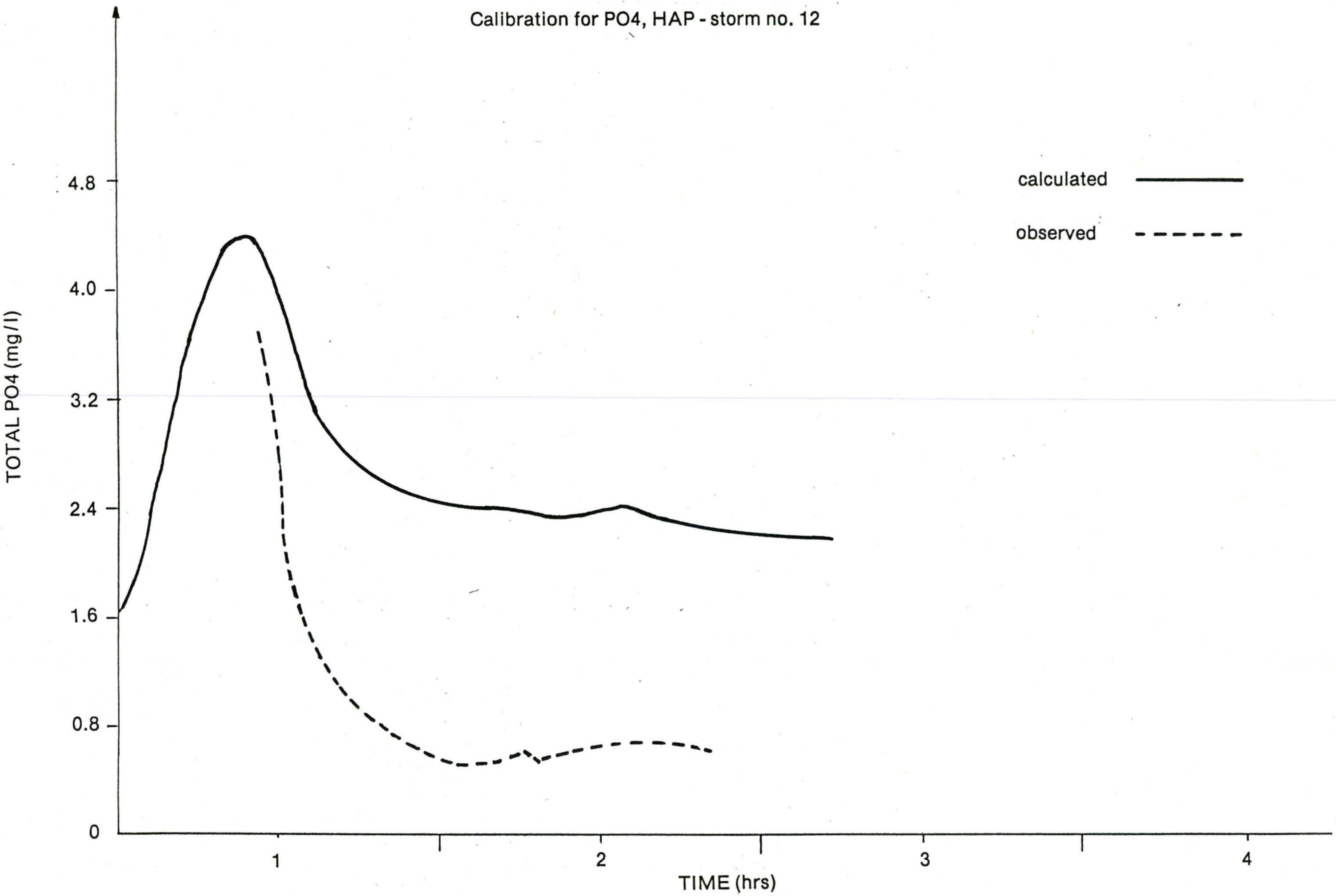


Figure 25: Calibration for PO4, HAP - Storm No. 12

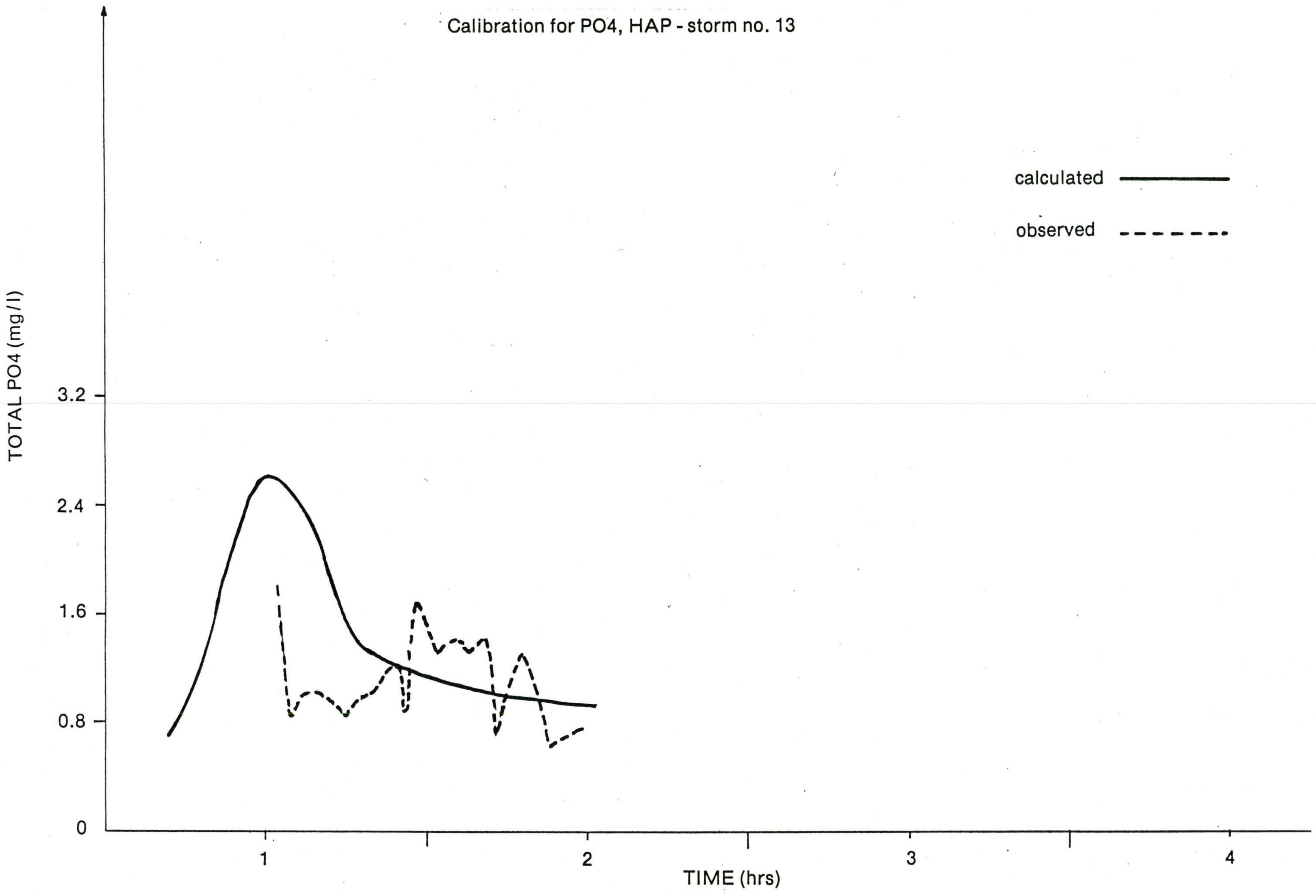


Figure 26: Calibration for PO4, HAP - Storm No. 13

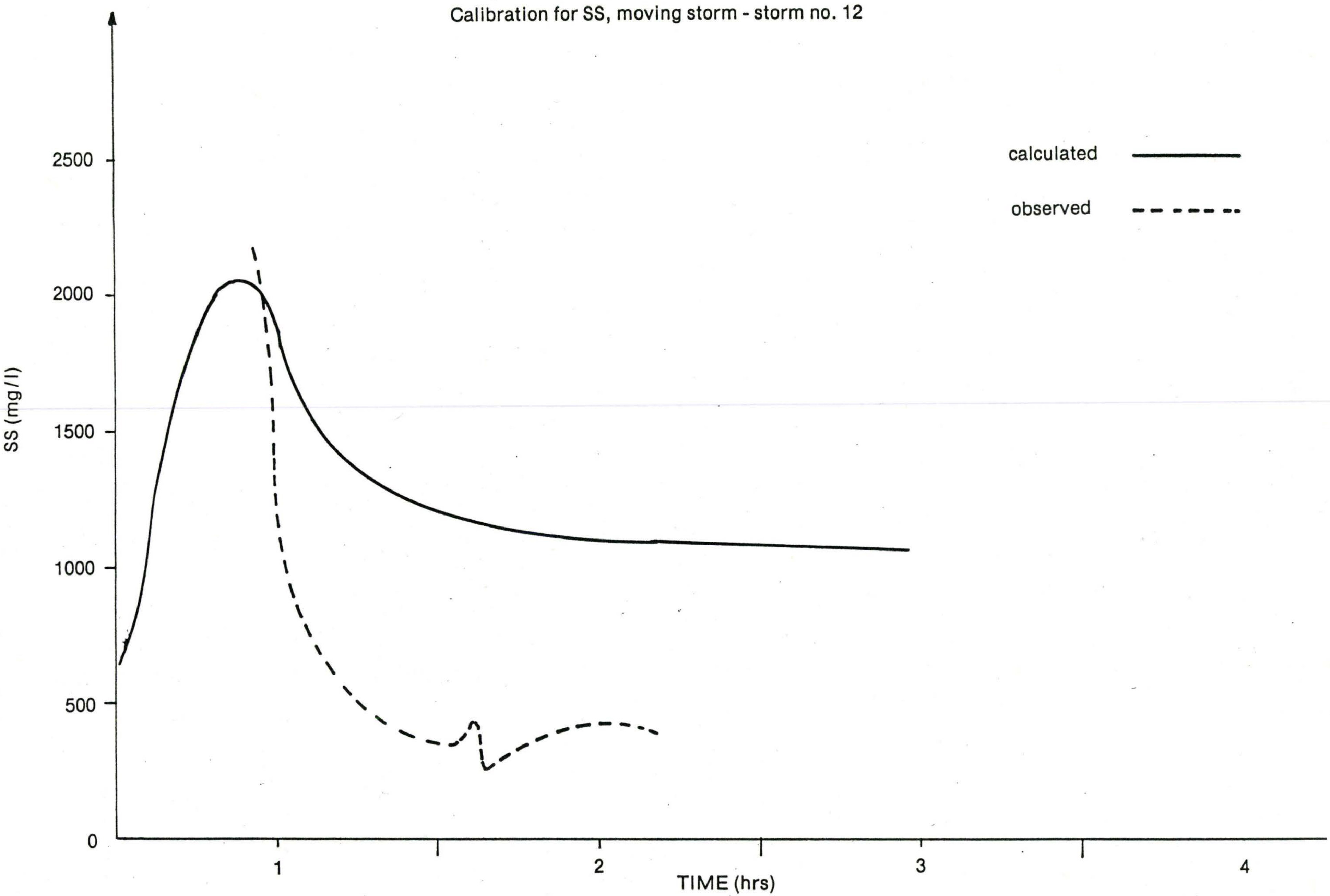


Figure 27: Calibration for S.S., Moving Storm, Storm No. 12

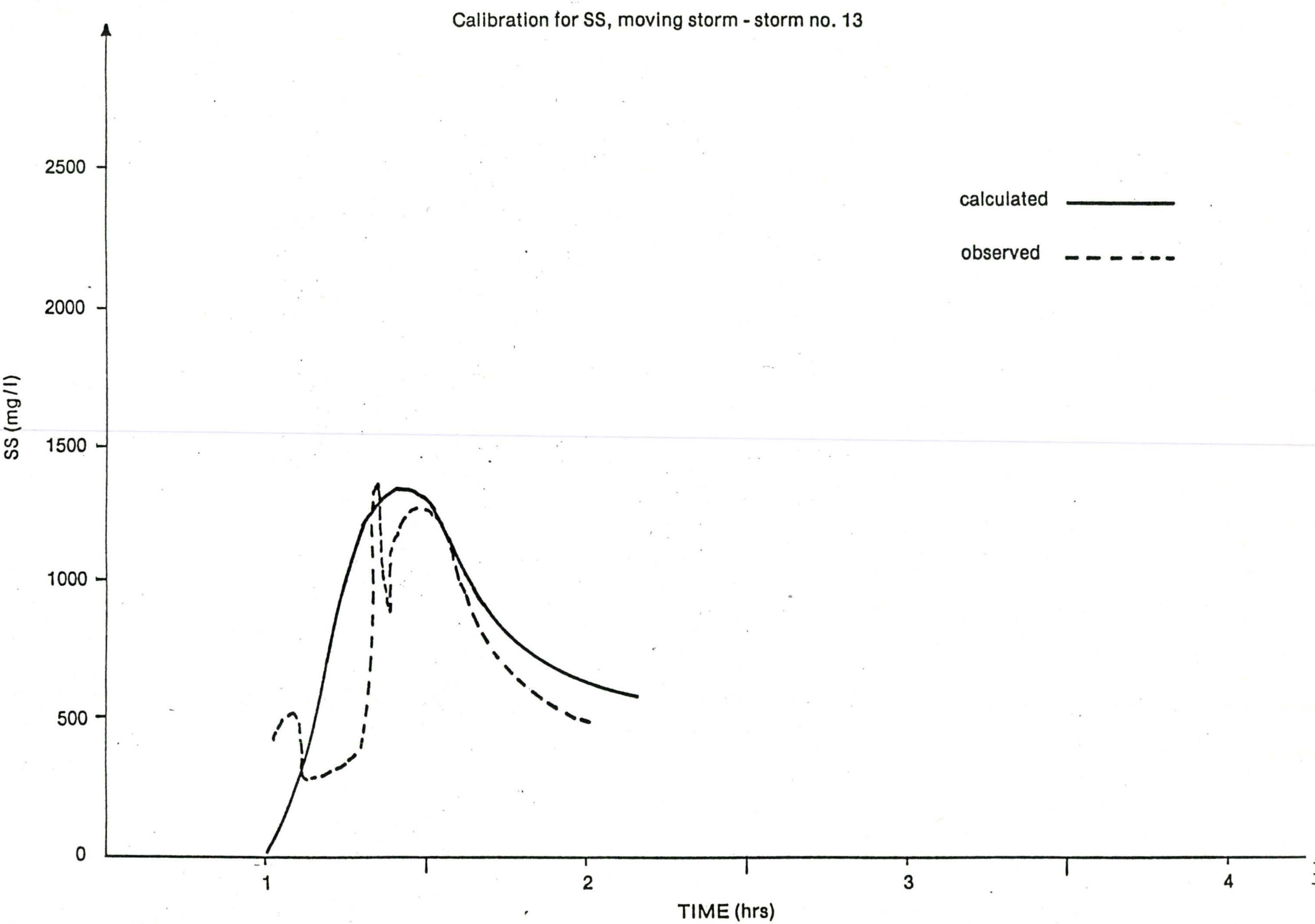


Figure 28: Calibration for S.S., Moving Storm, Storm No. 13

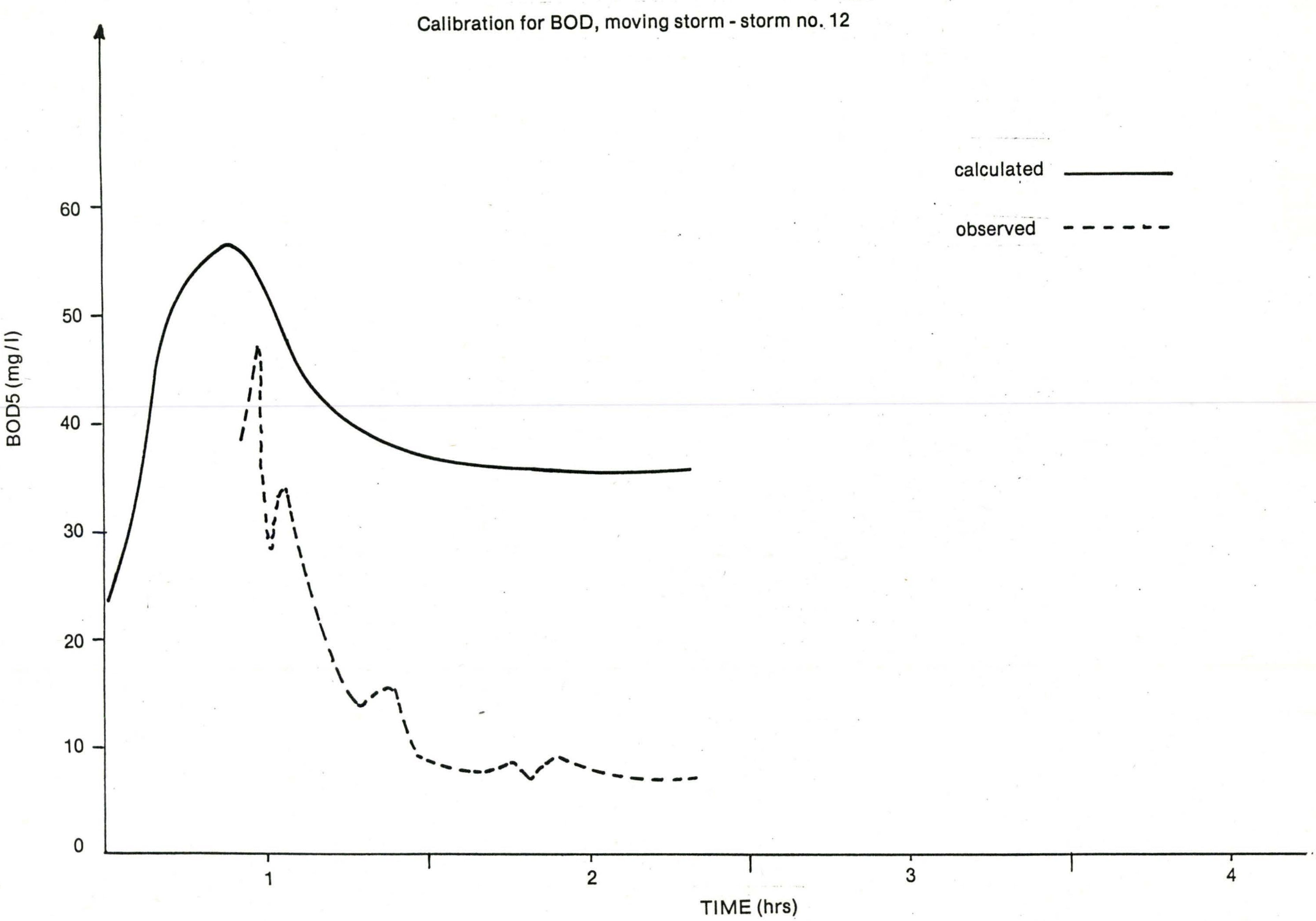


Figure 29: Calibration for BOD, Moving Storm, Storm No. 12

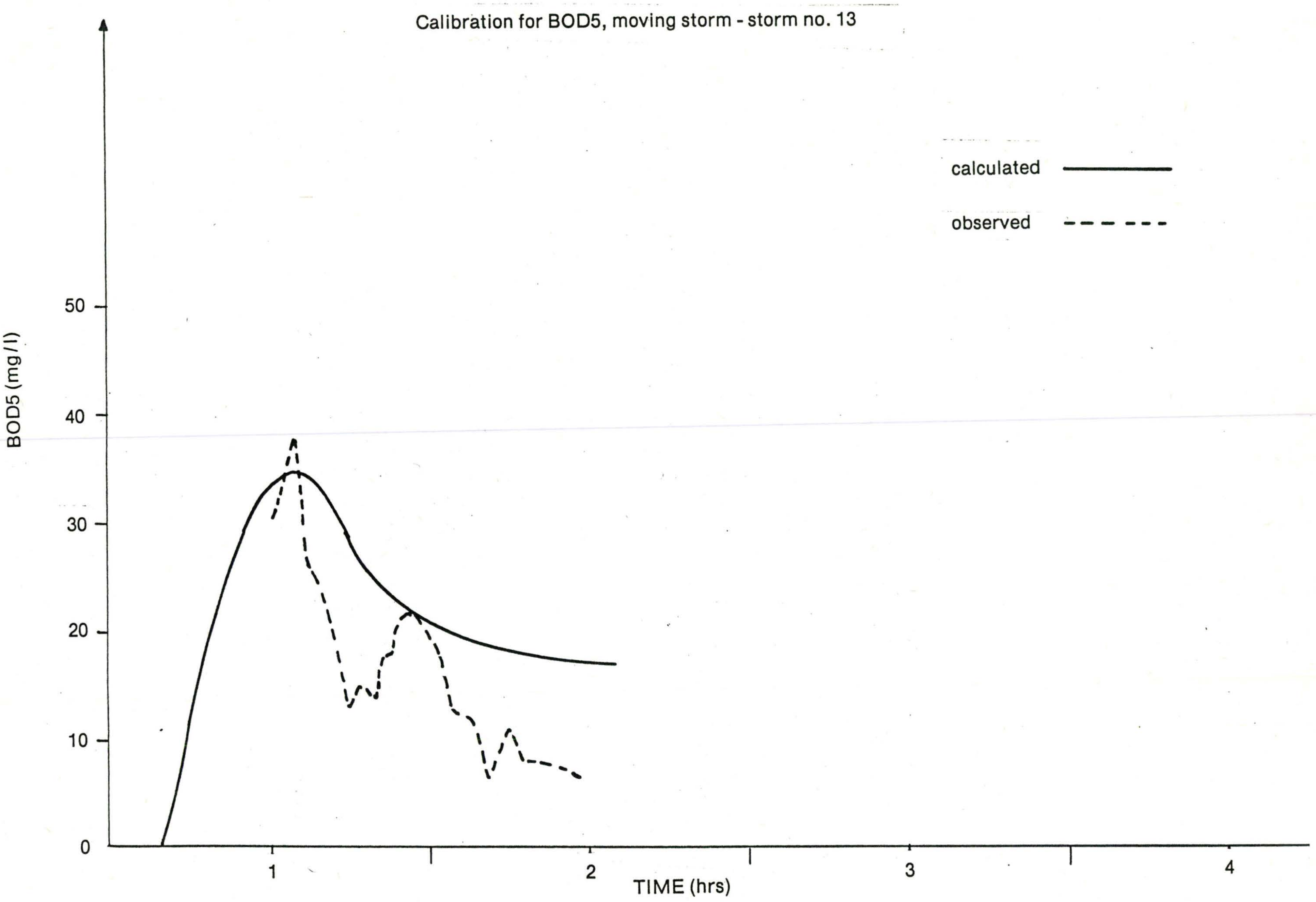


Figure 30: Calibration for BOD, Moving Storm, Storm No. 13

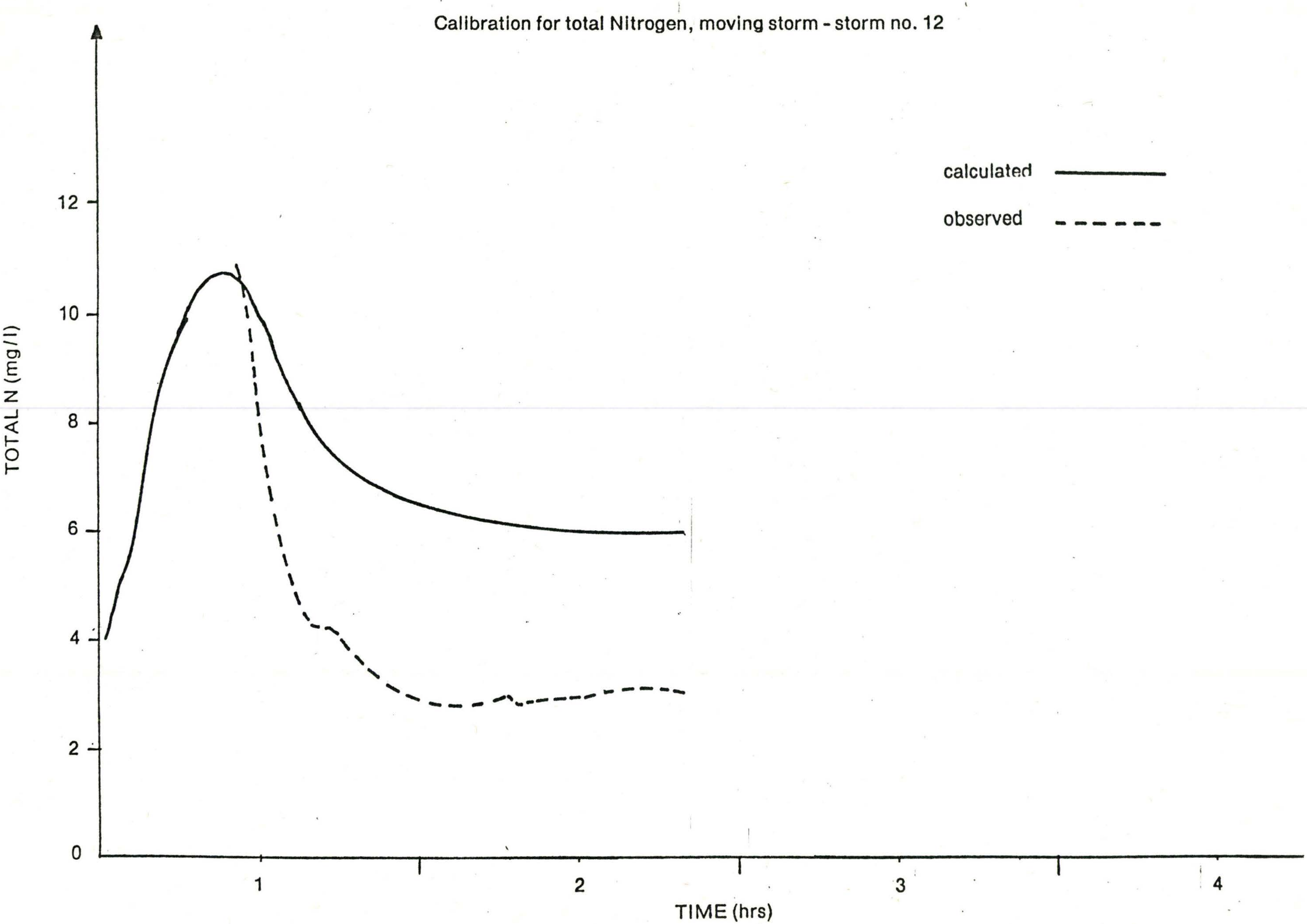


Figure 31: Calibration for Total Nitrogen, Moving Storm, Storm No. 12

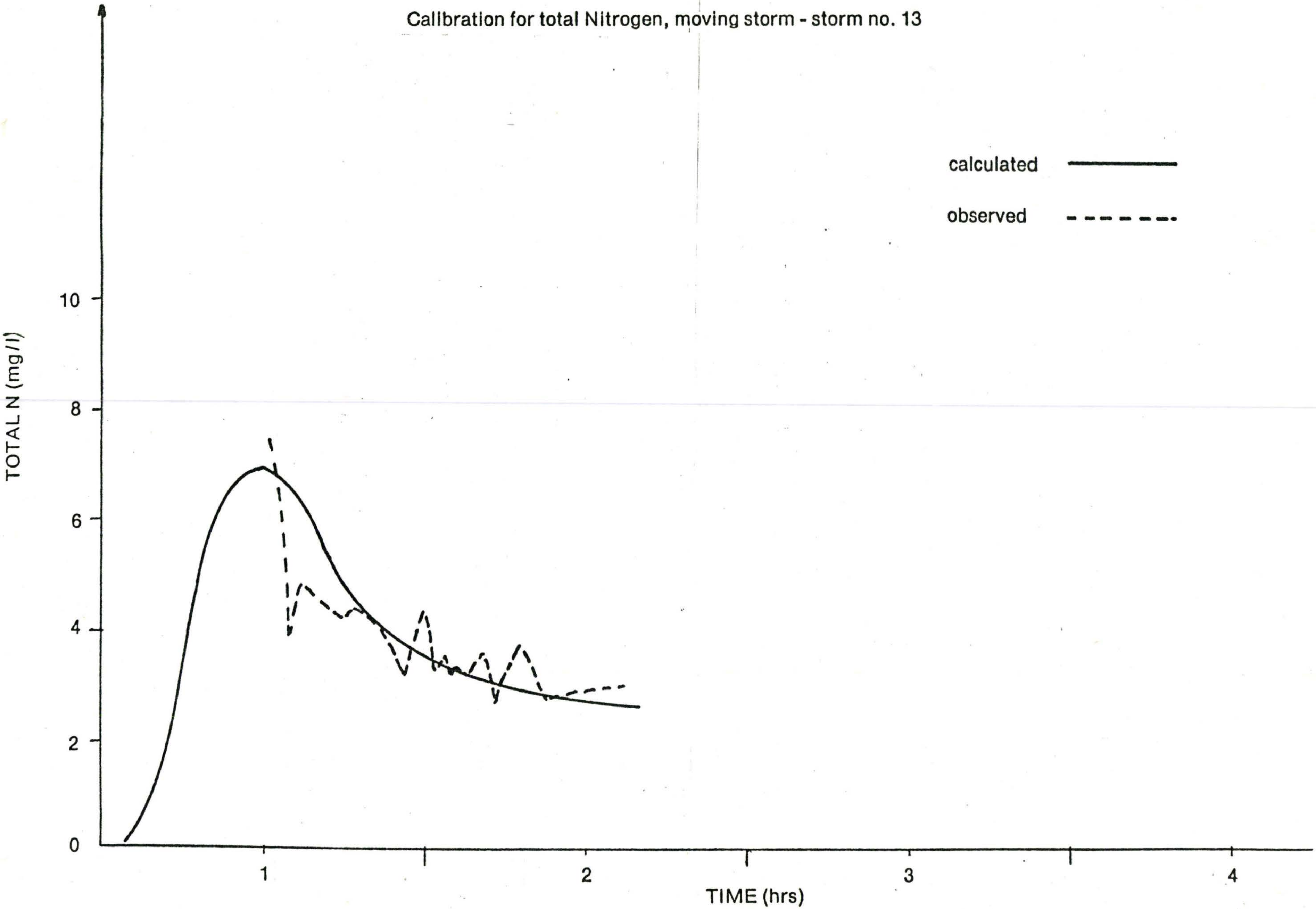


Figure 32: Calibration for Total Nitrogen, Moving Storm, Storm No. 13

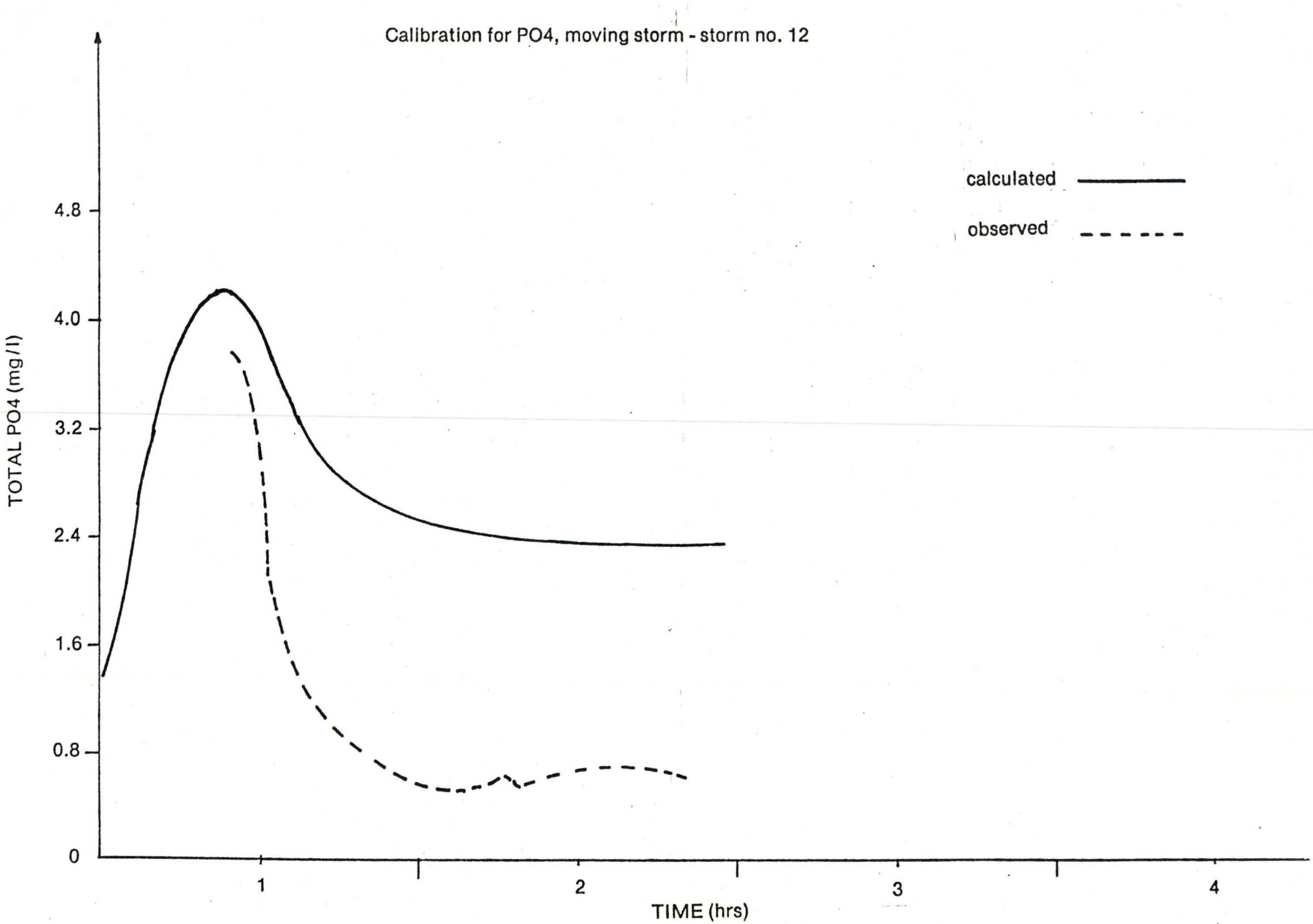


Figure 33: Calibration for PO4, Moving Storm, Storm No. 12

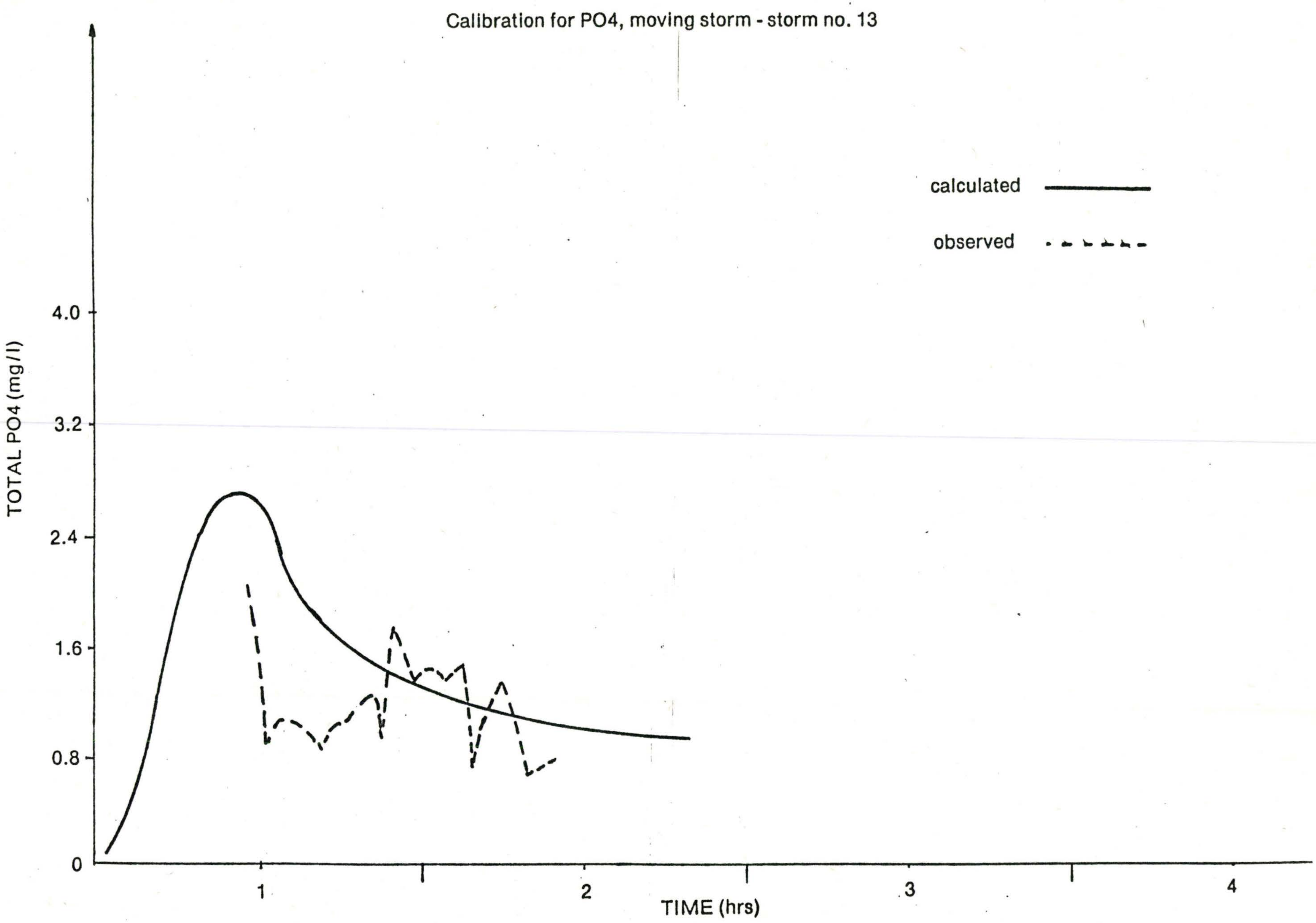


Figure 34: Calibration for PO4, Moving Storm, Storm No. 13

6.6 Comparison Between Observed and Computed Hydrographs and Pollutographs

As mentioned before the calibration procedure produced two sets of calibrated SWMM parameters: One set for observed Hamilton Airport hyetograph assumed to be stationary and uniform over the catchment and the second for the hyetographs generated by THOR, modelling spatially limited storms moving across the urban catchment. Three entirely different storms and responses, using the two sets of parameters, were then compared.

The final results after comparison between observed and computed flows are summarized in Tables 11 and 12.

Figures 35 to 49 present observed and computed hydrographs and pollutographs for the Chedoke Creek Catchment for Storms 8, 10 and 14. These storms were not used in the original calibration.

Table No. 11

Moving Storms
Comparison Between Observed and Computed Flows

	Peak flow (cfs)		Total Volume (ft3)	
Storm No.	Observed	SWMM	Observed	SWMM
8	233	282	939,000	633,000
10	175	260	1,171,500	819,500
14	340	345	1,173,000	820,500

Correlation Coefficient for Peak Flow = 0.995

R**2 = 99%

Correlation Coefficient for Total Volume = 1.0

R**2 = 100%

Table No. 12

Stationary Storm
Comparison Between Observed and Computed Flows

	Peak flow (cfs)		Total Volume (ft3)	
Storm No.	Observed	SWMM	Observed	SWMM
8	233	610	939,000	2,202,000
10	175	105	1,171,500	855,000
14	340	115	1,173,000	700,500

Correlation Coefficient for Peak Flow = 0.152

R**2 = 2.3%

Correlation Coefficient for Total Volume = 0.996

R**2 = 99.2%

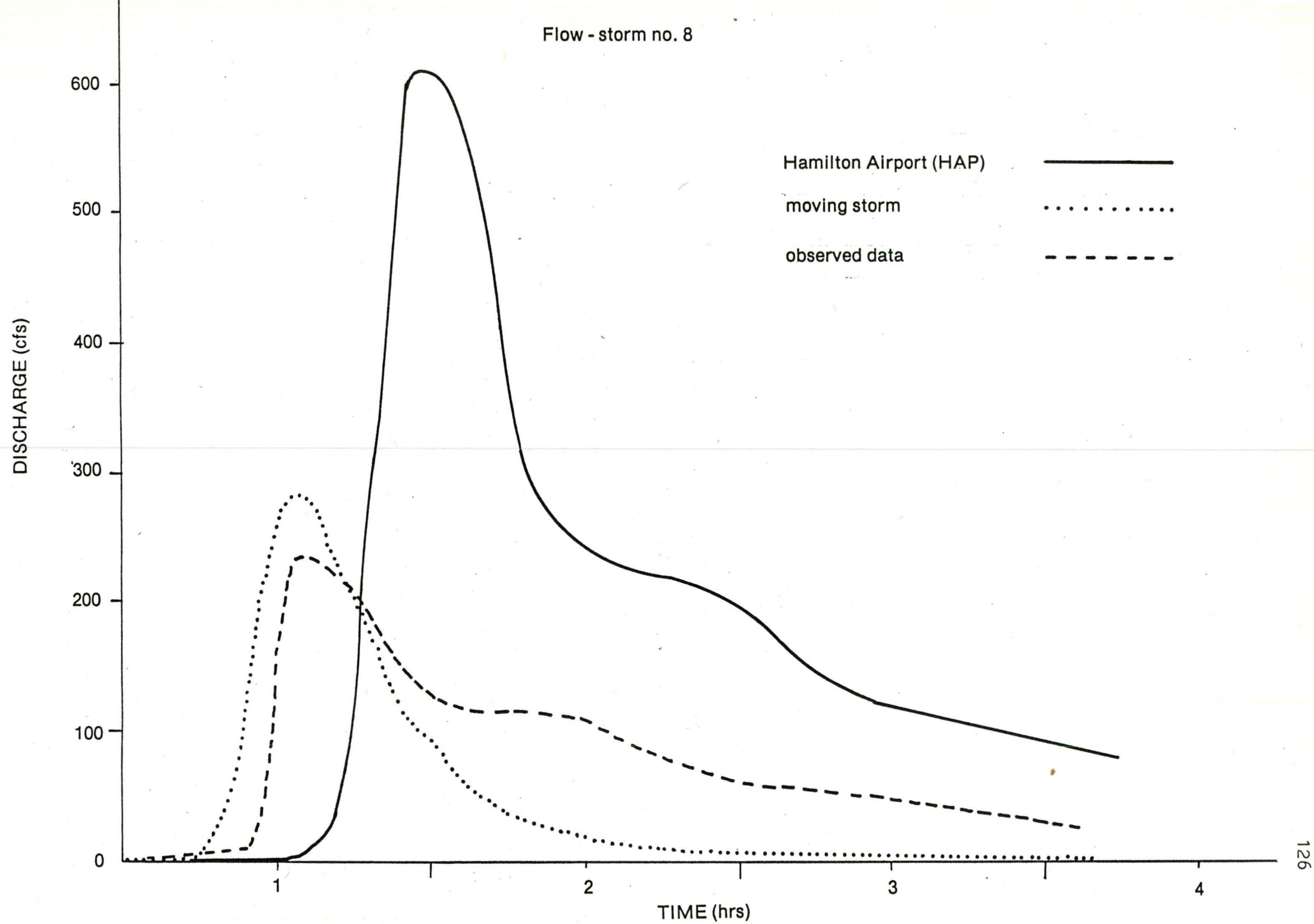


Figure 35: FLOW - Storm No. 8

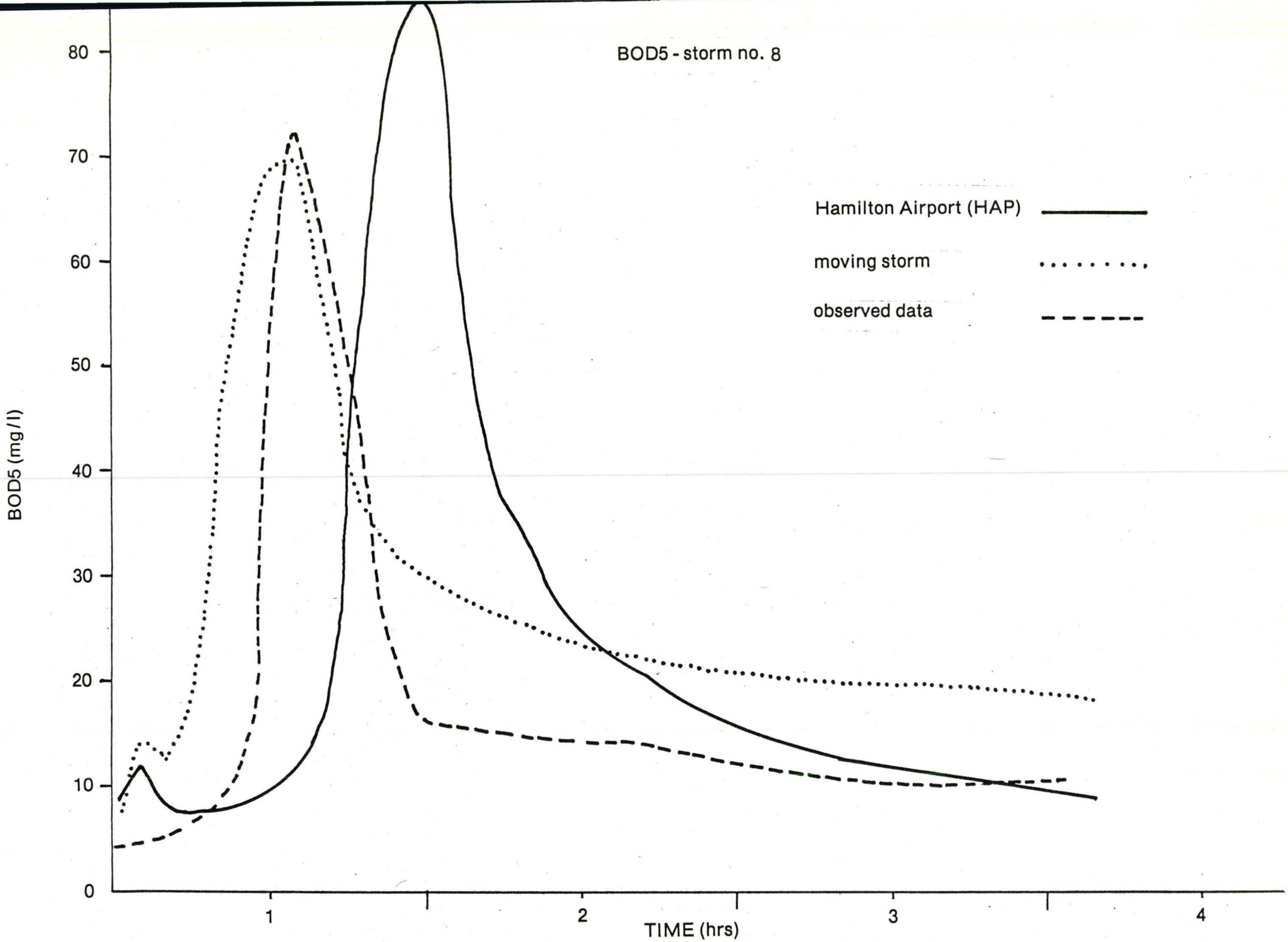


Figure 36: BOD - Storm No. 8

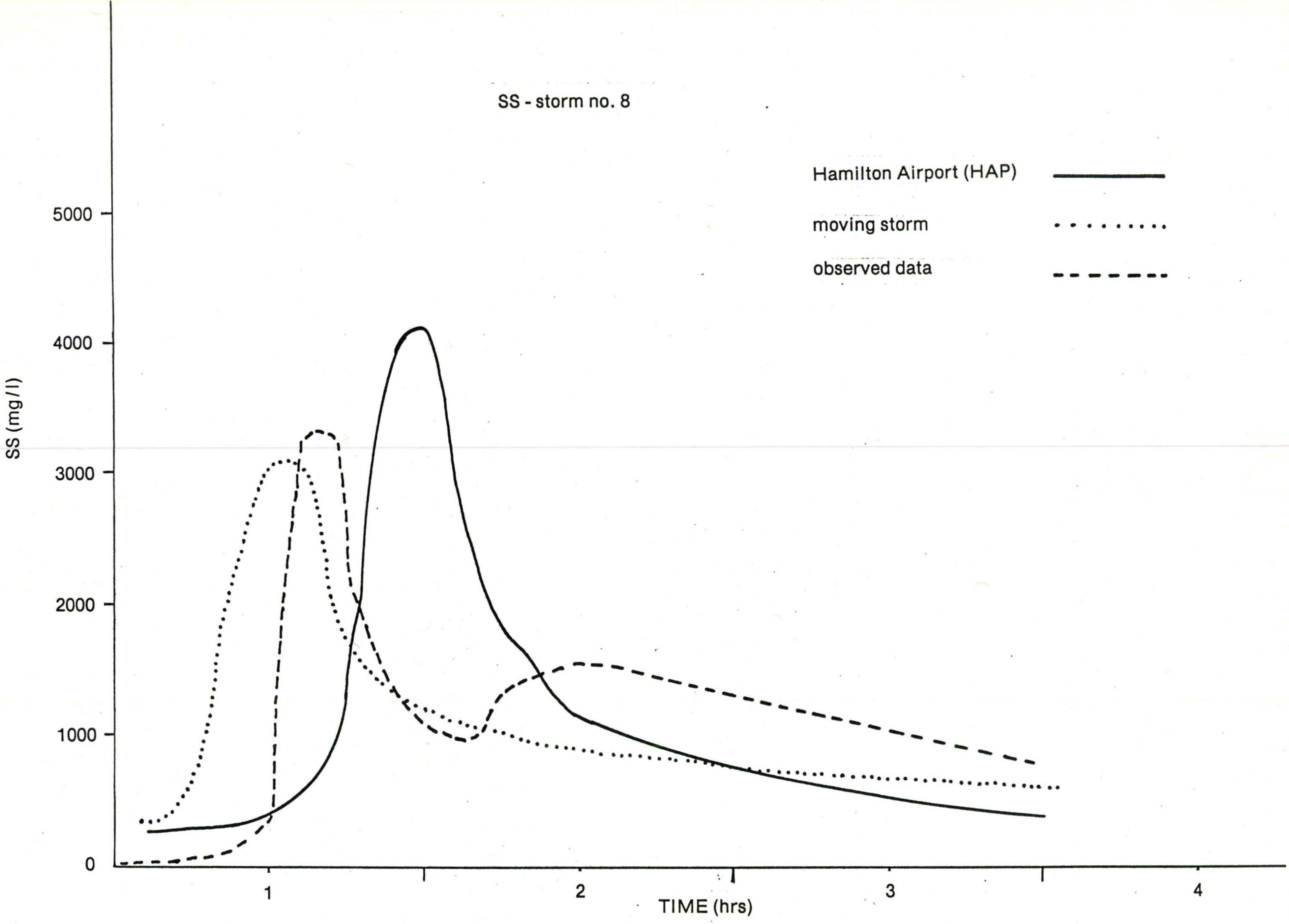


Figure 37: S.S. (Suspended Solids) - Storm No. 8

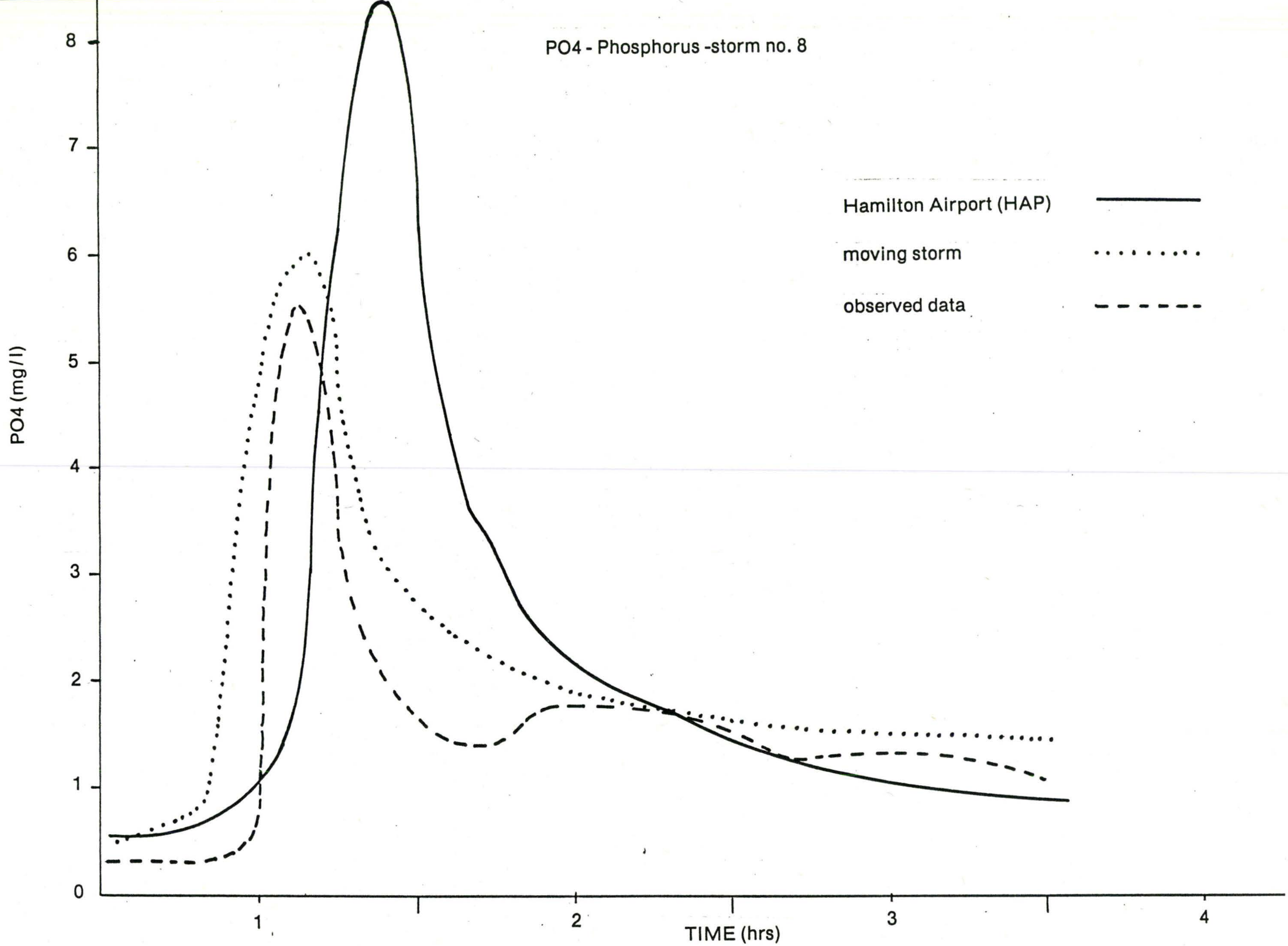


Figure 38: PO4 (Phosphorous) - Storm No. 8

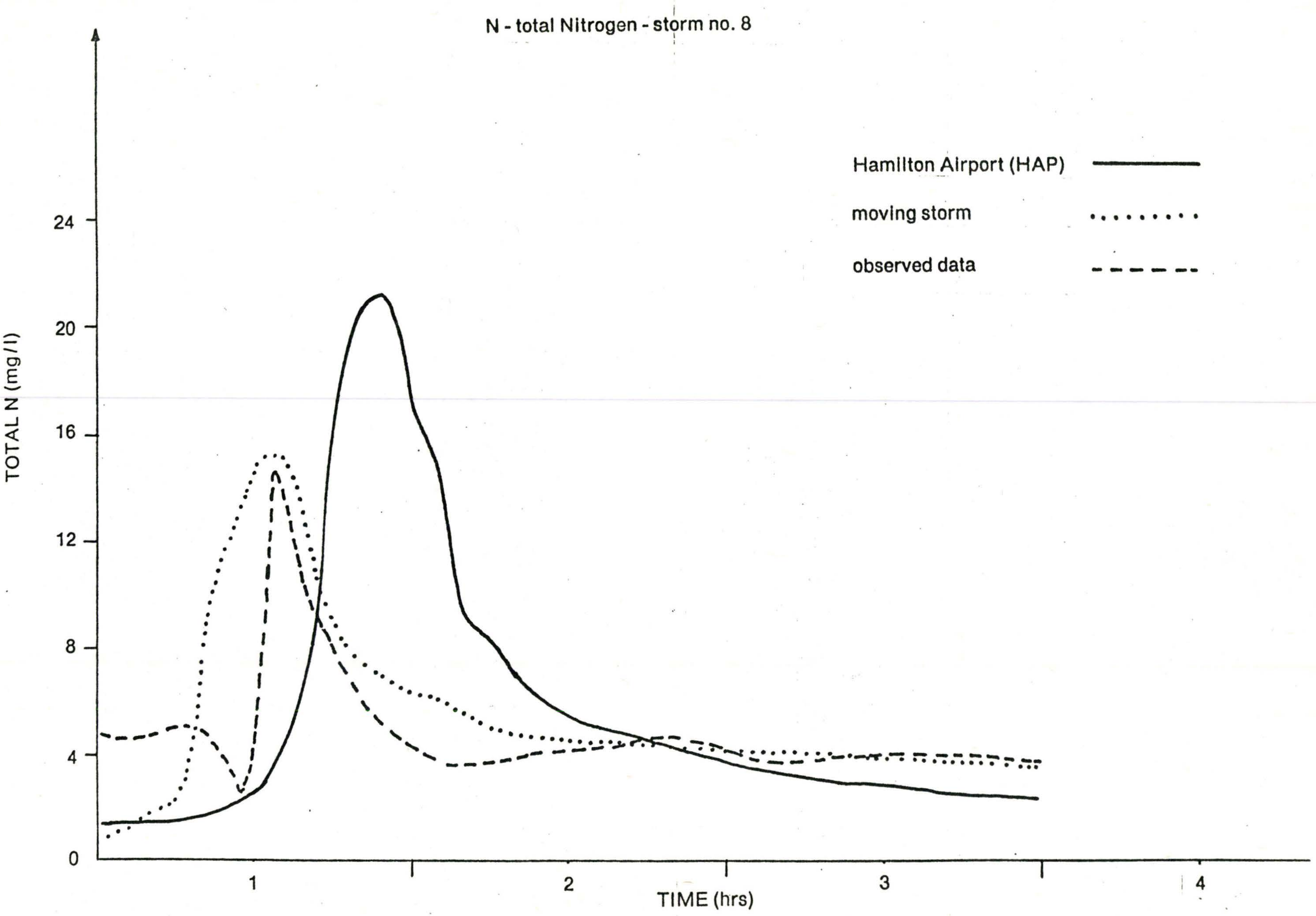


Figure 39: N (Total Nitrogen) - Storm No. 8

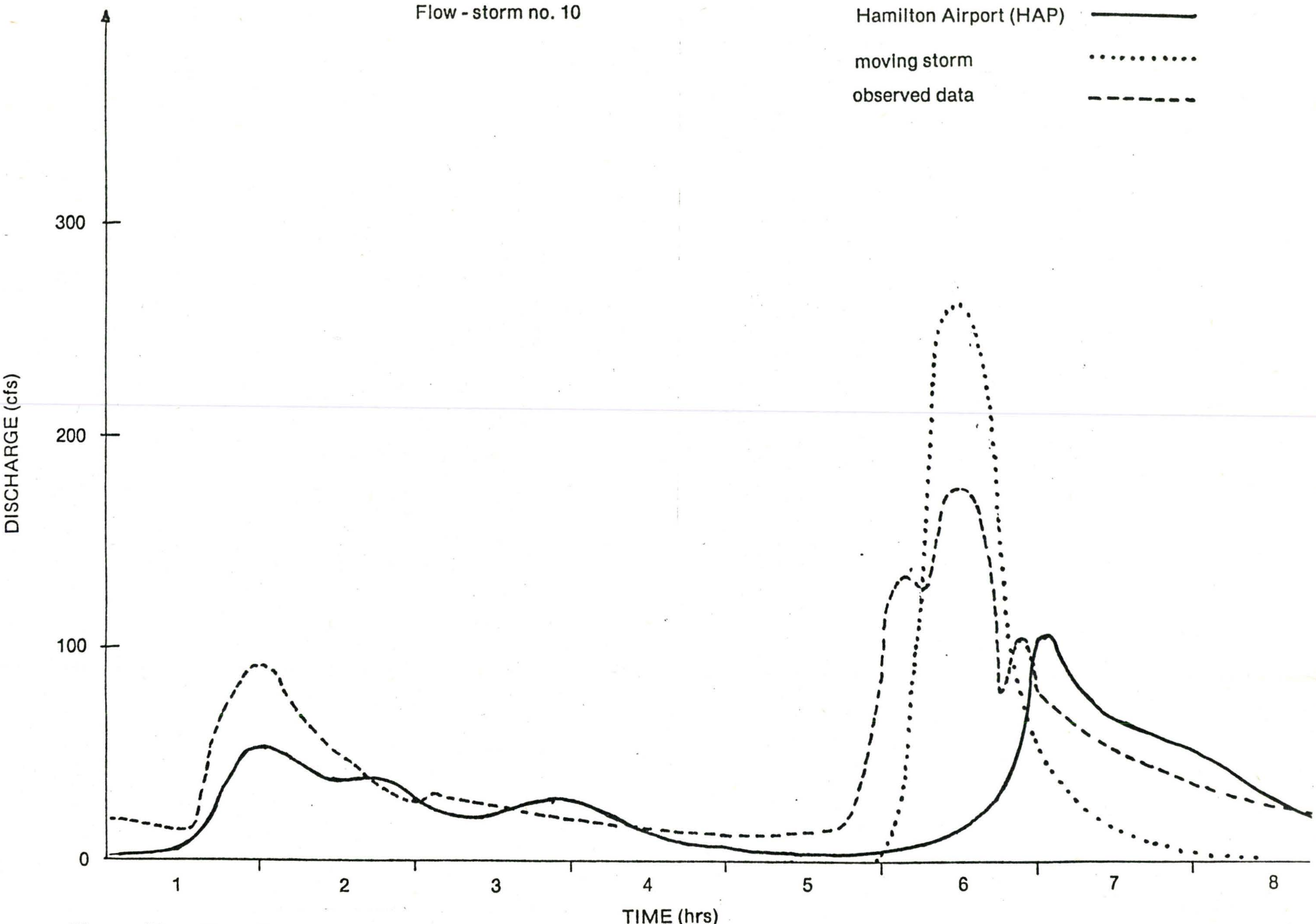


Figure 40: Flow Storm No. 10

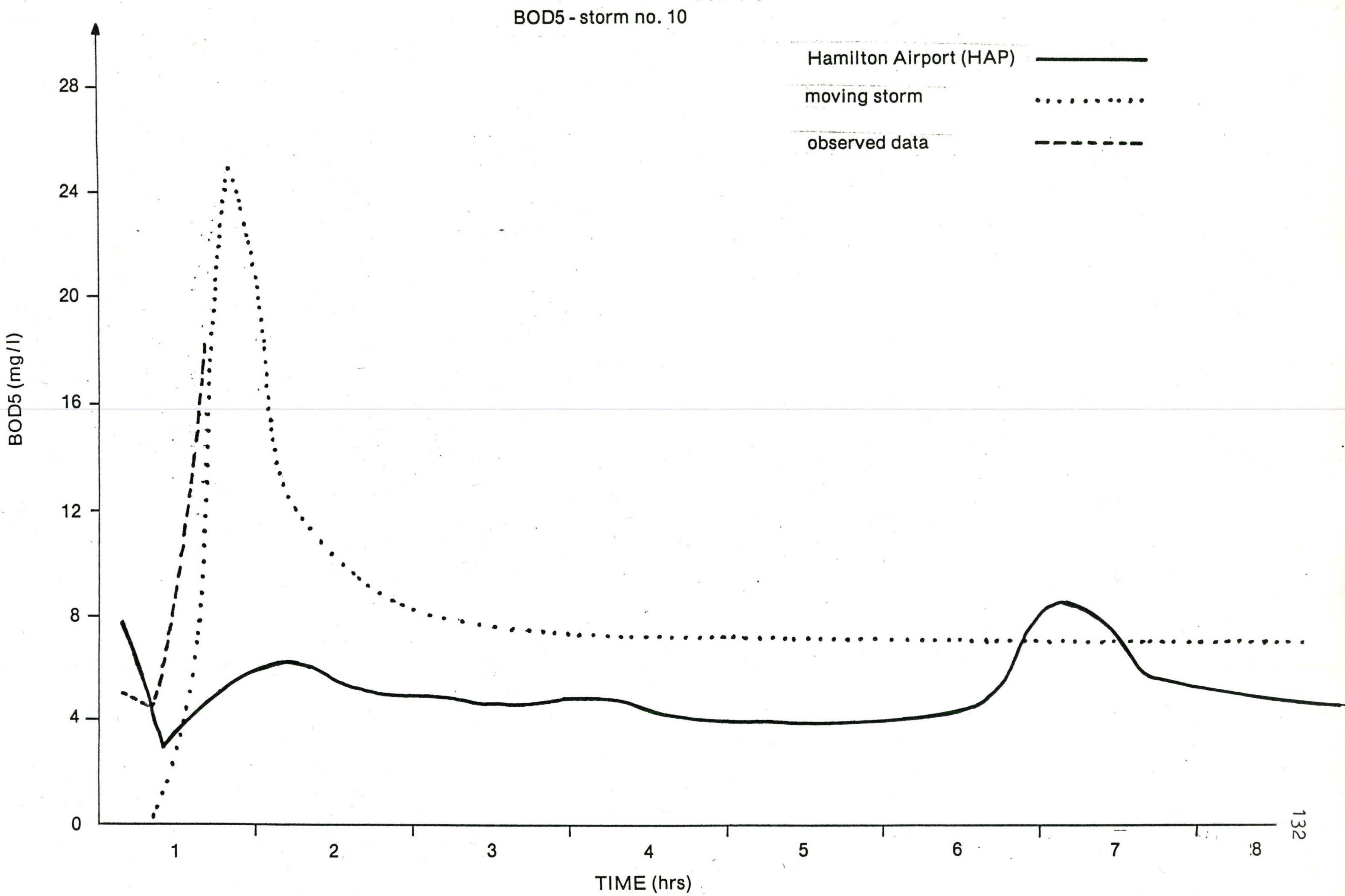


Figure 41: BOD Storm No. 10

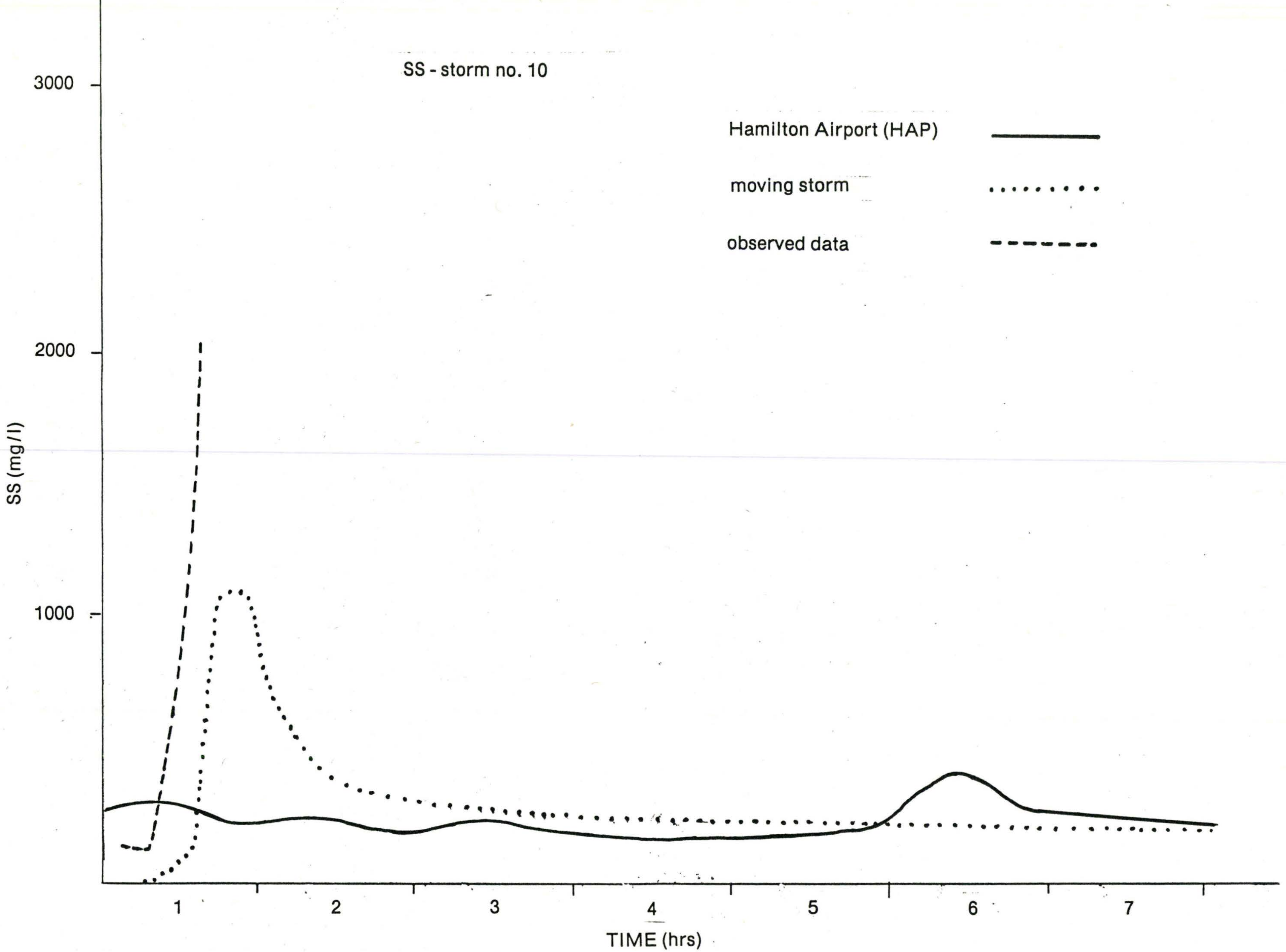


Figure 42: S.S. Storm No. 10

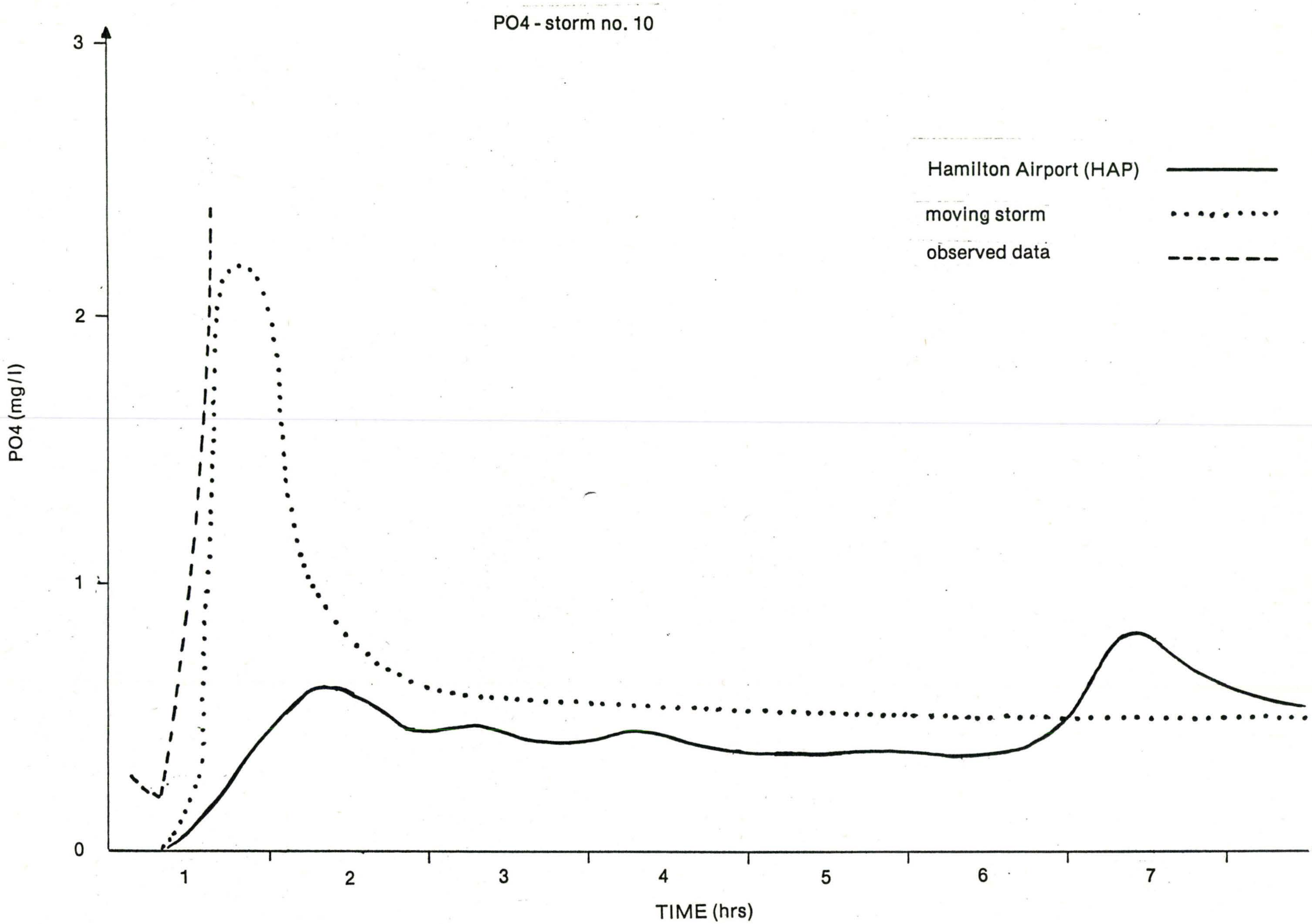


Figure 43: PO4 Storm No. 10

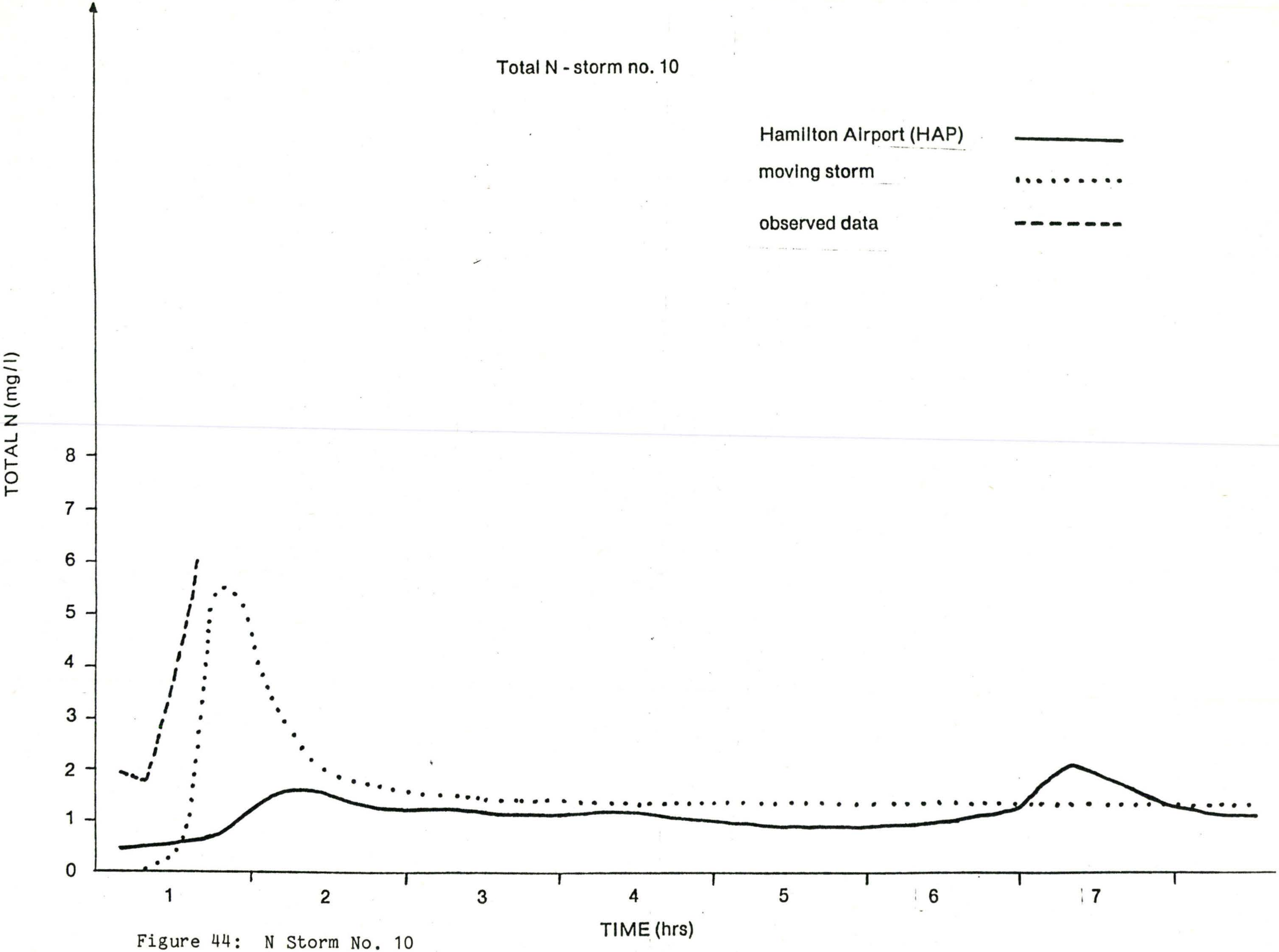


Figure 44: N Storm No. 10

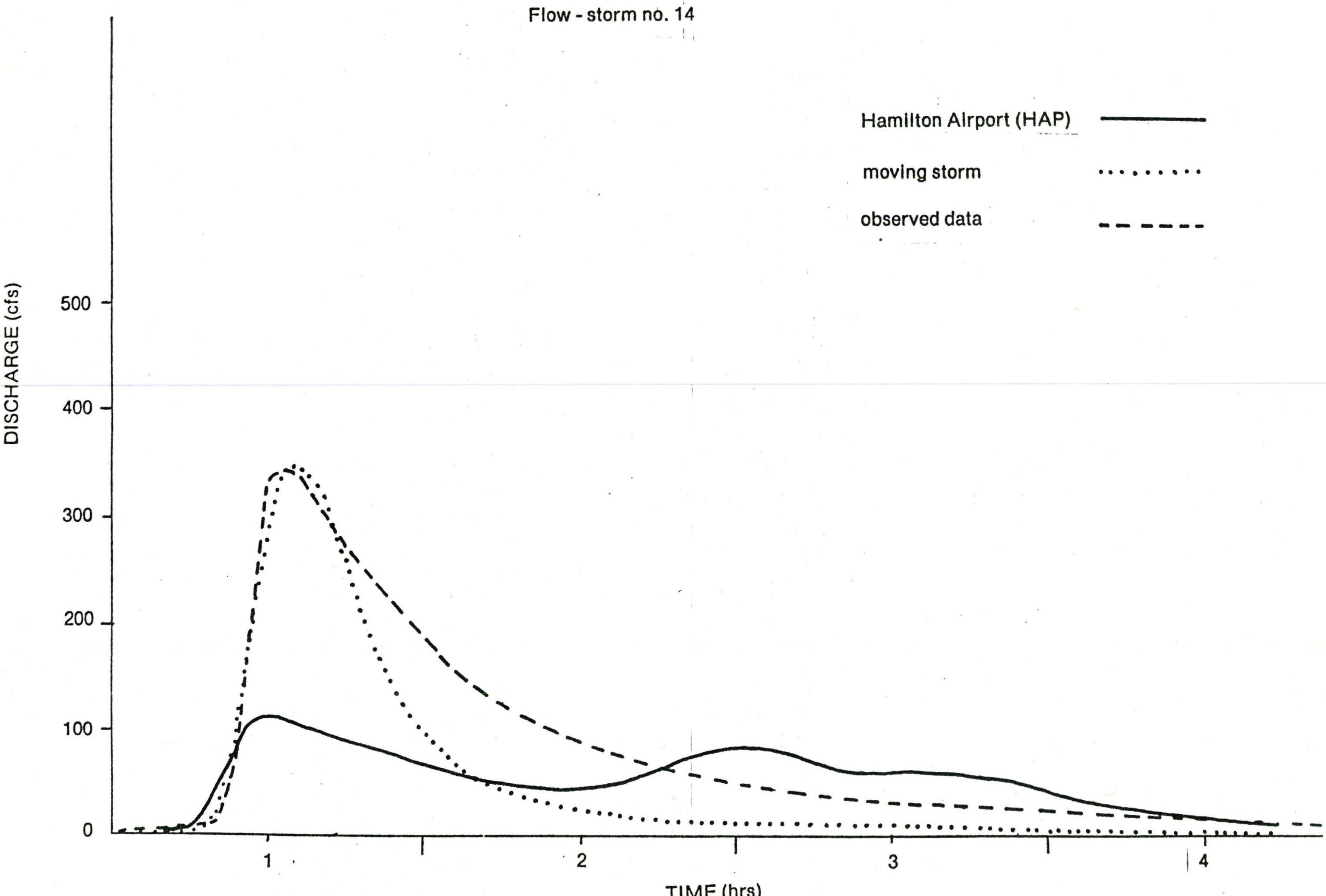


Figure 45: Flow Storm No. 14

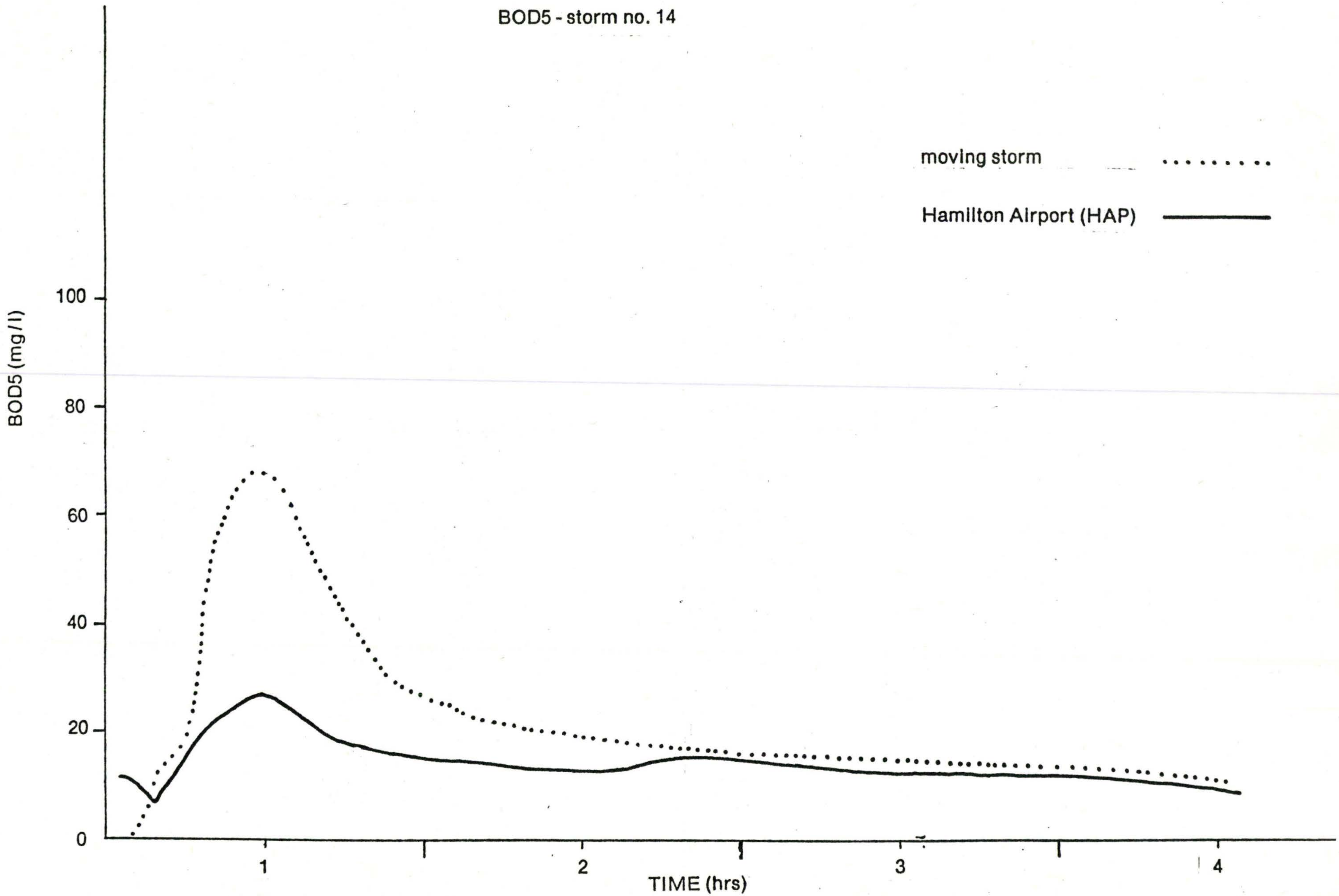


Figure 46: BOD Storm No. 14

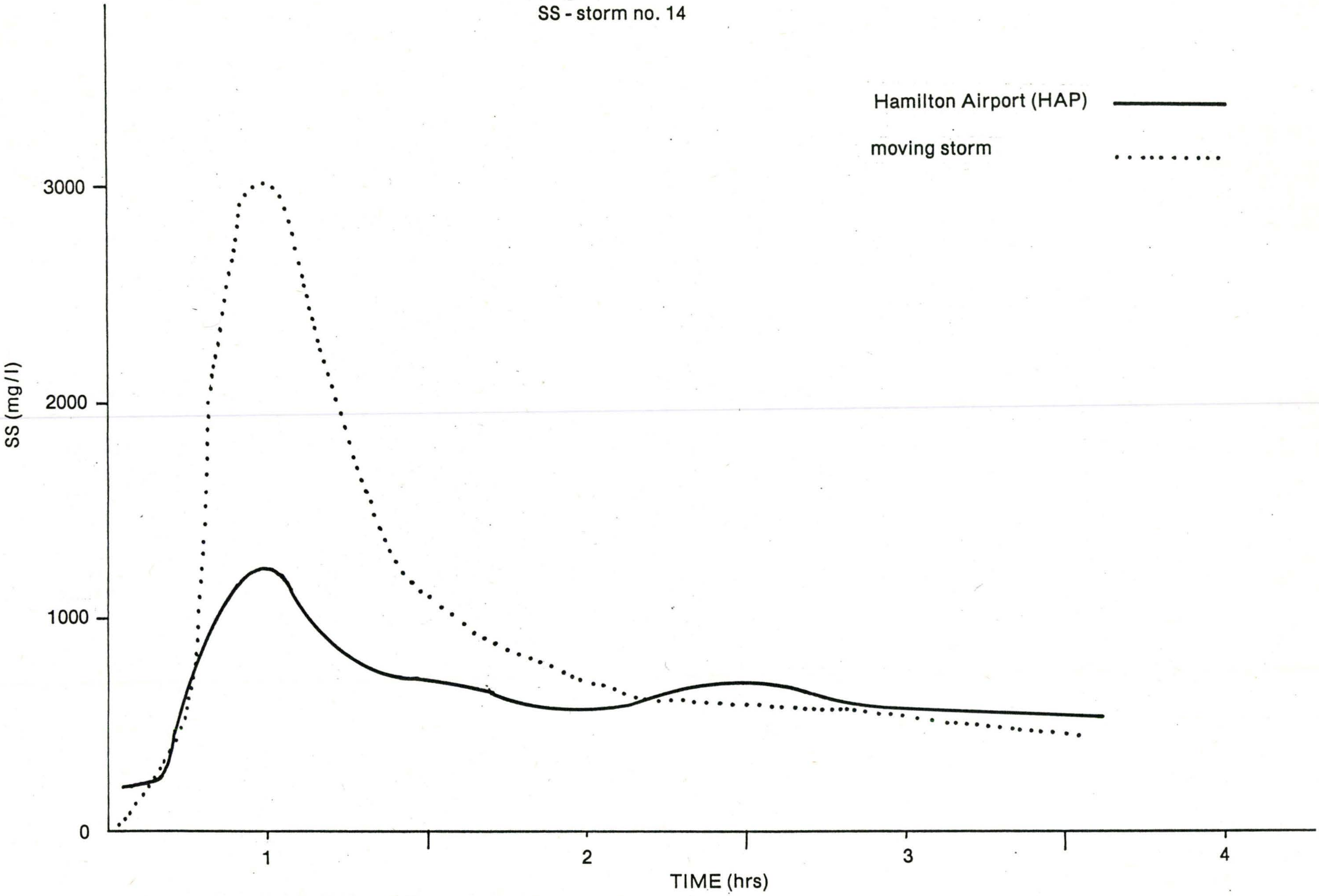


Figure 47: S.S. Storm No. 14

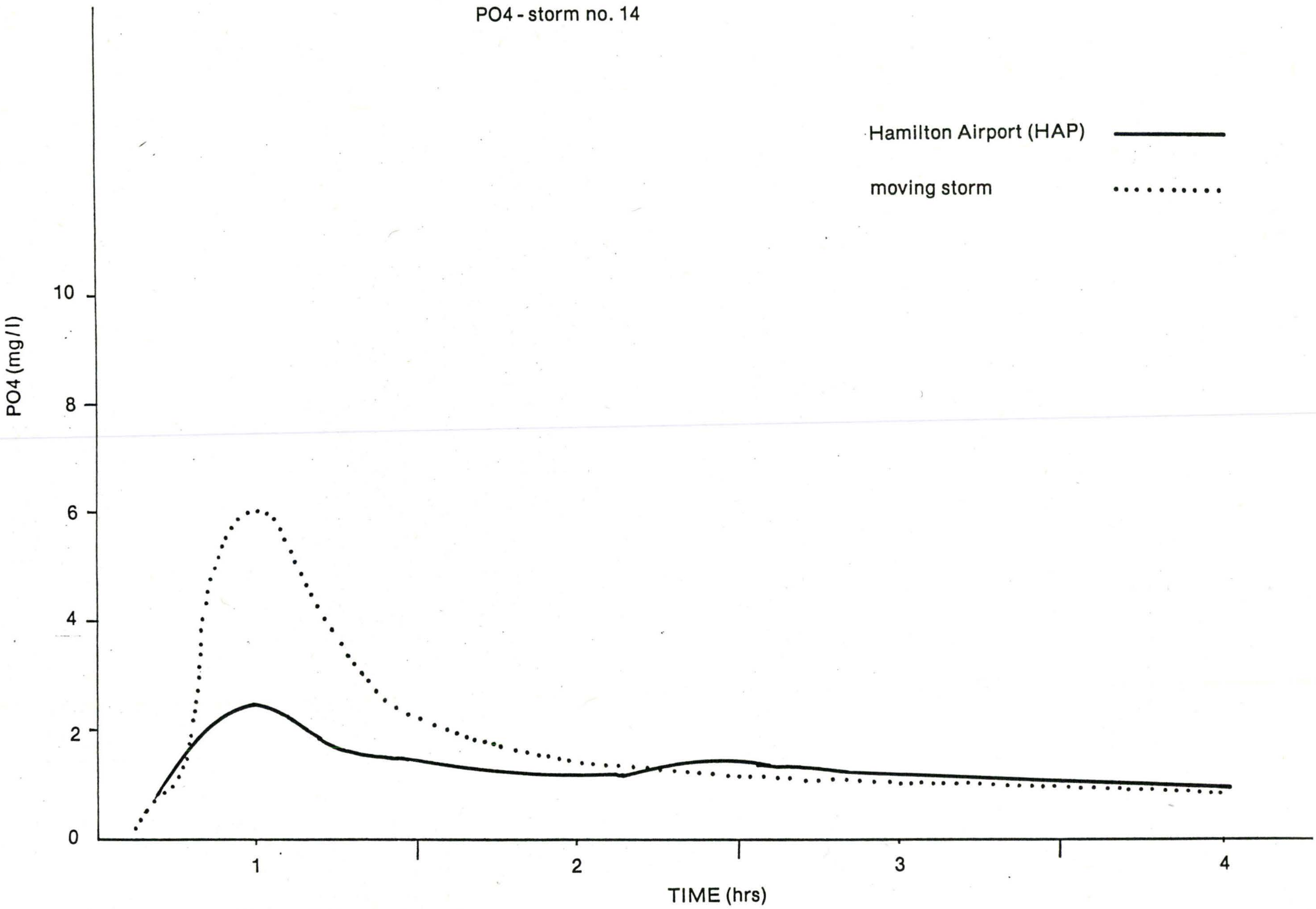


Figure 48: PO4 Storm No. 14

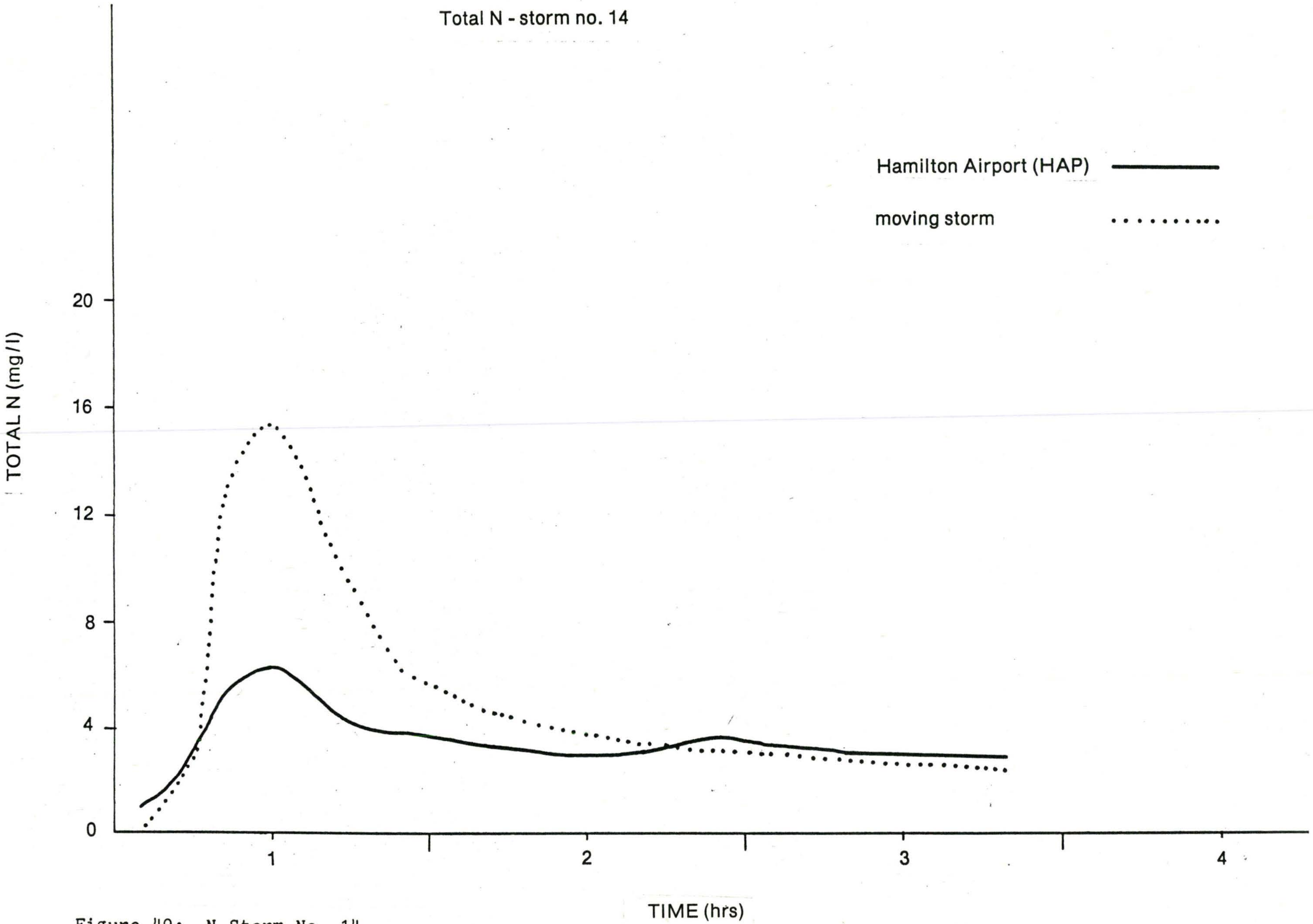


Figure 49: N Storm No. 14

6.7 Criticism of the Hamilton-Wentworth Design Storm

From Figures 9 and 10 we see the importance of adopting a moving storm model. A stationary storm model produces high peak flow, and very high total volume of flow, BOD, and S.S. The results produced for stationary storms are unlike the observed data and are extremely conservative. Thus using a stationary model for design will lead to unnecessarily high costs. Finally stationary design storms and moving storms were compared using: for a stationary storm, the "Hamilton-Wentworth Design Storm" applied uniformly across the catchment (produced 2.53 in. = 65 mm, total precipitation); for a moving storm, total precipitation of 65 mm, 130 mm and 200 mm and a wind velocity of 20 km/hr and direction 225.

The results are given in Figures 51 to 54. Large differences between peak flows and the volume of the flow are obtained even when using such extreme events as 200 mm total precipitation.

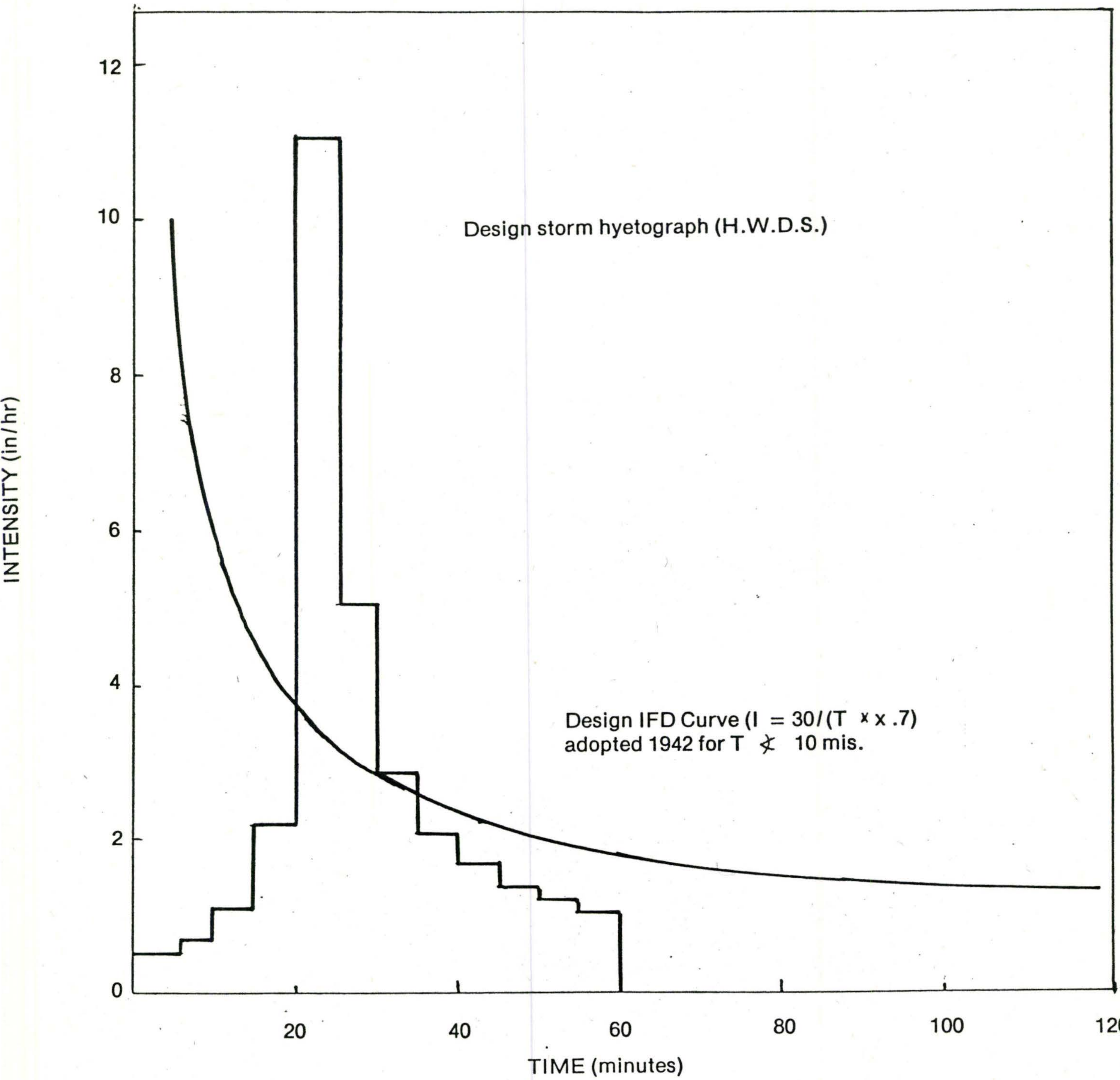


Figure 50: Hamilton-Wentworth Design Storm

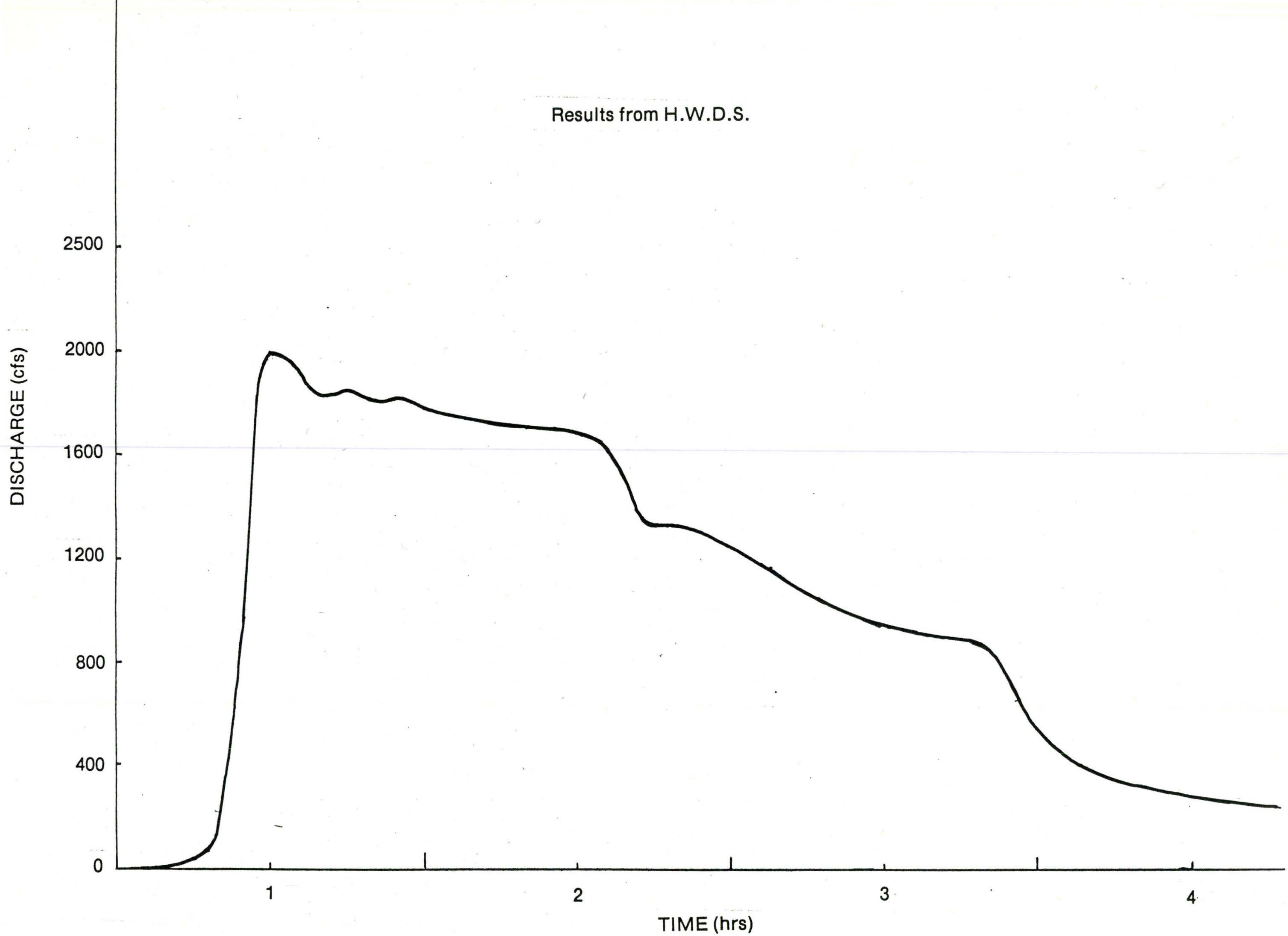


Figure 51: Results from H.W.D.S.

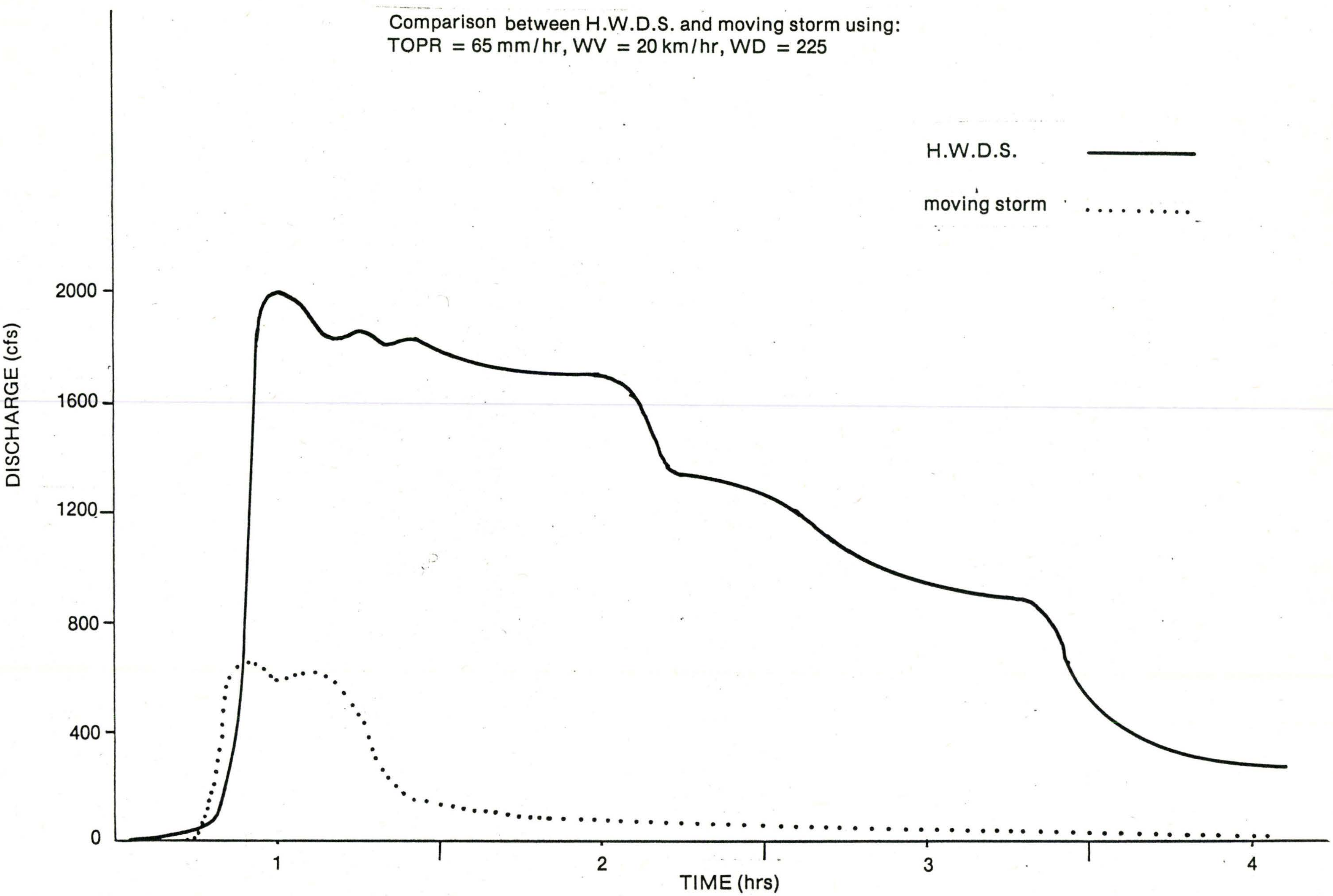


Figure 52: Comparison Between H.W.D.S. and Moving Storm Using TOPR = 65

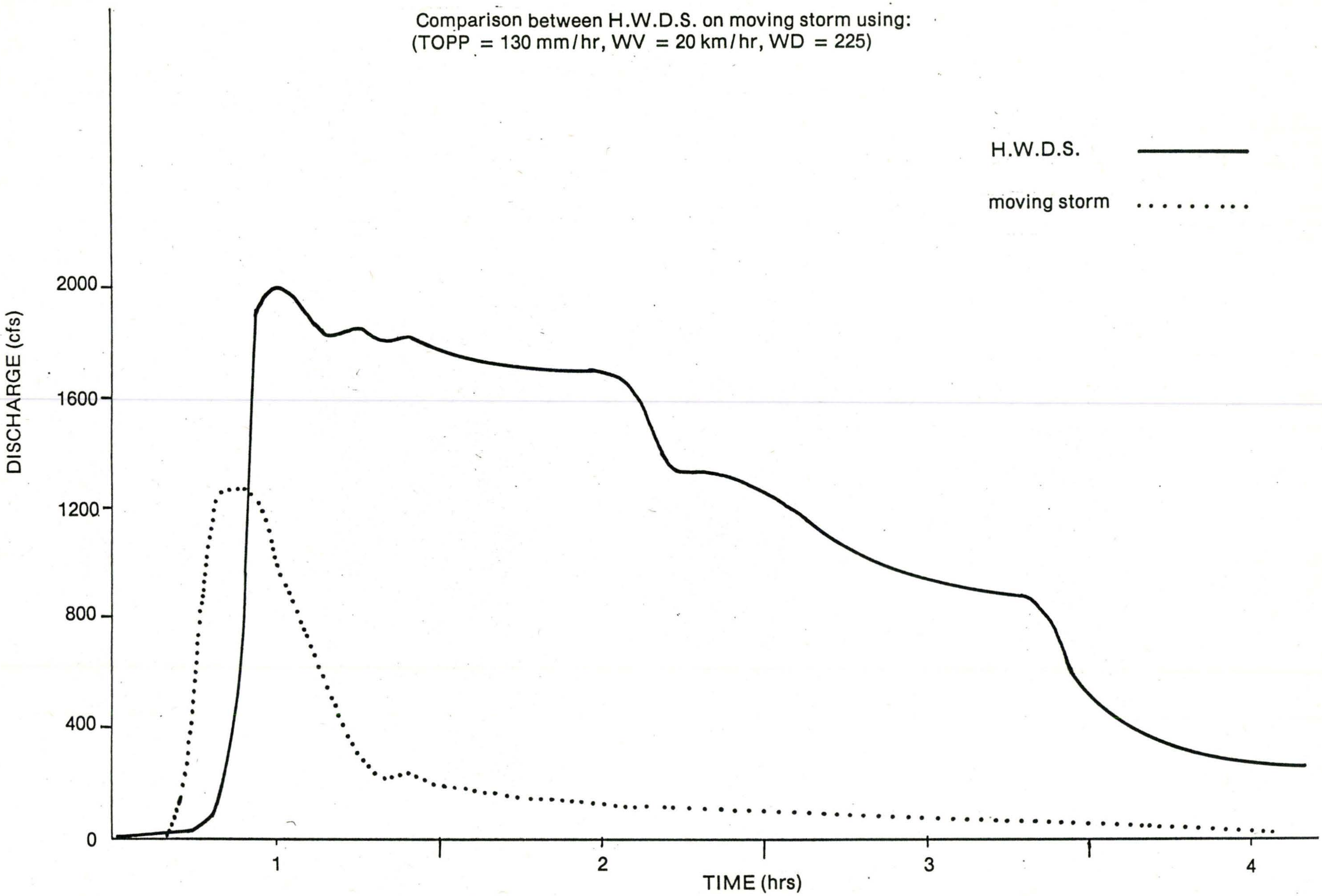


Figure 53: Comparison Between H.W.D.S. and Moving Storm Using TOPR = 130

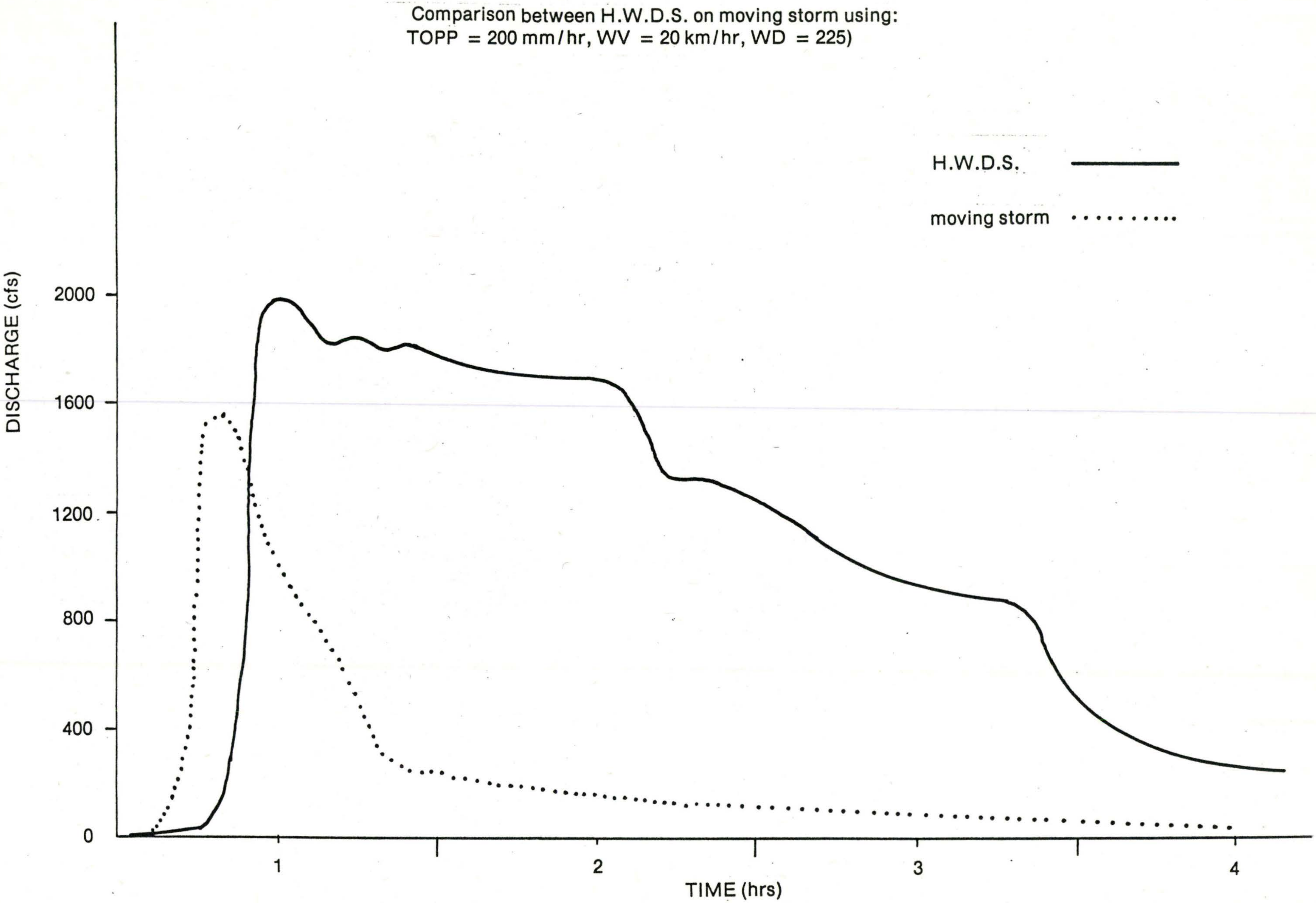


Figure 54: Comparison Between H.W.D.S. and Moving Storm Using TOPR = 200

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 General

Ground-based estimates of areal rainfall show considerably less attenuation from the maximum point value than do radar-based estimates (eg. Barge et al. (3)), because they do not take into account the translation of a storm across an area. The implication of these studies for operational urban hydrology is thus that a given total precipitation, wind velocity and duration from an extreme event in a given catchment may produce a smaller total volume of rain than is currently assumed from depth-area curves based on raingauge records, but that this volume may be distributed in space and time such that it gives rise to a greater peak runoff and rate-of-rise of the hydrograph (20, 21).

Simulations which make use of a stationary storm (20, 21) measured at a raingauge situated outside the study area can be improved by accounting for the movement of the storm across the catchment. THOR represents storms better than standard engineering practice does; computed hydrographs and pollutographs are much closer to observed data.

Nobody will dispute that the spatial distribution of rainfall has a direct effect on the amount of storm runoff, but the argument has been made in the past that errors in the rainfall input will be dampened when routed through a basin. The results from the runoff measured in this study do not support the above argument and point to the fact that using rainfall observed from one raingauge, assumed to be distributed uniformly all over the catchment leads to large discrepancies in the peak and volume of the observed output.

It seems clear from this study that the spatial and temporal distribution of rainfall has a marked influence on the behaviour of the runoff hydrograph and pollutographs. When storms are intense and of short duration they are more localized in space and time, i.e. storms of the convective type including thunderstorms. In these cases the application of rainfall runoff models without an appropriate description of the spatial and temporal character of the input may lead to unacceptable errors.

From this study a very good correlation was found between: wind direction and storm direction; wind velocity and storm velocity; peak intensity and total precipitation; and peak intensity, and the shape of the hyetograph.

7.2 Recommendations

Rainfall usually varies considerably from point to point within the watershed. In Hamilton, there is one raingauge located outside the urban catchments. For large catchments it becomes very important to have at least 2-3 raingauges located in the area. The "Hamilton-Wentworth Design Storm" can be improved by accounting for the movement of the storm across the catchment. It would be recommended to do future research on the model THOR in more than one dimension (space) as was presented in this study.

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APPENDIX A

THE MODEL THOR

A.1 List of THOR

```
STOHYS
.STOHYS
 PROGRAM STOHYS(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8,TAPE4)
 DIMENSION P(300),PX(300),PXT(300),PXT1(300),T(300),C(299,3)          A-2
 DIMENSION BPAR(4),DX1(10),DX2(10),DX3(10),DX4(10),XO1(10)
 DIMENSION XO2(10),XO3(10),XO4(10),XO5(10),XO6(10),XO7(10)
 DIMENSION XO8(10),TP(300),DX(10),XO(10),DXT(10),XOT(10),ISIZE(100)

 DATA BPAR/0.,0.,0.,0./
 DATA DX1/1.005,1.1,1.22,1.463,0.853,1.22,1.372,2.743,1.31,1.19/
 DATA DX2/0.915,0.49,0.43,0.49,0.55,0.366,0.305,1.036,0.975,1.097/
 DATA DX3/1.22,0.49,0.46,0.610,0.79,0.61,0.43,0.975,1.036,0.79/
 DATA DX4/1.47,1.524,1.067,0.915,0.915,1.22,0.99,1.22,1.372,0.915/
 DATA XO1/0.49,1.645,1.80,2.408,2.4,1.28,2.135,1.89,3.35,3.26/
 DATA XO2/0.565,1.085,0.823,1.645,2.012,1.524,2.25,2.59,2.77,1.83/
 DATA XO3/1.37,1.06,0.58,1.16,1.645,1.95,2.135,2.80,1.83,0.64/
 DATA XO4/3.17,2.1,1.68,1.59,1.92,3.02,2.56,3.26,1.43,0.61/
 DATA XO5/2.47,2.32,2.16,1.55,1.52,2.74,1.83,2.07,0.64,0.67/
 DATA XO6/3.14,2.56,2.804,1.92,1.646,2.135,1.27,1.066,0.85,1.80/
 DATA XO7/2.135,2.377,2.92,2.32,1.86,1.43,1.25,0.61,1.645,2.865/
 DATA XO8/1.066,2.135,2.59,2.65,2.286,1.19,1.646,1.005,2.80,3.6/

 DO 700 I=1,300
 PXT(I)=0.0
 PXT1(I)=0.0
 CONTINUE
 NTAPE=9
 NOUT=4

 WRITE(6,488)
 FORMAT(/,4X," THIS PROGRAM IS ONLY FOR THE CHEDOKE CREEK BASIN",/
$2X,"THE PROGRAM IS FOR DURATION OF RAINFALL LESS THEN 1.5HR",/
$2X,"FOR USE IN OTHER CATCHMENT THE USER SHOULD CHANGE THE",/
$2X,"SUBCATCHMENT CHARACTERISTICS (DX,XO) FOR THE 8 DIFFERENT ",/
$2X,"DIRECTIONS ...! ")

 WRITE(6,500)
 FORMAT(//,2X,"ENTER DATA MEASURED AT THE RAINGAGES: ")
 WRITE(6,505)
 FORMAT(//,2X,"TOPR(MM)",4X,"WIND VEL(KM/HR)",4X,"WIND DI.",3F7.2)
 READ(5,*)TOPR,VW,WD
 FORMAT(3F7.2)

 PO=((2.6)+1.81*TOPR)
 B1=(0.101+0.0025*PO)
 B2=(0.0818+0.299*B1)
 VS=(7.39+0.933*VW)
 SD=(33.+0.884*WD)
 TR=ALOG(20.)/(B1)
 TF=ALOG(20.)/(B2)
 TS=TR+TF
 XP=(TS*VS)/(60.)

 WRITE(6,520)
 FORMAT(//,2X,"STORM CHARACTERISTICS :")
 WRITE(6,525)PO
 FORMAT(//,2X,"MAX INTENSITY",2X,"PO= ",F7.2,2X,"MM/HR")
 WRITE(6,530)XP
 FORMAT(//,2X,"LENGTH (KM) BETWEEN 0.05PO MM/HR ",2X,"XP= ",F7.2)
 WRITE(6,535)TS
 FORMAT(//,2X,"TIME BASE (MIN) ",2X,"TS= ",F7.2)
 WRITE(6,805)TR
 FORMAT(//,2X,"TIME OF RISE (MIN)",2X,"TR= ",F7.2)
 WRITE(6,540)TF
 FORMAT(//,2X,"TIME OF FALL (MIN)",2X,"TF= ",F7.2)
```

```
WRITE(6,545)SD
FORMAT(//,2X,"STORM DIRECTION ",2X,"SD= ",F7.2)
WRITE(6,810)VS
FORMAT(/,2X,"THE VELOCITY OF THE STORM",2X,"VS= ",F7.2)
WRITE(6,550)
FORMAT(//,2X,"ENTER AVERAGING PERIOD(MIN).. ",F7.2)
READ(5,*)DT
FORMAT(F7.2)
WRITE(6,450)
FORMAT(/,X,"ENTER NUMBER OF RAINGAGES ...",F5.2)
READ(5,*) NRAGA
FORMAT(F5.2)
WRITE(6,560)DT
FORMAT(//,2X,"AVERAGE INTENSITY OVER DX",2X,"DT= ",F7.2,2X,"M")

DO 900 J=1,NRAGA
IF(SD.EQ.0.)GO TO 30
IF(SD.EQ.360.)GO TO 30
IF(SD.LT.90.0.AND.SD.GT.0.0)GO TO 40
IF(SD.EQ.90.)GO TO 50
IF(SD.LT.180.AND.SD.GT.90.)GO TO 60
IF(SD.EQ.180.)GO TO 70
IF(SD.LT.270.0.AND.SD.GT.180.)GO TO 80
IF(SD.EQ.270.)GO TO 10
IF(SD.LT.360.0.AND.SD.GT.270.)GO TO 20
DX(J)=DX1(J)
XO(J)=XO1(J)
GO TO 666
DX(J)=DX2(J)
XO(J)=XO2(J)
GO TO 666
DX(J)=DX3(J)
XO(J)=XO3(J)
GO TO 666
DX(J)=DX4(J)
XO(J)=XO4(J)
GO TO 666
DX(J)=DX1(J)
XO(J)=XO5(J)
GO TO 666
DX(J)=DX2(J)
XO(J)=XO6(J)
GO TO 666
DX(J)=DX3(J)
XO(J)=XO7(J)
GO TO 666
DX(J)=DX4(J)
XO(J)=XO8(J)
CONTINUE
DELT=0.5
XOT(J)=(XO(J)*60.)/VS
DXT(J)=(DX(J)*60.)/VS
TP(J)=XOT(J)+TP
NT=300

DO 200 I=1,NT
T(I)=(I-1)*DELT
IF(T(I).GT.TP(J))GO TO 210
P(I)=PO*EXP(-B1*(TP(J)-T(I)))
GO TO 200
P(I)=PO*EXP(-B2*(T(I)-TP(J)))
IF(P(I).LT.0.05*PO)P(I)=0.
```

```
DO 22 I=1,NT
PX(I)=P(I)
NP=NT
CALL ICSICU(T,P,NT,BPAR,C,NT-1,IER1)
NP=NT-(DXT(J)/DELT)-1.0
I1=((DXT(J)/DELT)/2.)+2.
DO 33 I=I1,NP
T1=T(I)-DXT(J)/2.
T2=T(I)+DXT(J)/2.
CALL DCSQDU(T,P,NT,C,NT-1,T1,T2,Q,IER2)
PX(I)=Q/DXT(J)
IF(PX(I).LT.0.)PX(I)=0.
CALL ICSICU(T,PX,NP,BPAR,C,NP-1,IER3)
NP1=NP*DELT/DT
ISIZE(J)=NP1
DO 55 I=1,NP1
T1=(I-1)*DT
T2=T1+DT
CALL DCSQDU(T,PX,NP,C,NP-1,T1,T2,Q,IER4)
PXT(I)=Q/DT
IF(PXT(I).LT.0.)PXT(I)=0.
DO 66 II=1,NP1
PXT1(II)=PXT(II)/25.4
CONTINUE
WRITE(NTAPE)(PXT1(I),I=1,NP1)
CONTINUE

DO 770 J=1,NRAGA
NP1=ISIZE(J)
IF(J.EQ.1)GO TO 771
IF(NP1.GT.IMAX)IMAX=NP1
GO TO 772
CONTINUE
IMAX=NP1
CONTINUE
CONTINUE

WRITE(NOUT,1001)IMAX,DT
FORMAT(3X,I5,1X,F6.2)
REWIND NTAPE
DO 773 J=1,NRAGA
NP1=ISIZE(J)
READ(NTAPE)(PXT1(I),I=1,NP1)
IF(NP1.LE.IMAX)GO TO 774
NPPLUS=NP1+1
DO 775 I=NPPLUS,IMAX
PXT1(I)=0.0
CONTINUE
CONTINUE

WRITE(6,570)J
FORMAT(/,2X,"FOR RAINGAGE NUMBER",I3, ", RAINFALL HISRORY IS")
NCOUNT=IMAX/10
NCHK=NCOUNT*10
IF(NCHK.LT.IMAX)NCOUNT=NCOUNT+1
DO 99 K=1,NCOUNT
IND1=((K-1)*10)+1
IND2=((K-1)*10)+10
WRITE(NOUT,707)(PXT1(I),I=IND1,IND2)
FORMAT(3X,10(F6.2,1X))
WRITE(6,708)(PXT1(I),I=IND1,IND2)
FORMAT(3X,10(F6.2,1X))
CONTINUE
CONTINUE
END
ENCOUNTERED.
```

A.2 List of Program Variables

1. PO - instantaneous point peak intensity
2. P - instantaneous point rain
3. PX - instantaneous average precipitation over a basin of length DX
4. PXT - instantaneous rain average over basin DX in length and duration DT
5. PXT1 = PXT in mm/hr
6. TP - time of peak at a point
7. T - time at a point
8. C - spline coefficients
9. BPAR - vector of length 4 containing the end condition parameter
10. DXi - the size of the subcatchment along the direction
11. XOi - the distance from general coordinate system to the subcatchment
12. TOPR - total precipitation
13. WV - wind velocity (km/hr)
14. WD - wind direction (km/hr)
15. BI=K1 - the shape exponent of the rising limb of the hyetograph
16. BI=K2 - the shape exponent of the falling limb of the hyetograph
17. VS = SV - storm velocity (km/hr)
18. SD - storm direction
19. TR - time of rise (min)
20. IF - time of fall (min)
21. TS - time base

22. XP - length of the storm (km)

23. XOT - time to reach subcatchment from the general coordinate system

24. DXT - time for storm motion along any subcatchment

25. T1, T2 - integral limits

26. NP, 11 - integral limits

27. Q - output from T1 to T2

28. DT - the basic timestep for the hyetograph

29. (NCount, NCHK, IND1, IND2) parameters to accept 10 data items in one line

A.3 Subroutines ICSICU and DSCQDU

A.3.1 ICSICU

This subroutine provides an interpolation approximation by cubic splines with arbitrary second derivative end conditions.

Usage - Call ICSICU (X, Y, NX, BPAR, C, IC, IER), = (T, P, NT, BPAR, C, NT-1, IER1) in the program

Arguments:

X - vector of length NX containing the abscissae of the NX data points (X(I), y(I) J=1...NX (input) must be ordered so that X(I).LT.X(I+1).

Y - vector of length NX containing the coordinates (or function values) of the NX data points (input).

NX - number of elements in X and Y (input) NX must be greater than or equal to 2.

BPAR - vector of length 4 containing the end condition parameters (input).

C - spline coefficients (output) c is an NX-1 by 3 matrix.

IC - Row dimension of matrix C exactly as specified in the dimension statement in the calling program (input).

IER - error parameter (output).

Note: the ISML routines VERTST and VGETIO are also required.

Notation - information on special notation and conventions is available in the introduction to the IMSL manual or through IMSL routine UHELP.

Algorithm

ICSICU computes an interpolatory approximation to a given set of points by cubic splines with arbitrary second derivative end conditions. The tridiagonal system defining the second derivatives of the spline interpolate for (x,y) is solved, producing the spline coefficients.

In the program ICSICU supply the instantaneous point precipitation along the size of the subcatchment.

$$P(t) = \int_{X-DX}^{X+DX} P(t)\,dx$$

A.3.2 DCSQDU

Purpose - cubic spline quadrature.

Usage - call DCSQDU (x,y,NX,C,IC,A,B,Q,IER) (T,P,NT,C,NT-1,T1,T2,Q,IERS) in the program

Arguments

X - vector of length Nx containing the abscissae of the Nx data points (X(I), Y(I)) I=1...NX. (input) x must be ordered so that X(I). LT.X(I+1).

Y - vector of lenth NX containing the ordinates (or function values) of the NX data points. (input).

NX - number of elements in X and Y. (input) NX must be greater than or equal to 2.

C - spline coefficients. (input) C is an NX-1 by 3 matrix.

IC - row dimension of matrix C exactly as specified in the dimension statement in the calling program (input).

A,B - limits of integration (input).

Q - integral from A to B (output).

IER - error parameter.

Note that the IMSL routines VERTST and VGETIO are also requested.

Notation - Information on special notation and conventions is available in the introduction or through IMSL routine UHELP.

Algorithm

DCSQDC integrates a cubic spline between limits A and B. IA and IB are determined so that min (A,B) is in {X(IA), X(IA+1)} and max (A,B) is in {X(IB), X(IB+1)}. The integral from X(IA) to min (A,B) is computed (call the value QA). Next the integral from X(IA) to X(IB) is computed (call this value QAB). The integral from X(IB) to max (A,B) is computed (call this value QB). Finally the integral from A to B (call this value Q) is computed by Q = QAB + QB-QA. If A is greater than B, Q = -Q.

$$P(t) = \int_{t-\Delta t}^{t+\Delta t} P(t)\,dt$$

The two subroutines are used twice.

$$P(t) = \int_{X-\Delta X}^{X+\Delta X}\int_{t-\Delta t}^{t+\Delta t} P(t)\,dt\,dx$$

Boths subroutines are available in the McMaster University Computer Library.

A.4 <u>Execution of THOR (sample program run)</u>

```
?HY

THIS PROGRAM IS ONLY FOR THE CHEDOKE CREEK BASIN
 PROGRAM IS FOR DURATION OF RAINFALL LESS THEN 1.5HR
 USE IN OTHER CATCHMENT THE USER SHOULD CHANGE THE
CATCHMENT CHARACTERISTICS (DX,X0) FOR THE 8 DIFFERENT
ECTIONS ...!

ER DATA MEASURED AT THE RAINGAGES:

R(MM)      WIND VEL(KM/HR)     WIND DI.
 , 0.1 ,45

RM CHARACTERISTICS :

 INTENSITY  PO=  120.25  MM/HR

GTH (KM) BETWEEN 0.05PO MM/HR   XP=    2.78

E BASE (MIN)   TS=  22.30

E OF RISE (MIN)  TR=    7.46

E OF FALL (MIN)  TF=   14.84

RM DIRECTION   SD=   72.78

 VELOCITY OF THE STORM  VS=    7.48

ER AVERAGING PERIOD(MIN)..

R NUMPER OF RAINGAGES ...


RAGE INTENSITY OVER DX  DT=    5.00  M

 RAINGAGE NUMBER  1, RAINFALL HISRORY IS
 .00    0.00    0.00     .00     .15    1.13    2.21    2.05     .87     .26
 .03     .00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00
 .00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00

 RAINGAGE NUMBER  2, RAINFALL HISRORY IS
 .00     .00     .07     .76    1.99    2.22    1.19     .33     .03     .00
 .00     .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00     .00    0.00    0.00    0.00

 RAINGAGE NUMBER  3, RAINFALL HISRORY IS
 .00     .00     .24    1.57    2.70    1.43     .51     .10     .00     .00
 .00     .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00     .00     .00    0.00    0.00
```

```
RAINGAGE NUMBER  4, RAINFALL HISRORY IS
 .00     .00     .29    1.95    2.73    1.24     .42     .05    0.00    0.
 .00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00
 .00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00

RAINGAGE NUMBER  5, RAINFALL HISRORY IS
 .00     .00     .05     .87    2.72    2.07     .77     .21     .01     .00
 .00     .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00     .00     .00    0.00    0.00

RAINGAGE NUMBER  6, RAINFALL HISRORY IS
 .00     .00     .00     .00     .15    1.25    2.50    1.39     .70     .18
 .01    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00
 .00    0.00    0.00    0.00    0.00    0.00    0.00     .00    0.00    0.00

RAINGAGE NUMBER  7, RAINFALL HISRORY IS
 .00     .00     .00     .05     .85    2.63    2.12     .30     .22     .01
 .00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00
 .00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00

RAINGAGE NUMBER  8, RAINFALL HISRORY IS
 .00    0.00    0.00     .00     .06     .79    2.31    2.26     .95     .23
 .03     .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00     .00    0.00    0.00    0.00

RAINGAGE NUMBER  9, RAINFALL HISRORY IS
 .00     .07     .73    2.15    2.27    1.06     .33     .05     .00     .00
 .00     .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00     .00    0.00    0.00    0.00

RAINGAGE NUMBER 10, RAINFALL HISRORY IS
 .05    1.06    2.90    1.90     .59     .17     .00    0.00    0.00    0.00
 .00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00
 .00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00
7.473 CP SECONDS EXECUTION TIME.
```

APPENDIX B

HYETOGRAPHS USED IN THE STUDY MOVING STORM,

HYETOGRAPHS FOR STORM NO. 8, 10, 14

B.1 Hyetographs for Storm No. 8

```
R(MM)      WIND VEL(KM/HR)      WIND DI.

9-9 TO STOP
.93 ,31.5 ,135

RM CHARACTERISTICS :

 INTENSITY  PD=    51.34  MM/HR

GTH (KM) BETWEEN 0.05PD MM/HR   XP=   20.22

E BASE (MIN)    TS=   32.98

E OF RISE (MIN)  TR=   13.06

E OF FALL (MIN)   TF=   19.92

RM DIRECTION   SD=  152.34

 VELOCITY OF THE STORM  VS=   36.78

ER AVERAGING PERIOD(MIN)..

ENTER RAINFALL HYETOGRAPH (CARDS 10)          ...
 RAINGAGE NUMBER  1,  RAINFALL HISTORY IS
.00    .18    .58   1.55   1.08    .51    .24    .08   0.00   0.00

 RAINGAGE NUMBER  2,  RAINFALL HISTORY IS
.02    .23    .72   1.62    .94    .44    .21    .05   0.00   0.00

 RAINGAGE NUMBER  3,  RAINFALL HISTORY IS
.01    .21    .66   1.60   1.00    .47    .22    .07   0.00   0.00

 RAINGAGE NUMBER  4,  RAINFALL HISTORY IS
.04    .28    .90   1.60    .81    .38    .18    .03   0.00   0.00

 RAINGAGE NUMBER  5,  RAINFALL HISTORY IS
.06    .32   1.02   1.54    .75    .35    .17    .02   0.00   0.00

 RAINGAGE NUMBER  6,  RAINFALL HISTORY IS
.04    .27    .94   1.62    .85    .40    .19    .03   0.00   0.00

 RAINGAGE NUMBER  7,  RAINFALL HISTORY IS
.08    .36   1.12   1.48    .70    .33    .16    .01   0.00   0.00

 RAINGAGE NUMBER  8,  RAINFALL HISTORY IS
.10    .40   1.25   1.38    .65    .31    .14   0.00   0.00   0.00

 RAINGAGE NUMBER  9,  RAINFALL HISTORY IS
.12    .44   1.33   1.31    .62    .29    .12   0.00   0.00   0.00

 RAINGAGE NUMBER 10,  RAINFALL HISTORY IS
.05    .31    .96   1.56    .79    .37    .17    .02   0.00   0.00
```

B.1.2 Hyetographs for Storm No. 10

```
NGTH (KM) BETWEEN 0.05PD MM/HR   XP=    7.80

ME BASE (MIN)    TS=   33.64

ME OF RISE (MIN)  TR=   13.45

ME OF FALL (MIN)  TF=   20.19

ORM DIRECTION    SD=  320.30

E VELOCITY OF THE STORM  VS=   13.92

TER AVERAGING PERIOD(MIN)..

. ENTER RAINFALL'CONTROL DATA (CARD 9)          ...
11 RAINFALL STEPS WITH TIME INTERVAL    5.00 MINS

. ENTER RAINFALL HYETOGRAPH (CARDS 10)         ...
OR RAINGAGE NUMBER  1,  RAINFALL HISTORY IS
  .07     .32     .97   1.43     .75     .35     .17     .03    0.00    0.00
 0.00

OR RAINGAGE NUMBER  2,  RAINFALL HISTORY IS
  .01     .19     .58   1.47    1.03     .49     .23     .08    0.00    0.00
 0.00

OR RAINGAGE NUMBER  3,  RAINFALL HISTORY IS
  .03     .25     .75   1.54     .87     .41     .20     .05    0.00    0.00
 0.00

OR RAINGAGE NUMBER  4,  RAINFALL HISTORY IS
 0.00     .08     .34   1.03    1.43     .70     .33     .16     .02    0.00
 0.00

OR RAINGAGE NUMBER  5,  RAINFALL HISTORY IS
 0.00     .03     .24    .73    1.52     .89     .42     .20     .05    0.00
 0.00

OR RAINGAGE NUMBER  6,  RAINFALL HISTORY IS
 0.00     .10     .38   1.15    1.35     .65     .31     .15     .01    0.00
 0.00

R RAINGAGE NUMBER  7,  RAINFALL HISTORY IS
 0.00     .01     .19    .58    1.48    1.03     .49     .23     .08    0.00
 0.00

R RAINGAGE NUMBER  8,  RAINFALL HISTORY IS
 0.00    0.00     .12    .43    1.21    1.27     .62     .30     .13     .01
 0.00

R RAINGAGE NUMBER  9,  RAINFALL HISTORY IS
 0.00    0.00     .09    .36    1.07    1.37     .69     .33     .15     .02
 0.00

R RAINGAGE NUMBER 10,  RAINFALL HISTORY IS
 0.00     .06     .29    .89    1.44     .80     .38     .18     .04    0.00
 0.00
```

B.1.3 Hyetographs for Storm No. 14

PR(MM)   WIND VEL(KM/HR)   WIND DI.

-9-9 TO STOP
1.6 ,11 ,330

ORM CHARACTERISTICS :

INTENSITY PO= 59.80 MM/HR

NGTH (KM) BETWEEN 0.05PO MM/HR XP= 9.14

ME BASE (MIN) TS= 31.08

ME OF RISE (MIN) TR= 11.96

ME OF FALL (MIN) TF= 19.12

RM DIRECTION SD= 324.72

VELOCITY OF THE STORM VS= 17.65

ER AVERAGING PERIOD(MIN)..

ENTER RAINFALL HYETOGRAPH (CARDS 10)   ...

R RAINGAGE NUMBER 1, RAINFALL HISTORY IS

.11	.52	1.61	1.38	.63	.29	.09	0.00	0.00	0.00

R RAINGAGE NUMBER 2, RAINFALL HISTORY IS

.04	.33	1.15	1.75	.83	.38	.17	0.00	0.00	0.00

R RAINGAGE NUMBER 3, RAINFALL HISTORY IS

.07	.41	1.41	1.57	.72	.33	.12	0.00	0.00	0.00

R RAINGAGE NUMBER 4, RAINFALL HISTORY IS

0.00	.19	.71	1.83	1.12	.51	.23	.05	0.00	0.00

R RAINGAGE NUMBER 5, RAINFALL HISTORY IS

0.00	.12	.52	1.65	1.36	.62	.28	.08	0.00	0.00

R RAINGAGE NUMBER 6, RAINFALL HISTORY IS

0.00	.22	.79	1.86	1.04	.48	.22	.03	0.00	0.00

R RAINGAGE NUMBER 7, RAINFALL HISTORY IS

0.00	.07	.42	1.46	1.54	.70	.32	.12	0.00	0.00

R RAINGAGE NUMBER 8, RAINFALL HISTORY IS

0.00	.04	.33	1.14	1.72	.85	.39	.17	.01	0.00

R RAINGAGE NUMBER 9, RAINFALL HISTORY IS

0.00	.02	.28	.98	1.79	.94	.43	.19	.01	0.00

R RAINGAGE NUMBER 10, RAINFALL HISTORY IS

0.00	.16	.63	1.71	1.24	.57	.26	.07	0.00	0.00

B.2 Rainfall from Hamilton Airport

Raingauge Storm 8, 10, 14

storm no. 8

FOR 22 RAINFALL STEPS, THE TIME INTERVAL IS 5.00 MINUTES

FOR RAINGAGE NUMBER 1, RAINFALL HISTORY IS

.09	.09	.28	.19	.28	.19	.28	1.13	1.86	2.94
.74	.38	.28	.28	.09	.09	.19	.28	.19	.09
.09	0.00								

storm no. 10

FOR 176 RAINFALL STEPS, THE TIME INTERVAL IS 5.00 MINUTES

FOR RAINGAGE NUMBER 1, RAINFALL HISTORY IS

.09	.09	.19	.38	.19	.19	.19	.38	.19	.09
.09	.09	.09	.09	0.00	.09	.09	.09	.19	.09
0.00	0.00	0.00	0.00	.09	0.00	.09	.09	.09	.09
.09	.09	.09	.09	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
.09	0.00	0.00	0.00	0.00	0.00	.09	0.00	.09	0.00
.09	0.00	.19	0.00	.09	.09	.19	0.00	.76	.49
.19	.19	.09	.09	.19	.09	.19	.09	.19	.19
.09	.09	.09	.09	.09	0.00	.09	0.00	.09	0.00
0.00	0.00	.09	0.00	0.00	.09	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	.09	0.00
0.00	0.00	0.00	.09	.76	.19	0.00	.09	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	.09	.49	.09
.38	.09	0.00	0.00	.09	0.00				

storm no 14

FOR 32 RAINFALL STEPS, THE TIME INTERVAL IS 5.00 MINUTES

FOR RAINGAGE NUMBER 1, RAINFALL HISTORY IS

.09	.79	1.50	.09	.19	.19	.09	.19	.19	.09
.09	.09	.09	.09	.09	.09	.19	.09	.38	.19
.19	.19	.19	.09	.09	0.00	.19	.09	.09	.19
.09	0.00								

APPENDIX C

SWMM INPUT DATA

C.1 SWMM Input Data, Stationary Storm

```
,HOODATT
PY,HOODATT
FPS
0
1
RUNOFF
CITY OF HAMILTON SWMM STUDY-COARSE SIMULATION - CHEDOKE CREEK
RAINFALL FROM FILD RAINGAGES :HAMILTON AIR PORT
3,90,0,0,5,1,25.,0,0.0,0
32 5
0.09 0.79 1.50 0.09 0.19 0.19 0.09 0.19 0.19 0.09
0.09 0.09 0.09 0.09 0.09 0.09 0.19 0.09 0.38 0.19
0.19 0.19 0.19 0.09 0.09 0.0 0.19 0.09 0.09 0.19
0.09 0.0
1005,265,1,6.,50,.0006,0,0,.018,6
265,276,1,15,762,.294,2,2,.018,2
276,290,2,4,642,.03
290,1004,1,5,1324,.019,3,3,.018,5
1004,1020,1,7,190,.057,3,3,.018,5
1006,353,2,5,420,.004
353,1014,2,5.03,200,.004
1014,1007,1,13.5,1700,.016,2,2,.018,2.5
1007,354,2,5,96,.037
354,365,1,5,292,.15,0,0,.018,5
365,451,1,6,910,.074,1.5,1.5,.018,2
1008,451,1,6.5,690,.009,0,0,.018,5.5
451,1017,1,7,1210,.009,0,0,.018,7
1017,1009,1,7.75,2055,.0025,0,0,.018,9.5
1009,1020,1,7.75,1030,.006,0,0,.018,9.5
1020,454,1,8,525,.0035,2,2,.018,6.5
454,455,1,12,302,.0034,0,0,.018,8
455,1010,1,8,1140,.012,1,1,.018,8
1001,155,2,6,950,.007
155,156,2,6,850,.0095
156,157,1,4,970,.011,2,2,.018,4
157,158,2,6,300,.0075
158,159,2,5.5,690,.0184
159,1002,1,6,2650,.0055,0,00,.018,6.5
1002,1011,1,6,400,0.0055,0,0,.018,6.5
1003,1011,2,5,625,.0046
1011,168,1,6,1427,.078,0,0,.018,5
168,1022,1,8,1050,.171,1,1,.018,8
1022,1010,1,5,325,.021,0,0,.018,12
1010,459,1,8,85,.0026,0,0,.018,9
459,460,1,8.0,800,0.0026,1.0,1.0,0.018,9.0
0
1,1,1001,5130,361,20,.03,0,0,0,0,0,0.05,0.01
1,2,1002,4270,244,24,0.025,0,0,0,0,0,0.05,0.01,
1,3,1003,3450,163,28,0.0125,0,0,0,0,0,0.05,0.01
1,4,1004,3020,235,6,0.035,0,0,0,0,0,0.05,0.01
1,5,1005,4720,164,22,0.0225,0,0,0,0,0,0.05,0.01
1,6,1006,3800,143,20,0.025,0,0,0,0,0,0.05,0.01
1,7,1007,3220,109,9,0.03,0,0,0,0,0,0.05,0.01
1,8,1008,4300,506,4,0.05,0,0,0,0,0,0.05,0.01
1,9,1009,4970,288,23,0.03,0,0,0,0,0,0.05,0.01
1,10,1010,4100,236,21,0.03,0,0,0,0,0,0.05,0.01
```

```
0
1,7,19,100,4.6,1,0
0.7,0,0,30,30,7,7,30,0.5,0.5,0.5,0.5,0.5
4.0,13.9,19.9,27.6,7.5
100,6500,1300000,1.5,40.0,0.12,0.02,1
100,1000,2700000,7.2,40.0,0.61,0.15,1
100,1000,1700000,7.7,39.0,0.41,0.07,1
100,1000,1000000,3,40,0.43,0.03,1
100,6500,0,1.5,20,0.01,0.02,1
0,0,0,100,310
0,0,0,0.02,0.02,0.01,0.001,0
0,0,0,0.02,0.05,0.005,0.002,0
1,1,150,376,0.0,0.0,0.0,0.0,0.0,0.0
2,1,239,604,0.0,0.0,0.0,0.0,0.0,0.0
3,1,220,559,0.0,0.0,0.0,0.0,0.0,0.0
4,5,7,68,0.0,0.0,0.0,0.0,0.0,0.0
5,1,142,359,0.0,0.0,0.0,0.0,0.0,0.0
6,1,130,329,0.0,0.0,0.0,0.0,0.0,0.0
7,1,39,92,0.0,0.0,0.0,0.0,0.0,0.0
8,5,43,203,0.0,0.0,0.0,0.0,0.0,0.0
9,1,47,665,0.0,0.0,0.0,0.0,0.0,0.0
10,5,21,146,0.0,0.0,0.0,0.0,0.0,0.0
12,1
1001,1002,1003,1004,1005,1006,1007,1008,1009,1010,459,460
ENDPROGRAM
 ENCOUNTERED.
```

C.2 SWMM Input Data, Moving Storm

```
Y,MOODATT
FPS
0
1
RUNOFF
CITY OF HAMILTON SWMM STUDY-COARSE SIMULATION - CHEDOKE CREEK
RAINFALL OF KINEMATIC STORM
8,120,0,0,5,10,25.,0,0.0,0
10 5
   .05      .35    1.19    1.66     .84     .38     .16     .01     .00     .00
   .00      .11     .50    1.49    1.45     .69     .31     .11     .00    0.00
   .00      .04     .33    1.14    1.72     .85     .39     .17     .01    0.00
   .00      .03     .31    1.09    1.76     .98     .40     .18     .01    0.00
  0.00     _.08     .42    1.41    1.56     .72     .33     .12     .00     .00
   .03      .31    1.09    1.73     .99     .41     .19     .02     .00     .00
   .00      .19     .73    1.79    1.13     .51     .23     .05     .00     .00
   .05      .36    1.24    1.65     .90     .37     .15     .01     .00     .00
  0.00     .02      .27     .99    1.75     .95     .44     .19     .03     .00
   .00      .00     .10     .49    1.54    1.45     .66     .30     .10    0.00
005,265,1,6.0,50,.0006,0,0,.019,6
65,276,1,15,762,.294,2,2,.019,2
76,290,2,4,642,.03
90,1004,1,5,1324,.019,3,3,.019,5
004,1020,1,7,190,.057,3,3,.019,5
006,353,2,5,420,.004
53,1014,2,5.03,200,.004
014,1007,1,13.5,1700,.016,2,2,.019,2.5
007,354,2,5,96,.037
54,365,1,5,292,.15,0,0,.019,5
65,451,1,6,910,.074,1.5,1.5,.019,2
009,451,1,6.5,690,.009,0,0,.019,5.5
51,1017,1,7,1210,.009,0,0,.019,7
017,1009,1,7.75,2055,.0025,0,0,.019,9.5
009,1020,1,7.75,1030,.006,0,0,.019,9.5
020,454,1,8,525,.0035,2,2,.019,6.5
54,455,1,12,302,.0034,0,0,.019,8
55,1010,1,9,1140,.012,1,1,.019,8
001,155,2,6,950,.007
55,156,2,6,850,.0095
56,157,1,4,970,.011,2,2,.019,4
57,158,2,6,300,.0075
58,159,2,5.5,680,.0184
59,1002,1,6,2650,.0055,0,00,.019,6.5
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003,1011,2,5,625,.0046
011,168,1,6,1427,.078,0,0,.019,5
68,1022,1,9,1050,.171,1,1,.019,8
022,1010,1,5,325,.021,00,0,.019,12
010,459,1,8,85,.0026,0,0,.019,9
59,460,1,8,800,0.0026,1,1,0.019,9

,1,1001,7060,361,25,.05,0,0,0,0,0,0.05,0.01
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,3,1003,5520,163,35,0.0125,0,0,0,0,0,0.05,0.01
,4,1004,4760,235,9,0.035,0,0,0,0,0,0.05,0.01
,5,1005,6340,164,26,0.0225,0,0,0,0,0,0.05,0.01
,6,1006,4900,143,23,0.025,0,0,0,0,0,0.05,0.01
,7,1007,5560,109,12,0.03,0,0,0,0,0,0.05,0.01
,8,1008,5480,506,8,0.05,0,0,0,0,0,0.05,0.01
,9,1009,6190,298,27,0.03,0,0,0,0,0,0.05,0.01
0,10,1010,5560,236,24,0.03,0,0,0,0,0,0.05,0.01
```

```
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0.7,0,0,30,30,7,7,30,0.5,0.5,0.5,0.5,0.5
4.5,13.8,19.8,27.6,9.0
100,6800,1300000,5.8,40.0,0.08,0.01,1
100,1000,2700000,7.2,40.0,0.61,0.05,1
100,1000,1700000,7.7,39.0,0.41,0.07,1
100,1000,1000000,3,40,0.43,0.03,1
100,6800,0,5.6,20,0.01,0.01,1
0,0,0,100,310
0,0,0,0.02,0.02,0.01,0.001,0
0,0,0,0.02,0.05,0.005,0.002,0
1,1,150,376,0.0,0.0,0.0,0.0,0.0,0.0
2,1,239,604,0.0,0.0,0.0,0.0,0.0,0.0
3,1,220,558,0.0,0.0,0.0,0.0,0.0,0.0
4,5,7,68,0.0,0.0,0.0,0.0,0.0,0.0
5,1,142,358,0.0,0.0,0.0,0.0,0.0,0.0
6,1,130,328,0.0,0.0,0.0,0.0,0.0,0.0
7,1,38,92,0.0,0.0,0.0,0.0,0.0,0.0
8,5,43,203,0.0,0.0,0.0,0.0,0.0,0.0
9,1,47,665,0.0,0.0,0.0,0.0,0.0,0.0
10,5,21,146,0.0,0.0,0.0,0.0,0.0,0.0
12,1
1001,1002,1003,1004,1005,1006,1007,1008,1009,1010,459,460
ENDPROGRAM
 ENCOUNTERED.
```

APPENDIX D

STATISTICAL BACKGROUND

D.1 Statistical Background (37)

D.1.1 Scatter Diagram Regression Lines

Suppose each pair of observed values of two variables x and y is represented by a dot at the point (x,y). The dots form a scatter diagram and there are three possibilities: (1) a marked relationship, (2) some degree of interdependence, (3) complete independence. This will be reflected in the scatter diagram as in Figure 55.

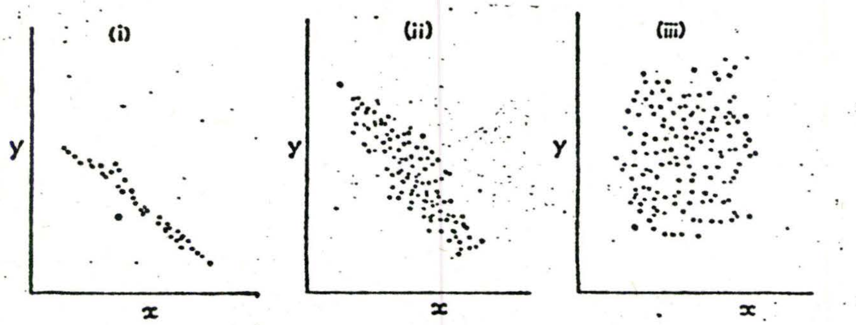

Figure 55: Scatter Diagrams (Source 37)

The main purpose of an empirical relation between two variables is to enable prediction of one when the other is given. It is clear that if, for a given value of x, the value of y can vary appreciably, we can at best only attempt to estimate the most probable value of y. This is usually taken to be the mean of the observed values. If ym is the mean value of y for given x, the locus of (X,Ym) is called "the line of regression of y on x", or the regression curve of y on x".

D.1.2 Linear Regression

The regression lines will in general be curved, but the simplest and most important case is where the locus of means approximates a straight line. The method of least squares is used to determine the equation the coefficients so obtained being "best" estimates on the basis of the observed values (x,y) (37).

Suppose there are N pairs of observed values. Let Ymi be the mean of ni values of y which occur with the value xi of x, so that Σ ni = N. In applying the method of least squares we give the pair (Xi, Ymi) a weight Ni, since the corresponding point represent the mean of ni observations. The best straight line passes through the centroid of the weighted observations, namely through (x,y) where:

$$\bar{X} = \frac{\sum n_i X_i}{\sum n_i} = \frac{\sum_{1}^{n} x}{N}$$

$$\bar{y} = \frac{\sum n_i y_{mi}}{\sum n_i} = \frac{\sum_{1}^{n} y}{N}$$

so that $(\bar{x},\bar{y})$ is simply the centroid of the original observed set of points, given equal weights. To find the gradient of the line of regression of y on x, suppose the origin shifted to $(\bar{x},\bar{y})$ and use capital letters for coordinates referred to the new origin.

$$b = \frac{\sum n_i X_i Y_{ni}}{\sum n_i X_i^2} = \frac{\sum_1^N XY}{\sum_1^n X^2} = \frac{\sum XY}{N \sigma_i^2} = \frac{\text{cov}(x,y)}{\text{Var } X}$$

where σ_1^2 is the variance of the x's.

The line regression of y on x is thus

$$y_M - \bar{y} = \frac{\text{cov}(x,y)}{\text{Var } X}(X - \bar{x})$$

The line of regression of x on y is similarly

$$x_M - \bar{x} = \frac{\text{cov}(x,y)}{\text{Var } y}(y - \bar{y})$$

The coefficients on the right hand side of these equations are called the regression coefficients and may be denoted by β_2 and β_1, respectively, the Greek letters (β_1, β_2) being applicable to the true values for the real or hypothetical complete population.

D.1.3 The Coefficient of Correlation

If X and Y denote deviations from the mean values of x and y, the equations of the regression lines may be written

$$\text{(I)} \qquad Y = \beta_2 X \quad \text{and} \quad X = \beta_1 Y$$

$$\beta_2 = \sum_{r=1}^{N} X_r Y_r / N\sigma_1^2 \quad , \quad \beta_1 = \sum_{r=1}^{N} X_r Y_r / N\sigma_2^2$$

These two gradients are unequal, but the equations above may be written

$$\text{(II)} \qquad \frac{Y}{\sigma_2} = \rho \frac{X}{\sigma_1} \quad \text{and} \quad \frac{X}{\sigma_1} = \rho \frac{Y}{\sigma_2}$$

$$\text{(III)} \qquad \rho = \frac{\sigma_1 \beta_2}{\sigma_2} = \frac{\sigma_2 \beta_1}{\sigma_1} = \frac{\sum_{r=1}^{n} X_r Y_r}{N\sigma_1\sigma_2}$$

The latter equations show that if the deviations from the means are standardized by dividing by the corresponding standard deviations, the regression lines with the standardized deviations as variable, make equal angles, the one with the horizontal and the other with the vertical axis. Numerically this means that, considering x as the independent variable, the mean value of y for given x in standaridized units, increases by ρ units for unit increase in x and a precisely similar statement is valid with y as the independent variable. The coefficient ρ is thus a measure of the tendency of either variable, expressed in standardized units, to increase when the other increases. ρ is called the coefficient of product-moment correlation. The corresponding quantity derived from a sample is denoted by r. Note that $\rho^2 = \beta_1 \beta_2$ and similarly r is the geometric mean of the regression coefficients derived from a sample. ρ^2 tells us what proportion of the variation of the y!s can be attributed to the linear relationship with x. Given a random sample of size - that is, paris of values (xi, yi) - it is customary to estimate ρ by means of sample correlation coefficient r in general form:

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} \qquad S_{xx} = n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2$$

$$S_{yy} = n \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2 \qquad S_{xy} = n \sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$$

D.1.4 Positive and Negative Correlation

Assuming approximately linear regression the trend of either variable may be either to increase or to decrease as the other increases. When each variable increases on the average and the other increases, the value of r is positive.

When on the average as either variable increases the other decreases, the value of r is negative.

Numerically ρ (or r) cannot exceed 1. If X, Y denote deviations from the means of x and y.

$$\rho^2 = \frac{(\Sigma XY)^2}{N^2 \sigma_1^2 \sigma_2^2} = \frac{(\Sigma XY)^2}{\Sigma x^2 \cdot \Sigma y^2}$$

Now

$$\Sigma X^2 \cdot \Sigma Y^2 - (\Sigma XY)^2$$

$$= (X_1^2 + X_2^2 + \ldots\ldots + X_N^2)(Y_1^2 + \ldots\ldots + Y_N^2) -$$

$$(X_1 Y_1 + X_2 Y_2 + \ldots\ldots + X_N Y_N)^2$$

$$= \Sigma (X_r Y_s - X_s Y_r)^2$$

Since the sum of square cannot be negative the denomination of ρ^2 $\geq$ the numerator i.e. $\rho^2 \leq 1$.

1. $\rho^2 = 1$ if and only if $Xr\ Ys - Xs\ Yr = 0$ for all values, r, s, i.e. $Xr/Yr = Xs/Ys$. In this case all points (Xr, Yr) lie on a straight line through the origin (which is the centroid). Correlation is perfect and the two regression lines coincide.
2. In general $-1 < \rho < 1$. With positive correlation the lines of regression lie in the first and third quadrants; with negative correlation the lines are in the second and fourth quadrants.
3. If $\rho = 0$, the variables are said to be uncorrelated. This implies that $\Sigma xy = 0$ and occurs if variables are independent, but it is not conversely true that if $\rho = 0$, the variables are independent.

D.1.5 Significance of the Correlation Coefficient

If the number n in a sample is small, it becomes quite probable that small value of r might be obtained (suggesting some degree of correlation) even if the sample is drawn from a normal distribution for which $\rho = 0$. In this case r has no significance and it might well have arisen as the result of random sampling from an uncorrelated distribution. The extreme example is afforded by n = 2, which is bound to give r = 1, suggesting perfect correlation: for we have only two points in the dot diagram and the line joining them represents both regression lines.

It follows that for small samples it is important to test significance of the calculated value of r by determining the probability that such a value could have arisen as the result of random sampling from an uncorrelated population. If this probability is low, the value of r is significant. The significance of a value of r could be tested by reference to a table of "critical values $r_{n,\alpha}$ of the linear correlation coefficient r". (See Section D.2). The table gives critical values for r in testing Ho: $\rho = 0$. Denoting a typical table entry by critical regions for testing at significance level α and against alternative hypotheses.

$H_1: \rho \neq 0, \quad \rho > 0, \quad \rho < 0$

$|r| \geq r_{n,\alpha}, \quad r \geq r_{n,2\alpha}, \quad r \leq -r_{n,2\alpha}$ respectively.

Inferences about x may also be made by using the Fisher Z transformation:

$$Z(r) = 1/2 \log (1+r/1-r) = 1.1513 \log 10 (1+r/1-r)$$

which is approximately normally distributed with mean Z(x) and variance 1/(n-3). For convenience however, tables have been prepared showing the values of r at various levels of significance for various values of n. Suppose the number of paird observations is 20 and the computed value of r is 0.35. The tables give $\alpha = 0.1$ (or 10%) with n = 20 and r = 0.378, which means that there is a probability of 10% that $r \geq 0.378$ as the result of random sampling from an uncorrelated normal population: the apparent correlation is therefore not significant. If however, we had found r = 0.53 it appears from the table that the probability of so large a value from an uncorrelated population is less than 0.02 (2%), we may, with considerable confidence, infer that the apparent correlation is real.

D.2 Table 6.2 Critical Values of the Linear Correlation Coefficient

6·2 critical values $r_{n,\alpha}$ of the linear correlation coefficient r

$H_0 : \rho = 0$; critical region: $|r| \geqslant r_{n,\alpha}$ for $H_1 : \rho \neq 0$,
$r \geqslant r_{n,2\alpha}$ for $H_1 : \rho > 0$,
$r \leqslant -r_{n,2\alpha}$ for $H_1 : \rho < 0$

n \ α	20%	10%	5%	2%	1%	0.2%
3	0.951	0.988	0.997	1.000	1.000	1.000
4	0.800	0.900	0.950	0.980	0.990	0.998
5	0.687	0.805	0.878	0.934	0.959	0.986
6	0.608	0.729	0.811	0.882	0.917	0.963
7	0.551	0.669	0.754	0.833	0.875	0.935
8	0.507	0.621	0.707	0.789	0.834	0.905
9	0.472	0.582	0.666	0.750	0.798	0.875
10	0.443	0.549	0.632	0.715	0.765	0.847
11	0.419	0.521	0.602	0.685	0.735	0.820
12	0.398	0.497	0.576	0.658	0.708	0.795
13	0.380	0.476	0.553	0.634	0.684	0.772
14	0.365	0.458	0.532	0.612	0.661	0.750
15	0.351	0.441	0.514	0.592	0.641	0.730
16	0.338	0.426	0.497	0.574	0.623	0.711
17	0.327	0.412	0.482	0.558	0.606	0.694
18	0.317	0.400	0.468	0.543	0.590	0.678
19	0.308	0.389	0.456	0.529	0.575	0.662
20	0.299	0.378	0.444	0.516	0.561	0.648
21	0.291	0.369	0.433	0.503	0.549	0.635
22	0.284	0.360	0.423	0.492	0.537	0.622
23	0.277	0.352	0.413	0.482	0.526	0.610
24	0.271	0.344	0.404	0.472	0.515	0.599
25	0.265	0.337	0.396	0.462	0.505	0.588
26	0.260	0.330	0.388	0.453	0.496	0.578
27	0.255	0.323	0.381	0.445	0.487	0.568
28	0.250	0.317	0.374	0.437	0.479	0.559
29	0.245	0.311	0.367	0.430	0.471	0.550
30	0.241	0.306	0.361	0.423	0.463	0.541

n \ α	20%	10%	5%	2%	1%	0.2%
32	0.233	0.296	0.349	0.409	0.449	0.526
34	0.225	0.287	0.339	0.397	0.436	0.511
36	0.219	0.279	0.329	0.386	0.424	0.498
38	0.213	0.271	0.320	0.376	0.413	0.486
40	0.207	0.264	0.312	0.367	0.403	0.474
42	0.202	0.257	0.304	0.358	0.393	0.463
44	0.197	0.251	0.297	0.350	0.384	0.453
46	0.192	0.246	0.291	0.342	0.376	0.444
48	0.188	0.240	0.285	0.335	0.368	0.435
50	0.184	0.235	0.279	0.328	0.361	0.427
55	0.175	0.224	0.266	0.313	0.345	0.408
60	0.168	0.214	0.254	0.300	0.330	0.391
65	0.161	0.206	0.244	0.288	0.317	0.376
70	0.155	0.198	0.235	0.278	0.306	0.363
75	0.150	0.191	0.227	0.268	0.296	0.351
80	0.145	0.185	0.220	0.260	0.286	0.340
85	0.140	0.180	0.213	0.252	0.278	0.331
90	0.136	0.174	0.207	0.245	0.270	0.322
95	0.133	0.170	0.202	0.238	0.263	0.313
100	0.129	0.165	0.197	0.232	0.256	0.305
110	0.123	0.158	0.187	0.222	0.245	0.292
120	0.118	0.151	0.179	0.212	0.234	0.279
130	0.113	0.145	0.172	0.204	0.225	0.269
140	0.109	0.140	0.166	0.196	0.217	0.259
150	0.105	0.135	0.160	0.190	0.210	0.250
160	0.102	0.131	0.155	0.184	0.203	0.243
180	0.096	0.123	0.146	0.173	0.192	0.229
200	0.091	0.117	0.139	0.164	0.182	0.217