# on the neasurement of the absolute viscosity of Nitrogen and air over the temperature range of $100 \mathrm{TO} 400^{\circ} \mathrm{K}$ AND AT PRESSURES OF 1 TO 150 ATOMSPHERES 

> by
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## A Thesis

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TITLE:

AUTHOR:

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SCOPE AND CONTENT:
A series capillary viscometer has been used to measure the viscosity of nitrogen and air over a temperature range of 100 to $400^{\circ} \mathrm{K}$ and to pressures of 150 atmospheres.

The low pressure results have been correlated using a modified Sutherland nquation of the form:

$$
n=\frac{C_{1} T^{C_{2}}}{T+C_{3}}
$$

Intermolecular force constants have been computed for nitrogen using the Chapman Enskog collision theory approach.

The viscosity in the dense gas resion was correlated using equations of the form

$$
n-n_{0}=A_{1} \rho+A_{2}{ }^{2}
$$

and thus overall equations for viscosity as a function of temnerature and density were formulated as

$$
\eta=\frac{C_{1} T^{C_{2}}}{T+C_{3}}+\Lambda_{1} \rho+\Lambda_{2}^{p}{ }^{2}
$$

These equations were used to generate tables of smoothed values of viscosity as a function of pressure and temperature.

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## NOMENCLATURE

Unless otherwise defined in the text, the symbols used in this work have the following meanings:

D capillary diameter
g gravitational constant
h height
K Boltzmann constant
L, $\ell \quad$ capillary length
m
n
P
Q
R
u
v
$\dot{\mathbf{v}}$
$z$
$\alpha$
$\varepsilon$

Hagenback correction factor
Couette correction factor
pressure
mass flow rate
capillary radius, D/2
absolute temperature, ${ }^{\circ} \mathrm{K}$
fluid flow velocity
volume
volume flow rate
compressibility factor
coefficient of thermal expansion
difference, differentiation sign
maximum potential energy of attraction
coefficient of viscosity
density
low velocity collision diameter
shear stress, time
weight, reduced density

Subscripts 1 and 2 refer to the upstream and downstream capillaries respectively.

## CHAPTER 1

## INTRODUCTION

The physical property that characterises the resistance to flow of simple (Newtonian) fluids, be they liquid or gas, is the viscosity. A concept of viscosity is the idea that the internal friction between the molecules of the fluid causes adjacent parts in the fluid to move when either of these parts are caused to move. In actual fact this effect is produced by a change of momentum from one layer in the fluid moving between finite boundaries. The resistance of the fluid to develop a velocity gradient is the essential feature and forms the quantitative assessment of the viscosity.

The viscosity is one of the properties of a fluid termed the "transport properties", the others being the coefficients of diffusion and thermal conductivity, all of which are physically similar in that they involve the transport of some physical property through the fluid.

In a simple fluid, the resisting force produced by the fluid flow is proportional to the velocity gradient and it may be defined by the equation

$$
\begin{equation*}
\tau=\eta \frac{d u}{d y} \tag{1.1}
\end{equation*}
$$

where $\tau$ is the shearing stress between adjacent parallel layers in a fluid having viscosity $\eta$ and a velocity gradient of du/dy normal to the axis of flow. This definition first propounded by Newton applies to steady laminar flow.

Accurate experimental viscosity data furnishes a worthwhile contribution to the study of the interrelationship between various transport coefficients of gases and intermolecular models as postulated by different theories. The theory is merely a method of describing certain phenomena which can be refined or replaced by a new one when it is no longer reliable. To justify such changes in a theoretical approach to the problem, experimental proof is important.

In order to improve understanding of molecular interaction in dilute and dense fluids accurate experimental data of the transport coefficients, especially that of viscosity of gases over broad temperature and pressure ranges are required.

Values for viscosity of gaseous substances are also of practical importance in engineering mechanics such as fluid dynamics and heat transfer in fluids, entering directly into Reynolds, Schmidt and Prandtl criteria.

Considering the aspects presented above the main objective of this research is to carry out accurate experimental viscosity measurements of nitrogen and air over a broad pressure and temperature range.

## CHAPTER 2

## LITERATURE SURVEY

Work relevant to the viscosity of nitrogen and air under dilute gas and dense gas conditions is to be discussed in this section. For a listing of the data presented by other authors the reader is referred to appendix ( $A^{3}$ ), also Figures (2.1) and (2.2) graphically illustrate the regionswhich other authors have covered.

LOW PRESSURE VISCOSITY WORK
Vogel [1] in 1914 carried out the first investigation of the viscosity of dilute nitrogen at low temperatures. Two oscillating disc viscometers, essentially of the same design but dimensionally different, were employed to obtain results for some twenty common gases. The viscometers were enclosed in sealed glass vessels, which in turn were located in constant temperature baths. The boiling point of liquid air, $81.6^{\circ} \mathrm{K}$ and the ice point of water, $273.15^{\circ} \mathrm{K}$ were the only two temperatures at which the viscosity of nitrogen was measured. Both apparatus were calibrated using dry air at the ice point of water and the measurements evaluated using the Coulomb-Maxwell theory for oscillating discs.

The absolute viscosity of nitrogen given as $n_{0}=1.678 \times 10^{-4}$
poise at $273.15^{\circ} \mathrm{K}$ appears to be some $1 \%$ above more recent determinations. However the other data point at $81.6^{\circ} \mathrm{K}$ is in good agreement with the works of Vasilesco [2], Clarke and Smith [3] and this present work, but since no statement of the accuracy of the apparatus has been given its reliability must be considered questionable.

In 1929 Trautz and Baumann [4] reported experimental viscosity data for several dilute gases and gas mixtures. The viscosities of nitrogen and air were obtained using a single capillary transpiration viscometer at fourteen different temperatures ranging from 195 to $583^{\circ} \mathrm{K}$. The formula given by Milliken, relating the reference dry air viscosity values to the calibrating temperatures T in ${ }^{\circ} \mathrm{C}\left[\mathrm{n}_{\mathrm{AIR}, \mathrm{T}}=1823.8-4 \times 10^{-8}(23-\mathrm{T})\right.$ poise $x 10^{7}$ ], was used by $T r a u t z$ and Baumann [4] to calculate their reference viscosities. This formula, which had been used as the standard for most of the work done in North America before 1940, gives the reference air viscosity as about $0.5 \%$ lower than that given by Vogel [1], which had been generally accepted as the standard in Europe. Therefore a recalculation of results would be necessary before a comparison could be attempted. Furthermore, Trautz and Baumann [4] apparently made no attempt to estimate the accuracy of these results.

An oscillating disc viscometer, calibrated with air at $22.6^{\circ} \mathrm{C}$ was used by Sutherland and Mass [5] in 1932 to investigate the viscosity of three common gases, including air, over the temperature range of 80 to $300^{\circ} \mathrm{K}$. The use of several heat transfer fluids in the temperature bath permitted a greater
number of data points to be obtained than had been possible with the viscometer used by Vogel. In all, eight data points were obtained over the temperature range, with a large gap between 90 and $170^{\circ} \mathrm{K}$. A great deal of care was taken to isolate the apparatus from any stray magnetic fields and also to eliminate any static electrical charges. The results are stated to be consistent to within $\pm .4 \%$.

Johnston and McCloskey [6] in 1940 reported on the viscosities of eight common gases including nitrogen and air over the temperature range of 90 to $300^{\circ} \mathrm{K}$. The apparatus employed was one very similar to, and in fact based on, that designed by Sutherland and Maass [5]. By means of a careful selection of heat transfer fluids used in the temperature bath, Johnston and McCloskey [6] were able to obtain data points evenly spaced over the entire reported temperature range. The smoothed results presented are stated to be accurate to within $\pm .7 \%$, however at the lowest temperatures they are 1.5 to $2.5 \%$ higher than the data obtained by all other authors. This would seem to indicate the existence of a systematic error at these temperatures.

In 1945 Vasilesco [2] presented several sets of gas viscosity data covering the broad temperature range of 90 to $1600^{\circ} \mathrm{K}$. The viscosity of nitrogen was determined at three points between 90 and $273^{\circ} \mathrm{K}$, and that of air at several temperatures in excess of $273^{\circ} \mathrm{K}$. The apparatus employed was an absolute Poiseuille type viscometer with platinum capillaries which had first been calibrated with the gas at $273.15^{\circ} \mathrm{K}$. In
this way the data were presented as a fraction of the ice point viscosity, i.e. $n / n_{0}$. Vasilesco [2] estimated that the data were accurate to within $\pm 0.15 \%$ with respect to the ice point viscosity. However no attempt to assess the absolute accuracy of the reference ice-point viscosities was reported.

A constant volume transpiration capillary viscometer was employed by Clarke and Smith [3] in 1967 to measure the viscosity of dilute nitrogen in the temperature range of 114 to $374^{\circ} \mathrm{K}$. The viscometer was composed of a helical capillary tube mounted between two vessels filled to different starting pressures. The resulting variable pressure drop over the capillary was read several times during each run in order to evaluate a integral-type working equation obtained from the Poiseuille equation. The data were obtained by taking an average of at least five individual determinations at a given temperature, this has the effect of reducing the experimental scatter but does not influence the absolute accuracy which is stated to be better than $\pm 1 \%$ at all but the lowest temperatures.

CONCLUDING COMMENTS ON LOW PRESSURE (DILUTE) VISCOSITY DATA FOR NITROGEN AND AIR.
(1) The available low temperature viscosity data is sparse even for one of the more thoroughly investigated gases such as nitrogen.
(2) In the lowest temperature regions, i.e. around $100^{\circ} \mathrm{K}$, the discrepancy between the nitrogen data of Clarke and Smith [3] and Johnston and McCloskey [6] reaches a maximum of 3\%. A similar condition exists between the only low



FIGURE 2.2 AVAILABLE AIR VISCOSITY DATA
temperature air data available, that of Sutherland and Maass [5] and Johnston and McCloskey [6].
(3) All dilute low temperature viscosity data for air and all but two determinations for nitrogen have employed oscilating disc viscometers; as recently as 1964 Kestin and Wang [7] stated that the theory of the oscillating disc had not been fully developed and therefore that the results of these earlier determinations were certainly in doubt.

## HIGH PRESSURE VISCOSITY WORK

In 1931 Michels and Gibson [8] presented data on the viscosity of nitrogen at pressures of up to 1000 atmospheres and at three isotherms of $298.15,323.15$, and $348.15^{\circ} \mathrm{k}$. A non-stcady flow capillary viscometer was used, the dense gas being driven through an 80 cm long glass capillary by the pressure produced by a falling column of mercury 30 cm in length. The progress of the dropping mercury was measured using a series of platinum contacts fused into a glass column. The repeatability of the data was reported as $\pm 0.5 \%$, although no actual details of the determination of this accuracy were given, the results appear to be in good agreement with other more recent work.

Ross and Brown [9] in 1957 presented data on the viscosity of nitrogen and two other gases at pressures from 35 to 680 atmospheres and over the temperature range of 223.35 to $298.15^{\circ} \mathrm{K}$. The apparatus used for the measurements was a single glass capillary non-steady flow viscometer. The glass capillary,
being mounted longitudinally between a pair of metal bellows, was displaced axially causing one bellow to expand and the other to contract, thus forcing gas through the tube. This device was mounted within a thermostated pressure bomb. The tube displacement was measured as a function of time and a careful calibration of the bellows spring constant facilitated the determination of a pressure time relationship and the calculation of the viscosity using the Poiseuille equation. The reported accuracy of the data was $\pm 1 \%$ and a comparison with the data obtained by Michels and Gibson [8] shows an average deviation of about $0.5 \%$.

Kestin and Wang [7] in 1958 presented a re-evaluation of viscosity measurements made and presented by Kestin and Pilarczyk [10] in 1954. The work included data on the viscosity of air and nitrogen at $298.15^{\circ} \mathrm{K}$ and at pressures up to 70 atmospheres. The re-evaluation was made using an improved theory for the oscillating disc viscometer formulated by the authors in 1957. The analysis consisted of the application of a semi-empirical edge correction factor; whereas previous evaluations had considered the disc an infinite flat plate with no edge effects. The correction had the effect of changing the previously published viscosity data by up to $1.7 \%$ for air and nitrogen.

The re-evaluated data were plotted as a function of pressure, which in the case of nitrogen exhibits excellent agreement with the work of Michels and Gibson [8]. However there were insufficient air data available to allow any meaningful comparison. The authors
state that their correlations for viscosity as a function of pressure at $25^{\circ} \mathrm{C}$ are accurate to within $\pm 0.6 \%$.

A determination of the viscosity of eleven gases including air and nitrogen, at temperatures of 293.15 and $298.15^{\circ} \mathrm{K}$ and at pressures of up to 70 atmospheres was published in 1960 by Kestin and Leidenfrost [11]. The apparatus employed an oscillating disc and was claimed to be an absolute viscometer on the basis of a more refined theory developed by Newell [12] in 1959. Although no statement of accuracy was made the data appear to be in excellent agreement with the data of Kestin and Wang [7] and Miche1s and Gibson [8].

In 1964 Kestin and Whitelaw [13] presented a set of data on the viscosity of dry and humid air, at pressures up to 140 atmospheres and for six isotherms between 298.15 and $523.15^{\circ} \mathrm{K}$. An oscillating disc viscometer was employed, which was calibrated with dry air at $293.15^{\circ} \mathrm{K}$. The results were stated not to be subject to an uncertainty in excess of $\pm 0.4 \%$. The data were correlated in the form of excess viscosity vs density, the excess viscosity being defined as $n$ excess $=n-n_{0}$ where $n_{0}$ is the viscosity at zero density for a specific temperature; $n_{0}(T)$. The authors' demonstrate that the isotherms below $373.15^{\circ} \mathrm{K}$ provide an excellent fit to a third order polynomial in density. The higher temperature isotherms, however, appear to produce values of viscosity up to $3.5 \%$ higher than previous work by Kestin and Leidenfrost [11] and Kestin and Wang [7]. This trend is not fully understood and cannot be fully accepted. The authors suggest that it may have been
caused at higher temperatures by a systematic error in the apparatus.

Whitelaw [14] in 1960 using a transpiration type capillary viscometer, obtained viscosity data on high pressure steam and nitrogen. The nitrogen data covered pressures up to 114 atmospheres and a temperature range of 289.15 to $296.15^{\circ} \mathrm{K}$. A detailed error analysis shows the results to be accurate to $\pm 1.1 \%$ at the most unfavorable conditions. Although the results show a fairly high degree of experimental scatter they are generally in good agreement with previous authors.

In 1963 Flynn, Hanks, Lemaire and Ross [15] presented viscosity measurements for four common gases, including nitrogen, at pressures of up to 200 atmospheres and over a temperature range of 194.65 to $373.15^{\circ} \mathrm{K}$. Previous to this work, there had been no accurate viscosity measurements made at pressures above 1 atmosphere and at temperatures below $273.15^{\circ} \mathrm{K}$. The authors used a single capillary steady flow viscometer, the driving pressure being produced by a pair of counter reciprocating pistons driven by a constant speed synchronous motor. A comparison of the work of these authors shows good agreement with that of previous authors; however the stated experimental accuracy of $\pm 0.1 \%$ does not appear to be justified or reasonable.

In 1963, Goldman [16] presented a paper covering work carried out exclusively on the viscosity of nitrogen in the dense gas region. Measurements were taken at temperatures of 194.7, 211.9 and $298.2^{\circ} \mathrm{K}$ and at pressures of up to 120 atmospheres. The apparatus
employed was a constant volume single capillary non-steady flow viscometer, the test gas was forced through the capillary by a pressure difference created by the oil in one side of a large U-tube manometer. The viscosity data was found to agree relatively well with the work of Ross and Brown [9] and was correlated using a standard derived formula based on Enskog's theory for dense gases. Although no estimate of the experimental accuracy was reported, the data points had a maximum deviation from the correlation of +5.0 and $-1.4 \%$.

The viscosity of nitrogen-helium gas systems was investigated by Kao and Kobayashi [17] in 1967. The results presented for pure nitrogen cover a temperature region from 183.15 to $323.15^{\circ} \mathrm{K}$ and pressures of up to 500 atmospheres. The apparatus employed was a single capillary steady flow viscometer equipped with a double plunger volumetric pump, which maintained a steady flow throughout a test run. The maximum probable error was reported to be $\pm 0.137 \%$, the overall precision of the results appears to be excellent.

Goring and Eagan [18] in 1971 presented data on the viscosity of air at the temperature of $423.15^{\circ} \mathrm{K}$ and pressures of 36.75 and 104.79 atmospheres. The viscosity was measured using a single capillary steady flow viscometer, which was callibrated with dry air. The experimental scatter in the determination of the end correction effect was extremely high, as much as $8 \%$ at the lowest flow rate. Also a temperature gradient in excess of $20^{\circ} \mathrm{C}$ was reported along the length of the 20 inch capillary, this is thought to cast dispersions on the accuracy of the data.

In 1966 Lo, Carrol and Stiel [19] presented a paper on the viscosity of air at high pressures, the work consisted of a correlation of work done by twelve previous authors. The excess viscosity $n$ - $n_{0}$ was plotted against the parameter $P_{T} / T$ where $P_{T}$ is defined as the thermal pressure; $T\left\{(\partial P / \partial T)_{V}\right\}$. The data were correlated using the equation $n-n_{0}=$ $5.76 \times 10^{-5}\left(\mathrm{P}_{\mathrm{T}} / \mathrm{T}\right)^{1.125}$ poise and the results were presented in a tabular form of viscosity as a function of temperature and pressure. The correlation is in agreement with the data at lower pressures to within $\pm 2.5 \%$, however at higher pressures, the deviation is as high as $\pm 10 \%$.

CONCLUDING COMMENTS ON HIGH PRESSURES (DENSE) VISCOSITY DATA FOR NITROGEN AND AIR
(1) High pressure low temperature viscosity data is virtually non-existent even for nitrogen.
(2) No viscosity data for air at densities in excess of $0.17 \mathrm{GM} / \mathrm{CC}$ is currently available.
(3) The medium pressure work done by Kestin and Whitelaw [13] exhibits a deviation of $+4 \%$ from the other medium pressure data available for air.
(4) The present data is the only available data to be produced over the wide pressure and temperature range for either nitrogen or air using a single primary apparatus.

## DENSITY

In order to calculate the viscosity from working equation
(3.15) it is necessary to have an accurate knowledge of the density of the measured gas at the appropriate thermodynamic state.

In 1971 Coleman and Stewart [20] published a paper on the thermodynamic properties of nitrogen. An equation of state was developed for the temperature range of 70 to $1000^{\circ} \mathrm{K}$ and at pressures of up to 1000 atmospheres. In correlating the available density data the authors examined the works of twenty nine authors and selected a shortened list of twelve sets of data. The final equation takes the form of pressure, as a function of density and temperature. The inaccuracy of the fit is stated not to exceed $\pm 0.15 \%$ at any temperature and pressure.

Vasserman, Kazavchenskii and Rabinovich [21] in 1971 published a book on the thermodynamic properties of air and air components. The equations of state for air and nitrogen were developed in much the same way as that of Coleman and Stewart [20]. The authors present the equation in the form of the compressability factor as a function of reduced temperature and reduced density. Although the equation for nitrogen was not used in this work, a comparison with the calculated densities obtained using the equation of Coleman and Stewart [20] was made and showed excellent agreement over the more dense region, but a deviation of $0.4 \%$ in the one atmosphere region. The equation of state for air is claimed to be in agreement with the data to within $\pm 0.2 \%$.

These two equations of state, along with the computer programmes necessary to use them, are presented in more detail in Appendix (A2).

## CHAPTER 3

## THEORY

The Hagen-Poiseuille equation can be arrived at by considering the forces acting on a fluid moving in a cylindrical pipe, as done by Schlichting [22], where (1) The axis of the tube is the $x$ direction. (2) The tangential and radial components of velocity are zero. (3) The fluid moves under the influence of a pressure gradient which acts in the direction of the $x$ axis. (4) There is no pressure gradient in the radial or tangential directions. Consider a coaxial fluid cylinder of length $\&$ and radius $y$. In order for the condition of equilibrium to be maintained in the $x$ direction the pressure forces $\left(P_{1}-P_{2}\right) \pi y^{2}$ acting on the faces of the cylinder must be equal to the shear forces $2 \pi y \ell \tau$ acting on the circumferential area, thus:

$$
\begin{equation*}
\tau=\frac{P_{1}-P_{2}}{\ell} \frac{y}{2} \tag{3.1}
\end{equation*}
$$

In the case of Newtonian fluids

$$
\begin{equation*}
\tau=-n d u / d y \tag{3.2}
\end{equation*}
$$

combining equation (3.1) and (3.2)

$$
\begin{equation*}
\frac{d u}{d y}=-\frac{P_{1}-P_{2}}{n} \frac{y}{2} \tag{3,3}
\end{equation*}
$$

and integrating

$$
\begin{equation*}
u(y)=\frac{P_{1}-P_{2}}{n \ell}\left(C-\frac{y^{2}}{4}\right) \tag{3.4}
\end{equation*}
$$

The boundary condition of no slip at the wall or $u=0$ at $y=R$ yields:

$$
\begin{equation*}
u(y)=\frac{P_{1}-P_{2}}{4 n \ell}\left(R^{2}-y^{2}\right) \tag{3.5}
\end{equation*}
$$

It can be seen that the velocity profile across the tube is parabolic, and therefore the mean velocity $\bar{u}$ is equal to half the maximum velocity.

$$
\begin{equation*}
\overline{\mathrm{u}}=\frac{\mathrm{P}_{1}-\mathrm{P}_{2}}{8 \eta \ell} \mathrm{R}^{2} \tag{3.6}
\end{equation*}
$$

now introducing the volumetric flow rate $\dot{\mathrm{V}}=\pi \mathrm{R}^{2} \overline{\mathrm{u}}$ equation (3.6) may be written as:

$$
\begin{equation*}
\dot{v}=\frac{\pi R^{4}}{8 \eta \ell}\left(P_{1}-P_{2}\right) \tag{3.7}
\end{equation*}
$$

This was first deduced empirically by G. Hagen in 1839, shortly thereafter by J. Poiseuille and is known as the Hagen-Poiseuille equation of laminar flow through a pipe.

This simple equation is inadequate for accurate work in the above form since the conditions of (a) steady flow, and (b) no slip at the walls, are not totally satisfied.

A correction for (b), the slip at the walls of the tube can be achieved by increasing the fourth power of the radius of the tube by a factor of $\left\{1+\frac{4 \varepsilon}{R}\right\}$ where $R$ is the tube radius and $\varepsilon$ is a function of the mean free path of the molecules of the fluid in the tube.

Knudsen [23] did some experimental work with flow in a
tube to find the coefficient of slip. He obtained the formula for the mean free path as

$$
\begin{equation*}
L=\frac{8}{3} n\{2 /(\pi \rho P)\}^{\frac{1}{2}}=\frac{2.128 n}{(\rho P)^{\frac{1}{2}}} \tag{3.8}
\end{equation*}
$$

He carried out a substantial number of experiments with several different fluids on solids and concluded that the slip correction coefficient could best be represented by the equation

$$
\begin{equation*}
\varepsilon=C \cdot \frac{8}{3} n\left\{2 / P_{\rho} \pi\right\}^{\frac{1}{2}} \tag{3.9}
\end{equation*}
$$

He found empirically that the average value of the constant 'C' was 0.81 . The viscosity equation with the slip correction is therefore,

$$
\begin{equation*}
n=\frac{\pi R^{4} \Delta P}{8 \ell V}(1+4 \varepsilon / R) \tag{3.10}
\end{equation*}
$$

For simple capillary flow conditions the above equation is still inadequate, since it neglects the acceleration of the flow at the entrance of the tube. The fluid must be accelerated to the parabolic profile it assumes some distance along the tube. It therefore follows that there must be a drop in pressure due to the fact that the fluid will have gained kinetic energy. This, in itself, would not affect the viscosity equation if the velocity could be fully recovered at the exit, however this is not possible and therefore allowances must be made for this loss of head. The pressure drop due to energy losses as the fluid is accelerated into the capillary may be derived from Bernoulli's equation,

$$
\begin{equation*}
\Delta P=\frac{1 / 2}{2} \rho u^{2}=\frac{\rho \dot{V}^{2}}{2 \pi^{2} R^{4}} \tag{3.11}
\end{equation*}
$$

This loss of pressure does not take into account the actual viscous friction in the transition length. In order to allow for this, the end correction is assumed to be of the form, $\frac{m \rho \dot{V}^{2}}{\pi^{2} R}$, where ' $m$ ' is a constant.

There is a considerable amount of contradictory data and conclusions in the literature concerning the magnitude of the dimensionless correction factor'm'. The values obtained for the constant ' $m$ ' vary from 1 to 1.21 from theoretical work. However the experimental work of Flynn et al. [15] stated that $m$ varied from 0.88 to 1.70 and was a function of both temperature and Reynolds number. Unfortunately there are not enough results of sufficient accuracy available to prove or disprove this theory.

The main advantage of the two capillary viscometer is that the end correction term is eliminated. The reader is therefore referred to references [14] and [31] for further information on the value of 'm'.

A further correction, first suggested by Couette [24], is the correction to allow for the viscous drag between the converging and diverging streamlines outside the entrance and exit of a capillary. This correction is usually achieved by increasing the length of the capillary by an amount 'nD', to ( $\ell+\mathrm{nD})$, where ' $n$ ' is a constant of the order of unity. The value of $n=0.6$, based on recent investigations, is often accepted for a capillary tube having squared ends. With a two capillary viscometer however the Couette correction is largely
eliminated and their is no need to discuss it in detail.
The coefficient of thermal expansion of the glass capillaries $\alpha$ is used to correct the capillary dimension. Therefore the final equation for flow through a single capillary is:

$$
\begin{equation*}
n=\frac{\pi V^{4} \rho g \Delta P}{128 Q(\ell+n D)} \cdot(1+8 \varepsilon / D) \cdot(1+3 \alpha \Delta t)-\frac{m Q}{8 \pi \ell(1+\alpha \Delta t)} \tag{3.12}
\end{equation*}
$$

The present work deals with a viscometer in which two capillaries are in series. The equation for the two capillaries may be expressed as,

$$
\begin{align*}
& n=\frac{\pi_{1}{ }^{4} \Delta P_{1} \rho_{1} g\left(1+8 \varepsilon / D_{1}\right)(1+3 \Delta t \alpha)}{128 Q\left(\ell 1_{1}+\mathrm{nD}_{1}\right)}-\frac{m Q}{8 \pi \ell{ }_{1}(1+\Delta t \alpha)} \\
& n=\frac{\pi D_{2}^{4} \Delta P_{2} \rho_{1} g\left(1+8 \varepsilon / D_{2}\right)(1+3 \Delta t \alpha)}{128 Q\left(\Omega_{2}+n D_{2}\right)}-\frac{m Q}{8 \pi \ell{ }_{2}(1+\Delta t \alpha)} \tag{3.13}
\end{align*}
$$

where suffixes 1 and 2 refer to capillaries 1 and 2 respectively. Treating the equations simultaneously to eliminate $m$ and n we get

$$
\begin{equation*}
n=\frac{\pi g D_{1}{ }^{4}(1+8 \varepsilon / D)(1+3 \Delta t \alpha)}{128 Q\left(l_{1}{ }^{-l}{ }_{2}\right)}\left\{\rho_{1} \Delta P_{1}-\left(\frac{D_{2}}{D_{1}}\right)^{4} \rho_{2} \Delta P_{2}\right\} \tag{3.14}
\end{equation*}
$$

The slip correction was calculated and found to be less than $0.2 \%$ at the least dense region, thus the working equation employed was:

$$
\begin{equation*}
n=\frac{\pi g D_{1}^{4}(1+3 \alpha \Delta t)}{128 Q\left(l_{1}-l_{2}\right)}\left\{\rho_{1} \Delta P_{1}-\left(\frac{D_{2}}{D_{1}}\right)^{4} \rho_{2} \Delta P_{2}\right\} \tag{3.15}
\end{equation*}
$$

## CHAPTER 4

## DESCRIPTION OF APPARATUS

## GENERAL SYSTEM ARRANGEMENT

The apparatus described was built by Hesoun [25] based on an earlier design developed by Latto [26]. The high and low pressure arrangements are described simultaneously, as they differ only in minor detail. The general flow diagram and photographs of various components of the viscometer are given in Figures 4.1 to 4.4 which are repeatedly referred to throughout this section.

The test gas was supplied from a gas bottle (A) to the viscometer plenum (E) after passing through a silica-gel dryer (D). A set of high and low pressure regulators (B) and (C) were employed to maintain a constant plenum pressure during the low pressure experiments. The high pressure regulator alone was initially used during high pressure runs. However this proved to produce a slight pressure oscillation caused by the regulator hunting and an arrangement whereby the gas supply bottle after having been filled to slightly above the desired operating pressure and connected directly to the system, was finally employed.

Three copper heat exchange coils were wrapped around the stainless steel high pressure vessel. A heat exchange fluid could be pumped through these coils in order to produce the


FIGURE 4.1 SCHEMATIC OF VISCOMETER SYSTEM

## LEGEND FOR FIGURE 4.1

| A | Gas Bottle |
| :---: | :---: |
| B | High Pressure Regulator |
| C | Low Pressure Regulator |
| D | Silica-Gel Dryer |
| E | Viscometer Plenum |
| F | Heat Exchangers |
| G | Micro-Metering Valve |
| H | Solenoid Valve |
| I | U-Tube Manometer |
| J | Volumeter |
| K | Photocell Relays |
| L | Timer |
| I | Cathetometer |
| N | High Pressure Three Limb Manometer |
| 0 | Manometer Oil Traps |
| P | Manometer Release Valves |
| Q | Safety Valve |
| R | High Pressure Gauge |
| S | High Pressure Gauge |
| T | Dead Weight Tester |
| U | U-Tube Manometer |
| V | Air Supply Line |
| W | Silica Gel Air Dryer |
| X | Air Pressure Regulator |
| Y | Coolant Mixing Chamber |
| 2 | Liquid Nitrogen Dewer |

AA Viscometer Insulation Box
$A B \quad$ Coolant Control Valves
TH1-9 Thermocouples 1 to 9
desired temperature witnin the viscometer cell. Both the stainless steel vessel and the viscometer cell were maintained at the required temperature.

The test gas enters the upstream (long) capillary directly from the vessel plenum then passes through an intermediate chamber entering the downstream (short) capillary, after this it discharges into the external system. In leaving the temperature controlled pressure vessel the gas flows through two heat exchangers, (F), which restore it to ambient temperature, and a micro-regulating valve (G), then to the low pressure displacement volumeter (J). The gas then exhausts through a three way solenoid valve ( $H$ ) to atmosphere.

The pressure drops across the capillaries were measured with a high pressure oil-in-glass three limb manometer (N). To protect the plenum during a possible blow out; oil traps (0) were situated in the manometer lines. The absolute pressure of the gas in the plenum was measured using a dead weight tester (T), sensitive to 0.5 psi . For the low pressure experiments a mercury manometer ( $U$ ) was used. Two high pressure gauges ( $R$ ) and (S) served as indicators.

The viscometer cell and high pressure vessel were kept at a given temperature by passing an appropriate heat exchange fluid through the heat exchanger coils (AA). Liquid nitrogen was pumped directly into the coils during very low temperature runs or to achieve rapid cooling of the viscometer plenum. A nitrogen rich mixture of precooled air was used to maintain the plenum at moderate to low temperature, and steam was passed through the coils during the high temperature runs. The
temperature could be controlled manually by monitoring thermocouples TH1 through TH5 and adjusting the heat exchanger exit valves ( $A B$ ). After some practice this proved to be a more or less suitable system; however a more convenient control arrangement would have been achieved by the addition of a more sophisticated feed back temperature controller.

A high precision system (not shown in Figure 4.1) was used to measure the thermocouple outputs. this system was composed of "Guildiine" Model 9461A Potentiometer together with Model 9460 Photocell Galvanometer Amplifier and Model 9461A Galvanometer.

THE VISCOMETER CELL AND HIGH PRESSURE VESSEL
The viscometer cell and high pressure vessel are shown in detail in figures 4.2 and 4.3. The brass end chambers are connected by $\frac{1}{4}$ " $0 . D$. stainless steel tubes to the outside Swagelok connections. Rubber tubing was used to seal the capillaries to the brassend connectors. This allowed for misalignment within the apparatus as well as considerable variation in the outside diameters of different sets of capillaries. In order to allow for the loss of elasticity of rubber at low temperatures, copper wire was tightly wound around the rubber seals. This simple arrangement proved to be more suitable than some elaborate connectors previously designed and tested.

The use of mixing motors, originally installed in the plenum to help establish a uniform temperature, was discontinued, since they tended to freeze atvery low temperatures and were


found to be unnecessary after a number of initial runs. The end chambers were designed with baffles in order to ensure that the flow did not appreciably core. This is important because the elimination of the Hagenbach and Couette corrections depend on similar entrance and exit conditions and therefore zero entrance velocity at both capillaries. THE HIGH PRESSURE DIFFERENTIAL MANOMETER

A drawing of the high pressure differential manometer is shown in figure 4.4. The oil-in-glass three limb manometer is fitted to a stainless steel back plate and is observed through a thick transparent plexiglass window. The window is sealed to the face of the backplate by means of a rubber gasket and is held in place with thirty-two screws located in two steel clamps. The high precision bore $1 / 8^{\prime \prime}$ I.D. glass tubes of the manometer limbs are about 30 cm long and were filled with manometer oil.

Two of the three 1 imbs were connected, by means of short plastic tubes, to copper lines soldered in the back plate. The third limb was open to the manometer chamber, which was at the working pressure of the viscometer plenum. Both the capillaries and glass manometer tubes are thus not subjected to high differential bursting pressures.

The manometer was read by means of a cathetometer mounted on the floor some two feet away. An additional $\frac{1}{2}$ inch thick plexiglass safety window was installed in front of the manometer window, which fortunately proved to be unnecessary during the entire period of experimentation.


FIGURE 4.4 HIGH PRESSURE THREE LIMB DIFFERENTIAL MANOMETER

TEMPERATURE CONTROL EQUIPMENT
The heat exchanger equipment consisted of three copper coils wrapped along the length of the high pressure vessel. The vessel and coils were surrounded by polyurethane foam an excellent insulation material. An appropriate heat exchange fluid was passed through these coils.

Over the broad temperature range covered, it was necessary to employ three different heat exchange fluids.
(1) If the desired temperature was very low or if it was desirable to lower the temperature quickly, liquid nitrogen could be forced directly from the nitrogen dewer into the three heat exchange coils.
(2) If the temperature desired was above $150^{\circ} \mathrm{K}$ and no rapid temperature changes were necessary a low temperature mixture of nitrogen and air was passed through the heat exchange coils. This mixture was attained by bubbling air through the liquid nitrogen dewer. The thermal capacity of gaseous nitrogen is relatively low and this coupled with the large heat capacity of the pressure vessel and viscometer cell resulted in a large thermal inertia, thus facilitating accurate temperature control and stability.
(3) For high temperature runs medium pressure steam was supplied directly to the coils.

The flow rate of heat exchange fluid was controlled by means of valves located in the exit lines of the heat exchange coils. By individually adjusting the quantity of heat exchange fluid passing through each coil it was possible to attain an
even temperature distribution along the length of the pressure vessel. Initially the control valves were located on the inlet side of the heat exchangers inside the insulated box, with valve stems and handles extended to the outside, however after some difficulty with valves freezing during low temperature operation they were abandoned and replaced with exit line valves externally mounted.

## FLOW MEASUREMENT

Flow measurement was achieved by means of a direct volume displacement method employing a "Brooks" Vol-U-Meter with a total capacity of 1200 cubic centimeters. A frictionless mercury ' ${ }^{\prime}$ ' ring sealed piston was displaced vertically up a precision diameter glass tube. Two photo electric cells, located such that the distance between them represented a precalculated volume, were used to start and stop a "Hewlett Packard" digital timer. On stopping the timer the second photo cell also activated a three way solenoid valve causing the volumeter to exhaust to atmosphere. The photo cells could be set so as to measure nominal volumes of 200,400 and 800 cubic centimeters.

In order to determine the mass flow rate it is not only necessary to measure the volume flow rate but a knowledge of the gas density is also required. This was achieved by measurement of the temperature and pressure within the volumeter. Thermocouples TH7 and TH8 were located at opposite ends of the volumeter cylinder and a small U-tube oil manometer was connected to the inlet line.

The viscometer cell was connected to the panel instrumentation and to the entrance and exit lines by means of $1 / 8^{\prime \prime}$ OD copper tubing and brass swagelok fittings. When access to the cell was desired to check or replace the capillaries, the insulation box and high pressure vessel was slid back and the viscometer cell supported on wooden blocks.

For a further description of the development and design criteria of the apparatus the reader is referred to reference [25].

## CHAPTER 5

## EXPERIMENTAL PROCEDURE (PART I)

CAPILLARY CALIBRATION
In the working equation the capillary diameter is raised to the fourth power. This caused an error in measurement of the diameter to be increased by a factor of four in the final viscosity calculation. It is therefore apparent that the diameter of the capillaries must be measured with the utmost precision.

The viscometer was designed to use "Veridia" glass capillaries having high precision bores ranging in diameter from 0.25 to approximately 2 mm . This present work employed capillaries having nominal bores of 0.5 and 0.3 mm for the 10 w and high pressure experiments respectively. The tubes were calibrated using a "mercury thread" method which, in previous works, has proved to be convenient and of sufficient accuracy. The selection of the capillaries and the calibration procedure was conducted as follows.

Initially the capillaries were thoroughly cleaned by soaking in sulphuric acid and rinsing in distilled water and ether. The ellipticity, conicallity and any other irregularities of the tube bores were checked by passing a small mercury pellet
along the length of the capillary and observing its variation in length. Graphical illustrations of the bore fluctuation appear in figures 5.1 to 5.4. Two capillaries having the most uniform bores were chosen. The mean diameter of each capillary was then obtained gravimetrically by filling the entire length of the capillary with a thread of mercury. The length and weight of the long thread was measured using a cathetometer equipped with a microscope, and a high precision balance respectively. The measurement of the pellet length was carried out while the capillary was immersed in a glycerine bath, this was done because glycerine which has approximately the same refractive index as glass allows an extremely clear and distortionless. view of the mercury pellet. The entire procedure of filling the tube with mercury, measuring and weighing it was carried out several times for each capillary, thus decreasing the influence of random weighing errors.

The capillaries were then cut to the design lengths, accurately faced off, and the lengths measured using the cathetometer. The uniformity of the tube bores was sufficient to consider the mean diameter obtained for the rough uncut tube to be the same as that for the shorter design length.

The theory for both the "short pellet" and "long pellet" calibration methods is straight forward and for further details the reader is referred to reference [26].

## VOLUMETER CALIBRATION

The volumeter consisted of a long vertical precision bore glass tube with a light plastic piston, sealed around its


FIGURE 5.1 SMALL MERCURY PELLET CALIBRATION CURVE


FIGURE 5.2 SMALL MERCURY PELLET CALIBRATION CURVE


FIGURE .5.3 SMALL MERCURY PELLET CALIBRATION CURVE


FIGURE 5.4 SMALL MERCURY PELLET CALIBRATION CURVE
diameter with a frictionless mercury ' 0 ' ring. The position of the piston within the tube was a measure of the volume of gas in the volumeter. As gas filled the volumeter the piston would move vertically upward, at a certain position the piston would interrupt one of two photo electric cells mounted along the volumeter, thus starting the timer. When the piston had risen a predetermined height it would then interrupt the second photo cell and stop the timer. The distance between these two photo cells was measured using the cathetometer and with the lower photo cell able to be mounted in any one of three positions, nominal timed volumes of 200,400 and 800 cc were attainable.

The diameter of the volumeter cylinder was measured gravimetrically by filling the tube with distilled water to a measured height and the quantity of water weighed. As in the case of the capillary calibration the procedure was repeated several times to eliminate random errors. MANOMETER CALIBRATION

The specific gravity of the manometer oil was measured using a standard specific gravity bottle. This proved quite satisfactory for low pressure work, however during high pressure experiments the oil is subjected to considerable pressure which affects its density as well as resulting in gas absorption. In order to obtain a quantitative analysis of the overall effects of gas pressure on the oil the following test was conducted. A given volume of oil was placed in the high pressure manometer and the length of $1 / 8^{\prime \prime}$ ID precision glass tubing it filled was calculated. The apparatus was then
pressurized in steps of 20 atmospheres up to 150 atmospheres and the change in length of oil recorded with the cathetometer. The ratio of change in length to original length, $\Delta \mathrm{L} / \mathrm{L}$, provided a measure of the change in density of the oil as a function of pressure.

The apparatus was further allowed to stand at high pressure for several days in order for gas absorption in the manometer oil to reach equilibrium. The manometer was then quickly decompressed and the manometer oil observed. It was estimated from the volume of disolved gas emitted from the oil that no significant quantity of gas has been absorbed and therefore any change in oil density due to gas absorption could be ignored.

THERMOCOUPLE CALIBRATION
The four most important thermocouples were TH4 and TH5, used to measure the temperatures of the gas entering the two capillaries, and TH7 and TH8 used in the volumeter. Thermocouples 4 and 5 were calibrated at the fixed points of boiling liquid nitrogen, the ice point, and the boiling point of water, also at various other intermediate temperatures against a standard number 705 copper constantan thermocouple which was calibrated by Thermo Electric Ltd., Canada using a reference NBS standard thermocouple APHSS-992/1 and APH 1421, which are claimed to be accurate to $\pm 0.10^{\circ} \mathrm{C}$ and $\pm 0.06^{\circ} \mathrm{C}$ respectively. The other two thermocouples, TH7 and TH8 were calibrated over a much smaller temperature range in a similar manner. The calibration curves appear in figure 5.5.


The calibration as well as the experimental temperature measurements were done using a high pressure "Guideline Instrument" ; Potentiometer Type 9160 together with Type 9461 A Galvanometer and Type 9460 Photocell Galvanometer Amplifier. This set up is capable of measuring potential difference as small as $10^{-7}$ volts The systematic error of the temperature measurement is believed to be very close to that of the number 705 copper constantan thermocouple or $\pm 0.1^{\circ} \mathrm{C}$.

EXPERIMENTAL PROCEDURE (PART 2)
The viscometer cell was thoroughly cleaned with ether, dried and the capillaries and rubber connecters assembled. The manometer lines to the two end chambers were plugged at the pressure vessel head and the unit was left out of the vessel in order to test the internal connections for leakage.

Since the pressure during normal operation is greater in the plenum than in the capillaries or end chambers, the long capillary was connected to a mercury manometer and the exit line to a vacuum pump. In this way a greater external pressure could be applied to the internal circuit than would actually be encountered during a run.

After the viscometer cell had been leak tested, attention was turned to testing the external circuit and pressure vessel for leakage. The manometer lines were assembled and the pressure vessel bolted into place. The apparatus was then pressurized to 150 atmospheres with nitrogen gas and all external connection tested with leak test fluid. It should also be noted that with the manometer release valves, and the exit valves closed there should be no change in the manometer fluid levels. This however is not the case if there is a leak in one of the manometer lines or in the exit line, and in fact a very slight leak was found to cause a visible pressure differential across the manometer. This provided a mechanism by which the external circuit could be checked after the apparatus had reached the desired operating temperature and proved very useful in preventing small leaks during actual runs.

The viscometer was repeatedly evacuated and flushed with test gas before use. The apparatus was then filled to the desired pressure. During this operation great care had to be exercised to ensure that the manometer release valves were open or the manometer oil would blow out. If a rather potent blow out occurred oil would get into the capillaries and pressure lines. This happened several times during the course of these experiments and necessitated the dismantling and cleaning of the entire apparatus.

The apparatus was brought to the desired operating temperature by pumping liquid nitrogen or steam through the heat exchanger coils while monitoring the four coolant circuit thermocouples and the two internal viscometer cell thermocouples. As soon as the temperature approached the desired value the exit line throttle valve was adjusted to give the desired pressure drop across the capillaries. The heat exchange fluid was then carefully controlled until a suitable temperature equilibrium had been reached, as indicated by the viscometer cell thermocouples. Then the volumeter was switched into the circuit and readings were taken. Firstly all temperature readings, then the atmospheric and the plenum pressures were recorded. The pressure drops across the capillaries were then measured using the cathetometer, and pressure and temperature were measured again then finally the volumeter time was recorded. The measurement of these parameters took approximately 10 to 15 minutes and was done during the time it took for the volumeter to fill. This procedure was repeated several times
until a good equilibrium condition had been established and the last set of readings constituted a run.

Considerable care was used to establish the temperature equilibrium simultaneously with an acceptable temperature distribution. The system was usually run for at least four hours before the apparatus was sufficiently stabilized to take any valid experimental readings. Stability of the system during the run is essential if high experimental precision of the viscosity measurements is to be obtained.

## ESTIMATION OF ERRORS

It is difficult to make an accurate estimate of the true absolute accuracy of the experimental results. In the foregoing chapter the word accuracy is used to define the relationships of a value to that of a given absolute standard, whereas the precision is defined as the reproduciblity of a given value. It is therefore obvious that one cannot have accuracy without precision, but one can have precision without accuracy. In this chapter both the precision and accuracy of the experimental work will be estimated.

There are several factors which limit the accuracy of the results obtained for the viscosity of nitrogen and air using the capillary method. These may be placed in four groups, (1) the reliability of the Hagen-Poiseuille equation, (2) the accuracy of the actual measurements, (3) the experimental corrections and (4) the accuracy of the subsidiary data required, i.e. the properties of the gases used. These four groups will not be discussed separately, but brought together for each assessment.

Commencing with the flow equation for a series capillary viscometer,

$$
\begin{equation*}
n=\frac{\pi g(1+3 \alpha \Delta T)}{128 Q\left(l_{1}-l_{2}\right)} \quad\left(z_{1}-z_{2}\right) \tag{6.1}
\end{equation*}
$$

where

$$
z_{1}=\Delta P_{1} \rho_{1}{ }_{1}{ }^{4}
$$

and

$$
z_{2}=\Delta P_{2} \rho_{2}{ }_{2}{ }^{4}
$$

and using the "most probable" method to ascertain the overall accuracy and precision of the experimental work i.e. taking the square root of the sum of the squares of the largest errors in each variable.

This method may be represented by the equation

$$
\begin{equation*}
\left.\mathrm{d} \eta=\sqrt{\sum_{i=1}^{J}\left(\frac{\partial \eta}{\partial x_{i}}\right.} \quad \mathrm{x} \quad \mathrm{~d} \mathrm{x}_{\mathrm{i}}\right)^{2} \tag{6.2}
\end{equation*}
$$

Partially differentiating equation (6.1) and putting the results into the form of equation (6.2) we get

$$
\begin{align*}
& \frac{d \eta}{\eta}=\sqrt{\left(\frac{d Q}{Q}\right)^{2}+\frac{\left(d \ell_{1}\right)^{2}+\left(d \ell_{2}\right)^{2}}{\left(l_{1}-\ell_{2}\right)^{2}}+} \begin{array}{l}
{\left[\left(\left(\frac{d \Delta P_{1}}{\Delta P_{1}}\right)^{2}+\left(\frac{\rho_{\rho}}{\rho_{1}}\right)^{2}+\left(\frac{4 \mathrm{dD}_{1}}{\mathrm{D}_{1}}\right)^{2}\right\}_{z_{1}}^{2}\right.} \\
\left.+\left\{\left(\frac{\mathrm{d} \Delta \mathrm{P}_{2}}{\Delta \mathrm{P}_{2}}\right)^{2}+\left(\frac{\mathrm{d}_{2}}{\rho_{2}}\right)^{2}+\left(\frac{4 \mathrm{~d} D_{2}}{\mathrm{D}_{2}}\right)^{2}\right\} z_{2}{ }^{2}\right] \\
x\left(\frac{1}{\left(z_{1}-z_{2}\right)^{2}}\right)
\end{array} \tag{6.3}
\end{align*}
$$

This may be illustrated more clearly by the following:
by putting

$$
\begin{equation*}
n=\frac{\cdots}{\left(l_{1}-l_{2}\right)} \tag{6.4}
\end{equation*}
$$

Then, applying equation (6.2):

$$
\begin{equation*}
\mathrm{d}_{\mathrm{n}}=\sqrt{\left(\frac{\mathrm{d} \nabla}{\ell_{1}-\ell_{2}}\right)^{2}+\left(\frac{-\nabla \mathrm{d} \ell_{1}}{\left(\ell_{1}-\ell_{2}\right)^{2}}\right)^{2}+\left(\frac{\nabla \mathrm{d} \ell_{2}}{\left(l_{1}-\ell_{2}\right)^{2}}\right)^{2}} \tag{6.5}
\end{equation*}
$$

and therefore,

$$
\begin{equation*}
\frac{d \eta}{n}=\sqrt{\left(\frac{d \nabla}{V}\right)^{2}+\frac{\left(d \ell_{1}\right)^{2}+\left(d \ell_{2}\right)^{2}}{\left(\ell_{1}-\ell_{2}\right)^{2}}} \tag{6:6}
\end{equation*}
$$

The following assessments of the individual errors will be used in conjunction with equation (6.3) to estimate the overall accuracy and precision.
dISCUSSION OF THE INDIVIDUAL VARIABLES
MASS FLOW RATE
The measurement of the mass flow rate is dependent on three factors, the volume and density of gas collected, and the time taken to collect it. That is

$$
\begin{equation*}
Q=V_{\rho} / \tau \tag{6.7}
\end{equation*}
$$

which leads to

$$
\begin{equation*}
\frac{d Q}{Q}=\sqrt{\left(\frac{d v}{v}\right)^{2}+\left(\frac{d \rho}{\rho}\right)^{2}+\left(\frac{d \tau}{\tau}\right)^{2}} \tag{6.8}
\end{equation*}
$$

The density was obtained from an equation of state, which is reported to be accurate to $\pm 0.1 \%$, and a knowledge of the state variables pressure and temperature, which were measured in the volumeter to accuracies of $\pm 0.05 \mathrm{~cm} \mathrm{Hg}$ and $\pm 0.3^{\circ} \mathrm{K}$ respectively. This results in an overall density accuracy of $\pm 0.142 \%$.

The volumeter volume was calibrated and is believed to be accurate to $\pm 0.20 \%$ and the Hewellet Packard digital timer is exact to five significant digits, or for the worst case $\pm 0.01 \%$.

Thus the overall uncertainty of the mass flow rate $Q$ is $\pm .245 \%$.

LENGTH OF CAPILLARIES
The long and short capillaries had nominal lengths of 52 and 6 cm respectively, these were measured with the Griffin and George cathetometer to an accuracy of $\pm 0.004 \mathrm{~cm}$. Thus the second term of equation (6.3) contributes to the overall error by $\pm .012 \%$.

PRESSURE DROPS
The determination of the differential pressures, $\Delta \mathrm{P}_{1}$ and $\Delta \mathrm{P}_{2}$, was dependent on the measurement of the height and density of the oil columns in the high pressure manometer. The error can be expressed by the equation,

$$
\begin{equation*}
\frac{d(\Delta P)}{\Delta P}=\sqrt{\left(\frac{d h}{h}\right)^{2}+\left(\frac{d_{\rho}}{\rho}\right)^{2}} \tag{6.9}
\end{equation*}
$$

The apparatus was operated such that the nominal pressure drops across capillaries 1 and 2 were 20 and 3 cm of oil respectively. These were measured to an accuracy of $\pm 0.006 \mathrm{~cm}$ using the Griffin and George cathetometer. The density of manometer oil was calibrated using a'mettler' balance and standard specific gravity bottle, also corrections for temperature and pressure dependent density changes were assessed and used. It is therefore believed that the corrected value of oil density is accurate to $\pm 0.12 \%$.

From the above observation it was determined that the absolute accuracy of the individual differential pressure measurements was $\pm .124$ and $\pm .233 \%$ for $\Delta \mathrm{P}_{1}$ and $\Delta \mathrm{P}_{2}$ respectively. TEMPERATURE MEASUREMENT

The temperature inside the viscometer was measured using two copper-constantan thermocouples which had been previously calibrated against two N.B.S. Standard thermocouples certified exact to $\pm 0.1$ and $\pm 0.06$ centigrade degrees over the temperature ranges of 90 to $270^{\circ} \mathrm{K}$ and 270 to $400^{\circ} \mathrm{K}$ respectively. Due to non-isothermal conditions along the length of the viscometer cell, as well as the uncertainty of N.B.S. circular 508 [27] thermocouple tables, the temperature within the apparatus is estimated to be accurate to $\pm 0.3$ centigrade degrees. This leads to errors of $\pm .3 \%$ and $\pm .075 \%$ at the temperature extremes of 100 and $400^{\circ} \mathrm{K}$ respectively.

## gas densities

The values of the densities of the nitrogen and air were calculated using equations of state given in references [20] and [21] respectively. These equations are stated to be accurate to $\pm 0.1 \%$ over the entire density range of this work.

As stated previously the temperatures are believed accurate to $\pm .3$ centigrade degrees. The pressure within the plenum was measured, over the dilute gas range, using a mercury U-tube manometer and at higher pressures with a dead weight tester, these measurements were accurate to $\pm .01 \%$ and $.067 \%$ respectively. Making the approximate assumption that for a gas $\rho=P / 2 R T$, then the equation for the effects of uncertanties in pressure and temperature may be expressed as

$$
\begin{equation*}
\frac{d \rho}{\rho}=\sqrt{\left(\frac{d p}{P}\right)^{2}+\left(\frac{d T}{T}\right)^{2}+\left(\frac{d Z}{2}\right)^{2}} \tag{6.10}
\end{equation*}
$$

where $d z / Z$ represents the uncertainty due to the equation of state used.

Using equation (6.11), table (6.1) has been generated listing the accuracy of gas density as a function of pressure and temperature.

CAPILLARY DIAMETERS
The diameters of the capillaries were measured using a gravimetric method using the equation

$$
\begin{equation*}
D=\frac{4}{\pi} \frac{\omega_{m}}{\rho_{m} l_{m}} \tag{6.11}
\end{equation*}
$$

where $\omega_{m}, \ell_{m}$ and $\rho_{m}$ represent the weight, length, and density of a long mercury pellet within the capillary.

The equation representing the accuracy of the capillary diameter is therefore:

$$
\begin{equation*}
\frac{d D}{D}=\frac{1}{2} \sqrt{\left(\frac{d \omega_{m}}{\omega_{m}}\right)^{2}+\left(\frac{d \ell_{m}}{\ell_{m}}\right)^{2}+\left(\frac{d \rho_{m}}{\rho_{m}}\right)^{2}} \tag{6.12}
\end{equation*}
$$

The density of the mercury was interpolated from Smithsonian tables [28] and believed to be determined accurately to $\pm 0.05 \%$.

The pellet length, nominally 50 centimeters, was measured by means of the Griffin and George cathetometer to an accuracy of $\pm .002 \mathrm{~cm}$ representing a relative error of $\pm .004 \%$.

A 'Mettler' balance accurate to $\pm .001$ grams was used to measure nominal mercury pellet weights of 0.6 and 1.5 grams for the 0.3 and 0.5 mm diameter capillaries respectively. This results in relative weighting errors of $\pm .167$ and $\pm .067 \%$ respectively.

The overall accuracies of the diameters of the high pressure, 0.3 mm and the low pressure, 0.5 mm capillaries were $\pm .087$ and $\pm .042 \%$ respectively.

## OVERALL ACCURACY

From the preceding sections and the use of equation (6.3) the experimental accuracy of viscosity as a function of pressure and temperature has been calculated and is presented in table 6.2.

It should be noted that this represents an assessment of the random error only and does not include systematic error. Errors due to flow instability, oscillation effect, small leaks and so forth are not quantifiable. It is believed however that

TABLE 6.1
ESTIMATED ERROR IN DENSITY
AS A FUNCTION OF TEMPERATURE
at high and low pressures

| $\frac{\mathbf{d} \rho}{\rho} \pm \%$ | 100 | 150 | 200 | 300 | 400 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Low <br> Pressure | .316 | .221 | .177 | .138 | .125 |
| High <br> Pressure |  | .233 | .190 | .154 | .142 |

TABLE 6.2
ESTIMATED ERROR IN VISCOSITY
AS A FUNCTION OF TEMPERATURE
AT HIGH AND LOW PRESSURES

| $\frac{\mathbf{d} n}{n} \pm \%$ | 100 | 150 | 200 | 300 | 400 |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Low <br> Pressures | .515 | .492 | .470 | .426 | .382 |
| High <br> Pressures |  | .604 | .589 | .561 | .533 |

these errors cannot be overlooked in a statement of the reliability of the results, and therefore based on the scatter of the data it is thought that the results have an overall reliability of better than $\pm 1.5 \%$ at the highest densities and better than $\pm 1 \%$ at densities lower than $0.25 \mathrm{gm} / \mathrm{cc}$.

## CHAPTER 7

RESULTS AND DISCUSSION

GENERAL
The absolute viscosity of nitrogen and air were experimentally measured over four nominal isobars of 1,50 , 100 and 150 atmospheres and over the general temperature range of 100 to $400^{\circ} \mathrm{K}$. The data are presented in tables 7.1 and 7.2.

LOW PRESSURE NITROGEN
It was decided not to use totally empirical methods of correlation, which often produce a good but physically unjustifiable fit of the experimental data. For example in the case of polynomial correlations, this results in an inability to interpolate outside the experimental temperature range, since an extension of only a few degrees may result in gross errors in the calculated viscosity.

Two semi-theoretical methods were used to correlate the low pressure nitrogen data, these are as follows:
CHAPMAN ENSKOG'S COLLISION THEORY
Presently the rigorous collision theory of gases provides a mathematical expression relating theory with experimental data. The viscosity of a dilute gas may be calculated using the

TABIE 7． 1 NTROCEN DATA THS WORK
$v \operatorname{ccostry}$
$(00 \mathrm{~S}$
TFNPEFATURE．
（OEGK）

PRFSSURE
（ATMOS）
UFASITY （GN／CC）

| －664247E－04 | 98．26 | 1.19 | ．427865E－02 |
| :---: | :---: | :---: | :---: |
| －829r31E－04 | 122.29 | 1.20 | －340711E－02 |
| ． $110363 \mathrm{E}-03$ | 171．31 | 1.20 | ．241085E－02 |
| ． $120244 \mathrm{E-03}$ | 188.00 | 1.15 | ． $210612 \mathrm{E}-02$ |
| ． $125605 \mathrm{E}-03$ | 196.75 | 1.21 | ． $211817 E-02$ |
| －］35718E－03 | 216.04 | 1.21 | －193153E－02 |
| ． $136312 \mathrm{E}-93$ | 217.91 | 1.21 | ． $191525 E-02$ |
| ． $129195 \mathrm{~F}-03$ | 224.24 | 1.20 | ．183787E－02 |
| ．148607E－03 | 238．32 | 1.20 | ．172856E－02 |
| －149252E－02 | 238．44 | 1.20 | －172734E－02 |
| ． $157433 \mathrm{E}-03$ | 255．09 | 1.21 | ．163043E－02 |
| －J57754E－03 | 256． 24 | 1.21 | －162172F－02 |
| －177536E－03 | 296.33 | 1.19 | －128073E－02 |
| ． $177111 \mathrm{~F}-03$ | 296.39 | 1.19 | ． $137938 \mathrm{E}-02$ |
| －178へ48E－03 | 296．98 | 1.20 | ． 13844 OE－02 |
| －17770EE－03 | 298.33 | 1.20 | ． $137512 \mathrm{E}-02$ |
| ．1780つFE－03 | 298.35 | 1.20 | ．127501E－02 |
| －178267E－03 | 298.37 | 1.20 | ． $1374915-0 \overline{2}$ |
| －1pnal1E－03 | 302.06 | 1.21 | ． $137451 E-02$ |
| －170035E－03 | 302.35 | 1.21 | ． $137818 \mathrm{~F}-02$ |
| －1P？613E－03 | 308.96 | 1.20 | ．133149E－02 |
| －19欠183t－03 | 330.41 | 1.19 | －123462E－02 |
| ． $193834 \mathrm{E}-0.2$ | 334.81 | 1.19 | ．121909E－02 |
| － $210426 \mathrm{E}-03$ | 371.64 | 1.19 | ．109830E－02 |
| ．2）0310E－0．3 | 371.99 | 1.19 | ． $109712 \mathrm{E}=02$ |
| －210204E－03 | 374.60 | 1.18 | ． $108245 \mathrm{E}-0$ ？ |
| －140R31E－03 | J53．65 | 51.27 | － $165655 E+00$ |
| －133520E－03 | 173.49 | 50.69 | －121829E＋00 |
| －1349078－03 | 185.02 | 50.94 | ． $109263 \mathrm{E}+00$ |
| －135883E－03 | 185.43 | 50.88 | －108608E＋00 |
| －139297E－03 | 193．12 | 49.73 | －पद1328E－01 |
| －137892E－03 | 193.76 | 49.73 | ．996222E－01 |

viscRgTty
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GEASTTY

| .146383E-03 | 208.49 | 51.09 | . $911674 E-01$ |
| :---: | :---: | :---: | :---: |
| .146047E-03 | 208.56 | 51.02 | .909990E-01 |
| - $154356 \mathrm{E}-03$ | 229.27 | 49.59 | -7e2191E-01 |
| -16500nE-03 | 250.54 | 51.64 | - 726212E-01 |
| .169725E-03 | 260.56 | 51.62 | - Es2626E-01 |
| .169461E-03 | cer. 96 | 51.62 | . $691352 \mathrm{E}-01$ |
| -17U?17E-03 | 264.81 | 49.96 | -657387E-01 |
| . $1844 \mathrm{~F} 4 \mathrm{~F}=03$ | 293.75 | 52.46 | .613105E-01 |
| . $186342 \mathrm{E}-03$ | 297.83 | 51.68 | . $59484 E E-01$ |
| . 1854 のSE-03 | 297.99 | 51.13 | . $588143 \mathrm{E}-01$ |
| - 2056356-03 | 351.35 | 49.33 | . $474642 \mathrm{E}-01$ |
| .206751 Em 0 | 351.48 | 49.33 | . $474501 E-01$ |
| -285143E-03 | 400.08 | 50.35 | -422699E-01 |
| .224299E-03 | 400.18 | 50.30 | - $422866 E-01$ |
| -270184E-03 | 160.9? | 99.66 | -342321E+00 |
| -1980FAE-0? | 169.26 | 100.09 | . $297177 E+00$ |
| -183406E-03 | 186.10 | 101.90 | -239366E+00 |
| -184790E-03 | 126.32 | 102.01 | - $239246 E+00$ |
| -17573nE-03 | 203.21 | 100.05 | . $196894 E+00$ |
| -17575EE-07 | 203.34 | 100.12 | - $156815 E+00$ |
| -102844E-n3 | 2.43 .44 | 101.86 | - $150492 \mathrm{E}+00$ |
| -104718E-03 | 243.53 | 101.79 | -150313E+00 |
| . 1985A9E-0.3 | 24.6 .47 | 100.24 | -114988E+00 |
| - 197892E-03 | 2वह.4 | 100.25 | . $114998 \mathrm{C}+00$ |
| . $197978 \mathrm{C}-03$ | 296.5? | 100.31 | . $115046 E+00$ |
| -199223E-03 | 296.52 | $100 \cdot 36$ | -115106E+00 |
| - 208250E-03 | 327.62 | 100.17 | . $102351 E+00$ |
| - 219964E-03 | 359.04 | 100.51 | -927603E-01 |
| - 22027 OE-03 | 359.50 | 100.51 | .926319E-01 |
| . 234 n4PE-0.3 | 403.25 | 100.38 | . $818064 \mathrm{E}-01$ |
| . $234022 \mathrm{E}-0.3$ | 403.42 | 100.16 | .816023E-01 |
| - $254105 E-03$ | 177.19 | 143.40 | . $3670906+00$ |
| - $252584 \mathrm{E}-07$ | 177.44 | 143.57 | . $366354 \mathrm{E}+00$ |
| . $213406 \mathrm{E}-03$ | 204.28 | 144.03 | . $279651 \mathrm{E}+00$ |


| VTSCOSIGY | $\begin{aligned} & \text { TENDFGATURF } \\ & \text { (OEGK) } \end{aligned}$ | FRFSSUHE （ 4 MOS ） | $\begin{aligned} & \text { DENSITY } \\ & \text { GN/CC } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| －211128E－03 | 2144.83 | 144.19 | ． $278607 \mathrm{E}+00$ |
| －201786t－03 | 227.06 | 144．12 | － $234340 \mathrm{E}+00$ |
| －2n3415F－0？ | 262.99 | 145.75 | －191412E＋00 |
| －C）1f06F－03 | 296．17 | 147.06 | ． $165797 E+00$ |
| －211507ビ－03 | 296.25 | 147.19 | ． $165876 F+00$ |
| －C106GGE－03 | रद6． 3 ？ | 145.32 | －163872E＋00 |
| －211207上゙03 | 298．35 | 145.43 | －163976E＋00 |
| －217909E－03 | 320.99 | 144.28 | ． $148059 \mathrm{E}+00$ |
| － 21797 Pr －03 | 321.13 | 144.21 | ．147867E＋00 |
| －2ว9） 6 2ビー02 | 362.13 | 145.27 | －129994E＋00 |
| － $233944 \mathrm{E}-0$ ？ | 374.94 | 141.80 | －122393E＋00 |
| － 243977 E－03 | 403.01 | 148.59 | ．118288E＋00 |
| ．24329］E－03 | 403.07 | 148.59 | ． $118269 E+00$ |

## TABIE 7. 2 AIR DATA THIS WORK

| $\begin{aligned} & \text { VISCOSITY } \\ & \text { (POTSE) } \end{aligned}$ | TENPERATURE (DEGK) | PRESSURF <br> (ATMOS) | $\begin{aligned} & \text { DENSITY } \\ & (G N / C C) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| . 703475E-04 | 100.78 | 1.18 | . $425069 \mathrm{E}-02$ |
| .923310E-04 | 132.7? | 1.21 | - 327473E-02 |
| -110125E-03 | 162.26 | 1.22 | . 267227E-02 |
| -130479E-03 | 199.79 | 1.20 | . $213384 \mathrm{E}-02$ |
| .149331F-03 | 2.32.63 | 1.21 | .184949E-02 |
| -156096t-03 | 243.04 | 1.22 | . $177536 \mathrm{E}-02$ |
| -1) $23844 \mathrm{~F}-03$ | ? 96.71 | 1.21 | . $144445 \mathrm{E}-02$ |
| - 2nG417E-03 | 349.48 | 1.21 | -123104E-02 |
| -227428F-03 | 396.55 | 1.22 | . $108614 \mathrm{E}-02$ |
| -142398E- 3 | 154.92 | 51.44 | . $177863 \mathrm{E}+00$ |
| -145556E-03 | 183.91 | 52.52 | -120836E+00 |
| -164733E-03 | 241.81 | 51.61 | -7द182IE-01 |
| -189947E-03 | 295.70 | 53.00 | . $639999 E-01$ |
| .214843E-03 | 349.73 | 54.19 | . $543680 E-01$ |
| . 234919 E -03 | 398.13 | 52.90 | -4E2632E-01 |
| - 249399F-n3 | [00.53 | 39.73 | -356020E+00 |
| -187178E-03 | 187.39 | 101.21 | - $253913 \mathrm{E}+00$ |
| -185354E-) 3 | 241.72 | 100.88 | . $158902 E+00$ |
| - $202344 \mathrm{~F}-03$ | 295.2.7 | 101.18 | -122182F+00 |
| -2?2?40E-0? | 351.51 | 99.99 | -985478E-01 |
| . $240925 \mathrm{E}-0$ ? | 398.21 | 99.68 | .857075E-01 |
| - $232038 \mathrm{ER-03}$ | 189.38 | 137.89 | . $340654 \mathrm{E}+00$ |
| . $207530 \mathrm{~F}-03$ | 242.71 | 140.07 | . $219275 \mathrm{~F}+00$ |
| -21523EE-03 | 275.65 | 145.21 | -172964E+00 |
| -233018E-03 | 351.55 | 143.20 | -138785E+00 |
| . 24543 2 E-03 | 398.29 | 139.76 | -118092E+00 |

Chapman Enskog equation for dilute gases

$$
\begin{equation*}
n=\frac{2.6693 \times 10^{-5} \sqrt{\mu T}}{\sigma^{2} \Omega_{22}} \tag{7.1}
\end{equation*}
$$

where $M$ is the molecular weight, $T$ the absolute temperature in ${ }^{\circ} \mathrm{K}, \sigma$ the low velocity collision diameter in $\AA$ (the distance at which the intermolecular potential is zero) and $\Omega_{22}$ the collision integral calculated from a suitable intermolecular potential function.

Since nitrogen is a non-polar gas, the collision integrals based on the Lennard-Jones [6-12] potential function, which have been found to accurately describe the force interactions of simple non-polar gas molecules, were used for this present work. The Lennard-Jones [6-12] potential assumes that the law of intermolecular potential energy is of the form:

$$
\begin{equation*}
E(r)=4 \varepsilon\left\{(\sigma / r)^{12}-(\sigma / r)^{6}\right\} \tag{7.2}
\end{equation*}
$$

where the potential energy, $E(r)$, is defined as a function of the distance of separation, $r$. The distance $\varepsilon$, at which maximum energy of attraction occurs, and $\sigma$ the collision diameter, are molecular constants.

The results of this theory, however, are applicable only to dilute gases, which can be defined as those whose properties are completely determined from the mechanism of binary collisions between gas molecules.

The collision integrals $\Omega_{22}$ which were used in this work were obtained from Liley's work [29] which tabulates $\Omega_{22}$ as a
function of the reduced temperature, $\quad T *=T /(\varepsilon / K)$, where $K$ is the Boltzman constant.

Over the entire temperature range covered, i.e. 100 to $400^{\circ} \mathrm{K}$ the optimum molecular parameters were found to be: $\sigma=3.630 \AA$ and $\varepsilon / K=101.14^{\circ} \mathrm{K}$. These produced a standard deviation from the 26 dilute nitrogen data points of $0.807 \%$ and a maximum deviation of $-1.65 \%$, which is considerably higher than the expected experimental precision. However the experimental data werealso fitted over the narrower temperature range of 170 to $400^{\circ} \mathrm{K}$ and parameters of $\sigma=3.567 \AA$ and $\varepsilon / K=114.08^{\circ} \mathrm{K}$ produced a standard and maximum deviation of 0.405 and $-0.956 \%$ respectively which is within the experimental precision expected, and is in close agreement with most existing data.

Unfortunately, there appears to be no unique value for the length parameter $\sigma$ and the energy parameter $\varepsilon / K$ which will result in a perfect fit of experimental data over a wide temperature range. This may be due to the inadequacy of the molecular model or the sensitivity of this approach to the experimental accuracy. However it is more likely to be due to the former reason. The above correlations were not used any further in this work.

MODIFIED SUTHERLAND EQUATION
The Sutherland viscosity equation is based on a more primitive model of molecular interaction than the Chapman-Enskog equation, and is of the form:

$$
\begin{equation*}
n=\frac{C_{1} T_{2}}{\left(T+C_{3}\right)} \tag{7.3}
\end{equation*}
$$

The constant $C_{2}$ is equal to 1.5 in the classical Sutherland equation, however allowing $C_{2}$ to be a variable in these present correlations, improved the closeness of fit considerably.

The modified Sutherland equation was used to correlate both the nitrogen and air low pressure data. Several high order polynomials were also tried but did not produce significantly better results.

The optimum fit for the 26 low pressure nitrogen data points was obtained with the following

$$
\begin{equation*}
n=\frac{5.58114 \times 10^{-4} \times T^{1.04322}}{T+900.670} \quad \text { POISE } \tag{7.4}
\end{equation*}
$$

which results in a standard deviation of $0.51 \%$ and a maximum deviation of $\mathbf{- 0 . 9 5 \%}$. Figure (7.1) gives a plot of viscosity vs temperature for this current nitrogen data, showing the fitted curve generated using equation (7.4). The high pressure data points are represented on this graph by symbols of different shape. A deviation plot of viscosity against temperature for all the available low pressure nitrogen data is given in figure (7.2). The deviation plot shows that the results of this work are in good agreement with those of previous authors above the temperature of $220^{\circ} \mathrm{K}$, but at the lower temperature of $200^{\circ} \mathrm{K}$ the viscosity equation appear to be some 1 to $2 \%$ lower than the data of Clarke and Smith [3], Johnston and McCloskey [6], Vasilesco [2] and Trautz and Bauman [4], at the lowest temperature of $100^{\circ} \mathrm{K}$ however all the data is in good agreement except that of Johnston and McCloskey [6]


FIGURE 7.1 VISCOSITY OF NITROGEN VS TEMPERATURE


FIGURE 7.2 LOW PRESSURE NITROGEN DEVIATION PLOT:
which is some $2 \%$ higher. There appears to be some systematic error in the low temperature work of Johnston and McCloskey [6], as stated in the literature survey.

HIGH PRESSURE NITROGEN
A general correlation of the available medium and high pressure nitrogen data including data given in references [7] to [17] is presented in figure (7.3) which shows a graph of excess viscosity $\left(\eta-\eta_{0}\right)$ against the density $\rho$. The data were correlated using the following equation:

$$
\begin{equation*}
n-n_{0}=9.84255 \times 10^{-5} \rho+6.98169 \times 10^{-4} \rho 2 \text { POISE } \tag{7.5}
\end{equation*}
$$

which is based on equation (7.4) for ${ }^{n}{ }_{0}$, the dilute viscosity of nitrogen. Thus a general equation for the viscosity of nitrogen as a function of temperature and density can be written in the form:

$$
\begin{equation*}
n=\frac{5.5811 \times 10^{-4} \mathrm{~T}^{1.0432}}{\mathrm{~T}+900.67}+9.8426 \times 10^{-5} \rho+6.9817 \times 10^{-4}{ }_{\rho} 2_{\text {POISE }} \tag{7.6}
\end{equation*}
$$

A deviation plot of viscosity $\left(n-\eta_{C A L}\right) / n$, against density p, where ${ }^{n} \mathrm{CAL}$ is the viscosity calculated from equation (7.6), is presented in figure (7.4). This plot indicates that there is a maximum scatter of the overall data from the fitted equation (7.6) of $\pm 4 \%$. However the standard deviation is only 1. $2 \%$.

There do not appear to be any discernable trends in the other data compared to the present data, or any apparent systematic discrepencies. The present data being the only data that has covered the entire density range shown with a single apparatus.


FIGURE 7.3 EXCESS VISCOSITY OF NITROGEN VS DENSITY


Graphical illustrations of nitrogen density and viscosity as functions of pressure and temperature using the equation of state from reference [20] and equation (7.6) are presented in figures (7.5) and (7.6) respectively. Also equation (7.6) was used to produce table (7.3), nitrogen viscosity as.a function of pressure and temperature.

LOW PRESSURE AIR
The modified Sutherland equation was the only correlating equation used for the air data presented in this work. The optimum fit to the 9 low pressure air data points was produced using the equation:

$$
\begin{equation*}
n=\frac{3.84475 \times 10^{-4} \mathrm{~T} 1.08224}{T+700.795} \quad \text { POISE } \tag{7.7}
\end{equation*}
$$

which has a standard deviation of $0.57 \%$ and a maximum deviation of $1.0 \%$ from the data. Figures (7.7) and (7.8) show respectively plots of viscosity against temperature for this work, with the low pressure curve drawn using correlating equation (7.7), and a deviation plot of all the available low pressure air data.

The deviation plot shows reasonably good agreement between this work and that of previous authors. However, as in the case of nitrogen, the work of Johnston and McCloskey [6] appears to be some 1 to $2 \%$ high in the lower temperature region. The only other data available at the lowest temperature was that of Sutherland and Mas [5] which is some 1 to $1.5 \%$ lower than the correlating equation.



FIGURE 7.6 THE VISCOSITY OF NITROGEN


figure 7.8 low pressure air deviation plot

HIGH PRESSURE AIR
A procedure similar to that for nitrogen was carried out for high pressure air, with all the data from references [7], [11], [13], [18] and [19] as well as the data from this work being plotted in the form of excess viscosity vs density. The correlating equation used for all the dense air data was:

$$
\begin{equation*}
n-n_{0}=1.02055 \times 10^{-4} \rho+5.96947 \times 10^{-4} \rho^{2} \text { POISE } \tag{7.8}
\end{equation*}
$$

Again a general equation for the viscosity of air as a function of temperature and density can be produced by the summation of equations (7.7) and (7.8) and is of the form:

$$
\begin{equation*}
n=\frac{3.84475 \times 10^{-4} \mathrm{~T} 1.08224}{\mathrm{~T}+700.795}+1.02055 \times 10^{-4} \rho+5.96947 \times 10^{-4}{ }_{\rho}^{2} \text { POISE } \tag{7.9}
\end{equation*}
$$

Figures (7.9) and (7.10) show graphical representations of excess viscosity vs density, with equation (7.8) used to draw the curve, and the deviation plot of $n-n_{C A L} / n$ against density. The deviation plot shows a standard deviation of $1.3 \%$ for equation (7.9) from the data, and an overall scatter of +4 and $-2 \%$.

The' data of Kestin and Whitelaw [13] appears to show a systematic discrepancy from the earlier work of Kestin et al. and from other more recent authors. The work of Lo, Carol and Stiel [19] correlates excess viscosity as a function of thermal pressure, $\partial P / \partial T$ as well as providing a tabulation of viscosity as a function of pressure and temperature. It was their tabulated values of viscosity rather than any real data points that were used in this present correlation, this was found to be necessary because of the shortage of high density air viscosity data.


FIGURE 7.9 EXCESS VISCOSITY OF AIR VS DENSITY


As in the case of nitrogen, graphical representation of air density and viscosity as functions of temperature and pressure are presented in figures (7.11) and (7.12) respectively, and computed values of air viscosity using equation (7.9) are given in table (7.4).

RECOMMENDATIONS:
If the present apparatus is to be used for further work on the measurement of the viscosity of gases, it is suggested that a thermostated temperature control system be installed. This would allow greater thermal stability to be achieved and would also facilitate the operation of the apparatus.
aIR DENSITY....VASSERMAN ET AL.


FIGURE 7.11 THE DENSITY OF AIR


TABLE 7.3 THE VISCOSITY OF NITROCEN

## TENPERATURE <br> (DEGK)

PRESSURE
(ATMOS)

5


TABLE 7.3.CONT'D

TEMPERATURE
(DEG K)

PRESSURE
(ATMOS)
30

35
40
45


TABLE 7.3 CONT ${ }^{\text {' }}$ D

TEMPERATURE
(DEG K)
PRESSURE
(ATMCS)

50
55
6]
65
70


TABLE 7.3 CONT' ${ }^{\text {D }}$

TEMPERATURE
(DEG K)

## PRESSURE <br> (ATMOS)

75
80
85
90
95


TEMPERATURE
（DEG K）

PRESSURE
（ATMOS）
105110
11.5

120

| $\begin{aligned} & 150 \\ & 168 \\ & 178 \\ & 180 \\ & 190 \\ & 200 \\ & 210 \end{aligned}$ | $\cdot$ $-2559 E-03$ -2 $-1996 E-03$ $-19615 E-03$ $-1793 E-03 ~$ $-1761 E-03 ~$ $-1753 E-03$ | $2754 E-03$ $\cdot 2338 E-03$ $-2074 E-03$ $-1924 E-03$ $-1343 E-03$ $-1835 E-03$ $-1788 E-03$ | $.2840 \mathrm{E}-03$ $.24<7 E-03$ $.2151 \mathrm{E}-03$ $.198 \mathrm{E}-03$ $.1895 E-03$ $.1846 E-03$ $.1824 E-03$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2？0 | ． $1759 \mathrm{E}=03$ | －1789E－03 | －1820E－03 | $\cdots 185 c \mathrm{E}-03$ | －18y5 $5-03$ |  |
| 2こ0 | －1774E－C3 | －18002－03 | －1828E－03 | －1856E－03 | －1885E－03 |  |
| 240 | －1795E－03 | －1819E－03 | $.1843 \mathrm{E}-03$ | －1868E－03 | ．1894E－U3 |  |
| 250 | －1921E－C3 | －1843E－03 | －1865E－03 | －1887E－03 |  |  |
| 2 EF | －1850E－03 | －1870E－03 | ． $1890 \mathrm{E}-03$ | －1910E－03 | ．1931E－33 |  |
| 270 | －1982E－ 13 | －1900E－03 | $.1918 \mathrm{E}-03$ | －1937E－03 | －1956E－J3 |  |
| 281 | －1315E－03 | －1932E－03 | $.1948 \mathrm{E}-03$ | －1966E－03 | $.1983 E-33$ |  |
| 290 | －1450E－ 3 | ． 1965 － 193 | ．1981E－03 | ． 1997 E－03 | －2013E－03 |  |
| 3 C |  | $\cdots 1999 \mathrm{E}-\mathrm{U}^{-3}$ | － $2014 \mathrm{E}-03$ | －2029E－ $0^{3}$ | － $2 \mathrm{~T} 44 \mathrm{E}=\mathrm{U}$ |  |
| 310 | －2］21E－03 | － $2034 \mathrm{E}-03$ | $.2048 \mathrm{E}-03$ | －2062E－03 | － $2076 E-03$ |  |
| 320 | － $2058 \mathrm{E}-03$ | －2070E－03 | ． $2083 E-03$ | －2496E－03 | －21u9E－03 |  |
| 3 C | － $2794 \mathrm{E}-13$ | －2106E－03 | － $2118 \mathrm{E}-03$ | －2131E－03 | － $2143 E-03$ |  |
| 340 | －2131E－03 | －2142E－03 | －2154E－03 | － $2166 \mathrm{E}-03$ | －2177E－03 |  |
| 350 | － $21685-03$ | －2179E－03 | $.2190 E-03$ | －22U1E－03 | －2212E－03 |  |
| 36 C | －2205E－03 | － 2215 －03 | ． $2226 \mathrm{E}-03$ | － 223 E E－03 | －2247E－33 |  |
| 370 | － $2242 \mathrm{E}-03$ | － 2 二52E－03 | － $2262 \mathrm{E}-03$ | － 227 CE－ 63 | － 228 CE－33 |  |
| 380 | － $2279 \mathrm{E}-03$ | － $2283 \mathrm{E}-03$ | － $2297 \mathrm{E}-13$ |  | －2317 E－${ }^{\text {－}}$ |  |
| 391 | －2315E－03 | － $2324 \mathrm{E}-03$ | － $2333 \mathrm{E}-03$ | －2342E－03 | － $2352 \mathrm{E}-03$ |  |
| 400 | －2352E－03 | － $2360 \mathrm{E}-03$ | － 2369 E－03 | －2378E－03 | －2387E－03 |  |
| 410 | －2388E－ 23 | － $2396 \mathrm{E}-03$ | $.2404 \mathrm{E}-03$ | －241JE－03 | －2421E－03 |  |
| 420 | － $2+24 E-03$ | －2432E－03 | －244JE－03 | － 2448 E－03 | － 245 EE－ 33 |  |
| 430 | － $2+59 E-03$ | － 2467 － 33 | ． $2475 \mathrm{E}-03$ | － $2482 \mathrm{E}-03$ | ． 2490 E－03 |  |
| 440 | － $2+95 \mathrm{E}-03$ | － 25 O2E－03 | .25 ［9E－03 | ． $2517 \mathrm{E}-03$ | ． 2525 㖪－3 |  |
| 450 | －2530E－03 | －2537E－03 | $.2544 \mathrm{E}-03$ | ．2551E－03 | ． $2559 \mathrm{E}-33$ |  |

TABLE 7.3 CONT ${ }^{1} \mathrm{I}$


TABLE 7.3 CONT'D


TABLE 7.4 THE VISCOSITY OF AIR

## (POISE)

TFMFFRATURE
(DEGK)

PRESSURF
(ATMOS)
5
$5 \quad 10$
15
20


TABLE 7.4 CONT'D

TFMPFRATURE
(DEGK)

## RRFSSUAF <br> ( $\triangle T M O S$ )

30


TABIE 7.4 CONT ${ }^{\prime} D$

PFFESURF
(ATMOS)
$55 \quad 60$


| . $2668 \mathrm{~F}-03$ | . 28 |
| :---: | :---: |
| -1742F-03 | 9 |
| -1534F-02 |  |
| . $1487 \mathrm{~F}-02$ | - |
| -1486F-03 |  |
| -1504F-0 ${ }^{\text {- }}$ | -1 |
| -1533F-03 |  |
| -1567E-03 | 15 |
| - 16055-03 | -16 |
| . $1645 \mathrm{~F}-0 \frac{1}{2}$ |  |
| -1597F-03 | - |
| -1729F-03 | - |
| -1773F-03 |  |
| .] 1 IGF-03 | 1 |
| . 1959F-03 | -1 |
| $.19025-0$ | . 19 |
| -1945F-0 | - |
| .190¢F-03 |  |
| - $2031 \mathrm{F-03}$ | - |
| - 2073F-03 | - |
| .2115F-02 |  |
| $156 F-03$ |  |
| . $2197 F-13$ | 2 |
| 237E-03 |  |
| 277F-03 |  |
| - ? $317 \mathrm{~F}-0$ こ |  |
| - $3356 \mathrm{~F}-03$ |  |
| - 3395 - 03 |  |
| - $24335-0 \frac{1}{}$ |  |
| - $74715-03$ |  |
| - 2504 F - 0 |  |
| - 2544 C - |  |
| - |  |


| .2854E-03 |
| :---: |
| :1845E-83 |
| -1547E=03 |
| -15406-03 |
| -1593E-03 |
| :1692E-03 |
| -1705E-03 |
| -1746E033 |
| -1930E-03 |
| -187EE03 |
| -1957E-03 |
| -1999E03 |
| - 20 28-03 |
| - 2165 -03 |
| - 2 20E03 |
| - 2224 |
| -2324E=03 |
| -2363F-03 |
| -2439E-03 |
| :2471E-O3 |
|  |
| -.2550E-03 |



TEMFFRATURF
(CEGK)

PRESTURF
(ATMOS)
$75-10$
85
90
95


TABLE 7.4 CONT'D


TABLE 7.4 CONT ${ }^{\text {D }}$

| TFMFFRATURE （CEGK） | $\begin{gathered} \text { PRESSURF } \\ \text { (ATMOS) } \end{gathered}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 150 | 155 | 160 | 165 | 170 |
| 170 | －2RROF－03 | ．2941F－03 | ． $2907 \mathrm{F-03}$ | ． 3051 E－03 |  |
| 180 | － $2617 \mathrm{~F}-03$ | － $2675 F-03$ | － $2732 \mathrm{~F}-03$ | － 2707 －03 | $\text { - } 24 \angle 0 F-03$ |
| 190 | .24 25F－ 03 | － 247 CF－03 | ． $2532 \mathrm{~F}-0{ }^{\text {a }}$ | ．2594E－03 | $2635 E-03$ |
| 200 | － $2296 \mathrm{~F}-03$ | － $23445-03$ | － $2392 \mathrm{~F}-03$ | －243cE－03 | － $24855-03$ |
| 210 | － $2213 \mathrm{~F}-03$ | － $22556-03$ | －？2976－03 | － $2339 E-03$ | － $2381 E-03$ |
| 220 | －？16フF－03 | － $2195 \mathrm{~F}-03$ | －2236F－03 | －2273E－03 | －2310E－03 |
| 230 240 | －2） $\begin{aligned} & 34 \mathrm{~F}-03 \\ & 21 F-n 3\end{aligned}$ | － $216 E E-n 3$ | － $1996-03$ | － $2232 E-03$ | $-22665-03$ $22395-03$ |
| 250 | －2 $12 n \mathrm{~F}-03$ | － 214 te－03 | － $2173 \mathrm{~F}-0 \mathrm{O}$ | －2 $290 \mathrm{C}=03$ | －2276F－03 |
| 260 | － 2 127F－03 | $\because 151 F-03$ | －2175F－03 | － $21995-03$ | － $2224 \mathrm{~F}-03$ |
| 270 | －2141，F－03 | －216\％F－03 |  | －22ncr－03 | － $2229 F-03$ |
| 290 | － 215 RF－03 | － 178 － 03 | － $2198 \mathrm{~F}-03$ | －2219E－03 | －224 CF－03 |
| 290 | －2180F－03 | － $198 E=03$ | －2217F－13 | －22360－03 | － $2255 F-03$ |
| 300 | － 22 NAF－03 | － $22 \mathrm{CLE}-03$ | －22 248003 | － $22566-03$ | － $22745-03$ |
| 310 320 |  | － 224 ¢F－ 23 | － 22 2arF－n3 | － $22796-03$ | －？ $245 F-03$ |
| 330 | － $22874-03$ | －a 3nz－o3 | － $2310 \mathrm{~F}-03$ | －2330E－03 | － $2345 \mathrm{~F}-03$ |
| 340 | －2319F－ก3 | －2371F－03 | ． $2344 \mathrm{~F}-03$ | －2358E－03 | － 2372 －03 |
| 350 | － $2344 \mathrm{~F}-03$ | －2361F－03 | ． $2374 \mathrm{~F}-03$ | － 238 － 2003 | － $2400 \mathrm{~F}-03$ |
| 360 | － 23 sinf－ 113 | ． 23 92F－03 | $.2404 \mathrm{~F}-03$ | $.2416 E-03$ | －2429F－03 |
| 370 | － $2412 \mathrm{~F}-03$ | －2423F－03 | $.2435 \mathrm{~F}-03$ | － 244 EE－03 | －2450F－03 |
| 380 | －2444F－03 | － $2455 F-03$ | － $2466 \mathrm{~F}-03$ | － $2477 \mathrm{E}-03$ | － 24 HRF－03 |
| 390 | －24766－n3 | － $2447 F-03$ | － $24975-03$ | － $2598 \mathrm{C}-03$ | －251PF－03 |
| 410 | － $2541 F-03$ | －25以1F－03 | －${ }^{\text {a } 560 F-03}$ | －2570E－03 | － 25 明F－03 |
| 420 | －2574F－03 |  | ． 2502 F － 03 | －26rat－03 | －2611F－03 |
| 430 | －2600F－03 | － $615 \mathrm{~F}-03$ |  | －2632F－03 | －2642F－03 |
| 440 | － $2639 \mathrm{~F}-13$ | － $2647 \mathrm{~F}-03$ | －26564－03 | － 26 Ğ4E－03 | －2613F－0．3 |
| 450 | $.2671 F-03$ | $.2675 \mathrm{~F}-03$ |  | $.2695 E-03$ | $.2704 \mathrm{~F}-03$ |

TABLE 7.4 CONT'D

TFMFFRATURE
(DFGK)

PREGGURF (ATMOS)
$125-130$
135
140
145


## CHAPTER 8

## CONCLUSIONS

1. A series capillary viscometer was used to measure the viscosity of nitrogen and air over the temperature of 100 to $400^{\circ} \mathrm{K}$ and up to a pressure of 150 atmospheres.
2. The results have been compared with results of previous workers and appear to agree quite favorably. An equation of the form:

$$
n=\frac{C_{1} T_{2}}{T+C_{3}}+C_{4} \rho+C_{5} \rho^{2}
$$

has been fitted to the two sets of data and is believed accurate to $\pm 1.5 \%$ at the most unfavorable conditions of high density.
3. The dilute nitrogen data have also been correlated using the Chapman - Enskog Collision theory, yielding intermolecular force constants.
4. It is apparent that the series capillary viscometer is a excellent primary instrument and is capable of producing accurate data over a broad range of pressures and temperatures.

## APPENDIX A1

## SAMPLE CALCULATIONS

An air viscosity reading taken at the test temperature of $T=162.26^{\circ} \mathrm{K}$ and at a pressure of 1.22 atmospheres was chosen arbitrarily to illustrate a typical calculation. Commencing with the flow equation,

$$
\begin{equation*}
n=\frac{\pi D_{1}{ }^{4} g(1+3 \alpha \Delta T)}{128 Q\left(L_{1}-L_{2}\right)}\left\{\Delta P_{1} \rho_{1}-\left(\frac{D_{2}}{D_{1}}\right)^{4} \Delta P_{2} \rho_{2}\right\} \tag{A1.1}
\end{equation*}
$$

the readings of the variables used were as follows:
THE CAPILLARY DIMENSIONS

$$
\begin{aligned}
& \mathrm{L}_{1}=53.162 \mathrm{~cm} \\
& \mathrm{~L}_{2}=5.161 \mathrm{~cm} \\
& \mathrm{D}_{1}=0.0500312 \mathrm{~cm} \\
& \mathrm{D}_{2}=0.0501366 \mathrm{~cm}
\end{aligned}
$$

THE ACCELERATION DUE TO GRAVITY
The acceleration due to gravity 'g' in the Hamilton area is $980.3941 \mathrm{~cm} / \mathrm{sec}^{2}$.

THERMAL EXPANSION CORRECTION
The coefficient of thermal expansion ' $\alpha$ ' of glass is $3.3 \times 10^{-6}$, while the temperature at which the glass capillaries were calibrated was $297.15^{\circ} \mathrm{K}$.

TEMPERATURE READINGS
Thermocouple readings, $T 4$ and $T 5$, were taken inside the viscometer cell, $T 7$ and $T 8$, in the volumeter and $T 9$ at the manometer. These were read twice during each experimental run, before and after the pressure readings, manually corrected and tabulated as follows:

TABLE (A1.1) TEMPERATURES DURING SAMPLE RUN

Thermocouple

| Thermocouple <br> number | 4 | 5 | 7 | 8 | 9 |
| :--- | ---: | :---: | :---: | :---: | :---: |
| EMF Before | -3.648 | -3.645 | 1.011 | 1.019 | 1.028 |
| EMF After | -3.652 | -3.639 | 1.012 | 1.019 | 1.029 |
| Average | -3.650 | -3.642 | 1.012 | 1.019 | 1.029 |
| Correction | +.002 | +.010 | +.006 | .005 |  |
| Correctod <br> EMF | -3.648 | -3.632 | 1.018 | 1.024 | 1.029 |
| Temperature <br> $\left({ }^{\circ} \mathrm{K}\right)$ | 162.26 | 162.85 | 298.92 | 299.12 |  |

The temperature at $T 4=162.26^{\circ} \mathrm{K}$ was used as the reference temperature for this run.

ATMOSPHERIC AND PLENUM PRESSURE READINGS
During this run the atmospheric pressure, automatically corrected for non-standard condition was 29.90 in Hg by converting to centimeters and using $13.5955 \mathrm{gm} / \mathrm{cm}^{3}$ as the density of mercury at $0^{\circ} \mathrm{C}$, the atmospheric pressure was found to be:

$$
\begin{aligned}
\mathbf{P}_{\mathbf{A}} & =29.90 \times 2.54 \times 13.5955 \\
& =1032.52 \mathrm{gmf} / \mathrm{cm}^{2}
\end{aligned}
$$

The pressure head between the viscometer plenum and ambient conditions was 17.60 cm Hg at $20^{\circ} \mathrm{C}$. The density of mercury at this temperature is $13.545 \mathrm{gm} / \mathrm{cm}^{3}$. Thus:

$$
\begin{aligned}
\Delta \mathrm{P}_{\mathrm{ENT}} & =17.60 \times 13.545 \\
& =238.39 \mathrm{gmf} / \mathrm{cm}^{2}
\end{aligned}
$$

Summing the atmospheric and the plenum gauge pressures the total plenum pressure can be determined as:

$$
\begin{aligned}
\mathrm{P}_{\mathrm{PLEN}} & =1032.53+238.39 \\
& =1270.91 \mathrm{gmf} / \mathrm{cm}^{2}
\end{aligned}
$$

## AIR DENSITY

The air densities were determined using an equation of state given in reference [21] and of the form $\rho=P / Z R T$ where $Z$, the compressibility factor, was a function of $\rho$ and $T$. Although this required an iterative solution, a unique value of density could be determined with a knowledge of the pressure and temperature. For a further description of the equation of state and of the method of computation the reader is referred to appendix A2.

MANOMETER AIR DENSITY
A manometer gas density of $\rho_{\mathrm{GAS}}=0.14263 \times 10^{-2} \mathrm{gm} / \mathrm{cm}^{3}$ was computed using $299.12^{\circ} \mathrm{K}$ and $1270.91 \mathrm{gmf} / \mathrm{cm}^{2}$ as the state variables.
differential pressure readings
The pressure drops across the capillaries were calculated from the individual manometer oil level measurements, that is

$$
\begin{aligned}
\Delta \mathrm{h}_{1} & =96.305-74.071 \\
& =22.234 \mathrm{~cm} \text { oil } \\
\Delta \mathrm{h}_{2} & =98.667-96.305 \\
& =2.362 \mathrm{~cm} \text { oil }
\end{aligned}
$$

The specific weight of the manometer oil was determined using the density of water of $0.9970 \mathrm{gmf} / \mathrm{cm}^{3}$ at the manometer temperature of $26^{\circ} \mathrm{C}$, and the specific gravity of .821 , for the manometer oil: thus:

$$
\begin{aligned}
{ }^{\rho_{\text {OIL }}} & =.9970 \times .821 \\
& =.8185
\end{aligned}
$$

The differential pressures, across capillaries one and two can now be calculated by means of the following equations:

$$
\begin{aligned}
A P_{1} & =\Delta h_{1} \times\left(\rho_{\mathrm{OLL}}-\rho_{\mathrm{GAS}}\right) \\
& =22.234 \times(.8185-.00142) \\
& =18.171 \mathrm{gmf} / \mathrm{cm}^{2} \\
\Delta P_{2} & =2.362 \times(.8185-.00142) \\
& =1.930 \mathrm{gmf} / \mathrm{cm}^{2}
\end{aligned}
$$

CAPILLARY GAS DENSITIES
In order to calculate the mean density of the gas in the two capillaries it is necessary to know the average pressure. ' $P_{A V}$ ' in each of the capillaries. Therefore:

$$
\begin{aligned}
P_{\text {AV1 }} & =P_{\text {PLEN }}-\Delta P_{1} / 2.0 \\
& =1270.91-18.171 / 2.0 \\
& =1261.83 \mathrm{gmf} / \mathrm{cm}^{2}
\end{aligned}
$$

$$
\begin{aligned}
P_{\text {AV } 2} & =P_{\text {PLEN }}-\Delta P_{1}-\Delta P_{2} / 2.0 \\
& =1270.91-18.171-1.930 / 20 \\
& =1251.78 \mathrm{gmf} / \mathrm{cm}^{2}
\end{aligned}
$$

Using the equation of state previously mentioned and T4 and $T 5$ as the temperatures in capillaries 1 and 2 respectively, the gas densities were computed as follows:

$$
\begin{aligned}
& \rho_{1}=0.267227 \times 10^{-2} \mathrm{gm} / \mathrm{cm}^{3} \\
& \rho_{2}=0.264103 \times 10^{-2} \mathrm{gm} / \mathrm{cm}^{3}
\end{aligned}
$$

THE MASS FLOW RATE
A pressure head of 5.0 cm of oil was measured between the volumeter and atmospheric conditions. Knowing the density of water and specific gravity of oil to be $0.9981 \mathrm{gm} / \mathrm{cm}^{3}$ and 0.821 respectively, as well as the ambient atmospheric pressure, the total volumeter pressure was determined to be:

$$
\begin{aligned}
\mathrm{P}_{\mathrm{VOL}} & =(5.0 \times .9981 \times .821)+1032.52 \\
& =1036.62 \mathrm{gmf} / \mathrm{cm}^{2}
\end{aligned}
$$

A gas density of $0.118495 \times 10^{-2} \mathrm{gm} / \mathrm{cm}^{3}$ was calculated using $298.92^{\circ} \mathrm{K}$ and the previously calculated pressure as the state variables.

Since the volume displaced was previously calibrated as $801.924 \mathrm{~cm}^{3}$ and the time taken for the volumeter piston to traverse this volume was measured at 768.56 sec . Then the mass flow rate could be determined as follows:

$$
\begin{aligned}
Q & =V \times \rho_{V O L} / \tau \\
& =801.924 \times .118495 \times 10^{-2} / 768.56 \\
& =0.123639 \times 10^{-2} \mathrm{gm} / \mathrm{sec}
\end{aligned}
$$

## ABSOLUTE VISCOSITY

The absolute viscosity can now be calculated from working equation (3.15) as:

$$
\begin{aligned}
\eta= & \frac{\pi \times .0500312^{4} \times 980.3941\left(1+3 \times 3.3 \times 10^{-6} \times(162.26-297.15)\right)}{128 \times .123639 \times 10^{-2}(53.162-5.161)} \\
& \left\{18.171 \times 0.267227 \times 10^{-2} \times \frac{(0.501366)^{4}}{(0.500312)} \times\right. \\
& \left.1.930 \times 0.264103 \times 10^{-2}\right\} \\
= & 0.110125 \times 10^{-3} \mathrm{gm} / \mathrm{cm} \sec
\end{aligned}
$$

REYNOLDS NUMBER
The mean Reynolds number may be estimated from:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{e}}=\frac{4 \mathrm{Q}}{\pi \mathrm{D}_{1}{ }^{\eta}} \\
& \mathrm{R}_{\mathrm{e}}=\frac{4 \times .123639 \times 10^{-2}}{\pi \times 0.0500312 \times 0.110125 \times 10^{-3}}=285.7
\end{aligned}
$$

This is well below the critical value for laminar flow in these capillaries.

DEVELOPMENT LENGTH
The development length for laminar flow was calculated from:

$$
\begin{aligned}
X_{1 \%} & =0.065 \times \mathrm{R}_{\mathrm{e}} \times \mathrm{D} \\
& =0.065 \times 285.70 \times 0.05 \\
& =0.929 \mathrm{~cm}
\end{aligned}
$$

A high speed CDC 6400 digital computer was used to perform these calculations. A listing of the programme along with the output for this sample run are attached to this section.





## IF(PIN.LT.20.)PM=PIN*13.545*PATM



MEAN GAS DENSITY IN MONOMETER (GM/CC)
$P A N=P M / 1033.2$
CALL DENSITG,PAM,RMON)
OIL COMPRESSIRILITY CORRECTION (GM/GM/ATM)
COMS $=0.6108 \mathrm{~F}-4$
PRESSURE LOSS ACROSS CAPILIARIES 1 AND 2 (GM/SQ CM)
SPGOIL= = . R ح 10 (1.+PAM\#COMS)
RWAT $=0.998203-.00021 *(T 9-293.15)$
DRENS ${ }^{\text {SPGOIIARWAT-HMON }}$

C MEAN CAPILIARY PRESSURES 1 AND 2 (GM/SQ CM)
PAVI=PM-DP1/2.0
PAVZ $=P M-D P 1-D P 2 / 2.0$
MFAN CHAMARE TEMPERATURE (DEG $K$ )
$T A V=T 4$
C. GAS DENSITY CAPS IAND 2 (GM/CC) R1,R2

PAI=PAV1/1033.2
PAD=PAV
$P A Z=P A V Z 1033.2$
$C A L E D E A S(T 4 \cdot D A 1, R 1)$
CALL DENS (T5,PAZ,R2)


PVOL $=P A T M+D P B * 0 ; 81944$
PPVV
PPVV=OVOL/1n33.2
210
C MASS FLOW RATE (GN/SEC) Q
C
F17val 00 1)
215

DELT=TAV-297.15




## APPENDIX A2

## GAS DENSITY COMPUTATION

The density of the gases were calculated using a computer subroutine of the form $\operatorname{DENS}(\mathrm{T}, \mathrm{P}, \rho$ ) where temperature, $T$, and pressure, $P$, are input arguments and the density, $\rho$, is an output argument.

NITROGEN DENSITY
In the case of nitrogen gas an equation of state obtained from reference [20] was used, that is:

$$
\begin{align*}
P & =\rho R T+\left(n_{1} T+n_{2}+n_{3} \cdot T^{2}+n_{4} / T^{4}\right) \rho^{2} \\
& +\left(n_{5} T^{2}+n_{6} T+n_{7}+n_{8} / T+n_{9} / T^{2}\right) \rho^{2} \\
& +\left(n_{10} T+n_{11}\right) \rho^{4}+\left(n_{12}+n_{13} / T\right) \rho^{5} \\
& +\left(n_{14} / T^{2}+n_{15} / T^{3}+n_{16} / T^{4}\right) \rho^{3} \operatorname{EXP}\left(n_{23} \rho^{2}\right) \\
& +\left(n_{17} / T^{2}+n_{18} / T^{3}+n_{19} / T^{4}\right) \rho^{5} \operatorname{EXP}\left(n_{23} \rho^{2}\right) \\
& +\left(n_{20} / T^{2}+n_{21} / T^{3}+n_{22} / T^{4}\right) \rho^{7} \operatorname{EXP}\left(n_{23} \rho^{2}\right) \tag{A2.1}
\end{align*}
$$

The subroutine started with an initial estimate of density in the form $\rho=P / R T$, then calculated a value of $P$ using equation (A2.1). It then corrected the initial value of density using $\rho_{i}=\rho_{i-1} \times \mathrm{P}_{\text {ACTUAL }} / \mathrm{P}_{\text {CALC }}$. This iterative procedure continued until convergence, defined by $\rho_{i}{ }^{-\rho}{ }_{i-1} \rightarrow 0$, was achieved.

The programme is listed at the end of this section.

## AIR DENSITY

The density of air was calculated in a similar manner, to that of nitrogen, using the equation of state obtained form reference [21] that was of the form:

$$
\begin{aligned}
& \rho=P / Z R T \\
& Z=a_{0} / T^{*}+\alpha_{1}+B / T^{*}+\gamma / T^{*}
\end{aligned}
$$

where

$$
\begin{align*}
& \alpha_{0}=n_{1} \omega+n_{2} \omega^{2}+n_{3} \omega^{3}+n_{4} \omega^{4}+n_{5} \omega^{5}+n_{6} \omega^{8} \\
& \alpha_{1}=1+n_{7} \omega+n_{8} \omega^{2}+n_{9} \omega^{3}+n_{10} \omega^{4}+n_{11} \omega^{6}+n_{12} \omega^{8} \\
& B_{2}=n_{13} \omega+n_{14} \omega^{2}+n_{15^{\omega}}{ }^{3}+n_{16^{\omega}}{ }^{4}+n_{17^{\omega}}{ }^{6}+n_{18} \omega^{8} \\
& Y=n_{19} \omega+n_{20^{\omega^{2}}}+n_{21} \omega^{3}+n_{22} \omega^{4}+n_{23^{\omega}}{ }^{6}+n_{24} \omega^{8} \tag{A2.2}
\end{align*}
$$

In the above equation $T$ is the reduced temperature, i.e. $T / T_{C R I T}$, and $\omega$ the reduced density, i.e. $\rho / \rho_{\text {CRIT }}$. The critical constants are given as $T_{C R I T}=132.55^{\circ} \mathrm{K}$ and $\rho_{\mathrm{CRIT}}=0.3128617464 \mathrm{gm} / \mathrm{cm}^{3}$. The ideal gas density was first calculated from the equation $\rho_{I D L}=P / R T$, this value of density was then used to compute a value of $Z$ from equation (A2.2) after which a new value of density was computed using $\rho=\rho_{I D L} / Z$. Each new value of density was then used to calculate a new value of $Z$ and the
iterative procedure continued until convergence occurred.
A listing of this subroutine is also presented at the end of this section.

```
    SURROUTINE DENS(T,P,ROI)
    x = =3:73762915RF-3
    x =-6.73'1 20906F
    x4=1.377nn68\capnE7
    x: =6:6&5763765F-8
    xG==-4,380719747F-5
    x = =% 457392454F-7
    * (4=-5.7063179n?
    X'7=4+31844145F2
    x10=5.934n89131F-5
    x 1]=-4,175n3^779F-3
    X12=-1 3 36RG5\capn9F=4
```



```
    x 17=-2,295834478F-3
```



```
    X|G=-5.63129つ4nत下7
    xl7=1.032002381E1
    x 1R=-2,7172775R8E
```



```
    x}\geqslant0=-3.343751654F-2.
    X?1=A.71045858%
    x 2?=-4.505295410-?
    xว3=-n nn5t
    F11=0.08アO5615593
    A=0
    F=P/(RU*T)
50 A=N+1
    FN=R*RU4T+(X1*T+XP+X3/T**? + X 4/T**4)*R*&2
```



```
    1+(x (2+x) 3/T) #0**5
```



```
    I+((x`n/T**2+x\1/T**3+x22/T**4)*R**7)*(EXP(X23*R**2))
    COLI=R
    Q=K&P/PN
```



```
    YY=ARS({R-RO(D)/R)
    IF(YY:LT:O.0nOl)GN_IO 99
    IF(A.Gİino)GO TO g%
99 GFTIJFN
FN(I
```



## APPENDIX A3

VISCOSITY DATA OF
OTHER AUTHORS

## VOGEL (1914)

$V T S C O S I T Y$
(POISE)
-56010E-04
$.167505-13$
81.600
273.150
VASILESCO (1945)
VISCOSITY
(FOISE)
$.61550 E-04$
$.12630:-03$
$.10637 E-03$

CLARK AMO SIITH (1968)


## TRAUTZ AND BAUMAHN (1929)

| $\cdots \underset{(P O I S E)}{\forall T S C O S I T}$ | $\begin{gathered} \text { TEYPEPATURE } \\ \text { (DEGK) } \end{gathered}$ |
| :---: | :---: |
| -12730E-03 | 195.650 |
| . 12750E-03 | 196.850 |
| -14640 -13 | -232.759 |
| -14650こ-03 | 235.250 |
| -17230E-03 | 289.250 |
| -17350E-03 | 290.950 |
| -18810E-03 | $-324.280$ |
| -208'0E-03 | 373.230 |

JOHNSTON ANO MC CLOSKEY (1946)


## MICHELS AND GIBSON (1931)



| VISCISITY <br> $($ POISE) | TEMPERATURE <br> $(O E G K)$ | PRESSURE <br> $(A T M O S)$ | DENSITY <br> $(G M / C C)$ |
| :--- | :---: | :---: | :---: |
| $.20830 E-03$ | 348.150 | 57.610 | $.55846 E-01$ |
| $.21750 E-03$ | 348.150 | $104.5-70$ | $.99584 E-01$ |
| $.24030 E-03$ | 348.150 | 212.430 | $.19089 E+00$ |
| $.27450 E-03$ | 348.150 | 320.370 | $.26692 E+00$ |
| $.30730 E-03$ | 348.150 | 430.230 | $.32992 E+00$ |
| $.34230 E-03$ | 348.150 | 541.730 | $.38222 E+00$ |

## ROSS AND BROWN (1957)

| $\begin{gathered} \text { VISCJSITY } \\ \text { (POISE) } \end{gathered}$ | $\begin{aligned} & \text { TEMPERATURE } \\ & \text { (OEGK) } \end{aligned}$ | PRESSYRE <br> (ATMOE) | DEASSTY <br> (Gi1/CC) |
| :---: | :---: | :---: | :---: |
| -185J0E-03 | 298.150 | 35.010 | . $40256 \mathrm{E}-01$ |
| .18670E-03 | 298.150 | 46.970 | . 54003E-01 |
| -19030E-03 | $-298.150$ | -69.0.30 | -792035-01 |
| . 20030E-03 | 298.150 | 103.040 | . $117315+00$ |
| -20850E-03 | 298.150 | 137.050 | . $15400 \mathrm{C}+00$ |
| -23230E-03 | 298.150 | 205.080 | . $22134 E+00$ |
| - 25640 E- 03 | 298.150 | 273.1 | $.27934 E+00$ |
| -286JGE-03 | 298.150 | 341.140 | . $32850 \mathrm{E}+00$ |
| . 31030E-03 | 298.150 | 409.150 | . $37029 E+00$ |
| -17270E-03 | 273.150 | 35.010 | . 44307 - 01 |
| -182-95-03 | 273.150 | -69.030 | -87755E-01 |
| . 19010E-03 | 273.150 | 103.040 | . $13060 E+00$ |
| - 20350E-03 | 273.150 | 137.050 | . $17186 \mathrm{E}+00$ |
| . 23080e-03 | 273.150 | 205.090 | . $24657 E+00$ |
| -26040E-03 | -273.150 | 273.170 | . $368995+00$ |
| - 29230E-03 | 273.150 | $341.1^{1}+0$ | . $36041 E+00$ |
| -16190こ-03 | 248.050 | 35.010 | . $49403 \mathrm{E}-01$ |
| -17330E-03 | 248.050 | 69.030 | . $98889 \mathrm{E}-01$ |
| -18180E-03 | 248.050 | -103.040 | $.14830 \mathrm{E}+00$ |
| -19710E-03 | 248.050 | 137.050 | . $19586 E+00$ |
| - 23130E-03 | 248.050 | 205.030 | . $27974 E+00$ |
| - 26270E-03 | 248.050 | 273.110 | . $34658 \mathrm{E}+00$ |


| $\begin{gathered} \text { vTscosety } \\ \text { (ooISE) } \end{gathered}$ | $\begin{gathered} \text { TEYPERATURE } \\ \text { (OEG K) } \end{gathered}$ | PRESSJRE <br> (ATMOS) | $\begin{aligned} & \text { OLNSITY } \\ & (G T 1 / C C) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| - $29810 \mathrm{E}-03$ | 248.050 | $341.1+10$ | . $39953 \mathrm{C}+00$ |
| -15270こ-03 | 223.350 | 35.010 | . 55926E-01 |
| . $15710=-23$ | 223.350 | 69.030 | . $11392 E+C 0$ |
| .17630-03 | 223.350 | $103.0+0$ | . $17302 E+C 0$ |
| -1984 $5-23$ | 223.359 | 137.073 | - $2 \subset 940 E+00$ |
| . $232908-83$ | 223.350 | 205.090 | - 32401E+00 |
| - 27120E-93 | 223.350 | 273.130 | . $39408 \mathrm{E}+00$ |

KESTEM AND WANG (1958)


KESTIN AND LEIDENFROST (1960)


| $\begin{gathered} \text { VTSCTSTTY } \\ (P O L S) \end{gathered}$ | $\begin{gathered} T=T P E R A T U R E \\ (D E G K) \end{gathered}$ | $\begin{aligned} & \text { PRESSTRE } \\ & \text { (ATMOS) } \end{aligned}$ | $\begin{aligned} & \text { पETSITY } \\ & (\mathrm{GY} / \mathrm{CC}) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| . 17854E-03 | 293.150 | 20.430 | . 23949E-01 |
| -17814E-33 | 293.150 | 18.0'0 | . 21088E-01 |
| . 17758:-03 | 293.150 | 14.590 | . 17046E-01 |
| -17696E-43 | -293.150 | 11.170 | -13042E-01 |
| . 17650E-03 | 293.150 | 7.790 | . 90890E-02 |
| . 19154=-03 | 298.150 | 69.040 | . $79214 \mathrm{E}-01$ |
| -168015-53 | 298.150 | 55.530 | . 63778E-01 |
| . 18479 - 03 | $-298.150$ | 41.890 | . $48134 \mathrm{E}-01$ |
| -18212三-03 | 298.150 | 28.210 | . 32428E-01 |
| -17989 - 03 | 298.150 | 14.630 | . 16761E-01 |
| -17833-83 | 298.150 | 7.830 | . $89450 \mathrm{E}-02$ |

WHITELAW (1960)


FLYN, HANKS, LEMAIRE AND ROSS (1953)

| VTSCOSETY | $\begin{aligned} & \text { APERATURE } \\ & \text { DEGK) } \end{aligned}$ | Pre55stz= <br> (ATMOS) |  |
| :---: | :---: | :---: | :---: |
| . 21327-033 | 373.150 | 26.350 | . 23 962E-01 |
| . $21634=-33$ | 373.150 | 53.140 | . 47928E-01 |
| - 22135-03 | 373.150 | 80.51 | . $71897 E-01$ |
| - 22638E-93 | 373.150 | 108.670 | . $95829 \mathrm{E}-01$ |
| . 23376E-03 | 373.150 | 137.750 | .11977E+00 |
| -23958=-03 | 373.150 | $168.0 \geq 0$ | . $14372 E+00$ |
| -17812=-93 | 298.154 | 6.770 | . $-77624 E-0$ |
| . 130285-03 | 298.150 | 20.850 | . 23965E-01 |
| .133935-03 | 298.150 | 41.670 | . 47916E-01 |
| -136345-03 | 298.150 | 52.210 | .60000E-01 |
| -18998-43 | 298.150 | 62.580 | .71864E-01 |
| -194228-03 | 298.150 | 83.720 | . $95800 \mathrm{E}-01$ |
| . 290512-33 | 298.150 | 105.250 | . $11974 \mathrm{E}+00$ |
| . $20723 \mathrm{E}-63$ | 298.150 | 127.340 | . $14370 E+00$ |
| - 21593-03 | -298.150 | 150.130 | .16707E+00 |
| - 2227ラニ-03 | 298.150 | 173.970 | . $19166 \mathrm{E}+00$ |
| - 16072-03 | 248.150 | 33.930 | . 47827E-01 |
| - 16538z-93 | 248.150 | 50.470 | . $71758 \mathrm{E}-01$ |
| . $17601-03$ | 248.150 | 83.430 | -11981E+ 50 |
| . 19U+3E-03 | 248.150 | 117.010 | .16807E+00 |
| - 20723E-03 | 248.150 | 152.540 | - $21631 E+00$ |
| . 21714E-03 | 248.150 | 171.530 | - $24035 \mathrm{E}+00$ |
| -14849 - 03 | 223.150 | 29.330 | . $47683=-01$ |
| . 15867E-03 | 223.150 | 58.250 | . $95386 \mathrm{E}-01$ |
| . 17135E-03 | 223.150 | 85.930 | . $14356 E+00$ |
| . 18633E-0.3 | 223.150 | 114.050 | . $192065+00$ |
| --20535E-73 | 223.150 | -143.790 | - $24+35-70$ |
| - 22639E-03 | 223.150 | 178.580 | - 2886 EE 9 + 00 |
| -13850E-63 | 194.650 | 37.110 | - $71035 E-01$ |
| . $14940 \mathrm{C}-03$ | 194.650 | 59.150 | -11884E+00 |
| - - 16.930 - 03 | 194.550 | $96.8-30$ | -19237E+00 |
| - 20046E-03 | 194.650 | 123.020 | - $26438 E+00$ |
| -22470E-03 | 194.650 | 147.130 | - $31134 \mathrm{E}+00$ |
| . 25350E-03 | 194.650 | 174.130 | . $35595 E+00$ |

GOLDTAN (19(3)


## KESTIN AND WHITELAW (1963)

| $\begin{gathered} \text { VISEOSITY } \\ \text { (FOTSE) } \end{gathered}$ | $\begin{aligned} & \text { MPERAFURE } \\ & (O E G K) \end{aligned}$ | -PRESS:IR <br> (ATMOS) | $\begin{aligned} & \text { DENSITY } \\ & \text { (G1/CC) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| - $22748 \mathrm{E}-03$ | 348.060 | 138.250 | . $12972 E+00$ |
| - 22553E-03 | 345.280 | 134.710 | -12777E+00 |
| -21954E-03 | 345.220 | 110.3 | -10588E+00 |
| - 21355-03 | 344.970 | 82.930 | . $80516 \mathrm{E}-01$ |
| - 20383E-03 | $3+4.750$ | 55.370 | - 54277E-01 |
| - 20417E-03 | 344.510 | 28.290 | . 27929E-01 |
| -1984'E-83 | -344.310 |  |  |
| - 23290E-03 | 372.230 | 131.850 | . $11528 E+00$ |
| - 22830E-03 | 372.160 | 109.870 | . 97111E-01 |
| . 22390 E-03 | 372.150 | $82.3: 0$ | . 75644E-01 |
| -22020 -03 | 372.140 | $-55.710$ | . $56345 E-01$ |
| . 21521E-03 | 372.100 | 27.670 | . $25227 \mathrm{E}-01$ |
| - 21022E-33 | 372.100 |  |  |

KAO ANO KOBAYASHI (1957)


TRAUIZ AND EAUMANN (1929)

| VISGSITY | TEMPERATURE |
| :---: | :---: |
| $($ (DEG K) |  |

JOHNSTON ANO MC CLOSKEY (1940)

| $\begin{gathered} \text { VISCOSTTY } \\ \text { (POISS) } \end{gathered}$ | $\begin{aligned} & \text { TEMPERATURE } \\ & \text { (DEG K) } \end{aligned}$ |
| :---: | :---: |
| . 64530E-34 | 90.220 |
| . $64530 \mathrm{E}-04$ | 90.280 |
| . 83530E-04 | 118.400 |
| - $929305-74$ | 132.780 |
| -10133E-03 | 146.190 |
| - 110?5E-03 | 160.890 |
| -12386E-03 | 183.270 |
| .134595-03 | 201.590 |
| -142295-03 | 215.230 |
| . 15032E-03 | 230.520 |
| . $15845 \mathrm{E}-03$ | 245.610 |
| -15611E-53 | -250.350 |
| . 17242E-03 | 273.290 |
| -17775E-03 | 284.600 |
| -1851+E-03 | 300.120 |

- vasilesco (1945)

$.172 J 0 E-03$
273.100
. 18030E-03
289.600
. 18240E-03
294.400

| $.221 J 0 E-03$ | 379.100 |
| :--- | :--- |
| $.22940 E-03$ | 399.850 |

- SUTHERLAND ANO MAASS (1932)

| VISCOSITY <br> $(P O T S E)$ | $T E M P E R A T U R E ~$ <br> $(O E G K)$ |
| :---: | :---: |
| $.12817 E-03$ | 194.650 |
| $.18115 E-03$ | 293.950 |
| $.17033 E-03$ | 273.150 |
| $.15392 E-03$ | 241.550 |
| $.13328 E-03$ | 203.750 |
| $.11295 E-03$ | 169.150 |
| $.62630 E-04$ | 90.050 |
| $.55110 E-04$ | 78.95 J |



KESTIV ANO HHITELAW (1964)

| VISCJSITY | TEMPERATURE | PRESSSRE | (OEGK) KSESTY |
| :---: | :---: | :---: | :---: |


| . 18440E-33 | 297.710 |  |  |
| :---: | :---: | :---: | :---: |
| -19635E-03 | 322.290 |  |  |
| - 20751E-03 | 347.120 |  |  |
| - $21952 \mathrm{c}=\mathrm{\square} 3$ | 374.570 |  |  |
| - 24077E-03 | 424.310 |  |  |
| - 22138E-93 | 298.250 | 139.470 | . $16466 E+00$ |
| - 21055z-03 | 297.880 | 109.870 | -13103E+00 |
| -20132-603 | 297.720 | 82.930 | -99361E-01 |
| .19305 - 03 | 297.690 | 55.130 | .60037E-01 |
| -18839E-03 | 297.670 | 27.950 | . $33381 \mathrm{E}-01$ |
| - 22227-03 | 322.340 | 124.840 | . $13479 E+00$ |
| -215395-03 | 322.330 | 100.820 | -10966E+00 |
| . 20981E-03 | 322.290 | 77.830 | . 85154E-01 |
| -20411e-03 | 322.320 | 52.130 | . 57134 E - ¢1 |
| . 19955E-03 | 322.340 | 25.830 | . $28307 \mathrm{E}-01$ |
| $-2.485 \leq-5$ | 347.300 | 142.330 | . $109925+00$ |
| - 22897E-93 | 347.200 | 116.270 | - $11554 E+00$ |
| - 22131E-63 | 347.160 | 85.510 | . 85832 E -01 |
| -2153'E-03 | 347.160 | 55.850 | . $56454 E-01$ |
| - 211312-03 | 347.170 | 28.550 | -20985E-01 |
| . 24032E-03 | 376.470 | 127.360 | . $11509 \mathrm{E}+00$ |
| -23551E-03 | 376.380 | 102.520 | . $93594 \mathrm{E}-01$ |
| . 23093E-03 | 375.340 | 78.440 | . $72438 \mathrm{E}-01$ |
| -22727E-03 | 375.030 | 52.650 | . $490375-01$ |
| - 22294E-93 | 374.570 | 25.030 | . $23547 \mathrm{E}-01$ |
| - 25400E-03 | 424.360 | 100.690 | . 80871E-01 |
| -25190E-03 | 424.430 | 80.830 | . 65487E-01 |
| -24975E-03 | 424.510 | 59.110 | . $48245 E-01$ |
| - 24639E-03 | 424.480 | 40.630 | . 33355 こ-01 |
| - 24259E-03 | 424.320 | 19.730 | -16363E-01 |
| - 27335E-03 | 469.810 | 104.230 | . 75196E-01 |
| -27127E-03 | 470.170 | 82.470 | . $59975 \mathrm{E}-01$ |
| - 26834E-03 | 470.340 | 62.440 | .45787E-01 |


| $\begin{gathered} \mathrm{VSCOSI} \\ (\operatorname{POIS})^{Y} \end{gathered}$ | $\begin{aligned} & \text { TEMPEPATURE } \\ & \text { (DEGK) } \end{aligned}$ | $\begin{aligned} & \text { PRESSTRE } \\ & \text { (ATHOS) } \end{aligned}$ | DENS ITY <br> (GA/CC) |
| :---: | :---: | :---: | :---: |
| . 2f573E-33 | 470.450 | 41.010 | . 30322E-01 |
| - $26248=-03$ | 470.460 | 21.410 | . 15945E-01 |
| - . 29050-3 | 523.970 | 102.850 | - $003025-01$ |
| -29425E-03 | 523.970 | 82.040 | . 53430E-01 |
| . 291118-33 | 523.920 | 61.550 | . $4 \mathrm{i} 458 \mathrm{E}-01$ |
| . $287545-33$ | 523.930 | 40.610 | . 26920E-01 |
| - $284325-13$ | 523.840 | 20.650 | . 13772E-01 |

GORING AND EAGAN (1971)

| VIScosity | TEMPERATURE (DEG K) | PRESSURE <br> (ATMOS) | $\begin{aligned} & \text { OENSITY } \\ & \text { (GUCC) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| . 23850-03 | 423.150 | 36.730 | . $30328 \mathrm{E}-01$ |
| . 24730503 | 423.150 | 104.790 | . $84272 \mathrm{E}-01$ |

IO , こARROLL AND STIEL (1966)

| $\begin{aligned} & \text { VISCOSITY } \\ & \text { (POESE) } \end{aligned}$ | TEMPERATURE (DEGK) | $\begin{aligned} & \text { PRESSURE } \\ & \text { (ATMOS) } \end{aligned}$ | $\begin{aligned} & \text { DENSITY } \\ & (\mathrm{GH} / \mathrm{CC}) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| . 14010E-03 | 203.150 | 20.010 | . $363555-01$ |
| . 15250E-33 | 203.150 | 50.030 | -97070E-01 |
| . 16540E-03 | 203.150 | 75.010 | . $15267 \mathrm{E}+00$ |
| -178+0E-03 | 203.150 | 100.030 | - $21045 E+00$ |
| . 214505-03 | 203.150 | 150.030 | . $31795 E+00$ |
| . $27430 \mathrm{~F}-03$ | 203.150 | 200.010 | . $40066 E+00$ |
| . 15000E-03 | 223.150 | 20.010 | . 3c618E-01 |
| - $160+0 E-03$ | 223.150 | 50.010 | . 85 U17E-01 |
| . 17250E-93 | 223.150 | 75.070 | . $13113 E+00$ |
| . 18550E-03 | 223.150 | 100.010 | . $17812 \mathrm{E}+00$ |
| -21220E-03 | 223.150 | 150.030 | . $26800 \mathrm{E}+00$ |
| $\because 254905-03$ | 223.150 | 200.010 | - 34L18E+00 |


| VTSESSTTY | $\begin{aligned} & \text { TEYPERATURE } \\ & \text { (OEGK) } \end{aligned}$ | $\begin{aligned} & \text { PRESSIRE } \\ & \text { (ATMOSS } \end{aligned}$ | $\begin{aligned} & \text { QEISSITY } \\ & \text { (GM/CC) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| . 29190E-03 | 223.150 | 250.030 | . $40481 E+00$ |
| -16290E-03 | 248.150 | 20.010 | . 28982E-01 |
| -17090E-03 | 248.150 | 50.030 | . 74178 E -01 |
| -17670\%-03 | 248.150 | 75.030 | -11284E+60 |
| -18710=-03 | 248.150 | 102.030 | . $15164 \mathrm{E}+00$ |
| . 214 30-03 | 248.150 | 150.030 | . $22643 E+00$ |
| . 25630E-03 | 248.150 | 200.030 | . $29325 E+00$ |
| -264502-03 | 248.150 | 250.030 | -35011E+00 |
| . 29330E-03 | 248.150 | 300.030 | . $39771 \mathrm{E}+00$ |
| .17430E-03 | 273.150 | 20.030 | . 26124E-01 |
| -18106を-03 | 273.150 | 50.010 | .66110E-01 |
| -187308-63 | 27-3.150 | 75.030 | . $99746 E-01$ |
| . 19510E-03 | 273.150 | 100.030 | $.13319 E+00$ |
| - 21320E-03 | 273.150 | 150.030 | . 19772E+00 |
| . 23300 - 03 | 273.150 | 200.030 | . $25674 \mathrm{E}+00$ |
| - $256+150-03$ | 273.150 | 250.010 | - 30888EF00 |
| . 27950-03 | 273.150 | 300.030 | . $35418 \mathrm{E}+40$ |
| . $30330 E-03$ | 273.150 | 350.030 | - $39339 E+00$ |
| - 32830E-03 | 273.150 | 400.030 | . $42751 E+00$ |
| . 185 - $08-03$ | 298.150 | 20.070 | -23805E-01 |
| -19120E-03 | 298.150 | 50.030 | . 59797E-01 |
| -196506-03 | 298.150 | 75.010 | . 89748E-01 |
| - 20330E-03 | 298.150 | 100.030 | . 11935E+00 |
| -21990迷03 | 298.150 | -150.030 | -17E44E+90 |
| - $23+308-03$ | 298.150 | 200.030 | -22932E+00 |
| . 25510E-03 | 298.150 | 250.010 | . 27707E+00 |
| . 27410:-93 | 298.150 | 300.030 | - $31959 \mathrm{E}+00$ |
| -295+0E- 03 | 298.150 | 350.010 | - $357245+00$ |
| -316305-03 | 298.150 | 400.030 | : 39062E+00 |
| -33750E-03 | 298.150 | 450.030 | . $42035 E+00$ |
| . 19530E-03 | 323.150 | 20.030 | . 21881E-01 |
| -20090e-03 | 323.150 | 50.010 | . 54.685E-01 |
| . 20540E-03 | 323.150 | 75.030 | . 81787E-01 |
| -21150E-93 | 323.150 | 100.030 | . $16847 \mathrm{E}+00$ |



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