EFFECT OF Λ-Σ CONVERSION

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THE Λ-PARTICLE BINDING

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NUCLEAR MATTER

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By

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The binding energy B of a Λ-particle in infinite nuclear matter has been estimated to be about 30 MeV by extrapolating the observed binding energies of hypernuclei. On the other hand, theoretical estimates so far done by various methods are generally much larger than 30 MeV. Various reasons for this descrepancy have been considered. We estimate the effect of the Λ-Σ conversion as one of the effects removing that descrepancy. In order to take account of the Λ-Σ conversion explicitly, it is convenient to use the so-called two-channel formalism. We calculate the binding energy B in the two-channel formalism (TCF) as well as in the more conventional one-channel formalism (OCF). It is found that B in the TCF can be substantially smaller than in the OCF. The difference of the values of B in the two formalisms is interpreted as due to the Pauli principle which suppresses the Λ-Σ conversion in nuclear matter. The relation between this effect in the TCF and three-body ΛNN forces in the OCF is clarified.

TABLE OF CONTENTS

		Page
ACKNOWLEDGEMENTS		ii
SCOPE AND CONTENT		iii
CHAPTER 1	INTRODUCTION	1
CHAPTER 2	ONE-AND TWO-CHANNEL FORMALISMS	7
CHAPTER 3	HYPERON-NUCLEON SCATTERING	11
CHAPTER 4	CALCULATION OF THE BINDING ENERGY OF HYPERNUCLEAR MATTER	18
CHAPTER 5	RESULTS AND DISCUSSION	24
APPENDIX A	DETAILS OF THE CALCULATION OF HYPERON-NUCLEON SCATTERING	32
APPENDIX B	DETAILS OF THE CALCULATION OF THE BINDING ENERGY OF HYPERNUCLEAR MATTER	41

CHAPTER 1

INTRODUCTION

The bound system of one or more hyperons and nucleons is called the hypernucleus. Hypernuclei containing one or two Λ-particles have been observed whereas none containing other hyperons like Σ and Ξ has been detected. In this thesis, we consider those which contain only one Λ-particle. They are denoted by ^{A}Z where Z is the nuclear charge and A is the total mass number. Since the first observation of such a system by Danysz and Pniewski (1), many hypernuclei have been reported. Hypernuclei with A=3 to 13 have been clearly identified, and also some heavier hypernuclei are known.

What kind of interactions are acting between the Λ-particle and nucleon? Let us consider first on the theoretical side. Since charge independence holds for nuclear interactions, the assignment I=0 for the isospin of the Λ-particle requires equality for the Λ-n and Λ-p interactions. Charge symmetry also forbids the process $\Lambda \rightarrow \Lambda + \pi^{o}$, since the π^{o} wavefunction reserves under the operation of charge symmetry. However, the following processes are allowed in the strong nuclear interactions;

$$\Lambda \leftrightarrow \Sigma + \pi$$

$$\Sigma \leftrightarrow \Lambda + \pi$$

$$\Sigma \rightarrow \Sigma + \pi$$

Hence, a Λ-particle can emit two or more pions. $\Lambda \to \Sigma + \pi \to \Lambda + \pi + \pi$, and the absorption of these pions by a neighboring nucleon will give rise to a Λ-nucleon force. This is the mechanism of giving rise to the forces of the longest-range possible for the Λ-N system (the range parameter = $(2m_\pi)^{-1} \simeq 0.7$ fm). Additional forces of shorter-ranges between Λ(or Σ) -particle and nucleon can also arise from the exchanges of more pions and/or heavy mesons. For instance, the one K-meson exchange yields a force of the range $(m_k)^{-1} = 0.4$ fm. Another possibility which must be considered in the analysis of binding energies is that three-body potentials may contribute appreciably. The long-range part of the three-body force will be generated when the two pions emitted by a Λ-particle are absorbed by different neighboring nucleons. (Fig. 1).

On the experimental side, a most direct source of the information about the Λ-N interaction is the free Λ-N scattering. The experimental data on the total cross sections of the Λ-p scattering at low energies have been reported (2,3). These data have been analysed by means of the effective range approximation, which has given us the s-wave scattering parameters in the singlet and triplet states, their typical values being $a_s = -1.8$, $r_s = 2.8$ and $a_t = -1.6$, $r_t = 3.3$ in fm. Assuming Λ-N potentials which fit the scattering parameters, the binding energies of light hypernuclei have been calculated. It has been found that the binding energies obtained in this way tend to be larger than the observed ones.

In particular for ${}^{5}_{\Lambda}$He, the calculated binding energy is $B_{cal} \simeq 6$ MeV whereas the experimental one is $B_{obs} = 3.08 \pm 0.03$ MeV.

There are various effects which may suppress the binding. (i) The effect of three-body ΛNN forces. Three-body forces in ordinary nuclei are believed to be unimportant. The situation in hypernuclei is quite different from that in ordinary nuclei because there is a Σ-component in the intermediate state of hypernuclei which has no counterpart in the nucleon-nucleon interactions in ordinary nuclei. (ii) The effect of the Pauli principle which suppresses the virtual transition $\Lambda N \rightarrow \Sigma N$ when N is imbedded in nuclear matter. Although this is usually discussed separately from the ΛNN force, these two effects are due to the same mechanism (4).

To see the effects of the three-body force and the Λ-Σ conversion explicitly, it is convenient to use the two-channel formalism (TCF) in which the wavefunction has two components corresponding to the Λ-N and Σ-N channels. The wavefunction in the more conventional one-channel formalism (OCF) consists of a single component.

The purpose of the work which is presented in this thesis is to investigate the two effects mentioned above. To this end, we calculate the binding energy of a Λ-particle in nuclear matter. Nuclear matter is an imaginary system consisting of equal, infinite number of protons and neutrons. Also the Coulomb interactions between protons are switched off.

The theoretical treatment of such an infinite homogeneous system is much simpler than that of a finite nucleus. Because of its simplicity, we hope that we can see some characteristic features of the ΛN interaction in a simple, clear-cut way.

Next, let us describe what has been done about hypernuclear matter on the both experimental and theoretical sides. Firstly, what is the "experimental" value of the binding energy of infinite hypernuclear matter? To determine the binding energy of a Λ-particle in nuclear matter phenomenologically, one assumes a single-particle potential $V(r)$ in which the Λ-particle moves. This potential has two parameters, namely the nuclear radius $R=r_o A^{1/3}$ and the well depth D_Λ. For a given analytical form of $V(r)$, one solves the Schrödinger equation for the Λ-particle moving in the potential and determines the lowest energy eigenvalues $E_o(A,D_\Lambda)$. Comparing $E_o(A,D_\Lambda)$ with the measured binding energy $B_\Lambda(A)$ for different hypernuclei, one can determine the value of D_Λ which gives the best overall agreement between $E_o(A,D_\Lambda)$ and $-B_\Lambda(A)$. In the following, we denote the binding energy of a Λ-particle in nuclear matter by B, which is equals to D_Λ in nuclear matter. In this way, B has been estimated to be about 30 MeV. This is regarded as an "experimental" value. The average binding energy for the nuclei in emulsion (mainly Br and Ag with the average $A \sim 70$) has been found to be $B \sim 24$ MeV. From this result, the well depth has been determined to be 27.7±0.6 MeV for a square well, and 30.6±0.6 MeV for a fermi shape potential (5,6).

Secondly, what kind of method is available to calculate B? There are three methods usually used for the calculation of B. (i) The perturbation expansion in powers of the strength of the ΛN potential. The convergence is not good (7,8). (ii) The variational calculation which uses correlated wavefunctions (9-15). (iii) The g-matrix approach which was developed by Brueckner and his collaborators (16), and later was reformulated in a simpler way by Gomes, Walecka, and Weisskopf (17). This method was first applied to calculation of B by Walecka (18), and later by many authors (15, 19-23). Recently, Bhaduri and Law attempted to estimate B directly from the Λ-p scattering cross section (24). In all the calculations so far done, except for the very recently one by Bodmer and Rote, one starts with ΛN potentials which fit the low-energy Λ-p scattering or the binding energies of s-shell hypernuclei or both, and calculate B without taking account of the possibility of Λ-Σ conversion nor the ΛNN force. A typical value one gets in this way is 50 MeV which is much larger than the "experimental" value 30 MeV.

In order to see the effect of the Λ-Σ conversion, let us calculate B in the one-channel formalism (OCF) and also in the two-channel formalism (TCF). In the following, Y stands for Λ or Σ. Now suppose that the YN potentials in the two formalisms, V(OCF) and V(TCF), are equivalent in the sense that they lead to the same low-energy ΛN scattering phase shift. For a many-body system like nuclear matter, however, these two potentials will yield different results because of the Pauli principle in the ΣN intermediate state, which is absent in the OCF. The two formalisms are equivalent in many-body problems only when many body-forces are introduced in the OCF.

Now let us assume that all the interactions are represented by separable potentials of Yamaguchi type (2, 5), which permit exact analytic solutions of the Bethe-Goldstone equation as well as those of the Schrodinger equation for the YN scattering. We consider only central forces in the s-state. The potentials are designed to reproduce the low-energy Λp scattering data, and the elastic and inelastic cross sections for Σ-p. We then can calculate B both in the OCF and TCF. It will be shown that if one artificially removes the restriction due to the Pauli principle in the ΣN virtual state in the TCF, the difference between the TCF and the OCF almost disappears. This clearly shows where the difference comes from.

The relation between one-and two-channel formalisms will be described in Chapter 2. The free scattering for the YN system will be solved in Chapter 3. The calculation of the binding energy of hypernuclear matter will be done in Chapter 4. The result will be shown in Chapter 5. Other related problems are also discussed in the last chapter.

CHAPTER 2

ONE-AND TWO-CHANNEL FORMALISMS

The reaction in the two-body YN system;

$$(a)\ \Lambda N\rightarrow\Lambda N,\quad (b)\ \Lambda N\leftarrow\rightarrow\Sigma N,\quad (c)\ \Sigma N\rightarrow\Sigma N \tag{2.1}$$

can be described in terms of a two-component wave functions;

$$\psi = \binom{\psi_\Lambda}{\psi_\Sigma} \tag{2.2}$$

where the suffixes Λ and Σ refer to the ΛN and ΣN channels, respectively. The Schrodinger equation takes the form;

$$(T_\Lambda+V_\Lambda)\ \psi_\Lambda+V_{\Lambda\Sigma}\psi_\Sigma = E\psi_\Lambda \tag{2.3}$$

$$(T_\Sigma+V_\Sigma)\ \psi_\Sigma+V_{\Sigma\Lambda}\psi_\Lambda = (E-\Delta)\psi_\Sigma \tag{2.4}$$

Where T_Λ and T_Σ are the kinetic energies in the ΛN and ΣN channels, respectively. The Σ-Λ mass difference is denoted by Δ. The units are taken as $\hbar = c = 1$. The potentials V_Λ, $V_{\Lambda\Sigma}$ and V_Σ correspond to the transitions (a), (b) and (c) in (2.1), respectively. If the energy is well below the ΣN threshold ($E<<\Delta$), one can reduce the above TCF to the OCF by eliminating ψ_Σ as follows. From (2.4) one obtains

$$\psi_\Sigma = -(T_\Sigma\ +V_\Sigma+\Delta-E)^{-1}\ V_{\Sigma\Lambda}\psi_\Lambda \tag{2.5}$$

Which is put into (2.3) to yield the Schrödinger equation in the OCF;

$$(T_\Lambda+V)\psi_\Lambda = E\psi_\Lambda \tag{2.6}$$

where we now deal with a one-component wavefunction ψ_Λ. The OCF potential V is related to V_Λ etc. by

$$V = V_\Lambda - V_{\Lambda\Sigma}(T_\Sigma+V_\Sigma+\Delta- E)^{-1}\, V_{\Sigma\Lambda} \tag{2.7}$$

For the sake of the following argument, let us expand V in terms of V_Σ to obtain

$$V = V_\Lambda - V_{\Lambda\Sigma}G_\Sigma V_{\Sigma\Lambda} + V_{\Lambda\Sigma}G_\Sigma V_\Sigma G_\Sigma V_{\Sigma\Lambda} + \cdots \tag{2.8}$$

where $G = (T_\Sigma+\Delta-E)^{-1}$

Next let us consider a many-body system consisting of one Λ and many nucleons. The OCF potential V is now related to V_Λ's in the TCF by

$$V = \sum_i V_\Lambda^{(i)} - \sum_i V_{\Lambda\Sigma}^{(i)}\, G_\Sigma V_{\Sigma V}^{(i)} - \sum_{i\neq j} V_{\Sigma V}^{(i)} G_\Sigma V_{\Sigma\Lambda}^{(j)} \tag{2.9}$$

where the superscript i indicates the nucleon with which Λ or Σ is interacting. The first and second terms in (2.9) are two body potentials but the third term represents a three body ΛNN force, which may well be as important as the second term. The expansion (2.9) is graphically shown in Fig. 2, where V and V_Λ etc. are represented by wavy and dashed lines, respectively.

The above formulae are only symbolic. To be more precise, one has to consider the isospin concerning the off-diagonal potential $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$. They can be written as

$$\begin{aligned} V^{(i)}_{\Lambda\Sigma} &\to \Lambda^{+}\underset{\sim}{\Sigma}\cdot\underset{\sim}{\tau}^{(i)} V_{\Lambda\Sigma}/\sqrt{3} \\ V^{(i)}_{\Sigma\Lambda} &\to \underset{\sim}{\tau}^{(i)}\cdot\underset{\sim}{\Sigma}^{+}\Lambda\ V_{\Lambda\Sigma}/\sqrt{3} \end{aligned} \tag{2.10}$$

where Σ annihilates a Σ and Λ^+ creates a Λ. The dot indicates a scalar product of two isovectors. $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$ are functions of positions, local or non-local. Then the second term in (2.9) remains the same whereas the third term now becomes

$$-\frac{1}{3} \sum_{i\neq j} \tau^{(i)}\cdot\tau^{(j)}\ V^{(i)}_{\Lambda\Sigma}\ G_\Sigma\ V^{(j)}_{\Sigma\Lambda} \tag{2.11}$$

where we have suppressed the factor $\Lambda^+\Lambda$. Because of the factor $\tau^{(i)}\cdot\tau^{(j)}$ the expectation value of this ΛNN force in an isospin-saturated system like ^{4}He or nuclear matter comes only from the exchange term, hence is repulsive.

Next, let us consider the perturbation calculation for the binding energy of a Λ in nuclear matter in the TCF. In the second order, we encounter the same diagrams as shown in figure 2 (b) and (c). However, unlike the expectation value of V in (2.9), now the diagram (c) gives no contribution in the TCF because of the isospin saturation and the Pauli principle in the ΣNN intermediate state. In the OCF the ΛNN force (2.11) is an instantaneous interaction, the ΣNN intermediate state having been eliminated. However, the diagram (c) in the TCF calculation represents a two step process. The contribution from the diagram (b) in the TCF is also different from that of the second term in (2.9), because the nucleon in the ΣN intermediate state must be above the Fermi surface. This difference, however, is found to be exactly the same as the expectation value of the ΛNN force (2.11) in nuclear matter. In other words, in the TCF the effective YN force in nuclear matter is modified from the free YN force because of the Pauli principle, but the same result is obtained by disregarding this modification if one includes an appropriate three-body ΛNN force in the OCF.

In addition to this kind of ΛNN force which arises from the Σ-channel coupling, there are other more genuine three-body forces which arises through intermediate states involving Y_1^*, etc. (4). We do not consider them in this thesis.

CHAPTER 3

HYPERON-NUCLEON SCATTERING

We take the TCF unless otherwise stated. The OCF follows as a special case of the TCF. We assume that the potential is given, in the momentum space, by

$$<p|V|p'> = -\begin{pmatrix} \lambda_\Lambda g_\Lambda(p) g_\Lambda(p') & \lambda_x g_\Lambda(p) g_\Sigma(p') \\ \lambda_x g_\Sigma(p) g_\Lambda(p') & \lambda_\Sigma g_\Sigma(p) g_\Sigma(p') \end{pmatrix} \tag{3.1}$$

where λ's are real constants, and g's are real functions, p and p' are relative momenta. To be precise, $g_\Lambda(p)$ and $g_\Sigma(p)$ should be written as $g_\Lambda(p_\Lambda)$ and $g_\Sigma(p_\Sigma)$, respectively.

If ΛN is the entrance channel, the wavefunction is obtained in the following form. All the detailed calculation for this chapter will be shown in Appendix A.

$$\psi_\Lambda(p,k) = \delta(\underset{\sim}{p}_\Lambda - \underset{\sim}{k}_\Lambda) + T_\Lambda(p,k)/[4\pi^2\mu_\Lambda e_\Lambda(p,k)] \tag{3.2}$$

$$\psi_\Sigma(p,k) = T_{\Sigma\Lambda}(p,k)/[4\pi^2\mu_\Sigma e_\Sigma(p,k)] \tag{3.3}$$

where k_Λ is the incident momentum, and also it is understood that $\psi_\Lambda(p,k) \equiv \psi_\Lambda(p_\Lambda,k_\Lambda)$, $f_\Lambda(p,k) \equiv f(p_\Lambda,k_\Lambda)$ etc. In the following, we shall often suppress the suffix Λ or Σ of the momentum. μ_Λ and μ_Σ are the reduced masses of the ΛN and

Σ N system, respectively and e_Λ and e_Σ are energy denominators given by

$$e_\Lambda(p,k) = (p_\Lambda^2 - k_\Lambda^2 - i\varepsilon)/2\mu_\Lambda \qquad (3.4)$$

$$e_\Sigma(p,k) = (p_\Sigma^2 + k_\Sigma^2)/2\mu_\Sigma \qquad (3.5)$$

In the coordinate space, the wavefunctions (3.2) and (3.3) have the following asymptotic forms:

$$\psi_\Lambda \sim e^{i\underset{\sim}{k}_\Lambda \cdot \underset{\sim}{r}_\Lambda} + T_\Lambda(k,k)e^{ik_\Lambda r_\Lambda}/r_\Lambda \qquad (3.6)$$

$$\psi_\Sigma \sim T_{\Sigma\Lambda}(k,k)e^{ik_\Sigma r_\Sigma}/r_\Sigma \qquad (3.7)$$

The amplitudes of the outgoing waves are given by

$$T_\Lambda(p,k) = 4\pi^2\mu_\Lambda g_\Lambda(p)\ g_\Lambda(k)\{\lambda_\Lambda - d(\lambda)\ J_\Lambda(k)\}/D(k) \qquad (3.8)$$

$$T_{\Sigma\Lambda}(p,k) = 4\pi^2\mu_\Sigma g_\Sigma(p)\ g_\Lambda(k)\ \lambda_x/D(k) \qquad (3.9)$$

with

$$d(\lambda) = \lambda_\Lambda\lambda_\Sigma - \lambda_x^2 \qquad (3.10)$$

$$D(k) = 1 - \lambda_\Lambda J_\Lambda(k) - \lambda_\Sigma J_\Sigma(k) + d(\lambda)\ J_\Lambda(k)\ J_\Sigma(k) \qquad (3.11)$$

$$J_\Lambda(k) = \int d^3p g_\Lambda^2(p)/e_\Lambda(p,k),\ J_\Sigma(k) = \int d^3p g_\Sigma^2(p)/e_\Sigma(p,k) \qquad (3.12)$$

The two momenta k_Λ and k_Σ are related by

$$E = k_\Lambda^2(2\mu_\Lambda)^{-1} = -k_\Sigma^2(2\mu_\Sigma)^{-1} + \Delta \qquad (3.13)$$

where $\Delta = m_\Sigma - m_\Lambda$. Below the ΣN threshold, $E<\Delta$, hence $k_\Sigma^2>0$.

The cross sections are given as follows.

$$\sigma(\Lambda N\to\Lambda N) = 4\pi|T_\Lambda|^2 \tag{3.14}$$

$$\sigma(\Lambda N\to\Sigma N) = 4\pi|T_{\Sigma\Lambda})^2 \ (V_\Sigma/V_\Lambda)\ \theta(E-\Delta) \tag{3.15}$$

where $V_\Lambda = k_\Lambda/\mu_\Lambda$ and $V_\Sigma = k_\Sigma/\mu_\Sigma$. If ΣN is the entrance channel,

$$\sigma(\Sigma N\to\Sigma N) = 4\pi|T_\Sigma|^2 \tag{3.16}$$

$$\sigma(\Sigma N\to\Lambda N) = 4\pi|T_{\Lambda\Sigma}|^2 \ (V_\Lambda/V_\Sigma) \tag{3.17}$$

where T_Σ and $T_{\Lambda\Sigma}$ are obtained from T_Λ and $T_{\Sigma\Lambda}$ by the interchanging $\Lambda\rightleftarrows\Sigma$ respectively. Note that for the ΛN only the I = 1/2 state enters in our calculation.

Now, for simplicity, let us assume that

$$g_\Lambda(k) = g_\Sigma(k) = (\beta^2+k^2)^{-1} \tag{3.18}$$

namely, we assume that all the elements of the potential (3.1) have the same shape. This is probably not a very good assumption because, from meson theoretical point of view, the interaction of $\Lambda N\to\Lambda N$ has the two pion exchange tail, but the interactions of $\Lambda N\to\Sigma N$ and $\Sigma N\to\Sigma N$ have the one-pion exchange tails. Below the ΣN threshold the real phase shift δ is given by $T_\Lambda = e^{i\delta}\sin\delta/k$. We introduce the scattering length a and effective range r by

$$k \cot \delta = -\frac{1}{a} + rk^2/2 \tag{3.19}$$

Then, we obtain

$$\frac{1}{a} = \frac{\beta}{2}\{1 - \frac{\beta^3[\beta(\beta+\kappa)^2 - \gamma_\Sigma]}{\gamma_\Lambda\beta(\beta+\kappa)^2 - d(\gamma)}\} \tag{3.20}$$

$$r = \frac{3}{\beta} - \frac{4}{a\beta^2} - \frac{\mu_\Sigma\gamma_x^2\beta^5(\beta+\kappa)}{\mu_\Lambda\kappa[\gamma_\Lambda\beta(\beta+\kappa)^2 - d(\gamma)]^2} \tag{3.21}$$

where $\gamma_\Lambda = 2\pi^2\mu_\Lambda\lambda_\Lambda$, $\gamma_\Sigma = 2\pi^2\mu_\Sigma\lambda_\Sigma$, $\gamma_x = 2\pi^2(\mu_\Lambda\mu_\Sigma)^{1/2}\lambda_x$, $d(\gamma) = \gamma_\lambda\gamma_\Sigma - \gamma_x^2$, and $\kappa = (2\mu_\Sigma\Delta)^{1/2}$. To determine the parameters β and γ's, we first assume γ_x and γ_Σ, and then solve the coupled equations (3.20) and (3.21) for given values of a and r. γ_Λ can be eliminated so that the equations are reduced to a single equation with respect to β. Having determined the parameters, we evaluate the cross sections $\sigma(\Sigma N \to \Sigma N)$ and $\sigma(\Sigma N \to \Lambda N)$ and compare them with experimental data. Formulae in OCF are obtained by putting $\lambda_x = \lambda_\Sigma = 0$. Then, β and λ_Λ are determined by a and r.

According to Alexander et al (2) the low-energy Λp scattering are well fitted by taking

$$\begin{aligned} a_s &= -1.8 \text{ fm} \qquad & a_t &= -1.6 \text{ fm} \\ r_s &= 2.8 \text{ fm} \qquad & r_t &= 3.3 \text{ fm} \end{aligned} \tag{3.22}$$

where the suffixes s and t refer to the spin singlet and triplet states, respectively. Because these parameters are nearly spin independent, one may replace them by their spin-average, $\bar{a}$ and $\bar{r}$, which give the same total cross section σ and its derivative $d\sigma/dE$ at $E=0$ as that obtained by using (3.22). This implies

$$\bar{a}^2 = (a_s^2+3a_t^2)/4 \tag{3.23}$$

$$\bar{a}^3(\bar{a}-\bar{r}) = \{a_s^3(a_s-r_s) + 3a_t^3(a_s-r_s)\}/4 \tag{3.24}$$

which yield

$$\bar{a} = -1.6523 \text{ fm}, \ \bar{r} = 3.1717 \text{ fm} \tag{3.25}$$

For masses, we take m_N = 938.9 MeV, m_Λ = 1115.6 MeV and m_Σ = 1193.1 MeV, except for the case of Schick and Toepher's potential which will be discussed later.

For the cross sections for $\Sigma^- p$ and also for $\Sigma^+ p$, data are available, although rather meager, at several momenta in the range p_Σ = 110 ∿ 170 MeV/c. For example at p_Σ∿150 MeV/c (26,27)

$$\begin{aligned} \sigma(\Sigma^- p \to xn) &= 147 \pm 19 \text{ mb} \\ \sigma(\Sigma^- p \to \Sigma^- p) &= 198 \pm 48 \text{ mb} \\ \sigma(\Sigma^- p \to \Sigma^0 n) &= 111 \pm 19 \text{ mb} \\ \sigma(\Sigma^+ p \to \Sigma^+ p) &= 203 \pm 117 \text{ mb} \end{aligned} \tag{3.26}$$

The cross sections for $\Sigma^- p$ quoted above cannot be compared with our calculated values as such, because the $\Sigma^- p$ state is not a pure I = 1/2 state. Let us denote the ΣN scattering amplitude in the pure isospin state by T_{2I}, I = 1/2 or 3/2. Then

$$\begin{aligned} T(\Sigma^- p \to \Sigma^- p) &= (T_3 + 2T_1)/3 \\ T(\Sigma^- p \to \Sigma^o p) &= \sqrt{2}(T_3 - T_1)/3 \end{aligned} \tag{3.27}$$

Hence, $\sigma(\Sigma^- P \to \Sigma^- p) + \sigma(\Sigma^- p \to \Sigma^o n) = (\sigma_3 + 2\sigma_1)/3$ where σ_{2I} is the elastic cross section in the pure isospin state. Combining this with $\sigma(\Sigma^+ p \to \Sigma^+ p) = \sigma_3$, we obtain

$$\sigma_1 = \{3[\sigma(\Sigma^- p \to \Sigma^- p) + \sigma(\Sigma^- p \to \Sigma^o n) - \sigma(\Sigma^+ p)\}/2 \tag{3.28}$$

Taking the face values in (3.26) without errors, we obtain

$$\sigma_1 = 362 \text{ mb} \tag{3.29}$$

Because of the errors in (3.26); σ_1 may be as small as 200 mb.

We consider three model potentials which are referred to as I, II and one of Schick and Toepfer's potentials (28) which will be referred to as ST. The potentials I and II both fit the spin averaged scattering parameters $\bar{a}$ and $\bar{r}$ given by (3.25). In I, all the parameters are spin-indendent, but in II they are spin-dependent. ST is fitted to the following Λp data:

$$\begin{aligned} a_s &= -2.46 \text{ fm} \quad & a_t &= -2.07 \text{ fm} \\ r_s &= 3.87 \text{ fm} \quad & r_t &= 4.50 \text{ fm} \end{aligned} \tag{3.30}$$

We adopt the pair D and E in ST's AEA potentials for the singlet and triplet states repsectively. Also in calculation concerning ST, we take the same mass values as in their calculation, namely $m_N = 939.9$ MeV, $m_\Lambda = 1115.4$ MeV and $m_\Sigma = 1193.0$ MeV.

The parameters of these potentials, both in the OCF and TCF, and also the calculated cross sections $\sigma(\Sigma N \to \Lambda N)$ and $\sigma(\Sigma N \to \Sigma N)$ at $p_\Sigma = 150$ MeV/c are listed in Table 1. The cross section $\sigma(\Sigma N \to \Sigma N)$ is much smaller than that of (3.29). We have varied our parameters in a wide range, but we could not find parameters which yield very large $\sigma(\Sigma N \to \Sigma N)$ and at the same time fit the data on $\Lambda N \to \Lambda N$ and $\Sigma N \to \Lambda N$. This may be because we assumed the same shape for all the elements of the potential. The values for $\sigma(\Sigma N \to \Lambda N)$ are in better agreement with experimental value.

CHAPTER 4

CALCULATION OF THE BINDING ENERGY OF HYPERNUCLEAR MATTER

The binding energy B of a Λ in nuclear matter or the well depth is given by

$$B = -\frac{4}{\alpha_\Lambda^3}\int_o^{\alpha_\Lambda k_F} G(k)\,d\underset{\sim}{k} \qquad (4.1)$$

where $\alpha_\Lambda = m_\Lambda/(m_\Lambda+m_N)$ and k_F, the Fermi momentum is taken to be 1.36 fm^{-1}. G(k) is the diagonal element of the reaction matrix G(k,k') for YN pair in nuclear matter, and defined by

$$G\Phi = V\psi^N \qquad (4.2)$$

where Φ and ψ^N are unperturbed and perturbed wavefunctions of the pair, respectively. The latter is obtained by solving the Bethe-Goldstone equation. In the TCF,

$$\Phi = \begin{pmatrix}\phi_\Lambda \\ o\end{pmatrix},\ \psi^N = \begin{pmatrix}\psi_\Lambda^N \\ \psi_\Sigma^N\end{pmatrix},\ V = \begin{pmatrix}V_\Lambda & V_{\Lambda\Sigma} \\ V_{\Sigma\Lambda} & V_\Sigma\end{pmatrix} \qquad (4.3)$$

Hence,

$$G(k,k') \equiv \langle\Phi(k)|G|\Phi(k')\rangle$$

$$= \langle\phi_\Lambda(k)|V_\Lambda|\psi_\Lambda^N(k')\rangle + \langle\phi_\Lambda(k)|V_{\Lambda\Sigma}|\psi_\Sigma^N(k')\rangle \qquad (4.4)$$

For our separable potential the Bethe-Goldstone equation can be readily solved:

$$\psi_\Lambda^N(p,k) = \delta(\underset{\sim}{p}-\underset{\sim}{k}_\Lambda) + T_\Lambda^N(p,k)Q/[4\pi^2\mu_\Lambda e_\Lambda^N(p,k)] \tag{4.5}$$

$$\psi_\Sigma^N(p,k) = T_{\Sigma\Lambda}^N(p,k)Q/[4\pi^2\mu_\Sigma e_\Sigma^N(p,k)] \tag{4.6}$$

Here Q is the Pauli operator which excludes excitations of the nucleon below the Fermi surface. For the energy denominators, we assume that the nucleon is initially moving in a potential $-U_N$, and is free in the intermedieat state. Then, we obtain

$$e_\Lambda^N(p,k) = e_\Lambda(p,k) + U_N,\quad e_\Sigma^N(p,k) = e_\Sigma(p,k) + U_N \tag{4.7}$$

Banerjee and Sprung (29), from their nuclear matter calculation with Reid's (30) soft-core potential, have found that U_N for $k_N<k_F$ is well represented by

$$U_N = 85.17 - 12.08\ k_N^2\ \text{MeV} \tag{4.8}$$

where the nucleon momentum k_N is in fm^{-1}. T_Λ^N and $T_{\Sigma\Lambda}^N$ are obtained from (3.8) and (3.9), respectively, by replacing J_Λ J_Σ by J_Λ^N and J_Σ^N, respectively, which are defined as follows.

$$\begin{aligned} J_\Lambda^N(k) &= \int d^3p\ Q\ g^2(p)/e_\Lambda^N(p,k) \\ J_\Sigma^N(k) &= \int d^3p\ Q\ g^2(p)/e_\Sigma^N(p,k) \end{aligned} \tag{4.9}$$

After performing the angular integrations in (4.9), we obtain

(see Appendix B)

$$J_\Lambda^N(k_\Lambda) = 4\pi\{\int_{k_F+\eta k_\Lambda}^{\infty} dp\ p + \frac{1}{4\eta k_\Lambda}\int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} dp[(p+\eta k_\Lambda)^2 - k_F^2]\}\frac{pg^2(p)}{e_\Lambda^N(p,k_\Lambda)} \tag{4.10}$$

$$J_\Sigma^N(k_\Sigma) = 4\pi\{\int_{k_F+\eta' k_\Lambda}^{\infty} dp\ p + \frac{1}{4\eta' k_\Lambda}\int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} dp[(p+\eta' k_\Lambda)^2 - k_F^2]\}\frac{pg^2(p)}{e_\Sigma^N(p,k_\Sigma)} \tag{4.11}$$

where $\eta = m_N/m_\Lambda$ and $\eta' = \eta(m_\Lambda+m_N)/(m_\Sigma+m_N)$. The G-matrix is then obtained as follows.

$$G(k,k') = g_\Lambda(k)\ g_\Lambda(k')\{\lambda_\Lambda - d(\lambda)\ J_\Sigma^N\}/D^N \tag{4.12}$$

with

$$D^N = 1 - \lambda_\Lambda J_\Lambda^N - \lambda_\Sigma J_\Sigma^N + d(\lambda)\ J_\Lambda^N J_\Sigma^N \tag{4.13}$$

and

$$\frac{k_\Sigma^2}{2\mu_\Sigma} = \Delta - \frac{k_\Lambda^2}{2\mu_\Lambda}$$

If we ignor the Pauli principle in all the intermediate states, J_Λ^N and J_Σ^N are reduced to Re J_Λ and J_Σ, and G becomes the free G-matrix, which is related to ΛN scattering phase shift δ by

$$G^F = -\tan\delta(k)/[4\pi^2\mu_\Lambda k] \tag{4.14}$$

Formulae for the OCF are obtained from those of the TCF by putting $\lambda_x = \lambda_\Sigma = 0$.

The detailed analytical calculations for the this chapter will be shown in Appendix B.

The results of the binding energy calculations are summarized in Table 2. The lowest order perturbation result is obtained by substituting $\lambda_\Lambda g^2(k)$ for $G(k)$ in (4.1). Also the result with the free G-matrix is obtained by using G^F (4.14) in place of the complete G (4.12). The values in the last column are obtained by replacing J_Σ^N in (4.12) by J_Σ. This means that the Pauli principle is artificially discarded in the ΣN intermediate state. Let us denote the suppression in the B by $\Delta B = B(OCF) - B(TCF)$. For the potentials I, II and ST, we obtain ΔB = 3.38, 16.68, and 27.74 MeV, respectively. Recall that these potentials are almost equivalent for the ΛN elastic scattering. As is seen from Table 1, they yield more or less the same values for $\sigma(\Sigma N \to \Lambda N)$ and $\sigma(\Sigma N \to \Sigma N)$. In particular the potential II and ST are almost equivalent. But they lead to very different values for ΔB. Among a number of potentials considered by ST (28), here we have deliberately chosen one which exhibits a drastic reduction. This large reduction takes place because the ST triplet potential consists of only off-diagonal elements as is seen in Table 1.

If one removes the reduction due to the Pauli principle by replacing J_Σ^N by J_Σ, one finds that the difference between the OCF and TCF almost disappears. Hence it is clear where the difference comes from.

It would be in order here to comment on the free G-matrix expansion method proposed by Bhaduri and Law (24). G and G^F satisfy the following equations,

$$G = V - V \frac{Q}{e^N} G, \quad G^F = V - V \frac{P}{e} G^F \tag{4.15}$$

where e^N is the energy denominator in nuclear matter and e is the free one. P represents the principal value. Then one can derive the relation

$$G = G^F + G^F \left(\frac{P}{e} - \frac{Q}{e^N}\right) G \tag{4.16}$$

The diagonal element of G^F is related to the ΛN scattering phase shift by (4.14). If one assumes that the ΛN interaction is spin independent, the phase shift can be obtained from the ΛN cross section. Replacing G in the second term on the R.H.S. of (4.16) by G^F and assuming that G^F is local and energy-independent, Bhaduri and Law estimated B in the OCF. This has been extended to the TCF by Law (31). A great advantage of their method is that one needs not to know the potentials, but the assumption of spin-independent of the interactions is essential. Law has estimated that $\Delta B<7$ MeV. In addition to our potential I we tried several similar spin-independent potentials and found that $3<\Delta B<6$ MeV which is consistent with Law's result.

If one allows spin-dependence as in our potential II or ST, ΔB can become very large. Also one can see then that the result with free G-matrix is very far from that with the complete G-matrix for the triplet state. Thus the free G-matrix expansion is not useful in these cases.

CHAPTER 5

RESULTS AND DISCUSSION

In all the previous calculations so far done for the binding energy of a Λ in nuclear matter, the ΛN interaction has been determined with respect to the low energy ΛN scattering and/or binding energies of very light hypernuclei. Our model potentials are all acceptable ones in that sense. Furthermore, all of our potentials yield reasonable values for $\sigma(\Sigma N \to \Lambda N)$, although $\sigma(\Sigma N \to \Sigma N)$ is underestimated. Our model calculation clearly shows that the effect of the Λ-Σ conversion on the binding energy, ΔB, can be alarmingly large. As was discussed in Chapter 2, this effect can be taken account of in the OCF by introducing many-body forces. The large uncertainty for the estimate of ΔB seems to arise because of our ignorance of the spin dependence of the interactions. The low-energy ΛN scattering parameters in our calculation are assumed to be spin-independent or nearly so. Nevertheless the parameters, β and λ's can be strongly spin-dependent as in the potential II and ST.

In our calculation, we took the spin-averaged ΛN scattering parameters (3.25) or the nearly spin-independent set (3.30). Although the most likely values of the ΛN scattering parameters are nearly spin-independent, they are still very far from being well established (3,32). If one increases the spin-dependence of the ΛN scattering parameters but keeping the ΛN cross section the same, the binding energy B tends to decrease. In the lowest order perturbation the ΛN scattering cross section and the binding energy B depend on the ΛN potential as follows:

$$\sigma \propto V_s^2 + 3V_t^2, \ B \propto V_s + 3V_t \tag{6.1}$$

Let us introduce the notations:

$$V_s^2 + 3V_t^2 = c^2, \ V_s + 3V_t = Y, \ V_t/V_s = x \tag{6.2}$$

Then we obtain

$$Y(x) = c(1+3x)(1+3x^2)^{-\frac{1}{2}} \tag{6.3}$$

If we vary x but keeping c, say the ΛN cross section, constant, Y becomes maximum at x=1 and Y decrease as x deviates from 1:

$$y(0) = c, \ y(1) = 2c, \ y(\infty) = \sqrt{3}\ c \tag{6.4}$$

Therefore, by assuming $V_s \gg V_t$, the binding energy can be substantially reduced. Though this is certainly a very crucial argument, we think this is enough to indicate the crucial importance of better knowledge on the spin-dependence of the ΛN interaction.

There are other relevant effects which have not been considered in our calculation. Namely, (i) the effect of a possible resonance in the Λp system just below the ΣN threshold, (ii) the effect of tensor force, (iii) the effect of effective mass of Λ. (iv) the effect of rearrangement. Let us make comments about these effects.

(i) Evidence for a possible resonance in the Λp system just below the ΣN threshold has been reported recently (33). A plausible interpretation is that this is a quasi-bound state of the ΣN system in an s-state. If this resonance is established and if its spin is known, it will greatly reduce the arbitrasiness in the choice of parameters in the TCF potentail.

(ii) Goodfellow and Nogami (34) have estimated the effect of the ΛN tensor force on B in the one channel formalism. For the same strength of the tensor force, the suppression of the binding energy increases as the range of the force increase. For reasonable values of the strength and range, however, the amount of suppression is found to be less than 1 MeV. Law et al (35) have examined the effect of the ΛN tensor force on the binding of ${}^{5}_{\Lambda}He$ and found that it was too small to resolve the difficulty of over-binding by itself. Thus it seems that the effect of the tensor force on the binding energy of a Λ-particle in finite nuclei and in nuclear matter is quite small. Bodmer and Rote (7) considered the Λ-Σ conversion caused by one-pion-exchange potential which has a strong tensor-component. They estimated that the suppression of the ΛN-ΣN tensor force in nuclear matter was perhaps as much as 10 MeV. However, this suppression is due to the channel coupling rather than to the tensor force.

(iii) The idea of the effective mass is as follows. Let us denote the single particle energy, kinetic energy and single particle potential by ε_k, $k^2/2m$, and $U(k)$, respectively.

$$\varepsilon_k = \frac{k^2}{2m} + U(k)$$

The effective mass is defined by assuming the form of $U(k)$ suitably, usually taking the quadratic form with respect to k and then lumping the quadratic term together with the kinetic energy. For instance, assuming $U(k) = U_o + k^2 U_2/k_F^2$, then $\varepsilon_k \simeq k^2/2m^* + U_o$, where $m^* = m/(1+2mU_2/\hbar^2 k_F^2)$.

In our calculation, we considered the effective mass of nucleon in nuclear matter by taking account of (4.8), however we did not mention about the effective mass of Λ-particle. Bodmer and Rote have taken account of it, however k_Λ-dependent on $U(k_\Lambda)$ was not shown clearly in their calculation. Now, Chong, Nogami and Satoh (36) have been estimating the effect of repulsive core in the Λ-N interaction on the binding energy of hypernuclear matter, in which the reasonable estimation for $U(k_\Lambda)$ would be shown.

(iv) The idea of the rearrangement energy is explained as follows. The binding energy of the Λ-particle in nuclear matter B is given by

$$-B = U = E(A+1_\Lambda) - E(A)$$

where $E(A+1_\Lambda)$ and $E(A)$ consist of potential and kinetic parts. In the ground state of hypernuclear matter, the Λ-particle occupies the state with zero momentum. Therefore

$$U = E_{pot}(A+1_\Lambda) - E_{pot}(A)$$

According to the Brueckner theory,

$$E_{pot}(A) = \sum_{\substack{k_1<k_F \\ k_2<k_F}} \sum_{S,T} (2S+1)(2T+1)(\underset{\sim}{k}_1,\underset{\sim}{k}_2|K(A)|\underset{\sim}{k}_1,\underset{\sim}{k}_2)$$

where $K(A)$ is the reaction matrix for the nucleon-nucleon interaction in nuclear matter, S and T are the total spin and the total isospin of the two interacting nucleons, respectively, and $\underset{\sim}{k}_1$, $\underset{\sim}{k}_2$ are momenta of two nucleons. Now $E_{pot}(A)$ and $E_{pot}(A+1_\Lambda)$ can be written as follows.

$$E_{pot}(A) = 6 \sum_{k_1<k_F,k_2<k_F} (\underset{\sim}{k}_1,\underset{\sim}{k}_2|K(A)|\underset{\sim}{k}_1,\underset{\sim}{k}_2)$$

$$E_{pot}(A+1_\Lambda) = 6 \sum_{k_1<k_F,k_2<k_F} (\underset{\sim}{k}_1,\underset{\sim}{k}_2|K(A+1_\Lambda)|\underset{\sim}{k}_1,\underset{\sim}{k}_2)$$

$$+ 4 \sum_{k_1<k_F} (\underset{\sim}{k}_1,\underset{\sim}{k}_\Lambda = 0|\kappa|\underset{\sim}{k}_1,\underset{\sim}{k}_\Lambda = 0)$$

where K is the reaction matrix for the Λ-N interaction.

$$U = E_{pot}(A+1_\Lambda) - E_{pot}(A)$$

$$= V + V_R$$

where

$$V = 4 \sum_{k_1<k_F} (\underset{\sim}{k}_1, \underset{\sim}{k}_\Lambda = 0|\kappa|\underset{\sim}{k}_1, \underset{\sim}{k}_\Lambda = 0)$$

$$V_R = 6 \sum_{k_1<k_F, k_2<k_F} (\underset{\sim}{k}_1, \underset{\sim}{k}_2|K(A+1_\Lambda) - K(A)|\underset{\sim}{k}_1, \underset{\sim}{k}_2)$$

Here V_R is so called "rearrangement potential".

The effects (iii) and (iv) have been discussed by Dabrowski and Kohler (21), who estimated suppression on the binding energy (iii) to be a few MeV, and the one by (iv) to be about 7 MeV. Their results, although not very reliable because of many approximations in their calculation, seem to indicate the importance of these effects. We plan to reinvestigate this problem in the near future.

Table 1

Potentials	I spin-independent	II Singlet	II Triplet	ST[16] (D&E) Singlet	ST[16] (D&E) Triplet
OCF: $\beta(fm^{-1})$		1.4663		1.1428	1.0685
OCF: $\lambda(10^{-2}\ fm^{-2})$		3.3867		1.7102	1.2563
TCF: $\beta(fm^{-1})$	1.4402	1.4623	1.3233	1.1364	0.9803
TCF: λ_Λ $(10^{-2}\ fm^{-2})$	2.8596	3.3030	0.7222	1.5963	0.
TCF: λ_x $(10^{-2}\ fm^{-2})$	2.5	1.0	5.5	1.0199	3.1978
TCF: λ_Σ $(10^{-2}\ fm^{-2})$	3.5	4.0	1.0	1.5963	0.
$\sigma(\Sigma N \rightarrow \Lambda N)$ (mb)	183	143		139	
$\sigma(\Sigma N \rightarrow \Sigma N)$ (mb)	105	121		120	

Table 2

Potentials			Perturbation	Free G	Complete G	$J_\Sigma' \to J_\Sigma$
I	OCF		46.36	74.12	50.85	
	TCF		41.69	74.39	47.47	51.32
	ΔB		4.67		3.38	-0.47
II	OCF		46.36	74.12	50.85	
	TCF	B_s	45.66	74.16	50.35	50.94
		B_t	14.11	75.64	28.78	53.52
		B	22.00	75.27	34.17	52.87
	ΔB		24.36		16.68	-2.02
ST	OCF	B_s	54.63	81.68	58.12	
		B_t	49.90	68.99	52.38	
		B	51.04	72.17	53.82	
	TCF	B_s	51.95	81.84	56.11	58.37
		B_t	0.	71.30	16.07	55.68
		B	12.99	73.93	26.08	56.35
	ΔB		38.05		27.74	-2.53

APPENDIX A

Schrödinger equation in the TCF is described as follows

$$(T_\Lambda + V_\Lambda)\psi_\Lambda + V_{\Lambda\Sigma}\psi_\Sigma = E\psi_\Lambda$$

$$(T_\Sigma + V_\Sigma)\psi_\Sigma + V_{\Sigma\Lambda}\psi_\Lambda = (E-\Delta)\psi_\Sigma$$

where $T_\Lambda = P_\Lambda^2/2\mu_\Lambda$, $T_\Sigma = P_\Sigma^2/2\mu_\Sigma$ are the kinetic energies in the ΛN and ΣN channels, respectively, and $\Delta \equiv m_\Sigma - m_\Lambda$.

Using the separable potential,

$$\left(\frac{p_\Lambda^2}{2\mu_\Lambda} - E\right)\psi_\Lambda(p_\Lambda, k_\Lambda) - \lambda_\Lambda g_\Lambda(p_\Lambda)\int g_\Lambda(p'_\Lambda)\ \psi_\Lambda(p'_\Lambda, k_\Lambda)d^3p'_\Lambda$$

$$- \lambda_x g_\Lambda(p_\Lambda)\int g_\Sigma(p'_\Sigma)\ \psi_\Sigma(p'_\Sigma, k_\Sigma)d^3p'_\Sigma = 0$$

$$\left(\frac{p_\Sigma^2}{2\mu_\Sigma} - E + \Delta\right)\psi_\Sigma(p_\Sigma, k_\Sigma) - \lambda_\Sigma g_\Sigma(p_\Sigma)\int g_\Sigma(p'_\Sigma)\ \psi_\Sigma(p'_\Sigma, k_\Sigma)d^3p'_\Sigma$$

$$- \lambda_x g_\Sigma(p_\Sigma)\int g_\Lambda(p'_\Lambda)\ \psi_\Lambda(p'_\Lambda, k_\Lambda)d^3p'_\Lambda = 0$$

Let us define k_Λ and k_Σ as follows,

$$E \equiv \frac{k_\Lambda^2}{2\mu_\Lambda}, \quad \Delta - E \equiv \frac{k_\Sigma^2}{2\mu_\Sigma}$$

For the ΛN scattering below the ΣN threshold, $\Delta - E > 0$ then

$k_\Sigma^2>0$. For the energies above the ΣN threshold, we take $k_\Sigma^2<0$.

Let us introduce x_Λ and x_Σ,

$$x_\Lambda \equiv \int g_\Lambda(p_\Lambda')\psi_\Lambda(p_\Lambda',k_\Lambda)d^3p_\Lambda' \tag{A.1}$$

$$x_\Sigma \equiv \int g_\Sigma(p_\Sigma')\psi_\Sigma(p_\Sigma',k_\Sigma)d^3p_\Sigma' \tag{A.2}$$

Then our Schrödinger equation becomes

$$\left(\frac{p_\Lambda^2}{2\mu_\Lambda} - \frac{k_\Lambda^2}{2\mu_\Lambda}\right)\psi_\Lambda - (\lambda_\Lambda X_\Lambda+\lambda_x X_\Sigma)\ g_\Lambda(p_\Lambda) = 0 \tag{A.3}$$

$$\left(\frac{p_\Sigma^2}{2\mu_\Sigma} + \frac{k_\Sigma^2}{2\mu_\Sigma}\right)\psi_\Sigma - (\lambda_\Sigma X_\Sigma+\lambda_x X_\Lambda)\ g_\Sigma(p_\Sigma) = 0 \tag{A.4}$$

The solutions are written as follows.

$$\psi_\Lambda(p_\Lambda,k_\Lambda) = \delta(\underset{\sim}{p}_\Lambda-\underset{\sim}{k}_\Lambda) + \frac{\lambda_\Lambda X_\Lambda+\lambda_x X_\Sigma}{e_\Lambda(p_\Lambda,k_\Lambda)}\ g_\Lambda(p_\Lambda) \tag{A.5}$$

$$\psi_\Sigma(p_\Sigma,k_\Sigma) = \frac{\lambda_\Sigma X_\Sigma+\lambda_x X_\Lambda}{e_\Sigma(p_\Sigma,k_\Sigma)}\ g_\Sigma(p_\Sigma) \tag{A.6}$$

where

$$e_\Lambda(p_\Lambda,k_\Lambda) = (p_\Lambda^2-k_\Lambda^2-i\varepsilon)/2\mu_\Lambda \tag{A.7}$$

$$e_\Sigma(p_\Sigma,k_\Sigma) = (p_\Sigma^2+k_\Sigma^2)/2\mu_\Sigma \tag{A.8}$$

X_Λ and X_Σ are obtained from (A.1), (A.2), (A.5) and (A.6),

$$X_\Lambda = [1-\lambda_\Sigma J_\Sigma(k_\Sigma)]\ g_\Lambda(k_\Lambda)/D(k_\Lambda,k_\Sigma) \tag{A.9}$$

$$X_\Sigma = \lambda_x J_\Sigma(k_\Sigma)\ g_\Lambda(k_\Lambda)/D(k_\Lambda,k_\Sigma) \tag{A.10}$$

where

$$D(k_\Lambda,k_\Sigma) = \begin{vmatrix} 1 - \lambda_\Lambda J_\Lambda(k_\Lambda) \,, & - \lambda_x J_\Lambda(k_\Lambda) \\ - \lambda_x J_\Sigma(k_\Sigma) \,, & 1 - \lambda_\Sigma J_\Sigma(k_\Sigma) \end{vmatrix} \tag{A.11}$$

with

$$J_\Lambda(k_\Lambda) = \int \frac{g_\Lambda^2(p'_\Lambda)}{e_\Lambda(p'_\Lambda,k_\Lambda)} d^3p'_\Lambda \tag{A.12}$$

$$J_\Sigma(k_\Sigma) = \int \frac{g^2(p'_\Sigma)}{e_\Sigma(p'_\Sigma,k_\Sigma)} d^3p'_\Sigma \tag{A.13}$$

Now, wavefunction (A.5) and (A.6) are written as follows.

$$\psi_\Lambda(p_\Lambda,k_\Lambda) = (\underset{\sim}{p}_\Lambda-\underset{\sim}{k}_\Lambda) + \frac{\{\lambda_\Lambda-d(\lambda)J_\Sigma(k_\Sigma)\}g_\Lambda(k_\Lambda)g_\Lambda(p_\Lambda)}{D(k_\Lambda,k_\Sigma)}\cdot\frac{1}{e_\Lambda(p_\Lambda,k_\Lambda)}$$

$$\sim \delta(\underset{\sim}{p}_\Lambda-\underset{\sim}{k}_\Lambda) + T_\Lambda(p_\Lambda,k_\Lambda)/[4\pi^2\mu_\Lambda e_\Lambda(p_\Lambda,k_\Lambda)] \tag{A.14}$$

$$\psi_\Sigma(p_\Sigma,k_\Sigma) = \frac{\lambda_x g_\Lambda(k_\Lambda)g_\Sigma(p_\Sigma)}{D(k_\Lambda,k_\Sigma)}\cdot\frac{1}{e_\Sigma(p_\Sigma,k_\Sigma)}$$

$$\sim T_{\Sigma\Lambda}(p_\Sigma,k_\Lambda)/[4\pi^2\mu_\Sigma e_\Sigma(p_\Sigma,k_\Sigma)] \tag{A.15}$$

where $d(\lambda) = \lambda_\Lambda\lambda_\Sigma - \lambda_x^2$, and $T_\Lambda(p_\Lambda,k_\Lambda)$, $T_{\Sigma\Lambda}(p_\Sigma,k_\Lambda)$ are the scattering amplitudes for $\Lambda N\to\Lambda N$ and $\Lambda N\to\Sigma N$, respectively, namely:

$$T_\Lambda(p_\Lambda,k_\Lambda) = 4\pi^2\mu_\Lambda\{\lambda_\Lambda-d(\lambda)\ J_\Sigma(k_\Sigma)\}\ g_\Lambda(k_\Lambda)\ g_\Lambda(p_\Lambda)/D(k_\Lambda,k_\Sigma) \tag{A.16}$$

$$T_{\Sigma\Lambda}(p_\Sigma,k_\Lambda) = 4\pi^2\mu_\Sigma\lambda_x g_\Lambda(k_\Lambda)\ g_\Sigma(p_\Sigma)/D(k_\Lambda,k_\Sigma) \tag{A.17}$$

The cross sections are given by

$$\begin{aligned} \sigma(\Lambda N \to \Lambda N) &= 4\pi |T_\Lambda|^2 \\ \sigma(\Lambda N \to \Sigma N) &= 4\pi |T_{\Sigma\Lambda}|^2 (V_\Sigma / V_\Lambda)\ \theta(E-\Delta) \end{aligned} \tag{A.18}$$

with $V_\Sigma = k_\Sigma/\mu_\Sigma$, $V_\Lambda = k_\Lambda/\mu_\Lambda$

If ΣN is the entrance channel

$$\begin{aligned} \sigma(\Sigma N \to \Sigma N) &= 4\pi |T_\Sigma|^2 \\ \sigma(\Sigma N \to \Lambda N) &= 4\pi |T_{\Lambda\Sigma}|^2 (V_\Lambda / V_\Sigma) \end{aligned} \tag{A.19}$$

where T_Σ, $T_{\Lambda\Sigma}$ are obtained from T_Λ and $T_{\Sigma\Lambda}$ by the interchanging $\Lambda \rightleftarrows \Sigma$ respectively.

Next, let us obtain the effective range and the scattering length in terms of the parameters of potential. In the following, J_Λ and J_Σ stand for $J_\Lambda(k_\Lambda)$ and $J_\Sigma(k_\Sigma)$ respectively, and D instead of $D(k_\Lambda, k_\Sigma)$.

$$A \equiv \lambda_\Lambda - d(\lambda) J_\Sigma .$$

Then

$$\frac{D}{A} = -J_\Lambda + \frac{1-\lambda_\Sigma J_\Sigma}{A}$$

For the ΛN scattering below the ΣN threshold, J_Λ is complex whereas J_Σ is real.

$$J_\Lambda = R_e(J_\Lambda) + i\, I_m(J_\Lambda)$$

$$T_\Lambda^{-1} = \frac{1}{4\pi^2 \mu_\Lambda g_\Lambda^2(k_\Lambda)} \frac{D}{A}$$

$$= \frac{1}{4\pi^2 \mu_\Lambda g_\Lambda^2(k_\Lambda)} \{-R_e(J_\Lambda) - i I_m(J_\Lambda) + \frac{1-\lambda_\Sigma J_\Sigma}{A}\}$$

Therefore

$$I_m(T_\Lambda^{-1}) = -\frac{I_m(J_\Lambda)}{4\pi^2\mu_\Lambda g_\Lambda^2(k_\Lambda)} \tag{A.20}$$

$$R_e(T_\Lambda^{-1}) = \frac{1}{4\pi^2\mu_\Lambda g_\Lambda^2(k_\Lambda)}\{-R_e(J_\Lambda) + \frac{1-\lambda_\Sigma J_\Sigma}{A}\} \tag{A.21}$$

Now

$$J_\Lambda = \int d^3p_\Lambda \; \frac{g_\Lambda^2(p_\Lambda)}{e_\Lambda(p_\Lambda,k_\Lambda)}$$

$$= 4\pi \; P \int dp_\Lambda \; \frac{p_\Lambda^2 g_\Lambda^2(p_\Lambda)}{e_\Lambda(p_\Lambda,k_\Lambda)} + i\{4\pi^2\mu_\Lambda k_\Lambda g_\Lambda^2(k_\Lambda)\}$$

Therefore

$$I_m(J_\Lambda) = 4\pi^2\mu_\Lambda k_\Lambda g_\Lambda^2(k_\Lambda) \tag{A.22}$$

$$R_e(J_\Lambda) = 4\pi \; P \int dp_\Lambda \; \frac{p_\Lambda^2 g_\Lambda^2(p_\Lambda)}{e_\Lambda(p_\Lambda,k_\Lambda)} \tag{A.23}$$

From (A.20) and (A.22), we obtain

$$I_m(T_\Lambda^{-1}) = -k_\Lambda$$

and, from (A.21) and (A.23)

$$R_e(T_\Lambda^{-1}) = \frac{1}{4\pi^2\mu_\Lambda g_\Lambda^2(k_\Lambda)}\left\{-4\pi \; P \int dp_\Lambda \; \frac{p_\Lambda^2 g_\Lambda^2(p_\Lambda)}{e_\Lambda(p_\Lambda,k_\Lambda)} + \frac{1-\lambda_\Sigma J_\Sigma}{A}\right\} \tag{A.25}$$

On the other hand, if we take only s-wave in partial wave analysis, the scattering amplitude is written like

$$T_\Lambda = e^{i\delta}\sin\delta/k$$

$$T_\Lambda^{-1} = -ik_\Lambda + k_\Lambda \cot \delta . \tag{A.26}$$

Therefore, from (A.25) and (A.26), we obtain

$$\begin{aligned} k_\Lambda \cot \delta &= R_e(T_\Lambda^{-1}) \\ &= \frac{1}{4\pi^2 \mu_\Lambda g_\Lambda^2(k_\Lambda)} \left\{ -4\pi \; P \int dp_\Lambda \frac{p_\Lambda^2 g_\Lambda^2(p_\Lambda)}{e_\Lambda(p_\Lambda, k_\Lambda)} + \frac{1-\lambda_\Sigma J_\Sigma}{A} \right\} \end{aligned} \tag{A.27}$$

with

$$A = \lambda_\Lambda - d(\lambda) J_\Sigma$$

$$d(\lambda) = \lambda_\Lambda \lambda_\Sigma - \lambda_x^2 .$$

Here, if we assume the separable potential of Yamaguchi type;

$$g_i(k_i) = \frac{1}{k_i^2+\beta^2} , \quad i = \Lambda \text{ or } \Sigma .$$

Then

$$P \int dp_\Lambda \frac{p_\Lambda^2 g_\Lambda^2(p_\Lambda)}{e_\Lambda(p_\Lambda, k_\Lambda)} = -\frac{\pi}{4} \frac{k_\Lambda^2 - \beta^2}{\beta(k_\Lambda^2+\beta^2)^2} \tag{A.28}$$

$$J_\Sigma = \int d^3 p_\Sigma \frac{g_\Sigma^2(p_\Sigma)}{e_\Sigma(p_\Sigma, k_\Sigma)} = \frac{2\pi^2 \mu_\Sigma}{\beta(k_\Sigma+\beta)^2} \tag{A.29}$$

Therefore, (A.27) becomes

$$k_\Lambda \cot \delta = -\beta + \frac{\beta^2+k_\Lambda^2}{2\beta} + \frac{(\beta^2+k_\Lambda^2)^2}{4\pi^2 \mu_\Lambda \lambda_\Lambda} \cdot H \tag{A.30}$$

where

$$H \equiv \frac{1-\lambda_\Sigma J_\Sigma}{1-\lambda' J_\Sigma} = \frac{\beta(\beta+k_\Sigma)^2 - 2\pi^2\mu_\Sigma\lambda_\Sigma}{\beta(\beta+k_\Sigma)^2 - 2\pi^2\mu_\Sigma\lambda'}$$

with

$$\lambda' = d(\lambda)/\lambda_\Lambda .$$

k_Λ and k_Σ are related to each other by following relation

$$\frac{k_\Sigma^2}{2\mu_\Sigma} = \Delta - \frac{k_\Lambda^2}{2\mu_\Lambda} ,$$

here, let us introduce κ_Λ and κ_Σ

$$\Delta \equiv \frac{\kappa_\Lambda^2}{2\mu_\Lambda} = \frac{\kappa_\Sigma^2}{2\mu_\Sigma} \tag{A.31}$$

Then

$$\begin{aligned} k_\Sigma &= \kappa_\Sigma\sqrt{1-x^2} \\ &= \kappa_\Sigma(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 + \ldots.) \end{aligned} \tag{A.32}$$

with

$$x \equiv \frac{k_\Lambda}{\kappa_\Lambda}$$

Using (A.32)

$$H = \frac{\beta\{\beta^2+2\beta\kappa_\Sigma(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 + \ldots.) + \kappa_\Sigma^2(1-x^2)\} - 2\pi^2\mu_\Sigma\lambda_\Sigma}{\beta\{\beta^2+2\beta\kappa_\Sigma(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 + \ldots.) + \kappa_\Sigma^2(1-x^2)\} - 2\pi^2\mu_\Sigma\lambda'}$$

$$= \frac{B}{C}\{1 - \frac{\beta\kappa_\Sigma(\beta+k_\Sigma)}{B}x^2 - \frac{\beta^2\kappa_\Sigma}{4B}x^4\}\{1 - \frac{\beta\kappa_\Sigma(\beta+k_\Sigma)}{C}x^2 - \frac{\beta^2\kappa_\Sigma}{4C}x^4\}^{-1}$$

$$= \frac{B}{C}\{1 - \frac{\beta\kappa_\Sigma(\beta+\kappa_\Sigma)}{B}\, x^2 - \frac{\beta^2\kappa_\Sigma}{4B}\, x^4\}$$

$$\{1 + \frac{\beta\kappa_\Sigma(\beta+\kappa_\Sigma)}{C}\, x^2 + (\frac{\beta^2\kappa_\Sigma^2(\beta+\kappa_\Sigma)^2}{C^2} + \frac{\beta^2\kappa_\Sigma}{4C})\, x^4\}$$

$$= \frac{B}{C}(1 - a_1x^2 - a_2x^4)(1 + a_3x^2 + a_4x^4)$$

where

$$B = \beta(\beta+\kappa_\Sigma)^2 - 2\pi^2\mu_\Sigma\lambda_\Sigma$$

$$C = \beta(\beta+\kappa_\Sigma)^2 - 2\pi^2\mu_\Sigma\lambda'$$

$$a_1 = \frac{\beta\ {}_\Sigma(\beta+\kappa_\Sigma)}{B}, \quad a_2 = \frac{\beta^2\kappa_\Sigma}{4B}$$

$$a_3 = \frac{\beta\kappa_\Sigma(\beta+\kappa_\Sigma)}{C}, \quad a_4 = \frac{\beta^2\kappa_\Sigma^2(\beta+\kappa_\Sigma)^2}{C^2} + \frac{\beta^2\kappa_\Sigma}{4C}$$

Thus, the third term of R.H.S. in (A.30) becomes

$$\frac{(\beta^2+k_\Lambda^2)^2}{4\pi^2\mu_\Lambda\lambda_\Lambda} H = \frac{1}{4\pi^2\mu_\Lambda\lambda_\Lambda}\frac{B}{C}\{\beta^4 + [2\beta^2\kappa_\Lambda^2 + (a_3-a_1)\beta^4]x^2$$

$$+ [(a_4-a_1a_3-a_2)\beta^4 + 2(a_3-a_1)\beta^2\kappa_\Lambda^2 + \kappa_\Lambda^4]x^4$$

$$+ [-(a_1a_4+a_2a_3)\beta^4 + 2\beta^2\kappa_\Lambda^2(a_4-a_1a_2-a_2) + (a_3-a_1)\kappa_\Lambda^4]x^6 + \ldots\}$$

In the effective range approximation

$$k_\Lambda \cot\delta = -\frac{1}{a} + \frac{1}{2}\, rk_\Lambda^2 - Pr^3k_\Lambda^4 + Qr^5k_\Lambda^6 + \ldots. \qquad (A.33)$$

By comparing the coefficients of (A.30) and (A.33),

$$\frac{1}{a} = \frac{\beta}{2}\{1 - \frac{\beta^3[\beta(\beta+\kappa)^2 - \gamma_\Sigma]}{\gamma_\Lambda\beta(\beta+\kappa)^2 - d(\gamma)}\} \tag{A.34}$$

$$r = \frac{3}{\beta} - \frac{4}{a\beta^2} - \frac{\mu_\Sigma\gamma_x\beta^5(\beta+\kappa)}{\mu_\Lambda\kappa[\gamma_\Lambda\beta(\beta+\kappa)^2 - d(\gamma)]^2} \tag{A.35}$$

where $\gamma_\Lambda = 2\pi^2\mu_\Lambda\lambda_\Lambda$, $\gamma_\Sigma = 2\pi^2\mu_\Sigma\lambda_\Sigma$, $\gamma_x = 2\pi^2(\mu_\Lambda\mu_\Sigma)^{1/2}\lambda_x$ and $d(\gamma) = \gamma_\Lambda\gamma_\Sigma - \gamma_x^2$, $\kappa = \kappa_\Sigma = (2\mu_\Sigma\Delta)^{1/2}$. We can get P and Q by comparing the coefficients of k_Λ^4 and k_Λ^6, respectively if we need.

APPENDIX B

Let us describe the functions in nuclear matter by superscript N,

$$G^N\Phi = V\psi^N$$

$$\Phi = \begin{pmatrix}\phi_\Lambda \\ o\end{pmatrix}, \ \psi^N = \begin{pmatrix}\psi^N_\Lambda \\ \psi^N_\Sigma\end{pmatrix}, \ V = \begin{pmatrix}V_\Lambda V_x \\ V_x V_\Sigma\end{pmatrix}$$

$$<\Phi|G^N|\Phi> = (\Phi_\Lambda, 0)\begin{pmatrix}V_\Lambda V_x \\ V_x V_\Sigma\end{pmatrix}\begin{pmatrix}\psi^N_\Lambda \\ \psi^N_\Sigma\end{pmatrix}$$

$$= (\phi_\Lambda, V_\Lambda\psi^N_\Lambda) + (\phi_\Lambda, V_x\psi^N_\Sigma)$$

$$<k'_\Lambda|G^N|k_\Lambda> = G^N_\Lambda + G^N_{\Lambda\Sigma} \qquad (B.1)$$

where

$$G^N_\Lambda = -\lambda_\Lambda g_\Lambda(k'_\Lambda) \int g_\Lambda(p'_\Lambda)\ \psi^N_\Lambda(p'_\Lambda, k_\Lambda)d\underset{\sim}{p}'_\Lambda \qquad (B.2)$$

$$G^N_{\Lambda\Sigma} = -\lambda_x g_\Lambda(k'_\Lambda) \int g_\Sigma(p'_\Sigma)\ \psi^N_\Sigma(p'_\Sigma, k_\Sigma)d\underset{\sim}{p}'_\Sigma \qquad (B.3)$$

wavefunctions in nuclear matter are,

$$\psi^N_\Lambda(p_\Lambda, k_\Lambda) = \delta(\underset{\sim}{p}_\Lambda - \underset{\sim}{k}_\Lambda) + (\lambda_\Lambda X^N_\Lambda + X^N_\Sigma) \quad Qg_\Lambda(p_\Lambda)/e^N_\Lambda(p_\Lambda, k_\Lambda) \qquad (B.4)$$

$$\psi^N_\Sigma(p_\Sigma, k_\Sigma) = (\lambda_\Sigma X_\Sigma + \lambda_x X_\Lambda)\ Qg_\Sigma(p_\Sigma)/e^N_\Sigma(p_\Sigma, k_\Sigma) \qquad (B.5)$$

Where Q is the Pauli operator which takes account of the exclusion principle for the nucleon, $e^N_\Lambda(p_\Lambda,k_\Lambda)$ and $e^N_\Sigma(p_\Sigma,k_\Sigma)$ are energy denominator in nuclear matter;

$$e^N_\Lambda(p_\Lambda,k_\Lambda) = e_\Lambda(p_\Lambda,k_\Lambda) + U_N$$

$$e^N_\Sigma(p_\Sigma,k_\Sigma) = e_\Sigma(p_\Sigma,k_\Sigma) + U_N$$

with

$$U_N = 85.17 - 12.08\ p_N^2 \text{ MeV}$$

where p_N is initial momentum of nucleon in fm unit.

$$X^N_\Lambda = (1-\lambda_\Lambda J^N_\Sigma)\ g_\Lambda(k_\Lambda)/D^N$$

$$X^N_\Sigma = \lambda_x J_\Sigma g_\Lambda(k_\Lambda)/D^N$$

$$D^N = \begin{vmatrix} 1 - \lambda_\Lambda J^N_\Lambda & -\lambda_x J^N_\Lambda \\ -\lambda_x J^N_\Sigma & 1 - \lambda_\Sigma J^N_\Sigma \end{vmatrix} \tag{B.6}$$

$$J^N = \int \frac{Qg^2_\Lambda(p_\Lambda)}{e_\Lambda(p_\Lambda,k_\Lambda)}\, dp_\Lambda \tag{B.7}$$

$$J^N_\Sigma = \int \frac{Qg^2_\Sigma(p_\Sigma)}{e_\Sigma(p_\Sigma,k_\Sigma)}\, dp_\Sigma \tag{B.8}$$

Substituting (B.4) into (B.2) and (B.5) into (B.3) respectively

$$G_\Lambda = -\lambda_\Lambda g_\Lambda(k'_\Lambda)\ g_\Lambda(k_\Lambda)(1-\lambda_\Sigma J^N_\Sigma)/D^N \tag{B.9}$$

$$G_{\Sigma\Lambda} = -\lambda_x^2 g_\Lambda(k'_\Lambda)\ g_\Lambda(k_\Lambda) J^N_\Sigma/D^N \tag{B.10}$$

Thus

$$G(k_\Lambda, k'_\Lambda) = G_\Lambda + G_{\Lambda\Sigma}$$

$$= -g_\Lambda(k_\Lambda)\ g_\Lambda(k'_\Lambda)(\lambda_\Lambda - d(\lambda)J^N_\Sigma)/D^N \tag{B.11}$$

with (B.16), (B.7) and (B.8) and $d(\lambda) = \lambda_\Lambda\lambda_\Sigma - \lambda_x^2$.

First of all, let us consider the restriction due to the Pauli operator Q, in the integrals J^N_Λ and J^N_Σ. Let us denote the initial momenta of nucleon and Λ-particle by $\mathbf{p}_N$ and $\mathbf{p}_\Lambda$ respectively, and intermediate momenta, by $\mathbf{p}'_N$ and $\mathbf{p}'_\Lambda$ respectively. From momentum conservation,

$$\mathbf{p}_N + \mathbf{p}_\Lambda = \mathbf{p}'_N + \mathbf{p}'_\Lambda\ . \tag{B.12}$$

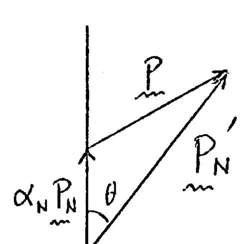
Let us denote the relative momenta for the Λ-nucleon pair in the initial and intermediate states by $\mathbf{k}_\Lambda$ and $\mathbf{p}$ respectively,

$$\mathbf{k}_\Lambda = \frac{m_\Lambda \mathbf{p}_N - m_N \mathbf{p}_\Lambda}{m_\Lambda + m_N} = \alpha_\Lambda \mathbf{p}_N - \alpha_N \mathbf{p}_\Lambda \tag{B.13}$$

$$\mathbf{p} = \frac{m_\Lambda \mathbf{p}'_N - m_N \mathbf{p}'_\Lambda}{m_\Lambda + m_N} = \alpha_\Lambda \mathbf{p}'_N - \alpha_N \mathbf{p}'_\Lambda \tag{B.14}$$

where $\alpha_\Lambda = m_\Lambda/(m_\Lambda + m_N)$, $\alpha_N = m_N/(m_\Lambda + m_N)$. For the ground state, $\mathbf{p}_\Lambda = 0$. From (B.12), (B.13) and (B.14) after considering $\mathbf{p}_\Lambda = 0$, we obtain

$$\mathbf{p}'_N = \mathbf{p} + \alpha_N \mathbf{p}_N \tag{B.15}$$



The nucleon momenta in initial and intermediate states should be below and above the fermi surface respectively.

$$p_N<k_F \quad \text{and} \quad p_N'>k_F \tag{B.16}$$

From (B.15) and (B.16)

$$p^2 + (\alpha_N p_N)^2 + 2\alpha_N p_N \, p \cos\theta > k_F^2 \, .$$

or

$$\cos\theta > X \equiv \frac{k_F^2 - p^2 - (\alpha_N p_N)^2}{2\alpha_N p_N p} \tag{B.17}$$

There are three cases.

a) $X > 1$. No angle is allowed.

The integral region about p in this case is determined from (B.17).

$$p<k_F - \alpha_N p_N$$

$$\int d(\cos\theta) = 0 \, .$$

b) $1 > X > -1$. Some angle are allowed.

The integral region about p in this case is from (B.17).

$$k_F + \alpha_N p_N > p > k_F - \alpha_N p_N$$

$$\int_X^1 d(\cos\theta) = 1 - X = \frac{(p+\alpha_N p_N)^2 - k_F^2}{2\alpha_N p_N p}$$

c) $X < -1$. Any angle is allowed.

The integral region about p in this case is from (B.17).

$$p > k_F + \alpha_N p_N \quad \text{or} \quad p < -k_F + \alpha_N p_N$$

Obviously the latter is impossible.

$$\int_{-1}^{1} d(\cos\theta) = 2 \; .$$

$$\begin{array}{ccccccc} & & X=1 & & X=-1 & & \\ & & \downarrow & & \downarrow & & \\ \hline \text{(a)} & & k_F-\alpha_N p_N & \text{(b)} & k_F+\alpha_N p_N & \text{(c)} & \rightarrow p \end{array}$$

$$\alpha_N p_N = \frac{m_N}{m_\Lambda + m_N}\, p_N = \frac{m_N}{m_\Lambda}\, \alpha_\Lambda p_N = \frac{m_N}{m_\Lambda}\, k_\Lambda \equiv \eta k_\Lambda$$

Thus (B.7) is written as follows.

$$J_\Lambda^N(k_\Lambda) = 4\pi\Big\{ \int_{k_F+\eta k_\Lambda}^{\infty} dp\; p + \frac{1}{4\eta k_\Lambda} \int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} dp\,[(p+\eta k_\Lambda)^2 - k_F^2]\Big\} \frac{p g_\Lambda^2(p)}{e_\Lambda(p,k_\Lambda)} \qquad (B.18)$$

The same procedure is available for the ΣN intermediate state. Let us denote the initial momenta of nucleon and Λ-particle by $\underset{\sim}{p}_N$ and $\underset{\sim}{p}_\Lambda = 0$, respectively, and the intermediate momenta of nucleon and Σ-particle by $\underset{\sim}{p}'_N$ and $\underset{\sim}{p}'_\Sigma$, respectively. From momentum conservation,

$$\underset{\sim}{p}_N = \underset{\sim}{p}'_N + \underset{\sim}{p}'_\Sigma$$

Let us denote the relative momenta of the ΛN apir in the initial state and the ΣN pair in the intermediate state by $\underset{\sim}{k}_\Lambda$ and $\underset{\sim}{p}$, respectively. Then

$$\underset{\sim}{k}_\Lambda = \alpha_\Lambda \underset{\sim}{p}_N$$

$$\underset{\sim}{p} = \beta_\Sigma \underset{\sim}{p}'_N - \beta_N \underset{\sim}{p}'_\Sigma$$

where $\beta_\Sigma = m_\Sigma/(m_\Sigma+m_N)$, $\beta_N = m_N/(m_\Sigma+m_N)$. Almost all the necessary formulae for the ΣN case are obtained from those for the ΛN case by replacing α's with β's.

$$\beta_N p_N = \frac{\beta_N}{\alpha_\Lambda} \alpha_\Lambda p_N = \frac{\beta_N}{\alpha_\Lambda} k_\Lambda \equiv \eta' k_\Lambda$$

$$\eta' = \frac{\beta_N}{\alpha_\Lambda} = \frac{m_N}{m_\Lambda} \cdot \frac{m_\Lambda+m_N}{m_\Sigma+m_N} = \eta \cdot \frac{m_\Lambda+m_N}{m_\Sigma+m_N}$$

Thus (B.8) is written as follows

$$J_\Sigma^N(k_\Sigma) = 4\pi\Big\{ \int_{k_F+\eta' k_\Lambda}^{\infty} dp\, p + \frac{1}{4\eta' k_\Lambda} \int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} dp\,[(p+\eta' k_\Lambda)^2 - k_F^2]\Big\} \frac{p g^2(p)}{e_\Sigma^N(p,k_\Sigma)} \tag{B.19}$$

<u>Calculation of J_Λ^N</u>

$$e_\Lambda^N = (p^2-k_\Lambda^2)/2\mu_\Lambda + U_N = (p^2+K_\Lambda^2)/2\mu_\Lambda$$

where

$$K_\Lambda^2 = 2\mu_\Lambda U_N - k_\Lambda^2$$

with $U_N = 85.17 - 12.08\ (k_\Lambda/\alpha_\Lambda)^2$ MeV, but k_Λ is in fm.

$$J_\Lambda^N = 8\pi\mu_\Lambda(I_{\Lambda 1} + \frac{1}{4\eta k_\Lambda}.I_{\Lambda 2})$$

where

$$I_{\Lambda 1} = \int_{k_F+\eta k_\Lambda}^{\infty} \frac{p^2 g^2(p)}{p^2 + K_\Lambda^2}\, dp$$

$$I_{\Lambda 2} = \int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} \frac{[(p+\eta k_\Lambda)^2 - k_F^2]\ pg^2(p)}{p^2 + K_\Lambda^2}\, dp$$

with

$$g(p) = \frac{1}{p^2 + \beta^2}$$

$$\frac{p^2}{(p+K_\Lambda^2)(p^2+\beta^2)^2} = \frac{A_\Lambda}{p^2 + K_\Lambda^2} + \frac{B_\Lambda p^2 + C_\Lambda}{(p^2+\beta^2)^2}$$

where

$$A_\Lambda = -\frac{K_\Lambda^2}{(\beta^2-K_\Lambda^2)^2}$$

$$B_\Lambda = -A_\Lambda$$

$$C_\Lambda = \frac{\beta^4}{(\beta^2-K_\Lambda^2)^2}$$

$$I_{\Lambda 1} = A_\Lambda S_1 + B_\Lambda S_2 + C_\Lambda S_3$$

$$I_{\Lambda 2} = (\eta^2 k_\Lambda^2 - k_F^2)S_4 + 2\eta k_\Lambda S_5 + S_6\ .$$

where

$$S_1 = \int_{k_F+\eta k_\Lambda}^{\infty} \frac{dp}{p^2 + K_\Lambda^2}$$

$$= \frac{1}{K_\Lambda}[\frac{\pi}{2} - \tan^{-1} \frac{k_F+\eta k_\Lambda}{K_\Lambda}]$$

$$S_2 = \int_{k_F+\eta k_\Lambda}^{\infty} \frac{p^2 dp}{(p^2+\beta^2)^2}$$

$$= \frac{1}{2}\{\frac{\pi}{2\beta} + f(+) - \frac{1}{\beta} \tan^{-1} \frac{k_F+\eta k_\Lambda}{\beta}\}$$

$$S_3 = \int_{k_F+\eta k_\Lambda}^{\infty} \frac{dp}{(p^2+\beta^2)^2}$$

$$= \frac{1}{2\beta^3}\left\{\frac{\pi}{2} - \beta f_\Lambda(+) - \tan^{-1} \frac{k_F+\eta k_\Lambda}{\beta}\right\}$$

$$S_4 = \int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} \frac{p dp}{(p^2+K_\Lambda^2)(p^2+\beta^2)^2}$$

$$= \frac{1}{2(\beta^2-K_\Lambda^2)} \{ \frac{f_\Lambda(+)}{k_F+\eta k_\Lambda} - \frac{f_\Lambda(-)}{k_F-\eta k_\Lambda} + \frac{1}{\beta^2 - K_\Lambda^2} \ln R_\Lambda\}$$

$$S_6 = \int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} \frac{p^3 dp}{(p^2+K_\Lambda^2)(p^2+\beta^2)^2}$$

$$= \frac{1}{2}\{\frac{f_\Lambda(-)}{k_F-\eta k_\Lambda} - \frac{f_\Lambda(+)}{k_F+\eta k_\Lambda}\} - K_\Lambda^2 S_4$$

$$S_5 = \int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} \frac{p^2 dp}{(p^2+K_\Lambda^2)(p^2+\beta^2)^2} \quad .$$

$$= A_\Lambda S_{51} + B_\Lambda S_{52} + C_\Lambda S_{53} \quad .$$

where

$$S_{51} = \int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} \frac{dp}{p^2 + K_\Lambda^2}$$

$$= \frac{1}{K_\Lambda}(\tan^{-1} \frac{k_F+\eta k_\Lambda}{K_\Lambda} - \tan^{-1} \frac{k_F-\eta k_\Lambda}{K_\Lambda})$$

$$S_{52} = \int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} \frac{p^2 dp}{(p^2+\beta^2)^2}$$

$$= \frac{1}{2}\{f_\Lambda(-) - f_\Lambda(+) + \frac{1}{\beta}(\tan^{-1} \frac{k_F+\eta k_\Lambda}{\beta} - \tan^{-1} \frac{k_F-\eta k_\Lambda}{\beta})\}$$

$$S_{53} = \int_{k_F-\eta k_\Lambda}^{k_F+\eta k_\Lambda} \frac{dp}{(p^2+\beta^2)^2}$$

$$= \frac{1}{2\beta^2}\{f_\Lambda(+) - f_\Lambda(-) + \frac{1}{\beta}(\tan^{-1} \frac{k_F+\eta k_\Lambda}{\beta} - \tan^{-1} \frac{k_F-\eta k_\Lambda}{\beta})\}$$

with

$$f_\Lambda(+) \equiv \frac{k_F+\eta k_\Lambda}{(k_F+\eta k_\Lambda)^2 + \beta^2}$$

$$f_\Lambda(-) \equiv \frac{k_F-\eta k_\Lambda}{(k_F-\eta k_\Lambda)^2 + \beta^2}$$

$$R_\Lambda \equiv \frac{[(k_F+\eta k_\Lambda)^2 + K_\Lambda^2][(k_F-\eta k_\Lambda)^2 + \beta^2]}{[(k_F+\eta k_\Lambda)^2 + \beta^2][(k_F-\eta k_\Lambda)^2 + K_\Lambda^2]}$$

Calculation of J_Σ^N

$$e_\Sigma^N = (p^2+k_\Sigma^2)/2\mu_\Sigma + U_N = (p^2+K_\Sigma^2)/2\mu_\Sigma$$

$$K_\Sigma^2 = 2\mu_\Sigma U_N + k_\Sigma^2$$

U_N = same in the case of J_Λ^N.

$$J_\Sigma^N = 8\pi\mu_\Sigma(I_{\Sigma 1} + \frac{1}{4\eta' k_\Lambda} I_{\Sigma 2})$$

where

$$I_{\Sigma 1} = \int_{k_F+\eta' k_\Lambda}^{\infty} \frac{p^2 dp}{(p^2+K_\Sigma^2)(p^2+\beta^2)^2}$$

$$= A_\Sigma S_{\Sigma 1} + B_\Sigma S_{\Sigma 2} + C_\Sigma S_{\Sigma 3}$$

$$I_{\Sigma 2} = \int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} \frac{[(p+\eta' k_\Lambda)^2 - k_F^2]p}{(p^2+K_\Lambda^2)(p^2+\beta^2)^2}$$

$$= (\eta'^2 k_\Lambda^2 - k_F^2)S_{\Sigma 4} + 2\eta' k_\Lambda S_{\Sigma 5} + S_{\Sigma 6}$$

where

$$A_\Sigma = -\frac{K_\Sigma^2}{(\beta^2-K_\Sigma^2)^2}$$

$$B_\Sigma = -A_\Sigma$$

$$C_\Sigma = \frac{\beta^4}{(\beta^2 - K_\Sigma^2)^2}$$

$$S_{\Sigma 1} = \int_{k_F+\eta' k_\Lambda}^{\infty} \frac{dp}{p^2 + K_\Sigma^2}$$

$$= \frac{1}{K_\Sigma} \left(\frac{\pi}{2} - \tan^{-1} \frac{k_F+\eta' k_\Lambda}{K_\Sigma}\right)$$

$$S_{\Sigma 2} = \int_{k_F+\eta' k_\Lambda}^{\infty} \frac{p^2 dp}{(p^2+\beta^2)^2}$$

$$= \frac{1}{2}\{\frac{\pi}{2\beta} + f_\Sigma(+) - \frac{1}{\beta} \tan^{-1} \frac{k_F+\eta' k_\Lambda}{\beta}\}$$

$$S_{\Sigma 3} = \int_{k_F+\eta' k_\Lambda}^{\infty} \frac{dp}{(p^2+\beta^2)^2}$$

$$= \frac{1}{2\beta^3}\{\frac{\pi}{2} - \beta f_\Sigma(+) - \tan^{-1} \frac{k_F+\eta' k_\Lambda}{\beta}\}$$

$$S_{\Sigma 4} = \int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} \frac{pdp}{(p^2+K_\Sigma^2)(p^2+\beta^2)^2}$$

$$= \frac{1}{2(\beta^2-K_\Sigma^2)}\{\frac{f_\Sigma(+)}{k_F+\eta' k_\Lambda} - \frac{f_\Sigma(-)}{k_F-\eta' k_\Lambda} + \frac{1}{\beta^2-K_\Sigma^2} \ln R_\Sigma\}$$

$$S_{\Sigma 6} = \int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} \frac{p^3 dp}{(p^2+K_\Sigma^2)(p^2+\beta^2)^2}$$

$$= \frac{1}{2}\left\{\frac{f_\Sigma(-)}{k_F-\eta' k_\Lambda} - \frac{f_\Sigma(+)}{k_F+\eta' k_\Lambda}\right\} - K_\Sigma^2 S_{\Sigma 4}$$

$$S_{\Sigma 5} = \int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} \frac{p^2 dp}{(p^2+K_\Sigma^2)(p^2+\beta^2)^2}$$

$$= A_\Sigma S_{\Sigma 51} + B_\Sigma S_{\Sigma 52} + C_\Sigma S_{\Sigma 53}$$

where

$$S_{\Sigma 51} = \int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} \frac{dp}{p^2+K_\Sigma^2}$$

$$= \frac{1}{K_\Sigma}\left(\tan^{-1} \frac{k_F+\eta' k_\Lambda}{K_\Sigma} - \tan^{-1} \frac{k_F-\eta' k_\Lambda}{K_\Sigma}\right)$$

$$S_{\Sigma 52} = \int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} \frac{p^2 dp}{(p^2+\beta^2)^2}$$

$$= \frac{1}{2}\{f_\Sigma(-) - f_\Sigma(+) + \frac{1}{\beta}(\tan^{-1} \frac{k_F+\eta' k_\Lambda}{\beta} - \tan^{-1} \frac{k_F-\eta' k_\Lambda}{\beta})\}$$

$$S_{\Sigma 53} = \int_{k_F-\eta' k_\Lambda}^{k_F+\eta' k_\Lambda} \frac{dp}{(p^2+\beta^2)^2}$$

$$= \frac{1}{2\beta^2}\{f_\Sigma(+) - f_\Sigma(-) + \frac{1}{\beta}(\tan^{-1} \frac{k_F+\eta' k_\Lambda}{\beta} - \tan^{-1} \frac{k_F-\eta' k_\Lambda}{\beta})\}$$

with

$$f_\Sigma(+) \equiv \frac{k_F+\eta' k_\Lambda}{(k_F+\eta' k_\Lambda)^2 + \beta^2}$$

$$f_\Sigma(-) \equiv \frac{k_F-\eta' k_\Lambda}{(k_F-\eta' k_\Lambda)^2 + \beta^2}$$

$$R_\Sigma \equiv \frac{[(k_F+\eta' k_\Lambda)^2 + K_\Sigma^2][(k_F-\eta' k_\Lambda)^2 + \beta^2]}{[(k_F+\eta' k_\Lambda)^2 + \beta^2][(k_F-\eta' k_\Lambda)^2 + K_\Sigma^2]}$$

Perturbation calculation in lowest order

Putting $D^N = 1.0$ and $d(\lambda) = 0$ in (B.11)

$$G(k_\Lambda,k_\Lambda) = -\lambda_\Lambda g_\Lambda^2(k_\Lambda) = -\frac{\lambda_\Lambda}{(k_\Lambda^2+\beta^2)^2}$$

Free G-matrix approximation

G^F is given by

$$G^F = \frac{1}{4\pi^2\mu_\Lambda}\ \frac{1}{k_\Lambda \cot\delta}$$

From (A.27)

$$k_\Lambda \cot\delta = \frac{1}{4\pi^2\mu_\Lambda g_\Lambda^2(k_\Lambda)}\{-R_e(J_\Lambda) + \frac{1 - \lambda_\Sigma J_\Sigma}{\lambda_\Lambda - d(\lambda)J_\Sigma}\}$$

where

$$R_e(J_\Lambda) = 8\pi\mu_\Lambda \; P \int \frac{p_\Lambda^2 g_\Lambda^2(p_\Lambda)}{e_\Lambda(p_\Lambda, k_\Lambda)} dp_\Lambda$$

$$= -\frac{2\pi^2\mu_\Lambda}{\beta} \quad \frac{k_\Lambda^2 - \beta^2}{(k_\Lambda^2+\beta^2)^2}$$

$$J_\Sigma = \int \frac{g_\Sigma^2(p_\Sigma)}{e_\Sigma(p_\Sigma, k_\Sigma)} d^3p_\Sigma = \frac{2\pi^2\mu_\Sigma}{\beta(k_\Sigma+\beta)^2}$$

$$d(\lambda) = \lambda_\Lambda \Lambda_\Sigma - \lambda_x^2$$

$$\frac{k_\Lambda^2}{2\mu_\Lambda} = \Delta - \frac{k_\Sigma^2}{2\mu_\Sigma}$$

$\underline{J_\Sigma^N \rightarrow J_\Sigma}$

In order to see the effect of the Σ-component, let us remove the restriction due to the Pauli principle in the ΣN virtual state. Taking J_Σ(A.29) instead of J_Σ^N(B.8) in $G(k_\Lambda, k_\Lambda)$ (B.11), whereas J_Λ^N keep the same.

$$G(k_\Lambda, k_\Lambda) = -\, g_\Lambda^2(k_\Lambda) \quad \frac{\lambda_\Lambda - d(\lambda) J_\Sigma}{D^N}$$

where

$$D^N = 1 - \lambda_\Lambda J_\Lambda^N - \lambda_\Sigma J_\Sigma + d(\lambda) J_\Lambda^N J_\Sigma$$

with

$$J_\Lambda^N = \int \frac{Qg_\Lambda^2(p_\Lambda)}{e_\Lambda^N(p_\Lambda,k_\Lambda)} d^3p_\Lambda \qquad \text{(Equivalent to (B.7) or (B.18))}$$

$$J_\Sigma = \int \frac{g_\Sigma^2(p_\Sigma)}{e_\Sigma(p_\Sigma,k_\Sigma)} d^3p_\Sigma \qquad \text{(Equivalent to (A.29))}$$

$$d(\lambda) = \lambda_\Lambda\lambda_\Sigma - \lambda_x^2$$

TABLE CAPTIONS

Table 1. Parameters for the three potentials I, II and ST, in the OCF and TCF. The potentials I and II fit the spin-averaged low-energy ΛN scattering parameters (3.25), while ST fit (3.30). The potential ST is a combination of AEA Spin O, D and Spin 1, E in Table II of Schick and Toepfer (28). The calculated cross sections for $\Sigma N \rightarrow \Lambda N$ and $\Sigma N \rightarrow \Sigma N$ at P_Σ = 150 MeV/c are also shown.

Table 2. Results of the binding-energy calculation of a Λ-particle in nuclear matter in MeV, in various methods as explained in the text. The analytical form of the G-matrix used is given in Appendix B. $\Delta B \equiv B(OCF) - B(TCF)$ represents the suppression of B due to the Λ-Σ conversion.

FIGURES CAPTIONS

Figure 1. (a) The diagram for the pionic component of the Λ-N potential.

(b) The diagram for the K-exchange component of the Λ-N potential.

(c) The diagram for the pionic component of the ΛNN three-body potential, N_1 and N_2 mean the different nucleons.

Figure 2. Relation between the OCF and TCF. The wavy line represents the OCF potential which the dashed line stands for the TCF potential. The detailed discussions for the diagram (a), (b) and (c) are given in Chapter 2.

Fig. 1

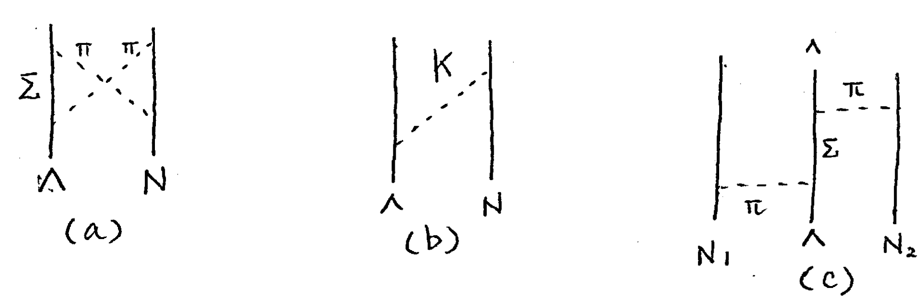
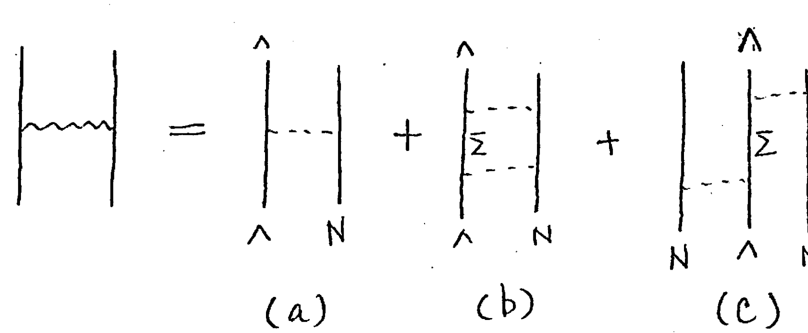


Fig. 2.



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