INFLUENCE OF SPHERICAL AND CYLINDRICAL CAPSULES ON PRESSURE TRANSIENTS IN HYDRAULIC PIPELINES

INFLUENCE OF SPHERICAL AND CYLINDRICAL CAPSULES ON PRESSURE TRANSIENTS IN HYDRAULIC PIPELINES

BY

RAJBIR SINGH SAMRA

A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Master of Engineering

McMaster University

November, 1970

MASTER OF ENGINEERING(1970)
(Mechanical Engineering)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Influence of Spherical and Cylindrical Capsules on Pressure Transients in Hydraulic Pipelines

AUTHOR: Rajbir Singh Samra

SUPERVISOR: Dr. G. F. Round

NUMBER OF PAGES: viii, 72

SCOPE AND CONTENTS:

The influence of capsules on pressure transients in hydraulic pipelines has been reported in this dissertation. This experimental investigation was carried out for flow velocities of 1 to 4 ft/sec. for both cylindrical and spherical capsules. Shock waves were generated by a fast closing valve in a 1 in. diameter pipe and pressure transients were measured for a train of cylindrical capsules of diameter 0.875 in. and 0.95 in. The spherical capsules were of diameter 0.75 in. and 0.875 in.

A theoretical analysis for this type of waterhammer problem using a perturbation technique has been obtained. The theoretical solution was found to be in agreement with the experimental observations for cylindrical capsules.

ACKNOWLEDGEMENTS

The author expresses his deep sense of gratitude and indebtness to Dr. G. F. Round, the Chairman of the department of Mechanical Engineering, for his invaluable guidance and constant encouragement throughout this work.

The research for this thesis was supported in part by the Defence Research Board of Canada, Grant No. 9550-42 and in part by the National Research Council of Canada, Grant No. A5183.

TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

w*	fluid velocity
ρ	density
ρ_m	mean density
P	pressure
t	time
a	speed of sound in the fluid
K	isothermal bulk compression modulus
x	axial co-ordinate
u	axial component of velocity
q	discharge
A	area of cross-section
L	length between solenoid value and the primary reflection site
A_o	area of cross-section of the pipe at the solenoid valve
Q	discharge at steady-state condition
M	Mach number
B(x)	function of capsule size and capsule train length
ε	perturbation parameter
α	an arbitrary constant
P_o, P_1, P_2	zero-order, first-order, second-order perturbation pressure

---

* quantities with a bar are physical quantities having dimensions.

q_o, q_1, q_2	zero-order, first-order and second-order perturbation discharge
f_1, f_2, f_3	arbitrary functions of time
c_1, c_2, c_3, c_4	functions of the geometry of the system
P_i	initial pressure
Q_i	initial discharge
P_c	zero-order pressure in a capsule-containing pipeline
P_f	zero-order pressure in a capsule-free pipeline

1. INTRODUCTION

The influence of capsules on pressure transients in a hydraulic pipeline (waterhammer waves) has been studied in this investigation. The transportation of capsules in pipelines has aroused interest in this area. The word 'capsule' in this context has come to mean a large regularly-shaped body, whose minor axis is comparable to the diameter of pipe in which it is travelling. The capsule may be hollow or solid, cast or extruded, coated or uncoated, rigid or non-rigid and cylindrical or spherical in shape.

The Research Council of Alberta has been largely responsible for work that has been done on capsule pipelining. In a series of papers having the general title "The pipeline flow of capsules" part (1-9)[8-16] the emphasis has been on the determination of capsule velocity or velocity ratio (capsule/free stream) and pressure drops entailed as a function of average velocity, diameter ratio and capsule/liquid density ratio. The capsules used in the experimental tests have been cylindrical or spherical, hollow or solid and of a wide range of diameter ratio; the cylindrical ones having a variety of lengths.

Part 1 of this series describes the emergence of the concept of the pipeline transport of solids by a stream of capsules [8]; part 2 presents a simplified theory of the flow of long concentric cylindrical capsules [9], and part 3 discusses an experimental investigation of the transport by water of cylinders and spheres of equal density [10]. Parts 4 and 5 are concerned with experimental results relating to cylindrical and spherical capsules of density greater than water [11,12]. Part 6 presents a numerical analysis of some variables affecting the flow of cylinders [13]. In part 7 experiments are repeated using water with equal density capsules in an oil carrier [14]. Part 8 is an experimental investigation with cylinders and spheres denser than an oil carrier [15]. Part 9 presents a theoretical study of a free flowing infinite long capsule [16].

However, to date, no significant work has been reported on the influence of capsules on the pressure transients in hydraulic pipelines. The propagation of pressure transient by the disturbance of steady conditions, either intentionally or by a system failure such as a pump stoppage, is a major consideration in the design of pipeline systems.

The basic concepts and equations of waterhammer analysis were developed and confirmed experimentally by Joukowsky [1] and Allievi [2] at the beginning of this century. It should be noted that all of the methods used in waterhammer analysis stem from the same basic equations of Joukowsky and Allievi. These basic relations have been amplified but still remain the foundation of all waterhammer studies. It has been definitely established that the same basic equations apply to the surge computations for any fluid in any type of closed conduit provided the physical factors are known.

Numerical and graphical methods have been used for the solution of these equations with friction omitted from the theory, but included in lumped form in the graphical methods. Some analytical solutions have been obtained by linearizing the equations with friction included [3,4,5].

Waterhammer, as applied to slightly compressible liquids in elastic conduits, may be classified into two categories: the fluid transient case in which the conditions change from one steady-state situation to another steady-state situation and the steady oscillating case, in which the same cycle is repeated periodically. The first case solution by the method of characteristics has been found to be more convenient in that it takes non-linearity into account and yields simple relationships for handling the most complex boundary conditions; in the second case is more conveniently handled by the impedence method in which friction is linearized and the boundary conditions are satisfied by taking a harmonic analysis of the exciting variable, such as head or discharge, then summing the solutions for each harmonic.

In recent years, Streeter and Wylie [6] have developed computer solutions of hydraulic system transients. They first took up the characteristics method and applied it to several situations varying from a simple pipe to a pipe network. Pump power failure with the resulting pressure transient have also been discussed by Streeter and Wylie. The impedence method was next developed for computer solution and applied to piping systems excited by reciprocating pumps or by waves on a reservoir and to self-excited systems.

The method of characteristics has been found to be successful for most of the problems and many physical phenomena can be fully explained by this method. But it is important to note that in this method one studies the flow by dividing the flow field into a network of characteristics (along which certain flow properties remain constant) and then satisfying the boundary conditions at the ends. The full equations are never solved. The method of characteristics may not be the best one when solving problems having internal disturbances in the flow field, such as having capsules in a pipeline. In the present investigation, a mathematical approach using the perturbation technique has been used for such a problem.

1.1 The Present Method

The one-dimensional problem of a slightly compressible, inviscid, unsteady flow is governed by two equations - continuity and momentum. The variation in density is small and the equation of state relates the velocity of sound to the compressibility of water. In the governing equations, velocity has been replaced by discharge as it is convenient to deal in terms of discharge for a varying area section. The governing equations are non-dimensionalized. A perturbation parameter, ϵ, exists and it is proportional to the square of appropriate Mach number. Asymptotic expansions for the flow variables (pressure and discharge) can be written as follows. If f denotes the flow variable, then

$$f(x,t;\epsilon) = f_0(x,t) + \epsilon f_1(x,t) + \epsilon^2 f_2(x,t) + \ldots\ldots$$

substituting such expansions in the governing equations and collecting in turn, terms independent of ε and first powers of ε, we get the zero order equations and the first order perturbation equations. The system of equations can be solved and the integration constants can be evaluated from the boundary conditions. Assuming an exponential function for valve closure, simple expressions for pressure transients may be obtained.

The analysis indicates that qualitatively, internal area changes affect the magnitude of the pressure transients and these changes result in magnification of the waterhammer wave in a pipeline containing capsules as compared to waves in a capsule-free pipeline. Magnification of the pressure in the presence of capsules might seem to be surprising but this has been also observed experimentally.

2. THEORY

2.1. Assumptions

1. The fluid (water) is inviscid.

2. The flow is one-dimensional.

3. The fluid (water) is slightly compressible, and variations in density are small.

4. Convective acceleration is negligible as compared to the local acceleration.

5. The flow is unsteady.

6. The boundaries of the flow are rigid.

7. There are no external forces.

2.2. Basic Equations

The flow of a slightly compressible liquid is completely described by the following equations [18].

Euler:

$$\bar{\rho} \frac{D\bar{W}}{D\bar{t}} = - \nabla\bar{p} \tag{1}$$

Continuity:

$$\nabla\cdot(\bar{\rho}\bar{W}) + \frac{\partial\bar{\rho}}{\partial\bar{t}} = 0 \tag{2}$$

Equation of State:

$$K = \bar{\rho} \frac{\partial\bar{p}}{\partial\bar{\rho}} \tag{3}$$

or

$$\bar{a}^2 = \frac{\partial\bar{p}}{\partial\bar{\rho}}$$

where: $\bar{w}$ is fluid velocity

$\bar{\rho}$ is density

$\bar{t}$ is time

$\bar{p}$ is pressure

$\bar{a}$ is speed of sound in the fluid, and

K is the isothermal bulk compression modulus.

Assumptions 3 and 4 above for liquids are valid for all reasonable velocities (say less than 40 ft/sec.) and pressure pulses of the order of 1000 psi. With these assumptions the Euler equation (1) becomes

$$\frac{\partial \bar{p}}{\partial \bar{x}} = - \bar{\rho}_m \frac{\partial \bar{u}}{\partial \bar{t}} \tag{4}$$

where: $\bar{x}$ is the axial co-ordinate

$\bar{\rho}_m$ is the mean density

$\bar{u}$ is the axial components of velocity.

Since one-dimensional (plane) flow has been assumed $\bar{p}$ and $\bar{u}$ are functions only of $\bar{x}$ and $\bar{t}$.

Averaging equation (4) across an arbitrary cross-section of area $\bar{A}(\bar{x})$ thus gives

$$\frac{\partial \bar{p}}{\partial \bar{x}} = - \frac{\bar{\rho}_m}{\bar{A}(\bar{x})} \frac{\partial \bar{q}}{\partial \bar{t}} \tag{5}$$

where

$$\bar{q} = \int_{\bar{A}} \bar{u} \, d\bar{A} \tag{6}$$

Averaging the continuity equation (2) over a cross-section of the pipeline and utilizing the equation of state (3) to eliminate $\bar{\rho}$ gives

$$\frac{\partial \bar{q}}{\partial \bar{x}} = - \frac{\bar{A}(\bar{x})}{K} \frac{\partial \bar{p}}{\partial \bar{t}} \qquad (7)$$

Therefore, we have the governing equations (5) and (7).

$\bar{x}$ and $\bar{q}$ are positive when both are measured in the same direction; that is, from reservoir to the gate (in this case from the reservoir R, to the solenoid valve B (c.f. Figs. 1 and 4)). However, for solutions of waterhammer problems involving a gate movement at the downstream end, it is more convenient to express the distance to a section of the pipe from the lower end of the pipe as a positive distance, since initial disturbances in the flow occur first at the downstream end of the pipeline and then move upstream.

Taking $\bar{x}$ as positive from the valve, with $\bar{x} = 0$ denoting the solenoid valve and $\bar{x} = L$ denoting the reflecting bend, the governing equations become

$$\frac{\partial \bar{p}}{\partial \bar{x}} = \frac{\bar{\rho}_m}{\bar{A}(\bar{x})} \frac{\partial \bar{q}}{\partial \bar{t}} \qquad (8)$$

$$\frac{\partial \bar{q}}{\partial \bar{x}} = \frac{\bar{A}(\bar{x})}{K} \frac{\partial \bar{p}}{\partial \bar{t}} \qquad (9)$$

For non-dimensionalisation, the standard length, discharge and cross-section have been taken as:

L = length between solenoid valve and the primary reflection site

A_o = area of cross-section of the pipe at the valve

Q = discharge at steady-state conditions.

2.3. Non-Dimensional Forms

Let us define the non-dimensional variables as follows:

$$x = \frac{\bar{x}}{L} \tag{10.a}$$

$$q = \frac{\bar{q}}{Q} \tag{10.b}$$

$$A = \frac{\bar{A}}{A_o} \tag{10.c}$$

$$p = \frac{\bar{p}}{\rho_m \frac{Q^2}{A_o^{\;2}}} = \frac{\bar{p}\, A_o^{\;2}}{\rho_m\, Q^2} \tag{10.d}$$

$$t = \frac{\bar{t}}{L/(Q/A_o)} = \frac{Q\, \bar{t}}{L\, A_o} \tag{10.e}$$

$$a = \frac{\bar{a}}{(Q/A_o)} = \frac{1}{M} \tag{10.f}$$

$$K' = \frac{K}{\rho_m (Q^2/A_o^{\;2})} = \frac{A_o^{\;2} K}{Q^2 \rho_m} \equiv \frac{\rho \bar{a}^2}{\rho_m\, Q^2/A_o^{\;2}} \sim \frac{1}{M^2} \tag{10.g}$$

$$= \frac{1}{\varepsilon} \text{ (say)}$$

M is the Mach number

$M \ll 1$ for this problem

ε is a very small quantity, which can be taken as a perturbation parameter

$$\varepsilon = 0(M^2) \ll 1 \tag{10.h}$$

Also

$$\frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial x} \quad \frac{dx}{d\bar{x}} \tag{11}$$

Using the non-dimensional scheme, we get, after simplification, the governing equations as follows:

$$\frac{\partial p}{\partial x} = \frac{1}{A} \frac{\partial q}{\partial t} \tag{12}$$

$$\frac{\partial q}{\partial x} = \varepsilon A \frac{\partial p}{\partial t} \tag{13}$$

2.4. Zero-Order and First-Order Perturbation Equations

Let us define asymptotic expansions for pressure and discharge as follows:

$$p(x,t;\varepsilon) = p_o(x,t) + \varepsilon p_1(x,t) + \varepsilon^2 p_2(x,t) + \ldots. \tag{14.a}$$

$$q(x,t;\varepsilon) = q_o(x,t) + \varepsilon q_1(x,t) + \varepsilon^2 q_2(x,t) + \ldots. \tag{14.b}$$

where p_o, p_1, p_2 denotes the zero-order, first-order and second-order perturbation pressure, and

q_o, q_1, q_2 refers to discharge.

Substituting the above expansions in equation (12) and (13) we get

$$\frac{\partial p_o}{\partial x} + \varepsilon \frac{\partial p_1}{\partial x} + \varepsilon^2 \frac{\partial p_2}{\partial x} + \ldots\ldots = \frac{1}{A}\left\{\frac{\partial q_o}{\partial t} + \varepsilon \frac{\partial q_1}{\partial t} + \varepsilon^2 \frac{\partial q_2}{\partial t} + \ldots\right\} \tag{15.a}$$

and

$$\frac{\partial q_o}{\partial x} + \varepsilon \frac{\partial q_1}{\partial x} + \varepsilon^2 \frac{\partial q_2}{\partial x} + \ldots = \varepsilon A\left\{\frac{\partial p_o}{\partial t} + \varepsilon \frac{\partial p_1}{\partial t} + \varepsilon^2 \frac{\partial p_2}{\partial t} + \ldots\right\} \tag{15.b}$$

Collecting, in turn, terms independent of ε; and first power of ε we have

$$\frac{\partial p_o}{\partial x} = \frac{1}{A}\frac{\partial q_o}{\partial t} \tag{16.a}$$

$$\frac{\partial q_o}{\partial x} = 0 \tag{16.b}$$

$$\frac{\partial p_1}{\partial x} = \frac{1}{A}\frac{\partial q_1}{\partial t} \tag{16.c}$$

$$\frac{\partial q_1}{\partial x} = A\frac{\partial p_o}{\partial t} \tag{16.d}$$

Equation (16.b) implies that q_o is not a function of x.

$q_o = q_o(t)$ and we can write $\frac{dq_o}{dt}$ instead of a partial derivative in (16.a).

$$q_o = q_o(t) \tag{17.a}$$

Solving equation (16.a)

$$\frac{\partial p_o}{\partial x} = \frac{1}{A(x)} \frac{dq_o}{dt}$$

integrating with respect to x

$$p_o(x,t) = \frac{dq_o}{dt} \int \frac{dx}{A(x)} + f_1(t)$$

It is quite reasonable to write $\int \frac{dx}{A(x)} = c_1 \frac{x}{A}$ in this case as the variation of A with respect to x is only significant for a small interval of x. c_1 is a function of the geometry of the system and = 1 for a capsule-free pipeline

> 1 for a capsule-containing pipeline.

$f_1(t)$ is some function of t and should be evaluated from the boundary conditions. The conditions at x = 0 may be used for evaluating $f_1(t)$.

We have

$$p_o(o,t) = p(0,t)$$

i.e. the total rise in pressure at the valve is just equivalent to the zero-order pressure. This is quite reasonable as the first-order perturbation pressure p_1 is not significant at the valve.

$$p(0,t) = 0 + f_1(t)$$

so

$$p_o(x,t) = c_1\frac{x}{A} \frac{dq_o}{dt} + p(0,t) \qquad (17.b)$$

The above equation may be interpreted as the zero-order pressure and is equal to the sum of the pressure at the valve plus the pressure generated due to change of momentum.

$$\left[\frac{x}{A} \frac{dq_o}{dt} \equiv x\frac{dU_o}{dt}, \right.$$ rate of change of momentum per unit area].

Substituting equation (17.b) into equation (16.d) we have

$$\frac{\partial q_1}{\partial x} = A\left(\frac{\partial p_o}{\partial t}\right) = A\left\{ \frac{c_1 x}{A} \frac{d^2 q_o}{dt^2} + \frac{dp(0,t)}{dt} \right\}$$

Integrating with respect to x we have

$$q_1 = \frac{d^2 q_o}{dt^2} \int c_1 x \, dx + \frac{dp(0,t)}{dt} \int A \, dx$$

$$= c_2 \frac{x^2}{2} \frac{d^2 q_o}{dt^2} + V \frac{dp(0,t)}{dt} + f_2(t)$$

It is reasonable to take $\int c_1 \, xdx = c_2 \frac{x^2}{2}$ in this case, and c_2 is the function of the geometry of the system.

$c_2 = 1$ for a capsule-free pipeline

> 1 for a capsule-containing pipeline

$\int Adx = V$, is the volume.

The first-order perturbation velocity will be zero at the valve ($x = 0$). For evaluating $f_2(t)$ we can use the boundary condition that perturbation is zero at $x = 0$. Also at $x = 0$, $V \to 0$. Therefore,

$$f_2(t) = 0$$

We have

$$q_1(x,t) = c_2 \frac{x^2}{2} \frac{d^2 q_o}{dt^2} + V \frac{dp(0,t)}{dt} \tag{17.c}$$

substituting in equation (16.c) we have an expression for the first-order perturbation pressure

$$\frac{\partial p_1}{\partial x} = \frac{1}{A(x)} \frac{\partial q_1}{\partial t}$$

$$= \frac{1}{A(x)} \{ c_2 \frac{x^2}{2} \frac{d^3 q_o}{dt^3} + V \frac{d^2 p(0,t)}{dt^2} \}$$

Integrating with respect to x

$$p_1 = \frac{d^3 q_o}{dt^3} \int \frac{c_2 x^2}{2A(x)} dx + \frac{d^2 p(0,t)}{dt^2} \int \frac{V dx}{A(x)} + f_3(t)$$

As variation of A with respect to x is for a short interval of x, we can take

$$\int \frac{c_2 x^2}{A(x)} dx = c_3 \frac{x^3}{3A(x)}$$

and

$$\int \frac{V}{A} dx = \frac{c_4 x}{A}$$

c_3 and c_4 are functions of the geometry of the system and

$c_3 = 1$ for a capsule-free pipeline

> 1 for a capsule-containing pipeline.

Also $f_3(t)$ be evaluated from the boundary condition that at the valve the first-order pressure is zero.

Then $f_3(t) = 0$. Therefore

$$p_1 = \frac{c_3 x^3}{6A} \frac{d^3 q_o}{dt^3} + \frac{c_4 x}{A} \frac{d^2 p(0,t)}{dt^2} \tag{17.d}$$

We have the expressions for zero-order and first-order perturbation quantities [Equations 17(a) - (d)] in terms of $q_o(t)$, $p(0,t)$ and $A(x)$. If we define $q_o(t)$ and $p(0,t)$ by some appropriate expressions, then it is possible to obtain q_o, p_o, q_1 and p_1. We can have a number of expressions for $q_o(t)$ and $p(0,t)$ depending on the type of valve, its generating characteristics and other physical dimensions.

In the analysis to follow, $p(0,t)$ is related to the discharge and then results are obtained by defining $q_o(t)$. The effect of area change can be seen from these expressions.

The pressure variation at the valve can be reasonably estimated from the well-known result for a waterhammer wave and this equation is

$$d\bar{H} = - \frac{\bar{a}}{g} \, d\bar{V} \tag{18}$$

$d\bar{H}$ denotes a rise in the head; $\bar{V}$ is the velocity of flow. Also

$$\bar{p} = \bar{\rho} \, \bar{g} \, \bar{H} \tag{19}$$

By using equation (19) the equation (18) is modified as

$$d\bar{p} = - \bar{a} \, \rho \, d\bar{V} \tag{20.a}$$

Averaging over the cross-section

$$d\bar{p} = - \frac{\bar{a}\rho_m}{\bar{A}(x)} \, d\bar{q} \tag{20.b}$$

Using the non-dimensional scheme as defined previously, we get

$$dp = -\frac{dq}{AM} \tag{21}$$

Now at the valve, i.e. $x = 0$, the pressure $= p(0,t)$; $A = 1$ and

$$q(0,t) = q_o; \qquad \text{as} \qquad q_1 \to 0 \qquad \text{at} \quad x = 0$$

therefore

$$dp(0,t) = -\frac{1}{M}\, dq_o \tag{22.a}$$

Also we can write

$$\frac{dp(0,t)}{dt} = -\frac{1}{M}\,\frac{dq_o}{dt} \tag{22.b}$$

$$\frac{d^2p(0,t)}{dt^2} = -\frac{1}{M}\,\frac{d^2q_o}{dt^2} \tag{22.c}$$

To find $p(0,t)$ we integrate equation (22.a)

$$\int_{p_i}^{p} dp(0,t) = -\frac{1}{M}\int_{Q_i}^{q_o} dq$$

where p_i and Q_i are the initial pressure and flow rate

p and q_o are the pressure and discharge at anytime during valve closure.

Therefore

$$p(0,t) - p_i = -\frac{1}{M}\,[q_o - Q_i] \tag{23}$$

Non-dimensional $Q_i = 1$. Substituting in equations (17 a-d) we get

$$q_o(x,t) = q_o(t) \tag{24.a}$$

$$p_o(x,t) = c_1 \frac{x}{A} \frac{dq_o}{dt} + \frac{1}{M}(1 - q_o) + p_i \tag{24.b}$$

$$q_1(x,t) = c_2 \frac{x^2}{2} \frac{d^2q_o}{dt^2} - \frac{V}{M} \frac{dq_o}{dt} \tag{24.c}$$

$$p_1(x,t) = \frac{c_3x^3}{A6} \frac{d^3q_o}{dt^3} - \frac{c_4x}{AM} \frac{d^2q_o}{dt^2} \tag{24.d}$$

The function $q_o(t)$ refers to the variation of the flow rate with respect to time and this depends on the type of the valve and its closure rate. Also, the function assumed should be such that:

At $t = 0$, q_o must be equal to the full value of flow, and for large t; $q_o \to 0$.

The function should be continuous and analytic throughout the range of interest. If the function or its derivatives are discontinuous, then the analysis becomes invalid. It is reasonable to assume an exponential function for the zero-order discharge for valve closure, taking

$$q_o(t) = Q\,\bar{e}^{\alpha t} \tag{25.a}$$

where α is an arbitrary constant; $Q = 1$ (non-dimensional)

$$q_o(t) = \bar{e}^{\alpha t} \tag{25.b}$$

It is possible to assume any other type of expression for $q_o(t)$, but this exponential form will illustrate the procedure in a simple way. As pointed out earlier, the choice of the function depends on the valve characteristics and physical dimensions. For $q_o = \bar{e}^{\alpha t}$ the equations (24 a-d) are modified as follows:

$$q_o = \bar{e}^{\alpha t} \qquad (26.a)$$

$$p_o(x,t) = - c_1 \frac{x}{A} \alpha \bar{e}^{\alpha t} + \frac{1}{M} (1 - \bar{e}^{\alpha t}) + p_i \qquad (26.b)$$

$$q_1(x,t) = \alpha \bar{e}^{\alpha t} (\alpha c_2 \frac{x^2}{2} + \frac{V}{M}) \qquad (26.c)$$

$$p_1(x,t) = - \alpha^2 \bar{e}^{\alpha t} \frac{x}{A} (\frac{\alpha c_3 x^3}{6} + \frac{c_4}{M}) \qquad (26.d)$$

2.5. Analysis of Pressure

We have

$$p_o(x,t) = - \frac{\alpha c_1 x}{A} \bar{e}^{\alpha t} + \frac{1}{M} (1 - \bar{e}^{\alpha t}) + p_i \qquad (26.e)$$

It is reasonable to assume that the zero-order pressure wave is travelling at the speed of sound in water. One may write then

$$\bar{x} = - \bar{a} \bar{t} \qquad (27.a)$$

and using non-dimensionalisation

$$x = - \frac{t}{M} \qquad (27.b)$$

This expression (equation 27.b) is applicable only after valve closure has started, as propagation of the wave is related to valve closure. Substitution of equation (27.b) into equation (26.b) gives

$$p_o = \frac{1}{M} [1 + \bar{e}^{\alpha t} (\frac{c_1 \alpha t}{A} - 1)] + p_i \tag{28}$$

For a capsule-free pipeline cross-section is constant (A = 1) throughout the length of the pipeline. But for a capsule-containing pipeline the flow area is reduced for the portion equivalent to the length of the capsule train. Let the capsule area be denoted by B(x). Then, for a capsule-containing pipeline, A = 1 - B(x). In the present investigation, variation in area is only for a small interval of x; say x = 0.01, i.e. capsules are present over one percent of the total length of the pipeline.

Let the pressure transient for a capsule-free pipeline be denoted by p_f and for a capsule-containing pipeline be p_c. For a capsule-free pipeline B(x) = 0 and c_1 = 1, but for a capsule-containing pipeline B(x) is positive for some interval of x and $c_1 > 1$. Then

$$p_f = \frac{1}{M} [1 + \bar{e}^{\alpha t}(\alpha t - 1)] + p_i \tag{29.a}$$

$$p_c = \frac{1}{M} [1 + \bar{e}^{\alpha t}(\frac{\alpha t c_1}{1 - B(x)} - 1)] + p_i \tag{29.b}$$

Therefore, the change in pressure due to capsules is:

$$p_c - p_f = \frac{\alpha t \, \bar{e}^{\alpha t}}{M} \left(\frac{c_1}{1 - B(x)} - 1 \right) \tag{30}$$

The above expression is always positive for a capsule-containing pipeline, c_1 and B(x) depend on the size of the capsules and length of the capsule train.

The above analysis indicates qualitatively that capsules present in a pipeline will produce magnification of the waterhammer wave. The amount of magnification will depend on the capsule size and train length.

The first-order perturbation pressure will also be slightly affected by the capsules but for short lengths, the cummulative effect will be negligible.

It is important to remember that the above analysis is true for an inviscid, slightly compressible fluid and the flow is one-dimensional and unsteady.

3. APPARATUS AND EXPERIMENTAL PROCEDURE

3.1. General Requirements

The system was designed and instrumented to meet the following requirements:

(1) provision for generation of waterhammer waves, i.e. a fast-closing valve at the downstream end of the line.

(2) the pipeline is to be sufficiently long so that valve closure in relation to the test section could be considered as instant closure. This means that valve closure time should have to be less than 2 L/a.

L = distance between test section and the primary reflection site

a = velocity of sound in fluid used (water)

(3) provision made so that the number of capsules mounted, their spacing, and capsule/pipe diameter ratio could be varied

(4) response time for the pressure transducer system to be small

(5) recording instrument should be able to record pressure transients continuously

(6) extraneous pressure variation should be minimum

(7) the system should not leak during pressure wave propagation.

3.2. Apparatus Description

The apparatus which was used in the experiments is shown schematically in Figure 1. The flow direction from the reservoir, R, to the solenoid valve is shown in Figure 1. The reservoir was a rectangular tank 36 in. x 36 in. x 30 in. fitted with a 2 1/2 in. diameter pipe leading to the pump; a 1 in. diameter drain line was provided. The pump was rated at 3 H.P. , delivering up to 100 I.G.P.M. at 50 ft. water head. The pump was directly coupled to an electric motor (3600 r.p.m.). The reservoir facilitated the removal of entrapped air in the pipeline.

A 2 in. diameter discharge line connected the pump to the surge chamber assembly. The surge chamber had an effective volume of 904 cu.in. and was connected to a nitrogen bottle through a 1 in. globe valve.

The 2 in. diameter line was connected to 1 in. diameter loop via a 90^{o} elbow. The total length of 1 in. diameter line was 245 ft. In order that this line could be accommodated in a laboratory approximately 50 ft. long, the pipe was built in three loops containing straight lengths of 37 ft. and semicircular bends each having a radius of curvature of 1.5 ft. (c.f. Figure 2). Swaffield [17] has indicated that if the ratio of bend radius to pipe inside diameter is greater than six then there will be practically complete transmission of a pressure transient at the bend. In the present instance, care was taken to maintain a bend radius >> 6, thus avoiding reflection at intermediate points.

The flow rate was controlled by the throttling valve near the junction of 1 in. and 2 in. diameter pipe. A rotameter, covering the range 0 - 10 I.G.P.M., indicated the flow rate.

The line was mounted on angle iron frames, which were bolted to the ground. Cross-bars were provided to reduce the vibrations of the frame. Although the vibrations of the line were reduced by firmly holding it with U clamps, this source of extraneous pressure waves was not completely removed however the magnitude was reduced to an acceptable level.

The shock wave was generated by a solenoid-actuated, pressure operated fast-closing valve located at the end of pipe loop. The valve selected had a closing time of less than 1/15 sec. and it ensured instantaneous closure condition at the test section. Six pressure points were provided on the test section and one near the primary reflection site. T_1, T_2 and T_3 in Figures 1 and 2 indicate the position of the pressure transducers. T_1 indicated the pressure history at the reflection bend. T_2 and T_3 indicated the pressure history at two points on the test section; on either side of the capsule train. The distance between T_1 and T_3 was 231 ft.

The pressure cells mounted have the following specifications and trace identification.

Position	Pressure Cell Rating		Trace Identification
	$Kg/cm^2/P.F.$	PSI/P.F.	
T_3	54.5	774	upper trace
T_2	40.0	568	middle trace
T_1	30.5	443	lower trace

A 6 ft. long, 1 in. diameter lucite tube was used as a test section, located near the final discharge end of the pipe loop. The test section could be easily connected or detached by means of

Johnson dresser couplings. A number of holes were provided in the test section for locating the capsules. The capsules could be placed at the desired position by a prong device and then held in position by small bolts.

The capsules used were:

(a) solid cylinders, length 5 in., diameter 7/8 in. and 0.95 in. prepared from lucite bars

(b) plastic spheres of 7/8 in. and 3/4 in. diameter

Pressures were measured by means of capacitive type transducers connected to tuning plugs, which in turn were connected to an oscillator-reactance converter (c.f. Figure 6). The sensing element in the type of transducer used was a stainless steel diaphram. Applied pressure varied the capacitance between the diaphram and a fixed electrode. This capacitance formed part of a resonant circuit that controlled the frequency of an oscillator. A frequency detector then converted the frequency variation into an analog D.C. voltage. Hence, a change in a mechanical displacement could be converted via a change in capacitance into a frequency change which, in turn, could be converted into a D.C. voltage in a reactance convertor.

Figure 5 shows a special adaptor for installation without a pressure channel for measuring high pressure variations. It is important to have a short pressure channel for measuring the transient pressure. The adapter made it possible to install the pressure cell flush with the walls of test section.

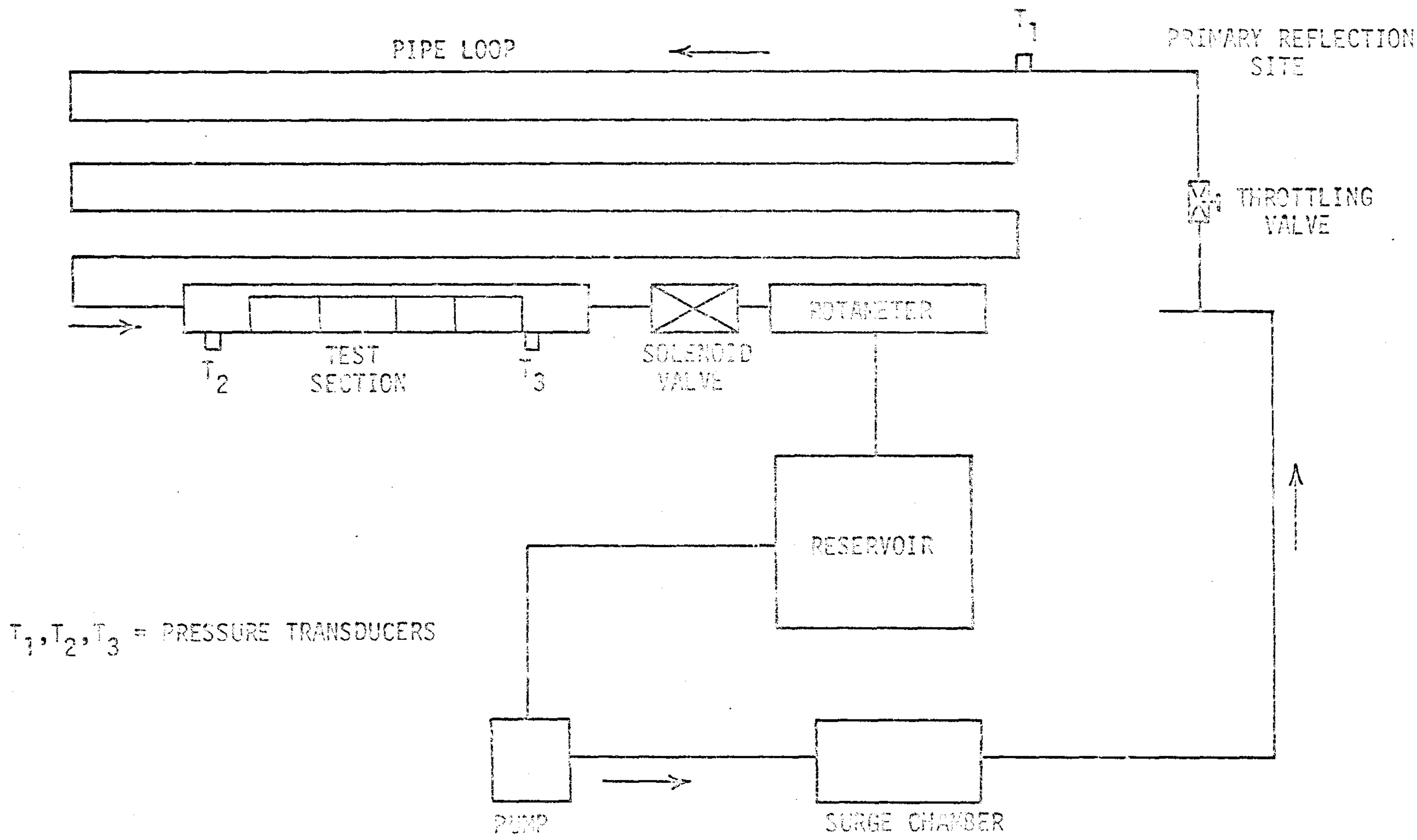


FIG. 1 SCHEMATIC DIAGRAM OF APPARATUS

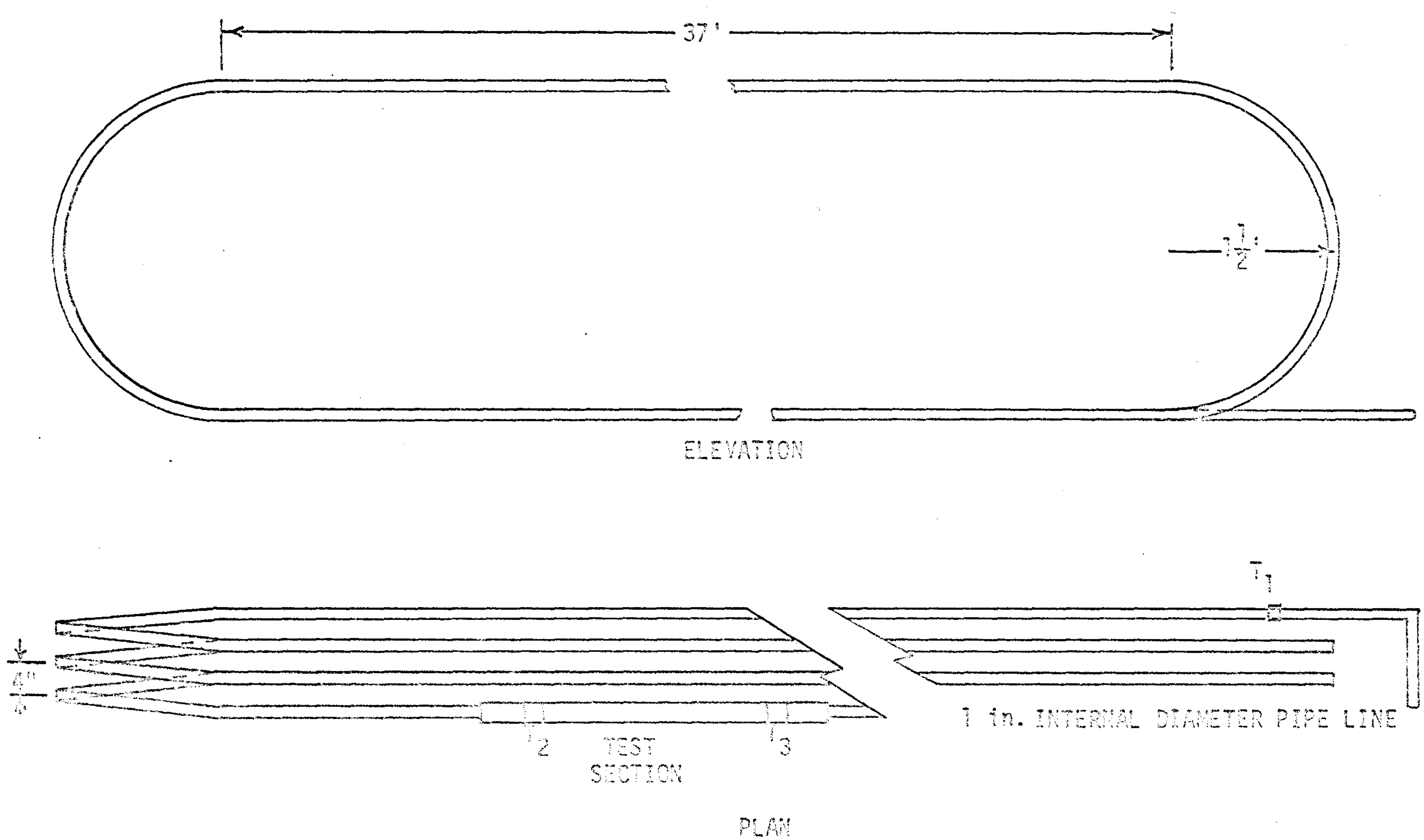


FIG. 2 DETAILS OF PIPE LOOP

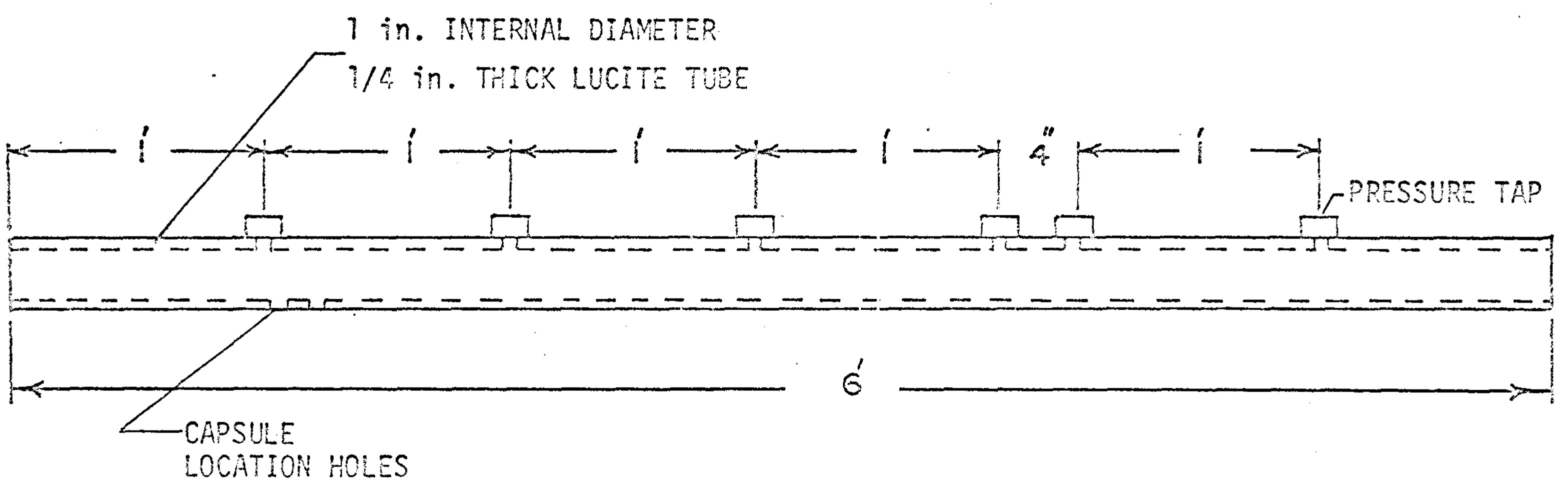


FIG. 3 DETAILS OF THE TEST SECTION

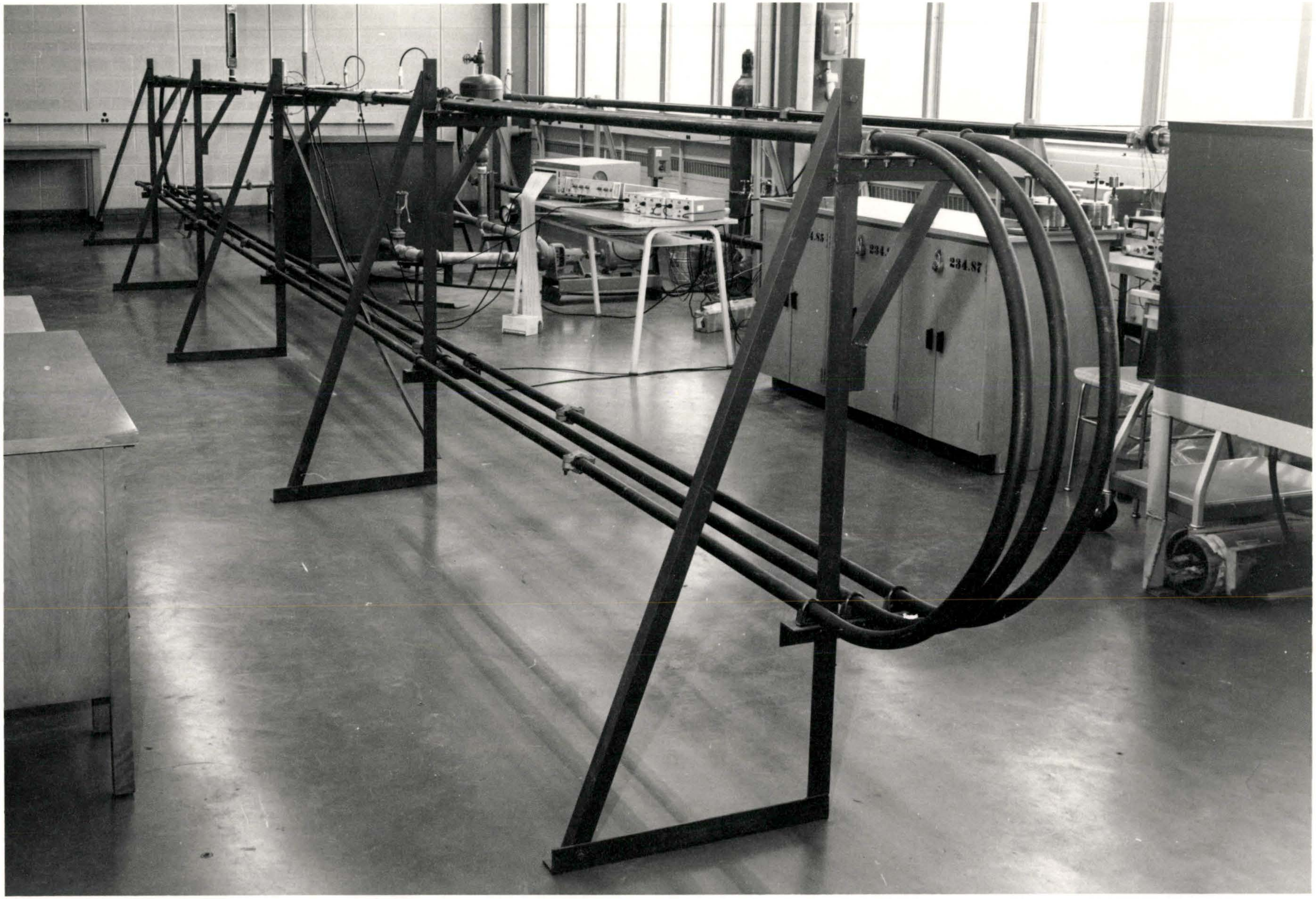


FIG. 4(a) OVERALL VIEW OF APPARATUS

DESCRIPTION OF SYMBOLS IN FIGURE 4(b)

A	pipe loop
B	solenoid-valve
C	rotameter
R	reservoir
S	surge chamber
D	throttling valve
T_3	transducer at the test section (62 in. from the solenoid-valve)
T_2	transducer at the test section; upstream from the capsules (102 in. from the solenoid-valve)
T_1	transducer 72 in. from the primary reflection site (231 ft. from transducer T_3)
O_1, O_2, O_3	oscillators
N_1, N_2, N_3	reactance converters
E	visicorder-oscillograph
F	multichannel D.C. amplifier

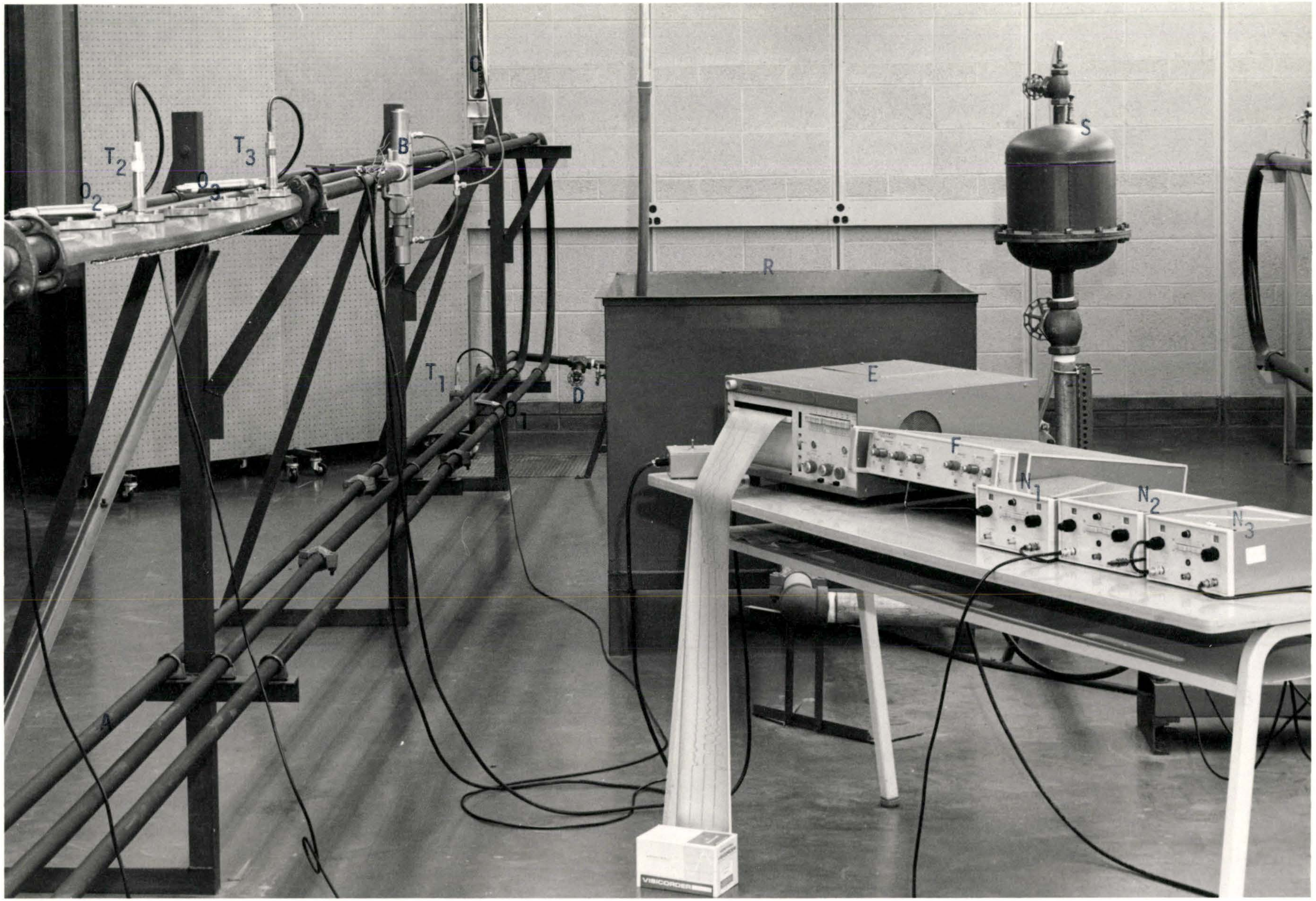


FIG. 4(b) OVERALL VIEW OF INSTRUMENTATION

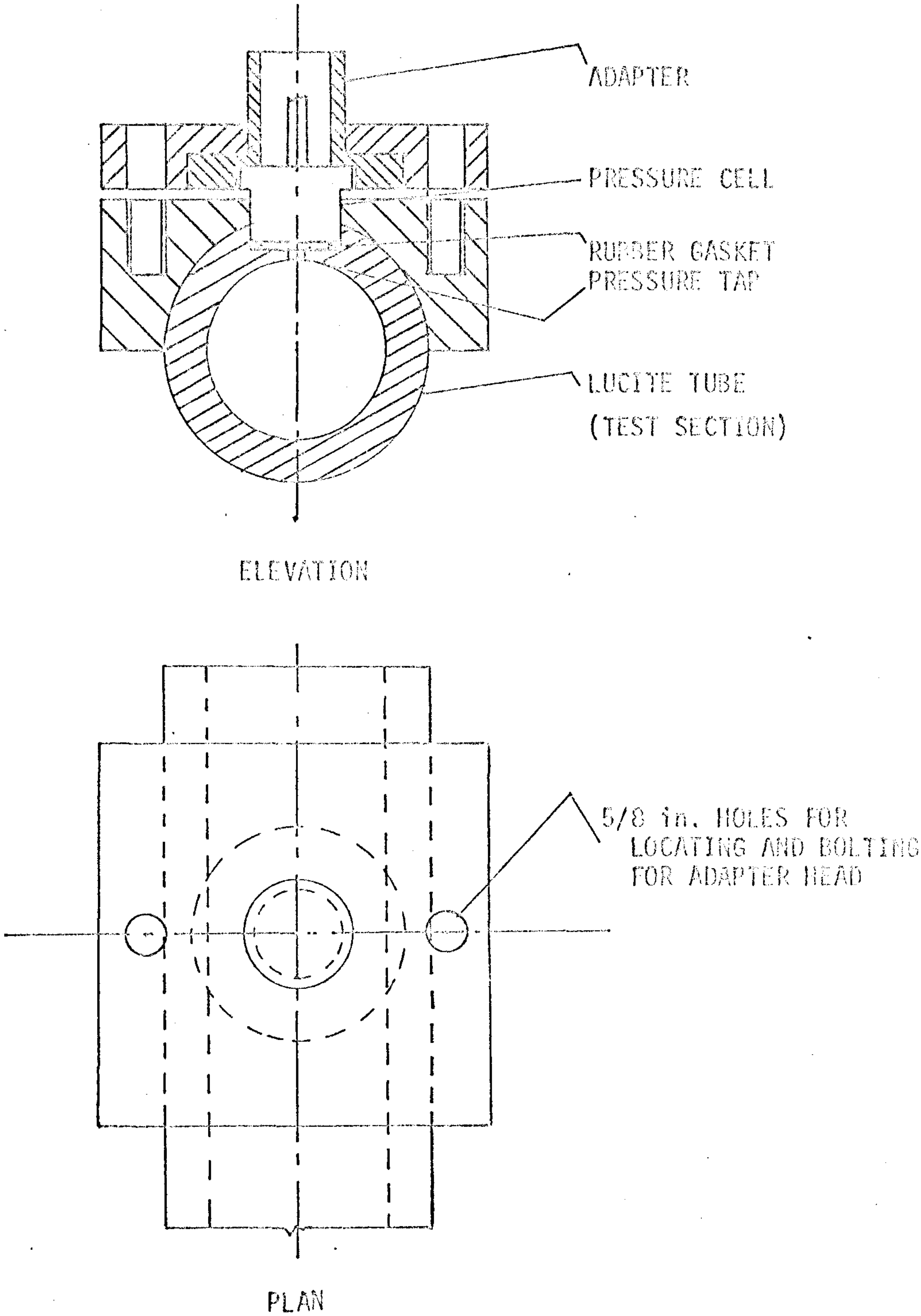


FIG. 5 PRESSURE CELL MOUNTING

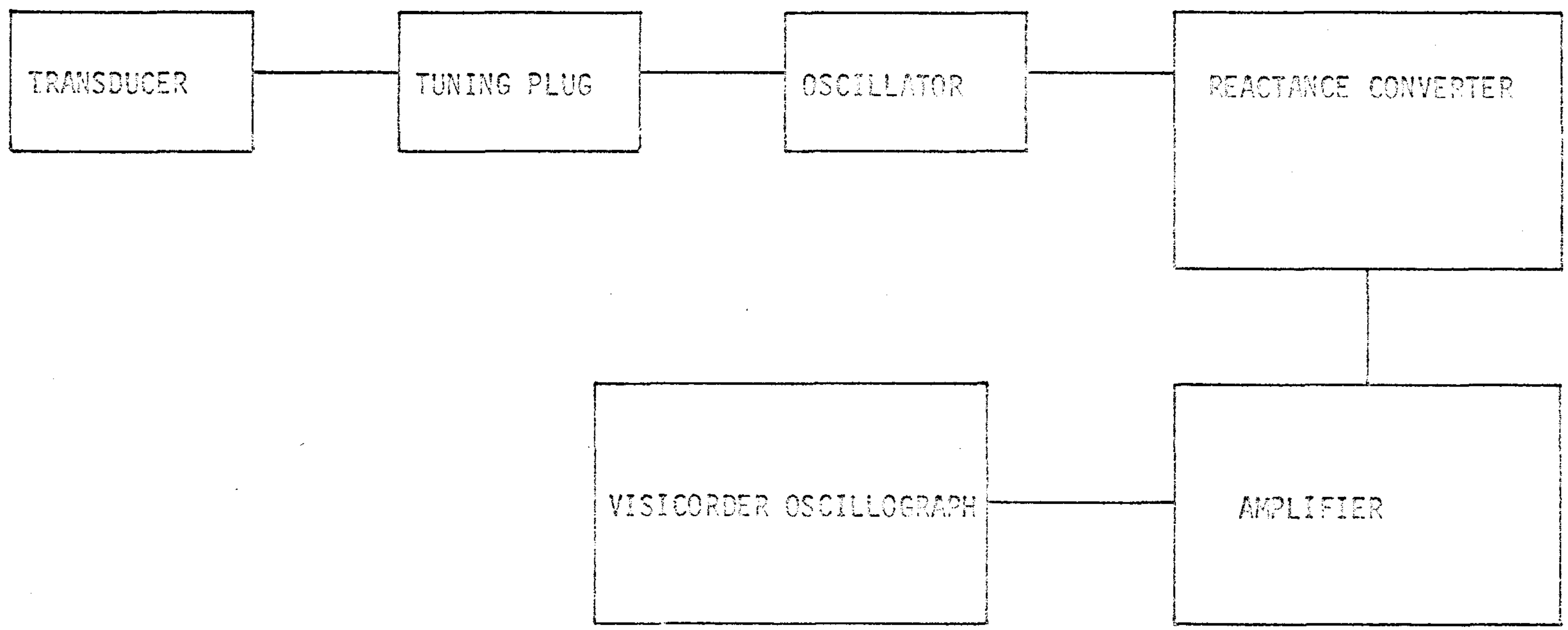


FIG. 6 SCHEMATIC DIAGRAM OF PRESSURE RECORDING SYSTEM

The output signal from the reactance converter could be displayed on an oscilloscope or recorded on a high speed chart recorder. A Honeywell 2106 visicorder-oscillograph in conjunction with a Honeywell accudata 117 multichannel D.C. amplifier for circuitry protection was used for recording the pressure history of the waterhammer waves.

The multichannel D.C. amplifier enabled voltage scale adjustment to be made, e.g. one vertical division (= 1 in.) displacement could be made to correspond to a voltage change of 0.1 to 100 volts depending on the selector position.

Initial difficulties were experienced in finding the right type of fast-closing solenoid Valve. The valve finally selected was a VERSA T.G.G. 2701.

The response time of the system, based on the rise time of the individual instruments was in millisecond range and so was well within the wave reflection times.

3.3. Experimental Procedure

The pressure cells were calibrated using a dead-weight tester. The rotameter was calibrated by weighing amounts of water collected in a fixed time.

The converter units were switched on to warm up for at least one hour before the start of each experiment. This helped in minimising drift.

The reservoir was filled with water to 1/2 capacity and the pump stuffing box nuts (2 off) were adjusted so that there was slight drip of water as the driving shaft was rotated slowly by hand. This indicated that the pump was well primed.

The capsules (cylinders or spheres) were located at the desired position in the test section. The test section was connected in the line by means of the Johnson dresser couplings. The valve at the junction of 1 in. and 2 in. diameter lines was closed. After the pump was started the valve was opened slowly and at full opening water was circulated for 15 minutes, so that air was swept from the system. The flow was adjusted to the required velocity.

The reactance converter for the appropriate transducer was adjusted so that the output voltage was zero. For a particular flow velocity, the scale on the amplifier was selected. Immediately after the visicorder-oscillograph drive was started at a selected paper speed, the solenoid-operated valve was closed. The waterhammer wave thus generated propagated through the line and pressure signals were recorded from all the transducers. Typical output signals, for three traces, were recorded indicating the transient pressure history at three points T_1, T_2 and T_3 on the pipeline. There was some time lag for the signal from the transducer T_1. The time lag was determined accurately by running at higher chart speeds.

The procedure described above was followed for the entire experimental program, i.e. for varying flow velocities, capsule diameters, shapes, number and position of capsules.

3.4. Error Analysis

The following analysis is based on estimates of error due to such factors as instrument error, calibration error and reading error. The quantities measured were:

(1) transient pressure

(2) flow velocity

(3) capsule and pipe dimensions

Error in Pressure

Pressure is the most significant variable in this investigation and error in its measurement may be due to error in instruments or air bubbles in the system.

(a) visicorder-oscillograph error

maximum error of ±1 psi in 25 psi

estimated error = ±4%

(b) calibration error. For calibration the dead weight tester was used and estimated error in 50 psi is ±0.25 psi.

estimated error = ±0.5%

(c) the combined error due to the electrical system (pressure cell-tuning plug-oscillator-reactance converter) may be estimated = ±2.5%

Total error in pressure = 4 + 0.5 + 2.5

= ±7%

It is not possible to have an estimate of error introduced, due to the air bubbles in the system. However, this source of error was recognized and care was taken to keep the system free of air.

Error in Flow Velocity

Velocity was measured by a rotameter.

(a) maximum error in rotameter reading = ±1 m.m.

minimum velocity measured = 1 ft/sec.

maximum error in velocity for 1 m.m. error in rotameter reading = ±0.05 ft/sec.

estimated error = ±5%

(b) calibration error

maximum error in weight of water collected = ±0.25 lbs.

minimum weight collected = 50 lbs.

estimated error in weight = ±0.5%

estimated error in time = ±0.5%

Total error in flow velocity = 5 + 0.5 + 0.5

= ±6%

Error in Dimension Measurement

Capsule dimensions were measured by a micrometer and error in 3/4 in. = ±.001 in.

estimated error = 0.13%

test section length was measured by a meter rod and error in 1 ft. length = ±1/8 in.

estimated error = ±1.0%

4. RESULTS AND DISCUSSION

The experiments were conducted using the following series of capsules all made of lucite:

0.875 in. diameter, 5 in. long cylinders

0.95 in. diameter, 5 in. long cylinders

0.75 in. diameter spheres

0.875 in. diameter spheres

The pressure transient history was recorded by the transducers mounted at T_1, T_2 and T_3. Initially a set of experiments was made in a capsule-free pipeline to investigate the effect of varying the flow velocity and to have the data available for comparison with the capsule located pipeline. Typical traces are shown in Figure 7.

A waterhammer wave generated by the solenoid valve reached point T_3 first and then travelled 231 ft. before reaching point T_1. The time lag between signals indicated the time taken by the wave to travel the distance between T_3 and T_1. The chart paper was run at higher speed (16 in/sec.) and the traces were obtained for measuring the time lag accurately. The speed of waterhammer wave was found to be 231/0.05 = 4620 ft/sec. and remained constant for all flow velocities.

The experimental data obtained for a capsule-free pipeline agreed very well with the theoretical values obtained from the well-known waterhammer equation $dH = \frac{a}{g} dV$. The excellent agreement (c.f. Figure 15) between the two results assured that:

(1) The instrumentation was working well.

(2) The valve closure could be considered to be instantaneous.

A large number of pressure traces were obtained for various combinations of:

(a) flow velocity (1-4 ft/sec.)

(b) capsule shape and diameter

(i) spheres 3/4 in. and 7/8 in. diameter

(ii) cylinders 5 in. long, 7/8 in. and 0.95 in. diameter

(c) capsule train length and configuration

(1) train of capsules with no gap between adjacent capsules

(i) 2,4,6, and 8 cylinders, diameter 7/8 in.

(ii) 1,2 and 3 cylinders, diameter 0.95 in.

(2) train of capsules with a gap between adjacent capsules

(i) 2,3,4, and 6 cylinders, diameter 7/8 in. and gap 5 in.

(ii) 2,3,4,5 and 6 cylinders, diameter 7/8 in. and gap 2 1/2 in.

(iii) 2 cylinders, diameter 0.95 in., gap 5 in. and 2 1/2 in.

(iv) 2,4,6,8 and 12 spheres, diameter 3/4 in. and centre to centre distance 1 1/4 in.

(v) 2,4,6,8 and 12 spheres, diameter 7/8 in. and centre to centre distance 1 1/4 in.

It has been observed that the pressure transient was slightly affected by the capsules of 0.875 diameter ratio. (capsule/pipe). But for 0.95 in. diameter cylinders the change in magnitude was significant. However the train length for such a diameter ratio was kept small because of the drop of flow velocity with the addition of each cylinder and corresponding pressure increase. Figures 8 - 14 have been selected to demonstrate different effects for different combinations of parameters.

Figures 7 - 14 show the rise in pressure during the transient wave propagation. The transient pressure recorded is the pressure above the initial steady state static pressure. After valve closure, maximum pressure is at the first peak and the magnitude, in the subsequent reflections, drops down; finally attaining the steady state situation. The initial steady state pressure is different for different flow situations, and in Figures 7 - 14 the base pressure is different for all the traces. Figure 22 illustrates this point clearly.

Figure 22 shows the initial steady state pressure (pressure before any transient wave has been generated in the system) for three flow situations.

(1) For a capsule-free pipeline and flow velocity = 1 ft/sec.

(2) For a capsule-free pipeline and flow velocity = 4 ft/sec.

(3) For a capsule-containing pipeline and flow velocity = 1 ft/sec.

The drop in pressure along the pipeline is due to two main factors.

(1) Friction in the pipeline.

(2) Throttling at the valve.

The following calculations can be used to obtain the head loss due to friction:

$$\text{Head Loss} = h_f = \frac{flu^2}{D2g}$$

where l = length of the pipe

D = diameter of the pipe

u = flow velocity

f = friction factor

Friction factor depends on the pipe roughness and Reynolds number.

$$\text{Reynolds number} = \frac{uD}{\nu}$$

$$= \frac{u}{12 \times 0.1217 \times 10^{-4}}$$

$$= 6{,}850\ u$$

The flow velocity varies from 1 - 4 ft/sec. Reynolds number is in the range of 6,850 to 27,400 so the flow is turbulent in the range of interest.

Friction factor for the pipe used in this range of Reynolds number = 0.07.

Length between the throttling valve and discharge end = 250 ft.

Diameter of the pipe = 1 in. Therefore

$$h_f = \frac{flu^2}{D2g}$$

$$= \frac{0.07 \times 250}{\frac{1}{12} \times 64.4} \times \frac{62.4}{144}\ u^2 \text{ in psi}$$

$$= 1.41\ u^2 \text{ psi}$$

The following table is for flow velocities 1 - 4 ft/sec.

Flow Velocity in ft/sec.	Head Loss in lbs/sq.in.
1	1.41
2	5.64
3	12.69
4	22.46

Pressure at the discharge end is atmospheric pressure and 30 psig just upstream the throttling valve for all flow situations. There is loss of head at the downstream end of the solenoid valve and has magnitude of 1.5 psig for all flow situations.

As pointed out earlier, the drop in pressure from 30 psig (upstream of the throttling valve) to 1.5 psig (at the solenoid valve) is either due to the throttling valve or due to friction in the pipe.

In Figure 22 the pressure at a and g is always the same and does not vary with different flow situations.

At a pressure = 30 psig

At g pressure = 1.5 psig

For flow velocity 1 ft/sec. the loss of head due to friction in the pipe is 1.41 psi. Therefore, just downstream the throttling valve pressure is 2.91 psig and upstream the throttling valve pressure is 30 psig. The pressure drop across the valve is about 27 psig. In Figure 22 it is represented by points a-b-c-d-e-1-f-g.

For a flow velocity 4 ft/sec. loss of head due to friction is 22.46 psi. So pressure just downstream of the throttling valve is 23.96 psi. The drop in pressure across the throttling valve is about 6 psi. In Figure 22 it is represented by a-b-c-d-2-g.

In a capsule-containing pipeline there is pressure drop across the capsule and in such a case the initial steady state pressure may be represented by points a-b-c-3-h-f-g (c.f. Figure 22).

The initial steady state pressure is different for different flow situations. The final steady state pressure (when the solenoid valve is fully closed) is indicated by the dotted line a-4-j in Figure 22. It is clear from Figure 22 that the pressure rise, when the system goes from the initial steady state to final steady state, depends on the flow situation.

Next, consider the pressure history at point T, which corresponds to point T_1 in the present experimental set-up. For flow velocity 1 ft/sec. rise in pressure is from points 1 to 4 and is about 27 psi. For 4 ft/sec. the rise in pressure is from points 2 to 4 about 7 psi. Similarily different amounts of pressure rise for different combinations of capsules as the system moves from the initial steady state situation to the final steady state situation.

In Figures 7 - 14 the pressure rise has been measured by the transducers T_1, T_2 and T_3. The rise in pressure during transient wave propagation is not affected by the final steady state or the initial steady state pressure. Also, it is important to note that the maximum pressure is the pressure of the primary wave generated by the solenoid valve. If the system is safe for this maximum pressure it will be safe for the subsequent pressure waves. So the designer need to have a look only at the magnitude of the primary peak.

The number of reflections is different for different flow situations. This may be due to resonance in the piping system. The magnitude gets attentuated in the subsequent reflections. This aspect is not investigated in the present work. Since the magnitude gets attentuated it will not present any serious problem in a pipeline network.

Pressure transient history in a capsule free pipeline for valve closure completed in a finite time (less than 2L/a seconds) is presented in the Appendix.

Figures 8 - 13 indicate the effect of 0.95 in. diameter cylinders on the waterhammer wave.

Figures 8 and 9 show that at position T_3 the pressure wave became magnified in the presence of cylindrical capsules in the pipeline.

Pressure at T_2 was the same as at T_3 for most cases, but reached a lower value for a larger capsule diameter ratio (Figure 13.)

Figures 10 and 11 indicate that there was attentuation of the waterhammer wave at point T_1 in a cylindrical capsule system.

Figure 14 shows how the pressure wave at T_3 became modified in the presence of 7/8 in. diameter cylindrical capsules. It may be noted that the overall effect is small but there is slight magnification of the first peak.

It can be seen that after valve closure there is a primary peak and many secondary reflections. The first peak has the greatest magnitude and is most significant from the design point of view. The first peak pressure for each set of experimental data has been plotted against number of capsules for flow velocities 1 - 4 ft/sec.

Figure 15 shows the pressure T_3 versus flow velocity for a capsule-free pipeline. There is good agreement between the experimental data and the theoretical prediction.

In Figures 16 and 18 the pressure at T_3 versus the number of cylinders has been plotted. The pressure becomes magnified in the presence of cylindrical capsules and magnification depends on the diameter of the capsule and length of the capsule train. This has been predicted theoretically in Chapter 2.

In Figures 17 and 19 the pressure at T_1 versus the number of cylinders has been plotted. The pressure at T_1 becomes attentuated in a capsule located line.

Figure 20 indicates the effect of spherical capsules on the pressure at T_3. The effect is not very significant but the pressure wave becomes attentuated. Figure 21 shows the effect of spheres of diameter 7/8 in. at T_1. In general, there is little effect of 7/8 in. diameter spheres and almost no effect for 3/4 in. diameter spheres on the pressure wave. But it is significant to note that the spherical capsules with gaps in between individual spheres produced attentuation of the waves and cylindrical capsules produced magnification of the wave at T_3 and attentuation at T_1.

Attentuation of the waterhammer wave may be due to the disturbance and friction introduced by the capsules. During valve closure the pressure wave seems to be affected by a higher velocity* change at capsule located section.

The pressure transient in a capsule-containing pipeline seems to be affected as follows:

(1) area change alone is responsible for magnification of the wave

(2) the added obstruction and the accompanying friction produce attentuation of the wave

For a large diameter cylindrical capsule magnification due to area change is greater than the attentuation due to friction and the net effect is one of magnification. But, for spherical capsules the magnification produced seems to be less than the attentuation and the net effect has been observed as attentuation.

In the present investigation, the capsules are present for a short interval of length of the pipeline as shown in Figure 23(a). Capsule train length is 0.5 to 1.5% of the total pipeline length. This may not be the case in a real situation. In a real case, the capsule train length may be 50 - 100% of the length of the

---

* i.e. for a capsule of 0.95 diameter ratio, the pipe area covered is 90%, thus increasing the fluid velocity by a factor of ten.

pipeline.

If a pipeline is 100% filled with the capsules as shown in Figure 23(b), the pressure transient may be analysed as follows:

Taking basis for comparison as a capsule-free pipeline, let

Flow velocity in a capsule-free pipeline = v_1

Pipe area of cross-section = A_1

Length of the pipe = l_1

Loss of head due to friction = L_{f_1}

Maximum pressure rise at valve closure = dH_1

Therefore;

$$dH_1 = - \frac{a}{g} \; dv_1 = - \frac{av_1}{g}$$

and let the length, discharge and area of cross-section be the same in the two cases. If the pipeline is 100% filled with the capsules, then it is equivalent to a pipe of smaller area of cross-section.

Let the flow area $A_2 = n \, A_1$

n depends on the geometry of the system. If the cross-section area ratio (capsule/pipeline) is 0.9, then n = 0.1. Assuming the loss of head due to friction = L_{f_2} and flow velocity $v_2 = v_1/n$ and if the maximum pressure rise at the valve closure = dH_2, therefore

$$\begin{aligned} dH_2 &= - \frac{a}{g} \; dv_2 \\ &= - \frac{av_2}{g} = - \frac{av_1}{gn} \\ &= \frac{dH_1}{n} \end{aligned}$$

The effective rise in pressure will be the sum of the pressure dH_2 and L_{f_2} (pressure increase due to area change and pressure drop due to friction). In the same way, a pipeline filled with any length of capsule train length can be analysed.

5. CONCLUSIONS

On the basis of the analyses and experimental data presented above, the following conclusions can be made:

(i) The waterhammer wave is slightly affected by the capsules located in a pipeline up to a diameter ratio 0.875. Only for long trains of capsules of large diameter ratio (say 0.95) is the effect significant.

(ii) In general, at the location of capsules there is a magnification of the waterhammer wave in the presence of cylindrical capsules and attentuation in the case of spherical capsules.

(iii) Pressure waves past the capsules are affected as follows:

(a) In the immediate neighborhood of the capsule, the pressure is the same as at the location or just before the capsule.

(b) At some distance upstream from the capsule, the wave becomes attentuated in a pipeline containing capsules.

(iv) Magnification or attentuation depends on the flow velocity, capsule size and train length.

(v) Pressure wave transient velocity remains constant for all flow velocity and capsule located line.

(vi) For a capsule-free pipeline, the magnitude of the primary wave can be obtained from the well-known expression, $dH = - a/g\ dv$ for waterhammer.

(vii) Gaps between the adjacent cylindrical capsules have no appreciable effect on the waterhammer wave.

T_1, T_2, T_3 INDICATES THE TRACES FROM THE TRANSDUCERS MOUNTED AT POSITIONS T_1, T_2, T_3 RESPECTIVELY (FIG. 1)

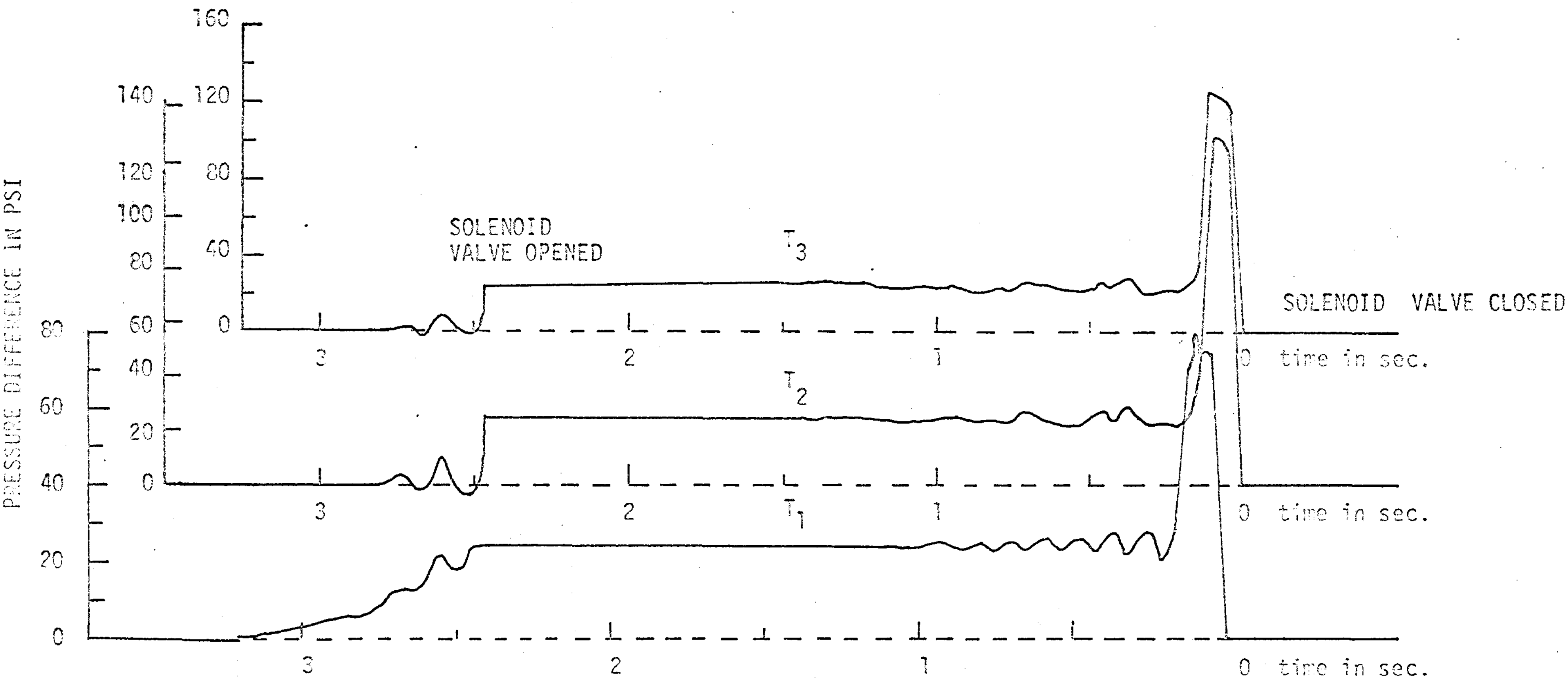


FIG. 7 TYPICAL TRACES FROM VISICORDER OSCILLOGRAPH

(VELOCITY OF FLOW = 2 ft/sec; CAPSULE-FREE PIPELINE)

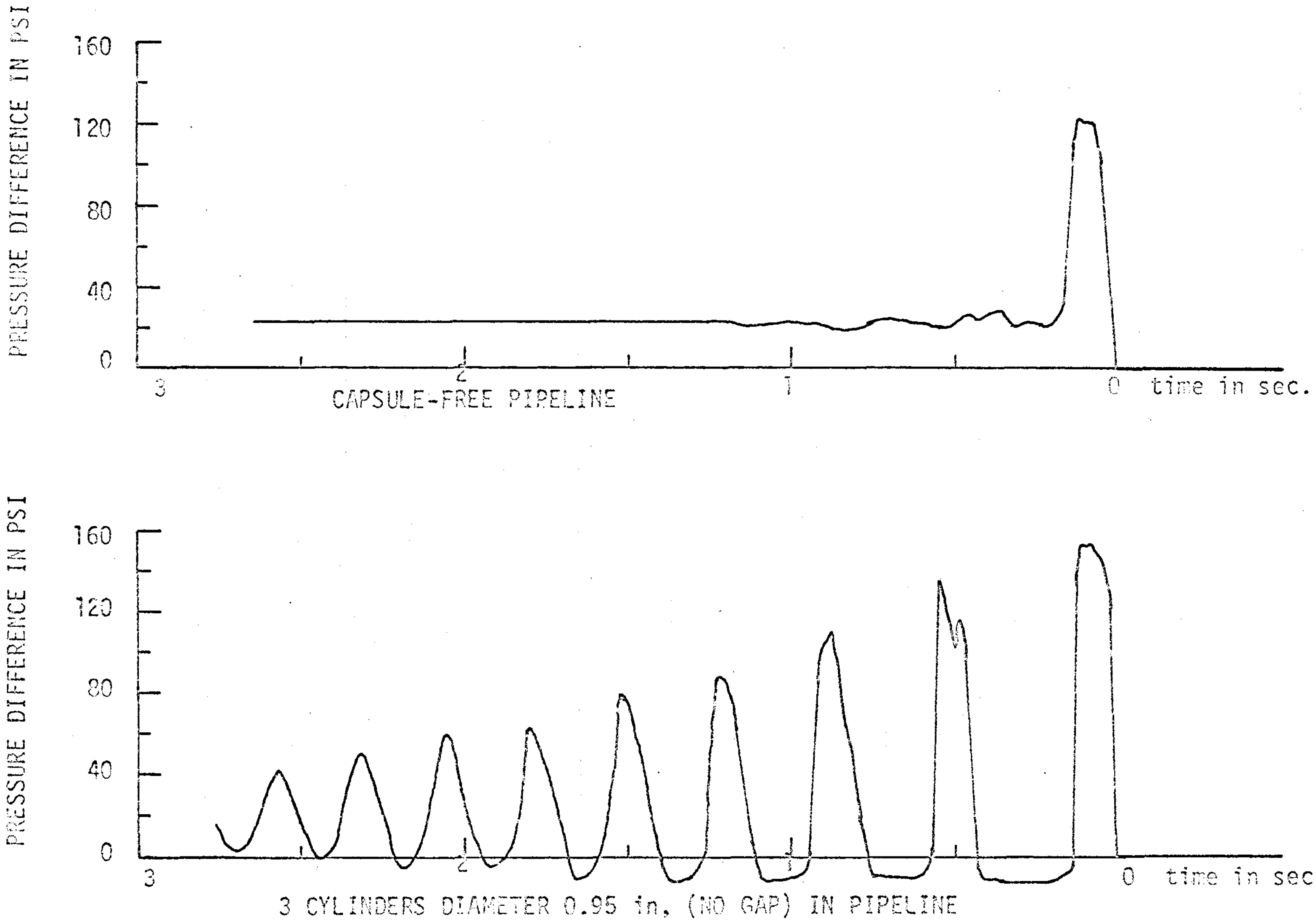
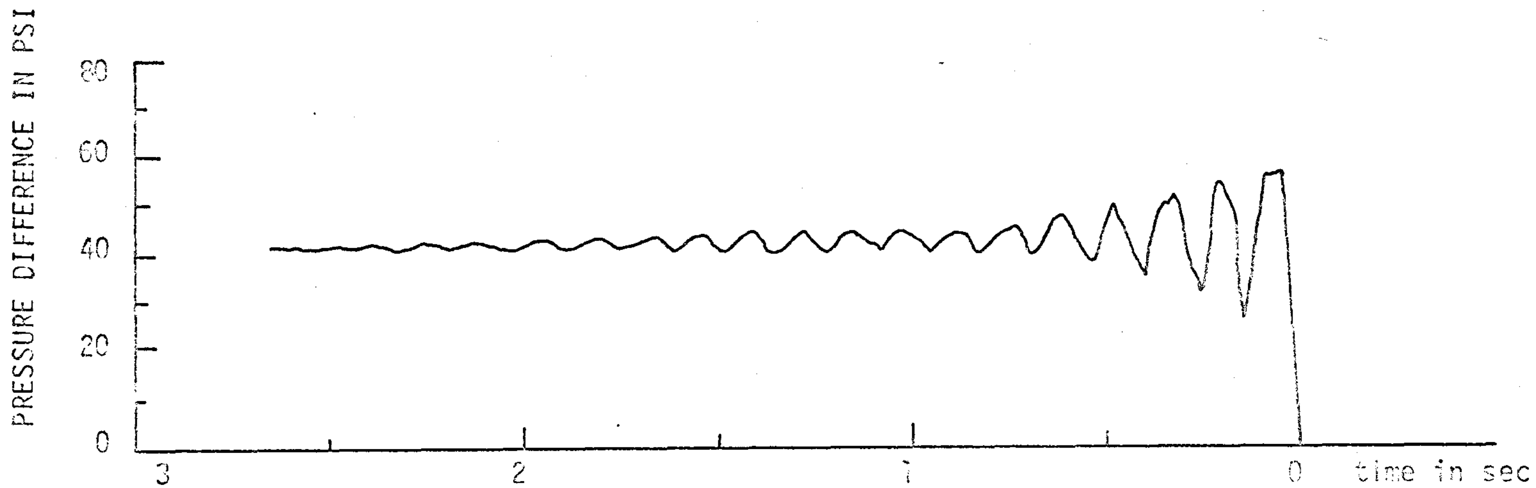
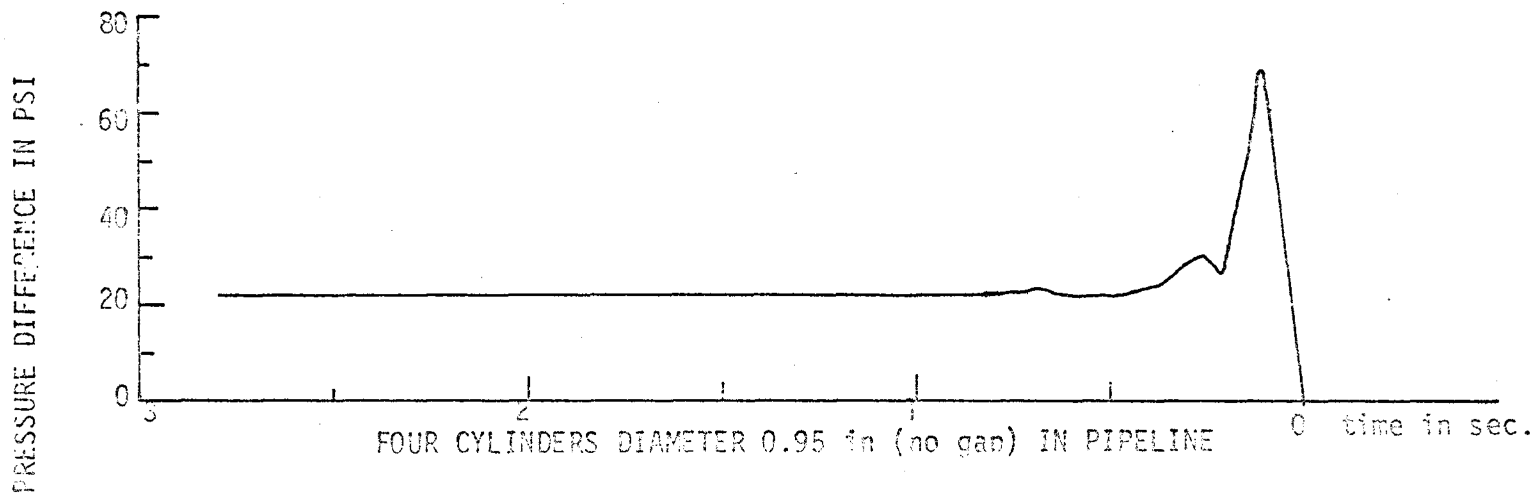


FIG. 8 TRACES FOR TRANSDUCER T_3; VELOCITY = 2 ft/sec



CAPSULE-FREE PIPELINE



FOUR CYLINDERS DIAMETER 0.95 in (no gap) IN PIPELINE

FIG. 9 TRACES FOR TRANSDUCERS T_3; VELOCITY = 1 ft/sec.

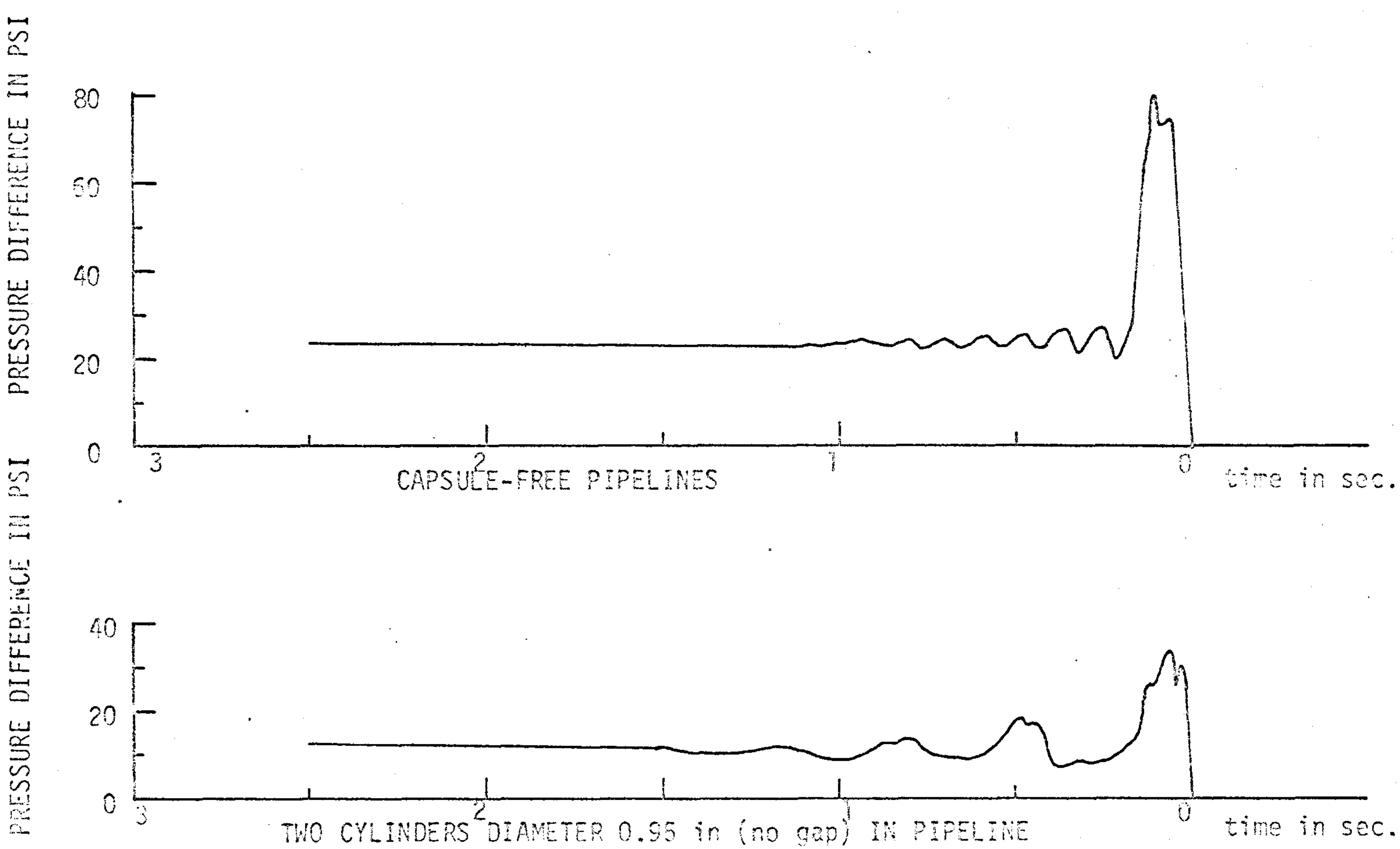


FIG. 10 TRACES FOR TRANSDUCER T_1; VELOCITY = 2 ft/sec.

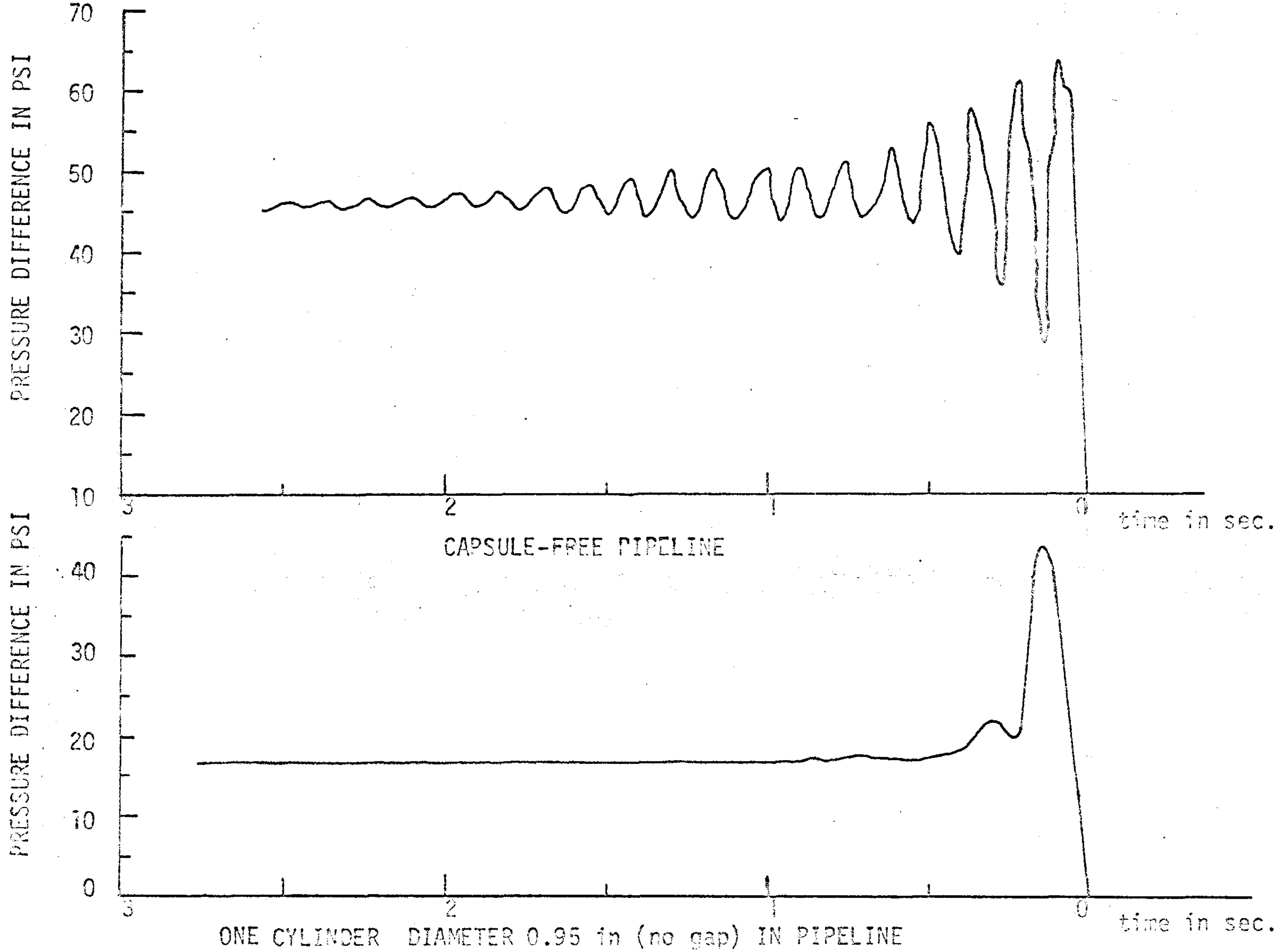


FIG. 11 TRACES FOR TRANSDUCER T_1; VELOCITY = 1 ft/sec.

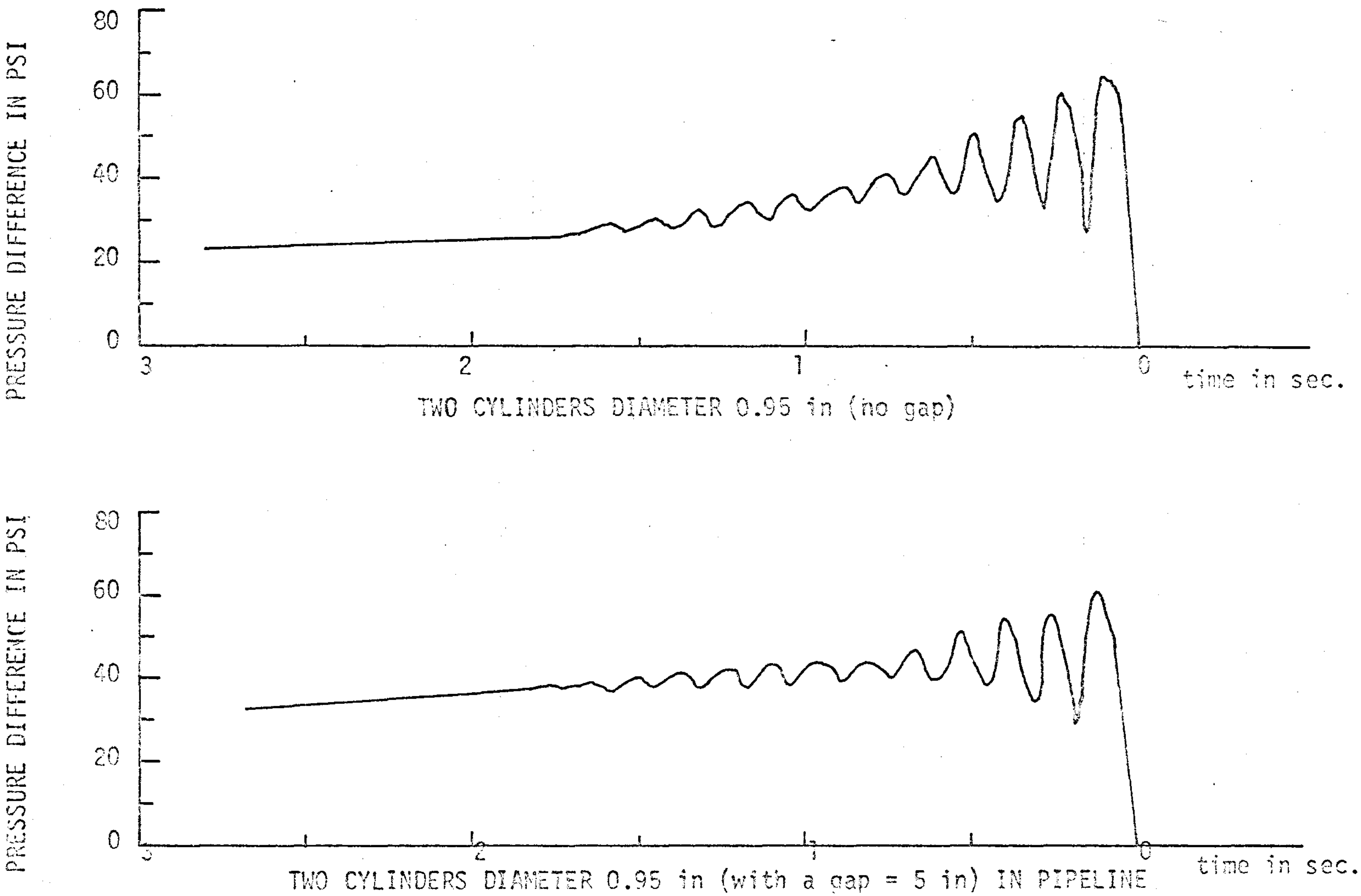


FIG. 12. TRACES FOR TRANSDUCERS T_3; VELOCITY = 1 ft/sec.

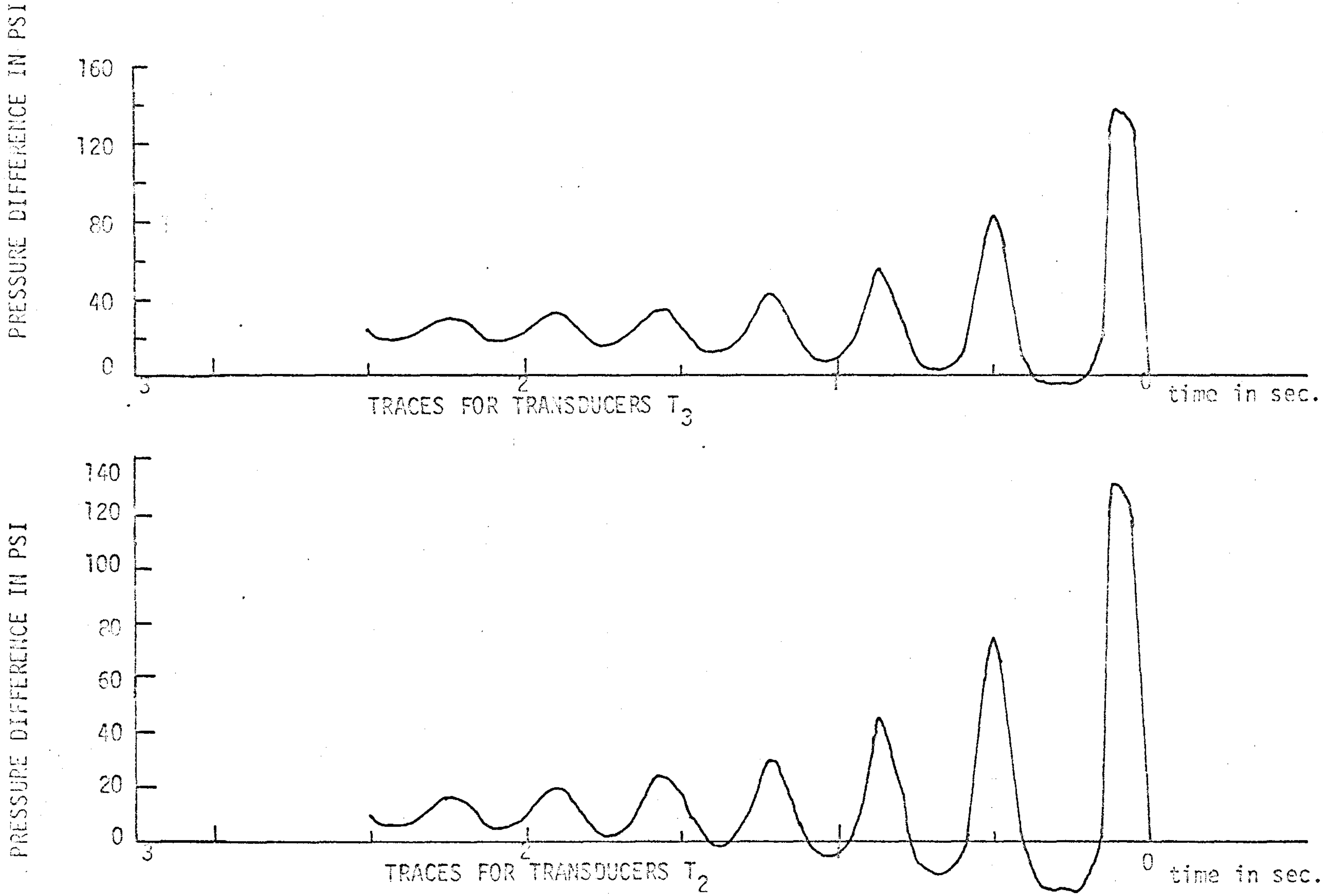


FIG. 13 TRACES FOR TRANSDUCERS T_3,T_2; VELOCITY = 2 ft/sec.

(two cylinders 0.95 in diameter in the tube)

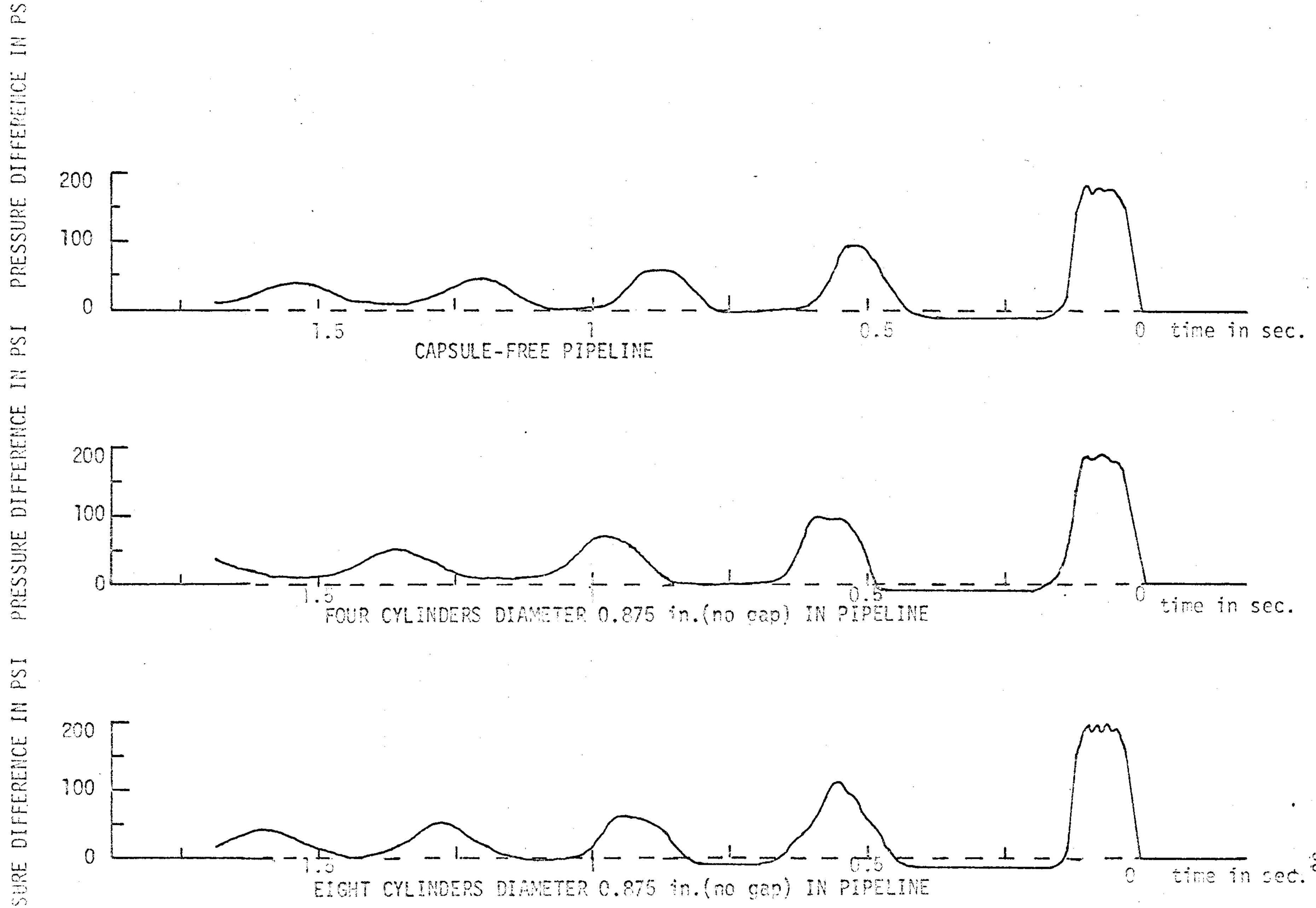


FIG. 14 TRACES FOR TRANSDUCER T_3; VELOCITY = 3 ft/sec.

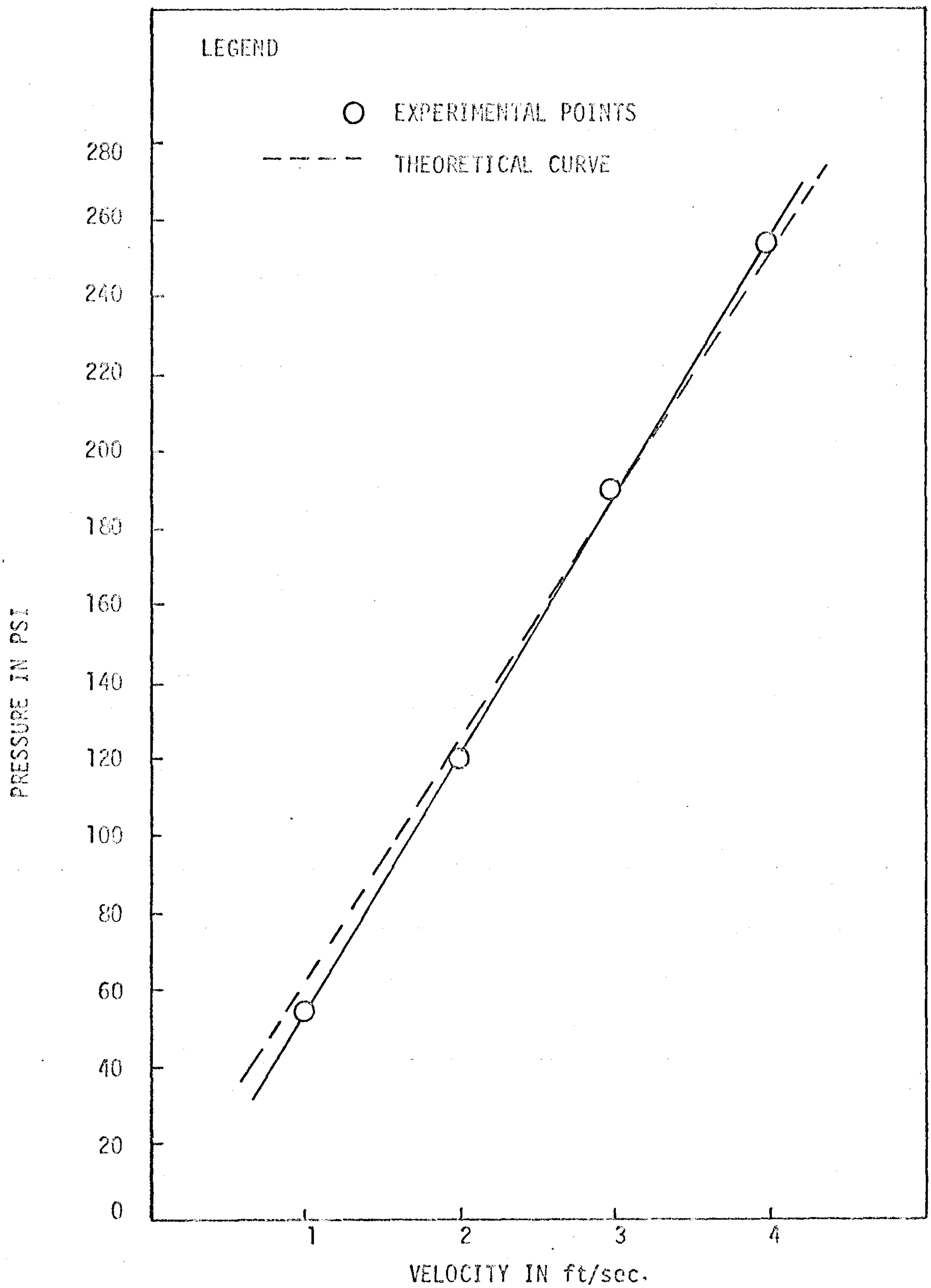


FIG. 15 PRESSURE AT T_3 VERSUS FLOW VELOCITY

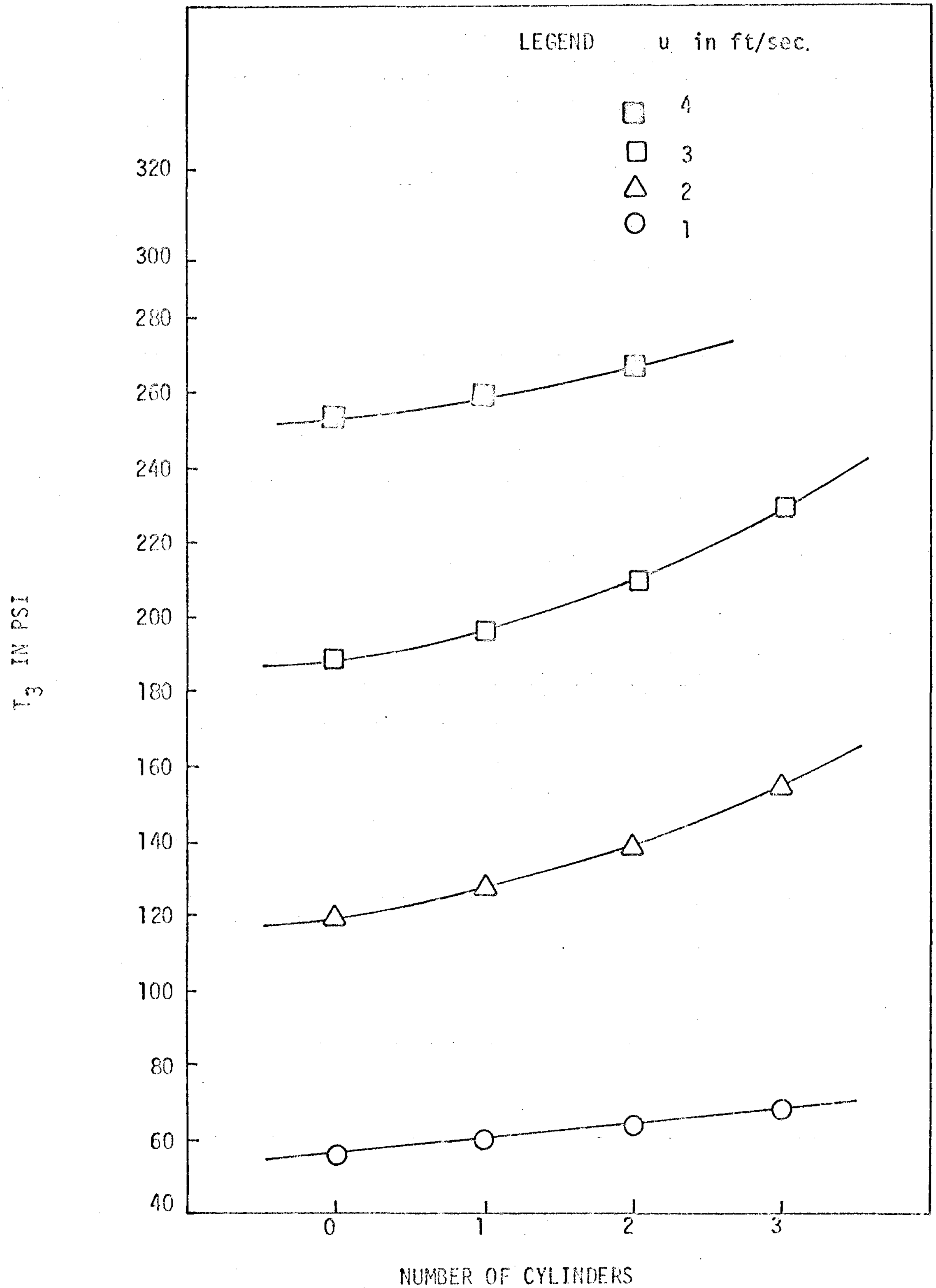


FIG. 16 PRESSURE AT T_3 VERSUS NUMBER OF CAPSULES

(0.95 in. DIAMETER CYLINDERS)

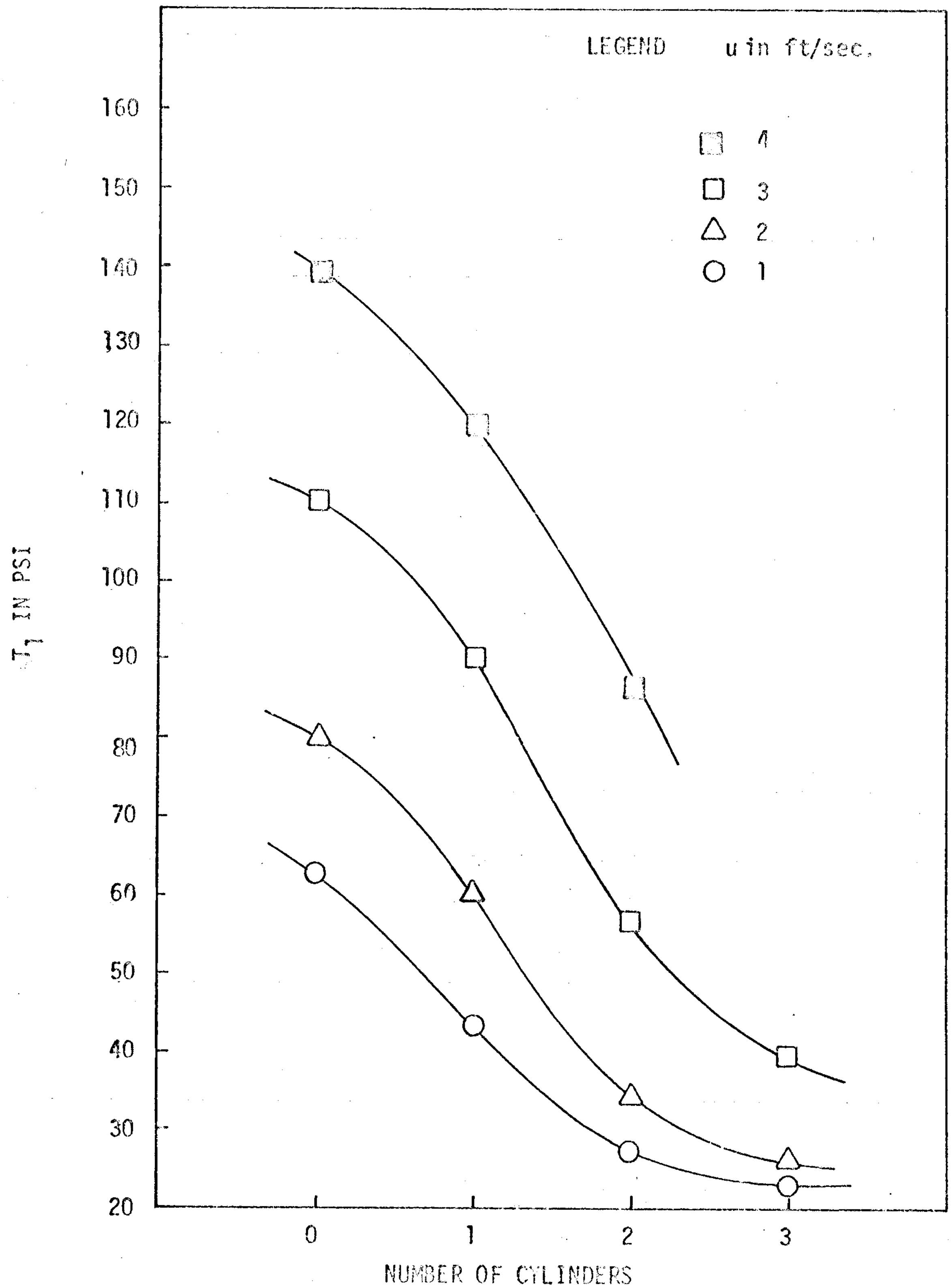


FIG. 17 PRESSURE AT T_1 VERSUS NUMBER OF CAPSULES

(0.95 in. DIAMETER CYLINDERS)

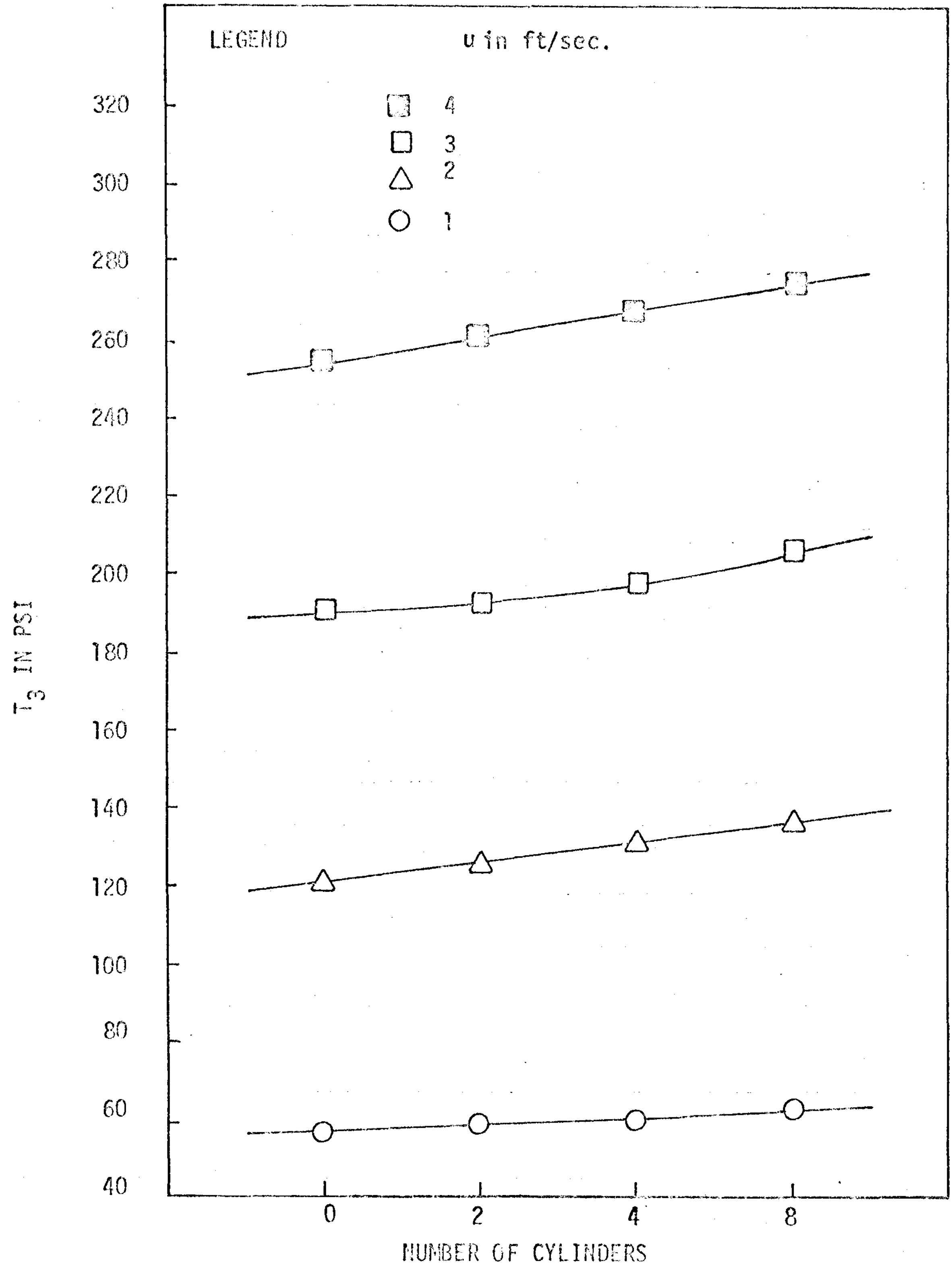


FIG. 18 PRESSURE AT T_3 VERSUS NUMBER OF CAPSULES (0.875 in. DIAMETER CYLINDERS)

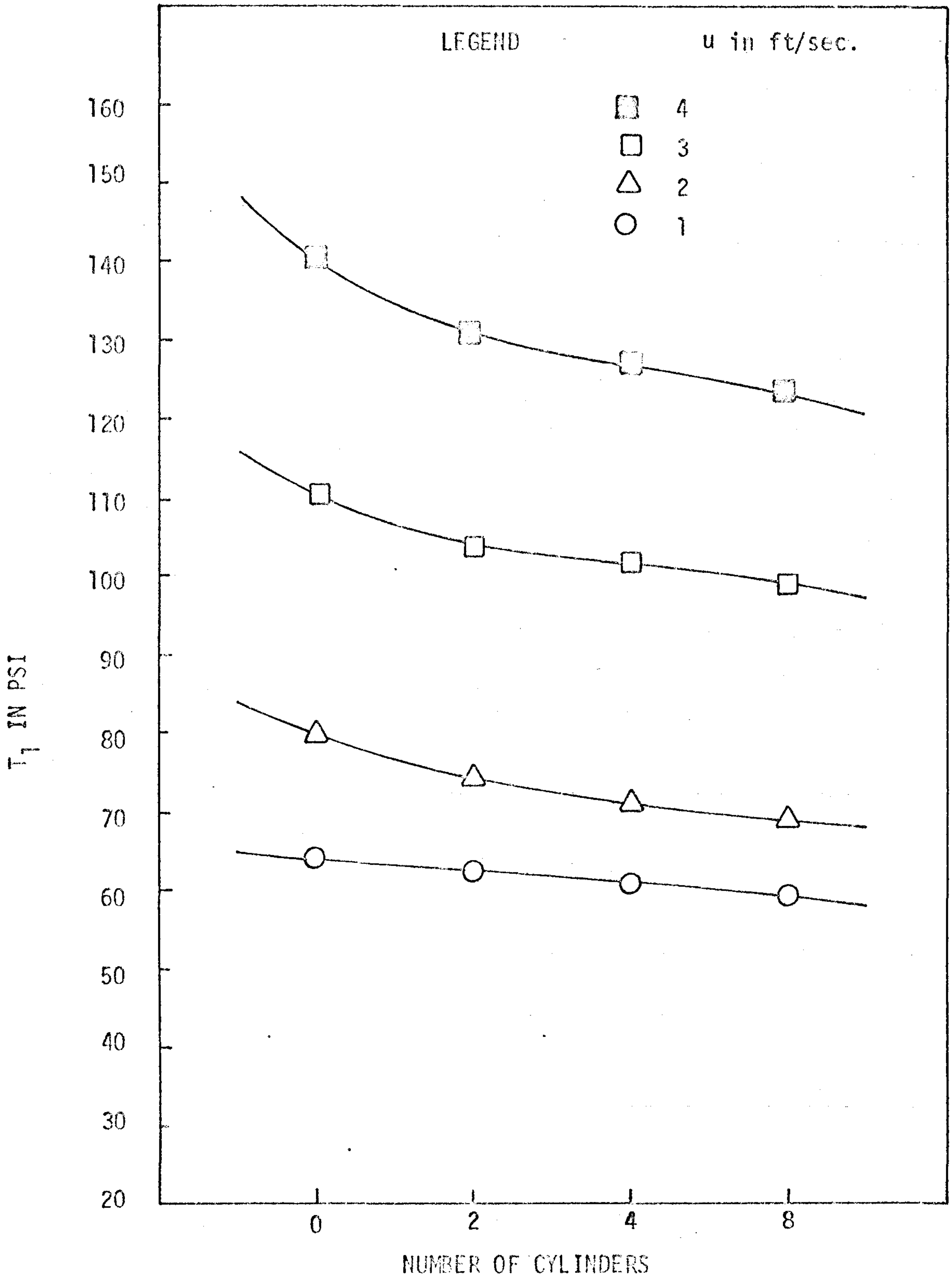


FIG. 19 PRESSURE AT T_1 VERSUS NUMBER OF CAPSULES

(0.875 in. DIAMETER CYLINDERS)

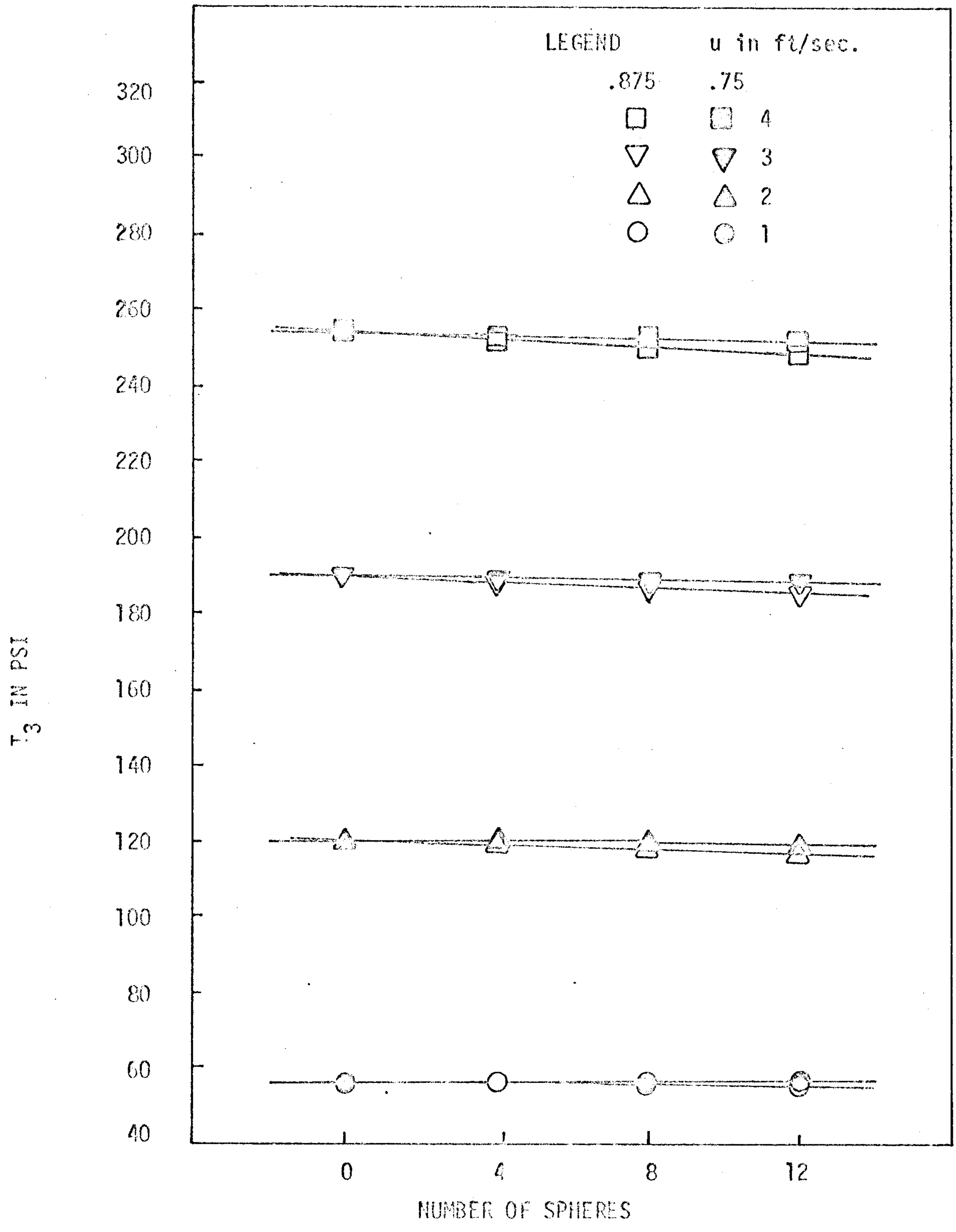


FIG. 20 PRESSURE AT T_3 VERSUS NUMBER OF CAPSULES

(.875 in. and 0.75 in. DIAMETER SPHERES)

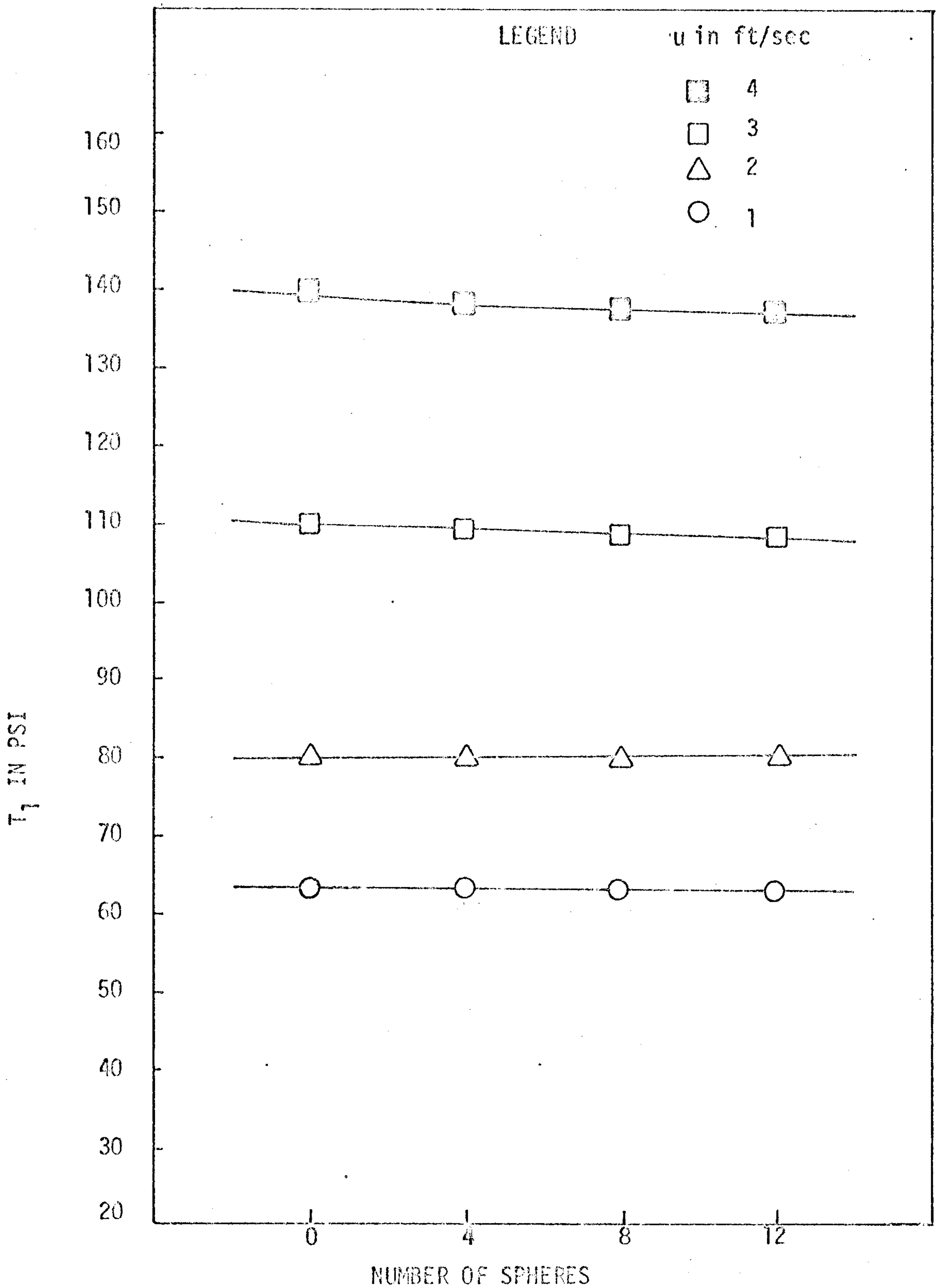


FIG. 21 PRESSURE AT T_1 VERSUS NUMBER OF CAPSULES

(0.875 in. DIAMETER SPHERES)

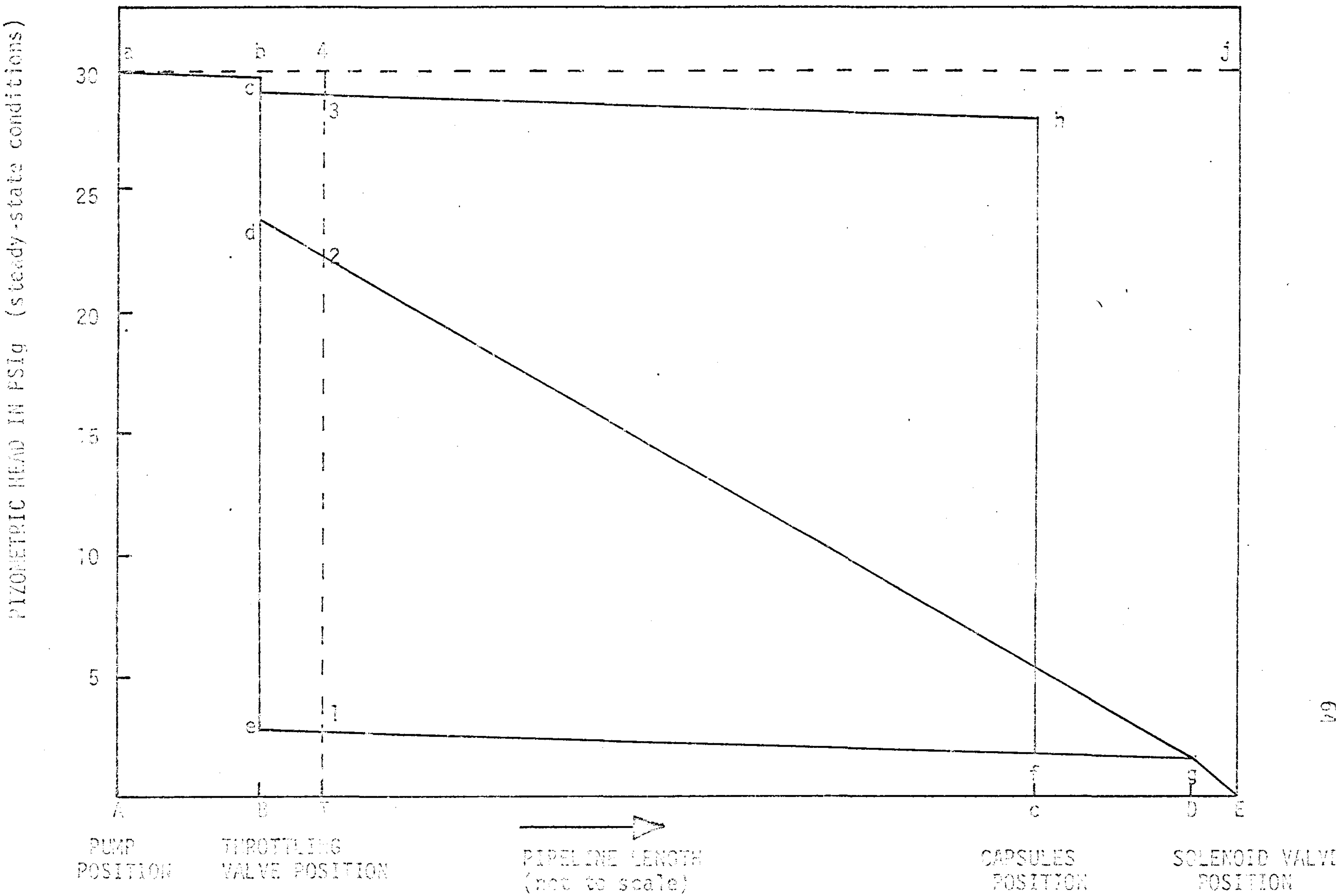


FIG. 22 HEAD LOSS VERSUS DISTANCE (steady-state condition)

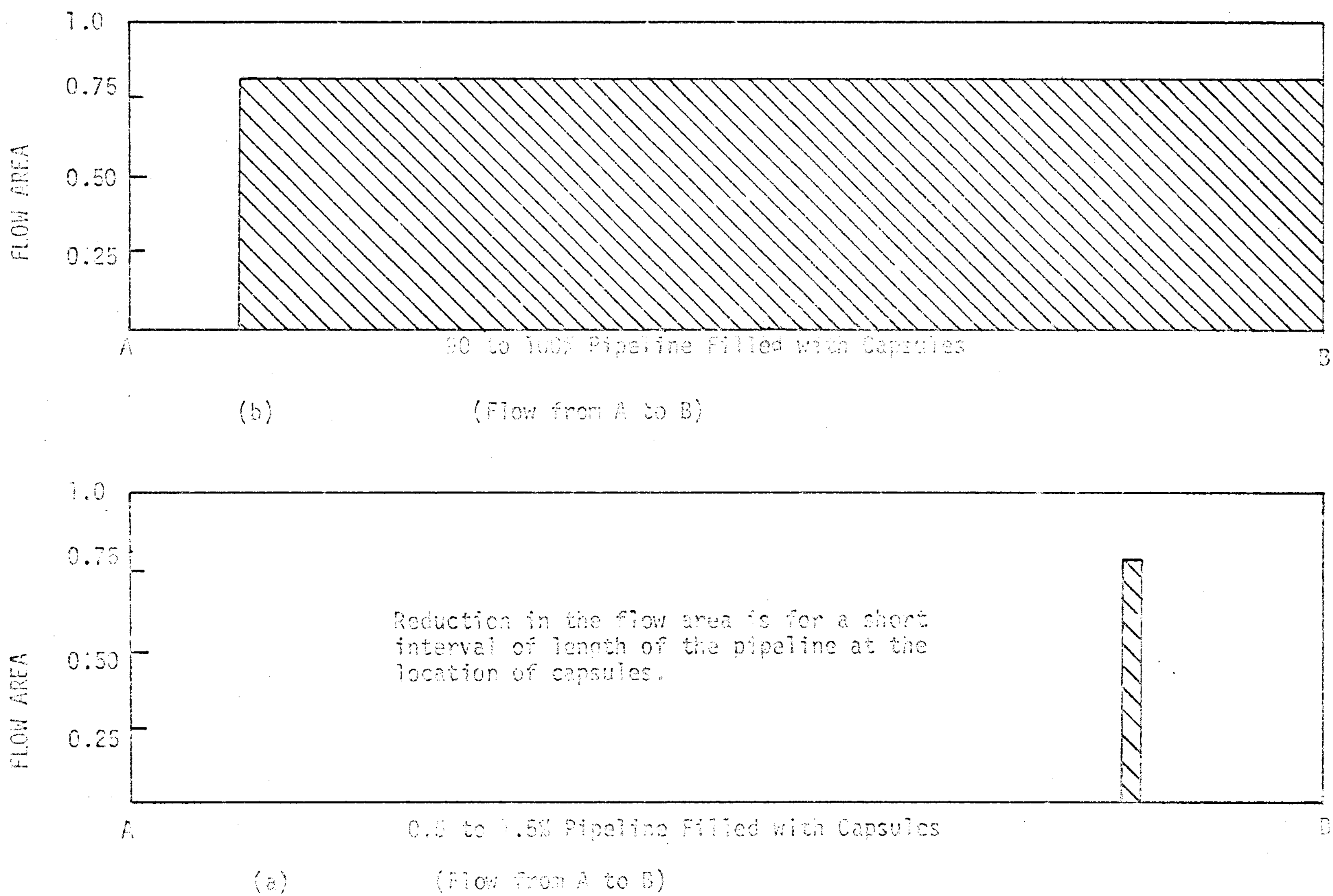


FIG. 23 FLOW AREA VERSUS DISTANCE ALONG THE PIPELINE

REFERENCES

1. Joukowsky, N.E.,"Waterhammer", (translated by O. Simis), Proc. Am. Waterworks Assoc., p. 335 (1904).
2. Allievi, L.,"Theory of Waterhammer", (translated by E.E. Halmos), (1925).
3. Wood, F.M.,"The Application of Heaviside Operational Calculus to the Solution of Waterhammer Problems", Trans. ASME, Vol. 59, Paper Hyd-59-15, November (1937).
4. Rich, G.R.,"Waterhammer Analysis by Laplace Mellin Transformation", Trans. ASME (1945).
5. Rich, G.R.,"Hydraulic Transient", Mc-Graw Hill Book Co., New York City, N.Y. (1951).
6. Streeter, V.L. and Wylie,E.B.,"Hydraulic Transients", Mc-Graw Hill Book Co., New York City, N.Y. (1967).
7. Parmakian, J.,"Waterhammer Analysis", Prentice-Hall, Inc., New York (1955).
8. Hodgson, G.W. and Charles, M.E.,"The Pipeline Flow of Capsules; Part 1: The Concept of Capsule Pipelining", Can. J. Chem. Eng., Vol. 41, 43-45, (1963).
9. Charles, M.E.,"The Pipeline Flow of Capsules; Part 2: Theoretical Analysis of the Concentric Flow of Cylindrical Forms", Can. J. Chem. Eng., Vol. 41 (2), 46-51 (1963).
10. Ellis, H.S.,"The Pipeline Flow of Capsules; Part 3: An Experimental Investigation of the Transport by Water of Single Cylindrical and Spherical Capsules with Density Equal to that of Water", Can. J. Chem. Eng., Vol. 42(1), 1-8 (1964).

11. Ellis, H.S.,"The Pipeline Flow of Capsules; Part 4: An Experimental Investigation of the Transport in Water of Single Cylindrical Capsules with Density Greater than that of Water". Can. J. Chem. Eng., Vol. 42(2), 69-76 (1964).

12. Ellis, H.S.,"The Pipeline Flow of Capsules: Part 5: An Experimental Investigation of the Transport by Water of Single Spherical Capsules with Density Greater than that of Water", Can. J. Chem. Eng., Vol. 42(4), 155-161 (1964).

13. Newton, R., Redberger, P.J. and Round, G.F.,"The Pipeline Flow of Capsules; Part 6: Numerical Analysis of Some Variables Determining Free Flow", Can. J. Chem. Eng., Vol. 42(4), 168-173 (1964).

14. Ellis, H.S, Bolt, L.H.,"The Pipeline Flow of Capsules; Part 7: An Experimental Investigation of the Transport by Two Oils of Single Cylindrical and Spherical Capsules with Density Equal to that of the Oil", Can. J. Chem. Eng., Vol. 42(5), 201-210 (1964).

15. Round, G.F. and Bolt, L.H.,"The Pipeline Flow of Capsules; Part 8: An Experimental Investigation of the Transport in Oil of Single, Denser-than-oil Spherical and Cylindrical Capsules", Can. J. Chem. Eng., Vol. 43(4), 197-205 (1965).

16. Kruyer, Jan. Redberger, P.J. and Ellis, H.S.,"The Pipeline Flow of Capsules; Part 9: J. Fluid Mechanics, Vol. 30(3), 513-531 (1967).

17. Swaffield, J.A.,"The Influence of Bends on Fluid Transients Propagated in Incompressible Pipe Flow", Proc., Inst. Mech. Eng., Vol. 183(29), 603-613 (1968).

18. Milne-Thomson, L.M.,"Theoretical Hydrodynamics", 4th ed. The MacMillan Co., New York, (1960).

19. Tarantine, F.J. and Roulea, W.T.,"Waterhammer Attentuation with a Tapered Line", Trans. ASME Paper No. 68-WAFE-6, (1968), (1-11).

APPENDIX

PRESSURE TRANSIENT HISTORY IN A CAPSULE-FREE PIPELINE FOR VALVE CLOSURE COMPLETED IN LESS THAN 2L/a SECONDS.

If the primary reflection site is at such a distance that a reflected wave cannot return to the solenoid valve before the valve motion is completed, maximum pressure change occurring at the valve is the same as if the valve closed instantaneously. For any valve closure, which takes place in less than 2L/a seconds, the maximum change in head extends from the valve to a certain limiting point along the pipeline. This limiting point may be obtained as follows. If x_2 is the distance from the limiting point to the primary reflection site, the wave-travel time from the solenoid valve to the limiting point is $(L - x_2)/a$ seconds. The time required for the pressure wave to travel from the solenoid valve to the primary reflection site and back to the limiting point is $(L + x_2)/a$ seconds. If the total valve closure is completed in time T, the elasped time from the start of the valve movement to the instant of arrival of the final incremental pressure change at the limiting point is $T + (L - x_2)/a$. Therefore, the limiting point is located along the pipe where the direct wave is met by the reflected wave. That is

$$\frac{L + x_2}{a} = T + \frac{L - x_2}{a}$$

$$x_2 = \frac{Ta}{2}$$

In the present investigation

$$T = \frac{1}{15} \text{ seconds}$$

$$L = 250 \text{ ft.}$$

$$a = 4620 \text{ ft/sec.}$$

Therefore

$$x_2 = \frac{1}{15} \times \frac{4620}{2}$$

$$= 154 \text{ ft.}$$

Pictorial representation of the head changes along the pipeline for valve closure completed in less than 2L/a seconds is shown in Figures 24(a - h). In Figure 24 the rise or drop in pressure above the steady state is indicated by the dotted line. Initial steady state velocity is represented by v_i and the velocity during the transient wave propagation is represented by v. v is always less than v_i for a valve closing in finite time.

The pressure rise takes place in a series of compression waves. As the valve closure starts the head of the compression wave starts propagating from the solenoid valve to the primary reflection site.

Figures 24 (a - b) show the compression wave travelling up to the reflection site and the valve, up to this time in the present problem, is not fully closed. The wave reflection takes place as shown in Figure 24 (b) and then the reflected wave gets superimposed on the compression wave. The valve becomes fully closed before the reflection wave reaches the valve. In the present case as shown above the reflected wave travels 154 ft. from the reflection site at the instant the solenoid valve is fully closed. So pipeline length 250 - 154 = 96 ft. is subjected to the same maximum pressure as if the valve closed instantaneously.

Figure 24(d) shows the pressure history at time 2L/a, when the head of the reflected wave reaches the solenoid valve. At the solenoid valve the expansion wave is reflected as an expansion wave and it travels back in the pipeline. The minimum pressure reached is the vapor pressure of the liquid and pressure history for this case is represented by Figure 24 (e-f). The head of the wave gets reflected from the primary reflection site and travels along the pipe. At time 4L/a the whole pipeline reaches the same conditions as they were at time t = 0. The cycle is repeated in the same fashion and the pressure drops in subsequent cycles and reaches the final steady state conditions.

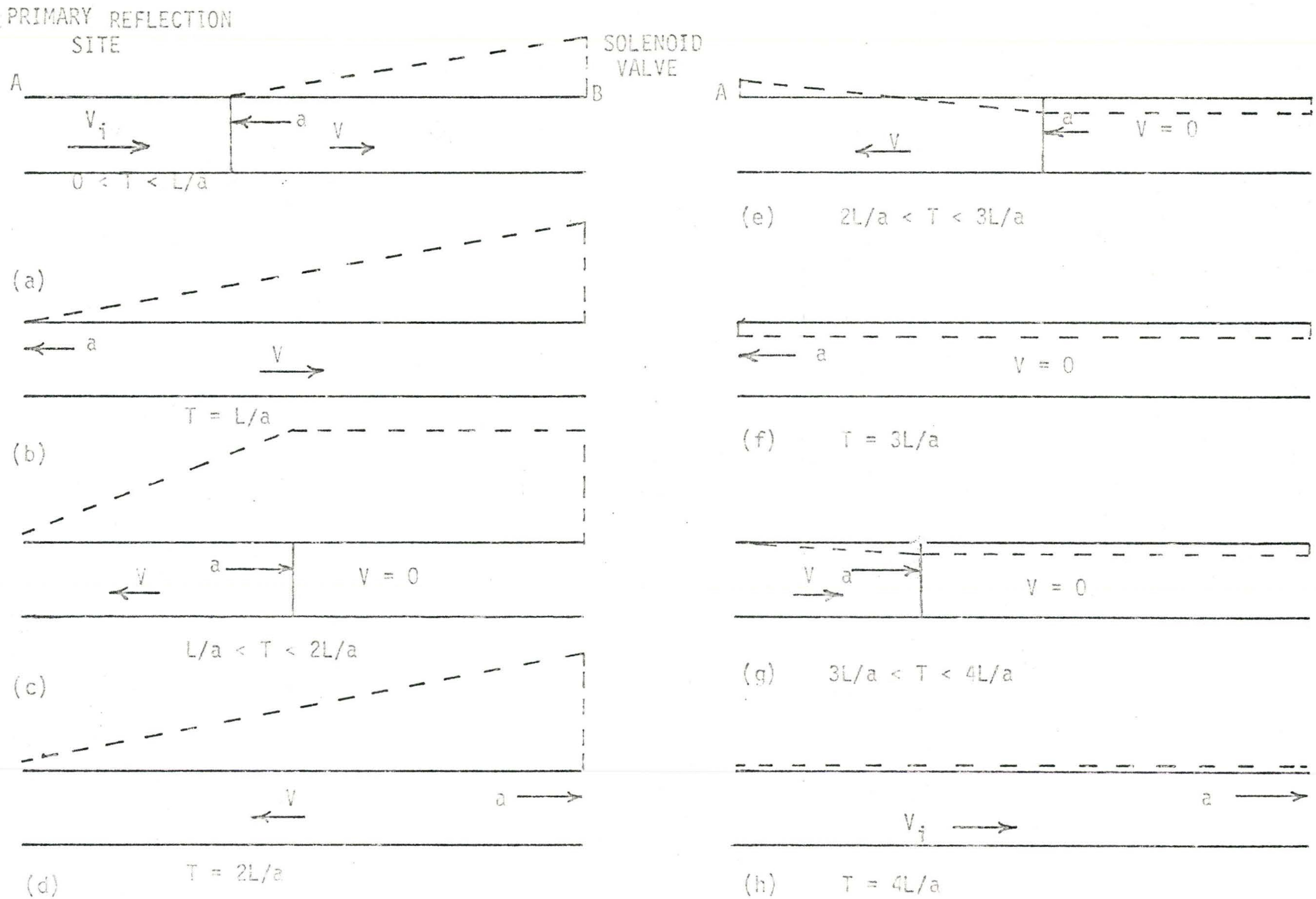


FIG. 24 PRESSURE TRANSIENT HISTORY FOR VALVE CLOSURE LESS THAN 2L/a second.