OUT-OF-PLANE VIBRATION OF PORTAL FRAMES

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By

PALAMUTHU RENGASAMI B.E.(Hons), (CIVIL), M.E.

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Master of Engineering

McMaster University
July 1970

MASTER OF ENGINEERING (Civil Engineering)

McMASTER UNIVERSITY Hamilton, Ontario.

TITLE: Out-of-Plane Vibration of Portal Frames.

AUTHOR: Palamuthu Rengasami, B.E. (Hons.) Annamalai University
M.E. (Indian Institute of Science)
India.

SUPERVISOR: Dr. A. C. Heidebrecht

NUMBER OF PAGES: xi ,95

SCOPE AND CONTENTS:

The theoretical and experimental investigations presented in this work are primarily related to the dynamic response of simple portal frames. The theoretical results obtained using beam elements with consistent mass matrix, are compared with experimental results. The interaction of the components in modes of the combined structure is investigated.

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ACKNOWLEDGEMENTS

I wish to express my sincere thanks to Dr. A. C. Heidebrecht for his constructive and valuable guidance in all phases of the investigation.

I wish to thank McMaster University for awarding me scholarship and teaching assistantship.

My thanks are also due to all the laboratory staff who helped me in carrying out the experimental work.

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NOTATIONS

[a]	Displacement distribution functions
[C]	Damping matrix
C_{ij}	Coefficient of damping matrix
{e}	Strain vector
{F(t)}	Column matrix of applied dynamic load
$\{F_i\}$	Column matrix of internal forces
i,j	Indices
[k]	Element stiffness matrix in member coordinate
[K]	Square symmetric matrix of stiffness of the entire system in global coordinate
[m]	Element mass matrix in member coordinate
[M]	Square symmetric matrix of the masses of the entire system in global coordinate
n	Number of joints
N	Number of elements
$\{R_i\}$	Column matrix of external load
$\{\ddot{U}\}$	Column matrix of acceleration in global system
$\{\dot{U}\}$	Column matrix of velocity in global system
{U}	Column matrix of displacement in global system

$\bar{x}_i, \bar{x}_j$	Strain response amplitude at ith and jth cycles under free vibration
$\bar{\delta}$	Logarithmic decrement
ζ	Damping factor

CHAPTER I

INTRODUCTION

1.1 General

Transverse vibration of simple beams have been widely discussed and well documented in the literature[1,2,3*]. It appears, from the literature available to date, that the out-of-plane vibration of simple framed structures have not drawn the same degree of attention of researchers. To gain some more knowledge in this field, it is desirable to investigate the dynamic response of simple frames subjected to base motion. Such investigation is necessary for the better understanding of the dynamic behaviour of structures, especially the tower type of structure, balcony supported structures and also in connection with the vibration of framework of machines.

A number of methods, based on simplifying assumptions have been devised to conduct the dynamic analysis of complex structural systems. At the present time, the basic need is not the development of additional similar computational

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*Numbers refer to the bibliography listings.

techniques, but is for more knowledge of the actual behaviour of real structural systems. Such knowledge will help to determine the validity of the assumptions made in the mathematical models used in the dynamic analysis.

The object of the present investigation is to apply both theoretical and experimental approaches to study the dynamic response of simple portal frames, beams and columns subjected to transverse base motion.

1.2 Historical Review

Extensive work has been done on the transverse vibration of simple beams. For other than simple loading and boundary conditions, the methods based on the exact differential equations are extremely tedious and inordinately involved. The advent of high speed digital computer has boosted the efforts of many researchers to develop various methods for studying in greater detail the dynamic behaviour of complex structural systems.

The energy method proposed by Rayleigh[1] has been extensively applied to laterally vibrating structures. Its' usefulness is somewhat limited to relatively simple problems. Although it is possible to apply it to members of variable stiffness, statically indeterminate members of more than one span are not susceptible to an easy solution by this method.

Stodola[2] has proposed an interative method for obtaining the normal modes of vibration. This method requires a trial eigenfunction in the computation of the eigenvalue. The convergence will be faster if the chosen original trial-eigenfunction is a good approximation to the exact fundamental eigenfunction. This approach can be extended to get the higher eigenfunctions and eigenvalues by using the property of orthogonality of normal modes. In the case of problems of complex nature, it will be difficult to choose a trial eigenfunction that is close to the exact fundamental eigenfunction and hence this method is not well suited for the analysis of complex structural systems.

Pestal[4] et al have generalized the well known methods of Holzer, Myklestad and Thompson and the method is described as the method of Transfer Matrices. A lumped-mass idealization is used in this approach and the computations require the trial value of frequency in the transfer matrices. Though this method, in principle, permits the analysis of all types of frame works, it has been reported in the literature[5] that this method takes a considerable amount of computer time when the number of lumped masses becomes large and when many frequencies are required.

The discrete-element stiffness matrix technique for static and dynamic problems has been widely discussed and well documented in the literature[6,7,8,9]. This approach has proved advantageous in obtaining approximate analyses of complex structural configuration that are difficult to handle by exact mathematical formulation.

The lack of a practical theoretical treatment consistent with the direct stiffness matrix approach has resulted in the use of gross lumping technique by many researchers with resulting inaccuracies[10,11] in the solution. In an attempt to improve the accuracy of the dynamic analysis as it is affected by the mass matrix, a consistent mass-matrix construction similar to standard stiffness matrix synthesis technique is investigated by Archer[11,12] which accounts for the actual distribution of mass throughout the structure. This approach is used by the above author in the dynamic analysis of simple beams and it is shown that the results obtained by the use of consistent mass matrix are upper bounds to the exact solution.

The consistent mass matrix construction investigated by Archer is limited to simple beam elements subjected to only inplane bending and hence the technique is extended, by Zienkiewicz[13] et al, to space elements. A general mass-matrix construction for various elements is presented in reference 8.

1.3 Present Work

The analytical and experimental investigations presented in this disseration are primarily related to the dynamic response of simple portal frames, columns and beam respectively subjected to dynamic base motion. The main purpose of the investigation is to determine both theoretically and experimentally the natural frequencies and the relative displacements of the structural system. In addition, the present investigation also includes the study of inter-action of components, namely, beam and column in modes of the combined structure (portal frame).

The physical models used in the theoretical and experimental investigations are described in Chapter II.

The theoretical analysis of a simple frame is briefly outlined in Chapter III. The equation of motion used for the dynamic analysis is also presented in this chapter. The analysis is carried out for the following cases:

(i) Response of the structure to free vibration.

- (a) Treating the joints as rigid.
- (b) Treating the joints as non-rigid .

(ii) Response of the structure to sinusoidal base motion.

The experimental investigation on the physical models is described in Chapter IV. The object of the investigation is to compare the experimentally obtained results with those obtained analytically. The natural frequencies obtained by the analytical method are compared with those observed experimentally. The joint test procedure for the evaluation of the rotational stiffness of the connections is also discussed in this chapter.

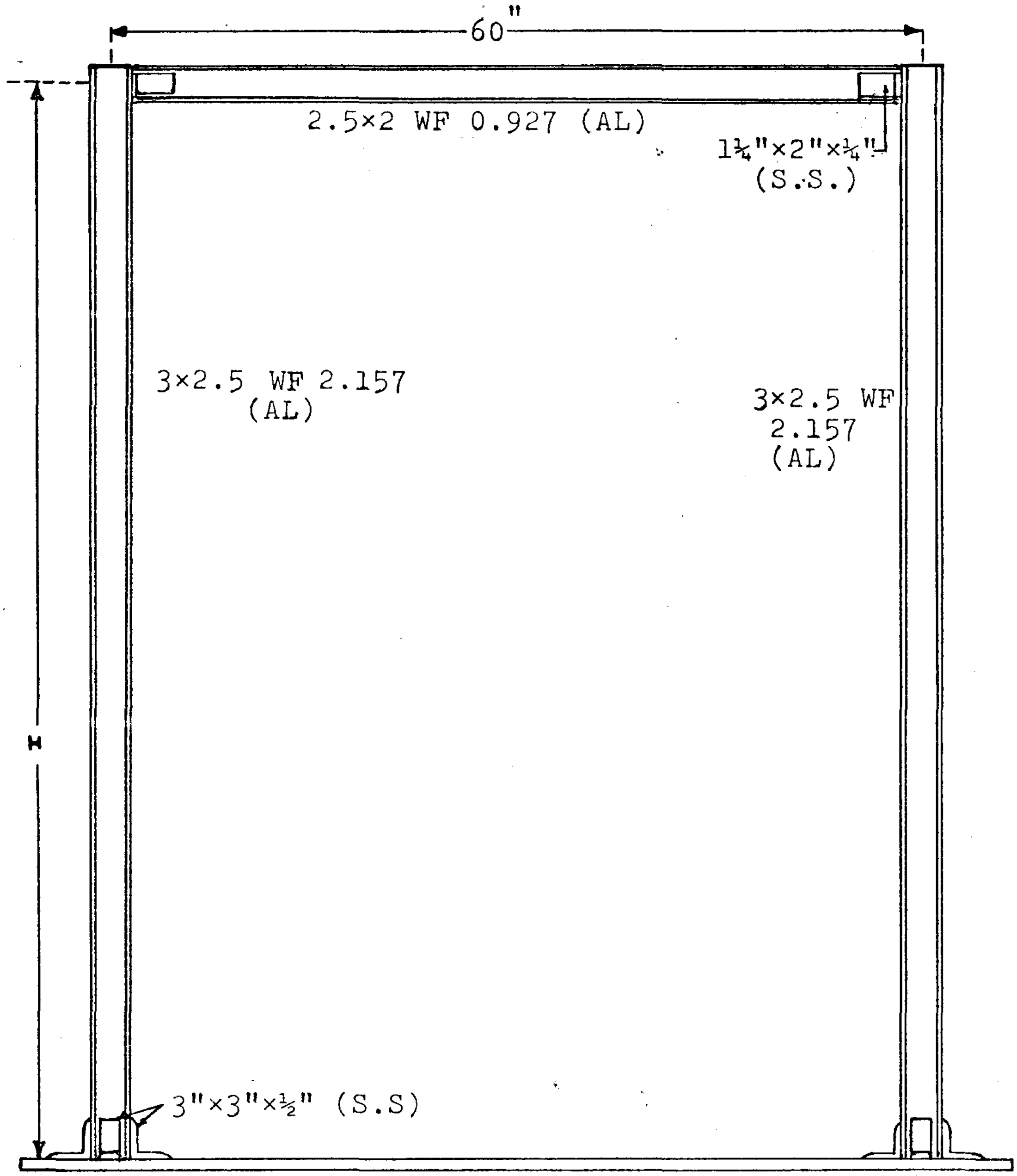
In Chapter V, comparison of the analytical and experimental results are presented and discussed. The interaction of components in modes of combined structure is also discussed in this chapter.

CHAPTER II

PHYSICAL MODEL

For the experimental verification of the theoretical results, it was decided to conduct dynamic tests on simple portal frames, beams and columns. Aluminium members were used in the fabrication of the models. This was done because the wide flange sections used for columns and beams were rolled from the same aluminium alloy thus ensuring a reasonable degree of homogeneity of the material. Column to beam and column to base connections were fabricated by using structural steel sections. This was done to ensure that both the above connections behave as rigid joints. Figures 2-1, 2-2 and 2-3 show the front-elevation of the physical models used in the investigations. It was decided to test the portal frame and column for three different heights (viz) 80", 60" and 40". First the model was fabricated with a height of 80" and the same model was used in the dynamic tests with heights of 60" and 40" respectively after cutting the columns to the required heights. As the present investigation is restricted to the elastic range, the use of the same model after cutting the columns should not significantly affect the results. After completing all the dynamic tests for the three levels, the columns were cut at 5" from bottom and the beam was fixed to the columns. This model was used for the beam test. After completing the dynamic test on this beam, it was used for the joint tests, so that the moment-rotation properties of the joints could be evaluated. The details of the column to the base and the beam to the column connections are respectively shown in Figures 2-4 and 2-5.

The stress-strain curve of the aluminium shown in Fig. 2-6 is taken from the reference 14. As the sections used in the present investigation and the ones used in the above reference are from the same batch of aluminum alloy, the stress-strain properties taken from this curve should be valid for the material used in this investigation.



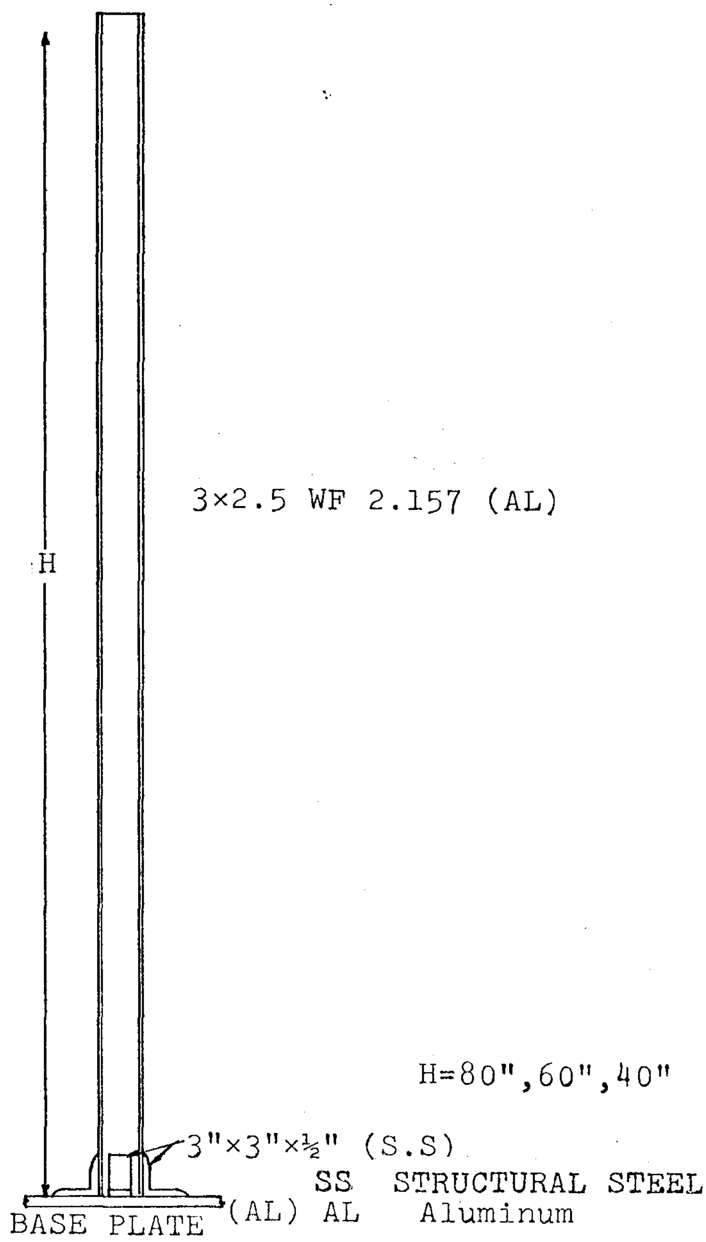
BASE PLATE (AL)

FRONT ELEVATION

AL ALUMINUM
SS STRUCTURAL STEEL

FIG. 2.1 PORTAL FRAME

H= HEIGHT
=80",60",40"



FRONT ELEVATION

FIG. 2.2 COLUMN

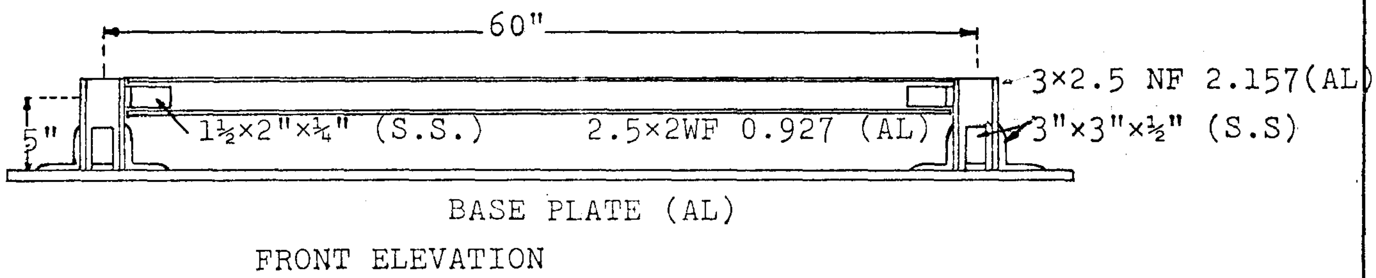

60"
3×2.5 NF 2.157(AL)
5"
1½×2"×¼" (S.S.)
2.5×2WF 0.927 (AL)
3"×3"×½" (S.S)
BASE PLATE (AL)
FRONT ELEVATION

FIG. 2.3 BEAM

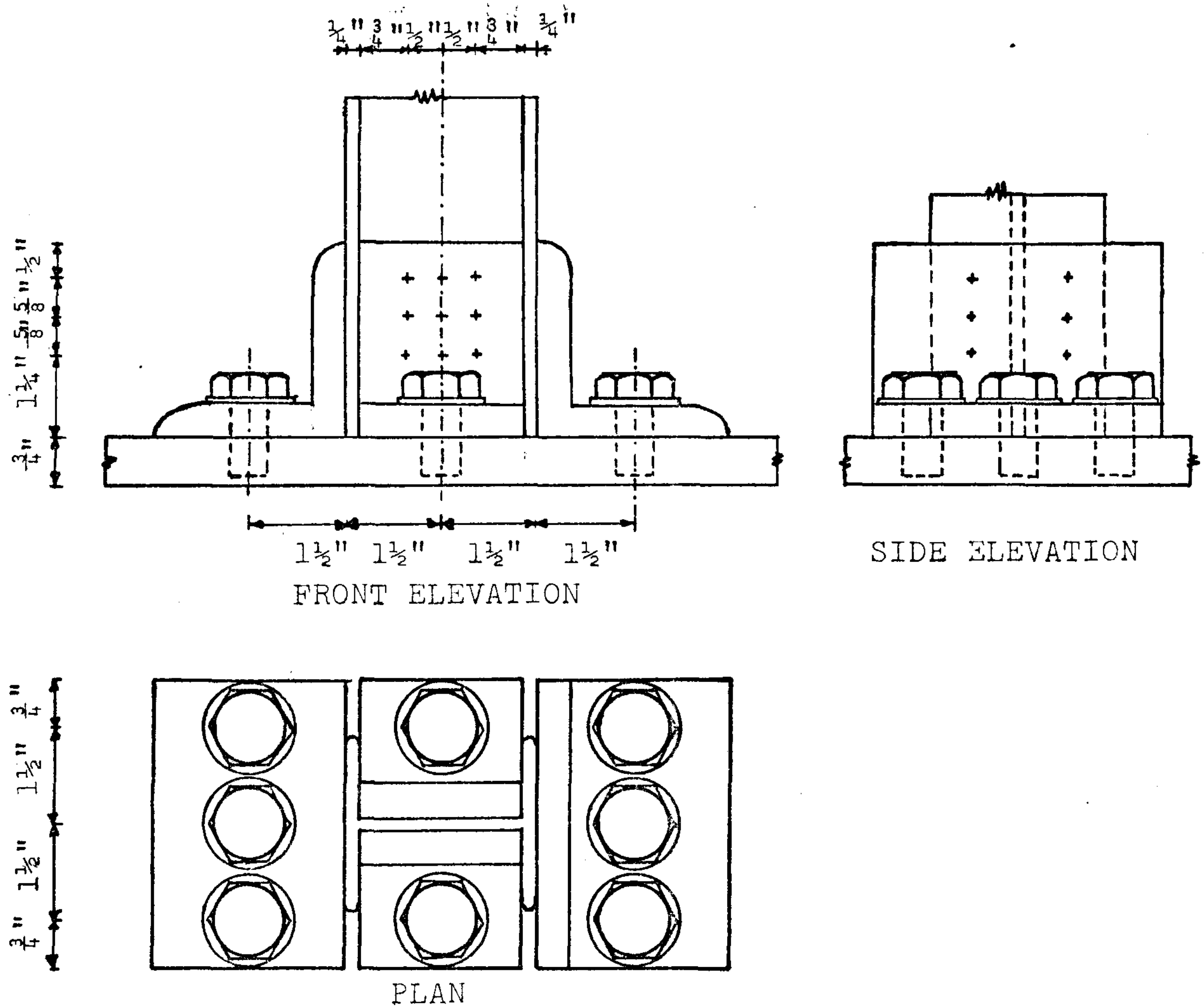
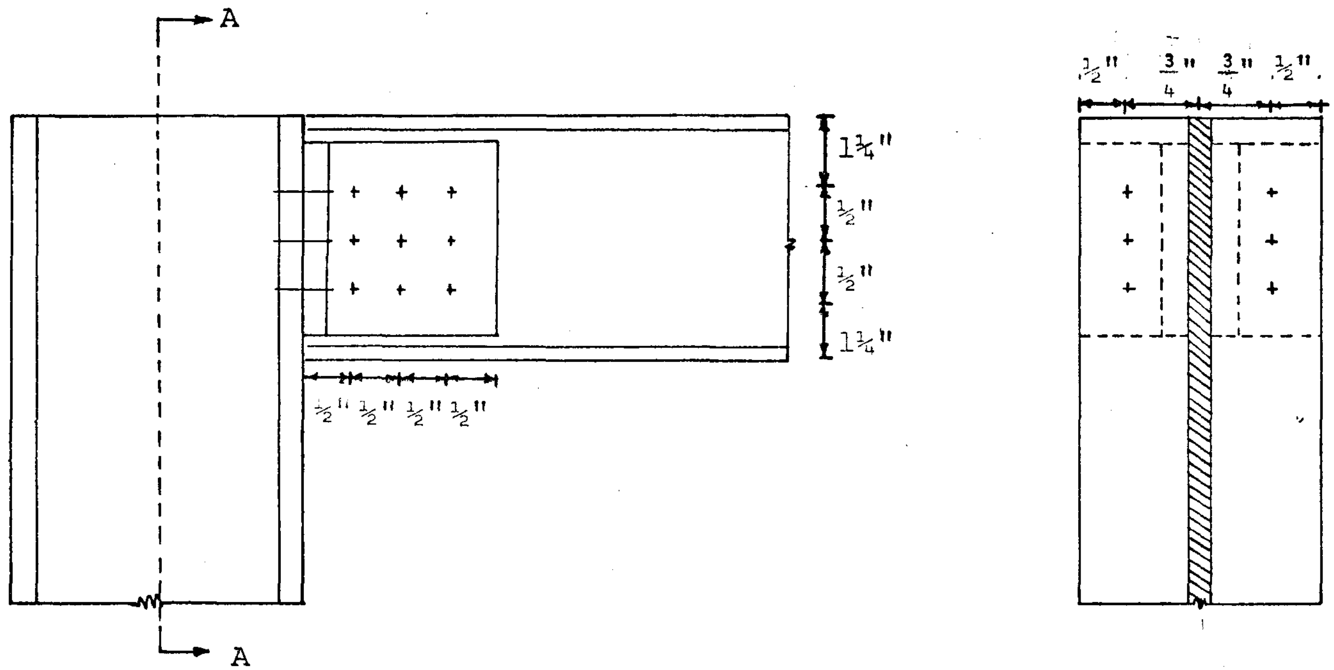


FIG. 2.4 DETAILS OF COLUMN - BASE CONNECTIONS



FRONT ELEVATION

SECTION AA

FIG. 2.5 DETAILS OF BEAM COLUMN CONNECTIONS

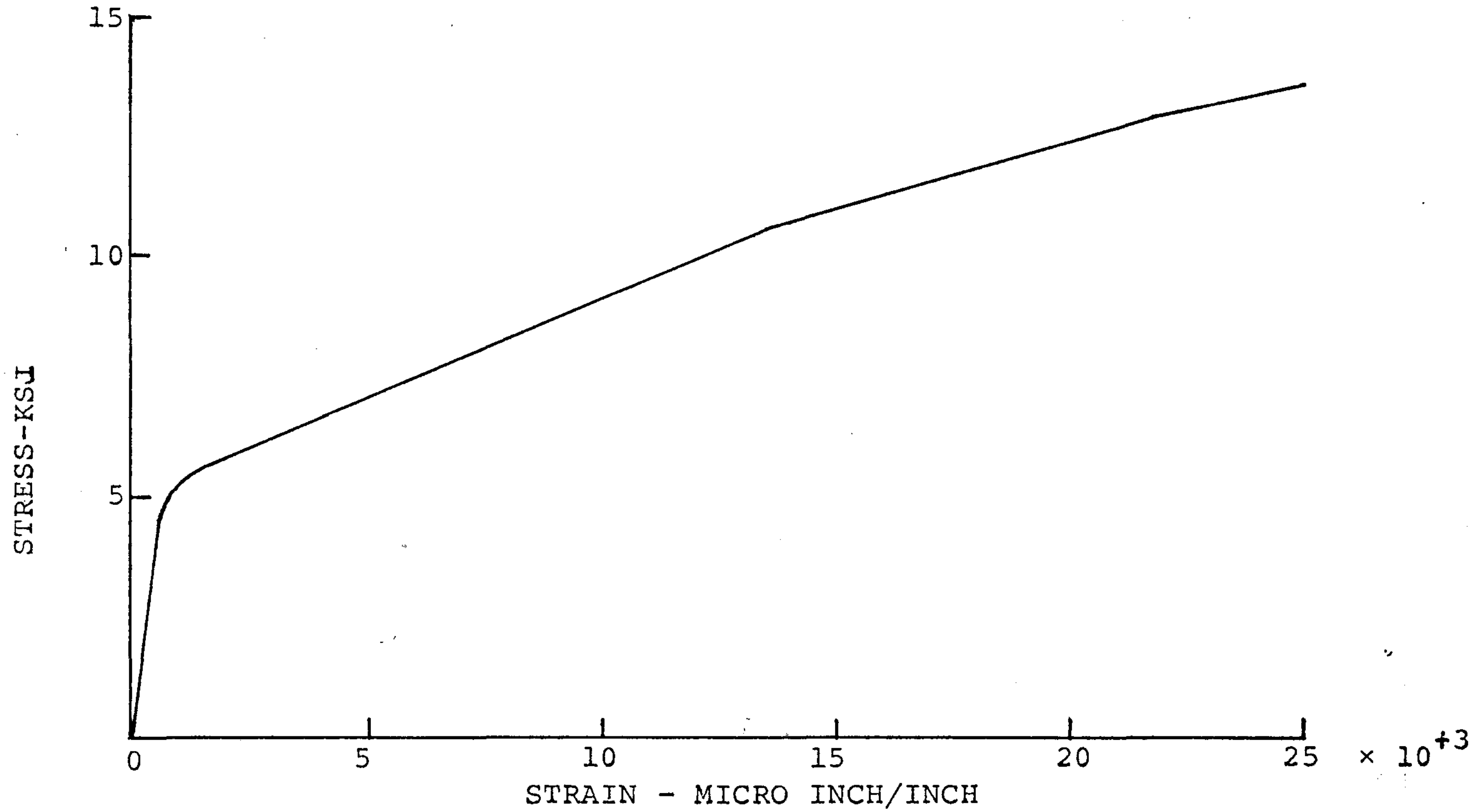


FIG. 2.6 ALUMINUM STRESS-STRAIN CURVE

CHAPTER III

THEORETICAL ANALYSIS

A method describing the dynamic analysis of simple beams is given by Archer[11]. The consistent matrix formulation using finite element technique is the basis for this type of analysis. In the present investigation, the above method is extended to compute the dynamic response of simple space frames.

3.1 Basic Assumptions

The following assumptions are made in the dynamic analysis of frames:

(i) The effect of axial strain is neglected. This has been assumed for the sake of simplicity, by various authors in this field. Based on experience, it has been reported in the literature[15] that if the height to width ratio of the frame is no larger than five, axial strains in columns may be neglected without appreciably affecting the dynamic response of structures.

(ii) The effect of shear deformation is neglected. This assumption is justified as the column lengths of a normally proportioned frame are approximately ten times larger than their depth or longer. Shear deformation can become appreciable when the length to depth ratio of member is small.

(iii) The dynamic analyses of building frames usually involve the assumption of rigid joints, but in many cases the connections are actually rather flexible. Both rigid and non-rigid connections are considered in the present investigation. Recently a method of analysis has been formulated to consider the non-rigidity of the connections in the dynamic analysis of inelastic multi-story building frames[16].

3.2 Basis of Analysis

The theoretical analysis, based on the above assumptions is carried out using the discrete element technique, in which a stiffness matrix defines the elastic characteristics and a consistent mass matrix, which accounts for the actual distribution of mass throughout the element, defines the inertial characteristics. This method is most general and one of the most powerful tools for the analysis of problems of complex nature.

Based on structural rather than mathematical approximations, the finite element method essentially seeks to idealize the structure into an assembly of a finite number of discrete elements connected at a finite number of points, and then proceeds to solve for the system's response on an exact mathematical basis.

The method of obtaining the element stiffness and consistent mass matrices is discussed in references 8,9. The basic steps may be described as follows.

i) A function (or functions) is chosen to uniquely define the displacement distribution inside each element in terms of the nodal displacements.

$$\{u\} = [a]\ \{U\} \tag{3-1}$$

where

$\{u\}$ = displacement of point inside the element

$\{U\}$ = nodal displacements of the element

$[a]$ = a function of the co-ordinate of the point and the assumed displacement function.

ii) The strain field can be obtained from the displacements.

$$\{e\} = [B]\ \{U\} \tag{3-2}$$

where

$[B] = [B(x,y,z)]$.

iii) The element mass and stiffness matrices can be obtained by equating external and internal virtual work.

$$[m] = \rho \int_{V} [a]^{T}[a]\ dv \tag{3-3}$$

$$[K] = \int_{V} [B]^{T}[\chi][B]\ dv \tag{3-4}$$

where

ρ = density of the material

$[\chi]$ = matrix of linear elastic coefficients in the relation between the stress and strain.

[m] = element mass matrix

[k] = element stiffness matrix.

3.3 The Consistent Mass Matrix

The simplest form of mathematical model for inertia properties of structural elements is the lumped-mass representation. In this idealization the mass of the element is lumped at nodes in the direction of the assumed element degrees of freedom. The resulting mass matrix is diagonal and leads to a simple formulation and solution. However, the computed natural frequencies and mode shapes may differ considerably from the exact values. To improve the accuracy of the dynamic analysis as it is affected by the mass matrix, a consistent mass matrix construction which accounts for the actual distribution of mass throughout the element was investigated by Archer[11] for simple beams subjected to bending and the method was generalized for the space elements by Zienkiewicz and Cheung. The name of 'consistent mass matrix' has been coined for this distributed mass element matrix and may be obtained from the expression 3.3.

Frequently the static displacement distributions are assumed to determine [a] as it is difficult to determine the same for structural systems subjected to a general dynamic loading. Thus the mass matrix obtained by this procedure will be an approximate one, however, when the discrete elements selected are small, the accuracy of such mass representation may be adequate for practical purposes.

3.4 Assembly of the Overall Matrix

The element mass and stiffness matrices are assembled to get the overall mass and stiffness matrices of the entire structure, based on the condition of equilibrium.

Each component of the external forces $\{R_i\}$ acting at a node must be equal to the sum of the component forces in the same direction shared by elements joining at that node.

$$\{R_i\} = \Sigma\{F_i\}. \tag{3-5}$$

the summation being taken over all the elements connected at node i, substituting for F_i,

$$\{R_i\} = \sum_{p=1}^{n} \sum_{q=1}^{N} ([m_{ip}]^q\{\ddot{U}_p\}+[k_{ip}]^q\{U_p\}) \tag{3-6}$$

in the absence of initial strains, and

n = number of nodes

N = number of elements in the structure.

The inside summation is taken over all the elements of the structure.

If a particular element is not connected to node i, it will not contain submatrices with an i suffix. Hence the submatrices $[M_{ip}]$ and $[K_{ip}]$ of the assembled mass and stiffness matrices are

$$[M_{ip}] = \sum_{p=1}^{n} \sum_{q=1}^{N} [m_{ip}]^q \tag{3-7}$$

$$[K_{ip}] = \sum_{p=1}^{n} \sum_{q=1}^{N} [k_{ip}]^q \qquad (3\text{-}8)$$

where the superscript q refers to the element number.

3.5 Equation of Motion for the Entire System

The equation of motions which are derived from the similar equations formulated for each one of the elements separately appears as follows in matrix form, for a viscously damped system.

$$[M]\ \{\ddot{U}\} + [C]\ \{\dot{U}\} + [K]\ \{U\} = \{F(t)\} \qquad (3\text{-}9)$$

where

[M] is a square symmetrix matrix of masses in global co-ordinate system.

[K] is a square symmetric matrix of stiffness in global co-ordinate system.

[C] is a square damping matrix.

$\{\ddot{U}\}$ is a column matrix of acceleration in global system.

$\{\dot{U}\}$ is a column matrix of velocity in global co-ordinate system

{U} is a column matrix of displacement in global system.

{F(t)} column matrix of applied dynamic load.

3.6 Analytical Predictions

The prediction of the dynamic characteristics of the structural system used for the experimental investigations was done by using the methods described in Section 3.2. The properties of sections and the material properties were taken from the properties of Alcan Extruded Shapes[17]. All the computations were done, at the McMaster University Data Processing and Computing Centre on a CDC6400 computer. The computer programme written for this purpose is given in the Appendix I.

The quantities predicted for comparison with those obtained experimentally (details follow in Chapter IV) are the following.

(a) the natural frequencies (for a limited number of modes)

(b) the relative displacements in each mode for which a frequency is computed.

3.6.1 Response of the structural system to free-vibration

The following assumptions were made to compute theoretically the natural frequencies and mode shapes.

(i) damping is absent

(ii) the amplitudes of transverse vibrations are small

(iii) the structure remains elastic.

Based on the above assumptions, the free vibration analysis is carried out by treating the joints as rigid and as well as non-rigid. The properties of the elements with rigid and non-rigid connections are taken respectively from the references 8,18 and the over-all stiffness matrix for the complete structure is obtained by using the direct assembly technique

The natural frequencies and the corresponding mode shapes were computed using the standard procedures available in structural dynamics[6,17] and are given in Tables 3.1 to 3.7, for the various structural systems considered.

3.6.2 Response of the structural system to sinusoidal base motion

The elastic response of the structures was predicted by using the method outlined in the reference[17]. A single sinusoidal pulse having a frequency equal to the first natural frequency was used as the excitation frequency to predict the response of the system. In all the predictions a damping value of 0.6% of the critical viscous type of damping was used. This value of damping was determined from the decay of amplitude under free-vibration.

The predicted relative displacements for the various structural systems are given in Table 3.8.

3.7 Convergence of Results

The most important items that govern the accuracy of the solutions are the physical approximation or idealization into a finite-element system and the manner in which the displacement patterns inside the elements are defined. In the present investigation only the beam elements are used for which the deflection's distributions are well documented in the literature. It is of interest to investigate the convergence of the results obtained by using these element properties.

To obtain convergence to the true results, it is necessary to increase the number of elements. The size of the elements or the number of nodal points necessary to get reasonably good approximation is not obvious. There are no fixed guide-lines to find out the size of the elements required. But an inference can be drawn by studying the convergence of the results. The variation of the results obtained for successively refined elements gives an indication as to the correctness of the results.

The portal-frame considered for illustration is shown in Fig. 2.1. The values of the first five frequencies obtained for four different idealizations, with respectively 2, 5, 11 and 13 nodal points (Fig. 3.1a,b,c,d) are tabulated in Table 3.9.

A plot of (w_n/w_{nbest}) vs number of nodes is shown in Fig. 3.2. The plot indicates that the convergence of the results is excellent even with 13 nodal points and hence further refinements of the elements is not required.

Mode Number	Frequency in C.P.S.			% Difference between experimental and theoretical frequencies	
	Rigid joint Analysis	Non-rigid joint analysis	Experimental values	Rigid case	Non-rigid case
1	8.2	8.2	7.5	+9.3	+9.3
2	9.7	9.7	8.9	+9.0	+9.0
3	42.9	42.2	42.3	+1.4	-0.2
4	60.8	59.6	65.4	-8.5	-8.9
5	71.3	70.6	79.0	-9.7	-10.1

Table 3.1 Comparison of theoretical and experimental frequencies of portal frame
Height: 80"

Mode Number	Frequency in C.P.S.			% Difference between experimental and theoretical frequencies	
	Rigid joint Analysis	Non-rigid joint analysis	Experimental Values	Rigid	Non-rigid
1	13.5	13.5	12.4	+8.9	+8.9
2	16.8	16.7	16.0	+4.9	+4.4
3	50.2	49.6	48.5	+3.6	+2.3
4	103.6	100.3	107.1	-3.3	-6.3

Table 3.2 Comparison of theoretical and experimental frequencies of portal frame
Height: 60"

Mode Number	Frequency in C.P.S.			% Difference between experimental and theoretical frequencies	
	Rigid joint Analysis	Non-rigid joint analysis	Experimental values	Rigid	Non-rigid
1	24.8	24.6	23.1	+7.3	+6.5
2	35.4	35.3	32.8	+8.0	+7.6
3	61.7	61.2	58.1	+6.2	+5.3

Table 3.3 Comparison of theoretical and experimental frequencies of portal frame
Height: 40"

S.NO.	Added weight lbs.	FREQUENCY C.P.S.							
		ω_1			% Difference between experimental & theoretical values	ω_2			% Difference between experimental & theoretical values
		Finite element method	Conventional method	Experimental values		Finite element method	Conventional method	Experimental values	
1	0	10.5	10.4*	10.4	+2.22	66.3	65.1*	64.3	+3.22
2	2.5†	8.1	8.1**	7.9	+2.02	55.4		56.1	+1.25

* Continuous analysis (reference 19)

** Approximate analysis (reference 20)

† Weight added to the column top to simulate portal frame conditions.

Table 3.4 Natural frequencies of column
Height: 80"

S. NO.	Added weight lbs.	FREQUENCY C.P.S.							
		ω_1			% Difference between experimental & theoretical values	ω_2			% Difference between experimental & theoretical values
		Finite element method	Conventional method	Experimental values		Finite element method	Conventional method	Experimental values	
1	0	18.8	18.5*	18.4	2.2	118.4	115.8*	117.3	0.9
2	2.5†	13.4	13.4**	12.9	3.8	95.97	-	99.7	3.7

* Continuous analysis (reference 19)

** Approximate analysis (reference 20)

† Weight added to the column top to simulate portal frame conditions

Table 3.5 Natural frequencies of column
Height: 60"

S.NO.	Added weight lbs.	FREQUENCY C.P.S.							
		ω_1			% Difference between experimental & theoretical values	ω_2			% Difference between experimental & theoretical values
		Finite element method	Conventional method	Experimental values		Finite element method	Conventional method	Experimental values	
1	0	41.7	41.6*	41.3	+0.9	261.3	260.5*	-	-
2	2.5†	26.8	27.0**	27.2	- 1.5	206.0	-	-	-

* Continuous analysis (reference 19)

** Approximate analysis (reference 20)

† Weight added to the column top to simulate portal frame conditions.

Table 3.6 Natural frequencies of column
Height: 40"

Mode No.	FREQUENCY IN C.P.S.					
	Fixed-Fixed Beam			Hinged-Hinged Beam		
	Finite element method	Conven-tional method *	% Dif-ference	Finite element method	Conven-tional method *	% Dif-ference
1	90.1	88.5	+1.8	39.9	39.0	+2.5
2	248.9	243.9	+2.1	159.7	156.1	+2.3
3	490.1			359.5		
4	819.1			639.5		
5	1229.0			925.1		

*Conventional method (reference 19)

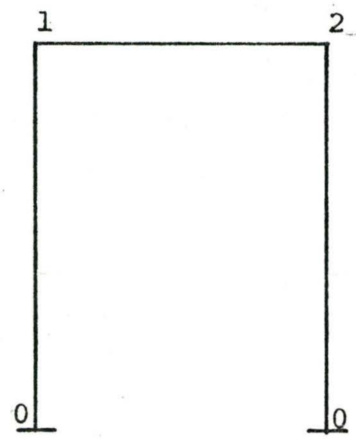
Table 3.7 Natural frequencies of fixed-fixed and hinged-hinged beams
Length of beam: 60"

Height	Mode Number	Relative modal displacement (at top of column)	
		Portal Frame	Column
80"	1	+0.42	+0.66
	2	±0.03	-0.20
	3	+0.01	-0.066
60"	1	+0.36	+0.55
	2	±0.024	-0.13
	3	+0.05	+0.04
40"	1	+0.23	+0.44
	2	±0.015	-0.066
	3	+0.11	+0.02

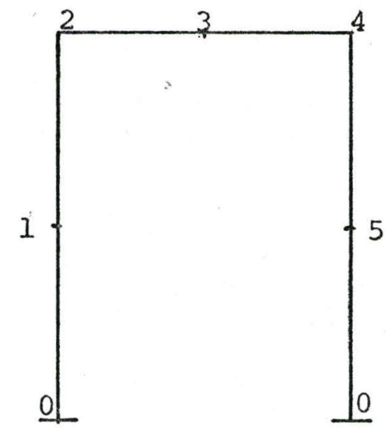
Table 3.8 Relative modal displacements at the top of the column

Frequency No.	NO. OF ELEMENTS 3 Fig. 7-a C.P.S.	6 Fig. 7-b C.P.S.	12 Fig. 7-c C.P.S.	14 Fig. 7-d C.P.S.
ω_1	8.2	8.2	8.2	8.2
ω_2	9.7	9.8	9.7	9.7
ω_3	44.5	43.1	42.9	42.9
ω_4	112.5	61.2	60.8	60.8
ω_5		71.7	71.3	71.3

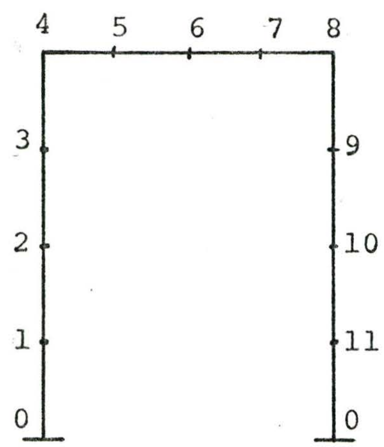
Table 3.9 Natural frequency of portal frame corresponding to the number of elements considered.
Height: 80"



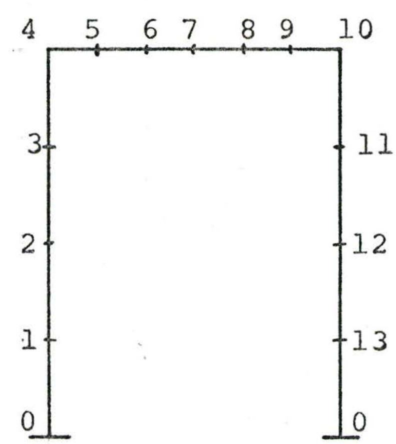
a) Number of nodes 2
Number of elements 3



a) No. of nodes 5
No. of elements 6



c) Number of nodes 11
Number of elements 12



d) Number of nodes 13
Number of elements 14

FIG. 3.1 FINITE-ELEMENT IDEALIZATIONS OF PORTAL FRAME
HEIGHT: 80" , WIDTH: 60"

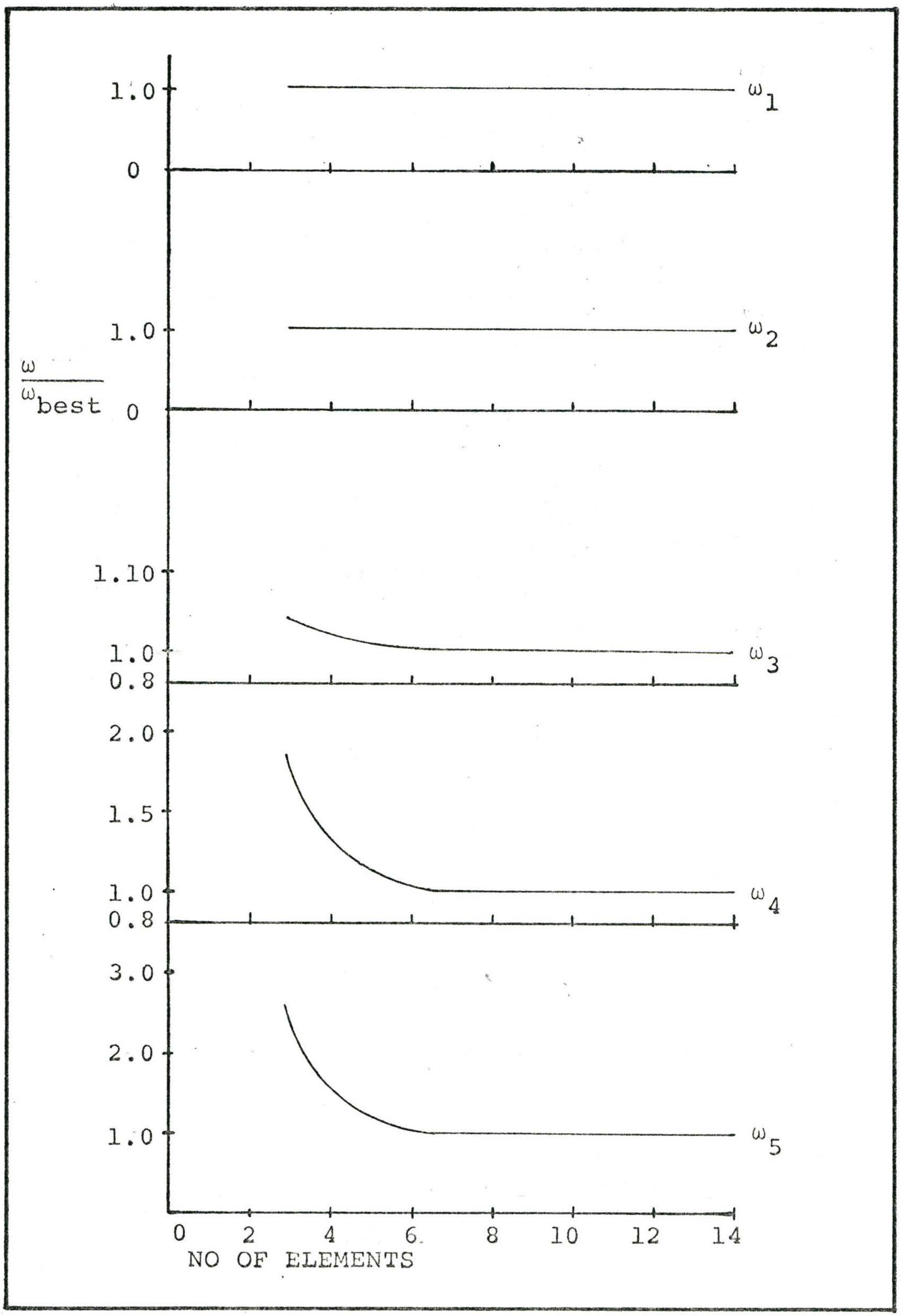


FIG. 3.2 NO. OF ELEMENTS VS $\frac{\omega}{\omega_{best}}$ (FOR PORTAL FRAME)
(HEIGHT = 80")

CHAPTER IV

EXPERIMENTAL PROGRAM

The main purpose of the experimental investigation is to determine experimentally the dynamic characteristics of the physical models. The determination of dynamic characteristics includes the determination of damping factor, the natural frequencies of the system, and the mode shapes associated with these frequencies. After the dynamic properties are determined experimentally, the experimental results can be compared with those predicted theoretically. Such a comparison will help to verify the validity of the theoretical approach used to predict the dynamic characteristics of the systems.

4.1 Experimental System for Dynamic Test

The experimental system consists of a shake table 6.5 ft wide and 7.0 ft long in plan dimension and having a live weight capacity of 3000 lbs. during dynamic loading. It is excited by a servo-controlled actuator which can apply base accelerations of 1g to the shaking table (with maximum live weight attached) at frequencies which may exceed 100 cycles per sec.

The structures loading system (M.T.S 903.03) supplied by the M.T.S. Corporation is used to excite the table and displacement is the control parameter. The programming section of the loading system is capable of accepting program inputs having velocity or acceleration dimensions and applying these inputs to the control portion of the system in corresponding displacement dimensions. This is accomplished by accepting velocity or acceleration input and integrating or double integrating the input as required to develop a displacement command for the control portion of the system.

The motion of the shake table is controlled by a servo-controlled loading feed-back system. The principle behind this system is the comparison of the shaking table displacement with the actual displacement of the shaking table at any instant of time. If any difference is detected between these two, a correction signal is sent to the servo-valve which adjusts the flow of hydraulic fluid into the actuator to eliminate the detected difference of the desired and existing displacements.

Strain Gauges

The strain gauges are mounted at the base and top of the columns and at intervals along the length of the beam. These gauges were of high elongation type H.E.141-B as manufactured by the Budd Company, Phoenixville, Pa. The resistance and the gauge factor of the gauges were 120±0.2 ohms and 2.05±0.5 % respectively. These were specified to be suitable for use up to a temperature of +200°F.

The strain gauges were connected to a direct writing type R Dynograph (Beckman Instruments, Inc., Offner Division, Schiller Park, Ill.).

Accelerometers

Two accelerometers were mounted, respectively, one near the top of the column and the other at the centre of the beam. One accelerometer was also mounted on the shake table to record the base acceleration applied to the experimental structure.

The accelerometer mounted on the shake table was a Universal Servo Accelerometer Model 305A, S/N 2477. Its sensitivity is 0.2 ma/g, 0.100 v/g. The output from this accelerometer was recorded on the type R Dynograph Direct Writing Recorder. The accelerometers mounted on the frame were Endevco Series 2200 Accelerometers. The accelerometers used on the column and at the centre of the beam were models 2221C Serial ED60 and 2221C serial EC54 respectively. The operating acceleration range varies from 0.001_g to 10,000 g's. The Endevco Accelerometer mounted on the column was connected to the Dynograph Direct Writing Recorder.

The output from the Endevco Accelerometer mounted at the centre of the beam was amplified by a Laboratory Amplifier model 2616B. The resulting output was fed to Model

CEC1-165D.C Amplifier. The output signal thus obtained was fed to the Direct Writing Oscillograph Recorder type RG32.12/15 This recorder uses an ultra-violet light source using pencil type mirror galvonometers focussed onto ultra-violet sensitive recording paper.

4.2 Damping

To determine the damping factor, the frame was given a push by hand and the strain responses were recorded. The strain response was plotted (Fig. 4.1) on a semi-logarithmic scale to find the damping factor. It may be seen that this plot is a straight line whcih indicates that the dmaping is constant and is of viscous type.

The value of the logarithmic decrement $\bar{\delta}$ is given by

$$\bar{\delta} = \ell n \, (\bar{x}_i/\bar{x}_j)/(j-i) \tag{4-1}$$

in which $\bar{x}_i$ and $\bar{x}_j$ are the maximum amplitudes of free vibrations in i^{th} and j^{th} cycles (j>i). From the plot shown in Fig. 4.1 $\ell n(\bar{x}_i/\bar{x}_j) = 0.693$ for $j-i = 17.8$. Thus the logarithmic decrement $\bar{\delta}$ is given by

$$\bar{\delta} = \frac{0.693}{17.8} = 0.0385.$$

Therefore the damping factor ζ is given by

$$\zeta = \frac{\bar{\delta}}{2\pi} = \frac{0.0385}{2\pi} = 0.006$$

$$= 0.6\% \text{ of critical damping.}$$

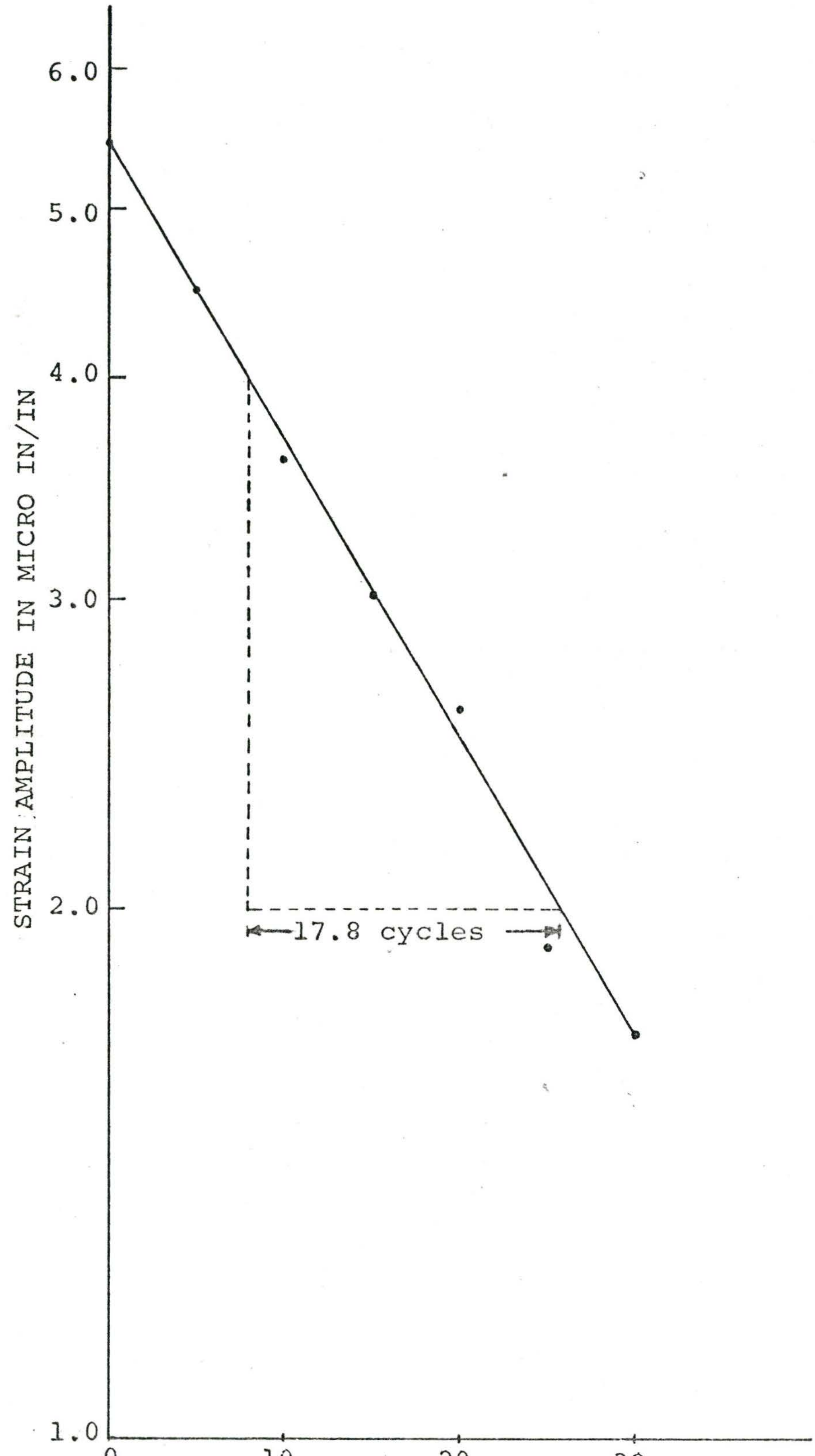


FIG. 4.1 LOGARITHMIC DECREMENT OF STRAIN RESPONSE MEASURED AT THE BASE OF THE COLUMN

4.3 Dynamic Test

The natural frequency of the various systems were determined by giving the base of the structure a sinusoidal displacement and varying the frequency. For convenience in observing the magnification of the response, the signal from an accelerometer mounted near the top of the column was fed to an oscilloscope. The amplitude of the base displacement was kept at ±0.005 in, in order to keep the response of the structure well within the elastic range. As soon as the natural frequency was reached while increasing the base frequency, a spontaneous amplification of amplitude was observed. It was decided not to keep the structure in resonance condition for a longer time to avoid any possibilities of damage resulting from resonance condition. This procedure was repeated several times to verify whether or not the frequency thus obtained remains stationary. It was observed that this remains reasonably constant. The natural frequencies thus observed are given in Tables 3.1 to 3.7.

4.4 Calibration

Beckman Oscillograph

A cantilever beam was used in the calibration test. Strain gauge form the cantilever beam was connected to one channel on the Beckman oscillograph. A known static load was applied at the tip of the beam and the strain recordings at all the gauges were recorded by the recorder. The strains were also measured with the help of strain indicator. On this basis a calibration table was prepared.

Load Cell

A load cell was constructed, using 3/4" diameter steel bar. Strain gauges were fixed on the circumference of the load cell.

The load cell was given a number of load cycles from 0 to 8000 lbs on a Tinius-Olsen testing machine. This was done in order to reach a stage where the load-deformation characteristics did not change appreciably. The load cell was used in the joint test.

4.5 Evaluation of Rotational Stiffness of Connections

The prediction of elastic response of non-rigid frame requires the knowledge of rotational stiffness of the connections. The rotational stiffness of the connection can be determined from the moment-rotation relationship of the connection. As the deformations in a dynamic process are likely to be cyclic, it is realized that the determination of the rotational stiffness of the connections should be done by the application of dynamic loading. However, it is of interest to investigate whether or not the rotational-stiffness of connections determined under static conditions and subsequently used to predict the dynamic properties of the frame gives a satisfactory correlation with the results observed experimentally.

Such a study is of interest as the rotational stiffness of the connections can be determined more easily under static loading rather than under dynamic loading.

Fig. 2.3 shows the experimental structure for the beam column joint test. The height of the column was about 5" and the full length of beam was used in the test. The load was applied through a load-cell and acting at the centre of the beam. The displacement gauges were mounted to measure horizontal rotation of the joints.

The moment-rotation characteristics determined from the column-beam joint test is shown in Fig. 4.2. Based on tangent modulus technique, the computed rotational stiffness is 12.24×10^5 in lbs per radian. This value is used to predict the dynamic properties of the frames with non-rigid connections, in Chapter III.

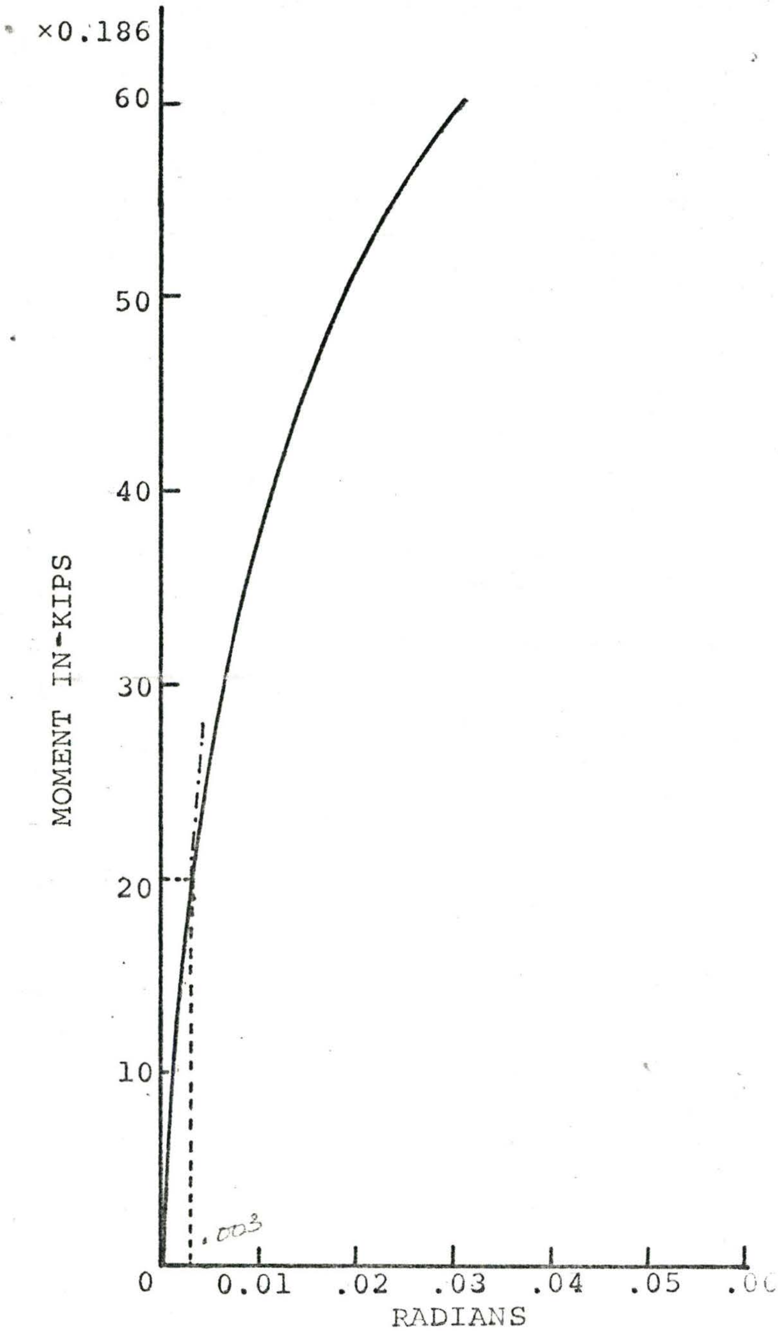


FIG. 4.2 MOMENT-ROTATION RELATIONSHIP OF JOINT FROM BEAM-COLUMN JOINT TEST.

CHAPTER V

DISCUSSION OF RESULTS

5.1 Comparison of Analytical and Experimental Results.

Natural frequencies.

The comparison of the computed and experimentally observed results for the various structural systems are given in Tables 3.1 to 3.7.

The observation of the frequencies of the portal frames computed by treating the connections as rigid and as non-rigid indicates they are nearly the same.

A comparison of the frequencies of the portal frames computed by finite element method with the experimentally observed values indicated that they are in good agreement. The difference between the actual and predicted natural frequencies is less than 10%.

The frequencies of the columns computed by finite element method and conventional methods[19,20] are in good agreement. The predicted values for the various cases are in good agreement with those observed experimentally.

The comparison of the frequencies of the beam computed by finite element method and conventional method shows that they are in good agreement. As the experimental structure (Fig. 2.3) used for the beam test behaved more like a portal frame rather than like fixed-fixed beam, the computed results of fixed-fixed beam could not be compared with the experimental results.

The first three mode shapes of the portal frames determined based on the strain response, agree with those computed theoretically. In the case of portal frame with a height of 60", the visually observed second mode shape agrees with the theoretically computed one.

5.2 Effect of Added Mass on Top of the Column

To investigate the effect of added mass on top of the column, the frequencies are computed for the various cases of added masses on the top of column. The computations of frequencies are also carried out for the three different column heights, namely, 80", 60" and 40". The added mass is varied from 0.0 lbs to 100 lbs. and the theoretically predicted frequencies are given in Tables 5-1, 5-2, and 5-3 respectively for the heights of 80", 60" and 40". The observation of the frequencies corresponding to various added masses indicates that the added mass has an appreciable effect on the first natural frequency compared to the higher frequencies. The different added masses results in an entirely different dynamical system with altogether different dynamic properties. Hence, it will be desirable to make proper estimation of the mass of the system preceding the analysis and design.

5.3 Interaction of the Components in Modes of the Combined System

The main purpose of this particular study is to investigate the interaction of the component structures, namely, column and beam, in modes of the combined structure .
To aid in the understanding of the interaction effect, the

Added wt. in. lb → / Mode No. ↓	Frequency in c.p.s.										
	0	10 lbs	20 lbs	30 lbs	40 lbs	50 lbs	60 lbs	70 lbs	80 lbs	90 lbs	100 lbs
1	10.5	5.3	4.0	3.4	3.0	2.7	2.4	2.3	2.1	2.0	1.92
2	66.3	49.8	48.2	47.7	47.4	47.2	47.1	47.0	46.9	46.9	46.8
3	185.9	154.3	152.5	151.9	151.6	151.4	151.2	151.1	151.1	151.0	151.0
4	364.9	318.4	316.5	315.9	315.5	315.3	315.2	315.0	315.0	315.0	314.9
5	605.4	543.4	541.6	540.9	540.6	540.4	540.2	540.1	540.1	540.0	540.0

Table 5.1 Computed frequencies of column
Height: 80"

Mode Number ↓ \ Wt. added in lbs →	Frequency in c.p.s.										
	0	10 lbs	20 lbs	30 lbs	40 lbs	50 lbs	60 lbs	70 lbs	80 lbs	90 lbs	100 lbs
1	18.8	8.5	6.3	5.3	4.6	4.1	3.8	3.5	3.3	3.1	3.0
2	118.4	87.2	85.0	84.2	83.8	83.6	83.3	83.3	83.2	83.1	83.1
3	330.9	272.8	270.3	269.4	269.0	268.7	268.5	268.4	268.3	268.2	268.2
4	651.5	564.4	561.8	561.0	560.5	560.2	560.0	559.9	559.8	559.7	559.7
5	1086.1	964.6	961.9	961.1	960.6	960.3	960.2	960.0	959.9	959.8	959.8

Table 5.2 Computed frequencies of column
Height: 60"

Wt. added in lbs → / Mode Number ↓	Frequency in c.p.s.										
	0 lbs	10 lbs	20 lbs	30 lbs	40 lbs	50 lbs	60 lbs	70 lbs	80 lbs	90 lbs	100 lbs
1	41.7	16.1	11.8	9.8	8.5	7.7	7.0	6.5	6.1	5.8	5.5
2	261.3	193.0	189.5	188.3	187.7	187.3	187.0	186.9	186.7	186.6	186.5
3	732.2	610.0	606.1	604.8	604.1	603.7	603.4	603.2	603.0	602.9	602.8
4	1437.1	1264.8	1260.9	1259.5	1258.8	1258.4	1258.1	1257.9	1257.8	1257.7	1257.6
5	2384.0	2160.2	2156.2	2154.8	2154.1	2153.7	2153.4	2153.3	2153.1	2152.9	2152.9

Table 5.3 Computed frequencies of column
Height: 40"

investigation is carried out for the various combinations of the column-beam stiffnesses. Such investigation will help to understand the effect of the variations of stiffnesses of the component structures on the behaviour of the combined structure.

To investigate the interaction effect, the frequencies and mode shapes of the structural systems were computed for the following combinations of column-beam frequencies,

- i column frequency << beam frequency
- ii column frequency < beam frequency
- iii column frequency $\simeq$ beam frequency
- iv column frequency > beam frequency
- v column frequency >> beam frequency.

The computed frequencies for the various combinations considered are given in Tables 5-4 to 5-9. The mode shapes of the combined systems were plotted with reference to the member axes (Figs. 5-1 to 5-6) so that these can be easily compared with those of the component systems (Figs. 5-7 to 5-14).

5.4.1 Column frequency << Beam frequency

Observation of the first mode of the combined system (Fig. 5-1a) indicates that this mode is primarily first mode column deformation deforming in the same direction with very little beam deformation. This indicates that the first mode

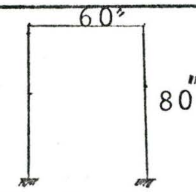
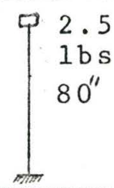
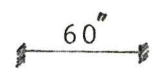
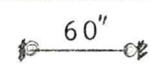
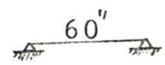
S.No.	FREQUENCY C.P.S.					DESCRIPTION
	1	2	3	4	5	
1	8.2	9.7	42.9	60.8	71.3	
2	8.1	55.4	162.2	327.5	553.3	
3	90.1	248.9	490.1	819.2	1229.0	
4	87.8	247.2	489.1	808.5	1198.7	
5	39.9	159.7	359.5	639.5	925.1	

Table 5.4 Column frequency << beam frequency

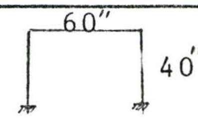
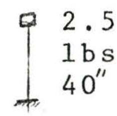
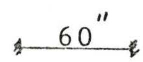
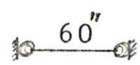
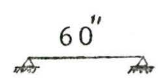
S NO	FREQUENCY C.P.S.					DESCRIPTION
	1	2	3	4	5	
1	24.7	35.4	61.6	167.1	238.6	
2	26.8	205.9				
3	90.1	248.9	490.0	819.1	1229.0	
4	87.8	247.2	489.1	808.5	1198.7	
5	39.9	159.7	359.55	639.5	925.1	

Table 5.5 Column frequency < beam frequency

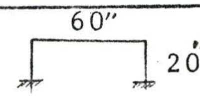
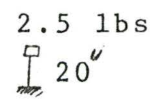
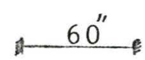
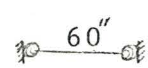
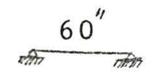
S NO.	FREQUENCY C.P.S.					DESCRIPTION
	1	2	3	4	5	
1	38.7	110.7	145.61	212.4	399.5	
2	85.4	796.3				
3	90.5	251.3	496.1	826.7	1237.5	
4	87.8	247.2	489.1	808.5	1198.7	
5	39.9	159.7	359.5	639.5	925.1	

Table 5.6 Column frequency $\sim$ beam frequency (fixed-fixed)

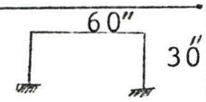
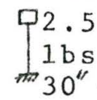
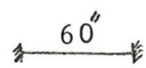
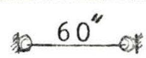
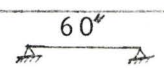
S. NO.	FREQUENCY C.P.S.					DESCRIPTION
	1	2	3	4	5	
1	32.7	59.0	81.0	182.6	356.6	
2	43.8	370.3				
3	90.5	251.3	496.1	826.7	1237.5	
4	87.8	247.2	489.1	808.5	1198.7	
5	39.9	159.7	359.5	639.5	925.1	

Table 5.7 Column frequency $\simeq$ beam frequency (hinged-hinged)

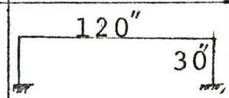
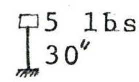
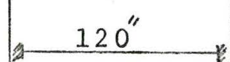
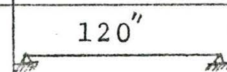
S. NO.	FREQUENCY C.P.S.					DESCRIPTION
	1	2	3	4	5	
1	10.1	35.4	56.8	70.5	106.7	
2	33.9	348.7				
3	22.5	62.0	121.7	201.4	301.4	
4	9.9	39.9	89.8	159.8	250.1	

Table 5.8 Column frequency > beam frequency

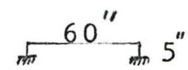
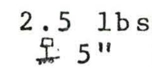
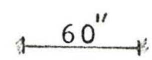
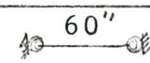
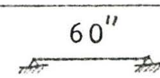
S. NO.	FREQUENCY C.P.S.					DESCRIPTION
	1	2	3	4	5	
1	44.9	164.4	361.7	634.2	924.6	
2	757.9	12130.3				
3	90.5	251.3	496.1	824.7	1237.5	
4	87.8	247.2	489.1	808.5	1198.7	
5	39.9	159.7	359.5	639.5	925.1	

Table 5.9 Column frequency >> beam frequency

of the combined system is similar to the first mode of the column.

The second mode of the combined system (Fig. 5-1b) is again essentially the first mode of the columns, but this time deforming in the opposite directions with zero beam deformation.

The first and the second natural frequencies of this combined structure is nearly equal to the first frequency of the column.

Fig. 5.1c shows the third mode of the combined system. This mode is primarily horizontal beam deformation with a relatively small amount of column deformation. Because of this reason the third frequency of the combined system is similar to the first frequency of the hinged-hinged beam.

Observation of the fourth mode indicates that this mode is essentially the second mode column deformation, deforming in the opposite directions with very little beam deformation. It is for this reason that this frequency is similar to the second frequency of the column.

The fifth mode of the combined structure (Fig. 5.1e) is the combination of the second mode of the columns, deforming in the same direction with first mode of the beam. Since the frequency of the combined structure depends on the coupling of the components, the fifth frequency of the combined structure is greater than the frequency of the component modes.

5.4.2 Column frequency < beam frequency

Fig. 5.2a shows the first mode of the combined structure. This is the combination of the first mode of the columns deforming in the same direction and the first mode of the beam. In this case the first frequency of the combined structure is nearly equal to the first frequency of the column.

The second mode of the combined structure (Fig. 5.2b) is the first mode of columns deforming in opposite directions with zero beam deformation. The second frequency of the combined structure is greater than that of the first frequency of the columns. This indicates that the coupling effect is more predominant in this case than in the previous case (5.4.1).

The observation of the third mode in Fig. 5.2c indicates that this is the combination of the first mode of the columns deforming in the same direction and the first mode of the beam. The frequency of the combined structure depends on the coupling of the components. The third frequency of the combined structure of this particular case, is comparable to the sum of the first frequency of column and the first frequency of beam.

The fourth mode of the combined structure is primarily the second mode deformation of the beam with very little column deformation. Consequently the fourth frequency of the combined structure is nearly equal similar to the second frequency of the beam.

The fifth mode of the combined structure is the combination of the second mode of the columns deforming in the same direction and the first mode of the beam. The sum of the second frequency of the column and the first frequency of the beam is comparable with the fifth frequency of the combined structure. This need not always be true, as the frequency of the combined structure depends on the coupling of the components.

5.4.3 Column frequency $\simeq$ beam frequency (fixed-fixed)

Fig. 5.3a shows the first mode of the combined structure. This mode is primarily the first mode of the hinged-hinged beam with very little column deformation. Hence the first frequency of the combined structure is nearly the same as the first frequency of the hinged-hinged beam.

The second mode of the combined structure is similar to the first mode of columns deforming in the opposite directions with small amount of second mode deformation of the beam. In this case, the second frequency of the combined structure is greater than the frequency of the component modes.

The third mode of the combined structure is the combination of the first mode of the columns deforming in the same direction and the first mode of the beam. The sum of the frequencies of the corresponding modes of the components is less than the frequency of the combined structure. This is probably due to the effect of the coupling of the components.

The fourth mode is the combination of the first mode of the columns deforming in opposite directions and the second mode of the beam. The fourth frequency is less than the sum of the frequencies of the corresponding modes of the component.

Fig. 5.3e shows the fifth mode of the combined structure. This mode is essentially the third mode of the beam with first mode of columns deforming in the same direction. The frequency of the combined structure is different from that of the frequencies of the component modes.

5.4.4 Column frequency $\simeq$ beam frequency (hinged-hinged)

The first mode of the combined structure (Fig. 5.4a) is primarily the first mode deformation of the beam with little column deformation. The frequency of the combined structure is less than that of the frequency of the component modes (Table 5.7).

Fig. 5.4b shows the second mode of the combined structure. This is essentially the first mode of the columns deforming in opposite directions with very little beam deformation. The frequency of it is higher than the component frequency.

Observation of the third mode (Fig. 5.4c) indicates that this is essentially the combination of the first mode of the columns deforming in the same direction and the first mode of the beam. In this case, the sum of the corresponding frequencies of the components is nearly equal to the third frequency of the combined structure.

The fourth mode (Fig. 5.4d) is primarily the second mode deformation of the beam with first mode of the columns deforming in opposite directions. This frequency is less than the sum of the corresponding frequencies of the component modes.

Observation of the fifth mode (Fig. 5.4e) indicates that this is essentially the third mode of the beam with very little column deformation. The fifth frequency is similar to the third frequency of the beam.

5.4.5 Column frequency > beam frequency

The first mode of the combined structure is essentially the first mode beam deformation with very little column deformation. The frequency of the combined structure is close to the first frequency of the hinged-hinged beam.

Fig. 5.5b shows the second mode of the combined structure. This is the combination of the first mode of the columns deforming in opposite directions and the second mode of the beam. In this case, the second frequency of the combined structure lies between the first frequency of the column and the second frequency of the beam.

The third mode is the combination of the first mode of the columns deforming in the same direction and the first mode of the beam. This frequency is greater than the sum of the corresponding frequencies of the component

The fourth mode is the combination of the first mode of the columns deforming in opposite directions and the second mode of the beam. In this case the sum of the corresponding frequencies of the components is comparable to the frequency of the combined structure.

Fig. 5.5e shows the fifth mode of the combined structure. This is essentially the combination of the first mode of the columns deforming in the same direction and the third mode of the beam. The fifth frequency of the combined structure is different from that of frequencies of the component modes.

5.4.6 Column frequency >> beam frequency

The first five modes of the combined structure are given in Figs. 5.6a to 5.6e with reference to the member x axis.

The observation of the mode shapes indicates that they are essentially the beam modes with very small amount of column deformation. The frequencies of the combined structure, in this case, are nearly equal to the frequencies of the hinged-hinged beam. This shows that when the frequency of the column is far greater than that of the beam, the frequencies of the combined structure is close to that of the hinged-hinged beam.

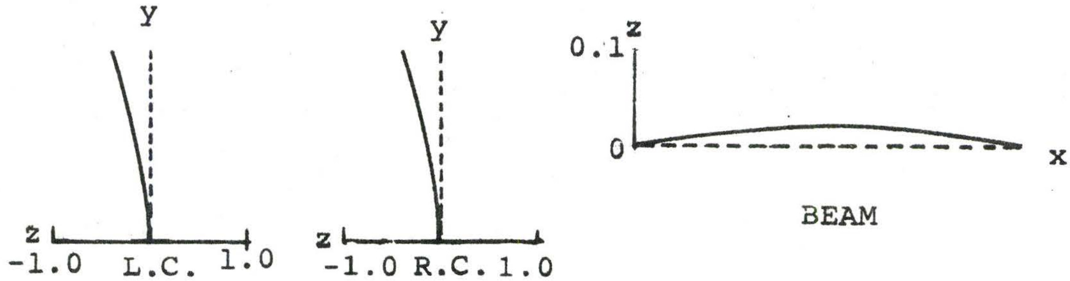


FIG. 5.1(a) MODE 1

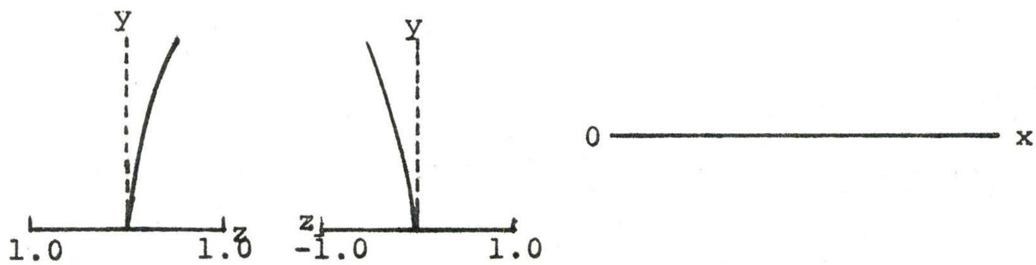


FIG. 5.1(b) MODE 2

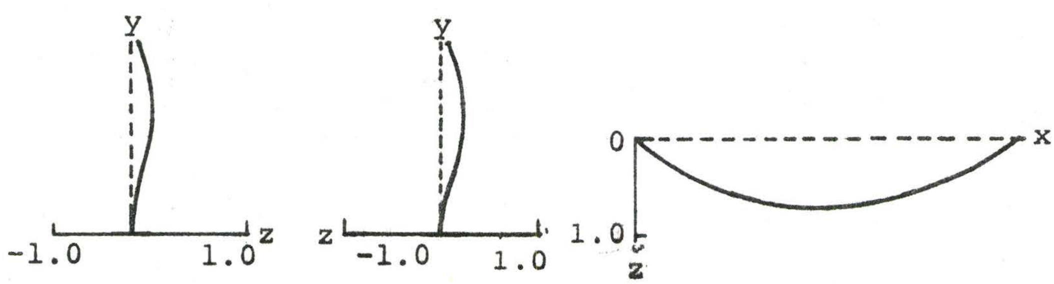


FIG. 5.1(c) MODE 3

L.C. LEFT COLUMN
R.C. RIGHT COLUMN

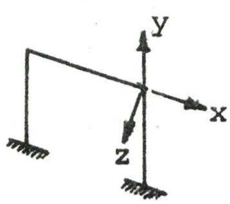


FIG. 5.1 MODE SHAPE OF COMBINED STRUCTURE
(HEIGHT: 80", LENGTH: 60")

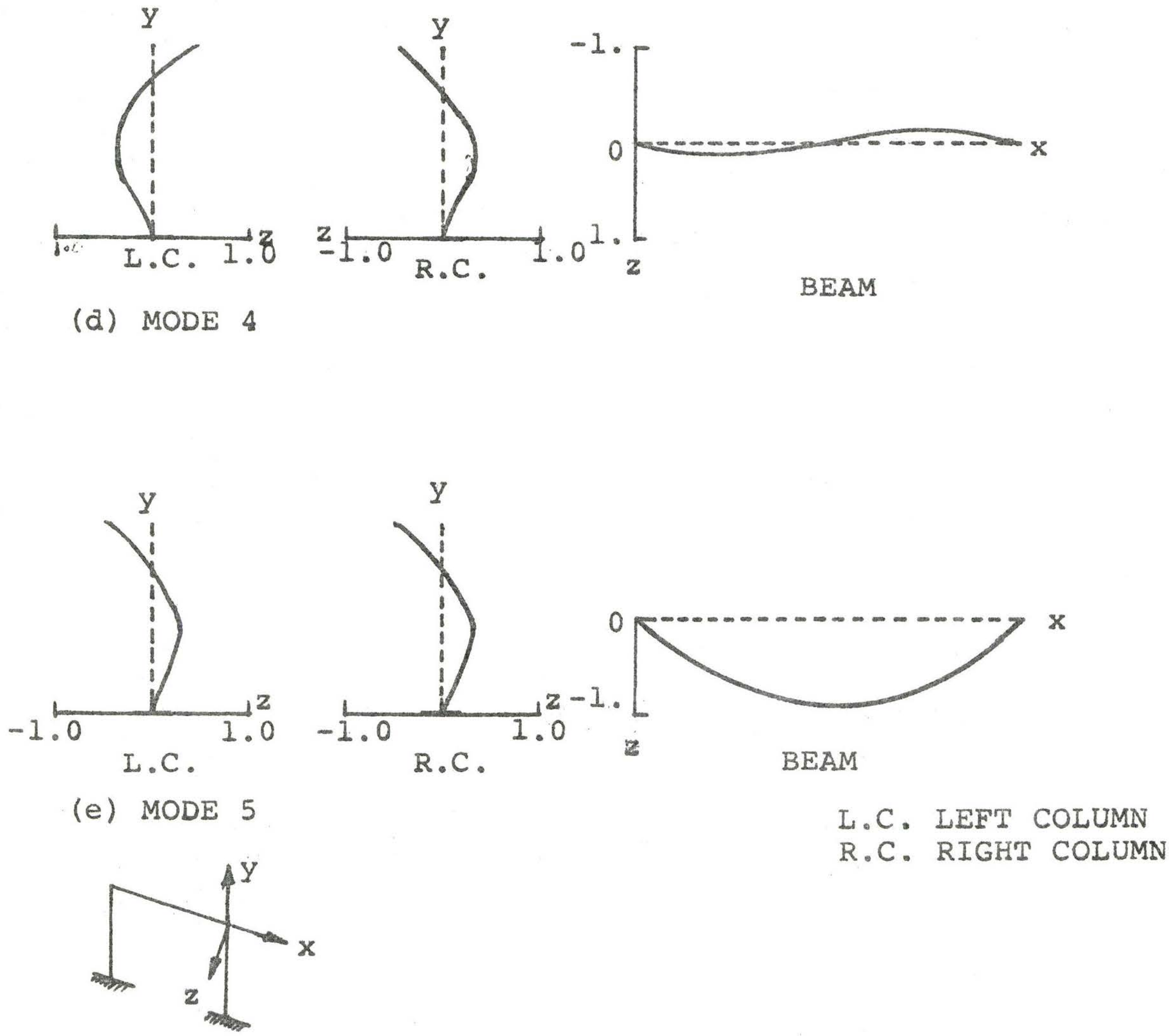


FIG. 5.1 MODE SHAPES OF COMBINED STRUCTURE WITH REFERENCE TO MEMBER x-axis (HEIGHT = 80", LENGTH OF 60")

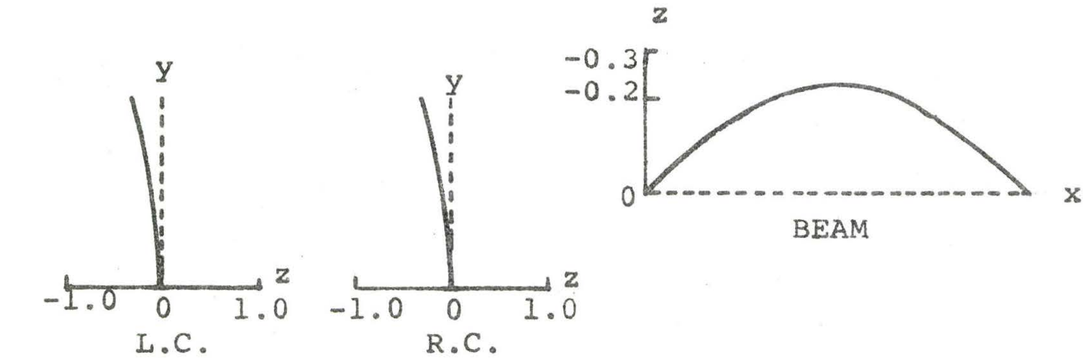


FIG.5.2(a) MODE 1

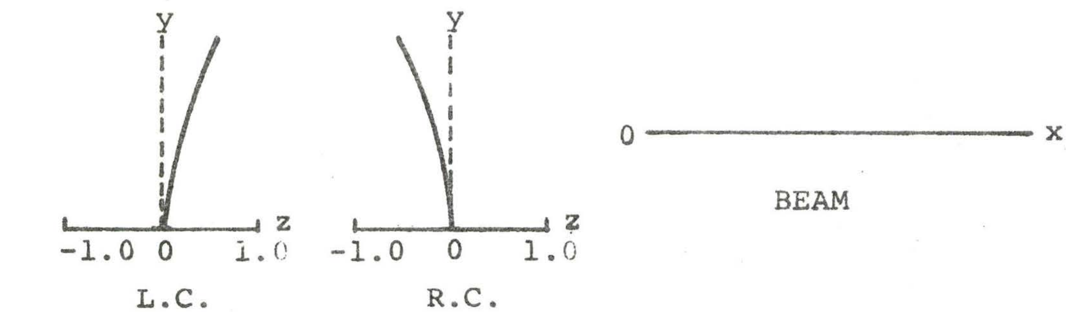


FIG.5.2(b) MODE 2

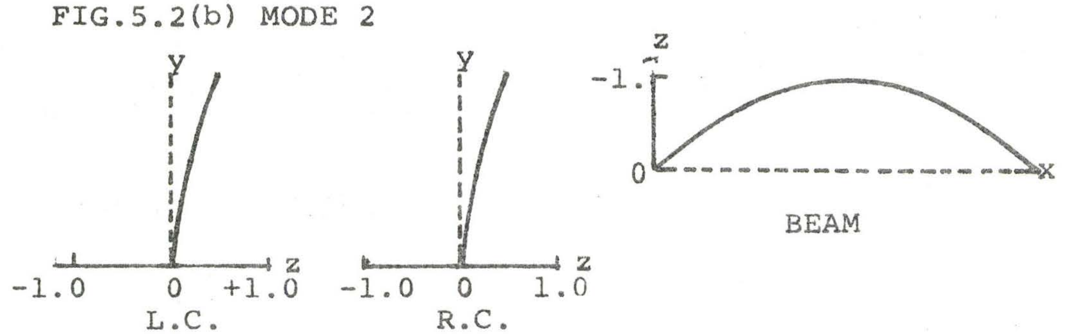


FIG.5.2(c) MODE 3

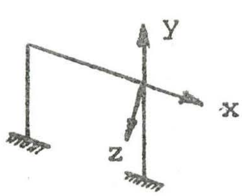


FIG. 5.2 MODE SHAPE OF COMBINED SYSTEM
HEIGHT: 40", LENGTH: 60")

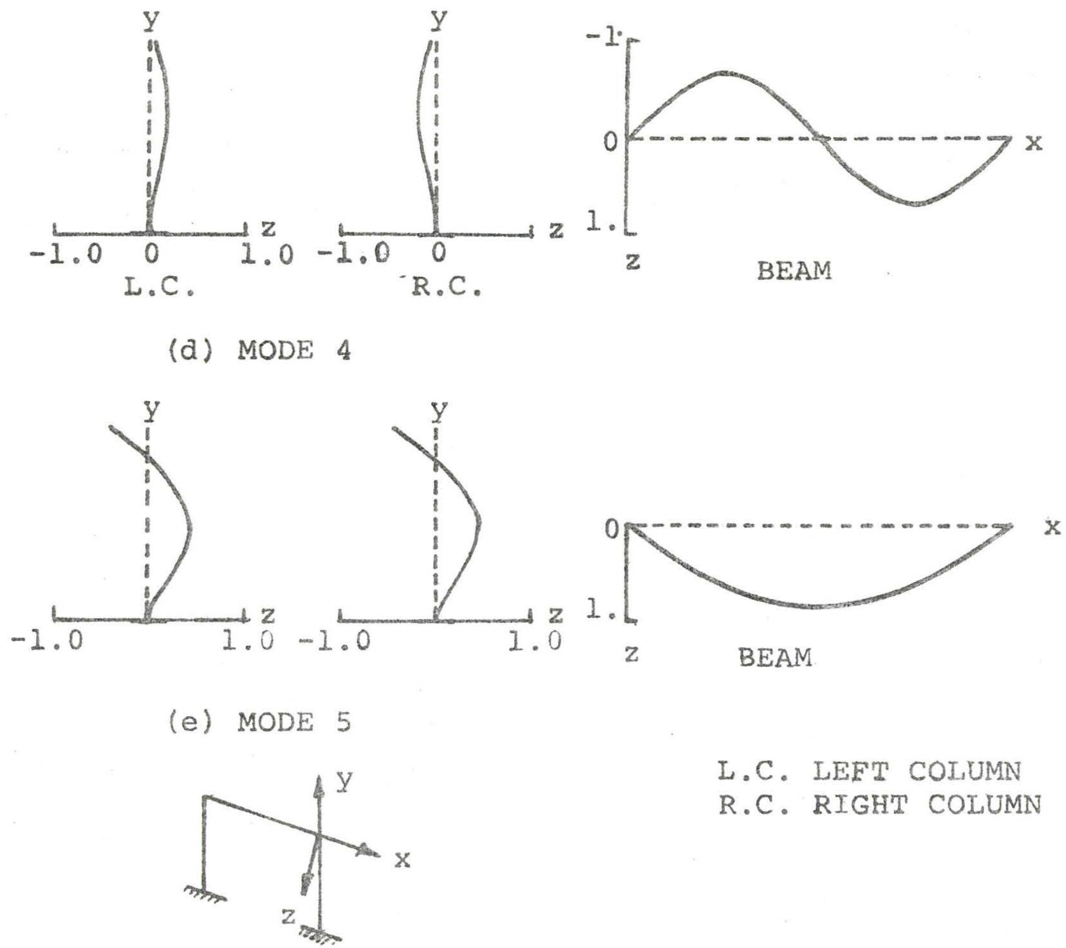


FIG. 5.2 MODE SHAPE OF COMBINED STRUCTURE WITH REFERENCE TO MEMBER x-AXIS (HEIGHT=40", LENGTH=60")

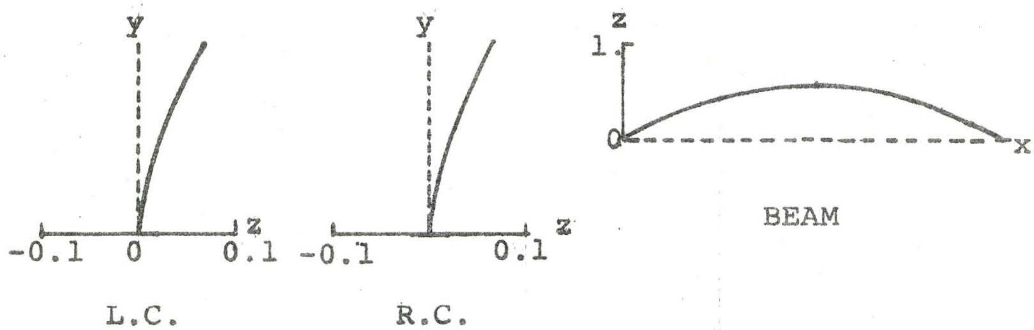


FIG.5.3(a) MODE 1

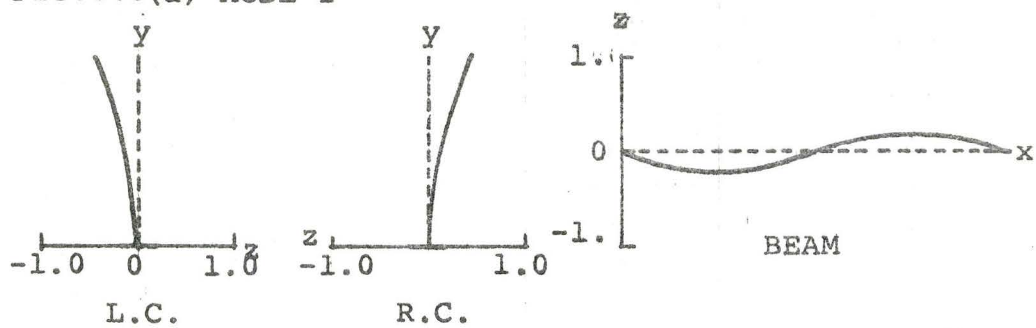


FIG.5.3(b) MODE 2

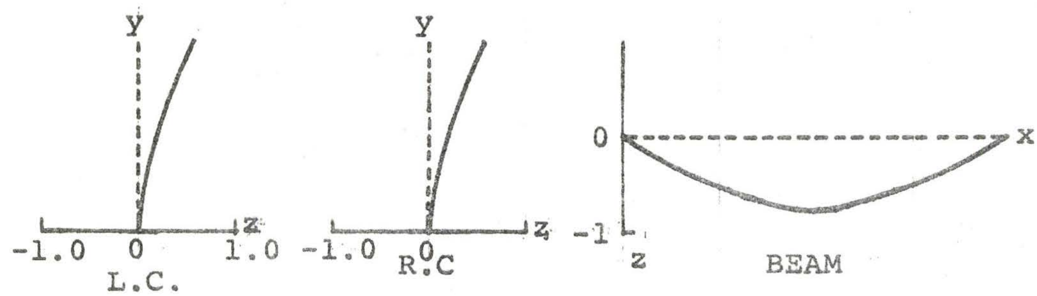


FIG.5.3(c) MODE 3

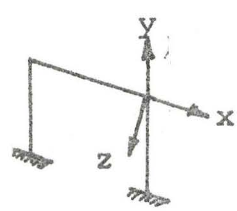


FIG. 5.3 MODE SHAPE OF COMBINED STRUCTURE
(HEIGHT: 20:, LENGTH: 60")

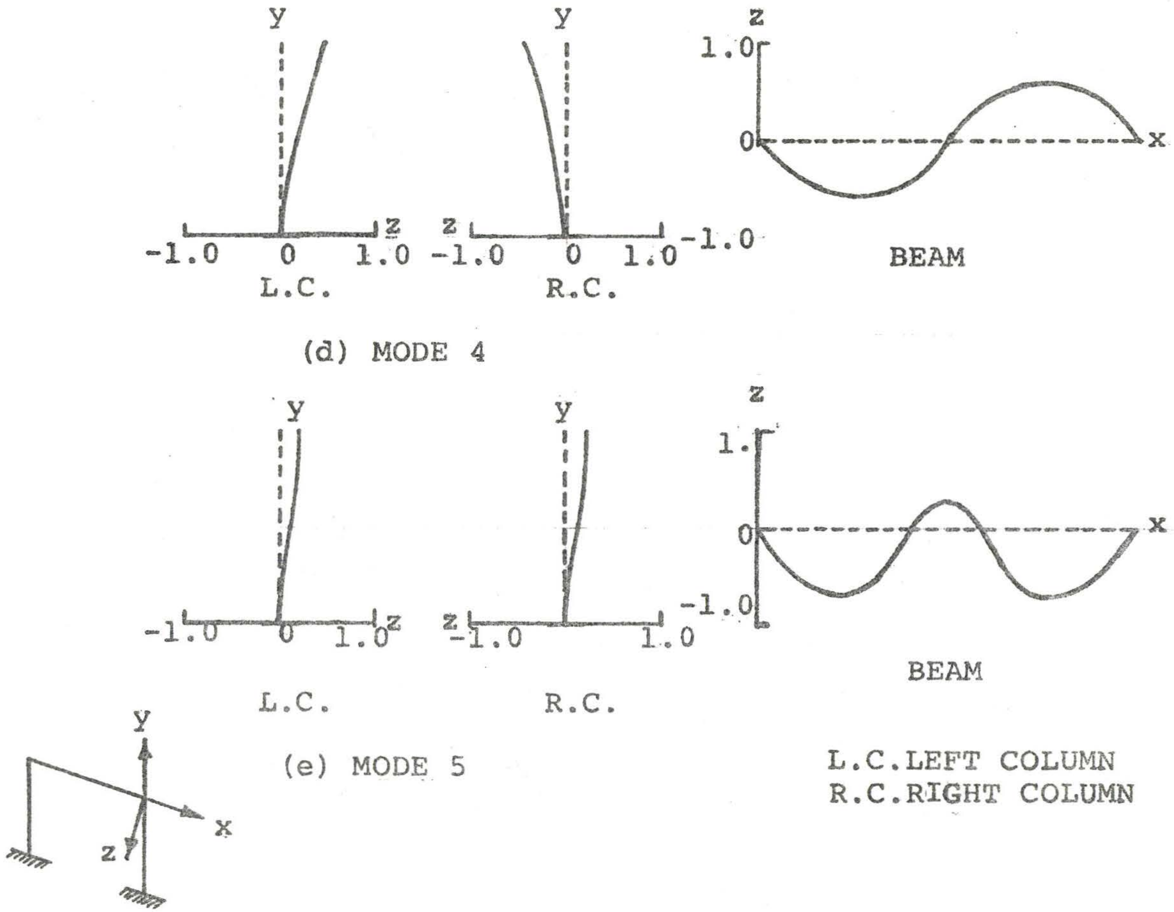


FIG. 5.3 MODE SHAPE OF COMBINED STRUCTURE WITH REFERENCE TO MEMBER x-AXIS (HEIGHT=20", LENGTH=60")

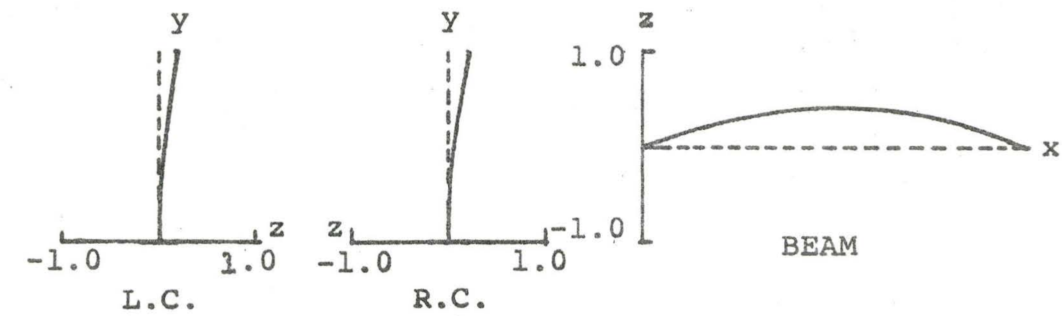


FIG.5.4(a) MODE 1

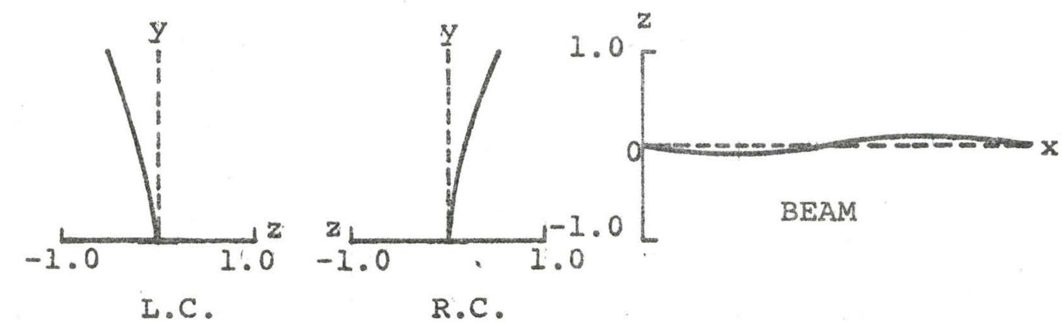


FIG.5.4(b) MODE 2

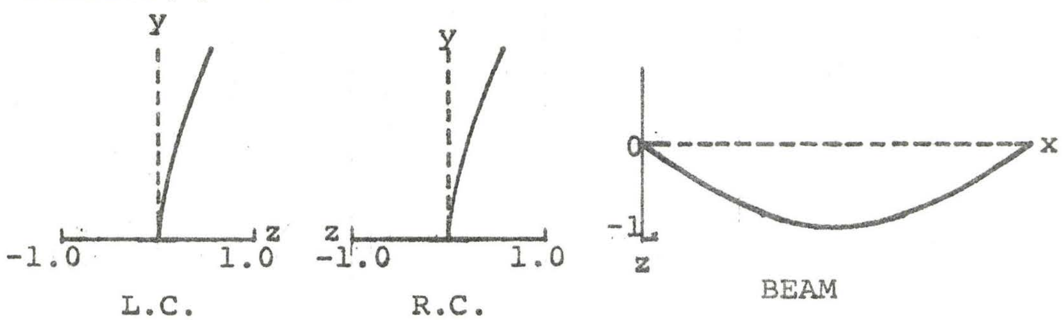


FIG.5.4(c) MODE 3

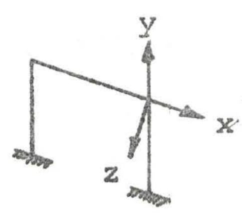


FIG. 5.4 MODE SHAPE OF COMBINED STRUCTURE
(HEIGHT: 30", LENGTH 60")

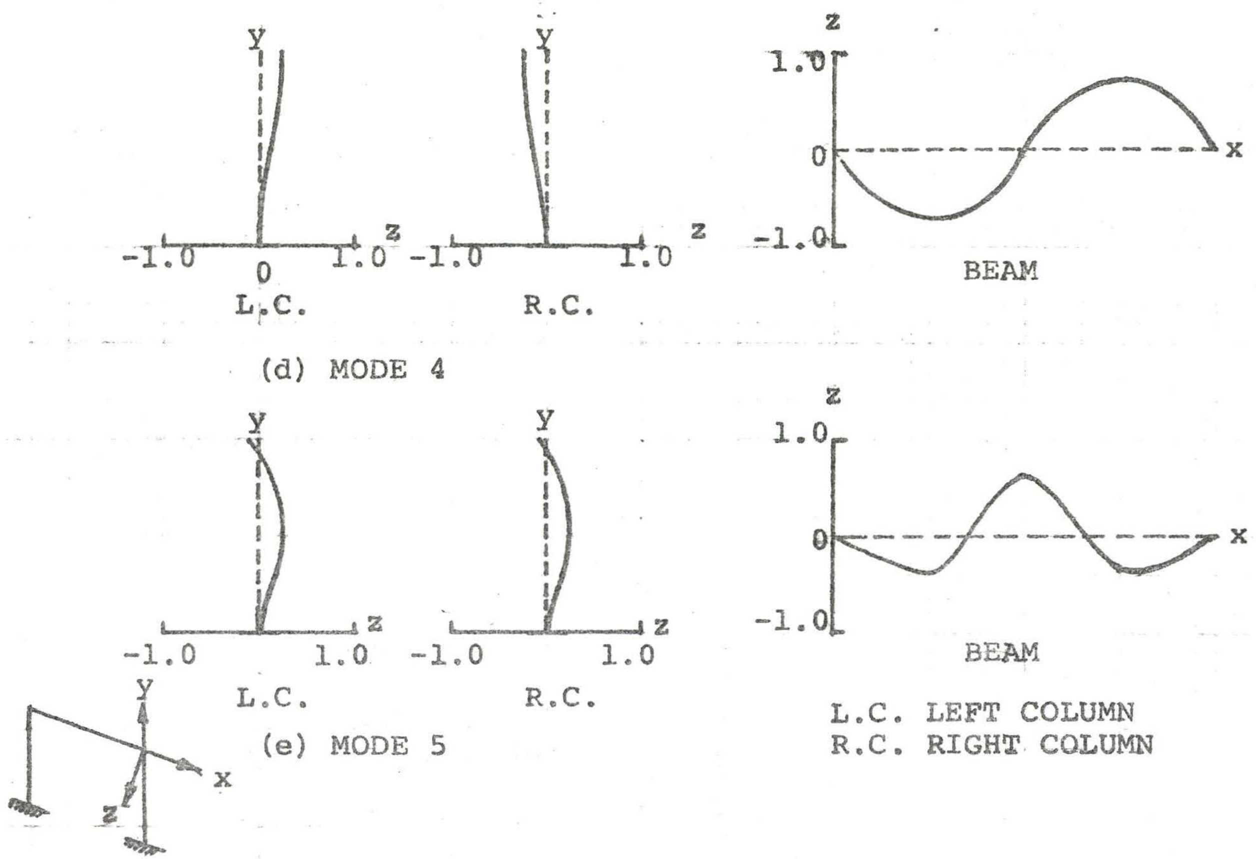


FIG. 5.4 MODE SHAPES OF COMBINED STRUCTURE WITH REFERENCE TO MEMBER x-AXIS (HEIGHT=30", LENGTH=60")

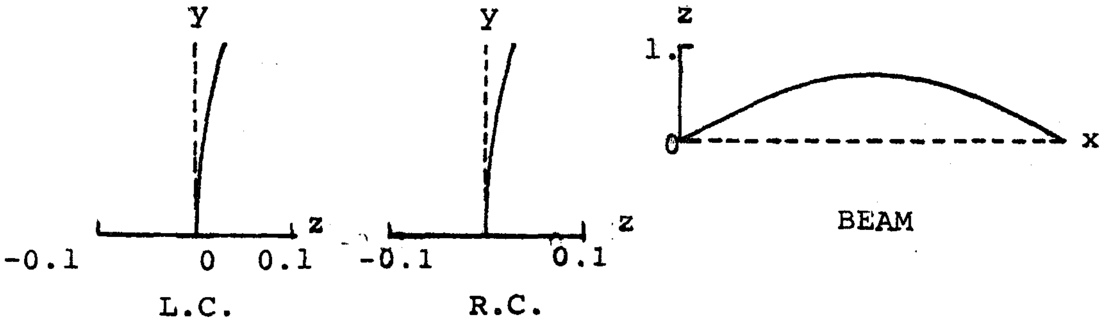


FIG.5.5(a) MODE 1

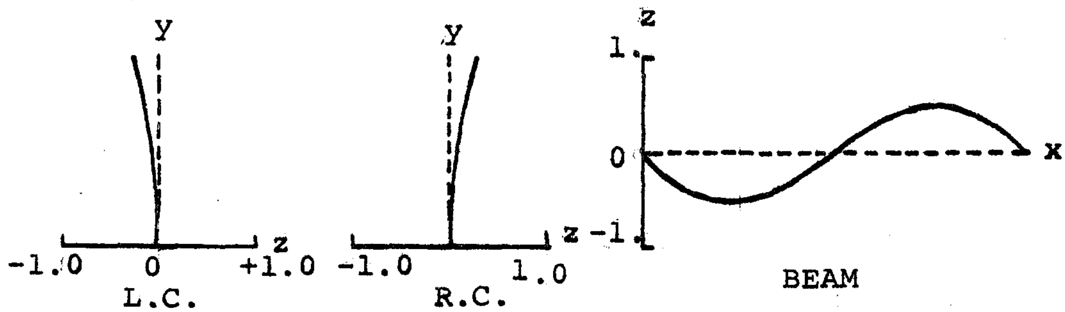


FIG.5.5(b) MODE 2

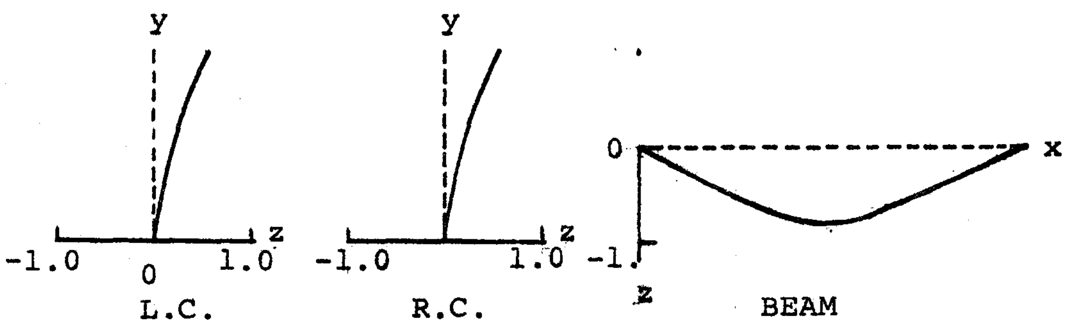


FIG.5.5(c) MODE 3

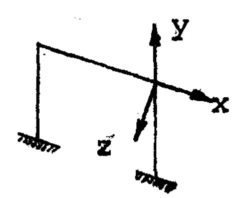
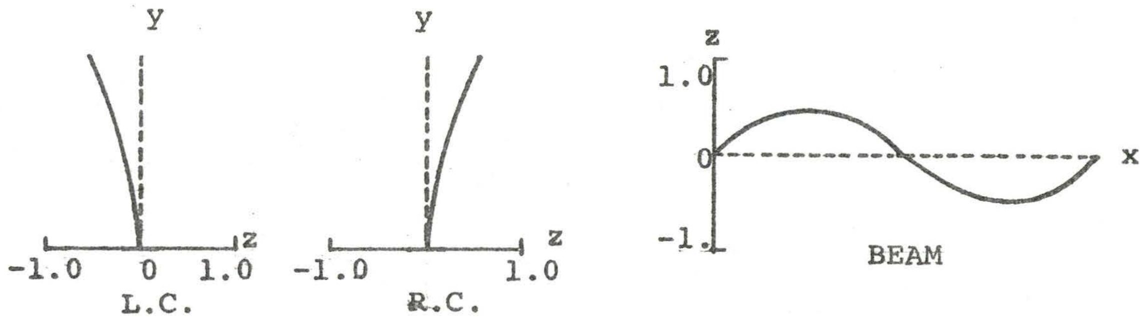
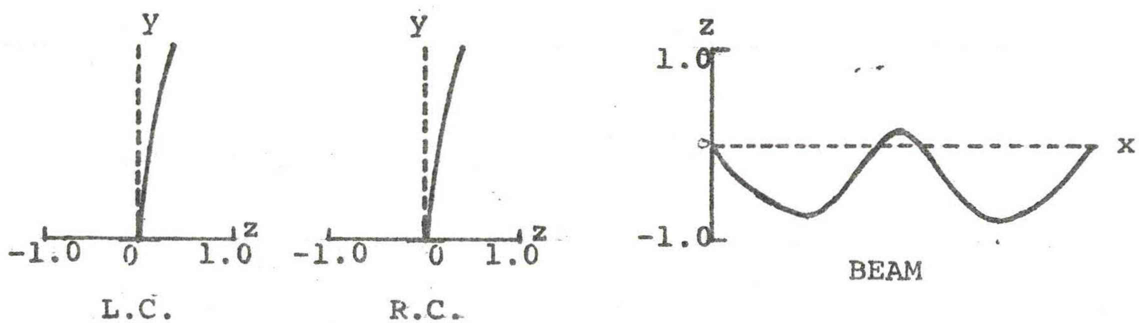


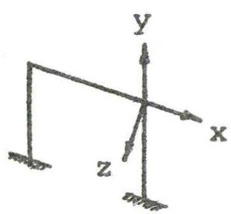
FIG. 5.5 MODE SHAPE OF COMBINED STRUCTURE
(HEIGHT: 30:, LENGTH: 120")



(d) MODE 4



(d) MODE 5



L.C. LEFT COLUMN
R.C. RIGHT COLUMN

FIG. 5.5 SHAPE OF COMBINED STRUCTURE WITH REFERENCE TO MEMBER x-AXIS (HEIGHT=30", LENGTH=120")

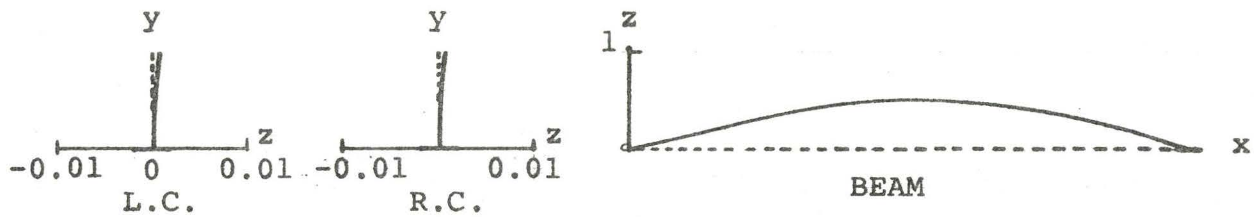


FIG.5.6(a) MODE 1

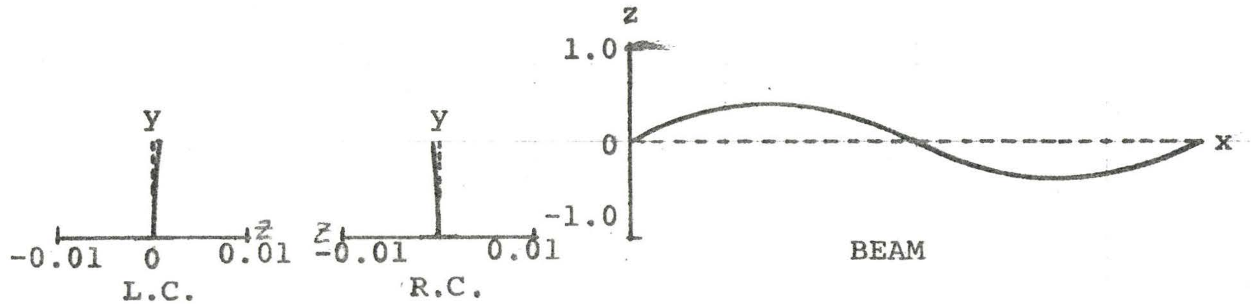
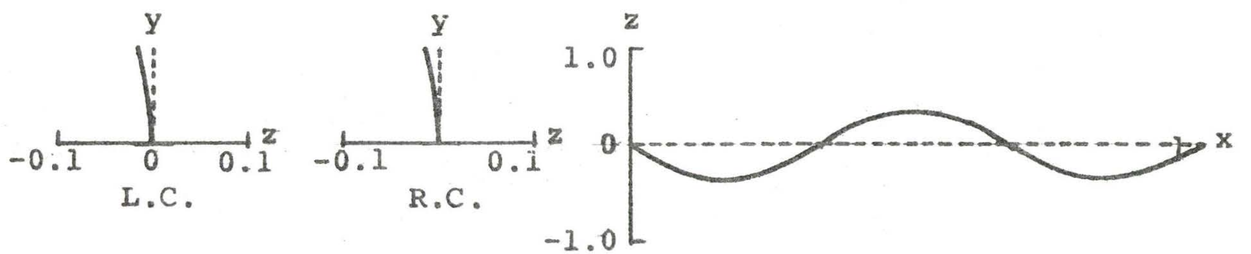


FIG.5.6(b) MODE 2



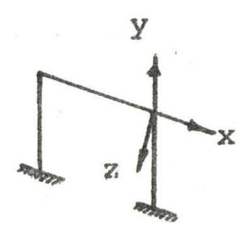
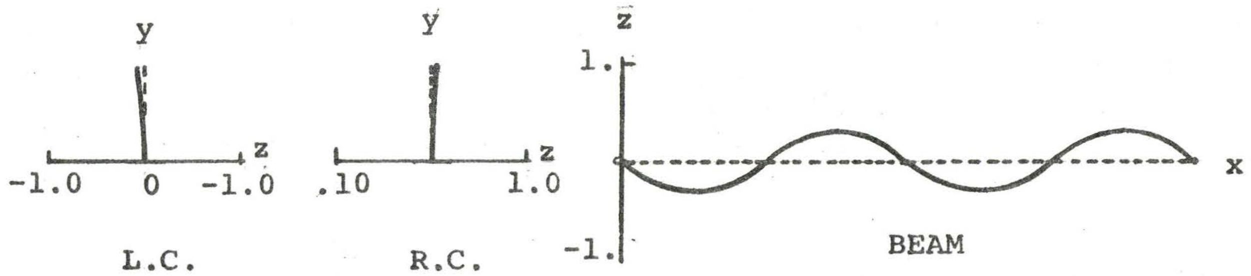
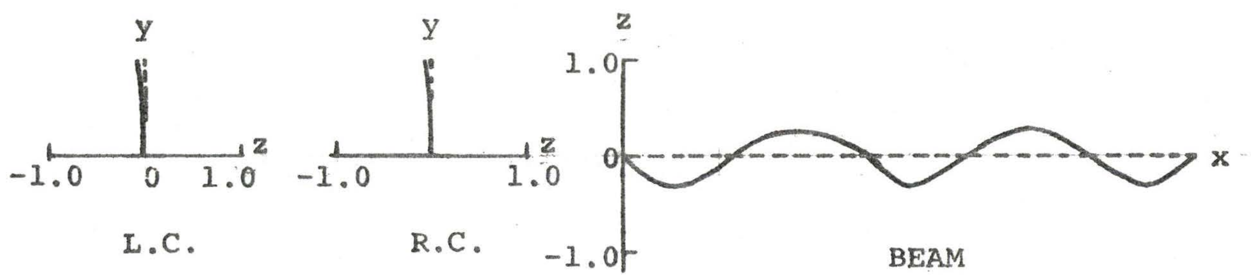


FIG.5.6(c) MODE 3

FIG. 5.6 MODE SHAPE OF COMBINED STRUCTURE
(HEIGHT: 5", LENGTH: 60")



(d) Mode 4



(e) MODE 5

L.C. LEFT COLUMN
R.C. RIGHT COLUMN

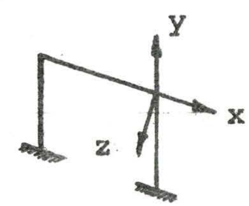


FIG. 5.6 MODE SHAPE OF COMBINED STRUCTURE WITH REFERENCE TO MEMBER AXES (HEIGHT=5", LENGTH=60")

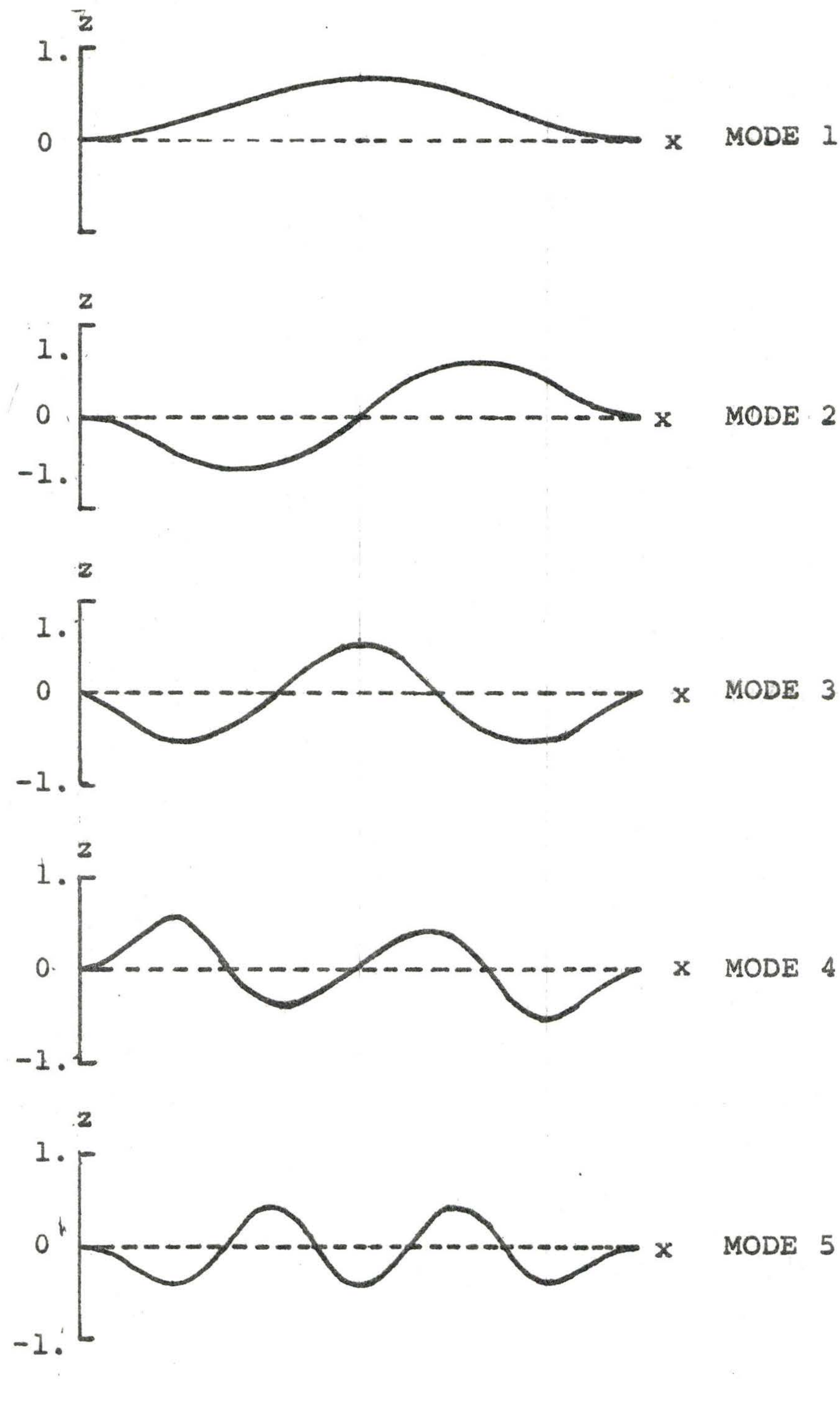


FIG. 5.7 MODE SHAPES OF FIXED-FIXED BEAM (LENGTH=60")

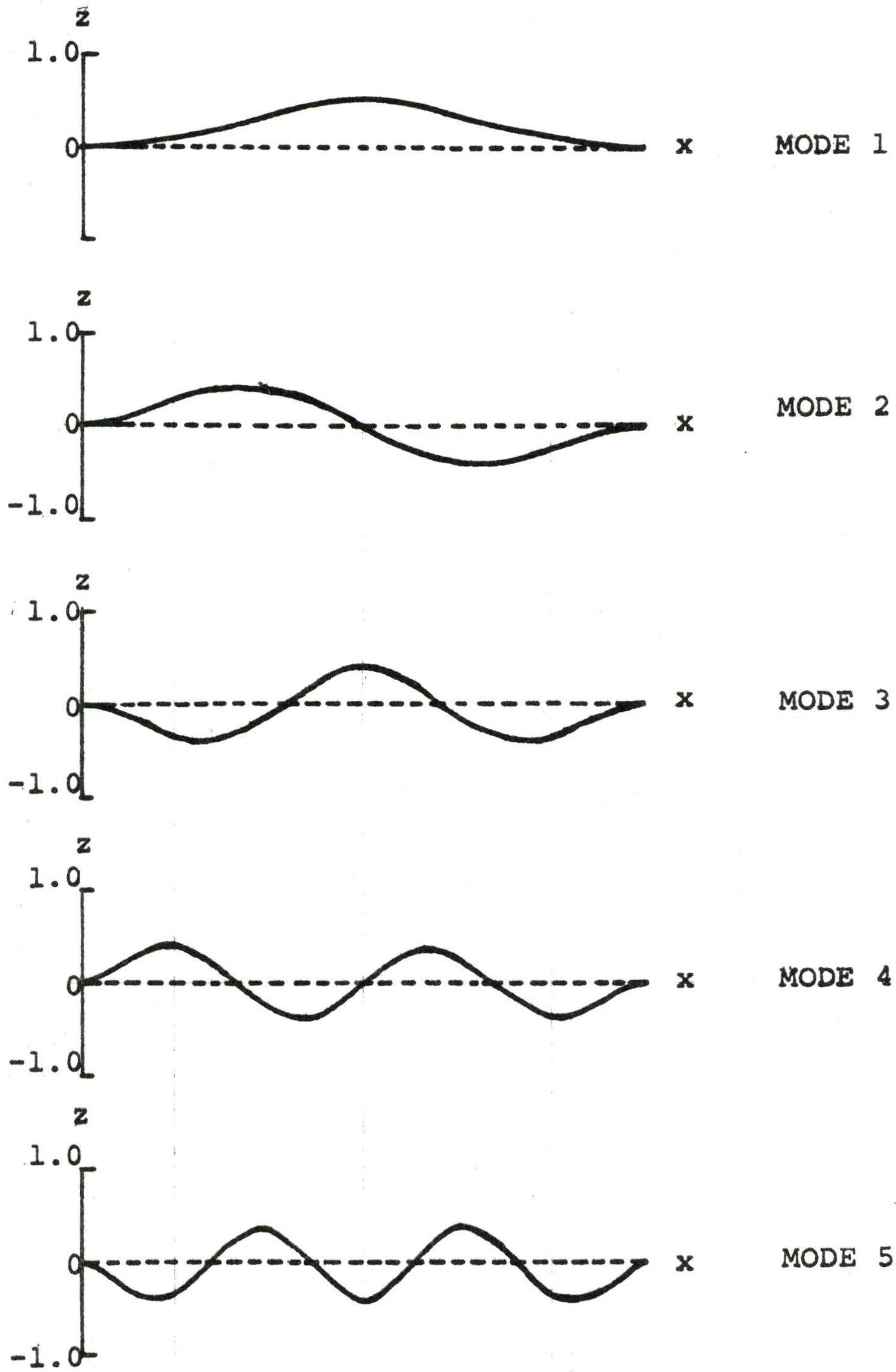


FIG. 5.8 MODE SHAPE OF BEAM
LENGTH=120"

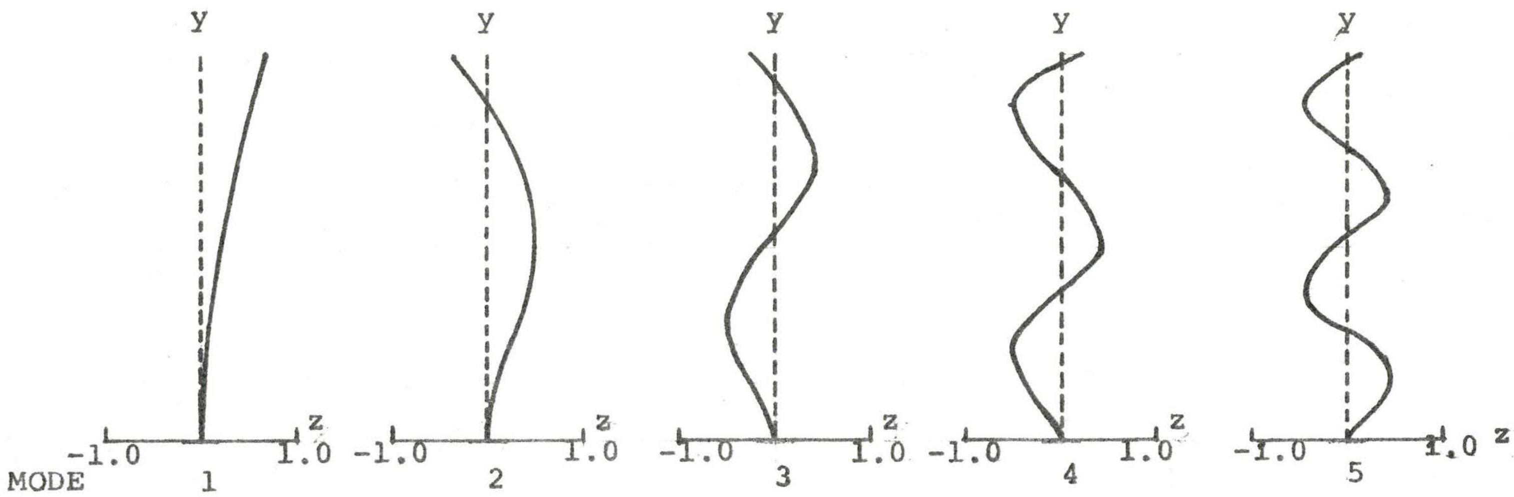


FIG. 5.9 MODE SHAPE OF COLUMN (HEIGHT=80")

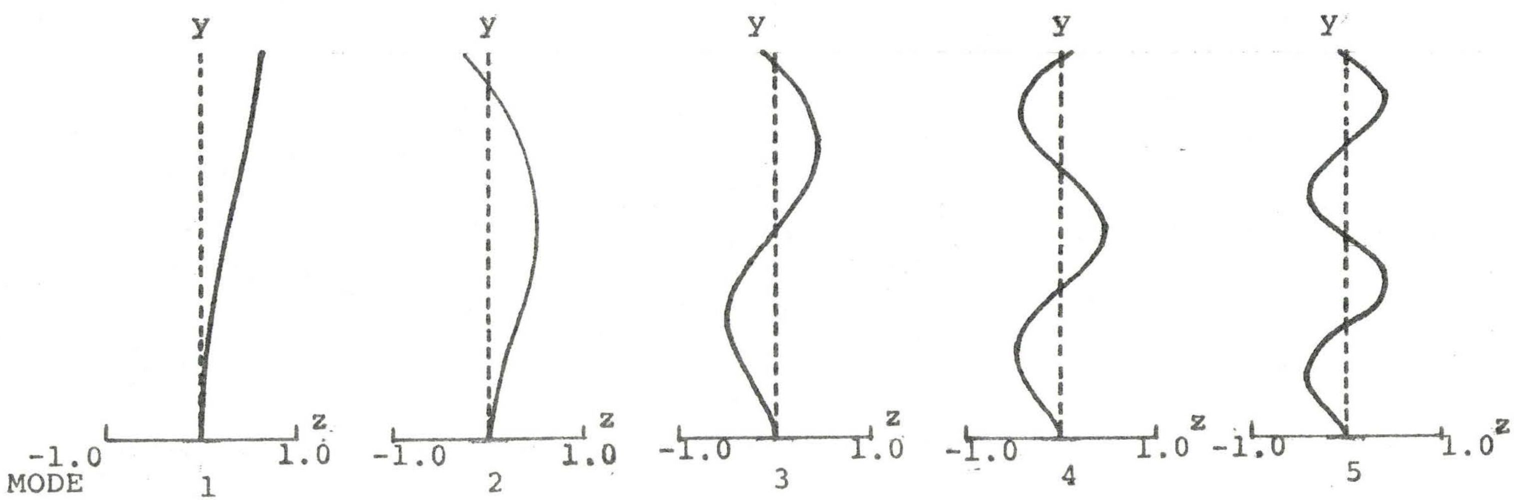


FIG. 5.10 MODE SHAPE OF COLUMN (HEIGHT=40")

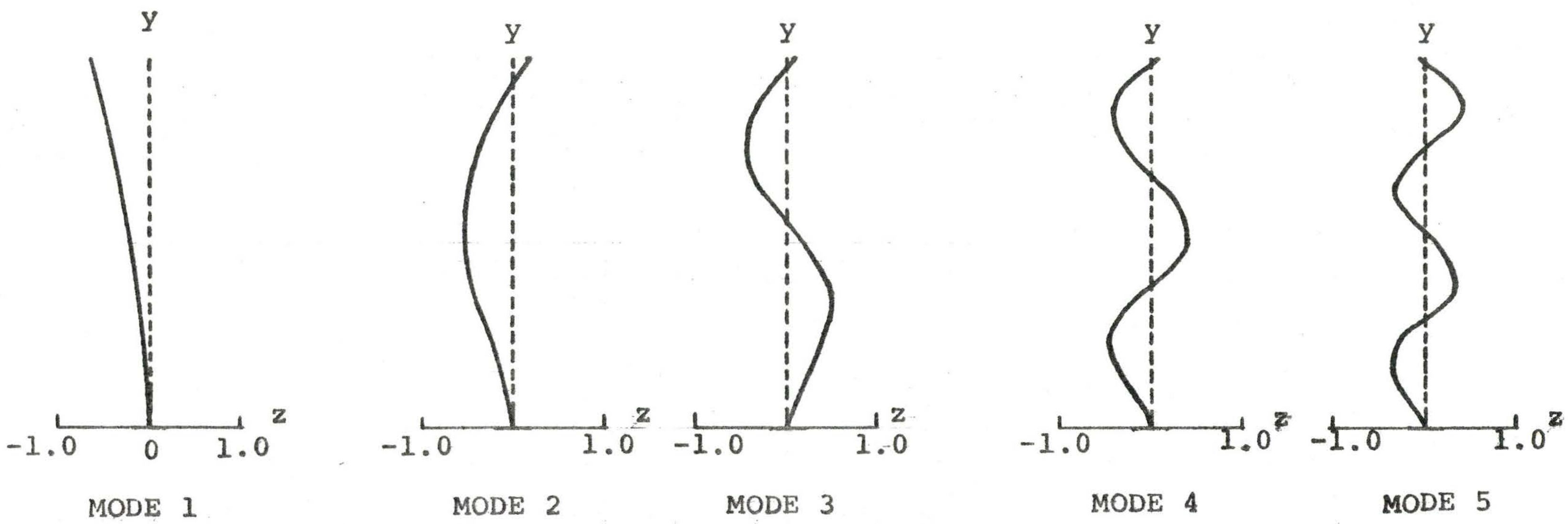


FIG. 5.11 MODE SHAPE OF COLUMN (HEIGHT=20")

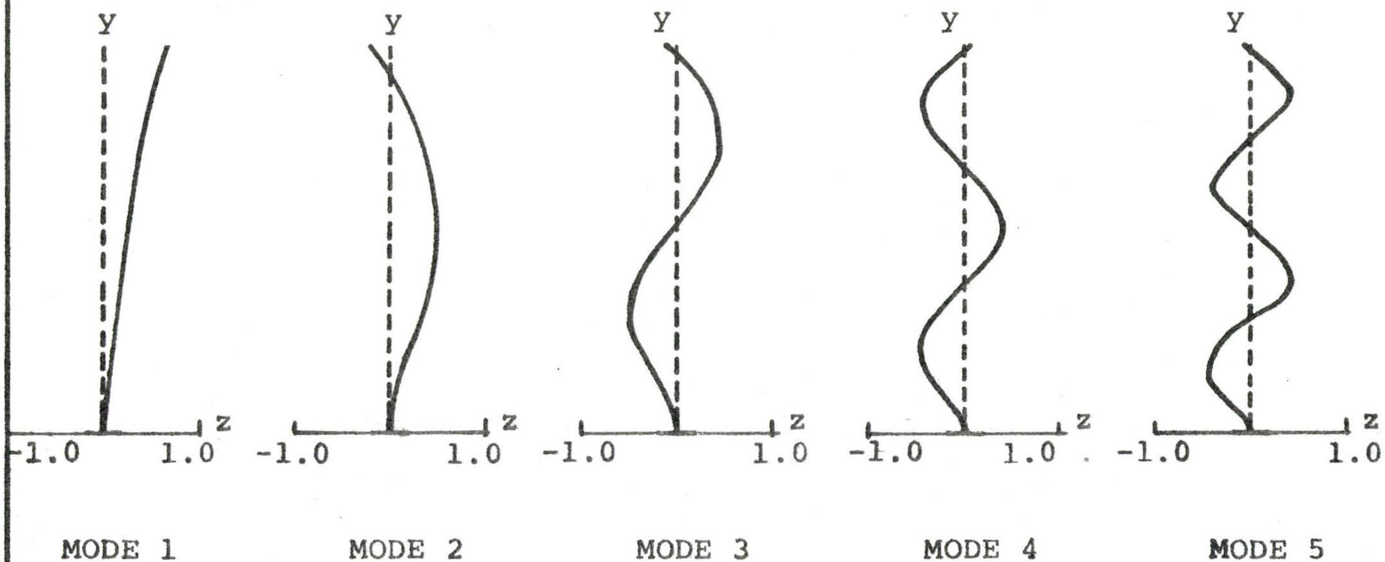
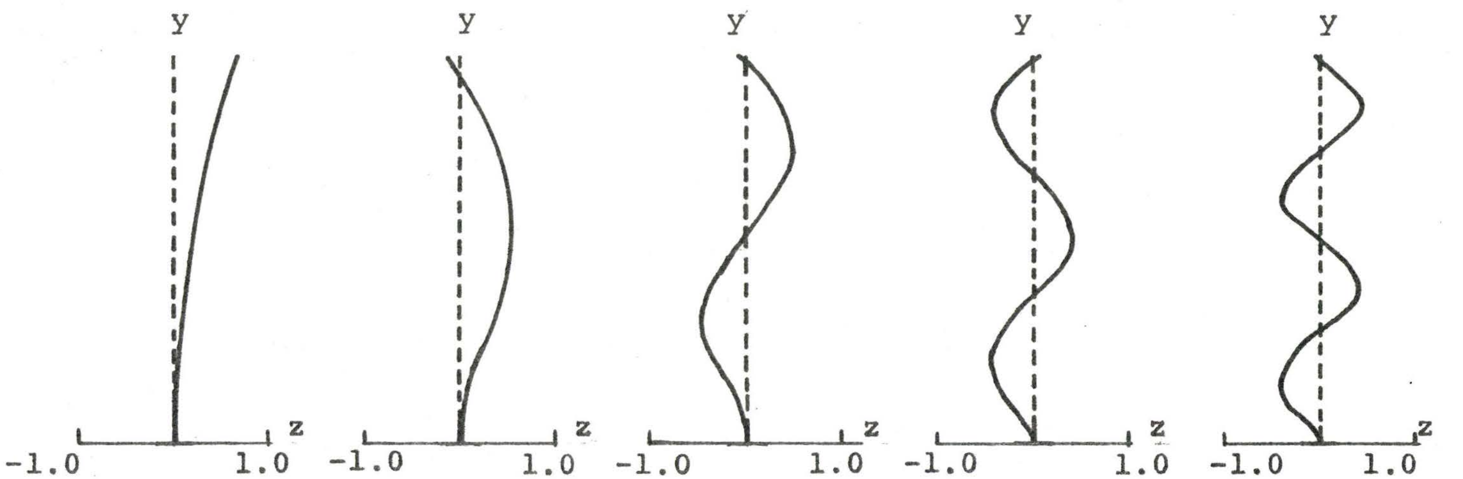


FIG. 5.12 MODE SHAPE OF COLUMN (HEIGHT=30")



MODE 1 MODE 2 MODE 3 MODE 4 MODE 5

FIG. 5.13 MODE SHAPE OF COLUMN (HEIGHT=30")

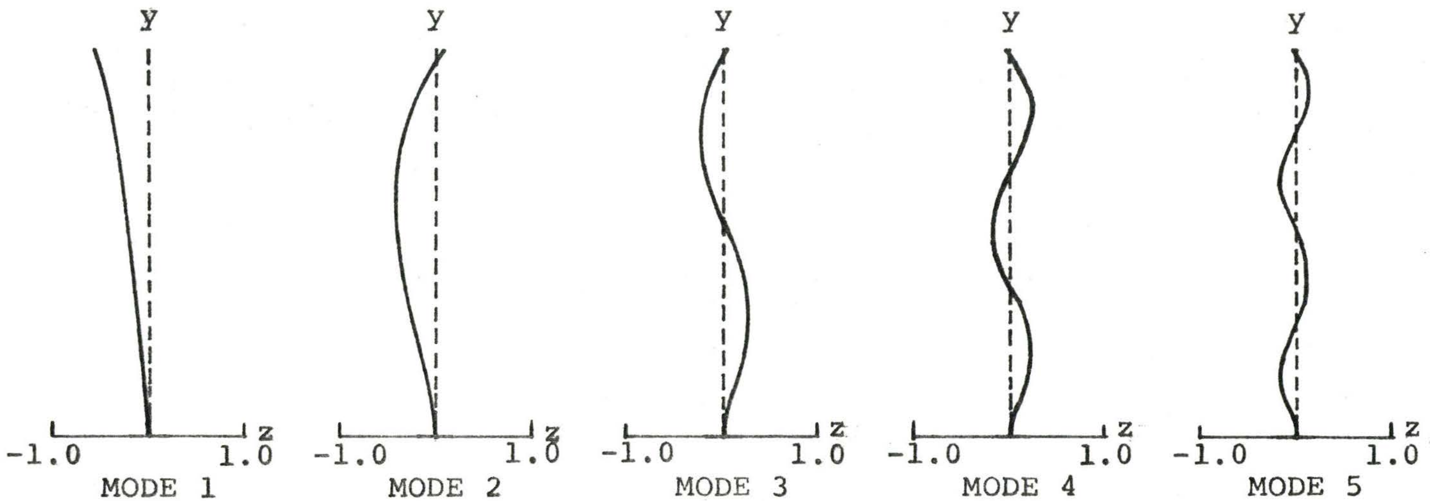


FIG. 5.14 MODE SHAPE OF COLUMN (HEIGHT=5")

CHAPTER VI

CONCLUSIONS

The following conclusions are arrived at, based on the theoretical and experimental investigations.

1. The comparison of the natural frequencies of the portal frames determined by finite element method and experiment indicates good agreement between the two sets. Hence it is concluded that the theoretical approach used is reasonably accurate for practical purposes.
2. The theoretical analysis by treating the structural connections as rigid gives reasonably accurate results in the working range.
3. The comparison of the frequencies computed by the finite element method and the conventional method indicates good agreement for both beam and column. Therefore the finite element method can be used to predict the dynamic properties of structural systems.
4. The effect of added mass on the top of the columns is found to have more influence on the fundamental frequency rather than on the higher frequencies. The different added masses result in different dynamical systems with different dynamic properties. This

indicates that the proper estimation of the mass of the structural system is important preceding the analysis and design.

5. When the column frequency is very small (Table 5.4) compared to the beam frequency, the first and second frequencies of the corresponding combined structure are nearly equal to the first frequency of the column.
6. When the column frequency is very large (Table 5.9) compared to that of the beam, the observation of the frequencies of the corresponding combined structure indicates that they are very close to the hinged-hinged beam frequencies.

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APPENDIX I

COMPUTER PROGRAM OUTLINE

A computer program was written to analyse a simple portal frame, by direct assembly technique. This program can be used for frames with rigid and non-rigid connections.

DATA INPUT:

The following data are needed in the analysis:

a) Total number of nodes and total number of elements.

b) Number of degrees of freedom for each element.

c) Material properties: Young's modulus of elasticity, etc.

d) Transformation matrix for each element.

The following computer programmes were used in the analysis

Subroutine SNARK:

Subroutine SNARK transforms the 3×3 element matrices from local co-ordinate systems to global co-ordinate system.

Subroutine STIFF:

This subroutine calculates the 6×6 stiffness matrix for each element. Expressions for the matrix are taken from reference 7. The 6×6 matrix is partitioned to 4 submatrices of sixe 3×3. Each submatrix is transformed from local co-ordinate system to global coordinate system by calling the subroutine SNARK. Then the submatrices are assembled to get the 6×6 element matrix in global system.

Subroutine AMASS:

Subroutine AMASS calculates the 6×6 mass matrix for each element, in global coordinate system; consistent mass matrix is taken from reference 7.

Subroutine INVMAT:

INVMAT is a library subroutine used for the inversion of the assembled stiffness or mass matrix.

Subroutine EBERVC:

The library subroutine EBERVC calculates the eigenvalues and eigenvectors of an unsymmetric matrix.

Main Program:

The main program utilizes the above programs to compute eigenvalues and mode shapes, and also displacements.

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RUN(S)
LOADER(PPLOADR)
SETINDF.
REDUCE.
LGO.
'          6400 END OF RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     ****************************************************************
C     *                                                              *
C     *                                                              *
C     *                                                              *
C     *ANALYSIS OF COMBINED SYSTEM WITH ROTATION SPRINGS IN BEAM     *
C     *                                                              *
C     *                                                              *
C     *                                                              *
C     ****************************************************************
      DIMENSION D(33,33), B(33,33),ALPHA(33),CN(33)
      DIMENSION DMAT(33,33)
      DIMENSION SAM(33,33),AMAS(33,33),AFREQ(33),VAL(33,33),NI(100)
      DIMENSION CC(60)
      DIMENSION RESAM(5,5),BMAS(5,5),WMAT(5,5),CMAS(5,5),IR(30)
      COMMON/BLOK1/T(3,3)/BLOK2/TT(3,3)/BLOK3/SF(6,6)/BLOK7/AM(6,6)
      READ(5,2)NELEM,NJOIN
    2 FORMAT(2I5)
      JOINT=3*NJOIN
      DO 130 I=1,JOINT
      DO 130 J=1,JOINT
      SAM(I,J)=0.0
      AMAS(I,J)=0.0
  130 CONTINUE
      KOUNT=1
    4 READ(5,5)XP1,YP1,XP2,YP2,E,G,Y,YJ,AREA,AL,IFROM,ITO
    5 FORMAT(4F10.2,2E9.2/4F10.4,2I5)
      WRITE(6,5)XP1,YP1,XP2,YP2,E,G,Y,YJ,AREA,AL,IFROM,ITO
      READ(5,25)RHO
   25 FORMAT(F10.4)
      READ(5,75)((T(I,J),J=1,3),I=1,3)
   75 FORMAT(3F10.3)
      CALL STIFFF(XP1,YP1,XP2,YP2,E,G,Y,YJ,AREA,AL,KOUNT)
      CALL AMASS(AL,AREA,YJ,RHO,KOUNT)
  134 IF(IFROM.EQ.0)GO TO 501
      DO 135 I=1,3
      DO 135 J=1,3
      IFROG=I+3*(IFROM-1)
      JFROG=J+3*(IFROM-1)
      ITOAD=I+3*(ITO-1)
      JTOAD=J+3*(ITO-1)
      SAM(IFROG,JFROG)=SAM(IFROG,JFROG)+SF(I,J)
      SAM(IFROG,JTOAD)=SAM(IFROG,JTOAD)+SF(I,J+3)
      SAM(ITOAD,JFROG)=SAM(ITOAD,JFROG)+SF(I+3,J)
      SAM(ITOAD,JTOAD)=SAM(ITOAD,JTOAD)+SF(I+3,J+3)
      AMAS(IFROG,JFROG)=AMAS(IFROG,JFROG)+AM(I  ,J  )
      AMAS(IFROG,JTOAD)=AMAS(IFROG,JTOAD)+AM(I  ,J+3)
      AMAS(ITOAD,JFROG)=AMAS(ITOAD,JFROG)+AM(I+3,J  )
      AMAS(ITOAD,JTOAD)=AMAS(ITOAD,JTOAD)+AM(I+3,J+3)
  135 CONTINUE
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      GO TO 15
  501 DO 502 I=1,3
      DO 502 J=1,3
      ITOAD=I+3*(ITO-1)
      JTOAD=J+3*(ITO-1)
      SAM(ITOAD,JTOAD)=SAM(ITOAD,JTOAD)+SF(I+3,J+3)
      AMAS(ITOAD,JTOAD)=AMAS(ITOAD,JTOAD)+AM(I+3,J+3)
  502 CONTINUE
   15 IF(KOUNT.EQ.NELEM)GO TO 160
      KOUNT=KOUNT+1
      GO TO 4
C 160 WRITE(6,9)SAM
C   9 FORMAT(1X,9E10.2)
C     WRITE(6,16)
C  16 FORMAT(1X,19X,*AMAS*)
C     WRITE(6,10)AMAS
C  10 FORMAT(1X,9E10.2)
  160 N=33
      CALL INVMAT(AMAS,33,33,1E-07,IERR,N1)
      IF(IERR.GT.0) GO TO 711
      DO 23 I=1,N
      DO 23 J=1,N
      DMAT(I,J)=0.0
      DO 23 K=1,N
      DMAT(I,J)=DMAT(I,J)+AMAS(I,K)*SAM(K,J)
   23 CONTINUE
  698 DO 703 I=1,N
      DO 703J=1,N
      VAL(I,J)=0.0
      IF(I.EQ.J)VAL(I,J)=1.0
  703 CONTINUE
      CALL EBERVC(DMAT,33,1,200,.01,.001,1000.,33,VAL,I)
      DO 704 I=1,N
  704 AFREQ(I)=SQRT(ABS(DMAT(I,I)))/6.2832
      PRINT 721
  721 FORMAT(1H0,*EIGEN VALUES IN RAD/SEC*,//)
      PRINT 706 ,(AFREQ(I),I=1,N)
  706 FORMAT(1X,10E12.4/)
      PRINT 707
  707 FORMAT(1H0,*EIGEN VECTOR*,/)
      PRINT 708,((VAL(I,J),I=1,N),J=1,N)
  708 FORMAT(1H0,11E12.4,/)
      CALL INVMAT(AMAS,33,33,1E-07,IERR,N1)
      DO 900 I=1,N
      DO 900 J=1,N
      ALPHA(I)=0.0
      B(I,J)=0.0
      D(I,J)=0.0
      DO 900 K=1,N
  900 B(I,J)=B(I,J)+VAL(K,I)*AMAS(K,J)
      PRINT 911,B
  911 FORMAT(1X,11E12.4,/)
      DO 901 I=1,N
      DO 901 J=1,N
  901 ALPHA(I)=ALPHA(I)+B(I,J)
      PRINT 912
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```
912 FORMAT(1X,*ALPHA IS*,/)
    PRINT 902,(ALPHA(I),I=1,N)
902 FORMAT(1X,10E13.5,/)
    DO 903 I=1,N
    DO 903 J=1,N
903 B(I,J)=0.0
    DO 904 I=1,N
    DO 904 J=1,N
904 B(I,J)=VAL(I,J)*ALPHA(J)
    PRINT 905,B
905 FORMAT(1X,11E12.4,/)
    DO 906 J=1,N
906 CN(J)=0.0
    BETA=0.006
    XSO2=0.5/100.0
    W1=AFREQ(21)
    DO 907 J=1,N
    W2=AFREQ(J)
    XXX=SQRT((1.0-W1**2/W2**2)**2+4.0*(BETA*W1/W2)**2)
    CN(J)=XSO2*(1.0/XXX-1.0)
907 CONTINUE
    PRINT 908,(CN(J),J=1,N)
908 FORMAT(1X,11E12.3,/)
    DO 909 I=1,N
    DO 909 J=1,N
909 D(I,J)=B(I,J)*CN(J)
    PRINT 910 ,((D(I,J),I=1,N),J=1,N)
910 FORMAT(1H1,/(11E12.4,/))
711 PRINT 712,IERR
712 FORMAT(I5)
    STOP
    END
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```
      SUBROUTINE STIFFF(X1,Y1,X2,Y2,E,G,Y,YJ,AR,AL,KNT)
      COMMON/BLOK1/T(3,3)/BLOK2/TT(3,3)/BLOK3/SF(6,6)/BLOK7/AM(6,6)
      DIMENSION RK(3,3),SK(3,3),QK(3,3),UK(3,3)
      WRITE(6,25)T
   25 FORMAT(1X,3E12.4)
      DO 7 I=1,3
      DO 7 J=1,3
      RK(I,J)=0.0
      SK(I,J)=0.0
      QK(I,J)=0.0
      UK(I,J)=0.0
    7 CONTINUE
      IF(KNT .EQ. 5)GO TO 601
      IF(KNT .EQ. 10)GO TO 602
      RK(1,1)=12.0*E*Y/AL/AL/AL
      RK(1,3)=6.0*E*Y/AL/AL
      RK(2,2)=G*YJ/AL
      RK(3,1)=RK(1,3)
      RK(3,3)=4.0*E*Y/AL
      SK(1,1)=-RK(1,1)
      SK(1,3)=RK(1,3)
      SK(2,2)=-RK(2,2)
      SK(3,1)=-RK(3,1)
      SK(3,3)=0.5*RK(3,3)
      QK(1,1)=SK(1,1)
      QK(1,3)=SK(3,1)
      QK(2,2)=SK(2,2)
      QK(3,1)=SK(1,3)
      QK(3,3)=SK(3,3)
      UK(1,1)=RK(1,1)
      UK(1,3)=SK(3,1)
      UK(2,2)=RK(2,2)
      UK(3,1)=SK(3,1)
      UK(3,3)=RK(3,3)
      GO TO 100
  601 EJ=E*Y/(AL*AK1)
      EJ=E*Y/(AL*AK1)
      A1=1.0+EJ
      A21=1.0+2.0*EJ
      A31=1.0+3.0*EJ
      A22=1.0
      A32=1.0
      AA=1.0+4.0*EJ
      GO TO 603
  602 EK=E*Y/(AL*AK2)
      EK=E*Y/(AL*AK2)
      A1=1.0+EK
      A21=1.0
      A31=1.0
      A22=1.0+2.0*EK
      A32=1.0+3.0*EK
      AA=1.0+4.0*EK
  603 RK(1,1)=(12.0*E*Y/AL**3)*(A1/AA)
      RK(1,3)=(6.0*E*Y/AL**2)*(A21/AA)
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```
      RK(3,1)=RK(1,3)
      RK(2,2)=G*YJ/AL
      RK(3,3)=(4.0*E*Y/AL)*(A31/AA)
      SK(1,1)=-RK(1,1)
      SK(1,3)=RK(3,1)*(A22/A21)
      SK(2,2)=-RK(2,2)
      SK(3,1)=-RK(3,1)
      SK(3,3)=2.0*E*Y/(AL*AA)
      QK(1,1)=SK(1,1)
      QK(1,3)=-RK(3,1)
      QK(2,2)=SK(2,2)
      QK(3,1)=SK(1,3)
      QK(3,3)=SK(3,3)
      UK(1,1)=RK(1,1)
      UK(1,3)=-SK(1,3)
      UK(2,2)=RK(2,2)
      UK(3,1)=-SK(1,3)
      UK(3,3)=(4.0*E*Y/AL)*(A32/AA)
  100 DO 8 I=1,3
      DO 8 J=1,3
      TT(J,I)=T(I,J)
    8 CONTINUE
      CALL SNARK(RK,1,1)
      CALL SNARK(SK,1,1)
      CALL SNARK(QK,1,1)
      CALL SNARK(UK,1,1)
      DO 20 I=1,3
      DO 20 J=1,3
      SF(I,J)=RK(I,J)
      SF(I,J+3)=SK(I,J)
      SF(I+3,J)=QK(I,J)
      SF(I+3,J+3)=UK(I,J)
   20 CONTINUE
      RETURN
      END
C
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```
      SUBROUTINE AMASS(AL,AR,YJ,RHO,KNT)
      COMMON/BLOK1/T(3,3)/BLOK2/TT(3,3)
      COMMON/BLOK7/AM(6,6)
      DIMENSION AK(3,3),BK(3,3),CK(3,3),DK(3,3)
      DO 7 I=1,3
      DO 7 J=1,3
      AK(I,J)=0.0
      BK(I,J)=0.0
      CK(I,J)=0.0
      DK(I,J)=0.0
    7 CONTINUE
      B=RHO*AL*AR/(420.0*386.0)
      AK(1,1)=156.0*B
      AK(1,3)=22.0*AL*B
      AK(2,2)=140.0*YJ*B/AR
      AK(3,1)=22.0*AL*B
      AK(3,3)=4.0*AL*AL*B
      BK(1,1)=54.0*B
      BK(1,3)=-13.0*AL*B
      BK(2,2)=70.0*YJ*B/AR
      BK(3,1)=-BK(1,3)
      BK(3,3)=-3.0*AL*AL*B
      CK(1,1)=BK(1,1)
      CK(1,3)=BK(3,1)
      CK(2,2)=BK(2,2)
      CK(3,1)=BK(1,3)
      CK(3,3)=BK(3,3)
      DK(1,1)=AK(1,1)
      DK(1,3)=-AK(1,3)
      DK(2,2)=AK(2,2)
      DK(3,1)=-AK(3,1)
      DK(3,3)=AK(3,3)
   60 CALL SNARK(AK,1,1)
      CALL SNARK(BK,1,1)
      CALL SNARK(CK,1,1)
      CALL SNARK(DK,1,1)
      DO 20 I=1,3
      DO 20 J=1,3
      AM(I,J)=AK(I,J)
      AM(I,J+3)=BK(I,J)
      AM(I+3,J)=CK(I,J)
      AM(I+3,J+3)=DK(I,J)
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SNARK(A,I1,J1)
      COMMON/BLCK7/AM(6,6)
      COMMON/BLCK1/T(3,3)/BLOK2/TT(3,3)/BLCK3/SH(6,6)
      DIMENSION A(3,3),B(3,3),D(3,3)
      DO 12 I=1,3
      DO 12 J=1,3
      B(I,J)=0.0
      D(I,J)=0.0
   12 CONTINUE
      DO 14 I=1,3
      DO 14 J=1,3
      DO 14 K=1,3
      B(I,J)=B(I,J)+A(I,K)*T(K,J)
   14 CONTINUE
      DO 16 I=1,3
      DO 16 J=1,3
      DO 16 K=1,3
      D(I,J)=D(I,J)+TT(I,K)*B(K,J)
   16 CONTINUE
      DO 17 I=1,3
      DO 17 J=1,3
      A(I,J)=D(I,J)
   17 CONTINUE
      RETURN
      END
```