MICROWAVE PROPAGATION IN SEMICONDUCTOR SUBJECT TO MAGNETIC FIELD

MICROWAVE PROPAGATION IN RECTANGULAR WAVEGUIDE CONTAINING SEMICONDUCTOR SUBJECT TO TRANSVERSE MAGNETIC FIELD

By

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SCOPE AND CONTENTS:

A theoretical and a practical study is presented of guided wave propagation through rectangular waveguide completely filled with a n-type germanium sample of conductivity 4.50 mho/meter subjected to an external transverse magnetic field of 10 Kilogauss.

Measurements of the reflection co efficients at the air-semiconductor interface for different values of the applied magnetic field were carried out at a frequency of 9.46 GHz.

ABSTRACT

A detailed theoretical analysis of the propagation constant and the field components in rectangular waveguide completely filled with a semiconductor subjected to an external transverse applied magnetic field, has been carried out. A numerical solution of the transcendental equation for the propagation constant has been obtained for the n-type germanium samples with different conductivities and magnetic fields.

An experimental verification of this theoretical analysis has been made with a 22.2 ohm-cm, n-type germanium sample at 9.46 GHz. The applied transverse magnetic field was varied from 0 to 10 Kilogauss. Measurements of the reflection co efficients at the air-semiconductor interface for different values of the applied magnetic field have been made with a high precision microwave reflection bridge. The experimental results agree well with the theoretical results.

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CHAPTER 1

1.1 INTRODUCTION

In recent years, in the microwave field, considerable attention is being paid to the problem of propagation of electromagnetic waves through semi-conductor subjected to an external steady magnetic field. In the presence of an external magnetic field, the semi-conductor, when propagating an electromagnetic wave shows such interesting phenomena as Hall effect, Faraday rotation and magneto-resistance which have drawn the attention of many investigators.

Ever since C. L. Hogan(1) and D. Polder(2) indicated that certain media with a complex tensor permeability have important applications at microwave frequencies, extensive investigations into the propagation of electromagnetic waves in anisotropic materials, particularly ferrites, have been carried out. As a result of these investigations, a large number of practical devices employing ferrites have been invented. Electromagnetic wave propagation in rectangular wave guides completely filled with magnetized ferrites, have been extensively studied and a thorough analysis is found in the literature. Ferrites become anisotropic when placed in a magnetostatic field and have a permeability which is a tensor quantity. The modes of propagation in these magnetized ferrites are found to have features which are quite distinct from those found in the case of isotropic media. In addition to the normal TE modes, anomalous gyromagnetic modes are known to exist in these cases.

In much the same way, the behaviour of semi-conductors, when subjected to an externally applied magnetic field, may be described in terms of a complex tensor permittivity. Thus, a semi-conductor placed in a magnetostatic field becomes anisotropic and is known as a gyroelectric medium, whereas it is essentially an isotropic medium in the absence of an external applied magnetic field.

When a steady magnetic field is applied to a semi-conductor filled wave guide, the propagating electric fields perpendicular to the applied magnetic field become coupled. Hence the characteristics of the wave guide modes are changed, as in the analogous case of magnetized ferrites where the microwave magnetic fields perpendicular to the steady magnetization are coupled. However, since the electric and magnetic fields are required to satisfy different conditions at the boundaries of the wave guide, the exact characteristics of the modes are likely to be different in the two cases.

A considerable number of papers dealing with the case when applied steady magnetic field is in the direction of the propagation of the electromagnetic wave, are available. However, relatively few papers concerning the electromagnetic wave propagation through a semi-conductor filled rectangular wave guide, in the presence of a steady transverse external magnetic field, have been published. In recent years, Hirota,[3,4] M. Toda,[5,6] Nag and Engineer,[7] Gabriel and Brodwin[8] have dealt with the effect of the transverse external magnetic field on the wave propagation through semi-conductor in rectangular wave guides.

Nag and Engineer[9] in one of their published papers have formulated a special form of Hall field expression which leads to a complex permittivity tensor for a semi-conductor in a transverse magnetic field. They have also analyzed theoretically the characteristics of electromagnetic waves propagating in a semi-conductor filled rectangular wave guide in the presence of a transverse magnetic field. An analysis of the same problem is presented in this thesis, together with experimental results and a modification of Nag's theoretical analysis has been made.

Following Nag and Engineer's suggestion of the special form of the Hall field, an expression for the complex permittivity tensor has been derived which agrees with their expression. Using Maxwell's equations and the appropriate boundary conditions, a rigorous and exact solution of the field components has been made in a different way, using a different procedure. In the analysis, it is shown that in the present case, no TM modes or TE modes other than those of the type TE_{on} may be excited. In addition, anomalous mode waves having all six E and H field components are shown to exist. Also it has been found that TE_{on} modes do not depend on the magnetostatic field. Hence in gyroelectric wave guide, TE_{on} modes hold no interest.

In practice, the wave within the filled wave guide is launched from an empty section supporting the dominant TE_{10} mode and the problem consists of determining the behaviour of this mode as it passes through the media. In gyroelectric media in the wave guide, if the wave is to depend on the magnetostatic field, all six components of the field are necessary and these must depend on both transverse co-ordinates. Since the TE_{10} mode is not admissible in the magnetized medium, it excites a mode which dependson both co-ordinates. Thus in a gyroelectric wave guide, there exists the TE_{10} mode in the absence of an external magnetic field B and an anomalous mode in the presence of B. In the analysis it has been found that the propagation characteristic is in general reciprocal.

In their theoretical analysis, Nag and Engineer assumed a sinusoidal field intensity variation in the direction of the magnetic field and then solved for the specific field components. However, in the present work, no such assumption has been made and the sinusoidal field intensity variation in the direction of the magnetic field has been proved mathematically by employing a special method of the separation of variables.[20] Proceeding in this completely different way, a resultant transcendental equation for the propagation constant has been obtained which agrees with the transcendental equation of the propagation constant, as obtained by Nag and Engineer.

This transcendental equation has been solved numerically by employing the Newton-Raphson method, and the variation of the attenuation constant and the phase constant with the applied magnetic field B, has been obtained.

An experimental verification of the theoretical analysis was performed on a 22.2 ohm-cm, n-type germanium sample at 9.46 GHz using a reflection type microwave bridge. A theoretical expression for the reflection coefficient at the semiconductor-air interface in the wave-guide was obtained and numerical values of the reflection coefficient as a function of the external magnetic field were calculated and plotted. The reflection bridge measurement employed the new technique for precision reflection bridge measurements suggested by Champlin, Holm and Armstrong[10]. The theoretical and experimental values of the reflection co efficients were in excellent agreement for lower values of the magnetic field. However, for higher values slight discrepancies exist and possible reasons for these discrepancies are discussed.

1.2 LITERATURE SURVEY

The propagation characteristics of electromagnetic waves in rectangular wave-guides loaded with magnetized ferrites have been extensively studied and are well discussed in the book written by Lax and Button(11). Also a general modal solution for electromagnetic wave propagation through a rectangular guide completely filled with transversely magnetized ferrite has been carried out by Mikaelian(12), Seidel(13) and Barzilai and Gerosa(14).

A considerable number of papers dealing with the guided wave propagation through semiconductor when the applied magnetic field is along the direction of the propagation of the electromagnetic wave are available. However, the following papers deal with guided wave propagation through semiconductor material subjected to a transverse magnetic field which is the subject of this thesis.

The Hall effect in semiconductors at microwave frequencies has been extensively studied by Prof. H. E. M. Barlow(15,16) and several other workers. In 1963, an experimental investigation into microwave propagation at 10 GHz through a rectangular waveguide partially filled by a thin semiconducting plate under a steady transverse magnetic field, was performed by Prof. Barlow and Koike(17). The applied transverse steady magnetic field was found to produce a change in the conductivity of the plate and a corresponding effect on the microwave propagation along the guide. Non-reciprocal transmission arising from asymmetry of the semiconductor over the cross section of the guide was also observed.

Microwave propagation at 24 GHz has been observed by M. Toda[5] in a wave-guide filled by InSb at 77^{o}K, in a transverse magnetic field. A transverse magnetic field in excess of 5 KOe was used and the transmitted power increased exponentially with increasing magnetic field intensity. The transmission showed a strong non-reciprocal character. Measurements of the phase shift in the InSb showed that the wave is transmitted only along one side of the wave-guide determined by the directions of magnetic field and of the power flow.

A theoretical analysis of the electromagnetic wave propagation through an n or p type semi-conductor with metal plated surfaces in a transverse magnetic field has been performed by Ryogo Hirota[3]. This theoretical analysis was in good agreement with the experimental results of M. Toda[5].

The high frequency transport properties of semi-conductors under a transverse magnetic field have been analyzed theoretically by several workers such as Y. Itikawa[18], H. Fujisada[19].

Propagation of waves in bounded solid state plasma in transverse magnetic fields have also been analyzed by Ryogo Hirota and K. Suzuki[4]. In their paper it has been reported that a new type of solid state plasma wave-guide has been constructed using n-type InSb and an insulator of similar dielectric constant, CuO. Its properties were measured in a transverse magnetic field. When all except the input and output surfaces of the plasma wave-guide were metal plated, for one direction of magnetic field, the microwave power was transmitted through the InSb with a transmission loss of about 20 db. For the reversed directed of the magnetic field, the microwave power was transmitted through the CuO with a transmission loss of about 5 db. When one surface of the plasma wave-guide was not metal plated, a strong magneto-plasma resonance at the frequency $\omega_0 = \omega p^2/\omega_c$ was observed for one direction of the external magnetic field, the transmission loss being about 38 db at the maximum. For the reversed direction of the magnetic field no resonance was observed.

Microwave field distributions were measured by M. Toda[6], when a thin plate of n-type InSb was placed vertically at the centre of a K-band wave-guide and also when the wave-guide was filled with n-type InSb. The field distributions were asymmetrically deformed in the applied magnetic fields and the propagation properties were demonstrated to be non-reciprocal.

The characteristics of electromagnetic waves propagating in a semi-conductor filled rectangular wave-guide in the presence of a transverse magnetic field have been theoretically analyzed by Nag & Engineer [7]. It has been shown that only TE mode waves having y-independent field components (y-being the direction of the steady magnetic field) and anomalous modes having all six field components can propagate.

A perturbation analysis of rectangular waveguide containing transversely magnetized semiconductor has been made by G. Gabriel and M. Brodwin[8]. The solution by first order theory is compared to the results of an experiment in which surface currents in the guide wall due to perturbed and unperturbed TE_{10} wave in n-type silicon are sampled and segregated. Theoretical and experimental results were found to be in reasonable agreement. Distinctions between the gyroelectric and gyromagnetic media in rectangular waveguide have been clearly analyzed by Gabriel and Brodwin[20].

The solution of guided waves in Inhomogenous anistropic media by perturbation and variational methods have also been performed by Gabriel and Brodwin[21].

Electromagnetic wave propagation in a plasma with non-linear electrical conductivity has been studied by Melvin Epstein[22]. An analysis of reflection and transmission of electromagnetic waves from a non-linear anistropic slab has been made by P. K. Kaw[23].

Starting from the general wave equation, Kaw has written appropriate non-linear equations describing the growth of the two modes of propagation of an electromagnetic wave in a non-linear medium. The applied magnetic field is taken in the direction of the propagation of the wave. The equations have been solved by a perturbation technique and the solutions have been used to obtain expressions for the linear and non-linear components of the reflected and transmitted parts of the electromagnetic wave. The paper was not directly applicable to the problem discussed here but the paper appears to be the only one available which deals with reflections from a non-linear anisotropic media.

1.3 MATERIALS EXHIBITING MAGNETO-RESISTANCE EFFECTS

When a magnetic field is applied transversely to an electric current in a semiconductor, containing both holes and electrons, a change in the conductivity of the semiconductor is observed. This fractional change in conductivity will be different for different types of the semiconductors, as it depends upon the electrical properties of the semiconductors. The choice of materials in which the change in conductivity will be most pronounced is discussed in this section. Attention is given mainly to the semiconductors at room temperature.

A semiconductor with spherical constant-energy surfaces should show no transverse magneto-resistance when relaxation time constant is constant and only one type of carrier is present. This is no longer true when a semiconductor has more than one type of carrier. For electrons, we have the following relations when an electric field is applied in x-y plane and magnetic field in z-direction[24] [19].

$$J_{ex} = A_e Ex - B_e Ey \tag{1.1}$$

$$J_{ey} = B_e Ex + A_e Ey \tag{1.2}$$

$$A_e \equiv \frac{ne^2}{m_e^*} \frac{\tau_e}{1+\omega_{ec}^2\tau_e^2} = \frac{\sigma_{oe}}{1+\mu_e^2B^2} \tag{1.3}$$

$$Be = \frac{ne^2}{m_e^*} \frac{\omega_{ec}\tau_e^2}{1+\omega_{ec}^2\tau_e^2} = \frac{\sigma_{oe}\mu_e B}{1+\mu_e^2B^2} \tag{1.4}$$

Where J_{ex} and J_{ey} are x and y components of current density due to electrons respectively, Ex and Ey are x and y components of electric field respectively, n is the electron density, e is the electronic charge, m_e^* is the effective mass of electrons, ω_{ec} is the cyclotron angular frequency of electrons, σ_{oe} is the conductivity due to electrons only for B=0, μ_e is the mobility of electrons, τ_e is the relaxation time of electrons and B is the magnetic flux density.

For holes, similarly, replacing the subscript e by h we have,

$$J_{hx} = A_h Ex - B_h Ey \quad (1.5)$$

$$J_{hy} = B_h Ex + A_h Ey \quad (1.6)$$

$$A_h = \frac{pe^2}{m_h^*} \frac{\tau_h}{1+\omega_{hc}^2 \tau_h^2} = \frac{\sigma_{oh}}{1+\mu_h^2 B^2} \quad (1.7)$$

$$B_h \equiv - \frac{pe^2}{m_h^*} \frac{w_{hc}\tau_h^2}{1+w_{hc}^2 \tau_h^2} = - \frac{\sigma_{oh}\mu_h B}{1+\mu_h^2 B^2} \quad (1.8)$$

where p is the hole density. When both electrons and holes are present, we must add their contributions to the total current

$$J_x = J_{ex} + J_{hx} = (A_e + A_h) Ex - (B_e + B_h)Ey \quad (1.9)$$

$$Jy = J_{ey} + J_{hy} = (B_e + B_h) Ex + (A_e + A_h)Ey \quad (1.10)$$

when

Jy = 0, we have

$$Ey = - \frac{(B_e + B_h)}{(A_e + A_h)} Ex \quad (1.11)$$

From equations (1.9) and (1.11) we have

$$\sigma = \frac{J_x}{E_x} = (A_e + A_h) + \frac{(B_e + B_h)^2}{A_e + A_h} \quad (1.12)$$

Using equations (1.3), (1.4), (1.7), (1.8) and (1.12) we have,

$$\sigma = \left\{ \frac{1 + Y\mu_e^{\,2}\, B^2}{1 + (X+Y)\mu_e^{\,2}\, B^2} \right\} \sigma_o \qquad (1.13)$$

where,

$$X = \frac{c(1+b)^2}{b(1+bc)^2} \qquad (1.14)$$

$$Y = \frac{(1-c)^2}{(1+bc)^2} \qquad (1.15)$$

$\sigma_o = \sigma_{oe} + \sigma_{oh}$; $b = {}^{\mu_e}/_{\mu_h}$; $c = {}^{n}/_{p}$

Putting $\rho_o = {}^{1}/_{\sigma_o}$; $\rho = \frac{1}{\sigma}$ and $\Delta\rho = \rho - \rho_o$

we get, $$\frac{\Delta\rho}{\rho_o} = \frac{X\,\mu_e^{\,2}\, B^2}{1+Y\mu_e^{\,2}\, B^2} = \frac{\frac{c(1+b)^2}{b(1+bc)^2}\,\mu_e^{\,2}\, B^2}{1 + \frac{(1-c)^2}{(1+bc)^2}\,\mu_e^{\,2}\, B^2}$$

$$\therefore \quad \frac{\Delta\rho}{\rho_o} = \frac{c(1+b)^2\mu_e^{\,2}\, B^2}{b\{(1+bc)^2 + (1-c)^2\mu_e^{\,2}B^2\}} = \frac{\frac{c}{b}\,(1+b)^2\,\mu_e^{\,2}B^2}{\left[(1+bc)^2 + (1-c)^2\mu_e^{\,2}B^2\right]}$$

$$= \frac{(n/p)\,(\frac{\mu_h}{\mu_e})\,(1+\frac{\mu_e}{\mu_h})^2\,\mu_e^{\,2}\, B^2}{\left[\{1+(\frac{\mu_e}{\mu_h})\,({}^{n}/_{p})\}^2 + (1-\frac{n}{p})^2\,\mu_e^{\,2}\, B^2\right]}$$

A similar expression has been used by Barlow and Koike[25] as follows:

$$\frac{\sigma_B - \sigma_o}{\sigma_B} = \frac{({}^{n}/_{p})\,(\frac{\mu_h}{\mu_e})\,(1+\frac{\mu_e}{\mu_h})^2\,\mu_e^{\,2}\, B^2}{(1 + \frac{\mu_e}{\mu_h}\frac{n}{p})^2 + (1-{}^{n}/_{p})^2\,\mu_e^{\,2}B^2}$$

$$\text{Let } \bar{x} = \frac{\Delta\rho}{\rho_o} = -\frac{\sigma_B - \sigma_o}{\sigma_B}$$

$$\therefore \bar{X} = \frac{\frac{n}{pb}(1+b)^2 \mu_e^2 B^2}{(1+b\frac{n}{p})^2 + (1-\frac{n}{p})^2 \mu_e^2 B^2} = \frac{np\,(1+b)^2 \mu_e^2 B^2}{b\left[(p+nb)^2 + (p-n)^2 \mu_e^2 B^2\right]}$$

For a n-type material, n >> p and approximating the above equation under this condition

$$\bar{X} \cong \frac{np\,(1+b)^2 \mu_e^2 B^2}{b\left[n^2b^2 + n^2\mu_e^2 B^2\right]} = \frac{p\,(1+b)^2 \mu_e^2 B^2}{nb\,(b^2 + \mu_e^2 B^2)}$$

$$= \frac{p}{n} \cdot \frac{(1+b)^2 \mu_e^2 B^2}{(b^3 + \mu_e^2 B^2 b)} = \frac{p}{n} \cdot D$$

For different values of B, ranging from .2 Wb/m^2 to 1.00 Wb/m^2, calculations of the values of D for different n-type materials can now be made using the values of the mobilities as shown in Table 1. Calculated values of D are tabulated in Table 2.

Since $\bar{X} = (\frac{p}{n})\cdot D$, the ratio (p/n) must be large in order to make $\bar{X}$ large. However, it must be noted that we can not change n arbitrarily, as the mobility varies with the resistivity. For resistivities above 1Ω-cm (Ge) and 10Ω-cm(Si), mobility is practically independent of resistivity. Below 1 ohm-cm for Ge, 10 ohm-cm for Si, the impurities begin to interfere with the carrier motion and the mobility drops with decreasing resistivity. This shows that higher resistivity material should be selected but at the same time it must remain n-type and not become intrinsic as the expression is only valid for n-type material.

Germanium

$$n_i\ (300°K) = 2.4 \times 10^{13}/cm^3$$

Let us suppose $n = 10^{14}/cm^3$ $\therefore$ resistivity $\simeq 20\Omega$-cm

$$p = \frac{n_i^2}{n} = 5.76 \times 10^{12}/cm^3$$

$$\therefore (\frac{p}{n}) = 5.76 \times 10^{-2}$$

Silicon

$$n_i\ (300°K) = 1.5 \times 10^{10}/cm^3$$

Let us suppose $n = 1.5 \times 10^{11}/cm^3$ resistivity $\simeq 2.98 \times 10^{+4}\ \Omega$-cm

$$\therefore p = 1.5 \times 10^9 \text{ and } (\frac{p}{n}) = 10^{-2}$$

InSb

$$n_i\ (300°K) = 1.35 \times 10^{16}/cm^3$$

Let us consider $n = 1.35 \times 10^{17}$ resistivity $\simeq 7.7 \times 10^{-4}\ \Omega$-cm

$$\therefore p = 1.35 \times 10^{15}/cm^3 \quad \therefore \frac{p}{n} = 10^{-2}$$

InAs

$$n_i\ (300°K) = 1.97 \times 10^{15}/cm^3$$

Let $n = 10^{16}/cm^3$ resistivity $\simeq .021\Omega$-cm

$$p = 3.86 \times 10^{14}/cm^3 \quad \therefore p/n = 3.86 \times 10^{-2}$$

For a particular p/n ratio in n-type material it can be seen from table 2 that the effect is most pronounced in InSb. The next preferable material is Ge. The order of these materials in terms of their magneto-resistance effect is as follows:-

(1) InSb (2) Ge (3) InAs (4) GaSb

(5) GaAs (6) Si (7) InP (8) AℓSb

An n-type Ge sample with a resistivity of 22.2 ohm-cm has been chosen for the experimental purpose in this thesis.

The Hall co-efficient is inversely proportional to the carrier concentration and the relative change in resistivity $\Delta\rho/\rho_o$ is found from the expression to contain terms in μ^2H^2. Therefore, in order to have a pronounced effect of transverse magnetic field on the resistivity or the conductivity of a semiconductor, carrier mobility must be high and the carrier concentration low. The geometrical magneto-resistance effects are large when the deviation of the carriers in the magnetic field is large and for this too a high mobility is needed. The mobility of a semiconductor is highest when all impurities have been removed, but unfortuantely, in this pure condition the carrier concentration is strongly temperature dependent. Doping, the intentional addition of impurities, will reduce the mobility but it also stabilizes the carrier concentration.

The element semiconductors do not generally have as high an electron mobility as the compounds studied. Also the hole mobility for the compounds is quite low, so that if the materials have to be doped to give temperature stability, they are normally made n-type and the magneto resistance effect is then determined by the electron mobility.

TABLE I

MATERIAL	μ_e (290°K)	μ_h (290°K)	$b = \frac{\mu_e}{\mu_h}$ (290°K)
	cm^2/V-sec	cm^2/V-sec	
Ge	3.8×10^3	1.8×10^3	2.11
Si	1.45×10^3	$.5 \times 10^3$	2.9
InSb	70×10^3	1.0×10^3	70
InAs	30×10^3	$.25 \times 10^3$	120
InP	3.4×10^3	$.05 \times 10^3$	68
GaSb	5×10^3	1×10^3	5
GaAs	7×10^3	$.4 \times 10^3$	17.5
AbSb	$.2 \times 10^3$	$.2 \times 10^3$	1

ELECTRON AND HOLE MOBILITIES AND THEIR RATIO FOR DIFFERENT TYPES OF MATERIALS

MATERIAL	$D = \frac{(1+b)^2 \mu_e^2 B^2}{b^3 + \mu_e^2 B^2 b}$									NET VARIATION IN THE $\frac{\Delta\rho}{\rho_o}$ AS THE MAG. FIELD VARIES FROM 0.2 TO 1.00 Wb/m^2
	B=0.2 Wb/m^2	B=0.3 Wb/m^2	B=0.4 Wb/m^2	B=0.5 Wb/m^2	B=0.6 Wb/m^2	B=0.7 Wb/m^2	B=0.8 Wb/m^2	B=0.9 Wb/m^2	B=1.00 Wb/m^2	
Ge	$.60\times10^{-2}$	1.33×10^{-2}	2.37×10^{-2}	3.69×10^{-2}	5.29×10^{-2}	7.17×10^{-2}	9.32×10^{-2}	11.7×10^{-2}	14.4×10^{-2}	$\frac{p}{n}(13.8\times10^{-2})$
Si	$.516\times10^{-3}$	1.16×10^{-3}	2.06×10^{-3}	3.23×10^{-3}	4.64×10^{-3}	6.32×10^{-3}	8.25×10^{-3}	10.4×10^{-3}	12.9×10^{-3}	$\frac{p}{n}(1.24\times10^{-2})$
InSb	2.88×10^{-2}	6.48×10^{-2}	11.5×10^{-2}	18×10^{-2}	25.8×10^{-2}	35.1×10^{-2}	45.8×10^{-2}	57.9×10^{-2}	71.3×10^{-2}	$\frac{p}{n}(68.42\times10^{-2})$
InAs	$.305\times10^{-2}$	$.686\times10^{-2}$	1.22×10^{-2}	1.91×10^{-2}	2.74×10^{-2}	3.74×10^{-2}	4.88×10^{-2}	6.17×10^{-2}	7.62×10^{-2}	$\frac{p}{n}(7.315\times10^{-2})$
InP	$.7\times10^{-4}$	1.58×10^{-4}	2.8×10^{-4}	4.38×10^{-4}	6.3×10^{-4}	8.58×10^{-4}	11.2×10^{-4}	14.2×10^{-4}	17.5×10^{-4}	$\frac{p}{n}(.168\times10^{-2})$
GaSb	$.288\times10^{-2}$	$.647\times10^{-2}$	1.15×10^{-2}	1.8×10^{-2}	2.58×10^{-2}	3.51×10^{-2}	4.58×10^{-3}	5.79×10^{-2}	7.13×10^{-2}	$\frac{p}{n}(6.84\times10^{-2})$
GaAs	1.25×10^{-3}	2.82×10^{-3}	5.01×10^{-3}	7.82×10^{-3}	11.3×10^{-3}	15.3×10^{-3}	20×10^{-3}	25.3×10^{-3}	31.2×10^{-3}	$\frac{p}{n}(2.995\times10^{-2})$
AlSb	$.64\times10^{-4}$	1.44×10^{-4}	2.56×10^{-4}	4×10^{-4}	5.76×10^{-4}	7.84×10^{-4}	10.2×10^{-4}	13×10^{-4}	16×10^{-4}	$\frac{p}{n}(.1536\times10^{-2})$

TABLE 2

VARIATION OF $\frac{\Delta\rho}{\rho_o}$ WITH MAGNETIC FIELD FOR DIFFERENT SEMICONDUCTOR MATERIALS.

CHAPTER II

THE COMPLEX TENSOR PERMITTIVITY OF A SEMICONDUCTOR IN A MAGNETIC FIELD

2.1 ROLE OF HALL FIELD IN MAXWELL'S EQUATIONS

The propagation characteristics of the electromagnetic waves in semiconductors, when an external steady magnetic field is applied in the direction of the propagation of the wave, have been analyzed by Rau and Caspari[26], and H. E. M. Barlow[16]. Maxwell's equations as written by these authors are given below-

AUTHOR	MAXWELL'S EQUATIONS
RAU & CASPARI	$\nabla \times H = J \; ; \nabla \times (E+E_H) = -\mu_o \left(\frac{\partial H}{\partial t}\right)$
	$J = J_c + \epsilon(\partial/\partial t)\,(E+E_H)$
	$J_c = \sigma E \; ; \; E_H = R_c \; B \times J_c$
H. E. M. BARLOW	$\nabla \times H = J \; ; \; \nabla \times E = -\mu_o \left(\frac{\partial H}{\partial t}\right)$
	$J = J_c + J_d \; ;$
	$Je = \sigma(E+E_H) \; ; \; J_d = \epsilon\left(\frac{\partial}{\partial t}\right)(E+E_H)$
	$E_H = R_c \; B \times J_c + \left(\frac{\epsilon - \epsilon_0}{\epsilon}\right) R_c \; B \times J_d$

where, E_H is the Hall field vector

R_c = Hall coefficient for conduction current

R_d = Hall coefficient for dielectric current

The other symbols have their usual meanings.

As a result of the differences in the role given to Hall field in Maxwell's equations, the expressions derived by the above authors for the propagation constants of circularly polarized waves were very different. Rau and Caspari's analysis predicts a Faraday rotation which does not agree with Barlow's analysis even when the dielectric current Hall coefficient Rd is assumed to be zero. Nag and Engineer[9] in their analysis of Faraday rotation in artificial dielectric have suggested a new role of the Hall field, E_H, in Maxwell's equations by considering the motion of free carriers in the presence of a magnetic field. The following derivation is based on the Nag and Engineer's suggestion for the proper form of the Hall field in Maxwell's equations.

Consider a semiconductor sample to which an electric field is applied in the x and y-direction and a magnetic field is applied in the z-direction. Assuming that the effect of the momentum relaxation time is negligible, the average velocities of the carriers v_x and v_y in the x and y directions respectively are given by

$$v_x = \mu[E_x + (\frac{\mu_H}{\mu})\, v_y\, B] \qquad (2.1)$$

$$v_y = \mu[E_y - (\frac{\mu_H}{\mu})\, v_x\, B] \qquad (2.2)$$

where μ is the conductivity mobility and μ_H is the Hall mobility. The above assumption is substantially correct even at microwave frequencies for conductors. If however, the effect of relaxation time is considerable in a particular sample one has only to substitute for μ and μ_H the complex values appropriate for the signal frequency[9].

Now the expression for the current components J_{ex} and J_{ey} are given by

$$J_{ex} = \sigma[E_x + R_c\ J_{cy}\ B] \tag{2.3}$$

$$J_{ey} = \sigma[E_y - R_c\ J_{cx}\ B] \tag{2.4}$$

When the electric field is applied in one direction only, one evaluates the Hall field from the above two equations by equating the current in the other direction to zero. But when electric fields are applied in two directions, one can not dissociate the Hall field by equating the transverse current to zero.

It therefore, appears that the current equations in Maxwell's equation should be written directly from (2.3) and (2.4). But from analogy with the expression for the Hall field when the applied electric field is in one direction, one may call E_H the equivalent Hall field as written below.

$$\vec{E}_H = R_c\ \vec{B} \times \vec{J}_c \tag{2.5}$$

Using equations (2.3), (2.4), (2.5), one may thus write

$$\vec{Jc} = \sigma(\vec{E} - \vec{E}_H) \tag{2.6}$$

2.2 PERMITTIVITY OF ANISOTROPIC MATERIAL

The electric flux density $\vec{D}$ is generally broken into two parts, the first related to the electric field intensity $\vec{E}$ by the free space permittivity, ε_0 and the second called the polarization vector $\vec{P}$. That is,

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \tag{2.7}$$

The polarization vector $\vec{P}$ arises from a distribution of electric dipoles in the material. For linear materials, $\vec{P}$ is related to the electric field intensity $\vec{E}$ by

$$\vec{P} = \varepsilon_0\chi_e \vec{E} \tag{2.8}$$

where, χ_e is the electric susceptibility and $\chi_e = (\hat{\varepsilon}_R - 1)$. In semiconductors $\hat{\varepsilon}_R$, the relative permittivity or dielectric constant is in general complex $\hat{\varepsilon}_R = \varepsilon_R' - j\varepsilon_R''$ in order to account for loss and phase shift in the material.

For isotropic materials, the polarization is parallel with the applied fields and is independent of the direction of the fields. For anisotropic materials, this is not the case. A helpful analogy is afforded by the strain in a solid body resulting from applied forces. Consider a rectangular solid having unequal edge lengths with a co-ordinate system oriented at an arbitrary angle as shown in Figure (2.1). If a force is applied along any one co-ordinate, strains result in all three directions with different magnitudes in each direction.

In the same way, an electric field applied to an anisotropic material along an axis of an arbitrarily oriented co-ordinate system leads to polarization which has components along each of the co-ordinate directions. If $\vec{E} = \vec{a}_x E_x$, then

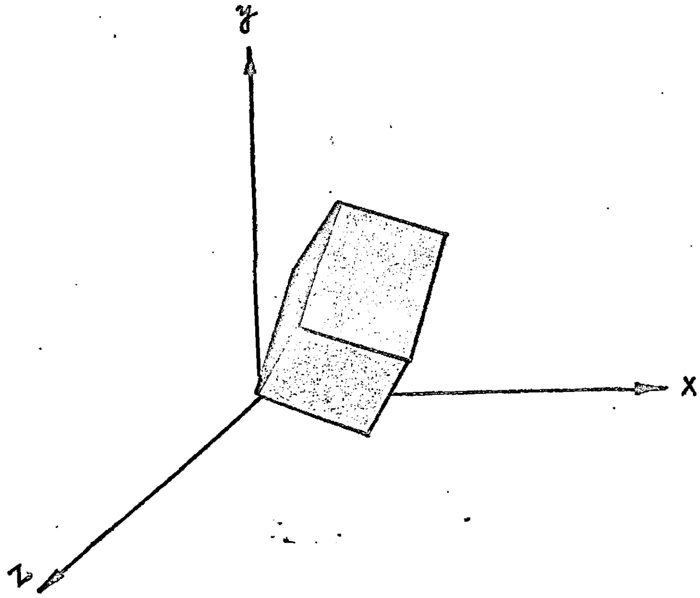


FIGURE 2.1

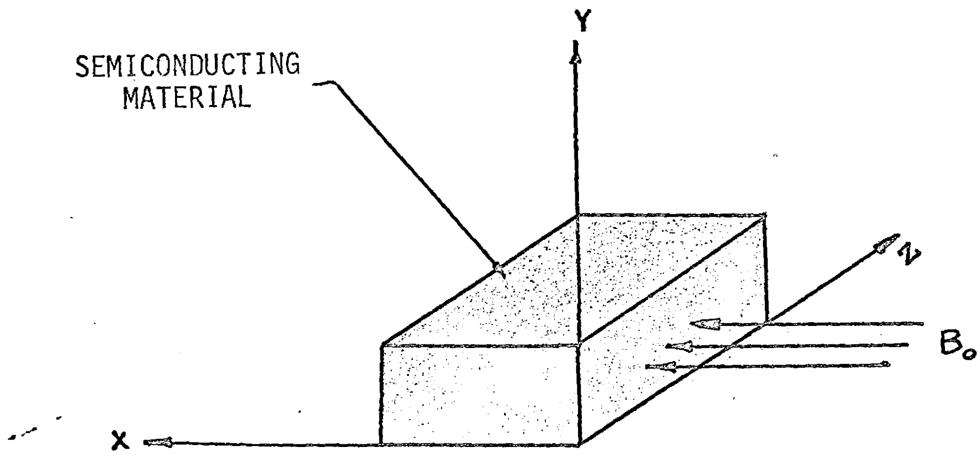


FIGURE 2.2

Semiconductor in a Magnetic Field

$$\vec{P} = \varepsilon_o(\vec{a}_x \chi_{e11}\, Ex + \vec{a}_y\, \chi_{e12}\, Ex + \vec{a}_z \chi_{e13} Ex) \tag{2.9}$$

where χ_e is the electric susceptibility and the added subscripts refer to x, y and z components of $\vec{P}$ and $\vec{E}$ respectively. Then if $\vec{E}$ has all three components E_x, E_y, E_z then

$$\begin{aligned} Px &= \varepsilon_o\,(\chi_{e11}\, Ex + \chi_{e12}\, Ey + \chi_{e13}\, Ez) \\ P_y &= \varepsilon_o\,(\chi_{e21}\, Ex + \chi_{e22}\, Ey + \chi_{e23}\, Ez) \\ P_z &= \varepsilon_o\,(\chi_{e31}\, Ex + \chi_{e32}\, Ey + \chi_{e33}\, Ez) \end{aligned} \tag{2.10}$$

Using the relation (2.7) we may obtain from (2.10)

$$\begin{aligned} D_x &= \varepsilon_{11}\, Ex + \varepsilon_{12}\, Ey + \varepsilon_{13}\, Ez \\ Dy &= \varepsilon_{21}\, Ex + \varepsilon_{22}\, Ey + \varepsilon_{23}\, Ez \\ D_z &= \varepsilon_{31}\, Ex + \varepsilon_{32}\, Ey + \varepsilon_{33}\, Ez \end{aligned} \tag{2.11}$$

where $\varepsilon_{11} = \varepsilon_o(1+\chi_{e11})$, $\varepsilon_{12} = \varepsilon_o \chi_{e12}$, etc.

It is convenient to write the equations (2.11) as an array in matrix form as follows

$$\begin{bmatrix} Dx \\ Dy \\ Dz \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{2.12}$$

2.3 DERIVATION OF THE COMPLEX TENSOR PERMITTIVITY

A rectangular waveguide is assumed to be completely filled with a semiconductor. In the presence of an electromagnetic wave in the guide, a dielectric and also a conduction current flows in the semiconductor. The dielectric current J_d is given by

$$\vec{J}_d = \varepsilon \frac{\partial \vec{E}}{\partial t} \tag{2.13}$$

where ε is the permittivity of the semiconductor and E is the electric field vector.

Professor H. E. M. Barlow in a paper(15) suggested that the dielectric current may be changed when a magnetic field is applied. However, this effect has not been experimentally observed and even if it exists, it would be small. Therefore, it is assumed that the dielectric current is unaffected by the magnetic field.

2.3.1 MODIFICATION OF THE CONDUCTION CURRENT BY EXTERNAL MAGNETIC FIELD

The conduction current in the absence of a magnetic field is σE, and if the signal frequency is assumed to be much less than the scattering frequency, σ is equal to the d. c. conductivity of the semiconductor.

When a magnetic field is applied, the conduction current is modified by the Hall effect. In the case of a semiconductor with spherical energy surfaces, the Hall field $\vec{E}_H$ is given by (2.5)

$$\vec{E}_H = R_c \vec{B}_o \times \vec{J}_{cH} \tag{2.14}$$

where R_c is the Hall coefficient and B_o is the steady magnetic field. The steady magnetic field is applied in the x direction as shown in Figure (2.2). The modified conduction current is thus,

$$\vec{J}_{cH} = \sigma(\vec{E} - \vec{E}_H) \tag{2.15}$$

Assuming the time dependence of the electromagnetic fields to be e^{jwt}, an expression for the total current J in terms of a complex permittivity tensor can be written in the following way.

$$\text{Total current} \quad \vec{J} = \vec{J}d + \vec{J}_{cH}$$
$$= \varepsilon\frac{\partial \vec{E}}{\partial t} + \sigma(\vec{E} - \vec{E}_H)$$
$$= jw\varepsilon\vec{E} + \sigma(\vec{E}-\vec{E}H) \quad (2.16)$$

But from equation (2.14) the three components of the Hall field, ie E_{Hx}, E_{Hy}, E_{Hz} in x, y, z direction are given by

$$E_{Hx} = 0$$
$$E_{Hy} = - R_c B_o J_{cHz} \quad (2.17)$$
$$E_{Hz} = R_c B_o J_{cHy}$$

where x, y, z components of J_{cH} are denoted by J_{cHx}, J_{cHy}, J_{cHz} respectively.

Now, from equation (2.15)

$$\vec{J}_{cH} = \sigma[\vec{a}_x E_x + \vec{a}_y E_y + \vec{a}_z E_z - \vec{a}_x E_{Hx} - \vec{a}_y E_{Hy} - \vec{a}_z E_H] \quad (2.18)$$

Therefore, from (2.17)aand (2.18)

$$J_{cHx} = \sigma[Ex - E_{Hx}] = \sigma Ex \quad (2.19)$$
$$J_{cHy} = \sigma[Ey - E_{Hy}] = \sigma[Ey + R_c B_o J_{cHz}] \quad (2.20)$$
$$J_{cHz} = \sigma[Ez - E_{Hz}] = \sigma[Ez - R_c B_o J_{cHy}] \quad (2.21)$$

From equation (2.20)

$$\sigma R_c B_o J_{cHy} = \sigma^2 R_c B_o E_y + \sigma^2 R_c^2 B_o^2 J_{cHz} \quad (2.22)$$

From equation (2.21)

$$\sigma R_c B_o J_{cHy} = \sigma E_z - J_{cHz} \quad (2.23)$$

Subtracting (2.23) and (2.22)

$$0 = \sigma^2 R_c B_o Ey - \sigma E_z + (\sigma^2 R_c^2 B_o^2 + 1) J_{cHz}$$

$$\therefore \quad J_{cHz} = \frac{\sigma}{(1+\sigma^2 R_c^2 B_o^2)} E_z - \frac{\sigma^2 R_c B_o}{(1+\sigma^2 R_c^2 B_o^2)} Ey \qquad (2.24)$$

From (2.20) and (2.24)

$$J_{cHy} = (\sigma - \frac{\sigma^3 R_c^2 B_o^2}{1+\sigma^2 R_c^2 B_o^2}) Ey + \frac{\sigma^2 R_c B_o}{1+\sigma^2 R_c^2 B_o^2} Ez$$

$$= \frac{\sigma}{1+\sigma^2 R_c^2 B_o^2} Ey + \frac{\sigma^2 R_c B_o}{1+\sigma^2 R_c^2 B_o^2} Ez \qquad (2.25)$$

2.3.2 THE MATRIX ELEMENTS OF THE COMPLEX PERMITTIVITY TENSOR

From equation (2.16)

$$\vec{J} = jw\varepsilon\vec{E} + \vec{J}_{cH}$$

$$\therefore Jx = jw\varepsilon Ex + J_{cHx} = jw\varepsilon Ex + \sigma Ex \quad \text{(from (2.19))}$$

$$= (\sigma + jw\varepsilon) Ex = jw\varepsilon(1 - \frac{j\sigma}{w\varepsilon}) Ex$$

$$\equiv \varepsilon_1 \, jwEx \qquad (2.26)$$

Using equation (2.25) and (2.16)

$$J_y = jw\varepsilon Ey + J_{cHy} = jw\varepsilon E_y + \frac{\sigma}{1+\sigma^2 R_c^2 B_o^2} Ey + \frac{\sigma^2 R_c B_o}{1+\sigma^2 R_c^2 B_o^2} Ez$$

$$= jw\varepsilon \left[1 - \frac{j\sigma}{w\varepsilon(1+\sigma^2 R_c^2 B_o^2)}\right] E_y + \frac{R_c B_o \sigma^2}{1+R_c^2 B_o^2 \sigma^2} E_z$$

$$= \varepsilon\left[1 - \frac{j\sigma}{w\varepsilon(1+R_c^2B_o^2\sigma^2)}\right] jwE_y - \frac{jR_cB_o\sigma^2}{w(1+R_c^2B_o^2\sigma^2)} \quad jwE_z$$

$$\equiv \varepsilon_2\, jwE_y - \varepsilon_3 jwE_z \tag{2.27}$$

$$J_z = jw\varepsilon\, E_z + J_{cHz}$$

$$= jw\varepsilon\, E_z + \frac{\sigma}{1+\sigma^2R_c^2B_o^2} E_z - \frac{\sigma^2R_cB_o}{1+\sigma^2R_c^2B_o^2} Ey$$

$$= jw\varepsilon\left[1 - \frac{j\sigma}{w\varepsilon(1+R_c^2B_o^2\sigma^2)}\right] E_z - \frac{\sigma^2R_cB_o}{1+\sigma^2R_c^2B_o^2} Ey$$

$$= \varepsilon\left[1 - \frac{j\sigma}{w\varepsilon(1+R_c^2B_o^2\sigma^2)}\right] jwE_z + \frac{jR_cB_o\sigma^2}{w(1+R_c^2B_o^2\sigma^2)} \quad jwEy$$

$$\equiv \varepsilon_2\, jwE_z + \varepsilon_3\, jwE_y \tag{2.28}$$

Let, $\quad J = [\varepsilon]\,\frac{\partial E}{\partial t}$

Therefore

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} jwE_x \\ jwE_y \\ jwE_z \end{bmatrix}$$

Comparing with equations (2.26), (2.27) and (2.28) it is seen that

$$\varepsilon_{11} = \varepsilon_1 = \varepsilon\left(1 - \frac{j\sigma}{w\varepsilon}\right)$$

$$\varepsilon_{12} = 0$$

$$\varepsilon_{13} = 0$$

$$\varepsilon_{21} = 0$$

$$\varepsilon_{22} = \varepsilon_2 = \varepsilon\{1 - \frac{j\sigma}{w\varepsilon(1+R_c^2B_o^2\sigma^2)}\}$$

$$\varepsilon_{23} = -\varepsilon_3$$

$$\varepsilon_{31} = 0$$

$$\varepsilon_{32} = \varepsilon_3 = \frac{jR_cB_o\sigma^2}{w(1+R_c^2B_o^2\sigma^2)}$$

$$\varepsilon_{33} = \varepsilon_2$$

Therefore, when a magnetic field is applied in the x-direction, the total current density in a semiconductor can be written as

$$J = [\varepsilon]\ \frac{\partial E}{\partial t} \tag{2.29}$$

and thus the elements of the complex permittivity tensor can be written as

$$[\varepsilon] = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & -\varepsilon_3 \\ 0 & \varepsilon_3 & \varepsilon_2 \end{bmatrix}$$

where $\varepsilon_1 = \varepsilon(1 - \frac{j\sigma}{W\varepsilon})$

$$\varepsilon_2 = \varepsilon\{1 - \frac{j\sigma}{w\varepsilon(1+R_c^2B_o^2\sigma^2)}\} = \varepsilon\left[1 - \frac{j\,\sigma}{W\varepsilon\ \{1+(R_cB_o\sigma)^2\}}\right]$$

$$\varepsilon_3 = \frac{jR_cB_o\sigma^2}{w(1+R_c^2B_o^2\sigma^2)}$$

$$= \frac{jR_cB_o\sigma^2}{w\ \{1+(R_cB_o\sigma)^2\}}$$

CHAPTER III

THEORETICAL ANALYSIS OF THE ELECTROMAGNETIC FIELD COMPONENTS

3.1 GENERAL CONSIDERATION

A rectangular waveguide as shown in Figure (3.1) is completely filled with a semiconductor. The coordinate system to be used in this analysis is shown in this figure. A steady transverse magnetic field B_0 is applied in the x-direction.

From equation (2.29) in section 2.3.2, the total current density in the semiconductor having a complex permittivity tensor can be written as

$$J = [\varepsilon] \frac{\partial E}{\partial t} \tag{3.1}$$

where,

$$[\varepsilon] = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & -\varepsilon_3 \\ 0 & \varepsilon_3 & \varepsilon_2 \end{bmatrix}$$

and

$$\varepsilon_1 = \varepsilon \left[1 - \frac{j\sigma}{W\varepsilon}\right]$$

$$\varepsilon_2 = \varepsilon \left[1 - \frac{j\sigma}{W\varepsilon\{1+(R_c B_0 \sigma)^2\}}\right] \tag{3.2}$$

$$\varepsilon_3 = \frac{j\, R_c B_0 \sigma^2}{W\{1+(R_c B_0 \sigma)^2\}}$$

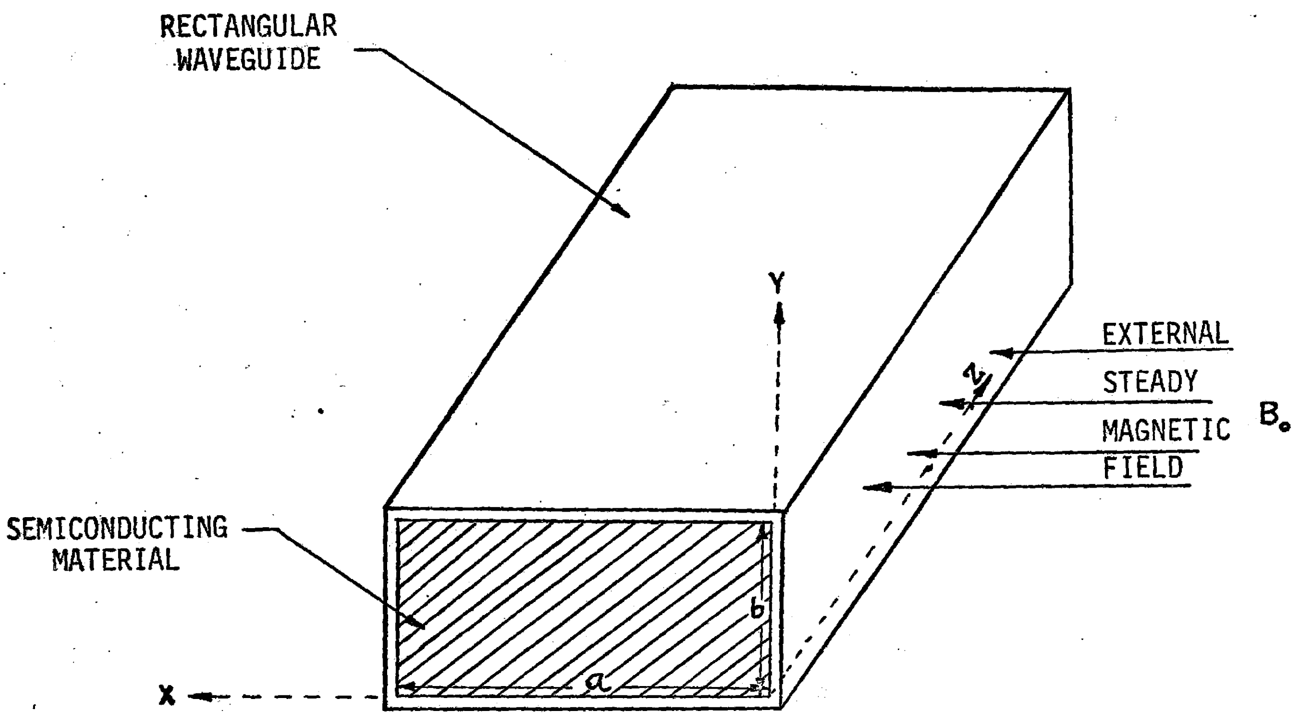


FIGURE 3.1 RECTANGULAR WAVEGUIDE COMPLETELY FILLED WITH SEMICONDUCTOR

Maxwell's curl equations in the case of the media having a complex permittivity tensor are

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \tag{3.3}$$

$$\nabla \times H = [\varepsilon] \frac{\partial E}{\partial t} \tag{3.4}$$

The time dependence of the electromagnetic fields is considered to be e^{jwt} and the z-dependence of the fields is assumed to be $e^{-\Gamma z}$

where Γ = propagation constant

$= \alpha + j\beta$

α = attenuation constant (nepers/meter)

β = phase constant (radians/meter)

Equations (3.3) and (3.4) are written in the following form to show the tensor permittivity.

$$\begin{bmatrix} 0 & \Gamma & \frac{\partial}{\partial y} \\ -\Gamma & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = -jw\mu \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \tag{3.5}$$

$$\begin{bmatrix} 0 & \Gamma & \frac{\partial}{\partial y} \\ -\Gamma & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = jw \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & -\varepsilon_3 \\ 0 & \varepsilon_3 & \varepsilon_2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{3.6}$$

From (3.5) and (3.6) the following six equations can be written as

$$\Gamma H_y + \frac{\partial H_z}{\partial y} = jw\ \varepsilon_1\ E_x \tag{3.7}$$

$$-\Gamma H_x - \frac{\partial}{\partial x} H_z = jw\ \varepsilon_2\ E_y - jw\ \varepsilon_3\ E_z \tag{3.8}$$

$$-\frac{\partial}{\partial y} H_x + \frac{\partial H_y}{\partial x} = jw\ \varepsilon_3\ E_y + jw\ \varepsilon_2\ E_z \tag{3.9}$$

$$\Gamma E_y + \frac{\partial E_z}{\partial y} = -\ jw\mu\ H_x \tag{3.10}$$

$$-\ \Gamma E_x - \frac{\partial E_z}{\partial x} = -\ jw\mu\ H_y \tag{3.11}$$

$$-\frac{\partial}{\partial y} E_x + \frac{\partial}{\partial x} E_y = -\ jw\mu\ H_z \tag{3.12}$$

From equations (3.7), (3.11) E_x can be expressed in terms of E_z and H_z, as

$$E_x = \frac{1}{(\Gamma^2+w^2\mu\varepsilon_1)} \left[-\Gamma \frac{\partial E_z}{\partial x} - jw\mu \frac{\partial H_z}{\partial y} \right] \tag{3.13}$$

Using equations (3.8), (3.10), E_y can be expressed in terms of E_z and H_z as

$$E_y = \frac{1}{(\Gamma^2+w^2\mu\varepsilon_2)} \left[(w^2\mu\varepsilon_3 - \Gamma \frac{\partial}{\partial y})\ E_z + jw\mu \frac{\partial H_z}{\partial x} \right] \tag{3.14}$$

From equations (3.10) and (3.14) H_x can be written as

$$H_x = \frac{1}{(\Gamma^2+w^2\mu\varepsilon_2)} \left[jw\ (\Gamma\varepsilon_3 + \varepsilon_2 \frac{\partial}{\partial y})\ E_z - \Gamma \frac{\partial}{\partial x} H_z \right] \tag{3.15}$$

From equations (3.11), (3.13) H_y can be written in terms of E_z and H_z as

$$H_y = -\ \frac{1}{(\Gamma^2+w^2\mu\varepsilon_1)} \left[jw\varepsilon_1 \frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y} \right] \tag{3.16}$$

Putting the values of E_x, E_y, H_x, H_y in equations (3.9) and (3.12), the following partial differential equations in E_z and H_z are obtained,

$$\left[\frac{\varepsilon_1}{(\Gamma^2+w^2\mu\varepsilon_1)}\frac{\partial^2}{\partial x^2}+\frac{\varepsilon_2}{(\Gamma^2+w^2\mu\varepsilon_2)}\frac{\partial^2}{\partial y^2}+\frac{\{\Gamma^2\varepsilon_2+w^2\mu(\varepsilon_2^2+\varepsilon_3^2)\}}{(\Gamma^2+w^2\mu\varepsilon_2)}\right]E_z$$

$$+\,jw\left[\frac{\Gamma(\varepsilon_1-\varepsilon_2)}{(\Gamma^2+w^2\mu\varepsilon_2)(\Gamma^2+w^2\mu\varepsilon_1)}\frac{\partial}{\partial y}+\frac{\varepsilon_3}{(\Gamma^2+w^2\mu\varepsilon_2)}\right]\mu\frac{\partial H_z}{\partial x}=0 \qquad (3.17)$$

$$-\,jw\left[\frac{(\varepsilon_2-\varepsilon_1)}{(\Gamma^2+w^2\mu\varepsilon_1)\,(\Gamma^2+w^2\mu\varepsilon_2)}\Gamma\frac{\partial}{\partial y}+\frac{\varepsilon_3}{(\Gamma^2+w^2\mu\varepsilon_2)}\right]\frac{\partial E_z}{\partial x}$$

$$+\left[\frac{1}{(\Gamma^2+w^2\mu\varepsilon_2)}\frac{\partial^2}{\partial x^2}+\frac{1}{(\Gamma^2+w^2\mu\varepsilon_1)}\frac{\partial^2}{\partial y^2}+1\right]H_z=0 \qquad (3.18)$$

3.1.1 ANALYSIS OF TE MODE PROPAGATION

If TE modes are considered, then by putting $E_z = 0$, equations (3.17) and (3.18) reduce to

$$\left[\frac{\Gamma(\varepsilon_1-\varepsilon_2)}{(\Gamma^2+w^2\mu\varepsilon_1)}\frac{\partial}{\partial y}+\varepsilon_3\right]\frac{\partial H_z}{\partial x}=0$$

$$\left[\frac{1}{(\Gamma^2+w^2\mu\varepsilon_2)}\frac{\partial^2}{\partial x^2}+\frac{1}{(\Gamma^2+w^2\mu\varepsilon_1)}\frac{\partial^2}{\partial y^2}+1\right]H_z=0 \qquad (3.19)$$

The above two equations can not be simultaneously satisfied by the same value of Γ unless $\frac{\partial}{\partial x} = 0$ or $H_z = 0$, even if $\varepsilon_1 = \varepsilon_2$.

When $\frac{\partial}{\partial x} = 0$, the equations (3.19) reduce to

$$\left[\frac{\partial^2}{\partial y^2} + (\Gamma^2 + w^2\mu\varepsilon_1)\right] H_z = 0$$

The solution of this equation is

$$H_z = C' \cos (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} y + c'' \sin (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} y$$

where, c' and c'' are arbitrary constants.

Boundary conditions are at $y = 0$, $\frac{\partial H_z}{\partial y} = 0$, $y = b$, $\frac{\partial H_z}{\partial y} = 0$

Hence,

$$\frac{\partial H_z}{\partial y} = - (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} c' \sin (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} y$$

$$+ c'' (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} \cos (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} y$$

$$0 = c'' (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}}$$

$$\therefore c'' = 0$$

$$H_z = c' \cos (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} y$$

At $y = b$, $\frac{\partial H_z}{\partial y} = 0$

$$\therefore 0 = - c'(\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} \sin (\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} b$$

or $$\sin(\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} b = 0$$

Therefore, $(\Gamma^2 + w^2\mu\varepsilon_1)^{\frac{1}{2}} = \frac{n\pi}{b}$

where n is an integer, $H_z = c' \cos \frac{n\pi}{b} y$

$$\therefore \Gamma^2 + w^2\mu\varepsilon_1 = \frac{n^2\pi^2}{b^2}$$

$$\text{or } \Gamma^2 = \frac{n^2\pi^2}{b^2} - w^2\mu\varepsilon_1$$

Since the above expression for ε_1 does not contain any term B (the applied magnetic field) the wave numbers and the associated fields do not depend on the magnetostatic field. Therefore, it is seen that only those TE modes which have x-independent field components can propagate in the semiconductor filled guide in the presence of an external magnetic field.

From equations (3.13), (3.14) it can be found that the TE_{on} modes which can propagate through the waveguide are characterized by having only the component of the electric field along the x direction, E_x. Since the external magnetic field is applied in the same direction, there will not be a Hall field produced and the characteristics of these modes should evidently remain unaltered by the application of the external magnetic field. Hence it can be concluded that TE_{on} modes hold no interest in this analysis.

3.1.2 ANALYSIS OF TM MODE PROPAGATION

TM modes can be investigated by putting $H_z = 0$ in equations (3.17) and (3.18), which then reduce to the following two equations.

$$\left[\frac{\epsilon_1}{(\Gamma^2+w^2\mu\varepsilon_1)}\frac{\partial^2}{\partial x^2}+\frac{\varepsilon_2}{(\Gamma^2+w^2\mu\varepsilon_2)}\frac{\partial^2}{\partial y^2}+\frac{\left\{\Gamma^2\varepsilon_2+w^2\mu(\varepsilon_2{}^2+\varepsilon_3{}^2)\right\}}{(\Gamma^2+w^2\mu\varepsilon_2)}\right]E_z = 0 \tag{3.20}$$

$$\left[\frac{(\varepsilon_2-\varepsilon_1)}{(\Gamma^2+w^2\mu\varepsilon_1)}\frac{\partial}{\partial y}+\varepsilon_3\right]\frac{\partial E_z}{\partial x} = 0$$

The above two equations can not be simultaneously satisfied for the same value of Γ, even if $\varepsilon_1 = \varepsilon_2$ unless $\frac{\partial}{\partial x} = 0$ or $E_z = 0$. However, if $\frac{\partial}{\partial x} = 0$, E_z must vanish everywhere since it must vanish at the boundaries.

Therefore, it is seen that in a rectangular waveguide filled with semiconductor, in the presence of an external steady transverse magnetic field, TM modes can not propagate.

3.1.3 ANALYSIS OF ANOMALOUS MODE PROPAGATION

The equations (3.17) and (3.18) however can be solved by the same value of Γ if $E_z \neq 0$ and $H_z \neq 0$. Hence, in addition to x-independent TE_{on} modes, the so called anomalous modes having all the six field components are possible. Therefore, if the wave is to depend on the external applied magnetic field, all six components of the field are necessary, and these must depend on both transverse coordinates.

3.2 RIGOROUS GENERAL SOLUTION USING SEPARATION OF VARIABLES METHOD

A rigorous general solution of the field components has been made by employing a special form of the separation of the variables method.

To separate the variables, let

$$E_z = u(x)\, e(y)$$

$$H_z = v(x)\, h(y)$$

Substituting these in equations (3.17) and (3.18) the following two equations are obtained,

$$\left[\frac{\varepsilon_1}{(\Gamma^2+w^2\mu\varepsilon_1)} u''e + \frac{\varepsilon_2}{(\Gamma^2+w^2\mu\varepsilon_2)} ue'' + \frac{\{\Gamma^2\varepsilon_2+w^2\mu(\varepsilon_2{}^2+\varepsilon_3{}^2)\}}{(\Gamma^2+w^2\mu\varepsilon_2)} ue\right.$$

$$\left. + \frac{jw\mu\,\Gamma(\varepsilon_1-\varepsilon_2)}{(\Gamma^2+w^2\mu\varepsilon_2)(\Gamma^2+w^2\mu\varepsilon_1)} v'h' + \frac{jw\mu\varepsilon_3}{(\Gamma^2+w^2\mu\varepsilon_2)} v'h\right] = 0 \qquad (3.21)$$

$$\left[\frac{-jw(\varepsilon_2-\varepsilon_1)\Gamma}{(\Gamma^2+w^2\mu\varepsilon_1)(\Gamma^2+w^2\mu\varepsilon_2)} u'e' - \frac{jw\varepsilon_3}{(\Gamma^2+w^2\mu\varepsilon_2)} u'e\right.$$

$$\left. + \frac{1}{(\Gamma^2+w^2\mu\varepsilon_2)} v''h + \frac{1}{(\Gamma^2+w^2\mu\varepsilon_1)} vh'' + vh\right] = 0 \qquad (3.22)$$

where primes denote the differentiation.

After dividing equation (3.21) by v'e and the equation (3.22) by u'h the following two equations are obtained.

$$F(x,y) = \frac{\varepsilon_1}{(\Gamma^2+w^2\mu\varepsilon_1)} \frac{u''}{v'} + \frac{\varepsilon_2}{(\Gamma^2+w^2\mu\varepsilon_2)} \frac{u}{v'} \cdot \frac{e''}{e} + \frac{\{\Gamma^2\varepsilon_2+w^2\mu(\varepsilon_2{}^2+\varepsilon_3{}^2)\}}{(\Gamma^2+w^2\mu\varepsilon_2)} \frac{u}{v'}$$

$$+ \frac{jw\mu\Gamma(\varepsilon_1-\varepsilon_2)}{(\Gamma^2+w^2\mu\varepsilon_1)(\Gamma^2+w^2\mu\varepsilon_2)} \frac{h'}{e} + \frac{jw\mu\varepsilon_3}{(\Gamma^2+w^2\mu\varepsilon_2)} \frac{h}{e}\Bigg] = 0$$

$$G(x,y) = \Bigg[\frac{jw(\varepsilon_1-\varepsilon_2)\Gamma}{(\Gamma^2+w^2\mu\varepsilon_1)(\Gamma^2+w^2\mu\varepsilon_2)} \frac{e'}{h} - \frac{jw\varepsilon_3}{(\Gamma^2+w^2\mu\varepsilon_2)} \frac{e}{h}$$

$$+ \frac{1}{(\Gamma^2+w^2\mu\varepsilon_2)} \frac{v''}{u'} + \frac{1}{(\Gamma^2+w^2\mu\varepsilon_1)} \frac{v}{u'} \cdot \frac{h''}{h} + \frac{v}{u'}\Bigg] = 0$$

The key to the separation of the variables is that the variations of F and G with respect to x and y must vanish. This follows from the fact that F and G are themselves zero for all values of x and y.

$$\text{Hence, } \frac{\partial^2 F}{\partial x \partial y} = 0 = \frac{\varepsilon 2}{(\Gamma^2+w^2\mu\varepsilon_2)} \cdot \frac{\partial^2}{\partial x \partial y} \frac{u}{v'} \cdot \frac{e''}{e}$$

$$\text{or } \frac{\partial^2}{\partial x \partial y} \left(\frac{u}{v'} \cdot \frac{e''}{e} \right) = 0$$

$$\therefore \frac{\partial}{\partial x} \left(\frac{u}{v'}\right) \frac{\partial}{\partial y} \left(\frac{e''}{e}\right) = 0$$

$$\therefore \frac{\partial}{\partial x} \left[\frac{u}{v'}\right] = 0$$

$$\frac{u}{v'} = \text{constant}$$

$$\therefore v'(x) = - c'u(x)$$

Similarly, $\frac{\partial^2 G}{\partial x \partial y} = 0 = \frac{\partial^2}{\partial x \partial y}\left[\frac{v}{u'} \cdot \frac{h''}{h}\right]$

$$\therefore \quad \frac{\partial}{\partial x}\left[\frac{v}{u'}\right] \frac{\partial}{\partial y}\left[\frac{h''}{h}\right] = 0$$

or $\frac{\partial}{\partial x}\left(\frac{v}{u'}\right) = 0$

$$\therefore \quad \frac{v}{u'} = \text{constant}$$

or, $u'(x) = cv(x)$

Therefore, the vanishing variations constrain u and v to the relations

$$v'(x) = -c'u(x) \tag{3.23}$$

$$u'(x) = cv(x) \tag{3.24}$$

where c and c' are separation constants.

The appropriate boundary conditions are

$$E_z = 0 \text{ at } x = 0,\ x = a$$

$$\therefore \quad u = 0 \text{ at } x = 0,\ x = a$$

Again $\frac{\partial H_z}{\partial x} = 0$ at $x = 0,\ x = a$

$$\therefore \; v' = 0 \quad \text{at } x = 0, a$$

From equations (3.23) and (3.24)

$$\frac{d^2u}{dx^2} + cc'u = 0$$

or, $u = A \cos\alpha_m x + B \sin\alpha_m x$

where $\alpha_m = (cc')^{\frac{1}{2}}$

$u = 0$, at $x = 0$ makes $A = 0$

$$\therefore \quad u = B \sin\alpha_m x$$

Again u = 0, x = a

$$\therefore\ 0 = B \sin\alpha_m a$$

$$\therefore\ \alpha_m = \frac{n\pi}{a}$$

Similarly putting the boundary conditions on the solutions of $v'' = -c'cv$ gives $v = \cos\frac{n\pi}{a}x$.

Therefore, the solutions of (3.23) and (3.24) give solutions u and v of the following form

$$u = \sin\alpha_m x \text{ and } v = \cos\alpha_m x \quad \text{where } \alpha_m = c = c' = \frac{n\pi}{a} \tag{3.25}$$

With the help of equations((3.23) and (3.24) and (3.25) u, and v can be eliminated from equations (3.21) and (3.22) to obtain the following two equations for e and h.

$$\begin{aligned} \left[a_1\frac{d^2}{dy^2}+b_1\right] e - \left[c_1\frac{d}{dy}+d_1\right] h = 0 \\ \left[c_2\frac{d}{dy}-d_2\right] e + \left[a_2\frac{d^2}{dy^2}+b_2\right] h = 0 \end{aligned} \tag{3.26}$$

where a_1, b_1, c_1, d_1, and a_2, b_2, c_2 and d_2 are given below.

$$a_1 = \frac{\varepsilon_2}{(\Gamma^2+w^2\mu\varepsilon_2)} \quad ; \qquad b_1 = \frac{(\Gamma^2+w^2\mu\varepsilon_1)\{\Gamma^2\varepsilon_2+w^2\mu(\varepsilon_2^2+\varepsilon_3^2)\}-(\Gamma^2+w^2\mu\varepsilon_2)\frac{n^2\pi^2}{a^2}\varepsilon_1}{(\Gamma^2+w^2\mu\varepsilon_2)\ (\Gamma^2+w^2\mu\varepsilon_1)}$$

$$c_1 = \frac{jw\mu\Gamma(\varepsilon_1-\varepsilon_2)\frac{n\pi}{a}}{(\Gamma^2+w^2\mu\varepsilon_2)(\Gamma^2+w^2\mu\varepsilon_1)} \quad ; \qquad d_1 = \frac{jw\mu\varepsilon_3\frac{n\pi}{a}}{(\Gamma^2+w^2\mu\varepsilon_2)}$$

$$a_2 = \frac{1}{(\Gamma^2+w^2\mu\varepsilon_1)} \quad ; \qquad b_2 = \frac{(\Gamma^2+w^2\mu\varepsilon_2 - \frac{n^2\pi^2}{a^2})}{(\Gamma^2+w^2\mu\varepsilon_2)}$$

$$c_2 = \frac{jw(\varepsilon_1-\varepsilon_2)\Gamma\,\frac{n\pi}{a}}{(\Gamma^2+w^2\mu\varepsilon_1)(\Gamma^2+w^2\mu\varepsilon_2)} \quad ; \qquad d_2 = \frac{jw\varepsilon_3\,\frac{n\pi}{a}}{(\Gamma^2+w^2\mu\varepsilon_2)}$$

Equations (3.26) are two second order linear differential equations in e and h.The determinantal equation for these two equations is

$$\left[\frac{d^4}{dy^4} + B\,\frac{d^2}{dy^2} + C\right]\begin{Bmatrix} e \\ h \end{Bmatrix} = 0 \tag{3.27}$$

where $B = \dfrac{a_1b_2 + a_2b_1 + c_1c_2}{a_1a_2}$

$$= \frac{2\Gamma^2\varepsilon_2 + w^2\mu(\varepsilon_2{}^2+\varepsilon_3{}^2+\varepsilon_1\varepsilon_2) - \frac{n^2\pi^2}{a^2}(\varepsilon_1+\varepsilon_2)}{\epsilon_2}$$

and $C = \dfrac{b_1b_2 - d_1d_2}{a_1a_2}$

$$= \frac{(\Gamma^2+w^2\mu\varepsilon_1)\{\Gamma^2\varepsilon_2+w^2\mu(\varepsilon_2{}^2+\varepsilon_3{}^2)\}-(\Gamma^2+w^2\mu\varepsilon_2)\frac{n^2\pi^2}{a^2}\varepsilon_1 + \frac{n^4\pi^4}{a^4}\varepsilon_1 - \frac{n^2\pi^2}{a^2}\varepsilon_2(\Gamma^2+w^2\mu\varepsilon_1)}{\epsilon_2}$$

Multiplying both sides by ε_2 the equation (3.27) can also be written in the following form

$$\left[P \frac{d^4}{dy^4} + Q \frac{d^2}{dy^2} + R\right]\begin{Bmatrix} e \\ h \end{Bmatrix} = 0 \qquad (3.28)$$

where $P = \varepsilon_2$

$$Q = 2\Gamma^2\varepsilon_2 + w^2\mu(\varepsilon_2{}^2 + \varepsilon_3{}^2 + \varepsilon_1\varepsilon_2) - \frac{n^2\pi^2}{a^2}(\varepsilon_1 + \varepsilon_2)$$

$$R = \Gamma^4\varepsilon_2 + \Gamma^2\left\{w^2\mu(\varepsilon_2{}^2 + \varepsilon_3{}^2 + \varepsilon_1\varepsilon_2) - \frac{n^2\pi^2}{a^2}(\varepsilon_1 + \varepsilon_2)\right\}$$

$$+ \left\{w^4\mu^2\varepsilon_1(\varepsilon_2{}^2 + \varepsilon_3{}^2) - 2w^2\mu\varepsilon_1\varepsilon_2\frac{n^2\pi^2}{a^2} + \frac{n^4\pi^4}{a^4}\varepsilon_1\right\}$$

$$\left[P \frac{d^4}{dy^4} + Q \frac{d^2}{dy^2} + R\right] e = 0$$

Now, the auxiliary equation of the above differential equation is

$$Pm^4 + Qm^2 + R = 0$$

The four roots of this equation are

$$\pm m_1 = \pm\left[-\frac{Q}{2P} + \left\{\left(\frac{Q}{2P}\right)^2 - \frac{R}{P}\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}} \qquad (3.29)$$

$$\pm m_2 = \pm\left[-\frac{Q}{2P} - \left\{\left(\frac{Q}{2P}\right)^2 - \frac{R}{P}\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

where, $\dfrac{Q}{2P} = \Gamma^2 + \dfrac{w^2\mu}{2}\left(\varepsilon_1 + \varepsilon_2 + \dfrac{\varepsilon_3^2}{\varepsilon_2}\right) - \dfrac{n^2\pi^2}{a^2}\dfrac{(\varepsilon_1 + \varepsilon_2)}{2\varepsilon_2}$

and $\left\{\left(\dfrac{Q}{2P}\right)^2 - \dfrac{R}{P}\right\}^{\frac{1}{2}}$

$$= \frac{1}{2}\left[\left\{\left(\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_2}\right)\left(w^2\mu\varepsilon_2 - \frac{n^2\pi^2}{a^2}\right) - w^2\mu\frac{\varepsilon_3{}^2}{\varepsilon_2}\right\}^2 - 4w^2\mu\frac{\varepsilon_1\varepsilon_3{}^2}{\varepsilon_2{}^2}\frac{n^2\pi^2}{a^2}\right]^{\frac{1}{2}}$$

Mikaelian[12] and H. Seidel[13] have shown that the boundary conditions at y = 0 and y = b can not be satisfied by either of the two independent birefringent modes corresponding to the four roots $\pm m_{1,2}$ but that a linear combination of these must be used.

Thus, $$e(y) = A e^{m_1 y} + Be^{-m_1 y} + Ce^{m_2 y} + De^{-m_2 y} \quad (3.30)$$

Therefore,

$$E_z = (Ae^{m_1 y} + Be^{-m_1 y} + Ce^{m_2 y} + De^{-m_2 y}) \sin \frac{n\pi x}{a} e^{(jwt-\Gamma z)} \quad (3.31)$$

where A, B, C, D are four arbitrary constants

The co-efficients of equation (3.28) do not contain any term containing odd powers of Γ, so that propagation in general will be of a reciprocal nature. Also from equation (3.2) the diagonal elements of the complex permittivity tensor are found to be equal if the value of B_0 is such that $(R_c B_0 \sigma)^2 << 1$. In the present analysis, the maximum value of $(R_c B_0 \sigma)^2$ is approximately 0.15 and therefore, the assumption $(R_c B_0 \sigma)^2 << 1$ is quite reasonable. This makes $\varepsilon_1 = \varepsilon_2$ and this assumption will be used to obtain the expressions for the field components. This simplifies the expressions of the field components and with this assumption, the expressions for m_1 and m_2 from equations (3.29) reduce to the following forms.

$$\left.\begin{aligned} m_1^2 &= \frac{n^2\pi^2}{a^2} - w^2\mu\varepsilon_2 - \Gamma^2 - \frac{w^2\mu\varepsilon_3^2}{2\varepsilon_2} + \frac{1}{2}\left\{\left(\frac{w^2\mu\varepsilon_3^2}{\varepsilon_2}\right)^2 - 4w^2\mu\frac{\varepsilon_3^2}{\varepsilon_2}\frac{n^2\pi^2}{a^2}\right\}^{\frac{1}{2}} \\ m_2^2 &= \frac{n^2\pi^2}{a^2} - w^2\mu\varepsilon_2 - \Gamma^2 - \frac{w^2\mu\varepsilon_3^2}{2\varepsilon_2} - \frac{1}{2}\left\{\left(\frac{w^2\mu\varepsilon_3^2}{\varepsilon_2}\right)^2 - 4w^2\mu\frac{\varepsilon_3^2}{\varepsilon_2}\frac{n^2\pi^2}{a^2}\right\}^{\frac{1}{2}} \end{aligned}\right\} \quad (3.32)$$

Let $a_3 = \frac{1}{2} w^2 \mu \frac{\varepsilon_3^2}{\varepsilon_2}$

$$b_3 = \frac{1}{2}\left[\left(\frac{w^2 \mu \varepsilon_3^2}{\varepsilon_2}\right)^2 - 4w^2 \mu \frac{\varepsilon_3^2}{\varepsilon_2} \frac{n^2 \pi^2}{a^2}\right]^{\frac{1}{2}}$$

$$\therefore \quad m_1^2 = \frac{n^2 \pi^2}{a^2} - w^2 \mu \varepsilon_2 - \Gamma^2 - a_3 + b_3$$

$$m_2^2 = \frac{n^2 \pi^2}{a^2} - w^2 \mu \varepsilon_2 - \Gamma^2 - a_3 - b_3$$

Now, substituting the expression of e(y) from (3.30) in equations (3.26) and solving for the expression of h(y) the following expression for h(y) is obtained

$$h(y) = \frac{\varepsilon_2}{jw\mu\varepsilon_3} \left(\frac{a}{n\pi}\right) \left\{A(a_3+b_3)e^{m_1 y} + B(a_3+b_3)e^{-m_1 y} + C(a_3-b_3)e^{m_2 y} + D(a_3-b_3)e^{-m_2 y}\right\}$$

Therefore, the expression for H_z is given by

$$H_z = \frac{\varepsilon_2}{jw\mu\varepsilon_3} \left(\frac{a}{n\pi}\right) \left\{A(a_3+b_3)e^{m_1 y} + B(a_3+b_3)e^{-m_1 y} + C(a_3-b_3)e^{m_2 y} + D(a_3-b_3)e^{-m_2 y}\right\} \cos \frac{n\pi x}{a}\, e^{(jwt-\Gamma z)}$$

Now substituting the expressions for E_z and H_z in equations (3.13), (3.14), (3.15) and (3.16) the expressions for E_x, E_y, H_x, H_y can be obtained. The expressions for all the six field components are written below.

$$E_x = \left(\frac{a}{n\pi}\right)\left[\frac{-\varepsilon_2}{\varepsilon_3(\Gamma^2+w^2\mu\varepsilon_2)}\right]\left[Ae^{m_1y}\left\{m_1(a_3+b_3)+\frac{\Gamma\epsilon_3}{\epsilon_2}\frac{n^2\pi^2}{a^2}\right\}\right.$$

$$+Be^{-m_1y}\left\{-m_1(a_3+b_3)+\Gamma\frac{n^2\pi^2}{a^2}\frac{\epsilon_3}{\epsilon_2}\right\}$$

$$+Ce^{m_2y}\left\{m_2(a_3-b_3)+\Gamma\frac{n^2\pi^2}{a^2}\frac{\epsilon_3}{\epsilon_2}\right\}$$

$$\left.+De^{-m_2y}\left\{-m_2(a_3-b_3)+\Gamma\frac{n^2\pi^2}{a^2}\frac{\epsilon_3}{\epsilon_2}\right\}\right]\cos\frac{n\pi x}{a}e^{(jwt-\Gamma z)} \quad (3.33)$$

$$E_y = \frac{1}{(\Gamma^2+w^2\mu\varepsilon_2)}\left[Ae^{m_1y}\left\{w^2\mu\varepsilon_3-\Gamma m_1-\frac{\epsilon_2}{\epsilon_3}(a_3+b_3)\right\}\right.$$

$$+Be^{-m_1y}\left\{w^2\mu\epsilon_3+\Gamma m_1-\frac{\epsilon_2}{\epsilon_3}(a_3+b_3)\right\} \quad (3.34)$$

$$+Ce^{m_2y}\left\{w^2\mu\epsilon_3-\Gamma m_2-\frac{\epsilon_2}{\epsilon_3}(a_3-b_3)\right\}$$

$$\left.+De^{-m_2y}\left\{w^2\mu\varepsilon_3+\Gamma m_2-\frac{\epsilon_2}{\epsilon_3}(a_3-b_3)\right\}\right]\sin\frac{n\pi x}{a}e^{(jwt-\Gamma z)}$$

$$E_z = (Ae^{m_1y}+Be^{-m_1y}+Ce^{m_2y}+De^{-m_2y})\sin\frac{n\pi x}{a}e^{(jwt-\Gamma z)} \quad (3.35)$$

$$H_x = \frac{\Gamma\varepsilon_2}{jw\mu\varepsilon_3(\Gamma^2+w^2\mu\varepsilon_2)}\left[Ae^{m_1y}\left\{-m_1\frac{w^2\mu\varepsilon_3}{\Gamma}-a_3+b_3\right\}\right.$$

$$+Be^{-m_1y}\left\{\frac{m_1w^2\mu\varepsilon_3}{\Gamma}-a_3+b_3\right\} \quad (3.36)$$

$$+Ce^{m_2y}\left\{-m_2\frac{w^2\mu\varepsilon_3}{\Gamma}-a_3-b_3\right\}$$

$$\left.+De^{-m_2y}\left\{m_2\frac{w^2\mu\varepsilon_3}{\Gamma}-a_3-b_3\right\}\right]\sin\frac{n\pi x}{a}e^{(jwt-\Gamma z)}$$

$$H_y = \left(\frac{a}{n\pi}\right)\left\{\frac{\Gamma\varepsilon_2}{jw\mu\varepsilon_3\,(\Gamma^2+w^2\mu\varepsilon_2)}\right\}\Bigg[Ae^{m_1y}\left\{\frac{w^2\mu\varepsilon_3}{\Gamma}\left(\frac{n^2\pi^2}{a^2}\right) - m_1(a_3+b_3)\right\}$$

$$+ Be^{-m_1y}\left\{\frac{w^2\mu\varepsilon_3}{\Gamma}\left(\frac{n^2\pi^2}{a^2}\right) + m_1(a_3+b_3)\right\} \tag{3.37}$$

$$+ Ce^{m_2y}\left\{\frac{w^2\mu\varepsilon_3}{\Gamma}\left(\frac{n^2\pi^2}{a^2}\right) - m_2(a_3-b_3)\right\}$$

$$+ De^{-m_2y}\left\{\frac{w^2\mu\varepsilon_3}{\Gamma}\left(\frac{n^2\pi^2}{a^2}\right) + m_2(a_3-b_3)\right\}\Bigg] \cos\frac{n\pi x}{a}\, e^{(jwt-\Gamma z)}$$

$$H_z = \frac{\varepsilon_2}{jw\mu\varepsilon_3}\left(\frac{a}{n\pi}\right)\Big[A(a_3+b_3)e^{m_1y} + B(a_3+b_3)e^{-m_1y}$$

$$+ C(a_3-b_3)e^{m_2y} + D(a_3-b_3)e^{-m_2y}\Big] \cos\frac{n\pi x}{a}\, e^{(jwt-\Gamma z)} \tag{3.38}$$

CHAPTER IV

THEORETICAL AND NUMERICAL SOLUTION FOR THE PROPAGATION CONSTANT

4.1 THEORETICAL SOLUTION

Since the expressions for all the six electric and magnetic field components have been derived in the previous chapter, a transcendental equation for the propagation constant can be obtained by implementing the four boundary conditions on these field components. The four boundary conditions in a rectangular waveguide completely filled with a semi-conductor for the electric field are given below.

$E_z = 0$ at $y = 0$, and also at $y = b$

$E_x = 0$ at $y = 0$, and also at $y = b$

Introducing the above four boundary conditions in equations (3.35) and (3.33) the following four linear homogeneous equations in A, B, C, and D are obtained.

$$[A + B + C + D] = 0$$

$$[Ae^{m_1 b} + Be^{-m_1 b} + Ce^{m_2 b} + De^{-m_2 b}] = 0$$

$$\Big[A\{m_1(a_3+b_3) + \frac{\Gamma\varepsilon_3}{\varepsilon_2}\frac{n^2\pi^2}{a^2}\} + B\{-m_1(a_3+b_3) + \Gamma\frac{n^2\pi^2}{a^2}\frac{\varepsilon_3}{\varepsilon_2}\}$$

$$+ C\{m_2(a_3-b_3) + \Gamma\frac{\varepsilon_3}{\varepsilon_2}\frac{n^2\pi^2}{a^2}\} + D\{-m_2(a_3-b_3) + \Gamma\frac{\varepsilon_3}{\varepsilon_2}\frac{n^2\pi^2}{a^2}\}\Big] = 0$$

$$\left[Ae^{m_1 b}\left\{ m_1(a_3+b_3) + \Gamma \frac{\varepsilon 3}{\varepsilon 2}\frac{n^2\pi^2}{a^2}\right\} + Be^{-m_1 b}\left\{ -m_1(a_3+b_3) + \Gamma \frac{\varepsilon 3}{\varepsilon 2}\frac{n^2\pi^2}{a^2}\right\} + Ce^{m_2 b}\left\{ m_2(a_3-b_3) + \Gamma \frac{\varepsilon 3}{\varepsilon 2}\frac{n^2\pi^2}{a^2}\right\} + De^{-m_2 b}\left\{ -m_2(a_3-b_3) + \Gamma \frac{\varepsilon 3}{\varepsilon 2}\frac{n^2\pi^2}{a^2}\right\}\right] = 0 \tag{4.1}$$

For the non-trivial case, the determinant of the co-efficients of the above set of equations must vanish.

Thus,

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ e^{m_1 b} & e^{-m_1 b} & e^{m_2 b} & e^{-m_2 b} \\ W_1 & W_2 & W_3 & W_4 \\ W_1 e^{m_1 b} & W_2 e^{-m_1 b} & W_3 e^{m_2 b} & W_4 e^{-m_2 b} \end{vmatrix} = 0 \tag{4.2}$$

where,

$$\left.\begin{aligned} W_1 &= m_1(a_3+b_3) + \Gamma \frac{\varepsilon_3}{\varepsilon_2}\frac{n^2\pi^2}{a^2} \\ W_2 &= -m_1(a_3+b_3) + \Gamma \frac{\varepsilon 3}{\varepsilon_2}\frac{n^2\pi^2}{a^2} \\ W_3 &= m_2(a_3-b_3) + \Gamma \frac{\varepsilon_3}{\varepsilon_2}\frac{n^2\pi^2}{a^2} \\ W_4 &= -m_2(a_3-b_3) + \Gamma \frac{\varepsilon 3}{\varepsilon 2}\frac{n^2\pi^2}{a^2} \end{aligned}\right\} \tag{4.2a}$$

The above determinantal equation (4.2) can also be written in the following form.

$$\begin{vmatrix} e^{-m_1 b} - e^{m_1 b} & e^{m_2 b} - e^{m_1 b} & e^{-m_2 b} - e^{m_1 b} \\ W_2 - W_1 & W_3 - W_1 & W_4 - W_1 \\ W_2 e^{-m_1 b} - W_1 e^{m_1 b} & W_3 e^{m_2 b} - W_1 e^{m_1 b} & W_4 e^{-m_2 b} - W_1 e^{m_1 b} \end{vmatrix} = 0 \qquad (4.3)$$

Expansion of the above determinantal equation leads to the following equation.

$$\Big[-2 \text{ Sinh } m_1 b \left\{- 2 W_3 W_4 \text{ Sinh } m_2 b + W_1 W_3 (e^{m_2 b} - e^{m_1 b}) + W_1 W_4 (e^{m_1 b} - e^{-m_2 b})\right\}$$
$$+ (e^{m_1 b} - e^{m_2 b}) \left\{- 2 W_1 W_2 \text{ Sinh } m_1 b + W_1 W_4 (e^{m_1 b} - e^{-m_2 b}) + W_4 W_2 (e^{-m_2 b} - e^{-m_1 b})\right\}$$
$$+ (e^{-m_2 b} - e^{m_1 b}) \left\{ W_2 W_3 (e^{m_2 b} - e^{-m_1 b}) - 2 W_1 W_2 \text{ Sinh } m_1 b + W_1 W_3 (e^{m_1 b} - e^{m_2 b})\right\}\Big]$$
$$= 0 \qquad (4.4)$$

which on simplifying after elaborate algebraic manipulations, reduces to the following form

$$4(W_1 W_2 + W_3 W_4) \text{ Sinh } m_1 b \text{ Sinh } m_2 b + 2(W_1 W_3 + W_4 W_2) \text{ Cosh}(m_1 b - m_2 b)$$
$$- 2(W_1 W_4 + W_2 W_3) \text{ Cosh}(m_1 b + m_2 b) - 2(W_1 W_3 - W_1 W_4 + W_2 W_4 - W_2 W_3) = 0$$

or,

$$2(W_1 W_2 + W_3 W_4) \text{ Sinh } m_1 b \text{ Sinh } m_2 b + (W_1 W_3 + W_2 W_4) \text{ Cosh } (m_1 b - m_2 b)$$
$$- (W_1 W_4 + W_2 W_3) \text{ Cosh}(m_1 b + m_2 b) + (W_1 W_4 + W_2 W_3) - (W_1 W_3 + W_2 W_4) = 0$$

or,

$$[2(W_1 W_2 + W_3 W_4) \text{ Sinh } m_1 b \text{ Sinh} m_2 b + (W_1 W_3 + W_2 W_4) \left\{\text{Cosh } m_1 b \text{ Cosh } m_2 b - \text{Sinh } m_1 b \text{ Sinh } m_2 b \right\}$$

$$- (W_1W_4 + W_2W_3)\{\text{Cosh } m_1b \text{ Cosh } m_2b + \text{Sinh } m_1b \text{ Sinh } m_2b\}$$

$$+ (W_1W_4 + W_2W_3) - (W_1W_3 + W_2W_4)\Big] = 0 \qquad (4.5)$$

or,

$$\Big[(\text{Sinh } m_1b \text{ Sinh } m_2b)\{2(W_1W_2 + W_3W_4) - (W_1W_3 + W_2W_4) - (W_1W_4 + W_2W_3)\}$$

$$+ (\text{Cosh } m_1b \text{ Cosh } m_2b)\{(W_1W_3 + W_2W_4) - (W_1W_4 + W_2W_3)\}$$

$$+ \{(W_1W_4 + W_2W_3) - (W_1W_3 + W_2W_4)\}\Big] = 0 \qquad (4.6)$$

However, from equation (4.2a),

$$\left.\begin{aligned}
(W_1W_2 + W_3W_4) &= 2\Gamma^2 \frac{\varepsilon_3^2}{\varepsilon_2^2} \frac{n^4\pi^4}{a^4} - m_1^2(a_3+b_3)^2 - m_2^2(a_3-b_3)^2 \\
(W_1W_3 + W_2W_4) &= 2m_1m_2(a_3^2 - b_3^2) + 2\Gamma^2 \frac{\varepsilon_3^2}{\varepsilon_2^2} \frac{n^4\pi^4}{a^4} \\
(W_1W_4 + W_2W_3) &= -2m_1m_2(a_3^2-b_3^2) + 2\Gamma^2 \frac{\varepsilon_3^2}{\varepsilon_2^2} \frac{n^4\pi^4}{a^4}
\end{aligned}\right\} \qquad (4.7)$$

Substitution of these expressions into the determinantal equation (4.5) gives the following equation,

$$(\text{Sinh } m_1b \text{ Sinh } m_2b)\left[-2\{m_1^2(a_3+b_3)^2 + m_2^2(a_3-b_3)^2\}\right]$$

$$+ (\text{Cosh } m_1b \text{ Cosh } m_2b)\{4m_1m_2(a_3^2-b_3^2)\}$$

$$+ \{-4m_1m_2(a_3^2-b_3^2)\} = 0 \qquad (4.8)$$

or,

$$- \{m_1^2(a_3+b_3)^2 + m_2^2(a_3-b_3)^2\}(\text{Sinh } m_1b \text{ Sinh } m_2b)$$

$$+ 2m_1m_2(a_3^2-b_3^2)\{\text{Cosh } m_1b \text{ Cosh } m_2b - 1\} = 0$$

or,

$$2m_1m_2(a_3^{\,2}-b_3^{\,2})\ (1-\text{Cosh}\ m_1b\ \text{Cosh}\ m_2b) = -\ \{m_1^{\,2}(a_3+b_3)^2 + m_2^{\,2}(a_3-b_3)^2\}$$
$$(\text{Sinh}\ m_1b\ \text{Sinh}\ m_2b) \qquad (4.9)$$

4.2 NUMERICAL SOLUTION

The solution of the transcendental equation (4.9) gives the required value of the propagation constant. The co-efficients B, C, and D can be evaluated in terms of A using equation (4.1) as follows,

$$\left.\begin{aligned} B &= A\ \frac{2m_1e^{m_1b} - (m_1+m_2)e^{m_2b} + (m_2-m_1)e^{-m_2b}}{\Delta} \\ C &= A\ \frac{(m_2-m_1)e^{m_1b} + (m_2+m_1)e^{-m_1b} - 2m_2e^{-m_2b}}{\Delta} \\ D &= A\ \frac{-(m_1+m_2)e^{m_1b} - (m_2-m_1)e^{-m_1b} + 2m_2e^{m_2b}}{\Delta} \\ \text{where,}\quad \Delta &= -\ 2m_1e^{-m_1b} + (m_1-m_2)e^{m_2b} + (m_2+m_1)e^{-m_2b} \end{aligned}\right\} \qquad (4.10)$$

Equation (4.9) is a transcendental equation in Γ and its solution gives the possible values of Γ. Knowing the value of Γ, the roots m_1 and m_2 can be evaluated from equation (3.32). As the coefficients B, C, and D have been obtained in terms of A, all the field components can be expressed in terms of the excitation represented by A. Since these expressions are very complicated and since solutions of equation (4.9) may be obtained only numerically, it is difficult to examine the characteristics of the field distribution in the general case.

4.2.1 THE PRACTICAL WAVEGUIDE SYSTEM

A numerical solution of the transcendental equation (4.9) has been obtained for a practical system as shown in fig. (4.a). A rectangular waveguide of internal dimensions a = .0228 meter and b = .01015 meter is completely filled with an n-type germanium sample with thickness d = .00188 meter. The calculations were made at a frequency of 9.46 GHz, the experimental frequency. The conductivity of the sample was varied from 4 mho/meter to 4.65 mho/meter and the magnetic field was varied from 0 to 10 K gauss, in steps of 1 K gauss. The other values of the parameters used in the calculations are

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ Farad/meter}, \ \varepsilon_r = 16.$$

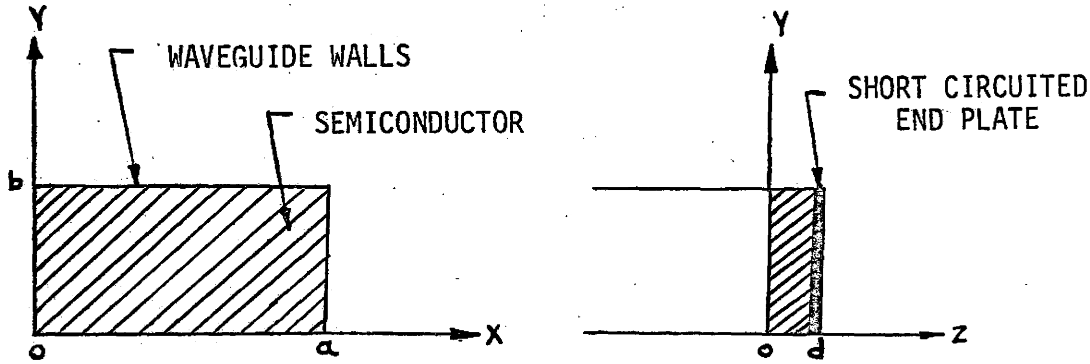
$$R_c\sigma = \mu H = .426 \ m^2/\text{volt-sec}$$ [29]

The solution of the transcendental equation gives the value of the propagation constant, the real and imaginary parts of which give the attenuation and phase constant respectively. The variation of the attenuation constant and the phase constant with the applied external magnetic field is given in Figure 4.b and 4.c.

4.2.2 NUMERICAL METHOD OF SOLUTION

The numerical solution of equation (4.9) was obtained with the Newton-Raphson iteration technique. To apply this technique the equation (4.9) may be put in the following form,

$$F(\Gamma) = 2m_1m_2(a_3{}^2-b_3{}^2)\ (1-\text{Cosh } m_1b \text{ Cosh } m_2b) + \{m_1{}^2(a_3+b_3)^2 + m_2{}^2(a_3-b_3)^2\}(\text{Sinh } m_1b \text{ Sinh } m_2b) = 0 \qquad (4.11)$$



frequency = 9.46 GHz

a = .0228 meter

b = .01015 meter

d = .00188 meter

σ = 4.50 mho/meter

$R_c\sigma$ = .426 m^2/v-sec.

ε_r = 16

n-type Germanium sample

FIGURE 4.a A PRACTICAL SYSTEM

where m_1 and m_2 are functions of Γ and their expressions are given in equation (3.32)

F(Γ) when expressed in terms of Γ becomes,

$$F(\Gamma) = \{(-\Gamma^2+G)^{\frac{1}{2}}(-\Gamma^2+H)^{\frac{1}{2}}\}\ \{1-\mathrm{Cosh}\ (-\Gamma^2+G)^{\frac{1}{2}}b\ \mathrm{Cosh}(-\Gamma^2+H)^{\frac{1}{2}}b\ \}$$

$$+ \{(-\Gamma^2+G)\ C_1 + (-\Gamma^2+H)C_2\}\ \ \mathrm{Sinh}\ (-\Gamma^2+G)^{\frac{1}{2}}b\ \mathrm{Sinh}\ (-\Gamma^2+H)^{\frac{1}{2}}b$$

$$= 0 \qquad (4.12)$$

where

$$G = \frac{n^2\pi^2}{a^2} - w^2\mu\varepsilon_1 = a_3+b_3$$

$$H = \frac{n^2\pi^2}{a^2} - w^2\mu\varepsilon_1 - a_3-b_3$$

$$C_1 = \frac{a_3+b_3}{2(a_3-b_3)} \qquad (4.13)$$

$$C_2 = \frac{a_3-b_3}{2(a_3+b_3)}$$

$$a_3 = \frac{1}{2}\, w^2\mu\, \frac{\varepsilon_3{}^2}{\varepsilon_1}$$

$$b_3 = \frac{1}{2}\ \left[(w^2\mu\, \frac{\varepsilon_3{}^2}{\varepsilon_1})^2 - 4w^2\mu\, \frac{\varepsilon_3{}^2}{\varepsilon_1}\, \frac{n^2\pi^2}{a^2}\right]^{\frac{1}{2}}$$

If Γ_i is an approximation to a root of a function F(Γ) = 0, a better approximation is given by

$$\Gamma_{i+1} = \Gamma_i - \frac{F(\Gamma_i)}{F'(\Gamma_i)} \qquad (4.14)$$

where the prime denotes the derivative. The expression for F'(Γ) is as follows.

$$F'(\Gamma) = \left[\left\{ \frac{(-\Gamma^2+G)(1-C_1) - (-\Gamma^2+H)C_2}{(-\Gamma^2+G)^{\frac{1}{2}}} \right\} \Gamma b \operatorname{Cosh}(-\Gamma^2+G)^{\frac{1}{2}}b \operatorname{Sinh}(-\Gamma^2+H)^{\frac{1}{2}}b \right.$$

$$+ \left\{ \frac{(-\Gamma^2+H)(1-C_2)-(-\Gamma^2+G)C_1}{(-\Gamma^2+H)^{\frac{1}{2}}} \right\} \Gamma b \operatorname{Sinh}(-\Gamma^2+G)^{\frac{1}{2}}b \operatorname{Cosh}(-\Gamma^2+H)^{\frac{1}{2}}b$$

$$+ \Gamma \left\{ \frac{(-\Gamma^2+G) + (-\Gamma^2+H)}{(-\Gamma^2+G)^{\frac{1}{2}}(-\Gamma^2+H)^{\frac{1}{2}}} \right\} \operatorname{Cosh}(-\Gamma^2+G)^{\frac{1}{2}}b \operatorname{Cosh}(-\Gamma^2+H)^{\frac{1}{2}}b$$

$$- 2\Gamma (C_1+C_2) \operatorname{Sinh}(-\Gamma^2+G)^{\frac{1}{2}}b \operatorname{Sinh}(-\Gamma^2+H)^{\frac{1}{2}}b$$

$$\left. -\Gamma \left\{ \frac{(-\Gamma^2+G) + (-\Gamma^2+H)}{(-\Gamma^2+G)^{\frac{1}{2}}(-\Gamma^2+H)^{\frac{1}{2}}} \right\} \right]$$

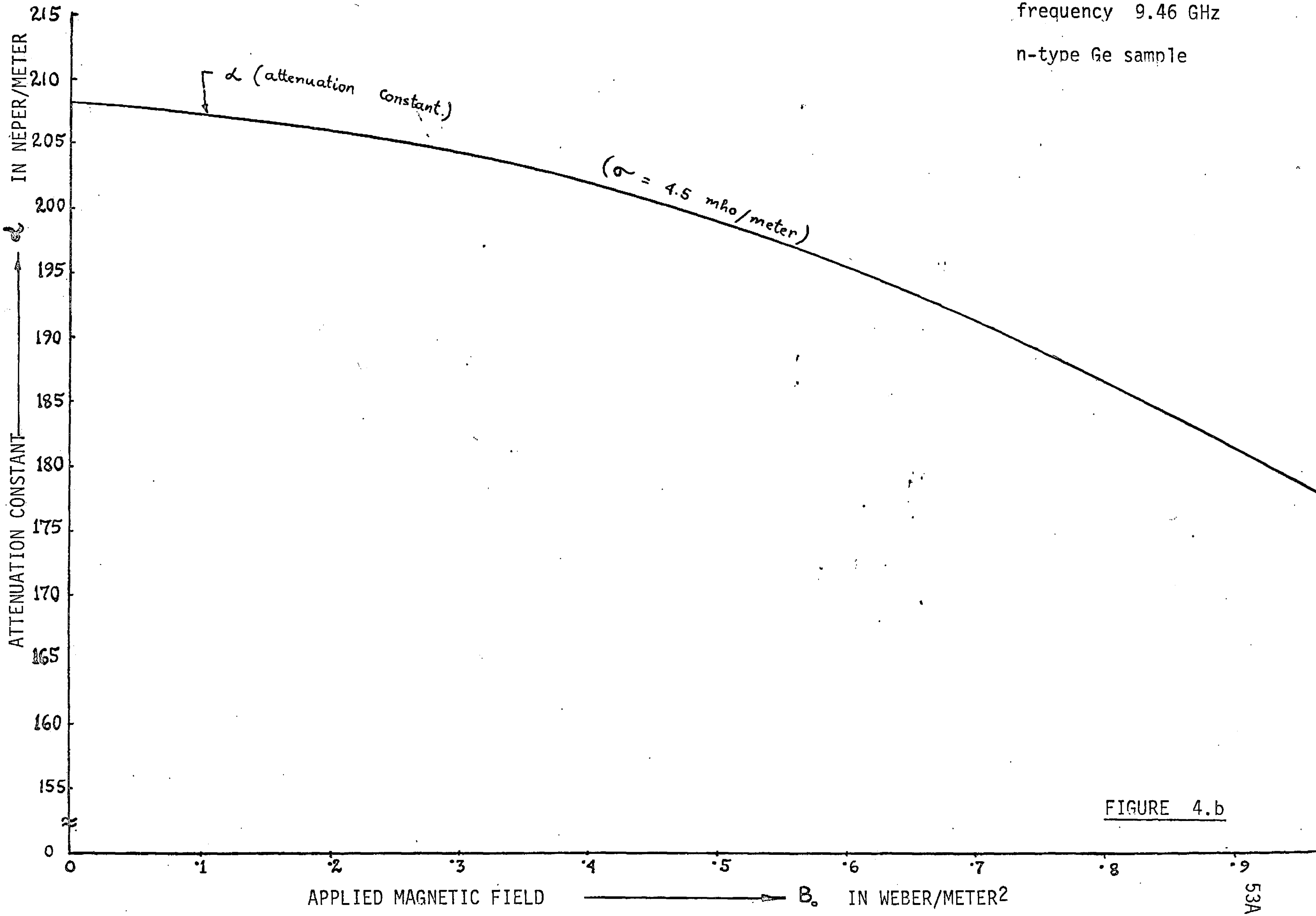
where the expressions for C_1, C_2, G, H are given in equation (4.13).

Using the relation (4.14), the numerical solution for the propagation constant has been made with an IBM 7040 computer and the computer programme for the solution of (4.9) is given at the end of the thesis.

4.2.3 NUMERICAL RESULTS.

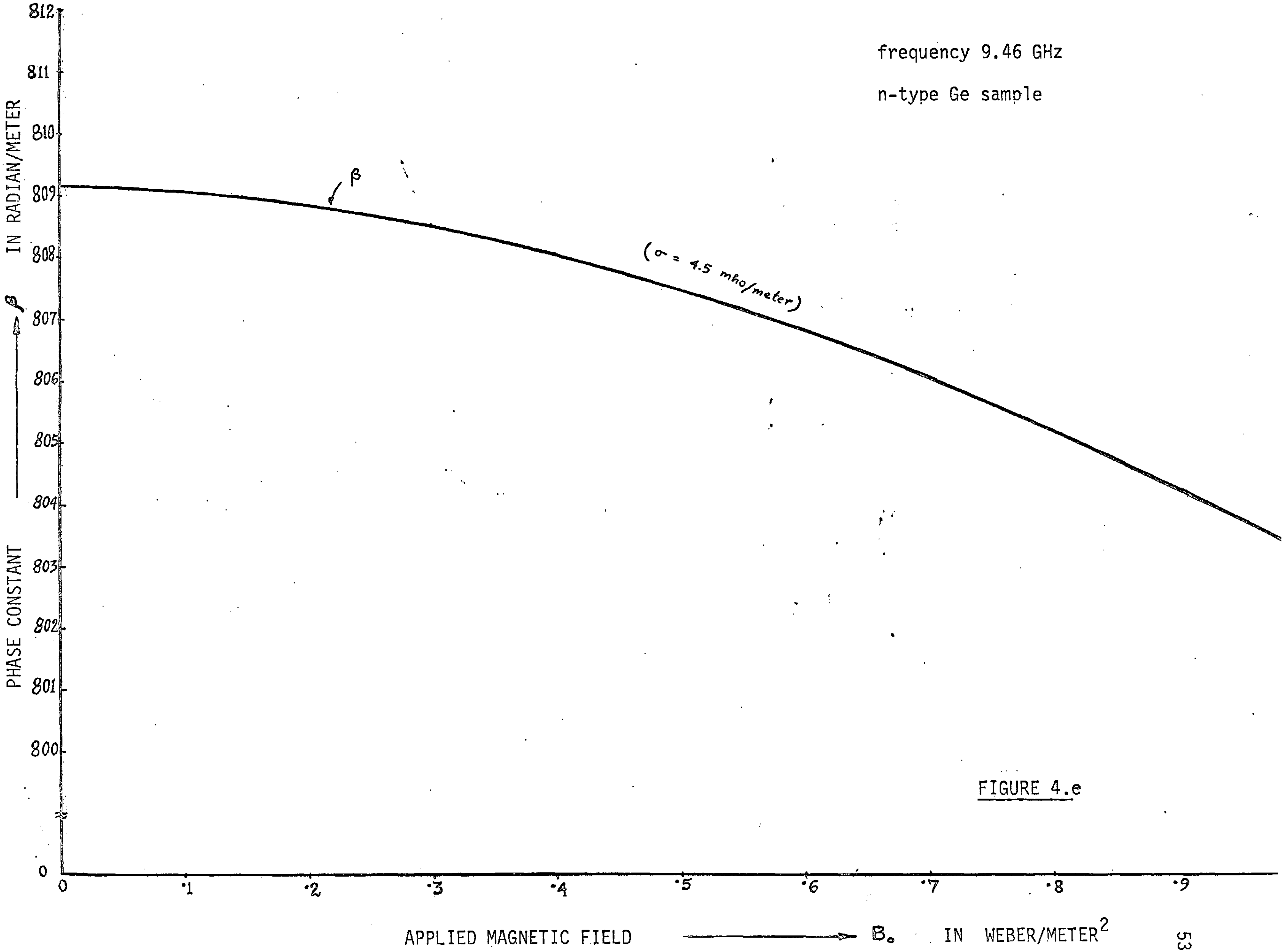
The variations of the attenuation constant and the phase constant with the variations of the external applied transverse magnetic field obtained from the calculations are shown in figure (4.b) and figure (4.c). The magnetic field was varied from 0 to 10 Kilogauss in steps of 1 Kgauss.

It was found that the phase constant decreases with increasing applied magnetic field. The attenuation constant also decreases with increasing magnetic field. The effect of the conductivity of the sample on the propagation constant was also calculated. The attenuation constant and the phase constant for different values of conductivity are also plotted in figure (4.d) and (4.e), as a function of the applied magnetic field.



frequency 9.46 GHz
n-type Ge sample
α (attenuation constant)
(σ = 4.5 mho/meter)
ATTENUATION CONSTANT α IN NEPER/METER
215
210
205
200
195
190
185
180
175
170
165
160
155
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
APPLIED MAGNETIC FIELD B₀ IN WEBER/METER2

FIGURE 4.b



frequency 9.46 GHz
n-type Ge sample
β
(σ = 4.5 mho/meter)
PHASE CONSTANT β IN RADIAN/METER
812
811
810
809
808
807
806
805
804
803
802
801
800
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
APPLIED MAGNETIC FIELD B₀ IN WEBER/METER²

FIGURE 4.e

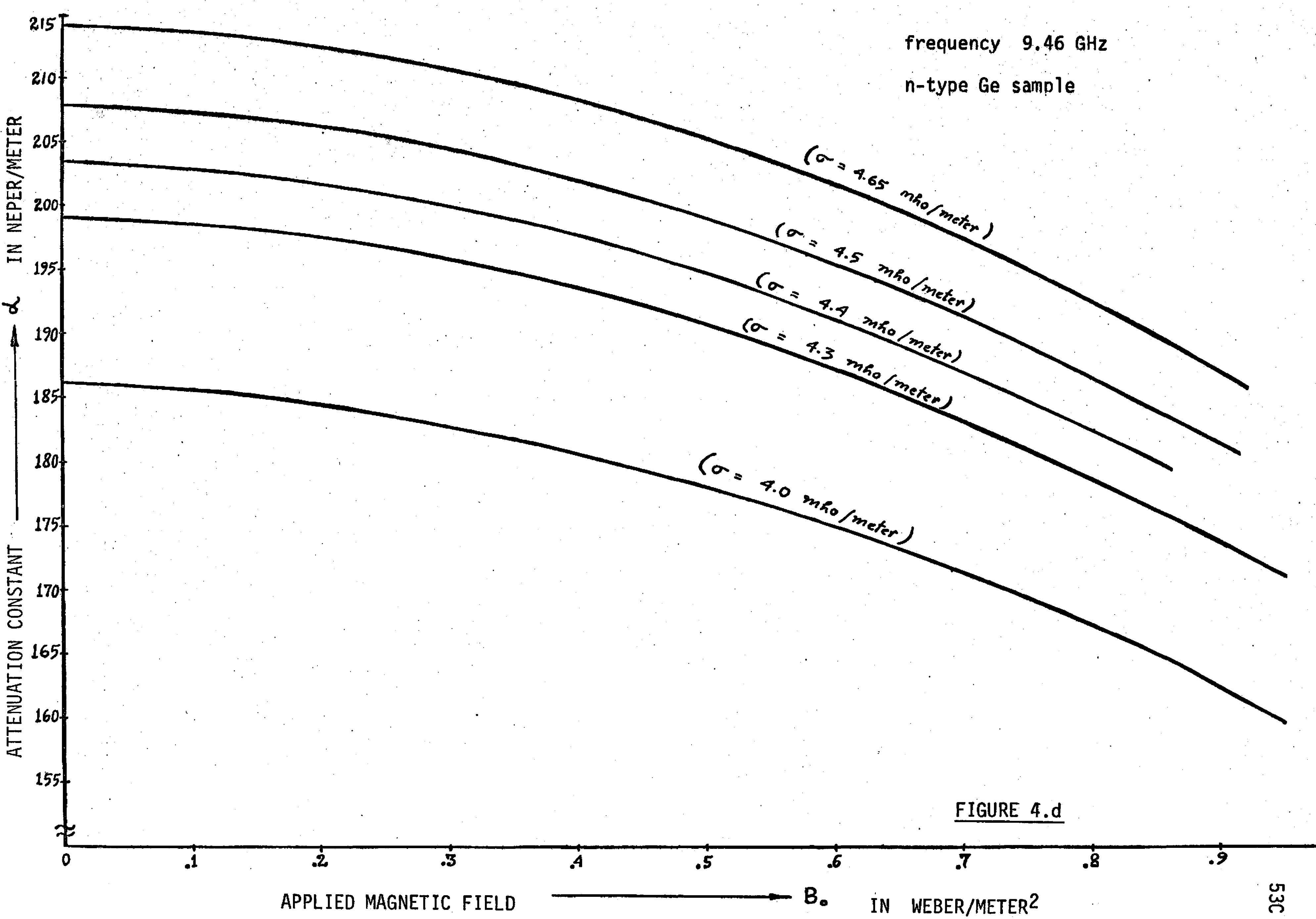

frequency 9.46 GHz
n-type Ge sample
(σ = 4.65 mho/meter)
(σ = 4.5 mho/meter)
(σ = 4.4 mho/meter)
(σ = 4.3 mho/meter)
(σ = 4.0 mho/meter)
ATTENUATION CONSTANT α IN NEPER/METER
215
210
205
200
195
190
185
180
175
170
165
160
155
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
APPLIED MAGNETIC FIELD B₀ IN WEBER/METER2

FIGURE 4.d

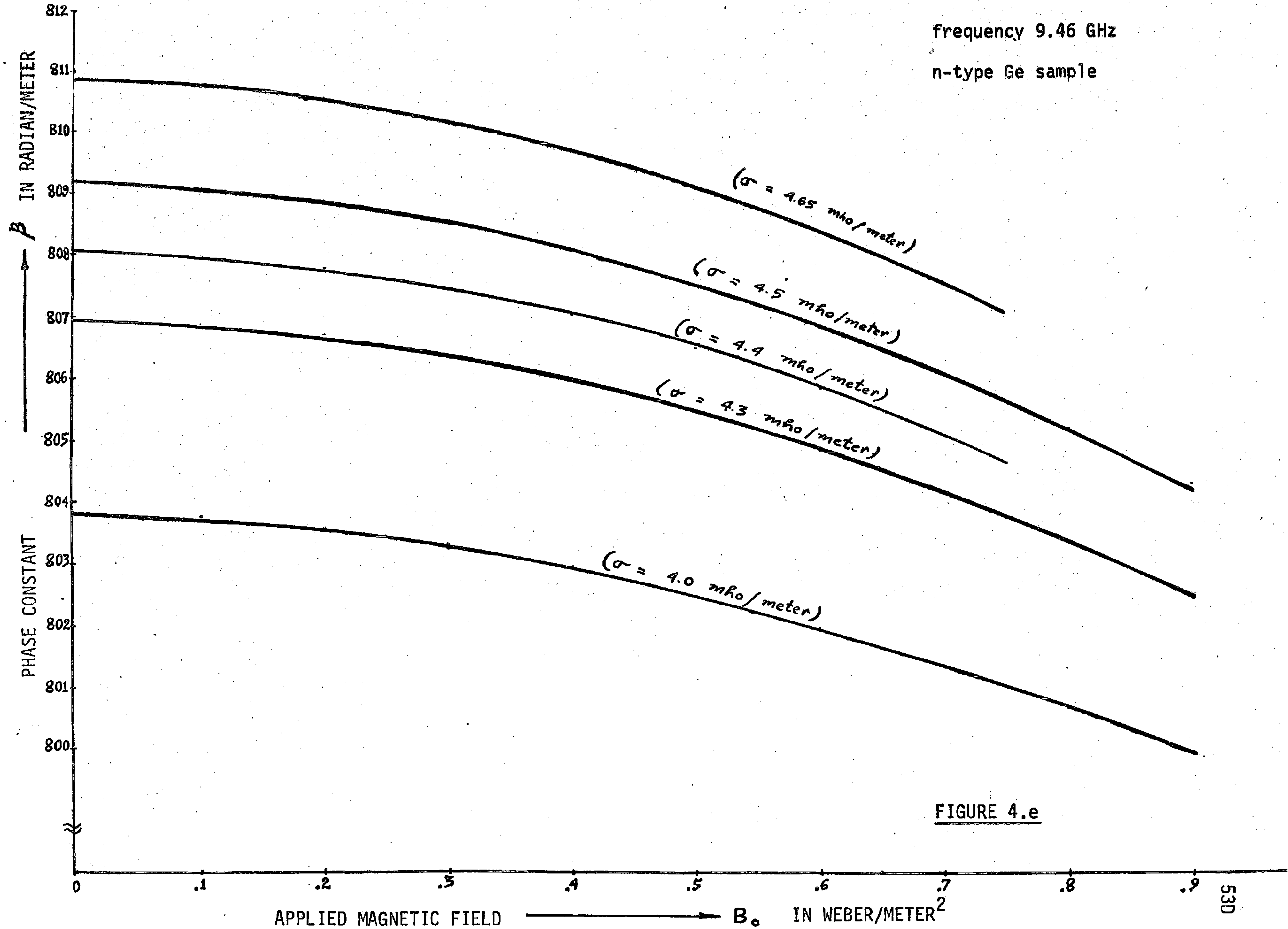

frequency 9.46 GHz
n-type Ge sample
PHASE CONSTANT → β IN RADIAN/METER
812
811
810
809
808
807
806
805
804
803
802
801
800
(σ = 4.65 mho/meter)
(σ = 4.5 mho/meter)
(σ = 4.4 mho/meter)
(σ = 4.3 mho/meter)
(σ = 4.0 mho/meter)
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
APPLIED MAGNETIC FIELD → B₀ IN WEBER/METER²

FIGURE 4.e

CHAPTER V

THEORY OF MEASUREMENT

5.1 THEORY OF THE REFLECTION BRIDGE

The schematic diagram of a high precision reflection bridge for directly measuring the magnitude and phase of reflection co efficient $|r|$ and θ respectively is shown in figure 5A. The sample is coupled to one side arm and the precision short and attenuator in the other are adjusted until the output is minimized.

The hybrid "tee" is converted into a magic "tee" by introducing slide screw tuners in the E and H arms and also in the side arms to compensate for asymmetry. The magic tee has the property that when the reflection co efficients at reference planes in the side arms are equal, the power fed to the detector in the E arm is zero.

In a practical system it is required to find the reflection co efficient r_x, at a particular measuring plane of the colinear arm of the hybrid tee which contains the semiconductor sample. This reflection co efficient $r_x = r_2 e^{2\gamma_s l_s}$ (5.1)

where r_2 = reflection co efficient at port 2

γ_s = propagation constant in the sample arm

l_s = distance between port 2 and the measuring plane

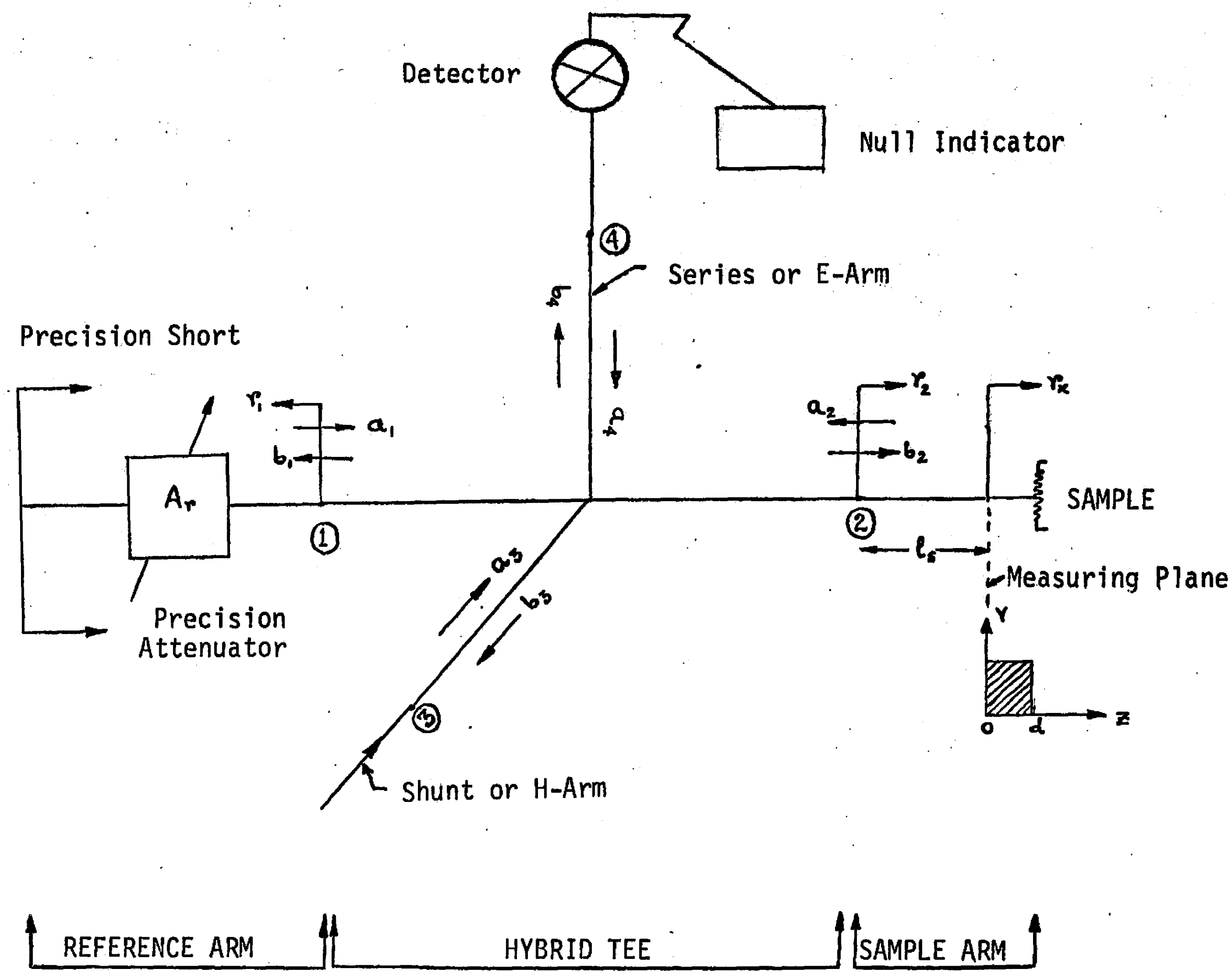


FIGURE 5A A MICROWAVE REFLECTION BRIDGE

5.1.1 DERIVATION OF BALANCE EQUATION"

In general the input and output relations of any passive four port (the hybrid tee) can be described by the equation

$$\underline{b} = \underline{s}\ \underline{a} \qquad (5.2)$$

where $\underline{a}$ and $\underline{b}$ are column matrices representing incident and scattered waves respectively and $\underline{s}$ is the scattering matrix given by

$$\underline{s} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{12} & s_{22} & s_{23} & s_{24} \\ s_{13} & s_{23} & s_{33} & s_{34} \\ s_{14} & s_{24} & s_{34} & s_{44} \end{bmatrix}$$

Because of reciprocity, s is symmetrical.

$$\left.\begin{aligned} &\text{The reflection co efficients } r_1 = \frac{a_1}{b_1} \\ &\text{and } r_2 = \frac{a_2}{b_2} \\ &\text{The null condition is } a_4 = b_4 = 0 \end{aligned}\right\} \qquad (5.3)$$

Using equations (5.2) and (5.3) the following sets of equations are obtained.

$$\left.\begin{aligned} b_1 &= s_{11}\, r_1\, b_1 + s_{12}\, r_2\, b_2 + s_{13}\, a_3 \\ b_2 &= s_{12}\, r_1\, b_1 + s_{22}\, r_2\, b_2 + s_{23}\, a_3 \\ b_3 &= s_{13}\, r_1\, b_1 + s_{23}\, r_2\, b_2 + s_{33}\, a_3 \\ b_4 &= s_{14}\, r_1\, b_1 + s_{24}\, r_2\, b_2 + s_{34}\, a_3 \end{aligned}\right\} \qquad (5.4)$$

The above four equations can be written in the unknowns b_1, b_2, b_3, a_3 as

$$
\begin{aligned}
(s_{11} r_1 - 1) b_1 + s_{12} r_2 b_2 + s_{13} a_3 &= 0 \\
s_{12} r_1 b_1 + (s_{22} r_2 - 1) b_2 + s_{23} a_3 &= 0 \\
s_{13} r_1 b_1 + s_{23} r_2 b_2 + s_{33} a_3 - b_3 &= 0 \\
s_{14} r_1 b_1 + s_{24} r_2 b_2 + s_{34} a_3 &= 0
\end{aligned}
$$

The first, second and the fourth equation form the homogeneous set

$$
\begin{aligned}
(s_{11} r_1 - 1) b_1 + s_{12} r_2 b_2 + s_{13} a &= 0 \\
s_{12} r_1 b_1 + (s_{22} r_2 - 1) b_2 + s_{23} a_3 &= 0 \\
s_{14} r_1 b_1 + s_{24} r_2 b_2 + s_{34} a_3 &= 0
\end{aligned}
$$

which has a unique solution if the determinant of the co-efficients is zero. Equating the determinant to zero gives

$$
\begin{vmatrix}
(s_{11} r_1 - 1) & s_{12} r_2 & s_{13} \\
s_{12} r_1 & (s_{22} r_2 - 1) & s_{23} \\
s_{14} r_1 & s_{24} r_2 & s_{34}
\end{vmatrix} = 0 \qquad (5.5)
$$

and after some algebraic manipulation of this determinantal equation, the relationships between r_1 and r_2 can be obtained and is given below

$$
r_2 = \frac{\left[\dfrac{s_{11} s_{34} - s_{13} s_{14}}{s_{22} s_{34} - s_{23} s_{24}}\right] r_1 - \left[\dfrac{s_{34}}{s_{22} s_{34} - s_{23} s_{24}}\right]}{\left[\dfrac{(s_{11}s_{22} - s_{12}^2)s_{34} + (s_{12}s_{13} - s_{11}s_{23})s_{24} + (s_{12}s_{23} - s_{13}s_{22})s_{14}}{s_{22}s_{34} - s_{23}s_{24}}\right] r_1 - 1}
$$

But $r_x = r_2 \, e^{2\gamma_s l_s}$

Therefore,

$$r_x = \frac{a\, r_1 - b}{c\, r_1 - 1} \tag{5.6}$$

where,

$$a = \left[\frac{s_{11}s_{34} - s_{13}s_{14}}{s_{22}s_{34} - s_{23}s_{24}}\right] e^{2\gamma_s l_s}$$

$$b = \left[\frac{s_{34}}{s_{22}s_{34} - s_{23}s_{24}}\right] e^{2\gamma_s l_s} \tag{5.6a}$$

$$c = \left[\frac{(s_{11}s_{22} - s_{12}^2)s_{34} + (s_{12}s_{13} - s_{11}s_{23})s_{24} + (s_{12}s_{23} - s_{13}s_{22})s_{14}}{s_{22}s_{34} - s_{23}s_{24}}\right]$$

The reflection co efficient r_1 of the reference arm has been shown by Champlin,K. S. and et. al.(10) to be equal to

$$r_1 = K\, e^{-2(A_r + j\, \beta_r\, l_r)} \tag{5.7}$$

where,

$K = K_A^2\, K_S$,

K_A = complex constant containing the residual attenuation and (fixed) phase shift of the attenuator

K_S = complex constant containing the zero setting and (fixed) loss of the short circuit.

A_r = reading of the precision attenuator in nepers

$\beta_r l_r$ = reading of the precision short in radians (measured from some arbitrary reference)

Substituting the above equation (5.7) in the expression for r_x in equation (5.6) gives the following equation

$$r_x = \frac{A\ e^{-2[A_r + j\beta_r l_r]} - B}{C\ e^{-2[A_r + j\beta_r l_r]} - 1} \tag{5.8}$$

where $A = aK$, $B = b$, $C = cK$.

Thus, under very general conditions the bridge is completely described by the three co-efficients A, B, C. Determination of these co efficients by means of preliminary measurements constitutes calibration of the bridge.

5.1.2 BRIDGE CALIBRATION

(i) The Matched and Symmetric Bridge: [27]

When the bridge is matched and compensated for asymmetry, the following symmetry conditions are satisfied,

$$s_{11} = s_{22}$$

$$s_{13} = s_{23}$$

$$s_{14} = -s_{24}$$

From these conditions and from the fact that the scattering matrix of the hybrid tee is unitary, one can show that the shunt&series arms are completely isolated.

i.e. $s_{34} = s_{43} = 0$

Substituting these conditions in the expressions for b and c in equation (5.6a), it is found that

$$B = 0$$

$$C = 0$$

Therefore, equation (5.8) reduces to

$$\gamma_x = - A\, e^{-2[A_r + J\beta_r l_r]} \tag{5.9}$$

The reflection co efficient γ_x at the air-semiconductor interface is shown in fig. 5A.

The coefficient A can be determined experimentally by balancing the bridge with a fixed short circuit terminating the sample arm. When a precision short cirucit is placed at d_1,

$$\gamma_{s1} = \gamma_x\, e^{2\gamma_s d_1}$$

$$\text{or} \quad -1 = \gamma_x\, e^{2\gamma_s d_1}$$

$$\text{or} \quad \gamma_x = - e^{-2\gamma_s d_1}$$

From equation (5.9)

$$-e^{-2\gamma_s d_1} = - A\, e^{-2[A_r(s_1) + J\beta_r l_r(s_1)]}$$

where $A_r(s_1)$ = attenuator reading with a fixed short

$l_r(s_1)$ = precision short reading with a fixed short

Therefore,

$$A = \frac{e^{-2\gamma_s d_1}}{e^{-2[A_r(s_1) + J\beta_r l_r(s_1)]}}$$

Hence from equation (5.9)

$$\gamma_x = - e^{-2[\{A_r - A_r(s_1)\} + J\beta_r\{l_r - l_r(s_1) + d_1\}]}$$

$$= - e^{-(A + J\emptyset)} \tag{5.10}$$

where $A = 2\{A_r - A_r(s_1)\}$ nepers

$$\phi = 2\beta_r \{l_r - l_r(s_1) + d_1\} \text{ radians} \qquad (5.10a)$$

Equation (5.10) has been used for the practical measurement of the magnitude and phase of reflection co efficient at the air-semiconductor interface.

(ii) The Matched and Unsymmetric Bridge:

When the input ports of hybrid tee are matched and the bridge is not compensated for asymmetry, then

$$s_{11} = s_{22} = 0$$

$$s_{12} = s_{21} = 0 \text{ at a single frequency}$$

and from equation (5.6a), $c = 0$.

Therefore,

$$\gamma_X = - A\, e^{-2(A_r + j\beta_r l_r)} + B \qquad (5.11)$$

With a precision short circuit termination at the sample arm,

$$-e^{-2\gamma_s d_1} = - A\, e^{-2[A_r(s_1) + j\beta_r l_r(s_1)]} + B \qquad (5.12)$$

With a matched termination Z_o at the sample arm

$$0 = - A\, e^{-2[A_r(Z_o) + j\beta_r l_r(Z_o)]} + B \qquad (5.13)$$

Solving equations (5.12) and (5.13) for A and B, and substituting in equation (5.11), the expression for γ_X becomes

$$\gamma_X = - \frac{e^{-2\gamma_s d_1}}{\left[1 - \frac{\gamma_1(Z_o)}{\gamma_1(s_1)}\right]} \left[e^{-2[A_r - A_r(s_1)] + j\beta_r[l_r - l_r(s_1)]} - \frac{\gamma_1(Z_o)}{\gamma_1(s_1)} \right]$$

where $$\frac{\gamma_1(Z_o)}{\gamma_1(s_1)} = \frac{e^{-2[A_r(Z_o)+J\beta_r l_r(Z_o)]}}{e^{-2[A_r(s_1)+J\beta_r l_r(s_1)]}}$$

The right hand side of the above equation contains known quantities and thus γ_x can be evaluated.

(iii) The General Bridge:

For a general bridge, equation (5.8) gives the expression for the reflection co efficient which is completely described by the three co efficients A, B, and C. These co efficients A, B, and C can be determined by calibrating the bridge with two shorts at two different distances d_1, d_2 from measuring plane and with one matched termination Z_o at the sample arm.

5.2 THEORETICAL CALCULATION OF THE REFLECTION CO EFFICIENT

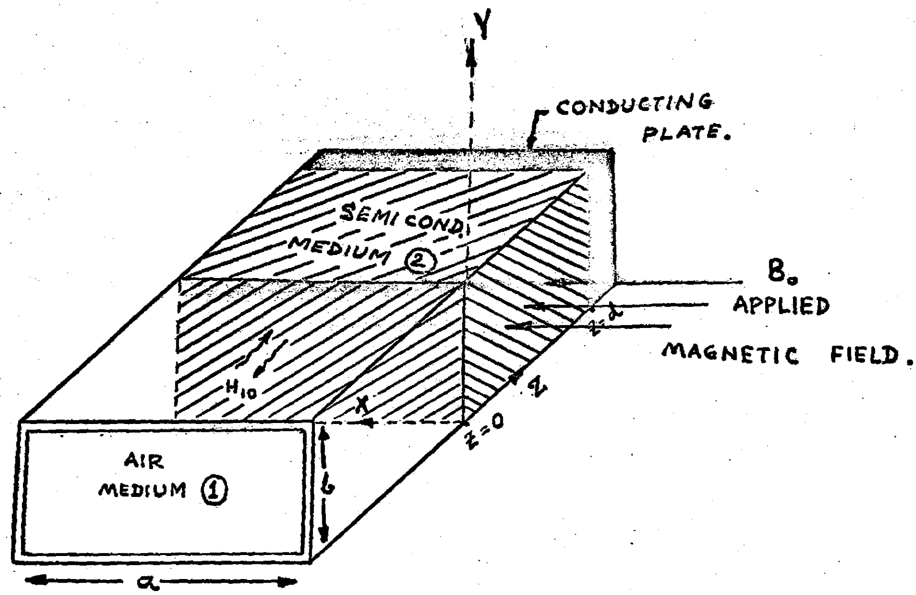
5.2.1 GENERAL SOLUTION.

An expression is derived for the reflection co efficient at the air-semiconductor interface of the rectangular wave guide system shown in figure 5.2A.

The transverse field components E_y and H_x in air (medium 1) at a distance Z from the air-semiconductor interface is given by

$$E_{y_1} = A_{i_1} e^{-\Gamma_1 Z} + A_{r_1} e^{\Gamma_1 Z}$$

$$= Ai_1 \left[e^{-\Gamma_1 Z} + \frac{A_{r_1}}{A_{i_1}} e^{\Gamma_1 Z} \right] \quad (5.2a)$$



n-type Ge sample

a = .0228 meter

b = .01015 meter

d = .00188 meter

frequency = 9.46 GHz

FIGURE 5.2A

Semiconductor in a waveguide

Where Γ_1 is the propagation constant in the air filled section of the waveguide i.e. in medium (1). A_{i_1} and A_{r_1} are the amplitudes of the incident and reflected waves at the air-semiconductor interface i.e. at the plane $Z = 0$.

The reflection coefficient is given by the ratio

$$\frac{A_{r_1}}{A_{i_1}} = \gamma \equiv e^{-2\emptyset} = |\gamma|\, e^{J\theta} \tag{5.2b}$$

where $\emptyset = u + j\, v$

which characterizes the terminating waveguide section containing the semiconducting medium 2.

Therefore, the equation (5.2a) can be written as

$$E_{y_1} = A_{i_1}\ [\, e^{-\Gamma_1 Z} + \gamma\, e^{\Gamma_1 Z}\,] \tag{5.2c}$$

$$H_{x_1} = H_{i_1}\, e^{-\Gamma_1 Z} + H_{r_1}\, e^{\Gamma_1 Z}$$

$$= \frac{A_{i_1}}{Z_1}\, e^{-\Gamma_1 Z} - \frac{A_{r_1}}{Z_1}\, e^{\Gamma_1 Z}$$

$$= \frac{A_{i_1}}{Z_1}\ [e^{-\Gamma_1 Z} - \gamma\, e^{\Gamma_1 z}] \tag{5.2d}$$

where $$Z_1 = \frac{-jw_\mu}{\Gamma_1} \tag{5.2e}$$

The wave impedance Z(o) given by the ratio of the transverse electric and magnetic field intensities at the air-semiconductor interface in medium 1, i.e.

$$Z(o) = \frac{E_{y_1}(o)}{H_{x_1}(o)} = Z_1\ \left(\frac{1+\gamma}{1-\gamma}\right) = Z_1 \frac{1+e^{-2\emptyset}}{1-e^{-2\emptyset}} = Z_1\ \mathrm{Coth}\emptyset \tag{5.2f}$$

In the semiconductor region, that is in medium (2)

$$E_{y2} = A_{i_2} e^{-\Gamma_2 Z} + A_{r_2} e^{\Gamma_2 Z}$$

$$= A_{i_2} [e^{-\Gamma_2 Z} + \frac{Ar_2}{A_{i_2}} e^{\Gamma_2 Z}] \qquad (5.2g)$$

$$H_{x2} = H_{i_2} e^{-\Gamma_2 Z} + H_{r_2} e^{\Gamma_2 Z}$$

$$= \frac{A_{i2}}{Z_2^+} e^{-\Gamma_2 Z} + \frac{Ar_2}{Z_2^-} e^{\Gamma_2 Z}$$

$$= \frac{A_{i2}}{Z_2^+} [e^{-\Gamma_2 Z} + \frac{A_{r2}}{A_{i_2}} \frac{Z_2^+}{Z_2^-} e^{\Gamma_2 Z}] \qquad (5.2h)$$

where $Z_2^+ = \frac{A_{i2}}{H_{i2}}$ = characteristic impedance for the +vely travelling wave

$Z_2^- = \frac{A_{r2}}{H_{r2}}$ = characteristic impedance for the -vely travelling wave

Γ_2 = propagation constant in medium (2).

At Z=d the waveguide is terminated by a metal plate. Introducing this boundary condition equation (5.2g) becomes

$$0 = A_{i_2} [e^{-\Gamma_2 d} + \frac{A_{r2}}{A_{i_2}} e^{\Gamma_2 d}]$$

or, $\frac{A_{r2}}{A_{i2}} = -e^{-2\Gamma_2 d}$

Therefore, the wave impedance at the air-semiconductor interface is given by

$$Z(o) = \frac{E_{y_2}(o)}{H_{x_2}(o)} = Z_2^{+} \frac{(1 + \frac{A_{r_2}}{A_{i_2}} e^{\Gamma_2 Z})}{(1 + \frac{A_{r_2}}{A_{i_2}} \frac{Z_2+}{Z_2-})}$$

$$= Z_2^{+} \frac{(1 - e^{-2\Gamma_2 d})}{(1 - e^{-2\Gamma_2 d} \frac{Z_2+}{Z_2-})} \tag{5.2i}$$

5.2.2 FIRST ORDER APPROXIMATE SOLUTION OF THE REFLECTION CO EFFICIENT.

With the application of the external magnetic field, the medium (2) shown in figure 5.2A has a complex tensor permittivity and in the analysis of the field components in Chapter 3 it has been shown that the medium (2) has all the six field components. However, to simplify the analysis only the E_y and H_x components of the electromagnetic wave will be considered here and a first order expansion of Z_2+, the characteristic impedance for the wave travelling in the +ve direction will be taken into consideration. It is seen that $Z_2+ \neq \frac{-jw\mu}{\Gamma 2}$, but

$$Z_2+ = \frac{-jw_\mu}{\Gamma_2} \left[1 - \frac{\varepsilon_3}{\varepsilon 2} (\Gamma_2{}^2 + w^2 \mu \varepsilon_2).F^+\right] \tag{5.2j}$$

where,

$$F^+ = \frac{\{m_1 A e^{m_1 y} - m_1 B e^{-m_1 y} + m_2 C e^{m_2 y} - m_2 D e^{-m_2 y}\}}{\left[\{\Gamma 2(a_3 - b_3) + w^2 \mu \varepsilon_3 m_1\} A e^{m_1 y} + \{\Gamma_2(a_3 - b_3) - w^2 \mu \varepsilon_3 m_1\} B e^{-m_1 y} + \{\Gamma_2(a_3 + b_3) + w^2 \mu \varepsilon_3 m_2\} C e^{m_2 y} + \{\Gamma_2(a_3 + b_3) - w^2 \mu \varepsilon_3 m_2\} D e^{-m_2 y}\right]}$$

where, the co efficients A, B, C and D are expressed in terms of A in equation (4.10)

Expanding F^+ into the Maclaurin's Series and taking only the first term

$$F^+ = F^+(0) + y\frac{F'^+(0)}{1!} + \frac{y^2F''^+(0)}{2!} + \ldots$$

Then, $F^+ \approx F^+(0)$

where $F^+(0) = \dfrac{m_1(A-B) + m_2(C-D)}{r_2(m_1{}^2-m_2{}^2)\ (C+D) + w^2\mu\varepsilon_3\{m_1(A-B)+m_2(C-D)\}}$

Similarly, for the wave travelling in the -ve z-direction

$$Z_2^- = \frac{E_{y_2^-}}{H_{x_2^-}}$$

and $Z_{\bar{2}} = \dfrac{jw\mu}{\Gamma_2}\{\ 1 - \dfrac{\varepsilon_3}{\varepsilon_2}(r_2{}^2 + w^2\mu\varepsilon_2)\ F^-\ \}$ (5.2k)

where,

$$F^- = \frac{\{m_1Ae^{m_1y} - m_1Be^{-m_1y} + m_2Ce^{m_2y} - m_2De^{-m_2y}\}}{\left[\{-r_2(a_3-b_3)+w^2\mu\varepsilon_3m_1\}Ae^{m_1y} + \{-r_2(a_3-b_3)-w^2\mu\varepsilon_3m_1\}Be^{-m_1y} + \{-r_2(a_3+b_3)+w^2\mu\varepsilon_3m_2\}Ce^{m_2y} + \{-r_2(a_3+b_3)-w^2\mu\varepsilon_3m_2\}De^{-m_2y}\right]}$$

Considering the first term of the series expansion of F^-, it is found that

$$F^- \approx F^-(0) = \frac{m_1(A-B) + m_2(C-D)}{-r_2(m_1{}^2-m_2{}^2)(C+D) + w^2\mu\varepsilon_3\{m_1(A-B)+m_2(C-D)\}}$$

where the co efficients A, B, C and D are expressed in terms of A in equations (4.10)

Equating equations (5.2i) and (5.2f) for Z(0) in the two regions, gives

$$Z_1 \text{ Coth } \emptyset = \frac{Z_2^+ (1 - e^{-2\Gamma_2 d})}{(1 - e^{-2\Gamma_2 d} \frac{Z_2^+}{Z_2^-})} \tag{5.2l}$$

But from equations (5.2j) and (5.2k)

$$\frac{Z_2^+}{Z_2^-} = - \frac{[1 - \frac{\varepsilon 3}{\varepsilon 2} (\Gamma_2{}^2 + w^2 \mu \varepsilon_2) F^+]}{[1 - \frac{\epsilon 3}{\epsilon 2} (\Gamma_2{}^2 + w^2 \mu \varepsilon_2) F^-]} \cong - \frac{K_1}{K_2}$$

where $K_1 = [1 - \frac{\varepsilon 3}{\varepsilon 2} (\Gamma_2{}^2 + w^2 \mu \varepsilon_2) . F^+(0)]$

$K_2 = [1 - \frac{\varepsilon 3}{\varepsilon 2} (\Gamma_2{}^2 + w^2 \mu \varepsilon_2) \; F^-(0)]$

From equation (5.2l)

$$\text{Coth } \emptyset = \frac{\frac{-jw\mu}{\Gamma_2} K_1 (1 - e^{-2\Gamma_2 d})}{\frac{-jw\mu}{\Gamma_1} (1 + \frac{K_1}{K_2} e^{-2\Gamma_2 d})}$$

$$= \frac{\Gamma_1 K_1 (1 - e^{-2\Gamma_2 d})}{\Gamma_2 (1 + \frac{K_1}{K_2} e^{-2\Gamma_2 d})}$$

or, $$\frac{1+\gamma}{1-\gamma} = \frac{j\beta_1 K_1 (1 - e^{-2\Gamma_2 d})}{\Gamma_2 (1 + \frac{K_1}{K_2} e^{-2\Gamma_2 d}}$$

or $$\gamma \equiv |\gamma| e^{j\theta} = \frac{j\beta_1 K_1 (1 - e^{-2\Gamma_2 d}) - \Gamma_2 (1 + \frac{K_1}{K_2} e^{-2\Gamma_2 d})}{j\beta_1 K_1 (1 - e^{-2\Gamma_2 d}) + \Gamma_2 (1 + \frac{K_1}{K_2} e^{-2\Gamma_2 d})} \tag{5.2m}$$

This approximate equation is plotted in figure (7.2A&B) as a function of applied magnetic field.

5.2.3 SIMPLE TE-MODE SOLUTION.

For the TE mode $Z_2^- = -Z_2 = \frac{jw\mu}{\Gamma_2}$ and equation (5.2i) becomes

$$Z(0) = \frac{Z_2^+ (1-e^{-2\Gamma_2 d})}{(1+e^{-2\Gamma_2 d})} = Z_2^+ \tanh \Gamma_2 d \qquad (5.2n)$$

Equating equations (5.2n) and (5.2f) gives

$$Z_1 \operatorname{Coth} \emptyset = Z_2^+ \tanh \Gamma_2 d$$

or

$$\operatorname{Coth} \emptyset = \frac{Z_2^+}{Z_1} \tanh \Gamma_2 d$$

$$= \frac{-jw\mu}{\Gamma_2} \times \frac{\Gamma_1}{-jw\mu} \tanh \Gamma_2 d \qquad (5.2o)$$

$$= \frac{\Gamma_1}{\Gamma_2} \tanh \Gamma_2 d$$

Normally, the attenuation in the air filled part of the guide can be neglected and the propagation constant reduces to $\Gamma_1 = j\beta_1$ where β_1 is the phase constant.

Therefore, the equation (5.2o) can be written as

$$\frac{1+\gamma}{1-\gamma} = \frac{j\beta_1 \tanh \Gamma_2 d}{\Gamma_2}$$

or

$$\gamma \equiv |\gamma| e^{j\theta} = \frac{j\beta_1 \tanh \Gamma_2 d - \Gamma_2}{j\beta_1 \tanh \Gamma_2 d + \Gamma_2} \qquad (5.2p)$$

Since the propagation constant in medium (2), Γ_2, corresponding to the different values of the applied magnetic field is known (figure 4(d), 4(e)) and also since β_1 and d are known quantities, the right hand side of the equation (5.21) is a known quantity. Calculated values of $\gamma = |\gamma|e^{j\theta}$ for the system of figure (5.2a) for different values of the applied magnetic field are given in figure 7.2(A), 7.2(B).

CHAPTER VI

EXPERIMENTAL PROCEDURES

6.1 PREPARATION OF THE SAMPLES

The samples of germanium were cut from a large block of 22.2 ohm-cm n-type Ge crystal <111> axial orientation of trapezoidal shape, by means of a diamond wheel cutter 0.020 inches thick. The samples were cut slightly larger than the waveguide cross section and reduced to the desired size by lapping on silicon carbide paper by hand. Great care was taken to insure that the x, y and z edges were perpendicular to each other and that the faces at z = 0 and z = ℓ are parallel.

The sample length ℓ was chosen to be approximately one quarter wavelength calculated from

$$\ell = \frac{\lambda g}{4} = \frac{2\pi}{4_\beta} = \frac{\pi}{2_\beta}$$

The phase constant (β) was obtained from the expression for the propagation constant $\Gamma = \alpha + j\beta = [(\frac{\pi}{a})^2 - w^2\mu_0\varepsilon_0\ (\varepsilon_r - j\frac{\sigma}{w\varepsilon_0})]^{\frac{1}{2}}$ for the case of a waveguide completely filled with semiconductor, with no magnetic field applied. The calculated value of ℓ was .07675" inch for a 22.2 ohm-cm germanium sample at a frequency of 9.46 GHz. Selection of ℓ to be equal to one quarter wavelength or in general an odd number of quarter wavelengths in the material gives maximum experimental accuracy[28].

Prior to measurement the faces of the sample at planes z = 0 and z = ℓ were ground on emery paper and washed with tap water. The dimensions of the sample used for the measurement were .893" x .397" x

.074" and gave an excellent fit to the waveguide section 0.900" x 0.400".

6.2 CALIBRATION OF THE ELECTROMAGNET.

The electromagnet used for the experiments was a "Spectromagnetic Industries" model 1019 which has 4" diameter adjustable poles. This magnet is designed to be used with its associated 1 Kilowatt power supply (model 6021).

The electromagnet was calibrated with a precision Gauss meter for a pole gap distance of 3 cm. The calibration curve relating magnet current to magnetic field produced is shown in Figure 6.1.

6.2.1 RESULTS OF CALIBRATION OF MAGNET.

The experimental readings for the magnetization curve of a 4" adjustable gap electromagnet with cylindrical pole geometry for a gap distance of 3 cm. is given in Table 6.1 below and the curve is plotted in Figure 6.1.

TABLE 6.1

EXPERIMENTAL VALUES OF THE MAGNETIZIATION CURVE

NO. OF READINGS	MAGNETIZING CURRENT	MAGNETIC FIELD IN THE GAP
	Amps	Kilogauss
1	1	.37
2	3	1.14
3	6	2.35
4	9	3.5
5	10	3.9
6	12	4.65
7	14	5.45
8	15	5.75
9	17	6.5
10	18	6.9
11	20	7.5
12	21	7.8
13	24	8.6
14	25	8.9
15	27	9.36

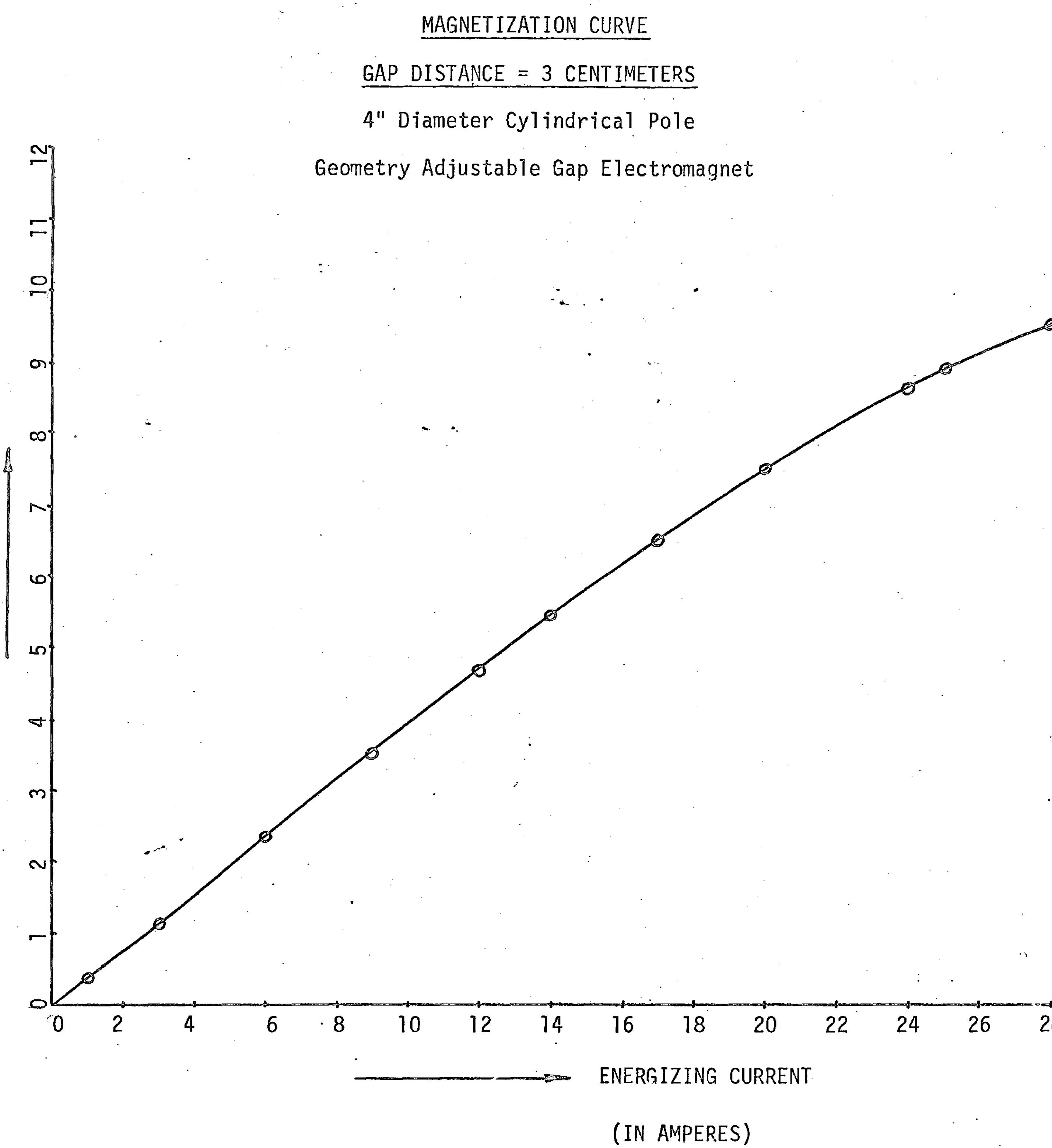


FIGURE 6.1 EXPERIMENTAL MAGNETIZATION CURVE OF THE ELECTROMAGNET

6.3 DESCRIPTION OF APPARATUS.

A schematic diagram of a reflection bridge for the precision measurement of the magnitude and the phase of the reflection co-efficient at the air-semiconductor interface is shown in Figure 6.2. The equipment used is as follows:

(i) Klystron.

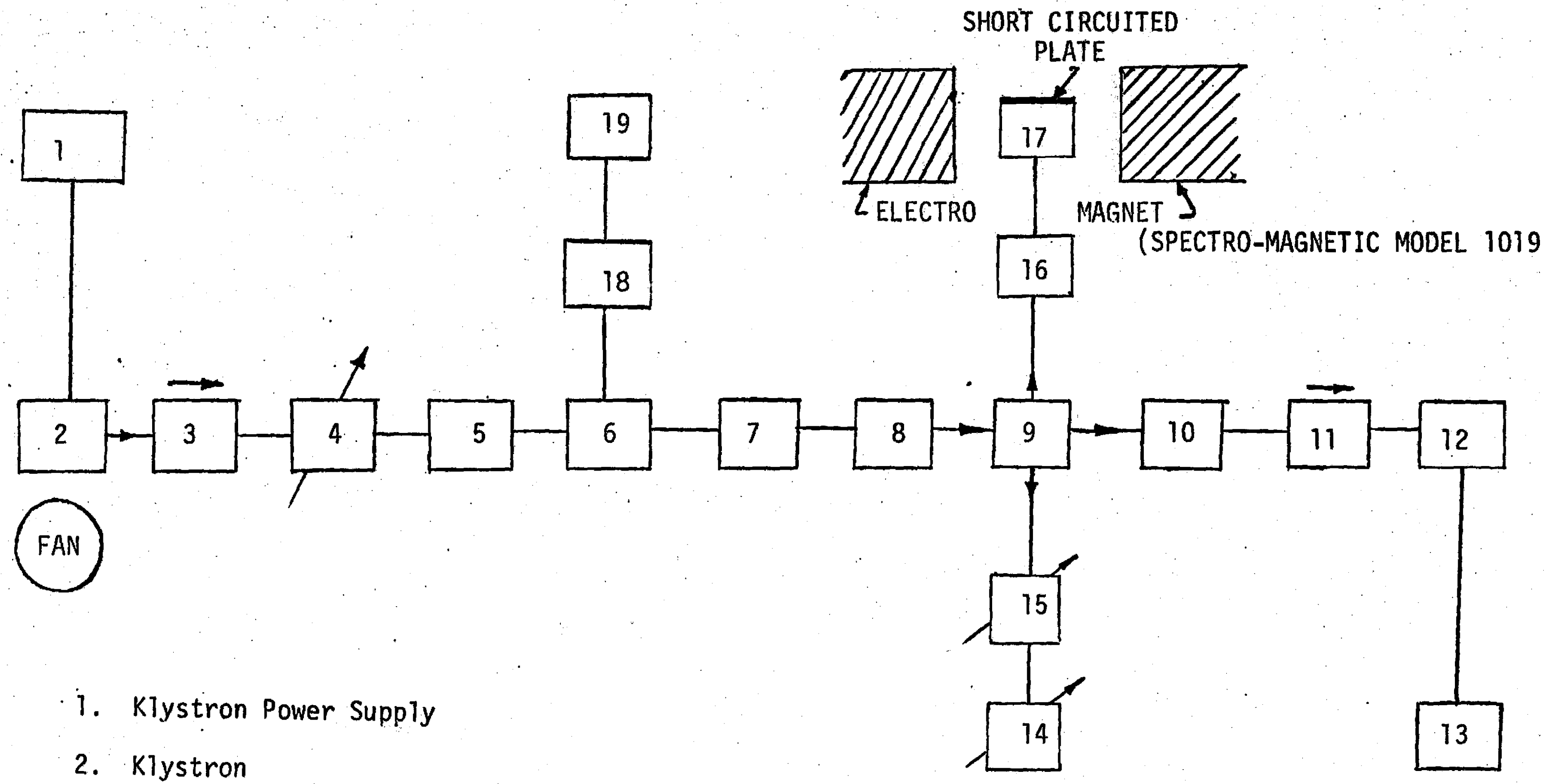
The Klystron used as a high frequency signal source is a reflex type, X-13 of Varian Associates of Canada Ltd. Its frequency is mechanically tunable over the range 8.2 to 12.4 GHz with its maximum power output for optimum load of 350 m watts. A frequency of 9.46 GHz was used for the experiments.

(ii) Klystron Power Supply.

The power to the Klystron was supplied from the P.R.D. power supply, type 890A. The reflector voltage can be modulated either from an external source or an internally produced square and sawtooth waves.

(iii) Ferrite Isolator.

This is an unidirectional element used to isolate the Klystron from the rest of the circuit as far as the reflected waves are concerned. Waves pass practically unattenuated in the direction of transmission of the isolator but in the opposite direction it has approximately 30db attenuation. Thus any change in the load will not affect the Klystron output power and frequency. Isolators manufactured by P.R.D. were used in the reflection bridge assembly.



1. Klystron Power Supply
2. Klystron
3. 11. Isolators
4. Attenuator
5. Wave meter
6. Directional Coupler
7. SW Indicator
8, 10, 16. Stub Tuners
9. Hybrid Tee
12. Crystal Detector
13. D.C. Micro Volt-ammeter
14. Precision Short
15. Precision Rotary Attenuator
17. Sample Holder
18. Crystal Detector
19. Indicator

FIGURE 6.2 AN EXPERIMENTAL SET UP OF A REFLECTION BRIDGE

(iv) Wave Meter.

The wave meter used was a Micro-line, Model 138A. This meter will read the frequency of the waves in the guide and its range is from 8430 to 9660 MC/S.

(v) Variable Attenuator.

This is used to adjust the power input to the circuit to a suitable value and the unit manufactured by P.R.D. was used in the bridge.

(vi) Directional Coupler.

This is a device consisting of two transmission lines coupled together in such a way that a wave travelling in one line in one direction excites a wave in the other guide ideally in one direction only. The normal attenuation in this process is 10-20 db.

(vii) Standing Wave Indicator.

The standing wave indicator is a model B433 (1637) of Elliott Brothers (London) Ltd. and is a high precision instrument. This has been used for the measurement of V.S.W.R. during the matching of the hybrid tee ports by means of the tuners.

(viii) Precision Attenuator, Precision Short and Hybrid Tee.

High quality equipments are needed for these components. Elliott instruments are used for the rotary vane attenuator type A1617/44 and the precision short. The hybrid tee was manufactured by Demornay Borandi.

(ix) Electromagnet.

The Spectromagnetic Industries' Model 1019 is a 4" diameter pole adjustable gap laboratory electromagnet. It may be operated continuously with currents up to 20 amperes to give 9.5 K gauss with pole gap of 3 cm. The poles, which are adjustable and reversible, are precision ground to enhance field homogeneity. The poles are chrome plated to resist wear and corrosion. Each pole has 4" diameter cylindrical pole geometry on one end and 1-1/2" diameter tapered pole geometry on the other. In addition, 6" variable flux rings may be attached to the 4" diameter cylindrical ends to increase field uniformity.

(x) Regulated Power Supply for the Electromagnet.

The model 6021 Spectromagnetic Industries' precision magnet power supply provides an extremely stable, accurate current regulated current. The output current is continuously adjustable over the entire range 0.1 to 30 Amps with a 10 turn potentiometer.

6.4 MEASURING PROCEDURE.

6.4.1 TUNING OF A HYBRID TEE.

The ports (1), (2) and (4) of the hybrid tee shown in Figure (5A) were terminated in matched loads and s_{33} was made zero by using the tuner in the arm (3). In practice a VSWR of about 1.02 was obtained. Similarly, s_{44} was made zero by the tuner in arm 4, with other ports terminated in matched loads. Then s_{34} was checked by measuring the output in the E-arm while feeding the H-arm with terminations placed on arms (1) and (2). If s_{34} is not zero, it can be reduced to a minimum by adjusting the tuner in arm (1). This adjustment slightly disturbs the tuning of E and H arms so these were again tuned and the process repeated till the effect of the side arm tuner on s_{33} and

s_{44} is negligible. Since the hybrid tee used was a high quality magic tee, only slight adjustments were needed for tuning the hybrid tee.

6.4.2 Measurement of Reflection Coefficient.

The sample arm was first shorted by a short circuit metal plate and the precision attenuator and the precision short in the reference arm were then adjusted for minimum output. Their readings $A_r(s_1)$ and $l_r(s_1)$ were recorded. The germanium sample was then placed in the waveguide in front of the short circuit plate. Again, the precision attenuator and the precision short were adjusted for minimum output and their readings A_r and l_r were noted. Care was taken to position the sample faces normal to the axis of the guide and the back face was completely in contact with the metal plate.

The external magnetic field B was then applied in a direction transverse to the direction of the wave propagation in the waveguide. The external magnetic field was varied by varying the magnetizing current from 0 to 27 amp in steps of 3 amps and the readings of the precision attenuator and the precision short A_r and l_r respectively for minimum output were recorded for each step. The direction of the magnetic field was then reversed and the entire procedure was repeated. Care was also taken to note that the temperature of the sample was not affected by the temperature rise of the electromagnet during the measurement.

6.4.3 MEASUREMENT OF THE D.C. RESISTIVITY OF THE SEMICONDUCTOR.

The conductivity of the semiconductor was measured by the four point probe method which has the advantage that it does not require any specific shape of the sample.

In this method, four probes are placed on the flat surface of the sample and the current I is passed through the outer probes and the voltage V is measured between the inner probes. If s_1, s_2, s_3 are the probe spacings then the conductivity σ is given by[29]

$$\sigma = \frac{I}{2\pi V}\left[\frac{1}{s_1} + \frac{1}{s_3} - \frac{1}{(s_1+s_2)} - \frac{1}{(s_1+s_3)}\right] \tag{6.1}$$

When the probe spacings are equal, the resistivity ρ of the sample is given by

$$\rho = \frac{1}{\sigma} = 2\pi s\left(\frac{V}{I}\right) \tag{6.2}$$

The equation 6.2 was used in calculating the d.c. resistivity of the sample and the resistivity of the sample was found to be 22.2 ohm-cm. When the dimensions of the sample are small compared to probe spacing, the corrections shown in figure (6.3) must be applied. The edge of the sample must be at least a distance 3s away from the probe.

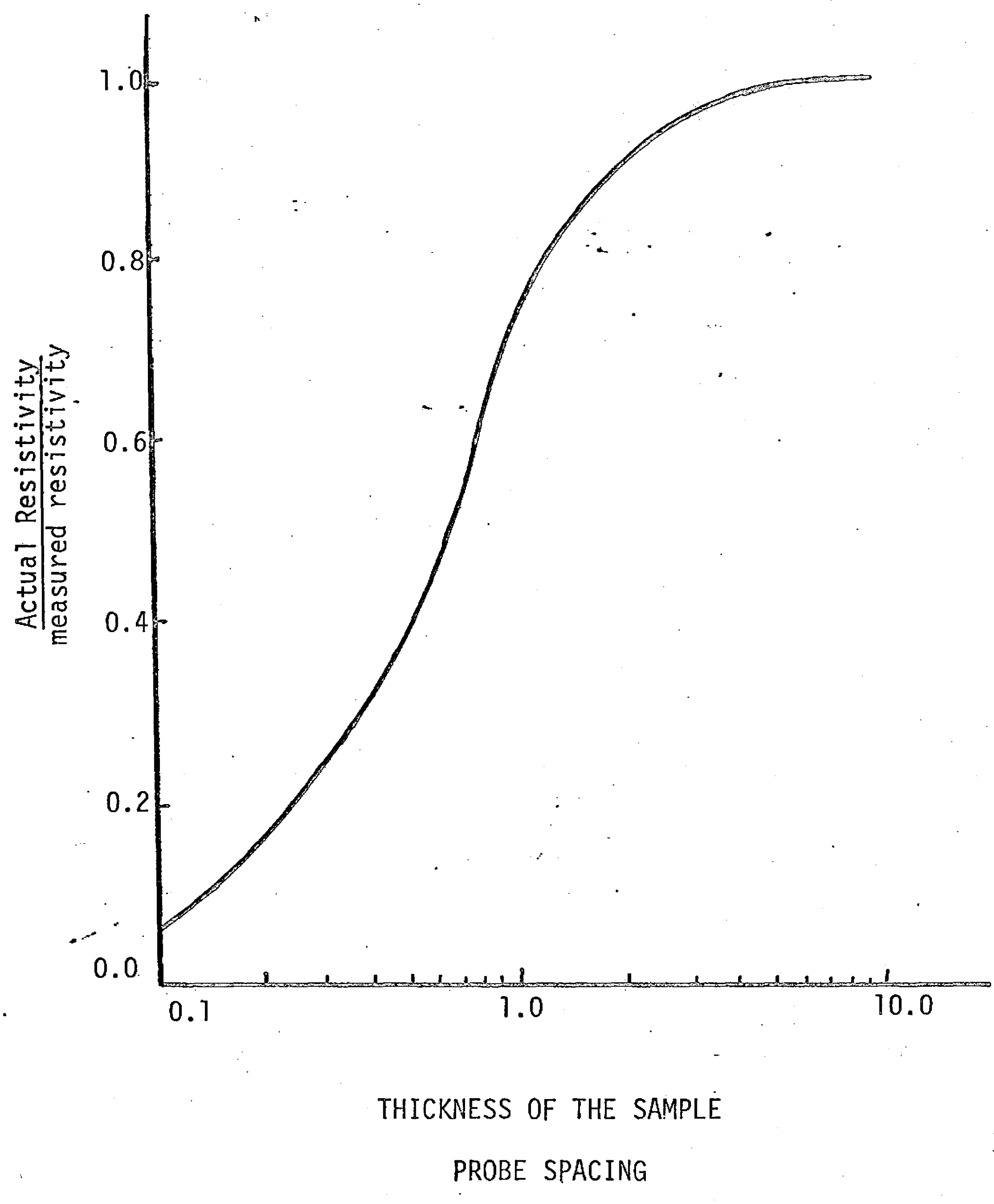


FIGURE 6.3 FOUR PROBE RESISTIVITY CORRECTION CURVE (AFTER HUNTER[30])

CHAPTER VII

RESULTS

7.1 THEORETICAL RESULTS

7.1.1 VARIATIONS IN PROPAGATION CONSTANT AND THE MATRIX ELEMENTS OF THE COMPLEX TENSOR PERMITTIVITY WITH "B".

The calculated variations in the values of the attenuation constant and the phase constant of the electromagnetic wave with the variation of the applied magnetic field as calculated from numerical solution of equation (4.9) are shown in tables 7.1(A), 7.1(B), 7.1(C), 7.1(D) and 7.1(E) for different values of the conductivity of the material. These tables are given in graphical form in figures 4.d and 4.e.

The calculated variations in the values of the matrix elements of the complex permittivity tensor using equation (3.2) with the variations of the applied magnetic field are also given in tables 7.1(A) to 7.1(E) and these variations have been plotted in the graph shown in figure 7.1 and 7.2.

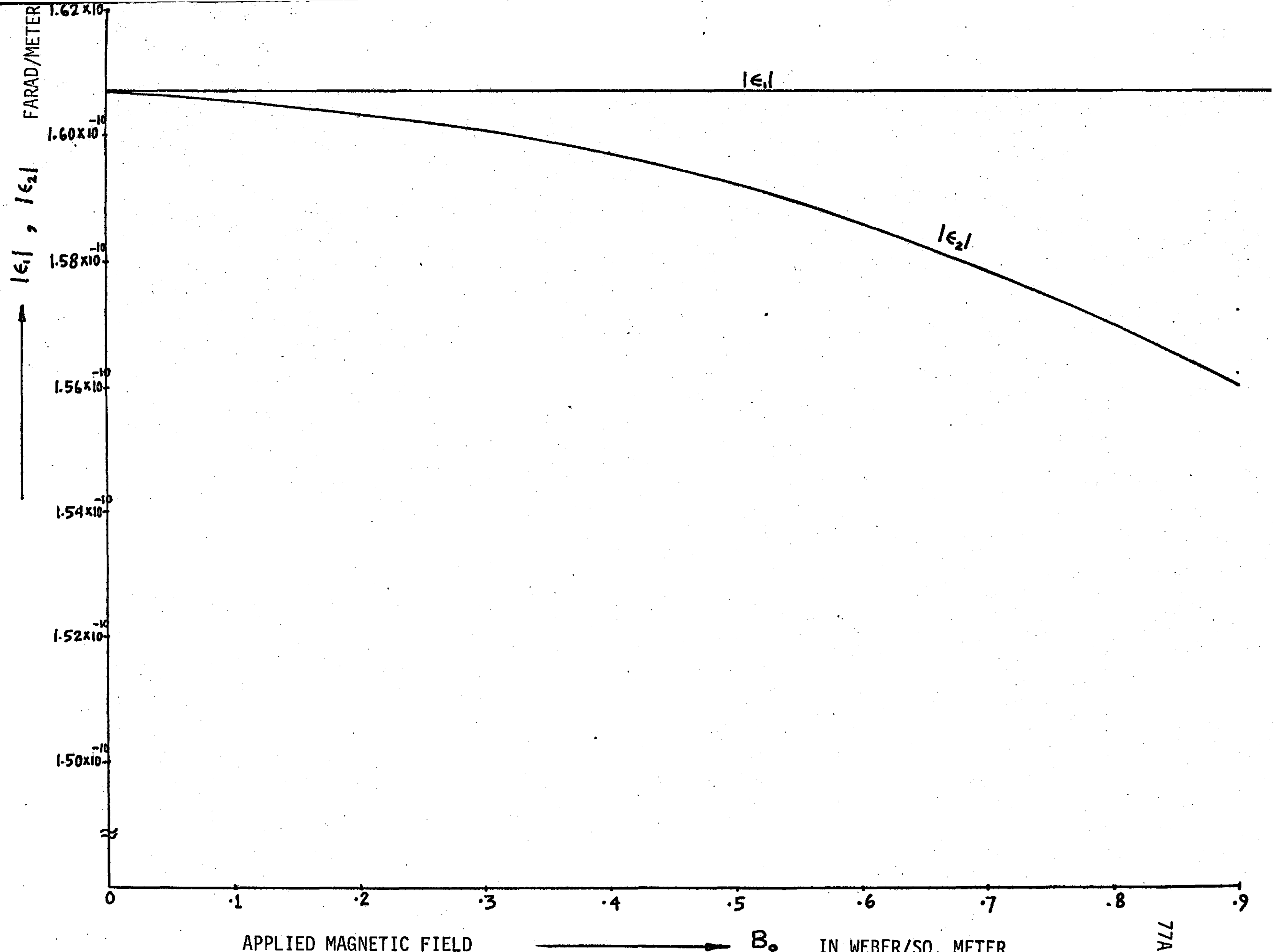
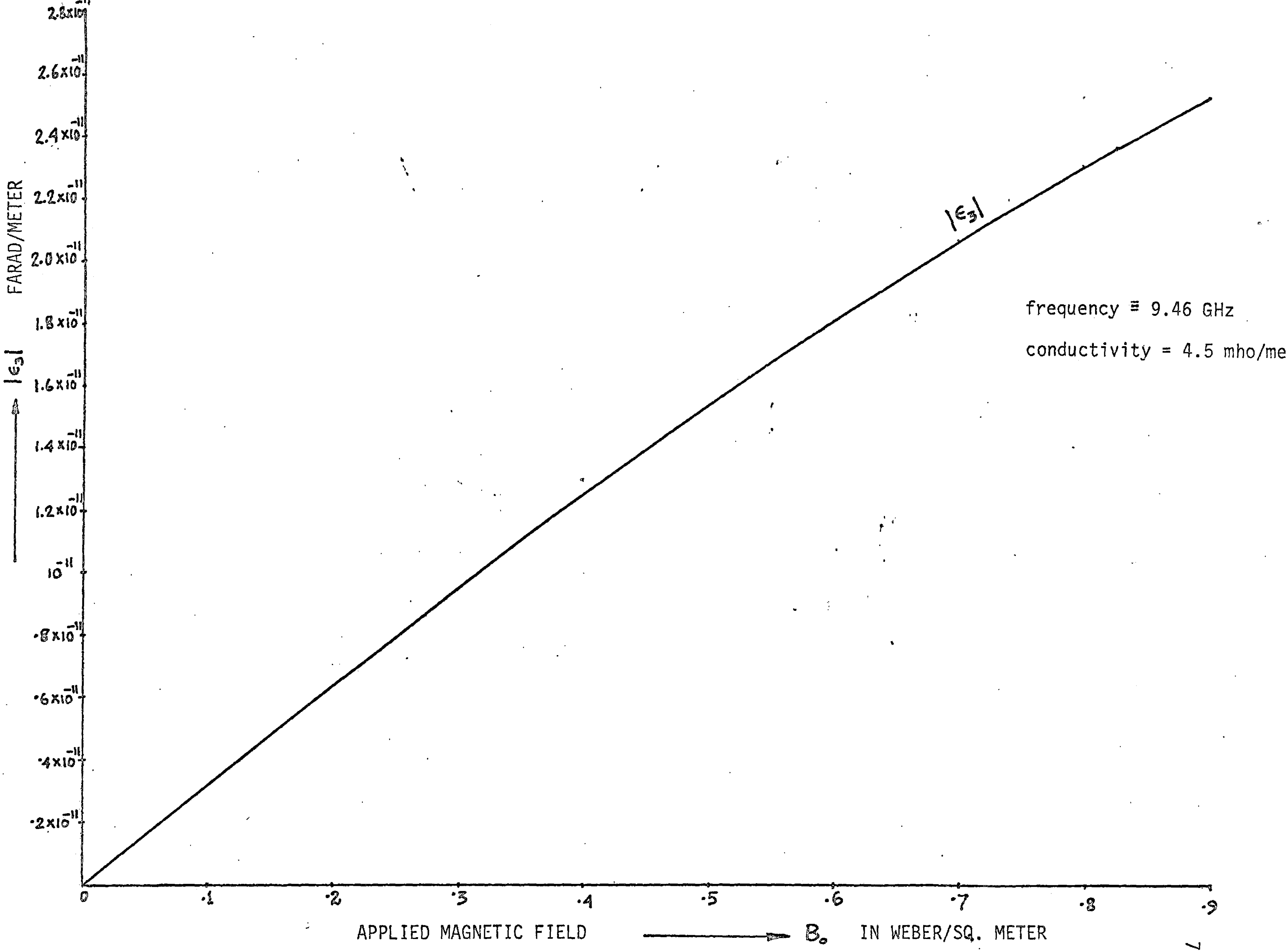

FARAD/METER
$|\epsilon_1|$, $|\epsilon_2|$
1.62 x 10
1.60 x 10^-10
1.58 x 10^-10
1.56 x 10^-10
1.54 x 10^-10
1.52 x 10^-10
1.50 x 10^-10
$|\epsilon_1|$
$|\epsilon_2|$
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
APPLIED MAGNETIC FIELD
B_o IN WEBER/SQ. METER

FIGURE 7.1



2.8x10^{-11}
2.6x10^{-11}
2.4x10^{-11}
2.2x10^{-11}
2.0x10^{-11}
1.8x10^{-11}
1.6x10^{-11}
1.4x10^{-11}
1.2x10^{-11}
10^{-11}
.8x10^{-11}
.6x10^{-11}
.4x10^{-11}
.2x10^{-11}
FARAD/METER
$|\epsilon_3|$
$|\epsilon_3|$
frequency ≡ 9.46 GHz
conductivity = 4.5 mho/me
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
APPLIED MAGNETIC FIELD
B_o IN WEBER/SQ. METER

TABLE 7.1A

THEORETICAL VARIATIONS IN PROPAGATION CONSTANT ϵ_1, ϵ_2 AND ϵ_3 WITH MAGNETIC FIELD

Frequency 9.46 GHz
Conductivity = 4.0mho/meter
n-type Ge sample

MAGNETIC FIELD B	PROPAGATION CONSTANT "Γ"=$\alpha+j\beta$		ϵ_1	ϵ_2	ϵ_3
	Attenuation Constant "α"	Phase Constant "β"			
Wb/m^2	neper/meter	radian/meter	farad/meter	farad/meter	farad/meter
0.0	186.01	803.8	$(1.417-j.672)\times10^{-10}$	$(1.417-j.672)\times10^{-10}$	0
0.1	185.68	803.75	"	$(1.417-j.670)\times10^{-10}$	$j\ 2.86\times10^{-12}$
0.2	184.69	803.59	"	$(1.417-j.667)\times10^{-10}$	$j\ 5.69\times10^{-12}$
0.3	183.06	803.31	"	$(1.417-j.661)\times10^{-10}$	$j\ 8.45\times10^{-12}$
0.4	180.84	802.94	"	$(1.417-j.653)\times10^{-10}$	$j\ 1.11\times10^{-11}$
0.5	178.06	802.48	"	$(1.417-j.643)\times10^{-10}$	$j\ 1.37\times10^{-11}$
0.6	174.78	801.93	"	$(1.417-j.631)\times10^{-10}$	$j\ 1.61\times10^{-11}$
0.7	171.07	801.31	"	$(1.417-j.617)\times10^{-10}$	$j\ 1.84\times10^{-11}$
0.8	166.99	800.64	"	$(1.417-j.602)\times10^{-10}$	$j\ 2.05\times10^{-11}$
0.9	162.59	799.92	"	$(1.417-j.586)\times10^{-10}$	$j\ 2.25\times10^{-11}$
1.00	166.76	782.34	"	$(1.417-j.569)\times10^{-10}$	$j\ 2.43\times10^{-11}$

TABLE 7.1B

VARIATIONS IN Γ, ε_1, ε_2 AND ε_3 WITH MAGNETIC FIELD

Frequency 9.46 GHz
n-type Ge sample
Conductivity = 4.3 mho/meter

MAGNETIC FIELD B	PROPAGATION CONSTANT "Γ" = $\alpha + j\beta$		ε_1	ε_2	ε_3
	Attenuation Constant "α"	Phase Constant "β"			
Wb/m^2	neper/meter	radian/meter	farad/meter	farad/meter	farad/meter
0.0	199.19	806.96	$(1.417-j.722)\times10^{-10}$	$(1.417-j.722)\times10^{-10}$	0
0.1	198.83	806.89	"	$(1.417-j.721)\times10^{-10}$	$j\ 3.07\times10^{-12}$
0.2	197.77	806.70	"	$(1.417-j.717)\times10^{-10}$	$j\ 6.11\times10^{-12}$
0.3	196.03	806.39	"	$(1.417-j.711)\times10^{-10}$	$j\ 9.09\times10^{-12}$
0.4	193.65	805.96	"	$(1.417-j.702)\times10^{-10}$	$j\ 1.20\times10^{-11}$
0.5	190.69	805.43	"	$(1.417-j.691)\times10^{-10}$	$j\ 1.47\times10^{-11}$
0.6	187.19	804.80	"	$(1.417-j.678)\times10^{-10}$	$j\ 1.73\times10^{-11}$
0.7	183.23	804.10	"	$(1.417-j.664)\times10^{-10}$	$j\ 1.98\times10^{-11}$
0.8	178.87	803.32	"	$(1.417-j.647)\times10^{-10}$	$j\ 2.21\times10^{-11}$
0.9	174.19	802.5	"	$(1.417-j.630)\times10^{-10}$	$j\ 2.41\times10^{-11}$
1.00	169.24	801.64	"	$(1.417-j.612)\times10^{-10}$	$j\ 2.61\times10^{-11}$

TABLE 7.1C

VARIATIONS IN Γ, ε_1, ε_2 AND ε_3 WITH MAGNETIC FIELD

Frequency 9.46 GHz
Conductivity = 4.4 mho/meter
n-type Ge sample

MAGNETIC FIELD B	PROPAGATION CONSTANT "Γ" = $\alpha + j\beta$ Attenuation Constant "α"	Phase Constant "β"	ε_1	ε_2	ε_3
Wb/m^2	neper/meter	radian/meter	farad/meter	farad/meter	farad/meter
0.0	203.54	808.04	$(.417-j.739)\times10^{-10}$	$(1.417-j.739)\times10^{-10}$	0
0.1	203.18	807.98	"	$(1.417-j.737)\times10^{-10}$	$j\ 3.14\times10^{-12}$
0.2	202.10	807.78	"	$(1.417-j.733)\times10^{-10}$	$j\ 6.25\times10^{-12}$
0.3	200.32	807.45	"	$(1.417-j.727)\times10^{-10}$	$j\ 9.30\times10^{-12}$
0.4	197.89	807.	"	$(1.417-j.718)\times10^{-10}$	$j\ 1.22\times10^{-11}$
0.5	194.87	806.45	"	$(1.417-j.707)\times10^{-10}$	$j\ 1.5\times10^{-11}$
0.6	191.3	805.8	"	$(1.417-j.694)\times10^{-10}$	$j\ 1.77\times10^{-11}$
0.7	187.25	805.06	"	$(1.417-j.679)\times10^{-10}$	$j\ 2.02\times10^{-11}$

TABLE 7.1D

VARIATIONS IN Γ, ε_1, ε_2 AND ε_3 WITH MAGNETIC FIELD

Frequency 9.46 GHz
Conductivity = 4.5 mho/meter
n-type GE sample

MAGNETIC FIELD B	PROPAGATION CONSTANT "Γ" = $\alpha+j\beta$		ε_1	ε_2	ε_3
	Attenuation Constant "α"	Phase Constant "β"			
Wb/m^2	neper/meter	radian/meter	farad/meter	farad/meter	farad/meter
0.0	207.89	809.15	$(1.417-j.756)x10^{-10}$	$(1.417-j.756)x10^{-10}$	0
0.1	207.51	809.08	"	$(1.417-j.754)x10^{-10}$	$j\ 3.21x10^{-12}$
0.2	206.41	808.87	"	$(1.417-j.75)x10^{-10}$	$j\ 6.40x10^{-12}$
0.3	204.60	808.53	"	$(1.417-j.743)x10^{-10}$	$j\ 9.51x10^{-12}$
0.4	202.12	808.06	"	$(1.417-j.735)x10^{-10}$	$j\ 1.3x10^{-11}$
0.5	199.03	807.48	"	$(1.417-j.723)x10^{-10}$	$j\ 1.54x10^{-11}$
0.6	195.39	806.8	"	$(1.417-j.709)x10^{-10}$	$j\ 1.81x10^{-11}$
0.7	191.27	806.03	"	$(1.417-j.694)x10^{-10}$	$j\ 2.07x10^{-11}$
0.8	186.73	805.19	"	$(1.417-j.6775)x10^{-10}$	$j\ 2.31x10^{-11}$
0.9	181.85	804.29	"	$(1.417-j.659)x10^{-10}$	$j\ 2.53x10^{-11}$

TABLE 7.1E

VARIATIONS IN Γ, ε_1, ε_2 AND ε_3 WITH MAGNETIC FIELD

Frequency 9.46 GHz
Conductivity = 4.65 mho/meter
n-type Ge Sample

MAGNETIC FIELD B	PROPAGATION CONSTANT "Γ"=α+jβ Attenuation Constant "α"	Phase Constant "β"	ε_1	ε_2	ε_3
Wb/m^2	neper/meter	radian/meter	farad/meter	farad/meter	farad/meter
0.0	214.37	810.84	$(1.417-j.781)x10^{-10}$	$(1.417-j.781)x10^{-10}$	0
0.1	213.98	810.76	"	$(1.417-j.780)x10^{-10}$	$j\ 3.32x10^{-12}$
0.2	212.85	810.54	"	$(1.417-j.776)x10^{-10}$	$j\ 6.61x10^{-12}$
0.3	210.98	810.18	"	$(1.417-j.769)x10^{-10}$	$j\ 9.82x10^{-12}$
0.4	208.43	809.68	"	$(1.417-j.759)x10^{-10}$	$j\ 1.29x10^{-11}$
0.5	205.25	809.07	"	$(1.417-j.747)x10^{-10}$	$j\ 1.59x10^{-11}$
0.6	201.50	808.34	"	$(1.417-j.733)x10^{-10}$	$j\ 1.88x10^{-11}$
0.7	197.26	807.53	"	$(1.417-j.718)x10^{-10}$	$j\ 2.14x10^{-11}$
0.8			"		
0.9			"	$(1.417-j.681)x10^{-10}$	$j\ 2.61x10^{-11}$

7.1.2 CALCULATED VALUES OF THE REFLECTION CO EFFICIENT.

The calculated theoretical values of the magnitude and phase of the reflection co efficients are shwon in Tables 7.1(F) and 7.1(G) The frequency used in the calculation is 9.46 GHz and the conductivity of the germanium sample is 4.5 mho/meter.

The theoretical values of the magnitude and phase of reflection co efficient have been plotted in Figure 7.2A and Figure 7.2B also along with the measured values.

TABLE 7.1(F)

REFLECTION CO EFFICIENT OBTAINED FROM

SIMPLE TE MODE CONSIDERATION

Frequency = 9.46 GHz
Conductivity = 4.50 mho/meter
n-type GE sample
Dimension = .893" x .397" x .074"

APPLIED MAGNETIC FIELD B	MAGNITUDE OF THE REFLECTION CO EFFICIENT $\|r\|$	PHASE ANGLE OF THE REFLECTION CO EFFICIENT θ
Wb/m^2		(IN DEGREES)
0.0	0.416	157.58°
0.1	0.415	157.51°
0.2	0.413	157.33°
0.3	0.410	157.03°
0.4	0.406	156.59°
0.5	0.400	156.02°
0.6	0.394	155.30°
0.7	0.386	154.43°
0.8	0.378	153.40°
0.9	0.370	152.19°
0.95	0.366	151.52°

TABLE 7.1(G)

REFLECTION CO-EFFICIENT OBTAINED FROM FIRST ORDER APPROX. SOLUTION OF THE CHARACTERISTIC IMPEDANCE

Frequency = 9.46 GHz
Conductivity = 4.50 mho/meter
n-type Ge sample
Dimension= .893" x .397" x .074"

APPLIED MAGNETIC FIELD B	MAGNITUDE OF THE REFLECTION CO-EFFICIENT $\|r\|$	PHASE ANGLE OF THE REFLECTION CO-EFFICIENT θ
Wb/m^2		(IN DEGREES)
0.0	0.416	157.58°
0.1	0.416	157.87°
0.2	0.414	158.03°
0.3	0.412	158.05°
0.4	0.408	157.94°
0.5	0.403	157.70°
0.6	0.397	157.35°
0.7	0.390	156.86°
0.8	0.383	156.25°
0.9	0.374	155.50°
0.95	0.369	155.07°

7.2 EXPERIMENTAL RESULTS OF THE MAGNITUDE AND PHASE OF THE REFLECTION CO EFFICIENT.

7.2.1 EXPERIMENTAL VALUES OF THE REFLECTION CO EFFICIENT.

The experimental readings for the magnetizing current, the precision attenuator (A_r) and the precision short (l_r) have been tabulated in Tables 7.2A & B. The corresponding calculations of the magnitude and phase of the reflection co efficient from these experimental readings have also been shown in the above tables. Two sets of readings have been obtained by changing the direction of the applied magnetic field.

The variations of the magnitude and phase of the reflection co efficient with the applied magnetic field B have been shown in Figures 7.2(A) and 7.2(B).

TABLE 7.2A

EXPERIMENTAL VALUES OF THE REFLECTION CO EFFICIENT

Frequency = 9.46 GHz

No sample and waveguide terminated by a conducting plate

$l_r(s_1) = 5440 \times 10^{-6}$meter

$A_r(s_1) = 8.8° \rightarrow 0.21$ dB = .02418 nepers

MAGNETIC FIELD→ (+B)

NO. OF READINGS	MAG. CURRENT IN AMPS.	B Wb/meter^2	A_r			$A_r - A_r(s_1)$	$A = 2[A_r - A_r(s_1)]$	$e^{-A} = \|r\|$
			Degree	Decibels	Nepers			
1	0	0	37.2°	3.95	.455	.43082	.86164	.423
2	1	.037	37.3°	3.97	.4575	.43332	.86664	.421
3	3	.114	37.4°	4.00	.460	.43582	.87164	.419
4	6	.235	37.5°	4.02	.4625	.43832	.87664	.417
5	9	.35	37.7°	4.07	.4675	.44332	.88664	.413
6	12	.465	38.°	4.14	.477	.45282	.90564	.405
7	15	.575	38.3°	4.21	.484	.45982	.91964	.399
8	18	.69	38.5°	4.26	.490	.46582	.93164	.394
9	21	.78	38.8°	4.33	.498	.47382	.94764	.388
10	24	.86	39.°	4.38	.504	.47982	.95964	.3835
11	27	.935	39.2°	4.43	.51	.48582	.97164	.379

TABLE 7.2B

EXPERIMENTAL VALUES OF THE REFLECTION CO EFFICIENT

Frequency = 9.46 GHz

With no sample and waveguide terminated by a conducting plate

$l_r(s_1) = 5440 \times 10^{-6}$ meter

$A_r(s_1) = 8.8° \rightarrow 0.21$dB = .02418 nepers

MAGNETIC FIELD → (-B)

NO. OF READINGS	MAG. CURRENT IN AMPS.	B Wb/meter^2	A_r Degree	A_r Decibels	A_r Nepers	$A_r - A_r(s_1)$	$A = 2[A_r - A_r(s_1)]$	$\lvert r\rvert = e^{-A}$
1	0	0	37.2°	3.95	.454	.42982	.85964	.424
2	1	.037	37.2°	3.95	.454	.42982	.85964	.424
3	3	.114	37.25°	3.96	.455	.43082	.86164	.423
4	6	.235	37.35°	3.99	.458	.43382	.86764	.420
5	9	.35	37.55°	4.03	.463	.43882	.87764	.416
6	12	.465	37.75°	4.08	.469	.44482	.88964	.411
7	15	.575	37.9°	4.12	.473	.44882	.89764	.408
8	18	.69	38.2°	4.19	.4825	.45832	.91664	.40
9	21	.78	38.5°	4.26	.490	.46582	.93164	.3945
10	24	.86	38.65°	4.29	.494	.46982	.93964	.391
11	27	.935	38.85°	4.34	.499	.47482	.94964	.3875

TABLE 7.2C

EXPERIMENTAL VALUES OF THE REFLECTION CO EFFICIENT

$r = e^{-A}(-e^{-j\phi}) = |r| \angle\theta$

$\phi = 2\beta_r\{l_r - l_r(s_1) + d_1\}$ in radian

β_r = 143 radian/meter

$l_r(s_1) = 5440 \times 10^{-6}$ meter

$d_1 = 1879.6 \times 10^{-6}$ meter

MAGNETIC FIELD → (+B)

NO. OF READINGS	MAG. CURRENT IN AMPS	B in Wb/meter^2	l_r (meter)	$\{l_r-l_r(s_1)+d_1\}$ (meter)	$\phi = 2\beta_r\{l_r-l_r(s_1)+d_1\}$ in radian	ϕ in Degrees	$\angle\theta$
1	0	0	5005×10^{-6}	1444.6×10^{-6}	.413	23.7°	156.3°
2	1	.037	5010×10^{-6}	1449.6×10^{-6}	.4145	23.75°	156.25°
3	3	.114	5015×10^{-6}	1454.6×10^{-6}	.4165	23.9°	156.1°
4	6	.235	5040×10^{-6}	1479.6×10^{-6}	.423	24.3°	155.7°
5	9	.35	5075×10^{-6}	1514.6×10^{-6}	.433	24.8°	155.2°
6	12	.465	5125×10^{-6}	1564.6×10^{-6}	.4475	25.65°	154.45°
7	15	.575	5180×10^{-6}	1619.6×10^{-6}	.4625	26.5°	153.5°
8	18	.69	5130×10^{-6}	1569.6×10^{-6}	.448	25.7°	154.3°
9	21	.78	5190×10^{-6}	1629.6×10^{-6}	.466	26.7°	153.3°
10	24	.86	5265×10^{-6}	1704.6×10^{-6}	.4875	27.9°	152.1°
11	27	.935	5325×10^{-6}	1764.6×10^{-6}	.505	28.9°	151.1°

TABLE 7.2D

EXPERIMENTAL VALUES OF THE REFLECTION CO EFFICIENT

$r = e^{-A}(-e^{-j\phi}) = |r| \angle\theta$

$\phi = 2\beta_r\{l_r - l_r(s_1) + d_1\}$ in radian

β_r = 143 radian/meter

$l_r(s_1) = 5440 \times 10^{-6}$ meter

$d_1 = 1879.6 \times 10^{-6}$ meter

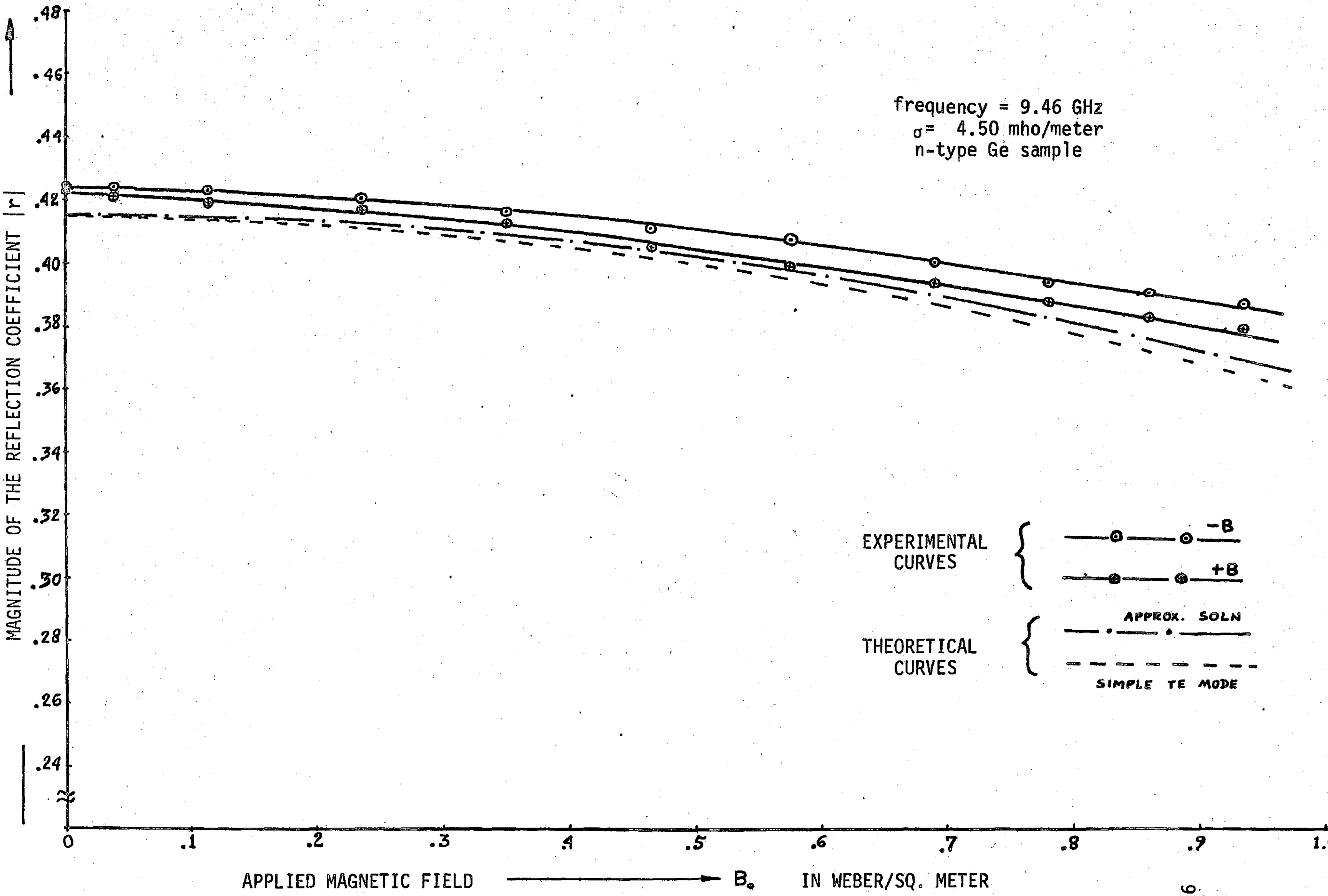
MAGNETIC FIELD → (-B)

NO. OF READINGS	MAG. CURRENT IN AMPS	B in Wb/meter^2	l_r (meter)	$\{l_r-l_r(s_1)+d_1\}$ (meter)	$\phi=2\beta_r\{l_r-l_r(s_1)+d_1\}$ (radian)	ϕ in degrees	$\angle\theta$
1	0	0	4900×10^{-6}	1339.6×10^{-6}	.383	21.9°	158.1°
2	1	.037	4900×10^{-6}	1339.6×10^{-6}	.383	21.9°	158.1°
3	3	.114	4910×10^{-6}	1349.6×10^{-6}	.386	22.1°	157.9°
4	6	.235	4925×10^{-6}	1364.6×10^{-6}	.39	22.35°	157.65°
5	9	.35	4960×10^{-6}	1399.6×10^{-6}	.40	22.9°	157.1°
6	12	.465	5000×10^{-6}	1439.6×10^{-6}	.412	23.6°	156.4°
7	15	.575	5050×10^{-6}	1489.6×10^{-6}	.426	24.4°	155.6°
8	18	.69	5105×10^{-6}	1544.6×10^{-6}	.442	25.3°	154.7°
9	21	.78	5165×10^{-6}	1604.6×10^{-6}	.458	26.3°	153.7°
10	24	.86	5220×10^{-6}	1659.6×10^{-6}	.474	27.15°	152.85°
11	27	.935	5265×10^{-6}	1704.6×10^{-6}	.4875	27.9°	152.1°

7.3 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF THE REFLECTION CO EFFICIENT

The experimental and the theoretical values of the magnitude of the reflection co efficient have been plotted in Figure 7.2(A). It is found that the experimental results are in excellent agreement with the theoretical results.

The experimental and the theoretical values of the phase of the reflection co efficient have been plotted in Figure 7.2(B). It is found that the theoretical values of the phase of the reflection co efficient obtained by simple TE mode consideration are in excellent agreement with the experimental values. The theoretical values of the phase of the reflection co efficient obtained by first order approx. solution are in close agreement with the experimental values for lower values of the magnetic field but at higher values of the magnetic field it differs slightly from the experimental curve.



frequency = 9.46 GHz
σ= 4.50 mho/meter
n-type Ge sample
MAGNITUDE OF THE REFLECTION COEFFICIENT |r|
.48
.46
.44
.42
.40
.38
.36
.34
.32
.30
.28
.26
.24
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
EXPERIMENTAL CURVES
-B
+B
THEORETICAL CURVES
APPROX. SOLN
SIMPLE TE MODE
APPLIED MAGNETIC FIELD
B₀ IN WEBER/SQ. METER

FIGURE 7.2A

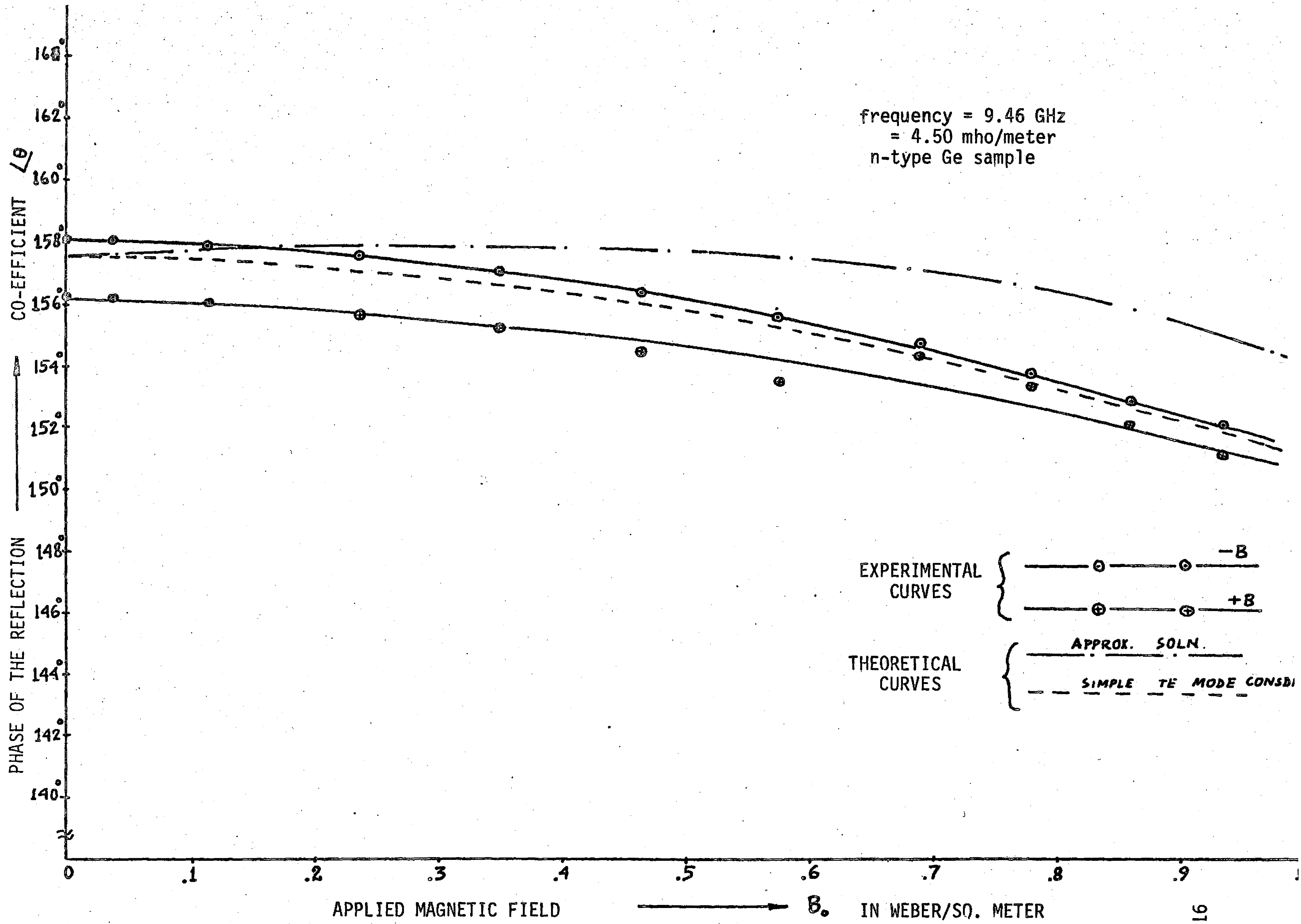

frequency = 9.46 GHz
= 4.50 mho/meter
n-type Ge sample
PHASE OF THE REFLECTION CO-EFFICIENT ∠θ
158°
156°
154°
152°
150°
148°
146°
144°
142°
140°
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
APPLIED MAGNETIC FIELD
B₀ IN WEBER/SQ. METER
EXPERIMENTAL CURVES
−B
+B
THEORETICAL CURVES
APPROX. SOLN.
SIMPLE TE MODE CONSD

FIGURE 7.2B

CHAPTER VIII

CONCLUSIONS

8.1 THEORETICAL WORK

In one of their published papers, Nag and Engineer[9] have formulated a special form of the Hall field expression, E_H. When a magnetic field is applied, the conduction current is modified due to the Hall effect. The Hall effect produces a field, E_H which in the case of a semiconductor with spherical energy surfaces is given by the following relations

$$E_H = R_c B_o \times J_{cH}$$

$$J_{cH} = \sigma \, (E - E_H)$$

where, R_c is the Hall co-efficient, B_o is the steady applied magnetic field, σ is the conductivity of the material and J_{cH} is the modified conduction current.

Following their suggestions of the Hall field expressions, a complex permittivity tensor characterizing the semiconductor in a transverse magnetic field has been derived. With this complex permittivity tensor, a detailed theoretical analysis of the electromagnetic wave propagation through a rectangular waveguide completely filled with a semiconductor subject to a transverse applied magnetic field has been carried out. In the analysis it has been clearly shown that no TM modes or TE modes other than those of the type TE_{on} can be excited in a rectangular waveguide completely filled with a semiconductor subjected to a transverse steady applied magnetic field. In the analysis, it has also been shown that TE_{on} modes do not depend on the magnetostatic field. Therefore, in this present analysis these modes were not considered in any detail. In addition to these TE_{on} modes, anomalous modes having all six E and H field components have been shown to exist. An investigation of these modes has been made.

Using Maxwell's equations and the appropriate boundary conditions, a rigorous and exact solution of all these six field components of the anomalous modes has been made by employing a different procedure from Nag and Engineer. In their theoretical analysis, a sinusoidal field intensity variation in the direction of the applied magnetic field was assumed. However, in the present work, no such assumption has been made and the sinusoidal field intensity variation in the direction of the magnetic field has been proved mathematically by employing a special method of separation of variables. Proceeding in this completely different way, a resultant transcendental equation for the propagation constant has been obtained which agrees with the transcendental equation of the propagation constant, as obtained by Nag and Engineer. It has been shown that the propagation characteristic is in general reciprocal.

A numerical solution of this transcendental equation has been made by employing the Newton-Raphson method. To the knowledge of the author, the numerical solution of this complicated transcendental equation has never been carried out before. The attenuation constant and the phase constant of the electromagnetic wave as a function of the applied magnetic field B which varies from 0 to 10 Kilogauss have been obtained numerically for n-type germanium with conductivity varying from 4 mho/meter to 4.65 mho/meter.

In this range of the magnetic field, both the attenuation constant and the phase constant are found to decrease with the increase of the applied transverse steady magnetic field. As the conductivity of the sample is increased the values of the attenuation constant and the phase constant are increased for a particular value of the magnetic field. Referring to figure (4.d) it is seen that as the magnetic field is increased from 0 to 0.9 Wb/m^2, the attenuation constant α decreases from 186.01 neper/meter to 162.5 neper/meter for the material having conductivity 4 mho/meter. For materials having conductivity 4.3 mho/meter the attenuation constant α decreases from 199.19 neper/meter to 174.19 neper/meter, in the same range of the magnetic field. The attenuation constant for materials of conductivity 4.5 mho/meter decreases from 207.89 neper/meter to 181.85 neper/meter as the magnetic field is increased from 0 to 0.9 Wb/m^2. This shows that with the increase of the conductivity of the sample the net change in the attenuation constant increases.

As the magnetic field is increased from 0 to 0.9 $Weber/m^2$ the phase constant β decreases from 803.8 rad/m to 799.9 rad/m for materials having conductivity 4 mho/m; from 806.96 rad/m to 802.5 rad/m for materials having conductivity 4.3 mho/meter and from 809.15 rad/m to 804.29 rad/m for materials having conductivity 4.5 mho/meter. Similarly, the net change in the phase constant is found to increase with the increase of the conductivity of the sample.

8.2 EXPERIMENTAL WORK

The experimental verification of the theoretical analysis was made on a 22.2 ohm-cm n-type germanium sample at 9.46 GHz using a reflection type microwave bridge(The d.c. resistivity of the sample was measured with a four point probe.).To the knowledge of the author, no experimental work employing this technique at this frequency range has been reported previously. The reflection bridge measurement technique followed that suggested by Champlin, Holm and Armstrong[10]. This gave the magnitude and phase of the reflection co-efficient for different values of the applied magnetic field which are plotted as a function of the magnetic field in figures 7.2(A) and 7.2(B).

8.3 COMPARISON OF THEORETICAL AND EXPERIMENTAL WORK

The measured values of the reflection co efficient at the air semiconductor interface were compared with the theoretical values of the reflection co efficient. As shown in figures 7.2(A) and 7.2(B) the agreement of the theoretical and the practical values of the reflection co efficient was excellent in both magnitude and phase.

Two expressions for the theoretical reflection co efficient have been derived. One expression is based on the simple TE mode analysis. The other expression is based on the considerations of E_y and H_x components of the electromagnetic field in the anisotropic material and on the first order approximation of the characteristic impedance in this material. The magnitudes of the reflection co efficient obtained theoretically by the above two expressions agree well with the measured magnitudes of the reflection co efficient. However, it is found that at higher magnetic fields the theoretical reflection co-efficient magnitude, based on the first order approximation, is closer to the experimental curve than those based on the simple TE mode analysis.

For the phase of the reflection co efficient the simple TE mode analysis gives theoretical values which almost coincide with the experimental values. However, the theoretical values of the phase of the reflection co-efficient based on the first order approximation, depart from the experimental values for higher magnetic field.

The calculation of reflection and transmission co efficients of an electromagnetic wave from an anisotropic medium such as in this case with all the six anomalous field components, is quite complicated. To the knowledge of the author an explicit expression for the reflection co efficient from such a surface has not yet been published. The analysis for the reflection co efficient which has been presented does not in general completely describe the behaviour of the practical system.

The analysis presented in this thesis for a completely filled guide can in principle be extended to the case of a waveguide partially filled with semiconductor subjected to a steady transverse magnetic field. A theoretical analysis would be very complicated but the analysis would be worthwhile, because of the practical importance of the partially filled structure.

APPENDIX

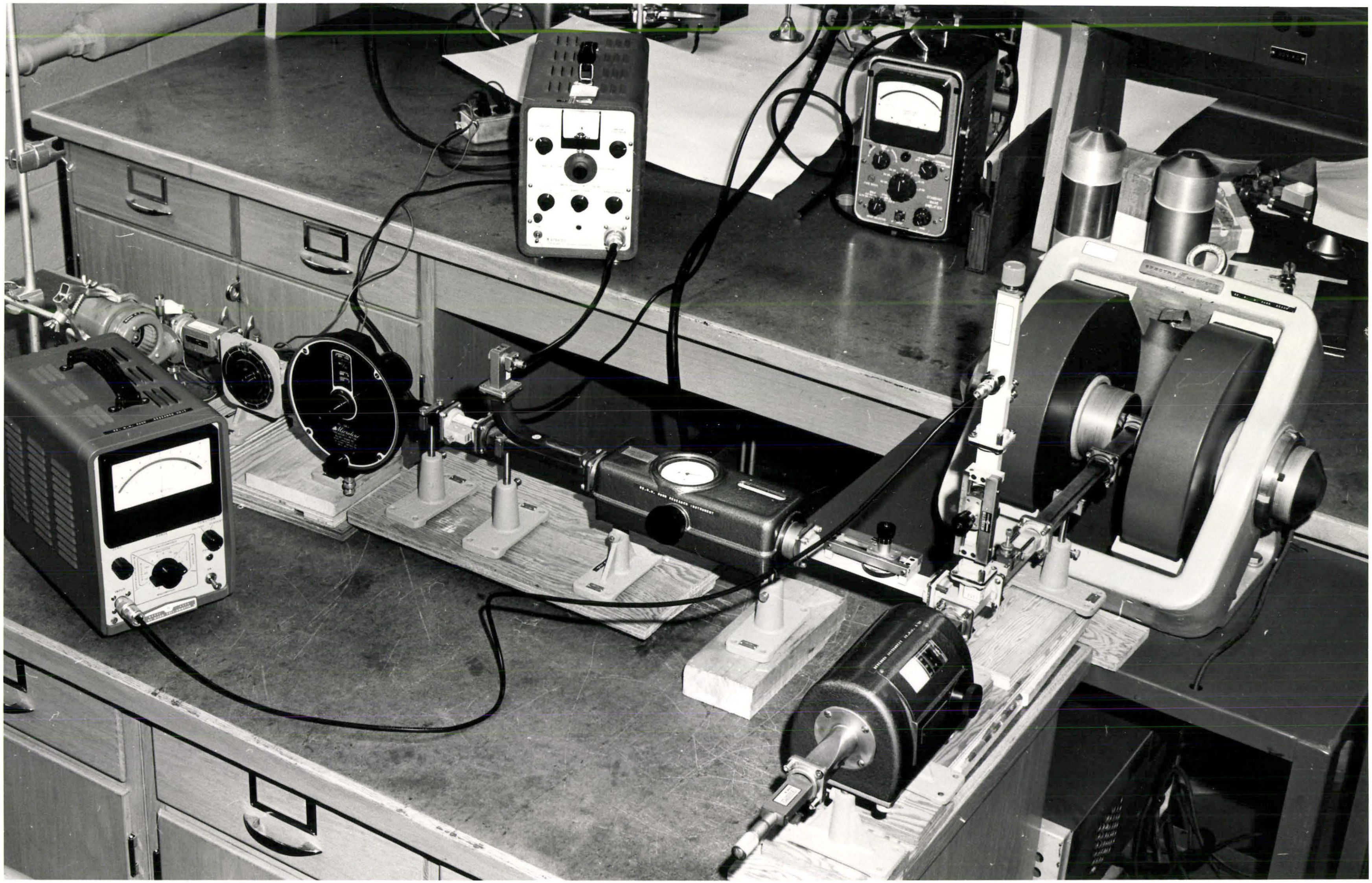
(COMPUTER PROGRAMMES)

```
$JOB      WATFOR   003515 ANIS                    100     010     030
$IBJOB             NODECK
$IBFTC
C        MICROWAVE PROPAGATION THROUGH SEMICONDUCTOR FILLED WAVEGUIDE
C        UNDER TRANSVERSE EXTERNAL MAGNETIC FIELD
C        FREQUENCY 9.46 GHZ ,N-TYPE GERMANIUM SAMPLE.
         COMPLEX EPS1,EPS2,EPS3,A3,B3,C1,C2,C3,FM,G,H,GAMMA,A4,B4,ROO
        1ROOT2,EM1B,D1,EM2B,D2,P,Q,R,S,V,W,T,D3,FGM,DFG,GAMNEW
         B=0.6
         SIGMA=4.5
    8    B2=B**2
         EPS1=CMPLX(1.41664E-10,-0.168E-10*SIGMA)
         EPS2=CMPLX(1.41664E-10,-0.168E-10*SIGMA/(1.0+0.181*B2))
         EPS3=CMPLX(0.0,7.16E-12*B*SIGMA/(1.0+0.181*B2))
         WRITE(6,16)B,SIGMA,EPS1,EPS2,EPS3
   16    FORMAT(2F5.2,6E15.6)
         A3=2.225E15*(EPS3**2)/EPS2
         B3=CSQRT(A3**2-3.78E4*A3)
         C1=0.5*(A3+B3)/(A3-B3)
         C2=0.25*(1.0/C1)
         C3=C1+C2
         FM=1.89E4-(4.45E15*EPS2)-A3
         G=FM+B3
         H=FM-B3
         GAMMA=CMPLX(0.1990305E+03,0.8074836E+03)
    9    A4=G-(GAMMA**2)
         B4=H-(GAMMA**2)
         ROOT1=CSQRT(A4)
         ROOT2=CSQRT(B4)
         EM1B=CEXP(ROOT1*0.01015)
         D1=1.0/EM1B
         EM2B=CEXP(ROOT2*0.01015)
         D2=1.0/EM2B
         P=0.5*(EM1B+D1)
         Q=0.5*(EM1B-D1)
         R=0.5*(EM2B+D2)
         S=0.5*(EM2B-D2)
         V=(A4*(1.0-C1)-B4*C2)/ROOT1
         W=(B4*(1.0-C2)-A4*C1)/ROOT2
         T=GAMMA*(A4+B4)/(ROOT1*ROOT2)
         D3=GAMMA*0.01015
         FGM=((ROOT1*ROOT2*(1.0-P*R))+((A4*C1)+(B4*C2))*Q*S)
         DFG=((D3*P*S*V)+(D3*Q*R*W)+(T*P*R)-(2.0*GAMMA*C3*Q*S)-T)
         GAMNEW=GAMMA-(FGM/DFG)
         X1=ABS(REAL(GAMMA-GAMNEW))
         X2=ABS(AIMAG(GAMMA-GAMNEW))
         IF(X1.LT.1E-1.AND.X2.LT.1E-1) GO TO 10
         GAMMA=GAMNEW
         GO TO 9
   10    WRITE(6,11) B,GAMNEW
   11    FORMAT(1F5.2,2E16.7)
         B=B+0.05
         IF(B.LE.1.0) GO TO 8
         STOP
         END
$ENTRY
$IBSYS
```

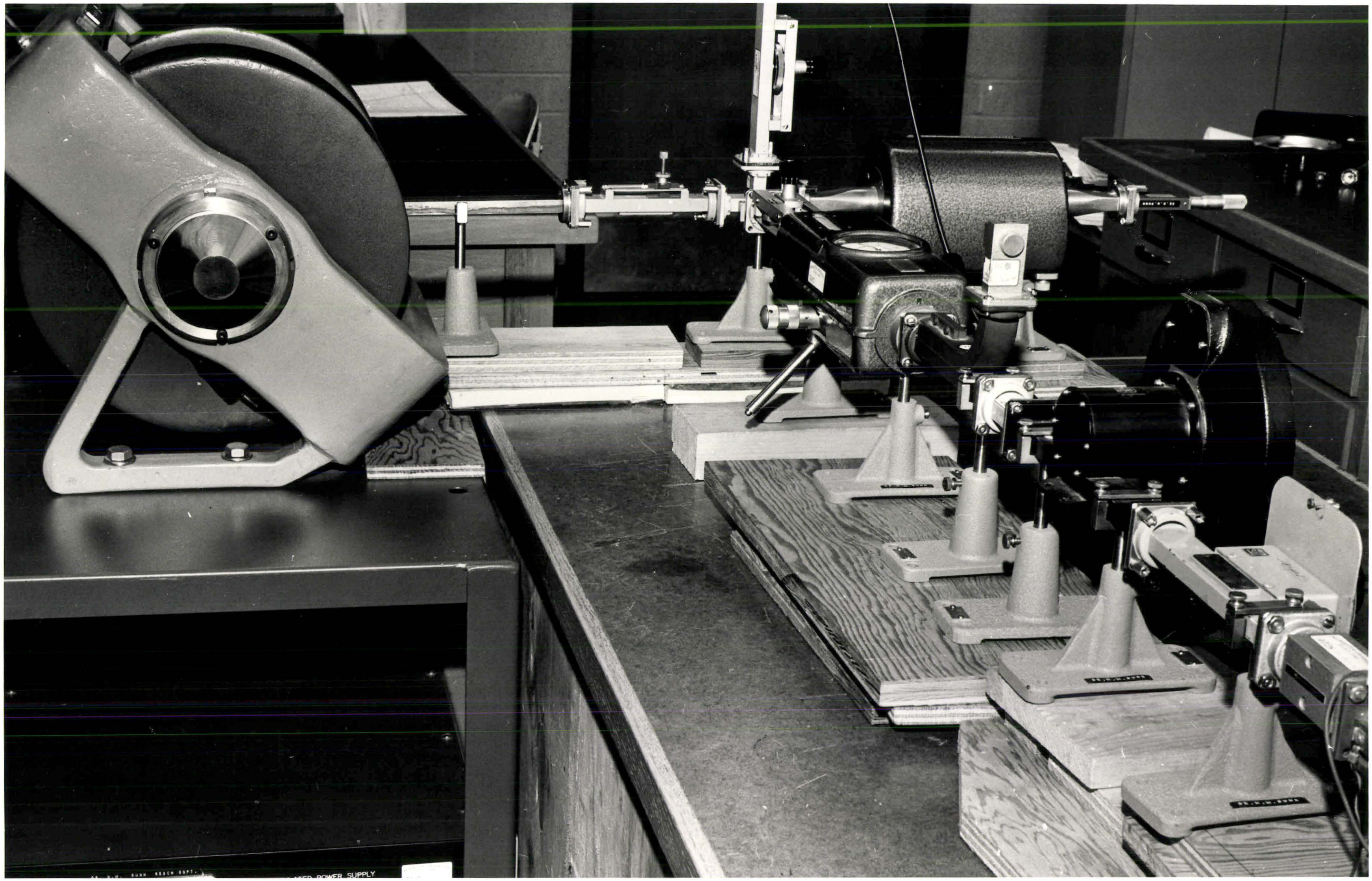
```
$JOB     WATFOR  003515 ANIS                        100    010    030
$IBJOB           NODECK
$IBFTC
C       REFLECTION COEFFICIENT CORRESPONDING TO THE CHANGE IN
C       PROPAGATION CONSTANT)
C       REFLECTION CO-EFFICIENT AT THE  SEMI-CONDUCTOR-AIR INTERFACE
C       IN A RECTANGULAR WAVE GUIDE UNDER A TRANSVERSE MAGNETIC FIELD
C       FREQUENCY 9.46 GC/S, N-TYPE GE SAMPLE, RESISTIVITY 25-OHM-CM
        COMPLEX GAMMA,A,B,V,P,Q,R
        REAL ABSVAL ,PHASE, D
   16   READ(5,17)GAMMA
   17   FORMAT(2E16.7)
        A=CMPLX(0.0,143. )
        D=1.8796E-3
        B=2.0 *GAMMA *D
        V=(CEXP(B)-1.0)/(CEXP(B)+1.0)
        P=(A*V-GAMMA)
        Q=(A*V+GAMMA)
        R=P/Q
        ABSVAL=CABS(R)
        PHASE=57.29578*ATAN2(AIMAG(R),REAL(R))
        WRITE(6,22) ABSVAL,PHASE,GAMMA
   22   FORMAT(4E16.7)
   25   GO TO 16
   27   STOP
        END
$ENTRY
    0.2078852E+03    0.8091483E+03
    0.2075141E+03    0.8090784E+03
    0.2064098E+03    0.8088704E+03
    0.2045983E+03    0.8085294E+03
    0.2021207E+03    0.8080636E+03
    0.1990305E+03    0.8074836E+03
    0.1953898E+03    0.8068020E+03
    0.1912663E+03    0.8060328E+03
    0.1867307E+03    0.8051912E+03
    0.1818537E+03    0.8042929E+03
    0.1793089E+03    0.8038274E+03
$IBSYS
```

```
$JOB   WATFOR  003515 ANIS                        1.00.   010    030
$IBJOB          NODECK
$IBFTC
C        REFLECTION COEFFICIENT  AT SEMICOND-AIR INTERFACE IN RECT GUI
C        UNDER TRANSVERSE EXTERNAL MAGNETIC FIELD
C        TAKING INTO CONSIDERATION OF THE FIRST TERM OF THE TAYLOR'S
C        SERIES EXPANSION OF THE CORRECTION TERM T.
         COMPLEX GAMMA,EPS2,EPS3,M1,M2,A1,A2,A3,B1,B2,B3,C3,D,GD2,BM1,
        1BM2,X1,X2,Y1,Y2,N1,N2,N3,N4,FO,P,BETA,R,FOD,K2,DMIN,V1,V2,K3,
        2K4,K5,S,NUM,DEN
         REAL ABSVAL,PHASE,D1,B,BS,SIGMA
   16    READ(5,17)B,GAMMA
   17    FORMAT(1F5.2,2E16.7)
         BS=B**2
         SIGMA=4.5
         EPS2=CMPLX(1.41664E-10,(-0.168E-10)*SIGMA/(1.0+0.181*BS))
         EPS3=CMPLX(0.0,(7.16E-12)*B*SIGMA/(1.0+0.181*BS))
         A3=2.225E15*(EPS3**2)/EPS2
         B3=CSQRT((A3**2)-(A3*3.78E4))
         C3=1.89E4-(4.45E15*EPS2)-(GAMMA**2)-A3
         M1=CSQRT(C3+B3)
         M2=CSQRT(C3-B3)
         BM1=M1*1.015E-2
         BM2=M2*1.015E-2
         A1=CEXP(BM1)
         B1=1.0/(A1)
         A2=CEXP(BM2)
         B2=1.0/(A2)
         X1=0.5*(A1+B1)
         X2=0.5*(A2+B2)
         Y1=0.5*(A1-B1)
         Y2=0.5*(A2-B2)
         N1=(X2-X1)
         N2=GAMMA*(M2*Y2-M1*Y1)
         N3=4.45E15*EPS3*N1
         FO=N1/(N2+N3)
         FOD=N1/(N3-N2)
         N4=((GAMMA**2)+4.45E15*EPS2)*(EPS3/EPS2)
         P=(1.0-(N4*FO))
         BETA=CMPLX(0.0,143. )
         D1=1.8796E-3
         GD2=2.0*GAMMA*D1
         D=CEXP(GD2)
         DMIN=1.0/D
         K2=(1.0-(N4*FOD))
         V1=(1.0-DMIN)
         V2=BETA*P*V1
         K3=P/K2
         K4=K3*DMIN
         K5=(1.0+K4)
         S=GAMMA*K5
         NUM=V2-S
         DEN=V2+S
         R=NUM/DEN
         ABSVAL=CABS(R)
         PHASE=57.29578*ATAN2(AIMAG(R),REAL(R))
         WRITE(6,22) ABSVAL,PHASE,GAMMA
   22    FORMAT (4E16.7)
   25    GO TO 16
```

```
   27  STOP
       END
$ENTRY
 0.10    0.2075141E+03    0.8090784E+03
 0.20    0.2064098E+03    0.8088704E+03
 0.30    0.2045983E+03    0.8085294E+03
 0.40    0.2021207E+03    0.8080636E+03
 0.50    0.1990305E+03    0.8074836E+03
 0.60    0.1953898E+03    0.8068020E+03
 0.70    0.1912663E+03    0.8060328E+03
 0.80    0.1867307E+03    0.8051912E+03
 0.90    0.1818537E+03    0.8042929E+03
 0.95    0.1793089E+03    0.8038274E+03
 0.00    0.2078852E+03    0.8091483E+03
$IBSYS
```



PHOTOGRAPH 1: The Experimental set up of the Microwave Reflection Bridge



PHOTOGRAPH 2: Rear View of the Microwave Reflection Bridge Showing the Electro Magnet and the Current Regulated Power Supply

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