MICROWAVE PROPAGATION IN SEMICONDUCTOR

SUBJECT TO MAGNETIC FIELD
MICROWAVE PROPAGATION IN RECTANGULAR WAVEGUIDE
CONTAINING SEMICONDUCTOR SUBJECT TO
TRANSVERSE MAGNETIC FIELD

By

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SCOPE AND CONTENTS:

A theoretical and a practical study is presented of guided wave propagation through rectangular waveguide completely filled with a n-type germanium sample of conductivity 4.50 mho/meter subjected to an external transverse magnetic field of 10 Kilogauss.

Measurements of the reflection coefficients at the air-semiconductor interface for different values of the applied magnetic field were carried out at a frequency of 9.46 GHz.
ABSTRACT

A detailed theoretical analysis of the propagation constant and the field components in rectangular waveguide completely filled with a semiconductor subjected to an external transverse applied magnetic field, has been carried out. A numerical solution of the transcendental equation for the propagation constant has been obtained for the n-type germanium samples with different conductivities and magnetic fields.

An experimental verification of this theoretical analysis has been made with a 22.2 ohm-cm, n-type germanium sample at 9.46 GHz. The applied transverse magnetic field was varied from 0 to 10 Kilogauss. Measurements of the reflection coefficients at the air-semiconductor interface for different values of the applied magnetic field have been made with a high precision microwave reflection bridge. The experimental results agree well with the theoretical results.
ACKNOWLEDGEMENTS

The author expresses his sincere gratitude to Dr. M. W. Gunn for his continued advice and encouragement. Thanks are also due to the National Research Council of Canada for a grant for the equipment and to the Government of Canada for the award of a scholarship under the Colombo Plan which made this investigation possible.
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CHAPTER 1

1.1 INTRODUCTION

In recent years, in the microwave field, considerable attention is being paid to the problem of propagation of electromagnetic waves through semi-conductor subjected to an external steady magnetic field. In the presence of an external magnetic field, the semi-conductor, when propagating an electromagnetic wave shows such interesting phenomena as Hall effect, Faraday rotation and magneto-resistance which have drawn the attention of many investigators.

Ever since C. L. Hogan (1) and D. Polder (2) indicated that certain media with a complex tensor permeability have important applications at microwave frequencies, extensive investigations into the propagation of electromagnetic waves in anisotropic materials, particularly ferrites, have been carried out. As a result of these investigations, a large number of practical devices employing ferrites have been invented. Electromagnetic wave propagation in rectangular wave guides completely filled with magnetized ferrites, have been extensively studied and a thorough analysis is found in the literature. Ferrites become anisotropic when placed in a magnetostatic field and have a permeability which is a tensor quantity. The modes of propagation in these magnetized ferrites are found to have features which are quite distinct from those found in the case of isotropic media. In addition to the normal TE modes, anomalous gyromagnetic modes are known to exist.
in these cases.

In much the same way, the behaviour of semi-conductors, when subjected to an externally applied magnetic field, may be described in terms of a complex tensor permittivity. Thus, a semi-conductor placed in a magnetostatic field becomes anisotropic and is known as a gyroelectric medium, whereas it is essentially an isotropic medium in the absence of an external applied magnetic field.

When a steady magnetic field is applied to a semi-conductor filled wave guide, the propagating electric fields perpendicular to the applied magnetic field become coupled. Hence the characteristics of the wave guide modes are changed, as in the analogous case of magnetized ferrites where the microwave magnetic fields perpendicular to the steady magnetization are coupled. However, since the electric and magnetic fields are required to satisfy different conditions at the boundaries of the wave guide, the exact characteristics of the modes are likely to be different in the two cases.

A considerable number of papers dealing with the case when applied steady magnetic field is in the direction of the propagation of the electromagnetic wave, are available. However, relatively few papers concerning the electromagnetic wave propagation through a semi-conductor filled rectangular wave guide, in the presence of a steady transverse external magnetic field, have been published. In recent years, Hirota, M. Toda, Nag and Engineer, Gabriel and Brodwin have dealt with the effect of the transverse external magnetic field on the wave propagation through semi-conductor in rectangular wave guides.

Nag and Engineer in one of their published papers have formulated a special form of Hall field expression which leads to a complex permittivity
tensor for a semi-conductor in a transverse magnetic field. They have also analyzed theoretically the characteristics of electromagnetic waves propagating in a semi-conductor filled rectangular wave guide in the presence of a transverse magnetic field. An analysis of the same problem is presented in this thesis, together with experimental results and a modification of Nag's theoretical analysis has been made.

Following Nag and Engineer's suggestion of the special form of the Hall field, an expression for the complex permittivity tensor has been derived which agrees with their expression. Using Maxwell's equations and the appropriate boundary conditions, a rigorous and exact solution of the field components has been made in a different way, using a different procedure. In the analysis, it is shown that in the present case, no TM modes or TE modes other than those of the type TE_{0n} may be excited. In addition, anomalous mode waves having all six E and H field components are shown to exist. Also it has been found that TE_{0n} modes do not depend on the magnetostatic field. Hence in gyroelectric wave guide, TE_{0n} modes hold no interest.

In practice, the wave within the filled wave guide is launched from an empty section supporting the dominant TE_{10} mode and the problem consists of determining the behaviour of this mode as it passes through the media. In gyroelectric media in the wave guide, if the wave is to depend on the magnetostatic field, all six components of the field are necessary and these must depend on both transverse co-ordinates. Since the TE_{10} mode is not admissible in the magnetized medium, it excites a mode which depends on both co-ordinates. Thus in a gyroelectric wave
guide, there exists the TE\textsubscript{10} mode in the absence of an external magnetic field \( B \) and an anomalous mode in the presence of \( B \). In the analysis it has been found that the propagation characteristic is in general reciprocal.

In their theoretical analysis, Nag and Engineer assumed a sinusoidal field intensity variation in the direction of the magnetic field and then solved for the specific field components. However, in the present work, no such assumption has been made and the sinusoidal field intensity variation in the direction of the magnetic field has been proved mathematically by employing a special method of the separation of variables\(^{(20)}\). Proceeding in this completely different way, a resultant transcendental equation for the propagation constant has been obtained which agrees with the transcendental equation of the propagation constant, as obtained by Nag and Engineer.

This transcendental equation has been solved numerically by employing the Newton-Raphson method, and the variation of the attenuation constant and the phase constant with the applied magnetic field \( B \), has been obtained.

An experimental verification of the theoretical analysis was performed on a 22.2 ohm-cm, n-type germanium sample at 9.46 GHz using a reflection type microwave bridge. A theoretical expression for the reflection coefficient at the semiconductor-air interface in the wave-guide was obtained and numerical values of the reflection coefficient as a
function of the external magnetic field were calculated and plotted. The reflection bridge measurement employed the new technique for precision reflection bridge measurements suggested by Champlin, Holm and Armstrong\(^{(10)}\). The theoretical and experimental values of the reflection coefficients were in excellent agreement for lower values of the magnetic field. However, for higher values slight discrepancies exist and possible reasons for these discrepancies are discussed.
1.2 LITERATURE SURVEY

The propagation characteristics of electromagnetic waves in rectangular wave-guides loaded with magnetized ferrites have been extensively studied and are well discussed in the book written by Lax and Button\textsuperscript{(11)}. Also a general modal solution for electromagnetic wave propagation through a rectangular guide completely filled with transversely magnetized ferrite has been carried out by Mikaelian\textsuperscript{(12)}, Seidel\textsuperscript{(13)} and Barzilai and Gerosa\textsuperscript{(14)}.

A considerable number of papers dealing with the guided wave propagation through semiconductor when the applied magnetic field is along the direction of the propagation of the electromagnetic wave are available. However, the following papers deal with guided wave propagation through semiconductor material subjected to a transverse magnetic field which is the subject of this thesis.

The Hall effect in semiconductors at microwave frequencies has been extensively studied by Prof. H. E. M. Barlow\textsuperscript{(15,16)} and several other workers. In 1963, an experimental investigation into microwave propagation at 10 GHz through a rectangular waveguide partially filled by a thin semiconducting plate under a steady transverse magnetic field, was performed by Prof. Barlow and Koike\textsuperscript{(17)}. The applied transverse steady magnetic field was found to produce a change in the conductivity of the plate and a corresponding effect on the microwave propagation along the guide. Non-reciprocal transmission arising from asymmetry of the semiconductor over the cross section of the guide was also observed.
Microwave propagation at 24 GHz has been observed by M. Toda\(^{(5)}\) in a wave-guide filled by InSb at 77\(^{0}\)K, in a transverse magnetic field. A transverse magnetic field in excess of 5 KOe was used and the transmitted power increased exponentially with increasing magnetic field intensity. The transmission showed a strong non-reciprocal character. Measurements of the phase shift in the InSb showed that the wave is transmitted only along one side of the wave-guide determined by the directions of magnetic field and of the power flow.

A theoretical analysis of the electromagnetic wave propagation through an n or p type semi-conductor with metal plated surfaces in a transverse magnetic field has been performed by Ryogo Hirota\(^{(3)}\). This theoretical analysis was in good agreement with the experimental results of M. Toda\(^{(5)}\).

The high frequency transport properties of semi-conductors under a transverse magnetic field have been analyzed theoretically by several workers such as Y. Itikawa\(^{(18)}\), H. Fujisada\(^{(19)}\).

Propagation of waves in bounded solid state plasma in transverse magnetic fields have also been analyzed by Ryogo Hirota and K. Suzuki\(^{(4)}\). In their paper it has been reported that a new type of solid state plasma wave-guide has been constructed using n-type InSb and an insulator of similar dielectric constant, CuO. Its properties were measured in a transverse magnetic field. When all except the input and output surfaces of the plasma wave-guide were metal plated, for one direction of magnetic field, the microwave power was transmitted through the InSb with a transmission loss of about 20 db. For the reversed directed of the magnetic field, the microwave power was transmitted through the CuO with
a transmission loss of about 5 db. When one surface of the plasma wave-guide was not metal plated, a strong magneto-plasma resonance at the frequency $\omega_0 = \omega_0^2/\omega_c$ was observed for one direction of the external magnetic field, the transmission loss being about 38 db at the maximum. For the reversed direction of the magnetic field no resonance was observed.

Microwave field distributions were measured by M. Toda(6), when a thin plate of n-type InSb was placed vertically at the centre of a K-band wave-guide and also when the wave-guide was filled with n-type InSb. The field distributions were asymmetrically deformed in the applied magnetic fields and the propagation properties were demonstrated to be non-reciprocal.

The characteristics of electromagnetic waves propagating in a semi-conductor filled rectangular wave-guide in the presence of a transverse magnetic field have been theoretically analyzed by Nag & Engineer (7). It has been shown that only TE mode waves having $y$-independent field components ($y$-being the direction of the steady magnetic field) and anomalous modes having all six field components can propagate.

A perturbation analysis of rectangular waveguide containing transversely magnetized semiconductor has been made by G. Gabriel and M. Brodwin(8). The solution by first order theory is compared to the results of an experiment in which surface currents in the guide wall due to perturbed and unperturbed $TE_{10}$ wave in n-type silicon are sampled and segregated. Theoretical and experimental results were found to be in reasonable agreement. Distinctions between the gyroelectric and gyromagnetic media in rectangular waveguide have been clearly analyzed
by Gabriel and Brodwin\textsuperscript{(20)}.

The solution of guided waves in Inhomogenous anisotropic media by perturbation and variational methods have also been performed by Gabriel and Brodwin\textsuperscript{(21)}.

Electromagnetic wave propagation in a plasma with non-linear electrical conductivity has been studied by Melvin Epstein\textsuperscript{(22)}. An analysis of reflection and transmission of electromagnetic waves from a non-linear anisotropic slab has been made by P. K. Kaw\textsuperscript{(23)}.

Starting from the general wave equation, Kaw has written appropriate non-linear equations describing the growth of the two modes of propagation of an electromagnetic wave in a non-linear medium. The applied magnetic field is taken in the direction of the propagation of the wave. The equations have been solved by a perturbation technique and the solutions have been used to obtain expressions for the linear and non-linear components of the reflected and transmitted parts of the electromagnetic wave. The paper was not directly applicable to the problem discussed here but the paper appears to be the only one available which deals with reflections from a non-linear anisotropic media.
1.3 MATERIALS EXHIBITING MAGNETO-RESISTANCE EFFECTS

When a magnetic field is applied transversely to an electric current in a semiconductor, containing both holes and electrons, a change in the conductivity of the semiconductor is observed. This fractional change in conductivity will be different for different types of the semiconductors, as it depends upon the electrical properties of the semiconductors. The choice of materials in which the change in conductivity will be most pronounced is discussed in this section. Attention is given mainly to the semiconductors at room temperature.

A semiconductor with spherical constant-energy surfaces should show no transverse magneto-resistance when relaxation time constant is constant and only one type of carrier is present. This is no longer true when a semiconductor has more than one type of carrier. For electrons, we have the following relations when an electric field is applied in x-y plane and magnetic field in z-direction\(^{(24)}\)\(^{(19)}\)

\[
J_{ex} = A_e E_x - B_e E_y \quad \text{and} \quad J_{ey} = B_e E_x + A_e E_y
\]

\[
A_e = \frac{n e^2}{m_e^*} \frac{\tau_e}{1 + \omega_{ec} \frac{\tau_e}{2}} = \frac{\sigma_{oe}}{1 + \mu_e 2B^2}
\]

\[
B_e = \frac{n e^2}{m_e^*} \frac{\omega_{ec} \tau_e}{2} \frac{\tau_e}{1 + \omega_{ec} \frac{\tau_e}{2}} = \frac{\sigma_{oe} \mu_e B}{1 + \mu_e 2B^2}
\]

Where \(J_{ex}\) and \(J_{ey}\) are x and y components of current density due to electrons respectively, \(E_x\) and \(E_y\) are x and y components of electric field respectively, \(n\) is the electron density, \(e\) is the electronic
charge, \( m^* \) is the effective mass of electrons, \( \omega_{ec} \) is the cyclotron angular frequency of electrons, \( \sigma_{oe} \) is the conductivity due to electrons only for \( B=0 \), \( \nu_e \) is the mobility of electrons, \( \tau_e \) is the relaxation time of electrons and \( B \) is the magnetic flux density.

For holes, similarly, replacing the subscript \( e \) by \( h \) we have,

\[
J_{hx} = A_h \, Ex - B_h \, Ey \tag{1.5}
\]

\[
J_{hy} = B_h \, Ex + A_h \, Ey \tag{1.6}
\]

\[
A_h = \frac{pe^2}{m_h} \, \frac{\tau_h}{1 + \omega_h^2 \tau_h^2} = \frac{\sigma_{oh}}{1 + \nu_h^2 \delta^2} \tag{1.7}
\]

\[
B_h = - \frac{pe^2}{m_h} \, \frac{w_h \tau^2}{1 + w_h \tau_h^2} = - \frac{\sigma_{oh} \nu_h \delta}{1 + \nu_h^2 \delta^2} \tag{1.8}
\]

where \( p \) is the hole density. When both electrons and holes are present, we must add their contributions to the total current

\[
J_x = J_{ex} + J_{hx} = (A_e + A_h) \, Ex - (B_e + B_h) \, Ey \tag{1.9}
\]

\[
J_y = J_{ey} + J_{hy} = (B_e + B_h) \, Ex + (A_e + A_h) \, Ey \tag{1.10}
\]

when

\[
J_y = 0, \text{ we have }
\]

\[
Ey = - \frac{(B_e + B_h)}{(A_e + A_h)} \, Ex \tag{1.11}
\]

From equations (1.9) and (1.11) we have

\[
\sigma = \frac{J_x}{Ex} = (A_e + A_h) + \frac{(B_e + B_h)^2}{A_e + A_h} \tag{1.12}
\]

Using equations (1.3), (1.4), (1.7), (1.8) and (1.12) we have,
\[ \sigma = \left\{ \frac{1 + \gamma \mu_e^2 B^2}{1 + (X+Y)\mu_e^2 B^2} \right\} \sigma_0 \]  
(1.13)

where,

\[ X = \frac{c(1+b)^2}{b(1+bc)^2} \]  
(1.14)

\[ Y = \frac{(1-c)^2}{(1+bc)^2} \]  
(1.15)

\[ \sigma_0 = \sigma_{oe} + \sigma_{oh} ; \ b = \mu_e/\mu_h ; \ c = n/p \]

Putting \( \rho_0 = 1/\sigma_0 \); \( \rho = \frac{1}{\sigma} \) and \( \Delta \rho = \rho - \rho_0 \)

we get,

\[ \frac{\Delta \rho}{\rho_0} = \frac{X \mu_e^2 B^2}{1+Y\mu_e^2 B^2} = \frac{c(1+b)^2}{b(1+bc)^2} \frac{\mu_e^2 B^2}{1 + (1-c)^2 \mu_e^2 B^2} \]

\[ \therefore \frac{\Delta \rho}{\rho_0} = \frac{c(1+b)^2 \mu_e^2 B^2}{b\{(1+bc)^2 + (1-c)^2 \mu_e^2 B^2\}} \]

\[ \frac{\Delta \rho}{\rho_0} = \frac{\mu_h}{\mu_e} \left( \frac{\mu_e}{\mu_h} \right)^2 \frac{\mu_e^2 B^2}{\left[ 1 + \left( \frac{\mu_e}{\mu_h} \frac{n}{p} \right)^2 + \left( 1 - \frac{n}{p} \right)^2 \mu_e^2 B^2 \right]} \]

A similar expression has been used by Barlow and Koike(25) as follows:

\[ - \frac{\sigma_B - \sigma_0}{\sigma_B} = \frac{(n/p) \left( \frac{\mu_h}{\mu_e} \right) \left( 1 + \frac{\mu_e}{\mu_h} \frac{n}{p} \right)^2 \mu_e^2 B^2}{(1 + \frac{\mu_e}{\mu_h} \frac{n}{p})^2 + (1 - \frac{n}{p})^2 \mu_e^2 B^2} \]
Let \( \bar{x} = \frac{\Delta \rho}{\rho_0} = -\frac{\sigma_B - \sigma_0}{\sigma_B} \)

\[
\bar{x} = \frac{n}{p_b} \frac{(1+b)^2 \mu_e^2 B^2}{(1+b \frac{n}{p})^2 + (1-n \frac{p}{p})^2 \mu_e^2 B^2} = \frac{np (1+b)^2 \mu_e^2 B^2}{b \left[ (p+nb)^2 + (p-n)^2 \mu_e^2 B^2 \right]}
\]

For a n-type material, \( n \gg p \) and approximating the above equation under this condition

\[
\bar{x} = \frac{np (1+b)^2 \mu_e^2 B^2}{b \left[ n^2 b^2 + n^2 \mu_e^2 B^2 \right]} = \frac{p (1+b)^2 \mu_e^2 B^2}{nb \left( b^2 + \mu_e^2 B^2 \right)}
\]

\[
= \frac{p}{n} \cdot (1+b)^2 \mu_e^2 B^2
\]

For different values of \( B \), ranging from .2 Wb/m^2 to 1.00 Wb/m^2, calculations of the values of \( D \) for different n-type materials can now be made using the values of the mobilities as shown in Table 1. Calculated values of \( D \) are tabulated in Table 2.

Since \( \bar{x} = \left( \frac{p}{n} \right) . D \), the ratio \( \frac{p}{n} \) must be large in order to make \( \bar{x} \) large. However, it must be noted that we can not change \( n \) arbitrarily, as the mobility varies with the resistivity. For resistivities above 1Ω-cm (Ge) and 10Ω-cm(Si), mobility is practically independent of resistivity. Below 1 ohm-cm for Ge, 10 ohm-cm for Si, the impurities begin to interfere with the carrier motion and the mobility drops with decreasing resistivity. This shows that higher resistivity material should be selected but at the same time it must remain n-type and not become intrinsic as the expression is only valid for n-type material.
Germanium

\[ n_i (300^\circ K) = 2.4 \times 10^{13}/cm^3 \]

Let us suppose \( n = 10^{14}/cm^3 \) \( \Rightarrow \text{ resistivity} = 20\Omega\cdot cm \)

\[ p = \frac{n_i^2}{n} = 5.76 \times 10^{12}/cm^3 \]

\( \therefore \left( \frac{p}{n} \right) = 5.76 \times 10^{-2} \)

Silicon

\[ n_i (300^\circ K) = 1.5 \times 10^{10}/cm^3 \]

Let us suppose \( n = 1.5 \times 10^{11}/cm^3 \) \( \Rightarrow \text{ resistivity} = 2.98 \times 10^{4} \Omega\cdot cm \)

\( \therefore p = 1.5 \times 10^{9} \) and \( \left( \frac{p}{n} \right) = 10^{-2} \)

InSb

\[ n_i (300^\circ K) = 1.35 \times 10^{16}/cm^3 \]

Let us consider \( n = 1.35 \times 10^{17} \) \( \Rightarrow \text{ resistivity} = 7.7 \times 10^{-4} \Omega\cdot cm \)

\( \therefore p = 1.35 \times 10^{15}/cm^3 \) \( \therefore \left( \frac{p}{n} \right) = 10^{-2} \)

InAs

\[ n_i (300^\circ K) = 1.97 \times 10^{15}/cm^3 \]

Let \( n = 10^{16}/cm^3 \) \( \Rightarrow \text{ resistivity} = 0.02\Omega\cdot cm \)

\[ p = 3.86 \times 10^{14}/cm^3 \] \( \therefore \frac{p}{n} = 3.86 \times 10^{-2} \)

For a particular \( \frac{p}{n} \) ratio in n-type material it can be seen from table 2 that the effect is most pronounced in InSb. The next preferable material is Ge. The order of these materials in terms of their magneto-resistance effect is as follows:

1. InSb
2. Ge
3. InAs
4. GaSb
5. GaAs
6. Si
7. InP
8. AlSb
An n-type Ge sample with a resistivity of 22.2 ohm-cm has been chosen for the experimental purpose in this thesis.

The Hall co-efficient is inversely proportional to the carrier concentration and the relative change in resistivity $\Delta \rho / \rho_0$ is found from the expression to contain terms in $\mu^2 H^2$. Therefore, in order to have a pronounced effect of transverse magnetic field on the resistivity or the conductivity of a semiconductor, carrier mobility must be high and the carrier concentration low. The geometrical magneto-resistance effects are large when the deviation of the carriers in the magnetic field is large and for this too a high mobility is needed. The mobility of a semiconductor is highest when all impurities have been removed, but unfortunately, in this pure condition the carrier concentration is strongly temperature dependent. Doping, the intentional addition of impurities, will reduce the mobility but it also stabilizes the carrier concentration.

The element semiconductors do not generally have as high an electron mobility as the compounds studied. Also the hole mobility for the compounds is quite low, so that if the materials have to be doped to give temperature stability, they are normally made n-type and the magneto resistance effect is then determined by the electron mobility.
<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>$\mu_e (290^\circ K)$</th>
<th>$\mu_h (290^\circ K)$</th>
<th>$b = \frac{\mu_e}{\mu_h (290^\circ K)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge</td>
<td>$3.8 \times 10^3$</td>
<td>$1.8 \times 10^3$</td>
<td>2.11</td>
</tr>
<tr>
<td>Si</td>
<td>$1.45 \times 10^3$</td>
<td>$0.5 \times 10^3$</td>
<td>2.9</td>
</tr>
<tr>
<td>InSb</td>
<td>$70 \times 10^3$</td>
<td>$1.0 \times 10^3$</td>
<td>70</td>
</tr>
<tr>
<td>InAs</td>
<td>$30 \times 10^3$</td>
<td>$0.25 \times 10^3$</td>
<td>120</td>
</tr>
<tr>
<td>InP</td>
<td>$3.4 \times 10^3$</td>
<td>$0.05 \times 10^3$</td>
<td>68</td>
</tr>
<tr>
<td>GaSb</td>
<td>$5 \times 10^3$</td>
<td>$1 \times 10^3$</td>
<td>5</td>
</tr>
<tr>
<td>GaAs</td>
<td>$7 \times 10^3$</td>
<td>$0.4 \times 10^3$</td>
<td>17.5</td>
</tr>
<tr>
<td>AbSb</td>
<td>$0.2 \times 10^3$</td>
<td>$0.02 \times 10^3$</td>
<td>1</td>
</tr>
</tbody>
</table>

ELECTRON AND HOLE MOBILITIES AND THEIR RATIO FOR DIFFERENT TYPES OF MATERIALS
\[ D = \frac{(1+b)^2 \mu e^2 B^2}{b^3 + \mu e^2 B^2 b} \]

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>( D ) (Wb/m²)</th>
<th>( B=0.2 )</th>
<th>( B=0.3 )</th>
<th>( B=0.4 )</th>
<th>( B=0.5 )</th>
<th>( B=0.6 )</th>
<th>( B=0.7 )</th>
<th>( B=0.8 )</th>
<th>( B=0.9 )</th>
<th>( B=1.00 )</th>
<th>NET VARIATION IN THE ( \Delta P ) AS THE MAG. ( \rho_0 ) FIELD VARIES FROM 0.2 TO 1.00 Wb/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge</td>
<td>(.60 \times 10^{-2})</td>
<td>(.133 \times 10^{-2})</td>
<td>(.237 \times 10^{-2})</td>
<td>(.369 \times 10^{-2})</td>
<td>(.529 \times 10^{-2})</td>
<td>(.717 \times 10^{-2})</td>
<td>(.932 \times 10^{-2})</td>
<td>(.117 \times 10^{-2})</td>
<td>(.14.4 \times 10^{-2})</td>
<td>(D = (13.8 \times 10^{-2}))</td>
<td></td>
</tr>
<tr>
<td>Si</td>
<td>(.516 \times 10^{-3})</td>
<td>(.116 \times 10^{-3})</td>
<td>(.206 \times 10^{-3})</td>
<td>(.323 \times 10^{-3})</td>
<td>(.464 \times 10^{-3})</td>
<td>(.632 \times 10^{-3})</td>
<td>(.825 \times 10^{-3})</td>
<td>(.10.4 \times 10^{-3})</td>
<td>(.12.9 \times 10^{-3})</td>
<td>(D = (124 \times 10^{-2}))</td>
<td></td>
</tr>
<tr>
<td>InSb</td>
<td>(.288 \times 10^{-2})</td>
<td>(.648 \times 10^{-2})</td>
<td>(.11.5 \times 10^{-2})</td>
<td>(.18 \times 10^{-2})</td>
<td>(.25.8 \times 10^{-2})</td>
<td>(.35.1 \times 10^{-2})</td>
<td>(.45.8 \times 10^{-2})</td>
<td>(.57.9 \times 10^{-2})</td>
<td>(.71.3 \times 10^{-2})</td>
<td>(D = (68.42 \times 10^{-2}))</td>
<td></td>
</tr>
<tr>
<td>InAs</td>
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<td>(.686 \times 10^{-2})</td>
<td>(.1.22 \times 10^{-2})</td>
<td>(.1.91 \times 10^{-2})</td>
<td>(.2.74 \times 10^{-2})</td>
<td>(.3.74 \times 10^{-2})</td>
<td>(.4.88 \times 10^{-2})</td>
<td>(.6.17 \times 10^{-2})</td>
<td>(.7.62 \times 10^{-2})</td>
<td>(D = (7.315 \times 10^{-2}))</td>
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</tr>
<tr>
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<td>(.58 \times 10^{-4})</td>
<td>(.2.8 \times 10^{-4})</td>
<td>(.4.38 \times 10^{-4})</td>
<td>(.6.3 \times 10^{-4})</td>
<td>(.8.58 \times 10^{-4})</td>
<td>(.1.2 \times 10^{-4})</td>
<td>(.1.4 \times 10^{-4})</td>
<td>(.1.7 \times 10^{-4})</td>
<td>(D = (1.68 \times 10^{-2}))</td>
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</tr>
<tr>
<td>GaSb</td>
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<td>(.647 \times 10^{-2})</td>
<td>(.1.15 \times 10^{-2})</td>
<td>(.1.8 \times 10^{-2})</td>
<td>(.2.58 \times 10^{-2})</td>
<td>(.3.51 \times 10^{-2})</td>
<td>(.4.58 \times 10^{-2})</td>
<td>(.5.79 \times 10^{-2})</td>
<td>(.7.13 \times 10^{-2})</td>
<td>(D = (6.84 \times 10^{-2}))</td>
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<tr>
<td>GaAs</td>
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<td>(.2.82 \times 10^{-3})</td>
<td>(.5.01 \times 10^{-3})</td>
<td>(.7.82 \times 10^{-3})</td>
<td>(.1.13 \times 10^{-3})</td>
<td>(.1.53 \times 10^{-3})</td>
<td>(.2.0 \times 10^{-3})</td>
<td>(.2.5 \times 10^{-3})</td>
<td>(.3.1 \times 10^{-3})</td>
<td>(D = (2.995 \times 10^{-2}))</td>
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<tr>
<td>A(\delta)Sb</td>
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<td>(.1.44 \times 10^{-4})</td>
<td>(.2.56 \times 10^{-4})</td>
<td>(.4 \times 10^{-4})</td>
<td>(.5.76 \times 10^{-4})</td>
<td>(.7.84 \times 10^{-4})</td>
<td>(.1.02 \times 10^{-4})</td>
<td>(.1.3 \times 10^{-4})</td>
<td>(.1.6 \times 10^{-4})</td>
<td>(D = (1.536 \times 10^{-2}))</td>
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</tr>
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**TABLE 2**

VARIATION OF \( \Delta P/\rho_0 \) WITH MAGNETIC FIELD FOR DIFFERENT SEMICONDUCTOR MATERIALS.
CHAPTER II
THE COMPLEX TENSOR PERMITTIVITY OF A SEMICONDUCTOR
IN A MAGNETIC FIELD

2.1 ROLE OF HALL FIELD IN MAXWELL'S EQUATIONS

The propagation characteristics of the electromagnetic waves in semiconductors, when an external steady magnetic field is applied in the direction of the propagation of the wave, have been analyzed by Rau and Caspari (26), and H. E. M. Barlow (16). Maxwell's equations as written by these authors are given below:

**MAXWELL'S EQUATIONS**

**AUTHOR**

RAU & CASPARI

\[ \nabla \times \mathbf{H} = \mathbf{J} ; \nabla \times (\mathbf{E} + \mathbf{E}_H) = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \]

\[ \mathbf{J} = \mathbf{J}_c + \varepsilon (\frac{\partial}{\partial t}) (\mathbf{E} + \mathbf{E}_H) \]

\[ \mathbf{J}_c = \sigma \mathbf{E} ; \mathbf{E}_H = \mathbf{R}_c \mathbf{B} \times \mathbf{J}_c \]

H. E. M. BARLOW

\[ \nabla \times \mathbf{H} = \mathbf{J} ; \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \]

\[ \mathbf{J} = \mathbf{J}_c + \mathbf{J}_d \]

\[ \mathbf{J}_e = \sigma (\mathbf{E} + \mathbf{E}_H) ; \mathbf{J}_d = \varepsilon (\frac{\partial}{\partial t}) (\mathbf{E} + \mathbf{E}_H) \]

\[ \mathbf{E}_H = \mathbf{R}_c \mathbf{B} \times \mathbf{J}_c + \left( \frac{e}{\varepsilon} \right) \mathbf{R}_d \mathbf{B} \times \mathbf{J}_d \]

where, \( \mathbf{E}_H \) is the Hall field vector

\( \mathbf{R}_c = \) Hall coefficient for conduction current

\( \mathbf{R}_d = \) Hall coefficient for dielectric current

The other symbols have their usual meanings.
As a result of the differences in the role given to Hall field in Maxwell's equations, the expressions derived by the above authors for the propagation constants of circularly polarized waves were very different. Rau and Caspari's analysis predicts a Faraday rotation which does not agree with Barlow's analysis even when the dielectric current Hall coefficient Rd is assumed to be zero. Nag and Engineer\(^{(9)}\) in their analysis of Faraday rotation in artificial dielectric have suggested a new role of the Hall field, \(E_H\), in Maxwell's equations by considering the motion of free carriers in the presence of a magnetic field. The following derivation is based on the Nag and Engineer's suggestion for the proper form of the Hall field in Maxwell's equations.

Consider a semiconductor sample to which an electric field is applied in the x and y-direction and a magnetic field is applied in the z-direction. Assuming that the effect of the momentum relaxation time is negligible, the average velocities of the carriers \(v_x\) and \(v_y\) in the x and y directions respectively are given by

\[
\begin{align*}
v_x &= \mu [E_x + (\frac{\mu_H}{\mu}) v_y B] \\
v_y &= \mu [E_y - (\frac{\mu_H}{\mu}) v_x B]
\end{align*}
\]

(2.1) (2.2)

where \(\mu\) is the conductivity mobility and \(\mu_H\) is the Hall mobility. The above assumption is substantially correct even at microwave frequencies for conductors. If however, the effect of relaxation time is considerable in a particular sample one has only to substitute for \(\mu\) and \(\mu_H\) the complex values appropriate for the signal frequency\(^{(9)}\).

Now the expression for the current components \(J_{ex}\) and \(J_{ey}\) are given by
\[ J_{ex} = \sigma [E_x + R_c J_{cy} B] \quad (2.3) \]
\[ J_{ey} = \sigma [E_y - R_c J_{cx} B] \quad (2.4) \]

When the electric field is applied in one direction only, one evaluates the Hall field from the above two equations by equating the current in the other direction to zero. But when electric fields are applied in two directions, one cannot dissociate the Hall field by equating the transverse current to zero.

It therefore appears that the current equations in Maxwell's equation should be written directly from (2.3) and (2.4). But from analogy with the expression for the Hall field when the applied electric field is in one direction, one may call \( E_H \) the equivalent Hall field as written below.

\[ \vec{E}_H = R_c \vec{B} \times \vec{J}_c \quad (2.5) \]

Using equations (2.3), (2.4), (2.5), one may thus write

\[ \vec{J}_c = \sigma (\vec{E} - \vec{E}_H) \quad (2.6) \]
2.2 **PERMITTIVITY OF ANISOTROPIC MATERIAL**

The electric flux density \( \mathbf{D} \) is generally broken into two parts, the first related to the electric field intensity \( \mathbf{E} \) by the free space permittivity, \( \varepsilon_0 \) and the second called the polarization vector \( \mathbf{P} \). That is,

\[
\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \tag{2.7}
\]

The polarization vector \( \mathbf{P} \) arises from a distribution of electric dipoles in the material. For linear materials, \( \mathbf{P} \) is related to the electric field intensity \( \mathbf{E} \) by

\[
\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} \tag{2.8}
\]

where, \( \chi_e \) is the electric susceptibility and \( \chi_e = (\varepsilon_R - 1) \).

In semiconductors \( \varepsilon_R \), the relative permittivity or dielectric constant is in general complex \( \varepsilon_R = \varepsilon_R' - j\varepsilon_R'' \) in order to account for loss and phase shift in the material.

For isotropic materials, the polarization is parallel with the applied fields and is independent of the direction of the fields. For anisotropic materials, this is not the case. A helpful analogy is afforded by the strain in a solid body resulting from applied forces. Consider a rectangular solid having unequal edge lengths with a co-ordinate system oriented at an arbitrary angle as shown in Figure (2.1). If a force is applied along any one co-ordinate, strains result in all three directions with different magnitudes in each direction.

In the same way, an electric field applied to an anisotropic material along an axis of an arbitrarily oriented co-ordinate system leads to polarization which has components along each of the co-ordinate directions. If \( \mathbf{E} = \hat{\mathbf{a}}_x E_x \), then
Semiconductor in a Magnetic Field

FIGURE 2.1

FIGURE 2.2

Semiconductor in a Magnetic Field
$P = \varepsilon_0 (\hat{a}_x \chi_{e11} E_x + \hat{a}_y \chi_{e12} E_y + \hat{a}_z \chi_{e13} E_z)$ \hfill (2.9)

where $\chi_e$ is the electric susceptibility and the added subscripts refer to $x$, $y$ and $z$ components of $P$ and $E$ respectively. Then if $E$ has all three components $E_x$, $E_y$, $E_z$ then

$P_x = \varepsilon_0 \left( \chi_{e11} E_x + \chi_{e12} E_y + \chi_{e13} E_z \right)$

$P_y = \varepsilon_0 \left( \chi_{e21} E_x + \chi_{e22} E_y + \chi_{e23} E_z \right)$ \hfill (2.10)

$P_z = \varepsilon_0 \left( \chi_{e31} E_x + \chi_{e32} E_y + \chi_{e33} E_z \right)$

Using the relation (2.7) we may obtain from (2.10)

$D_x = \varepsilon_{11} E_x + \varepsilon_{12} E_y + \varepsilon_{13} E_z$

$D_y = \varepsilon_{21} E_x + \varepsilon_{22} E_y + \varepsilon_{23} E_z$ \hfill (2.11)

$D_z = \varepsilon_{31} E_x + \varepsilon_{32} E_y + \varepsilon_{33} E_z$

where $\varepsilon_{11} = \varepsilon_0 (1 + \chi_{e11})$, $\varepsilon_{12} = \varepsilon_0 \chi_{e12}$, etc.

It is convenient to write the equations (2.11) as an array in matrix form as follows

$$
\begin{bmatrix}
D_x \\
D_y \\
D_z
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
$$

\hfill (2.12)
2.3 DERIVATION OF THE COMPLEX TENSOR PERMITTIVITY

A rectangular waveguide is assumed to be completely filled with a semiconductor. In the presence of an electromagnetic wave in the guide, a dielectric and also a conduction current flows in the semiconductor.

The dielectric current \( J_d \) is given by

\[
J_d = \varepsilon \frac{\partial \vec{E}}{\partial t}
\]  

(2.13)

where \( \varepsilon \) is the permittivity of the semiconductor and \( \vec{E} \) is the electric field vector.

Professor H. E. M. Barlow in a paper suggested that the dielectric current may be changed when a magnetic field is applied. However, this effect has not been experimentally observed and even if it exists, it would be small. Therefore, it is assumed that the dielectric current is unaffected by the magnetic field.

2.3.1 MODIFICATION OF THE CONDUCTION CURRENT BY EXTERNAL MAGNETIC FIELD

The conduction current in the absence of a magnetic field is \( \sigma \vec{E} \), and if the signal frequency is assumed to be much less than the scattering frequency, \( \sigma \) is equal to the d. c. conductivity of the semiconductor.

When a magnetic field is applied, the conduction current is modified by the Hall effect. In the case of a semiconductor with spherical energy surfaces, the Hall field \( \vec{E}_H \) is given by (2.5)

\[
\vec{E}_H = R_c B_o \times \vec{J}_{cH}
\]  

(2.14)

where \( R_c \) is the Hall coefficient and \( B_o \) is the steady magnetic field. The steady magnetic field is applied in the x direction as shown in Figure (2.2). The modified conduction current is thus,

\[
\vec{J}_{cH} = \sigma(\vec{E} - \vec{E}_H)
\]  

(2.15)
Assuming the time dependence of the electromagnetic fields to be $e^{j\omega t}$, an expression for the total current $\mathbf{J}$ in terms of a complex permittivity tensor can be written in the following way.

\[
\text{Total current } \quad \mathbf{J} = \mathbf{J}_d + \mathbf{J}_{CH} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \sigma(\mathbf{E} - \mathbf{E}_H)
\]

But from equation (2.14) the three components of the Hall field, i.e.

$E_{Hx}, E_{Hy}, E_{Hz}$ in $x, y, z$ direction are given by

\[
\begin{align*}
E_{Hx} &= 0 \\
E_{Hy} &= -R_c B_0 J_{CHz} \\
E_{Hz} &= R_c B_0 J_{CHy}
\end{align*}
\]

where $x, y, z$ components of $J_{CH}$ are denoted by $J_{CHx}, J_{CHy}, J_{CHz}$ respectively.

Now, from equation (2.15)

\[
\mathbf{J}_{CH} = \sigma[\hat{a}_x E_x + \hat{a}_y E_y + \hat{a}_z E_z - \hat{a}_x E_{Hx} - \hat{a}_y E_{Hy} - \hat{a}_z E_{Hz}] (2.18)
\]

Therefore, from (2.17) and (2.18)

\[
\begin{align*}
J_{CHx} &= \sigma[E_x - E_{Hx}] = \sigma E_x \\
J_{CHy} &= \sigma[E_y - E_{Hy}] = \sigma[E_y + R_c B_0 J_{CHz}] \\
J_{CHz} &= \sigma[E_z - E_{Hz}] = \sigma[E_z - R_c B_0 J_{CHy}]
\end{align*}
\]

From equation (2.20)

\[
\sigma R_c B_0 J_{CHy} = \sigma^2 R_c B_0 E_y + \sigma^2 R_c^2 B_0^2 J_{CHz} (2.22)
\]

From equation (2.21)

\[
\sigma R_c B_0 J_{CHy} = \sigma E_z - J_{CHz} (2.23)
\]
Subtracting (2.23) and (2.22)

\[ 0 = \sigma^2 R_c B_o E_y - \sigma E_z + (\sigma^2 R_c^2 B_o^2 + 1) J_{CHz} \]

\[ J_{CHz} = \frac{\sigma}{(1 + \sigma^2 R_c^2 B_o^2)} E_z - \frac{\sigma^2 R_c B_o}{(1 + \sigma^2 R_c^2 B_o^2)} E_y \]  \hspace{1cm} (2.24)

From (2.20) and (2.24)

\[ J_{CHy} = (\sigma - \frac{\sigma^2 R_c B_o^2}{1 + \sigma^2 R_c^2 B_o^2}) E_y + \frac{\sigma^2 R_c B_o}{1 + \sigma^2 R_c^2 B_o^2} E_z \]

\[ = \frac{\sigma}{1 + \sigma^2 R_c^2 B_o^2} E_y + \frac{\sigma^2 R_c B_o}{1 + \sigma^2 R_c^2 B_o^2} E_z \]  \hspace{1cm} (2.25)

### 2.3.2 The Matrix Elements of the Complex Permittivity Tensor

From equation (2.16)

\[ \mathbf{J} = j\omega \mathbf{E} + \mathbf{J}_{CH} \]

\[ J_x = j\omega E_x + J_{CHx} = j\omega E_x + \sigma E_x \quad \text{(from (2.19))} \]

\[ = (\sigma + j\omega) E_x = j\omega (1 - \frac{j\sigma}{\omega}) E_x \]

\[ = \varepsilon_1 j\omega E_x \]  \hspace{1cm} (2.26)

Using equation (2.25) and (2.16)

\[ J_y = j\omega E_y + J_{CHy} = j\omega E_y + \frac{\sigma}{1 + \sigma^2 R_c^2 B_o^2} E_y + \frac{\sigma^2 R_c B_o}{1 + \sigma^2 R_c^2 B_o^2} E_z \]

\[ = j\omega \left[ 1 - \frac{j\sigma}{\omega (1 + \sigma^2 R_c^2 B_o^2)} \right] E_y + \frac{R_c B_o \sigma^2}{1 + R_c^2 B_o^2 \sigma^2} E_z \]
\[ = \varepsilon \left[ 1 - \frac{j\sigma}{w \varepsilon (1 + \frac{R_c^2 B_0^2}{\sigma^2})} \right] \begin{bmatrix} jwE_y \\ jwE_y - \frac{jR_c B_0 \sigma^2}{w (1 + \frac{R_c^2 B_0^2}{\sigma^2})} \end{bmatrix} \]

\[ \equiv \varepsilon_2 jwE_y - \varepsilon_3 jwE_z \]

\[ J_z = jwE E_z + J_{CHz} \]

\[ = jwE E_z + \frac{\sigma}{1 + \sigma^2 R_c^2 B_0^2} E_z - \frac{\sigma^2 R_c B_0}{1 + \sigma^2 R_c^2 B_0^2} E_y \]

\[ = jwE \left[ 1 - \frac{j\sigma}{w \varepsilon (1 + \frac{R_c^2 B_0^2}{\sigma^2})} \right] E_z - \frac{\sigma^2 R_c B_0}{1 + \sigma^2 R_c^2 B_0^2} E_y \]

\[ = \varepsilon \left[ 1 - \frac{j\sigma}{w \varepsilon (1 + \frac{R_c^2 B_0^2}{\sigma^2})} \right] jwEz + \frac{jR_c B_0 \sigma^2}{w (1 + \frac{R_c^2 B_0^2}{\sigma^2})} jwEy \]

\[ \equiv \varepsilon_2 jwE_z + \varepsilon_3 jwE_y \]

Let, \[ J = [\varepsilon] \frac{\partial E}{\partial t} \]

Therefore

\[
\begin{bmatrix}
J_x \\
J_y \\
J_z
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}
\end{bmatrix}
\begin{bmatrix}
jwE_x \\
jwE_y \\
jwE_z
\end{bmatrix}
\]

Comparing with equations (2.26), (2.27) and (2.28) it is seen that

\[ \varepsilon_{11} = \varepsilon_1 = \varepsilon (1 - \frac{j\sigma}{w \varepsilon}) \]

\[ \varepsilon_{12} = 0 \]

\[ \varepsilon_{13} = 0 \]
\[ \varepsilon_{21} = 0 \]
\[ \varepsilon_{22} = \varepsilon_2 = \varepsilon \left\{ 1 - \frac{j \sigma}{w \varepsilon (1 + R_C R_B \sigma^2)} \right\} \]
\[ \varepsilon_{23} = - \varepsilon_3 \]
\[ \varepsilon_{31} = 0 \]
\[ \varepsilon_{32} = \varepsilon_3 = \frac{j R_C R_B \sigma^2}{w (1 + R_C R_B \sigma^2)} \]
\[ \varepsilon_{33} = \varepsilon_2 \]

Therefore, when a magnetic field is applied in the x-direction, the total current density in a semiconductor can be written as

\[ J = [\varepsilon] \frac{\partial E}{\partial t} \quad (2.29) \]

and thus the elements of the complex permittivity tensor can be written as

\[ [\varepsilon] = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & -\varepsilon_3 \\ 0 & \varepsilon_3 & \varepsilon_2 \end{bmatrix} \]

where

\[ \varepsilon_1 = \varepsilon \left\{ 1 - \frac{j \sigma}{w \varepsilon (1 + R_C R_B \sigma^2)} \right\} \]
\[ \varepsilon_2 = \varepsilon \left\{ 1 - \frac{j \sigma}{w \varepsilon (1 + R_C R_B \sigma^2)} \right\} = \varepsilon \left\{ 1 - \frac{j \sigma}{w \varepsilon (1 + (R_C R_B \sigma)^2)} \right\} \]
\[ \varepsilon_3 = \frac{j R_C R_B \sigma^2}{w (1 + R_C R_B \sigma^2)} \]
\[ = \frac{j R_C R_B \sigma^2}{w \{1 + (R_C R_B \sigma)^2\}} \]
CHAPTER III
THEORETICAL ANALYSIS OF THE ELECTROMAGNETIC FIELD COMPONENTS

3.1 GENERAL CONSIDERATION

A rectangular waveguide as shown in Figure (3.1) is completely filled with a semiconductor. The coordinate system to be used in this analysis is shown in this figure. A steady transverse magnetic field \( B_0 \) is applied in the x-direction.

From equation (2.29) in section 2.3.2, the total current density in the semiconductor having a complex permittivity tensor can be written as

\[
J = [\varepsilon] \frac{\partial E}{\partial t} \tag{3.1}
\]

where,

\[
[\varepsilon] = \begin{bmatrix}
\varepsilon_1 & 0 & 0 \\
0 & \varepsilon_2 & -\varepsilon_3 \\
0 & \varepsilon_3 & \varepsilon_2
\end{bmatrix}
\]

and

\[
\varepsilon_1 = \varepsilon \left[ 1 - \frac{j\sigma}{\omega \varepsilon} \right] \\
\varepsilon_2 = \varepsilon \left[ 1 - \frac{j\sigma}{\omega \varepsilon (1 + (R_c B_0 \sigma)^2)} \right] \\
\varepsilon_3 = \frac{j R_c B_0 \sigma^2}{\omega (1 + (R_c B_0 \sigma)^2)}
\]

(3.2)
FIGURE 3.1  RECTANGULAR WAVEGUIDE COMPLETELY FILLED WITH SEMICONDUCTOR
Maxwell's curl equations in the case of the media having a complex permittivity tensor are

\[ \nabla \times E = -\mu \frac{\partial H}{\partial t} \tag{3.3} \]
\[ \nabla \times H = [\varepsilon] \frac{\partial E}{\partial t} \tag{3.4} \]

The time dependence of the electromagnetic fields is considered to be \( e^{j\omega t} \) and the z-dependence of the fields is assumed to be \( e^{-r z} \)

where \( r = \) propagation constant

\[ = \alpha + j\beta \]

\( \alpha = \) attenuation constant (nepers/meter)
\( \beta = \) phase constant (radians/meter)

Equations (3.3) and (3.4) are written in the following form to show the tensor permittivity.

\[
\begin{bmatrix}
0 & r & \frac{\partial}{\partial y} \\
-r & 0 & -\frac{\partial}{\partial x} \\
-\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
= -j\omega
\begin{bmatrix}
H_x \\
H_y \\
H_z
\end{bmatrix}
\tag{3.5}
\]

\[
\begin{bmatrix}
0 & r & \frac{\partial}{\partial y} \\
-r & 0 & -\frac{\partial}{\partial x} \\
-\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y \\
H_z
\end{bmatrix}
= j\omega
\begin{bmatrix}
\varepsilon_1 & 0 & 0 \\
0 & \varepsilon_2 & -\varepsilon_3 \\
0 & \varepsilon_3 & \varepsilon_2
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
\tag{3.6}
\]
From (3.5) and (3.6) the following six equations can be written as

\[ rH'_y + \frac{\partial H_z}{\partial y} = jw \varepsilon_1 E_x \quad (3.7) \]

\[-rH_x - \frac{\partial H_z}{\partial x} = jw \varepsilon_2 E_y - jw \varepsilon_3 E_z \quad (3.8)\]

\[-sH_x + \frac{\partial H_y}{\partial x} = jw \varepsilon_3 E_y + jw \varepsilon_2 E_z \quad (3.9)\]

\[ rE_y + \frac{\partial E_z}{\partial y} = -jw \mu H_x \quad (3.10)\]

\[-rE_x + \frac{\partial E_z}{\partial x} = -jw \mu H_y \quad (3.11)\]

\[-\frac{\partial}{\partial y} E_x + \frac{\partial}{\partial x} E_y = -jw \mu H_z \quad (3.12)\]

From equations (3.7), (3.11), \( E_x \) can be expressed in terms of \( E_z \) and \( H_z \) as

\[ E_x = \frac{1}{(r^2+w^2\mu_\varepsilon_1)} \left( -r \frac{\partial E_z}{\partial x} - jw \mu \frac{\partial H_z}{\partial y} \right) \quad (3.13) \]

Using equations (3.8), (3.10), \( E_y \) can be expressed in terms of \( E_z \) and \( H_z \) as

\[ E_y = \frac{1}{(r^2+w^2\mu_\varepsilon_2)} \left( w^2 \mu_\varepsilon_3 - r \frac{\partial}{\partial y} E_z + jw \mu \frac{\partial H_z}{\partial x} \right) \quad (3.14) \]

From equations (3.10) and (3.14), \( H_x \) can be written as

\[ H_x = \frac{1}{(r^2+w^2\mu_\varepsilon_2)} \left( jw (r \varepsilon_3 + \varepsilon_2 \frac{\partial}{\partial y}) E_z - r \frac{\partial}{\partial x} H_z \right) \quad (3.15) \]

From equations (3.11), (3.13), \( H_y \) can be written in terms of \( E_z \) and \( H_z \) as

\[ H_y = -\frac{1}{(r^2+w^2\mu_\varepsilon_1)} \left[ jw \varepsilon_1 \frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y} \right] \quad (3.16) \]
Putting the values of $E_x$, $E_y$, $H_x$, $H_y$ in equations (3.9) and (3.12), the following partial differential equations in $E_z$ and $H_z$ are obtained,

\[
\begin{align*}
E_z &= \left[ \frac{\varepsilon_1}{(r^2+w^2\mu_1)} \frac{\partial^2}{\partial x^2} + \frac{\varepsilon_2}{(r^2+w^2\mu_2)} \frac{\partial^2}{\partial y^2} + \frac{\left\{ \frac{r^2 e_2 + w^2 (e_2^2 + e_3^2)}{r^2 + w^2 \mu_2} \right\}}{r^2 + w^2 \mu_2} \right] E_z \\
&+ jw \left[ \frac{\varepsilon_1 - \varepsilon_2}{(r^2+w^2\mu_1)(r^2+w^2\mu_2)} \frac{\partial}{\partial y} + \frac{e_3}{(r^2+w^2\mu_2)} \right] \frac{\partial H_z}{\partial x} = 0 \quad (3.17) \\
&- jw \left[ \frac{(e_2-e_1)}{(r^2+w^2\mu_1)(r^2+w^2\mu_2)} \frac{r}{(r^2+w^2\mu_1)} \frac{\partial}{\partial y} + \frac{e_3}{(r^2+w^2\mu_2)} \right] \frac{\partial E_z}{\partial x} \\
&+ \left[ \frac{1}{(r^2+w^2\mu_2)} \frac{\partial^2}{\partial x^2} + \frac{1}{(r^2+w^2\mu_1)} \frac{\partial^2}{\partial y^2} + 1 \right] H_z = 0 \quad (3.18)
\end{align*}
\]

3.1.1 ANALYSIS OF TE MODE PROPAGATION

If TE modes are considered, then by putting $E_z = 0$, equations (3.17) and (3.18) reduce to

\[
\begin{align*}
\left[ \frac{\varepsilon_1 - \varepsilon_2}{(r^2+w^2\mu_1)} \frac{\partial}{\partial y} + e_3 \right] \frac{\partial H_z}{\partial x} &= 0 \\
\left[ \frac{1}{(r^2+w^2\mu_2)} \frac{\partial^2}{\partial x^2} + \frac{1}{(r^2+w^2\mu_1)} \frac{\partial^2}{\partial y^2} + 1 \right] H_z &= 0 \quad (3.19)
\end{align*}
\]

The above two equations can not be simultaneously satisfied by the same value of $r$ unless \( \frac{\partial}{\partial x} = 0 \) or $H_z = 0$, even if \( \varepsilon_1 = \varepsilon_2 \).

When \( \frac{\partial}{\partial x} = 0 \),
the equations (3.19) reduce to
\[
\left[ \frac{\partial^2}{\partial y^2} + (r^2 + w^2 \mu e_1) \right] H_z = 0
\]

The solution of this equation is

\[
H_z = c' \cos \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} y + c'' \sin \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} y
\]

where, \( c' \) and \( c'' \) are arbitrary constants.

Boundary conditions are at \( y = 0 \), \( \frac{\partial H_z}{\partial y} = 0 \), \( y = b \), \( \frac{\partial H_z}{\partial y} = 0 \)

Hence,

\[
\frac{\partial H_z}{\partial y} = - \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} c' \sin \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} y
\]

\[
+ c'' \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} \cos \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} y
\]

\[0 = c'' \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} \]

\[\therefore c'' = 0 \]

\[H_z = c' \cos \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} y \]

At \( y = b \), \( \frac{\partial H_z}{\partial y} = 0 \)

\[0 = - c' \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} \sin \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} b \]

or \( \sin \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} b = 0 \)

Therefore, \( \left( r^2 + w^2 \mu e_1 \right)^{\frac{1}{2}} = \frac{n\pi}{b} \)

where \( n \) is an integer, \( H_z = c' \cos \frac{n\pi}{b} \)

\[\therefore r^2 + w^2 \mu e_1 = \frac{n^2 \pi^2}{b^2} \]

or \( r^2 = \frac{n^2 \pi^2}{b^2} - w^2 \mu e_1 \)
Since the above expression for $\varepsilon_1$ does not contain any term B (the applied magnetic field) the wave numbers and the associated fields do not depend on the magnetostatic field. Therefore, it is seen that only those TE modes which have $x$-independent field components can propagate in the semiconductor filled guide in the presence of an external magnetic field.

From equations (3.13), (3.14) it can be found that the $\text{TE}_{on}$ modes which can propagate through the waveguide are characterized by having only the component of the electric field along the $x$ direction, $E_x$. Since the external magnetic field is applied in the same direction, there will not be a Hall field produced and the characteristics of these modes should evidently remain unaltered by the application of the external magnetic field. Hence it can be concluded that $\text{TE}_{on}$ modes hold no interest in this analysis.

### 3.1.2 ANALYSIS OF TM MODE PROPAGATION

TM modes can be investigated by putting $H_z = 0$ in equations (3.17) and (3.18), which then reduce to the following two equations.

\[
\begin{align*}
\left[ \frac{\varepsilon_1}{(\Gamma^2 + w^2 \mu \varepsilon_1)} \frac{\partial^2}{\partial x^2} + \frac{\varepsilon_2}{(\Gamma^2 + w^2 \mu \varepsilon_2)} \frac{\partial^2}{\partial y^2} + \frac{\left( \Gamma^2 \varepsilon_2 + w^2 \mu (\varepsilon_2 + \varepsilon_3^2) \right)}{(\Gamma^2 + w^2 \mu \varepsilon_2)} \right] E_z &= 0 \\
\left[ \frac{(\varepsilon_2 - \varepsilon_1)}{(\Gamma^2 + w^2 \mu \varepsilon_1)} \frac{\partial}{\partial y} + \varepsilon_3 \right] \frac{\partial E_z}{\partial x} &= 0
\end{align*}
\]

(3.20)

The above two equations can not be simultaneously satisfied for the same value of $\Gamma$, even if $\varepsilon_1 = \varepsilon_2$ unless $\frac{\partial}{\partial x} = 0$ or $E_z = 0$. However, if $\frac{\partial}{\partial x} = 0$, $E_z$ must vanish everywhere since it must vanish at the boundaries.
Therefore, it is seen that in a rectangular waveguide filled with semiconductor, in the presence of an external steady transverse magnetic field, TM modes can not propagate.

3.1.3 ANALYSIS OF ANOMALOUS MODE PROPAGATION

The equations (3.17) and (3.18) however can be solved by the same value of $\Gamma$ if $E_z \neq 0$ and $H_z \neq 0$. Hence, in addition to $x$-independent $TE_{0n}$ modes, the so called anomalous modes having all the six field components are possible. Therefore, if the wave is to depend on the external applied magnetic field, all six components of the field are necessary, and these must depend on both transverse coordinates.
3.2 **RIGOROUS GENERAL SOLUTION USING SEPARATION OF VARIABLES METHOD**

A rigorous general solution of the field components has been made by employing a special form of the separation of the variables method.

To separate the variables, let

\[ E_z = u(x) e(y) \]
\[ H_z = v(x) h(y) \]

Substituting these in equations (3.17) and (3.18) the following two equations are obtained,

\[
\left[ \frac{\varepsilon_1}{(\Gamma^2 + \omega \varepsilon_1)} u'' e + \frac{\varepsilon_2}{(\Gamma^2 + \omega \varepsilon_2)} u e'' + \frac{\{\Gamma \varepsilon_2 + \omega (\varepsilon_2^2 + \varepsilon_3^2)\}}{(\Gamma^2 + \omega \varepsilon_2)} u e \right. \\
+ \frac{j \omega \mu}{(\Gamma^2 + \omega \varepsilon_2)(\Gamma^2 + \omega \varepsilon_1)} v' h' + \frac{j \omega \varepsilon_3}{(\Gamma^2 + \omega \varepsilon_2)} v' h \right] = 0 \quad (3.21)
\]

\[
\left[ \frac{-j \omega (\varepsilon_2 - \varepsilon_1)}{(\Gamma^2 + \omega \varepsilon_1)(\Gamma^2 + \omega \varepsilon_2)} u' e' - \frac{j \omega \varepsilon_3}{(\Gamma^2 + \omega \varepsilon_2)} u' e \\
+ \frac{1}{(\Gamma^2 + \omega \varepsilon_2)} v'' h + \frac{1}{(\Gamma^2 + \omega \varepsilon_1)} v h' + v h \right] = 0 \quad (3.22)
\]

where primes denote the differentiation.

After dividing equation (3.21) by \(v'e\) and the equation (3.22) by \(u'h\) the following two equations are obtained.

\[
F(x,y) = \frac{\varepsilon_1}{(\Gamma^2 + \omega \varepsilon_1)} u'' + \frac{\varepsilon_2}{(\Gamma^2 + \omega \varepsilon_2)} u e'' + \frac{\{\Gamma \varepsilon_2 + \omega (\varepsilon_2^2 + \varepsilon_3^2)\}}{(\Gamma^2 + \omega \varepsilon_2)} u e
\]

\[
+ \frac{j \omega \mu}{(\Gamma^2 + \omega \varepsilon_2)(\Gamma^2 + \omega \varepsilon_1)} v' h' + \frac{j \omega \varepsilon_3}{(\Gamma^2 + \omega \varepsilon_2)} v' h
\]

\[
\frac{-j \omega (\varepsilon_2 - \varepsilon_1)}{(\Gamma^2 + \omega \varepsilon_1)(\Gamma^2 + \omega \varepsilon_2)} u' e' - \frac{j \omega \varepsilon_3}{(\Gamma^2 + \omega \varepsilon_2)} u' e \\
+ \frac{1}{(\Gamma^2 + \omega \varepsilon_2)} v'' h + \frac{1}{(\Gamma^2 + \omega \varepsilon_1)} v h' + v h = 0
\]
The key to the separation of the variables is that the variations of \( F \) and \( G \) with respect to \( x \) and \( y \) must vanish. This follows from the fact that \( F \) and \( G \) are themselves zero for all values of \( x \) and \( y \).

\[
G(x,y) = \left[ \frac{jw(\varepsilon_1 - \varepsilon_2)\Gamma}{(\Gamma^2 + W^2\mu_\varepsilon_1)(\Gamma^2 + W^2\mu_\varepsilon_2)} \frac{e'}{h} - \frac{j\varepsilon_3}{(\Gamma^2 + W^2\mu_\varepsilon_2)} \frac{e}{h} \right]
+ \left[ \frac{1}{(\Gamma^2 + W^2\mu_\varepsilon_2)} \frac{v'''}{u'} + \frac{1}{(\Gamma^2 + W^2\mu_\varepsilon_1)} \frac{v'}{u'} \cdot \frac{h'''}{h} + \frac{v'}{u'} \right] = 0
\]

Hence, \( \frac{\partial^2 F}{\partial x \partial y} = 0 = \frac{\varepsilon_2}{(\Gamma^2 + W^2\mu_\varepsilon_2)} \cdot \frac{\partial^2 u}{\partial x \partial y} \cdot \frac{e'''}{e} \)

or \( \frac{\partial^2}{\partial x \partial y} \left( \frac{u}{v'} \cdot \frac{e'''}{e} \right) = 0 \)

\( \frac{\partial}{\partial x} \left( \frac{u}{v'} \right) \cdot \frac{\partial}{\partial y} \left( \frac{e'''}{e} \right) = 0 \)

\( \therefore \frac{\partial}{\partial x} \left[ \frac{u}{v'} \right] = 0 \)

\( \frac{u}{v'} = \text{constant} \)

\( \therefore v'(x) = - c'u(x) \)
Similarly, \[ \frac{\partial^2 G}{\partial x \partial y} = 0 = \frac{\partial^2}{\partial x \partial y} \left[ \frac{v}{u'} \cdot \frac{h'''}{h} \right] \]

\[ \therefore \frac{\partial}{\partial x} \left( \frac{v}{u'} \right) - \frac{\partial}{\partial y} \left( \frac{h'''}{h} \right) = 0 \]

or \[ \frac{\partial}{\partial x} \left( \frac{v}{u'} \right) = 0 \]

\[ \therefore \frac{v}{u'} = \text{constant} \]

or, \[ u'(x) = cv(x) \]

Therefore, the vanishing variations constrain \( u \) and \( v \) to the relations

\[ v'(x) = -c'u(x) \quad (3.23) \]
\[ u'(x) = cv(x) \quad (3.24) \]

where \( c \) and \( c' \) are separation constants.

The appropriate boundary conditions are

\[ E_z = 0 \text{ at } x = 0, x = a \]

\[ \therefore u = 0 \text{ at } x = 0, x = a \]

Again \[ \frac{\partial H_z}{\partial x} = 0 \text{ at } x = 0, x = a \]

\[ \therefore v' = 0 \text{ at } x = 0, a \]

From equations (3.23) and (3.24)

\[ \frac{d^2 u}{dx^2} + cc'u = 0 \]

or, \[ u = A \cos \alpha_m x + B \sin \alpha_m x \]

where \( \alpha_m = (cc')^{1/2} \)

\[ u = 0, \text{ at } x = 0 \text{ makes } A = 0 \]

\[ \therefore u = B \sin \alpha_m x \]
Again \( u = 0, \ x = a \)
\[
\therefore \ 0 = B \sin \alpha_m a
\]
\[
\therefore \alpha_m = \frac{n\pi}{a}
\]

Similarly putting the boundary conditions on the solutions of \( v''' = -c'cv \)
gives \( v = \cos \frac{n\pi}{a} \).

Therefore, the solutions of (3.23) and (3.24) give solutions \( u \) and \( v \)
of the following form
\[
u = \sin \alpha_m x, \quad v = \cos \alpha_m x
\]
where \( \alpha_m = c = c' = \frac{n\pi}{a} \) (3.25)

With the help of equations ((3.23) and (3.24) and (3.25)) \( u \), and
\( v \) can be eliminated from equations (3.21) and (3.22) to obtain the
following two equations for \( e \) and \( h \).
\[
\begin{bmatrix}
a_1 \frac{d^2}{dy^2} + b_1
\end{bmatrix} e - \begin{bmatrix}
c_1 \frac{d}{dy} + d_1
\end{bmatrix} h = 0
\]
\[
\begin{bmatrix}
c_2 \frac{d}{dy} - d_2
\end{bmatrix} e + \begin{bmatrix}
a_2 \frac{d^2}{dy^2} + b_2
\end{bmatrix} h = 0
\]

(3.26)

where \( a_1, b_1, c_1, d_1 \), and \( a_2, b_2, c_2 \) and \( d_2 \) are given below.
\[
a_1 = \frac{c_2}{(\Gamma^2 + w^2 \mu \varepsilon_2)} \quad ; \quad b_1 = \frac{\Gamma^2 \varepsilon_2 + w^2 \mu (\varepsilon_2 + \varepsilon_3^2)}{(\Gamma^2 + w^2 \mu \varepsilon_2)} - (\Gamma^2 + w^2 \mu \varepsilon_2) \frac{n\pi}{a} \frac{\Gamma^2 \varepsilon_2}{\varepsilon_1} \\
\]
\[
c_1 = \frac{j\omega \mu \varepsilon_1 (\varepsilon_1 - \varepsilon_2) \frac{n\pi}{a}}{(\Gamma^2 + w^2 \mu \varepsilon_2)(\Gamma^2 + w^2 \mu \varepsilon_1)} \quad ; \quad d_1 = \frac{j\omega \mu \varepsilon_3 \frac{n\pi}{a}}{(\Gamma^2 + w^2 \mu \varepsilon_2)}
\]
Equations (3.26) are two second order linear differential equations in $e$ and $h$. The determinantal equation for these two equations is

\[
\begin{bmatrix}
\frac{d^4}{dy^4} + B \frac{d^2}{dy^2} + C
\end{bmatrix}
\begin{bmatrix}
e \\
\{h\}
\end{bmatrix} = 0
\]

where

\[
B = \frac{a_1 b_2 + a_2 b_1 + c_1 c_2}{a_1 a_2}
\]

\[
C = \frac{b_1 b_2 - d_1 d_2}{a_1 a_2}
\]

\[
= \left(\frac{\pi}{a_1^2}\right)^2 - n^2 \pi^2 \epsilon_2 \left(\frac{\pi}{a_1^2}\right)
\]

Multiplying both sides by $\epsilon_2$ the equation (3.27) can also be written in the following form
\[
\left[ p \frac{d^4}{dy^4} + Q \frac{d^2}{dy^2} + R \right] \{e\} = 0
\]

(3.28)

where

\[
P = \epsilon_2
\]

\[
Q = 2r^2 \epsilon_2 + w^2 \mu (\epsilon_2^2 + \epsilon_3^2 + \epsilon_1 \epsilon_2) - \frac{n_2^2 \pi^2}{a^2} (\epsilon_1 + \epsilon_2)
\]

\[
R = r^4 \epsilon_2 + r^2 \left\{ w^2 \mu (\epsilon_2^2 + \epsilon_3^2 + \epsilon_1 \epsilon_2) - \frac{n_2^2 \pi^2}{a^2} (\epsilon_1 + \epsilon_2) \right\}
\]

\[
+ \left\{ w^4 \mu^2 \epsilon_1 (\epsilon_2^2 + \epsilon_3^2) - 2w^2 \mu \epsilon_1 \epsilon_2 \frac{n_2^2 \pi^2}{a^2} + \frac{n_4 \pi^4}{a^4} \epsilon_1 \right\}
\]

\[
\left[ p \frac{d^4}{dy^4} + Q \frac{d^2}{dy^2} + R \right] e = 0
\]

Now, the auxiliary equation of the above differential equation is

\[
P m^4 + Q m^2 + R = 0
\]

The four roots of this equation are

\[
\pm m_1 = \pm \left[ - \frac{Q}{2P} + \left\{ \frac{Q}{2P} - \frac{R}{P} \right\}^{\frac{1}{2}} \right]^{\frac{1}{2}}
\]

(3.29)

\[
\pm m_2 = \pm \left[ - \frac{Q}{2P} - \left\{ \frac{Q}{2P} - \frac{R}{P} \right\}^{\frac{1}{2}} \right]^{\frac{1}{2}}
\]

where

\[
\frac{Q}{2P} = r^2 + \frac{w^2 \mu}{2} (\epsilon_1 + \epsilon_2 + \epsilon_3^2) - \frac{n_2^2 \pi^2}{a^2} \frac{(\epsilon_1 + \epsilon_2)}{2\epsilon_2}
\]

and

\[
\left\{ \frac{Q}{2P} - \frac{R}{P} \right\}^{\frac{1}{2}} = \frac{1}{2} \left[ \left\{ \frac{\epsilon_1 - \epsilon_2}{\epsilon_2} \right\} \left( w^2 \mu \epsilon_2 - \frac{n_2^2 \pi^2}{a^2} \right) - w^2 \mu \frac{\epsilon_3^2}{\epsilon_2} \right]^{2} - 4w^2 \mu \frac{\epsilon_1 \epsilon_3^2}{\epsilon_2^2} \frac{n_2^2 \pi^2}{a^2} \right]^{\frac{1}{2}}
\]
Mikaelian\(^{(12)}\) and H. Seidel\(^{(13)}\) have shown that the boundary conditions at \(y = 0\) and \(y = b\) can not be satisfied by either of the two independent birefringent modes corresponding to the four roots \(\pm m_1, 2\) but that a linear combination of these must be used. Thus,

\[
e(y) = A e^{m_1 y} + B e^{-m_1 y} + C e^{m_2 y} + D e^{-m_2 y}
\]

Therefore,

\[
E_z = (A e^{m_1 y} + B e^{-m_1 y} + C e^{m_2 y} + D e^{-m_2 y}) \sin \frac{\pi x}{a} e^{j\omega t - yz} \tag{3.31}
\]

where \(A, B, C, D\) are four arbitrary constants.

The co-efficients of equation (3.28) do not contain any term containing odd powers of \(r\), so that propagation in general will be of a reciprocal nature. Also from equation (3.2) the diagonal elements of the complex permittivity tensor are found to be equal if the value of \(B_o\) is such that \((R_c B_o \sigma)^2 < 1\). In the present analysis, the maximum value of \((R_c B_o \sigma)^2\) is approximately 0.15 and therefore, the assumption \((R_c B_o \sigma)^2 < 1\) is quite reasonable. This makes \(\epsilon_1 = \epsilon_2\) and this assumption will be used to obtain the expressions for the field components. This simplifies the expressions of the field components and with this assumption, the expressions for \(m_1\) and \(m_2\) from equations (3.29) reduce to the following forms.

\[
\begin{align*}
m_1^2 &= \frac{n^2 \pi^2}{a^2} - w^2 \mu \varepsilon_2 - r^2 - \frac{w^2 \mu \varepsilon_3}{2 \varepsilon_2} + \frac{1}{2} \left\{ \left( \frac{w^2 \mu \varepsilon_3}{\varepsilon_2} \right)^2 - 4w^2 \mu \varepsilon_3^2 \frac{n^2 \pi^2}{a^2} \right\}^{1/2} \\
m_2^2 &= \frac{n^2 \pi^2}{a^2} - w^2 \mu \varepsilon_2 - r^2 - \frac{w^2 \mu \varepsilon_3}{2 \varepsilon_2} - \frac{1}{2} \left\{ \left( \frac{w^2 \mu \varepsilon_3}{\varepsilon_2} \right)^2 - 4w^2 \mu \varepsilon_3^2 \frac{n^2 \pi^2}{a^2} \right\}^{1/2}
\end{align*}
\]
Let
\[ a_3 = \frac{1}{2} w^2 \frac{\varepsilon_3}{\varepsilon_2} \]
\[ b_3 = \frac{1}{2} \left( \frac{w^2 \mu_3}{\varepsilon_2} \right)^2 - 4w^2 \frac{\varepsilon_3^2}{\varepsilon_2} \frac{n^2 \pi^2}{a^2} \]
\[ m_1^2 = \frac{n^2 \pi^2}{a^2} - w^2 \mu_2 - r^2 - a_3 + b_3 \]
\[ m_2^2 = \frac{n^2 \pi^2}{a^2} - w^2 \mu_2 - r^2 - a_3 - b_3 \]

Now, substituting the expression of \( e(y) \) from (3.30) in equations (3.26) and solving for the expression of \( h(y) \) the following expression for \( h(y) \) is obtained
\[ h(y) = \frac{e_2}{jw \mu e_3} \left( \frac{a}{n \pi} \right) \left\{ A(a_3 + b_3) e^{m_1 y} + B(a_3 + b_3) e^{-m_1 y} \right. \]
\[ + C(a_3 - b_3) e^{m_2 y} + D(a_3 - b_3) e^{-m_2 y} \}

Therefore, the expression for \( H_z \) is given by
\[ H_z = \frac{e_2}{jw \mu e_3} \left( \frac{a}{n \pi} \right) \left\{ A(a_3 + b_3) e^{m_1 y} + B(a_3 + b_3) e^{-m_1 y} + C(a_3 - b_3) e^{m_2 y} \right. \]
\[ + D(a_3 - b_3) e^{-m_2 y} \} \cos \frac{n \pi x}{a} e^{j(wt - rz)} \]

Now substituting the expressions for \( E_z \) and \( H_z \) in equations (3.13), (3.14), (3.15) and (3.16) the expressions for \( E_x, E_y, H_x, H_y \) can be obtained. The expressions for all the six field components are written below.
\[ E_x = \left( \frac{a}{n\pi} \right) \left[ \frac{-\varepsilon_2}{\varepsilon_3 (\Gamma^2 + w^2 \mu \varepsilon_2)} \right] \left[ \begin{array}{c} -m_1 y \\ \left( a_3 + b_3 \right) \end{array} \right] + \frac{\Gamma \varepsilon_3 n_2^2}{\varepsilon_2 a^2} \]

\[ + B e^{-m_1 y} \left\{ m_1 (a_3 + b_3) + \frac{\Gamma \varepsilon_3 n_2^2}{\varepsilon_2 a^2} \right\} \]

\[ + C e^{m_2 y} \left\{ m_2 (a_3 - b_3) + \frac{\Gamma \varepsilon_3 n_2^2}{\varepsilon_2 a^2} \right\} \]

\[ + D e^{-m_2 y} \left\{ -m_2 (a_3 - b_3) + \frac{\Gamma \varepsilon_3 n_2^2}{\varepsilon_2 a^2} \right\} \cos \frac{n\pi x}{a} \sqrt{(j\omega t - \tau)} \quad (3.33) \]

\[ E_y = \frac{1}{(\Gamma^2 + w^2 \mu \varepsilon_2)} \left[ \begin{array}{c} A e^{-m_1 y} \left\{ -w^2 \mu \varepsilon_3 - \Gamma m_1 - \frac{\varepsilon_2}{\varepsilon_3} (a_3 + b_3) \right\} \\
+ B e^{-m_1 y} \left\{ \frac{w^2 \mu \varepsilon_3}{\varepsilon_3} + \Gamma m_1 - \frac{\varepsilon_2}{\varepsilon_3} (a_3 + b_3) \right\} \end{array} \right] \quad (3.34) \]

\[ + C e^{m_2 y} \left\{ \frac{w^2 \mu \varepsilon_3}{\varepsilon_3} - \Gamma m_2 - \frac{\varepsilon_2}{\varepsilon_3} (a_3 - b_3) \right\} \]

\[ + D e^{-m_2 y} \left\{ \frac{w^2 \mu \varepsilon_3}{\varepsilon_3} + \Gamma m_2 - \frac{\varepsilon_2}{\varepsilon_3} (a_3 - b_3) \right\} \cos \frac{n\pi x}{a} \sqrt{(j\omega t - \tau)} \sin \frac{n\pi x}{a} \sqrt{(j\omega t - \tau)} \quad (3.35) \]

\[ E_z = (A e^{-m_1 y} + B e^{-m_1 y} + C e^{m_2 y} + D e^{-m_2 y}) \sin \frac{n\pi x}{a} \sqrt{(j\omega t - \tau)} \quad (3.35) \]

\[ H_x = \frac{\mu \varepsilon_2}{j\omega \mu \varepsilon_3 (\Gamma^2 + w^2 \mu \varepsilon_2)} \left[ \begin{array}{c} A e^{-m_1 y} \left\{ -m_1 \frac{w^2 \mu \varepsilon_3}{\Gamma} - a_3 + b_3 \right\} \\
+ B e^{-m_1 y} \left\{ \frac{m_1 w^2 \mu \varepsilon_3}{\Gamma} - a_3 + b_3 \right\} \end{array} \right] \quad (3.36) \]

\[ + C e^{m_2 y} \left\{ -m_2 \frac{w^2 \mu \varepsilon_3}{\Gamma} - a_3 - b_3 \right\} \]

\[ + D e^{-m_2 y} \left\{ m_2 \frac{w^2 \mu \varepsilon_3}{\Gamma} - a_3 - b_3 \right\} \cos \frac{n\pi x}{a} \sqrt{(j\omega t - \tau)} \sin \frac{n\pi x}{a} \sqrt{(j\omega t - \tau)} \]
\begin{align}
H_y &= \left(\frac{a}{n} \right) \left\{ \frac{r \varepsilon_2}{jw \mu \varepsilon_3 \left(r^2 + w^2 \mu \varepsilon_2\right)} \right\} \left[ A e^{m_1 y} \left\{ \frac{w^2 \mu \varepsilon_3}{r} \left(\frac{n^2 \pi^2}{a^2} - m_1 (a_3 + b_3)\right) \right\} \\
&+ B e^{-m_1 y} \left\{ \frac{w^2 \mu \varepsilon_3}{r} \left(\frac{n^2 \pi^2}{a^2} + m_1 (a_3 + b_3)\right) \right\} \\
&+ C e^{m_2 y} \left\{ \frac{w^2 \mu \varepsilon_3}{r} \left(\frac{n^2 \pi^2}{a^2} - m_2 (a_3 - b_3)\right) \right\} \\
&+ D e^{-m_2 y} \left\{ \frac{w^2 \mu \varepsilon_3}{r} \left(\frac{n^2 \pi^2}{a^2} + m_2 (a_3 - b_3)\right) \right\} \right] \cos \frac{n \pi x}{a} e^{jwt - rz} \tag{3.37}
\end{align}

\begin{align}
H_z &= \frac{\varepsilon_2}{jw \mu \varepsilon_3} \left(\frac{a}{n} \right) \left[ A (a_3 + b_3) e^{m_1 y} + B (a_3 + b_3) e^{-m_1 y} \\
&+ C (a_3 - b_3) e^{m_2 y} + D (a_3 - b_3) e^{-m_2 y} \right] \cos \frac{n \pi x}{a} e^{jwt - rz} \tag{3.38}
\end{align}
CHAPTER IV

THEORETICAL AND NUMERICAL SOLUTION FOR THE PROPAGATION CONSTANT

4.1 THEORETICAL SOLUTION

Since the expressions for all the six electric and magnetic field components have been derived in the previous chapter, a transcendental equation for the propagation constant can be obtained by implementing the four boundary conditions on these field components. The four boundary conditions in a rectangular waveguide completely filled with a semiconductor for the electric field are given below.

\[ E_z = 0 \quad \text{at} \quad y = 0, \quad \text{and also at} \quad y = b \]
\[ E_x = 0 \quad \text{at} \quad y = 0, \quad \text{and also at} \quad y = b \]

Introducing the above four boundary conditions in equations (3.35) and (3.33) the following four linear homogeneous equations in \( A, B, C, \) and \( D \) are obtained.

\[
\begin{align*}
[A + B + C + D] &= 0 \\
[Ae^{m_1b} + Be^{-m_1b} + Ce^{m_2b} + De^{-m_2b}] &= 0 \\

\left[ A(m_1(a_3+b_3) + \frac{\epsilon_3}{\epsilon_2} \frac{n^2\pi^2}{a^2} \right] + B \left\{ -m_1(a_3+b_3) + \frac{n^2\pi^2}{a^2} \frac{\epsilon_3}{\epsilon_2} \right\} \\
+ C \left\{ m_2(a_3-b_3) + \frac{\epsilon_3}{\epsilon_2} \frac{n^2\pi^2}{a^2} \right\} + D \left\{ -m_2(a_3-b_3) + \frac{\epsilon_3}{\epsilon_2} \frac{n^2\pi^2}{a^2} \right\} &= 0
\end{align*}
\]
For the non-trivial case, the determinant of the co-efficients of the above set of equations must vanish.

Thus, \[
\begin{vmatrix}
1 & 1 & 1 & 1 \\
\frac{m_1 b}{e} & \frac{-m_1 b}{e} & \frac{m_2 b}{e} & \frac{-m_2 b}{e} \\
W_1 & W_2 & W_3 & W_4 \\
W_1 e^{m_1 b} & W_2 e^{-m_1 b} & W_3 e^{m_2 b} & W_4 e^{-m_2 b}
\end{vmatrix} = 0
\]

where,
\[
W_1 = m_1 (a_3 + b_3) + \frac{r \varepsilon_3}{\varepsilon_2} \frac{n^2 \pi^2}{a^2}
\]
\[
W_2 = -m_1 (a_3 + b_3) + \frac{r \varepsilon_3}{\varepsilon_2} \frac{n^2 \pi^2}{a^2}
\]
\[
W_3 = m_2 (a_3 - b_3) + \frac{r \varepsilon_3}{\varepsilon_2} \frac{n^2 \pi^2}{a^2}
\]
\[
W_4 = -m_2 (a_3 - b_3) + \frac{r \varepsilon_3}{\varepsilon_2} \frac{n^2 \pi^2}{a^2}
\]

The above determinantal equation (4.2) can also be written in the following form.
Expansion of the above determinantal equation leads to the following equation.

\[
\begin{vmatrix}
-e^{m_1b} - e^{m_2b} & e^{m_1b} & e^{m_1b} & -e^{m_2b} & e^{m_1b} \\
W_2 - W_1 & W_3 - W_1 & W_4 - W_1 & W_2e - W_1e & W_4e - W_1e \\
\end{vmatrix} = 0
\]

(4.3)

\[\begin{align*}
-2 \text{Sinh } m_1b \left\{ -2 W_3W_4 \text{ Sinh } m_2b + W_1W_3 (e^{m_2b} - e^{m_1b}) + W_1W_4 (e^{m_1b} - e^{-m_2b}) \right\} \\
+ \left( e^{m_1b} - e^{m_2b} \right) \left\{ -2 W_1W_2 \text{ Sinh } m_1b + W_1W_4 (e^{m_1b} - e^{-m_2b}) + W_4W_2 (e^{m_1b} - e^{m_2b}) \right\} \\
+ \left( e^{m_2b} - e^{m_1b} \right) \left\{ W_2W_3 (e^{m_2b} - e^{m_1b}) - 2W_1W_2 \text{ Sinh } m_1b + W_1W_3 (e^{m_1b} - e^{m_2b}) \right\}
\end{align*}
\]

which on simplifying after elaborate algebraic manipulations, reduces to the following form

\[4(W_1W_2 + W_3W_4) \text{ Sinh } m_1b \text{ Sinh } m_2b + 2(W_1W_3 + W_4W_2) \text{ Cosh}(m_1b - m_2b)
- 2(W_1W_4 + W_2W_3) \text{ Cosh}(m_1b + m_2b) - 2(W_1W_3 - W_1W_4 + W_2W_4) = 0\]

or,

\[2(W_1W_2 + W_3W_4) \text{ Sinh } m_1b \text{ Sinh } m_2b + (W_1W_3 + W_2W_4) \text{ Cosh}(m_1b - m_2b)
- (W_1W_4 + W_2W_3) \text{ Cosh}(m_1b + m_2b) + (W_1W_4 + W_2W_3) - (W_1W_3 + W_2W_4) = 0\]

or,

\[2(W_1W_2 + W_3W_4) \text{ Sinh } m_1b \text{ Sinh } m_2b + (W_1W_3 + W_2W_4) \left\{ \text{ Cosh } m_1b \text{ Cosh } m_2b
- \text{ Sinh } m_1b \text{ Sinh } m_2b \right\}\]
\[-(W_1 W_4 + W_2 W_3) \{ \cosh m_1 b \cosh m_2 b + \sinh m_1 b \sinh m_2 b \} + (W_1 W_4 + W_2 W_3) - (W_1 W_3 + W_2 W_4) \] = 0 \quad (4.5)

or,
\[
(W_1 W_4 + W_2 W_3) \{ \cosh m_1 b \cosh m_2 b \} \{ (W_1 W_3 + W_2 W_4) - (W_1 W_4 + W_2 W_3) \} + (W_1 W_4 + W_2 W_3) - (W_1 W_3 + W_2 W_4) \] = 0 \quad (4.6)

However, from equation (4.2a),
\[
\begin{align*}
(W_1 W_2 + W_3 W_4) &= 2r^2 \frac{\epsilon_3^2 n^4 \pi^4}{\frac{4}{a^2}} - m_1^2 (a_3 + b_3)^2 - m_2^2 (a_3 - b_3)^2 \\
(W_1 W_3 + W_2 W_4) &= 2m_1 m_2 (a_3^2 - b_3^2) + 2r^2 \frac{\epsilon_3^2 n^4 \pi^4}{\frac{2}{a^2}} \\
(W_1 W_4 + W_2 W_3) &= -2m_1 m_2 (a_3^2 - b_3^2) + 2r^2 \frac{\epsilon_3^2 n^4 \pi^4}{\frac{2}{a^2}}
\end{align*}
\quad (4.7)
\]

Substitution of these expressions into the determinantal equation (4.5) gives the following equation,
\[
(W_1 W_4 + W_2 W_3) \{ \cosh m_1 b \cosh m_2 b \} \{ (W_1 W_3 + W_2 W_4) - (W_1 W_4 + W_2 W_3) \} + \{ (W_1 W_4 + W_2 W_3) - (W_1 W_3 + W_2 W_4) \} = 0
\quad (4.8)
\]

or,
\[
(W_1 W_4 + W_2 W_3) \{ \cosh m_1 b \cosh m_2 b \} \{ (W_1 W_3 + W_2 W_4) - (W_1 W_4 + W_2 W_3) \} + \{ (W_1 W_4 + W_2 W_3) - (W_1 W_3 + W_2 W_4) \} = 0
\quad (4.9)
\]

or,
\[
(W_1 W_4 + W_2 W_3) \{ \cosh m_1 b \cosh m_2 b \} \{ (W_1 W_3 + W_2 W_4) - (W_1 W_4 + W_2 W_3) \} + \{ (W_1 W_4 + W_2 W_3) - (W_1 W_3 + W_2 W_4) \} = 0
\quad (4.10)
\]
The solution of the transcendental equation (4.9) gives the required value of the propagation constant. The coefficients $B$, $C$, and $D$ can be evaluated in terms of $A$ using equation (4.1) as follows,

\[
\begin{align*}
B &= A \frac{2m_1 e^{m_1 b} - (m_1 + m_2) e^{m_2 b} + (m_2 - m_1) e^{-m_2 b}}{\Delta} \\
C &= A \frac{(m_2 - m_1) e^{m_1 b} + (m_2 + m_1) e^{-m_1 b} - 2m_2 e^{-m_2 b}}{\Delta} \\
D &= A \frac{-(m_1 + m_2) e^{m_1 b} - (m_2 - m_1) e^{-m_1 b} + 2m_2 e^{m_2 b}}{\Delta}
\end{align*}
\]

where, \( \Delta = -2m_1 e^{-m_1 b} + (m_1 - m_2) e^{m_2 b} + (m_2 + m_1) e^{-m_2 b} \)

Equation (4.9) is a transcendental equation in $\Gamma$ and its solution gives the possible values of $\Gamma$. Knowing the value of $\Gamma$, the roots $m_1$ and $m_2$ can be evaluated from equation (3.32). As the coefficients $B$, $C$, and $D$ have been obtained in terms of $A$, all the field components can be expressed in terms of the excitation represented by $A$. Since these expressions are very complicated and since solutions of equation (4.9) may be obtained only numerically, it is difficult to examine the characteristics of the field distribution in the general case.
4.2.1 THE PRACTICAL WAVEGUIDE SYSTEM

A numerical solution of the transcendental equation (4.9) has been obtained for a practical system as shown in fig. (4.a). A rectangular waveguide of internal dimensions \( a = 0.0228 \) meter and \( b = 0.01015 \) meter is completely filled with an n-type germanium sample with thickness \( d = 0.00188 \) meter. The calculations were made at a frequency of 9.46 GHz, the experimental frequency. The conductivity of the sample was varied from 4 mho/meter to 4.65 mho/meter and the magnetic field was varied from 0 to 10 K gauss, in steps of 1 K gauss. The other values of the parameters used in the calculations are

\[
\varepsilon_0 = 8.854 \times 10^{-12} \text{ Farad/meter}, \quad \varepsilon_r = 16.
\]

\[
R_e \sigma = \frac{M H}{\lambda} = 0.426 \text{ m}^2/\text{volt-sec}[29]
\]

The solution of the transcendental equation gives the value of the propagation constant, the real and imaginary parts of which give the attenuation and phase constant respectively. The variation of the attenuation constant and the phase constant with the applied external magnetic field is given in Figure 4.b and 4.c.

4.2.2 NUMERICAL METHOD OF SOLUTION

The numerical solution of equation (4.9) was obtained with the Newton-Raphson iteration technique. To apply this technique the equation (4.9) may be put in the following form,

\[
F(r) = 2m_1m_2(a_3^2-b_3^2) (1-Cosh m_1b Cosh m_2b) + \{m_1^2(a_3+b_3)^2

+ m_2^2(a_3-b_3)^2\} \{Sinh m_1b Sinh m_2b\} = 0 \quad (4.11)
\]
frequency = 9.46 GHz
a = 0.0228 meter
b = 0.01015 meter
d = 0.00188 meter
\( \sigma = 4.50 \text{ mho/meter} \)
\( R_c \sigma = 0.426 \text{ m}^2/\text{v-sec.} \)
\( \varepsilon_r = 16 \)
n-type Germanium sample

FIGURE 4.a A PRACTICAL SYSTEM
where \( m_1 \) and \( m_2 \) are functions of \( r \) and their expressions are given in equation (3.32)

\[
F(r) \quad \text{when expressed in terms of } r \text{ becomes,}
\]

\[
F(r) = \{-r^2+G\}^{1/2} \{-r^2+H\}^{1/2} \{1-Cosh(-r^2+G)^{1/2} \ Cosh(-r^2+H)^{1/2} \}
\]

\[
+ \{-r^2+G\} \ C_1 \ + \{-r^2+H\} \ C_2 \} \ Sinh (-r^2+G)^{1/2} \ Sinh (-r^2+H)^{1/2} \quad = 0
\]

(4.12)

where

\[
G = \frac{\pi^2}{\alpha^2} - \omega^2 \mu \varepsilon \ 1 = a_3 + b_3
\]

\[
H = \frac{\pi^2}{\alpha^2} - \omega^2 \mu \varepsilon \ 1 = a_3 - b_3
\]

\[
C_1 = \frac{a_3 + b_3}{2(a_3 - b_3)} \quad (4.13)
\]

\[
C_2 = \frac{a_3 - b_3}{2(a_3 + b_3)}
\]

\[
a_3 = \frac{1}{2} \frac{\omega^2 \mu \varepsilon^2}{\varepsilon_1}
\]

\[
b_3 = \frac{1}{2} \left[ \left( \frac{\omega^2 \mu \varepsilon^2}{\varepsilon_1} \right)^2 - 4 \omega^2 \mu \varepsilon^2 \frac{n^2 \pi^2}{a^2} \right]^{1/2}
\]

If \( r_i \) is an approximation to a root of a function \( F(r) = 0 \), a better approximation is given by

\[
F_{i+1} = F_i - \frac{F(r_i)}{F'(r_i)} \quad (4.14)
\]

where the prime denotes the derivative. The expression for \( F'(r) \) is as follows.
\[ F'(r) = \left\{ \frac{(-r^2+G)(1-C_1) - (-r^2+H)C_2}{(-r^2+G)^{1/2}} \right\} rb \cosh(-r^2+G)^{1/2}b \sinh(-r^2+H)^{1/2}b \\
+ \left\{ \frac{(-r^2+H)(1-C_2) - (-r^2+G)C_1}{(-r^2+H)^{1/2}} \right\} rb \sinh(-r^2+G)^{1/2}b \cosh(-r^2+H)^{1/2}b \\
+ \frac{\Gamma}{\left( \frac{(-r^2+G)^{1/2}}{(-r^2+H)^{1/2}} \right)} \cosh(-r^2+G)^{1/2}b \cosh(-r^2+H)^{1/2}b \\
- 2\Gamma (C_1+C_2) \sinh(-r^2+G)^{1/2}b \sinh(-r^2+H)^{1/2}b \\
\left\{ \frac{(-r^2+G)^{1/2}}{(-r^2+H)^{1/2}} \right\} \right] \]

where the expressions for \( C_1, C_2, G, H \) are given in equation (4.13).

Using the relation (4.14), the numerical solution for the propagation constant has been made with an IBM 7040 computer and the computer programme for the solution of (4.9) is given at the end of the thesis.

4.2.3 NUMERICAL RESULTS.

The variations of the attenuation constant and the phase constant with the variations of the external applied transverse magnetic field obtained from the calculations are shown in figure (4.b) and figure (4.c). The magnetic field was varied from 0 to 10 Kilogauss in steps of 1 Kgauss.

It was found that the phase constant decreases with increasing applied magnetic field. The attenuation constant also decreases with
increasing magnetic field. The effect of the conductivity of the sample on the propagation constant was also calculated. The attenuation constant and the phase constant for different values of conductivity are also plotted in figure (4.d) and (4.e), as a function of the applied magnetic field.
$\mu$ (attenuation constant)

($\sigma = 4.5$ mho/meter)

frequency 9.46 GHz

n-type Ge sample
Figure 4.e

Phase constant in radian/meter

Applied magnetic field in weber/meter²

Frequency 9.46 GHz

n-type Ge sample
FIGURE 4.d

frequency 9.46 GHz
n-type Ge sample

IN NEPER/METER

ATTENUATION CONSTANT

APPLIED MAGNETIC FIELD $B_0$ IN WEBER/METER$^2$
frequency 9.46 GHz
n-type Ge sample

FIGURE 4.e

APPLIED MAGNETIC FIELD \( B_0 \) IN WEBER/METER^2
CHAPTER V

THEORY OF MEASUREMENT

5.1 THEORY OF THE REFLECTION BRIDGE

The schematic diagram of a high precision reflection bridge for directly measuring the magnitude and phase of reflection coefficient \( r \) and \( \theta \) respectively is shown in figure 5A. The sample is coupled to one side arm and the precision short and attenuator in the other are adjusted until the output is minimized.

The hybrid "tee" is converted into a magic "tee" by introducing slide screw tuners in the E and H arms and also in the side arms to compensate for asymmetry. The magic tee has the property that when the reflection coefficients at reference planes in the side arms are equal, the power fed to the detector in the E arm is zero.

In a practical system it is required to find the reflection coefficient \( r_x \), at a particular measuring plane of the colinear arm of the hybrid tee which contains the semiconductor sample. This reflection coefficient \( r_x = r_2 e^{2\gamma_s l_s} \)  

\[ r_x = r_2 e^{2\gamma_s l_s} \quad (5.1) \]

where  
\( r_2 \) = reflection coefficient at port 2
\( \gamma_s \) = propagation constant in the sample arm
\( l_s \) = distance between port 2 and the measuring plane
FIGURE 5A  A MICROWAVE REFLECTION BRIDGE
5.1.1 DERIVATION OF BALANCE EQUATION

In general the input and output relations of any passive four port (the hybrid tee) can be described by the equation

\[ b = sa \]  \hspace{1cm} (5.2)

where \( a \) and \( b \) are column matrices representing incident and scattered waves respectively and \( s \) is the scattering matrix given by

\[
\begin{bmatrix}
  s_{11} & s_{12} & s_{13} & s_{14} \\
  s_{12} & s_{22} & s_{23} & s_{24} \\
  s_{13} & s_{23} & s_{33} & s_{34} \\
  s_{14} & s_{24} & s_{34} & s_{44}
\end{bmatrix}
\]

Because of reciprocity, \( s \) is symmetrical.

The reflection coefficients

\[
\begin{align*}
  r_1 &= \frac{a_1}{b_1} \\
  r_2 &= \frac{a_2}{b_2}
\end{align*}
\]  \hspace{1cm} (5.3)

The null condition is

\[ a_4 = b_4 = 0 \]

Using equations (5.2) and (5.3) the following sets of equations are obtained.

\[
\begin{align*}
  b_1 &= s_{11} r_1 b_1 + s_{12} r_2 b_2 + s_{13} a_3 \\
  b_2 &= s_{12} r_1 b_1 + s_{22} r_2 b_2 + s_{23} a_3 \\
  b_3 &= s_{13} r_1 b_1 + s_{23} r_2 b_2 + s_{33} a_3 \\
  b_4 &= s_{14} r_1 b_1 + s_{24} r_2 b_2 + s_{34} a_3
\end{align*}
\]  \hspace{1cm} (5.4)
The above four equations can be written in the unknowns \( b_1, b_2, b_3, a_3 \) as

\[
\begin{align*}
(s_{11} r_1 - 1) b_1 + s_{12} r_2 b_2 + s_{13} a_3 &= 0 \\
(s_{12} r_1 b_1 + (s_{22} r_2 - 1) b_2 + s_{23} a_3 &= 0 \\
(s_{13} r_1 b_1 + s_{23} r_2 b_2 + s_{33} a_3 - b_3 &= 0 \\
(s_{14} r_1 b_1 + s_{24} r_2 b_2 + s_{34} a_3 &= 0 \\

\end{align*}
\]

The first, second and the fourth equation form the homogeneous set

\[
\begin{align*}
(s_{11} r_1 - 1) b_1 + s_{12} r_2 b_2 + s_{13} a_3 &= 0 \\
(s_{12} r_1 b_1 + (s_{22} r_2 - 1) b_2 + s_{23} a_3 &= 0 \\
(s_{14} r_1 b_1 + s_{24} r_2 b_2 + s_{34} a_3 &= 0 \\

\end{align*}
\]

which has a unique solution if the determinant of the coefficients is zero. Equating the determinant to zero gives

\[
\begin{vmatrix}
(s_{11} r_1 - 1) & s_{12} r_2 & s_{13} \\
(s_{12} r_1) & (s_{22} r_2 - 1) & s_{23} \\
(s_{14} r_1) & s_{24} r_2 & s_{34}
\end{vmatrix} = 0 
\]

(5.5)

and after some algebraic manipulation of this determinantal equation, the relationships between \( r_1 \) and \( r_2 \) can be obtained and is given below

\[
r_2 = \frac{s_{11} s_{34} - s_{13} s_{14}}{s_{22} s_{34} - s_{23} s_{24}} r_1 - \frac{s_{34}}{s_{22} s_{34} - s_{23} s_{24}}
\]

\[
r_2 = \frac{-(s_{11} s_{22} - s_{12} s_{23}) s_{34} + (s_{12} s_{13} - s_{11} s_{23}) s_{24} + (s_{12} s_{23} - s_{13} s_{22}) s_{14}}{s_{22} s_{34} - s_{23} s_{24}} r_1 - 1
\]
But \( r_x = r_2 e^{2y s l_s} \)

Therefore, \( r_x = \frac{a r_1 - b}{c r_1 - 1} \) \( (5.6) \)

where,

\[
a = \left[ \frac{s_{11}s_{34} - s_{13}s_{14}}{s_{22}s_{34} - s_{23}s_{24}} \right] e^{2y s l_s}
\]

\[
b = \left[ \frac{s_{34}}{s_{22}s_{34} - s_{23}s_{24}} \right] e^{2y s l_s} \quad \text{(5.6a)}
\]

\[
c = \left[ \frac{(s_{11}s_{22} - s_{12})s_{34} + (s_{12}s_{13} - s_{11}s_{23})s_{24} + (s_{12}s_{23} - s_{13}s_{22})s_{14}}{s_{22}s_{34} - s_{23}s_{24}} \right]
\]

The reflection coefficient \( r_1 \) of the reference arm has been shown by Champlin, K. S. and et al. \( (10) \) to be equal to

\( r_1 = K e^{-2(A_r + j \beta_r l_r)} \) \( (5.7) \)

where,

\( K = K_A^2 K_S \),

\( K_A \) = complex constant containing the residual attenuation and (fixed) phase shift of the attenuator

\( K_S \) = complex constant containing the zero setting and (fixed) loss of the short circuit.

\( A_r \) = reading of the precision attenuator in nepers

\( \beta_r l_r \) = reading of the precision short in radians (measured from some arbitrary reference)

Substituting the above equation (5.7) in the expression for \( r_x \) in equation (5.6) gives the following equation
\[
\frac{r_x}{C e^{-2[A_r+J_{Br}]r}} - B = A e^{-2[A_r+J_{Br}]r} - 1 \quad (5.8)
\]

where \( A = aK \), \( B = b \), \( C = cK \).

Thus, under very general conditions the bridge is completely described by the three co-efficients \( A, B, C \). Determination of these co-efficients by means of preliminary measurements constitutes calibration of the bridge.

5.1.2 \textbf{BRIDGE CALIBRATION}

(i) The Matched and Symmetric Bridge: \(^{(27)}\)

When the bridge is matched and compensated for asymmetry, the following symmetry conditions are satisfied,

\[
\begin{align*}
S_{11} &= S_{22} \\
S_{13} &= S_{23} \\
S_{14} &= -S_{24}
\end{align*}
\]

From these conditions and from the fact that the scattering matrix of the hybrid tee is unitary, one can show that the shunt-series arms are completely isolated.

i.e. \( S_{34} = S_{43} = 0 \)

Substituting these conditions in the expressions for \( b \) and \( c \) in equation (5.6a), it is found that

\[
B = 0 \\
C = 0
\]

Therefore, equation (5.8) reduces to
The reflection coefficient \( \gamma_x \) at the air-semiconductor interface is shown in fig. 5A.

The coefficient \( A \) can be determined experimentally by balancing the bridge with a fixed short circuit terminating the sample arm.

When a precision short circuit is placed at \( d_1 \),

\[
\gamma_{s1} = \gamma_x e^{2\gamma_s d_1}
\]

or

\[-1 = \gamma_x e^{2\gamma_s d_1}
\]

or

\[\gamma_x = -e^{-2\gamma_s d_1}\]

From equation (5.9)

\[-e^{-2\gamma_s d_1} = -A e^{-2[A_r(s_1) + J\beta_r(s_1)]}\]

where \( A_r(s_1) \) = attenuator reading with a fixed short \( l_r(s_1) \) = precision short reading with a fixed short

Therefore,

\[A = \frac{e^{-2\gamma_s d_1}}{e^{-2[A_r(s_1) + J\beta_r(s_1)]}}\]

Hence from equation (5.9)

\[\gamma_x = -e^{-2[\{A_r-A_r(s_1)\} + J\beta_r(l_r-l_r(s_1)+d_1)]}\]

\[= -e^{-(A+J\beta)}\]  

(5.10)
where \[ A = 2(A_r - A_r(s_1)) \text{ nepers} \] \[ \theta = 2\beta_r \{1_r - 1_r(s_1) + d_1\} \text{ radians} \]

Equation (5.10) has been used for the practical measurement of the magnitude and phase of reflection coefficient at the air-semiconductor interface.

(ii) **The Matched and Unsymmetric Bridge:**

When the input ports of hybrid tee are matched and the bridge is not compensated for asymmetry, then

\[ s_{11} = s_{22} = 0 \]
\[ s_{12} = s_{21} = 0 \text{ at a single frequency} \]

and from equation (5.6a), \( c = 0 \).

Therefore,

\[ \gamma_x = - A e^{-2(A_r+J\beta_r l_r)} + B \] (5.11)

With a precision short circuit termination at the sample arm,

\[ -2s_{d1} e^{-2[A_r(s_1)+J\beta_r l_r(s_1)]} = - A e^{-2[A_r(Z_0)+J\beta_r l_r(Z_0)]} + B \] (5.12)

With a matched termination \( Z_0 \) at the sample arm

\[ 0 = - A e^{-2[A_r(Z_0)+J\beta_r l_r(Z_0)]} + B \] (5.13)

Solving equations (5.12) and (5.13) for \( A \) and \( B \), and substituting in equation (5.11), the expression for \( \gamma_x \) becomes

\[ \gamma_x = - \frac{-2s_{d1} e^{-2[A_1(Z_0)]}}{1 - \frac{\gamma_1(Z_0)}{\gamma_1(s_1)}} - \frac{-2[A_r-A_r(s_1)]}{\gamma_1(s_1)} \]

\[ + J\beta_r[l_r-1_r(s_1)] - \frac{\gamma_1(Z_0)}{\gamma_1(s_1)} \]
where \( r_1(Z_0) = \frac{e^{-2[A_r(Z_0) + \text{Im} r_1(Z_0)]}}{e^{-2[A_r(s_1) + \text{Im} r_1(s_1)]}} \)

The right hand side of the above equation contains known quantities and thus \( r_x \) can be evaluated.

(iii) The General Bridge:

For a general bridge, equation (5.8) gives the expression for the reflection coefficient which is completely described by the three coefficients \( A, B, \) and \( C \). These coefficients \( A, B, \) and \( C \) can be determined by calibrating the bridge with two shorts at two different distances \( d_1, d_2 \) from measuring plane and with one matched termination \( Z_0 \) at the sample arm.

5.2 THEORETICAL CALCULATION OF THE REFLECTION COEFFICIENT

5.2.1 GENERAL SOLUTION.

An expression is derived for the reflection coefficient at the air-semiconductor interface of the rectangular wave guide system shown in figure 5.2A.

The transverse field components \( E_y \) and \( H_x \) in air (medium 1) at a distance \( Z \) from the air-semiconductor interface is given by

\[
E_y = A_{i1} e^{-r_1 Z} + A_{r1} e^{r_1 Z}
\]

\[
= A_{i1} \left( e^{-r_1 Z} + \frac{A_{r1}}{A_{i1}} e^{r_1 Z} \right)
\]

(5.2a)
n-type Ge sample

a = .0228 meter
b = .01015 meter
d = .00188 meter

frequency = 9.46 GHz

Semiconductor in a waveguide
Where $\Gamma_1$ is the propagation constant in the air filled section of the waveguide i.e. in medium (1). $A_{i_1}$ and $A_{r_1}$ are the amplitudes of the incident and reflected waves at the air-semiconductor interface i.e. at the plane $Z = 0$.

The reflection coefficient is given by the ratio

\[\frac{A_{r_1}}{A_{i_1}} = \gamma = e^{-2\theta} \quad (5.2b)\]

where $\theta = \mu + j \nu$

which characterizes the terminating waveguide section containing the semiconducting medium 2.

Therefore, the equation (5.2a) can be written as

\[E_{y_1} = A_{i_1} \left[ e^{-\Gamma_1 Z} + \gamma e^{\Gamma_1 Z} \right] \quad (5.2c)\]

\[H_{x_1} = H_{i_1} e^{-\Gamma_1 Z} + H_{r_1} e^{\Gamma_1 Z} \]

\[= \frac{A_{i_1}}{Z_1} e^{-\Gamma_1 Z} - \frac{A_{r_1}}{Z_1} e^{\Gamma_1 Z} \]

\[= \frac{A_{i_1}}{Z_1} \left[ e^{-\Gamma_1 Z} - \gamma e^{\Gamma_1 Z} \right] \quad (5.2d)\]

where $Z_1 = \frac{-j\omega \mu}{\Gamma_1} \quad (5.2e)$

The wave impedance $Z(o)$ given by the ratio of the transverse electric and magnetic field intensities at the air-semiconductor interface in medium 1, i.e.

\[Z(o) = \frac{E_{y_1}(o)}{H_{x_1}(o)} = Z_1 \left( \frac{1 + \gamma}{1 - \gamma} \right) = Z_1 \frac{1 + e^{-2\theta}}{1 - e^{-2\theta}} = Z_1 \text{Coth}\theta \quad (5.2f)\]
In the semiconductor region, that is in medium (2)
\[ E_{y2} = A_{i2} e^{-r_2Z} + A_{r2} e^{r_2Z} = A_{i2} \left[ e^{-r_2Z} + \frac{A_{r2}}{A_{i2}} e^{r_2Z} \right] \]  
(5.2g)

\[ H_{x2} = H_{i2} e^{-r_2Z} + H_{r2} e^{r_2Z} = \frac{A_{i2}}{Z_2^+} e^{-r_2Z} + \frac{A_{r2}}{Z_2^-} e^{r_2Z} \]

\[ = \frac{A_{i2}}{Z_2^+} \left[ e^{-r_2Z} + \frac{A_{r2} Z_2^+}{A_{i2} Z_2^-} e^{r_2Z} \right] \]  
(5.2h)

where \( Z_2^+ = \frac{A_{i2}}{H_{i2}} \) = characteristic impedance for the +vely travelling wave

\( Z_2^- = \frac{A_{r2}}{H_{r2}} \) = characteristic impedance for the -vely travelling wave

\( r_2 = \) propagation constant in medium (2).

At \( Z=d \) the waveguide is terminated by a metal plate. Introducing this boundary condition equation (5.2g) becomes

\[ 0 = A_{i2} \left[ e^{-r_2d} + \frac{A_{r2}}{A_{i2}} e^{r_2d} \right] \]

or, \( \frac{A_{r2}}{A_{i2}} = -e^{-2r_2d} \)

Therefore, the wave impedance at the air-semiconductor interface is given by
\[ Z(0) = \frac{E_{y2}(0)}{H_{x2}(0)} = Z_2^+ \left(1 + \frac{A_{r2}}{A_{i2}} e^{2z} \right) \]

\[ = Z_2^+ \left(1 - e^{-2r_2d} \right) \]

\[ (1 - e^{-2r_2d} \frac{Z_2^+}{Z_2^-}) \]

5.2.2 FIRST ORDER APPROXIMATE SOLUTION OF THE REFLECTION COEFFICIENT.

With the application of the external magnetic field, the medium (2) shown in figure 5.2A has a complex tensor permittivity and in the analysis of the field components in Chapter 3 it has been shown that the medium (2) has all the six field components. However, to simplify the analysis only the \( E_y \) and \( H_x \) components of the electromagnetic wave will be considered here and a first order expansion of \( Z_2^+ \), the characteristic impedance for the wave travelling in the +ve direction will be taken into consideration. It is seen that \( Z_2^+ \neq \frac{-j\omega\mu}{r_2} \), but

\[ Z_2^+ = \frac{-j\omega\mu}{r_2} \left[1 - \frac{e_3}{\varepsilon_2} (r_2^2 + w^2\mu_\varepsilon_2).F^+ \right] \]

where,

\[ F^+ = \left\{ \begin{array}{c}
m_1^y A e^{-m_1^y} + m_2^y C e^{-m_2^y} \\
\{r_2(a_3-b_3)+w^2\mu_\varepsilon_3^m_1\} A e^{m_1^y} + \{r_2(a_3-b_3)-w^2\mu_\varepsilon_3^m_2\} B e^{-m_2^y} \\
+ \{r_2(a_3+b_3)+w^2\mu_\varepsilon_3^m_2\} C e^{m_2^y} + \{r_2(a_3+b_3)-w^2\mu_\varepsilon_3^m_1\} D e^{-m_1^y} \\
\{r_2(a_3-b_3)+w^2\mu_\varepsilon_3^m_1\} A e^{-m_1^y} + \{r_2(a_3-b_3)-w^2\mu_\varepsilon_3^m_2\} D e^{m_2^y} \end{array} \right\} \]

where, the coefficients A, B, C and D are expressed in terms of A in equation (4.10).
Expanding $F^+$ into the Maclaurin's Series and taking only the first term

$$F^+ = F^+(0) + y \frac{F''^+(0)}{1!} + \frac{y^2 F'''^+(0)}{2!} + \ldots$$

Then, $F^+ \approx F^+(0)$

where

$$F^+(0) = \frac{m_1 (A - B) + m_2 (C - D)}{r_2 (m_1^2 - m_2^2) (C + D) + \mu_\varepsilon_3 (m_1 (A - B) + m_2 (C - D))}$$

Similarly, for the wave travelling in the -ve z-direction

$$Z_2^- = \frac{E_y^-}{H_x^-}$$

and

$$Z_2^- = \frac{j \omega \mu_\varepsilon}{\Gamma_2} \{ 1 - \frac{\varepsilon_3}{\varepsilon_2} (r_2^2 + \omega^2 \mu_\varepsilon_2) F^- \} \quad (5.2k)$$

where

$$F^- = \frac{m_1 A e^{m_1 Y} - m_1 B e^{-m_1 Y} + m_2 C e^{m_2 Y} - m_2 D e^{-m_2 Y}}{\left[ -r_2 (a_3 - b_3) + \omega^2 \mu_\varepsilon_3 m_1 \right] A e^{m_1 Y} + \left[ -r_2 (a_3 - b_3) - \omega^2 \mu_\varepsilon_3 m_1 \right] B e^{-m_1 Y} + \left[ -r_2 (a_3 + b_3) + \omega^2 \mu_\varepsilon_3 m_2 \right] C e^{m_2 Y} + \left[ -r_2 (a_3 + b_3) - \omega^2 \mu_\varepsilon_3 m_2 \right] D e^{-m_2 Y}}$$

Considering the first term of the series expansion of $F^-$, it is found that

$$F^- \approx F^- (0) = \frac{m_1 (A - B) + m_2 (C - D)}{-r_2 (m_1^2 - m_2^2) (C + D) + \omega^2 \mu_\varepsilon_3 (m_1 (A - B) + m_2 (C - D))}$$

where the coefficients $A$, $B$, $C$ and $D$ are expressed in terms of $A$ in equations (4.10)
Equating equations (5.2i) and (5.2f) for $Z(0)$ in the two regions, gives

$$Z_1 \text{ Coth } \phi = \frac{Z_2^+ (1 - e^{-2r_2d})}{(1 - e^{-2r_2d} \frac{Z_2^+}{Z_2})} \quad (5.21)$$

But from equations (5.2j) and (5.2k)

$$\frac{Z_2^+}{Z_2} = \frac{[1 - \frac{e^3}{\epsilon_2} (r_2^2 + w^2 \mu_\epsilon_2) \hat{\beta}^+]}{[1 - \frac{e^3}{\epsilon_2} (r_2^2 + w^2 \mu_\epsilon_2) \hat{\beta}^-]} = \frac{K_1}{K_2}$$

where $K_1 = [1 - \frac{e^3}{\epsilon_2} (r_2^2 + w^2 \mu_\epsilon_2) \hat{\beta}^+]^+ (0)$

$$K_2 = [1 - \frac{e^3}{\epsilon_2} (r_2^2 + w^2 \mu_\epsilon_2) \hat{\beta}^-]^- (0)$$

From equation (5.21)

$$\text{Coth } \phi = \frac{-j\omega \mu_\epsilon_1 K_1 (1 - e^{-2r_2d})}{\frac{-j\omega \mu_\epsilon_1}{r_1} (1 + \frac{K_1}{K_2} e^{-2r_2d})}$$

$$= \frac{r_1 K_1 (1 - e^{-2r_2d})}{r_2 (1 + \frac{1}{K_2} e^{-2r_2d})}$$

or, \( \frac{1 + \gamma}{1 - \gamma} = \frac{j\beta_1 K_1 (1 - e^{-2r_2d})}{r_2 (1 + \frac{1}{K_2} e^{-2r_2d})} \)

or \( \gamma = |\gamma| e^{j\theta} = \frac{j\beta_1 K_1 (1 - e^{-2r_2d}) - r_2 (1 + \frac{1}{K_2} e^{-2r_2d})}{j\beta_1 K_1 (1 - e^{-2r_2d}) + r_2 (1 + \frac{1}{K_2} e^{-2r_2d})} \quad (5.2m) \)
This approximate equation is plotted in figure (7.2A&B) as a function of applied magnetic field.

5.2.3 SIMPLE TE-MODE SOLUTION.

For the TE mode \( Z_2 = -Z_1 \) and equation (5.2f) becomes

\[
Z(0) = Z_2^+ (1 - e^{-2r_2d})
\]

Equating equations (5.2n) and (5.2f) gives

\[
Z_1 \text{ Coth } \theta = Z_2^+ \tanh r_2d
\]

or

\[
\text{ Coth } \theta = \frac{Z_2^+ \tanh r_2d}{Z_1}
\]

\[
= \frac{-j\omega\mu}{r_2} x \frac{r_1}{-j\omega\mu} \text{ tanh } r_2d
\]

\[
= \frac{r_1}{r_2} \text{ tanh } r_2d
\]

Normally, the attenuation in the air filled part of the guide can be neglected and the propagation constant reduces to \( \Gamma_1 = j\beta_1 \)

where \( \beta_1 \) is the phase constant.

Therefore, the equation (5.2o) can be written as

\[
\frac{1 + \gamma}{1 - \gamma} = \frac{j\beta_1 \text{ tanh } r_2d}{\Gamma_2}
\]

or

\[
\gamma = |\gamma| e^{j\theta} = \frac{j\beta_1 \text{ tanh } r_2d - r_2}{j\beta_1 \text{ tanh } r_2d + r_2}
\]
Since the propagation constant in medium (2), $\Gamma_2$, corresponding to the different values of the applied magnetic field is known (figure 4(d), 4(e)) and also since $\beta_1$ and $d$ are known quantities, the right hand side of the equation (5.21) is a known quantity. Calculated values of $\gamma = |\gamma|e^{j\theta}$ for the system of figure (5.2a) for different values of the applied magnetic field are given in figure 7.2(A), 7.2(B).
CHAPTER VI
EXPERIMENTAL PROCEDURES

6.1 PREPARATION OF THE SAMPLES

The samples of germanium were cut from a large block of 22.2 ohm-cm n-type Ge crystal (111) axial orientation of trapezoidal shape, by means of a diamond wheel cutter 0.020 inches thick. The samples were cut slightly larger than the waveguide cross section and reduced to the desired size by lapping on silicon carbide paper by hand. Great care was taken to insure that the x, y and z edges were perpendicular to each other and that the faces at z = 0 and z = L are parallel.

The sample length L was chosen to be approximately one quarter wavelength calculated from

$$L = \frac{\lambda_0}{4} = \frac{2\pi}{4\beta} = \frac{\pi}{2\beta}$$

The phase constant (\beta) was obtained from the expression for the propagation constant \( r = \alpha + j\beta = \left(\frac{\pi}{a}\right)^2 - \omega^2\varepsilon_0\varepsilon_r (\varepsilon_r - j\frac{\alpha}{\omega\mu_0})^{1/2} \) for the case of a waveguide completely filled with semiconductor, with no magnetic field applied. The calculated value of L was 0.07675" inch for a 22.2 ohm-cm germanium sample at a frequency of 9.46 GHz. Selection of L to be equal to one quarter wavelength or in general an odd number of quarter wavelengths in the material gives maximum experimental accuracy (28).

Prior to measurement the faces of the sample at planes z = 0 and z = L were ground on emery paper and washed with tap water. The dimensions of the sample used for the measurement were .893" x .397" x
and gave an excellent fit to the waveguide section 0.900" x 0.400".

6.2 CALIBRATION OF THE ELECTROMAGNET.

The electromagnet used for the experiments was a "Spectromagnetic Industries" model 1019 which has 4" diameter adjustable poles. This magnet is designed to be used with its associated 1 Kilowatt power supply (model 6021).

The electromagnet was calibrated with a precision Gauss meter for a pole gap distance of 3 cm. The calibration curve relating magnet current to magnetic field produced is shown in Figure 6.1.

6.2.1 RESULTS OF CALIBRATION OF MAGNET.

The experimental readings for the magnetization curve of a 4" adjustable gap electromagnet with cylindrical pole geometry for a gap distance of 3 cm. is given in Table 6.1 below and the curve is plotted in Figure 6.1.
<table>
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<th>NO. OF READINGS</th>
<th>MAGNETIZING CURRENT</th>
<th>MAGNETIC FIELD IN THE GAP</th>
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</thead>
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<td>Amps</td>
<td>Kilogauss</td>
</tr>
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<td>.37</td>
</tr>
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<td>3</td>
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</tr>
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<td>9</td>
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<td>15</td>
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</tr>
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</table>
MAGNETIZATION CURVE

GAP DISTANCE = 3 CENTIMETERS

4" Diameter Cylindrical Pole

Geometry Adjustable Gap Electromagnet

FIGURE 6.1 EXPERIMENTAL MAGNETIZATION CURVE OF THE ELECTROMAGNET
6.3 **DESCRIPTION OF APPARATUS.**

A schematic diagram of a reflection bridge for the precision measurement of the magnitude and the phase of the reflection co-efficient at the air-semiconductor interface is shown in Figure 6.2. The equipment used is as follows:

(i) **Klystron.**

The Klystron used as a high frequency signal source is a reflex type, X-13 of Varian Associates of Canada Ltd. Its frequency is mechanically tunable over the range 8.2 to 12.4 GHz with its maximum power output for optimum load of 350 m watts. A frequency of 9.46 GHz was used for the experiments.

(ii) **Klystron Power Supply.**

The power to the Klystron was supplied from the P.R.D. power supply, type 890A. The reflector voltage can be modulated either from an external source or an internally produced square and sawtooth waves.

(iii) **Ferrite Isolator.**

This is an unidirectional element used to isolate the Klystron from the rest of the circuit as far as the reflected waves are concerned. Waves pass practically unattenuated in the direction of transmission of the isolator but in the opposite direction it has approximately 30db attenuation. Thus any change in the load will not affect the Klystron output power and frequency. Isolators manufactured by P.R.D. were used in the reflection bridge assembly.
1. Klystron Power Supply
2. Klystron
3. 11. Isolators
4. Attenuator
5. Wave meter
6. Directional Coupler
7. SW Indicator
8, 10, 16. Stub Tuners
9. Hybrid Tee
12. Crystal Detector

FIGURE 6.2 AN EXPERIMENTAL SET UP OF A REFLECTION BRIDGE
(iv) Wave Meter.  
The wave meter used was a Micro-line, Model 138A. This meter will read the frequency of the waves in the guide and its range is from 8430 to 9660 MC/S.

(v) Variable Attenuator.  
This is used to adjust the power input to the circuit to a suitable value and the unit manufactured by P.R.D. was used in the bridge.

(vi) Directional Coupler.  
This is a device consisting of two transmission lines coupled together in such a way that a wave travelling in one line in one direction excites a wave in the other guide ideally in one direction only. The normal attenuation in this process is 10-20 db.

(vii) Standing Wave Indicator.  
The standing wave indicator is a model B433 (1637) of Elliott Brothers (London) Ltd. and is a high precision instrument. This has been used for the measurement of V.S.W.R. during the matching of the hybrid tee ports by means of the tuners.

(viii) Precision Attenuator, Precision Short and Hybrid Tee.  
High quality equipments are needed for these components. Elliott instruments are used for the rotary vane attenuator type A1617/44 and the precision short. The hybrid tee was manufactured by Demornay Borandi.

(ix) Electromagnet.  
The Spectromagnetic Industries' Model 1019 is a 4" diameter pole adjustable gap laboratory electromagnet. It may be operated
continuously with currents up to 20 amperes to give 9.5 K gauss with pole gap of 3 cm. The poles, which are adjustable and reversible, are precision ground to enhance field homogeneity. The poles are chrome plated to resist wear and corrosion. Each pole has 4" diameter cylindrical pole geometry on one end and 1-1/2" diameter tapered pole geometry on the other. In addition, 6" variable flux rings may be attached to the 4" diameter cylindrical ends to increase field uniformity.

(x) **Regulated Power Supply for the Electromagnet.**

The model 6021 Spectromagnetic Industries' precision magnet power supply provides an extremely stable, accurate current regulated current. The output current is continuously adjustable over the entire range 0.1 to 30 Amps with a 10 turn potentiometer.

6.4 **MEASURING PROCEDURE.**

6.4.1 **TUNING OF A HYBRID TÉÉ.**

The ports (1), (2) and (4) of the hybrid tee shown in Figure (5A) were terminated in matched loads and $s_{33}$ was made zero by using the tuner in the arm (3). In practice a VSWR of about 1.02 was obtained. Similarly, $s_{44}$ was made zero by the tuner in arm 4, with other ports terminated in matched loads. Then $s_{34}$ was checked by measuring the output in the E-arm while feeding the H-arm with terminations placed on arms (1) and (2). If $s_{34}$ is not zero, it can be reduced to a minimum by adjusting the tuner in arm (1). This adjustment slightly disturbs the tuning of E and H arms so these were again tuned and the process repeated till the effect of the side arm tuner on $s_{33}$ and
$s_{44}$ is negligible. Since the hybrid tee used was a high quality magic tee, only slight adjustments were needed for tuning the hybrid tee.

6.4.2 **Measurement of Reflection Coefficient.**

The sample arm was first shorted by a short circuit metal plate and the precision attenuator and the precision short in the reference arm were then adjusted for minimum output. Their readings $A_r(s_1)$ and $l_r(s_1)$ were recorded. The germanium sample was then placed in the waveguide in front of the short circuit plate. Again, the precision attenuator and the precision short were adjusted for minimum output and their readings $A_r$ and $l_r$ were noted. Care was taken to position the sample faces normal to the axis of the guide and the back face was completely in contact with the metal plate.

The external magnetic field $B$ was then applied in a direction transverse to the direction of the wave propagation in the waveguide. The external magnetic field was varied by varying the magnetizing current from 0 to 27 amp in steps of 3 amps and the readings of the precision attenuator and the precision short $A_r$ and $l_r$ respectively for minimum output were recorded for each step. The direction of the magnetic field was then reversed and the entire procedure was repeated. Care was also taken to note that the temperature of the sample was not affected by the temperature rise of the electro-magnet during the measurement.
6.4.3 MEASUREMENT OF THE D.C. RESISTIVITY OF THE SEMICONDUCTOR.

The conductivity of the semiconductor was measured by the four point probe method which has the advantage that it does not require any specific shape of the sample.

In this method, four probes are placed on the flat surface of the sample and the current $I$ is passed through the outer probes and the voltage $V$ is measured between the inner probes. If $s_1, s_2, s_3$ are the probe spacings then the conductivity $\sigma$ is given by

$$\sigma = \frac{1}{2 \pi V} \left[ \frac{1}{s_1} + \frac{1}{s_3} - \frac{1}{(s_1+s_2)} - \frac{1}{(s_1+s_3)} \right]$$

(6.1)

When the probe spacings are equal, the resistivity $\rho$ of the sample is given by

$$\rho = \frac{1}{\sigma} = 2 \pi s \left( \frac{V}{I} \right)$$

(6.2)

The equation 6.2 was used in calculating the d.c. resistivity of the sample and the resistivity of the sample was found to be 22.2 ohm-cm. When the dimensions of the sample are small compared to probe spacing, the corrections shown in figure (6.3) must be applied. The edge of the sample must be at least a distance $3s$ away from the probe.
FIGURE 6.3  FOUR PROBE RESISTIVITY CORRECTION CURVE
(AFTER HUNTER(30))
CHAPTER VII

RESULTS

7.1 THEORETICAL RESULTS

7.1.1 VARIATIONS IN PROPAGATION CONSTANT AND THE MATRIX ELEMENTS OF THE COMPLEX TENSOR PERMITTIVITY WITH "B".

The calculated variations in the values of the attenuation constant and the phase constant of the electromagnetic wave with the variation of the applied magnetic field as calculated from numerical solution of equation (4.9) are shown in tables 7.1(A), 7.1(B), 7.1(C), 7.1(D) and 7.1(E) for different values of the conductivity of the material. These tables are given in graphical form in figures 4.d and 4.e.

The calculated variations in the values of the matrix elements of the complex permittivity tensor using equation (3.2) with the variations of the applied magnetic field are also given in tables 7.1(A) to 7.1(E) and these variations have been plotted in the graph shown in figure 7.1 and 7.2.
frequency = 9.46 GHz
conductivity = 4.5 mho/meter
### TABLE 7.1A

**THEORETICAL VARIATIONS IN PROPAGATION CONSTANT $\varepsilon_1$, $\varepsilon_2$ AND $\varepsilon_3$ WITH MAGNETIC FIELD**

Frequency 9.46 GHz  
Conductivity = 4.0 ohm/meter  
n-type Ge sample

<table>
<thead>
<tr>
<th>MAGNETIC FIELD $B$ (Wb/m²)</th>
<th>PROPAGATION CONSTANT $\Gamma^* = \alpha + j\beta$</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\varepsilon_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Attenuation Constant $\alpha$</td>
<td>Phase Constant $\beta$</td>
<td>farad/meter</td>
<td>farad/meter</td>
</tr>
<tr>
<td>0.0</td>
<td>186.01</td>
<td>803.8</td>
<td>$(1.417 - j.672) \times 10^{-10}$</td>
<td>$(1.417 - j.672) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.1</td>
<td>185.68</td>
<td>803.75</td>
<td>&quot;</td>
<td>$(1.417 - j.670) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.2</td>
<td>184.69</td>
<td>803.59</td>
<td>&quot;</td>
<td>$(1.417 - j.667) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.3</td>
<td>183.06</td>
<td>803.31</td>
<td>&quot;</td>
<td>$(1.417 - j.661) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.4</td>
<td>180.84</td>
<td>802.94</td>
<td>&quot;</td>
<td>$(1.417 - j.653) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.5</td>
<td>178.06</td>
<td>802.48</td>
<td>&quot;</td>
<td>$(1.417 - j.643) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.6</td>
<td>174.78</td>
<td>801.93</td>
<td>&quot;</td>
<td>$(1.417 - j.631) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.7</td>
<td>171.07</td>
<td>801.13</td>
<td>&quot;</td>
<td>$(1.417 - j.617) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.8</td>
<td>166.99</td>
<td>800.64</td>
<td>&quot;</td>
<td>$(1.417 - j.602) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.9</td>
<td>162.59</td>
<td>799.92</td>
<td>&quot;</td>
<td>$(1.417 - j.586) \times 10^{-10}$</td>
</tr>
<tr>
<td>1.00</td>
<td>166.76</td>
<td>782.34</td>
<td>&quot;</td>
<td>$(1.417 - j.569) \times 10^{-10}$</td>
</tr>
</tbody>
</table>
**TABLE 7.1B**

**VARIATIONS IN \( \gamma, \varepsilon_1, \varepsilon_2 \) AND \( \varepsilon_3 \) WITH MAGNETIC FIELD**

Frequency 9.46 GHz  
*n*-type Ge sample  
Conductivity = 4.3 mho/meter

<table>
<thead>
<tr>
<th>MAGNETIC FIELD B (Wb/m²)</th>
<th>PROPAGATION CONSTANT ( \gamma = \alpha + j \beta )</th>
<th>( \varepsilon_1 )</th>
<th>( \varepsilon_2 )</th>
<th>( \varepsilon_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Attenuation Constant ( \alpha )</td>
<td>Phase Constant ( \beta )</td>
<td>fF/meter</td>
<td>fF/meter</td>
</tr>
<tr>
<td>0.0</td>
<td>199.19</td>
<td>806.96</td>
<td>((1.417-j.722)\times10^{-10})</td>
<td>((1.417-j.722)\times10^{-10})</td>
</tr>
<tr>
<td>0.1</td>
<td>198.83</td>
<td>806.89</td>
<td>&quot;</td>
<td>((1.417-j.721)\times10^{-10})</td>
</tr>
<tr>
<td>0.2</td>
<td>197.77</td>
<td>806.70</td>
<td>&quot;</td>
<td>((1.417-j.717)\times10^{-10})</td>
</tr>
<tr>
<td>0.3</td>
<td>196.03</td>
<td>806.39</td>
<td>&quot;</td>
<td>((1.417-j.711)\times10^{-10})</td>
</tr>
<tr>
<td>0.4</td>
<td>193.65</td>
<td>805.96</td>
<td>&quot;</td>
<td>((1.417-j.702)\times10^{-10})</td>
</tr>
<tr>
<td>0.5</td>
<td>190.69</td>
<td>805.43</td>
<td>&quot;</td>
<td>((1.417-j.693)\times10^{-10})</td>
</tr>
<tr>
<td>0.6</td>
<td>187.19</td>
<td>804.80</td>
<td>&quot;</td>
<td>((1.417-j.678)\times10^{-10})</td>
</tr>
<tr>
<td>0.7</td>
<td>183.23</td>
<td>804.10</td>
<td>&quot;</td>
<td>((1.417-j.664)\times10^{-10})</td>
</tr>
<tr>
<td>0.8</td>
<td>178.87</td>
<td>803.32</td>
<td>&quot;</td>
<td>((1.417-j.647)\times10^{-10})</td>
</tr>
<tr>
<td>0.9</td>
<td>174.19</td>
<td>802.5</td>
<td>&quot;</td>
<td>((1.417-j.630)\times10^{-10})</td>
</tr>
<tr>
<td>1.00</td>
<td>169.24</td>
<td>801.64</td>
<td>&quot;</td>
<td>((1.417-j.612)\times10^{-10})</td>
</tr>
</tbody>
</table>
TABLE 7.1C

VARIATIONS IN $\gamma$, $\varepsilon_1$, $\varepsilon_2$ AND $\varepsilon_3$ WITH MAGNETIC FIELD

Frequency 9.46 GHz  
Conductivity = 4.4 mho/meter  
n-type Ge sample

<table>
<thead>
<tr>
<th>MAGNETIC FIELD B (Wb/m²)</th>
<th>PROPAGATION CONSTANT $\gamma = \alpha + j\beta$</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\varepsilon_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Attenuation Constant $\alpha$</td>
<td>Phase Constant $\beta$</td>
<td>farad/meter</td>
<td>farad/meter</td>
</tr>
<tr>
<td>0.0</td>
<td>203.54 neper/meter</td>
<td>808.04 radian/meter</td>
<td>$(1.417-j.739) \times 10^{-10}$</td>
<td>$(1.417-j.739) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.1</td>
<td>203.18 neper/meter</td>
<td>807.98 radian/meter</td>
<td>&quot;</td>
<td>$(1.417-j.737) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.2</td>
<td>202.10 neper/meter</td>
<td>807.78 radian/meter</td>
<td>&quot;</td>
<td>$(1.417-j.733) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.3</td>
<td>200.32 neper/meter</td>
<td>807.45 radian/meter</td>
<td>&quot;</td>
<td>$(1.417-j.727) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.4</td>
<td>197.89 neper/meter</td>
<td>807.0 radian/meter</td>
<td>&quot;</td>
<td>$(1.417-j.718) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.5</td>
<td>194.87 neper/meter</td>
<td>806.45 radian/meter</td>
<td>&quot;</td>
<td>$(1.417-j.707) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.6</td>
<td>191.3 neper/meter</td>
<td>805.8 radian/meter</td>
<td>&quot;</td>
<td>$(1.417-j.694) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.7</td>
<td>187.25 neper/meter</td>
<td>805.06 radian/meter</td>
<td>&quot;</td>
<td>$(1.417-j.679) \times 10^{-10}$</td>
</tr>
</tbody>
</table>
### TABLE 7.1D

**VARIATIONS IN $\gamma$, $\varepsilon_1$, $\varepsilon_2$ AND $\varepsilon_3$ WITH MAGNETIC FIELD**

Frequency 9.46 GHz  
Conductivity = 4.5 mho/meter  
n-type GE sample

<table>
<thead>
<tr>
<th>MAGNETIC FIELD B (Wb/m²)</th>
<th>PROPAGATION CONSTANT $\gamma = \alpha + j\beta$</th>
<th>$\varepsilon_1$ (farad/meter)</th>
<th>$\varepsilon_2$ (farad/meter)</th>
<th>$\varepsilon_3$ (farad/meter)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Attenuation Constant $\alpha$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Phase Constant $\beta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>207.89  neper/meter 809.15</td>
<td>(1.417-j.756)x10⁻¹⁰</td>
<td>(1.417-j.756)x10⁻¹⁰</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>207.51  809.08</td>
<td>&quot;</td>
<td>(1.417-j.754)x10⁻¹⁰</td>
<td>j 3.21x10⁻¹²</td>
</tr>
<tr>
<td>0.2</td>
<td>206.41  808.87</td>
<td>&quot;</td>
<td>(1.417-j.75)x10⁻¹⁰</td>
<td>j 6.40x10⁻¹²</td>
</tr>
<tr>
<td>0.3</td>
<td>204.60  808.53</td>
<td>&quot;</td>
<td>(1.417-j.743)x10⁻¹⁰</td>
<td>j 9.51x10⁻¹²</td>
</tr>
<tr>
<td>0.4</td>
<td>202.12  808.06</td>
<td>&quot;</td>
<td>(1.417-j.735)x10⁻¹⁰</td>
<td>j 1.3x10⁻¹¹</td>
</tr>
<tr>
<td>0.5</td>
<td>199.03  807.48</td>
<td>&quot;</td>
<td>(1.417-j.723)x10⁻¹⁰</td>
<td>j 1.54x10⁻¹¹</td>
</tr>
<tr>
<td>0.6</td>
<td>195.39  806.8</td>
<td>&quot;</td>
<td>(1.417-j.709)x10⁻¹⁰</td>
<td>j 1.81x10⁻¹¹</td>
</tr>
<tr>
<td>0.7</td>
<td>191.27  806.03</td>
<td>&quot;</td>
<td>(1.417-j.694)x10⁻¹⁰</td>
<td>j 2.07x10⁻¹¹</td>
</tr>
<tr>
<td>0.8</td>
<td>186.73  805.19</td>
<td>&quot;</td>
<td>(1.417-j.6775)x10⁻¹⁰</td>
<td>j 2.31x10⁻¹¹</td>
</tr>
<tr>
<td>0.9</td>
<td>181.85  804.29</td>
<td>&quot;</td>
<td>(1.417-j.659)x10⁻¹⁰</td>
<td>j 2.53x10⁻¹¹</td>
</tr>
</tbody>
</table>
**TABLE 7.1E**

VARIATIONS IN $\Gamma$, $\varepsilon_1$, $\varepsilon_2$ AND $\varepsilon_3$ WITH MAGNETIC FIELD

Frequency 9.46 GHz  
Conductivity = 4.65 mho/meter  
n-type Ge Sample

<table>
<thead>
<tr>
<th>MAGNETIC FIELD B (Wb/m²)</th>
<th>PROPAGATION CONSTANT $\Gamma = \alpha + j\beta$</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\varepsilon_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Attenuation Constant $\alpha$</td>
<td>Phase Constant $\beta$</td>
<td>Farad/meter</td>
<td>Farad/meter</td>
</tr>
<tr>
<td>0.0</td>
<td>214.37 neper/meter</td>
<td>810.84 radian/meter</td>
<td>$(1.417-j.781) \times 10^{-10}$</td>
<td>$(1.417-j.781) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.1</td>
<td>213.98 neper/meter</td>
<td>810.76 radian/meter</td>
<td>$\cdot$</td>
<td>$(1.417-j.780) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.2</td>
<td>212.85 neper/meter</td>
<td>810.54 radian/meter</td>
<td>$\cdot$</td>
<td>$(1.417-j.776) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.3</td>
<td>210.98 neper/meter</td>
<td>810.18 radian/meter</td>
<td>$\cdot$</td>
<td>$(1.417-j.769) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.4</td>
<td>208.43 neper/meter</td>
<td>809.68 radian/meter</td>
<td>$\cdot$</td>
<td>$(1.417-j.759) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.5</td>
<td>205.25 neper/meter</td>
<td>809.07 radian/meter</td>
<td>$\cdot$</td>
<td>$(1.417-j.747) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.6</td>
<td>201.50 neper/meter</td>
<td>808.34 radian/meter</td>
<td>$\cdot$</td>
<td>$(1.417-j.733) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.7</td>
<td>197.26 neper/meter</td>
<td>807.53 radian/meter</td>
<td>$\cdot$</td>
<td>$(1.417-j.718) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.8</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$(1.417-j.681) \times 10^{-10}$</td>
</tr>
<tr>
<td>0.9</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
</tr>
</tbody>
</table>
7.1.2 **CALCULATED VALUES OF THE REFLECTION COEFFICIENT.**

The calculated theoretical values of the magnitude and phase of the reflection coefficients are shown in Tables 7.1(F) and 7.1(G). The frequency used in the calculation is 9.46 GHz and the conductivity of the germanium sample is 4.5 mho/meter.

The theoretical values of the magnitude and phase of reflection coefficient have been plotted in Figure 7.2A and Figure 7.2B also along with the measured values.
TABLE 7.1(F)

REFLECTION COEFFICIENT OBTAINED FROM SIMPLE TE MODE CONSIDERATION

Frequency = 9.46 GHz  
Conductivity = 4.50 mho/meter  
n-type GE sample  
Dimension = .893" x .397" x .074"

<table>
<thead>
<tr>
<th>APPLIED MAGNETIC FIELD B Wb/m²</th>
<th>MAGNITUDE OF THE REFLECTION COEFFICIENT</th>
<th>PHASE ANGLE OF THE REFLECTION COEFFICIENT θ (IN DEGREES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.416</td>
<td>157.58°</td>
</tr>
<tr>
<td>0.1</td>
<td>0.415</td>
<td>157.51°</td>
</tr>
<tr>
<td>0.2</td>
<td>0.413</td>
<td>157.33°</td>
</tr>
<tr>
<td>0.3</td>
<td>0.410</td>
<td>157.03°</td>
</tr>
<tr>
<td>0.4</td>
<td>0.406</td>
<td>156.59°</td>
</tr>
<tr>
<td>0.5</td>
<td>0.400</td>
<td>156.02°</td>
</tr>
<tr>
<td>0.6</td>
<td>0.394</td>
<td>155.30°</td>
</tr>
<tr>
<td>0.7</td>
<td>0.386</td>
<td>154.43°</td>
</tr>
<tr>
<td>0.8</td>
<td>0.378</td>
<td>153.40°</td>
</tr>
<tr>
<td>0.9</td>
<td>0.370</td>
<td>152.19°</td>
</tr>
<tr>
<td>0.95</td>
<td>0.366</td>
<td>151.52°</td>
</tr>
</tbody>
</table>
### TABLE 7.1(G)

**Reflection Coefficient Obtained from First Order Approx. Solution of The Characteristic Impedance**

Frequency = 9.46 GHz  
Conductivity = 4.50 mho/meter  
n-type Ge sample  
Dimension = .893 x .397 x .074

| APPLIED MAGNETIC FIELD $B$ (Wb/m²) | MAGNITUDE OF THE REFLECTION COEFFICIENT $|\tau|$ | PHASE ANGLE OF THE REFLECTION COEFFICIENT $\theta$ (IN DEGREES) |
|-----------------------------------|---------------------------------------------|---------------------------------------------------------------|
| 0.0                              | 0.416                                       | 157.58°                                                      |
| 0.1                              | 0.416                                       | 157.87°                                                      |
| 0.2                              | 0.414                                       | 158.03°                                                      |
| 0.3                              | 0.412                                       | 158.05°                                                      |
| 0.4                              | 0.408                                       | 157.94°                                                      |
| 0.5                              | 0.403                                       | 157.70°                                                      |
| 0.6                              | 0.397                                       | 157.35°                                                      |
| 0.7                              | 0.390                                       | 156.86°                                                      |
| 0.8                              | 0.383                                       | 156.25°                                                      |
| 0.9                              | 0.374                                       | 155.50°                                                      |
| 0.95                             | 0.369                                       | 155.07°                                                      |
7.2 EXPERIMENTAL RESULTS OF THE MAGNITUDE AND PHASE OF THE REFLECTION COEFFICIENT.

7.2.1 EXPERIMENTAL VALUES OF THE REFLECTION COEFFICIENT.

The experimental readings for the magnetizing current, the precision attenuator \( (A_r) \) and the precision short \( (l_r) \) have been tabulated in Tables 7.2A & B. The corresponding calculations of the magnitude and phase of the reflection coefficient from these experimental readings have also been shown in the above tables. Two sets of readings have been obtained by changing the direction of the applied magnetic field.

The variations of the magnitude and phase of the reflection coefficient with the applied magnetic field \( B \) have been shown in Figures 7.2(A) and 7.2(B).
TABLE 7.2A

EXPERIMENTAL VALUES OF THE REFLECTION COEFFICIENT

Frequency = 9.46 GHz

No sample and waveguide terminated by a conducting plate

\( r_{1} = 5.44 \times 10^{-6} \text{ meter} \)

\( A_{r}(s_{1}) = 8.8^\circ \pm 0.21 \text{ dB} = 0.02418 \text{ nepers} \)

| NO. OF READINGS | MAG. CURRENT IN AMPS. | B Wb/meter\(^2\) | \( A_{r} \) | \( A_{r} - A_{r}(s_{1}) \) | \( A = 2[A_{r} - A_{r}(s_{1})] \) | \( e^{-A} = |r| \) |
|-----------------|------------------------|-----------------|----------|-----------------|-----------------|-----------------|
| 1               | 0                      | 0               | 37.2°    | 3.95            | .455            | .43082          | .86164          | .423            |
| 2               | 1                      | .037            | 37.3°    | 3.97            | .4575           | .43332          | .86664          | .421            |
| 3               | 3                      | .114            | 37.4°    | 4.00            | .460            | .43582          | .87164          | .419            |
| 4               | 6                      | .235            | 37.5°    | 4.02            | .4625           | .43832          | .87664          | .417            |
| 5               | 9                      | .35             | 37.7°    | 4.07            | .4675           | .44332          | .88664          | .413            |
| 6               | 12                     | .465            | 38.0°    | 4.14            | .477            | .45282          | .90564          | .405            |
| 7               | 15                     | .575            | 38.3°    | 4.21            | .484            | .45982          | .91964          | .399            |
| 8               | 18                     | .69             | 38.5°    | 4.26            | .490            | .46582          | .93164          | .394            |
| 9               | 21                     | .78             | 38.8°    | 4.33            | .498            | .47382          | .94764          | .388            |
| 10              | 24                     | .86             | 39.0°    | 4.38            | .504            | .47982          | .95964          | .3835           |
| 11              | 27                     | .935            | 39.2°    | 4.43            | .51             | .48582          | .97164          | .379            |
TABLE 7.28

EXPERIMENTAL VALUES OF THE REFLECTION COEFFICIENT

Frequency = 9.46 GHz

With no sample and waveguide terminated by a conducting plate

\[ l_r(s_1) = 5440 \times 10^{-6} \text{ meter} \]
\[ A_r(s_1) = 8.8^\circ + 0.21 \text{ dB} = 0.02418 \text{ nepers} \]

| NO. OF READINGS | MAG. CURRENT IN AMPS. | B Wb/meter² | \( A_r \) Degree | \( A_r \) Decibels | \( A_r - A_r(s_1) \) | \( A = 2[A_r - A_r(s_1)] \) | \(| r | = e^{-A} \) |
|-----------------|-----------------------|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1               | 0                     | 0           | 37.2°           | 3.95            | 0.454           | 0.42982         | 0.85964         | 0.424           |
| 2               | 1                     | .037        | 37.2°           | 3.95            | 0.454           | 0.42982         | 0.85964         | 0.424           |
| 3               | 3                     | .114        | 37.25°          | 3.96            | 0.455           | 0.43082         | 0.86164         | 0.423           |
| 4               | 6                     | .235        | 37.35°          | 3.99            | 0.458           | 0.43382         | 0.86764         | 0.420           |
| 5               | 9                     | .35         | 37.55°          | 4.03            | 0.463           | 0.43882         | 0.87764         | 0.416           |
| 6               | 12                    | .465        | 37.75°          | 4.08            | 0.469           | 0.44482         | 0.88964         | 0.411           |
| 7               | 15                    | .575        | 37.9°           | 4.12            | 0.473           | 0.44882         | 0.89764         | 0.408           |
| 8               | 18                    | .69         | 38.2°           | 4.19            | 0.4825          | 0.45832         | 0.91664         | 0.40            |
| 9               | 21                    | .78         | 38.5°           | 4.26            | 0.490           | 0.46582         | 0.93164         | 0.3945          |
| 10              | 24                    | .86         | 38.65°          | 4.29            | 0.494           | 0.46982         | 0.93964         | 0.391           |
| 11              | 27                    | .935        | 38.85°          | 4.34            | 0.499           | 0.47482         | 0.94964         | 0.3875          |
TABLE 7.2C

EXPERIMENTAL VALUES OF THE REFLECTION COEFFICIENT

\[ r = e^{-A(-e^{-j\theta})} = |r| e^{-j\theta} \]
\[ \theta = 2\beta_r(1_r - 1_r(s_1) + d_1) \text{ in radian} \]

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<tr>
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<th>MAG. CURRENT IN AMPS</th>
<th>B in Wb/meter²</th>
<th>( 1_r ) (meter)</th>
<th>( (1_r - 1_r(s_1) + d_1) ) (meter)</th>
<th>( \theta = 2\beta_r(1_r - 1_r(s_1) + d_1) ) in radian</th>
<th>( \theta ) in Degrees</th>
<th>( \beta_r ) in radian/meter</th>
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<td>0</td>
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<td>156.3°</td>
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<tr>
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<td>156.1°</td>
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<td>.235</td>
<td>5040 x 10⁻⁶</td>
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<td>155.7°</td>
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<td>5075 x 10⁻⁶</td>
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<td>153.5°</td>
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<tr>
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<tr>
<td>11</td>
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<td>.935</td>
<td>5325 x 10⁻⁶</td>
<td>1764.6 x 10⁻⁶</td>
<td>.505</td>
<td>28.9°</td>
<td>151.1°</td>
</tr>
</tbody>
</table>

MAGNETIC FIELD + (+B)
**TABLE 7.2D**

EXPERIMENTAL VALUES OF THE REFLECTION COEFFICIENT

\[ r = e^{-A(-e^{-j\beta})} = |r| e^{-j\beta} \]

\[ \beta_r = 143 \text{ radian/meter} \]

\[ \theta = 2\beta_r(l_r-l_r(s_1)+d_1) \text{ in radian} \]

\[ l_r(s_1) = 5440 \times 10^{-6} \text{ meter} \]

\[ d_1 = 1879.6 \times 10^{-6} \text{ meter} \]

MAGNETIC FIELD: \((-B)\)

| NO. OF READINGS | MAG. CURRENT IN AMPS | B in Wb/meter\(^2\) | \(l_r\) (meter) | \(|l_r-l_r(s_1)+d_1|\) (meter) | \(\theta = 2\beta_r(l_r-l_r(s_1)+d_1)\) (radian) | \(\theta\) in degrees | \(\theta\) |
|-----------------|----------------------|---------------------|----------------|-----------------|---------------------------------|----------------|------|
| 1               | 0                    | 0                   | 4900\times10^{-6} | 1339.6\times10^{-6} | .383                           | 21.9°       | 158.1°|
| 2               | 1                    | .037                | 4900\times10^{-6} | 1339.6\times10^{-6} | .383                           | 21.9°       | 158.1°|
| 3               | 3                    | .114                | 4910\times10^{-6} | 1349.6\times10^{-6} | .386                           | 22.1°       | 157.9°|
| 4               | 6                    | .235                | 4925\times10^{-6} | 1364.6\times10^{-6} | .39                            | 22.35°      | 157.65°|
| 5               | 9                    | .35                 | 4960\times10^{-6} | 1399.6\times10^{-6} | .40                            | 22.9°       | 157.1°|
| 6               | 12                   | .465                | 5000\times10^{-6} | 1439.6\times10^{-6} | .412                           | 23.6°       | 156.4°|
| 7               | 15                   | .575                | 5050\times10^{-6} | 1489.6\times10^{-6} | .426                           | 24.4°       | 155.6°|
| 8               | 18                   | .69                 | 5105\times10^{-6} | 1544.6\times10^{-6} | .442                           | 25.3°       | 154.7°|
| 9               | 21                   | .78                 | 5165\times10^{-6} | 1604.6\times10^{-6} | .458                           | 26.3°       | 153.7°|
| 10              | 24                   | .86                 | 5220\times10^{-6} | 1659.6\times10^{-6} | .474                           | 27.15°      | 152.85°|
| 11              | 27                   | .935                | 5265\times10^{-6} | 1704.6\times10^{-6} | .4875                          | 27.9°       | 152.1°|
7.3 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF THE REFLECTION COEFFICIENT

The experimental and the theoretical values of the magnitude of the reflection coefficient have been plotted in Figure 7.2(A). It is found that the experimental results are in excellent agreement with the theoretical results.

The experimental and the theoretical values of the phase of the reflection coefficient have been plotted in Figure 7.2(B). It is found that the theoretical values of the phase of the reflection coefficient obtained by simple TE mode consideration are in excellent agreement with the experimental values. The theoretical values of the phase of the reflection coefficient obtained by first order approximation solution are in close agreement with the experimental values for lower values of the magnetic field but at higher values of the magnetic field it differs slightly from the experimental curve.
Figure 7.2A

Frequency = 9.46 GHz

$\sigma = 4.50$ mho/meter

n-type Ge sample
frequency = 9.46 GHz
= 4.50 mho/meter
n-type Ge sample

EXPERIMENTAL CURVES
\{ -B \}
\{ +B \}

THEORETICAL CURVES
\{ APPROX. SOLN. \}
\{ SIMPLE TE MODE \}

FIGURE 7.2B
CHAPTER VIII

CONCLUSIONS

8.1 THEORETICAL WORK

In one of their published papers, Nag and Engineer\textsuperscript{(9)} have formulated a special form of the Hall field expression, $E_H$. When a magnetic field is applied, the conduction current is modified due to the Hall effect. The Hall effect produces a field, $E_H$ which in the case of a semiconductor with spherical energy surfaces is given by the following relations

$$E_H = R_C B_0 \times J_{CH}$$

$$J_{CH} = \sigma (E - E_H)$$

where, $R_C$ is the Hall co-efficient, $B_0$ is the steady applied magnetic field, $\sigma$ is the conductivity of the material and $J_{CH}$ is the modified conduction current.

Following their suggestions of the Hall field expressions, a complex permittivity tensor characterizing the semiconductor in a transverse magnetic field has been derived. With this complex permittivity tensor, a detailed theoretical analysis of the electromagnetic wave propagation through a rectangular waveguide completely filled with a semiconductor subject to a transverse applied magnetic field has been carried out. In the analysis it has been clearly shown that no TM modes or TE modes other than those of the type $TE_{0n}$ can be excited in a rectangular waveguide completely filled with a semiconductor subjected...
to a transverse steady applied magnetic field. In the analysis, it has also been shown that $TE_{0n}$ modes do not depend on the magnetostatic field. Therefore, in this present analysis these modes were not considered in any detail. In addition to these $TE_{0n}$ modes, anomalous modes having all six E and H field components have been shown to exist. An investigation of these modes has been made.

Using Maxwell's equations and the appropriate boundary conditions, a rigorous and exact solution of all these six field components of the anomalous modes has been made by employing a different procedure from Nag and Engineer. In their theoretical analysis, a sinusoidal field intensity variation in the direction of the applied magnetic field was assumed. However, in the present work, no such assumption has been made and the sinusoidal field intensity variation in the direction of the magnetic field has been proved mathematically by employing a special method of separation of variables. Proceeding in this completely different way, a resultant transcendental equation for the propagation constant has been obtained which agrees with the transcendental equation of the propagation constant, as obtained by Nag and Engineer. It has been shown that the propagation characteristic is in general reciprocal.

A numerical solution of this transcendental equation has been made by employing the Newton-Raphson method. To the knowledge of the author, the numerical solution of this complicated transcendental equation has never been carried out before. The attenuation constant and the phase constant of the electromagnetic wave as a function of the applied magnetic field $B$ which varies from 0 to 10 Kilogauss.
have been obtained numerically for n-type germanium with conductivity varying from 4 mho/meter to 4.65 mho/meter.

In this range of the magnetic field, both the attenuation constant and the phase constant are found to decrease with the increase of the applied transverse steady magnetic field. As the conductivity of the sample is increased the values of the attenuation constant and the phase constant are increased for a particular value of the magnetic field. Referring to figure (4.d) it is seen that as the magnetic field is increased from 0 to 0.9 Wb/m², the attenuation constant $\alpha$ decreases from 186.01 neper/meter to 162.5 neper/meter for the material having conductivity 4 mho/meter. For materials having conductivity 4.3 mho/meter the attenuation constant $\alpha$ decreases from 199.19 neper/meter to 174.19 neper/meter, in the same range of the magnetic field. The attenuation constant for materials of conductivity 4.5 mho/meter decreases from 207.89 neper/meter to 181.85 neper/meter as the magnetic field is increased from 0 to 0.9 Wb/m². This shows that with the increase of the conductivity of the sample the net change in the attenuation constant increases.

As the magnetic field is increased from 0 to 0.9 Weber/m² the phase constant $\beta$ decreases from 803.8 rad/m to 799.9 rad/m for materials having conductivity 4 mho/m; from 806.96 rad/m to 802.5 rad/m for materials having conductivity 4.3 mho/meter and from 809.15 rad/m to 804.29 rad/m for materials having conductivity 4.5 mho/meter. Similarly, the net change in the phase constant is found to increase with the increase of the conductivity of the sample.
8.2 EXPERIMENTAL WORK

The experimental verification of the theoretical analysis was made on a 22.2 ohm-cm n-type germanium sample at 9.46 GHz using a reflection type microwave bridge. The d.c. resistivity of the sample was measured with a four point probe. To the knowledge of the author, no experimental work employing this technique at this frequency range has been reported previously. The reflection bridge measurement technique followed that suggested by Champlin, Holm and Armstrong. This gave the magnitude and phase of the reflection coefficient for different values of the applied magnetic field which are plotted as a function of the magnetic field in figures 7.2(A) and 7.2(B).

8.3 COMPARISON OF THEORETICAL AND EXPERIMENTAL WORK

The measured values of the reflection coefficient at the air semiconductor interface were compared with the theoretical values of the reflection coefficient. As shown in figures 7.2(A) and 7.2(B) the agreement of the theoretical and the practical values of the reflection coefficient was excellent in both magnitude and phase.

Two expressions for the theoretical reflection coefficient have been derived. One expression is based on the simple TE mode analysis. The other expression is based on the considerations of the $E_y$ and $H_x$ components of the electromagnetic field in the anisotropic material and on the first order approximation of the characteristic impedance in this material. The magnitudes of the reflection coefficient obtained theoretically by the above two expressions agree well with the
measured magnitudes of the reflection coefficient. However, it is found that at higher magnetic fields the theoretical reflection coefficient magnitude, based on the first order approximation, is closer to the experimental curve than those based on the simple TE mode analysis.

For the phase of the reflection coefficient the simple TE mode analysis gives theoretical values which almost coincide with the experimental values. However, the theoretical values of the phase of the reflection coefficient based on the first order approximation, depart from the experimental values for higher magnetic field.

The calculation of reflection and transmission coefficients of an electromagnetic wave from an anisotropic medium such as in this case with all the six anomalous field components, is quite complicated. To the knowledge of the author an explicit expression for the reflection coefficient from such a surface has not yet been published. The analysis for the reflection coefficient which has been presented does not in general completely describe the behaviour of the practical system.

The analysis presented in this thesis for a completely filled guide can in principle be extended to the case of a waveguide partially filled with semiconductor subjected to a steady transverse magnetic field. A theoretical analysis would be very complicated but the analysis would be worthwhile, because of the practical importance of the partially filled structure.
APPENDIX

(COMPUTER PROGRAMMES)
MICROWAVE PROPAGATION THROUGH SEMICONDUCTOR FILLED WAVEOUGHS UNDER TRANSVERSE EXTERNAL MAGNETIC FIELD

FREQUENCY 9.46 GHz, N-TYPE GERMANIUM SAMPLE.

COMPLEX EPS1, EPS2, EPS3, A3, B3, C1, C2, C3, FM, F, R, S, V, T, D3, FGM, DFG, GAMMA:

B = 0.6
SIGMA = 4.5

8
B2 = B**2
EPS1 = CMPLX(1.41664E-10, -0.168E-10*SIGMA)
EPS2 = CMPLX(1.41664E-10, -0.168E-10*SIGMA / (1.0 + 0.181**2))
EPS3 = CMPLX(1.0*716E-12, 0.181**2*SIGMA / (1.0 + 0.181**2))
WRITE(6, 16) B, SIGMA, EPS1, EPS2, EPS3

16
FORMAT(2F5.2, 6E15.6)
A3 = 2.2E15*(EPS3**2) / EPS2
B3 = CMPLX(A3**2 - 3.784E*A3)
C1 = 0.5*(A3 + B3) / (A3 - B3)
C2 = 0.25*(1.0 / C1)
C3 = C1 + C2
FM = 1.89E4*C4.45E15*EPS2**A3
G = FM + A3
H = FM - A3
GAMMA = CMPLX(0.1990305E+03, 0.8074836E+03)
A4 = G - (GAMMA**2)
B4 = H - (GAMMA**2)
ROOT1 = CSQRT(A4)
ROOT2 = CSQRT(B4)

EM1B = CEXP(ROOT1*0.01015)
D1 = 1.0/EM1B
EM2B = CEXP(ROOT2*0.01015)
D2 = 1.0/EM2B
P = 0.5*(EM1B + D1)
Q = 0.5*(EM1B - D1)
R = 0.5*(EM2B + D2)
S = 0.5*(EM2B - D2)
V = A4*(1.0 - C1) - B4*C2) / ROOT1
W = B4*(1.0 - C2) - A4*C1) / ROOT2
T = GAMMA*(A4 + B4) / (ROOT1*ROOT2)
D3 = GAMMA**0.01015
FGM = ((ROOT1*ROOT2*(1.0 - P*R)) + (A4*C1) + (B4*C2) * C*S)
DFG = ((D3*P*S*V) + (D3*O*R*W) + (T*P*R) - (2.0*GAMMA*C3*O*S) - T)
GAMMA = GAMMA - (FGM / DFG)
X1 = ABS(REAL(GAMMA - GAMMA))
X2 = ABS(IMAG(GAMMA - GAMMA))
IF(X1.LE.1E-1. AND. X2.LE.1E-1) GO TO 10
GAMMA = GAMMA
GAMMA = GAMMA
GO TO 9

10
WRITE(6, 11) B, GAMMA
11
FORMAT(1F5.2, 2E16.7)
R = 0.05
IF(R.LE.1.0) GO TO 8
STOP
END
C REFLECTION COEFFICIENT CORRESPONDING TO THE CHANGE IN
C PROPAGATION CONSTANT.
C REFLECTION COEFFICIENT AT THE SEMI-CONDUCTOR-AIR INTERFACE
C IN A RECTANGULAR WAVE GUIDE UNDER A TRANSVERSE MAGNETIC FIELD
C FREQUENCY 9.46 GC/SEC, N-TYPE GE SAMPLE, RESISTIVITY 25-Ohm-cm.
C COMPLEX GAMMA, A, B, V, P, R, R
C REAL ABSVAL, PHASE, D
16 READ(5,17) GAMMA
17 FORMAT(2E16.7)
   A=CMPLX(0.0,143. )
   D=1.8796E-3
   B=2.0 *GAMMA *D
   V=(CEXP(B)-1.0)/(CEXP(B)+1.0)
   P=(A*V+GAMMA)
   Q=(A*V+GAMMA)
   R=P/Q
   ABSVAL=CABS(R)
   PHASE=57.29578*ATAN2(AIMAG(R),REAL(R))
   WRITE(6,22) ABSVAL, PHASE, GAMMA
22 FORMAT(4E16.7)
25 GO TO 16
27 STOP
END
$ENTRY
 0.2078852E+03  0.8091483E+03
 0.2075141E+03  0.8090784E+03
 0.2064098E+03  0.8088704E+03
 0.2045983E+03  0.8085294E+03
 0.2021207E+03  0.8080636E+03
 0.1990305E+03  0.8074836E+03
 0.1953898E+03  0.8068020E+03
 0.1912663E+03  0.8060328E+03
 0.1867307E+03  0.8051912E+03
 0.1818537E+03  0.8042929E+03
 0.1793089E+03  0.8038274E+03
$IBSYS
C REFLECTION COEFFICIENT AT SEMICONDUCTOR-AIR INTERFACE IN PLOTTING
C UNDER TRANSVERSE EXTERNAL MAGNETIC FIELD
C TAKING INTO CONSIDERATION OF THE FIRST TERM OF THE TAYLOR'S
C SERIES EXPANSION OF THE CORRECTION TERM T.
C
COMPLEX GAMMA, EPS2, EPS3, M1, M2, A1, A2, A3, B1, B2, B3, C3, D, FOD, N1, N2, N3, N4, FO, P, BETA, R, FOD, K2, DMIN, V1, V2, S,
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PHOTOGRAPH 1: The Experimental set up of the Microwave Reflection Bridge
PHOTOGRAPH 2: Rear View of the Microwave Reflection Bridge Showing the Electro Magnet and the Current Regulated Power Supply
REFERENCES


