THREE-DIMENSIONAL RELATIONSHIPS AMONG TRAFFIC FLOW THEORY VARIABLES:
A COMPARATIVE STUDY

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AMONG
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Abstract

This paper is a further investigation of Gilchrist and Hall's work on the three-dimensional relationships in traffic flow theory variables (Gilchrist and Hall, 1989). The investigation examines the three variables of traffic flow (speed, volume, and occupancy). Gilchrist and Hall's data is then compared with data used in this analysis. The traffic variables are first examined by the traditional two-dimensional model and then they are examined obliquely in three-dimensional space. The resulting oblique views of the data are reviewed for points of agreement with conclusions made by Gilchrist and Hall. Resultant views of the data suggest that there is a plane along which all of the uncongested data fall. However, this phenomena only occurs when there is a wide range of speed values in the data set. The results also suggest that low-speed congested data do not lie on the same plane as do the uncongested data. It is concluded that low-speed congested data do not lie on a plane at all.
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In the past, uninterrupted traffic flow was examined only in two-dimensional space by the traditional theory (Gilchrist and Hall, 1989, p.2). This paper is a further investigation of Gilchrist and Hall's work on the three-dimensional relationships among traffic flow theory variables (Gilchrist and Hall, 1989). The first section of the paper gives a brief description of the research, gives some background information on the topic, discusses conclusions made by Gilchrist and Hall, and outlines the purposes of the investigation. The second section indicates how, where, and when the data were collected. There will also be a description of the nature of the data in this section. In the third section, the plotting process will be described, as will an explanation of how to interpret the data plots. Traditional and oblique views of the data will also be observed in this section. Following that, a discussion of the relationship of the data to the catastrophe theory will be given. Finally, a number of conclusions are presented.
Unfortunately, little work has been done on the three-dimensional relationships among traffic flow theory variables as it is a relatively new study. This section will briefly describe the difficulties researchers have had in the past with matching traffic flow theory with data. It will also describe an alternative model for viewing traffic flow.

In the past, uninterrupted traffic flow was measured only in two-dimensional space using the traditional traffic flow theory. However, recent improvements in data collection methods have created concern over the validity of the traditional theory. The problem that arises is the inability of the traditional theory to completely and accurately explain the data obtained by the new equipment and techniques that are now available. Today, several recording systems can now accurately collect speed as well as volume and occupancy data. In the past speed had to be calculated based on the relationships between the volume and occupancy data.

Researchers of today and the past have experienced difficulty in matching traffic flow theory with data (Gilchrist and Hall, 1989). Gilchrist and Hall (1989) discuss two examples of this inability to match data with
traffic flow theory in their paper. The examples given are from work done by Ceder, and Hurdle and Datta.

An example of this theory modification occurs in Ceder's work (1975, 1976). His data did not agree with the conventional theory very well so he modified the theory to create a better match. The way he did this was by using a two-regime model. Ceder used continuous as well as discontinuous functions in traffic flow theory to try to explain the existence of gaps in the data patterns. However, he found that different sets of data collected at the same location required different sets of parameters. Even though Ceder made modifications to theory, complete success was still not obtained.

Similarly, Hurdle and Datta (1983) attempted to fit a curve to data. This attempt was not successful because of the considerable scatter in the data. Hurdle and Datta tried to fit the data they were working with to various "theoretical" curves. Their attempts were also frustrated because they were unaware of the shape the curve was supposed to take.

These approaches have attempted to model the data using some form of a curve in two-dimensional space. Hall and others believe that speed-flow-density relationships are better explained using a three-dimensional model defined by the cusp catastrophe theory (Hall, 1987, Hall and Dillon, 1987, Persaud and Hall, 1988). Navin (1986) first suggested
that the catastrophe theory might provide the necessary approach to explain why gaps occur in the data patterns.

The catastrophe theory takes its name from the sudden discrete changes that occur in one variable of interest while other related variables are exhibiting smooth and continuous change (Persaud and Hall, 1988). For instance, a sudden discrete change might occur in speed while flow and occupancy exhibit smooth and continuous change. This is usually the case with these three variables.

The catastrophe theory was developed by Thom (1975), and elaborated on by Zeeman (1977). According to Gilchrist and Hall (1989), cusp catastrophe can be characterized graphically by a partially torn sheet of paper. At the centre of the paper (at the tear) operations do not occur. The top portion of the page represents high speed uncongested activities while the lower section of the page contains low speed congested operations. Therefore, when operations cross the folded area they "drop off" the so called edge and move from uncongested conditions to congested conditions and visa versa. According to Gilchrist and Hall (1989), transitions between congested and uncongested conditions can occur either by traversing the tear or by going around the end of the tear. Movement around the tear indicates a sudden jump in all of the three variables. If the transition occurs by traversing the fold, one of the three variables experience a sudden jump while
the other two undergo continuous change.

Persaud and Hall (1988) provide three points of support for the catastrophe theory model. The first is that occupancy and flow change gradually while speed changes suddenly. The second important point of support is agreement between data taken at different time intervals (30-second and 5-minute intervals) with regard to the amount of scatter in the congested data. The third point of support is that the uncongested operations are confined to a tightly defined line, with little variance from it.

The main advantage of the catastrophe theory, according to Persaud and Hall (1988), is that it provides an understanding of the "jumps" (transitions) from congested to uncongested conditions and visa versa. The main disadvantage of the theory is that some observations still do not fit. Another problem of the theory is that testing has relied upon two-dimensional projections, despite the fact that the theory is explicitly three-dimensional. New computer programs, such as the Macintosh Macspin software (used in this analysis), should reduce this problem as they are able to represent the data explicitly in three-dimensional space.

The catastrophe model provides a strong basis for an alternative approach to incident detection on freeways. The results are also of theoretical importance. Gilchrist and Hall have recently tested the catastrophe theory model on
data obtained from the Queen Elizabeth Way in Toronto. Their study was somewhat successful, however, they were faced with some peculiarities in the data. Data from other freeway systems are needed to test the overall catastrophe theory model.

2.1 CONCLUSIONS MADE BY GILCHRIST AND HALL

The conclusions made by Gilchrist and Hall are as follows:

There is a plane along which all of the uncongested data fall; and

Low speed congested data do not lie in the same plane as do the uncongested data.

Gilchrist and Hall's data are based on recurring traffic congestion on the Mississauga QEW. According to Gilchrist (1989, p.32) "east bound travel into Toronto generates recurrent, ongoing congestion between about 7:00 a.m. and 9:00 a.m.". The data used in this analysis represent times in which incidents or accidents, occurring on the Skyway, create traffic congestion. Data from recurring congestion may differ somewhat from incident-caused data. For example, in the case of recurring congestion, motorists are usually aware that there will be a slowdown in traffic at specific times. As a result, drivers who are aware of possible congested conditions (recurring congestion) may tend to be more prepared to slowdown than
drivers that are unaware of approaching congestion (incident/accident-caused). A more drastic slowdown of traffic in the case of incident-caused congestion will generally occur. An analysis of incident caused congestion is needed to compare with Gilchrist and Hall's analysis of recurring traffic congestion.

2.2 PURPOSE OF THE INVESTIGATION

The purposes of this investigation are as follows:

To examine empirically the three-dimensional relationships among speed, volume, and occupancy for uninterrupted traffic flow;

To compare Gilchrist and Hall's data with data used in this analysis; and

To provide evidence to either support or refute conclusions made by Gilchrist and Hall's study.

CHAPTER THREE

THE DATA

3.1 DATA ACQUISITION

Data for this analysis were obtained from the Burlington Freeway Traffic Management System (F.T.M.S.) located on the Burlington Skyway, just east of Hamilton,
Ontario. The portion of the highway under study is the part of the Skyway that is situated to the southeast of the Burlington canal. The data presented in this paper are from north-bound stations seven and eight and south-bound stations six and seven. This information was taken from the FTMS computer tapes for July 30, August 4, and August 6, 1989. Unlike Gilchrist and Hall's data, which were based on recurring traffic congestion, data for this analysis represent times in which incidents or accidents, occurring on the Skyway, created traffic congestion. Seven separate incidents are analyzed and compared with the data found by Gilchrist and Hall.

The FTMS uses video cameras, changeable message signs, and traffic monitoring stations to continuously manage and record freeway operations occurring on the Burlington Skyway. Induction loops are placed in each lane at various counting stations along the Skyway to record some of the freeway operations. Each loop is able to collect flow and occupancy data. Speed is calculated for each vehicle as it passes over successive loops which are about six meters apart. The speed data are averaged for all vehicles passing over the detectors every 30 seconds. Volume is a measure of the actual number of vehicles that pass over each loop at 30-second intervals. Occupancy is a measure of the percentage of time that vehicles are passing over the loop detectors. This is also measured at every loop at 30-second
intervals.

The data were extracted from the Burlington FTMS computer tapes for each incident via the "sscvax" system at McMaster University. A program called "Resky" reads the summary files obtained from the FTMS and puts them in a form which allows easy use. Each data file was then transferred from the "vax" to a 3.5 inch disk for use on the MacIntosh Apple computer. Three-dimensional representations of the data are created using the "Macspin" software offered by this system. Graphical representations of speed, flow, and occupancy are then analyzed and interpreted. These graphs are then compared and contrasted with the ones based on data extracted from the Mississauga FTMS (QEW, Toronto) by Gilchrist and Hall.

3.2 REVIEW OF THE DATA

3.2.1 JULY 30, 1989

On this day, incidents were detected at two different stations, north-bound seven (NB7) and north-bound eight (NB8). NB7 and NB8 are located to the southeast of the Burlington canal. NB7 is upstream of NB8. (Traffic passes station NB7 before it passes station NB8.) The data used for plotting purposes included 721 30-second intervals for each station. In each case, the first point plotted is 4:00 p.m. (interval 1921). The last point plotted is 10:00 p.m.
At the NB7 station, speeds were consistently over 80 km/h between interval 1921 and interval 2284 (7:01:30). At about interval 2285 (7:02), there was a sudden drop in speed as traffic operations became congested. Between intervals 2307 (7:33) and 2321 (7:40), speeds climbed slowly and slightly over 80 km/h. From interval 2322 (7:40:30) to interval 2455 (8:47), speeds were again consistently well below 80 km/h, thus indicating congestion. At interval 2456 (8:47:30) speed began to climb above 80 km/h. From about that interval onward, the freeway again operated under uncongested conditions.

Although speed values are usually available for most of the 30-second intervals, they are sometimes not recorded. Values for volume and occupancy were also not recorded at various times. When a value for any of the three traffic variables (speed, flow, and occupancy) is not sent from the station to the FTMS, a value of negative one (-1) is substituted. Of the 721 intervals for which data were obtained and plotted, 57 had negative one values. This represents 7.9 percent of the data set. The problem of missing data will be discussed in the next section.

Slightly downstream of the NB7 station, the NB8 station recorded speeds consistently above 80 km/h between interval 1921 (4:00) and interval 2281 (7:00). After interval 2281
there was a slight drop in speed as traffic operations became slightly congested. However, congestion only lasted for one and a half minutes (3 intervals), compared with an eleven minute duration at the NB7 station. Operations then hovered around the 80km/h to the 100 km/h range until about interval 2457 (8:28). Between intervals 2285 (7:02) and 2457 there were only a few occasions when speed dropped below 80 km/h. At interval 2457 speeds climbed well over the 80 km/h mark (100 km/h +). There was however, a slight drop in speed below 80 km/h at interval 2594 (9:36:30). Generally, the freeway operated under uncongested conditions from approximately interval 2285 onward. However, for uncongested conditions, traffic operations were relatively slower than normal (80-100 km/h). It was not until after interval 2456 (8:27:30) that the station witnessed "true" uncongested conditions (100 km/h +). At station NB8, a total of 11 of the 721 intervals had negative one values. This represents about 1.5 percent of the data set.

3.2.2 AUGUST 4, 1989

On August 4, 1989 two incidents were detected on the Skyway. The incidents were detected by south-bound station six (SB6) and south-bound station seven (SB7) at approximately 4:00 p.m. The data used for plotting purposes, in both cases, included 721 intervals of 30-seconds each. The first point plotted for each incident is
1:00 p.m. (interval 15961). The last point plotted in each case is 7:00 p.m. (interval 16631).

At station SB6, speeds were consistently over 80 km/h between interval 15961 and interval 16287 (3:43). Speeds took periodic drops below 80 km/h from interval 16288 (3:43:30) to interval 16320 (3:59:30). At about interval 16323 (4:01) there was a sudden drop in speed as traffic operations became congested. From interval 16351 (4:15) onward, the freeway operated under uncongested conditions with only periodic drops in speed below 80 km/h. Station SB6 was unable to record speed in only one of the 721 intervals. This represents .14 percent of the data set.

Slightly downstream of SB6, another incident was detected by the SB7 station (probably the same incident in both cases). At this station, speeds were consistently over 80 km/h between interval 15961 (1:00) and interval 16315 (3:57). Periodically, speeds dropped slightly below 80 km/h after interval 16315. From interval 16342 (4:10:30) onward, uncongested conditions continued. In this data set, 27 of the 721 intervals did not contain values for speed. This represents 3.7 percent of the data set.

3.2.3 AUGUST 6, 1989

On August 6, 1989 station NB7 detected two incidents. One was at 6:00 p.m. and the other was at 9:00 p.m. Since the two incidents occurred on the same day, they will be
grouped and plotted collectively. The data used for plotting purposes included 839 30-second intervals. The first point plotted is 5:00 p.m. (interval 2041). The last point plotted is 11:59 p.m. (interval 2879). Between intervals 2185 (6:12) and 2191 (6:15) the first congested traffic conditions occur. From interval 2192 to interval 2523 (9:01), traffic was uncongested with speeds above 80 km/h. It is at interval 2524 (9:01:30) where the second congested traffic conditions occur. Traffic remains congested until interval 2539 (9:09). After that speeds again climbed above 80 km/h.

It should be noted that the congested periods experienced relatively high speeds (60-70 km/h). It should also be mentioned that at interval 2211 (6:25), a speed of 43 km/h was recorded. This speed is unusual because it occurs well after the first congested period (10 minutes) and also because it is surrounded by a number of intervals with speeds exceeding 100 km/h. Analysis was done with and without the interval to see if there would be any difference in output. The results arrived at were roughly the same in both cases. Therefore, it was determined that interval 2211 should be included in the data set as it did not significantly affect the data.

A total of 49 of the 839 intervals for which data were obtained and plotted had negative one values. This represents 5.8 percent of the data set. However, of the 49
intervals that were absent of speed values, 10 occurred in a span of 11 consecutive intervals. This leads to considerable concern for the reliability of the data points.

Finally, the last incident under investigation is from station NB8. This incident was detected on August 6, 1989 at approximately 9:01 p.m. The data used for plotting purposes included 719 30-second intervals. The first point plotted is 6:00 p.m. (interval 2161). The last point plotted is 11:59 p.m. (interval 2879). Between interval 2161 and interval 2517 (8:58) speeds were consistently over 80 km/h. Between interval 2518 (8:58:30) and interval 2570 (9:24:30) traffic operations became congested. That is, speeds were below 80 km/h. From that interval onward, the freeway operated under uncongested conditions. However, at interval 2671 (10:15), interval 2810 (11:24:30), and interval 2820 (11:29:30) speeds dropped to 59 km/h, 46 km/h, and 58 km/h, respectively. These sudden drops in speed are considered to be peculiar occurrences. Of the 719 intervals observed, 7 were not given a value for speed. This represents .97 percent of the data set.

3.3 RELIABILITY OF THE DATA

The absence of speed values leads to some concern for the reliability of the data set. These absent values may have contained some potentially important information,
especially in the case of NB7 station data on August 6, 1989 where ten absent values occurred in a span of eleven consecutive intervals. However, Gilchrist and Hall decided that missing data did not significantly affect their investigation (Gilchrist & Hall, 1989, p.19). Therefore, in keeping consistent with Gilchrist and Hall's study, intervals which did not contain speed values (speed value = -1)) were replaced with the data from the previous interval.

3.4 DIFFERENCES BETWEEN GILCHRIST AND HALL'S DATA AND THE DATA USED IN THIS ANALYSIS

A difference that arises between the two studies is that the data are taken from two different locations; the Mississauga QEW; and the Burlington Skyway. This means that there are physical differences between the two highways. Gilchrist (1989, p.6) explains that the freeway he obtained his data from is situated on generally level terrain. On the other hand, data used in this analysis were obtained from the Skyway which is located on a relatively steep grade. The topography of the highway may have an influence on the speed of traffic. For instance, a steep incline (NB8) would tend to slow traffic down, a steep decline (SB6 and SB7) would tend to speed it up, and level terrain would tend to keep traffic at constant speeds.
The fact that the data are obtained from two separate highways also means that each highway is travelled by a different assortment of drivers. Those riding on the Mississauga QEW may have a totally different style of driving than those on the Skyway. For instance, "QEW drivers" may drive faster, slower, or more or less cautiously than "Skyway drivers". Differences in driving styles may result in differences between the data sets.

Another difference between the data sets has to do with volume of traffic. That is, recurring traffic congestion (Gilchrist's analysis) is generally caused by an increase in traffic volume. When volume becomes to great for the highway to handle, a slowdown in traffic occurs. On the other hand, incident-caused congestion (this analysis) precedes volume congestion. That is, increased volume is created after an incident occurs. As a result, with incident-caused congestion there is an initial slowdown as a result of the incident and then, as volume builds, further slowdown occurs because of the increased volume.

Although information is accumulated under differing conditions in each study, there should not be a problem when comparing the data sets. Data sets from both investigations contain the necessary congested and uncongested conditions. As a result, differences in the data sets may be expected.
CHAPTER FOUR
ANALYSIS

4.1 THE PLOTTING PROCESS

Once the data were extracted from the Burlington FTMS, they were down-loaded onto a Macintosh computer system. It was here where graphical representations of the data were created. The various data sets were plotted using a three-dimensional rectangular coordinate system. In this scheme, the positive x-axis extends horizontally to the right; the positive y-axis extends vertically upwards; and the positive z-axis extends into the plane of the page away from the viewer (Gilchrist, 1989, p.21-22).

Data will first be presented in two dimensions as views in the plane of the axis system. Following that, parallel oblique projections of the data will be presented. In these views, one of the planes in the "XYZ" coordinate system is placed in the plane of the page, and the third dimension is viewed obliquely.

The majority of the plots will represent the data from the July 30th incident on north-bound station seven (NB7). This data set contains the widest range of information and will therefore provide the best possible picture of the results. An explanation of how the data plots are transformed from the traditional views to the oblique
projections will also be demonstrated using the July 30th information. Final oblique projections will be plotted for each incident.

Projections of data used to obtain traditional views are outlined in figure 1. The views of the traditional theory are of speed-volume, speed-occupancy, and flow-occupancy. Each view is represented by a reference number. Reference point "1" corresponds to the volume-occupancy view, point "2" represents the speed-occupancy view, and point "3" corresponds to the speed-volume view. The July 30th, north-bound station seven (NB7) data are represented by each of these views.

In order to arrive at the oblique projections of the data, the traditional views had to be rotated. Rotation was done on the speed-occupancy plots because they offered the best oblique projection results. The traditional plot was rotated to the left, downward, and then to the right in order to arrive at the oblique projections. This process will be further analyzed in the following section using the July 30th NB7 data.
FIGURE 1: PROJECTIONS USED TO OBTAIN TRADITIONAL VIEWS

4.2 TRADITIONAL VIEWS OF THE NORTH-BOUND STATION SEVEN DATA

In this section several traditional views of the "0730NB7" data set will be discussed and illustrated. The data from this station contain a great number of congested and uncongested conditions and therefore provide an excellent view of the incident. Traditional views of the other incidents will not be illustrated as a view very similar to that of the July 30th NB7 incident is created. It is felt that the traditional views of the July 30th NB7 incident provides a reliable representation of the other incidents.

Each plot is represented by three traffic variables; speed, volume, and occupancy. In each figure the "volup" label depicts volume data, the "occup" label depicts occupancy data, and the "spd" label depicts speed data. The plus sign ("+"") represents speeds in excess of 80km/h. These data points are assumed to represent uncongested traffic conditions. The box symbol ("□") represents speeds that are between 50km/h and 80km/h. These speeds are indicative of traffic conditions that are in transition from a congested to an uncongested state and visa versa. Finally, the multiplication symbol ("x") represents congested freeway conditions (speeds below 50km/h).

Figure 2 shows the traditional volume-occupancy view.
FIGURE 2: TRADITIONAL VOLUME-OCCUPANCY VIEW
To the left of the plot are the uncongested data ("+") , to
the right are the congested data ("x") and between the two
groups are data in transition from congested to uncongested
("□") conditions and visa versa. The uncongested area (to
the left of the plot) is characterized by a relatively
narrow band width. The data extend from a point close to
the origin upwards at an angle of about 67 degrees.

The congested data with speeds less than 50km/h ("x"),
are characterized by a much wider band width. The extent of
this band is partially due to the wide range of occupancy
rates. The group of data that appear between the high-speed
uncongested conditions and the low-speed congested
conditions ("□") are also regarded as congested traffic
conditions. This group of congested data, with speeds
between 50km/h and 80km/h, appear to contain a band width
that is wider than the band width of the uncongested data
and narrower than the band width of the congested data.
This will become more apparent in the oblique views of the
data.

Figure 3 demonstrates the traditional speed-occupancy
view of the data set. In this view a better indication of
speed range is given because speed is defined by the
vertical axis ("y-axis"). This view creates no overlap of
the speed stratifications. Figure 4 shows the traditional
speed-volume view of the data set. A wider variation in
volume is noticed in this view because volume is defined by
FIGURE 3: TRADITIONAL SPEED-OCCUPANCY VIEW
FIGURE 4: TRADITIONAL SPEED-VOLUME VIEW
the horizontal axis ("x-axis"). In figures 3 and 4 a very wide band width is noticed.

From the three traditional views, a few generalizations may be made. First, high speed values are characterized by very wide variations in volume and relatively low variations in occupancy rates. On the other hand, low speed values are characterized by wide variations in occupancy rates and relatively low variations in volume. Mention should be made at this point about the high-speed value that is located at the far right of figures 2, 3, 5a,b,c,d, and 6 (denoted by a plus sign ("+")). This point is assumed to be a mistake as it occurs between a series of congested conditions. On either side of the point, speed values of less than 40km/h occur (24km/h and 35km/h). It is unlikely that speeds increased or decreased this much (approximately 100km/h) over such a short time period (4 30-second intervals).

4.3 OBLIQUE VIEWS OF THE DATA

Oblique views of the data are provided in the following section. These views were obtained by rotating the traditional plots. An example of this process is shown using the "0730NB7" (July 30th) data and illustrated in figure 5. Figure 5a shows the traditional speed-occupancy view of the "0730NB7" data. This plot is rotated to the left to arrive at figure 5b. Figure 5c shows the plot after
FIGURE 5: PROGRESSION FROM A TRADITIONAL TO AN OBLIQUE SPEED-OCCUPANCY VIEW
it was rotated downward. Finally, the end-product, as shown in figure 5d and the remaining views, were arrived at by rotating the plot back to the right.

As indicated in the previous section, the traditional view of the "0730NB7" speed-occupancy plot (figures 3 and 5a) reveals a wide range of high speed data. The traditional view indicates that the data are partially situated on their side but are tending more towards lying flat. When the plot is rotated to the left (figure 5b), the uncongested data spread out even more, thereby further increasing the band width. If the plot was further rotated to the left, the high speed data would begin to cluster again. This view (figure 5b) indicates that the data are totally spread and lying flat on a surface.

Rotating the plot downward or forward tends to spread out the high speed variables horizontally while condensing them somewhat vertically (figure 5c). This view of the plot is from the top portion of the graph where the viewer is looking down at the picture. Rotation of the plot back to the right creates an image shown in figure 5d. This oblique view is also used to represent the remaining incidents. In these views, the plot is tilted forward and to the right. Both the volume and the occupancy axes are pointing downward and toward the viewer.

In this view (figure 5d), even more so than the traditional volume-occupancy view, a very narrow band width
is noticed of the high-speed uncongested data. Figure 6 depicts sequential lines connecting the points of data. This picture provides an even more obvious indication of the cluster of the uncongested data and the scatter of most of the congested data. The uncongested data in figure 6 tend to be oscillating vertically. These data points are characterized by substantial vertical movement perpendicular to the horizontal axis. Speeds in this area are in excess of 80 km/h. At speeds less than 50 km/h ("x") there seems to be a more prominent movement along the horizontal axis. That is, the congested data are oscillating horizontally. At speeds between 50 km/h and 80 km/h ("□") there appears to be a transition in the data from the vertical oscillations of the higher speeds to the horizontal oscillations of the lower speeds.

The presence of the narrow band of points to the left of the graph, indicates that the uncongested data fall on a tilted plane. This plane is viewed on its edge. Using an analogy of paper with writing on it may provide a better indication of how these data are viewed. Viewing a document that is lying flat creates a spread out impression of the words on it. The traditional speed-occupancy and speed-volume plots characterize this occurrence. When this same document is viewed on its side, the words begin to clutter or group together, as seen in figures 5d and 6.
FIGURE 6: SEQUENTIAL LINE REPRESENTATION OF OBLIQUE PROJECTION OF 0730NB7
The oblique views of data in figures 7 through 11 are not typical of the oblique projections of the "0730NB7" data. Particularly in figures 7, 8, and 9, the uncongested data are not grouped within a narrow band like is noticed in figures 5d and 6. The uncongested data ("+") in figures 7 through 9 tend to be very spread out. This is especially noticeable near the middle of the graphs. As a result, it is very difficult to conclude that the uncongested data fall on a plane.

On the other hand, the uncongested data of the "0806NB7" incident (figure 10) are characterized by a relatively narrow band width, similar to that of the "0730NB7" incident (figures 5d and 6). Although not as tightly clustered as the uncongested data of the "0730NB7" incident, it is apparent that the uncongested data in figure 10 show little scatter. The presence of the narrow band of uncongested data indicates that these data points fall on a plane.

The uncongested data in figure 11 are characterized by a relatively wide scatter. However, unlike the wide dispersal of data that was noticed in figures 7 through 9, the uncongested data in figure 11 tend to cluster vertically in columns. The data appear to take the form of five distinct tightly clustered columns. It is difficult to understand why these data points cluster in this manner. It is apparent, however, that the wide scatter of points
FIGURE 7: OBLIQUE VIEW OF 0730NB8
FIGURE 8: OBLIQUE VIEW OF 0804SB6
FIGURE 9: OBLIQUE VIEW OF 0804SB7
FIGURE 10: OBLIQUE VIEW OF 0806NB7
FIGURE 11: OBLIQUE VIEW OF 0806NB8
indicates that the uncongested data do not fall on a plane.

The general shape of the plots also fail to follow a consistent pattern. The data in figures 5d and 6 appear to take the form of an stretched "C" (\(\text{"("}\)), whereas the data in figures 7 through 10 follow a somewhat straight line. These phenomenon may be occurring because of the absence or abundance of congested data points. In figures 5d and 6, there are an abundance of low-speed congested ("x") and uncongested data points. It is the presence of the low-speed congested points that tend to create a "bend" in the data. The deficiency of low-speed congested points in figures 7 through 10 result in a "linear" appearance of the data. In figure 11, a very slight bend in the data is noticed. This may be a result of the occasional appearance of low-speed congested data points.

In general, the uncongested data in figures 5d and 6 ("0730NB7") seem to be much more tightly clustered than the uncongested data of the other incidents. This is assumed to be a result of spatial scaling. Only the "0730NB7" incident contains a wide variety of both congested and uncongested conditions. The lack of congested variables in the data set results in a broadening of the scale in which the data are plotted. This tends to create a greater spread of the data. Variations in the range of speed values tend to create completely different plots. This seems to be a limitation of using the Macspin program to create plots of the three
traffic variables. Some of these plots, especially those that are lacking a wide range of speed values (figures 7-11), do not effectively describe the three traffic variables. Traffic variables that contain a small range of speed values would probably be better described using a two-dimensional model.

In figures 5d and 6, uncongested data ("+") and high-speed congested data ("□") tend to exhibit very narrow band widths. The uncongested data are characterized by vertical oscillations. Similarly, the high-speed congested data also tend towards vertical oscillations (although not as distinct). It seems as though all data with speeds exceeding 80km/h fall on the same tilted plane. When data from the 50km/h to 80km/h range are included with the uncongested data, the plane becomes twisted or bent.

The data plotted for speeds below 50km/h are not as clearly defined. These data form a very wide band that extends horizontally to the right of the uncongested data (figures 5d and 6). In figure 6, an apparent shift from vertical oscillations on the left, to very pronounced horizontal oscillations on the right of the plot are noticed. Congested data with relatively high speeds (50km/h to 80km/h) tend toward vertical oscillations, while the data containing very low speeds tend toward horizontal oscillations. The band widths of the lower speed congested data are, once again, very wide. At no time do these bands
converge or taper. This is contrary to deductions made by Gilchrist and Hall. In their study they found a gradual tapering of the band width at lower speeds (Gilchrist and Hall, p.14).

As previously mentioned, the uncongested and high-speed congested data of the "0730NB7" (figures 5d and 6) and "0806NB7" (figure 10) tend to fall on a tilted plane. This plane runs parallel to the vertical axis ("speed") and then tends to bend toward the right at the middle of the graph. Evidence that uncongested data and high-speed congested data fall on a plane was demonstrated by the clustering of data points. It is, however, difficult to determine whether the low-speed congested data also fall on a plane. No clustering of the low-speed congested data was noticed. This would indicate that the low-speed congested data do not fall on a plane. Gilchrist and Hall (1989, p.23) concluded from their study that low-speed congested data may or may not lie on a plane. No evidence was found in this research to support the proposal that these data form a plane themselves.
CHAPTER FIVE
DISCUSSION

5.1 RELATIONSHIP OF THE DATA TO THE CATASTROPHE THEORY

Like Gilchrist and Hall's analysis, this study provides more support for the catastrophe theory model of traffic flow. This section outlines some of the points of agreement and disagreement with the catastrophe theory model. There are three main points of agreement and two main points of disagreement with the model. These five points were also made by Gilchrist and Hall (1989, p.20).

One point of agreement with the model is the presence of a plane in which the uncongested data fall. However, the presence of this plane was not evident in all of the plots. Figures 7 through 9 and figure 11 did not clearly show that a plane existed for uncongested data. Another point of agreement with the model is the fact that uncongested operations and low-speed congested conditions do not lie on the same plane. In some cases (figures 7, 8, 9, and 11), it was difficult to see the presence of any plane at all. In figures 5d, 6, and 10, where the existence of a plane was noticed for the uncongested and high-speed congested data, it was clear that the low-speed congested data did not fall on the same plane (or any plane at all) as the uncongested and high-speed congested data. The final point of agreement
with the model is the presence of discontinuities between the congested and uncongested data. It is clear that the uncongested data and the congested data do not act in the same manner. For instance, uncongested data oscillate vertically while low-speed congested data oscillate horizontally.

One point of disagreement with the model is the fact that the congested data do not clearly lie on a plane. Evidence of this was created by the relatively wide band widths of the congested data. It is also apparent that, in some plots, uncongested data do not clearly lie on a plane. The other point of disagreement with the model is that high-speed congested operations are occurring on the same plane as the uncongested conditions. Evidence of this was created by the relative difference in the band widths of the two data sets.

CHAPTER SIX
CONCLUSION

Throughout this paper the three-dimensional relationships among speed, volume, and occupancy have been empirically examined for uninterrupted traffic flow. From this study, evidence has been provided to help support or refute conclusions by Gilchrist and Hall, who concluded that
there is a plane along which all of the uncongested data fall and that the low-speed congested data do not lie in the same plane as do the uncongested data.

At least for uncongested data, there does seem to be a plane along which all of the data fall. However, a plane is only noticed when there is a wide range of speed values (both congested and uncongested). This plane is observed when the traditional views are rotated to create a clustering of the uncongested data points, illustrated in two of the oblique views ("0730NB7" and "0806NB7"). Data in this plane possess a very narrow band width and are characterized by vertical oscillations (figure 6).

With a wide range of speed values, high-speed congested data also seem to fall on a plane, the same plane as the uncongested data. The combination of the high-speed congested and uncongested data creates a slight twist in the plane. However, this twist is only witnessed in figures 5d and 6 ("0730NB7"). Figures 7 through 11 indicate a linear relationship of the data (probably due to the relative absence of low-speed congested data). The presence of a slight twist in the uncongested and high-speed congested data in figures 5d and 6 ("0730NB7") provides a strong indication that all data with speeds above 50km/h fall on a plane or surface. However, due to the deficiency of low-speed congested data in figures 7 through 11, the presence of a twisted plane was not noticed.
The plane of uncongested and high-speed congested data in figures 5d and 6 are characterized by narrow band widths and vertical oscillations. Low-speed congested data are characterized by horizontal oscillations and relatively wide band widths. It is therefore evident that the low-speed congested data do not lie on the same plane as do the uncongested data or the high-speed congested data. These characteristic were common to both studies.

However, unlike Gilchrist and Hall’s findings, a gradual tapering of the band width of low-speed congested data was not noticed. Rather, the band width remained wide, even at very low speeds. Also, little indication was provided in this study to support the claim made by Gilchrist and Hall that the low speed congested data may or may not form a plane themselves. No evidence that low-speed congested data form a plane was found in this research.

It is evident that more work is needed to clear up the contradictions between the two studies. It is also apparent that, with wide ranging speed values, speed, volume, and occupancy relationships are described very well using three-dimensional oblique projections of the data. In the absence of a wide range of speed values, the three-dimensional oblique projection does not provide a good representation of the data. Perhaps a two-dimensional approach would provide a better representation of the three traffic variables in this case.
REFERENCES


