FLOW OF A TWO-LAYERED VISCOUS FLUID

TOWARDS A LINE SINK

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By

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A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Master of Engineering

McMaster University
October 1965

Master of Engineering (1965) McMaster University

(Mechanical Engineering) Hamilton, Ontario

Title: Flow of a Two-Layered Viscous Fluid Towards a Line Sink

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Number of Pages: (vii) 65

Scope and Contents: The thesis contains an experimental verification of a paper by D. G. Huber dealing with a solution to the problem of the irrotational motion of two fluid strata towards a line sink. Friction was assumed negligible in the theoretical analysis and, as a result, the experimental values deviated somewhat from the theoretical expectations. The largest deviation occurred at the point of incipient drawdown where the theoretical and experimental values of the Froude number differed by 48%. The trends obtained in the experiment verify the theoretical solution. The effect of viscosity at the point of incipient drawdown for two different interface height to width ratios was determined.

The work of Harleman et al. in submerged sluice control was extended and showed that the Froude number of the lower layer at the point of incipient drawdown with high rates of flow was much larger than expected. The reason for the change was explained using the Khafagi-Hammad relationships.

Acknowledgements

The author gratefully acknowledges the help and direction of Dr. D. G. Huber in the planning, building and performance of this project. The author also wishes to express his appreciation to Miss. M. Corkum for the typing of the thesis.

This project was sponsored by the National Research Council of Canada.

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1.1 Introduction

The ability to selectively withdraw fluid from a region in which the density varies in the vertical direction is a significant advance which has been brought about by an understanding of the mechanics of stratified flow[1]. Density differences may be caused by a variety of reasons such as dissolved salts, suspended sediment and temperature differentials, creating density gradients which lead to density currents. Because of the advantages gained by the knowledge of the mechanics of stratified flow, there are many technological fields finding applications for the control of stratified fluids.

The control of density currents has long been considered economically desirable. The use of thermal stratification, which exists in tropical seas, as a source of energy has raised the question of the degree of selectivity which exists at an intake in a fluid having a vertical density gradient. Thermal power plants, whose efficiency is increased as the intake water to the condenser becomes cooler, have employed control structures to provide the cooler water. Knowledge of the mechanics of stratified flow, the existing density gradient and the required rate of flow are important since by careful design an optimum condition can be obtained. Lakes or artificial reservoirs are used as large heat exchangers for nuclear power plants, making it necessary to design intakes to withdraw the low-level cool water and to distribute the inflow of warm water near the surface. The reduction of reservoir sedimentation is a means of lengthening the life of major structures.

The control of density currents has also been considered economically desirable for the proposed control of salt water intrusion in rivers, canals, and tidal estuaries by the erection of various barriers.

In the cases described, a common feature of the applications is that the fluids involved are miscible, essentially incompressible, similar in viscosity, and the density differences are small. Additional applications in cases of immiscible fluids of different viscosities could arise in the separation of petroleum products and in other liquid=liquid separation processes.

Although there have been a number of papers dealing with the selective withdrawal of a fluid from a two-layered system, few papers have actually dealt with the case where both fluids are caused to flow.

The purpose of this thesis was to verify, by experimental means, the paper of D. G. Huber[2*]. This paper dealt with the irrotational motion of the two liquid strata of equal depth towards a line sink located in the bottom corner of a rectangular channel with the upstream end extending to infinity. The paper was entirely analytical and a solution was obtained using relaxation techniques. Relationships between the Froude numbers in the two layers were determined and the critical condition, when the lighter fluid began or ceased to flow, was estimated. Two extreme conditions were studied, a critical one at low discharges when the lighter fluid began or ceased to flow, and a limiting condition at high discharges when gravity effects became negligible.

As well as investigating Dr. Huber's paper, an extension of D. R. F. Harleman's experimental paper "Submerged Sluice Control of

* A note on Huber's solution to the problem is given in the Appendix.

Stratified Flow"[3] was developed. Higher values of some parameters were used to obtain information of use in liquid-liquid separation processes.

Since the model used for the experiments was a small-scale one, it was expected that viscosity would have an important effect and since this was not taken into account in the analytical paper, various parameters were altered to determine how important a role viscosity plays. D. G. Huber assumed that slip could take place at the interface. This was not the case in actuality and it was expected that this additional boundary condition as well as those at the structural boundaries, would affect the theoretical result. The accuracy of the assumption of immiscibility of the two fluids was checked by measuring the amount of interfacial mixing.

An experimental investigation was carried out for the cases where $H_1 = 3H_2$ and $H_2 = 3H_1$ with the same boundary conditions as the case where the two heights were equal. H_1 is the height of the upper fluid at the entrance to the test section while H_2 is the height of the lower fluid at the entrance to the test section.

Since the sink height and the height of the heavier fluid at the entrance to the test section could be varied; it was decided to correlate the results of these tests with those of Harleman[3]. The only tests in the experiment of value in the extension of Harleman's work are those in which a maximum sugar solution flow is obtained with the upper fluid just on the point of being drawn down, the point of incipient drawdown.

1.2 Problem

The following is a description of the theoretical problem under investigation. In Figure 1.1, the line sink was located at the bottom corner of a rectangular channel in which two fluid strata were extended to infinity in the (-x) direction.

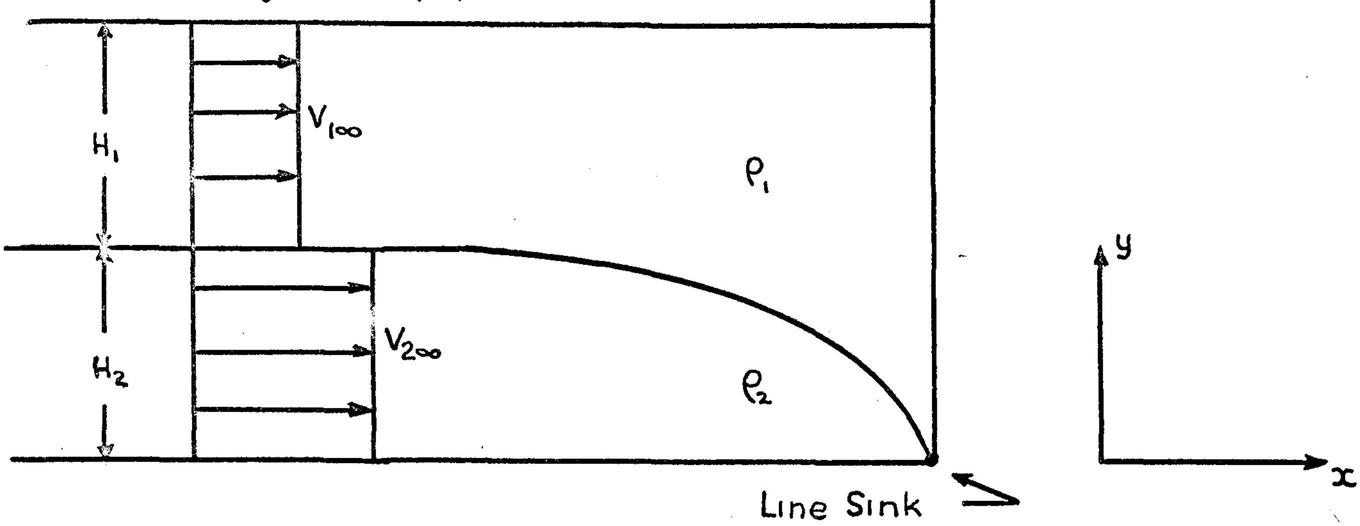


FIGURE 1.1 DEFINITION SKETCH

V_1 and V_2 , the velocity vectors at infinity, were assumed to be oriented in the x-direction and of constant magnitude in their respective layers. H_1 and H_2, the depths of each layer for upstream are considered to be incompressible, inviscid, and homogeneous, and the flow was assumed to be irrotational. Because of these stipulations, slip was allowed to take place at the interface and thus there was a discontinuity in the velocity and density gradients at the interface.

If the fluids were at rest, the interface would be horizontal. If the strength of the sink was increased from zero, fluid 2 would begin to flow and accelerate, until at some critical condition, known as incipient drawdown, the interface would be down to the sink and the lighter fluid would take part in the flow. Further increases in the strength of the sink would cause increases in the magnitude of the total discharge and in the proportions of the two fluids in the total flow. It was found in the analytical paper that, as the total flow was increased past that required for incipient drawdown, the rate of flow of the lower fluid decreased, while that of the upper fluid increased. As the magnitude of the total flow was increased still further, the rate of flow of the lower fluid began to increase along with that of the upper fluid.

It was apparent that two of the factors of paramount importance in the determination of the interfacial shape and the proportions of flow were inertia and gravity. Therefore the Froude number would be expected to be of considerable value as a parameter. At large Froude numbers the inertial effect predominated, while at low Froude numbers the gravity effects would be most important.

In D. G. Huber's paper, the objective was the determination of the relationships between the discharge from each layer, the densities and the interfacial shape. The present thesis was concerned with the experimental verification of these relationships as presented in the analytical paper. The theoretical curves are shown in Figs. 5.2, 5.4 and 5.5.

The densimetric Froude number was defined as a parameter in which the gravitational acceleration, g, was replaced by a reduced gravitational acceleration, g', taking into account the density difference of the two fluid strata. The equation

$$g' = g \frac{\Delta\rho}{\rho}$$

related the reduced gravitational acceleration to the usual gravitational acceleration.

The densimetric Froude number then could be written:

$$F = \frac{V}{\sqrt{g^{i}H}}$$

The densimetric Froude number at the point of incipient drawdown was found by D. G. Huber to be $F_2 = 2.76$ where

$$F_{2\infty} = \frac{V_{2\infty}}{g^{i}H_2}$$

The significance of this result is that for any pair of fluids in any geometrically similar potential flow situation, the velocity at infinity of the lower fluid at which the upper fluid either starts or ceases to participate in the flow can be calculated.

The shape of the interface and the streamline pattern were shown to be dependent on the Froude number in the upper layer, since the angle of the interface at the sink was shown to depend on the Froude number ratio and it was found that there is only one Froude number ratio possible for each value other than zero of the Froude number in the upper stratum. Whenever the Froude number is mentioned in this thesis it means the Froude number at the entrance to the test section (a point "infinitely" far upstream where the flow is steady and horizontal) whether it is written as $F_{1\infty}$ and $F_{2\infty}$ or F_1 and F_2.

2. Literature Survey

Craya[4] has treated, analytically, the withdrawal of fluid from a two-layer system with a horizontal intake located on a vertical boundary such as would be found on the upstream face of a dam. The intake was located above the initially horizontal interface as shown in the accompanying figure.

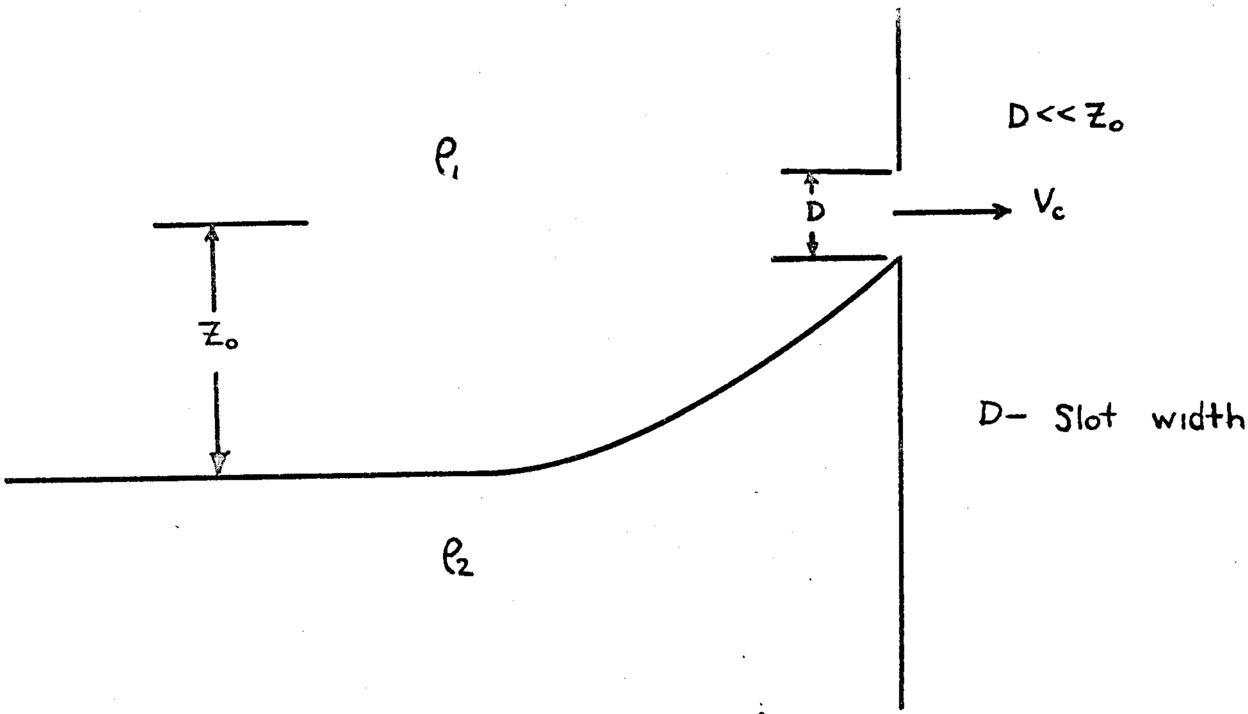


FIG. 2.1

Consideration was given to the flow of the upper fluid necessary to raise the interface locally to the level of the intake at which time discharge from the lower layer would begin. The efflux velocity at this critical condition was designated V_c . Intakes in the form of horizontal line sinks and three-dimensional flows to point sink were considered for the case in which the vertical extent of both the upper and lower layers was unlimited. It was assumed that the size of the slit or orifice would be small in comparison to the height of the opening above the original interface (Z_o). The critical-efflux velocity equations for both the two- and three-dimensional conditions are shown below:

For the orifice:

$$\frac{V_c}{\sqrt{g' Z_o}} = 3.25 \left(\frac{Z_o}{D}\right)^2$$

For the slot:

$$\frac{V_c}{\sqrt{g' Z_o}} = 1.52 \left(\frac{Z_o}{D}\right)$$

The equations presented by Craya have been substantiated experimentally by Gariel[6] for the same boundary conditions.

As a brief exploratory study, Rouse[8], Davidian et al. presented some experimental results for the case of a vertical axis, circular intake pape withdrawing the lighter of two stratified liquids from a point above the fluid interface as shown in the following figure:

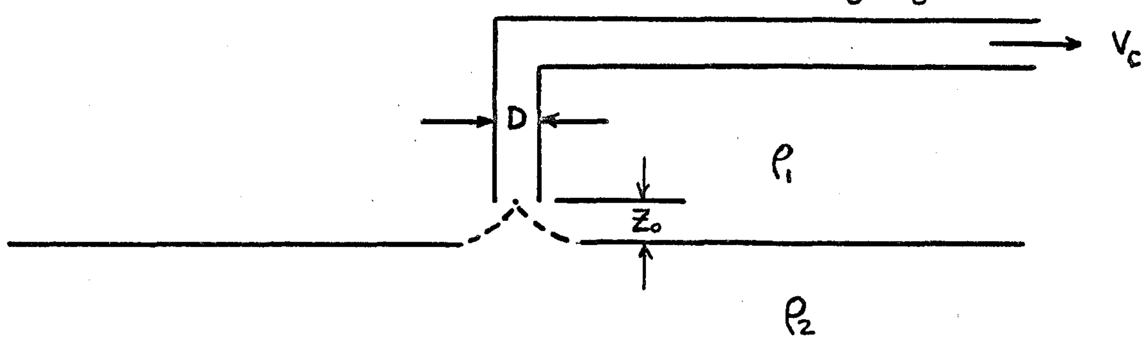


FIG. 2.2

Their investigation covered two cases (with respect to the upper and lower fluids), air-water and fresh water-saline water (S.G. = 1.1). Both the upper and lower fluids were, as in the previous case, of unlimited extent. The results obtained enabled them to develop the following equation for the critical efflux velocity.

$$\frac{V_c}{\sqrt{g' Z_o}} = 5.70 \left(\frac{Z_o}{D}\right)^{3/2}$$

Considering the case in which the lower layer is limited in vertical extent, Harleman et al.[9] have investigated, analytically and experimentally, the efflux from a vertical circular intake in the bottom boundary as shown in the following figure.

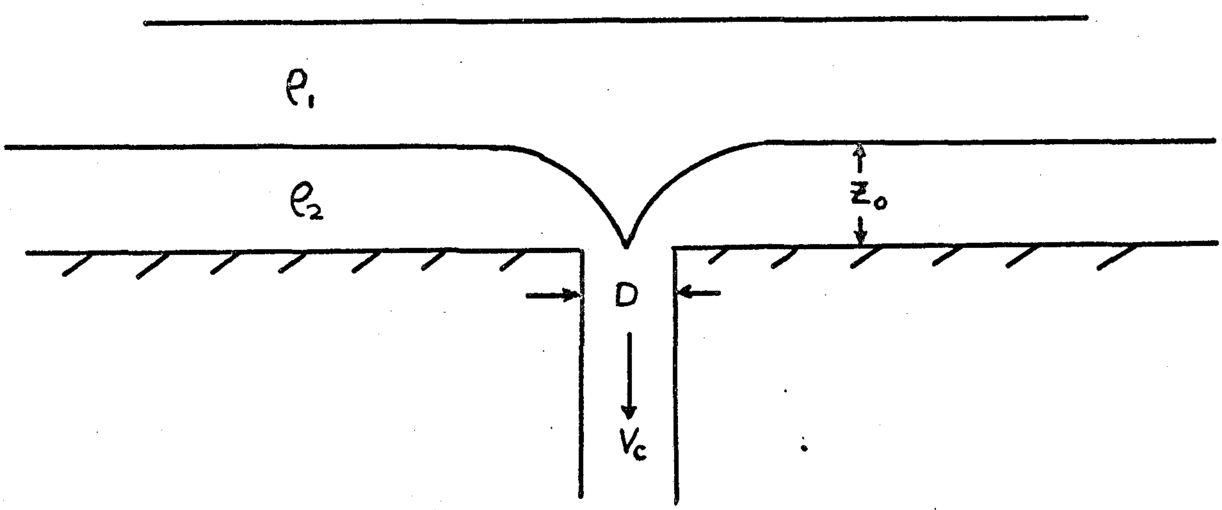


FIG.2.3

Fluid was drawn into the intake from the lower layer only until a critical discharge was reached. At this condition, the limit of selective withdrawal, the upper layer was in a state of incipient drawdown.

The following critical efflux-velocity equation has been verified by their analytical and experimental work:

$$\frac{V_c}{\sqrt{g^1 Z_o}} = 2.05 \left(\frac{Z_o}{D}\right)^2$$

Rates of flow greater than the critical discharge would cause increasing amounts of the upper liquid to be drawn into the intake. The intakes tested would be useful in situations in which it is desirable to withdraw liquid from a stratified layer of limited vertical extent.

A more general boundary configuration than the one described by Huber was studied by Harleman[3], Ippen, and Gooch. The boundary conditions are shown in the following figure:

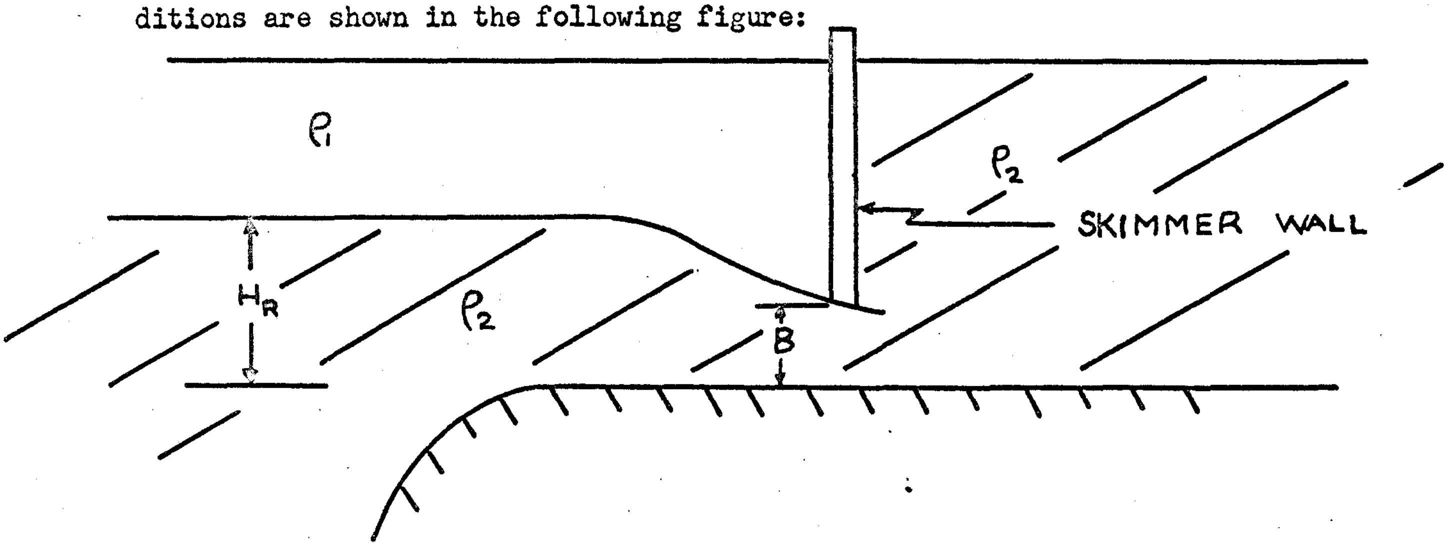


FIG. 2.4

The model involved was designed in connection with the design of condenser water intakes for thermal power plants. It was desired to determine, for a given gate opening, B, and interface elevation, H_R, the limiting discharge for the lower stratum. The vertical dimension of the intake opening was made comparable to the initial depth of the lower layer.

It was found necessary in the analysis, based on the one-dimensional energy equation, to account for both the non-uniform velocity distribution and the non-hydrostatic pressure distribution in the plane of the vertical gate. The critical Froude number for the lower layer flow is shown as a function of H_R/B and the kinetic energy parameter α in the following figure:

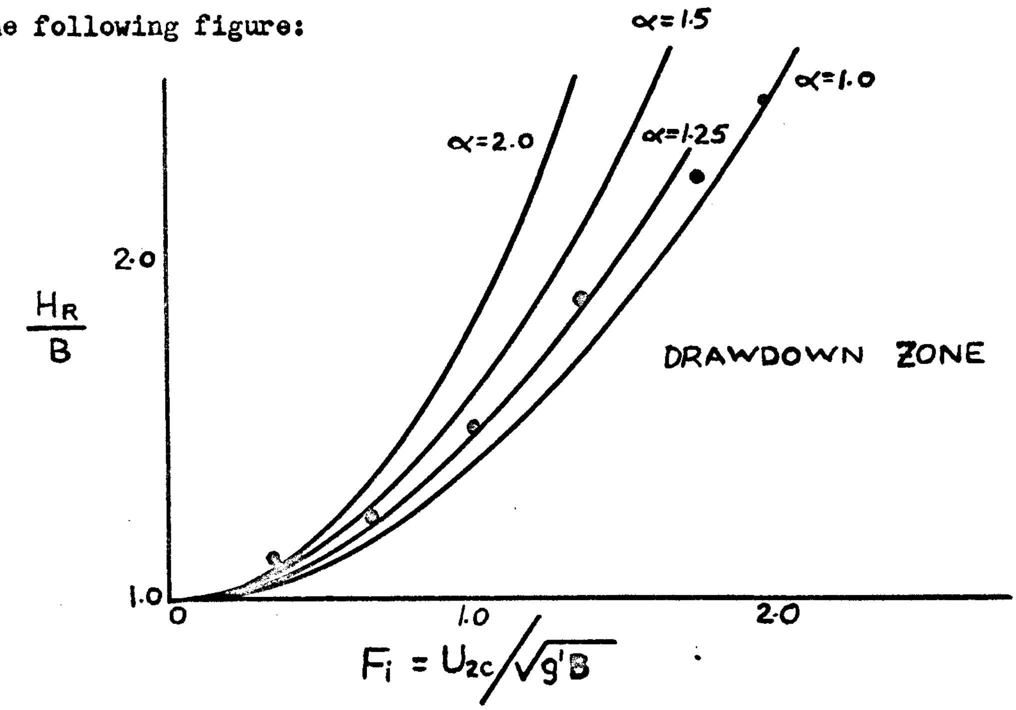


FIG. 2.5

Laboratory tests have essentially verified the curves of the high values of α associated with small-scale flume tests.

Engineering applications of this type of control structure for cooling water intakes have been described by Elder[11] et al. in their paper concerning the design of the condenser water intake for the

Kingston steam power plant of the Tennessee Valley Authority. Elder and Dougherty[10] discussed the Kingston skimmer wall in detail in their paper. Field data on water-intake temperatures obtained before and after completion of a skimmer wall were compared and an accurate economic study could thus be made.

Angelin and Flagestad[12] have advanced the knowledge of control structures in their report on a model study of a particular intake structure designed to take advantage of thermal stratification.

Schijf and Schönfeld[13] have given the one-dimensional equations of motion for the steady, non-uniform flow of a two-layered system if the vertical accelerations in the fluids can be neglected and only the mean velocities in the respective layers are considered. Bata[14] applied these equations to the problem of the location of interfaces in connection with the re-circulation of cooling water between the intake and outlet of a thermoelectric power plant.

The approximate solution of Keulegan[15] for the steady flow of a stream of viscous incompressible fluid over another at rest has been extended by Potter[16] to the case where both fluids are moving cocurrently but at different velocities. His solution utilized a sextic polynomial for the velocity distribution in the boundary layers. He found that the solutions depended only on the ratio u_2/u_1 of the velocities of the two streams and on the product of their corresponding viscosity and density ratios. These solutions gave rise to corresponding solutions for mass transfer between the two fluid phases. However, since one of the prime factors in fluid-fluid interaction and in the behaviour of fluids with a free surface is the question of stability of the interface or free surface, these solutions are of limited value. Yih[17] has considered various aspects of this question and has confirmed the expectation that instability sets in at rather low values of Reynolds numbers.

Knowledge of multi-layered stratified fluids and fluids with continuous density gradients is less well developed than that of two-layered systems.

Yih[18] has obtained an analytical solution for the steady two-dimensional flow of a stratified fluid toward a line sink located at the corner formed by the channel bottom and a vertical wall, for the case of an initially constant density gradient in the vertical direction. He has shown that the continuous-density variation can be analyzed as the limiting case of the multi-layered system.

Yih[19] has pointed out the remarkable property of the stiffening of the streamlines of constant density against vertical displacements when the motion is laminar in continuously stratified flows. He thus showed that vertical motion was inhibited by the density variation and the presence of a gravitational field if the disturbance was weak. Gariel first demonstrated this phenomenon in an experimental investigation of the flow produced by a thin slit in the vertical boundary of a rectangular channel for a liquid with a linear density variation. The vertical thickness of the principal horizontal current created by the discharge through the slit was shown to be a function of the density gradient, discharge rate, and fluid viscosity.

For the same boundary conditions as in the case of Dr. Huber, Debler[20] has reported some experiments on the discharge of a linearly stratified liquid through a horizontal slit at the bottom corner of a rectangular channel. Debler defined the densimetric Froude number as:

$$F = \frac{q}{h^2} \sqrt{\frac{h\rho_o}{g\,\Delta\rho}}$$

where

q = slit discharge

h = total depth

ρ_o = density at channel bottom

$\Delta\rho$ = density difference between surface and bottom.

Debler's experimental work showed that when F was less than 0.28, the flow pattern was divided into two horizontal regions: an upper one, essentially stagnant, and a lower region in which the entire discharge was concentrated. The entire depth of the fluid took part in the flow toward the outlet at Froude numbers greater than this critical value. Yih[18] had predicted, analytically, a critical densimetric Froude number of $1/\pi = 0.32$ for the same boundary conditions.

3. Test Facility

A photograph of the test facility used in performing the experimental study is shown in Figure 3.1. A labelled diagram of the components comprising the test facility, arranged to form an open system is shown in Figure 3.2.

Two fluids of slightly different density, approximately similar in viscosity, and essentially immiscible were required. After considering such fluid systems as salt water-fresh water, fuel oil-fresh water, varsol-fresh water and sugar water-fresh water, it was decided to use the latter system for reasons of economy and corrosion prevention. By adding a vegetable dye to the sugar solution it was found that the interface between the two fluids could be easily observed.

For purposes of description, the test facility may be considered to be comprised of two assemblies, the test equipment and the test section. The test equipment consisted of all the equipment necessary for the experimental study except for the test section. The function of the test equipment was to provide the test section with the proper levels of sugar water solution and fresh water and to provide the test section with the desired flow rate of each strata. The function of the test section was to establish various flow situations and from the resulting steady-state patterns, relationships between the Froude numbers in each layer could be determined.

3.1 Test Equipment

The following is a description of each piece of apparatus of the test equipment in reference to the schematic diagrams.

The tank which contained the sugar-water solution, labelled the storage tank, is shown in Figure 3.1. Figure 3.2 shows how it was connected to the rest of the test equipment. The dimensions of the tank were 82.0" long, 40.0" wide and 31.5" high. It was in this tank in which the sugar and water were mixed. A dye, crystal violet, was added to the sugar solution in this tank so that a clear distinction could be made between the sugar solution and the fresh water. It was found that the sugar took a long time to dissolve in the water if it was simply put in the tank and allowed to dissolve of its own accord. To speed the process, a long-handled steel scraper was used to scrape the bottom of the tank, thus mechanically mixing the sugar into solution and creating a uniform density throughout.

Directly above the tank previously considered was another tank used as a reservoir for both the fresh water and sugar solution during the testing. Its dimensions were 72.0" long, 120.0" wide and 24.0" high and was supported by six steel legs 42.0" high. It is shown in Figure 3.1 as well as schematically in Figure 3.2. The latter shows the route that the sugar solution followed to the reservoir tank and also where the fresh water was pumped into the tank. Connected to this tank were four taps in a vertical line 2 1/2 inches apart, starting from the bottom of the tank. Samples could be obtained from these taps and the uniformity of the sugar solution with regard to density could be checked.

Since the theoretical solution to the problem stipulated that the fluids used be immiscible, the sugar solution had to be admitted to the reservoir tank from the storage tank in such a way that the two fluids did not mix appreciably. This was accomplished by making the diameter of the pipe feeding the sugar solution to the reservoir tank increase as the distance to the top tank decreased, so that the velocity of the sugar solution decreased as it came nearer the entrance to the top tank, thus decreasing the tendency of the sugar solution to flow up through the fresh water and mix with it. The arrangement of the supply lines is shown in Figure 3.2 and a detailed drawing of the diffusers in Figure 3.3.

A circular steel baffle (shown in Figure 3.3) 6.00" in diameter was fixed at a height of 0.75" above the entrance to the top tank to prevent the sugar solution from moving upwards into the fresh water because of its momentum.

The tank below and to the right of the glass channel seen in Figure 3.2 was known as the collector tank. It was in this tank that the air was evacuated to cause higher discharge rates. Because there was a vacuum in the tank it had to be air-tight. The only openings were for the entry of the test solution, the vacuum control regulator, the pipe withdrawing the air and a rectangular door which was closed by four clamps and made air-tight with a rubber gasket.

Inside the collector tank was a plexiglass tank which was open at the top and moved back and forth on two rails. The rails extended the length of the tank and the plexiglass tank was propelled along the rails by means of a cog gear that was fastened to a long shaft, perpendicular to the rails. The bottom of the plexiglass tank had a rack gear attached so that by rotating the shaft with a handle located outside the collector tank, the plexiglass tank could be moved. The latter was known as the test tank and is shown in Figure 3.6.

As seen in Figure 3.1, the collector tank had two windows. The one on the left was inserted to check the level of the mixture which flowed onto the bottom of the collector tank before the actual test. The window on the right was to check the level of the test tank after a test since the door of the collector tank could not be opened if there were a vacuum inside.

The two pumps used were 1/3 horsepower capacitor start gear pumps. One of the pumps was used to pump the sugar solution from the bottom to the top tank, a distance of 46.0 inches. The other pump was used to pump the sugar solution-fresh water mixture from the collector tank to either the sugar solution tank or to the drain. The placing of the pumps is shown in Figure 3.2. When desired, the direction of pumping could be reversed.

The camera used in the experiment to obtain the interfacial angles was a Graphic View 11 with a variable length tripod. The exposure times and aperture settings depended upon the type and time of day. A normal setting during the day would be an exposure time of 1/100 second and an opening f 5.6. The type of film used was Ilford HP3. The required specific gravities of the fresh water, sugar solution and test solution were obtained using a Westphal specific gravity balance. This instrument measured accurately to within 0.0003 of the correct specific gravity.

Air was evacuated from the collector tank by a standard single-stage air ejection system located one floor below the test facility. Essentially, the system consisted of a four-stage centrifugal pump, a large water reservoir, and a converging-diverging nozzle.

The vacuum control regulator was a Honeywell system and is shown in Figure 3.2. In order to obtain a desired amount of vacuum, a needle was set on the dial shown in the figure to the desired reading in inches of mercury. The regulator would then allow the air ejector system to operate until the required vacuum had been reached at which point the line to the vacuum generator would be closed. Vacuums of 28.5 inches of mercury were obtained quite rapidly.

The valves used in this project were gate and globe valves. Figure 3.2 shows the location of the valves in each line.

3.2 Test Section

The test section was located in a rectangular horizontal plexiglass channel attached to the bottom of the reservoir tank. The inside dimensions of the plexiglass channel, seen in Figure 3.2, were 6.00 feet long, 1.00 feet high and 2.0 inches wide. The channel was made of 3/8 inch plexiglass and fastened by bolts and glue to the top tank. A cork gasket was inserted to make the connection leak-proof. Attached to the channel but inside the reservoir tank and resting on the bottom was a bell mouth entrance to ensure a smooth flow. The bell mouth entrance is seen in Figure 3.4.

The actual test section consisted of the downstream half of the plexiglass channel. Attached to the front side of the test section was a thin transparent plastic sheet three feet by one foot on which there was a grid, shown in Figure 3.2. Since the test section was transparent and pictures were to be taken of the phenomenon, a length of paper was used to cover the back side of the channel so that the phenomenon would show more distinctly. Five small valves were inserted on the top of the channel so that entrapped air could be released.

At the lower right-hand corner of the test section was a line sink of the same width as the test section, with a height of 0.3 inches. Below the line sink was a gate valve which regulated the flow into the collector tank located beneath and downstream of the test section.

During the tests it was decided to measure the amount of interfacial mixing in the test section. It was noted from visual observations that there was a slight fuzziness at the interface indicating a continuous density gradient. As can be seen in Figure 3.3, holes were drilled 1/16" apart vertically, although slightly offset horizontally, to allow space for the tubing. Twenty-four of these holes were drilled through the test section, covering a vertical distance of 1 1/2". A plexiglass plate was attached to the side of the test section and copper connectors were fitted through the holes in the plate and extended into the side of the test section. Plastic tubing was connected to each of these holes and the solution allowed to pass through the tubes where the specific gravity could be measured. The twenty-four samples provided an accurate density gradient from one uniform liquid to the other.

These same holes were used to obtain a velocity gradient by connecting the tubes to a reservoir filled with coloured water, located above the reservoir tank. By allowing dye to pass through the tubes and recording its movement on a motion picture camera, it was possible to determine the thickness of the interfacial boundary layer. The motion picture camera used was a Bell and Howell 35 mm. which had a speed adjuster attached. The speed used to determine the thickness of the velocity gradient was two frames per second.

To determine the effect of varying the sink height, a 1/8" aluminum plate was inserted through a slit drilled in the top of the plexiglass test section. This plate was exactly the width of the channel, two inches, so very little leakage could take place. The bottom of the plate was rounded off to ensure that there was no obstruction to the flow at the sink. To vary the height of the sink, the plate was simply moved up or down. The hydrostatic pressure of the two liquids held the plate against the end of the channel. A pointer, as seen in Figure 3.3, indicated the height of the sink.

A one-half inch thick plexiglass sheet one foot high and six feet long was fastened to the inside of the test section to determine the effect of changing the width of the test section. The sheet was inserted into the test section through the bell mouth entrance and fastened tightly to the side of the test section with six screws. The bell mouth entrance was also reduced to take into account the reduction in channel width.

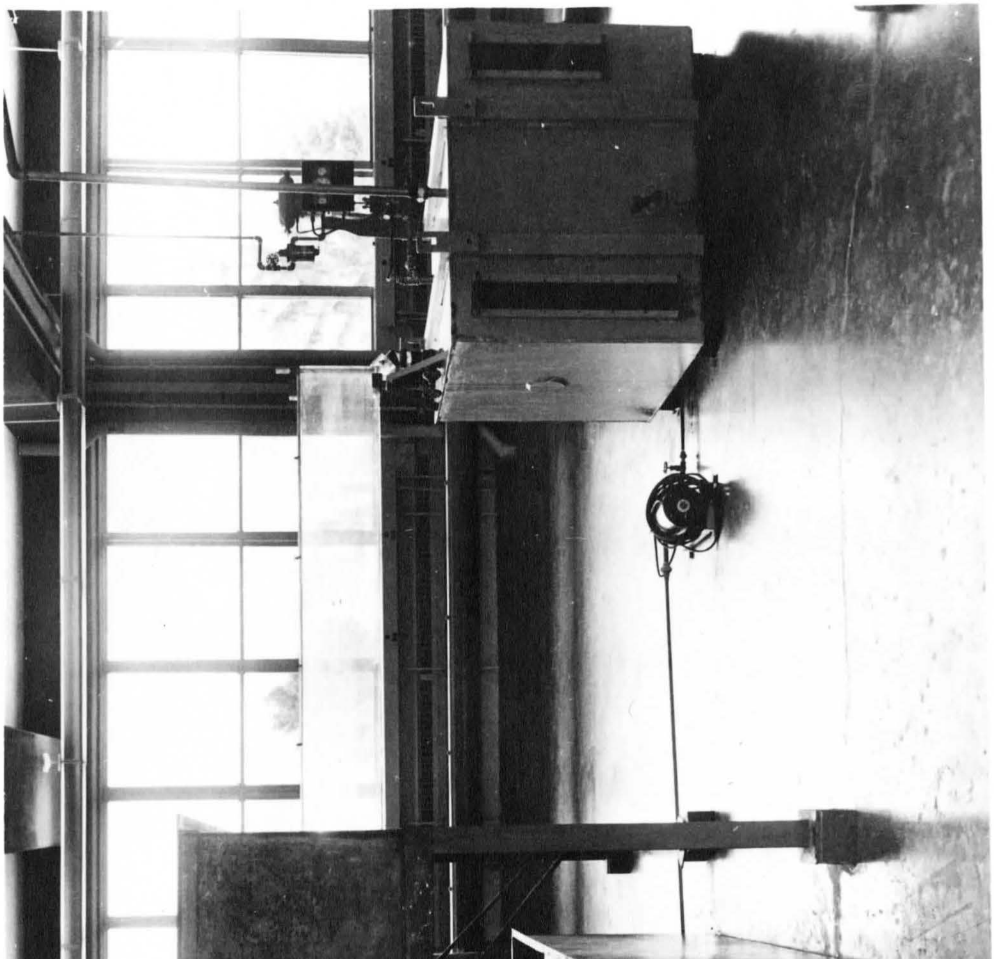


FIGURE 3.1

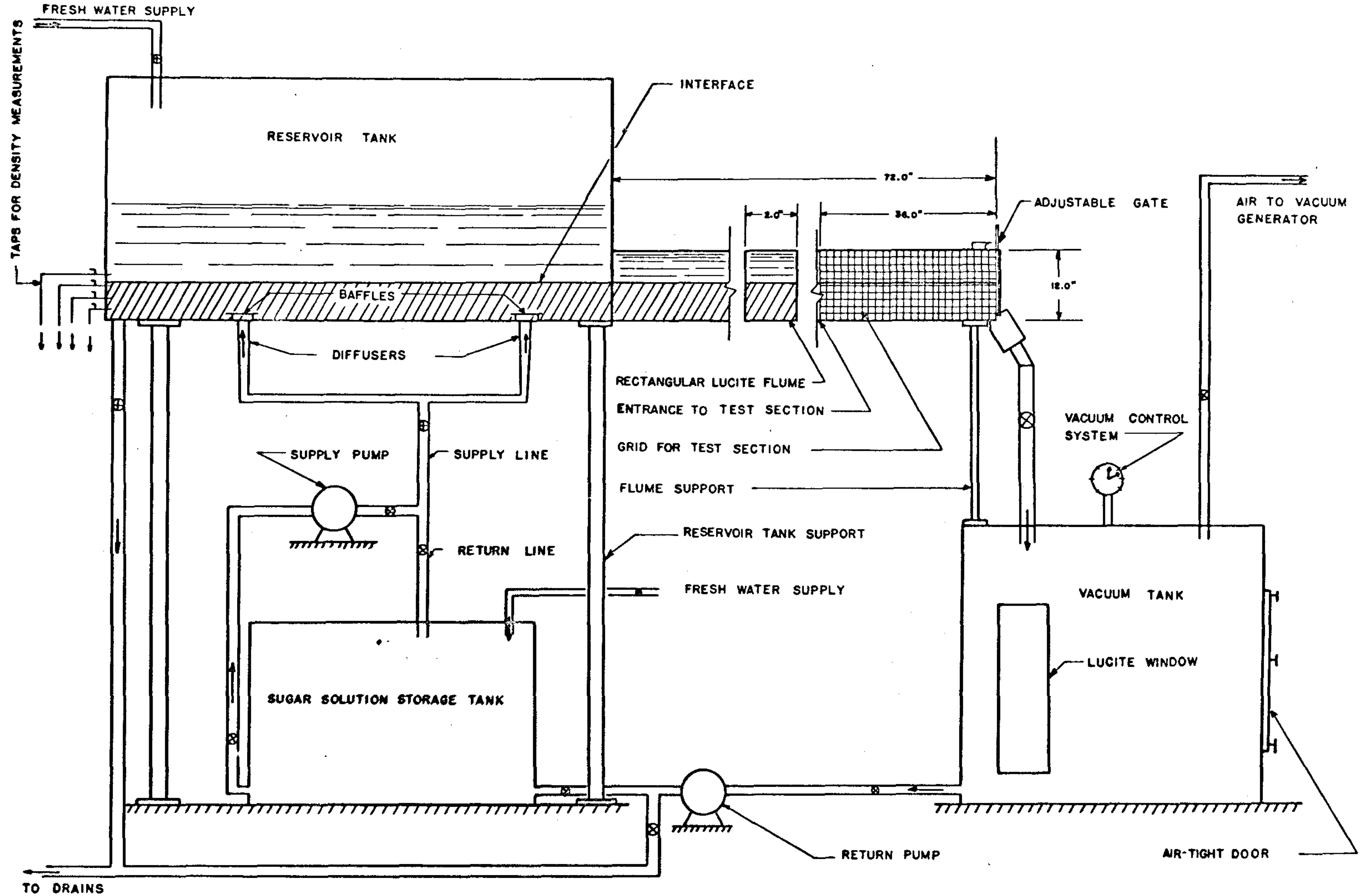
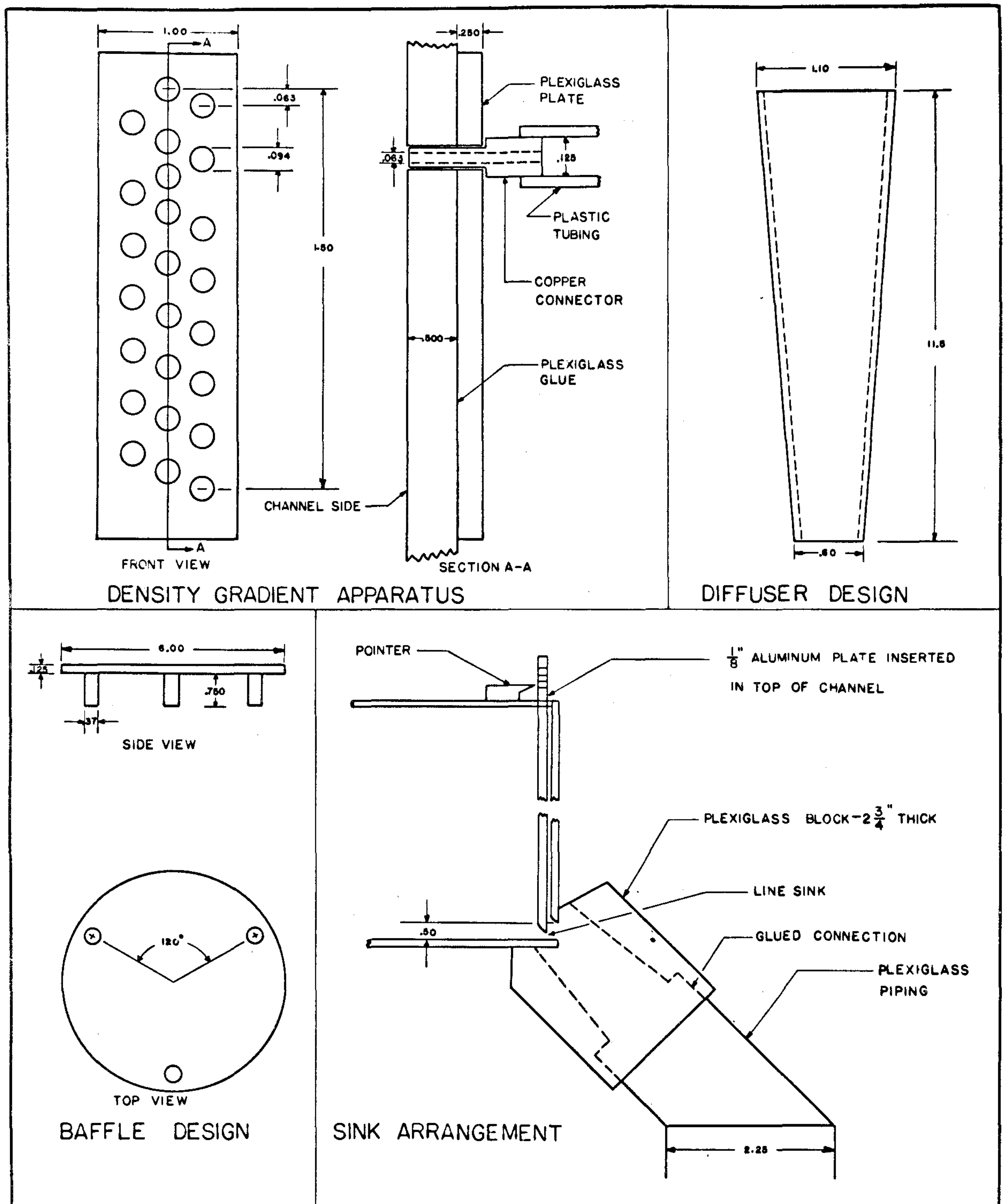


DIAGRAM OF EXPERIMENTAL EQUIPMENT

FIGURE 3·2



NOTE: ALL MEASUREMENTS ARE IN INCHES

FIGURE 3·3

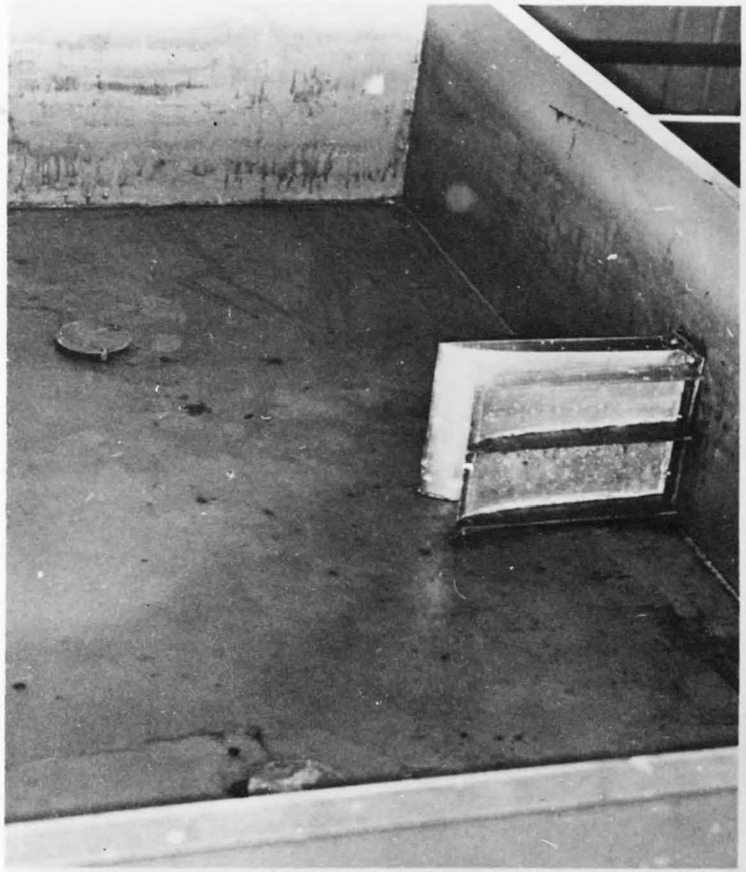


Fig.3.4

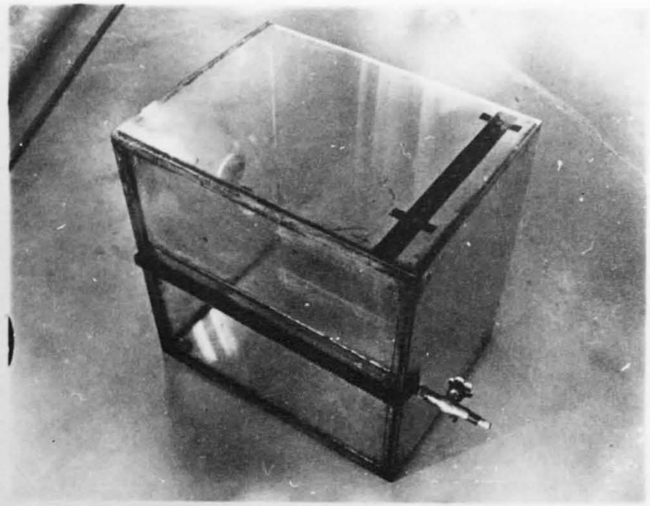


Fig. 3.6

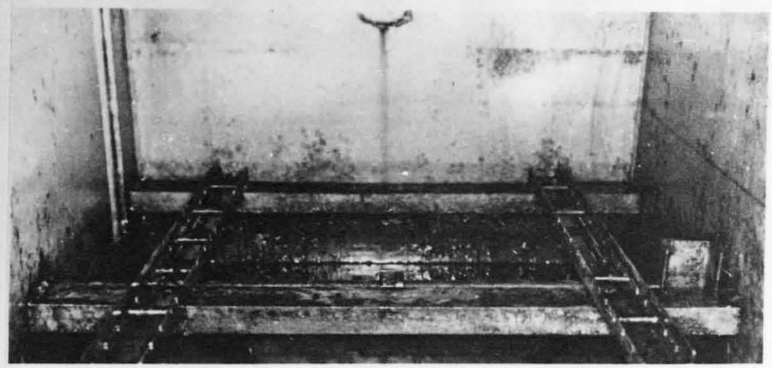


Fig. 3.5

4. Experimental Procedure

The required levels at the entrance to the test section at which the tests were carried out were marked on the plastic grid to aid in establishing the correct heights during the test. These heights were one-quarter, one-half, and three-quarters of the total channel height. A series of tests was carried out at each of these three heights to establish the relationship between the flow of fresh water and the sugar solution.

The top tank was filled with fresh water to a height of nine or ten inches above the bottom of the glass channel for the one-half and three-quarter test heights. The water was usually pumped to a height of eleven or twelve inches for the one-quarter series of tests. For the one-half test level, the sugar solution was pumped up to a height above the channel bottom of six to nine inches depending on whether high or low discharges were required. Care was taken in pumping the sugar solution to the top tank to ensure that a minimum of mixing took place between the fresh water and the sugar solution.

The desired vacuum was obtained by using a Honeywell vacuum control system. The steel door on the collector tank was clamped shut and the vacuum generator allowed to evacuate the tank. A camera was set up close to the sink so that photographs could be taken showing the angle that the interface made with the channel bottom for various rates of flow. Relationships between the interfacial shape at the sink and the Froude number of the lower fluid were obtained from these photographs.

The height of the sugar solution at the e ntrance to the test section had to be exactly at the one-quarter, one-half or the three-quarter level during a test depending on which series was being investigated. The level at the start of the test would be appreciably above these heights. The valve between the line sink and the collector tank was the one which controlled the test, and the valve setting determined whether high or low discharges would be obtained. If a low discharge were desired for establishing points in the low flow region, the valve would be slowly opened until the level of the sugar solution at the entrance to the test section was exactly on one of the three test levels. If a high discharge were required, the sugar solution would be pumped up to a level just above the required test level, usually within an inch of it. The valve was then opened quickly and the interface at the entrance to the test section would remain where it was except for a slight continuing downward movement since the top tank was being emptied of its sugar solution. When results were being recorded, in the one-half test level for instance, it was customary to commence the testing when the level at the entrance to the test section was 0.505 and finish when the level was 0.495.

The tests usually lasted from twenty to forty-five seconds depending on the amount of time taken to fill the glass test tank to a reasonable height. In order that a test be valid, the sugar solution at the entrance to the test section had to be at the proper level and the flow had to be steady. The test started when the glass tank was moved along its rails until it had reached a point directly under the flow of the two liquids coming through the spout at the top of the collector tank. Timing of the test began at the instant the flow entered the tank. When a reasonable height had been reached in the test tank, the tank was moved from the path of the flow, the timing stopped and the test completed.

The test tank had been calibrated so that the volume of liquid in the tank could be found for any given height on the scale attached to the tank. The calibration curve for the test tank is shown in Figure 5.1. Knowing the volume collected in the test tank and the time taken for the collection, the discharge rate for the mixture of sugar solution and fresh water could be calculated. However, the discharge rate for each was required. Since the two fluids go into solution, it was impossible to separate them and their discharge rates could not be obtained by simply reading the scale at two levels. The discharge rates were therefore obtained using specific gravity measurements of the fresh water, the sugar solution in the top tank and the combination of the fluids in the test tank. The development of the equation relating the discharge rate of the sugar solution, to the total discharge rate and the specific gravities of the fresh water, sugar solution and combination of both, is given in the Appendix.

The velocity of each fluid at the entrance to the test section was easily calculated since the discharge rate of each fluid and the area through which it flows at this point was known. The specific gravity of the sugar solution was found using a Westphal balance. Samples for the determination of the specific gravity of the sugar solution were obtained from taps in the top tank and a tap at the line sink. The sample for the fresh water specific gravity measurement was obtained from the highest tap in the top tank. All the variables necessary for the determination of the Froude number in each layer were then available.

5. Results

The main purpose of this project was the experimental verification of the theoretical results obtained by Huber in his paper and also to attempt an extension of the work of Harleman on submerged sluice gate control. Figure 5.1 is simply a calibration curve for the test tank and was used in each test to determine the volume of flow. The scale reading was not zero when the volume was zero because the scale was inserted into the crack in the bottom of the tank along one side and so a reading of one-half inch would be about the minimum reading possible. If the tables of results are checked closely, it will be apparent that for some tests the volume did not agree with the scale for this curve. These were the tests at which a sugar solution flow rate was desired with no fresh water flow. To obtain these tests, a high start level was required. Therefore the interface took a long time to reach the proper level at the entrance to the test section. Because the mixture was continually flowing into the vacuum tank during the interfacial letdown period, the height of the mixture in the vacuum tank above the floor would become so great that it would lift the plexiglass test tank off its rails and ruin a test because the tank could not be moved under the spout to collect the test mixture. The problem was solved by putting two cast iron weights in the tank, totalling 42.0 pounds, to delay the test tank in lifting off its rails. Since the weight density of the cast iron weights were known, a simple calculation gave the volume taken up by the weights. This volume was 0.09 feet^3 and accounts for the discrepancy between volume obtained from a given scale reading on the calibration curve and the volume used in the tables of results. Figure 5.2 shows graphically the experimental results obtained as compared to the theoretical results for the case where the heights of the two liquids were equal at the entrance to the test section. Forty-two tests were run in this part of the project and it was felt that this was a sufficient number to establish a curve and obtain the trend. As can be seen from the curve in Figure 5.2, the point of incipient drawdown (the point where the water just begins or ceases to flow, or the point where F_1 becomes greater than zero) as obtained experimentally was quite different from the theoretical point of incipient drawdown. The experimental value of the Froude number for the sugar solution at the point of incipient drawdown was 0.76 as compared to a theoretical value of 2.76. The difference between these two values will be explained in the following chapter. It can be seen from Figure 5.2 that there was a scattering of data points near the point of incipient drawdown. There was some thought as to the manner in which the curve should be drawn from $F_1 = 0.4$ to $F_1 = 0.0$. A curve could justifiably be drawn in the manner shown in Figure 5.2 because the point of incipient drawdown is the maximum F_2 that could be obtained with no water flow and could be obtained only by repeated testing. It was thought that three valid tests at the point of incipient drawdown would be sufficient for an end point on the curve. One reason for the scattering of data points in this low-flow region was that the errors introduced into the measuring of the Froude numbers was quite high as can be seen from the discussion of errors in chapter 6.

Referring to Figure 5.2, it can be seen that at high discharges, the experimental values agreed with the theoretical values quite closely. The general trend of the two curves was similar also in that after the point of incipient drawdown, there was a decrease in the Froude number of the sugar solution with an increase in the Froude number of the fresh water until a point was reached where the Froude numbers increased together.

A series of tests was carried out for the case where the height of the sugar solution at the entrance to the test section was three times that of the fresh water. The results of these tests are tabulated in Table 5.3 and presented in Figure 5.3. The only part of the graph that could be used was the region of relatively high flow since in producing the condition of incipient drawdown, a relatively high start level was required as shown in the appendix. Although there was a large scattering of data points as can be seen from Figure 5.3 the slope of the line at high discharges is approximately 3. The bottom portion of the curve is shown by a broken line to indicate the probable path of the curve in that region. The sugar solution storage tank would only fill the reservoir tank to a height of 0.90 feet. A higher level was required, so the search for experimental results of incipient drawdown had to be abandoned. However, as shown in these tests, at high discharges there was a straight line relationship between F_1 and F_2, and the slope of this line was approximately 3 which agrees with the theoretical solution.

Figure 5.4 shows the experimental relationship between θ_2, the angle of the interface at the sink between the interface and the horizontal, and F_2. It can be seen from this curve that there was a large variation between experimental and theoretical values but that the trend of the two curves was the same. The largest deviation between the experimental and theoretical values again occurred at the point of incipient drawdown. At relatively high Froude numbers there was a very close correlation between experimental and theoretical values. Although the accuracy of the angles obtained from experiment was not perfect due to instabilities and fuzziness at the interface, it was thought that enough tests were made so that the average curve could be drawn. It can be seen from the figure that the experimental curve between $\theta_2 = 50°$ and $\theta_2 = 85°$ was much steeper than the theoretical curve.

Figure 5.5 shows the relationship between the ratio of the Froude numbers, F_1/F_2 and the sum of the square roots of the two Froude numbers, F_1 and F_2 . The two curves had much the same trend and agreed quite closely at very low and relatively high discharges. There is, of course, a difference between experimental and theoretical results at the point of incipient drawdown. At $F_1/F_2 = 0$, signifying no fresh water flow, the experimental maximum sum of the square roots was 0.95 as compared to a theoretical result of 1.66.

Figure 5.6 shows the results of the tests made in the case in which the ratio of the level of sugar solution at the entrance to the test section was one-third that of the fresh water. The curve obtained was quite similar in shape to the case in which the levels of the two liquids at the entrance to the test section were equal.

It can be seen from the figure that as F_2 became larger than zero, the curve describing the relationship between the two Froude numbers proceeded along the x-axis until a certain F_2 was reached (0.46) whereupon a further increase in total flow resulted in a decrease in F_2 and an increase in F_1. This trend continued until a point was reached beyond which both Froude numbers increased together. An interesting feature of this curve was that at high discharges the slope of the F_1 vs. F_2 curve was 1/3 whereas the slope for the F_1 vs. F_2 curve at high discharges for the case of $H_1 = H_2$ was 1.0. These results agree with the theoretical solution and are discussed in the next section.

Figure 5.7 shows the curve of H_R/B vs. F_1 where $F_1 = \frac{Q/wB}{\sqrt{g\frac{\Delta\gamma}{\gamma}B}}$. This curve is actually an extension of the curve shown in Figure 2.5 which covers small values of H_R/B. If Figure 2.5 is examined, it can be seen that the value of F_1 seems to be increasing parabolically whereas the extension of the curve shown in Figure 5.7 indicates a somewhat more linear relationship between the two non-dimensional parameters. Explanations for this occurrence will be found in the following chapter. The experimental values of H_R/B measured in this project varied from $H_R/B = 6$ to $H_R/B = 40$. To obtain the maximum H_R/B, a reservoir height of six inches was used with a 0.15" sink height. To obtain the minimum

H_R/B used in this project, a 3" reservoir height was used with a sink height of 0.50".

Figures 5.8 and 5.9 show the density gradient for two cases. For the case shown in Figure 5.8, the liquids were at rest and a fresh sugar solution had just been pumped up from the storage tank, representing a minimum of interfacial mixing. In Figure 5.9, a test had been run and the interface allowed to settle. There was not a large change in the effective mixing thicknesses between the two cases. The effective mixing thickness, δ , was defined in these tests as the vertical distance between the points at which the density of the fresh water reached 101% of its bulk value and the point where the density of the sugar solution reached 99% of its bulk value. In the tests done by Gariel in France on a fresh water and a salt water solution, he reported that the passage of fresh water to the salt water took place less than 1/2 millimeter immediately after the preparation of a test. He did not report however if he had observed this visually or had taken an actual density profile. Harleman does not mention the thickness of the mixing layer in his papers.

In Huber's[2] theoretical analysis of this project, he assumed that slip would take place at the interface. This meant that there would be no mixing layers and that discontinuities could occur at the interface. By injecting dye into the channel at the entrance to the test section and allowing just the sugar solution to flow, the thickness of the velocity gradient was obtained. Motion pictures were taken of these tests and it was found that the distance between the average velocity in one layer to that in the other layer took place in a vertical distance of 1/4 to 1/2 inch. By allowing just the sugar solution to flow, the dye injected into the top layer would not flow downstream except at the interface and one could then obtain the distance between the relatively rapidly moving dye in the lower layer to the stagnant dye in the top layer.

Table 5.4 shows the results obtained in tests designed to determine the effect of varying the sink height. It was found that the same value of Froude number for the condition of incipient drawdown was obtained for all values of sink height below 0.50 inches. Therefore the assumption that the slit used was a sink when the height was 0.30 inches, was verified.

Table 5.5 shows the effect of varying the channel width. It was found that the maximum Froude number of the sugar solution with no water flow was 0.542 with a 1.5 inch channel width as compared to a Froude number of 0.765 with a channel width of 2.0 inches, representing a significant difference. The only variable changed was the width of the channel and thus the alpha effect was found to be of considerable importance.

Figures 5.10, 5.11 and 5.12 are photographs of the test section taken during actual testing of the series where $H_1 = H_2$. Figure 5.10 shows that the angle of the interface at the sink, θ_2, is very close to 45° and thus the test is one in which the Froude numbers at the entrance to the test section are equal. Figure 5.11 indicates that θ_2 has increased and thus F_2 must be higher than F_1. The extreme left hand side of the grid is the entrance to the test section and, as can be seen from the photograph, the flow of both layers is horizontal at this point and $H_1 = H_2$. Figure 5.12 shows a test in which a maximum

F_2 is obtained with no flow of the upper fluid, the point of incipient drawdown. Figures 5.13 and 5.14 are tests in the series where $H_1 = 3H_2$. Figure 5.13 is a test at the point of incipient drawdown while Figure 5.14 is a test in the high discharge region as indicated by the smaller angle of the interface at the sink. Figures 5.15 and 5.16 are tests in the $H_2 = 3H_1$ series. Figure 5.15 shows a test in the lower portion of the curve where both fluids are flowing but inertia forces do not predominate over gravity forces. Figure 5.16 shows a test in the high discharge region. The actual Froude numbers of both layers for all of these photographs are given in the list of figures in the front of the thesis.

The viscosity of the sugar solution was measured at different specific gravities and it was found that there was a large increase in viscosity with increase in specific gravity. The viscosity of a sugar solution with a specific gravity of 1.026 was 2.0 centipoises while that of a sugar solution with a specific gravity of 1.068 was 6.4 centipoises.

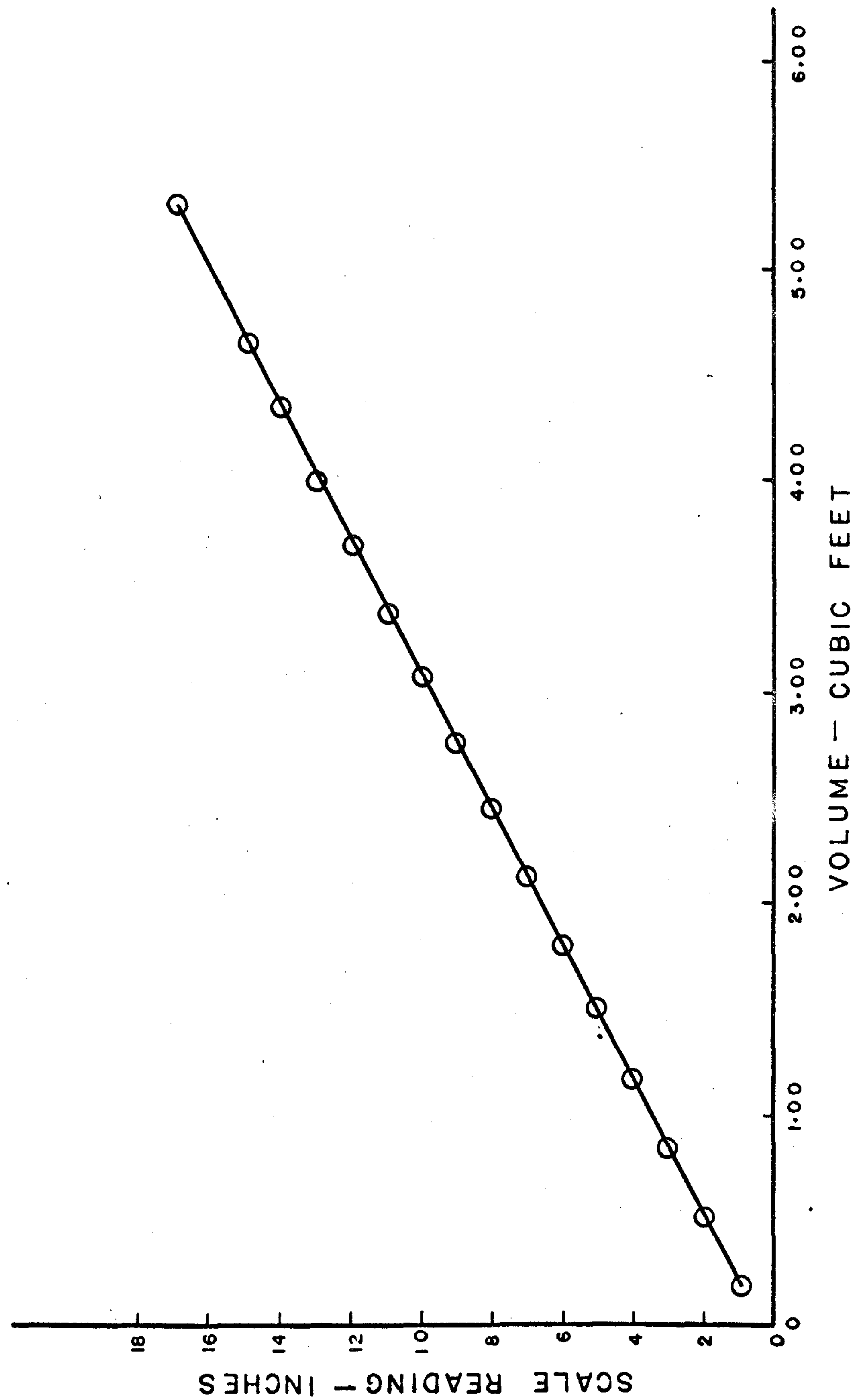
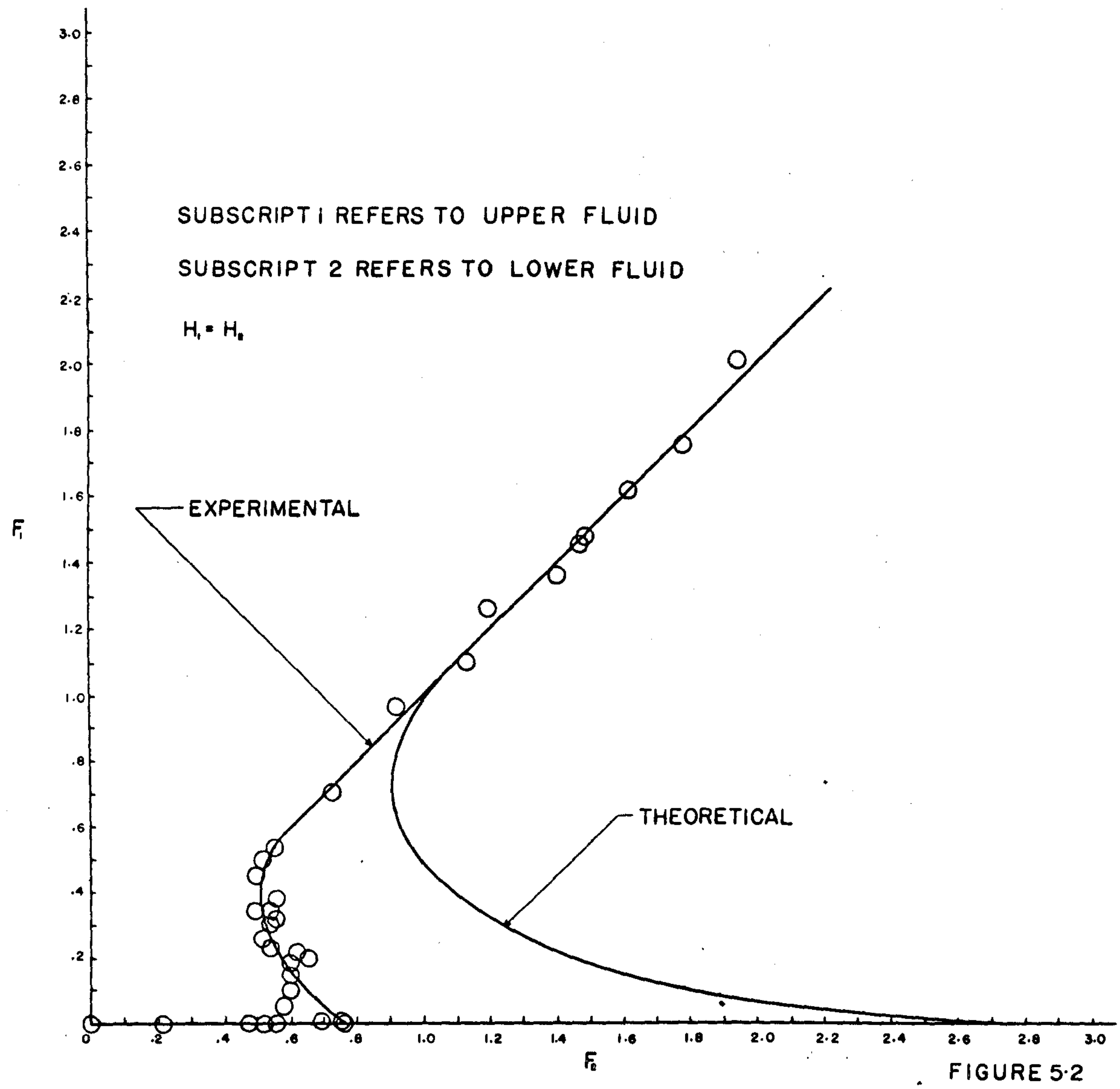
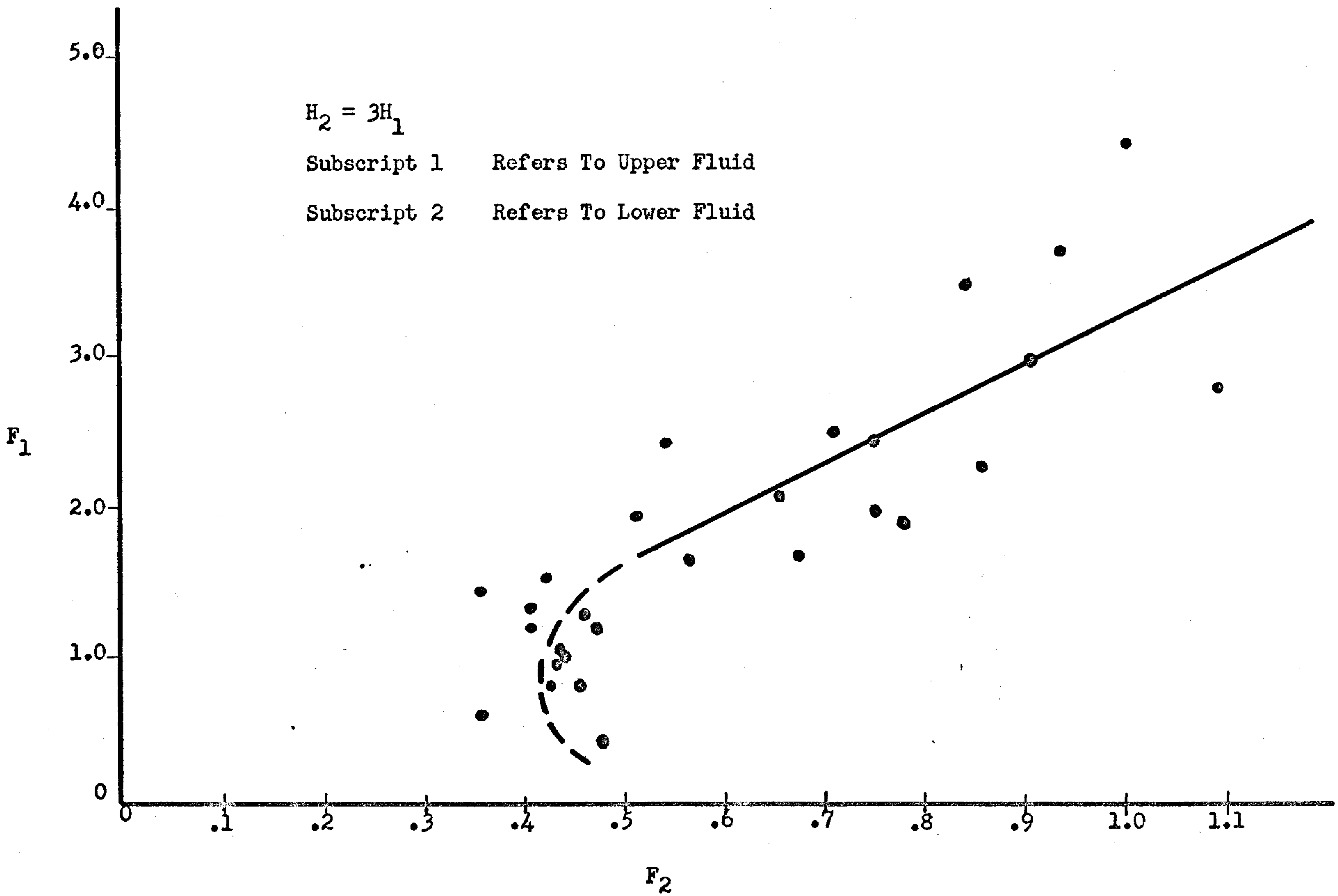

SCALE READING — INCHES
0
2
4
6
8
10
12
14
16
18
VOLUME — CUBIC FEET
1·00
2·00
3·00
4·00
5·00
6·00

FIGURE 5·1



3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
SUBSCRIPT 1 REFERS TO UPPER FLUID
SUBSCRIPT 2 REFERS TO LOWER FLUID
$H_1 = H_2$
EXPERIMENTAL
THEORETICAL
F_1
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
F_2

FIGURE 5·2



5.0
4.0
3.0
2.0
1.0
0
F_1
$H_2 = 3H_1$
Subscript 1 Refers To Upper Fluid
Subscript 2 Refers To Lower Fluid
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
F_2

Fig. 5.3

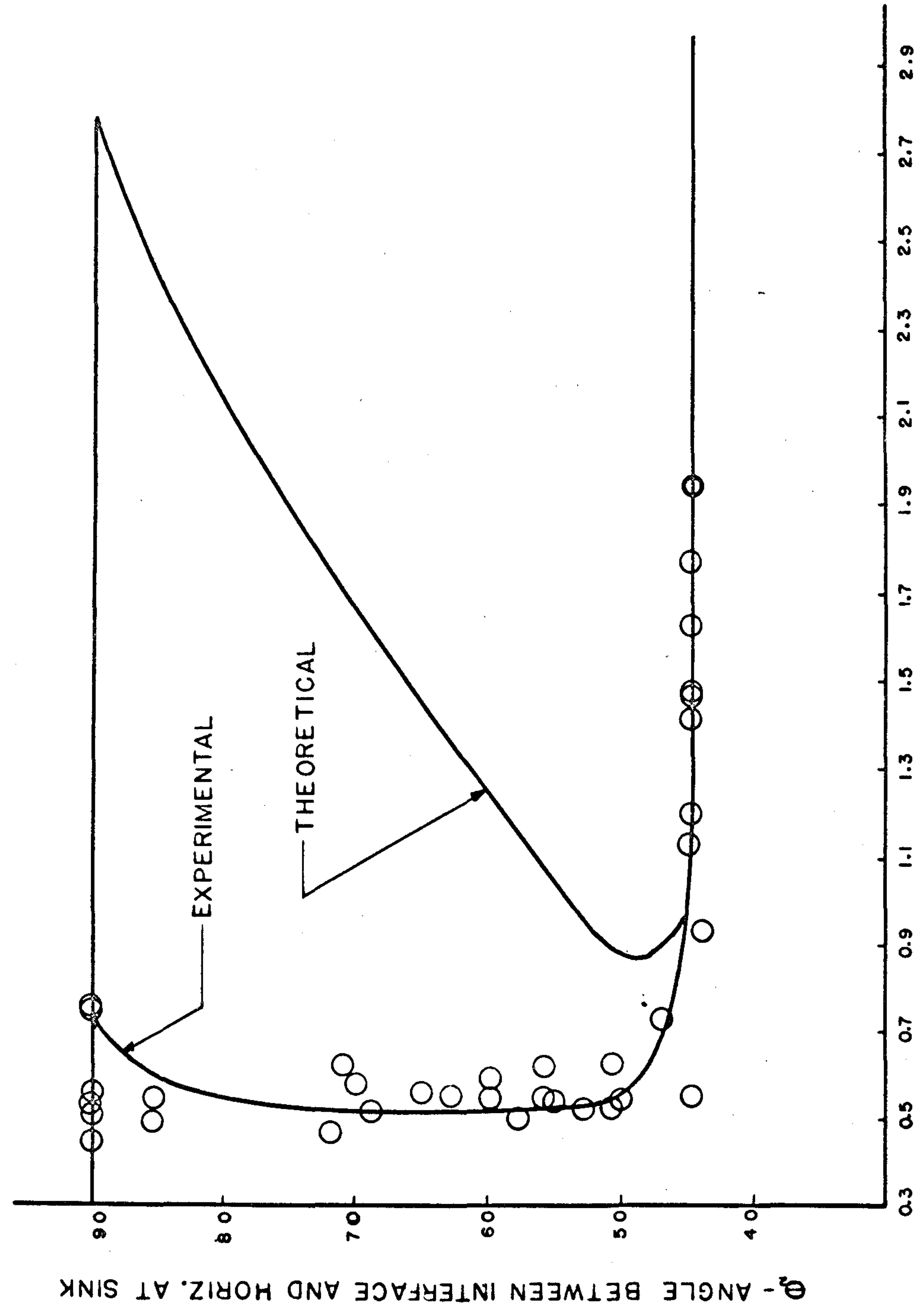
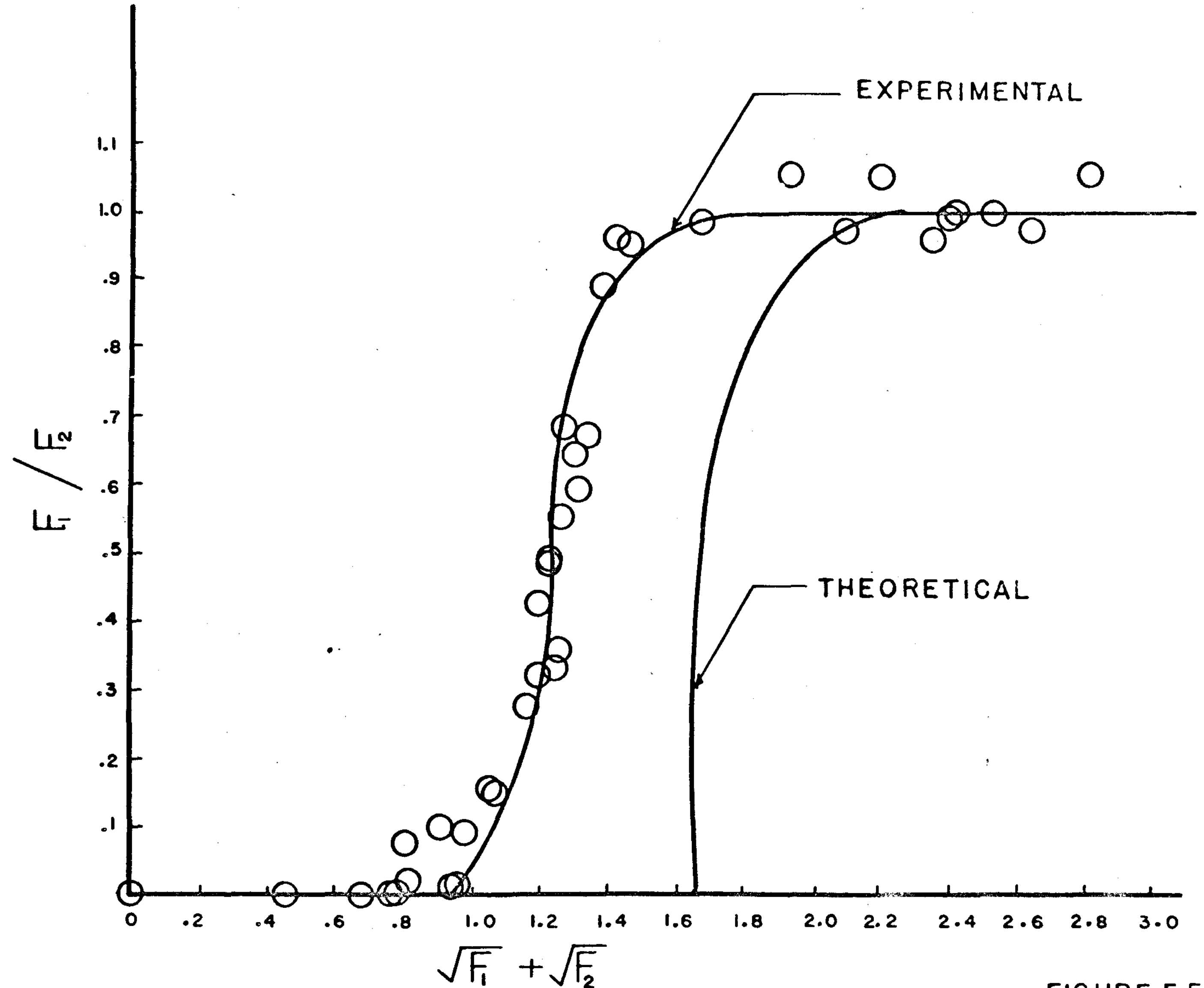

EXPERIMENTAL
THEORETICAL
Θ_2 - ANGLE BETWEEN INTERFACE AND HORIZ. AT SINK
90
80
70
60
50
40
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7
2.9
F_2

FIGURE 5-4



EXPERIMENTAL
THEORETICAL
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
F_1 / F_2
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
$\sqrt{F_1} + \sqrt{F_2}$

FIGURE 5·5

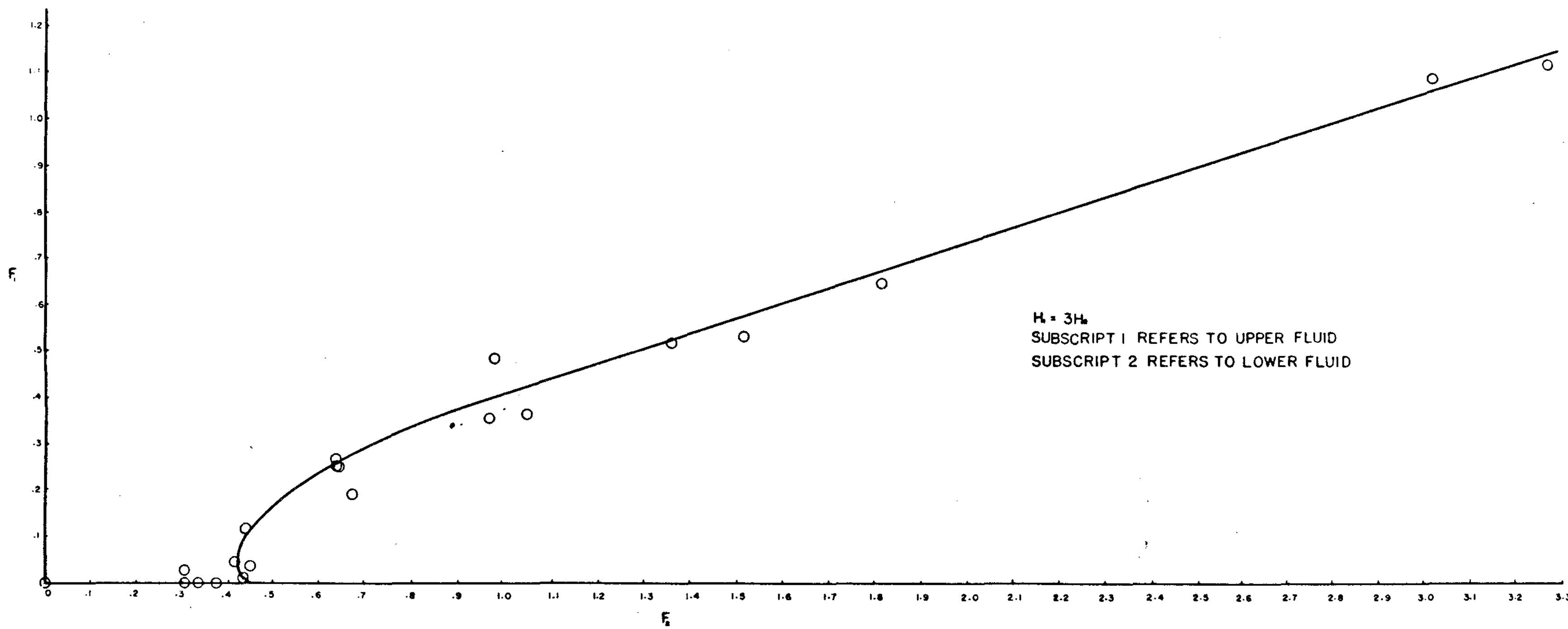

$H_1 = 3H_2$
SUBSCRIPT 1 REFERS TO UPPER FLUID
SUBSCRIPT 2 REFERS TO LOWER FLUID
F_1
F_2
0 ·1 ·2 ·3 ·4 ·5 ·6 ·7 ·8 ·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0 2·1 2·2 2·3 2·4 2·5 2·6 2·7 2·8 2·9 3·0 3·1 3·2 3·3
·1 ·2 ·3 ·4 ·5 ·6 ·7 ·8 ·9 1·0 1·1 1·2

FIGURE 5·6

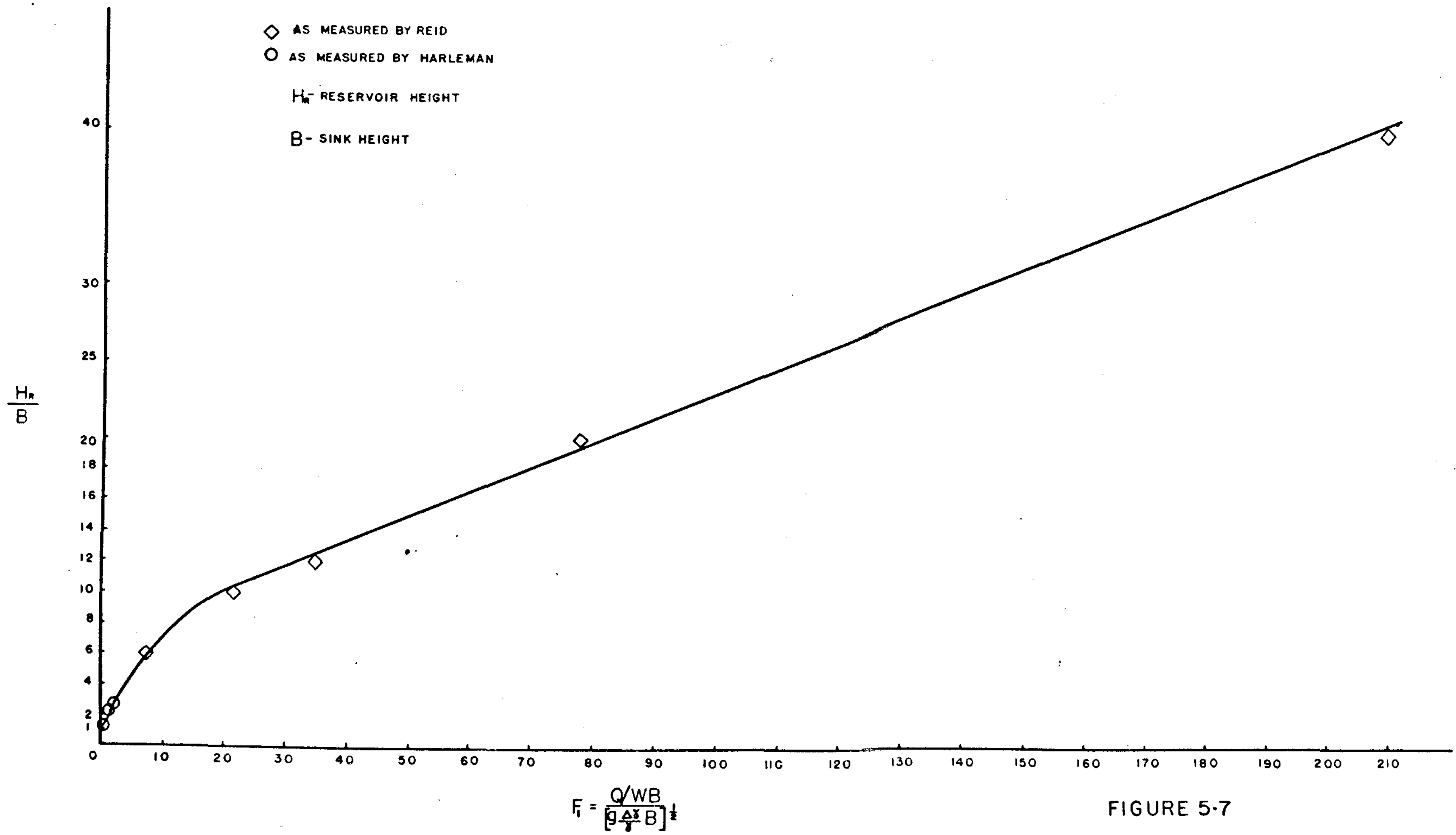

◇ AS MEASURED BY REID
○ AS MEASURED BY HARLEMAN
H_R - RESERVOIR HEIGHT
B - SINK HEIGHT
$\frac{H_R}{B}$
$F_i = \frac{Q/WB}{\left[g \frac{\Delta\gamma}{\gamma} B\right]^{1/2}}$

FIGURE 5-7

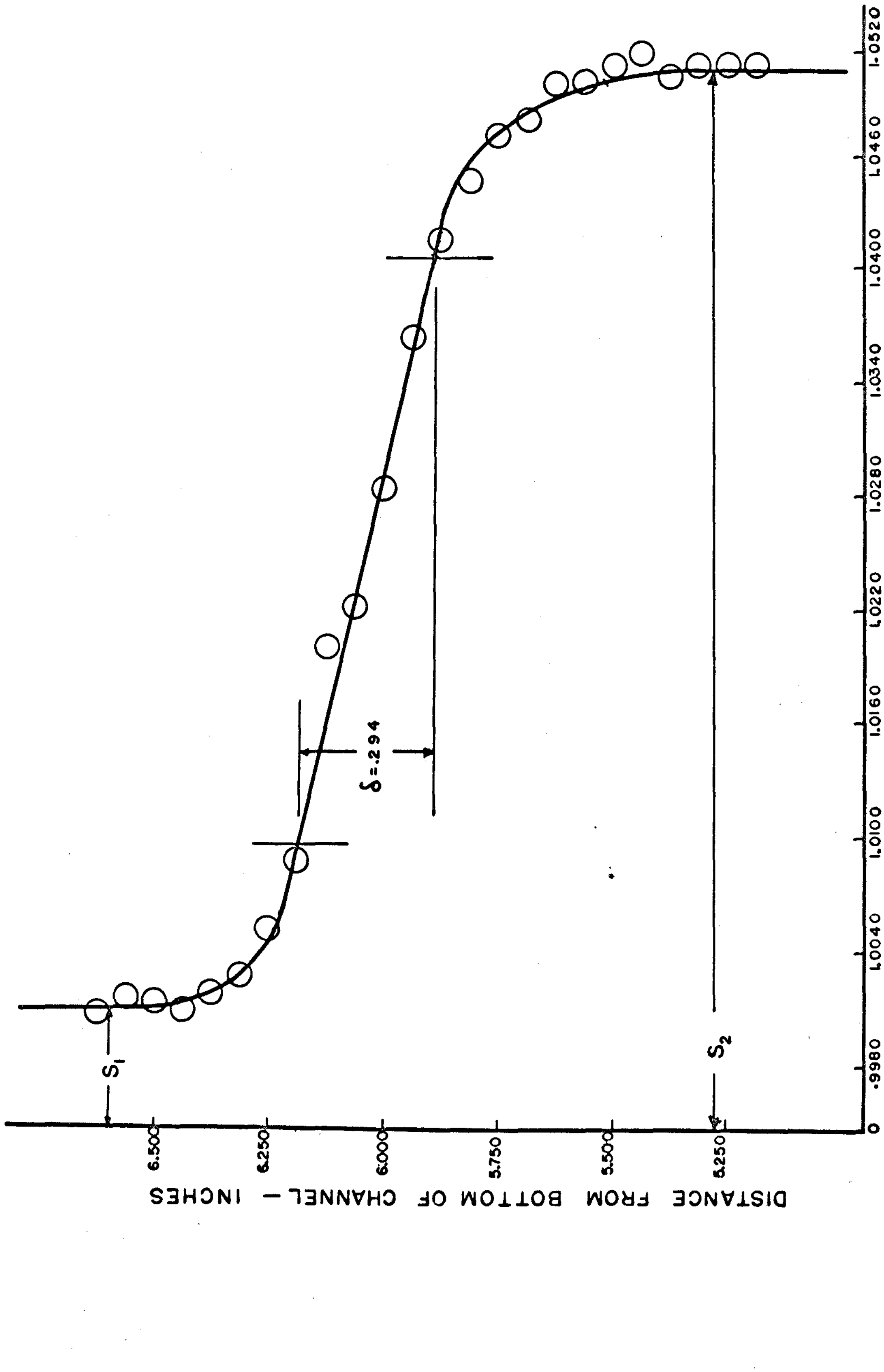
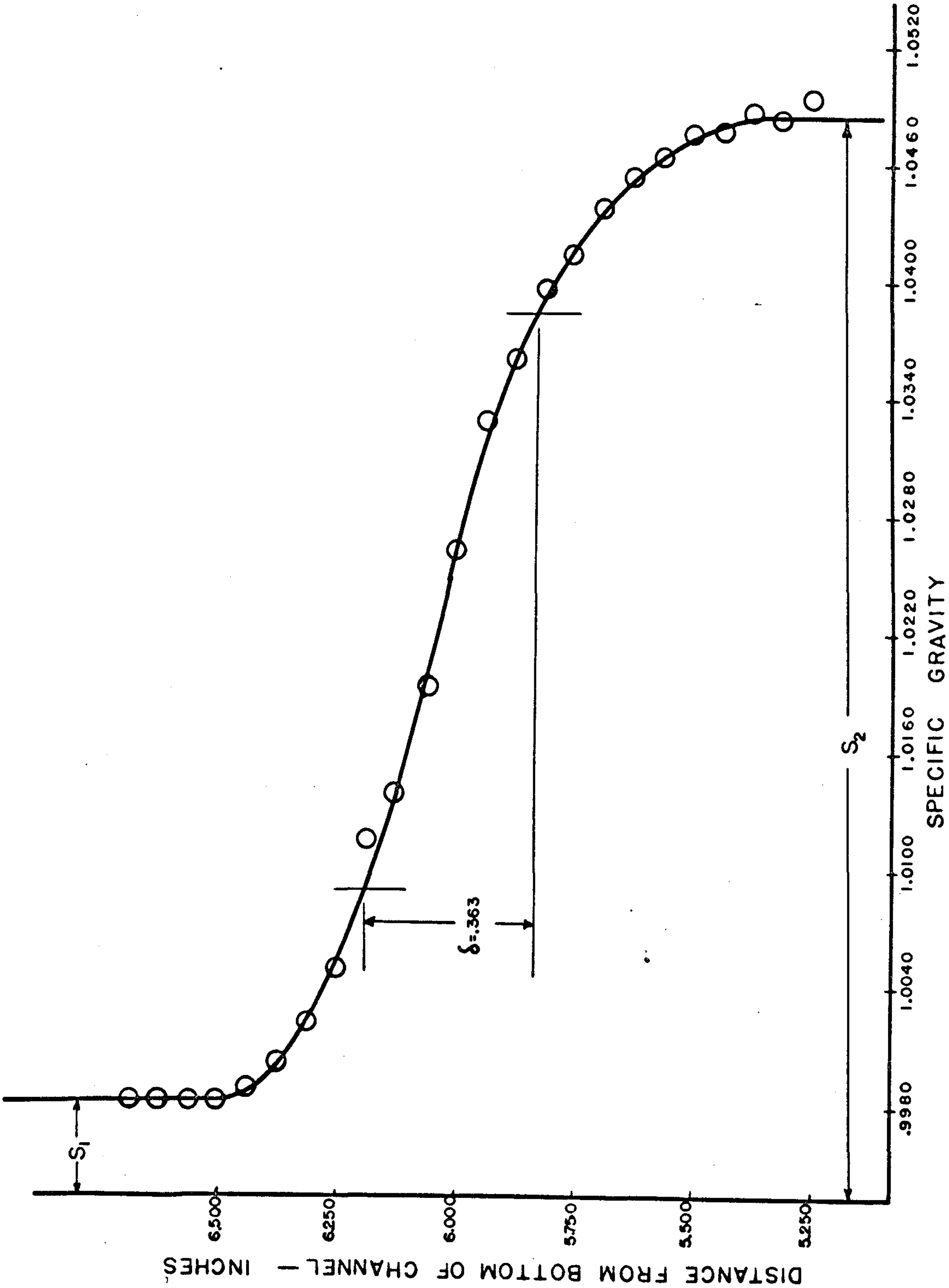

DISTANCE FROM BOTTOM OF CHANNEL — INCHES
SPECIFIC GRAVITY
S_1
S_2
$\delta = .294$
6.500
6.250
6.000
5.750
5.500
5.250
0
.9980
1.0040
1.0100
1.0160
1.0220
1.0280
1.0340
1.0400
1.0460
1.0520

FIGURE 5·8



SPECIFIC GRAVITY
.9980
1.0040
1.0100
1.0160
1.0220
1.0280
1.0340
1.0400
1.0460
1.0520
DISTANCE FROM BOTTOM OF CHANNEL — INCHES
6.500
6.250
6.000
5.750
5.500
5.250
S_1
S_2
$\delta = .363$

FIGURE 5·9

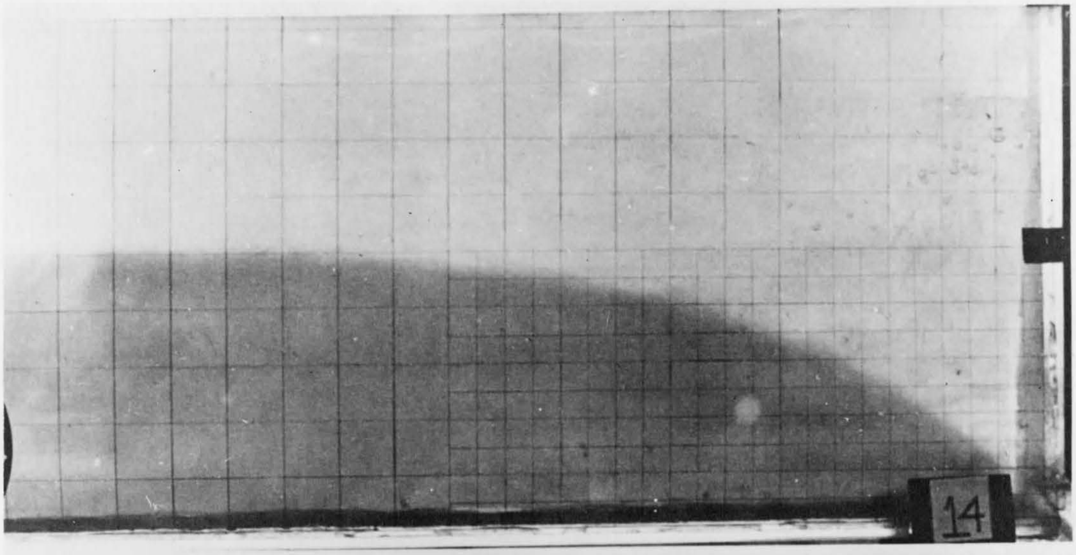


Fig.5.10

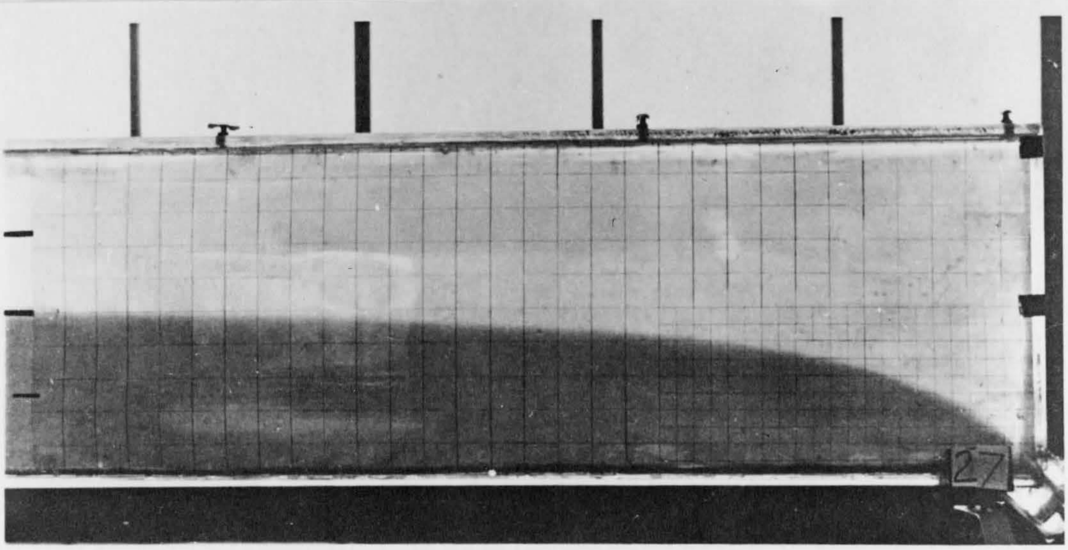


Fig.5.11

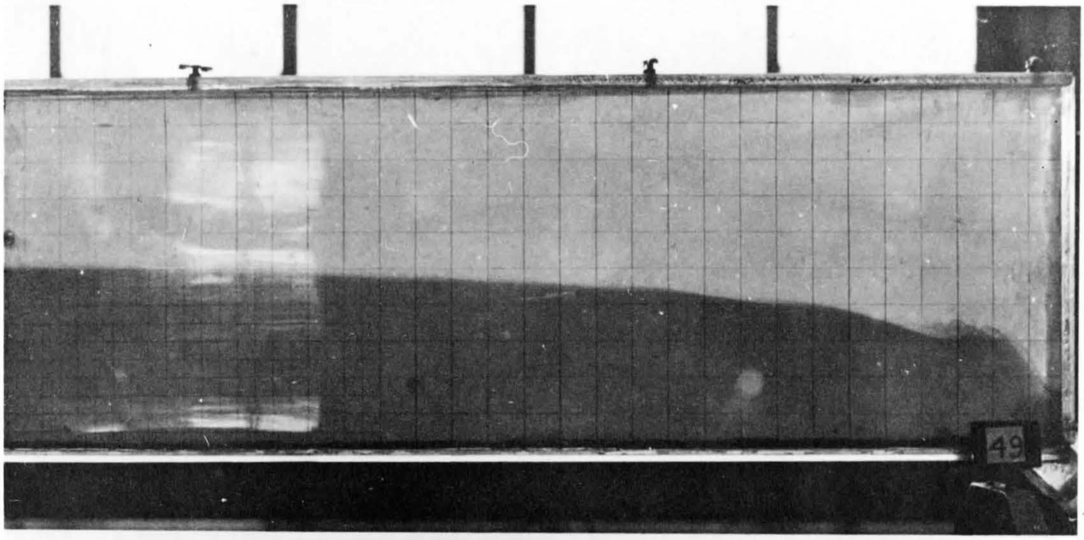


Fig.5.12

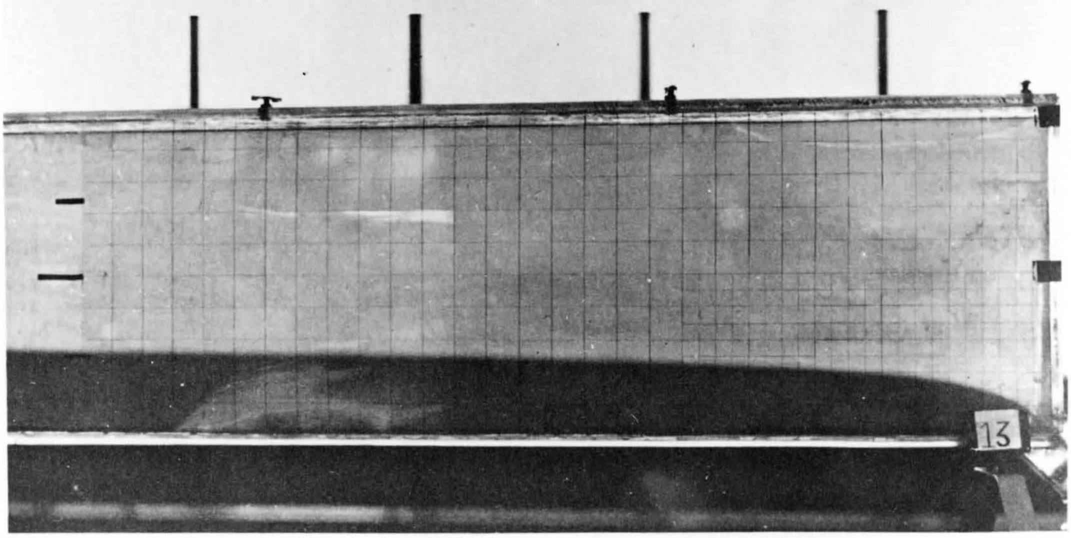


Fig.5.13

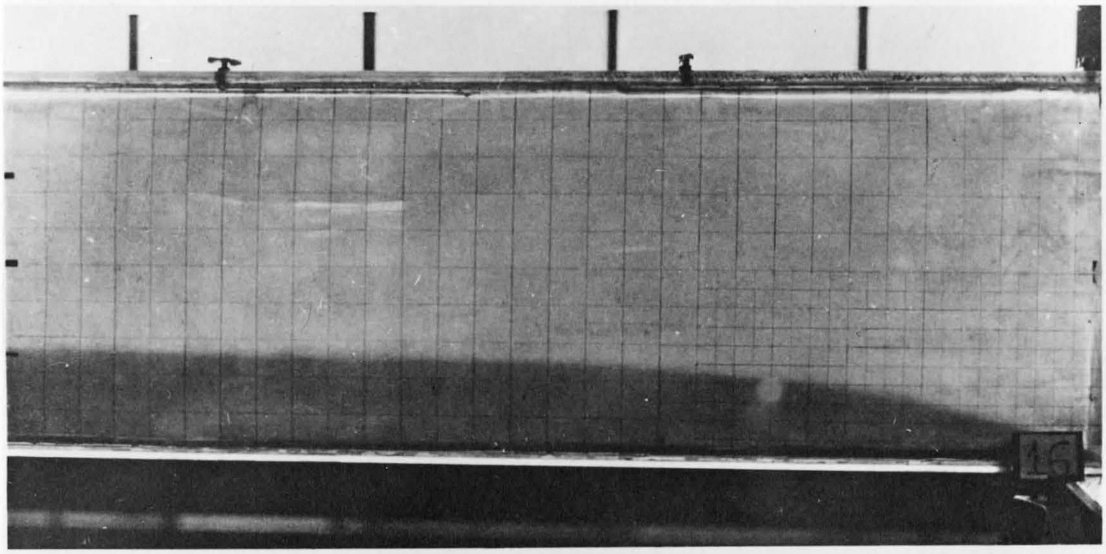


Fig.5.14

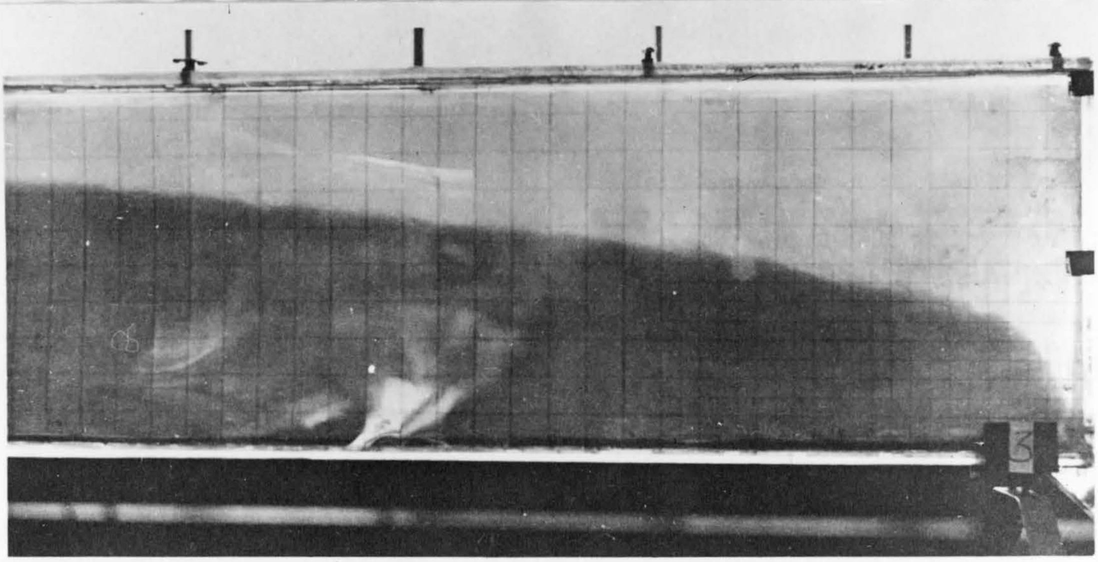


Fig.5.15

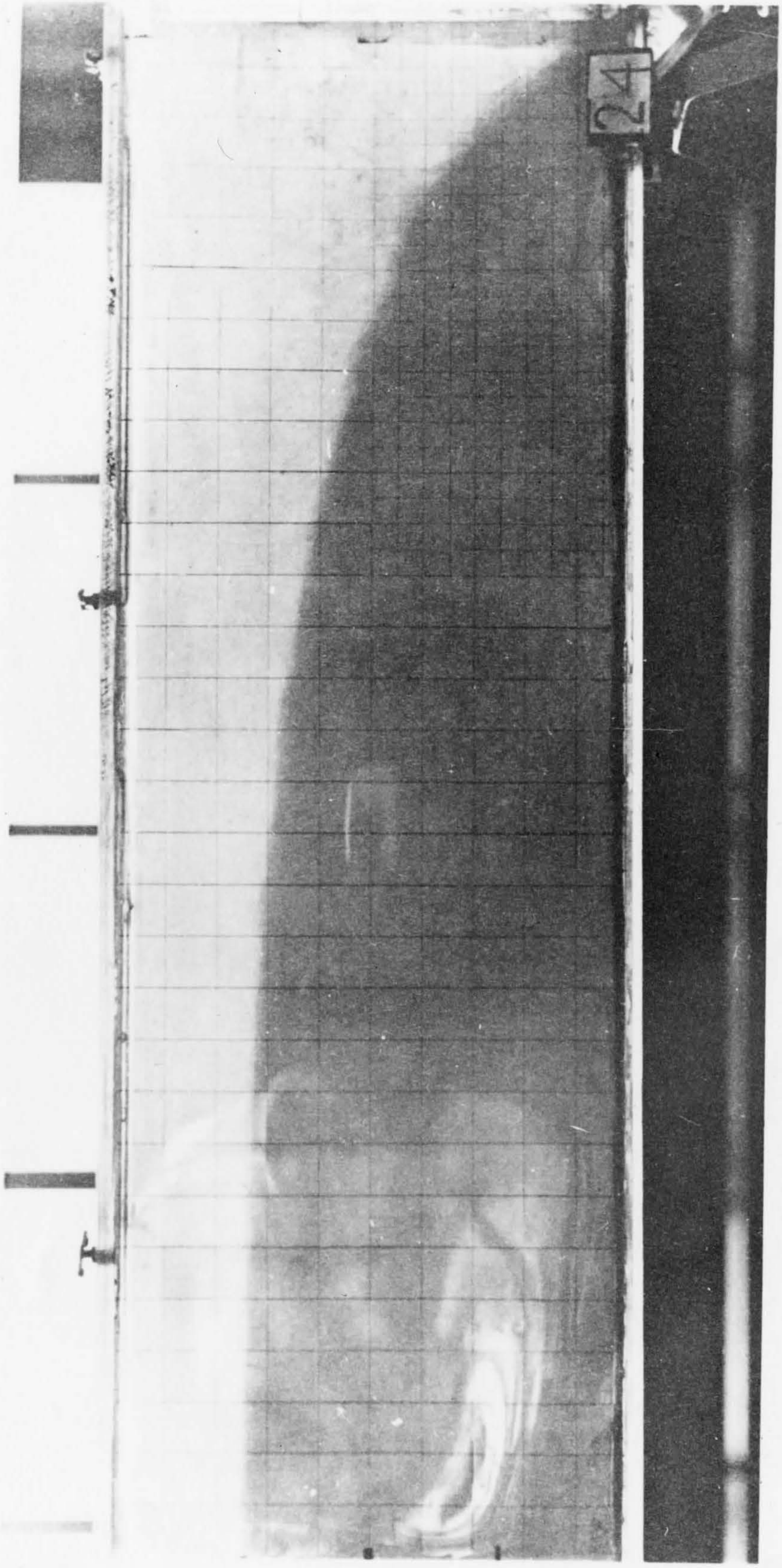


Fig.5.16

TABLE 5·1

RESULTS OF THE CASE WHERE $H_1 = H_2$, $W = 2"$, $B = .3"$

TEST NO.	START LEVEL FT.	VACUUM "OF HG	S.G. OF WATER	S.G. OF SUGAR SOL'N.	S.G. OF MIXTURE	SCALE READING INCHES	VOLUME $FT.^3$	TIME SEC.	Q_T $FT.^3/SEC.$	Q_2 $FT.^3/SEC.$	Q_1 $FT.^3/SEC.$	V_2 FT/SEC.	V_1 FT/SEC.	F_2	F_1
4	0.56	26	0.9990	1.0461	1.0230	11.45	3.52	35.6	.0989	.0503	.0486	.604	.583	.603	.448
6	0.62	28	1.0000	1.0415	1.0225	11.85	3.65	42.2	.0865	.0469	.0396	.563	.475	.494	.338
8	0.58	29	1.0000	1.0370	1.0186	8.45	2.56	28.2	.0908	.0454	.0454	.545	.545	.517	.499
9	0.55	28	1.0000	1.0350	1.0175	10.00	3.07	21.0	.1462	.0731	.0731	.877	.877	1.414	1.366
10	0.54	28	1.0000	1.0245	1.0120	11.55	3.55	24.2	.1467	.0719	.0719	.863	.898	1.936	2.046
11	0.52	28	1.0000	1.0240	1.0118	10.15	3.12	27.4	.1139	.0560	.0560	.672	.695	1.198	1.251
12	0.52	28	1.0000	1.0250	1.0125	9.63	2.95	21.2	.1392	.0696	.0696	.835	.835	1.777	1.734
13	0.52	28	1.0000	1.0320	1.0160	9.50	2.90	23.2	.1250	.0625	.0625	.750	.750	1.128	1.093
14	0.54	27	1.0000	1.0320	1.0159	10.35	3.17	22.0	.1441	.0716	.0716	.859	.870	1.479	1.470
15	0.51	27	1.0000	1.0310	1.0152	9.00	2.75	24.3	.1132	.0555	.0555	.666	.692	.917	.960
16	0.58	16	1.0000	1.0760	1.0378	9.60	2.93	22.0	.1332	.0662	.0662	.794	.804	.555	.529
19	0.77	28	1.0007	1.0640	1.0555	8.30	2.52	38.0	.0663	.0574	.0574	.689	.107	.496	.011
20	0.77	28	1.0000	1.0585	1.0535	7.50	2.28	36.4	.0626	.0572	.0054	.686	.065	.529	.004
21	0.79	28	1.0000	1.0585	1.0555	7.85	2.37	38.0	.0624	.0592	.0032	.710	.038	.567	.002
22	0.77	28	1.0000	1.0461	1.0435	7.20	2.17	39.6	.0548	.0517	.0031	.620	.037	.542	.002
23	0.69	28	1.0000	1.0461	1.0300	8.25	2.50	30.0	.0833	.0542	.0291	.650	.349	.596	.164
24	0.72	28	1.0000	1.0389	1.0275	7.80	2.36	34.6	.0682	.0482	.0200	.578	.240	.555	.092
25	0.74	28	1.0000	1.0370	1.0280	8.45	2.57	44.6	.0576	.0436	.0140	.523	.168	.477	.047
26	0.65	27	1.0000	1.0370	1.0216	8.40	2.55	32.8	.0777	.0454	.0323	.545	.388	.517	.253
27	0.64	28	1.0009	1.0292	1.0171	7.05	2.12	29.6	.0716	.0409	.0307	.491	.368	.544	.297
28	0.50	28	1.0009	1.0292	1.0150	7.70	2.33	16.4	.1421	.0707	.0714	.848	.857	1.623	1.612
29	0.74	27	1.0000	1.0250	1.0178	8.90	2.73	48.6	.0562	.0400	.0162	.480	.194	.587	.094
30	0.70	28	1.0004	1.0385	1.0240	8.90	2.73	33.5	.0815	.0505	.0310	.606	.372	.622	.226
31	0.62	28	1.0004	1.0385	1.0214	9.45	2.88	33.8	.0852	.0470	.0382	.564	.458	.539	.342
32	0.68	28	1.0000	1.0330	1.0210	9.30	2.83	39.0	.0726	.0462	.0264	.554	.317	.597	.189
33	0.59	28	1.0000	1.0330	1.0180	9.75	2.98	36.4	.0819	.0447	.0372	.536	.446	.559	.375
34	0.54	28	1.0000	1.0330	1.0165	11.00	3.38	33.2	.1018	.0509	.0509	.611	.611	.726	.703
35	0.51	28	1.0016	1.0360	1.0187	9.15	2.82	19.0	.1484	.0738	.0746	.886	.895	1.470	1.450
36	0.69	27	1.0023	1.0518	1.0338	9.00	2.75	30.3	.0908	.0578	.0330	.694	.396	.636	.198
37	0.68	28	1.0010	1.0510	1.0310	7.95	2.41	27.0	.0892	.0535	.0357	.642	.428	.541	.228
38	0.76	27	1.0010	1.0410	1.0406	4.65	1.36	30.6	.0445	.0441	.0004	.529	.005	.455	0
39	0.66	27	1.0010	1.0410	1.0400	4.10	1.18	38.4	.0308	.0300	.0008	.360	.009	.211	0
40	0.80	28	1.0018	1.0388	1.0348	4.35	1.27	25.0	.0508	.0453	.0055	.544	.066	.518	.007
41	0.72	27	1.0010	1.0320	1.0206	2.35	2.35	32.8	.0717	.0454	.0263	.545	.316	.616	.200
42	0.62	28	1.0010	1.0320	1.0184	2.69	2.69	35.0	.0769	.0432	.0337	.518	.405	.558	.330
43	0.83	28	1.0010	1.0275	1.0274	1.15	1.15	30.0	.0384	.0383	.0001	.460	.001	.512	0
44	0.65	28	1.0010	1.0275	1.0165	2.33	2.33	35.0	.0666	.0390	.0276	.468	.331	.530	.258
45	0.80	27	1.0006	1.0256	1.0218	1.71	1.71	39.6	.0432	.0366	.0066	.440	.079	.496	.016
46	0.69	27	1.0006	1.0256	1.0165	2.04	2.04	31.2	.0655	.0416	.0239	.499	.288	.638	.206
47	0.75	28	1.0005	1.0247	1.0190	9.55	2.83	55.6	.0508	.0389	.0119	.466	.143	.576	.053
48	0.85	28	1.0006	1.0565	1.0525	6.60	1.88	26.2	.0719	.0667	.0052	.800	.063	.754	.004
49	0.87	28	1.0006	1.0560	1.0509	6.40	1.82	24.9	.0731	.0665	.0066	.798	.079	.756	.007

TABLE 5·2

RESULTS OF THE CASE WHERE $H_1 = 3H_2$, $W = 2''$, $B = .3''$

TEST NO.	START LEVEL FT.	VACUUM " OF HG	S.G. OF WATER	S.G. OF SUGAR SOL'N.	S.G. OF MIXTURE	SCALE READING INCHES	VOLUME FT^3	TIME SEC.	Q_T $FT^3/SEC.$	Q_2 $FT^3/SEC.$	Q_1 $FT^3/SEC.$	V_2 FT/SEC.	V_1 FT/SEC.	F_2	F_1
1	0.29	28	1.0000	1.0550	1.0143	8.30	2.52	29.6	.0851	.0221	.0630	.530	.504	.670	.191
2	0.27	27	1.0000	1.0550	1.0127	8.35	2.53	27.0	.0937	.0216	.0721	.518	.577	.640	.251
3	0.26	28	1.0000	1.0550	1.0130	11.25	3.46	30.9	.1120	.0265	.0855	.636	.684	.965	.353
4	0.25	27	1.0009	1.0482	1.0109	9.20	2.80	23.8	.1176	.0248	.0928	.595	.742	.974	.482
5	0.35	27	1.0009	1.0510	1.0205	7.05	2.12	48.0	.0442	.0173	.0269	.415	.215	.449	.038
6	0.35	28	1.0000	1.0460	1.0415	4.50	1.32	83.0	.0159	.0143	.0016	.343	.013	.333	.0002
7	0.32	28	1.0000	1.0460	1.0123	9.25	2.83	46.2	.0613	.0164	.0449	.394	.359	.439	.116
8	0.34	28	1.0000	1.0330	1.0120	7.70	2.33	62.2	.0375	.0136	.0239	.326	.191	.414	.046
9	0.25	27	1.0000	1.0330	1.0080	10.70	3.28	36.8	.0891	.0216	.0675	.518	.540	1.044	.366
10	0.34	28	1.0009	1.0292	1.0208	2.75	0.76	49.2	.0154	.0108	.0046	.259	.037	.303	.002
11	0.25	25	1.0009	1.0292	1.0078	8.55	2.60	17.9	.1453	.0354	.1099	.850	.879	3.261	1.130
12	0.38	28	1.0000	1.0235	1.0216	4.15	1.20	100.0	.0120	.0110	.0010	.264	.008	.377	.0001
13	0.36	27	1.0004	1.0380	1.0215	5.75	1.70	64.5	.0264	.0148	.0116	.355	.093	.433	.010
14	0.34	26	1.0010	1.0307	1.0127	6.65	1.98	71.0	.0279	.0111	.0168	.266	.134	.305	.025
15	0.28	28	1.0010	1.0300	1.0077	11.25	3.47	50.9	.0682	.0158	.0524	.379	.419	.634	.251
16	0.26	27	1.0010	1.0300	1.0080	14.00	4.35	30.2	.1440	.0348	.1092	.835	.874	3.079	1.093
17	0.27	28	1.0006	1.0510	1.0120	10.85	3.32	36.4	.0912	.0206	.0706	.494	.565	.633	.263
18	0.25	27	1.0006	1.0440	1.0110	8.80	2.68	19.8	.1354	.0324	.1030	.778	.824	1.810	.649
19	0.25	28	1.0003	1.0460	1.0110	9.00	2.75	22.3	.1233	.0288	.0945	.691	.756	1.359	.518
20	0.25	27	1.0003	1.0470	1.0115	9.35	2.85	22.3	.1278	.0307	.0971	.737	.777	1.514	.536

TABLE 5·3

RESULTS OF THE CASE WHERE $H_2=3H_1$, W=2", B=.3"

TEST NO.	START LEVEL FT.	VACUUM "OF HG	S.G. OF WATER	S.G. OF SUGAR SOL'N.	S.G. OF MIXTURE	SCALE READING INCHES	VOLUME $FT.^3$	TIME SEC.	Q_T $FT.^3/SEC.$	Q_2 $FT.^3/SEC.$	Q_1 $FT.^3/SEC.$	V_2 FT./SEC.	V_1 FT./SEC.	F_2	F_1
1	0.77	28	1.0009	1.0292	1.0225	8.00	2.44	17.4	.1402	.1068	.0334	.854	.802	1.097	2.823
2	0.83	28	1.0000	1.0250	1.0185	8.10	2.45	29.6	.0828	.0613	.0215	.490	.516	.408	1.324
3	0.87	27	1.0000	1.0195	1.0155	7.50	2.27	35.6	.0638	.0507	.0131	.406	.314	.357	.629
4	0.77	27	1.0000	1.0195	1.0140	7.90	2.39	33.8	.0707	.0508	.0199	.406	.478	.357	1.457
5	0.90	27	1.0000	1.0185	1.0156	9.95	3.05	44.8	.0681	.0574	.0107	.459	.257	.481	.444
6	0.88	28	1.0004	1.0385	1.0305	8.60	2.62	26.4	.0992	.0784	.0208	.627	.499	.444	.813
7	0.76	28	1.0004	1.0385	1.0286	9.40	2.87	22.4	.1281	.0948	.0333	.758	.799	.649	2.084
8	0.84	28	1.0000	1.0330	1.0246	8.25	2.50	26.7	.0936	.0698	.0238	.558	.571	.404	1.228
9	0.75	28	1.0000	1.0330	1.0236	11.10	3.42	24.3	.1407	.1006	.0401	.805	.962	.841	3.487
10	0.85	28	1.0000	1.0260	1.0200	9.95	3.05	36.6	.0833	.0641	.0192	.513	.461	.430	1.016
11	0.76	28	1.0000	1.0260	1.0185	9.30	2.83	20.3	.1394	.0992	.0402	.794	.965	1.039	4.453
12	0.85	28	1.0000	1.0443	1.0340	9.35	2.85	26.3	.1084	.0832	.0252	.666	.605	.433	1.027
13	0.80	28	1.0000	1.0445	1.0335	8.95	2.74	24.0	.1142	.0860	.0282	.688	.677	.460	1.280
14	0.79	28	1.0000	1.0445	1.0340	8.75	2.67	18.2	.1467	.1121	.0346	.897	.830	.783	1.925
15	0.89	28	1.0000	1.0430	1.0333	9.75	2.98	28.3	.1053	.0815	.0238	.652	.571	.427	.943
16	0.79	28	1.0000	1.0430	1.0318	9.50	2.90	19.8	.1465	.1083	.0382	.866	.917	.754	2.431
17	0.83	28	1.0000	1.0430	1.0325	8.80	2.68	23.7	.1131	.0855	.0276	.684	.662	.470	1.267
18	0.79	28	.9988	1.0394	1.0295	9.50	2.90	20.9	.1388	.1050	.0338	.840	.811	.749	2.012
19	0.81	28	.9988	1.0393	1.0297	10.85	3.35	25.6	.1309	.0999	.0310	.799	.744	.679	1.697
20	0.80	27	1.0020	1.0383	1.0284	9.05	2.76	20.9	.1321	.0961	.0360	.769	.864	.712	2.562
21	0.82	28	1.0020	1.0382	1.0290	9.05	2.76	23.8	.1160	.0865	.0295	.692	.708	.569	1.725
22	0.79	28	1.0020	1.0380	1.0292	8.75	2.67	19.0	.1405	.1062	.0343	.850	.823	.863	2.344
23	0.87	28	1.0020	1.0380	1.0302	8.25	2.50	26.5	.0943	.0739	.0204	.591	.490	.417	.831
24	0.81	28	1.0016	1.0360	1.0265	8.30	2.52	22.7	.1110	.0803	.0307	.642	.737	.514	1.966
25	0.79	28	1.0016	1.0360	1.0270	8.05	2.44	16.9	.1444	.1066	.0378	.853	.907	.908	2.978
26	0.80	28	1.0002	1.0342	1.0243	7.60	2.30	19.9	.1156	.0819	.0337	.655	.809	.541	2.394
27	0.79	28	1.0002	1.0342	1.0246	8.10	2.45	16.4	.1494	.1072	.0422	.858	1.013	.928	3.753
28	0.80	28	1.0006	1.0510	1.0370	7.50	2.26	18.9	.1196	.0864	.0332	.691	.797	.413	1.568

TABLE 5·4

RESULTS OF VARYING SINK HEIGHT B, $H_1 = H_2$ W=2"

TEST NO.	START LEVEL FT.	B INCHES	VACUUM "OF HG	S.G. OF WATER	S.G. OF SUGAR SOL'N.	S.G. OF MIXTURE	SCALE READING INCHES	VOLUME $FT.^3$	TIME SEC.	Q_T $FT.^3/SEC.$	Q_2 $FT.^3/SEC.$	Q_1 $FT.^3/SEC.$	V_2 FT/SEC.	V_1 FT/SEC.	F_2	F_1
50	0.78	.15	28	1.0000	1.0568	1.0540	4.30	1.16	20.9	.0555	.0528	.0027	.634	.032	.467	.001
51	0.83	.15	28	1.0020	1.0530	1.0500	6.90	1.98	31.5	.0629	.0592	.0037	.710	.044	.653	.002
52	0.85	.15	28	1.0000	1.0550	1.0523	7.75	2.26	34.0	.0665	.0633	.0032	.760	.038	.693	.002
53	0.85	.15	28	1.0008	1.0540	1.0510	6.75	1.94	29.8	.0652	.0616	.0036	.740	.043	.680	.002
54	0.81	.50	28	1.0008	1.0540	1.0470	7.60	2.21	31.8	.0696	.0605	.0091	.726	.109	.655	.014
55	0.80	.50	28	1.0008	1.0540	1.0505	6.60	1.89	29.4	.0644	.0602	.0040	.723	.048	.646	.003
56	0.83	.50	28	1.0010	1.0515	1.0490	6.85	1.97	30.4	.0648	.0616	.0032	.740	.038	.714	.002
57	0.83	.50	28	1.0010	1.0515	1.0485	6.75	1.94	29.8	.0652	.0614	.0038	.736	.046	.705	.003

TABLE 5·5

RESULTS OF REDUCING CHANNEL WIDTH W, $H_1 = H_2$, W=1.5", B=.3"

TEST NO.	START LEVEL FT.	VACUUM "OF HG	S.G. OF WATER	S.G. OF SUGAR SOL'N.	S.G. OF MIXTURE	SCALE READING INCHES	VOLUME $FT.^3$	TIME SEC.	Q_T $FT.^3/SEC.$	Q_2 $FT.^3/SEC.$	Q_1 $FT.^3/SEC.$	V_2 FT/SEC.	V_1 FT/SEC.	F_2	F_1
58	0.83	28	1.0006	1.0586	1.0498	6.10	1.82	37.6	.0485	.0412	.0073	.659	.117	.494	.014
59	0.83	28	1.0010	1.0576	1.0515	5.25	1.55	32.8	.0472	.0421	.0051	.674	.082	.527	.007
60	0.84	28	1.0000	1.0560	1.0530	6.30	1.88	43.6	.0432	.0408	.0024	.653	.038	.500	.002
61	0.83	28	1.0005	1.0519	1.0500	5.75	1.71	42.0	.0407	.0392	.0015	.627	.024	.503	.001
62	0.84	28	1.0012	1.0552	1.0515	6.50	1.95	43.6	.0447	.0417	.0030	.667	.048	.542	.003

6. Discussion

There was a notable discrepancy in the theoretical and experimental values at the point of incipient drawdown for the case where $H_1=H_2$. The experimental value of F_2 was 0.756 while the theoretical value was 2.76. In this respect, it must be remembered that the Froude number used in Huber's paper was a generalized Froude number, utilizing the square of the velocity. This fact tends to exaggerate any discrepancy in velocity. Since the only variable in the Froude number which could differ from the theoretical value was the velocity, (the densities and heights at the entrance to the test section being reasonably accurate) a comparison was made between the theoretical and experimental values of $\sqrt{F_2}$ to determine the validity of the analysis. This gave a theoretical value of $\sqrt{F_2}$ at incipient drawdown of 1.66 and an experimental value of 0.87, a discrepancy of 47.5%. It was suspected that the presence of the side boundaries had a large effect on the flow, so tests were conducted with a narrowed channel width. A significant difference was found in the results at incipient drawdown. By decreasing the width of the channel by 25%, there was a resulting decrease in the value of F_2 by 15.5%. It was hypothesized that further decreases in width would result in further decreases of F_2 indicating that by enlarging the channel width, the theoretical value at incipient drawdown would be approached. It would have been desirable to continue the tests using increased channel widths but this extension to the programme was not possible with the present apparatus. As well as the side boundaries, the bottom and interfacial boundaries also had to be considered. They tended to reduce the average velocity in the bottom layer for the condition of incipient drawdown and added to the discrepancy. Craya[4] has shown that because the density difference between layers was small, the interface was much more sensitive to the forces of friction than had there been a large density difference since the gravity term in the flow equation was reduced by $\Delta\rho/\rho$ and made the effect of friction larger. This has been shown in the experiments. It was found that for a greater density difference, a larger Froude number at the point of incipient drawdown could be obtained. Therefore, if the density difference was made much larger, the theoretical value would be more closely approached because the interfacial friction term in the flow equation would be of less significance. To verify this point experimentally, several tests were made with increasing density differences between the sugar solution and the fresh water. It was found that when the density difference was made sufficiently large to permit an increase in the flow of the bottom layer at the point of incipient drawdown a surface disturbance, as shown in Fig. 5.12, occasionally disrupted the steady state and caused a temporary reduction in the flow of the lighter fluid. As a consequence considerable scatter is evident on the experimental curves. This instability of the interface was more pronounced at high flow rates. It was for this reason that further tests were not carried out using different combinations of working fluids.

The results of the tests indicate that the main effect of viscosity was to lower the Froude number at which incipient drawdown occurred. Taking the influence of viscosity into consideration, the correlation between the experimental and theoretical values at the point of incipient drawdown appears to be reasonable and since this is the point at which the theoretical and experimental values differ the most the trends predicted from the theoretical solution have been verified by experiment.

The discrepancies between the theoretical and experimental curves of Figure 5.5 can be explained in the above manner since these curves use the same results as Figure 5.2, presented in a different manner. The experimental curve of Figure 5.4 also deviates from the theoretical, at the point of incipient drawdown, for the same reasons. When there was no water flow, θ_2 must have been 90°, the reason for the large discrepancy at this point.

In the previous section it was noted that at high discharges for the cases of $H_1 = 3H_2$ and $H_2 = 3H_1$, the slopes of the lines on the F_2 vs. F_1 graph were 1/3 and 3 respectively. Yih[19] proved that under conditions of high discharge of a stratified fluid in the absence of vorticity and when gravity effects may be neglected, every steady irrotational flow corresponds to a stratified flow, the velocity of which is that for the irrotational flow divided by the square root of the density. Thus the following relationship can be written:

$$V_{1\infty}\sqrt{\rho_1} = V_{2\infty}\sqrt{\rho_2}$$

or

$$V_{1\infty}^2 \rho_1 = V_{2\infty}^2 \rho_2$$

Now, $F_{1\infty} = \dfrac{V_{1\infty}^{2} \rho_1}{g\Delta\rho H_1}$ and $F_{2\infty} = \dfrac{V_{2\infty}^{2} \rho_2}{g\Delta\rho H_2}$

Therefore,

$$\frac{F_{2\infty}}{F_{1\infty}} = \frac{H_1}{H_2} = 3$$

for the case where $H_1 = 3H_2$

and, $\dfrac{F_{2\infty}}{F_{1\infty}} = \dfrac{H_1}{H_2} = \dfrac{1}{3}$

for the case where $H_2 = 3H_1$.

thus it has been shown that the experimental and theoretical results agree for high discharges in the three series of tests.

The other part of this project was to extend Harleman's work on submerged sluice control. Studying Figure 2.5, it was thought that the curve would continue to rise parabolically with an increase in H_R/B. However, in the present experiments, it was found that the curve followed a straight line after a certain H_R/B ratio had been reached. The Khafagi-Hammad[24] relationships explain this phenomenon. The curves shown in Figure 2.5 were drawn using the following relationships and by assuming that h_f, the friction head, was zero.

$$\frac{H_R}{B} = \eta + \frac{\alpha F_1^{2}}{2} \qquad (1)$$

where H_R = reservoir height

B = sink height

$$\eta = 1 - \frac{F_1^{2}}{2} \left(f_1 (R) + f_2 (\theta_2)\right) \qquad (2)$$

α = kinetic energy correction factor*

$$f_1(R) = 1 - \left(\frac{3}{1+2e^{k}}\right)^2 \qquad (3)$$

where $k = \frac{B}{2.12\ R}$

and R was the radius of curvature

$$f_2(\theta) = 1 - \frac{1}{\beta^2} \qquad (4)$$

where $\beta = \frac{1}{\theta} \ln \tan \left(\frac{\theta}{2} + \frac{\pi}{4}\right)$

and θ was the angle of inclination of the interface.

If η were essentially a constant, the curve of H_R/B vs. F_1 would be a parabola. This was the case for $F_1 < 1.0$ since η only varied between 1.0 and 0.9 in Harleman's tests. However since the values of F_1 used in the tests were larger than 1.0, it has been found from earlier research work that η was not essentially constant and became large negatively as F_1 became large tending to reduce H_R/B in equation (1).

* The kinetic energy correction factor is a factor which, when applied to the kinetic energy term found by using the average velocity over a section, will determine the average kinetic energy passing that section. The equation for the kinetic energy correction factor is derived in most elementary Fluid Mechanics textbooks and is found to be $\alpha = \frac{1}{A} \int_A \left(\frac{v}{V}\right)^3 dA$ where A is the cross-sectional area, V is the average velocity across the section and v is the velocity at any point in that section.

At the point of incipient drawdown, θ was 90° and R the radius of curvature of the lower fluid at the sink was very much larger than B, the sink height.

Thus, from equation (4):

$$f_2(\theta) = 1 - \frac{1}{\beta^2} = 1 - 0 = 1.$$

From equation (3):

$$f_1(R) = 0$$

since R >> B, making K = 0. Inserting these values into equation (2) gave:

$$\eta = 1 - \frac{F_1^2}{2} .$$

Substituting this value of into equation (1) for this case:

$$\frac{H_R}{B} = 1 - \frac{F_1^2}{2} + \frac{\alpha F_1^2}{2} \qquad (5)$$

By comparing equation (1), where was essentially constant for Harleman's case of very low flow rates, with equation (5), it is apparent that the curve could not rise as steeply parabolically. Also, if there were no friction, the curve of H_R/B vs. F_1 would be a horizontal straight line at $H_R/B = 1.0$, providing the Khafagi-Hammad functions held true. However, since this was approaching the limits of the case, there were discrepancies and this solution would no longer be valid. The Khafagi-Hammad relationships were introduced to show how Harleman obtained his results and to demonstrate why the results obtained in this project did not follow an extension of the original curve for the low flow rates. The Khafagi-Hammad relations were useful in demonstrating the necessity for a more gradual curve than that of Harleman.

The utility of the results of the above series of tests is elaborated upon in the appendix.

Error Analysis

Variables in which there were possible errors were the following:

1. density of fresh water
2. density of sugar solution
3. density of mixture in test tank
4. timing of test
5. height of interface at entrance to test section
6. volume of mixture

The specific gravity measurements were accurate to within 0.0003. The error was of no importance when considering the specific gravities of fresh water and sugar solution separately, but of significance when considering their difference. If there were a difference in specific gravities of 0.0300, the error would then be 1%. When calculating the volume rates of flow, the difference in specific gravities was also used. For example, suppose the difference in specific gravity between the mixture and the fresh water was 0.0150. The error involved would be 2%. Because of the density error, the volume rate of flow had an error of 2%. This was not the only error in the volume rate of flow; there was a further deviation involved in the reading of the scale, which could be read to within ± 0.125 inch. With a scale reading of 5.00 inches, the error in the reading would be 2.5%. The error involved in reading the stopwatch was estimated at 0.3 seconds. With a time of 20 seconds, the error would be 1.5%. Therefore the volume rates of flow were accurate only within the accuracy of the measurements of the density, stopwatch and scale reading. These errors combined to give, for the above representative example, a total error of 6% in the volume rates of flow. This gave the velocity at the entrance to the test section, an error of 6% without considering the fact that the height of the sugar solution at this point may not be exactly 0.50. An error in height of ± 0.1 inch would be typical, constituting a deviation of 2%. The total error in velocity at the entrance to the test section, then would be 8%.

The Froude numbers had errors associated with the velocity, the height of the interface and the difference in specific gravities of the two fluids. These errors would be 16%, due to the velocity squared term, 2% due to the height of the interface, and 1% due to the density, giving a total maximum error of 19%.

7. Conclusions

The theoretical paper of D. G. Huber on the irrotational motion of two fluid strata towards a line sink was verified experimentally. It was found that there was excellent agreement at high flow rates when the inertia forces predominated and the friction forces were of relatively slight significance. The Froude number at the point of incipient drawdown was found to be 0.765, indicating the maximum amount of lower fluid which could be evacuated without the upper fluid taking part in the flow. This result has significance in practice making it possible, in any geometrically similar condition, to determine the velocity at infinity of the lower fluid at which the upper fluid either starts or ceases to flow and thus, would be of value in a separation process if the various densities were known.

It was found that the largest deviation between theoretical and experimental results occurred at the point of incipient drawdown. The results obtained at relatively high Froude numbers showed an excellent agreement with the theory. The deviations between experimental and theoretical results were shown to be due to frictional effects caused by the fact that the fluids used were not inviscid.

An extension of Harleman's work was carried out and unexpected results obtained. Again, these results are of significance in separation processes if the densities of the two strata, the sink height and the reservoir height of the lower layer are known, since the maximum discharge per foot of width, can be calculated. If utilizing the results of this experiment, it must be known that because of the high depth to width ratio used, the alpha factor will be quite high. Since in most practical cases where large dimensions are used, the alpha factor is lower than that used in this experiment, the maximum amount of lower fluid evacuated will not be precise. This simply introduces a larger safety factor since as the alpha factor was decreased the Froude number at the point of incipient drawdown increased.

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APPENDIX

1. NOMENCLATURE

SYMBOL	QUANTITY	UNITS
B	Height of gate opening	Ft.
F_1, F_2	Densimetric Froude Numbers in upper and lower fluid respectively	-
F_i	Densimetric Froude Number of the lower fluid at point of incipient drawdown as used by Harleman	-
g	Gravitational acceleration	$Ft/sec.^2$
H_1, H_2	Fluid depths far upstream in upper and lower fluids respectively	Ft.
H_R	Interface elevation	Ft.
R	Radius of curvature	Ft.
V_1, V_2	Fluid velocities far upstream in upper and lower fluid respectively	Ft/sec.
W	Width of channel	Ft.
α	Kinetic energy correction factor	-
ρ_1, ρ_2	Fluid density of upper and lower respectively	$Slugs/ft^3$
S_1, S_2, S_T	Specific gravities of upper, lower and a mixture of the two fluids respectively	-
θ_1, θ_2	Angles made by the interface with channel end boundary and channel bottom repectively	°

2. Derivation of Formula Used For Calculation of Flow Rates

From the equation of continuity the following can be written:

$$Q_2S_2 + Q_1S_1 = Q_TS_T \quad (1)$$

And since it is assumed that the volume change due to reaction is zero, then;

$$Q_1 + Q_2 = Q_T \quad (2)$$

And $Q_2S_2 + (Q_T - Q_2)S_1 = Q_TS_T$

$\therefore Q_2(S_2 - S_1) = Q_T(S_T - S_1)$

And $Q_2 = Q_T \dfrac{S_T - S_1}{S_2 - S_1}$

where Q_1 = fresh water flow rate (ft.^{3}/sec.)

Q_2 = sugar solution flow rate (ft.^{3}/sec.)

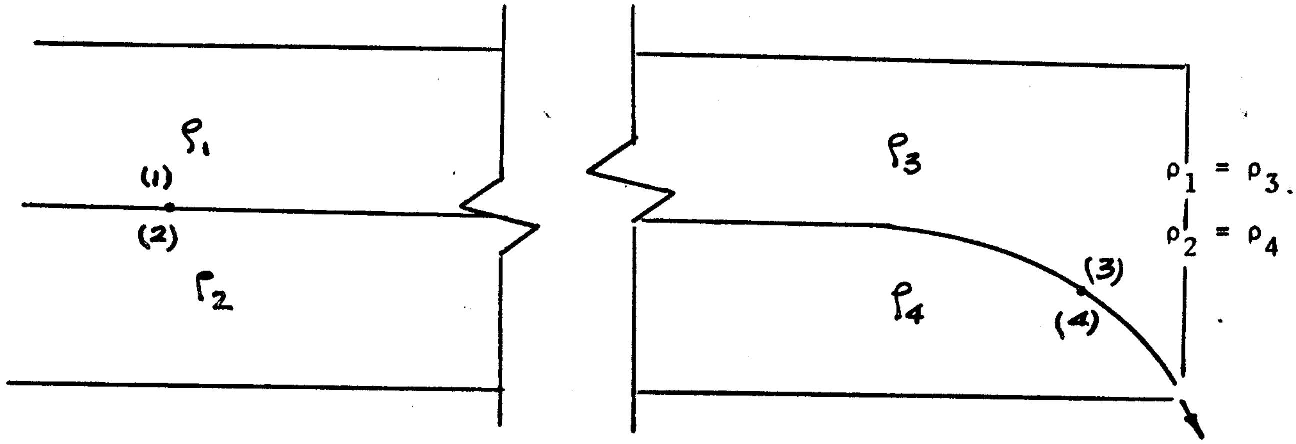
Q_T = combined flow rate (ft.^{3}/sec.)

S_1 = Specific gravity of fresh water

S_2 = Specific gravity of sugar solution

S_T = Specific gravity of mixture

3. Derivation of Equation Showing Relationship Between Elevation Difference, Velocity Difference and Specific Gravity Difference



The effect of friction is neglected in this analysis.

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_3}{\rho_3} + \frac{V_3^2}{2} + gz_3$$

$$\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 = \frac{P_4}{\rho_4} + \frac{V_4^2}{2} + gz_4$$

Now, $P_1 = P_2$, $P_3 = P_4$, $Z_1 = Z_2$, $Z_3 = Z_4$

$V_1 = 0$ and $V_2 = 0$ since points (1) and (2) are considered to be "infinitely" far upstream.

Then $P_1 = P_3 + \frac{\rho_1 V_3^2}{2} + g\rho_1 Z_3 - g\rho_1 Z_1$

and $P_2 = P_4 + \frac{\rho_2 V_4^2}{2} + g\rho_2 Z_4 - g\rho_2 Z_2$

Combining the above two equations gives

$$(P_1 - P_2) = (P_3 - P_4) + \left(\frac{\rho_1 V_3^2}{2} - \frac{\rho_2 V_4^2}{2}\right) + g\rho_1(Z_3 - Z_1) - g\rho_2(Z_4 - Z_2)$$

But $(Z_4 - Z_2) = (Z_3 - Z_1)$

$$\therefore \quad g(Z_3 - Z_1)(\rho_1 - \rho_2) = \frac{\rho_2 V_4^2 - \rho_1 V_3^2}{2}$$

and
$$(Z_3 - Z_1) = \frac{\rho_2 V_4^2 - \rho_1 V_3^2}{2g(\rho_1 - \rho_2)}$$

or
$$Z_3 - Z_1 = \frac{S_2 V_4^2 - S_1 V_3^2}{2g(S_1 - S_2)}$$

since $S_1 < S_2$ and, at low discharge, $V_4 \gg V_3$, then $Z_3 < Z_1$.

4. Illustrative Example

Suppose there was a large reservoir in which there were two immiscible fluids which were to be separated by causing a flow through a rectangular hole at a side of the reservoir. It is assumed that $H_R = 10$ ft., $B = 0.5$ ft., $\frac{\Delta\rho}{\rho} = .050$ and $W = 1.0$ ft.

From Fig. 5.7 at $H_R/B = 20$, $F_i = 77.8$

Then
$$F_i = \frac{Q/WB}{\sqrt{g\frac{\Delta\rho}{\rho}B}} = 77.8$$

$$\therefore \quad Q = 77.8 \sqrt{32.2\ (.050)\ (0.5)}\ (1.0)\ (0.5) = 34.8 \text{ cfs.}$$

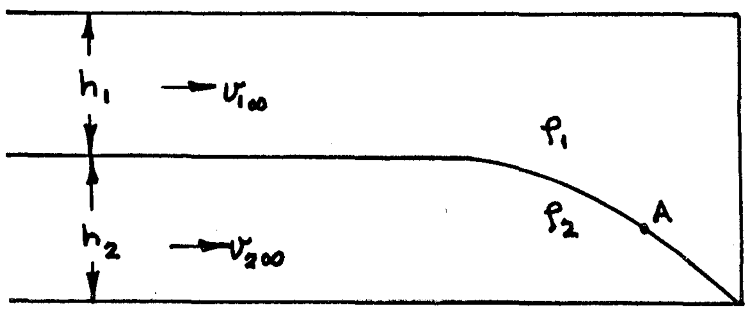
The maximum flow rate of the lower fluid with no flow of the upper fluid is then determined.

§. A Note On The Theoretical Solution To The Problem Of Irrotational Motion Of Two Fluid Strata Towards A Line Sink As Probosed By Huber[2]

One of the objectives of the present experiment was to obtain a value of the Froude number at the point of incipient drawdown which could be compared to that predicted by Huber[2] who offered an analytical solution to the problem.

Huber showed that because the boundary conditions were complicated a rigourous mathematical analysis of the critical condition when the lighter fluid either began or ceased to flow was not readily obtained. However, an approximate numerical analysis technique was available as desecribed by McNown, Hsu and Yih in Reference 25, for the equations of motion, and using this relaxation technique Huber obtained a solution to the problem.

The method used for the solution by relaxation procedures necessitated, first, the assumption of an interfacial shape and, second, the estimation of the stream function ψ throughout the two potential fields. By relaxation techniques the assumed values of ψ were altered until they were correct at every point for the assumed boundary conditions. Velocities were calculated at a number of points along the interface and the interfacial equation as presented below was checked to determine if the boundary conditions were satisfied.

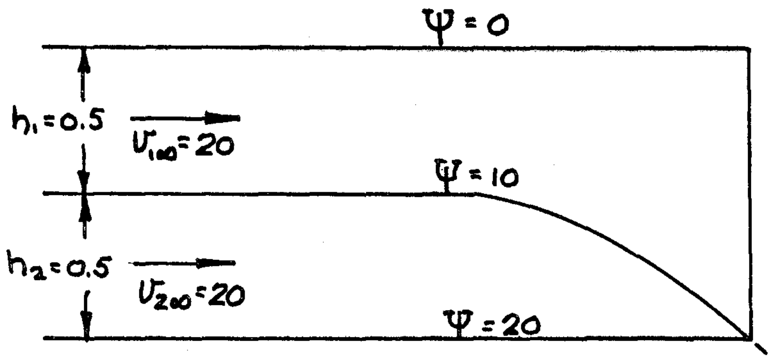


By combining the flow equations for the upper and lower fluids the equation of the interface becomes

$$\frac{\rho_1}{2}(v_{1A}^2 - v_{1\infty}^2) - \frac{\rho_2}{2}(v_{2A}^2 - v_{2\infty}^2) = y_A \Delta\gamma \quad \cdots\cdots\cdots \quad (1)$$

where $\Delta\gamma$ is $\rho_1 g - \rho_2 g$

By the use of constants in the equation it was possible to calculate the two velocities at every point, and thus the Froude number, once the correct interfacial shape was obtained.



In the above figure the assumed values of the stream function and velocity are given symbols Ψ and v respectively, to distinguish them from the true values of ψ and V. The numerical values chosen were as shown for two of the three cases attempted. For the third case the value of Ψ on the lower boundary was 19. The grid pattern used was one of 0.1 unit squares progressively reduced to 0.05 units in the vicinity of the interface, and finally to 0.025 units close to the sink.

Each case necessitated the assumption of an interfacial curve so drawn that the first and second derivatives when plotted also gave smooth curves. The value of ψ at each grid intersection was estimated and residues calculated using the standard techniques. When the residues were relaxed to a very small value at each intersection, the assumed interfacial shape was checked by the calculation of velocities on each side of the interface at a number of points spaced along the interfacial curve, and a calculation to determine if these velocities satisfied the interfacial boundary condition. Two factors were introduced into Equation 1.

$$k = \frac{v_1}{V_1}$$

and

$$J = \frac{v_2}{V_2}$$

to give

$$\frac{\rho_1}{2} K^2 (V_{1A}^2 - V_{1\infty}^2) - \frac{\rho_2}{2} J^2 (V_{2A}^2 - V_{2\infty}^2) = y_A \Delta\gamma \quad \ldots\ldots\ (2)$$

A pair of points on the interface was selected and values of V_{1A} and V_{2A} as found from the relaxed values of ψ were substituted into the above equation along with corresponding values of y_A. Values were assigned to ρ_1 and ρ_2. The resulting pair of equations was solved simultaneously for K and J. A correctly assumed interfacial curve would give constant values of K and J regardless of the pair of points selected for the calculation. If K and J were not found reasonably constant by calculation involving successive pairs of points, it was necessary to redraw the interfacial curve and repeat the relaxation procedures.

Three interfacial curves were determined in this manner to produce three pairs of Froude numbers for the upper and lower fluids and enabled a plot of F_2 vs. F_1 to be drawn. Huber's results of this plot are shown in Fig. 5.2. The results were rearranged and presented in Fig. 5.5 as F_1/F_2 vs. $\sqrt{F_1} + \sqrt{F_2}$ to define the point of incipient drawdown ($F_1/F_2 = 0$) since the curve as shown in Fig. 5.2 approaches the abscissa in an asymptotic manner.

Rearranging equation 1 as

$$\frac{\rho_1 v_{1A}^2}{2} - \frac{\rho_2 v_{2A}^2}{2} = \frac{\rho_1 v_{1\infty}^2}{2} - \frac{\rho_2 v_{2\infty}^2}{2} + y_A \Delta\gamma$$

and multiplying through by $2/v_{2A}^2$ gives

$$\frac{\rho_1 v_{1A}^2}{v_{2A}^2} - \rho_2 = \frac{\rho_1 v_{1\infty}^2 - \rho_2 v_{2\infty}^2}{v_{2A}^2} + \frac{2 y_A \Delta\gamma}{v_{2A}^2}$$

If point A is taken close to the sink where v_{2A}^2 becomes very large compared to the other terms in the equation then the following equation results.

$$\rho_1 v_{1A}^2 = \rho_2 v_{2A}^2 \qquad \ldots\ldots\ldots \qquad (3)$$

By pursuing the analysis close to the sink a relationship between the interfacial angle, the velocities and the densities may be derived. The angle that the tangent to the interface at the sink makes with the vertical is θ_1 and with the horizontal θ_2. If an arc of small radius r is drawn, the discharge through the area $r\theta_1$ is $v_{1A}r\theta_1$, and that through the area $r\theta_2$ is $v_{2A}r\theta_2$. The ratio of the two discharges is then $v_{1A}r\theta_1/v_{2A}r\theta_2$ or $v_{1A}\theta_1/v_{2A}\theta_2$. The discharge may also be written as $v_{1\infty}h_1$ and $v_{2\infty}h_2$, and the ratio becomes $v_{1\infty}/v_{2\infty}$ since h_1 is equal to h_2. When the two expressions for the discharge ratio are equated, and equation 3 substituted, the following relationship results:

$$\frac{v_{1\infty}\sqrt{\rho_1}}{v_{2\infty}\sqrt{\rho_2}} = \frac{\theta_1}{\theta_2} = \frac{\sqrt{F_1}}{\sqrt{F_2}}$$

To plot a graph of F_2 vs. θ_2 it was necessary to first find F_1 corresponding to the required F_2 and then substitute into the above equation remembering that $\theta_1 + \theta_2$ must equal 90° and that θ_2 could be no less than 45°. Huber's theoretical results are shown in Fig. 5.4.