STUDY OF NUMERICAL SOLUTIONS FOR DEFORMATIONS OF A BOURDON TUBE

STUDY OF NUMERICAL SOLUTIONS FOR DEFORMATIONS OF A BOURDON TUBE

by

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TITLE: Study of Numerical Solutions for Deformations of A Bourdon Tube. AUTHOR: Govind Prashad, B. Tech. (I.I.T., Delhi, India) SUPERVISOR: Professor G. Kardos NUMBER OF PAGES: vii, 102 SCOPE AND CONTENTS:

The objective of this study is to compute numerically the deformations on an elliptical cross-sectional bourdon tube by solving the partial differential equations as presented by Lee [Reference 12].

The partial differential equations and boundary conditions are reduced to a set of simultaneous linear equations by approximating partial derivatives to the corresponding difference quotients using finite difference techniques.

The next step involves the solution of this set of linear equations. The direct method of inverting the matrix was not possible due to the memory limitations of the computer. Therefore, a block iteration technique was used, but it was found that convergence was not possible. The next method evolved was that of double iteration. For this method a convergence test was applied which indicated that convergence was possible, but the rate of convergence was very low.

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It is not practical to use this method, unless the convergence rate is improved. At the present no method is available to improve the convergence rate effectively. Therefore the study concludes with suggestions that either the convergence rate should be improved by evolving new methods or an entirely new formulation of the problem should be made.

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THE TEXT

CHAPTER 1

INTRODUCTION

The bourdon tube is the basic element of a pressure measuring instrument of such fundamental importance and widespread application, it is not surprising that it has been the subject of many studies. However, most of these analyses are highly approximate because of many simplifications. Flackbarth [1], Wuest [2], Goitein [3], Clark and Reisenner [4] in their studies used a strength of material approach or a compounded known solution of simpler problems. Jennings [5] compared some of these results together with his own theory. While Mason [6] has compiled sensitivity and life data, and has compared this data against some simplified theories. Kardos [7], [8], [9], [10] has carried out some tests on bourdon tubes and has given some design considerations.

For the first time Dressler [11] presented an exact formulation of the problem in terms of elastic shell theory. Lee [12] carried out an extensive literature survey in his Master's thesis. He also reanalysed Dressler's theory with a different approach, checked and completed the formulations. He gave the final forms of all necessary equations, boundary conditions, etc. for the solution of the three governing partial differential equations of the bourdon tube with an elliptical crosssection.

The aim of this thesis, has been to solve these partial differential equations with boundary conditions as presented by Lee, by numerical analysis and computer techniques.

CHAPTER 2

ELASTIC SHELL THEORY FORMULATION

2.1 GEOMETRY OF THE BOURDON TUBE:

Various shapes are used for the cross-sections of bourdon tubes. The elliptical cross-section poses the most complicated condition, and an attempt has been made to attain solutions for this shape which, if successful, will ensure success with other shapes. The tube can be taken as a section of a torus with an elliptical cross-section and a constant wall thickness, and clamped to a rigid wall at its opening. Internal fluid pressure causes a complicated overall distortion of the tube where the deflection at the free end serves as a measurement for the pressure. The free end of the tube is closed by a plug.

Any point on the central shell of the bourdon tube can be defined [Fig.2.1] with parameters α , representing the angle sweeping out the ellipse, and β defining a section of the torus, from the entrance at $\beta=0$ to the plug at $\beta=\beta_{\rm L}$. The radius of the torus is ρ , and the ellipse has semidiameters \overline{a} and \overline{b} .





Fig.2.1 Geometry of a bourdon tube

Fig. 2.2 Infinitesimal element of a tube

Lines of principal curvatures are defined by lines α and β . The metric coefficients of these lines are given by

 $|\bar{g}_{\beta}| = (g_{\beta\beta})^{1/2} = \rho + \bar{a} \cdot \cos \alpha$

$$|\bar{g}_{\alpha}| = (g_{\alpha\alpha})^{1/2} = (\bar{a}^2 \sin^2 \alpha + \bar{b}^2 \cos^2 \alpha)^{1/2}$$
 [2.1]

$$g_{\beta\alpha} = g_{\alpha\beta} = 0$$

Curvature of the parametric lines are given as:

A. Line β : $k_{\beta}^{(n)} = \overline{b} \cdot \cos \alpha / (|\overline{g}_{\alpha}| \cdot |\overline{g}_{\beta}|)$

$$k_{\beta}^{(g)} = \overline{a}.sin\alpha/(|\overline{g}_{\alpha}|.|\overline{g}_{\beta}|)$$

B. Line α : $k_{\alpha}^{(n)} = \overline{a} \cdot \overline{b} / (g_{\alpha \alpha})^{3/2}$

[2.2]

An infinitesimal element of the tube can be represented as shown in fig. 2.2. h' being half thickness of the tube.



Fig. 2.3 Resultant forces and moments

Fig. 2.3 illustrates an infinitesimal element of the central shell of the tube. The postive directions of the resultant forces (per unit of length along the shell edge) are as indicated.

$$\begin{split} & F_{\alpha\alpha} \ , F_{\beta\beta} \ \text{are tensile forces.} \\ & F_{\alpha\beta} \ , F_{\beta\alpha} \ \text{are shear forces.} \\ & F_{\alpha3} \ , F_{\beta3} \ \text{are transverse shear forces.} \\ & M_{\alpha\beta} \ , M_{\beta\alpha} \ \text{are bending moments.} \\ & M_{\alpha\alpha} \ , M_{\beta\beta} \ \text{are twisting moments.} \end{split}$$

The third moment components $(M_{\alpha3}, M_{\beta3})$ are zero for shells. Local moving axes x, y and z are taken as indicated. The x-axis, tangent to the line β =constant, the y-axis, tangent to α =constant, and z forming a right handed system. The local displacement along x,y and z are u, v and w respectively.

The expressions for the above forces and couples are as follow:[12]

Stress Resultants:

$$F_{\beta\beta} = E' \cdot [(g_{\beta\beta})^{-1/2} \cdot u_{,\beta} - k_{\beta}^{(g)} \cdot v - (k_{\beta}^{(n)} + v \cdot k_{\alpha}^{(n)}) \cdot w$$
$$+ v \cdot (g_{\alpha\alpha})^{-1/2} \cdot v_{,\alpha}]$$

$$F_{\alpha\alpha} = E' \cdot [(g_{\alpha\alpha})^{-1/2} \cdot u_{\alpha} - (k_{\alpha}^{(n)} + v \cdot k_{\beta}^{(n)}) \cdot w - v \cdot k_{\beta}^{(g)} \cdot u$$
$$+ v \cdot (g_{\beta\beta})^{-1/2} \cdot v_{\beta}]$$

$$F_{\beta\alpha} = F_{\alpha\beta} = E' \cdot (1-\nu) \cdot [(g_{\beta\beta})^{-1/2} \cdot u_{,\beta} + (g_{\alpha\alpha})^{-1/2} \cdot v_{,\alpha} + k_{\beta}^{(g)} \cdot v]/2$$
[2.3]

Stress couples:

$$M_{\beta\alpha} = D \cdot \left[\frac{1}{g_{\beta\beta}} \cdot w_{\beta\beta} - \frac{k_{\beta}^{(g)}}{g_{\alpha\alpha}} \cdot \frac{(g_{\alpha\alpha})^{-1/2} + v(\overline{a}^2 - \overline{b}^2) \sin\alpha \cdot \cos\alpha}{(g_{\alpha\alpha})^2} \cdot w_{\alpha} + \frac{v}{g_{\alpha\alpha}} \cdot w_{\alpha\alpha} \right]$$

 $M_{\alpha\beta} = -D \cdot \left[\frac{1}{g_{\alpha\alpha}} \cdot w \cdot_{\alpha\alpha} - \left[v \cdot k_{\beta}^{(g)} \cdot \left(g_{\alpha\alpha}\right)^{-1/2} + \left(\overline{a}^2 - \overline{b}^2\right) \sin\alpha \cdot \cos\alpha \right] \right]$ $/ \left(g_{\alpha\alpha} \right)^2 \cdot w \cdot_{\alpha} + \sqrt{g_{\beta\beta}} \cdot w \cdot_{\beta\beta} \right]$

$$M_{\beta\beta} = D(1-\nu) \cdot \left[-(g_{\alpha\alpha} \cdot g_{\beta\beta})^{-1/2} \cdot w_{\beta\alpha} - k_{\beta}^{(g)} \cdot (g_{\beta\beta})^{-1/2} \cdot w_{\beta\beta}\right]$$

$$M_{\alpha\alpha} = -M_{\beta\beta}$$
 [2.4]

Transverse shears:

$$F_{\alpha 3} = (g_{\beta \beta})^{-1/2} \cdot M_{\beta \beta} \cdot \beta^{+} (g_{\alpha \alpha})^{-1/2} \cdot M_{\alpha \beta} \cdot \beta^{-k} \cdot \beta^{(g)} \cdot (M_{\alpha \beta} + M_{\beta \alpha})$$

$$F_{\beta 3} = -(g_{\beta \beta})^{-1/2} \cdot M_{\beta \alpha} \cdot \beta^{-} (g_{\alpha \alpha})^{-1/2} \cdot M_{\alpha \alpha} \cdot \alpha^{+k} \cdot \beta^{(g)} \cdot (M_{\beta \beta} - M_{\alpha \alpha})$$

$$[2.5]$$

Where

$$E' = 2Eh'/(1-v^{2})$$
$$D = 2Eh'^{3}/[3(1-v^{2})]$$

E and v are the coefficient of elasticity and Poisson's ratio respectively of the material of the tube.

2.2 EQUILIBRIUM EQUATIONS:

By considering the shell element to be in equilibrium under the external forces and moment resultants, three equilibrium equations can be obtained in terms of the displacement components (u, v and w) and their partial derivatives with respect to α and β . This system of three partial differential equations are arranged in a matrix form as presented in the table [2.1] corresponding to the general form:

 $f_1(u) + f_2(v) + f_3(w) + F = 0$

Table 3 2.1 - Governing Equations of the BOURDON Gage $u(\alpha, \beta)$ $w(\alpha, \beta)$ Free $v(\alpha,\beta)$ Terms $\frac{E(1-\nu)}{2}\frac{\partial^{2}}{\partial \beta^{2}} + \frac{E'}{g_{\alpha\alpha}}\frac{\partial^{2}}{\partial \alpha^{2}} + \frac{E'}{g_{\alpha\alpha}}\frac{\partial^{2}}{\partial \alpha^{2}} + \frac{E'}{g_{\alpha\alpha}}\frac{\partial^{2}}{\partial \alpha^{2}} + \frac{E'(a^{2}-b)\sin\alpha\cos\alpha}{(g_{\alpha\alpha})^{2}}\frac{\partial^{2}}{\partial b}$ D Kat $\frac{E(1+\nu)}{2/\bar{g}_{\times}//\bar{g}_{\beta}/}\frac{\partial^{2}}{\partial \alpha \partial \beta} - \frac{E'k_{\beta}^{(9)}(\nu-3)\partial}{2/\bar{g}_{\beta}/}\frac{\partial}{\partial \beta}$ + 2D kaka $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{$ Dka $+ \frac{D \kappa_{\alpha}}{(g_{\alpha\alpha})^{3/2}} \frac{\partial}{\partial \alpha^{3}} + \frac{D \kappa_{\alpha}}{/\tilde{g}_{\alpha\beta}} \frac{\partial}{\partial \beta^{3}} \frac{\partial}{\partial \alpha} + \frac{2D \kappa_{\alpha} k_{\beta}}{g_{\beta\beta}} \frac{\partial}{\partial \beta^{3}} - \frac{D \kappa_{\alpha}}{g_{\beta\beta}} \left[\frac{k_{\beta}}{k_{\beta}} + \frac{3(\tilde{a}-\tilde{b})\sin\alpha\cos\alpha}{(g_{\alpha\alpha})^{3/2}} \right] \frac{\partial}{\partial \alpha^{3}} - \left[\frac{E}{/\tilde{g}_{\alpha}} (k_{\alpha}^{*} + \nu k_{\beta}) + \frac{D \kappa_{\alpha}}{(g_{\alpha\alpha})^{3/2}} \right] \frac{\partial}{\partial \alpha^{3}} - \left[\frac{E}{/\tilde{g}_{\alpha}} (k_{\alpha}^{*} + \nu k_{\beta}) + \frac{D \kappa_{\alpha}}{(g_{\alpha\alpha})^{3/2}} \right] \frac{\partial}{\partial \alpha^{3}} - \left[\frac{E}{/\tilde{g}_{\alpha}} (k_{\alpha}^{*} + \nu k_{\beta}) + \frac{D \kappa_{\alpha}}{(g_{\alpha\alpha})^{3/2}} \right] \frac{\partial}{\partial \alpha^{3}} + \frac{\kappa_{\beta}}{(g_{\alpha\alpha})^{3/2}} + \frac{\delta}{(g_{\alpha\alpha})^{3/2}} + \frac$ dis $\frac{\left(g_{\alpha\alpha}\right)^{2}}{\sqrt{g_{\alpha}}}\frac{\int_{\partial\alpha}}{\sqrt{g_{\alpha}}} - \frac{K_{\beta}}{K_{\beta}} \frac{1}{(\bar{a}-\bar{b})}}{K_{\beta}}$ sinacosall F ·sinacosa]] $\frac{1}{\partial \alpha} - \left[E' k_{\beta}^{(g)} (k_{\beta}^{(a)} + J k_{\alpha}^{(a)}) - E' k_{\beta}^{(g)} k_{\alpha}^{(m)} - \frac{E' J}{g_{\alpha \alpha}} \frac{\delta g_{\alpha \alpha}}{g_{\alpha \alpha}} \frac{1}{g_{\alpha \alpha}} \frac{J}{g_{\alpha \alpha}} \frac{J}{g_{\alpha$ BaulEst $\frac{E(1+\nu)}{2/\bar{g}_{\alpha}//\bar{g}_{\beta}/}\frac{\partial^{2}}{\partial_{\beta}\partial_{\alpha}}+\frac{E'E_{\beta}^{(2)}}{2/\bar{g}_{\beta}}(\nu-3)\frac{\partial}{\partial_{\beta}}$ $\frac{\mathbf{E}'}{\mathbf{E}_{A3}} \frac{\partial^{2}}{\partial \beta^{2}} + \frac{\mathbf{E}(1-\nu)}{2\mathbf{g}_{\alpha\alpha}} \frac{\partial^{2}}{\partial \alpha^{2}} - \frac{\mathbf{E}(1-\nu)}{2/\bar{\mathbf{g}}_{\alpha}/} + \frac{\mathbf{D} \mathbf{k}_{A}}{2/\bar{\mathbf{g}}_{\alpha}/} \frac{\partial^{2}}{\partial \beta^{3}} + \frac{\mathbf{D} \mathbf{k}_{A}}{\mathbf{E}_{\alpha'\alpha}/\bar{\mathbf{E}}_{\beta}/} \frac{\partial^{3}}{\partial \beta \partial \alpha^{2}} - \frac{\mathbf{D} \mathbf{k}_{A}'}{/\bar{\mathbf{g}}_{\alpha}//\bar{\mathbf{E}}_{A}/} \mathbf{k}_{A}' - \frac{\mathbf{E}(1-\nu)}{2/\bar{\mathbf{g}}_{\alpha}//\bar{\mathbf{E}}_{\alpha}/} \mathbf{k}_{A}' - \frac{\mathbf{E}(1-\nu)}{2/\bar{\mathbf{g}}_{\alpha}//\bar{\mathbf{E}}_{\alpha}/} \mathbf{k}_{A}' - \frac{\mathbf{E}(1-\nu)}{2} \mathbf{k$ - K3 + FA $-\frac{\mathbf{E}'\left(\mathbf{k}_{\alpha}^{(m)}+\mathbf{v}\mathbf{k}_{\beta}^{(m)}\right)}{/\overline{\mathbf{E}}_{\alpha}/(m)}\frac{\partial}{\partial\alpha}+\mathbf{E}'\mathbf{k}_{\beta}^{(3)}}$ $\cdot\left(\mathbf{k}_{\beta}^{(m)}+\mathbf{v}\mathbf{k}_{\alpha}^{(m)}\right)$ $\frac{\mathbf{E}'(\mathbf{k}_{\beta}^{(n)}+\mathbf{j}\mathbf{k}_{\alpha}^{(n)})}{\overline{fg}_{\beta}}\frac{\partial}{\partial\beta}$ $+ \frac{D}{(g_{\beta\beta})^{2}} \frac{\partial^{4}}{\partial \beta^{4}} + \frac{D}{(g_{\alpha\beta})^{2}} \frac{\partial^{4}}{\partial \alpha^{2}} + \frac{2D}{g_{\alpha\alpha}} \frac{\partial^{4}}{\partial \beta^{2}} \frac{\partial^{4}}{\partial \beta^{2}} + \frac{2D}{g_{\alpha\alpha}} \frac{\partial^{4}}{\partial \beta^{2}} \frac{\partial^{4}}$ $\frac{d^{2}}{d\beta^{2}} - \left[\frac{D}{k_{\beta}k_{\beta}} + \frac{D}{\beta} \frac{1}{(1+\gamma)} + \frac{D}{D} \frac{(1+\gamma)}{(1+\gamma)} + \frac{D}{k_{\beta}} \frac{(1+\gamma)}{(1+\gamma)} + \frac{D}{k_{\beta}} \frac{(1+\gamma)}{(1+\gamma)} + \frac{D}{(1+\gamma)} + \frac{D}{$ Fz $-\frac{19D(\bar{a}-\bar{b})^{2}\sin^{2}\alpha\cos^{2}\alpha}{(g_{\alpha\alpha})^{4}} \frac{1}{2} - \left[\frac{3D}{k_{\beta}k_{\beta}}^{(g)}(\bar{a}-\bar{b})\right]$ $\cdot\sin^{\alpha}\cos^{\alpha} + \frac{D}{k_{\beta}k_{\beta}}^{(g)}(\bar{a}) + \frac{2D}{k_{\beta}}^{(g)}(\bar{a}-\bar{b})\sin^{\alpha}(1+\nu) - \frac{1}{2}$ $-\frac{D}{k_{3}^{(q)}(\ddot{a}-\dot{b})\cos(\alpha (1+\nu))} + \frac{D}{k_{3}^{(q)}(\ddot{a}-\dot{b})\sin(\alpha (1+\nu))} + \frac{D}{k_{3}^{(q)}(\ddot{a}-\dot{b})\cos(\alpha (1+\nu))} + \frac{D}{(g_{\alpha\alpha})^{5/2}} \frac{C}{(g_{\alpha\alpha})^{1/2}/\ddot{g}_{\beta}} + \frac{D}{(g_{\alpha\alpha})^{1/2}/\ddot{g}_{\beta}} + \frac{D}{(g_{\alpha\alpha})^{1/2}/\ddot{g}_{\beta}} + \frac{D}{(g_{\alpha\alpha})^{1/2}} + \frac{28(\ddot{a}-\ddot{b})^{3}\sin^{3}\cos^{3}c}{(g_{\alpha\alpha})^{1/2}} - \frac{D}{(g_{\alpha\alpha})^{1/2}} + \frac{D}{(g_{\alpha\alpha})^{1/2}} + \frac{E}{k_{\alpha}^{(n)}k_{\alpha}^{(n)}} + \frac{E}{k_{\alpha}^{(n)}k_{\alpha}^{(n)}}$

In the table [2.1] the unknown functions $u(\alpha,\beta)$, $v(\alpha,\beta)$ and $w(\alpha,\beta)$ are the headings. The first three columns include the linear differential operators. The fourth column contains the free terms of differential equations determined by components F_{α} , F_{β} and F_{z} of external boundary or surface load. In this case:

$$F_{\alpha} = F_{\beta} = 0$$
$$F_{z} = -p$$

where p is the fluid pressure.

2.3 BOUNDARY CONDITIONS:

To solve the system of equations, appropriate relations between the forces, moments, displacement or functions of these quantities at the edge or boundary of the shell must be specified.

Dressler [11] and Lee [12] derived the boundary conditions as follow:

(a) At the clamped edge or rigidly fixed edge of the tube (i.e. at $\beta=0$):

(i)
$$u = 0$$

(ii) $v = 0$
(iii) $w = 0$
(iv) $w_{10} = 0$

[2.6]



Fig. 2.4 The α , β -domain

(b) The tube can be considered symmetrical about $\alpha=0$ and $\alpha=\pi$; u being antisymmetrical and v and w symmetrical about these boundaries. From this symmetry condition, boundary conditions at $\alpha=0$ and $\alpha=\pi$ are given as

(i)
$$u = 0$$

(ii) $v_{\alpha} = 0$
(iii) $w_{\alpha} = 0$
(iv) $w_{\alpha\alpha\alpha} = 0$

[2.7]

(c) The Free end conditions:

By assuming the plug to be completely flexible the boundary conditions will be simplified. This simplification won't effect displacement appreciably, however, this may not yield correct stresses. Boundary conditions with these assumptions at $\beta = \beta_{L}$ are given below:

(i) $(g_{\beta\beta})^{-1/2} \cdot v_{,\beta} + v \cdot (g_{\alpha\alpha})^{-1/2} \cdot u_{,\alpha} - [k_{\beta}^{(n)} + v \cdot k_{\alpha}^{(n)}] \cdot w = R/(CIR.E')$

Where $R = p\pi a \overline{b}$

and $CIR = \pi \cdot [2 \cdot (\overline{a}^2 + \overline{b}^2)]^{1/2}$

(ii)
$$(g_{\beta\beta})^{-1} \cdot w_{\beta\beta}^{+\nu} (g_{\alpha\alpha})^{-1} \cdot w_{\alpha\alpha}^{-} [k_{\beta}^{(g)} \cdot (g_{\alpha\alpha})^{-1/2} + \nu (a^{2} - b^{2}) \cdot sin\alpha \cdot cos\alpha} (g_{\alpha\alpha})^{-2}] \cdot w_{\alpha} = 0.$$

(iii) $(g_{\beta\beta})^{-1/2} u_{\beta}^{+} (g_{\alpha\alpha})^{-1/2} \cdot v_{\alpha}^{+} k_{\beta}^{(g)} \cdot v = 0$

(iv)
$$(g_{\beta\beta})^{-3/2} w_{,\beta\beta\beta} + (2-\nu) \cdot (g_{\alpha\alpha})^{-1} \cdot (g_{\beta\beta})^{-1/2} \cdot w_{,\alpha\alpha\beta}$$

 $-(g_{\alpha\alpha},g_{\beta\beta})^{-1/2} \cdot [k_{\beta}^{(g)} \cdot (2\nu-1) + (2-\nu) \cdot (\overline{a}^2 - \overline{b}^2) \sin\alpha \cdot \cos\alpha \cdot (g_{\alpha\alpha})^{-3/2}] \cdot w_{,\beta\alpha} - 2(1-\nu) \cdot (g_{\beta\beta})^{-1/2} \cdot [k_{\beta}^{(g)} \cdot (\overline{a}^2 - \overline{b}^2) \cdot ($

2.4 DEFLECTION OF A POINT ON THE CENTRAL CROSS-SECTIONAL AXIS

Let the cosines of the angles between the moving, local x,y,z-axes and fixed X,Y,Z-axes be denoted by table 2.2

$$\begin{array}{c|cccc} X & Y & Z \\ \hline x & \ell_1 & m_1 & n_1 \\ y & \ell_2 & m_2 & n_2 \\ z & \ell_3 & m_3 & n_3 \end{array}$$

TABLE 2.2

These cosines are known functions of α and β for any point on the central shell of the bourdon tube.



Fig. 2.5 A typical cross-section of the bourdon tube

A typical cross-section at angle β is illustrated in Fig. 2.5.

Let u',v' and w' be components of the displacement of point A on the central shell in terms of the fixed co-ordinate system, while u,v and w are displacement components in terms of the moving system. The relation between the displacement vectors of the two systems is given as:

$$u' = \ell_{1}u + \ell_{2}v + \ell_{3}w$$

$$v' = m_{1}u + m_{2}v + m_{3}w$$

$$w' = n_{1}u + n_{2}v + n_{3}w$$
[2.9]

For $\alpha=90^{\circ}$ (i.e. at point E), the displacement vectors are (u_{90}, v_{90}, w_{90}) and $(u'_{90}, v'_{90}, w'_{90})$. Due to symmetry of the system the displacement vector at point L (centre point of

cross-section of the tube) can be given as $(u'_{90}, v'_{90}, 0)$.

The total deflection l of point L is given as:

$$\ell = (u_{90}^{\prime 2} + v_{90}^{\prime 2})^{1/2}$$
 [2.10]

At point E, the cosines of the angle between the moving axes and the fixed axes is denoted by table 2.3



TABLE 2.3

From equation (2.9) the relation between the local and fixed displacement vector is given as

 $u'_{90} = u_{90} \cos\beta + v_{90} \sin\beta$ $v'_{90} = u_{90} \sin\beta + v_{90} \cos\beta$ $w'_{90} = -w_{90}$ [2.11]

From equations [2.10] and [2.11] we get

$$\ell = (u_{90}^2 + v_{90}^2 + 4 \cdot u_{90} \cdot v_{90} \cdot \sin\beta \cdot \cos\beta)^{1/2}$$
 [2.12]

For the given angle β , let $\Delta \rho$ be the change in radius of curvature of central axis of cross-section of the bourdon tube. Kardos [8] give a relation

$$\frac{2}{\Delta \rho} = [(\beta . \cos\beta - \sin\beta)^2 + (\beta \sin\beta + \cos\beta - 1)^2]^{1/2}$$
[2.13]

The sensitivity of the bourdon tube is defined as

Sen. =
$$\frac{\Delta \rho}{\rho} \cdot \frac{E}{\rho}$$
 [2.14]

Equations [2.12], [2.13] and [2.14] can be combined to find the sensitivity of the bourdon tube for any value of β .

$$Sen = \frac{(u_{90}^{2} + v_{90}^{2} + 4.u_{90} \cdot v_{90} \cdot Sin\beta \cdot cos\beta)^{1/2}}{[(\beta \cdot cos\beta - sin\beta)^{2} + (\beta \cdot sin\beta + cos\beta - 1)^{2}]^{1/2}} \cdot \frac{E}{\rho \cdot p}$$
[2.15]

2.5 TRANSFORMATION TO DIMENSIONLESS VARIABLES:

It is more convenient to formulate the problem in dimensionless variables. The parameters of the bourdon tube in dimensionless form are given as:

$$RO = \rho/h'$$

$$A = \bar{a}/h'$$

$$B = \bar{b}/h'$$
[2.16]

Similarly metric coefficients and curvature can also be defined in dimensionless form as given below:

$$GB = |\overline{g}_{\beta}|/h' = RO + A.\cos\alpha$$

$$GA = |\overline{g}_{\alpha}|/h' = (A^{2}.\sin^{2}\alpha + \beta^{2}.\cos^{2}\alpha)^{1/2}$$

$$GBB = (GB)^{2}$$

$$GAA = (GA)^{2}$$

$$CBN = k_{\beta}^{(n)}.h' = B\cos\alpha/(GA.GB)$$

$$CBG = k_{\beta}^{(g)}.h' = A.Sin\alpha/(GA.GB)$$

$$(2.18]$$

$$CAN = k_{\alpha}^{(n)}.h' = A.B/(GAA)^{3/2}$$

The local displacement components can also be defined as

dimensionless quantities as:

$$U = u/h'$$

 $V = v/h'$ [2.19]
 $W = w/h'$

The given PDE's can also be written in dimensionless form, by dividing these equations by E'/h' (i.e. by $2E/(1-v^2)$) These equations in dimensionless form are:

(1)
$$\frac{(1-\nu)}{2GBB} \cdot U_{,\beta\beta} + \frac{1}{GAA} \cdot U_{,\alpha\alpha} + \left[-\frac{CBG}{GA} - \frac{(A^2-B^2).\sin\alpha.\cos\alpha}{(GAA)^2}\right] \cdot U_{,\alpha} - \left[CBG.CBG + \frac{\nu}{GA} \cdot \left[\frac{A.\cos\alpha}{GA,GB} - \frac{CBG.(A^2-B^2).\sin\alpha.\cos\alpha}{GAA}\right]\right] \cdot U_{,\alpha} - \left[CBG.CBG + \frac{\nu}{GA} \cdot \left[\frac{A.\cos\alpha}{GA,GB} - \frac{CBG.(A^2-B^2).\sin\alpha.\cos\alpha}{GAA}\right]\right] \cdot U_{,\alpha} + \frac{(1+\nu)}{2.GA.GB} \cdot V_{,\alpha\beta} - \frac{CBG.(\nu-3)}{2.GB} \cdot V_{,\beta} + \frac{CAN}{3(GAA)^{3/2}} \cdot W_{,\alpha\alpha\alpha} + \frac{CAN}{3.GA.GBB} \cdot W_{,\beta\beta\alpha} + \frac{2.CAN.CBG}{3.GBB} \cdot W_{,\beta\beta} - \frac{CAN}{3GAA} \cdot \left[CBG + \frac{3.(A^2-B^2).\sin\alpha.\cos\alpha}{(GAA)^{3/2}}\right] \cdot W_{,\alpha\alpha} - \left[\frac{CAN+\nu.CBN}{GA} + \frac{3.(A^2-B^2).(\cos^2\alpha-\sin^2\alpha)}{(GAA)^2} - \frac{CBG.(1+\nu).(A^2-B^2)}{(GAA)^{3/2}}\right] \cdot \frac{(A^2-B^2)^2.\sin^2\alpha.\cos^2\alpha}{(GAA)^{3/2}} + \frac{\nu.A.\cos\alpha}{(GAA)GB} \cdot \left[\frac{4(A^2-B^2)^2.\sin^2\alpha.\cos^2\alpha}{(GAA)^3} + \frac{\nu.A.\cos\alpha}{(GAA,GB}\right] \cdot W_{,\alpha} - \left[CBG.(CBN+\nu.CAN)-CBG.CAN - \frac{\nu.B.Sin\alpha}{GAA.GB} - \frac{(A^2-B^2).sin\alpha.\cos\alpha}{GA} \cdot \left[\frac{3.A.B}{(GAA)^{5/2}} + \frac{\nu.CBN}{GAA}\right] \right] \cdot W_{,\alpha} = 0$$

(2)
$$\frac{(1+\nu)}{2\text{GA.GB}}$$
. $U_{,\alpha\beta} + \frac{\text{CBG}}{2\text{GB}}$. $(\nu-3)$. $U_{,\beta} + \frac{1}{\text{GBB}}$. $V_{,\beta\beta} + \frac{(1-\nu)}{2\text{GAA}}$. $V_{,\alpha\alpha} - \frac{(1-\nu)}{2\text{GA}}$ [CBG + $(\frac{A^2-B^2}{B}) \cdot \frac{\sin\alpha \cdot \cos\alpha}{3/2}$]. $V_{,\alpha}$

$$\begin{array}{l} + \frac{(1-\nu)}{2} & \left[\frac{A.\cos\alpha}{GAA.GB} - CBG.CBG - \frac{CBG.(A^2-B^2).\sin\alpha.\cos\alpha}{(GAA)^{3/2}}\right] \cdot V \\ + \frac{CBN}{3(GBB)^{3/2}} & W,_{\beta\beta\beta} + \frac{CBN}{3GAA.GB} & W,_{\beta\alpha\alpha} - \frac{CBN}{3GA.GB} & [CBG + \frac{(A^2-B^2).\sin\alpha.\cos\alpha}{(GAA)^{3/2}}\right] W,_{\alpha\beta} - \left[\frac{CBN+\nu.CAN}{GB} + \frac{(1-\nu).CBN}{3GAA.GB}\right] \\ & \left[\frac{(A^2-B^2).\sin\alpha.\cos\alpha}{GA}\right] \cdot W,_{\alpha\beta} - \left[\frac{CBN+\nu.CAN}{GB}\right] + \frac{(1-\nu).CBN}{3GAA.GB} \\ & \left[\frac{CBG.(A^2-B^2).\sin\alpha.\cos\alpha}{GA} - \frac{A.Cos\alpha}{GB}\right] \right] W,_{\beta} \\ = 0 \\ & \left[2.21\right] \\ (3) - \frac{(CAN+\nu.CBN)}{GA} & U,_{\alpha} + CBG.(CBN+\nu.CAN), U - \frac{(CBN+\nu.CAN)}{GB} \\ & \cdot V,_{\beta} + \frac{1}{3(GBB)^2} W,_{\beta\beta\beta\beta} + \frac{1}{3(GAA)^2} \cdot W,_{\alpha\alpha\alpha\alpha} + \frac{2}{3GAA.GEB} \\ & W,_{\alpha\alpha\beta\beta} - \frac{2}{3GEB} \cdot \left[\frac{(A^2-B^2).\sin\alpha.\cos\alpha}{(GAA)^2} - \frac{CBG}{GA}\right] \\ & \cdot W,_{\alpha\beta\beta} - \frac{1}{3.(GAA)^{3/2}} \cdot \left[\frac{6.(A^2-B^2).\sin\alpha.\cos\alpha}{(GAA)^{3/2}} + 2CBG\right] \\ & \cdot W,_{\alpha\alpha\alpha} - \left[\frac{CBG.(3-\nu).(A^2-B^2)}{3.(GAA)^{3/2}.GBB} + \sin\alpha.\cos\alpha - \frac{2.CBG.CBG}{3.GBB} \\ & \cdot (2+3\nu) - \frac{(3-\nu).A.\cos\alpha}{3.GAA.(GBB)^{3/2}}\right] \cdot W,_{\beta\beta} - \left[\frac{CBG.CBG}{3.GAA} + \frac{A.Cos\alpha}{3.(GAA)^2.GB} - \frac{CBG.(A^2-B^2).\sin\alpha.\cos\alpha}{3.(GAA)^{5/2}} + \frac{4.(A^2-B^2).(\cos^2\alpha-\sin^2\alpha)}{3.(GAA)^3} - \frac{19.(A^2-B^2)^2.\sin^2\alpha.\cos^2\alpha}{3.(GAA)^4} \\ & \cdot W,_{\alpha\alpha} - \left[-\frac{CBG.CBG}{(GAA)^2} - \frac{(2BA.(A^2-B^2).\sin\alpha.\cos\alpha}{3.(GAA)^{6/2}} + \frac{4.(A^2-B^2).(\cos^2\alpha-\sin^2\alpha)}{3.(GAA)^3} - \frac{19.(A^2-B^2)^2.\sin^2\alpha.\cos^2\alpha}{3.(GAA)^4} \\ & \cdot W,_{\alpha\alpha} - \left[-\frac{CBG.CBG}{(GAA)^2} - (A^2-B^2).\sin\alpha.\cos\alpha + \frac{(CBG)^3}{3.(GA)} + \frac{2.CBG.(A^2-B^2).\sin\alpha.\cos\alpha}{3.(GAA)^{5/2}} - \frac{2BA.(CBB)^{3/2}}{3.(GAA)^{4}} \\ & \cdot W,_{\alpha\alpha} - \left[-\frac{CBG.CBG}{(GAA)^2} - (A^2-B^2).\sin\alpha.\cos\alpha + \frac{(CBG)^3}{3.(GAA)^4} + \frac{2.CBG.(A^2-B^2).\sin\alpha.\cos\alpha}{3.(GAA)^{5/2}} - \frac{2BA.(CBC)^3}{3.(GAA)^{5/2}} + \frac{2.CBG.(A^2-B^2).\sin\alpha.\cos\alpha}{3.(GAA)^{5/2}} - \frac{CBG.(A^2-B^2).\cos^2\alpha.(3+5\nu)}{3.(GAA)^{5/2}} \\ \end{array} \right]$$

$$+ \frac{CBG.A.Cos\alpha}{3.(GAA)^{3/2}.GB} \cdot (1+v) + \frac{CBG.(A^2-B^2)^2.sin^2\alpha.cos^2\alpha}{3.(GAA)^{7/2}}$$

$$\cdot (9+5v) + \frac{1}{3GA} \cdot \left[-\frac{4.(A^2-B^2)}{(GAA)^{5/2}} \cdot Sin\alpha.Cos\alpha - \frac{13.(A^2-B^2)^2}{(GAA)^{7/2}}\right]$$

$$\cdot Sin\alpha.Cos\alpha.(Cos^2\alpha - Sin^2\alpha) + \frac{28.(A^2-B^2)^3}{(GAA)^{9/2}} \cdot Sin^3\alpha.Cos^3\alpha]$$

$$- \frac{CBG.v}{3.(GAA)^{3/2}} \cdot W_{,\alpha} + \left[CAN.CAN + CBN.CBN + 2.v.CAN.CBN\right]$$

$$\cdot W = p.(1-v^2)/2E \qquad [2.22]$$

The boundary conditions are written in dimensionless form and given below

Along
$$\beta = 0$$
;
(i) $U = 0$
(ii) $V = 0$
(iii) $W = 0$
(iv) $W_{,\beta} = 0$
Along $\alpha = 0^{\circ}$ and $\alpha = 180^{\circ}$;
(i) $U = 0$
(ii) $V_{,\alpha} = 0$
(iii) $V_{,\alpha} = 0$
(iv) $W_{,\alpha\alpha\alpha} = 0$
Along $\beta = \beta_{L}$;
(i) $\frac{1}{GE} \cdot V_{,\beta} + \frac{v}{GA} \cdot U_{,\alpha} - CBG \cdot U - (CBN + v \cdot CAN) \cdot W$
 $= \frac{p \cdot (1 - v^{2}) \cdot A \cdot B}{2E[2(A^{2} + B^{2})]^{1/2}}$

(ii)
$$\frac{1}{GBB} \cdot W_{,\beta\beta} + \frac{v}{GAA} \cdot W_{,\alpha\alpha} - \left[\frac{CBG}{GA} + \frac{v \cdot (A^2 - B^2) \cdot Sin\alpha \cdot Cos\alpha}{(GAA)^2}\right] \cdot W_{,\alpha} = 0$$

(iii)
$$\frac{1}{GB} \cdot U_{,\beta} + \frac{1}{GA} \cdot V_{,\alpha} + CBG \cdot V = 0$$

(iv)
$$\frac{1}{(GBB)^{3/2}} \cdot W_{,\beta\beta\beta} + \frac{(2 - v)}{GAA \cdot GB} \cdot W_{,\alpha\alpha\beta} - \frac{1}{GA \cdot GB} \cdot [CBG \cdot (2v - 1)] + \frac{(2 - v) \cdot (A^2 - B^2) \cdot Sin\alpha \cdot Cos\alpha}{(GAA)^{3/2}} \cdot W_{,\alpha\beta} - \frac{2 \cdot (1 - v)}{GB} \cdot [CBG \cdot (2v - 1)] + \frac{(A^2 - B^2) \cdot Sin\alpha \cdot Cos\alpha}{(GAA)^{3/2}} - \frac{A \cdot Cos\alpha}{GAA \cdot GB} - CBG \cdot CBG \cdot CBG \cdot W_{,\beta}$$

$$= 0 \qquad [2.25]$$

The deflection of a point of central axis of cross-section of bourdon tube can be expressed as a dimensionless variable as below:

$$AL = \ell/h' \qquad [2.26]$$

From equations [2.10], [2.19] and [2.26] we get

AL =
$$(U_{90}^2 + V_{90}^2 + 4 \cdot U_{90} \cdot V_{90} \cdot \text{Sin}\beta \cdot \text{Cos}\beta)^{1/2}$$
 [2.27]

CHAPTER 3

COMPUTATIONS OF THE DEFORMATION OF A BOURDON TUBE

3.1 FORMATION OF A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS:

The deformation of a bourdon tube with an elliptical cross-section can be obtained by solving the three partial differential equations (2.20), (2.21), (2.22) and the sixteen boundary conditions (2.23), (2.24) and (2.25).

The best available technique, to solve PDE's, is finite difference apprximations as given in detail in Appendix A. With the help of this method PDE's can be transformed to a set of simultaneous linear equations.

The domain over which PDE's hold good consists of an area over which α varies from 0° to 180° and β varies from 0° to β_L . This domain can be covered with rectangular spacings by dividing the α -side into n intervals of width 'h' each and dividing the β -side into m intervals of width 'k' each.

As the PDE's have partial derivatives, which are fourth order in terms of W, so two more imaginary lines are drawn around the domain as shown in Fig. 3.1 by dashed lines.

At all the points in the domain, the three given PDE's hold good and on the boundaries the sixteen given boundary conditions hold good. The following euqations are the given PDE's and BC's transformed into linear equations by using finite difference approximation, simplified and grouped.





(1)
$$U_{0,-1} \cdot (A1) + U_{-1,0} \cdot (A2-A3) + U_{0,0} \cdot (-2.A1-2.A2+A4)$$

+ $U_{1,0} \cdot (A2+A3) + U_{0,1} \cdot (A1) + V_{-1,-1} \cdot (B1) + V_{0,-1} \cdot (-B2)$
+ $V_{1,-1} \cdot (-B1) + V_{-1,1} \cdot (-B1) + V_{0,1} \cdot (B2) + V_{1,1} \cdot (B1)$
+ $W_{0,-2} \cdot (-F3) + W_{-1,-1} \cdot (-F2) + W_{0,-1} \cdot (16.F3) + W_{1,-1} \cdot (F2)$
+ $W_{-2,0} \cdot (-F1-F4+F5) + W_{-1,0} \cdot (2.F1+2.F2+16.F4-8.F5)$
+ $W_{0,0} \cdot (-30.F3-30.F4+F6) + W_{1,0} \cdot (-2.F1-2.F2+16.F4+8.F5)$
+ $W_{2,0} \cdot (F1-F4-F5) + W_{-1,1} \cdot (-F2) + W_{0,1} \cdot (16.F3)$
+ $W_{1,1} \cdot (F2) + W_{0,2} \cdot (-F3) = 0$ [3.1]
(2) $U_{-1,-1} \cdot (B1) + U_{0,-1} \cdot (B2) + U_{1,-1} \cdot (-B1) + U_{-1,1} (-B1) + U_{0,1} \cdot (-B2)$

$$-1,-1 = 0,-1 = 1,-1 = -1,1 = 0,1$$

$$+ U_{1,1} \cdot (B1) + V_{0,-1} \cdot (D1) + V_{-1,0} \cdot (D2 - D3) + V_{0,0} \cdot (-2D1 - 2.D2)$$

$$+D4) + V_{1,0} \cdot (D2 + D3) + V_{0,1} \cdot (D1) + W_{0,-2} \cdot (-G1 + G4)$$

$$+ W_{-1,-1} \cdot (-G2 + G3) + W_{0,-1} \cdot (2.G1 + 2.G2 - 8.G4) + W_{1,-1} \cdot (-G2)$$

$$-G3) + W_{-1,1} \cdot (G2 - G3) + W_{0,1} \cdot (-2.G1 - 2.G2 + 8.G4)$$

$$+ W_{1,1} \cdot (G2 + G3) + W_{0,2} \cdot (G1 - G4) = 0 \qquad [3.2]$$

(3)
$$U_{-1,0} \cdot (-C1) + U_{0,0} \cdot (C2) + U_{1,0} \cdot (C1) + V_{0,-1} \cdot (-E1) + V_{0,1} \cdot (E1)$$

+ $W_{0,-2} \cdot (H1-H6) + W_{-1,-1} \cdot (H3-H4) + W_{0,-1} \cdot (-4.H1-2H3)$
+ $H6.H6) + W_{1,-1} \cdot (H3+H4) + W_{-2,0} \cdot (H2-H5-H7+H8)$

+ $W_{-1,0}$ (-4.H2-2.H3+2.H4+2.H5+16.H7-8.H8) + $W_{0,0}$ (6.H1 +6.H2 + 4.H3 - 30.H6 - 30.H7 + H9) + $W_{1,0}$ (-4.H2-2.H3 -2.H4-2.H5+16.H7+8.H8) + $W_{2,0}$ (H2+H5-H7-H8) + $W_{-1,1}$ (H3-H4) + $W_{0,1}$ (-4.H1-2.H3+16.H6) + $W_{1,1}$ (H3+H4)+ $W_{0,2}$ (H1-H6) = p. (1- v^2)/2E [3.3]

BC's along
$$\beta=0^{\circ}$$
 are:
(i) $U_{0,0} = 0$
(ii) $V_{0,0} = 0$
(iii) $W_{0,1} = W_{0,-1}$
BC's along $\alpha=0$ and $\alpha=180^{\circ}$ are:
(i) $U_{0,0} = 0$
(ii) $V_{-1,0} = V_{1,0}$
(iii) $W_{-1,0} = W_{1,0}$
(iv) $W_{-2,0} = W_{2,0}$
BC's along $\beta=\beta_{L}$ are
(i) $U_{-1,0} \cdot [-02] + U_{0,0}[03] + U_{1,0} \cdot [02] + V_{0,-1} \cdot [-01] + V_{0,1} \cdot [01]$
 $+ W_{0,0} \cdot [04] = p \cdot (1-v^{2}) \cdot A \cdot B / [2 \cdot E \cdot (2(A^{2}+B^{2}))^{2}]$
(ii) $W_{0,-2} \cdot [-P1] + W_{0,-1} \cdot [16 \cdot P1] + W_{-2,0} \cdot [-P2+P3] + W_{-1,0} (16 \cdot P2$
 $-8 \cdot P3] + W_{0,0} \cdot [-30 \cdot P1 - 30 \cdot P2] + W_{1,0} \cdot [16 \cdot P2 + 8P3]$
 $+ W_{2,0} \cdot [-P2 - P3] + W_{0,1} \cdot [16 \cdot P1] + W_{0,2} \cdot [-P1] = 0$

(iii)
$$U_{0,-1} \cdot [-R1] + U_{0,1} \cdot [R1] + V_{-1,0} \cdot [-R2] + V_{0,0} \cdot [R3] + V_{1,0} \cdot [R2] = 0$$

(iv) $W_{0,-2} \cdot [-Q1+Q4] - W_{-1,-1} \cdot [Q2-Q3] + W_{0,-1} \cdot [2.Q1+2.Q2-8.Q4] + W_{1,-1} \cdot [-Q2-Q3] + W_{-1,1} \cdot [Q2-Q3] + W_{0,1} \cdot [-2.Q1-2.Q2+8.Q4] + W_{1,-1} \cdot [-Q2-Q3] + W_{-1,1} \cdot [Q2-Q3] + W_{0,1} \cdot [-2.Q1-2.Q2+8.Q4] + W_{1,-1} \cdot [-Q2-Q3] + W_{-1,1} \cdot [Q2-Q3] + W_{0,1} \cdot [-2.Q1-2.Q2+8.Q4]$
(3.6]

$$+$$
 1, -1 · $(2^{2} + 2^{3}) + (0, 2^{2} + 2^{4}) = 0$ [3.0]

In the above equations, Al, Bl etc. are coefficients of the PDEs and the BCs and are defined below:

$$\begin{aligned} \text{Al} &= \frac{(1-\nu)}{2\text{GBB}} \cdot \frac{1}{k^2} \\ \text{A2} &= \frac{1}{\text{GAA} \cdot h^2} \\ \text{A3} &= \frac{1}{2\text{h}} \cdot \left[-\frac{\text{CBG}}{\text{GA}} - \frac{(A^2 - B^2) \cdot \text{Sin}\alpha \cdot \text{Cos}\alpha}{(\text{GAA})^2} \right] \\ \text{A4} &= -\left[\text{CBG} \cdot \text{CBG} + \frac{\nu}{\text{GA}} \cdot \left(\frac{A \cdot \text{Cos}\alpha}{\text{GA} \cdot \text{GB}} - \frac{\text{CBG} \cdot (A^2 - B^2) \cdot \text{Sin}\alpha \cdot \text{Cos}\alpha}{\text{GAA}} \right) \right] \\ \text{B1} &= \frac{1}{2 \cdot \text{GA} \cdot \text{GB}} \cdot \frac{1}{4\text{h} \cdot \text{k}} \\ \text{B2} &= -\frac{\text{CBG} \cdot (\nu - 3)}{2 \cdot \text{GB}} \cdot \frac{1}{2 \cdot \text{k}} \\ \text{C1} &= -\frac{\text{CAN} + \nu \cdot \text{CBN}}{\text{GA}} \cdot \frac{1}{2 \cdot \text{h}} \\ \text{C2} &= \text{CBG} \cdot (\text{CBN} + \nu \cdot \text{CAN}) \\ \text{D1} &= \frac{1}{\text{GBE}} \cdot \frac{1}{k^2} \\ \text{D2} &= \frac{(1 - \nu)}{2 \cdot \text{GA}} \cdot \frac{1}{h^2} \\ \text{D3} &= -\frac{(1 - \nu)}{2 \cdot \text{GA}} \cdot \left[\text{CBG} + \frac{(A^2 - B^2) \cdot \text{Sin}\alpha \cdot \text{Cos}\alpha}{(\text{GAA})^{3/2}} \right] \cdot \frac{1}{2 \cdot \text{h}} \end{aligned}$$

$$D4 = \frac{(1-v)}{2} \cdot \left[\frac{A \cdot \cos \alpha}{GAA \cdot GB} - CBG \cdot CBG - \frac{CBG \cdot (A^2 - B^2) \cdot Sin\alpha \cdot Cos\alpha}{(GAA)^{3/2}}\right]$$

$$E1 = -\frac{CBN + v \cdot CAN}{GB} \cdot \frac{1}{2 \cdot k}$$

$$F1 = \frac{CAN}{3 \cdot (GAA)^{3/2}} \cdot \frac{1}{2 \cdot h^3}$$

$$F2 = \frac{CAN}{3 \cdot (GAA)^{3/2}} \cdot \frac{1}{2 \cdot h \cdot k^2}$$

$$F3 = \frac{2 \cdot CAN \cdot CBG}{3 \cdot GBB} \cdot \frac{1}{2 \cdot h \cdot k^2}$$

$$F4 = -\frac{CAN}{3 \cdot GAA} \cdot \left[CBG + \frac{3 \cdot (A^2 - B^2) \cdot Sin\alpha \cdot Cos\alpha}{(GAA)^{3/2}}\right] \cdot \frac{1}{12h^2}$$

$$F5 = -\left[\frac{1}{GA} \cdot (CAN + v \cdot CBN) + \frac{CAN}{3 \cdot GA} \cdot \left[\frac{(A^2 - B^2) \cdot (Cos^2\alpha - Sin^2\alpha)}{(GAA)^2} - \frac{CBG \cdot (1 + v) \cdot (A^2 - B^2) \cdot Sin\alpha \cdot Cos\alpha}{(GAA)^{3/2}} + CBG \cdot CBG} - \frac{4 \cdot (A^2 - B^2) \cdot Sin\alpha \cdot Cos\alpha}{(GAA)^{3/2}} + \frac{v \cdot A \cdot Cos\alpha}{(GAA)^{3/2}}\right] \cdot \frac{1}{12h}$$

$$F6 = -\left[CBG \cdot (CEN + v \cdot CAN) - CBG \cdot CAN - \frac{v \cdot B \cdot Sin\alpha}{GAA \cdot GB} - \frac{(A^2 - B^2) \cdot Sin\alpha \cdot Cos\alpha}{GA} \cdot \left[\frac{3 \cdot A \cdot B}{(GAA)^{5/2}} + \frac{v \cdot CBN}{GAA}\right]\right]$$

$$G1 = \frac{CBN}{3 \cdot (GBB)^{3/2}} \cdot \frac{1}{2k^3}$$

$$C2 = \frac{CBN}{3 \cdot GAA \cdot GB} \cdot \frac{1}{2h^2 \cdot k}$$

$$G3 = -\frac{CBN}{3.GA.GB} \cdot [CBG + \frac{(A^2 - B^2.Sin\alpha.Cos\alpha}{(GAA)^{3/2}}] \cdot \frac{1}{4hk}$$

$$\begin{aligned} \mathsf{G4} &= - \left[\frac{\mathsf{CBN} + \upsilon \cdot \mathsf{CAN}}{\mathsf{GB}} + \frac{(1 - \upsilon) \cdot \mathsf{CBN}}{3 \cdot \mathsf{GAA}, \mathsf{GB}} \cdot \left[\frac{\mathsf{CBG} \cdot (A^2 - B^2)}{\mathsf{GA}} \right] \cdot \mathsf{Sina} \cdot \mathsf{Cosa} \right. \\ &\quad - \frac{\mathsf{A} \cdot \mathsf{Cosa}}{\mathsf{GB}} \mathsf{I} \right] \cdot \frac{1}{12\mathsf{K}} \\ \mathsf{H1} &= \frac{1}{3 \cdot (\mathsf{GBB})^2 \cdot \mathsf{k}^4} \\ \mathsf{H2} &= \frac{1}{3 \cdot (\mathsf{GAA})^2 \cdot \mathsf{h}^4} \\ \mathsf{H3} &= \frac{2}{3 \cdot \mathsf{GBB}} \cdot \left[\frac{(A^2 - B^2) \cdot \mathsf{Sina} \cdot \mathsf{Cosa}}{(\mathsf{GAA})^2} - \frac{\mathsf{CBG}}{\mathsf{GA}} \right] \cdot \frac{1}{2\mathsf{h} \cdot \mathsf{k}^2} \\ \mathsf{H4} &= -\frac{2}{3 \cdot \mathsf{GBB}} \cdot \left[\frac{(A^2 - B^2) \cdot \mathsf{Sina} \cdot \mathsf{Cosa}}{(\mathsf{GAA})^2} - \frac{\mathsf{CBG}}{\mathsf{GA}} \right] \cdot \frac{1}{2\mathsf{h} \cdot \mathsf{k}^2} \\ \mathsf{H5} &= -\frac{1}{3 \cdot (\mathsf{GAA})^{3/2}} \cdot \left[\frac{6 \cdot (A^2 - B^2) \cdot \mathsf{Sina} \cdot \mathsf{Cosa}}{(\mathsf{GAA})^{3/2} - \mathsf{GBG}} + 2 \cdot \mathsf{CBG} \right] \cdot \frac{1}{2\mathsf{h}^3} \\ \mathsf{H6} &= - \frac{\mathsf{f} \frac{\mathsf{CBG} \cdot (3 - \upsilon) \cdot (A^2 - B^2)}{3 \cdot (\mathsf{GAA})^{3/2} \cdot \mathsf{GBB}} \cdot \mathsf{Sina} \cdot \mathsf{Cosa} - \frac{2 \cdot \mathsf{CBG} \cdot \mathsf{CBG}}{3 \cdot \mathsf{GBB}} \\ \cdot (2 + 3 \upsilon) - \frac{(3 - \upsilon) \cdot \mathsf{A} \cdot \mathsf{Cosa}}{3 \cdot \mathsf{GAA}} \cdot (\mathsf{GBB})^{3/2} \right] \cdot \frac{1}{12\mathsf{k}^2} \\ \mathsf{H7} &= - \left[\frac{\mathsf{CBG} \cdot \mathsf{CBG}}{3 \cdot \mathsf{GAA}} + \frac{\mathsf{A} \cdot \mathsf{Cosa} \cdot (1 + \upsilon)}{3 \cdot (\mathsf{GAA})^2 \cdot \mathsf{GB}} - \frac{\mathsf{CBG} \cdot (A^2 - B^2) \cdot \mathsf{Sina} \cdot \mathsf{Cosa} \cdot (7 + \upsilon)}{3 \cdot (\mathsf{GAA})^{5/2}} \\ &+ \frac{4 \cdot (A^2 - B^2) \cdot (\mathsf{cos}^2 \alpha - \mathsf{Sin}^2 \alpha)}{3 \cdot (\mathsf{GAA})^3} - \frac{\mathsf{19} \cdot (A^2 - B^2)^2 \cdot \mathsf{Sin}^2 \alpha \cdot \mathsf{Cos}^2}{3 \cdot (\mathsf{GAA})^4} \\ \mathsf{I} \cdot \frac{1}{12\mathsf{h}^2} \\ \mathsf{H8} &= - \left[-\frac{\mathsf{CBG} \cdot (\mathsf{CBG} \cdot (A^2 - B^2) \cdot \mathsf{Sina} \cdot \mathsf{Cosa}}{(\mathsf{GAA})^2} - \frac{\mathsf{CBG} \cdot (A^2 - B^2)}{3 \cdot (\mathsf{GAA})^{5/2}} - \mathsf{Cos}^2 \alpha \\ \cdot (3 + 5 \cdot \upsilon) + \frac{\mathsf{CBG} \cdot \mathsf{A} \cdot \mathsf{Cosa} \cdot (1 + \upsilon)}{3 \cdot (\mathsf{GAA})^{3/2} \cdot \mathsf{GB}} + \frac{\mathsf{CBG} \cdot (\mathsf{A}^2 - \mathsf{B}^2)^2}{3 \cdot (\mathsf{GAA})^{5/2}} \cdot \mathsf{Cos}^2 \alpha \\ \cdot (3 + 5 \cdot \upsilon) + \frac{\mathsf{CBG} \cdot \mathsf{A} \cdot \mathsf{Cosa} \cdot (1 + \upsilon)}{3 \cdot (\mathsf{GAA})^{3/2} \cdot \mathsf{GB}} + \frac{\mathsf{CBG} \cdot (\mathsf{A}^2 - \mathsf{B}^2)^2}{3 \cdot (\mathsf{GAA})^{7/2}} \cdot \mathsf{Cos}^2 \alpha \\ \cdot (3 + 5 \cdot \upsilon) + \frac{\mathsf{CBG} \cdot \mathsf{A} \cdot \mathsf{Cosa} \cdot (1 + \upsilon)}{3 \cdot (\mathsf{GAA})^{3/2} \cdot \mathsf{GB}} + \frac{\mathsf{CBG} \cdot (\mathsf{A}^2 - \mathsf{B}^2)^2}{3 \cdot (\mathsf{GAA})^{7/2}} \cdot \mathsf{Cos}^2 \alpha \\ \cdot \mathsf{Cas}^2 + \mathsf{CBS} \cdot \mathsf{Cos}^2 + \mathsf{CBS} \cdot \mathsf{Cos}^2 + \mathsf{CBS} \cdot \mathsf{COs}^2 + \mathsf{CS}^2 + \mathsf{$$

$$\begin{aligned} \sin^{2} \alpha \cdot \cos^{2} \alpha \cdot (9+5.\nu) + \frac{1}{3 \cdot GA} \cdot \left[-\frac{4 \cdot (A^{2}-B^{2}) \cdot \sin \alpha \cdot \cos \alpha}{(GAA)^{5/2}} \right] \\ &- \frac{13 \cdot (A^{2}-B^{2})^{2}}{(GAA)^{7/2}} \cdot \sin \alpha \cdot \cos \alpha \cdot (\cos^{2} \alpha - \sin^{2} \alpha) + \frac{28 \cdot (A^{2}-B^{2})^{3}}{(GAA)^{9/2}} \\ &\cdot \sin^{3} \alpha \cdot \cos^{3} \alpha] - \frac{\nu \cdot CBG}{3 \cdot (GAA)^{3/2}} \right] \cdot \frac{1}{12h} \end{aligned}$$

$$\begin{aligned} &H9 = \left[CAN \cdot CAN + CBN \cdot CBM + 2\nu \cdot CAN \cdot CBN \right] \\ &01 = \frac{1}{GB \cdot 2k} \\ &02 = \frac{\nu}{GA \cdot 2h} \\ &03 = -CEG \\ &04 = - (CEN + \nu \cdot CAN) \\ &P1 = \frac{1}{GEB} \cdot \frac{1}{12h^{2}} \\ &P2 = \frac{\nu}{GAA} \cdot \frac{1}{12h^{2}} \\ &P3 = -\left[\frac{CBG}{GA} + \frac{\nu \cdot (A^{2}-B^{2}) \cdot \sin \alpha \cdot \cos \alpha}{(GAA)^{2}} \right] \cdot \frac{1}{12h} \\ &R1 = \frac{1}{GB} \cdot \frac{1}{2k} \\ &R2 = \frac{1}{GA} \cdot \frac{1}{2h} \\ &R3 = CBG \\ &Q1 = \frac{1}{(GE)^{3}} \cdot \frac{1}{2k^{3}} \\ &Q2 = \left(\frac{2(-\nu)}{GAA \cdot GB} - \frac{1}{2h^{2} \cdot k} \right) \\ &Q3 = - \frac{1}{GA \cdot GB} \cdot \left[CBG \cdot (2 \cdot \nu - 1) + \frac{(2 - \nu) \cdot (A^{2} - B^{2})}{(GAA)^{3/2}} \cdot \sin \alpha \cdot \cos \alpha \right] \cdot \frac{1}{4h \cdot k} \\ &Q4 = - \frac{2 \cdot (1 - \nu)}{GB} \cdot \left[\frac{CBG \cdot (A^{2} - B^{2}) \cdot \sin \alpha \cdot \cos \alpha}{(GAA)^{3/2}} - \frac{A \cdot \cos \alpha}{CAA \cdot GB} - CBG \cdot CEG \right] \cdot \frac{1}{12k} \end{aligned}$$

It can be observed that these coefficients are functions of the parameters of the bourdon tube, grid widths h and k and value of α . To compute these coefficients, for the given bourdon tube, h, k and α , subroutine 'COE' is written and described in detail in Appendix D.

From the equations [3.5] (i.e. BC's along $\alpha=0$ and $\alpha=180^{\circ}$) it can be noted that these boundaries serve as a mirror for W and V (i.e. for a given B, values of V and W on points inside the domain are the same as those on the equidistant points outside the domain lying on imaginary lines). Similarly from the equations [3.4] (i.e. BC's long $\beta=0$], it is observed that this boundary (i.e. $\beta=0$) serves as a mirror for W. Also values of all the deformations (i.e. U, V and W) are zero on this boundary. While studying the equations [3.6] (i.e. at BC's along $\beta=\beta_L$), it can be noted that these are complicated and one has to solve for the values of U, V and W on the first imaginary line after $\beta=\beta_L$ and for values of W on the second imaginary line as well. (because PDE's contain partial derivatives, which are third and fourth order in terms of W).

So now the domain over which unknowns are to be solved has been increased in area. The domain over which the values of U and V are to be solved consists of α varying from 0° to 180° and β varying from k to $(\beta_L + k)$, and that over which the values of W are to be solved consists of α varying from 0° to 180° and β varying from k to $(\beta_L + 2k)$. Therefore the total number of unknowns are [2(n+1).(m+1)+(n+1).(m+2)] or (n+1).(3m+4).
At all the points in the given domain except at $\beta=0$, all the three difference equations [3.1], [3.2] and [3.3] are written, and at boundary $\beta=\beta_L$ all the four difference equations [3.6] are written. This produces (n+1)(3m+4) linear equations to be solved for the same number of unknowns.

This system of simultaneous linear equations can be written in the usual from of

$$Ax = b$$
 [3.7]

Where A is a matrix of order (n+1). (3m+4) and b and x are vectors of length (n+1). (3m+4).

3.2 ARRANGEMENT OF THE SYSTEM IN BLOCKS

If this sytem is solved directly (i.e. by inverting the matrix). The storage required in the computer is more than the square of the order of the matrix A. This will be very large for a reasonable grid size (e.g. for n=18; m=30, the order of the matrix A is 1786). Again if this problem of storage is solved by storing the matrix in the discs or the magnetic tapes the number of multiplications and divisions is the cube of the order of the matrix A, and also a lot of time is required in transferring data from discs or tapes to the computer memory, which will require much computer time which is not advisable for any practical purpose (e.g. for the matrix A of the order 1786, it will take about 40 computer hours to solve it).

The best available technique to solve such a large but sparse matrix is the block iteration method, which is discussed in detail in Appendix B. To utilise the method to its full extent, subsystems are made in a way, which minimize the storage requirements. This is described below.

The information, that the coefficients of PDE's, thereby those of difference equations are functions of α only (i.e. they are independent of β), is of maximum use here. Because if a subsystem is written by writing a difference equation with any value of β and α varying from 0° to 180° and another subsystem is written by writing the same difference equation with any other value of β and α varying from 0° to 180°; the blocks i.e. submatrices involved will be identical. So if one of the many identical blocks is stored in the computer memory, others need not be stored.

It can be observed that many blocks contain the elements, all of which have values equal to zero. These blocks need not be stored. All the rest of the blocks contain non-zero elements lying on a few diagonals. These blocks can be stored as rectangular matrices with their rows being non-zero diagonals of the original blocks (e.g. say a (19x19) block has all the elements which have values equal to zero except the elements on three diagonals, this block can be stored as a (3x19) matrix).

Also these subsystems must be written in a fashion so that the diagonal blocks must have values which are bigger in magnitude than those in other blocks.

A15 A14 A16 0 A13 1 A11 A12 +A16 -A13 0 A13 A14 A15 A14 A16 2 A12 A11 A12 -A13 0 A13 Al6 Al4 Al5 Al4 Al6 3 Al2 All Al2 . -A13 0 A13 Al6 Al4 Al5 Al4 Al6 A12 A11 A12 m--1 -A13 0 A13 A16 A14 A15 A14 A16 Al2 All Al2 m A45 0 0 0 0 0 m+1 A46 0 A44 0 A20 A22 A21 -A23 A25 A23 m+2-A20 0 A20 A21 A22 A21 -A25 0 A25 A23 ·m+3 -A20 0 A20 A21 A22 A21 -A23 -A25 0 A25 A23 m+4 . -A20 0 A20 A21 A22 A21 -A23-A25 0 A23 A25 2m-A20 0 A21 A22 A21 A20 -A23 -A25 0 A23 A25 2m+1 -A55 0 A55 A54 0 A56 0 0 2m+2A31 0 -A32 A36 A34 A35 2m+3 A31 A32 0 -A32 A34 A33 A34 A35 2m+4 A31 A32 0 -A32 A35 A34 A33 A34 A35 2m+5 . . A31 A32 0 -A32 A35 A34 A33 A34 A35 3m+1 A31 A32 0 -A32 A35 A34 A33 A34 A35 3m+20 0 0 A67 A66 A65 A66 A67 3m+3-0 . 0 -A77-A76 0 A76 A77 0 3m+4 ω Fig. 3.1 Matrix A

VECTOR	X Number V	ECTO	DR b
	1	[0]	
U2	2	0	
•	•		
	•		
Um	m	0	
U _{m+1}	m+1	0	
v ₁	m+2	0	2
v ₂	m+ 3	0	1 11 11 11 11 11 11 11 11 11 11 11 11 1
•		•	
	•	•	
v _m	2m+1	0	
v _{m+1}	2m+2	В4	
Wl	2m+3	B3	
W2	2m+4	В3	
•	•	•	
Wm	3m+2	B3	
Wm+1	3m+3	0	
W _{m+2}	3m+4	(0)	X

Fig. 3.3

Keeping all this in mind the system of simultaneous linear equations is divided in a fashion as illustrated by matrix A in fig. [3.2], vector x and vector b in fig. 3.3.

These are obtained by writing the subsystems for equation [3.1] in a way as discussed before for β varying from k to β_L . Thereby forming m subsystems of (n+1) equations each. The (m+1)th subsystem is formed by using Equation [3.6(iii)] at $\beta = \beta_L$. Similarly m more subsystems are formed for equation [3.2] and the (2m+2)th subsystem is formed by using equation [3.6(i)]. Again another m subsystems are formed for equation [3.3]. Then (3m+3)th and (3m+4)th subsystems are written for equations [3.6(ii)] and [3.6(iv)] respectively. All these subsystems consist of (n+1)linear equations. Hence all the submatrices and subvectors are of order (n+1). All summatrices required in matrix $\overline{\overline{\lambda}}$ (i.e. All, Al2 etc) and subvectors required in vector \overline{b} (i.e. B3 and B4) are defined and computed in subroutine 'MATRIX' described in detail in Appendix D.

3.3 ITERATION PROCESS:

Once all the submatrices and subvectors are defined the iteration can be started after assuming initial values $U_i^{(0)}$, $V_i^{(0)}$ and $W_i^{(0)}$ for the deformations.

Intermediate deformations UI, VI and WI are defined after (k+1)th iteration as below:

$$\begin{array}{l} (i) \quad \mathrm{UI}_{1}^{(k+1)} = \mathrm{All}^{-1} \cdot [-\mathrm{Al2} \cdot \mathrm{U}_{2}^{(k)} - \mathrm{Al3} \cdot \mathrm{V}_{2}^{(k)} - \mathrm{Al5} \cdot \mathrm{W}_{1}^{(k)} - \mathrm{Al6} \cdot \mathrm{W}_{1}^{(k)} \\ & -\mathrm{Al4} \cdot \mathrm{W}_{2}^{(k)} - \mathrm{Al6} \cdot \mathrm{W}_{3}^{(k)}] \\ (ii) \quad \mathrm{UI}_{2}^{(k+1)} = \mathrm{All}^{-1} \cdot [-\mathrm{Al2} \cdot \mathrm{U}_{1}^{(k)} - \mathrm{Al2} \cdot \mathrm{U}_{3}^{(k)} + \mathrm{Al3} \cdot \mathrm{V}_{1}^{(k)} - \mathrm{Al3} \cdot \mathrm{V}_{3}^{(k)} \\ & -\mathrm{Al4} \cdot \mathrm{W}_{1}^{(k)} - \mathrm{Al5} \mathrm{W}_{2}^{(k)} - \mathrm{Al4} \cdot \mathrm{W}_{3}^{(k)} - \mathrm{Al6} \cdot \mathrm{W}_{4}^{(k)}] \\ (iii) \quad \mathrm{UI}_{1}^{(k+1)} = \mathrm{Al1}^{-1} \cdot [-\mathrm{Al2} \cdot \mathrm{U}_{1-1}^{(k)} - \mathrm{Al2} \cdot \mathrm{U}_{1+1}^{(k)} + \mathrm{Al3} \cdot \mathrm{V}_{1-1}^{(k)} \\ & -\mathrm{Al3} \cdot \mathrm{V}_{1+1}^{(k)} - \mathrm{Al6} \cdot \mathrm{W}_{1-2}^{(k)} - \mathrm{Al4} \cdot \mathrm{W}_{1-1}^{(k)} - \mathrm{Al5} \mathrm{W}_{1}^{(k)} \\ & -\mathrm{Al4} \cdot \mathrm{W}_{1+1}^{(k)} - \mathrm{Al6} \cdot \mathrm{W}_{1+2}^{(k)}] \qquad i=3,4,\ldots,m \end{array}$$

$$(iv) \quad \mathrm{UI}_{m+1}^{(k+1)} = \mathrm{A44}^{-1} \cdot [-\mathrm{A46} \cdot \mathrm{U}_{m-1}^{(k)} - \mathrm{A45} \cdot \mathrm{V}_{m}^{(k)}] \\ (v) \quad \mathrm{VI}_{1}^{(k+1)} = \mathrm{A22}^{-1} \cdot [-\mathrm{A20} \cdot \mathrm{U}_{2}^{(k)} - \mathrm{A21} \cdot \mathrm{V}_{2}^{(k)} + \mathrm{A23} \cdot \mathrm{W}_{1}^{(k)} - \mathrm{A25} \cdot \mathrm{W}_{2}^{(k)} \\ & -\mathrm{A23} \cdot \mathrm{W}_{3}^{(k)}] \\ (vi) \quad \mathrm{VI}_{2}^{(k+1)} = \mathrm{A22}^{-1} \cdot [\mathrm{A20} \cdot \mathrm{U}_{1}^{(k)} - \mathrm{A23} \cdot \mathrm{W}_{4}^{(k)}] \\ (vii) \quad \mathrm{VI}_{1}^{(k+1)} = \mathrm{A22}^{-1} \cdot [\mathrm{A20} \cdot \mathrm{U}_{1-1}^{(k)} - \mathrm{A23} \cdot \mathrm{W}_{4}^{(k)}] \\ (vii) \quad \mathrm{VI}_{1}^{(k+1)} = \mathrm{A22}^{-1} \cdot [\mathrm{A20} \cdot \mathrm{U}_{1-1}^{(k)} - \mathrm{A23} \cdot \mathrm{W}_{4}^{(k)}] \\ (vii) \quad \mathrm{VI}_{1}^{(k+1)} = \mathrm{A22}^{-1} \cdot [\mathrm{A20} \cdot \mathrm{U}_{1-1}^{(k)} - \mathrm{A23} \cdot \mathrm{W}_{4}^{(k)}] \\ (vii) \quad \mathrm{VI}_{1}^{(k+1)} = \mathrm{A22}^{-1} \cdot [\mathrm{A20} \cdot \mathrm{U}_{1-1}^{(k)} - \mathrm{A23} \cdot \mathrm{W}_{4}^{(k)}] \\ (vii) \quad \mathrm{VI}_{1}^{(k+1)} = \mathrm{A22}^{-1} \cdot [\mathrm{A20} \cdot \mathrm{U}_{1-1}^{(k)} - \mathrm{A23} \cdot \mathrm{W}_{4}^{(k)}] \\ (\mathrm{VII}^{(k+1)} = \mathrm{A22}^{-1} \cdot [\mathrm{A20} \cdot \mathrm{U}_{1-1}^{(k)} - \mathrm{A23} \cdot \mathrm{W}_{1}^{(k)} - \mathrm{A21} \cdot \mathrm{W}_{1-1}^{(k)} - \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} + \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} + \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} + \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} + \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{(k)} + \mathrm{W}_{1}^{(k)} - \mathrm{W}_{1}^{$$

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•

$$\begin{aligned} -&\lambda 23.W_{i+2}^{(k)}] & i=3,4,\ldots,m \\ (\text{viii}) \quad &VI_{m+1}^{(k+1)} = \lambda 55^{-1} \cdot \left[-\lambda 54.U_{m}^{(k)} +\lambda 55.V_{m-1}^{(k)} -\lambda 56.W_{m}^{(k)} +B4\right] \\ (\text{ix}) \quad &WI_{1}^{(k+1)} = \lambda 36^{-1} \cdot \left[-\lambda 31.U_{1}^{(k)} +\lambda 32.V_{2}^{(k)} -\lambda 34.W_{2}^{(k)} -\lambda 35.W_{3}^{(k)} +B3\right] \\ (\text{x}) \quad &WI_{2}^{(k+1)} = \lambda 33^{-1} \cdot \left[-\lambda 31.U_{2}^{(k)} -\lambda 32.V_{1}^{(k)} +\lambda 32.V_{3}^{(k)} -\lambda 34.W_{1}^{(k)} \right] \\ & -\lambda 34.W_{3}^{(k)} -\lambda 35.W_{4}^{(k)} +B3\right] \\ (\text{xi}) \quad &WI_{i}^{(k+1)} = \lambda 33^{-1} \cdot \left[-\lambda 31.U_{i}^{(k)} +\lambda 32.V_{i-1}^{(k)} +\lambda 32.V_{i+1}^{(k)} \right] \\ & -\lambda 35.W_{i-2}^{(k)} -\lambda 34.W_{i-1}^{(k)} +\lambda 34.W_{i+1}^{(k)} \\ & -\lambda 35.W_{i+2}^{(k)} +B3\right] \\ (\text{xii}) \quad &WI_{m+1}^{(k+1)} = \lambda 66^{-1} \cdot \left[-\lambda 67.W_{m-2}^{(k)} -\lambda 66.W_{m-1}^{(k)} -\lambda 65.W_{m}^{(k)} \right] \\ & -\lambda 67.W_{m+2}^{(k)}\right] \\ (\text{xiii}) \quad &WI_{m+2}^{(k+1)} = \lambda 77^{-1} \cdot \left[\lambda 77.W_{m-2}^{(k)} +\lambda 76.W_{m-1}^{(k)} -\lambda 76.W_{m+1}^{(k)}\right] \\ & \dots \left[3.8\right] \end{aligned}$$

The (k+1)th approximation for deformations is given as below:

(i)
$$U_{i}^{(k+1)} = U_{i}^{(k)} + G[UI_{i}^{(k+1)} - U_{i}^{(k)}]$$
 $i=1,2,...,m+1$
(ii) $V_{i}^{(k+1)} = V_{i}^{(k)} + G[VI_{i}^{(k+1)} - V_{i}^{(k)}]$ $i=1,2,...,m+1$
(iii) $W_{i}^{(k+1)} = W_{i}^{(k)} + G[WI_{i}^{(k+1)} - W_{i}^{(k)}]$ $i=1,2,...,m+2$
[3.9]

Where G is the matrix of convergence. Usually G is a diagonal matrix. The most common form of G is:

 $G = \alpha I$

Where α is a scalor and is chosen to get the fast convergence for the process, as explained in section B.4 of Appendix B.

Variables $\mu^{(k+1)}$, Ver^(k+1), ERR^(k+1) and RER^(k+1) which are required to find the rate of convergence and to test the criterion of convergence, as discussed in section B.2 of appendix B, are computed next.

3.4 EIGENVALUES OF JACOBI ITERATION MATRIX:

As defined in Appendix B, Jacobi iteration matrix is $J = -D^{-1} [L+U]$

The biggest and the smallest eigenvalues of a matrix can be computed with the help of power method as discussed in detail in Appendix C.

It can be observed from equation [C.2] that two equations for the iteration of the eigenvalues can be written as below:

$$xi^{(k+1)} = J.x^{(k)}$$
 [3.10]

$$x^{(k+1)} = \frac{1}{AMAX^{(k+1)}} \cdot xi^{(k+1)} [3.11]$$

Where AMAX (k+1) is the largest element in vector xi (k+1).

Comparing equations [3.10] and [B.2] it can be noted that the two iteration processes are similar except in the former case vector b is a null vector.

The equations [3.8] can be used for this iteration as well, except in equations[3.8(viii)],[3.8(ix)],[3.8(x)] and [3.8(xi)], B3 and B4 are taken as null vectors. After iteration process [3.8] is carried out, the biggest element in UI_i^(k+1) $VI_i^{(k+1)}$ and $WI_i^{(k+1)}$ is searched and given a value AMAX^(k+1). From the equation [3.11] the (k+1)th approximation for U_i, V_i and W_i is given as (i) U_i^(k+1) = UI_i^(k+1)/AMAX^(k+1) i=1,2,...,m+1 (ii) V_i^(k+1) = $VI_i^{(k+1)}$ /AMAX^(k+1) i=1,2,...,m+1 (iii) W_i^(k+1) = $WI_i^{(k+1)}$ /AMAX^(k+1) i=1,2,...,m+2

A quantity DX is defined

[3.12]

$$DX = \left| \frac{AMAX^{(k+1)} - AMAX^{(k)}}{AMAX^{(k+1)}} \right|$$
 [3.13]

If this quantity DX is very small, of order 10^{-p} (where p is an integer), the biggest eigenvalue of the matrix J is taken as AMAX^(k+1), otherwise the iteration is carried on until the value of DX comes down to order of 10^{-p} .

Once the biggest eigenvalue of the matrix J is computed, its smallest eigenvalue can be calculated, as discussed in Appendix C, by finding the biggest eigenvalue of the matrix $(J-\lambda_{j,b}I)$, where $\lambda_{J,b}$ is the biggest eigenvalue of the matrix J. For this the iteration equation [3.10] can be written as

$$xi^{(k+1)} = J.x^{(k)} - \lambda_{J,b} x^{(k)}$$
 [3.14]

The same iteration process [3.8] can be used with a little modification in values of $UI_i^{(k+1)}$, $VI_i^{(k+1)}$ and $WI_i^{(k+1)}$, as given below.

$$UI_{i}^{(k+1)} = UI_{i}^{(k+1)} |_{[3.8]}^{-\lambda}J, bU_{i}^{(k)}$$

$$VI_{i}^{(k+1)} = VI_{i}^{(k+1)} |_{[3.8]}^{-\lambda}J, bV_{i}^{(k)}$$

$$WI_{i}^{(k+1)} = WI_{i}^{(k+1)} |_{[3.8]}^{-\lambda}J, bW_{i}^{(k)}$$

$$(3.15)$$

These modified values of UI_i^(k+1), VI_i^(k+1) and WI_i^(k+1) are used in equations [3.12] to complete the iteration. Say λ_1 is the biggest eigenvalue of this matrix (i.e. matrix $(J-\lambda_{J,b}I)$), then the smallest eigenvalue $(\lambda_{J,s})$ of matrix J is given as

$$\lambda_{\mathbf{J},\mathbf{S}} = \lambda_{\mathbf{1}} + \lambda_{\mathbf{J},\mathbf{b}}$$
[3.16]

These values of $\lambda_{J,b}$ and $\lambda_{J,s}$ can be used to find the value of α , which gives the fastest convergence. Also these values can indicate whether the convergence is possible or not (convergence is not possible if the values of $\lambda_{J,s}$ and $\lambda_{J,b}$ lies on different side of 1. (i.e. if $\lambda_{J,b}$ >1 and $\lambda_{J,s}^{<1}$).

The parameters of a typical elliptical crosssectional bourdon tube are taken and the eigenvalues of Jacobi iteration matrix are computed. It was found that $\lambda_{J,b}$ >1 and $\lambda_{J,s}$ <1, which means that convergence is not possible. Different arrangements of matrix A were tried to find ill conditioned region. It was found that ill conditions were due to the subsystem arising from boundary conditions at $\beta=\beta_L$ (i.e. from the equations [3.6]). In the iteration equations [3.8] there are four equations due to these boundary conditions, namely equations [3.8(iv)], [3.8(viii)], [2.8(xii)], and [3.8(xiii)]. These equations are deleted from the iteration process, thereby keeping values of U_{m+1} , V_{m+1} , W_{m+1} and W_{m+2} at the level of initial approximation.

After deleting these subsystems from the system of equations, eigenvalues of the Jacobi iteration matrix were computed. Here the eigenvalues were better i.e. both $\lambda_{J,s}$ and $\lambda_{J,b}$ are less than 1. But these values indicate very slow convergence [value of $\lambda_{J,b}$ is approximately equal to 1 and value of $\lambda_{J,s}$ is less than -1]. A computer subroutine 'EIGEN' is written to find eigenvalues and described in detail in Appendix D.

3.5 DOUBLE ITERATION METHOD:

Because the subsystems at the boundary $\beta = \beta_L$ are deleted from the iteration process it requires double iteration to solve for the deformations at the boundary.

In this process some initial values for the deformations at all the points are taken as before and after deleting the four iteration equations from [3.8] the iteration is carried for deformations at all points except for U and V at $\beta = \beta_L + k$ and for W at $\beta = \beta_L + k$ and at $\beta = \beta_L + 2k$. When this iteration gets converged, deformations at boundaries are calculated as described below.

From equation [3.7] and figs. (3.2), (3.3) and (3.4) we get

$$U_{m+1} = A44^{-1} \cdot [-A46 \cdot U_{m-1} - A45 \cdot V_{m}^{(k)}]$$
 [3.17]

$$V_{m+1} = A55^{-1} \cdot [-A54 \cdot U_m + A55 \cdot V_{m-1} - A56 \cdot W_m + B4]$$
 [3.18]

$$W_{m+1} = A66^{-1} \cdot [-A67 \cdot W_{m-2} - A66 \cdot W_{m-1} - A65 \cdot W_{m} - A67 \cdot W_{m+2}] [3.19]$$

$$W_{m+2} = A77^{-1} \cdot [A77 \cdot W_{m-2} + A76 \cdot W_{m-1} - A76 \cdot W_{m+1}]$$
 [3.20]

Substituting [3.20] into [3.19] and grouping we get,

$$W_{m+1} = A80^{-1} \cdot [2A67 \cdot W_{m-2} + 2A66 \cdot W_{m-2} + A80 \cdot W_{m-1} + A65 \cdot W_{m}] [3.21]$$

Where $A80 = A67 \cdot A77^{-1} \cdot A76 - A66$

The equations [3.17], [3.18], [3.21] and [3.20] are used in sequence to compute the deformations at the boundary of the free end (i.e. at $\beta = \beta_T$). A subroutine 'FREEND' is written to compute the deformations at the free end and described in detail in appendix D.

When the deformations at the boundary $\beta = \beta_L$ are computed, these and the values of deformations at other points, as obtained at the end of last iteration process, are taken as the initial approximation and the modified iteration process is carried out again until convergence, and after the convergence values of deformations at the boundary $\beta = \beta_L$ are obtained. The process goes on repeating itself until deformations at the free end also converge. A subroutine 'BORDON' is written to carry out this process and described in detail in Appendix D.

It was found that the process is progressing in the right direction, (i.e. toward convergence), but it is very slow, as indicated from the eigenvalues. A very large amount of computer time will be required to obtain convergence, and this is not a practical result. Thus, the iteration process has failed so far in finding the solution to the problem.

CHAPTER 4

CONCLUSIONS

As explained in the Chapter 3, when attempting a solution of the system of linear equations, which were formed by using finite difference approximations on the given PDEs and BCs, the method of block iteration using single iteration failed as convergence was not possible Then the double iteration method was in that case. applied which was not a complete failure as it was found that convergence was possible, but the convergence rate was extremely slow. Again a few techniques were used (i.e. use of a reduced grid size and the use of different relaxation factors for differetn subsystems) to increase the convergence rate, but it was of no use. As a last resort the computer was booked for three hours. Aqain a few arrangements were tried and also the iteration process was carried for a longer time. However, the result was the same i.e. convergence was possible but the convergence rate If the iteration was carried on with the present was low. rate of convergence it would have taken approximately forty hours of computer time before it would have converged. Therefore, this method was discarded because of impractability. Work on this project was discontinued at this point because it was considered that the defined scope had been covered, and although no practical method of solution has been

developed some possible alternative methods of approach are suggested for further work.

- Use a variable grid size on the domain of the bourdon tube. This may give an increased rate of convergence. It is a trial and error method and it will take many trials to obtain the right variation of grid size along the domain.
- Use the matrix of convergence G of the form shown in the equation [B.22], but as it is pointed out there, it is not economical to find the right combination of relaxation factors, which will give the fastest convergence, because the number of unknowns is approximately 2000.
- Evolve some new technique to increase the rate of convergence.
- Formulate the problem from an entirely different approch, for instance instead of elastic shell theory, use a finite element technique.

APPENDIX A

FINITE DIFFERENCE APPROXIMATION

difference representations for partial Finite derivatives are the most frequent approach to the numerical integration of PDE's. The partial derivatives are replaced by the difference quotients in the independent variables and the result used for an approximation of derivatives. In general the domain over which the PDE holds has dimensions equal to the independent variables. For the special case of only two independent variables x and y, the domain is two-dimensional and can be represented on a plane surface [fig. A.1]. The domain can be covered with a network of rectangular spacings. The spacing between any two adjacent vertical lines (the x-direction) is taken as h, a constant, and between any two horizontal points (the y-direction) as k, a constant. The value of the dependent variable f (f=f(x,y)) is specified at all the points within the domain. In particular if one point on the rectangular grid spacing is given the co-ordinate x,y, then the points around have co-ordinates (x+h,y+k), (x,y+k), (x-h,y+k), (x-h,y), (x-h,y-k), (x,y-k), (x+h,y-k) and (x+h,y). The corresponding values of the function f are f 1.1, f 0.1, f-1,1, $f_{-1,0}$, f_{-1-1} , $f_{0,-1}$, $f_{1,-1}$ and $f_{1,0}$ respectively.



Fig. A.1 Rectangular grid superimposed on the domain of interest.

The method of replacing the partial derivatives by finite differences follows directly from Taylor's series expansion, which is given in two dimensions as,

$$f(x+h,y+k) = f(x,y) + h.f_{x} + k.f_{y} + \frac{h^{2}}{2!} \cdot f_{xx}$$

+ h.k. $f_{xy} + \frac{k^{2}}{2!} \cdot f_{yy} + \frac{h^{3}}{3!} \cdot f_{xxx}$
+ $\frac{h^{2}k}{2!} \cdot f_{xxy} + \frac{hk^{2}}{2!} \cdot f_{xyy} + \frac{k^{3}}{3!} \cdot f_{yyy} + \dots$
[A.1]

From this series finite differences in central difference form are obtained as:

$$f_{,xxxx} = \frac{1}{h^4} \cdot [f_{-2,0} - 4f_{-1,0} + 6f_{0,0} - 4f_{1,0} + f_{2,0}]$$

$$f_{,xxx} = \frac{1}{2h^3} \cdot [-f_{-2,0} + 2f_{-1,0} - 2f_{1,0} + f_{2,0}]$$

$$f_{,xx} = \frac{1}{12h^2} \cdot [-f_{-2,0} + 16f_{-1,0} - 30f_{0,0} + 16f_{1,0} - f_{2,0}] \cdot$$

$$f_{,x} = \frac{1}{12h} \cdot [f_{-2,0} - 8f_{-1,0} + 8f_{1,0} - f_{2,0}]$$

First and second derivatives can also be written in less accurate form using three points.

$$f_{,xx} = \frac{1}{h^2} [f_{-1,0} - 2f_{0,0} + f_{1,0}]$$
$$f_{,x} = \frac{1}{2h} [-f_{-1,0} + f_{1,0}]$$

Mixed derivative can be written by a combination of pure derivatives:

$$f_{,xy} = \frac{1}{4hk} [f_{-1,-1} - f_{1,-1} - f_{-1,1} + f_{1,1}]$$

$$f_{,xxy} = \frac{1}{2h^2k} [-f_{-1,-1} + 2f_{0,-1} - f_{1,-1} + f_{-1,1} - 2f_{0,1} + f_{1,1}]$$

$$f_{,xxyy} = \frac{1}{h^2k^2} \cdot [f_{-1,-1} - 2f_{0,-1} + f_{1,-1} - 2f_{-1,0} + 4f_{0,0}]$$

$$-2f_{1,0} + f_{-1,1} - 2f_{0,1} + f_{1,1}$$

Other derivatives can also be written in the same fashion.

APPENDIX B

B.1 BLOCK ITERATION METHOD:

An iterative method is a rule for operating on a previous approximate solution to obtain the improved solution. Iterative methods are prefered for solving large sparse systems (such as those arising from finite difference approximation for PDE's), because they can take advantage of zeros in the matrix and tend to be self correcting, and hence tend to minimize round off errors. One of the biggest drawbacks of the iterative method is the possibility of slow or irregular convergence.

In the block iterative scheme several unknowns are connected together in the iteration formulae in such a way that a linear subsystem must be solved before anyone of them can be determined. The equations are divided into groups and the subsystem of equations belonging to a given group is solved for the corresponding unknown using approximate values of other unknowns.

Say the given system of linear equations is $\overline{Ax}=\overline{b}$, then the unknowns are divided into N groups so that x_1, \ldots, x_m , belong to group \overline{x}_1 ; x_{m_1+1}, \ldots, x_m belong to group \overline{x}_2 ; etc. In general $x_{m_{k-1}+1}, \ldots, x_m$ belong to group \overline{x}_k . The matrix \overline{A} is similarly divided into blocks \overline{A}_{ij} , where the submatrix \overline{A}_{ij} has m_i rows and $(m_j - m_{j-1})$ columns, and the \overline{b} vector is divided into N groups $\overline{\beta}_1, \ldots, \overline{\beta}_N$. Then the system can be written



Naturally, the blocks are chosen so that soving each subsystem is as simple as possible.

Approximate values of unknown subvectors $\bar{x}_1, \dots \bar{x}_N$ are given as $\bar{x}_1^{(k)}, \dots \bar{x}_N^{(k)}$ after the kth iteration.

A vector \overline{xi} of the same length as vector \overline{x} , divided into subvecotrs \overline{xI} ;... \overline{xI}_N , has its (k+1)th approximation defined as

$$\overline{XI}_{i}^{(k+1)} = \overline{\overline{A}}_{11}^{-1} \cdot [\overline{\beta}_{i} - \sum_{\substack{j=1\\ j\neq 1}}^{N} A_{ij} \overline{X}_{j}^{(k)}] \qquad [B.1]$$

 $\vec{x}i^{(k+1)} = \vec{J} \vec{x}^{(k)} + \vec{D}^{-1} \vec{b}$ [B.2]

Where \overline{J} is known as the Jacobi iteration matrix and is defined as

 $\bar{J} = - [\bar{D}^{-1} (\bar{L} + \bar{U})]$ [B.3]

and \overline{D} , \overline{L} and $\overline{\overline{U}}$ are defined as

$$\overline{\overline{A}} = \overline{\overline{L}} + \overline{\overline{D}} + \overline{\overline{U}}$$
 [B.4]

Where

or

$$E = L = \begin{pmatrix} \frac{0}{\bar{A}}_{21} & 0 & & \\ \vdots & \vdots & \vdots & \vdots & \\ \frac{-1}{\bar{A}}_{N1} & \vdots & \vdots & \vdots & \frac{0}{\bar{A}}_{N,N-1} & 0 \end{pmatrix}$$



and
$$U = \begin{pmatrix} 0 & \bar{A}_{12} & \cdots & \bar{A}_{1N} \\ & & & & \bar{A}_{2N} \\ & & & & \bar{A}_{2N} \\ & & & & \bar{A}_{2N} \\ & & & & & \bar{A}_{N-1,N} \\ & & & & & 0 \end{pmatrix}$$

then (k+1)th approximation for unknown vector \overline{x} is given as

$$\bar{x}^{(k+1)} = \bar{x}i^{(k+1)}$$
 [B.5]

It is desired that $\bar{x}^{(k)} \rightarrow h = \bar{A}^{-1} \bar{b}$ as $k \rightarrow \infty$.

B.2 CRITERIA OF CONVERGENCE:

(i) The error vector \overline{er} , divided into N groups $\overline{ER}_{1}, \ldots \overline{ER}_{N}$, after the kth iteration is defined as

$$\overline{ER}_{i}^{(k)} = \beta_{i} - \sum_{j=1}^{N} \overline{\overline{A}}_{ij} \overline{\overline{X}}_{j}^{(k)}$$
[B.6]

or $\overline{er}^{(k)} = \overline{b} - \overline{A} \overline{x}^{(k)}$ [B.7]

A quantity ERR^(k) is defined at kth iteration as

ERR^(k) =
$$\begin{pmatrix} N & (k) \\ \Sigma & (er_{i})^{2} \\ \frac{i=1}{N} \end{pmatrix}^{1/2}$$
 [B.8]

The value of ERR^(k) would become zero, when the solution vector acquires the desired value. But for practical purposes convergence is assumed when ERR becomes very small (order of 10^{-p} were p is any pre-assigned iteger).

The quantity RER^(k) defined below at kth iteration will indicate the rate of convergence.

$$RER^{(k)} = ERR^{(k)} / ERR^{(k-1)}$$
 [B.9]

The smaller the value of RER, the faster is the convergence. If RER has a value equal to 1, the system is static. If it is more than 1, the system is diverging.

(ii) Another criterion of convergence is obtained by determining the change in the value of the solution vector after each iteration.

A vector $dx^{(k)}$ is defined at the kth iteration as

$$\overline{dx}^{(k)} = (\overline{Xi}^{(k+1)} - \overline{x}^{(k)}) / \overline{xi}^{(k+1)}$$
 [B.10]

and a quantity Ver^(k) is defined at the kth iteration $Ver^{(k)} = \begin{pmatrix} N \\ \Sigma (dx_i)^2 \\ \frac{i=1}{N} \end{pmatrix}$ [B.11]

As $k \rightarrow \infty$, Ver^(k) $\rightarrow 0$. But for practical purposes again convergence is assumed when Ver^(k) is very small (of order of 10^{-p} where p is any pre-assigned integer).

. The rate of convergence can also be defined at the $k^{\mbox{th}}$ iteration by the quantity $\mu^{\mbox{(k)}}$ as

$$\mu^{(k)} = Ver^{(k)} / Ver^{(k-1)}$$
 [B.12]

This will behave similarly as RER.

B.3 CONDITION OF CONVERGENCE:

To obtain the convergence of the block iteration process with any initial vector $\bar{\mathbf{x}}^{(0)}$ and any value of the vector $\bar{\mathbf{b}}$, it is necessary and sufficient that all the eigenvalues of the Jacobi iteration matrix $\overline{\mathbf{j}}$ be less than unity in modulus, thus

B.4 CONVERGENCE PROMOTION TECHNIQUE:

The frequently used technique [18], is to form a general equation of convergence promotion which replaces the successive iteration equation [B.5]. This general equation of convergence promotion is applied at every iteration, and becomes a part of the iteration procedure. This general equation is

 $\bar{x}^{(k+1)} = \bar{x}^{(k)} + t \bar{\bar{G}} \cdot (\bar{xi}^{(k+1)} - x^{(k)})$ [B.14] Where $\bar{\bar{G}}$ is the matrix of convergence promotion coefficients and t is a relaxation factor. In a simple approach $\bar{\bar{G}}$ is taken as a diagonal matrix.

By substituting [B.2] into [B.14] we get

$$\bar{x}^{(k+1)} = \bar{x}^{(k)} + t \,\bar{\bar{G}} \,(\bar{\bar{J}} \,\bar{x}^{(k)} + \bar{\bar{D}}^{-1} \,\bar{\bar{D}} - \bar{x}^{k})$$
$$\bar{x}^{(k+1)} = \{\bar{\bar{I}} + t \,\bar{\bar{G}} \,(\bar{\bar{J}} - \bar{\bar{I}})\} \,\bar{x}^{(k)} + t \,\bar{\bar{G}} \,, \bar{\bar{D}}^{-1} \,\bar{\bar{D}}$$

or

A matrix \overline{B} is defined as

$$\bar{B} = \bar{I} + t \bar{G}.(\bar{J} - \bar{I})$$
 [B.16]

Then equation [B.15] becomes

$$\bar{x}^{(k+1)} = \bar{B}.\bar{x}^{(k)} + t \bar{G}.\bar{D}^{-1}.\bar{D}$$
 [B.17]

[B.17] is a new iteration process, the rate of convergence depends upon the eigenvalues of the matrix $\overline{\overline{B}}$. The biggest eigenvalue of B must be as small as possible.

The most usual form of the matrix of convergence \overline{G} is $\overline{\overline{G}} = \alpha \overline{\overline{I}}$ and t = 1 [B.18]

where α is a scalor.

Substituting [B.18] into [B.16] we get

$$\vec{B} = \vec{I} + \alpha \vec{I} (\vec{J} - \vec{I})$$

or
 $\vec{B} = (1 - \alpha) \vec{I} + \alpha \vec{J}$ [B.19]

Now we want to find α , so that the eigenvalues of matrix $\overline{\tilde{B}}$ are as small as possible in modulus. As it can be observed from [B.19] that $\overline{\tilde{B}}$ is a matrix polynomial of $\overline{\tilde{J}}$, therefore, each eigenvalue of $\overline{\tilde{B}}$ is the same scalor polynomial of the respective eigenvalue of $\overline{\tilde{J}}$, i.e.

$$\lambda_{\rm B} = (1-\alpha) + \alpha \lambda_{\rm T} \qquad [B.20]$$

If $\lambda_{J,b}$ and $\lambda_{B,b}$ are the biggest eigenvalues of \overline{J} and \overline{B} respectively and $\lambda_{J,s}$ and $\lambda_{B,s}$ are the smallest eigenvalue of $\overline{\overline{J}}$ and $\overline{\overline{B}}$ respectively then for any α

$$A_{B,b} = (1-\alpha) + \alpha \lambda_{J,b}$$

$$B,s = (1-\alpha) + \alpha \lambda_{J,s}$$

$$(B.21)$$

All other eigenvalues of the two matrices lie inbetween.

$$\lambda_{B,s} < \lambda_{B} < \lambda_{B,b}$$

$\lambda_{J,s} < \lambda_{J} < \lambda_{J,b}$

Fig. (B.1) illustrates the variation of eigenvalues of the matrix \overline{B} . The upper line indicates $\lambda_{B,b}$ and the lower line $\lambda_{B,s}$. Obviously when $\alpha=1$, the eigenvalues of matrix $\overline{\overline{B}}$ are equal to those of matrix $\overline{\overline{J}}$.

For accelerating the convergence the best α that may be chosen is to make absolute eigenvalues of \overline{B} as small as possible. In the case illustrated in Fig. B.1 (a), the value of α must be more than 1, to accelerate the convergence.

For the case illustrated in Fig. B.1(b), when $\lambda_{J,s}$ is less than -1. The value of α must be less than 1, so that values of $\lambda_{B,b}$ and $\lambda_{B,s}$ are small.

In the third case illustrated in Fig. B.1(c), where both $\lambda_{J,b}$ and $\lambda_{J,s}$ are greater than 1, the value of α must be negative to make absolute value of all eigenvalues of matrix \overline{B} less than 1.

In the last case illustrated in the Fig. B.1(d), when $\lambda_{J,b} > 1$ and $\lambda_{J,s} < 1$, there is no value of α , which can transfer the non-convergent iterative process to convergent one. This is the worst case and for this case convergence is not possible.







ą.

Variation of the biggest and the smallest eigenvalue of matrix B with coefficient $\boldsymbol{\alpha}$

(b)



(c)





Fig.B.l

105

Variation of the biggest and the smallest eigenvalue of matrix B with coefficient $\boldsymbol{\alpha}$

54

Ο λ_{J,b}

Ο^λJ,s

It can be noted that the relaxation factor t, and the coefficient α play the same role and it is possible to use the relaxation factor t instead of α in the above development, when matrix $\overline{\overline{G}}$ is considered as the unit matrix $\overline{\overline{\overline{I}}}$.

The matrix of convergence \overline{G} can also be assumed to have the form given below:

[B.22]

In this case to find the best combination of $\alpha_1, \ldots, \alpha_N$ for the fastest convergence, it is a matter of trial and error, and it may not be practical when N is greater than three [18].

APPENDIX C

POWER METHOD

An important iterative procedure used to find one eigenvalue at a time is the power method [14]. To illustrate the procedure, let the eigenvalues of the matrix $\overline{\overline{A}}$ be real and distinct, and adopt the convention

$$|\lambda_1| > |\lambda_2| > |\lambda_3| \ge \dots \ge |\lambda_N| \qquad [C.1]$$

Where λ_1 is the largest eigenvalue. Staring with an almost arbitrary vector $\bar{\mathbf{x}}^{(0)}$ a sequence is generated, which after (k+1)th operation is

$$\bar{x}^{(k+1)} = R^{(k+1)} \bar{\bar{A}} \bar{x}^{(k)}$$
 [C.2]

where $R^{(k+1)}$ is chosen to make the component in $\bar{x}^{(k+1)}$ of largest absolute value equal to 1. In other words $\frac{1}{R^{(k+1)}}$ is merely the largest element in $\bar{x}^{(k+1)}$. As shown by Von Mises, this sequence will converge to the vector \bar{x}_1 (i.e. eigenvector), $\bar{x}^{k+1} + \bar{x}_1$, corresponding to the eigenvalue λ_1 , when convergence has occured.

$$\bar{\mathbf{x}}^{(k+1)} = \bar{\bar{\mathbf{A}}} \bar{\mathbf{x}}^{(k)}$$
 [C.3]

It can be used to calculate the eigenvalue λ_1 , since $\bar{x}^{(k+1)}$ and $\bar{x}^{(k)}$ must be a scalor multiple λ_1 of each other. The test of convergence is usually.

$$|\lambda_{1}^{(k+1)} - \lambda_{1}^{(k)}| < 10^{-p}$$

or

$$\begin{vmatrix} \lambda_{1}^{(k+1)} - \lambda_{1}^{(k)} \\ \frac{\lambda_{1}^{(k+1)}}{\lambda_{1}} \end{vmatrix} < 10^{-p}$$
 [C.4]

Where p is an integer.

The rate of convergence of $\bar{x}^{(k)}$ to \bar{x}_1 is dictated largely by the ratio of λ_2/λ_1 ; if this ratio is approximately equal to 1.0, convergence is slow, whereas wide separation of λ_1 and λ_2 leads to a rapid convergence. Also, the initial choice of $\bar{x}^{(0)}$ controls the rate of convergence. If it is almost orthogonal to the dominating eigenvector, convergence will be a slow process. The advantage of the method is that it yields $\bar{x}^{(k+1)}$ as a function of $\bar{x}^{(k)}$ and not of $\bar{x}^{(k-1)}$, $\bar{x}^{(k-2)}$,... An error made in one step of calculation is self correcting by the continued use of approximations.

This method can be extended to find the lowest eigenvalue (i.e. λ_N) as well. This can be done by finding the eigenvalues of the matrix $[\overline{A}-\lambda_1\overline{I}]$. All eigenvalues of this new matrix will be less in magnitude than eigenvalues of matrix \overline{A} , by a quantity λ_1 . i.e. eigenvalues of this matrix are:

$$0, \lambda_2 - \lambda_1, \lambda_3 - \lambda_1, \dots, \lambda_N - \lambda_1$$

It can be observed that the absolute largest eigenvalue of this new matrix is $|\lambda_N - \lambda_1|$. This can be found by the power method, as described above. Once this is known λ_N can be computed.

APPENDIX D

COMPUTER PROGRAM

D.1 PURPOSE AND SCOPE

The purpose of the computer programs, described in this appendix, is to set up a system of linear equations in the form of the submatrices as explained in Chapter 3 and then to solve this system. For this purpose one main program and a few subroutines are written. The program is of a very general nature, and can be used for any bourdon tube with an elliptical cross-section, and any given number of intervals in both directions. When a different number of intervals is used, changes must be made in the common statements to change the dimensions of the variables, as described in section D.3.

D.2 MAIN PROGRAM:

The purpose of this program is, to read the given information about the parameters of the bourdon tube and the number of invervals in which the domain is devided, and to ask the computer to execute the subroutine BORDON, if the deformations are to be computed, or to execute the subroutine 'EIGEN' if the eigenvalues are to be computed.

Variable Notations:

A - Semi-minor axis of the elliptical cross-section of the bourdon tube in inches.



Fig. D.1. Flow chart of the main program.

B - Semi-major axis in inches.

ANEW - Poisson's ratio of the materials of the bourdon tube.

C - Coefficient of elasticity of the materials in psi.

RO - Radius of curvature of the central axis of the crosssection of the tube in inches.

BL - Coiling angle of the bourdon tube in degrees.

AH - Thickness of the bourdon tube in inches.

P - Fluid pressure in the tube in psi.

- N Number of intervals in which the domain is divided in the $\alpha-$ direction.
- M Number of intervals in which the domain is divided in the $\beta-$ direction.
- I I=0 if deformations are to be computed
 - I=1 if eigenvalue of the Jacobi iteration matrix is
 required.

Fig. D.1 shows how the main program works.

D.3 COMMON STATEMENTS:

The purpose of a common statement is to transfer value of an argument from a subprogram to another subprogram, or from a subprogram to main program or vice versa. This can also be used in the case, when dimensions of a given variable changes with changes in data. In that case dimensions in the common statements have to be changed only. The remainder of the program remains the same.

In the program, four labelled common blocks are used, namely MATRIX, DEFORM, WORK and REL. The variables used have notation as given below.

All, Al2 etc : The different submatrices arranged in matrix

A as explained in Fig. (3.2), the values of their elements are calculated in the subroutine MATRIX. The first parameter of the dimensions is always the same with any change in the values of N or M. But the second parameter must

be given a value equal to the value of (N+1).

B3,B4 - Subvectors as indicated in fig. 3.3 and calculated in the subroutine MATRIX. Dimensions of these subvectors must be given a value equal to the value of (N+1).
U,V,W - Deformations at each point on the bourdon tube after each iteration. The first parameter of dimension of these variables must be given a value equal to the

value of (N+1) and the second parameter of dimension of U and V must be given a value of (M+1) and the value of (M+2) is given to that of W.

VI, VI, WI - Intermediate deformations in the process of

iteration as calculated with the help of equation [3.8]. The dimension of these variables behaves similar to those of U, V and W.

X, Y - Work vectors with dimension equal to (N+1).

D, E, F - Work matrices, with dimension having the first parameter constant and the second with value equal to the value of M.

REL - Relaxation vector, usually all the elements have the same

value. But they can be given different values if required for faster convergence. The dimension of this vector is equal to the value of M. EIG - Work vector with dimension equal to that of REL.

Whenever values of N or M are changed, dimensions of these variables must be changed as indicated above.

D.4 SUBROUTINE 'BORDON'

The purpose of this subroutine is to find deformations on the bourdon tube by solving the system of simultaneous linear equations, which are formed-due to use of finite difference approximation on the given PDE's and BC's. The block iteration method, as described in Appendix B, is used to solve this system. This subroutine puts the double iteration method, as discussed in Chapter 3, in computer language.

The notation of the variables used is the same as used in sections D.2 and D.3. Notation for other variables is given below:

PI - Value of π

- H The spacing between two adjacent points on a constant β -grid.
- Ak The spacing between two adjacent points on a constant . α-grid.
- NI The number of iterations for finding deformation at all points except those at the free end.
- KI The number of iterations for finding deformations of the points on the free end.

- INT The maximum number of iterations to be carried out before iteration process is stopped, in the case of no convergénce.
- IKT The maximum number of computation of deformations at free end is to be done before the computer is stopped, in the case of no convergence.
- RMAX Value of the element of the error vector which has the highest absolute value.
- EMAX Absolute value of RMAX.
- ESUM Sum of the square of the elements of the error vector.
- AMAX Absolute value of the element of vector dx: (as defined by equation [B.10]), which has the highest absolute value.

SUM - Sum of the square of the elements of the vector dx.DX - An element of the vector dx.

KZ - Total number of the elements in the vector dx. VERN - Value of Ver^(k+1) as defined by equation [B.11]. ERR - Value of ERR^(k+1) as defined by equation [B.8] AMEW - Value of $\mu^{(k+1)}$ as defined by equation [B.12] RER - Value of RER^(k+1) as defined by equation [B.9] ER - Value of ERR^(k).

VER - Value of Ver^(k).

BETA - Value of β at different grid point along α=90°.
AL - Deflection of centre point of cross-section in dimensionless form.

AF - Ratio $\ell/\Delta \rho$.
SEN - Sensitivity ratio of the bourdon tube.

Other variables are work variables.

The working of this subroutine is explained in the flow chart [fig. D.2]. The first step is to calculate elements of the subvectors and the submatrices after defining the grid size in which the domain is divided. This is accomplished in the subroutine "MATRIX". [This subroutine is explained in section D.6]. After all the submatrices and subvectors are set, the block iteration method is used as explained in the Chapter 3. In case of divergence the computer will stop. In case of no convergence in a specified number of iterations, the computer is stopped again. If convergence occurs, deformations at the free end are computed and the whole iteration process is repeated. If convergence at the free end does not occur in a specified number of iterations again the computer is stopped. If it converges, deflections at points on the central axis of the cross-section of the bourdon tube are computed. In all cases, when the program is diverging, or it is not converging in a specified number of iterations, or it has converged, a punched output is obtained for deformations at all points, so that they can be used in any way required. For instance, if the iteration process is to be continued, these values can be used for initial approximations.



D.5 SUBROUTINE "FREEND"

The purpose of this subroutine is to find deformations at the free end with the help of deformations at other points using equations [3.17], [3.18], [3.21] and [3.20]. The error vector at the end point is also computed. Deviations of these values of deformation from the last values of deformation is computed as well.

The variables used in this subroutine have the same notations as used in the subroutine "BORDON". The working of this subroutine can be explained by the flow chart [Fig. D.3].





D.6 SUBROUTINE "EIGEN"

The purpose of this subroutine is to compute the biggest or the smallest eigenvalue of the Jacobi iteration matrix. The iteration is carried out as explained in Chapter 3. The element having the biggest absolute value is searched. The vector is divided by this value and iteration is continued until convergence occurs. The notation of the variables used is the same as used in the subroutine "BORDON". The working of this subroutine is explained by the flow chart in fig. D.4.

D.7 SUBROUTINE "MATRIX"

The subroutine "MATRIX" is used to determine the elements of the submatrices and the subvectors. As most of the block matrices are banded matrices, they are stored in a fashion which conserves the memory space in the computer. For example an nxn matrix, which has a total of m non-zero diagonals, is stored as an mxm matrix. The notation for the variables, used in this subroutine, is the same as used in the subroutine "BORDON". The working of this subroutine can be explained with the flow chart, illustrated in Fig. D.5. Here first the coefficients Al, A2 etc. are computed with the help of the subroutine "COE" [As explained in section D.8]. Then the values of the elements of the subvectors and submatrices are computed, and these values are transferred to other subroutine's, where they are used, with the help of the COMMON statement, labelled MATRIX.





D.8 SUBROUTINE"COE"

The purpose of this subroutine is to calculate the coefficients (i.e. Al, A2, etc) of the PDE's and the BC's, for the given bourdon tube and for given values of h, k and α . The notation of the variables used is the same as used in the subroutine "MATRIX", except for the notations given below:

ALPHA - The given value of α, corresponding to which the coefficients are calculated.

 $SI - Sin\alpha$

 $CO - Cos\alpha$

DI - value of $(A^2 - B^2)$

GB, GA, GAA, GBB, CAN, CBN and CBG are as defined in Chapter 2. The working of this subroutine is explained in the flow chart in fig. D.6. The expression for all these coefficients (i.e. Al, A2, etc) are defined in Chapter 3.

D.9 SERVICE SUBROUTINES

A few service subroutines are also written. The purpose of these subroutines is to multiply submatrices, or to subtract vectors or to solve a system of equations. These subroutines are as given below:

D.9.1 Subroutine 'BNDPRD' :

The purpose of this subroutine is to multiply a banded matrix, as stored in the subroutine "MATRIX", to a vector which is a column of a matrix. The working of this subroutine is illustrated in fig. D.7 with the help of a flow chart.





D.9.2 Subroutine 'ADDVEC':

The purpose of this subroutine is to add the scalor multiple of two vectors to get a resultant vector. The flow chart of this subroutine is illustrated in fig. D.8.

D.9.3 Subroutine 'SOLVE':

The purpose of this subroutine is to obtain a soluation vector X, of a set of simultaneous linear equations.

A.X = B

Where A is a banded matrix as stored in the subroutine 'MATRIX', and B is a given vector. Two library subroutines DIAG3 and BNDSOL are used to assist the subroutine. Also,



Fig. D.7. Flow chart for the subroutine "BNDPRD"

 $Z(I) = Z(I) + X(KJ, I) \cdot Y(LM, J)$



Fig. D.8. Flow chart for the subroutine 'ADDVEC'

matrix A is not destroyed during the computation. The flow chart for this subroutine is illustrated in fig. D.9.

D.10 ARRANGEMENT AND LISTING OF THE COMPUTER PROGRAM;

The program deck is arranged as shown in fig. D.10. The listings for the computer programs are given in the following pages.



 $\tilde{T}_{k,r}^{k,q}$

Fig. D.9 Flow chart for the subroutine 'SOLVE'

RETURN



Fig. D.10. Setting of the program deck.

PROGRAM TST(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, PUNCH)

COMMON /MATRIX/ A11(3,19),A12(1,19),A13(3,19),A14(3,19),A15(5,19), 1A2U(3,19),A21(1,19),A22(3,19),A23(1,19),A16(1,19),A25(3,19),A46(1, 219),A31(3,19),A32(1,19),A33(5,19),A34(3,19),A35(1,19),A44(1,19),A4 35(3,19),A54(3,19),A55(1,19),A56(1,19),A36(5,19),A65(5,19),A66(1,19) 4),A67(1,19),A76(3,19),A77(1,19),B3(19),B4(19),A80(3,19) COMMON /DEFORM/ U(19,31),V(19,31),W(19,32),UI(19,31),VI(19,31),WI(119,32),X(19),Y(19) COMMON /WORK/ D(3,19),E(5,19),F(1,19) COMMON /REL/ REL(30),EIG(30)

READ (5,3) A,B,ANEW,RO,BL READ (5,4) AH,P,C,N,M,I

DEFORMATIONS ARE COMPUTED, IF I=0, OTHERWISE EIGENVALUE OF THE ITERATION MATRIX IS COMPUTED.

IF (I.EQ.0) GO TO 1 CALL EIGEN (A,B,ANEW,RO,BL,N,M,AH,P,C) GO TO 2 CALL BORDON (A,B,ANEW,RO,BL,N,M,AH,P,C) STOP

FORMAT (5E16.7) FORMAT (3E16.7,3I3) END 76

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C

PURPOSE TO COMPTE THE DEFORMATIONS ON A BOURDON TUBE BY SOLVING THE SYSTEM OF SIMULTANEOUS LENEAR EQUATIONS, WHICH ARISES DUE TO USING FINITE DIFFERENCE TECHNIQUE ON THE GIVEN PARTIAL DIFFEREN-TIAL EQUATIONS AND BOUNDARY CONDITIONS, USING BLOCK ITERATION TECHNIQUE. COMMON /MATRIX/ A11(3,19),A12(1,19),A13(3,19),A14(3,19),A15(5,19), 1A2U(3,19),A21(1,19),A22(3,19),A23(1,19),A16(1,19),A25(3,19),A46(1,

219),A31(3,19),A32(1,19),A33(5,19),A34(3,19),A35(1,19),A44(1,19),A4 35(3,19),A54(3,19),A55(1,19),A56(1,19),A36(5,19),A65(5,19),A66(1,19) 4),A67(1,19),A76(3,19),A77(1,19),B3(19),B4(19),A80(3,19) COMMON /DEFORM/ U(19,31),V(19,31),W(19,32),UI(19,31),VI(19,31),WI(119,32), X(19), Y(19)COMMON /WORK/ D(3,19),E(5,19),F(1,19) COMMON /REL/ REL(30), EIG(30)

SUBROUTINE BORDON (A,B,ANEW,RO,BL,N,M,AH,P,C)

DETERMINE THE GRID SIZE AND DIMENSIONLESS PARAMETERS OF BOURDON TUBE.

 $PI=4 \cdot *ATAN(1 \cdot)$ H=PI/FLOAT(N) $AK = BL * PI / (180 \cdot * FLOAT(M))$ A=2.*A/AHB=2.*B/AH RO=2.*RO/AH NN = N + 1DETERMINE THE ELEMENTS OF THE BLOCK MATRICES AND THE BLOCK VECTORS CALL MATRIX (A, B, RO, H, AK, P, C, N, NN, ANEW) MJ = M - 2MK = M - 1ML = M + 1MM = M + 2FEED THE INITIAL VALUES OF DEFORMATION TO START ITERATION. DO 2 I=1,NN DO 1 J=1,ML U(I,J) = 0.0V(I,J) = 0.0W(I,J) = 0.0CONTINUE W(I, MM) = 0.0CONTINUE START ITERATION

С C

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2 С C

```
KI = 0
FEED IN THE VALUE OF INT AND IKT.
INT = 100
IKT = 1
CONTINUE
NI = NI + 1
COMPUTE THE INTERMEDIATE DEFORMATIONS OF THE TUBE. MAXIMUM
ABSOLUTE VALUE IN THE ERROR VECTOR AND THE NORMALISED SUM OF
ELEMENTS OF ERROR VECTOR IS ALSO COMPUTED.
FMAX=0.0
ESUM=0.0
CALL BNDPRD (A12, U, X, 1, U, NN, ML, 2)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, 2)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A15,W,Y,5,2,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16,W,Y,1,U,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, 2)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16, W, Y, 1, C, NN, MM, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A11, U, Y, 3, 1, NN, ML, 1)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A11:X,NN:3,D)
DO 4 J=1,NN
ESUM=ESUM+Y(J)**2
IF (ABS(Y(J)).LE.EMAX) GO TO 4
EMAX = ABS(Y(J))
RMAX = Y(J)
UI(J,1) = X(J)
CONTINUE
CALL BNDPRD (A12, U, X, 1, 0, NN, ML, 1)
CALL BNDPRD (A12, U, Y, 1, U, NN, ML, 3)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A13,V,Y,3,1,NN,ML,1)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, 1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A15,W,Y,5,2,NN,MM,2)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16,W,Y,1,C,NN,MM,4)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A11, U, Y, 3, 1, NN, ML, 2)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A11,X,NN,3,D)
DO 5 J=1,NN
```

```
IF (ABS(Y(J)).LE.EMAX) GO TO 5
EMAX = ABS(Y(J))
RMAX = Y(J)
UI(J,2)=X(J)
CONTINUE
DO 6 I=3.M
II = I - 2
I J = I - 1
IK = I + 1
IL = I + 2
CALL BNDPRD (A12, U, X, 1, 0, NN, ML, IJ)
CALL BNDPRD (A12, U, Y, 1, U, NN, ML, IK)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, IJ)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, IK)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16,W,Y,1,0,NN,MM,II)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, IJ)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A15,W,Y,5,2,NN,MM,I)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, IK)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16, W, Y, 1, U, NN, MM, IL)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A11, U, Y, 3, 1, NN, ML, I)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A11,X,NN,3,D)
DO 6 J=1,NN
ESUM=ESUM+Y(J)**2
IF (ABS(Y(J)).LE.EMAX) GO TO 6
FMAX = ABS(Y(J))
RMAX = Y(J)
\cup I(J,I) = X(J)
CONTINUE
CALL BNDPRD (A20, U, X, 3, 1, NN, ML, 2)
CALL BNDPRD (A21, V, Y, 1, 0, NN, ML, 2)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A23,W,Y,1,U,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A25,W,Y,3,1,NN,MM,2)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A23,W,Y,1,U,NN,MM,3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A22, V, Y, 3, 1, NN, ML, 1)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A22,X,NN,3,D)
DO 7 J=1,NN
ESUM = ESUM + Y(J) * * 2
IF (ABS(Y(J)).LE.EMAX) GO TO 7
EMAX = ABS(Y(J)).
RMAX = Y(J)
VI(J,1) = X(J)
```

6

CONTINUE

```
CALL BNDPRD (A20, U, X, 3, 1, NN, ML, 1)
CALL BNDPRD (A20, U, Y, 3, 1, NN, ML, 3)
CALL ADDVĖC (X,Y,NN,1.,-1.)
CALL BNDPRD (A21, V, Y, 1, 0, NN, ML, 1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A21, V, Y, 1, 0, NN, ML, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A25,W,Y,3,1,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A25, w, Y, 3, 1, NN, MM, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A23,W,Y,1,U,NN,MM,4)
CALL ADDVEC (X:Y,NN,1.,-1.)
CALL BNDPRD (A22, V, Y, 3, 1, NN, ML, 2)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A22, X, NN, 3, D)
DO 8 J=1,NN
FSUM = ESUM + Y(J) * * 2
IF (ABS(Y(J)).LE.EMAX) GO TO 8
EMAX = ABS(Y(J))
RMAX = Y(J)
VI(J,2) = X(J)
CONTINUE
DO 9 I=3,M
I J = I - 2
IK = I - 1
II = I + I
IM = I + 2
CALL BNDPRD (A20, U, X, 3, 1, NN, ML, IK)
CALL BNDPRD (A20,U,Y,3,1,NN,ML,IL)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A21, V, Y, 1, 0, NN, ML, IK)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A21, V, Y, 1, 0, NN, ML, IL)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A23,W,Y,1,0,NN,MM,IJ)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A25, W, Y, 3, 1, NN, MM, IK)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A25,W,Y,3,1,NN,MM,IL)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A23,W,Y,1,0,NN,MM,IM)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A22,V,Y,3,1,NN,ML,I)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A22,X,NN,3,D)
DO 9 J=1,NN
ESUM=ESUM+Y(J)**2
IF (ABS(Y(J)).LE.EMAX) GO TO 9
EMAX = ABS(Y(J))
RMAX = Y(J)
\forall I(J,I) = X(J)
CONTINUE
CALL BNDPRD (A31, U, X, 3, 1, NN, ML, 1)
CALL BNDPRD (A32, V, Y, 1, 0, NN, ML, 2)
```

8

9

CALL ADDVEC (X,Y,NN,-1.,1.)

```
CALL BNDPRD (A34, W, Y, 3, 1, NN, MM, 2)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A35, W, Y, 1, U, NN, MM, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL ADDVEC (X,B3,NN,1.,1.)
CALL BNDPRD (A36,W,Y,5,2,NN,MM,1)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A36,X,NN,5,E)
DO 10 J=1,NN
ESUM=ESUM+Y(J)**2
IF (ABS(Y(J)).LE.EMAX) GO TO 10
EMAX = ABS(Y(J))
RMAX = Y(J)
WI(J,1) = X(J)
CONTINUE
CALL BNDPRD (A31, U, X, 3, 1, NN, ML, 2)
CALL BNDPRD (A32,V,Y,1,U,NN,ML,1)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A32,V,Y,1,U,NN,ML,3)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A34,W,Y,3,1,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A34, W, Y, 3, 1, NN, MM, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A35,W,Y,1,U,NN,MM,4)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL ADDVEC (X,B3,NN,1.,1.)
CALL BNDPRD (A33, W, Y, 5, 2, NN, MM, 2).
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A33,X,NN,5,E)
DO 11 J=1,NN
ESUM=ESUM+Y(J)**2
IF (ABS(Y(J)).LE.EMAX) GO TO 11
EMAX = ABS(Y(J))
RMAX = Y(J)
WI(J,2)=X(J)
CONTINUE
DO 12 I=3,M
II = I - 2
I J = I - 1
IK = I + 1
IL = I + 2
CALL BNDPRD (A31, U, X, 3, 1, NN, ML, I)
CALL BNDPRD (A32, V, Y, 1, 0, NN, ML, IJ)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A32, V, Y, 1, U, NN, ML, IK)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A35,W,Y,1,U,NN,MM,II)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A34, W, Y, 3, 1, NN, MM, IJ)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A34, W, Y, 3, 1, NN, MM, IK)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A35,W,Y,1,0,NN,MM,IL)
CALL ADDVEC (X,Y,NN,1.,-].)
```

CALL ADDVEC (X,B3,NN,1.,1.)

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11
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```
CALL BNDPRD (A33,W,Y,5,2,NN,MM,I)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A33,X,NN,5,E)
DO 12 J=1,NN
ESUM=ESUM+Y(J)**2
IF (ABS(Y(J)).LE.EMAX) GO TO 12
EMAX = ABS(Y(J))
RMAX=Y(J)
WI(J,I) = X(J)
CONTINUE
NORMALISED SUM OF DEVIATION OF THE NEW VALUES FROM THE OLD VALUES
AND THE BIGGEST ABSOLUTE VALUE IN THE DEVATION VECTOR IS COMPUTED.
AMAX=0.0
SUM=0.0
DO 14 I=1,NN
DO 13 J=1,M
DX=0.0
IF (UI(I,J).NE.0.0) DX=(UI(I,J)-U(I,J))/UI(I,J)
SUM=SUM+DX*DX
IF (ABS(DX).GT.AMAX) AMAX=ABS(DX)
DX=0.0
IF (VI(I,J).NE.0.0) DX=(VI(I,J)-V(I,J))/VI(I,J)
SUM=SUM+DX*DX
IF (ABS(DX).GT.AMAX) AMAX=ABS(DX)
DX=0.0
IF (WI(I,J) \cdot NE \cdot O \cdot O) DX = (WI(I,J) - W(I,J))/WI(I,J)
SUM=SUM+DX*DX
IF (ABS(DX).GT.AMAX) AMAX=ABS(DX)
CONTINUE
CONTINUE
KZ=3*M*NN
VERN=SQRT(SUM/FLOAT(KZ))
ERR=SQRT(ESUM/FLOAT(KZ))
AMEW=0.0
IF (NI.GE.2) AMEW=VERN/VER
RER=0.0
IF (NI.GE.2) RER=ERR/ER
FR=FRR
VER=VERN
WRITE (6,24) NI, VER, AMEW, AMAX
WRITE (6,25) ERR, RMAX, RER
IN CASE OF DIVERGENCE PROGRAM IS STOPPED.
IF (ERR.GT.1.E+03) GO TO 21
NEW VALUE OF DEFORMATIONS AFTER EACH ITERATION IS DETERMINED.
DO 15 I=1,M
REL(I)=0.6
CONTINUE
DO 17 I=1,NN
DO 16 J=1,M
UI(I,J) = UI(I,J) - U(I,J)
```

81.

12 C C C

C

13

14

C

```
VI(I,J) = VI(I,J) - V(I,J)
      WI(I,J) = WI(I,J) - W(I,J)
      U(I,J)=U(I,J)+REL(J)*UI(I,J)
      V(I,J) = V(I,J) + REL(J) * VI(I,J)
      W(I,J) = W(I,J) + REL(J) * WI(I,J)
      CONTINUE
16
17
      CONTINUE
      CONDITION OF CONVERGENCE IS APPLIED.
С
      IF (VER.LT.1.E-4) GO TO 18
      IF (NI.LT.INT) GO TO 3
      GO TO 22
      CALL FREEND (N,NN,M,MJ,MK,ML,MM,EVER)
18
      END OF ONE ITERATION.
C
      NI = 0
      KI = KI + 1
      IF (EVER.LT.1.E-04) GO TO 19
      IF (KI.LT.IKT) GO TO 3
      GO TO 22
      DETERMINE DEFLECTION OF CENTRE OF CROSS-SECTION AND SENSITIVITY
      AT EACH GRID POINT ALONG BETA-DIRECTION.
С
19
      WRITE (6,26)
      I = N/2 + 1
      BETA=0.0
      DO 20 J=1,MM
      BETA=BETA+AK
      AL=SQRT((U(I,J))**2+(V(I,J))**2+4.*U(I,J)*V(I,J)*SIN(BETA)*COS(BET
     1A))
      AF=SQRT((BETA*COS(BETA)-SIN(BETA))**2+(BETA*SIN(BETA)+COS(BETA)-1.
     1) * * 2)
      SEN=AL*C/(AF*RO*P)
      WRITE (6,27) J,AL,SEN.
      CONTINUE
20
      GO TO 23
      WRITE (6,28)
21 .
С
      PROGRAM IS TERMINATED IF CONVERGENCE DOES NOT OCCUR IN A GIVEN
С
      NUMBER OF ITERATIONS.
С
С
22
      WRITE (6,29)
С
      VALUES OF DEFORMATIONS AFTER THE LAST ITERATION ARE PUNCHED AND
С
      WRITTEN, SO THAT IT CAN BE USED IN ANY WAY REQUIRED.
С
С
23
      CONTINUE
      PUNCH 30, ((U(I,J),I=1,NN),J=1,ML)
      PUNCH 30, ((V(I,J),I=1,NN),J=1,ML)
      PUNCH 30, ((W(I,J), I=1, NN), J=1, MM)
      WRITE (6,31)
      WRITE (6,34) ((U(I,J),I=1,NN,2),J=1,ML)
      WRITE (6,32)
```

С

C

CC

C

С

С

```
WRITE (6,34) ((V(I,J),I=1,NN,2),J=1,ML)
      WRITE (6.33)
      WRITE (6,34) ((W(I,J),I=1,NN,2),J=1,MM)
      RETURN
С
24
      FORMAT (1H , 15, 3E14.3)
25
      FORMAT (1H ,47X,3E14.6)
      FORMAT (1H1,*BETA DEFLECTION SENSITIVITY*)
26
27
      FORMAT (1HU, 13, 2E16.6)
      FORMAT (1H0,*ITERATION IS DIVERGING*)
28
      FORMAT (1HU, *NO CONVERGENCE*)
29
30
      FORMAT (5E16.9)
31
      FORMAT (1H1, *VALUES OF 'U'*)
32
      FORMAT (1H1, *VALUES OF 'V'*)
33
      FORMAT (1H1, *VALUES OF 'W'*)
      FORMAT (1H0,10E13.3)
34
      END
```

SUBROUTINE FREEND (N,NN,M,MJ,MK,ML,MM,VERN)

```
PURPOSE
    TO DETERMINE DEFORMATIONS FOR THE POINTS LYING OUTSIDE THE
    BOUNDARY AT FREE END OF THE TUBE, GIVEN THE DEFORMATIONS AT THE
    POINTS LYING INSIDE THE BOUNDARY.
 COMMON /MATRIX/ A11(3,19),A12(1,19),A13(3,19),A14(3,19),A15(5,19),
1A20(3,19),A21(1,19),A22(3,19),A23(1,19),A16(1,19),A25(3,19),A46(1,
219),A31(3,19),A32(1,19),A33(5,19),A34(3,19),A35(1,19),A44(1,19),A4
35(3,19),A54(3,19),A55(1,19),A56(1,19),A36(5,19),A65(5,19),A66(1,19
4),A67(1,19),A76(3,19),A77(1,19),B3(19),B4(19),A80(3,19)
COMMON /DEFORM/ U(19,31), V(19,31), W(19,32), UI(19,31), VI(19,31), WI(
119,32), X(19), Y(19)
 COMMON /WORK/ D(3,19),E(5,19),F(1,19)
 EMAX=0.0
 ESUM=0.0
 CALL BNDPRD (A66,W,X,1,U,NN,MM,MK)
 CALL BNDPRD (A67, W, Y, 1, 0, NN, MM, MJ)
 CALL ADDVEC (X,Y,NN,2.,2.)
 CALL BNDPRD (A65, W, Y, 5, 2, NN, MM, M)
 CALL ADDVEC (X,Y,NN,1.,1.)
 CALL BNDPRD (A80, W, Y, 3, 1, NN, MM, MK)
 CALL ADDVEC (X,Y,NN,1.,1.)
 CALL BNDPRD (A80, W, Y, 3, 1, NN, MM, ML)
 CALL ADDVEC (Y,X,NN,1.,-1.)
 CALL SOLVE (A80,X,NN,3,D)
 DO 1 J=1,NN
 ESUM=ESUM+Y(J)**2
 IF (ABS(Y(J)).LE.EMAX) GO TO 1
 EMAX = ABS(Y(J))
 RMAX = Y(J)
 WI(J,ML) = X(J)
 CONTINUE
 CALL BNDPRD (A77, W, X, 1, U, NN, MM, MJ)
 CALL BNDPRD (A76, W, Y, 3, 1, NN, MM, MK)
 CALL ADDVEC (X,Y,NN,1.,1.)
 CALL BNDPRD (A76, WI, Y, 3, 1, NN, MM, ML)
 CALL ADDVEC (X,Y,NN,1.,-1.)
 CALL BNDPRD (A77, W, Y, 1, 0, NN, MM, MM)
 CALL ADDVEC (Y,X,NN,-1.,1.)
 CALL SOLVE (A77,X,NN,1,F)
 DO 2 J=1,NN
 ESUM = ESUM + Y(J) * * 2
 IF (ABS(Y(J)).LE.EMAX) GO TO 2
 EMAX = ABS(Y(J))
 RMAX = Y(J)
 WI(J,MM) = X(J)
 CONTINUE
 CALL BNDPRD (A55, V, X, 1, 0, NN, ML, MK)
 CALL BNDPRD (A54, U, Y, 3, 1, NN, ML, M)
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CALL BNDPRD (A56,W,Y,1,0,NN,MM,M)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL ADDVEC (X, B4, NN, 1., 1.)
CALL BNDPRD (A55,V,Y,1,0,NN,ML,ML)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A55, X, NN, 1, F)
DO 3 J=1,NN
ESUM=ESUM+Y(J)**2
IF (ABS(Y(J)).LE.EMAX) GO TO 3
EMAX = ABS(Y(J))
RMAX = Y(J)
VI(J,ML) = X(J)
CONTINUE
CALL BNDPRD (A46,U,X,1,0,NN,ML,MK)
CALL BNDPRD (A45,V,Y,3,1,NN,ML,M)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A44, U, Y, 1, 0, NN, ML, ML)
CALL ADDVEC (Y,X,NN,-1.,1.)
CALL SOLVE (A44,X,NN,1,F)
DO 4 J=1,NN
ESUM=ESUM+Y(J)**2
IF (ABS(Y(J)).LE.EMAX) GO TO 4
EMAX=ABS(Y(J))
RMAX = Y(J)
UI(J,ML) = X(J)
CONTINUE
NORMSLISED SUM OF DEVIATION OF THE NEW VALUES FROM THE OLD VALUES
IS COMPUTED.
AMAX=0.0
SUM=0.0
DO 5 I=1.NN
DX=0.0
IF (UI(I,ML).NE.0.0) DX=(UI(I,ML)-U(I,ML))/UI(I,ML)
SUM=SUM+DX*DX
IF (ABS(DX).GT.AMAX) AMAX=ABS(DX)
DX=0.0
IF (VI(I,ML).NE.0.0) DX=(VI(I,ML)-V(I,ML))/VI(I,ML)
SUM=SUM+DX*DX
IF (ABS(DX).GT.AMAX) AMAX=ABS(DX)
DX=0.0
IF (WI(I,ML).NE.0.0) DX=(WI(I,ML)-W(I,ML))/WI(I,ML)
SUM=SUM+DX*DX
IF (ABS(DX).GT.AMAX) AMAX=ABS(DX)
DX=0.0
IF (WI(I,MM).NE.0.0) DX=(WI(I,MM)-W(I,MM))/WI(I,MM)
SUM=SUM+DX*DX
IF (ABS(DX).GT.AMAX) AMAX=ABS(DX)
CONTINUE
KZ = 4 \times NN
ERR=SQRT(ESUM/FLOAT(KZ))
VERN=SQRT(SUM/FLOAT(KZ))
OM=0.1
DO 6 I=1,NN
U(I,ML)=U(I,ML)+OM*(UI(I,ML)-U(I,ML))
```

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V(I,ML)=V(I,ML)+OM*(VI(I,ML)-V(I,ML)) W(I,ML)=W(I,ML)+OM*(WI(I,ML)-W(I,ML)) W(I,MM)=W(I,MM)+OM*(WI(I,MM)-W(I,MM)) CONTINUE WRITE (6,7) AMAX,VERN,ERR,RMAX RETURN

FORMAT (1H0,* AT FREE END*4E14.6/) END

6

С

SUBROUTINE EIGEN (A,B,ANEW,RO,EL,N,M,AH,P,C) PURPOSE TO OBTAIN THE BIGGEST EIGENVALUE OF THE ITERATION MATRIX USING POWER METHOD.

COMMON /MATRIX/ A11(3,19),A12(1,19),A13(3,19),A14(3,19),A15(5,19), 1A20(3,19),A21(1,19),A22(3,19),A23(1,19),A16(1,19),A25(3,19),A46(1, 219),A31(3,19),A32(1,19),A33(5,19),A34(3,19),A35(1,19),A44(1,19),A4 35(3,19),A54(3,19),A55(1,19),A56(1,19),A36(5,19),A65(5,19),A66(1,19) 4),A67(1,19),A76(3,19),A77(1,19),B3(19),B4(19),A8U(3,19) COMMON /DEFORM/ U(19,31),V(19,31),W(19,32),UI(19,31),VI(19,31),WI(119,32),X(19),Y(19)

```
COMMON /WORK/ D(3,19),E(5,19),F(1,19)
COMMON /REL/ REL(3,),EIG(30)
```

DETERMINE THE GRID SIZE AND DIMENSIONLESS PARAMETERS OF BOURDON TUBE.

```
PI=4.*ATAN(1.)
H=PI/FLOAT(N)
AK=BL*PI/(180.*FLOAT(M))
A=2.*A/AH
B=2.*B/AH
RO=2.*RO/AH
NN=N+1
```

DETERMINE THE ELEMENTS OF THE BLOCK MATRICES AND THE BLOCK VECTORS

CALL MATRIX (A, B, RO, H, AK, P, C, N, NN, ANEW) MJ = M - 2MK = M - 1ML = M+1MM = M + 2BEIG=-1.0 DO 1 I=1,M REL(I)=1.0 $EIG(I) = 1 \cdot - REL(I) + BEIG$ CONTINUE FEED THE INITIAL VALUES OF THE VECTOR TO START ITERATION. DO 3 I=1,NN DO 2 J=1,ML U(I,J)=0.0V(I,J) = 0.0W(I,J) = 1.0CONTINUE W(I,ML)=0.0

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C

W(I, MM) = 0.0

CONTINUE

START ITERATION

C

4

5

```
NI = 0
INT = 100
CONTINUE
NI = NI + 1
CALL BNDPRD (A12, U, X, 1, U, NN, ML, 2)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, 2)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A15,W,Y,5,2,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16,W,Y,1,0,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, 2)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16,W,Y,1,0,NN,MM,3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A11, U, Y, 3, 1, NN, ML, 1)
CALL ADDVEC (X,Y,NN,REL(1),EIG(1))
CALL SOLVE (A11,X,NN,3,D)
DO 5 J=1,NN
UI(J,1)=X(J)
CONTINUE
CALL BNDPRD (A12, U, X, 1, U, NN, ML, 1)
CALL BNDPRD (A12, U, Y, 1, U, NN, ML, 3)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, 1)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14,W,Y,3,1,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A15,W,Y,5,2,NN,MM,2) .
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16,W,Y,1,C,NN,MM,4)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A11, U, Y, 3, 1, NN, ML, 2)
CALL ADDVEC (X,Y,NN,REL(2),EIG(2))
CALL SOLVE (A11,X,NN,3,D)
DO 6 J=1.NN
UI(J,2)=X(J)
CONTINUE
DO 7 I=3,M
II = I - 2
I J = I - 1
IK = I + 1
IL = I + 2
CALL BNDPRD (A12, U, X, 1, 0, NN, ML, IJ)
CALL BNDPRD (A12, U, Y, 1, U, NN, ML, IK)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, IJ)
CALL ADDVEC (X;Y,NN,1.,1.)
CALL BNDPRD (A13, V, Y, 3, 1, NN, ML, IK)
CALL ADDVEC (X,Y,NN,1.,-1.)
```

```
CALL BNDPRD (A16,W,Y,1,U,NN,MM,II)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, IJ)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A15,W,Y,5,2,NN,MM,I)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A14, W, Y, 3, 1, NN, MM, IK)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A16, W, Y, 1, U, NN, MM, IL)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A11, U, Y, 3, 1, NN, ML, I)
CALL ADDVEC (X,Y,NN,REL(I),EIG(I))
CALL SOLVE (A11, X, NN, 3, D)
DO 7 J=1,NN
\bigcup I(J,I) = X(J)
CONTINUE
CALL BNDPRD (A20,0,X,3,1,NN,ML,2)
CALL BNDPRD (A21, V, Y, 1, U, NN, ML, 2)
CALL ADDVEC (X,Y,NN,-1.,-1.)
CALL BNDPRD (A23,W,Y,1,0,NN,MM,1)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A25,W,Y,3,1,NN,MM,2)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A23, W, Y, 1, U, NN, MM, 3)
CALL ADDVEC (X,Y,NN,1.,-].)
CALL BNDPRD (A22, V, Y, 3, 1, NN, ML, 1)
CALL ADDVEC (X,Y,NN,REL(1),EIG(1))
CALL SOLVE (A22,X,NN,3,D)
DO 8 J=1,NN
VI(J,1) = X(J)
CONTINUE
CALL BNDPRD (A20, U, X, 3, 1, NN, ML, 1)
CALL BNDPRD (A20, U, Y, 3, 1, NN, ML, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A21, V, Y, 1, U, NN, ML, 1)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A21, V, Y, 1, 0, NN, ML, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A25, W, Y, 3, 1, NN, MM, 1)
CALL ADDVEC (X,Y,NN,1.,1.)
CALL BNDPRD (A25, W, Y, 3, 1, NN, MM, 3)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A23,W,Y,1,0,NN,MM,4)
CALL ADDVEC (X,Y,NN,1.,-1.)
CALL BNDPRD (A22, V, Y, 3, 1, NN, ML, 2)
CALL ADDVEC (X,Y,NN,REL(2),EIG(2))
CALL SOLVE (A22,X,NN,3,D)
DO 9 J=1.NN
VI(J,2) = X(J)
CONTINUE
DO 10 I=3.M
I J = I - 2
IK = I - 1
II = I + 1
```

```
IM=I+2
CALL BNDPRD (A20,0,X,3,1,NN,ML,IK)
```

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CALL BNDPRD (A21, V, Y, 1, U, NN, ML, IK) CALL ADDVEC (X:Y:NN,1.,-1.) CALL BNDPRD (A21, V, Y, 1, 0, NN, ML, IL) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A23,W,Y,1,U,NN,MM,IJ) CALL ADDVEC (X,Y,NN,1.,1.) CALL BNDPRD (A25, W, Y, 3, 1, NN, MM, IK) CALL ADDVEC (X,Y,NN,1.,1.) CALL BNDPRD (A25, W, Y, 3, 1, NN, MM, IL) CALL ADDVEC (X,Y,NN,1..-1.) CALL BNDPRD (A23,W,Y,1,U,NN,MM,IM) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A22, V, Y, 3, 1, NN, ML, I) CALL ADDVEC (X,Y,NN,REL(I),EIG(I)) CALL SOLVE (A22,X,NN,3,D) DO 10 J=1,NN $\forall I(J,I) = X(J)$ CONTINUE CALL BNDPRD (A31, U, X, 3, 1, NN, ML, 1) CALL BNDPRD (A32, V, Y, 1, 0, NN, ML, 2) CALL ADDVEC (X,Y,NN,-1.,1.) DO 11 J=1,NN WI(J,1) = X(J)CONTINUE DO 12 J=1,NN

12

CALL BNDPRD (A34,W,Y,3,1,NN,MM,2) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A35,W,Y,1,0,NN,MM,3) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A36,W,Y,5,2,NN,MM,1) CALL ADDVEC (X,Y,NN,REL(1),EIG(1)) CALL SOLVE (A36, X, NN, 5, E) CALL BNDPRD (A31, U, X, 3, 1, NN, ML, 2) CALL BNDPRD (A32, V, Y, 1, 0, NN, ML, 1) CALL ADDVEC (X,Y,NN,-1.,-1.) CALL BNDPRD (A32, V, Y, 1, 0, NN, ML, 3) CALL ADDVEC (X,Y,NN,1.,1.) CALL BNDPRD (A34,W,Y,3,1,NN,MM,1) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A34, W, Y, 3, 1, NN, MM, 3) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A35,W,Y,1,0,NN,MM,4) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A33,W,Y,5,2,NN,MM,2) CALL ADDVEC (X,Y,NN,REL(2),EIG(2)) CALL SOLVE (A33,X,NN,5,E) WI(J,2) = X(J)CONTINUE DO 13 I=3,M II = I - 2I J = I - 1IK = I + 1IL = I + 2CALL BNDPRD (A31, U, X, 3, 1, NN, ML, I)

CALL BNDPRD (A20,U,Y,3,1,NN,ML,IL)

CALL ADDVEC (X,Y,NN,1.,-1.)

CALL BNDPRD (A32, V, Y, 1, U, NN, ML, IJ) CALL ADDVEC (X,Y,NN,-1.,-1.) CALL BNDPRD (A32, V, Y, 1, U, NN, ML, IK) CALL ADDVEC (X,Y,NN,1.,1.) CALL BNDPRD (A35,W,Y,1,U,NN,MM,II) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A34, W, Y, 3, 1, NN, MM, IJ) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A34, W, Y, 3, 1, NN, MM, IK) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A35,W,Y,1,0,NN,MM,IL) CALL ADDVEC (X,Y,NN,1.,-1.) CALL BNDPRD (A33,W,Y,5,2,NN,MM,I) CALL ADDVEC (X,Y,NN,REL(I),EIG(I)) CALL SOLVE (A33,X,NN,5,E) DO 13 J=1,NN WI(J,I) = X(J)13 CONTINUE FLEMENT WITH THE BIGGEST ABSOLUTE VALUE IS SEARCHED AND THE NEW VECTOR IS COMPUTED BY DIVIDING THE INTERMEDIATE VECTOR BY THIS VALUE. RMAX=0.0DO 17 I=1,NN DO 16 J=1.M IF (ABS(UI(I,J)).LE.RMAX) GO TO 14 RMAX = ABS(UI(I,J))AMAX=UI(I,J) CONTINUE 14 IF (ABS(WI(I,J)).LE.RMAX) GO TO 15 RMAX = ABS(WI(I,J))AMAX = WI(I,J)15 CONTINUE IF (ABS(VI(I,J)).LE.RMAX) GO TO 16. RMAX=ABS(VI(I,J)) AMAX=VI(I,J) CONTINUE 16 CONTINUE 17 DO 19 I=1,NN DO 18 J=1,M U(I,J) = UI(I,J) / AMAXV(I,J) = VI(I,J) / AMAXW(I,J) = WI(I,J) / AMAXCONTINUE 18 19 CONTINUE IF (NI.EQ.1) AMA=0.0 DX=ABS((AMAX-AMA)/AMAX) WRITE (6,23) NI,AMAX AMA = AMAXIF (DX.LE.1.E-04) GO TO 20 IF (NI.GE.INT) GO TO 21 END OF ONE ITERATION. GO TO 4

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20	AMA=AMA-BEIG WRITE (6,24) AMA
	GO TO 22
21	WRITE (6,25)
C	IN CASE OF NO CONVERGENCE GET THE PUNCHED OUTPUT OF THE VALUES
C	OF THE ELEMNIS OF THE VECTOR . OBTAINED AFTER THE LAST ITERATION.
c	of the letting of the feetok , obtained in the end fremention
C	PUNCH 26. $((U(1,1), I=1, NN), I=1, MI)$
	$PUNCH 26 \cdot ((V(1 \cdot 1) \cdot 1 = 1 \cdot NN) \cdot 1 = 1 \cdot M1)$
	$DUNCH 263 ((V(1)0)) = 1 \times NN (0) = 1 \times NN (0)$
22	CONTINUE
22	CONTINUE
	RETORN
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23	FORMAT (1H ,15,E21.9)
24	FORMAT (2H0, *THE EIGENVALUE OF THE ITERATION MATRIX =*, E14.7)
25	FORMAT (1H0,*NO CONVERGENCE*)
26	FORMAT (5E16.8) END

SUBROUTINE MATRIX (A,B,RO,H,AK,P,C,N,NN,ANEW)

```
PURPOSE
   TO DETERMINE THE ELEMENTS OF BLOCK MATRICES AND BLOCK VECTORS
   STORED IN A GIVEN MANNER.
 COMMON /MATRIX/ A11(3,19),A12(1,19),A13(3,19),A14(3,19),A15(5,19),
1A20(3,19),A21(1,19),A22(3,19),A23(1,19),A16(1,19),A25(3,19),A46(1,
219),A31(3,19),A32(1,19),A33(5,19),A34(3,19),A35(1,19),A44(1,19),A4
35(3,19),A54(3,19),A55(1,19),A56(1,19),A36(5,19),A65(5,19),A66(1,19)
4),A67(1,19),A76(3,19),A77(1,19),B3(19),B4(19),A80(3,19)
ALPHA=0.0
 DO 5 I=1.NN
 CALL COE (ALPHA, ANEW, A, B, RO, H, AK, A1, A2, A3, A4, B1, B2, C1, C2, D1, D2, D3,
1D4,E1,F1,F2,F3,F4,F5,F6,G1,G2,G3,G4,H1,H2,H3,H4,H5,H6,H7,H8,H9,O1,
202,03,04,P1,P2,P3,Q1,Q2,Q3,Q4,R1,R2,R3)
 A11(1,I)=A2-A3
 A11(2,I)=-2.*A1-2.*A2+A4
 A11(3,I) = A2 + A3
 A12(1,I) = A1
 A13(1,I)=-B1
 A13(2,I) = B2
 A13(3,I)=B1
 A14(1,I) = -F2
 A14(2,I) = F3 \times 16.
 A14(3 \cdot I) = F2
 A15(1,I) = -F1 - F4 + F5
 A15(2,1)=2.*F1+2.*F2+F4*16.-8.*F5
 A15(3,I) = -30 \cdot F3 - 30 \cdot F4 + F6
 A15(4,I)=-2.*F1-2.*F2+F4*16.+8.*F5
 A15(5,I) = F1 - F4 - F5
 A16(1,I) = -F3
 A20(1,I) = -B1
 A20(2,I) = -B2
A20(3,I)=B1
A21(1,I)=D1
 A22(1,I) = D2 - D3
 A22(2,1) = -2 \cdot D1 - 2 \cdot D2 + D4
A22(3,I)=D2+D3
A_{23}(1, I) = G_{1} - G_{4}
 A25(1,I) = G2 - G3
A25(2,I)=-2.*G1-2.*G2+G4*8.
A25(3,I)=G2+G3
A31(1,I) = -C1
A31(2,I)=C2
A31(3,I)=C1
A32(1,I) = -E1
A33(1,I)=H2-H5-H7+H8
A33(2,1)=-4.*H2-2.*H3+2.*H4+2.*H5+H7*16.-8.*H8
A33(3,I)=6.*H1+6.*H2+4.*H3-30.*H6-30.*H7+H9
A33(4,I)=-4.*H2-2.*H3-2.*H4-2.*H5+H7*16.+8.*H8
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A33(5,I) = H2 + H5 - H7 - H8
A34(1,I)=H3-H4
A34(2,I)=-4.*H1-2.*H3+H6*16.
A34(3,I)=H3+H4
A35(1,1) = H1 - H6
A44(1,I) = R1
A45(1,I) = -R2
A45(2,I)=R3
A45(3,I) = R2
A46(1,I) = -R1
A54(1,I) = -02
A54(2,1)=03
A54(3,I)=02
A55(1,I)=01
A56(1,I)=04
A65(1,I) = -P2 + P3
A65(2,I)=16.*P2-8.*P3
A65(3,I)=-30.*P1-30.*P2
A65(4,I)=16.*P2+8.*P3
A65(5,I) = -P2 - P3
A66(1,I)=16.*P1
A67(1,I) = -P1
A76(1, I) = Q2 - Q3
A76(2,I)=-2.*Q1-2.*Q2+8.*Q4
A76(3, I) = Q2 + Q3
A77(1,I) = Q1 - Q4
B3(I)=P*(1.-ANEW**2)/(2.*C)
B4(I)=B3(I)*A*B/(2•*SQRT((A*A+B*B)/2•))
ELEMENTS AT THE BOUNDARIES ARE DETERMINED
IF (I.NE.2) GO TO 1
A15(3,I)=A15(3,I)+A15(1,I)
A33(3,I)=A33(3,I)+A33(1,I)
A65(3,I)=A65(3,I)+A65(1,I)
IF (I.NE.N) GO TO 2
A15(3,I)=A15(3,I)+A15(5,I)
A33(3,I)=A33(3,I)+A33(5,I)
A65(3,I)=A65(3,I)+A65(5,I)
 IF (I.NE.1) GO TO 3
 A11(2,I)=1.0
 A11(3,I)=0.0
 A12(1,I)=0.0
 A13(2,I)=0.0
 A13(3,I)=0.0
 A14(2,I)=0.0
 A14(3,I)=0.0
 A15(3,I)=0.0
 A15(4,I)=0.0
 A15(5,I)=0.0
 A16(1,I)=0.0
 A20(3,I) = A20(3,I) - A20(1,I)
 A22(3,I) = A22(3,I) + A22(1,I)
 A25(3,I)=A25(3,I)+A25(1,I)
 A31(3,I)=A31(3,I)-A31(1,I)
 A33(4,I)=A33(4,I)+A33(2,I)
```

C C

С

1

2

```
A33(5,I) = A33(1,I) + A33(5,I)
A34(3,I) = A34(3,I) + A34(1,I)
A44(1,1)=1.0
A45(2,I)=0.0
A45(3,I)=0.0
A46(1,I)=0.0
A54(3,I) = A54(3,I) - A54(1,I)
A65(4,I) = A65(4,I) + A65(2,I)
A65(5,I) = A65(1,I) + A65(5,I)
A76(3,I) = A76(3,I) + A76(1,I)
IF (I.NE.NN) GO TO 4
A11(1,I)=0.0
A11(2,I)=1.0
A12(1,I)=0.0
A13(1,I)=0.0
A13(2,I)=0.0
A14(1,I)=0.0
A14(2,I)=0.0
A15(1,I)=0.0
A15(2,I)=0.0
A15(3,I)=0.0
A16(1,I)=0.0
A20(1,I) = A20(1,I) - A20(3,I)
A_{22}(1,I) = A_{22}(3,I) + A_{22}(1,I)
A_{25}(1,I) = A_{25}(3,I) + A_{25}(1,I)
A31(1,I) = A31(1,I) - A31(3,I)
A33(1,I) = A33(1,I) + A33(5,I)
A33(2,I) = A33(4,I) + A33(2,I)
A34(1,I) = A34(3,I) + A34(1,I)
A44(1,I)=1.0
A45(1,I)=0.0
A45(2,I)=0.0
A46(1,1)=0.0
A54(1,I) = A54(1,I) - A54(3,I)
A65(1,I) = A65(1,I) + A65(5,I)
A65(2,I) = A65(4,I) + A65(2,I)
A76(1,I) = A76(3,I) + A76(1,I)
CONTINUE
A36(1,I) = A33(1,I)
A36(2,I) = A33(2,I)
A36(3,I)=A33(3,I)+A35(1,I)
A36(4,I) = A33(4,I)
A36(5,I) = A33(5,I)
A80(1,I) = A76(1,I) * A67(1,I) / A77(1,I)
A8U(2,I) = A76(2,I) * A67(1,I) / A77(1,I) - A66(1,I)
A80(3,1)=A76(3,1)*A67(1,1)/A77(1,1)
ALPHA=ALPHA+H
CONTINUE
RETURN
```

END

3

4

EVALUATION OF METRIC COEFFICIENT AND CURVATURE AS DIMENSIONLESS QUANTITIES. SI = SIN(ALPHA)CO=COS(ALPHA) DI=A**2-B**2 GB=RO+A*CO GA=SQRT((A*SI)**2+(B*CO)**2) GBB=GB**2GAA=GA**2 CBN=B*CO/(GA*GB) CBG=A*SI/(GA*GB) CAN=(A*B)/(GA**3)

PURPOSE

EVALUATION OF ALL CO-EFFIICIENTS INVOLVED IN THE GIVEN EQUATIONS AND BOUNDARY CONDITIONS.

SUBROUTINE COE (ALPHA, ANEW, A, B, RO, H, AK, A1, A2, A3, A4, B1, B2, C1, C2, D1, 1D2,D3,D4,E1,F1,F2,F3,F4,F5,F6;G1,G2,G3,G4,H1,H2,H3,H4,H5,H6,H7,H8,

TO FIND ALL THE COEFFICIENTS INVOLVED IN THE PARTIAL DIFFEREN-

2H9,01,02,03,04,P1,P2,P3,Q1,Q2,Q3,Q4,R1,R2,R3)

TIAL EQUATIONS AND BOUNDARY CONDITIONS.

A1=(1.0-ANEW)/(2.0*GBB)/(AK*AK) $A2=1 \cdot U/GAA/(H*H)$ A3=(CBG/GA+DI*SI*CO/(GAA**2))/(-2.*H) A4=-(CBG**2+ANEW/GA*(A*CO/(GA*GB)-CBG*DI*CO*SI/GAA)) B1=(1.0+ANEw)/(2.0*GA*GB)/(4.*H*AK) B2=-CBG*(ANEW-3.0)/(4.*GB*AK) C1=-(CAN+ANEW*CBN)/GA/(2.*H) C2=CBG*(CBN+ANEW*CAN) D1=1.0/(GBB*AK*AK)D2=(1.-ANEW)/(2.0*GAA*H*H) D3=-(1.0-ANEW)/(2.0*GA)*(CBG+DI*SI*CO/(GA**3))/(2.*H) D4=(1.0-ANEW)/2.0*(A*CO/(GAA*GB)-CBG**2-CBG*DI*SI*CO/(GA**3)) E1=-(CBN+ANEW*CAN)/(GB*2.*AK) F1=CAN/(3.0*(GA**3)*2.0*H**3) $F_2=CAN/(3.0*GA*GBB*2.*H*AK*AK)$ F3=(2./3.*CAN*CBG/GBB)/(12.*AK*AK) F4=-CAN/3./GAA*(CBG+3.0*DI*SI*CO/(GA**3))/(12.*H*H) F5=-((CAN+ANEW*CBN)/GA+CAN/3•/GA*(DI*(CO**2-SI**2)/(GAA**2)-CBG*(1 1.0+ANEW)*DI*SI*CO/(GA**3)+CBG**2-4.0*(DI*SI*CO)**2/(GAA**3)+ANEW*A 2*CO/(GAA*GB)))/(12•*H) F6=-(CBG*(CBN+ANEW*CAN)-CBG*CAN-ANEW*B*SI/(GAA*GB)-DI*SI*CO/GA*(3. $1 \times A \times B / (GA \times 5) + ANEW \times CBN / GAA))$ G1=CBN/3.0/(GB**3)/(2.*AK**3) $G_2 = CBN/3 \cdot U/(GB * GAA)/(2 \cdot *H * H * AK)$ G3=-CBN/(3.0*GA*GB)*(CBG+DI*SI*CO/(GA**3))/(4.*H*AK) G4=-((CBN+ANEW*CAN)/GB+(1.0-ANEW)*CBN/(3.*GAA*GB)*(CBG*DI*SI*CO/GA 1-A*CO/GB))/(12.*AK) H1=1.0/3.0/(GBB**2)/(AK**4)

```
H3=2 \cdot U/(3 \cdot U \times GAA \times GBB)/(H \times H \times AK \times AK)
 H4=-2.0/(3.*GBB)*(DI*SI*CO/(GAA**2)-CBG/GA)/(2.*H*AK*AK)
 H5=-1 • / (GA**3) / 3 • * (6 • * DI*SI*CO/(GA**3) + 2 • 0 * CBG) / (2 • * H**3)
 H6=-(CBG*(3.-ANEW)*DI*SI*CO/(3.*GBB*GA**3)-2.*CBG**2*(2.0+3.*ANEW)
1/(3.*GBB)-(3.-ANEW)*A*CO/(3.*GAA*GB**3))/(12.*AK*AK)
 H7=-(CBG**2/GAA+A*CO*(1.+ANEW)/(GAA**2*GB)-CBG*DI*SI*CO*(7.+ANEW)/
1(GA**5)+4.0*DI*(CO**2-SI**2)/(GAA**3)-19.*(DI*SI*CO)**2/(GAA**4))/
2(36.*H*H)
 H8=-(-3.*CBG**2*DI*SI*CO/(GAA**2)+CBG**3/GA+2.*CBG*DI*SI**2*(1.+AN
1EW)/(GA**5)-CBG*DI*CO**2*(3.+5.*ANEW)/(GA**5)+CBG*A*CO*(1.+ANEW)/(
2GA**3*GB)+CBG*(DI*SI*CO)**2*(9.+5.*ANEW)/(GA**7)+(-4.*DI*SI*CO/GA*
3*5-13.*DI**2*SI*CO*(CO**2-SI**2)/(GA**7)+28.*(DI*SI*CO)**3/(GA**9)
4)/GA-CBG*ANEW/(GA**3))/(36.0*H)
 H9=CAN**2+CBN**2+2.*ANEW*CAN*CBN
 01=1./(GB*AK*2.)
 02=ANEW/(GA*2.*H)
 03=-CBG
 O4 = -(CBN + ANEW * CAN)
 P1=1./(GBB*12.*AK*AK)
 P2 = ANEW / (GAA + H + H + 12)
 P3=-(CBG/GA+ANEW*DI*SI*CO/(GAA**2))/(12.*H)
 R1=1.0/(GB*AK*2.)
R2=1.0/(GA*2.*H)
R3 = CBG
Q1=1./(GB**3*2.*AK**3)
Q_2=(2 - ANEW)/(GAA + GB + H + AK + 2 )
Q3=-(CBG*(2•*ANEW-]•)+(2•-ANEW)*DI*SI*CO/(GA**3))/(GA*GB*4•*H*AK)
 Q4=-2•*(1•-ANEW)*(CBG*DI*SI*CO/(GA**3)-A*CO/(GAA*GB)-CBG**2)/(GB*A
1K*12.)
RETURN
END
```

SUBROUTINE BNDPRD (X,Y,Z,M,L,N,K,J)

SUBROUTINE BNDPRD PURPOSE TO MULTIPLY A BANDED MATRIX WITH A SPECIFIED COLUMN OF ANOTHER GIVEN MATRIX. USAGE CALL BNDPRD(X,Y,Z,M,L,N,K,J) DESCRIPTION OF PARAMETERS - THE GIVEN BANDED MATRIX STORED AS STATED IN REMARKS. X Y - THE OTHER GIVEN MATRIX WHOSE ONE SPECIFIED COLUMN IS TO BE MULTIPLIED WITH THE BANDED MATRIX X. Z - THE RESAULTANT VECTOR. L - THE NUMBER OF NON-ZERO DIAGONALS ABOVE OR BELOW OF THE DIAGONAL OF X. M - THE NUMBER OF TOTAL NON-ZERO DIAGONALS OF X, M=2*L+1 - NUMBER OF ROWS OF MATRIX X OR Y OR VECTOR Z. N - NUMER OF COLUMNS OF MATRIX Y. K - THE SPECIFIED COLUMN OF MATRIX Y. J REMARKS A GIVEN MATRIX A IS STORED AS X IN A MANNER AS GIVEN BELOW A(I,J) IS STORED IN LOCATION X(L+1+J-I,I) AND A(I,J) IS ZERO IF ABS(I-J) IS GREATER THAN L. METHOD COLUMN J OF MATRIX Y IS MULTIPLIED WITH BANDED MATRIX X AND THE RESULT IS STORED AS VECTOR Z. DIMENSION X(1), Y(1), Z(1)DO 1 I=1.N Z(I) = 0.0DO 1 KJ=1,M LM = I + KJ - L - 1IF (LM.LT.1.OR.LM.GT.N) GO TO 1 LL = (I - 1) * M + KJLK = (J-1) * N + LMZ(I) = Z(I) + X(LL) * Y(LK)CONTINUE RETURN END

С
SUBROUTINE ADDVEC (X,Y,N,A,B)

SUBROUTINE ADDVEC

PURPOSE

TO FIND THE SUM OF TWO VECTORS OBTAINED BY MULTIPLYING TWO GIVEN VECTORS WITH TWO GIVEN CONSTANS RESPECTIVELY. SUCH THAT X=A*X+B*Y

USAGE

CALL ADDVEC(X,Y,N,A,B)

DESCRIPTION OF PARAMETERS

Х	-	THE FIRST GIVEN VECTOR WHICH IS DESTROYED AND	THE
		RESAULTANT VECTOR IS STORED IN THE LOCATION.	
Y	-	THE SECOND GIVEN VECTOR.	
N	-	LENGTH OF THE VECTOR X OR Y.	
А		THE FIRST CONSTANT.	
B	-	THE SECOND CONSTANT.	

METHOD

THE VECTOR X IS MULTIPLIED WITH CONSTANT A, AND VECTOR Y IS MULTIPLIED WITH CONTANT B, THE RESAULTANTS ARE ADDED AND STORED AS VECTOR X.

DIMENSION X(1), Y(1)

DO 1 I=1,N X(I)=A*X(I)+B*Y(I) CONTINUE RETURN END

SUBROUTINE SOLVE (A,X,N,L,D)

SUBROUTINE SOLVE PURPOSE TO SOLVE A SET OF SIMULTANEOUS LINEAR EQUATIONS AX=B, WHERE A IS A BANDED MATRIX OF L DIAGONALS, WITHOUT GETTING A DESTROYED. USAGE CALL SOLVE (A, X, N, L, D) DESCRIPTION OF PARAMETERS -THE GIVEN BANDED MATRIX OF L DIAGONALS, STORED AS STATED A BELOW IN REMARKS. -THE VECTOR OF ORIGINAL CONSTANTS, DESTROYED IN THE Х PROCESS AND REPLACED BY SOLUTION VECTOR. -THE LENGTH OF VECTOR X. N -TOTAL NUMBER OF NON-ZERO DIAGONALS OF THE MATRIX A . L -WORK MATRIX OF SIZE L BY N. D NAME OF SUBROUTINES USED DIAG3, BNDSOL REMARKS THE MATRIX A IS STORED AS A MATRIX OF DIMENSION L BY N, SUCH THAT ANY ELEMENT A(I,J) OF ORIGINAL MATRIX IS STORED AS A(M,I), WHERE M = J - I + 1 + (L - 1)/2METHOD AFTER STORING MATRIX A IN WORK MATRIX D , THE GENERAL METHODS OF SOLVING A SET OF SIMULTANEOUS EQUATIONS WITH BANDED MATRIX ARE APPLIED. DIMENSION A(L,1), D(L,1), X(1) DO 1 I=1.L DO 1 J=1.N D(I,J) = A(I,J)CONTINUE IF (L.NE.1) GO TO 3 DO 2 I=1 .N X(I) = X(I)/D(I)CONTINUE GO TO 5 IF (L.NE.3) GO TO 4 CALL DIAG3 (D,X,N) GO TO 5 M=L/2 CALL BNDSOL (D,X,L,M,N) RETURN END

5

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