Refractor Plates Optimization and Data Analysis of a Correlation Spectrometer, used as a remote sensing instrument.

by

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PART A: Industrial Project\*

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\* One of the two project reports. The other part is desingated PART B: McMaster ( On-Campus ) Project.

### PREFACE

The work described in this report was conducted at Atmospheric Enviroment Service of Canada from May 28th to September 28th, 1973 under a cooperative arrangement between Atmospheric Enviroment Service of Canada and McMaster University, Hamilton. The aim was to provide an M. Eng. student from McMaster with an opportunity to gain experience in an industrial environment while fulfilling a need generated by industry. This report was part of a project on a modified correlation spectrometer, developed to be used as a point sample. TABLE OF CONTENTS

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### SUMMARY

The Barringer Refractor Plate Correlation Spectrometer has been used as a remote sensing instrument for the detection of SO<sub>2</sub> and NO<sub>2</sub>. Its efficiency is given by the equation of the signal to noise ratio, with the gate waveform duty cycle as one of its parameter.

This report is divided into two parts. Part A is a theoretical study of optimization of the refractor plates, which affects the gate waveform duty cycle. A computer program is developed which tabulates the angle between the refractor plates, the width, the area, and its length. The corresponding gate waveform duty cycle is also given.

Part B is an analysis of the data taken with the correlation spectrometer in a Sudbury project. This project was carried out by the Air Research Quality Branch of Atmospheric Environment Service. SO<sub>2</sub> emission from stackplumes near Sudbury area were measured and the results were analysed . Thus, the feasibility of remote sensing using a correlation spectrometer is demonstrated.

# PART A: REFRACTOR PLATE OPTIMIZATION OF A CORRELATION SPECTROMETER, USED AS A REMOTE SENSING INSTRUMENT.

## 1). Instrumental Description and Background.

### 1.1 Introduction

A correlation spectrometer, using the property of molecular band absorption, for the remote sensing of air-polluting gases of SO<sub>2</sub> and NO<sub>2</sub> was developed by the Barringer Research Limited. The theory, applications and results were clearly described by Millan and Newcomb (1). A more detailed and extended theoretical and experimental description was given by Millan (2).

In its simplest form, a schematic diagram of the correlation spectrometer is given in Fig. 1. The incoming light is reflected through mirrors M1, M2, then passes through the field lens onto the entrance slit. It is then refracted through a pair of moving refractor plates onto mirror M3, which directs the light towards a grating. The dispersed light from the grating strikes the mirror M3 again and is reflected towards the exit slit. A correlation mask is situated at the exit plane for sampling the desired region of the output spectrum. The resultant light through the mask is collected by a photomultiplier tube and analysed through an electronic circuit. The final result of the detected concentration of the polluting gas in parts per million per meter path-length (ppm-m) along the line of observation is indicated by a meter and graphical display is available through a chart recorder. Typical output graph is shown in Part B.

### 1.2 Modes of Operation

There are two different modes of operating the correlation spectrometer. In the active mode, a light source of known spectral intensity is used. It has the advantage of minimizing background interferences and gives a better signal to noise ratio during measurements. However, in remote sensing, the setting up of a light source is not always feasible and passive mode of observation has to be used.



FIG. 1 SCHEMATIC DIAGRAM OF THE CORRELATION SPECTROMETER

In the passive mode, the radiation from the sun is used as a source. It is relatively difficult to calculate the sun's spectral intensity because it is changing with time, zenith angle, and surface albedo. Besides these, the scattering and reflection of light from dispersed particles in the air and clouds cause extra interference. These variables must be known for accurate calculation of absolute concentration.

In both cases, the basic equation used is the Beer-Lambert Law, given by

$$H(\Lambda) = H_{o}(\Lambda) \exp(-\mathcal{O}(\Lambda)nx)$$

where  $H_o(\Lambda)$  is the spectral radiance of the light source,

 $\sigma(\lambda)$  is the absorption coefficient of the polluting gas molecules at wavelength  $\lambda$ ,

n is the number of absorbing molecules per unit volume,

x is the pathlength along line of observation,

 $H(\lambda)$  is the observed spectral intensity.

1.3 Reference Cell and Grating

The reference cell provides a standard for measurements. A fixed amount of gas sample that is to be detected is sealed inside the cell. It can be rotated into the path of view and withdrawn at any time. Prior to each measurement, the reference cell is used to calibrate the concentration scale on the chart recorder against a clear background. As long as the background radiation does not change rapidly, the instrument needs no recalibration for each measurements.

The dispersive grating is a reflective type. It separates the incoming light into its various wavelength components so that the output spectrum is a function of wavelength position. By means of this, the correlation mask is able to sample at the required spectral region to get the best signal to noise ratio. The grating can be rotated within a limited range and can be locked into position.

### 1.4 Entrance Slit and Exit Mask

According to the calculation given by Millan (2), the entrance slit width must be equal to the exit slit width for the best signal to noise ratio. For  $SO_2$ , it is about 8 Angstrom. Its physical dimension depends on the dispersive power of the spectrometer. For increasing signal throughput, multiple entrance slits are used.

The exit mask is made up of several slits spacing differently. At one position of the refractor plates, the slits are at the peaks of the absorption spectrum, and are approximately at the troughs for the other position. The location of the slits with respect to the two spectrum positions are shown in FIG. 2. The detailed mechanism and amount of shift in the two spectrum positions through the moving refractor plates will be given later. For optimum signal to noise ratio, there is a certain number of slits, with different width and spacing, per mask. Different masks are used for different gases.

### 1.5 Electronic Processing

Depending on whether the instrument is working on bistable or multistable measurements, there will be one or two automatic gain control (AGC) circuits. These help to adjust the input signal from the photomultiplier tube to a standard reference current level, thus control the photomultiplier gain within the operating range. The signal waveforms from the different measurements are multiplied by a generated synchronous gate waveform and the resultant is passed through an integrator. Some algebric manipulation is performed and a numerical value is given to indicate the relative amount of gas detected in ppm-m when compared with the reference cell.

## 2). Modulation Technique and Refractor Plates Orientation.

### 2.1 Modulation Technique

By means of modulation, the output spectrum can be sampled at various desirable wavebands. Therefore it provides the correlation between the exit mask and the output spectrum. In the following, two types of modulation technique are considered and their respective merits are discussed.

In the refractor plate modulation, as shown in FIG. 2, the refractor plates are mounted on a tuning fork which is vibrating at its resonant frequency. The refractor plates are located directly behind the entrance slit. As the refractor plates are moving, the incoming light is refracted into two positions, separated by distance 5, after passing through the plates. A correlation mask, having one set of slits, is placed at the exit plane and samples at the two output spectrums at different positions. In this method, the moving mechanism provided by the resonant tuning fork is very stable and has a long usable life time. The gate duty cycle is good and the noise level is relatively low. Besides that, the slits on the mask are easily etched. However, the best signal is not obtained because each slit is not sampling at the maximum and minimum positions of the spectrum respectively in the two positions. A compromise jump, S, can be found to get an optimum signal.

In the rotating disk modulation, as shown in FIG. 3, the output spectrum is fixed in position at the exit plane. Two sets of slits are etched along the surface of a rotating disk, driven by a small motor. The first set of slits is sampling at the maximum positions of the absorption output spectrum, and the second set of slits is at the minimum positions. In this way, the best signal is obtained. However, the mechanical motion provided by the motor is not always uniform and the disk wobbles as it turns. This accounts for the observed gate jitter noise. The etching of the slits along the surface of ths disk is relatively difficult to achieve.

In the prototype of the Barringer Correlation Spectrometer, refractor plates was used. But in later developement, rotating disk modulation has





FIG. 3 ROTATING DISK MODULATION

been employed. In this report, refractor plates are studied and its parameter optimized to obtain a good duty cycle.

2.2 Consideration in Refractor Plates' Orientation

Two positions of the refractor plates are possible with respect to the incoming light. Because of the width of the entrance slit, distance of source from slit, distance of refractor plates from entrance slit, and spreading of the source, the incoming light is divergent. This accounts for the transient period where a constant signal cannot be obtained as the refractor plates move across the entrance slit.

In the inverted V-shape position, as shown in FIG. 4A, when the light is within plate positions a and a', some of it will be refracted into the other plate. Depends on the angle of divergent source, the refractor plate angle, and distance of slit from plate, the refracted light into the other plate may be totally internal reflected and lost. This orientation was used in the prototype of the Barringer Correlation Spectrometer.

In the V-shape position, as shown in FIG. 4B, a similar transition zone is between b and b', but now the chance of refracted light into the other plate is less than in the above case. It can be seen clearly that if the incoming light is narrow and parallel, there is no chance of crossing over into the other plate for the V-shape orientation, but not for the case of the inverted V-shape orientation.

The V-shape orientation is studied in the following and the relation between refractor plate positions and gate duty cycle is obtained.



3.1 Area Calculation of Refractor Plates

The V-shape orientation of the refractor plates is considered. From the mirror symmetry of the two refractor plates, we can consider only one of them. By tracing the incident light through one of the refractor plate, as shown in FIG. 5, the relation between the parameters can be obtained from simple geometrical consideration, as following;

$$\frac{SIN i}{SIN r} = n$$
(2)

where i is the angle of incident,

r is the angle of refraction,

- 0 is the angle of the refractor plate,
- n is the refractive index of the material of the refractor

plate at the wavelength of the incoming light.

From the property and position of the refractor plates, the height of it does not affect the spectral jump, S; therefore the weight of the refractor plates is proportional to the area. For simplicity of derivation, the incoming light is assumed to be monochromatic, therefore the refractive index, n, is a constant. In actual calculation, different values of n must be used for different wavelength of incident light.

Area of plate 
$$=\frac{td}{SIN\Theta} + \frac{t^2 COT \Theta}{2}$$
 (3)

where t is the thickness of the refractor plates,

d is the horizontal distance of the refractor plate as shown in FIG. 5.

(4)

From geometrical optics, the refracted light through a parallel slab of substance is parallel to the incident light.

Therefore,  

$$\frac{S}{2} = p SIN (i-r)$$

$$= \frac{t}{COS r} (SIN i COS r - COS i SIN r)$$
or  

$$S = 2t SIN i (1 - \frac{COS i SIN r}{COS r SIN i})$$

- 11-



# FIG. 5 LIGHT TRACING THROUGH REFRACTOR PLATES

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From (1),  $SIN i = COS \theta$ ,  $COS i = SIN \theta$ , and from (2), equation (4) can be written as;

$$S = 2t \cos \theta \left( 1 - \frac{\sin \theta}{n \sqrt{1 - \sin^2 r}} \right)$$
$$= 2t \cos \theta \left( 1 - \frac{\sin \theta}{\sqrt{n^2 - \cos^2 \theta}} \right)$$
(5)

From (3) and (5), we can eliminate t,

$$\operatorname{area} = \frac{S}{4 \operatorname{SIN} \theta \cos \theta \left(1 - \frac{\operatorname{SIN} \theta}{\sqrt{n^2 - \cos^2 \theta}}\right)} \left\{ 2d + \frac{S \cos \theta}{2 \cos \theta \left(1 - \frac{\operatorname{SIN} \theta}{\sqrt{n^2 - \cos^2 \theta}}\right)} \right\}$$
(6)

From equation (6), if d is given as a constant, and the spectral jump, S, is specified, the area of the plate is a function of the angle,  $\theta$ , of the plate only.

The refractor plates are mounted on a tuning fork and vibrated, and a light weight is more desirable. Therefore a minimum area, which is proportional to the weight, of the refractor plate is required. This can be solved graphically, analytically, or with computed tabulation.

### 3.2 Gate Duty Cycle Calculation

In the Introduction, it is stated that the gate duty cycle is a parameter in the signal to noise ratio equation. The greater the gate duty cycle, the better the signal to noise ratio. The following will give the derivation of the gate duty cycle equation and the parameters involved.

A schematic drawing of the positions of the refractor plates with respect to the incident light is given in FIG. 6, and their corresponding locations in the sine wave output of the tuning fork, and the gate duty cycle waveform are shown. The tuning fork has a simple harmonic motion and can be approximated by a sine wave. Therefore, the refractor plates, which are mounted on the tuning fork, have a sine wave motion. At position a, the light falls equally on the two plates and a low signal is indicated by the output gate wave form. At position b, all the light passes through one of the plates and a saturation signal is obtained. In between positions a and b, the signal starts to increase. At position c, the refractor plates begin to move in the opposite direction until position d is reached. The maximum amplitude moved by the plates is made such that all the light is catched through the refractor plates, as shown in position c. It explains the constant signal output obtained in the gate waveform in between positions c and d. Positions e,f,g,and h show the incident light passing through the other plate. The difference in signal output by the gate wave form is that a different part of the spectrum is sampled. After position h, the whole cycle is repeated, starting from position a again.

The gate duty cycle is defined as the time spent in saturation signal in any one of the plates as compared to the total time for a cycle.

Therefore from symmetry of the plates, and from the output gate wave form in FIG. 6, we have;

Gate duty cycle = 
$$\frac{2 bc}{4 ac}$$
  
=  $\frac{1}{2} \left( \frac{ac - ab}{ac} \right)$   
=  $\frac{1}{2} \left( 1 - \frac{ab}{ac} \right)$  (7)

However ab and ac are in time scale and must be related to amplitude moved by the refractor plates. Correspondence between the sine wave and the output gate waveform is made, as following;

At a, amplitude of sine wave is zero, At b, amplitude of sine wave is bb', At c, amplitude of sine wave is cc',

Taking position a as the starting position from rest,











FIG. 6 OUTPUT GATE WAVEFORM W. T. t. REFRACTOR

## PLATE POSITIONS

or

$$(ab) = SIN^{-1} \frac{bb}{cc}$$

$$(ac) = SIN^{-1} \frac{cc^{\dagger}}{cc^{\dagger}} = \frac{\pi}{2}$$

$$\frac{(ab)}{(ac)} = \frac{2}{\pi} SIN^{-1} \frac{bb^{\circ}}{cc^{\circ}}$$

(8)

Putting equation (8) into equation (7),

Gate duty cycle, D , 
$$=\frac{1}{2}(1-\frac{2}{\pi}SIN^{-1}\frac{bb!}{cc!})$$
 (9)

3.3 Amplitude Measurement w.r.t. Refractor Plate Parameters

As shown in FIG. 7, three positions A,B, and C, of the refractor plate are given with respect to the incoming light and the entrance slit. Position A is the start of a cycle, corresponding to position a in FIG. 6. Position B is the start of the time when all the light are collected, as position b in FIG. 6. Position C is the maximum displacement acquired by the refractor plate, corresponding to position c in FIG. 6.

Distance moved between positions A and B corresponds to bb' in (9), distance moved between positions A and C corresponds to cc' in (9).

$$bb' = (s+m+h) \operatorname{TAN} \alpha$$

$$cc' = d - (s+m) \operatorname{TAN} \alpha$$
(10)

Putting equation (10) into equation (9), and substitute  $h=d \text{ COT } \theta$ , we have,

$$D = \frac{1}{2} \left\{ 1 - \frac{2}{\pi} \operatorname{SIN}^{-1} \left[ \frac{(s+m+d \operatorname{COT} \theta) \operatorname{TAN} \alpha}{d - (s+m) \operatorname{TAN} \alpha} \right] \right\}$$
$$= \frac{1}{2} \left\{ 1 - \frac{2}{\pi} \operatorname{SIN}^{-1} \left[ \frac{d \operatorname{COT} \theta \operatorname{TAN} \alpha + (s+m) \operatorname{TAN} \alpha}{d - (s+m) \operatorname{TAN} \alpha} \right] \right\}$$
(11)



FIG. 7 GATE DUTY CYCLE CALCULATION W. T. t. REFRACTOR PLATES' POSITIONS MCMASIER UNIVERSITY LIBRARY

where s is the light source's distance from the entrance slit,

- m is the distance of the front edge of the refractor plate from the entrance slit,
  - & is the divergent angle of the source from its central line.

In arriving at equation (11), some assumptions have been made, leading to some constrained conditions. As shown in position B of FIG. 7, the mostleft ray, after refraction, must lie within the same plate,

i.e. 
$$\angle r \leq (90^{\circ} - \theta)$$
  
or  $SIN r \leq SIN(90^{\circ} - \theta)$  for  $0^{\circ} \leq \theta \leq 90^{\circ}$   
 $SIN r \leq COS \theta$  (12)  
 $\frac{SIN i}{SIN r} = n$ 

but

and

 $i = (90^{\circ} - \theta) + \alpha$   $SIN \{ \alpha + (90^{\circ} - \theta) \} = n SIN r$   $SIN \alpha COS(90^{\circ} - \theta) + COS \alpha SIN(90^{\circ} - \theta) = n SIN r$  $SIN \alpha SIN \theta + COS \alpha COS \theta = n SIN r$ (13)

From equations (12) and (13),

$$n \cos \theta \ge n \sin r$$
  

$$\ge \sin \alpha \sin \theta + \cos \alpha \cos \theta$$
  
or  $n \ge \sin \alpha \tan \theta + \cos \alpha$  (14)  
evotion (11) to hold

For equation (11) to hold,

 $\frac{d \operatorname{COT} \theta \operatorname{TAN} \mathcal{L} + (s+m) \operatorname{TAN} \mathcal{L}}{d - (s+m) \operatorname{TAN} \mathcal{L}} \leq 1$ 

d	COT	9	TAI	A d	+ (s	+m)	TAN	£ :	∠ d	-	(s-	rm)	TAN &	
	d	(	1	•	COT 0	TAI	(	≽	2	(s+	m)	TAI	Sol	(15)

3.4 Equations Consideration

The relevant equations are collected from the above paragraphs and discussed to obtain the necessary parameters for best optimum results.

Area = 
$$\frac{S}{4 \operatorname{SIN}\theta \cos\theta \left(1 - \frac{\operatorname{SIN}\theta}{\sqrt{n^2 - \cos^2\theta}}\right)} \left\{ 2d + \frac{S \cos\theta}{2 \cos\theta \left(1 - \frac{\operatorname{SIN}\theta}{\sqrt{n^2 - \cos^2\theta}}\right)} \right\}$$
$$D = \frac{1}{2} \left\{ 1 - \frac{2}{\pi} \operatorname{SIN}^{-1} \left[ \frac{d \cot\theta \tan \omega + (s+m) \tan \omega}{d - (s+m) \tan \omega} \right] \right\}$$

subjected to conditions

 $n \geqslant SIN \mathcal{L} TAN \theta + COS \mathcal{L}$ 

d ( 1 - COTO TAN  $\mathcal{L}$  )  $\geq$  2 (s+m) TAN  $\mathcal{L}$ 

From the equation of area, we can see that when  $\theta = 90^{\circ}$ ,  $0^{\circ}$ , the area will go to infinity. A minimum area will occur for a certain  $\theta$ , lying between  $90^{\circ}$  and  $0^{\circ}$ . From the equation of the gate duty cycle, it is a monotonic increasing function of  $\theta$ , for  $\theta$  lying between 90 and 0. The parameter d, as it becomes larger, will increase the gate guty cycle as well as the area. Usually the divergent angle  $\mathcal{L}$ , of the source with respect to the entrance slit is fixed for a chosen instrument. A small  $\mathcal{L}$  is desirable, but the intensity obtained must be strong enough for a good signal output.

Comparatively speaking, the gate duty cycle is more important than the area ( proportional to the weight ) of the refractor plates, as long as the weight of the refractor plates lie within the load limit of the tuning fork. A computer program is developed to tabulate the values of the angle  $\theta$ , the area, the gate duty cycle D, and its thickness t, for a given set of values of n, $\alpha$ ,d,s,and m. Then decisions can be made from the tabulated values. An output program will be shown for some calculations.

3.5 Theoretical Estimation and Computation

For practical application,  $SO_2$  and  $NO_2$  are chosen and their optimum refractor plates are calculated.

 For SO<sub>2</sub> detection, a filter is used to filter away all the wavelengths, except those between 2920 Å to 3120 Å, centering approximately at 3000 Å. The required spectral jump S, is 11 Å with an entrance slit width of 8 Å. From experimental determination, the beam spreads to 64 mm. at a distance of 250 mm. from the slit. Therefore the distance s of the virtual source from the entrance slit and the divergent angle , can be evaluated.



2).For NO<sub>2</sub> detection, the waveband region used is from 4260 Å to 4500 Å, centering at 4400 Å. The required spectral jump S, is 20 Å with an entrance slit of 14 Å.



The following is a sample program listing, with some typical output. The grating has a dispersive power of 33 A/mm. @ FR5 MAIN

COMPILED BY LEVEL 23.8 CSCX FORTRAN V ON D8 AUG 73 AT 14:00 00100 C THIS PROGRAM IS USED FOR OPTIMIZATION OF REFRACTOR PLATES USED IM CORRELATION 1 \* 00100 C SPECTROMETER. A MINIMUM WEIGHT AND A MAXIMUN GATE DUTY CYCLE ARE WANTED WITH 2\* 00100 3\* C RESPECT TO ANGLE. THICKNESS, AND LENGTH OF PLATE 00101 DIMENSION  $A(50) \cdot T(50) \cdot D(50) \cdot F(50) \cdot O(50) \cdot D1(50) \cdot D2(50)$ 4 \* 00101 C A IS THE AREA OF THE REFRACTOR PLATE IN SQ. MM. 5\* 00101 C T IS THE THICKNESS OF THE REFRACTOR FLATE IN MM. 6\* 00101 7\* C F IS ANGLE OF REFRACTOR PLATE IN DEGREE C D TS THE LENGTH OF THE REFRACTOR PLATE IN MM. 00101 3\* 00101 C G IS THE GATE DUTY CYCLE 9\* 00101 C D1 IS THE DISPLACEMENT CORRESPONDING TO RE1 10\* 00101 11\* C D2 IS THE DISPLACEMENT CORRESPONDING TO RE2 READ (5+10) RE+DE+S+XM+AL+RE1+RE2 00103 12\* 150 00114 13\* 10 FORMAT (7F10.6) 00115 IF (RE.EQ.0.0) GO TO 200 14\* 00115 15\* C RE IS THE REFRACTIVE INDEX OF THE PLATE FOR A PARTICULAR CHOSEN WAVELENGTH 00115 16 \* C DE IS THE REQUIRED SHIFT OF THE SPECTRUM IN MM. 00115 C S IS THE DISTANCE OF SUPPOSED SOURCE FROM SLIT IN MM 17\* C XM IS THE TOP OF REFRACTOR PLATE FROM SLIT IN MM. 00115 13\* 00115 C AL IS THE DIVERGENT ANGLE OF SUPPOSED SOURCE IN DEGREE 13\* 00115 20\* C TO ESTIMATE THE RANGE OF ANGLE F USING EQT 4 00117 21\* XAL=(AL\*3.14159265)/180.0 00120 27\* TANE=(RE-COS(XAL))/SIN(XAL) 00121 23\* XF1=(ATAN(TANF)/3.14159265)\*180.0 00121 24 \* C XF1 IS THE MAXIMUM ANGLE OF PLATE FOR THE FOLLOWING CALCULATION 00121 25\* C TO ESTIMATE THE MINIMUM LENGTH OF PLATE WHICH IS XD IN MM 00122 26\* TA=SIN(XAL)/COS(XAL) 00123 27\* XD = (S + XM) \* TA00124 28\* X= TA 00124 29\* C PRINT OUT GIVEN DATA AND ESTIMATIONS. 00125 30 \* WRITE (6,80) XF1 00130 FORMAT(1X, MAXIMUM ANGLE OF PLATE=', F12.6) 31 \* 80 00131 32\* WRITE (6,100) XD 00134 33\* FORMAT(1X\*\*MINIMUM LENGTH OF PLATE=\*\*F12.6) 100 00135 34 \* WRITE (6,142) RE FORMAT(1X, "REFRACTIVE INDEX OF PLATE="+F12.6) 00140 35\* 142 00141 36 \* WRITE (6,85) RE1

N

F

00144	37*	85	FORMAT(1X, 'MINIMUM REFRACTIVE INDEX=', F12.6)	
00145	38*		WRITE (6,86) RE2	
00150	39*	36	FORMAT(1X, MAXIMUM REFRACTIVE INDEX=", F12.6)	
00151	40*		WRITE (6+143) DE	
00154	41*	143	FORMAT(1X, 'REQUIRED SHIFT OF SPECTRUM=', F12.6)	
00155	42*		WRITE (6,144) S	
00160	43*	144	FORMAT(1X, 'DISTANCE OF SUPPOSED SOURCE FROM SLIT=', F12.6)	
00161	44*		WRITE (6,145) XM	
00164	45*	145	FORMAT(1X, 'TOP OF PLATE FROM SLIT =', F12.6)	
00165	46*		WRITE (6,146) AL	
00170	47*	145	FORMAT(1X, 'DIVERGENCE OF ANGLE OF SOURCE='.F12.6)	
00170	48*	C BY	VARYING D AND F, A.G.T ARE TABULATED	
00171	49*		C=1.0	
00172	50*		E=0.0	
00173	51*		A1=1.5707238	
00174	52*		A2=-0.2121144	
00175	53*		A3=0.0742610	
00176	54 *		A4=-0.0187293	
00176	55*	C A1	A2,A3,A4 ARE COEFFICIENTS USED IN THE POLYNOMIALS APPROXIAMATION FOR THE	N
00176	55*	C ARI	C SINE EXPANSION	
00176	57*	C BY	VARYING I, LENGTH OF PLATE ARE SPECIFIED	· ·
00177	58*		DO 20 I=2.7	
00202	59*		C=C+1.0	
00203	60*		D(T)=C	
00204	61*		WRITE (6,40) D(I)	
00207	62*	40	FORMAT(1X, 'LENGTH OF PLATE=', F12.6)	
00210	63*		XF2=(ATAN((D(I)*TA)/(D(I)-(2.0*TA*(S+XM))))/3.14159265)*180.0	
00210	64*	C XF	2 IS THE MINIMUM ANGLE OF PLATE	
00211	65*		WRITE (6,90) XF2	
00214	66*	90	FORMAT(1X, "MINIMUM ANGLE OF PLATE=', F12.6)	
00215	67* .		AM=D(T)-((S+XM)*TA)	
00216	68 *		WRITE (5,17) AM	
00221	63*	17	FORMAT(1X, 'AMPLITUDE OF VIBRATION=', F12.6)	
00222	70*		WRITE (6,50)	
00224	71*	50	FORMAT(5X, 'ANGLE=',10X, 'THICKNESS=',5X, 'AREA=',10X, 'GATE CYCLE=',5	
00224	72*		1X, *DISPLACEMENT1=*,5X, *DISPLACEMENT2=*)	

00224	73*	C BY VARYING K, THE ANGLE OF PLATE ARE VARIED
00225	74 *	DO 30 K=1+44
00230	75*	E=E+2.0
00231	76*	XF=E*3.14159265/180.0
00232	77*	T(K)=(DE/(COS(XF)*(1.D-SIN(XF)/SQRT(RE*RE-COS(XF)*COS(XF)))))*0.5
00233	78*	A(K)=T(K)/(2.0*SIN(XF))*(2.0*C+T(K)*COS(XF))
00234	79*	F(K) = F
00235	30*	D1(K)=T(K)*(COS(XF)*(1.D-(SIN(XF)/SQRT(RE1*RE1-COS(XF)*COS(XF)))))
00235	81*	1*2.0
00236	82*	D2(K)=T(K)*(COS(XF)*(1.0-(SIN(XF)/SORT(RE2*RE2-COS(XF)*COS(XF)))))
00236	83*	1*2.0
00236	84 *	TO CALCULATE GATE DUTY CYCLE G
00237	85*	R=(C*(COS(XF)/SIN(XF))*X+(S+XM)*X)/(C-(S+XM)*X)
00240	86*	IF (R.GE.1.0.0R.R.LE.0.C) GO TO 30
00242	87*	ASN=3.14159265/2.0-SQRT(1.0-R)*(A1+A2*R+A3*R*R+A4*R*R*R)
00243	\$8*	G(K)=0.5*(1.0-(2.0/3.14159265*ASN))
00244	39*	WRITE (6,60) $F(K)$ , $T(K)$ , $A(K)$ , $G(K)$ , $D1(K)$ , $D2(K)$
00254	<b>90</b> *	60 FORMAT(1X,F12.6,8X,F12.6,2X,F12.6,4X,F12.6,5X,F10.6,5X,F10.6)
00255	91*	30 CONTINUE
00257	92*	E=0.0
00260	93*	WRITE (6.70)
00262	94 *	70 FORMAT(1H1)
00263	95 *	20 CONTINUE
00263	95*	C READ THE SECOND SET OF DATA
00265 .	97*	GO TO 15U
00266	93*	200 ST 0P
00267	99*	END
END COMPI	LATION	** DIMESSAGES **

DATA	CARDS					
1.4664	0. 60606	1.6683	1-0	7.15	1.4656	1.4676
1-4664	0.60606	1-6683	2.0	7.15	1.4656	1.4676
1.4664	0.60606	1.6683	3.0	7.15	1.4656	1.4676
1.4878	0.33334	0.9506	1.0	7.2667	1.4848	1.4901
1.4878	0.33334	0.9506	2.0	7.2667	1.4848	1.4901
1.4878	0.33334	0.9506	3.0	7.2667	1.4848	1.4901

## NO2 OUTPUT DATA

MAXIMUM ANGLE OF PLATE: 75.292134 MINIMUM LENGTH OF PLATE: .334719 REFRACTIVE INDEX OF PLATE: 1.465400 MINIMUM REFRACTIVE INDEX: 1.465600 MAXIMUM REFRACTIVE INDEX: 1.467500 REQUIRED SHIFT OF SPECTRUM: .606060 DISTANCE OF SUPPOSED SOURCE FROM SLIT: 1.668300 TOP OF PLATE FROM SLIT = 1.000000 DIVERGENCE OF ANGLE OF SOURCE: 7.150000 LENGTH OF PLATE: 2.000000 MINIMUM ANGLE OF PLATE: 10.678110 AMPLITUDE OF VIBRATION: 1.665231

## 50, OUTPUT DATA

MAXIMUM ANGLE OF PLATE: 75.638918 MINIMUM LENGTH OF PLATE: .243725 REFRACTIVE INDEX OF PLATE: 1.437800 MINIMUM REFRACTIVE INDEX: 1.434300 MAXIMUM REFRACTIVE INDEX: 1.490100 REQUIRED SHIFT OF SPECTRUM: .333340 DISTANCE OF SUPPOSED SOURCE FROM SLIT: .950600 TOP OF PLATE FROM SLIT = 1.000000 DIVERGENCE OF ANGLE OF SOURCE: 7.266700 LENGTH OF PLATE: 2.000000 MINIMUM ANGLE OF PLATE: 9.632893 AMPLITUDE OF VIBRATION: 1.751275

ENGTH OF PLATE	6.00000		10		
INIMUM ANGLE OF	PLATE= 8.03684	12	NUZ		
MPLITUDE OF VIB	RATION= 5.6652	31			
ANGLE=	THICKNESS=	AREA=	GATE CYCLE=	DISPLACEMENT1=	DISPLACEMENT2=
10.000000	.366238	13.034835	.193095	.605945	.606232
12.000000	.382615	11.386010	.260191	.605920	.606270
14.000000	.400410	10.252257	.293358	.605394	.606308
16.000000	.419711	9.443334	.325023	.605868	.606348
18.000000	.440610	8.853819	.344971	.605841	.606387
20.000000	.463208	8.420725	.360576	.605315	.606427
22.000000	.487613	8.104252	. 373184	.605789	.606466
24.000000	.513947	7.878161	.383626	.605763	.606505
26.000000	.542343	7.724604	.392446	.605737	.606543
23.000000	.572349	7.631174	.400020	.605712	.606530
30.000000	.605932	7.539144	.406612	.605688	.606616
32.000000	.641478	7.592383	.412418	.605665	.606652
34.000000	.679801	7.636657	.417584	.605642	.606686
36.000000	.721143	7.719186	. 422223	.605620	.606719
38.000000	.765786	7.833356	.426421	.605599	.606750
40.000000	.314054	7.993539	.430249	.605579	.606781
42.000000	.866326	8.184990	. 433762	.605559	.606810
44.000000	.923046	8.413808	.437005	.605541	.606837
46.000000	. 984743	8.581944	.440016	.605524	.606363
48.000000	1.052043	8.992266	.442826	.605507	.606388
50.000000	1.125704	9.348675	.445460	.605492	.606911
52.000000	1.206643	9.756280	.447940	.605477	.606933
54.000000	1.295932	10.221668	.450236	.605463	.606954
55.999999	1.395112	10.753269	.452512	.605450	.606973
58.000000	1.505777	11.361880	.454635	.605438	.606991
59.999999	1.530186	12.061415	.456664	.605427	.607007
52.000000	1.771187	12.369974	.453611	.605417	.607023
64.000000	1.932509	13.811419	.460436	.605408	.607037
66.000000	2.119129	14.917752	.462297	.605399	.607050

16.232783

17.813009

19.762429

22.199693

25.339999

29.533850

35.412347

44.237909

58.956492

88.406270

176.779020

2.337844

2.598174

2.913883

3.305625

3.805869

4:468749

5.391720

6.769717

9.057623

13.620122

27.230755

68.000000

70.000000

72.000000

73.099999

77.9999999

80.000000

82.000000

84.000000

86.000000

83.000000

.464051

.465755

.467415

.469037

.470626

.472186

.473723

.475241

.476742

.478233

.479715

1 N

S

1

.607062

.607073

.607082

.607091

.607098

.607105

.607110

.607115

.607118

.607121

.607122

.605391

.605384

.605377

.605372

.605367

.605362

.605359

.605356

.605354

.605352

.605351

NGTH OF PLA	ATE= 6.00	0000		60				
NIMUM ANGLE	E OF PLATE=	7.91564	8	202				
PLITUDE OF	VIBRATION=	5.75127	5 υ			x		
ANGLEE	THIC	KNESS=	AREA=	GATE	CYCLE=	DISPLACEMENT1=	DISPLACEMENT2=	
8.000000		.192427	3.427539		.045542	.333166	.333472	
10.000000		.200453	7.040132		.206070	.333118	.333509	
12.000000		.209190	6.139823		.255685	.333069	.333546	
14.000000		.213678	5.519413		.304297	.333019	.333584	
16.000000	•	.228963	5.075413	2.4	.330679	.332959	.333523	
18.000000		.240094	4.750464		.350459	.332918	.333661	
20.000000		.252124	4.510289		.365955	.332367	.333700	
22.000000		.265111	4.333217	1 a 1	.378489	.332816	.333739	
24.000000		.279121	4.204956		.383877	.332766	.333777	
26.000000		.794222	4.115772		.397657	.332717	.333315	
28.000000		.310495	4.059838		.405200	.332668	.333852	
30.00000		.328028	4.029527		.411768	.332621	.333888	
32.000000		.346922	4.024316		.417555	.332575	.333923	
34.000000		.367238	4.040913		.422706	.332531	.333956	
36.000000	v	.389259	4.077758		.427333	.332488	.333939	1
38.000000		.412983	4.133921		.431521	.332447	.334021	N
40.000000		. 438633	4.203998		.435340	.332408	.334051	01
42.000000		.466412	4.303056		.433846	.332370	.334080	1
44.000000		.496558	4.416607		.442083	.332334 .	.334107	
46.000000		.529352	4.550616		.445033	•332300	•334133	
48.000000		.565130	4.706522		.447893	.332268	.334153	
50.000000		.504234	4.836306		.450523	.332237	.334181	
52.000000		. 647335	5.092577		.453000	.332209	.334203	
54.000000		.694853	5.323706		.455343	.332182	.334224	
55.9999999		•747589	5.599013		.457587	.332157	.334243	
58.000000		.806474	5.903066		.459687	.332133	.334261	
59.999999		.872637	6.266003		.461715	.332111	.334278	
62.000000		.947748	6.679146		.463660	.332091	.334294	
64.000000		1.033647	7.160731		.465534	.332072	.334308	
66.000000		1.133040	7.727391		.467343	•332055	.334321	
68.000000		1.249554	8.401544		.469096	.332039	.334333	
70.000000		1.383268	9.214921		.470799	.332025	.334344	
72.000000		1.556526	10.213375		.472458	.332012	.334354	
73.399399		1.765350	11.465769		.474080	.332001	.334363	
76.000000		2.032063	13.080403		.475668	. 331991	.334370	
77.393933		2.385549	15.237875		.477228	.331983	.334377	
80.000000		2.377307	18.253358		.473765	.331975	.334382	
82.000000		3.612844	22.807314		.480282	.331970	.334387	
84.000000		4.833369	30.387551		.491783	.331965	.334390	
86.000000		7.267518	45.558244		. 483274	.331962	.334393	
88% 000000	1	4.555038	91.043940		. 484756	.331960	. 334394	

### 4). Results and Discussion.

The computer outprint tabulates the angle  $\Theta$ , in steps of every two degrees. The maximum and minimum angles of the refractor plates are printed so that the gate duty cycle equation holds within these range. The steps in evaluate these are given in the Appendix. From the data cards, there is a change in the distance m of the front surface of the refractor plates from the entrance slit. The closer it is to the entrance slit, the better is the gate duty cycle.

Since the refractive index is a function of the incoming wavelengths, there is a spread of the output spectrum. In our calculation, the spectral jump,  $\zeta$ , is taken to be that from the center of the detected wavelength region. From the computer outprint, there are two columns, displacement 1 and displacement 2. These are obtained from the different refractive index n, corresponding to the two ends of the detected wavelength region, and is an indication of the smearing of the output spectrum from its central position.

A compromise between the gate duty cycle and its weight is required and the dimensions of the refractor plates for  $NO_2$  and  $SO_2$  have been chosen, as shown in FIG. 8A and FIG. 8B. These give a gate duty cycle of 0.47 and 0.48 for  $NO_2$  and  $SO_2$  respectively. They are close to the ideal gate duty cycle of 0.5.

The distance d of the refractor plates does not represent the amplitude of the vibrating tuning fork. Because of the divergence of the source, the amplitude of the fork should be a bit less, given by

## d - (s+m) TANd

The material chosen for the refractor plates is fused quartz, which has more than 90 % transmissivity in the ultra-violet region where the SO<sub>2</sub> is being detected. It has good optical quality and can be manufactured at any specified shape and size.

 $EF = 6 \cdot 2 \pm 0 \cdot 1$  mm.  $FH = 3 \cdot 4 \pm 0 \cdot 1$  mm.  $GH = 7 \cdot 2 \pm 0 \cdot 1$  mm.







 $CD = 6 \cdot 1 \pm 0 \cdot 1$  mm.  $DB = 3 \cdot 6 \pm 0 \cdot 1$  mm.  $AB = 6 \cdot 6 \pm 0 \cdot 1$  mm.



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An estimation of the possible error in the gate duty cycle D, is given in the Appendix. It amounts to about 2% for some typical values of the refractor plates.

### 5). Conclusion.

The dimensions of the optimum refractor plates for  $SO_2$  and  $NO_2$  are calculated and given in FIG. 8A and FIG. 8B. These are going to be used in a modified correlation spectrometer using as a point sampler. The gate duty cycle of 0.47 and 0.48, are very efficient as compared to the prototype.

### ACKNOWLEDGEMENTS

The author would like to express his thanks to Dr. M. M. Millan for his supervising and patience in explaining the working of the instrument. - 30 -

- 'Theory, Applications, and Results of the Long-Line Correlation Spectrometer' ---- M.M.Millan and G.S.Newcomb.
   IEEE Transactions on Geoscience Electronics, vol. GE-8, No. 3, July, 1970. page 149.
- (2) 'A study of the operational characteristics and optimization procedures of dispersive correlation spectrometer for the detection of trace gases in the atmosphere.' ---- M.M.Millán. PH. D. thesis, University of Toronto, 1972.

APPENDIX

1). Evaluation on Constrained Conditions

From first condition.

 $n \ge SIN \& TAN \Theta + COS \&$ for  $so_2$ ,  $\mathcal{L} = 7.2667^{\circ}$ n = 1.4878 at 3000 Å for quartz :. TAN  $\theta \leq \frac{n - \cos \alpha}{\operatorname{STN} \alpha}$ ≤ 3.92 θ ≤ 75° 36° for NO<sub>2</sub>,  $\& = 7.15^{\circ}$ n = 1.4664 at 4400 Å for quartz TAN  $\theta \leq 3.8$ 

θ ≤ 75°12'

From second condition,

d  $(1 - COT \Theta TAN \mathcal{L}) \ge 2 (s+m) TAN \mathcal{L}$ 

...

or  $COT \theta \leq \left[\frac{d - 2 (s+m) TAN \mathcal{L}}{d TAN \mathcal{L}}\right]$ for SO2, TAN & = 0.1275 d = 6 mm.s+m = 3 mm. COT  $\theta \leq 6.86$ 

for NO2,

. .

Ø≥8°18• TAN  $\mathcal{L} = 0.1255$ , d = 6 mm., s+m = 3.6683 mm. COT  $\theta \leq 6.74$ 

0 28 241

The above estimation for the maximum and minimum angles of the refractor plates give an idea of the magnitudes involved. Different values of n, s+m, d, and & will yield some other angles.

2). Error Estimation

It is stated that the gate duty cycle is more important than the weight of the refractor plates, and there are tolerances in manufacturing the required dimensions of the refractor plates. Therefore an estimation of the error introduced, if the tolerances are accounted for, will give an insight of the change of the gate duty cycle performance.

From the gate duty cycle equation, we have;

$$D = \frac{1}{2} \left\{ 1 - \frac{2}{\pi} \operatorname{SIN}^{-1} \left[ \frac{d \operatorname{COT} \sigma \operatorname{TAN} \omega + (s+m) \operatorname{TAN} \omega}{d - (s+m) \operatorname{TAN} \omega} \right] \right\}$$

From differential calculas, total error in D is given by,

$$(m+a) \gtrsim \left| \frac{\partial D}{(m+a) \odot} \right| + \lambda \approx \left| \frac{\partial D}{\lambda \odot} \right| + \theta \approx \left| \frac{\partial C}{\partial \odot} \right| + \delta \approx \left| \frac{\partial C}{\delta \odot} \right| = db$$

each of the partial derivatives is calculated as following;

a). 
$$\frac{\partial D}{\partial d} = -\frac{1}{\pi} \left\{ 1 - \left[ \frac{dCOT0TANd + (s+m)TANd^2}{d - (s+m)TANd} \right]^{-\frac{1}{2}} \frac{d - (s+m)TANd COT0TANd - g}{[d - (s+m)TANd]^2} \right\}$$

$$= \frac{(s+m) TAN\alpha [1+COT \theta TAN\alpha]}{\pi [d-(s+m) TAN\alpha] \sqrt{[d-(s+m) TAN\alpha]^2 - g^2}}$$

where g = dCOT9TANd+(s+m)TANd

b). 
$$\frac{\partial D}{\partial \theta} = -\frac{1}{\pi} \left\{ 1 - \left[ \frac{dCOT \theta TANd + (s+m)TANd}{d - (s+m)TANd}^2 \right]^{-\frac{1}{2}} \left\{ \frac{dTANd(-COSEC^2\theta)}{d - (s+m)TANd} \right\} \right\}$$

$$= \frac{1}{\pi} \left\{ \frac{a \cos 2\theta \tan \alpha}{\sqrt{d} - (s+m) \tan \alpha} \right\}^2 - g^2 \right\}$$

c). 
$$\frac{\partial D}{\partial \alpha} = -\frac{1}{\pi} \left\{ 1 - \left[ \frac{g}{d - (s+m)TAN\alpha} \right]^2 \right\}^{-\frac{1}{2}} \left[ \frac{g(s+m)SEC^2 \alpha}{[d - (s+m)TAN\alpha]^2} \right]^{-\frac{1}{2}} \left[ \frac{d - (s+m)TAN\alpha}{[d - (s+m)TAN\alpha]^2} \right]^{-\frac{1}{2}} \right]^{-\frac{1}{2}} \left[ \frac{d - (s+m)TAN\alpha}{[d - (s+m)TAN\alpha]^2} \right]^{-\frac{1}{2}} \left[ \frac{d - (s+m)TAN\alpha}{[d - (s+m)TAN\alpha} \right]^{-\frac{1}{2}} \left[ \frac{d - (s+m)TAN\alpha}{[d - (s+m)TAN\alpha} \right]^{-\frac{1}{2}} \left[ \frac{d - (s+m)TAN\alpha}{[d - (s+m)TAN\alpha} \right]^{-\frac{1}{2}} \right]^{-\frac{1}{2}} \left[ \frac{d - (s+m)TAN\alpha}{[d - (s+m)TAN\alpha} \right]^{-\frac{1}{2}} \left[ \frac{d - (s+m)TAN\alpha}{[d - (s+m)TAN\alpha} \right]^{-\frac{1}{2}} \left[ \frac{d - (s+m)TAN\alpha}{[d - (s+m)TAN\alpha} \right]^{-\frac{1}{2$$

$$= -\frac{1}{\pi} \frac{d \operatorname{SEC}^2 \mathscr{A} \operatorname{k}}{[d - (s + m) \operatorname{TAN} \mathscr{A}] \sqrt{[d - (s + m) \operatorname{TAN} \mathscr{A}]^2 - g^2}}$$

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where  $k = d COT \theta + (s+m)$ 

d). 
$$\frac{\partial D}{\partial (s+m)} = -\frac{1}{\pi} \left\{ 1 - \left[ \frac{g}{d - (s+m)TANd} \right]^2 \right\}^{\frac{1}{2}} \left\{ \frac{[d - (s+m)TANd]TANd + g TANd}{[d - (s+m)TANd]^2} \right\}^{\frac{1}{2}}$$
$$= -\frac{1}{\pi} \frac{d TANd [1 + COT \theta TANd]}{[d - (s+m)TANd]} \left[ \frac{d TANd [1 + COT \theta TANd]}{[d - (s+m)TANd]^2 - g^2} \right]^{\frac{1}{2}}$$

Equations a) to d) give the partial errors in the respective parameters. Assuming, 0 ~

the followings are obtained,

$$\frac{\partial D}{\partial d} = 0.0065$$

$$\frac{\partial D}{\partial \theta} = 0.062$$

$$\frac{\partial D}{\partial c} = -0.455$$

$$\frac{\partial D}{\partial (s+m)} = 0.0107$$

$$dD = 0.008$$

From the computer print out, let values of D = 0.45

relative error in 
$$D = \frac{0.008}{0.45} = 2\%$$
# PART B: DATA ANALYSIS OF CONCENTRATION PROFILE TAKEN WITH A CORRELATION SPECTROMETER.

#### A). Introduction

The instrumental description of the refractor plates correlation spectrometer has been given in PART A. The instrument is able to measure the concentration of polluting gas in parts per million per meter pathlength along the line of observation ( ppm-m ).

In the following analysis, the data are taken from a project measurement near the Sudbury area. It was carried out by the Air Reasearch Quality Branch of Atmosperic Environment Service around June ,1973. The main purpose of the project is to investigate the workability of remote sensing of polluting gases using correlation spectrometer.

The correlation spectrometer used in this project is a COSPEC N from the Barringer Research Limited. Disk modulation was used instead of the refractor plates, with some modification in the electronics. The working and positioning of the disk has been given in PART A. For details, consult manuel of COSPEC N. However, the principles behind are the same for both types of modulation. Multistable measurement was used, with four sets of slits along the surface of the disk to reduce background interference.

B). Recording of Sudbury Data

The COSPEC II was mounted in a station wagon and its telescope was pointing out of the window. A reflector adapter was connected with the telescope so that the instrument was looking overhead at the sky. A constant power inverter, a chart recorder and a mileage indicator were connected with the COSPEC IV. When the station wagon was travelling along the road, a continuous measurement was recorded on the chart, indicating the concentration of pollutant overhead. A typical output is shown in FIG. 9. There are three simultaneously recording graphs on the chart. The signal output gives the concentration profile of the gas overhead along the road. The automatic gain control (AGC) output indicates the optimum operating condition.



FIG. 9 TYPICAL CHART RECORDER OUTPUT

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The mileage indicator gives the distance travelled, and a click is shown for every one hundredth of a mile.

In this project, SO<sub>2</sub> emission from the stack-plume was measured. On a clear day, the plumes can be seen and tracked for miles. By choosing various sections of roads which ran under the plume, the concentration of SO<sub>2</sub> across the plume section was monitored. Due to the plume size, the vehicle took several minutes for each run across the plume. Calibration against a standard cell was done before each run commenced, and all readings are compared linearly with this standard. The calibration had to be carried out against the sky beyond the plume region. Normally several runs across the same plume section were made consecutively. This gives a better averaging value.

C). Estimation of SO, emission from stack

In order to estimate the emission rate of SO<sub>2</sub> from stack, several procedures must be known, as following;

- The concentration profile of the SO<sub>2</sub> along the road is obtained with the COSPEC output in ppm-m. The starting time and location of each run have to be marked alongside the chart, so that the exact section of the road travelled is known.
- 2) The wind velocity at the altitude of the plume should be known at the time when measurements were taken. In the Sudbury project, a meteorological balloon was used. From the elevation of the balloon, time elapsed and azimnthal angle of observation, the wind direction and speed (in meter per second) was calculated at different altitudes.
- 3) From the regional map, the portion of the road being travelled was traced. The shape, scale and direction of the road was obtained.

After the above procedures, the required informations are obtained and steps are taken to give an numerical estimation of SO<sub>2</sub> emission. The following steps leads to an equation;-

- a). Because of the variation in the speed of the vehicle, the original concentration profile on the chart, as FIG. 9, does not represent the true profile along the road. Therefore it must be reduced to the true concentration profile on a distance scale along the road. This is done by averaging the concentration within each hundredth of a mile and then replot them on another graph, as shown in FIG.10. Notice that there are conspicuous spikes shown in FIG. 9, during measurement. These were due to the interference on the electronics whenever the instrument went under cables or overpasses. Therefore they must be eliminated during replotting.
- b). From the obtained road section profile, it was then divided into i approximately straight line portions. Then the true distance of each ith portion was marked on the replotted graph. As shown in FIG. 10, i is equal to 5. The area under each portion of the graph was counted and denoted by A<sub>i</sub>.
- c). From the wind direction and direction of each portion of the road section, the angle between the wind vector and the normal to the road was measured geometrically. This angle,  $\theta_i$ , was related to the normal flow of the pollutant.
- d). The wind speed was measured as v m/s.
- e). The conversion factor from ppm to grams for each gas has to be calculated. For SO<sub>2</sub>,

1 ppm of  $SO_2 = \frac{64 \text{ gm}}{22400}$  of  $SO_2$  in  $10^6$  c.c. of air

 $=\frac{64}{22400}$  gm. of SO<sub>2</sub> in 1 m<sup>3</sup> of air

f). Now the true concentration profile of the plume cross-section was summed up, taken into account of the angle  $\theta_i$ .

true cross-section concentration = ≤ A<sub>i</sub> COS O<sub>i</sub> sq. units
f). From the standard cell calibration, a real conversion factor is
required for relating each square unit area under curve into ppm-m<sup>2</sup>.
Let 1 sq. unit = C ppm-m<sup>2</sup>

From the above,

rate of flow of  $SO_2 = C \times v \times \frac{1}{2} A_1 \cos \theta_1 \times \frac{64}{22400}$  gm./sec.



FIG. 10 CONCENTRATIONS V. S. ROAD DISTANCE

D). Example and Result

One of the runs is taken as an examples to illustrate the above estimation of SO<sub>2</sub>. The road section chosen is at Highway 144 and Clarabelle Road, travelling east along Hwy 144. The total distance is a mile, and the road section is shown below.



The wind speed, v=8.6m/sec. Wind direction = 223°

From the above, the road is divided into five portions for calculation. The following table gives  $\geq A_i \cos \Theta_i$ ,

Section	Distance in miles	i	A <sub>i</sub> (sq. units)	Ai COS i
1	0.429	12	11.30	11.04
2	0.143	3	41.00	40.80
3	0.090	31	31.30	24.40
4	0.162	8	20.30	20.20
5	0.176	47	1.00	0.68
				₹=97.12

From the scale of the graph,

 $1 \text{ sq. unit} = 16150 \text{ ppm-m}^2$ 

... rate of flow of 
$$SO_2 = 16150 \times 8.6 \times 97.12 \times \frac{64}{22400}$$
 gm./sec.

= 38.5 Kg./sec.

This is equivalent to 3300 metric tons per day.

#### E). Discussion

From FIG. 10, there are three concentration profiles, indicating three different consecutive runs on the same section of the road. The values of the flow rate of SO<sub>2</sub> are 38.5,32.2, and 44.6 Kg./sec. respectively. The average is 38.4 Kg./sec. and the maximum deviation from this average value is about 6.2 Kg./sec. or approximately 15 %. This fluctation is expected because the wind velocity is varying all the time, and what we measure is an instantaneous value. More runs are needed for a better average value. The following points should be observed for accurate results;

- 1. Conspicuous spikes, caused by overhanging cables, must be smoothed out. Distinction between real concentration peak and faulty spikes must be recognised. For a short response time of the instrument, the spikes are long and sharp, and background fluctuation are measured into the signal output. A medium response time, together with a moderate driving, are desirable.
- 2. The wind velocity at the plume height should be measured at the same time as the readings are taken. If possible, a continuous recording of the wind velocity should be made so that any sudden fluctuation can be noticed and accounted for in the output data analysis.
- 3. In calibration and subsequent measurements, the background radiation and changes are assumed to be constant. Therefore a steady, or slow changing, background gives a more reliable reading of the true concentration.
- 4. In arriving at the emission rate per day, a constant output from the stack is assumed. This is not very true. Therefore sets of readings at different time of the day should be taken, and at least several runs should be made for each set.

The above method of chasing the plume with a vehicle is tedious and the results in analysing are time consuming, especially a lot of data have to be analysed. Method of driving the spectrometer to sweep across the sky has been considered. With a driving mechanism, the reflector adapter can be rotated in a vertical plane and lights can be reflected into the instrument. If possible, a data acquisition system can be incorporated so that readings are taken and stored automatically for every degree of the angle swept. These readings can then be computed using a computer program.

# F). Conclusion

From the result, the emission rate of  $SO_2$  was found to be approximately. 3300 metric tons per day. This is comparable to the several thousand tons obtained with the in-stack sampling. Values of some more data and results from Sudbury at the same time will be given in some other report. However, their values are within the several thousand tons range. Therefore the using of the correlation spectrometer as a remote sensing is quite justified, at least in the case of  $SO_2$ . A Study of Thermoelectrets

by

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A project report submitted in partial fulfillment of the requirements for the degree of Master of Engineering

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\* One of two project reports. The other part is designated PART A: Industrial Project.

## ABSTRACT

The thermoelectret properties of Mylar, employed in the form of a membrane, was studied. An optical method was used to measure the residual surface charge on the electret. Investigation were made of the variation of surface charge with temperature, charging voltage, charging time, and cooling time. These were related to various proposed mechanisms for dipole orientation, ionic displacement, ionic trapping and direct transfer of charges during air breakdown. From the displacement of the membrane surface under an applied field, the surface charges can be related to the applied field.

#### FIGURE INDEX

- FIG. 1 -- Dimensions of assembly of membrane and base stand.
- FIG. 2 -- Heating coil assembly.
- FIG. 3 -- Interferometer layout.
- FIG. 4 -- Electrical connection of assembly.
- FIG. 5 -- Typical chart recorder output.
- FIG. 6 -- Graph of measured membrane voltage v s. time voltage applied during cooling cycle.
- FIG. 7 -- Graph of measured membrane voltage v s. charging time.
- FIG. 8 -- Graph of measured membrane voltage v s. temperation.

FIG. 9 -- Graph of measured membrane voltage v s. applied voltages.

## TABLE INDEX

Table 1 -- Tabulation of temperature corresponding to applied voltages. Table 2 -- Voltage across Mylar membrane due to applied voltage across electrodes.

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1). REVIEW OF THERMOELECTRETS.

# A). Introduction

The term " electret " was first introduced by Oliver Heaviside (1) in 1885 to denote a "permanently" polarized dielectric. Thus it is an electrical analogue of a permanent magnet. However, the first serious study of electrets was carried out by Eguchi (2),(3), a Japanese physicist, in 1919. By using a molten mixture of carnauba wax: and resin, and subsequently cooling them under an applied electric field, he was able to produce an internal electric polarization in the solidified carnauba wax. The polarization remained when the applied field was removed. The degree of electric polarization is usually evaluated by measurement of the free charge produced on the electret surface.

The electrets prepared by Eguchi were termed "thermoelectrets" because they were produced by heating up the dielectric thermally, applying an electric field and then cooling with the field applied. However, electrets can also be formed by applying an electric field alone. These are called "electroelectrets". In 1937, Nadzhakov (4) discovered that permanent charges could be produced on certain dielectrics after application of an electric field together with illumination by light. He called this type of material a "photoelectret"; later many applications were made using this phenomena, e.g. in electrophotography. "Radioelectrets" have been studied by Gross (5). These were produced by application of an electric field together with x-ray radiation. In 1964, Bhatnagar (6) discovered electrets produced by electromagnetic induction. These are called "magnetoelectrets".

Eguchi noticed that the charges on the electrets decayed to a "steady" value after removal of the applied field. Most of the time, a charge opposite to the charging electrode would result and thus it was given the name of "heterocharge". But when the initial applied field was strong, the heterocharge would decay and finally reverse polarity. This produced a "homocharge", which has the same polarity as the charging electrode.

# B). Methods of producing Surface Charges

In Eguchi's carnauba wax preparation, direct contact was made between the electrodes and the wax. Various temperature and charging voltages were used and the resultant surface charges were different for different samples. In order to have reproducibility, Gerson and Rohrbaugh (7) purified the wax and made them into disks having smooth finished surfaces. Under controlled temperatures in a dessicator, and using fixed charging times and cooling rates, the results were reproducible.

Because a homocharge has a longer life than a heterocharge, it is more desirable. Perlman and Reedyk (8) obtained very high homocharges on dielectrics by using appropriate dielectric materials on the charging electrodes. The electret sample was then sandwiched in between the dielectric bonded electrodes, but separated by an air gap on either side of the electret, thus forming a "five-layer" capacitor. During break down of the air gap under a high applied electric

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field, charges were transferred onto the sample surfaces, thus lowering the field and a further breakdown could occur. This process went on until the charges on the electret produced a sufficiently high field to counteract further charging.

Creswell and Perlman (9) charged Mylar by rolling it under the very high field produced by a pointed electrode. Charges were deposited on the Mylar surface by the resulting corona discharge. This method gave reasonably uniform charges on the surfaces, particularly if a number of electrodes were used.

Recently, Chudleigh (10) had introduced a very simple way of charging polymer foils by using a liquid contact. A thin film of polar-liquid is used between the electrode and the polymer. After voltage is applied for a given time, the liquid is drained away and the electrode removed. This gives precise control of the initial polarization voltage and produces highly uniform charge deposition.

The following report will be concerned only with thermoelectrets. The electret material employed was Mylar in the form of a membrane. This was charged by induction. One electrode was separated from the Mylar by an air gap; the other was in contact with the Mylar. The Mylar surface had a very thin vacuum deposited layer of aluminum.

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C). Methods of measuring surface charges

Eguchi (11) used the induction plate method to measure the free charge of a permanently electrified dielectric. A circular plate, together with an adjacent guard ring, was placed on the surface of the electret. The charge induced on this plate was then transferred to a capacitor. Next, the potential of the charged capacitor was measured with a calibrated gold-leaf electroscope. From measurement of the potential, the capacitance of the capacitor, and the size of the induction plate, the surface charge density and therefore the degree of polarization could be calculated. This could also allow calculation of surface charge density for any other size of the particular dielectric material.

Saito (12) suggested the use of an electron beam passing near the charged electret. The charge density can be calculated from the angle of deflection of the beam. Similarly Kleint (13) measured the potential distribution using an electron beam probe.

Gross (14) used a dissectible capacitor to measure simultaneously the external current flow ( when electrodes were shortcircuited ) and the induced charges on the electrodes of the capacitor containing the dielectric. After the charged dielectric capacitor was short-circuited, the electric charge found on the electrodes is due to induction by the internal field of the dielectric. We made use of Maxwell's equation,

J(t) = i(t) + dq/dt

where J(t) is the external current,

i(t) is the conduction current across the dielectricelectrode interface,

dq/dt is a measure for the displacement of charge carriers within the dielectric.

We can calculate the component of the conduction current i(t) when values of the external current J(t) and the induced charge q are known. This gives more insight into the dielectric behaviour. However, it has the drawback that a discharge can occur through the air gap and therefore measurements must be made in a vacuum to achieve accurate results.

Freedman (15) had suggested vibrating an electrode in the field of the electret while measuring the short circuit ac current or open circuit ac voltage. The surface charge can then be calculated. However, ideal short or open circuit conditions can never be achieved and the results must be corrected for circuit parameters.

Reedyk (16) and Perlman suggested vibrating an electrode in the surface field while observing the electrode signal voltage on an oscilloscope. A bias voltage is applied across the electrodes and then adjusted until the electrode ac signal voltage becomes zero. Both the surface charge and its polarity can be related to the bias. The technique has the advantages of being non-destructive, is independent of circuit parameters, and does not produce discharges at atmospheric pressure.

Ballik (17) proposed a new technique for the

measurement of a charged membrane by making optical measurement of the displacement of a reflecting membrane. It was used as the movable reflector in a Michelson interferometer. Simple calculation and analysis relate the electric potential in terms of displacement. It is non-destructive, simple to use, has high resolution and accuracy. Furthermore it can be used to investigate fast electret charge decay. Details of the technique will be given later.

In the following experiments, charges on the dielectric surface were always measured using this optical method.

# D). Theory and Mechanism of Charge Production and Decay

Many workers have attempted to explain the nature of the homocharge and heterocharge. Different explanations have been given for the change from a heterocharge to a homocharge after some time interval. The following presents some of the various theories and suggestions, and differences between them.

Adam (18) was the first to give some theoretical basis to electrets. He assumed that the heterocharge is due to the orientation of dipoles in the dielectrics, and that homocharge is due to free charges which compensate the heterocharge. If the relaxation time of the dipoles is faster than the decay time of the free charges, then a conversion occurs. The transition from heterocharge to homocharge is explained by the decay of the heterocharge which then produces an uncompensated free homocharge on the electret surface. Gemant (19) objected to the above theory because the initial high polarization observed experimentally could not be accounted for even when all the dipoles are orientated. However dipole orientation is a complicated mechanism, as shown by Groetzinger (20) in thermal conductivity experiment, and by Ewing (21) in x-ray diffraction analysis of the structure of thermoelectrets.

Gemant (19) then proposed that heterocharge is due to the displacement of ions, forming space charges near the electrodes, and that homocharge is due to piezoelectric effect caused by the deformation of oriented crystals in a thermoelectret. This deformation is at right angles to the direction of polarization, and it appears during the cooling of a dielectric. However Theissen (22) et al carried out stress experiments and found that an applied stress had no effect on a homocharge. However the displacement of ions to produce a heterocharge was confirmed.

Nakata (23) had tried to explain homocharge on the basis of a separation of materials near the surface. The material subsequently acquired charges from direct contact at the dielectric-metal interfaces. This was disproved by Thiessen (22) et al, who proposed that there are both internal and external polarizations. The internal component is due to ion displacements, which are frozen in at the end of the polarizing process. The external polarization results in a homocharge, and is due to ionization in the air gap produced by a large field.

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Later, Gross (24), by measuring surface charges and discharge current simultaneously, was able to distinguish between homocharge and heterocharge. He concluded that heterocharge are formed by all the processes of charge absorption in dielectrics, and that homocharges are due to breaksowns at the dielectric-electrode interface. Conversion of heterocharge into homocharge is due to a gradual decay of the internal polarization while the surface charge is retained. Dipole orientation, ion displacements, and the appearance of charges at macroscopic inhomogeneities were all considered as intrinsic heterocharge. Charge formed by conduction currents flowing through the dielectricelectrode interface were considered as intrinsic homocharges. Therefore a surface measurement is a combination of the two ( homocharge and heterocharge ). A rapid decay of a heterocharge, relative to the decay of a homocharge, will result in a net homocharge at the surface after some time interval.

Gemant (25) then developed a theory based on three processes:

- a) Displacement of ions and dipole orientation give the heterocharge,
- b) as the field intensity in the electrode-dielectric interface increases, there is a drift of ions from the gap and the oriented dipoles form into groups.
- c) Later, the dipole fields neutralize ions in the interior of the dielectric and the ions from the discharge gap are held at the surface, giving a net homocharge.

Gemant's theory explains the observation that dielectrics with high electrical conductivity predominantly have heterocharges, and that the stability of homocharges are due to lack of decay of free charges in materials having negligible conductivity.

Gerson (26) and Rohrbaugh found that stable thermoelectrets can be made from nonpolar dielectrics. The electrets are due to localization of electrons at deep traps. Perlman (27) has done extensive thermal current studies in carnauba wax and found that a uniform polarization existed inside the electret. This rule out space charge and macroscopic ion displacement. He proposed that heterocharge is due to the trapping of ions after a microscopic displacement. This is found to be consistent with the observed dependence of thermal currents on temperature.

From the above evidence, the two-charge ( homocharge and heterocharge ) theory has been established and confirmed. Its main feature is a volume effect associated with dielectric absorption. Phenomenological theories of electrets are given by Gubkin (28) and Wiseman (29).

## E). Applications

An electret is simply a source of electric field. This field depends on the charge distribution in the electret itself, and also depends on the distribution of neighbouring charges and conductors. The main advantages of thermoelectrets for a wide range of applications (e.g. microphones) are their high internal resistance and their small dimensions. It has the disadvantage of having charge decay at high

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temperature and humidity. However, this can be minimized by the use of ceramic electret materials.

Thermoelectrets have been combined with photoconducting layers for use in electrophotography. Electrets were used for capacitor microphones in telephones and for standard microphones by the Japanese army during World War Two. They are currently in limited use by the Bell telephone system. The decrease of surface charge of an electret, which occurs after irradiation by gamma rays, is used to measure the exposure dosage in dosemeters for radiative sources. Electrets are used as low-power, high-voltage sources for instrumentation. Electrets, in particular thermoelectrets, are used in numerous other applications. The field of electrets has revived recently after being dormant for a considerable length of time. A considerable research effort is underway internationally to better understand both the theory and application of electrets.

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#### 2). EXPERIMENTAL ARRANGEMENT.

# A). Reflector and Membrane Assembly

The whole assembly is similar to the prototype of Ballik (17). The original materials used in the construction were aluminum (for support and conductors) and Delrin (for insulators). Later, invar was used instead of the aluminum. The much lower thermal expansion provided a more stable optical system.

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The 25 micron thick polyester film, (Mylar\*) aluminized on one surface, was first mounted on a 4" diameter temporary support ring. A 2" aluminum (or invar) ring, of greater height, was placed under the Mylar and inside the Mylar temporary support ring. Using a slight presure, the Mylar surface was stretched and examined to find the best optically smooth area. A 2" diameter clamping ring was placed on top of the first 2" diameter ring, sandwiching the Mylar in between. Screws were used to secure the Mylar membrane between the two rings. The assembly was then cut away from the 4" ring. A gentle stream of hot air was blown over the Mylar surface, which when cooled, formed a tight, flat and stabilized membrane surface. In fabrication, the surfaces of the two 2" rings must be flat and parallel to within 0.0005" tolerance. The aluminized surface was used as the movable reflector in the Michelson interferometer and also as one of the charging electrodes. The completed assembly is shown in FIG. 1.

\* known as polyethylene terepthalate, supplied by Hastings and Co., Inc., Philadephia. B). Charging electrode and Support stand for the membrane assembly

A small solid aluminum (or invar) plate, 1" in diameter, was used as the back electrode. It is secured to a Delrin plate of 2" diameter, which is fastened to the back of the Mylar membrane assembly. The separation between the charging electrode and the Mylar membrane was 0.005". The flatness of the electrode surface had a tolerance of 0.0005". Note that the non-aluminized surface of the Mylar was adjacent to the back electrode.

An aluminum support stand was formed by parts A, B, C, as shown in FIG. 1. A Delrin plate, H, then held the Mylar assembly to the base stand. A 0.25" diameter hole at the centre of the Delrin plate provides access for the laser beam.

# C). Heating up elements and Assembly

A cyclindrical aluminum cylinder was fitted outside the Mylar assembly. A thin layer of fiber-glass tape was wound around the cylinder to provide electrical insulation. Insulated copper wire (#20) was then wound tightly around the cylinder for nearly the full length. After one layer of wire, another thin layer of fiber-glass tape was wrapped around the copper coil. A second layer of copper wire was then wound in the opposite direction to the first, such that the direction of current flow in the two layers were opposite to each other. Thus the magnetic field from the individual coils cancel out. Otherwise mechanical vibration would occur when ac current is passed through the coil because

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of the magnetic properties of the Invar (ac was the most convenient method of heating available at the time). Finally a thick layer of fiberglass tape was wrapped around the second copper layer in order to prevent heat loss to the surroundings.

The front and back surfaces of the plates on the cylinder were insulated by cork sheet to minimize additional heat loss. The resistance of the copper coil can be found by measuring the current and applied voltage when the coil is at room temperature. The change in resistance from the room temperature resistance provides a means of measuring the cylinder temperature. The whole assembly is shown as in FIG. 2.

# D). Interferometer Arrangement

The experimental arrangement is shown in FIG. 3. A Helium-Neon laser, having a 2 milli-watts output at 6328Å (Metrologic, Model 360), was used as a source for the Michelson interferometer. It passed through a beam splitter set at 45° to the laser beam axis. One beam was directed towards a fixed reference reflector and the other went to the Mylar assembly. Both reflected beams were made incident onto a type 6199 photomultiplier. For high visibility the axes and wavefronts of the two incident beams were made coincident. The whole arrangement was enclosed in a black box (literally!) to minimize acoustic vibration and stray light. E). Electrical supply and Recording of data

The photomultiplier was powered by a stable high voltage supply (Fluke model 415B). Its output was connected to the Yaxis of an X-Y chart recorder (Hewlett Packard, Model 2FA). The heating coil was powered by a low voltage transformer (Hammond type 1125×60), operated in conjunction with a Variac. A voltmeter (part of Simpson 270) and an ammeter (part of Simpson 260 and a current adaptor) were used for measuring voltage and current respectively, in order to get a good estimate of temperature.

The complete circuitry is shown in FIG. 4. Two regulated power supplies were used to provide dc charging and measurement voltages; a Harrison ( Model 6209B ) and a Heathkit ( Model 1P-17 ) were connected through a double-pole, double-throw switch arrangement to provide the appropiate voltages across the membrane and charging electrode. The maximum output of each supply was about 400 volts. The switches were arranged so that the voltages from the two power supply can be added either in series or in opposition to each other. Fhrthermore the polarities at the membrane assembly could be reversed. The voltage applied across the electrodes was also applied to the X input of the X-Y recorder.

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3). EXPERIMENTAL RESULTS.

# A). Preliminary Test

The voltage across the electrodes ( i.e. between the aluminized coating and the back electrode ) was varied from -200 volts to +200 volts and the X-Y recorder was used to observe the photomultiplier output as a function of the applied voltages. The positions and orient-ations of the reflectors were adjusted to give maximum visibility. In addition, the observed fringes should be symmetrical about the centre of symmetry. A typical fringe pattern is shown in FIG. 5a. Furthermore the pattern should remain stable during the period required for a measurement. This requires that the mechanical stability is kept within a very small fraction of a fringe ( say< 100/ $\lambda$  ) during the measurement interval. Moreover the voltage sweeps have to be somewhat slower than the time constants of the X-Y recorder. Otherwise the sweep in opposite direction is displaced relative to one another, which results in an absolute displacement of the pattern. Note that the centre of symmetry corresponds to zero potential present across the electret material.

In the series aiding position, the voltage was adjusted and the fringe pattern observed to obtain an estimate of collapse voltage of the membrane (17). Care was taken during the rest of the measurements not to exceed this collapse voltage.

When aluminum was used for the assembly, it was found that the forward sweep and the reversed sweep of the graph did not coincide when the assembly was heated or cooled. This results from changes of path-length in the interferometer arm due to thermal expansion. A great improvement occured when the aluminum was replaced by invar.

When the membrane was heated by passing current through the windings, great care was taken to ensure that the temperature was considerably below the melting pointing of Mylar, for obvious reasons. The relation ,

$$V = I R \cdot [1 + \alpha (T - 20^{\circ})]$$

provides the relationship between the winding voltage V, current I, and temperature T. R.is the resistance at room temperature, and  $\alpha$  is the coefficient of resistivity. For copper,  $\alpha = 0.004/$  °C., at room temperature, T=20 °C, the observed voltages and current parameters was V=5 volts and I= 1.68×5 amperes. Measurements were taken sufficiently quickly that negligible heating of the winding occured.

$$R = \frac{5}{5 \times 1.68} = 0.595$$
 ohms

and

or

V

$$= I \times 0.595 [1 + 0.004 (T - 20)]$$

 $\mathbf{T} = \left[ \left( \frac{\mathbf{V}}{0.595\mathbf{I}} - \mathbf{1} \right) \frac{\mathbf{1}}{0.004} + 20 \right] ^{\circ} \mathbf{C}$ 

Table 1 gives typical data for converting the various parameters to winding temperatures. Note that several minutes are required before the membrane temperature approaches the winding temperature because of the thermal inertia provided by the aluminum in the membrane assembly. The actual voltage across the Mylar will be equal to the applied voltage only at the voltage corresponding to the centre of symmetry of the fringes of the typw shown in FIG. 5b. It can however, be calculated for other applied voltages. Consider the following arrangement,



a = dielectric thickness
d = air gap distance
V = total applied voltage

From the diagram,

 $V = dE + aE_d$  $= E_d (dK + a)$ 

Where E, is the electric field of the dielectric,

K is the dielectric constant,

: voltage across dielectric,  $V_d = E_d$  a

$$=\frac{Va}{Kd+a}$$

For the Mylar, K = 3.3

a = 25 microns

at V = 0 volts, d = 0.005''

and  $\lambda = 6328$  Å = 0.0000249"

From curves similar to that shown in FIG. 5c, the number of fringes moved can be related to applied voltages because the spacing between adjacent peaks corresponds to  $\frac{1}{2}$ . This allow calculation of the actual air separation d. From theoretical calculation given by Ballik (30), collapse voltage corresponds to 1/3 of the distance of the zero field separation between membrane and electrode.

: collapse voltage 
$$=\frac{1}{3} \times 0.005"$$
  
= 0.001667"  
= 67  $\lambda$  at 6328 Å

Corresponding voltage is given by  $V^2 = \frac{4 D^3}{27 k}$ 

where D is the separation of air gap in meters,

k is the proportionality constant.

However the value of k was found somewhat difficult to measure. Values ranged from 2.4 to 4.0 (working in units of wavelength) resulting in a collapse voltage in the range from 670 volts to 540 volts. Values of the voltage across the dielectric are tabulated in Table 2, as a function of the applied voltages across the electrodes.

# B). Membrane Heating Assembly

The transformer input voltage was adjusted by means of the Variac to provide the necessary voltage shown in Table 1, corresponding to the required temperature. During warm-up, the voltage was kept constant. However the current decreased, indicating an increase in resistance and therefore a rise in temperature, as expected. After approximately one hour, the current stabilized to a fixed value, indicating that a steady temperature had been reached. The voltage was then applied across the electrodes. After a predetermined time ( from several minutes to several hours ), the power to the heating coil was removed; the membrane voltage however, continued to be applied. Approximately two hours were reauired for the system to come to room temperature. The electrode voltage was then switched off.

#### C). Duration of Applied Electrode Voltage during Cooling Cycle

A series of measurements were carried out in which the Mylar membrane assembly was heated to a fixed temperature of approximately 145°C and then cooled. A voltage of 400 volts was applied across the electrodes. From Table 2, this corresponds to 27 volts across the membrane. Cooling commenced half an hour later after application of the electrode voltage. Measurements of membrane potential were made, in successive runs, 16 1/2, 12, 5, and 2.5 hours after cooling commenced. During these cooling cycles periods the electrode voltages were continuously applied. The measured voltage in each case was 22 volts, as shown in FIG. 6. This indicates that the interval the voltage is applied at room temperature does not affect the measured membrane voltage or surface charge. Thus once the sample is at room temperature ( after approximately 2 hours ), the applied voltage has negligible effect on the surface charge.

## D). Variation of Charging Time

500 volts ( from Table 2, this was 39 volts of membrane voltage ) was applied to the electrodes at a membrane temperature of about 110 °C for a series of charging times, previous to cooling, of 4, 2, 1, 0.45, 0.25, 0.20, 0.10, and 0.0 hours. The winding voltage was switched off at the end of each charging time ( the applied electrode voltage remained ) and the assembly cooled down to room temperature. The electrode voltage was removed approximately 10 hours after cooling commenced. Then the electrect potential was measured. Results are shown in FIG. 7. It can be seen that the induced voltage increases quickly during the first two hours interval and reaches a value which changes very slowly with large increase in the duration of the applied voltage.

#### E). Variation of Electret Potential with Charging Temperature

500 volts was applied across the Mylar for one hour. A series of measurements were made at different temperatures. These were, in order, approximately 160, 135, 125, 120, 111, 106, 104, 93, and 86 °C. The measured electret potentials are shown in FIG. 8. A large increase in measured potential occured between 80 °C and 130 °C, which then gradually stabilized to a fixed value beyond 130 °C. Below 80 °C, the applied voltage is not very effective in producing an electret. The maximum effectiveness is achieved by 130 °C. Further increases in temperature would risk damage to the Mylar. Note that the corresponding curves for materials other than Mylar would be different.

#### F). Variation of Electret Potential with Applied Voltages

A series of electrode voltages were applied at a charging temperature of 145 °C. Values of 100, 200, 300, 400, and 500 volts were applied for two hours for one sample of Mylar and for one hour for

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another sample, previous to cooling. The electrode voltages remained during the cooling cycle of greater than 6 hours. Both gave a linear equation within the collapse voltage range, as shown in FIG. 9. From the graph, we can see that different samples gave different proportionality constants. 4). DISCUSSION AND RECOMMENDATION FOR FURTHER WORK.

1). From the data in FIG. 6, we can see that the duration of the applied voltage after most of the cooling occurs does not affect the residual charges on the electret. At 145 °C, the thermal energy is great enough to produce an intrinsic volume effect under the applied field. Dipole orientation, displacement of ionic charges and ionic trapping are possible. Since the applied field used in the experiments described here are strong enough for an air breakdown, direct transfer of charges across the air gap can occur. These have been discussed in the section D of 1).

In our experiment, the membrane took approximately two hours to reach room temperature. Once at room temperature, there is insufficient thermal energy to provide further charging mechanism. This explains why the membrane has the terminal charge if the electrode voltage is left on until the membrane reaches room temperature, independent of the duration of the applied voltage at room temperature. Some measurements were carried out in which the electret potential was measured before the assembly reached room temperature. Smaller electret potentials were observed if the applied voltage was removed at higher temperatures. It would seem that, without the applied field, thermal motion at higher temperatures provides enough energy for dipole-disorientation, trap releasing, etc.

A future experiment could investigated many different samples of Mylar,

- 2). The data in FIG. 7 has an exponential which can be explained by the fact that there is a certain relaxation time for each charging mechanism. This depends on the duration of applied voltage because the same voltage and temperature were used for all measurements. Dipole orientation in a field requires a finite time. Once they are approximately ordered, they will be inert to changes. The longer the charging time, the better are the chances of the random dipoles being ordered. But once saturation is reached; additional charging time has negligible effect. The life time of trapping levels of ions are small and equilibrium will be reached in seconds, provided the thermal energy is great enough. Therefore it is not important here. The effect of direct transfer of charges across the air-gap is always present provided we have a high enough applied field. The latter can be investigated by using much lower applied fields. Such experiments would determine the importance of this mechanism.
- 3). The temperature effect in FIG. 8 shows an sudden increse in measured electret charge between 80°C and 130°C. Dipole orientation surely needs a minimum energy ( provided by temperature ) to start aligning in the applied field. From the trapping mechanisms, there were different trap levels corresponding to different energies. Therefore once a certain temperature ( kinetic energy ) is reached, that corresponding trap level will be activated and releasing or trapping of ions can

occur. There is some suspicion that the electret voltage does not change continuously but in small steps.

For further experiments, much smaller temperature increments should be used between 80 °C and 130 °C. This might help to single out the trapping mechanism and their energy levels. Different applied voltages and different samples would help establish if other mechanisms are contributing to charge production.

4). From FIG. 9, the measured electret potential is proportional to the applied voltage. Since a temperature of 145 °C has been used, it is in the saturation range as seen from FIG. 8. Therefore a series of measurements should be attempted at lower temperatures to see the effect of operating below the saturation range.

For more reliable temperature measurement, a thermistor ( or other device ), located near the membrane surface, should be used. Other dielectric materials should be tried to see if the same general relations hold. Different sample thickness and different air gap could be another variable parameter.
5). CONCLUSION.

The mechanism of dipole orientation, ionic trapping, and air gap discharge can be explained by the surface charge measured on the membrane. The strong dependence on temperature was shown by the sudden rise in measured membrane voltage. At saturation temperature, a linear relation links the measured electret charge to the applied voltage. However, the duration of applied voltage at room temperature does not affect the measured electret potential. Charging time at elevated temperatures has an exponential increase on measured electret voltage, reaching a saturation value after about two hours ( at least for the case of Mylar ). More experiments under various condition should be carried out to separate and study each mechanism.

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FIG. 1 ASSEMBLY OF MEMBRANE AND BASE STAND

- 26

- COPPER WIRES
- CORK SHEET
- DELRIN
- B CONDUCTING SCREW



# FIG. 2 HEATING COIL ASSEMBLY



FIG. 3 INTERFEROMETER LAYOUT







( STINU YAAATIBAA ) TUGTUO MG

OUTPUT RECORDER CHART TYPICAL FIG. 5



FIG. 6

- 31 -



- 32 -





FIG. 9

## TABLE 1

 $\mathbf{T} = \begin{bmatrix} \mathbf{v} \\ \mathbf{0.595I} - \mathbf{1} \end{bmatrix} \mathbf{x} \frac{\mathbf{1}}{\mathbf{0.004}} + 20^{\circ} \mathbf{C}$ 

V(volts)	I( 5amperes)	Z= <u>V</u> -1	<u>Z</u> 0.004	T(°C)
5.5	1.19	0.56	140	160
5.0	1.12	0.50	125	145
4.5	1.04	0.46	115	135
4.0	0.95	0.42	105	125
3.8	0.915	0.40	100	120
3.7	0.910	0.364	91	111
3.6	0.90	0.344	85	106
3.5	0.88	0.336	84	104
3.0	0.78	0.29	73	93
2.8	0.74	0.27	68	88
2.0	0.58	0.16	40	60

TABULATION OF TEMPERATURE CORRESPONDING TO APPLIED VOLTAGES.

## TABLE 2

$$V_d = \frac{Va}{Kd+a}$$

Applied (volts) Voltage	Fringes moved(in.)	Real air gap(in.)	Kd+a	Vd
100	2 =0.00005	0.00495	0.0173	5.8
200	8 =0.0002	0.0048	0.0168	11.9
300 ·	17 =0.000448	0.00456	0.0160	18.8
400	33 =0.00082	0.00419	0.0148	27.0
450	44 =0.001095	0.00391	0.0139	32.4
475	50 =0.001245	0.00375	0.0134	35.5
500	57 =0.00142	0.00358	0.0128	39.0
530	67 =0.00167	0.00343	0.0123	43.2

VOLTAGE ACROSS MYLAR MEMBRANE DUE TO APPLIED VOLTAGE ACROSS ELECTRODE.

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