THE PLANETARY ROLLING MILL

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SCOPE AND CONTENTS:

A survey was made of the available literature on Planetary Rolling and Planetary Rolling Mills.

From this a distillation of information was made which included a description of the mill and the material flow when rolling with it. Some independent work was done in the development of rolling formulas.

The end result was the assembly of a design procedure which can be used to calculate roll forces and power requirements in a given Planetary Mill. With this information the capability of a given Mill to perform as required in a rolling situation can be determined.

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SUMMARY

The work presented herein traces a short history of the Planetary Rolling Mill and gives a full description of the physical characteristics of the mill. Material flow in the Planetary Mill is described and a brief comparison made with flow in classical rolling practise.

Thorough attention is given to the calculation of strip speeds, roll forces and power requirements in the Planetary Rolling Mill so that the knowledge developed there may be assembled into a working procedure for pre selecting rolling speeds or trouble shooting machinery failures in the Planetary Mill.

Information given here is nearly all from available literature. The exceptions occur in sections 5.4 and 5.7 where the development is based on the same assumptions as the references but is the work of this author.

Section 6. gathers the formulas of section 5 into a working procedure for Planetary Mill calculations. An example appears in Appendix 5.

The Work gathers together most of the available references on the Planetary Mill since its invention a short twenty years ago.

LIST OF FIGURES

NOMENCLATURE

Symbol	Definition	Units
D	Diameter of the back-up roll	ins.
d	Diameter of the planetary work roll	ins.
H	Thickness of the slab at entry	ins.
h	Thickness of the strip at exit	ins.
B	Width of the slab at entry	ins.
b	Width of the strip at exit	ins.
b_{av}	Average width of the material in the mill throat (classical theory)	ins.
R	Radius of the outer orbit of planetary work rolls	ins.
r	Work roll radius	ins.
V	Speed of slab at entry	fpm.
v	Speed of the strip at exit	fpm.
N	Rotational speed of back-up roll	rpm.
n	Rotational speed of work roll	rpm.
NN	Cage speed	rpm.
S	Surface speed of back-up roll	fpm.
s	Surface speed of work roll	fpm.
s_1	Peripheral speed of work rolls relative to its centre	fpm.
S_c	Speed of cage	fpm.
F	Instantaneous roll force	lbs.
M	Instantaneous torque	in.-lbs.
P	Power	HP
α	Angle of bite	degrees

Symbol	Definition	Units
β	Work roll position from vertical	degrees
ψ	No slip angle (Designates neutral plane classical theory)	degrees
l	Length of the Arc of Contact	ins.
δ	Radial draft	ins.
h	Thickness at neutral plane (classical theory)	ins.
p	Deformation resistance	psi.
θ	Angle of friction	degrees
μ	Coefficient of friction	
ϕ_G	Angle of contact for planetary roll	degrees
a	Advance of planetary slab between roll contacts	ins.
z	Advance of strip due to rolling by one pair of planetary rolls	ins.
z*	Calculated strip advance in iterative process for determination of 'z'	ins.
e, f, g, j, k	Constants used in derivation of z*	
Nr	The number of work rolls about each back-up roll	
Tm	Mean single roll torque	in lbs
Mc	Mean combined torque	in lbs

1. INTRODUCTION

Leonardo Da Vinci made sketches for a mill to roll flat products. Until about twenty years ago all rolling mills were variations of his original concept. They consisted of two work rolls in contact with the metal and varying numbers of support rolls to ensure flatness of the product.

In 1948 T. Sendzimir filed a patent application in Britain for a Planetary Rolling Mill. The mill consisted of a pair of support rolls and some eighteen to twenty-four pairs of work rolls held in orbit about the support rolls by two cages. Each of these pairs of work rolls acts on the slab to reduce its thickness by as much as ninety-eight percent of the original in one passage through the mill.

The following work will describe the mill and its rolling process. The Planetary Mill's action will be compared to conventional mills and some conclusions drawn to relate material properties to the planetary rolling process.

The Planetary Mill is a medium output mill for rolling of steel strip. Its output is less than continuous mills of many stands but more than the reversing two-high or the three-high mill.

This mill can be used to roll all metals but it is especially useful for those which have low ductility.

All forces in the mill are compressive so there are no discontinuities such as sometimes occur in normal rolling practise when the mill rolls pull the metal into the roll throat.

The deformation of metal in the Planetary Mill is very rapid and so the temperature of the incoming metal can be raised as much as 300°F while in the mill throat. Because of this increase metals can be introduced into the mill at less than critical temperature and raised to proper rolling temperature in the mill itself.

The advantage of lower than critical entry temperature is reduced scale formation on the slab. This results in material saving, lower heating costs, and more importantly, less chance of rolled in scale on the strip surface.

In operating the Planetary Mill some care must be taken in preparing the slabs for rolling. A convex edge is required on the slab so that when spread occurs the strip has a square finished edge. The slab surface must be carefully prepared to remove imperfections. Any marks on the slab surface will be magnified on the strip.

As a medium producer the Planetary Mill is a good investment. It can produce continuous heavy weight coils at a rate of approximately one ton, per inch of width, per hour. Due to ease of "change over" of mill guides and screw down gear the mill can be used to do small orders.

Strip produced can be maintained to a tolerance of ± .002 in. on all thicknesses between .060 and .250 in. (Baker 1958). This is very close tolerance for hot rolled strip.

One Planetary Mill can effect the same percent reduction as a continuous mill of eight stands each of which produced a thirty percent reduction.

While this dissertation primarily concerns the Planetary Rolling Mill, in many instances a very brief coverage of the comparable situation in the classical rolling mill theory has been given. This will provide an understanding of the unique qualities of the mill under discussion.

The end product of the investigation is a formulation of mill criteria so that knowing sizes, required tonnage and the material to be rolled, the roll forces and power requirements to roll that material at those speeds may be calculated and thus the feasibility of the operation determined.

2. HISTORICAL DEVELOPMENT OF THE PLANETARY MILL

The schematic drawing of the Planetary Mill, Figure 1, shows that the mill is entirely different from the classical type. The small work rolls revolve, planet-like, around the large inner backing rolls. Each pair of small rolls squeezes a small portion of the slab down to strip thickness. The cage revolves so that approximately 100 pairs of rolls come in contact with the strip per second.

The original Planetary Mill was evolved by an English engineer named Picken who applied for a patent in 1941. (Picken, 1941) Picken's application was for an improved means of rolling metal. The important section of the specification is:

"Rolling means, according to this invention, comprise a pair of rolling units consisting of a set of small rolls freely mounted and arranged in cylindrical form, that is to say having their axes disposed in the surface of a common imaginary cylinder. These rolls are journalled in suitable bearings of the respective rolling unit, and this latter is rotated at a relatively high speed while the billet or other work is fed forward."

Slightly further on in the text, rolling is described as:

"The effect is that the rolls of the two units, acting first on the forward end of the billet, 'stroke' it down to a reduced thickness depending on the distance apart of the roll of one unit from the corresponding roll of the other unit when they are nearest together."

No significant developments took place until 1948 when Sendzimir put forward his own independently conceived ideas.

Sendzimir summarized part of his claim as follows:

"A process of producing metal strip which comprises subjecting flat slab to the action of a procession of pairs of work rolls acting on said slab in rapid succession, each roll entering into contact with the surface of the slab at an oblique angle thereto while leaving said contact almost parallel to its surface, each roll taking a relatively small reduction and producing conjointly with the others a heavy single pass reduction, concurrently forcing said slab with a force sufficient to overcome the horizontal component of the rolling pressure exerted by the work rolls to move forward at substantially uniform speed in the direction of movement of said rolls thereby causing said work rolls, in combination with said feeding force to subject the metal within the major portion of the relatively short zone of plastic reduction to two sided compression, thus eliminating danger of internal cracks." (Sendzimir, 1948)

Sendzimir's first mill was installed by Peugeot, in France, in 1950, for rolling 6.5 inch strip. The second was in America and the third at Ductile Steels in Wolverhamton England. In Canada, two Planetary Mills are owned and operated by Atlas Steels division of Rio-Algom Mines Limited at Welland Ontario and Tracy Quebec.

3. FULL DESCRIPTION OF THE MILL

The Planetary Mill equipment consists of a slab heating furnace of the roller hearth type, a pair of sizing and feeding rolls, the planetary roll assembly, a planishing stand and a coiler.

These are arranged as shown in Figure 2. (Baker, 1958)

The slab is heated to a uniform temperature during passage through the furnace. Descaling is done with high pressure water jets which provide even cooling while washing the scale away.

The feed rolls reduce the slab thickness about 20 percent. These rolls size the material for entry into the planetary assembly and force the slab into the mill throat against the counteracting forces of the planetary work rolls. The feed rate is low; from 4-9 fpm.; so high torque is required for the feed roll drives. Special rolls are used with lateral grooves to provide grip and minimum contact for heat transfer. Cooling is provided to reduce thermal shock and heat checking of the feed rolls.

Between the feed rolls and the planetary assembly the slab moves through a set of water cooled guides. These guides are faced with stellite which only contacts the metal when the tag end is being pushed from feed rolls to the planetary assembly by the succeeding slab. (Walter, 1957)

The Planetary Assemblies typically consist of two 20 inch diameter back-up rolls each surrounded by twenty-six 2 inch diameter work rolls. The rolls have a width of 18 inches which allows 15 inch strip to be rolled.

The work rolls are mounted in a cage which holds them in place and carries them around the back-up rolls.

The "chocks" housing the individual work roll bearings are held into pockets in the annular cage by spring loaded clamps. Figure 3 shows a cross-section of the roll system.

The pressure between work rolls and the back-up roll is preset to counteract centrifugal force and ensure positive drive when the cage is rotating.

The work roll cages are driven by a ring gear which ensures that pairs of rolls pass through the roll bite in accurate alignment with one another. Two oil systems provide lubrication, a flush through system for the back-up roll and a high pressure oil pulsing system for the work roll bearings.

The cages are driven initially through a ring gear and clutch. When strip has emerged from the mill the clutch is disengaged and cage drive comes entirely from contact with the back-up roll. The speed of the work rolls is determined by the total reduction, the input speed, and the back-up roll speed as fully described later in chapter five. Figure 4 gives a simplified description of the roll and material speed conditions at the planetary mill throat. The back-up rolls rotate at 500 rpm. in this example and thus have a peripheral speed of about 2600 fpm.. If the cages were kept from rotating the work rolls would have a peripheral speed of 2600 fpm., conversly, if the cages rotated at 500 rpm. the work rolls peripheral speed, relative to their centres, would be zero. The operating speed is between these two extremes and depends on the rate of advance of the slab.

For example, referring to Figure 4 with a slab speed of 9 fpm. and a back-up roll peripheral speed of 2615 fpm. the ideal cage speed would be 1312 fpm. (227 rpm.). This would mean that 100 pairs of rolls would pass through the roll bite each second. (Larke 1954)

The theoretical cage speed changes from entry to exit at D. (Figure 4) At B the speed is 1312 fpm., while at D the speed is 1413 fpm.. This speed increase is compensated for by placing rubber pads in the bearing mounts. In this way the roll is able to move slightly relative to the cage during contact and this allows the cage to move at constant velocity.

The Planetary Mill is a truly continuous mill in which each slab can be lightly welded to its predecessor and rolled continuously. Each slab pushes the one in front of it into the planetary assembly.

The work rolls are only in contact with the metal for one twentieth of the rolling time. Because of this they remain cool and are not subject to conventional heat checking faults.

It has been found that from 700 to 800 tons can be rolled without regrinding the work rolls. The back-up rolls last much longer. (Baker 1958)

The large reductions achieved by the Planetary Mill generate considerable heat. This is sufficient, at times, to raise the temperature of the metal as much as $300^{O}F$ during passage from entry to exit. Thus the formation of scale can be reduced because the metal can be heated to less than critical temperature in the furnace and then raised to rolling temperature by expenditure of energy due to plastic deformation in the mill throat.

A special field of application is the rolling of hard metals. To roll metals which are resistant to plastic deformation at elevated temperatures it is easier to achieve reduction to light guages by operating the mill to produce an elevated temperature at exit where the guage is least.

Conventional mills are severely handicapped in the above operation because the metal loses heat between stands and is thus coolest at the lightest guage where high temperature is most required.

Since the Planetary Mill can be operated continuously the temperatures are constant at the various stages of rolling. This allows very close strip guage control.

With relatively low entry temperature and contact with the cold work rolls the strip produced by the Planetary Mill has a good surface finish. Work rolls are only in contact with the metal for one onehundredth of a second and after each contact the work roll travels around the support roll where it is cooled by water sprays.

4. MATERIAL FLOW IN ROLLING - CLASSICAL AND PLANETARY MILL COMPARISON

One of the earliest investigations of material flow in classical mills was carried out by A. Hollenberg in 1883 (Underwood). His technique was to place wrought iron dowels in iron bars and then roll the bars to observe the flow of material. Hollenberg's technique was improved by N. Metz who used threaded plugs. A measurement of the change in thread pitch of different parts of the plug showed details of the flow zones. The results of these investigations and others using composite bars with scribed faces at the centre determined the material flow as shown in Figure 5.

The results of these earlier tests indicated three things:

1. In general, straight lines on the side of the bar, perpendicular to the rolling direction do not remain straight.
2. That straight lines on the top surface of the bar do remain straight.
3. That a network of orthogonal lines on the cross-section remains so after rolling.

The rolling of steel in a two-high mill moves the surface ahead of the centre of the bar. This is to be expected since the two work rolls are also the rolls that apply the forward force to the metal.

Orowan made tests using layered plasticine for material and wooden work rolls. (Underwood) The deformations studied were classed as in-homogeneous since the "neutral point" was revealed to be a zone. Figure 6 is taken directly from Underwood and shows the results obtained by Orowan with the plasticine.

The initially plane vertical surfaces became markedly curved in the direction opposite to the direction of rolling. The extention of the surface of the bar took place mainly near the plane of entry and the ends of the layers remained at nearly constant spacing throughout the arc of contact. It is worth noting that the deformation penetrated more deeply as the material passed between the rolls. The zones of deformation are shown in Figure 7.

A similar sort of experimental work was carried out for planetary rolling by H. M. Walter of Ductile Steels Limited. (Walter, 1955 and 1957)

Walter's first experiments involved placing plugs in a slab which was then rolled. The plugs were of different material than the slab and therefore, when etched, showed distinctly.

The forces generated by planetary rolling oppose the motion of the steel through the roll gap. Thus the surface is forced backward while the rest of the steel section behaves somewhat the same as in classical rolling; i.e. the centre lagging behind. Figure 9.

Tests were also carried out with plasticine using a full scale mill model.

The factors which affect the material flow in a Planetary Mill are the sizes of the work roll, the back-up roll, the slab and the strip, as well as the temperature, feeding speed and the amount of slip. Walter's paper discusses the above factors as they affect the material flow. (Walter 1957)

Walter showed that higher feeding speed in a given mill gave a more equal movement of surface and centre. Practical limitations are set by the strength of the rolling mill and the amount of internal heating acceptable in the metal being rolled.

A series of tests were made, varying the roll sizes with a fixed outer orbit size of 24 inches. This meant varying between a 1 inch diameter work roll with a 22 inch back-up roll and a 3 inch diameter work roll with an 18 inch back-up roll. In these tests the larger work roll produced a more uniform flow of rolled material in the bite. The larger work roll diameter meant that material was more deeply affected toward the centre since the metal could not flow to the front or back as easily. Thus, under the same conditions the 3 inch work roll produced a flow of material more closely resembling a conventional rolling mill.

A second result was that small diameter work rolls produced a back-fin. Under the same material conditions and slab speeds, increase of the work roll size eliminated the back-fin in every case. A section of slab showing back-fin is shown in Figure 10.

Back-fin is undesirable because, once formed, the slivers of metal could break off and cause damage to the mill by passage through the rolls. There are conflicting opinions about the reasons for back-fin formation. Trinks states that the plunging of work rolls into a zone of plastic deformation produces a peculiar kneading action in which the metal tries to escape in the direction of least resistance. By this means a wave escapes both before and behind the roll. (Sendzimir 1957)

A larger work roll cannot push material up as easily since the pressure cone gradient under the roll is less extreme. This reduces back-fin formation.

A reduced slab thickness reduces the angle of bite which produces more uniform material flow.

The plasticity of a material changes with change in its temperature. This change can be reproduced with plasticine. Walter's experiments showed that more uniform rolling occurred with a cool slab. Of course, this increased the work roll load but the resultant strip was better.

A chilled surface, such as occurs from water spray descaling of steel produced a more even flow of plasticine during tests.

Under certain circumstances, a slip forward or backward could occur and influence the success of the rolling. Slip backwards tends to retard the surface and act the same as increased feeding speed. Slip forward produces the reverse effect. Backward slip cannot be used as a production technique because it tends to produce defects such as surface overlap and heavy wear on the mill. Large bite angles have the same result as reduced feeding speed, they retard the surface. The large angles also give a greater tendency toward back-fin formation.

The other significant fact stated by Walter was that back-fin is a pressure formed phenomenon. The tests on plasticine were done at slow speed so no impact or slip occurred. Thus, when a back-fin formed it must have been due to pressure. This conclusion appears to negate the statement attributed to Trinks, (Sendzimir, 1957) which describes back-fin formation as wave formed.

Walter's conclusions from experiments on planetary rolling of plasticine are:

1. The dominant factor is the influence of feeding speed.
2. Higher feeding speeds give deeper penetration of rolling pressure toward the centre and more equal flow of material.

3. Bigger work rolls give more benefit than smaller.
4. An increase of slab thickness has the same effect as a decrease of feeding speed or the use of smaller work rolls.
5. Cold and less deformable material improves the material flow.
6. Hot, or more plastic, material reverses the effect.
7. A chilled surface, of different plasticity does not have an adverse effect.
8. A backward slip has a retarding action on the surface of the slab.
9. A forward slip has the same effect as having a more plastic material.
10. The maximum slab thickness for a given feeding speed is dependent on the outer orbit of the work roll.
11. The maximum slab thickness is also dependent on the work roll size in conjunction with feeding speed.
12. Increased feeding speed diminishes the spread in the width.
13. Bigger work roll diameters also decrease the spread in the width.
14. Bigger work roll diameters and higher feeding speeds are beneficial in avoiding the rolling in of scale.

(Walter 1957)

During rolling, best conditions are developed in the two-high mill with high feed, tension on strip, and hot working. The Planetary Mill, by contrast, rolls better strip with large work rolls, a chilled surface, high feed rate and generally cooler, less deformable material.

5. ANALYSIS OF THE KINEMATIC AND DYNAMIC FACTORS IN THE ROLLING PROCESS

5.1 CLASSICAL MILL THEORY

Classical two-high rolling mill theory will be examined briefly in order to permit an informed comparison with the Planetary Mill.

For the classical mill, (Underwood, 1950) shown in cross section in Figure 11, the formulation of speed conditions is subject to the following premises: that the material does not change volume during plastic deformation, that forward slip occurs, that pressure is constant over the arc of contact and that spread is negligible.

The known constant in formulating the speed conditions in the classical mill is the peripheral speed of the rolls... s.

$$s = \frac{2 \pi R n}{12} \qquad (1)$$

R and n are the outer radius and the work roll angular velocity respectively.

At the neutral plane there is no slip between the metal and the rolls so the surface speed is 's'. Thus the axial speed of the metal being rolled is

$$v_{\psi} = s \cos \psi \qquad (2)$$

The angle ψ can be calculated from the expression: (Underwood, 1950)

$$\psi = \frac{\alpha}{2} - \frac{1}{\mu}\left(\frac{\alpha}{2}\right)^2 \qquad (3)$$

ψ is known as the no-slip angle and as such determines the position of the neutral plane. The angle α is the angle of contact, μ is the coefficient of friction for hot steel on steel.

From the roll throat geometry the value of h_ψ, the thickness at the neutral point, can be related to exit thickness by,

$$h_\psi = h + 2R(1 - \cos\psi) \qquad (4)$$

Knowing the constant volume relationships existing in plastic deformation we see that,

$$v h = v_\psi h_\psi \qquad (5)$$

$$v h = s \cos\psi \left[h + 2R(1 - \cos\psi)\right] \qquad (6)$$

In a similar way an expression for velocity at any position x before the exit plane can be developed. This will be valid to the entry plane where $x = R \sin\alpha$.

$$v_x h_x = v h \qquad (5)$$

$$h_x = h + 2R\left[1 - \cos\left(\sin^{-1}\frac{x}{R}\right)\right] \qquad (7)$$

Therefore

$$v_x = \frac{s \cos\psi\left[h + 2R(1 - \cos\psi)\right]}{h + 2R\left[1 - \cos\left(\sin^{-1}\frac{x}{R}\right)\right]} \qquad (8)$$

The rate of deformation of a two-high mill changes throughout the bite but a formula has been developed for the mean rate of deformation where slip occurs at all points but the neutral plane. (Larke 1958)

$$M_s = \frac{s\, h\, \theta\, \cos\theta}{H h} \sqrt{\frac{2(H-h)}{D}} \qquad (9)$$

$$\theta = \tan^{-1}\mu \qquad (10)$$

where Ms = mean rate of deformation
and θ is the angle of friction.

The roll separating force in a two-high mill is made up of two parts, the force due to resistance to plastic deformation and the force due to friction.

The resistance of a material to plastic deformation is determined by temperature, rate of deformation and other factors.

The force distribution along the arc of contact is shown in Figure 11. The length of the arc of contact projected into a horizontal plane is

$$L = R \sin \alpha \qquad (11)$$

If the widths are measured before and after rolling the average width will be given by

$$b_{av} = (B + b)/2 \qquad (12)$$

Using the force distribution shown in Figure 11 and the average width given by equation 12 an expression for roll separating force can be derived.

$$FORCE = \left[L \cdot p \cdot b_{av} + \frac{L}{4}(R \sin \psi \cdot \tan \theta)\right] \qquad (13)$$

where ψ is obtained from equation 3.

5.2 FREE RUNNING SPEEDS FOR PLANETARY MILL

The mill speeds have been partly described using the Ductile Steels Mill example in section three.

For no metal in the bite, no slip, no load, with back-up roll speed N and peripheral speed S

$$S = \frac{N \pi D}{12} \qquad (14)$$

At idling conditions, the work roll peripheral speed s is equal to S. If the angular speed of the cage, NN is known then the speed of the cage, which will be defined as the velocity of the work roll centres is given by

$$S_c = \frac{NN \pi (D+d)}{12} \qquad (15)$$

Thus, $\dot{s}_1$, the peripheral velocity of the work roll relative to its own centre is

$$s_1 = S - S_c \qquad (16)$$

So the angular speed of the work roll is given by

$$n = \frac{(S - S_c)\,12}{\pi d} \qquad (17)$$

But substitution of (14) and (15) into (17) also gives

$$n = \frac{D}{d}(N - NN) - NN \qquad (18)$$

Conversley, if n and N are known then the angular speed of the cage is

$$NN = \frac{S_c \cdot 12}{\pi (D+d)} \qquad (15)$$

5.3 PLANETARY MILL SPEEDS WITH METAL IN BITE

The angle of bite, α , is determined by the roll diameters and metal thicknesses.

$$\cos\alpha = \left[\frac{D+2d}{2} - \frac{H-h}{2}\right] \Big/ \left[\frac{D+2d}{2}\right] \qquad (19)$$

$$R = \frac{D+2d}{2} \qquad (20)$$

$$\alpha = \cos^{-1}\left[1 - \left(\frac{H-h}{2R}\right)\right] \qquad (21)$$

This formulation is identical to that of the classical mill except that in the Planetary Mill case R is the outer orbit radius of the mill and in the classical theory R is the work roll radius.

At the entry point the speed of the back-up roll at the outer orbit is V cos α. (Larke, 1954)

At the exit point, with no slip and no volume change, the work roll speed at the outer orbit is VH / h .

With a slab height reduction of 95%

$$H = 20h$$

$$v = 20V$$

The change of work roll centre velocities between points A and B, Figure 12, part (c) is

$$Sc_A = (S + V\cos\alpha)/2$$

$$Sc_B = (S + v)/2$$

$$\frac{Sc_A}{Sc_B} = \frac{S + V\cos\alpha}{S + 20V}$$

The discontinuity only occurs while the work rolls are in contact $(\alpha \approx 20°)$ so the total angular precession of the rolls relative to their idle speed position is small and is taken up by the rubber mounting blocks. If the rolls are fixed, slip occurs but this is indeterminate.

5.4 STRIP SPEED IN MILL THROAT

A simplified analysis leads naturally to the same formula as classical rolling for the exit speed of the strip. (Gelegi, 1967)

$$VH = vh$$

$$v = \frac{VH}{h} \qquad (5)$$

As an overall analysis this is correct but it gives no insight into the true motion of the strip in Planetary Rolling.

To analyse the strip motion, an intermittent advance of the slab and single roll contact must be considered.

Starting with the following assumptions, (Tovini, 1960),

1. All cross sections of the slab remain truly plane during rolling.
2. No elastic deformation of the rolls occurs.
3. Mill housings are rigid.
4. No slip occurs.
5. The back-up roll spindle and the work roll spindles remain parallel during rolling.
6. The speeds of the back-up roll and the cage are constant.
7. The advance of the slab is intermittent.
8. Advance occurs between successive impacts of the work rolls.

The following derivation will give strip speed in the mill throat and the arc of contact of the work rolls. Referring to Figure 13 the roll preceeding the one shown has just pressed curve 2 into the slab. At the end of the preceeding pass curve 2 was in position 1. Between roll impacts the curve has moved forward a distance 'a'. The curve 2 was a circular arc with centre C at the instant it was generated. As a result of advance 'a' the centre of 2 is at B.

Thus, an instant before the planetary roll contacts the slab the conditions are:

- The circular arc 2 touches the surface of the slab and has its centre at B.
- "C" represents the centre of the back-up roll.
- The curve 1 which is not yet formed represents the outer orbit of the planetary rolls which is about to make contact with the slab. The envelope is a circular arc with centre C.

This argument applies to both the top and bottom rolls.

The position of the roll along the arc of contact is as shown. At each instant the plastic deformation takes place between the exit and entry points G and I. The material outside these limits is not subject to deformation.

When the work roll has moved to angle β the surface of the slab is assumed to have advanced to curve 3 which has the same radius as curve 2 and is a distance z + a ahead of curve 1. The advance 'a' to position 2 is the component due to the slab advance by the feed rolls. The advance 'z' is that due to reduction of the slab by the planetary rolls. The value of 'z' will be different for each position of the work roll. The value of 'z' governs the speed of the strip end and the radial depth of the pass through the slab and consequently the force and rolling power required from the mill.

The value of 'z' depends on the volume displaced by the planetary rolls. This volume can be represented by areas in the metal cross section. (Figure 13) The area EHIF represents the volume which has been displaced. The areas JKMN plus GHP represent the volume restored to the left of the work roll. The value of 'z' may be determined by equating these areas.

To do the calculations a trial value for 'z' must be chosen before any area equalities are determined. This trial value can be improved by successive iterations. Set:

$$EHIF = JKMN + GHP \qquad (22)$$

Consider the point A the origin of a polar coordinate system. All the curves are circular arcs, and all points on the cross section can be expressed in polar coordinates with reference to A(0,0).

The equations needed to approximate 'z' are derived in Appendix 1.

With angle β and a trial value of the strip advance 'z' the strip advance z* is obtained from,

$$z^* = \frac{Ra}{h}\left[\cos\beta + 2\cos\beta_H + \cos\beta_G - 2(1-\cos\alpha)\right] \qquad (23)$$

If z* is not too different from 'z' then z* represents the strip advance, If z* differs appreciably from 'z' recalculate using z* for assumed value of 'z'.

The terms of equation (23) are calculated from,

$$\cos\alpha = \left[1 - \left(\frac{H-h}{2R}\right)\right] \qquad (9)$$

$$\cos\beta_G = \frac{gf + \sqrt{g^2f^2 - (g^2-e^2)(e^2+f^2)}}{(e^2+f^2)} \qquad (24)$$

$$\cos\phi_G = \frac{R\cos\beta_G - f}{r} \qquad (25)$$

$$\cos\beta_H = \frac{kf + \sqrt{k^2f^2 - (k^2-j^2)(j^2+f^2)}}{(j^2+f^2)} \qquad (26)$$

$$\cos\phi_H = \frac{R\cos\beta_H - f}{r} \qquad (27)$$

$$e = z + a + (R - r) \sin \beta \tag{28}$$

$$f = (R - r) \cos \beta \tag{29}$$

$$g = \left[\frac{R^2 + e^2 + f^2 - r^2}{2R} \right] \tag{30}$$

$$j = e - z \tag{31}$$

$$k = \left[\frac{R^2 + j^2 + f^2 - r^2}{2R} \right] \tag{32}$$

The exit speed of the strip can be determined by differentiating z* with respect to time.

$$v_\beta = \frac{dz}{dt} \tag{33}$$

The overall speed v is equal to the mean of the displacement curve divided by the time for one pair of rolls to make contact. (Figure 14-A). Figure 14-A shows the strip displacement and output velocity as the work roll moves through its working arc.

The contact of a second pair of work rolls before release of the slab by the previous pair must also be considered. Combining the actions due to two pairs of rolls at the appropriate angular spacing the combined displacement is determined by superimposing the displacement curves. (Tovini, 1960). The combination results in an increase of about 2.5% over the displacement of a single roll pair.

5.5 RADIAL DRAFT

A convenient parameter in Planetary Mill calculations involving rolling force and power required is the radial draft.

For classical rolling radial draft is defined as the thickness of the layer that is removed in rolling two sides of the slab. Analagously, the draft of the Planetary Mill is the layer of metal which is progressively removed between curves 3 and 2 of Figure 13 measured normal to arc 3 at the plane of entry. (Tovini, 1960).

The draft in this case is not a constant but varies with the roll position. Tovini gives the formula for draft as:

$$\delta = (z + a) \sin \beta_G \qquad (34)$$

The angle β_G is shown in Figure 13. The angle is calculated from equation (24).

As in the ordinary rolling mill the length of the arc of contact is a function of the draft. Consequently the rolling force and other dynamic factors are also draft dependent.

"The draft is therefore a governing factor; its maximum value will determine the maximum value of the force and power required for rolling, and its range of variation in the pass will provide an indication as to the amplitude of forces in play and thus the vibrations set up in the mill." (Tovini, 1960)

5.6 ROLLING FORCE BACKTHRUST AND ROLLING POWER

The rolling force depends on the arc of contact, the width and the unit resistance to deformation of the material at that temperature, percent reduction and rate of deformation.

The length of the arc of contact is given by;

$$\ell = \sqrt{2 r \delta} \qquad (35)$$

Appendix 2 gives the verification of equation (35).

Strip width is defined as b. The unit resistance to deformation is defined as p and depends on chemical composition, temperature of the metal, percent reduction, rate of deformation and thickness to roll diameter ratio.

The instantaneous value of the rolling force is given by;

$$F = pb\ell = pb\sqrt{2 r \delta} \qquad (36)$$

This relationship is similar to that for the classical rolling mill with the difference that δ is not constant along the working arc.

The variation of p may be substantial, increasing as the thickness decreases. This variation was measured by Orowan and Pascoe. (Larke, 1963)

Equation (36) indicates, that for identical material conditions, the force varies with the square root of the draft. F bisects the arc of contact and acts through the centre of the work roll as shown in Figure 15. The force acts at an angle $\mathcal{E}$ to the vertical.

$$\mathcal{E} = \beta - \frac{\phi_G}{2} = \beta - \sqrt{\frac{\delta}{2\lambda}} \quad \text{(Tovini 1960)} \quad (37)$$

The component of force opposing forward motion due to one roll is;

$$F \; \mathrm{Sin}(\mathcal{E}) \quad (38)$$

The total back thrust T which must be overcome by the feed rolls is;

$$T = 2F \; \mathrm{Sin}(\mathcal{E}) \quad (39)$$

In classical mill rolling a pressure zone developes as shown in Figure 11. The mean force resulting from the friction hill times the moment arm from the exit plane gives the roll torque required. The power required depends on the torque and the roll speed.

For the torque and power required for a single planetary work roll we consider rotation about centre C as in Figure 15. The moment arm CT is given by;

$$CT = (R-\lambda) \sin\left(\frac{\phi_G}{2}\right) \tag{40}$$

The back torque for a single roll is;

$$M = CT \cdot F = pb\,(R-\lambda)\sqrt{2\lambda\delta} \cdot \sin\left(\frac{\phi_G}{2}\right) \tag{41}$$

The power required for one roll is;

$$P = \frac{2\,\pi\,M \cdot NN}{12 \cdot 33000} \tag{42}$$

The power required is the sum of the requirements of all the rolls in contact with the strip. The strip displacement versus time is shown in Figure 14. The variation of instantaneous roll force through the working arc is shown on Figure 16. The maximum values of roll separating force and counter torque occur at approximately $\beta = 4^\circ$ to 6°

As the work roll approaches the vertical there is a point where the back thrust is reversed. This occurs when $\beta = \frac{\phi_G}{2}$. The reversal of this thrust does not make the mill self propelling since the contact of the next roll will negate this.

5.7 RATE OF DEFORMATION IN THE PLANETARY MILL

The rate of deformation is defined as the rate of reduction of thickness per inch of thickness at any point in the working arc.

For the Planetary Mill this reduction must be taken from geometry of the mill throat and material speed at the point in question. The approach cannot be the same as in classical rolling theory, section 5.1, equation (9), because there is no neutral point where the roll speeds are equal to the material speed.

The nominal rate of deformation at position ϕ in the throat is the rate of thickness reduction λ, divided by the thickness h_ϕ (Figure 17). Integrating from $\phi = \alpha$ to $\phi = 0$ and dividing by the arc of contact gives the mean rate of deformation.

The rate of thickness reduction λ is;

$$\lambda = 2\left[\frac{v_\phi \ \tan\phi}{h_\phi}\right] \tag{43}$$

The mean speed of the slab at ϕ is v_ϕ ;

$$v_\phi = \frac{v h}{h_\phi} = \frac{v h}{h + 2R(1 - \cos\phi)} \tag{44}$$

Therefore,

$$\lambda = \frac{2 v h \cdot \tan \phi}{[h + 2R(1 - \cos \phi)]^2} \tag{45}$$

The mean rate of deformation is therefore;

$$M_\lambda = \frac{1}{5\alpha} \int_0^{\alpha} \frac{2 v h \cdot \tan \phi}{[h + 2R(1 - \cos \phi)]^2} \, d\phi \tag{46}$$

6. DESIGN APPLICATION

The planetary rolling theory developed above can be applied to test the practicality of applying the rolling process to a new material.

Usually the Planetary Mill exists with fixed detail geometry so variation of the mill design will not be considered here.

In the application of the mill the following are fixed sizes;

- D - the back-up roll diameter
- d - the work roll diameter
- H - the slab thickness
- N - the back-up roll speed
- h - the strip thickness
- Nr - the number of work rolls

For a material to be considered for processing by the Planetary Mill the important economic consideration is output tonnage.

The factors which limit output will be;

- Roll forces generated which represent an upper bound.
- Power required which also represents an upper bound.

The following step by step sequence of parameters will permit the determination of roll forces and power required for a desired output tonnage.

1. Determine the exit speed of the strip to produce the required output.

$$v = \frac{12 \cdot \text{Output}}{5\ bh\ (\text{Sp. Wt.})} \qquad (47)$$

2. Determine the angle of bite.

$$R = \frac{D + 2d}{2}$$

$$\alpha = \cos^{-1}\left[1 - \left(\frac{H-h}{2R}\right)\right] \qquad (16)$$

3. Determine the rate of deformation in the Planetary Mill required to produce the desired output velocity.

$$M_\lambda = \frac{1}{5\alpha} \int_0^\alpha \frac{2 v h \tan\phi}{\left[h + 2R(1 - \cos\phi)\right]^2}\, d\phi \qquad (46)$$

(This can be evaluated numerically)

4. Determine the rolling temperature to be used.
5. Determine the percent reduction of the Planetary Mill.

$$\text{Percent Reduction} = \left[\frac{H-h}{H}\right] 100 \qquad (48)$$

6. Determine the resistance to deformation p for the metal. Knowing the rolling temperature, rate of deformation and percent reduction the deformation resistance of

the metal can be determined from curves such as Figure 18.

The general trend for p versus percent reduction was demonstrated by Alder and Phillips (Larke, 1963) The curves determined for p versus percent reduction at constant temperature and rate of deformation have the general shape of the example in Figure 18.

Because of this the extrapolation appears valid. In the case of an untried alloy a laboratory procedure for obtaining the curve in Figure 18 is suggested in Appendix .

7. Calculate the cage speed, NN, from;

$$S_c = \left[\frac{S + \upsilon}{2}\right]$$

$$NN = \frac{12\, S_c}{\pi(D + d)} \qquad (15)$$

8. Calculate the slab advance between impacts;

The number of roll impacts per minute is;

(No of rolls) NN

The advance of the slab per minute is;

$$V = \frac{\upsilon h}{H} \qquad (5)$$

The advance of the slab between roll impacts is;

$$a = \frac{12\, V}{NN\,(\text{No. of Rolls})} \qquad (49)$$

9. Estimate a trial value for 'z'.

10. Calculate the displacements at intervals along the working arc from;

$$e = z + a + (R - r)\ Sin\,\beta \qquad (28)$$

$$f = (R - r)\ Cos\,\beta \qquad (29)$$

$$g = \left[\frac{R^2 + e^2 + f^2 - r^2}{2R} \right] \qquad (30)$$

$$j = e - Z \qquad (31)$$

$$k = \left[\frac{R^2 + j^2 + f^2 - r^2}{2R} \right] \qquad (32)$$

$$Cos\,\beta_G = \frac{gf + \sqrt{g^2 f^2 - (g^2 - e^2)(e^2 + f^2)}}{(e^2 + f^2)} \qquad (24)$$

$$Cos\,\beta_H = \frac{kf + \sqrt{k^2 f^2 - (k^2 - j^2)(j^2 + f^2)}}{(j^2 + f^2)} \qquad (26)$$

$$\phi_G = Cos^{-1}\left[\frac{R\,Cos\,\beta_G - f}{r} \right] \qquad (25)$$

$$Z^* = \frac{Ra}{h}\left[Cos\beta + 2\,Cos\,\beta_H + Cos\,\beta_G - 2(1 - Cos\,\alpha) \right] \qquad (23)$$

If z* is much different from "z' use z* for new trial.

11. Calculate the roll forces from;

$$\delta = (z + a) \sin \beta_G \qquad (34)$$

$$\ell = \sqrt{2 \lambda \delta} \qquad (35)$$

$$F = p\, b\, \ell \qquad (36)$$

12. The instantaneous torque and back thrust are given by;

$$\varepsilon = \beta - \frac{\phi_G}{2} \qquad (37)$$

$$THRUST = 2 F \sin \varepsilon \qquad (39)$$

$$TORQUE = p b (R - \lambda) \sqrt{2 \lambda \delta} \sin \frac{\phi_G}{2} \qquad (41)$$

13. From the instantaneous values of torque determined in step 12, find the mean torque for a single roll by averaging the values obtained at the different roll positions. Graphically this is shown in Figure 19.

14. The mean combined torque is that required because of more than one roll being in contact at one time. The torque curves overlap as shown in Figure 19 and the combined mean torque is calculated thus;

$$\text{COMBINED MEAN TORQUE} = T_M \left[\frac{\alpha (\text{No. of Rolls})}{360} \right] \qquad (50)$$

$$= M_c$$

15. The power required for metal deformation is determined from;

$$\text{Power} = \frac{2 \pi M_c \cdot NN}{12 \cdot (33000)} \qquad (42)$$

Using the value of M_c obtained in step 14.

If any requirements exceed the strengths given in the mill specifications then trouble may be expected if production rolling is attempted.

An example calculation based on this procedure appears in Appendix 5.

7. CONCLUSIONS

The theory of Planetary Mill Rolling has been investigated in order to allow prediction of a specific mill's performance with the procedure outlined in Section 6.

The results can also be used to investigate the reasons for mill failures or breakdowns.

In the case where the mill is to be used for a completely new alloy the procedure outline in Appendix 5 will permit the determination of p, the deformation resistance, so the procedure in Section 6 can be used.

The next stages in the investigation of the planetary rolling process is to formulate the slip line field for the rolling process. From the slip line field and the accompanying velocity discontinuity diagram the zones of temperature generation could be determined. To date there has not been any investigation into the quantitative aspects of heat generation and temperature rise in the planetary rolling process. The ability to predict the temperature variation during rolling would allow better consideration of metallurgical factors in rolling a particular alloy.

8. FIGURES

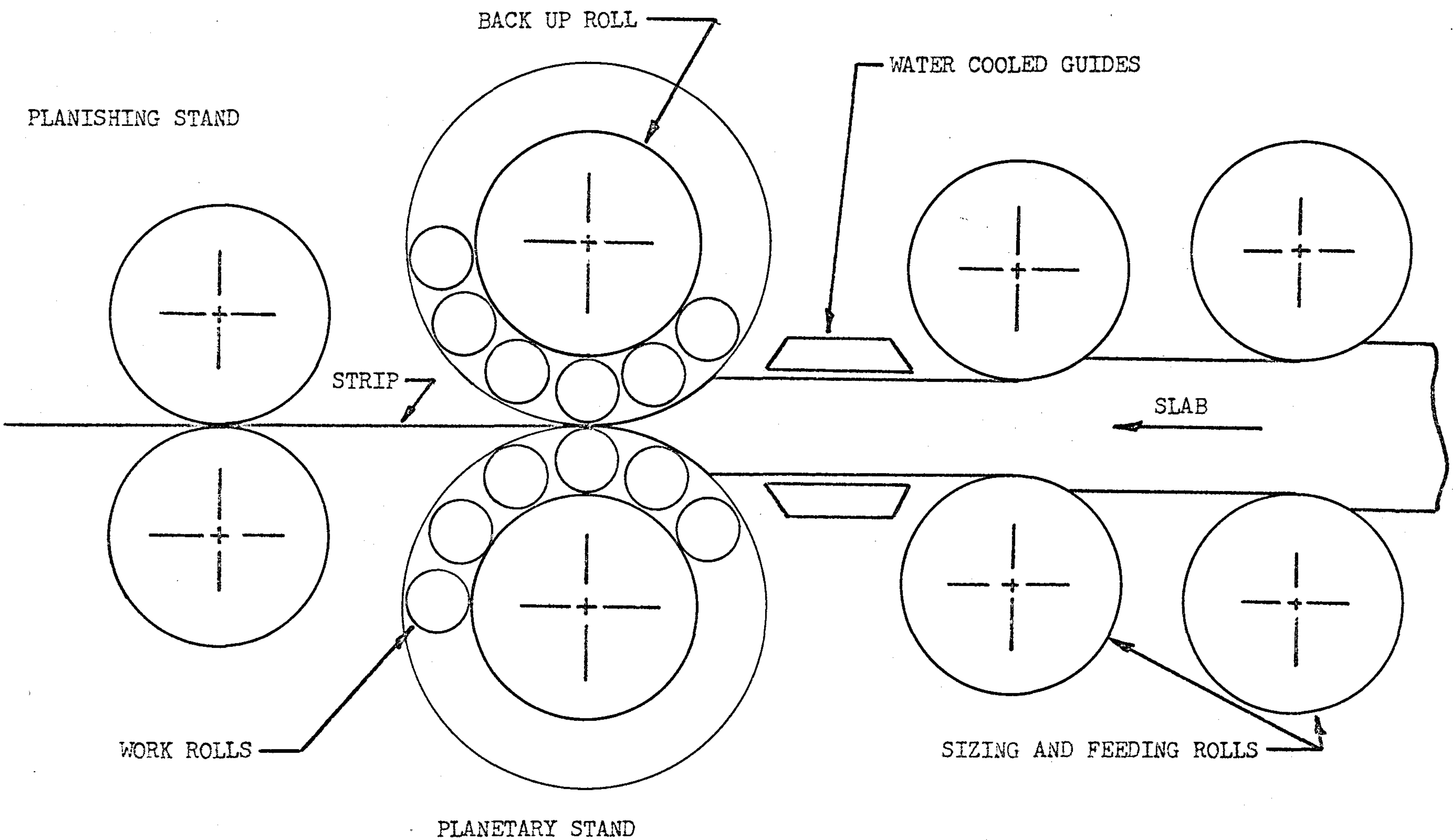


FIGURE 1 SCHEMATIC CROSS SECTION OF PLANETARY MILL

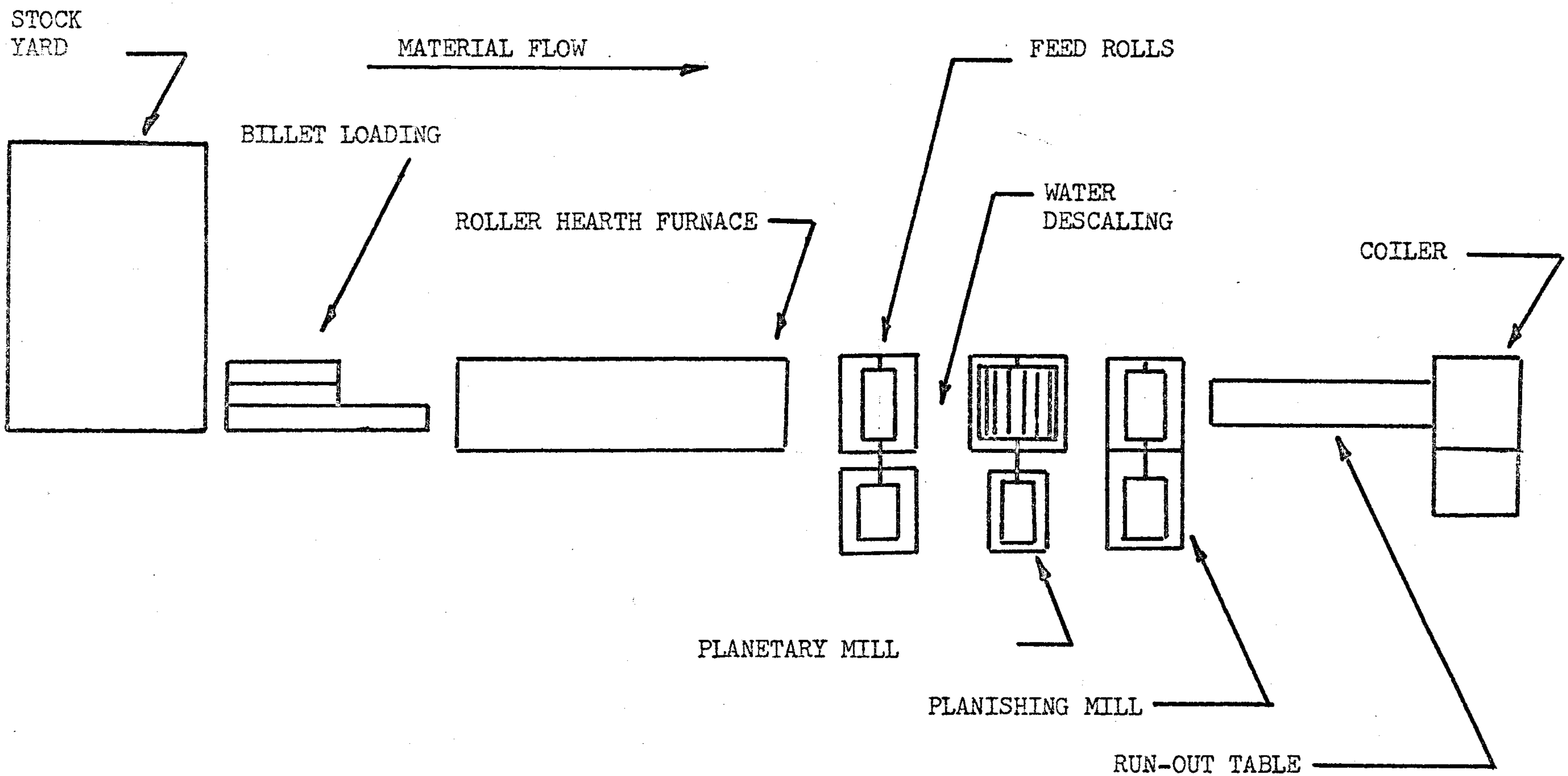


FIGURE 2 TYPICAL PLANETARY MILL EQUIPMENT PLAN

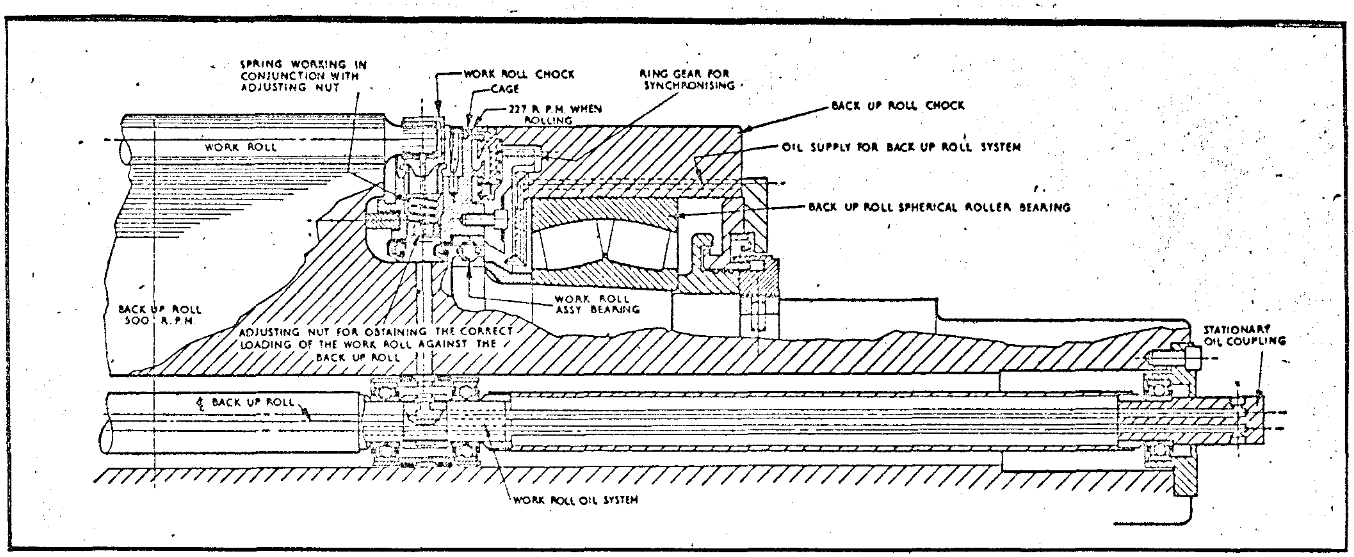
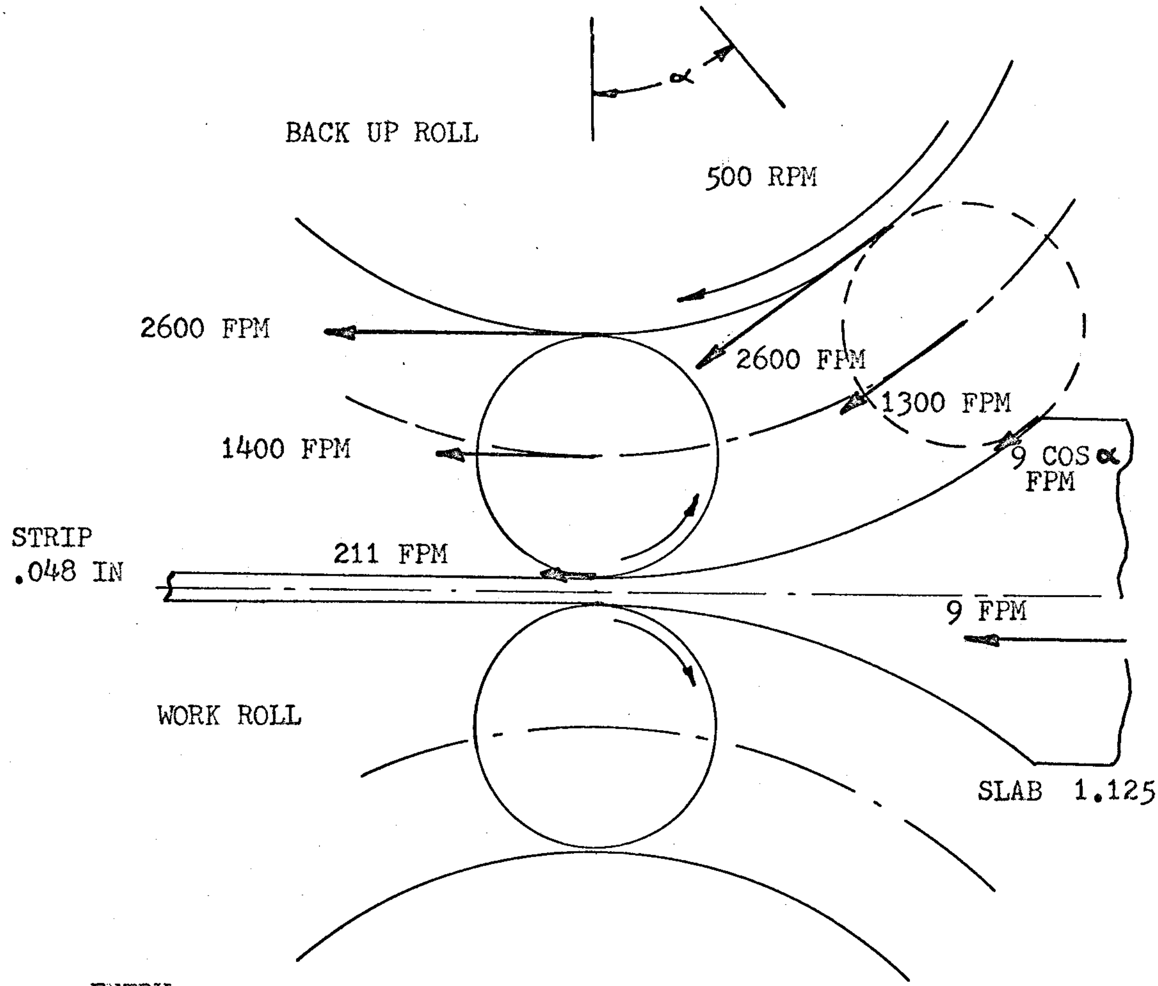


FIGURE 3 ROLL SYSTEM CROSS - SECTION (BAKER 1958)



ENTRY

SPEED OF BACK UP ROLLS = 500 RPM = 2600 FPM = X
ENTRY SPEED OF MATERIAL = 9 COS α FPM = Y
SPEED OF CAGE = (X+Y)/2 = 1300 FPM APPROX.

EXIT

SPEED OF BACK UP ROLLS = 2600 FPM = X
EXIT SPEED OF MATERIAL = 211 FPM = Z
SPEED OF CAGE = (X+Z)/2 = 1400 FPM APPROX.

FIGURE 4 SIMPLIFIED SPEED CONDITIONS IN PLANETARY MILL (LARKE 1954)

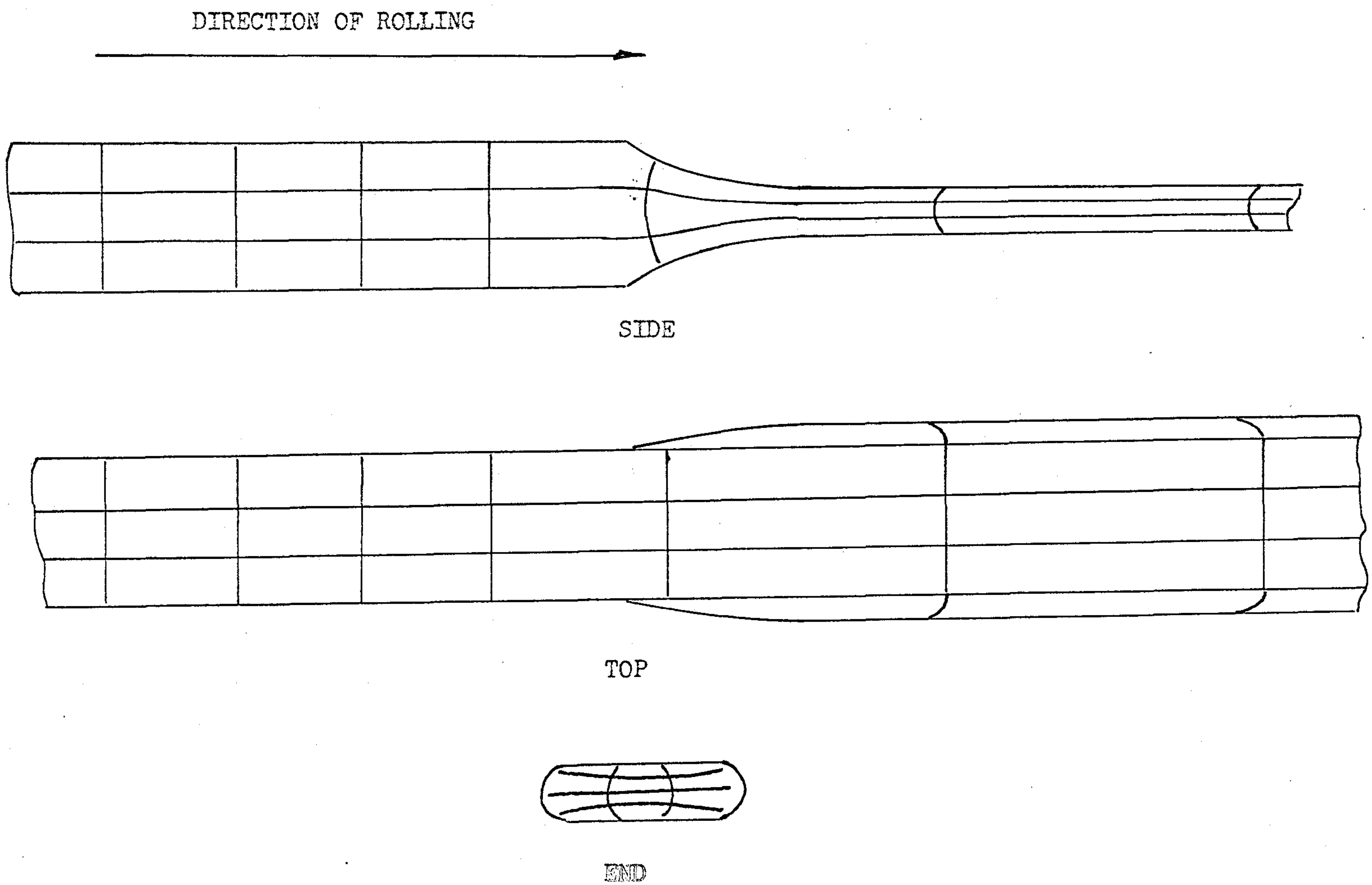


FIGURE 5 FLOW PATTERN IN CLASSICAL HOT ROLLING PRACTISE (UNDERWOOD)

ALTERNATE LAYERED PLASTICINE BAR

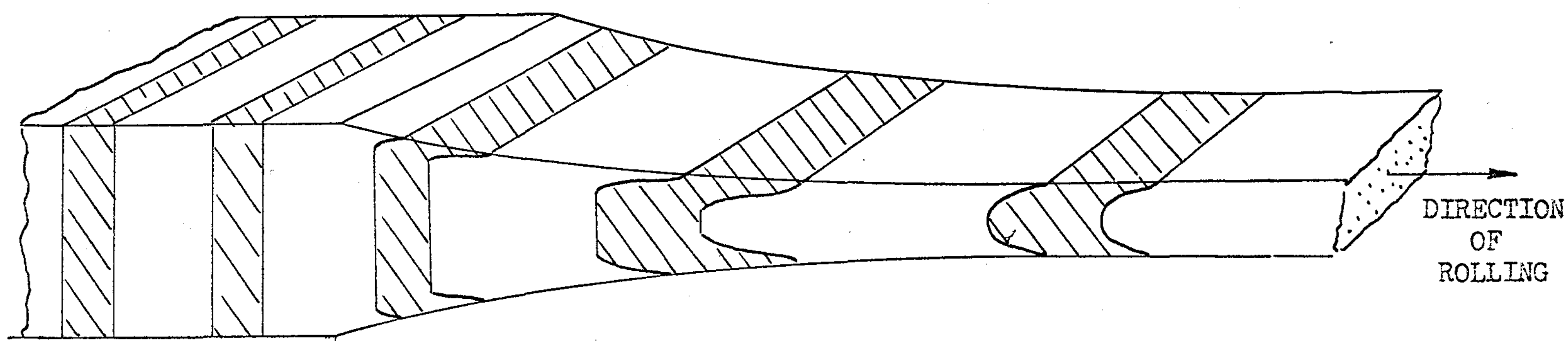


FIGURE 6 FLOW OF MATERIAL IN PLASTICINE BAR (UNDERWOOD)

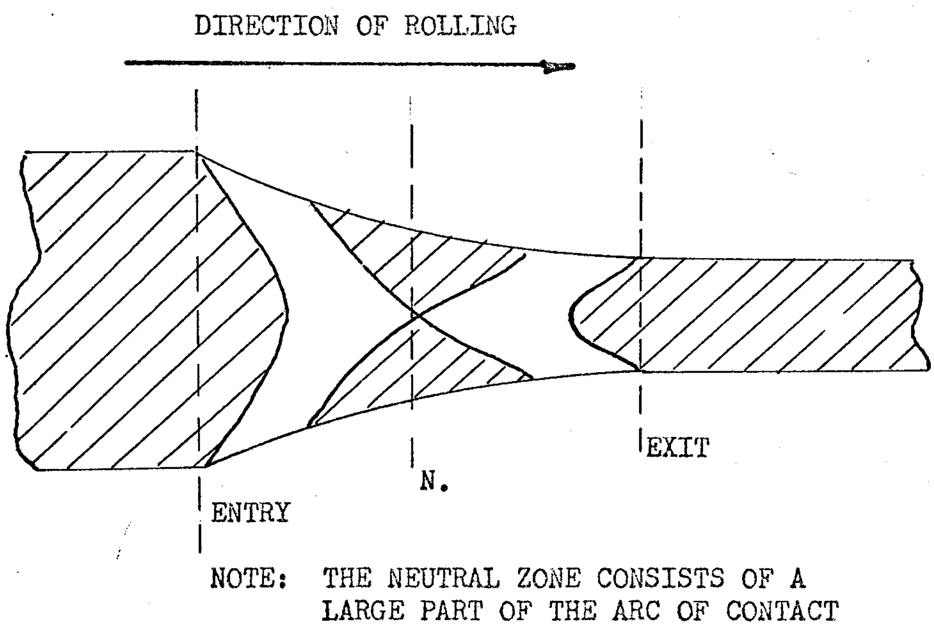


FIGURE 7 DIAGRAMATIC REPRESENTATION OF IN-HOMOGENEOUS DEFORMATION OF MATERIAL BETWEEN ROLLS

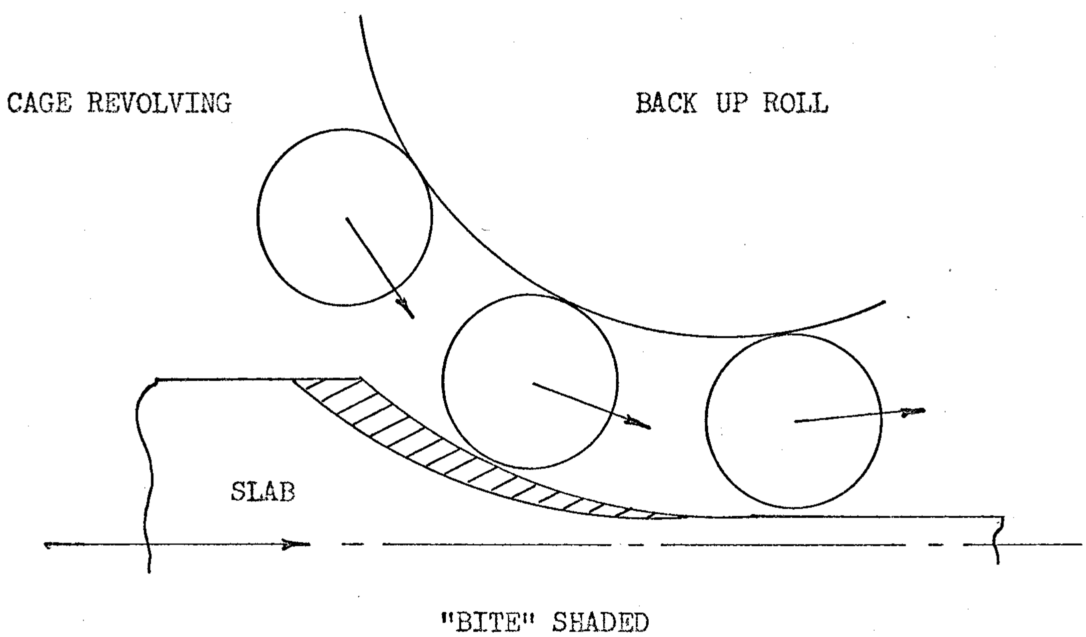
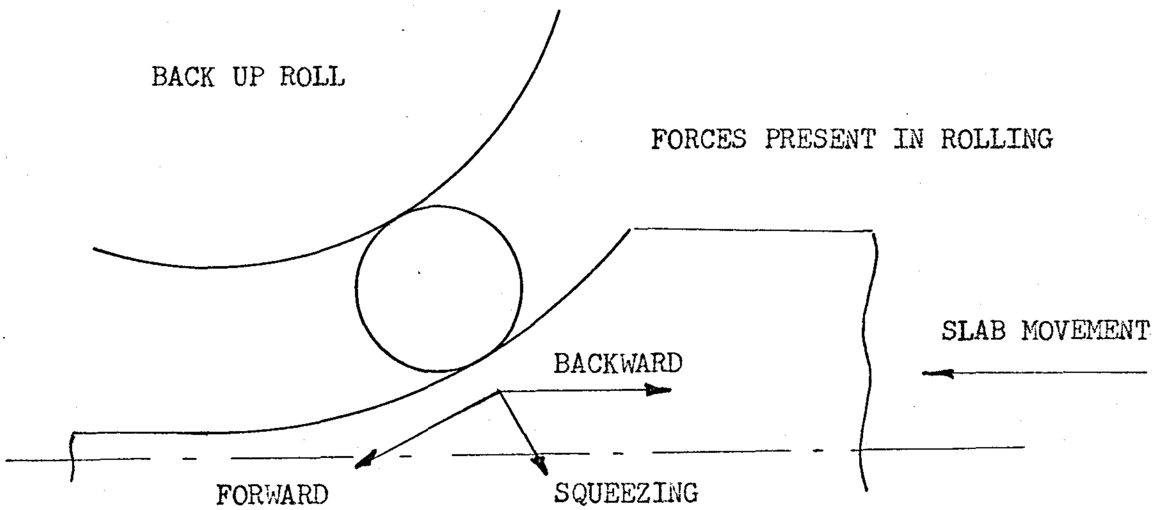


FIGURE 8 "BITE" OF INDIVIDUAL WORK ROLL



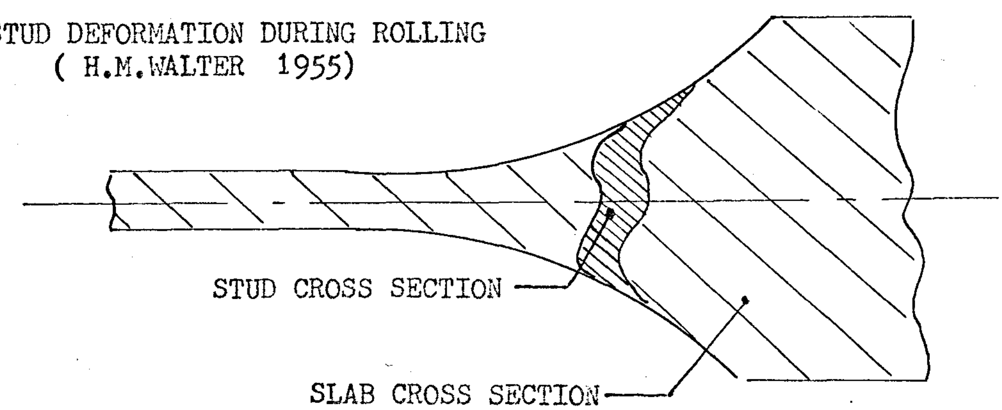
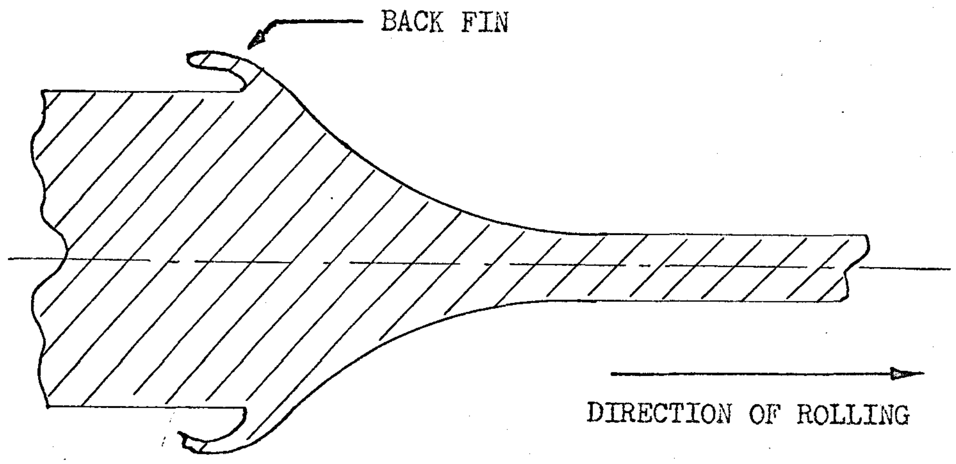
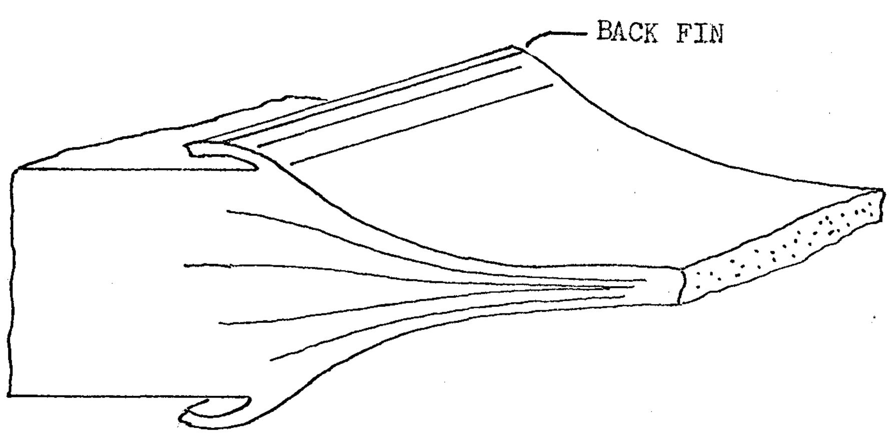


FIGURE 9 ROLLING FORCES IN PLANETARY MILL
AND
STUD DEFORMATION DURING ROLLING (WALTER 1955)



CROSS SECTION OF SLAB



SLAB PICTORIAL WITH BACK FIN

FIGURE 10 BACK FIN ON PLANETARY MILL SLAB

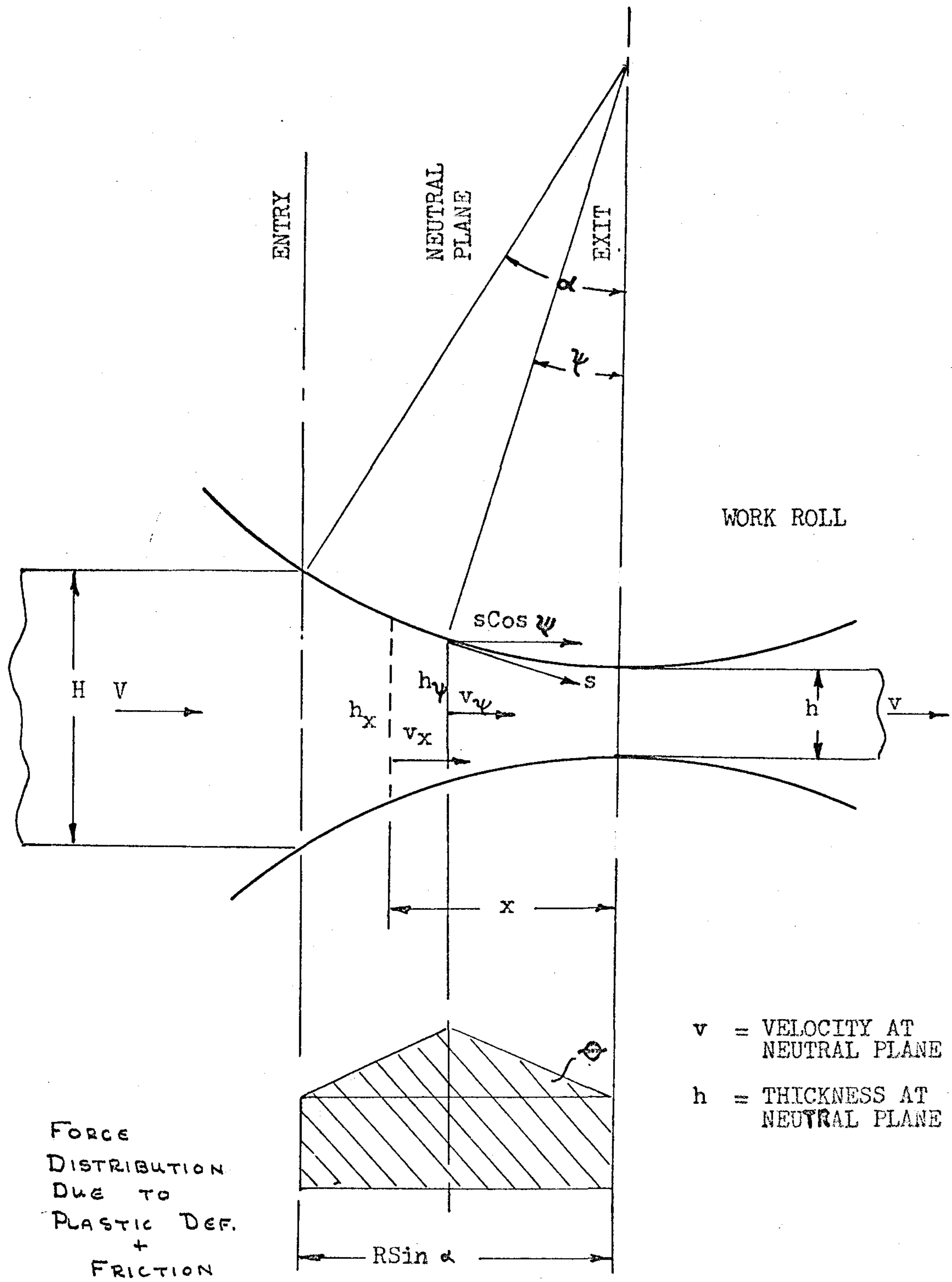


FIGURE 11 SPEED CONDITIONS IN CLASSICAL MILL THROAT

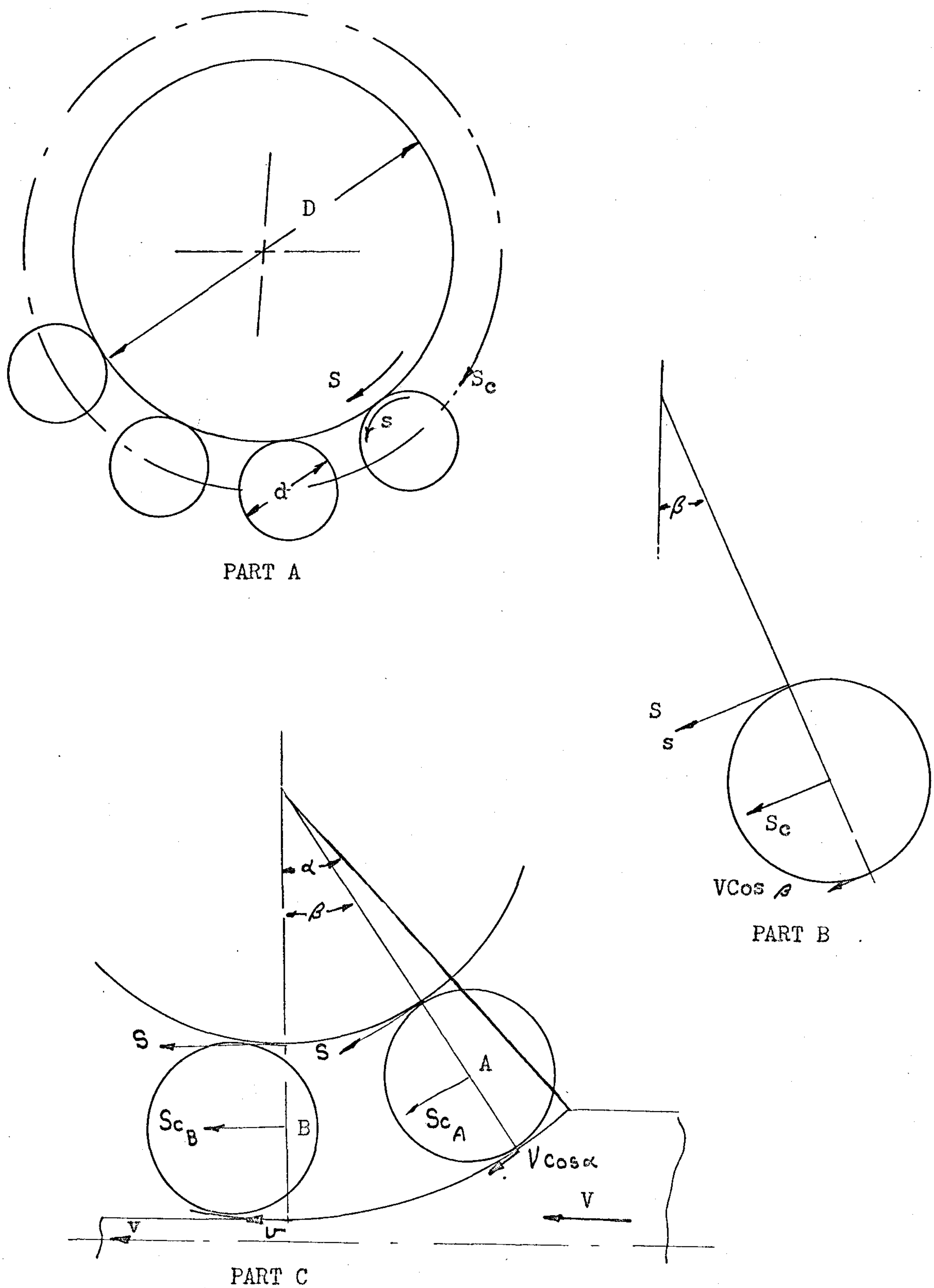


FIGURE 12 REPRESENTATION OF PLANETARY MILL KINEMATICS

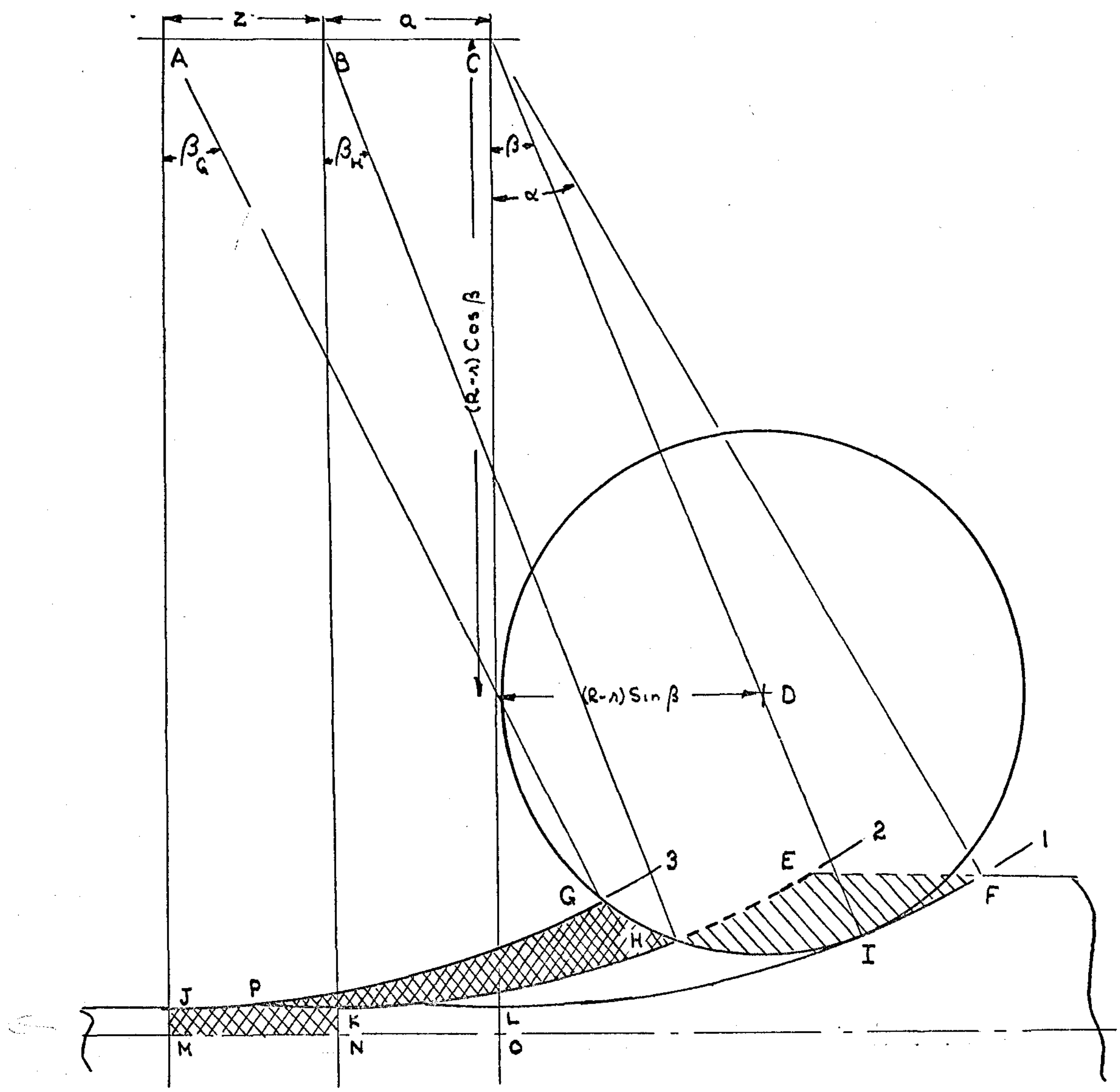
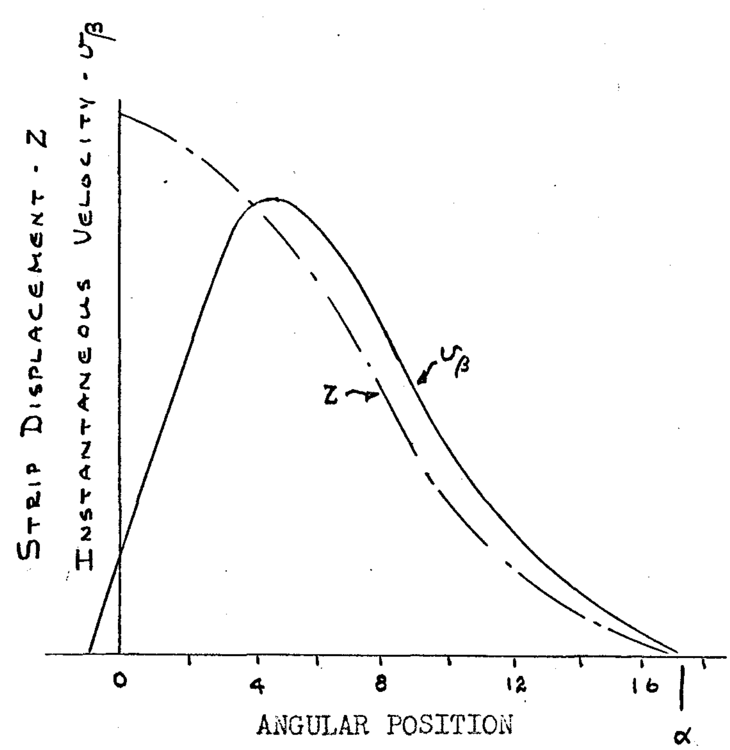
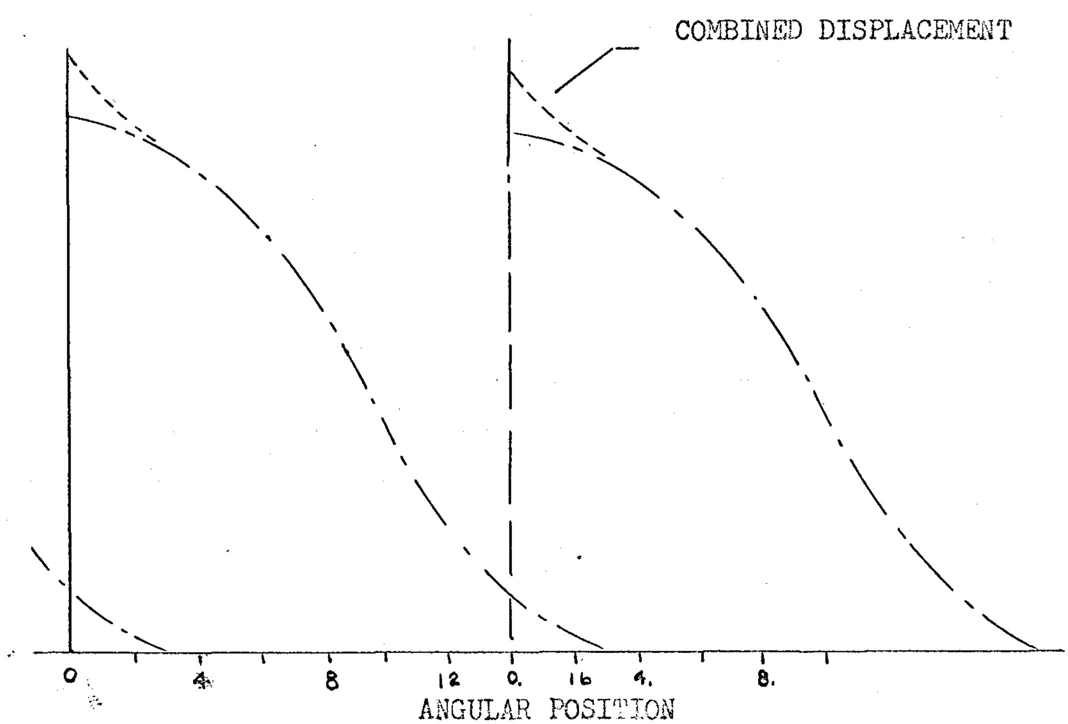


FIGURE 13 PLANETARY MILL CROSS SECTION



PART A EXAMPLE CONFIGURATION FOR STRIP DISPLACEMENT AND RUNOUT SPEED



PART B COMBINATION OF SIMULTANEOUS STRIP DISPLACEMENTS

FIGURE 14

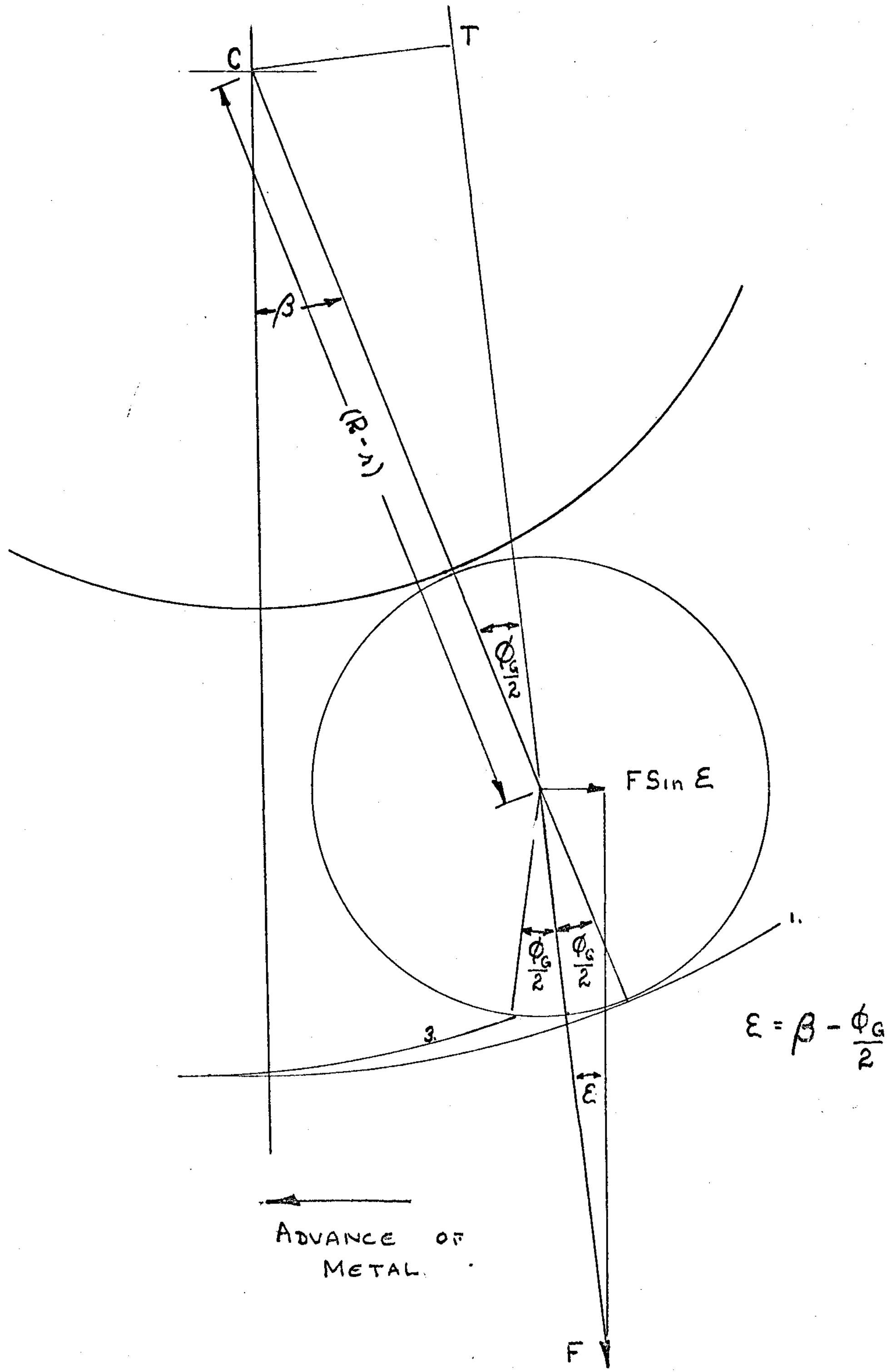


FIGURE 15 INSTANTANEOUS FORCES AND MOMENT ARM AT POSITION β

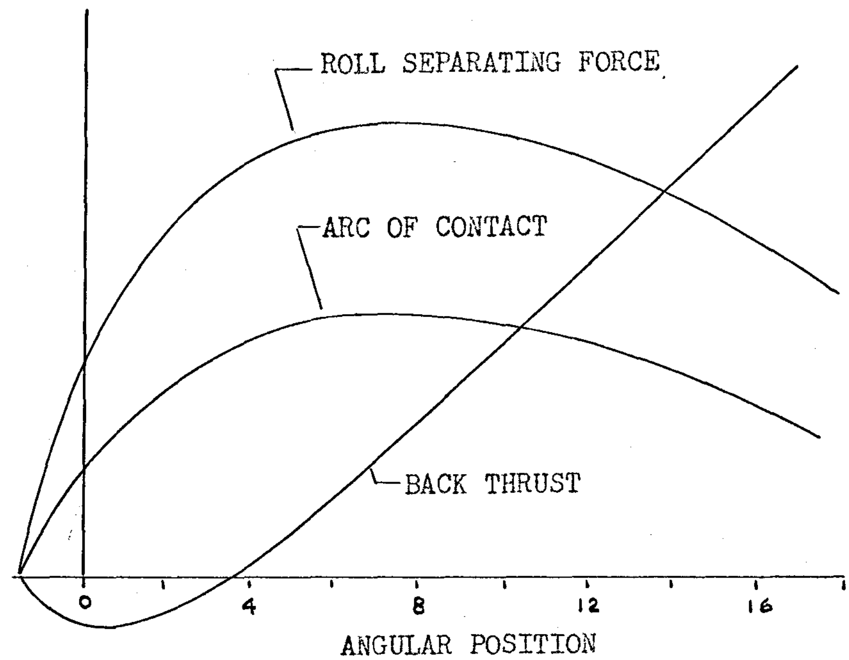


FIGURE 16 MILL FORCES VERSUS ANGULAR POSITION (GENERAL TREND)

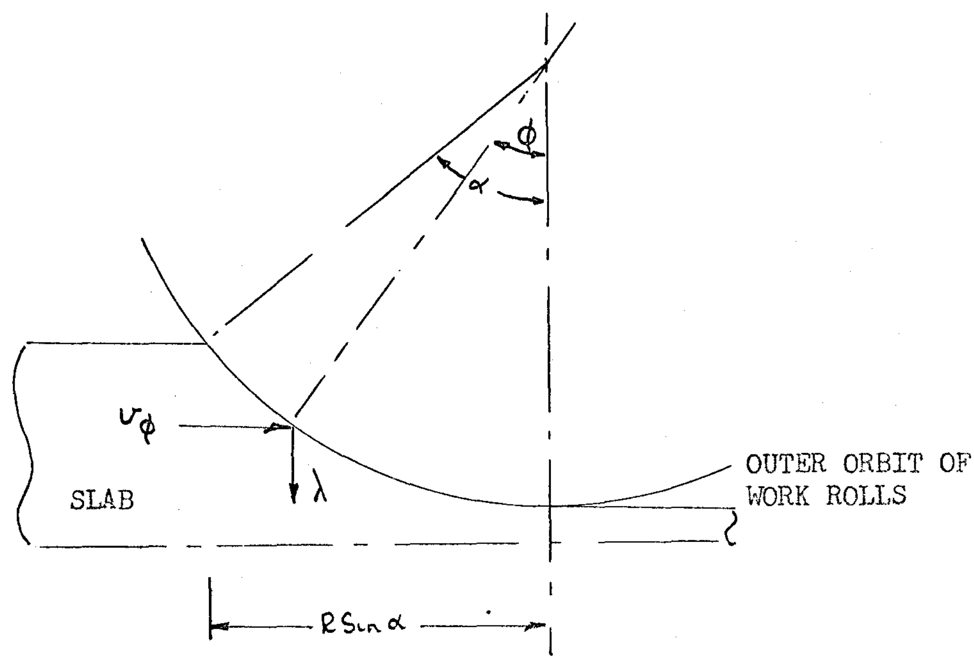


FIGURE 17 SECTION FOR RATE OF DEFORMATION DETERMINATION
IN PLANETARY MILL.

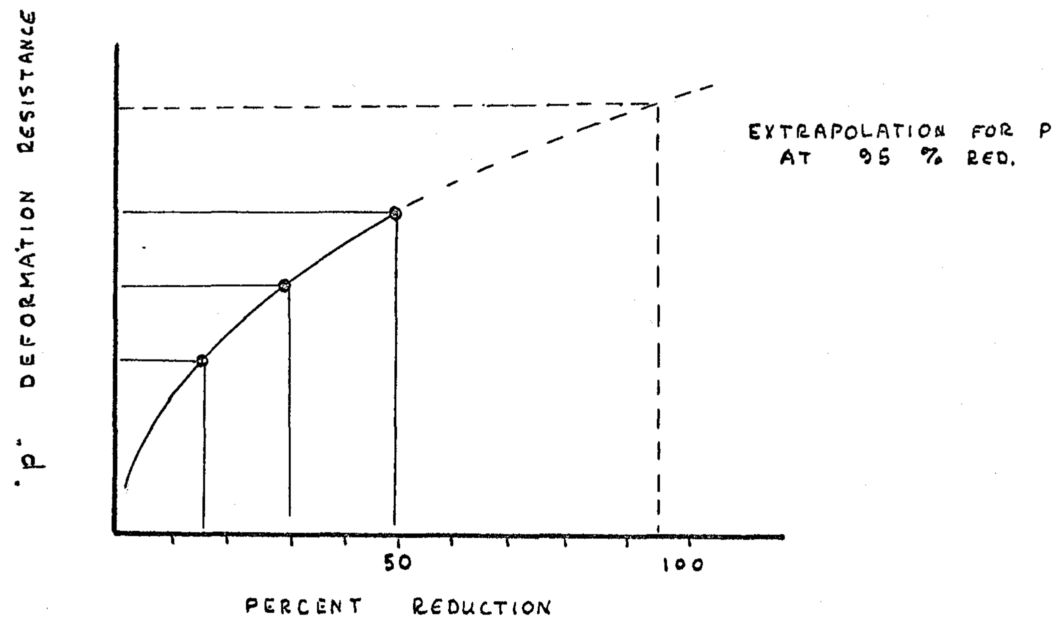


FIGURE 18 DEFORMATION RESISTANCE VERSUS PERCENT REDUCTION FROM ALDER AND PHILLIPS (LARKE 1963)

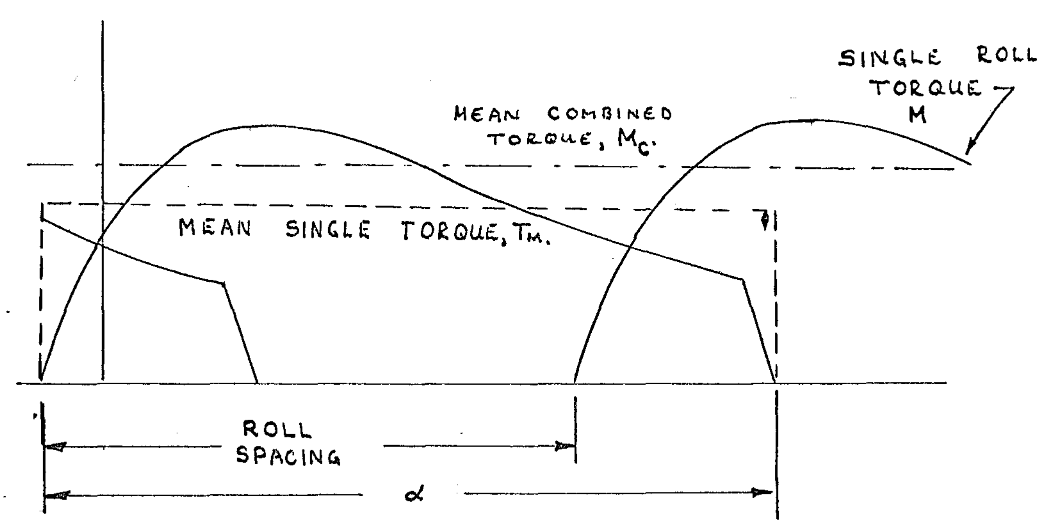


FIGURE 19 MEAN COMBINED TORQUE FROM SINGLE ROLL VALUE

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APPENDICES

1. A detailed development of the strip speed formulas of section 5.4.

2. Derivation of arc length formula from radial draft and explanation of Tovini's formula for radial draft.

3. Results of a test of the iterative nature of the strip displacement formula (23).

4. A suggested procedure for determination of "p".

5. An example calculation using the procedure of section 6.

APPENDIX 1. A DETAILED DEVELOPMENT OF THE STRIP DISPLACEMENT FORMULA IN SECTION 5.4

The assumptions made in formulating this method are,

1. All cross sections of the slab remain truly plane during rolling.
2. No elastic deformation of the rolls occurs.
3. Mill housings are rigid.
4. No slip occurs.
5. All roll axes remain parallel during rolling.
6. The speeds of the back-up roll and the cage are constant.
7. The slab advance is intermittent.
8. The advance occurs between successive work roll impacts.

This section is not an explanation of Tovini's work but another formulation toward the same result.

In order to determine the value of the slab advance 'z' (Figure AP-1) the area equality

$$JKMN + GHKL = EFIH \tag{22}$$

must be determined.

By setting up polar equalities for the coordinates of G and H the values of $\cos\beta_G$, $\cos\beta_H$, ϕ_G and ϕ_H can be determined.

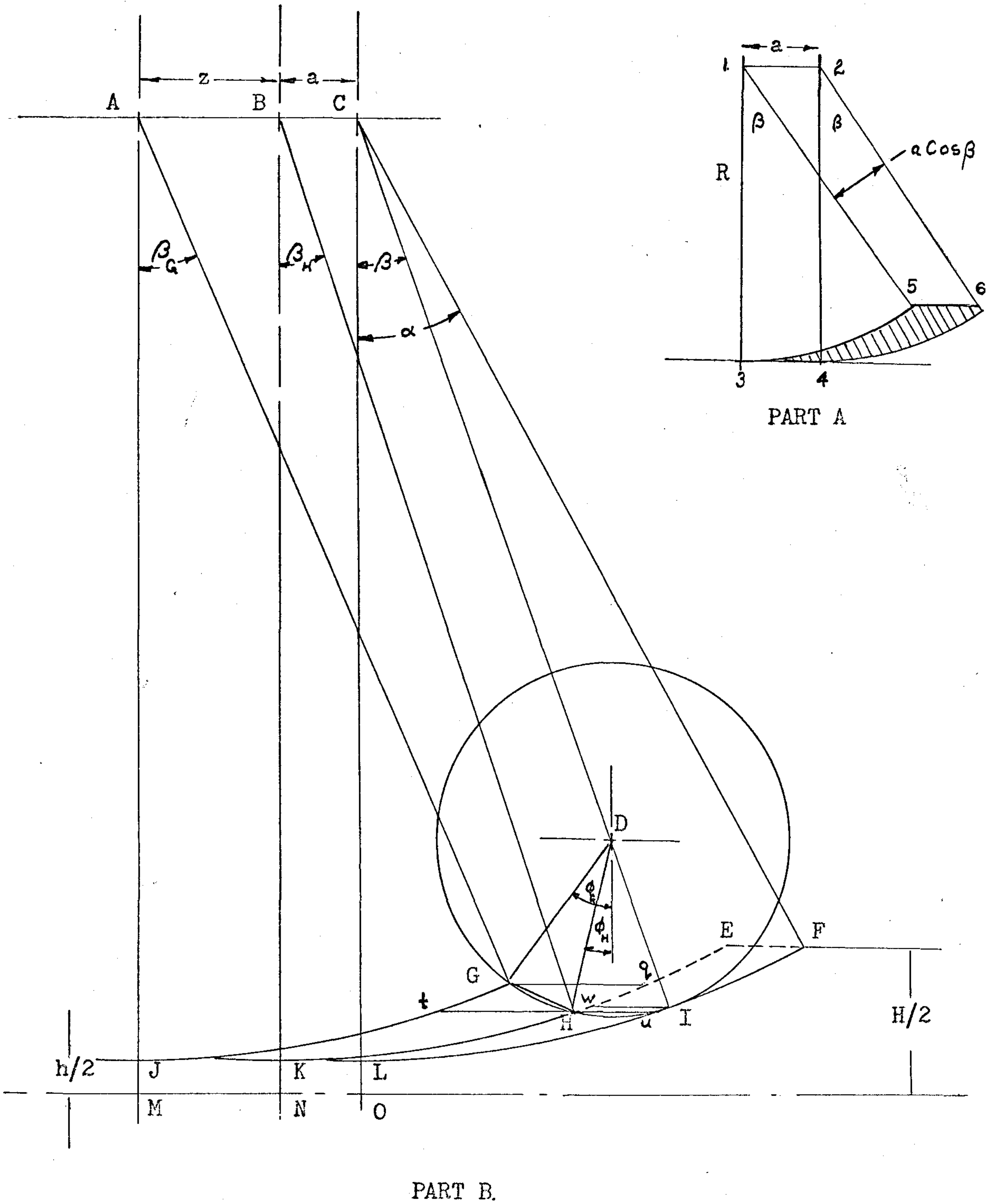


FIGURE A1-1 PLANETARY MILL CROSS SECTION AS IN FIGURE 13

For point G the values of $\cos\beta_G$ and ϕ_G are required. β is fixed.

In the horizontal direction;

$$R \sin\beta_G = z + a + (R - r)\sin\beta - r\sin\phi_G$$

In the vertical direction;

$$R\cos\beta_G = (R - r)\cos\beta + r\cos\phi_G$$

Form the constants e and f;

$$e = z + a + (R - r)\sin\beta \qquad (28)$$

$$f = (R - r)\cos\beta \qquad (29)$$

Therefore, by substitution;

$$R\sin\beta_G = e - r\sin\phi_G$$

$$R\cos\beta_G = f + r\cos\phi_G$$

Therefore;

$$R^2\sin^2\beta_G - 2eR\sin\beta_G + e^2 = r^2\sin^2\phi_G$$

$$R^2\cos^2\beta_G - 2fR\cos\beta_G + f^2 = r^2\cos^2\phi_G$$

Add and separate Cos and Sin terms to opposite sides of the equality;

$$R^2(\sin^2\beta_G + \cos^2\beta_G) - 2R(e \sin\beta_G + f\cos\beta_G) + (e^2+f^2)$$
$$= r^2(\sin^2\phi_G + \cos^2\phi_G)$$

$$(R^2 + e^2 + f^2 - r^2) - 2Re\sin\beta_G = 2Rf\cos\beta_G$$

Let;

$$g = \left[\frac{R^2 + e^2 + f^2 - r^2}{2R}\right] \qquad (30)$$

Therefore;

$$g - f\cos\beta_G = e\sin\beta_G$$

From $\cos^2(A) + \sin^2(A) = 1$;

$$g^2 - 2f\cos\beta_G + f^2\cos^2\beta_G = e^2\sin^2\beta_G$$
$$= e^2(1 - \cos^2\beta_G)$$

$$(e^2+f^2)\cos^2\beta_G - 2fg\cos\beta_G + (g^2 - e^2) = 0$$

Solve for;

$$\cos\beta_G = \frac{gf + \sqrt{g^2f^2 - (g^2-e^2)(e^2+f^2)}}{(e^2+f^2)} \qquad (24)$$

$$\cos\phi_G = \frac{R\cos\beta_G - f}{r} \qquad (25)$$

By a similar procedure the values for Cos   and   can be found.

Using the horizontal and vertical equalities;

$$z + R \sin\beta_H = z + a + (R-r)\sin\beta - r\sin\phi_H$$

$$R\cos\beta_H = (R-r)\cos\beta + r\cos\phi_H$$

Let;

$$j = a + (R-r)\sin\beta = e - z \qquad (31)$$

$$f = (R-r)\cos\beta \qquad (29)$$

$$k = \left[\frac{R^2 + j^2 + f^2 - r^2}{2R}\right] \qquad (32)$$

Therefore;

$$\cos\beta_H = \frac{kf + \sqrt{k^2 f^2 - (k^2 - j^2)(j^2 + f^2)}}{(j^2 + f^2)} \qquad (26)$$

$$\cos\phi_H = \frac{R\cos\beta_H - f}{r} \qquad (27)$$

The basis for the area equality determination is made by referring to figure A1-1 Part A.

$$3\text{-}4\text{-}6\text{-}5 + 1\text{-}2\text{-}6\text{-}5 + 1\text{-}5\text{-}3 = 1\text{-}2\text{-}4\text{-}3 + 2\text{-}6\text{-}4$$

$$1\text{-}5\text{-}3 = 2\text{-}6\text{-}4$$

$$\therefore \quad 3\text{-}4\text{-}6\text{-}5 = aR(1 - \cos\beta)$$

In Figure A1-1 B the areas EFIH and GHP can be very closely approximated in terms of areas similar to 3-4-6-5. The angles are;

$$\alpha = \cos^{-1}\left[1 - \left(\frac{H-h}{2R}\right)\right]$$

$$\beta$$

$\cos\beta_G$
$\cos\beta_H$ } CALCULATED ABOVE:

With the construction lines Gq and Ht, Hu and Iw drawn parallel to the slab axis and the chords GH and HI drawn we can consider tGqH a parallelogram and thus

GHKJ = The average area of the shapes. + The area of the sector on chord GH

$$GHKJ = \frac{1}{2}\left[GqKJ + tHKJ\right] - r^2\left[\frac{\pi(\phi_G - \phi_H)}{360} - \frac{\sin(\phi_G - \phi_H)}{2}\right]$$

The second term, the sector area, can be neglected since it is extremely small.

Also;

$$EHIF = EFKL - HIKL$$

$$= EFKL - \left\{\frac{1}{2}\left[HtKL + IwKL\right] - \text{SECTOR AREA}\right\}$$

By using the relationship developed for 3-4-6-5 in Figure A1-1 the area equality (22) can be expressed as follows,

$$JKMN = \frac{zh}{2}$$

$$GHKI = \frac{1}{2}\left[Ra\,(1 - \cos\beta_G) + Ra\,(1 - \cos\beta_H)\right]$$

$$EHIF = Ra\,(1 - \cos\alpha) - \frac{Ra}{2}\left[(1 - \cos\beta_H) + (1 - \cos\beta)\right]$$

Therefore (22) becomes;

$$\frac{zh}{2} + \frac{Ra}{2}\left[2 - \cos\beta_G - \cos\beta_H\right] = Ra\,(1 - \cos\alpha) - \frac{Ra}{2}\left[2 - \cos\beta_H - \cos\beta\right]$$

and, $$z^{*} = \frac{Ra}{h}\left[\cos\beta + 2\cos\beta_H + \cos\beta_G - 2\,(1 - \cos\alpha)\right] \qquad (23)$$

If z* is not equal to the value of 'z' which was used to begin the calculation set 'z' = z* and start again.

APPENDIX 2. DERIVATION OF ARC LENGTH FORMULA FROM THE DRAFT

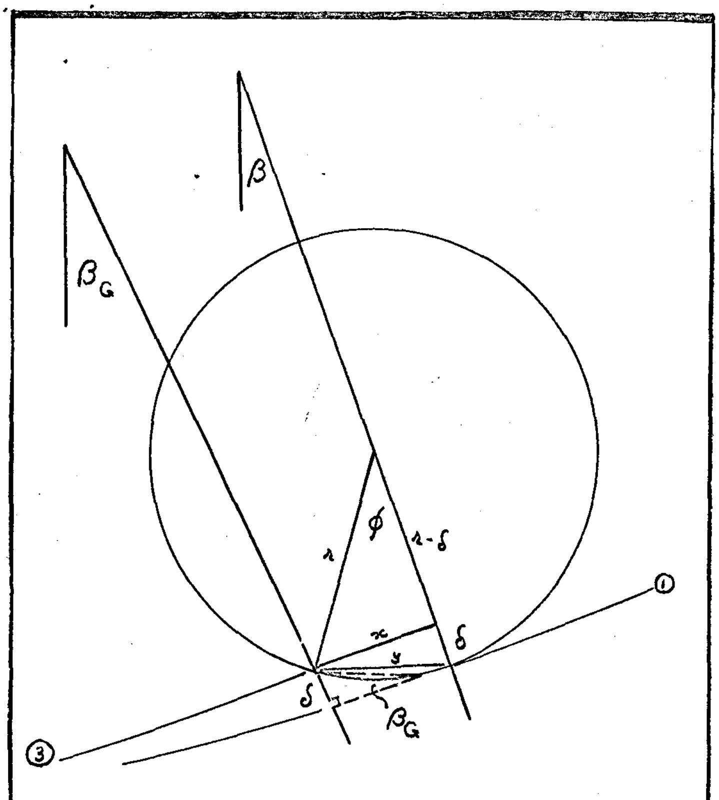


FIGURE A2-1

From Figure A2-1

$$x = \sqrt{r^2 - (r-\delta)^2} = \sqrt{2r\delta - \delta^2}$$

$$y = \sqrt{x^2 + \delta^2}$$

$$y = \sqrt{2r\delta}$$

For small values of ϕ the chord is almost equal to the arc length.

ϕ is small;

$$\therefore \quad l = \sqrt{2r\delta} \qquad (35)$$

TOVINI'S DERIVATION OF

Tovini's formula for Radial Draft is;

$$\delta = (z + a) \sin \beta_G \qquad (34)$$

This shows in Figure A2-1 where the shaded triangle has a horizontal line between curves 1 and 3 for a hypotenuse.

The formula is accurate enough considering the small values of z and a which occur in practise.

APPENDIX 3. A TEST OF THE ITERATIVE NATURE OF THE STRIP DISPLACEMENT FORMULA

To test the iterative nature of the strip displacement formulation, section 5.4, equations 23 to 32, a computor program was written incorporating those equations.

For the test the input parameters were, Beta = 6, R = 12., r = 2., H = 1.5, h = .075, b = 15., a = .012, p = 20000., and the first trial 'z'= .15.

In the first test each trial after the first was started with the average of 'z' and z*. the calculation took seven trials before an acceptable value of z* was reached.

In the second test each trial was initiated by the value of z* from the preceding calculation. This test took two iterations before arriving at the same value of z* as the first test.

For each test the calculation was carried on until the difference between z* and z for that particular trial was less than .001.

In both cases the value of z* obtained was .1895 correct to four decimal places.

The results are tabulated next page.

Trial	Initial z	z*
Test No. 1.		
1	.1500	.18693
2	.16981	.18955
3	.17969	.18951
4	.18460	.18949
5	.18705	.18948
6	.18827	.18947
7	.18887	.18947
Test No. 2.		
1	.1500	.18963
2	.18963	.18947

APPENDIX 4. A LABORATORY PROCEDURE FOR DETERMINATION OF 'p'

The outline presented here is a suggested laboratory procedure for producing the deformation resistance curve of Figure 18 for an untried alloy.

1. Equip a laboratory two-high mill with load cells under the roll screw-downs in order to measure the roll separating force during rolling.

2. Measure the coefficient of sticking friction between the rolls of the laboratory mill and the alloy by placing a block of it, at rolling temperature, on a steel plate of equal surface roughness as the mill rolls and tilting the plate. The angle θ to which the plate is tilted at the moment sliding occurs is the angle of sticking friction.

3. Choose three different percent reductions and in each case, calculate the laboratory mill surface speed to give the same rate of deformation as the Planetary Mill would have.

$$\mu = \tan\theta \qquad (10)$$

$$\alpha = \cos^{-1}\left[1 - \left(\frac{H-h}{2R}\right)\right] \qquad (16)$$

$$\psi = \frac{\alpha}{2} - \frac{1}{\mu}\left(\frac{\alpha}{2}\right)^2 \qquad (3)$$

$$M_s = \frac{\alpha h \psi \cos\psi}{H h} \cdot \sqrt{\frac{2(H-h)}{D}} \qquad (9)$$

$$\therefore \Delta = \frac{M_s\, h\, H}{\left[h\,\psi\, Cos\,\psi \cdot \sqrt{\frac{2(H-h)}{D}}\right]} \qquad (9)$$

4. Roll samples at the three different reductions, with their respective surface speeds, and record the separating force in each case. Measure the sample width before and after rolling.

Knowing;

$$FORCE = 2\left[L \cdot p \cdot b_{av} + \frac{L}{4}\left(R\, Sin\,\psi \cdot Tan\,\theta\right)\right] \qquad (13)$$

$$L = R\, Sin\,\alpha \qquad (11)$$

$$b_{av} = \frac{B + b}{2} \qquad (12)$$

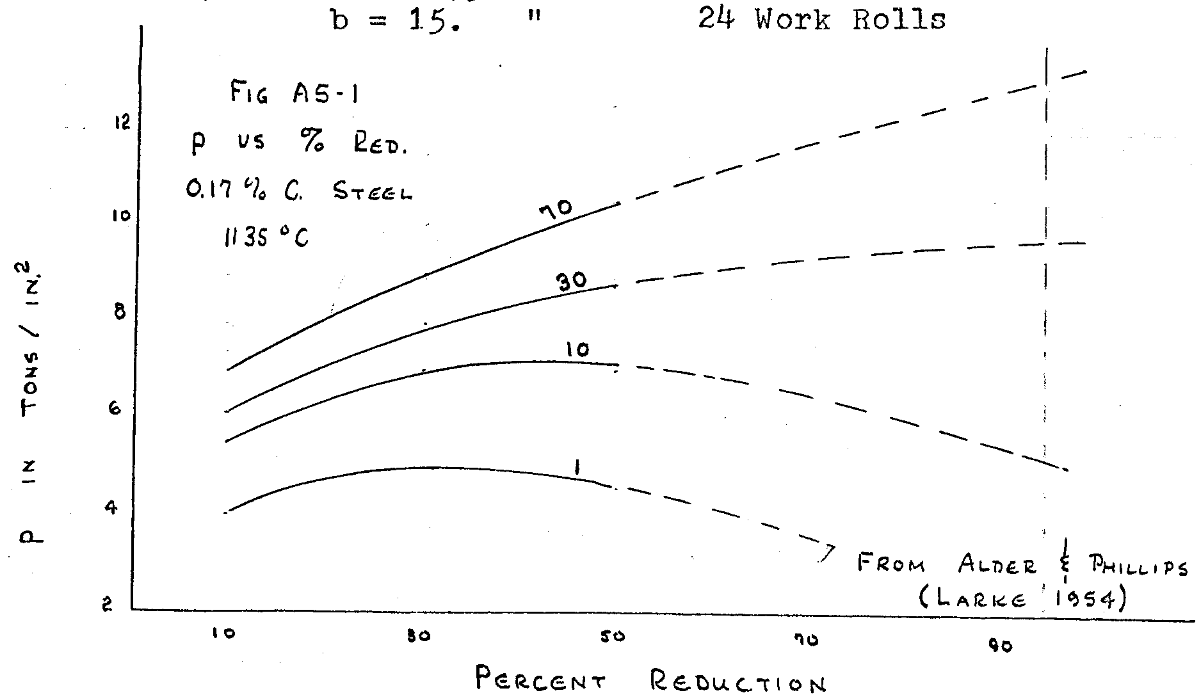
The values of p can be calculated from,

$$p = \frac{1}{2 \cdot L \cdot b_{av}}\left[FORCE - \frac{L}{2}\left(R\, Sin\,\psi \cdot Tan\,\theta\right)\right] \qquad (13)$$

5. Plot the values of p obtained against the percent reduction as shown in Figure 18.

APPENDIX 5 EXAMPLE CALCULATION AS PER SECTION 6.

Given; $D = 20.$ ins. $N = 500$ rpm.
$r = 2.$ " Sp. Wt. of 0.17% C Steel
$H = 1.5$ " $= .25$T. per ft^3
$h = .075$ "
$b = 15.$ " 24 Work Rolls



Required output tonnage is 13 T. per hour.

...

Solution; As per Section 6.

1. For 13 T./hr. the exit speed of the strip is,

$$v = \frac{12 \cdot (\text{OUTPUT})}{5 \cdot b \cdot h \, (\text{Sp. Wt.})} = \frac{12\,(13)}{5 \cdot 15 \cdot .075\,(.25)} = 111 \text{ fpm.} \qquad (47)$$

2. The angle of bite is,

$$\alpha = \cos^{-1}\left[1 - \left(\frac{H-h}{2R}\right)\right] = \cos^{-1}\left[1.\ \left(\frac{1.5 - .075}{2 \cdot 12}\right)\right] = 19^\circ \qquad (16)$$

3. The rate of deformation in the mill throat is;

$$M_1 = \frac{1}{5\alpha} \int_0^1 \frac{2 v h \tan\phi}{[h + 2R(1 - \cos\phi)]^2} \, d\phi \qquad (46)$$

$$M_\lambda = \frac{1}{5 \cdot 19} \int_0^{19} \frac{2(111)\,.075 \,\mathrm{Tan}\,\phi}{[.075 + 24(1 - \cos\phi)]^2}\, d\phi \doteq 5.5 \ \frac{\mathrm{in/sec}}{\mathrm{in.}}$$

4. The rolling temperature is to be 1135 C.

5. The percent reduction is,

$$\%\ \mathrm{Red.} = \left[\frac{H-h}{H}\right] \cdot 100 = \left[\frac{1.5 - .075}{1.5}\right] \cdot 100 = 95. \qquad (48)$$

6. From Figure A5-1 the resistance to plastic deformation is 5 tons per square inch.

7. The cage speed is calculated from,

$$S_c = \left[\frac{S+v}{2}\right] = \left[\frac{500\,\pi \cdot \frac{20}{12} + 111}{2}\right] = 1410 \ \mathrm{fpm.}$$

$$NN = \frac{12 \cdot 1410}{\pi\ (20+2)} = 245 \ \mathrm{RPM.} \qquad (15)$$

8. Thus the slab advance between roll impacts is,

$$a = \frac{12\,V}{NN\ (\mathrm{No.\ of\ Rolls})} = \frac{12 \cdot 5.6}{245(24)} = .012 \ \mathrm{in.} \qquad (49)$$

9. The strip advance at all points along the working arc may be calculated in the same manner as the example following.

Example, $\beta = 8°$

TRIAL $z = .15$ INS.

$$e = (z + a) + (R - r)\sin\beta = 1.55373 \quad (28)$$

$$f = (R - r)\cos\beta = 9.90268 \quad (29)$$

$$g = \left[\frac{R^2 + e^2 + f^2 - r^2}{2R}\right] = 10.01988 \quad (30)$$

$$j = e - z = 1.40373 \quad (31)$$

$$k = \left[\frac{R^2 + j^2 + f^2 - r^2}{2R}\right] = 10.00140 \quad (32)$$

$$\cos\beta_G = \frac{gf + \sqrt{g^2 f^2 - (g^2 - e^2)(e^2 + f^2)}}{(e^2 + f^2)} = .99187 \quad (24)$$

$$\cos\beta_H = \frac{kf + \sqrt{k^2 f^2 - (k^2 - j^2)(j^2 + f^2)}}{(j^2 + f^2)} = .99112 \quad (26)$$

$$\phi_G = \cos^{-1}\left[\frac{R\cos\beta_G - f}{r}\right] = 8.53^\circ \quad (25)$$

$$z^* = \frac{R_a}{h}\left[\cos\beta + 2\cos\beta_H + \cos\beta_G - 2(1 - \cos\alpha)\right] = .15963 \quad (23)$$

A SECOND TRIAL GAVE $z^* = .15961$

10. The roll forces at $\beta = 8^\circ$ are calculated from,

$$\delta = (z + a) \sin \beta_G = .02185 \text{ ins.} \qquad (34)$$

$$l = \sqrt{2 r \delta} = .29561 \text{ ins} \qquad (35)$$

$$F = p b l = 10{,}000 (15) .29561 = 44341 \text{ lbs.} \qquad (36)$$

11. The instantaneous torque and back thrust are given by,

$$\varepsilon = \beta - \frac{\phi_G}{2} = 8 - \frac{8.5}{2} = 3.75^\circ \qquad (37)$$

$$\text{THRUST} = 2 F \sin \varepsilon = 2912 \text{ lbs} \qquad (39)$$

$$\text{TORQUE} = p b (R - r) \sqrt{2 r \delta} \sin \frac{\phi_G}{2} = 2728 \text{ in-lbs} \qquad (41)$$

The values for all points along the working arc may be calculated and combined as shown in equation 50.