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COMPUTER ANALYSIS OF STRATIFIED FLOW PHENOMENA

TOWARDS A PROBLEM-ORIENTED LIBRARY FOR THE COMPUTER ANALYSIS OF STRATIFIED FLOW PHENOMENA

By

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ABSTRACT

Flows in channels or estuaries may exhibit variations in density arising from differences in temperature, salinity or suspended solids. In the absence of significant vertical mixing, stable, discrete layers may form with distinct density interfaces.

This thesis presents a computational approach for the analysis of two-layer, vertically stratified, one-dimensional horizontal flows in open channels. A variety of such problems are identified and a critical survey of the existing literature is presented. A framework is defined against which these problems are classified and decomposed into analytical problems of the simplest possible scope. Based on the conditions that lead to changes in flow characteristics, four research areas are examined. These are energy balance, interfacial hydraulic jump, lock exchange flows, and long transitions. Although restricted to essentially one-dimensional flows, the analytical study of these four areas is extended to allow for non-uniform velocity distribution by the introduction of boundary-layer displacement thicknesses and correction factors for kinetic energy and momentum. Also, a significant feature of the study is the ability to handle channels of arbitrary cross-sectional geometry.

The basic philosophy of the approach followed in this study is to develop a relatively simple and computationally economical procedure which is applicable to a wide variety of problems involving channels systems of arbitrary geometry and boundary conditions. A library of computer subroutines provides a convenient means of developing an open-ended system of computational techniques for the solution of a wide range of problems. Such a library of computational algorithms may also promote cooperation and collaboration among researchers and engineers concerned with stratified flow hydraulics. Such algorithms should provide solutions for frequently recurring problems, should be mutually compatible and allow the construction of relatively complex analytical models in a modular fashion. A comprehensive library of routines is developed which consists of fourty-four subroutines and functions. This evolves as a well-defined hierarchy of algorithms in which the most basic algorithms are nested within the more sophisticated ones to the sixth or seventh level.

The computational algorithms are tested for theoretical and computational performance. Numerical predictions are compared with available experimental and field data. Moreover, an experimental program is described which is designed and carried out to verify the numerical predictions obtained for the first of the above-mentioned four topics.

An important aspect of the study is the illustration of the application of the routines in the solution of typical practical problems such as selective withdrawal from stratified water bodies and recirculation of cooling water from power plants. In addition, to facilitate utilization of the programs by others, complete documentation and listings are provided.

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CHAPTER 1

SCOPE AND OBJECTIVES OF THE STUDY

1.1 INTRODUCTION

In a large body of water, the density may vary vertically because of the variations in temperature, dissolved salts (e.g. salinity), or suspended sediment load. The density may vary continuously throughout the depth to form a continuous density gradient, or, stable, discrete layers with distinct density interfaces may form. Horizontal gradients may also exist. In many applications of stratified-flow theory to natural phenomena, it has been assumed that a fluid system with continuous density gradient may be approximated by a layered system. Multi-layered systems have been found in the oceans, where observations suggest that it is typical to find fluid consisting of well-mixed layers separated by sharp interfaces. It therefore seems that multi-layer systems may occur more frequently than previously thought and an investigation into the flow of multi-layer systems is of general importance. Homogeneous flows are special cases of stratified systems, where the free surface marks the boundary between one fluid and another of negligible inertia and viscosity; its presence implies the presence of non-homogeneity, and, in fact, non-homogeneity in an extreme form. Therefore, the methods used in the analysis of homogeneous flows can be extended to some simple cases of the flow of multi-layer systems.

Although fluid motion in a stratified system was investigated by many researchers more than a century ago, it is only during recent years that it has attracted the serious attention of oceanographers, meteorologists, and hydraulicians. Individual cases of stratified flows have received considerable attention. However, what is lacking is a unified approach to study the phenomena as a whole.

1.2 OBJECTIVES AND SUMMARY

This study identifies a wide range of stratified flow problems and presents a critical survey of the existing literature. Generally, changes in flow characteristics result from one (or more) of the following conditions:

(1) Variations in boundary geometry with negligible change in total energy.

(2) Variations in flow characteristics due to discontinuities in the boundary and/or the interface accompanied by significant energy losses.

(3) Energy losses due to boundary and/or interfacial friction (long transitions).

(4) Time-dependent boundary conditions.

Methods of analysis generally involve application of one or more of the three principles of conservation, i.e.

(i) conservation of mass

(ii) conservation of energy

(iii) conservation of momentum

This study seeks to define a framework against which a wide variety of problems may be classified and decomposed into analytical problems of the simplest possible scope. For each of these, a computational module (i.e. a subroutine) is developed which may find frequent application in a wide variety of different solution types.

Areas of research contributions are examined which tend to focus on only one of the four conditions defined above in the hope that this may serve to identify computational algorithms which represent implementation of one or more of the three conservation principles. Such algorithms should provide solutions for frequently recurring problems, should be mutually compatible and allow the construction of relatively complex analytical models in a modular fashion.

Existing programs frequently do not provide an appropriate solution method for practicing engineers either because of:

(i) the size and computational cost of programs,

or (ii) the difficulty of introducing practical (i.e. real) boundary conditions,

or (iii) the effort necessary to comprehend the background research contributions in the literature on which the programs are based.

Among the numerical techniques currently applied to stratified flow problems are Finite Difference methods (Stigter and Siemons (1967), Thatcher and Harleman (1972), and Apelt, Gout and Szewczyk (1973)), Marker-and-Cell (Hwang and Slotta (1968), Slotta (1969) and Williams and Holmes (1974)) and Finite Element methods (King, Norton and Orlob (1973), Adey and Brebbia (1973), Gallagher (1975), Pearce and Christodoulou (1975), Futagami (1975), Liggett (1975) and Farraday, O'Connor and Smith (1975)).

The basic philosophy of the approach followed in this study is to develop a relatively simple and computationally economical procedure which is applicable to a wide range of problems involving systems of arbitrary geometry and boundary conditions.

The library approach is a natural one for an open-ended approach. A library (or framework) of computational algorithms may provide a means of promoting cooperation and collaboration among researchers and engineers concerned with stratified flow hydraulics. Whereas the improvement or augmentation of a large "all-embracing" program is a task of some magnitude, it should be possible and reasonably easy to incorporate in a properly designed library package, a new or modified routine concerned with a relatively elementary problem type or solution technique.

An important aspect of the study is the illustration of the application of the routines in the solution of typical practical problems. In addition, to facilitate utilization of the programs by others, complete documentation is provided.

As a first step in the development of such a library of elementary routines, four research areas are examined, each of which involves only one of the four conditions defined above. This should serve to focus attention on the major computational problems and aid in the identification of the necessary algorithms for their solution. The four topics considered are defined below.

(1) Steady stratified flow through a streamlined transition. Negligible energy losses allow the use of energy balance techniques in this case.

(2) Interfacial hydraulic jump. This is probably one of the more well-researched areas of steady stratified flow in the vicinity of boundary and/or interfacial discontinuities.

(3) Determination of surface profiles in long transitions. Present knowledge is restricted mainly to laminar flows which, however, are of greater significance in densimetric phenomena than in homogeneous flow resistance problems.

(4) Lock exchange flow. The classic case of unsteady lock exchange flow may frequently find application in the analysis of quasi-steady phenomena such as the arrested wedge.

Each of these problems is developed as a computational algorithm and tested for theoretical and computational performance. Where possible the numerical predictions are compared with available experimental and field data. An experimental program is described which was designed and carried out to verify the theoretical predictions obtained for the first of the above-mentioned four topics.

The development of each of the four solution algorithms involves a number of subroutines and it is found that a significant degree of computational interaction occurs. For example:-

(a) The shape of the interface during lock exchange flow represents a condition of unsteady gradually varied flow in which one of the dominant force actions occurs as interfacial shear.

(b) The selection of real solutions in the analysis of conjugate depths involves the estimation of energy losses.

(c) Situations of two-dimensional velocity distribution in a wide range of stratified flow problems may be reduced to one-dimensional approximations by means of the concept of boundary layer displacement thickness.

(d) The application of momentum balance in the analysis of abrupt discontinuities may require evaluation of force actions arising from interfacial shear.

The analytical study is intended to deal with two-layer systems using a one-dimensional approach with improvements to allow for non-uniform velocity distributions (e.g. boundary layer displacement thicknesses, energy and momentum correction factors). As stated earlier one of the basic advantages of this study is the ability to handle channels or arbitrary geometry.

With reference to the previously-mentioned conditions that lead to variations in the flow characteristics, a classification of stratified flow regimes is suggested which is illustrated in Figure 1.1.

1.3 ORGANIZATION OF SOLUTION ALGORITHMS

In attempting to set up a library of computational algorithms for the solution of the problems defined earlier, it is useful to consider a relatively small number of situations which correspond approximately to the basic reasons for changes in flow characteristics and which are outlined in Section 1.2

In each of these solution algorithms, a number of independent variables are involved many of which may be selected as the dependent variable. Consequently there may be several forms of the same basic solution algorithm depending on the formulation of the problem. In the case of two-layer systems this results in a substantial increase in the number of computational routines required to handle the various forms of a solution algorithm.

Typically it is found that the selection of a particular dependent variable in a functional relation allows an explicit - or at least a less complex - solution. The subroutine developed for this particular solution may then frequently be used to facilitate an iterative solution of another dependent variable.

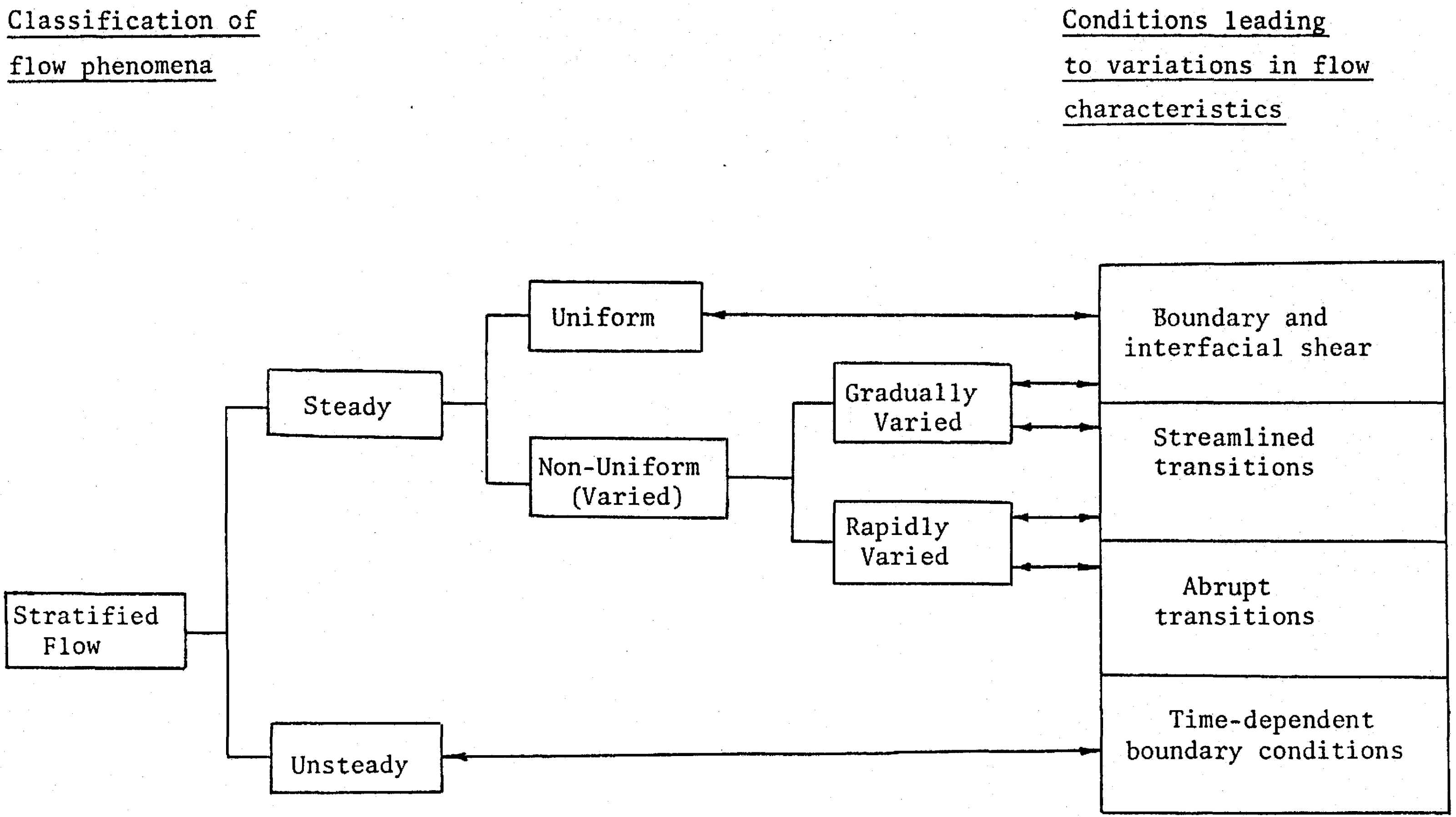


Fig. 1.1 - Classification of Stratified Flow Phenomena

For many problems it is possible to identify and separate out certain basic problems concerning the definition of system geometry, processing of cross-section properties, solution of simultaneous non-linear equations and the like. Some of these draw in turn from subroutines already available in a library of sub-programs for homogeneous flow problems (C.E.P.L.). Thus the development of a comprehensive library of routines evolves as a well-defined hierarchy of algorithms in which the most basic algorithms are nested within the more sophisticated ones to the sixth or seventh level.

In general, the organization of a repository or library of complete programs or sub-programs has become one of the most frequently voiced proposals for the dissemination of program information in various research fields.

In the following chapters, a general literature review of the subject is presented which concludes with a problem classification (Chapter 2). The next four chapters include presentation, analysis and development of the four topics discussed earlier. Chapter 7 illustrates the application of the developed computer library in the solution of typical practical problems.

CHAPTER 2

CLASSIFICATION AND BIBLIOGRAPHIC STUDY OF STRATIFIED FLOW PHENOMENA

2.1 INTRODUCTION

In this chapter, a wide range of stratified flow problems are identified with a review of the existing literature. These problems are then classified within the flow regimes defined in Chapter 1 accompanied by the corresponding references. Some of the problems may be included in more than one division depending on the flow conditions and the theoretical considerations.

2.2 PROBLEM IDENTIFICATION

In this sub-section seven classes of problems are considered. These are:

a) Selective withdrawal

b) Interfacial resistance

c) Internal hydraulics of a stratified flow system

d) Density wedges

e) Submerged jets and plumes

f) Stratified flow in circular pipes

g) Wind effect on a stable density interface.

For all but the last two topics - which are treated only very briefly - the problem is considered under the headings of (i)

Introduction, (ii) Literature Review, and (iii) Discussion.

2.2.1 Selective Withdrawal

2.2.1.1 Introduction

Significant efforts have been directed towards improving the quality of water abstracted from lakes and reservoirs. Selective withdrawal from stratified reservoirs is a means of providing water of desired quality for downstream municipal, agricultural, and recreational use. When discrete layers of distinct density differences exist, it is possible to withdraw from only one, or from several, of these layers, and the term "selective withdrawal" is used to describe this process.

One practical application of this and similar problems is encountered in the cooling of thermo-electric generating stations. In certain of such installations it has been observed that the coolant discharge water, by virtue of its lower density (i.e. higher temperature), flowed into the proximity of the coolant inlet from where it was subsequently recirculated through the cooling system with an attendant loss in operating efficiency. Other examples in which stratification phenomena are present are in the drawing off of crude petroleum from underground reservoirs, and the removal of salt water that has encroached upon a supply of fresh water. This latter problem has assumed considerable importance in the agriculture of many countries.

2.2.1.2 Literature Review

Analytical research was originally motivated by the need for a basic understanding of density-stratified flow as related to selective withdrawal from lakes and reservoirs for water quality control. Most of the work has dealt with the case of withdrawal from a reservoir containing a stable density gradient. In power-station cooling ponds, control structures are designed so that only the coolest water is used. In this case the pond most frequently consists of two distinct layers and it is therefore not surprising that some investigations are for the withdrawal of one layer from a two-layer system.

Yih (1958) investigates the problem of a two-dimensional flow of an inviscid, non-diffusive, density stratified fluid into a bottom line sink. He assumes that the density gradient far upstream is linear and that the velocity distribution far upstream is given by

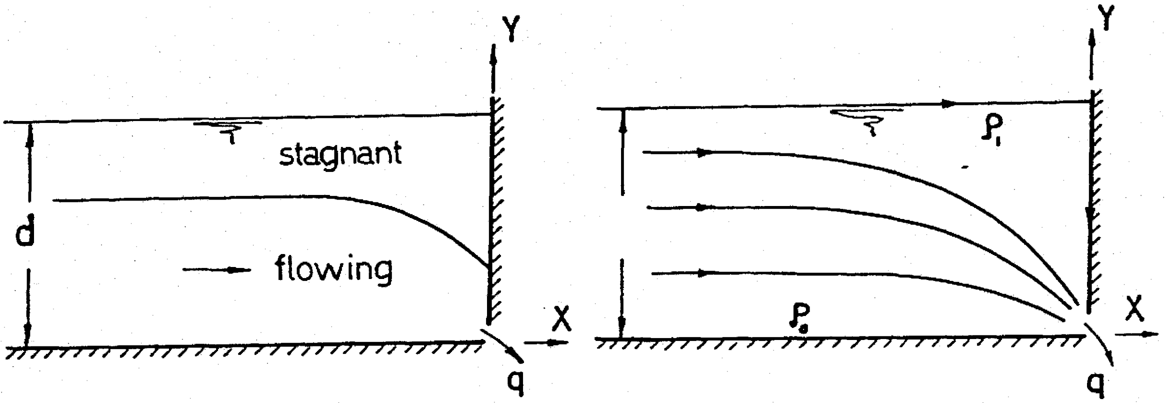
$$\rho^{1/2}U = A = \text{constant} \qquad 2.1$$

where ρ is the density of the fluid and U is its horizontal velocity, both functions of the vertical coordinate y. He points out that equation (2.1) is valid when the flow issues horizontally from a large reservoir. With this boundary condition a complete solution is obtained. It is shown that this solution ceases to be valid for densimetric Froude numbers (F) equal to or less than a critical value of $1/\pi (\simeq 0.32)$. F is defined as:

$$F = \frac{A}{d\sqrt{g\beta}} = \frac{U}{\sqrt{gd\Delta\rho/\rho}} \qquad 2.2$$

with reference to Figure 2.1(b), d is the depth of the fluid, $\beta = (\rho_0 - \rho_1)/d$ is the density gradient, where ρ_0 is the density at the bottom, ρ_1 is the density at the top (i.e. free surface) and $\Delta\rho$ is the density difference. Yih's theoretical results show that there is no tendency for flow separation (i.e. discharge of the heavier fluid to the exclusion of the lighter portions) for Froude numbers greater than $1/\pi$. Any separation will be expected to occur when the Froude number is equal to or less than this critical value. Debler (1959) investigates the problem experimentally. He demonstrates that, when F is near $1/\pi$, the flow is characterized by the presence of a stagnant layer which is separated from the flow region by a line of velocity discontinuity (see Figure 2.1(a)). Kao (1965) extended Yih's solution to include the case where F is less than $1/\pi$ by introducing a fictitious uniform sink distribution on the vertical wall at the outlet.

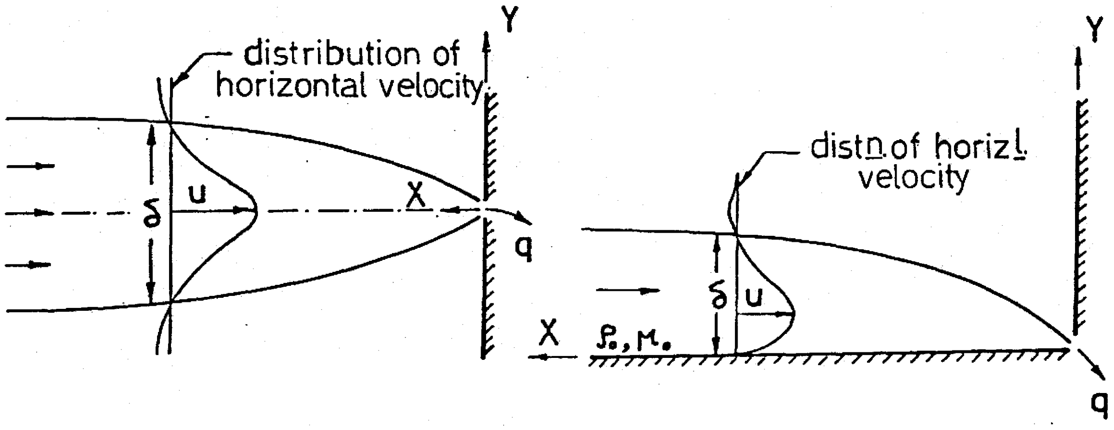
Koh (1966) considers the effect of viscosity in stratified flow towards a sink. He develops a viscous diffusive model to describe selective withdrawal at an intermediate depth (see Figure 2.2(a)). A boundary-layer-type assumption is made and a perturbation method is used to solve the governing differential equations. The analytical solutions obtained are applicable for



(a) stagnant zone form

(b) flow from full depth

Fig. 2.1 - Bottom Withdrawal From a Density-Stratified Reservoir (Inviscid Fluid)



(a) withdrawal at an intermediate depth

(b) withdrawal at the bottom

Fig. 2.2 - Withdrawal From a Density-Stratified Reservoir (Viscous Fluid)

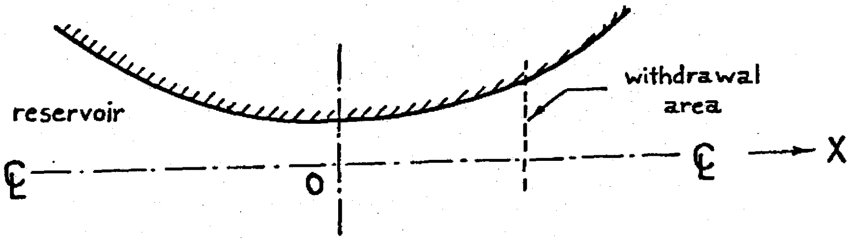
very small discharges. The results obtained agree closely with laboratory results of experiments carried out within the range of applicability of the analytical solution.

A more recent contribution is a study by Walesh and Monkmeyer (1973) on viscous, non-diffusive, bottom selective withdrawal (see Figure 2.2(b)). No particular density structure, e.g. linear, is specified. Their analysis applies to that portion of the withdrawal layer sufficiently far from the sink so that the withdrawal layer thickness is small compared to that distance. This stipulation facilitates the neglect of inertial effects, and is also the basis for application of a boundary-layer-type simplification of the governing equations. They provide mathematical expressions for horizontal and vertical velocities, horizontal shear, withdrawal layer thickness, and the distribution of flow within the withdrawal layer. It is confirmed that normalized profiles of vertical velocity, horizontal velocity, and horizontal shear are similar; i.e. each has a form independent of distance from the sink, density gradient, viscosity, and volumetric discharge. Previous experimental investigation indicates that the creeping flow assumption made in this study is valid for Reynolds number less than 1.0, where the Reynolds number is given by $(u.x/\nu_0).(\delta/x)^2$, in which u is the maximum horizontal velocity component at a distance x from the sink, δ is the withdrawal layer thickness at the same point, and ν_0 is the kinematic viscosity of the fluid at the channel bottom which is assumed together with its density ρ_0 to be constant along the bottom (see Figure 2.2(b)).

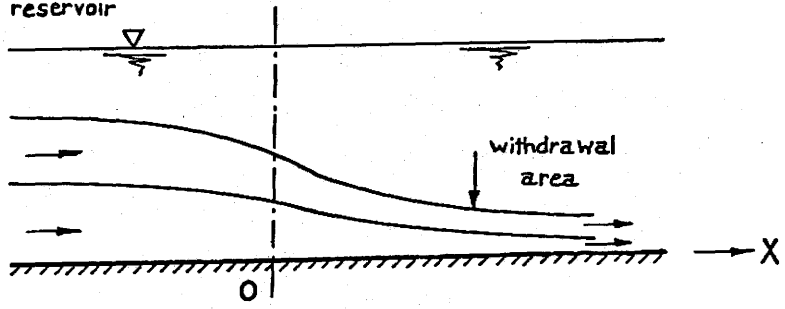
Ho and Monkmeyer (1975) investigated the problem of bottom withdrawal from a linearly-stratified reservoir. They consider a two-dimensional, viscous, non-diffusive flow. Inertial effects are considered although fluid flow, as in previous literature, is assumed to be laminar. The theoretical results obtained are confirmed by laboratory experiments for the withdrawal layer thickness and the maximum horizontal velocity.

The problem of selective withdrawal of a viscous, non-diffusive linearly-stratified fluid is extended to cover the axisymmetric case with the withdrawl being at the bottom of the impoundment (Ho, Monkmeyer and Clark (1976)) or at an intermediate depth (Clark, Monkmeyer, Ho and Hoopes (1976)). In the bottom withdrawal case, laminar flow is considered and inertial effects are retained so that non-creeping flows can be studied. In withdrawal from an intermediate depth, inertial effects are neglected. In both cases, experimental data are provided for velocity profiles in the withdrawal layer.

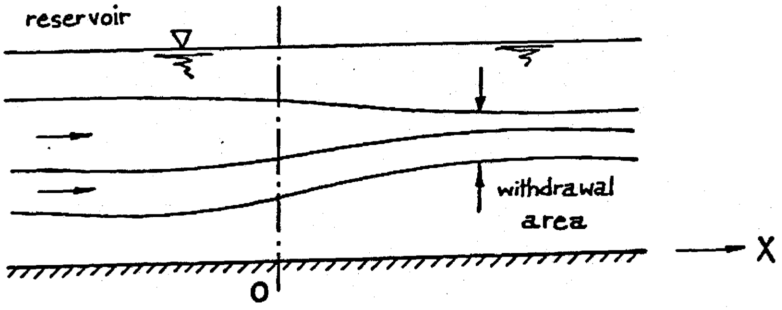
Wood (1968) considered a reservoir connected through a horizontal contraction to a channel. Both the reservoir and the channel are assumed to contain a stable, multi-layer system of fluids (see Figure 2.3(b)). When there is flow in only one layer, the volume discharge calculations are carried out at a single section (the section of minimum width). Where there are



Plan view



Elevation
(a) Bottom Withdrawal



Elevation
(b) Withdrawal at an intermediate depth

Fig. 2.3 - Withdrawal of Two-Layers Downstream of a Contraction

velocities in only two layers, the theory involves computations at two sections in the flow. These are the section of minimum width and a section upstream of the position of minimum width (the virtual point of control). It is shown that the depths of the layers at the point of maximum contraction are two-thirds of those far upstream. Similar results are shown for any stable continuous or discrete density stratification in the reservoir. Theoretical predictions are confirmed by experimental work carried out with a contraction in a flume for the withdrawal of two discrete layers from a three-layer system and the withdrawal from a fluid with a linear density gradient. Wood and Lai (1972) and Lai and Wood (1975) extended this work to show the variations of flow profiles in the withdrawal from a reservoir of two layers.

Hwang and Slotta (1968) applied an extension of the Marker-and-Cell method (MACE) for density stratified flow to problems of optimum selective withdrawal concerning two-dimensional, unsteady, non-diffusive, viscous, laminar, two-layered stratified fluid flow involving free surfaces. Cases of multi-layered fluids, continuous density stratification, obstacles introduced into the flow region are not included in the method adopted. No experimental evidence is provided.

Elder and Wunderlich (1969) proposed empirical relations for the withdrawal layer thickness and its internal velocity distribution based on field measurements made by the Tennessee Valley Authority in two of its large reservoirs. A more recent investigation by the same authors (1972) has dealt with density currents caused by inflows in three different TVA reservoirs.

Brooks and Koh (1969) presented a review of analyses and experiments for withdrawal layer flows from linearly stratified fluids. They include a brief review of discrete layer systems and propose an extension to turbulent flow.

2.2.1.3 Discussion

In reviewing the previous literature, the following points may be made:

1- In dealing with stratified flows towards a sink, the basic assumptions involve viscosity, diffusion, and inertial effects. In the region near the sink, the velocities are suffficiently large that inertia and gravity are the dominant factors, with viscous and diffusive effects being secondary. However, if all variations in density $\Delta\rho$ are very much smaller than any reference density ρ_0 (Boussinesq assumption), the density variations are significant only in the gravity force terms in the equations of motion, involving $\Delta\rho$, and have negligible effect in inertial terms (involving $\rho_0+\Delta\rho\approx\rho_0$). On the other hand, if the analysis is applied to that portion of the withdrawal layer sufficiently far from the sink, inertial effects may be neglected. Also, under the assumption of small density differences which implies small temperature or concentration gradient, heat and mass diffusion may be ignored. In the experiments carried out by Koh (1966) with salt as the stratifying agent, the ratio of viscosity to diffusivity is 760. Therefore, it is appropriate to seek a solution which includes viscosity while ignoring diffusion. For the thermal stratification cases, where the ratio is about 5, the diffusion may probably not be neglected. Yih (1958) and Kao (1965) do not show the flow characteristics when it is dominated by viscous effects. The analysis of Koh (1966) and Walesh and Monkmeyer (1972) in which viscous effects are considered, predicts - and laboratory experiments verify - the existence of a withdrawal layer characterized by a principal current moving towards the sink bounded by small reverse currents. The critical Froude number determined experimentally by Debler (1959) ($\simeq 0.28$), is somewhat less than the theoretical value of $1/\pi$ determined by Yih (1958) for flow separation which is based upon the assumption of an inviscid fluid. This is apparently due partly to the neglect of viscous effects which will tend to make the depth of the stagnant zone much smaller.

2- Most of the work deals with steady flows. In selective withdrawal downstream of a contraction, it is assumed that the reservoir is sufficiently large so that the time of travel of a particle through the contraction is short compared to the time for the streamline patterns to change due to the withdrawal of fluid from the reservoir. For an unsteady inviscid non-diffusive flow towards a sink in a linearly stratified fluid, Koh (1966A), (summarized by Brooks and Koh (1969)), presents solutions for a sink flow of infinite extent which starts suddenly at a constant rate at time t=0. He derives mathematical expressions for the withdrawal layer thickness and the horizontal and vertical velocities as functions of space and time. A similar problem is studied by Kao, Pao and Wei (1972) of the unsteady inviscid non-diffusive flow of stratified fluid in a channel of finite depth towards a line sink. The flow is assumed to start from rest and the fluid to be linearly-stratified. The flow and density fields are determined as a function of space and time. Their linearized theory is invalidated when the velocity away from the sink becomes excessively large. Their experimentally-determined velocity profiles are found to be in good agreement with theory. The attempt by Hwang and Slotta (1968) to use the MACE numerical technique for unsteady stratified flows has several severe limitations which are described in the previous section. It is foreseen that the programming codes would become more complex and lengthy with added features and layers which would make the use of this method impractical.

3- In all previous work, the flow is assumed to be laminar except for the attempt by Brooks and Koh (1969) to modify Koh's viscous diffusive results by using turbulent exchange coefficients for momentum and mass (or heat) in place of the corresponding molecular quantities. Yet, there is a need to define a transition criterion between laminar and turbulent flows. In reservoirs, it is expected that the flows associated with selective withdrawal are turbulent because of the large scale or large Reynolds numbers. For example, typical TVA field measurements, (Elder and Wunderlich (1969)), are given in Table 2.1. The test results are from two reservoirs of different geometry and operation conditions. The measurements are taken 3170 ft (970 m) and 8980 ft (2740 m) upstream from the dam in Fontana and Cherokee Reservoirs respectively. By almost any definition of Reynolds number for the abstracted layer, it is clear that turbulent flow is more or less assured.

Reservoir	Withdrawal layer thickness	Max. velocity
Fontana	105 ft (32.0 m)	0.10 ft/sec (3.05 cm/sec)
Cherokee	100 ft (30.5 m)	0.18 ft/sec (5.49 cm/sec)

Table 2.1 - Typical Field Measurements in Two TVA Reservoirs

4- In several works, the flow is assumed to be either one- or two-dimensional. In selective withdrawal downstream of a contraction, it is further assumed that the contraction is sufficiently gradual for the curvature of the streamlines to be small enough for the one-dimensional assumption to be used. For viscous diffusive flow towards a point sink, Koh (1966) has derived a solution for the axisymmetric flow of a linearly stratified fluid. Qualitatively, the flow field is similar to the two-dimensional case. However, reservoirs are usually irregular in shape and conform to neither the purely two-dimensional nor axisymmetric flow assumptions. However, in a general sense, reservoirs following a river valley more nearly approximate to a two-dimensional strip than a radial space about a point. Near an outlet in a dam the flow is locally radial, but at a distance upstream, the flow is expected to be reasonably well distributed across the reservoir. For some reservoirs where the plan geometry is very complex, a three-dimensional solution may be desirable.

5- In addition to the different approximations described above, there is an additional fact that may account in part for the discrepancies observed when comparing theoretical predictions with experimental observations. In one-dimensional analyses, the effect of the boundary layers at the bottom of the channel and at the interfaces for discrete layer systems, is neglected. Also, in both one- and two-dimensional analyses, the side-wall boundary layer is ignored.

6- No mention is made in the available literature of non-linear stratification which may be of practical importance.

7- In stratified flow towards a sink, if the vertical dimension of the outlet opening is comparable to the predicted thickness of the withdrawal layer or even larger, then clearly the analyses based on the line sink or point sink do not apply.

8- Following personal communictions with engineers of H.G. Acres (Niagara Falls), it is felt that there is a practical need to analyze theoretically cases when the density gradient is not continuous. This is encountered in composite thermal-salt density differences which lead to a step-like variation of density. There is also a need for three-dimensional analysis of the problem to account for the complex geometry of real reservoirs.

2.2.2 Interfacial Resistance

2.2.2.1 Introduction

When a fluid flows into another fluid of different density, stratification may occur and a distinct interface may be formed. One of the major uncertainties in practical cases lies in the explanation of the phenomena located near the interface. The problem of interfacial shear has been treated both theoretically and experimentally by several authors and from different aspects. The question in practical problems is chiefly in the determination of the interfacial resistance if one layer intrudes into another and stratification occurs.

2.2.2.2 Literature Review

In two-layer stratified flow systems, a boundary layer is developed in both layers along the interface starting from the point of contact of the two fluids. Depending upon the depths of both layers, the boundary layers will eventually expand over the whole cross-section of flow, if the region is long enough, and the flow may be said to be established.

Keulegan (1944) considers laminar flow at the interface of two liquids which have semi-infinite depths, i.e. the flow is non-established in both layers. He determines the shear stress at the interface based on Prandtl's boundary-layer theory. The evaluation is carried out by a method of successive approximations. Lock (1951) considers the same case and gives a method for obtaining the solution of the laminar boundary layer equations. An approximate solution is also obtained by means of the momentum equation. Both Keulegan (1944) and Lock (1951) in independent works show that the solution depends on the ratio of the velocities of the two streams and on the product of the corresponding density and viscosity ratios. Their results are found by the author to be approximately the same; Lock's solution however, is more general.

The theory presented by Lock (1951) is examined experimentally by Wang (1975). The measured shear stress coefficient is shown to have higher values than those predicted theoretically. The slight variation of the interfacial velocity in the horizontal direction is described as a probable reason since this velocity is assumed to be independent of x in the theory.

Bata (1959) analyzes the case of a moving lower layer where the flow in this layer is assumed to be both laminar and established. He applies the methods of boundary-layer theory and, using numerical integration, shows that the intensity of the shear on the interface τ_i, in dimensionless form, is given by:

$$\frac{\tau_i}{\rho u_i^2} = \frac{-0.4435}{\sqrt{R_x}} \qquad 2.3$$

where

u_i = interfacial velocity

ρ = density of the lower (moving) layer

R_x = Reynolds number = $u_i . x/\nu$

x = distance along the interface measured from the point of initial contact of both layers

ν = kinematic viscosity of the lower layer

To determine the total resistance in the moving layer, he obtains the form

$$\frac{(384-\lambda R_e)^{3/2}}{\lambda R_e-384} = \frac{31.2}{\sqrt{R_e M}} \qquad 2.4$$

where

λ = the usual total resistance coefficient in the Darcy-Weissbach equation

Re = Reynolds number = $4\bar{u}h/\nu$

$\bar{u}$ = mean velocity of the moving lower layer

h = depth of the lower layer

M = $(h/L).(\mu_1/\mu).(\rho_1/\rho)$

L = total length of the lower layer

μ = dynamic viscosity of the lower layer

Subscript 1 denotes the upper layer

For established laminar flow in both layers (uniform flow), the following expression holds: (Bata (1957))

$$\lambda = 384\,\frac{3+N}{3+4N}\cdot\frac{1}{R_e} \qquad 2.5$$

where

$$N = \frac{h_1}{h}\cdot\frac{\mu}{\mu_1}$$

Abraham and Eysink (1971) express the interfacial shear in stratified flows as

$$\tau_i = \frac{1}{8}\,\lambda_i \rho u_{rel}^2 \qquad 2.6$$

where

τ_i = interfacial shear

λ_i = dimensionless interfacial shear stress coefficient

ρ = mean density of both fluids involved

u_{rel} = relative velocity between both layers (algebraic difference of mean velocities).

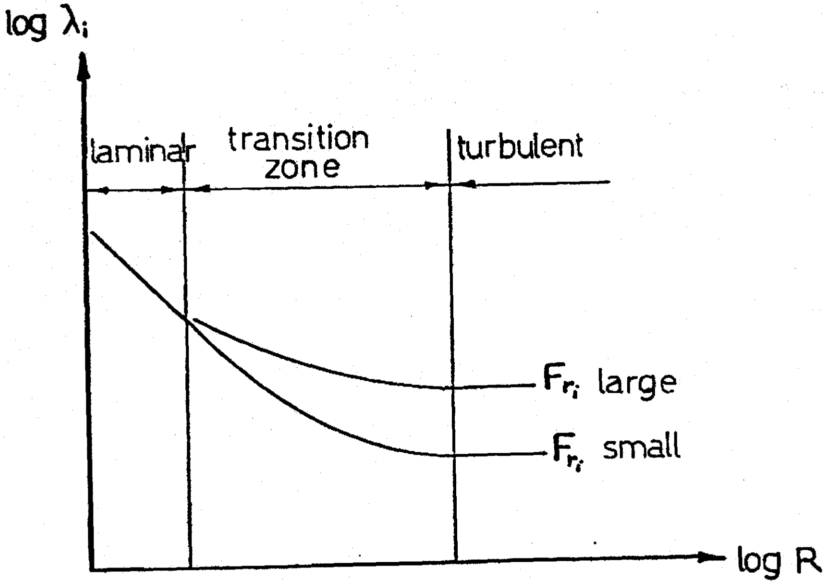


Fig. 2.4 - Schematic Representation of Relationship between R, F_{ri} and λi (after Abraham and Eysink (1971))

They show that, for laminar flow, the shear stress coefficient is inversely proportional to Reynolds number. In the case of turbulent flow it is shown using energy consideratons that the coefficient depends on the internal Froude number (equation 2.9) as well. This is illustrated in Figure 2.4. Reynolds number R is defined as

$$R = \frac{\bar{u}}{\nu} \cdot \frac{h.B}{2(h+B)} \qquad 2.7$$

where $\frac{h.B}{2(h+B)}$ = hydraulic radius of the lower layer

B = width of flume

In the laminar range, Harleman and Ippen (1952) derive the following relationship both analytically and experimentally (referred to by Abraham and Eysink (1971):

$$\lambda_i = \frac{11.3}{R} \qquad 2.8$$

In the turbulent range, Figure 2.4 show that a unique relationship between the interfacial shear stress coefficient and Reynolds number may be expected in a stratified flow phenomena characterized by a constant value of the internal Froude number. For this reason, the study of Abraham and Eysink (1971) is restricted to lock exchange flows which approximately satisfy this requirement. They consider initial flow conditions where the internal Froude number F_{r_i} is equal to about 0.8. F_{r_i} is defined as

$$F_{r_i} = \frac{u_{rel}^2}{(\Delta\rho/\rho)gh} \qquad 2.9$$

where

$\Delta\rho$ = density difference

g = gravitational acceleration

h = total water depth

They derive an experimental relationship between λ_i and R on the basis of energy considerations. This relationship shows that λ_i does not depend upon R for R greater than about 10,000.

Dick and Marsalek (1973) present a review of various methods for the estimation of the interfacial shear stress coefficient in density wedges. Based on previous experimental data, they recommend that the shear stress coefficient be estimated from the Moody diagram for pipe flow assuming the relative roughness as "smooth" and defining the Reynolds number as

$$R_i = \frac{4\bar{u}h}{\nu} \qquad 2.10$$

For the arrested salt wedge where the mean velocity $\bar{u} = o$, they state that the upper layer can be treated in the same way. They suggest that the Froude number is not a dominant parameter and may for practical purposes be safely ignored. They provide a useful list of previous studies concerning interfacial shear stress coefficient. This list is reproduced in Table 2.2 and the corresponding references which are not included in the primary reference list are given with the Secondary References.

Lepetit and Rogan (1970) provide a useful bibliographic study of salt wedges which include a review of studies in interfacial shear.

2.2.2.3 Discussion

1- In a two-layer system, the question of flow establishment as described earlier depends upon the depths of both layers and the length of the flow region. For small depths, the flow in both layers is expected to be established sufficiently far downstream from the point of contact (Bata (1957)). When the depths are large, the boundary layers do not occupy the whole region of flow for some distance downstream from the point of contact and then the flow is non-established in both layers (Keulegan (1944) and Lock (1951)). On the other hand, if, for instance, the depth of the lower layer is much smaller than the depth of the upper layer, the flow is established in the lower fluid but not in the upper one. This is nearly always the case in laboratory flume investigations (e.g., Bata (1959)). This can be an important factor in the discrepancy between the laboratory flume results by Wang (1975) and the theory developed by Lock (1951) in which the liquids involved are assumed to have semi-infinite depths. Moreover, solutions given for cases when the flow is established

Table 2.2 - List of Studies Dealing with Interfacial Shear Stress (after Dick and Marsalek (1973))

AUTHORS	TYPE OF A DENSITY CURRENT	METHOD FOR ESTIMATING f_i	RANGE OF Re	RANGE OF Fr_{dens}	COMMENTS
Abraham and Eysink, 1971	Lock exchange flow analogous to the wedge with counter currents	Calculating from equations (1) - (4)	1,000 - 40,000	~.8	Laboratory Study
Barr, 1963	Lock exchange flow	Calculating from eqs. (1)-(4) (Analyzed by Abraham & Eysink)	115 - 14,000	~.8	Laboratory Study
Bata, 1957	Arrested thermal wedge Arrested salt wedge Wedge with counter-currents & co-currents	Assuming hydraulically smooth interface & using Moody diagram	$\sim 10^4$	$\frac{.14}{.50}$	Laboratory Study
Bata and Knezevich, 1953	Silt laden under-flow in a pool of lighter fluid	Using Darcy - Weisbach equation	10^4 - 10^5	Not Known	Laboratory Study
Cross and Hoult, 1971	Oil slick arrested by a boom in river	Fitting eqs. of motion to experimental data	$\sim 10^5$	~.5 (Deduced)	Laboratory Study
Dick and Marsalek, 1972	Thermal wedge with counter currents	Assuming hydraulically smooth interface and using Moody diagram	$\sim 10^6$	~.5	Field Study
Dick and Marsalek, 1973	Thermal wedge with counter currents	Fitting eqs. of motion to experimental data	$\sim 10^6$	~.5	Field Study
Hendrikse, 1965	Arrested salt wedge	Fitting eqs. of motion to experimental data	10^4	.3	Laboratory Study
Keulegan, 1957	Lock exchange flow	Calculated from eqs. (1)-(4) by Abraham & Eysink	1,300 - 5,000	~.8	Laboratory Study
Lofquist, 1960	Saline under-flow in a pool of fresh water	Using an equation for shear stress	10^3 - 10^4	$\frac{.03}{.38}$	Laboratory Study
Macagno and Rouse 1962	Counter flow in a closed cross-section	Calculated from motion equations	300 - 4,000	>.5	Laboratory Study
Polk, Benedict & Parker, 1971	Arrested thermal wedge	Assuming hydraulically smooth interface & using Moody diagram	$\frac{10^6}{10^7}$	$\frac{.3}{.65}$	Field Study
TVA, 1966	Arrested thermal wedge	Assuming hydraulically smooth interface & using Moody diagram	Not Known	Not Known	Field Study

in one or both layers must be strictly applied for long canals, i.e. for prototype conditions, where the zone of establishment is negligible in comparison with the total length. For short laboratory flumes, a combination of solutions would give a better agreement of the experimental results with the theory.

2- All the previous literature is concerned with laminar flows near the interface. For the determination of the interfacial resistance in turbulent flows, Bata (1959) referred to Schijf and Schonfeld (1953) and Bata and Knezevich (1953) (see Secondary References). He comments that "however, this problem is still far from the definite solution".

In the experimental investigation of Wang (1975), the case of a turbulent interface is contrasted photographically with a laminar interface. It is apparent that the interfacial stress for the turbulent case needs new definition because there is no clearly defined interface observable from the photographic evidence presented. Moreover, there is uncertainty regarding the definition of the laminar/turbulent transition.

3- The normal hydraulic one-dimensional approach to the problem of interfacial resistance in density wedges is not entirely satisfactory. Improvements of the various theoretical approaches towards the two- or three-dimensional consideration would be useful.

4- The conclusion by Dick and Marsalek (1973) that the internal Froude number is not a significant parameter in determining the interfacial shear stress coefficient in density wedges, which contradicts the idea of Abraham and Eysink (1971), seems to be weakly supported and is based on an entirely arbitrary procedure.

5- If the results of different authors are compared for the case of a flowing layer with non-established flow in both layers, the general relation is:

$$\lambda_i = \frac{K}{\sqrt{R_x}}$$

where $R_x = \bar{u}x/\nu$, K is a constant and λ_i is defined by equation (1.6).

If the product of the density and viscosity ratios of both layers is assumed to be unity, the calculated results are as follows:

(a) Keulegan (1944)	K = 1.5668
(b) Lock (1951)	K = 1.5968
(c) Bata (1959)	K = 1.5997 using equation (2.3) and substituting an average value for the interfacial velocity ($0.588\bar{u}$) obtained from Keulegan and Lock.

2.2.3 Internal Hydraulics of a Stratified Flow System

2.2.3.1 Introduction

Several studies describe the analogy between the flow in an open channel and either internal or surface currents in large bodies of stratified fluid and show that the methods of analysis may be extended to some simple cases of the flow of multi-layer systems. In spite of the encouraging advance of recent years in the studies of fluid motion in a stratified system, much has still to be learned about the subject and many questions at issue are yet to be settled. Among these is the application of the energy and momentum principles and the validity of the concept of the critical regime. This is relevant to problems such as the interfacial hydraulic jump and multi-layer flow through a contraction. Nature is replete with examples of stratified flows that are controlled by a horizontal or a vertical contraction in the boundary geometry. Among the more important ones are flows in river estuaries (horizontal contraction) and mountain winds (vertical contraction).

In this section, the application of conservation theorems to stratified flow systems is discussed and the concept of a critical regime is analyzed. Following this, the problems of stratified flow through a transition and the interfacial hydraulic jump are presented.

2.2.3.2 Literature Review

A. Application of Conservation Theorems to Stratified Flows

Mass, energy and momentum conservation theorems may be applied to stratified flow systems in a similar way to homogeneous flows. Several authors have applied Bernoulli's equation for energy balance to multi-layer flows where one equation is written for each individual layer on the interfacial streamline.

Yih (1965, pp.122-124) considers the simple case of a single moving layer in a two-fluid system assuming a hydrostatic pressure distribution in the stagnant layer. Long (1954) studies a two-layer system with both layers flowing over a barrier with the same approach velocities. He draws a hydraulic analogy between the flow of a single fluid over a barrier and that of a two-fluid system. He obtains critical curves which, for given upstream conditions, give the possible regimes of motion. His analysis is based on the idea of minimizing the total energy transport with respect to the horizontal distance assuming that the conditions at the top of the barrier are just critical.

Wood (1968) considers two layers flowing through a horizontal (i.e. lateral) contraction with stationary fluids above and below these layers. He shows that two control points occur, one at the minimum width and the other at a position upstream of the minimum width (the virtual point of control).

Wood (1969) extends the study to the flow of two layers either in the same direction or in opposite directions (lock exchange flow) under a third stagnant layer through a horizontal contraction (a channel connecting two reservoirs).

Cases of multi-layer flows through contractions as applied to selective withdrawal from stratified reservoirs are further invesigated by Wood and Lai (1972 and 1972A) and Lai and Wood (1975).

Yih and Guha (1955) consider the internal hydraulic jump in a fluid system of two layers and apply the momentum principle to each layer neglecting the shear and assuming hydrostatic distribution of pressure.

The question of a critical regime in a stratified system is discussed in some studies. It is evident that the transition from the one degree of freedom of open-channel homogeneous flows to the many degrees of freedom of stratified flows introduces unavoidable difficulties and obscurities.

In open channel flow, the critical condition is defined as the condition at which:

$$\left.\frac{dE}{dy}\right|_{Q=\text{const.}} = 0 \text{ or } \left.\frac{dM}{dy}\right|_{Q=\text{const.}} = 0 \text{ or } \left.\frac{dQ}{dy}\right|_{E=\text{const.}} = 0$$

where

$$E = y + \frac{Q^2}{2gA^2}, \qquad M = A\bar{y} + \frac{Q^2}{gA}$$

Q = discharge

y = depth of flow

$\bar{y}$ = depth of the centroid of the flow section below the free surface

A = cross-sectional area

If this definition is applied to simple cases of a single-layer flow in a two-fluid system (Pedersen (1972) and Elsayed (1975)), the condition for a critical section in a rectangular channel is

$$F_i^2 = 1 \qquad 2.11$$

where F_i is the densimetric Froude number defined as

$$F_i^2 = \frac{q^2}{g' y^3} \qquad 2.12$$

where

q = discharge per unit width

y = depth of the flowing layer

g' = reduced gravitational acceleration = $g \cdot \Delta\rho/\rho_2$

g = gravitational acceleration

$\Delta\rho$ = density difference between both fluids

ρ_2 = density of the lower fluid

Other workers - e.g. Barr - have employed the notation F_Δ to describe the Froude number with respect to reduced gravitational acceleration g'.

The same relationship is obtained by Hamada (1969) using small amplitude theory (interfacial linear long wave) which states that in any channel transition which acts as a control, a long wave of infinitesimal amplitude is stationary at the critical section.

Equation (2.11) is similar to that for open-channel homogeneous flow except for the reduction in gravitational acceleration due to buoyancy effect.

Stommel and Farmer (1952) determine the critical condition for the more general case of two flowing layers using long wave theory. For a stationary interfacial wave, they derive the following approximate relationship ($\Delta\rho/\rho_2 << 1$).

$$F_{i1}^2 + F_{i2}^2 = 1 \qquad 2.13$$

where the subscript indices 1 and 2 refer to the upper and lower layers respectively and the definition of g' in both terms is the same as previously defined (2.12). They provide experimental results for the case of flow in the upper layer which show agreement with the theory within the accuracy of measurement.

Equation (2.13) without approximation is:

$$\beta F_{i1}^2 F_{i2}^2 - F_{i1}^2 - F_{i2}^2 + 1 = 0 \qquad 2.14$$

where

$$\beta = \frac{\Delta\rho}{\rho_2}$$

Mehrotra (1973) studies boundary contractions as controls in two-layer flows with the upper fluid either bounded by a rigid wall or having a free surface. Using long wave theory, he determines the critical conditions. For a free upper layer, the relationship obtained for either horizontal or vertical contractions is the same as equation (2.14) but in a different form. For a bounded upper layer, the critical condition in a modified form is

$$rF_{i1}^2 + F_{i2}^2 - 1 = 0 \qquad 2.15$$

where

$$r = \frac{\rho_1}{\rho_2} .$$

In a two-layer flow through a contraction with an upper free surface, Wood (1968), Wood (1969), Wood and Lai (1972), Wood and Lai (1972A) and Lai and Wood (1975) obtain equation (2.14) in another form by assuming that the slopes of the interfaces of the two flowing layers are continuous. This equation applies at the section of minimum width. They also show that there exists another control point upstream of the position of minimum width which is called "the point of virtual control". At this point, the following conditions, after some modifications, hold

$$F_{i1}^2\, y_1 - F_{i2}^2\, y_2 - \beta\, y_1\, F_{i1}^2\, F_{i2}^2 = 0 \qquad 2.16$$

and

$$F_{i1}^2\, y_1\, r - F_{i2}^2\, y_2 + \beta\, y_2\, F_{i1}^2\, F_{i2}^2 = 0 \qquad 2.17$$

Armi (1975) shows that in the plane of Froude numbers (F_{i1}^2, F_{i2}^2), there are two lines along which equation (2.14) holds.

For $\beta = \Delta\rho/\rho_2 << 1$, the first line is given by equation (2.13) which represents critical conditions with respect to the internal long wave. The second line represents critical flow with respect to the free surface long wave and is given by

$$(1 - F_1^2)\ (1 - F_2^2) = 1, \qquad F_n^2 = \frac{q_n^2}{g y_n^3} \quad n = 1,\ 2 \qquad 2.18$$

Numerical verification of these conclusions obtained by the writer are presented in Chapter 3.

In all the previously-mentioned studies, the flow is assumed to be frictionless. Hsu and Stolzenbach (1975) consider bottom and interfacial friction in a two-layer gradually-varied stratified flow through a contraction. They show that a critical condition ($dn/dx \to \infty$) exists when equation (2.13) is satisfied, where $n = y_2/H$ and H is the total depth.

For the flow of one layer in a two-layer system, Craya (1951) shows that the critical condition given by equation (2.11) may be obtained in terms of either the energy principle (minimum energy flux) or the momentum principle (minimum force plus momentum).

The flow of a continuously-stratified fluid bounded above and below by rigid surfaces is analyzed by Long (1953) using long wave theory. He assumes a uniform velocity U and a density distribution given by $\rho = \rho_o \cdot \exp(-\beta_1 y_o)$. He shows that the critical internal Froude number is

$$F_i = \frac{U}{(g\beta_1 H^2)^{1/2}} = \frac{1}{\pi} \qquad 2.19$$

where H is the total depth.

He also shows that for (n+1) layers of equal depth and a constant and very small density difference from one layer to the next, the critical Froude number is given by:

$$F_i^2 = \frac{U^2}{gH\Delta\rho/\rho_b} = \frac{1}{4n(n+1)} \cdot \sec^2 \frac{n\pi}{2(n+1)} \qquad 2.20$$

where

$\Delta\rho$ = the overall density difference from top to bottom

ρ_b = the density at the bottom

For small density gradients ($\beta_1 H \simeq \Delta\rho/\rho_b$), equation (2.20) reduces to equation (2.19) as $n \to \infty$. Also, equation (2.20) agrees with equation (2.15) when the density difference is very small ($\rho_1/\rho_2 \simeq 1$) and n=1.

It should be noted that the equations presented or mentioned in this sub-section have been modified by the author from the forms originally presented in order to obtain some consistency in notation and presentation.

B. Stratified Flow Through a Transition

In the previous sub-section, the question of a critical regime in stratified flows is discussed. This is an essential feature when considering multi-layer flows through horizontal or vertical transitions which act as control points. In this sub-section, previous studies dealing with stratified flow through transitions are described and the general conclusions are presented. This problem is related to the problem of selective withdrawal from stratified reservoirs which is discussed in Section 2.2.1.

Stommel and Farmer (1952) show both theoretically and experimentally that an abrupt widening in a channel can act as a control on the regime of two-layer flow immediately upstream. The case of an upper layer flow is chosen for the experiments and equation (2.11) is used to calculate the critical depth of the flowing layer. They also consider the case of an abrupt contraction where the channel transition does not act as a control. In this case, they use the Bernoulli's principle to analyze the flow in the upper layer assuming uniform velocity distributions and a hydrostatic pressure distribution. In comparison with experiments, the energy losses are neglected.

Long (1953, 1954 and 1955) provides theoretical and experimental evidence for the problem of stratified flow over an obstacle. He describes an experimental investigation of a two-layer flow over a barrier. The obstacle is drawn by a motor drive at a uniform speed along the bottom of the channel. Three regimes of motion are observed. If the velocities of the fluids are sufficiently small, the interface is little disturbed except for a slight depression over the barrier. If the velocities are sufficiently high, the interface swells symmetrically over the obstacle. At intermediate speeds, a hydraulic jump occurs in the lee of the barrier and the lower layer increases in depth upstream. If the obstacle is small compared to the depth of the lower layer, weak lee waves appear at low speeds. When the speed of the fluid is moderately high, the obstacle large and the upper fluid relatively thin, a hydraulic "drop" appears in the lee. Long (1954) presents a theoretical discussion which provides an explanation of the observed behaviour.

Wood (1968) considers a multi-layer flow from a reservoir connected through a horizontal contraction to a channel. Wood (1969) studies a two-layer flow through a horizontal contraction (a channel connecting two reservoirs) with both layers flowing either in the same direction or in opposite directions. This problem of a two-layer flow through a contraction is also analyzed by Wood and Lai (1972) and Lai and Wood (1975). All these studies are described earlier in Sections 2.2.1.2 and A. They consider rectangular channels and use the critical conditions at the minimum width as well as at the virtual point of control to determine the discharges in the flowing layers. They also show that the depths of the layers at the point of maximum contraction are two-thirds of those far upstream. Wood and Lai (1972A) apply the same principle to the flow of a layered fluid over a broad-crested weir. They assume that the fluid flows from a reservoir through a gradually contracting channel in which there is a definite minimum width and that the crest of the round-crested weir is at this minimum width.

Mehrotra (1973) describes a two-layer flow controlled by horizontal or vertical contractions with the upper fluid either bounded by a rigid wall or having a free surface. He shows that for both forms of contractions with bounded upper layer, only the section at the maximum contraction can act as a control. When the upper layer is free, this is true only with regard to vertical obstructions. Two-layer flows past horizontal contractions with free upper layer can be controlled at two sections - one at the throat and the other away from it. However, when the flow throughout is subcritical, vertical and horizontal contractions are equivalent, the only control being at the maximum contraction.

Armi (1975) presents theoretical and experimental results for cases of a two-layer flow through a horizontal contraction and flow over a broad-crested weir. He demonstrates the existence of two unique solutions for flow through a contraction, the actual solution observed being determined by the downstream state.

Smith and Elsayed (1976) describe an analytical and experimental study of a lower moving layer in a two-layer system over a broad-crested weir. The analysis employs the notion of boundary layer displacement thicknesses to construct a model in which the flow is essentially one-dimensional at the critical section. At fixed boundaries, such as the weir crest and side walls, both laminar and turbulent boundary layers are included, but at the density interface only laminar boundary layers are considered.

King, Norton and Iceman (1975) apply their two-dimensional finite element model to the flow of a two-layer system over a broad-crested weir.

C. Interfacial Hydraulic Jump

The occurrence of internal hydraulic jumps and their different forms are described by Long (1954) for a two-fluid system. The question of determining the state downstream from a hydraulic jump for a completely specified state upstream is analyzed for a two-layer system by Yih and Guha (1955) and by Yih (1965, pp.130-133). They apply the momentum principle to each layer neglecting the shear and assuming hydrostatic distribution of pressure. They show that the momentum equations can have at most nine real solutions, one of which is obviously the given upstream state. But of these nine solutions five are not entirely positive (i.e. one or both of the depths is negative). Hence there can be at most only four positive solutions representing four mutually conjugate states. If the conjugate state is unique, energy considerations are used to decide whether or not a hydraulic jump can occur. If there are more than one conjugate states, it may not be possible to determine uniquely the downstream state merely from momentum and energy considerations. The uniqueness of the downstream state is demonstrated theoretically and verified experimentally for three special cases. These are a stagnant upper layer, a stagnant lower layer and equal downstream velocities in both layers.

Mehrotra and Kelly (1972) extend the previous work and show that the conjugate state to a given state is uniquely determinable for a two-fluid system for both the cases when the upper fluid is bounded by a rigid top and when it has a free surface, purely internal shocks being considered in the latter case.

Macagno and Macagno (1975) extend the analysis of Yih and Guha (1955) to include mixing at the interface. They assume that the lower layer entrains fluid from the upper layer at the foot of the jump. Their idea is to determine the drop in energy for the ideal hydraulic jump without mixing, and suppose then that a certain fraction of this energy is the energy available for entrainment. A model is then introduced to determine the change that the entrainment induces in the entering flow. This leads to new inlet values for depths, discharges and densities, which are used for a calculation of the final conjugate quantities. A comparison with laboratory measurements is made.

2.2.3.3 Discussion

In general, most of the previous studies dealing with the internal hydraulics of a stratified system are based on the assumption of one-dimensional flows. Uniform velocity profiles are assumed at vertical sections as well as a hydrostatic distribution of pressures. Only Long (1953, 1954 and 1955) and Pedersen (1972) consider a two-dimensional model.

Of the general assumptions that are common among all researchers are those of negligible frictional forces at the rigid boundaries as well as at the interfaces. Hamada (1969) incorporates the effects of bottom and interfacial resistance by using friction coefficients. Smith and Elsayed (1976) take into consideration the effect of the boundary layers formed at the rigid boundaries as well as at the interface.

The assumptions of one-dimensional frictionless flow probably account for some of the discrepancies between theoretical predictions and laboratory experimental observations where the effect of boundary layers is normally significant.

Steady-state flows are also assumed in all theoretical approaches.

The approach of Macagno and Macagno (1975) to consider mixing at the interface of an internal hydraulic jump is discussed and criticized in chapter 4 (Sections 4.2.3 and 4.3.2).

2.2.4 Density Wedges

2.2.4.1 Introduction

When a fluid flows into another fluid of different density, stratification may occur and a distinct interface may be formed. As the interface is usually sloping the phenomenon is often called a density wedge. There are many examples of density wedges that occur frequently in nature. One form may occur when a lock gate or other such division separates bodies of still water which differ slightly in density. While the opening of the gate may result in local disturbances, the predominant effect will be a continuing exchange pattern of flow which is caused by the density difference. This phenomenon is called "lock exchange flow" (Figure 2.5(a)). Density wedges of either overflow or underflow type arise from "lock exchange" phenomena which are fundamentally unsteady. When salt water enters a fresh water estuary, salt wedges are formed (Figure 2.5(b)). The need for water for irrigation, hydropower, municipal and industrial use results in increased abstraction of fresh water from rivers; in addition, the demand for increased food production necessitates the utilisation of cultivable land even in the estuarial region. These two requirements are contradictory in the sense that increased abstraction of fresh water draws salt water further inland, thus damaging fertile soil that could otherwise be used for cultivation.

Another example of density wedges occur when heated effluents enter colder bodies of water (thermal wedges, Figure 2.5(c)). When cooling water from power plants is discharged into rivers and lakes, a surface density layer may be created upstream of the point of discharge. This wedge of warm water may extend upstream beyond the cooling water intake point and thus the chance of recirculation has always been an important consideration in the design and location of cooling water inlet and outlet structures. Increased concern during recent years about the thermal pollution

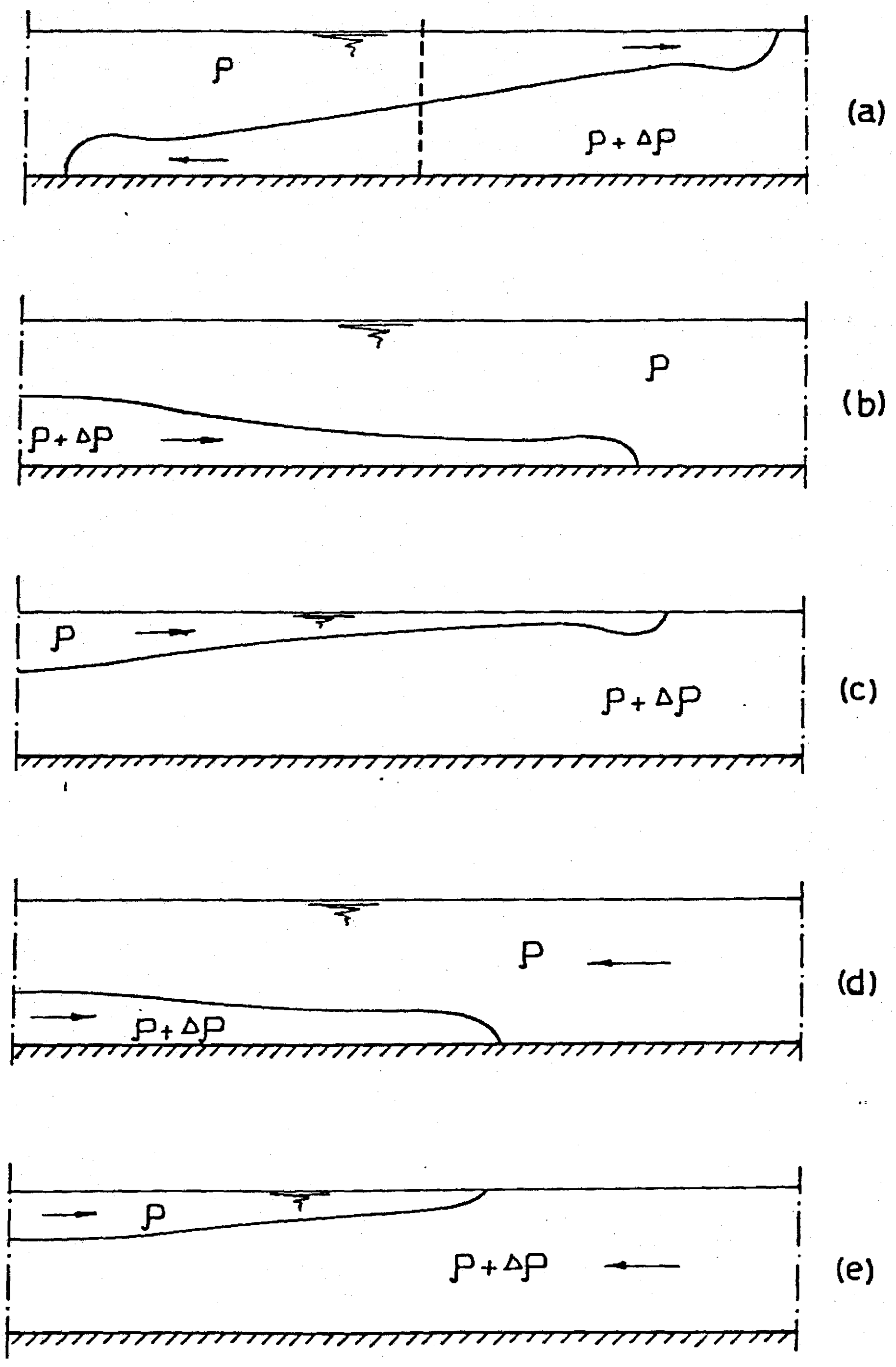


Fig. 2.5 - Diagrammatic Illustrations of Some Types of Density Wedges

of streams has focused additional attention upon the effects of thermal wedges on the temperature regime of the receiving stream.

When either the salt wedge or the thermal wedge penetrates an opposing current, the moving frame of reference may result in a stagnant wedge (i.e. no net flow) being formed. This phenomenon is often called the "arrested wedge" (Figures 2.5(d) and (e)).

Other forms of density wedges may occur when oil spills are retained by booms and when a silt-laden river enters a reservoir.

2.2.4.2 Literature Review

Theories of density wedges have been developed by a number of investigators and verified experimentally mainly by means of laboratory experiments. In density wedge circumstances, the capacity of the one body of liquid to extend relative to the other depends to some considerable extent on interfacial friction which is discussed in section 2.2.2. Also, density wedges in general are characterized by the unsteadiness of flow except for the arrested-wedge-type flows.

Salt water intrusion in estuaries takes a variety of different forms, ranging from fully-stratified to well-mixed. Stigter and Siemons (1967) present a one-dimensional numerical model to calculate the longitudinal salt-distribution in well-mixed estuaries as a function of time using the finite-difference method. The model is calibrated and verified using both flume and field data. Thatcher and Harleman (1972) also present a one-dimensional numerical model for the prediction of unsteady salinity intrusion in partially-mixed estuaries as well as the well-mixed ones as the limiting case by formulating the problem in finite-difference terms. The model is also verified using both flume and field data. Dazzi and Tomasino (1975) consider fully-stratified estuaries and develop a two-layer, one-dimensional numerical model using a finite-difference scheme. Mixing at the wedge front is introduced as an entrainment process coupled with the Richardson number as stability criterion. The lower layer overall Richardson number is defined as

$$Ri_2 = \frac{g' y_2}{(u_1 - u_2)^2} \qquad 2.21$$

where

g' = reduced gravitational acceleration

y_2 = thickness of the lower layer

u_1, u_2 = mean current velocities of the upper and lower layers respectively

Several authors have investigated the problem of steady (arrested) density wedges. Keulegan (1966), using dimensional reasoning, analyses a large number of data from experiments in laboratory flumes. He formulates expressions for the length of intrusion and shape of arrested saline wedges in rectangular channels. Concerning the shape of the wedge, he concludes that the ratio h_s/h_{sc} depends solely on the realtive distance x/L_i,

where

h_s = thickness of wedge at a distance x from the river mouth

h_{sc} = thickness at the river mouth

L_i = length of the wedge.

That is, he asserts that the wedge shape is geometrically similar when normalized with respect to h_{sc} and L_i. He defines a criterion for mixing in the form of a critical velocity of the fresh water current. He also provides an expression for the total amount of salt transported to the sea by fresh water. Expressions are also provided for the average interfacial and bottom stresses along the wedge.

Shi-Igai and Sawamoto (1969) study saline wedges in stratified estuaries. The intrusion length and the shape of an arrested salt wedge in a rectangular channel are formulated based on previous studies which assume critical conditions at the river mouth. Semi-empirical formulae for the interfacial resistance coefficient, which is a factor in their relationships, are presented as a function of the Reynolds number and the densimetric Froude number of the fresh water flow and also of a parameter which is assumed to be a characteristic of each river. They also recommend and provide guidelines for a laboratory model in the case of moderate or strong mixing conditions.

Similar results for the arrested salt wedge length and shape are discussed by Lepetit and Rogan (1970) in their useful bibliographic study of salt wedges. They show general agreement among different authors.

Partheniades, Dermisis and Mehta (1975), referring to the experimental study of Keulegan (1966), show analytically that x/L_i must be a function of both (h_s/h_{sc}) and the densimetric Froude number F_o which is given by

$$F_o = \frac{V_o}{\sqrt{gh_o(\Delta\rho/\rho)}} \qquad 2.22$$

where

V_o = the fresh water velocity upstream of the wedge

ρ = the fresh water density

h_o = the total depth of flow (assumed to be constant).

They obtain a relationship relating these parameters and use it to modify Keulegan's experimental relationship for the total rate of salt entrainment over the entire wedge. Accordingly too, they provide modified expressions for the average interfacial and bottom stresses.

Studies concerning the disposal of heated circulating water from thermal power-stations has stimulated interest in the problem of correctly simulating the rate of spread, extent and shape of thermal wedges. This increased concern is demonstrated by the large-scale physical model studies carried out by Ontario Hydro for the proposed nuclear generating stations (e.g. Bruce and Darlington) where no field measurements are available due to the huge costs involved.* The thermal wedge phenomena is very similar to the salt wedge and is characterized by the transfer of heat (nonconservative) at the interface and the free surface rather than the transfer of salt (conservative) at the interface as in the case of a salt wedge. Another effect which may frequently be of importance is the interaction of the wedge front with the mass transport at the surface resulting from wave action. This effect is studied experimentally by Smith (1965) where the overflow front is produced by means of a simple two-dimensional lock exchange. He shows that for a moderate wave action, the front continues as a discrete body of water as long as the mass transport acting in opposition to the front does not exceed a certain critical value. As soon as this value is passed, the mechanism of the front undergoes a dramatic change and a plunging roller develops which effectively mixes and breaks down the foremost section of the front. The coupling effect of the overflow and underflow front movements is not discussed.

Sharp (1971) investigates the overall characteristics of the unsteady spread of a buoyant surface discharge. Using dimensional analysis, he develops similarity criteria for the rate of spread, concentration and shape. Experimental work is described and data are presented to illustrate the relationship among the relevant parameters. Again, fresh water is used as an

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* Personal communications with Dr. C.K. Jonys (Hydraulic Model Laboratory-Ontario Hydro-Toronto.

effluent discharged from a circular nozzle and salt water as the receiving fluid. It is shown that viscous effects become important as the spread moves away from the source. The author expects that his results can be applied to thermal spreads with little error.

Stefan (1972) describes an experimental study of heated water discharge from a channel into a deep lake or reservoir wihtout allowing for lateral spread due to the presence of side walls. He introduces an overall entrainment ratio which is the ratio of volumetric flow rates measured respectively downstream and upstream from the outlet mixing region. Mixing can be produced either by a turbulent-jet type flow or by an internal hydraulic jump. The amount of entrainment produced is found to depend on the depth of the heated water layer in the reservoir and also on the densimetric Froude number of the heated layer.

Polk, Benedict and Parker (1971) study arrested thermal wedges. They provide an analytical relationship for the wedge length and show that the most critical parameter is the densimetric Froude number, $F' = u/[(\Delta\rho/\rho_2)gH]^{1/2}$, where u is the ambient stream velocity and H is the total depth. The initial mixing of the heated discharge prior to wedge development is shown to have a significant effect. Temperature reductions of 30% to 75% are observed in the field and are concluded to be the result of mixing of the warmer and colder layers, with cooling to the atmosphere playing a significant role. In the field observations, the velocity and temperature profiles do not exhibit a sharp interface. Therefore, the stipulation of zero average velocity in the upper layer is utilized to define the interface between the two layers.

Koh (1971) studies the mixing and dispersion of a two-dimensional surface buoyant jet discharged horizontally into a stagnant environment. He analyzes the interplay of source buoyancy, source momentum, entrainment, interfacial shear, and surface heat exchange. The analysis presented allows the determination of the flow field given the source characteristics and the ambient heat exchange. It is found that the relative magnitudes of source Froude number F_o, source Reynolds number R, and the dimensionless heat exchange coefficient k play an important role not only in the detailed quantitative description of the flow field but also in determining the type of flow field. F_o, R, and k are defined by

$$F_o = \frac{U_o}{g(\Delta\rho/\rho_o)h_o}, \quad R = \frac{U_o\, h_o}{\varepsilon}, \quad k = \frac{K}{U_o} \qquad 2.23$$

where

U_o = discharge velocity

ρ_o = density of the ambient fluid

h_o = initial thickness of surface jet

$\Delta\rho$ = source density difference

ε = shear coefficient

K = surface heat exchange coefficient

An experimental study of a two-dimensional turbulent jet of buoyant fluid spreading horizontally on the free surface of a stationary ambient fluid is presented by Chu, Baddour and Vanvari (1975). The experiments are performed using salt water and fresh water. Different types of flow field noted by Koh (1971) are observed in the experiments including the "density jump" formed at the interface.

Waldrop and Farmer (1975) present a three-dimensional finite- difference model to describe the fluid dynamics of a heated effluent discharging into a flowing river. The model may be applied to either steady or unsteady flows. The isotherms and velocity vectors predicted by the model agree reasonably well with measured field data.

The problem of lock exchange flow, being the classical case of unsteady non-uniform flow in the field of small density difference phenomena, has been investigated by many authors. Barr (1963) discusses the significance of the densimetric Froude-Reynolds number $\overline{F_{\Delta}R}$ defined by

$$\overline{F_{\Delta}R} = \frac{H^{3/2}\, g'^{1/2}}{\nu} \qquad 2.24$$

where H is the total depth. It has been customary to compare lock exchange flows on the basis of $K.\overline{F_{\Delta}R}$ where K is the initial velocity (V_o) coefficient:

$$K = \frac{V_o}{\sqrt{g'H}} = \frac{V_o}{V_\Delta} \qquad 2.25$$

The results are usually shown on a diagram relating $K.\overline{F_\Delta R}$ to L/H where L is the horizontal extent of the front measured from the barrier (see Figure 2.5(a)) with V/V_o being a third parameter where V is the front velocity.

Barr (1963A) and Barr and Hassan (1963) show experimentally that the initial velocities of the fronts of the underflow and overflow appear uniform for a greater or lesser relative distance depending on the scale of the experiment, and are unaffected by the channel width to depth ratio (B/H) except for extreme cases where (B/H) is well under 0.5.

The results of such investigations have been applied to large scale prototype spreads (Frazer, Barr and Smith (1967), 1968)) in order to estimate the appropriate degree of vertical scale exaggeration which is required to simulate surface spread correctly. New diagrams for lock exchange flow are utilized (plots of L/H against $\overline{F_\Delta R}$ for different values of the non-dimensional time t/T_Δ where $T_\Delta = (H/g')^{1/2}$). It is noted by P. Ackers in the discussion of the latter paper that when considering the implications of the separate zones in the diagrams, there are three zones analogous to the laminar, transition and turbulent zones in flow resistance tests. In the turbulent zone, the phenomena is independent of viscosity, and the data confirm, within the limits of experimental accuracy, the expected relationship between the velocity of spread and the density difference and depth, namely,

$$\frac{L/t}{\sqrt{g'H}} = \text{constant} \qquad 2.26$$

where for underflow the constant is about 0.5 and for overflow it is about 0.6. The limit to the "turbulent" zone, using the word turbulent to denote independence of viscosity, can be represented approximately by the equation

$$\overline{F_{\Delta}R} \cdot \frac{H}{L} \equiv \frac{\sqrt{g'}\, H^{5/2}}{\nu\, L} \geq 150 \qquad 2.27$$

Wood (1969 and 1970) considers theoretically and experimentally the interchange between two reservoirs connected by a contraction and containing fluid of different densities. The theory is described in Sections 2.2.1.2 and A. It is shown that there are two points of control, one at the position of minimum width and one (the virtual point of control) away from this position of minimum width. The theory is based on the assumptions of one-dimensional, steady flow, inviscid fluids and hydrostatic distribution of pressure.

The problem of interfacial shear in lock exchange flows is presented by Abraham and Eysink (1971) and discussed in Section 2.2.2.2.

Vasiliev and Chernyshova (1975) consider the unsteady problem of lock exchange flow, in a horizontal channel of rectangular section. The flow is assumed two-dimensional and the pressure distribution hydrostatic over the depth. A numerical algorithm is suggested using finite-difference approximations to the governing differential equations. Calculated velocity and density profiles are compared with experimental observations.

2.2.4.3 Discussion

1- In summary, it can be said that analytical work, hydraulic model studies, and the collection of field data have served to produce a good working knowledge of the principal physical phenomena of density wedges. Predictions of prototype behaviour can be made with some success using a combination of physical and mathematical modelling. However, the accuracy which can be obtained in predicting prototype behaviour is not really known in many cases. Uncertainty results both from the assumptions that have to be made in the modelling and the difficulty in obtaining field data which correspond to model assumptions. Relatively little information has been published comparing the predictions of any wedge model to actual field data although Barr's (1967) large scale flume experiments certainly approach prototype dimensions.

2- Wedge shape must be dependent to some degree on the existence of boundary layers in the receiving water. These boundary layers, while non-existing in lock exchange flows, (where the fluids are initially stagnant) are expected to have a significant effect on the shape of arrested wedges.

3- In personal communications with engineers of H.G. Acres (Niagara Falls), a practical problem related to irrigation systems is mentioned. This problem occurs at the withdrawl of fresh water from several points along the length of an estuary which is fed from an upstream reservoir and the corresponding formation of a salt wedge at the downstream end. A study is needed to demonstrate the sensitivity of the salt wedge to the rate of fresh water withdrawal as a means of controlling the water quality. Also, a description is sought for the effect of changes in the hydrological input on the geometry of the salt wedge.

4- Table 2.2 in Section 2.2.2.2 (reproduced after Dick and Marsalek (1973)) provides a useful list of additional studies dealing with density wedges. The corresponding references which are not included in the primary reference list are given with the Secondary References.

2.2.5 Submerged Jets and Plumes

2.2.5.1 Introduction

The increase in the production of electric power has resulted in the attendant generation of large quantities of waste heat. This waste heat is usually disposed of either in the atmosphere through cooling towers or ponds or to adjacent bodies of water. For the latter case, two limiting schemes can be envisioned for the method of discharge of heated water. First, to float the warm water on the surface, resulting in a minimum of initial dilution while maximizing the rate of heat loss to the atmosphere. This case is discussed in the previous section. Alternatively, the other extreme would be to employ diffusers submerged at some depth to promote much initial dilution such as is done for sewage. This technique is expected to be used more in the future because of the environmental considerations.* This is the subject of this section.

In this survey, all jet-flows are referred to as jets. If the initial momentum of the jet has a negligible effect upon its diffusion pattern, it is referred to as a plume.

2.2.5.2 Literature Review

When heated water from the cooling system of a power station is discharged from an outfall into a calm sea, it rises to the surface mainly because it is less dense than the surrounding sea water (buoyant jets). Previous literature dealing with buoyant jets have usually simulated the motion in the rising column by a convectional plume above a point source. The motion in the rising plume can be turbulent or laminar. Anwar (1972) examines the theory of a turbulent vertical plume in stagnant ambient fluid and determines theoretically the region below which the flow in the plume is laminar. He shows that a turbulent plume

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* Personal communication with Dr. C.K. Jonys (Hydraulic Model Laboratory - Ontario Hydro - Toronto).

exists when the source densimetric Froude number $F_s \geq 5^{1/2}/2$ (≈ 1.12), where F_s is defined by

$$F_s = \left[\frac{\alpha}{\lambda^2} \cdot \frac{1}{b_o} \cdot \frac{\rho_a}{g . \Delta\rho_o}\right]^{1/2} . U_o \qquad 2.28$$

where

α = entrainment coefficient

λ = ratio of lateral spread between velocity and density profiles

b_o = characteristic radius of finite source

ρ_a = mass density of ambient fluid

$\Delta\rho_o = \rho_a - \rho_o$

ρ_o = mass density of fluid in nozzle

g = gravitational acceleration

U_o = mean velocity in nozzle

When $F_s < 5^{1/2}/2$ a real solution for the turbulent plume can not be obtained. This is examined experimentally where it is found that the motion in the plume is laminar and becomes unstable at a certain distance above the nozzle where a turbulent plume develops. It is found that the height of the laminar plume above the nozzle depends very much upon the jet Reynolds number defined by

$$R = \frac{U_o D}{\nu} \qquad 2.29$$

where

D = diameter of nozzle

ν = kinematic viscosity of fluid at nozzle exit.

Anwar (1972A) extends this work to study the radial spreading of the rising plume when it impinges on the free surface of the stagnant ambient fluid. He defines three different zones in the surface layer. Initially, an "entrainment zone" where the surface layer entrains the underlying fluid. He shows that the entrainment decreases with the increasing distance from the point where the rising column impinges on the free surface and also that the radius of the entrainment zone depends on the initial buoyancy flux from the nozzle and also on submergence height. The surface layer passes through a "transition zone" and then reaches a final stage within which its density and its thickness remain unchanged - the so-called "zone of homogeneous flow". The ultimate thickness of the surface layer, and the radius at which this final stage is reached are shown to depend upon the depth of submergence. The theoretical results are compared with experimental observations. Fair agreement is obtained.

A numerical solution is presented by Trent and Welty (1973) of steady-flow, turbulent, free momentum jets (similar jet and ambient fluids) and forced plumes (buoyant driving forces involved) issuing vertically into a stagnant receiving medium. The problem is solved in axisymmetric coordinates using finite-difference techniques. In comparison with experiments, it is found that although a constant value of diffusivity will permit accurate computation in the case of a momentum jet, a variable coefficient is needed for buoyant plumes or large errors will result.

If a buoyant jet issues vertically into stratified surroundings, its mean density increases as it rises and mixes with the environment until at a certain distance above the point of discharge the density difference between the jet and the ambient fluid is reduced to zero "equilibrium level". Above this level, the jet fluid is heavier than the ambient fluid. Accordingly, the jet fluid can penetrate above this level only if it has sufficient kinetic energy to do so when reaching this level. During the penetration the jet fluid gradually loses the kinetic energy, and so cannot penetrate in a vertical direction beyond a certain "ceiling level". Jet fluid which has reached the ceiling level descends. In this respect, distinction must be made between the case of stagnant receiving fluid and that of receiving fluid flowing in a horizontal direction. This problem is analyzed theoretically by Abraham (1972) where he considers three-dimensional axisymmetical jets issuing vertically upwards into a density gradient considering both cases of a stagnant and moving ambient fluid.

Maxworthy (1972) studies the unsteady spread of a horizontal jet in a stratified fluid. The problem is investigated experimentally by injecting a constant density salt solution at its equilibrium level into a long, linearly-stratified water tank. Length and width of the jet flow are measured as a function of time and compared with those obtained theoretically assuming that the wave speed of internal waves is much greater than the flow speed. Good agreement is observed for sufficiently small rates of effluent inflow.

Hecker and Medeiros (1976) discuss physical scale models that are frequently used to develop diffusers for thermal discharges, and to predict temperature rise pattern which will exist in the field. Such models are typically designed and operated according to the principles of Froude scaling due to the predominance of gravity and inertial forces in free surface and density flows. The influence of the reduction in Reynolds number on scaling of jet entrainment or dilution is assumed to be small once the jet is turbulent from its origin. Their experimental study is aimed at investigating the effects of turbulent flow Reynolds number on single buoyant jet dilution. Heated water is discharged horizontally from a single jet into a large tank. The densimetric Froude number (F_i) is held constant and the Reynolds number (R) is varied. These numbers are defined by

$$F_i = \frac{U_o}{\sqrt{(\Delta\rho/\rho_a)gD}} , \quad R = \frac{U_o D}{\nu} . \tag{2.30}$$

It is found that close to the jet, lower turbulent flow Reynolds numbers produce lower centreline concentrations than at higher Reynolds numbers, for the same densimetic Froude number.

2.2.5.3 Discussion

1- Previous theoretical and experimental investigations have always been directed to the simple case of a single jet. The outlined results and conclusions cannot be translated directly to multi-jet discharges such as in a diffuser since other factors, notably geometric constraints, jet interference and the available flow path of entrained water, may dominate diffuser induced dilution.

2- No information has been found comparing the predictions of any jet model to actual field data.

3- In practical situations, as mentioned by engineers of H.G. Acres (Niagara Falls) in personal communications, the available theoretical predictions proved to be of limited validity. The effects of solid boundaries, jet interference and spacing of diffusers seem to be significant and need to be investigated. Also, the behaviour of buoyant liquid jets under the action of wind shear (at the surface of the receiving water) appears to be of practical importance. Their present approach is mainly empirical based on dimensional analysis and laboratory data.

2.2.6 Stratified Flow in Circular Pipes

Gemmell and Epstein (1962) consider stratified laminar flow of two immiscible Newtonian liquids in a circular pipe. They use a two- dimensional finite difference method to determine the velocity profiles. The theoretical results for laminar flow are compared with experimental data in the literature for the horizontal flow of a mineral oil and water in a circular pipe. Good agreement is obtained when the flow of both water and oil is laminar.

The most common practical example is the pipeline pumping of petroleum oils laden with water.

2.2.7 Wind Effect on a Stable Density Interface

An experimental study is presented by Wu (1975) with two layers of stably stratified fluids in a wind-wave tank. The slope of the density interface is related to the wind stress, the density difference of the fluids, and the depth of the interface.

2.3 Problem Classification

It is shown from the previous identification and review of several stratified flow problems that considerable attention has been given by different researchers to individual cases of stratified flows. It is also apparent that there exists some kind of hydraulic analogy between stratified flows and conventional homogeneous flows.

Therefore, an attempt is made in this section to classify and relate a wide range of stratified flow problems in a manner similar to the classification of homogenous flow regimes. This is a necessary and possibly significant step towards a unified approach to study stratified flow phenomena.

The suggested classification is presented in Table 2.3 with the relevant reference numbers attached to each problem. Table 2.4 identifies these references. Some of the problems may be included in more than one division depending on the flow conditions and the theoretical considerations. It should be noted that the judgement of whether the non-uniform flows are rapidly-varied or gradually-varied is in some cases arbitrary and depends on the rate of variation and the scale of the problem under consideration.

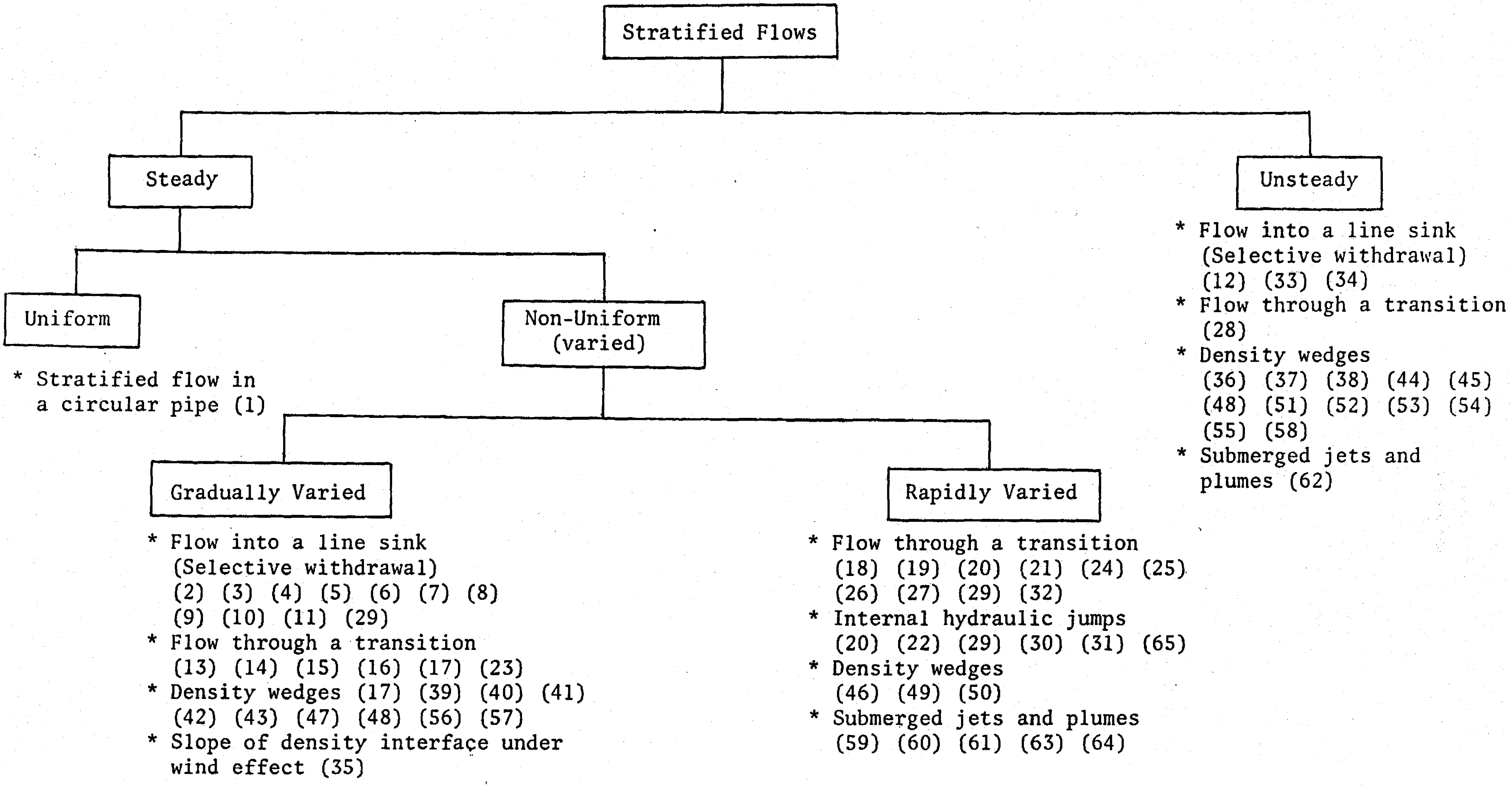


Table 2.3 - Classification of Stratified Flow Problems

(references are indicated by numbers and identified in Table 2.4)

Ref. No.	Author(s)	Year	Ref. No.	Author(s)	Year
1	Gemmell and Epstein	1962	36	Stigter and Siemons	1967
2	Yih	1958	37	Thatcher and Harleman	1972
3	Debler	1959	38	Dazzi and Tomasino	1975
4	Kao	1965	39	Keulegan	1966
5	Koh	1966	40	Shi-Igai and Sawamoto	1969
6	Walesh and Monkmeyer	1973	41	Lepetit and Rogan	1970
7	Ho and Monkmeyer	1975	42	Dick and Marsalek	1973
8	Clark, Monkmeyer, Ho and Hooper	1976	43	Partheniades, Dermisis and Mehta	1975
9	Ho, Monkmeyer and	1976	44	Smith	1965
10	Elder and Wunderlich	1969	45	Sharp	1971
11	Brooks and Koh	1969	46	Stefan	1972
12	Hwang and Slotta	1968	47	Polk,Benedict and Parker	1971
13	Wood	1968	48	Waldrop and Farmer	1975
14	Wood and Lai	1972	49	Koh	1971
15	Wood and Lai	1972A	50	Chu, Baddour and Vanvari	1975
16	Lai and Wood	1975	51	Barr	1963
17	Wood	1969	52	Barr	1963A
18	Stommel and Farmer	1952	53	Barr and Hassan	1963
19	Long	1953	54	Frazer, Barr and Smith	1967
20	Long	1954	55	Frazer, Barr and Smith	1968
21	Long	1955	56	Wood	1970
22	Yih and Guha	1955	57	Abraham and Eysink	1971
23	Pedersen	1972	58	Vasiliev and Chernyshova	1975
24	Smith and Elsayed	1976	59	Anwar	1972
25	Mehrotra	1973	60	Anwar	1972A
26	Armi	1975	61	Abraham	1972
27	Hsu and Stolzenbach	1975	62	Maxworth	1972
28	King, Norton and Iceman	1975	63	Trent and Welty	1973
			64	Hecker and Medeiros	1976
29	Yih	1965	65	Hayakawa	1970
30	Mehrotra and Kelly	1972			
31	Macagno and Macagno	1975			
32	Hamada	1969			
33	Koh	1966A			
34	Kao, Pao and Wei	1972			
35	Wu	1975			

Table 2.4 - Reference Identification

CHAPTER 3

ENERGY BALANCE IN STREAMLINED TRANSITIONS

3.1 INTRODUCTION

This chapter describes the analysis of transitional two-layer flow problems involving negligible energy losses. Theoretical analysis of the problem is presented followed by a description of the computer routines developed and the numerical techniques employed. A combination of literature search and experimental investigation are employed to provide data and thus check the validity of the results obtained by the routines.

3.2 THEORY

Solutions may be obtained by application of the principles of conservation of energy and mass. Two important subdivisions of these solutions must be treated, viz:

a) Transitional flows which are subcritical throughout.

b) Transitional flows which involve a critical flow section or control at some point in the system.

It should be noted that in the latter class of solutions critical flow may occur in one or both layers. The rather rare situation of transitional flow which is supercritical throughout has not

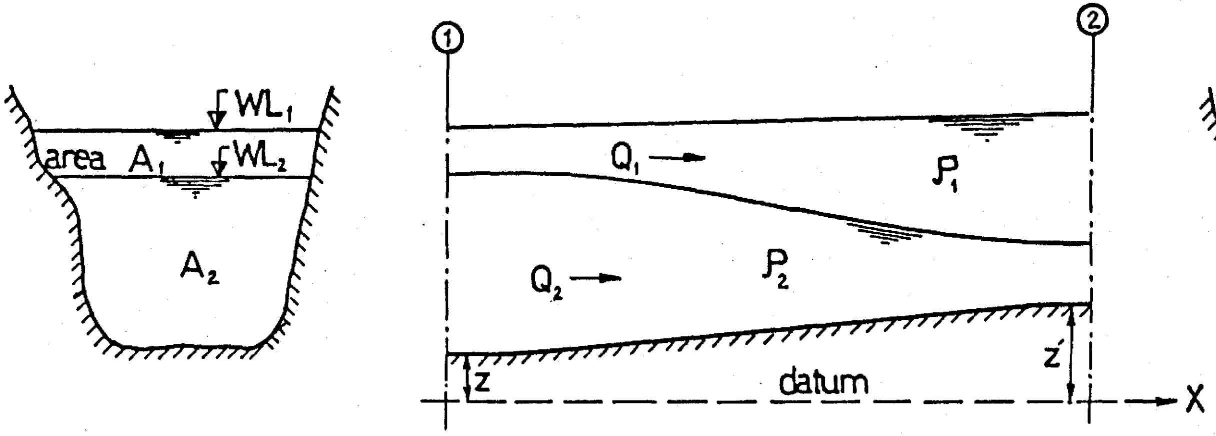


Fig. 3.1 - Two-Layer Transitional Flow

been considered here since in practice the very high velocity gradients give rise to interfacial instabilities and transverse mixing which is difficult to quantify.

With reference to the definition sketch of Figure 3.1 and assuming hydrostatic pressure distribution, Bernoulli's equations may be applied to each layer between sections 1 and 2 as follows

For the upper layer

$$\rho_1 \, g \, (WL_1 - WL_2) + \rho_1 \, g \, (WL_2 - z) + \rho_1 \, g \, z + \alpha_1 \, \rho_1 \frac{Q_1^2}{2A_1^2} =$$

$$\rho_1 \, g \, (WL_1' - WL_2') + \rho_1 \, g \, (WL_2' - z') + \rho_1 \, g \, z' + \alpha_1 \, \rho_1 \frac{Q_1^2}{2A_1'^2} \qquad 3.1$$

where

g = acceleration due to gravity

α_1 = kinetic energy correction factor for the upper layer flow

Subscripts 1 and 2 are used for the upper and lower layers respectively. Non-primed and primed quantities refer to sections 1 and 2 respectively.

Dividing equation 3.1 by ρ_1 g yields

$$WL_1 + \alpha_1 \frac{Q_1^2}{2gA_1^2} = WL_1' + \alpha_1 \frac{Q_1^2}{2gA_1'^2} \qquad 3.2$$

For the lower layer

$$\rho_1 g (WL_1 - WL_2) + \rho_2 g (WL_2 - z) + \rho_2 g z + \alpha_2 \rho_2 \frac{Q_2^2}{2A_2^2} =$$

$$\rho_1 g (WL_1' - WL_2') + \rho_2 g (WL_2' - z') + \rho_2 g z' + \alpha_2 \rho_2 \frac{Q_2^2}{2A_2'^2} \qquad 3.3$$

where

α_2 = kinetic energy correction factor for the lower layer flow

Dividing equation 3.3 by ρ_2 g yields

$$\frac{\rho_1}{\rho_2} WL_1 + \frac{\Delta\rho}{\rho_2} WL_2 + \alpha_2 \frac{Q_2^2}{2gA_2^2} = \frac{\rho_1}{\rho_2} WL_1' + \frac{\Delta\rho}{\rho_2} WL_2' + \alpha_2 \frac{Q_2^2}{2gA_2'^2} \qquad 3.4$$

where

$\Delta\rho = \rho_2 - \rho_1$ = density difference

If the critical flow section is defined as the section at which

$$\frac{d(WL_1)}{dx} \rightarrow \infty \quad \text{and/or} \quad \frac{d(WL_2)}{dx} \rightarrow \infty \quad ,$$

and with reference to equations 6.16 and 6.18 (Chapter 6), the critical condition is shown to be

$$F_{i1}^2 + F_{i2}^2 - \beta\, F_{i1}^2 \cdot F_{i2}^2 - 1 = 0 \qquad 3.5$$

where

F_{i1}^2 = densimetric Froude number of the upper layer at the critical section

$$= \alpha_1 \frac{Q_1^2 \; T_1}{g' \; A_1^3}$$

F_{i2}^2 = densimetric Froude number of the lower layer at the critical section

$$= \alpha_2 \frac{Q_2^2 \; T_2}{g' \; A_2^3}$$

g' = reduced gravitational acceleration = $g.\beta$

β = $\Delta\rho/\rho_2$

T = top width of the layer

subscripts 1 and 2 refer to the upper and lower layers respectively.

Equations 3.2 and 3.4 apply to the case where transitional flows are subcritical throughout. If the flows involve a critical section at some point of the system, equation 3.5 may be applied at that section.

It is clear that these equations reduce to the well-known equations for homogeneous flow (air-water interface) when $\rho_1 = 0$.

It should be noted that equations 3.2, 3.4 and 3.5 may be reduced to the same equations developed by other authors [e.g. Wood (1968), Lai and Wood (1975), Stommel and Farmer (1952) and Armi (1975)], if they are subjected to the same assumptions made by these researchers, namely

a) rectangular cross-sectional strip

b) horizontal channel bottom

c) uniform velocity distributions

d) small density differences

None of these assumptions are used in the present analysis and the more generalized version of the energy equations presented here have not been treated before.

3.3 DEVELOPMENT OF COMPUTER ROUTINES

Complete documentation and listings of the developed subroutines are contained in Appendix (2). Table 3.1 describes briefly the energy balance routines under different conditions.

Considering the six variables shown in Table 3.1 for the transitional subcritical flow, equations 3.2 and 3.4 may be solved for two of these variables as unknowns. This results in a large number (15) of permutations. Also, for transitional flow with a critical section, 20 different problems may be idenified. However, some of these problems may not be practically significant and those presented in Table 3.1 are chosen accordingly.

In the following section, the numerical methods employed in three of these routines (BERNWL2, WLCRIT1 AND WLCRIT) are described. As shown in Table 3.1, the methods used for the other seven routines are either explicit solutions of the corresponding equations (e.g. BERNQ2 AND QCRIT) or iterative procedures using the other routines. Problem formulation and convergence criteria are discussed for all routines in Appendix (2).

CONDITION	SUBROUTINE	Q_1	Q_2	WL_1	WL_2	WL_1'	WL_2'	EQNS. USED	METHOD
Transitional Subcritical Flow	BERNQ2	+	+	*	*	*	*	3.2 & 3.4	explicit
	BERNWL2	*	*	+	+	*	*	3.2 & 3.4	Newton-Raphson
		*	*	*	*	+	+		
Critical Section	QCRIT	+	*	-	-	*	*	3.5	explicit
		*	+	-	-	*	*		
	WLCRIT	*	*	-	-	+	*	3.5	secant method
	WLCRIT1	*	*	-	-	*	+	3.5	Newton-Raphson
Transitional Flow with a Critical Section	WLUCR	*	*	+	+	+	*	3.2,3.4&3.5	using WLCRIT & BERNWL2
	WLQCR	+	*	*	*	+	+	3.2,3.4&3.5	ZSYSTM1 (Brown's Method)#
		*	+	*	*	+	+		
	QCRIT1	+	*	+	+	*	*	3.2,3.4&3.5	using QCRIT & BERNWL2
		*	+	+	+	*	*		
	QCRIT12	+	+	*	*	+	*	3.2,3.4&3.5	iterative using BERNQ2 & WLCRIT
	QCRIT22	+	+	*	*	*	+	3.2,3.4&3.5	iterative using BERNQ2 & WLCRIT1

[ρ_1, ρ_2, α_1, α_2, geometry of cross-sections are given in all cases]

* given
\+ computed
\# see documentation (Appendix 2)

Table 3.1 Energy Balance Subroutines

3.3.1 Subroutine BERNWL2

This routine analyzes two-layer stratified subcritical flow in a streamlined transition and determines the free surface and interface elevations at one section in terms of those at the other section (see Table 3.1, p. 79 and documentation Appendix 2, p. 289). The two-dimensional Newton-Raphson method is used to solve equations 3.2 and 3.4 for the unknowns WL_1, WL_2 (or WL_1' and WL_2'). These equations may be written in the form

$$\phi(WL_1, WL_2) = (WL_1 - WL_1') + \frac{\alpha_1 Q_1^2}{2g}\left[\frac{1}{A_1^2} - \frac{1}{A_1'^2}\right] = 0 \qquad 3.6$$

and

$$\psi(WL_1, WL_2) = \frac{\rho_1}{\rho_2}(WL_1 - WL_1') + \frac{\Delta\rho}{\rho_2}(WL_2 - WL_2') + \frac{\alpha_2 Q_2^2}{2g}\left[\frac{1}{A_2^2} - \frac{1}{A_2'^2}\right] = 0 \qquad 3.7$$

Using the Newton-Raphson method [Scarborough (1962)], the successive approximations are given by

$$(WL_1)_{r+1} = (WL_1)_r + h \qquad 3.8$$

and

$$(WL_2)_{r+1} = (WL_2)_r + k \qquad 3.9$$

where the corrections h and k are given by

$$h = \frac{A}{B}, \qquad k = \frac{C}{B}$$

and where A, B and C are defined in matrix form as follows.

$$A = \begin{vmatrix} -\phi[(WL_1)_r, (WL_2)_r] & [\frac{\partial\phi}{\partial(WL_2)}]_r \\ & \\ -\psi[(WL_1)_r, (WL_2)_r] & [\frac{\partial\phi}{\partial(WL_2)}]_r \end{vmatrix} , \qquad 3.10$$

$$C = \begin{vmatrix} \left[\frac{\partial\phi}{\partial(WL_1)}\right]_r & -\phi[(WL_1)_r, (WL_2)_r] \\ & \\ \left[\frac{\partial\psi}{\partial(WL_1)}\right]_r & -\psi[(WL_1)_r, (WL_2)_r] \end{vmatrix} \qquad 3.11$$

and $$B = \begin{vmatrix} \left[\frac{\partial\phi}{\partial(WL_1)}\right]_r & \left[\frac{\partial\phi}{\partial(WL_2)}\right]_r \\ & \\ \left[\frac{\partial\psi}{\partial(WL_1)}\right]_r & \left[\frac{\partial\psi}{\partial(WL_2)}\right]_r \end{vmatrix} \qquad 3.12$$

From equations 3.6 and 3.7,

$$\frac{\partial\phi}{\partial(WL_1)} = 1 + \frac{\alpha_1\ Q_1^2}{2g}\left[\frac{-2}{A_1^3} \cdot \frac{\partial\ A_1}{\partial(WL_1)}\right]$$

It should be pointed out that the term $\partial A_i/\partial(WL_j)$ i, j = 1 or 2, may take different values depending on the corresponding subscripts.

Substituting $\partial A_1/\partial(WL_1) \approx T_1$ where T_1 is the top width of the upper layer yields

$$\frac{\partial\phi}{\partial(WL_1)} = 1 - \frac{\alpha_1\ Q_1^2\ T_1}{g\ A_1^3} \qquad 3.13$$

Also,

$$\frac{\partial\phi}{\partial(WL_2)} = \frac{\alpha_1 Q_1^2}{2g}\left[\frac{-2}{A_1^3} \cdot \frac{\partial A_1}{\partial(WL_2)}\right]$$

substituting $\partial A_1/\partial(WL_2) \simeq -T_2$ where T_2 is the top width of the lower layer (or the "bottom width" of the upper layer) yields

$$\frac{\partial\phi}{\partial(WL_2)} = \frac{\alpha_1 Q_1^2 T_2}{g A_1^3} \qquad 3.14$$

Similarly,

$$\frac{\partial\psi}{\partial(WL_1)} = \frac{\rho_1}{\rho_2} + \frac{\alpha_2 Q_2^2}{2g}\left[\frac{-2}{A_2^3} \cdot \frac{\partial A_2}{\partial(WL_1)}\right]$$

Substituting $\frac{\partial A_2}{\partial(WL_1)} = 0$ gives

$$\frac{\partial\psi}{\partial(WL_1)} = \frac{\rho_1}{\rho_2} \qquad 3.15$$

and $$\frac{\partial\psi}{\partial(WL_2)} = \frac{\Delta\rho}{\rho_2} + \frac{\alpha_2 Q_2^2}{2g}\left[\frac{-2}{A_2^3} \cdot \frac{\partial A_2}{\partial(WL_2)}\right]$$

substituting $\frac{\partial A_2}{\partial(WL_2)} = T_2$ gives

$$\frac{\partial\psi}{\partial(WL_2)} = \frac{\Delta\rho}{\rho_2} - \frac{\alpha_2 Q_2^2 T_2}{g A_2^3} \qquad 3.16$$

Equations 3.6 to 3.16 form the basis of the solution method used in subroutine BERNWL2.

Subroutine BERNWL2 is used later in this chapter to make comparison with the experimental observations described in Section 3.4. A detailed illustration of the use of the routine is contained in the documentation (Appendix 2, p. 289).

3.3.2 Subroutine WLCRIT1

Both the routines WLCRIT1 and WLCRIT (see 3.3.3) are concerned with the analysis of critical flow at a single section containing two-layer stratified flow. The problem may be stated in general terms as

$$\phi\ [Q_1,\ Q_2,\ WL_1',\ WL_2'] = 0,$$

other factors such as section and fluid properties being essentially constant. Subroutine WLCRIT1 is concerned with the solution in which WL_2' is the dependent (i.e. unknown) quantity whereas WLCRIT is designed to solve for an unknown free surface WL_1'. Both methods are based on solution of equation 3.5. Special properties of the solution are discussed in Section 3.3.5(d).

With reference to Figure 3.1 (p. 74), section 2 is assumed to contain critical flow where the free surface elevation WL_1' is given.

The Newton-Raphson method is used to solve equation 3.5 for the unknown WL_2'. Equation 3.5 may be written in the form

$$f(WL_2') = \frac{\Delta\rho}{\rho_2} \cdot \frac{\alpha_1 Q_1^2 T_1'}{g' A_1'^3} \cdot \frac{\alpha_2 Q_2^2 T_2'}{g' A_2'^3} - \frac{\alpha_1 Q_1^2 T_1'}{g' A_1'^3} - \frac{\alpha_2 Q_2^2 T_2'}{g' A_2'^3} + 1 = 0 \qquad 3.17$$

Using the Newton-Raphson method, the successive approximations are given by

$$(WL_2')_{r+1} = (WL_2')_r - \frac{f[(WL_2')_r]}{f'[(WL_2')_r]} \text{ where } f' = \frac{df}{d(WL_2')} \qquad 3.18$$

For more rapid convergence with reasonable linearity, equation 3.17 may be expressed as

$$f(WL_2') = \frac{1}{Y^{1/3}} + 1 = 0 \qquad 3.19$$

where

$$Y = \frac{\Delta\rho}{\rho_2} \cdot \frac{\alpha_1 Q_1^2 T_1'}{g' A_1'^3} \cdot \frac{\alpha_2 Q_2^2 T_2'}{g' A_2'^3} - \frac{\alpha_1 Q_1^2 T_1'}{g' A_1'^3} - \frac{\alpha_2 Q_2^2 T_2'}{g' A_2'^3} \qquad 3.20$$

Therefore,

$$f'(WL_2') = \frac{-1/3\ Y^{-2/3}\ (Y')}{Y^{2/3}} \text{ where } Y' = \frac{dY}{d(WL_2')}$$

$$= -\frac{1}{3}\frac{Y'}{Y^{4/3}} \qquad 3.21$$

$$Y' = \frac{\Delta\rho}{\rho_2}\left[\frac{\alpha_1 Q_1^2 T_1'}{g' A_1'^3}\left(\frac{\alpha_2 Q_2^2}{g'} \cdot \frac{A_2'^3 \dfrac{dT_2'}{d(WL_2')} - T_2' . 3A_2'^2 \dfrac{dA_2'}{d(WL_2')}}{A_2'^6}\right) + \right.$$

$$\left. \frac{\alpha_2 Q_2^2 T_2'}{g' A_2'^3}\left(\frac{\alpha_1 Q_1^2}{g'} \cdot \frac{A_1'^3 \dfrac{dT_1'}{d(WL_2')} - T_1' . 3A_1'^2 \dfrac{dA_1'}{d(WL_2')}}{A_1'^6}\right)\right] -$$

$$\frac{\alpha_1 Q_1^2}{g'} \cdot \frac{A_1'^3 \dfrac{dT_1'}{d(WL_2')} - T_1' \cdot 3A_1'^2 \dfrac{dA_1'}{d(WL_2')}}{A_1'^6} -$$

$$\frac{\alpha_2 Q_2^2}{g'} \cdot \frac{A_2'^3 \cdot \dfrac{dT_2'}{d(WL_2')} - T_2' \cdot 3\, A_2'^2 \dfrac{dA_2'}{d(WL_2')}}{A_2'^6}$$

Rearranging terms and substituting $dT_1'/d(WL_2') = 0$ gives

$$Y' = \frac{\alpha_1 Q_1^2}{g'} \cdot \frac{3T_1' T_2'}{A_1'^4}\left[\frac{\Delta\rho}{\rho_2}\, \frac{\alpha_2 Q_2^2 T_2'}{g' A_2'^3} - 1\right] + \frac{\alpha_2 Q_2^2}{g'} \cdot \frac{A_2' \dfrac{dT_2'}{d(WL_2')} - 3\, T_2'^2}{A_2'^4} \cdot$$

$$\left[\frac{\Delta\rho}{\rho_2}\, \frac{\alpha_1\, Q_1^2\, T_1'}{g'\, A_1'^3} - 1\right] \qquad 3.22$$

The term $dT_2'/d(WL_2')$ depends on the cross-sectional geometry and is approximated in the subroutine as $\Delta T_2'/\Delta(WL_2')$ [see Appendix (2), p. 359]. Equations 3.18 to 3.22 are the basis of the solution method used in subroutine WLCRIT1.

3.3.3 Subroutine WLCRIT

As mentioned in the previous section, subroutine WLCRIT is designed to obtain solutions of the general form

$$WL_1' = \phi\ [Q_1,\ Q_2,\ WL_2']$$

With reference to Figure 3.1, section 2 is assumed to contain critical flow where the interface elevation WL_2' is given.

The secant method is used to solve equation 3.5 for the unknown WL_1'. This method requires that the function be real, continuous and single-valued within the interval defined by two initial values of the argument [Smith (1970) and James, Smith and Wolford (1977)). If the solution lies outside the range of the initial values, it is further required that no turning value exists between the solution and the initial values. With reference to Fig. 3.2, and if the equation to be solved is $F(Y) = C$,the successive approximations may be described by the equation

$$Y_{r+1} = Y_r + (Y_r - Y_{r-1}) \ .\ [C - F(Y_r)]/[F(Y_r) - F(Y_{r-1})] \qquad 3.23$$

Alternatively

$$Y_{r+1} - Y_r = (Y_r - Y_{r-1}) \ .\ [C - F(Y_r)]/[F(Y_r) - F(Y_{r-1})] \qquad 3.24$$

Equation 3.5 may be expressed as

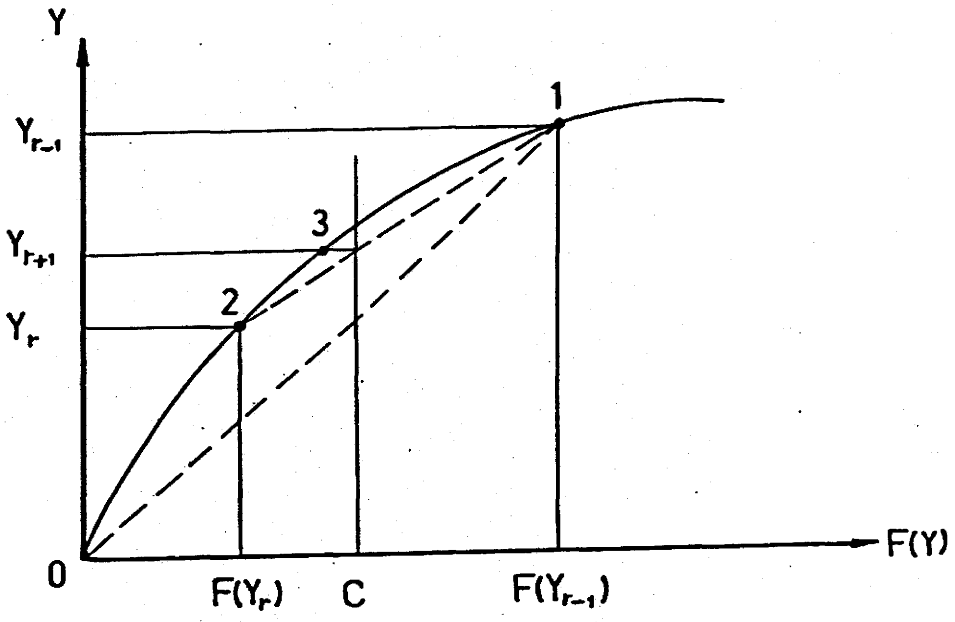


Fig. 3.2 - The Secant Method [after Smith (1970)]

$$F(WL_1') = \frac{Q_1^2 \left[\dfrac{\Delta\rho}{\rho_2} \dfrac{Q_2^2 T_2'}{g' A_2'^3} - 1 \right]}{\dfrac{Q_2^2\ T_2'}{g' A_2'^3} - 1} = \frac{g'\ A_1'^3}{T_1'} \qquad 3.25$$

Equation 3.25 in a simpler form will be

$$F(WL_1') = \frac{\beta\ F_{i2}^2 - 1}{F_{i2}^2 - 1}\ \alpha_1\ Q_1^2 = \frac{g'\ A_1'^3}{T_1'} \qquad 3.26$$

where

$$\beta = \frac{\Delta\rho}{\rho_2}$$

$$F_{i2}^2 = \frac{\alpha_2\ Q_2^2\ T_2'}{g'\ A_2'^3}$$

For computational efficiency a more linear functional form is adopted for F, given by

$$F(WL_1') = \left[\frac{\beta\ F_{i2}^2 - 1}{F_{i2}^2 - 1}\ \alpha_1\ Q_1^2 \right]^{1/2} = \left[\frac{g'\ A_1'^3}{T_1'} \right]^{1/2} \qquad 3.27$$

Equations 3.24 and 3.27 are used in the solution method in subroutine WLCRIT.

Different numerical methods are used in subroutines WLCRIT and WLCRIT1 in order to demonstrate that different possibilities exist in the choice of a numerical technique used for a specific purpose.

3.3.4 Comparison with Published Experimental Data

In order to check the validity of the results obtained by the routines, a combination of literature search and experimental investigation is employed. In this section, experimental results published by Keulegan (1966) regarding arrested salt wedges are examined quantitatively and compared with predictions using subroutine WLCRIT1.

Keulegan (1966) performed a number of laboratory experiments on arrested saline wedges. Fig. 3.3 shows Keulegan's experimental measurements for the salt water depth at the river mouth compared with the computed results, assuming critical condition at the river mouth [Stommel and Farmer (1952)], and using subroutine WLCRIT1 to compute the critical interface elevation for a given free surface level. Referring to equation 3.5, the relationship between $2F_o$ and h_{2C}/H for the conditions of Keulegan's experiments (rectangular cross-section and $Q_2 = 0$), may be derived as follows.

With the simplifying assumptions of a rectangular cross-section and zero net flow within the wedge, equation 3.5 reduces to

$$F_{i1}^2 = \frac{\alpha_1 q_1^2}{g'(H-h_{2c})^3} = 1 \qquad 3.28$$

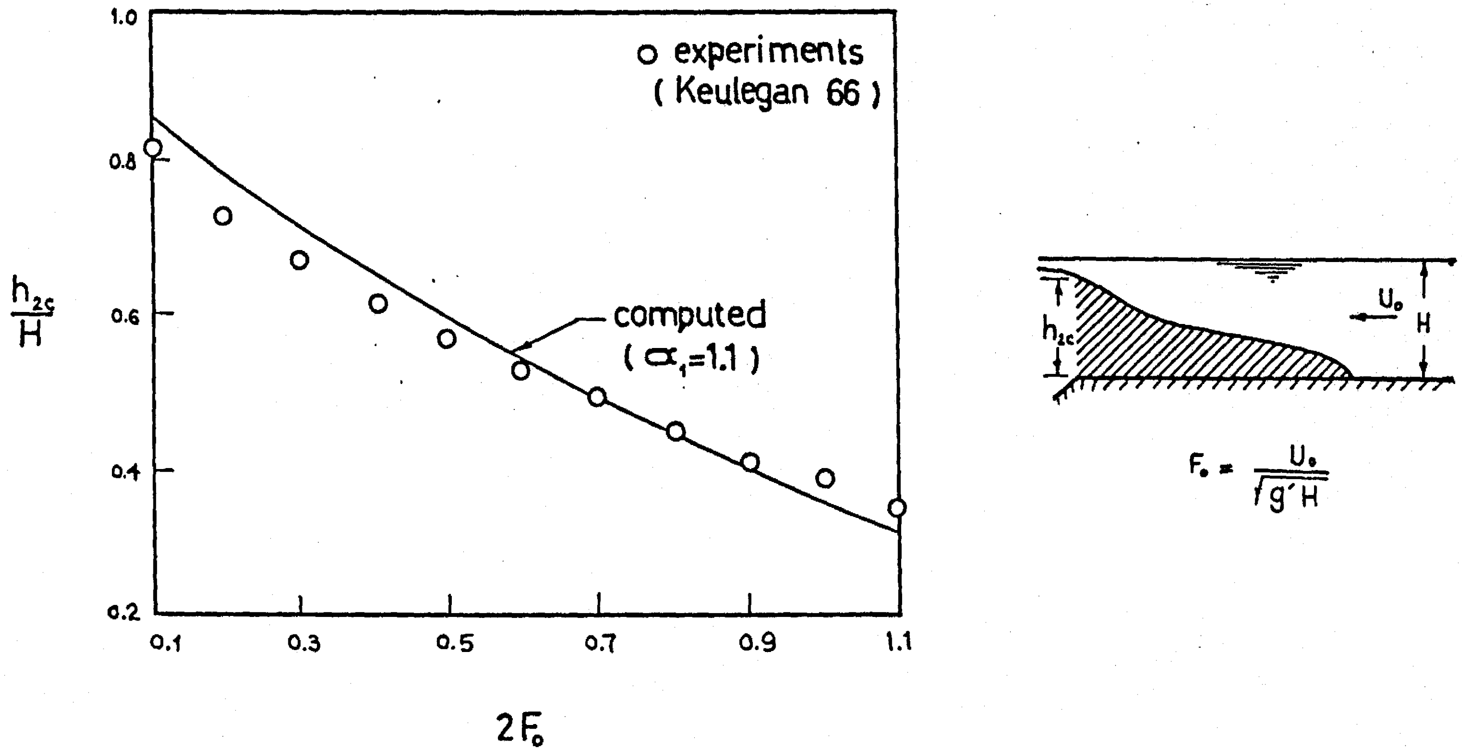


Fig. 3.3 - Depth of Salt Water at River Mouth (arrested salt wedge)
[after Keulegan (1966)]

where q_1 is the discharge of the upper layer per unit width of the flume.

This results in

$$q_1 = U_o H$$

which gives

$$\frac{\alpha_1 U_0^2 H^2}{g' H^3 (1 - h_{2c}/H)^3} = 1$$

$$\frac{\sqrt{\alpha_1} U_0}{\sqrt{g' H}} = \left[1 - \frac{h_{2C}}{H}\right]^{3/2}$$

or $$\sqrt{\alpha_1} \cdot 2 F_0 = 2 \left[1 - \frac{h_{2C}}{H}\right]^{3/2} \qquad 3.29$$

which is the same relationship derived by Keulegan using the principle of internal waves except that he neglected the effect of the non-uniformity of the velocity distribution (i.e. α_1). Using a typical velocity profile measured by Keulegan, the Author estimated the value of α_1 to be about 1.10. This value is used in the computations of Fig. 3.3.

3.3.5 Comments

a) Equation 3.5 may be written in the from

$$F_{i1}^2 = \frac{1 - F_{i2}^2}{1 - \beta F_{i2}^2} \qquad 3.30$$

In order that F_{i1} be real, either

i) $F_{i2}^2 < 1$ and $\beta F_{i2}^2 < 1$ which gives $F_{i2}^2 < 1$ because β is always less than one.

or ii) $F_{i2}^2 > 1$ and $\beta F_{i2}^2 > 1$ which gives

$$F_{i2}^2 > \frac{1}{\beta} \text{ i.e. } F_{i2}^2 > \frac{\rho_2}{\Delta\rho}$$

Similarly the same conditions may be shown to apply for F_{i2} to be real.

It may further be shown from equation 3.30 that if F_{i2}^2 is less than one, F_{i1}^2 must also be less than one because $\beta F_{i2}^2 < F_{i2}^2$.

Therefore, it may be concluded that for equation 3.5 to have real solutions, one of the following sets of conditions must be satisfied

$$F_{i1}^2 < 1 \text{ and } F_{i2}^2 < 1 \tag{3.31}$$

$$\text{or} \quad F_{i1}^2 > \frac{\rho_2}{\Delta\rho} \text{ and } F_{i2}^2 > \frac{\rho_2}{\Delta\rho} \tag{3.32}$$

b) Referring to the previous section, similar conditions are examined at the "point of virtual control". When two layers flow through a lateral contraction with an upper free surface, Wood (1968, 1969), Wood and Lai (1972, 1972A) and Lai and Wood (1975) show that there exists another control point upstream of the position of minimum width which they called "the point of virtual control". They show that at this point the following conditions hold

$$F_{i1}^2\, y_1 - F_{i2}^2\, y_2 - \beta\, y_1\, F_{i1}^2\, F_{i2}^2 = 0 \qquad 3.33$$

and

$$F_{i1}^2\, y_1 \frac{\rho_1}{\rho_2} - F_{i2}^2\, y_2 + \beta\, y_2\, F_{i1}^2\, F_{i2}^2 = 0 \qquad 3.34$$

where y_1 and y_2 are the depths of the upper layer and the lower layer respectively.

Equations 3.33 and 3.34 may be solved for F_{i1}^2 and F_{i2}^2 which yields

$$F_{i1}^2 = \frac{y_2}{y_1(\rho_1/\rho_2) + y_2} \qquad 3.35$$

and

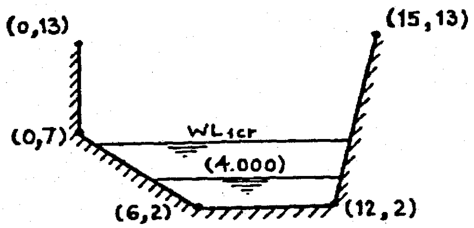
$$F_{i2}^2 = \frac{y_1}{y_1 + y_2} \qquad 3.36$$

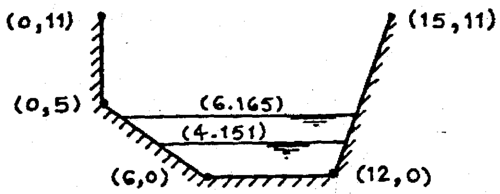
It may easily be seen from equations 3.35 and 3.36 that $F_{i1}^2 < 1$ and $F_{i2}^2 < 1$.

By substituting equations 3.35 and 3.36 in equation 3.5, it can be shown that equation 3.5 also applies at the point of virtual control.

c) In this section, the question of the solution uniqueness of equation 3.5 is discussed. From the above discussion, it is shown that equation 3.5 may have two solutions corresponding to conditions 3.31 and 3.32. Following is a typical example that shows this possibility.

Given





critical section

$\rho_1 = 1.00 \quad \rho_2 = 1.02 \quad g = 9.81\ \text{m/sec}^2 \quad \alpha_1 = 1.10 \quad \alpha_2 = 1.20$

Computed

solution I : $Q_1 = 10.000\ \text{m}^3/\text{sec}$ $Q_2 = 5.000\ \text{m}^3/\text{sec}$ $WL_{1cr} = 6.162$ m

solution II: $Q_1 = 48.962\ \text{m}^3/\text{sec}$ $Q_2 = 69.202\ \text{m}^3/\text{sec}$ $WL_{1cr} = 5.042$ m

Solution II is obtained using subroutine QCRIT12. Solution I is obtained iteratively using subroutine WLUCR.

Although the solutions are independent of the directions of flows, a typical example would be from the right cross-section to the left cross-section (into the critical section). By examining these two solutions it may be shown that

i) For solution I: $F_{i1}^2 \approx 0.587$, $F_{i2}^2 \approx 0.418$

i.e. both are less than one

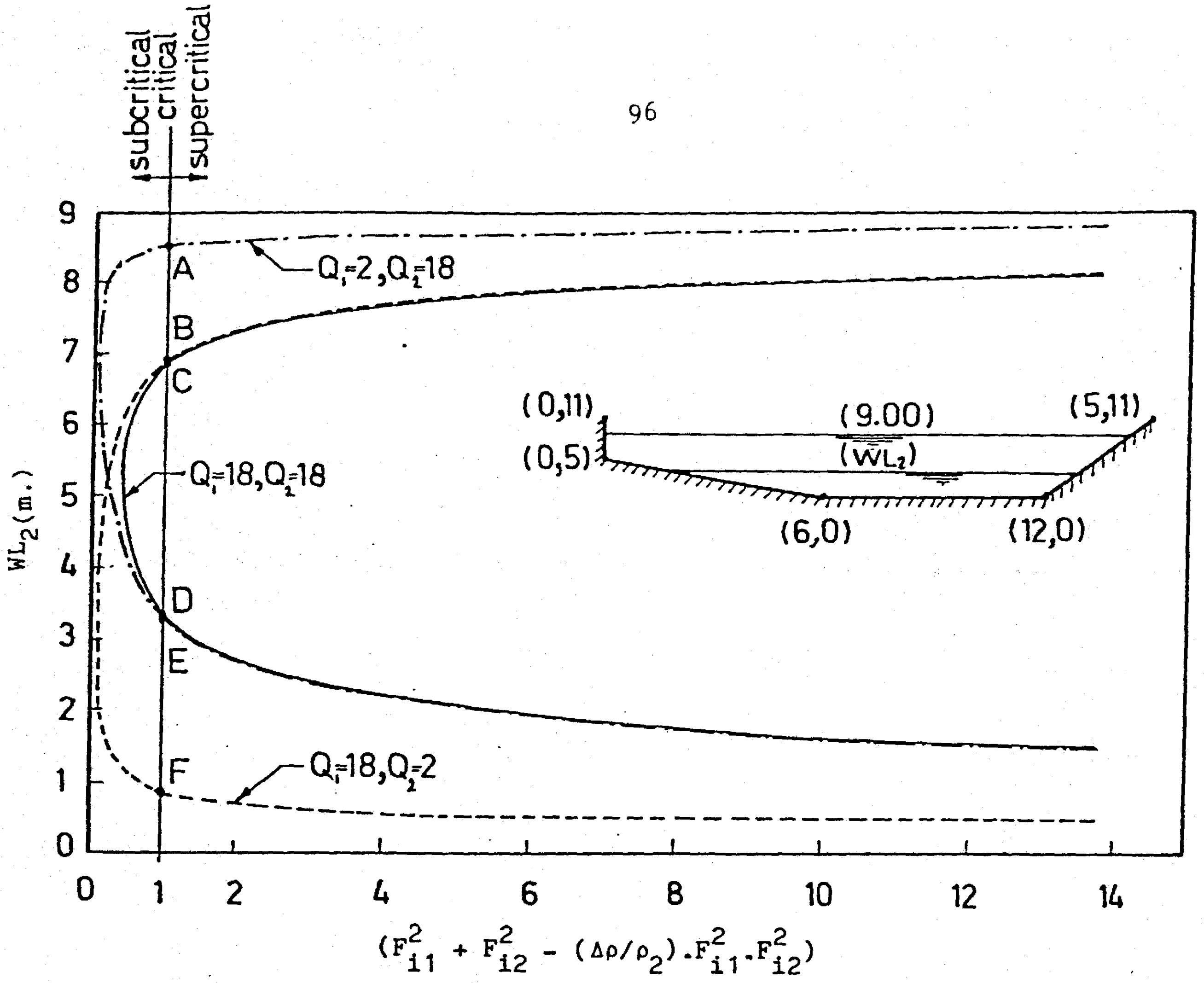
ii) For solution II: $F_{i1}^2 = 138.760$, $F_{i2}^2 = 80.056$

i.e. both are greater than $\rho_2/\Delta\rho$ (= 51 in this example)

These results confirm those of Armi (1975) who shows that equation 3.5 has two solutions. The first is given by $F_{i1}^2 + F_{i2}^2 \approx 1$ which represents critical conditions with respect to the internal long wave. The second is given by $(1 - F_{i1}^2 \cdot (\Delta\rho/\rho_2)) \cdot (1 - F_{i2}^2 \cdot (\Delta\rho/\rho_2)) \simeq 1$, which represents critical conditions with respect to the free surface long wave.

d) Consider the solution of equation 3.5 in which both densimetric Froude numbers are less than unity. For the problem analyzed by subroutine WLCRIT1, in which the critical interface elevation is computed for given discharges and free surface level, it may be shown that equation 3.5 may yield two positive roots. Each of these roots is related to the critical condition in one of the two layers. For the special cases of one stagnant layer, the solution to this problem is unique. The following example of Figure 3.22 illustrates the general case for different flow conditions. Subroutine WLCRIT1 is designed to obtain both roots by starting the search once near the free surface and the other time near the bottom of the cross-section.

In the following section, it is shown that the root observed in the experimental investigations when both layers are flowing is the one which is closer to the flume bottom (refer to Figures 3.22 p. 96 and 3.23 p. 128). This is because of the experimental set-up and the way in which the flows are started and is discussed in sub-section 3.4.7.



$\rho_1 = 1.0$, $\rho_2 = 1.02$, $\alpha_1 = 1.10$, $\alpha_2 = 1.20$, discharges in m^3/sec.

Point	Q_1(m^3/sec)	Q_2(m^3/sec)	F_{i1}^2	F_{i2}^2
A	2	18	0.969	0.032
B	18	2	0.999	0.001
C	18	18	0.930	0.071
D	18	18	0.063	0.938
E	2	18	0.001	0.999
F	18	2	0.028	0.973

Fig. 3.22 - A Typical Example of Alternative Solutions for Equation 3.5

3.3.6 Higher Order Routines

With reference to Table 3.1 (p. 79), the last group of subroutines is concerned with the anlysis of a two-layer stratified flow in a streamlined transition. The transition is defined by two cross-sections, one of which is assumed to contain critical flow. The problem may be stated in general terms as

$$\phi\ [Q_1,\ Q_2,\ WL_1,\ WL_2,\ WL_1',\ WL_2'] = 0$$

where

Q_1, Q_2 = discharges in the upper and lower layers respectively.

WL_1', WL_2' = free surface and interface elevations respectively at the critical section.

WL_1, WL_2 = free surface and interface elevations respectively at the other section,

other factors such as section and fluid properties being essentially constant.

The energy balance relationships (equations 3.2 and 3.4) as well as the critical condition (equation 3.5) allow the introduction of three dependent (i.e. unknown) quantities in each of these subroutines. Solutions are obtained either by explicit use of the lower order routines (e.g. WLUCR and QCRIT1), by iterative use of the lower order routines (e.g. QCRIT12 and QCRIT22), or by using a numerical technique for the solution of non-linear simultaneous equations (e.g. WLQCR). Listings and documentation of these routines are included in Appendix (2).

3.4 EXPERIMENTAL APPARATUS AND PROCEDURE

3.4.1 Objectives of Experiments

An experimental programme was designed and carried out to verify the numerical computations of the energy balance routines. This experimental work is concerned with the study of the flow characteristics of a two-layer system passing through a streamlined contraction. It is convenient to consider four separate sets of experiments as described below.

A) Lower layer moving - upper layer stagnant (Set I).

B) Upper layer moving - lower layer stagnant (Set II).

C) Both layers moving in the same direction (Set III).

D) Both layers moving in opposite directions (Set IV).

3.4.2 Scope and Limitation of Apparatus

The recirculating flume used for the experiments is sketched in Figure 3.4 (see also the photographs of Figures A1.1 and A1.2 (Appendix 1)). It is about 35 feet (10.67 m) long with a working cross-section 23 5/16" inches (59.2 cm) wide by 18 inches (45.7 cm) deep.

A discharge of fresh water may be introduced from either end of the flume through an entry box of plexiglass with a front screen to promote uniform flow across the width of the flume. This box is designed so that it may be used as either a discharging box or a receiving box. One of these boxes is fixed to the sides of the flume at each end and at one of three different possible levels. These boxes may also be tilted around

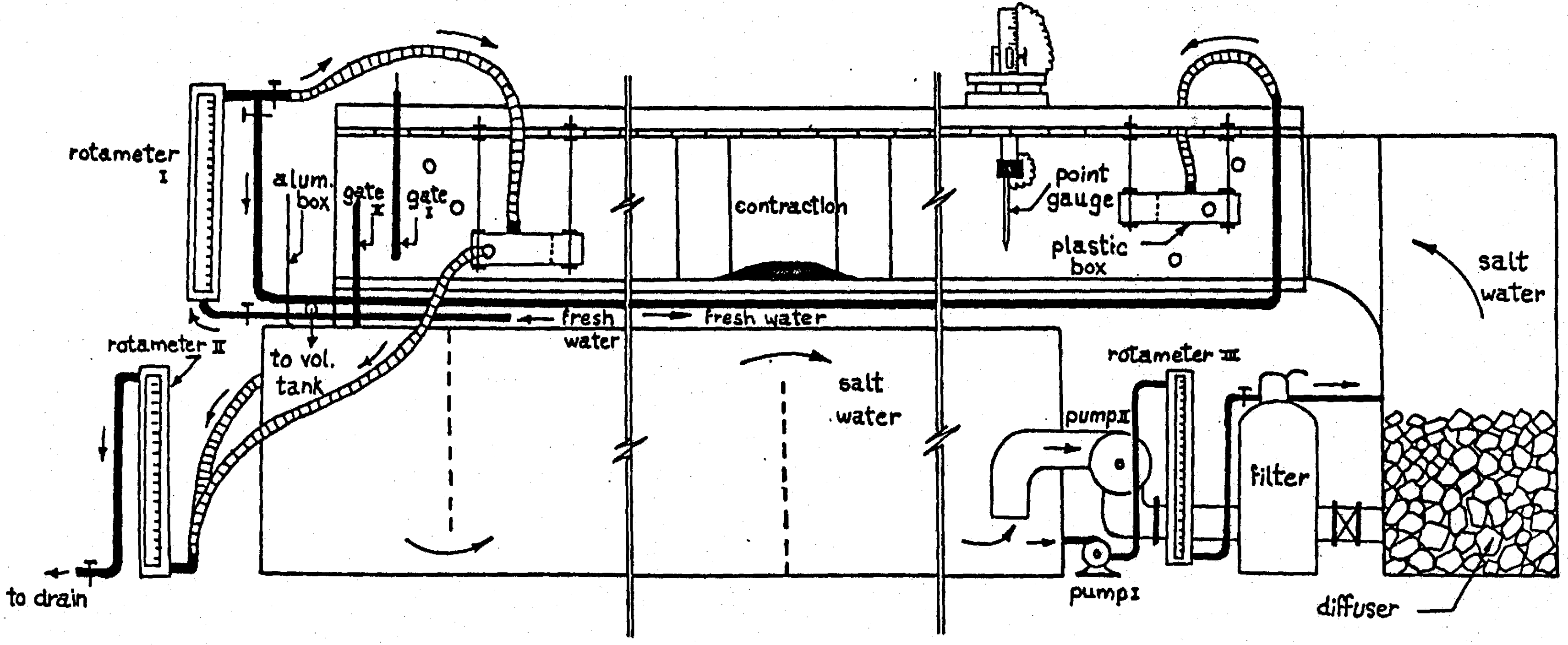
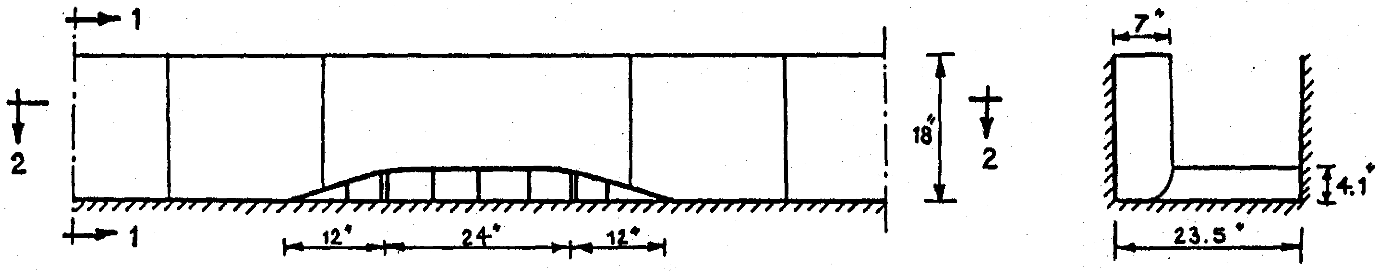


Fig. 3.4 - Sketch of the Experimental Apparatus

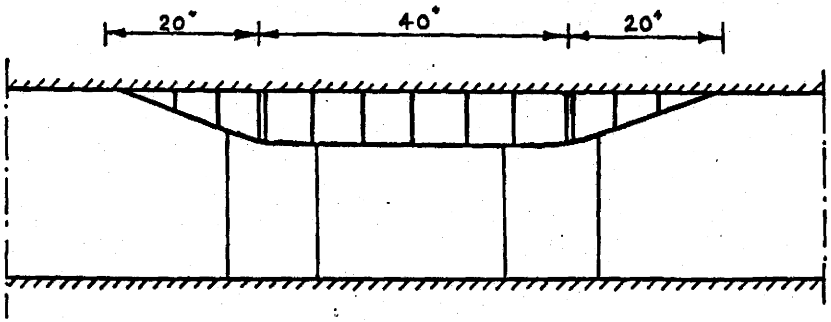
the side openings when located at any of these levels in order to adjust the edge of the box to the desired elevation. These boxes are shown in the photographs of Figures A1.3 and A1.4 (Appendix 1). The fresh water discharge may be measured at both the entry and the exit points using calibrated rotameters (rotameters I and II - Figure 3.4) which have a capacity of about 0.05 cfs (1416 cm^3/sec).

The salt water flow is pumped from a long sump using a 0.5 H.P. pump (pump I - Figure 3.4) where it can be measured using a third calibrated rotameter (rotameter III - Figure 3.4). This sump is sufficiently long [about 27 feet (8.23 m)] and is provided with vertical baffles to allow for a complete mixing of diluted return flow. The contents of the sump can be passed through a sand filter to remove any suspended impurities. At the other end of the flume, the salt water flow may be measured using a volumetric tank 30" x 30" (76.2 x 76.2 cm) in cross-section and with a working height of about 3 feet (91.4 cm), so that it has a capacity of about 18 ft^3 (0.510 m^3). The salt water is directed from the flume by a flow collecting box of aluminium and thence through a 3" (7.6 cm) diameter flexible pipe to the tank. This box is designed for a capacity of about 0.25 cfs (7079 cm^3/sec). Before flowing into the aluminium box, the salt water passes underneath gate I and then spills over gate II into the box (Figure 3.4). Gate I is made of aluminium with rubber sides and is adjusted manually. Gate II is connected to a 0.25 H.P.



(a) Longitudental section of the flume at the contraction

(b) Section 1.1



(c) Section 2.2

Fig. 3.5 - Sketch of the Flume Contraction

reversible motor so that it can be adjusted mechanically during the experiment. This arrangement is shown by the photograph of Figure A1.4 (Appendix 1).

The contraction used consists of two plastic streamlined inserts, one of them being fixed to the bottom of the flume to form a vertical contraction and the other is fixed to one side of the flume to form a horizontal (lateral) contraction. The dimensions of these inserts are shown on the sketch of Figure 3.5 (see also the photograph of Figure A1.5, Appendix 1).

In all the experiments, the two-layer system consists of a fresh water layer and a salt water layer which is dyed with Uranine that gives to it a green colour with a flourescent surface to help distinguish the interface clearly. While the colour is green with reflected light, it appears to change to yellow with transmitted light (See the photographs of Appendix (1)). The salt water density may be adjusted by adding common fine salt to the sump and circulating salt water using a 15 H.P. pump (pump II - Figure 3.4).

Densities (or specific gravities) are measured using a hydrometer of 0.0005 division. Levels are measured using a point gauge of 0.01 inch (0.025 cm) division. For discharge measurements, fresh water flow is measured at both inlet and exit points using two rotameters with an accuracy of about 0.0001 cfs (2.83 cm^3/sec). The exit rotameter (rotameter II in Figure 3.4) is mounted on a moveable stand which can be moved from one end of the flume to the other. Salt water discharge is measured at the inlet point using a rotameter with an accuracy of 0.0001 cfs (2.83 cm^3/sec). At the exit, salt water flow is measured using a volumetric tank which has a scale of 0.05 ft^3 division and a stop watch of 0.1 second division.

Free surface and interface levels are measured at three stations along the flume. Station (1) is about 10 ft (3 m) upstream of the centre of the flume where the flume bed level is -0.19 inches (-0.48 cm). Station (2) is at the centre of the flume and also in the middle of the contraction. The bed level at station (2) is 3.91 inches (9.93 cm). Station (3) is about 10 ft (3 m) downstream of station (2) where the bed level is -0.19 inches (-0.48 cm). Upstream and downstream directions are defined with respect to the direction of salt water flow which is the same for all experiments.

3.4.3 Description and Results of Experimental Set I

3.4.3.1 Procedure

The experimental arrangement for this set is shown in Figure 3.6 and the sequence of operations to establish steady flow is illustrated in Figure 3.7. The flow pattern of salt water through the contraction is shown by the photograph of Figure A1.6 (Appendix 1). Salt water discharge is measured at the inlet using rotameter III (Figure 3.4) and at the outlet using the volumetric tank.

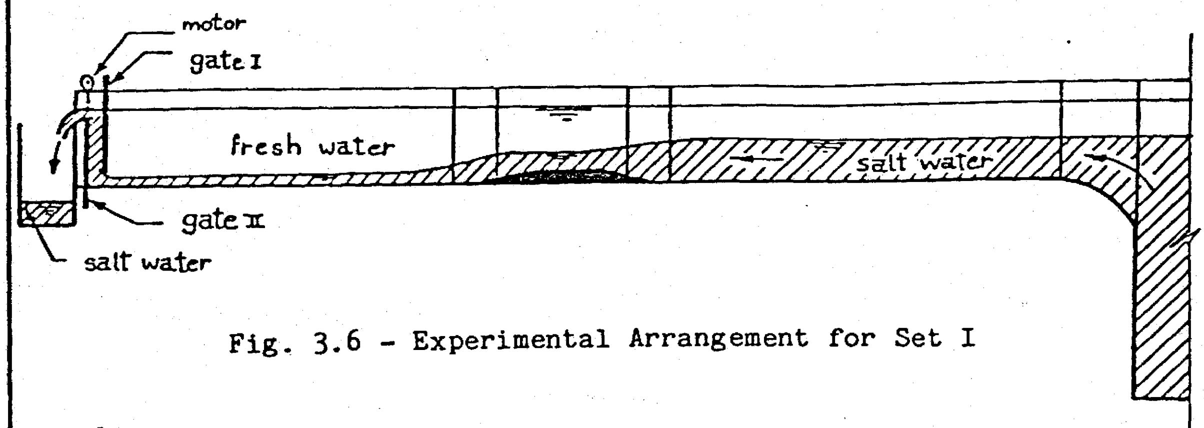


Fig. 3.6 - Experimental Arrangement for Set I

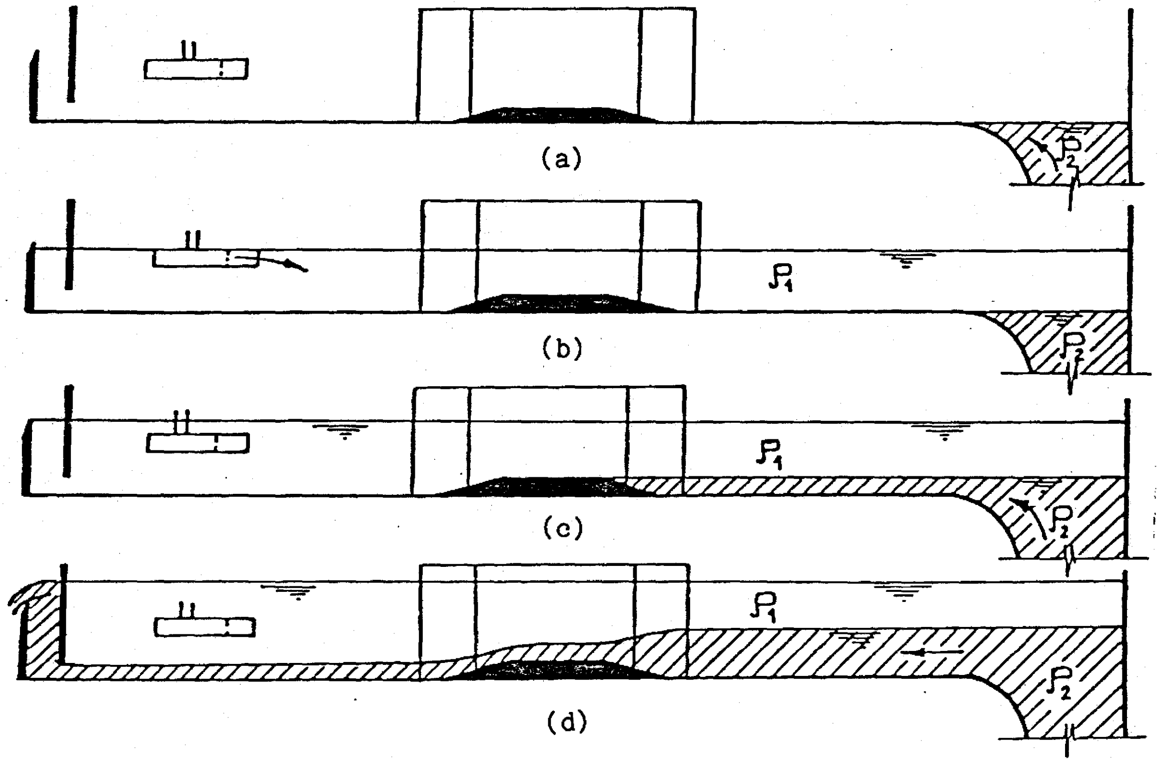


Fig. 3.7 - Sequence of Operations in Establishing Steady Flow for Set I.

(a) entry tank filled with salt water.
(b) flume filled with fresh water up to crest of downstream weir.
(c) salt water pool established upstream of transition.
(d) salt water underflow established.

3.4.3.2 Results

The results of this set which consists of four experiments are presented in Table 3.2.

The measured quantities are:

a) specific gravaity of fresh water (ρ_1)

b) specific gravity of salt water (ρ_2)

c) salt water discharge at inlet in cfs (Q_2)

d) salt water discharge at exit in cfs (Q_2')

e) free surface elevation at stations 1, 2 and 3 (WL_{11}, WL_{12}, WL_{13})

f) interface elevation at stations 1, 2, and 3 (WL_{21}, WL_{22}, WL_{23})

Table 3.2 also shows the computed values of WL_{11}, WL_{21} and WL_{22} using subroutines WLCRIT1 and BERNWL2 given the flume cross-section geometry at stations (1) and (2), $Q_1 = 0$, Q_2, WL_{12}, ρ_1, ρ_2 and α_2 where α_2 is the kinetic energy correction factor for the salt water flow. Figures 3.8 and 3.9 show the comparison between measured and computed values of WL_{21} and WL_{22} for values of 1.00 and 1.70 for α_2 respectively. The choice of the 1.70 value for α_2 is discussed in Section 3.4.4.

3.4.4 Description and Results of Experimental Set II

3.4.4.1 Procedure

The experimental arrangement for this set is shown in Figure 3.10 and the sequence of operations to establish steady flow is illustrated in Figure 3.11. A long view of the flow pattern is shown in the photograph of Figure A1.9 (Appendix 1).

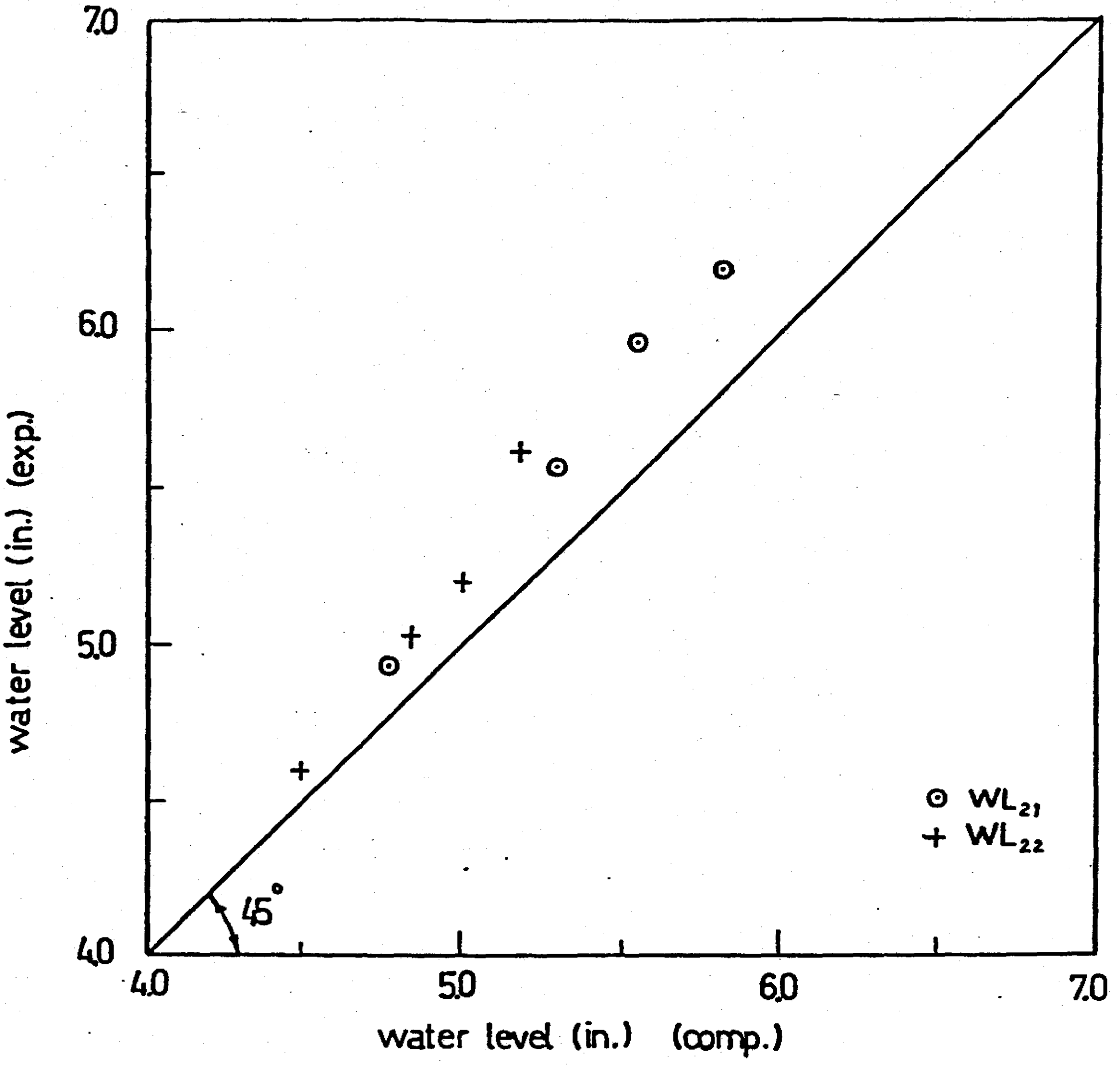
	Expt.	1	2	3	4
Measured	Gate I opening (in)	1.00	1.00	1.00	1.00
	Gate II height (in)	9.00	9.00	9.00	9.00
	ρ_1	0.9982	0.9982	0.9982	0.9982
	ρ_2	1.0137	1.0130	1.0262	1.0262
	Q_2 (cfs)	0.0100	0.0200	0.0350	0.0440
	V_i (ft^3)	1.07	2.00	4.12	4.12
	V_f (ft^3)	2.00	3.97	1.23	7.27
	t (sec.)	92.80	98.70	82.55	71.60
	Q_2' (cfs)	0.0100	0.0200	0.0350	0.0440
	WL_{11} (in)	9.12	9.18	9.30	9.36
	WL_{12} (in)	9.12	9.18	9.30	9.36
	WL_{13} (in)	9.12	9.18	9.30	9.36
	WL_{21} (in)	4.93	5.56	5.96	6.19
	WL_{22} (in)	4.60	5.02	5.20	5.61
	WL_{23} (in)	1.70	3.74	3.87	4.98
Computed α_2=1.00	WL_{11} (in)	9.12	9.18	9.30	9.36
	WL_{21} (in)	4.77	5.29	5.54	5.80
	WL_{22} (in)	4.48	4.84	5.00	5.18
Computed α_2=1.70	WL_{11} (in)	9.12	9.18	9.30	9.36
	WL_{21} (in)	4.94	5.56	5.85	6.17
	WL_{22} (in)	4.60	5.02	5.21	5.43

V_i = initial volume in volumetric tank

V_f = final volume in volumetric tank

t = collection time $\qquad Q_2' = (V_f - V_i)/t$

Table 3.2 Measured and Computed Results of Experimental Set I



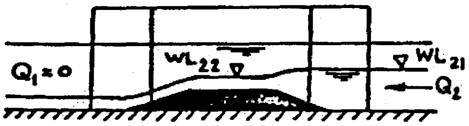


Fig. 3.8 - Comparison of Measured and Computed Water Levels for Set I (α_2 = 1.00)

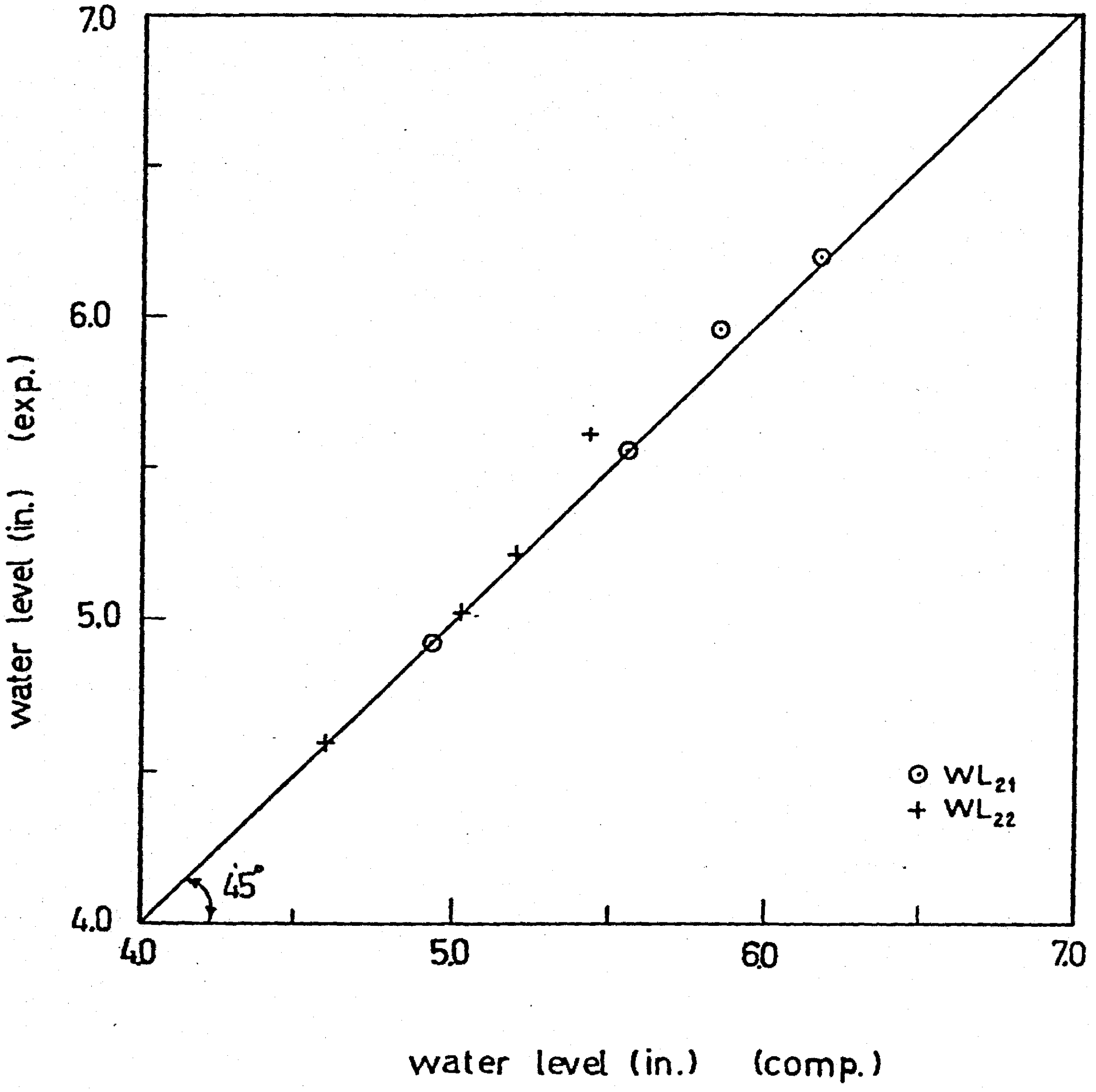


Fig. 3.9 - Comparison of Measured and Computed Water Levels for Set I (α_2 = 1.70)

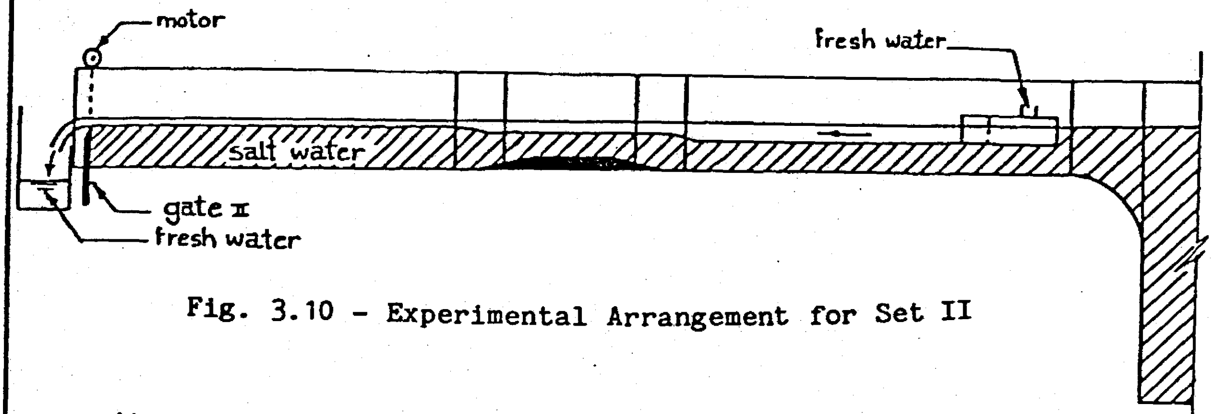
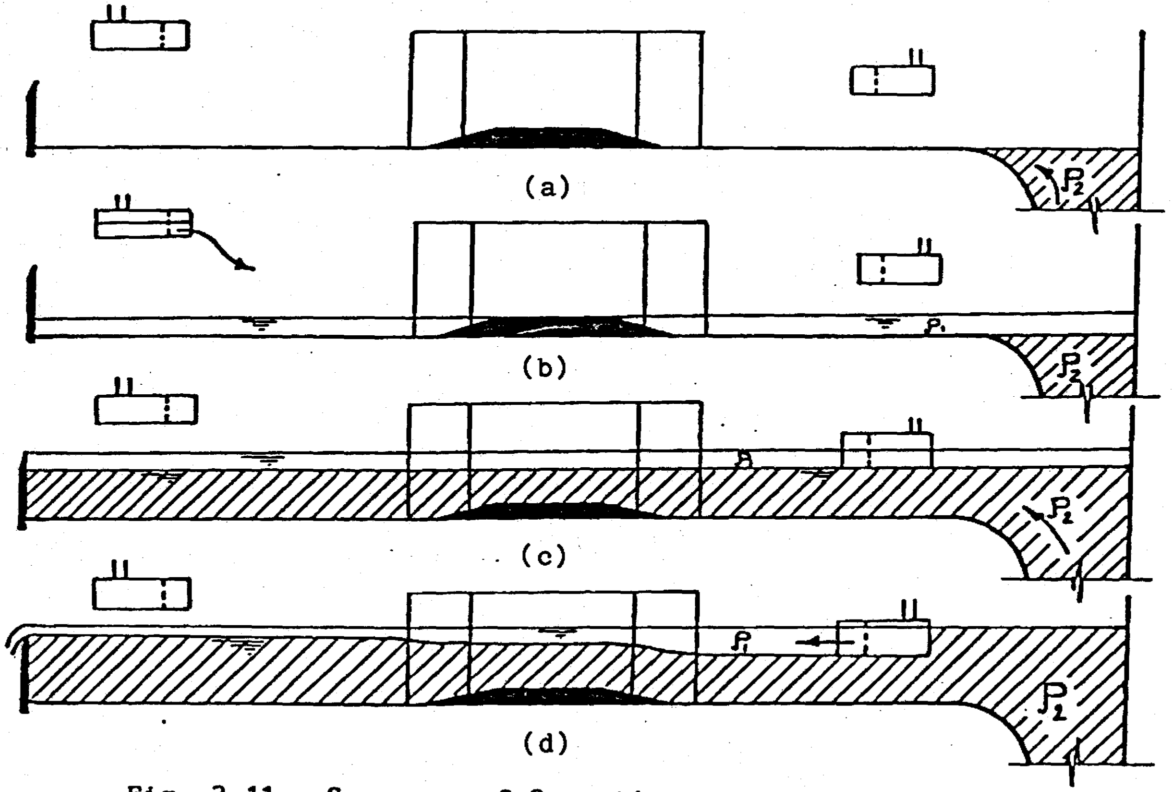


Fig. 3.10 - Experimental Arrangement for Set II



(a)

(b)

(c)

(d)

Fig. 3.11 - Sequence of Operations in Establishing Steady Flow for Set II

(a) entry tank filled with salt water.
(b) flume filled with fresh water to crest of middle weir.
(c) salt water pool established to bottom level of entry box.
(d) fresh water overflow established.

The photograph of Figure A1.10 (Appendix 1) shows the entry box where fresh water flow enters the flume.

Fresh water discharge is measured at the inlet using rotameter I (Figure 3.4) and at the exit using the volumetric tank.

3.4.4.2 Results

The results of this set which consists of four experiments are presented in Table. 3.3.

The measured quantities are:

a) specific gravity of fresh water (ρ_1)

b) specific gravity of salt water (ρ_2)

c) fresh water discharge at inlet in cfs (Q_1)

d) fresh water discharge at exit in cfs (Q_1')

e) free surface elevations at stations 1, 2 and 3 (WL_{11}, WL_{12}, WL_{13})

f) interface elevations at stations 1, 2 and 3 (WL_{21}, WL_{22}, WL_{23})

Table 3.3 also shows the computed values of WL_{11}, WL_{21} and WL_{22} using subroutines WLCRIT1 and BERNWL2 given the flume cross-section geometry at stations (1) and (2), $Q_2 = 0$, Q_1, WL_{12}, ρ_1, ρ_2 and α_1 where α_1 is the kinetic energy correction factor for the fresh water flow. Figures 3.12 and 3.13 show the comparison between measured and computed values of WL_{21} and WL_{22} for values of 1.00 and 1.30 for α_2 respectively. The choice of the 1.30 value for α_1 is discussed in Section 3.4.4.

3.4.5 Description and Results of Experimental Set III

3.4.5.1 Procedure

The experimental arrangement for this set is shown in Figure 3.14 and the sequence of operations to establish steady flow is illustrated in Figure 3.15. A long view of the flow pattern is shown in the photograph of Figure A1.11 (Appendix 1). The flow profile at the contraction is shown in the photograph of Figure A1.12 (Appendix 1). Conditions at both the entry box and the exit box are shown in Figures A1.15 and A1.16 (Appendix 1) respectively.

Fresh water discharge is measured at the inlet using rotameter I and at the exit using rotameter II (Figure 3.4). Salt water flow is measured at the inlet using rotameter III and at the outlet using the volumetric tank.

3.4.5.2 Results

The results of this set which consists of five experiments are presented in Table 3.4.

The measured quantities are:

a) specific gravity of fresh water (ρ_1)

b) specific gravity of salt water (ρ_2)

c) fresh water discharge at inlet in cfs (Q_1)

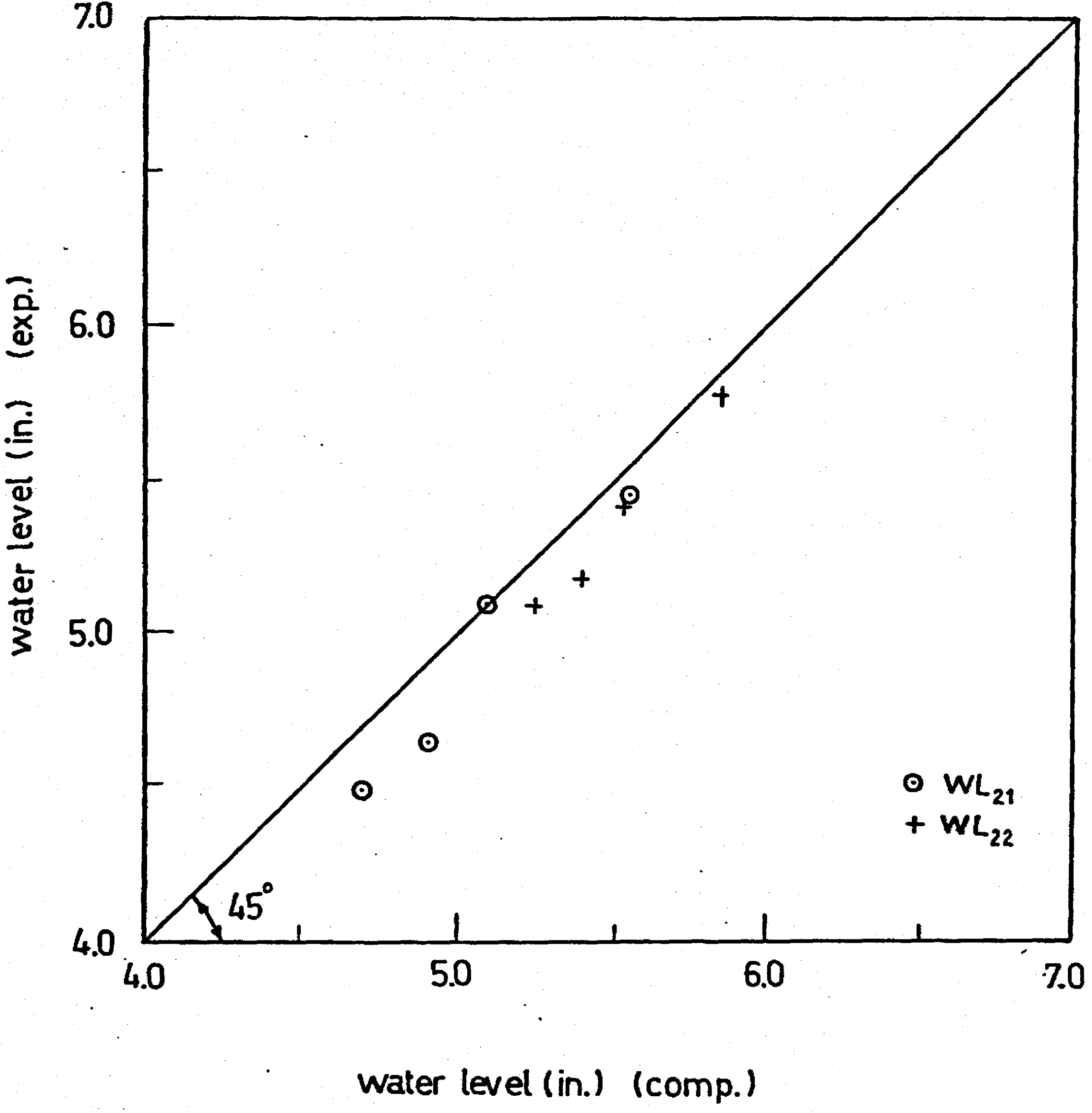
d) fresh water discharge at exit in cfs (Q_1')

e) salt water discharge at inlet in cfs (Q_2)

f) salt water discharge at exit in cfs (Q_2')

	Expt.	1	2	3	4
Measured	Gate II height (in.)	6 9/16	6 9/16	6 9/16	6 9/16
	ρ_1	0.9992	0.9992	0.9990	0.9990
	ρ_2	1.0210	1.0210	1.0155	1.0155
	Q_1 (cfs)	0.0196	0.0490	0.0290	0.0359
	V_i (ft3)	1.02	2.83	1.87	3.12
	V_f (ft^3)	2.83	5.96	3.12	5.46
	t (sec)	94.20	63.80	43.15	65.25
	Q_1' (cfs)	0.0192	0.0491	0.0290	0.0359
	WL_{11} (in.)	6.64	6.72	6.66	6.71
	WL_{12} (in.)	6.64	6.72	6.66	6.71
	WL_{13} (in.)	6.64	6.72	6.66	6.71
	WL_{21} (in.)	5.45	4.49	5.13	4.64
	WL_{22} (in.)	5.77	5.09	5.41	5.18
	WL_{23} (in.)	5.89	4.93	5.45	5.25
Computed $\alpha_1=1.00$	WL_{11} (in.)	6.65	6.73	6.67	6.72
	WL_{21} (in.)	5.55	4.70	5.10	4.91
	WL_{22} (in.)	5.84	5.24	5.52	5.39
Computed $\alpha_1=1.30$	WL_{11} (in.)	6.65	6.73	6.67	6.72
	WL_{21} (in.)	5.45	4.52	4.96	4.75
	WL_{22} (in.)	5.77	5.10	5.41	5.27
Q_1 used in computations (cfs)		0.0194	0.0490	0.0290	0.0359

Table 3.3 Measured and Computed Results of Experimental Set II



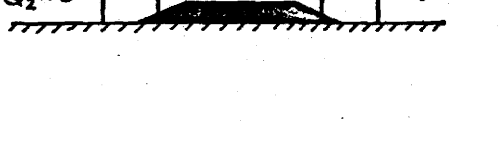


Fig. 3.12 - Comparison of Measured and Computed Water Levels for Set II (α_1 = 1.00)

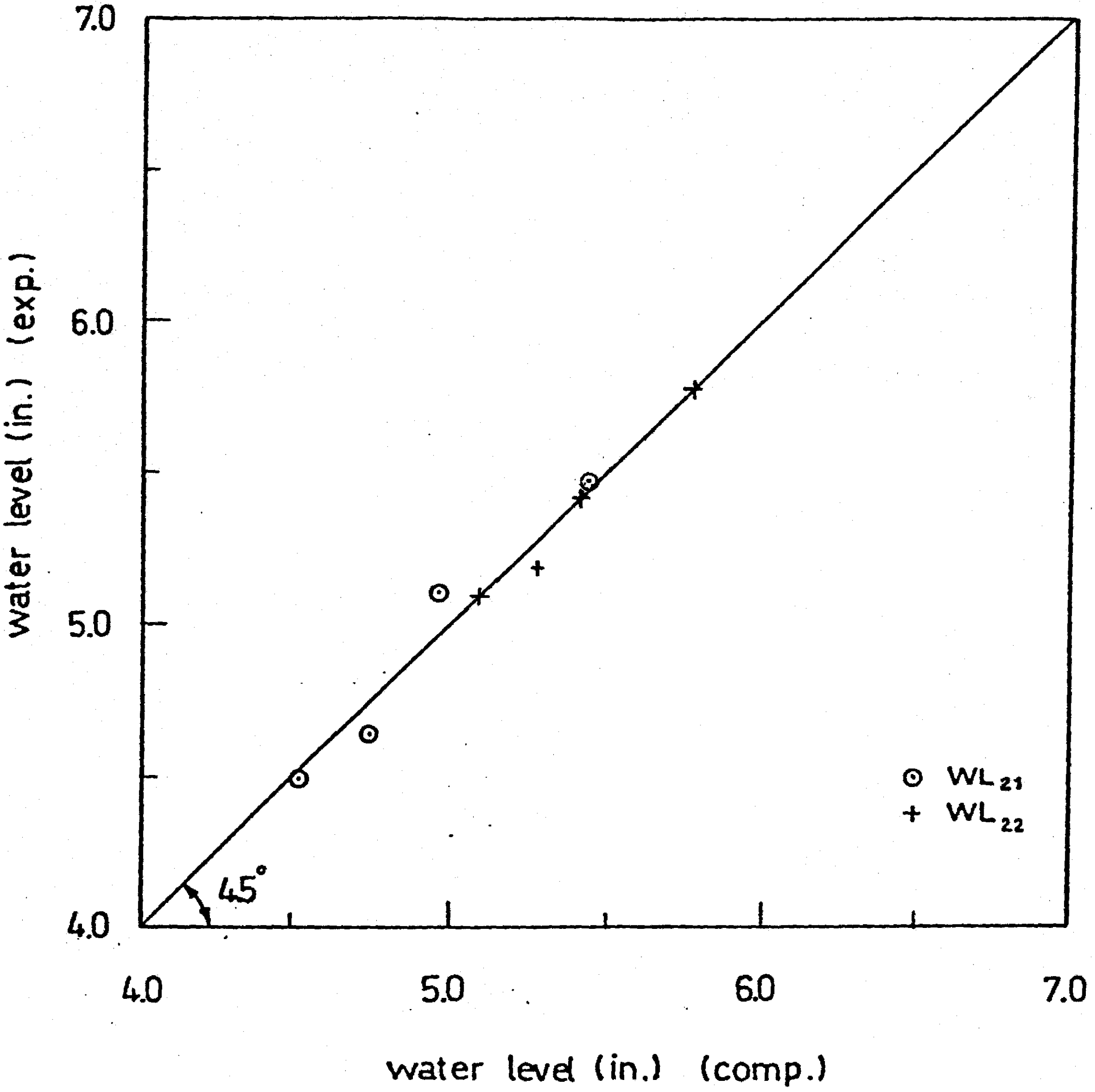


Fig. 3.13 - Comparison of Measured and Computed Water Levels for Set II (α_1 = 1.30)

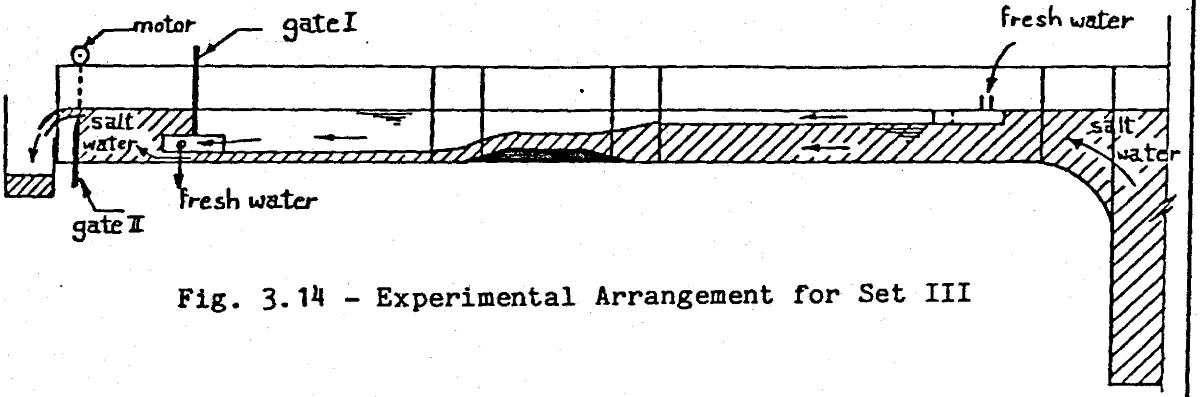


Fig. 3.14 - Experimental Arrangement for Set III

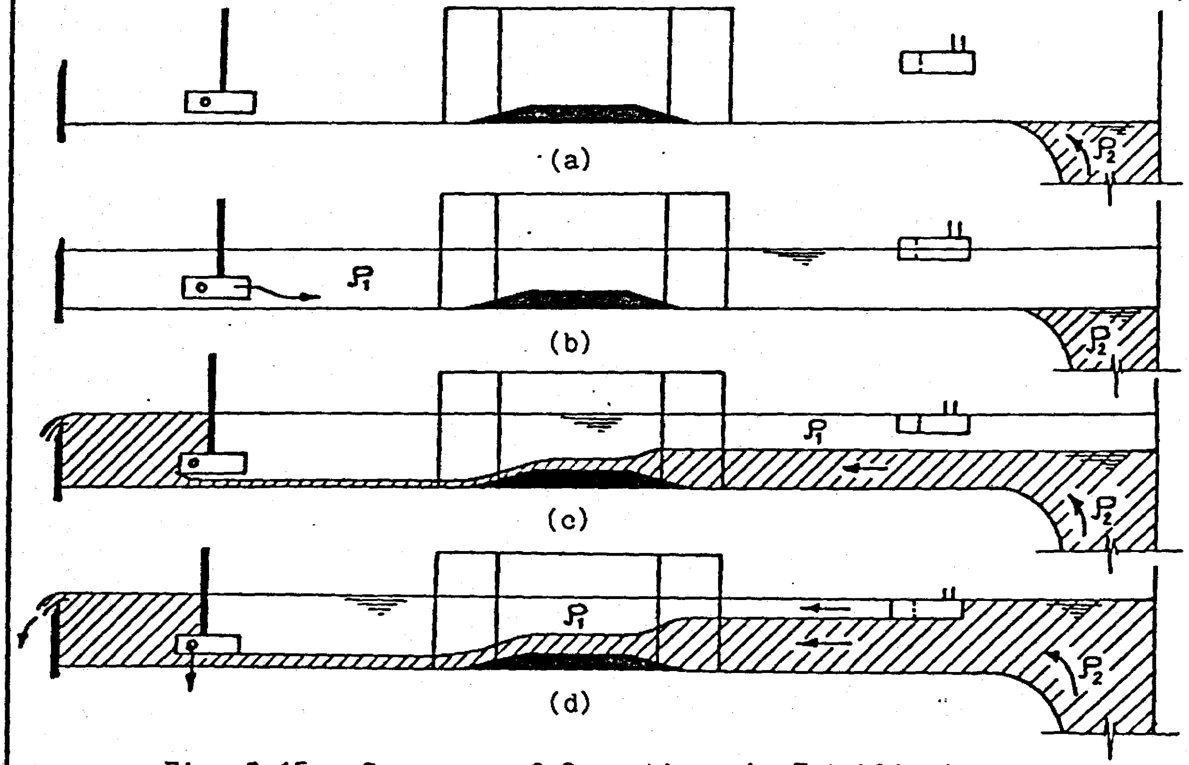


Fig. 3.15 - Sequence of Operations in Establishing Steady Flow for Set III

(a) entry tank filled with salt water.
(b) flume filled with fresh water up to crest of downstream weir.
(c) salt water pool established upstream of transition, then, salt water underflow established.
(d) fresh water overflow established in the same direction.

g) free surface elevations at stations 1, 2 andd 3 (WL_{11}, WL_{12}, WL_{13})

h) interface elevations at stations 1, 2 and 3 (WL_{21}, WL_{22}, WL_{23})

Table 3.4 also shows the computed values of WL_{11}, WL_{21} and WL_{22} using subroutines WLCRIT1 and BERNWL2 given the flume cross-section geometry at stations (1) and (2), Q_1, Q_2, WL_{12}, ρ_1, ρ_2, α_1 and α_2. Figure 3.16 and 3.17 show the comparison between measured and computed values of WL_{21} and WL_{22} for $\alpha_1 = \alpha_2 = 1.00$ and for $\alpha_1 = 1.30$, $\alpha_2 = 1.70$ respectively.

3.4.6 Description and Results of Exerimental Set IV

3.4.6.1 Procedure

The experimental arrangement for this set is shown in Figure 3.18 and the sequence of operations to establish steady flow is illustrated in Figure 3.19. A long view of the flow pattern is shown in the photograph of Figure A1.17 (Appendix 1). The flow profile at the contraction is shown in the photograph of Figure A1.18 and the velocity profile at that section is illustrated in Figure A1.19 (Appendix 1) using Potassium Permanganate crystals. Conditions at both the downstream box and the upstream box are shown in Figures A1.20 and A1.21 (Appendix 1) respectively.

Fresh water discharge is measured at the inlet using rotameter I and at the exit using rotameter II (Figure 3.4). Salt water flow is measured at the inlet using rotameter III and at the outlet using the volumetric tank.

	Expt.	1	2	3	4	5
Measured	Gate II height (in)	8.0	7 19/32	8.0	8.0	8.0
	ρ_1	0.9982	0.9980	0.9980	0.9988	0.9990
	ρ_2	1.0193	1.0205	1.0197	1.0190	1.0157
	Q_1 (cfs)	0.0200	0.0101	0.0508	0.0300	0.0401
	Q_1' (cfs)	0.0200	0.0100	0.0508	0.0300	0.0401
	Q_2 (cfs)	0.0199	0.0407	0.0100	0.0400	0.0300
	V_i (ft^3)	1.39	1.48	0.82	1.10	1.77
	V_f (ft3)	3.52	5.22	2.00	3.05	3.50
	t (sec)	107.00	91.90	118.00	48.80	57.70
	Q_2' (cfs)	0.0199	0.0407	0.0100	0.0400	0.0300
	WL_{11} (in)	8.24	7.91	8.15	8.35	8.26
	WL12 (in)	8.24	7.91	8.15	8.35	8.26
	WL_{13} (in)	8.24	7.91	8.15	8.35	8.26
	WL_{21} (in)	5.30	6.13	4.75	6.20	5.90
	WL_{22} (in)	4.86	5.49	4.58	5.51	5.36
	WL_{23} (in)	2.23	4.00	1.51	2.63	2.19
Computed $\alpha_1=\alpha_2=1.00$	WL_{11} (in)	8.24	7.91	8.15	8.35	8.26
	WL_{21} (in)	5.13	5.84	4.62	5.87	5.61
	WL_{22} (in)	4.74	5.21	4.44	5.26	5.11
Computed $\alpha_1=1.30$ $\alpha_2=1.70$	WL_{11} (in)	8.24	7.91	7.91	8.35	8.26
	WL_{21} (in)	5.36	6.21	4.75	6.25	5.93
	WL_{22} (in)	4.90	5.46	4.55	5.54	5.38
Q_1 used in computations (cfs)		0.0200	0.0101	0.0508	0.0300	0.0401
Q_2 used in computations (cfs)		0.0199	0.0407	0.0100	0.0400	0.0300

Table 3.4 Measured and Computed Results of Experimental Set III

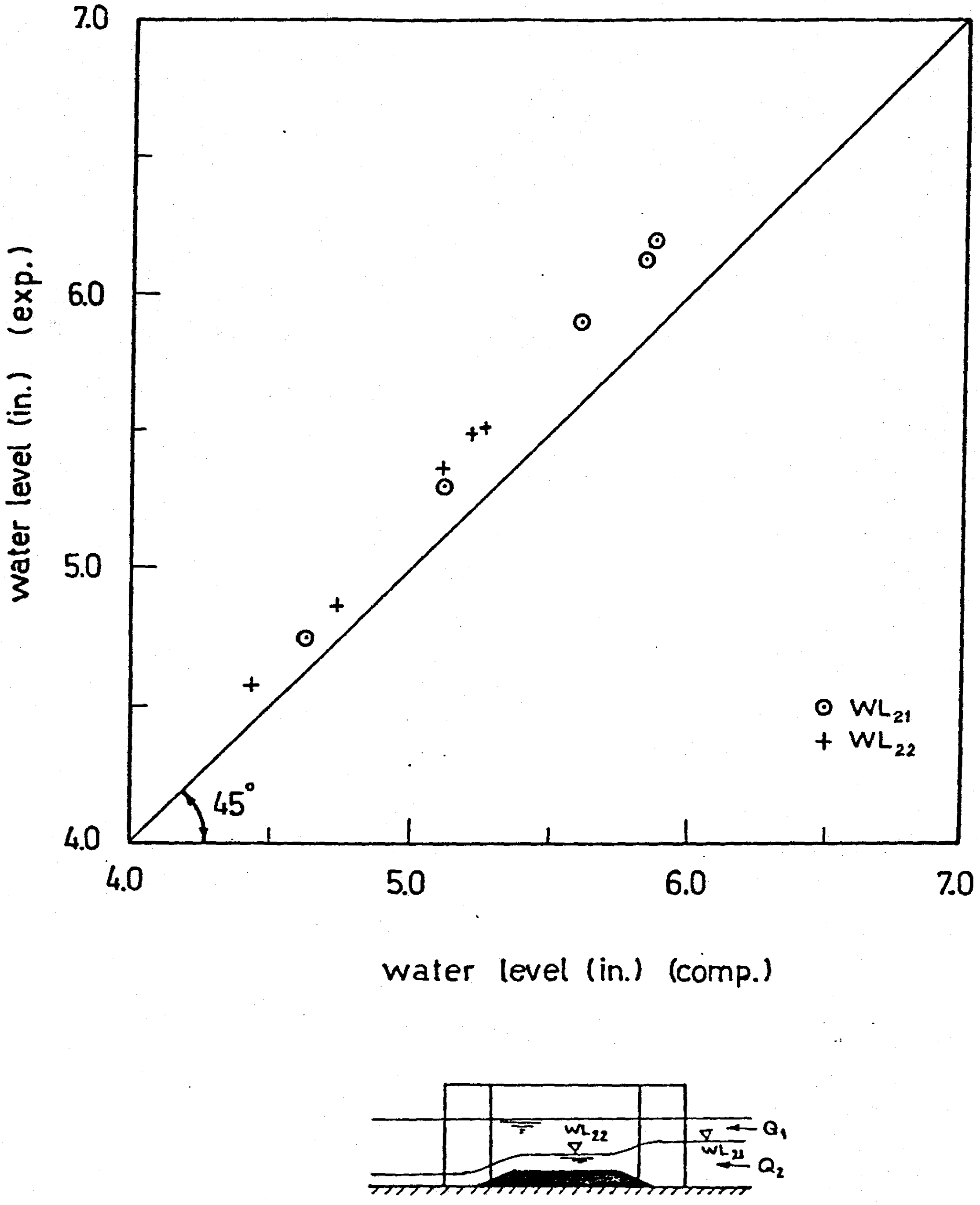


Fig. 3.16 - Comparison of Measured and Computed Water Levels for Set III ($\alpha_1=\alpha_2=1.00$)

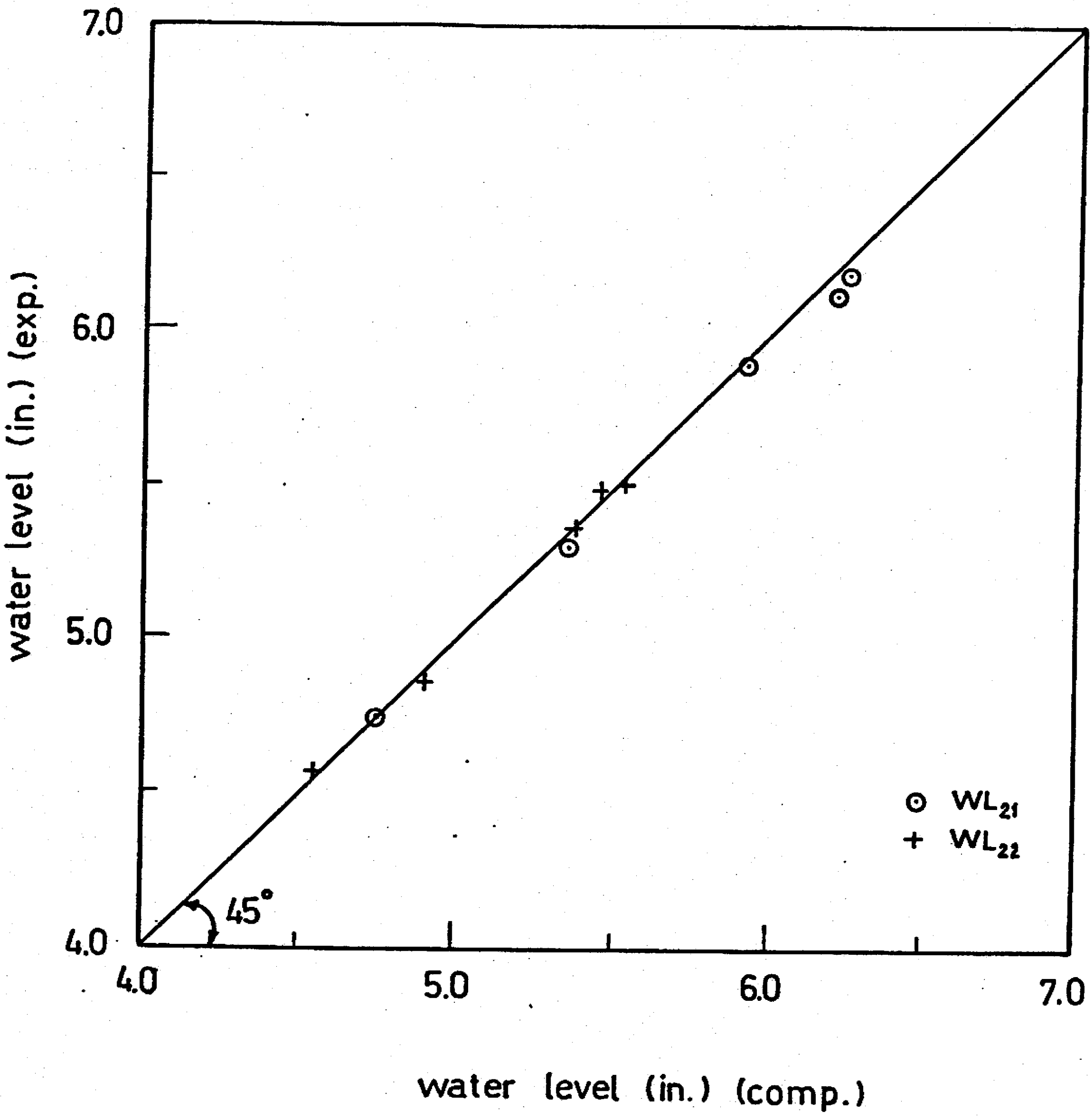


Fig. 3.17 - Comparison of Measured and Computed Water Levels for Set III (α_1=1.30-α_2=1.70)

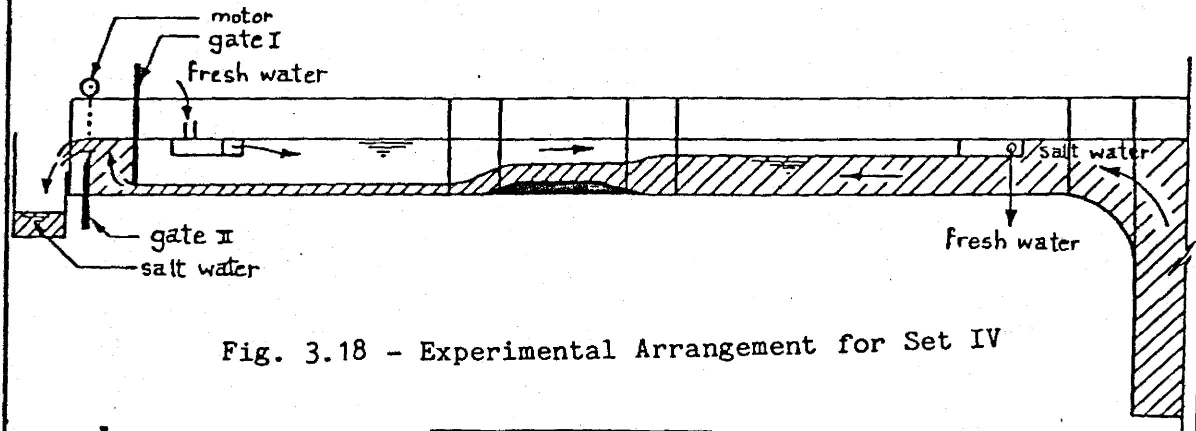


Fig. 3.18 - Experimental Arrangement for Set IV

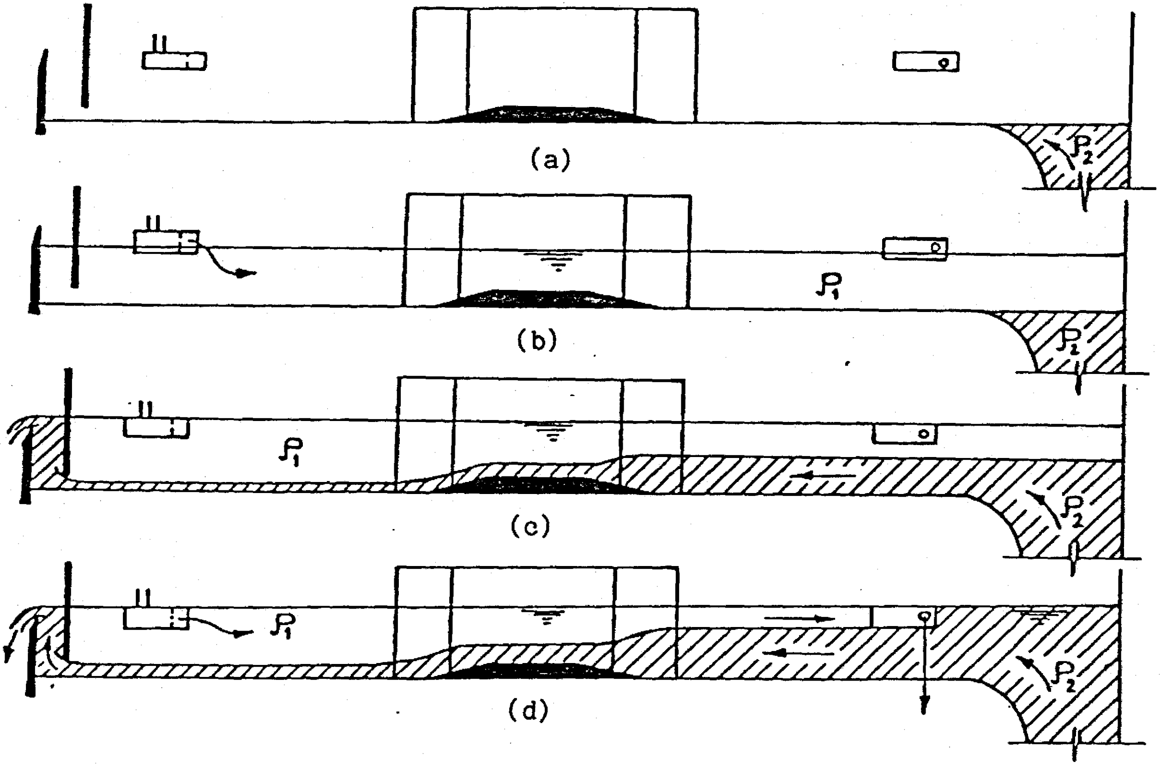


Fig. 3.19 - Sequence of Operations in Establishing Steady Flow for Set IV

(a) entry tank filled with salt water.

(b) flume filled with fresh water up to crest of downstream weir.

(c) salt water pool established upstream of transition, then, salt water underflow established.

(d) fresh water overflow established in opposite direction.

3.4.6.2 Results

The results of this set which consists of five experiments are presented in Table 3.5.

The measured quantities are the same as those of set III (Section 3.4.5.2).

Table 3.5 also shows the computed values of WL_{11}, WL_{21} and WL_{22} using subroutines WLCRIT1 and BERNWL2 given the flume cross-section geometry at stations (1) and (2), Q_1, Q_2, WL_{12}, ρ_1, ρ_2, α_1 and α_2. Figures 3.20 and 3.21 show the comparison between measured and computed values of WL_{21} and WL_{22} for $\alpha_1 = \alpha_2 = 1.00$ and for $\alpha_1 = 1.30$ and $\alpha_2 = 1.70$ respectively.

3.4.7 Discussion

1. In the experimental sets I, III and IV, due to the relatively high velocity of the salt water layer at the downstream part of the vertical contraction, entrainment of the fresh water by the salt water flow occurs. This is illustrated in the photographs of Figures A1.7 and A1.8 (Appendix 1). Subsequently the depth of the salt water layer downstream of the contraction increases gradually until the interface elevation downstream gets closer to the interface elevation in the contraction. At this stage it starts to affect the upstream conditions and the control does not exist anymore. However, this process is very slow due to the relatively low flows used in these experiments which makes it possible to obtain all measurements for a

	Expt.	1	2	3	4	5
Measured	Gate I opening (in)	1.0	1.0	1.0	1.0	1.0
	Gate II height (in)	8 11/31	7.5	8 5/16	8.0	8.0
	ρ_1	0.9982	0.9985	0.9985	0.9982	0.9982
	ρ_2	1.0140	1.0200	1.0198	1.0178	1.0135
	Q_1 (cfs)	0.0200	0.0101	0.0508	0.0301	0.0401
	Q_1' (cfs)	0.0200	0.0101	0.0508	0.0299	0.0401
	Q_2 (cfs)	0.0200	0.0410	0.0100	0.0400	0.0300
	V_i (ft^3)	1.25	1.43	1.06	1.29	1.87
	V_f (ft^3)	3.02	5.15	3.73	2.76	3.53
	t (sec)	88.50	90.75	267.00	36.80	55.30
	Q_2' (cfs)	0.0200	0.0410	0.0100	0.0399	0.0300
	WL_{11} (in)	8.36	7.89	8.46	8.30	8.31
	WL_{12} (in)	8.36	7.89	8.46	8.30	8.31
	WL_{13} (in)	8.36	7.89	8.46	8.30	8.31
	WL_{21} (in)	5.53	6.26	4.82	6.29	6.05
	WL_{22} (in)	5.01	5.50	4.58	5.57	5.43
	WL_{23} (in)	3.25	3.76	1.53	3.48	2.20
Computed $\alpha_1=\alpha_2=1.00$	WL_{11} (in)	8.36	7.89	8.46	8.30	8.31
	WL_{21} (in)	5.25	5.88	4.64	5.89	5.65
	WL_{22} (in)	4.82	5.24	4.44	5.27	5.15
Computed $\alpha_1=1.30$ $\alpha_2=1.70$	WL_{11} (in)	8.36	7.89	8.46	8.30	8.31
	WL_{21} (in)	5.51	6.26	4.77	6.28	5.99
	WL_{22} (in)	5.00	5.50	4.54	5.56	5.43
Q_1 used in computations (cfs)		0.0200	0.0101	0.0508	0.0300	0.0401
Q_2 used in computations (cfs)		0.0200	0.0410	0.0100	0.0400	0.0300

Table 3.5 Measured and Computed Results of Experimental Set IV

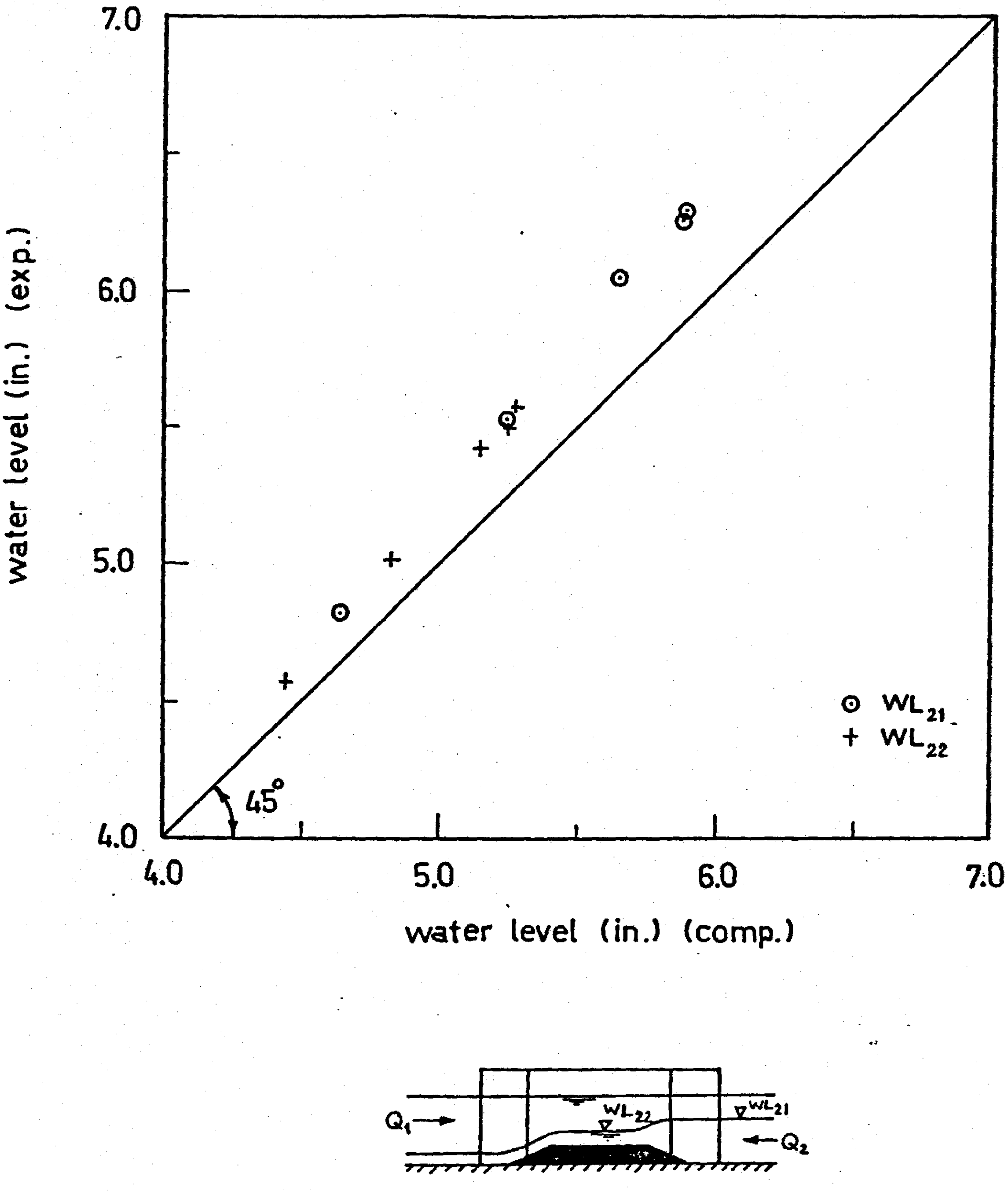


Fig. 3.20 - Comparison of Measured and Computed Water Levels for Set IV ($\alpha_1 = \alpha_2 = 1.00$)

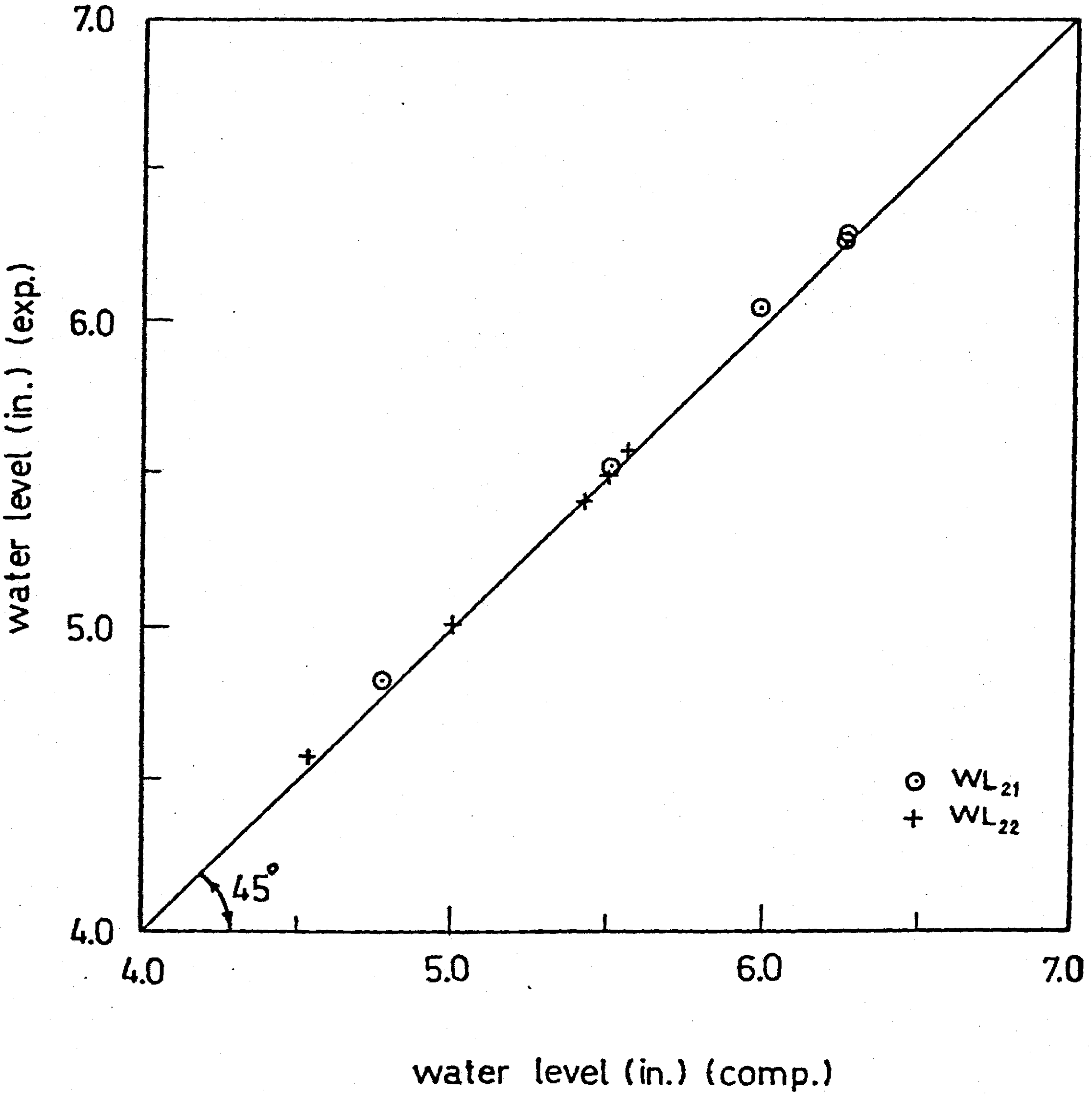


Fig. 3.21 - Comparison of Measured and Computed Water Levels For Set IV (α_1=1.30,α_2=1.70)

particular experiment (and sometimes for two successive experiments) before the control is submerged.

2. In set II, the variations of the interface elevation between the upstream section (station 1) and the control section (station 2) are very small because the fresh water flow in this case is not affected by the vertical contraction and is only subject to the effect of the lateral contraction. However, these small variations are easy to measure due to the extremely clear and sharp interface. In this set, some disturbances are noted at the free surface and at the interface close to the discharging box [Figure A1.10 - Appendix 1]. These disturbances vanish at station (1) which is about 5 feet from the edge of the box. These disturbances are not noticed in the other sets [Figure A1.15 and A1.21] due to the relatively larger depth of the fresh water layer.

3. The measured interface levels at station (3) are not accurate because of the previously-mentioned entrainment in the downstream half of the flume. Due to the continuous erosion of the fresh water layer as well as mixing at the interface, the interface becomes unsteady and unclear at station (3). The measurements given at that station are approximate and are taken at the time when steady-state conditions are reached at stations (1) and (2).

4. The non-uniformity of the velocity distributions across the

flume is shown to have a considerable effect on the computed water levels through the use of the kinetic energy correction factors α_1 and α_2 for the upper and lower layers respectively. The values of 1.30 and 1.70 for α_1 and α_2 respectively are chosen arbitrarily but shown to give reasonable agreement with the measured water levels for all experiments. These factors are not expected to vary significantly from one experiment to the other or from one station to the other for the range of discharges used.

Velocity profiles were not measured due to the lack of the appropriate instruments that can be used for such low flows.

The lower layer is surrounded by solid boundaries at the bottom and sides as well as a moveable boundary (interface) at the top. Therefore, α_2 is expected to have a relatively large value. Harleman, Gooch and Ippen (1958) report values for α_2 up to 2.00 in experiments similar to Set I. The upper layer has solid boundaries at the sides, the interface at the bottom and a free surface at the top. Therefore, α_1 may be expected to be close to unity. However, it is noticed throughout the experiments that a thin layer of dust forms at the free surface and causes a boundary layer to exist at the top. This significantly affects the velocity profiles in the upper layer which tends to increase the value of α_1 considerably. This is illustrated in the photographs of Figures A1.13 and A1.14 (Appendix 1) which show the progress of a dye streak showing the velocity profile just upstream of the contraction in one of the set III experiments.

5. As described in section 3.3.5, when both layers are flowing, equation 3.5 indicates two possible positions for the interface at the critical section. One of them is closer to the free surface while the other is closer to the bottom. In the experimental sets III and IV, the one observed is the position which is near the bottom. The experimental set-up and procedure described previously for these sets of experiments show that the flows start from a position when the interface is at the weir crest level (bottom level at the critical section). Also, the salt water flow is withdrawn at the flume bottom in the downstream end of the flume. Because of this procedure the interface position observed is the one which is closer to the weir crest. Accordingly, the measured elevations are compared to the ones computed for that position. To illustrate the other possibility, consider experiment 4 in set III and experiment 4 in set IV (refer to Tables 3.4 and 3.5). Using the other root obtained from subroutine WLCRIT1 in subroutine BERNWL2, the following water levels are obtained for $\alpha_1 = 1.30$ and $\alpha_2 = 1.70$:

Experiment 4 - Set III:

$WL_{11} = 8.36$, $WL_{21} = 6.95$, $WL_{22} = 7.10$

Experiment 4 - Set IV:

$WL_{11} = 8.31$, $WL_{21} = 6.90$, $WL_{22} = 7.03$

No attempts have been made to obtain this solution due to the difficulties involved in the experimental procedure.

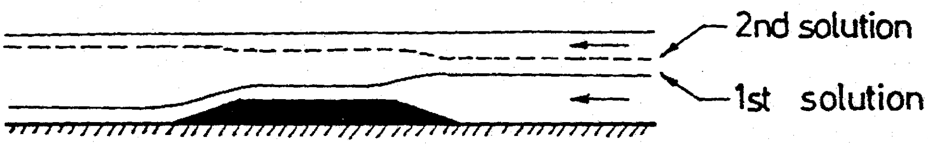


Fig. 3.23 - Alternative Interface Profiles

3.5 CONCLUSIONS

This chapter presents a generalized formulation of the energy equations considering channels of arbitrary geometry and accounting for the non-uniformity of velocity distributions. Critical condition also is defined in the general sense. Based on this theoretical analysis, a number of computer routines are developed for problems that have practical significance. These routines are tested for theoretical and computational performance using data from a specially designed experimental investigation as well as from published literature. The comparison shows good agreement for different flow conditions and illustrates the sensitivity of the results to the non-uniformity of velocity distributions. More applications of the energy equations are shown in Chapter (7).

CHAPTER 4

INTERFACIAL HYDRAULIC JUMP

4.1 INTRODUCTION

This chapter includes the analysis of a hydraulic jump (or drop) in a fluid system of two layers. Theoretical and numerical analysis is aimed at considering a channel of arbitrary geometry and determining the state at one end of a hydraulic jump for a completely specified state at the other end taking mixing at the interface into consideration. The question of the uniqueness of solution is discussed by means of the momentum and energy principles. Solutions which are peculiar to a two-layer system are presented. This chapter also includes description of the developed computational model followed by typical examples and discussion.

Literature review and background of the subject is contained in Section 2.2.3.2 - part C - Chapter 2.

4.2 THEORY

4.2.1 Momentum Principle

With reference to the definition sketch of Figure 4.1, the momentum principle may be applied to each layer between sections 1 and 2 neglecting the shear and assuming hydrostatic distribution

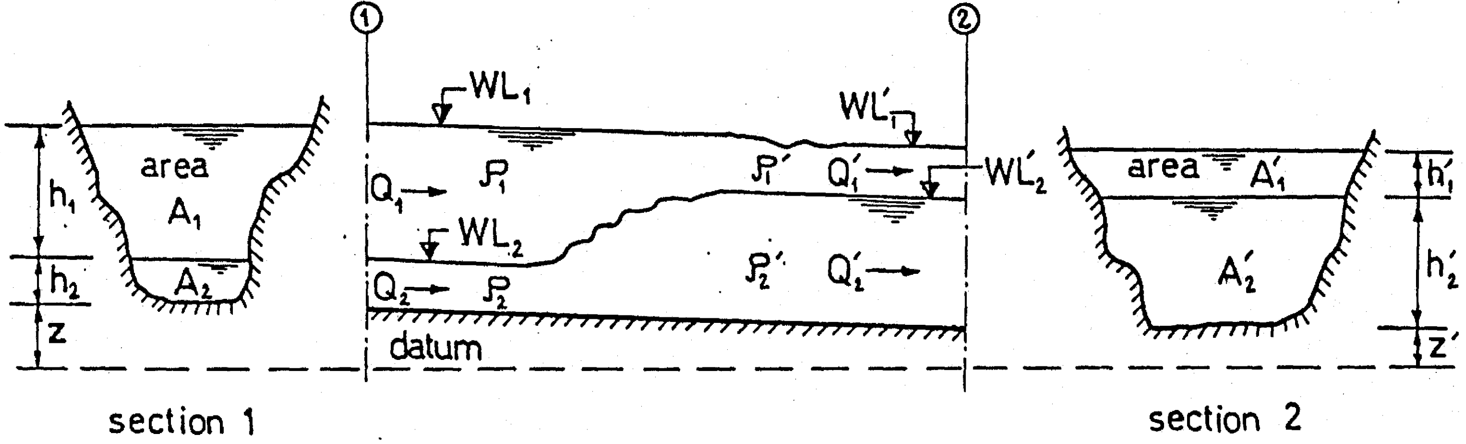


Fig. 4.1 - Definition Sketch of the Interfacial Hydraulic Jump

of pressure.

Lower Layer

Figure 4.2 shows the forces acting on the lower layer between sections 1 and 2. The momentum equation for this layer may be written as follows by equating the rate of change of momentum in the direction of flow to the algebraic sum of the components of all the external forces acting in the same direction (refer also to Figure 4.1)

$$\beta_2 \left[\rho_2' \frac{Q_2'^2}{A_2'} - \rho_2 \frac{Q_2^2}{A_2} \right] = F_1 - F_2 + F_3 + F_4 \qquad 4.1$$

where

β_2 = momentum correction factor of the lower layer

F_1 = hydrostatic pressure force on section 1 at the lower layer

= $\rho_1 \, g \, h_1 \, A_2 + \rho_2 \, g \, A_2 \, \bar{y}_2$

F_2 = hydrostatic pressure force on section 2 at the lower layer

= $\rho_1' \, g \, h_1' \, A_2' + \rho_2' \, g \, A_2' \, \bar{y}_2'$

g = gravitational acceleration

$\bar{y}_2$, $\bar{y}_2'$ = centroidal depths of the areas A_2 and A_2' respectively (Figure 4.1) below the interface

F_3 = hydrostatic pressure force on the interface (area A_b in Figure 4.2)

= $\rho_{1\text{average}} \cdot g \cdot A_b \cdot y_c$

F_3 may be negative if the jump is inverted.

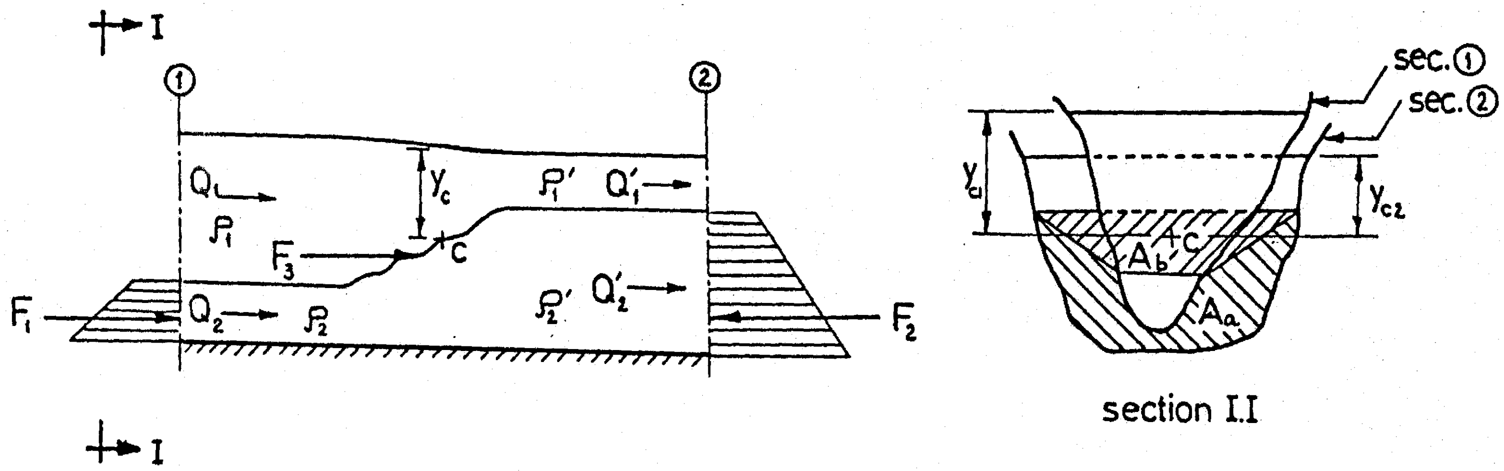


Fig. 4.2 - Forces Acting on the Lower Layer between Sections 1 and 2

where c is the centroid of the area A_b.

$$\rho_{1average} \approx \frac{\rho_1 + \rho_1'}{2}$$

$$y_c \approx \frac{y_{c1} + y_{c2}}{2}$$

A_b is a function of the top widths of the interface at sections 1 and 2 as well as the difference in the interface elevations at both sections.

F_4 = hydrostatic pressure force exerted on the lower layer by the solid boundaries (area A_a in Figure 4.2).

$$\approx \frac{1}{2}\left[\frac{F_1}{A_2} + \frac{F_2}{A_2'}\right] \cdot A_a$$

$$A_a = A_2' - A_2 - A_b$$

F_4 may be positive or negative depending on whether the lower layer area is expanding or contracting in the flow direction respectively.

It should be noted that the discharges Q_1' and Q_2' may not be equal to Q_1 and Q_2 respectively either due to entrainment or because of abstraction or augmentation. In the case of entrainment the density at the downstream section of the entraining layer will also be different from that at the upstream section (e.g. $\rho_1' \neq \rho_1$ or $\rho_2' \neq \rho_2$).

Upper Layer

Figure 4.3 shows the forces acting on the upper layer between sections 1 and 2. The momentum equation for this layer may be written as follows (refer also to Figure 4.1) in which as before the components along the direction of flow are used.

$$\beta_1 \left[\rho_1' \frac{Q_1'^2}{A_1'} - \rho_1 \frac{Q_1^2}{A_1} \right] = F_5 - F_6 - F_3 + F_7 \qquad 4.2$$

where

β_1 = momentum correction factor of the upper layer.

F_3 = as in equation 4.1

F_5 = hydrostatic pressure force on section 1 at the upper layer

$= \rho_1 \, g \, A_1 \, \bar{y}_1$

F_6 = hydrostatic pressure force on section 2 at the upper layer

$= \rho_1' \, g \, A_1' \, \bar{y}_1'$

$\bar{y}_1, \bar{y}_1'$ = centroidal depths of the areas A_1 and A_1' respectively (Figure 4.1) below the free surface.

F_7 = hydrostatic pressure force exerted on the upper layer by the solid boundaries (area A_d in Figure 4.3).

$$\simeq \frac{1}{2} \left[\frac{F_5}{A_1} + \frac{F_6}{A_1'} \right] \cdot A_d$$

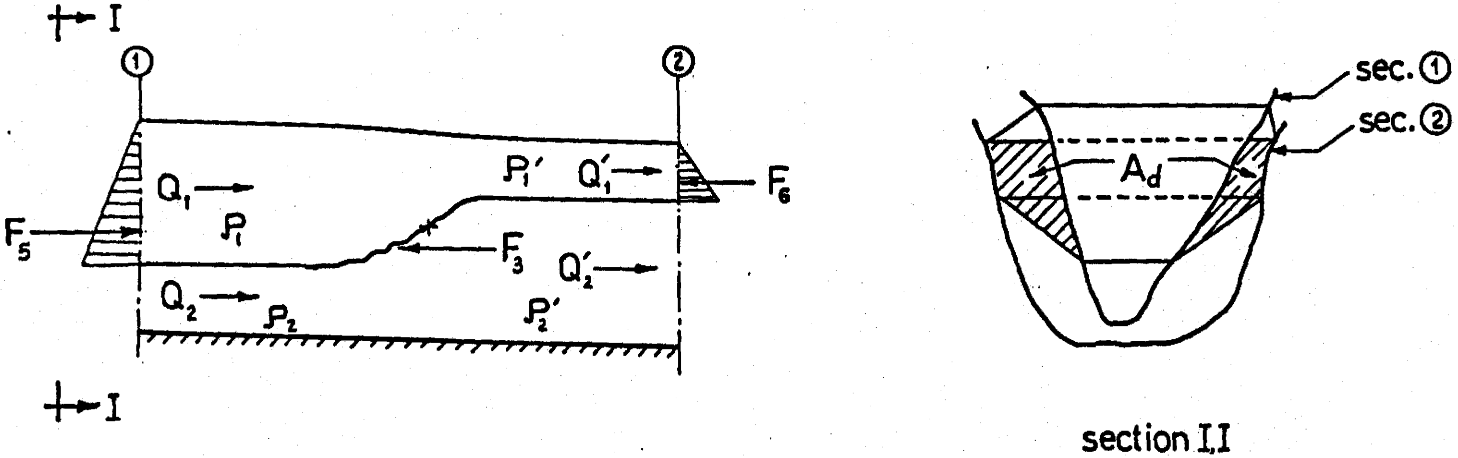


Fig. 4.3 - Forces Acting on the Upper Layer between Sections 1 and 2

A_d is a function of the top widths of the free surface at sections 1 and 2, the difference in the free surface elevations at both sections ($WL_1 - WL_1'$) and the areas A_1', A_b and A_1.

4.2.2 Energy Principle

In addition to the momentum equations, it is necessary to develop an expression for the change in energy ΔE between the two sections. This quantity is used later to check on the validity of alternative solutions of the momentum equations and also as a measure of the energy available for the mechanism of entrainment at the interface.

The drop in mechanical energy from one side of the jump to the other may be expressed as follows with reference to Figure 4.1

$$\begin{aligned}\Delta E = {} & Q_1 \left[\rho_1 g h_1 + \rho_1 g h_2 + \alpha_1 \frac{\rho_1 g Q_1^2}{2 g A_1^2} + \rho_1 g z\right] - \\ & Q_1' \left[\rho_1' g h_1' + \rho_1' g h_2' + \alpha_1 \frac{\rho_1' g Q_1'^2}{2 g A_1'^2} + \rho_1' g z'\right] + \\ & Q_2 \left[\rho_1 g h_1 + \rho_2 g h_2 + \alpha_2 \frac{\rho_2 g Q_2^2}{2 g A_2^2} + \rho_2 g z\right] - \\ & Q_2' \left[\rho_1' g h_1' + \rho_2' g h_2' + \alpha_2 \frac{\rho_2' g Q_2'^2}{2 g A_2'^2} + \rho_2' g z'\right] \end{aligned} \qquad 4.3$$

Equation 4.3 may be rewritten as

$$\Delta E = Q_1 \, \rho_1 \, g \, [WL_1 + \frac{\alpha_1 \, Q_1^2}{2 \, g \, A_1^2}] - Q_1' \, \rho_1' \, g \, [WL_1' + \frac{\alpha_1 \, Q_1'^2}{2 \, g \, A_1'^2}] +$$

$$Q_2 \, \rho_2 \, g \, [\frac{\rho_1}{\rho_2} WL_1 + \frac{\Delta\rho}{\rho_2} WL_2 + \frac{\alpha_2 \, Q_2^2}{2 \, g \, A_2^2}] -$$

$$Q_2' \, \rho_2' \, g \, [\frac{\rho_1'}{\rho_2'} WL_1' + \frac{\Delta\rho'}{\rho_2'} WL_2' + \frac{\alpha_2 \, Q_2'^2}{2 \, g \, A_2'^2}] \qquad 4.3a$$

where

α_1 and α_2 are the kinetic energy correction factors for the upper and lower layers respectively.

$\Delta\rho = \rho_2 - \rho_1$ = density difference at section 1

$\Delta\rho' = \rho_2' - \rho_1'$ = density difference at section 2

4.2.3 Mixing at the Interface

For a rectangular, prismatic channel, Macagno and Macagno (1975) investigate the mechanisms of entrainment and mixing at the interface of an internal hydraulic jump in a two-layer system. They apply the criterion that in an interfacial jump practically all the entrainment is accomplished over a short portion of the interface at the foot of the jump. They assume that the lower layer entrains fluid from the upper layer at the foot of the jump (refer to Figure 4.4). This assumption is discussed in Section 4.3.2.

Their approach is to determine the drop in energy (ΔE) for the ideal hydraulic jump without mixing, and suppose then that a certain fraction of this energy ($\alpha . \Delta E$) is the power available for entrainment. A model is then introduced to determine the change that the entrainment induces in the flow. This leads to new values of depths, discharges and densities, which are used for a calculation of the final conjugate quantities. No consideration is given to second-order changes in ΔE.

They also show experimentally that the ratio α varies from 0.30 to 0.22 as the ratio of conjugate depths of the lower layer goes from 2.4 to 8.

This approach of Macagno and Macagno is used in this study and generalized to consider channels of arbitrary geometry.

Figure 4.4 shows a sketch of the idealized scheme of entrainment and mixing assuming a regular jump in which the lower layer is initially supercritical - i.e. the interface is below the lower critical level for the specified discharges. A fraction βQ_1 of the discharge of the upper layer is assumed to be rapidly entrained by the lower layer and form a modified stream with a new density ρ_2'' and a new depth h_2''.

We can set

$$\beta Q_1 + Q_2 = Q_2'' \qquad 4.4$$

Q_2'' is the discharge of the modified lower stream, the density of which can be determined by the conservation of mass equation:

$$\rho_1 \ \beta \ Q_1 + \rho_2 \ Q_2 = \rho_2'' \ Q_2''$$

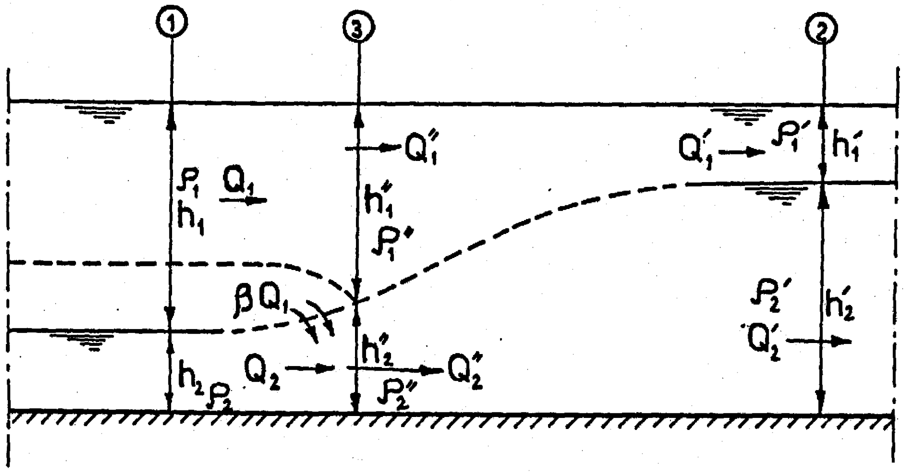


Fig. 4.4 - Definition Sketch for Mixing at the Foot of the Jump [after Macagno and Macagno (1975)]

thus $$\rho_2'' = (\rho_1\ \beta\ Q_1 + \rho_2\ Q_2)/Q_2'' \qquad 4.5$$

Also, $$Q_1'' = (1 - \beta)\ Q_1,\ \rho_1'' = \rho_1,\ h_1'' = h_1 + h_2 - h_2'' \qquad 4.6$$

The coefficient β used in equation 4.4 should not be confused with the momentum correction factors β_1 and β_2 defined in equations 4.1 and 4.2.

To determine the depth h_2'' one can neglect the possible change in total depth and in total area between sections 1 and 3, and write the momentum equation:

$$\beta_1 \frac{\rho_1\ Q_1^2}{A_1} + \beta_2 \frac{\rho_2\ Q_2^2}{A_2} + g\ \rho_2\ A_2\ \overline{y}_2 + \rho_1\ g\ (A_1\ \overline{y}_1 + A_2\ h_1)$$

$$= \beta_1 \frac{\rho_1''\ Q_1''^2}{A_1''} + \beta_2 \frac{\rho_2''\ Q_2''^2}{A_2''} + g\ \rho_2''\ A_2''\ \overline{y}_2'' + \rho_1''\ g\ (A_1''\ \overline{y}_1'' + A_2''\ h_1'') \qquad 4.7$$

In Figure 4.4, the double primes refer to section 3; h_1'' and thus A_1'' are determined by the assumed condition that $h_1 + h_2 = h_1'' + h_2''$. Equation 4.7 may be replaced by two similar equations (one for each layer) if one does not want to assume a constant total depth.

For different values of β one can determine with equations 4.4, 4.5, 4.6 and 4.7 the values of Q_2'', ρ_2'', Q_1'', ρ_1'', h_2'' and h_1'' (equation 4.7 is solved for the unknown h_2'').

Once the double-primed quantities are known, their conjugates (primed quantities) can be calculated using the momentum balance equations 4.1 and 4.2, but with the double-primed quantities replacing the non-primed ones.

Using these calculated quantities, the energy drop $\Delta E''$ between sections 3 and 2 (Figure 4.4) can be computed.

We have

$$\frac{\Delta E''}{\Delta E} = 1 - \alpha \ . \qquad 4.8$$

For a given value of α, the value of β may be obtained which satisfies equation 4.8. As mentioned previously, α may be selected in terms of the ratio of conjugate depths. The fraction α is distinct from the kinetic energy correction factors α_1 and α_2 (equation 4.3).

One of the shortcomings of the approach taken by Macagno and Macagno is that only one special case of interfacial hydraulic jump is considered. This is the situation illustrated in Figure 4.4 in which flow in the lower layer is initially supercritical and in which entrainment occurs from the upper slower layer into the lower faster layer. In such a case, the interfacial hydraulic jump is manifested by an increase in the elevation of the interface. As discussed later, the case of the inverted interfacial hydraulic jump may also be considered, as long as assumptions concerning the direction of entrainment and change of interface elevation are reversed. Therefore, in the present study, equations 4.4, 4.5 and 4.6 are applied only in the former case (lower layer entraining). In the latter case (upper layer entraining), the corresponding equations are:

$$\beta Q_2 + Q_1 = Q_1'' \qquad 4.9$$

$$\rho_2 \beta Q_2 + \rho_1 Q_1 = \rho_1'' Q_1'' \qquad 4.10$$

$$Q_2'' = (1 - \beta) Q_2, \; \rho_2'' = \rho_2, \; h_1'' = h_1 + h_2 - h_2'' \qquad 4.11$$

Other reservations concerning the approach followed by Macagno and Macagno are discussed in Section 4.3.2. Also, numerical examples of different possible solutions in a two-layer system are presented in Section 4.3.1.

4.2.4 Solution Uniqueness of Equations 4.1 and 4.2

Yih and Guha (1955) and Yih (1965, pp. 130-133) discuss the question of determining the state downstream from a hydraulic jump in a two-layer system for a completely specified state upstream. They show that the momentum equations can have at most nine real solutions, one of which is obviously the given upstream state. But of these nine solutions five are not entirely positive. Hence there can be at most only four positive solutions representing four mutually conjugate states.

Mehrotra and Kelly (1972) demonstrate that of these four solutions, only one solution is possible for purely internal shocks. With reference to Figure 4.1, excluding the solution $h_1'/h_1 = 1$ and $h_2'/h_2 = 1$ (the given state), there are three solutions one of which is characterized by $h_1'/h_1 < 1$ and $h_2'/h_2 < 1$ (i.e. in both layers the computed conjugate depth h' is less than the given depth h). Hayakawa (1970) shows that this solution is always unacceptable from energy consideration. Mehrotra and Kelly (1972) demonstrate that of the remaining two admissible conjugate states, only the one which is closer to the given state is physically realizable. They also bring up certain features that are peculiar to a two-fluid system such as the existence of "drop" solutions (see Section 4.3.1 - example C). All the above mentioned studies consider the case in which the upstream state is given and the downstream state calculated. In the opposite situation the energy consideration is reversed, so that the solution $h_1'/h_1 < 1$ and $h_2'/h_2 < 1$ may be acceptable.

4.3 DEVELOPMENT OF COMPUTER ROUTINES

A computer model is developed to determine the state downstream (or upstream) from a hydraulic jump (or drop) at the interface and/or the free surface of a two-layer system for a completely specified state upstream (or downstream). The state is taken to mean the free surface and interface elevations.

The numerical model is characterized by the following features:

a) The model is designed for channels of arbitrary cross-sections. Each of the upstream and downstream sections is defined by a series of straight lines between points, the coordinates of which are referred to some arbitrary vertical and horizontal axes. This feature requires not only extensive modifications to the momentum and energy equations, but also special considerations in the numerical procedure.

b) The model allows the discharge and the density of each layer to vary from one cross-section to the other. This feature allows the analysis of abrupt transitions in the vicinity of lateral inflows or abstractions.

c) Mixing at the interface is accounted for and may be taken into consideration at the option of the user. The approach of Macagno and Macagno (1975) is used and generalized with respect to channel geometry and specification of the entraining layer. However, this approach does not apply to the special cases when one of the layers is stagnant. In such cases, due to entrainment of the stagnant layer at the interface, the hydraulic jump is expected to move in the upstream direction.

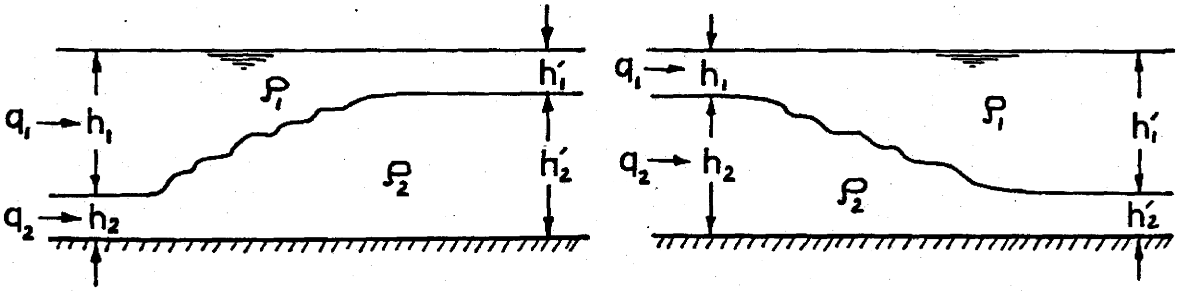
d) Although the solution is one-dimensional, the non-uniformity of velocity distributions in a vertical section within each layer is accounted for by applying momentum and energy correction factors to the governing equations.

e) The occurrence of a hydraulic jump (or drop) and the uniqueness of the solution of the momentum equations are decided from energy considerations. If more than one conjugate state appears to be possible from consideration of the momentum and energy equations, only the closest solution to the given state is considered. In addition, a diagnostic message is printed if the occurrence of a hydraulic jump is not possible.

The computational model takes the form of an organizational sub-program INTJMP which uses an additional eight subroutines and functions. Complete documentation and listings of the developed subroutines and functions are contained in Appendix (2).

4.3.1 Typical Results

A) The model results are compared to the theoretical results presented by Yih and Guha (1955) for the special cases of one moving layer in a channel of rectangular section.



(1) normal jump (2) inverted jump

h_1, h_2 - given depths

h_1', h_2' - computed depths

(1) $q_1 = 0$ (<u>stagnant upper layer-normal jump</u>)

$q_2 = 50$ cm^3/sec/cm, $\rho_1 = 1.00$, $g = 981.0$ cm/sec^2

$$F_{i2}^2 = \frac{q_2^2}{g' \, h_2^3}$$

Given			Computed			
h_1 (cm)	h_2 (cm)	ρ_2	h_1' (cm)	h_2' (cm)	h_2'/h_2	F_{i2}
3.00	1.47	1.25	0.98	3.49	2.37	2.00
4.00	1.12	1.25	0.88	4.24	3.79	3.01
5.00	0.927	1.25	1.13	4.80	5.18	4.00
6.00	0.799	1.25	1.54	5.26	6.58	5.00
3.00	1.62	1.177	0.78	3.84	2.37	2.00
4.00	1.24	1.177	0.60	4.64	3.74	2.99

$$h_1 - h_1' = h_2' - h_2 \text{ (level free surface)}$$

(2) $q_2 = 0$ (stagnant lower layer-inverted jump)

$q_1 = 50$ cm^3/sec/cm, $\rho_1 = 1.00$, $g = 981.0$ cm/sec^2

$$F_{i1}^2 = \frac{q_1^2}{g' h_1^3}$$

Given			Computed			
h_1 (cm)	h_2 (cm)	ρ_2	h_1' (cm)	h_2' (cm)	h_1'/h_1	F_{i1}
1.042	5.00	1.333	3.93	2.83	3.77	3.00
0.860	6.00	1.333	4.46	3.30	5.19	4.00
0.742	7.00	1.333	4.89	3.89	6.59	5.00
0.657	8.00	1.333	5.25	4.55	7.99	6.00
0.593	9.00	1.333	5.58	5.26	9.41	6.99
1.236	5.00	1.177	4.65	2.10	3.76	3.00
1.020	6.00	1.177	5.28	2.38	5.18	4.00
0.879	7.00	1.177	5.79	2.83	6.59	5.00
0.779	8.00	1.177	6.22	3.38	7.98	5.99
0.703	9.00	1.177	6.60	3.99	9.39	6.99
1.782	5.00	1.053	6.70	0.33	3.76	3.00
1.471	6.00	1.053	7.59	0.19	5.16	4.00
1.268	7.00	1.053	8.32	0.30	6.56	5.00

$$\frac{\rho_1}{\rho_2}(h_1 - h_1') = h_2' - h_2 \text{ (free surface is not level)}$$

These results obtained by INTJMP are identical to those presented by Yih and Guha. It is confirmed that in these special cases the solution is unique and dependent on the densimetric Froude number of the moving layer. Also, the condition of a level free surface in case (1) - (i.e. stagnant upper layer, regular jump) - is confirmed within close limits of accuracy, whereas in case (2) the free surface is clearly not level.

B) With reference to Section 3.3.5, part (d) - (Chapter 3), it is shown that when both layers are flowing, and for given flows and free surface elevation, there are two possible positions for the interface for which a critical condition exists at a cross-section (equation 3.5). This, as related to the interfacial hydraulic jump, is illustrated by the following example:

A prismatic rectangular channel is considered which is 10 cm wide and 20 cm deep and has a bottom elevation of 0.0 cm. Refer to Figure 4.5 for definition of notation.

Given

$Q_1 = Q_1' = 100$ cm^3/sec.

$Q_2 = Q_2' = 100$ cm^3/sec.

$\rho_1 = \rho_1' = 1.00$

$\rho_2 = p_2' = 1.02$

$g = 981$ cm^3/sec.

$WL_1 = 15.0$ cm

$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1.00$

no mixing considered

Given	Computed		Comments
WL_2 (cm)	WL_1' (cm)	WL_2' (cm)	
0.4	15.0	4.9	Jump
1.5	15.0	2.0	Jump
1.7	15.0	1.8	Jump
1.8	--	-	No solution
13.2	--	-	No solution
13.5	15.0	13.0	Inverted jump
14.0	15.0	12.3	Inverted jump

It should be noted that the two critical interface levels, computed using subroutine WLCRIT1 are 1.73 cm and 13.27 cm.

C) A stretch of a channel is defined by the two end cross-sections shown in Figure 4.5. Each cross-section is defined by a series of straight lines between points, the coordinates of which are referred to some arbitrary vertical and horizontal axes. The following examples illustrate the effect of the different parameters on the flow characteristics. Reference is made to Figure 4.5 for definition of notation.

General Input

$Q_1 = Q_1' = 430\ m^3/sec$

$Q_2 = Q_2' = 23\ m^3/sec$

$\rho_1 = \rho_1' = 1.00$

$\rho_2 = \rho_2' = 1.02$

$g = 9.81\ m/sec^2$

Example (1)

Input

$WL_1 = 15.00\ m$

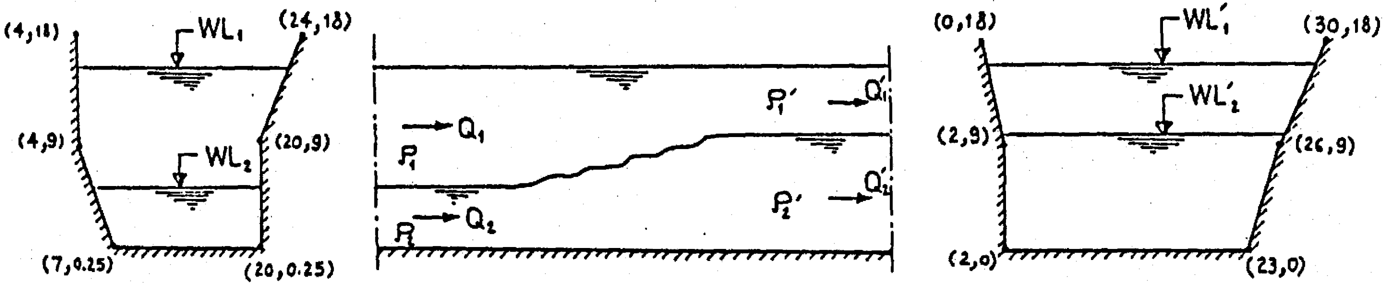


Fig. 4.5 - Typical Example - Interfacial Hydraulic Jump

$WL_2 = 5.00$ m

$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1.00$

where

α_1, α_2 = kinetic energy correction factors for the upper and lower layers respectively.

β_1, β_2 = momentum correction factors for the upper and lower layers respectively.

These factors are defined as:

$$\alpha = \frac{1}{A} \int_A \left(\frac{u}{U}\right)^3 dA, \quad \beta = \frac{1}{A} \int_A \left(\frac{u}{U}\right)^2 dA$$

where

u = local velocity

U = average velocity

A = cross-sectional area

No mixing is considered in this example.

Output

$WL_1' = 14.81$ m

$WL_2' = 9.42$ m

Example (2)

Input

$WL_1 = 15.00$ m

$WL_2 = 5.00$ m

$\alpha_1 = 1.1$, $\alpha_2 = 1.2$

$\beta_1 = 1.05$, $\beta_2 = 1.07$

No mixing considered

Output

$WL_1' = 14.81$ m

$WL_2' = 9.39$ m

Example (3)

Input

$WL_1 = 15.00$ m

$WL_2 = 5.00$ m

$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1.0$

Mixing is considered ($\alpha = 0.22$) where α is the fraction of energy used for entrainment.

Output

$WL_1' = 14.78$ m

$WL_2' = 10.34$ m

$\beta = 0.098$

where β is the fraction of the lower layer flow entrained by the upper layer.

Example (4)

Input

The same as example (3) but with $\alpha = 0.30$

Output

$WL_1' = 14.78$ m

$WL_2' = 10.55$ m

$\beta = 0.125$

Example (5)

Input

$WL_1' = 14.81$ m

$WL_2' = 9.42$ m

$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1.0$

No mixing considered

<u>Output</u>

Two solutions are obtained

i) $WL_1 = 15.00$ m

$WL_2 = 5.00$ m

ii) $WL_1 = 1.73$ m

$WL_2 = 0.25$ m

Referring to the results obtained in the previous examples, the following points may be made:

1 - the effect of mixing at the interface is demonstrated in examples 3 and 4. In example 3, it is assumed that 22% of the energy losses are available for entrainment. This changes the conjugate depths of the upper and lower layers by -17.63% and +9.77% respectively. 9.8% of the lower layer discharge is entrained by the upper layer. In example 4, 30% of the energy losses are assumed to be available for entrainment. The changes in depths are -21.52% and +12.00% respectively. In this case 12.5% of the lower layer flow is entrained by the upper layer.

2 - The effect of the non-uniformity of velocity distributions in vertical sections is examined in example 2. Fixing all other parameters and assuming values of $\alpha_1 = 1.1$, $\alpha_2 = 1.2$,

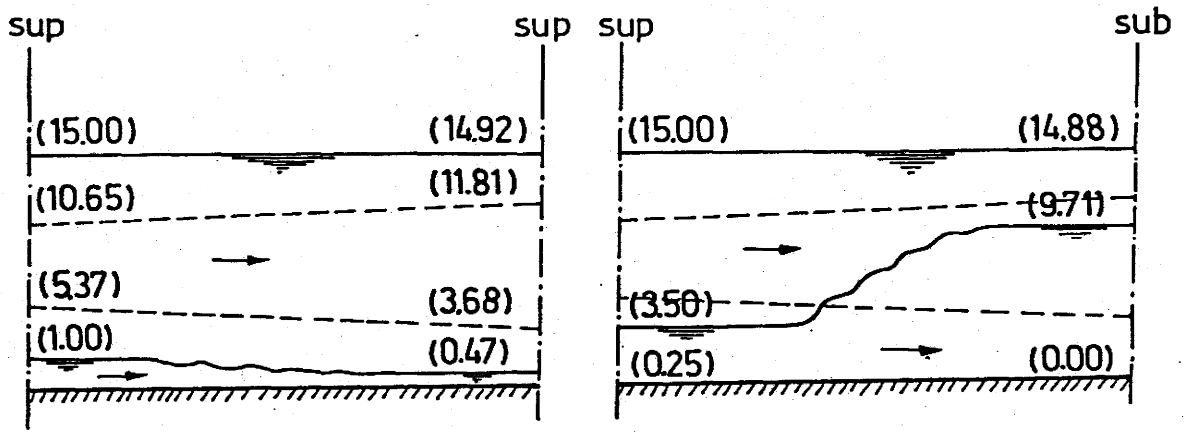
$\beta_1 = 1.05$, $\beta_2 = 1.07$, the conjugate depths of the upper and lower layers show a change of +0.56% and -0.32% respectively, relative to the values obtained in example 1.

3 - Example 5 raises the question of non-uniqueness of solution. In solution (ii), the computed conjugate depths are both less than the given depths. Hayakawa (1970) demonstrates that this solution is always unacceptable from energy consideration. However, he only considered the case where the downstream state is determined for a specified upstream state. This conclusion does not apply to this example in which the upstream state is computed as a function of a given downstream state. In other words, solution (ii) is theoretically possible in this case from energy consideration. However, this solution does not seem to be practically possible because the interface would break down in the presence of a surface jump.

4 - Subroutine WLCRIT1 is used in an attempt to determine the critical interface elevations for the conditions defined in example 1 (i.e. discharges, specific gravities, free surface levels and kinetic energy correction factors) at both the upstream and downstream sections. In fact, it was found that no solutions could be obtained either at the upstream or the downstream sections. This indicates that at either section the flow is supercritical at any position of the interface. Thus the interfacial jump analyzed in

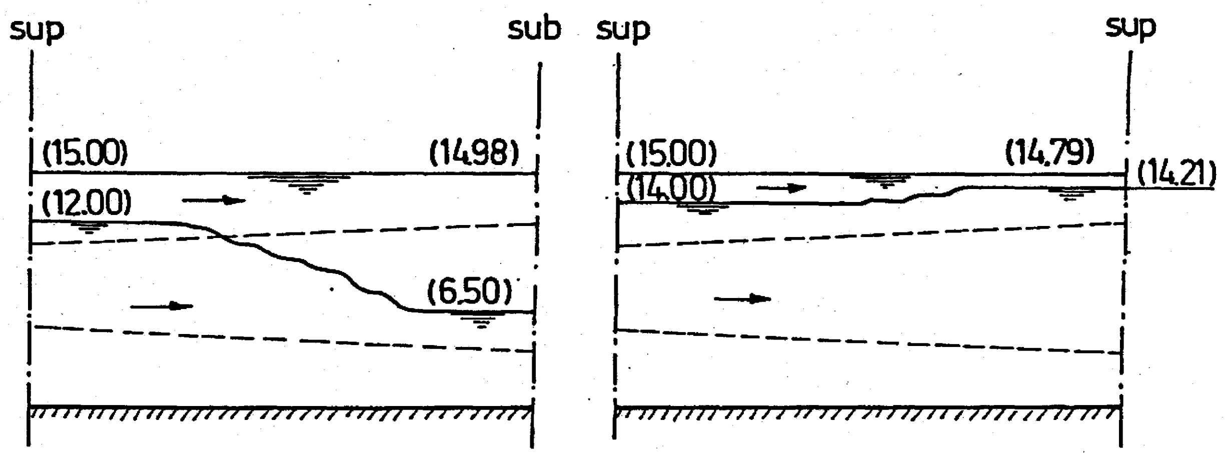
these examples (1 to 5) show an abrupt interfacial transition from an initially supercritical state to another supercritical state downstream (e.g. for the solution of example 1 the evaluation of equation 3.5 (Chapter 3) is approximately 8.6 - a solution well within the supercritical regime).

5 - In order to illustrate the different possible and theoretically acceptable solutions of the momentum equations, the discharges of the upper and lower layers are changed to 65 m^3/sec in each layer keeping all the other parameters unchanged. Mixing is not considered and the kinetic energy and momentum correction factors are assumed to be unity. The following diagrams of Figure 4.6 illustrate the different possibilities for different interface elevations. The dotted lines represent the critical interface elevations at each cross-section. In two cases the existance of a hydraulic drop or an inverted hydraulic drop is shown to be mathematically possible. In both cases the transition occurs entirely within the supercritical zone. In this respect the unusual circumstances of the previous illustration (example 1) is parallelled by the case of the inverted drop shown here.



Hydraulic Drop

Hydraulic Jump



Inverted Jump

Inverted Drop

Fig. 4.6 - Different Possible Forms of Interfacial Discontinuity

sup≡supercritical flow
sub≡subcritical flow

4.3.2 Discussion

In this section the approach followed by Macagno and Macagno (1975) to consider mixing at the interface of an internal jump is discussed in more detail. With reference to the procedure described in Section 4.2.3, the following points may be presented:

1) The entrainment process is assumed by Macagno and Macagno to be uni-directional, the upper layer fluid being always entrained by the lower layer. As described previously in Section 4.2.3, this is modified in the computer model to be dependent on the velocities of flow.
2) With reference to Figure 4.4, Macagno and Macagno calculate the modified discharges, densities and depths at section 3 and use these values to calculate the final conjugate quantities at section 2 using the momentum equations. In applying the momentum balance between sections 2 and 3, they apparently neglect the variations in the discharges and the densities at section 2 due to entrainment. In other words, they use the original quantities at section 2.
3) The energy drop between sections 1 and 2 (ΔE) and thus ($\alpha.\Delta E$) is assumed by Macgno and Macagno to be constant. In other words, the second-order changes in ΔE, due to the changes caused by entrainment at section 2, are not considered.
4) The effect of the entrainment process on the total energy at section 2 and 3 may be illustrated using the following

example.

A rectangular prismatic channel is considered which is 10 m wide and 20 m deep has a horizontal bed elevation of 0.0 m. With reference to Figure 4.4 for the definition of notation, the free surface and interface elevations at section 1 are assumed to be 15 and 2 m respectively. Specific gravities of the upper and lower layers are 1.00 and 1.02 respectively. The discharge is assumed to be 40 m^3/sec in each layer. The computed conjugate elevations at section 3 without allowing for mixing are 14.99 and 8.68 m for the free surface and interface respectively. The corresponding power drop ΔE is 29499 Kg.m/sec. If 5% of the upper layer discharge is assumed to be entrained by the lower layer (i.e. $\beta = 0.05$), the calculated lower layer depth at section 3 using equation 4.7 is 2.15 m. This results in an increase in the total power at section 3 so that the power drop between sections 1 and 3 becomes negative (-290 Kg.m/sec) which seems unreasonable. For the same case, if the discharges and densities at section 2 are adjusted (i.e. $Q_1' = Q_1'' = 38$ m^3/sec, $Q_2' = Q_2'' = 42$ m^3/sec, $\rho_1' = \rho_1'' = 1.00$, $\rho_2' = \rho_2'' = 1.01905$), the solution of the momentum equations between sections 3 and 2 yields values of 14.99 and 8.99 m for the free surface and interface elevations respectively at section 2. The power drop ($\Delta E''$) between sections 3 and 2 is found to be 28231 Kg.m/sec.

The rather odd result of a negative energy drop between sections 1 and 3 may be due to one or more of the following reasons:

i) The total energy at a certain cross-section is defined as the sum of the potential and kinetic energies. The conversion of energy due to turbulence is not taken into consideration since it is difficult to quantify. Rouse, Siao and Nagaratnam (1971) discuss the process of energy conversion through turbulence for a hydraulic jump in a homogeneous flow. However, no attempt has been made for a similar anlaysis in a stratified system. Rouse and Dodu (1971) suggest that a certain percentage of the power of the turbulence generated by a mixing device would be used in increasing the potential energy of a density-stratified fluid. Therefore, the increase of energy from section 1 to section 3 may be due to the conversion of turbulence energy available at section 1 to additional potential energy at section 3.

ii) The assumption of constant total depth between sections 1 and 3 is examined. Equation 4.7 is replaced by two equations to check the possible variation in the free surface elevation between the two sections. It is found that the computed free surface drop at section 3 (0.001 m) does not affect the total energy at that section significantly.

iii) In the previous calculations, the momentum and energy correction factors are assumed to be unity. However, for the purpose of illustration, if the kinetic energy correction factor of the lower layer at section 1 is increased to 1.3 keeping that at section 3 equal to unity, the energy drop between sections 1 and 3 is significantly affected and becomes about +24480 Kg.m/sec (compared to -290 Kg.m/sec). This shows that the energy difference in a stratified system is very sensitive to the values of these correction factors. However, it is difficult to incorporate in the numerical model values for these factors which are different for each layer and also different from one cross-section to the other since there is no experimental basis available. Therefore, experimental investigations are certainly needed to further check the theory and to determine the variation of velocity profiles between different sections of the jump.

CHAPTER 5

LOCK EXCHANGE FLOW

5.1 INTRODUCTION

This chapter deals with the phenomena of lock exchange flow, being the classical case of unsteady non-uniform flow in the field of small density difference hydraulics. This phenomena occurs when a lock gate or other such division separates bodies of still water with the same surface elevation but which differ slightly in density. While the opening of the gate may result in local distrurbances, the predominent effect will be a continuing exchange pattern of flow which is caused by the density difference. Reference is made to the definition sketch of Figure 5.1.

This chapter includes theoretical analysis of the problem with reference to the experimental investigations by different authors. It also describes the computer subroutine designed to simulate this phenomena.

5.2 THEORY AND BACKGROUND

A detailed literature review of the subject is presented in Chapter 2. It is shown that various experimental investigations indicate that the expected realtionship between the velocity of spread V of either the overflow front or the underflow front and

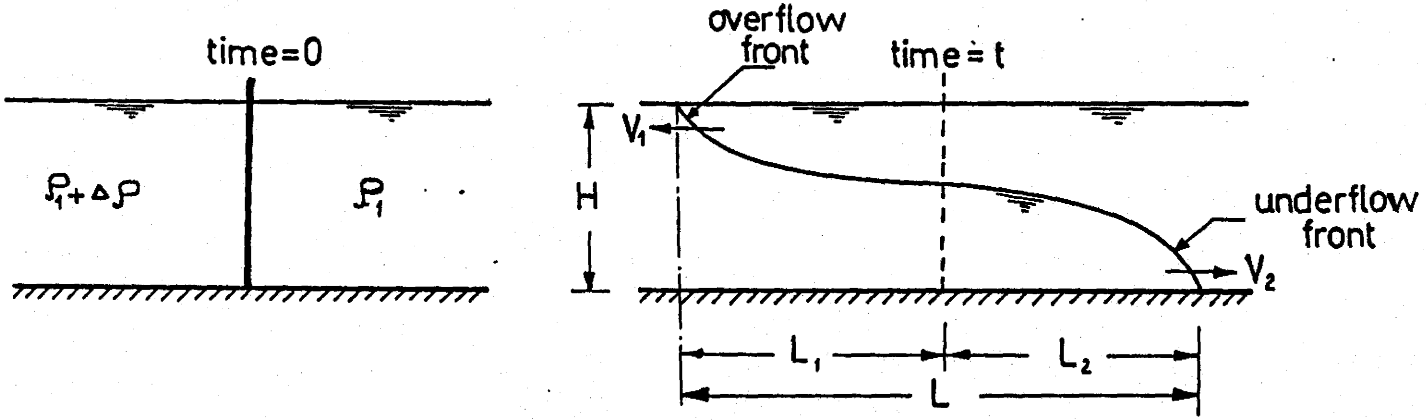


Fig. 5.1 - Definition Sketch of Lock Exchange Flow

the densimetric velocity V_Δ is:

$$\frac{V_i}{V_\Delta} \equiv \frac{L_i/t}{\sqrt{g' H}} = \text{constant } i = 1,\ 2 \qquad 5.1$$

where
$$g' = g \frac{\Delta\rho}{\bar{\rho}}$$

g = gravitational acceleration

$\Delta\rho$ = density difference

$\bar{\rho}$ = mean density

This constant is found experimentally by Barr (1963A) and reported by Frazer, Barr and Smith (1967 and 1968) to be about 0.5 for underflow and 0.6 for overflow. Equation 5.1 is found to be true in the "turbulent" zone, using the word turbulent to denote independence of viscosity. The limit to the turbulent zone is represented approximately by the equation

$$\overline{F_\Delta R} \cdot \frac{H}{L_i} \equiv \frac{g' H^{5/2}}{\nu L_i} \geq 150 \qquad 5.2$$

where

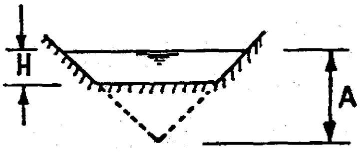
$\overline{F_\Delta R}$ = densimetric Froude-Reynolds number

$$= \frac{H^{3/2} \cdot g'^{1/2}}{\nu}$$

ν = kinematic viscosity

Condition 5.2 is expected to be satisfied in most practical situations. For example, if the total depth is 1.0 m, the specific gravities of the two fluids are 1.00 and 1.02 (e.g. fresh and salt waters), the kinematic viscosity is 0.116 x 10^{-5} m^2/sec, the condition 5.2 is valid for a distance from the barrier $\leq$ 2533 m.

The previously-mentioned experimental studies are performed in rectangular flumes. However, Barr (1964) studies the effect of the channel cross-sectional geometry on overflow and underflow velocities. Based on an experimental study using channels of different cross-sectional shapes, he shows that for an open trapezoidal section, the constant of equation 5.1 depends on the ratio A/H.



He illustrates that for large A/H (> 10), which is the case in most actual water courses, the values of the constant approach those of an open rectangular section, namely, 0.465 for underflow and 0.590 for overflow. Barr concludes that the fact that in a rectangular section the overflow front moves at 1.27 times the speed of the underflow front may be of greater practical significance. Consequently, with reference to Figure 5.1,

$$\frac{L_1}{L} = 0.559, \quad \frac{L_2}{L} = 0.441 \qquad 5.3$$

Bache (1976) derives the following relationship on the basis of inviscid flow theory:

$$V = \frac{dL}{dt} = \frac{\alpha\ t}{(H^2+\alpha t^2)^{1/2}}, \quad \alpha = \frac{1}{2} g' H \qquad 5.4$$

where

V = relative velocity of overflow and underflow fronts.

$= \frac{dL}{dt}$

With reference to equation 5.4, for small times, V increases linearly with time and at large times ($t^2 >> H^2/\alpha$):

$$V = \alpha^{1/2} = 0.71 \sqrt{g' H} \qquad 5.5$$

Based on a following discussion of that paper (Journal of Hydraulic Research 14, No. 8, 1976, pp. 251-254), and personal communications with Dr. D.H. Bache, equation 5.4 is corrected to

$$V = \frac{dL}{dt} = \frac{2\ \alpha\ t}{(H^2+2\alpha t^2)^{1/2}} \qquad 5.6$$

which for large times will reduce to

$$V = (2\ \alpha)^{1/2} = \sqrt{g' H} \qquad 5.7$$

If the overflow and underflow velocities are assumed to be equal, then

$$\frac{V_i}{V_\Delta} = 0.5 \text{ for large } t,\ i = 1,\ 2 \qquad 5.8$$

which is in accordance with the result obtained using the energy approach [O'Brien and Cherno (1934)].

5.3 DEVELOPMENT OF COMPUTER ROUTINE

Subroutine LKEXFL computes the overflow and underflow front velocities of a lock exchange flow at any specified time after the removal of the barrier as well as the distances travelled by both fronts from the barrier. These are computed as functions of densities and total depth as well as a specified time increment.

Equation 5.6 is used to compute for any specified time the relative velocity V by direct substitution as well as the total frontal distance L using the Runge-Kutta-Merson method (a differential equation solver algorithm) as follows:

Equation 5.6 may be expressed in the form

$$\frac{dL}{dt} = \mathrm{FUNC}(t)$$

If L_a is the frontal distance at time t and L_b is the distance at time t+Δt, then

$$L_a = L_b + (F_1 + 4F_2 + F_3) \cdot \Delta t/2$$

where

$F_1 = \mathrm{FUNC}(t)/3$

$F_2 = \mathrm{FUNC}(t + \Delta t/2)/3$

$F_3 = \mathrm{FUNC}(t + \Delta t)/3$

The truncation error is estimated by

$$\mathrm{ERROR} = (2F_1 - 9F_4 + 8F_2 - F_3).\Delta t/10$$

where

$F_4 = \mathrm{FUNC}(t + \Delta t/3)/3$

This error is not a correction term but merely a measure of the truncation error.

If this error is greater than a certain allowable tolerence, then Δt is halved and the process repeated. On the other hand, if the error is less than 1/32 of the allowable tolerence, Δt is doubled for the next step.

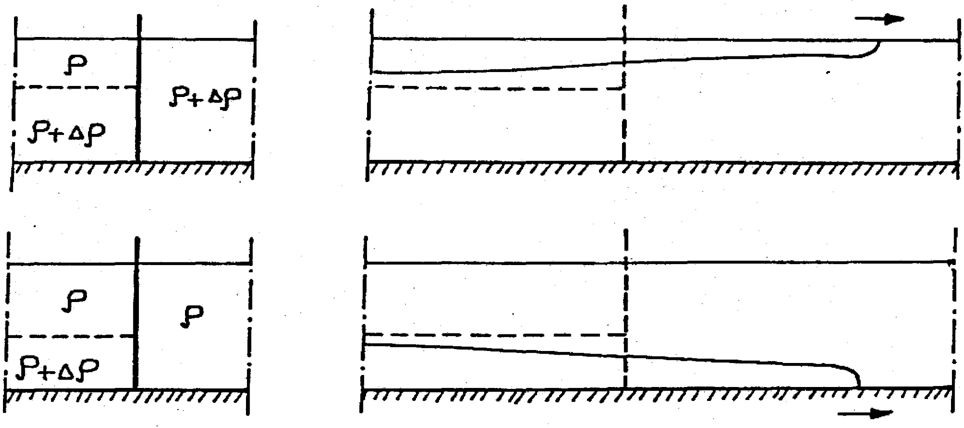
The relative velocity is used to compute both the overflow and underflow front velocities using the ratio 1.27 suggested by Barr as mentioned eariler. Also, the frontal distances from the barrier are estimated from the total distance using the relationships 5.3.

Complete documentation and listing of the subroutine together with an example are contained in Appendix (2).

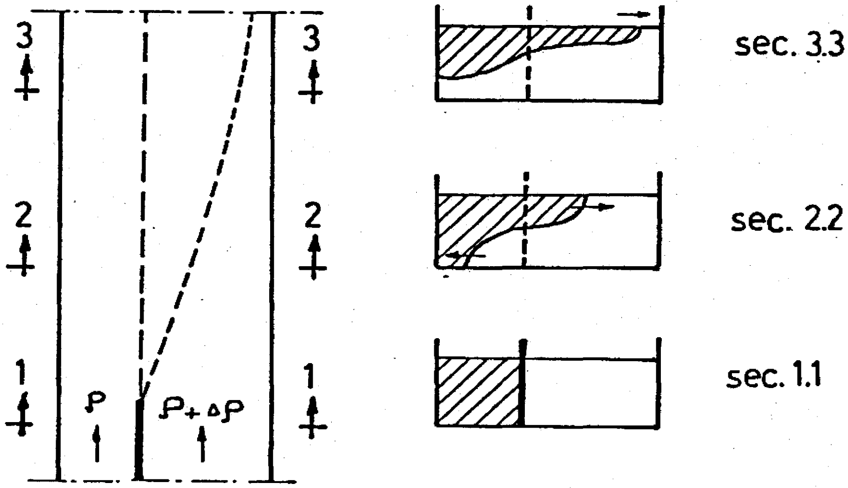
5.4 DISCUSSION

Lock exchange flows as described in this chapter form the general case of unsteady non-uniform density flows. The theoretical analysis available is limited to the idealized problem of prismatic horizontal channels containing fluids that have zero initial velocities and equal initial depths. However, the computer routine developed for these conditions serves as a starting point to analyze other problems of practical significance such as the following.

a) Non-equal depths of fluids (e.g. thermal and saline wedges).



b) Non-zero velocity of the ambient fluid. The ambient fluid may be flowing in the longitudinal direction (with or against the density flow) or in the transverse direction (see sketch below).



Special cases of steady density flows are also frequently encountered in practical situations. Density wedges of either the overflow type (thermal wedges) or the underflow type (salt wedges) arise from the "lock exchange" phenomena. These cases are analyzed and discussed in the following chapter.

CHAPTER 6

LONG TRANSITIONS

[INTERFACIAL RESISTANCE]

6.1 INTRODUCTION

This chapter deals with gradually-varied flows in a two-layer system where significant energy losses are due to boundary and/or interfacial friction. Relationships are derived for the energy gradients and surface slopes of the upper and lower layers, in terms of the shear stresses at the solid boundaries as well as at the interface. Evaluation of these stresses requires an estimation of the corresponding friction factors. The determination of these factors under different flow and boundary conditions is also discussed in this chapter.

The present version of the algorithms does not include for any shear stress at the free surface (e.g. wind stress). This additional factor might be included wihtout any major difficulty, as long as some appropriate means of defining the magnitude and direction of the stress is available.

The computer routines developed for this area are presented accompanied by comparison of the computations with published laboratory and field measurements.

Reference is made to Chapter 2 for detailed literature review of the subject. Also, complete documentation and listings of the developed

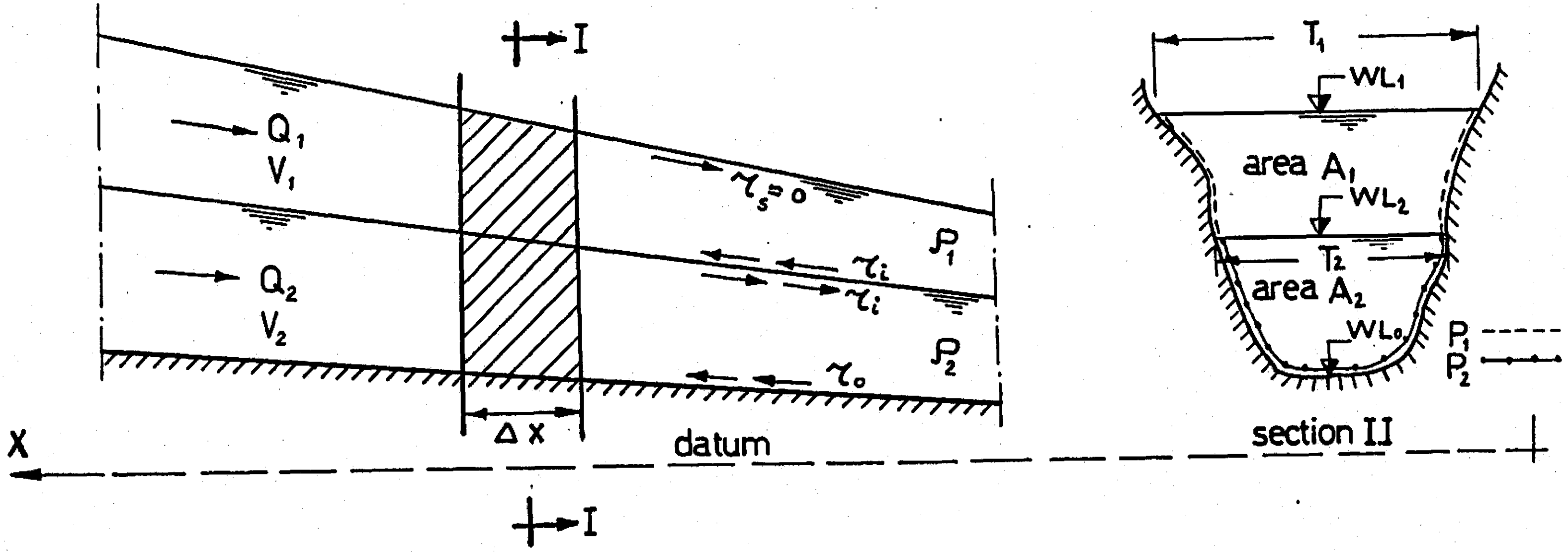


Fig. 6.1 - Definition Sketch of a Two-Layer System

routines with illustrative examples are included in Appendix 2.

6.2 THEORY

The strategy adopted by the author in this study may be described in the following procedure:

i) define energy and surface slopes and relate them.

ii) relate energy slopes and shear stresses.

iii) define shear stresses in terms of shear stress coefficients and discuss the evaluation of these coefficients as presented in a variety of other contributions.

6.2.1 Energy Gradients and Surface Slopes

With reference to the definition sketch of Fig. 6.1, the total energy for each layer may be expressed as follows:

$$H_1 = WL_1 + \alpha_1 \frac{Q_1^2}{2gA_1^2} , \qquad 6.1$$

$$H_2 = \frac{\rho 1}{\rho_2} (WL_1) + \frac{\Delta\rho}{\rho_2} (WL_2) + \alpha_2 \frac{Q_2^2}{2gA_2^2} \qquad 6.2$$

where

H_1, H_2 = total energy of the upper and lower layers respectively.

A_1, A_2 = cross-sectional areas of the upper and lower layers respectively.

Q_1, Q_2 = discharges in the upper and lower layers respectively.

ρ_1, ρ_2 = densities of the upper and lower fluids respectively.

$\Delta\rho$ = $\rho_2 - \rho_1$ = density difference

α_1, α_2 = kinetic energy correction factors of the upper and lower layers respectively.

g = gravitational acceleration.

The energy gradient of the upper layer in the x-direction is then

$$\frac{dH_1}{dx} = S_{1e} = \frac{\partial H_1}{\partial WL_1} \cdot \frac{dWL_1}{dx} + \frac{\partial H_1}{\partial A_1} \cdot \frac{dA_1}{dx} \qquad 6.3$$

dWL_1/dx and dWL_2/dx are the free surface and interface slopes respectively and are subsequently denoted by S_1 and S_2 respectively.

It should be noted that the positive x-coordinate is taken opposite to the flow direction to avoid complication with the signs of energy and surface slopes. In homogeneous flows, the reverse convention is normally adopted i.e. x is positive in the flow direction and the sustaining slopes are positive (e.g. $S_e = - dH/dx$).

From equation 6.1

$$\frac{\partial H_1}{\partial WL_1} = 1 \qquad 6.4$$

and

$$\frac{\partial H_1}{\partial A_1} = - \alpha_1 \frac{Q_1^2}{gA_1^3} \qquad 6.5$$

dA_1/dx may be expressed as

$$\frac{dA_1}{dx} = \frac{\partial A_1}{\partial WL_1} \cdot \frac{dWL_1}{dx} + \frac{\partial A_1}{\partial WL_2} \cdot \frac{dWL_2}{dx} \tag{6.6}$$

With reference to Fig. 6.1, $\partial A_1/\partial WL_1$ and $\partial A_1/\partial WL_2$ may be replaced by T_1 and $-T_2$ respectively.

Equation 6.3 may therefore be rewritten as

$$S_{1e} = S_1 - \frac{\alpha_1 Q_1^2}{g\ A_1^3} (T_1 S_1 - T_2 S_2) \tag{6.7}$$

Similarly, for the lower layer

$$\frac{dH_2}{dx} = S_{2e} = \frac{\partial H_2}{\partial WL_1} \cdot \frac{dWL_1}{dx} + \frac{\partial H_2}{\partial WL_2} \cdot \frac{dWL_2}{dx} + \frac{\partial H_2}{\partial A_2} \cdot \frac{dA_2}{dx} \tag{6.8}$$

$$\frac{\partial H_2}{\partial WL_1} = \frac{\rho_1}{\rho_2} \quad , \qquad \frac{\partial H_2}{\partial WL_2} = \frac{\Delta\rho}{\rho_2} \tag{6.9}$$

$$\frac{\partial H_2}{\partial A_2} = -\ \alpha_2 \frac{Q_2^2}{gA_2^3} \tag{6.10}$$

$$\frac{dA_2}{dx} = \frac{\partial A_2}{\partial WL_2} \cdot \frac{dWL_2}{dx} + \frac{\partial A_2}{\partial WL_0} \cdot \frac{dWL_0}{dx} \tag{6.11}$$

where $\partial A_2/\partial WL_2$ and $\partial A_2/\partial WL_0$ may be replaced by T_2 and $-T_2$ respectively. Therefore,

$$S_{2e} = \frac{\rho_1}{\rho_2} S_1 + \frac{\Delta\rho}{\rho_2} S_2 - \frac{\alpha_2 Q_2^2 T_2}{g\ A_2^3} (S_2 - S_0) \tag{6.12}$$

where S_0 is the bottom slope in the x-direction.

For simplicity, the following notation are used:

$$F_1^2 = \frac{\alpha_1 Q_1^2 T_1}{g\ A_1^3}\ ,\quad F_{11}^2 = \frac{\alpha_1 Q_1^2 T_2}{g\ A_1^3}\ ,\quad F_{i2}^2 = \frac{\alpha_2 Q_2^2 T_2}{g'\ A_2^3}\ ,\quad F_2^2 = \frac{\alpha_2 Q_2^2 T_2}{g\ A_2^3}$$

where $g' = g\ \Delta\rho/\rho_2$.

Equations 6.7 and 6.12 may then be rewritten as:

$$S_{1e} = (1 - F_1^2)\ S_1 + F_{11}^2\ S_2 \tag{6.13}$$

$$S_{2e} = \frac{\rho_1}{\rho_2}\ S_1 + \frac{\Delta\rho}{\rho_2}\ (1 - F_{i2}^2)\ S_2 + F_2^2\ S_0 \tag{6.14}$$

From equation 6.13

$$S_1 = \frac{S_{1e} - F_{11}^2\ S_2}{1 - F_1^2} \tag{6.15}$$

Substituting 6.15 into 6.14 yields

$$S_{2e} = \frac{\rho_1}{\rho_2}\left[\frac{S_{1e} - F_{11}^2\ S_2}{1 - F_1^2}\right] + \frac{\Delta\rho}{\rho_2}\ (1 - F_{i2}^2)\ S_2 + F_2^2\ S_0$$

Rearranging terms, S_2 may be expressed as follows

$$S_2 = \frac{-\frac{\rho_1}{\Delta\rho}\ S_{1e} + \frac{\rho_2}{\Delta\rho}\ (1 - F_1^2)\ S_{2e} - \frac{\rho_2}{\Delta\rho}\ F_2^2\ (1 - F_1^2)\ S_0}{(1 - F_1^2)\ (1 - F_{i2}^2) - \frac{\rho_1}{\Delta\rho}\ F_{11}^2} \tag{6.16}$$

The denominator of the right-hand side of equation 6.16 may be rearranged as follows:

$$(1 - F_1^2)\ (1 - F_{i2}^2) - \frac{\rho_1}{\Delta\rho} F_{11}^2 = \beta\ F_{i1}^2 \cdot F_{i2}^2 - F_{i1}^2 - F_{i2}^2 + 1 \qquad 6.17$$

where

$$F_{i1}^2 = \frac{\alpha_1 Q_1^2 T_1}{g'\ A_1^3} \quad \text{and} \quad \beta = \frac{\Delta\rho}{\rho_2}$$

The right-hand side of equation 6.17 may be called Y for convenience.

Substituting equation 6.16 into equation 6.15, S_1 may be expressed as

$$S_1 = \frac{(1 - F_{i2}^2)\ S_{1e} - \frac{\rho_2}{\Delta\rho} F_{11}^2\ S_{2e} + \frac{\rho_2}{\Delta\rho} F_{11}^2\ F_2^2\ S_0}{Y} \qquad 6.18$$

From equations 6.16 and 6.18,

$$S_1 \text{ and } S_2 \to \infty \quad \text{when} \quad Y = 0$$

which is shown previously to be the critical flow condition (Chapter 3, equation 3.5).

Equations 6.13 and 6.14 relate the energy gradients to the surface slopes while equations 6.16 and 6.18 define the slopes of the interface and free surface respectively in terms of the energy gradients of the two layers.

For uniform flow (i.e. $dA_1/dx = dA_2/dx = 0$, Q_1 and $Q_2 \neq 0$), the following relationships may be derived:

from equation 6.7 $$S_{1e} = S_1 \quad , \quad S_1 = S_2 \frac{T_2}{T_1} \qquad 6.19$$

from equation 6.12 $$S_2 = S_0 \qquad 6.20$$

also from equation 6.12 $$S_{2e} = \frac{\rho_1}{\rho_2} S_1 + \frac{\Delta\rho}{\rho_2} S_2 = S_1 \left(\frac{\rho_1}{\rho_2} + \frac{\Delta\rho}{\rho_2} \cdot \frac{T_1}{T_2}\right) \qquad 6.21$$

if $T_1 \approx T_2$ (e.g. rectangular section), $S_1 = S_2 = S_{1e} = S_{2e} = S_0$.

In the special case when the lower layer is moving and the upper layer is stagnant, the uniform flow condition is $dA_2/dx = 0$ which gives the following relations:

from equation 6.7 and for $Q_1 = 0$; $S_{1e} = S_1$ 6.19a

from equation 6.12: $S_2 = S_0$ 6.20a

also from equation 6.12:

$$S_{2e} = \frac{\rho_1}{\rho_2} S_1 + \frac{\Delta\rho}{\rho_2} S_2 \qquad 6.21a$$

6.2.2 Equations of Motion

Considering the fluid element shown in Fig. 6.1, the equation of motion for each layer may be written as follows.

For the upper layer

$$\rho_1 g A_1 \cdot \Delta x \cdot S_1 - \tau_i T_2 \cdot \Delta x - \tau_{01} P_1 \cdot \Delta x = -\rho_1 A_1 \cdot \Delta x \cdot V_1 \frac{dV_1}{dx} \qquad 6.22$$

where

τ_i = shear stress at the interface

τ_{01} = shear stress at the solid boundary for the upper layer

V_1 = mean velocity of the upper layer.

Equation 6.22 may be rewritten as

$$\tau_{01}\ P_1 + \tau_i\ T_2 = \rho_1\ g\ A_1\ [S_1 + \frac{d}{dx}\ (\frac{V_1^2}{2g})]$$

$$= \rho_1\ g\ A_1\ .\ S_{1e} \qquad 6.23$$

Thus,

$$S_{1e} = \frac{\tau_{01}\ P_1 + \tau_i\ T_2}{p_1\ g\ A_1} \qquad 6.24$$

Similarly for the lower layer

$$(\rho_2-\rho_1)\ g\ A_2\ .\ \Delta x\ .\ S_2 + \rho_1\ g\ A_2\ .\ \Delta x\ .\ S_1 + \tau_i\ T_2\ .\ \Delta x - \tau_{02}\ P_2\ .\ \Delta x$$

$$= -\ \rho_2\ A_2\ .\ \Delta x\ .\ V_2\ .\ \frac{dV_2}{dx} \qquad 6.25$$

where

τ_{02} = shear stress at the solid boundary for the lower layer

V_2 = mean velocity of the lower layer.

The second term on the left-hand side of equation 6.25 represents the differential hydrostatic force exerted by the upper layer on the fluid element of the lower layer.

Equation 6.25 may be expressed as

$$\tau_{02} P_2 - \tau_i T_2 = \rho_2 g A_2 \left[\frac{\Delta\rho}{\rho_2} S_2 + \frac{\rho_1}{\rho_2} S_1 + \frac{d}{dx}\left(\frac{V_2^2}{2g}\right)\right]$$

$$= \rho_2 g A_2 \cdot S_{2e} \qquad 6.26$$

Therefore,

$$S_{2e} = \frac{\tau_{02} P_2 - \tau_i T_2}{\rho_2 g A_2} \qquad 6.27$$

6.2.3 Shear Stresses and Shear Stress Coefficients

The shear stresses τ_{01}, τ_{02} and τ_i are given by

$$\tau_{01} = f_{01} \frac{\rho_1}{8} |V_1| V_1 \qquad 6.28$$

$$\tau_{02} = f_{02} \frac{\rho_2}{8} |V_2| V_2 \qquad 6.29$$

$$\tau_i = f_i \frac{\bar{\rho}}{8} |V_1 - V_2| (V_1 - V_2) \quad , \quad \bar{\rho} = \frac{\rho_1 + \rho_2}{2} \qquad 6.30$$

where

f_{01} = solid boundary shear stress coefficient for the upper layer

f_{02} = solid boundary shear stress coefficient for the lower layer

f_i = interfacial shear stress coefficient.

With reference to the literature review presented in Chapter 2, the following relationships are used to calculate the shear stress coefficients:

6.2.3.1 Boundary Coefficients

The case is considered when one layer is flowing while the other layer is stagnant. The moving layer is treated as a homogeneous flow case.

The Reynolds number is defined by

$$R_e = \frac{4VR}{\nu'} \qquad 6.31$$

where

V = mean velocity of flow

R = hydraulic radius of the moving layer

ν' = kinematic viscosity of the moving layer

With reference to Streeter (1971):

for laminar flow ($R_e \leq 2000$) $$f_0 = \frac{64}{R_e} \qquad 6.32$$

for turbulent flow ($R_e > 2000$):

a) smooth boundary $$f_0 = \frac{0.316}{R_e^{0.25}} \qquad 6.33$$

b) rough boundary ($D \neq 0$) $$\frac{1}{\sqrt{f_0}} = -0.86 \ln \left[\frac{D_r}{3.7} + \frac{2.51}{R_e \sqrt{f_0}}\right] \qquad 6.34$$

where D_r = relative roughness = $D/4R$

D = roughness height

It should be noted that, for turbulent flow, three different conditions may be defined with respect to the boundary:

i) smooth $f_0 = \phi(R_e)$

ii) transitional $f_0 = \phi(R_e, D_r)$

and iii) fully-developed rough $f_0 = \phi(D_r)$

The three conditions are adequately described by Colebrook equation (6.34). The Blasius formula (6.33) is interpreted by many as an approximation to 6.34 for $R_e \leq 100{,}000$.

6.2.3.2 Interfacial Coefficient

In the following sub-sections, transition criteria are presented which define the types of flow and interfacial boundary layers. Then, based on previous contributions, several relationships are presented to describe the interfacial shear stress coefficient under different flow and boundary layer conditions.

Transition Criteria (Keulegan - 1949)

The case is considered when one layer is flowing while the other layer is stagnant. θ is a parameter that determines the type of the interfacial boundary layer (i.e. laminar or turbulent) and is defined by:

$$\theta^3 = \frac{\nu g(\Delta\rho/\rho)}{V^3} \qquad 6.35$$

where

ν = kinematic viscosity of the stagnant layer

ρ = density of the moving layer

Also, the Reynolds number R_1 is defined by

$$R_1 = \frac{V.R}{\nu} \qquad 6.36$$

$\theta_c = 0.178$ for $R_1 > 450$ (turbulent flow)

$\theta_c = 0.127$ for $R_1 \leq 450$ (laminar flow)

If $\theta < \theta_c$, the boundary layers are turbulent.

If $\theta \geq \theta_c$, the boundary layers are laminar.

In order to evaluate the interfacial shear stress coefficient f_i, the following cases are considered:

a) laminar flow/laminar boundary layers

b) turbulent flow/laminar boundary layers

  i) flow established in one layer

  ii) flow established in both layers

  iii) flow non-established in both layers

c) turbulent flow/turbulent boundary layers

a) Laminar Flow/Laminar Boundary Layers

[Ippen and Harleman (1952) referenced by Abraham and Eysink (1971)]

For the case of a flowing lower layer:

$$f_i = \frac{11.3}{R_2} \qquad 6.37$$

where

$R_2 = h_r.V/\nu$ and $h_r = A/(P+T)$

A = cross-sectional area of the lower layer

P = wetted perimeter of the lower layer

T = top width of the lower layer

b) Turbulent Flow/Laminar Boundary Layers

In a two-layer stratified flow system, a boundary layer is developed in both layers along the interface starting from the point of contact of the two fluids. Depending upon the depths of both layers, sooner or later the boundary layers will expand over the whole region of flow, if the region is long enough, and the flow will be established.

In this part, the case is considered where one layer is flowing and the other layer is stagnant under the following conditions:

i) <u>Flow established in the flowing layer</u> [Bata - 1959]

The interfacial shear stress coefficient is given by:

$$\frac{(384 - f_i \cdot R_1)^{3/2}}{4 f_i \cdot R_1 - 384} = \frac{31.2}{\sqrt{R_1 . M}} \qquad 6.38$$

where

M = $(h/L) \cdot (\mu_1/\mu) \cdot (\rho_1/\rho)$

h = depth of the moving layer

L = total length of the moving layer from the ponit of contact of the two fluids.

μ_1 = dynamic viscosity of the stagnant layer

μ = dynamic viscosity of the moving layer

ρ_1 = density of the stagnant layer

ρ = density of the moving layer

R_1 = Reynolds number = $4Vh/\nu'$

ii) <u>Flow established in both layers (uniform flow)</u> [Bata - 1959]

$$f_i = 384 \frac{3 + N}{3 + 4N} \cdot \frac{1}{R_1} \qquad 6.39$$

where

N = $(h_1/h) \cdot (\mu/\mu_1)$

h_1 = depth of the stagnant layer

iii) <u>Flow non-established in both layers</u> [Keulegan - 1944, Lock - 1951]

$$f_i = \frac{K}{\sqrt{R_x}} \qquad 6.40$$

where

K = coefficient

R_x = Reynolds number = $V.x/\nu'$

x = distance from the point of contact of the two fluids.

The coefficient K as well as the thicknesses of the interfacial boundary layers which determine the type of flow establishment are calculated using the method of successive approximations developed by Keulegan (1944) based on Prandtl's boundary-layer theory. This method is also described by Elsayed (1975).

c) Turbulent Flow/Turbulent Boundary Layers

[Dick and Marsalek (1973)]

In the case when the lower layer is flowing while the upper layer is stagnant,

$$f_i = \frac{0.316}{R^{0.25}} \qquad 6.41$$

where

R = $4Vh'/\nu'$

h' = hydraulic mean depth of the lower layer.

6.3 DEVELOPMENT OF COMPUTER ROUTINES

Complete documentation and listings of the developed subroutines are contained in Appendix (2). Figure 6.2 describes briefly the functions of the basic routines.

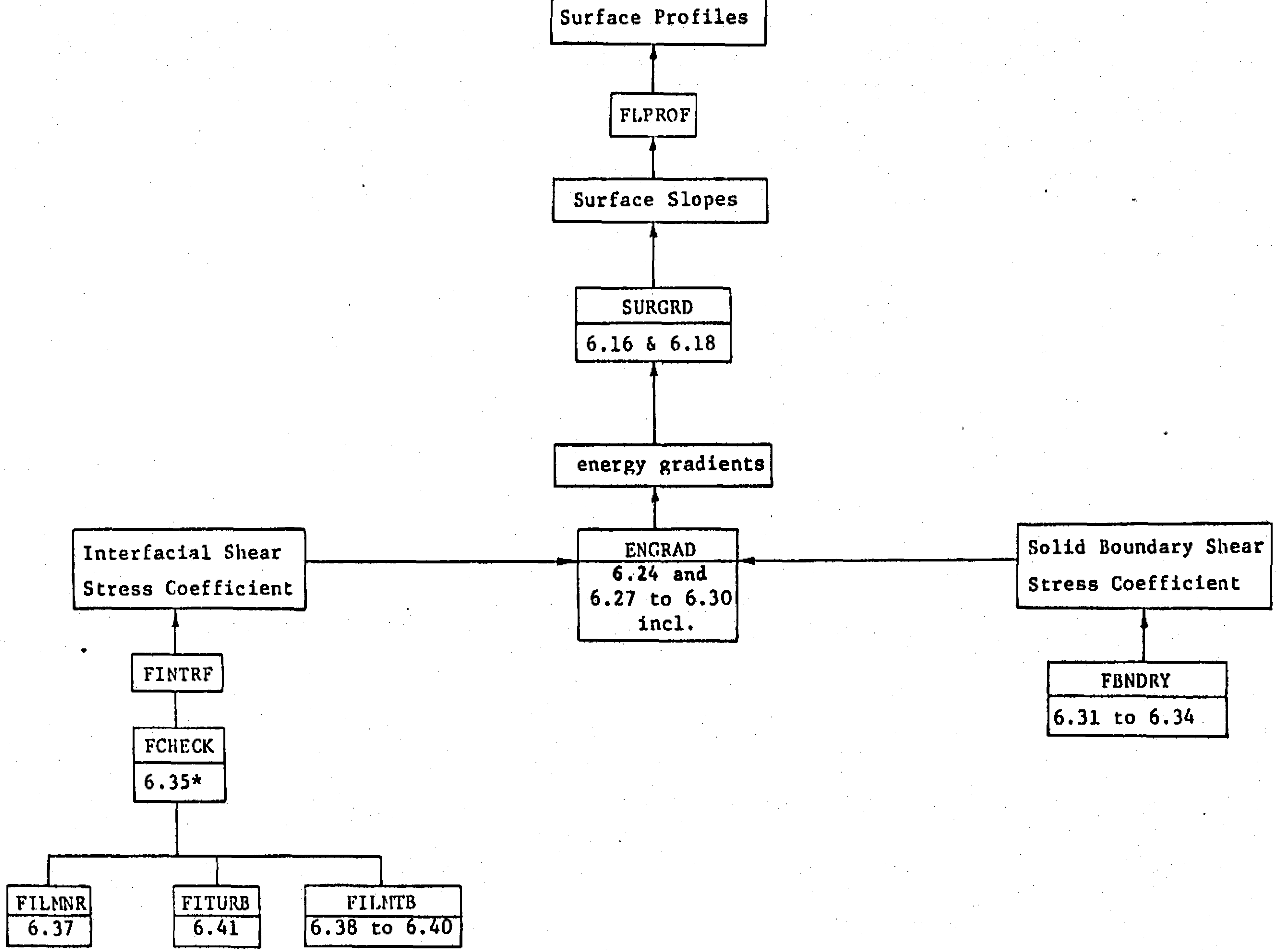


Fig. 6.2 - Subroutines Developed for Long Transitions

* equations used

6.3.1 Subroutine FLPROF

A stretch of a river containing stratified flow is described by a sequence of cross-sections. All sections are defined by the same number of coordinate pairs. Subroutine FLPROF operates on the system thus defined and for a selected reach calculates the free surface and the interface elevations as well as energy levels for both layers at one end of the reach as functions of specified discharges and water levels at the other end with a specified incremental distance appropriately signed. The subroutine is intended for successive applications working in either the upstream or the downstream direction.

The surface slopes given by equation 6.16 and 6.18 are integrated along an elementary reach of the river using the Runge-Kutta-Merson technique. This differential equation solver algorithm is described as follows.

Equations 6.16 and 6.18 may be expressed in the form

$$\frac{dWL_1}{dx} = \phi_1 \, (WL_1, \, WL_2, \, x) \qquad 6.42$$

$$\frac{dWL_2}{dx} = \phi_2 \, (WL_1, \, WL_2, \, x) \qquad 6.43$$

The unknown elevations WL_1 and WL_2 at section 2 which is distant Δx from section 1 that has a chainage x_1 are explicitly defined by the following relationships.

$$(WL_1)_2 = (WL_1)_1 + (F_{11} + 4 \, F_{41} + F_{51}) \cdot \frac{\Delta x}{2} \qquad 6.44$$

$$(WL_2)_2 = (WL_2)_1 + (F_{12} + 4 \, F_{42} + F_{52}) \cdot \frac{\Delta x}{2} \qquad 6.45$$

where

$$F_{11} = \phi_1\,[x_1,\ (WL_1)_1,\ (WL_2)_1]/3$$

$$F_{12} = \phi_2\,[x_1,\ (WL_1)_1,\ (WL_2)_1]/3$$

$$F_{21} = \phi_1\,[(x_1 + \frac{\Delta x}{3}),\ ((WL_1)_1 + F_{11} \cdot \Delta x),\ ((WL_2)_1 + F_{12} \cdot \Delta x)]/3$$

$$F_{22} = \phi_2\,[(x_1 + \frac{\Delta x}{3}),\ ((WL_1)_1 + F_{11} \cdot \Delta x),\ ((WL_2)_1 + F_{12} \cdot \Delta x)]/3$$

$$F_{31} = \phi_1\,[(x_1 + \frac{\Delta x}{3}),\ ((WL_1)_1 + F_{21} \cdot \frac{\Delta x}{2} + F_{11} \cdot \frac{\Delta x}{2}),$$
$$((WL_2)_1 + F_{22} \cdot \frac{\Delta x}{2} + F_{12} \cdot \frac{\Delta x}{2})]/3$$

$$F_{32} = \phi_2\,[(x_1 + \frac{\Delta x}{3}),\ ((WL_1)_1 + F_{21} \cdot \frac{\Delta x}{2} + F_{11} \cdot \frac{\Delta x}{2}),$$
$$((WL_2)_1 + F_{22} \cdot \frac{\Delta x}{2} + F_{12} \cdot \frac{\Delta x}{2})]/3$$

$$F_{41} = \phi_1\,[(x_1 + \frac{\Delta x}{2}),\ ((WL_1)_1 + 9 \cdot F_{31} \cdot \frac{\Delta x}{8} + 3 \cdot F_{11} \cdot \frac{\Delta x}{8}),$$
$$((WL_2)_1 + 9 \cdot F_{32} \cdot \frac{\Delta x}{8} + 3 \cdot F_{12} \cdot \frac{\Delta x}{8})]/3$$

$$F_{42} = \phi_2\,[(x_1 + \frac{\Delta x}{2}),\ ((WL_1)_1 + 9 \cdot F_{31} \cdot \frac{\Delta x}{8} + 3 \cdot F_{11} \cdot \frac{\Delta x}{8}),$$
$$((WL_2)_1 + 9 \cdot F_{32} \cdot \frac{\Delta x}{8} + 3 \cdot F_{12} \cdot \frac{\Delta x}{8})]/3$$

$$F_{51} = \phi_1 \, [(x_1 + \Delta x), ((WL_1)_1 + 6 \cdot F_{41} \cdot \Delta x - 9 \cdot F_{31} \cdot \frac{\Delta x}{2} + 3 \cdot F_{11} \cdot \frac{\Delta x}{2}), ((WL_2)_1 + 6 \cdot F_{42} \cdot \Delta x - 9 \cdot F_{32} \cdot \frac{\Delta x}{2} + 3 \cdot F_{12} \cdot \frac{\Delta x}{2})]/3$$

$$F_{52} = \phi_2 \, [(x_1 + \Delta x), ((WL_1)_1 + 6 \cdot F_{41} \cdot \Delta x - 9 \cdot F_{31} \cdot \frac{\Delta x}{2} + 3 \cdot F_{11} \cdot \frac{\Delta x}{2}), ((WL_2)_1 + 6 \cdot F_{42} \cdot \Delta x - 9 \cdot F_{32} \cdot \frac{\Delta x}{2} + 3 \cdot F_{12} \cdot \frac{\Delta x}{2})]/3$$

The method also yields a measure of the truncation errors given by:

$$\text{ERROR1} = [2 \cdot F_{11} - 9 \cdot F_{31} + 8 \, F_{41} - F_{51}] \cdot \frac{\Delta x}{10}$$

$$\text{ERROR2} = [2 \cdot F_{12} - 9 \cdot F_{32} + 8 \, F_{42} - F_{52}] \cdot \frac{\Delta x}{10}$$

These are not correction terms, but merely measures of the error. If either error is greater than a certain allowable tolerence, then Δx is halved and the surface elevations recalculated for this reduced step size. On the other hand, if the error is less than 1/32 of the allowable tolerence, Δx is doubled for the next step.

6.3.2 Typical Examples

The following examples show typical results obtained from two of the high-level routines. Examples of the driver programs needed to operate these and all the other routines are included in the documentation (Appendix 2, p. 330 and 337).

a) Subroutine FLPROF

A prismatic channel has the cross-section shown in Fig. 6.3. It consists of three reaches (four sections) which have the following characteristics.

Section	Chainage (m)	roughness height (m)
1	30	0.02
2	100	0.05
3	300	0.04
4	500	0.03

Upper layer characteristics:

discharge = 0.0, specific gravity = 1.00, kinematic viscosity = 1×10^{-6} m^2/sec.

Lower layer characteristics:

discharge = 5.0 m^3/sec., specific gravity = 1.05, kinematic viscosity = 1.1×10^{-6} m^2/sec., kinetic energy correction factor = 1.20

Δx = -30.00 m

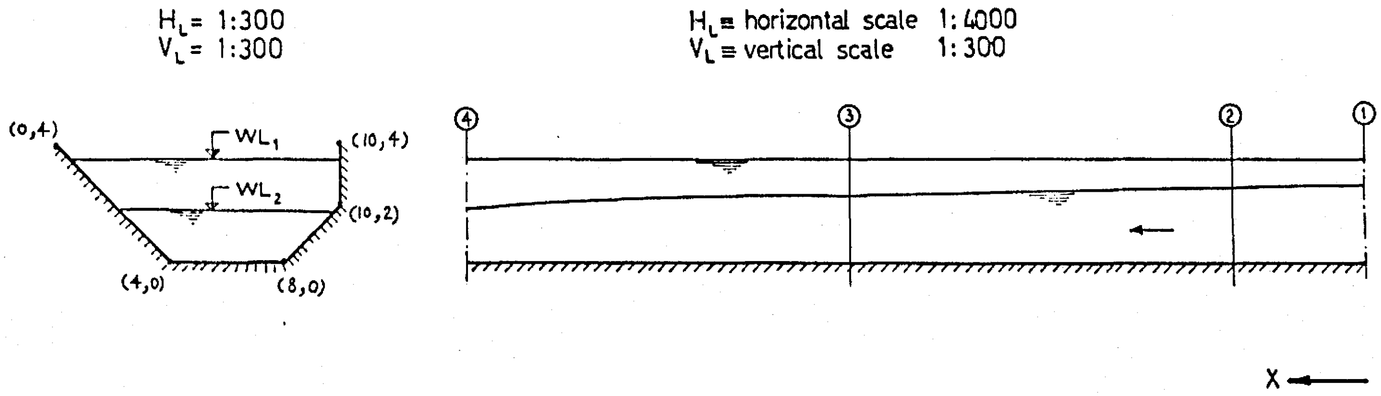


Fig. 6.3 - Example (a) Profile Computations

Free surface and interface elevations at section 4 are 3.50 and 1.80 m respectively.

The computed profiles are listed in Table 6.1 and shown in Fig. 6.3.

It should be noted that for WL_1 = 3.50 m, the critical interface elevation is 1.40 m (using WLCRIT1) which means that the flow is subcritical throughout. The free surface profile is nearly horizontal.

b) Subroutine ARSWDG

This subroutine computes the profiles for an arrested wedge (either overflow or underflow) assuming critical condition at the river mouth (or the warm water outlet).

i) Underflow Wedge (Salt Wedge)

A channel reach is defined by the two end cross-sections shown in Fig. 6.4. These sections have the following characteristics.

	Section	chainage (m)	roughness height (m)
	1	6000	0.01
River mouth →	2	10000	0.01

Upper layer characteristics:

discharge = 6.0 m^3/sec., specific gravity = 1.0,

kinematic viscosity = 1.02 x 10^{-6} m^2/sec.,

kinetic energy correction factor = 1.10

Section	Chainage (m)	WL_1 (m)	WL_2 (m)
4	500	3.5000	1.8000
	470	3.4996	1.9100
	440	3.4993	1.9944
	410	3.4990	2.0644
	350	3.4983	2.1781
3	300	3.4978	2.2565
3	300	3.4978	2.2565
	270	3.4975	2.3011
	210	3.4969	2.3813
2	100	3.4958	2.5069
2	100	3.4958	2.5069
	70	3.4955	2.5361
1	30	3.4951	2.5735

Table 6.1 Results of Example (a) - Profile Computations

Lower layer characteristics:

discharge = 0.0, specific gravity = 1.02

kinematic viscosity = 1.11×10^{-6} m^2/sec.

$\Delta x = -100$ m, $f_i = 0.003$

Free surface elevation at the river mouth (section 2) is 2.50 m.

The computed profiles are listed in Table 6.2 and shown in Fig. 6.4.

It should be noted that a value for f_i has to be defined by the user in this example since there is no theoretical basis available to determine f_i under these conditions (i.e. turbulent flow-turbulent interfacial boundary layers - flowing upper layer - stagnant lower layer).

ii) <u>Overflow Wedge (Thermal Wedge)</u>

A prismatic channel has the cross-section shown in Fig. 6.5.

Section	chainage (m)	roughness height (m)
1	6000	0.01
2	10000	0.01

Warm water outlet

Characteristics of both layers are the same as the previous example except that the upper and lower layer discharges are 0.0 and 5.0 m^3/sec. respectively.

Free surface elevation at section 2 is 2.50 m.

The computed profiles are listed in Table 6.3 and shown in Fig. 6.5.

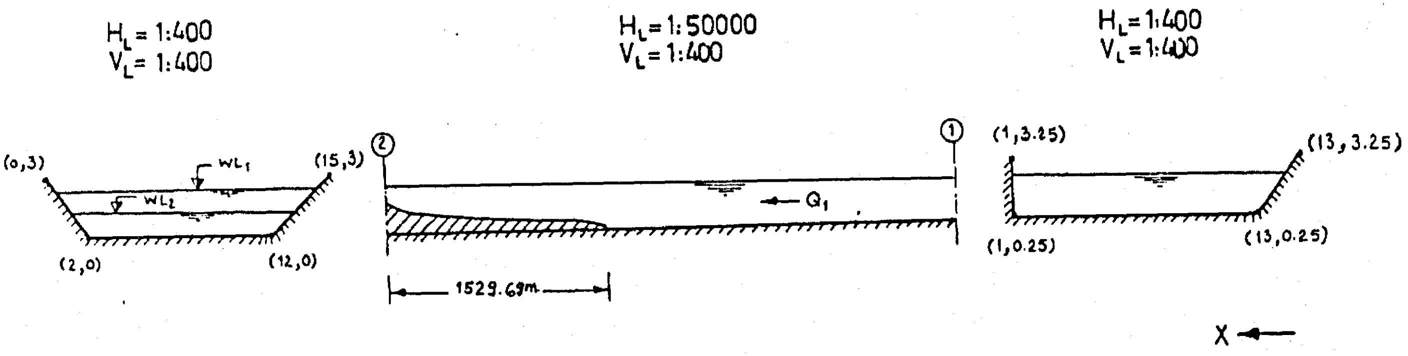


Fig. 6.4 - Example (b-i) Arrested Salt Wedge

Section	Chainage (m)	WL_1 (m)	WL_2 (m)
2	10000.00	2.5000	1.4219
	9996.88	2.5004	1.3995
	9993.75	2.5007	1.3828
	9990.63	2.5010	1.3688
	9984.38	2.5015	1.3454
	9978.13	2.5018	1.3258
	9965.63	2.5024	1.2931
	9953.13	2.5029	1.2655
	9928.13	2.5037	1.2194
	9903.13	2.5044	1.1805
	9853.13	2.5055	1.1152
	9803.13	2.5064	1.0601
	9753.13	2.5071	1.0113
	9653.13	2.5084	.9260
	9553.13	2.5095	.8512
	9453.13	2.5105	.7833
	9253.13	2.5121	.6601
	9053.13	2.5135	.5458
	8853.13	2.5146	.4324
	8753.13	2.5152	.3730
	8653.13	2.5157	.3084
	8603.13	2.5159	.2721
	8553.13	2.5162	.2307
	8528.13	2.5163	.2064
	8503.12	2.5165	.1771
	8490.63	2.5165	.1587
	8484.37	2.5166	.1475
	8478.13	2.5166	.1335
	8475.00	2.5166	.1244
	8473.44	2.5167	.1187
	8471.87	2.5167	.1110

Wedge length = 1529.69 m

$f_{01} = 0.0219$, $f_i = 0.0030$

Table 6.2 Results of Example (b-i) - Arrested Salt Wedge

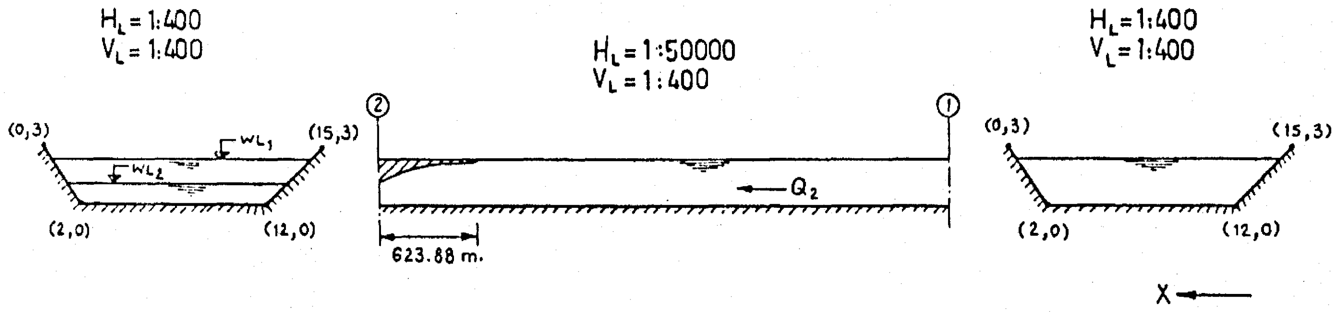


Fig. 6.5 - Example (b-ii) Arrested Thermal Wedge

Section	Chainage (m)	WL_1 (m)	WL_2 (m)
2	10000.00	2.5000	1.1341
	9999.94	2.5000	1.1410
	9999.88	2.5000	1.1458
	9999.75	2.5000	1.1533
	9999.63	2.5000	1.1593
	9999.38	2.5000	1.1688
	9999.13	2.5000	1.1767
	9998.63	2.5000	1.1896
	9996.13	2.5000	1.2003
	9997.13	2.5000	1.2180
	9996.12	2.5000	1.2327
	9994.13	2.4999	1.2570
	9992.13	2.4999	1.2773
	9988.13	2.4999	1.3109
	9964.12	2.4998	1.3387
	9976.13	2.4997	1.3848
	9968.13	2.4997	1.4229
	9952.13	2.4995	1.4858
	9936.12	2.4993	1.5378
	9920.13	2.4992	1.5831
	9888.13	2.4989	1.6605
	9856.13	2.4986	1.7268
	9792.13	2.4980	1.8393
	9728.13	2.4974	1.9363
	9664.13	2.4967	2.0246
	9536.13	2.4951	2.1925
	9472.13	2.4940	2.2821
	9440.13	2.4934	2.3335
	9408.13	2.4924	2.3982

wedge length = 623.88 m

$f_{02} = 0.0220$, $f_i = 0.0094$

Table 6.3 Results of Example (b-ii) - Arrested Thermal Wedge

It should be mentioned that in subroutine ARSWDG, the interface profile computations start at either 0.999 WL_{2cr} or 1.001 WL_{2cr} depending on whether the stagnant layer is the lower or the upper respectively (WL_{2cr} is the critical interface elevation). It is shown from the previous two examples that the interface profile moves such that the conditions upstream of the control are subcritical. The computation of the wedge profile is terminated when the depth of the wedge is less than 4×10^{-5}.

6.3.3 Comparison with Published Experimental and Field Data

Figures 6.6 to 6.9 show comparisons between published laboratory-flume data and computed results. Figure 6.10 shows a comparison with measured field data from four different steam plant sites. Table 6.4 shows the basic data for these four sites.

It should be noted that the non-dimensional terms used in plotting these measurements are those used in the original plots. While some authors use these quantities to suggest some functional relationships, it is shown that this may not be correct. For example, in Fig. 6.10, it is shown that the relative length of the thermal wedge is not solely a function of Froude number. It depends also on other parameters such as the channel geometry. Therefore, in Figs. 6.6 to 6.10, the actual data given by the authors have been used for computations rather than simply checking the validity of the suggested functional relationships.

In plotting the experimental results of Fig. 6.8, Bata (1957)

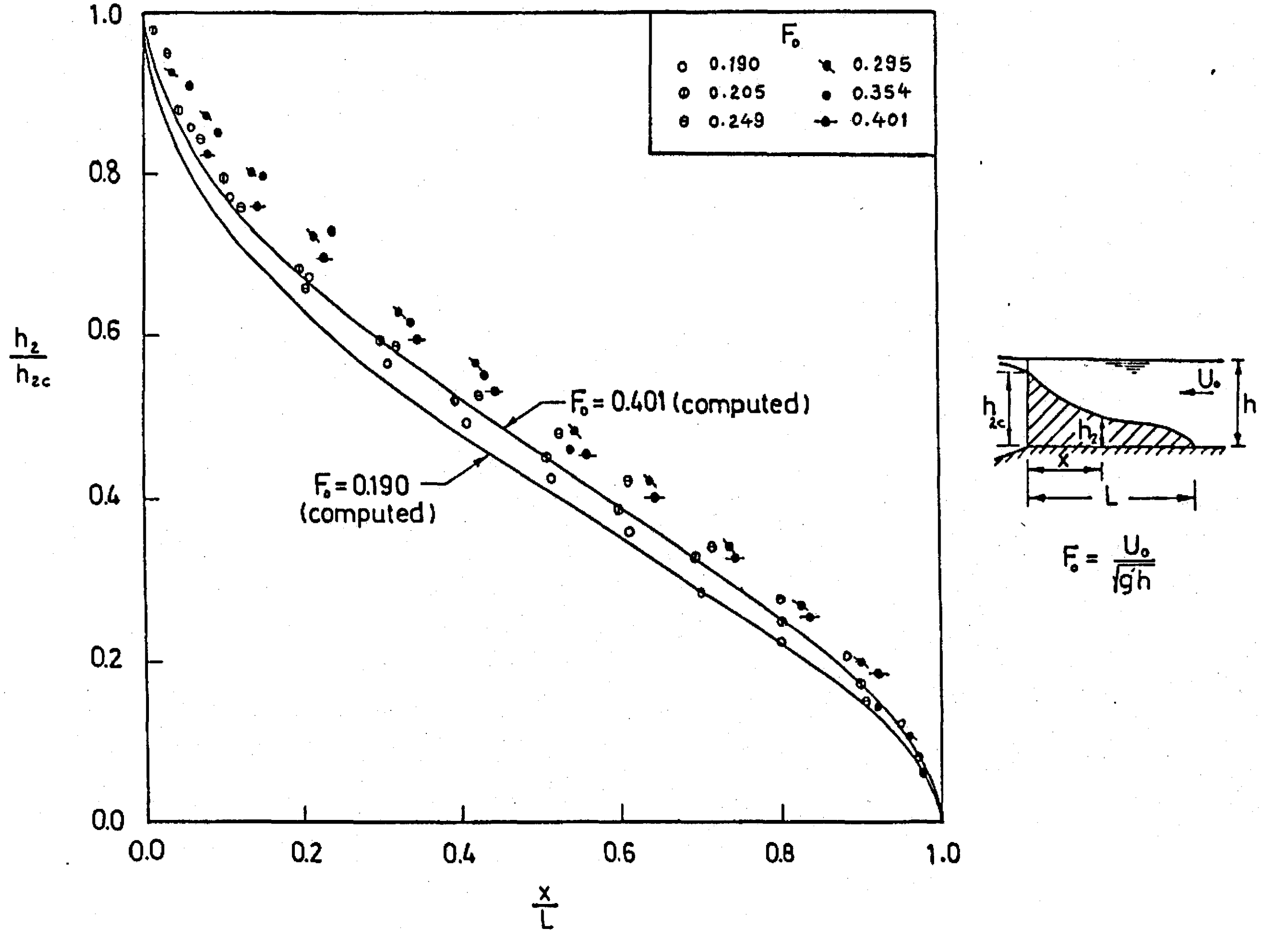


Fig. 6.6 - Relative Shape of Arrested Salt Wedge
[Experiments after Keulegan (1952)]

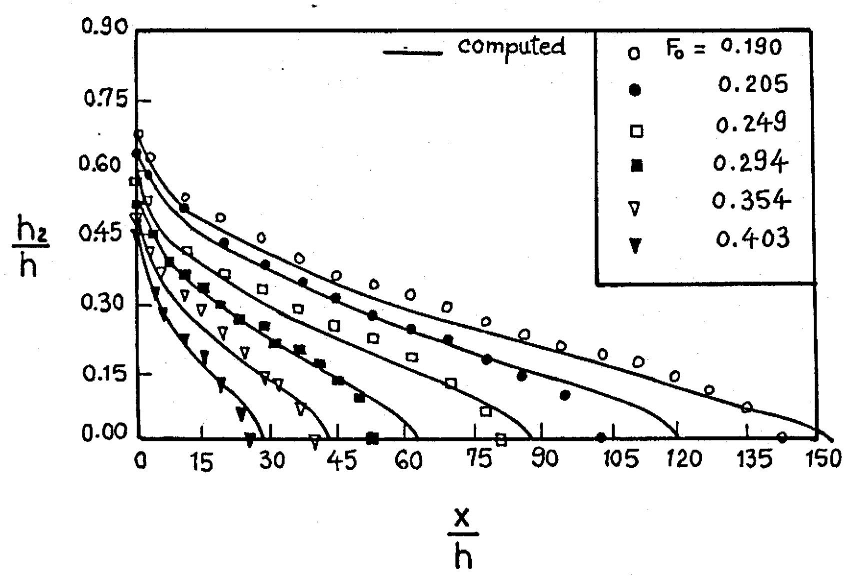


Fig. 6.7 - Form of Arrested Salt Wedge
[Experiments after Keulegan (1966)]

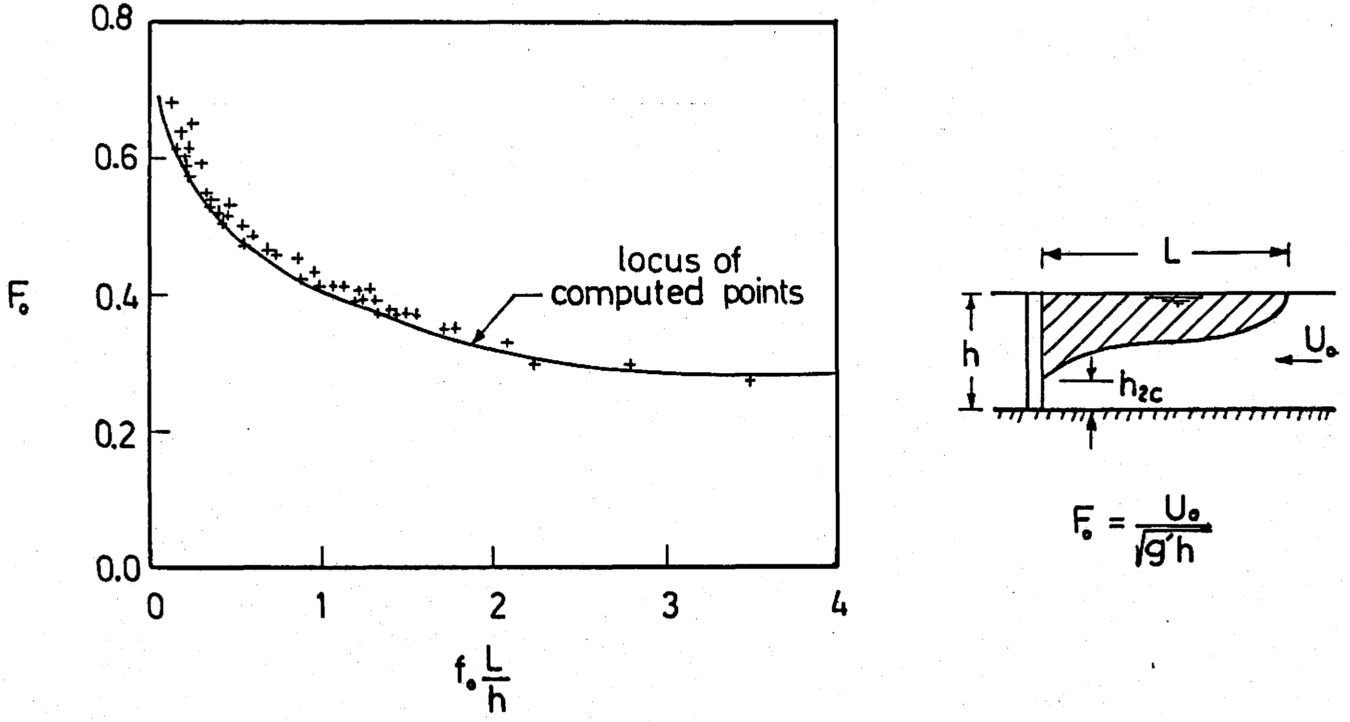


Fig. 6.8 - Length of Thermal Wedge
[Experiments after Bata (1957)]

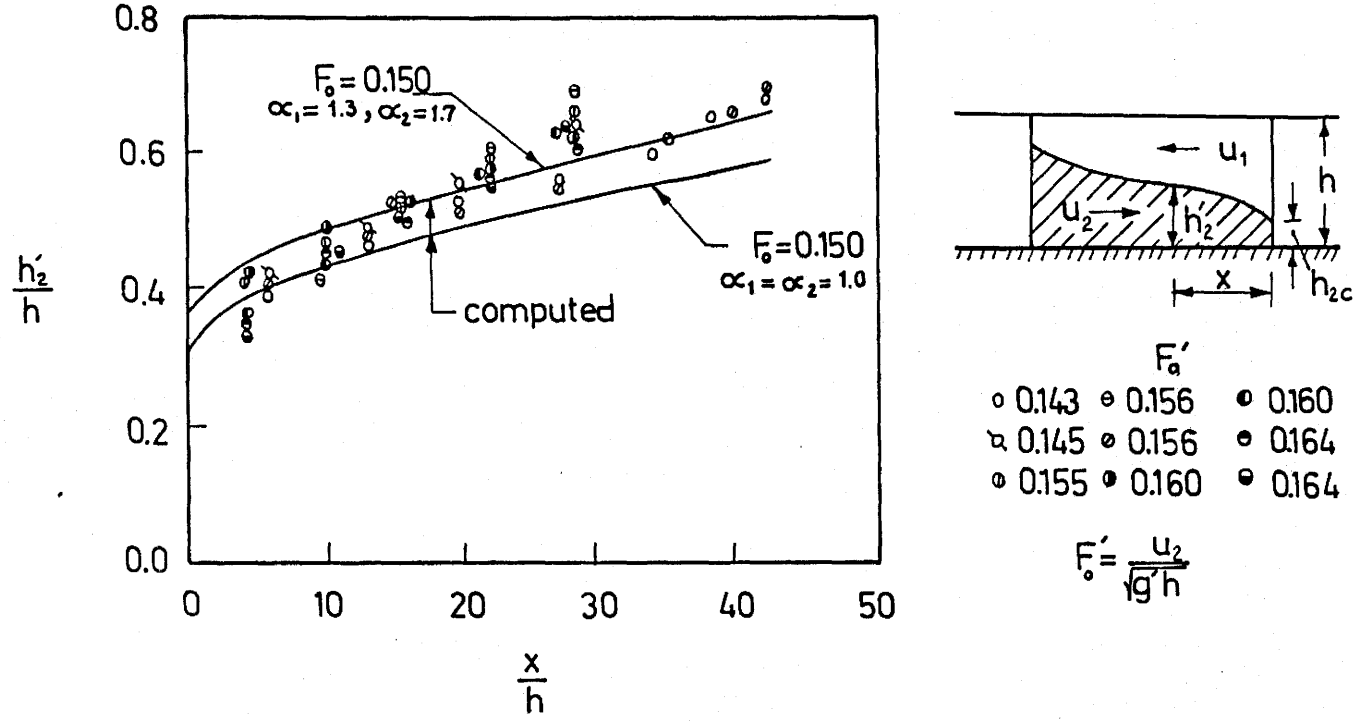
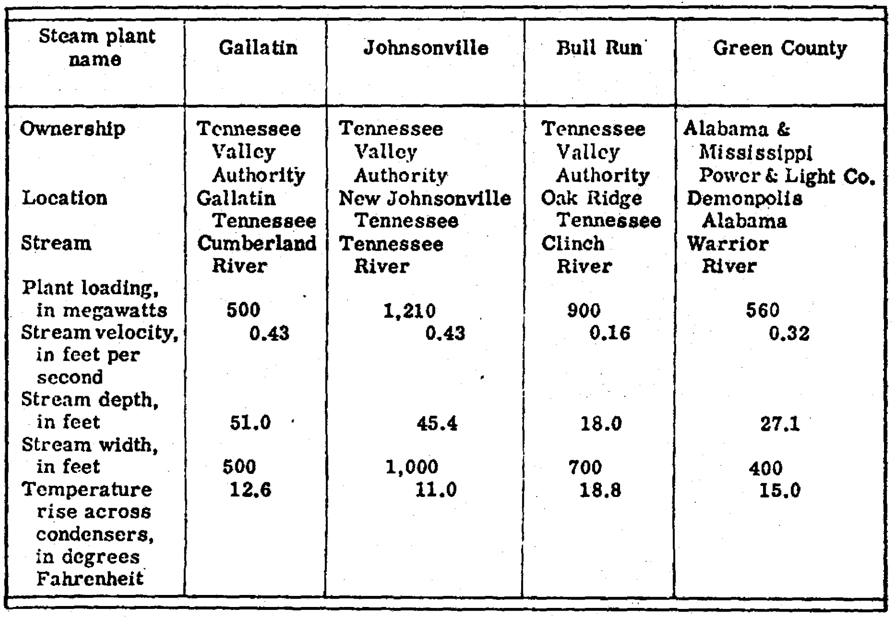


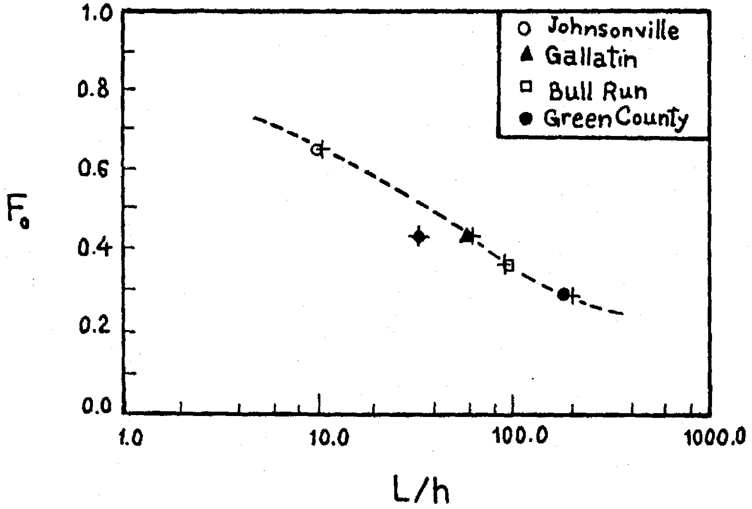
Fig. 6.9 - Shape of Thermal Wedge When Both Layers are Flowing
[Experiments after Bata (1957)]

Table 6.4 Basic Data for Steam Plant Sites Surveyed

[after Polk, Benedict and Parker (1971)]



Steam plant name	Gallatin	Johnsonville	Bull Run	Green County
Ownership	Tennessee Valley Authority	Tennessee Valley Authority	Tennessee Valley Authority	Alabama & Mississippi Power & Light Co.
Location	Gallatin Tennessee	New Johnsonville Tennessee	Oak Ridge Tennessee	Demonpolis Alabama
Stream	Cumberland River	Tennessee River	Clinch River	Warrior River
Plant loading, in megawatts	500	1,210	900	560
Stream velocity, in feet per second	0.43	0.43	0.16	0.32
Stream depth, in feet	51.0	45.4	18.0	27.1
Stream width, in feet	500	1,000	700	400
Temperature rise across condensers, in degrees Fahrenheit	12.6	11.0	18.8	15.0



✦ Gallatin – computed using arbitrary data keeping $F_o = 0.45$.

\+ computed using actual data for each of the four stations.

Fig. 6.10 - Length of Thermal Wedge [Field data - after Polk, Benedict and Parker (1971)]

assumes that f_0 (the boundary shear stress coefficient) has a constant value along the wedge. In the computed results, the calculated f_0 varies along the wedge. Therefore, to plot the computed results, an average value of f_0 is considered for each wedge. For example, for $F_0 = 0.4$, f_0 varies from 0.0293 at the critical section to 0.0281 at the upstream leading edge.

6.4 DISCUSSION

1. The theoretical analysis presented in this chapter concerning energy gradients and surface profiles considers channels of arbitrary geometry with the purpose of possible application in practical problems. Previous theoretical work [e.g. Bata (1957) - Harleman (1961) - Partheniades, Dermisis and Mehta (1975)] is based on several simplifying assumptions such as:

   a) Very small density differences.

   b) The variation in the total depth of flow is negligible.

   c) Horizontal bottom.

   d) Idealized rectangular strip of the channel cross-section (infinite width).

   e) Uniform velocity distributions in the vertical direction.

   None of these limiting assumptions are required in the present treatment.

2. There is still some uncertainty in the evaluation of the shear stress coefficients under some flow conditions, e.g. when both

layers are flowing. In the present study, and under such circumstances, estimated values for these coefficients have to be supplied by the user. These could be based on laboratory or field measurements. Also, the method adopted for the evaluation of the interfacial shear stress coefficient when both the flow and the interfacial boundary layers are turbulent is suggested by Dick and Marsalek (1973). Their approach is completely arbitrary and is supported only by experimental evidence. One of the advantages of the modular nature of a computer library is the possibility to incorporate in a properly-designed library package, a new or modified routine concerned with a relatively elementary problem type or solution technique. For example, subroutine FITURB may be modified or replaced by another whenever a more justifiable approach is available. Although it is relatively simple to devise subroutines for the prediction of f_i in certain of these as yet undefined situations, these would of necessity be based on assumptions which might be more ingeneous than justifiable. This has not been done because of the danger that any algorithm encoded as part of a library may be assumed by the user to be theoretically or experimentally substantiated.

3. Good agreement is shown to exist between the computed results and the published laboratory and field data. It is also shown that the channel geometry is a significant parameter which makes the assumptions of horizontal bottom and infinite width not

justifiable. Also, the non-uniformity of the velocity distributions in the vertical direction which is taken into account through the use of kinetic energy correction factors is shown to have a significant effect [Fig. 6.9].

CHAPTER 7

PRACTICAL APPLICATIONS

7.1 INTRODUCTION

This chapter includes some examples which illustrate the applicability of the developed computer sub-programs to the solution and analysis of typical practical problems. Each example is accompanied by typical computations using laboratory and field data when available. These examples demonstrate that the computational modules (i.e. subroutines) may provide solutions for frequently recurring problems, are mutually compatible and allow the construction of relatively complex analytical models in a modular fashion which may be used in the analysis and design of large water resources projects.

7.2 EXAMPLES

7.2.1 Example (1)

Selective withdrawal at an intermediate depth.

Selective withdrawal from a stratified body of water is a means of providing water of desired quality for municipal, agricultural, or recreational use. When discrete layers of distinct density difference exist, it is possible to withdraw from only one, or from several, of these layers, and the term "selective withdrawal" is used to describe this process.

In this example, a fluid is withdrawn from a two-layer system with a horizontal intake located on a vertical boundary, (Fig. 7.1) as in the case of the upstream face of a dam. The intake is located above the initially horizontal interface. It is desired to determine an upper limit to the rate of abstraction of the upper fluid at which flow the interface will be raised to the level of the intake, thus resulting in entrainment from the lower layer. With reference to Fig. 7.1, subroutine FLPROF may be used to compute the surface elevations at section I.I for a specified upper layer flow given the elevations at section II-II. Thus, for a specified interface level at section I.I, the limiting upper layer flow may be computed by trial using subroutine FLPROF. In this problem, the abstraction is assumed to be along the entire width of the downstream section (SectionII.II).

Given

- Geometry of the end cross-sections (Fig. 7.1). (More complex geometry may be described by as many cross-sections as necessary)
- Water levels at section II.II (Fig. 7.1).
- Upper layer:

  discharge = 100 m^3/sec. (assumed trial value), specific gravity = 1.00, kinematic viscosity = 1.02×10^{-6} m^2/sec., kinetic energy correction factor = 1.1.
- Lower layer:

  discharge = 0.0, specific gravity = 1.05, kinematic viscosity = 1.11×10^{-6} m^2/sec.
- Roughness height at both cross-sections is 0.02 m.

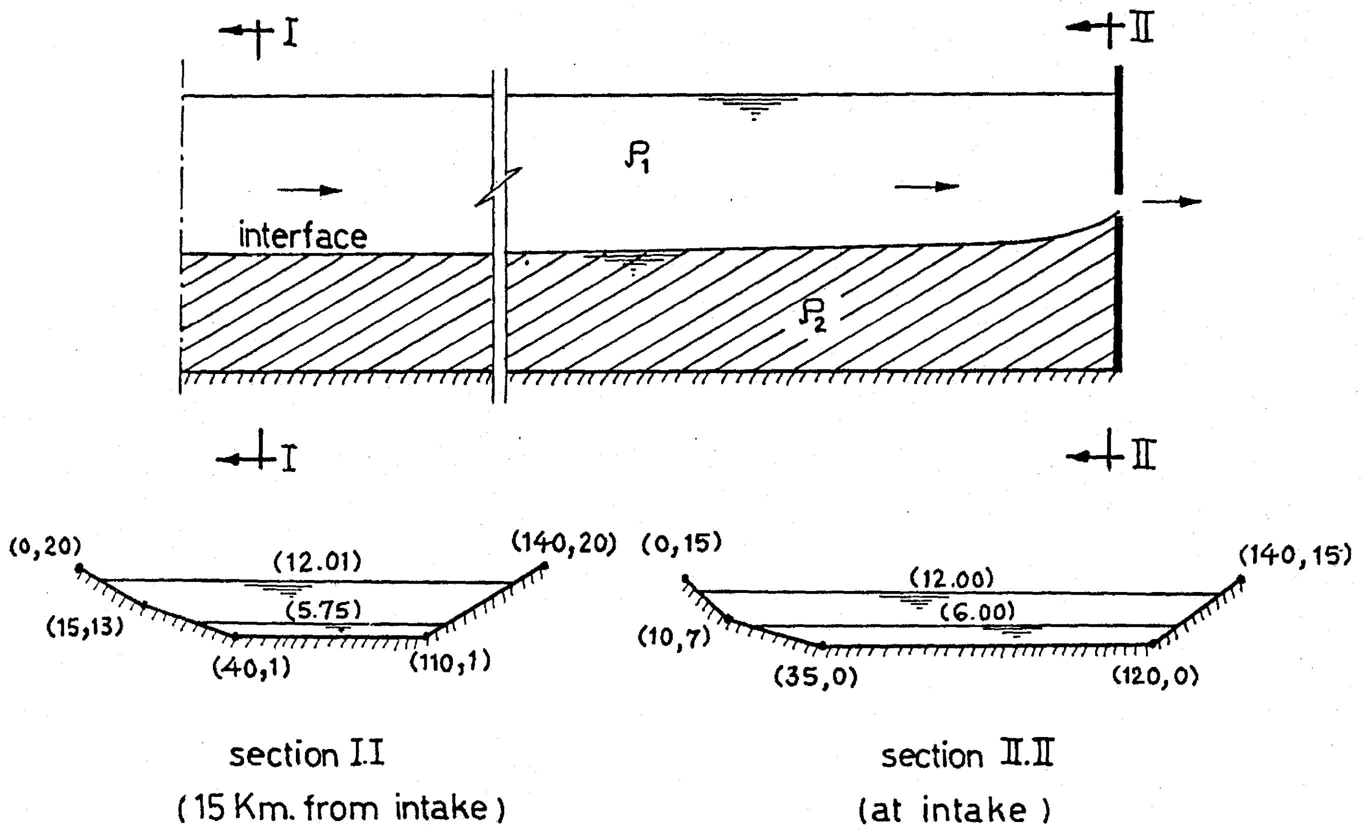


Fig. 7.1 - Selective Withdrawal at an Intermediate Depth (Example 1)

- Interfacial shear stress coefficient = 0.006.

Computed (using subroutine FLPROF)

- Water levels at section I.I. (Fig. 7.1).
- Boundary shear stress coefficient for the upper layer = 0.0197.

It should be noted that in this problem, a slight reduction in the initially-horizontal interface level at section I.I may be expected if the stagnant layer is of finite volume. However, this reduction is expected to be small since the "draw-up" volume near section II.II is relatively local. Also, this error gives a slightly lower value for the limiting upper layer flow which is on the conservative side. This correction may be computed either by modifying subroutine FLPROF or by adding a new subroutine to calculate the volume of the stagnant layer.

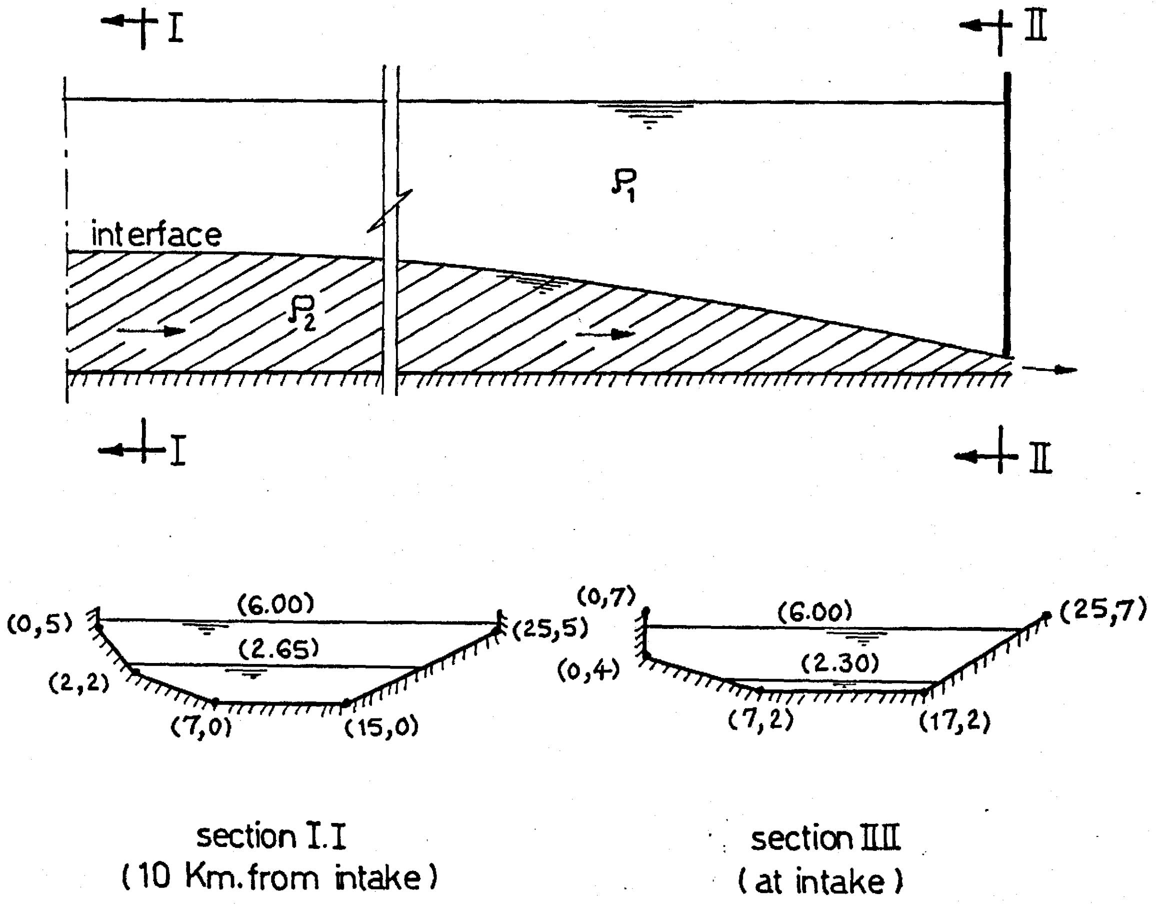
7.2.2 Example (2)

Selective withdrawal at the bottom.

With reference to Fig. 7.2, if the fluids are at rest, the interface will be horizontal. As the withdrawn discharge is increased, the lower layer flows until at some limiting discharge, the interface is drawn down to the bottom intake and the upper fluid begins to be entrained in the abstracted flow.

Given

- Geometry of the end cross-sections (Fig. 7.2).
- Water levels at section II.II (Fig. 7.2).



section I.I
(10 Km. from intake)

section II.II
(at intake)

Fig. 7.2 - Selective Withdrawal at the Bottom (Example 2)

- Upper layer:

  discharge = 0.0, specific gravity = 1.00, kinematic viscosity = 1.02×10^{-6} m^2/sec.

- Lower layer:

  discharge = 0.5 m^3/sec., specific gravity = 1.05, kinematic viscosity = 1.11×10^{-6} m^2/sec., kinetic energy correction factor = 1.2.

- Roughness height at both cross-sections = 0.02 m.

Computed (using subroutine FLPROF)

- Water levels at section I.I (Fig. 7.2).
- Boundary shear stress coefficient for the lower layer = 0.0429.
- Interfacial shear stress coefficient = 0.0276.

For a specified interface level at section I.I, the required lower layer flow may be computed iteratively using subroutine FLPROF.

7.2.3 Example (3)

In example (2), if the vertical height of the slot is large relative to the lower layer depth, energy balance may be applied to determine, for a given gate opening and interface elevation, the limiting discharge of the lower layer.

Given

- Geometry of cross-sections (Fig. 7.3).
- Water levels at both cross-sections (Fig. 7.3).

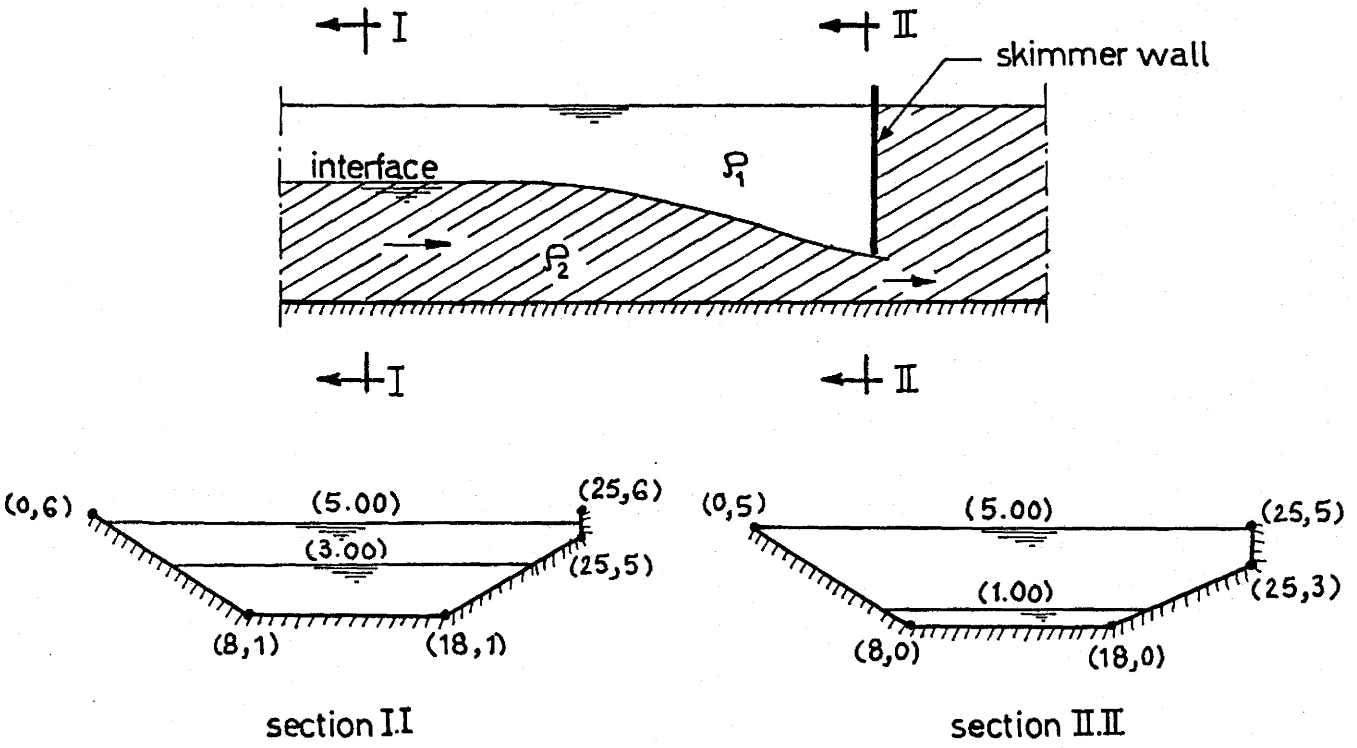


Fig. 7.3 - Selective Withdrawal at the Bottom (Example 3)

- Upper layer:

  specific gravity = 1.00, discharge = 0.0.

- Lower layer:

  specific gravity = 1.05, kinetic energy correction factor = 1.2.

Computed (using subroutine SWLBOT)

- The limiting discharge of the lower layer = 16.7 m^3/sec.

Also, given water levels at one section and the lower layer discharge, water levels at the other section may be computed using subroutine BERNWL2.

Examples 2 and 3 may be combined so that subroutine SWLBOT (example 3) is used to compute to local draw-down near the skimmer wall which will provide a starting point to compute the interface profile in the upstream direction using subroutine FLPROF (example 2).

7.2.4 Example (4)

Recirculation of cooling water in rivers and canals.

The recirculation of water after it has been used as a coolant in thermo-electric plants and returned to the river or canal from which it is originally drawn is of great importance to the economy of operation of such plants. Bata (1957) shows that the amount of recirculation is directly and simply related to the areas occupied by the warm water and the cool water, respectively, at the intake i.e. to the position of the common interface of the warm and cool layers. Thus, the problem of recirculation reduces to the problem of determination of the form of the warm wedge. The results can be used either for prediction of recirculation or for selection of the distance between the intake and the outlet.

Bata, in his analysis based on experimental work, divides the flow into three zones: that upstream from the intake, that between the intake and the outlet, and that downstream from the outlet (see Fig. 7.4).

In the first zone the warm wedge is stagnant and the fluid in it has no mean velocity. Bata shows that critical conditions occur at the intake and outlet, thus providing the starting points needed for the upstream and the middle zones, respectively. He assumes that the middle zone has to end with the critical "depth" or with supercritical flow (occurence of an interfacial jump). By using simplified momentum equations, he suggests that the first assumption is more logical. However, at the intake, the transition occurs in such a way that a "supercritical flow of the lower layer" is established downstream from the intake, followed by a jump. The depth of the lower layer immediately upstream from the intake has to be critical, he concludes. Bata also assumes that the amount of recirculation is proportional to the mean value of the upstream and downstream depths at the intake. The upstream depth, h_{2c}, is the critical depth for a stagnant upper layer. The downstream depth, t, is governed by the jump. He expresses the amount of recirculation as:

$$r = \frac{(h-h_{2c}) + (h-t)}{2h}$$

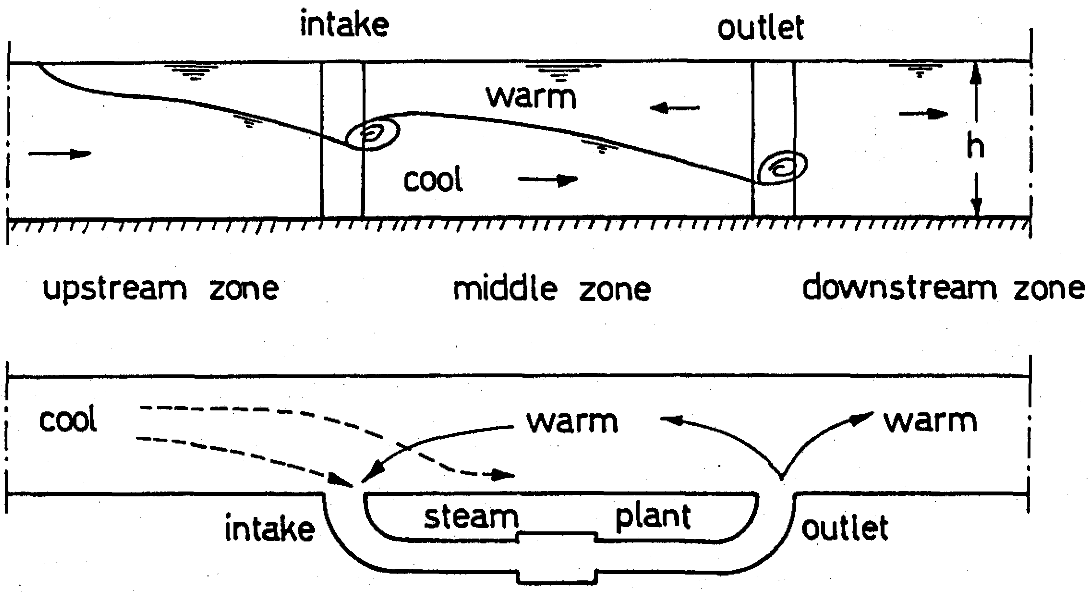
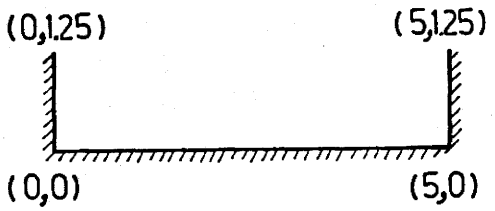


Fig. 7.4 - The Cool and Warm Layers Near the Intake and the Outlet of a Steam Plant [after Bata (1957)]

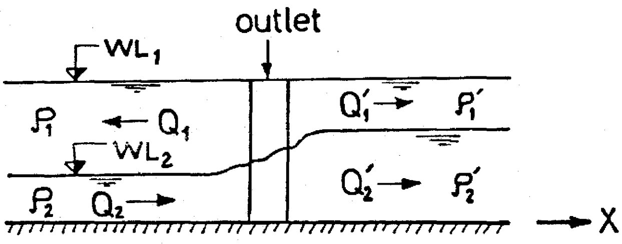
Bata's experimental results are compared with the computer results in Figs. 6.8 and 6.9 (Chapter 6) for the length of the warm wedge in the upstream zone and the shape of the wedge in the middle zone.

Following is an illustration of the use of the available subroutines to handle this problem using typical data from Bata's experiments:

The flume cross-section is shown below (dimensions in feet)



a) Downstream zone



(using subroutine INTJMP)

Given

$Q_1 = -0.109$ cfs $Q_1' = 0.091$ cfs

$Q_2 = 0.159$ cfs $Q_2' = 0.159$ cfs

$WL_1 = 0.71$ ft $WL_2 = 0.213$ ft $\rho_1 = 0.9936$ $\rho_2 = 0.9980$

$\rho_1' = 0.9936$ $\rho_2' = 0.9980$ $g = 32.2$ ft/sec^2

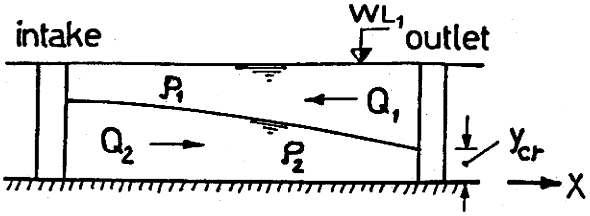
momentum correction factors for the upper and lower layers respectively are $\beta_1 = 1.1$, $\beta_2 = 1.5$

kinetic energy correction factors for the upper and lower layers respectively are $\alpha_1 = 1.3$, $\alpha_2 = 1.7$

Result No jump possible

Therefore, a critical condition is assumed at the end of the middle zone.

b) Middle zone



(using subroutines WLCRIT1 and FLPROF)

Given

$Q_1 = -0.109$ cfs $Q_2 = 0.159$ cfs $WL_1 = 0.71$ ft

ρ_1 = 0.9941 ρ_2 = 0.9980 g = 32.2 ft/sec^2

α_1 = 1.3 α_2 = 1.7

Boundary shear stress coefficients for the upper and lower layers are 0.03 and 0.03.

Kinematic viscosities of the upper and lower layers are 0.739 x 10^{-5} and 0.930 x 10^{-5} ft^2/sec respectively.

Distance between the intake and the outlet = 30 ft

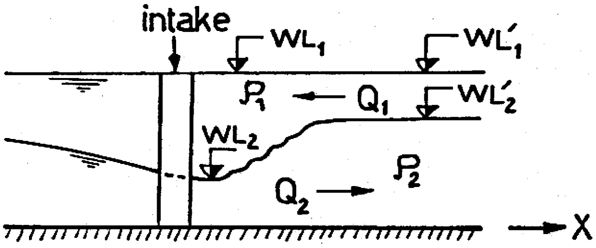
Results

The computed profiles are listed in Table 7.1

CHAINAGE	FREE SURFACE LEVEL	INTERFACE LEVEL
30.00	.7100	.2454
29.98	.7100	.2478
29.97	.7100	.2493
29.95	.7100	.2506
29.92	.7100	.2526
29.86	.7100	.2556
29.80	.7100	.2581
29.67	.7100	.2621
29.55	.7100	.2654
29.30	.7100	.2708
29.05	.7100	.2754
28.55	.7100	.2830
28.05	.7100	.2894
27.05	.7100	.3002
26.05	.7100	.3094
24.05	.7100	.3251
22.05	.7099	.3386
20.05	.7099	.3508
16.05	.7099	.3731
12.05	.7099	.3938
4.05	.7098	.4351
.01	.7097	.4584

Table 7.1 - Interface Profile in the Middle Zone (Example 4)

c) Hydraulic jump at the intake



(using subroutine INTJMP)

Given

$WL'_1 = 0.7097$ ft $\quad WL'_2 = 0.4584$ ft (from step b)

$Q_1 = -0.109$ cfs $\quad Q_2 = 0.159$ cfs

$\rho_1 = 0.9941$ $\quad \rho_2 = 0.9980$

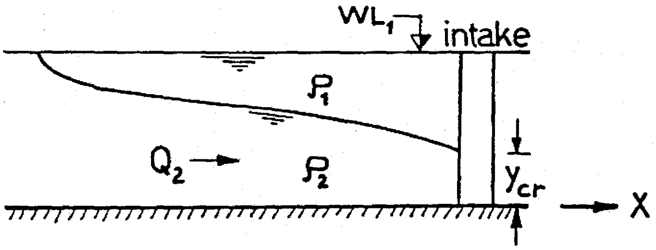
$g = 32.2$ ft/sec^2

$\beta_1 = 1.1$ $\quad \beta_2 = 1.5$ $\quad \alpha_1 = 1.3$ $\quad \alpha_2 = 1.7$

Result

$WL_1 = 0.71$ ft , $WL_2 = 0.10$ ft

d) Upstream zone



(using subroutine ARSWDG)

Given

$Q_1 = 0.0$ $Q_2 = 0.25$ cfs

$WL_1 = 0.7098$ ft $\rho_1 = 0.9941$ $\rho_2 = 0.9980$

$g = 32.2$ ft/sec^2

$\alpha_1 = 1.3$ $\alpha_2 = 1.7$

Roughness height = 0.003 ft

Kinematic viscosities for the upper and lower layers are 0.739×10^{-5} and 0.930×10^{-5} ft^2/sec respectively.

Results

Boundary shear stress coefficient for the lower layer is 0.0301.

Interfacial shear stress coefficient is 0.0261.

The computed wedge profile is listed in Table 7.2.

CHAINAGE	FREE SURFACE LEVEL	INTERFACE LEVEL
90.00	.7098	.3265
89.98	.7098	.3288
89.97	.7098	.3304
89.94	.7098	.3329
89.91	.7098	.3348
89.84	.7098	.3379
89.78	.7098	.3404
89.66	.7098	.3446
89.53	.7098	.3481
89.28	.7098	.3538
89.03	.7098	.3586
88.53	.7098	.3665
88.03	.7098	.3732
87.03	.7098	.3843
86.03	.7098	.3936
84.03	.7098	.4090
82.03	.7098	.4219
78.03	.7097	.4436
74.03	.7097	.4618
70.03	.7097	.4779
62.03	.7097	.5062
54.03	.7096	.5313
38.03	.7095	.5770
22.03	.7094	.6223
14.03	.7094	.6481
10.03	.7093	.6634
6.03	.7093	.6839

DEPTH = 0.0 AT A CHAINAGE OF ABOUT 2.03

Table 7.2 - Interface Profile in the Upstream Zone (Example 4)

Based on the previous results, when the distance between the intake and the outlet is 30 ft., the amount of recirculation as defined by Bata is:

$$r = \frac{(0.7098 - 0.3265) + (0.7098 - 0.100)}{2 \times 0.7098} = 69.96\%$$

This amount may be reduced by increasing the distance between the intake and the outlet.

7.2.5 Example (5)

Thermal density underflow diversion, Kingston Steam Plant

This project is described by Elder and Dougherty (1958) and concerns the design of the Tennessee Valley Authority's Kingston Steam Plant. As shown in the general site plan of Fig. 7.5, the plant is located on the neck of a peninsula formed by the Emory and the Clinch Rivers. During warm weather, the flow in the Clinch River may be vertically stratified due to the release of relatively cold water from a dam some distance upstream. In developing the design for the circulating (cooling) water for the thermal plant it appeared to be desirable to abstract the cold water underflow by causing it to be diverted as a reversed underflow up the Emory River to a point where it could be pumped through the plant. To ensure that a reasonably large fraction of the cold water underflow may be diverted up the Emory River, a submerged dam had to be designed at a point in the Clinch River just downstream of the confluence. In this way, a cold underflow current is forced up the Emory River to a point some 2 miles upstream of the confluence. From there to the pump intake, the flow has to pass over ground some 20 to 25 feet higher than the invert of the Emory River. By constructing a skimmer wall at this position, only cold water is admitted to a one mile reach of man-made canal which leads to the pump intake of the power station. The entire system is represented schemati-

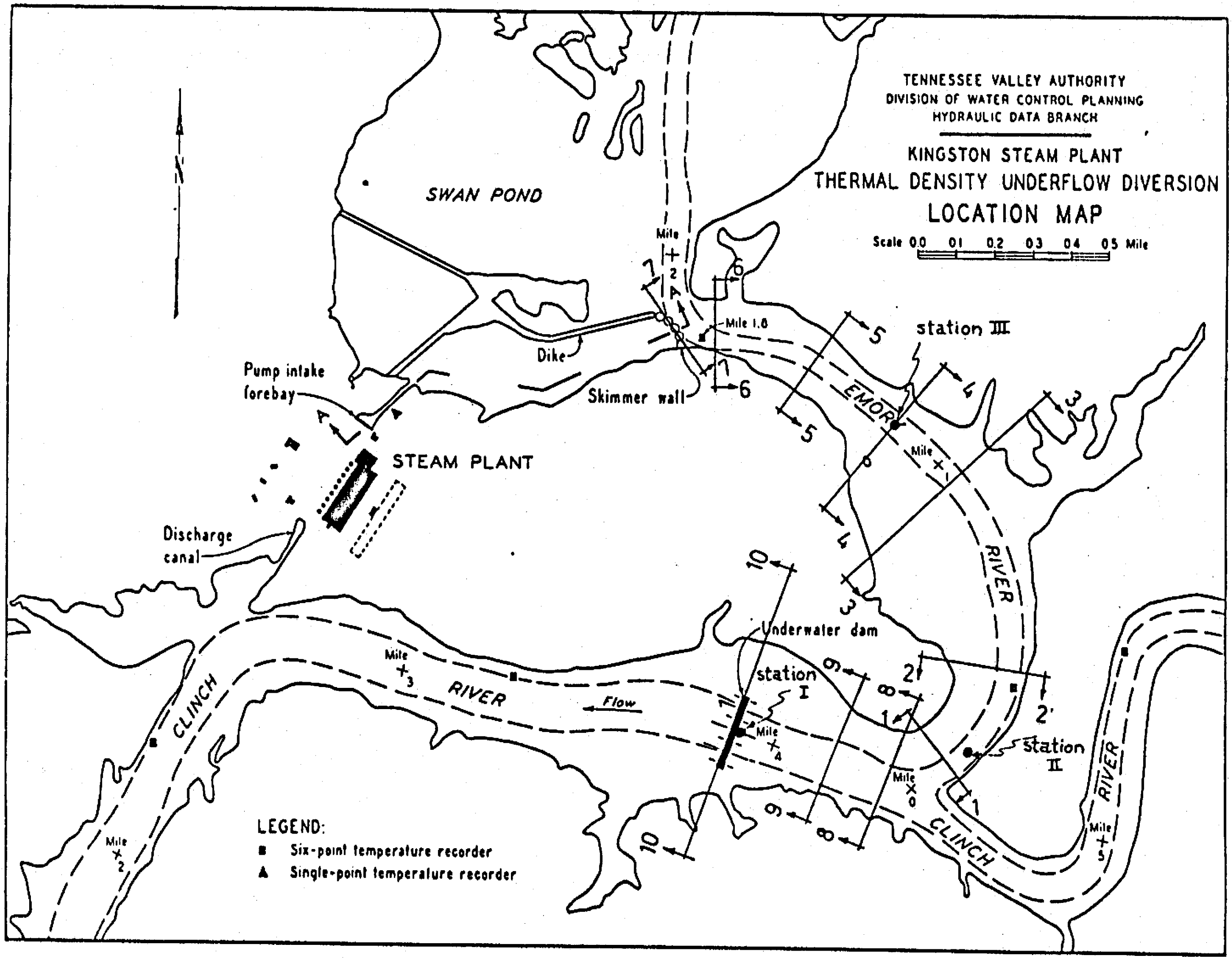


Fig. 7.5 - Location Map of Underflow Diversion Works

cally in Figure 7.6 showing qualitatively the stratification in the various channel reaches and the location of the principle structures such as the underwater dam and the skimmer wall.

During the summer months the only appreciable flows in the reservoir occur as thermal density underflows in the Clinch River arm. The source of these underflowing waters is TVA's Norris Reservoir some 78 miles upstream. Because of stratification in this large storage reservoir, relatively cold water is discharged through the Norris hydro-electric turbines from spring until late fall. Over-all steam plant considerations at the Kingston site required that the condensing waters be drawn from the north or Swan Pond side of the peninsula and discharged on the south or Clinch River side. Under normal conditions the Emory River flows are negligible during the summer months. As a consequence the cold Clinch underflows push their way up the original Emory River channel.

Figure 7.6 shows a section taken along the southern edge of Swan Pond at section A.A of Fig. 7.5. This section is typical for Swan Pond and shows that the water depth in this area is comparatively shallow and the water is thus susceptible to stagnation and marked temperature increases due to solar heating. In its natural state, the high bottom elevation of this area in essence forms a dam which would block out the cold underflowing Clinch River water. Under this condition, only the hot surface waters could be drawn into the pumps.

The plant as originally conceived was to have a maximum condenser water demand of 2470 cubic feet per second. Later changes reduced the

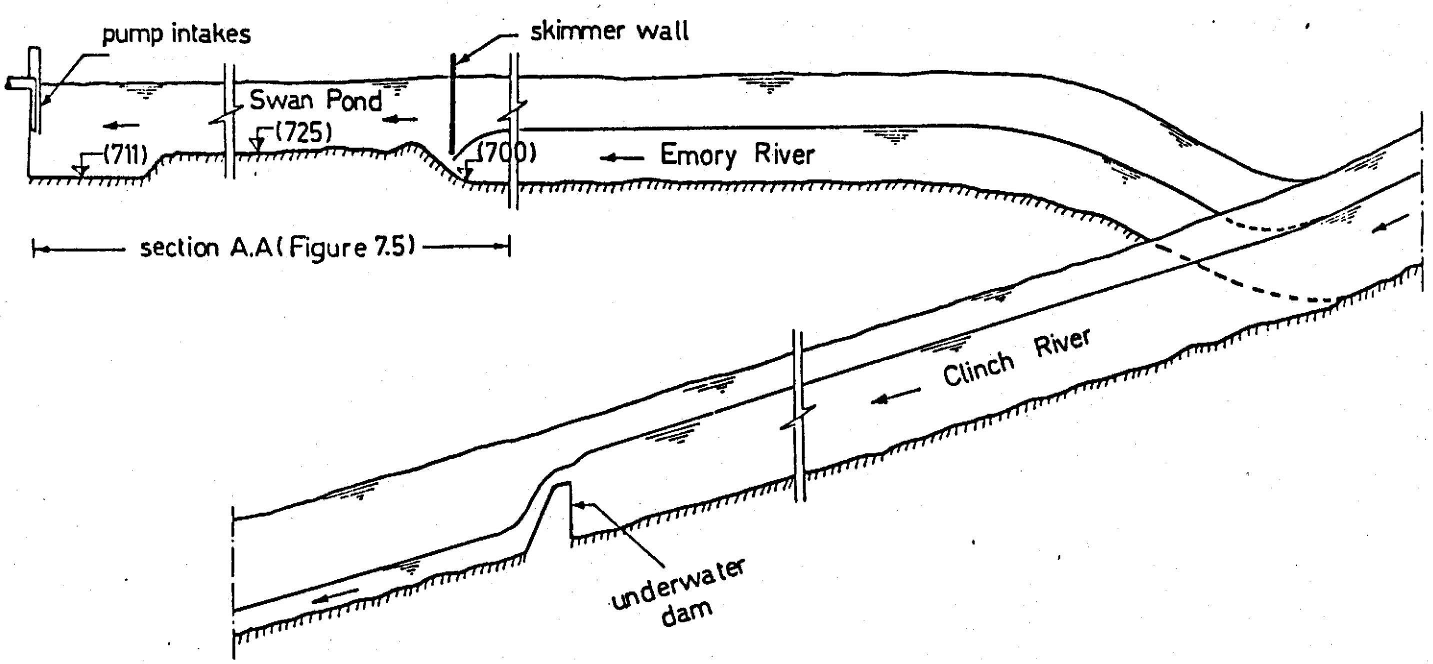


Fig. 7.6 - Flow Regime with Skimmer Wall and Underwater Dam - Kingston Steam Plant

maximum demand to 2310 cubic feet per second. There was no question about the normal Clinch flows being sufficient and of the proper temperature to meet the plant demand.

The problem was thus one of developing a design which would economically make available to the pumps the cold underflowing Clinch River waters by bringing these up the Emory River channel and across Swan Pond to the condenser water pump intakes.

A) Proposed design

One obvious solution to the problem would have been to excavate a channel across Swan Pond so that the cold Clinch waters could be reached. This solution was not economical since over 1.5 million cubic yards of material, a high percentage of which would have been rock, would have had to be excavated under water. It was proposed, therefore, to provide an intake canal across the high Swan Pond area and into the original Emory River channel by means of dikes such as shown in Fig. 7.5. A wall structure, as shown on Figs. 7.5 and 7.6, would close the east end of the canal except for openings along its bottom. These openings would allow the cold bottom water to flow into the canal to replace water pumped from the canal for condensing purposes. In effect the wall would skim off the hot top waters and it was therefore termed a skimmer wall.

In addition, it was desired to investigate the effectiveness of an underwater dam located on the Clinch River just downstream from the Emory River mouth. Since the releases from Norris Dam are subject to extreme regulation, especially during the summer months, it appeared that such a dam might be required to divert the Clinch flow into the Emory channel at a high enough level to make full use of the available cold water.

B) Available field data

Field data indicated a value of 0.016 for the boundary shear stress coefficient (Elder and Dougherty, 1958). The average density difference between the two layers for the mid-summer period is $\Delta\rho/\rho$ = 0.0016 where $\Delta\rho$ is the density difference due to temperature variation and ρ is the average density.

In addition, more detailed field measurements were obtained through personal correspondence with Dr. W.R. Waldrop (Water Systems Development Branch - Tennessee Valley Authority). These included the following:

1) A navigation chart shows the river bottom topography of the Clinch and Emory Rivers in the vicinity of the Kingston Steam Plant. This chart provides an estimate of the bottom topography at locations where cross-sections are not available.

2) Cross-section surveys were obtained at Clinch River Mile 3.85 (underwater dam), 4.17, and 4.35 (sections 10.10, 9.9, and 8.8 - Fig. 7.5) and Emory River Mile 0.1 (section 1.1 - Fig. 7.5). These cross-sections are shown in Fig. 7.7. With sufficient accuracy, all cross-sections are represented by thirteen points

each, the coordinates of which are shown. The other cross-sections indicated in Figure 7.5 are estimated from the navigation chart and are shown in Fig. 7.8. Section 7.7 at the skimmer wall is based on the wall dimensions given by Elder and Dougherty (1958).

The chainage of each cross-section is shown on the corresponding figure. The cross-section chainages are referred to the confluence point (Emory River Mile 0 in Fig. 7.5) and are taken to be increasing positively in the direction of the cold underflow - i.e. downstream in the Clinch River and "upstream" in the Emory River.

3) Velocity and temperature profiles were obtained after the construction of the underwater dam and the skimmer wall. These data were recorded during a field study in May and June, 1973. The corresponding Emory River discharges are also given and are found to have an average value of about 2310 cubic feet per second which is the maximum condenser water demand. This is confirmed by the power plant operating conditions during this period which show the condenser coding water flow rates in cubic feet per second.

4) The free surface elevation at the skimmer wall is assumed to have an average value of 738 feet above sea level. This is estimated

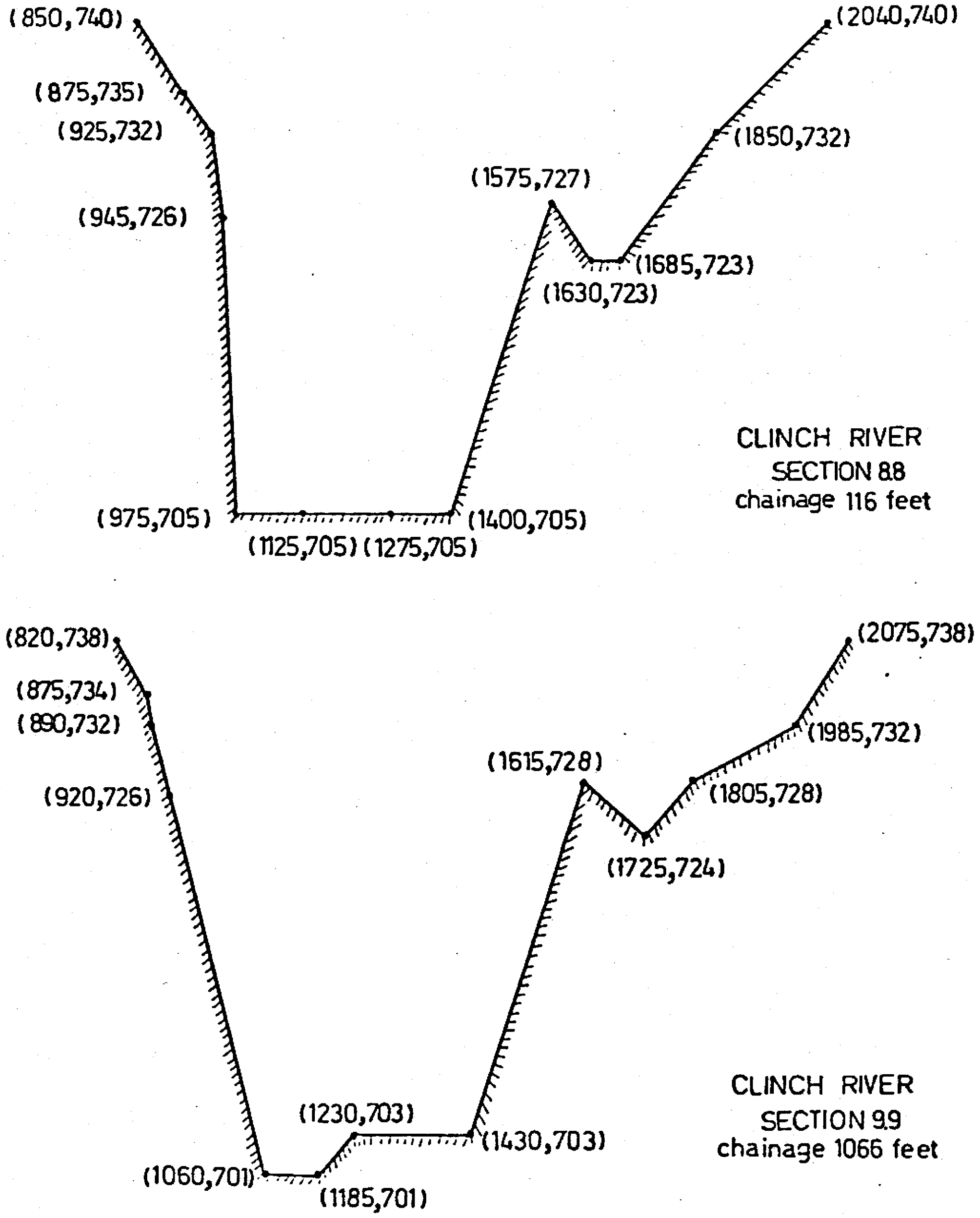


Fig. 7.7(a) - Cross-Sections at Clinch River

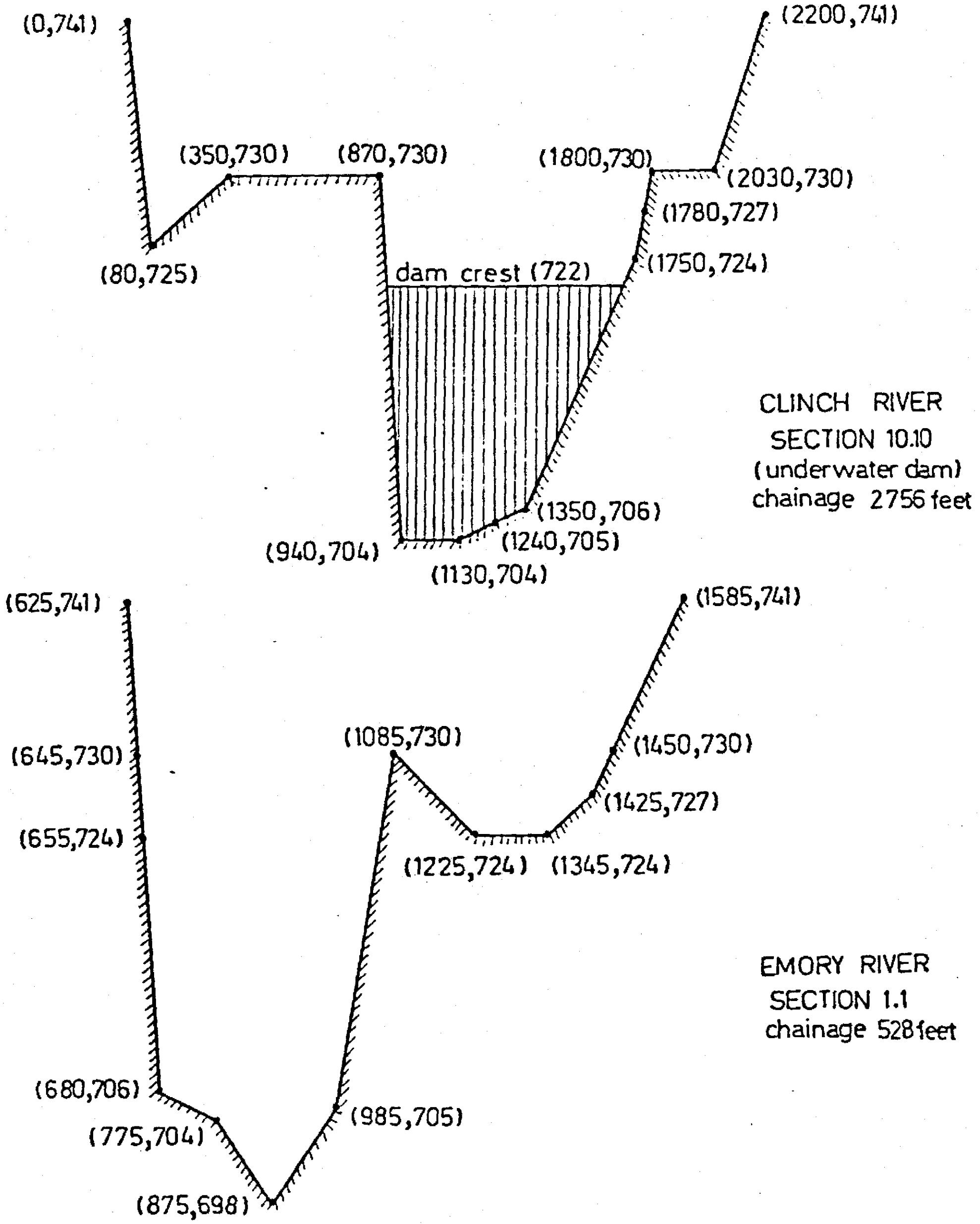


Fig. 7.7(b) - Cross-Sections at Clinch and Emory Rivers

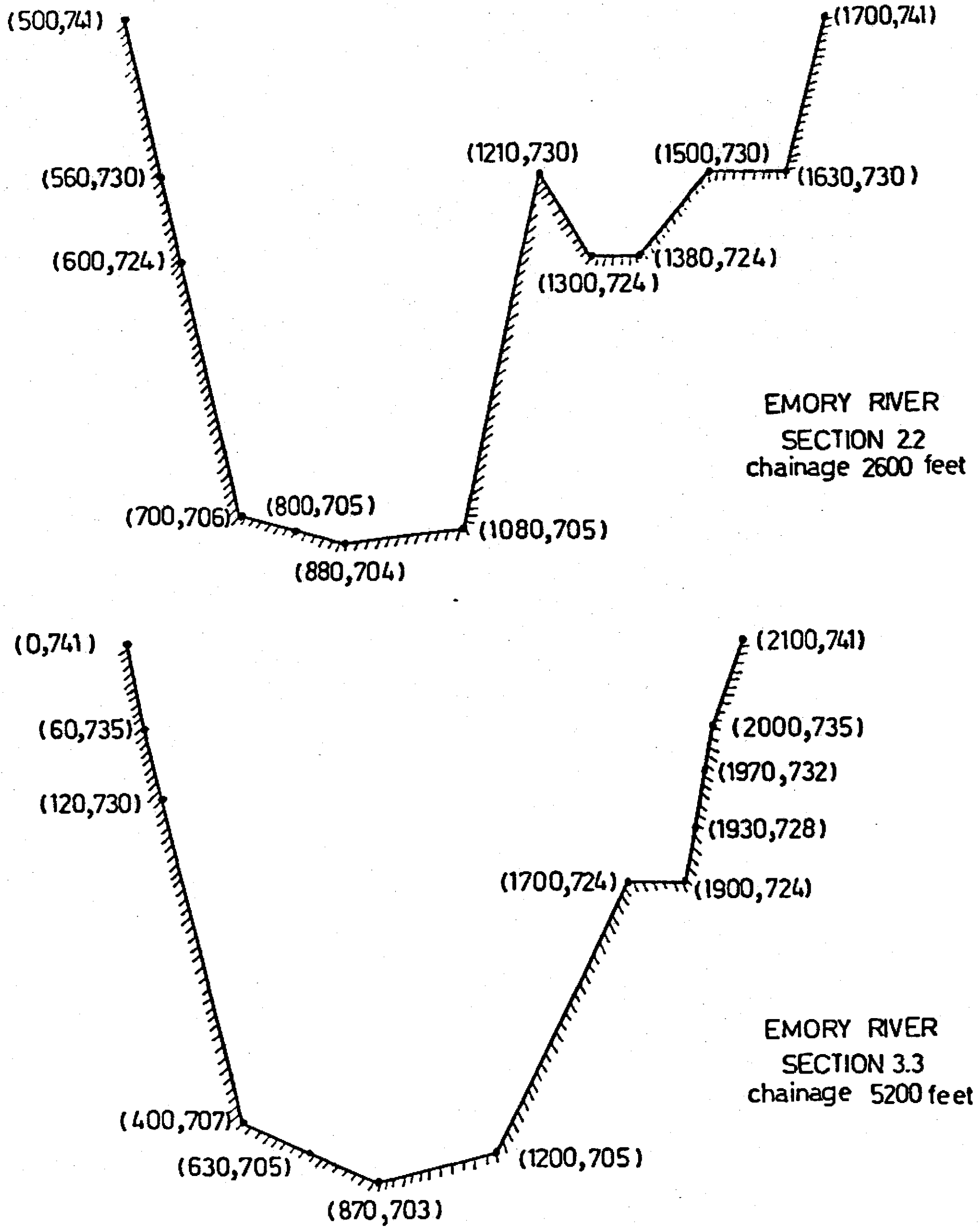


Fig. 7.8(a) - Cross-Sections at Emory River

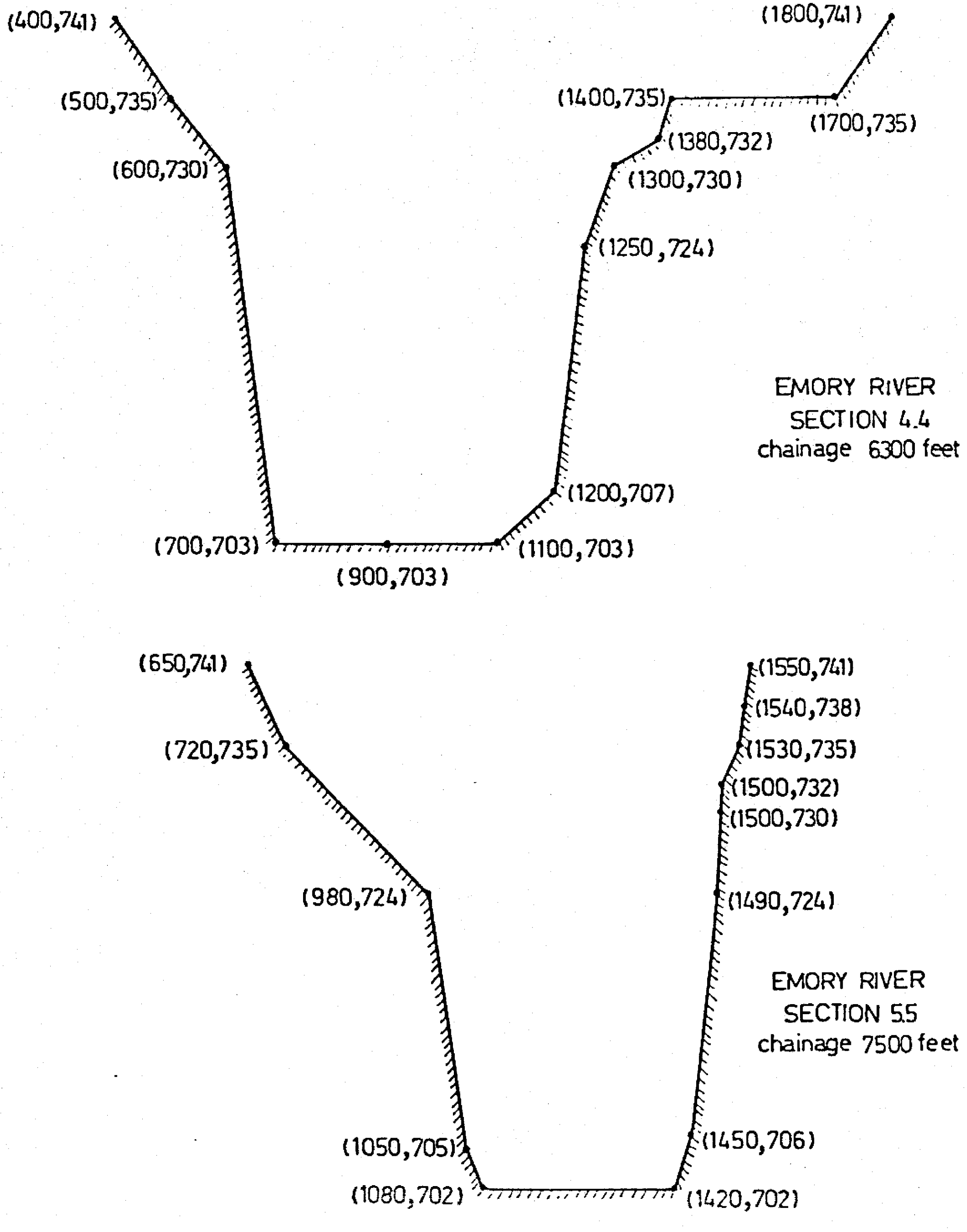


Fig. 7.8(b) - Cross-Sections at Emory River

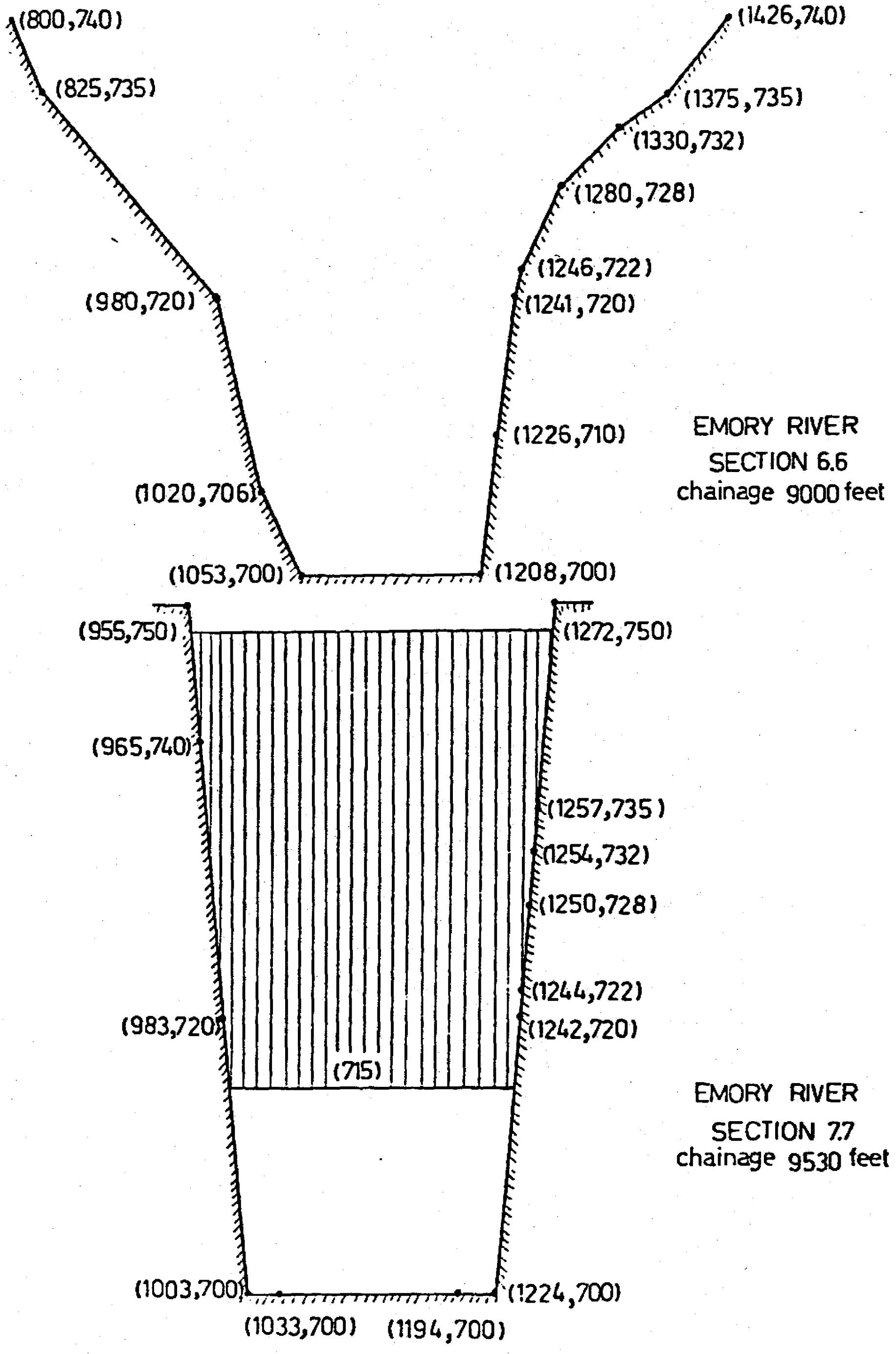


Fig. 7.8(c) - Cross-Sections at Emory River

from the provided velocity and temperature profiles.

C) Computer analysis

The developed subroutines are used to analyze this project. The purpose of this analysis may be summarised under the following headings.

a) Illustrate the use of these sub-programs in dealing with relatively complex practical problems. It also shows the possibility of constructing complex analytical models in a modular fashion using the mutually-compatible subroutines.

b) Compare the computational results to the measured field data when possible.

c) Illustrate the feasibility and the effect of constructing the underwater dam. The necessity of the skimmer wall is obvious based on the previously-mentioned facts.

The computer analysis is performed in the following steps using one overall driving program.

1) Free surface and interface profiles are computed along the Emory River starting at the skimmer wall and proceeding to the confluence of the Emory and Clinch Rivers. Seven cross-sections (six reaches) are used to represent this part of the River (sections 1.1 to 7.7 inclusive - Figs. 7.5, 7.7, and 7.8). This part is done using subroutine FLPROF.

2) It is assumed that the free surface and interface elevations at the confluence of the two rivers (as computed in step 1) are

coincident with the free surface and interface profiles which obtain in the Clinch River. This provides a basis for the estimation of the cold underflow discharge in the Clinch River under different conditions. Two different cases are considered:

a) Without underwater dam: In this case the flow in the lower Clinch river is obtained assuming the flow to be normal flow and the computed interface depth to be the normal depth. This part is done using subroutine NORMQ2.

b) With underwater dam: Once again the problem is one of unknown discharge and must be solved by successive trials. For an assumed lower layer discharge, the critical interface elevation over the crest of the dam is computed using subroutine WLCRIT1. Thus, assuming the dam crest to be a control section, the computation may be started at this point and proceed upstream. Then the local transition (drawdown) in the interface elevation at the dam location is computed using subroutine BERNWL2. Having obtained the free surface and interface elevations just upstream of the dam for the assumed underflow, the free surface and interface profiles are computed along the Clinch River starting at the dam location and proceeding upstream to the confluence of the Emory and Clinch Rivers. Three cross-sections (two reaches) are used to represent this part of the River (sections 8.8, 9.9, and 10.10 - Figs. 7.5 and 7.7).

This part is done using subroutine FLPROF.

Step (2.b) is repeated in an iterative fashion for different values of the underflow in order to determine the lower layer flow which would cause the free surface and interface elevations at the confluence of the two rivers computed from step (1) along the Emory River and from step (2.b) along the Clinch River to match.

3) The sum of the underflow in the Emory River (given) and that in the Clinch River (computed in either step (2.a) or (2.b)) gives the required upper Clinch River underflow, i.e., the required release from the Norris Dam to supply the maximum plant demand from the cold underflowing waters. Thus, this release is computed with and without the underwater dam, which determines the effect of the dam.

4) The computed profiles from steps (1) and (2.b) allow the plotting of the computed elevations on the measured velocity and temperature profiles. These profiles are measured at one station along the Clinch River (Clinch River Mile 3.85 - underwater dam) and at two stations along the Emory River (Emory River Miles 0.26 and 1.19). The locations of these stations are estimated from the maps provided by the TVA and are shown on Fig. 7.5 (Stations I, II, and III respectively).

Computer Results

1) Profiles along the Emory River

Input data

- Geometry of each of the seven cross-sections (Figs. 7.7 and 7.8).
- Chainage of each cross-section. In the computations, chainages are referred to Norris Dam (i.e. a distance of 78 miles (411840 ft) is added to the chainages shown in Figs. 7.7 and 7.8). The reason for this is that under certain conditions of flow the horizontal distances have to be referred to the point of contact of the two layers (e.g. when the flow is turbulent and the interfacial boundary layers are laminar - see subsection 6.2.3.2 - Chapter 6).
- Roughness height at each cross-section (assumed 0.03 feet).
- Free surface and interface elevations at section 6.6 (skimmer wall). These are 738 (observed free surface) and 715 (lower edge of skimmer wall) respectively.
- Upper layer:

  discharge = 0.0, specific gravity = 1.0000, kinematic viscosity = 1.1×10^{-5} ft^2/sec.
- Lower layer:

  discharge = 2310 cfs, specific gravity = 0.9984, kinematic viscosity = 1.13×10^{-5} ft^2/sec., kinetic energy correction factor = 1.10.

Output (subroutine FLPROF)

- Computed profiles in the six reaches are shown in Table 7.3.

CROSS-SECTION	CHAINAGE	FREE SURFACE LEVEL	INTERFACE LEVEL
7.7	421370.00	738.0000	715.0000
	421360.00	738.0000	715.0623
	421340.00	738.0000	715.1841
	421300.00	738.0000	715.4183
	421220.00	737.9999	715.8568
	421060.00	737.9999	716.6534
6.6	420840.00	737.9998	717.6474
6.6	420840.00	737.9998	717.6474
	420830.00	737.9998	717.6728
	420810.00	737.9998	717.7210
	420770.00	737.9998	717.8085
	420690.00	747.9997	717.9564
	420530.00	737.9997	718.1826
	420210.00	737.9996	718.4895
	419570.00	737.9995	718.8598
5.5	419340.00	737.9995	718.9546
5.5	419340.00	737.9995	718.9546
	419330.00	737.9995	718.9593
	419310.00	737.9995	718.9687
	419270.00	737.9995	718.9873
	419190.00	737.9994	719.0239
	419030.00	737.9994	719.0947
	418710.00	737.9994	719.2267
4.4	418140.00	737.9993	719.4312
4.4	418140.00	737.9993	719.4312
	418130.00	737.9993	719.4354
	418110.00	737.9993	719.4435
	418070.00	737.9993	719.4593
	417990.00	737.9993	719.4887
	417830.00	737.9993	719.5402
	417510.00	737.9993	719.6215
3.3	417040.00	737.9992	719.7067
3.3	417040.00	737.9992	719.7067
	417030.00	737.9992	719.7080
	417010.00	737.9992	719.7108
	416970.00	737.9992	719.7163
	416890.00	737.9992	719.7277
	416730.00	737.9992	719.7516
	416410.00	737.9992	719.8047
	415770.00	737.9992	719.9385
	414490.00	737.9991	720.4291
2.2	414440.00	737.9991	720.4600

Table 7.3 - Computed Profiles in the Emory River

CROSS-SECTION	CHAINAGE	FREE SURFACE LEVEL	INTERFACE LEVEL
2.2	414440.00	737.9991	720.4600
	414430.00	737.9991	720.4729
	414410.00	737.9991	720.4986
	414370.00	737.9991	720.5495
	414290.00	737.9991	720.6497
	414130.00	737.9990	720.8443
	413810.00	737.9990	721.2137
	413170.00	737.9989	721.8963
1.1	412368.00	737.9988	722.6925

Table 7.3 (cont.) - Computed Profiles in the Emory River

- Computed shear stress coefficients at the end of each reach are shown in Table 7.4.

Cross-section	f_{02}	f_i
6.6	0.0174	0.0074
5.5	0.0174	0.0087
4.4	0.0175	0.0092
3.3	0.0186	0.0113
2.2	0.0179	0.0090
1.1	0.0168	0.0084

f_{02} = boundary shear stress coefficient for the lower layer
f_{01} = boundary shear stress coefficient for the upper layer = 0
f_i = interfacial shear stress coefficient

Table 7.4 - Computed shear stress coefficients for the Emory River

2) <u>Discharge in the Clinch River</u>

a) <u>Without underwater dam</u>:

Input data

- Geometry of section 8.8 (Figs. 7.5 and 7.7)
- Free surface and interface elevations at section 8.8. These are 738.00 and 722.69 respectively (from step 1 - Table 7.3).
- Boundary and interfacial shear stress coefficients (assumed to be the same as those at section 1.1).
- Bed slope of the Clinch River in the flow direction (estimated to be -0.0004, where the negative sign indicates descending slope in the flow direction).
- Specific gravities of the upper and lower layers. These are 1.0000 and 0.9984 respectively.
- There is assumed to be no flow in the upper layer.

Output (subroutine NORMQ2)

- Normal lower layer flow = 2559.75 cfs (2560 cfs)

b) With underwater dam (Clinch River)

Critical condition over the dam crest

Input data

- Geometry of a cross-section across the body of the dam (the upper part of section 10.10 - Fig. 7.7).
- Free surface elevation above the dam crest is 738 ft.
- Upper layer:

  discharge = 0.0, specific gravity = 1.0000.
- Lower layer:

  specific gravity = 0.9984, kineetic energy correction factor =

1.10.

Output (subroutine WLCRIT1)

- For a lower layer discharge of 55 cfs, the critical interface elevation above the dam crest is 722.46 ft.

Local transition (drawdown) just upstream of the dam

Input data

- Geometry of two cross-sections, one across the body of the dam (previous step) and the other is section 10.10 - Fig. 7.7 (just upstream of the dam).
- Free surface and interface elevations over the dam crest (from previous step).
- Upper layer:

  discharge = 0.0, specific gravity = 1.0000.
- Lower layer:

  discharge = 55 cfs, specific gravity = 0.9984, kinetic energy correction factor = 1.10.

Output (subroutine BERNWL2)

- Free surface and interface elevations just upstream of the dam are 738.00 and 722.69 respectively.

Profiles along the Clinch River

Input data

- Geometry of each of the three cross-sections (Fig. 7.7).
- Chainage of each cross-section. In the computations, chainages

are referred to Norris Dam (i.e. a distance of 78 miles (411840 ft) is added to the chainages shown in Fig. 7.7).

- Roughness heigh at each cross-section (assumed 0.03 ft).
- Free surface and interface elevations just upstream of the underwater dam (obtained in the previous step).
- Upper layer:

  discharge = 0.0, specific gravity = 1.0000, kinematic viscosity = 1.10×10^{-5} ft^2/sec.
- Lower layer:

  discharge = 55 cfs, specific gravity = 0.9984, kinematic viscosity = 1.13×10^{-5} ft^2/sec., kinetic energy correction factor = 1.10.

Output (subroutine FLPROF)

- Computed profiles in the two reaches are shown in Table 7.5.
- Computed shear stress coefficients at the end of each reach are shown in Table 7.6

It should be noted that the lower layer discharge of 55 cfs is obtained as a result of successive trials using different discharges in this step. Within sufficient accuracy, a flow of 55 cfs gives free surface and interface elevations at section 8.8 (Table 7.5) equal to those at section 1.1 (Table 7.3). In these trials, the incremental discharge used is 1 cfs.

CROSS SECTION	CHAINAGE	FREE SURFACE LEVEL	INTERFACE LEVEL
10.10	414596.00	738.0000	722.6900
	414586.00	738.0000	722.6900
	414566.00	738.0000	722.6900
	414526.00	738.0000	722.6900
	414446.00	738.0000	722.6900
	414286.00	738.0000	722.6901
	413966.00	738.0000	722.6901
	413326.00	738.0000	722.6903
9.9	412906.00	738.0000	722.6904
9.9	412906.00	738.0000	722.6904
	412896.00	738.0000	722.6904
	412876.00	738.0000	722.6904
	412836.00	738.0000	722.6904
	412756.00	738.0000	722.6904
	412596.00	738.0000	722.6904
	412276.00	738.0000	722.6904
8.8	411956.00	738.0000	722.6904

Table 7.5 - Computed Profiles in the Clinch River

Cross-Section	f_{02}	f_i
9.9	0.0251	0.0047
8.8	0.0248	0.0054

f_{02} = boundary shear stress coefficient for the lower layer

f_{01} = boundary shear stress coefficient for the upper layer = 0

f_i = interfacial shear stress coefficient

Table 7.6 - Computed Shear Stress Coefficients for the Clinch River

D) Discussion

1. From the previous computations, the release required from the Norris Dam to supply the maximum plant demand from the cold underflowing waters is as follows.

   a) Without underwater dam:

   release = 2310 + 2560 = 4870 cfs

   b) With underwater dam:

   release = 2310 + 55 = 2365 cfs

   This shows the significance of constructing the underwater dam based on the fact that the releases from Norris Dam are subject to extreme regulation especially during the summer months for which these computations are made.

2. The primary purpose of the previous analysis is to demonstrate the applicability of the developed computational modules in the analysis of complex practical problems which arise in the design of such large-scale projects. The computed elevations at the previously-mentioned three measuring stations are shown on the measured velocity and temperature profiles in Figs. 7.9 and 7.10. Although the comparison shows good correspondence, this comparison involves some uncertainty due to the following reasons:

   a) The number of cross-sections provided for the Emory and Clinch Rivers is not sufficient. The other sections estimated from the navigation chart are subject to some error of approximations due to the lack of sufficiently-accurate information on the chart. Moreover, the provided cross-sections were surveyed in the period 1960-1961 while the temperature and velocity profiles were measured in 1973. The charts provided indicate cross-sectional variations due to sedimentation. For example, the bottom elevations of sections 8.8 (Clinch River) and 1.1 (Emory River) increased by 4 feet and 3 feet respectively between October 1946 and May 1961.

b) The time-histories of river discharges are provided in a graphical form using a rather inaccurate scale (1 inch = 40,000 cfs) which causes the interpolated flow values to be very approximate.

c) No information is available as to what the free surface elevations are along both rivers. Therefore, the free surface level has to be estimated roughly. However, this parameter does not significantly affect the computations since there is no flow in the upper layer.

d) The number of stations where the temperature and velocity profiles are measured is not sufficient and the locations of these stations are not indicated accurately.

3. The shape of the measured velocity profiles confirm the assumption that there is no net flow in the upper layer. Instead, circulation occurs in the upper layer due to the effect

of the interfacial shear.

4. The value of 1.10 which is adopted in the computations for the kinetic energy correction factor for the lower layer is an average value estimated from the measured velocity profiles.

5. For the best over-all plant design it is desirable to set the maximum allowable condenser water temperature at 75°F. Figs. 7.9 and 7.10 show that this condition is always satisfied.

6. If sufficient information were to be made available, the computer routines could be used to analyze the project in more detail (e.g. alternative designs and locations of the skimmer wall and the underwater dam). The example illustrates the feasibility of analyzing a number of alternatives and bring into the design quesitons of economic significance. In particular, the sensitivity of the Norris Dam releases to variations in the height (and thus the cost) of the underwater dam could be explored. Similarly, the cost of lost power may be related to the geometry of the skimmer wall as a result of varying the temperature of the condensing water.

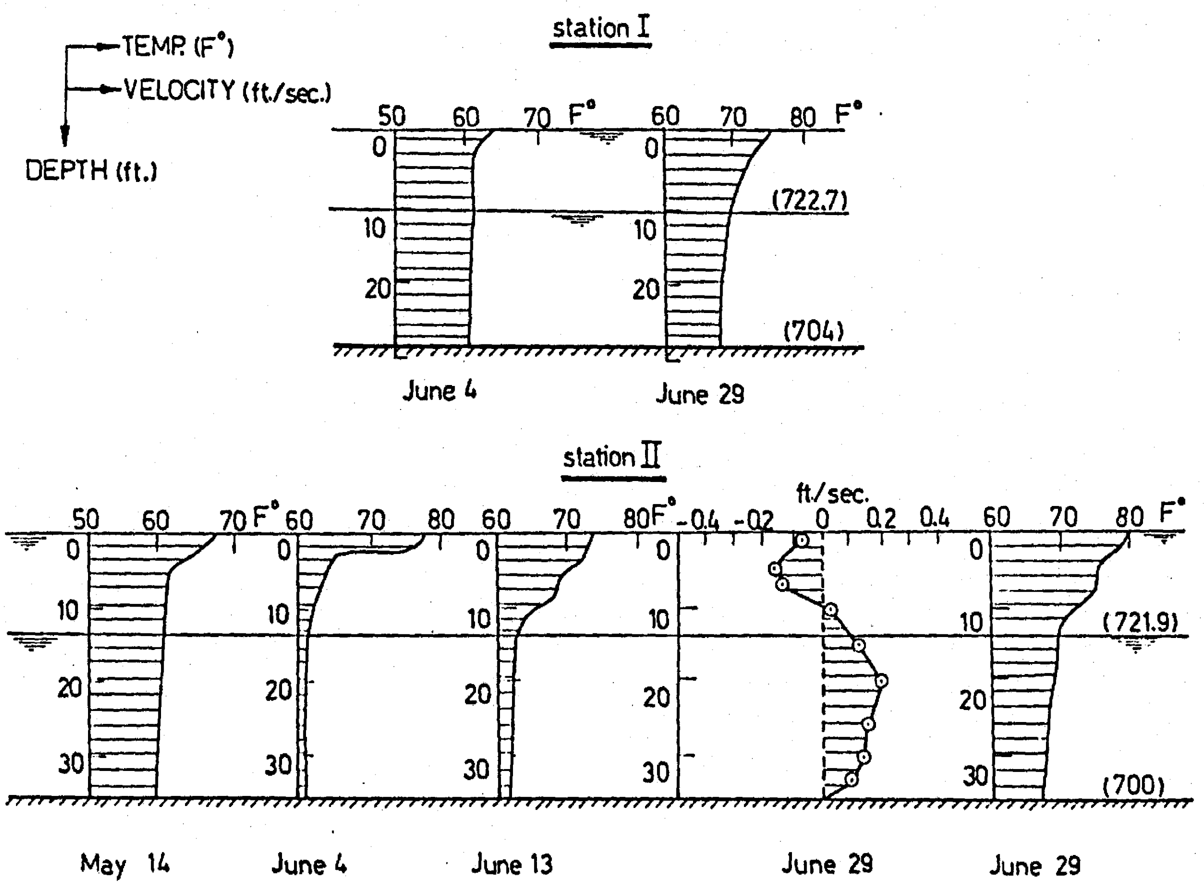


Fig. 7.9 - Measured Temperature and Velocity Profiles with Computed Elevations [Stations I & II]

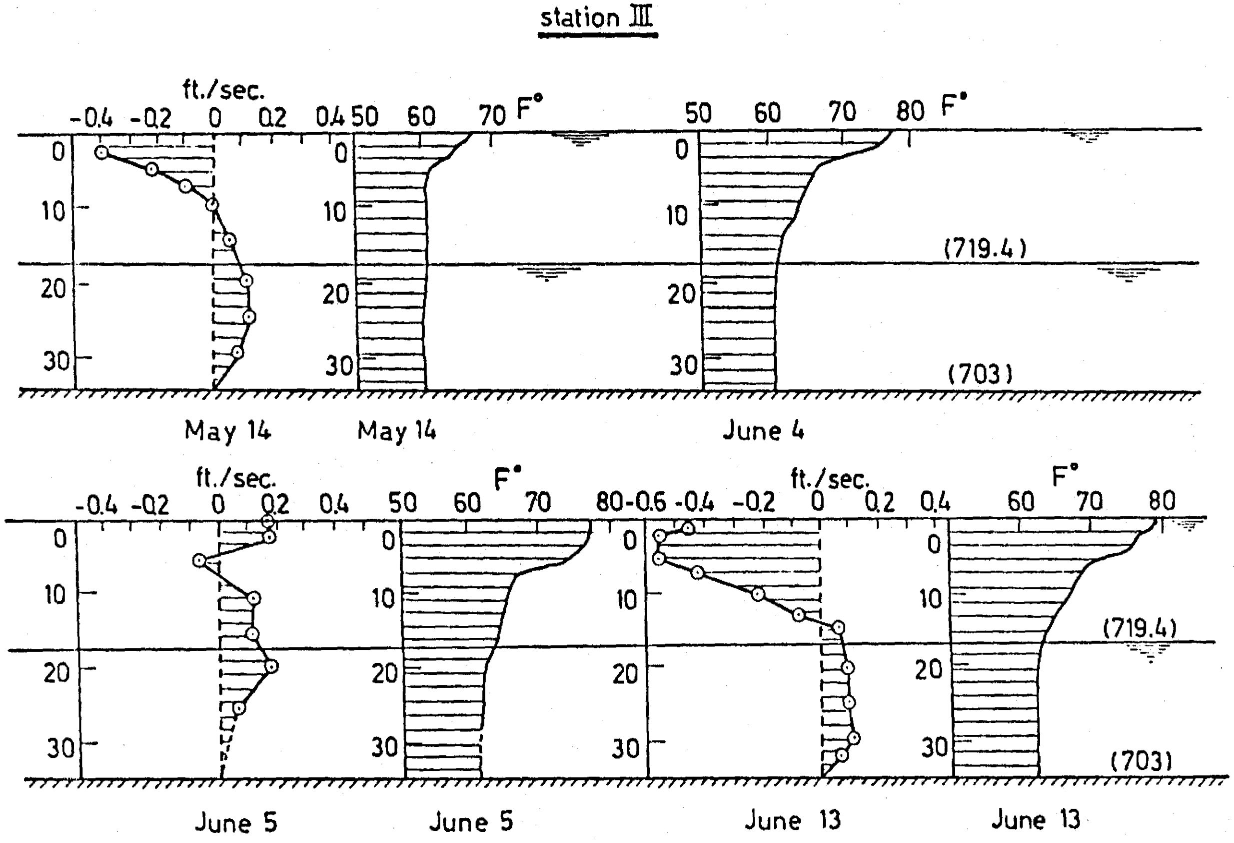


Fig. 7.10 - Measured Temperature and Velocity Profiles with Computed Elevations [Station III]

CHAPTER 8

SUMMARY AND CONCLUSIONS

This chapter includes a brief summary of this study with reference to the general findings. It also includes a discussion as to the extent to which the objectives of the thesis have been achieved and the scope for future research.

1. Individual cases of stratified flow have received considerable attention by many researchers. However, what is lacking is a unified approach to study the phenomena as a whole. Therefore, the first step of the present study comprises an extensive literature survey of the stratified flow phenomena which leads to a general classification based on variations of flow characteristics. A wide range of stratified flow problems are classified within the previously-defined regimes.

2. Based on this classification, four research areas are examined, each of which involves one of the conditions that lead to variations of flow characteristics. The analytical study of these four areas deals with two-layer systems using a one-dimensional approach with improvements to allow for non-uniform velocity distributions (e.g. boundary layer displacement

thicknesses, energy and momentum corrections factors). One of the basic advantages of the analytical study is that it considers channels of arbitrary geometry. The four topics considered are defined below.

a) Steady stratified flow through streamlined transitions. Negligible energy losses allow the use of energy balance techniques in this case. The analytical study includes two subdivisions:

   i) Transitional flows which are subcritical throughout.

   ii) Transitional flow which involve a critical flow section or control at some point in the system.

b) Interfacial hydraulic jumps. These represent steady stratified flow in the vicinity of boundary and/or interfacial discontinuities. The analytical study is aimed at determining the state at one end of a hydraulic jump (or drop) for a completely specified state at the other end taking mixing at the interface into consideration. The question of the uniqueness of solution is discussed by means of the momentum and energy principles.

c) Lock exchange flows. This is the classical case of unsteady stratified flows which may frequently find application in the analysis of quasi-steady phenomena such as the arrested wedge. The analytical study involves the determination of the front velocities as a function of time and is designed to be applicable in practical situations.

d) Long transitions. Gradually-varied flow involves significant energy losses due to boundary and interfacial friction. The analytical study is aimed at the determination of boundary and interfacial shear stresses and subsequently the evaluation of energy gradients and surface slopes. This allows the determination of the free surface and/or interface profiles in a stratified two-layer system.

3. Thus, this study defines a framwork against which a variety of stratified flow problems may be classified and decomposed into analytical problems of the simplest possible scope. For each of these, a computational module (i.e. subroutine) is developed which may find frequent application in a wide variety of different solution types. As a result, a library (or framework) of computational algorithms is developed which consists of 44 subroutines and functions. Such algorithms provide solutions for frequently-recurring practical problems, are mutually compatible and allow the construction of relatively complex analytical models in a modular fashion. Complete documentation and listings of these subroutines and functions are provided in Appendix (2). Each documentation is presented in identical format including

purpose, method of solution, method of use, and an example.

4. These computational algorithms are tested for theoretical characteristics and computational performance. The numerical predictions are compared to available laboratory and field data. These comparisons show excellent agreement. In addition, an experimental programme is designed and carried out to verify the numerical computations of the energy balance routines. Again, comparison of the numerical predictions to the experimental results show very good agreement.

5. An important aspect of this study is the illustration of the application of the routines in the solution of typical practical problems. Five examples are provided accompanied by typical computations using laboratory and field data when available. These examples demonstrate the capabilities of the computational algorithms as well as the advantages of the library concept.

6. Throughout the thesis comparisons are drawn between laboratory and field observations and numerical predictions for a number of widely different situations. In addition, the examples used for illustration of the method cover a variety of practical circumstances. It is suggested therefore that the simplified approach used here may be successfully applied to a wide range of stratified flow problems of practical engineering significance.

The approach presented here is capable of solving relatively complex practical problems with an accuracy and reliability that is consistent with the assumptions and availability of data commonly associated with engineering design. In contrast, a more rigorous and complex approach to the solution of particular problems is generally limited to idealized circumstances and is therefore of little practical advantage.

Other existing programs frequently do not provide an appropriate solution method for practicing engineers either because of:

a) the size and computational cost of programs,

b) the difficulty of introducing practical (i.e. real) boundary conditions, or

c) the effort necessary to comprehend the background research contributions in the literature on which the programs are based.

7. This study represents only the first step in the development of a computer library dealing with stratified flow problems. With reference to the literature review presented at the beginning of the thesis, many more stratified flow problems that are of practical significance may be treated and analyzed in a similar fashion to those presented in this study. However, the four research areas chosen for this investigation cover a wide range of the more significant and basic problems.

Also, further investigation is needed in some areas where there is still some uncertainty. Examples of these are the determination of shear stress coefficients when the two layers are flowing and the evaluation of the interfacial shear stress coefficient when both the flow and the inerfacial boundary layers are turbulent. In the latter case, the approach used is arbitrary and is supported only by experimental evidence. In this area, present knowledge is restricted mainly to laminar flows which, however, are of greater significance in densimetric phenomena than in homogeneous flow resistance problems.

In this respect, a library of computational algorithms may provide a means of promoting cooperation and collaboration among researchers and engineers concerned with stratified flow hydraulics. Whereas the improvement or augmentation of a large "all-embracing" program is a task of some magnitude, it should be possible to incorporate in a properly-designed library package, a new or modified routine concerned with a relatively elementary problem type or solution technique.

APPENDIX (1)

PHOTOGRAPHS OF THE ENERGY BALANCE EXPERIMENTS

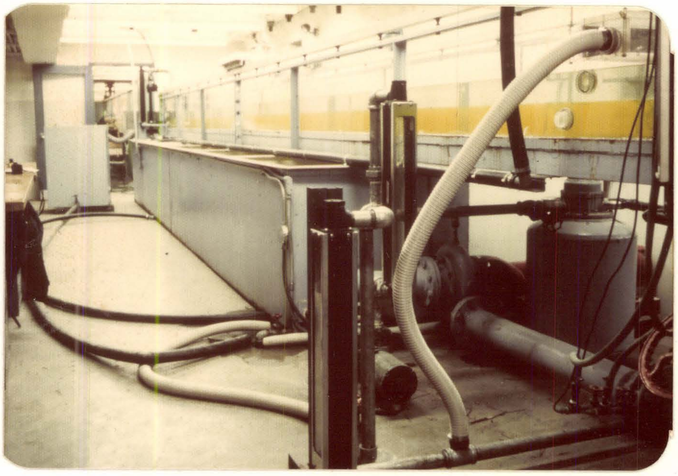


Fig. A1.1 - Long View of the Flume from the Upstream End

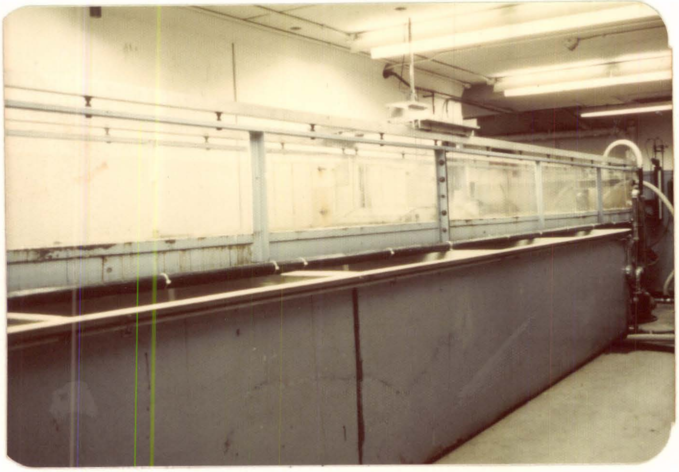


Fig. A1.2 - Long View of the Flume from the Downstream End

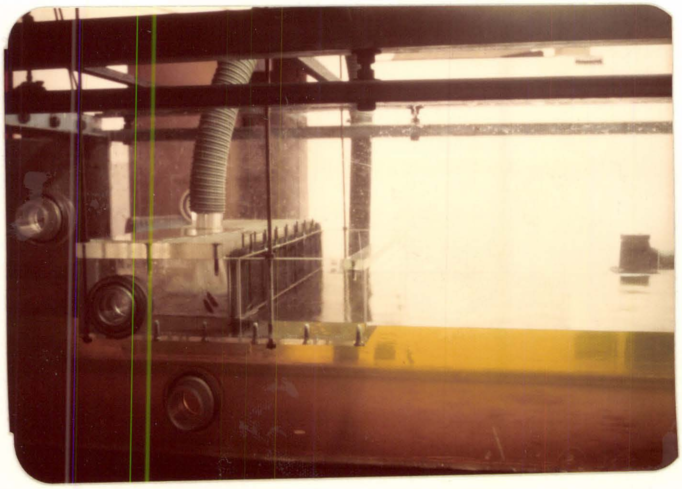


Fig. A1.3 - Upstream Plastic Box (taken from the rear of the flume)

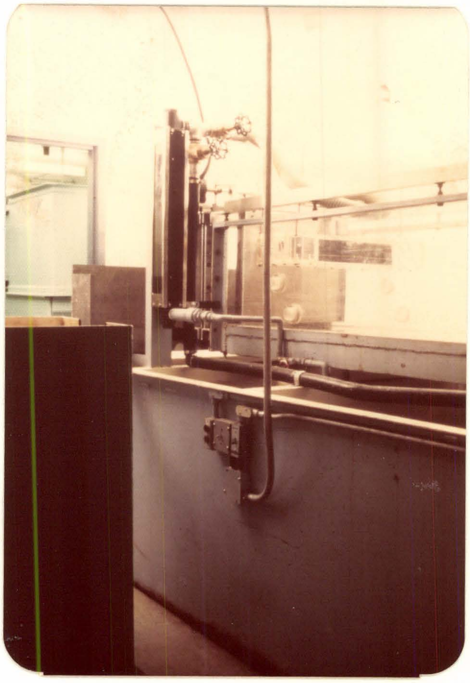


Fig. A1.4 - Downstream End of the Flume

Fig. A1.5 - Flume Contraction (taken from inside the flume)

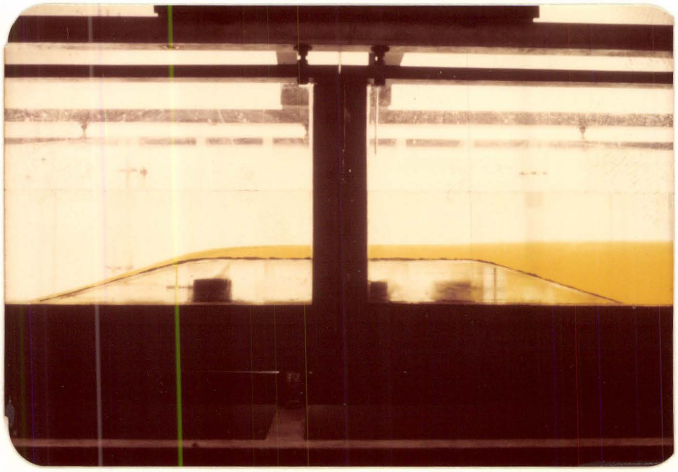


Fig. A1.6 - Underflow Pattern of Salt Water Through the Contraction (Set I)

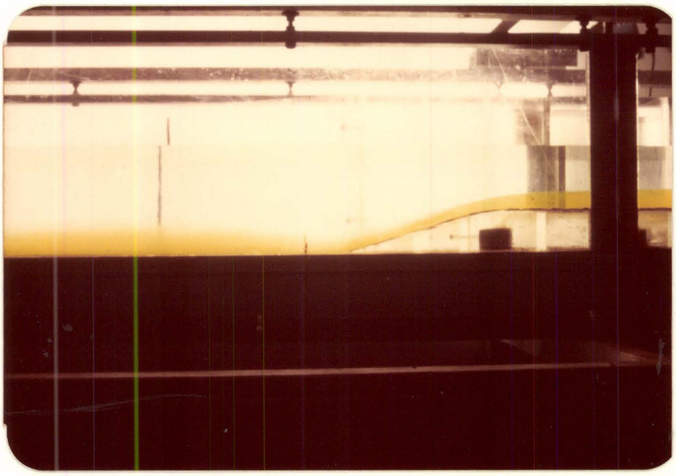


Fig. A1.7 - Entrainment of Fresh Water Downstream of the Contraction (Set I)

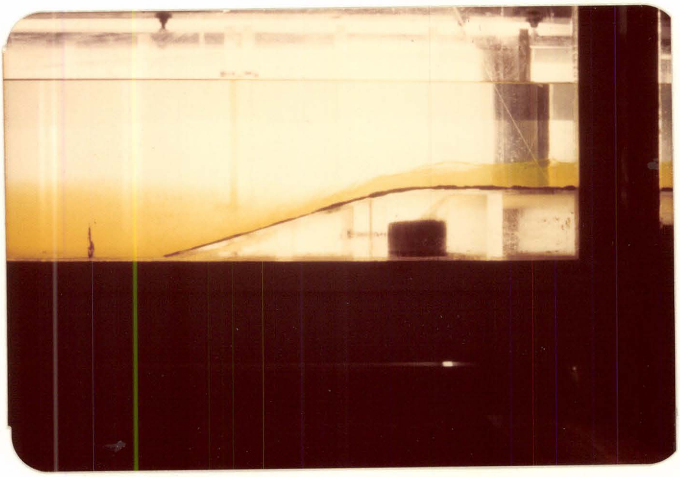


Fig. A1.8 - Submergence of Control Due to Entrainment of Fresh Water (Set I)

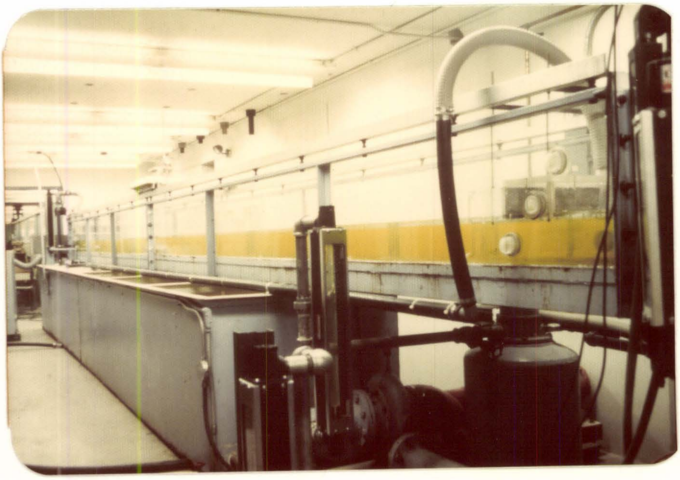


Fig. A1.9 - Long View of Fresh Water Flow Over Stagnant Salt Water (Set II).

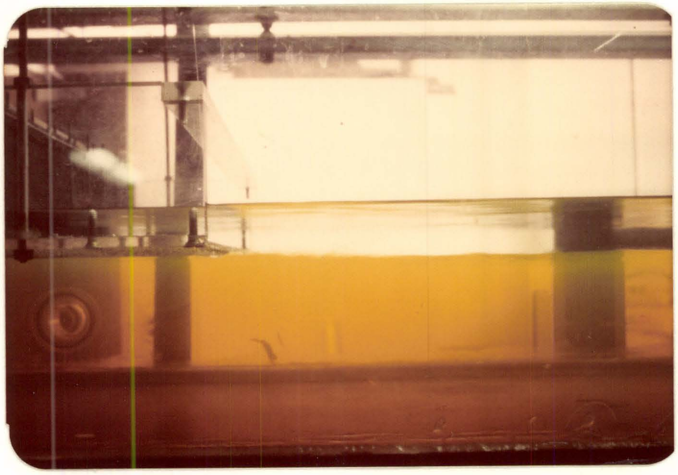


Fig. A1.10 - Fresh Water Flowing from the Upstream Box Over the Stagnant Salt Water Layer - Set II (taken from the rear of the flume)

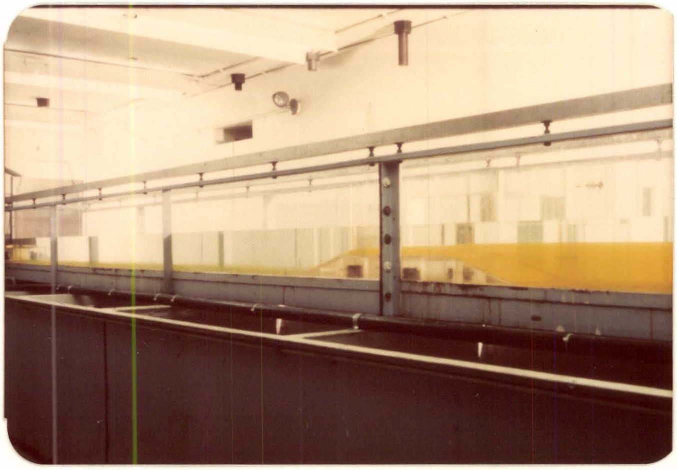


Fig. A1.11 - Long View of Both Layers Flowing in the Same Direction (Set III). Direction of Flow from Right to Left

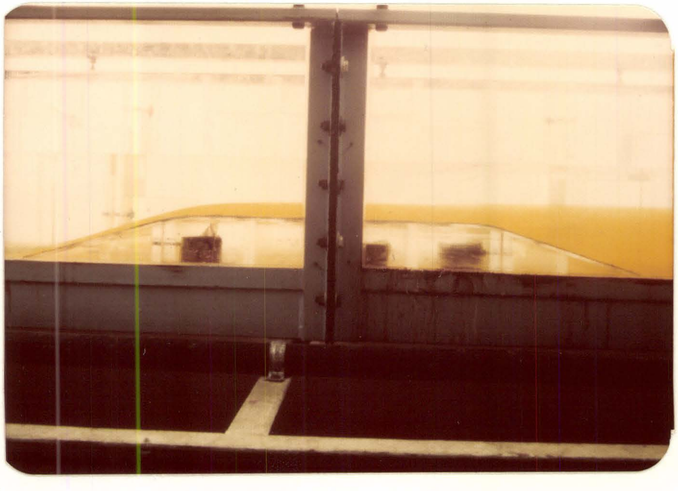


Fig. A1.12 - Flow Pattern of Both Layers Moving in the Same Direction Through the Contraction (Set III)

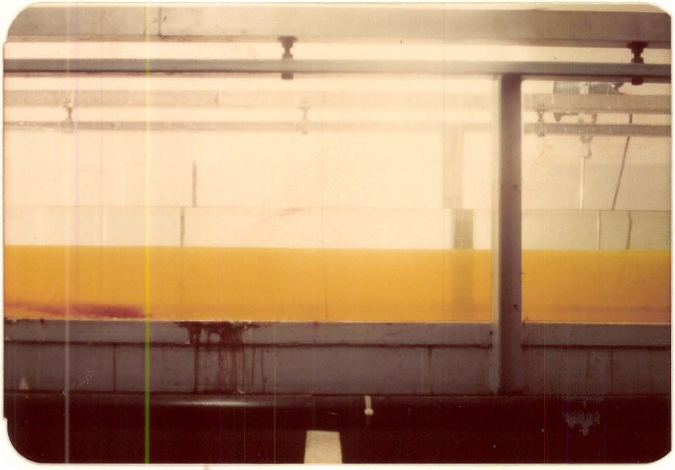


Fig. A1.13 - Velocity Profile Just Upstream of the Contraction (Set III)

Fig. A1.14 - Progress of Dye Streak a Few Seconds After Fig. A1.13 (Set III)

Fig. A1.15 - Upstream End of the Flume Where Both Layers Start Moving in the Same Direction - Set III (taken from the rear of the flume)

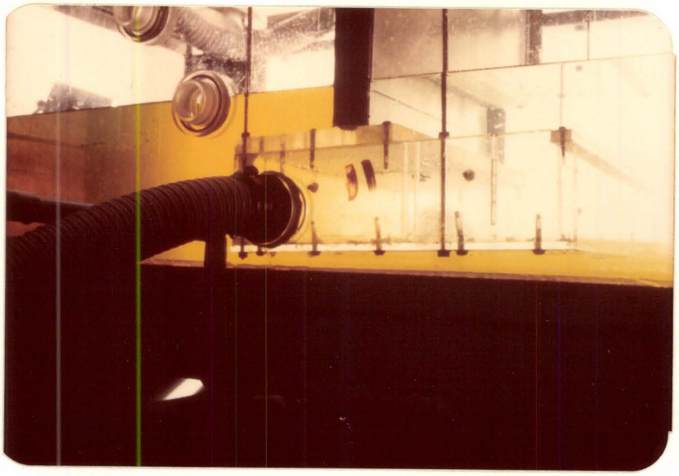


Fig. A1.16 - Downstream End of the Flume Where Both Layers are Extracted (Set III)

Fig. A1.17 - Long View of Both Layers Flowing in Opposite Directions (Set IV). Salt Water Flowing from Right to Left. Fresh Water from Left to Right.

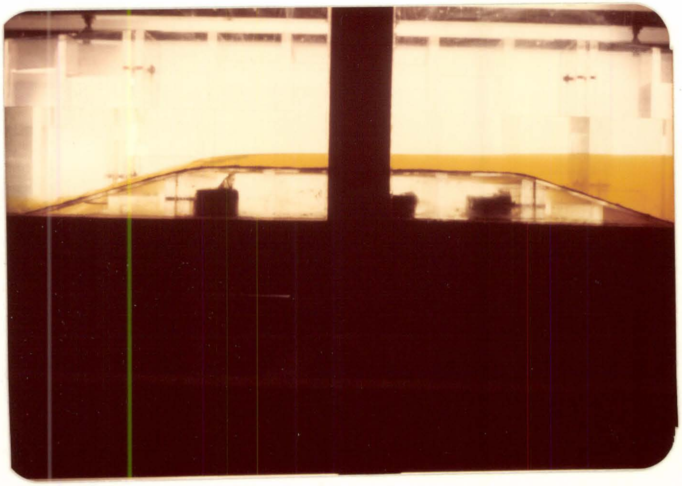


Fig. A1.18 - Flow Pattern of Both Layers Moving in Opposite Directions Through the Contraction (Set IV)

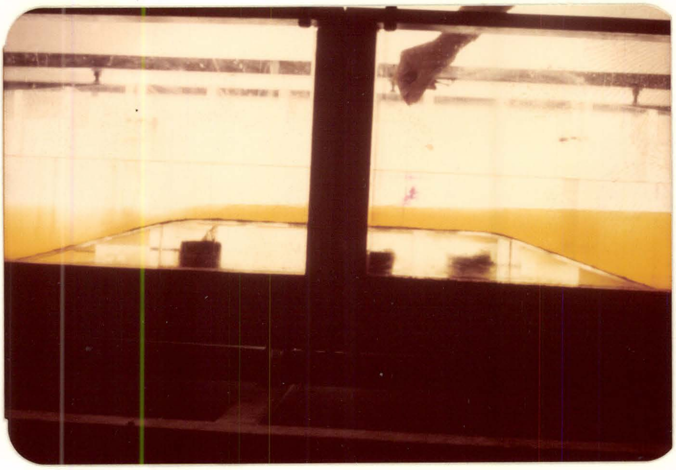


Fig. A1.19 - Velocity Profile at the Middle of the Contraction (Set IV)

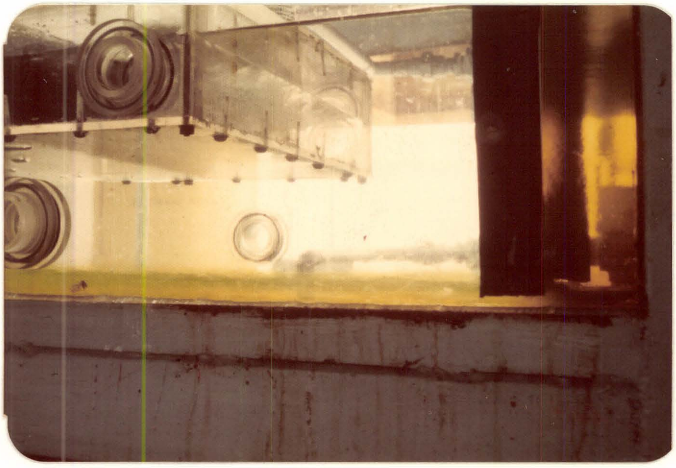


Fig. A1.20 - Downstream End of the Flume Where Salt Water is Leaving the Flume and Fresh Water is Discharged from the Shown Box in the Other Direction - Set IV (taken from the rear of the flume)

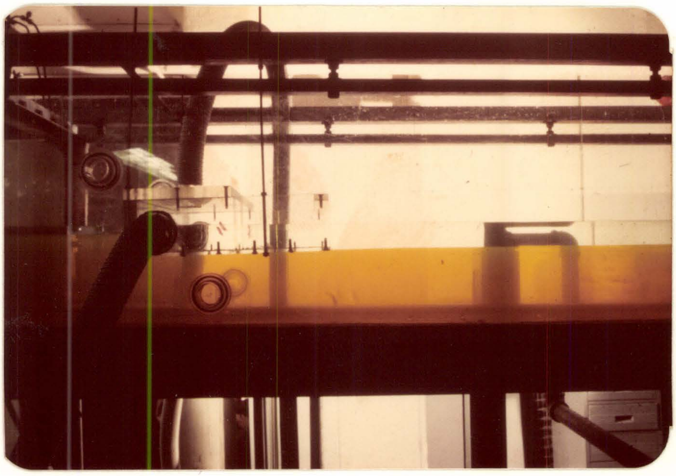


Fig. A1.21 - Upstream End of the Flume Where Fresh Water is Leaving the Flume and Salt Water is Discharged from Left to Right - Set IV (taken from the rear of the flume)

APPENDIX (2)

DOCUMENTATION AND LISTINGS OF THE COMPUTER SUBROUTINES

INTRODUCTION

THIS VOLUME CONTAINS AN INDEX AND DOCUMENTATION OF SUBROUTINES WRITTEN FOR THE SOLUTION OF STRATIFIED FLOW PROBLEMS. THE ROUTINES HAVE BEEN DESIGNED TO BE MUTUALLY COMPATIBLE IN ORDER TO FACILITATE THE COMBINATION OF SUB-PROGRAMS IN THE SOLUTION OF COMPLEX PROBLEMS.

THE INDEX CONTAINS A LIST OF ALL SUBROUTINES AND FUNCTIONS TOGETHER WITH A BRIEF DESCRIPTION OF THE PURPOSE AND OTHER ROUTINES INVOLVED.

THE MAIN BODY OF THE TEXT CONSISTS OF TWO PARTS. THE FIRST PART COMRISES DETAILED DOCUMENTATION OF EACH SUB-PROGRAM. EACH DOCUMENTATION IS PRESENTED IN IDENTICAL FORMAT INCLUDING PURPOSE, METHOD OF SOLUTION, METHOD OF USE, AND AN EXAMPLE. THE SECOND PART CONTAINS COMPLTE LISTINGS OF ALL SUBROUTINES.

INDEX

NAME	PURPOSE	OTHER ROUTINES USED
PROPS	EVALUATES THE CROSS-SECTIONAL AREA, SURFACE WIDTH, WETTED PERIMETER, AND THE PRODUCT OF AREA AND CENTROIDAL DEPTH FOR A SPECIFIED WATER LEVEL AT A CROSS-SECTION OF A CHANNEL OF ARBITRARY GEOMETRY CONTAINING HOMOGENEOUS ONE-LAYER FLOW.	NONE
BOTTOM	DETERMINES THE LOWEST POINT(INVERT LEVEL) IN A CHANNEL CROSS-SECTION OF ARBITRARY SHAPE.	NONE
SELSEC	A WATER COURSE IS DESCRIBED BY A SERIES OF CROSS-SECTIONS. THIS INFORMATION IS STORED IN THE TWO-DIMENSIONAL ARRAYS HORZAR AND VERTAR. THIS ROUTINE SELECTS FROM THESE ARRAYS THE COORDINATES RELATING A PARTICULAR CROSS-SECTION AND ASSIGN THEM TO THE ONE-DIMENSIONAL ARRAYS.	NONE
ZSYSTM1	SOLVES N SIMULTANEOUS NON-LINEAR EQUATIONS IN N UNKNOWNS (USING BROWN METHOD).	AUXILIARY FUNCTION(S
PROPS2	EVALUATES THE PROPERTIES OF A CHANNEL CROSS-SECTION OF ARBITRARY GEOMETRY CONTAINING TWO FLUIDS.	PROPS
BERNQ2	APPLIES THE PRINCIPLE OF ENERGY CONSERVATION BETWEEN TWO CROSS-SECTIONS OF A CHANNEL OF ARBITRARY GEOMETRY CONTAINING A TWO-LAYER SYSTEM. IT COMPUTES DISCHARGES IN BOTH LAYERS GIVEN ELEVATIONS OF THE FREE SURFACE AND THE INTERFACE AT BOTH SECTIONS.	PROPS2
BERNWL2	COMPUTES ELEVATIONS OF THE FREE SURFACE AND THE INTERFACE AT ONE SECTION GIVEN ELEVATIONS AT THE OTHER SECTION AND DISCHARGES IN BOTH LAYERS. USES NEWTON-RAPHSON METHOD.	PROPS2
QCRIT	CALCULATES CRITICAL DISCHARGE IN THE UPPER(OR LOWER) LAYER GIVEN THE DISCHARGE IN THE LOWER(OR UPPER) LAYER AND THE ELEVATIONS OF THE FREE SURFACE AND THE INTERFACE.	PROPS2
WLCRIT	CALCULATES THE FREE SURFACE ELEVATION AT A CRITICAL SECTION GIVEN THE ELEVATION OF THE INTERFACE AND DISCHARGES IN BOTH LAYERS. USES THE SECANT METHOD.	PROPS2-PROPS-BOTTOM
WLCRIT1	CALCULATES THE INTERFACE ELEVATION(S) AT A CRITICAL SECTION GIVEN THE FREE SURFACE ELEVATION AND DISCHARGES IN BOTH LAYERS. USES NEWTON-RAPHSON METHOD.	PROPS2-PROPS-BOTTOM

NAME	PURPOSE	OTHER ROUTINES USED
WLUCR	APPLIES THE PRINCIPLE OF ENERGY CONSERVATION BETWEEN TWO CROSS-SECTIONS OF A CHANNEL OF ARBITRARY GEOMETRY WHEN ONE OF THESE SECTIONS ACTS AS A CONTROL(CRITICAL SECTION). IT COMPUTES THE FREE SURFACE ELEVATION AT THE CRITICAL SECTION AS WELL AS BOTH THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE OTHER SECTION GIVEN THE INTERFACE ELEVATION AT THE CRITICAL SECTION AND THE DISCHARGES IN BOTH LAYERS.	WLCRIT-BERNWL2
WLQCR	COMPUTES THE DISCHARGE IN THE UPPER(OR LOWER) LAYER AND THE ELEVATIONS OF THE FREE SURFACE AND THE INTERFACE AT THE CRITICAL SECTION GIVEN THE DISCHARGE OF THE LOWER(OR UPPER) LAYER AND THE ELEVATIONS AT THE OTHER SECTION.	ZSYSTM1(AUXX)-PROPS2
QCRIT1	SIMILAR TO WLQCR EXCEPT THAT THE ELEVATIONS AT THE CRITICAL SECTION ARE GIVEN AND THOSE AT THE OTHER SECTION ARE COMPUTED.	QCRIT-BERNWL2
QCRIT12	COMPUTES DISCHARGES IN BOTH LAYERS AND THE FREE SURFACE ELEVATION AT THE CRITICAL SECTION GIVEN THE INTERFACE ELEVATION AT THE CRITICAL SECTION AS WELL AS THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE OTHER SECTION.	PROPS-BERNQ2-WLCRIT
QCRIT22	SIMILAR TO QCRIT12 EXCEPT THAT THE FREE SURFACE ELEVATION AT THE CRITICAL SECTION IS GIVEN AND THE INTERFACE ELEVATION AT THAT SECTION IS COMPUTED. TWO SETS OF SOLUTIONS MAY BE OBTAINED.	BOTTOM-BERNQ2-WLCRIT1
SOLVE	SEARCHES ALL POSSIBLE SOLUTIONS OF THE MOMENTUM EQUATIONS FOR AN INTERFACIAL HYDRAULIC JUMP IN A TWO-LAYER SYSTEM.	ZSYSTM1(AUX)
SOLVE1	SOLVES THE MOMENTUM EQUATION AT THE MIXING REGION OF AN INTERFACIAL HYDRAULIC JUMP.	ZSYSTM1(AUX1)
INTJMP	DETERMINES THE STATE UPSTREAM(OR DOWNSTREAM) FROM A HYDRAULIC JUMP AT THE INTERFACE OF TWO FLOWING LAYERS FOR A COMPLETELY SPECIFIED STATE DOWNSTREAM(OR UPSTREAM) IN A CHANNEL OF ARBITRARY CROSS-SECTION. PROVISION IS MADE TO COSIDER MIXING AT THE INTERFACE.	SOLVE-SOLVE1-PROPS2 BOTTOM
LKEXFL	CALCULATES THE OVERFLOW AND THE UNDERFLOW FRONT VELOCITIES OF A LOCK EXCHANGE FLOW AT ANY GIVEN TIME AFTER THE START OF THE FLOW WITH SPECIFIED TIME INCREMENT. IT ALSO CALCULATES, AT THE SAME TIME INTERVALS, THE DISTANCES TRAVELLED BY THE OVERFLOW AND UNDERFLOW FRONTS. USES RUNGE-KUTTA-MERSON METHOD.	NONE

NAME	PURPOSE	OTHER ROUTINES USED
FCHECK	DETERMINES THE TYPES OF FLOW AND OF THE INTERFACIAL BOUNDARY LAYERS (I.E. LAMINAR OR TURBULENT) IN A TWO-LAYER SYSTEM WITH ONE LAYER MOVING THROUGH A CHANNEL OF ARBITRARY GEOMETRY.	PROPS2-PROPS
FILMNR	CALCULATES THE ITERFACIAL FRICTION COEFFICIENT WHEN BOTH FLOW AND INTERFACIAL BOUNDARY LAYERS ARE LAMINAR. THE SYSTEM CONSIDERED IS A TWO- LAYER SYSTEM WITH THE LOWER LAYER MOVING IN A CHANNEL OF ARBITRARY GEOMETRY.	PROPS
FITURB	CALCULATES THE INTERFACIAL FRICTION COEFFICIENT WHEN BOTH FLOW AND INTERFACIAL BOUNDARY LAYERS ARE TURBULENT. THE SYSTEM CONSIDERED IS THE SAME AS FILMNR.	PROPS
FBNDRY	CALCULATES THE FRICTION COEFFICIENT AT THE SOLID BOUNDARIES IN A TWO-LAYER SYSTEM WITH ONE LAYER FLOWING THROUGH A CHANNEL OF ARBITRARY GEOMETRY.	PROPS2-PROPS-ZSYSTM1(AUX)
FILMTB	CALCULATES THE INTERFACIAL FRICTION COEFFICIENT WHEN THE FLOW IS TURBULENT AND THE INTERFACIAL BOUNDARY LAYERS ARE LAMINAR. THE SYSTEM CONSIDERED IS THE SAME AS FBNDRY. USES SUCCESSIVE APPROXIMATIONS BASED ON PRANDTL BOUNDARY-LAYER THEORY.	PROPS2-BOTTOM-ZSYSTM1(AUX1-AUX2-AUX3-AUX4-AUX5)
FINTRF	CALCULATES THE INTERFACIAL FRICTION COEFFICIENT IN A TWO-LAYER SYSTEM WITH ONE LAYER MOVING THROUGH A CHANNEL OF ARBITRARY GEOMETRY.	FCHECK-FILMNR-FITURB FILMTB
ENGRAD	CALCULATES THE ENERGY GRADIENT OF EACH LAYER IN A TWO-LAYER SYSTEM AT A CROSS-SECTION OF ARBITRARY GEOMETRY.	FBNDRY-FINTRF-PROPS2
SURGRD	CALCULATES THE FREE SURFACE SLOPE AND THE INTERFACE SLOPE IN A TWO-LAYER SYSTEM AT A CROSS-SECTION OF ARBITRARY GEOMETRY.	ENGRAD-PROPS2-BOTTOM PROPS
FLPROF	A STRETCH OF RIVER IS DESCRIBED BY A SEQUENCE OF CROSS-SECTIONS. THE ROUTINE OPERATES ON THE SYSTEM THUS DEFINED AND FOR A SELECTED REACH CALCULATES THE FREE SURFACE AND THE INTERFACE ELEVATIONS AS WELL AS THE ENERGY LEVELS FOR EACH LAYER AT ONE END AS A FUNCTION OF SPECIFIED DISCHARGES AND CONTROL LEVELS AT THE OTHER END WITH A SPECIFIED INCREMENTAL DISTANCE ADEQUATELY SIGNED. THE ROUTINE IS INTENDED FOR SUCCESSIVE APPLICATION WORKING IN EITHER UPSTREAM OR DOWNSTREAM DIRECTION. USES RUNGE-KUTTA-MERSON METHOD.	SELSEC-WLCRIT1-SURGRD-PROPS2-BOTTOM
NORMQ2	CALCULATES NORMAL DISCHARGES IN A TWO-LAYER SYSTEM AT A CROSS-SECTION OF ARBITRARY SHAPE.	PROPS2-ZSYSTEM1 (FUNN)
NORMWL	CACULATES NORMAL WATER LEVELS IN A TWO-LAYER SYSTEM AT A CROSS-SECTION OF ARBITRARY SHAPE.	PROPS2-ZSYSTM1 (FUNM)

NAME	PURPOSE	OTHER ROUTINES USED
ARSWDG	CALCULATES THE PROFILE AND THE LENGTH OF AN ARRESTED WEDGE (SALINE WEDGE OR THERMAL WEDGE) IN A STRETCH OF RIVER DESCRIBED BY A SEQUENCE OF CROSS-SECTIONS. IT EVALUATES THE FREE SURFACE AND THE INTERFACE ELEVATIONS AS A FUNCTION OF A SPECIFIED DISCHARGE AND FREE SURFACE ELEVATION AT THE INLET WITH A SPECIFIED INCREMENTAL DISTANCE ADEQUATELY SIGNED. CRITICAL CONDITION IS ASSUMED AT THE INLET.	WLCRIT1-FLPROF-SELSEC
SWLMID	CALCULATES THE DISCHARGE OF THE UPPER LAYER AND THE FREE SURFACE ELEVATION AT ONE END OF A REACH AS A FUNCTION OF THE FREE SURFACE ELEVATION AT THE OTHER END AND THE INTERFACE ELEVATIONS AT BOTH ENDS AS WELL AS THE DISCHARGE OF THE LOWER LAYER. THE REACH IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS. TRANQUIL FLOW IS ASSUMED THROUGHOUT WITHOUT LOSSES.	PROPS2-PROPS
SWLBOT	CALCULATES THE DISCHARGE OF THE LOWER LAYER AND THE FREE SURFACE ELEVATION AT ONE END OF A REACH AS A FUNCTION OF THE FREE SURFACE ELEVATION AT THE OTHER END AND THE INTERFACE ELEVATIONS AT BOTH ENDS IN THE CASE OF A STAGNANT UPPER LAYER. THE REACH IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS. TRANQUIL FLOW IS ASSUMED THROUGHOUT WITHOUT LOSSES.	PROPS

****
* DOCUMENTATION *
****

PROPS
=====

PROPERTIES OF A CHANNEL CROSS-SECTION.

(1) PURPOSE THE CROSS-SECTION OF A RIVER CHANNEL IS DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE REFERRED TO ARBITRARY DATUMS FOR LEVEL AND TRANSVERSE DISTANCE. FOR A SPECIFIED HORIZONTAL WATER LEVEL (REFERRED TO THE SAME COORDINATE SYSTEM) THE ROUTINE EVAUATES THE AREA, SURFACE WIDTH, WETTED PERIMETER AND THE PRODUCT OF AREA AND CENTROIDAL DEPTH.

(2) METHOD THE SECTION IS SUBDIVIDED INTO A SERIES OF TRIANGLES AND TRAPEZIA BY VERTICALS THROUGH EACH OF THE POINTS BELOW THE WATER SURFACE. THE VARIOUS PROPERTIES OF EACH SUB-AREA ARE ACCUMULATED, TAKING ACCOUNT OF SIGN IN THE EVENT OF OVERHANGING SIDES BEING ENCOUNTERED. IF THE WATER LEVEL IS HIGHER THAN THE TOPMOST POINTS, THE PROPERTIES ARE EVALUATED AS THOUGH THE SECTION WERE EXTENDED BY VERTICALS THROUGH THE END POINTS. IF THE WATER LEVEL IS BELOW THE LOWERMOST POINT, ZERO VALUES ARE RETURNED FOR THE PROPERTIES.
IF THE CHANNEL IS DIVIDED BY A MIDDLE BANK THE PROPERTIES OF THE SEPARATE CHANNELS THUS FORMED ARE AGGREGATED AND TOTAL VALUES RETURNED.

(3) PROGRAM

(A) DECK NAME PROPS

(B) CALLING SEQUENCE CALL PROPS(HORZ,VERT,NPTS,WL,AREA,TOPW,PERIM,AY)

WHERE

HORZ = ONE-DIMENSIONAL ARRAY CONTAINING THE HORIZONTAL COORDINATES OF THE POINTS DESCRIBING THE CROSS-SECTION.

VERT = ONE-DIMENSIONAL ARRAY CONTAINING THE VERTICAL COORDINATES OF THE POINTS DESCRIBING THE CROSS-SECTION.

NPTS = THE NUMBER OF COORDINATE POINTS DESCRIBING THE CROSS-SECTION.

WL = WATER SURFACE ELEVATION.

AREA = LIQUID CROSS-SECTIONAL AREA.

TOPW = SECTION TOP WIDTH.

PERIM = WETTED PERIMETER.

AY = PRODUCT OF THE CROSS-SECTIONAL AREA AND CENTROIDAL DEPTH.

(C) OUTPUT FORM THE COMPUTED VALUES ARE ASSIGNED TO AREA, TOPW, PERIM AND AY.

(D) RESTRICTIONS THE COORDINATES OF THE POINTS MUST BE LISTED IN CONSISTENT ORDER STARTING WITH THE HIGHEST POINT ON ONE BANK AND FINISHING WITH THE HIGHEST POINT ON THE OTHER. NO RESTRICTIONS OF SIGN ARE IMPOSED.

(E) OTHER DECKS REQUIRED NONE.

(4) EXAMPLE

(A) INPUTS DATA DESCRIBING A SECTION WITH SIX POINTS IS GIVEN BELOW, THE INTEGER NPTS BEING GIVEN FIRST FOLLOWED BY SUCCESSIVE PAIRS OF THE COORDINATES HORZ(1), VERT(1),HORZ(2) ETC.

```
6
 0.0      100.0     -20.0      90.0     -10.0      90.0
10.0       87.0      20.0      80.0      40.0     105.0
```

THE SPECIFIED WATER LEVEL IS 100.0.

(B) CODE

```
      DIMENSION B(6),H(6)
      READ(5,10) NPTS
   10 FORMAT(I3)
      READ(5,20) (B(I),H(I),I=1,NPTS)
   20 FORMAT(6F10.2)
```

```
          CALL PROPS(B,H,NPTS,100.0,A,T,P,AY)
          WRITE(6,30) A,T,P,AY
       30 FORMAT(4F12.3)
          STOP
          END

(C) OUTPUT   555.000      36.000       90.403     3945.000
(5) SOURCE   A.A. SMITH
             C.E.P.L.
             MCMASTER UNIVERSITY
             HAMILTON, ONTARIO.
```

BOTTOM
======

LOWEST LEVEL IN A CROSS-SECTION.

(1) PURPOSE A RIVER CROSS-SECTION IS DESCRIBED BY SERIES OF POINTS, THE VERTICAL COORDINATES OF WHICH ARE REFERRED TO AN ARBITRARY DATUM. THE SUBROUTINE DETERMINES THE LOWEST POINT IN THE SECTION (INVERT LEVEL) AND THE HIGHEST POSSIBLE WATER LEVEL (LOWER OF THE TWO BANK ELEVATIONS) BOTH QUANTITIES BEING REFERRED TO THE SAME DATUM

(2) METHOD THE FIRST AND LAST POINT ELEVATIONS ARE COMPARED AND THE LOWER VALUE ASSIGNED TO THE MAXIMUM POSSIBLE WATER LEVEL. ALL THE POINTS ARE SCANNED SUCCESSIVELY AND THE LOWEST VALUE ASSIGNED TO THE INVERT LEVEL.

(3) PROGRAM

(A) DECK NAME BOTTOM

(B) CALLING SEQUENCE CALL BOTTOM(VERT,NPTS,BOT,WLMAX)

WHERE
VERT = ONE-DIMENSIONAL ARRAY CONTAINING THE VERTICAL COORDINATES OF THE POINTS DESCRIBING THE CROSS-SECTION.
NPTS = THE NUMBER OF COORDINATE POINTS DESCRIBING THE CROSS-SECTION.
BOT = THE LOWEST LEVEL IN THE CROSS-SECTION.
WLMAX = MAXIMUM WATER SURFACE ELEVATIPN.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO BOT AND WLMAX.

(D) RESTRICTIONS THE VERTICAL COORDINATES MUST BE LISTED IN SEQUENCE, THE FIRST AND LAST POINTS THUS DEFINED CORRESPONDING TO THE HIGHEST LEVEL ON EACH BANK.

(E) OTHER DECKS REQUIRED NONE.

(4) EXAMPLE

(A) INPUTS A SECTION IS DEFINED BY THE SIX POINTS WHOSE COORDINATES ARE LISTED IN PAIR BELOW. ONLY THE VERTICAL COORDINATES ARE RELEVANT BUT SECTION DATA WILL NORMALLY BE STORED WITH BOTH COORDINATES.

```
0,100     -20,90     -10,79      10,90      20,80      40,105
```

(B) CODE

```
      DIMENSION VERT(6)
      NPTS=6
      READ(5,10) (X,VERT(I),I=1,NPTS)
   10 FORMAT(2F10.1)
C     HORZ COORDS IGNORED
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)
      WRITE(6,20) BOT,WLMAX
   20 FORMAT(2F12.3)
      STOP
      END
```

(C) OUTPUT

```
79.000          100.000
```

(5) SOURCE
A.A. SMITH
C.E.P.L.
MCMASTER UNIVERSITY
HAMILTON, ONTARIO.

SELSEC

SELECTION OF CROSS-SECTION COORDINATES.

(1) PURPOSE A WATERCOURSE IS DESCRIBED BY A SERIES OF CROSS-SECTIONS, EACH OF WHICH IS, IN TURN, DESCRIBED BY THE COORDINATES OF A NUMBER OF POINTS ON IT. THIS INFORMATION IS STORED IN THE TWO-DIMENSIONAL ARRAYS HORZAR(NSEC,MAXPTS) AND VERTAR(NSEC,MAXPTS), WHERE HORZAR CONTAINS THE HORIZONTAL COORDINATES, VERTAR THE VERTICAL COORDINATES, NSEC THE TOTAL NUMBER OF CROSS-SECTIONS AND MAXPTS THE MAXIMUM NUMBER OF POINTS IN ANY CROSS-SECTION. THIS SUBROUTINE SELECTS FROM THE ABOVE ARRAYS THE COORDINATES RELATING A PARTICULAR CROSS-SECTION NN AND ASSIGNS THEM TO THE ONE-DIMENSIONAL ARRAYS.

(2) METHOD THE SUBROUTINE ASSIGNS TO TWO ONE-DIMENSIONAL ARRAYS THAT PORTION OF THE CORRESPONDING TWO-DIMENSIONAL ARRAYS WHICH CONTAIN THE DATA FOR THE DESIRED CROSS-SECTION.

(3) PROGRAM
(A) DECK NAME SELSEC
(B) CALLING SEQUENCE CALL SELSEC(HORZAR,VERTAR,NSEC,MAXPTS,NN,HORZ,VERT)

WHERE
HORZAR = TWO-DIMENSIONAL ARRAY CONTAINING THE HORIZONTAL COORDINATES FOR THE POINTS DESCRIBING THE SERIES OF CROSS-SECTIONS. THE FIRST VALUE OF THE ARRAY REPRESENTS THE SECTION NUMBER AND THE SECOND VALUE REPRESENTS ONE OF THE HORIZONTAL COORDINATES.
VERTAR = TWO-DIMENSIONAL ARRAY CONTAINING THE VERTICAL COORDINATES FOR THE POINTS DESCRIBING THE SERIES OF CROSS-SECTIONS. THE FIRST VALUE OF THE ARRAY REPRESENTS THE SECTION NUMBER AND THE SECOND VALUE REPRESENTS ONE OF THE VERTICAL COORDINATES.
NSEC = THE NUMBER OF CROSS-SECTIONS IN THE RIVER CHANNEL
MAXPTS = THE MAXIMUM NUMBER OF COORDINATE POINTS REQUIRED TO DESCRIBE ANY CROSS-SECTION IN THE SERIES OF CROSS-SECTIONS.
NN = NUMBER OF REQUIRED CROSS-SECTION.
HORZ = ONE-DIMENSIONAL ARRAY CONTAINING THE HORIZONTAL COORDINATES OF THE POINTS DESCRIBING THE CROSS-SECTION.
VERT = ONE-DIMENSIONAL ARRAY CONTAINING THE VERTICAL COORDINATES OF THE POINTS DESCRIBING THE CROSS-SECTION.

(C) OUTPUT FORM CALCULATED VALUES ARE ASSIGNED TO HORZ AND VERT.
(D) RESTRICTIONS ARRAY BOUNDS MUST BE OBSERVED BY THE USER.
(E) OTHER DECKS REQUIRED NONE.

(4) EXAMPLE TRIVIAL.

(5) SOURCE A.A. SMITH
C.E.P.L.
MCMASTER UNIVERSITY
HAMILTON, ONTARIO.

ZSYSTM1

DETERMINATION OF A ROOT OF A SYSTEM OF N SIMULTANEOUS NONLINEAR EQUATIONS IN N UNKNOWNS, F(X)=0, IN VECTOR FORM. N CAN BE 1.

(1) PURPOSE THE SUBROUTINE SOLVES A SYSTEM OF N SIMULTANEOUS NONLINEAR EQUATIONS IN N UNKNOWNS.

(2) METHOD THE SUBROUTINE USES BROWN METHOD WHICH REQUIRES N**2/2+3*N/2 FUNCTION EVALUATIONS PER ITERATIVE STEP. A ROOT IS ACCEPTED IF TWO SUCCESSIVE APPROXIMATIONS TO A GIVEN ROOT AGREE IN THE FIRST NSIG DIGITS. A ROOT IS ALSO ACCEPTED IF THE ABSOLUTE VALUE OF F(X,K,PAR,...) IS LESS THAN EPS FOR EVERY K=1,...,N.

(3) PROGRAM
(A) DECK NAME ZSYSTEM1
(B) CALLING SEQUENCE

```
CALL ZSYSTM1(F,EPS,NSIG,N,X,ITMAX,F1,F2,R,B,HORZD,VERTD,NPTSD,
*BETAU,BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,
*Q2,RHO1,RHO2,AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,
*AY12,AY22,TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22,H22,WA,PAR,IER)
```

WHERE

F = THE NAME OF THE FUNCTION CALLED BY ZSYSTM1 TO FURNISH THE VALUES OF THE FUNCTIONS WHICH DEFINE THE SYSTEM OF EQUATIONS BEING SOLVED. THE USER SPECIFIES F BY WRITING A FUNCTION SUBPROGRAM F(X,K,PAR,...) WHICH COMPUTES THE K-TH COMPONENT OF F EVALUATED AT X.

EPS = FIRST STOPPING CRITERION. A ROOT X(1),...,X(N) IS ACCEPTED IF THE MAXIMUM ABSOLUTE VALUE OF F(X,K, PAR,...) IS LESS THAN OR EQUAL TO EPS, WHERE K=1, ...,N.

NSIG = SECOND STOPPING CRITERION. A ROOT IS ACCEPTED IF TWO SUCCESSIVE APPROXIMATIONS TO A GIVEN ROOT AGREE IN THE FIRST NSIG DIGITS.
NOTE- IF EITHER, OR BOTH, OF THE STOPPING CRITERIA ARE FULFILLED, THE ROOT IS ACCEPTED.

N = THE NUMBER OF EQUATIONS (= NO. OF UNKNOWNS).

X = THE VECTOR X OF LENGTH N, AS INPUT, IS THE INITIAL GUESS OF THE ROOT. AS OUTPUT, IT IS THE COMPUTED SOLUTION.

ITMAX = ON INPUT = THE MAXIMUM ALLOWABLE NUMBER OF ITERATIONS AND ON OUTPUT = THE NUMBER OF ITERATIONS USED IN FINDING THE COMPUTED SOLUTION.

WA = A WORKING ARRAY OF SIZE ((N+2)*(N-1))/2+(3*N) SUPPLIED BY THE USER.

IER = ERROR PARAMETER
TERMINAL ERROR = 128+NN
NN=1 INDICATES FAILURE TO CONVERGE WITHIN ITMAX ITERATIONS.
NN=2 SINGULAR SYSTEM (JACOBIAN).

ALL THE OTHER PARAMETERS ARE PASSED TO THE USER SUPPLIED FUNCTION F AND MAY BE USED TO PASS ANY AUXILIARY PARAMETERS NECESSARY FOR COMPUTATION OF THE FUNCTION F.
ZSYSTM1 IS A MODIFIED VERSION OF ZSYSTM(I.M.S.L. LIBRARY-MCMASTER UNIVERSITY, HAMILTON, ONTARIO, CANADA). MODIFICATIONS GENERALLY INVOLVE FUNCTION FORMAT AND OUTPUT FORM IN ORDER TO MAKE IT MORE SUITABLE FOR SOLUTIONS OF THE TYPE OF PROBLEMS INVOLVED IN THIS STUDY.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO X, ITMAX, IER
(D) RESTRICTIONS NONE.
(E) OTHER DECKS REQUIRED NONE.

(4) EXAMPLE

THE FOLLOWING THREE EQUATIONS NEED TO BE SOLVED FOR THE THREE UNKNOWNS X(1),X(2) AND X(3)

```
                    X(1)+EXP(X(1)-1.0)+(X(2)+X(3))**2-27.0 = 0.0
                    X(1)*EXP(X(2)-2.0)+X(3)**2-10.0        = 0.0
                    X(3)+SIN(X(2)-2.0)+X(2)**2-7.0         = 0.0
```

```
(B) CODE            DIMENSION X(3),WA(14)
                    EXTERNAL AUX
                    X(1)=0.75
                    X(2)=1.5
                    X(3)=4.0
                    EPS=1.0E-6
                    NSIG=5
                    N=3
                    ITMAX=100
                    CALL ZSYSTM1(AUX,EPS,NSIG,N,X,ITMAX,F1,F2,R,B,HORZD,VERTD,1,
                   *BETAU,BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,
                   *Q2,RHO1,RHO2,AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,
                   *AY12,AY22,TOPW12,TOPW22,HORZ,VERT,1,RHO22,Q22,H22,WA,PAR,IER)
                    WRITE(6,10) (X(I),I=1,3)
                 10 FORMAT (3F10.3)
                    WRITE(6,20) ITMAX,IER
                 20 FORMAT (2I10)
                    STOP
                    END
              C
              C     THE FUNCTION SUBPROGRAM AUX IS AS FOLLOWS
              C
                    FUNCTION AUX(X,K,PAR,F1,F2,R,B,HORZD,VERTD,1,BETAU,BETAL,BOTD,
                   *HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,RHO2,
                   *AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,AY12,AY22,
                   *TOPW12,TOPW22,HORZ,VERT,1,RHO22,Q22)
                    DIMENSION X(1)
                    GO TO (5,10,15),K
                  5 AUX=X(1)+EXP(X(1)-1.0)+(X(2)+X(3))**2-27.0
                    RETURN
                 10 AUX=X(1)*EXP(X(2)-2.0)+X(3)**2-10.0
                    RETURN
                 15 AUX=X(3)+SIN(X(2)-2.0)+X(2)**2-7.0
                    RETURN
                    END
```

```
(C) OUTPUT          1.000     2.000     3.000
                        6         0
```

PROPS2

PROPERTIES OF A CHANNEL CROSS-SECTION CONTAINING TWO FLUIDS.

(1) PURPOSE THE CROSS-SECTION OF A RIVER CHANNEL IS DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE REFERRED TO ARBITRARY DATUMS FOR LEVEL AND TRANSVERSE DISTANCE. FOR SPECIFIED HORIZONTAL WATER LEVELS OF THE FREE SURFACE AND THE INTERFACE (REFERRED TO THE THE SAME COORDINATE SYSTEM), THE ROUTINE EVALUATES AREAS, SURFACE WIDTHS, WETTED PERIMETERS AND PRODUCTS OF AREA AND CENTROIDAL DEPTH FOR EACH LAYER.

(2) METHOD PROPERTIES OF THE LOWER LAYER ARE COMPUTED USING WL2 IN SUBROUTINE PROPS. PROPERTIES OF THE WHOLE SECTION ARE COMPUTED USING WL1 IN SUBROUTINE PROPS. PROPERTIES OF THE UPPER LAYER ARE THEN CALCULATED USING THE ABOVE PROPERTIES AS FOLLOWS

AREA1=TOTAL AREA-AREA2
TOPW1=TOP WIDTH OF THE TOTAL SECTION.
PERIM1=WETTED PERIMETER OF THE TOTAL SECTION-PERIM2+TOPW2
AY1=AY(TOTAL)-AY2-AREA2*(WL1-WL2)

(3) PROGRAM
(A) DECK NAME PROPS2
(B) CALLING SEQUENCE

```
CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,AREA1,AREA2,TOPW1,TOPW2,PERIM1
*,PERIM2,AY1,AY2)
```

WHERE
HORZ = ONE-DIMENSIONAL ARRAY CONTAINING THE HORIZONTAL COORDINATES OF THE POINTS DESCRIBING THE CROSS-SECTION.
VERT = ONE-DIMENSIONAL ARRAY CONTAINING THE VERTICAL COORDINATES OF THE POINTS DESCRIBING THE CROSS-SECTION.
NPTS = THE NUMBER OF COORDINATE POINTS DESCRIBING THE CROSS-SECTION.
WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY.
AREA1,AREA2 = LIQUID CROSS-SECTIONAL AREAS OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY.
TOPW1,TOPW2 = SECTION TOP WIDTH OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY.
PERIM1,PERIM2 = WETTED PERIMETER OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY. PERIM1 INCLUDES INTERFACE.
AY1,AY2 = PRODUCT OF THE CROSS-SECTIONAL AREA AND CENTROIDAL DEPTH FOR THE UPPER AND THE LOWER LAYERS RESPECTIVELY. CENTROIDAL DEPTH IS MEASURED FROM FREE SURFACE FOR THE UPPER LAYER AND FROM THE INTERFACE FOR THE LOWER LAYER.

(C) OUTPUT FORM THE COMPUTED VALUES ARE ASSIGNED TO AREA1,AREA2,TOPW1,TOPW2, PERIM1,PERIM2,AY1 AND AY2.

(D) RESTRICTIONS THE COORDINATES OF THE POINTS MUST BE LISTED IN CONSISTENT ORDER STARTING WITH THE HIGHEST POINT ON ONE BANK AND FINISHING WITH THE HIGHEST POINT ON THE OTHER. NO RESTRICTIONS OF SIGN ARE IMPOSED.

(E) OTHER DECKS REQUIRED PROPS

(4) EXAMPLE
(A) INPUTS DATA DESCRIBING A SECTION WITH SIX POINTS IS GIVEN BELOW. THE INTEGER NPTS BEING GIVEN FIRST FOLLOWED BY SUCCESSIVE PAIRS OF THE COORDINATES HORZ(1), VERT(1),HORZ(2) ETC.

```
6
 0.0      100.0     -20.0       90.0     -10.0       90.0
10.0       87.0      20.0       80.0      40.0      105.0
```

THE SPECIFIED WATER LEVELS FOR THE FREE SURFACE AND THE INTERFACE

ARE 100.0 AND 90.0 RESPECTIVELY.

(B) CODE

```
      DIMENSION B(6),H(6)
      READ(5,10) NPTS
   10 FORMAT( I3)
      READ(5,20) (B( I),H( I),I=1,NPTS)
   20 FORMAT(6F10.2)
      CALL PROPS2(B,H,NPTS,100.0,90.0,A1,A2,T1,T2,P1,P2,AY1,AY2)
      WRITE(6,30) A1,T1,P1,AY1,A2,T2,P2,AY2
   30 FORMAT(4F12.3,/,4F12.3)
      STOP
      END
```

(C) OUTPUT

```
    420.000      36.000      83.167    2200.000
    135.000      38.000      45.237     395.000
```

BERNQ2

======

DISCHARGE IN A TWO-LAYER SYSTEM THROUGH A STREAMLINED TRANSITION

(1) PURPOSE THE ROUTINE FINDS THE DISCHARGE OF BOTH LAYERS IN A TRANSITION AS A FUNCTION OF THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT TWO CROSS-SECTIONS. THE TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS WHICH ARE DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN. TRANQUIL FLOW IS ASSUMED THROUGHOUT WITH NO LOSSES.

(2) METHOD THE ROUTINE INITIALLY CALLS ON SUBROUTINE PROPS2 TO EVALUATE AREAS AT EACH SECTION. THE DISCHARGES ARE THEN CALCULATED USING BERNOULLI EQUATIONS (EQUATIONS 3.2 AND 3.4- CHAPTER 3).

(3) PROGRAM

(A) DECK NAME BERNQ2

(B) CALLING SEQUENCE

```
CALL BERNQ2(WL1,WL2,WL11,WL21,RHO1,RHO2,G,ALPHA1,ALPHA2,HORZ,VERT
*,NPTS,HORZD,VERTD,NPTSD,Q1,Q2)
```

WHERE

WL1,WL11 = FREE SURFACE ELEVATIONS AT THE FIRST AND SECOND CROSS-SECTIONS RESPECTIVELY.

WL2,WL21 = INTERFACE ELEVATIONS AT THE FIRST AND SECOND CROSS-SECTIONS RESPECTIVELY.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

ALPHA1,ALPHA2 = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE FIRST CROSS-SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.

HORZD,VERTD = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES OF THE POINTS DEFINING THE SECOND CROSS-SECTION. THE COORDINATES ARE REFERRED TO THE SAME AXES AS HORZ,VERT.

NPTS,NPTSD = NUMBER OF POINTS DEFINING THE FIRST AND THE SECOND CROSS-SECTIONS RESPECTIVELY.

Q1,Q2 = DISCHARGES IN THE UPPER AND THE LOWER LAYERS RESPECTIVELY.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO Q1 AND Q2. IF THERE ARE NO REAL SOLUTIONS TO THE CASE COSIDERED (I.E. Q1**2 OR Q2**2 NEGATIVE) A VALUE OF -1.0 IS ASSIGNED TO THE CORRESPONDING DISCHARGE.

(D) RESTRICTIONS NONE.

(E) OTHER DECKS REQUIRED PROPS2

(4) EXAMPLE

(A) INPUTS A CHANNEL DESCRIBED BY 5 POINTS AT BOTH UPSTREAM AND DOWNSTREAM SECTIONS HAS UPSTREAM LEVELS OF 10.00 AND 5.00 FOR THE FREE SURFACE AND THE INTERFACE RESPECTIVELY AND DOWNSTREAM LEVELS OF 9.97 AND 7.00 RESPECTIVELY. SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY ARE 1.00 AND 1.02.

UPSTREAM COORDINATES	0,11	0,5	6,0	12,0	15,11
DOWNSTREAM COORDINATES	0,11	6,0.5	12,0.5	15,6	15,11

(B) CODE

```
DIMENSION HORZ(5),VERT(5),HORZD(5),VERTD(5)
DATA HORZ/0.0,0.0,6.0,12.0,15.0/
```

```
      DATA VERT/11.0,5.0,0.0,0.0,11.0/
      DATA HORZD/0.0,6.0,12.0,15.0,15.0/
      DATA VERTD/11.0,0.5,0.5,6.0,11.0/
      WL1=10.0
      WL2=5.0
      WL11=9.97
      WL21=7.0
      RHO1=1.00
      RHO2=1.02
      G=9.81
      ALPHA1=1.1
      ALPHA2=1.2
      CALL BERNQ2(WL1,WL2,WL11,WL21,RHO1,RHO2,G,ALPHA1,ALPHA2,HORZ,VERT
     *,5,HORZD,VERTD,5,Q1,Q2)
      WRITE(6,10) Q1,Q2
   10 FORMAT(2F12.2)
      STOP
      END
```

```
(C) OUTPUT       35.97        30.77
```

BERNWL2

WATER LEVELS IN A TWO-LAYER SYSTEM THROUGH A STREAMLINED TRANSITION

(1) PURPOSE THE ROUTINE FINDS THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT ONE CROSS-SECTION IN A TRANSITION AS A FUNCTION OF THE DISCHARGES IN BOTH LAYERS AND THE ELEVATIONS AT THE OTHER SECTION. THE TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS WHICH ARE DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN. TRANQUIL FLOW IS ASSUMED THROUGHOUT WITH NO LOSSES.

(2) METHOD TWO-DIMENSIONAL NEWTON-RAPHSON METHOD (SEE SECTION 3.3.1- CHAPTER 3) IS USED TO SOLVE BENOULLI EQUATIONS (EQUATIONS 3.2 AND 3.4- CHAPTER 3). PROPS2 IS USED FIRST TO EVALUATE THE PROPERTIES OF THE CROSS-SECTION WHERE ELEVATIONS ARE GIVEN. THE SOLUTION STARTS WITH WL11=WL1 AND WL21=WL2. CONVERGENCE IS ASSUMED WHEN THE CHANGE IN BOTH WATER LEVELS IS LESS THAN OR EQUAL TO 0.001 PERCENT OF THE CURRENT WATER LEVELS.

(3) PROGRAM
(A) DECK NAME BERNWL2
(B) CALLING SEQUENCE

```
CALL BERNWL2(Q1,Q2,WL1,WL2,RHO1,RHO2,G,ALPHA1,ALPHA2,HORZ,VERT,
*NPTS,HORZD,VERTD,NPTSD,WL11,WL21)
```

WHERE
Q1,Q2 = DISCHARGES IN THE UPPER AND THE LOWER LAYERS RESPECTIVELY.
WL1,WL2 = GIVEN FREE SURFACE AND INTERFACE ELEVATIONS AT THE FIRST CROSS-SECTION.
RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
G = GRAVITATIONAL ACCELERATION.
ALPHA1,ALPHA2 = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE FIRST CROSS-SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.
HORZD,VERTD = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES OF THE POINTS DEFINING THE SECOND CROSS-SECTION. THE COORDINATES ARE REFERRED TO THE SAME AXES AS HORZ,VERT.
NPTS,NPTSD = NUMBER OF POINTS DEFINING THE FIRST AND THE SECOND CROSS-SECTIONS RESPECTIVELY.
WL11,WL21 = COMPUTED FREE SURFACE AND INTERFACE ELEVATIONS AT THE SECOND CROSS-SECTION.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO WL11 AND WL21. ZERO VALUES ARE ASSIGNED TO THESE WATER LEVELS IF (EITHER) WL21 IS VERY SMALL OR NEGATIVE (OR) WHEN AN IMPRACTICAL SOLUTION IS OBTAINED (I.E. WL21 GREATER THAN WL11) WHICH IS MATHEMATICALLY POSSIBLE.
(D) RESTRICTIONS NONE.
(E) OTHER DECKS REQUIRED PROPS2

(4) EXAMPLE
(A) INPUTS A CHANNEL DESCRIBED BY 5 POINTS AT BOTH UPSTREAM AND DOWNSTREAM SECTIONS HAS UPSTREAM LEVELS OF 10.00 AND 5.00 FOR THE FREE SURFACE AND THE INTERFACE RESPECTIVELY. DISCHARGES OF THE UPPER AND THE LOWER LAYERS ARE 35.97 AND 30.77 RESPECTIVELY. SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY ARE 1.00 AND 1.02.
UPSTREAM COORDINATES

```
               0,11      0,5       6,0       12,0      15,11
               DOWNSTREAM COORDINATES
               0,11      6,0.5     12,0.5    15,6      15,11

(B) CODE       DIMENSION HORZ(5),VERT(5),HORZD(5),VERTD(5)
               DATA HORZ/0.0,0.0,6.0,12.0,15.0/
               DATA VERT/11.0,5.0,0.0,0.0,11.0/
               DATA HORZD/0.0,6.0,12.0,15.0,15.0/
               DATA VERTD/11.0,0.5,0.5,6.0,11.0/
               CALL BERNWL2(35.97,30.77,10.0,5.0,1.0,1.02,9.81,1.1,1.2,HORZ,VERT
              *,5,HORZD,VERTD,5,WL11,WL21)
               WRITE(6,10) WL11,WL21
            10 FORMAT(2F13.2)
               STOP
               END

(C) OUTPUT         9.97         7.00
```

QCRIT
=====

CRITICAL DISCHARGE IN A TWO-LAYER SYSTEM

(1) PURPOSE THE ROUTINE FINDS THE CRITICAL DISCHARGE IN ONE LAYER IN AN ARBITRARY CROSS-SECTION AS A FUNCTION OF THE DISCHARGE OF THE OTHER LAYER AS WELL AS THE FREE SURFACE AND THE INTERFACE ELEVATIONS. THE CROSS-SECTION IS DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN.

(2) METHOD THE DISCHARGE IS CALCULATED USING EQUATION 3.5 (CHAPTER 3).

(3) PROGRAM
(A) DECK NAME QCRIT
(B) CALLING SEQUENCE

```
CALL QCRIT(WL1CR,WL2CR,QA,RHO1,RHO2,G,ALPHAU,ALPHAL,N1,HORZC,
*VERTC,NPTSC,QB)
```

WHERE
WL1CR, WL2CR = CRITICAL FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY.
QA = GIVEN DISCHARGE.
RHO1, RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
G = GRAVITATIONAL ACCELERATION.
ALPHAU, ALPHAL = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.
N1 = 1 IF THE DISCHARGE OF THE UPPER LAYER IS GIVEN AND THE DISCHARGE OF THE LOWER LAYER IS R5QUIRED.
= 2 IF THE DISCHARGE OF THE LOWER LAYER IS GIVEN AND THE DISCHARGE OF THE UPPER LAYER IS REQUIRED.
HORZC, VERTC = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE CRITICAL SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.
NPTSC = NUMBER OF POINTS DESCRIBING THE CROSS-SECTION.
QB = COMPUTED DISCHARGE.

(C) OUTPUT FORM THE CALCULATED VALUE IS ASSIGNED TO QB. IF NEITHER CONDITION GIVEN BY EQUATIONS 3.31 AND 3.32 (CHAPTER 3) IS SATISFIED, A VALUE OF ZERO IS ASSIGNED TO QB (NO SOLUTION).
(D) RESTRICTIONS NONE.
(E) OTHER DECKS REQUIRED PROPS2

(4) EXAMPLE
(A) INPUTS A CROSS-SECTION IS DESCRIBED BY 5 POINTS THE COORDINATES OF WHICH ARE LISTED BELOW

0,11   0,5   6,0   12,0   15,11

THE DISCHARGE OF THE UPPER LAYER IS SET AT 40.0 AND THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT 9.25 AND 4.00 RESPECTIVELY. SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.02 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.

(B) CODE

```
      DIMENSION B(5),H(5)
      READ(5,10) (B(I),H(I),I=1,5)
   10 FORMAT(2F20.2)
      CALL QCRIT(9.25,4.0,40.0,1.0,1.02,9.81,1.1,1.2,1,B,H,5,Q)
      WRITE(6,20) Q
   20 FORMAT(F10.2)
      STOP
      END
```

```
(C) OUTPUT    20.00
```

WLCRIT

CRITICAL FREE SURFACE ELEVATION IN A TWO-LAYER SYSTEM

(1) PURPOSE THE ROUTINE FINDS THE CRITICAL FREE SURFACE ELEVATION IN AN ARBITRARY CROSS-SECTION AS A FUNCTION OF THE INTERFACE ELEVATION AND THE DISCHARGES IN BOTH LAYERS. THE CROSS-SECTION IS DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN.

(2) METHOD EQUATION 3.5 (CHAPTER 3) IS SOLVED USING THE SECANT METHOD (SEE SECTION 3.3.3- CHAPTER 3). CONVERGENCE IS ASSUMED WHEN THE CHANGE IN THE DEPTH OF THE UPPER LAYER IS LESS THAN 0.01 PERCENT OF THE CURRENT DEPTH.

(3) PROGRAM

(A) DECK NAME WLCRIT

(B) CALLING SEQUENCE

```
CALL WLCRIT(Q1,Q2,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZ,VERT,NPTS,
*WL1CR)
```

WHERE

Q1,Q2 = DISCHARGES OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY.

WL2CR = CRITICAL INTERFACE ELEVATION.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

ALPHAU,ALPHAL = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE CRITICAL SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.

NPTS = NUMBER OF POINTS DESCRIBING THE CROSS-SECTION.

WL1CR = CRITICAL FREE SURFACE ELEVATION.

(C) OUTPUT FORM THE CALCULATED VALUE IS ASSIGNED TO WL1CR. IF NEITHER CONDITION GIVEN BY EQUATIONS 3.31 AND 3.32 (CHAPTER 3) IS SATISFIED, A VALUE OF ZERO IS ASSIGNED TO THE UPPER LAYER DEPTH (NO SOLUTION).

(D) RESTRICTIONS NONE.

(E) OTHER DECKS REQUIRED PROPS2,PROPS,BOTTOM

(4) EXAMPLE

(A) INPUTS A CROSS-SECTION IS DESCRIBED BY 5 POINTS THE COORDINATES OF WHICH ARE LISTED BELOW

0,11 0,5 6,0 12,0 15,11

THE DISCHARGES OF THE UPPER AND THE LOWER LAYERS ARE SET AT 40.0 AND 20.0 RESPECTIVELY. THE INTERFACE ELEVATION IS 4.00. SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.02 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.

(B) CODE

```
      DIMENSION B(5),H(5)
      READ(5,10) (B(I),H(I),I=1,5)
   10 FORMAT(2F20.2)
      CALL WLCRIT(40.0,20.0,4.0,1.0,1.02,9.81,1.1,1.2,B,H,5,WL1CR)
      WRITE(6,20) WL1CR
   20 FORMAT(F10.2)
      STOP
      END
```

(C) OUTPUT

```
9.25
```

WLCRIT1

=======

CRITICAL INTERFACE ELEVATION(S) IN A TWO-LAYER SYSTEM

(1) PURPOSE THE ROUTINE FINDS CRITICAL INTERFACE ELEVATION(S) IN AN ARBITRARY CROSS-SECTION AS A FUNCTION OF THE FREE SURFACE ELEVATION AND THE DISCHARGES IN BOTH LAYERS. THE CROSS-SECTION IS DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN.

(2) METHOD EQUATION 3.5 (CHAPTER 3) IS SOLVED USING NEWTON-RAPHSON METHOD (SEE SECTION 3.3.2- CHAPTER 3). THE UNKNOWN IS THE LOWER LAYER DEPTH. TWO DIFFERENT STARTING VALUES ARE USED TO DETERMINE THE TWO POSSIBLE SOLUTIONS (SEE SECTION 3.3.5-PART D-CHAPTER 3). THE FIRST STARTING VALUE IS 1 PERCENT OF THE TOTAL DEPTH AND THE SECOND IS 99 PERCENT OF THE TOTAL DEPTH. CONVERGENCE IS ASSUMED WHEN THE CHANGE IN DEPTH IS LESS THAN OR EQUAL TO 0.001 PERCENT OF THE CURRENT DEPTH.

(3) PROGRAM

(A) DECK NAME WLCRIT1

(B) CALLING SEQUENCE

```
CALL WLCRIT1(Q1,Q2,WL1CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZ,VERT,NPTS
*,WL2CR1,WL2CR2)
```

WHERE

Q1,Q2 = DISCHARGES OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY.

WL1CR = CRITICAL FREE SURFACE ELEVATION.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

ALPHAU,ALPHAL = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE CRITICAL SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.

NPTS = NUMBER OF POINTS DESCRIBING THE CROSS-SECTION.

WL2CR1,WL2CR2 = CRITICAL INTERFACE ELEVATIONS(TWO POSSIBLE ROOTS)

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO WL2CR1 AND WL2CR2. IF THE LOWER LAYER DEPTH IS LESS THAN ZERO, A VALUE OF -1.0 IS ASSIGNED TO THE INTERFACE ELEVATION(S). WL2CR1 AND WL2CR2 MAY BE IDENTICAL IF THERE IS ONLY ONE ROOT.

(D) RESTRICTIONS NONE.

(E) OTHER DECKS REQUIRED PROPS2,PROPS,BOTTOM

(4) EXAMPLE

(A) INPUTS A CROSS-SECTION IS DESCRIBED BY 5 POINTS THE COORDINATES OF WHICH ARE LISTED BELOW

0,11 0,5 6,0 12,0 15,11

THE DISCHARGES OF THE UPPER AND THE LOWER LAYERS ARE SET AT 40.0 AND 20.0 RESPECTIVELY. THE FREE SURFACE ELEVATION IS 9.25. SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.02 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.

(B) CODE

```
      DIMENSION B(5),H(5)
      READ(5,10) (B(I),H(I),I=1,5)
   10 FORMAT(2F20.2)
      CALL WLCRIT1(40.0,20.0,9.25,1.0,1.02,9.81,1.1,1.2,B,H,5,WL2CR1,
     *WL2CR2)
      WRITE(6,20) WL2CR1,WL2CR2
   20 FORMAT(2F10.2)
```

```
                STOP
                END

(C) OUTPUT      4.00      5.25
```

WLUCR
=====

WATER LEVELS IN A TRANSITION WITH A CRITICAL SECTION FO A TWO-LAYER SYSTEM

(1) PURPOSE THE TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THE TWO SECTIONS ARE DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN. THE SUBROUTINE FINDS THE FREE SURFACE ELEVATION AT THE CRITICAL SECTION AS WELL AS BOTH THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE OTHER SECTION AS A FUNCTION OF THE INTERFACE ELEVATION AT THE CRITICAL SECTION AND THE DISCHARGES IN BOTH LAYERS.

(2) METHOD TWO CALL STATEMENTS OF WLCRIT AND BERNWL2

(3) PROGRAM

(A) DECK NAME WLUCR

(B) CALLING SEQUENCE

```
 CALL WLUCR(Q1,Q2,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,VERTC,
*NPTSC,HORZ,VERT,NPTS,WL1CR,WL1,WL2)
```

WHERE

Q1,Q2 = DISCHARGES OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY.

WL2CR = INTERFACE ELEVATION AT THE CRITICAL SECTION.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

ALPHAU,ALPHAL = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

HORZC,VERTC = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE CRITICAL SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.

NPTSC = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE OTHER SECTION. COORDINATES ARE REFERRED TO THE SAME AXES AS HORZC AND VERTC.

NPTS = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.

WL1CR = FREE SURFACE ELEVATION AT THE CRITICAL SECTION.

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE OTHER SECTION.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO WL1CR,WL1,WL2. VALUES OF WL2CR,0.0,0.0 ARE ASSIGNED TO THESE PARAMETERS RESPECTIVELY IF NO SOLUTION IS POSSIBLE.

(D) RESTRICTIONS NONE.

(E) OTHER DECKS REQUIRED WLCRIT,BERNWL2

(4) EXAMPLE

(A) INPUTS A TRANSITION IS DESCRIBED BY 5 POINTS AT BOTH UPSTREAM AND DOWNSTREAM (CRITICAL)SECTIONS HAS AN INTERFACE LEVEL AT THE CRITICAL SECTION OF 4.00. THE DISCHARGES OF THE UPPER AND THE LOWER LAYERS RESPECTIVELY ARE 40.0 AND 20.0.
SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.02 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.

UPSTREAM COORDINATES

0,11	6,0.5	12,0.5	15,6	15,11

DOWNSTREAM COORDINATES (CRITICAL SECTION)

0,11	0,5	6,0	12,0	15,11

```
(B) CODE          DIMENSION HORZC(5),VERT(5),HORZ(5),VERT(5)
                  READ(5,10) (HORZC(I),VERTC(I),I=1,5)
                  READ(5,10) (HORZ(J),VERT(J),J=1,5)
               10 FORMAT(10F5.1)
                  CALL WLUCR(40.0,20.0,4.0,1.0,1.02,9.81,1.1,1.2,HORZC,VERTC,5,HORZ
                 *,VERT,5,WL1CR,WL1,WL2)
                  WRITE(6,20) WL1CR,WL1,WL2
               20 FORMAT(3F10.2)
                  STOP
                  END

(C) OUTPUT        9.25      9.23      5.77
```

WLQCR

CRITICAL WATER LEVELS AND DISCHARGE IN A TRANSITION FOR A TWO- LAYER SYSTEM.

(1) PURPOSE THE TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THE TWO SECTIONS ARE DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN. THE SUBROUTINE FINDS THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE CRITICAL SECTION AS WELL AS THE DISCHARGE IN ONE LAYER AS A FUNCTION OF THE CORRESPONDING ELEVATIONS AT THE OTHER SECTION AND THE DISCHARGE IN THE OTHER LAYER.

(2) METHOD SUBROUTINE ZSYSTM1 IS USED TO SOLVE EQUATIONS 3.2, 3.4, AND 3.5 (CHAPTER 3) SIMULTANEOUSLY (USING BROWN METHOD).

(3) PROGRAM

(A) DECK NAME WLQCR

(B) CALLING SEQUENCE

```
CALL WLQCR(WL1,WL2,QA,RHO1,RHO2,G,ALPHA1,ALPHA2,N1,HORZC,VERTC,
*NPTSC,HORZ,VERT,NPTS,WL1CR,WL2CR,QB)
```

WHERE

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE OTHER SECTION.

QA = GIVEN DISCHARGE.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

ALPHA1,ALPHA2 = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

N1 = 1 IF THE DISCHARGE OF THE UPPER LAYER IS GIVEN AND THAT OF THE LOWER LAYER IS COMPUTED
= 2 IF THE DISCHARGE OF THE LOWER LAYER IS GIVEN AND THAT OF THE UPPER LAYER IS COMPUTED.

HORZC,VERTC = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE CRITICAL SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.

NPTSC = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE OTHER SECTION. COORDINATES ARE REFERRED TO THE SAME AXES AS HORZC AND VERTC.

NPTS = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.

WL1CR,WL2CR = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE CRITICAL SECTION.

QB = COMPUTED DISCHARGE.

(C) OUTPUT FORM THE COMPUTED VALUES ARE ASSIGNED TO WL1CR,WL2CR,QB. ZERO VALUES ARE ASSIGNED TO THESE PARAMETERS WITH A PRINTED MESSAGE IF NO SOLUTION IS OBTAINED (NO CONVERGENCE).

(D) RESTRICTIONS NONE.

(E) OTHER DECKS REQUIRED PROPS2,ZSYSTM1(AUXX)

(4) EXAMPLE

(A) INPUTS A TRANSITION IS DESCRIBED BY 5 POINTS AT BOTH UPSTREAM AND DOWNSTREAM (CRITICAL) SECTIONS HAS WATER LEVELS OF 6.1646 AND 4.1509 FOR THE FREE SURFACE AND THE INTERFACE RESPECTIVELY AT THE NON-CRITICAL SECTION.THE DISCHARGE OF THE UPPER LAYER IS 10.0. SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.02 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.
UPSTREAM COORDINATES

```
0,11        0,5         6,0         12,0        15,11
DOWNSTREAM COORDINATES (CRITICAL SECTION)
0,13        0,7         6,2         12,2        15,13
```

(B) CODE

```
      DIMENSION HORZC(5),VERT(5),HORZ(5),VERT(5)
      EXTERNAL AUXX
      READ(5,10) (HORZC(I),VERTC(I),I=1,5)
      READ(5,10) (HORZ(J),VERT(J),J=1,5)
   10 FORMAT(10F5.1)
      CALL WLQCR(6.1646,4.1509,10.0,1.0,1.02,9.81,1.1,1.2,1,HORZC,VERTC
     *,5,HORZ,VERT,5,WL1CR,WL2CR,Q2)
      WRITE(6,20) WL1CR,WL2CR,Q2
   20 FORMAT(3F10.4)
      STOP
      END
```

(C) OUTPUT

```
6.1617    3.9996    5.0003
```

QCRIT1

WATER LEVELS AND DISCHARGES IN A TRANSITION WITH A CRITICAL SECTION FOR A TWO-LAYER SYSTEM.

(1) PURPOSE THE TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THE TWO SECTIONS ARE DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN. THE SUBROUTINE FINDS THE DISCHARGE IN ONE LAYER AS WELL AS THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE NON-CRITICAL SECTION AS A FUNCTION OF THE CORRESPONDING ELEVATIONS AT THE CRITICAL SECTION AND THE DISCHARGE IN THE OTHER LAYER.

(2) METHOD TWO CALL STATEMENTS OF QCRIT AND BERNWL2

(3) PROGRAM
(A) DECK NAME QCRIT1
(B) CALLING SEQUENCE

```
CALL QCRIT1(WL1CR,WL2CR,QA,RHO1,RHO2,G,ALPHAU,ALPHAL,N1,HORZC,
*VERTC,NPTSC,HORZ,VERT,NPTS,WL1,WL2,QB)
```

WHERE

WL1CR,WL2CR = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE CRITICAL SECTION.
QA = GIVEN DISCHARGE.
RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
G = GRAVITATIONAL ACCELERATION.
ALPHAU,ALPHAL = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.
N1 = 1 IF THE DISCHARGE OF THE UPPER LAYER IS GIVEN AND THAT OF THE LOWER LAYER IS COMPUTED
= 2 IF THE DISCHARGE OF THE LOWER LAYER IS GIVEN AND THAT OF THE UPPER LAYER IS COMPUTED.
HORZC,VERTC = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE CRITICAL SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.
NPTSC = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE OTHER SECTION. COORDINATES ARE REFERRED TO THE SAME AXES AS HORZC AND VERTC.
NPTS = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE OTHER SECTION.
QB = COMPUTED DISCHARGE.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO WL1,WL2,QB. ZERO VALUES ARE ASSIGNED TO THESE PARAMETERS IF NO SOLUTION IS POSSIBLE.
(D) RESTRICTIONS NONE.
(E) OTHER DECKS REQUIRED QCRIT,BERNWL2

(4) EXAMPLE
(A) INPUTS A TRANSITION IS DESCRIBED BY 5 POINTS AT BOTH UPSTREAM AND DOWNSTREAM (CRITICAL) SECTIONS HAS WATER LEVELS OF 6.16 AND 4.00 FOR THE FREE SURFACE AND THE INTERFACE RESPECTIVELY. THE DISCHARGE OF THE LOWER LAYER IS 5.0.
SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.02 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.
UPSTREAM COORDINATES

```
          0,11      0,5       6,0       12,0      15,11
          DOWNSTREAM COORDINATES (CRITICAL SECTION)
          0,13      0,7       6,2       12,2      15,13
```

```
(B) CODE         DIMENSION HORZC(5),VERT(5),HORZ(5),VERT(5)
                 READ(5,10) (HORZC( I),VERTC( I),I=1,5)
                 READ(5,10) (HORZ(J),VERT(J),J=1,5)
              10 FORMAT( 10F5.1)
                 CALL QCRIT1(6.16,4.00,5.0,1.0,1.02,9.81,1.1,1.2,2,HORZC,VERTC,5,
                *HORZ,VERT,5,WL1,WL2,Q1)
                 WRITE(6,20) WL1,WL2,Q1
              20 FORMAT(3F10.2)
                 STOP
                 END
```

```
(C) OUTPUT       6.16      4.15      10.00
```

QCRIT12

CRITICAL FREE SURFACE LEVEL AND DISCHARGES IN A TRANSITION FOR A TWO-LAYER SYSTEM.

(1) PURPOSE THE TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THE TWO SECTIONS ARE DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN. THE SUBROUTINE FINDS THE DISCHARGES IN BOTH LAYERS AND THE FREE SURFACE ELEVATION AT THE CRITICAL SECTION AS A FUNCTION OF THE INTERFACE ELEVATION AT THE CRITICAL SECTION AS WELL AS THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE OTHER SECTION.

(2) METHOD ITERATIVE PROCEDURE ARE USED WHERE THE FREE SURFACE ELEVATION AT THE CRITICAL SECTION IS ASSUMED (STARTING VALUES AND INCREMENTS DEPEND ON CROSS-SECTIONAL AREAS). THEN BERNQ2 IS CALLED TO COMPUTE Q1 AND Q2 WHICH ARE IN TURN USED TO COMPUTE A NEW VALUE FOR THE ASSUMED ELEVATION (USING WLCRIT), WHICH IS THEN USED FOR THE NEXT ITERATION AND SO ON. CONVERGENCE IS ASSUMED WHEN THE CHANGE IN THE UPPER LAYER DEPTH IS LESS THAN 0.1 PERCENT OF THE CURRENT DEPTH.

(3) PROGRAM

(A) DECK NAME QCRIT12

(B) CALLING SEQUENCE

```
CALL QCRIT12(WL1,WL2,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,VERTC,
*NPTSC,HORZ,VERT,NPTS,WL1CR,Q1CR,Q2CR)
```

WHERE

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE NON-CRITICAL SECTION.

WL2CR = INTERFACE ELEVATION AT THE CRITICAL SECTION.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

ALPHAU,ALPHAL = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

HORZC,VERTC = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE CRITICAL SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.

NPTSC = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE OTHER SECTION. COORDINATES ARE REFERRED TO THE SAME AXES AS HORZC AND VERTC.

NPTS = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.

WL1CR = FREE SURFACE ELEVATION AT THE CRITICAL SECTION.

Q1CR,Q2CR = DISCHARGES IN THE UPPER AND THE LOWER LAYERS RESPECTIVELY.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO WL1CR,Q1CR,Q2CR.

(D) RESTRICTIONS NONE

(E) OTHER DECKS REQUIRED PROPS,BERNQ2,WLCRIT

(4) EXAMPLE

(A) INPUTS A TRANSITION IS DESCRIBED BY 5 POINTS AT BOTH UPSTREAM AND DOWNSTREAM (CRITICAL) SECTIONS HAS AN INTERFACE LEVEL AT THE CRITICAL SECTION OF 4.00 AND LEVELS OF 6.16 AND 4.15 AT THE OTHER SECTION FOR THE FREE SURFACE AND THE INTERFACE RESPECTIVELY.
SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.02 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.

```
UPSTREAM COORDINATES
0,11        0,5         6,0         12,0        15,11
DOWNSTREAM COORDINATES (CRITICAL SECTION)
0,13        0,7         6,2         12,2        15,13
```

(B) CODE

```
      DIMENSION BC(5),HC(5),B(5),H(5)
      READ(5,10) (BC(I),HC(I),I=1,5)
      READ(5,10) (B(I),H(I),I=1,5)
   10 FORMAT(10F5.1)
      CALL QCRIT12(6.16,4.15,4.00,1.0,1.02,9.81,1.1,1.2,BC,HC,5,B,H,5,
     *WL1CR,Q1CR,Q2CR)
      WRITE(6,20) WL1CR,Q1CR,Q2CR
   20 FORMAT(3F10.2)
      STOP
      END
```

(C) OUTPUT

```
5.04      48.96      69.20
```

QCRIT22
=======

CRITICAL INTERFACE LEVEL AND DISCHARGES IN A TRANSITION FOR A TWO-LAYER SYSTEM.

(1) PURPOSE THE TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THE TWO SECTIONS ARE DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN. THE SUBROUTINE FINDS THE DISCHARGES IN BOTH LAYERS AND THE INTERFACE ELEVATION AT THE CRITICAL SECTION AS A FUNCTION OF THE FREE SURFACE ELEVATION AT THE CRITICAL SECTION AS WELL AS THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE OTHER SECTION. TWO DIFFERENT SETS OF SOLUTIONS MAY BE OBTAINED (SEE SECTION 3.3.5-PART D-CHAPTER 3).

(2) METHOD ITERATIVE PROCEDURE ARE USED WHERE THE INTERFACE ELEVATION AT THE CRITICAL SECTION IS ASSUMED (STARTING VALUES AND INCREMENTS DEPEND ON FREE SURFACE LEVELS AT BOTH SECTIONS). THEN BERNQ2 IS CALLED TO COMPUTE Q1 AND Q2 WHICH ARE USED IN WLCRIT1 TO COMPUTE A NEW VALUE FOR THE INTERFACE ELEVATION WHICH IS THEN USED FOR THE NEXT ITERATION AND SO ON. CONVERGENCE IS ASSUMED WHEN THE CHANGE IN BOTH DISCHARGES IS LESS THAN OR EQUAL TO ONE PERCENT OF THE CURRENT DISCHARGES.
THIS PROCEDURE IS REPEATED TWICE USING THE TWO ROOTS OBTAINED FROM SUBROUTINE WLCRIT1 IN ORDER TO COMPUTE THE CORRESPONDING TWO SETS OF SOLUTIONS.

(3) PROGRAM

(A) DECK NAME QCRIT22

(B) CALLING SEQUENCE

```
CALL QCRIT22(WL1,WL2,WL1CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,VERTC,
*NPTSC,HORZ,VERT,NPTS,WL2CR1,Q1CR1,Q2CR1,WL2CR2,Q1CR2,Q2CR2)
```

WHERE

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE NON-CRITICAL SECTION.

WL1CR = FREE SURFACE ELEVATION AT THE CRITICAL SECTION.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

ALPHAU,ALPHAL = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

HORZC,VERTC = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE CRITICAL SECTION. THE HORIZONTAL COORDINATES ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDINATES ARE REFERRED TO THE SAME DATUM AS THE FLUID LEVELS.

NPTSC = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND THE VERTICAL COORDINATES R5SPECTIVELY OF THE POINTS DEFINING THE OTHER SECTION. COORDINATES ARE REFERRED TO THE SAME AXES AS HORZC AND VERTC.

NPTS = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.

WL2CR1,WL2CR2 = INTERFACE ELEVATION AT THE CRITICAL SECTION (TWO POSSIBLE ROOTS).

Q1CR1,Q2CR1 = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECTIVELY(FIRST SOLUTION).

Q1CR2,Q2CR2 = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECTIVELY(SECOND SOLUTION).

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO WL2CR1,Q1CR1,Q2CR1,WL2CR2, Q1CR2,Q2CR2

(D) RESTRICTIONS NONE

(E) OTHER DECKS REQUIRED BOTTOM,BERNQ2,WLCRIT1

(4) EXAMPLE

(A) INPUTS

A TRANSITION IS DESCRIBED BY 5 POINTS AT BOTH UPSTREAM AND DOWNSTREAM (CRITICAL) SECTIONS HAS A FREE SURFACE LEVEL AT THE CRITICAL SECTION OF 6.1646 AND LEVELS OF 6.1617 AND 4.1509 AT THE OTHER SECTION FOR THE FREE SURFACE AND THE INTERFACE RESPECTIVELY SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.02 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.

```
UPSTREAM COORDINATES
0,11        0,5         6,0         12,0        15,11
DOWNSTREAM COORDINATES (CRITICAL SECTION)
0,13        0,7         6,2         12,2        15,13
```

(B) CODE

```
      DIMENSION BC(5),HC(5),B(5),H(5)
      READ(5,10) (BC(I),HC(I),I=1,5)
      READ(5,10) (B(I),H(I),I=1,5)
   10 FORMAT(10F5.1)
      CALL QCRIT22(6.1617,4.1509,6.1646,1.0,1.02,9.81,1.1,1.2,BC,HC,5,
     *B,H,5,WL2CR1,Q1CR1,Q2CR1,WL2CR2,Q1CR2,Q2CR2)
      WRITE(6,20) WL2CR1,Q1CR1,Q2CR1,WL2CR2,Q1CR2,Q2CR2
   20 FORMAT(6F10.4)
      STOP
      END
```

(C) OUTPUT

```
3.9541     9.9936     4.9915
```

SOLVE

SOLUTION OF THE MOMENTUM EQUATIONS FOR AN INTERFACIAL HYDRAULIC JUMP IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE SUBROUTINE SEARCHES ALL POSSIBLE SOLUTIONS OF THE MOMENTUM EQUATIONS FOR AN INTERFACIAL HYDRAULIC JUMP IN A TWO-LAYER SYSTEM THE EQUATIONS ARE APPLIED BETWEEN TWO-CROSS-SECTIONS IN A CHANNEL OF ARBITRARY GEOMETRY. THE TWO UNKNOWNS ARE H11/H1 AND H21/H2 WHERE H1,H2 ARE THE DEPTHS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE AND H11,H21 ARE THE CORRESPONDING DEPTHS AT THE COMPUTED STATE (CONJUGATE DEPTHS). THE TWO EQUATIONS HAVE NINE PAIRS OF SOLUTIONS AT THE MOST. REFER TO EQUATIONS 4.1 AND 4.2- CHAPTER 4.

(2) METHOD TEN DIFFERENT PAIRS OF STARTING VALUES FOR THE TWO UNKNOWNS ARE GIVEN AND FOR EACH PAIR OF VAUES, SUBROUTINE ZSYSTM1 IS CALLED TO SEARCH A SOLUTION USING BROWN METHOD.
IF THE EQUATIONS ARE BEING SOLVED IN THE MIXING ZONE, ONE PAIR OF STARTING VALUES IS GIVEN AND ONE SOLUTION ONLY IS SOUGHT.

(3) PROGRAM

(A) DECK NAME SOLVE

(B) CALLING SEQUENCE

```
CALL SOLVE(Q1,Q2,Q11,Q21,HORZD,VERTD,NPTSD,HORZ,VERT,NPTS,BETAU,
*BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,A1D,A2D,AREA12,
*AREA22,AY12,AY22,TOPW12,TOPW22,X,WA,X1,X2,NNN,Y111,Y121,RHO1,
*RHO2,RHO11,RHO21,G,NN)
```

WHERE

Q1,Q2 = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE.

Q11,Q21 = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE COMPUTED STATE.

HORZD,VERTD = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING THE HORIZONTAL AND VERTICAL COORDS. RESPECTIVELY OF THE POINTS DESCRIBING THE CROSS-SECTION AT THE COMPUTED STATE. THESE COORDS. ARE REFERRED TO THE SAME AXES AS HORZ,VERT.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE CROSS-SECTION AT THE GIVEN STATE.
THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.

NPTS,NPTSD = NUMBER OF POINTS DESCRIBING THE CROSS-SECTIONS AT THE GIVEN STATE AND THE COMPUTED STATE RESPECTIVELY.

BETAU,BETAL = MOMENTUM CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

BOTD = VERTICAL COORDINATE OF THE LOWEST POINT AT THE CROSS-SECTION AT THE COMPUTED STATE.

HH1,HH2 = DEPTHS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE.

AA1,AA2 = CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE.

TT1,TT2 = TOP WIDTHS OF THE FREE SURFACE AND THE INTERFACE RESPECTIVELY AT THE GIVEN STATE.

AAY1,AAY2 = PRODUCTS OF CROSS-SECTIONAL AREA AND CENTROIDAL DEPTH FOR THE UPPER LAYER(DEPTH BELOW FREE SURFACE) AND THE LOWER LAYER(DEPTH BELOW INTERFACE) RESPECTIVELY AT THE GIVEN STATE.

A1D,A2D = ONE-DIMENSIONAL ARRAYS OF SIZE 10 CONTAINING CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE COMPUTED STATE FOR ALL THE SOLUTIONS OBTAINED.

AREA12,AREA22 = CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AFTER THE MIXING REGION.

AY12,AY22 = PRODUCTS OF CROSS-SECTIONAL AREA AND CENTROIDAL

DEPTH FOR THE UPPER AND LOWER LAYERS RESPECTIVELY AFTER THE MIXING REGION.

TOPW12,TOPW22 = TOP WIDTHS OF THE FREE SURFACE AND THE INTERFACE RESPECTIVELY AFTER THE MIXING REGION.

X = ONE-DIMENSIONAL ARRAY OF SIZE 2 CONTAINING A SOLUTION OF THE MOMENTUM EQUATIONS.

WA = ONE-DIMENSIONAL WORKING ARRAY OF SIZE 8.

X1,X2 = ONE-DIMENSIONAL ARRAYS OF SIZE 10 CONTAINING ALL THE OBTAINED SOLUTIONS OF THE MOMENTUM EQUATIONS.

NNN = 0 IF THE EQUATIONS ARE BEING SOLVED AT THE MIXING ZONE IN WHICH CASE ONLY ONE SOLUTION IS SOUGHT. OTHERWISE ALL POSSIBLE SOLUTIONS ARE SEARCHED.

Y111,Y121 = THE SOLUTION OBTAINED FOR THE IDEAL JUMP WITHOUT MIXING (IF THERE IS ONE AND ONLY ONE FEASIBLE SOLUTION). THESE ARE USED ONLY IF MIXING IS CONSIDERED (IF NNN=0).

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE.

RHO11,RHO21 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE COMPUTED STATE.

G = GRAVITATIONAL ACCELERATION.

NN = A DIRECTION PARAMETER.
= +1 IF THE UPSTREAM STATE IS GIVEN AND THE DOWNSTREAM STATE IS COMPUTED.
= -1 IN THE OPPOSITE CASE.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO X1,X2,A1D,A2D. IF THE EQUATIONS ARE SOLVED IN THE MIXING ZONE, ONLY ONE VALUE FOR EACH IS OBTAINED.

(D) RESTRICTIONS NONE

(E) OTHER DECKS REQUIRED ZSYSTM1(USES FUNCTION AUX WHICH CONTAINS THE MOMENTUM EQUATIONS AND IS CALLED BY ZSYSTM1 TO EVALUATE THE EQUATIONS BEING SOLVED USING TRIAL VALUES FOR THE UNKNOWNS)-PROPS2-BOTTOM

(4) EXAMPLE

(A) INPUTS A CHANNEL STRETCH IS DESCRIBED BY 6 POINTS AT BOTH UPSTREAM AND DOWNSTREAM SECTIONS.

UPSTREAM COORDINATES (GIVEN STATE)
4,18 4,9 7,0.25 20,0.25 20,9 24,18
DOWNSTREAM COORDINATES (COMPUTED STATE)
0,18 2,9 2,0 23,0 26,9 30,18
ALL COORDS ARE IN METERS.

UPPER LAYER

DISCHARGE=430,SPECIFIC GRAVITY=1.0,MOMENTUM CORRECTION FACTOR=1.0
GIVEN STATE
DEPTH=10,AREA=165.26,TOP WIDTH=18.67,PRODUCT OF AREA AND CENTROIDAL DEPTH=792.23

LOWER LAYER

CORRESPONDING VALUES ARE 23,1.02,1.0,4.75,65.62,14.63,152.78 RESPECTIVELY.

GRAVITATIONAL ACCELERATION IS 9.81 AND MIXING IS NOT CONSIDERED.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6),HORZD(6),VERTD(6),A1D(10),A2D(10),
     *X1(10),X2(10),X(2),WA(8)
      EXTERNAL AUX
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
      READ(5,10) (HORZD(I),VERTD(I),I=1,6)
   10 FORMAT(12F5.2)
      CALL SOLVE(430.0,23.0,430.0,23.0,HORZD,VERTD,6,HORZ,VERT,6,1.0,
     *1.0,0.0,10.0,4.75,165.26,65.62,18.67,14.63,792.23,152.78,A1D,A2D,
     *AREA12,AREA22,AY12,AY22,TOPW12,TOPW22,X,WA,X1,X2,1,Y111,Y121,1.0,
     *1.02,1.0,1.02,9.81,1)
      DO 20 I=1,10
      WRITE(6,30) I,X1(I),X2(I),A1D(I),A2D(I)
   30 FORMAT(I10,4F10.3)
      STOP
```

```
        END

(C) OUTPUT        1      0.059      0.001     12.544      0.115
                  2      0.539      1.984    140.448    212.743
                  3      0.539      1.984    140.448    212.743
                  4      0.000      0.000      0.000      0.000
                  5      0.000      0.000      0.000      0.000
                  6      0.000      0.000      0.000      0.000
                  7      0.000      0.000      0.000      0.000
                  8      0.000      0.000      0.000      0.000
                  9      0.000      0.000      0.000      0.000
                 10      0.000      0.000      0.000      0.000
```

NOTE- X1,X2,A1D,A2D ARE GIVEN ZERO VALUES AT THE BEGINNING OF THE PROGRAM WHICH MEANS THAT ONLY TWO SOLUTIONS ARE OBTAINED IN THIS EXAMPLE.

SOLVE1
======

SOLUTION OF A MOMENTUM EQUATION AT THE MIXING REGION OF AN INTERFACIAL HYDRAULIC JUMP IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE SUBROUTINE SEARCHES A SOLUTION OF THE MOMENTUM EQUATION AT THE MIXING REGION (EQUATION 4.7-CHAPTER 4) OF AN INTERFACIAL HYDRAULIC JUMP IN A TWO-LAYER SYSTEM. THE DEPTH OF THE LOWER LAYER AT THE DOWNSTREAM SIDE OF THE MIXING REGION IS DETERMINED FOR A SPECIFIED UPSTREAM STATE.

(2) METHOD USING THE LOWER LAYER DEPTH AT THE UPSTREAM STATE AS A STARTING VALUE FOR THE UNKNOWN, SUBROUTINE ZSYSTM1 IS CALLED TO SEARCH A SOLUTION USING BROWN METHOD.

(3) PROGRAM
(A) DECK NAME SOLVE1
(B) CALLING SEQUENCE

```
CALL SOLVE1(H1,H2,BOTU,Q1,Q2,RHO1,RHO2,AREA1,AREA2,G,BETA,BETAU,
*BETAL,AY1,AY2,HORZ,VERT,NPTS,HORZD,VERTD,NPTSD,NN,AREA12,AREA22,
*AY12,AY22,TOPW12,TOPW22,RHO22,Q22,Q12,H22)
```

WHERE

H1,H2 = DEPTHS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE UPSTREAM STATE.

BOTU = VERTICAL COORDINATE OF THE LOWEST POINT AT THE CROSS-SECTION OF THE REQUIRED STATE.

Q1,Q2 = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE UPSTREAM STATE.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE UPSTREAM STATE.

AREA1,AREA2 = CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE UPSTREAM STATE.

G = GRAVITATIONAL ACCELERATION.

BETA = FRACTION OF THE SLOWER LAYER DISCHARGE ENTRAINED BY THE FASTER LAYER AT THE MIXING REGION.

BETAU,BETAL = MOMENTUM CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

AY1,AY2 = PRODUCTS OF CROSS-SECTIONAL AREA AND CENTROIDAL DEPTH FOR THE UPPER LAYER(DEPTH BELOW FREE SURFACE) AND THE LOWER LAYER(DEPTH BELOW INTERFACE) RESPECTIVELY AT THE UPSTREAM STATE.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE CROSS-SECTION AT THE UPSTREAM STATE.
THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.

HORZD,VERTD = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING THE HORIZONTAL AND VERTICAL COORDS. RESPECTIVELY OF THE POINTS DESCRIBING THE CROSS-SECTION AT THE DOWNSTREAM END OF THE JUMP. THE COORDS. ARE REFERRED TO THE SAME AXES AS HORZ AND VERT.

NPTS,NPTSD = NUMBER OF POINTS DESCRIBING THE CROSS-SECTIONS AT THE UPSTREAM AND DOWNSTREAM STATES RESPECTIVELY

NN = A DIRECTION PARAMETER.
= +1 IF THE UPSTREAM STATE IS GIVEN AND THE DOWNSTREAM STATE OF THE JUMP IS COMPUTED.
= -1 IN THE OPPOSITE CASE.

AREA12,AREA22 = CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AFTER THE MIXING REGION.

AY12,AY22 = PRODUCTS OF CROSS-SECTIONAL AREA AND CENTROIDAL DEPTH FOR THE UPPER AND LOWER LAYERS RESPECTIVELY AFTER THE MIXING REGION.

TOPW12,TOPW22 = TOP WIDTHS OF THE FREE SURFACE AND THE INTERFACE RESPECTIVELY AFTER THE MIXING REGION.

RHO22 = LOWER LAYER DENSITY (OR SPECIFIC GRAVITY) AFTER THE MIXING REGION.

Q22,Q12 = DISCHARGES OF THE LOWER AND UPPER LAYERS RESPECTIVELY AFTER THE MIXING REGION.
H22 = LOWER LAYER DEPTH AFTER THE MIXING REGION.

(C) OUTPUT FORM THE CACULATED VALUE IS ASSIGNED TO AREA12,AREA22,AY12,AY22,TOPW12 TOPW22,H22
A DIAGNOSTIC STATEMENT IS PRINTED IF THE SOLUTION FAILS TO CONVERGE.

(D) RESTRICTIONS NONE

(E) OTHER DECKS REQUIRED ZSYSTM1(USES FUNCTION AUX1 WHICH CONTAINS THE MOMENTUM EQUATION AND IS CALLED BY ZSYSTM1 TO EVALUATE THE EQUATION BEING SOLVED USING TRIAL VALUES FOR THE UNKNOWN)-PROPS2

(4) EXAMPLE

(A) INPUTS A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS. THIS CROSS-SECTION IS LOCATED JUST UPSTREAM OF AN INTERFACIAL HYDRAULIC JUMP. THE COORDINATES OF THE POINTS ARE
4,18 4,9 7,0.25 20,0.25 20,9 24,18

UPPER LAYER

DISCHARGE=430,SPECIFIC GRAVITY=1.0,MOMENTUM CORRECTION FACTOR=1.0 DEPTH=10,AREA=165.26,PRODUCT OF AREA AND CENTROIDAL DEPTH=792.23.

LOWER LAYER

CORRESPONDING VALUES ARE 23,1.02,1.0,4.75,65.62,152.78 RESPECTIVELY.

GRAVITATIONAL ACCELERATION IS 9.81.
IT SHOULD BE NOTED THAT THE TOTAL DEPTH AND THE CROSS-SECTIONAL GEOMETRY ARE ASSUMED TO BE CONSTANT WITHIN THE MIXING REGION.

AT THE END OF THE MIXING REGION

IT IS ASSUMED THAT 12.5 PERCENT OF THE SLOWER LAYER(LOWER) DISCHARGE IS ENTRAINED BY THE FASTER LAYER (UPPER) IN THE MIXING REGION. THIS WILL RESULT IN VALUES OF 432.88 AND 20.13 FOR THE DISCHARGES OF THE UPPER AND LOWER LAYERS RESPECTIVELY AFTER THE MIXING REGION.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6),HORZD(6),VERTD(6),WA1(3)
      EXTERNAL AUX1
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.2)
      CALL SOLVE1(10.0,4.75,0.25,430.0,23.0,1.0,1.02,165.26,65.62,9.81,
     *.125,1.0,1.0,792.23,152.78,HORZ,VERT,6,HORZD,VERTD,6,1,AREA12,
     *AREA22,AY12,AY22,TOPW12,TOPW22,1.02,20.13,432.88,H22)
      WRITE(6,20) H22
   20 FORMAT(F10.2)
      STOP
      END
```

(C) OUTPUT

```
 7.07
```

INTJMP

INTERFACIAL HYDRAULIC JUMP IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE SUBROUTINE DETERMINES THE STATE DOWNSTREAM (OR UPSTREAM) FROM A HYDRAULIC JUMP (OR DROP) AT THE INTERFACE AND/OR THE FREE SURFACE OF A TWO-LAYER SYSTEM FOR A COMPLETELY SPECIFIED STATE UPSTREAM (OR DOWNSTREAM) IN A CHANNEL OF ARBITRARY GEOMETRY. EACH OF THE UPSTREAM AND DOWNSTREAM SECTIONS IS DESCRIBED BY A SERIES OF POINTS THE COORDINATED OF WHICH ARE GIVEN. PROVISION IS ALLOWED TO CONSIDER MIXING AT THE INTERFACE.

(2) METHOD THE MOMENTUM EQUATIONS FOR THE UPPER AND LOWER LAYERS ARE SOLVED

SIMULTANIOUSLY USING BROWN METHOD (EQUATIONS 4.1 AND 4.2- CHAPTER 4). THE UNKNOWNS ARE H11/H1 AND H21/H2 WHERE H1 AND H2 ARE THE DEPTHS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE. H11 AND H21 ARE THE DEPTHS AT THE COMPUTED STATE(CONJUGATE DEPTHS).
THE SOLUTIONS OBTAINED ARE CHECKED FOR FEASIBILITY USING ENERGY CONSIDERATION (EQUATION 4.3- CHAPTER 4) AFTER ELIMINATING TRIVIAL AND NEGATIVE SOLUTIONS. IF MIXING AT THE INTERFACE IS TO BE CONSIDERED, THE GENERALIZED APPROACH OF MACAGNO AND MACAGNO IS USED (SEE SECTION 4.2.3- CHAPTER 4).

(3) PROGRAM
(A) DECK NAME INTJMP
(B) CALLING SEQUENCE

```
CALL INTJMP(N,NN,Q1,Q2,Q11,Q21,WL1,WL2,RHO1,RHO2,RHO11,RHO21,G,
*BETAU,BETAL,ALPHAU,ALPHAL,NPTS,NPTSD,BETA,RHO22,Q22,HORZ,VERT,
*HORZD,VERTD,A1D,A2D,X1,X2,X,WA,Y1,Y2,ALPHA,WL11,WL21)
```

WHERE
N = A MIXING PARAMETER.
= 0 IF INTERFACIAL MIXING IS TO BE NEGLECTED. OTHERWISE MIXING WILL BE CONSIDERED.
NN = A DIRECTION PARAMETER.
= +1 IF THE UPSTREAM STATE IS GIVEN AND THE DOWNSTREAM STATE IS COMPUTED.
= -1 IN THE OPPOSITE CASE.
Q1,Q2 = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE.
Q11,Q21 = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE COMPUTED STATE.
WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE GIVEN STATE.
RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE.
RHO11,RHO21 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE COMPUTED STATE.
G = GRAVITATIONAL ACCELERATION.
BETAU,BETAL = MOMENTUM CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.
ALPHAU,ALPHAL = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.
NPTS,NPTSD = NUMBER OF POINTS DESCRIBING THE CROSS-SECTIONS AT THE GIVEN STATE AND THE COMPUTED STATE RESPECTIVELY.
BETA = FRACTION OF THE SLOWER LAYER DISCHARGE ENTRAINED BY THE FASTER LAYER (IF MIXING IS CONSIDERED).
RHO22 = LOWER LAYER DENSITY (OR SPECIFIC GRAVITY) AT THE MIXING REGION.
Q22 = LOWER LAYER DISCHARGE AT THE MIXING REGION.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING THE HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE CROSS-SECTION AT THE GIVEN STATE.
THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE

REFERRED TO THE SAME DATUM AS WATER LEVELS.

HORZD,VERTD = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING THE HORIZONTAL AND VERTICAL COORDS. RESPECTIVELY OF THE POINTS DESCRIBING THE CROSS-SECTION AT THE COMPUTED STATE. THESE COORDS. ARE REFERRED TO THE SAME AXES AS HORZ,VERT.

A1D,A2D = ONE-DIMENSIONAL ARRAYS OF SIZE 10 CONTAINING CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE COMPUTED STATE FOR ALL THE SOLUTIONS OBTAINED.

X1,X2 = ONE-DIMENSIONAL ARRAYS OF SIZE 10 CONTAINING ALL THE OBTAINED SOLUTIONS OF THE MOMENTUM EQUATIONS.

X = ONE-DIMENSIONAL ARRAY OF SIZE 2 CONTAINING A SOLUTION OF THE MOMENTUM EQUATIONS.

WA = ONE-DIMENSIONAL WORKING ARRAY OF SIZE 8.

Y1,Y2 = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE 10 CONTAINING ALL FEASIBLE SOLUTIONS OF THE MOMENTUM EQUATIONS.

ALPHA = FRACTION OF THE ENERGY DROP AVAILABLE FOR ENTRAINMENT (IF MIXING IS CONSIDERED).

WL11,WL21 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE COMPUTED STATE REFERRED TO THE SAME DATUM AS WL1,WL2.

(C) OUTPUT FORM

THE CALCULATED VALUES ARE ASSIGNED TO BETA,RHO22,Q22,A1D,A2D,X1, X2,Y1,Y2,WL11,WL21

PRINTOUT OF DIAGNOSTIC STATEMENTS IS INCLUDED FOR THE FOLLOWING CASES-

* IF NO FEASIBLE SOLUTIONS ARE OBTAINED FOR THE MOMENTUM EQUATIONS. (NO JUMP POSSIBLE).

* IF MORE THAN ONE FEASIBLE SOLUTIONS ARE OBTAINED. IN THIS CASE IF MIXING IS NOT TO BE CONSIDERED, ALL THE SOLUTIONS ARE PRINTED. IF MIXING IS CONSIDERED, THE PROGRAM PROCEEDS WITH THE SOLUTION THAT IS CLOSEST TO THE GIVEN STATE.

* IF ONE OF THE LAYERS IS STAGNANT AND MIXING IS TO BE CONSIDERED IN WHICH CASE THE PROGRAM PRINTS A MESSAGE AND RETURNS WITH THE SOLUTION OBTAINED WITHOUT MIXING.

* IF NO SOLUTION IS OBTAINED FOR THE MOMENTUM EQUATION IN THE MIXING REGION. IN THIS CASE A MESSAGE IS PRINTED AND THE PROGRAM RETURNS WITH THE SOLUTION OBTAINED WITHOUT MIXING.

(D) RESTRICTIONS

NO MIXING CAN BE CONSIDERED IF ONE LAYER IS STAGNANT (UNSTABLE JUMP).

(E) OTHER DECKS REQUIRED

SOLVE,SOLVE1,PROPS2,BOTTOM

(4) EXAMPLE

(A) INPUTS

A CHANNEL STRETCH IS DESCRIBED BY 6 POINTS AT BOTH UPSTREAM AND DOWNSTREAM SECTIONS.

UPSTREAM COORDINATES (GIVEN STATE)

4,18	4,9	7,0.25	20,0.25	20,9	24,18

DOWNSTREAM COORDINATES (COMPUTED STATE)

0,18	2,9	2,0	23,0	26,9	30,18

ALL COORDS ARE IN METERS.

UPPER LAYER

DISCHARGE=430,SPECIFIC GRAVITY=1.0,KINETIC ENERGY AND MOMENTUM CORRECTION FACTORS ARE 1.0,1.0 RESPECTIVELY.

LOWER LAYER

CORRESPONDING VALUES ARE 23,1.02,1.0,1.0

GRAVITATIONAL ACCELERATION IS 9.81 AND MIXING IS TO BE TAKEN INTO CONSIDERATION. FREE SURFACE AND INTERFACE ELEVATIONS AT THE GIVEN STATE ARE 15 AND 5 RESPECTIVELY. THE FRACTION OF ENERGY DROP USED FOR ENTRAINMENT IS 22 PERCENT.

(B) CODE

```
       DIMENSION HORZ(6),VERT(6),HORZD(6),VERTD(6),A1D(10),A2D(10)
       DOMENSION X1(10),X2(10),Y1(10),Y2(10),X(2),WA(8)
       EXTERNAL AUX,AUX1
       READ(5,10) (HORZ(I),VERT(I),I=1,6)
       READ(5,10) (HORZD(I),VERTD(I),I=1,6)
    10 FORMAT(12F5.2)
       CALL INTJMP(1,1,430.0,23.0,430.0,23.0,15.0,5.0,1.0,1.02,1.0,1.02,
      *9.81,1.0,1.0,1.0,1.0,6,6,BETA,RHO22,Q22,HORZ,VERT,HORZD,VERTD,
      *A1D,A2D,X1,X2,X,WA,Y1,Y2,0.22,WL11,WL21)
       WRITE(6,20) WL11,WL21,BETA
```

```
            20 FORMAT(3F10.2)
               STOP
               END
```

```
(C) OUTPUT        14.78       10.34       0.10
```

OTHER EXAMPLES ARE GIVEN IN SECTION 4.3.1- CHAPTER 4.

LKEXFL

LOCK EXCHANGE FLOW

(1) PURPOSE THE SUBROUTINE CALCULATES THE OVERFLOW AND UNDERFLOW FRONT VELOCITIES OF A LOCK EXCHANGE FLOW AT ANY SPECIFIED TIME AFTER THE OPENING OF THE LOCK GATE OR OTHER SUCH DIVISION. IT ALSO CALCULATES THE DISTANCES TRAVELLED BY BOTH FRONTS AWAY FROM THE GATE AT THE SAME SPECIFIED TIME.

(2) METHOD EQUATION 5.6 (CHAPTER 5) IS SOLVED USING THE RUNGE-KUTTA-MERSON METHOD (SEE SECTION 5.3- CHAPTER 5).THE ALLOWABLE TOLERENCE FOR THE TRUNCATION ERROR IS 0.01 PERCENT.

(3) PROGRAM

(A) DECK NAME LKEXFL

(B) CALLING SEQUENCE CALL LKEXFL(D,RHO1,RHO2,G,TIME,DT,NPRINT,ELO,ELU,VO,VU)

WHERE
D = TOTAL DEPTH.
RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE LIGHTER AND HEAVIER FLUIDS RESPECTIVELY.
G = GRAVITATIONAL ACCELERATION.
TIME = SPECIFIED TIME AFTER THE GATE OPENING FOR WHICH VELOCITIES AND LENGTHS ARE COMPUTED.
DT = TIME INCREMENT SPECIFIED BY THE USER (MAY BE MODIFIED INSIDE THE PROGRAM DEPENDING ON THE TRUNCATION ERROR).
NPRINT = 1 IF A PRINTOUT OF TIME, VELOCITIES, AND LENGTHS AT THE INTERMEDIATE STEPS IS REQUIRED. OTHERWISE NO SUCH PRINTOUT IS PROVIDED.
ELO,ELU = LENGTH AWAY FROM GATE OF OVERFLOW AND UNDERFLOW WEDGES RESPECTIVELY AT SPECIFIED TIME.
VO,VU = FRONT VELOCITIES OF OVERFLOW AND UNDERFLOW RESPECTIVELY AT SPECIFIED TIME.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO ELO,ELU,VO,VU. IF NPRINT=1, A PRINTOUT OF TIME, VELOCITIES, AND LENGTHS AT THE INTERMEDIATE STEPS IS PROVIDED.

(D) RESTRICTIONS FLOE IS ASSUMED TO BE HORIZONTAL.

(E) OTHER DECKS REQUIRED NONE.

(4) EXAMPLE

(A) INPUTS A LOCK GATE SEPARATES TWO BODIES OF WATER OF SPECIFIC GRAVITIES 0.9997 AND 1.0042 RESPECTIVELY. DEPTH OF WATER ON BOTH SIDES OF THE GATE BEFORE OPENING IS 0.5. SPECIFIED TIME IS 200 AND THE TIME INCREMENT IS 5. GRAVITATIONAL ACCELERATION IS 32.2.

(B) CODE

```
      CALL LKEXFL(0.5,0.9997,1.0042,32.2,200.0,5.0,1,ELO,ELU,VO,VU)
      WRITE(6,10) ELO,ELU,VO,VU
   10 FORMAT(4F10.2)
      STOP
      END
```

(C) OUTPUT

```
TIME OVERFLOW VEL. OVERFLOW LENGTH UNDERFLOW VEL. UNDERFLOW LENGTH

0.00     0.0000          0.0000          0.0000            0.0000
 .16      .0126           .0010           .0099             .0008
 .31      .0249           .0039           .0196             .0031
 .47      .0368           .0088           .0290             .0069
 .63      .0479           .0154           .0377             .0121
 .94      .0677           .0335           .0533             .0264
1.25      .0839           .0573           .0661             .0451
1.88      .1068           .1175           .0841             .0925
2.50      .1207           .1890           .0951             .1488
3.13      .1293           .2673           .1018             .2105
3.75      .1348           .3500           .1061             .2756
```

FCHECK

CHECK OF THE TYPES OF FLOW AND INTERFACIAL BOUNDARY LAYERS IN A TWO-LAYER SYSTEM WITH ONE LAYER FLOWING.

(1) PURPOSE THE ROUTINE DETERMINES THE TYPES OF THE FLOW AND THE INTERFACIAL BOUNDARY LAYERS (I.E. LAMINAR OR TURBULENT) IN A TWO-LAYER SYSTEM WITH ONE LAYER FLOWING THROUGH A CHANNEL OF ARBITRARY GEOMETRY.

(2) METHOD USES THE CRITERION DEFINED BY EQUATIONS 6.35 AND 6.36 (CHAPTER 6)

(3) PROGRAM
(A) DECK NAME FCHECK
(B) CALLING SEQUENCE

```
          CALL FCHECK(Q,HORZ,VERT,NPTS,WL1,WL2,N,VISC1,VISC2,G,RHO1,RHO2,
         *RE,THETA,BL,FLOW)
```

WHERE

Q = DISCHARGE OF THE FLOWING LAYER.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID LEVELS.
NPTS = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-SECTION.
WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY. IF THE LOWER LAYER IS FLOWING, WL1 MAY BE GIVEN ANY ARBITRARY VALUE.
N = 1 IF THE UPPER LAYER IS FLOWING.
= 2 IF THE LOWER LAYER IS FLOWING.
VISC1,VISC2 = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
G = GRAVITATIONAL ACCELERATION.
RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
RE = REYNOLDS NUMBER OF THE FLOWING LAYER.
THETA = A PARAMETER DEFINING THE TYPE OF THE INTERFACIAL BOUNDARY LAYERS.
BL = 1.0 IF THE BOUNDARY LAYERS ARE LAMINAR.
= 2.0 IF THE BOUNDARY LAYERS ARE TURBULENT.
FLOW = 1.0 IF THE FLOW IS LAMINAR.
= 2.0 IF THE FLOW IS TURBULENT.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO RE,THETA,BL,FLOW
(D) RESTRICTIONS THIS ROUTINES CAN NOT BE USED IF BOTH LAYERS ARE FLOWING.
(E) OTHER DECKS REQUIRED PROPS2,PROPS

(4) EXAMPLE
(A) INPUTS A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,4	2,2	4,0	8,0	10,2	10,4

UPPER LAYER
DISCHARGE=0,SPECIFIC GRAVITY=1.00,KINEMATIC VISCOSITY=1.0E-6.
LOWER LAYER
DISCHARGE=5,SPECIFIC GRAVITY=1.05,KINEMATIC VISCOSITY=1.1E-69

FREE SURFACE AND INTERFACE ELEVATIONS ARE 3.5 AND 1.8 RESPECTIVELY. GRAVITATIONAL ACCELERATION IS 9.81.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6)
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.1)
      CALL FCHECK(5.0,HORZ,VERT,6,3.5,1.8,2,1.0E-6,1.1E-6,9.81,1.0,1.05
```

```
  4.38        .1385           .4354          .1090            .3429
  5.63        .1428           .6115          .1125            .4815
  6.88        .1452           .7917          .1144            .6234
  9.38        .1476          1.1581          .1162            .9119
 11.88        .1486          1.5285          .1170           1.2036
 16.88        .1495          2.2744          .1177           1.7908
 21.88        .1499          3.0231          .1180           2.3804
 31.88        .1502          4.5238          .1183           3.5620
 51.88        .1503          7.5295          .1184           5.9288
 91.88        .1504         13.5451          .1184          10.6654
171.88        .1504         25.5793          .1185          20.1411
200.00        .1504         29.8103          .1185          23.4727
    29.81     23.47       .15       .12
```

```
     *,RE,THETA,BL,FLOW)
      WRITE(6,20) RE,THETA,BL,FLOW
   20 FORMAT(4F15.4)
      STOP
      END
```

```
(C) OUTPUT       499985.7151       0.0162        2.0000        2.0000
```

FILMNR

INTERFACIAL SHEAR STRESS COEFFICIENT FOR LAMINAR FLOW AND LAMINAR INTERFACIAL BOUNDARY LAYERS IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE ROUTINE CALCULATES THE INTERFACIAL SHEAR STRESS COEFFICIENT WHEN THE FLOW AND THE INTERFACIAL BOUNDARY LAYERS ARE LAMINAR. THE SYSTEM CONSIDERED IS A TWO-LAYER SYSTEM WITH FLOW IN THE LOWER LAYER ONLY THROUGH A CHANNEL OF ARBITRARY GEOMETRY.

(2) METHOD USING EQUATION 6.37 (CHAPTER 6).

(3) PROGRAM
(A) DECK NAME FILMNR
(B) CALLING SEQUENCE CALL FILMNR(Q,HORZ,VERT,NPTS,WL2,VISC2,FI)

WHERE
Q = DISCHARGE OF THE LOWER LAYER.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID LEVELS.
NPTS = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-SECTION.
WL2 = ELEVATION OF THE INTERFACE.
VISC2 = KINEMATIC VISCOSITY OF THE LOWER FLUID.
FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

(C) OUTPUT FORM THE CALCULATED VALUE IS ASSIGNED TO FI.
(D) RESTRICTIONS THIS ROUTINE IS USED ONLY WHEN THE LOWER LAYER IS FLOWING AND THE UPPER LAYER IS STAGNANT.
(E) OTHER DECKS REQUIRED PROPS

(4) EXAMPLE
(A) INPUTS A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,4	2,2	4,0	8,0	10,2	10,4

LOWER LAYER DISCHARGE IS 0.2 AND KINEMATIC VISCOSITY IS 1.1E-6 THE ELEVATION OF THE INTERFACE IS 1.8. GRAVITATIONAL ACCELERATION IS 9.81.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6)
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.1)
      CALL FILMNR(0.2,HORZ,VERT,6,1.8,1.1E-6,FI)
      WRITE(6,20) FI
   20 FORMAT(F10.4)
      STOP
      END
```

(C) OUTPUT

```
0.0010
```

FITURB

INTERFACIAL SHEAR STRESS COEFFICIENT FOR TURBULENT FLOW AND TURBULENT INTERFACIAL BOUNDARY LAYERS IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE ROUTINE CALCULATES THE INTERFACIAL SHEAR STRESS COEFFICIENT WHEN THE FLOW AND THE INTERFACIAL BOUNDARY LAYERS ARE TURBULENT. A TWO-LAYER SYSTEM IS CONSIDERED WITH FLOW IN THE LOWER LAYER ONLY THROUGH A CHANNEL OF ARBITRARY GEOMETRY.

(2) METHOD USING EQUATION 6.41 (CHAPTER 6).

(3) PROGRAM

(A) DECK NAME FITURB

(B) CALLING SEQUENCE CALL FITURB(Q,HORZ,VERT,NPTS,WL2,VISC2,FI)

WHERE

Q = DISCHARGE OF THE LOWER LAYER.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID LEVELS.

NPTS = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-SECTION.

WL2 = ELEVATION OF THE INTERFACE.

VISC2 = KINEMATIC VISCOSITY OF THE LOWER FLUID.

FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

(C) OUTPUT FORM THE CALCULATED VALUE IS ASSIGNED TO FI.

(D) RESTRICTIONS THIS ROUTINE IS USED ONLY WHEN THE LOWER LAYER IS FLOWING AND THE UPPER LAYER IS STAGNANT.

(E) OTHER DECKS REQUIRED PROPS

(4) EXAMPLE

(A) INPUTS A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,4 2,2 4,0 8,0 10,2 10,4

LOWER LAYER DISCHARGE IS 5.0 AND KINEMATIC VISCOSITY IS 1.1E-6 THE ELEVATION OF THE INTERFACE IS 1.8. GRAVITATIONAL ACCELERATION IS 9.81.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6)
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.1)
      CALL FITURB(5.0,HORZ,VERT,6,1.8,1.1E-6,FI)
      WRITE(6,20) FI
   20 FORMAT(F10.4)
      STOP
      END
```

(C) OUTPUT

```
0.0080
```

FBNDRY

BOUNDARY SHEAR STRESS COEFFICIENT IN A TWO-LAYER SYSTEM WITH ONE FLOWING LAYER.

(1) PURPOSE THE ROUTINE CALCULATES THE SHEAR STRESS COEFFICIENT AT THE SOLID BOUNDARIES FOR A TWO-LAYER SYSTEM WITH ONE FLOWING LAYER THROUGH A CHANNEL OF ARBITRARY GEOMETRY.

(2) METHOD USING EQUATIONS 6.31,6.32,6.33 AND 6.34 (CHAPTER 6).
FOR ROUGH TURBULENT FLOW, EQUATION 6.34 IS SOLVED USING BROWN METHOD (SUBROUTINE ZSYSTM1).
THE OTHER RELATIONSHIPS ARE EXPLICIT.

(3) PROGRAM
(A) DECK NAME FBNDRY
(B) CALLING SEQUENCE CALL FBNDRY(Q,HORZ,VERT,NPTS,WL1,WL2,N,VISC1,VISC2,D,FO1,FO2)

WHERE
Q = DISCHARGE OF THE FLOWING LAYER.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID LEVELS.
NPTS = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-SECTION.
WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY. IF THE LOWER LAYER IS FLOWING, WL1 MAY BE GIVEN ANY ARBITRARY VALUE.
N = 1 IF THE UPPER LAYER IS FLOWING.
= 2 IF THE LOWER LAYER IS FLOWING.
VISC1,VISC2 = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
D = ROUGHNESS HEIGHT.
FO1,FO2 = BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO FO1 AND FO2.
THE COEFFICIENT THAT CORRESPONDS TO THE STAGNANT LAYER IS GIVEN A ZERO VALUE.

(D) RESTRICTIONS THIS ROUTINE MAY NOT BE USED IF BOTH LAYERS ARE FLOWING.

(E) OTHER DECKS REQUIRED PROPS2,PROPS,ZSYSTM1(WHICH USES THE AUXILIARY FUNCTION AUX).

(4) EXAMPLE
(A) INPUTS A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,4 2,2 4,0 8,0 10,2 10,4

FREE SURFACE AND INTERFACE ELEVATIONS ARE 3.5 AND 1.8 RESPECTIVELY. KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS ARE 1.0E-6 AND 1.1E-6 RESPECTIVELY.
BOUNDARY SHEAR STRESS COEFFICIENTS ARE REQUIRED FOR THE FOLLOWING CASES-

A) Q1=0.2,Q2=0.0,D=0.05 (N=1,Q=0.2)
B) Q1=0.2,Q2=0.0,D=0.0 (N=1,Q=0.2)
C) Q1=0.0,Q2=0.2,D=0.05 (N=2,Q=0.2)
D) Q1=0.0,Q2=0.2,D=0.0 (N=2,Q=0.2)

(B) CODE

```
      DIMENSION HORZ(6),VERT(6)
      EXTERNAL AUX
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.1)
```

```
      DO 20 J=1,4
      READ(6,30) N,Q,D
   30 FORMAT(I10,2F10.2)
      CALL FBNDRY(Q,HORZ,VERT,6,3.5,1.8,N,1.0E-6,1.1E-6,D,FO1,FO2)
      WRITE(6,40) FO1,FO2
   40 FORMAT(2F10.4)
   20 CONTINUE
      STOP
      END
```

(C) OUTPUT

```
    0.0396    0.0000
    0.0196    0.0000
    0.0000    0.0405
    0.0000    0.0188
```

FILMTB

INTERFACIAL SHEAR STRESS COEFFICIENT IN A TWO-LAYER SYSTEM WITH ONE FLOWING LAYER- TURBULENT FLOW AND LAMINAR INTERFACIAL BOUNDARY LAYERS.

(1) PURPOSE THE ROUTINE CALCULATES THE SHEAR STRESS COEFF. AT THE INTERFACE OF A TWO-LAYER SYSTEM WITH ONE FLOWING LAYER THROUGH A CHANNEL OF ARBITRARY GEOMETRY. THIS ROUTINE CONSIDERES THE CASE WHEN THE FLOW IS TURBULENT AND THE INTERFACIAL BOUNDARY LAYERS ARE LAMINAR

(2) METHOD USING EQUATIONS 6.38,6.39 AND 6.40 (CHAPTER 6).
THE COEFFICIENT K IN EQUATION 6.40 AS WELL AS THE THICKNESSES OF THE INTERFACIAL BOUNDARY LAYERS WHICH DETERMINE THE TYPE OF FLOW ESTABLISHMENT ARE CALCULATED USING THE METHOD OF SUCCESSIVE APPROXIMATIONS DEVELOPED BY KEULEGAN(1944) BASED ON PRANDTL BOUNDARY-LAYER THEORY. THIS METHOD IS ALSO DESCRIBED BY ELSAYED (1975).

(3) PROGRAM

(A) DECK NAME FILMTB

(B) CALLING SEQUENCE CALL FILMTB(Q,HORZ,VERT,NPTS,WL1,WL2,RHO1,RHO2,VISC1,VISC2,NINCR1
*,N,ELA,X,UREL1,UREL2,UI,YS,YS1,FI)

WHERE

Q = DISCHARGE OF THE FLOWING LAYER.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID LEVELS.

NPTS = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-SECTION.

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

VISC1,VISC2 = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

NINCR1 = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFACIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION OF THE VELOCITY PROFILE IN EACH LAYER.
NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED NOT BE CALCULATED.

N = 1 IF THE UPPER LAYER IS FLOWING.
= 2 IF THE LOWER LAYER IS FLOWING.

ELA = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS.

X = DISTANCE FROM THE POINT OF CONTACT OF THE TWO FLUIDS TO THE POINT UNDER COSIDERATION.

UREL1,UREL2 = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).

UI = VELOCITY AT THE INTERFACE RELATIVE TO THE MEAN VELOCITY OF FLOW.

YS,YS1 = THICKNESSES OF THE INTERFACIAL BOUNDARY LAYERS IN THE FLOWING AND THE STAGNANT LAYERS RESPECTIVELY.

FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO UREL1,UREL2(IF NINCR1.NE.0) UI,YS,YS1,FI

(D) RESTRICTIONS USE OF THIS ROUTINE IS RESTRICTED TO LAMINAR BOUNDARY LAYERS WITH ONLY ONE LAYER FLOWING.

(E) OTHER DECKS REQUIRED BOTTOM,PROPS2,ZSYSTM1(WHICH USES THE AUXILIARY FUNCTIONS AUX1, AUX2,AUX3,AUX4,AUX5).

(4) EXAMPLE

(A) INPUTS A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,4	2,2	4,0	8,0	10,2	10,4

FREE SURFACE AND INTERFACE ELEVATIONS ARE 3.5 AND 1.8 RESPECTIVELY.

UPPER LAYER
DISCHARGE=0.2,KINEMATIC VISCOSITY=1.0E-6,SP. GRAVITY=1.00
LOWER LAYER
DISCHARGE=0.0,KINEMATIC VISCOSITY=1.1E-6,SP. GRAVITY=1.05

TOTAL LENGTH OF THE UPPER LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS IS 200 AND THE CROSS-SECTION UNDER CONSIDERATION IS AT THE END OF THE FLOW REGION.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6),UREL1(10),UREL2(10)
      EXTERNAL AUX1,AUX2,AUX3,AUX4,AUX5
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.1)
      CALL FILMTB(0.2,HORZ,VERT,6,3.5,1.8,1.0,1.05,1.0E-6,1.1E-6,10,
     *200.0,200.0,UREL1,UREL2,UI,YS,YS1,FI)
      DO 20 J=1,10
      WRITE(6,30) UREL1(J),UREL2(J)
   30 FORMAT(2F10.4)
   20 CONTINUE
      WRITE(6,40) UI,YS,YS1,FI
   40 FORMAT(4F10.4)
      STOP
      END
```

(C) OUTPUT

```
    0.5678    0.5678
    0.6760    0.3946
    0.7749    0.2480
    0.8577    0.1391
    0.9205    0.0679
    0.9626    0.0274
    0.9865    0.0083
    0.9970    0.0015
    0.9998    0.0001
    1.0000    0.0000
    0.5678    0.5852    1.1009    0.0010
```

NOTE- IF X=5.0 THE VALUES OF YS,YS1 AND FI WILL BE 0.0925,0.1741 AND 0.0063 RESPECTIVELY.

FINTRF

INTERFACIAL SHEAR STRESS COEFFICIENT IN A TWO-LAYER SYSTEM WITH ONE FLOWING LAYER.

(1) PURPOSE THE ROUTINE CALCULATES THE INTERFACIAL SHEAR STRESS COEFF. FOR A TWO-LAYER SYSTEM WITH ONE LAYER FLOWING THROUGH A CHANNEL OF ARBITRARY GEOMETRY.

(2) METHOD THE TYPES OF FLOW AND INTERFACIAL BOUNDARY LAYERS ARE DETERMINED USING SUBROUTINE FCHECK. THEN, THE APPROPRIATE ROUTINE(FILMNR, FITURB OR FILMTB) IS CALLED TO CALCULATE THE SHEAR STRESS COEFF.

(3) PROGRAM
(A) DECK NAME FINTRF
(B) CALLING SEQUENCE

```
CALL FINTRF(Q,HORZ,VERT,NPTS,WL1,WL2,VISC1,VISC2,G,RHO1,RHO2,RE,
*THETA,BL,FLOW,NINCR1,N,ELA,X,UREL1,UREL2,UI,FI)
```

WHERE
Q = DISCHARGE OF THE FLOWING LAYER.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID LEVELS.
NPTS = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-SECTION.
WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY.
VISC1,VISC2 = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
G = GRAVITATIONAL ACCELERATION.
RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
RE = REYNOLDS NUMBER OF THE FLOWING LAYER.
THETA = A PARAMETER DEFINING THE TYPE OF THE INTERFACIAL BOUNDARY LAYERS.
BL = 1.0 IF THE BOUNDARY LAYERS ARE LAMINAR.
= 2.0 IF THE BOUNDARY LAYERS ARE TURBULENT.
FLOW = 1.0 IF THE FLOW IS LAMINAR.
= 2.0 IF THE FLOW IS TURBULENT.
NINCR1 = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFACIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION OF THE VELOCITY PROFILE IN EACH LAYER.
NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED NOT BE CALCULATED.
N = 1 IF THE UPPER LAYER IS FLOWING.
= 2 IF THE LOWER LAYER IS FLOWING.
ELA = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS.
X = DISTANCE FROM THE POINT OF CONTACT OF THE TWO FLUIDS TO THE POINT UNDER COSIDERATION.
UREL1,UREL2 = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).
UI = VELOCITY AT THE INTERFACE RELATIVE TO THE MEAN VELOCITY OF FLOW.
FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO RE,THETA,BL,FLOW,UREL1, UREL2,UI,FI. NOTE THAT UI,UREL1,UREL2 ARE CALCULATED ONLY IN THE CASE OF TURBULENT FLOW AND LAMINAR INTERFACIAL BOUNDARY LAYERS, I.E., BL=1.0 AND FLOW=2.0 (IN THIS CASE UREL1 AND UREL2 ARE COMPUTED ONLY WHEN NINCR1 IS SET LARGER THAN ZERO).

(D) RESTRICTIONS THIS ROUTINE IS USED ONLY WHEN ONE LAYER IS FLOWING AND THE OTHER

LAYER IS STAGNANT. IF THE UPPER LAYER IS FLOWING, A SOLUTION IS OBTAINED ONLY IN THE CASE OF TURBULENT FLOW AND LAMINAR INTERFACIAL BOUNDARY LAYERS. IN OTHER CASES, FI IS SET EQUAL TO ZERO.

(E) OTHER DECKS REQUIRED FCHECK,FILMNR,FITURB,FILMTB

(4) EXAMPLE

(A) INPUTS A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,4 2,2 4,0 8,0 10,2 10,4

FREE SURFACE AND INTERFACE ELEVATIONS ARE 3.5 AND 1.8 RESPECTIVELY.
UPPER LAYER
DISCHARGE=0.2,KINEMATIC VISCOSITY=1.0E-6,SP. GRAVITY=1.00
LOWER LAYER
DISCHARGE=0.0,KINEMATIC VISCOSITY=1.1E-6,SP. GRAVITY=1.05

TOTAL LENGTH OF THE UPPER LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS IS 200 AND THE CROSS-SECTION UNDER CONSIDERATION IS AT A DISTANCE 5 FROM THE POINT OF CONTACT.
GRAVITATIONAL ACCELERATION IS 9.81.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6)
      EXTERNAL AUX1,AUX2,AUX3,AUX4,AUX5
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.1)
      CALL FINTRF(0.2,HORZ,VERT,6,3.5,1.8,1.0E-6,1.1E-6,9.81,1.00,1.05,
     *RE,THETA,BL,FLOW,0,1,200.0,5.0,UREL1,UREL2,UI,FI)
      WRITE(6,40) FI
   40 FORMAT(1F10.4)
      STOP
      END
```

(C) OUTPUT

```
0.0063
```

ENGRAD

======

ENERGY GRADIENTS FOR A TWO-LAYER SYSTEM.

(1) PURPOSE THE ROUTINE CALCULATES THE ENERGY GRADIENTS OF EACH LAYER OF A TWO-LAYER SYSTEM AT A CROSS-SECTION OF ARBITRARY GEOMETRY.

(2) METHOD USING EQUATIONS 6.24 AND 6.27 (CHAPTER 6). THE SIGN OF THE RIGHT-HAND SIDES OF THESE EQUATIONS ARE CHANGED IN THE PROGRAM SO THAT THE GRADIENTS ARE COMPUTED IN THE FLOW DIRECTION. SHEAR STRESSES ARE COMPUTED USING EQUATIONS 6.28,6.29 AND 6.309 SHEAR STRESS COEFFICIENTS ARE COMPUTED USING SUBROUTINES FBNDRY AND FINTRF

(3) PROGRAM
(A) DECK NAME ENGRAD
(B) CALLING SEQUENCE

```
CALL ENGRAD(HORZ,VERT,NPTS,RHO1,RHO2,VISC1,VISC2,G,Q1,Q2,WL1,WL2,
*D,FO1,FO2,BL,FLOW,NINCR1,ELA,X,UREL1,UREL2,UI,FI,S1E,S2E)
```

WHERE

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID LEVELS.

NPTS = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-SECTION.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

VISC1,VISC2 = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

Q1,Q2 = DISCHARGES OF THE UPPER AND LOWER LAYERS RESPECTIVELY. SHOULD HAVE DIFFERENT SIGNS IF OPPOSITE.

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY.

D = CROSS-SECTIONAL ROUGHNESS HEIGHT.

FO1,FO2 = SHEAR STRESS COEFFICIENTS AT THE SOLID BOUNDARY FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

BL = 1.0 IF INTERFACIAL BOUNDARY LAYERS ARE LAMINAR.
= 2.0 IF THE BOUNDARY LAYERS ARE TURBULENT.

FLOW = 1.0 IF THE FLOW IS LAMINAR.
= 2.0 IF THE FLOW IS TURBULENT.

NINCR1 = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFACIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION OF THE VELOCITY PROFILE IN EACH LAYER. NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED NOT BE CALCULATED.

ELA = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS.

X = DISTANCE FROM THE POINT OF CONTACT OF THE TWO FLUIDS TO THE POINT UNDER COSIDERATION.

UREL1,UREL2 = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).

UI = VELOCITY AT THE INTERFACE RELATIVE TO THE MEAN VELOCITY OF FLOW.

FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

S1E,S2E = ENERGY GRADIENTS AT THE CROSS-SECTION UNDER CONSIDERATION OF THE UPPER AND LOWER LAYERS RESPECTIVELY. THESE GRADIENTS ARE CALCULATED IN THE POSITIVE FLOW DIRECTION,I.E.,IF THE COMPUTED GRADIENT IS NEGATIVE, IT INDICATES THAT THE ENERGY OF THE CORRESPONDING LAYER IS DECREASING IN THE POSITIVE FLOW DIRECTION (THE POSITIVE DIRECTION AS DEFINED BY THE GIVEN DISCHARGES).

(C) OUTPUT

FORM

1) IF BOTH LAYERS ARE FLOWING, THE CALCULATED VALUES ARE ASSIGNED TO S1E AND S2E. IN THIS CASE THE USER HAS TO SUPPLY VALUES FOR FO1,FO2 AND FI. THE VALUES OF D,ELA,X,VISC1 AND VISC2 DO NOT HAVE TO BE DEFINED.

2) IF THE UPPER LAYER ONLY IS FLOWING, THE USER HAS TO SUPPLY A VALUE FOR FI. IF THE FLOW IS FOUND TO BE TURBULENT AND THE INTERFACIAL BOUNDARY LAYERS LAMINAR, A VALUE FOR FI IS COMPUTED AND THE CALCULATED VALUES ARE ASSIGNED TO FO1,FO2,BL,FLOW, UREL1,UREL2,UI,FI,S1E AND S2E. OTHERWISE, THE VALUE OF FI SUPPLIED BY THE USER IS USED AND THE CALCULATED VALUES ARE ASSIGNED TO FO1,FO2,BL,FLOW,S1E AND S2E.

3) IF THE LOWER LAYER ONLY IS FLOWING, THE CALCULATED VALUES ARE ASSIGNED TO FO1,FO2,BL,FLOW,FI,S1E AND S2E.

(D) RESTRICTIONS SEE ABOVE.

(E) OTHER DECKS REQUIRED PROPS2,FBNDRY,FINTRF (THE LAST TWO ROUTINES ARE NEEDED ONLY IN THE CASE OF ONE FLOWING LAYER).

(4) EXAMPLE

(A) INPUTS A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,4 2,2 4,0 8,0 10,2 10,4

FREE SURFACE AND INTERFACE ELEVATIONS ARE 3.5 AND 1.8 RESPECTIVELY. ROUGHNESS HEIGHT IS 0.05.
UPPER LAYER
DISCHARGE=0.0,KINEMATIC VISCOSITY=1.0E-6,SP. GRAVITY=1.00
LOWER LAYER
DISCHARGE=0.2,KINEMATIC VISCOSITY=1.1E-6,SP. GRAVITY=1.05

TOTAL LENGTH OF THE UPPER LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS IS 200 AND THE CROSS-SECTION UNDER CONSIDERATION IS AT A DISTANCE 15 FROM THE POINT OF CONTACT.
GRAVITATIONAL ACCELERATION IS 9.81.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6)
      EXTERNAL AUX1,AUX2,AUX3,AUX4,AUX5,AUX
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.1)
      CALL ENGRAD(HORZ,VERT,6,1.0,1.05,1.0E-6,1.1E-6,9.81,0.0,0.2,3.5,
     *1.8,0.05,FO1,FO2,BL,FLOW,0,200.0,15.0,UREL1,UREL2,UI,FI,S1E,S2E)
      WRITE(6,20) FO1,FO2,BL,FLOW
   20 FORMAT(4F10.4)
      WRITE(6,30) UI,FI,S1E,S2E
   30 FORMAT(2F10.4,2E10.2)
      STOP
      END
```

(C) OUTPUT

```
    0.0000    0.0405    1.0000    2.0000
    0.6102    0.0029    .10E-07   -.17E-06
```

SURGRD

SURFACE SLOPES IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE ROUTINE CALCULATES THE FREE SURFACE SLOPE AND THE INTERFACE SLOPE FOR A TWO-LAYER SYSTEM AT A CROSS-SECTION OF ARBITRARY GEOMETRY.

(2) METHOD USING EQUATIONS 6.16 AND 6.18 (CHAPTER 6). THE ENERGY GRADIENTS OF THE TWO LAYERS ARE CALCULATED USING SUBROUTINE ENGRAD.

(3) PROGRAM
(A) DECK NAME SURGRD
(B) CALLING SEQUENCE

```
CALL SURGRD(HORZ,VERT,NPTS,VERTT,SO,RHO1,RHO2,VISC1,VISC2,G,Q1,
*Q2,ALPHA1,ALPHA2,WL1,WL2,D,FO1,FO2,BL,FLOW,NINCR1,ELA,X,UREL1,
*UREL2,UI,FI,S1E,S2E,S1,S2)
```

WHERE

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID LEVELS.

NPTS = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-SECTION.

VERTT = ONE-DIMENSIONAL WORKING ARRAY OF SIZE NPTS.

SO = CHANNEL BED SLOPE AT THE CROSS-SECTION UNDER CONSIDERATION. NEGATIVE IF SLOPING IN THE POSITIVE FLOW DIRECTION (SEE DEFINITION OF S1E AND S2E).

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

VISC1,VISC2 = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

Q1,Q2 = DISCHARGES OF THE UPPER AND LOWER LAYERS RESPECTIVELY. SHOULD HAVE DIFFERENT SIGNS IF OPPOSITE.

ALPHA1,ALPHA2 = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY.

D = CROSS-SECTIONAL ROUGHNESS HEIGHT.

FO1,FO2 = SHEAR STRESS COEFFICIENTS AT THE SOLID BOUNDARY FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

BL = 1.0 IF INTERFACIAL BOUNDARY LAYERS ARE LAMINAR.
= 2.0 IF THE BOUNDARY LAYERS ARE TURBULENT.

FLOW = 1.0 IF THE FLOW IS LAMINAR.
= 2.0 IF THE FLOW IS TURBULENT.

NINCR1 = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFACIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION OF THE VELOCITY PROFILE IN EACH LAYER.
NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED NOT BE CALCULATED.

ELA = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS.

X = DISTANCE FROM THE POINT OF CONTACT OF THE TWO FLUIDS TO THE POINT UNDER COSIDERATION.

UREL1,UREL2 = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).

UI = VELOCITY AT THE INTERFACE RELATIVE TO THE MEAN VELOCITY OF FLOW.

FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

S1E,S2E = ENERGY GRADIENTS AT THE CROSS-SECTION UNDER CONSIDERATION OF THE UPPER AND LOWER LAYERS RESPECTIVELY. THESE GRADIENTS ARE CALCULATED IN THE POSITIVE FLOW DIRECTION,I.E.,IF THE COMPUTED GRADIE-

NT IS NEGATIVE, IT INDICATES THAT THE ENERGY OF THE CORRESPONDING LAYER IS DECREASING IN THE POSITIVE FLOW DIRECTION (THE POSITIVE DIRECTION AS DEFINED BY THE GIVEN DISCHARGES).

S1,S2 = FREE SURFACE AND INTERFACE SLOPES RESPECTIVELY AT THE CROSS-SECTION UNDER CONSIDERATION. SIGN CONVENTION IS THE SAME AS SO,S1E AND S2E.

(C) OUTPUT FORM

CALCULATED VALUES ARE ASSIGNED TO S1 AND S2. IF THE DENOMINATOR OF EQUATIONS 6.16 AND 6.18 (CHAPTER 6) IS EQUAL TO ZERO(CRITICAL FLOW), S1 AND S2 ARE GIVEN VERY LARGE VALUES(1.0E20).
FOR OTHER INPUT AND OUTPUT PARAMETERS SEE SECTION 3(C) OF THE DOCUMENTATION OF SUBROUTINE ENGRD.

(D) RESTRICTIONS SEE ABOVE.

(E) OTHER DECKS REQUIRED PROPS2,ENGRAD

(4) EXAMPLE

(A) INPUTS

A CHANNEL CROSS-SECTION IS DESCRIBED BY 6 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,4 2,2 4,0 8,0 10,2 10,4

FREE SURFACE AND INTERFACE ELEVATIONS ARE 3.5 AND 1.8 RESPECTIVELY. ROUGHNESS HEIGHT IS 0.05 AND BED SLOPE IS -0.001.
UPPER LAYER
DISCHARGE=0.0,KINEMATIC VISCOSITY=1.0E-6,SP. GRAVITY=1.00,KINETIC ENERGY CORRECTION FACTOR=1.1
LOWER LAYER
DISCHARGE=0.2,KINEMATIC VISCOSITY=1.1E-6,SP. GRAVITY=1.05,KINETIC ENERGY CORRECTION FACTOR=1.2

TOTAL LENGTH OF THE UPPER LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS IS 200 AND THE CROSS-SECTION UNDER CONSIDERATION IS AT A DISTANCE 15 FROM THE POINT OF CONTACT.
GRAVITATIONAL ACCELERATION IS 9.81.

(B) CODE

```
      DIMENSION HORZ(6),VERT(6),VERTT(6)
      EXTERNAL AUX1,AUX2,AUX3,AUX4,AUX5,AUX
      READ(5,10) (HORZ(I),VERT(I),I=1,6)
   10 FORMAT(12F5.1)
      CALL SURGRD(HORZ,VERT,6,VERTT,-0.001,1.0,1.05,1.0E-6,1.1E-6,9.81,
     *0.0,0.2,1.1,1.2,3.5,1.8,0.05,FO1,FO2,BL,FLOW,0,200.0,15.0,UREL1,
     *UREL2,UI,FI,S1E,S2E,S1,S2)
      WRITE(6,20) S1,S2
   20 FORMAT(2E20.2)
      STOP
      END
```

(C) OUTPUT

```
              .73E-08             -.31E-05
```

FLPROF

SURFACE PROFILES IN A TWO-LAYER SYSTEM.

(1) PURPOSE A STRETCH OF RIVER IS DESCRIBED BY A SEQUENCE OF CROSS-SECTIONS EACH OF WHICH IS DEFINED BY THE SAME NUMBER OF POINTS THE COORDINATES OF WHICH ARE REFERRED TO ARBITRARY DATUMS FOR LEVEL AND HORIZONTAL DISTANCE. IN ADDITION, VALUES OF THE CHAINAGE AND ROUGHNESS HEIGHT ARE ASSIGNED TO EACH CROSS-SECTION. THIS ROUTINE OPERATES ON THE SYSTEM THUS DEFINED AND FOR A SELECTED REACH, CALCULATES THE FREE SURFACE AND INTERFACE ELEVATIONS AS WELL AS ENERGY LEVELS FOR EACH LAYER AT ONE END OF THE REACH AS FUNCTIONS OF SPECIFIED DISCHARGES AND CONTROL LEVELS AT THE OTHER END WITH A SPECIFIED INCREMENTAL DISTANCE ADEQUATELY SIGNED. THE ROUTINE IS INTENDED FOR SUCCESSIVE APPLICATION WORKING IN EITHER UPSTREAM OR DOWNSTREAM DIRECTION.

(2) METHOD RUNGE-KUTTA-MERSON METHOD (A DIFFERENTIAL EQUATION SOLVER ALGORITHM) IS USED TO INTEGRATE THE FLOW PROFILES ALONG AN ELEMENTARY REACH OF CHANNEL. THE METHOD IS DESCRIBED IN SECTION 6.3.1 (CHAPTER 6). THE ALLOWABLE TOLERENCE USED FOR THE TRUNCATION ERRORS IS 0.01 PERCENT OF THE CORRESPONDING WATER LEVEL.

(3) PROGRAM
(A) DECK NAME FLPROF
(B) CALLING SEQUENCE

```
CALL FLPROF(HORZAR,VERTAR,CHAINR,NXSEC,NPTS,NSEC1,WL1,WL2,RHO1,
*RHO2,VISC1,VISC2,G,Q1,Q2,ALPHA1,ALPHA2,DAR,FO1,FO2,FI,UREL1,
*UREL2,HORZ1,VERT1,HORZ2,VERT2,HORZ,VERT,VERTT,NINCR1,ELA,DX,
*NPRINT,WL11,WL21,ENLV1,ENLV2)
```

WHERE

HORZAR,VERTAR = TWO-DIMENSIONAL ARRAYS OF SIZE (NXSEC,NPTS) CONTAINING THE HORIZONTAL AND VERTICAL COORDINATES OF THE POINTS DESCRIBING EACH OF A SERIES OF CROSS-SECTIONS DEFINING THE CHANNEL. THE FIRST VALUE OF THE ARRAY REPRESENTS THE SECTION NUMBER AND THE SECOND VALUE REPRESENTS ONE OF THE HORIZONTAL (OR VERTICAL) COORDINATES.

CHAINR = ONE-DIMENSIONAL ARRAY OF SIZE NXSEC CONTAINING THE CHAINAGE VALUES FOR THE SERIES OF CROSS-SECTIONS.
CHAINAGE VALUES ARE REFERRED TO THE POINT OF CONTACT OF THE TWO LAYERS AND INCREASE IN THE DOWNSTREAM DIRECTION. UPSTREAM AND DOWNSTREAM DIRECTIONS ARE DEFINED ACCORDING TO THE SIGNE OF THE GIVEN DISCHARGES (I.E. IF THE TWO LAYERS ARE FLOWING IN OPPOSITE DIRECTIONS, THE UPSTREAM DIRECTION IS THE DIRECTION OF FLOW OF THE LAYER THAT HAS THE POSITIVE DISCHARGE).

NXSEC = THE NUMBER OF CROSS-SECTIONS IN THE HYDRAULIC PROFILE.

NPTS = NUMBER OF POINTS DESCRIBING EACH OF THE CROSS-SECTIONS.

NSEC1 = SECTION NUMBER WHERE THE ELEVATIONS ARE SOUGHT.

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT ONE END OF THE REACH. THE REACH IS THE PORTION OF THE CHANNEL BETWEEN TWO SUCCESSIVE CROSS-SECTIONS.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

VISC1,VISC2 = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

Q1,Q2 = DISCHARGES OF THE UPPER AND LOWER LAYERS RESPECTIVELY. SHOULD HAVE DIFFERENT SIGNS IF OPPOSITE.

ALPHA1,ALPHA2 = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

DAR = ONE-DIMENSIONAL ARRAY OF SIZE NXSEC CONTAINING THE ROUGHNESS HEIGHT FOR EACH CROSS-SECTION.

FO1,FO2 = SHEAR STRESS COEFFICIENTS AT THE SOLID BOUNDARY FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

UREL1,UREL2 = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).

HORZ1,VERT1 = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.

HORZ2,VERT2 = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.

HORZ ,VERT = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.

VERTT = ONE-DIMENSIONAL WORKING ARRAY OF SIZE NPTS.

NINCR1 = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFACIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION OF THE VELOCITY PROFILE IN EACH LAYER. NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED NOT BE CALCULATED.

ELA = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS.

DX = INCREMENTAL DISTANCE TO BE USED FOR THE INTEGRTION. THE SIGN OF DX DEPENDS ON THE DIRECTION OF INTEGRATION (POSITIVE IF THE INTEGRATION IS IN THE DOWNSTREAM DIRECTION AND VISE VERSA).

NPRINT = AN INTEGER VARIABLE SET EQUAL TO ONE IF A PRINTOUT OF PROFILES COORDINATES AT THE INTERMEDIATE STEPS IS REQUIRED. IF NOT IT MAY BE SET TO ANY OTHER VALUE.

WL11,WL21 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE OTHER END OF THE REACH.

ENLV1,ENLV2 = ENERGY LEVELS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE OTHER END OF THE REACH.

(C) OUTPUT FORM

THE COMPUTED VALUES ARE ASSIGNED TO WL11,WL21,ENLV1,ENLV2. IF NPRINT IS SET EQUAL TO ONE, A PRINTOUT OF THE PROFILE COORDS. AT THE INTERMEDIATE STEPS IS GIVEN. A DIAGNOSTIC STATEMENT IS PRINTED OUT WHEN THE DEPTH OF EITHER LAYER REACHES EITHER CRITICAL DEPTH OR ZERO. ALSO, WHEN THERE IS AN ABRUPT CHANGE IN THE CROSS-SECTIONAL GEOMETRY (CASCADE REACH), CRITICAL CONDITION IS ASSUMED WITH THE FREE SURFACE ELEVATION REMAINING THE SAME AND A DIAGNOSTIC STATEMENT IS PRINTED OUT. CASCADE REACHES ARE REPRESENTED BY TWO CROSS-SECTIONS HAVING THE SAME CHAINAGE. IN THIS CASE, TWO SOLUTIONS FOR THE CRITICAL INTERFACE ELEVATION MAY BE OBTAINED(SEE SECTION 3.3.5-PART D-CHAPTER 3). THEREFORE, THE SECOND SOLUTION IS PRINTED BETWEEN BRACKETS BESIDE THE FIRST ONE IF N IS SET EQUAL TO ONE. IF NOT, ONLY THE FIRST SOLUTION IS PRINTED

FOR OTHER INPUT AND OUTPUT PARAMETERS SEE SECTION 3(C) OF THE DOCUMENTATION OF SUBROUTINE ENGRAD.

(D) RESTRICTIONS SEE ABOVE

(E) OTHER DECKS REQUIRED SELSEC,WLCRIT1,SURGRD,PROPS2,BOTTOM

(4) EXAMPLE

(A) INPUTS

A PRISMATIC CHANNEL HAS A CROSS-SECTION DESCRIBED BY 4 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,1.25 0,0 5,0 5,1.25

THE CHANNEL IS CONSIDERED AS ONE REACH (TWO CROSS-SECTIONS) WHICH HAVE THE FOLLOWING CHARACTERISTICS-

SECTION NO.	CHAINAGE
1	0.0
2	30.0

FREE SURFACE AND INTERFACE ELEVATIONS AT SECTION 2 ARE 0.71 AND 0.2454 RESPECTIVELY (CRITICAL CONDITION-COMPUTED USING SUBROUTINE WLCRIT1). BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY ARE 0.03 AND 0.03. INTERFACIAL SHEAR STRESS COEFF. IS 0.02. GRAVITATIONAL ACCELERATION IS 32.2.

UPPER LAYER
DISCHARGE=-0.109,SPECIFIC GRAVITY=0.9941,KINETIC ENERGY CORRECTION FACTOR=1.3.

LOWER LAYER.
DISCHARGE=0.159,SPECIFIC GRAVITY=0.9980,KINETIC ENERGY CORRECTION=1.7.

THE INCREMENTAL DISTANCE DX IS CHOSEN TO BE -1.0.

(B) CODE

```
      DIMENSION HORZAR(2,4),VERTAR(2,4),CHAINR(2),HORZ1(4),VERT1(4),
     *HORZ2(4),VERT2(4),HORZ(4),VERT(4),VERTT(4),DAR(2)
      EXTERNAL AUX,AUX1,AUX2,AUX3,AUX4,AUX5
      WL1=0.71
      WL2=0.2454
      NXSEC=2
      DO 20 N=1,NXSEC
      READ(5,30) I,CHAINR(I)
   30 FORMAT(I10,F10.2)
      READ(5,40) (HORZAR(I,M),VERTAR(I,M),M=1,4)
   40 FORMAT(8F10.3)
   20 CONTINUE
      WLA=WL1
      WLB=WL2
      NXSC1=NXSEC-1
      DO 60 K=1,NXSC1
      I=NXSEC-K
      DX=-1.0
      FO1=FO2=0.03
      FI=0.02
      CALL FLPROF(HORZAR,VERTAR,CHAINR,NXSEC,4,I,WLA,WLB,0.9941,0.9980,
     *VISC1,VISC2,32.2,-0.109,0.159,1.3,1.7,DAR,FO1,FO2,FI,UREL1,UREL2,
     *HORZ1,VERT1,HORZ2,VERT2,HORZ,VERT,VERTT,0,ELA,DX,1,WL11,WL21,
     *ENLV1,ENLV2)
      WRITE(6,50) WL11,WL21,ENLV1,ENLV2
   50 FORMAT(4F15.2)
      WLA=WL11
      WLB=WL21
   60 CONTINUE
      STOP
      END
```

(C) OUTPUT

```
 CHAINAGE       FREE SURFACE LEVEL        INTERFACE LEVEL

   30.00              .7100                    .2454
   29.98              .7100                    .2478
   29.97              .7100                    .2493
   29.95              .7100                    .2506
   29.92              .7100                    .2526
   29.86              .7100                    .2556
   29.80              .7100                    .2581
   29.67              .7100                    .2621
   29.55              .7100                    .2654
   29.30              .7100                    .2708
   29.05              .7100                    .2754
   28.55              .7100                    .2830
   28.05              .7100                    .2894
   27.05              .7100                    .3002
   26.05              .7100                    .3094
   24.05              .7100                    .3251
   22.05              .7099                    .3386
   20.05              .7099                    .3508
   16.05              .7099                    .3731
   12.05              .7099                    .3938
    4.05              .7098                    .4351
     .00              .7097                    .4584
           .71            .46            .71            .71
```

NORMQ2

NORMAL DISCHARGES IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE ROUTINE CALCULATES THE NORMAL VELOCITIES AND NORMAL DISCHARGES IN A TWO-LAYER SYSTEM THROUGH A CHANNEL OF ARBITRARY GEOMETRY. TWO CASES ARE CONSIDERED WHICH ARE OF PRACTICAL SIGNIFICANCE. THE FIRST CASE IS WHEN BOTH LAYERS ARE FLOWING IN THE SAME DIRECTION AND THE SECOND CASE WHEN THE LOWER LAYER ONLY IS FLOWING WHILE THE UPPER LAYER IS STAGNANT.

(2) METHOD FIRST CASE

THE ENERGY GRADIENTS OF BOTH LAYERS ARE CALCULATED USING EQUATIONS 6.19, 6.20 AND 6.21 (CHAPTER 6). THEN EQUATIONS 6.24 AND 6.27 ARE SOLVED SIMULTANEOUSLY USING SUBROUTINE ZSYSTM1 (BROWN METHOD) THE UNKNOWNS BEING THE VELOCITIES IN THE UPPER AND LOWER LAYERS. EQUATIONS 6.28,6.29 AND 6.30 ARE USED FOR THE SHEAR STRESSES.

SECOND CASE

THE ENERGY GRADIENT OF THE LOWER LAYER IS CALCULATED USING EQUATIONS 6.19A,6.20A AND 6.21A IN TERMS OF THE FREE SURFACE SLOPE S1. THEN EQUATIONS 6.24 AND 6.27 ARE SOLVED SIMULTANEOUSLY USING SUBROUTINE ZSYSTM1 (BROWN METHOD), THE UNKNOWNS BEING THE FREE SURFACE SLOPE AND THE LOWER LAYER VELOCITY(NOTE THAT THE UPPER LAYER VELOCITY=0.0).

(3) PROGRAM

(A) DECK NAME NORMQ2

(B) CALLING SEQUENCE

```
CALL NORMQ2(SO,N,RHO1,RHO2,G,WL1,WL2,FO1,FO2,FI,HORZ,VERT,VERTT,
*NPTS,U1,U2,Q1,Q2)
```

WHERE

SO = CHANNEL BED SLOPE. NEGATIVE IF BED ELEVATION DECREASES IN FLOW DIRECTION.

N = 1 IF THE LOWER LAYER ONLY IS FLOWING AND THE UPPER LAYER IS STAGNANT. OTHERWISE IT IS ASSUMED THAT BOTH LAYERS ARE FLOWING IN THE SAME DIRECTION.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY.

FO1,FO2 = BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY AXIS. VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.

VERTT = ONE-DIMENSIONAL WORKING ARRAY OF SIZE NPTS.

NPTS = NUMBER OF POINTS DEFINING THE CHANNEL CROSS-SECTION.

U1,U2 = NORMAL VELOCITIES (CORRESPONDING TO UNIFORM FLOW) OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

Q1,Q2 = NORMAL DISCHARGES OF THE UPPER AND LOWER LAYERS RESPECTIVELY.

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO U1,U2,Q1,Q2
A MESSAGE IS PRINTED IF THE SOLUTION DOES NOT CONVERGE.

(D) RESTRICTIONS THIS ROUTINE MAY BE USED ONLY IN THE TWO CASES SPECIFIED ABOVE.

(E) OTHER DECKS REQUIRED PROPS2,PROPS,BOTTOM,ZSYSTM1(USES THE AUXILIARY FUNCTION FUNN).

(4) EXAMPLE

(A) INPUTS A CHANNEL CROSS-SECTION IS DEFINED BY 4 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

0,15   0,0   50,0   50,15

THE BED SLOPE IS -0.001. SPECIFIC GRAVITIES OF THE UPPER AND LOWER FLUIDS ARE 1.0 AND 1.02 RESPECTIVELY. BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER AND LOWER LAYERS ARE 0.02 AND 0.03 RESPECTIVELY. INTERFACIAL SHEAR STRESS COEFFICIENT IS 0.01. FREE SURFACE AND INTERFACE ELEVATIONS ARE 10 AND 5 RESPECTIVELY. IT IS REQUIRED TO CALCULATE NORMAL VELOCITIES AND NORMAL DISCHARGES IN THE FOLLOWING TWO CASES

A- BOTH LAYERS ARE FLOWING IN THE SAME DIRECTION.
B- LOWER LAYER ONLY IS FLOWING.

(B) CODE

```
      DIMENSION HORZ(4),VERT(4),VERTT(4),XX(2),WAX(8)
      EXTERNAL FUNN
      READ(5,10) (HORZ(I),VERT(I),I=1,4)
   10 FORMAT(8F10.2)
      DO 40 N=1,2
      CALL NORMQ2(-0.001,N,1.0,1.02,32.2,10.0,5.0,0.02,0.03,0.01,HORZ,
     *VERT,VERTT,4,U1,U2,Q1,Q2)
      WRITE(6,30) U1,U2,Q1,Q2
   30 FORMAT(4F15.2)
   40 CONTINUE
      STOP
      END
```

(C) OUTPUT

```
           0.00           0.67           0.00         168.18
          14.05           7.03        3512.45        1756.26
```

NORMWL

NORMAL WATER LEVELS IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE ROUTINE CALCULATES THE NORMAL FREE SURFACE AND INTERFACE ELEVATIONS (LEVELS WHICH CORRESPOND TO UNIFORM FLOW) IN A TWO-LAYER SYSTEM THROUGH A CHANNEL OF ARBITRARY GEOMETRY. TWO CASES ARE CONSIDERED WHICH ARE OF PRACTICAL SIGNIFICANCE. THE FIRST CASE IS WHEN BOTH LAYERS ARE FLOWING IN THE SAME DIRECTION AND THE SECOND CASE WHEN THE LOWER LAYER ONLY IS FLOWING WHILE THE UPPER LAYER IS STAGNANT.

(2) METHOD FIRST CASE
EQUATIONS 6.24 AND 6.27 (CHAPTER 6) ARE SOLVED SIMULTANEOUSLY USING SUBROUTINE ZSYSTM1 (BROWN METHOD) THE UNKNOWNS BEING THE FREE SURFACE AND INTERFACE ELEVATIONS. FOR EACH TRIAL VALUES OF THE UNKNOWNS, THE ENERGY GRADIENTS OF BOTH LAYERS ARE CALCULATED USING EQUATIONS 6.19, 6.20 AND 6.21. SHEAR STRESSES ARE CALCULATED USING EQUATIONS 6.28, 6.29 AND 6.30.
SECOND CASE
EQUATIONS 6.24 AND 6.27 ARE SOLVED SIMULTANEOUSLY USING SUBROUTINE ZSYSTM1, THE UNKNOWNS BEING THE FREE SURFACE SLOPE AND THE INTERFACE ELEVATION. FOR EACH TRIAL VALUES OF THE UNKNOWNS,ENERGY GRADIENTS ARE CALCULATED USING EQUATIONS 6.19A,6.20A AND 6.21A.

(3) PROGRAM
(A) DECK NAME NORMWL
(B) CALLING SEQUENCE

```
CALL NORMWL(SO,RHO1,RHO2,G,WL1S,WL2S,Q1,Q2,FO1,FO2,FI,HORZ,VERT,
*HORZD,VERTD,NPTS,WL1,WL2)
```

WHERE
SO = CHANNEL BED SLOPE. NEGATIVE IF BED ELEVATION DECREASES IN FLOW DIRECTION.
RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
G = GRAVITATIONAL ACCELERATION.
WL1S,WL2S = INITIAL GUESS FOR THE NORMAL FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY. IF THE LOWER LAYER ONLY IS FLOWING, WL1S IS THE ACTUAL FREE SURFACE ELEVATION (INPUT).
Q1,Q2 = NOSCHARGES IN THE UPPER AND LOWER LAYERS RESPECTIVELY.
FO1,FO2 = BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.
FI = INTERFACIAL SHEAR STRESS COEFFICIENT.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES OF THE POINTS DESCRIBING THE CHANNEL CROSS-SECTION. HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY AXIS. VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.
HORZD,VERTD = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
NPTS = NUMBER OF POINTS DEFINING THE CHANNEL CROSS-SECTION.
WL1,WL2 = NORMAL FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY (CORRESPONDING TO UNIFORM FLOW).

(C) OUTPUT FORM THE CALCULATED VALUES ARE ASSIGNED TO WL1,WL2
A MESSAGE IS PRINTED IF THE SOLUTION DOES NOT CONVERGE.

(D) RESTRICTIONS THIS ROUTINE MAY BE USED ONLY IN THE TWO CASES SPECIFIED ABOVE.

(E) OTHER DECKS REQUIRED PROPS2,PROPS,BOTTOM,ZSYSTM1(USES THE AUXILIARY FUNCTION FUNM).

(4) EXAMPLE
(A) INPUTS A CHANNEL CROSS-SECTION IS DEFINED BY 4 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

```
0,15        0,0        50,0        50,15
```

THE BED SLOPE IS -0.001. SPECIFIC GRAVITIES OF THE UPPER AND LOW-

ER FLUIDS ARE 1.0 AND 1.02 RESPECTIVELY. BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER AND LOWER LAYERS ARE 0.02 AND 0.03 RESPECTIVELY. INTERFACIAL SHEAR STRESS COEFFICIENT IS 0.01.
IT IS REQUIRED TO CALCULATE NORMAL WATER LEVELS FOR THE FOLLOWING TWO CASES

A- BOTH LAYERS ARE FLOWING IN THE SAME DIRECTION WITH DISCHARGES OF 3512 AND 1756 FOR THE UPPER AND LOWER LAYERS RESPECTIVELY. INITIAL GUESS FOR THE NORMAL FREE SURFACE AND INTERFACE ELEVATIONS ARE 9 AND 4 RESPECTIVELY.

B- LOWER LAYER ONLY IS FLOWING (I.E. Q1=0). LOWER LAYER DISCHARGE IS 168 AND FREE SURFACE ELEVATION IS 10. INITIAL GUESS FOR THE NORMAL INTERFACE ELEVATION IS 4.

(B) CODE

```
      DIMENSION HORZ(4),VERT(4),HORZD(4),VERTD(4),XX(2),WAX(8)
      EXTERNAL FUNM
      READ(5,10) (HORZ(I),VERT(I),I=1,4)
   10 FORMAT(8F10.2)
      CALL NORMWL(-0.001,1.0,1.02,32.2,9.0,4.0,3512.0,1756.0,0.02,0.03,
     *0.01,HORZ,VERT,HORZD,VERTD,4,WL1,WL2)
      WRITE(6,20) WL1,WL2
   20 FORMAT(2F20.2)
      CALL NORMWL(-0.001,1.0,1.02,32.2,10.0,4.0,0.0,168.0,0.02,0.03,
     *0.01,HORZ,VERT,HORZD,VERTD,4,WL1,WL2)
      WRITE(6,20) WL1,WL2
      STOP
      END
```

(C) OUTPUT

```
               10.00                5.00
               10.00                5.10
```

ARSWDG

ARRESTED WEDGE PROFILE IN A TWO-LAYER SYSTEM.

(1) PURPOSE THE ARRESTED WEDGE MAY BE A SALT WEDGE(LOWER LAYER DISCHARGE =0) OR A THERMAL WEDGE(UPPER LAYER DISCHARGE=0). THE WEDGE IS ARRESTED(BROUGHT TO ZERO NET VELOCITY) DUE TO AN OPPOSING CURRENT IN THE OTHER LAYER.

A STRETCH OF RIVER IS DESCRIBED BY A SEQUENCE OF CROSS-SECTIONS EACH OF WHICH IS DEFINED BY THE SAME NUMBER OF POINTS THE COORDINATES OF WHICH ARE REFERRED TO ARBITRARY DATUMS FOR LEVEL AND HORIZONTAL DISTANCE. IN ADDITION, VALUES OF THE CHAINAGE AND ROUGHNESS HEIGHT ARE ASSIGNED TO EACH CROSS-SECTION.

THIS ROUTINE CALCULATES THE WEDGE PROFILE STARTING AT THE RIVER MOUTH (OR WARM WATER OUTLET) WHICH IS THE SECTION OF LARGEST CHAINAGE TO THE SECTION WHERE THE DEPTH OF THE WEDGE BECOMES ZERO. IN ADDITION TO THE WEDGE PROFILE THIS ALSO GIVES THE LENGTH OF THE WEDGE.

(2) METHOD GIVEN THE FREE SURFACE ELEVATION AT THE RIVER MOUTH (OR WARM WATER OUTLET), THE ROUTINE CALCULATES THE INTERFACE ELEVATION AT THAT SECTION ASSUMING CRITICAL CONDITION. THEN, USING SUBROUTINE FLPROF THE ROUTINE COMPUTES THE WEDGE PROFILE UNTIL THE DEPTH OF THE WEDGE REACHES ZERO.

(3) PROGRAM

(A) DECK NAME ARSWDG

(B) CALLING SEQUENCE

```
CALL ARSWDG(HORZAR,VERTAR,CHAINR,NXSEC,NPTS,WL1,Q1,Q2,RHO1,RHO2,
*G,VISC1,VISC2,ALPHA1,ALPHA2,DAR,FO1,FO2,FI,UREL1,UREL2,HORZ1,
*VERT1,HORZ2,VERT2,HORZ3,VERT3,HORZ,VERT,VERTT,ELA,DX,NPRINT)
```

WHERE

HORZAR,VERTAR = TWO-DIMENSIONAL ARRAYS OF SIZE (NXSEC,NPTS) CONTAINING THE HORIZONTAL AND VERTICAL COORDINATES OF THE POINTS DESCRIBING EACH OF A SERIES OF CROSS-SECTIONS DEFINING THE CHANNEL. THE FIRST VALUE OF THE ARRAY REPRESENTS THE SECTION NUMBER AND THE SECOND VALUE REPRESENTS ONE OF THE HORIZONTAL (OR VERTICAL) COORDINATES.

CHAINR = ONE-DIMENSIONAL ARRAY OF SIZE NXSEC CONTAINING THE CHAINAGE VALUES FOR THE SERIES OF CROSS-SECTIONS. CHAINAGE VALUES INCREASE IN THE DOWNSTREAM DIRECTION.

NXSEC = THE NUMBER OF CROSS-SECTIONS IN THE HYDRAULIC PROFILE.

NPTS = NUMBER OF POINTS DESCRIBING EACH OF THE CROSS-SECTIONS.

WL1 = FREE SURFACE ELEVATION AT THE RIVER MOUTH (OR WARM WATER OUTLET).

Q1,Q2 = DISCHARGES OF THE UPPER AND LOWER LAYERS RESPECTIVELY. ONE OF THEM SHOULD BE ZERO.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

G = GRAVITATIONAL ACCELERATION.

VISC1,VISC2 = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

ALPHA1,ALPHA2 = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

DAR = ONE-DIMENSIONAL ARRAY OF SIZE NXSEC CONTAINING THE ROUGHNESS HEIGHT FOR EACH CROSS-SECTION.

FO1,FO2 = SHEAR STRESS COEFFICIENTS AT THE SOLID BOUNDARY FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.

FI = INTERFACIAL SHEAR STRESS COEFFICIENT.

UREL1,UREL2 = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW). IN THIS ROUTINE, NINCR1 IS SET EQUAL TO ZERO. THEREFORE, UREL1 AND UREL2 ARE DUMMY PARAMETERS.

HORZ1,VERT1 = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.

HORZ2,VERT2 = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
HORZ3,VERT3 = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
HORZ ,VERT = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
VERTT = ONE-DIMENSIONAL WORKING ARRAY OF SIZE NPTS.
ELA = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT OF CONTACT OF THE TWO FLUIDS.
DX = INCREMENTAL DISTANCE TO BE USED FOR THE INTEGRTION. THE SIGN OF DX IS ALWAYS NEGATIVE SINCE THE INTEGRATION IS IN THE UPSTREAM DIRECTION.
NPRINT = AN INTEGER VARIABLE SET EQUAL TO ONE IF A PRINTOUT OF PROFILES COORDINATES AT THE INTERMEDIATE STEPS IS REQUIRED. IF NOT IT MAY BE SET TO ANY OTHER VALUE.

(C) OUTPUT FORM

A) IF THE DEPTH OF THE WEDGE REACHES ZERO VALUE WITHIN THE REGION DEFINED, A MESSAGE IS PRINTED GIVING THE CHAINAGE AT WHICH THIS OCCURED.
B) IF THE WEDGE DEPTH DID NOT REACH ZERO VALUE WITHIN THE DEFINED REGION, A MESSAGE IS PRINTED TO THAT EFFECT.

IF NPRINT IS SET EQUAL TO ONE, A PRINTOUT OF PROFILE COORDS. AT THE INTERMEDIATE STEPS IS GIVEN. ALSO, THE CRITICAL INTERFACE LEVEL AT THE RIVER MOUTH IS PRINTED AS WELL AS THE WATER ELEVATIONS, ENERGY LEVELS, AND SHEAR STRESS COEFFICIENTS AT THE END OF EACH REACH (THE REACH BEING THE PORTION OF THE RIVER BETWEEN TWO SUCCESSIVE CROSS-SECTIONS).
FOR OTHER INPUT AND OUTPUT PARAMETERS SEE SECTION 3(C)-PARTS 2 AND 3 OF THE DOCUMENTATION OF SUBROUTINE ENGRAD.

(D) RESTRICTIONS SEE ABOVE

(E) OTHER DECKS REQUIRED WLCRIT1,FLPROF,SELSEC

(4) EXAMPLE

(A) INPUTS A CHANNEL REACH IS DEFINED BY TWO END CROSS-SECTIONS EACH OF THEM IS DEFINED BY 4 POINTS THE COORDINATES OF WHICH ARE AS FOLLOWS

SECTION NO. 1			
1,3.25	1,0.25	13,0.25	13,3.25
SECTION NO. 2 (RIVER MOUTH)			
0,3	2,0	12,0	15,3

THESE TWO SECTIONS HAVE THE FOLLOWING CHARACTERISTICS

SECTION NO.	CHAINAGE	ROUGHNESS HEIGHT
1	6000	0.01
2	10000	0.01

FREE SURFACE ELEVATION AT THE RIVER MOUTH IS 2.5. INTERFACIAL SHEAR STRESS COEFFICIENT IS 0.003.
GRAVITATIONAL ACCELERATION IS 9.81.

UPPER LAYER
DISCHARGE=6.0,SPECIFIC GRAVITY=1.00,KINEMATIC VISCOSITY=1.02E-6,
KINETIC ENERGY CORRECTION FACTOR=1.10.

LOWER LAYER
DISCHARGE=0.0,SPECIFIC GRAVITY=1.02,KINEMATIC VISCOSITY=1.11E-6,
KINETIC ENERGY CORRECTION FACTOR=1.20.

THE INCREMENTAL DISTANCE DX IS CHOSEN TO BE -100.

(B) CODE

```
      DIMENSION HORZAR(2,4),VERTAR(2,4),CHAINR(2),HORZ1(4),VERT1(4),
     *HORZ2(4),VERT2(4),HORZ(4),VERT(4),HORZ3(4),VERT3(4),VERTT(4),
     *DAR(2)
      EXTERNAL AUX,AUX1,AUX2,AUX3,AUX4,AUX5
      NXSEC=2
      DO 20 N=1,NXSEC
      READ(5,30) I,CHAINR(I),DAR(I)
   30 FORMAT(I10,2F10.3)
      READ(5,40) (HORZAR(I,M),VERTAR(I,M),M=1,4)
   40 FORMAT(8F10.2)
   20 CONTINUE
      DX=-100.0
```

```
 FI=0.003
 CALL ARSWDG(HORZAR,VERTAR,CHAINR,NXSEC,4,2.5,6.0,0.0,1.0,1.02,
*9.81,1.02E-6,1.11E-6,1.1,1.2,DAR,FO1,FO2,FI,UREL1,UREL2,HORZ1,
*VERT1,HORZ2,VERT2,HORZ3,VERT3,HORZ,VERT,VERTT,10000.0,DX,1)
 STOP
 END
```

(C) OUTPUT

```
 CRITICAL INTERFACE LEVEL AT THE RIVER MOUTH IS    1.4233

CHAINAGE       FREE SURFACE LEVEL       INTERFACE LEVEL

10000.0             2.5000                  1.4219
 9996.88            2.5004                  1.3995
 9993.75            2.5007                  1.3828
 9990.63            2.5010                  1.3688
 9984.38            2.5015                  1.3454
 9978.13            2.5018                  1.3258
 9965.63            2.5024                  1.2931
 9953.13            2.5029                  1.2655
 9928.13            2.5037                  1.2194
 9903.13            2.5044                  1.1805
 9853.13            2.5055                  1.1152
 9803.13            2.5064                  1.0601
 9753.13            2.5071                  1.0113
 9653.13            2.5084                   .9260
 9553.13            2.5095                   .8512
 9453.13            2.5105                   .7833
 9253.13            2.5121                   .6601
 9053.13            2.5135                   .5458
 8853.13            2.5146                   .4324
 8753.13            2.5152                   .3730
 8653.13            2.5157                   .3084
 8603.13            2.5159                   .2721
 8553.13            2.5162                   .2307
 8528.13            2.5163                   .2064
 8503.12            2.5165                   .1771
 8490.63            2.5165                   .1587
 8484.37            2.5166                   .1475
 8478.13            2.5166                   .1335
 8475.00            2.5166                   .1244
 8473.44            2.5167                   .1187
 8471.87            2.5167                   .1110

 DEPTH=0.0 AT A CHAINAGE OF ABOUT   8470.31
 FRICTION FACTORS FO1,FO2,FI ARE   .0219   0.0000   .0030
```

SWLMID

UPPER LAYER DISCHARGE AND SURFACE LEVEL IN A TWO-LAYER SYSTEM THROUGH A STREAMLINED TRANSITION.

(1) PURPOSE — THE TRANSITION IS DEFINED BY TWO CROSS-SECTIONS OF ARBITRARY GEOMETRY. THIS ROUTINE CALCULATES THE DISCHARGE OF THE UPPER LAYER AND THE FREE SURFACE ELEVATION AT ONE SECTION OF THE TRANSITION AS FUNCTIONS OF THE FREE SURFACE ELEVATION AT THE OTHER SECTION AND THE INTERFACE ELEVATIONS AT BOTH SECTIONS AS WELL AS THE DISCHARGE IN THE LOWER LAYER. TRANQUIL FLOW IS ASSUMED THROUGHOUT WITHOUT LOSSES.
THIS PROBLEM HAS AN APPLICATION IN SELECTIVE WITHDRAWAL OF A FLUID (SEE EXAMPLE).

(2) METHOD — EXPLICIT USE OF ENERGY EQUATIONS (EQUATIONS 3.2 AND 3.4-CHAPTER 3

(3) PROGRAM
(A) DECK NAME — SWLMID
(B) CALLING SEQUENCE

```
CALL SWLMID(WL1,WL2,Q2,WL21,ALPHA1,ALPHA2,G,RHO1,RHO2,HORZ,VERT,
*NPTS,HORZD,VERTD,NPTSD,Q1,WL11)
```

WHERE
WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE GIVEN SECTION.
Q2 = DISCHARGE IN THE LOWER LAYER.
WL21 = INTERFACE ELEVATION AT THE OTHER SECTION.
ALPHA1,ALPHA2 = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.
G = GRAVITATIONAL ACCELERATION.
RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.
HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE GIVEN CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO SOME ARBITRARY AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.
NPTS = NUMBER OF POINTS DESCRIBING THE GIVEN CROSS-SECTION.
HORZD,VERTD = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING THE HORIZONTAL AND VERTICAL COORDS. OF THE POINTS DEFINING THE OTHER CROSS-SECTION. THESE ARE REFERRED TO THE SAME AXES AS HORZ,VERT.
NPTSD = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
Q1 = DISCHARGE OF THE UPPER LAYER.
WL11 = FREE SURFACE ELEVATION AT THE OTHER SECTION.

(C) OUTPUT FORM — THE CALCULATED VALUES ARE ASSIGNED TO Q1,WL11
(D) RESTRICTIONS — NONE.
(E) OTHER DECKS REQUIRED — PROPS2,PROPS

(4) EXAMPLE
(A) INPUTS — A TRANSITION IS DESCRIBED BY 4 POINTS AT BOTH UPSTREAM AND DOWNSTREAM SECTIONS.A HORIZONTAL INTAKE IS LOCATED ON A VERTICAL BOUNDARY AT THE DOWNSTREAM SECTION (AS IN THE CASE OF THE UPSTREAM FACE OF A DAM). THE INTAKE IS LOCATED AT ELEVATION 20.0. THE FREE SURFACE ELEVATION AT THE UPSTREAM SECTION IS 40.0 AND THE INTERFACE ELEVATION AT THAT SECTION IS 13.0 WHICH IS ALSO THE INITIAL INTERFACE ELEVATION AT THE DOWNSTREAM SECTION. IT IS DESIRED TO DETERMINE THE FLOW OF THE UPPER LAYER NECESSARY TO RAISE THE INTERFACE LOCALLY TO THE LEVEL OF THE INTAKE, AT WHICH TIME DISCHARGE FROM THE LOWER LAYER BEGINS.

SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.05 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER AND LOWER LAYERS ARE 1.1 AND 1.2 RESPECTIVELY.

```
UPSTREAM COORDINATES
0,50       0,0        300,0      300,50
DOWNSTREAM COORDINATES
0,50       0,0        300,0      300,50
```

(B) CODE

```
      DIMENSION B(4),H(4),BD(4),HD(4)
      READ(5,10) (B(I),H(I),I=1,4)
      READ(5,10) (BD(I),HD(I),I=1,4)
   10 FORMAT(8F10.2)
      CALL SWLMID(40.0,13.0,0.0,20.0,1.1,1.2,32.2,1.0,1.05,B,H,4,BD,HD,
     *4,Q1,WL11)
      WRITE(6,20) Q1,WL11
   20 FORMAT(2F10.2)
      STOP
      END
```

(C) OUTPUT

```
38909.77     39.65
```

SWLBOT

LOWER LAYER DISCHARGE AND FREE SURFACE LEVEL IN A TWO-LAYER SYSTEM WITH A STAGNANT UPPER LAYER THROUGH A STREAMLINED TRANSITION.

(1) PURPOSE

THE TRANSITION IS DEFINED BY TWO CROSS-SECTIONS OF ARBITRARY GEOMETRY. THIS ROUTINE CALCULATES THE DISCHARGE OF THE LOWER LAYER AND THE FREE SURFACE ELEVATION AT ONE SECTION OF THE TRANSITION AS FUNCTIONS OF THE FREE SURFACE ELEVATION AT THE OTHER SECTION AND THE INTERFACE ELEVATIONS AT BOTH SECTIONS IN THE CASE OF A STAGNANT UPPER LAYER. TRANQUIL FLOW IS ASSUMED THROUGHOUT WITHOUT LOSSES.

THIS PROBLEM HAS AN APPLICATION IN SELECTIVE WITHDRAWAL OF A FLUID (SEE EXAMPLE).

(2) METHOD

EXPLICIT USE OF ENERGY EQUATIONS(EQUATIONS 3.2 AND 3.4-CHAPTER 3)

(3) PROGRAM

(A) DECK NAME

SWLBOT

(B) CALLING SEQUENCE

```
CALL SWLBOT(WL1,WL2,WL21,ALPHA2,G,RHO1,RHO2,HORZ,VERT,NPTS,HORZD,
*VERTD,NPTSD,Q2,WL11)
```

WHERE

WL1,WL2 = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVELY AT THE GIVEN SECTION.

WL21 = INTERFACE ELEVATION AT THE OTHER SECTION.

ALPHA2 = KINETIC ENERGY CORRECTION FACTOR FOR THE LOWER LAYER.

G = GRAVITATIONAL ACCELERATION.

RHO1,RHO2 = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER AND LOWER FLUIDS RESPECTIVELY.

HORZ,VERT = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY OF THE POINTS DEFINING THE GIVEN CROSS-SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO SOME ARBITRARY AXIS AND THE VERTICAL COORDS. ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.

NPTS = NUMBER OF POINTS DESCRIBING THE GIVEN CROSS-SECTION.

HORZD,VERTD = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING THE HORIZONTAL AND VERTICAL COORDS. OF THE POINTS DEFINING THE OTHER CROSS-SECTION. THESE ARE REFERRED TO THE SAME AXES AS HORZ,VERT.

NPTSD = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.

Q2 = DISCHARGE OF THE LOWER LAYER.

WL11 = FREE SURFACE ELEVATION AT THE OTHER SECTION.

(C) OUTPUT FORM

THE CALCULATED VALUES ARE ASSIGNED TO Q2,WL11

(D) RESTRICTIONS

NONE.

(E) OTHER DECKS REQUIRED

PROPS

(4) EXAMPLE

(A) INPUTS

A TRANSITION IS DESCRIBED BY 4 POINTS AT BOTH UPSTREAM AND DOWNSTREAM SECTIONS. A HORIZONTAL INTAKE IS LOCATED AT THE CHANNEL BOTTOM AT THE DOWNSTREAM SECTION. THE SLOT HEIGHT IS 0.5. THE FREE SURFACE ELEVATION AT THE UPSTREAM SECTION IS 10.0 AND THE INTERFACE ELEVATION AT THAT SECTION IS 2.0 WHICH IS ALSO THE INITIAL INTERFACE ELEVATION AT THE DOWNSTREAM SECTION. IT IS DESIRED TO DETERMINE THE FLOW OF THE LOWER LAYER NECESSARY TO LOWER THE INTERFACE LOCALLY TO THE LEVEL OF THE INTAKE, AT WHICH TIME DISCHARGE FROM THE UPPER LAYER BEGINS.

SPECIFIC GRAVITIES OF THE UPPER AND THE LOWER FLUIDS ARE 1.00 AND 1.05 RESPECTIVELY. THE KINETIC ENERGY CORRECTION FACTOR FOR THE LOWER LAYER IS 1.2.

UPSTREAM COORDINATES

0,15	0,0	50,0	50,15

DOWNSTREAM COORDINATES

0,15	0,0	50,0	50,15

(B) CODE

```
      DIMENSION B(4),H(4),BD(4),HD(4)
      READ(5,10) (B(I),H(I),I=1,4)
      READ(5,10) (BD(I),HD(I),I=1,4)
   10 FORMAT(8F10.2)
      CALL SWLBOT(10.0,2.0,0.5,1.2,32.2,1.0,1.05,B,H,4,BD,HD,4,Q2,WL11)
      WRITE(6,20) Q2,WL11
   20 FORMAT(2F10.2)
      STOP
      END
```

(C) OUTPUT

```
     50.55     10.00
```

****
* LISTINGS *
****

```
C     ****************************************************************
      SUBROUTINE PROPS(HORZ,VERT,NPTS,WL,AREA,TOPW,PERIM,AY)
C     ****************************************************************
C
C     CALCULATES THE CROSS-SECTIONAL PROPERTIES IN AN OPEN CHANNEL OF
C     ARBITRARY SHAPE.
C
C     HORZ            = ARRAY OF SIZE NPTS HOLDING HORIZONTAL COORDINATES
C                       OF POINTS DEFINING SECTION.
C     VERT            = ARRAY OF VERTICAL COORDS.
C     NPTS            = NO. OF POINTS DEFINING SECTION.
C     WL              = SPECIFIED WATER SURFACE ELEVATION REFERRED TO
C                       SAME DATUM AS VERT.
C     AREA            = COMPUTED CROSS-SECTION AREA.
C     TOPW            = COMPUTED CROSS-SECTION TOP WIDTH.
C     PERIM           = COMPUTED WETTED PERIMETER.
C     AY              = COMPUTED PRODUCT OF AREA AND CENTROIDAL DEPTH.
C     HANDLES OVERHANGING BANKS. IF WL IS HIGHER THAN BANKS, SECTION IS
C     ASSUMED EXTENDED BY VERTICALS THROUGH END POINTS.
C     ****************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS)
      AREA=TOPW=PERIM=AY=DA=DT=DP=DAY=0.0
      N=NPTS-1
      DO 10 I=1,N
      IF(WL.GT.VERT(I).AND.WL.GT.VERT(I+1)) GO TO 20
      IF(WL.GT.VERT(I).OR.WL.GT.VERT(I+1)) GO TO 40
      DA=DT=DP=DAY=0.0
      GO TO 60
C
C     PROPERTIES OF THE TRAPEZOIDAL SEGMENTS
C
   20 DT=HORZ(I+1)-HORZ(I)
      X1=WL-VERT(I)
      X2=VERT(I)-VERT(I+1)
      DA=DT*(X1+X2/2.0)
      DP=SQRT(DT*DT+X2*X2)
      IF(I.EQ.1) DP=DP+(WL-VERT(1))
      IF((I+1).EQ.NPTS) DP=DP+(WL-VERT(NPTS))
      DAY=DT*X1*X1/2.0+0.5*DT*X2*(X1+X2/3.0)
      GO TO 60
C
C     PROPERTIES OF THE TRIANGULAR SEGMENTS
C
   40 S=(HORZ(I+1)-HORZ(I))/(VERT(I+1)-VERT(I))
      X1=VERT(I+1)-WL
      IF(X1.GE.0.0) X1=WL-VERT(I)
      DT=S*X1
      X2=ABS(X1)
      DA=DT*X2/2.0
      DP=X2*SQRT(1.0+S*S)
      DAY=DA*X2/3.0
C
C     SUMMATION OF PROPERTIES
C
   60 AREA=AREA+DA
      TOPW=TOPW+DT
      PERIM=PERIM+DP
      AY=AY+DAY
   10 CONTINUE
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE BOTTOM(VERT,NPTS,BOT,WLMAX)
C     ******************************************************************
C
C     FINDS THE LOWEST POINT IN AN ARRAY OF VERTICAL COORDINATES. AND
C     THE LOWEST OF THE TWO END POINTS (E.G. BANK LEVELS).
C
C     VERT              = ARRAY OF VERTICAL COORDINATES OF POINTS
C                         DESCRIBING A CROSS-SECTION.
C     NPTS              = NUMBER OF POINTS DESCRIBING SECTION.
C     BOT               = COMPUTED LOWEST POINT IN SECTION.
C     WLMAX             = COMPUTED VALUE OF LOWER OF TWO BANK LEVELS
C                         VERT(1) AND VERT(NPTS).
C     ******************************************************************
C
      DIMENSION VERT(NPTS)
      IF(VERT(1).LE.VERT(NPTS)) GO TO 20
      WLMAX=VERT(NPTS)
      GO TO 30
   20 WLMAX=VERT(1)
   30 BOT=VERT(1)
      DO 40 J=1,NPTS
      IF(VERT(J).LT.BOT) BOT=VERT(J)
   40 CONTINUE
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE SELSEC(HORZAR,VERTAR,NSEC,MAXPTS,NN,HORZ,VERT)
C     ******************************************************************
C
C     SELRCTS FROM 2-DIM. ARRAYS OF COORDINATES THE COORDS. OF POINTS
C     DESCRIBING A SPECIFIC CROSS-SECTION AND STORES THESE IN 1-DIM.
C     ARRAYS FOR USE IN SUBSEQUENT OPERATIONS.
C
C     HORZAR          = 2-DIM. ARRAY OF SIZE (NSEC,MAXPTS) USED TO STORE
C                       HORIZONTAL COORDS. OF POINTS DEFINING A SERIES OF
C                       CROSS-SECTIONS.
C     VERTAR          = 2-DIM. ARRAY OF VERTICAL COORDS.
C     NSEC            = NO. OF CROSS-SECTIONS DEFINING CHANNEL.
C     MAXPTS          = MAXIMUM NUMBER OF POINTS USED TO DEFINE A SECTION
C     NN              = SPECIFIED CROSS-SECTION NUMBER.
C     HORZ            = ARRAY OF SIZE MAXPTS HOLDING HORIZONTAL COORDS.
C                       OF REQUIRED SECTION.
C     VERT            = ARRAY OF REQUIRED VERTICAL COORDS.
C     NOTE THAT LAST ELEMENTS OF ARRAYS HORZ AND VERT MAY NOT HOLD
C     MEANINGFUL DATA IF SECTION USES LESS THAN MAXPTS POINTS.
C     ******************************************************************
C
      DIMENSION HORZAR(NSEC,MAXPTS),VERTAR(NSEC,MAXPTS),HORZ(MAXPTS),
     1VERT(MAXPTS)
      DO 10 I=1,MAXPTS
      HORZ(I)=HORZAR(NN,I)
   10 VERT(I)=VERTAR(NN,I)
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE ZSYSTM1(F,EPS,NSIG,N,X,ITMAX,F1,F2,R,B,HORZD,VERTD,
     1NPTSD,BETAU,BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,
     1BOTU,Q1,Q2,RHO1,RHO2,AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,
     1AREA22,AY12,AY22,TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22,H22,WA,PAR
     1,IER)
C     ******************************************************************
C
C     DETERMINES A ROOT OF A SYSTEM OF N SIMULTANEOUS NONLINEAR EQUAT-
C     IONS IN N UNKNOWNS, F(X)=0, IN VECTOR FORM. N CAN BE 1
C
C     F               = THE NAME OF THE FUNCTION CALLED BY ZSYSTM1 TO
C                       FURNISH THE VALUES OF THE FUNCTIONS WHICH DEFINE
C                       THE SYSTEM OF EQUATIONS BEING SOLVED. THE USER
C                       SPECIFIES F BY WRITING A FUNCTION SUBPROGRAM
C                       F(X,K,PAR,....) WHICH COMPUTES THE K-TH COMPONENT
C                       OF F EVALUATED AT X.
C     EPS             = FIRST STOPPING CRITERION. A ROOT X(1),...,X(N) IS
C                       ACCEPTED IF THE MAXIMUM ABSOLUTE VALUE OF F(X,K,
C                       PAR,...) IS LESS THAN OR EQUAL TO EPS, WHERE K=1,
C                       ...,N.
C     NSIG            = SECOND STOPPING CRITERION. A ROOT IS ACCEPTED IF
C                       TWO SUCCESSIVE APPROXIMATIONS TO A GIVEN ROOT
C                       AGREE IN THE FIRST NSIG DIGITS.
C                       NOTE- IF EITHER, OR BOTH, OF THE STOPPING CRITER-
C                       IA ARE FULFILLED, THE ROOT IS ACCEPTED.
C     N               = THE NUMBER OF EQUATIONS (= NO. OF UNKNOWNS).
C     X               = THE VECTOR X OF LENGTH N, AS INPUT, IS THE
C                       INITIAL GUESS TO THE ROOT. AS OUTPUT, IT IS THE
C                       COMPUTED SOLUTION.
C     ITMAX           = ON INPUT = THE MAXIMUM ALLOWABLE NUMBER OF ITERA-
C                       TIONS AND ON OUTPUT = THE NUMBER OF ITERATIONS
C                       USED IN FINDING THE COMPUTED SOLUTION.
C     WA              = A WORKING ARRAY OF SIZE ((N+2)*(N-1))/2+(3*N)
C                       SUPPLIED BY THE USER.
C     IER             = ERROR PARAMETER
C                       TERMINAL ERROR = 128+NN
C                       NN =1 INDICATES FAILURE TO CONVERGE WITHIN ITMAX
C                             ITERATIONS.
C                       NN =2 SINGULAR SYSTEM (JACOBIAN)
C     ALL THE OTHER PARAMETERS ARE PASSED TO THE USER SUPPLIED FUNCTION
C     F AND MAY BE USED TO PASS ANY AUXILIARY PARAMETERS NECESSARY FOR
C     COMPUTATION OF THE FUNCTION F.
C     ROUTINES USED- NONE.
C     ZSYSTM1 IS A MODIFIED VERSION OF ZSYSTM (I.M.S.L. LIBRARY,MCMASTER
C     UNIVERSITY, HAMILTON, ONTARIO, CANADA).
C     ******************************************************************
C
C   LATEST REVISION     - SEPTEMBER 28, 1973
C
      DIMENSION         X(1),WA(1),PAR(1)
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTSD),VERTD(NPTSD)
      DATA              PREC,DELTA/5.E-12,5.E-9/
      DATA              ZERO,PM1,PT1,P2/0.0,.1,.0001,.002/
      IER=0
C                                  PREC IS A FUNCTION OF THE MACHINE
C                                  SIGNIFICANCE, SIG, AND SHOULD BE
C                                  COMPUTED AS PREC=5.*10.**(-SIG+2).
C                                  IN THIS INSTANCE WE WERE DEALING
C                                  WITH A 14 DIGIT MACHINE.
C                                  DELTA SHOULD BE TAKEN AS
C                                  5.*10.**(-(SIG+4)/2), FOR SIG EVEN,
C                                  AND 16.*10.**(-(SIG+5)/2, FOR SIG ODD
      N2 = N+N
      RELCON=10.0**(-NSIG)
      JTEST = 1
      IERROR=0
      IPART=((N+2)*(N-1))/2
      ITMP=IPART+N
      LKSUB=ITMP+N
      DO 130 M = 1, ITMAX
         IQUIT=0
         FMAX=ZERO
```

```
         M1 = M-1
         K1 = LKSUB + 1
         KMIN = LKSUB + N
         XTEMP = ZERO
C                                  THE ARRAY WA(LKSUB+1),...,WA(LKSUB+N)
C                                  PERMITS A PARTIAL PIVOTING EFFECT
C                                  WITHOUT HAVING TO PHYSICALLY
C                                  INTERCHANGE ROWS OR COLUMNS.
         DO 5 J = K1,KMIN
            XTEMP = XTEMP+1.0
            WA(J) = XTEMP
    5    CONTINUE
         K = 1
   10    IF(K .LE. 1) GO TO 30
         KMIN = K-1
C                                  THE FOLLOWING CODE BACK-SOLVES THE
C                                  FIRST KMIN ROWS OF A TRIANGULARIZED
C                                  LINEAR SYSTEM FOR IMPROVED X VALUES
C                                  IN TERMS OF PREVIOUS ONES.
         KK = 1
   15    DO 25 K1=1,KMIN
            ISUB=K-K1
            MM=((ISUB-1)*(N2 -ISUB))/2
            LIM=N-ISUB
            KPOINT = WA(LKSUB+ISUB)+PM1
C                                  THE ADDITION OF .1 (PM1) IN THE LAST
C                                  STATEMENT (AND OTHERS LIKE IT
C                                  BELOW) IS ESSENTIAL, SINCE WA
C                                  CONTAINS INTEGERS AS WELL AS FLOATING
C                                  POINT NUMBERS.  FOR EXAMPLE, SUPPOSE
C                                  THE INTEGER 3 WAS STORED AS
C                                  2.99999999999998
            ISUB1 = ISUB-1
            X(KPOINT)=ZERO
            DO 20 L1=1,LIM
               JS1=ISUB1+L1
               LKJSUB=LKSUB + JS1 +1
               IJ=MM+JS1
               JPOINT= WA(LKJSUB) + PM1
               X(KPOINT)=X(KPOINT) + WA(IJ)*X(JPOINT)
   20       CONTINUE
            X(KPOINT)=X(KPOINT) + WA(MM+N)
   25    CONTINUE
         GO TO (30,45,105), KK
C
C                                  SET UP PARTIAL DERIVATIVES OF
C                                  KTH FUNCTION..
C
C
   30           E=F(X,K,PAR,F1,F2,R,B,HORZD,VERTD,NPTSD,BETAU,BETAL,BOTD
     1,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,RHO2,
     1AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,AY12,AY22,
     1TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22)
         FMAX =AMAX1(FMAX,ABS(E))
         IF( ABS(E) .GE. EPS) GO TO 35
         IQUIT=IQUIT+1
         IF(IQUIT .EQ. N) GO TO 140
   35    I = K
   40    IP=IPART+I
         ITEMP = WA(LKSUB+I) + PM1
         HOLD = X(ITEMP)
         ETA=.001*ABS(HOLD)
         IF (ABS(HOLD) .LT.PREC)  ETA=DELTA
         H =AMIN1(FMAX,ETA)
         IF(H .LT. PREC) H=PREC
         X (ITEMP)=HOLD+H
         IF (K .LE. 1) GO TO 45
         KK = 2
         GO TO 15
C
   45       FPLUS=F(X,K,PAR,F1,F2,R,B,HORZD,VERTD,NPTSD,BETAU,BETAL,BOTD
     1,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,RHO2,
     1AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,AY12,AY22,
     1TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22)
```

```
      TOP=FPLUS-E
   50 WA(IP)=TOP/H
   55 X(ITEMP)=HOLD
      I = I + 1
      IF (I .LE. N) GO TO 40
      IF (K .LT. N) GO TO 60
      IP=IPART+N
      IF (ABS( WA(IP)) .EQ. ZERO)   GO TO 80
      X(ITEMP) = -E/ WA(IP) + X(ITEMP)
      GO TO 100
C
C                                 FIND PARTIAL DERIVATIVE OF LARGEST
C                                 ABSOLUTE VALUE..
C
   60 KL=LKSUB+K
      LOOK= WA(KL) + PM1
      KMAX=LOOK
      IP=IPART+K
      DERMAX= ABS( WA(IP))
      KPLUS = K+1
      DO 65 I = KPLUS,N
         TEST= ABS( WA(IPART+I))
         IF(TEST .LE. DERMAX) GO TO 65
         DERMAX = TEST
         KMAX=I
   65 CONTINUE
      IF(LOOK .EQ. KMAX) GO TO 75
      LKMAX=LKSUB+KMAX
      WA(KL)=WA(LKMAX)
      WA(LKMAX)=LOOK
      IP=IPART+KMAX
      XTEMP= WA(IP)
      IPK=IPART+K
      WA(IP)=WA(IPK)
      WA(IPK)=XTEMP
      IF(K .LT. 2) GO TO 75
      KMIN=K-1
      I1 = 0
      DO 70 I=1,KMIN
         L=((I1 )*(N2 -I))/2-1
         J=L+KMAX
         XTEMP= WA(J)
         JJ=L+K
         WA(J)=WA(JJ)
         WA(JJ)=XTEMP
         I1 = I
   70 CONTINUE
   75 IF ( ABS(WA(IPART+K)).NE. ZERO) GO TO 90
   80 IF(IERROR .EQ. 1) GO TO 135
      DO 85  I=1,N
C                                 IF THE MODIFIED JACOBIAN IS SINGULAR
C                                 AT X, CHANGE THE COMPONENTS OF X AND
C                                 PROCEED WITH THE ITERATIONS.  IF IT
C                                 HAPPENS A SECOND TIME, TERMINATE.
         X(I) = 0.9*X(I) + .12345
   85 CONTINUE
      IERROR=1
      GO TO 105
C
C                                 SET UP COEFFICIENTS FOR KTH ROW
C                                 OF TRIANGULAR LINEAR SYSTEM USED
C                                 TO BACK-SOLVE FOR THE FIRST K X(I)
C                                 VALUES...
C
   90 L=((K-1)*(N2 -K))/2
      KN=L+N
      I1 = L-1
      WA(KN)=ZERO
      IPK=IPART+K
      DO 95  J = KPLUS,N
         JSUB= WA(LKSUB+J) + PM1
         JJ=I1+J
         IPJ=IPART+J
         WA(JJ)=-WA(IPJ)/WA(IPK)
```

```
            WA(KN)=WA(KN)+WA(IPJ)*X(JSUB)                               ***2220
  95     CONTINUE                                                       ***2230
         LK= WA(LKSUB+K) + PM1                                          ***2240
         WA(KN)=(WA(KN)-E)/WA(IPK) + X(LK)                              ***2250
         K = K+1                                                        ***2260
         IF (K .LE. N) GO TO 10                                         ***2270
C                                 BACK SUBSTITUTE TO OBTAIN NEXT        ***2280
C                                 APPROXIMATION TO X                    ***2290
  100    IF( N .EQ. 1) GO TO 105                                        ***2300
         KMIN=N-1                                                       ***2310
         KK = 3                                                         ***2320
         GO TO 15                                                       ***2330
  105    IF (M .LE. 1) GO TO 120                                        ***2340
C                                                                       ***2350
C                                 TEST FOR CONVERGENCE..                ***2360
C                                                                       ***2370
         DO 110 I = 1,N                                                 ***2380
            IF (ABS(WA(ITMP+I)-X(I)) .GT. ABS(X(I))*RELCON) GO TO 115   ***2390
  110    CONTINUE                                                       ***2400
         JTEST = JTEST+1                                                ***2410
         IF (JTEST-3)120,140,140                                        ***2420
  115    JTEST = 1                                                      ***2430
  120    DO 125 I = 1,N                                                 ***2440
            WA(ITMP+I)=X(I)                                             ***2450
  125    CONTINUE                                                       ***2460
  130 CONTINUE                                                          ***2470
      M=ITMAX                                                           ***2480
      IER = 129                                                         ***2490
      GO TO 240                                                         ***2500
  135 IER = 130                                                         ***2510
  140 FMAX=ZERO                                                         ***2520
      TEST=1.0E+15                                                      ***2530
      IF (N .GT. 1) GO TO 145                                           ***2540
C                                                                       ***2550
      WA(IPART+2)=F(X,1,PAR,F1,F2,R,B,HORZD,VERTD,NPTSD,BETAU,BETAL,BOTD ***2560
     1,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,RHO2,    ***2570
     1AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,AY12,AY22,     ***2580
     1TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22)                           ***2590
      FMAX =AMAX1(FMAX,ABS(WA(IPART+2)))                                ***2600
      GO TO 155                                                         ***2610
  145 DO 150 I = 1,N                                                    ***2620
         IP=IPART+I                                                     ***2630
C                                                                       ***2640
           WA(IP)=F(X,I,PAR,F1,F2,R,B,HORZD,VERTD,NPTSD,BETAU,BETAL,BOTD ***2650
     1,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,RHO2,    ***2660
     1AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,AY12,AY22,     ***2670
     1TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22)                           ***2680
         FMAX=AMAX1(FMAX, ABS(WA(IP)))                                  ***2690
  150 CONTINUE                                                          ***2700
C                                 CHECK TO SEE IF SMALL COMPONENTS ARE  ***2710
C                                 ACTUALLY ZERO                         ***2720
  155 K=1                                                               ***2730
      DO 160 I=1,N                                                      ***2740
         WA(I)=X(I)                                                     ***2750
         IF (ABS(X(I)) .GT. P2) GO TO 160                               ***2760
         K=2                                                            ***2770
         WA(I)=ZERO                                                     ***2780
  160 CONTINUE                                                          ***2790
      IF(K .EQ. 1) GO TO 195                                            ***2800
      KK = 1                                                            ***2810
      GO TO 205                                                         ***2820
  165 IF(FMAX .LT. TEST) GO TO 190                                      ***2830
C                                 NOTE THAT SMALL COMPONENTS ARE SET    ***2840
C                                 TO ZERO ONLY IF THE NORM OF THE       ***2850
C                                 FUNCTION VECTOR IS REDUCED AS A       ***2860
C                                 RESULT OF THIS PROCESS.               ***2870
      DO 170 I=1,N                                                      ***2880
         X(I)= WA(I)                                                    ***2890
  170 CONTINUE                                                          ***2900
      IF (N .GT. 1) GO TO 175                                           ***2910
      WA(IPART+2) = WA(ITMP+2)                                          ***2920
      GO TO 185                                                         ***2930
  175 DO 180 I = 1,N                                                    ***2940
         WA(IPART+I) = WA(ITMP+I)                                       ***2950
```

```
  180 CONTINUE                                                         ***2960
  183 FMAX=TEST                                                        ***2970
C                                CHECK FOR INTEGER COMPONENTS          ***2980
  190 K=1                                                              ***2990
  195 ITEST=0                                                          ***3000
      DO 200 I=1,N                                                     ***3010
         WA(I)=X(I)                                                    ***3020
         IF (ABS(X(I)) .LE. P2) GO TO 200                              ***3030
         L=X(I)+PT1                                                    ***3040
         J=X(I)-PT1                                                    ***3050
         IF(L .EQ. J) GO TO 200                                        ***3060
         WA(I) =ISIGN(1,J)*MAX0(IABS(L),IABS(J))                       ***3070
         K=2                                                           ***3080
  200 CONTINUE                                                         ***3090
      IF(K .EQ. 1) GO TO 235                                           ***3100
      KK = 2                                                           ***3110
  205 TEST=ZERO                                                        ***3120
      IF (N .GT. 1) GO TO 210                                          ***3130
C                                                                      ***3140
      WA(ITMP+2)=F(WA,1,PAR,F1,F2,R,B,HORZD,VERTD,NPTSD,BETAU,BETAL,BOTD ***3150
     1,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,RHO2,     ***3160
     1AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,AY12,AY22,     ***3170
     1TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22)                           ***3180
      TEST =AMAX1(TEST,ABS(WA(ITMP+2)))                                ***3190
      GO TO 220                                                        ***3200
  210 DO 215 I=1,N                                                     ***3210
         IT=ITMP+I                                                     ***3220
C                                                                      ***3230
          WA(IT)=F(WA,I,PAR,F1,F2,R,B,HORZD,VERTD,NPTSD,BETAU,BETAL,BOTD ***3240
     1,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,RHO2,     ***3250
     1AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,AY12,AY22,     ***3260
     1TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22)                           ***3270
         TEST=AMAX1(TEST, ABS(WA(IT)))                                 ***3280
  215 CONTINUE                                                         ***3290
  220 GO TO (165,225), KK                                              ***3300
  225 IF(FMAX .LT. TEST) GO TO 235                                     ***3310
C                                NOTE THAT NEAR-INTEGER COMPONENTS     ***3320
C                                ARE SET TO BE INTEGERS ONLY IF THE    ***3330
C                                NORM OF THE FUNCTION VECTOR IS        ***3340
C                                REDUCED AS A RESULT OF THIS PROCESS.  ***3350
      DO 230 I=1,N                                                     ***3360
         X(I)= WA(I)                                                   ***3370
  230 CONTINUE                                                         ***3380
      ITEST=1                                                          ***3390
C                                TEST FOR CONVERGENCE                  ***3400
  235 IF(FMAX .LT. EPS .OR. TEST .LT. EPS)  IER = 0                    ***3410
  240 ITMAX=M1 + 1                                                     ***3420
 9000 CONTINUE                                                         ***3430
C                                                                      ***3440
 9005 RETURN                                                           ***3450
      END                                                              ***3460
```

```
C     ******************************************************************
      SUBROUTINE PROPS2(HORZ,VERT,NPTS,WL1,WL2,AREA1,AREA2,TOPW1,TOPW2,
     1PERIM1,PERIM2,AY1,AY2)
C     ******************************************************************
C

C     CALCULATES THE CROSS-SECTIONAL PROPERTIES IN AN OPEN CHANNEL OF
C     ARBITRARY SHAPE CONTAINING TWO FLUIDS.
C
C     HORZ            = ARRAY OF SIZE NPTS HOLDING HORIZONTAL COORDINATES
C                       OF POINTS DEFINING SECTION.
C     VERT            = ARRAY OF VERTICAL COORDS.
C     NPTS            = NO. OF POINTS DEFINING SECTION.
C     WL1             = SPECIFIED FREE SURFACE ELEVATION REFERRED TO SAME
C                       DATUM AS VERT.
C     WL2             = SPECIFIED INTERFACE ELEVATION REFERRED TO SAME
C                       DATUM AS VERT.
C     AREA1,AREA2     = COMPUTED CROSS-SECTIONAL AREAS OF THE UPPER AND
C                       THE LOWER LAYERS RESPECTIVELY.
C     TOPW1,TOPW2     = COMPUTED CROSS-SECTION TOP WIDTHS FOR THE UPPER
C                       AND THE LOWER LAYERS RESPECTIVELY.
C     PERIM1,PERIM2   = COMPUTED WETTED PERIMETERS FOR THE UPPER AND THE
C                       LOWER LAYERS RESPECTIVELY. WETTED PERIMETER OF
C                       THE UPPER LAYER INCLUDES INTERFACE.
C     AY1,AY2         = COMPUTED PRODUCTS OF AREA AND CENTROIDAL DEPTH
C                       FOR THE UPPER AND THE LOWER LAYERS RESPECTIVELY.
C                       CENTROIDAL DEPTH IS MEASURED FROM THE FREE
C                       SURFACE FOR THE UPPER LAYER AND FROM THE INTERF-
C                       ACE FOR THE LOWER LAYER.
C     HANDLES OVERHANGING BANKS. IF WATER LEVELS ARE HIGHER THAN BANKS,
C     SECTION IS ASSUMED EXTENDED BY VERTICALS THROUGH END POINTS.
C     ******************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS)
C
C     PROPERTIES OF THE TOTAL SECTION
C
      CALL PROPS(HORZ,VERT,NPTS,WL1,AREA,TOPW,PERIM,AY)
C
C     PROPERTIES OF THE LOWER LAYER
C
      CALL PROPS(HORZ,VERT,NPTS,WL2,AREA2,TOPW2,PERIM2,AY2)
C
C     PROPERTIES OF THE UPPER LAYER
C
      AREA1=AREA-AREA2
      TOPW1=TOPW
      PERIM1=PERIM-PERIM2+TOPW2
      AY1=AY-AY2-AREA2*(WL1-WL2)
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE BERNQ2(WL1,WL2,WL11,WL21,RHO1,RHO2,G,ALPHAU,ALPHAL,
     1HORZ,VERT,NPTS,HORZD,VERTD,NPTSD,Q1,Q2)
C     ******************************************************************
C
C     FINDS DISCHARGES OF BOTH LAYERS IN A TRANSITION AS A FUNCTION OF
C     THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT TWO CROSS-
C     SECTIONS. THE TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-
C     SECTIONS WHICH ARE DESCRIBED BY A SERIES OF POINTS THE COORDINATES
C     OF WHICH ARE GIVEN. TRANQUIL FLOW IS ASSUMED THROUGHOUT WITH NO
C     LOSSES.
C
C     WL1,WL11        = FREE SURFACE ELEVATIONS AT THE FIRST AND THE
C                       SECOND CROSS-SECTIONS RESPECTIVELY.
C     WL2,WL21        = INTERFACE ELEVATIONS AT THE FIRST AND THE SECOND
C                       CROSS-SECTIONS RESPECTIVELY.
C     RHO1,RHO2       = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND THE LOWER FLUIDS RESPECTIVELY.
C     G               = GRAVITATIONAL ACCELERATION.
C     ALPHA1,ALPHA2   = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                       AND THE LOWER LAYERS RESPECTIVELY.
C     HORZ,VERT       = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                       HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DEFINING THE FIRST CROSS-SECTION.
C     HORZD,VERTD     = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING
C                       HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DEFINING THE SECOND CROSS-SECTION.
C     NPTS,NPTSD      = NUMBER OF POINTS DEFINING THE FIRST AND THE SEC-
C                       OND CROSS-SECTIONS RESPECTIVELY.
C     Q1,Q2           = DISCHARGES IN THE UPPER AND THE LOWER LAYERS
C                       RESPECTIVELY.
C     SAME DATUM MUST BE USED FOR VERT,VERTD,WL1,WL2,WL11,WL21
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2
C     ******************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTSD),VERTD(NPTSD)
C
C     EVALUATE CROSS-SECTIONAL AREAS
C
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A1,A2,T1,T2,P1,P2,AY1,AY2)
      CALL PROPS2(HORZD,VERTD,NPTSD,WL11,WL21,A11,A21,T11,T21,P11,P21,
     1AY11,AY21)
C
C     DISCHARGE OF THE UPPER LAYER
C
      IF(A1.EQ.A11) GO TO 10
      X1=1.0/(A1*A1)-1.0/(A11*A11)
      X2=ALPHAU*X1/(2.0*G)
      X3=WL11-WL1
      X4=X3/X2
      IF(X4.LT.0.0) GO TO 10
      Q1=SQRT(X4)
      GO TO 20
   10 Q1=-1.0
C
C     DISCHARGE OF THE LOWER LAYER
C
   20 DRHO=RHO2-RHO1
      G1=G*DRHO/RHO2
      X4=WL21-WL2
      IF(A2.EQ.A21) GO TO 40
      X1=1.0/(A2*A2)-1.0/(A21*A21)
      X2=ALPHAL*X1/(2.0*G1)
      X3=X3*RHO1/DRHO
      X5=(X3+X4)/X2
      IF(X5.LT.0.0) GO TO 40
      Q2=SQRT(X5)
      RETURN
   40 Q2=-1.0
      RETURN
      END
```

```
C     ****************************************************************
      SUBROUTINE BERNWL2(Q1,Q2,WL1,WL2,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZ,
     1VERT,NPTS,HORZD,VERTD,NPTSD,WL11,WL21)
C     ****************************************************************
C
C     FINDS THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT ONE CROSS-
C     SECTION IN A TRANSITION AS A FUNCTION OF THE DISCHARGES IN BOTH
C     LAYERS AND THE ELEVATIONS AT THE OTHER SECTION. THE TRANSITION IS
C     DEFINED BY TWO ARBITRARY CROSS-SECTIONS WHICH ARE DESCRIBED BY A
C     SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN. TRANQUIL FLOW
C     IS ASSUMED THROUGHOUT WITH NO LOSSES.
C
C     Q1,Q2            = DISCHARGES IN THE UPPER AND THE LOWER LAYERS
C                        RESPECTIVELY.
C     WL1,WL2          = GIVEN FREE SURFACE AND INTERFACE ELEVATIONS AT
C                        THE FIRST CROSS-SECTION.
C     RHO1,RHO2        = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                        AND THE LOWER FLUIDS RESPECTIVELY.
C     G                = GRAVITATIONAL ACCELERATION.
C     ALPHA1,ALPHA2    = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                        AND THE LOWER LAYERS RESPECTIVELY.
C     HORZ,VERT        = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE FIRST CROSS-SECTION.
C     HORZD,VERTD      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE SECOND CROSS-SECTION.
C     NPTS,NPTSD       = NUMBER OF POINTS DEFINING THE FIRST AND THE SEC-
C                        OND CROSS-SECTIONS RESPECTIVELY.
C     WL11,WL21        = COMPUTED FREE SURFACE AND INTERFACE ELEVATIONS AT
C                        THE SECOND CROSS-SECTION.
C     SAME DATUM MUST BE USED FOR VERT,VERTD,WL1,WL2,WL11,WL21
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2
C     ******************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTSD),VERTD(NPTSD)
      DRHO=RHO2-RHO1
      G1=G*DRHO/RHO2
      RATIO=RHO1/DRHO
      X1=ALPHAU*Q1*Q1/G
      X2=ALPHAL*Q2*Q2/G1
      DF2WL1=-RATIO
      DWL1=DWL2=0.0
C
C     EVALUATE CROSS-SECTIONAL PROPERTIES WHERE THE ELEVATIONS ARE GIVEN
C
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A1,A2,T1,T2,P1,P2,AY1,AY2)
      X3=WL1+X1/(2.0*A1*A1)
      X4=RATIO*WL1+WL2+X2/(2.0*A2*A2)
   30 WL11=WL1
      WL21=WL2
C
C     USE NEWTON-RAPHSON METHOD
C
   10 WL11=WL11+DWL1
      WL21=WL21+DWL2
      CALL PROPS2(HORZD,VERTD,NPTSD,WL11,WL21,A11,A21,T11,T21,P11,P21,
     1AY11,AY21)
C
C     RETURN WITH ZERO VALUES IF AREA IS VERY SMALL OR NEGATIVE
C
      IF(A21.LT.1.0E-12) GO TO 50
C
C     EVALUATE THE FUNCTIONS
C
      FUNC1=X3-WL11-X1/(2.0*A11*A11)
      FUNC2=X4-RATIO*WL11-WL21-X2/(2.0*A21*A21)
      A13=A11*A11*A11
C
C     EVALUATE THE DERIVATIVES
C
```

```
      DF1WL1=X1*T11/A13-1.0
      DF1WL2=-X1*T21/A13
      DF2WL2=X2*T21/(A21*A21*A21)-1.0
      A=-FUNC1*DF2WL2+FUNC2*DF1WL2
      C=-DF1WL1*FUNC2+DF2WL1*FUNC1
      B=DF1WL1*DF2WL2-DF1WL2*DF2WL1
      DWL1=A/B
      DWL2=C/B
C
C     CONVERGENCE CRITERIA
C
      IF(ABS(DWL1/WL11).GT.0.00001.OR.ABS(DWL2/WL21).GT.0.00001)GO TO 10
C
C     RETURN WITH ZERO VALUES IF AN IMPRACTICAL SOLUTION IS OBTAINED
C
      IF(WL21.LT.WL11) GO TO 20
   50 WL11=0.0
      WL21=0.0
   20 RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE QCRIT(WL1CR,WL2CR,QA,RHO1,RHO2,G,ALPHAU,ALPHAL,N1,
     1HORZC,VERTC,NPTSC,QB)
C     ******************************************************************
C
C     FINDS CRITICAL DISCHARGE IN ONE LAYER IN AN ARBITRARY CROSS-
C     SECTION AS A FUNCTION OF THE DISCHARGE OF THE OTHER LAYER AND THE
C     FREE SURFACE AND THE INTERFACE ELEVATIONS. THE CROSS-SECTION IS
C     DESCRIBED BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN
C
C     WL1CR,WL2CR     = CRITICAL FREE SURFACE AND INTERFACE ELEVATIONS
C                       RESPECTIVELY.
C     QA              = GIVEN DISCHARGE.
C     RHO1,RHO2       = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND LOWER LAYERS RESPECTIVELY.
C     G               = GRAVITATIONAL ACCELERATION.
C     ALPHAU,ALPHAL   = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                       AND THE LOWER LAYERS RESPECTIVELY.
C     N1              = 1  IF THE DISCHARGE OF THE UPPER LAYER IS GIVEN
C                          AND THAT OF THE LOWER LAYER IS REQUIRED.
C                     = 2  IF THE DISCHARGE OF THE LOWER LAYER IS GIVEN
C                          AND THAT OF THE UPPER LAYER IS REQUIRED.
C     HORZC,VERTC     = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING
C                       HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DEFINING THE CRITICAL SECTION.
C     NPTSC           = NUMBER OF POINTS DESCRIBING THE CROSS-SECTION.
C     QB              = COMPUTED DISCHARGE.
C     SAME DATUM MUST BE USED FOR VERTC,WL1CR,WL2CR
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2
C     ******************************************************************
C
      DIMENSION HORZC(NPTSC),VERTC(NPTSC)
      DRHO=RHO2-RHO1
      RATIO=DRHO/RHO2
      G1=G*RATIO
C
C     CROSS-SECTIONAL PROPERTIES
C
      CALL PROPS2(HORZC,VERTC,NPTSC,WL1CR,WL2CR,A1,A2,T1,T2,P1,P2,AY1,
     1AY2)
      IF(N1.EQ.1) GO TO 50
      F2=ALPHAL*QA*QA*T2/(G1*A2*A2*A2)
      GO TO 60
   50 F2=ALPHAU*QA*QA*T1/(G1*A1*A1*A1)
C
C     CHECK DENSIMETRIC FROUDE NUMBER
C
   60 IF(F2.LT.1.0) GO TO 10
      IF(F2.GT.(1.0/RATIO)) GO TO 10
      QB=0.0
      RETURN
   10 IF(N1.EQ.2) GO TO 70
C
C     CALCULATE DISCHARGE
C
      X1=(G1*A2*A2*A2/T2)*(F2-1.0)
      X2=ALPHAL*(RATIO*F2-1.0)
      QB=SQRT(X1/X2)
      GO TO 80
   70 X1=(G1*A1*A1*A1/T1)*(F2-1.0)
      X2=ALPHAU*(RATIO*F2-1.0)
      QB=SQRT(X1/X2)
   80 RETURN
      END
```

```
C     *****************************************************************
      SUBROUTINE WLCRIT(Q1,Q2,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZ,
     1VERT,NPTS,WL1CR)
C     *****************************************************************
C
C     FINDS THE CRITICAL FREE SURFACE ELEVATION IN AN ARBITRARY CROSS-
C     SECTION AS A FUNCTION OF THE INTERFACE ELEVATION AND THE DISCHAR-
C     GES IN BOTH LAYERS. THE CROSS-SECTION IS DESCRIBED BY A SERIES OF
C     POINTS THE COORDINATES OF WHICH ARE GIVEN.
C
C     Q1,Q2            = DISCHARGES OF THE UPPER AND THE LOWER LAYERS
C                        RESPECTIVELY.
C     WL2CR            = CRITICAL INTERFACE ELEVATION.
C     RHO1,RHO2        = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                        AND LOWER LAYERS RESPECTIVELY.
C     G                = GRAVITATIONAL ACCELERATION.
C     ALPHAU,ALPHAL    = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                        AND THE LOWER LAYERS RESPECTIVELY.
C     HORZ,VERT        = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS  CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE CRITICAL SECTION.
C     NPTS             = NUMBER OF POINTS DESCRIBING THE CROSS-SECTION.
C     WL1CR            = CRITICAL FREE SURFACE ELEVATION.
C     SAME DATUM MUST BE USED FOR WL2CR,VERT,WL1CR
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2,PROPS,BOTTOM
C     ******************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS)
      DRHO=RHO2-RHO1
      BETA=DRHO/RHO2
      G1=G*BETA
C
C     CROSS-SECTIONAL PROPERTIES OF THE LOWER LAYER
C
      CALL PROPS(HORZ,VERT,NPTS,WL2CR,A21,T21,P21,AY21)
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)
C
C     CHECK OF LOWER LAYER DENSIMETRIC FROUDE NUMBER
C
      X1=ALPHAL*Q2*Q2*T21/(G1*A21*A21*A21)
      IF(X1.LT.1.0) GO TO 70
      IF(X1.GT.(1.0/BETA)) GO TO 70
      WL1CR=WL2CR
      RETURN
C
C     THE SECANT METHOD
C
   70 C=Q1*SQRT(ALPHAU*(X1*BETA-1.0)/(X1-1.0))
      FUNC2=Y=0.0
      DY=WLMAX-WL2CR
      IF(DY.LT.0.001) DY=0.1
   10 Y=Y+DY
C
C     CONVERGENCE CRITERIA
C
      IF(ABS(DY/Y).LT.0.0001) GO TO 30
      FUNC1=FUNC2
      IF(Y.LE.0.0) GO TO 50
      CALL PROPS2(HORZ,VERT,NPTS,Y+WL2CR,WL2CR,A11,A21,T11,T21,P11,P21,
     1AY11,AY21)
      FUNC2=A11*SQRT(A11*G1/T11)
      GO TO 60
   50 FUNC2=0.0
      Y=Y*(-1.0)
      DY=-Y/2.0
      GO TO 10
   60 DY=DY*(C-FUNC2)/(FUNC2-FUNC1)
      GO TO 10
   30 WL1CR=Y+WL2CR
      RETURN
      END
```

```
C     ****************************************************************** ***
      SUBROUTINE WLCRIT1(Q1,Q2,WL1CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZ,VERT ***
     1,NPTS,WL2CR1,WL2CR2)                                               ***
C     ****************************************************************** ***
C                                                                        ***
C     FINDS THE CRITICAL INTERFACE ELEVATION(S) IN AN ARBITRARY CROSS-   ***
C     SECTION AS A FUNCTION OF THE FREE SURFACE ELEVATION AND THE DISCH- ***
C     ARGES IN BOTH LAYERS. THE CROSS-SECTION IS DESCRIBED BY A SERIES   ***
C     OF POINTS THE COORDINATES OF WHICH ARE GIVEN.                      ***
C                                                                        ***
C     Q1,Q2            = DISCHARGES OF THE UPPER AND THE LOWER LAYERS    ***
C                        RESPECTIVELY.                                   ***
C     WL1CR            = CRITICAL FREE SURFACE ELEVATION.                ***
C     RHO1,RHO2        = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER  ***
C                        AND LOWER LAYERS RESPECTIVELY.                  ***
C     G                = GRAVITATIONAL ACCELERATION.                     ***
C     ALPHAU,ALPHAL    = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER  ***
C                        AND THE LOWER LAYERS RESPECTIVELY.              ***
C     HORZ,VERT        = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS  CONTAINING ***
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY***
C                        OF THE POINTS DEFINING THE CRITICAL SECTION.    ***
C     NPTS             = NUMBER OF POINTS DESCRIBING THE CROSS-SECTION.  ***
C     WL2CR1,WL2CR2    = CRITICAL INTERFACE ELEVATIONS (TWO ROOTS).      ***
C     SAME DATUM MUST BE USED FOR WL1CR,VERT,WL2CR1,WL2CR2               ***
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF  ***
C     G USED.                                                            ***
C     ROUTINES USED- PROPS2,PROPS,BOTTOM                                 ***
C     ****************************************************************** ***
C                                                                        ***
      DIMENSION HORZ(NPTS),VERT(NPTS)                                    ***
      DRHO=RHO2-RHO1                                                     ***
      BETA=DRHO/RHO2                                                     ***
      G1=G*BETA                                                          ***
      JJ=1                                                               ***
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)                                   ***
   50 IF(JJ.EQ.1) GO TO 30                                               ***
      WL2CR1=WL2CR                                                       ***
      Y=(WL1CR-BOT)*0.99                                                 ***
      GO TO 40                                                           ***
   30 Y=(WL1CR-BOT)*0.01                                                 ***
   40 DY=0.0                                                             ***
      X1=ALPHAU*Q1*Q1/G1                                                 ***
      X2=ALPHAL*Q2*Q2/G1                                                 ***
   10 Y=Y+DY                                                             ***
      IF(Y.LT.0.0) GO TO 20                                              ***
      N=0                                                                ***
C                                                                        ***
C     APPLY NEWTON-RAPHSON METHOD                                        ***
C                                                                        ***
      CALL PROPS2(HORZ,VERT,NPTS,WL1CR,Y+BOT,A1,A2,T1,T2,P1,P2,AY1,AY2)  ***
      YPLUS=1.01*Y                                                       ***
      YMINUS=0.99*Y                                                      ***
      CALL PROPS(HORZ,VERT,NPTS,YPLUS+BOT,A21,T21,P21,AY21)              ***
      CALL PROPS(HORZ,VERT,NPTS,YMINUS+BOT,A22,T22,P22,AY22)             ***
      F1=X1*T1/A1**3                                                     ***
      F2=X2*T2/A2**3                                                     ***
C                                                                        ***
C     EVALUATE THE FUNCTION                                              ***
C                                                                        ***
      FUNC=BETA*F1*F2-F1-F2                                              ***
      IF(FUNC.LT.0.0) N=1                                                ***
      IF(N.EQ.1) FUNC=-FUNC                                              ***
      FUNCC=1.0/FUNC**(1.0/3.0)                                          ***
      FUNC1=FUNCC+1.0                                                    ***
      IF(N.EQ.1) FUNC1=FUNC1-2.0*FUNCC                                   ***
      Y1=Y*0.01                                                          ***
C                                                                        ***
C     EVALUATE THE DERIVATIVE                                            ***
C                                                                        ***
      DTDWL1=(T21-T2)/Y1                                                 ***
      DTDWL2=(T2-T22)/Y1                                                 ***
      DTDWL=(DTDWL1+DTDWL2)/2.0                                          ***
      DFUNC=3.0*X1*T1*T2/A1**4*(BETA*F2-1.0)+X2*(A2*DTDWL-3.0*T2*T2)/(A2 ***
```

```
     1**4)*(BETA*F1-1.0)
      DFUNC1=-DFUNC/(3.0*(FUNC**(4.0/3.0)))
      DY=-FUNC1/DFUNC1
C
C     CONVERGENCE CRITERIA
C
      IF(ABS(DY/Y).GT.0.00001) GO TO 10
      WL2CR=Y+BOT
      IF(JJ.EQ.2) GO TO 70
      JJ=2
      GO TO 50
   70 WL2CR2=WL2CR
      RETURN
   20 WL2CR=-1.0
      IF(JJ.EQ.2) GO TO 80
      JJ=2
      GO TO 50
   80 WL2CR2=WL2CR
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE WLUCR(Q1,Q2,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,
     1VERTC,NPTSC,HORZ,VERT,NPTS,WL1CR,WL1,WL2)
C     ******************************************************************
C
C     A TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF
C     THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THIS SUBROUTINE FINDS
C     THE FREE SURFACE ELEVATION AT THE CRITICAL SECTION AS WELL AS BOTH
C     THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE OTHER SECTION
C     AS A FUNCTION OF THE INTERFACE ELEVATION AT THE CRITICAL SECTION
C     AND THE DISCHARGES IN BOTH LAYERS.
C
C     Q1,Q2          = DISCHARGES OF THE UPPER AND THE LOWER LAYERS
C                      RESPECTIVELY.
C     WL2CR          = INTERFACE ELEVATION AT THE CRITICAL SECTION.
C     RHO1,RHO2      = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                      AND LOWER LAYERS RESPECTIVELY.
C     G              = GRAVITATIONAL ACCELERATION.
C     ALPHAU,ALPHAL  = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                      AND TH+ LOWER LAYERS RESPECTIVELY.
C     HORZC,VERTC    = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING
C                      HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                      OF THE POINTS DEFINING THE CRITICAL SECTION.
C     NPTSC          = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.
C     HORZ,VERT      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS  CONTAINING
C                      HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                      OF THE POINTS DEFINING THE OTHER SECTION.
C     NPTS           = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
C     WL1CR          = FREE SURFACE ELEVATION AT THE CRITICAL SECTION.
C     WL1,WL2        = FREE SURFACE AND INTERFACE ELVATIONS RESPECTIVELY
C                      AT THE OTHER SECTION.
C     SAME DATUM MUST BE USED FOR WL2CR,VERTC,VERT,WL1CR,WL1,WL2
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- WLCRIT,BERNWL2
C     ******************************************************************
C
      DIMENSION HORZC(NPTSC),VERTC(NPTSC),HORZ(NPTS),VERT(NPTS)
C
C     CALCULATE WL1CR
C
      CALL WLCRIT(Q1,Q2,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,VERTC,
     1NPTSC,WL1CR)
      IF(WL1CR.NE.WL2CR) GO TO 10
      WL1=WL2=0.0
      RETURN
C
C     CALCULATE CORRESPONDING WL1 AND WL2
C
   10 CALL BERNWL2(Q1,Q2,WL1CR,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,
     1VERTC,NPTSC,HORZ,VERT,NPTS,WL1,WL2)
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE WLQCR(WL1,WL2,QA,RHO1,RHO2,G,ALPHA1,ALPHA2,N1,HORZC,
     1VERTC,NPTSC,HORZ,VERT,NPTS,WL1CR,WL2CR,QB)
C     ******************************************************************
C
C     A TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF
C     THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THIS SUBROUTINE FINDS
C     THE FREE SURFACE AND THE INTERFACE ELEVATIONS AT THE CRITICAL
C     SECTION AS WELL AS THE DISCHARGE IN ONE LAYER AS A FUNCTION OF THE
C     CORRESPONDING ELEVATIONS AT THE OTHER SECTION AND THE DISCHARGE IN
C     THE OTHER LAYER.
C
C     WL1,WL2          = FREE SURFACE AND INTERFACE ELVATIONS RESPECTIVELY
C                        AT THE NON-CRITICAL SECTION.
C     QA               = GIVEN DISCHARGE.
C     RHO1,RHO2        = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                        AND LOWER LAYERS RESPECTIVELY.
C     G                = GRAVITATIONAL ACCELERATION.
C     ALPHA1,ALPHA2    = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                        AND THE LOWER LAYERS RESPECTIVELY.
C     N1               = 1 IF THE DISCHARGE OF THE UPPER LAYER IS GIVEN
C                          AND THAT OF THE LOWER LAYER REQUIRED.
C                      = 2 IF THE DISCHARGE OF THE LOWER LAYER IS GIVEN
C                          AND THAT OF THE UPPER LAYER IS REQUIRED.
C     HORZC,VERTC      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE CRITICAL SECTION.
C     NPTSC            = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.
C     HORZ,VERT        = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS  CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE OTHER SECTION.
C     NPTS             = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
C     WL1CR,WL2CR      = FREE SURFACE AND INTERFACE ELEVATIONS AT THE
C                        CRITICAL SECTION RESPECTIVELY.
C     QB               = COMPUTED DISCHARGE.
C     SAME DATUM MUST BE USED FOR WL1CR,WL2CR,VERTC,VERT,WL1,WL2
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2,ZSYSTM1(AUXX)
C     ******************************************************************
C
      EXTERNAL AUXX
      DIMENSION HORZC(NPTSC),VERTC(NPTSC),HORZ(NPTS),VERT(NPTS),WAA(14),
     1XA(3)
      EN1=FLOAT(N1)
      DRHO=RHO2-RHO1
      BETA=DRHO/RHO2
      BETA1=RHO1/DRHO
      G1=G*BETA
      XA(1)=WL1
      XA(2)=WL2
      XA(3)=QA
      CALL ZSYSTM1(AUXX,1.0E-6,5,3,XA,100,WL1,WL2,DRHO,BETA1,HORZC,VERTC
     1,NPTSC,ALPHA1,ALPHA2,QA,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,G1
     1,Q1,Q2,RHO1,RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,
     1T22,HORZ,VERT,NPTS,RHO22,EN1,H22,WAA,PAR,IER)
      IF(IER.NE.129.AND.IER.NE.130) GO TO 10
      WRITE(6,20)
   20 FORMAT(5X,*NO SOLUTION FROM WLQCR*)
      WL1CR=WL2CR=QB=0.0
      RETURN
   10 WL1CR=XA(1)
      WL2CR=XA(2)
      QB=XA(3)
      RETURN
      END
```

```
C     ******************************************************************
      FUNCTION AUXX(XA,K,PAR,WL1,WL2,DRHO,BETA1,HORZC,VERTC,NPTSC,ALPHA1
     1,ALPHA2,QA,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,G1,Q1,Q2,RHO1,
     1RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,
     1NPTS,RHO22,EN1)
C     ******************************************************************
C
      DIMENSION XA(3),HORZC(NPTSC),VERTC(NPTSC),HORZ(NPTS),VERT(NPTS)
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,AA1,AA2,TT1,TT2,P1,P2,AAY1,
     1AAY2)
      CALL PROPS2(HORZC,VERTC,NPTSC,XA(1),XA(2),A1D,A2D,T11,T12,P11,P12,
     1AY12,AY22)
      IF(A1D.GT.1.0E-12.AND.A2D.GT.1.0E-12) GO TO 40
      WRITE(6,50)
   50 FORMAT(5X,*NO SOLUTION FROM AUXX*)
      STOP
   40 IF(EN1.EQ.2.0) GO TO 100
C
C     CASE OF GIVEN UPPER DISCHARGE
C
      GO TO (5,10,15),K
    5 AUXX=WL1-XA(1)+(ALPHA1*QA*QA/(2.0*G))*(1.0/(AA1*AA1)-1.0/(A1D*A1D)
     1)
      RETURN
   10 AUXX=BETA1*(WL1-XA(1))+(WL2-XA(2))+(ALPHA2*XA(3)*XA(3)/(2.0*G1))*(
     11.0/(AA2*AA2)-1.0/(A2D*A2D))
      RETURN
   15 F12=ALPHA1*QA*QA*T11/(G1*A1D*A1D*A1D)
      F22=ALPHA2*XA(3)*XA(3)*T12/(G1*A2D*A2D*A2D)
      AUXX=F12+F22-BETA*F12*F22-1.0
      RETURN
C
C     CASE OF GIVEN LOWER DISCHARGE
C
  100 GO TO (20,25,30),K
   20 AUXX=WL1-XA(1)+(ALPHA1*XA(3)*XA(3)/(2.0*G))*(1.0/(AA1*AA1)-1.0/(
     1A1D*A1D))
      RETURN
   25 AUXX=BETA1*(WL1-XA(1))+(WL2-XA(2))+(ALPHA2*QA*QA/(2.0*G1))*(1.0/(
     1AA2*AA2)-1.0/(A2D*A2D))
      RETURN
   30 F12=ALPHA1*XA(3)*XA(3)*T11/(G1*A1D*A1D*A1D)
      F22=ALPHA2*QA*QA*T12/(G1*A2D*A2D*A2D)
      AUXX=F12+F22-BETA*F12*F22-1.0
      RETURN
      END
```

```
C     *****************************************************************
      SUBROUTINE QCRIT1(WL1CR,WL2CR,QA,RHO1,RHO2,G,ALPHAU,ALPHAL,N1,
     1HORZC,VERTC,NPTSC,HORZ,VERT,NPTS,WL1,WL2,QB)
C     *****************************************************************
C
C     A TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF
C     THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THIS SUBROUTINE FINDS
C     THE DISCHARGE IN ONE LAYER AND THE FREE SURFACE AND THE INTERFACE
C     ELEVATIONS AT THE NON-CRITICAL SECTION AS A FUNCTION OF THE CORRE-
C     SPONDING ELEVATIONS AT THE CRITICAL SECTION AND THE DISCHARGE IN
C     THE OTHER LAYER.
C
C     WL1CR,WL2CR     = FREE SURFACE AND INTERFACE ELEVATIONS AT THE CRI-
C                       TICAL SECTION RESPECTIVELY.
C     QA              = GIVEN DISCHARGE.
C     RHO1,RHO2       = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND LOWER LAYERS RESPECTIVELY.
C     G               = GRAVITATIONAL ACCELERATION.
C     ALPHAU,ALPHAL   = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                       AND THE LOWER LAYERS RESPECTIVELY.
C     N1              = 1  IF THE DISCHARGE OF THE UPPER LAYER IS GIVEN
C                          AND THAT OF THE LOWER LAYER IS COMPUTED
C                     = 2  IF THE DISCHARGE OF THE LOWER LAYER IS GIVEN
C                          AND THAT OF THE UPPER LAYER IS COMPUTED.
C     HORZC,VERTC     = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING
C                       HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DEFINING THE CRITICAL SECTION.
C     NPTSC           = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.
C     HORZ,VERT       = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS  CONTAINING
C                       HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DEFINING THE OTHER SECTION.
C     NPTS            = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
C     WL1,WL2         = FREE SURFACE AND INTERFACE ELVATIONS RESPECTIVELY
C                       AT THE OTHER SECTION.
C     QB              = COMPUTED DISCHARGE.
C     SAME DATUM MUST BE USED FOR WL1CR,WL2CR,VERTC,VERT,WL1,WL2
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- QCRIT,BERNWL2
C     *****************************************************************
C
      DIMENSION HORZC(NPTSC),VERTC(NPTSC),HORZ(NPTS),VERT(NPTS)
C
C     CALCULATE THE REQUIRED DISCHARGE
C
      CALL QCRIT(WL1CR,WL2CR,QA,RHO1,RHO2,G,ALPHAU,ALPHAL,N1,HORZC,VERTC
     1,NPTSC,QB)
      IF(QB.NE.0.0) GO TO 10
      WL1=WL2=0.0
      RETURN
   10 IF(N1.EQ.1) GO TO 20
      Q1=QB
      Q2=QA
      GO TO 30
   20 Q1=QA
      Q2=QB
C
C     CALCULATE THE CORRESPONDING WL1 AND WL2
C
   30 CALL BERNWL2(Q1,Q2,WL1CR,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,
     1VERTC,NPTSC,HORZ,VERT,NPTS,WL1,WL2)
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE QCRIT12(WL1,WL2,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,
     1HORZC,VERTC,NPTSC,HORZ,VERT,NPTS,WL1CR,Q1CR,Q2CR)
C     ******************************************************************
C
C     A TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF
C     THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THIS SUBROUTINE FINDS
C     THE DISCHARGES IN BOTH LAYERS AND THE FREE SURFACE ELEVATION AT
C     THE CRITICAL SECTION AS A FUNCTION OF THE INTERFACE ELEVATION AT
C     THE CRITICAL SECTION AS WELL AS THE FREE SURFACE AND THE INTERFACE
C     ELEVATIONS AT THE OTHER SECTION.
C
C     WL1,WL2          = FREE SURFACE AND INTERFACE ELVATIONS RESPECTIVELY
C                        AT THE NON-CRITICAL SECTION.
C     WL2CR            = INTERFACE ELEVATION AT THE CRITICAL SECTION.
C     RHO1,RHO2        = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                        AND LOWER LAYERS RESPECTIVELY.
C     G                = GRAVITATIONAL ACCELERATION.
C     ALPHAU,ALPHAL    = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                        AND THE LOWER LAYERS RESPECTIVELY.
C     HORZC,VERTC      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE CRITICAL SECTION.
C     NPTSC            = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.
C     HORZ,VERT        = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS  CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE OTHER SECTION.
C     NPTS             = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
C     WL1CR            = FREE SURFACE ELEVATION AT THE CRITICAL SECTION.
C     Q1CR,Q2CR        = DISCHARGES IN THE UPPER AND THE LOWER LAYERS RES-
C                        PECTIVELY.
C     SAME DATUM MUST BE USED FOR WL1CR,WL2CR,VERTC,VERT,WL1,WL2
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS,BERNQ2,WLCRIT
C     ******************************************************************
C
      DIMENSION HORZC(NPTSC),VERTC(NPTSC),HORZ(NPTS),VERT(NPTS)
C
C     CALCULATE CROSS-SECTIONAL PROPERTIES OF THE LOWER LAYER AT BOTH
C     SECTIONS
C
      CALL PROPS(HORZ,VERT,NPTS,WL2,A2,T2,P2,AY2)
      CALL PROPS(HORZC,VERTC,NPTSC,WL2CR,A21,T21,P21,AY21)
C
C     ASSUME WL1CR
C
      IF(A21.LT.A2) GO TO 40
      WL11=WL1*1.001
      GO TO 20
   40 WL11=WL1*0.999
C
C     CALCULATE CORRESPONDING Q1 AND Q2
C
   20 CALL BERNQ2(WL1,WL2,WL11,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZ,VERT
     1,NPTS,HORZC,VERTC,NPTSC,Q1,Q2)
      IF(Q1.NE.-1.0.AND.Q2.NE.-1.0) GO TO 30
   60 IF(A21.GT.A2) GO TO 50
      WL11=WL11-(WL1-WL2)*0.001
      GO TO 20
   50 WL11=WL11+(WL1-WL2)*0.001
      GO TO 20
C
C     CALCULATE CORRESPONDING WL1CR
C
   30 CALL WLCRIT(Q1,Q2,WL2CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,VERTC,
     1NPTSC,WL12)
      IF(WL12.EQ.WL2CR) GO TO 60
C
C     CHECK CONVERGENCE
C
      IF(ABS((WL12-WL11)/(WL12-WL2CR)).LT.0.001) GO TO 10
      WL11=WL12
```

```
      GO TO 20
   10 WL1CR=WL12
      Q1CR=Q1
      Q2CR=Q2
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE QCRIT22(WL1,WL2,WL1CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,
     1VERTC,NPTSC,HORZ,VERT,NPTS,WL2CR1,Q1CR1,Q2CR1,WL2CR2,Q1CR2,Q2CR2)
C     ******************************************************************
C
C     A TRANSITION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS ONE OF
C     THEM IS ASSUMED TO CONTAIN CRITICAL FLOW. THIS SUBROUTINE FINDS
C     THE DISCHARGES IN BOTH LAYERS AND THE INTERFACE ELEVATION AT THE
C     CRITICAL SECTION AS A FUNCTION OF THE FREE SURFACE ELEVATION AT
C     THE CRITICAL SECTION AS WELL AS THE FREE SURFACE AND THE INTERFACE
C     ELEVATIONS AT THE OTHER SECTION. TWO DIFFERENT SETS OF SOLUTIONS
C     MAY BE OBTAINED(SEE SECTION 3.3.5-PART D-CHAPTER 3).
C     WL1,WL2          = FREE SURFACE AND INTERFACE ELVATIONS RESPECTIVELY
C                        AT THE NON-CRITICAL SECTION.
C     WL1CR            = FREE SURFACE ELEVATION AT THE CRITICAL SECTION.
C     RHO1,RHO2        = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                        AND LOWER LAYERS RESPECTIVELY.
C     G                = GRAVITATIONAL ACCELERATION.
C     ALPHAU,ALPHAL    = KINETIC ENERGY CORRECTION FACTORS OF THE UPPER
C                        AND THE LOWER LAYERS RESPECTIVELY.
C     HORZC,VERTC      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSC CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE CRITICAL SECTION.
C     NPTSC            = NUMBER OF POINTS DESCRIBING THE CRITICAL SECTION.
C     HORZ,VERT        = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS  CONTAINING
C                        HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                        OF THE POINTS DEFINING THE OTHER SECTION.
C     NPTS             = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
C     WL2CR1,WL2CR2    = INTERFACE ELEVATION AT THE CRITICAL SECTION (TWO
C                        POSSIBLE ROOTS).
C     Q1CR1,Q2CR1      = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECT-
C                        IVELY(FIRST SOLUTION).
C     Q1CR2,Q2CR2      = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECT-
C                        IVELY(SECOND SOLUTION).
C     SAME DATUM MUST BE USED FOR WL1CR,WL2CR1,WL2CR2,VERTC,VERT,WL1,WL2
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- BOTTOM,BERNQ2,WLCRIT1
C     ******************************************************************
C
C
      DIMENSION HORZC(NPTSC),VERTC(NPTSC),HORZ(NPTS),VERT(NPTS)
      QA=0.0
      QB=0.0
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)
C
C     FIRST SET OF SOLUTIONS
C
C     ASSUME WL2CR
C
      IF(WL1CR.GT.WL1) GO TO 40
      WL21=WL2*1.01
      GO TO 20
   40 WL21=WL2*0.99
C
C     CALCULATE CORRESPONDING Q1 AND Q2
C
   20 CALL BERNQ2(WL1,WL2,WL1CR,WL21,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZ,VERT
     1,NPTS,HORZC,VERTC,NPTSC,Q1,Q2)
      IF(Q1.NE.-1.0.AND.Q2.NE.-1.0) GO TO 30
   60 IF(WL1CR.GT.WL1) GO TO 50
      WL21=WL21+(WL2-BOT)*0.001
      GO TO 20
   50 WL21=WL21-(WL2-BOT)*0.001
      GO TO 20
C
C     CALCULATE CORRESPONDING WL2CR
C
   30 CALL WLCRIT1(Q1,Q2,WL1CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,VERTC,
     1NPTSC,WL221,WL222)
      IF(WL221.GT.0.0) GO TO 80
      QA=Q1
      QB=Q2
```

```
      GO TO 60
C
C     CHECK CONVERGENCE
C
   80 IF(ABS((QA-Q1)/Q1).LE.0.01.AND.ABS((QB-Q2)/Q2).LE.0.01) GO TO 10
      QA=Q1
      QB=Q2
      IF(WL21.GT.WL221) GO TO 70
      WL21=WL221*0.99999
      GO TO 20
   70 WL21=WL221*1.00001
      GO TO 20
   10 WL2CR1=WL221
      Q1CR1=Q1
      Q2CR1=Q2
      WRITE(6,1000) WL2CR1,Q1CR1,Q2CR1
 1000 FORMAT(3F20.4)
C
C     SECOND SET OF SOLUTIONS
C
C     ASSUME WL2CR
C
      IF(WL1CR.GT.WL1) GO TO 140
      WL21=WL2*0.99
      GO TO 120
  140 WL21=WL2*1.01
C
C     CALCULATE CORRESPONDING Q1 AND Q2
C
  120 CALL BERNQ2(WL1,WL2,WL1CR,WL21,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZ,VERT
     1,NPTS,HORZC,VERTC,NPTSC,Q1,Q2)
      IF(Q1.NE.-1.0.AND.Q2.NE.-1.0) GO TO 130
  160 IF(WL1CR.GT.WL1) GO TO 150
      WL21=WL21-(WL2-BOT)*0.001
      GO TO 120
  150 WL21=WL21+(WL2-BOT)*0.001
      GO TO 120
C
C     CALCULATE CORRESPONDING WL2CR
C
  130 CALL WLCRIT1(Q1,Q2,WL1CR,RHO1,RHO2,G,ALPHAU,ALPHAL,HORZC,VERTC,
     1NPTSC,WL221,WL222)
      IF(WL222.GT.0.0) GO TO 180
      QA=Q1
      QB=Q2
      GO TO 160
C
C     CHECK CONVERGENCE
C
  180 IF(ABS((QA-Q1)/Q1).LE.0.01.AND.ABS((QB-Q2)/Q2).LE.0.01) GO TO 110
      QA=Q1
      QB=Q2
      IF(WL21.GT.WL222) GO TO 170
      WL21=WL222*1.00001
      GO TO 120
  170 WL21=WL222*0.99999
      GO TO 120
  110 WL2CR2=WL222
      Q1CR2=Q1
      Q2CR2=Q2
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE SOLVE(Q1,Q2,Q11,Q21,HORZD,VERTD,NPTSD,HORZ,VERT,NPTS,
     1BETAU,BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,A1D,A2D,AREA12,
     1AREA22,AY12,AY22,TOPW12,TOPW22,X,WA,X1,X2,NNN,Y111,Y121,RHO1,RHO2,
     1RHO11,RHO21,G,NN)
C     ******************************************************************
C
C     SEARCHES ALL POSSIBLE SOLUTIONS OF THE MOMENTUM EQUATIONS FOR AN
C     INTERFACIAL HYDRAULIC JUMP IN A TWO-LAYER SYSTEM. THE EQUATIONS
C     ARE APPLIED BETWEEN TWO CROSS-SECTIONS IN A CHANNEL OF ARBITRARY
C     GEOMETRY. THE TWO UNKNOWNS ARE H11/H1 AND H21/H2 WHERE H1,H2 ARE
C     THE DEPTHS OF THE UPPER AND LOWER LAYERS RESPECTIVELY AT THE GIVEN
C     STATE AND H11,H21 ARE THE CORRESPONDING DEPTHS AT THE COMPUTED
C     STATE(CONJUGATE DEPTHS). THE TWO EQUATIONS HAVE NINE PAIRS OF
C     SOLUTIONS AT THE MOST. REFER TO EQUATIONS 4.1 AND 4.2- CHAPTER 4.
C
C     Q1,Q2            = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECT-
C                        IVELY AT THE GIVEN STATE.
C     Q11,Q21          = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECT-
C                        IVELY AT THE COMPUTED STATE.
C     HORZD,VERTD      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING
C                        THE HORIZONTAL AND VERTICAL COORDINATES RESPECTI-
C                        VELY OF THE POINTS DESCRIBING THE CROSS-SECTION
C                        AT THE COMPUTED STATE. THESE COORDINATES ARE
C                        REFERRED TO THE SAME AXES AS HORZ,VERT.
C     HORZ,VERT        = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                        THE HORIZONTAL AND VERTICAL COORDINATES RESPECT-
C                        IVELY OF THE POINTS DEFINING THE CROSS-SECTION AT
C                        THE GIVEN STATE.
C                        THE HORIZONTAL COORDINATES ARE REFERRED TO AN
C                        ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS.
C                        ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.
C     NPTS,NPTSD       = NUMBER OF POINTS DESCRIBING THE CROSS-SECTIONS AT
C                        THE GIVEN STATE AND THE COMPUTED STATE RESPECTIV-
C                        ELY.
C     BETAU,BETAL      = MOMENTUM CORRECTION FACTORS FOR THE UPPER AND
C                        LOWER LAYERS RESPECTIVELY.
C     BOTD             = VERTICAL COORDINATE OF THE LOWEST POINT AT THE
C                        CROSS-SECTION OF THE COMPUTED STATE.
C     HH1,HH2          = DEPTHS OF THE UPPER AND LOWER LAYERS RESPECTIVELY
C                        AT THE GIVEN STATE.
C     AA1,AA2          = CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER
C                        LAYERS RESPECTIVELY AT THE GIVEN STATE.
C     TT1,TT2          = TOP WIDTHS OF THE FREE SURFACE AND THE INTERFACE
C                        RESPECTIVELY AT THE GIVEN STATE.
C     AAY1,AAY2        = PRODUCTS OF CROSS-SECTIONAL AREA AND CENTROIDAL
C                        DEPTH FOR THE UPPER LAYER(DEPTH BELOW FREE
C                        SURFACE) AND THE LOWER LAYER (DEPTH BELOW INTERF-
C                        ACE) RESPECTIVELY AT THE GIVEN STATE.
C     A1D,A2D          = ONE-DIMENSIONAL ARRAYS OF SIZE 10 CONTAINING
C                        CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER
C                        LAYERS RESPECTIVELY AT THE COMPUTED STATE FOR
C                        ALL THE SOLUTIONS OBTAINED.
C     AREA12,AREA22    = CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER
C                        LAYERS RESPECTIVELY AFTER THE MIXING REGION.
C     AY12,AY22        = PRODUCTS OF CROSS-SECTIONAL AREA AND CENTROIDAL
C                        DEPTH FOR THE UPPER AND LOWER LAYERS RESPECTIVELY
C                        AFTER THE MIXING REGION.
C     TOPW12,TOPW22    = TOP WIDTHS OF THE FREE SURFACE AND THE INTERFACE
C                        RESPECTIVELY AFTER THE MIXING REGION.
C     X                = ONE-DIMENSIONAL ARRAY OF SIZE 2 CONTAINING A SOL-
C                        UTION OF THE MOMENTUM EQUATIONS.
C     WA               = ONE-DIMENSIONAL WORKING ARRAY OF SIZE 8.
C     X1,X2            = ONE-DIMENSIONAL ARRAYS OF SIZE 10 CONTAINING ALL
C                        THE OBTAINED SOLUTIONS OF THE MOMENTUM EQUATIONS.
C     NNN              = 0  IF THE EQUATIONS ARE BEING SOLVED AT THE
C                           MIXING ZONE IN WHICH CASE ONLY ONE SOLUTION IS
C                           SOUGHT.
C                        OTHERWISE ALL POSSIBLE SOLUTIONS ARE SEARCHED.
C     Y111,Y121        = THE SOLUTION OBTAINED FOR THE IDEAL JUMP WITHOUT
C                        MIXING (IF THERE IS ONE AND ONLY ONE FEASIBLE
C                        SOLUTION). USED ONLY IF MIXING IS COSIDERED (IF
C                        NNN=0).
```

```
C     RHO1,RHO2        = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                        AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE.
C     RHO11,RHO21      = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                        AND LOWER LAYERS RESPECTIVELY AT THE COMPUTED
C                        STATE.
C     G                = GRAVITATIONAL ACCELERATION.
C     NN               = A DIRECTION PARAMETER.
C                      = +1  IF THE UPSTREAM STATE IS GIVEN AND THE DOWN-
C                            STREAM STATE IS COMPUTED.
C                      = -1  IN THE OPPOSITE CASE.
C     ALL UNITS MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- ZSYSTM1(USES FUNCTION AUX)-PROPS2-BOTTOM
C     FUNCTION AUX CONTAINS THE MOMENTUM EQUATIONS AND IS CALLED BY
C     ZSYSTM1 TO EVALUATE THE EQUATIONS BEING SOLVED FOR CERTAIN TRIAL
C     VALUES OF THE UNKNOWNS. SEE DOCUMENTATION OF ZSYSTM1.
C     *****************************************************************
C
      DIMENSION HORZD(NPTSD),VERTD(NPTSD),A1D(10),A2D(10),HORZ(NPTS),
     1VERT(NPTS)
      DIMENSION X1(10),X2(10),X(2),WA(8)
      EXTERNAL AUX
      L=0
      J=0
      DO 10 K=1,10
      X1(K)=0.0
      X2(K)=0.0
      A1D(K)=0.0
      A2D(K)=0.0
   10 CONTINUE
C
C     IF SOLUTION IS SOUGHT IN THE MIXING REGION START WITH THE SOLUTION
C     OBTAINED PREVIOUSLY
C
      IF(NNN.NE.0) GO TO 150
      IF(NN.EQ.1) GO TO 170
      X(1)=1.0/Y111
      X(2)=1.0/Y121
      GO TO 20
  170 X(1)=Y111
      X(2)=Y121
      GO TO 20
C
C     IN THE GENERAL CASE TRY TEN DIFFERENT PAIRS OF STARTING VALUES
C
  150 X(1)=0.0
      X(2)=0.0
      GO TO 20
   60 X(1)=0.5
      X(2)=1.5
      GO TO 20
   70 X(1)=0.0
      X(2)=4.0
      GO TO 20
   80 X(1)=1.5
      X(2)=0.5
      GO TO 20
   90 X(1)=2.0
      X(2)=2.0
      GO TO 20
  100 X(1)=0.0
      X(2)=3.0
      GO TO 20
  110 X(1)=4.0
      X(2)=0.0
      GO TO 20
  120 X(1)=3.0
      X(2)=0.0
      GO TO 20
  130 X(1)=2.0
      X(2)=4.0
      GO TO 20
  140 X(1)=4.0
      X(2)=2.0
```

```
   20 EPS=1.0E-6
      NSIG=5
      N=2
      ITMAX=100
C
C     SOLVE THE MOMENTUM EQUATIONS
C
      CALL ZSYSTM1(AUX,EPS,NSIG,N,X,ITMAX,Q11,Q21,RHO11,RHO21,HORZD,
     1VERTD,NPTSD,BETAU,BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,
     1H2,BOTU,Q1,Q2,RHO1,RHO2,AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,
     1AREA12,AREA22,AY12,AY22,TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22,H22
     1,WA,PAR,IER)
      IF(IER.NE.129.AND.IER.NE.130) GO TO 30
      IF(NNN.EQ.0) GO TO 160
      L=L+1
C
C     STORE ALL THE SOLUTIONS OBTAINED AND THE CORRESPONDING CROSS-SECTI
C     AREAS.
C
      IF(L.GT.9) GO TO 40
      GO TO (60,70,80,90,100,110,120,130,140),L
   30 J=J+1
      L=L+1
      X1(J)=X(1)
      X2(J)=X(2)
      A1D(J)=AREA1D
      A2D(J)=AREA2D
      IF(NNN.EQ.0) GO TO 40
      IF(L.GT.9) GO TO 40
      GO TO (60,70,80,90,100,110,120,130,140),L
  160 X1(1)=0.0
      X2(1)=0.0
   40 RETURN
      END
C     ******************************************************************
      FUNCTION AUX(X,K,PAR,Q11,Q21,RHO11,RHO21,HORZD,VERTD,NPTSD,BETAU,
     1BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1
     1,RHO2,AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,AY12,AY22
     1,TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22)
C     ******************************************************************
      DIMENSION X(2),HORZD(NPTSD),VERTD(NPTSD),HORZ(NPTS),VERT(NPTS)
      H11=X(1)*HH1
      H21=X(2)*HH2
      WL2D=BOTD+H21
      WL1D=WL2D+H11
C
C     CALCULATE SECTION PROPERTIES FOR THE GIVEN TRIAL VALUE OF THE UNKN
C
      CALL PROPS2(HORZD,VERTD,NPTSD,WL1D,WL2D,AREA1D,AREA2D,TOPW1D,
     1TOPW2D,PERIM1D,PERIM2D,AY1D,AY2D)
      IF(ABS(AREA1D).LE.1.0E-12.OR.ABS(AREA2D).LE.1.0E-12) GO TO 20
      ARR1=AREA1D
      ARR2=AREA2D
C
C     SOLVE MOMENTUM EQUATIONS
C
   20 CALL BOTTOM(VERT,NPTS,BOTU,WLU)
      WL2=HH2+BOTU
      WL1=WL2+HH1
      WLDF=WL2D-WL2
      AB=(TT2+TOPW2D)*WLDF/2.0
      IF(AB.NE.0.0) GO TO 70
      YCC=0.0
      GO TO 80
   70 YCC=(TT2*WLDF**2/2.0+WLDF**2*(TOPW2D-TT2)/6.0)/AB
   80 YC1=YCC+(WL1-WL2D)
      YC2=YCC+(WL1D-WL2D)
      YC=(YC1+YC2)/2.0
      F3=((RHO1+RHO11)/2.0)*AB*YC
      AA=AREA2D-AA2-AB
      AD1=(TT1+TOPW1D)*(WL1-WL1D)/2.0
      AD=AD1+AREA1D+AB-AA1
      F1=RHO1*HH1*AA2+RHO2*AAY2
      F2=RHO11*H11*AREA2D+RHO21*AY2D
```

```
      F5=RHO1*AAY1
      F6=RHO11*AY1D
      IF(ABS(AREA1D).GT.1.0E-12.AND.ABS(AREA2D).GT.1.0E-12) GO TO 50
      F4=F1*AA/(2.0*AA2)
      F7=F5*AD/(2.0*AA1)
      GO TO 60
   50 F4=(F1/AA2+F2/AREA2D)*AA/2.0
      F7=(F5/AA1+F6/AREA1D)*AD/2.0
   60 IF (K.EQ.1) GO TO 5
      IF(K.EQ.2) GO TO 10
    5 AUX=BETAU*(RHO11*Q11**2*AA1-RHO1*Q1**2*AREA1D)-G*AA1*AREA1D*(F5-F6
     1-F3+F7)
      IF(ABS(AREA1D).GT.1.0E-12.AND.ABS(AREA2D).GT.1.0E-12) GO TO 30
      AREA1D=ARR1
      AREA2D=ARR2
   30 RETURN
   10 AUX=BETAL*(RHO21*Q21**2*AA2-RHO2*Q2**2*AREA2D)-G*AA2*AREA2D*(F1-F2
     1+F3+F4)
      IF(ABS(AREA1D).GT.1.0E-12.AND.ABS(AREA2D).GT.1.0E-12) GO TO 40
      AREA1D=ARR1
      AREA2D=ARR2
   40 RETURN
      END
```

```
C     ****************************************************************
      SUBROUTINE SOLVE1(H1,H2,BOTU,Q1,Q2,RHO1,RHO2,AREA1,AREA2,G,BETA,
     1BETAU,BETAL,AY1,AY2,HORZ,VERT,NPTS,HORZD,VERTD,NPTSD,NN,AREA12,
     1AREA22,AY12,AY22,TOPW12,TOPW22,RHO22,Q22,Q12,H22)
C     ****************************************************************
C
C     SEARCHES A SOLUTION OF THE MOMENTUM EQUATION AT THE MIXING REGION
C     (EQUATION 4.7 CHAPTER 4) OF AN INTERFACIAL HYDRAULIC JUMP IN A
C     TWO-LAYER SYSTEM. THE DEPTH OF THE LOWER LAYER AT THE DOWNSTREAM
C     SIDE OF THE MIXING REGION IS DETERMINED FOR A SPECIFIED UPSTREAM
C     STATE.
C
C     H1,H2            = DEPTHS OF THE UPPER AND LOWER LAYERS RESPECTIVELY
C                        AT THE UPSTREAM STATE.
C     BOTU             = VERTICAL COORDINATE OF THE LOWEST POINT AT THE
C                        CROSS-SECTION OF THE REQUIRED STATE.
C     Q1,Q2            = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECT-
C                        IVELY AT THE UPSTREAM STATE.
C     RHO1,RHO2        = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                        AND LOWER LAYERS RESPECTIVELY AT UPSTREAM STATE.
C     AREA1,AREA2      = CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER
C                        LAYERS RESPECTIVELY AT THE UPSTREAM STATE.
C     G                = GRAVITATIONAL ACCELERATION.
C     BETA             = FRACTION OF THE SLOWER LAYER DISCHARGE ENTRAINED
C                        BY THE FASTER LAYER AT THE MIXING REGION.
C     BETAU,BETAL      = MOMENTUM CORRECTION FACTORS FOR THE UPPER AND
C                        LOWER LAYERS RESPECTIVELY.
C     AY1,AY2          = PRODUCTS OF CROSS-SECTIONAL AREA AND CENTROIDAL
C                        DEPTH FOR THE UPPER LAYER(DEPTH BELOW FREE
C                        SURFACE) AND THE LOWER LAYER (DEPTH BELOW INTERF-
C                        ACE) RESPECTIVELY AT THE UPSTREAM STATE.
C     HORZ,VERT        = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                        THE HORIZONTAL AND VERTICAL COORDINATES RESPECT-
C                        IVELY OF THE POINTS DEFINING THE CROSS-SECTION AT
C                        THE UPSTREAM STATE.
C                        THE HORIZONTAL COORDINATES ARE REFERRED TO AN
C                        ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS.
C                        ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.
C     HORZD,VERTD      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING
C                        THE HORIZONTAL AND VERTICAL COORDINATES RESPECTI-
C                        VELY OF THE POINTS DESCRIBING THE CROSS-SECTION
C                        AT THE DOWNSTREAM END OF THE JUMP. THE COORDS ARE
C                        REFERRED TO THE SAME AXES AS HORZ,VERT.
C     NPTS,NPTSD       = NUMBER OF POINTS DESCRIBING THE CROSS-SECTIONS AT
C                        THE UPSTREAM AND THE DOWNSTREAM STATES RESPECTIV-
C                        ELY.
C     NN               = A DIRECTION PARAMETER.
C                      = +1  IF THE UPSTREAM STATE IS GIVEN AND THE DOWN-
C                            STREAM STATE OF THE JUMP IS COMPUTED.
C                      = -1  IN THE OPPOSITE CASE.
C     AREA12,AREA22    = CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER
C                        LAYERS RESPECTIVELY AFTER THE MIXING REGION.
C     AY12,AY22        = PRODUCTS OF CROSS-SECTIONAL AREA AND CENTROIDAL
C                        DEPTH FOR THE UPPER AND LOWER LAYERS RESPECTIVELY
C                        AFTER THE MIXING REGION.
C     TOPW12,TOPW22    = TOP WIDTHS OF THE FREE SURFACE AND THE INTERFACE
C                        RESPECTIVELY AFTER THE MIXING REGION.
C     RHO22            = LOWER LAYER DENSITY (OR SPECIFIC GRAVITY) AFTER
C                        THE MIXING REGION.
C     Q22,Q12          = DISCHARGES OF THE LOWER AND UPPER LAYERS RESPECT-
C                        IVELY AFTER THE MIXING REGION.
C     H22              = LOWER LAYER DEPTH AFTER THE MIXING REGION.
C     ALL UNITS MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- ZSYSTM1(USES FUNCTION AUX1)-PROPS2
C     FUNCTION AUX1 CONTAINS THE MOMENTUM EQUATION AND IS CALLED BY
C     ZSYSTM1 TO EVALUATE THE EQUATION  BEING SOLVED FOR CERTAIN TRIAL
C     VALUES OF THE UNKNOWN.  SEE DOCUMENTATION OF ZSYSTM1.
C     ****************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTSD),VERTD(NPTSD),WA1(3)
      EXTERNAL AUX1
      H22=H2
```

```
      EPS=1.0E-6
      NSIG=5
      N=1
      ITMAX=100
C
C     SOLVE THE MOMENTUM EQUATION FOR THE LOWER LAYER DEPTH JUST AFTER
C     THE MIXING REGION
C
      CALL ZSYSTM1(AUX1,EPS,NSIG,N,H22,ITMAX,Q11,Q12,RHO11,RHO21,HORZD,
     1VERTD,NPTSD,BETAU,BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AY1,H1,
     1H2,BOTU,Q1,Q2,RHO1,RHO2,AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,
     1AREA12,AREA22,AY12,AY22,TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22,H22
     1,WA1,PAR,IER1)
      IF(IER1.NE.129.AND.IER1.NE.130) GO TO 10
      WRITE(6,30)
   30 FORMAT(5X,*SOLVE1 DID NOT CONVERGE*,///)
      STOP
   10 RETURN
      END
C     ******************************************************************
      FUNCTION AUX1(H22,K,PAR,Q11,Q21,RHO11,RHO21,HORZD,VERTD,NPTSD,
     1BETAU,BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AY1,H1,H2,BOTU,Q1,
     1Q2,RHO1,RHO2,AREA1,AREA2,AREA1D,AREA2D,G,BETA,AY2,AREA12,AREA22,
     1AY12,AY22,TOPW12,TOPW22,HORZ,VERT,NPTS,RHO22,Q22)
C     ******************************************************************
C
      DIMENSION HORZD(NPTSD),VERTD(NPTSD),HORZ(NPTS),VERT(NPTS)
      WL22=H22+BOTU
      H12=H1+H2-H22
      WL12=WL22+H12
C
C     CALCULATE SECTION PROPERTIES FOR THE GIVEN TRIAL VALUE OF THE
C     UNKNOWN
C
      CALL PROPS2(HORZ,VERT,NPTS,WL12,WL22,AREA12,AREA22,TOPW12,TOPW22,
     1PERIM12,PERIM22,AY12,AY22)
C
C     SOLVE MOMENTUM EQUATION
C
      AUX1=(BETAU*RHO1*Q1**2*AREA12*AREA22)/AREA1+(BETAL*RHO2*Q2**2*
     1AREA12*AREA22)/AREA2+G*AREA12*AREA22*(RHO2*AY2+RHO1*(AY1+AREA2*H1)
     1)-(BETAU*RHO1*(1.0-BETA)**2*Q1**2*AREA22)-(BETAL*RHO22*Q22**2*
     1AREA12)-G*AREA12*AREA22*(RHO22*AY22+RHO1*(AY12+AREA22*H12))
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE INTJMP(N,NN,Q1,Q2,Q11,Q21,WL1,WL2,RHO1,RHO2,RHO11,RHO21
     1,G,BETAU,BETAL,ALPHAU,ALPHAL,NPTS,NPTSD,BETA,RHO22,Q22,HORZ,VERT,
     1HORZD,VERTD,A1D,A2D,X1,X2,X,WA,Y1,Y2,ALPHA,WL11,WL21)
C     ******************************************************************
C
C     DETERMINES THE STATE UPSTREAM (OR DOWNSTREAM) FROM A HYDRAULIC
C     JUMP AT THE INTERFACE OF TWO LAYERS FOR A COMPLETELY SPECIFIED
C     STATE DOWNSTREAM (OR UPSTREAM) IN A CHANNEL OF ARBITRARY GEOMETRY.
C     EACH OF THE UPSTREAM AND DOWNSTREAM SECTIONS IS DEFINED BY A
C     SERIES OF STRAIGHT LINES BETWEEN POINTS, THE COORDINATES OF WHICH
C     ARE REFERRED TO SOME ARBITRARY VERTICAL AND HORIZONTAL AXES.
C     PROVISION IS ALLOWED TO CONSIDER MIXING AT THE INTERFACE.
C
C     N               = A MIXING PARAMETER.
C                     = 0  IF INTERFACIAL MIXING IS TO BE NEGLECTED.
C                       OTHERWISE MIXING WILL BE CONSIDERED.
C     NN              = A DIRECTION PARAMETER.
C                     = +1  IF THE UPSTREAM STATE IS GIVEN AND THE DOWN-
C                           STREAM STATE IS COMPUTED.
C                     = -1  IN THE OPPOSITE CASE.
C     Q1,Q2           = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECT-
C                       IVELY AT THE GIVEN STATE.
C     Q11,Q21         = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECT-
C                       IVELY AT THE COMPUTED STATE.
C     WL1,WL2         = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                       LY AT THE GIVEN STATE.
C     RHO1,RHO2       = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND LOWER LAYERS RESPECTIVELY AT THE GIVEN STATE.
C     RHO11,RHO21     = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND LOWER LAYERS RESPECTIVELY AT THE COMPUTED
C                       STATE.
C     G               = GRAVITATIONAL ACCELERATION.
C     BETAU,BETAL     = MOMENTUM CORRECTION FACTORS FOR THE UPPER AND
C                       LOWER LAYERS RESPECTIVELY.
C     ALPHAU,ALPHAL   = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER
C                       AND LOWER LAYERS RESPECTIVELY.
C     NPTS,NPTSD      = NUMBER OF POINTS DESCRIBING THE CROSS-SECTIONS AT
C                       THE GIVEN STATE AND THE COMPUTED STATE RESPECTIV-
C                       ELY.
C     BETA            = FRACTION OF THE SLOWER LAYER DISCHARGE ENTRAINED
C                       BY THE FASTER LAYER (IF MIXING IS CONSIDERED).
C     RHO22           = LOWER LAYER DENSITY (OR SPECIFIC GRAVITY) AT THE
C                       MIXING REGION.
C     Q22             = LOWER LAYER DISCHARGE AT THE MIXING REGION.
C     HORZ,VERT       = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                       THE HORIZONTAL AND VERTICAL COORDINATES RESPECT-
C                       IVELY OF THE POINTS DEFINING THE CROSS-SECTION AT
C                       THE GIVEN STATE.
C                       THE HORIZONTAL COORDINATES ARE REFERRED TO AN
C                       ARBITRARY VERTICAL AXIS AND THE VERTICAL COORDS.
C                       ARE REFERRED TO THE SAME DATUM AS WATER LEVELS.
C     HORZD,VERTD     = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING
C                       THE HORIZONTAL AND VERTICAL COORDINATES RESPECTI-
C                       VELY OF THE POINTS DESCRIBING THE CROSS-SECTION
C                       AT THE COMPUTED STATE. THESE COORDINATES ARE
C                       REFERRED TO THE SAME AXES AS HORZ,VERT.
C     A1D,A2D         = ONE-DIMENSIONAL ARRAYS OF SIZE 10 CONTAINING
C                       CROSS-SECTIONAL AREAS OF THE UPPER AND LOWER
C                       LAYERS RESPECTIVELY AT THE COMPUTED STATE FOR
C                       ALL THE SOLUTIONS OBTAINED.
C     X1,X2           = ONE-DIMENSIONAL ARRAYS OF SIZE 10 CONTAINING ALL
C                       THE OBTAINED SOLUTIONS OF THE MOMENTUM EQUATIONS.
C     X               = ONE-DIMENSIONAL ARRAY OF SIZE 2 CONTAINING A SOL-
C                       UTION OF THE MOMENTUM EQUATIONS.
C     WA              = ONE-DIMENSIONAL WORKING ARRAY OF SIZE 8.
C     Y1,Y2           = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE 10 CONTAI-
C                       NING ALL FEASIBLE SOLUTIONS OF THE MOMENTUM EQUA-
C                       TIONS.
C     ALPHA           = FRACTION OF THE ENERGY DROP AVAILABLE FOR ENTRAI-
C                       NMENT (IF MIXING IS CONSIDERED).
C     WL11,WL21       = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                       LY AT THE COMPUTED STATE REFERRED TO THE SAME
```

```
C                     DATUM AS WL1,WL2.
C
C     ALL UNITS MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- SOLVE,SOLVE1,PROPS2,BOTTOM
C     *****************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTSD),VERTD(NPTSD),A1D(10),
     1A2D(10)
      DIMENSION X1(10),X2(10),Y1(10),Y2(10),X(2),WA(8)
      EXTERNAL AUX,AUX1
      BETA=0.05
      BETA1=0.0
      BETA2=0.05
      L=0
C
C     CALCULATE SECTION PROPERTIES FOR THE GIVEN STATE
C
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,AREA1,AREA2,TOPW1,TOPW2,PERIM1,
     1PERIM2,AY1,AY2)
      CALL BOTTOM(VERT,NPTS,BOTU,WLMAXU)
      CALL BOTTOM(VERTD,NPTSD,BOTD,WLMAXD)
      H1=WL1-WL2
      H2=WL2-BOTU
      HH1=H1
      HH2=H2
      AA1=AREA1
      AA2=AREA2
      TT1=TOPW1
      TT2=TOPW2
      AAY1=AY1
      AAY2=AY2
C
C     FIND ALL SOLUTIONS FOR THE MOMENTUM EQUATIONS
C
      CALL SOLVE(Q1,Q2,Q11,Q21,HORZD,VERTD,NPTSD,HORZ,VERT,NPTS,BETAU,
     1BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,A1D,A2D,AREA12,AREA22
     1,AY12,AY22,TOPW12,TOPW22,X,WA,X1,X2,1,Y111,Y121,RHO1,RHO2,RHO11,
     1RHO21,G,NN)
      M=0
      DO 10 I=1,10
C
C     IGNORE NEGATIVE SOLUTIONS
C
      IF(X1(I).LE.1.0E-8.OR.X2(I).LE.1.0E-8) GO TO 10
C
C     IGNORE TRIVIAL SOLUTION (GIVEN STATE)
C
      IF(ABS(1.0-X1(I)).LE.0.0001.OR.ABS(1.0-X2(I)).LE.0.0001) GO TO 10
      A1=X1(I)*H1
      A2=X2(I)*H2
C
C     CALCULATE ENERGY DROP
C
      DE=(Q1*(RHO1*G*(H1+H2+BOTU)+ALPHAU*0.5*RHO1*Q1**2*(1.0/(AREA1**2))
     1)-Q11*(RHO11*G*(A1+A2+BOTD)+ALPHAU*0.5*RHO11*Q11**2*(1.0/(A1D(I)**
     12)))+Q2*(RHO1*G*H1+RHO2*G*(H2+BOTU)+ALPHAL*0.5*RHO2*Q2**2*(1.0/(
     1AREA2**2)))-Q21*(RHO11*G*A1+RHO21*G*(A2+BOTD)+ALPHAL*0.5*RHO21*Q21
     1**2*(1.0/(A2D(I)**2))))*FLOAT(NN)
C
C     CHECK THE SIGN OF THE ENERGY DROP
C
      IF(DE.LE.0.0) GO TO 10
      M=M+1
      Y1(M)=X1(I)
      Y2(M)=X2(I)
   10 CONTINUE
C
C     IF THERE ARE NO FEASIBLE SOLUTIONS STOP AND PRINT A MESSAGE
C
      IF(M.EQ.0) GO TO 1000
C
C     IF THERE IS ONLY ONE SOLUTION PROCEED
C
```

```
      IF(M.EQ.1) GO TO 60
C
C     IF THERE ARE MORE THAN ONE DIFFERENT SOLUTIONS PRINT THEM AND STOP
C     IF MIXING IS CONSIDERED CARRY ON WITH THE SOLUTION CLOSEST TO THE
C     GIVEN STATE
C
      IF(N.NE.0) GO TO 180
      DO 70 I=1,M
      IF(ABS(Y1(1)-Y1(I)).GT.1.0E-2) GO TO 1300
   70 CONTINUE
   60 Y111=Y1(1)
      Y121=Y2(1)
      H11=Y111*H1
      H21=Y121*H2
C
C     CALCULATE CONJUGATE WATER LEVELS FROM THE ONE SOLUTION OBTAINED
C
      WL21=H21+BOTD
      WL11=WL21+H11
      WNO1=WL11
      WNO2=WL21
C
C     RETURN IF MIXING IS NOT TO BE CONSIDERED
C
      IF(N.EQ.0) GO TO 1700
C
C     RETURN WITH THE SOLUTION OBTAINED WITHOUT MIXING IF ONE OF THE LAY
C     IS STAGNANT AND MIXING IS TO BE CONSIDERED AND PRINT A MESSAGE TO
C     THAT EFFECT
C
  180 IF(N.NE.0.AND.Q1.EQ.0.0) GO TO 1800
      IF(N.NE.0.AND.Q2.EQ.0.0) GO TO 1800
      IF(M.EQ.1) GO TO 190
      IMIN=1
      ZZ=SQRT((Y1(1)-1.0)**2+(Y2(1)-1.0)**2)
      DO 210 KKK=2,M
      ZZ1=SQRT((Y1(KKK)-1.0)**2+(Y2(KKK)-1.0)**2)
      IF(ZZ1.LT.ZZ) IMIN=KKK
  210 CONTINUE
      Y111=Y1(IMIN)
      Y121=Y2(IMIN)
      H11=Y111*H1
      H21=Y121*H2
      WL21=H21+BOTD
      WL11=WL21+H11
      WNO1=WL11
      WNO2=WL21
  190 CALL PROPS2(HORZD,VERTD,NPTSD,WL11,WL21,ARE11,ARE21,TO11,TO21,PE11
     1,PE21,AY11,AY21)
C
C     PROCEED IF MIXING IS TO BE TAKEN INTO CONSIDERATION
C
   20 IF(NN.EQ.1) GO TO 100
      U1=Q11/ARE11
      U2=Q21/ARE21
      IF(ABS(U1).GT.ABS(U2)) GO TO 140
      Q12=(1.0-BETA)*Q11
      Q1BET=ABS(Q11*BETA)
      Q2ABS=ABS(Q21)
      Q22=(Q2ABS+Q1BET)*Q21/Q2ABS
      RHO22=(RHO11*Q1BET+RHO21*Q2ABS)/(ABS(Q22))
      RHO12=RHO11
      GO TO 150
  140 Q22=(1.0-BETA)*Q21
      Q2BET=ABS(Q21*BETA)
      Q1ABS=ABS(Q11)
      Q12=(Q1ABS+Q2BET)*Q11/Q1ABS
      RHO12=(RHO21*Q2BET+RHO11*Q1ABS)/(ABS(Q12))
      RHO22=RHO21

      SOLVE MOMENTUM EQUATION TO DETERMINE DEPTH OF THE LOWER LAYER AT
      THE MIXING REGION

  150 CALL SOLVE1(H11,H21,BOTD,Q11,Q21,RHO11,RHO21,ARE11,ARE21,G,BETA,
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```
     1BETAU,BETAL,AY11,AY21,HORZD,VERTD,NPTSD,HORZ,VERT,NPTS,NN,AREA12,
     1AREA22,AY12,AY22,TOPW12,TOPW22,RHO22,Q22,Q12,H22)
      GO TO 110
  100 U1=Q1/AREA1
      U2=Q2/AREA2
      IF(ABS(U1).GT.ABS(U2)) GO TO 160
      Q12=(1.0-BETA)*Q1
      Q1BET=ABS(Q1*BETA)
      Q2ABS=ABS(Q2)
      Q22=(Q2ABS+Q1BET)*Q2/Q2ABS
      RHO22=(RHO1*Q1BET+RHO2*Q2ABS)/(ABS(Q22))
      RHO12=RHO1
      GO TO 170
  160 Q22=(1.0-BETA)*Q2
      Q2BET=ABS(Q2*BETA)
      Q1ABS=ABS(Q1)
      Q12=(Q1ABS+Q2BET)*Q1/Q1ABS
      RHO12=(RHO2*Q2BET+RHO1*Q1ABS)/(ABS(Q12))
      RHO22=RHO2
  170 CALL SOLVE1(H1,H2,BOTU,Q1,Q2,RHO1,RHO2,AREA1,AREA2,G,BETA,BETAU,
     1BETAL,AY1,AY2,HORZ,VERT,NPTS,HORZD,VERTD,NPTSD,NN,AREA12,AREA22,
     1AY12,AY22,TOPW12,TOPW22,RHO22,Q22,Q12,H22)
C
C     CALCULATE ALL PARAMETERS AT THE MIXING REGION
C
  110 IF(NN.EQ.1) GO TO 120
      H12=H11+H21-H22
      GO TO 130
  120 H12=H1+H2-H22
  130 HH1=H12
      HH2=H22
      AA1=AREA12
      AA2=AREA22
      TT1=TOPW12
      TT2=TOPW22
      AAY1=AY12
      AAY2=AY22
C
C     SOLVE MOMENTUM EQUATIONS FOR THE NEW CASE USING THE PREVIOUSLY-
C     OBTAINED SOLUTION
C
      IF(NN.EQ.1) GO TO 200
      CALL SOLVE(Q12,Q22,Q1,Q2,HORZ,VERT,NPTS,HORZD,VERTD,NPTSD,BETAU,
     1BETAL,BOTU,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,A1D,A2D,AREA12,AREA22
     1,AY12,AY22,TOPW12,TOPW22,X,WA,X1,X2,0,Y111,Y121,RHO12,RHO22,RHO1,
     1RHO2,G,NN)
      GO TO 300
  200 CALL SOLVE(Q12,Q22,Q11,Q21,HORZD,VERTD,NPTSD,HORZ,VERT,NPTS,BETAU,
     1BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,A1D,A2D,AREA12,AREA22
     1,AY12,AY22,TOPW12,TOPW22,X,WA,X1,X2,0,Y111,Y121,RHO12,RHO22,RHO11,
     1RHO21,G,NN)
C
C     IF A SOLUTION IS OBTAINED PROCEED,IF NOT CHANGE BETA
C
  300 IF(X1(1).EQ.0.0.AND.X2(1).EQ.0.0) GO TO 30
      A1=X1(1)*H12
      A2=X2(1)*H22
C
C     CALCULATE NEW ENERGY DROP
C
      IF(NN.EQ.1) GO TO 400
      DE2=(Q12*(RHO12*G*(H12+H22+BOTD)+ALPHAU*0.5*RHO12*Q12**2*(1.0/
     1(AREA12**2)))-Q1*(RHO1*G*(A1+A2+BOTU)+ALPHAU*0.5*RHO1*Q1**2*(1./
     1(A1D(1)**2)))+Q22*(RHO12*G*H12+RHO22*G*(H22+BOTD)+ALPHAL*0.5*RHO22
     1*Q22**2*(1.0/(AREA22**2)))-Q2*(RHO1*G*A1+RHO2*G*(A2+BOTU)+ALPHAL*
     10.5*RHO2*Q2**2*(1.0/(A2D(1)**2))))
      GO TO 500
  400 DE2=(Q12*(RHO12*G*(H12+H22+BOTU)+ALPHAU*0.5*RHO12*Q12**2*(1.0/(
     1AREA12**2)))-Q11*(RHO11*G*(A1+A2+BOTD)+ALPHAU*0.5*RHO11*Q11**2*(1.
     1/(A1D(1)**2)))+Q22*(RHO12*G*H12+RHO22*G*(H22+BOTU)+ALPHAL*0.5*RHO2
     12*Q22**2*(1.0/(AREA22**2)))-Q21*(RHO11*G*A1+RHO21*G*(A2+BOTD)+
     1ALPHAL*0.5*RHO21*Q21**2*(1.0/(A2D(1)**2))))
C
C     CHECK THE SIGN OF THE ENERGY DROP
```

```
C
  500 IF(DE2.LE.0.0) GO TO 2000
      H11=X1(1)*H12
      H21=X2(1)*H22
C
C     CHECK THE VALIDITY OF THE ASSUMED BETA,IF VALID RETURN,IF NOT
C     PROCEED
C
      C=DE2/(DE*(1.0-ALPHA))
      WL21=H21+BOTD
      WL11=WL21+H11
   30 IF(ABS(1.0-C).LE.0.010) GO TO 1700
      IF(C.LT.1.0) GO TO 40
      IF(L.EQ.1) GO TO 50
C
C     USE INTERVAL HALVING TO OBTAIN THE CORRECT BETA
C
      BETA1=BETA
      BETA2=BETA+0.05
      BETA=BETA2
      IF(BETA.GE.1.0) GO TO 2000
      GO TO 20
   40 BETA=(BETA1+BETA)/2.0
      L=1
      GO TO 20
   50 BETA1=BETA
      BETA=(BETA2+BETA)/2.0
      GO TO 20
 1000 WRITE(6,1100)
 1100 FORMAT(5X,* NO  JUMP POSSIBLE *,//)
      GO TO 1200
 1300 JJJ=1
      HH1=Y1(1)*H1
      HH2=Y2(1)*H2
      WWL2=HH2+BOTD
      WWL1=WWL2+HH1
      WRITE(6,1500) JJJ,WWL1,WWL2
      DO 1600 K=2,M
      IF(ABS(Y1(K)-Y1(K-1)).LE.1.0E-4) GO TO 1600
      HH1=Y1(K)*H1
      HH2=Y2(K)*E2
      WWL2=HH2+BOTD
      WWL1=WWL2+HH1
      JJJ=JJJ+1
      WRITE(6,1500) JJJ,WWL1,WWL2
 1500 FORMAT(5X,*SOLUTION NUMBER*,I3,5X,*WL11 = *,F10.3,10X,*WL21 = *,
     1F10.3,///)
 1600 CONTINUE
      WRITE(6,1400) JJJ
 1400 FORMAT(5X,* THERE  ARE *,I3,* CONJUGATE  STATES (NO MIXING CONSIDE
     1RED) *,//)
 1200 STOP
 1800 WRITE(6,1900)
 1900 FORMAT(5X,*MIXING IS NOT CONSIDERED IN THIS SPECIAL CASE OF A STAG
     1NANT LAYER*,//,5X,*THIS WILL BE AN UNSTABLE SITUATION*,//,5X,
     1*THE SOLUTION IS OBTAINED WITHOUT ALLOWING FOR MIXING*,//)
      GO TO 1700
 2000 WRITE(6,2100)
 2100 FORMAT(5X,*NO SOLUTION COULD BE OBTAINED IF MIXING IS CONSIDERED*,
     1//,5X,*THE SOLUTION IS OBTAINED WITHOUT ALLOWING FOR MIXING*,//)
      WL11=WNO1
      WL21=WNO2
 1700 RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE LKEXFL(D,RHO1,RHO2,G,TIME,DT,NPRINT,ELO,ELU,VO,VU)
C     ******************************************************************
C
C     CALCULATES THE OVERFLOW AND UNDERFLOW FRONT VELOCITIES OF A LOCK
C     EXCHANGE FLOW AT ANY SPECIFIED TIME AFTER THE OPENING OF THE LOCK
C     GATE AS WELL AS THE DISTANCES TRAVELLED BY BOTH FRONTS AWAY FROM
C     THE GATE.
C
C     D               = TOTAL DEPTH OF WATER.
C     RHO1,RHO2       = DENSITIES (OR SPECIFIC GRAVITIES) OF THE LIGHTER
C                       AND HEAVIER FLUIDS RESPECTIVELY.
C     G               = GRAVITATIONAL ACCELERATION.
C     TIME            = SPECIFIED TIME AFTER THE GATE OPENING FOR WHICH
C                       VELOCITIES AND LENGTHS ARE COMPUTED.
C     DT              = TIME INCREMENT SPECIFIED BY THE USER (MAY BE
C                       MODIFIED INSIDE THE PROGRAM DEPENDING ON THE
C                       TRUNCATION ERROR).
C     NPRINT          = 1  IF A PRINTOUT OF TIME, VELOCITIES, AND LENGTHS
C                       AT THE INTERMEDIATE STEPS IS REQUIRED. OTHERWISE
C                       NO SUCH PRINTOUT IS PROVIDED.
C     ELO,ELU         = LENGTH AWAY FROM GATE OF OVERFLOW AND UNDERFLOW
C                       WEDGES RESPECTIVELY AT SPECIFIED TIME.
C     VO,VU           = FRONT VELOCITIES OF OVERFLOW AND UNDERFLOW
C                       RESPECTIVELY AT SPECIFIED TIME.
C     ALL UNITS MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- NONE.
C     ******************************************************************
C
      TOLER=0.0001
      DRHO=RHO2-RHO1
      RHOM=(RHO1+RHO2)/2.0
      ALPHA=G*DRHO*D/(2.0*RHOM)
      TO=VO=VU=ELO=ELU=ELI=0.0
      IF(NPRINT.NE.1) GO TO 10
      WRITE(6,20)
   20 FORMAT(1H1,10X,*TIME*,5X,*OVERFLOW VEL.*,5X,*OVERFLOW LENGTH*,5X,*
     1UNDERFLOW VEL.*,5X,*UNDERFLOW LENGTH*,///)
      WRITE(6,30) TO,VO,ELO,VU,ELU
   30 FORMAT(5X,F10.2,5X,F13.4,5X,F15.4,5X,F14.4,5X,F16.4)
      GO TO 10
   40 CONTINUE
C
C     EVALUATE RELATIVE VELOCITY AT DIFFERENT TIME INCREMENTS
C
      FUN=2.0*ALPHA*T/((D*D+2.0*ALPHA*T*T)**0.5)
C
C     APPLY RUNGE-KUTTA-MERSON ALGORITHM
C
      GO TO (101,102,103,104),NENTRY
   50 DT=DT/2.0
   10 T=TO
      NENTRY=1
      GO TO 40
  101 F1=FUN/3.0
      T=TO+DT/3.0
      NENTRY=2
      GO TO 40
  102 F3=FUN/3.0
      T=TO+DT/2.0
      NENTRY=3
      GO TO 40
  103 F4=FUN/3.0
      T=TO+DT
      NENTRY=4
      GO TO 40
  104 F5=FUN/3.0
      V=FUN
      EL=ELI+(F1+4.0*F4+F5)*DT/2.0
C
C     CHECK TRUNCATION ERROR
C
```

```
      ERROR=ABS((2.0*F1-9.0*F3+8.0*F4-F5)*DT/(10.0*EL))
      IF(ERROR.LT.(TOLER/32.0)) DT=2.0*DT
      IF(ERROR.GT.TOLER) GO TO 50
C
C     CALCULATE OVERFLOW AND UNDERFLOW VELOCITIES
C
      VO=1.27*V/2.27
      VU=V-VO
C
C     CALCULATE OVERFLOW AND UNDERFLOW WEDGE EXTENSIONS
C
      ELO=1.27*EL/2.27
      ELU=EL-ELO
      IF(NPRINT.NE.1) GO TO 60
      WRITE(6,30) T,VO,ELO,VU,ELU
C
C     CHECK TIME
C
   60 IF(ABS(T-TIME).LE.0.4) GO TO 70
      IF(T+DT-TIME) 80,80,90
   80 TO=T
      ELI=EL
      GO TO 10
   90 DT=TIME-T
      TO=T
      ELI=EL
      GO TO 10
   70 RETURN
      END
```

```
C     *****************************************************************
      SUBROUTINE FCHECK(Q,HORZ,VERT,NPTS,WL1,WL2,N,VISC1,VISC2,G,RHO1,
     1RHO2,RE,THETA,BL,FLOW)
C     *****************************************************************
C
C     CHECKS THE TYPES OF FLOW AND INTERFACIAL BOUNDARY LAYERS IN A TWO-
C     LAYER SYSTEM WITH ONE LAYER FLOWING.
C
C     Q                  = DISCHARGE OF THE FLOWING LAYER.
C     HORZ,VERT          = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                          HORIZONTAL  AND VERTICAL COORDINATES RESPECTIVELY
C                          OF THE POINTS DESCRIBING THE CHANNEL CROSS-
C                          SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO
C                          AN ARBITRARY VERTICAL AXIS AND THE VERTICAL
C                          COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID
C                          LEVELS.
C     NPTS               = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-
C                          SECTION.
C     WL1,WL2            = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                          LY. IF THE LOWER LAYER IS FLOWING, WL1 MAY BE
C                          GIVEN ANY ARBITRARY VALUE.
C     N                  = 1  IF THE UPPER LAYER IS FLOWING.
C                        = 2  IF THE LOWER LAYER IS FLOWING.
C     VISC1,VISC2        = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER
C                          FLUIDS RESPECTIVELY.
C     G                  = GRAVITATIONAL ACCELERATION.
C     RHO1,RHO2          = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                          AND LOWER FLUIDS RESPECTIVELY.
C     RE                 = REYNOLDS NUMBER OF THE FLOWING LAYER.
C     THETA              = A PARAMETER DEFINING THE TYPE OF THE INTERFACIAL
C                          BOUNDARY LAYERS.
C     BL                 = 1.0  IF THE BOUNDARY LAYERS ARE LAMINAR.
C                        = 2.0  IF THE BOUNDARY LAYERS ARE TURBULENT.
C     FLOW               = 1.0  IF THE FLOW IS LAMINAR.
C                        = 2.0  IF THE FLOW IS TURBULENT.
C     SAME DATUM MUST BE USED FOR WL1,WL2,VERT
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2,PROPS
C     *****************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS)
      DRHO=RHO2-RHO1
      G11=G*DRHO
      IF(N.EQ.2) GO TO 50
C
C     CASE OF A MOVING UPPER LAYER
C
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A1,A2,T1,T2,P1,P2,AY1,AY2)
      U=Q/A1
      HR1=A1/P1
      RE=HR1*U/VISC1
      G1=G11/RHO1
      THETA=(VISC2*G1/U**3)**(1.0/3.0)
      GO TO 60
C
C     CASE OF A MOVING LOWER LAYER
C
   50 CALL PROPS(HORZ,VERT,NPTS,WL2,A2,T2,P2,AY2)
      U=Q/A2
      HR1=A2/P2
      RE=HR1*U/VISC2
      G1=G11/RHO2
      THETA=(VISC1*G1/U**3)**(1.0/3.0)
C
C     CHECK FLOW TYPE (FLOW=1.0 IF LAMINAR ,=2.0 IF TURBULENT)
C
   60 IF(RE.LE.450.0) GO TO 10
      FLOW=2.0
      THETAC=0.178
      GO TO 20
   10 FLOW=1.0
      THETAC=0.127
```

```
C                                                                 *** 740
C     CHECK INTERFACIAL BOUNDARY LAYER (BL=1.0 IF LAMINAR,=2.0 IF TURB.) *** 750
C                                                                 *** 760
   20 IF(THETA.LT.THETAC) GO TO 30                                *** 770
      BL=1.0                                                      *** 780
      GO TO 40                                                    *** 790
   30 BL=2.0                                                      *** 800
   40 RETURN                                                      *** 810
      END                                                         *** 820
```

```
C     ******************************************************************
      SUBROUTINE FILMNR(Q,HORZ,VERT,NPTS,WL2,VISC2,FI)
C     ******************************************************************
C
C     INTERFACIAL SHEAR STRESS COEFFICIENT FOR LAMINAR FLOW AND LAMINAR
C     INTERFACIAL BOUNDARY LAYERS IN A TWO-LAYER SYSTEM.
C
C     Q              = DISCHARGE IN THE LOWER LAYER.
C     HORZ,VERT      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                      HORIZONTAL  AND VERTICAL COORDINATES RESPECTIVELY
C                      OF THE POINTS DESCRIBING THE CHANNEL CROSS-
C                      SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO
C                      AN ARBITRARY VERTICAL AXIS AND THE VERTICAL
C                      COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID
C                      LEVELS.
C     NPTS           = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-
C                      SECTION.
C     WL2            = ELEVATION OF THE INTERFACE.
C     VISC2          = KINEMATIC VISCOSITY OF THE LOWER FLUID.
C     FI             = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     SAME DATUM MUST BE USED FOR WL2 AND VERT
C     UNITS USED MUST BE COSISTENT THROUGHOUT.
C     ROUTINES USED- PROPS
C     ******************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS)
      CALL PROPS(HORZ,VERT,NPTS,WL2,A2,T2,P2,AY2)
      HR=A2/(P2+T2)
      U2=Q/A2
C
C     CALCULATE REYNOLDS NUMBER
C
      RE=HR*U2/VISC2
C
C     CALCULATE THE INTERFACIAL FRICTION FACTOR
C
      FI=11.3/RE
      RETURN
      END
```

```
C     *****************************************************************
      SUBROUTINE FITURB(Q,HORZ,VERT,NPTS,WL2,VISC2,FI)
C     *****************************************************************
C
C     INTERFACIAL SHEAR STRESS COEFFICIENT FOR TURBULENT FLOW AND TURBU-
C     LENT INTERFACIAL BOUNDARY LAYERS IN A TWO-LAYER SYSTEM.
C
C     Q             = DISCHARGE IN THE LOWER LAYER.
C     HORZ,VERT     = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                     HORIZONTAL  AND VERTICAL COORDINATES RESPECTIVELY
C                     OF THE POINTS DESCRIBING THE CHANNEL CROSS-
C                     SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO
C                     AN ARBITRARY VERTICAL AXIS AND THE VERTICAL
C                     COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID
C                     LEVELS.
C     NPTS          = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-
C                     SECTION.
C     WL2           = ELEVATION OF THE INTERFACE.
C     VISC2         = KINEMATIC VISCOSITY OF THE LOWER FLUID.
C     FI            = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     SAME DATUM MUST BE USED FOR WL2 AND VERT
C     UNITS USED MUST BE COSISTENT THROUGHOUT.
C     ROUTINES USED- PROPS
C     *****************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS)
      CALL PROPS(HORZ,VERT,NPTS,WL2,A2,T2,P2,AY2)
      V2=Q/A2
      H2=A2/T2
C
C     CALCULATE REYNOLDS NUMBER
C
      RE=4.0*V2*H2/VISC2
C
C     CALCULATE THE INTERFACIAL FRICTION FACTOR
C
      FI=0.316/(RE**0.25)
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE FBNDRY(Q,HORZ,VERT,NPTS,WL1,WL2,N,VISC1,VISC2,D,FO1,
     1FO2)
C     ******************************************************************
C
C     BOUNDARY SHEAR STRESS COEFFICIENT IN A TWO-LAYER SYSTEM WITH ONE
C     FLOWING LAYER.
C
C     Q               = DISCHARGE OF THE FLOWING LAYER.
C     HORZ,VERT       = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                       HORIZONTAL  AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DESCRIBING THE CHANNEL CROSS-
C                       SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO
C                       AN ARBITRARY VERTICAL AXIS AND THE VERTICAL
C                       COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID
C                       LEVELS.
C     NPTS            = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-
C                       SECTION.
C     WL1,WL2         = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                       LY. IF THE LOWER LAYER IS FLOWING, WL1 MAY BE
C                       GIVEN ANY ARBITRARY VALUE.
C     N               = 1  IF THE UPPER LAYER IS FLOWING.
C                     = 2  IF THE LOWER LAYER IS FLOWING.
C     VISC1,VISC2     = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER
C                       FLUIDS RESPECTIVELY.
C     D               = ROUGHNESS HEIGHT.
C     FO1,FO2         = BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER
C                       AND LOWER LAYERS RESPECTIVELY.
C     SAME DATUM MUST BE USED FOR WL1,WL2,VERT
C     UNITS USED MUST BE CONSISTENT THROUGHOUT.
C     ROUTINES USED- PROPS2,PROPS,ZSYSTM1(WHICH USES THE AUXILIARY FUN-
C     CTION AUX).
C     ******************************************************************
C
      EXTERNAL AUX
      DIMENSION HORZ(NPTS),VERT(NPTS),WA(3)
      IF(N.EQ.2) GO TO 10
C
C     CASE OF A MOVING UPPER LAYER
C
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A1,A2,T1,T2,P1,P2,AY1,AY2)
      U=Q/A1
      R=A1/P1
      RE=4.0*U*R/VISC1
      GO TO 20
C
C     CASE OF A MOVING LOWER LAYER
C
   10 CALL PROPS(HORZ,VERT,NPTS,WL2,A2,T2,P2,AY2)
      U=Q/A2
      R=A2/P2
      RE=4.0*U*R/VISC2
   20 IF(RE.GT.2000.0) GO TO 30
C
C     LAMINAR FLOW
C
      E=64.0/RE
      GO TO 40
   30 IF(D.EQ.0.0) GO TO 50
C
C     ROUGH TURBULENT FLOW
C
      DR=D/(4.0*R)
      EE=0.2
      EPS=1.0E-6
      NSIG=5
      M=1
      ITMAX=100
      CALL ZSYSTM1(AUX,EPS,NSIG,M,EE,ITMAX,DR,RE,R,B,HORZD,VERTD,1,BETAU
     1,BETAL,BOTD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,
     1RHO1,RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,
     1VERT,NPTS,RHO22,Q22,H22,WA,PAR,IER)
      E=EE**2.0
```

```
      IF(IER.NE.129.AND.IER.NE.130) GO TO 40
      WRITE(6,70)
   70 FORMAT(5X,*NO SOLUTION OBTAINED FROM FBNDRY*)
      FO1=FO2=0.0
      RETURN
C
C     SMOOTH TURBULENT FLOW
C
   50 E=0.316/(RE**0.25)
   40 IF(N.EQ.2) GO TO 60
      FO1=E
      FO2=0.0
      RETURN
   60 FO1=0.0
      FO2=E
      RETURN
      END
C     ******************************************************************
      FUNCTION AUX(EE,K,PAR,DR,RE,R,B,HORZD,VERTD,NPTSD,BETAU,BETAL,BOTD
     1,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,RHO2,A1,
     1A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,NPTS,
     1RHO22,Q22)
C     ******************************************************************
C
      AUX=(1.0/EE)+0.86*ALOG(DR/3.7+2.51/(RE*EE))
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE FILMTB(Q,HORZ,VERT,NPTS,WL1,WL2,RHO1,RHO2,VISC1,VISC2,
     1NINCR1,N,ELA,X,UREL1,UREL2,UI,YS,YS1,FI)
C     ******************************************************************
C
C     INTERFACIAL SHEAR STRESS COEFFICIENT FOR A TWO-LAYER SYSTEM IN A
C     CHANNEL OF ARBITRARY GEOMETRY. TURBULENT FLOW AND LAMINAR INTER-
C     FACIAL BOUNDARY LAYERS.
C
C     Q                  = DISCHARGE OF THE FLOWING LAYER.
C     HORZ,VERT          = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                          HORIZONTAL  AND VERTICAL COORDINATES RESPECTIVELY
C                          OF THE POINTS DESCRIBING THE CHANNEL CROSS-
C                          SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO
C                          AN ARBITRARY VERTICAL AXIS AND THE VERTICAL
C                          COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID
C                          LEVELS.
C     NPTS               = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-
C                          SECTION.
C     WL1,WL2            = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                          LY.
C     RHO1,RHO2          = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                          AND LOWER FLUIDS RESPECTIVELY.
C     VISC1,VISC2        = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER
C                          FLUIDS RESPECTIVELY.
C     NINCR1             = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFA-
C                          CIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION
C                          OF THE VELOCITY PROFILE IN EACH LAYER.
C                          NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PRO-
C                          FILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED
C                          NOT BE CALCULATED.
C     N                  = 1  IF THE UPPER LAYER IS FLOWING.
C                        = 2  IF THE LOWER LAYER IS FLOWING.
C     ELA                = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT
C                          OF CONTACT OF THE TWO FLUIDS.
C     X                  = DISTANCE FROM THE POINT OF CONTACT OF THE TWO
C                          FLUIDS TO THE POINT UNDER CONSIDERATION.
C     UREL1,UREL2        = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING
C                          VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY
C                          LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).
C     UI                 = VELOCITY AT THE INTERFACE RELATIVE TO THE MEAN
C                          VELOCITY OF FLOW.
C     YS,YS1             = THICKNESSES OF THE INTERFACIAL BOUNDARY LAYERS IN
C                          THE FLOWING AND THE STAGNANT LAYERS RESPECTIVELY.
C     FI                 = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     SAME DATUM MUST BE USED FOR WL1,WL2,VERT
C     UNITS USED MUST BE CONSISTENT THROUGHOUT.
C     ROUTINES USED- BOTTOM,PROPS2,ZSYSTM1(WHICH USES THE AUXILIARY FUN-
C     CTIONS AUX1,AUX2,AUX3,AUX4,AUX5).
C     ******************************************************************
C
      EXTERNAL AUX1,AUX2,AUX3,AUX4,AUX5
      DIMENSION HORZ(NPTS),VERT(NPTS),X1(6),WA1(38),X2(2),WA2(8),X3(2),
     1WA3(8),X4(6),WA4(38),WA5(3)
      DIMENSION UREL1(NINCR1),UREL2(NINCR1)
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A1,A2,T1,T2,P1,P2,AY1,AY2)
      H1=WL1-WL2
      H2=WL2-BOT
      IF(N.EQ.2) GO TO 10
C
C     CASE OF A MOVING UPPER LAYER
C
      RHOA=RHO1
      RHOB=RHO2
      VISCA=VISC1
      VISCB=VISC2
      U=Q/A1
      HA=H1
      HB=H2
      GO TO 20
C
C     CASE OF A MOVING LOWER LAYER
```

```
C
   10 RHOA=RHO2
      RHOB=RHO1
      VISCA=VISC2
      VISCB=VISC1
      U=Q/A2
      HA=H2
      HB=H1
C
C     APPLY KEULEGAN,S METHOD
C
   20 R=(RHOB/RHOA)*SQRT(VISCB/VISCA)
      X1(1)=0.60
      X1(2)=3.00
      X1(3)=-0.30
      X1(4)=4.00
      EPS=1.0E-6
      NSIG=5
      N1=4
      ITMAX=100
      CALL ZSYSTM1(AUX1,EPS,NSIG,N1,X1,ITMAX,F1,F2,R,B,HD,VD,1,
     1BETAU,BETAL,BOTD,S1,S2,S3,S4,S5,S6,S7,AAY2,H1,H2,BOTU,Q1,Q2,RHO1,
     1RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,
     1NPTS,RHO22,Q22,H22,WA1,PAR1,IER1)
      X1(5)=-X1(3)*R
      X1(6)=X1(1)
      X2(1)=-0.06
      N2=1
      CALL ZSYSTM1(AUX2,EPS,NSIG,N2,X2,ITMAX,F1,F2,R,B,HD,VD,1,
     1BETAU,BETAL,BOTD,S1,S2,S3,S4,S5,S6,S7,AAY2,X1,H2,BOTU,Q1,Q2,RHO1,
     1RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,
     1NPTS,RHO22,Q22,H22,WA2,PAR2,IER2)
      X2(2)=-0.10-8.0*X2(1)/15.0
      X3(1)=-0.30
      N3=1
      CALL ZSYSTM1(AUX3,EPS,NSIG,N3,X3,ITMAX,F1,F2,R,B,HD,VD,1,
     1BETAU,BETAL,BOTD,S1,S2,S3,S4,S5,S6,S7,AAY2,X1,H2,BOTU,Q1,Q2,RHO1,
     1RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,
     1NPTS,RHO22,Q22,H22,WA3,PAR3,IER3)
      X3(2)=-0.10-8.0*X3(1)/15.0
      S1=0.5*(1.0+16.0*X2(1)/15.0+X2(2))
      S2=0.5*(1.0+16.0*X3(1)/15.0+X3(2))
      S3=(0.15+0.26667*X2(1)+0.21425*X2(2))/(1.0+X2(2))
      S4=(0.15+0.26667*X3(1)+0.21425*X3(2))/(1.0+X3(2))
      S5=(0.029365+0.07619*X2(1)+0.07625*X2(2)+0.0308*X2(1)**2+
     10.0758*X2(1)*X2(2)+0.04*X2(2)**2)/(1.0+X2(2))
      S6=(0.029365+0.07619*X3(1)+0.07625*X3(2)+0.0308*X3(1)**2+
     10.0758*X3(1)*X3(2)+0.04*X3(2)**2)/(1.0+X3(2))
      S7=(1.0+X3(2))/(1.0+X2(2))
      X4(1)=0.60
      X4(2)=3.00
      X4(3)=-0.30
      X4(4)=6.00
      N4=4
      CALL ZSYSTM1(AUX4,EPS,NSIG,N4,X4,ITMAX,F1,F2,R,B,HD,VD,1,
     1BETAU,BETAL,BOTD,S1,S2,S3,S4,S5,S6,S7,AAY2,X1,H2,BOTU,Q1,Q2,RHO1,
     1RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,
     1NPTS,RHO22,Q22,H22,WA4,PAR4,IER4)
      X4(5)=-(X4(3)*S7*R)
      X4(6)=X4(1)
      EM=X4(2)*SQRT(2.0)
      EM1=X4(4)*SQRT(2.0)
C
C     CALCULATE THICKNESSES OF THE INTERFACIAL BOUNDARY LAYERS
C
      SQ=SQRT(X/U)
      YS=EM*SQRT(VISCA)*SQ
      YS1=EM1*SQRT(VISCB)*SQ
      XXA=HA**2*U/(VISCA*EM**2)
      XXB=HB**2*U/(VISCB*EM1**2)
      IF(NINCR1.EQ.0) GO TO 80
C
C     CALCULATE VELOCITY PROFILES
C
```

```
      FF=1.0/(FLOAT(NINCR1-1))
      DO 30 I=1,NINCR1
      NNN=I-1
      Y=NNN*FF
      A1=Y-Y**3.0+0.5*Y**4.0
      A2=16.0*Y**3.0/3.0-8.0*Y**4.0+16.0*Y**5.0/5.0
      A3=Y-10.0*Y**3.0+55.0*Y**4.0/2.0-27.0*Y**5.0+9.0*Y**6.0
      UREL1(I)=X4(1)+X4(5)*X4(2)*(A1+X2(1)*A2+X2(2)*A3)
   30 UREL2(I)=X4(6)+X4(3)*X4(4)*(A1+X3(1)*A2+X3(2)*A3)
C
C     CALCULATE INTERFACIAL VELOCITY
C
      UI=UREL1(1)
C
C     CHECK FLOW ESTABLISHMENT
C
   80 IF(XXA.GT.X.AND.XXB.GT.X) GO TO 50
      IF(XXA.LT.X.AND.XXB.GT.X) GO TO 60
      IF(XXA.GT.X.AND.XXB.LT.X) GO TO 60
C
C     FLOW ESTABLISHED IN BOTH LAYERS(UNIFORM FLOW)
C
      RR=4.0*U*HA/VISCA
      EN=HA*VISCA*RHOB/(HB*VISCB*RHOA)
      FI=384.0*(3.0+EN)/((3.0+4.0*EN)*RR)
      RETURN
C
C     FLOW ESTABLISHED IN ONE LAYER ONLY
C
   60 EMM=HA*VISCB*RHOB**2.0/(ELA*VISCA*RHOA**2.0)
      RR=4.0*U*HA/VISCA
      FI=0.01
      N5=1
      CALL ZSYSTM1(AUX5,EPS,NSIG,N5,FI,ITMAX,RR,EMM,R,B,HD,VD,1,
     1BU,BL,BD,HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,H1,H2,BU,Q1,Q2,RHO1,
     1RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,
     1NPTS,RHO22,Q22,H22,WA5,PAR5,IER5)
      RETURN
C
C     FLOW NON-ESTABLISHED IN BOTH LAYERS
C
   50 S=X4(5)*(1.0+X2(2))*SQRT(2.0)
      SS=4.0*S
      RX=U*X/VISCA
      FI=SS/SQRT(RX)
      RETURN
      END
C     *****************************************************************
      FUNCTION AUX1(X1,K,PAR1,F1,F2,R,B,HD,VD,ND,BU,BL,BD,
     1HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,HM,HN,BOTU,Q1,Q2,RHO1,RHO2,A1,A2
     1,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,NPTS,
     1RHO22,Q22)
C     *****************************************************************
C
      DIMENSION X1(6)
      GO TO (5,10,15,20),K
    5 AUX1=X1(1)-0.5*X1(2)*X1(3)*R-1.0
      RETURN
   10 AUX1=X1(1)+0.5*X1(3)*X1(4)
      RETURN
   15 AUX1=X1(3)**2-0.15*X1(1)*X1(3)**2*X1(4)**2-0.023936*X1(3)**3*
     1X1(4)**3
      RETURN
   20 AUX1=(X1(3)*R)**2-0.15*X1(1)*X1(2)**2*(X1(3)*R)**2+0.023936*X1(2)
     1**3*(X1(3)*R)**3
      RETURN
      END
C     *****************************************************************
      FUNCTION AUX2(X2;K;PAR2;F1;F2;R;B;HD;VD;ND;BU;BL;BD;
     1HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,X1,HN,BOTU,Q1,Q2,RHO1,RHO2,A1,A2
     1,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,NPTS,
     1RHO22,Q22)
C     *****************************************************************
C
```

```
      DIMENSION X2(2),X1(6)
      AUX2=64.0*X2(1)+(4.0*X1(1)*(1.0-X1(1))**2/X1(5)**2)*((1.0-0.1-8.0
     1*X2(1)/15.0)/(1.0+16.0*X2(1)/15.0-0.1-8.0*X2(1)/15.0)**2)
      RETURN
      END
C     *****************************************************************
      FUNCTION AUX3(X3,K,PAR3,F1,F2,R,B,HD,VD,ND,BU,BL,BD,
     1HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,X1,HN,BOTU,Q1,Q2,RHO1,RHO2,A1,A2
     1,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,NPTS,
     1RHO22,Q22)
C     *****************************************************************
C
      DIMENSION X3(2),X1(6)
      AUX3=64.0*X3(1)+(4.0*X1(6)**3/X1(3)**2)*((1.0-0.1-8.0*X3(1)/15.0)
     1/(1.0+16.0*X3(1)/15.0-0.1-8.0*X3(1)/15.0)**2)
      RETURN
      END
C     *****************************************************************
      FUNCTION AUX4(X4,K,PAR4,F1,F2,R,B,HD,VD,ND,BU,BL,BD,
     1S1,S2,S3,S4,S5,S6,S7,AAY2,X1,HN,BOTU,Q1,Q2,RHO1,RHO2,A1,A2,A1D,A2D
     1,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,NPTS,
     1RHO22,Q22)
C     *****************************************************************
C
      DIMENSION X4(6),X1(6)
      GO TO (5,10,15,20),K
    5 AUX4=X4(1)-S1*X4(2)*X4(3)*S7*R-1.0
      RETURN
   10 AUX4=X4(1)+S2*X4(3)*X4(4)
      RETURN
   15 AUX4=X4(3)**2-S4*X4(1)*X4(3)**2*X4(4)**2-S6*X4(3)**3*X4(4)**3
      RETURN
   20 AUX4=(X4(3)*S7*R)**2-S3*(X4(3)*S7*R)**2*X4(1)*X4(2)**2+S5*(X4(3)
     1*S7*R)**3*X4(2)**3
      RETURN
      END
C     *****************************************************************
      FUNCTION AUX5(FI,K,PAR5,F1,F2,R,B,HD,VD,ND,BU,BL,BD,
     1HH1,HH2,AA1,AA2,TT1,TT2,AAY1,AAY2,HM,HN,BOTU,Q1,Q2,RHO1,RHO2,A1,A2
     1,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,NPTS,
     1RHO22,Q22)
C     *****************************************************************
C
      AUX5=SQRT(F1*F2)*(384.0-FI*F1)**1.5-31.2*(4.0*FI*F1-384.0)
      RETURN
      END
```

```
C     ****************************************************************
      SUBROUTINE FINTRF(Q,HORZ,VERT,NPTS,WL1,WL2,VISC1,VISC2,G,RHO1,RHO2
     1,RE,THETA,BL,FLOW,NINCR1,N,ELA,X,UREL1,UREL2,UI,FI)
C     ****************************************************************
C
C      INTERFACIAL SHEAR STRESS COEFFICIENT FOR A TWO-LAYER SYSTEM IN
C      A CHANNEL OF ARBITRARY GEOMETRY.
C
C      Q              = DISCHARGE OF THE FLOWING LAYER.
C      HORZ,VERT      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                       HORIZONTAL  AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DESCRIBING THE CHANNEL CROSS-
C                       SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO
C                       AN ARBITRARY VERTICAL AXIS AND THE VERTICAL
C                       COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID
C                       LEVELS.
C      NPTS           = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-
C                       SECTION.
C      WL1,WL2        = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                       LY.
C      VISC1,VISC2    = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER
C                       FLUIDS RESPECTIVELY.
C      G              = GRAVITATIONAL ACCELERATION.
C      RHO1,RHO2      = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND LOWER FLUIDS RESPECTIVELY.
C      RE             = REYNOLDS NUMBER OF THE FLOWING LAYER.
C      THETA          = A PARAMETER DEFINING THE TYPE OF THE INTERFACIAL
C                       BOUNDARY LAYERS.
C      BL             = 1.0  IF THE BOUNDARY LAYERS ARE LAMINAR.
C                     = 2.0  IF THE BOUNDARY LAYERS ARE TURBULENT.
C      FLOW           = 1.0  IF THE FLOW IS LAMINAR.
C                     = 2.0  IF THE FLOW IS TURBULENT.
C      NINCR1         = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFA-
C                       CIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION
C                       OF THE VELOCITY PROFILE IN EACH LAYER.
C                       NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PRO-
C                       FILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED
C                       NOT BE CALCULATED.
C      N              = 1  IF THE UPPER LAYER IS FLOWING.
C                     = 2  IF THE LOWER LAYER IS FLOWING.
C      ELA            = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT
C                       OF CONTACT OF THE TWO FLUIDS.
C      X              = DISTANCE FROM THE POINT OF CONTACT OF THE TWO
C                       FLUIDS TO THE POINT UNDER CONSIDERATION.
C      UREL1,UREL2    = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING
C                       VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY
C                       LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).
C      UI             = VELOCITY AT THE INTERFACE RELATIVE TO THE MEAN
C                       VELOCITY OF FLOW.
C      FI             = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     SAME DATUM MUST BE USED FOR WL1,WL2,VERT
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- FCHECK,FILMNR,FITURB,FILMTB
C     ****************************************************************
C
      EXTERNAL AUX1,AUX2,AUX3,AUX4,AUX5
      DIMENSION HORZ(NPTS),VERT(NPTS),UREL1(NINCR1),UREL2(NINCR1)
C
C     CHECK TYPES OF FLOW AND INTERFACIAL BOUNDARY LAYERS
C
      CALL FCHECK(Q,HORZ,VERT,NPTS,WL1,WL2,N,VISC1,VISC2,G,RHO1,RHO2,RE,
     1THETA,BL,FLOW)
      IF(BL.EQ.1.0.AND.FLOW.EQ.1.0) GO TO 10
      IF(BL.EQ.1.0.AND.FLOW.EQ.2.0) GO TO 20
      IF(BL.EQ.2.0.AND.FLOW.EQ.2.0) GO TO 30
   10 IF(N.NE.2) GO TO 40
C
C     LAMINAR FLOW/LAMINAR BOUNDARY LAYER
C
      CALL FILMNR(Q,HORZ,VERT,NPTS,WL2,VISC2,FI)
      RETURN
C
```

```
C     TURBULENT FLOW/LAMINAR BOUNDARY LAYER
C
   20 CALL FILMTB(Q,HORZ,VERT,NPTS,WL1,WL2,RHO1,RHO2,VISC1,VISC2,NINCR1,
     1N,ELA,X,UREL1,UREL2,UI,YS,YS1,FI)
      RETURN
   30 IF(N.NE.2) GO TO 40
C
C     TURBULENT FLOW/TURBULENT BOUNDARY LAYER
C
      CALL FITURB(Q,HORZ,VERT,NPTS,WL2,VISC2,FI)
      RETURN
   40 UI=FI=0.0
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE ENGRAD(HORZ,VERT,NPTS,RHO1,RHO2,VISC1,VISC2,G,Q1,Q2,WL1
     1,WL2,D,FO1,FO2,BL,FLOW,NINCR1,ELA,X,UREL1,UREL2,UI,FI,S1E,S2E)
C     ******************************************************************
C
C     ENERGY GRADIENTS OF A TWO-LAYER SYSTEM.
C
C     HORZ,VERT       = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                       HORIZONTAL  AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DESCRIBING THE CHANNEL CROSS-
C                       SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO
C                       AN ARBITRARY VERTICAL AXIS AND THE VERTICAL
C                       COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID
C                       LEVELS.
C     NPTS            = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-
C                       SECTION.
C     RHO1,RHO2       = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND LOWER FLUIDS RESPECTIVELY.
C     VISC1,VISC2     = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER
C                       FLUIDS RESPECTIVELY.
C     G               = GRAVITATIONAL ACCELERATION.
C     Q1,Q2           = DISCHARGE OF THE UPPER AND LOWER LAYERS RESPECTI-
C                       IVELY. SHOULD HAVE DIFFERENT SIGNS IF OPPOSITE.
C     WL1,WL2         = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                       LY.
C     D               = CROSS-SECTIONAL ROUGHNESS HEIGHT.
C     FO1,FO2         = SHEAR STRESS COEFFICIENTS AT THE SOLID BOUNDARY
C                       FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.
C     BL              = 1.0  IF INTERFACIAL BOUNDARY LAYERS ARE LAMINAR.
C                     = 2.0  IF THE BOUNDARY LAYERS ARE TURBULENT.
C     FLOW            = 1.0  IF THE FLOW IS LAMINAR.
C                     = 2.0  IF THE FLOW IS TURBULENT.
C     NINCR1          = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFA-
C                       CIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION
C                       OF THE VELOCITY PROFILE IN EACH LAYER.
C                       NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PRO-
C                       FILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED
C                       NOT BE CALCULATED.
C     ELA             = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT
C                       OF CONTACT OF THE TWO FLUIDS.
C     X               = DISTANCE FROM THE POINT OF CONTACT OF THE TWO
C                       FLUIDS TO THE POINT UNDER CONSIDERATION.
C     UREL1,UREL2     = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING
C                       VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY
C                       LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).
C     UI              = VELOCITY AT THE INTERFACE RELATIVE TO THE MEAN
C                       VELOCITY OF FLOW.
C     FI              = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     S1E,S2E         = ENERGY GRADIENTS AT THE CROSS-SECTION UNDER CONS-
C                       IDERATION OF THE UPPER AND LOWER LAYERS RESPECTI-
C                       VELY. THESE GRADIENTS ARE CALCULATED IN THE POSI-
C                       TIVE FLOW DIRECTION,I.E., IF THE COMPUTED GRADIE-
C                       NT IS NEGATIVE, IT INDICATES THAT THE ENERGY OF
C                       THE CORRESPONDING LAYER IS DECREASING IN THE
C                       POSITIVE FLOW DIRECTION (THE POSITIVE DIRECTION
C                       DEFINED BY THE GIVEN DISCHARGES).
C     SAME DATUM MUST BE USED FOR WL1,WL2,VERT
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2,FBNDRY,FINTRF (THE LAST TWO ROUTINES ARE
C     NEEDED ONLY IN THE CASE OF ONE FLOWING LAYER).
C     ******************************************************************
C
      EXTERNAL AUX,AUX1,AUX2,AUX3,AUX4,AUX5
      DIMENSION HORZ(NPTS),VERT(NPTS),UREL1(NINCR1),UREL2(NINCR1)
      IF(Q1.NE.0.0.AND.Q2.NE.0.0) GO TO 30
      IF(Q1.EQ.0.0) GO TO 10
      Q=Q1
      N=1
      GO TO 20
   10 Q=Q2
      N=2
C
```

```
C     CALCULATE BOUNDARY FRICTION FACTORS
C
   20 CALL FBNDRY(Q,HORZ,VERT,NPTS,WL1,WL2,N,VISC1,VISC2,D,FO1,FO2)
C
C     CALCULATE INTERFACIAL FRICTION FACTORS
C
      CALL FINTRF(Q,HORZ,VERT,NPTS,WL1,WL2,VISC1,VISC2,G,RHO1,RHO2,RE,
     1THETA,BL,FLOW,NINCR1,N,ELA,X,UREL1,UREL2,UI,FII)
      IF(FII.EQ.0.0) GO TO 30
      FI=FII
C
C     CALCULATE CROSS-SECTION PROPERTIES
C
   30 CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A1,A2,T1,T2,P1,P2,AY1,AY2)
      U2=Q2/A2
      U1=Q1/A1
C
C     CALCULATE BOUNDARY SHEAR STRESS
C
      TO1=(FO1*RHO1/8.0)*U1*ABS(U1)
      TO2=(FO2*RHO2/8.0)*U2*ABS(U2)
      A=((RHO1+RHO2)/16.0)*(U1-U2)*ABS(U1-U2)
C
C     CALCULATE INTERFACIAL SHEAR STRESS
C
      TI=FI*A
      A11=(P1-T2)*TO1
      A22=RHO1*G*A1
      A33=TO2*P2
      A44=RHO2*G*A2
C
C     CALCULATE ENERGY GRADIENTS
C
      S1E=-(A11+TI*T2)/A22
      S2E=-(A33-TI*T2)/A44
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE SURGRD(HORZ,VERT,NPTS,VERTT,SO,RHO1,RHO2,VISC1,VISC2,G,
     1Q1,Q2,ALPHA1,ALPHA2,WL1,WL2,D,FO1,FO2,BL,FLOW,NINCR1,ELA,X,UREL1,
     1UREL2,UI,FI,S1E,S2E,S1,S2)
C     ******************************************************************
C
C     SURFACE SLOPES OF A TWO-LAYER SYSTEM.
C
C     HORZ,VERT      = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                      HORIZONTAL  AND VERTICAL COORDINATES RESPECTIVELY
C                      OF THE POINTS DESCRIBING THE CHANNEL CROSS-
C                      SECTION. THE HORIZONTAL COORDS. ARE REFERRED TO
C                      AN ARBITRARY VERTICAL AXIS AND THE VERTICAL
C                      COORDS. ARE REFERRED TO THE SAME DATUM AS FLUID
C                      LEVELS.
C     NPTS           = NUMBER OF POINTS DESCRIBING THE CHANNEL CROSS-
C                      SECTION.
C     VERTT          = ONE-DIMENSIONAL WORKING ARRAY OF SIZE NPTS.
C     SO             = CHANNEL BED SLOPE AT THE CROSS-SECTION UNDER CON-
C                      SIDERATION. NEGATIVE IF SLOPING IN THE POSITIVE
C                      FLOW DIRECTION (SEE DEFINITION OF S1E AND S2E).
C     RHO1,RHO2      = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                      AND LOWER FLUIDS RESPECTIVELY.
C     VISC1,VISC2    = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER
C                      FLUIDS RESPECTIVELY.
C     G              = GRAVITATIONAL ACCELERATION.
C     Q1,Q2          = DISCHARGE OF THE UPPER AND LOWER LAYERS RESPECTI-
C                      IVELY. SHOULD HAVE DIFFERENT SIGNS IF OPPOSITE.
C     ALPHA1,ALPHA2  = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER
C                      AND LOWER LAYER RESPECTIVELY.
C     WL1,WL2        = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                      LY.
C     D              = CROSS-SECTIONAL ROUGHNESS HEIGHT.
C     FO1,FO2        = SHEAR STRESS COEFFICIENTS AT THE SOLID BOUNDARY
C                      FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.
C     BL             = 1.0  IF INTERFACIAL BOUNDARY LAYERS ARE LAMINAR.
C     BL             = 2.0  IF THE BOUNDARY LAYERS ARE TURBULENT.
C     FLOW           = 1.0  IF THE FLOW IS LAMINAR.
C                    = 2.0  IF THE FLOW IS TURBULENT.
C     NINCR1         = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFA-
C                      CIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION
C                      OF THE VELOCITY PROFILE IN EACH LAYER.
C                      NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PRO-
C                      FILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED
C                      NOT BE CALCULATED.
C     ELA            = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT
C                      OF CONTACT OF THE TWO FLUIDS.
C     X              = DISTANCE FROM THE POINT OF CONTACT OF THE TWO
C                      FLUIDS TO THE POINT UNDER CONSIDERATION.
C     UREL1,UREL2    = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING
C                      VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY
C                      LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).
C     UI             = VELOCITY AT THE INTERFACE RELATIVE TO THE MEAN
C                      VELOCITY OF FLOW.
C     FI             = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     S1E,S2E        = ENERGY GRADIENTS AT THE CROSS-SECTION UNDER CONS-
C                      IDERATION OF THE UPPER AND LOWER LAYERS RESPECTI-
C                      VELY. THESE GRADIENTS ARE CALCULATED IN THE POSI-
C                      TIVE FLOW DIRECTION,I.E., IF THE COMPUTED GRADIE-
C                      NT IS NEGATIVE, IT INDICATES THAT THE ENERGY OF
C                      THE CORRESPONDING LAYER IS DECREASING IN THE
C                      POSITIVE FLOW DIRECTION (THE POSITIVE DIRECTION
C                      DEFINED BY THE GIVEN DISCHARGES).
C     S1,S2          = FREE SURFACE AND INTERFACE SLOPES RESPECTIVELY AT
C                      THE CROSS-SECTION UNDER CONSIDERATION. SIGN CON-
C                      VENTION IS THE SAME AS SO,S1E AND S2E.
C     SAME DATUM MUST BE USED FOR WL1,WL2,VERT
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2,ENGRAD
C     ******************************************************************
C
      EXTERNAL AUX,AUX1,AUX2,AUX3,AUX4,AUX5
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```
      DIMENSION HORZ(NPTS),VERT(NPTS),UREL1(NINCR1),UREL2(NINCR1),VERTT
     1(NPTS)
C
C     CALCULATE ENERGY GRADIENTS
C
      CALL ENGRAD(HORZ,VERT,NPTS,RHO1,RHO2,VISC1,VISC2,G,Q1,Q2,WL1,WL2,
     1D,FO1,FO2,BL,FLOW,NINCR1,ELA,X,UREL1,UREL2,UI,FI,S1E,S2E)
      DRHO=RHO2-RHO1
      BETA=RHO2/DRHO
      BETA1=RHO1/DRHO
      G1=G/BETA
C
C     CALCULATE SECTION PROPERTIES
C
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A1,A2,T1,T2,P1,P2,AY1,AY2)
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)
      DWLO=(WL2-BOT)*0.001
      DO 20 I=1,NPTS
      VERTT(I)=VERT(I)+DWLO
   20 CONTINUE
      CALL PROPS(HORZ,VERTT,NPTS,WL2,ADA,TDT,PDP,AYDY)
      F12=ALPHA1*Q1*Q1*T1/(G*A1*A1*A1)
      FI22=ALPHA2*Q2*Q2*T2/(G1*A2*A2*A2)
      F112=F12*T2/T1
      X1=1.0-F12
      X2=1.0-FI22
      XC=(X1*X2)-BETA1*F112
      IF(XC.EQ.0.0) GO TO 10
C
C     CALCULATE SURFACE SLOPES
C
      TO=(A2-ADA)/DWLO
      F202=FI22*TO/T2
      S1=(X2*S1E-BETA*F112*S2E+F112*F202*SO)/XC
      S2=(-S1E*BETA1+BETA*X1*S2E-X1*F202*SO)/XC
      RETURN
C
C     ASSIGN LARGE VALUES FOR SURFACE SLOPES IF FLOW IS CRITICAL
C
   10 S1=S2=1.0E20
      RETURN
      END
```

```
C     *****************************************************************
      SUBROUTINE FLPROF(HORZAR,VERTAR,CHAINR,NXSEC,NPTS,NSEC1,WL1,WL2,
     1RHO1,RHO2,VISC1,VISC2,G,Q1,Q2,ALPHA1,ALPHA2,DAR,FO1,FO2,FI,UREL1,
     1UREL2,HORZ1,VERT1,HORZ2,VERT2,HORZ,VERT,VERTT,NINCR1,ELA,DX,NPRINT
     1,WL11,WL21,ENLV1,ENLV2)
C     *****************************************************************
C
C     SURFACE PROFILES IN A TWO-LAYER SYSTEM- SINGLE REACH.
C
C     HORZAR,VERTAR  = TWO-DIMENSIONAL ARRAYS OF SIZE (NXSEC,NPTS) CONT-
C                      AINING THE HORIZONTAL AND VERTICAL COORDINATES
C                      OF THE POINTS DESCRIBING EACH OF A SERIES OF
C                      CROSS-SECTIONS DEFINING THE CHANNEL. THE FIRST
C                      VALUE OF THE ARRAY REPRESENTS THE SECTION NUMBER
C                      AND THE SECOND VALUE REPRESENTS ONE OF THE HORIZ-
C                      ONTAL (OR VERTICAL) COORDINATES.
C     CHAINR         = ONE-DIMENSIONAL ARRAY OF SIZE NXSEC CONTAINING
C                      THE CHAINAGE VALUES FOR THE SERIES OF CROSS-
C                      SECTIONS.
C                      CHAINAGE VALUES ARE REFERRED TO THE POINT OF
C                      CONTACT OF THE TWO LAYERS AND INCREASE IN THE
C                      DOWNSTREAM DIRECTION. UPSTREAM AND DOWNSTREAM
C                      DIRECTIONS ARE DEFINED ACCORDING TO THE SIGNS OF
C                      THE GIVEN DISCHARGES (I.E. IF THE TWO LAYERS ARE
C                      FLOWING IN OPPOSITE DIRECTIONS, THE UPSTREAM
C                      DIRECTION IS THE DIRECTION OF FLOW OF THE LAYER
C                      THAT HAS THE POSITIVE DISCHARGE).
C     NXSEC          = THE NUMBER OF CROSS-SECTIONS IN THE HYDRAULIC
C                      PROFILE.
C     NPTS           = NUMBER OF POINTS DESCRIBING EACH OF THE CROSS-
C                      SECTIONS.
C     NSEC1          = SECTION NUMBER WHERE THE ELEVATIONS ARE SOUGHT.
C     WL1,WL2        = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                      LY AT ONE END OF THE REACH. THE REACH IS THE
C                      PORTION OF THE CHANNEL BETWEEN TWO SUCCESSIVE
C                      CROSS-SECTIONS.
C     RHO1,RHO2      = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                      AND LOWER FLUIDS RESPECTIVELY.
C     VISC1,VISC2    = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER
C                      FLUIDS RESPECTIVELY.
C     G              = GRAVITATIONAL ACCELERATION.
C     Q1,Q2          = DISCHARGE OF THE UPPER AND LOWER LAYERS RESPECTI-
C                      IVELY. SHOULD HAVE DIFFERENT SIGNS IF OPPOSITE.
C     ALPHA1,ALPHA2  = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER
C                      AND LOWER LAYER RESPECTIVELY.
C     DAR            = ONE-DIMENSIONAL ARRAY OF SIZE NXSEC CONTAINING
C                      THE ROUGHNESS HEIGHT FOR EACH CROSS-SECTION.
C     FO1,FO2        = SHEAR STRESS COEFFICIENTS AT THE SOLID BOUNDARY
C                      FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.
C     FI             = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     UREL1,UREL2    = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING
C                      VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY
C                      LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).
C     HORZ1,VERT1    = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
C     HORZ2,VERT2    = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
C     HORZ  ,VERT    = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
C     VERTT          = ONE-DIMENSIONAL WORKING ARRAY  OF SIZE NPTS.
C     NINCR1         = NUMBER OF INCREMENTAL DEPTHS WITHIN THE INTERFA-
C                      CIAL BOUNDARY LAYERS THICKNESSES FOR CALCULATION
C                      OF THE VELOCITY PROFILE IN EACH LAYER.
C                      NINCR1 IS SET EQUAL TO ZERO IF THE VELOCITY PRO-
C                      FILES WITHIN THE INTERFACIAL BOUNDARY LAYERS NEED
C                      NOT BE CALCULATED.
C     ELA            = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT
C                      OF CONTACT OF THE TWO FLUIDS.
C     DX             = INCREMENTAL DISTANCE TO BE USED FOR THE INTEGRA-
C                      TION. THE SIGN OF DX DEPENDS ON THE DIRECTION OF
C                      INTEGRATION (POSITIVE IF THE INTEGRATION IS IN
C                      THE DOWNSTREAM DIRECTION AND VISE VERSA).
C     NPRINT         = AN INTEGER VARIABLE SET EQUAL TO 1 IF A PRINTOUT
C                      OF PROFILES COORDINATES AT THE INTERMEDIATE STEPS
C                      IS REQUIRED.
C     WL11,WL21      = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIV-
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C                   ELY AT THE OTHER END OF THE REACH.
C     ENLV1,ENLV2     = TOTAL ENERGY LEVELS FOR THE UPPER AND LOWER LAY-
C                       ERS RESPECTIVELY AT THE OTHER END OF THE REACH.
C     SAME DATUM MUST BE USED FOR VERTAR,WL1,WL2,WL11,WL21,ENLV1,ENLV2
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2,SELSEC,WLCRIT1,SURGRD,BOTTOM
C     ******************************************************************
C
      EXTERNAL AUX,AUX1,AUX2,AUX3,AUX4,AUX5
      DIMENSION HORZAR(NXSEC,NPTS),VERTAR(NXSEC,NPTS),CHAINR(NXSEC),
     1HORZ1(NPTS),VERT1(NPTS),HORZ2(NPTS),VERT2(NPTS),HORZ(NPTS),
     1VERT(NPTS),DAR(NXSEC),VERTT(NPTS)
      TOLER=0.0001
      TOLER1=TOLER/32.0
      E1=ALPHA1*Q1*Q1*0.5/G
      E2=ALPHA2*Q2*Q2*0.5/G
      DRHO=RHO2-RHO1
      IF(DX.LT.0.0) GO TO 10
      M=-1
      GO TO 20
   10 M=1
   20 IF(NPRINT.NE.1) GO TO 30
      WRITE(6,40)
   40 FORMAT(1H1,5X,*CHAINAGE*,6X,*FREE SURFACE LEVEL*,6X,*INTERFACE LEV
     1EL*,///)
   30 IPLUS1=NSEC1+M
C
C     SELECT COORDINATES OF END SECTIONS OF THE REACH
C
      CALL SELSEC(HORZAR,VERTAR,NXSEC,NPTS,IPLUS1,HORZ2,VERT2)
      CALL SELSEC(HORZAR,VERTAR,NXSEC,NPTS,NSEC1,HORZ1,VERT1)
      XO=CHAINR(IPLUS1)
      Z12=M*(CHAINR(IPLUS1)-CHAINR(NSEC1))
      D=(DAR(NSEC1)+DAR(IPLUS1))/2.0
C
C     COORDINATES OF LOWEST POINTS FOR END SECTIONS
C
      BOT1=VERT1(1)
      HBOT1=HORZ1(1)
      BOT2=VERT2(1)
      HBOT2=HORZ2(1)
      DO 700 J=1,NPTS
      IF(VERT1(J).GT.BOT1) GO TO 700
      BOT1=VERT1(J)
      HBOT1=HORZ1(J)
  700 CONTINUE
      DO 710 J=1,NPTS
      IF(VERT2(J).GT.BOT2) GO TO 710
      BOT2=VERT2(J)
      HBOT2=HORZ2(J)
  710 CONTINUE
      IF(Z12.GT.0.0) GO TO 50
C
C     ASSUME CRITICAL CONDITION AT CASCADE REACHES
C
      X1=XO
      WL11=WL1
      IF(BOT1.GT.BOT2) GO TO 600
      CALL WLCRIT1(Q1,Q2,WL11,RHO1,RHO2,G,ALPHA1,ALPHA2,HORZ2,VERT2,NPTS
     1,WL21,WL211)
      GO TO 610
  600 CALL WLCRIT1(Q1,Q2,WL11,RHO1,RHO2,G,ALPHA1,ALPHA2,HORZ1,VERT1,NPTS
     1,WL21,WL211)
  610 IF(NPRINT.NE.1) GO TO 300
      WRITE(6,310) X1,WL11,WL21,WL211
  310 FORMAT(5X,F8.2,6X,F18.2,6X,F15.2,2X,*(*,F15.2,*)*,/,5X,*ABRUPT CHA
     1NGE-CRITICAL CONDITION IS ASSUMED*,///)
  300 CALL PROPS2(HORZ1,VERT1,NPTS,WL11,WL21,A1,A2,T1,T2,P1,P2,AY1,AY2)
      ENLV1=WL11+E1/(A1*A1)
      ENLV2=WL11*RHO1/RHO2+WL21*DRHO/RHO2+E2/(A2*A2)
      RETURN
C
C     CALCULATE BED SLOPE OF REACH
```

```
C
   50 SO=M*(BOT2-BOT1)/SQRT(Z12**2+(HBOT1-HBOT2)**2)
      IF(NPRINT.NE.1) GO TO 60
      WRITE(6,70) XO,WL1,WL2
   70 FORMAT(5X,F10.2,6X,F18.4,6X,F15.4)
      GO TO 60
   80 CONTINUE
C
C     CALCULATE COORDINATES AT EACH INCREMENTAL DISTANCE BY LINEAR
C     INTERPOLATION
C
      DO 90 I=1,NPTS
      HORZ(I)=HORZ2(I)+M*(CHAINR(IPLUS1)-X)*(HORZ1(I)-HORZ2(I))/Z12
   90 VERT(I)=VERT2(I)+M*(CHAINR(IPLUS1)-X)*(VERT1(I)-VERT2(I))/Z12
C
C     EVALUATE SURFACE SLOPES AT EACH INCREMENTAL DISTANCE
C
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)
      IF((WL1A-WL2A).GT.4.0E-5.AND.(WL2A-BOT).GT.4.0E-5) GO TO 500
      WRITE(6,260) X
      WRITE(6,1000) FO1,FO2,FI
      STOP
  500 CALL SURGRD(HORZ,VERT,NPTS,VERTT,SO,RHO1,RHO2,VISC1,VISC2,G,Q1,Q2,
     1ALPHA1,ALPHA2,WL1A,WL2A,D,FO1,FO2,BL,FLOW,NINCR1,ELA,X,UREL1,UREL2
     1,UI,FI,S1E,S2E,S1,S2)
C
C     CHECK CRITICAL CONDITION
C
      IF(S1.LT.1.0E20) GO TO 400
      WRITE(6,410) WL1A,WL2A,X
  410 FORMAT(///,5X,*CRITICAL CONDITION  WATER LEVELS ARE*,2F15.2,*AT A
     1CHAINAGE OF ABOUT*,F13.2,///)
      STOP
C
C     APPLY RUNGE-KUTTA-MERSON ALGORITHM
C
  400 GO TO(101,102,103,104,105),NENTRY
  100 DX=DX/2.0
   60 X=XO
      WL1A=WL1
      WL2A=WL2
      NENTRY=1
      GO TO 80
  101 F1=S1/3.0
      F11=S2/3.0
      X=XO+DX/3.0
      WL1A=WL1+F1*DX
      WL2A=WL2+F11*DX
      NENTRY=2
      GO TO 80
  102 F2=S1/3.0
      F21=S2/3.0
      WL1A=WL1+F2*DX/2.0+F1*DX/2.0
      WL2A=WL2+F21*DX/2.0+F11*DX/2.0
      NENTRY=3
      GO TO 80
  103 F3=S1/3.0
      F31=S2/3.0
      X=XO+DX/2.0
      WL1A=WL1+9.0*F3*DX/8.0+3.0*F1*DX/8.0
      WL2A=WL2+9.0*F31*DX/8.0+3.0*F11*DX/8.0
      NENTRY=4
      GO TO 80
  104 F4=S1/3.0
      F41=S2/3.0
      X=XO+DX
      WL1A=WL1+6.0*F4*DX-9.0*F3*DX/2.0+3.0*F1*DX/2.0
      WL2A=WL2+6.0*F41*DX-9.0*F31*DX/2.0+3.0*F11*DX/2.0
      NENTRY=5
      GO TO 80
  105 F5=S1/3.0
      F51=S2/3.0
      WL11=WL1+(F1+4.0*F4+F5)*DX/2.0
      WL21=WL2+(F11+4.0*F41+F51)*DX/2.0
```

```
      X1=X
C
C     CHECK TRUNCATION ERRORS
C
      ERROR1=ABS((2.0*F1-9.0*F3+8.0*F4-F5)*DX/(10.0*WL11))
      ERROR2=ABS((2.0*F11-9.0*F31+8.0*F41-F51)*DX/(10.0*WL21))
      IF(ERROR1.LT.TOLER1.AND.ERROR2.LT.TOLER1) DX=2.0*DX
      IF(ERROR1.GT.TOLER.OR.ERROR2.GT.TOLER) GO TO 100
      IF(NPRINT.NE.1) GO TO 110
      WRITE(6,70) X1,WL11,WL21
  110 IF((WL11-WL21).LE.4.0E-5.OR.(WL21-BOT).LE.4.0E-5) GO TO 200
C
C     CHECK DISTANCE
C
      IF(ABS(X1-CHAINR(NSEC1)).LE.0.1) GO TO 210
      IF(X1+DX-CHAINR(NSEC1)) 220,230,230
  220 DX=CHAINR(NSEC1)-X1
  230 XO=X1
      WL1=WL11
      WL2=WL21
      GO TO 60
  210 CALL PROPS2(HORZ1,VERT1,NPTS,WL11,WL21,A1,A2,T1,T2,P1,P2,AY1,AY2)
C
C     CALCULATE ENERGY LEVELS
C
      ENLV1=WL11+E1/(A1*A1)
      ENLV2=WL11*RHO1/RHO2+WL21*DRHO/RHO2+E2/(A2*A2)
      GO TO 250
  200 WRITE(6,260) X1
  260 FORMAT(///,5X,*DEPTH=0.0 AT A CHAINAGE OF ABOUT*,F13.2)
      WRITE(6,1000) FO1,FO2,FI
 1000 FORMAT(5X,*FRICTION FACTORS FO1,FO2,FI ARE*,3F20.4)
      STOP
  250 RETURN
      END
```

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C     **********************************************************************
      SUBROUTINE NORMQ2(SO,N,RHO1,RHO2,G,WL1,WL2,FO1,FO2,FI,HORZ,VERT,
     1VERTT,NPTS,U1,U2,Q1,Q2)
C     **********************************************************************
C
C     NORMAL DISCHARGES IN A TWO-LAYER SYSTEM.
C
C     SO                 = CHANNEL BED SLOPE. NEGATIVE IF BED ELEVATION DEC-
C                          REASES IN FLOW DIRECTION.
C     N                  = 1  IF THE LOWER LAYER ONLY IS FLOWING AND THE
C                             UPPER LAYER IS STAGNANT.
C                          OTHERWISE IT IS ASSUMED THAT BOTH LAYERS ARE FLO-
C                          WING IN THE SAME DIRECTION.
C                          THESE ARE THE TWO CASES THAT HAVE PRACTICAL
C                          SIGNIFICANCE.
C     RHO1,RHO2          = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                          AND LOWER FLUIDS RESPECTIVELY.
C     G                  = GRAVITATIONAL ACCLERATION.
C     WL1,WL2            = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                          LY.
C     FO1,FO2            = BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER
C                          AND LOWER LAYERS RESPECTIVELY.
C     FI                 = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     HORZ,VERT          = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                          HORIZONTAL AND VERTICAL COORDINATES OF THE POINTS
C                          DESCRIBING THE CHANNEL CROSS-SECTION.
C                          HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY
C                          AXIS. VERTICAL COORDS. ARE REFERRED TO THE SAME
C                          DATUM AS WATER LEVELS.
C     VERTT              = ONE-DIMENSIONAL WORKING ARRAY OF SIZE NPTS.
C     NPTS               = NUMBER OF POINTS DEFINING THE CHANNEL CROSS-
C                          SECTION.
C     U1,U2              = NORMAL VELOCITIES OF THE UPPER AND LOWER LAYERS
C                          RESPECTIVELY.
C     Q1,Q2              = NORMAL DISCHARGES OF THE UPPER AND LOWER LAYERS
C                          RESPECTIVELY.
C     SAME DATUM MUST BE USED FOR WL1,WL2,VERT
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2,PROPS,BOTTOM,ZSYSTM1(USES THE AUXILIARY FUN-
C     CTION FUNN).
C     **********************************************************************
C
      EXTERNAL FUNN
      DIMENSION HORZ(NPTS),VERT(NPTS),XX(2),WAX(8),VERTT(NPTS)
      DRHO=RHO2-RHO1
      BETA1=FLOAT(N)
C
C     CALCULATE SECTION PROPERTIES
C
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A11,A21,T1,T2,P1,P2,AY1,AY2)
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)
      DWLO=(WL2-BOT)*0.001
      DO 30 I=1,NPTS
      VERTT(I)=VERT(I)+DWLO
   30 CONTINUE
      CALL PROPS(HORZ,VERTT,NPTS,WL2,ADA,TDT,PDP,AYDY)
      TO=(A21-ADA)/DWLO
      S2=SO*TO/T2
      S2E=S2
      IF(N.EQ.1) GO TO 40
      S1E=S2*T2/T1
      S2E=S1E*(RHO1/RHO2+DRHO*T1/(RHO2*T2))
      P11=P1-T2
   40 XX(1)=0.0
      XX(2)=0.0
      CALL ZSYSTM1(FUNN,1.0E-6,5,2,XX,100,S1E,S2E,DRHO,BETA1,HORZD,VERTD
     1,1,ALPHA1,ALPHA2,FO1,FO2,FI,A11,A21,T1,T2,P11,P2,H1,H2,G1,Q1,Q2,
     1RHO1,RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,
     1VERT,NPTS,RHO22,Q22,H22,WAX,PAR,IER)
      IF(IER.NE.129.AND.IER.NE.130) GO TO 10
      WRITE(6,20)
   20 FORMAT(5X,*NO SOLUTION FROM NORMQ2*)
```

```
      U1=U2=Q1=Q2=0.0
      RETURN
   10 IF(N.EQ.1) GO TO 50
      U1=XX(1)
      U2=XX(2)
      Q1=U1*A11
      Q2=U2*A21
      RETURN
   50 U1=Q1=0.0
      U2=XX(2)
      Q2=U2*A21
      RETURN
      END
C     *********************************************************************
      FUNCTION FUNN(XX,K,PAR,S1E,S2E,DRHO,BETA1,HORZD,VERTD,NPSD,ALPHA1,
     1ALPHA2,FO1,FO2,FI,A11,A21,T1,T2,P11,P2,H1,H2,G1,Q1,Q2,RHO1,RHO2,
     1A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,NPTS,
     1RHO22,Q22)
C     *********************************************************************
C
      DIMENSION XX(2),HORZ(NPTS),VERT(NPTS)
      IF(BETA1.EQ.1.0) GO TO 20
C
C     BOTH LAYERS FLOWING. UNKNOWNS ARE UPPER AND LOWER LAYERS
C     VELOCITIES RESPECTIVELY.
C
      GO TO (5,10),K
    5 FUNN=S1E*RHO1*G*A11+(FO1*RHO1*XX(1)*ABS(XX(1))/8.0)*P11+(FI*((RHO1
     1+RHO2)/16.0)*(XX(1)-XX(2))*ABS(XX(1)-XX(2)))*T2
      RETURN
   10 FUNN=S2E*RHO2*G*A21+(FO2*RHO2*XX(2)*ABS(XX(2))/8.0)*P2-(FI*((RHO1+
     1RHO2)/16.0)*(XX(1)-XX(2))*ABS(XX(1)-XX(2)))*T2
      RETURN
C
C     LOWER LAYER ONLY FLOWING. UNKNOWNS ARE UPPER LAYER ENERGY GRADIENT
C     AND LOWER LAYER VELOCITY RESPECTIVELY.
C
   20 GO TO (25,30),K
   25 FUNN=XX(1)*RHO1*G*A11-(FI*((RHO1+RHO2)/16.0)*XX(2)*ABS(XX(2))*T2)
      RETURN
   30 FUNN=(S2E*DRHO/RHO2+RHO1*XX(1)/RHO2)*RHO2*G*A21+(FO2*RHO2*XX(2)*
     1ABS(XX(2))/8.0)*P2+(FI*((RHO1+RHO2)/16.0)*XX(2)*ABS(XX(2))*T2)
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE NORMWL(SO,RHO1,RHO2,G,WL1S,WL2S,Q1,Q2,FO1,FO2,FI,HORZ,
     1VERT,HORZD,VERTD,NPTS,WL1,WL2)
C     ******************************************************************
C
C     NORMAL WATER LEVELS IN A TWO-LAYER SYSTEM.
C
C     SO                = CHANNEL BED SLOPE. NEGATIVE IF BED ELEVATION DEC-
C                         REASES IN FLOW DIRECTION.
C     RHO1,RHO2         = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                         AND LOWER FLUIDS RESPECTIVELY.
C     G                 = GRAVITATIONAL ACCLERATION.
C     WL1S,WL2S         = INITIAL GUESS FOR THE FREE SURFACE AND INTERFACE
C                         ELEVATIONS RESPECTIVELY. IF THE LOWER LAYER ONLY
C                         IS FLOWING, WL1S IS THE ACTUAL FREE SURFACE ELEV-
C                         ATION.
C     Q1,Q2             = DISCHARGES IN THE UPPER AND LOWER LAYERS RESPECT-
C                         IVELY.
C     FO1,FO2           = BOUNDARY SHEAR STRESS COEFFICIENTS FOR THE UPPER
C                         AND LOWER LAYERS RESPECTIVELY.
C     FI                = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     HORZ,VERT         = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                         HORIZONTAL AND VERTICAL COORDINATES OF THE POINTS
C                         DESCRIBING THE CHANNEL CROSS-SECTION.
C                         HORIZONTAL COORDS. ARE REFERRED TO AN ARBITRARY
C                         AXIS. VERTICAL COORDS. ARE REFERRED TO THE SAME
C                         DATUM AS WATER LEVELS.
C     HORZD,VERTD       = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
C     NPTS              = NUMBER OF POINTS DEFINING THE CHANNEL CROSS-
C                         SECTION.
C     WL1,WL2           = NORMAL FREE SURFACE AND INTERFACE ELEVATIONS RES-
C                         PECTIVELY (FOR UNIFORM FLOW).
C     SAME DATUM MUST BE USED FOR WL1S,WL2S,WL1,WL2,VERT
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- PROPS2,PROPS,BOTTOM,ZSYSTM1(USES THE AUXILIARY FUN-
C     CTION FUNM).
C     ******************************************************************
C
      EXTERNAL FUNM
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTS),VERTD(NPTS),XX(2),
     1WAX(8)
      IF(Q1.NE.0.0) GO TO 30
      XX(1)=0.0
      XX(2)=WL2S
      WL1=WL1S
      GO TO 40
   30 XX(1)=WL1S
      XX(2)=WL2S
   40 CALL ZSYSTM1(FUNM,1.0E-6,5,2,XX,100,S1E,S2E,WL1,BETA1,HORZD,VERTD,
     1NPTS,ALPHA1,ALPHA2,FO1,FO2,FI,A11,A21,T1,T2,P11,P2,H1,H2,G1,Q1,Q2,
     1RHO1,RHO2,A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,
     1VERT,NPTS,RHO22,SO,H22,WAX,PAR,IER)
      IF(IER.NE.129.AND.IER.NE.130) GO TO 10
      WRITE(6,20)
   20 FORMAT(5X,*NO SOLUTION FROM NORMWL*)
      WL1=WL2=0.0
      RETURN
   10 IF(Q1.NE.0.0) GO TO 50
      WL1=WL1S
      WL2=XX(2)
      RETURN
   50 WL1=XX(1)
      WL2=XX(2)
      RETURN
      END
C     ******************************************************************
      FUNCTION FUNM(XX,K,PAR,S1E,S2E,WL1,BETA1,HORZD,VERTD,NPTSD,ALPHA1,
     1ALPHA2,FO1,FO2,FI,A11,A21,T1,T2,P11,P2,H1,H2,G1,Q1,Q2,RHO1,RHO2,
     1A1,A2,A1D,A2D,G,BETA,AY2,A12,A22,AY12,AY22,T12,T22,HORZ,VERT,NPTS,
     1RHO22,SO)
C     ******************************************************************
C
```

```
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTS),VERTD(NPTS),XX(2)
      DRHO=RHO2-RHO1
      CALL BOTTOM(VERT,NPTS,BOT,WLMAX)
      DWLO=(XX(2)-BOT)*0.001
      DO 20 I=1,NPTS
      VERTD(I)=VERT(I)+DWLO
   20 CONTINUE
      CALL PROPS(HORZ,VERTD,NPTS,XX(2),ADA,TDT,PDP,AYDY)
      IF(Q1.EQ.0.0) GO TO 30
C
C     BOTH LAYERS FLOWING. UNKNOWNS ARE FREE SURFACE AND INTERFACE
C     ELEVATIONS RESPECTIVELY.
C
      CALL PROPS2(HORZ,VERT,NPTS,XX(1),XX(2),A11,A21,T1,T2,P1,P2,AY1,
     1AY2)
      TO=(A21-ADA)/DWLO
      S2=SO*TO/T2
      S1E=S2*T2/T1
      S2E=S1E*(RHO1/RHO2+DRHO*T1/(RHO2*T2))
      P11=P1-T2
      U1=Q1/A11
      U2=Q2/A21
      GO TO (5,10),K
    5 FUNM=S1E*RHO1*G*A11+(FO1*RHO1*U1*ABS(U1)/8.0)*P11+(FI*((RHO1+RHO2)
     1/16.0)*(U1-U2)*ABS(U1-U2))*T2
      RETURN
   10 FUNM=S2E*RHO2*G*A21+(FO2*RHO2*U2*ABS(U2)/8.0)*P2-(FI*((RHO1+RHO2)/
     116.0)*(U1-U2)*ABS(U1-U2))*T2
      RETURN
C
C     LOWER LAYER ONLY FLOWING: UNKNOWNS ARE FREE SURFACE SLOPE AND
C     INTERFACE ELEVATION RESPECTIVELY.
C
   30 CALL PROPS2(HORZ,VERT,NPTS,WL1,XX(2),A11,A21,T1,T2,P1,P2,AY1,AY2)
      TO=(A21-ADA)/DWLO
      S2=SO*TO/T2
      U2=Q2/A21
      GO TO (35,40),K
   35 FUNM=XX(1)*RHO1*G*A11-(FI*((RHO1+RHO2)/16.0)*U2*ABS(U2)*T2)
      RETURN
   40 FUNM=(RHO1*XX(1)/RHO2+DRHO*S2/RHO2)*RHO2*G*A21+(FO2*RHO2*U2*
     1ABS(U2)/8.0)*P2+(FI*((RHO1+RHO2)/16.0)*U2*ABS(U2)*T2)
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE ARSWDG(HORZAR,VERTAR,CHAINR,NXSEC,NPTS,WL1,Q1,Q2,RHO1,
     1RHO2,G,VISC1,VISC2,ALPHA1,ALPHA2,DAR,FO1,FO2,FI,UREL1,UREL2,HORZ1,
     1VERT1,HORZ2,VERT2,HORZ3,VERT3,HORZ,VERT,VERTT,ELA,DX,NPRINT)
C     ******************************************************************
C

C     ARRESTED WEDGE PROFILE IN A TWO-LAYER SYSTEM.
C
C     HORZAR,VERTAR  = TWO-DIMENSIONAL ARRAYS OF SIZE (NXSEC,NPTS) CONT-
C                      AINING THE HORIZONTAL AND VERTICAL COORDINATES
C                      OF THE POINTS DESCRIBING EACH OF A SERIES OF
C                      CROSS-SECTIONS DEFINING THE CHANNEL. THE FIRST
C                      VALUE OF THE ARRAY REPRESENTS THE SECTION NUMBER
C                      AND THE SECOND VALUE REPRESENTS ONE OF THE HORIZ-
C                      ONTAL (OR VERTICAL) COORDINATES.
C     CHAINR         = ONE-DIMENSIONAL ARRAY OF SIZE NXSEC CONTAINING
C                      THE CHAINAGE VALUES FOR THE SERIES OF CROSS-
C                      SECTIONS.
C                      CHAINAGE VALUES INCREASE IN THE DOWNSTREAM
C                      DIRECTION.
C     NXSEC          = THE NUMBER OF CROSS-SECTIONS IN THE HYDRAULIC
C                      PROFILE.
C     NPTS           = NUMBER OF POINTS DESCRIBING EACH OF THE CROSS-
C                      SECTIONS.
C     WL1            = FREE SURFACE ELEVATION AT THE BEGINNING OF THE
C                      WEDGE(RIVER MOUTH OR WARM WATER OUTLET).
C     Q1,Q2          = DISCHARGE OF THE UPPER AND LOWER LAYERS RESPECTI-
C                      IVELY. ONE OF THEM SHOULD BE ZERO.
C     RHO1,RHO2      = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                      AND LOWER FLUIDS RESPECTIVELY.
C     G              = GRAVITATIONAL ACCELERATION.
C     VISC1,VISC2    = KINEMATIC VISCOSITIES OF THE UPPER AND LOWER
C                      FLUIDS RESPECTIVELY.
C     ALPHA1,ALPHA2  = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER
C                      AND LOWER LAYER RESPECTIVELY.
C     DAR            = ONE-DIMENSIONAL ARRAY OF SIZE NXSEC CONTAINING
C                      THE ROUGHNESS HEIGHT FOR EACH CROSS-SECTION.
C     FO1,FO2        = SHEAR STRESS COEFFICIENTS AT THE SOLID BOUNDARY
C                      FOR THE UPPER AND LOWER LAYERS RESPECTIVELY.
C     FI             = INTERFACIAL SHEAR STRESS COEFFICIENT.
C     UREL1,UREL2    = ONE-DIMENSIONAL ARRAYS OF SIZE NINCR1 CONTAINING
C                      VELOCITY PROFILES WITHIN THE INTERFACIAL BOUNDARY
C                      LAYERS (RELATIVE TO THE MEAN VELOCITY OF FLOW).
C                      IN THIS ROUTINE, NINCR1 IS SET EQUAL TO ZERO.
C                      THEREFORE, UREL1 AND UREL2 ARE DUMMY PARAMETERS.
C     HORZ1,VERT1    = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
C     HORZ2,VERT2    = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
C     HORZ3,VERT3    = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
C     HORZ ,VERT     = ONE-DIMENSIONAL WORKING ARRAYS OF SIZE NPTS.
C     VERTT          = ONE-DIMENSIONAL WORKING ARRAY  OF SIZE NPTS.
C     ELA            = TOTAL LENGTH OF THE FLOWING LAYER FROM THE POINT
C                      OF CONTACT OF THE TWO FLUIDS.
C     DX             = INCREMENTAL DISTANCE TO BE USED FOR THE INTEGRA-
C                      TION. THE SIGN OF DX IS ALWAYS NEGATIVE SINCE
C                      THE INTEGRATION IS IN THE UPSTREAM DIRECTION.
C     NPRINT         = AN INTEGER VARIABLE SET EQUAL TO 1 IF A PRINTOUT
C                      OF PROFILES COORDINATES AT THE INTERMEDIATE STEPS
C                      IS REQUIRED.
C     SAME DATUM MUST BE USED FOR VERTAR,WL1
C     UNITS USED MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF
C     G USED.
C     ROUTINES USED- WLCRIT1,FLPROF,SELSEC
C     ******************************************************************
C
      EXTERNAL AUX,AUX1,AUX2,AUX3,AUX4,AUX5
      DIMENSION HORZAR(NXSEC,NPTS),VERTAR(NXSEC,NPTS),CHAINR(NXSEC),
     1HORZ1(NPTS),VERT1(NPTS),HORZ2(NPTS),VERT2(NPTS),HORZ3(NPTS),
     1VERT3(NPTS),HORZ(NPTS),VERT(NPTS),DAR(NXSEC),VERTT(NPTS)
      CALL SELSEC(HORZAR,VERTAR,NXSEC,NPTS,NXSEC,HORZ3,VERT3)
C
C     CALCULATE CRITICAL CONDITION AT THE RIVER MOUTH
C
```

```
      CALL WLCRIT1(Q1,Q2,WL1,RHO1,RHO2,G,ALPHA1,ALPHA2,HORZ3,VERT3,
     1NPTS,WL2,WL2)
      IF(NPRINT.NE.1) GO TO 10
      WRITE(6,20) WL2
   20 FORMAT(1H1,5X,*CRITICAL INTERFACE LEVEL AT THE RIVER MOUTH IS*,
     1F20.4)
   10 WLA=WL1
      IF(Q2.EQ.0.0) GO TO 70
      WLB=WL2*1.001
      GO TO 80
   70 WLB=WL2*0.999
   80 NXSC1=NXSEC-1
      DO 30 K=1,NXSC1
      I=NXSEC-K
C
C     CALCULATE THE WEDGE PROFILE
C
      CALL FLPROF(HORZAR,VERTAR,CHAINR,NXSEC,NPTS,I,WLA,WLB,RHO1,RHO2,
     1VISC1,VISC2,G,Q1,Q2,ALPHA1,ALPHA2,DAR,FO1,FO2,FI,UREL1,UREL2,
     1HORZ1,VERT1,HORZ2,VERT2,HORZ,VERT,VERTT,0,ELA,DX,NPRINT,WL11,WL21,
     1ENLV1,ENLV2)
      IF(NPRINT.NE.1) GO TO 60
      WRITE(6,50) WL11,WL21,ENLV1,ENLV2,FO1,FO2,FI
   50 FORMAT(5X,*WL11,WL21,ENLV1,ENLV2,FO1,FO2,FI*,7F14.4,///)
   60 WLA=WL11
      WLB=WL21
   30 CONTINUE
      WRITE(6,40)
   40 FORMAT(5X,*THE WEDGE IS LONGER THAN THE GIVEN STRETCH*)
      STOP
      END
```

```
C     ****************************************************************
      SUBROUTINE SWLMID(WL1,WL2,Q2,WL21,ALPHA1,ALPHA2,G,RHO1,RHO2,HORZ,
     1VERT,NPTS,HORZD,VERTD,NPTSD,Q1,WL11)
C     ****************************************************************
C
C     CALCULATION OF THE DISCHARGE OF THE UPPER LAYER AS WELL AS THE
C     FREE SURFACE ELEVATION AT ONE SECTION OF A TRANSITION AS FUNCTIONS
C     OF THE FREE SURFACE ELEVATION AT THE OTHER SECTION AND THE INTERF-
C     ACE ELEVATION AT BOTH SECTIONS AS WELL AS THE DISCHARGE OF THE
C     LOWER LAYER (SELECTIVE WITHDRAWAL-SEE DOCUMENTATION). THE TRANSIT-
C     ION IS DEFINED BY TWO ARBITRARY CROSS-SECTIONS WHICH ARE DESCRIBED
C     BY A SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN.
C
C     WL1,WL2         = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                       LY AT THE GIVEN SECTION.
C     Q2              = DISCHARGE OF THE LOWER LAYER.
C     WL21            = INTERFACE ELEVATION AT THE OTHER SECTION.
C     ALPHA1,ALPHA2   = KINETIC ENERGY CORRECTION FACTORS FOR THE UPPER
C                       AND LOWER LAYERS RESPECTIVELY.
C     G               = GRAVITATIONAL ACCELERATION.
C     RHO1,RHO2       = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND LOWER FLUIDS RESPECTIVELY.
C     HORZ,VERT       = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                       HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DEFINING THE GIVEN CROSS-SECTION.
C                       THE HORIZONTAL COORDS. ARE REFERRED TO SOME ARBI-
C                       TRARY AXIS AND THE VERTICAL COODS. ARE REFERRED
C                       TO THE SAME DATUM AS WATER LEVELS.
C     NPTS            = NUMBER OF POINTS DESCRIBING THE GIVEN CROSS-
C                       SECTION.
C     HORZD,VERTD     = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING
C                       THE HORIZONTAL AND VERTICAL COORDS OF THE POINTS
C                       DEFINING THE OTHER CROSS-SECTION.
C                       THESE ARE REFERRED TO THE SAME AXES AS HORZ AND
C                       VERT.
C     NPTSD           = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
C     Q1              = DISCHARGE OF THE UPPER LAYER.
C     WL11            = FREE SURFACE ELEVATION AT THE OTHER SECTION.
C     SAME DATUM MUST BE USED FOR WL1,WL2,WL21,VERT,VERTD,WL11
C     UNITS MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF G
C     USED.
C     ROUTINES USED-PROPS2,PROPS
C     ****************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTSD),VERTD(NPTSD)
      DRHO=RHO2-RHO1
C
C     CALCULATE SECTION PROPERTIES
C
      CALL PROPS2(HORZ,VERT,NPTS,WL1,WL2,A1,A2,T1,T2,P1,P2,AY1,AY2)
      CALL PROPS(HORZD,VERTD,NPTSD,WL21,A21,T21,P21,AY21)
C
C     APPLY BERNOULLI EQUATION FOR THE LOWER LAYER
C
      X1=WL1*RHO1/RHO2+(DRHO/RHO2)*(WL2-WL21)+(ALPHA2*Q2*Q2/(2.0*G))*(
     11.0/(A2*A2)-1.0/(A21*A21))
      WL11=X1*RHO2/RHO1
      CALL PROPS2(HORZD,VERTD,NPTSD,WL11,WL21,A11,A211,T11,T211,P11,P211
     1,AY11,AY211)
      X2=WL1-WL11
C
C     APPLY BERNOULLI EQUATION FOR THE UPPER LAYER
C
      X3=(X2*2.0*G)/(ALPHA1*(1.0/(A11*A11)-1.0/(A1*A1)))
      Q1=SQRT(X3)
      RETURN
      END
```

```
C     ******************************************************************
      SUBROUTINE SWLBOT(WL1,WL2,WL21,ALPHA2,G,RHO1,RHO2,HORZ,VERT,NPTS,
     1HORZD,VERTD,NPTSD,Q2,WL11)
C     ******************************************************************
C
C     CALCULATION OF THE DISCHARGE OF THE LOWER LAYER AS WELL AS THE
C     FREE SURFACE ELEVATION AT ONE SECTION OF A TRANSITION AS FUNCTIONS
C     OF THE FREE SURFACE ELEVATION AT THE OTHER SECTION AND THE INTERF-
C     ACE ELEVATION AT BOTH SECTIONS IN THE CASE OF A STAGNANT UPPER
C     LAYER (SELECTIVE WITHDRAWAL-SEE DOCUMENTATION). THE TRANSITION IS
C     DEFINED BY TWO ARBITRARY CROSS-SECTIONS WHICH ARE DESCRIBED BY A
C     SERIES OF POINTS THE COORDINATES OF WHICH ARE GIVEN.
C
C     WL1,WL2         = FREE SURFACE AND INTERFACE ELEVATIONS RESPECTIVE-
C                       LY AT THE GIVEN SECTION.
C     WL21            = INTERFACE ELEVATION AT THE OTHER SECTION.
C     ALPHA2          = KINETIC ENERGY CORRECTION FACTOR FOR THE LOWER
C                       LAYER.
C     G               = GRAVITATIONAL ACCELERATION.
C     RHO1,RHO2       = DENSITIES (OR SPECIFIC GRAVITIES) OF THE UPPER
C                       AND LOWER FLUIDS RESPECTIVELY.
C     HORZ,VERT       = ONE-DIMENSIONAL ARRAYS OF SIZE NPTS CONTAINING
C                       HORIZONTAL AND VERTICAL COORDINATES RESPECTIVELY
C                       OF THE POINTS DEFINING THE GIVEN CROSS-SECTION.
C                       THE HORIZONTAL COORDS. ARE REFERRED TO SOME ARBI-
C                       TRARY AXIS AND THE VERTICAL COODS. ARE REFERRED
C                       TO THE SAME DATUM AS WATER LEVELS.
C     NPTS            = NUMBER OF POINTS DESCRIBING THE GIVEN CROSS-
C                       SECTION.
C     HORZD,VERTD     = ONE-DIMENSIONAL ARRAYS OF SIZE NPTSD CONTAINING
C                       THE HORIZONTAL AND VERTICAL COORDS OF THE POINTS
C                       DEFINING THE OTHER CROSS-SECTION.
C                       THESE ARE REFERRED TO THE SAME AXES AS HORZ AND
C                       VERT.
C     NPTSD           = NUMBER OF POINTS DESCRIBING THE OTHER SECTION.
C     Q2              = DISCHARGE OF THE LOWER LAYER.
C     WL11            = FREE SURFACE ELEVATION AT THE OTHER SECTION.
C     SAME DATUM MUST BE USED FOR WL1,WL2,WL21,VERT,VERTD,WL11
C     UNITS MUST BE CONSISTENT THROUGHOUT AND APPROPRIATE VALUE OF G
C     USED.
C     ROUTINES USED-PROPS
C     ******************************************************************
C
      DIMENSION HORZ(NPTS),VERT(NPTS),HORZD(NPTSD),VERTD(NPTSD)
      DRHO=RHO2-RHO1
C
C     BERNOULLI EQUATION FOR THE UPPER LAYER
C
      WL11=WL1
      X1=(WL2-WL21)*DRHO/RHO2
C
C     CALCULATE SECTION PROPERTIES
C
      CALL PROPS(HORZ,VERT,NPTS,WL2,A2,T2,P2,AY2)
      CALL PROPS(HORZD,VERTD,NPTSD,WL21,A21,T21,P21,AY21)
C
C     BERNOULLI EQUATION FOR THE LOWER LAYER
C
      X2=1.0/(A21*A21)-1.0/(A2*A2)
      X3=X1*2.0*G/(ALPHA2*X2)
      Q2=SQRT(X3)
      RETURN
      END
```

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