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FUEL TRAJECTORIES OF NUCLEAR REACTORS

by

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ABSTRACT

The fissile fuel inventory of an external stockpile that supplies a nuclear reactor is shown to be characterised by time dependent mean residence time of fuel in the core, the load factor for the reactor, and the breeding gains. The time dependent forms of these parameters have been considered and quantitative evaluations of the fissile fuel inventory have been carried out. The results show the substantial potentials of efficient reactors in extending the duration of fissile fuel for Nuclear industries.

The inventory concept has also been extended to CANDU reactors and the result clearly depicts the effect of a net fissile fuel consuming system on the nuclear fuel reserves.

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LIST OF FIGURES

LIST OF TABLES

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

The fissile fuel inventory of a nuclear reactor can be represented as a function containing time dependent parameters which include the mean residence time of fuel in the core and blanket, the capacity or load factor, and the breeding gains. An accurate assessment of the temporal variation of the bred fissile fuel therefore rests on the establishment of the time dependent forms of these determining factors.

In this analysis, the fissile fuel inventory for a nuclear reactor based on earlier work, (Ref. 1 and 4), is presented and evaluations have been made for the case of asymptotic constant parameters. The time dependent forms of the mean residence time and load factor have been considered from a practical point of view while rational approximations for the breeding gains have been made in other to provide an explicit integral form for the fuel inventory. The integrands involved however pose mathematical problems; further evaluation have therefore been based on a quasi-static scheme which incorporates the time dependent forms of the mean residence time and the load factor.

The analysis begins with a specification of the fissile fuel balance for a nuclear reactor and its peripheral support facility, followed by a detailed time dependent accounting of the fissile fuel production and consumption over the reactor life. The resulting function then represents the fissile fuel trajectory from which we can obtain the net available fissile fuel to supply other reactors. The inventory concept has also been extended to CANDU reactors to demonstrate the effect of net fuel consuming systems on the available fissile fuel.

A traditional problem encountered in the fissile fuel breeding analysis is the incorporation of various fertile and fissile nuclei into a sufficiently clear and tractable formulation, this problem has been circumvented by the use of the concept of "equivalent worth" of the various nuclei. This concept which is credited to Baker and Ross (1963) assigns some weight w_i, to a given nucleus to account for its contribution to the maintenance of fissile fuel consumption processes in a given system. The mass which appears in the balance equation thus denotes the weighted sum of all the masses of the various nuclei in the fissile material. A detailed discussion of this "equivalent worth" concept is listed in the Appendix.

CHAPTER 2

FISSILE FUEL INVENTORY

The system of interest consists of a nuclear reactor and an associated stockpile of fissile fuel (Fig. 2.1.). The fuel stockpile initially contains m_i kg of (equivalent) fissile fuel. Prior to the reactor start-up at $t = 0$, m_c kg of this amount is removed to load the core. The fissile fuel inventory in the external stockpile which will be designated by $m_{ext}^{(t)}$ is then given by

$$m_{ext}(t) = m_i - m_c + \int_0^t \{(\frac{dm}{dt})_c + (\frac{dm}{dt})_b\}dt \tag{1}$$

where $(\frac{dm}{dt})c$ and $(\frac{dm}{dt})_b$ are the net fissile fuel flows to or from the external stockpile associated with the core and blanket processes respectively.

Equation (1) is basically a statement of mass conservation which is solvable by different approaches depending on the extent of physical detail incorporated. As an initial observation, we note that the fissile fuel bred in the core or blanket is usually not instantaneously available in its desired form because of reprocessing; accounting for this will require the insertion of time-delays in the mass flow. Except for very small fractions of process losses/retentions, these considerations do not change the total availability of the fissile fuel but only its availability in a preferred form. The time delays are therefore not included in the formulation and attention is focussed on the physical processes in the reactor that effect the production and consumption of fissile fuel over the reactor life-time.

In Eq. (1) the integrand terms are obtained by the following considerations. The net fissile fuel flows for the core term, $(\frac{dm}{dt})_c$, consists of two main components: a constant equilibrium component and a transient component during the early part of the reactor life; this latter part accounts for the extra fissile fuel added to the core to compensate for the initial accumulation of neutron absorbing fission products. The core term can thus be written as

$$(\frac{dm}{dt})_c = (\frac{dm}{dt})_c^{\infty} + (\frac{dm}{dt})_c^{T} \qquad (2)$$

The net fissile fuel requirements for the core at equilibrium is taken to be proportional to the fissile fuel destruction rate R(t). Hence

$$(\frac{dm}{dt})_c^{\infty} = G_c(t)R(t), \ -1 \leq G_c(t) < 0 \qquad (3)$$

where the core breeding gain $G_c(t)$ is obviously a negative quantity. The transient contribution is taken to be a time dependent fraction of the equilibrium term and can be written as

$$(\frac{dm}{dt})_c^{T} = T(t)G_c(t)R(t), \ 0 \leq T(t) \leq 1 \qquad (4)$$

Here, T(t) is the transient-fraction function; it is required that $T(t) \to 0$ as $t \to t_{\infty}$. The core term can thus be written as

$$(\frac{dm}{dt})_c = G_c(t)R(t)[1 + T(t)] \qquad (5)$$

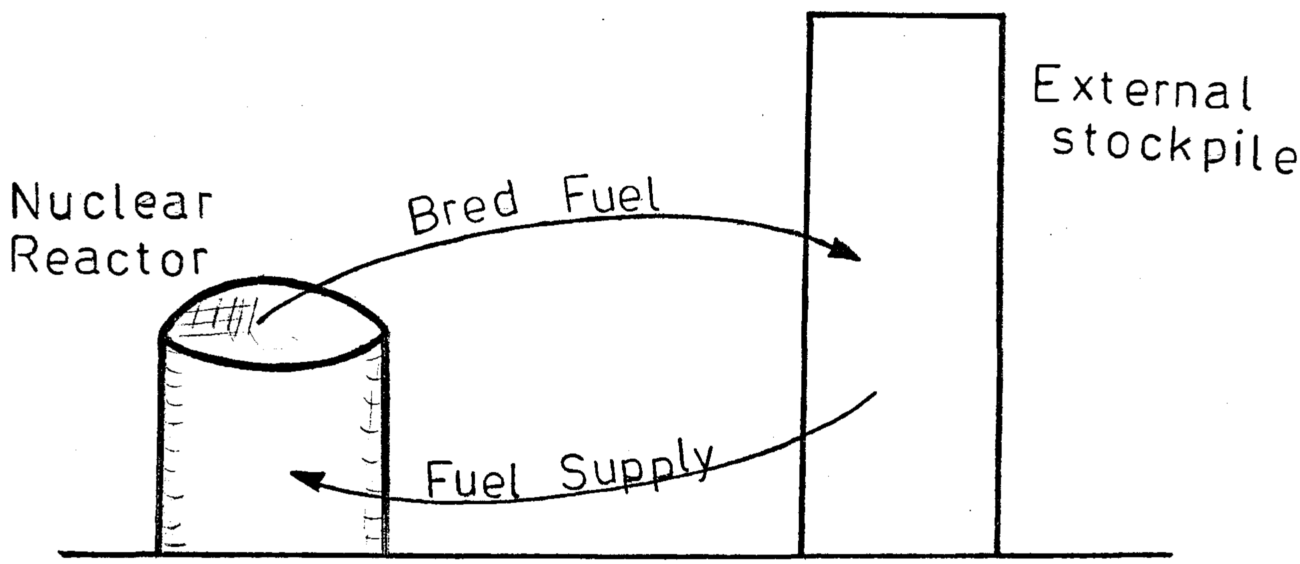

Fig. 2.1: A schematic illustration of fissile fuel flow between a nuclear reactor and its affilitated external stockpile of fissile fuel.

The blanket is taken to consist of radial and axial regions each subject to a distinct fuel management scheme. The net fissile fuel production in each region is given in terms of a specified residence time λ, by

$$\left(\frac{dm}{dt}\right)_{b,r} = \frac{m_{b,r}(t)}{\lambda_r(t)} \tag{6}$$

and

$$\left(\frac{dm}{dt}\right)_{b,a} = \frac{m_{b,a}(t)}{\lambda_a(t)} \tag{7}$$

where the subscripts "r" and "a" are used to identify the radial and axial blankets respectively. The residence times $\lambda_r(t)$ and $\lambda_a(t)$ specify how long a blanket element remains in each of the blanket zones before being replaced. The blanket term in Eq. (1) is thus given by

$$\left(\frac{dm}{dt}\right)_{b} = \left(\frac{dm}{dt}\right)_{b,r} + \left(\frac{dm}{dt}\right)_{b,a} \tag{8}$$

The fissile fuel inventories $m_{b,r}(t)$ and $m_{b,a}(t)$ in the blankets appear with time according to their production rate by transmutation and removal rates with the specified residence times. This requires that $m_{b,r}(t)$ and $m_{b,a}(t)$ in Eq. (6) and Eq. (7) satisfy the condition

$$\frac{dm_{b,r}(t)}{dt} = G_{b,r}(t)R(t) - \frac{m_{b,r}(t)}{\lambda_r(t)}\ ,\ m_{b,r}(0) = m_{b,r} \tag{9}$$

and

$$\frac{dm_{b,a}(t)}{dt} = G_{b,a}(t)R(t) - \frac{m_{b,a}(t)}{\lambda_a(t)}\ ,\ m_{b,a}(0) = m_{b,a} \tag{10}$$

where the net breeding gains $G_{b,r}(t)$ and $G_{b,a}(t)$ apply to the radial and axial blankets respectively. The initial conditions represent the initial fissile fuel mass in the blankets, usually in the form of heavily depleted fissile materials.

Solutions of Eq. (9) and Eq. (10) are given by

$$m_{b,j}(t) = \exp\left(-\int \frac{dt}{\lambda_j(t)}\right) \int_0^t G_{b,j}(t)R(t)\exp\left(\int \frac{dt}{\lambda_j(t)}\right)dt + m_{b,j}(0)$$

$$j = r,a \tag{11.a}$$

which can also be written as

$$\begin{aligned} m_{b,j}(t) = {} & \exp\left(-\int \frac{dt}{\lambda_j(t)}\right)\{[\int G_{b,j}(t)R(t)\exp\left(\int\frac{dt}{\lambda_j(t)}\right)dt]_{t=t} \\ & + [m_{b,j}(t)\exp\left(\int \frac{dt}{\lambda_j(t)}\right) - \int G_{b,j}(t)R(t)\exp\left(\int \frac{dt}{\lambda_j(t)}\right)dt]_{t=0}\} \end{aligned} \tag{11.b}$$

Substutition of Eq. (11.b) into Eq. (1) gives the final form for the fissile fuel inventory of the external stockpile in terms of time dependent quantities to be specified; we thus have

$$\begin{aligned} m_{ext}(t) & = m_i - m_c + \int_0^t \{G_c(t)R(t)[1+T(t)] + \frac{m_{b,r}(t)}{\lambda_r(t)} + \frac{m_{b,a}(t)}{\lambda_a(t)}\}\, dt \\ & = m_i - m_c + \int_0^t \{G_c(t)R(t)[1+T(t)] \\ & + \frac{1}{\lambda_r(t)} \exp\left(-\int\frac{dt}{\lambda_r(t)}\right)\{ \int G_{b,r}(t)R(t)\exp\left(\int \frac{dt}{\lambda_r(t)}\right)dt \\ & + [m_{b,r}(t)\exp\left(\int \frac{dt}{\lambda_r(t)}\right) - \int G_{b,r}(t)R(t)\exp\left(\int \frac{dt}{\lambda_r(t)}\right)dt]_{t=0}\} \end{aligned}$$

$$+ \frac{1}{\lambda_a(t)} \exp\left(-\int\frac{dt}{\lambda_a(t)}\right)\left\{\int G_{b,a}(t)R(t)\exp\left(\int \frac{dt}{\lambda_a(t)}\right)dt\right.$$

$$\left. + \left[m_{b,a}(t)\exp\left(\int \frac{dt}{\lambda_a(t)}\right) - \int G_{b,a}(t)R(t)\exp\left(\int \frac{dt}{\lambda_a(t)}\right)dt\right]_{t=0}\right\}\}dt \tag{12}$$

Equation (12) is the basic equation of interest and provides for the evaluation of the amount of fissile fuel in the stockpile, on specification of the time dependent quantities.

CHAPTER 3

ASYMPTOTIC PARAMETER SOLUTION OF THE FISSILE FUEL TRAJECTORY EQUATION

The amount of fissile fuel available in the external stockpile can be obtained from Eq. (12) providing the various integrand functions are known. Before attempting to find time dependent forms for some of the integrand functions, a numerical test for the formulation can be provided by making rational approximations for the quantities involved.

The functions $\lambda_r(t)$ and $\lambda_a(t)$ represent the length of time that a fuel element remains in the radial and axial blankets respectively. These time intervals may vary from a maximum of three to a maximum of six years respectively depending on the fuel management scheme and cycle lengths. If these times are designated as mean residence times, it is obvious that after a long time or after many cycles, the mean residence time will approach the maximum time that a fuel element can reside in the blanket and we can thus make the approximations $\lambda_r(t) = \lambda_r$ and $\lambda_a(t) = \lambda_a$ where λ_r and λ_a are the asymptotic residence times.

The breeding gains are the ratios of the net fissile fuel produced to the fissile fuel consumed as appropriate to the core and each of the blanket zones, and can be related to the breeding ratio BR(t) by

$$G_c(t) + G_{b,r}(t) + G_{b,a}(t) = BR(t) - 1 \tag{13}$$

Under equilibrium condition which follows a relatively short initial transient period of the reactor life, these breeding gains are essentially constants. It is known[3] that even during the pre-equilibrium period, the variations of the breeding ratio do not generally exceed ∿10%. Such small variations suggest that these time dependent functions can be taken as constants. Hence we will use $G_c(t) = G_c$, $G_{b,r}(t) = G_{b,r}$ and $G_{b,a}(t) = G_{b,a}$.

The remaining functions to be specified are the destruction rate R(t) and the initial transient-fraction T(t). The function T(t) accounts for such nuclear effects as fission product poisioning. These effects are of very short duration when considered on the time scale of years and, in addition, in terms of the total reactivity capacity of the core, this fraction is not predominant. The approximation to be made here is then $1 + T(t) \approx 1$. The variation in the fissile fuel destruction rate will be incorporated by the introduction of the station load factor L(t) defined by

$$R(t) = RL(t) \tag{14}$$

where R is a constant fissile fuel destruction rate. The load factor can vary typically from a low of about 0.3 in the first year of operation to attain an equilibrium value of about 0.75 after one or two refuelling periods. Using the functions as specified above Eq. (12) reduces to

$$\begin{aligned} m_{ext}(t) = {} & m_i - m_c + G_c R \int_0^t L(t)dt \\ & + m_{b,r}(1-e^{-t/\lambda_r}) + m_{b,a}(1-e^{-t/\lambda_a}) \\ & + \frac{G_{b,r}R}{\lambda_r} \int_0^t e^{-t/\lambda_a}[\int L(t)e^{t/\lambda_r}dt - \{\int L(t)e^{t/\lambda_r}dt\}_{t=0}]dt \\ & + \frac{G_{b,r}R}{\lambda_a} \int_0^t e^{-t/\lambda_a}[\int L(t)e^{t/\lambda_a}dt - \{\int L(t)e^{t/\lambda_a}dt\}_{t=0}]dt \end{aligned} \tag{15}$$

For the present purpose the asymptotic value of the load factor will be used thus enabling us to write $L(t) = L$. This reduces Eq. (15) to the simple form

$$\begin{aligned} m_{ext}(t) &= m_i - m_c + [G_c + G_{b,r} + G_{b,a}]RLT \\ &+ [m_{b,r} - \lambda_r G_{b,r} RL][1 - e^{-t/\lambda_r}] \\ &+ [m_{b,a} - \lambda_a G_{b,a} RL][1 - e^{-t/\lambda_a}] \end{aligned} \tag{16}$$

Further evaluation requires typical values for the parameters contained in Eq. (16) and the most relevant and complete data available for this purpose are those for the Clinch River Breeder Reactor[4] which are contained in Table I. If for reasons of algebraic convenience we let $m_i = m_c$, the use of the data in Table I leads to

$$m_{ext}(t) = 66.0t - 702(1 - e^{t/6}) - 198(1 - e^{t/3}) \tag{17}$$

Equation (17) is the asymptotic fissile fuel trajectory equation with the graphical form shown in Fig. 3.1.

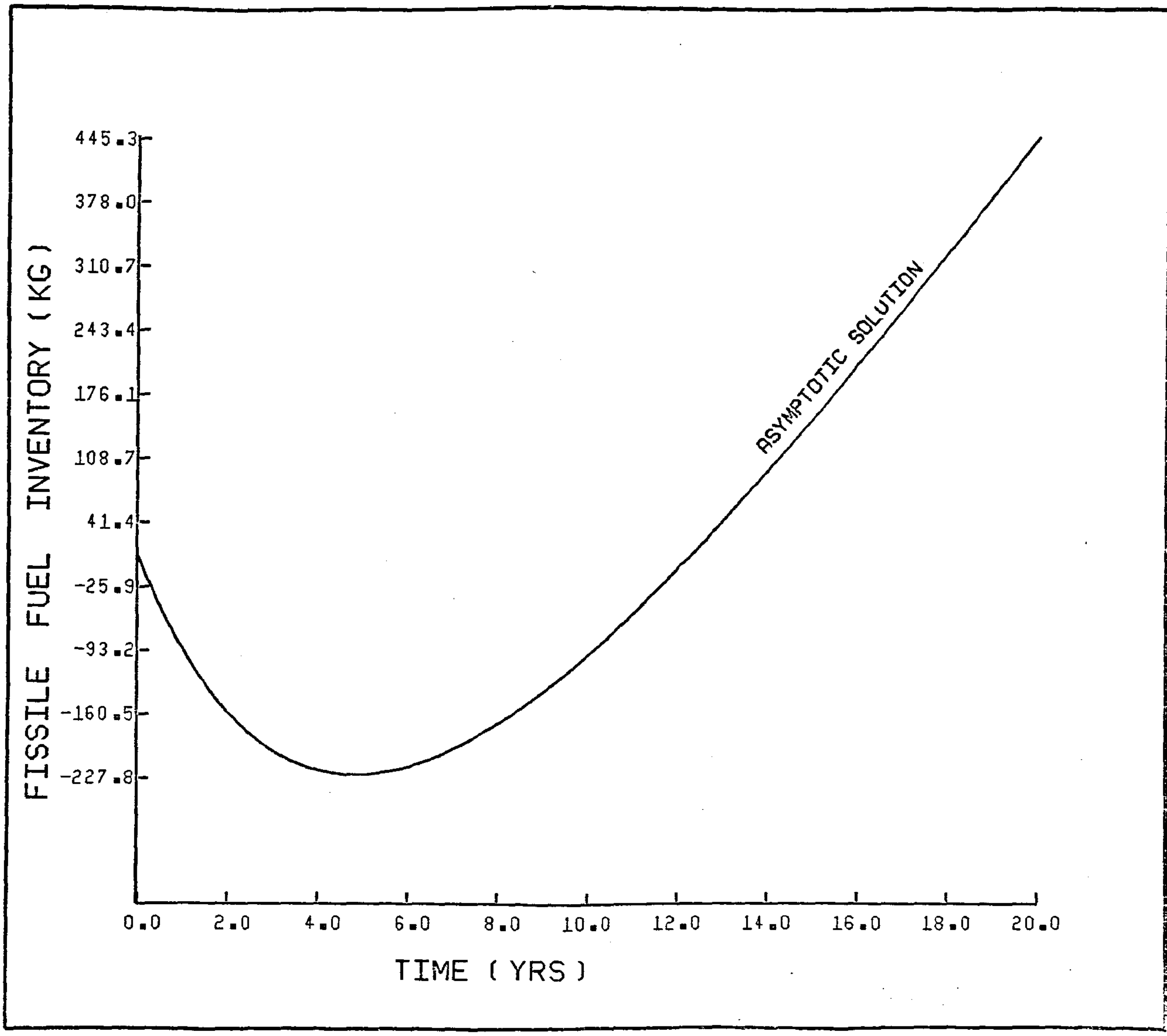

Fig. 3.1: The trajectory of fuel inventory of external stockpile for a breeder reactor obtained with asymptotic values of reactor parameters. The parametric values are based on data for the CRBR.

TABLE I

Parametric Values for the Calculation of Net Fissile Fuel Flows". The data corresponds to those for the Clinch River Breeder Reactor.[5]

Parameter	Numerical Value Used
Total breeding gain $G_c+G_{b,r}+G_{b,a}$	BR-1 = 0.22
Equilibrium residence time for radial blanket, λr	6 yrs
Equilibrium residence time for axial blanket, λ_r	3 yrs
Radial blanket breeding gain, $G_{b,r}$	.39
Axial blanket breeding gain, $G_{b,a}$	.22
Constant fissile fuel destruction rate, R	400 kg/y
Equilibrium load factor, L	.75
Refuelling Interval, t_c	Annual
Equilibrium reload fraction for axial blanket, α_a	1/3
Equilibrium reload fraction for radial blanket, α_r	1/6
Blanket shuffling	in$\rightarrow$out
Initial fissile fuel in radial blanket, $m_{b,r}$	$\sim$0
Initial fissile fuel in axial blanket, $m_{b,a}$	$\sim$0

CHAPTER 4

TIME DEPENDENT PARAMETERS

4.1 Time Dependent Average Residence Time for a Reactor Fuel

The objective in this section is to obtain the average residence time for all the fissile fuel fed into a reactor core up to and including those in the core at any point in time of the reactor life. The formulation will be based on fuel management scheme which allows replacement of some fuel bundles during each of many refuelling periods. In the CRBR[2] for example, 1/3 of the core fuel assemblies are replaced on a yearly refuelling basis. In this case all the initial fuel load are replaced at the end of the third year. During the next refuelling period the whole one-third of the core fuel used for the first refuelling is now replaced and this sequence is maintained for the subsequent cycles.

It is important to note that the result we shall obtain for the core will be generally true for average residence times in the blanket regions since the difference in the refuelling schemes are primarily due to the fraction of fuel replaced in each cycle, and the cycle lengths which are specified quantities.

If we now adopt a method of lumping together all fuel bundles replaced in each cycle, we shall have 3 lumps of fuel for the first core load of the above example. The number of fuel lumps then corresponds to the number of refuelling periods required to replace all the initial fuel loads as illustrated schematically in Fig. 4.1.

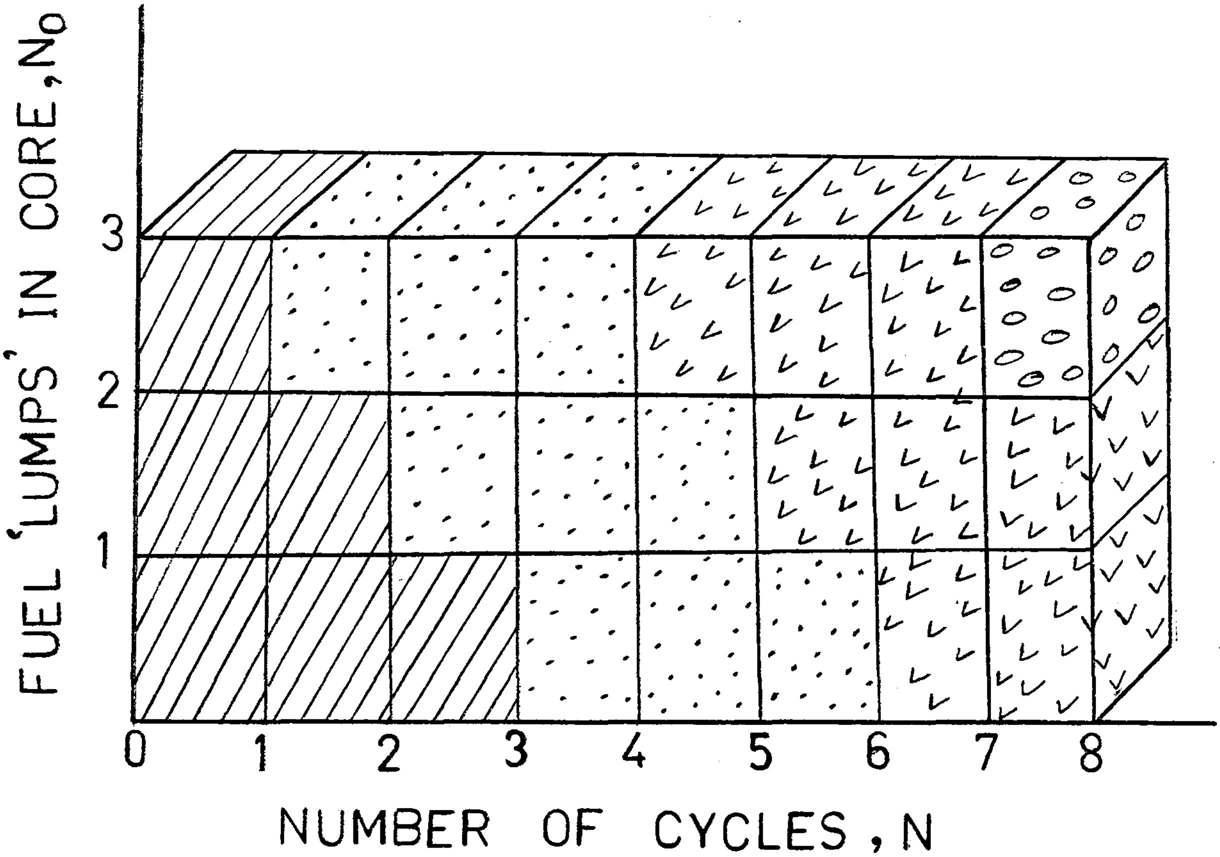

Fig. 4.1: A model showing fractions of the core fuel replaced per cycle and the duration of each fraction or "fuel lump" in the core. The model assumes one-third of the core fuel is replaced at the end of each cycle, i.e. $N_o = 3$.

In the formulation the following parameters will be employed:

t_c: Reactor refuelling time interval specified.

N_o: Number of refuelling periods or cycle required to replace all initial fuel loads; this is related to the fraction α of the core fuel assemblies replaced each time by $N_o = 1/\alpha$.

t: Point in time of the reactor life.

N: Number of completed cycles at time t.

$\overline{\lambda}(t)$: Average residence time for all the core fuel used by the time t.

A notion for the value of $\overline{\lambda}(t)$ can be obtained through the following considerations. Since all initial core load remains in the reactor till the first refuelling time t_c, this specifies a lower bound for the average residence time while the upper bound is determined by the number of cycles N_o, required to replace all the initial loads. Hence,

$$t_c < \overline{\lambda}(t) < N_o t_c \qquad (18)$$

A bookkeeping procedure which counts all the fuel lumps that have been used in the core at the end of each cycle and the total time they have spent in the core is shown in Table II. In Table II, the average residence time for all the fuel elements that have been used and removed from the core together with those in the core during each of the refuelling periods have been obtained from the ratio of the cumulative time to cumulative number of fuel lumps in the storage bays and core. We can thus obtain directly from the last column of Table II a general form for the average residence time $\overline{\lambda}(t)$ and this is given by

TABLE II

Fuel "lumps" used during the operation of a reactor in which one-third of the total core fuel is replaced per cycle.

No. of cycles or refuelling periods (N)	1	2	3	4	5	6	7	8	N
Time (tc yrs)	1	2	3	4	5	6	7	8	N
No. of fuel lumps in core (No)	3	3	3	3	3	3	3	3	N_o
Time for various fuel lumps in core (t_c yrs)	1,1,1	2,2,1	3,2,1	1,3,2	2,1,3	3,2,1	1,3,2	2,1,3	
No. of fuel lumps in storage bay	0	1	2	3	4	5	6	7	N-1
Time for various fuel lumps in storage (t_c yrs)	0	1	1,2	1,2,3	1,2,3,3	1,2,3,3,3	1,2,3,3,3,3	1,2,3,3,3,3,3	
Cumulative No. of fuel lumps in core and storage bay	3	4	5	6	7	8	9	10	N_o+N-1
Cumulative time (t_c yrs)	3	6	9	12	15	18	21	24	N_o N
Average residence time of each fuel lump at the end of each cycle (t_c yrs)	1	1.5	1.8	2.0	2.14	2.25	2.33	2.4	$\frac{N_o N}{N_o+N-1}$

$$\overline{\lambda}(t) = \frac{N_o N}{N_o + N - 1} \; t_c \tag{19}$$

Since the number of cycles N are related to the time t and the cycle length tc by N = t/tc we then have

$$\overline{\lambda}(t) = \frac{N_o \; t_c \; t}{t + (N_o - 1)t_c} \tag{20}$$

The average residence time $\overline{\lambda}(t)$ thus depends on the fraction of fuel N_o^{-1} replaced at the end of each cycle and the refuelling time t_c. To obtain $\overline{\lambda}(t)$ in terms of the fraction α, of the fuel assemblies replaced in each cycle we make the substitution

$$N_o = 1/\alpha$$

and this leads to

$$\overline{\lambda}(t) = \frac{\frac{1}{\alpha} \; t_c \; t}{t + (\frac{1}{\alpha} - 1)t} \tag{21}$$

i.e.

$$\overline{\lambda}(t) = \frac{t_c \; t}{t + (1 - \alpha)t_c} \tag{22}$$

For the case where 1/3 of the fuel is replaced in each cycle on a yearly refuelling basis the average residence time of all the fissile fuel used at any time t is thus given by

$$(t) = \frac{t}{0.333t + 0.6667} \tag{23}$$

In Table III the values of $\overline{\lambda}(t)$ given by Eq. (23) at the end of each year of the reactor operation are shown. The values obtained in Table III correspond to those in Table II as it should, since the model for our formulation was based on the same refuelling conditions, and the average residence times of Table III have been calculated at times t corresponding to successive cycle periods used in the model. This calculation thus crosschecks that the final form of the average residence time given by Eq. (22) agrees with the counting procedure adopted in Table II.

It is obvious from Eq. (22) that the average residence time will vary for different values of cycle length t_c, even for the case when the same fraction α, of fuel is replaced in each cycle. These variations are shown in Fig. 4.2. The maximum value of the mean residence time corresponding to $t = \infty$ for each value of t_c is equal to t_c/α.

4.2 Time Dependent Load Factor

The load factor as noted earlier can vary from 0.3 in the early cycles to 0.75 during the equilibrium cycles. In the first one or two cycles the fissile fuel loadings in the reactor can be set to provide

TABLE III

Sample values of mean residence time of Eq. (23)

t (yrs)	$\overline{\lambda}(t)$ (yrs)
1	1
2	1.5
3	1.8
4	2.0
5	2.14
6	2.25
7	2.33
8	2.4
9	2.46
10	2.5
∞	3.0

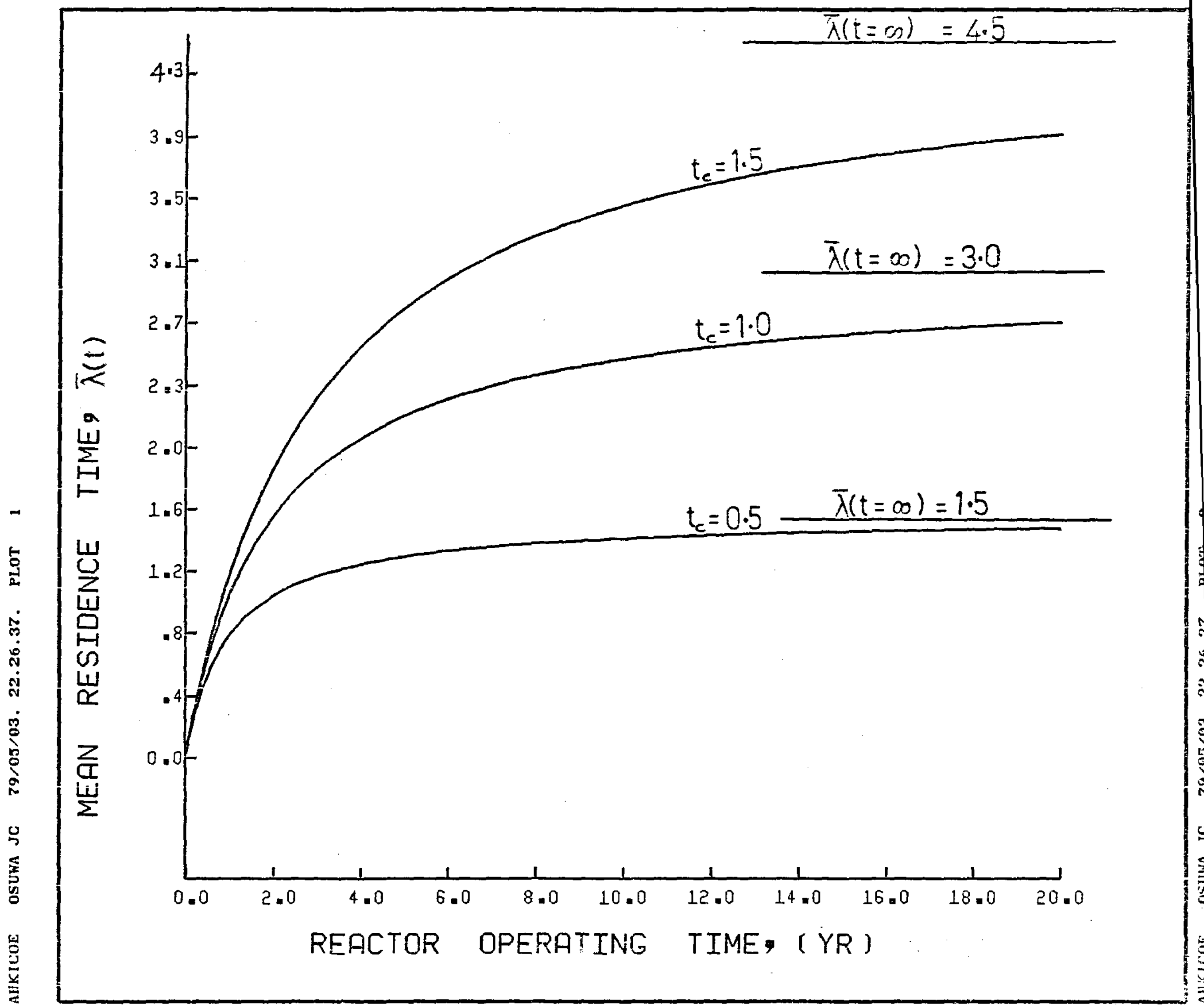

Fig. 4.2: Time dependent mean residence time of fuel in a reactor core in which one-third of the core fuel is replaced per cycle. The t_c denotes cycle length.

sufficient excess reactivity for 128 full power days of operation. For a cycle length of 366 days, this will correspond to a load factor of 128/366 or 0.35. In the subsequent cycles the fissile fuel loading may have enrichment capable of providing 275 full power days of operation corresponding to a load factor of 0.75. The load factor thus varies from cycle to cycle in a stepwise manner shown in Fig. 4.3.

It is important to note that the fuel burnup will vary accordingly as the load factor changes. Since higher load factor requires higher fuel enrichment, the fuel burnup will therefore increase with load factor.

One form of time dependent load factor that can be used is to approximate the stepwise variations by a linear function which simply leads to

$$L(t) = \begin{cases} c + at\ , & t \leq t_\infty \\ \text{constant}, & t > t_\infty \end{cases} \tag{24}$$

where t_∞ denotes the time at which the load factor attains a constant value. The load factor in the form of Eq. (24) obviously introduces as much errors as indicated in the shaded portion of Fig. 4.3. A more accurate form of the load factor can be obtained by the use of the Heaviside function which precisely defines a step function by the relation

$$\theta(t - t') = \begin{cases} 1, & t > t' \\ 0, & t \leq t' \end{cases} \tag{25}$$

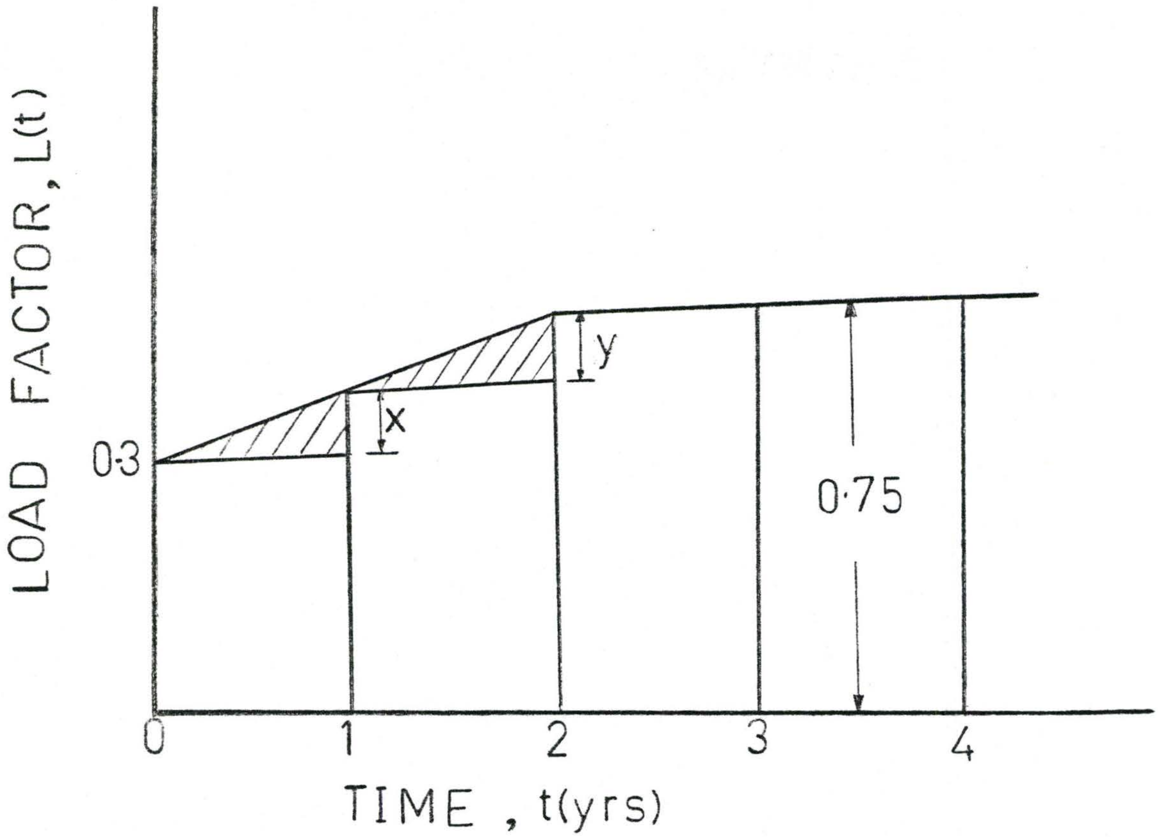

Fig. 4.3: A possible time dependent load factor representation.

We can immediately write the load factor as

$$
\begin{aligned}
L(t) &= c + x\theta(t-t_1) + y\theta(t-t_2) + \ldots\ldots \; z\theta(t-t_N) \\
&= c + \sum_{i=1}^{N} a_i\theta(t-t_i)
\end{aligned}
\tag{26}
$$

where c is the initial load factor and a_1=x, a_2=y etc., are the increments in the load factor at times t_i up to the equilibrium value.

CHAPTER 5

MATHEMATICAL ANALYSIS OF THE FUEL TRAJECTORY EQUATION

5.1 Detailed Integral Analysis

With the time dependent forms of the average residence time and the load factor obtained, we can now proceed with a more rigorous mathematical evaluation of the fuel trajectory of Eq. (12) which is of the form

$$m_{ext}(t) = m_i - m_c + \int \{G_c(t)R(t)[1+T(t)] + \frac{m_{b,r}(t)}{\lambda_r(t)} + \frac{m_{b,a}(t)}{\lambda_a(t)}\}dt \tag{27}$$

where the parameters are as previously defined, and the fissile fuel inventories $m_{b,r}(t)$ and $m_{b,a}(t)$ in the two blanket regions are given by

$$m_{b,j}(t) = \exp(-\int \frac{dt}{f(t)}) \int G_{b,j}(t)R(t)\exp(\int\frac{dt}{\lambda_j(t)})dt + m_{b,j}(0) \qquad j = r,a \tag{28}$$

The approximations used in the asymptotic parameter calculations for the breeding gains $G_c(t)$, $G_{b,r}(t)$, $G_{b,a}(t)$ and the transient fraction function $T(t)$ will be retained for the same reasons that their variations are very small and non predominant. Hence $[1 + T(t)]$ will be approximated by unity and the breeding gains taken as constants.

We now require to evaluate $m_{b,j}(t)$ analytically from Eq. (28) before substituting into Eq. (27). Using the average residence time given by Eq. (22) and the load factor of Eq. (26) we can then write Eq. (28) as

$$m_{b,j}(t) = \exp(-\int \frac{\alpha t + (1-\alpha)t_c}{t\ tc} dt)\int_0^t \{G_{b,j}R[c + \Sigma a_i \theta(t-t_i)]$$
$$\exp(\int \frac{\alpha t + (2-\alpha)t_c}{t\ tc} dt)\}dt + m_{b,j}(0) \quad (29)$$

For further simplification we will base our calculation on the data available on Table I for the CRBR and we can thus take the cycle length to be one year. Writing $(1-\alpha)$ as β will then lead to

$$m_{b,j}(t) = \exp(-\int (\alpha+\beta/t)\int_0^t G_{b,j}R\{c + \Sigma a_i \theta(t-t_i)\}$$
$$\exp(\int(\alpha+\beta/t)dt)dt + m_{b,j}(0) \quad (30.a)$$

$$= G_{b,j}Re^{-\alpha t}t^{-\beta}\int_0^t \{c + \sum_i a_i \theta(t-t_i)\}e^{\alpha t}t^{\beta}dt + m_{b,j}(0) \quad (30.b)$$

$$= G_{b,j}Re^{-\alpha t}t^{-\beta}[\int_0^t ce^{\alpha t}t^{\beta}dt + \sum_i a_i \int_0^t \theta(t-t_i)e^{\alpha t}t^{\beta}dt]$$
$$+ m_{b,j}(0) \quad (30.c)$$

From the property of the Heaviside function $\theta(t-t_i)$ has a value equal to unity if $t>t_i$ but zero otherwise and this allows us to write

$$m_{b,j}(t) = G_{b,j}Re^{-\alpha t}t^{-\beta}[\int_0^t ce^{\alpha t}t^{\beta}dt + \sum_i a_i \int_{ti}^t e^{\alpha t}t^{\beta}dt] + m_{b,j}(0) \quad (31)$$

For the purpose of simplifying the notations, the subscript j has been omitted from α and β which represent the fraction of fuel replaced per cycle in each of the blanket regions.

In Eq. (31) the second term in the bracket can be broken into two terms and the equation then reduces to

$$m_{b,j}(t) = G_{b,j}Re^{-\alpha t}t^{-\beta}[(c + \sum_i a_i)\int_0^t e^{\alpha t}t^{\beta}dt - \sum_i a_i\int_0^{t_i} e^{\alpha t}t^{\beta}dt] \tag{32}$$
$$+ m_{b,j}(0)$$

Equation (32) accurately describes the fissile fuel inventory in the blanket regions, but we are now faced with the problem of integrating function of the form

$$F(t) = \int_0^t e^{\alpha t}t^{\beta}dt \tag{33}$$

where α and β are non-integers.

Further evaluation of Eq. (32) involves the use of hypergeometric functions and ultimately leads to infinite series solutions. Rather than pursuing this solution formalism we undertake some specific approaches which lead to more tractable representations and systems descriptions.

5.2 Quasi-Static Solution of the Fuel Trajectory Equation

In order to study the effect of the time dependent forms of the mean residence time and the load factor we will adopt a quasi-static approach aimed at improving the result obtained for the asymptotic parameter solution.of the fissile fuel trajectory equation. This requires that instead of the asymptotic values used for the mean residence time and the load factor in the evaluation of the external stockpile inventory, the appropriate values of the mean residence time and the load factor corresponding to the point in time at which the inventory is required will be used.

We consider once more the specific case of a breeder reactor with a refueling interval t_c, of one year and reload fractions, $\alpha_a = 1/3$

for the axial blanket and $\alpha_r = 1/6$ for the radial blanket. With the mean residence time given by Eq. 22 as:

$$\bar{\lambda}(t) = \frac{t_c t}{t + (1-\alpha)tc} \tag{34}$$

we then have

$$\bar{\lambda}_a(t) = \frac{t}{.333t + .667} \tag{35}$$

and

$$\bar{\lambda}_r(t) = \frac{t}{.1667t + .833} \tag{36}$$

The operating requirements for annual refuelling are as shown in Table (IV).

Using Eq. (26) for the load factor we then have

$$L(t) = 0.35 + 0.2\theta(t-2) + 0.2\theta(t-3)$$

$$= \begin{cases} 0.35, & t < 2 \\ 0.55, & 2 \leqslant t < 3 \\ 0.75, & t \geqslant 3 \end{cases} \tag{37}$$

For our case study the quasi-static fuel trajectory equation which follows from Eq. (16) takes the form

TABLE IV

Operating and Fuel Performance Requirements for CRBR

Cycle Number	Capacity Factor (full-power-days)	Maximum Allowable Burn-up Limit for Reload Core Assemblies Charged at the Start-of-Cycle (m WD/Tonne)
1	0.35 (128 FPD)	80,000
2	0.55 (200 FPD)	100,000
3	0.75 (274 FPD)	125,000
4	0.75 (274 FPD)	125,000
5	0.75 (274 FPD)	150,000

$$m_{ext}(t) = [.35 + 0.2\theta(t-2) + 0.2\theta(t-3)]GRT$$
$$+ \{m_{b,a} - G_{b,a}[.35+0.2\theta(t-2)+0.2\theta(t-3)]\frac{t}{.333t+.667}\}$$
$$\{1-e^{-(.333t+.6667)}\}$$
$$+ \{m_{b,r} - G_{b,r}[.35+0.2\theta(t-2)+0.2\theta(t-3)\frac{t}{.1667+.833}\} \qquad (38)$$
$$\{1-e^{-(.1667+.833)}\}$$

where G is the total breeding gain and all the other parameters are as previously defined. Substutitng the parametric values contained in Table I into Eq. (38) leads to the following result:

$$m_{ext}(t) = \begin{cases} 30.8t - \frac{30.8t}{.333t+.667}(1-e^{-(.333t+.667)}) - \frac{54.6t}{.1667+.833} \\ \quad (1 - e^{(1.667t-.833)}), \ t < 2 \\ 48.4t - \frac{48.4t}{.333t+.667}(1-e^{-(.333t+.667)}) \\ \quad - \frac{85.8t}{.1667+.833}(1-e^{-(.1667t+.833)}), \ 2 \leq t < 3 \\ 66t - \frac{.66t}{.33t+.667}(1-e^{-(.333t+.667)}) \\ \quad - \frac{117}{.1667t+.833}(1-e^{-(.1667t+.833)}), \ t \geq 3 \end{cases} \qquad (39)$$

The graphical form of Eq. (39) is shown in Fig. 5.1 in which the effect of the discrete form of the load factor is quite pronounced with a higher load factor depicting a higher fuel destruction rate.

Figure 5.2 is a comparison of the two results obtained in this work with the discrete space-time calculation by Paik et al.[5] for the CRBR case.

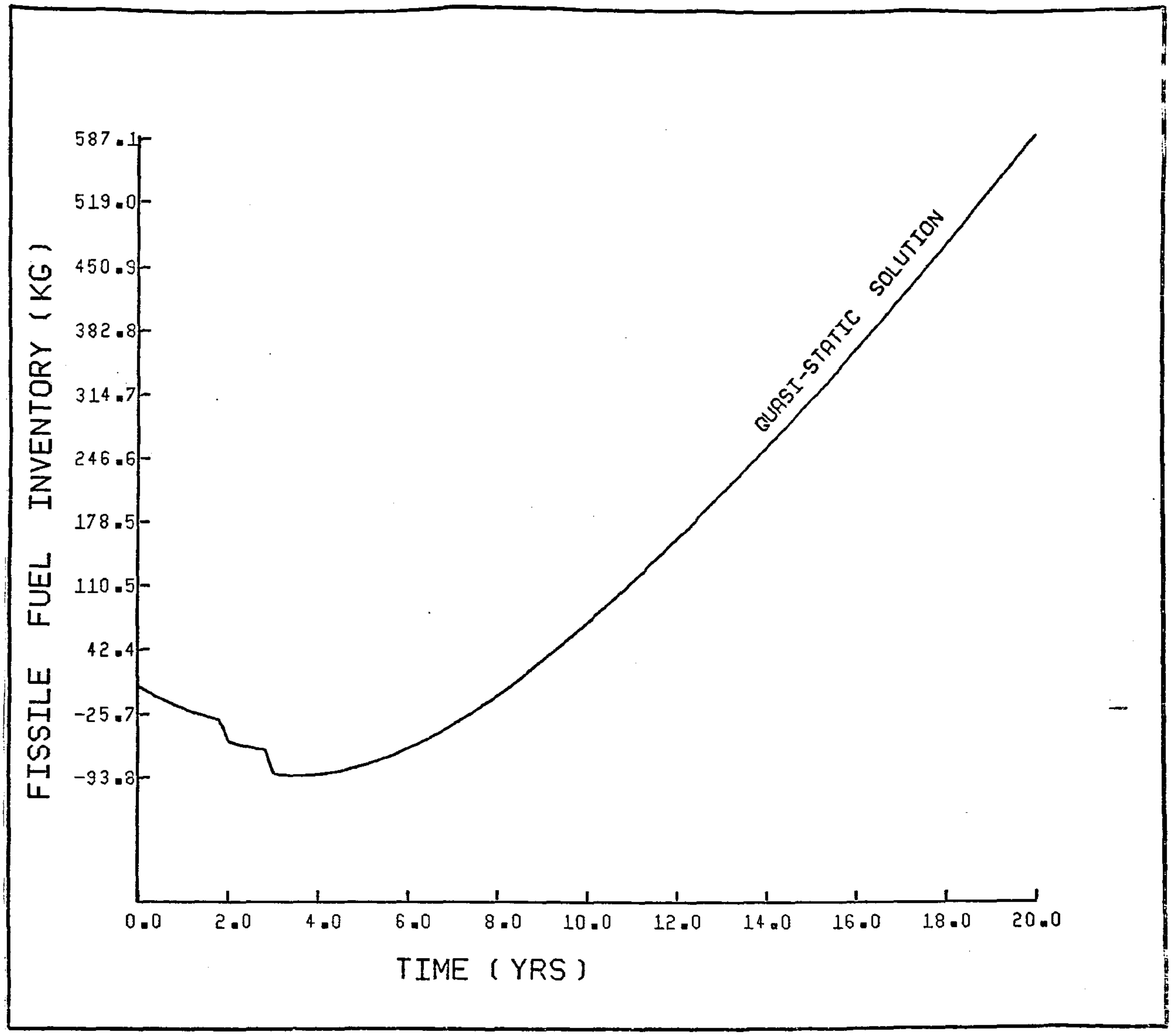

Fig. 5.1: The trajectory of the fuel inventory of external stockpile for a breeder reactor with time dependent forms of the mean residence time and the load factor incorporated.

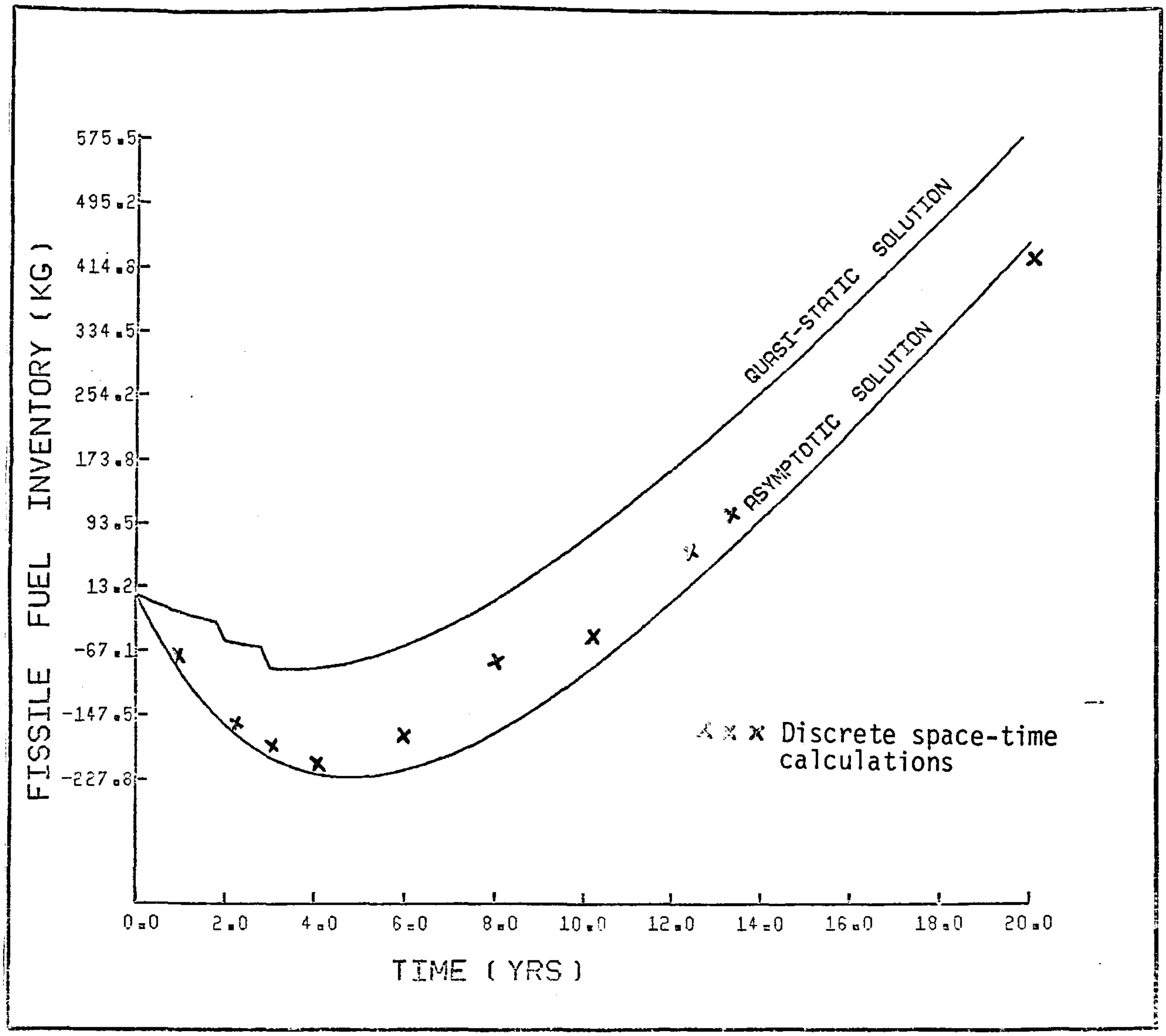

Fig. 5.2: The asymptotic and quasi-static results obtained in this work compared with the discrete space-time calculation for the CRBR by Paik et al.[5].

CHAPTER 6

APPLICATION OF THE FUEL INVENTORY FORMALISM TO CANDU REACTOR

6.1 Introduction

In this section the fuel inventory formalism developed will be extended to CANDU reactors. First, we enumerate the various ways in which the fuel requirements of the CANDU which differ from those of other reactors can effect the formulation and results. In a CANDU reactor all the fissile fuel is accommodated in the core and no breeding blankets need be included in the formulation. The refuelling process in a CANDU is continuous and this implies that the spent fuel is being discharged at the same rate as fresh fuel is fed into the core. The fuel residence time in the core will therefore be determined by the refuelling rate.

Since the CANDU is not basically designed to breed fissile fuel, our result should reflect the rate of depletion of an external stockpile of natural uranium which supplies the fuel used for the continuous charging of the CANDU core, and to which is added only a smaller amount of fuel which acrews from the conversion of some fertile nuclei to fissile material inside the core. Thus, contrary to the present practice of storing spent fuel with the accumulated bred fuel in the storage bays, we assume that the bred fuel in its fissile feed "equivalent" form[10] is added directly to the external stockpile to conform with the system of Fig. 2.1.

6.2 Fuel Residence Time for a CANDU Reactor

The continuous on power fuelling of a CANDU core proceeds in a calandria which contains 380 fuel channels. Each fuel channel contains 12 fuel bundles which are continuously moved along the channel during a refuelling operation. The most common fuelling scheme replaces roughly two thirds of the channel with fresh fuel at each refuelling. In one refuelling operation two alternate channels are continuously fed with fresh fuel bundles from opposite directions, while an equal number of spent fuel is discharged at the apposite ends of the channels. Figure 6.1 shows a schematic form of the Calandria with an open section illustrating the fuelling directions in the channels.

For the purpose of this analysis we assume that all the fuel bundles in a channel are replaced during a refuelling operation and that the fuel is fed at a constant rate through the channels. The length of each channel will be represented by L and the constant rate at which the fuel moves through each channel by U. The time taken by an element of fuel to move through the channel will then be given by

$$\lambda' = L/u \tag{40}$$

There are 380 channels in the Calandria with n channels refuelled each day, where n is equal or greater than 2. After the first round of refuelling session each channel will be refuelled once in every 380/n daily operations. Except for the initial fuel bundles used to load the channels, all the fuel bundles used for refuelling will therefore have residence time given by

$$\lambda = \frac{380}{n}(\tau + \lambda') \tag{41}$$

where τ is the time interval between refuelling operations.

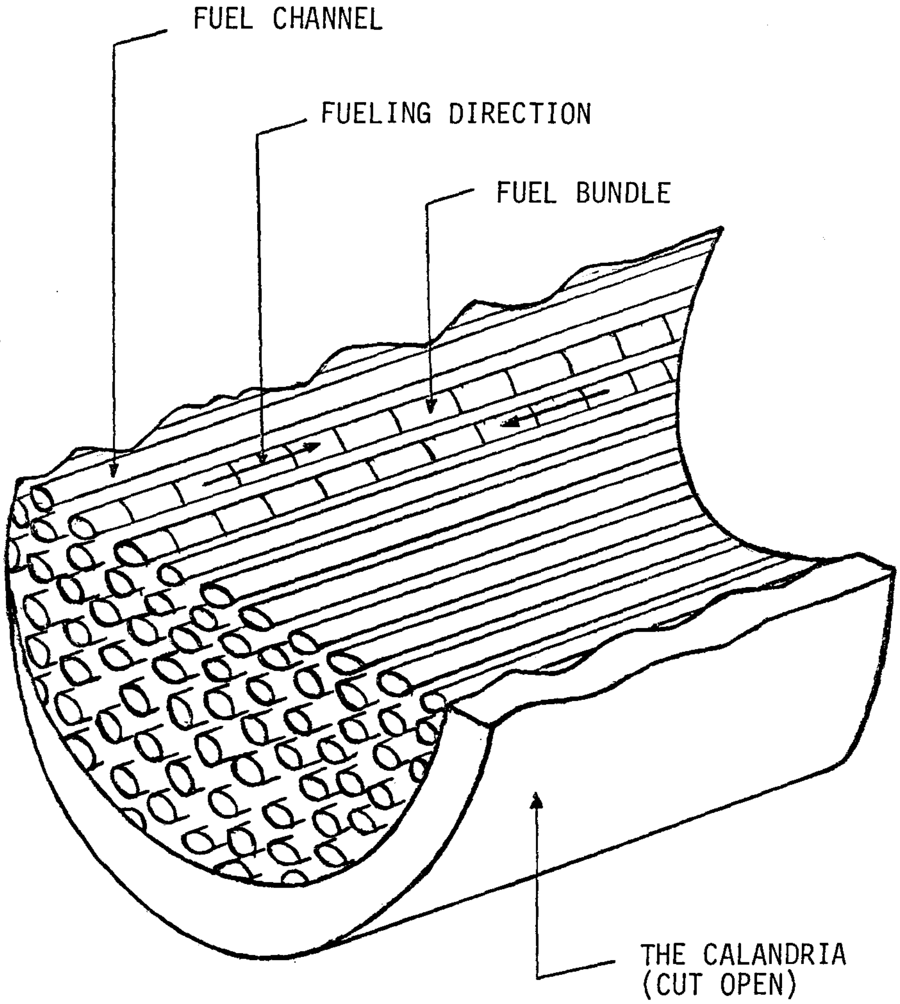

Fig. 6.1: A CANDU Calandria showing fuel channels, a section of fuel bundles, and the directions of fuel movement during a refueling operation.

The fuel bundles used for the initial loading will reside in the core for some time between $n\lambda/380$ to λ. The mean residence time can thus be taken to be given by Eq. (41), an approximation which is more valid as the number of fuelling operations increases.

6.3 External Stockpile Trajectory for a CANDU Reactor

The rate of change of fissile fuel contained in external stock-ile that supplies a CANDU reactor is obviously determined by the amount of fuel fed into the core per unit time and the amount of bred fuel discharged per unit time from the core into the stockpile. As stated earlier we assume that all the bred fuel discharged goes directly into the stockpile; we can then write

$$\frac{dm_{ext}(t)}{dt} = -\text{ charge rate of fresh fuel + discharge rate of bred fuel}$$

$$= -\frac{m_o}{\lambda} + \frac{m_b(t)}{\lambda} \qquad (42)$$

where $m_b(t)$ is the bred fuel discharged from the core in a refuelling session, and m_o is the amount of fuel fed into the core in each refuelling session, λ is of course the mean residence time.

To obtain the quantity of bred fuel discharged from the core, we note that $m_b(t)$ must satisfy the following condition inside the core:

$$\frac{dm_b(t)}{dt} = CR - \frac{m_b(t)}{\lambda}, \quad m_b(0) = 0 \qquad (43)$$

where R is the fissile fuel destruction rate in the core which is given by

$$R = \iint_{rE} \Sigma_a^{fi}(r,E)\phi(r,E)drdE \tag{44}$$

and C is the conversion ratio for the reactor which is defined by

$$C = \frac{\text{production rate of fissile material}}{\text{consumption rate of fissile material}} \tag{45}$$

The solution of Eq. (43) directly gives the available bred fuel in the core as

$$m_b(t) = CR\lambda\{1 - e^{-t/\lambda}\} \tag{46}$$

Using Eqs. (42) and (46) the fissile fuel inventory for the external stockpile is then

$$\frac{dm_{ext}(t)}{dt} = \frac{m_o}{\lambda} + CR\{1 - e^{-t/\lambda}\} \tag{47}$$

By direct integration we have

$$m_{ext}(t) = -\frac{m_o}{\lambda}t + CRt + CR\lambda(e^{-t/\lambda} - 1) \tag{48}$$

Obviously the fissile fuel destruction rate R, determines the rate at which fuel is fed into the core to supplement it, and these two quantities can thus be set equal. Hence,

$$R = m_o/\lambda \tag{49}$$

Substituting Eq. (49) into Eq. (48) then leads to

$$m_{ext}(t) = \frac{m_o}{\lambda}(C-1)t + Cm_o(e^{-t/\lambda} - 1) \tag{50}$$

i.e.

$$\frac{m_{ext}(t)}{m_o} = \frac{1}{\lambda}(C-1)t + C(e^{-t/\lambda} - 1) \tag{51}$$

Equation (51) is the final form of the trajectory equation of an external stockpile for a CANDU reactor given in units of m_o, the amount of fissile fuel fed into the core in one refuelling session. The graphical forms of Eq. (51) are shown in Figs. 6.2 and 6.3 for various values of λ with constant values of 0.6 and 1.0 for the conversion ratio respectively, and in Fig. 6.4 for various values of C with a constant value of 1.0 yr. for the mean residence time. The two graphs cover typical ranges of mean residence time and conversion ratio that can characterize the system of interest.

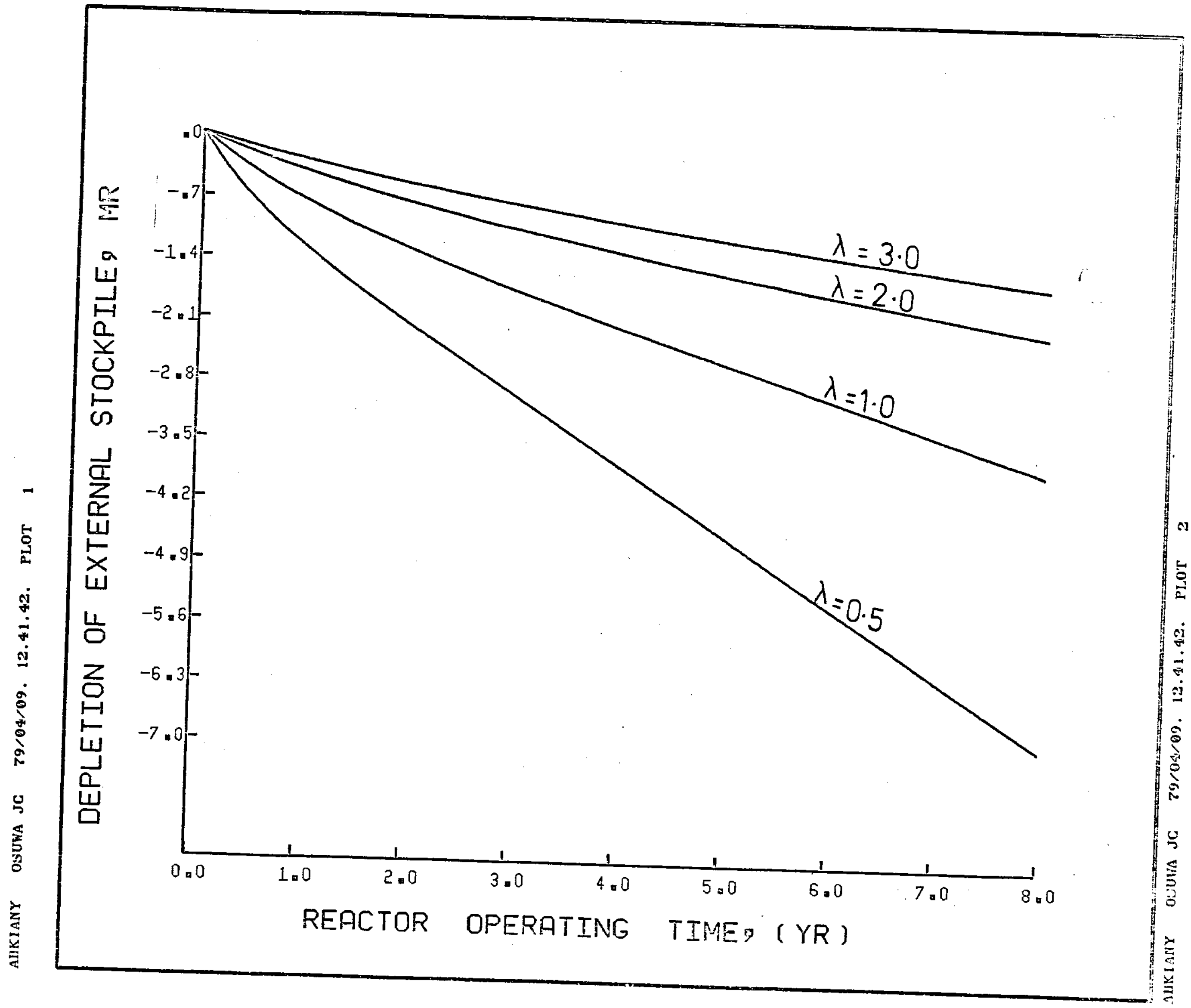

Fig. 6.2: External stockpile inventory associated with a CANDU reactor for various values of mean residence time with a conversion ratio of 0.6. MR denotes mass ratio $m_{ext}(t)/m_o$.

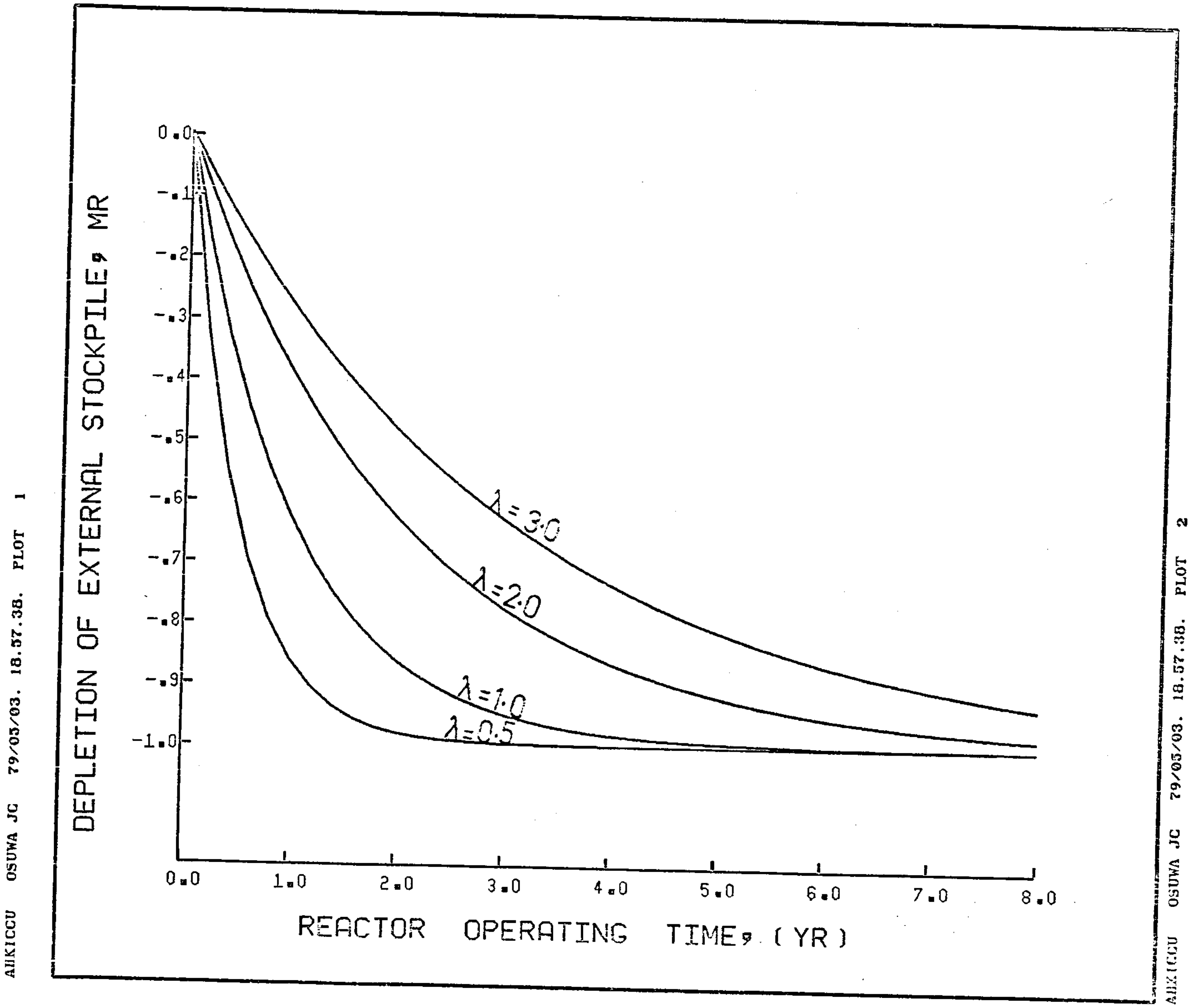

Fig. 6.3: External stockpile inventory associated with a CANDU reactor for various values of mean residence time with a conversion ratio of 1.0. MR denotes mass ratio $m_{ext}(t)/m_o$.

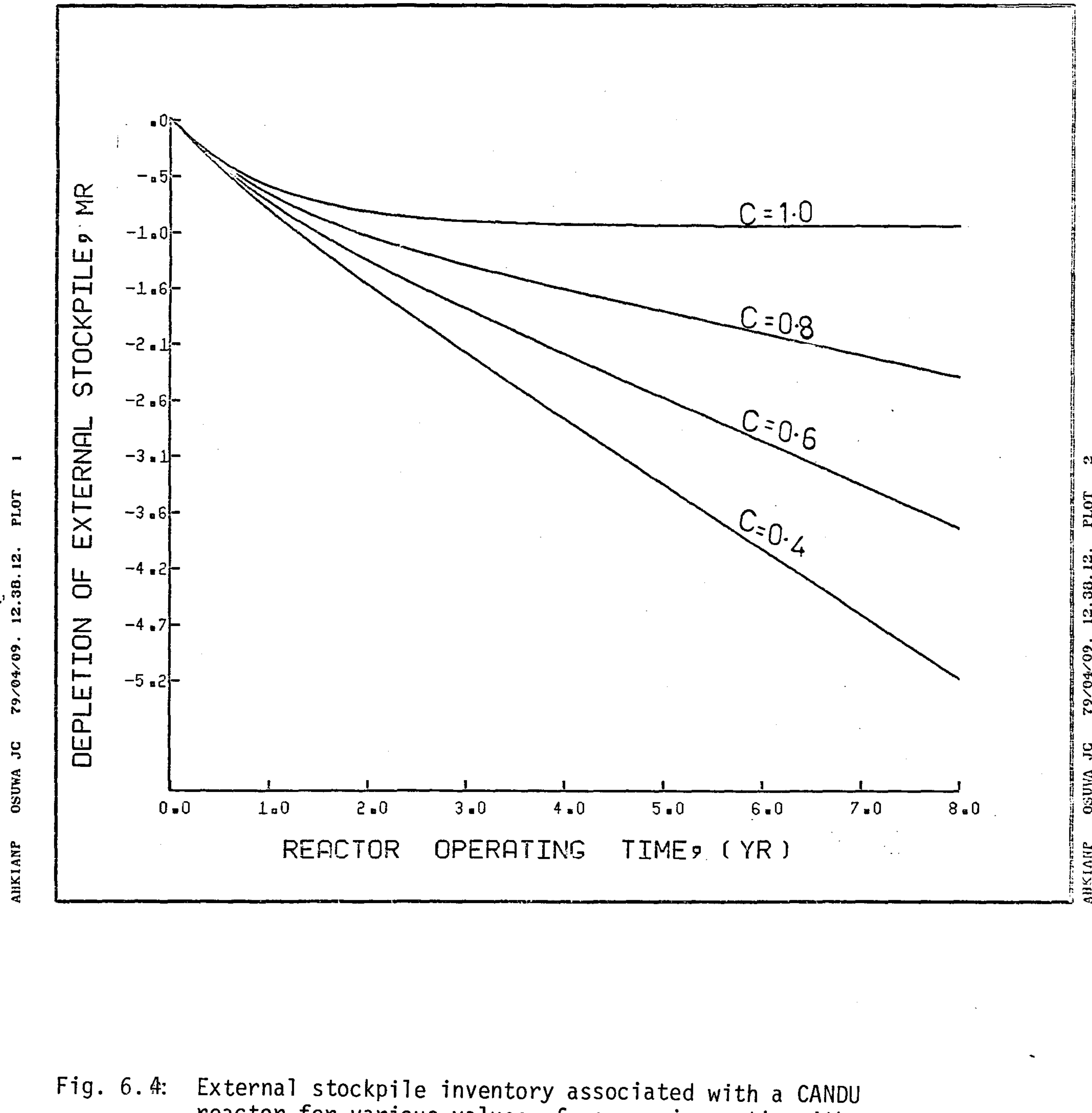

Fig. 6.4: External stockpile inventory associated with a CANDU reactor for various values of conversion ratio with a mean residence time of 1.0 year. MR denotes mass ratio $m_{ext}(t)/m_o$.

CHAPTER 7

CONCLUSION

Starting with a relatively simple mass balance equation, the final basic fuel inventory equation for a nuclear reactor takes on a complicated form. The first step in the solution of this equation which involves obtaining the forms of the time dependent reactor parameters that appear in the formulation has been successfully accomplished. However, a complete detailed evaluation of the inventory equation that incorporates the various forms of these parameters proved mathematically intractable due to the nature of the integrals involved.

Two alternative solutions which are sufficiently instructive have been provided. One of these solutions is based on the asymptotic values of the various parameters involved as provided in a data for the CRBR. The second solution utilizes the forms of the mean residence time and the load factor obtained here in a quasi-static approach. In both cases, the results provide satisfactory information on the trajectory of an external fuel stockpile that supplies a nuclear reactor. The results also compare favourably with the discrete space-time calculation by Paik et al.[5] for the Clinch River Breeder Reactor.

As is evident in these results, a breeder reactor is obviously a net fissile fuel consumer in the early stages of its life time but has an enormous potential of extending the duration of our limited fissile fuel which is a basic commodity for Nuclear Industries.

An extension of the inventory concept to CANDU reactors provides a great deal of insight on the effect of net fissile fuel consuming systems on the nuclear fuel reserves. The results obtained using various values of mean residence time and conversion ratios clearly indicates that fuel conservation strategy for such systems can be acheived by judiciously incorporating these two parameters in the fuel management scheme.

APPENDIX

"EQUIVALENT WORTH" OF REACTOR FUEL

Suppose the fuel fed into a reactor were composed of only one type of fissile isotope (index j) and one type of fertile isotope (index j-1) which is a precursor of the fissile isotope j, we would then appropriately write

$$(j-1) + \text{neutron} \rightarrow j \qquad \text{(A-1)}$$

and this would make the definition of fissile fuel particularly simple: the isotope j would be the fissile fuel, and the amount of fuel, m_j, would be given by the amount of isotope j. A trivial consequence of this assumption is that the fuel fed into the reactor would be indistinguishable from the fissile fuel discharged from the reactor.

The above "monoisotopic" concept of fissile fuel which enjoys a widespread application is clearly unrealistic since the predominant fissile feed isotope which is ^{235}U has no fertile precursor. The "equivalent worth" concept about to be discussed takes care of the consequent notational and conceptual difficulties of the real situation. Let (j-1) again denote the naturally occurring fertile fuel, then the fissile fuel isotope j is bred from (j-1) (as in Th_{232}-U_{233}-cycle) so also can other higher isotopes j+1, j+2, etc. be bred (^{235}U-^{239}Pu-cycle). Let n_k be the atomic density of isotope k in the fuel, n_5 representing ^{235}U and let $\bar{n}$ be a fissile fuel composition vector

$$\bar{n} = (n_5, n_k, n_{k+2} \ldots) \qquad \text{(A-2)}$$

Since the fuel consumption and breeding processes affect the isotopic composition of the fissile fuel, the fissile feed fuel, $\bar{n}_f$, will differ in isotopic composition from the discharged fuel, $\bar{n}_d$:

$$\text{i.e.} \quad \bar{n}_f \neq \bar{n}_d \tag{A-3}$$

This makes direct comparison of amounts of fuel feed, m_f, and discharged fuel m_d impossible; the definition of parameters based on amounts of consumed and produced fuel would thus be meaningless except for the case of "equilibrium composition" when $\bar{n}_f = \bar{n}_d = \bar{n}_{equil}$ (Ref. 10 and 11).

This problem is overcome by the use of a generalised notion of fissile fuel that is based on "equivalence weight factors" w_i (ref. 10). defined for each isotope i. These factors account for the "worth" of the isotope i with respect to any one of the fuel isotopes taken as reference isotope, preferably to the main fissile isotope, preferably the main fissile isotope. Thus taking ^{235}U as the reference isotope, we then have $w_5 = 1$. Then 1 kg of ^{235}U would have the same worth as $1/w_i$ kg of isotope i if

$$m_5 = m_i w_i \tag{A-4}$$

By the use of this concept the definition of fuel includes all the fissile and fertile isotopes, occurring in the fuel cycle with (j-1) representing the naturally occurring fertile isotope and the "fissile fuel" represented by all the other isotopes. The fissile fuel inventory, m, of a reactor or a stockpile containing $m_5$kg of ^{235}U, m_i kg of isotope i etc. will be defined in terms of "Equivalent Inventory" of only ^{235}U as

$$m = m_5 + \sum_i w_j m_j \tag{A-5}$$

The reactor feed and discharge rates will be similarly defined as

$$R = R_5 + \sum_i w_j R_j \tag{A-6}$$

The fissile fuel isotopic composition vector of Eq. (A-1) will now be replaced by

$$\bar{n} = (n_5, w_k n_k, w_{k+1} n_{k+1} \cdots) \tag{A-7}$$

The use of Eqs. (A-5) and (A-6) allows different inventories and rates to be compared and added to each other even for the case of different isotopic compositions since they are given in terms of ^{235}U equivalent quantities.

We note that the kind of "worth" has not been specified in the above discussion because of the generalized approach taken. The kind of worth is obviously not arbitrary but has to account for the worth of each isotope relevant in a given analysis. In thepresent study, the focus is on the amount of fuel that is consumed and produced by a reactor. The worth of each isotope is therefore determined by its contribution to the net amount of fuel in the stockpile:

$$w_j = \text{relative mass of isotope j with respect to the mass of the reference isotope, } ^{235}U \tag{A-8}$$

Hence, fissile fuel inventories, reaction rates, and transfer rates can be treated as if the fissile fuel were monoisotopic.

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