A MOVEABLE ARMTROUGH
FOR HEMIPLEGICS

MONK

A MOVEABLE ARMTROUGH FOR HEMIPLEGICS

By

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TO MY WIFE, BRENDA JEAN

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PREFACE

Patients with totally or partially dysfunctional arms, (hemiplegics or stroke patients) require a base on which to rest their afflicted arms. Presently, at Chedoke Hospitals, this consists of a padded, wooden support, rigidly secured to the wheelchair arm. This stationary support proved to be an obstacle when in close proximity to a work bench or at dinner tables.

A definite need was indicated for a type of moveable support, which could be rotated inward or outward when at a table.

This report is the evolution of a moveable armtrough developed at Chedoke Hospitals. The main features include:

1) simultaneous horizontal and vertical motion with locking.
2) soft sling-type support for afflicted arm, adjustable for varying arm sizes.
3) adjustment for various distances from body to suit individual patients.
4) detachable cone to be used for hand support, which can be positioned along the base for various lengths of forearms.

Costs, of both in manhours and dollars are discussed, detailed construction details have been included for the various parts and an initial patient response can be found in Chapter 7, in which the general acceptance of the armtrough was excellent.

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CHAPTER 1

INTRODUCTION

1.1 Definition of Problems

Patients who have suffered a stroke are usually paralyzed on one side of the body, with little or no voluntary movement of the affected limbs. With respect to the arms, the usual position of the affected arm is to be folded into the upper body with the palm of the hand facing the floor. Initially, severe pain or discomfort is encountered by the patient, when attempting to rotate the arm away from the body, but with physiotherapy, this manouvre can be accomplished with little or no discomfort.

All patients, whose arms have become immobile tend to develop a state of edema in the hand or upper arm, due to any number of reasons. However, such localized edema is generally caused by poor drainage through the lymph vessels of the arm, thus allowing the lymph (which is about 85% water) to accumulate in the tissues. This swelling may be relieved by elevating the afflicted limb, to encourage better drainage and by application of a cold compress.

The existing armtrough interferes when the patient is sitting at a bench or table, such as during work periods, at mealtimes and during recreational activities. The trough is securely fastened to the arm of the wheelchair and cannot be moved, unless it is unfastened. For more obese patients, the present trough presents a problem when they attempt to get in or out of their wheelchair, since it overlaps the sides of the wheelchair arm by about three or four inches on the inside.

When an immobile limb is subjected to support from a rigid object, such as the present trough, there is the possibility of lesions forming due to "pressure points" on the limb. This is an inherent problem since the arm or leg is not of a uniform shape. However, in discussing this matter with the Occupational Therapists, this is not a serious problem even with the present trough, since the arm rests on an acrylic fur material, which is quite plush, thus allowing the support member to conform somewhat to the shape of the arm.

1.2 Design Criteria

Considering the problems stated above, a modified armtrough was designed subject to the criteria summarized below:

1) The trough section must be able to rotate in the horizontal plane to a maximum of 90° on either side of the stationary wheelchair arm, and must be able to elevate in the vertical

plane to an angle of about 40° to the horizontal.

2) The trough must be easily adjustable by the patient or a physio-therapist to account for the variety of arm sizes and shapes.

3) The trough must not interfere with the patient's activities at a bench or table, or when the patient is getting in or out of the wheelchair.

4. The supporting section for the arm must not cause lesions of the arm due to contact points on the arm.

5) The armtrough assembly, as a whole, must have good aesthetic value, i.e., it must be pleasing to the eye.

CHAPTER 2

EXISTING ARMTROUGH

2.1 Description

The armtrough presently in use by the patients in the Holbrook Pavilion is essentially a single unit. It consists of a pattern which is cut out of a sheet of 3/4" plywood, with side walls approximately 2" high. Over this plywood base is attached a piece of acrylic fur material, to offer some means of comfort.

The entire unit is secured to the stainless steel arm of the wheelchair by means of two bolts, which protrude through the existing bolt holes in the metal arm of the chair. This means of fastening the trough prevents any possible rotation or elevation of the arm. Some elevation of the arm can be achieved by placing a rolled-up towel under the hand and wrist, but this elevation is 15° or 20° at best.

The arm itselt is secured to the trough by means of velcro straps, which proved to be uncomfortable to the patients if any appreciable movement of the arm was to be elimated. Quite often, the arm was allowed merely to rest freely on the acrylic fur, prevented from being accidently knocked out of the trough by the two inch high walls, the height of which was further reduced by the presence of the acrylic fur. To be sure, this type of accident was infrequent, if not rare, but the possibility remained.

2.2 Drawbacks of Existing Model

As stated in the previous section, there are many drawbacks in the armtrough presently in use. These will be outlined and briefly discussed in this section.

The foremost problem, is that the trough and hence the arm is restricted to one position, that being the position of the wheelchair arm and directed towards the front part of the chair.

There are no facilities for movement since it is securely fastened to the arm of the wheelchair by two bolts. Referring to section 1.1, this position is quite painful if not completely impossible for stroke patients or others who cannot attain this positioning of the arm except through extensive sessions in physiotherapy. The number of patients capable of using this present device is automatically restricted to those who are able to maintain this arm position in relative comfort over a period of hours.

A second problem is that the existing trough presents an obstacle when attempting to sit at a comfortable distance from a table or work bench. The end of the trough protrudes approximately six inches beyond the front of the wheelchair, and this extra distance from the table is unnecessary.

A similar problem is evident when relatively obese patients attempt to get in or out of their wheelchairs, with the help of a therapist. The trough contacts part of the abdomen during this manoeuvre, resulting from the trough extending into the sitting area by approximately three inches. This presents no problem to patients of smaller proportions.

This type of armtrough provides no easy means of adjustment, in either height or length, except by complete removal of the entire unit, or by placing padding under the arm. The latter, however, increases the probability of the arm being accidently knocked out of the trough, since the effective height of the walls is reduced.

The arm, of course, can be secured to the trough by means of velcro strapping, but this tends to be rather uncomfortable for the patients and the possibility exists for prolonged contact to cause minor skin irritations on the arm.

The above are the major drawbacks of the existing armtrough used by a wide number of patients. Considering these and the design criteria summarized in section 1.2, a modified armtrough was designed and constructed, the details of which appear in the next section.

CHAPTER 3

INITIAL MODIFICATIONS

3.1 Design Features

The initial design, in concept, was much improved over its predecessor, having separate facilities for horizontal rotation and vertical elevation. The major modifications are outlined in the next sections.

3.1.1 Vertical and Horizontal Motion

Rotation in the horizontal plane was to be accomplished by means of a strip of metal, such that when folded upon itself, would consist of a slot on the topside, and a series of holes on the bottom. Through a strip of metal on the underside of the armtrough a bolt would protrude through a hole. This bolt would go through the slot in the folded section and would be capable of fitting into one of the holes in the bottom portion. A nut located on this bolt would be located between the top and bottom of the folded section, such that the bolt and hence the armtrough could be raised enough to clear the holes, but could not be removed from the slot. In this way, the trough could be positioned on either side of the wheelchair arm.

Vertical elevation would also incorporate a slotted section. The slotted strip of metal would be secured rigidly to the bottom of the trough and another strip of metal would be secured at one point on the base plate, with a bolt on the other end to slide along the slot upon elevation. The trough would be locked in this vertically elevated position by means of a wing nut.

3.1.2 Trough Section

Pursuing the criterion that the armtrough be easily adjustable, the initial design of the trough was such that the two halves of the trough be connected by three plastic strips with a series of holes along the lengths. A bolt would pass through these connecting strips, through one of the sections and would be secured by a wing nut underneath the trough.

Considering the time of fabrication and the overall complexity of this design, it was abandoned in lieu of a much simpler idea.

The second trough design was cut from a sheet of ½" acrylic plastic. It was cut such that a central rectangular piece had two protruding "arms" along it's sides, separated by 5½ inches. These arms were to be bent 90° to the central plate by heating the plastic to a pliable temperature.

The patient's arm was to rest between these bent arms in the trough section. However, as will be shown in section 4, this design was rejected and replaced by a much simpler design.

3.1.3 Trough Padding For Arm

Employing the same idea as in the existing model, the patient's arm would be resting on a layer of acrylic fur material, while the bent arms of the trough would secure the arm from being knocked out of the trough. However, a much better method of employing this acrylic fur will be discussed in section 4, when discussing the final prototype.

3.2 Advantages Over Existing Armtrough

The advantages of this initial design over the existing design are clear when the two are compared.

The fact that this model has facilities for horizontal rotation and vertical elevation, however crude, makes this design far superior. The patient can easily control the device for his convenience, rather than have to modify his own actions to suit the device. He can more closely approach a table for work or pleasure, rather than having to reach the extra six inches, as with the existing model. The armtrough is more readily available to a wider variety of patients, since it can be rotated to a position in which the patient feels most comfortable.

The trough section itself offers more protection for the arm, since if it is accidently knocked, the patient's arm will be kept in the trough by the bent arms of the trough. This reduces the probability of the arm falling out of the device to practically zero.

3.3 Drawbacks of Initial Design

The initial design, although having distinct advantages over the existing armtrough, has many drawbacks. These will be discussed in this section.

Ironically, the most important advantage of this initial design is also its most serious drawback. This is the rotational and elevating apparatus.

The reader should appreciate the many steps in the fabrication of these parts. The cutting, drilling and bending will all contribute to the cost in manhours needed to make these. This, plus the fact that the apparatus did not operate as efficiently as was hoped, resulted in this design feature, per se, being rejected. Although the rotational and elevating motions could be achieved, it was done only with extreme difficulty, as the bolt had to be manually placed in the appropriate holes in the vast majority of trials. The apparatus also had a slight wobble, even when locked. The reader will appreciate the crudeness of this apparatus when the final rotational apparatus is discussed in section 4.

The idea of incorporating a hammock-type support for the patients arm was suggested. Seeing the many advantages offered by this idea, the trough section was slightly modified, so as to incorporate the sling effect. Thus, the patients arm would not be in contact with a rigid base, which was the idea with this initial design.

One other drawback that was considered in the rejection of this initial model was its appearance. The device looked too mechanical with the metal and plastic being clearly visible. Since it was so displeasing to the author, it was quite conceivable that the patients would find it similarly displeasing, if not more so.

CHAPTER 4

FINAL MODIFICATIONS

4.1 Design Features

The basic concept of Engineering is to utilize simpler concepts to achieve a goal with as much, if not better efficiency, i.e., to minimize input effort while maximizing the output benefits. With this in mind, the final modifications are discussed, which prove to be much simpler, yet better than the initial design of the arm-trough.

4.1.1 Vertical and Horizontal Motion

The one unique and central feature of this design is the incorporation of a ball-joint clamp to accomplish the desired horizontal rotation and vertical elevation. A nylon collar exerts pressure on the steel ball by way of a screw located on the underside of the steel housing. To change the positioning of the trough, one loosens the clamping screw such that the unit can be rotated to another position. This being done, the clamping screw is tightened, and the unit remains fixed in this position. Comparison of this apparatus to the initial modifications in section 3.1.1, enables the reader to appreciate the simplicity of operation of the ball-joint clamp. Whereas the previous model required two separate operations for the horizontal and vertical motions, the ball-joint clamp permits this with one operation.

4.1.2 Trough Section

The trough section was modified so as to make fabrication and assembly simpler. It consists of four rectangular pieces; a base (3/4" plywood), two side panels (½" acrylic plastic) and one end (½" acrylic plastic). The three plastic panels are screwed into the plywood base as shown in figures 9.1 to 9.5 inclusive.

Prior to this, a series of twelve holes, (four rows of three holes), were drilled into each side panel, and one row of three holes drilled into the end panel. The purpose of these holes is to accomodate the arm support, which is discussed in the next section.

The reader may refer to section 9 for technical data.

4.1.3 Arm Support

Rather than permit the arm to rest on a rigid base, it was decided to incorporate a sling in which the patients arm is placed. Since the acrylic fur material proved satisfactory for patient comfort, it was decided to use for the sling.

From a large piece of this material, a pattern is cut as shown in figure 9.6. A series of eight eyelets are attached along both sides, and four are attached on the apex end of the pattern. Through these eyelets, a section of 1/8" rope is woven, as in the second illustration, and knotted on either end. Cotter pins, slipped over the rope between each pair of eyelets on the back of the material are the means of attaching the sling to the trough section.

The cotter pins are pushed through the holes in the plastic side and end panels in the trough, and when the acrylic material between these panels is depressed, the sling appears. The trough is now ready to accept the patients arm. Again, the reader is urged to refer to section 6 for construction details.

4.1.4 Hand Support

On the occasions when it is not satisfactory for the patients hand merely to rest in the sling, separate facilities have been added for better support. This consists of a conic section, flattened on both ends.

To accomodate this hand support, a slot was cut through the front end of the plywood base. The wooden cone rests on the upper side of the base with the attachment bolt extending through to the underside. A wingnut is used on the bolt to secure this section from underneath.

To attach this unit, it is necessary to detach the sling from the front plastic panel and roll it such that it can be neatly tucked under the sling still formed between the side panels. The hand support can then be attached while the arm support remains.

The purpose of the slot in the plywood base is to accomodate the variety of lengths in patients arms. By loosening the wing nut, the support can be easily slid along the slot until the most comfortable position for the patient is attained.

4.1.5 Trough Attachment to Ball-Joint Clamp

Welded to the steel ball of the clamp is a rectangular bar of stainless steel, which serves as the base for the entire armtrough.

Attached along the central axis of the plywood base of the trough section are two stainless steel brackets, which consist of two stainless steel rectangular plates connected on either side by a "sandwich" of stainless steel strips that have been tack-welded together. The result is a box structure, open at both ends.

Once these have been attached to the base of the trough section, the bar from the ball-joint is slipped through these brackets, and it is locked by means of a central wingnut on each bracket.

The result is a structure, which can be slid along the bar, to be secured at a comfortable distance for the patient. Once the wingnuts are tightened, the trough section cannot be moved further. Again, pertinent data can be found in section 9.

4.2 Advantages Over Initial Modifications

Comparing the initial design of section 3.1 to the one just discussed, it is evident that the latter has many distinct advantages over its immediate predecessor. These are outlined in this section.

Although the final trough section is comprised of four separate parts, the fabrication and construction is simpler, since there are only straight cuts to be made. The heating and bending of the previous plastic trough, which proved to be cumbersome and time consuming, is eliminated in the final design.

Greater efficiency can be observed in the manner in which the trough may be adjusted. The height of the sling may be varied by using different sets of horizontal holes on the side and end panels. Proximity to the patient is also variable by sliding the trough along the bar welded to the ball-joint. The hand-support option is adjustable along the length of the trough by virtue of the slotted plywood base. The facilities for horizontal and vertical motions are much simpler and more efficient.

By utilizing the ball-joint clamp, the patient may attain an "infinite" variety of positions, bounded only by his own physical limitations or the structural limitations of the ball-joint. Together with the fact that both horizontal rotation and vertical elevation may be accomplished with one motion, makes this design simpler, yet more efficient than the apparatus described in section 3.1. The locking of the ball-joint is complete once the screw is tightened, and can only be moved when the locking-screw is loosened.

Another distinct advantage can be observed in the sling support for the arm. Rather than constraining the arm to comply with a rigid padded base, the sling will conform with the patient's arm. Thus, no one portion of the arm is in contact with the support more than another, hence there is no chance for lesions to occur due to these pressure points. The sling provides a firm, yet pliable arm rest.

The height of the sling is also adjustable, by using the three different rows of horizontal holes in the side panels of the trough section. Thus, for patients with relatively short upper arms, the sling may be raised, and for those with longer arms, the sling may be lowered.

The ease with which such an adjustment is made provides one other advantage. By simply extracting the cotter pins from one row of holes and transferring them to another row, the adjustment can be made in a matter of seconds.

4.3 Drawbacks of Final Design

One minor drawback with this armtrough is that over a period of time, the nylon pad pressing against the steel ball may tend to wear down. However, this part can be replaced easily and inexpensively. Estimates of this eventual wearing process will depend on the amount of adjustments made, but a minimum of five years would elapse before any noticeable wearing would appear.

Another source of potential wear is the eyelets attached to the sling section. During use of the sling, these may tend to detach themselves from the material. However, they are also simple and inexpensive to replace, and exercising care when attaching the eyelets will keep this potential problem to a minimum.

CHAPTER 5

ESTIMATION OF COSTS

5.1 Cost in Manhours

The time required for fabrication and construction of the assembly are listed below, in tabular form:

ITEM	CONSTRUCTION DETAILS		TIME REQ. (min.)
Trough Section	i)	Cut side and end panels and round corners	15
	ii)	Measure and drill holes	10
	iii)	Cut plywood base and slot	8
	iv)	Attach panels to base	2
		TOTAL	35
Arm Support	i)	Cut pattern	2
	ii)	Measure and cut slots for eyelets	2
	iii)	Secure eyelets to pattern	15
	iv)	Weave rope through eyelets and knot	3
	v)	Attach cotter pins (cleaned)	3
		TOTAL	25

5.1 Cost in Manhours (cont)

ITEM	CONSTRUCTION DETAILS	TIME REQ.(min)
Brackets for trough	i) Cut stainless steel pieces	10
	ii) Tack-weld strip "sandwiches"	10
	iii) Attach wing-nut lock	10
	iv) Drill holes for attachment to trough	10
	v) Secure to base of trough	5
	TOTAL	45
Ball-joint Clamp	Single unit- no fabrication	-
	Collar to secure to wheel-chair	5
	Front section to fit wheel-chair	15
	Cut and weld lever arm to ball	15
	TOTAL	35

Total time required = 2 hours, 45 minutes.

Thus, as a conservative estimate, one trough could be initiated and completed in 4½ hours.

5.2 Material Cost in Dollars

ITEM	QUANTITY	UNIT COST	ITEM COST	LOCATION
Wood Screw (1"x.6)	10	.35/pkg.12	$ .30	Hardware Stores
Wood Screw (1"x.10)	8	.35/pkg.12	.24	Hardware Stores
Bolt - (1½"x¼")	1	.05 each	.05	Hardware Stores
Cotter Pins (1/8")	9	.25/pkg.20	.12	Hardware Stores
Lumber-Pine (3/4" x 4½")	2 feet	.21/ft.	.42	Holbrook Work Shop
Twine-Corticelli Type CBM 8398	3 feet	$31/500 yd. spool	.06	Mohawk Hospital Services
Acrylic Sheet (5/8")	75 ft.^{2}	$4.19/ft^2	3.15	
Stainless Steel Tube (7/8" O.D.)	1½ ft.	$128.15/100ft.	1.92	Biomed. Eng. Dept.
Stainless Steel Sheet (.05")			10.00	Biomed. Eng. Dept.
Decubitus Padding	28" x 12"	$15.75/yd.	5.25	Hamilton Surgical
Eyelets	20	.79/pkg. 50	.40	Hardware Stores
Hand Support Cone			10.00	
Aluminum Bracket for cone	1"x2"x1/8"	$15/6ft. (1" x 1/8")	.42	
TOTAL			$32.33	

Locking Ball Joint

Locking ball joints were not available as separate units in Canada or the U.S.A., but were incorporated in accessories for surgical tables and were quite expensive as such.

They can be imported from the U.K. as separate units but the cost of a dozen ball joints would tend to be rather expensive also, an estimate being $75.00-$100.00 per unit.

CHAPTER 6

CONSTRUCTION DETAILS

6.1

Most of the construction can be accomplished with reference to the Engineering drawings in Chapter 9, as they are straightforward procedures. The construction details presented here are basically procedures which simplify the fabrication of the arm support.

6.1.1 Details of Arm Support

The decubitus padding, from which the arm support is fabricated, is cut easily with a sharp scalpel or razor blade, after the pattern is drawn on the plain back. The cut is very neat and the acrylic fur is not trimmed or damaged, as the fur separates easily once the backing is cut. Once the pattern is completed, the eyelets must be inserted.

This is most easily done by making tiny slits in the backing, where the eyelets will be, (1/8" is sufficient). The eyelet is inserted through these slits from the back with the smooth end on the backing. The fur is separated around the shank and hand tools and mallet are used to spread the shank, in order to secure the eyelet to the pattern. Once this is done, the fur can be arranged to cover the spread section, so that only the smooth part of the eyelet is visible on the backing.

When one group of eyelets are securely in place, the twine is weaved through. A knot is made in the twine, which is then passed through the first eyelet, from the fur side. The knot is then covered by the fur, and is not visible through the fur. Once through the series of eyelets, the twine should be pulled tight, but not so tight as to pull the material. It is then knotted in the same manner as was the opposite end. The cotter pins are slid over the twine where it protrudes through the backing.

This procedure is repeated on the other side and the smaller end of the pattern. Once complete, the are support is ready to be attached to the trough support, by inserting the row of cotter pins in one of the horizontal rows of holes in each side and front end of the support.

6.1.2 Edges of Rigid Support

All edges, which are visible or which may be contacted, must be rounded, thus preventing any lacerations which may result if they are left sharp.

CHAPTER 7

INITIAL PATIENT RESPONSE

7.1 The protype armtrough was tested on a patient who had suffered a stroke with resulting paraplegia of the right side of the body. This section lists his observations, and possible alterations required to improve the device.

COMFORT ....GOOD
HAND SUPPORT (CONE).................GOOD
ATTACHMENT (COTTER PINS)...........GOOD
AESTHETIC VALUE ....GOOD
LOCKING OF BALL JOINT ....DIFFICULT
TRANSFER OF PATIENT FROM CHAIR .....NONE

The latter two points merit discussion, as these presented difficulties with effective operation of the device.

The patient controlled locking of the ball-joint clamp was not as effective as was hoped, as he required assistance in accomplighing this. Apparently, the locking mechanism requires more force than can be delivered with one functional arm. This problem has been referred to our Mechanical Engineering Technologist for assessment and improvement.

Transfer of the patient into and out of the wheelchair was impossible when the patient attempted to support his body weight on the trough. This is not a valid criticism of the armtrough, however, since it was not designed to support the entire body during transfer, only to support an arm. Had this been one of the initial criteria, this facility would have been incorporated in the overall engineering design.

This does not prevent, however, incorporation of this feature once the problem is encountered, as is the case here.

One simple method of making this manouvre possible is to attach a piece of tubular stainless steel to the bottom brace of the ball-joint assembly near the front of the wheelchair. This would swing up and be secured when transfer of the patient is necessary, and would swing away when not required, to allow for the full range of motion of the trough. One possible design is presented in the engineering drawings.

Overall, the moveable armtrough presented was met with great acceptance, by the Occupational Therapy Department but more importantly, by the patient himself.

Realizing that a single patient does not constitute a general attitude, each patient testing the armtrough will be required to make observations on all facets of the device, which will be kept in a file for future reference and consideration.

CHAPTER 8

STRENGTH ANALYSIS OF ARMTROUGH

8.1 Assumptions

Three assumptions have been made before calculating the strength of the armtrough. These are all "worst case" conditions, in that during usage, these conditions will not be encountered. However, to simplify calculations, these assumptions are necessary:

1) The forearm is of uniform weight and density.
2) The armtrough is of uniform weight and density.
3) The ball-joint is clamped such that it provides a rigid support for the bar.

Shear Force, Bending Moment and Yield Strength

The situation is illustrated in figure 1 below:

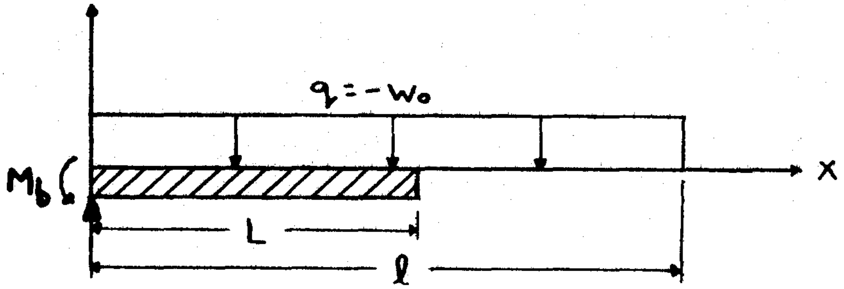


Figure 8.1

where, q = loading intensity (arm and trough) (lb/in)

Mb = clamping moment (in - lb)

Ra = reaction at clamped end (lb)

L = length of support bar (in)

l = length of trough (in)

By vertical force equilibrium, Ra may be expressed in terms of more useful parameters.

$$\Sigma Fy = 0 = Ra - Wol \rightarrow \therefore Ra = Wol \quad \ldots\ldots\ldots\ldots\ldots\ldots 1$$

Now, by definition, the shear force, V(x), is related to the loading intensity, q (x), by the differential equation,

$$\frac{dV(x)}{dx} + q(x) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

$$\therefore \frac{dV(x)}{dx} - Wo = \rightarrow V(x) - WoX = C_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

To determine the value of C_1, consider the boundary conditions, when

$$X = 0,\ V(o) = -Ra \rightarrow \therefore C_1 = -Ra = -Wol$$

Substituting this value into equation 3, an analytical expression emerges for the shear force along the bar, i.e.;

$$V(x) = Wo\ (X-l) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

A similar definition exists relating the bending moment, Mb, to the shear force,

$$\frac{dMb\ (x)}{dx} + V(x) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

Substituting equation 4 into 5 results in,

$$\frac{dMb\ (x)}{dx} + Wo\ (x-1) = 0$$

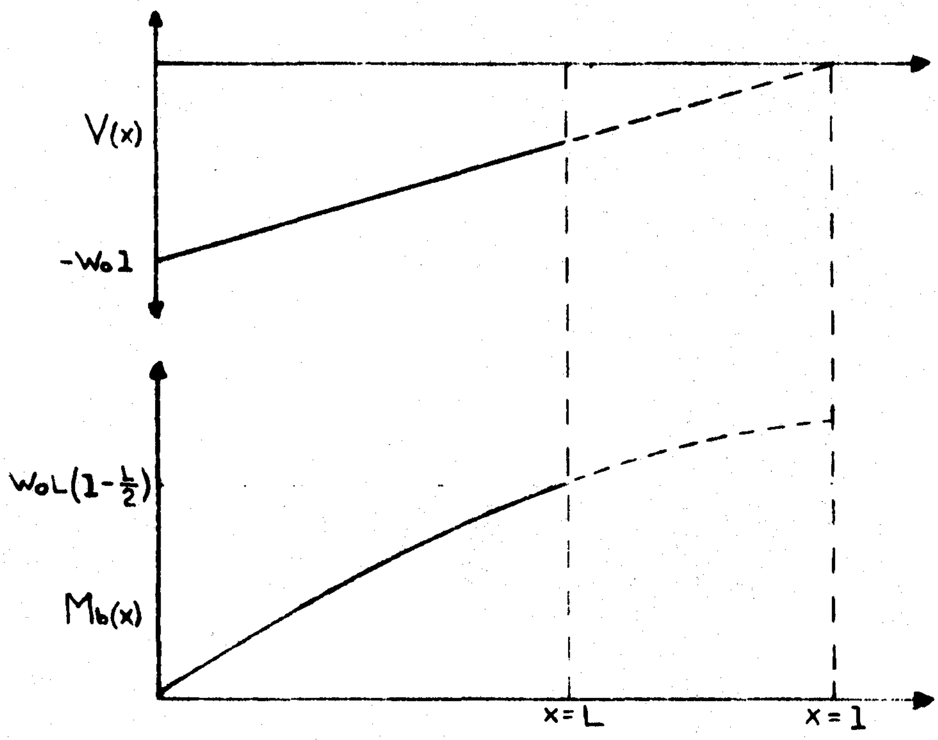
$$\therefore Mb(x) + Wo\left(\frac{x^2}{2} - 1x\right) = C_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

Again to calculate C_2, consider another boundary condition, when $X = 0$, $Mb\ (o) = 0 \rightarrow \therefore C_2 = 0$ 7

Thus, an expression for the bending moment along the bar is,

$$Mb\ (x) - WoX\left(1 - \frac{x}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

Equations 4 and 8 may be better visualized by means of shear force and bending moment diagrams; as in Figure 8.2.



NOTE:
Shear force and bending moment do not reach zero and maximum respectively, since the load overhangs the free end of the bar.

SHEAR FORCE AND BENDING MOMENT AS A FUNCTION OF DISTANCE ALONG THE BAR

Figure 8.2

Numerical Values

Substituting actual values for the parameters in equations 4 and 8 will result in numerical quantities. The parameters are:

length of bar → $L = 12$ in.
length of uniform load → $l = 18$ in.
loading intensity → $Wo = 6$ lb./18in. = .33 lb/in.

As illustrated in figure 2, the maximum shear force occurs at $x = 0$, i.e., $V(o) = -Wol$

$$= .33 \text{ lb/in.} \times 18 \text{ in.}$$

$$= -6 \text{ lb.}$$

$$V\ max = 6 \text{ lb.}$$

The maximum bending moment occurs at the free end of the bar, i.e., at $X = L$.

$$Mb\ (l) = WoL\ (l - \frac{L}{2})$$

$$= .33 \times 12'' \ (18-6)$$

$$- 48 \text{ in. - lb.}$$

$$M_{b\ max} = 48 \text{ in-lb.}$$

Deflection and Yield Strength of Bar

The maximum deflection for a cantilever beam is given by the equation,

$$\delta\ max = \frac{Pa^2}{6EI_{yy}} (3L-a) \qquad \text{...............9}$$

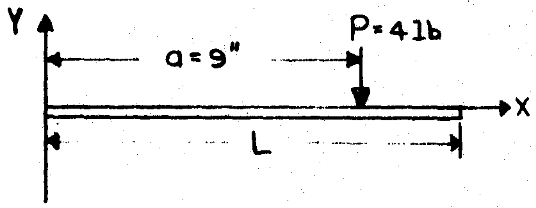


Fig. 8.3

Now, I_{yy} is defined by the integral,

$$I_{yy} = Y^2 \ b \ dy$$

$$-\frac{h}{2}$$

$$= \left.\frac{Y^3 b}{3}\right._{-\frac{h}{2}}^{\frac{h}{2}}$$

$$= \frac{bh^3}{12}$$

$$1.6 \times 10^{-4} \text{ in}^4$$

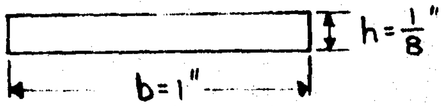


Figure 8.4

And for stainless steel → $E \sim 3 \times 10^7$ lb/in^2

Using these values, the maximum deflection is,

δ max $\sim .3$ in.

Yielding of the bar will occur when a sufficient bending moment is present along the bar, and will occur at the surface farthest from the neutral axis, which is an imaginary line through the mid-section of the bar.

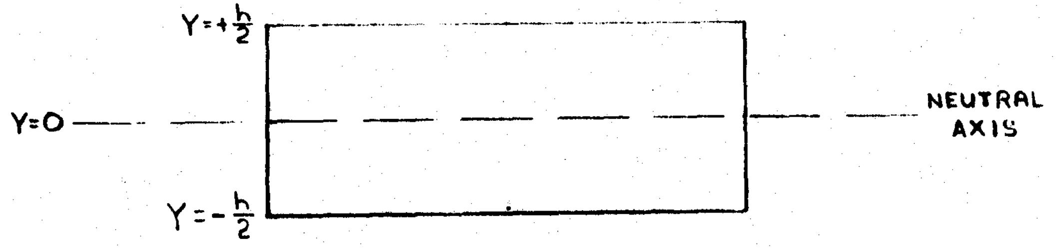


Figure 8.5

Yielding occurs when the yield strength, S_y, is reached, which for the bar, (type 304 stainless steel) is rated at 35,000 psi.

There is an equation which relates the bending moment to the yield strength, at the onset of yielding, i.e.;

$$M_{by} = \frac{Sy\ I_{yy}}{Y} \quad \ldots\ldots 10$$

where, I_{yy} = moment of inertia of area $\sim 1.6 \times 10^{-4}$ in ^{4}

Y = maximum distance from neutral axis = 6.25×10^{-2}in.

S_y = rated yield strength of bar $\sim 3.5 \times 10^4$ lb/in^2

$$Mb_y = \frac{S_y\ I_{yy}}{Y} \sim 89.6 \text{ in - lb.}$$

Thus, yielding of the bar will occur when Mb ∿ 90 in-lb. From equation 8, this bending moment can be translated into a loading intensity,

$$W_o = \frac{Mbx}{L(1-\frac{L}{2})} \sim .625 \text{ lb/in.}$$

Translating this quantity into an actual load value,

$$P_L = W01 = 11.25 \text{ lb.}$$

Thus, yielding of the bar will occur when it is subjected to a load of 11.25 lb.

Inspection of this last result, and comparing it to the expected maximum load, (V max = 6 lb), one observes that the load resulting in yielding occurs at only twice the maximum load expected.

Now, to be sure that yielding does not occur as a result of an exceptionally heavy arm or by an accident, consider the yielding load to be 10 times the expected load, and calculate the required thickness of the bar, retaining other parameters as they are, i.e.;

maximum shear force expected → V max = 16 lb.

∴ PL = Wol = 60 lb. = 10 Vmax.

∴ loading intesity → $Wo = \frac{PL}{1} = 3.33$ lb/in.

∴ maximum bending moment is,

$$Mb \max = WoL\,(1-\frac{L}{2}) \sim 240 \text{ in-lb.}$$

Recall equation 10 for the bending moment at the onset of yielding,

$$Mby = \frac{S_y\ I_{yy}}{Y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 10$$

Since we are calculating the thickness of the bar, h, equation 10 may be simplified for this purpose, recalling that,

$$I_{yy} = \frac{bh^3}{12}$$

and in equation 10 → $Y = \frac{h}{2}$

Thus, equation 10 becomes,

$$M_{by} = \frac{S_y\ (\frac{bh^3}{12})}{(\frac{h}{2})} = \frac{S_y\ bh^2}{6}$$

Solving for h, we have,

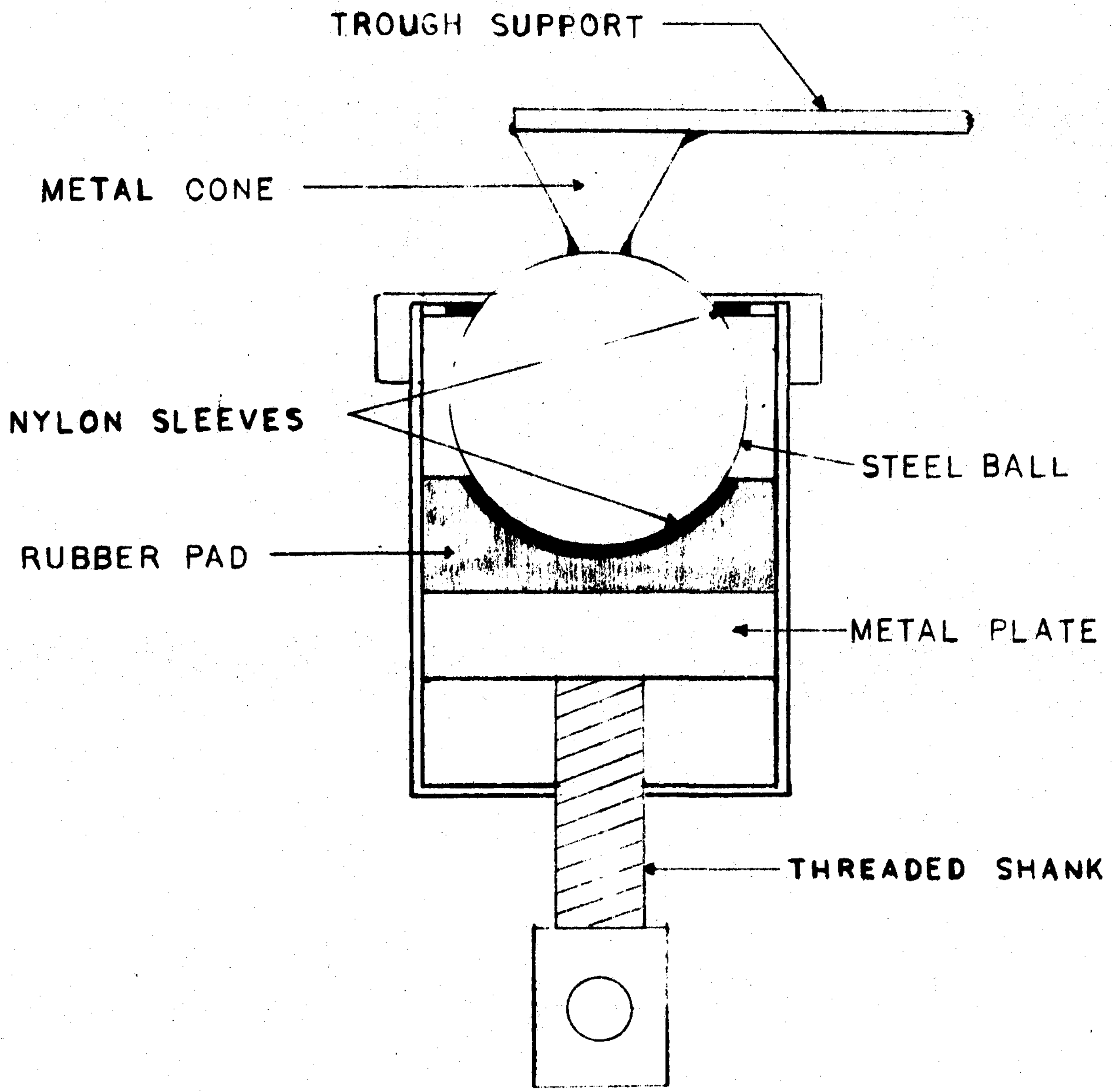
$$h = \frac{6\ M_{by}^{1/2}}{b\ S_y}$$

$$\sim .2 \text{ in.}$$

Thus, increasing the thickness of the bar from .125 in., (1/8 in.), to .2 in., will increase the maximum load before yielding to 10 times the expected load.

CHAPTER 9

TECHNICAL DRAWINGS



CROSS-SECTION OF BALL JOINT

SCALE APPROX. 1:1

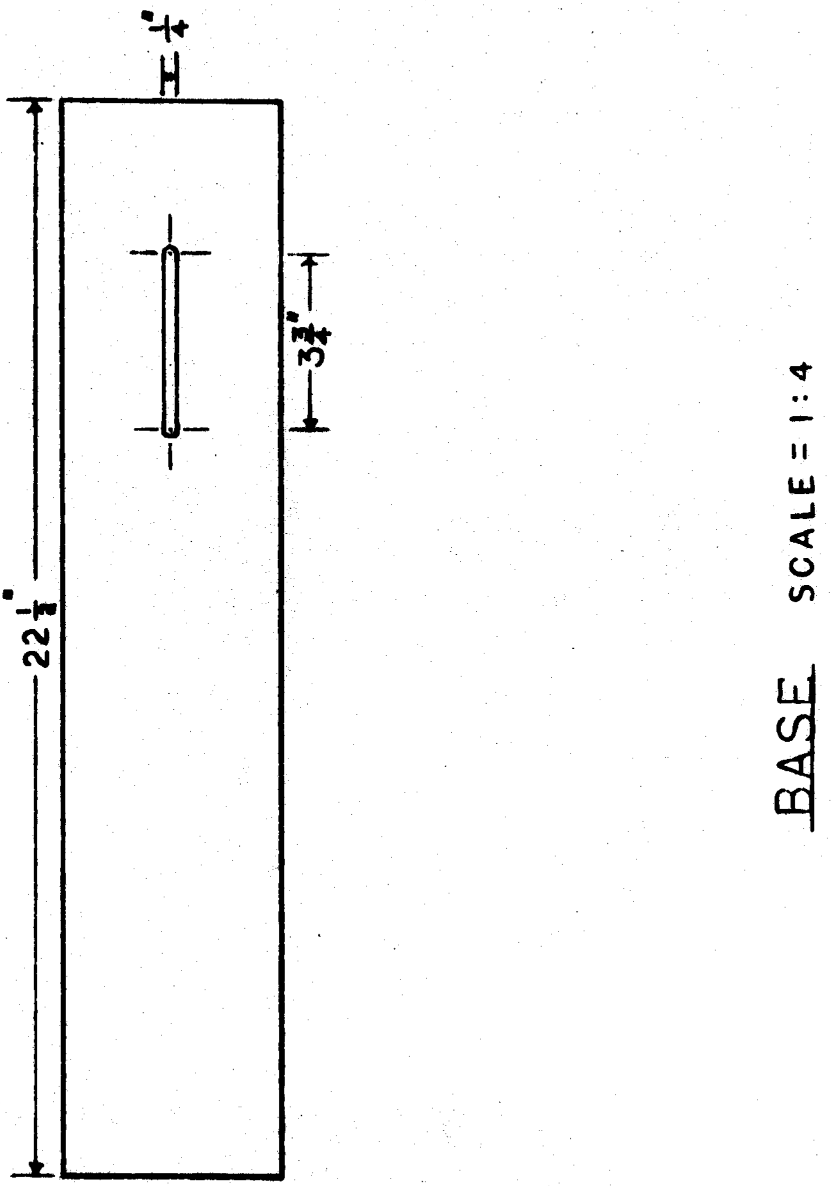


Figure 9.2

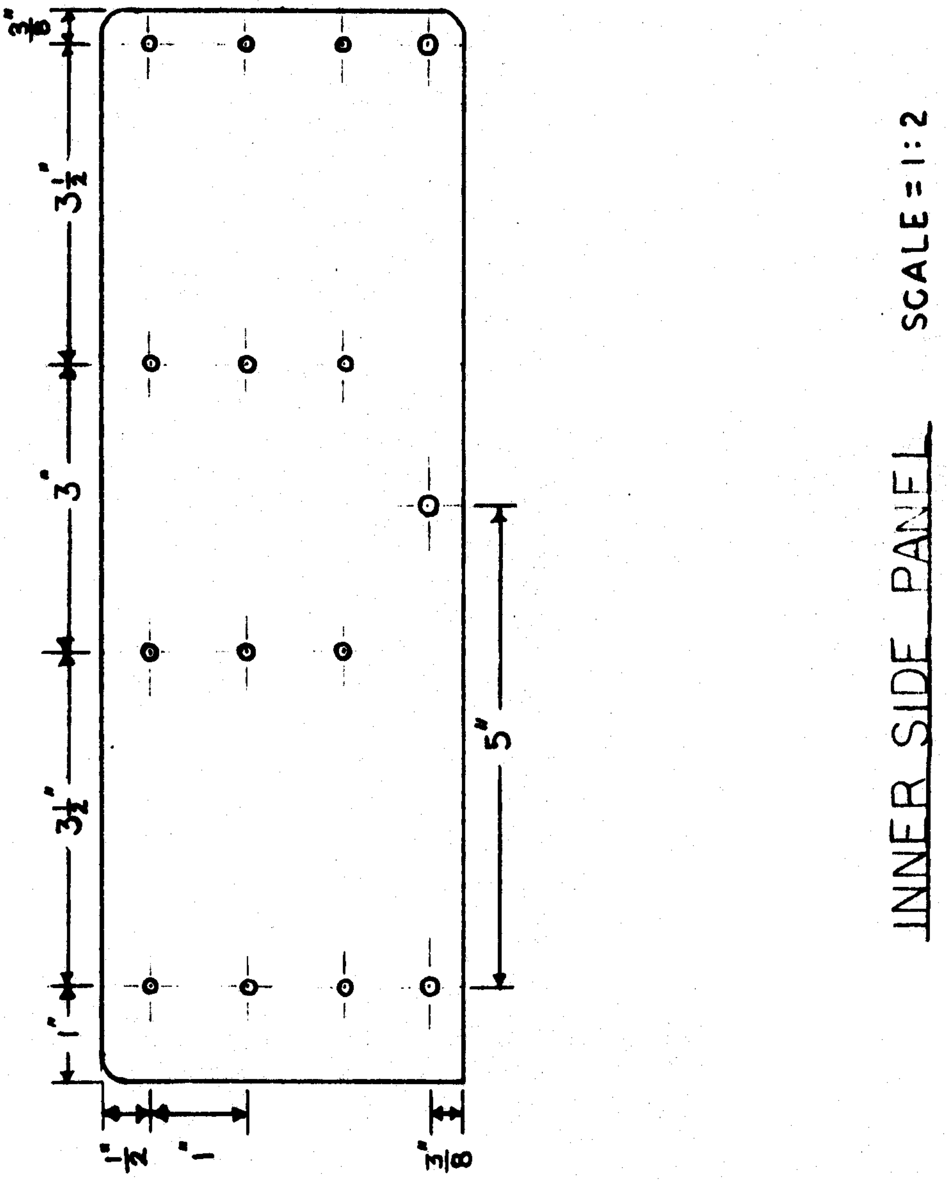
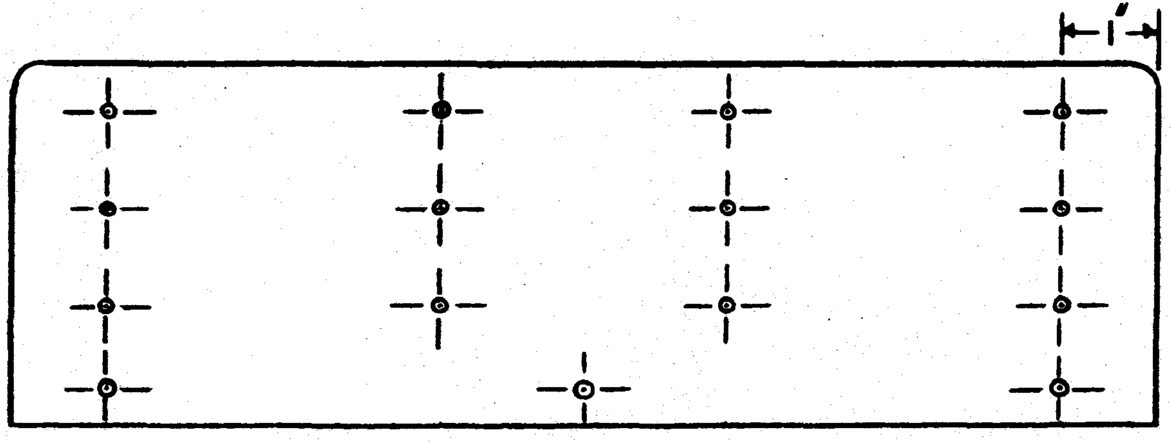
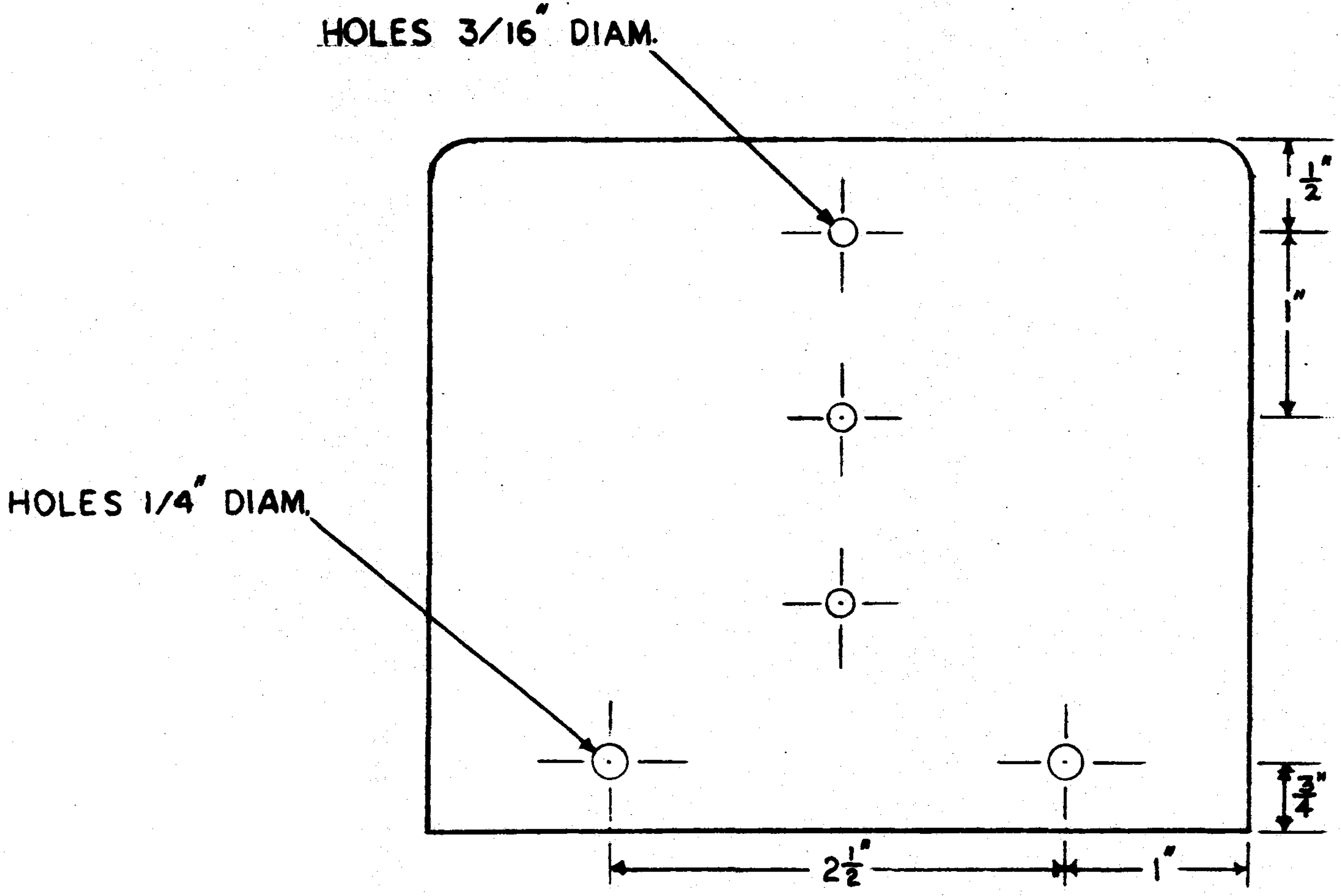


Figure 9.3



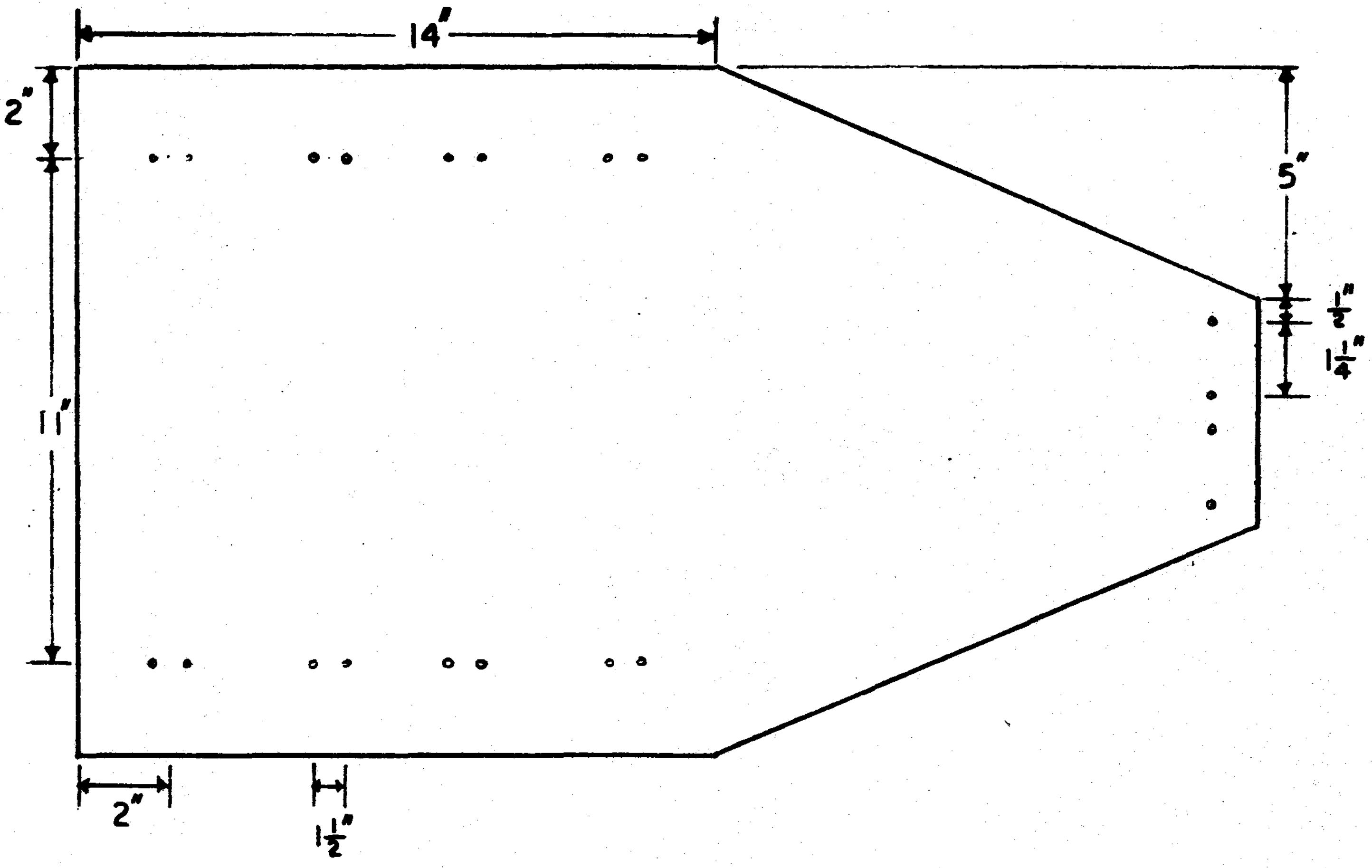
OUTER SIDE PANEL SCALE = 1:2

Figure 9.4



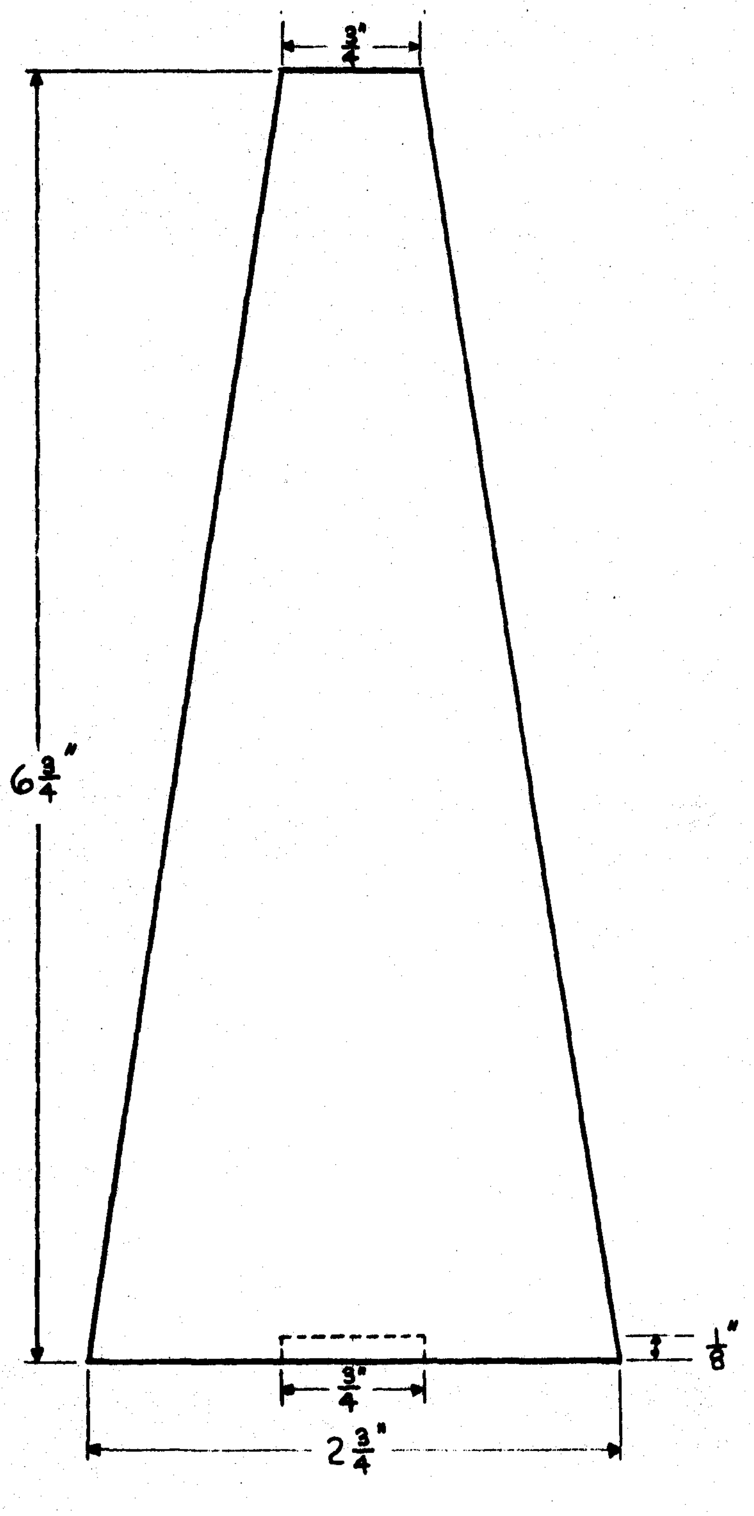
END PANEL SCALE = 1:1

Figure 9.5



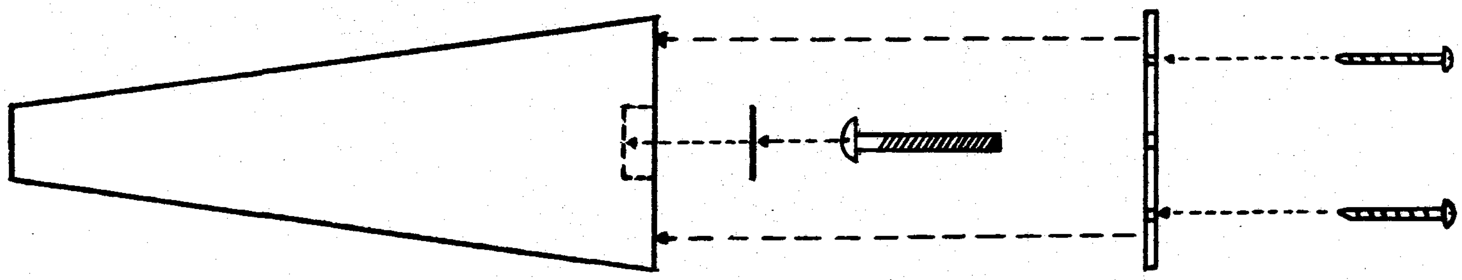
PATTERN FOR ARMSLING SCALE = 1:4

Figure 9.6



HAND SUPPORT SCALE = 1:1

Figure 9.7



ASSEMBLY OF HAND SUPPORT SCALE = 1:2

Figure 9.8

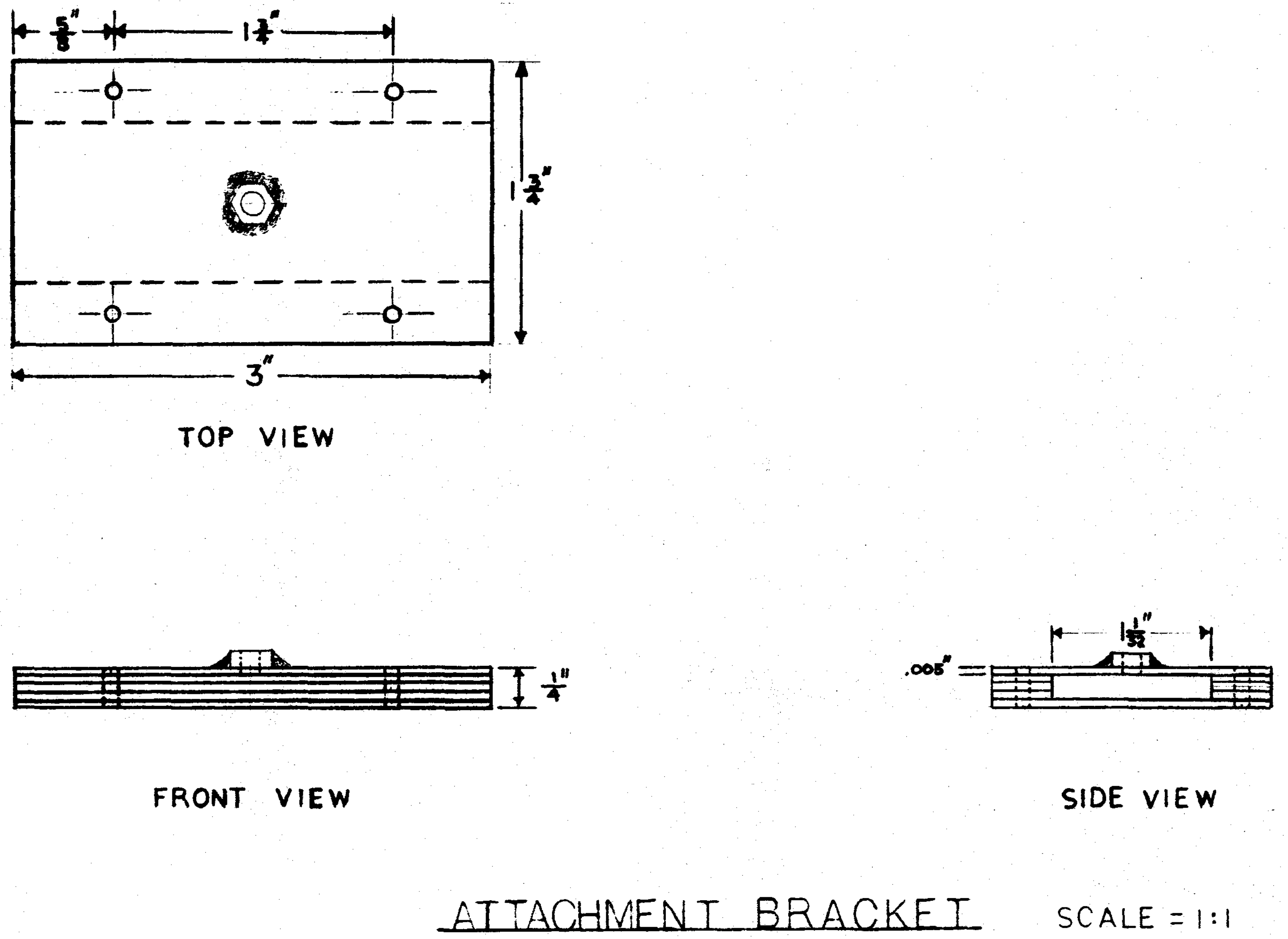

5/8"
1 1/4"
1 3/4"
3"
TOP VIEW
1/4"
FRONT VIEW
1 1/32"
.005"
SIDE VIEW
ATTACHMENT BRACKET
SCALE = 1:1

Figure 9.9