SOLID STATE SPEED CONTROL OF A TWO PHASE GENERALIZED MACHINE

by

H. Moussadji, B. Eng.

A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Master of Engineering

McMaster University

October 1969

MASTER OF ENGINEERING (1969)
(Electrical Engineering)

McMASTER UNIVERSITY
Hamilton, Ontario.

TITLE: Solid State Speed Control of a Two Phase Generalized Machine

AUTHOR: Haim Moussadji, B.Eng. (McGill University)

SUPERVISOR: Professor C. D. diCenzo

NUMBER OF PAGES: vii, 86

ABSTRACT

The voltage, current and torque equations of a two phase generalized machine are reviewed and the transformation are derived for the equivalent two phase commutator primitive of the three-phase slip-ring machine. The design of an SCR controlled inverter for speed variation of the two phase machine is presented and a method of comparing theoretical with practical values described.

ACKNOWLEDGEMENT

I am thankful for the guidance and encouragement of my supervisor Professor C. D. diCenzo and for the assistance and interest of Mr. J. M. Wylie in making this work possible. I wish to acknowledge the technical assistance given by Mr. B. Williams as well as the encouragement given by my wife, Mireille, who typed this thesis.

A special acknowledgement is made of the financial assistance given by the Canadian Westinghouse Company Limited in support of this work.

TABLE OF CONTENTS

L I S T   O F   I L L U S T R A T I O N S

INTRODUCTION

The speed of an induction motor can be controlled by; variation of the supply voltage, changing the number of poles, introduction of external rotor resistance, or varying the supply frequency. Variation of the supply voltage or changing the number of poles gives a very limited speed range and incorporation of rotor resistance is inefficient because of the large loss. Speed control using a variable frequency inverter is by far the most superior method.

In this thesis an SCR bridge inverter is designed for the provision of a two-phase variable frequency supply. The equations for a two-phase generalized machine are reviewed and transformation matrices developed to relate the more common three phase machine to the two-phase commutator primitive machine.

CHAPTER ONE

EQUATIONS OF A TWO PHASE GENERALIZED MACHINE

General:

The generalized machine is an assembly of standard and special components. It was first conceived by Professors D. C. White and H. H. Woodson from the earlier work of Professor Gabriel Kron. Because of its flexibility it can be used as any practical machine by appropriate connection of and supplies to its windings.

The theoretical treatment starts from a consideration of the two phase slip-ring primitive and the transformation leading to a two phase commutator primitive.

The dynamic circuit theory is extended to cover the practical three phase slip ring and the transformation to a three phase commutator primitive. The latter is replaced by a two phase commutator primitive plus a zero sequence component.

Modification of the general equations to conform with the circuit configuration then yields the performance equations of an induction motor.

Primitive machines are essentially theoretical concepts and have no practical significance since the compromise in their design attendant on their need to perform all tasks, means that they can do none well. However, such machines can be built and the two Westinghouse Generalized machines are practical examples.

Rotating And Stationary Axis :

Slip rings as well as a commutator can be interposed on the same shaft of a rotor, and the equivalence of both requires that the overall flux distribution in both cases be identical at any instant in time.

If it is assumed that the flux distributions of the individual windings are sinusoidal then we can obtain the relationship between the rotating and the stationary systems as shown in Fig. 1-1 and Fig. 1-2.

$$\phi_{rd} = \phi_{r\alpha} \cos\theta + \phi_{r\beta} \sin\theta$$
$$\phi_{rq} = \phi_{r\beta} \cos\theta - \phi_{r\alpha} \sin\theta$$

The above relationships can be expressed in matrix form

$$\begin{bmatrix} \phi_{rd} \\ \phi_{rq} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \phi_{r\alpha} \\ \phi_{r\beta} \end{bmatrix}$$

or

$$\begin{bmatrix} \phi_{r\alpha} \\ \phi_{r\beta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \phi_{rd} \\ \phi_{rq} \end{bmatrix} \qquad \text{----- 1}$$

where rd and rq define the fixed coils

$r\alpha$ and $r\beta$ define the rotating coils

If we include the stator windings as shown in Fig. 1-3 and Fig. 1-4 and consider the magnetomotive force to be equivalent, then matrix 1 expressed in terms of currents becomes

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3' \\ i_4' \end{bmatrix}$$

or

$$I_o = C\, I_n \qquad \text{----- 2}$$

where o stands for old and n stands for new.

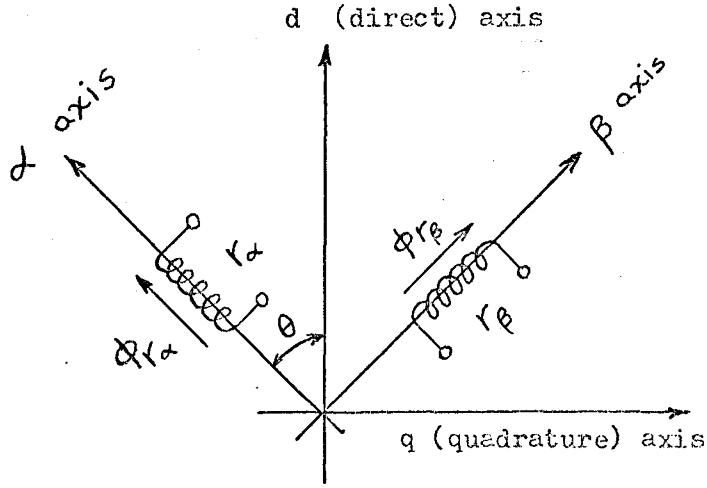


FIG. 1-1 ROTATING SYSTEM

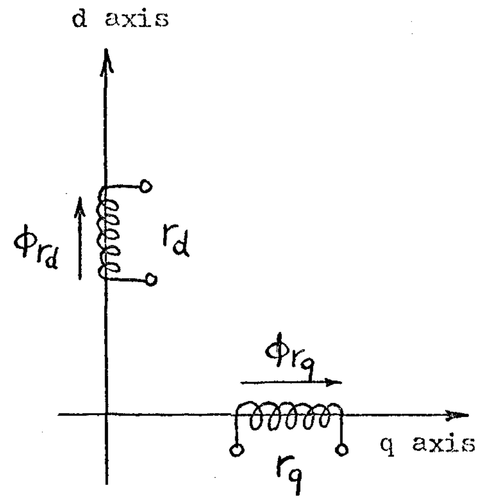


FIG. 1-2 STATIONARY SYSTEM

where $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix}$

and $C^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$

The voltage relation can be deduced from the fact that the power flow in both machines is equal

$$E_n^T I_n = E_o^T I_o$$
$$= E_o^T C I_n \qquad ------3$$

Transposing both sides

$$(E_n^T)^T = (E_o^T C)^T = C^T E_o$$
$$E_n = C^T E_o$$

The next step is to establish interrelations between the winding impedances, which would satisfy the power invariance

$$E_o = R_o I_o + p (L_o I_o) \qquad -----4$$

where $p = \frac{d}{dt}$

Substituting for E_o and I_o in equation 4 from equations 2 and 3

$$E_n = C^T E_o = C^T (R_o C I_n + p (L_o C I_n))$$
$$E_n = C^T R_o C I_n + C^T L_o C pI_n + C^T p (L_o C) I_n$$
$$E_n = R_n I + L_n pI + G_c \omega_m I \qquad -----5$$

where
$$R_n = C^T R_o C$$
$$L_n = C^T L_o C$$

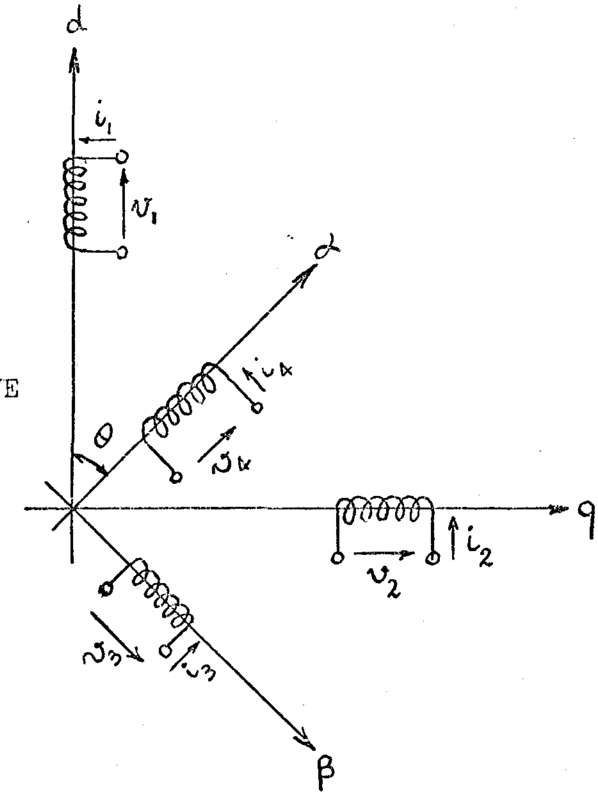


FIG. 1-3

THE SLIP RING PRIMITIVE

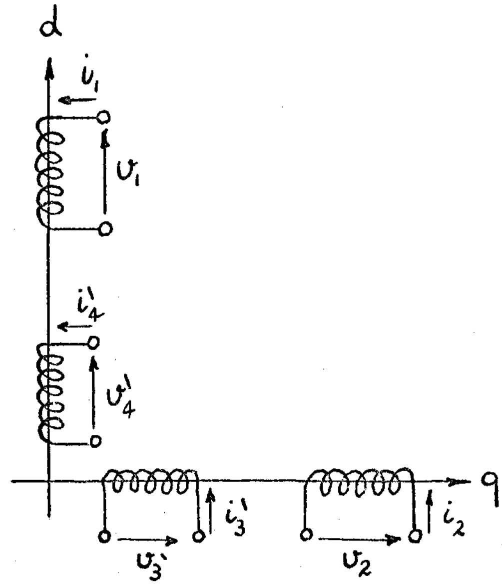


FIG. 1-4

THE COMMUTATOR PRIMITIVE

$$G_C\,\omega_m = C^T\left(\frac{\partial}{\partial\psi}(L_o\ C)\right)\frac{d\psi}{dt} = \left(mC^T L_o \frac{\partial C}{\partial\theta} + C^T G\ C\right)\omega_m$$

where $G = \dfrac{\partial L}{\partial\psi}$ and $G_c = C^T G\ C$

For a machine with a uniform air gap the resistance and inductance matrices are given by

$$R_o = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix}$$

$$L_o = \begin{bmatrix} L_s & 0 & -M\sin\theta & M\cos\theta \\ 0 & L_s & M\cos\theta & M\sin\theta \\ -M\sin\theta & M\cos\theta & L_r & 0 \\ M\cos\theta & M\sin\theta & 0 & L_r \end{bmatrix}$$

The resistance matrix for fixed coil then becomes

$$R_n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_n = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix} \qquad \text{-----}\ 6$$

The inductance matrix is deduced similarly

$$L_n = \begin{bmatrix} L_s & 0 & 0 & M \\ 0 & L_s & M & 0 \\ 0 & M & L_r & 0 \\ M & 0 & 0 & L_r \end{bmatrix} \qquad \text{-----}\ 7$$

The motional inductance term is

$$G_c = m \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \frac{\partial}{\partial\theta} \begin{bmatrix} L_s & 0 & -M\sin\theta & M\cos\theta \\ 0 & L_s & M\cos\theta & M\sin\theta \\ -M\sin\theta & M\cos\theta & L_r & 0 \\ M\cos\theta & M\sin\theta & 0 & L_r \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$G_c = m \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -M & 0 & 0 & -L_r \\ 0 & M & L_r & 0 \end{bmatrix} \qquad ------8$$

Substituting matrices 6, 7 and 8 in equation 5 the impedance matrix becomes

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} R_s + L_s p & 0 & 0 & M p \\ 0 & R_s + L_s p & M p & 0 \\ -m\,\omega_m M & M p & R_r + L_r p & -m\,\omega_m L_r \\ M p & m\,\omega_m M & m\,\omega_m L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

where ω_m is the rotational speed of rotor

m is the number of pair of poles

Three Phase To Two Phase Transformation:

The method of dynamic circuit theory for a two phase commutator system has been derived. Although the resulting simplification in the matrix manipulations are quite handy to use, it is far removed from reality in which two phase system are practically non-existent. The next logical step is the consideration of the three phase system which is universal and the means of transforming the three phase system to a two phase system and then analyze as previously described. If one would consider the problem in the three phase system, the result would be a complex matrix with 36 elements as against 16 of the corresponding two phase machine. In addition, since the windings are spacially distributed at 120 degrees rather than 90 degrees intervals, the mutual coupling are more complex. The impedance matrix for a three phase slip ring primitive as illustrated in Fig. 1-5 is given in matrix 9

$$Z = \begin{bmatrix} R_s+L_sp & M_sp & M_sp & Mp\cos\theta & Mp\cos(\theta+120) & Mp\cos(\theta+240) \\ M_sp & R_s+L_sp & M_sp & Mp\cos(\theta+240) & Mp\cos\theta & Mp\cos(\theta+120) \\ M_sp & M_sp & R_s+L_sp & Mp\cos(\theta+120) & Mp\cos(\theta+240) & Mp\cos\theta \\ Mp\cos\theta & Mp\cos(\theta+240) & Mp\cos(\theta+120) & R_r+L_rp & M_rp & M_rp \\ Mp\cos(\theta+120) & Mp\cos\theta & Mp\cos(\theta+240) & M_rp & R_r+L_rp & M_rp \\ Mp\cos(\theta+240) & Mp\cos(\theta+120) & Mp\cos\theta & M_rp & M_rp & R_r+L_rp \end{bmatrix} \quad \text{--9}$$

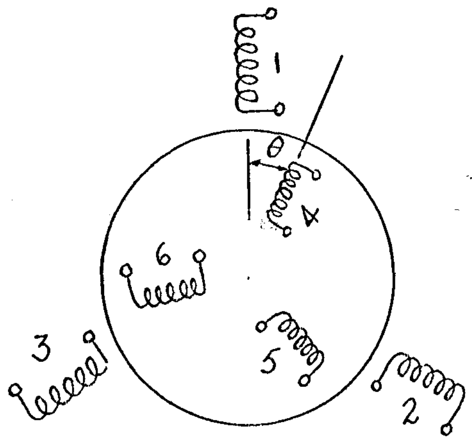
<u>The Three Phase Commutator Primitive :</u>

The condition for equivalence of rotor M.M.F's leads to two equations which can be obtained by resolving the currents along the axis of winding 4 and perpendicular to it

$$i_4+i_5\cos 240 + i_6\cos 120 = i_4'\cos\theta + i_5'\cos(\theta+240) + i_6'\cos(\theta+120) \quad \text{---- 10}$$

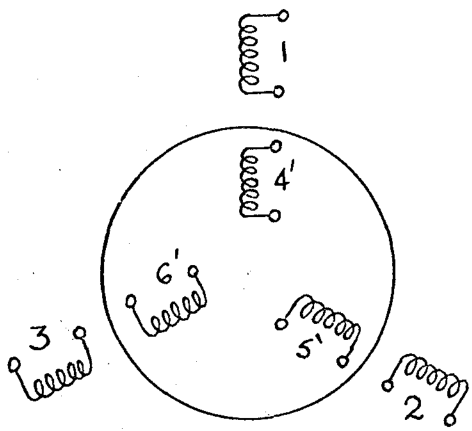
$$i_5\sin 240 + i_6\sin 120 = i_4'\sin\theta + i_5'\sin(\theta+240) + i_6'\sin(\theta+120) \quad \text{---- 11}$$

the third relation is derived from the equality of the neutral or



(a) SLIP-RING

FIG. 1-5 THREE PHASES PRIMITIVES



(b) COMMUTATOR

quence current i.e.

$$i_4 + i_5 + i_6 = i_4' + i_5' + i_6' \qquad \text{---------} \quad 12$$

tions 10, 11 and 12 may be written in matrix form as follow:

$$\begin{bmatrix} 1 & 1 \\ \cos 240 & \cos 120 \\ \sin 240 & \sin 120 \end{bmatrix} \begin{bmatrix} i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \cos\theta & \cos(\theta+240) & \cos(\theta+120) \\ \sin\theta & \sin(\theta+240) & \sin(\theta+120) \end{bmatrix} \begin{bmatrix} i_4' \\ i_5' \\ i_6' \end{bmatrix}$$

which by inversion of the appropriate matrix, the following

onships are obtained

$$= \frac{1}{3} \begin{bmatrix} 1 + 2\cos\theta & 1 + 2\cos(\theta+240) & 1 + 2\cos(\theta+120) \\ 1 + 2\cos(\theta+120) & 1 + 2\cos\theta & 1 + 2\cos(\theta+240) \\ 1 + 2\cos(\theta+240) & 1 + 2\cos(\theta+120) & 1 + 2\cos\theta \end{bmatrix} \begin{bmatrix} i_4' \\ i_5' \\ i_6' \end{bmatrix}$$

or $$I_{ro} = C\ I_{rn}$$

$$= \frac{1}{3} \begin{bmatrix} 1 + 2\cos\theta & 1 + 2\cos(\theta+120) & 1 + 2\cos(\theta+120) \\ 1 + 2\cos(\theta+240) & 1 + 2\cos\theta & 1 + 2\cos(\theta+120) \\ 1 + 2\cos(\theta+120) & 1 + 2\cos(\theta+240) & 1 + 2\cos\theta \end{bmatrix} \begin{bmatrix} i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

or $$I_{rn} = C^{-1}\ I_{ro}$$

be noted that $C^T = C^{-1}$, therefore the connection matrix

the property of orthogonality.

The complete current matrix has the form

$$\begin{bmatrix} i_{so} \\ i_{ro} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} i_{sn} \\ i_{rn} \end{bmatrix}$$

The impedance matrix Z_c of the commutator primitive is given by

$$Z_c = \begin{bmatrix} 1 & 0 \\ 0 & C_t \end{bmatrix} \begin{bmatrix} Z_s & Z_m \\ Z_{mt} & Z_r \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & C \end{bmatrix}$$

$$= \begin{bmatrix} Z_s & Z_m\,C \\ C_t\,Z_{mt} & C_t\,Z_r\,C \end{bmatrix} \qquad \text{------- 13}$$

The evaluation of Z_c is simplified by partitioning in this way. The problem is reduced to determine the three submatrices $Z_m\,C$; $C_t\,Z_{mt}$, $C_t\,Z_r\,C$. For simplicity let us consider the C matrix as a summation of two matrices C_1 and C_2 such that

$$C_1 = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \text{------- 14}$$

$$C_2 = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta+240) & \cos(\theta+120) \\ \cos(\theta+120) & \cos\theta & \cos(\theta+240) \\ \cos(\theta+240) & \cos(\theta+120) & \cos\theta \end{bmatrix} \qquad \text{------- 15}$$

The problem will involve the products $C_{1t}\,C_1$, $C_{1t}\,C_2$ and $C_{1t}\,\frac{dC_2}{d\theta}$

From 14 and 15

$$\frac{dC_1}{dt} = 0$$

$$\frac{dC_2}{d\theta} = -\frac{2}{3} \begin{bmatrix} \sin\theta & \sin(\theta+240) & \sin(\theta+120) \\ \sin(\theta+120) & \sin\theta & \sin(\theta+240) \\ \sin(\theta+240) & \sin(\theta+120) & \sin\theta \end{bmatrix}$$

$$C_{1t}\,C_1 = C_1$$

$$C_{1t}\,C_2 = C_{2t}\,C_1 = C_{1t}\,\frac{dC_2}{d\theta} = \frac{dC_{2t}}{d\theta}\,C_1 = 0 \qquad \text{------ 16}$$

$$C_{2t}\, C_2 = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \qquad \text{------17}$$

$$C_{2t}\, \frac{dC_2}{d\theta} = \frac{1}{3} \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \qquad \text{------18}$$

__ evaluation of $Z_m\, C$:__

From inspection of submatrix Z_m, it can be rewritten as

$$Z_m = \frac{3}{2} M\, p\, C_{2t} \qquad \text{-------19}$$

$$Z_m\, C = \frac{3}{2} M\, p\, C_{2t}\, (C_1 + C_2)$$

From equation 16 $C_{2t}\, C_1 = 0$

$$Z_m\, C = \frac{3}{2} M\, p\, C_{2t}\, C_2$$

$$= \frac{3}{2} M\, (C_{2t}\, C_2\, p + C_{2t}\, (p\, C_2) + (p\, C_{2t})\, C_2)$$

$$= \frac{3}{2} M\, C_{2t}\, C_2\, p + \frac{3}{2}\, m\omega_m M \left(C_{2t} \left(\frac{dC_2}{d\theta}\right) + \left(\frac{dC_{2t}}{d\theta}\right) C_2 \right)$$

From 17 and 18

$$Z_m\, C = \frac{3}{2} M\, C_{2t}\, C_2\, p$$

$$= \begin{bmatrix} M\,p & -\tfrac{1}{2}M\,p & -\tfrac{1}{2}M\,p \\ -\tfrac{1}{2}M\,p & M\,p & -\tfrac{1}{2}M\,p \\ -\tfrac{1}{2}M\,p & -\tfrac{1}{2}M\,p & M\,p \end{bmatrix} \qquad \text{------20}$$

__The evaluation of $C_t\, Z_{mt}$:__

By transposing equation 19

$$Z_{mt} = \frac{3}{2} M\, p\, C_2$$

Hence $C_t\, Z_{mt} = \frac{3}{2} M\, (C_{1t} + C_{2t})\, (C_2\, p + p\, C_2))$

$$= \frac{3}{2} M\, C_{2t}\, C_2\, p + \frac{3}{2} m \quad M\, C_{2t} \frac{dC_2}{d\theta}$$

$$= \begin{bmatrix} Mp & -\tfrac{1}{2}Mp + \frac{\sqrt{3}}{2}m \quad M & -\tfrac{1}{2}Mp - \frac{\sqrt{3}}{2}m\omega_m M \\ -\tfrac{1}{2}Mp - \frac{\sqrt{3}}{2}m\,\omega_m M & Mp & -\tfrac{1}{2}Mp + \frac{\sqrt{3}}{2}m\omega_m M \\ -\tfrac{1}{2}Mp + \frac{\sqrt{3}}{2}m\omega_m M & -\tfrac{1}{2}Mp - \frac{\sqrt{3}}{2}m\,\omega_m M & Mp \end{bmatrix} \quad \text{--- 21}$$

<u>The evaluation of $C_t\, Z_r\, C$:</u>

This will be accomplished in three parts, the resistance, the self inductance, and the mutual inductance components being treated separately.

a) <u>Rotor Resistance</u>

$$C_t\, R_r\, C = R_r\, C_t\, C$$

$$= R_r\, (C_{1t}\, C_1 + C_{2t}\, C_2)$$

$$= \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix} \quad \text{-------- 22}$$

b) <u>Rotor Self Inductance</u>

$$C_t\, L_r\, p\, C = L_r\, C_t\, C\, p + m\,\omega_m L_r\, C_t \frac{dC}{d\theta}$$

$$= \begin{bmatrix} L_r\, p & \frac{1}{\sqrt{3}}m\omega_m L_r & -\frac{1}{\sqrt{3}}m\,\omega_m L_r \\ -\frac{1}{\sqrt{3}}m\omega_m L_r & L_r\, p & \frac{1}{\sqrt{3}}m\,\omega_m L_r \\ \frac{1}{\sqrt{3}}m\,\omega_m L_r & -\frac{1}{\sqrt{3}}m\,\omega_m L_r & L_r\, p \end{bmatrix} \quad \text{------- 23}$$

c) <u>Mutual Inductance</u>

Similarly $\quad C_t\, M_r\, p\, C = 3\, M_r\, C_{1t}\, C_1\, C_1\, p$

$$C_t\, M_r\, p\, C = \begin{bmatrix} 0 & M_r p - \frac{1}{\sqrt{3}} m\omega_m M_r & M_r p + \frac{1}{\sqrt{3}} m\omega_m M_r \\ M_r p + \frac{1}{\sqrt{3}} m\omega_m M_r & 0 & M_r p - \frac{1}{\sqrt{3}} m\omega_m M_r \\ M_r p - \frac{1}{\sqrt{3}} m\omega_m M_r & M_r p + \frac{1}{\sqrt{3}} m\omega_m M_r & 0 \end{bmatrix} \quad \text{---}24$$

The impedance matrix for a three phase commutator primitive with a uniform air gap is obtained by substituting matrices 20, 21, 22, 23 and 24 in 13

$$= \begin{bmatrix} R_s+L_s p & M_s p & M_s p & Mp & -\tfrac{1}{2}Mp & -\tfrac{1}{2}Mp \\ M_s p & R_s+L_s p & M_s p & -\tfrac{1}{2}Mp & Mp & -\tfrac{1}{2}Mp \\ M_s p & M_s p & R_s+L_s p & -\tfrac{1}{2}Mp & -\tfrac{1}{2}Mp & Mp \\ Mp & -\tfrac{1}{2}Mp + \frac{\sqrt{3}}{2} m\omega_m M & -\tfrac{1}{2}Mp - \frac{\sqrt{3}}{2} m\omega_m M & R_r+L_r p & M_r p + \frac{1}{\sqrt{3}} \times m\omega_m (L_r - M_r) & M_r p - \frac{1}{\sqrt{3}} m\omega_m (L_r - M_r) \\ -\tfrac{1}{2}Mp - \frac{\sqrt{3}}{2} m\omega_m M & Mp & -\tfrac{1}{2}Mp + \frac{\sqrt{3}}{2} m\omega_m M & M_r p - \frac{1}{\sqrt{3}} m\omega_m (L_r - M_r) & R_r+L_r p & M_r p + \frac{1}{\sqrt{3}} m\omega_m (L_r - M_r) \\ -\tfrac{1}{2}Mp + \frac{\sqrt{3}}{2} m\omega_m M & -\tfrac{1}{2}Mp - \frac{\sqrt{3}}{2} m\omega_m M & Mp & M_r p + \frac{1}{\sqrt{3}} m\omega_m (L_r - M_r) & M_r p - \frac{1}{\sqrt{3}} \times m\omega_m (L_r - M_r) & R_r+L_r p \end{bmatrix} \quad 25$$

Three To Two Phase Transformation :

The next move is the transformation of a three phase commutator primitive to a two phase commutator primitive as illustrated in Fig. 1-6. The conditions for equivalence of magnetomotive forces are obtained by resolution along the direct and quadrature axes

$$\begin{aligned} i'_1 &= i_1 - i_2 \cos 60 - i_3 \cos 60 \\ i'_2 &= \quad\;\; i_2 \cos 30 - i_3 \cos 30 \end{aligned} \quad \text{------}\ 26$$

These equations are sufficient to define the two phase system

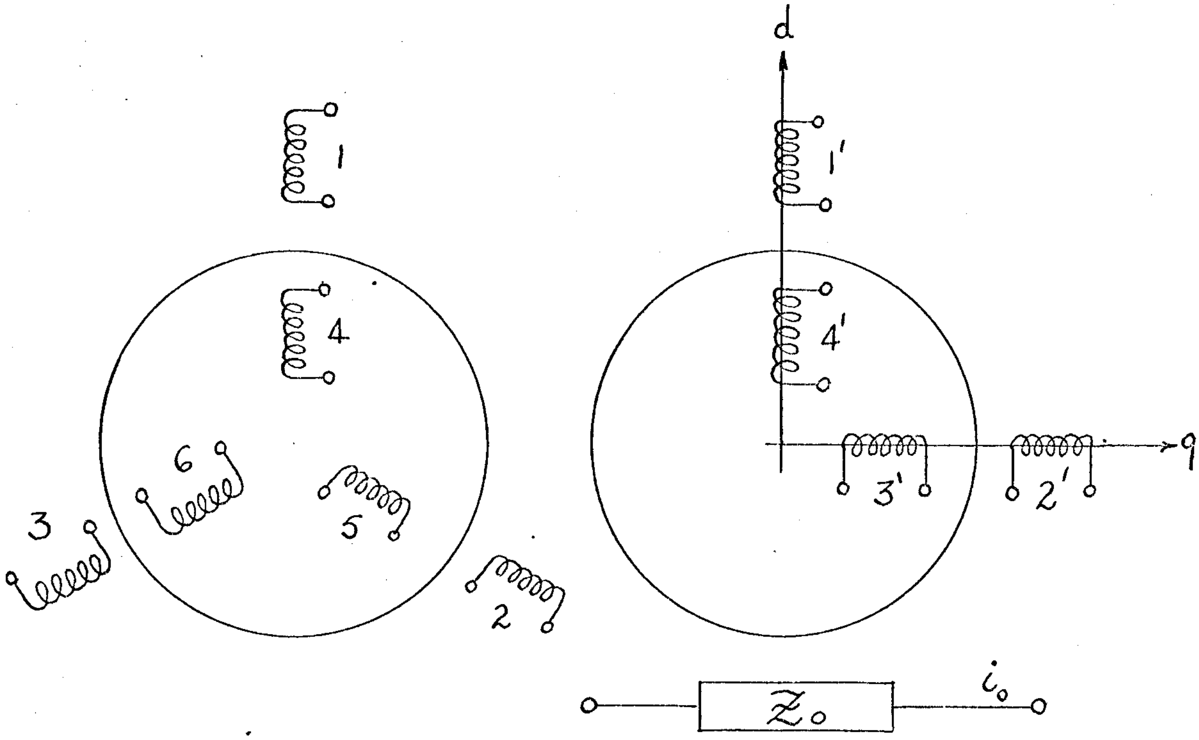


FIG. 1-6 THREE AND TWO PHASE EQUIVALENCE

in terms of the three phase system, but the extra degree of freedom possessed by the latter prohibits the inverse relationship, which is necessary for the impedance transformation $Z_n = C_t \; Z_o \; C$. The two phase system must be given an additional degree of freedom which does not affect the magnetomotive force relationships of equation 26 and a new independent relationship between the variables must be established. The extra degree of freedom is obtained by introducing an impedance Z_o, carrying a current i_o, which is external to the two phase machine and hence make no contribution to its M.M.F's and defined by

$$i_o = i_1 + i_2 + i_3 \qquad \text{--------}27$$

Equation 26 and 27 can be combined in a matrix form as follow

$$\begin{bmatrix} i_o \\ i_1' \\ i_2' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\cos 60 & -\cos 60 \\ 0 & \cos 30 & \cos 30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Similarly a relationship for the rotor current can be derived and the connection matrix between the two and three phase becomes

$$\begin{bmatrix} i_{os} \\ i_1' \\ i_2' \\ i_{or} \\ i_3' \\ i_4' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -\cos 60 & -\cos 60 & 0 & 0 & 0 \\ 0 & \cos 30 & -\cos 30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & \cos 30 & -\cos 30 \\ 0 & 0 & 0 & 1 & -\cos 60 & -\cos 60 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} \quad -28$$

A computer program was written to invert the connection matrix 28. The result is tabulated in matrix 29

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 & 0 & 0 & 0 & 0 \\ 1/3 & -1/3 & 1/\sqrt{3} & 0 & 0 & 0 \\ 1/3 & -1/3 & -1/\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 0 & 2/3 \\ 0 & 0 & 0 & 1/3 & 1/\sqrt{3} & -1/3 \\ 0 & 0 & 0 & 1/3 & -1/\sqrt{3} & -1/3 \end{bmatrix} \begin{bmatrix} i'_{os} \\ i'_1 \\ i'_2 \\ i_{or} \\ i'_3 \\ i'_4 \end{bmatrix} \qquad \text{----29}$$

or $\quad I_o = C \; I_n$

The voltage relationship may be directly written down using the invariance of power condition $V_n = C_t \; V_o$. The impedance matrix Z_2 for the two phase equivalent is obtained from the three phase system Z_3 by using the transformation $C_t \; Z_3 \; C$ where Z_3 is given by the matrix 25. Partitioning is used in this case because of the complexity of the matrices.

$$Z_2 = \begin{bmatrix} C_{1t} & 0 \\ 0 & C_{2t} \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{22} \\ Z_{33} & Z_{44} \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$

$$= \begin{bmatrix} C_{1t} \; Z_{11} \; C_1 & C_{1t} \; Z_{22} \; C_2 \\ C_{2t} \; Z_{33} \; C_1 & C_{2t} \; Z_{44} \; C_2 \end{bmatrix} \qquad \text{--------30}$$

The problem is reduced to determine the four submatrices of matrix 30

<u>The evaluation of $C_{1t} \; Z_{11} \; C_1$</u> :

This will be accomplished in two parts, the resistance and the terms which have the operator p

a) <u>Resistance</u>

$$C_{1t}\,R_{11}\,C_1 = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & -1/3 & -1/3 \\ 0 & 1/\sqrt{3} & -1/\sqrt{3} \end{bmatrix} \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & 0 \\ 1/3 & -1/3 & 1/\sqrt{3} \\ 1/3 & -1/3 & -1/\sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1/3\,R_s & 0 & 0 \\ 0 & 2/3\,R_s & 0 \\ 0 & 0 & 2/3\,R_s \end{bmatrix}$$

b) Terms in p

Similarly the terms in p are reduced to

$$C_{1t}\,X_{11}\,C_1 = \begin{bmatrix} (1/3\,L_s + 2/3\,M_s)\,p & 0 & 0 \\ 0 & 2/3(L_s-M_s)p & 0 \\ 0 & 0 & 2/3(L_s-M_s)p \end{bmatrix}$$

Combining a and b we obtain

$$C_{1t}\,Z_{11}\,C_1 = \begin{bmatrix} 1/3\,R_s+(1/3\,L_s+2/3\,M_s)p & 0 & 0 \\ 0 & 2/3\,R_s+2/3(L_s-M_s)p & 0 \\ 0 & 0 & 2/3R_s+2/3(L_s-M_s)p \end{bmatrix} \qquad -31$$

The Evaluation of $C_{2t}\,Z_{44}\,C_2$:

This will also be accomplished in two parts, the terms with and without p

a) Terms without p

$$C_{2t}\,R_{44}\,C_2 = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1/\sqrt{3} & -1/\sqrt{3} \\ 2/3 & -1/3 & -1/3 \end{bmatrix} \begin{bmatrix} R & F & -F \\ -F & R & F \\ F & -F & R \end{bmatrix} \begin{bmatrix} 1/3 & 0 & 2/3 \\ 1/3 & 1/\sqrt{3} & -1/3 \\ 1/3 & -1/\sqrt{3} & -1/3 \end{bmatrix}$$

where $F = 1/\sqrt{3}\; m\,\omega_m\,(L_r - M_r)$

$$C_{2t}\, R_{44}\, C_2 = \begin{bmatrix} 1/3\ R_r & 0 & 0 \\ 0 & 2/3\ R_r & -2/\sqrt{3}\ F \\ 0 & 2/\sqrt{3}\ F & 2/3\ R_r \end{bmatrix}$$

b) Terms with p

Similarly the terms in p are reduced to

$$C_{2t}\, X_{44}\, C_2 = \begin{bmatrix} (1/3\ L_r + 2/3\ M_r)p & 0 & 0 \\ 0 & 2/3\ (L_r - M_r)p & 0 \\ 0 & 0 & 2/3\ (L_r - M_r)p \end{bmatrix}$$

Combining a and b

$$C_{2t} Z_{44} C_2 = \begin{bmatrix} 1/3R_r + (1/3L_r + 2/3M_r)p & 0 & 0 \\ 0 & 2/3R_r + 2/3(L_r - M_r)p & -2/3\ m\,\omega_m (L_r - M_r) \\ 0 & 2/3\ m\,\omega_m (L_r - M_r) & 2/3R_r + 2/3(L_r - M_r)p \end{bmatrix} \tag{32}$$

The Evaluation Of $C_{2t}\ Z_{33}\ C_1$:

The multiplication of the above submatrix will be done in two parts, terms with p and terms without p

a) Terms without p

$$C_{2t}\ R_{33}\ C_1 = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1/\sqrt{3} & -1/\sqrt{3} \\ 2/3 & -1/3 & -1/3 \end{bmatrix} \begin{bmatrix} 0 & F & -F \\ -F & 0 & F \\ F & -F & 0 \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & 0 \\ 1/3 & -1/3 & 1/\sqrt{3} \\ 1/3 & -1/3 & -1/\sqrt{3} \end{bmatrix}$$

where $F = \sqrt{3}/2\ m\,\omega_m M$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -m\,\omega_m M & 0 \\ 0 & 0 & m\,\omega_m M \end{bmatrix}$$

b) Terms with p

$$C_{2t}\, X_{33}\, C_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & Mp & 0 \end{bmatrix}$$

By adding a and b we obtain:

$$C_{2t}\, Z_{33}\, C_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -m\omega_m M & 0 \\ 0 & Mp & m\omega_m M \end{bmatrix} \quad \text{--------- 33}$$

The Evaluation of $C_{1t}\, Z_{22}\, C_2$:

$$C_{1t}\, Z_{22}\, C_2 = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & -1/3 & -1/3 \\ 0 & 1/\sqrt{3} & -1/\sqrt{3} \end{bmatrix} M \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 & 2/3 \\ 1/3 & 1/\sqrt{3} & -1/3 \\ 1/3 & 1/\sqrt{3} & -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & Mp \\ 0 & Mp & 0 \end{bmatrix} \quad \text{------------ 34}$$

The impedance matrix for the two phase commutator primitive with a uniform air gap is obtained by substituting submatrices 31, 32, 33 and 34 in matrix 30

$$\begin{bmatrix} v'_1 \\ v'_2 \\ v'_3 \\ v'_4 \end{bmatrix} = \begin{bmatrix} R_{sl}+L_{sl}p & 0 & 0 & Mp \\ 0 & R_{sl}+L_{sl}p & Mp & 0 \\ -m\omega_m M & 0 & R_{rl}+L_{rl}p & -m\omega_m L_{rl} \\ Mp & m\omega_m M & m\omega_m L_{rl} & R_{rl}+L_{rl}p \end{bmatrix} \begin{bmatrix} i'_1 \\ i'_2 \\ i'_3 \\ i'_4 \end{bmatrix} \quad \text{-- 35}$$

$$v_{os} = (1/3\, R_s + (1/3\, L_s + 2/3\, M_s)\, p)\, i_{os}$$

$$v_{or} = (1/3\, R_r + (1/3\, L_r + 2/3\, M_r)\, p)\, i_{or} \quad \text{----------- 36}$$

where $R_{s1} = 2/3\ R_s$

$L_{s1} = 2/3\ (L_s - M_s)$

$R_{r1} = 2/3\ R_r$

$L_{r1} = 2/3\ (L_r - M_r)$

The matrix 35 is identical to the two phase commutator primitive derived before. Hence the above lengthy calculations led to the familiar four by four impedance matrix plus two independent zero sequence equations 36. The resistances and inductances are adjusted from three to two phase.

Torque Equation:

In a rotating machine the mechanical output power is equal to the product of output torque and speed of rotation

$$T_e \frac{d\Psi}{dt} = P_{out}$$

And from Appendix A

$$P_{out} = \tfrac{1}{2}\, i_{ot} \frac{dL}{dt}\, i_o$$

$$\therefore\ T_e = \tfrac{1}{2}\, i_{ot} \frac{dL}{d\Psi}\, i_o = \tfrac{1}{2}\, i_{ot}\, G\, i_o \qquad \text{-------}\ 37$$

where T_e is the electromagnetic output torque

ψ is the angle of the rotor relative to the stator

$G = \frac{dL}{d\psi}$

The current in the above equations are instantaneous values and the torque is the instantaneous torque. Phasor currents may be used provided i_t is replaced by the conjugate of I i.e. by I_t^*

$$T = \tfrac{1}{2}\, I_t^*\, G\, I$$

From equation 5, $G_c = \left[m\, C_t\, L\, \frac{\partial C}{\partial \theta} + C_t\, G\, C \right]$

Equation 37 in the fixed axes becomes:

$$T_e = \tfrac{1}{2}\, i_{nt}\, C_t\, G_o\, C\, i_n = \tfrac{1}{2}\, i_{nt}\, G_n\, i_n$$

By inspection the matrix G_n can be rewritten as $G_c + G_{ct}$

$$\therefore\ T_e = \tfrac{1}{2}\, i_{nt}\, G_c\, i_n + \tfrac{1}{2}\, i_{nt}\, G_{ct}\, i_n$$

but $(i_{nt}\, G_{ct}\, i_n)_t = i_{nt}\, G_c\, i_n = i_{nt}\, G_{ct}\, i_n$

Since torque is a scalar whose transpose is equal to itself

$$\therefore T_e = i_{nt}\, G_c\, i_n$$

$$= m \begin{bmatrix} i_1' & i_2' & i_3' & i_4' \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -M_d & 0 & 0 & -L_d \\ 0 & M_q & L_q & 0 \end{bmatrix} \begin{bmatrix} i_1' \\ i_2' \\ i_3' \\ i_4' \end{bmatrix} \qquad \text{-----}38$$

By omitting the factor ½, the same rule as for the slip-ring machine may be used for deriving the torque equation of the commutator machine.

Equation 38 reduces to:

$$T_e = m\,(-M_d\, i_1\, i_3 - L_d\, i_4\, i_3 + M_q\, i_4\, i_2 + L_q\, i_3\, i_4) \quad \text{--}39$$

The torque which is delivered to the Torquemeter by the generalized machine is given by:

$$T_m = T_e - \left(J\, \frac{d\omega}{dt} + D\, \omega + C \right) \qquad \text{-----}\ 40$$

where J is the moment of inertia

D is the coefficient of viscous drag (windage)

C is the Coulomb friction torque

T_e is defined in equation 39

CHAPTER TWO

INVERTER DESIGN

General :

In chapter one, the equations which express the voltage, current and torque of a two phase induction motor have been derived. The object of this chapter and the following chapter is the generation of a variable frequency, two phase waveshape, by a parallel bridge type-inverter, to power the two phase induction motor. The flux density for an induction machine is directly proportional to the voltage and inversely proportional to the frequency. Consequently, to maintain a constant flux density, a constant Volts/Hertz ratio must be maintained. This means changing the applied voltage with a change in frequency.

Single Phase Inverter :

A single phase bridge inverter with feedback circuitry is shown in Fig. 2-1. The output terminals are labeled A_1 and A_2. A blocking rectifier (R_i) is in series with each SCR, upper and lower commutating capacitors are indicated between SCR and diodes, and feedback diodes (F_i) are connected to each output terminal. The series blocking diodes prevent the charge stored on the commutating capacitors from discharging through the load. The voltage of the commutating capacitor is charged to a maximum immediately before the commutation of an SCR. After commutation of an SCR,

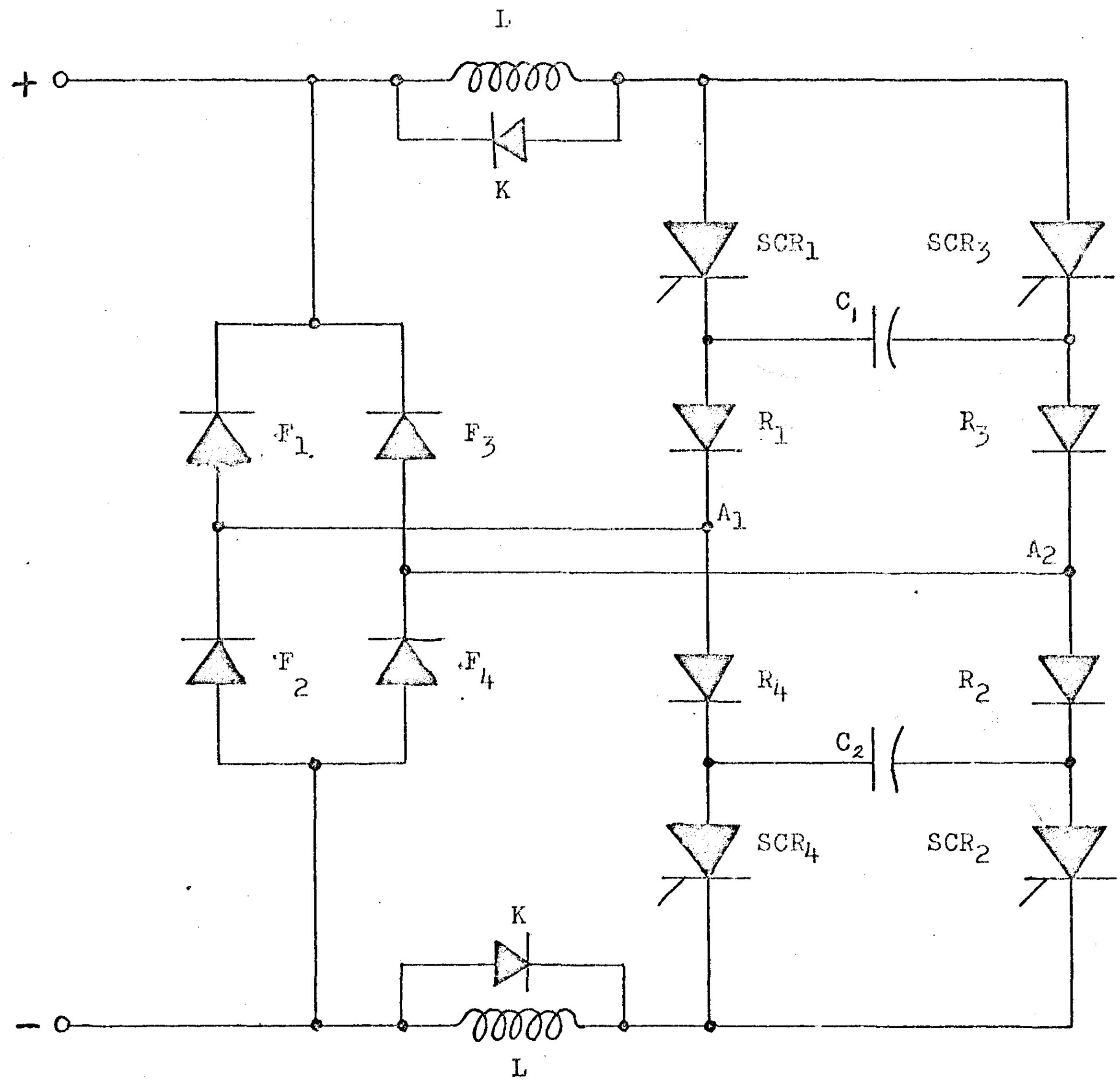


FIG. 2-1 SINGLE PHASE PARALLEL CAPACITOR COMMUTATED INVERTER

the residual current of the load flows through one of the feedback rectifiers to the line. Two reactors (L), along with their shunt diodes (K), suppress the surge current due to the commutation transient, assuring a constant potential at the SCR bus. Consider the case where SCR_1 and SCR_2 are triggered; the capacitors C_1 and C_2 are charged to a voltage equal to the source voltage. When trigger pulses are applied to SCR_3 and SCR_4, causing them to conduct SCR_3 places the negative end of the charged capacitor C_1 on the positive DC bus, and SCR_4 places the positive end of the charged capacitor on the negative DC bus. Capacitor C_1 discharge current flows through the path R_1, F_1, L and SCR_3 while the C_2 path is L , F_4, R_2 and SCR_4. The potential drop thus developped across L reverse bias SCR_1 and SCR_2 simultaneously. Each succeeding mode repeats in a similar fashion.

One of the major disadvantages of this construction is that an instantaneous switch must be made from one polarity to the other. This means a simultaneous turn-on of one SCR, say SCR_4, and turn-off of the other, say SCR_1. Because of the very fast turn-on (typically one microsecond) and relatively long turn-off time, the SCR's would present a direct short across the supply, and would result in component failure. This led us to investigate the SCR turn-off time, and the SCR turnoff mechanism in the following sections.

SCR Turn-Off Mechanism :

When a thyristor is in the conducting state, each of the three junctions of Fig. 2-2a are forward biased and the two base regions (B_p and B_n) are heavily saturated with holes and electrons

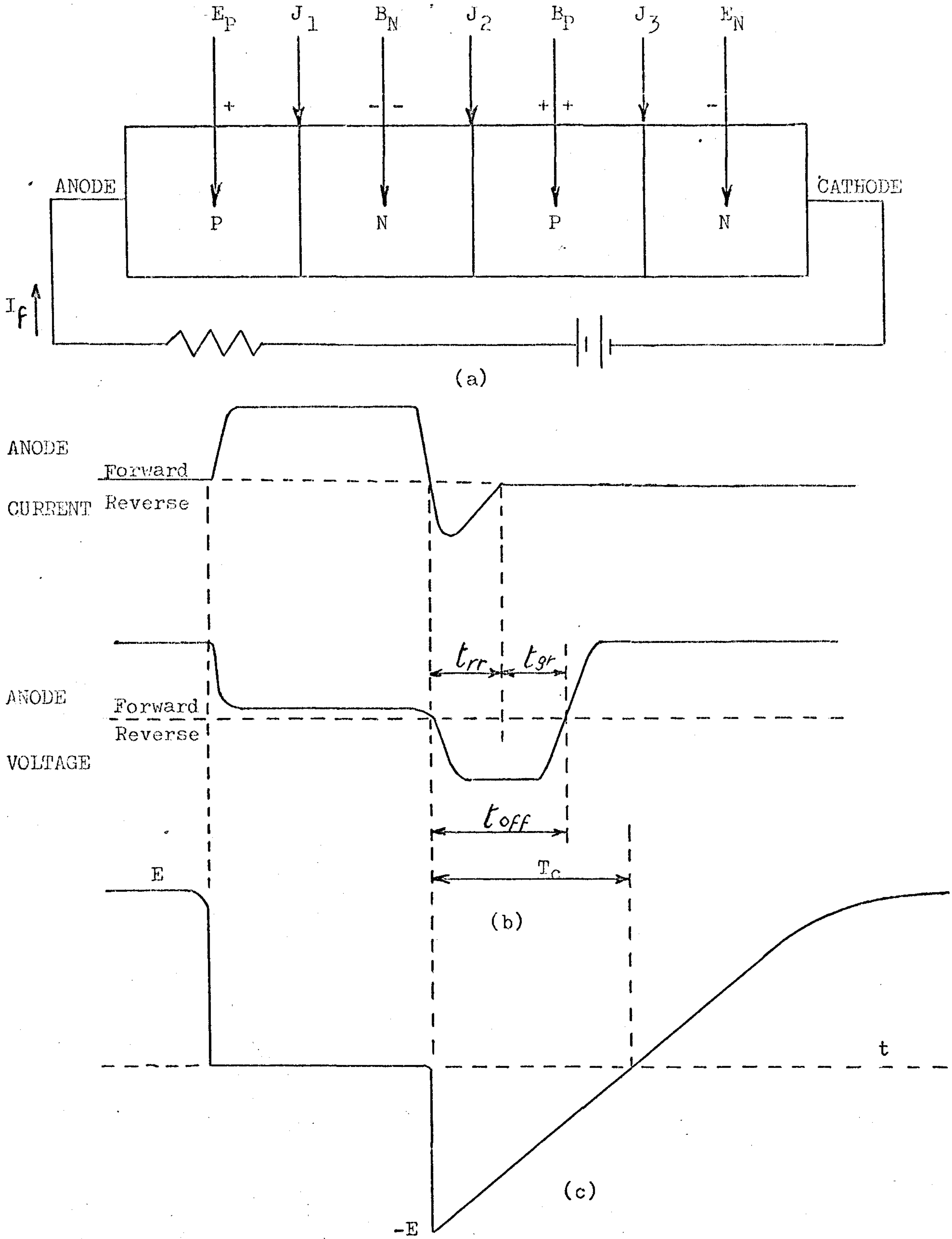


FIG. 2-2 SCR WAVEFORMS FOR TURN OFF TIME MEASUREMENTS

(stored charge). To turn-off the thyristor in a minimum time, it is necessary to apply a reverse voltage. When this reverse voltage is applied the holes and electrons in the vicinity of the two end junctions (J_1, J_3) will diffuse to these junctions and result in a reverse current in the external circuit. The voltage drop across the device will remain at about +0.7 volts as long as an appreciable reverse current flows. After the holes and electrons in the vicinity of J_1 and J_3 have been removed, the reverse current will cease and the junction J_1 and J_3 will assume a blocking state. This interval is represented by t_{rr} (reverse recovery time) in Fig. 2-2 b. The reverse voltage across the device now increases to a value determined by the external circuit. Recovery of the SCR is, however, not yet complete, since a high concentration of holes and electrons still exists in the vicinity of the center junction J_2. These carriers are removed by recombination which is independent of external bias. When these carriers have nearly completely recombined the junction J_2 can then regain its blocking state. If the carriers are not sufficiently recombined, they can cause J_1 and J_3 to inject as soon as they are forward biased during the forward blocking cycle. This interval is represented by t_{gr} (gate recovery time) in Fig. 2-2 b. The turn-off time t_{off} is defined as $t_{rr} + t_{gr}$. It is not a constant but is a function of several parameters as increase in forward current, rate of decay of forward current, peak reverse current, increase in forward blocking voltage.

Forced Commutation :

The commutating principle of the parallel inverter is illustrated in Fig. 2-3. The term parallel capacitor-commutated inverter is used to indicate an inverter which is commutated by a capacitor connected in parallel with the load. This circuit illustrates the commutation action obtained in more efficient parallel capacitor inverters.

When SCR_1 and SCR_2 are gated on simultaneously, in Fig. 2-3, with switch 3 closed, the capacitor C will charge exponentially, with the polarity shown, approaching the DC source voltage. Then switch 5 is closed to discharge the capacitor c, switch 2 is moved from A to B, and switch 3 opened. SCR_3 is gated-on, connecting the capacitor C across SCR_1 in a direction to provide a negative anode to cathode voltage, thereby diverting the load current through the capacitor C, turning off SCR_1. The capacitor C, resistance R and the voltage to which C is charged must be sufficient to divert the maximum load current from the SCR for the time interval (t_{off}) required for the SCR to regain ability to hold off forward voltage.

The voltage across the capacitor is given by:

$$v = V\,(1 - 2\,e^{-t/RC}) \qquad \text{--------} \; 1$$

For v in equation 1 to be zero, t should be equal to .7 RC. The circuit turn-off time T_c is equal to .7 times RC and must always be greater than the turn-off time of the SCR otherwise the SCR will turn-on. The values of R and C were decreased to a value that did not turn SCR off. The value just above this combination can be taken as t_{off} and was in the order of 30 microseconds.

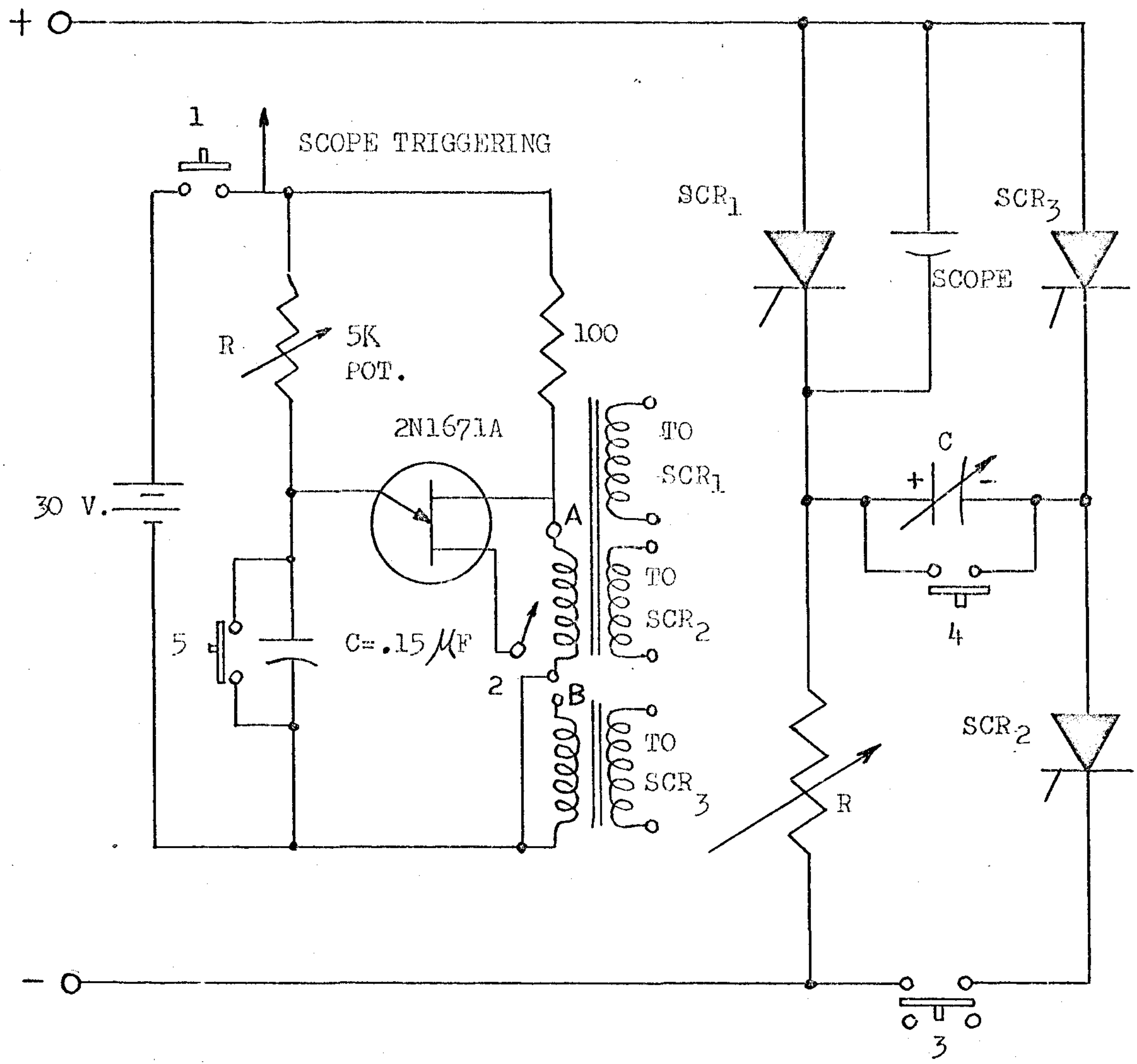


FIG. 2-3 CIRCUIT TO MEASURE TURN OFF TIME OF SCR

The delay between the scope triggering and the voltage measurement across SCR_1 is determined by RC of the firing circuit and is given by $T = RC \ln 1/(1-\eta)$

where η is the intrinsic standoff ratio

For an approximate value of $\eta = .63$, $R=1.5K\Omega$ and $C=.15\mu f$ the value of T is equal to 225μ seconds.

LC Forced Commutation :

In the circuit illustrated in Fig. 2-4 (a), SCR_1 and SCR_2 are gated-on with switch 1 closed. The capacitor C charges as indicated to a voltage E_i equal to E. When steady state is reached, switch 1 is opened and SCR_3 is triggered on, which reduces Fig. 2-4 (a) to Fig. 2-4 (b) with initial voltage on the capacitor $E_i = E$ and initial currents $i_1(0^+) = I_i$ and $i_2(0^+) = 0$

Replace diode D_2 by a resistance $R_2 \simeq 0.2\Omega$

The differential equation for the circuit is:

$$i_2 R_2 = i_1 R_1 + L \frac{di_1}{dt} - E \qquad \text{-------- } 2$$

$$-i_2 R_2 = L \frac{di_1}{dt} + L \frac{di_2}{dt} + \frac{1}{C}\int i_1 \, dt + \frac{1}{C}\int i_2 \, dt \qquad \text{------- } 3$$

The Laplace Transforms of equations 2 and 3 are:

$$(R_1 + L S) I_1 - \quad R_2 I_2 \quad = \frac{E}{S} + L I_o \qquad \text{---- } 4$$

$$\left(LS + \frac{1}{CS}\right) I_1 + \left(L S + \frac{1}{CS} + R_2\right) I_2 = \frac{E_i}{S} + L I_o \qquad \text{---- } 5$$

Solving equations 4 and 5 the Laplace Transform of i_1 and i_2 are:

$$I_1(S) = \frac{R(4) \quad (CS1\, S^3 + B1\, S^2 + A1\, S + 1)}{S \quad (Y(1)\, S^3 + Y(2)\, S^2 + Y(3)\, S + 1)} \qquad \text{----- } 6$$

where $R(4) = \dfrac{E}{R_1 + R_2}$

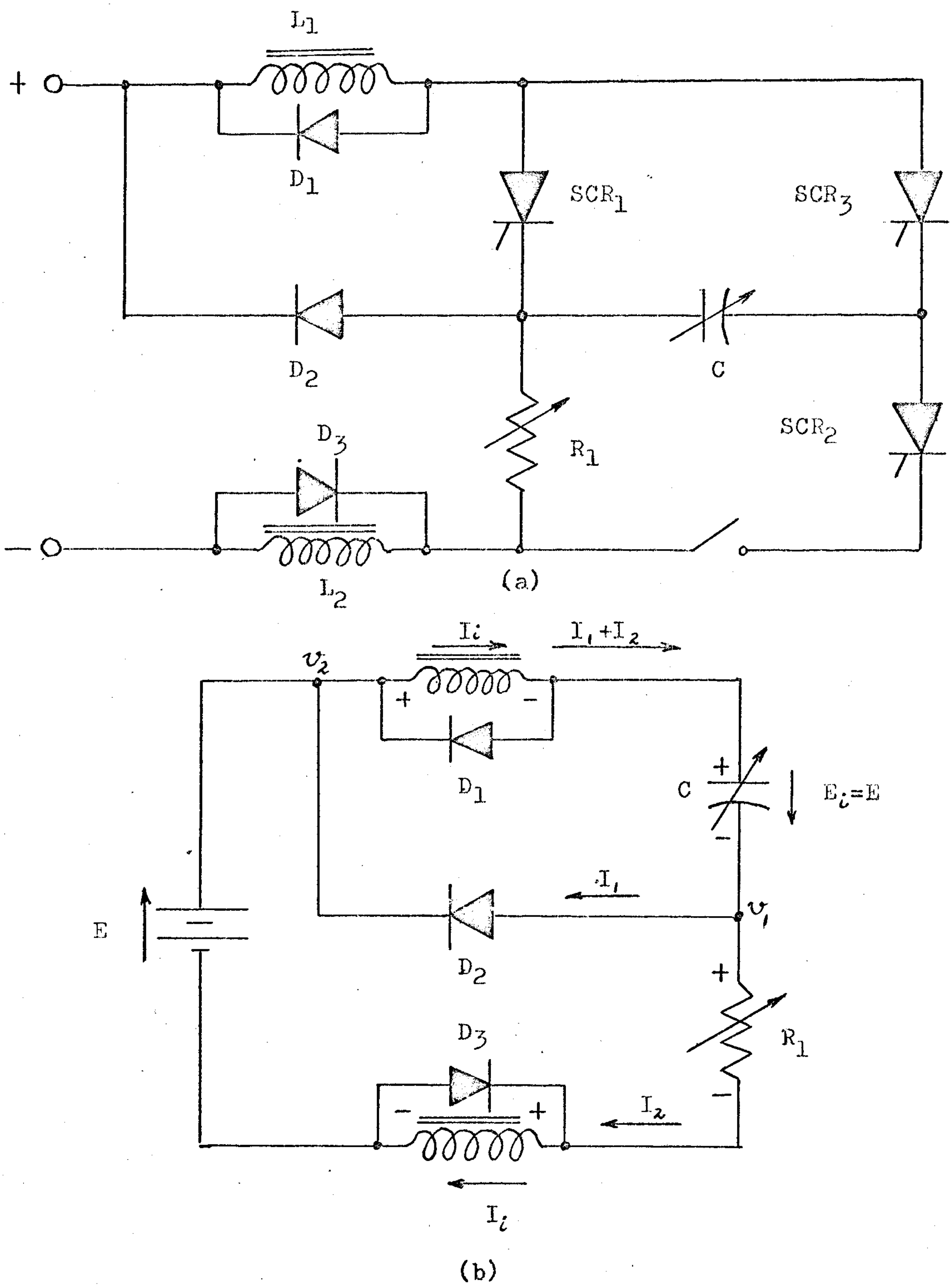


FIG. 2-4 LC FORCED COMMUTATION CIRCUIT

$$CS1 = \frac{I_o\ L^2\ C}{E}$$

$$B1 = (E\ L\ C + 2\ I_o\ L\ R_2\ C)\ /\ E$$

$$A1 = (I_o\ L + 2\ E\ R_2\ C)\ /\ E$$

$$Y(1) = L^2\ C\ /\ (R_1 + R_2)$$

$$Y(2) = (R_1\ L\ C + 2\ R_2\ L\ C)\ /\ (R_1 + R_2)$$

$$Y(3) = (R_1\ R_2\ C + L)\ /\ (R_1 + R_2)$$

and $$I_2(S) = \frac{Y(4)\ (B\ S^2 + A\ S + 1)}{S\ (Y(1)\ S^3 + Y(2)\ S^2 + Y(3)\ S + 1)} \qquad \text{-----7}$$

where $Y(4) = -E\ /\ (R_1 + R_2)$

$B = -(I_o\ L\ R_1\ C)\ /\ E$

$A = -(E\ R_1\ C - I_o\ L)\ /\ E$

Factoring the denominator of equations 6 and 7 yields:

$$=(1 + TS)\left[\frac{Y(1)}{T}\ S^2 + \frac{1}{T}\left(Y(2) - \frac{Y(1)}{T}\right) S + \frac{1}{T}\left(Y(3) - \frac{1}{T}\left(Y(2) - \left(\frac{Y(1)}{T}\right)\right)\right)\right] \text{--}8$$

where $$R = 1 - \frac{1}{T}\left(Y(3) - \frac{1}{T}\left(Y(2) - \frac{Y(1)}{T}\right)\right) \qquad \text{------ 9}$$

For the remainder to be zero equation 9 becomes:

$$T^3 - Y(3)\ T^2 + Y(2)\ T - Y(1) = 0 \qquad \text{----- 10}$$

Equation 10 has at least one real root and two immaginary roots. Equation 9 is solved on a digital computer using Bairstow technique. The value of T is substituted in equation 8, then R is zero.

Substituting equation 8 for the denominator of 6 and 7:

$$I_1(S) = \frac{R(5)\ (CS1\ S^3 + B1\ S^2 + A1\ S + 1)}{S\ (1 + TS)\ (Y(6)\ S^2 + Y(7)\ S + 1)} \qquad \text{----- 11}$$

where $$Y(5) = Y(4).T \Big/ \left(Y(3) - \frac{1}{T}\left(Y(2) - \frac{Y(1)}{T}\right)\right)$$

$$Y(6) = Y(1) \Big/ \left(Y(3) - \frac{1}{T}\left(Y(2) - \frac{Y(1)}{T}\right)\right)$$

$$Y(7) = \left(Y(2) - \frac{Y(1)}{T}\right) \Big/ \left(Y(3) - \frac{1}{T}\left(Y(2) - \frac{Y(1)}{T}\right)\right)$$

and $$I_2(S) = \frac{Y(5)\ (B\,S^2 + A\,S + 1)}{S\ (1 + TS)\ (Y(6)\ S^2 + Y(7)\ S + 1)} \qquad \text{------} 12$$

where $$Y(5) = Y(4).T \Big/ \left(Y(3) - \frac{1}{T}\left(Y(2) - \frac{Y(1)}{T}\right)\right)$$

Equation 12 expressed in partial forms:

$$I_2(S) = \frac{AA}{S} + \frac{BB}{1 + TS} + \frac{DS + E}{Y(6)\ S^2 + Y(7)\ S + 1} \qquad \text{------} 13$$

where $$AA = SI_2(S)\big|_{S=0}$$

$$BB = (1 + TS)\ I_2(S)\big|_{S=-1/T}$$

D and E are found by equating 12 and 13:

$$D = -\ (T.Y(5).Y(6) + BB\ Y(6))\ /\ T$$

$$E = ((B\ Y(5)) - (Y(5).Y(6) + TY(5).Y(7) + BB.Y(7) + D))/T$$

All the above constants were calculated by using a digital computer for the following set of data

E=30 volts L=1 milliHenry R_1 =2.72 ohms R_2 =.2 ohm

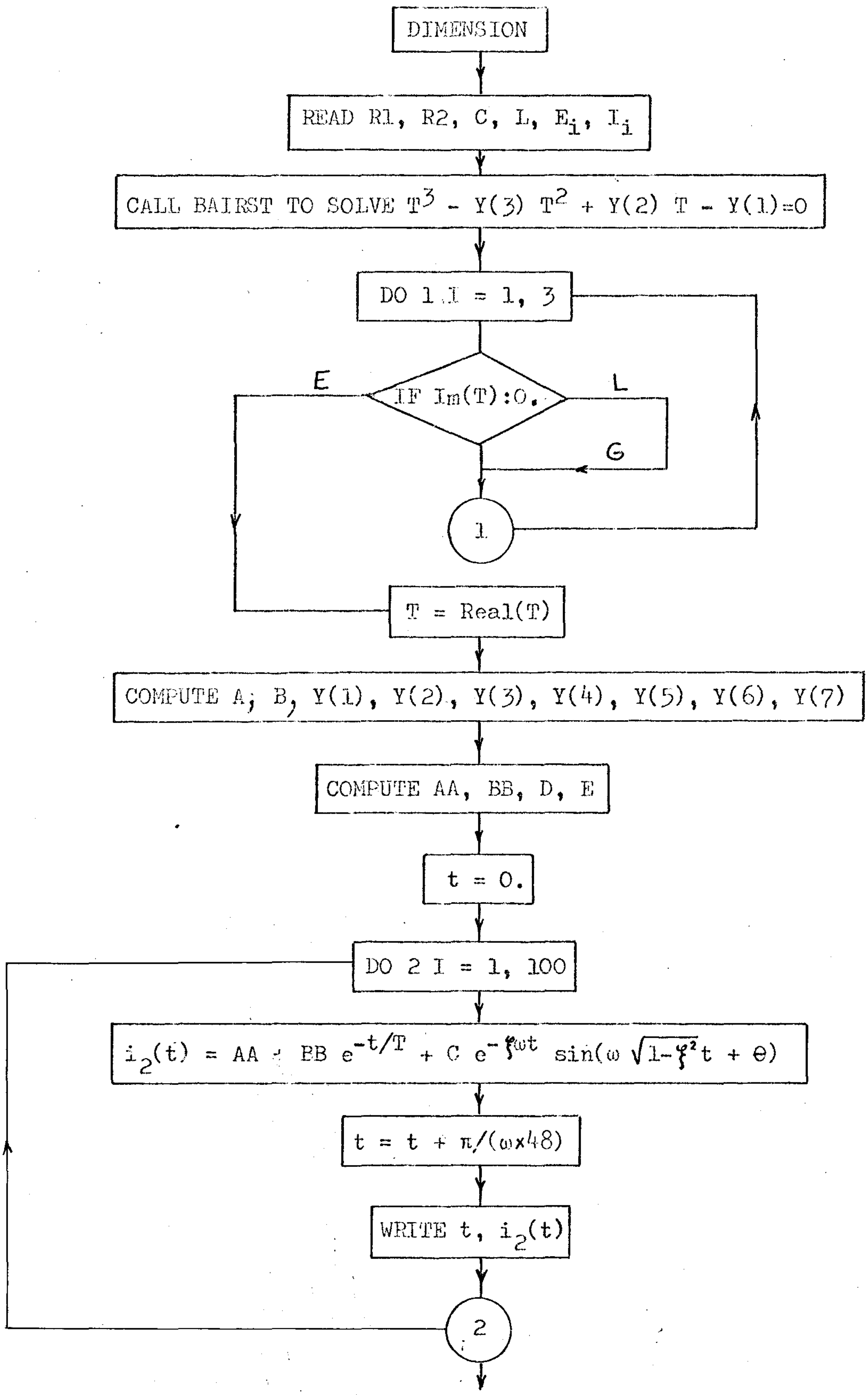
I_o =11 amperes E_i =30 volts

$$\therefore\ I_2(S) = \frac{-10.27}{S} + \frac{.2514\ 10^{-3}}{1+0.342\times10^{-3}\ S} + \frac{2.75\times10^{-7}\ S + 7.5\times10^{-4}}{0.25\times10^{-7}\ S^2 + 0.494\times10^{-3}\ S + 1}$$

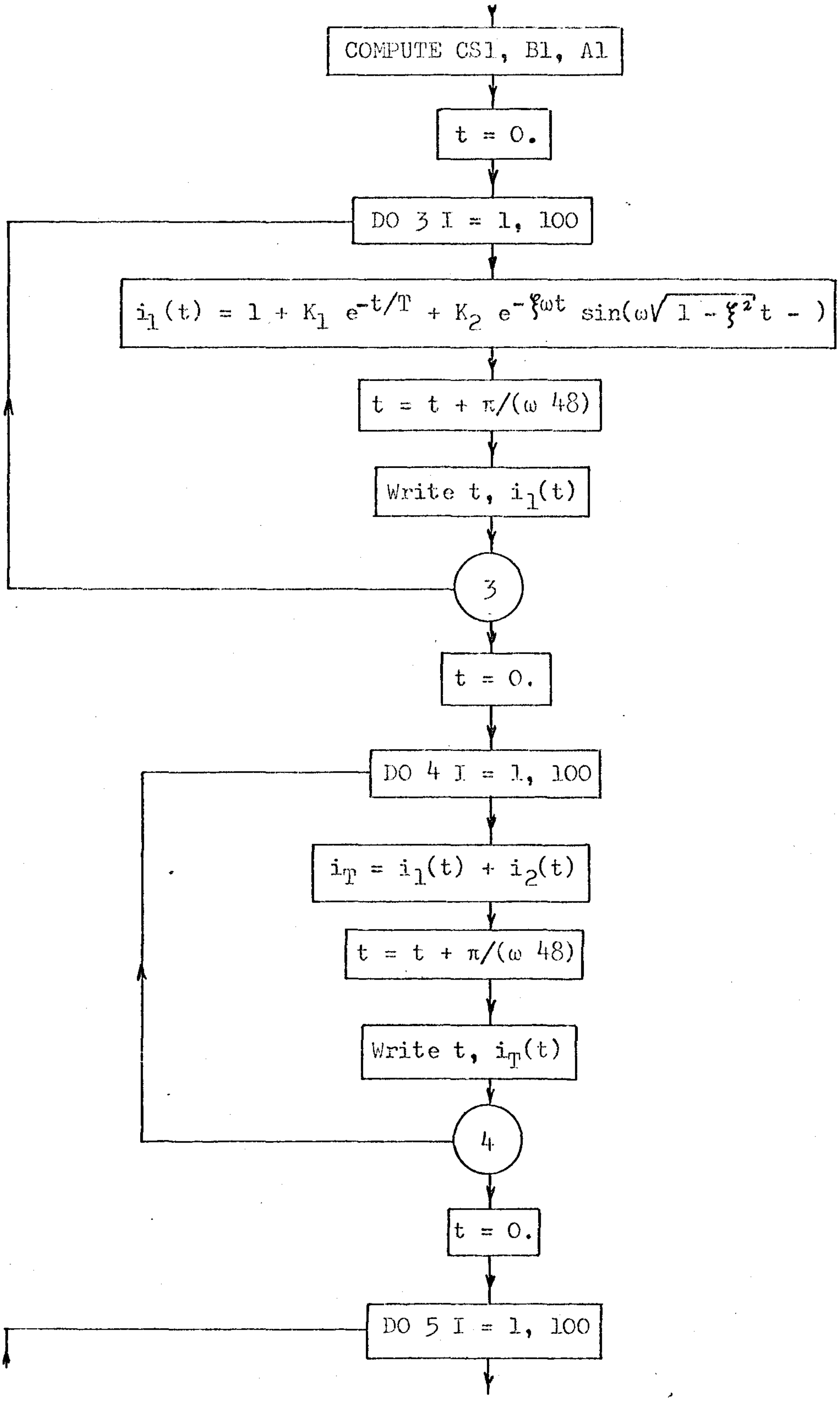
By using the tables the inverse transform of $I_2(S)$ is:

$$i_2(t) = -10.274 - 0.7348\ e^{-t/0.342\times10^{-3}} + 11.9\ e^{-98.9t}\ \sin(6321t+67.4^\circ)$$

By using the tables the inverse transform of $I_1(S)$ is:



DIMENSION
READ R1, R2, C, L, E_i, I_i
CALL BAIRST TO SOLVE $T^3 - Y(3)\,T^2 + Y(2)\,T - Y(1)=0$
DO 1 I = 1, 3
IF Im(T):0.
E
L
G
1
T = Real(T)
COMPUTE A, B, Y(1), Y(2), Y(3), Y(4), Y(5), Y(6), Y(7)
COMPUTE AA, BB, D, E
t = 0.
DO 2 I = 1, 100
$i_2(t) = AA + BB\,e^{-t/T} + C\,e^{-\zeta\omega t}\,\sin(\omega\sqrt{1-\zeta^2}\,t + \theta)$
$t = t + \pi/(\omega\times 48)$
WRITE t, $i_2(t)$
2


COMPUTE CS1, B1, A1
t = 0.
DO 3 I = 1, 100
$i_1(t) = 1 + K_1\, e^{-t/T} + K_2\, e^{-\xi\omega t} \sin(\omega\sqrt{1-\xi^2}\, t -)$
$t = t + \pi/(\omega\ 48)$
Write t, $i_1(t)$
3
t = 0.
DO 4 I = 1, 100
$i_T = i_1(t) + i_2(t)$
$t = t + \pi/(\omega\ 48)$
Write t, $i_T(t)$
4
t = 0.
DO 5 I = 1, 100

$$C_3 = C_1 \sin(\omega\sqrt{1-\zeta^2}\,t + \theta) + C_2 \sin(\omega\sqrt{1-\zeta^2}\,t + \psi)$$

$$C_4 = C_1 \cos(\omega\sqrt{1-\zeta^2}\,t + \theta) + C_2 \cos(\omega\sqrt{1-\zeta^2}\,t + \psi)$$

$$v_L = C_5 e^{-t/T} + C_6\, e^{-\zeta\omega t}\, C_3 + C_9\, e^{-\zeta\omega t}\, C_4$$

$t = t + \pi/(\omega \times 48)$

Write t, v_L

5

Compute AAA, BBB, DD, EE

$t = 0.$

DO 6 I = 1, 100

$$v_C = (AAA - E_i) + (BBB/T)\; e^{-t/T} + C\; e^{-\zeta\omega t}\, \sin(\omega\sqrt{1-\zeta^2}\,t + \phi)$$

$t = t + \pi/(\omega \times 48)$

WRITE t, $v_C(t)$

6

STOP

END

FIG. 2-6 FLOW DIAGRAM FOR OBTAINING THE VOLTAGES AND THE CURRENTS OF FIG. 2-4

$$i_1(t) = 10.274 + 0.726 \times e^{-t/0.342\ 10^{-3}} + 0.348\ e^{-98.9t} \sin(6321t)$$

The current in the top branch of Fig. 2-4 (b) is the summation of $i_1(t) + i_2(t)$

$$i_T(t) = -0.085\ e^{-2920t} + e^{-98.9t}\ (3.48 \sin(6321t) + 11.92 \sin(6321t+67.4^{\circ}))$$

The voltage across the inductor is obtained by differentiating the total current i_T:

$$v_L = L \frac{di_T}{dt}$$

$$= -25.9\ e^{-2920t} + e^{-98.9}\Big[-0.989\ (3.48 \sin(6321t) + 11.92 \times \sin(6321t + 67.4^{\circ})) + 6.321\ (3.48 \cos(6321t) + 11.92 \cos(6321t+67.4^{\circ}))\Big]$$

The expression for the voltage on the capacitor C is obtained in a similar manner:

$$e_C = -\left|E_i\right| + \frac{1}{C}\int_0^t i(t)\ dt \qquad \text{------ } 14$$

The Laplace Transform of equation 14 is:

$$E_C(S) = \frac{3.767\ 10^{-6}\ S^2 + 2.202\ 10^{-2}\ S + 32.05}{S\ (1+0.34\ 10^{-3}\ S)\ (0.25\ 10^{-7}\ S^2 + 0.49\ 10^{-5}\ S + 1)} - \frac{E_i}{S} \qquad \text{--} 15$$

Equation 15 expressed in partial form:

$$E_C(S) = \frac{32.05 - E_i}{S} + \frac{4.144\ 10^{-5}}{1 + 0.34\ 10^{-3}\ S} + \frac{-8.05\ 10^{-7}\ S + 1.086\ 10^{-2}}{0.25\ 10^{-7}\ S^2 + 0.49\ 10^{-5}\ S + 1}$$

By using the tables, the inverse transform of $E_C(S)$ is

$$e_C = 2.05 + .12\ e^{-2920t} + 76.1\ e^{-98.9t} \sin(6321t - 25^{\circ}) \qquad \text{----} 16$$

All the above equations are applicable for i_2 positive only. The results from the digital computer for i_1, i_2, i_T, v_L and v_C are plotted in Fig. 2-5.

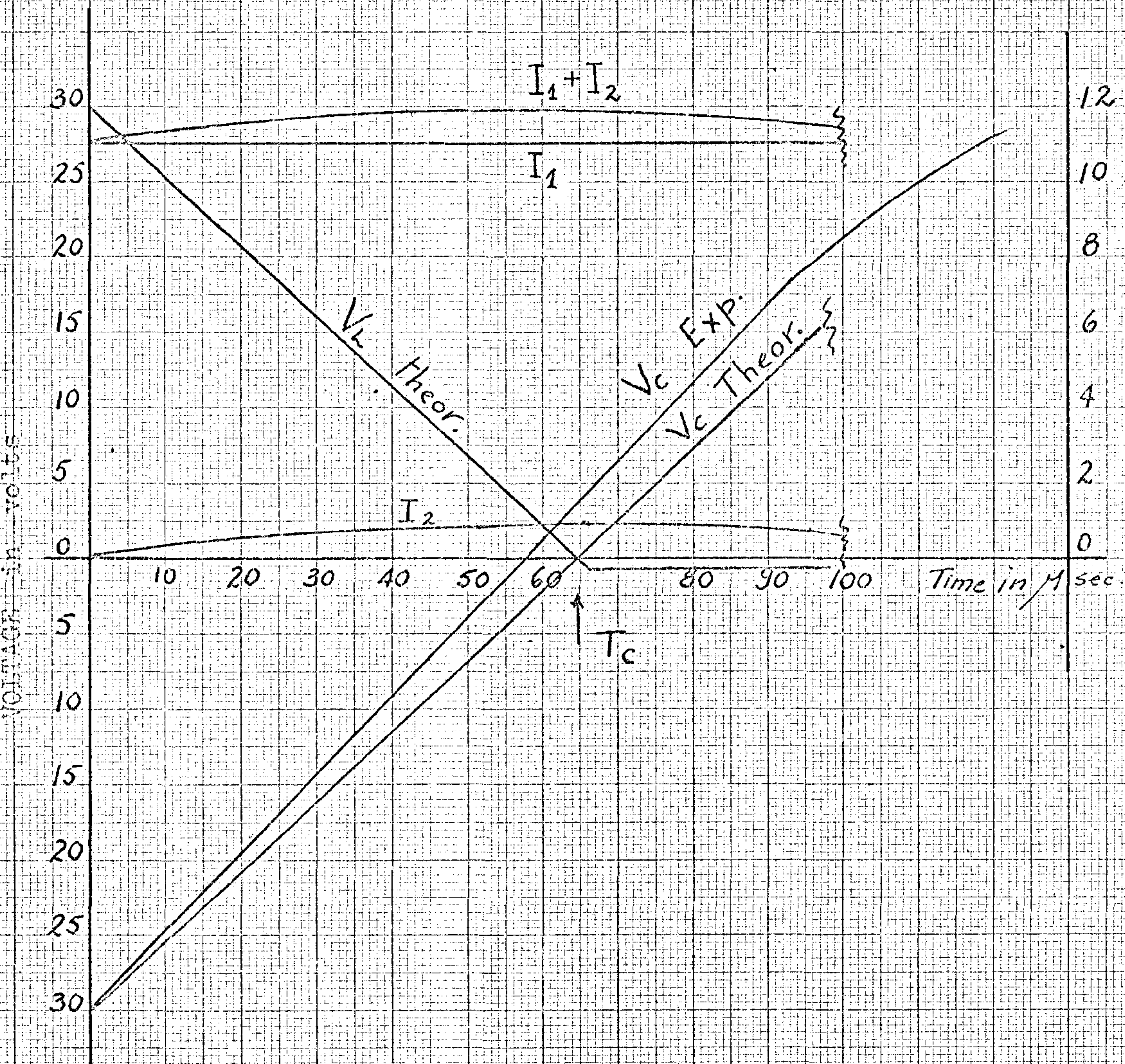


FIG. 2-5 SCR COMMUTATED CUTOFF TIME

The point of interest of this whole analysis is the intersection of V_C with the time axis. As previously discussed this interval must always be greater than the turn-off time of SCR .

The experimental curve (V_C Exp.) is shown in Fig. 2-5. It crosses the time axis at 58 microseconds compared to 66 microseconds for the calculated one (V_C Theor.). This is due to many simplifiying approximations as follows:

a) neglecting the forward voltage drop of diode D_2 .

b) neglecting the leakage current and the forward voltage drop of SCR_1 and SCR_3

c) the application of the mathematical model for a certain range only

d) the initial voltage across the capacitor is less than E due to leakage

e) the resistance of the inductor was neglected

f) the diode D_2 was considered as a perfect switch

When the current is flowing in the forward direction, the circuit is an R, L and C network which is easily solved.

Inverter Specifications:

The main components of the bridge are the SCR's and the diodes. They must be capable of withstanding the rated voltage and the rated current during continuous operation. They must have reserve capacity for large motor starting currents; typically seven times the full load current.

When two SCR's on the same leg are turned-on, a direct short circuit is placed across the DC supply. An induction reactor, L, must be supplied (Fig. 2-1) to limit the rate-of-rise of current within the specifications of the SCR's until one of the two SCR's is turned-off. The value of the reactor inductance is given by:

$$L = \tfrac{1}{2}\sqrt{2}\ V \Big/ \frac{di}{dt}$$

where V is the RMS voltage

Note the ½ is used because there are two inductors in series, the root of two for peak voltage.

If a failure-to-commutate should occur, the inverter will hang-up, and the SCR's will burn-out before conventional fuses can melt. Consequently, a high speed electronic circuit breaker is necessary to avert SCR burn-out. To bring the starting current to a value within the electronic breaker capacity (20 amperes), an external rotor resistor should be added during starting only, then shorted-out for continuous operation.

Provision must be made for dissipation of the heat generated in the SCR's. The major sources will be those due to conduction and high rates of switching. The conduction dissipation is computed by integrating the products of anode currents and forward voltage drops for the waveshape of conduction. Meaningful dissipations for high repetition rates must be obtained by actual thermal measurements. For the switching speeds used in the proposed inverter configuration, the dissipation due to switching can be approximately the same as the conduction dissipation.

Design of a 1.65 KVA Inverter:

The design of a two phase inverter to drive the generalized machine will be considered.

The characteristics of the generalized machine are:

Stator winding:

Standard two pole, two phase distributed winding, 230 volts, 3.6 amperes AC or DC (series) or 115 volts, 7.2 amperes AC or DC (parallel)

Rotor winding:

Standard two pole, continuous lap wound armature with commutator, 230 volts, 8 amperes AC or DC.

The synchronous speed of the motor is given by:

$$N_s = 120\ f\ /\ p$$

where N_s = synchronous speed in RPM

f = supply frequency in Hertz

p = number of poles

To operate in the speed range of 300 to 4800 RPM the inverter must have a frequency capability of 5 to 80 Hertz.

The DC supply will consist of a bridge rectifier operating from the 230 volts, 3 phase, 60 Hertz power line. The average DC rectifier output is given by:

$$E_d = \frac{\sqrt{2}\ E \sin \pi/p}{\pi/p}$$

where E_d = average DC output in volts

E = phase voltage

p = number of phases

Therefore $E_d = \dfrac{230 \times \sqrt{2} \times \sqrt{3}/2}{\pi/3}$

$= 269$ volts

The peak of the ripple voltage will be

$$E_{max} = 230 \times \sqrt{2}$$

$= 326$ volts

The SCR's should have a blocking capability in the range 600-650 volts (85-100 percent overshoot)

The line current of each inverter will be 7.2 amperes for the parallel connection of the stator. Therefore the SCR's should have a minimum current capability of 25 amperes for operation with external rotor resistance to limit the starting current to 25 amperes.

The diodes in series with the SCR should have similar ratings to the SCR's, 600 volts and 25 amperes.

The SCR's, diodes and bridge rectifier used in reference 1 are used as components for this inverter. It will be noted that the rectifier diodes are rated about twice the inverter SCR's, because they feed two single phase inverter bridgesin parallel.

Development of the triggering sequence and design of the associated circuitry are covered in chapter 3. It is established in chapter 3 that each leg of the inverter conducts for half cycle of the basic frequency. Assume that, at the maximum repetition rate, the leg sees continuous rated current for half cycle of the basic frequency; hence the percent duty cycle is 50 percent.

From the 2N690 SCR forward V-I characteristic, the forward voltage drop, at the rated current of 7.2 amperes, is 1.25 volts, therefore the conduction dissipation is

$= \frac{1}{2} \times 7.2 \times 1.25$

$= 4.5$ watts

The predominant dissipation for the 2N690, at repetition rates below 400 Hertz, is that due to conduction. The highest repetition rate of each SCR in the inverter is 80 Hertz. Hence a heat sink of 5 watts is required for each SCR.

Similar heat dissipating radiators are used for the diodes.

The current limiting reactors are designed next. The reactor should not saturate over its operating range. During the short-circuit commutation across the DC supply, the rate-of-rise of current should be limited to 2 amps in 25 microseconds which is equal to 8×10^4 amps/sec.

$$L = \frac{V / 2}{\frac{di}{dt}}$$

$$L = \frac{165 / 2}{8\ 10^4} = 1 \text{ mH.}$$

The reactor should be capable of continuous operation at 7.2 amperes.

The computer program developed is used to determine the value of the capacitor to commutate the SCR. A value of 48 micro-farads is found to be satisfactory with a hundred percent factor of safety.

Current overload protection must be provided to detect commutation failures and disconnect the DC supply from the inverter.

The design of an electronic breaker with a ten microseconds fault-to-interrupt time is covered in reference 1. The resistance R to trigger the Unijunction Transistor for a 20 amperes load is choosen to be .01 ohm.

CHAPTER THREE

TIMING AND TRIGGERING

General :

The logic unit generates the signals which are transmitted to the inverter for gating the different SCR's in a sequential manner. The logic unit has the capability of setting the operating frequency. The input voltage, however, is adjusted by using a Variac.

The inverter generates a square wave voltage from a DC source. A closer approximation to a sinusoidal waveshape is discussed and a typical circuit is included at the end of this chapter.

Generation Of A Single Phase Waveform :

The square waveshape can be generated by step triggering of SCR's in a bridge configuration. Consider the bridge configuration illustrated in Fig. 3-1, SCR_{1-2} and SCR_{5-6} conduct the current during the positive half cycles of phase A and phase B respectively, while SCR_{3-4} and SCR_{7-8} conduct the current during the negative half cycles.

Consider a system with a four mode operation as illustrated in Fig. 3-2. Establish an operation for phase A such that SCR_1 and SCR_2 conduct during modes 1 and 2, which represent the positive portion of waveform. SCR_3 and SCR_4 conduct during modes 3 and 4 which represent the negative portion of the waveform.

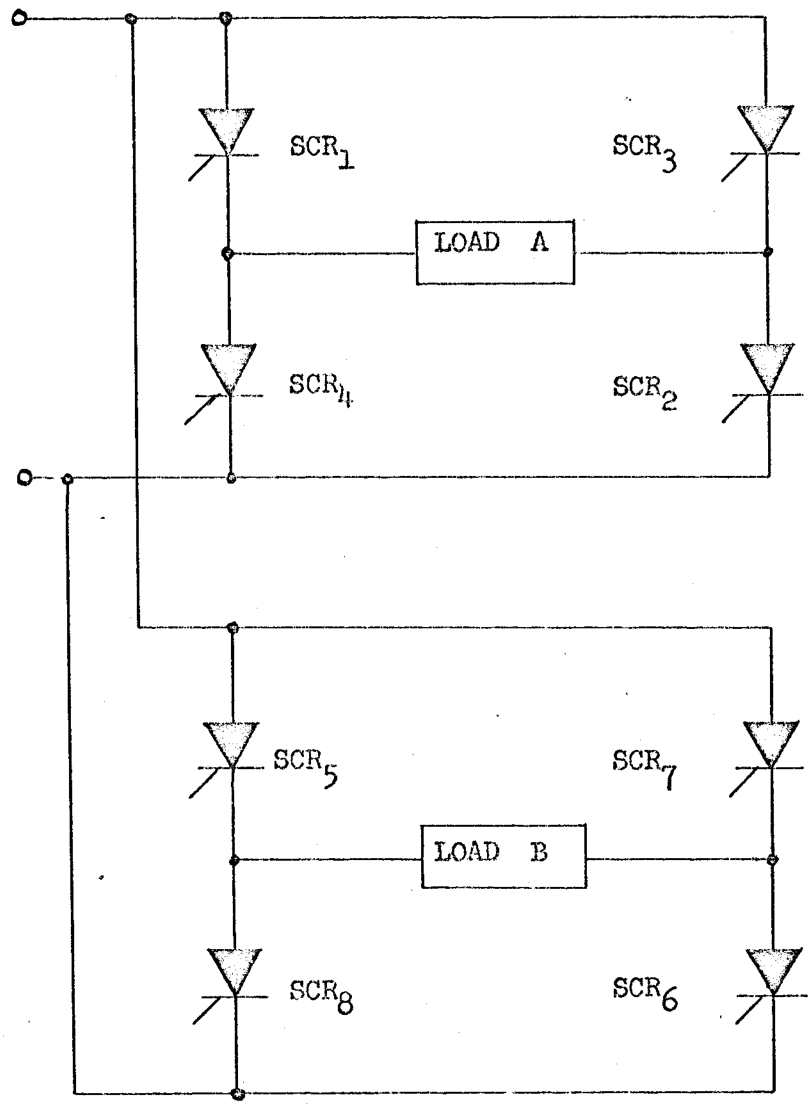


FIG. 3-1 TWO PHASE INVERTER CIRCUIT

Phase B is displaced by 90° from phase A, consequently, SCR_5 and SCR_6 conduct during modes 1 and 4 to form the positive part of phase B, SCR_7 and SCR_8 conduct during modes 2 and 3 to form the negative portion of the waveform.

Phase A and B are isolated from each other to form two phases in quadrature.

The highest frequency of operation is 80 Hertz, therefore the shortest period of one mode is:

$$= \frac{1}{4} \times \frac{1}{80} = 3.125 \text{ milliseconds}$$

Two Bit Register :

It has been established that two single phases in quadrature can be constructed from a four mode operation. The two bit register generates the necessary outputs for such an operation. The construction of this register involvesthe use of two binary counting units (flip-flop) and two inverters.

Consider the operation as illustrated in Fig. 3-3. The input C represents a clock which generates a continuous train of positive input triggers to the first binary. Q_1 and Q_2 represent the outputs of the first and second binary units respectively. The binary units are connected in cascade and respond only to changes from a one to a zero level.

The realization of such a circuit is illustrated in Fig. 3-4. In this circuit four outputs are available Q_1, $\overline{Q_1}$, Q_2 and $\overline{Q_2}$. The J's, K's and reset terminals are connected to a high level (+3volts). The high level is obtained from a NAND gate with the

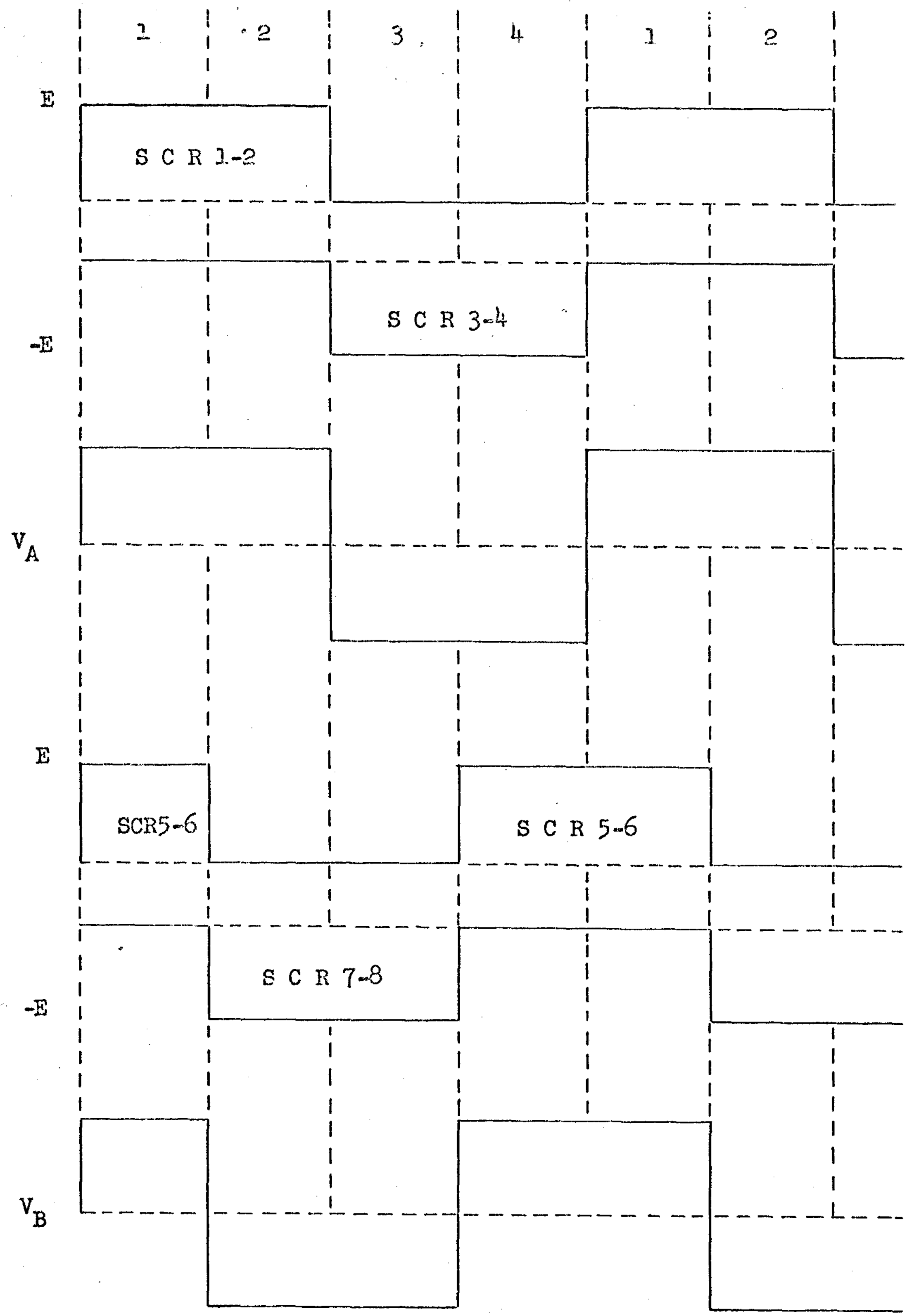


FIG. 3-2 4 MODE INVERTER WAVEFORMS FOR TWO PHASES

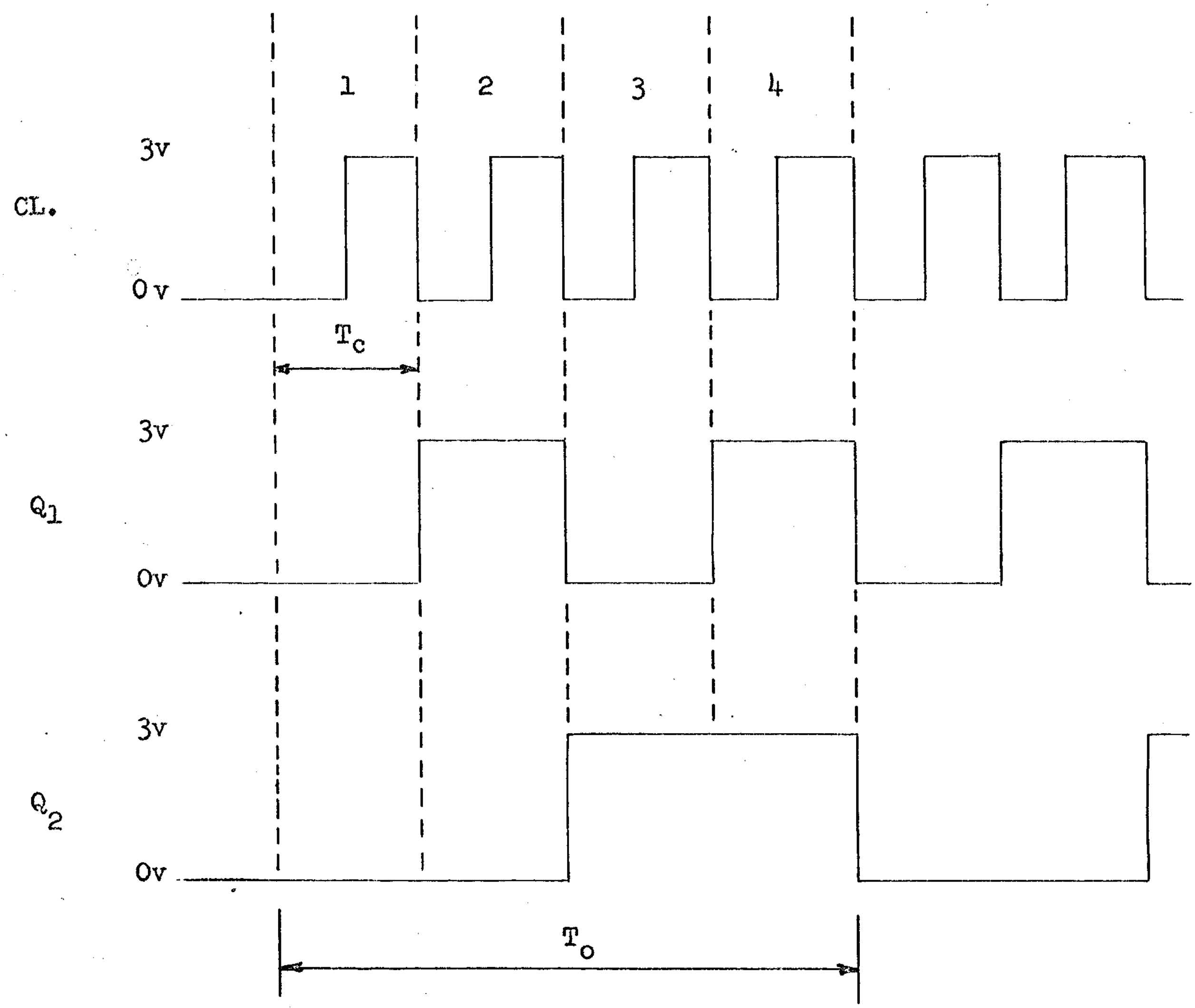


FIG. 3-3 TWO BIT REGISTER WAVEFORMS

input grounded. The micro-switch shown in the normal position will not affect the circuit, but when pushed down will put the reset terminals at low level, consequently, Q_1 and Q_2 will go to zero level and will not respond to the clock input until the micro-switch is released. The micro-switch clears the flip-flops and identifies the starting point.

Logic Equations :

The two bit register defines a four mode operation which provides for sequential triggering of the inverter SCR's. Each of the modes can be uniquely defined by wri' ing logic equations from the register waveforms. Referring to Fig. 3-5, mode 1 is represented by $\overline{Q_1} = 1$ and $\overline{Q_2} = 1$. The logic equation for mode 1 can be written as:

$$1 = \overline{Q_1} \text{ and } \overline{Q_2} = \overline{Q_1} \cdot \overline{Q_2}$$

Proceeding in a similar fashion the remaining modes can be uniquely defined as shown in Fig. 3-5 (a). Now that the four modes have been defined, the logic equations defining the triggering of the various SCR's can be established. Referring to Fig. 3-2 it will be noted that SCR_1 and SCR_2 conduct for both modes 1 and 2. This represent a logic OR operation, the logic equation for $V_A{}^+$ can be written as:

$$V_A{}^+ = 1 \text{ OR } 2 = 1 + 2$$

The logic equation for $V_A{}^+$ referring to Fig. 3-5 (a) can be written as:

$$V_A{}^+ = \overline{Q_1} \cdot \overline{Q_2} + Q_1 \cdot Q_2$$

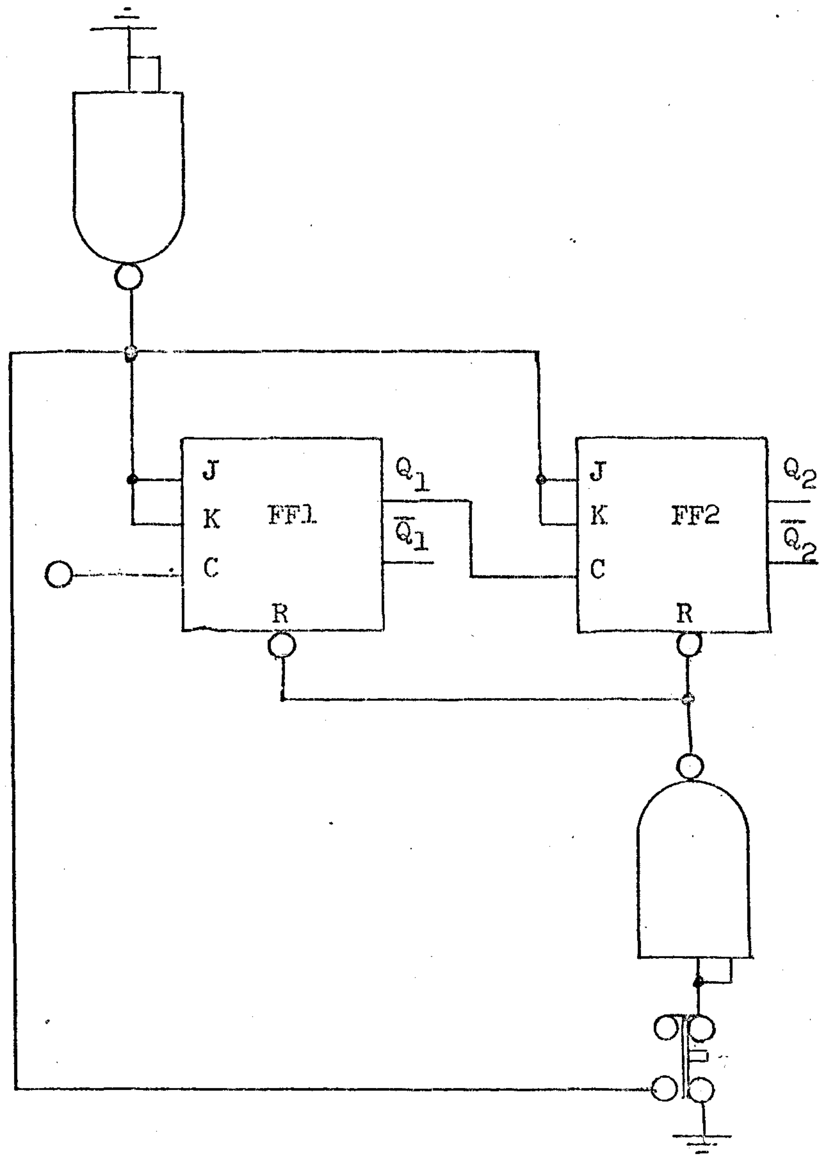


FIG. 3-4 Two bit register block diagram

$$1 = \bar{Q}_1 \cdot \bar{Q}_2$$

$$2 = Q_1 \cdot \bar{Q}_2$$

$$3 = \bar{Q}_1 \cdot Q_2$$

$$4 = Q_1 \cdot Q_2$$

(a)

$$V_A^{+} = 1 + 2 = \bar{Q}_1 \cdot \bar{Q}_2 + Q_1 \cdot \bar{Q}_2$$

$$V_A^{-} = 3 + 4 = \bar{Q}_1 \cdot Q_2 + Q_1 \cdot Q_2$$

$$V_B^{+} = 1 + 4 = \bar{Q}_1 \cdot \bar{Q}_2 + Q_1 \cdot Q_2$$

$$V_B^{-} = 2 + 3 = Q_1 \cdot \bar{Q}_2 + \bar{Q}_1 \cdot Q_2$$

(b)

$$V_A^{+} = \bar{Q}_2$$

$$V_A^{-} = Q_2$$

$$V_B^{+} = \overline{(\overline{\bar{Q}_1 \cdot \bar{Q}_2}) \cdot (\overline{Q_1 \cdot Q_2})}$$

$$V_B^{-} = (\overline{\bar{Q}_1 \cdot \bar{Q}_2}) \cdot (\overline{Q_1 \cdot Q_2})$$

(c)

FIG. 3-5 LOGIC EQUATIONS

Proceeding in a similar fashion, the remaining logic equations can be established for both phases. These are summerized in Fig. 3-5 (b) V_A^+ and V_A^- can be modified using Boolean identities as follow :

$$V_A^+ = \overline{Q_1} \cdot \overline{Q_2} + Q_1 \cdot \overline{Q_2} = (\overline{Q_1} + Q_1) \cdot \overline{Q_2} = \overline{Q_2}$$

$$V_A^- = \overline{Q_1} \cdot Q_2 + Q_1 \cdot Q_2 = (\overline{Q_1} + Q_1) \cdot Q_2 = Q_2$$

Circuit realization of Fig. 3-5 (b) is easily established using AND and OR gates, but these gates are not readily available on the commercial market. The logic equations must be transformed for the use of NAND gates only; which is easily accomplished by using deMorgan's theorem and Boolean identities:

$$V_B^+ = \overline{Q_1} \cdot \overline{Q_2} + Q_1 \cdot Q_2$$

$$= \overline{\overline{(\overline{Q_1} \cdot \overline{Q_2})} \cdot \overline{(Q_1 \cdot Q_2)}} \qquad ----- 1$$

and $$V_B^- = Q_1 \cdot \overline{Q_2} + Q_2 \cdot \overline{Q_1}$$

$$= (\overline{Q_1} + \overline{Q_2}) \cdot (Q_1 + Q_2)$$

where $$Q_1 \cdot \overline{Q_1} = Q_2 \cdot \overline{Q_2} = 0$$

$$V_B^- = \overline{(\overline{Q_1} \cdot \overline{Q_2})} \cdot \overline{(Q_1 \cdot Q_2)} \qquad ----- 2$$

Equation 2 represents the compliment of equation 1 which can be realized by an inverter circuit. The overall logic circuit realization, using NAND gates, is shown in Fig. 3-6. It will be noticed that Q_2 and $\overline{Q_2}$ are passed through additional NAND gates to provide input-output isolation.

<u>The Astable Multivibrator Used As A Clock:</u>

The frequency of the logic system is determined by the clock input to the first flip-flop. An astable multivibrator, illustrated in Fig. 3-7 (a), is used as a variable frequency

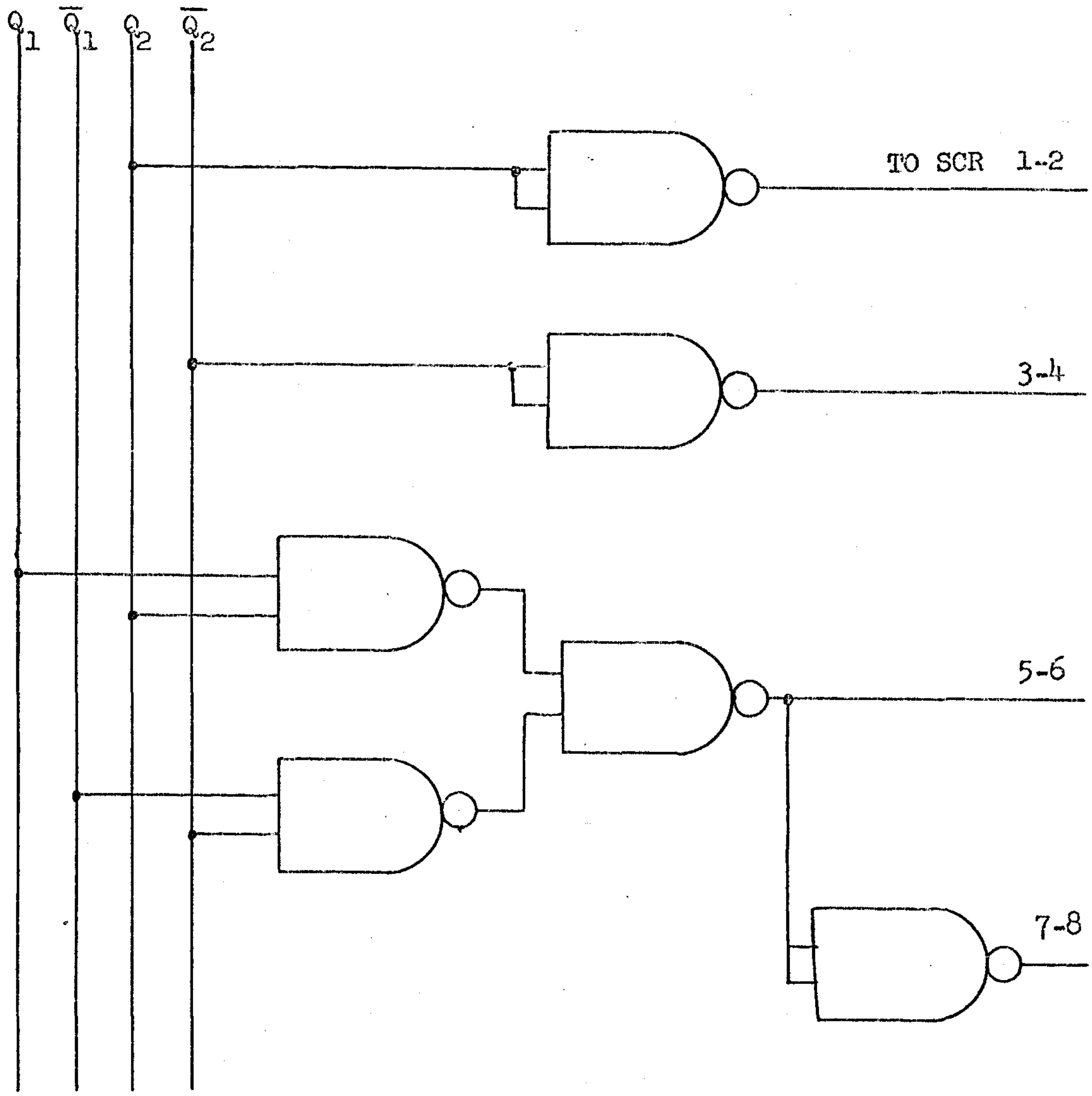


FIG. 3-6 OVERALL LOGIC REALIZATION

source. The time for each portion of the cycles shown in Fig. 3-7 (b) is given by:

$$T = T_1 + T_2$$
$$= R_1 C_1 \ln\left(1 + \frac{V_{cc}}{V}\right) + R_2 C_2 \ln\left(1 + \frac{V_{cc}}{V}\right)$$

For a symmetrical circuit with $R_1 = R_2 = 9.2$ KΩ, $C_1 = C_2 = .33\,\mu f$ and $V = V_{cc} = 3.2$ volts

$$T = 1.38\, R_1 C_1 = 4.2 \times 10^{-3} \text{ seconds} \qquad \text{-----} \; 3$$

$$f = \frac{1}{T} = 240 \text{ Hertz}$$

It will be noticed that varying R and C in equation 3 varies the period T which in turn varies the frequency. In Fig. 3-7 (b) it is noticed that there is a transient τ' associated with the waveforms of the transistor when it is driven into saturation. Each collector waveform has one rounded edge because of the time required for this transient to die down. The transient of constant time τ' is given by:

$$\tau' = (R_c + r_{bb}')\, C \qquad \text{-----} \; 4$$

where $r_{bb}' \approx 200$ ohms

$$\tau' = 150 \text{ microseconds}$$

The transient τ' should be in the range of 200 microseconds for the trailing edge of the clock to trigger the flip-flop. Therefore R_c in equation 4 must be kept low but not so low so as to upset the saturation condition $I_B \gg \frac{I_C}{\beta}$

For this case $I_B = V / R_1 = .348$ ma.

$I_C / \beta = V / (R_c \times 100) = .135$ ma.

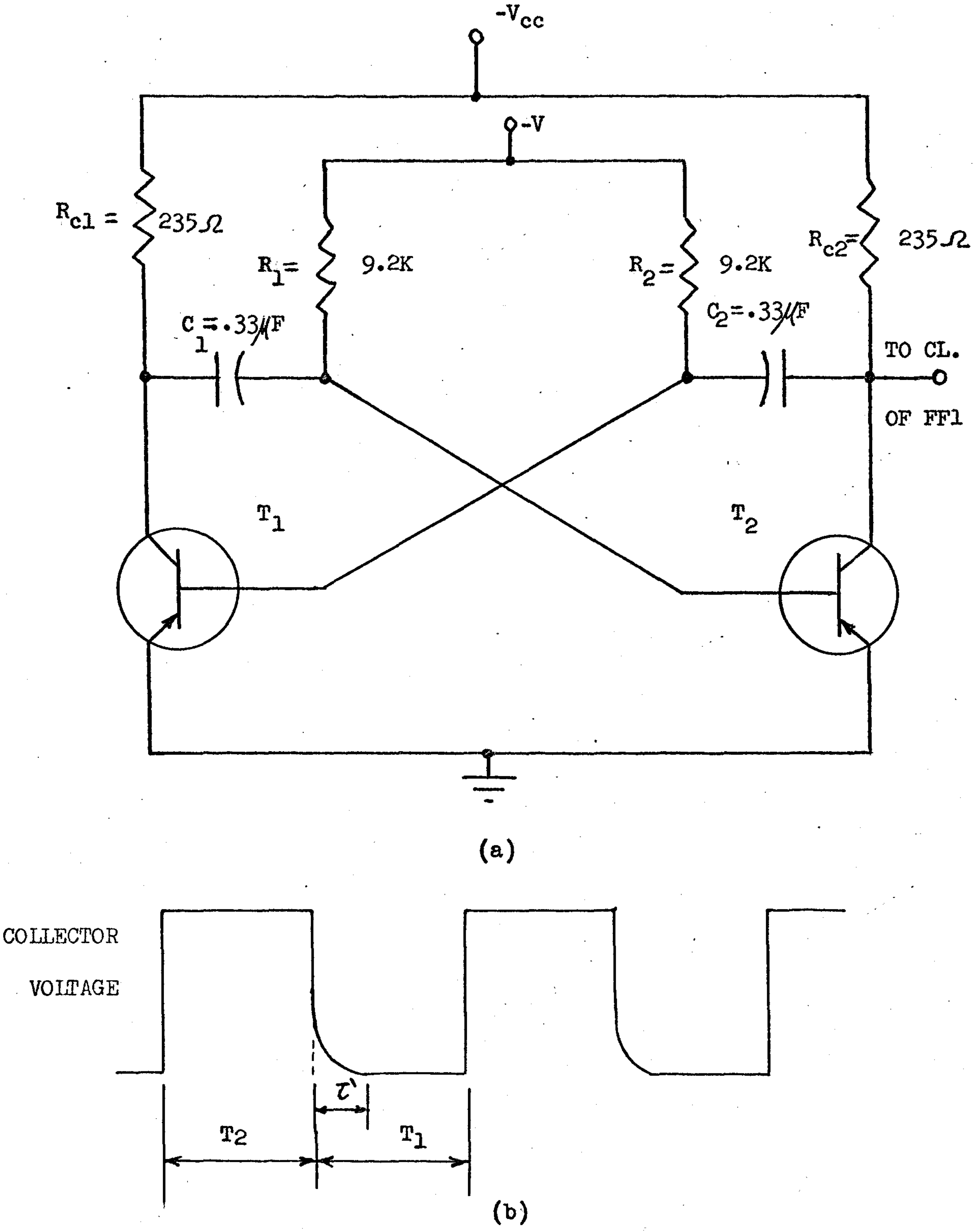


FIG. 3-7 ASTABLE MULTIVIBRATOR

UJT Pulse Trigger Circuit:

The desired waveform for triggering the SCR's is shown in Fig. 3-8 (a). The pulse should be at least 50 microseconds wide and have an amplitude of from 6 to 10 volts to ensure positive triggering action. The desired waveshape can be obtained from a Unijunction Relaxation Oscillator as illustrated in Fig. 3-8 (b).

The resistor R_2 is used to prevent the Oscillator from free running. The value of R_2 is selected to hold the emitter at 2.8 volts below its peak-point-voltage, which is defined by:

$$V = \eta V_{cc} + V_s$$

where V_s is the equivalent emitter diode voltage in the order of ½ volt at 25°C

η is the intrinsic standoff ratio of the Unijunction Transistor and varies from 0.47 to 0.67 for the 2N1671A Unijunction Transistor.

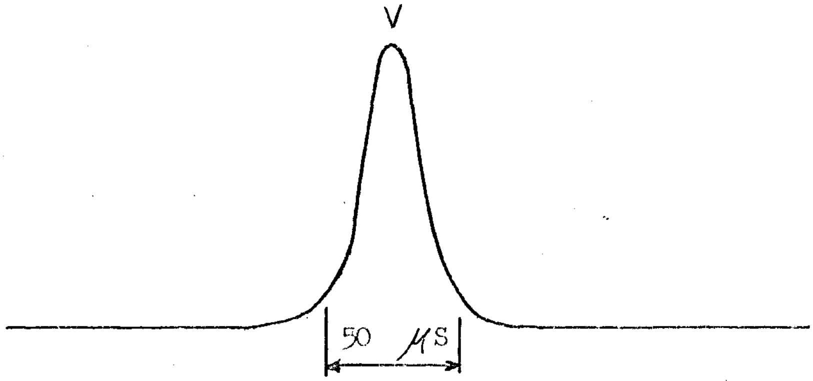
By selecting $R_1 = R_2 = 18\ K\Omega$, $C_1 = .15\ \mu f$ and $V_{cc} = 25$ volts then the voltage at point P is equal to 12.5 volts. By choosing $R_2 = 18\ K\Omega$, the voltage at point P is kept at 2.8 volts below the peak-point-voltage

$$V = 12.5 + 2.8 = 15.3 = \eta \times 25 + .5 \qquad ----- \ 5$$

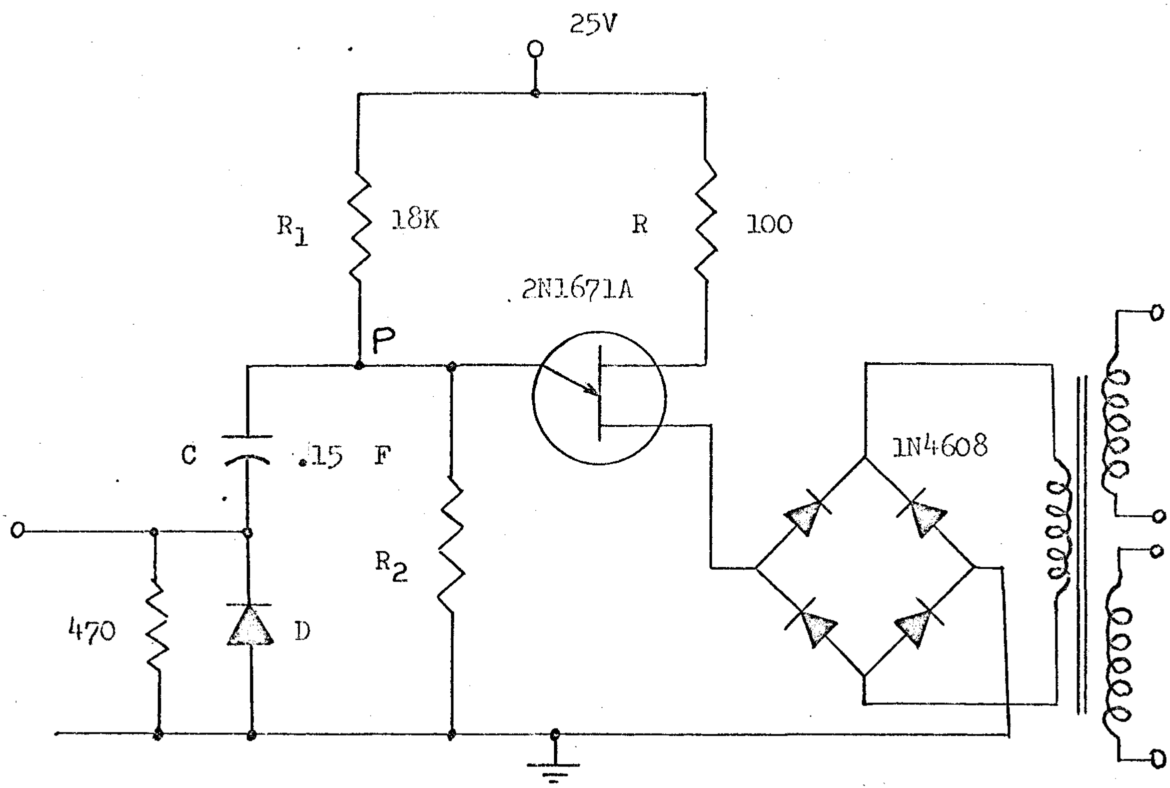
Solving equation 5 in η yiels:

$$\eta = .59$$

The unit is triggered by raising the emitter voltage above its peak-point-voltage. The logic circuit develops 3 volts across the diode which raises the emitter voltage and the capacitor



(a)



(b)

FIG. 3-8 UJT PULSE TRIGGER CIRCUIT

discharges through the UJT, the pulse transformer and the diode D (which bypasses the output impedance of the NAND gate and the 470 ohms shunt resistor).

The use of the 470 ohms resistor is to shunt the large output impedance of the NAND gate (about 14 KΩ) and therefore speed up the charging of the capacitor.

The time constant of charging voltage should be compatible with the highest operating frequency. The oscillation time constant is calculated as follow:

$$V_C = V\left(\frac{R_2}{R_1 + R_2}\right)\left[1 - e^{-t / \left(R_1 R_2 C / (R_1 + R_2)\right)}\right]$$

where V_c is the voltage across the capacitor in Fig. 3-8 (b).

For $R_1 = R_2 = 18$ KΩ, $C_1 = .15\mu f$ and $V_{cc} = 25$ volts

$$V_C = 12.5\left(1 - e^{-t / 1.35\times10^{-3}}\right) \qquad \text{-------}6$$

The UJT fires when the voltage at point P is greater than or equal to 15.3 volts, or the voltage across the capacitor is greater or equal to the voltage at point P less the voltage across the diode. Expressed in mathematical form

$$V_C \geqslant 15.3 - 3.0 = 12.3 \text{ volts} \qquad \text{-------}7$$

Substitute equation 7 in 6 to obtain the oscillation time constant:

T = 5.5 milliseconds

Consider, for instance, an operating frequency of 100 Hertz then from the four mode inverter waveforms illustrated in Fig. 3-3, SCR_1 and SCR_2 will be on for two modes or 5 milliseconds. Hence the UJT will fire only once in the total period of 10 milliseconds.

The output from the Unijunction circuit is sufficient to drive the SCR gates directly in the proposed inverter.

Decoupling UJT Circuit Against SCR Gate Transients :

With the inverter configuration selected, pulse transformers must be employed in order to obtain electrical isolation between the two circuits and the firing of two SCR's at the same time. However, when transformers are employed, a negative current flows through the gate when the SCR is commutated. The negative voltage transient appearing between the gate and the cathode of the SCR's when transmitted to the UJT can cause erratic triggering. Also the negative pulse can cause ringing in the secondary of the transformer, when the stray capacitances are considered. These transients can be eliminated by using a diode bridge in the gate circuit of the SCR as illustrated in Fig. 3-8 (b).

The diodes selected should be of high conductance, and short recovery, type. The use of high conductance diodes reduces the gate drive requirements because the gate characteristics of the SCR are that of a very high conductance diode. Fast recovery diodes are essential because the frequency of ringing is high, due to the relatively small stray capacitance. The bridge is placed before the primary of the pulse transformer to cut the number of bridges by two.

Generation Of A Stepped Waveform:

The square wave considered has a disadvantage caused by all the odd harmonics present in its fourier series, which cause an increase in iron and copper losses.

A waveshape closer to a sine wave is illustrated as V_A and V_B in Fig. 3-12.

The inverter to generate such a waveshape is illustrated in Fig. 3-11. It consists of four single phase bridges with the output of pairs of bridges added through isolating transformers to give phase A and B. The single phase inverters are operated so that their outputs are as shown in the four waveforms V_{A1}, V_{A2}, V_{B1} and V_{B2} of Fig. 3-12. The summation of the two waveforms results in the load voltage waveform shown as V_A and V_B in Fig. 3-12.

The above system requires a 12 mode operation. Consider mode 6, for instance, SCR 1-2, SCR 11-12 and SCR 15-16 will be gated-on, then another set of SCR's will be gated-on for the next mode as shown in Fig. 3-12.

The use of four flip-flops will result in a 16 mode operation, hence feedback must be used to reduce the 16 mode to a 12 mode operation. Consider the operation as illustrated in Fig. 3-13. The register behaves like a conventional four bit register up to pulse twelve at which time feedback is employed to change Q_4 and to maintain Q_3 at a zero level. The regular synchronous 16 mode operation has neither a NAND gate 3 nor $\overline{Q_4}$ at NAND gate 1 and NAND gate 4 is just an inverter.

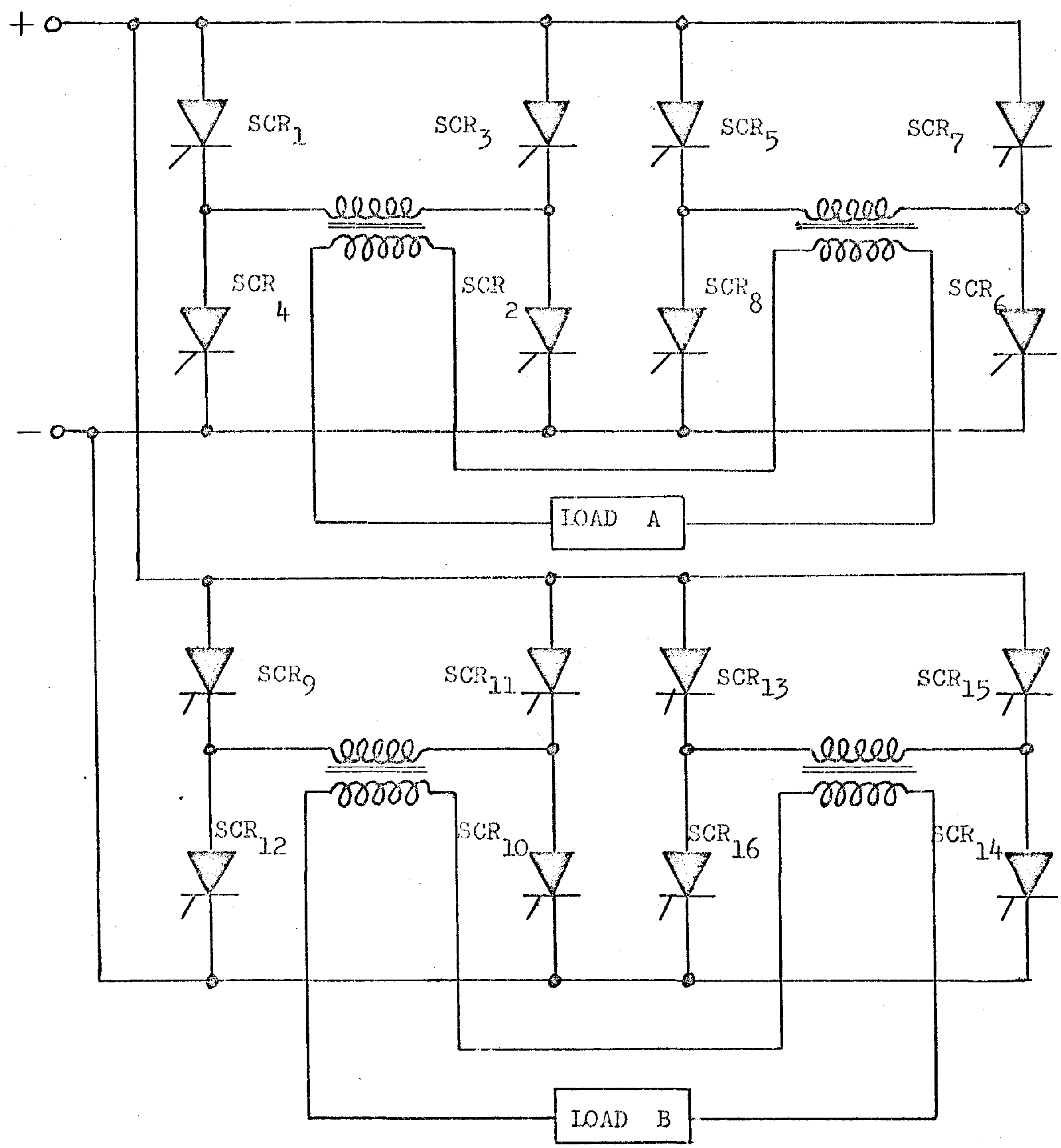


FIG. 3-11 TWO PHASE BRIDGE INVERTER

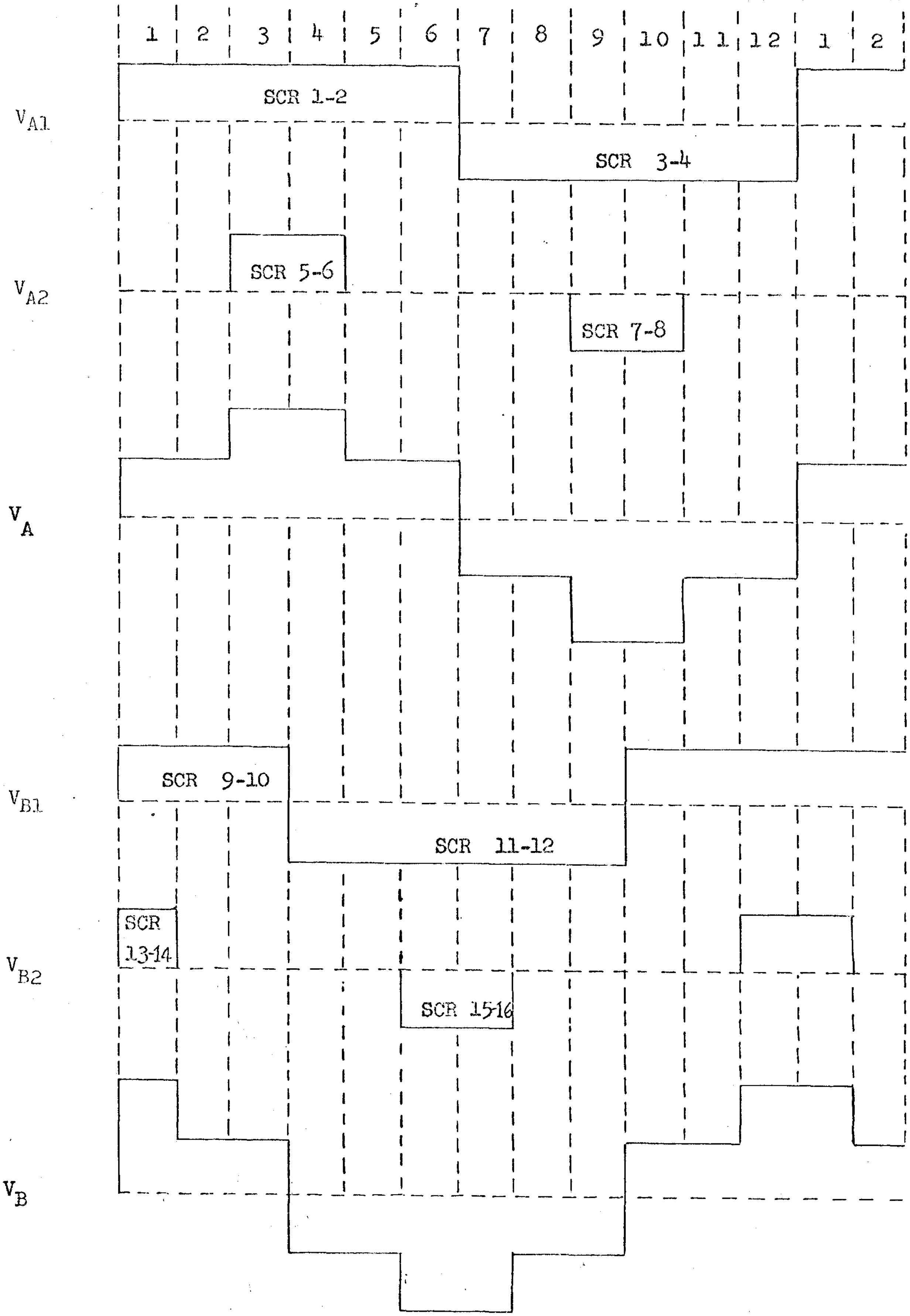


FIG. 3-12 12 MODE INVERTER WAVEFORMS

For a better understanding of the feedback operation let us tabulate the output in a table as represented in Fig 3-14. From the table the J's and K's of flip-flop 3 will be at a zero level in mode 12, therefore the flip-flop will remain in its present condition when a clock pulse occurs. On the contrary, J's and K's of flip-flop 4 are at a one level in the same mode and the flip-flop will go to a zero level on a clock pulse. Consequently, the four flip-flops will be at zero level in mode 12 which is the same as mode 1.

The four bit register defines a 12 mode operation which provides for sequential triggering of the inverter SCR's. The logic equations defining the triggering can be written using Fig. 3-12 and Fig. 3-13 for each pair of SCR's as shown below:

SCR_1 and SCR_2 are gated-on from modes 1 to 6:

$$\text{Mode } 1+2+3+4+5+6 = \overline{Q_1}\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot\overline{Q_4} + Q_1\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot\overline{Q_4} + \overline{Q_1}\cdot Q_2\cdot\overline{Q_3}\cdot\overline{Q_4} + Q_1\cdot Q_2\cdot\overline{Q_3}\cdot\overline{Q_4} + \overline{Q_1}\cdot\overline{Q_2}\cdot Q_3\cdot\overline{Q_4} + Q_1\cdot\overline{Q_2}\cdot Q_3\cdot\overline{Q_4}$$

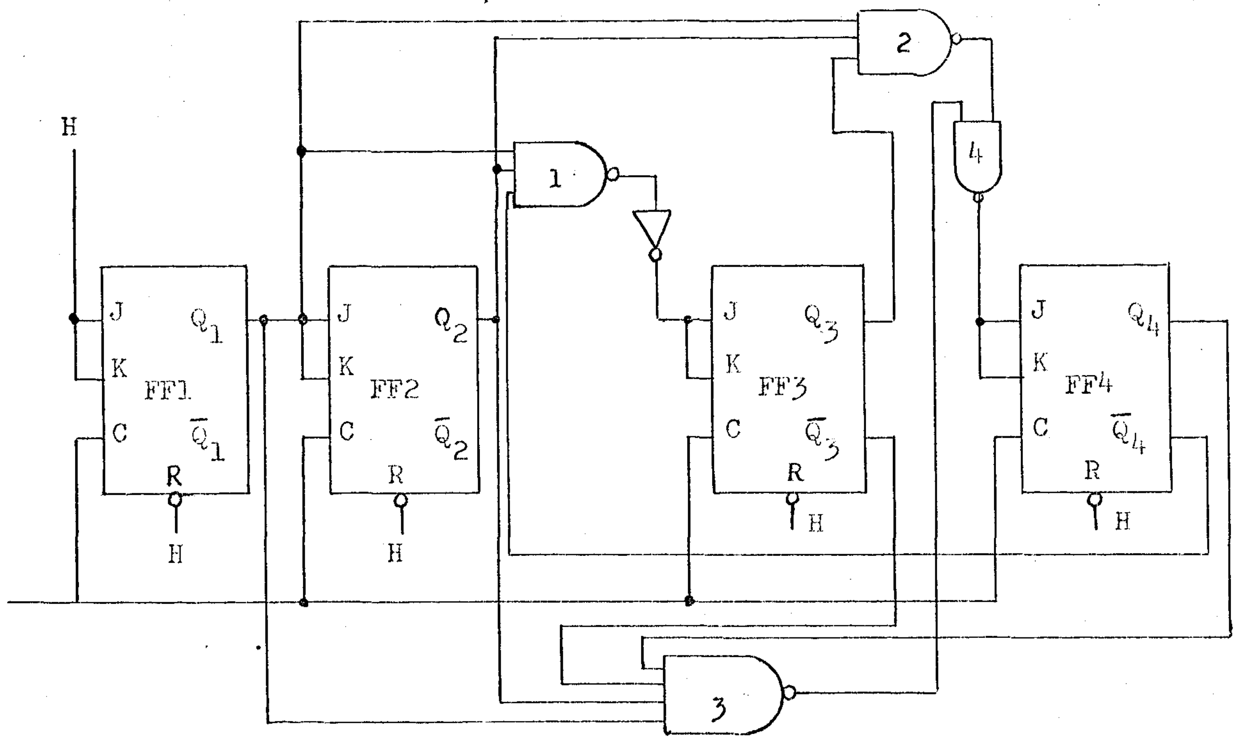
By using the Karnaugh Mapping Technique the above expression is reduced to:

$$= \overline{Q_3}\cdot\overline{Q_4} + \overline{Q_2}\cdot Q_3\cdot\overline{Q_4}$$

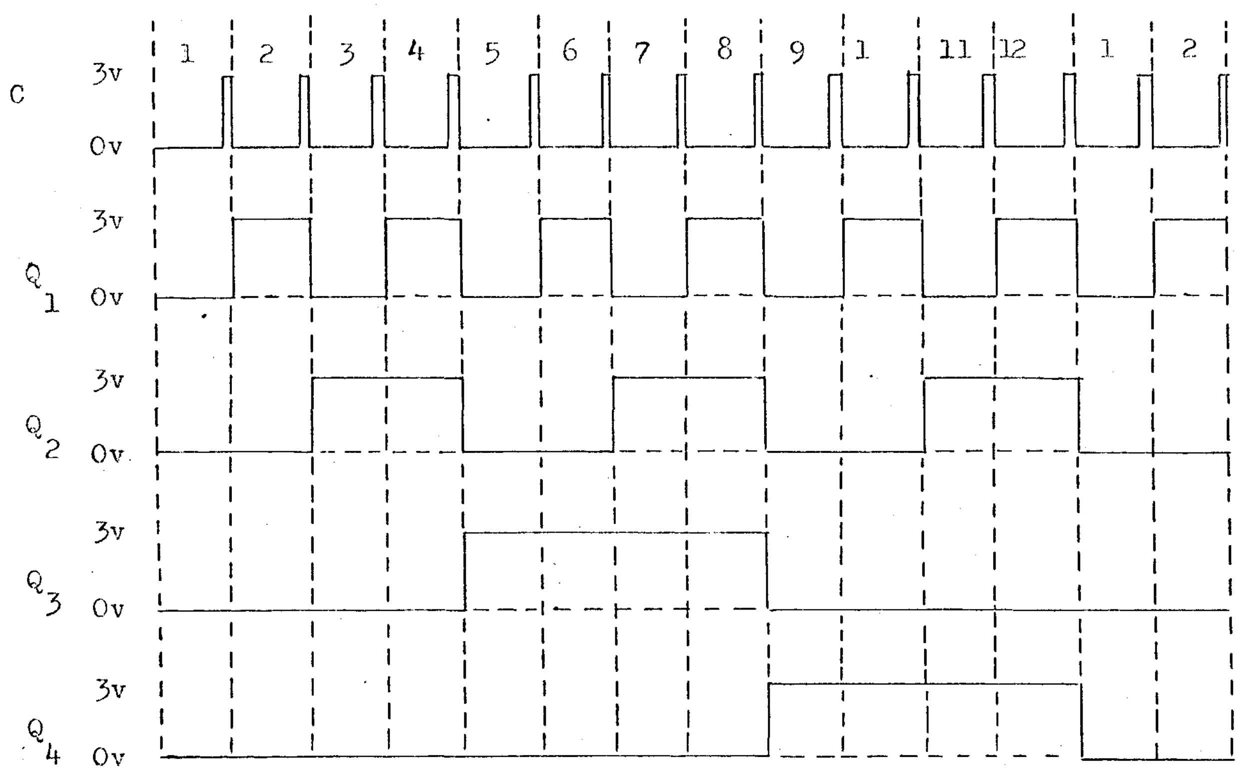
SCR_3 and SCR_4 are gated-on from modes 7 to 12:

$$\text{Mode } 7+8+9+10+11+12 = \overline{Q_1}\cdot Q_2\cdot Q_3\cdot\overline{Q_4} + Q_1\cdot Q_2\cdot Q_3\cdot\overline{Q_4} + \overline{Q_1}\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot Q_4 + Q_1\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot Q_4 + \overline{Q_1}\cdot Q_2\cdot\overline{Q_3}\cdot Q_4 + Q_1\cdot Q_2\cdot\overline{Q_3}\cdot Q_4$$

Similarly it is reduced to $= \overline{Q_3}\cdot Q_4 + Q_2\cdot Q_3\cdot\overline{Q_4}$



(a)



(b)

FIG.3-13 SYNCHRONOUS RECYCLING MODULO 12 COUNTER

OUTPUT	FLIP FLOP				NAND GATES			
MODE	Q_4	Q_3	Q_2	Q_1	1	2	3	4
1	0	0	0	0	1	1	1	0
2	0	0	0	1	1	1	1	0
3	0	0	1	0	1	1	1	0
4	0	0	1	1	0	1	1	0
5	0	1	0	0	1	1	1	0
6	0	1	0	1	1	1	1	0
7	0	1	1	0	1	1	1	0
8	0	1	1	1	0	0	1	1
9	1	0	0	0	1	1	1	0
10	1	0	0	1	1	1	1	0
11	1	0	1	0	1	1	1	0
12	1	0	1	1	1	1	0	1
1	0	0	0	0	1	1	1	0

FIG. 3-14 MODULO 12 COUNTER TRUTH TABLE

SCR_5 and SCR_6 are gated-on during modes 3 and 4:

$$\text{Mode } 3 + 4 = \overline{Q_1}\cdot Q_2\cdot Q_3\cdot\overline{Q_4} + Q_1\cdot Q_2\cdot\overline{Q_3}\cdot\overline{Q_4} = Q_2\cdot\overline{Q_3}\cdot\overline{Q_4}$$

SCR_7 and SCR_8 are gated-on during modes 9 and 10:

$$\text{Mode } 9 + 10 = \overline{Q_1}\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot Q_4 + Q_1\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot Q_4 = \overline{Q_2}\cdot\overline{Q_3}\cdot Q_4$$

SCR_9 and SCR_{10} are gated-on during modes 1 to 3 and 10 to 12:

$$\begin{aligned}\text{Mode } 1 + 2 + 3 + 10 + 11 + 12 &= \overline{Q_1}\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot\overline{Q_4} + Q_1\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot\overline{Q_4} + \\ &\quad \overline{Q_1}\cdot Q_2\cdot\overline{Q_3}\cdot\overline{Q_4} + Q_1\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot Q_4 + \\ &\quad \overline{Q_1}\cdot Q_2\cdot\overline{Q_3}\cdot Q_4 + Q_1\cdot Q_2\cdot\overline{Q_3}\cdot\overline{Q_4} \\ &= \overline{Q_1}\cdot\overline{Q_2}\cdot\overline{Q_4} + Q_2\cdot\overline{Q_3}\cdot Q_4 + Q_1\cdot\overline{Q_2}\cdot\overline{Q_3}\end{aligned}$$

SCR_{11} and SCR_{12} are gated-on during modes 4 to 9:

$$\begin{aligned}\text{Mode } 4 + 5 + 6 + 7 + 8 + 9 &= Q_1\cdot Q_2\cdot\overline{Q_3}\cdot\overline{Q_4} + \overline{Q_1}\cdot\overline{Q_2}\cdot Q_3\cdot\overline{Q_4} + \\ &\quad Q_1\cdot\overline{Q_2}\cdot Q_3\cdot\overline{Q_4} + \overline{Q_1}\cdot Q_2\cdot Q_3\cdot\overline{Q_4} + \\ &\quad Q_1\cdot Q_2\cdot Q_3\cdot\overline{Q_4} + \overline{Q_1}\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot Q_4 \\ &= Q_3\cdot\overline{Q_4} + Q_1\cdot Q_2\cdot\overline{Q_4} + \overline{Q_1}\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot Q_4\end{aligned}$$

SCR_{13} and SCR_{14} are gated-on during modes 1 and 12:

$$\text{Mode } 1 + 12 = \overline{Q_1}\cdot\overline{Q_2}\cdot\overline{Q_3}\cdot\overline{Q_4} + Q_1\cdot Q_2\cdot\overline{Q_3}\cdot Q_4$$

SCR_{15} and SCR_{16} are gated-on during modes 6 and 7:

$$\text{Mode } 6 + 7 = Q_1\cdot\overline{Q_2}\cdot Q_3\cdot\overline{Q_4} + \overline{Q_1}\cdot Q_2\cdot Q_3\cdot\overline{Q_4}$$

The overall logic circuit realization using NAND gates is shown in Fig. 3-15. This circuit was tested on the Digital Computer Lab and performed satisfactorely according to the specifications described above. This circuit was not built due to lack of fund. It is interesting to note that four times as many NAND gates are required in a 12 mode operation than are required in a 4 mode operation.

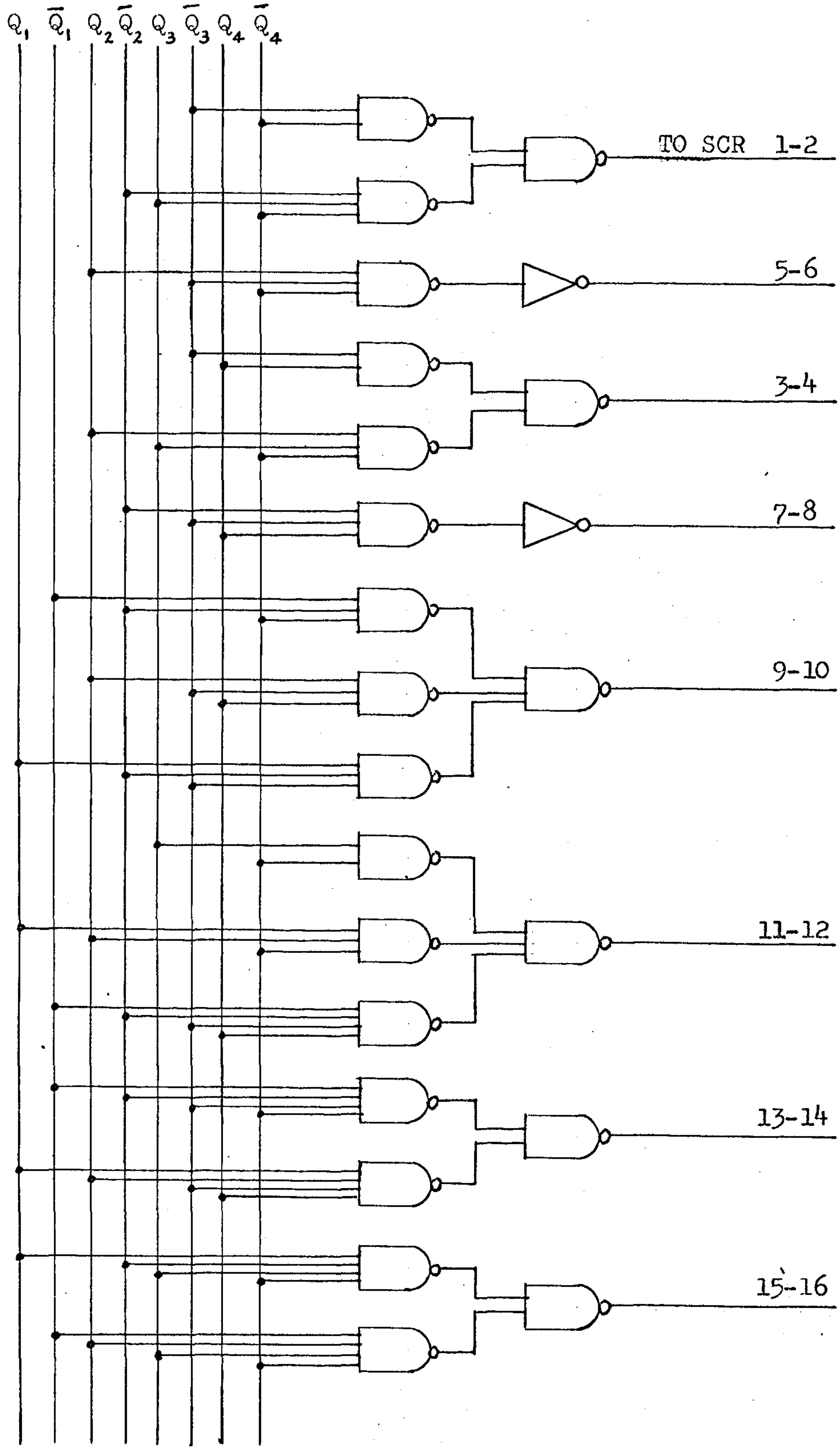


FIG. 3-15 OVERALL LOGIC REALIZATION

Summary:

The timing and triggering requirements are summarized in block diagram form in Fig. 3-9. The output waveshapes are summarized in Fig. 3-10.

The Astable Multivibrator provides the basic frequency control. The two bit register provides a four mode operation. The logic unit assimilates the output of the register and generates a set of sequential timing pulses. The firing circuits provide the necessary gate drive for reliable operation of the SCR's. The Variac changes the input voltage to the bridge rectifier to maintain a constant flux density. The bridge inverters, by gating the SCR's on and off, invert the DC voltage to two phase voltagesin quadrature.

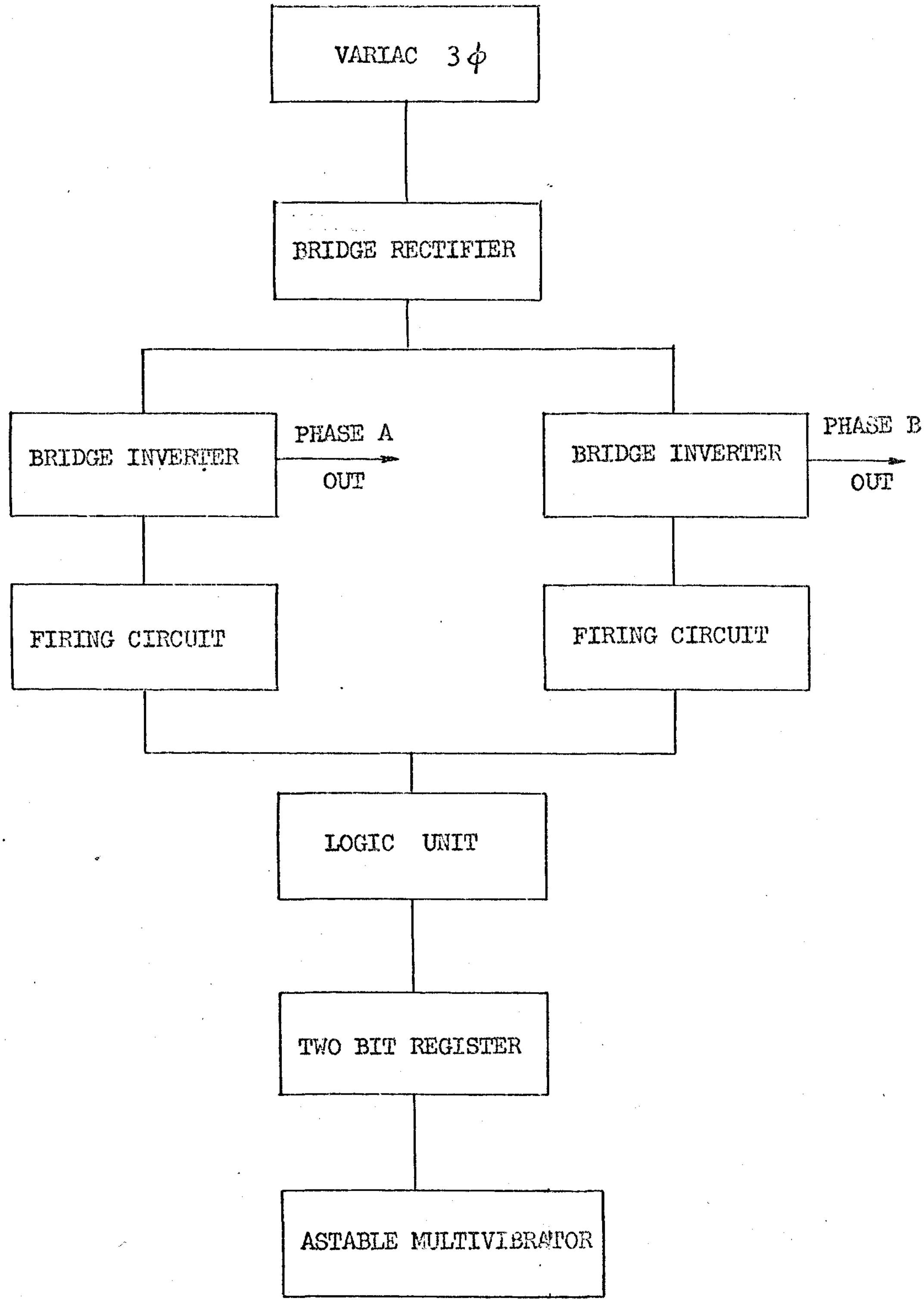


FIG. 3-9 OVERALL BLOCK DIAGRAM

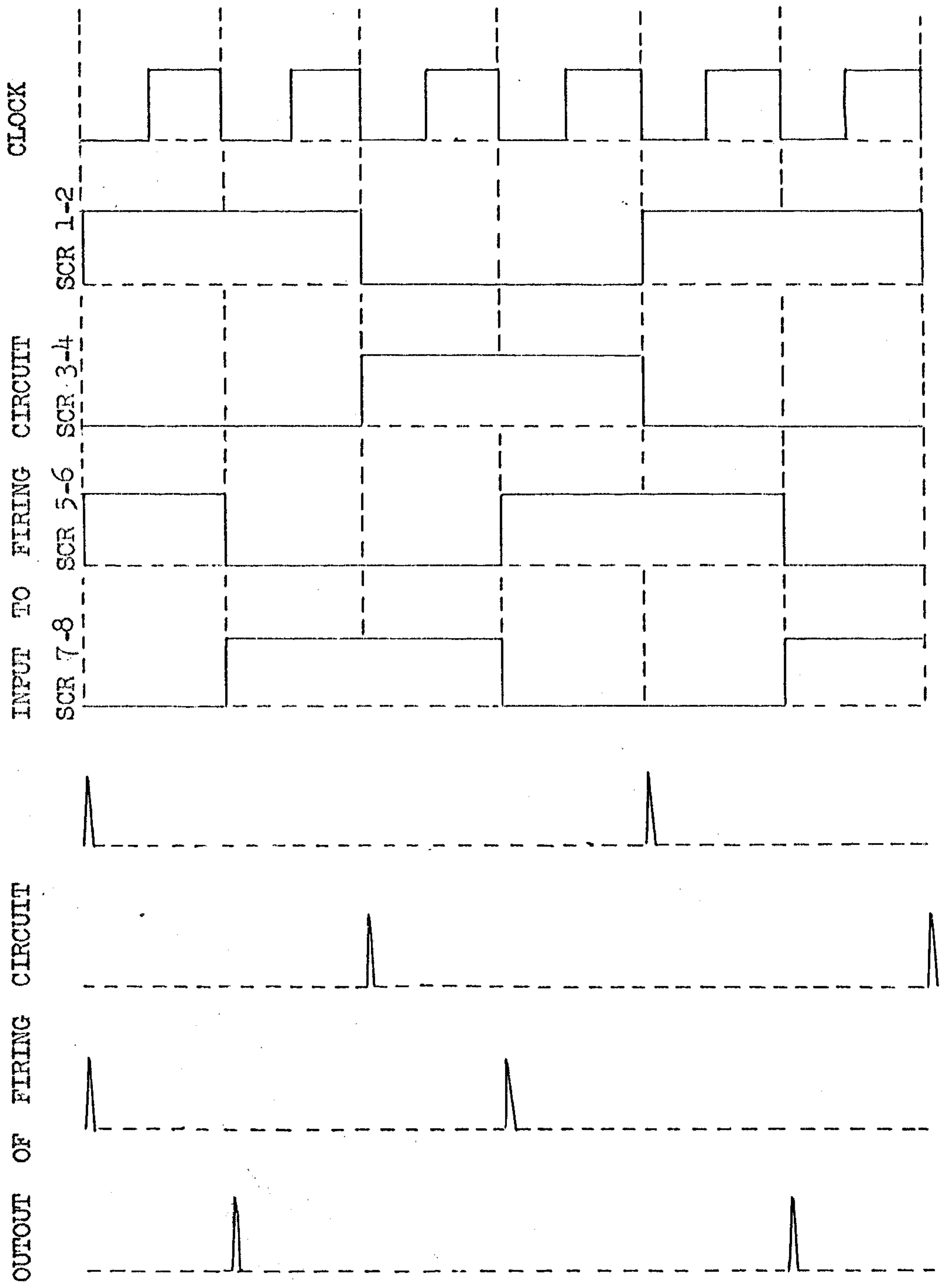


FIG. 3-10 OVERALL FIRING CIRCUIT WAVEFORMS

CHAPTER FOUR

DIGITAL COMPUTER ANALYSIS OF INDUCTION MACHINES STARTING CURRENTS

General:

The AC induction machine equations have been derived in chapter one in a stationary frame. The voltage and current relationship is given by matrix 35 which is rewritten as follow:

$$\begin{bmatrix} V\cos\omega_o t \\ V\sin\omega_o t \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & m\omega_m M_d & R_r & m\omega_m L_d \\ -m\omega_m M & 0 & -m\omega_m M_q & R \end{bmatrix} \begin{bmatrix} i_2 \\ i_1 \\ i_3 \\ i_4 \end{bmatrix} + \begin{bmatrix} L_s & 0 & M_q & 0 \\ 0 & L_s & 0 & M_d \\ M_d & 0 & L_q & 0 \\ 0 & M_q & 0 & L_d \end{bmatrix} \begin{bmatrix} pi_2 \\ pi_1 \\ pi_3 \\ pi_4 \end{bmatrix}$$

where p is $\frac{d}{dt}$

The above matrix form can be written as:

$$V = R\,I + Z\,p\,I \qquad \text{--------- 1}$$

Equation 1 can be rearranged to:

$$p\,I = Z^{-1}\,V - Z^{-1}\,R\,I \qquad \text{--------- 2}$$

The inverse of Z, Z^{-1}, is obtained by a matrix inversion subroutine. After Z has been numerically inverted; equation 2 can be expanded to give the explicit form for the differential equations:

$$pi_1 = A(2,2)\,V\sin\omega_o t + A(2,2)\,R_s\,i_1 + A(2,4)\,m\omega_m L_q\,i_3$$
$$- A(2,4)\,R_r\,i_4 + A(2,4)\,m\omega_m M_q\,i_2$$

$$\left.\begin{aligned}
pi_2 &= A(1,1)\ V \cos \omega_o t - A(1,1)\ R_s\ i_2 - A(1,3)\ m\omega_m M_d\ i_1 \\
&\quad - A(1,3)\ R_r\ i_3 - A(1,3)\ m\omega_m\ L_d\ i_4 \\
pi_3 &= A(3,1)\ V \cos \omega_o t - A(3,1)\ R_s\ i_2 - A(3,3)\ m\omega_m M_d\ i_1 \\
&\quad - A(3,3)\ R_r\ i_3 - A(1,3)\ m\omega_m\ L_d\ i_4 \\
pi_4 &= A(4,2)\ V \sin \omega_o t - A(4,4)\ m\omega_m M_q\ i_2 - A(4,2)\ R_s\ i_1 \\
&\quad + A(4,4)\ m\omega_m L_q\ i_3 - A(4,4)\ R_r\ i_4
\end{aligned}\right\} 3$$

The torque is given by equation 40:

$$T_m = J\ p\omega + D\ \omega + C - m(-M_d\ i_1\ i_3 - L_d\ i_4\ i_3 + M_q\ i_4\ i_2 + L_q\ i_3\ i_4) \quad \text{-----}\ 4$$

Equation 4 can be rearranged to:

$$p\omega = \frac{T_m}{J} + \frac{m}{J}(-M_d\ i_1\ i_3 - L_d\ i_4\ i_3 + M_q\ i_4\ i_2 + L_q\ i_3\ i_4) - \frac{D}{J}\omega - C \quad \text{-----}\ 5$$

Equation 5 is then in suitable form to be included with the four differential equations of the phase currents. A digital computer is used to solve the five differential equations using numerical approximations. Two of the most commonly used techniques are the Runga Kutta and the predictor-corrector (Milne) methods.

Runga Kutta Method:

The Runga Kutta method was selected for use in this investigation. Essentially this method involves using information available at the n, n-1, n-2 .....,etc., increments to predict values of the desired function at the (n+1) increment. The predicted value at the (n+1) increment is then combined with previous values of the function, to obtain a corrected value at the (n+1) increment.

The equations used in the differential equation algorithm being discussed here are:

$$\frac{di_1}{dt}=f_1\ (t,\ i_1,\ i_2,\ i_3,\ i_4,\ \omega)$$

$$\frac{di_2}{dt}=f_2\ (t,\ i_1,\ i_2,\ i_3,\ i_4,\ \omega) \qquad \text{etc.}$$

The increment in i_1 for the first interval is found from:

$$k_1 = f_1\ (t_0,\ i_{10},\ i_{20},\ i_{30},\ i_{40},\ \omega_0)$$

$$k_2 = f_1\ (t_0+\Delta t/2,\ i_{10}+k_1/2,\ i_{20}+l_1/2,\ i_{30}+m_1/2,\ i_{40}+n_1/2, \omega_0+q_1/2)\Delta t$$

$$k_3 = f_1\ (t_0+\Delta t/2,\ i_{10}+k_2/2,\ i_{20}+l_2/2,\ i_{30}+m_2/2,\ i_{40}+n_2/2, \omega_0+q_2/2)\Delta t$$

$$k_4 = f_1\ (t_0+\Delta t,\ i_{10}+k_3,\ i_{20}+l_3,\ i_{30}+m_3,\ i_{40}+n_3,\ \omega_0+q_3)\ \Delta t$$

$$\Delta i_1 = (k_1 + 2k_2 + 2k_3 + k_4)/6$$

Each of the above expressions *is* applicable for the other variables i_2, i_3, i_4 and ω to obtain l, m, n and q. Note that the initial conditions for the currents and speed are zero.

Mechanical Parameters:

The constants J, D and C of equation 5 are determined using the Running Down technique. When the Generalized Machine is unexcited and driven by the Prime Mover, equation 4 is reduced to:

$$T = J\ p\omega + D\ \omega\ C \qquad \text{-------- 6}$$

The motor torque is suddenly interrupted and the machine will slow down. During this movement equation 6 is reduced to:

$$J\ p\omega + D\ \omega + C = 0 \qquad \text{-------- 7}$$

The solution of equation 7 is:

$$(D\ \omega + C) = (D\ \omega_{or} + C)\ e^{-\ D\ t\ /\ J} \qquad \text{-------- 8}$$

where ω_{or} is the speed at shut down

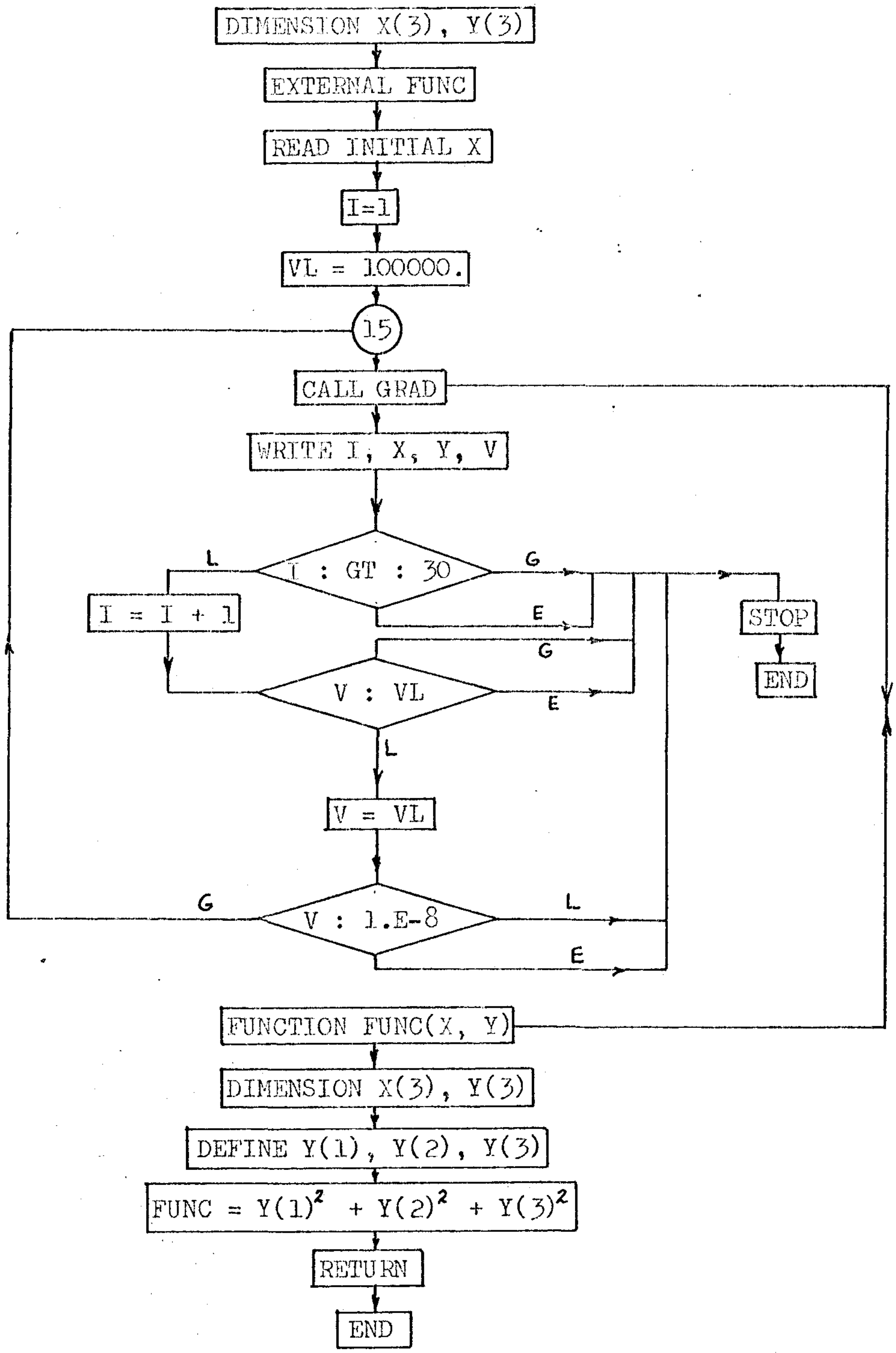


FIG. 4-1 Block Diagram For Solving 3 Simultaneous Non-Linear Equations

Three run down tests at different speeds were performed and the times taken for the machine to stop recorded.

Substituting for ω_{or} and t in equation 8, for the three run down tests, gives three simultaneous nonlinear equations.

Fig. 4-1 describes the solution of these three simultaneous nonlinear equations using a digital computer.

From the program, the constants are:

J = 0.028		Kg-Meter^2
D = 0.00026		Newton-Meter/ radian per second
C = 0.386		Newton-Meter

Electrical Parameters:

The DC resistance was measured at room temperature. The stator resistance R_s is 1.4 ohms (1.6 ohms at 60 Hertz)*. The rotor resistance excluding brush contact resistance is 0.46 ohms (0.53 ohms at 60 Hertz)*.

The self and mutual inductances were taken from the results of experiments performed on a similar generalized machine using the Operational Amplifier technique in Reference 4.

Due to saturation the curves for self and mutual inductances are nonlinear. A curve fitting subroutine using the least square technique was used to fit the curves of inductances to a polynomial of a degree n. For n=5 the mutual is:

---

* Knowlton, A. E. , Standard Handbook For Electrical Engineers, McGraw Hill, New York, 1961, page 127 of section 4 and table 4-7.

$$M = .18 + .00327\, i - .00004\, i^2 - .00024\, i^3 + .0003\, i^4 - .00001\, i^5$$

For n=8 the self inductance is:

$$L = .48 + .0425\, i + .115\, i^2 - .484\, i^3 + .694\, i^4 - .492\, i^5 + .18\, i^6 - .032\, i^7 + .0023\, i^8$$

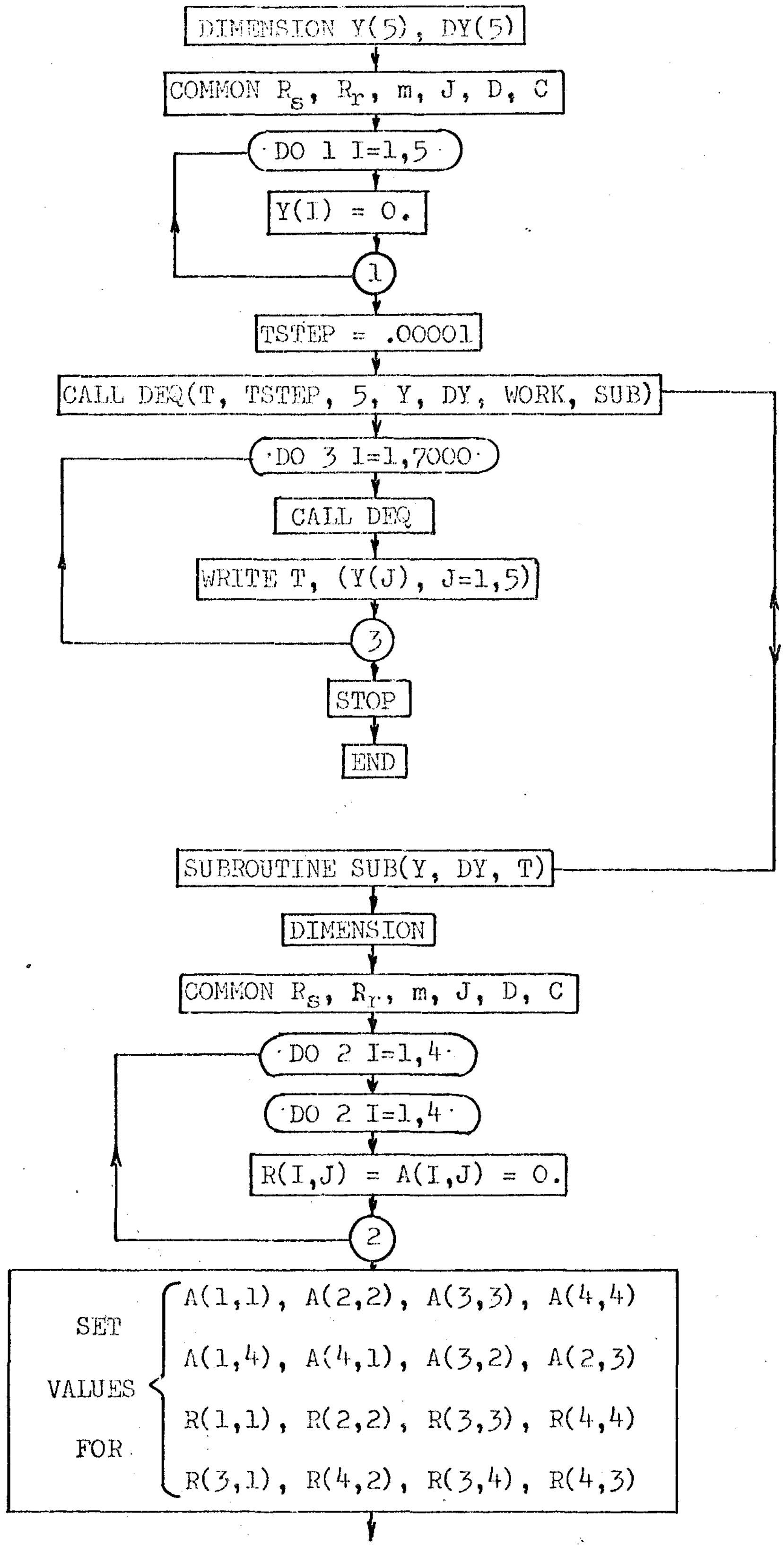
Results:

The five differential equations were solved on a digital computer and the block diagram is shown in Fig. 4-2.

The peak starting currents for the various windigs versus external rotor resistances are illustrated in Fig. 4-3. From these curves, a four ohms external resistors will limit the maximum starting current to 10 amperes. This is within the limitation of the Electronic Breaker.

The accuracy of the Runga Kutta method is largely dependent on the size of increment Δt. This method, however, gave satisfactory results for this investigation.

If a greater accuracy is required the predictor-corrector method should be used because it has an automatic adjustment of Δt to keep the truncation error within prescribed limits.


DIMENSION Y(5), DY(5)
COMMON R_s, R_r, m, J, D, C
DO 1 I=1,5
Y(I) = 0.
1
TSTEP = .00001
CALL DEQ(T, TSTEP, 5, Y, DY, WORK, SUB)
DO 3 I=1,7000
CALL DEQ
WRITE T, (Y(J), J=1,5)
3
STOP
END
SUBROUTINE SUB(Y, DY, T)
DIMENSION
COMMON R_s, R_r, m, J, D, C
DO 2 I=1,4
DO 2 I=1,4
R(I,J) = A(I,J) = 0.
2
SET VALUES FOR
A(1,1), A(2,2), A(3,3), A(4,4)
A(1,4), A(4,1), A(3,2), A(2,3)
R(1,1), R(2,2), R(3,3), R(4,4)
R(3,1), R(4,2), R(3,4), R(4,3)

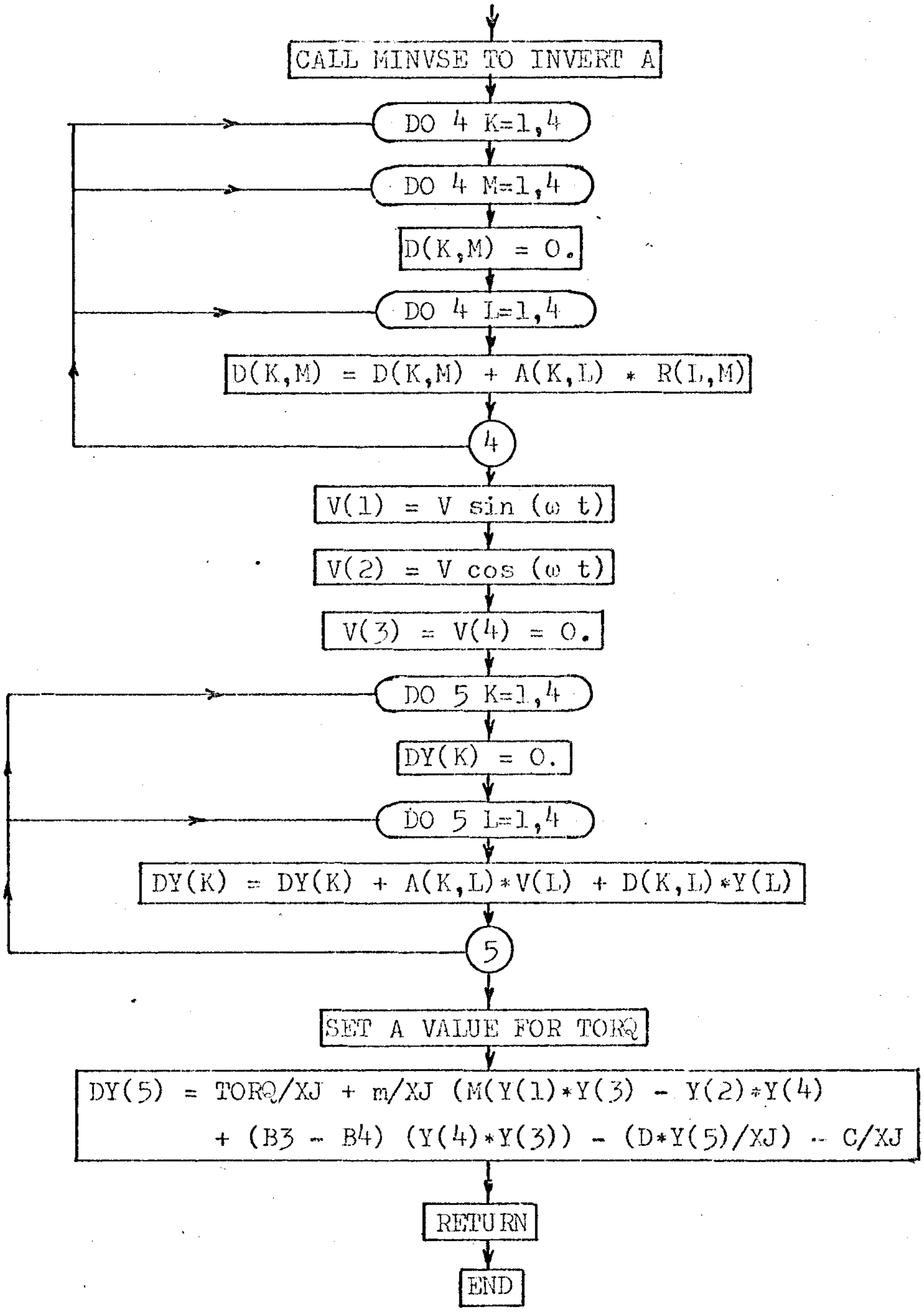


FIG. 4-2 BLOCK DIAGRAM FOR SOLVING 5 SIMULTANEOUS DIFFERENTIAL EQUATIONS

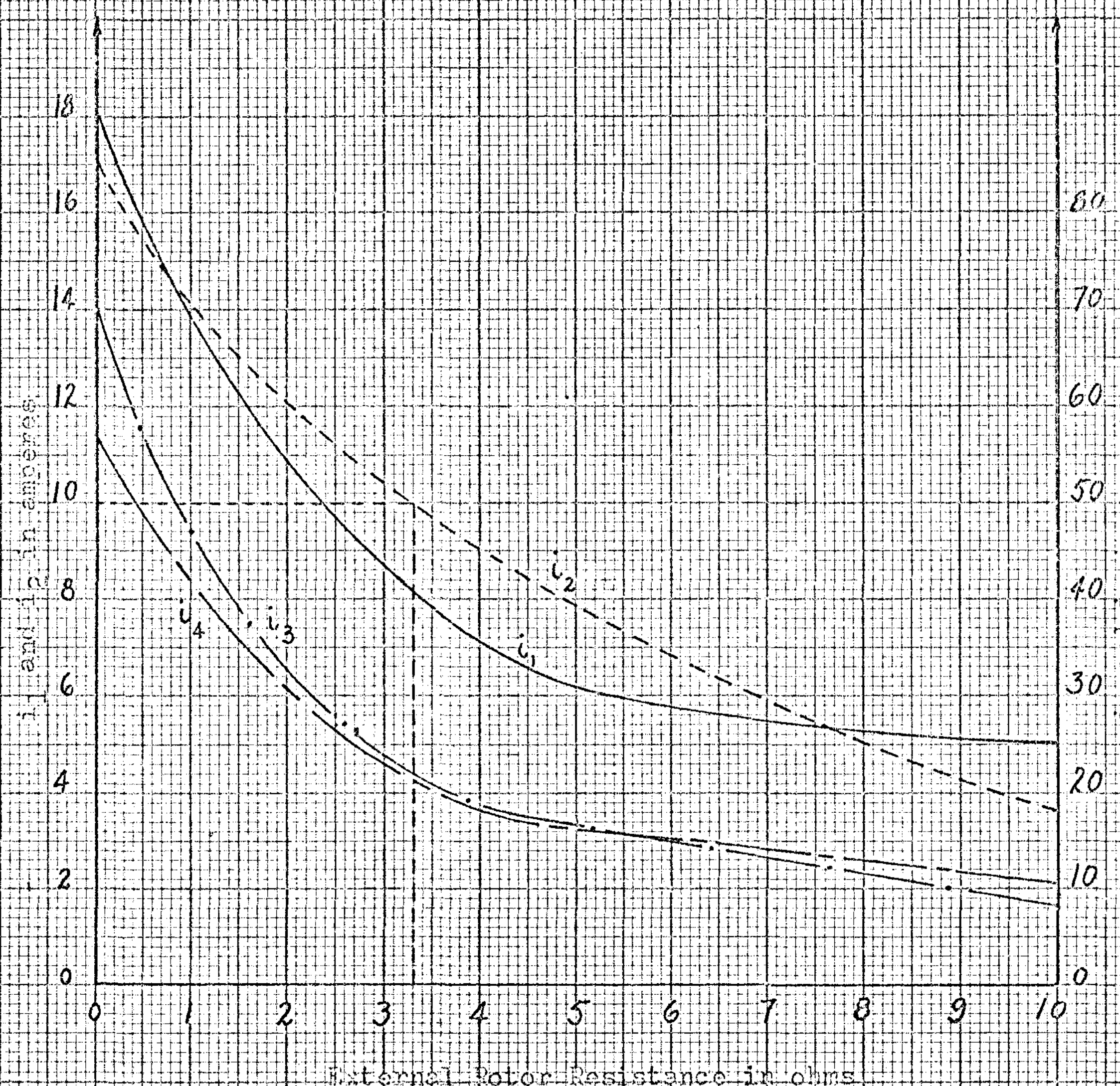


FIG. 4-3 Phase Currents vs External Rotor Resistance

CHAPTER FIVE

CONCLUSION

The current, voltage and torque equations for a two phase generalized machine are reviewed. The transformation, from a three phase slip-ring machine to a three phase commutator primitive, is derived. The equations for the three phase commutator primitive are simplified to an equivalent two phase commutator primitive plus zero sequence components.

Runga-Kutta method is used to investigate the starting currents of the two phase machine. The currents for a three phase machine can be determined by using the connection matrices derived in chapter one.

From the results obtained in chapter 4, it was found that a four ohms external resistor would limit the starting current to ten amperes.

The speed of a two phase generalized machine can be controlled by varying the applied frequency. Two single phase bridge inverters are designed to provide a variable frequency supply.

To achieve the most efficient operation of the induction motor the magnetizing flux should be maintained at its designed value. The applied voltage must be varied in proportion to a variation of the frequency; a constant volts/hertz ratio must be maintained. The average voltage input to the inverter can be varied using a Variac.

It has been established in chapter 3 that a 4-mode operation of the inverter provides a square wave output. Such a 4-mode operation can be established by utilizing logic circuits.

An astable multivibrator constitutes the variable frequency clock to the two bit register.

The firing circuits provide the necessary trigger signals for reliable operation of the SCR's. Pulse transformers are used for isolation and coupling of the trigger sources and the SCR gates.

When forced commutation of the SCR's is employed, additional circuitry is necessary to eliminate parasitic operation of the inverter. The negative voltage transient, appearing between the gate and the cathode of the SCR's, when transmitted to the UJT can cause erratic triggering. This situation is overcome by utilizing diode bridge circuits in the primaries of the pulse transformers.

It was concluded, at the end of chapter 3, that a 12 mode operation provides a more sinusoidal waveshape. A four bit register defines the 12 mode operation, which provides for sequential triggering of the inverter SCR's.

APPENDIX A

Output Power:

Consider a set of n mutually coupled circuits. The circuit equation is:

$$v = R\,i + \frac{d}{dt}\,L\,i$$

where $v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ $i = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix}$ $R = \begin{bmatrix} R_1 & & \\ & R_2 & \\ & & \ddots \; R_n \end{bmatrix}$ $L = \begin{bmatrix} L_1 & M_{12} \cdots & M_{1n} \\ M_{21} & L_2 \cdots & M_{2n} \\ M_{1n} & M_{2n} \cdots & L_n \end{bmatrix}$

The instantaneous electrical input power P_1 is:

$$P_1 = i_t\,v = i_t\left(R + \frac{d}{dt}L\right)i$$

$$P_1 = i_t\,R\,i + i_t\,L\,\frac{di}{dt} + i_t\,\frac{dL}{dt}\,i$$

The rate of dissipation of energy as heat by the circuit resistance is:

$$P_h = i_t\,R\,i$$

Energy storage $U_s = \tfrac{1}{2}\,i_t\,L\,i$

The rate of increase of stored energy is:

$$\frac{dU_s}{dt} = \tfrac{1}{2}\,i_t\,L\,\frac{di}{dt} + \tfrac{1}{2}\,i_t\,\frac{dL}{dt}\,i + \tfrac{1}{2}\,\frac{di_t}{dt}\,L\,i \qquad \text{-----}\,1$$

Each of the three terms in the expression is a scalar so that each may be transposed without affecting its value

$$\tfrac{1}{2}\,\frac{di_t}{dt}\,L\,i_{\;t} = \tfrac{1}{2}\,i_t\,L_t\,\frac{di}{dt} = \tfrac{1}{2}\,i_t\,L\,\frac{di}{dt} \qquad \text{-----}\,2$$

Substituting equation 2 in 1 yields:

$$\frac{dU_s}{dt} = i_t \, L \frac{di}{dt} + \frac{1}{2} \, i_t \frac{dL}{dt} \, i$$

The mechanical output power is:

$$P_{out} = P_l - P_h - \frac{dU_s}{dt}$$

$$P_{out} = \frac{1}{2} \, i_t \frac{dL}{dt} \, i$$