SOME THEORETICAL ASPECTS

OF THE OPTIMUM LOCATION OF THE FIRM

SOME THEORETICAL ASPECTS OF THE IMPACT OF SELECTED DEMAND AND TECHNOLOGICAL CONDITIONS ON THE OPTIMUM LOCATION OF THE FIRM

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SCOPE AND CONTENTS:

The Objective of this study is to investigate analytically the impact of certain technological and market conditions on the optimum location of the firm. The existing location models may be divided into those which consider both supply and demand aspects and those which concentrate on supply factors alone. Traditionally, the former group of models define equilibrium as the profit maximizing location and assume both a linear-homogeneous production function and a linear demand function. The latter class of models assume only the linear-homogeneity of production, and equilibrium is found at the cost-minimizing site.

In this paper two cases are examined. Firstly, the influence of a general non-linear homogeneous production function on a simple cost minimizing model is considered. Secondly, the effect of non-linear demand functions and non-linear homogeneous technology on a profit maximizing model are assessed. The results indicate that the optimum location in the cost minimizing situation does not vary with the level of output, whatever the degree of homogeneity of the production function. This

directly contradicts the common belief regarding the effects of production. Furthermore, in the profit-maximizing problem, and with non-linear homogeneous production, the solution is unaffected by the shape of the demand function.

Suggestions for refining and extending this analysis include the use of general rather than specific demand, transportation and production functions: the employment of exhaustible inputs, and generalization to the three-dimensional situation.

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CHAPTER I

INTRODUCTION
AND REVIEW OF EXISTING MODELS

Two schools of thought have developed in the location theory of the firm: the "point" location models, which assume that the market supplied by a producer may be represented by a point; and the "areal" models, which envisage a market distributed around the firm (see, for example, Churchill, 1966). The analysis undertaken below is concerned exclusively with the former school of thought. These models have traditionally focussed on the spatial behaviour of the individual firm, and are essentially normative in their structure; that is, they prescribe an optimum location for the firm arising from the maximization or minimization of some function. All proceed along the lines of partial rather than general equilibrium analysis, that is, they focus on a very small proportion of the pertinent relationships, and assume the others as given. (For a review of partial equilibrium in a spatial context, see Hoover, 1968). Successive relaxation of the assumptions hopefully brings the model closer to reality, and most theoretical research in point location analysis has proceeded along these lines.

The brief critical review of point location models which follows may be useful in highlighting some of their deficiencies and indicating the need for research of the type pursued in succeeding chapters.

1.1 The Classical Models

One may distinguish two phases in this line of inquiry. In the earlier phase the existence of a linear homogeneous production function is explicitly or implicitly accepted; in the latter phase the role of different technological conditions is discussed. The original concept stems from the work of Weber (1909) although some of his ideas may have been anticipated by Launhardt (1885). The Weberian theory is based upon three factors of location; transportation costs, labour costs, and agglomeration forces. The first two are fundamentally spatial in their impact; the third is primarily a technical consideration. Weber reduces the various determinants of the price of the finished product to transportation costs and labour costs alone. Initially, Weber assumes equal and constant labour costs everywhere and concentrates on the effect of transportation costs on the locational decision. Given a fixed market point C, and the location of the inputs, he forms a locational polygon. For simplicity we assume merely two factors of production, located at M_1 and M_2 respectively, yielding a locational triangle M_1M_2C.

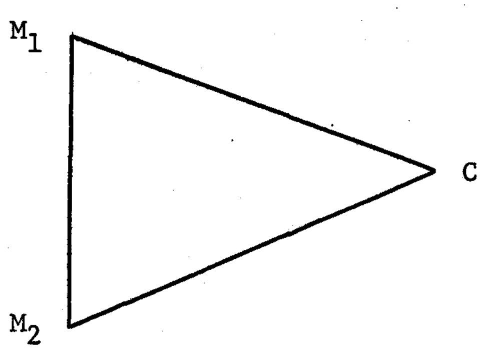

FIG. 1. The Weberian Location Triangle

If a uniform transport system and a homogeneous land surface are assumed then transportation costs will ultimately depend on weight and distance alone. Each apex of the triangle draws the location of the firm with a force proportionate to its own weight. The point of equilibrium represents the best location for production. Next, Weber relaxes the assumption of constant and equal labour costs. He conceives of labour costs as a point phenomenon, and so the location of lowest wages will not induce the firm to approach it, but rather offers itself as an alternative to the point of minimum transport costs. To determine which of the two points is preferable isodapanes (lines of equal unit freight charges) are constructed around the point of minimum transport costs. The value of the isodapane will naturally increase away from the transport cost minimum. Obviously a change will only take place if the savings in labour costs more than offset the increased transport costs. That is, if the optimum location from a labour cost standpoint lies on a lower isodapane than that on which labour savings equal transport costs, then the firm will be attracted there, and conversely. Agglomeration economies are treated in identical manner; a critical isodapane again being constructed where transport cost increases exactly cancel out gains from agglomeration.

Hoover (1937), considers the activities of the firm in three parts (A) the procurement of raw materials, (B) their processing, (C) the distribution of the product. The relative importance of these stages varies with the type of production involved. Procurement and distribution costs both vary systematically with distance from the raw material sources and the market, respectively, due to the role of transport costs. These costs generally increase with distance but less rapidly, giving a tapering effect, which is more pronounced where terminal charges are considerable.

Rates are frequently grouped by sections so that a step-like progression occurs as distance increases. When the volume of the shipment increases, transport costs per unit usually decrease, since terminal costs typically do not increase. To minimize the cost of procurement, better access to material sources is sought; to minimize the costs of distribution the firm will seek better access to the market. As these objectives may be opposed to each other, the problem rests in balancing them so that aggregate costs are minimized.

Hoover's strategy is as follows. Firstly, holding everything constant except transport costs, the relative attractive forces of the materials and markets are examined. If the product loses considerable weight in processing it is more likely to be located close to the material source. This latter also attracts the location, if for any of the reasons listed above, the procurement costs per ton-mile exceed the distribution costs. On the other hand, if there is weight gain during processing or if transport costs are higher per ton-mile for distribution, the plant will be oriented towards the market. As goods become more processed they usually become more fragile and hence more market oriented. To determine the loci of points of equal transport costs, Hoover introduces the concept of the "isotim" - a line joining points of equal delivered price. A continuous family of isotims surrounds the location of each input and the location of the market. Total transport costs at any point may be determined by summing the value of the three isotims running through that point. Points with equal isotim totals are connected by isodapanes and the minimum transport cost region will be contained by the lowest isodapane.

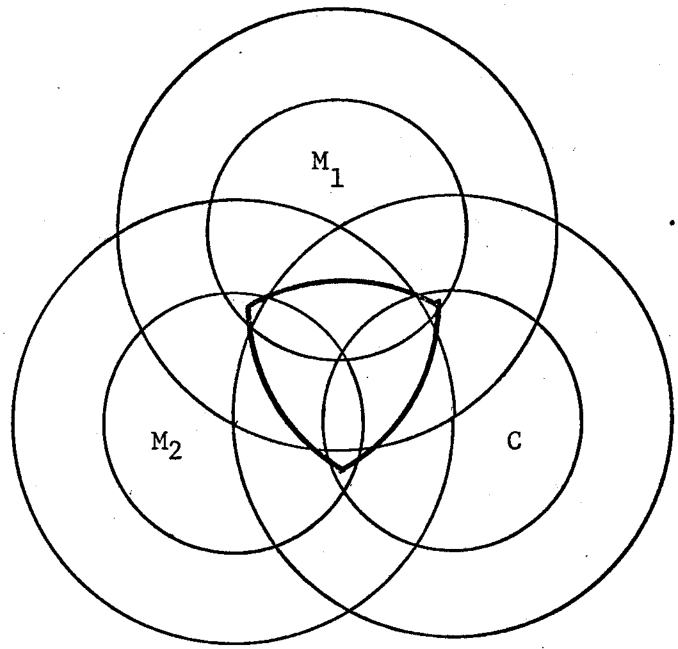

FIG. 2. The Hooverian Isodapane Map

Following this the market supply areas are considered. Assuming a standardised product, each market point will buy from whichever producer supplies it most cheaply. If production and distribution costs are everywhere equal, market areas will also be equal. If costs are higher at any given center the boundary will move closer to it. In certain industries transport costs vary little in comparison with operating cost, so the latter would appear to be a significant locating factor in these instances. Factor price differences may arise from immobility, especially as regards "land inputs". The appropriate combination of inputs at any given site will depend on the relative prices there. The optimum site regarding these aspects is then compared with the optimum as regards transport costs. The former will be preferred only if it lies on an isodapane lower than that on which savings from processing costs balance increased costs of transportation. A similar treatment is applied to economies arising from agglomeration.

Because of the broad similarity in the approaches of Hoover and Weber, the same criticisms may be applied to both. Isard (1956), Alonso (1967), and Greenhut (1967), among others, have pointed out that this approach is assymetric in that all location factors are defined in terms of cost. A profit-maximising equilibrium, will, of course, depend on the interaction of demand and supply. Secondly, the Weber-Hoover approach assumes that market price and quantity sold are known and fixed, and this requires a state of perfect competition at the market. Given that every product in a given industry is differentiated by its spatial origin from all others this assumption appears unreasonable. Isard (1956) has indicated that Weber has committed a logical error when he considers variations in transport costs. Such variations, Weber claims, may be reduced to variations in the weights and distances. However, the distances concerned are those from the proposed plant location to the raw material sources and the market, and calculating these presupposes that the plant has in fact been located, which is incorrect. Another error is Weber's assumption that when a factor is highly priced at a given site then that site may be considered farther away from the proposed location. The location of this input will to a large extent determine the least cost location itself. Predohl (1927) has shown that the Weberian scheme does not permit exhaustible resources, whose prices vary with the firm's consumption of them. A further defect is that once the assumption of fixed factor proportions (which implies that technology and factor endowment are constant through space) is removed, then the solution becomes indeterminant. Weber also utilises an unrealistic linear transport function. Both Weber and Hoover ignore any long run changes in the economic environment (e.g., technological progress) and their analyses are completely static in nature.

Isard (1956) attempts to synthesize the "point" and "areal" types of location models, again using the Weber-Hoover polygonal situation. He likewise begins by assuming all costs but transport, but he uses these in a substitution framework. He analyses transport orientation in two phases: (1) he assumes fixed factor proportions and constant market demand and so reduces all variations in transport inputs to variations in distance. (2) realising that the amounts of raw materials used may vary he frames the corresponding substitution analysis in terms of transportation inputs. In the first phase, if a single mobile unit m_1 is used, then location may be anywhere along M_1C, where C is the location of the market.

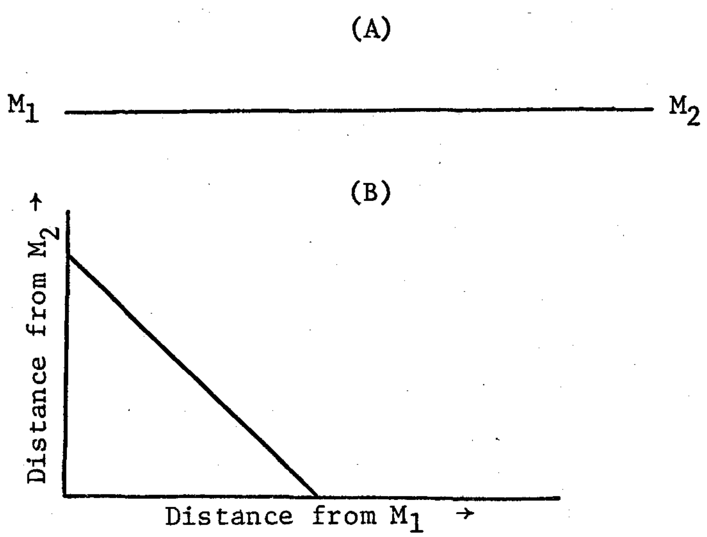

FIG. 3. (A) Single Input, (B) Two Input Situations in Isard's Model

The introduction of another material m_2 at M_2 in the process yields the locational triangle, and instead of a single transformation line there is a series of them.

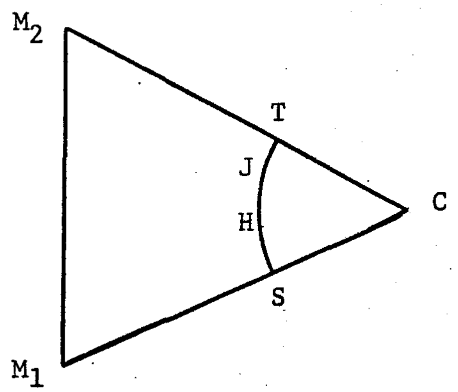

FIG. 4. Finite Location Possibilities in Isard's Locational Triangle

The transformation line TJHS between m_1 and m_2 is drawn on the basis that the site is to be located three miles from C. T,J,H, and S represent the finite number of possible locations along the arc TS. Since factors are used in fixed proportions, the relationship of freight rate to distance is the most important one. Assuming that freight rates are proportional to distance and that one ton of each material is used per unit of output then the iso-outlay curves EF, GK, LN, may be constructed.

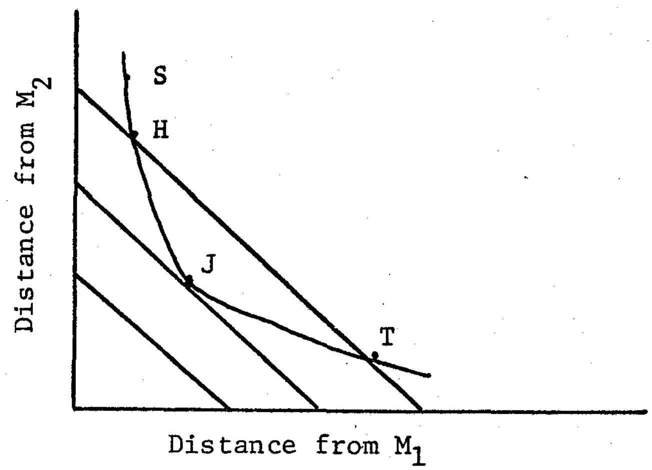

FIG. 5. Tangency of Location Arc TS to an Iso-Outlay Curve.

Since the transport rate on both inputs and final product is equal the price ratio lines for each must be straight and have a slope of -1. The tangency point J gives the proper location on arc TS, always assuming that the plant must be located three miles from C.

Allowing C to vary we can consider all conceivable transformation lines and their respective points of tangency. This is done by the partial equilibrium approach -- taking distance from M_2 consistent with point J as fixed, Isard constructs transformation lines for variable distance from M_2, and from C. Knowing the transport rate structure he can construct price ratio lines and determine the partial equilibrium for these two points. As a result the transformation line between the variables, distance from M_1 and distance from M_2, changes and therefore it may be necessary to find a new partial equilibrium with respect to these two variables. A full equilibrium is reached when partial equilibria between distance from C and M_1, distance from C and M_2, and distance from M_2 and M_1, all coincide.

Next Isard begins to introduce complexities, and he reframes the problem recognizing that usage of any given factor depends on the location of the plant. He uses transport inputs which encompass both the distance variable and the weight variable. He now assumes that the producing site is located 5 miles from C, and the feasible locations are A,B,D,E,F.

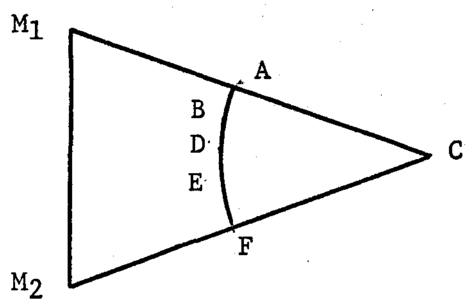

FIG. 6. Isard's Locational Triangle with Variable Factor Proportions

Since usage of a factor depends on the location of the plant, a location at A, for example, will employ more of m_1 than a location at F. The proportions of m_1 and m_2 required to produce 1 ton of output at each location may be calculated, as may the distance in miles from the alternate sites to m_1 and m_2. These may be combined to produce transport ton-miles. This information is plotted on transformation lines ABDEF.

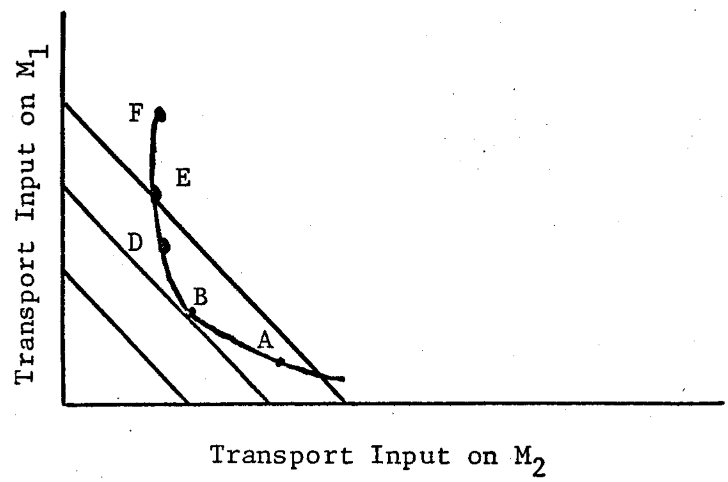

FIG. 7. Isard's Tangency Solution with Variable Proportions

Assuming that it costs the same to move 1 ton of m_1 and 1 ton of m_2, B is the point of minimum transport cost. When the weight ratio changes, the price ratio lines remain unchanged, and a new transport-input transformation line becomes relevant. A full equilibrium is again attained when the three partial equilibria, this time expressed in terms of transport inputs rather than distance, coincide.

The model proposed by Isard is again faulted on the issue that price differentials are assumed beyond the power of the firm, implying therefore that demand is infinitely elastic and the firm is a perfect competitor, which is essentially the Weber postulate. Like its predecessors, this model also assumes infinite factor supplies and ignores long-run or dynamic aspects of the location problem. As argued by Moses (1957), this theory is likewise based on the acceptance of a linear homogeneous production function, wherein substitution between the factor inputs is not permitted. The substitution which Isard is considering here is that between transport expenditures for inputs. This linearity assumption leads Isard to conclude that there is a single locational optimum, which occurs where the marginal rate of substitution between any two transportation inputs equals the reciprocal of the corresponding freight rates.

1.2 The Modern Models

Genuine reformulations of the Weber-type location problem begin with Moses (1958). This model again considers two transportable inputs located at M_1 and M_2, and a point form market at C. Initially location is restricted to the arc IJ, some fixed distance from C.

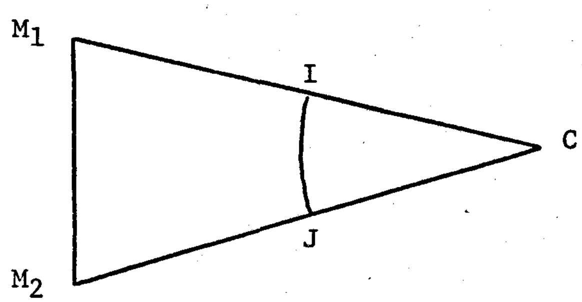

FIG. 8. Moses' Locational Triangle

If the firm locates at I, m_1 will be cheaper, while at J, m_2 is less expensive. The various combinations the producer can buy at I with his fixed income are given by the isocost function $m_1^I\ m_2^I$. The various combinations he can buy at J are represented by the isocost $m_2^J\ m_1^J$. If

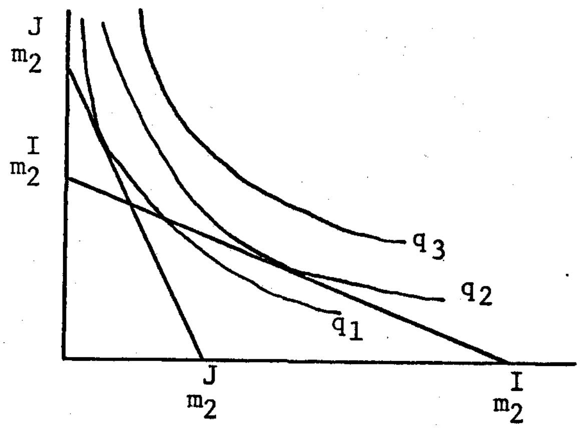

FIG. 9. Tangency Between the Isoquants and Fixed Isocosts in Moses' Model

q_1, q_2, q_3 are samples of isoquants from the firm's production function, then at a level of output corresponding to q_1, I is the optimum location, since q_1 is tangent to $m_1^J\ m_2^J$ but not $m_1^I\ m_2^I$. If, however, the level of output rises to q_2 then J becomes the optimum location since J's isocost curve is tangent to q_2. Now if location is possible anywhere along the arc IJ then the kinked line $m_1^J\ m_2^I$ will become a smooth curve, since each location will have its unique isocost, and therefore makes a unique tangency solution with the set of isoquants. Each point on this smoothed curve therefore corresponds to a particular location, and shows the combination of factors the firm will use at that location. If the level of expenditure is allowed to vary then a series of such smoothed isocosts

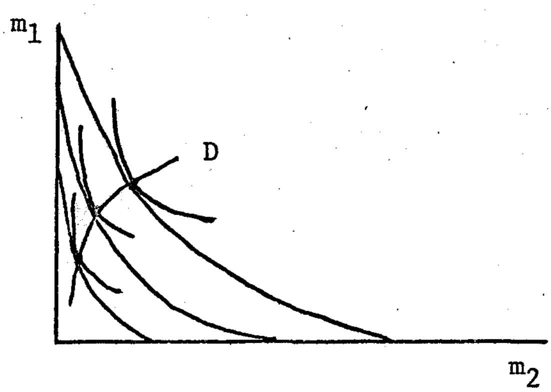

FIG. 10. Tangency Between Continuous Locational Isocosts and the Isoquants in Moses' Model

is generated. If the firm wishes to produce a given level of output, then optimum input proportions are indicated by the point of tangency of the relevant isoquant to one of the isocosts (e.g., D in Fig. 10). However, D now represents not just the input level, but also a particular location along IJ. This leads Moses to conclude "If the production function is not homogeneous of the first degree there is no single optimum location along the arc IJ. The optimum location varies with the level of output".

As Sakashita (1967) has pointed out, this conclusion is partially incorrect. The situation Moses envisaged when location does not change with output is actually one wherein the inputs are not substitutable, i.e., a fixed factor production function. The conclusion of this model is, therefore, not pertinent to the most common production function - the linear homogeneous <u>and</u> input substitutable one. The Moses model may also be criticised for ignoring market considerations. Although in the latter part of his article Moses adds the impact of transport costs to a linear demand function, he does not show in any analytical or potentially analytical sense how the firm incorporates a demand function in its optimum production decision, and hence in its locational decision.

Churchill (1966) attempts to remedy certain of the defects of the location models by 1) introducing more realistic transport costs; 2) by including imperfectly competitive factor markets; 3) by incorporating plant size as a decision variable; 4) by treating production technology directly; 5) by introducing monopolistic competition arising from product differences. He posits the familiar situation with two inputs V_1 and V_2 located at m_1 and m_2 respectively, and a market located at C.

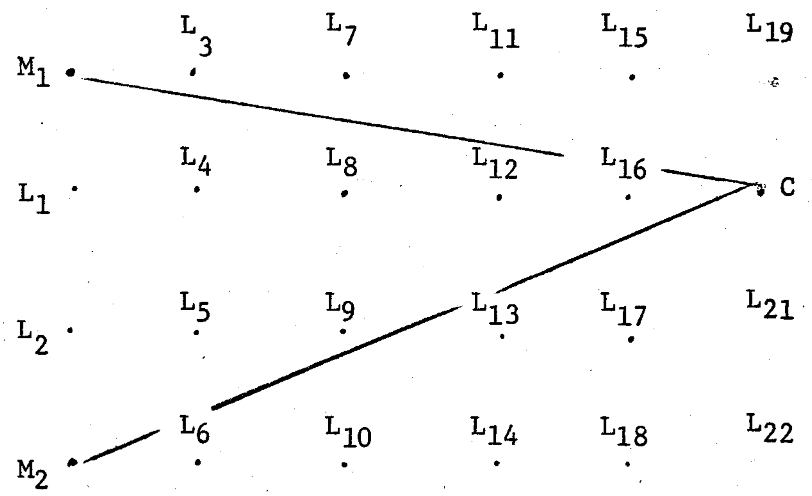

FIG. 11. Churchill's Location Map

He then expresses the cost of the inputs as functions of the quantities of them consumed, as well as their basic prices and transport costs. Similarily, he develops a transport function for the finished product which reflects volume and the tapering effect of distance as well as quantity shipped.

Churchill now comes to the crux of his argument, which is that before the physical plant is installed, both plant size and location are decision variables for the firm. This, he reasons, implies that location theorists should employ a stock-flow production function in which capital is treated explicitly as an input. The function he employs is of the form

$$q = V_1^{\alpha_1} V_2^{\alpha_2} V_3^{\alpha_3}$$

where q is output, V_1 and V_2 the current inputs, V_3 the capital inputs, and the α's are the output elasticities. Churchill claims that site to site productivity differences may be represented by changes in the α's, and returns to scale may be varied by changing $(\alpha_1 + \alpha_2 + \alpha_3)$. V_3 is assumed to be non-transportable, and available only at a finite number of points (the L's in Fig. 11). The Lagrangean L is then formed from the cost and production functions, where the P's are the unit prices of the

$$L = P_1V_1 + P_2V_2 + P_3V_3 + \lambda[q - V_1^{\alpha_1} V_2^{\alpha_2} V_3^{\alpha_3}]$$

inputs. The partial derivatives of L with respect to the inputs are equal to zero, the resulting simultaneous equations are solved, and the cost-output expression is obtained.

Next Churchill assumes that the demand for the product at the market may be represented by a linear function

$$P = a - bq \quad .$$

From this the total revenue function (Pq) is readily constructed. The profit equation (total revenue minus total cost) then follows automatically, and this relates the level of output to profit at each particular site. The maximum profits available at each site are then compared to discover the maximum maximorum.

Apart from including a better treatment of transport costs and factor prices, this model is open to the same criticisms as its predecessors. The idea that plant size and location are variable for the firm in the long run is very correct, and undoubtedly this factor should be considered in location theory, but the manner in which Churchill tries to incorporate this idea in his theory is highly questionable. The use of a finite number of possible locations is unnecessary and destroys the elegance of the analysis. Since his objective is to find the long run cost curve, the correct method would appear to be through constructing the mathematical envelope of the short-run cost curves, which may then be used in conjunction with the total revenue equation to find the long run profit maximizing location.

Sakashita (1967) states his objective as that of investigating the case of an input substitutable and linear homogeneous production function in the location problem. He employs two models, a cost-minimizing one and a profit-maximizing one. In the former he uses an even simpler situation than is generally assumed. Two inputs V_1 and V_2, are located a fixed distance s apart. Cost is defined in terms of the distance which each input must be carried to the chosen location, an unknown distance x from m_2. Given the transport rates on the inputs as m_1 and m_2 respectively, and their base prices as r_1 and r_2, we may form the cost equation for the firm.

$$c = (r_1 + m_1(s - x))V_1 + (r_2 + m_2 x)V_2 \quad .$$

Given a linear homogeneous production function $q = f(V_1, V_2)$ then the Lagrangean is formed

$$L = f(V_1, V_2) + \lambda[c - (r_1 + m_1(s - x))]V_1 - [(r_2 + m_2 x)V_2]$$

which may be solved for the optimum factor proportions, eventually getting V_1 and V_2 in terms of x. From this he is able to minimise c in terms of x -- that is, to find the cost-minimizing location between M_1 and M_2. He discovers that the optimised cost function is strictly concave with respect to x. This means that the preferred location will always be at one of the input sources in this class of problems. Furthermore, he concludes that the optimum location does depend on the base prices of the inputs, and is not affected by the level of output.

The second model proposed by Sakashita is a profit maximizing one. The market for the final product is concentrated at some point C, V_1 is still obtainable only at M_1, but V_2 is available everywhere along CM_1. C and M_1 are a distance s apart, and the distance between the firm's location and the market is the unknown x. Given some demand function P(q) for the product and its transport rate h, the firm's profit function is

$$\pi = [P(q) - hx]q - [(r_1 + m_1(s - x)]V_1 - r_2V_2 .$$

Maximization of this function with respect to x leads to the conclusion that the optimised profit function is strictly convex with respect to x, which implies that intermediate locations are excluded. As before, he shows that the optimum location does depend on the prices of the inputs, and in addition, the shape and position of the demand function does not influence the profit-maximizing site.

The two models of Sakashita are both liable to criticisms of the sort levied by Churchill, eg., unrealistic transport rates and factor prices, no accounting for plant size, etc. However, they do appear to be the most promising models so far developed in this approach. Their usefulness lies not in any new insight they have given location theory, (in fact, they are little more than rigorous restatements of pre-existing models), but rather in their structure, which reveals avenues for future research far more explicitly than did that of their predecessors.

1.3 Summary and Conclusions

The development of the classical location problem proceeded along the lines of sophisticated redefinitions of the Weber problem up to the contribution by Moses. He made the critical connection between the theory of production and the theory of location. Since then, efforts have been directed at introducing standard microeconomic notions to the location problem and re-interpreting their significance from the heretofore neglected spatial perspective. One of the most recent investigations has been that of Sakashita, who has formulated a simple theory using the most restrictive assumption about production and demand (linear-homogeneity and linearity, respectively). This paper attempts to continue this modern tradition by using some common economic analysis to examine the role of different technological and market conditions in a Sakashita-type model.

CHAPTER II

THE NON-LINEAR COBB-DOUGLAS COST-MINIMIZATION LOCATION MODEL

2.1 Introduction

Sakashita (1967) indicates Moses' error in assuming that a linear homogeneous production function with fixed input coefficients is necessary to ensure that the optimum location of the firm depends only on transport costs and the location of the factor inputs. Moses claims that if production conditions are otherwise, several other influences, including the level of output, will come to bear upon the optimum plant site. Sakashita proceeds to demonstrate that the fixed coefficient assumption is not essential, and he derives an optimum location determined by transport rates and input locations on the basis of a linear-homogeneous and factor-substitutable production function. In this chapter the effect of variable homogeneity in the production function is investigated to determine whether the linearity assumption of Sakashita, Moses, and others is necessary for reaching these conclusions.

2.2 The Model

Assume the production function

$$q = V_1^{\alpha} V_2^{\beta} \qquad 1 > \alpha, \beta > 0 \qquad (1)$$

where q represents outputs, V_1 and V_2 are the inputs, and α and β are the technological parameters (more precisely, α and β are the input elasticities). This is the general form of the Cobb-Douglas production function, which is homogeneous of any degree $(\alpha + \beta)$ (see Ferguson 1969).

The inputs V_1 and V_2 are located some arbitrary distance s miles apart at M_1 and M_2 respectively. r_1 and r_2 are their base prices, and m_1 and m_2 are their corresponding transport rates.

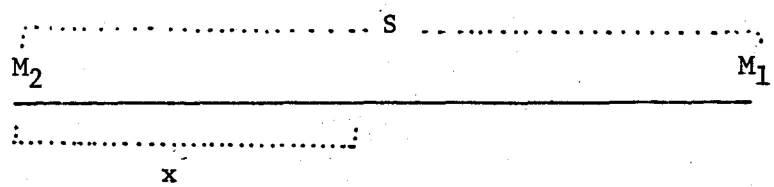

FIG. 12. The Locational Cost Minimising Situation

The plant is located at some unknown distance x from M_2. The cost function (c) of the firm associated with a same level of output q will, therefore, be

$$c = (r_1 + m_1(s - x))V_1 + (r_2 + m_2)V_2 \quad . \tag{2}$$

Firstly, we wish to establish the optimum input ratio at any location. To this end we form the Lagrangean (L)

$$L = V_1^{\alpha} V_2^{\beta} + \lambda\{c - (r_1 + m_1(s - x))V_1 - (r_2 + m_2x)V_2\} \quad , \tag{3}$$

therefore,

$$L_1 = \alpha q V_1^{-1} - (r_1 + m_1(s - x)) = 0 \quad , \tag{4}$$

$$L_2 = \beta q V_2^{-1} - (r_2 + m_2 x) = 0 \quad . \tag{5}$$

Consequently,

$$\frac{\alpha}{\beta}\,\frac{V_2}{V_1} = \left[\frac{r_1 + m_1(s - x)}{r_2 + m_2 x}\right] \quad , \tag{6}$$

$$V_2 = \left[\frac{r_1 + m_1(s - x)}{r_2 + m_2 x}\right] \frac{\beta}{\alpha}\, V_1 \quad , \tag{7}$$

$$V_1 = \left[\frac{r_2 + m_2 x}{r_1 + m_1(s - x)}\right] \frac{\alpha}{\beta}\, V_2 \quad . \tag{8}$$

From (1)

$$q = V_1^{\alpha}\, V_2^{\beta} = V_1^{\alpha + \beta} \left[\frac{r_1 + m_1(s - x)}{r_2 + m_2 x}\right]^{\beta} \frac{\beta}{\alpha}^{\beta} \quad , \tag{9}$$

$$V_1 = (q)^{\frac{1}{\alpha + \beta}} \left[\frac{\beta}{\alpha} \cdot \frac{r_2 + m_2 x}{r_1 + m_1(s - x)}\right]^{\frac{\beta}{\alpha + \beta}} \quad . \tag{10}$$

Similarly,

$$V_2 = (q)^{\frac{1}{\alpha + \beta}} \left[\frac{\alpha}{\beta} \; \frac{r_1 + m_1(s - x)}{r_2 + m_2 x} \right]^{\frac{\alpha}{\alpha + \beta}} . \tag{11}$$

We wish to minimize c with respect to x, so from (2),

$$\frac{dc}{dX} = \frac{\partial c}{\partial X} = - m_1 V_1 + m_2 V_2 \qquad \text{(Samuelson, 1948).} \tag{12}$$

From (10), (11), and (12),

$$\frac{dc}{dX} = - m_1 (q)^{\frac{1}{\alpha + \beta}} \left[\frac{\beta}{\alpha} \cdot \frac{r_2 + m_2 x}{r_1 + m_1(s - x)} \right]^{\frac{\beta}{\alpha + \beta}}$$

$$+ m_2 (q)^{\frac{1}{\alpha + \beta}} \left[\frac{\alpha}{\beta} \cdot \frac{r_1 + m_1(s - x)}{r_2 + m_2 x} \right]^{\frac{\alpha}{\alpha + \beta}} . \tag{13}$$

$$\frac{d^2 c}{dX^2} = - m_1 (q)^{\frac{1}{\alpha + \beta}} \left[\frac{\beta}{\alpha} \right]^{\frac{\beta}{\alpha + \beta}} \left[\frac{(r_1 + m_1(s - x))m_2 + (r_2 + m_2 x)m_1}{(r_1 + m_1(s - x))^2} \right]$$

$$\cdot \frac{\beta}{\alpha + \beta} \left[\frac{r_2 + m_2 x}{r_1 + m_1(s - x)} \right]^{\frac{\beta}{\alpha + \beta} - 1} + m_2 (q)^{\frac{1}{\alpha + \beta}} \left[\frac{\alpha}{\beta} \right]^{\frac{\alpha}{\alpha + \beta}}$$

$$\cdot \frac{\alpha}{\alpha + \beta} \left[\frac{r_1 + m_1(s - x)}{r_2 + m_2 x} \right]^{\frac{\alpha}{\alpha + \beta} - 1}$$

$$- \left[\frac{(r_2 + m_2 x)m_1 - (r_1 + m_1(s - x))m_2}{(r_2 + m_2 x)^2} \right] . \tag{14}$$

That is,

$$\frac{d^2c}{dx^2} = -K_1 \left[\frac{(r_1 + m_1(s - x))m_2 + (r_2 + m_2x)m_1}{(r_1 + m_1(s - x))^2}\right] \cdot \left[\frac{r_2 + m_2x}{r_1 + m_1(s - x)}\right]^{\frac{\beta}{\alpha + \beta} - 1}$$

$$- K_2 \left[\frac{(r_2 + m_2x)m_1 + (r_1 + m_1(s - x))m_2}{(r_2 + m_2)^2}\right] \cdot \left[\frac{r_1 + m_1(s - x)}{r_2 + m_2x}\right]^{\frac{\alpha}{\alpha + \beta} - 1} . \tag{15}$$

Since α, β, q, m_1, and m_2 are all positive, we may conclude that

$$\frac{d^2c}{dx^2} < 0 \qquad \text{for all} \quad 0 \leq x \leq s . \tag{16}$$

Equation (16) implies that the locational cost function is concave, and the cost-minimizing location must, therefore, be at one of the terminals.

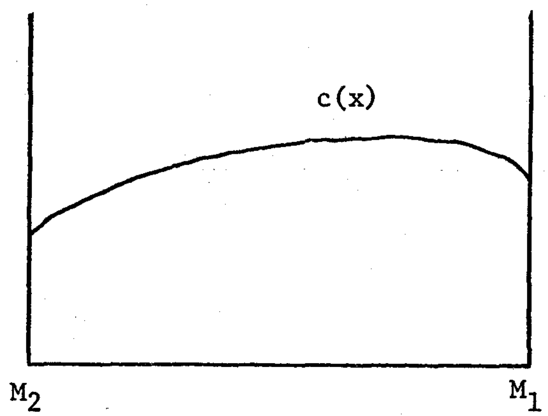

FIG. 13. The Optimized Location Cost Function

The problem now rests in choosing between the two input sites.

From (12),

$$\frac{dc}{dx} = (-\, m_1 V + m_2) V_2$$

where

$$V = \frac{V_1}{V_2} \quad .$$

Let

$$(-\, m_1 V + m_2) = \phi \tag{17}$$

From (6),

$$V = \frac{\alpha}{\beta} \left[\frac{r_2 + m_2 x}{r_1 + m_1 (s - x)} \right] , \tag{18}$$

therefore,

$$\phi = - m_1 \frac{\alpha}{\beta} \left[\frac{r_2 + m_2 x}{r_1 + m_1 (s - x)} \right] + m_2 \qquad . \tag{19}$$

Consequently,

$$\phi(0) = - m_1 \frac{\alpha}{\beta} \left[\frac{r_2}{r_1 + m_1 s} \right] + m_2 \qquad , \tag{20}$$

$$\phi(s) = - m_1 \frac{\alpha}{\beta} \left[\frac{r_2 + m_2 s}{r_1} \right] + m_2 \qquad . \tag{21}$$

If $\phi(0) \leq 0$, then the locational cost function will be as shown in Fig. 14, and the optimum location will be at M_1.

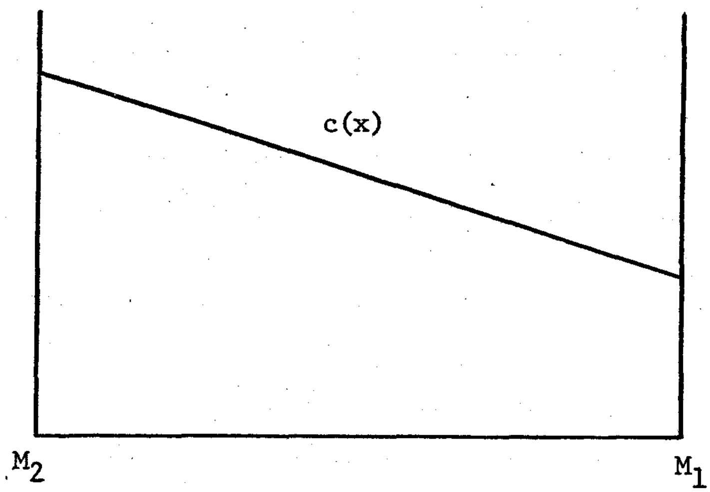

FIG. 14. Optimized Locational Cost Function where $\phi(s) \geq 0$

If $\phi(s) \geq 0$, then c(x) behaves as in Fig. 15, and M_2 is the optimum solution.

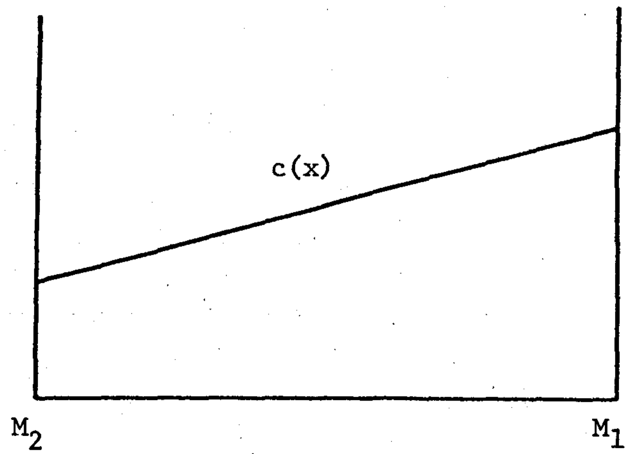

FIG. 15. Optimized Locational Cost Function where $\phi(s) \geq 0$

If $\phi(0) > 0$ and $\phi(s) < 0$, then the solution is not readily apparent, as may be seen from Fig. 16.

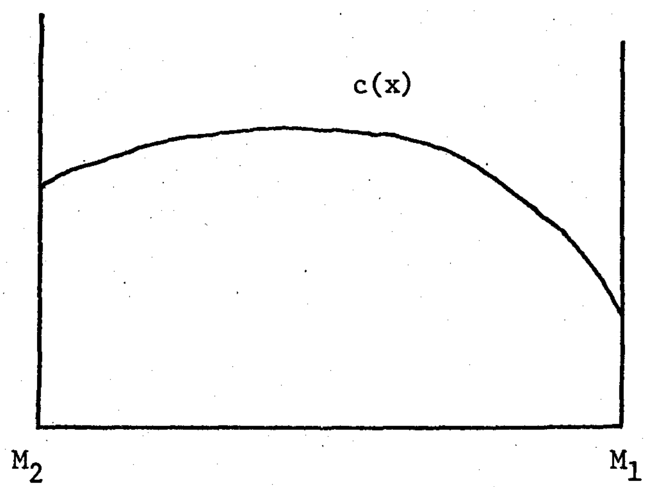

FIG. 16. Optimized Locational Cost Function where $\phi(0) > 0$, $\phi(s) < 0$

To deduce the superior site we must compare the average cost (AC) obtaining at both

$$AC(0) = \frac{c(0)}{q} = \frac{(r_1 + m_1(s - x))V_1 + r_2V_2}{V_1^{\alpha}\ V_2^{\beta}} \quad . \quad (22)$$

$$AC(s) = \frac{c(s)}{q} = \frac{r_1V_1 + (r_2 + m_2s)V_2}{V_1^{\alpha}\ V_2^{\beta}} \quad . \quad (23)$$

2.3 Conclusions

As with Sakashita, the optimum location must be at one of the terminals, there is no possibility of an intermediate location. The optimum location does depend on the base factor prices and transport costs, and considerations of the level of output, returns to scale, etc., have no impact upon the cost minimizing location. These conclusions have thus been shown by the above analysis to extend to the case of a non-linear homogeneous production function, which is a direct refutation of Moses' claim that the optimum location varies with output if the production function is not homogeneous of degree one.

CHAPTER III

PROFIT MAXIMIZING MODEL

3.1 Introduction

In his second model, Sakashita considers the firm in a somewhat different linear spatial context. One input (V_1) is obtainable only at location M_1 as before, while the other is available at any point, effectively reducing transport costs for V_2 to zero. At some point C, representing the market, the firm is faced with a demand function p(q) in which market price in some way depends upon quantity sold. The symbol s now represents the distance between the fixed input and the market, and x

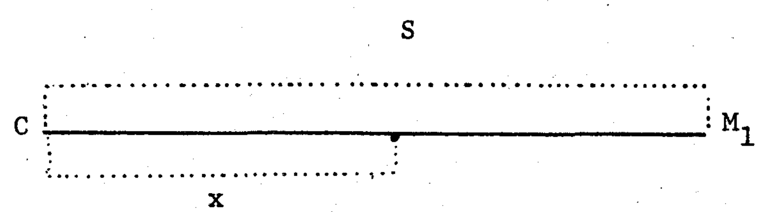

FIG. 17. The Profit Maximizing Situation

represents the distance separating the unknown location of the plant and the market. In his example, Sakashita uses a linear demand function with a linear homogeneous production function, and concludes that under these circumstances the optimum location will be independent of the demand conditions. However, he also claims that once the linear-homogeneity assumption is relaxed, demand conditions do exert an influence on the firm's decision.

3.2 The Model

It is consequently proposed to test this latter assertion by using the h-homogeneous production function employed in the previous chapter with a simple demand relationship of the form

$$p = a - bq \qquad a, b > 0 \tag{24}$$

Since

$$\pi = pq - c$$

where π is profit and c is total cost,

then

$$\pi = (a - bq)q - [r_1 + m_1(s - x)]V_1 - r_2V_2 \quad . \tag{25}$$

That is

$$\pi = (a - bq)q - \frac{r_2}{\beta}(\alpha + \beta)\left[\frac{\beta}{\alpha}\,\frac{r_1 + m_1(s - x)}{r_2}\right]^{\frac{\alpha}{\alpha + \beta}} \cdot q^{\frac{1}{\alpha + \beta}} \quad . \tag{26}$$

Now let us assume for convenience, the special case of decreasing returns to scale where $\alpha + \beta = {}^1/_2$.

i.e.

$$\pi = (a - bq)q - \frac{r_2}{2\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{2\alpha} q^2 \qquad (27)$$

(Notice in the above that the cost of transporting the final product to the market from the plant has not been incorporated in the cost function; it is not considered critical to the analysis and may, it is felt, be omitted without altering the logic of the argument). The general conditions for a maximum of the profit function at any location are

$$\pi_q = 0, \qquad \pi_{qq} < 0.$$

From (27),

$$\pi_q = a - 2bq - \frac{r_2}{\beta}\, q \left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{2\alpha} = 0 \quad , \qquad (28)$$

$$\pi_{qq} = -\, 2b - \frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{2\alpha} . \qquad (29)$$

Since $a, b, r_2, \beta > 0$, it is reasonable to assume that the maximising conditions are fulfilled. We thus have a maximising value of q for a given x, and the problem is to find the optimum x.

$$\pi_x = m_1 \left[\frac{\beta}{\alpha} \; \frac{r_1 + m_1(s - x)}{r_2} \right]^{2\alpha - 1} q^2 \qquad , \qquad (30)$$

$$\pi_{xx} = -(2\alpha - 1)m_1 \left[\frac{\beta}{\alpha} \; \frac{r_1 + m(s - x)}{r_2} \right]^{2\alpha - 2} \frac{\beta}{\alpha}\frac{m_1}{r_2}$$

$$+ \, m_1 \left[\frac{\beta}{\alpha} \; \frac{r_1 + m_1(s - x)}{r_2} \right]^{2\alpha - 1} 2q \frac{dq}{dx} \qquad . \qquad (31)$$

From (28),

$$q = \left[\frac{a}{2b + \frac{r_2}{\beta}\frac{\beta}{\alpha}\left[\frac{r_1 + m_1(s - x)}{r_2}\right]^{2\alpha}} \right] \qquad , \qquad (32)$$

therefore,

$$\frac{dq}{dx} = -\, a \quad 2b + \frac{r_2}{\beta} \left[\frac{\beta}{\alpha} \; \frac{r_1 + m_1(s - x)}{r_2} \right]^{2\alpha \; -2} \cdot \frac{-m_1}{\alpha} \; . \qquad (33)$$

Since $2\alpha < 1$ given $\alpha + \beta = {}^1/_2$ and $\alpha,\beta > 0$, then we conclude that

$$\pi_{xx} > 0 \qquad .$$

This implies a strictly convex profit function with respect to distance from the market. If $\frac{d\pi}{dx} > 0$ at $x = 0$, then m_1 is the preferred location. If $\frac{d\pi}{dx} < 0$ at $s = x$ then c is the optimum.

From (32),

$$q(0) = \left[\frac{a}{2b + \frac{r_2}{\beta} \; \frac{\beta}{\alpha} \left[\frac{r_1 + m_1 s}{r_2} \right]^{2\alpha}} \right] , \tag{34}$$

$$q(s) = \left[\frac{a}{2b + \frac{r_2}{\beta} \left[\frac{\beta}{\alpha} \; \frac{r_1}{r_2} \right]^{2\alpha}} \right] . \tag{35}$$

$$\text{Since } \pi(0) = (a - bq)q - \frac{r_2}{2\beta} \left[\frac{\beta}{\alpha} \; \frac{r_1 + m_1 s}{r_2} \right]^{2\alpha} q^2 , \tag{36}$$

$$\text{therefore } \pi(0) = a^2 \left[\frac{1}{2b + \frac{r_2}{\beta} \left[\frac{\beta}{\alpha} \; \frac{r_1 + m_1 s}{r_2} \right]^{2\alpha}} \right] - {}^1/_2 . \tag{37}$$

$$\text{Similarily, } \pi(s) = a^2 \left[\frac{1}{2b + \frac{r_2}{\beta} \left[\frac{\beta}{\alpha} \; \frac{r_1}{r_2} \right]^{2\alpha}} \right] - {}^1/_2 . \tag{38}$$

If $\frac{d\pi}{dx} < 0$ at $x = 0$ and $\frac{d\pi}{dx} > 0$ at $x = 3$, then the optimum location depends on which of (37) and (38) is the larger. This essentially implies comparing

$$\frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1 s}{r_2}\right]^{2\alpha} \qquad \text{with} \qquad \frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1}{r_2}\right]^{2\alpha} .$$

Therefore, we conclude that selection of the preferred site is independent of the demand conditions.

3.3 Conclusions

Sakashita states: "Since the total delivery cost is a linear function of q [in the linear homogeneous case] a homogeneity assumption about the production function other than a linear one, will change this result, i.e., the demand conditions are generally relevant to the locational decision unless a linear homogeneous production function is assumed by contrast". The above analysis demonstrates that, at least in the instance of a production function homogeneous of degree ($^1/2$), the demand conditions are also irrelevant to the locational decision.

CHAPTER IV

THE ROLE OF DEMAND FACTORS IN THE PROFIT-MAXIMISING MODEL

4.1 Introduction

In the preceding chapter the optimum location of the firm in a linear situation is discussed using a linear demand function. In this section the spatial impact of various non-linear classes is examined within the same model. It is emphasized that this is not a treatment of spatially induced demand as envisaged by location theorists from Lösch (1938) to Long (1970), all of which are concerned with an areal distribution of consumers and the effect of this on the product demand, and consequently the location of the firm. What we are concerned with here is essentially a non-spatial demand at single market point. The different demand functions, therefore, correspond to different assumptions about the consumers' preference for the particular good being marketed rather than their attitude to the disutility of travelling or paying delivery costs to obtain it. The impact of demand in such a situation is, therefore, indirect (if it exists at all), operating through the constrained optimization process on the production decision and thence on the locational choice.

The procedure adopted in this chapter is to select certain of those demand curves discussed by Stevens and Rydell (1966) and apply them to the profit-maximising model of Chapter III, using the most convenient assumption about the homogeneity of the production function in an attempt to derive an optimal location dependent upon the demand parameters.

4.2 Selected Demand Functions

From the seven demand functions discussed by Stevens and Rydell, the linear example is identical to that utilised in the above chapter. From the remainder, the convex, concave, and rectangular-hyperbola types are chosen for study in this context. A demand curve is defined as convex if and only if

$$q''(P) > 0 \quad .$$

The condition is satisfied by the inverse function

$$P = bq^{1/2} \quad ; \; b > 0 \quad ,$$

$$q''(P) = \frac{2}{b^2} \quad .$$

A concave demand function exists where

$$q''(P) < 0 \quad .$$

This is satisfied by

$$P = a - bq^2 \quad ; \; a,b, > 0 \quad ,$$

since

$$q'' = \frac{-1}{2b}\left[\frac{a - P}{b}\right]^{-1/2} .$$

A demand schedule which is a rectangular hyperbola is defined where

$$q''(P) = \frac{2[q'(P)]^2}{q} .$$

4.3 The Convex Demand Model

Given $P = bq^{1/2}$, then the firm's profit function is

$$\pi = bq^{3/2} - \frac{r_2}{\beta}(\alpha + \beta)\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{\frac{\alpha}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}} . \quad (39)$$

Assuming $(\alpha + \beta) = {}^1/_3$, then

$$\pi = bq^{3/2} - \frac{r_2}{3\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha} q^3 . \quad (40)$$

$$\pi_q = \frac{3}{2}\,bq^{1/2} - \frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha} q^2 = 0 . \quad (41)$$

Therefore,

$$q^{3/2} = \frac{3b}{2}\frac{\beta}{r_2}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{-3\alpha} , \quad (42)$$

$$q = \frac{3b}{2}\frac{\beta}{r_2}^{2/3}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{-2\alpha} . \quad (43)$$

Therefore,

$$\frac{dq}{dx} = \frac{2m_1\beta}{r_2}\left[\frac{3b}{2}\frac{\beta}{r_2}\right]^{2/3}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{-2\alpha-1} > 0. \quad (44)$$

From (40), and (43),

$$\pi_x = q^3 m_1\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha-1} . \quad (45)$$

$$\pi_{xx} = 3q^2\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha-1} m_1\frac{dq}{dx}$$

$$- (3\alpha-1)m_1 q^3\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha-1}\frac{\beta}{\alpha}\frac{m_1}{r_2} . \quad (46)$$

Since $(\alpha_1\beta) > 0$ and $(\alpha + \beta) = {}^1/_3$, then $(3\alpha-1)$ must be negative and so

π_{xx} is positive. This implies, as in the linear demand case discussed in Chapter III, that the optimised profit function of the firm is strictly convex with respect to x, and the selection of the best location returns once again to a choice between the market and the source of m_1. This again involves comparing the profits obtainable at each of the two. From (40), and (43),

$$\pi = b\frac{3b}{2}\frac{\beta}{r_2}\left[\frac{\beta}{\alpha}\frac{r_1 + m_1(s-x)}{r_2}\right]^{-3\alpha} - \frac{r_2}{3\beta}\left[\frac{\beta}{\alpha}\frac{r_1 + m_1(s-x)}{r_2}\right]^{3\alpha}$$

$$\cdot\left[\frac{3b}{2}\frac{\beta}{r_2}\right]^{2}\frac{\beta}{\alpha}\left[\frac{r_1 + m_1(s-x)}{r_2}\right]^{-6\alpha}$$

$$= \left[\frac{\beta}{\alpha}\frac{r_1 + m_1(s-x)}{r_2}\right]^{-3\alpha}\frac{3b^2}{4}\frac{\beta}{r_2} \quad . \quad (47)$$

$$\pi(0) = \left[\frac{\beta}{\alpha}\frac{r_1 + m_1(s-x)}{r_2}\right]^{-3\alpha}\frac{3b^2\beta}{4r_2} \quad . \quad (48)$$

$$\pi(x) = \left[\frac{\beta}{\alpha}\frac{r_1}{r_2}\right]^{-3\alpha}\frac{3b^2\beta}{4r_2} \quad . \quad (49)$$

As before, the decision reduces to comparing the relative magnitude of

$$\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{-3\alpha} \quad \text{and} \quad \left[\frac{\beta}{\alpha}\;\frac{r_1}{r_2}\right]^{-3\alpha} ,$$

and the demand characteristics drop out of consideration.

4.4 The Concave Demand Model

Given $P = (a - bq^2)$,

Then we can calculate profits,

$$\pi = aq - bq^3 - \frac{r_2}{\beta}(\alpha+\beta)q^{\frac{1}{\alpha+\beta}} \left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{\frac{\alpha}{\alpha+\beta}} . \quad (50)$$

Assuming $(\alpha + \beta) = {}^1/_3$, then

$$\pi = aq - bq^3 - \frac{r_2}{3\beta}\, q^3 \left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha} . \quad (51)$$

$$\pi_q = a - 3bq^2 - \frac{r_2}{3\beta}\, q^3 \left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha} . \quad (52)$$

Therefore,

$$q = a^{1/2} \left[3b + \frac{r_2}{\beta} \left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}\right]^{-1/2} \quad (53)$$

$$\frac{dq}{dx} = \frac{3a^{1/2}}{2} m_1 \left[3b + \frac{r_2}{3\beta}\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}\right]^{-3/2} > 0 \quad (54)$$

From (51),

$$\pi_x = q^3 m_1 \left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha-1} , \quad (55)$$

$$\pi_{xx} = 3q^2 m_1 \frac{dq}{dx}\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha-1} - q^3 m_1(3\alpha-1)\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha-2} \frac{\beta}{\alpha}\frac{m_1}{r_2} . \quad (56)$$

Given $(\alpha + \beta) = {}^1/_3$, then $\alpha < {}^1/_3$ and $(3d-1) > 0$, therefore since $\frac{dg}{dx} > 0$, we conclude $\pi_{xx} > 0$. Again we have a strictly convex optimised function and the preferred location is an end point.

From (51) and (53),

$$\pi = a - b\left[a^{1/2}\left[3b + \frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}\right]^{-1/2}\right]^2 . \ a^{1/2}\left[3b + \frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}\right]^{-1/2}$$

$$- \frac{r_2}{3\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}$$

$$a^{3/2}\left[3b + \frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 - m_1(s - x)}{r_2}\right]^{3/2}\right]^{-3/2} , \quad (57)$$

$$= \; a^{3/2}\left[3b + \left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}\right]^{-1/2}$$

$$- \, {}^{1}/_{3}\left[3b + \frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}\right]$$

$$\cdot \; a^{3/2}\left[3b + \frac{r_2}{\beta}\left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}\right]^{-3/2} . \quad (58)$$

$$= \left[3b + \left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1(s - x)}{r_2}\right]^{3\alpha}\right]^{-1/2} {}^{2}/_{3}a^{3/2}\left[\frac{2a}{3}\right]^{3/2} . \quad (59)$$

The profits at the fixed input source and the market are respectively

$$\pi(x) \; = \; \left[3b + \left[\frac{\beta}{\alpha}\;\frac{r_1 + m_1 s}{r_2}\right]^{3\alpha}\right]^{-1/2} \left[\frac{2a}{3}\right]^{3/2} , \quad (60)$$

$$\pi(0) \; = \; \left[3b + \left[\frac{\beta}{\alpha}\;\frac{r_1}{r_2}\right]^{3\alpha}\right]^{-1/2} \left[\frac{2a}{3}\right]^{3/2} . \quad (61)$$

As before, this reduces to comparing

$$\left[\frac{\beta}{\alpha}\ \frac{r_1}{r_2}\right]^{3\alpha} \quad \text{and} \quad \left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1 s}{r_2}\right]^{3\alpha} .$$

the demand factors play no part in the selection process.

4.5 The Rectangular Hyperbola Demand Model

Given $P = b + aq^{-1}$, then

$$\pi = bq + a - \frac{r_2}{\beta}(\alpha+\beta)\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{\frac{\alpha}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}} . \quad (62)$$

Assuming $(\alpha + \beta) = {}^1/2$, then

$$\pi = bq + a - \frac{r_2}{2\beta}\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{2\alpha} q^2 . \quad (63)$$

$$\pi_q = b - \frac{r_2}{\beta} q \left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{2\alpha} = 0 , \quad (64)$$

therefore,

$$q = \frac{b\beta}{r_2}\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{-2\alpha-1} \cdot \frac{\beta}{\alpha}\frac{m_1}{r_2} > 0 . \quad (65)$$

$$\pi_x = m_1\left[\frac{\beta}{\alpha}\ \frac{r_1 + m_1(s - x)}{r_2}\right]^{2\alpha-1} q^2 . \quad (66)$$

$$\pi_{xx} = - m_1^2 (2\alpha - 1) \left[\frac{\beta}{\alpha} \frac{r_1 + m_1(s - x)}{r_2} \right]^{2\alpha-2} \cdot q^2 \frac{\beta}{\alpha} \frac{1}{r_2}$$

$$+ m_1 \left[\frac{\beta}{\alpha} \frac{r_1 + m_1(s - x)}{r_2} \right]^{2\alpha-1} \cdot 2q \frac{dq}{dx} \quad . \quad (67)$$

Since $(\alpha + \beta) = {}^{1}/2$, and $\alpha, \beta > 0$ then $\alpha < {}^{1}/2$ and $(2\alpha - 1) < 0$. Since from (33) $\frac{dq}{dx} > 0$, we conclude $\pi_{xx} > 0$. The conclusion once more is that the optimised profit function is convex with respect to x and the optimum location is at one of the terminals.

Then from (63) and (64),

$$\pi = \frac{b^2\beta}{r_2} \left[\frac{\beta}{\alpha} \frac{r_1 + m_1(s - x)}{r_2} \right]^{-2\alpha} + a - \frac{r_2}{2\beta} \left[\frac{\beta}{\alpha} \frac{r_1 + m_1(s - x)}{r_2} \right]^{2\alpha}$$

$$\cdot \frac{b\beta}{r_2}^{2} \left[\frac{\beta}{\alpha} \frac{r_1 + m_1(s - x)}{r_2} \right]^{-4\alpha} \quad , \quad (68)$$

$$= a + \frac{b^2\beta}{2r_2} \left[\frac{\alpha}{\beta} \frac{r_2}{r_1 + m_1(s - x)} \right]^{2\alpha} \quad . \quad (69)$$

Comparing profits at the two extrema,

$$\pi(0) = a + \frac{b^2\beta}{2r_2} \left[\frac{\alpha}{\beta} \frac{r_1}{r_1 + m_1 s} \right]^{2\alpha} \quad , \quad (70)$$

$$\pi(x) = a + \frac{b^2\beta}{2r_2}\left[\frac{\alpha}{\beta}\ \frac{r_2}{r_1}\right]^{2\alpha} \quad . \quad (71)$$

This implies comparing,

$$\left[\frac{\alpha}{\beta}\ \frac{r_2}{r_1}\right]^{2\alpha} \quad \text{and} \quad \frac{\alpha}{\beta}\left[\frac{r_2}{r_1 + m_1(s - x)}\right]^{2\alpha} \quad .$$

This means, as in all the casea analysed above, that the demand function is irrelevant to the locational decision of the firm.

4.6 Conclusions

Obviously, the deductions which one can make from the above analysis are very limited. We have examined the effects of three different demand functions on the location model using two different homogeneities of the production function, and have found that the optimum location is always at a terminal point, and that the demand function plays no role in determining the comparative profitability of these poles. We are not justified in concluding that such solutions hold for all demand and production conditions, but further calculations, including several with increasing returns to scale [$(\alpha + \beta) > 1$], indicate an absolute consistency in this outcome.

CHAPTER V

CONCLUSIONS

5.1 Summary

The objectives of this research were: 1) to determine the effect, if any, of a non-linear homogeneous assumption about the production function on the cost-minimising location, 2) to investigate the impact of such non-linear conditions on the firm's locational decision in the context of a simple profit-maximising model, employing a linear demand function, 3) to examine the effects of various non-linear demand situations on the optimum location in the context of the non-linear homogeneous technology. It has been discovered that a production function homogeneous of any degree will generate an end-point solution in the Sakashita-type model. Furthermore, such a result holds true for the profit maximising case also, at least for the special case chosen. Finally, the end point optimum appears to stand with variable market conditions.

5.2 Limitations of the Analysis

The main deficiency in the strategy adopted in the above research is the reliance on special cases rather than general formulations. The conclusions one may draw from the results, therefore, are strictly speaking, limited to these particular situations. It was hoped that a general model with the production function homogeneous of any degree and a general demand function $q = q(p)$ could be used in Chapters III and IV. However, attempts in this direction proved analytically intractable. Given this defect in the methodology, the number of special cases examined (most of which are not included in the text) with different degrees of homogeneity in the production process prompt some intuitive comments on the efficiency of this general framework. In all instances the degree of returns to scale do not affect the choice of site, and so it appears that this type of model is not appropriate for examining such topics as the impact of technological progress on the optimum location. Similarily, the uniformity of results for different demand conditions leads one to suspect that this is not an adequate basis for discussing the role of dynamic demand on location, etc.

5.3 Possible Extensions of the Analysis

Nevertheless, certain means of improving the model are apparent. Sakashita has suggested, as one possible course of generalisation of his model, a conversion to a two dimensional form from the linear one. However, such a model will be analytically inelegant, and it is doubtful whether an algorithmic approach will be a fruitful medium for developing the theory along the lines attempted in this work.

Another possibility for improving the model is the incorporation of a non-homogeneous production function, for example, the general Solow function $q = [v_1^{\alpha} + v_2^{\beta}]^{\delta}$, for which according to Hilhorst (1964), could prove to be a more useful tool for examining the nature of technical progress than its homogeneous counterparts.

An obvious alteration, which will overcome many of the objections to the established location theory literature, is to treat the inputs

V_1 and V_2 as exhaustible in supply by expressing their values as functions of the level of output. Similarily, transport costs may be expressed as a non-linear function of x for both inputs, reflecting the empirical evidence for tapering carrying rates. A four-dimensional approach may then be adopted for the locational strategy of the firm, with the various combinations of technical, market, factor supply, and transport conditions generating a vastly more realistic locational profit function.

Another development, within the general context of the above model, is the use of a stock flow production function (i.e., a function treating capital as well as the variable factors as an input) as suggested by Churchill (1967). Churchill, however, does not try to use this innovation to bring the location decision into the general investment theory of the firm, which appears to be the most advantageous course to take.

Some of the simpler extensions of the model include the use of a larger number of resource supply points and several market points. As long as we remain in a linear setting such changes merely involve adding additional expressions for cost and revenue (corresponding to each new supply point or market) into the equations described above for the one-market, one-fixed input case.

At a more advanced level, given that the locational process may be subsumed in the investment decision, some established methods of introducing dynamics to the problem appear feasible. For instance, multiperiod production and investment functions (Henderson and Quandt, 1958) may be employed to derive solutions for optimum production decisions through time. The optimum location through time will follow as a consequence of the latter.

However, given the serious drawbacks mentioned above, it may be more rewarding to commence with a Hotelling-Smithies-Chamberlain (e.g., Smithies 1941) type problem with two or more rival producers interacting along a linear market, and examine how cost, production, and demand factors affect a competitive locational solution. It is felt that the lack of useful results from the operations described in the preceding chapters is not due to the unimportance of the questions attempted, but rather to the sterility of the specific locational situation posited.

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