THE UNSYMMETRIC TWO IMPACTS PER CYCLE STEADY STATE MOTION OF THE IMPACT DAMPER

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SCOPE AND CONTENTS:

Steady state response of a single degree of freedom system provided with an impact damper assuming two unsymmetric impacts per cycle motion, and its asymptotic stability criterion are derived analytically. Stability regions are also determined for a wide range of parameters of impact damper by using a digital computer.

An experimental study is also made to verify the assumptions taken in the analytical solution and to obtain a general response of the system for a wide range of parameters of the impact damper.

ABSTRACT

Steady state response of a single degree of freedom system with impact damper, with the main emphasis of two impacts (symmetric or unsymmetric)/cycle motion, and its asymptotic stability criterion are derived analytically. Stability regions are determined for wide range of parameters of the impact damper by using digital computer.

Experimental study is also made to verify the assumptions taken in the analytical solution and to obtain general response of the system for wide range of parameters of the impact damper.

As a result, it is found that unsymmetric two impacts per cycle motion exists and is stable for a wide range of parameters of the impact damper.

Also, it is found that three and four impacts/cycle motions exist and are stable.

Stability boundaries are found to be a complicated function of the impact damper parameters.

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NOMENCLATURE

A	displacement of the primary system in the absence of the impact damper
c	damping constant
d	clearance in which the particle is free to oscillate
e	coefficient of restitution
Fo	maximum force of excitation
k	spring constant
M	mass of primary system
m	mass of particle
P	perturbation matrix
R	remainder term
n	ratio of smaller time taken by the particle to travel from one end of the container to the other to the total time of the cycle $(\frac{2\pi}{\Omega})$ $[0 \leq n \leq 1]$
N	n/2
w	natural frequency, $\sqrt{K/M}$
Ω	forcing frequency
r	ratio, forcing frequency/natural frequency
t	time
V_1	absolute velocity of the particle $(0_+ \leq t \leq \frac{n\pi}{\Omega_-})$
V_2	absolute velocity of the particle $(\frac{n\pi}{\Omega_+} \leq t \leq \frac{2\pi}{\Omega_-})$
x	displacement of M
x_a	displacement immediately after impact(at t = 0)
x_n	displacement immediately after impact (at $t = \frac{n\pi}{\Omega}$)
x_b	displacement immediately before impact (at t = 0)

x_g	displacement immediately before impact (at $t = \frac{n\pi}{\Omega}$)
y_1	displacement of particle
y	relative displacement of particle with respect to M
α	phase angle (initially unknown)
δ	ratio of critical damping
μ	mass ratio, m/M
ξ	perturbation vector
ρ	ratio d/A
ψ	phase angle (due to damping)
τ	phase angle, $\tau = \alpha - \psi$

1. INTRODUCTION

1.1. Historical Review of Impact Damper

The impact damper is a device for reducing the vibration amplitude of a mechanical system through the mechanism of momentum transfer by collision and conversion of mechanical energy into heat.

Paget (1)* was the pioneer in making qualitative study of this damper.

The idea of reducing the vibration of a mechanical system by attaching to it a container in which a solid particle is constrained to oscillate was conceived and investigated in 1944 by Lieber and Jensen (2).

In that investigation, the authors assumed that the motion of a undamped single degree of freedom oscillator with an operating damper (referred to as an "acceleration damper") was still simple harmonic; that the impact of the primary system with the particle was completely plastic (i.e. no rebound); and that during a period of the sinusoidal forcing function two impacts occur at equal time intervals and at opposite sides of the container (i.e. symmetric two impacts per cycle motion). As a result, they determined that for maximum energy dissipation per cycle, the clearance of the particle should be π times the amplitude of response.

Grupin (3), in his investigation of this device, assumed the existence of symmetric two impacts/cycle motion (henceforth, unless otherwise specified, the motion will be assumed to be symmetric) and he determined the behaviour of the viscously damped primary

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* Numbers in parenthesis designate references at the end of the thesis.

system, by adding the effects of many impacts. It was shown that these assumptions result in two possible solutions, but it could not be shown which one of these prevails without solving the problem by a more exact but longer numerical impact to impact method.

By introducing an unknown phase angle into the applied harmonic force and assuming steady state of two equispaced impacts per cycle and neglecting the inherent damping in the system, Arnold (4) analysed the problem. His experimental evidence did not agree with the theory.

A considerably simpler method for deriving the solution for two impacts/cycle motion, which requires only the consideration of two successive impacts, was suggested by Warburton (5), and he used it to obtain the solution for the special case of an undamped primary system forced at resonance.

Kaper (6) investigated the influence of impact vibration absorber (referred to as "discontinuous dynamic vibration absorber") on the motion of a vertical vibrating system. Attention was paid to the effectiveness in the case of free vibrations and of vibrations due to sinusoidal excitations, where structural damping was also taken into account. For certain configurations numerical results were given.

Sadek (7) obtained steady state solution, assuming two unsymmetric impacts per cycle. The impact force-time curve is assumed to be of rectangular shape and of infinitismal duration. He used Fourier series to represent impact cycle.

Periodic symmetric two impacts per cycle were sought and their asymptotic stability boundaries were determined analytically be Masri (8). The stability analysis involved a perturbation of the phase space trajectory of the motion, and it is indicated that the solution was stable if the modulus of all eigenvalues of a certain matrix is less than unity. This matrix continuously related the perturbations immediately after each of the two consecutive impacts. Results of the anaylsis were verified by: (a) numerical step-by-step construction of solutions for all types of motion, (b) experiments with an electric analog computer (c) experiments with a mechanical model.

The effectiveness of the impact damper on nonlinear systems is to be found in a recent work by Dokainish and Jha (9).

A very simple stability criterion for these solutions, neglecting the inherent damping in the system, was developed by Egle (10), and was used to determine the dependence of the stability boundaries on the parameters of the system.

Steady state response of a system of two degrees of freedom with impact damper and its asymptotic stability criterion are derived analytically, numerically and experimentally as was investigated by Dokainish and Agrawal (11).

Dampers containing two particles in a single container and the effect of filling the container with a fluid were investigated by Dokainish and Shah (12).

On the experimental side, the feasibility of using impact damping to reduce the vibrations of such diverse systems as ship hulls, cantilever beams, single degree of freedom systems, and turbine buckets was investigated by McGoldrick (13), Lieber and Tripp (14), Sankey (15), and Duckwald (16), respectively. Estabrook and Plunkett (17) made an analytical study of impact damping in turbine buckets.

Also, this type of damping was employed in reducing the vibrations of telephone switching relays.

All the previous investigators have reported excessive noise level while the impact damper is in operation.

1.2. Objective

The objective of the present study is to investigate the behaviour of a single degree of freedom system provided with an impact damper, when the system is subjected to a sinusoidal excitation, and to study the effects of parameters variations on the response and stability of the primary system. It is assumed that two unsymmetric impacts occur per cycle.

The theoretical soluti is derived in Chapter 2, and its stability boundaries are determined in Chapter 3. The experimental studies that were conducted in the course of this investigation with a mechanical model, supplemented by numerical studies with a digital computer, are described in Chapter 4. The experimental results as well as theoretical results are discussed in Chapter 5. The conclusions drawn from this research work and recommendations for future work are also stated in Chapter 5.

The digital computer programs to obtain the steady state solution and stability region are given in Appendix IX

2. STEADY STATE SOLUTION WITH TWO UNEQUALLY SPACED IMPACTS PER CYCLE

2.1. Introduction

Experimentally, it was observed that a single degree of freedom system, provided with an impact damper, may exhibit a steady state motion with nonsymmetric two impacts per cycle. This type of motion exists for certain combinations of the parameters of the system. Hence, analytical model is constructed which allows for two nonsymmetric impacts per cycle. The smaller time interval = $\frac{n\pi}{\Omega}$, and the other interval = $\frac{(2 - n)\pi}{\Omega}$, where $0 \leq n \leq 1$, figure 2.2.

2.2. Unequally-spaced-two-impacts/cycle solution

A model of the system under discussion is shown in figure 2.1. The free mass m is essentially a frictionless solid particle constrained to oscillate with clearance d in a container attached to the primary mass. The equation of motion of the primary mass M, between impacts, is

$$M\ddot{x} + c\dot{x} + kx = F_o \sin \Omega t \qquad (2.1)$$

Following the method suggested by Warburton (5), assume the disturbing force to be Fo sin $(\Omega t + \alpha)$ where α is an unknown phase angle.

Equation (2.1) now becomes

$$M\ddot{x} + c\dot{x} + kx = F_o \sin(\Omega t + \alpha) \qquad (2.2)$$

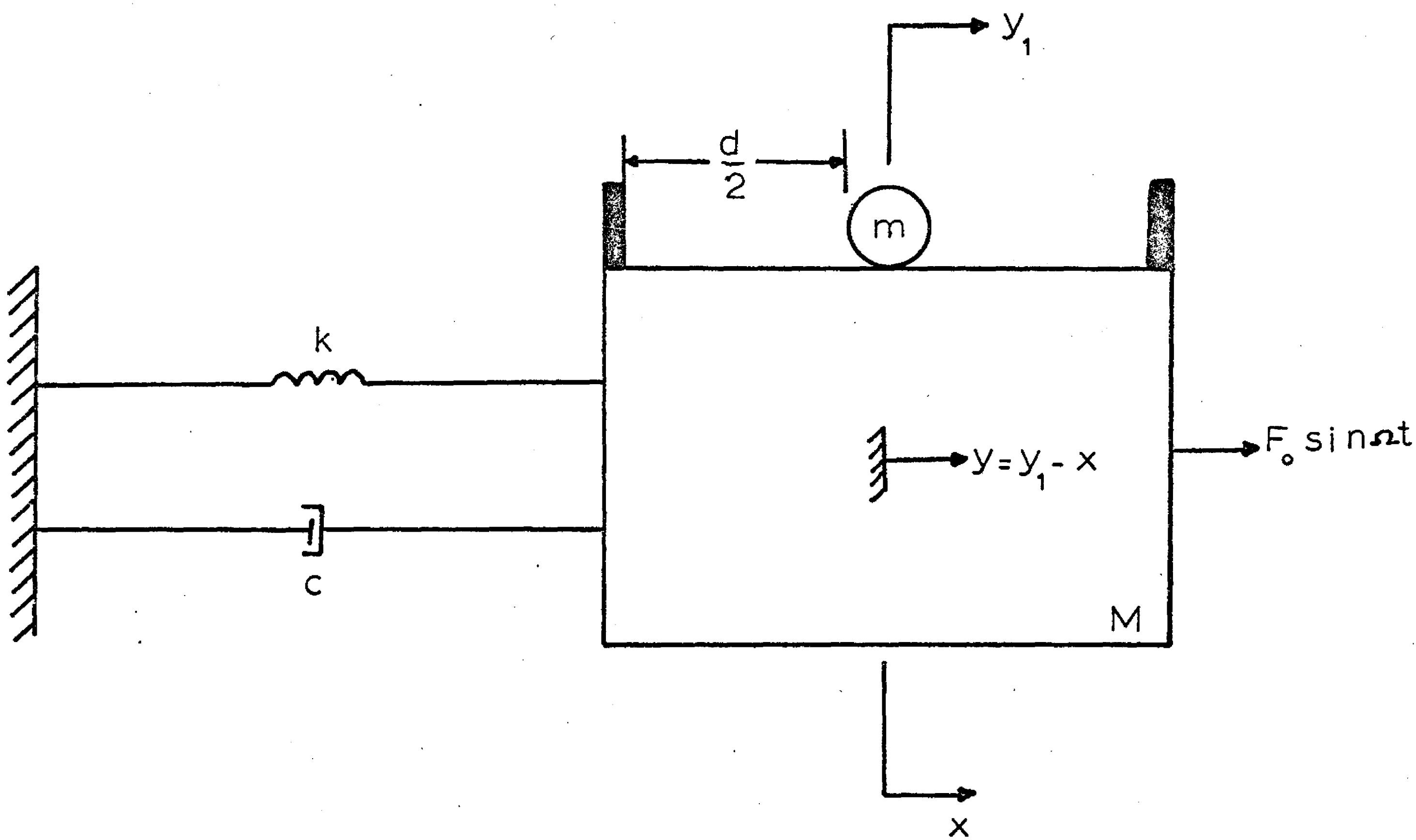


Fig.(2.1) Model of system

The complete solution of equation (2.2) is

$$x = e^{-\delta\omega t}\left(B_1 \sin(\eta\omega t) + B_2 \cos(\eta\omega t)\right) + A \sin(\Omega t + \tau) \qquad (2.3)$$

where

$$\delta = c / c_{cr}$$

$$c_{cr} = 2\sqrt{KM}$$

$$\omega = \sqrt{K/M}$$

$$\eta = \sqrt{1 - \delta^2}$$

$$r = \Omega / \omega$$

$$A = \frac{F_o / K}{\sqrt{(1 - r^2)^2 + (2\delta r)^2}}$$

$$\tau = \alpha - \psi$$

$$\psi = \tan^{-1} \frac{2\delta r}{1 - r^2} \qquad 0 < \psi < \pi$$

For steady state motion with two unequally spaced impacts per cycle, if one impact is assumed to occur at time $t = 0$, then the next impact will occur at $t = \frac{n\pi}{\Omega}$, where $0 \leq n \leq 1$, and the following impact at $t = \frac{2\pi}{\Omega}$. In the case of equally spaced impacts, n would be equal to 1. Then as shown in figure 2.2, immediately

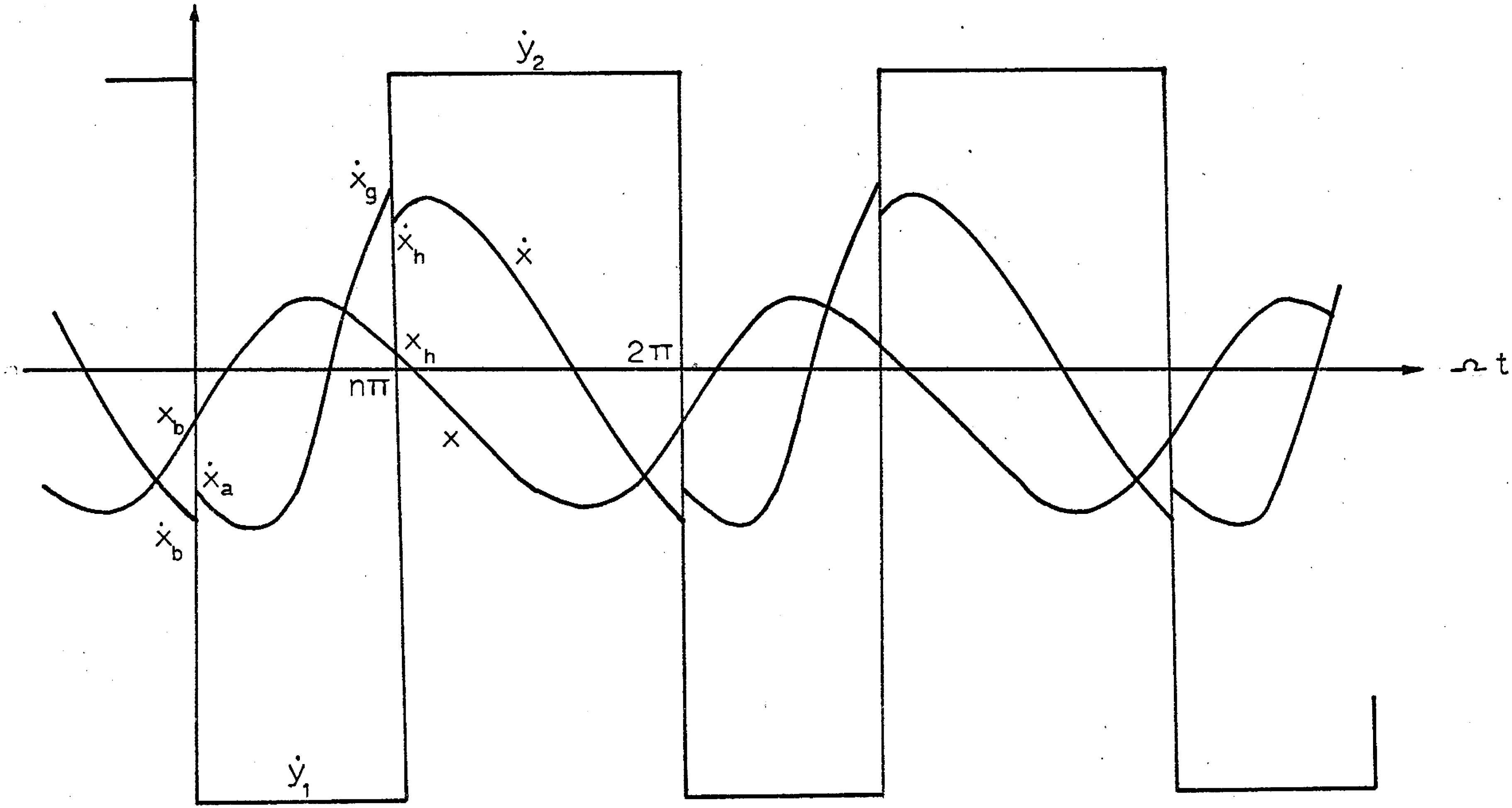


Fig.(2.2) Wave form of unsymmetric 2 impacts / cycle

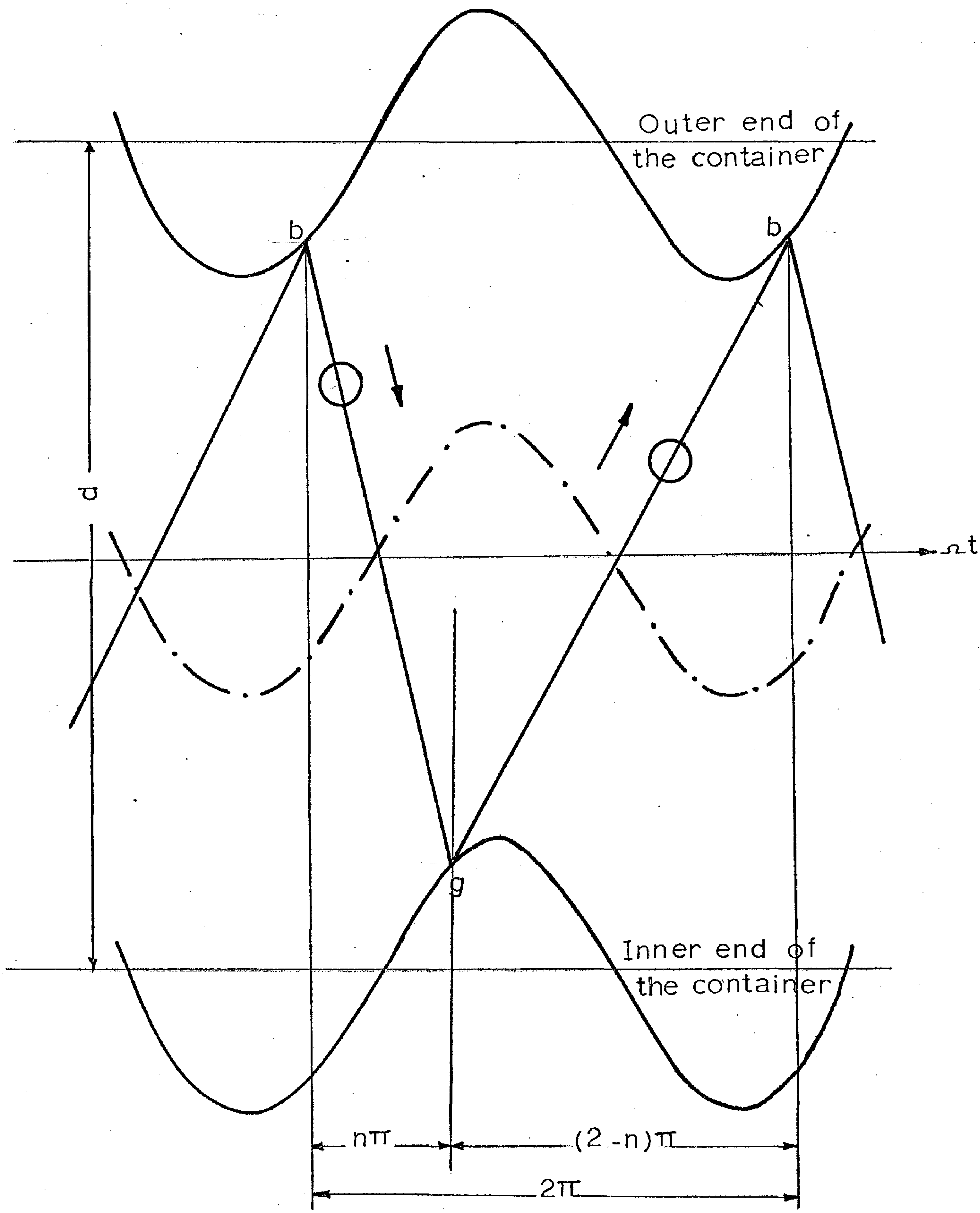


Fig. (2.3) Unsymmetric motion of the impact damper preceding the impact at $t = 0_-$ (t_- denotes the time immediately preceding the impact and t_+ denotes the time immediately following the impact), $x = x_b$, $\dot{x} = \dot{x}_b$, $y = \frac{d}{2}$ and the absolute velocity of the particle $= v_2$.

The duration of the impact is very small compared to the natural frequency of the primary system, hence it is reasonable to assume that at $t = 0_+$, the position of M and m remain unchanged, i.e.

$$x_a = x_b \tag{2.4}$$

while their respective absolute velocities are discontinuously changed to $\dot{x}_a$ and v_1, respectively.

Similarily at $t = (\frac{n\pi}{\Omega})_-$; $x = x_g$, $\dot{x} = \dot{x}_g$, and $y = -\frac{d}{2}$. After the impact, at $t = (\frac{n\pi}{\Omega})_+$, $x = x_h$, $\dot{x} = \dot{x}_h$ and $y = -\frac{d}{2}$. In this case, also

$$x_g = x_h \tag{2.5}$$

To summarize, the system should satisfy the following conditions:

at	$t = 0_-$	$x = x_b$	$y = \frac{d}{2}$	$\dot{x} = \dot{x}_b$	$V = V_2$
	$t = 0_+$	$x = x_a$	$y = \frac{d}{2}$	$\dot{x} = \dot{x}_a$	$V = V_1$
	$t = (\frac{n\pi}{\Omega})_-$	$x = x_g$	$y = -\frac{d}{2}$	$\dot{x} = \dot{x}_g$	$V = V_1$
	$t = (\frac{n\pi}{\Omega})_+$	$x = x_h$	$y = -\frac{d}{2}$	$\dot{x} = \dot{x}_h$	$V = V_2$

The equation of motion of M from $t = 0_+$ to $t = (\frac{n\pi}{\Omega})_-$ is

$$x = e^{-\delta\omega t}(B_1 \sin(\eta\omega t) + B_2 \cos(\eta\omega t)) + A \sin(\Omega t + \tau) \tag{2.6}$$

and the equation of motion of M from $t = (\frac{n\pi}{\Omega})_+$ to $t = (\frac{2\pi}{\Omega})_-$ is

$$x = e^{-\delta\omega(t - \frac{n\pi}{\Omega})}\left[B_1' \sin(\eta\omega(t - \frac{n\pi}{\Omega})) + B_2' \cos(\eta\omega(t - \frac{n\pi}{\Omega}))\right] + A \sin(\Omega t + \tau) \tag{2.7}$$

From equation (2.6), at $t = 0_+$

$$x(0_+) = x_a = B_2 + A \sin\tau \tag{2.8}$$

and at $t = (\frac{n\pi}{\Omega})_-$

$$x(\frac{n\pi}{\Omega}_-) = x_g = e^{-\frac{\delta\omega n\pi}{\Omega}}(B_1 \sin(\frac{\eta\omega n\pi}{\Omega}) + B_2 \cos(\frac{\eta\omega n\pi}{\Omega})) + A \sin(n\pi + \tau) \tag{2.9}$$

An expression describing the velocity of M (from $t = (0)_+$ to $t = (\frac{n\pi}{\Omega})_-$) can be obtained by differentiating equation (2.6) with respect to t, thus

$$\dot{x} = e^{-\delta\omega t}(B_1\eta\omega\cos(\eta\omega t) - B_2\eta\omega\sin(\eta\omega t)) -$$

$$\delta\omega\, e^{-\delta\omega t}\,(B_1 \sin(\eta\omega t) + B_2 \cos(\eta\omega t)) + A\Omega\cos(\Omega t + \tau) \qquad (2.10)$$

Equations (2.6) and (2.10) describe the displacement and velocity of M during the time interval $0_+ \leqslant t \leqslant (\frac{n\pi}{\Omega})_-$.

From equation (2.10) we get

$$\dot{x}(0_+) = \dot{x}_a = -\delta\omega B_2 + \eta\omega B_1 + A\Omega\cos\tau \qquad (2.11)$$

$$\dot{x}\left(\frac{n\pi}{\Omega}_-\right) = \dot{x}_g = -\delta\omega\, e^{-\frac{\delta\omega n\pi}{\Omega}}\left(B_1 \sin\left(\eta\omega\frac{n\pi}{\Omega}\right) + B_2\cos\left(\eta\omega\frac{n\pi}{\Omega}\right)\right) + \eta\omega\, e^{-\frac{\delta\omega n\pi}{\Omega}}\left(B_1 \cos\left(\eta\omega\frac{n\pi}{\Omega}\right) - B_2 \sin\left(\eta\omega\frac{n\pi}{\Omega}\right)\right) + A\Omega\cos(n\pi + \tau) \qquad (2.12)$$

Similiarly, an expression describing the velocity of M for $t = (\frac{n\pi}{\Omega})_+$ to $t = (\frac{2\pi}{\Omega})_-$ can be obtained by differentiating equation (2.7) with respect to time, thus

$$\dot{x} = \eta\omega\, e^{-\delta\omega(t - \frac{n\pi}{\Omega})}\left[B_1' \cos\left(\eta\omega\left(t - \frac{n\pi}{\Omega}\right)\right) - B_2' \sin\left(\eta\omega\left(t - \frac{n\pi}{\Omega}\right)\right)\right] - \delta\omega\, e^{-\delta\omega(t - \frac{n\pi}{\Omega})}\left[B_1' \sin\left(\eta\omega\left(t - \frac{n\pi}{\Omega}\right)\right) + B_2' \cos\left(\eta\omega\left(t - \frac{n\pi}{\Omega}\right)\right)\right] + A\Omega\cos(\Omega t + \tau) \qquad (2.13)$$

So, at $t = (\frac{n\pi}{\Omega})_+$

$$x(\frac{n\pi}{\Omega_+}) = x_h = \grave{B}_2 + A\sin(n\pi + \tau) \quad (2.14)$$

$$\dot{x}(\frac{n\pi}{\Omega_+}) = \dot{x}_h = \grave{B}_1 \eta\omega - \grave{B}_2 \delta\omega + A\Omega\cos(n\pi + \tau) \quad (2.15)$$

and at $t = (\frac{2\pi}{\Omega})_-$

$$x(\frac{2\pi}{\Omega_-}) = x_b = e^{-\delta\omega\frac{(2\pi - n\pi)}{\Omega}} \left[\grave{B}_1 \sin(\eta\omega\frac{(2\pi-n\pi)}{\Omega}) + \grave{B}_2 \cos(\eta\omega\frac{(2\pi-n\pi)}{\Omega})\right] + A\sin\tau \cdot \quad (2.16)$$

$$\dot{x}(\frac{2\pi}{\Omega_-}) = \dot{x}_b = e^{-\delta\omega\frac{(2\pi - n\pi)}{\Omega}} \cdot \eta\omega \left[\grave{B}_1 \cos(\eta\omega\frac{(2\pi-n\pi)}{\Omega}) - \grave{B}_2 \sin(\eta\omega\frac{(2\pi-n\pi)}{\Omega})\right] - \delta\omega e^{-\delta\omega\frac{(2\pi-n\pi)}{\Omega}} \left[\grave{B}_1 \sin(\eta\omega\frac{(2\pi-n\pi)}{\Omega}) + \grave{B}_2 \cos(\eta\omega\frac{(2\pi-n\pi)}{\Omega})\right] + A\Omega\cos\tau \quad (2.17)$$

Since the motion of the system during impact must satisfy the momentum equation, then

$$M\dot{x}_- + mv_- = M\dot{x}_+ + mv_+ \quad (2.18)$$

and from the definition of the coefficient of restitution e,

$$\dot{x}_+ - v_+ = -e(\dot{x}_- - v_-) \quad (2.19)$$

From equations (2.18) and (2.19) the following relations that will be useful later on are obtained:

$$\dot{x}_+ = \frac{(1-\mu e)\dot{x}_- + \mu(1+e)V_-}{(1+\mu)} \qquad (2.20)$$

$$V_- = \frac{(1+e)\dot{x}_- + (\mu - e)V_-}{(1+\mu)} \qquad (2.21)$$

$$\dot{x}_- = \frac{(e-\mu)V_- + (1+\mu)V_+}{(1+e)} \qquad (2.22)$$

$$\dot{x}_+ = \frac{e(1+\mu)V_- + (1-\mu e)V_+}{(1+e)} \qquad (2.23)$$

Also, from equations (2.18) and (2.19) we get

$$\dot{x}_a - \dot{x}_b = \mu(V_2 - V_1) \qquad (2.24)$$

$$\dot{x}_h - \dot{x}_g = \mu(V_1 - V_2) \qquad (2.25)$$

$$\dot{x}_a - V_1 = -e(\dot{x}_b - V_2) \qquad (2.26)$$

$$\dot{x}_h - V_2 = -e(\dot{x}_g - V_1) \qquad (2.27)$$

In the steady state motion, the absolute velocity of the particle is constant, and it is equal to:

$$V_1 = -\frac{\Omega}{n\pi}(d + x_a - x_g) \qquad 0_+ \leq t \leq \frac{n\pi}{\Omega_-} \qquad (2.28)$$

$$V_2 = \frac{\Omega}{(2-n)\pi}(d + x_a - x_g) \qquad \frac{n\pi}{\Omega_+} \leq t \leq \frac{2\pi}{\Omega} \qquad (2.29)$$

Thus,

$$V_2 = -\frac{n}{(2-n)} V_1 \tag{2.30}$$

From (2.24)

$$\dot{x}_a = \dot{x}_b + \mu (V_2 - V_1)$$

$$= \dot{x}_b - \mu V_1 \frac{2}{(2-n)} \tag{2.31}$$

Substituting for (2.26) in (2.31), we get

$$\dot{x}_b - \mu V_1 \frac{2}{(2-n)} - V_1 = -e\dot{x}_b - \frac{en}{(2-n)} V_1$$

Thus

$$\dot{x}_b(1+e) = V_1\left[\frac{2\mu}{(2-n)} + 1 - \frac{en}{(2-n)}\right]$$

or

$$\dot{x}_b = V_1 \frac{(2\mu + 2 - n - en)}{(2-n)(1+e)} \tag{2.32}$$

Substituting from (2.32) in (2.26) we get

$$\dot{x}_a = V_1\left[\frac{(2\mu + 2 - n - en)}{(2-n)(1+e)} - \frac{2\mu}{(2-n)}\right]$$

Thus

$$\dot{x}_a = V_1 \frac{(2 - n - en - 2\mu e)}{(2-n)(1+e)} \tag{2.33}$$

Similarly from (2.25)

$$\dot{x}_h = \dot{x}_g + \mu (V_1 - V_2)$$

$$\dot{x}_h = \dot{x}_g + \mu V_1 \left[1 + \frac{n}{(2-n)}\right]$$

$$= \dot{x}_g + \mu V_1 \frac{2}{(2-n)} \qquad (2.34)$$

and from (2.27)

$$\dot{x}_h = V_2 - e(\dot{x}_g - V_1) \qquad (2.35)$$

Substituting from (2.35) in (2.34) we get

$$\dot{x}_g + \mu V_1 \frac{2}{(2-n)} = -V_1 \frac{n}{(2-n)} - e(\dot{x}_g - V_1)$$

or

$$\dot{x}_g(1+e) = V_1 \left[-\frac{2\mu}{(2-n)} - \frac{n}{(2-n)} + e\right]$$

Thus

$$\dot{x}_g = V_1 \frac{(-2\mu - n + 2e - ne)}{(1+e)(2-n)} \qquad (2.36)$$

Substituting from (2.36) in (2.35), we get

$$\dot{x}_h = V_1 \frac{(-2\mu - n + 2e - ne)}{(1+e)(2-n)} + \mu V_1 \left[1 + \frac{n}{(2-n)}\right]$$

Thus

$$\dot{x}_h = V_1 \frac{(-n + 2e - ne + 2\mu e)}{(1+e)(2-n)} \qquad (2.37)$$

Equations (2.8), (2.11), (2.9), (2.12), (2.14), (2.15), (2.16), (2.17), (2.4), (2.5), (2.33), (2.32), (2.36), (2.37), (2.28) can be written as*

$$x_a = B_2 + A \sin\tau \tag{2.38}$$

$$\dot{x}_a = B_1 G_1 - B_2 G_2 + A\Omega\cos\tau \tag{2.39}$$

$$x_g = G_3(B_1 \sin G_4 + B_2 \cos G_4) + AG_5 \cos\tau + AG_6 \sin\tau \tag{2.40}$$

$$\dot{x}_g = G_3 G_1 (B_1 \cos G_4 - B_2 \sin G_4) - G_2 G_3 (B_1 \sin G_4 + B_2 \cos G_4) + A\Omega(G_6 \cos\tau - G_5 \sin\tau) \tag{2.41}$$

$$x_h = B_2' + AG_6 \sin\tau + AG_5 \cos\tau \tag{2.42}$$

$$\dot{x}_h = B_1' G_1 - B_2' G_2 + A\Omega G_6 \cos\tau - A\Omega G_5 \sin\tau \tag{2.43}$$

$$x_b = G_7(B_1' \sin G_8 + B_2' \cos G_8) + A \sin\tau \tag{2.44}$$

$$\dot{x}_b = G_1 G_7 (B_1' \cos G_8 - B_2' \sin G_8) - G_2 G_7 (B_1' \sin G_8 + B_2' \cos G_8) + A\Omega \cos\tau \tag{2.45}$$

$$x_a = x_b \tag{2.46}$$

$$x_g = x_h \tag{2.47}$$

$$\dot{x}_a = G_9 V_1 \tag{2.48}$$

$$\dot{x}_b = G_{10} V_1 \tag{2.49}$$

* The values of $G_1, \ldots G_{87}$ are given in Appendix 1.

$$\dot{x}_g = G_{11} V_1 \qquad (2.50)$$

$$\dot{x}_h = G_{12} V_1 \qquad (2.51)$$

$$-d = V_1 \frac{n\pi}{\Omega} + x_a - x_g \qquad (2.52)$$

Equations (2.38) to (2.52) are 15 equations in the 15 unknowns: x_a, $\dot{x}_a$, x_g, $\dot{x}_g$, x_h, $\dot{x}_h$, x_b, $\dot{x}_b$, B_1, B_2, $\grave{B}_1$, $\grave{B}_2$, n, τ, V_1.

By a series of substitutions from each equation into some of the others, they finally could be reduced to two equations in τ and n.

Substituting from equations (2.46), (2.47) into (2.42), (2.44) and from (2.48) into (2.49), (2.50), (2.51) and (2.52) we get:

$$x_g = B_1 G_{21} + B_2 G_{22} + AG_5 \cos\tau + AG_6 \sin\tau \qquad (2.53)$$

$$\dot{x}_g = B_1 G_{23} - B_2 G_{24} - B_1 G_{25} - B_2 G_{26} + AG_{17} \cos\tau - AG_{18} \sin\tau \qquad (2.54)$$

$$x_g = \grave{B}_2 + AG_6 \sin\tau + AG_5 \cos\tau \qquad (2.55)$$

$$\dot{x}_h = \grave{B}_1 G_1 - \grave{B}_2 G_2 + AG_{17} \cos\tau - AG_{18} \sin\tau \qquad (2.56)$$

$$x_a = \grave{B}_1 G_{27} + \grave{B}_2 G_{28} + A \sin\tau \qquad (2.57)$$

$$\dot{x}_b = B_1' G_{29} - B_2' G_{30} - B_1' G_{31} - B_2' G_{32} + A\Omega\cos\tau \quad (2.58)$$

$$\dot{x}_b = G_{33}\,\dot{x}_a \quad (2.59)$$

$$\dot{x}_g = G_{34}\,\dot{x}_a \quad (2.60)$$

$$\dot{x}_h = G_{35}\,\dot{x}_a \quad (2.61)$$

$$-d = G_{36}\,\dot{x}_a + x_a - x_g \quad (2.62)$$

Substituting from equations (2.59), (2.60), (2.61) in (2.54), (2.56), (2.58) we get

$$\dot{x}_a = B_1 G_{52} - B_2 G_{53} + AG_{41}\cos\tau - AG_{42}\sin\tau \quad (2.63)$$

$$\dot{x}_a = B_1' G_{43} - B_2' G_{44} + AG_{45}\cos\tau - AG_{46}\sin\tau \quad (2.64)$$

$$\dot{x}_a = B_1' G_{54} - B_2' G_{55} + AG_{51}\cos\tau \quad (2.65)$$

Substituting from equations (2.38), (2.39), (2.55), in (2.53), (2.63), (2.64), (2.65), (2.57), (2.62), we get

$$B_2' = B_1 G_{21} + B_2 G_{22} \quad (2.66)$$

$$B_1 G_{59} - B_2 G_{60} + AG_{61}\cos\tau + AG_{42}\sin\tau = 0 \quad (2.67)$$

$$B_1 G_1 - B_2 G_2 + AG_{62}\cos\tau - B_1' G_{43} + B_2' G_{44} + AG_{46}\sin\tau = 0 \quad (2.68)$$

$$B_2 = B_1' G_{27} + B_2' G_{28} \quad (2.69)$$

$$B_1 G_1 - B_2 G_2 + AG_{63} \cos\tau - \grave{B}_1 G_{54} + \grave{B}_2 G_{55} = 0 \qquad (2.70)$$

$$-d = B_1 G_{56} - B_2 G_{64} - \grave{B}_2 + AG_{65} \cos\tau + AG_{66} \sin\tau \qquad (2.71)$$

Substituting from (2.66) into (2.69) we get

$$B_2 = \grave{B}_1 G_{27} + G_{28} (B_1 G_{21} + B_2 G_{22})$$

Thus

$$B_2 = \grave{B}_1 G_{67} + B_1 G_{68} \qquad (2.72)$$

Substituting from (2.72) into (2.66) we get

$$\grave{B}_2 = B_1 G_{21} + G_{22} (\grave{B}_1 G_{67} + B_1 G_{68})$$

Thus

$$\grave{B}_2 = B_1 G_{69} + \grave{B}_1 G_{70} \qquad (2.73)$$

Substituting from (2.72) and (2.73) into (2.66), (2.67), (2.70), (2.71), we get

$$B_1 G_{71} - \grave{B}_1 G_{72} + A(G_{61} \cos\tau + G_{42} \sin\tau) = 0 \qquad (2.74)$$

$$B_1 G_{73} + \grave{B}_1 G_{74} + A(G_{62} \cos\tau + G_{46} \sin\tau) = 0 \qquad (2.75)$$

$$B_1 G_{75} + \grave{B}_1 G_{76} + AG_{63} \cos\tau = 0 \qquad (2.76)$$

$$B_1 G_{77} + \grave{B}_1 G_{78} + A(G_{65} \cos\tau + G_{66} \sin\tau) = -d \qquad (2.77)$$

Equations (2.74), (2.75), (2.76), (2.77) are four equations in four unknowns B_1, B_1', τ, and n. From (2.74) and (2.75) we get

$$B_1 = -A(G_{80}\cos\tau + G_{81}\sin\tau) \tag{2.78}$$

$$B_1' = -A(G_{82}\cos\tau + G_{83}\sin\tau) \tag{2.79}$$

Substituting from (2.78) and (2.79) in (2.76) and (2.77) we get

$$G_{84}\cos\tau + G_{85}\sin\tau = 0 \tag{2.80}$$

$$G_{86}\cos\tau + G_{87}\sin\tau = -\rho \tag{2.81}$$

Equations (2.80) and (2.81) are two equations in two unknowns (τ, n).

Solution of equation (2.81) for τ results in

$$\sin\tau = \frac{-\rho G_{87} \pm \sqrt{\rho^2 G_{87}^2 - (G_{86}^2 + G_{87}^2)(\rho^2 - G_{86}^2)}}{2(G_{86}^2 + G_{87}^2)}$$

and

$$\cos\tau = \frac{-\rho G_{86} \mp \sqrt{\rho^2 G_{86}^2 - (G_{86}^2 + G_{87}^2)(\rho^2 - G_{87}^2)}}{2(G_{86}^2 + G_{87}^2)}$$

Thus

$$\tau = \tan^{-1}\frac{-\rho G_{87} \pm \sqrt{\rho^2 G_{87}^2 - (G_{86}^2 + G_{87}^2)(\rho^2 - G_{86}^2)}}{-\rho G_{86} \mp \sqrt{\rho^2 G_{86}^2 - (G_{86}^2 + G_{87}^2)(\rho^2 - G_{87}^2)}} \tag{2.82}$$

The solutions of (2.81) satisfying equation (2.80) are the required solutions.

In order to have real values for sin τ and cos τ the clearance d cannot be arbitrarily large, it should satisfy the relation

$$\rho^2 \leq (G_{86}^2 + G_{87}^2)$$

The physical interpretation of this restriction is that for d exceeding this limit, the actual system will not have a two-impact-per-cycle (equally spaced or not) steady state motion.

The two sets of signs appearing in equation (2.82) correspond to two distinct steady state solutions.

Since the conditions that were used to obtain equation (2.82) are the exact conditions that must be met by the system if it is in the steady state motion with two impacts per cycle, then, as seen from equation (2.82), there are two possible steady state solutions. The analytical criteria of deciding which solution will be valid, if any, will be furnished by the stability analysis of the solutions. Such an analysis is carried out in Chapter 3.

With the values of τ and n determined from (2.80) and (2.82), B_1, B_1', B_2 and B_2' can be found from (2.78), (2,79), (2.72) and (2.73). Since

$$x = e^{-\delta\omega t}\left(B_1 \sin(\eta\omega t) + B_2 \cos(\eta\omega t)\right) + A\sin(\Omega t + \tau) \qquad 0_+ \leqslant t \leqslant \left(\frac{n\pi}{\Omega}\right)_-$$

and

$$x = e^{-\delta\omega(t - \frac{n\pi}{\Omega})}\left[\grave{B}_1 \sin\left(\eta\omega\left(t - \frac{n\pi}{\Omega}\right)\right) + \grave{B}_2 \cos\left(\eta\omega\left(t - \frac{n\pi}{\Omega}\right)\right)\right] + A\sin(\Omega t + \tau) \qquad \left(\frac{n\pi}{\Omega}\right)_+ \leqslant t \leqslant \left(\frac{2\pi}{\Omega}\right)_-$$

then the motion of the primary system is determined.

2.3. Special Case;

For the special case of n = 1 (symmetric two impact-cycle motion), the steady state solution (8) is considerably simpler. In this case,

$$x_a = x_b = -x_g = -x_h \;, \quad \dot{x}_h = -\dot{x}_a \;, \quad \dot{x}_g = -\dot{x}_b \;,$$
$$\grave{B}_1 = -B_1 \;, \quad \grave{B}_2 = -B_2$$

The relation between x_b, x_b, x_a, B_1, B_2, A and τ can be put in the matrix form

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & -1 & -S \\ 0 & 0 & 1 & -\eta\omega & \delta\omega & -C \\ 1 & 0 & 0 & h_1 & h_2 & -S \\ 0 & 1 & 0 & \theta_1 & \theta_2 & -C \\ 1 & \sigma_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \sigma_2 & 0 & 0 & 0 \end{array}\right] \left\{\begin{array}{c} x_b \\ \dot{x}_b \\ \dot{x}_a \\ B_1 \\ B_2 \\ A \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{d}{2} \\ -\frac{d}{2} \end{array}\right\} \quad (2.83)$$ ◆

From the solution of (2.83), expressions for A, B_1, B_2, in terms of the known parameters can be obtained. Thus,

$$A = \frac{N(\Delta)}{\Delta} \quad (2.84)$$

$$B_1 = \frac{N(B_1)}{\Delta} \quad (2.85)$$

$$B_2 = \frac{N(B_2)}{\Delta} \quad (2.86)$$

where

$$N(A) = \frac{d}{2}\left[h_1(\sigma_1\theta_2 - \sigma_2\omega\delta) - (\sigma_1\theta_1 + \eta\sigma_2\omega)(1+h_2)\right]$$

$$N(B_1) = \frac{d}{2}(1+h_2)(\sigma_2 - \sigma_1)\,C$$

$$N(B_2) = \frac{d}{2}\,h_1(\sigma_1 - \sigma_2)\,C$$

◆ Symbols in (2.83) are given in Appendix V.

$$\Delta = h_1\left[C(\sigma_2 - \sigma_1) - (S + C\sigma_2)\sigma_1\theta_1 + (S + C\sigma_1)\delta\omega\sigma_2\right] + (1+h_2)\left[(S + C\sigma_2)\sigma_1\theta_1 + (S + C\sigma_1)\eta\omega\sigma_2\right]$$

Equation (2.84) can be put in the form

$$2\sin\tau + H\cos\tau = -\rho \qquad (2.87)$$

where

$$H = 2\Omega\left[\frac{((\sigma_2-\sigma_1)+\sigma_1\sigma_2(\delta\omega-\theta_2))h_1+(\sigma_1\sigma_2(\theta_1+\eta\omega))(1+h_2)}{(\delta\sigma_2\omega-\sigma_1\theta_2)h_1+(\sigma_1\theta_1+\eta\sigma_2\omega)(1+h_2)}\right]$$

solution of Equation (2.87) for τ results in:

$$\sin\tau = \frac{-2\rho \pm H\sqrt{H^2+4-\rho^2}}{H^2+4}$$

$$\cos\tau = \frac{-\rho H \mp 2\sqrt{H^2+4-\rho^2}}{H^2+4}$$

$$\tau = \tan^{-1}\left[\frac{-2\rho \pm H\sqrt{H^2+4-\rho^2}}{-\rho H \mp 2\sqrt{H^2+4-\rho^2}}\right] \qquad (2.88)$$

Again, in order to have real values of sin τ and cos τ , the clearance d cannot be arbitrarily large; it should satisfy the relation $\rho^2 \leq H^2 + 4$.

CHAPTER 3

3. STABILITY

3.1. Theoretical considerations :

Now that we have a particular steady state solution for the system under consideration, we can proceed to investigate the stability of this solution. The type of stability that we are concerned with in this case is asymptotic stability.

Let the differential equation of motion of our system be expressed in the form

$$\dot{\vec{Z}} = \vec{F}(Z_1, \ldots, Z_4, t) \tag{3.1}$$

and let a particular solution of (3.1) be

$$\vec{Z} = \vec{S}(t) \tag{3.2}$$

If this solution is perturbed slightly, so that

$$\vec{Z}_p = \vec{S}(t) + \vec{\xi}(t) \tag{3.3}$$

the solution is said to be asymptotically stable if

$$\lim_{t \to \infty} \left| \xi_i(t) \right| = 0 \quad \text{for} \quad i = 1, \ldots, 4 \tag{3.4}$$

So we perturb our solution immediately after one impact, and then determine the deviation of the resulting solution from the steady state conditions immediately after the following impact. By repeating this process over and over again, we can determine the propagation (similar to change with time) of the initial perturbations in the solution. The stability or instability of the solution is determined by whether or not the deviations from the steady state solution decay or grow, as the number of impacts is increased indefinitely (i.e. $t \to \infty$).

The differential equations of motion of our system, between impacts

$$\ddot{x} = -\omega^2 x - 2\delta\omega\dot{x} + \frac{F_o}{M}\sin\Omega t$$

$$\ddot{y}_1 = 0$$

can be put in the form

$$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x}, t) \qquad (3.5)$$

where

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad \vec{f}(\vec{x}, t) = \begin{pmatrix} f_1(x, t) \\ f_2(x, t) \\ f_3(x, t) \\ f_4(x, t) \end{pmatrix}$$

and

$$x_1 = x \qquad f_1 = x_2$$

$$x_2 = \dot{x} \qquad f_2 = -\omega^2 x_1 - 2\delta\omega x_2 + \frac{F_o}{M}\sin\Omega t$$

$$x_3 = y_1 \qquad f_3 = x_4$$

$$x_4 = \dot{y}_1 \qquad f_4 = 0$$

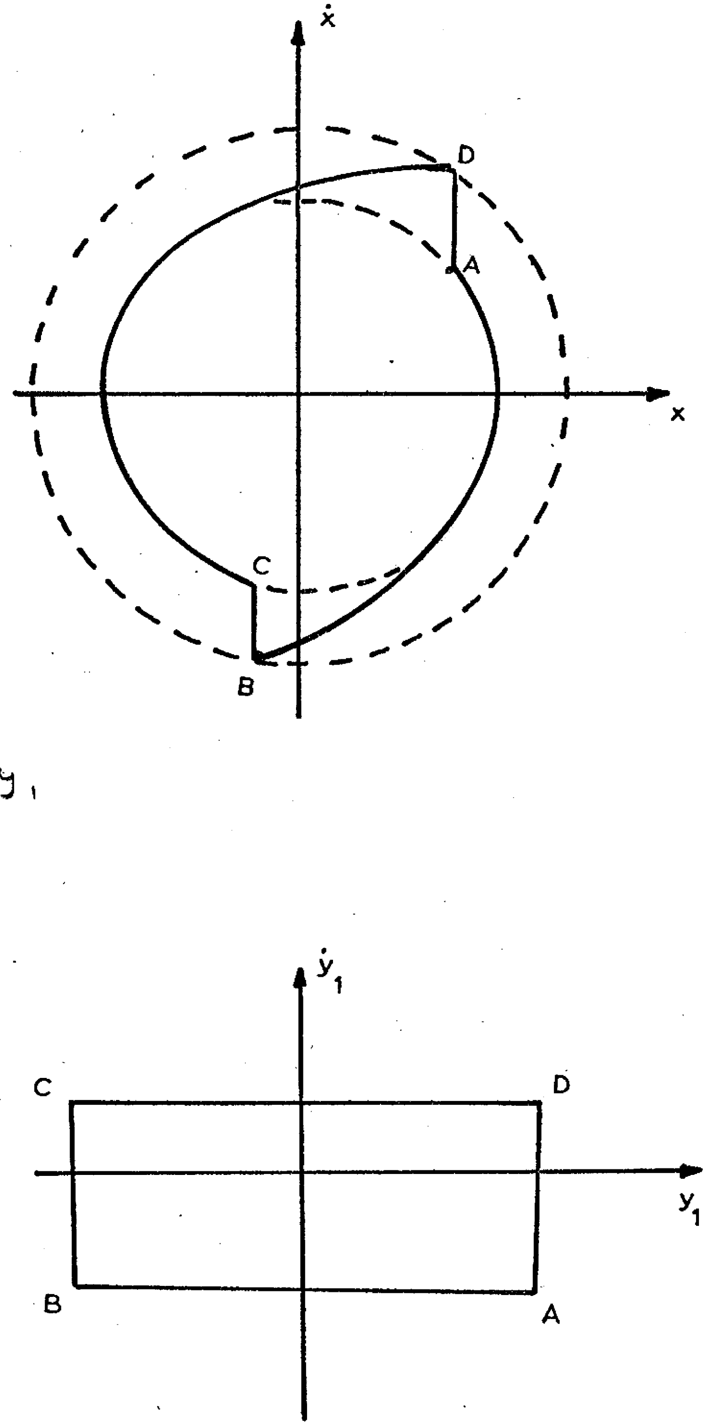
The first partial derivatives of the four functions $f_i(\vec{x}, t)$ $i = 1,\ldots,4$ with respect to their five variables, $x_1, \ldots x_4$, t, exist and are continuous (8) and if the initial conditions are specified then the solution exists and is unique.

This type of motion can be represented in the phase plane by a periodic process as shown in Fig. 3.1. On the analytic trajectories AB and CD, the motion of the system is governed by equation (3.5). On the stretches BC and DA, where the small impact time is idealized to be infinitesimally small, equation (3.5) does not apply, but the motion of the system is determined by the impact conditions, equations (2.18) and (2.19). These equations relate the conditions at C and A to those at B and D, respectively.

Now let the solution curve be perturbed slightly right after an impact, e.g. at A, then the perturbations at point A are continuously related to the perturbations at point C.

If at $\Omega t = (\Delta t_o)_+$ the steady state solution is perturbed by a small amount $\vec{\xi}(0)$, then the time of the next impact, $\Omega t = n\pi + \Delta t_o'$ is determined by a relation of the form

$$\Delta t_o' = g(\vec{S}, \vec{\xi}_{(0)}, \Delta t_o')$$



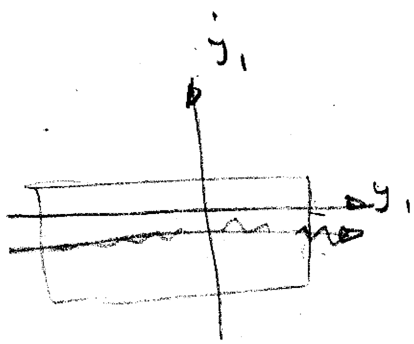


Fig. (3.1) Phase plane representation of periodic 2 unsymmetric impacts per cycle motion or

$$G(\vec{S}, \vec{\xi}_{(0)}, \Delta t_o') = 0 \qquad (3.6)$$

The deviation of the solution from the steady state at $\Omega t = (n\pi + \Delta t_o')_+$ can be put in the form

$$\vec{\xi}_{(1)} = P_1 \vec{\xi}_{(0)} + \vec{R}_1(\vec{\xi}_{(0)})$$

The deviation of the solution from steady state at $\Omega t = (2\pi + \Delta t_o'')_+$ can be put in the form

$$\vec{\xi}_{(2)} = P_2 \vec{\xi}_{(1)} + \vec{R}_2(\vec{\xi}_{(1)})$$
$$= P_2 P_1 \vec{\xi}_{(0)} + \vec{R}'(\vec{\xi}_{(0)})$$

where P_1 and P_2 are constant matrices, and $\vec{R}_1$, $\vec{R}_2$ contain all terms of ξ_i higher than the first power.

Since the two impacts per cycle solution repeats itself after intervals of $\Omega t = 2\pi$, the perturbation at $t = (4\pi + \Delta t_o''')_+$ will be

$$\vec{\xi}_{(4)} = P_2 P_1 \vec{\xi}_{(2)} + \vec{R}''(\vec{\xi}_{(0)})$$

By following the perturbed solution from one impact to the next one, we obtain the continuous transformation

$$\vec{\xi}_{(2n+2)} = P_2 P_1 \vec{\xi}_{(2n)} + \vec{R}(\vec{\xi}_{(2n)}) \qquad (3.7)$$

$$= (P_2 P_1)^{n+1} \vec{\xi}_{(0)} + \vec{R}' (\vec{\xi}_{(0)}) \qquad (3.8)$$

It is worth noting that if the sign of P did change after each impact, the net effect would be to multiply P in (3.7) by $(-1)^{n+1}$. This has no effect on the stability criteria, which depends on (8) the modulus of the eigenvalues of P. $(P = P_2 . P_1)$

Consider the linear part of equation (3.8), i.e.

$$\vec{\xi}_{(2n+2)} = (P_2 P_1)^{n+1} \vec{\xi}_{(0)} \qquad (3.9)$$

Equation (3.9) will be asymptotically stable if

$$\lim_{n \to \infty} P^n = 0$$

The requirement that

$$\lim_{n \to \infty} P^n = 0$$

is satisfied if and only if (8) all the eigenvalues of P have modulus less than unity, i.e. if

$$|\lambda_i| < 1 \qquad (i = 1, \ldots, 4)$$

Thus, our problem is to determine P and to examine its eigenvalues.

Since our system has two degrees of freedom, $\vec{\xi}$ should be 4-component vector. A proper choice of the components can be the two displacements and the two velocities. However, it is more natural for this system if the phase angle between the motion of the particle and M is used as one of the components, instead of the displacement of the particle. In other words, the initial perturbation will consist of small variation of the steady state value of $x, \dot{x}, v$ and τ.

The conditions that we have are:

Steady State	Perturbed
At $\Omega t = 0_+$	At $\Omega t = (0 + \Delta t_o)_+$
$x = x_{a_o}$	$x = x_{a_o} + \Delta x_{a_o}$
$y = \frac{d}{2}$	$y = \frac{d}{2}$
$\dot{x} = \dot{x}_{a_o}$	$\dot{x} = \dot{x}_{a_o} + \Delta \dot{x}_{a_o}$
$v = -V_{1_o}$	$v = -(V_{1_o} + \Delta V_{1_o})$
$\tau = \tau_o$	$\tau = \tau_o + \Delta \tau_o$
At $\Omega t = (n\pi)_+$	At $\Omega t = (n\pi + \Delta t_o')_+$
$x = -x_{h_o}$	$x = -(x_{h_o} + \Delta x_{h_o})$
$y = -\frac{d}{2}$	$y = -\frac{d}{2}$
$\dot{x} = -\dot{x}_{h_o}$	$\dot{x} = -(\dot{x}_{h_o} + \Delta \dot{x}_{h_o})$
$v = V_{2_o}$	$v = V_{2_o} + \Delta V_{2_o}$

$\tau = \tau_o$ $\qquad\qquad$ $\tau = \tau_o + \Delta\tau_o'$

At $\Omega t = (2\pi)_+$	At $\Omega t = (2\pi + \Delta t_o'')_+$
$x = x_{a_o}$	$x = x_{a_o} + \Delta x_{a_o}'$
$y = \frac{d}{2}$	$y = \frac{d}{2}$
$\dot{x} = \dot{x}_{a_o}$	$\dot{x} = \dot{x}_{a_o} + \Delta\dot{x}_{a_o}'$
$v = -V_{1_o}$	$v = -(V_{1_o} + \Delta V_{1_o}')$
$\tau = \tau_o$	$\tau = \tau_o + \Delta\tau_o''$

3.2 . Determination of P_1

Without any loss of generality, ω and $\frac{F_o}{k}$ can each be taken as unity. Then for the general case ($\Omega \neq 1$, $\delta \neq 0$)

$$x = e^{-\delta t}(B_1 \sin \eta t + B_2 \cos \eta t) + A \sin(\Omega t + \tau) \qquad 0_+ \leq t \leq \frac{n\pi}{\Omega} \tag{3.10}$$

$$\dot{x} = e^{-\delta t}(-\delta \sin \eta t + \eta \cos \eta t) B_1 + e^{-\delta t}(-\delta \cos \eta t - \eta \sin \eta t) B_2 + A\Omega \cos(\Omega t + \tau) \qquad 0_+ \leq t \leq \frac{n\pi}{\Omega} \tag{3.11}$$

Now

$$x(0_+) = x_{a_o} = B_{2_o} + A\sin\tau_o \qquad (3.12)$$

$$\dot{x}(0_+) = \dot{x}_{a_o} = \eta B_{1_o} - \delta B_{2_o} + A\Omega\cos\tau_o \qquad (3.13)$$

where the 0 subscript refers to the unperturbed conditions (i.e., $x_{oa} = x_a$, $\dot{x}_{oa} = \dot{x}_a$, etc.)

From (3.12) and (3.13)

$$B_{2_o} = x_{a_o} - A\sin\tau_o \qquad (3.14)$$

and

$$B_{1_o} = \frac{1}{\eta}(\dot{x}_{a_o} + \delta B_{2_o} - A\Omega\cos\tau_o) \qquad (3.15)$$

In finding the perturbed values of B_{1_o} and B_{2_o} the quantities with 0 subscript in the above equations should be replaced by their perturbed values. Thus,

$$B_2(0+\Delta t_o) = (x_{a_o} + \Delta x_{a_o}) - A\sin(\tau_o + \Delta\tau_o) \qquad (3.16)$$

and

$$B_1(0+\Delta t_o) = \frac{1}{\eta}\Big[(\dot{x}_{a_o} + \Delta\dot{x}_{a_o}) + \delta B_2(0+\Delta t_o) - A\Omega\cos(\tau_o + \Delta\tau_o)\Big] \qquad (3.17)$$

Since $\Delta\tau_o$ is a small quantity of order ϵ, then to first order approximation.

$$B_2(0+\Delta t_o) = B_{2_o} + \Delta x_{a_o} - A\,\Delta\tau_o \cos\tau_o \qquad (3.18)$$

and

$$B_1(0+\Delta t_o) = B_{1_o} + \frac{\delta}{\eta}\Delta x_{a_o} + \frac{1}{\eta}\Delta \dot{x}_{a_o} + a\,\Delta\tau_o \qquad (3.19)$$

where

$$a \equiv \frac{A}{\eta}(\Omega \sin\tau_o - \delta\cos\tau_o)$$

Equation (3.10) describes the motion of the primary system immediately after $t = \frac{0 + \Delta t_o}{\Omega}$ to immediately prior to $t = \frac{n\pi + \Delta t'_o}{\Omega}$. Thus, the time during which equation (3.10) is applicable is $\Omega t = n\pi + \Delta T$ where $\Delta T = (\Delta t'_o - \Delta t_o)$. Hence,

$$-(x_{h_o} + \Delta x_{h_o}) = e^{-\delta\left(\frac{n\pi+\Delta T}{\Omega}\right)}\Big[B_1(0+\Delta t_o)\sin\Big(\eta\Big(\frac{n\pi+\Delta T}{\Omega}\Big)\Big) + B_2(0+\Delta t_o)\cos\Big(\eta\Big(\frac{n\pi+\Delta T}{\Omega}\Big)\Big)\Big] + A\sin(n\pi+\Delta T+\tau_o+\Delta\tau_o)$$

$$= C_o + C_1\Delta x_{a_o} + C_2\Delta\dot{x}_{a_o} + C_3\Delta T + C_4\Delta\tau_o \qquad (3.20)$$

where

$$C_0 \equiv e^{-\frac{\delta n\pi}{\Omega}} (B_{1_0} \sin(\tfrac{n\pi}{\Omega}\eta) + B_{2_0} \cos(\tfrac{n\pi}{\Omega}\eta)) + A \sin(n\pi + \tau_0)$$

$$C_1 \equiv e^{-\frac{\delta n\pi}{\Omega}} (\frac{\delta}{\eta} \sin(\tfrac{n\pi}{\Omega}\eta) + \cos(\tfrac{n\pi}{\Omega}\eta))$$

$$C_2 \equiv e^{-\frac{\delta n\pi}{\Omega}} (\frac{1}{\eta} \sin(\tfrac{n\pi}{\Omega}\eta))$$

$$C_3 \equiv b_3 + (b_1 + b_2) e^{-\frac{\delta n\pi}{\Omega}} + A \cos(n\pi + \tau_0)$$

$$C_4 \equiv e^{-\frac{\delta n\pi}{\Omega}} (a \sin(\tfrac{n\pi}{\Omega}\eta) - A \cos\tau_0 \cos(\tfrac{n\pi}{\Omega}\eta)) + A \cos(n\pi + \tau_0)$$

$$b_1 \equiv B_{1_0} \frac{\eta}{\Omega} \cos(\tfrac{n\pi}{\Omega}\eta)$$

$$b_2 \equiv -B_{2_0} \frac{\eta}{\Omega} \sin(\tfrac{n\pi}{\Omega}\eta)$$

$$b_3 \equiv (-\frac{\delta}{\Omega} e^{-\frac{\delta n\pi}{\Omega}})(B_{1_0} \sin(\tfrac{n\pi}{\Omega}\eta) + B_{2_0} \cos(\tfrac{n\pi}{\Omega}\eta))$$

Since

$$C_0 = x(\tfrac{n\pi}{\Omega}) = -X_{h_0}$$

equation (3.20) reduces to

$$\Delta X_{h_0} = -C_1 \Delta x_{a_0} - C_2 \Delta \dot{x}_{a_0} - C_3 \Delta T - C_4 \Delta \tau_0 \qquad (3.21)$$

The time required by the particle to travel from one end of the container (where $y = \frac{d}{2}$) to the other end (where $y = -\frac{d}{2}$) is the absolute distance travelled divided by the absolute velocity. Hence,

$$\frac{(n\pi+\Delta \grave{t}_o) - \Delta t_o}{\Omega} = \frac{y(n\pi+\Delta \grave{t}_o) + x(n\pi+\Delta \grave{t}_o) - (y(\Delta t_o) + x(\Delta t_o))}{-(V_{1_o} + \Delta V_{1_o})} \tag{3.22}$$

substituting for the values of the quantities on the left hand side of (3.22) then

$$\Delta T = -\frac{n\pi}{V_{1_o}}\Delta V_{1_o} + \frac{\Omega}{V_{1_o}}\Delta X_{h_o} + \frac{\Omega}{V_{1_o}}\Delta X_{a_o} \tag{3.23}$$

Replacing ΔT in (3.21) by its value form (3.23) we obtain

$$\Delta X_{h_o} = d_5 \Delta X_{a_o} + \frac{V_{1_o}}{\Omega} d_2 \Delta \dot{X}_{a_o} - C_3 d_3 \Delta V_{1_o} + \frac{V_{1_o}}{\Omega} d_4 \Delta \tau_o \tag{3.24}$$

where

$$d_5 = -\frac{\Omega C_3 + C_1 V_{1_o}}{V_{1_o} + \Omega C_3}$$

$$d_2 = -\frac{\Omega C_2}{V_{1_o} + \Omega C_3}$$

$$d_3 = -\frac{n\pi}{V_{1_0} + \Omega C_3}$$

$$d_4 = -\frac{\Omega C_4}{V_{1_0} + \Omega C_3}$$

If (3.24) is now substituted in (3.23), then

$$\Delta T = d_1 \Delta x_{a_0} + d_2 \Delta \dot{x}_{a_0} + d_3 \Delta V_{1_0} + d_4 \Delta \tau_0 \qquad (3.25)$$

where

$$d_1 \equiv \frac{\Omega(1 - C_1)}{V_{1_0} + \Omega C_3}$$

since

$$\Delta \tau_0' = \Delta \tau_0 + \Delta T$$

Then by using (3.25)

$$\Delta \tau_0' = d_1 \Delta x_{a_0} + d_2 \Delta \dot{x}_{a_0} + d_3 \Delta V_{1_0} + (1 + d_4) \Delta \tau_0 \qquad (3.26)$$

Using equation (3.11) to find the velocity at $t = \left(\frac{n\pi + \Delta t_0'}{\Omega}\right)_-$, then

$$\dot{x}\left(\frac{n\pi + \Delta t_0'}{\Omega}\right)_- = e^{-\delta\left(\frac{n\pi + \Delta T}{\Omega}\right)} \left[-\delta \sin\left(\eta\left(\frac{n\pi + \Delta T}{\Omega}\right)\right) + \eta \cos\left(\eta\left(\frac{n\pi + \Delta T}{\Omega}\right)\right)\right] B_1(0 + \Delta t_0) +$$

$$e^{-\delta\left(\frac{n\pi+\Delta T}{\Omega}\right)}\left[-\delta\cos\left(\eta\left(\frac{n\pi+\Delta T}{\Omega}\right)\right)-\eta\sin\left(\eta\left(\frac{n\pi+\Delta T}{\Omega}\right)\right)\right]B_2(0+\Delta t_0)+A\Omega\cos(n\pi+\Delta T+\tau_0+\Delta\tau_0)$$

$$=\rho_0+\rho_1\Delta x_{3_0}+\rho_2\Delta\dot{x}_{3_0}+\rho_3\Delta T+\rho_4\Delta\tau_0 \qquad (3.27)$$

where

$$\rho_0\equiv g_0 e^{-\delta\frac{n\pi}{\Omega}}+A\Omega\cos(n\pi+\tau_0)$$

$$\rho_1\equiv g_1 e^{-\delta\frac{n\pi}{\Omega}}$$

$$\rho_2\equiv g_2 e^{-\delta\frac{n\pi}{\Omega}}$$

$$\rho_3\equiv e^{-\delta\frac{n\pi}{\Omega}}\left(g_3-\frac{\delta}{\Omega}g_0\right)-A\Omega\sin(n\pi+\tau_0)$$

$$\rho_4\equiv g_4 e^{-\delta\frac{n\pi}{\Omega}}-A\Omega\sin(n\pi+\tau_0)$$

$$g_0\equiv-(f_1B_{1_0}+f_2B_{2_0})$$

$$g_1\equiv-\left(f_1\frac{\delta}{\eta}+f_2\right)$$

$$g_2\equiv-\frac{f_1}{\eta}$$

$$g_3\equiv\frac{\eta}{\Omega}(f_1B_{2_0}-f_2B_{1_0})$$

$$g_4 \equiv f_2 A \cos \tau - a f_1$$

$$f_1 \equiv \delta \sin\left(\eta \frac{n\pi}{\Omega}\right) - \eta \cos\left(\eta \frac{n\pi}{\Omega}\right)$$

$$f_2 \equiv \delta \cos\left(\eta \frac{n\pi}{\Omega}\right) + \eta \sin\left(\eta \frac{n\pi}{\Omega}\right)$$

From equations (2.18) and (2.19)

$$\dot{x}_+ = (k_1 \dot{x}_-) + (k_2 v_-)$$

and

$$\dot{v}_+ = (k_3 \dot{x}_-) + (k_4 v_-)$$

where

$$k_1 \equiv \frac{(1-\mu e)}{(1+\mu)}$$

$$k_2 \equiv \frac{\mu(1+e)}{(1+\mu)}$$

$$k_3 \equiv \frac{(1+e)}{(1+\mu)}$$

$$k_4 \equiv \frac{(\mu - e)}{(1+\mu)}$$

Hence,

$$\dot{x}\left(\frac{n\pi+\Delta t_0}{\Omega}\right)_+ = -\dot{x}_{h_0} - \Delta\dot{x}_{h_0}$$

$$= k_1 (p_1 \Delta x_{a_0} + p_2 \Delta \dot{x}_{a_0} + p_3 \Delta T + p_4 \Delta \tau_0)$$

$$- k_2 \Delta V_{1_0} + k_1 p_0 - k_2 V_{1_0}$$

Also,

$$V\left(\frac{n\pi + t_o'}{\Omega}\right)_- = V_{2_o} + \Delta V_{2_o}$$

$$= k_3(p_1 \Delta x_{a_o} + p_2 \Delta \dot{x}_{a_o} + p_3 \Delta T + p_4 \Delta \tau) - k_4 \Delta V_{1_o} + k_3 p_o - k_4 V_{1_o}$$

noting that

$$k_1 p_o - k_2 V_{1_o} = k_1 \dot{x}\left(\frac{n\pi}{\Omega}\right)_- + k_2(-V_{1_o}) = -\dot{x}_{h_o}$$

and

$$k_3 p_o - k_4 V_{1_o} = k_3 \dot{x}\left(\frac{n\pi}{\Omega}\right)_- + k_4\left(V\left(\frac{n\pi}{\Omega}\right)\right) = V_{2_o}$$

and replacing ΔT by its value from (3.25), then

$$\Delta \dot{x}_{h_o} = -k_1(p_1 + p_3 d_1)\Delta x_{a_o} - k_1(p_2 + p_3 d_2)\Delta \dot{x}_{a_o} + (k_2 - k_1 p_3 d_3)\Delta V_{1_o} - k_1(p_4 + p_3 d_4)\Delta \tau \qquad (3.28)$$

and

$$\Delta V_{2_o} = k_3(p_1 + p_3 d_1)\Delta x_{a_o} + k_3(p_2 + p_3 d_2)\Delta \dot{x}_{a_o} + (k_3 p_3 d_3 - k_4)\Delta V_{1_o} + k_3(p_4 + p_3 d_4)\Delta \tau \qquad (3.29)$$

Equations (3.24),(3.28), (3.29) and (3.26) can be expressed in a matrix form as:

$$\begin{Bmatrix} \Delta x_{h_o} \\ \Delta \dot{x}_{h_o} \\ \Delta V_{2_o} \\ \Delta \tau_o \end{Bmatrix} = \begin{bmatrix} d_5 & \frac{V_{1o}}{\Omega} d_2 & -C_3 d_3 & \frac{V_{1o}}{\Omega} d_4 \\ -k_1(p_1 + p_3 d_1) & -k_1(p_2 + p_3 d_2) & k_2 - k_1 p_3 d_3 & -k_1(p_4 + p_3 d_4) \\ k_3(p_1 + p_3 d_1) & k_3(p_2 + p_3 d_2) & k_3 p_3 d_3 - k_4 & k_3(p_4 + p_3 d_4) \\ d_1 & d_2 & d_3 & 1 + d_4 \end{bmatrix} \begin{Bmatrix} \Delta x_{a_o} \\ \Delta \dot{x}_{a_o} \\ \Delta V_{1_o} \\ \Delta \tau_o \end{Bmatrix} \tag{3.30}$$

3.3. Determination of P_2

For the general case

$$x = e^{-\delta(t - \frac{n\pi}{\Omega})} \left[B_1' \sin(\eta(t - \frac{n\pi}{\Omega})) + B_2' \cos(\eta(t - \frac{n\pi}{\Omega})) \right] + A \sin(\Omega t + \tau) \qquad (3.31)$$

$$\dot{x} = e^{-\delta(t - \frac{n\pi}{\Omega})} \left[-\delta \sin(\eta(t - \frac{n\pi}{\Omega})) + \eta \cos(n(t - \frac{n\pi}{\Omega})) \right] B_1' + e^{-\delta(t - \frac{n\pi}{\Omega})} \left[-\delta \cos(\eta(t - \frac{n\pi}{\Omega})) - \eta \sin(\eta(t - \frac{n\pi}{\Omega})) \right] B_2' + A\Omega \cos(\Omega t + \tau) \qquad (3.32)$$

Now

$$x_{(n\pi_+)} = -x_{h_0} = B_{2_0}' + A \sin(n\pi + \tau_0) \qquad (3.33)$$

and

$$\dot{x}_{(n\pi_+)} = -\dot{x}_{h_0} = \eta B_{1_0}' - \delta B_{2_0}' + A\Omega \cos(n\pi + \tau_0) \qquad (3.34)$$

From (3.33) and (3.34) we get

$$B_{2_0}' = -x_{h_0} - A \sin(n\pi + \tau_0) \qquad (3.35)$$

and

$$\grave{B}_{1_o} = \frac{1}{\eta}\left(-\dot{x}_{h_o} + \delta \grave{B}_{2_o} - A\Omega\cos(n\pi + \tau_o)\right) \qquad (3.36)$$

In finding the perturbed values of $\grave{B}_{1_o}$ and $\grave{B}_{2_o}$ the quantities with 0 subscripts in the above equation should be replaced by their perturbed values. Thus,

$$\grave{B}_2(n\pi + \Delta\grave{t}_o) = -(x_{h_o} + \Delta x_{h_o}) - A\sin(n\pi + \tau_o + \Delta\grave{\tau}_o) \qquad (3.37)$$

and

$$\grave{B}_1(n\pi + \Delta\grave{t}_o) = \frac{1}{\eta}\left[-(\dot{x}_{h_o} + \Delta\dot{x}_{h_o}) + \delta \grave{B}_{2(n\pi + \Delta\grave{t}_o)} - A\Omega\cos(n\pi + \tau_o + \Delta\grave{\tau}_o)\right] \qquad (3.38)$$

since $\Delta\tau_o$ is a small quantity of order ϵ, then to first order approximation,

$$\grave{B}_2(n\pi + \Delta\grave{t}_o) = \grave{B}_{2_o} - \Delta x_{h_o} - A\Delta\grave{\tau}_o\cos(n\pi + \tau_o) \qquad (3.39)$$

and

$$\grave{B}_1(n\pi + \Delta\grave{t}_o) = \grave{B}_{1_o} - \frac{\delta}{\eta}\Delta x_{h_o} - \frac{1}{\eta}\Delta\dot{x}_{h_o} + \grave{a}\,\Delta\grave{\tau}_o \qquad (3.40)$$

where

$$\grave{a} \equiv \frac{A}{\eta}\left[-\delta\cos(n\pi + \tau_o) + \Omega\sin(n\pi + \tau_o)\right]$$

Equation (3.31) describes the motion of the primary system immediately after $t = \frac{n\pi + \Delta\grave{t}_o}{\Omega}$ to immediately prior to $t = \frac{2\pi + \Delta t_o''}{\Omega}$. Thus, the time during which equation (3.31) is applicable is $\Omega t = (2\pi - n\pi) + \Delta T'$ where $\Delta T' = (\Delta t_o'' - \Delta t_o')$

From equation (3.31) we get

$$X_{a_o} + \Delta X_{a_o} = e^{-\delta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)} \Big[B'_{1(n\pi + \Delta t_o')} \sin\left(\eta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)\right) + B'_{2(n\pi + \Delta t_o')} \cos\left(\eta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)\right)\Big] + A \sin(2\pi + \Delta T + \tau_o + \Delta\tau_o')$$

$$= C'_o + C'_1 \Delta X_{h_o} + C'_2 \Delta \dot{X}_{h_o} + C'_3 \Delta T' + C'_4 \Delta \tau_o' \qquad (3.41)$$

where

$$C'_o \equiv e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \Big[B'_{1_o} \sin\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right) + B'_{2_o} \cos\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right)\Big] + A \sin \tau_o$$

$$C'_1 \equiv e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \Big[-\frac{\delta}{\eta} \sin\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right) - \cos\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right)\Big]$$

$$C'_2 \equiv e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \Big[-\frac{1}{\eta} \sin\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right)\Big]$$

$$C'_3 \equiv -\Big[e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} (b'_1 + b'_2) + b'_3 + A \cos \tau \Big]$$

$$\dot{C}_4 = e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \left(\dot{a} \sin\left(\eta\left(\frac{2\pi + n\pi}{\Omega}\right)\right) - A\cos(n\pi + \tau_o)\cos\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right)\right) + A\cos\tau$$

$$\dot{b}_1 \equiv \dot{B}_{1_o}\frac{\eta}{\Omega}\cos\left(\frac{2\pi - n\pi}{\Omega}\right)$$

$$\dot{b}_2 \equiv -\dot{B}_{2_o}\frac{\eta}{\Omega}\sin\left(\frac{2\pi - n\pi}{\Omega}\right)$$

$$\dot{b}_3 \equiv -\frac{\delta}{\Omega} e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \dot{B}_{1_o}\sin\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right) + \dot{B}_{2_o}\cos\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right)$$

Since

$$C_o = x\left(\frac{2\pi}{\Omega}\right)_+ = x_{a_o}$$

equation (3.41) reduces to

$$\Delta \dot{x}_{a_o} = -\dot{C}_1 \Delta x_{h_o} - \dot{C}_2 \Delta \dot{x}_{h_o} - \dot{C}_3 \Delta \dot{T} - \dot{C}_4 \Delta \dot{\tau}_o \qquad (3.42)$$

The time required by the particle to travel from one end of the container (where $y = -\frac{d}{2}$) to the other end (where $y = \frac{d}{2}$) equals the absolute distance travelled, divided by the absolute velocity. Hence,

$$\frac{(2\pi - n\pi + \Delta t''_o) - \Delta \dot{t}_o}{\Omega} = \frac{[y(2\pi + \Delta t''_o) + x(2\pi + \Delta t''_o)] - [y(n\pi + \Delta \dot{t}_o) + x(n\pi + \Delta \dot{t}_o)]}{V_{2_o} + \Delta V_{2_o}}$$

Thus,

$$\frac{2\pi - n\pi + \Delta T'}{\Omega} = \frac{d + X_{a_o} + \Delta X'_{a_o} - (-X_{h_o} + \Delta X_{h_o})}{V_{2_o} + \Delta V_{2_o}} \qquad (3.43)$$

substituting for the values of the quantities on the left hand side of (3.43), then

$$\Delta T' = \frac{\Omega}{V_{2_o}} \Delta X_{a_o} + \frac{\Omega}{V_{2_o}} \Delta X_{h_o} - \frac{2\pi - n\pi}{V_{2_o}} \Delta V_{2_o} \qquad (3.44)$$

Replacing $\Delta T'$ in (3.42) by its value from (3.44), we obtain

$$\Delta X'_{a_o} = d'_5 \Delta X_{h_o} + \frac{V_{2_o}}{\Omega} d'_2 \Delta \dot{X}_{h_o} - C'_3 d'_3 \Delta V_{2_o} + \frac{V_{2_o}}{\Omega} d'_4 \Delta \tau'_o \qquad (3.45)$$

where

$$d'_5 \equiv -\frac{C'_1 V_{2_o} + C'_3 \Omega}{V_{2_o} + \Omega C'_3}$$

$$d'_2 \equiv -\frac{\Omega C'_2}{V_{2_o} + \Omega C'_3}$$

$$d'_3 \equiv -\frac{2\pi - n\pi}{V_{2_o} + \Omega C'_3}$$

$$d'_4 \equiv -\frac{C'_4}{V_{2_o} + \Omega C'_3}$$

If (3.45) is now substituted in (3.44), then

$$\Delta T' = d'_1 \Delta x_{h_o} + d'_2 \Delta \dot{x}_{h_o} + \dot{d}'_3 \Delta V_{2_o} + d'_4 \Delta \tau''_o \qquad (3.46)$$

where

$$d'_1 = \frac{\Omega(1 - C'_1)}{V_{2_o} + \Omega C'_3}$$

Since

$$\Delta \tau''_o = \Delta \tau'_o + \Delta T'$$

then by using (3.46)

$$\Delta \tau''_o = d'_1 \Delta x_{h_o} + d'_2 \Delta \dot{x}_{h_o} + \dot{d}'_3 \Delta V_{2_o} + (1 + d'_4)\Delta \tau'_o \quad (3.47)$$

Using equation (3.32) to find the velocity at $t = (\frac{2\pi + \Delta t''_o}{\Omega})_-$, then

$$\dot{x}\left(\frac{2\pi + \Delta t''_o}{\Omega}\right)_- = e^{-\delta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)} \left[-\delta \sin\left(\eta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)\right) + \eta \cos\left(\eta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)\right)\right] B'_{1(n\pi + \Delta t'_o)} +$$

$$e^{-\delta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)} \left[-\delta \cos\left(\eta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)\right) - \eta \sin\left(\eta\left(\frac{2\pi - n\pi + \Delta T'}{\Omega}\right)\right)\right] B'_{2(n\pi + \Delta t'_o)}$$

$$+ A\Omega \cos(2\pi + \Delta T + \Delta \tau'_o + \tau'_o)$$

$$= \rho'_o + \rho'_1 \Delta x_{h_o} + \rho'_2 \Delta \dot{x}_{h_o} + \dot{\rho}'_3 \Delta T' + \rho'_4 \Delta \tau'_o \quad (3.48)$$

where

$$\grave{\rho}_0 \equiv e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \grave{g}_0 + A\Omega\cos\tau_0$$

$$\grave{\rho}_1 \equiv e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \grave{g}_1$$

$$\grave{\rho}_2 \equiv e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \grave{g}_2$$

$$\grave{\rho}_3 \equiv e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} \left(-\frac{\delta}{\Omega}\grave{g}_0 + \grave{g}_3\right) - A\Omega\sin\tau_0$$

$$\grave{\rho}_4 \equiv e^{-\delta\left(\frac{2\pi - n\pi}{\Omega}\right)} g_4 - A\Omega\sin\tau_0$$

$$\grave{g}_0 \equiv -\grave{f}_1 \grave{B}_{1_0} - \grave{f}_2 \grave{B}_{2_0}$$

$$\grave{g}_1 \equiv \grave{f}_1 \frac{\delta}{\eta} + \grave{f}_2$$

$$\grave{g}_2 \equiv \frac{\grave{f}_1}{\eta}$$

$$\grave{g}_3 \equiv -\grave{f}_2 \frac{\eta}{\Omega} \grave{B}_{1_0} + \grave{f}_1 \frac{\eta}{\Omega} \grave{B}_{2_0}$$

$$\grave{g}_4 \equiv -\grave{a}\,\grave{f}_1 + A\grave{f}_2\cos(\tau_0 + 2\pi)$$

$$\grave{f}_1 \equiv \delta\sin\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right) - \eta\cos\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right)$$

$$\grave{f}_2 \equiv \delta\cos\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right) + \eta\sin\left(\eta\left(\frac{2\pi - n\pi}{\Omega}\right)\right)$$

From equations (2.18) and (2.19) we get,

$$\dot{x}_+ = k_1 \dot{x}_- + k_2 V_-$$

and

$$V_+ = k_3 \dot{x}_- + k_4 V_-$$

Hence,

$$\dot{x}\frac{(2\pi + \Delta t_0'')}{\Omega}_+ = \dot{x}_{a_0} + \Delta \dot{x}'_{a_0}$$

$$= k_1(\rho'_1 \Delta x_{h_0} + \rho'_2 \Delta \dot{x}_{h_0} + \rho'_3 \Delta T' + \rho'_4 \Delta \tau'_0) + k_2 \Delta V_{2_0} + k_1 \rho'_0 + k_2 V_{2_0}$$

and

$$V\frac{(2\pi + \Delta t_0'')}{\Omega} = -(V_{1_0} + \Delta V_{1_0})$$

$$= k_3(\rho'_1 \Delta x_h + \rho'_2 \Delta \dot{x}_h + \rho'_3 \Delta T' + \rho'_4 \Delta \tau'_0) + k_4 \Delta V_{2_0} + k_3 \rho'_0 + k_4 V_{2_0}$$

Noting that

$$k_1 \rho'_0 + k_2 V_{2_0} = k_1 \dot{x}\left(\frac{2\pi}{\Omega}\right)_- + k_2(V_{2_0}) = \dot{x}_{a_0}$$

and

$$k_3 \rho'_0 + k_4 V_{2_0} = k_3 \dot{x}\left(\frac{2\pi}{\Omega}\right)_- + k_4 V\left(\frac{2\pi}{\Omega}\right)_- = -V_{1_0}$$

and replacing $\Delta T'$ by its value from (3.46), then

$$\Delta \dot{x}_{a_o} = k_1 (\rho'_1 + \rho'_3 d'_1) \Delta x_{h_o} + k_1 (\rho'_2 + \rho'_3 d'_2) \Delta \dot{x}_{h_o} + (k_2 + k_1 \rho'_3 d'_3) \Delta V_{2_o} + k_1 (\rho'_4 + \rho'_3 d'_4) \Delta \tau'_o \quad (3.49)$$

and

$$-\Delta V_{1_o} = k_3 (\rho'_1 + \rho'_3 d'_1) \Delta x_{h_o} + k_3 (\rho'_2 + \rho'_3 d'_2) \Delta \dot{x}_{h_o} + (k_3 \rho'_3 d'_3 + k_4) \Delta V_{2_o} + k_3 (\rho'_4 + \rho'_3 d'_4) \Delta \tau'_o \quad (3.50)$$

Equations (3.45), (3.49), (3.50) and (3.47) can be expressed in a matrix form as:

$$
\begin{Bmatrix} \Delta x'_{a_0} \\ \Delta \dot{x}'_{a_0} \\ \Delta V'_{1_0} \\ \Delta \tau''_0 \end{Bmatrix}
=
\begin{bmatrix}
d'_5 & \frac{V_{2_0}}{\Omega} d'_2 & C'_3 d'_3 & \frac{V_{2_0}}{\Omega} d'_4 \\
k_1(\rho'_1 + \rho'_3 d'_1) & k_1(\rho'_2 + \rho'_3 d'_2) & (k_2 + k_1 \rho'_3 d'_3) & k_1(\rho'_4 + \rho'_3 d'_4) \\
-k_3(\rho'_1 + \rho'_3 d'_1) & -k_3(\rho'_2 + \rho'_3 d'_2) & -k_3 \rho'_3 d'_3 - k_4 & -k_3(\rho'_4 + \rho'_3 d_4) \\
d'_1 & d'_2 & d'_3 & 1 + d'_4
\end{bmatrix}
\begin{Bmatrix} \Delta x_{h_0} \\ \Delta \dot{x}_{h_0} \\ \Delta V_{2_0} \\ \Delta \tau'_0 \end{Bmatrix}
\tag{3.51}
$$

From (3 30) and (3 51) we get

$$\begin{bmatrix} \Delta x'_{a_o} \\ \Delta \dot{x}'_{a_o} \\ \Delta V'_{1_o} \\ \Delta \tau''_o \end{bmatrix} = \begin{bmatrix} d'_5 & \frac{V_{2o}}{\Omega} d'_2 & -C'_3 d'_3 & \frac{V_{2o}}{\Omega} d'_4 \\ k_1(\rho'_1 + \rho'_3 d'_1) & k_1(\rho'_2 + \rho'_3 d'_2) & (k_2 + k_1 \rho'_3 d'_3) & k_1(\rho'_4 + \rho'_3 d'_4) \\ -k_3(\rho'_1 + \rho'_3 d'_1) & -k_3(\rho'_2 + \rho'_3 d'_2) & -k_3 \rho'_3 d'_3 - k_4 & -k_3(\rho'_4 + \rho'_3 d'_4) \\ d'_1 & d'_2 & d'_3 & 1 + d'_4 \end{bmatrix} \begin{bmatrix} d_5 & \frac{V_{1o}}{\Omega} d_2 & -C_3 d_3 & \frac{V_{2o}}{\Omega} d_4 \\ -k_1(\rho_1 + \rho_3 d_1) & -k_1(\rho_2 + \rho_3 d_2) & (k_2 - k_1 \rho_3 d_3) & -k_1(\rho_4 + \rho_3 d_4) \\ k_3(\rho_1 + \rho_3 d_1) & k_3(\rho_2 + \rho_3 d_2) & k_3 \rho_3 d_3 + k_4 & k_3(\rho_4 + \rho_3 d_4) \\ d_1 & d_2 & d_3 & 1 + d_4 \end{bmatrix} \begin{bmatrix} \Delta x_{a_o} \\ \Delta \dot{x}_{a_o} \\ \Delta V_{1_o} \\ \Delta \tau_o \end{bmatrix}$$

OR

$$\begin{bmatrix} \Delta x'_{a_o} \\ \Delta \dot{x}'_{a_o} \\ \Delta V'_{1_o} \\ \Delta \tau''_o \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \begin{bmatrix} \Delta X_{a_o} \\ \Delta \dot{X}_{a_o} \\ \Delta V_{1_o} \\ \Delta \tau_o \end{bmatrix} \qquad (3.52)$$

Special Case

In the special case of symmetric two impacts/cycle motion (i.e. n = 1), let the perturbations at $t = 0_+$ be ΔX_o, $\Delta \dot{X}_o$, ΔV_o, $\Delta \tau_o$ and the resulting perturbations at $t = (\frac{\pi}{\Omega})_+$ be $\Delta X_o'$, $\Delta \dot{X}_o'$, $\Delta V_o'$, $\Delta \tau_o'$.

In this case, the stability matrix will be (8):

♦ $$\begin{Bmatrix} \Delta x_0' \\ \Delta \dot{x}_0' \\ \Delta V_0' \\ \Delta \tau_0' \end{Bmatrix} = \begin{bmatrix} d_5 & \frac{V_0}{\Omega} d_2 & -C_3 d_3 & \frac{V_0}{\Omega} d_4 \\ -k_1(\rho_1 + \rho_3 d_1) & -k_1(\rho_2 + \rho_3 d_2) & k_2 - k_1 \rho_3 d_3 & -k_1(\rho_4 + \rho_3 d_4) \\ k_3(\rho_1 + \rho_3 d_1) & k_3(\rho_2 + \rho_3 d_2) & k_3 \rho_3 d_3 - k_4 & k_3(\rho_4 + \rho_3 d_4) \\ d_1 & d_2 & d_3 & 1 + d_4 \end{bmatrix} \begin{Bmatrix} \Delta x_0 \\ \Delta \dot{x}_0 \\ \Delta V_0 \\ \Delta \tau_0 \end{Bmatrix}$$

♦ Symbols are given in Appendix VI

3.4. Stability Boundaries

The stability boundaries are the curves on which the modulus of the largest eigenvalue (S) equals unity.

The characteristic polynomial of the matrix P is

$$\begin{vmatrix} P_{11}-\lambda & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22}-\lambda & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33}-\lambda & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44}-\lambda \end{vmatrix} = 0 \quad (3.53)$$

which can be put in the form

$$\varphi(\lambda) = \lambda^4 - a_1\lambda^3 + a_2\lambda^2 - a_3\lambda + a_4 = 0 \qquad (3.54)$$

from the theory of matrices, it is known that if the eigenvalues of P are

$$\left.\begin{aligned} a_1 &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\ a_2 &= \lambda_1(\lambda_2+\lambda_3+\lambda_4) + \lambda_2(\lambda_3+\lambda_4) + \lambda_3\lambda_4 \\ a_3 &= \lambda_1(\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4) + \lambda_2\lambda_3\lambda_4 \\ a_4 &= \lambda_1\lambda_2\lambda_3\lambda_4 \end{aligned}\right\} \qquad (3.55)$$

Guided by some knowledge about the behaviour of the eigenvalues of P, we will assume that on the boundary, one of two cases occurs:

a) $|\lambda_1| = 1$

b) $|\lambda_1| = |\lambda_2| = 1 \quad ; \quad \lambda_1 = \bar{\lambda}_2$

Case a):

If one of the eigenvalues (e.g., λ_1) is real and equal to ± 1, then from (3.54) and (3.55)

$$\frac{1 + a_2 + a_4}{a_1 + a_3} = \lambda_1 = \pm 1 \tag{3.56}$$

Case b):

$$\lambda_1 = a_o + i b_o \quad , \lambda_2 = a_o - i b_o \quad ; a_o^2 + b_o^2 = 1$$

By using (3.54) and (3.55), it is found that

$$(a_2 - 1 - a_4)(a_4 - 1)^2 = (a_3 - a_1)(a_1 a_4 - a_3) \tag{3.57}$$

CHAPTER 4

EXPERIMENTAL STUDIES

4.1. Introduction

The objectives of the present experimental study are:

(a) to show the existence of two unsymmetric impacts per cycle motion.

(b) to study the variation of the value of (N) due to variations of $\mu, r, \frac{d}{F_o/k}$.

(c) to show the existence of motions, other than two impacts per cycle motion

(d) to study the general response of the system for a wide range of parameters of the impact damper

(e) to verify the assumptions made in determining steady state solutions.

In order to obtain some information relevant to these matters, the following experiments and studies were conducted as described below:

1 - experiments with a mechanical model

2 - numerical studies involving a digital computer

4.2. Experimental Model

A schematic diagram of the experimental model is shown in Figure (2.1). The photographs of the test rig and actual model are shown in Figures (4.1) and (4.3), respectively.

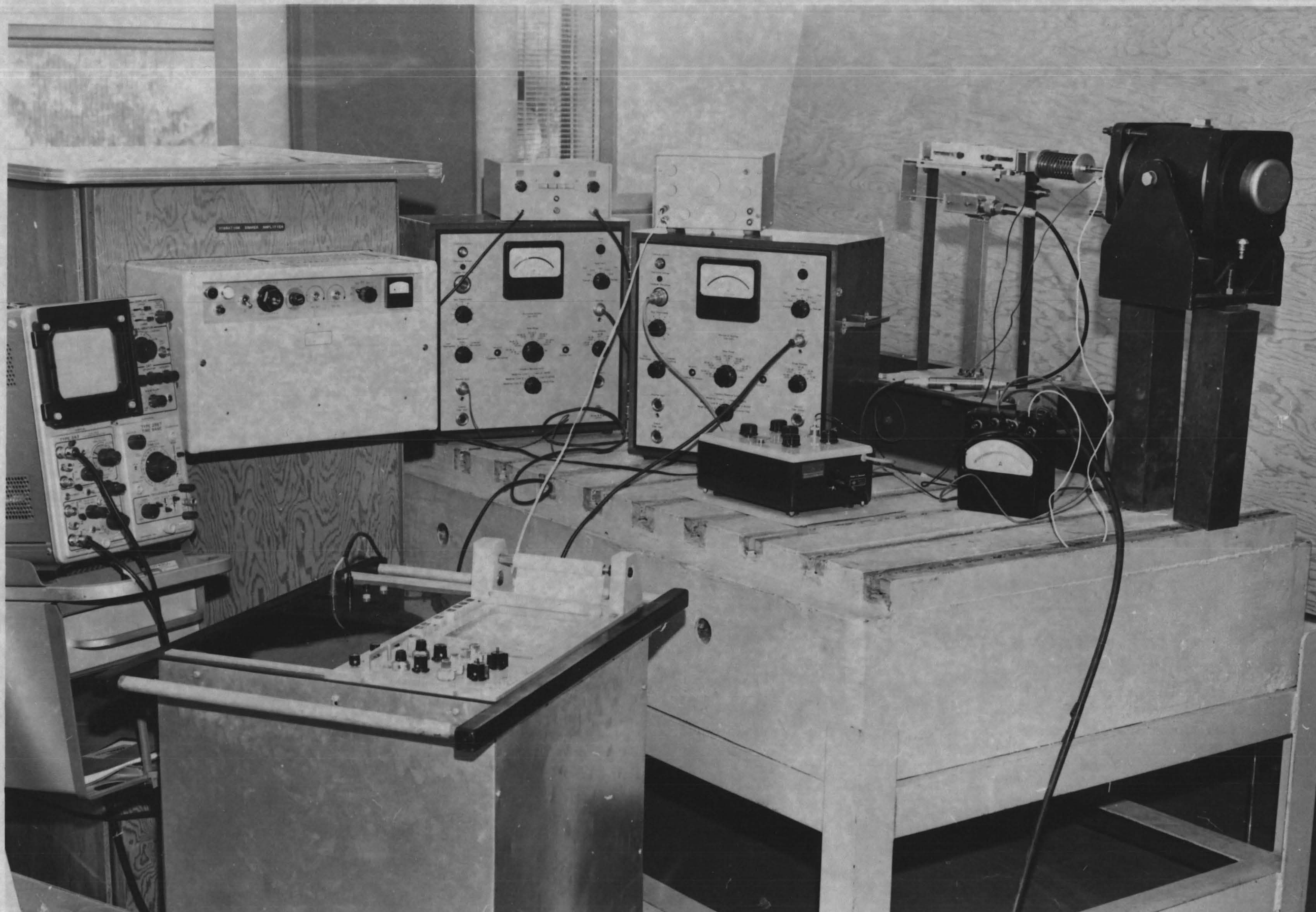


Fig. (4.1) GENERAL VIEW OF EXPERIMENTAL SET UP

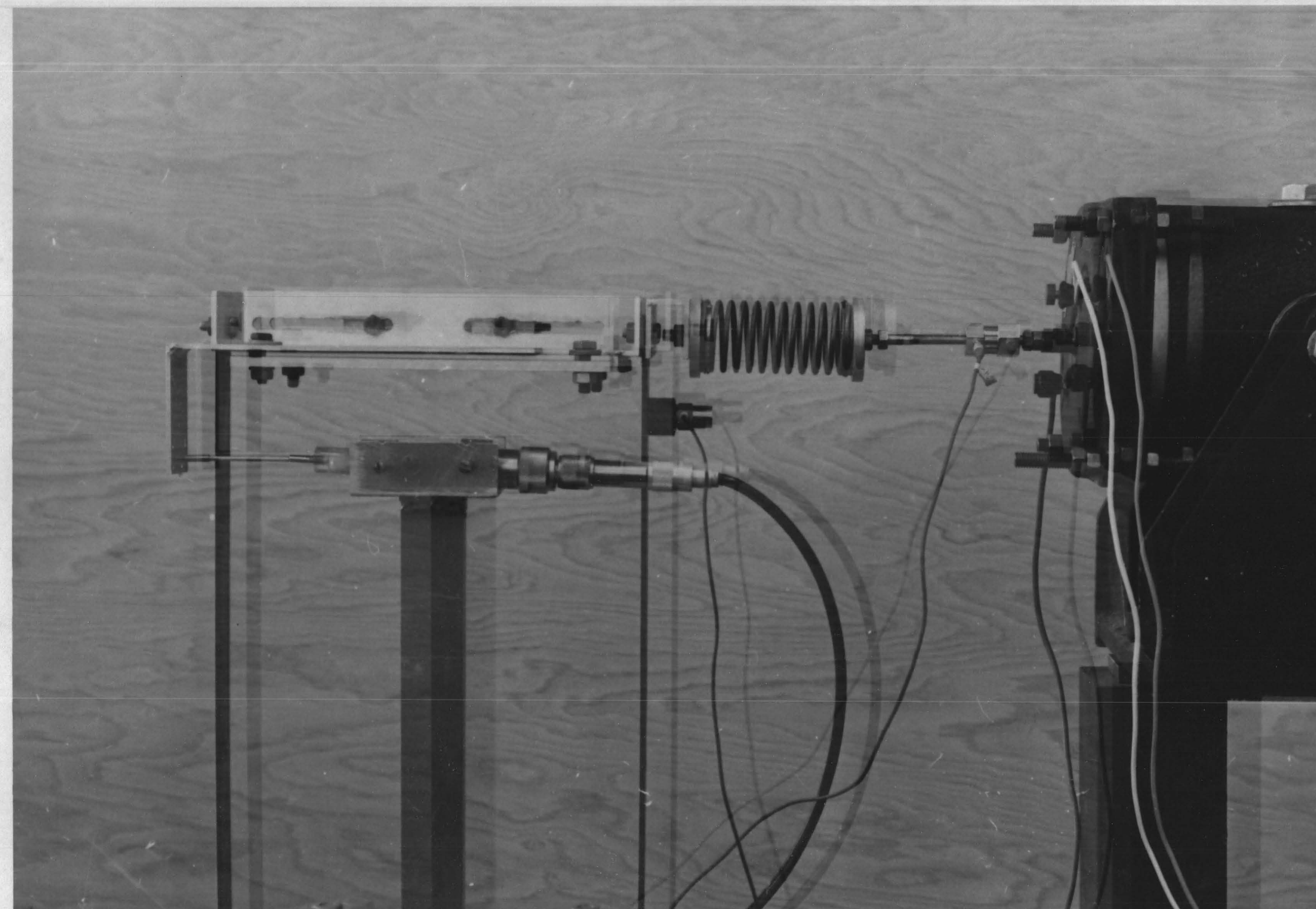


Fig. (4 . 2) FRONT VIEW OF THE MECHANICAL MODEL

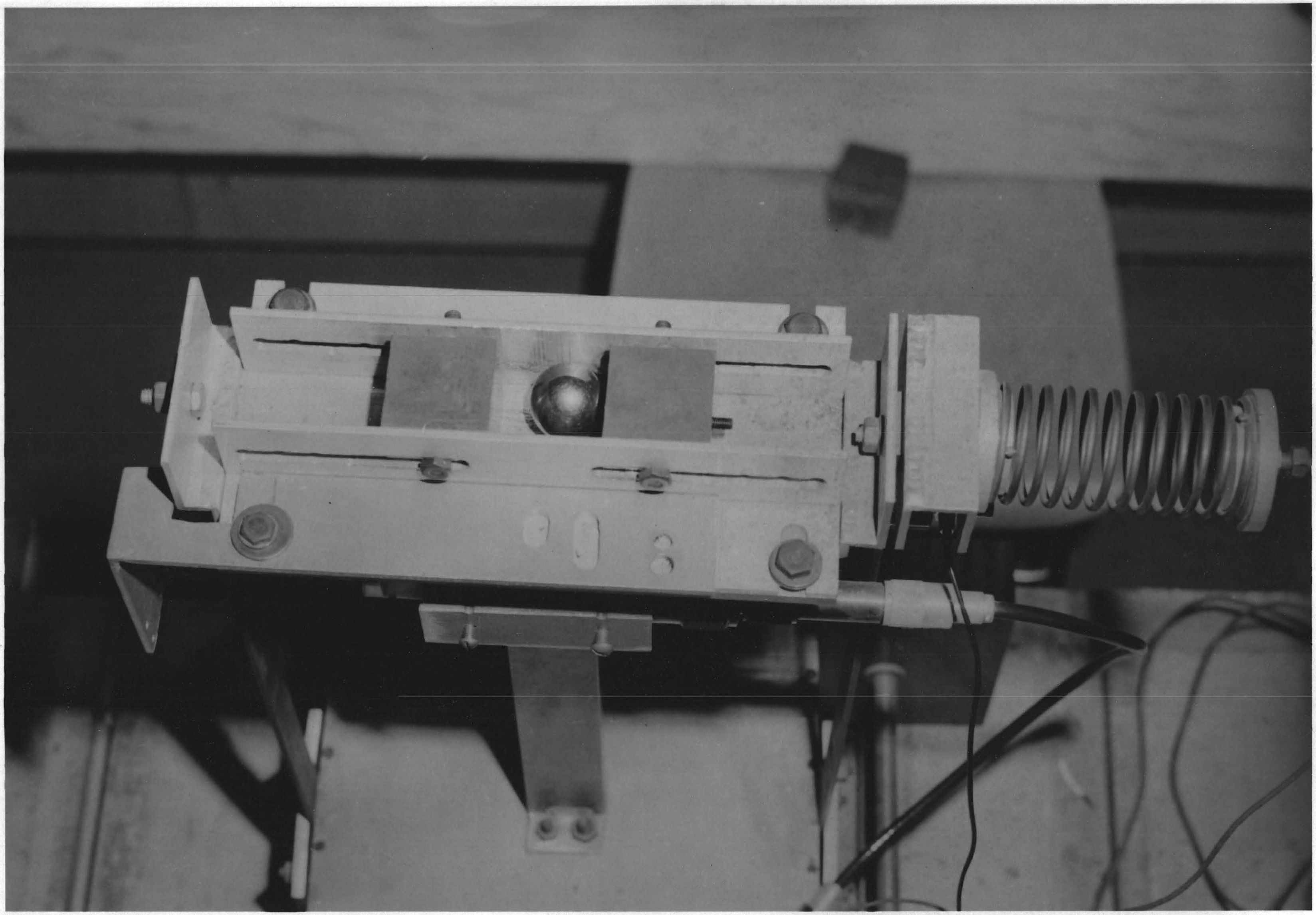


Fig. (4.3) TOP VIEW OF THE MECHANICAL MODEL

The primary mass is a simple rectangular box with rigid stops at the ends to constrain the movement of the frictionless solid particle (m) to oscillate horizontally within a certain clearance (d). The particle is a hardened steel ball that is usually used in ball bearings. The stops upon which the ball made collision were made of mild steel but had been case hardened so as to obtain high coefficient of restitution. Two springs are used, the first is a leaf spring which supports the primary mass and produce restoring force and the other one is a helical spring.

4.3. Electronic Measurement

A capacitance pick-up,with associated electronic equipment, is used to find the displacement of the primary mass. Velocity and acceleration of the primary mass are obtained by integrating with respect to time the output of an accelerometer attached to the primary mass. This integration is accomplished by using an integrating network in conjunction with an operation amplifier. A force gauge is used to keep F_o constant.

4.4. Experimental Results

(a) characteristic of the system without impact damper.

The equivalent characteristic constants of the system are:

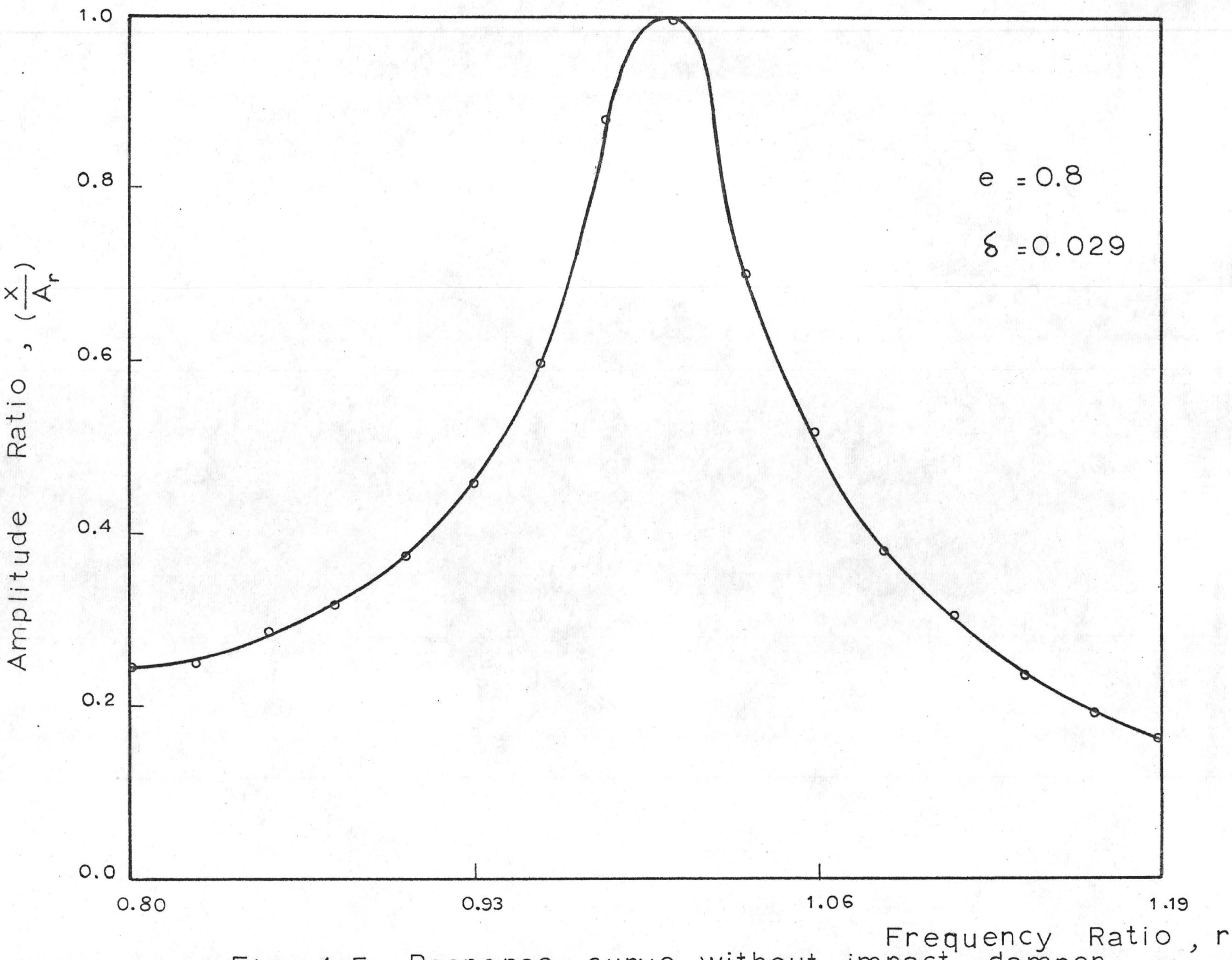


Fig. (4.5) Response curve without impact damper

$M = 4.5$ lb

$K = 24$ lb/in

$\omega = 7.4$ cycles/sec

$\delta = 0.029$

(b) Characteristic of the System with Impact Damper in Action:

The effect of various parameters of impact damper namely mass ratio and gap ratio on the system responses is investigated.'

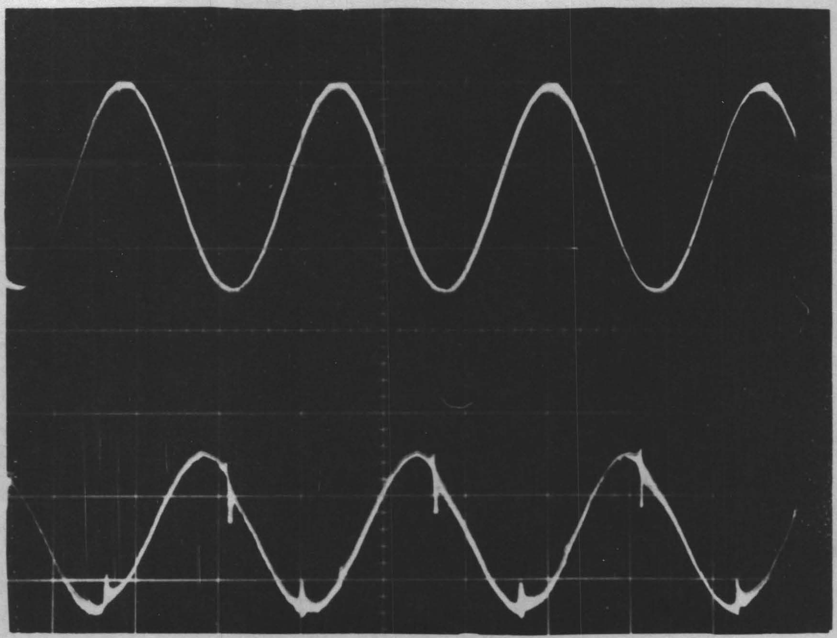
Six free mass ratios ranging from $\mu = 0.0136$ to $\mu = 0.084$ are used. The free mass strokes are varied from $d = 0.0625$ inch to $d = 0.1875$ inch.

The ratio of vibration amplitude (with the impact damper in action) to A_r which is the maximum amplitude obtained (with impact damper removed) is plotted versus frequency ratio. The results are shown in Figures (4.6) to (4.14).

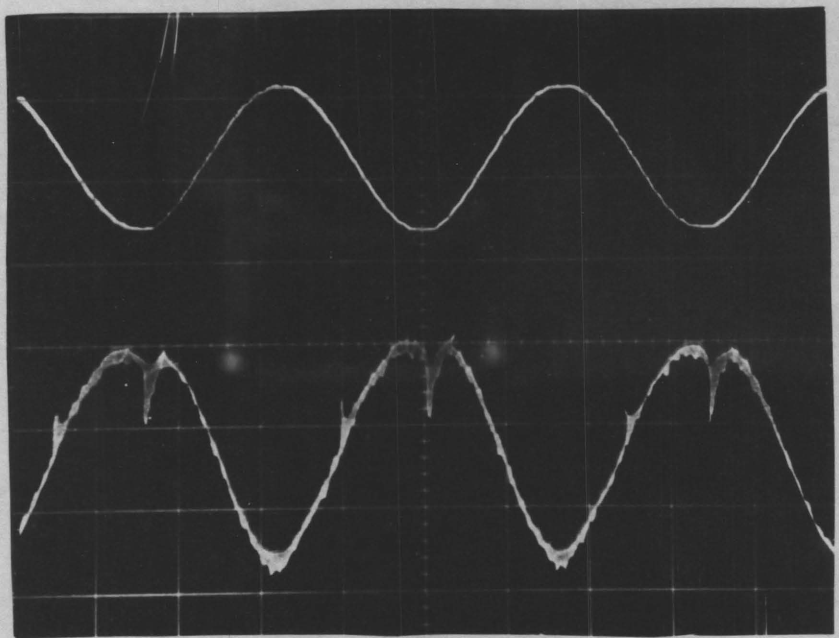
The variation of the unsymmetry ratio N is plotted also against the frequency ratio. The results are shown in Figures (4.15) to (4.21).

The velocities, displacements and accelerations of the primary system, with the impact damper in action are shown in Figures (4.4-a) to (4.4-f).

The displacement of the primary mass versus its velocity, with impact damper in it, is shown in Figures (4.4-h). Determination of δ is shown in Appendix IV.

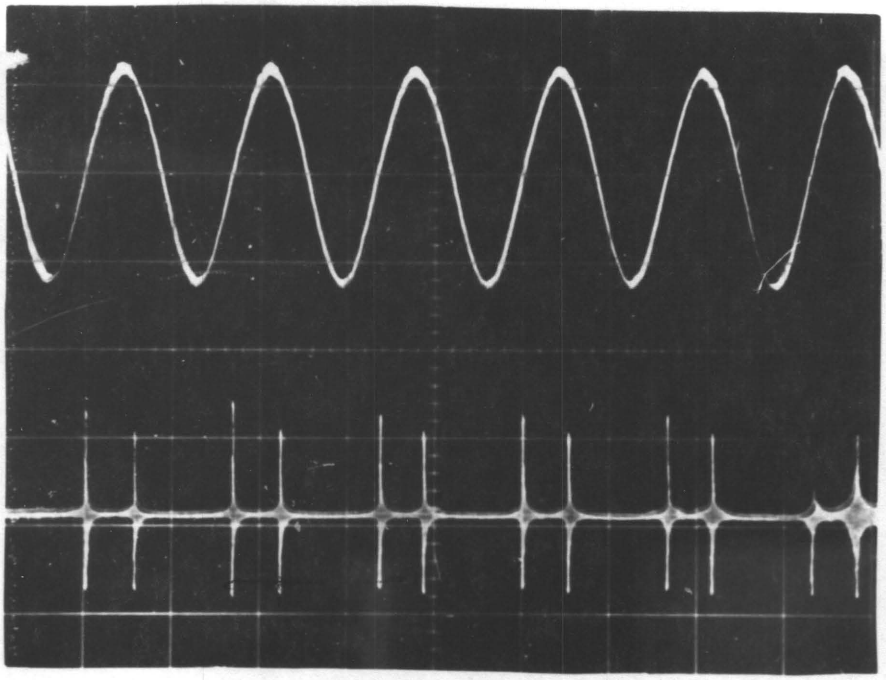


(a)

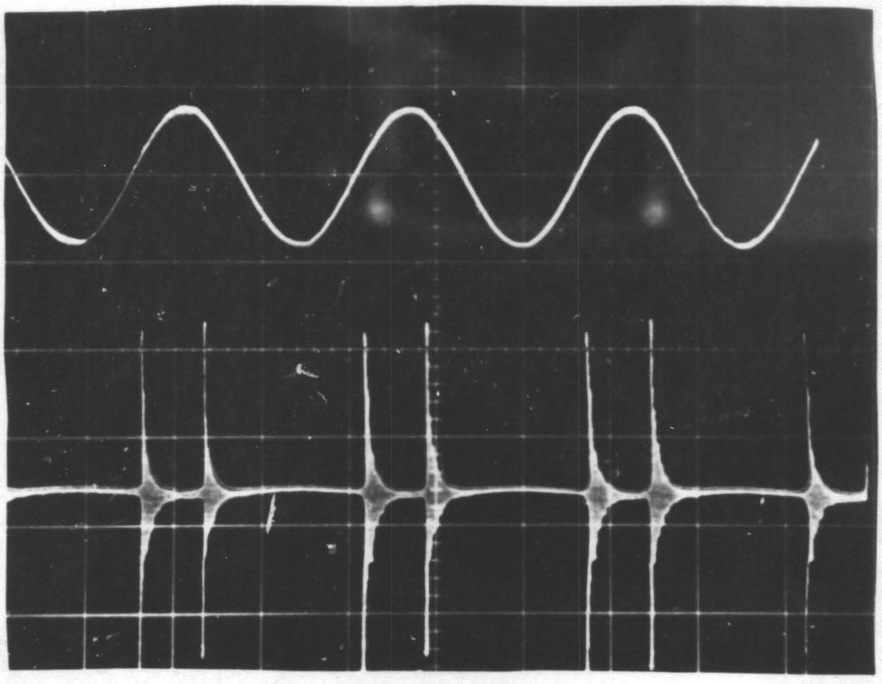


(b)

Fig. (4.4) (a),(b) Displacement and velocity curves for different parameters

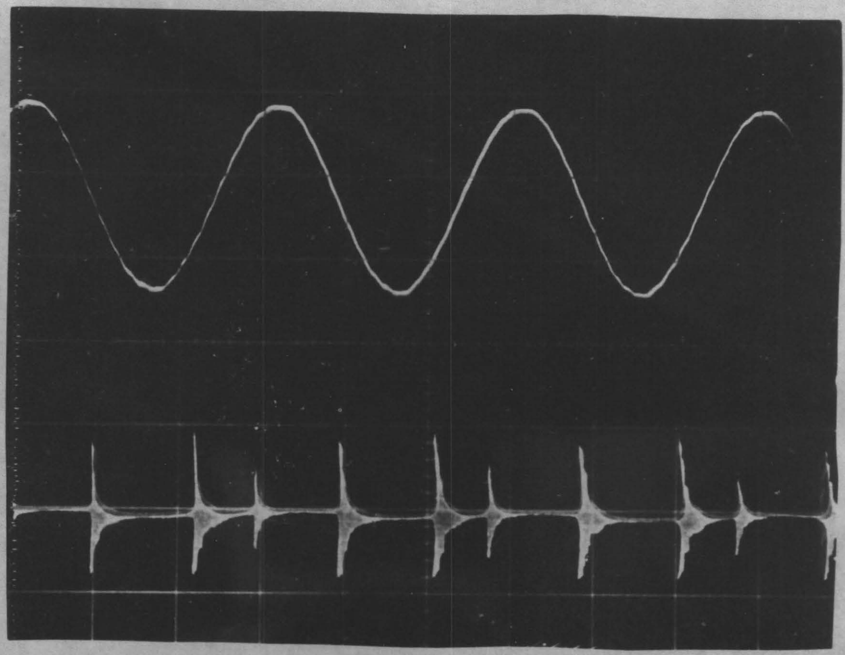


(c)

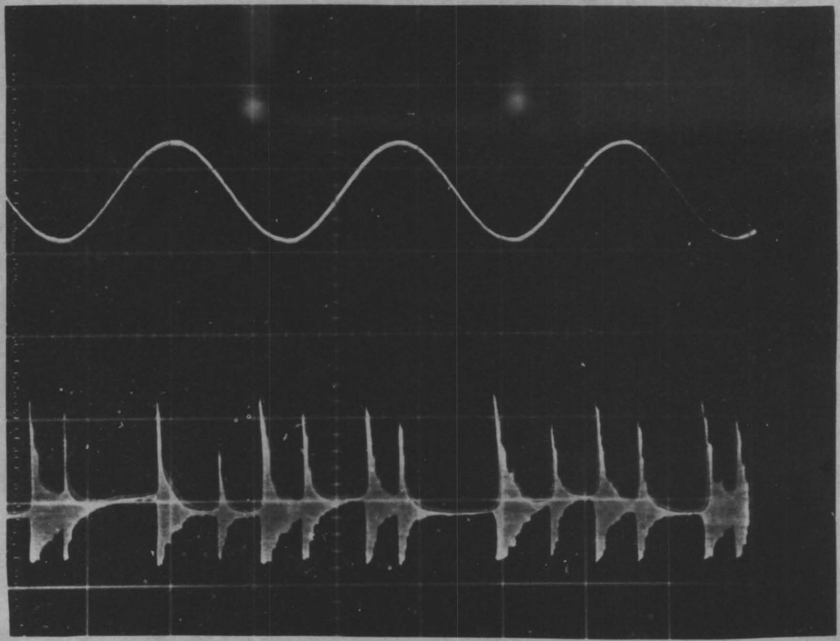


(d)

Fig. (4 . 4) (c),(d) Displacement and acceleration curves for different parameters



(e)



(f)

Fig.(4.4) Displacement and acceleration curves for(e)3 impacts/cycle (f)multiple impacts/cycle

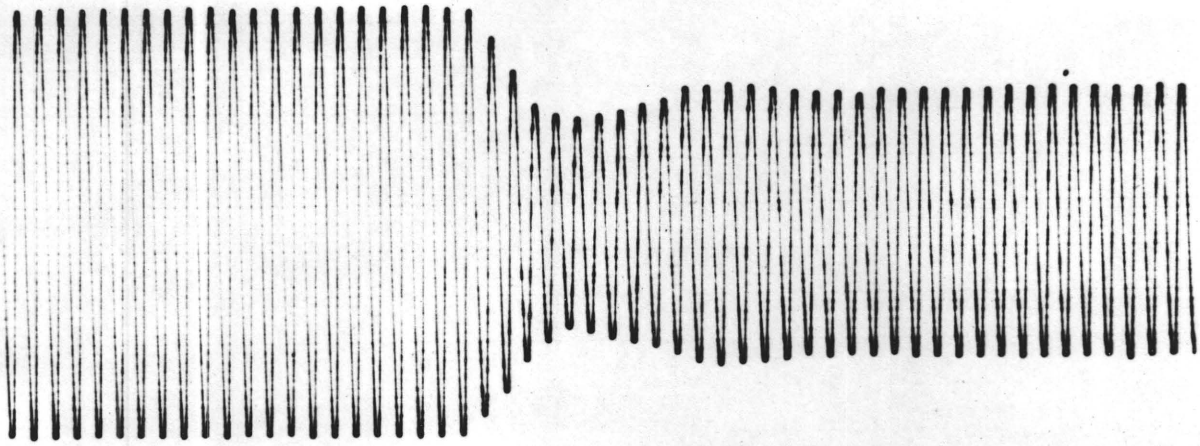


Fig. (4 . 4) (g) Displacement curve ; effect of the impact damper

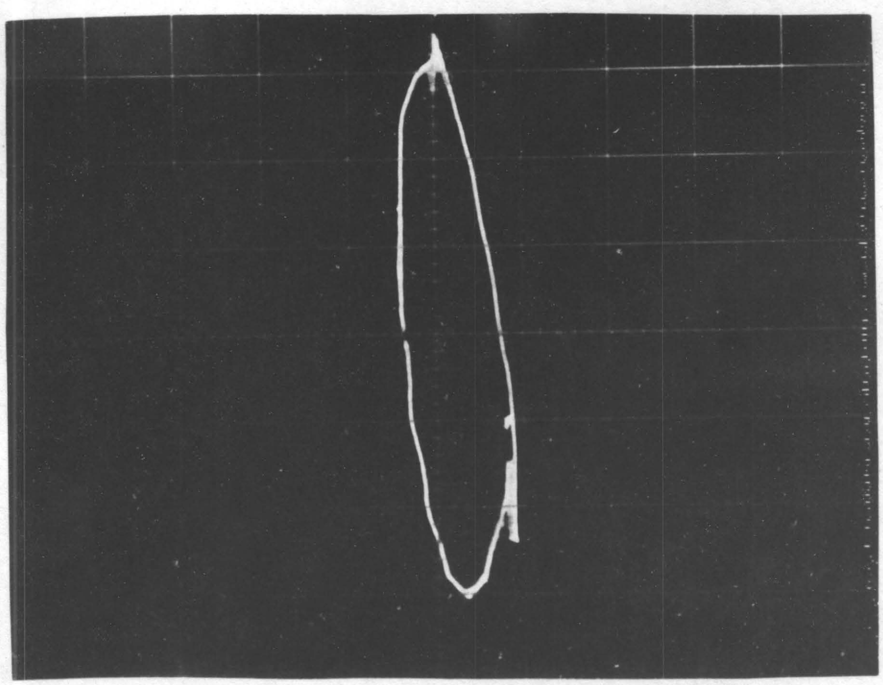


Fig. (4 . 4)(h) $x - \dot{x}$ curve for the impact damper

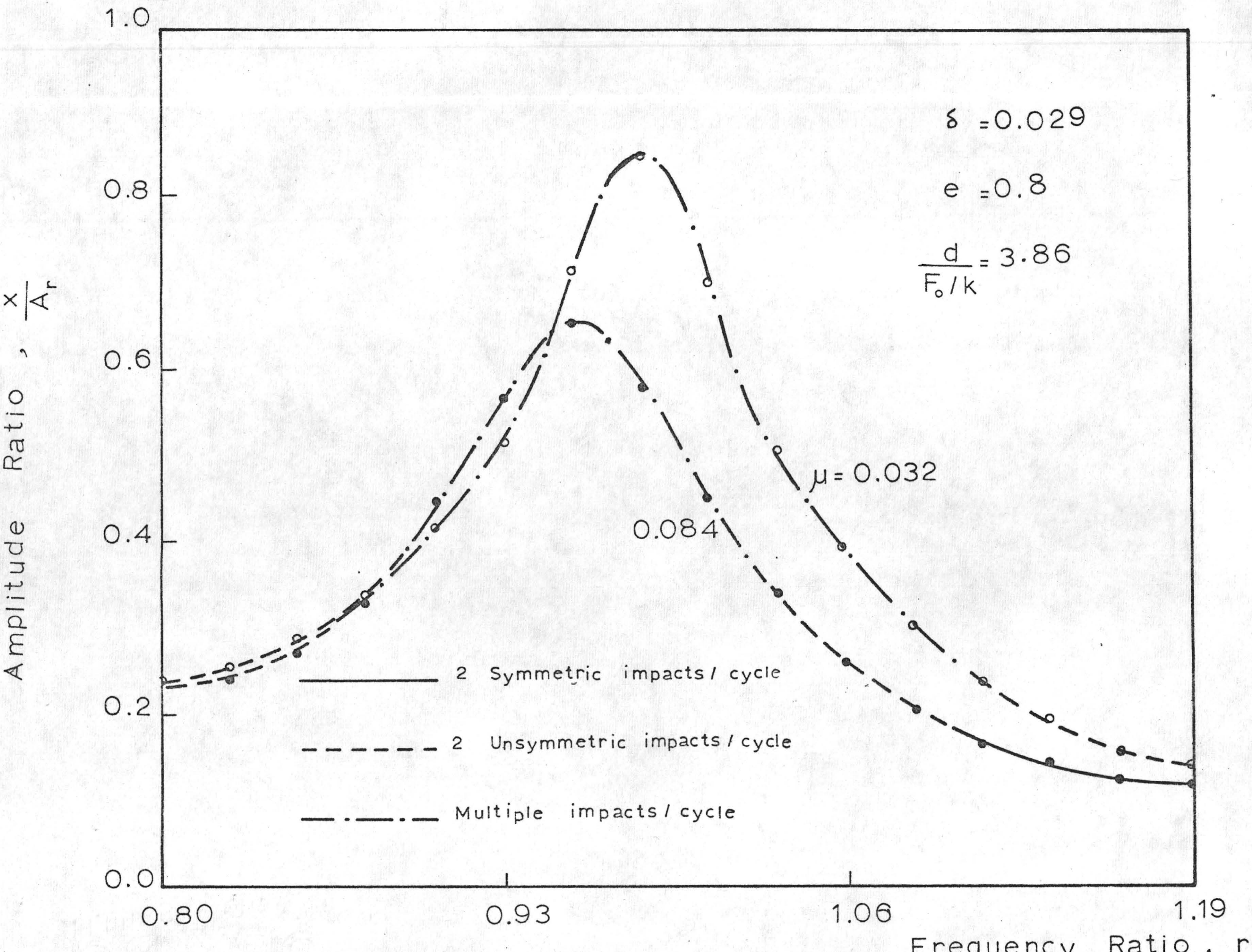


Fig (4.6) Response Curves

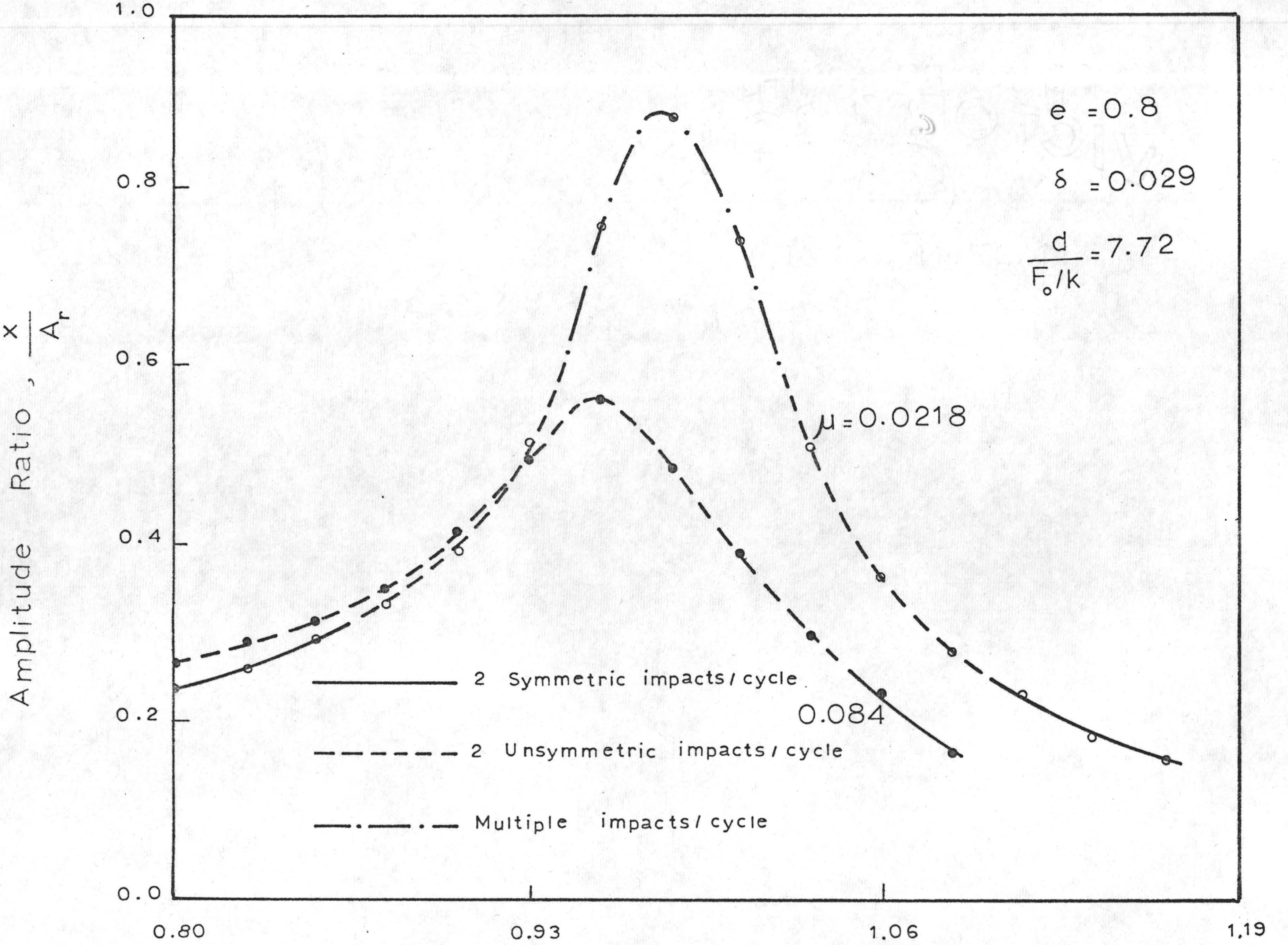


Fig (4.7) Response Curves

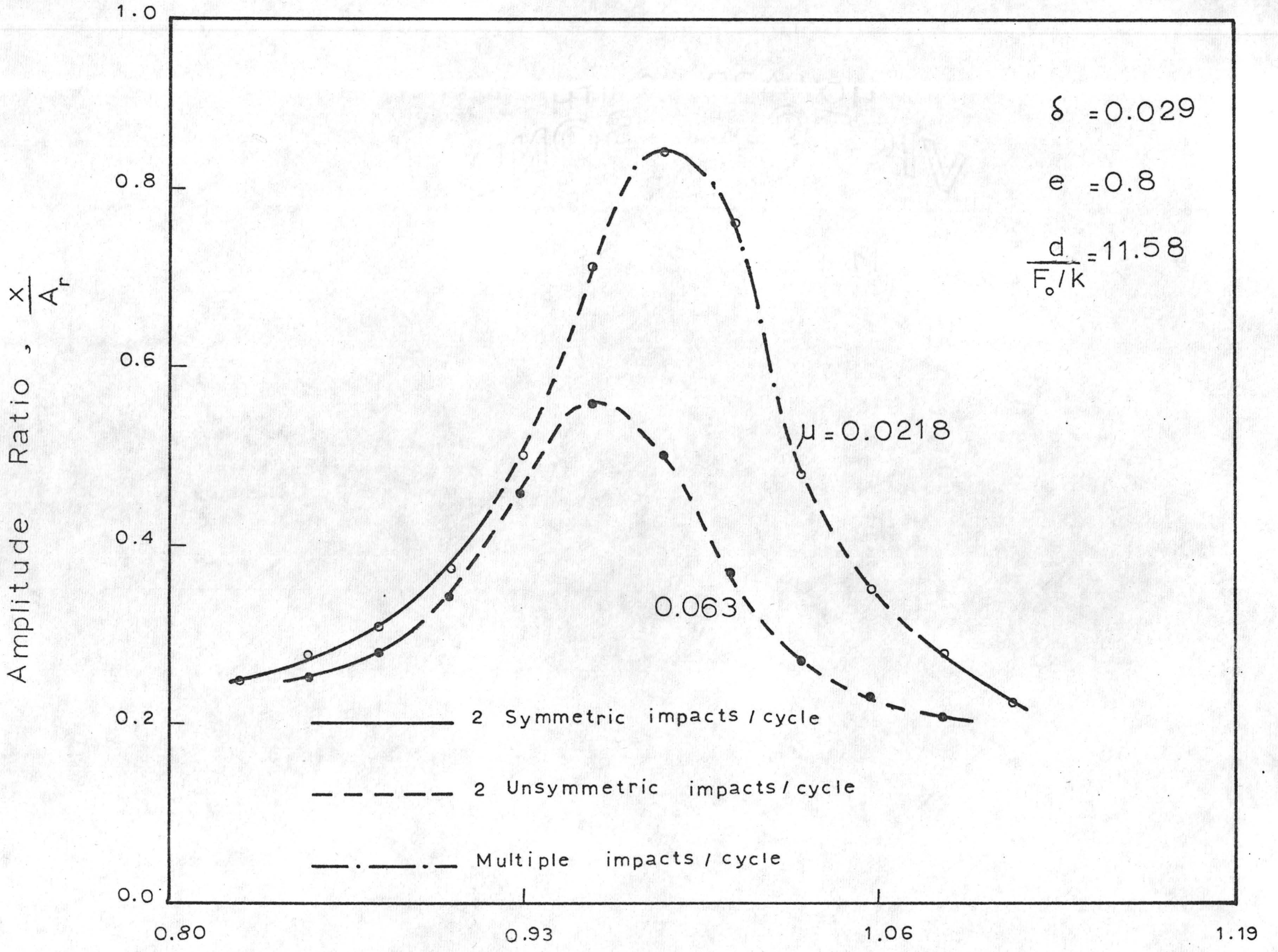


Fig (4.8) Response Curves

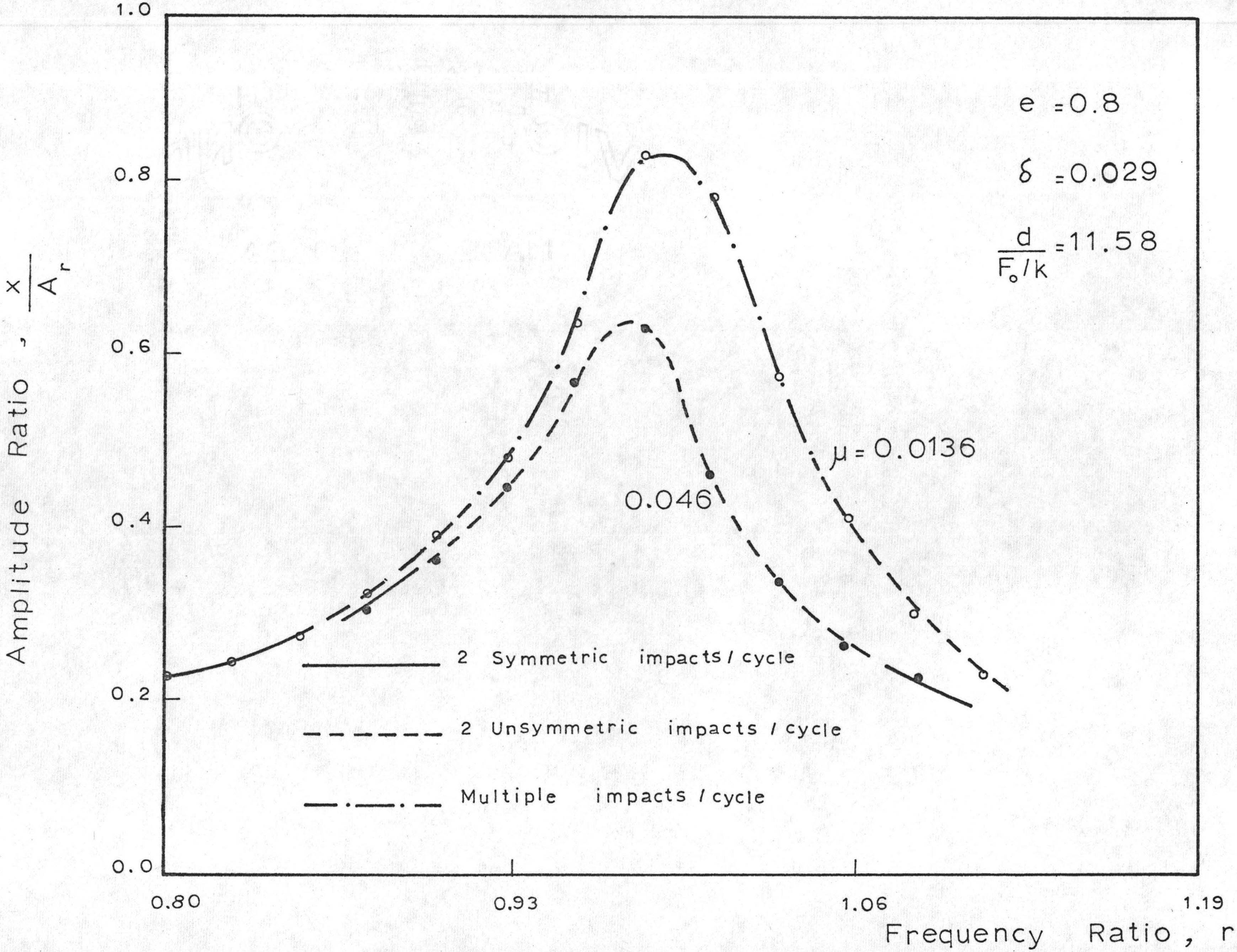


Fig (4.9) Response Curves

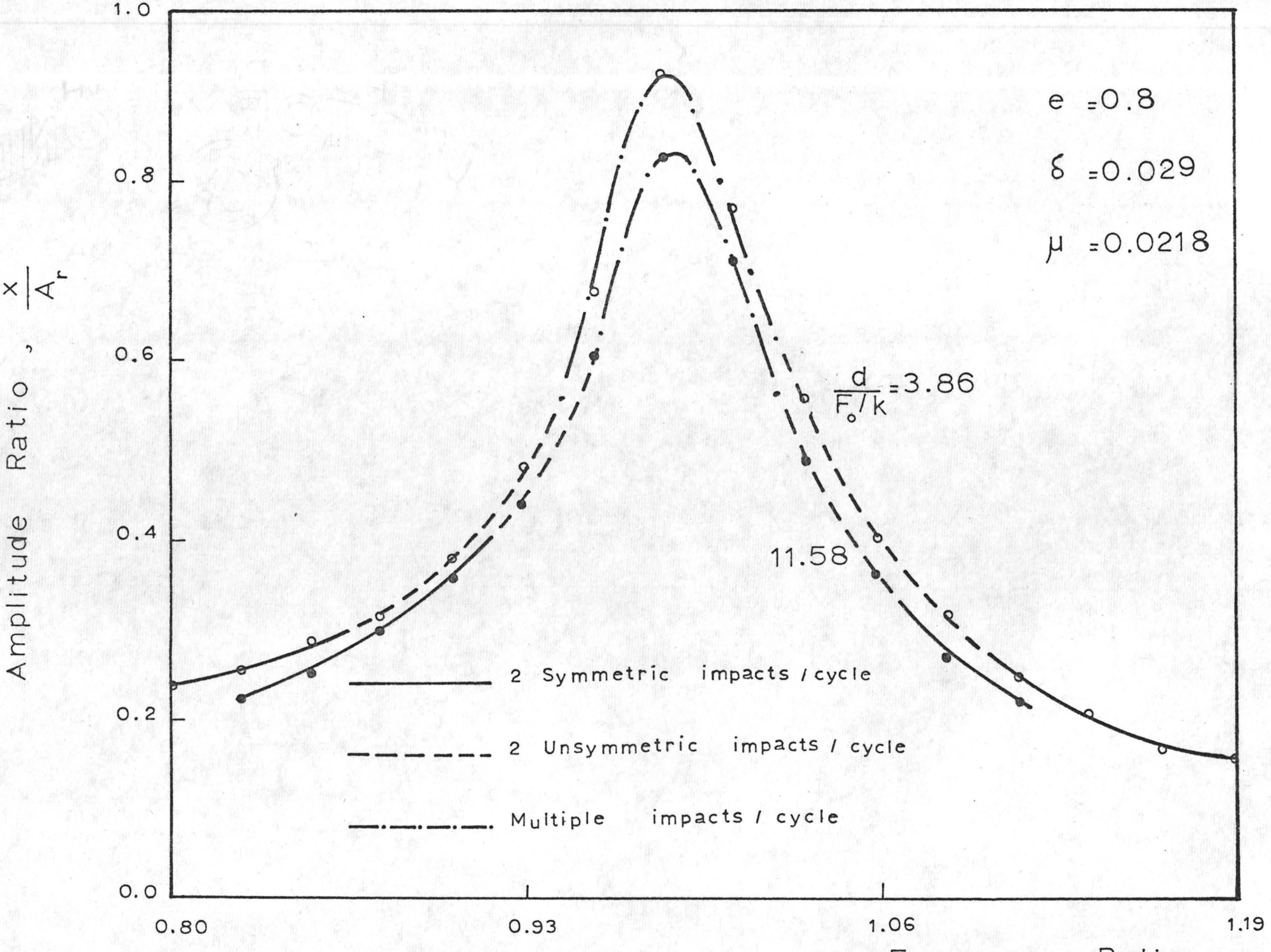


Fig (4.10) Response Curves

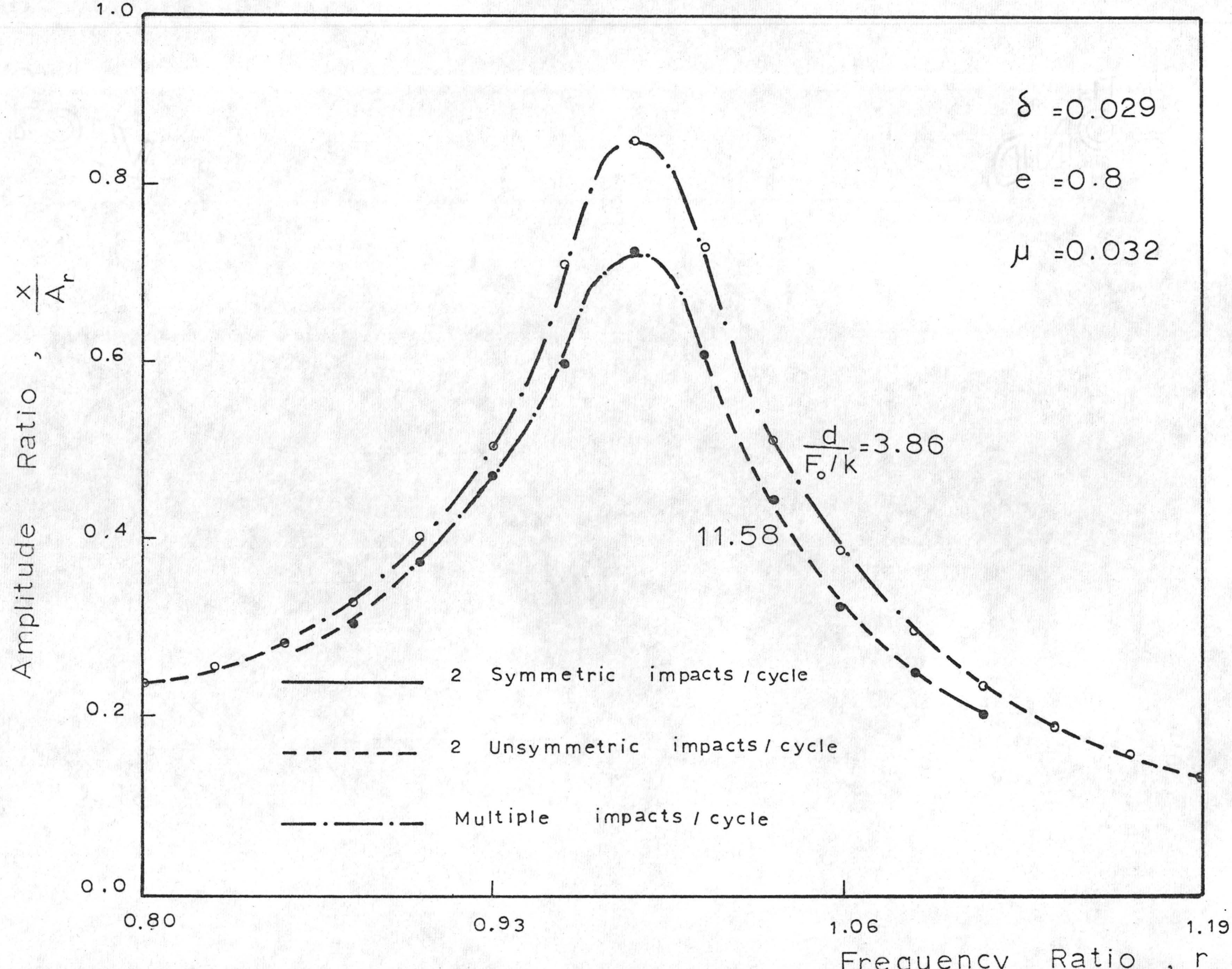


Fig.(4.11) Response Curves

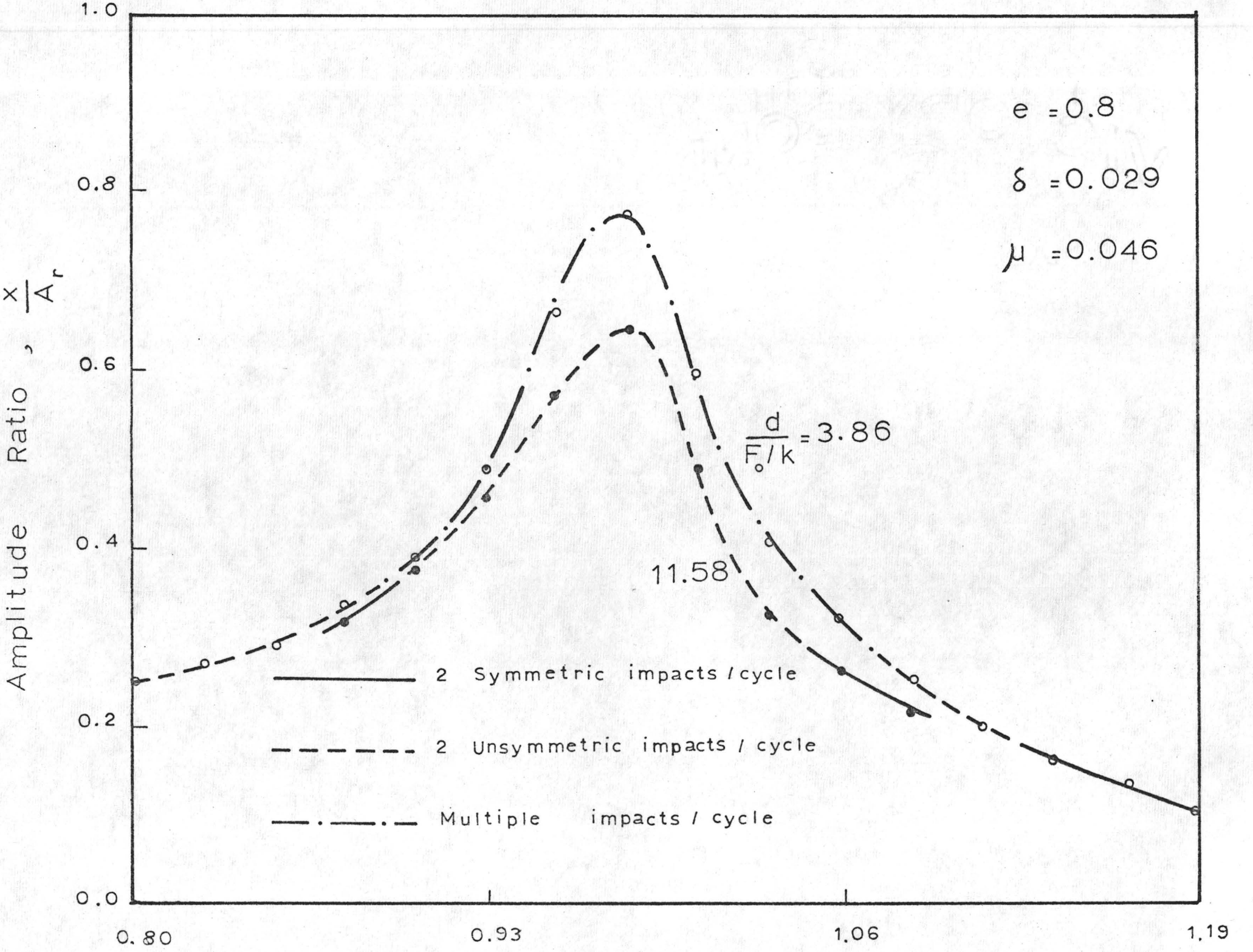


Fig. (4.12) Response Curves

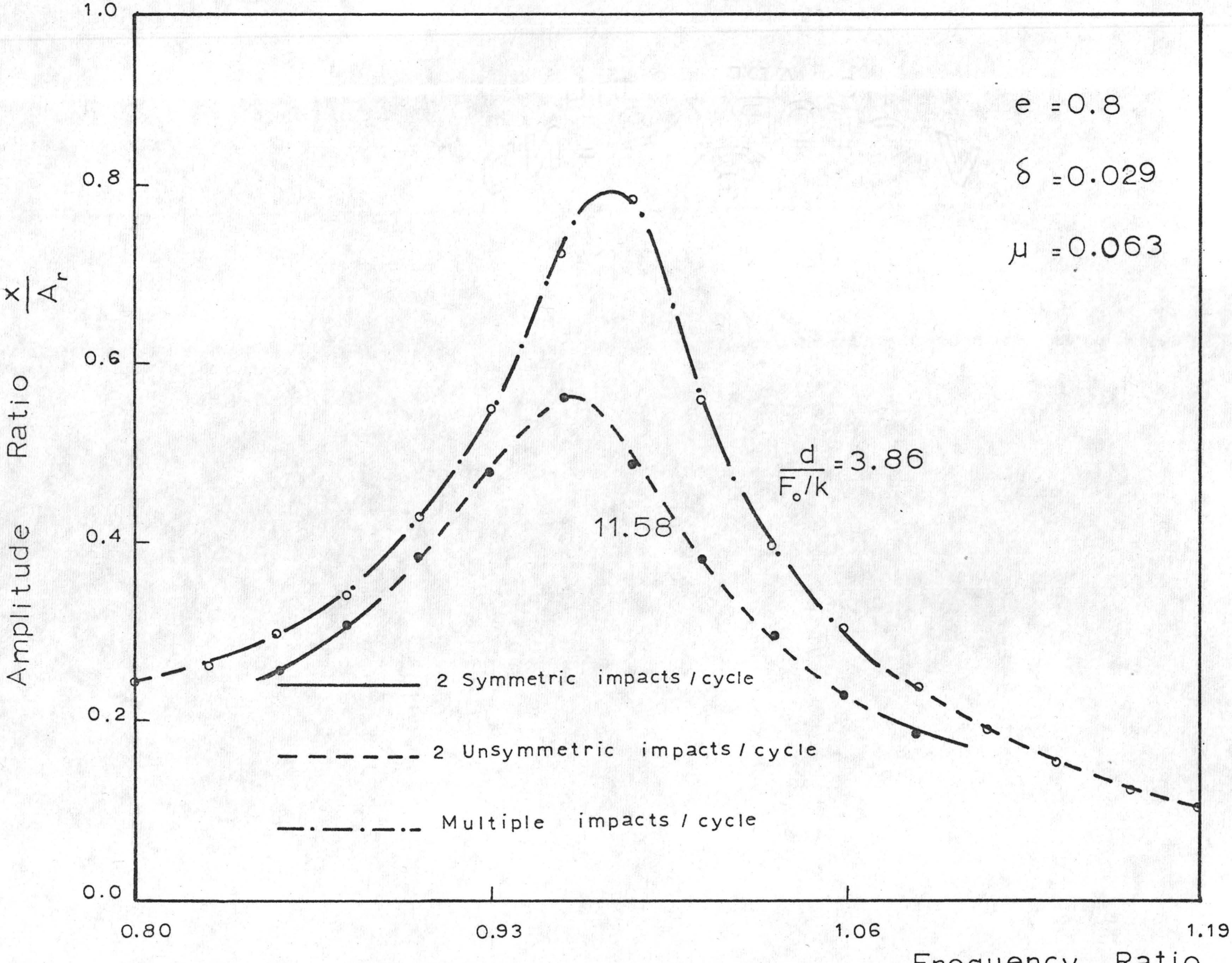


Fig. (4.13) Response Curves

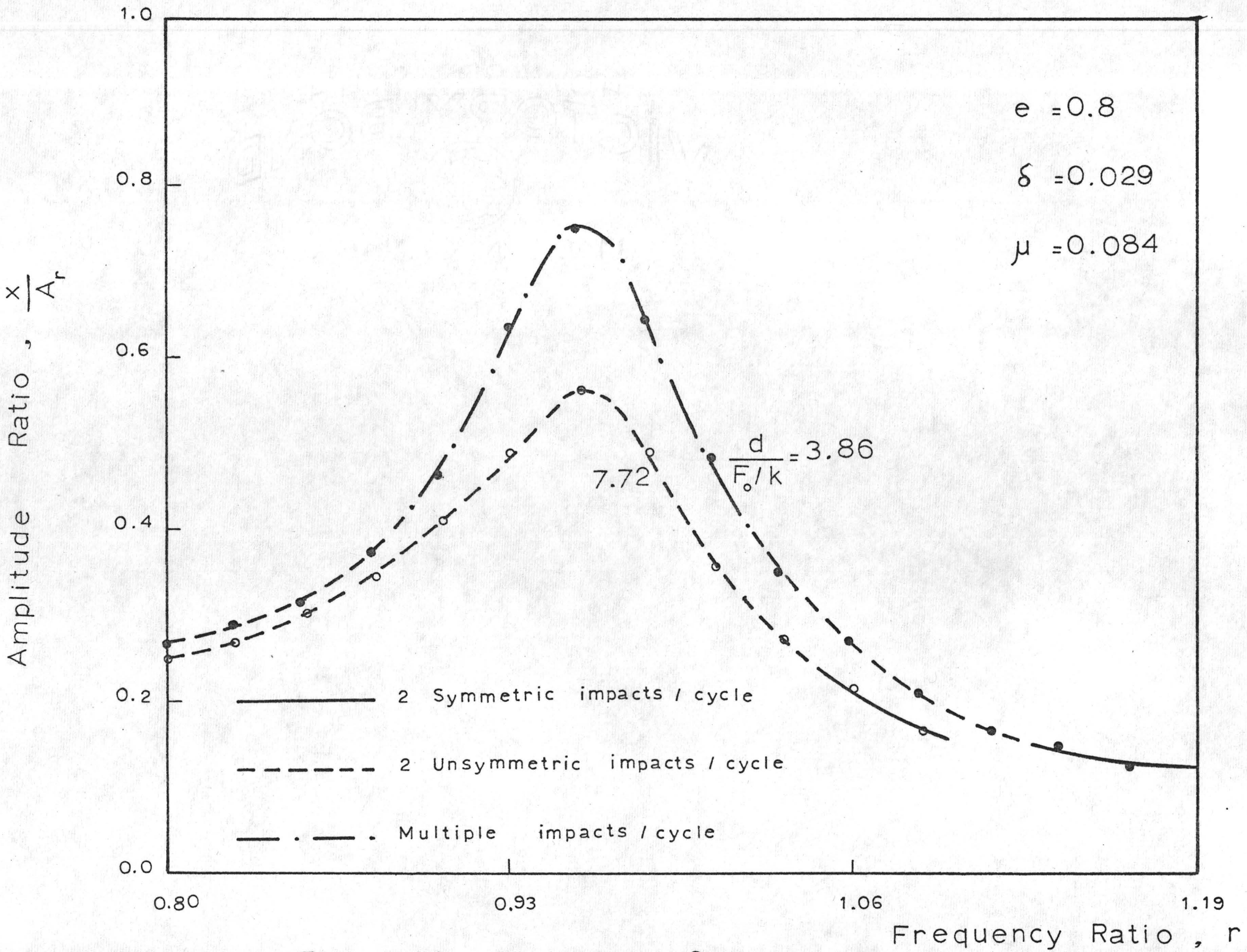


Fig (4.14) Response Curves

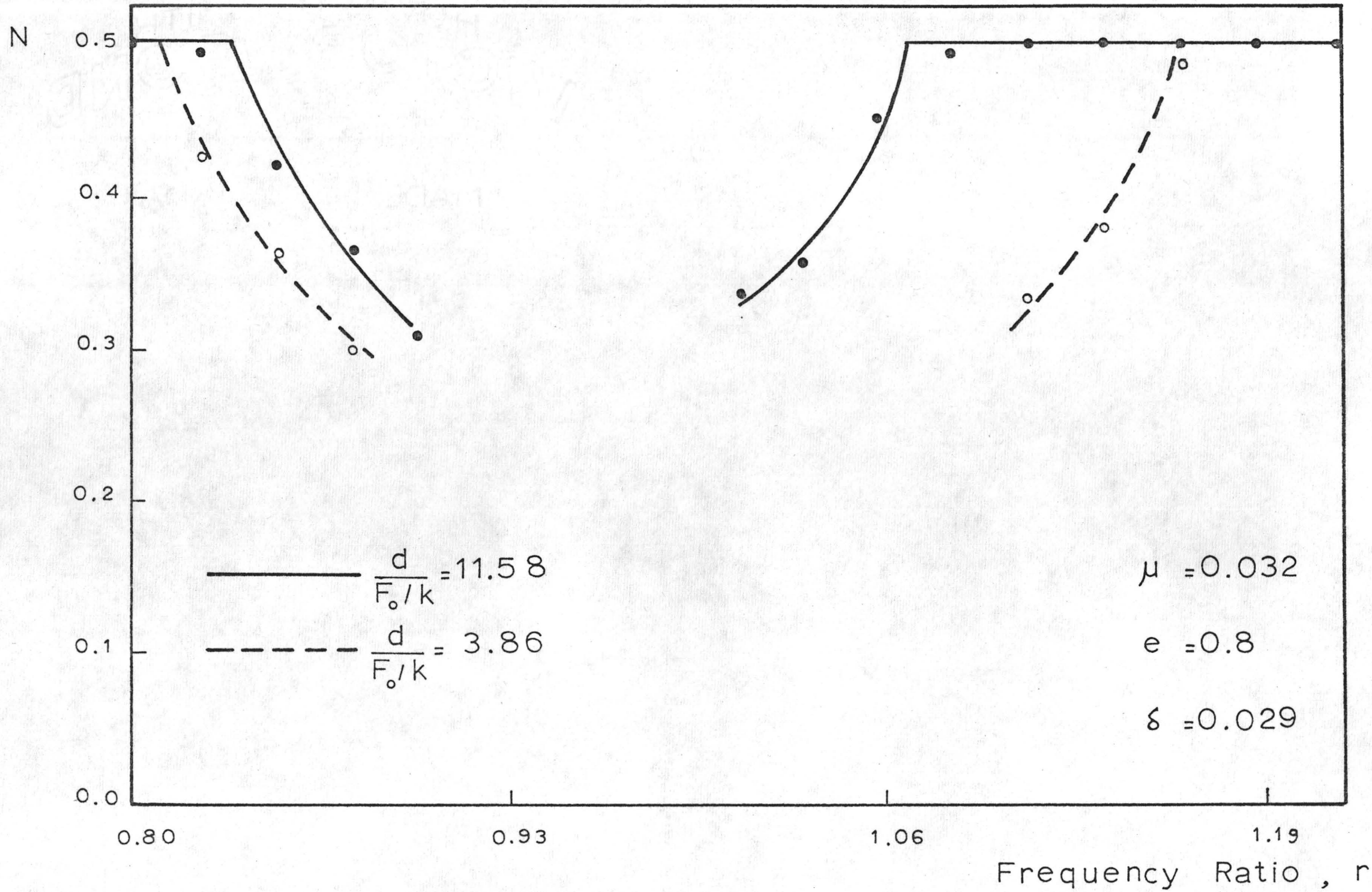


Fig.(4.15) Effect of frequency ratio on the unsymmetry ratio, N

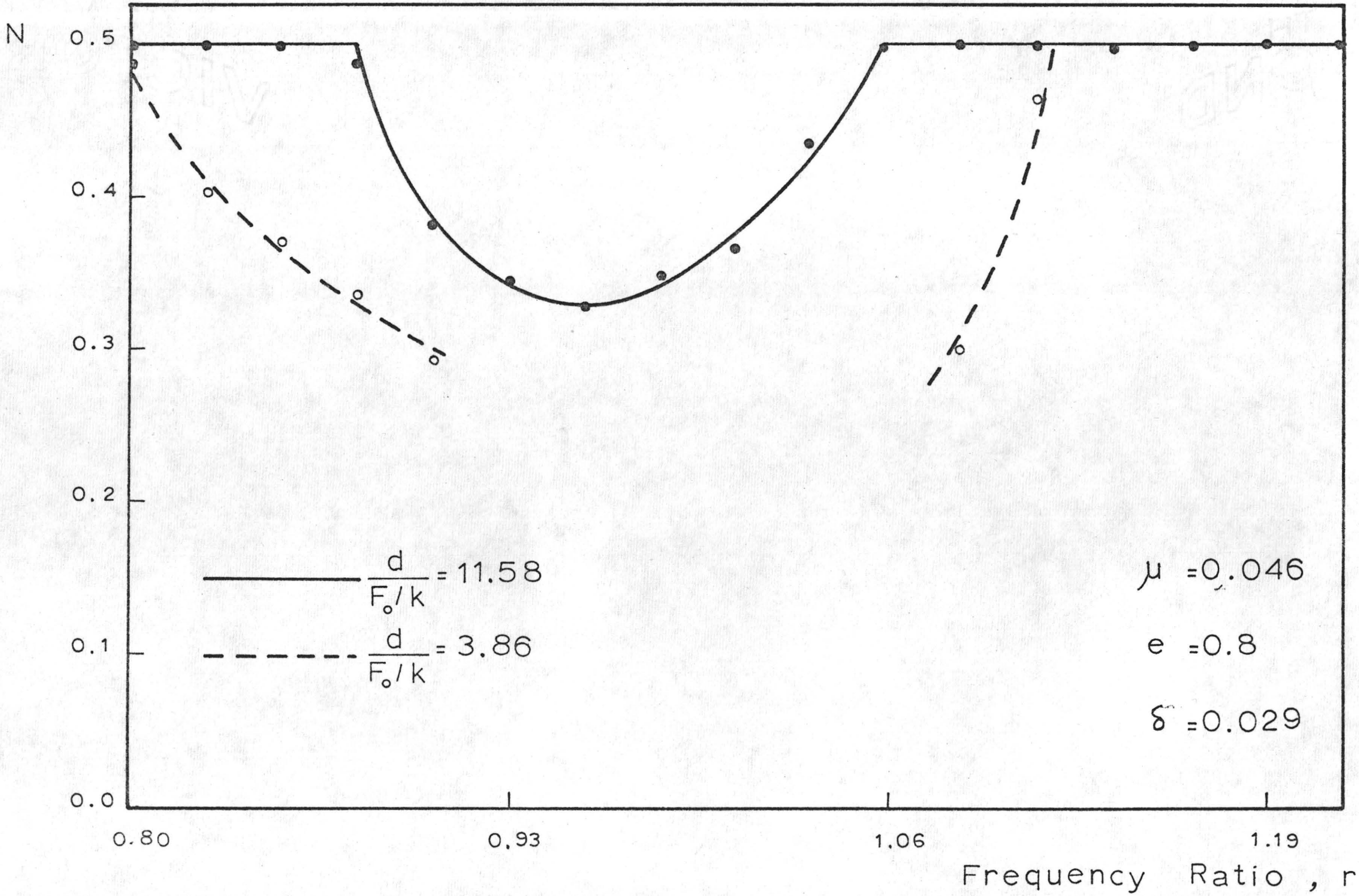


Fig (4.16) Effect of frequency ratio on the unsymmerty ratio , N

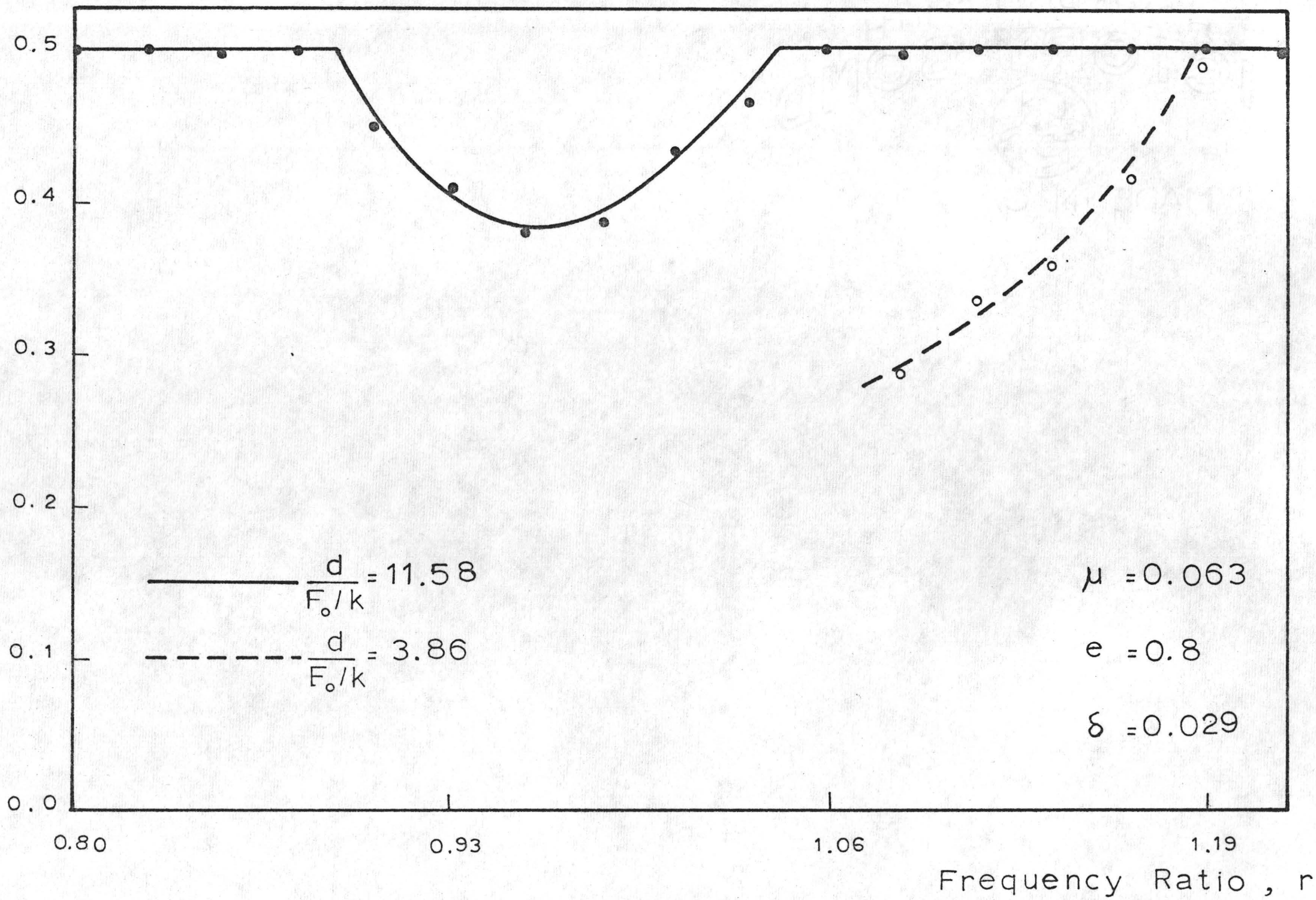


Fig. (4.17) Effect of frequency ratio on the unsymmetry ratio , N

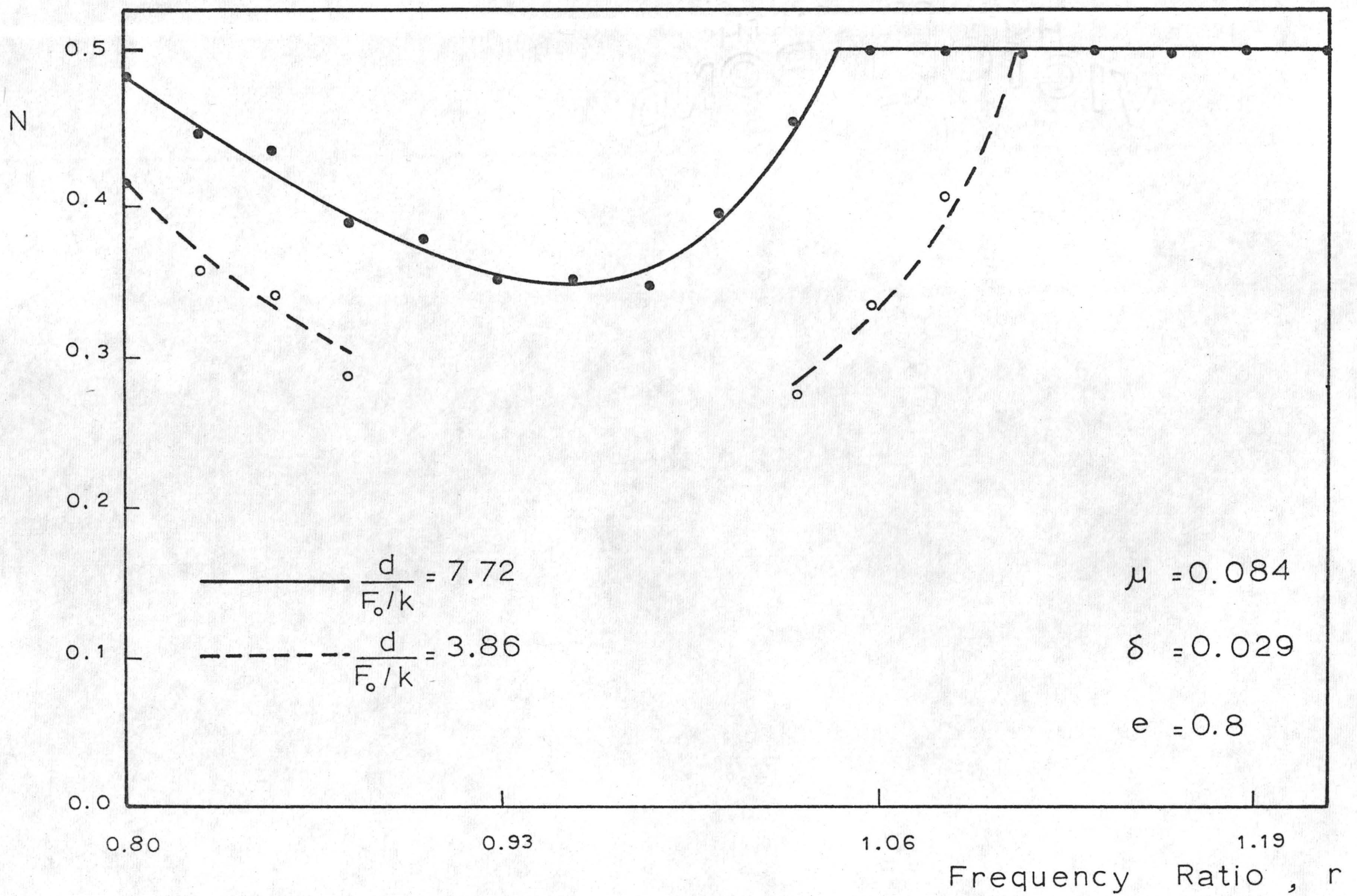


Fig. (4 .18) Effect of frequency Ratio on the unsymmetry ratio , N

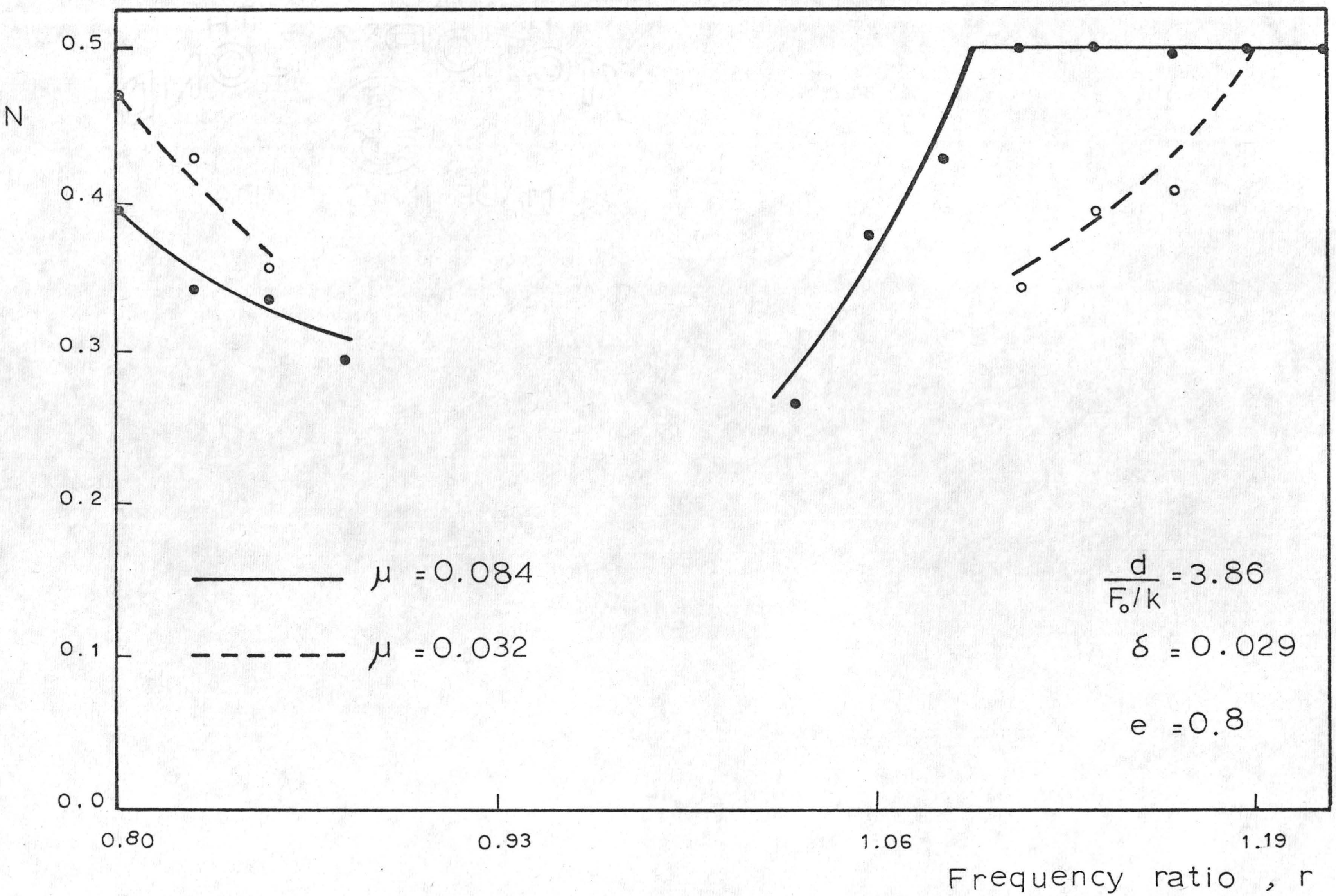


Fig. (4.19) Effect of frequency ratio on the unsymmetry ratio , N

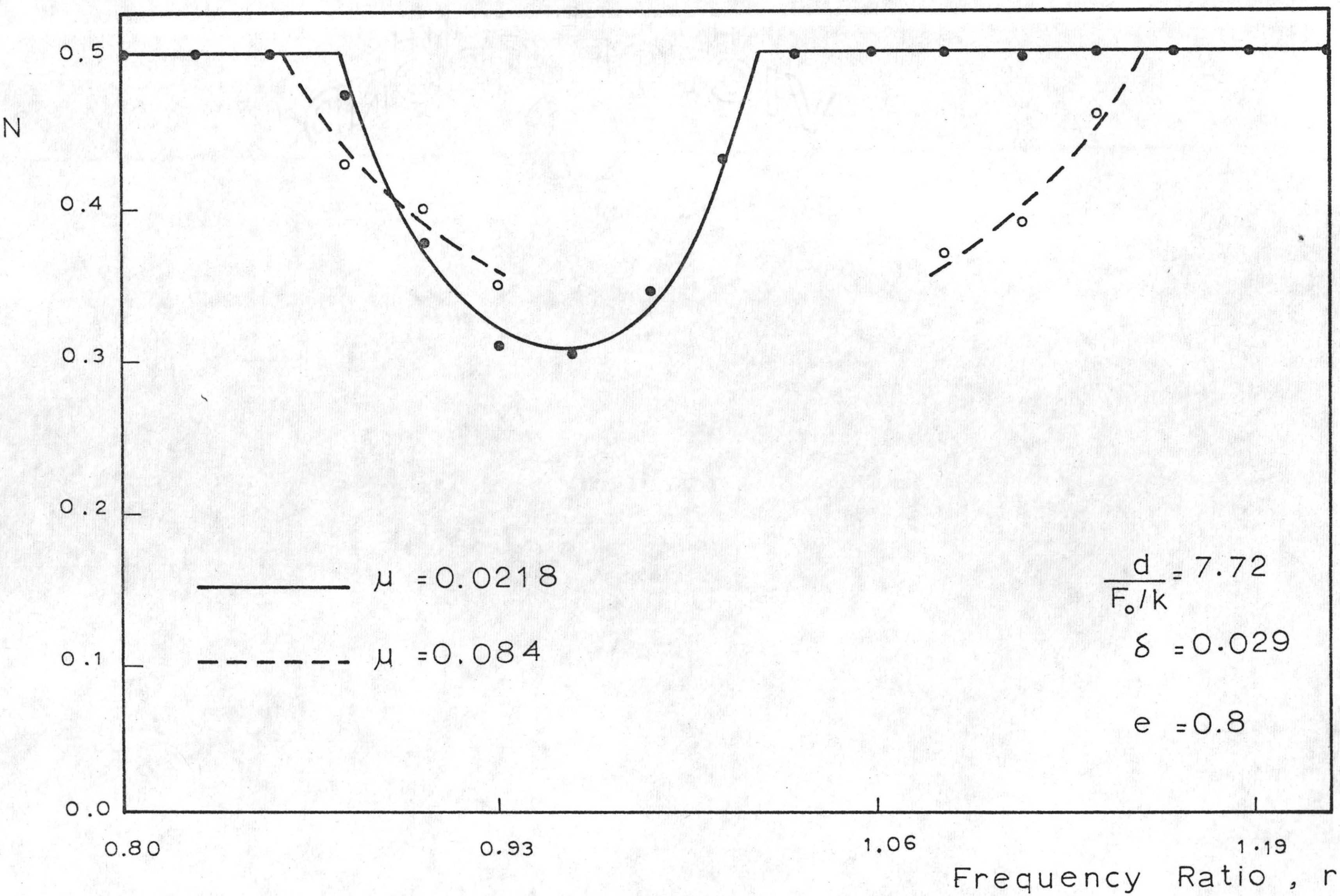


Fig. (4.20) Effect of frequency ratio on the unsymmetry ratio , N

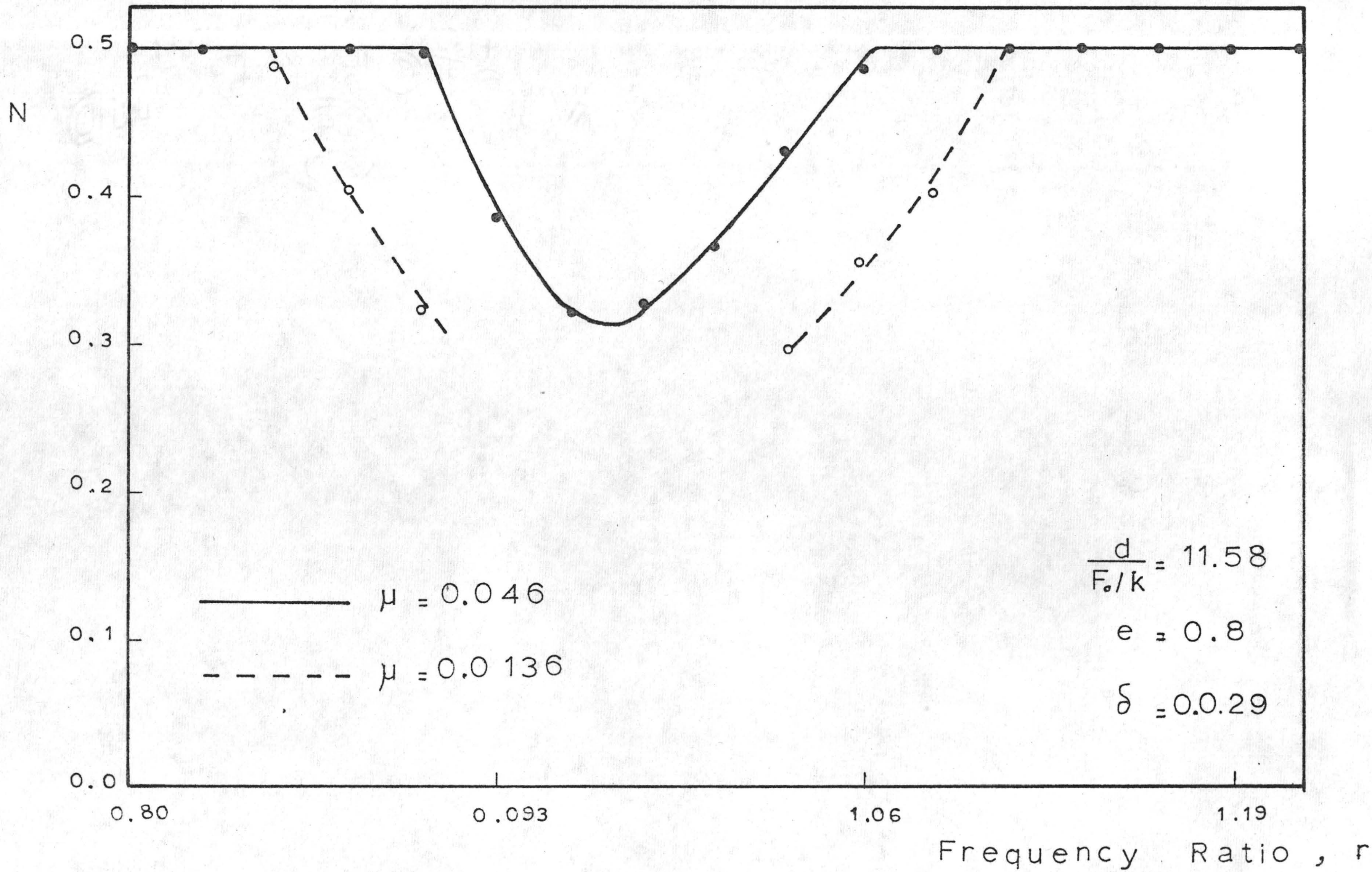


Fig.(4.21) Effect of frequency ratio on the unsymmetry ratio , N

4.5. DIGITAL COMPUTER STUDIES

An alternative approach to the above analytical studies is the direct step-by-step solution of the basic equation of motion on a digital computer. The disadvantages of this digital computer approach are the difficulty of exhibiting the results in a general form and certain computational problems in the investigation of marginal stability regions. The advantage is the fact that at least for specific cases, a complete picture of system response can be obtained to any desired degree of accuracy.

The equation of motion of the mathematical model Fig. (1.1), between impacts, is

$$\left.\begin{aligned} M\ddot{x} + c\dot{x} + kx &= F_o \sin(\Omega t) \\ \blacklozenge\ \ddot{y} &= -\ddot{x} \end{aligned}\right] \quad (4.1)$$

If immediately after the ith impact at $t = t_i$

$$x(t_{i_+}) = x_i$$

$$y(t_{i_+}) = y_i$$

♦ Derivation is in Appendix II

$$\dot{x}(t_{i_+}) = \dot{x}_i$$

$$\dot{y}(t_{i_+}) = \dot{y}_i$$

Then the motion of M and m is described during the time interval from t_{i_+} to the time immediately preceeding the next impact, $t_{(i+1)_-}$ by

$$\left.\begin{array}{l} x = e^{-\delta\omega(t-t_i)}(D_i \sin\eta\omega(t-t_i) + \\ \quad E_i \cos\eta\omega(t-t_i)) + A\sin(\Omega t - \Psi) \\ \\ y = -x + (x_i + y_i) + (\dot{x}_i + \dot{y}_i)(t-t_i) \\ \qquad\qquad t_{i_+} \leqslant t \leqslant t_{(i+1)_-} \end{array}\right] \qquad (4.2)$$

where

$$E_i = x_i - A\sin(\Omega t_i - \psi)$$

$$D_i = \frac{1}{\eta}(\delta E_i + \frac{\dot{x}_i}{\omega} - A r\cos(\Omega t_i - \psi))$$

From the impact conditions at $t_{(i+1)_+}$

$$\left.\begin{array}{l} x(t_{(i+1)_+}) = x(t_{(i+1)_-}) \\ \\ y(t_{(i+1)_+}) = y(t_{(i+1)_-}) \qquad ; |y| = \frac{d}{2} \end{array}\right]$$

$$\left.\begin{aligned}\dot{x}(t_{(i+1)_+}) &= \dot{x}(t_{(i+1)_-}) + k_2\,\dot{y}(t_{(i+1)_-})\\ \dot{y}(t_{(i+1)_+}) &= -e\,\dot{y}(t_{(i+1)_-})\end{aligned}\right] \qquad (4.3)$$

Conditions (4.3) can now be used as new initial conditions in equations (4.2) for the time interval $t_{(i+1)_+}$ to $t_{(i+2)_-}$. This process can be repeated over and over again so as to obtain the time behaviour of the model.

A digital computer program to find the "exact" sequence of initial conditions and the resulting motion according to (4.2), for any given set of parameters and "initial" initial conditions was written in FORTRAN IV language, and executed by means of the CDC 6400 computer, in the Computing Center of McMaster University.

Besides furnishing further checks on the validity of the data obtained from the theoretical 2 impacts/cycle solution, it provides also (by propagation of round off errors) convenient means of simulating the actual propagation of small perturbations in the steady state solution.

Among the basic features of this program were the following ones:

(a) the R.H.S. of equation (4.2) was evaluated at $t = (t_i + j \times \Delta t)$ repeatedly (with j increasing by unity each time) until the quantity $(\frac{d}{2} - |y|)$ becomes negative. Then the Newton-Raphson method was used to find t_{i+1} for which

$$\left| \frac{d}{2} - |y| \right| \leq \epsilon$$

where ϵ was usually chosen to be 0 (10^{-6}).

(b) In the case of equally spaced impacts, when a periodic solution would pass the test designed to determine if it had reached steady state conditions, the program would then discontinue that solution and start constructing a new one corresponding to a new set of the parameters ω, r, $\frac{Fo}{k}$, μ, δ, e and $\frac{d}{Fo/k}$.

Solutions that did not pass the unequally spaced two impacts-per-cycle steady state test (including the equally spaced two-impacts per cycle steady state) were terminated after reaching a specified number of impacts.

(c) Single precision arithmetic was employed throughout the program which required, for $\Delta t = \frac{2\pi}{60\Omega}$ an execution time of approximately 1 sec/100 impacts.

Table (4.1) shows a typical digital computer output.

TABLE 4.1 (a)

DIGITAL COMPUTER OUTPUT

CASE(1) SYMMETRIC 2 IMPACTS/CYCLE

D=.1 F0/K=1.00 E=.2 WN=1.00

W=1.25 R=1.25 D0=3.00 U=.4

T=6.28 A=1.62

IMPACT # (i)	T_i	X_i	Y_i	$\dot{X}_{i+}$	$\dot{Y}_{i+}$	$\frac{X_{max}}{A}$
–	–	–	–	–	–	–
1	4.88	-1.5000	1.50	-.8160	-.8578	-.9246
2	6.52	-.3079	-1.50	1.0210	.6468	-1.0500
3	8.93	.7072	1.50	-.3582	-.6165	.7856
4	12.27	.4467	-1.50	.5965	.4782	-.5437
5	14.33	-.3433	1.50	-.5449	-.4929	.4444
6	16.92	.2044	-1.50	.5949	-.4969	-.5026
7	19.18	-.0778	1.50	-.6011	-.5152	.5511
8	21.74	.0674	-1.50	.5839	.5175	-.5437
9	24.29	-.1226	1.50	-.5691	-.5084	.5125
10	26.81	.1665	-1.50	.5703	.5015	-.4970
11	29.30	-.1617	1.50	-.5791	-.5024	.5045
12	31.80	.1355	-1.50	.5825	.5065	-.5166
13	34.32	-.1237	1.50	-.5809	-.5083	.5197
14	36.84	.1297	-1.50	.5778	.5073	-.5153
15	39.35	-.1394	1.50	-.5768	-.5085	.5111
16	41.87	.1422	-1.50	.5780	.5054	-.5108
17	44.38	-.1386	1.50	-.5791	-.5060	.5129
18	46.89	.1351	-1.50	.5792	.5065	-.5143
19	49.40	-.1349	1.50	-.5786	-.5065	.5140
20	51.92	.1366	-1.50	.5783	.5063	-.5131
21	54.43	-.1377	1.50	-.5783	-.5061	.5127
22	56.94	.1375	-1.50	.5786	.5061	-.5130
23	59.46	-.1368	1.50	-.5787	-.5062	.5133
24	61.97	.1365	-1.50	.5786	.5063	-.5134
25	64.48	-.1367	1.50	-.5785	-.5062	.5133
26	67.00	.1370	-1.50	.5785	.5062	-.5133
27	69.51	-.1370	1.50	-.5785	-.5062	.5131
28	72.02	.1369	-1.50	.5786	.5062	-.5132
29	74.54	-.1368	1.50	-.5786	-.5062	.5132

30	77.05	.1368	-1.50	.5785	.5062	-.5132
31	79.56	-.1369	1.50	-.5785	-.5062	.5132
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
92	232.87	.1369	-1.50	.5785	.5062	-.5132
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.

TABLE 4.1 (b)

DIGITAL COMPUTER OUTPUT

CASE(2) UNSYMMETRIC 2 IMPACTS/CYCLE

D=0.10 FO/K=1.00 E=0.80 WN=1.00

W=1.00 R=1.00 DO=10.00 U=0.10

T=6.28 A=5.00

IMPACT # (i)	T_i	X_i	Y_i	$\dot{X}_{i+}$	$\dot{Y}_{i+}$	$\frac{X_{max}}{A}$
1	.86	3.5658	5.00	4.1288	-5.6160	.7915
2	3.78	-.7852	5.00	-3.9426	-2.3487	1.1139
3	5.84	-3.7604	-5.00	.4597	6.4575	-.8491
4	8.26	2.9412	5.00	3.2653	-3.4932	-.7208
5	10.15	2.5096	5.00	-3.0902	-2.7378	.8957
6	12.94	-3.7535	-5.00	.7182	6.2617	-.8485
7	15.46	3.8058	5.00	2.4531	-4.3301	.7848
8	17.54	-.1092	5.00	-4.1944	-2.2166	.8959
9	19.63	-3.4858	-5.00	.9333	7.0250	-.8325
10	21.69	2.8638	5.00	3.4829	-4.2808	-.6441
11	23.71	1.2471	5.00	-3.7904	-2.8624	.8713
12	25.94	-3.5544	-5.00	.3241	6.6736	-.7800
13	28.34	3.2692	5.00	2.6298	-4.1780	.7087
14	30.38	.1066	5.00	-3.8697	-2.2206	.8150
15	32.52	-2.9081	-5.00	1.0240	6.8049	-.7346
16	34.49	2.5158	5.00	3.0995	-4.5238	-.5267
17	36.98	-1.0310	5.00	-3.5179	-2.0025	.7575
18	39.01	-2.2227	-5.00	1.5250	6.7391	-.7001
19	40.77	2.3053	5.00	2.9487	-5.0844	.4766
20	46.15	.8004	-5.00	2.9120	4.8282	-.7293
21	47.72	2.9027	5.00	.4583	-6.9653	.6069
22	50.08	-2.4520	-5.00	-2.9216	3.4295	.5864
23	51.92	-1.5158	-5.00	3.0834	2.4636	-.7317
24	54.48	2.7069	5.00	-.9846	-6.2475	.6856
25	56.62	-2.7504	-5.00	-2.4890	4.5369	-.5684
26	62.07	-2.5990	5.00	-2.7343	-4.5742	.7371
27	63.59	-2.7338	-5.00	.2098	7.1914	-.6329
28	65.57	1.9327	5.00	3.3462	-3.8787	-.5249
29	67.80	.7464	5.00	-3.3343	-2.6799	.7347
30	70.06	-2.8650	-5.00	.6559	6.3800	-.6789
31	72.29	2.7817	5.00	2.5460	-4.2947	.6050
32	74.81	-1.6320	5.00	-3.2704	-1.4556	.7261
33	76.90	-1.5133	-5.00	2.0878	6.5176	-.7044

34	78.53	2.4695	5.00	2.6054	-5.7391	.5353
35	83.11	-1.8982	-5.00	2.2304	5.1309	-.7199
36	85.14	3.0795	5.00	1.5972	-5.5135	.6211
37	89.16	-2.6345	-5.00	1.4992	5.1800	-.7087
38	91.53	3.1988	5.00	1.3701	-5.0782	.6414
39	95.65	-2.0993	-5.00	1.9715	5.4327	-.7095
40	97.69	2.9586	5.00	1.8391	-5.3231	.6071
41	101.97	-1.9726	-5.00	2.0605	5.3035	-.7044
42	104.00	2.9598	5.00	1.7154	-5.4030	.6028
43	108.14	-2.2971	-5.00	1.7562	5.2071	-.6955
44	110.34	3.0373	5.00	1.5372	-5.1902	.6124
45	114.49	-2.1138	-5.00	1.8928	5.3047	-.6919
46	116.58	2.9575	5.00	1.6821	-5.2756	.6004
47	120.77	-2.1006	-5.00	1.9009	5.2555	-.6916
48	122.88	2.9645	5.00	1.6318	-5.2845	.6002
49	127.02	-2.1833	-5.00	1.8204	5.2352	-.6888
50	129.17	2.9838	5.00	1.5889	-5.2290	.6001
51	133.32	-2.1310	-5.00	1.8600	5.2609	-.6878
52	135.44	2.9629	5.00	1.6267	-5.2553	.5992
53	139.61	-2.1356	-5.00	1.8559	5.2460	-.6881
54	141.73	2.9673	5.00	1.6098	-5.2533	.5995
55	145.88	-2.1532	-5.00	1.8383	5.2435	-.6873
56	148.02	2.9710	5.00	1.6024	-5.2412	.5996
57	152.17	-2.1392	-5.00	1.8494	5.2459	-.6872
58	154.30	2.9659	5.00	1.6117	-5.2487	.5991
59	158.45	-2.1424	-5.00	1.8468	5.2453	-.6873
60	160.58	2.9676	5.00	1.6065	-5.2471	.5993
61	164.73	-2.1458	-5.00	1.8433	5.2454	-.6871
62	166.87	2.9681	5.00	1.6057	-5.2446	.5993
63	171.02	-2.1422	-5.00	1.8463	5.2467	-.6871
64	173.15	2.9669	5.00	1.6079	-5.2467	.5992
65	177.30	-2.1434	-5.00	1.8452	5.2466	-.6871
66.	179.43	2.9675	5.00	1.6064	-5.460	.5992
67	183.58	-2.1440	-5.00	1.8446	5.2458	-.6871
68	185.72	2.9675	5.00	1.6064	-5.2456	.5992
69	189.87	-2.431	-5.00	1.8454	5.2461	-.6871
70	192.00	2.9672	5.00	1.6069	-5.2461	.5992
71	196.15	-2.1435	-5.00	1.8450	5.2458	-.6871
72	198.28	2.9674	5.00	1.6065	-5.2458	.5992
73	202.43	-2.1436	-5.00	1.8450	5.2459	-.6871
74	204.57	2.9674	5.00	1.6066	-5.2458	.5992
75	208.72	-2.1434	-5.00	1.8451	5.2459	-.6871
76	210.85	2.9673	5.00	1.6067	-5.2459	.5992
77	215.00	-2.1435	-5.00	1.8450	5.2458	-.6871
78	217.13	2.9674	5.00	1.6066	-5.2458	.5992
79	221.28	-2.1435	-5.00	1.8450	5.2459	-.6871
80	223.41	2.9674	5.00	1.6066	-5.2458	.5992
81	227.57	-2.1434	-5.00	1.8451	5.2495	-.6871
82	229.70	2.9674	5.00	1.6066	-5.2458	.5992
83	233.85	-2.1435	-5.00	1.8450	5.2458	-.6871

84	235.98	2.9674	5.00	1.6066	-5.2459	.5992
85	240.13	-2.1435	-5.00	1.8450	5.2459	-.6871
86	242.26	2.9674	5.00	1.6066	-5.2459	.5992
87	246.42	-2.1435	-5.00	1.8450	5.2459	-.6871
88	248.55	2.9674	5.00	1.6066	-5.2459	.5992
89	252.70	-2.1435	-5.00	1.8450	5.2459	-.6871
90	254.83	2.9674	5.00	1.6066	-5.2459	.5992
91	258.98	-2.1435	-5.00	1.8450	5.2459	-.6871
92	261.11	2.9674	5.00	1.6066	-5.2459	.5992
93	265.27	-2.1435	-5.00	1.8450	5.2459	-.6871
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199	598.27	-2.1435	-5.00	1.8450	5.2459	-.6871
200	600.41	2.9674	5.00	1.6066	-5.2459	.5992
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TABLE 4.1 (c)

DIGITAL COMPUTER OUTPUT

CASE(3) 3 IMPACTS/CYCLE

D=.1	FO/K=1.	E=.8	WN=1.
W=1.	R=1.	DO=8.	U=.1
T=6.28	A=5.		

IMPACT # (i)	T_i	X_i	Y_i	$\dot{X}_{i+}$	$\dot{Y}_{i+}$	$\frac{X_{max}}{A}$
1	.71	3.0898	4.00	4.53	-5.2294	.6784
2	3.29	1.2896	4.00	-3.90	-3.0629	1.1161
3	5.22	-4.1454	-4.00	-1.46	5.2607	-.8328
4	9.46	3.9488	-4.00	.97	-4.5575	-.8870
5	12.40	-4.3122	-4.00	-.95	4.3753	-.8617
6	16.73	2.5020	4.00	-2.67	-5.8298	.9121
7	18.31	-2.9292	-4.00	-3.58	4.7062	-.5960
8	20.60	-.3483	-4.00	4.10	2.8386	-.9064
9	22.38	4.0482	4.00	.55	-6.1059	.8206
10	25.29	-4.0722	-4.00	-1.71	3.6768	-.8290
11	26.98	-.7523	-4.00	4.10	2.0452	-.8756
12	29.01	3.7715	4.00	-.26	-6.1300	.8045
13	31.36	-3.2574	-4.00	-2.87	3.3659	.7265
14	32.89	-2.5021	-4.00	3.08	2.4770	-.8425
15	35.42	3.5676	4.00	-.43	-5.7280	.7737
16	37.84	-3.3389	-4.00	-2.34	3.6450	-.7124
17	39.59	-1.0674	-4.00	3.63	2.2310	-.7938
18	41.71	3.3618	4.00	-.24	-5.8394	.7236
19	44.08	-3.0386	-4.00	-2.50	3.4288	.6466
20	45.77	-1.4624	-4.00	3.31	2.2749	-.7614
21	48.03	3.2058	4.00	-.33	-5.6634	.6961
22	50.40	-2.9949	-4.00	-2.33	3.5055	.6123
23	52.20	-.8875	-4.00	3.42	2.1489	-.7355
24	54.35	3.0623	4.00	-.41	-5.7194	.6775
25	56.60	-2.7699	-4.00	-2.54	3.4385	-.5999
26	58.39	-1.1517	-4.00	3.25	2.2474	-.7223
27	60.61	3.0421	4.00	-.37	-5.6106	.6703
28	62.94	-2.8610	-4.00	-2.34	3.4747	-.5945
29	64.77	-.7855	-4.00	3.35	2.1269	-.7154
30	66.92	2.9730	4.00	-.46	-5.6815	.6634

31	69.15	-2.7077	-4.00	-2.54	3.4462	-.5950
32	70.97	-1.0471	-4.00	3.25	2.2363	-.7128
33	73.17	3.0088	4.00	-.38	-5.6048	.6651
34	75.49	-2.8337	-4.00	-2.36	3.4646	-.5926
35	77.33	-.8009	-4.00	3.34	2.1345	-.7130
36	79.48	2.9717	4.00	-.46	-5.6691	.6632
37	81.71	-2.7216	-4.00	-2.52	3.4476	-.5952
38	83.54	-1.0256	-4.00	3.26	2.2280	-.7133
39	85.74	3.0109	4.00	-.38	-5.6110	.6659
40	88.05	-2.8311	-4.00	-2.37	3.4620	-.5935
41	89.88	-.8329	-4.00	3.33	2.1453	-.7142
42	92.04	2.9830	4.00	-.44	-5.6644	.6649
43	94.29	-2.7405	-4.00	-2.50	3.4481	-.5957
44	96.11	-1.0174	-4.00	3.27	2.2211	-.7146
45	98.30	3.0138	4.00	-.39	-5.6170	.6668
46	100.61	-2.8281	-4.00	-2.38	3.4609	-.5942
47	102.44	-.8558	-4.00	3.33	2.1533	-.7150
48	104.60	2.9895	4.00	-.44	-5.6610	.6657
49	106.86	-2.7520	-4.00	-2.49	3.4487	-.5958
50	108.68	-1.0074	-4.00	3.27	2.2153	-.7151
51	110.87	3.0134	4.00	-.39	-5.6213	.6671
52	113.17	-2.8229	-4.00	-2.39	3.4598	-.5945
53	115.00	-.8711	-4.00	3.32	2.1590	-.7153
54	117.17	2.9924	4.00	-.43	-5.6576	.6660
55	119.43	-2.7589	-4.00	-2.48	3.4495	-.5957
56	121.25	-.9967	-4.00	3.28	2.2104	-.7152
57	123.44	3.0117	4.00	-.39	-5.6244	.6671
58	125.74	-2.8175	-4.00	-2.40	3.4588	-.5946
59	127.57	-.8825	-4.00	3.32	2.1635	-.7153
60	129.73	2.9940	4.00	-.43	-5.6546	.6661
61	132.00	-2.7640	-4.00	-2.48	3.4501	-.5956
62	133.82	-.9870	-4.00	3.28	2.2063	-.7152
63	136.01	3.0100	4.00	-.40	-5.6269	.6670
64	138.30	-2.8128	-4.00	-2.41	3.4579	-.5946
65	140.13	-.8918	-4.00	3.32	2.1673	-.7152
66	142.29	2.9953	4.00	-.43	-5.6521	.6662
67	144.56	-2.7683	-4.00	-2.47	3.4507	-.5955
68	146.39	-.9789	-4.00	3.28	2.2029	-.7152
69	148.57	3.0087	4.00	-.40	-5.6290	.6669
70	150.87	-2.8089	-4.00	-2.41	3.4572	-.5947
71	152.70	-.8996	-4.00	3.31	2.1704	-.7152
72	154.87	2.9964	4.00	-.42	-5.6500	.6663
73	157.13	-2.7719	-4.00	-2.47	3.4512	-.5954
74	158.96	-.9721	-4.00	3.29	2.2001	-.7152
75	161.14	3.0076	4.00	-.40	-5.6308	.6669
76	163.43	-2.8058	-4.00	-2.42	3.4566	-.5948
77	165.26	-.9061	-4.00	3.31	2.1730	-.7152
78	167.43	2.9974	4.00	-.42	-5.6483	.6663
79	169.70	-2.7749	-4.00	-2.46	3.4516	-.5953
80	171.53	-.9665	-4.00	3.29	2.1978	-.7152

81	173.71	3.0067	4.00	-.40	-5.6323	.6668
82	176.00	-2.8031	-4.00	-2.42	3.4562	-.5948
83	177.82	-.9114	-4.00	3.31	2.1752	-.7152
84	180.00	2.9982	4.00	-.42	-5.6469	.6664
85	182.27	-2.7774	-4.00	-2.46	3.4520	-.5953
86	184.09	-.9618	-4.00	3.29	2.1958	-.7152
87	186.28	3.0060	4.00	-.40	-5.6336	.6668
88	188.56	-2.8009	-4.00	-2.42	3.4558	-.5948
89	190.39	-.9159	-4.00	3.31	2.1770	-.7152
90	192.56	2.9989	4.00	-.42	-5.6457	.6664
91	194.83	-2.7795	-4.00	-2.45	3.4523	-.5953
92	196.66	-.9579	-4.00	3.29	2.1942	-.7152
93	198.84	3.0054	4.00	-.41	-5.6346	.6668
94	201.13	-2.7991	-4.00	-2.43	3.4555	-.5949
95	202.96	-.9196	-4.00	3.31	2.1785	-.7152
96	205.13	2.9995	4.00	-.42	-5.6447	.6665
97	207.40	-2.7812	-4.00	-2.45	3.4525	-.5952
98	209.23	-.9546	-4.00	3.29	2.1928	-.7152
99	211.41	3.0049	4.00	-.41	-5.6355	.6667
100	213.69	-2.7975	-4.00	-2.43	3.4552	-.5949
101	215.52	-.9227	-4.00	3.30	2.1797	-.7152
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198	418.76	3.0022	4.00	-.41	-5.6401	.6666
199	421.03	-2.7894	-4.00	-2.44	3.4538	-.5951
200	422.86	-.9388	-4.00	3.30	2.1863	-.7152
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TABLE 4.1 (d)

DIGITAL COMPUTER OUTPUT

CASE(4) 4 IMPACTS/CYCLE

D=.1 FO/K=1. E=.8 WN=1.

W=1. R=1. DO=3.1 U=.1

T=6.28 A=5.

IMPACT # (i)	T_i	X_i	Y_i	$\dot{X}_{i+}$	$\dot{Y}_{i+}$	$\frac{X_{max}}{A}$
1	.30	1.4291	1.55	5.4653	-4.3662	.3763
2	2.58	3.8660	1.55	-2.6747	-3.5810	1.1070
3	3.78	-.5863	-1.55	-4.3429	1.8296	.6557
4	5.37	-4.5770	-1.55	-.3194	2.0985	-.9151
5	6.55	-2.4833	-1.55	3.2324	1.3901	-.9128
6	8.66	4.1597	1.55	1.5170	-2.9704	.8456
7	10.21	1.8959	1.55	-3.5017	-1.9593	.8906
8	11.81	-3.7070	-1.55	-2.6088	2.7281	-.7440
9	13.13	-3.5497	-1.55	2.3966	2.1783	-.9120
10	15.55	4.4356	1.55	1.2432	-3.1869	.8942
11	17.02	1.5755	1.55	-3.9986	-1.9656	.9184
12	18.44	-3.8053	-1.55	-2.8116	3.0154	-.8266
13	19.76	-3.5373	-1.55	2.6729	2.3618	-.9391
14	21.94	4.3179	1.55	1.8701	-3.0270	.8693
15	23.21	2.8477	1.55	-3.3927	-2.1387	.9359
16	25.01	-4.0116	-1.55	-2.4748	2.9238	-.8598
18	28.39	4.2840	1.55	1.6861	-3.1710	.8628
17	26.23	-3.4644	-1.55	2.7757	2.2255	-.9283
19	29.74	2.2877	1.55	-3C6929	-2.1121	C9133
20	31.30	-3.6980	-1.55	-2.7589	2.9136	-.7542
21	32.54	-3.5061	-1.55	2.5321	2.2741	-.9059
22	34.79	4.1953	1.55	1.4606	-3.2002	.8529
23	36.20	1.7485	1.55	-3.8397	-2.0088	.8783
24	37.64	-3.5679	-1.55	-2.6954	3.0169	-.7700
25	38.95	-3.1437	-1.55	2.7320	2.3048	-.8757
26	40.96	3.8712	1.55	2.0117	-2.8935	.8127
27	42.22	2.7628	1.55	-3.0576	-2.0812	.8614
28	44.10	-3.7895	-1.55	-2.0631	2.9419	-.7872
29	45.39	-2.6525	-1.55	3.0812	2.1066	-.8503
30	47.21	3.6876	1.55	2.1752	-2.8816	.7811

32	48.49	2.7855	1.55	-2.9108	-2.1086	.8403
33	50.41	-3.7417	-1.55	-1.9705	2.9163	-.7680
34	53.49	3.6180	1.55	2.1670	-2.8738	.7285
35	54.79	2.7048	1.55	-2.9013	-2.0991	.8268
36	66.68	-3.6821	-1.55	-1.9790	2.8899	-.7606
37	57.99	-2.4923	-1.55	3.0588	2.0546	-.8238
38	58.79	3.6017	1.55	2.1121	-2.8708	.7209
39	61.09	2.6138	1.55	-2.8347	-2.0814	.8165
40	62.95	-3.6357	-1.55	-2.0127	2.8728	-.7583
41	64.26	-2.5118	-1.55.	3.0089	2.0553	-.8180
42	66.08	3.6017	1.55	2.0651	-2.8687	.7604
43	67.39	2.5512	1.55	-2.9646	-2,670	.8159
44	69.23	-3.6069	-1.55	-2.0354	2.8659	-.7577
45	70.54	-2.5235	-1.55	2.9819	2.0579	-.8150
46	72.37	3.6019	1.55	2.0432	-2.8663	.7580
47	73.67	2.5241	1.55	-2.9767	-2.0599	.8142
48	75.51	-3.5999	-1.55	-2.0423	2.8641	-.7575
49	76.82	-2.5242	-1.55	2.9741	2.0587	-.8138
50	78.65	3.6007	1.55	2.0380	-2.8646	.7573
51	79.96	2.5179	1.55	-2.9779	-2.0578	.8136
52	81.79	-3.5983	-1.55	-2.0417	2.8638	-.7573
53	83.10	-2.5217	-1.55	2.9741	2.0585	-.8135
54	84.93	3.5998	1.55	2.0385	-2.8639	.7572
55	86.24	2.5186	1.55	-2.9766	-2.0577	.8135
56	88.07	-3.5991	-1.55	-2.0403	2.8638	-.7573
57	89.38	-2.5203	-1.55	2.9753	2.0581	-.8135
58	91.21	3.5998	1.55	2.0394	-2.8638	.7573
59	92.52	2.5200	1.55	-2.9759	-2.0580	.8136
60	94.35	-3.6000	-1.55	-2.0397	2.8639	-.7574
61	95.66	-2.5202	-1.55	2.9759	2.0581	-.8136
62	97.50	3.6003	1.55	2.0399	-2.8639	.7574
63	98.81	2.5210	1.55	-2.9758	-2.0582	.8137
64	100.64	-3.6006	-1.55	-2.0398	2.8640	-.7575
65	101.95	-2.5211	-1.55	2.9760	2.0582	-.8138
66	103.78	3.6008	1.55	2.0400	-2.8641	.7575
67	105.09	2.5216	1.55	-2.9759	-2.0583	.8138
68	106.92	-3.6011	-1.55	-2.0400	2.8641	-.7576
69	108.23	-2.5217	-1.55	2.9761	2.0583	-.8139
70	110.06	3.6012	1.55	2.0401	-2.8642	.7576
71	111.37	2.5220	1.55	-2.7761	-2.0584	.8139
72	113.20	-3.6013	-1.55	-2.0401	2.8642	-.7576
73	114.51	-2.5221	-1.55	2.9761	2.0584	-.8139
74	116.35	3.6014	1.55	2.0401	-2.8652	.7576
75	117.65	2.5222	1.55	-2.9761	-2.0585	.8139
76	119.49	-3.6015	-1.55	-2.0402	2.8642	-.7576
77	120.80	-2.5223	-1.55	2.9761	2.0585	-.8140
78	122.63	3.6015	1.55	2.0402	-2.8643	.7576
79	123.94	2.5224	1.55	-2.9762	-2.0585	.8140
80	125.77	-3.6016	-1.55	-2.0402	2.8643	-.7576
81	127.08	-2.5224	-1.55	2.9762	2.0585	-.8140

82	128.91	3.6016	1.55	2.0402	-2.8643	.7576
83	130.22	2.5224	1.55	-2.9762	-2.0585	.8140
84	132.05	-3.6016	-1.55	-2.0402	2.8643	-.7576
85	133.36	-2.5224	-1.55	2.9762	2.0585	-.8140
86	135.19	3.6016	1.55	2.0402	-2.8643	.7576
87	136.50	2.5224	1.55	-2.9762	-2.0585	.8140
88	138.34	-3.6016	-1.55	-2.0402	2.8643	-.7576
.	.	.	.	.	.	.
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.	.	.	.	.	.	.
197	309.29	-2.5224	-1.55	2.9762	2.0585	-.8140
198	311.12	3.6016	1.55	2.0402	-2.8643	.7576
199	312.43	2.5224	1.55	-2.9762	-2.0585	.8140
200	314.27	-3.6016	-1.55	-2.0404	2.8643	-.7576
.	.	.	.	.	.	.
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TABLE 4.2(a)

THEORETICAL RESULTS

CASE(1) SYMMETRIC 2 IMPACTS/CYCLE

D=.1	F0/K=1.	E=.2	WN=1.
W=1.25	R=1.25	DO=3.	U=.1
T=6.28	A=1.62		

A) FIRST THEORETICAL SOLUTION (STABLE)

Z1=-7.10543E-15	Z2=-1.25865E-14
N= 5.00000E-01	THET= 2.48175E+00
B1= 9.17221E-01	B2=-1.13270E+00
B11=-9.17221E-01	B21= 1.13270E+00
XA=-1.36862E-01	XG= 1.36862E-01
DXA=-5.78534E-01	DXB=-1.44633E+00
DXG= 1.44633E+00	DXH= 5.78534E-01
V1=-1.08475E+00	V2= 1.08475E+00
XMAXA1=-5.13211E-01	XMAX1=-8.31380E-01
XMAXA2= 5.13211E-01	XMAX2= 8.31380E-01

B) SECOND THEORETICAL SOLUTION (UNSTABLE)

Z3= 7.10543E-15	Z4= 1.60804E-14

N= 5.00000E-01	THET=-1.53175E+00
B1=-4.53185E-02	B2= 5.59650E-02
B11= 4.53185E-02	B21=-5.59650E-02
XA=-1.56735E+00	XG= 1.56735E+00
DXA= 2.85845E-02	DXB= 7.14612E-02
DXG=-7.14612E-02	DXH=-2.85845E-02
V1= 5.35959E-02	V2=-5.35959E-02
XMAXA3=-9.65402E-01	XMAX3=-1.46395E+00
XMAXA4= 9.65402E-01	XMAX4= 1.46395E+00

TABLE 4.2(b)

THEORETICAL RESULTS

CASE(2) UNSYMMETRIC 2 IMPACTS/CYCLE

D=.1	F0/K=1.	E=.8	WN=1.
W=1.	R=1.	DO=10.	U=.1
T=6.28	A=5.		

A) FIRST THEORETICAL SOLUTION (STABLE)

Z1= 1.56319E-13	Z2=-1.81330E-11
N= 3.39162E-01	THET= 2.94365E+00
B1= 3.18933E+00	B2= 1.16020E+00
B11=-3.22397E+00	B21= 1.70810E+00
XA=-2.14346E+00	XG= 2.96736E+00
DXA= 1.84505E+00	DXB= 2.91806E+00
DXG= 5.33583E-01	DXH= 1.60660E+00
V1= 7.09090E+00	V2=-3.63926E+00
XMAXA1=-6.87111E-01	XMAX1=-3.43550E+00
XMAXA2= 5.99201E-01	XMAX2= 2.99502E+00

B) SECOND THEORETICAL SOLUTION

Z3= 2.84217E-14	Z4=-5.25548E-01

THUS SECOND THEORETICAL SOLUTION DOES NOT EXIST

5. DISCUSSIONS AND CONCLUSIONS

5.1. Discussion of Theoretical Results

The theoretical two impacts per cycle solution derived in Chapter 2 is compared in Fig.(5.1) with the results obtained through digital computer.

In regard to Fig.(5.1) the following remarks can be made:

(a) in the case of symmetric two impacts per cycle motion (which is a special case of unequally spaced two impacts per cycle with N = 0.5), we get the two distinct curves, abe and eca, corresponding to the two theoretical solutions obtained from equation (2.81) by using for the value of τ in one case

$$\tau = \tau_1 = \tan^{-1} \frac{-\rho G_{87} + \sqrt{\rho^2 G_{87}^2 - (G_{86}^2 + G_{87}^2)(\rho^2 - G_{86}^2)}}{-\rho G_{86} - \sqrt{\rho^2 G_{86}^2 - (G_{86}^2 + G_{87}^2)(\rho^2 - G_{87}^2)}}$$

and in the other

$$\tau = \tau_2 = \tan^{-1} \frac{+\rho G_{87} - \sqrt{\rho^2 G_{87}^2 - (G_{86}^2 + G_{87}^2)(\rho^2 - G_{86}^2)}}{-\rho G_{86} + \sqrt{\rho^2 G_{86}^2 - (G_{86}^2 + G_{87}^2)(\rho^2 - G_{87}^2)}}$$

and both solutions satisfied equation (2.80).

(b) in the case of unequally spaced two impacts per cycle, we get the curve cd, corresponding to the one theoretical solution of equation (2.81) for τ which satisfied equation (2.80). The other solution for τ did not satisfy equation (2.80). It should be noted that the value of N varies gradually from 0.5 (at c) to 0.312 (at d).

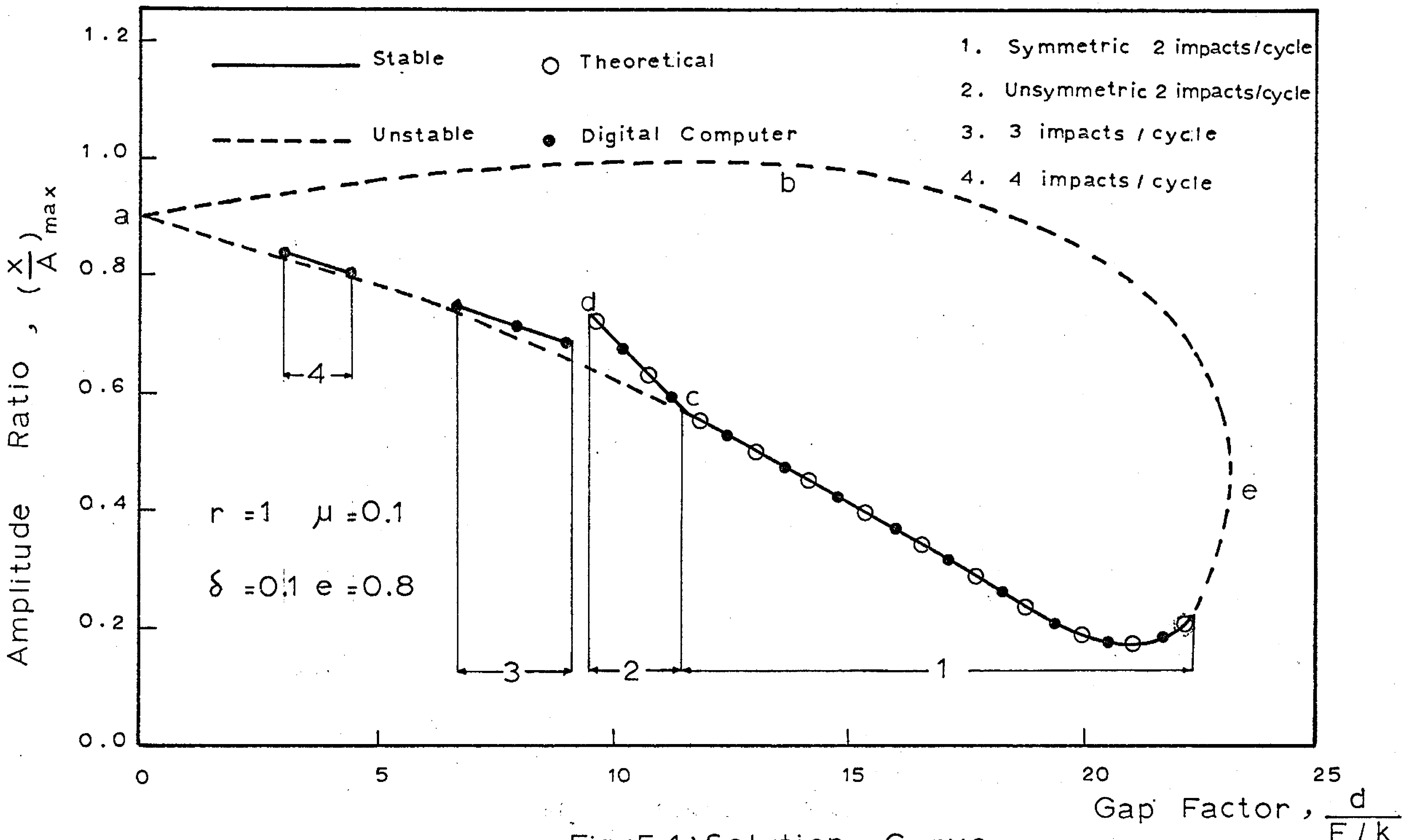


Fig.(5.1) Solution Curve

(c) the ratio $\frac{x_{max}}{A} = (\frac{x}{A})_{max}$ was found by evaluating $(\frac{x}{A})$ from

$$\frac{x}{A} = \frac{e^{-\delta\omega t}}{A}\left[B_1 \sin(\eta\omega t) + B_2 \cos(\eta\omega t)\right] + \sin(\Omega t + \tau) \qquad 0 \leq t \leq \frac{n\pi}{\Omega}$$

at $\Omega t = j \times \Delta t$, $(j = 0,1,2,\ldots)$, with $\Delta t = 0.01$ up to $\Omega t = n\Pi$ and by evaluating $(\frac{x}{A})$ from

$$\frac{x}{A} = \frac{e^{-\delta\omega(t-\frac{n\pi}{\Omega})}}{A}\left[B_1' \sin\eta\omega(t-\frac{n\pi}{\Omega}) + B_2' \cos\eta\omega(t-\frac{n\pi}{\Omega})\right] + \sin(\Omega t + \tau) \qquad \frac{n\pi}{\Omega} \leq t \leq \frac{2\pi}{\Omega}$$

at $\Omega t = n\Pi + j_x \Delta t$, $(j = 0,1,2,\ldots)$, with $\Delta t = 0.01$ up to $\Omega t = 2\pi$.

(d) in the case of symmetric two impacts per cycle, the two solutions corresponding to τ_1 and τ_2 coalesce at the extremes d = 0 for which

$$\tau_1 = \tau_2 = \tan^{-1}(-\frac{H}{2})$$

and at $\frac{d}{Fo/k} = 23.197$ (where $H^2 + 4 - \rho^2 = 0$), since then

$$\tau_1 = \tau_2 = \tan^{-1}(\frac{2}{H})$$

For $\frac{d}{Fo/k} > 23.197$, τ is complex; consequently our two impacts per cycle solution does not exist.

(e) at d = 0, the same value for $(\frac{x}{A})_{max}$ will be found as if the system is treated as a single degree-of-freedom oscillator with a natural frequence $\omega' = \frac{\omega}{\sqrt{1 + \mu}}$

(f) the stability analysis indicates that the τ_2 curve is entirely unstable, the τ_1 curve is only partly stable and the curve cd is completely stable.

The stability boundaries were determined by the method described in Chapter 3.

(g) stable solutions obtained through the digital computer agreed with the theoretical solution and the stability analysis. Also, no two impacts per cycle solutions (symmetric or not) were found outside the stable region.

(h) outside the stable region, the digital computer results show that for $\frac{d}{Fo/k}$ 22.25, the resulting motion is irregular.

(i) it is obvious from Figures (5.2), (5.3), (5.4) and (5.5) that for some parameters for which two impacts/cycle motion was not stable, stable periodic solutions with multiple impacts/cycle could be shown to exist.

Even for cases in which no stable periodic motions were established, the impact damper was often effective in reducing vibration amplitudes. On the other hand, for some stable periodic solutions, the impact damper resulted in an increase of vibration amplitude instead of a decrease. Stability alone is not the critical parameter deciding the effectiveness of the impact damper.

(j) when the free mass is locked to the vibrating system, the number of impacts/cycle will be infinite.

As the gap (d) is increased slightly, the impact damper begins to operate and a reduction in the amplitude of vibration occurs. The number of impacts/cycle starts to decrease from infinity to a large value. Generally, these impacts will be distributed at random over the cycle.

This state of affairs continues until an optimum condition is reached, after which this random distribution stops and a new state of a few impacts/cycle starts. The number of impacts continues to decrease until the steady state of two impacts/cycle prevails.

(k) it is obvious from Figure (5.6) that the increase in the mass ratio (μ) caused a decrease in $(x/A)_{max}$ (i.e. caused an increase in the efficiency of the impact damper), for the same gap (d). This is to be expected since increasing the free mass weight will cause higher dissipation of energy from the vibrating system in order to traverse the free mass through the specific gap (d). However, there exists an upper limit of the gap, for each mass ratio, which eventually leads to the ceasing of the state of two impacts/cycle motion and the start of irregular motion.

(l) Figures (5.6) and (5.7) show the stable two impacts/cycle motion for different parameters. It is noted that the unsymmetric two impacts/cycle does not necessarily appear in all the responses for different parameters. Actually, in Figure (5.7) the case (2) does not have any two impacts/cycle motion.

(m) Figures (5.8), (5.9) and (5.10) show the variation of the different parameters. The value of (N) is constant and equal to 0.5 in the case of symmetric two impacts/cycle motion, then N decreases gradually in the range of unsymmetric two impacts/cycle. The minimum value of N obtained during this work is 0.28.

From Figure (5.9) it is clear that for certain parameters, the unsymmetric two impacts/cycle motion does not exist and the motion is only symmetric two impacts/cycle.

Also, from Figures (5.11), (5.12), (5.13) and (5.14) it is obvious that a small change in mass ratio, damping factor, coefficient of restitution or frequency ratio does not affect the value of N appreciably.

Actually, the value of N is very sensitive to small changes in the gap factor.

Figures (5.15), (5.16) and (5.17) show the effect of frequency ratio on the value of N for different gap factors. It is observed that for the same gap factor the curve is not continuous due to the existence of multiple impacts/cycle motion.

(n) Figures (5.18), (5.19) and (5.20) show the stable-solution curves indicating regions of symmetric and unsymmetric 2 impacts/cycle motion, for different values of damping factor, mass ratio and coefficient of restitution.

(o) from Figure (5.21) it is obvious that the impact damper becomes most efficient (i.e. the amplitude ratio $(x/A)_{max}$ becomes minimum) at a certain gap factor (d/Fo/k). At this gap factor, the velocities of both primary mass and free mass are maximum, thus causing maximum dissipation of energy from the vibrating system.

(p) Figures (5.22) and (5.23) show good agreement between theoretical and experimental results.

(q) strictly speaking, if the mathematical model of Figure (2.1) is started from a state of rest with the particle in the middle of its container, the impact impact damper will not operate if the ratio of the container clearance to the original amplitude of the primary system is less than two. In actual situations, this condition will be remedied by the inevitable presence of friction between the particle and the primary mass or the initial displacement of the particle from the center of its container.

(r) The dependence of the stability boundaries, for any given set of the parameters, on the frequency ratio is complicated. In the immediate vicinity of resonance (where the impact damper would be normally used) the stability boundaries enclose within them a sufficient range of system parameters to make the two impacts/cycle motion practically realizable.

(s) the theoretical solutions and stability analysis for periodic motions with a different number of impacts/cycle, or with a different period than the one treated in this thesis, may be obtained, with some effort, by extending the methods used here.

(t) since in practical applications the resulting amplitude rather than the existence of stable periodic motions is of prime concern, the impact damper fulfilled its role even when its motion was not steady.

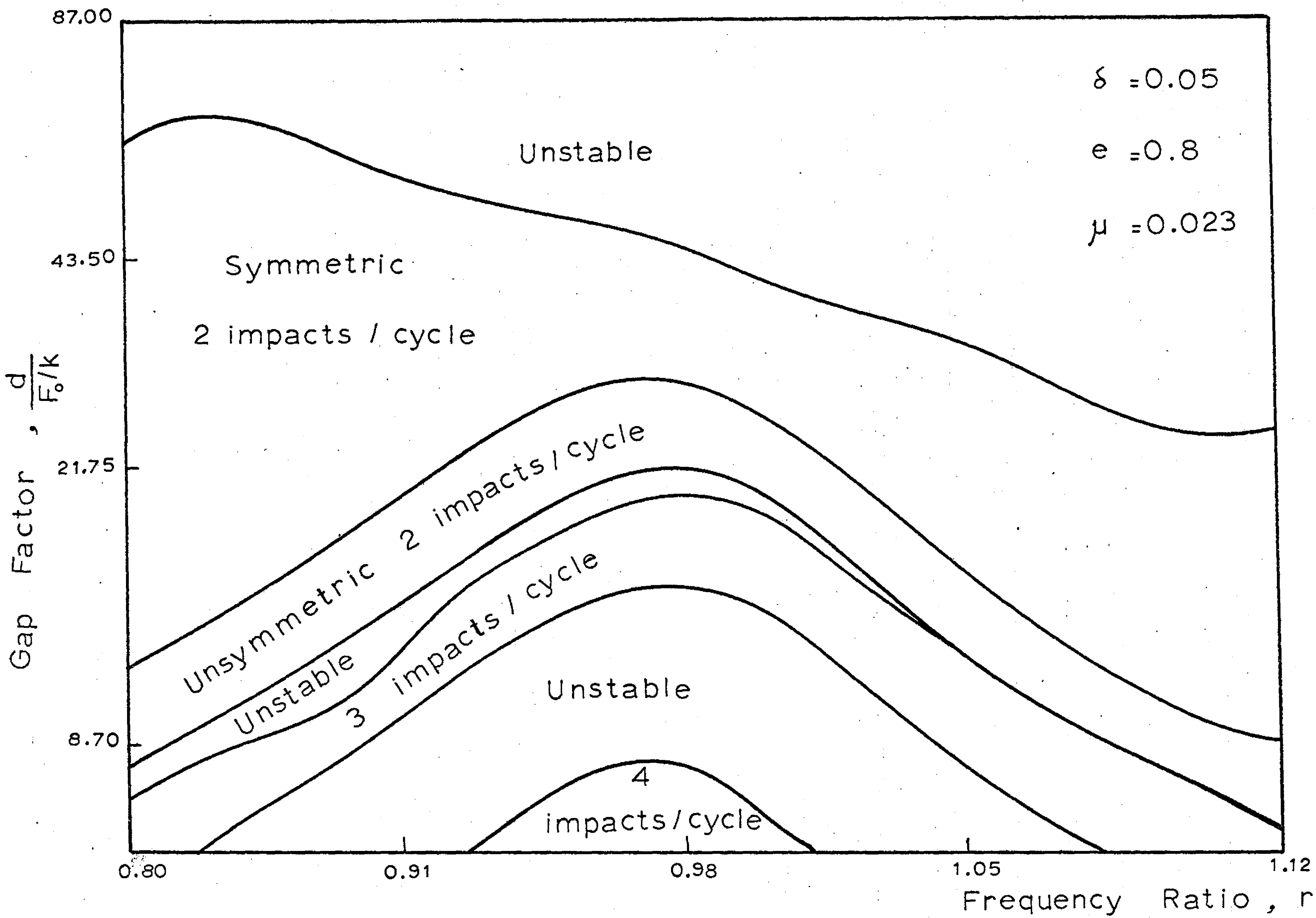


Fig. (5. 2) Stability Boundaries

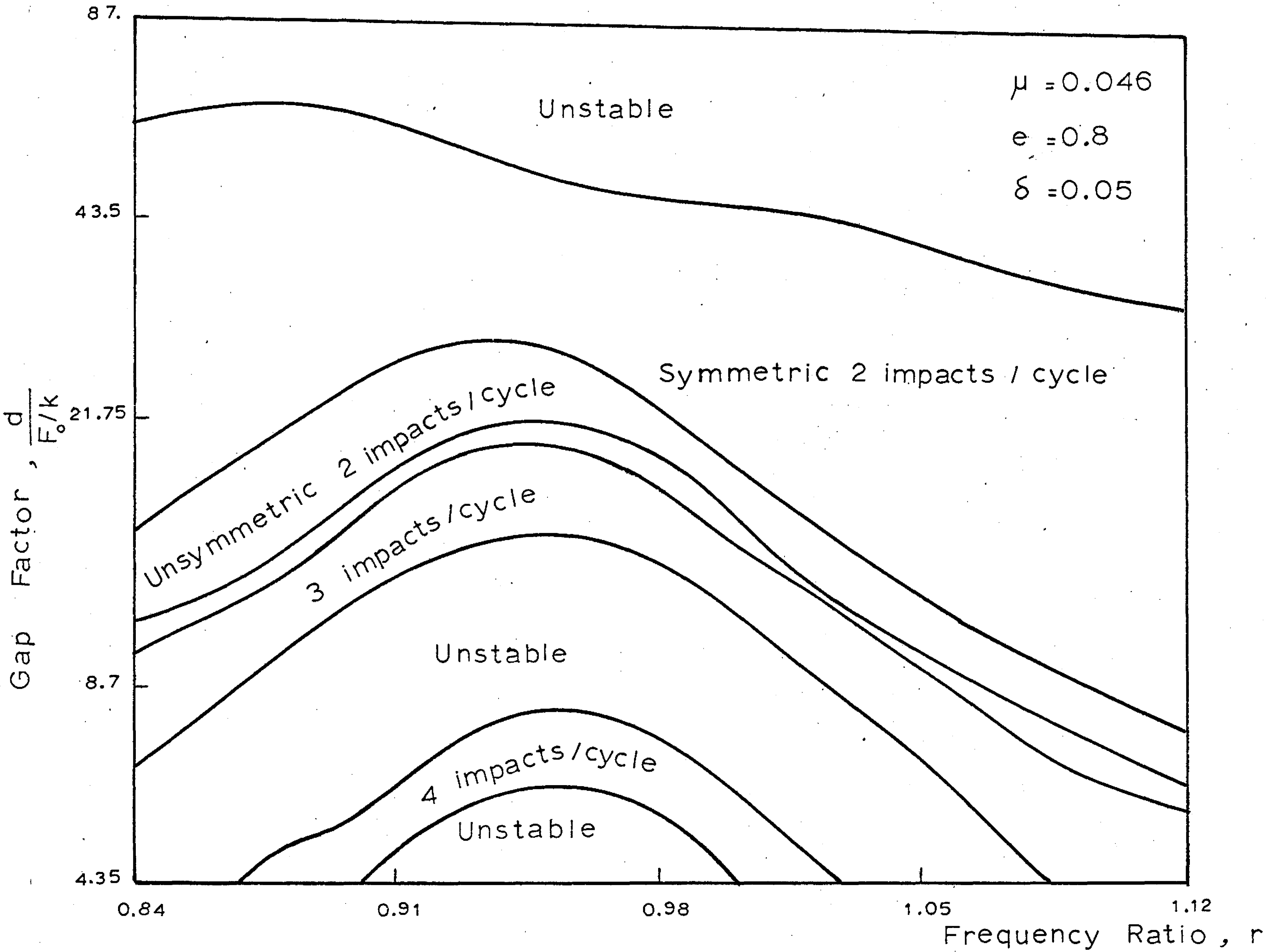


Fig. (5. 3) Stability Boundaries

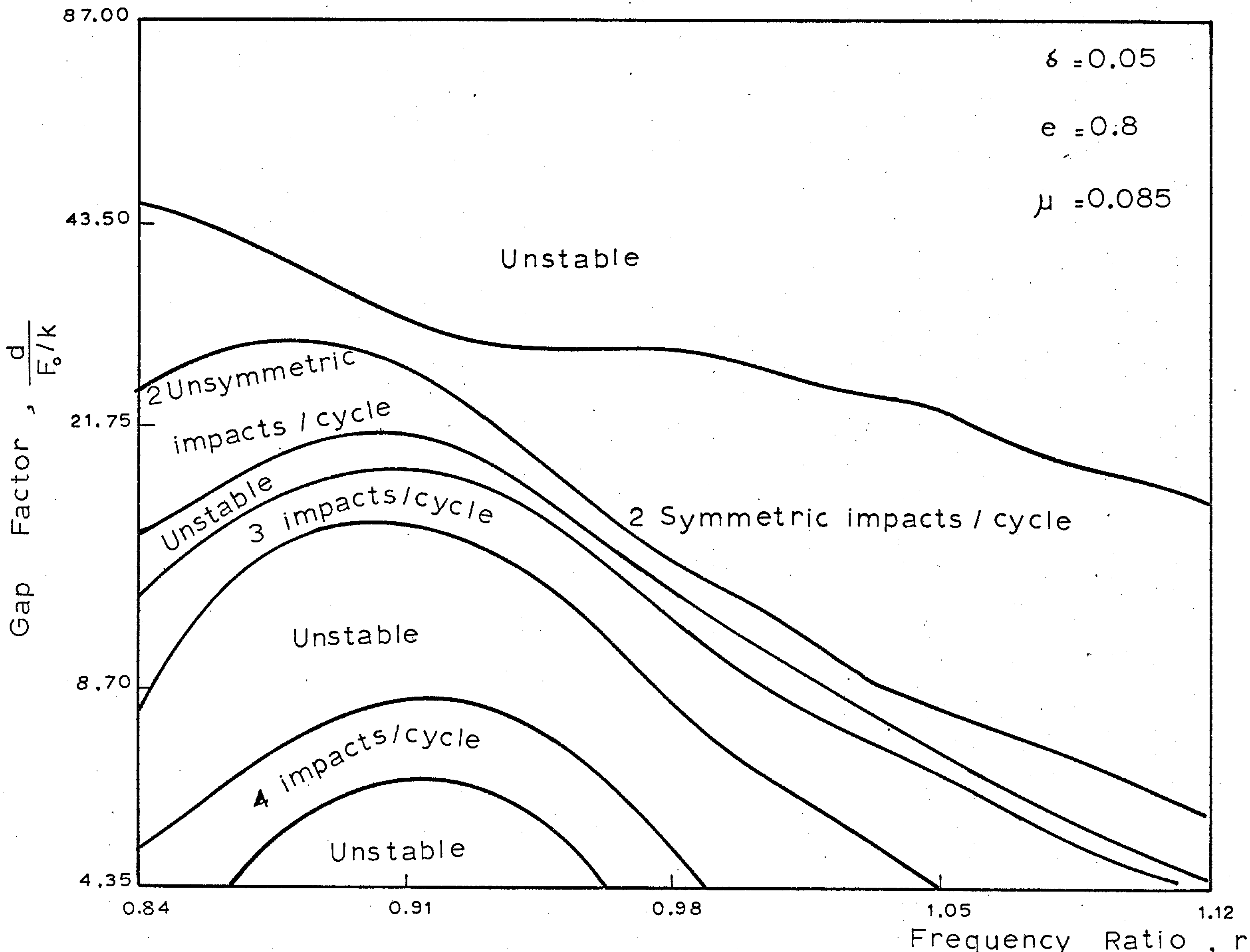


Fig. (5.4) Stability Boundaries

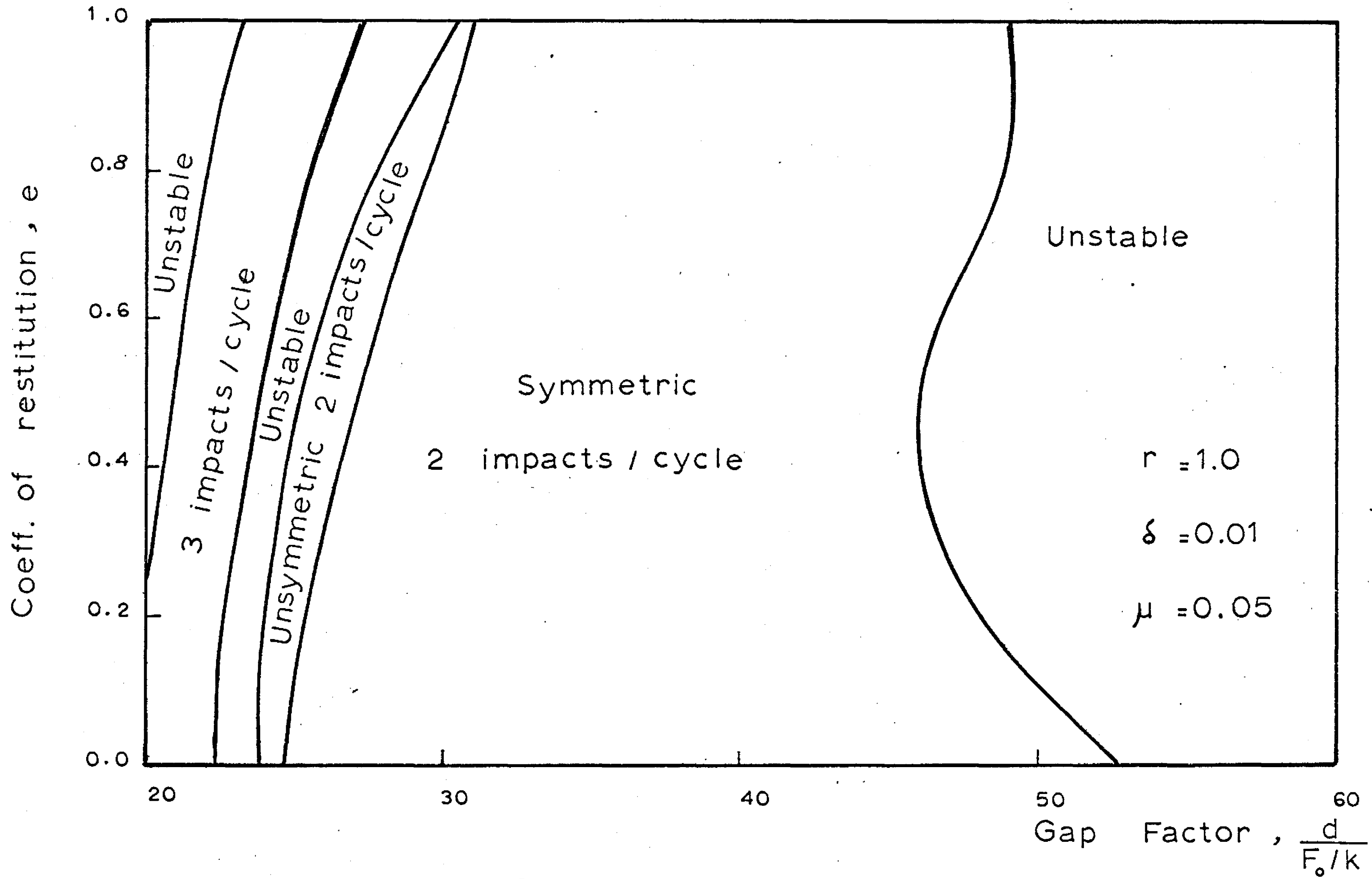


Fig (5.5) Stability boundaries

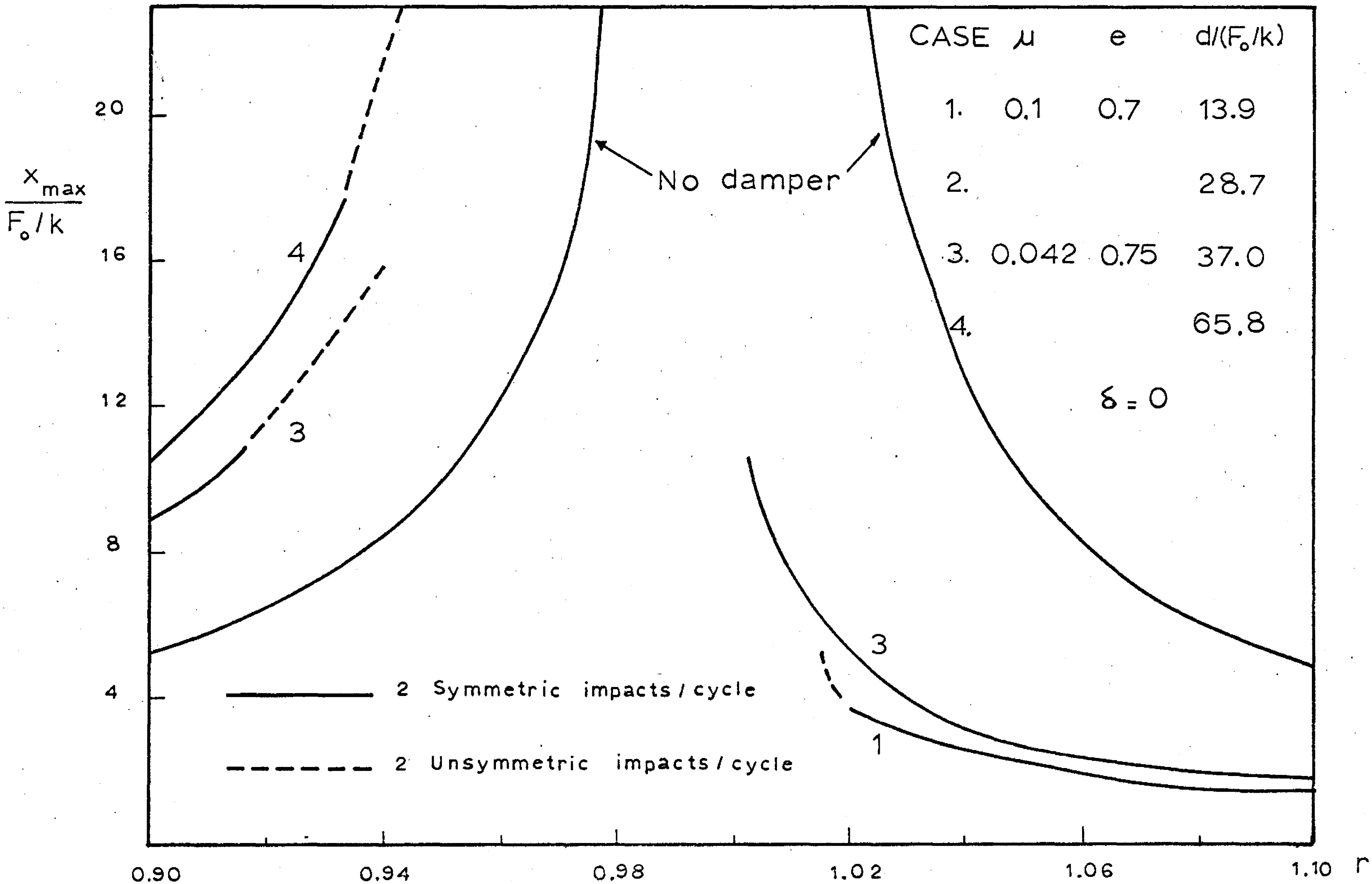


Fig. (5.6) Stable two impacts / cycle motion

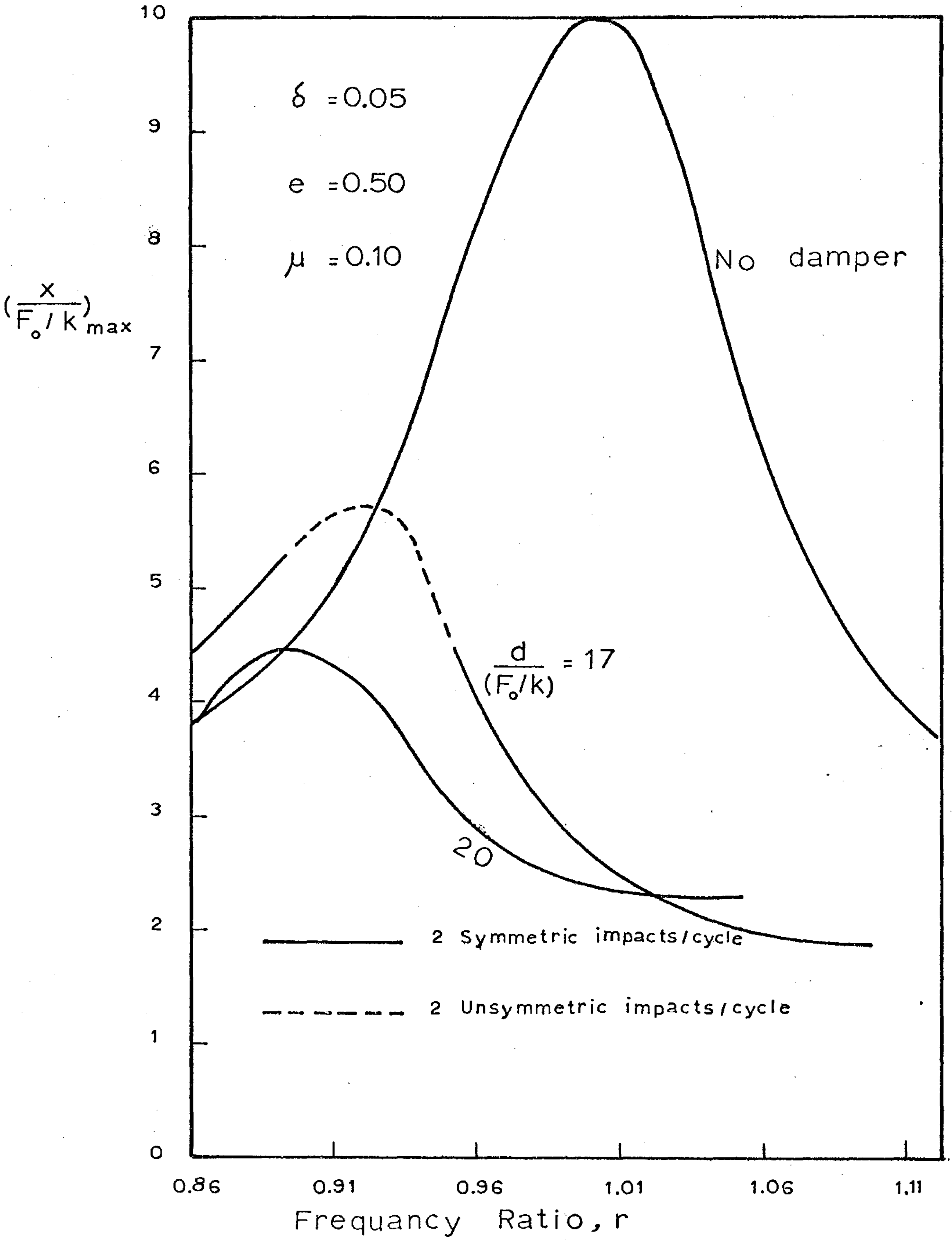


Fig. (5.7) Stable two impacts/cycle motion

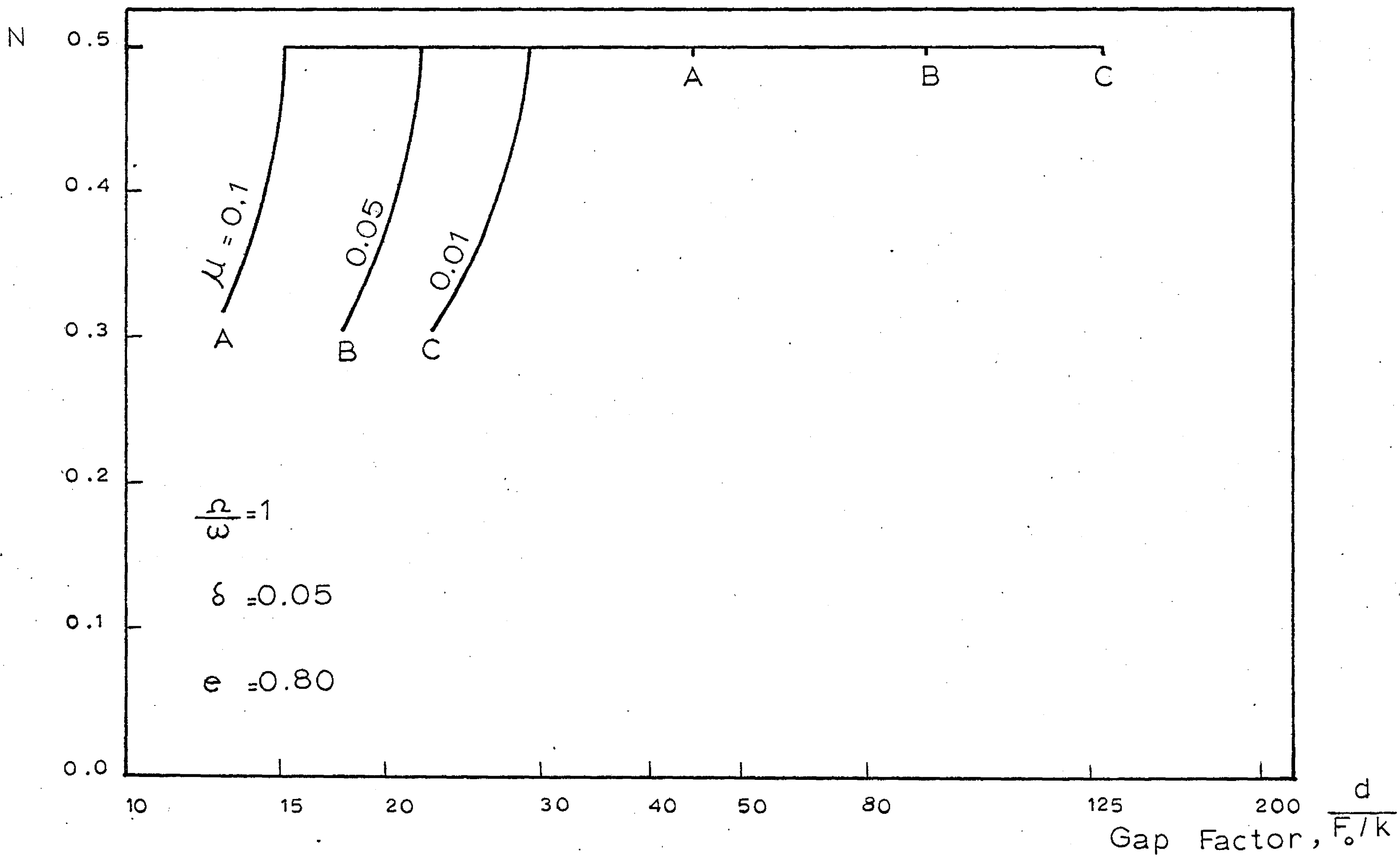


Fig.(5.8) Effect of mass ratio on the unsymmetry ratio, N

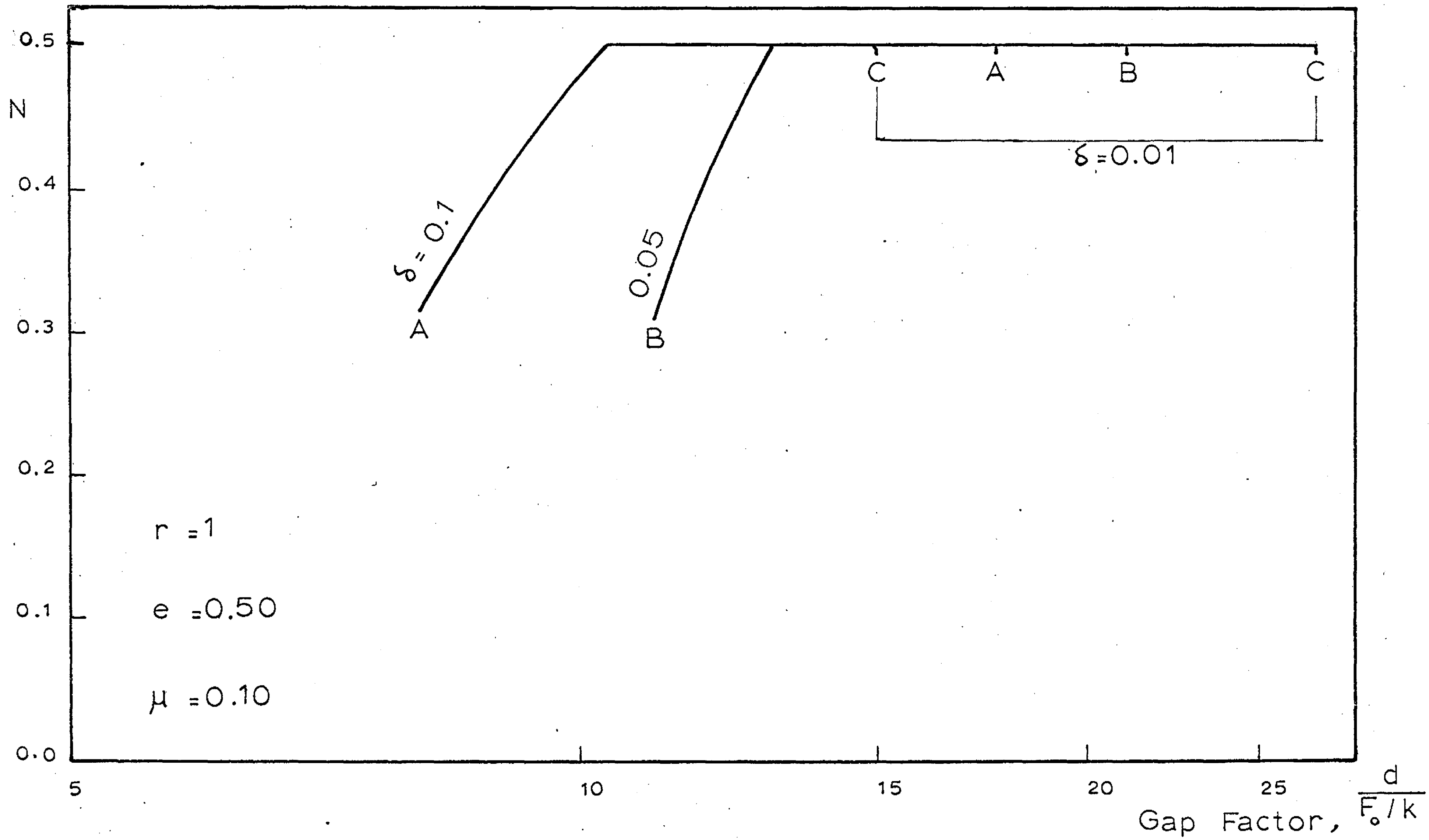


Fig.(5.9) Effect of viscous damping on unsymmetry ratio, N

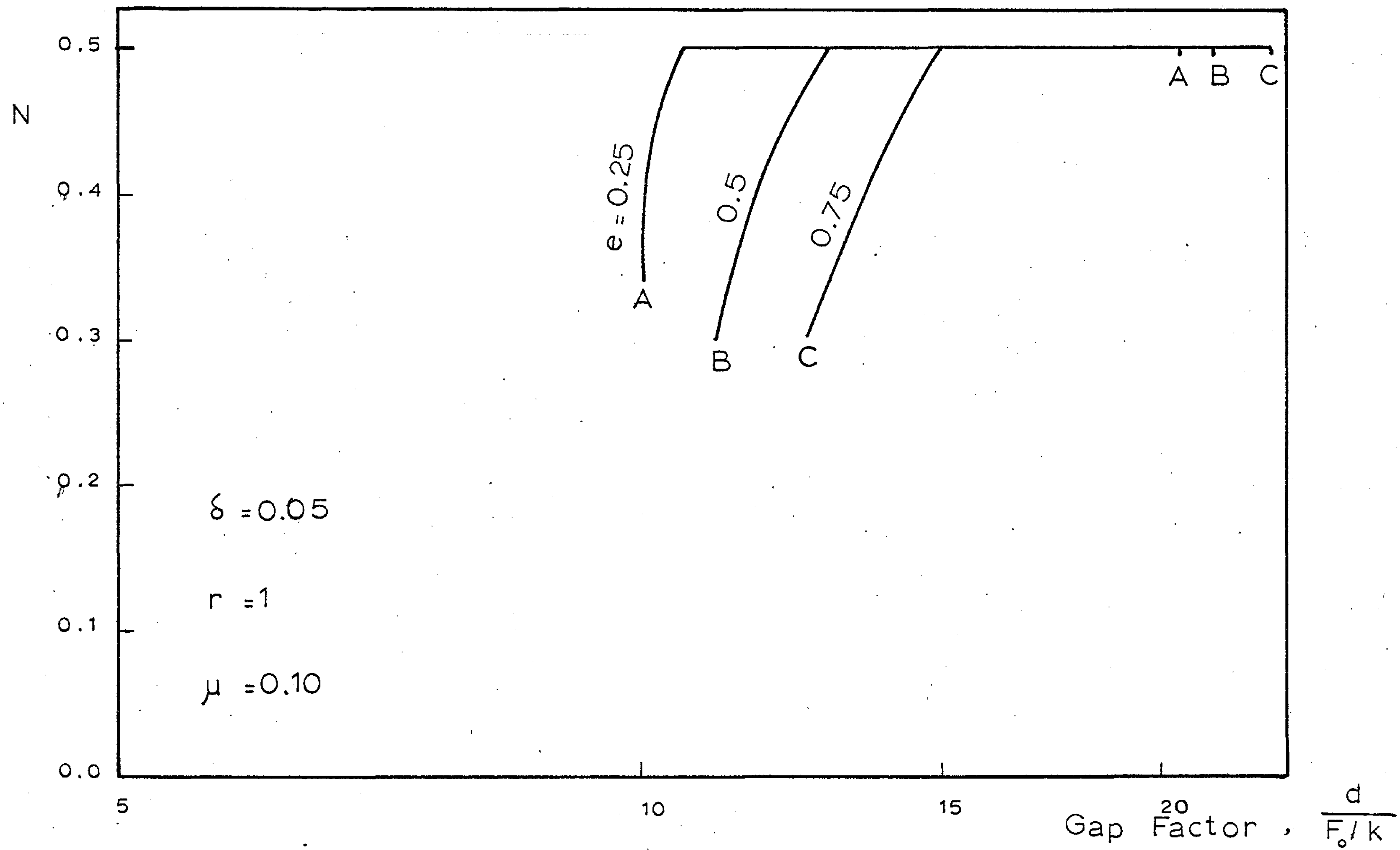


Fig. (5.10) Effect of coeff. of restitution on the unsymmetry ratio, N

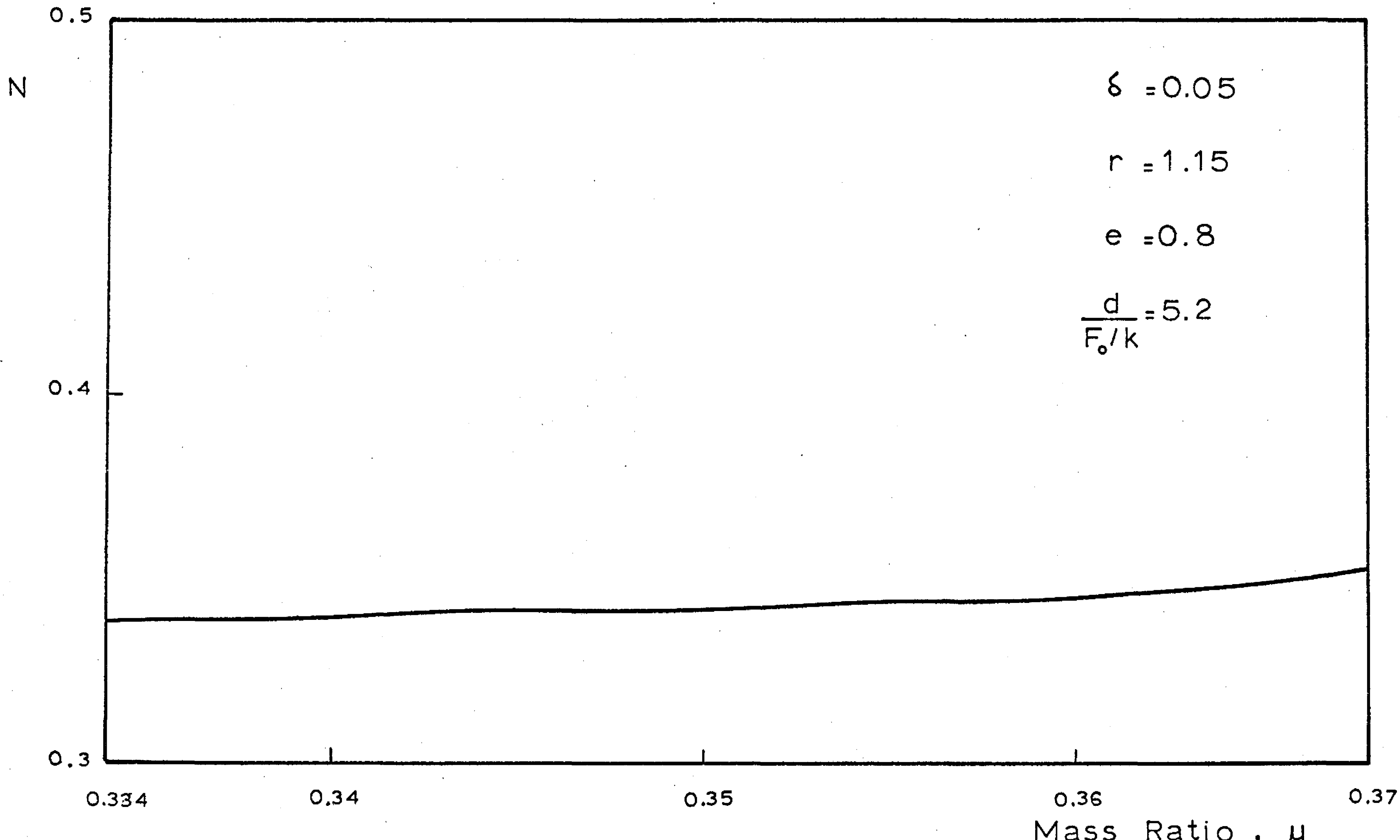


Fig.(5.11) Effect of mass ratio on unsymmetry ratio, N

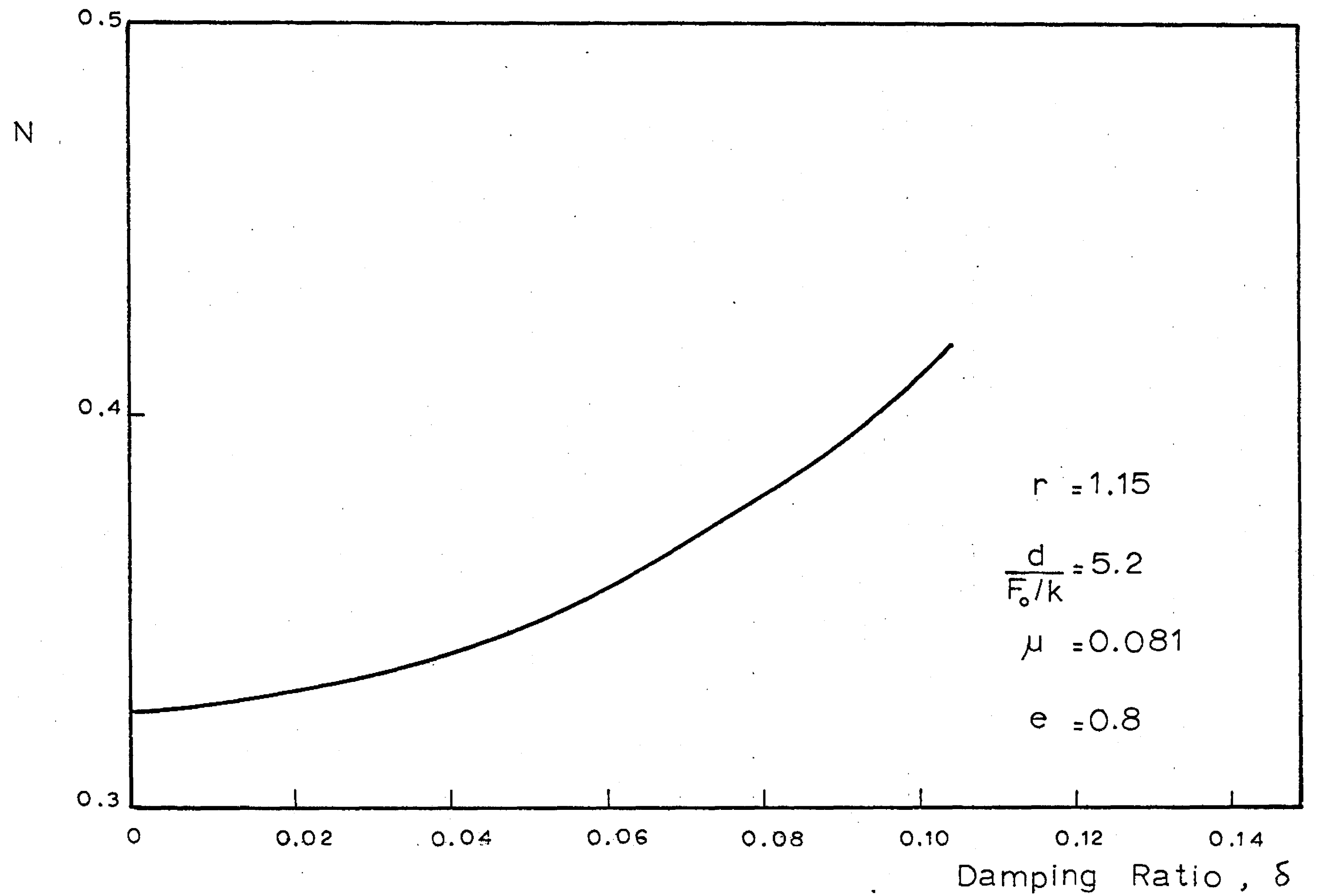


Fig.(5.12) Effect of damping ratio on unsymmetry ratio , N

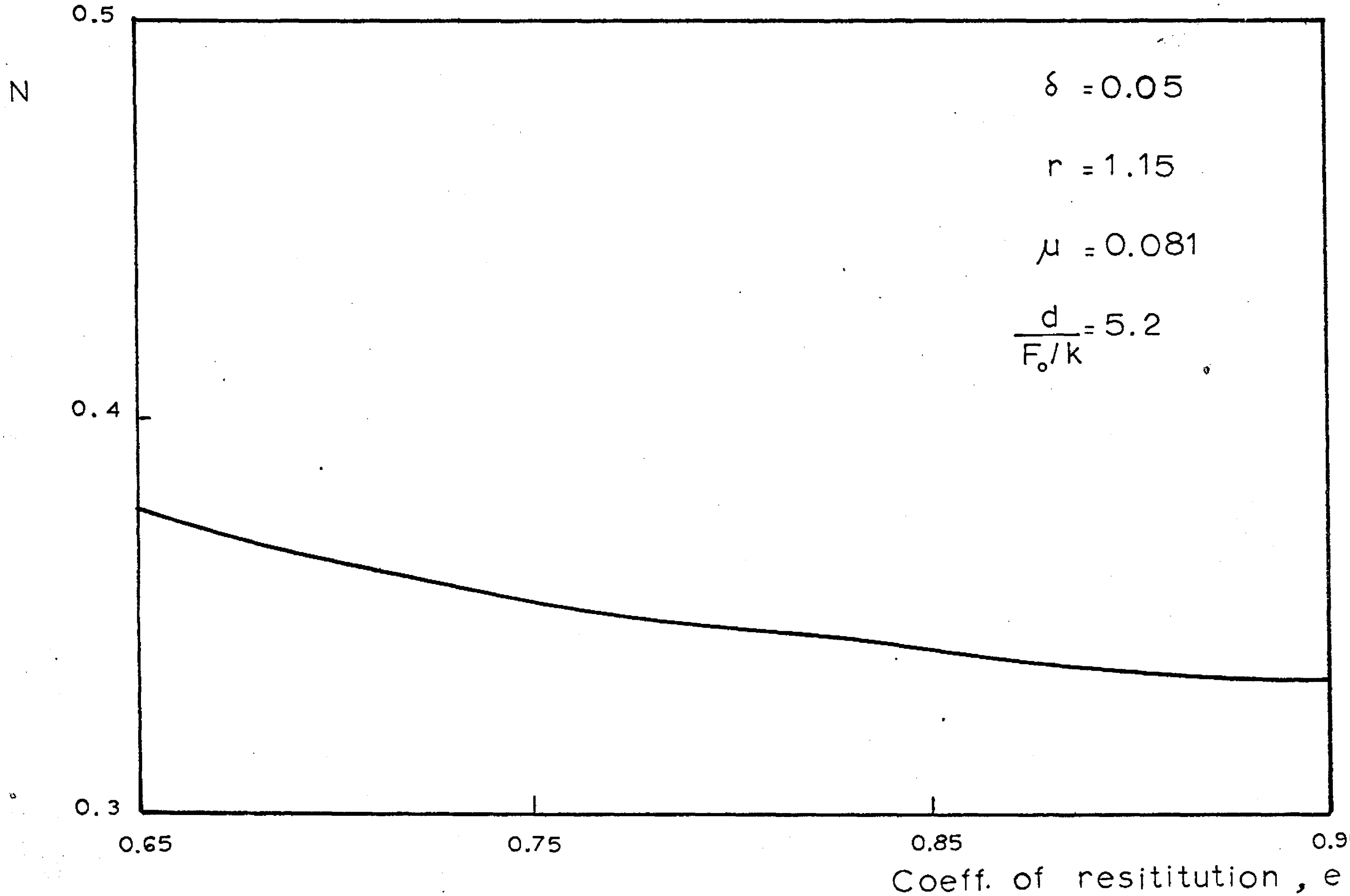


Fig. (5.13) Effect of coeff. of restitution on the unsymmetry ratio, N

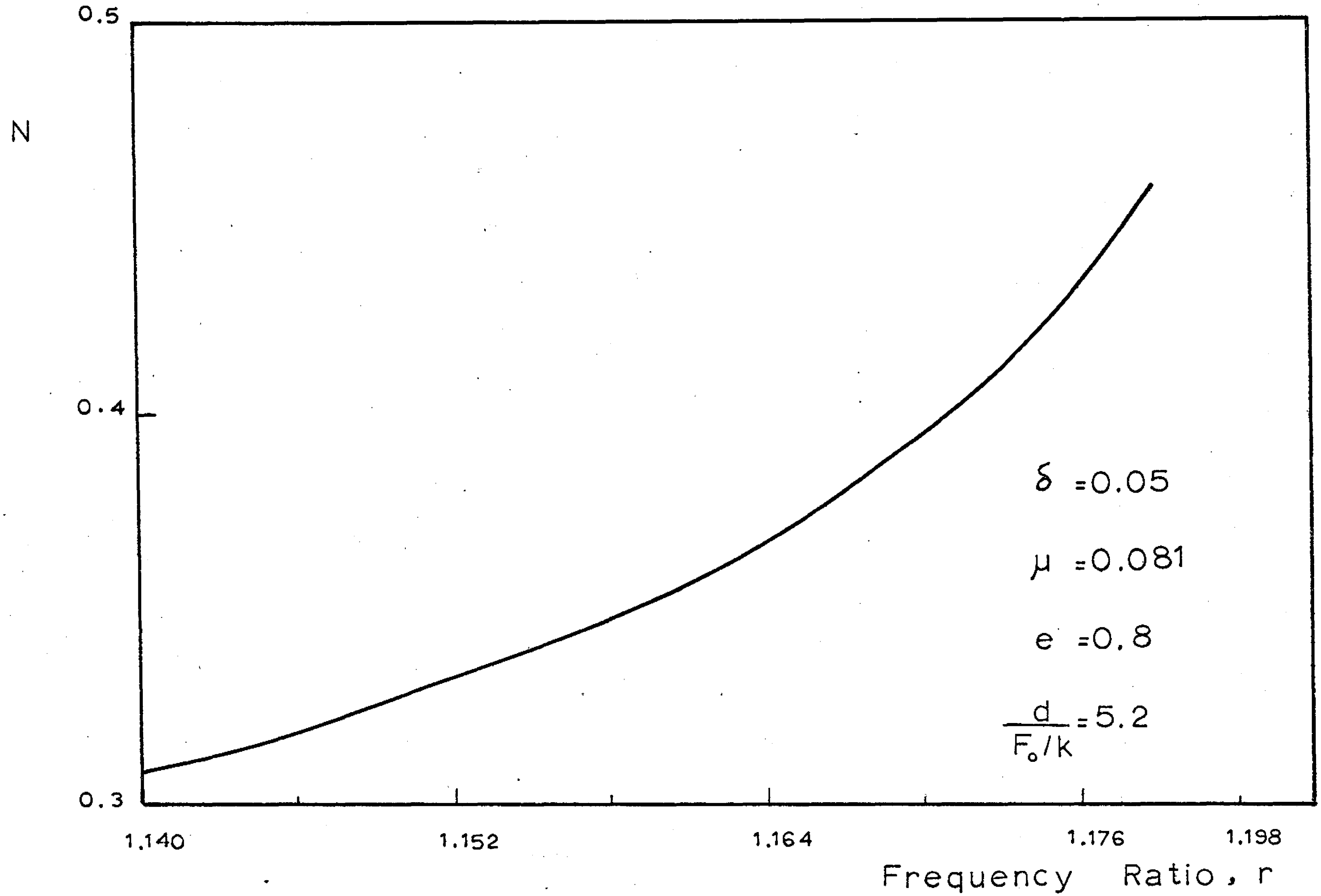


Fig.(5.14) Effect of frequency ratio on unsymmetry ratio, N

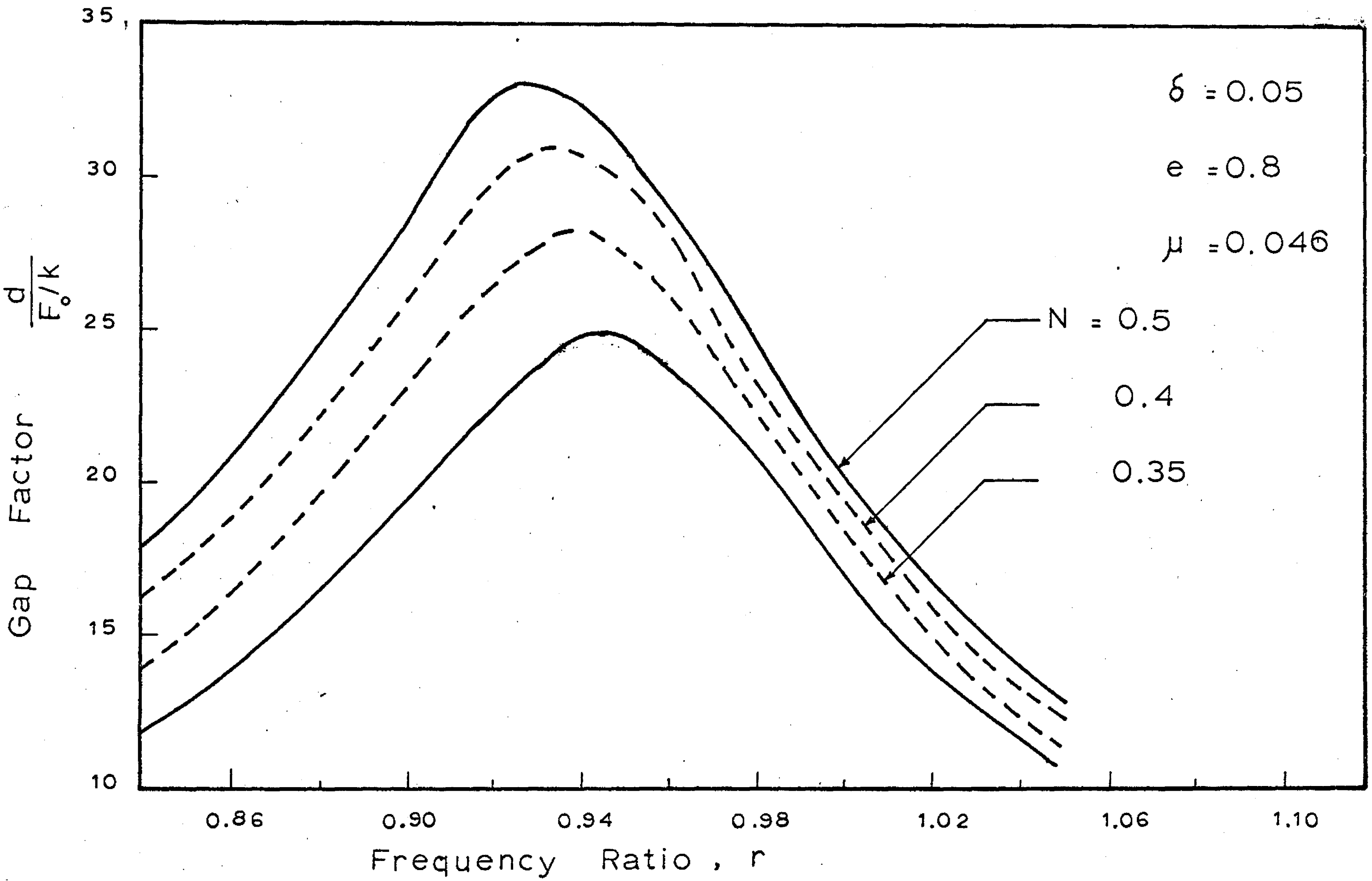
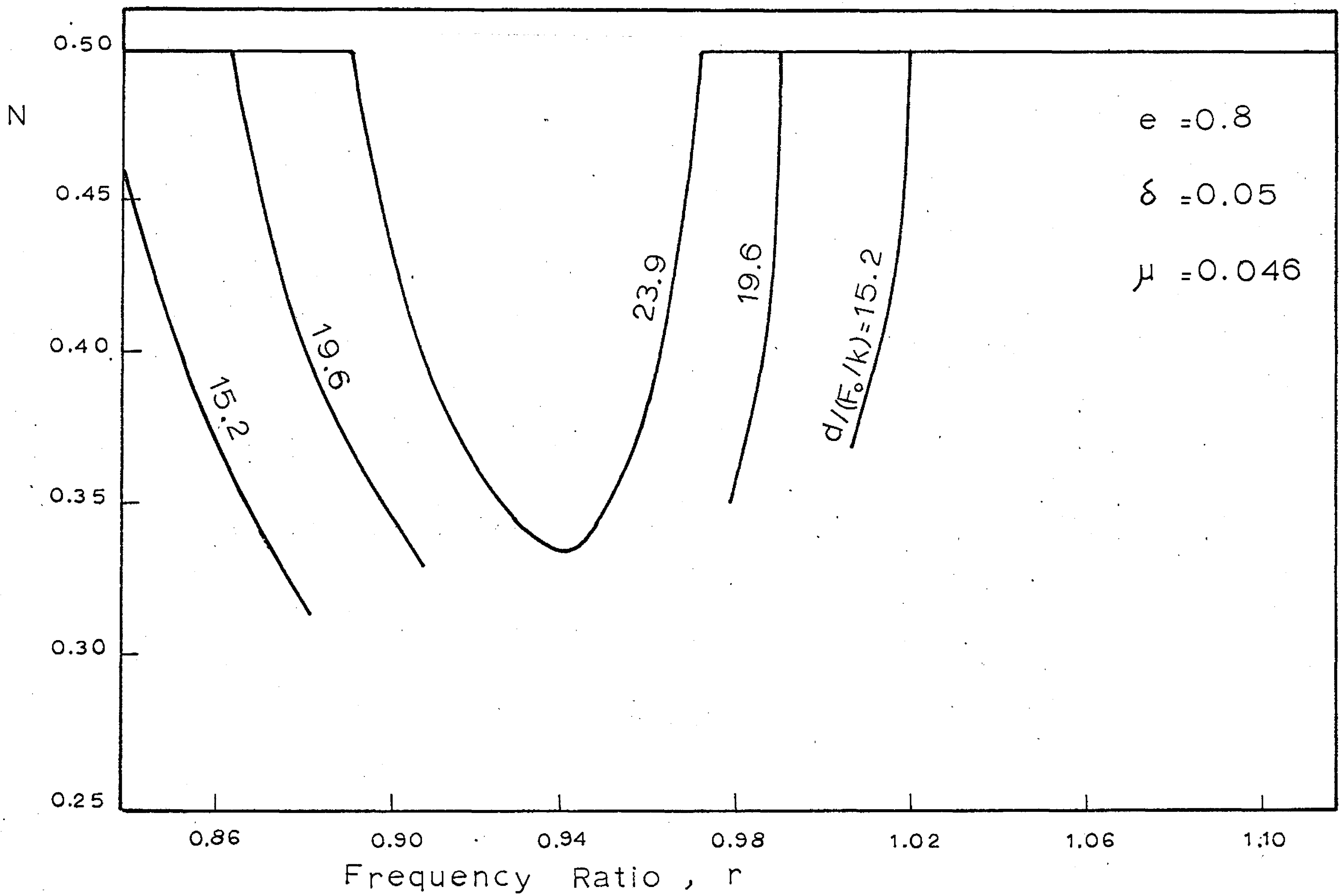
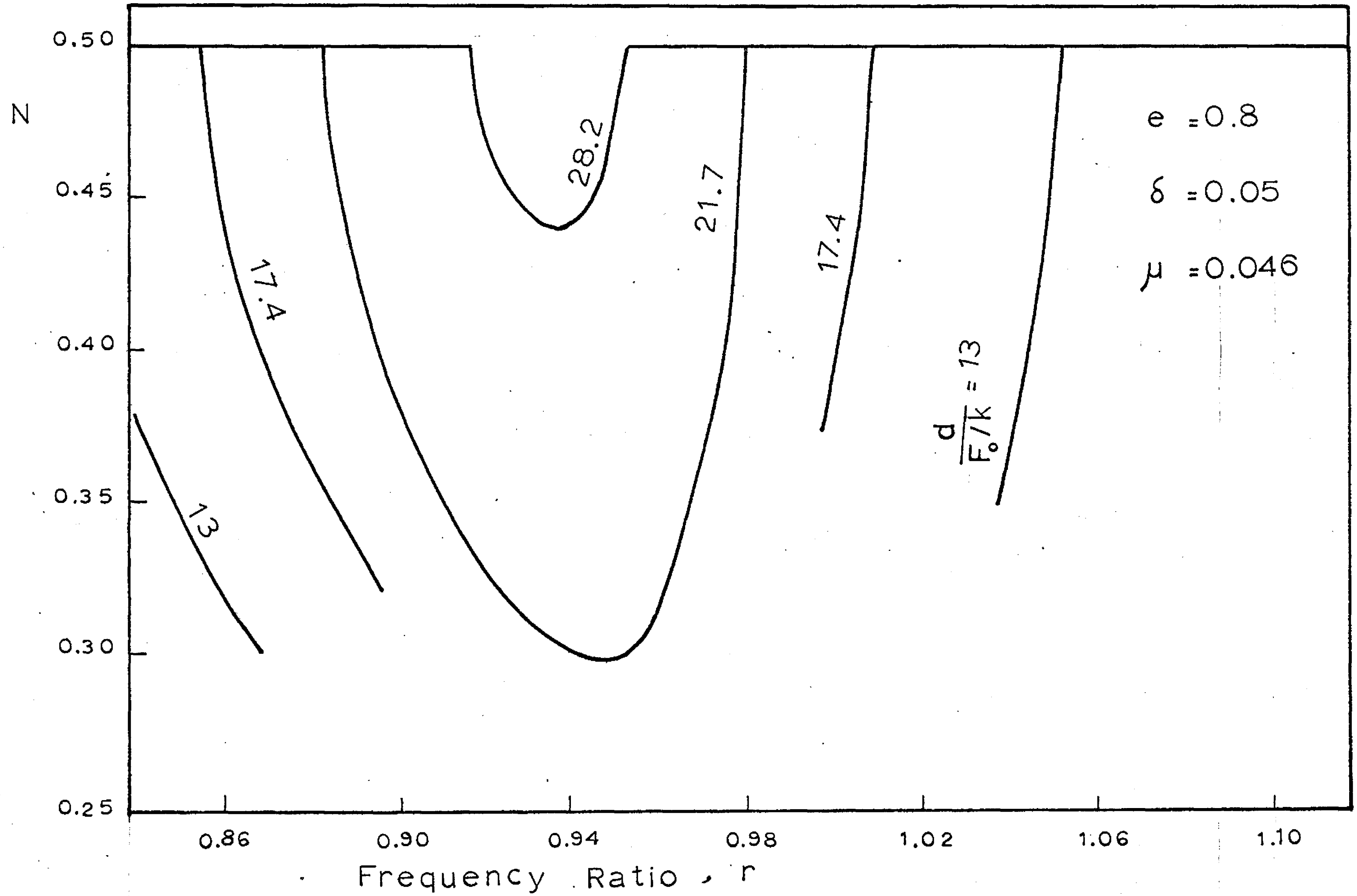


Fig (5.15) Effect of frequency ratio & gap factor on the unsymmetry ratio, N



Fig(5.16) Effect of frequency ratio on the unsymmetry ratio, N



Fig(5.17)Effect of frequency ratio on the unsymmetry ratio, N

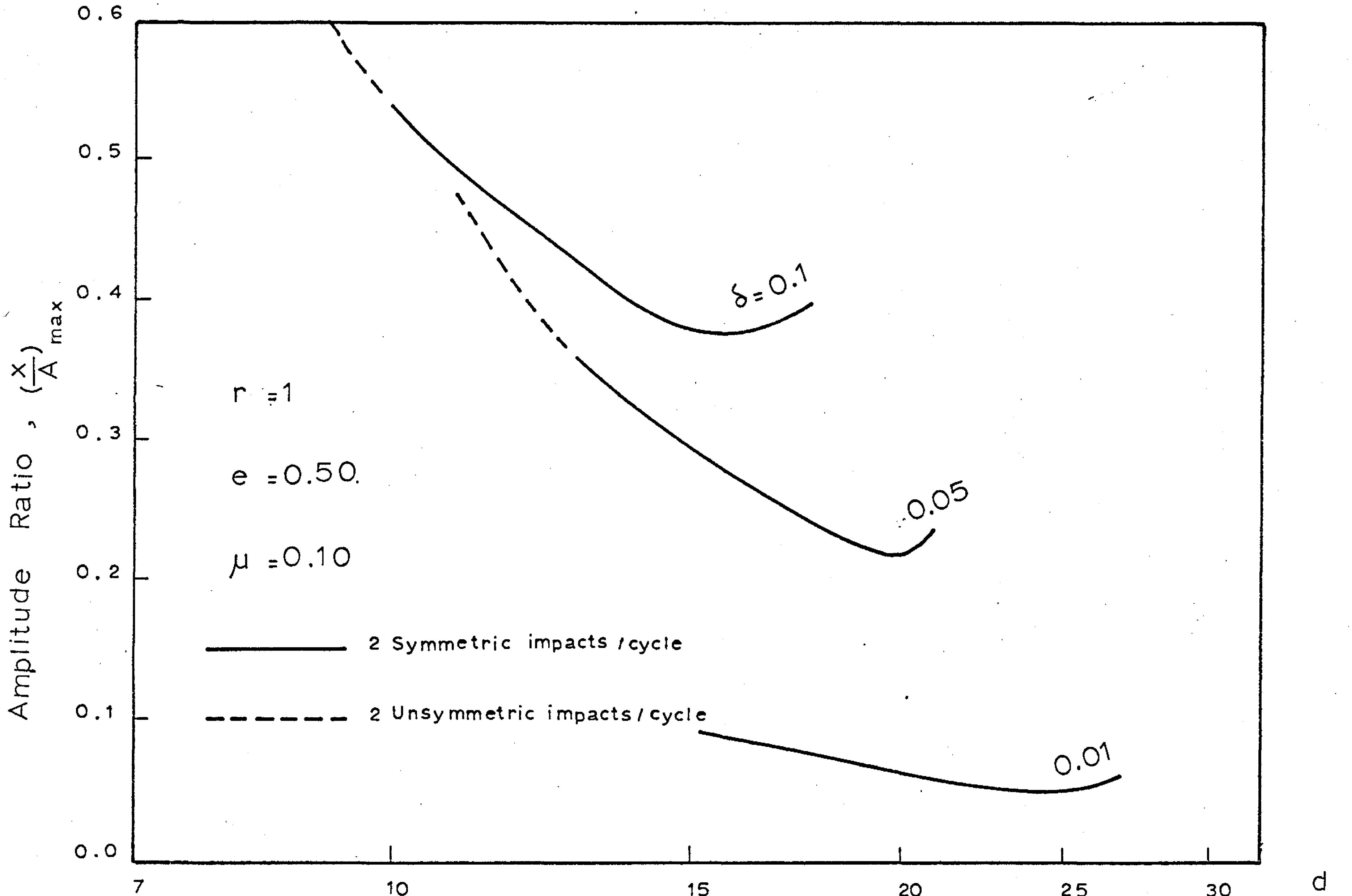


Fig. (5.18) Stable - Solution Curves

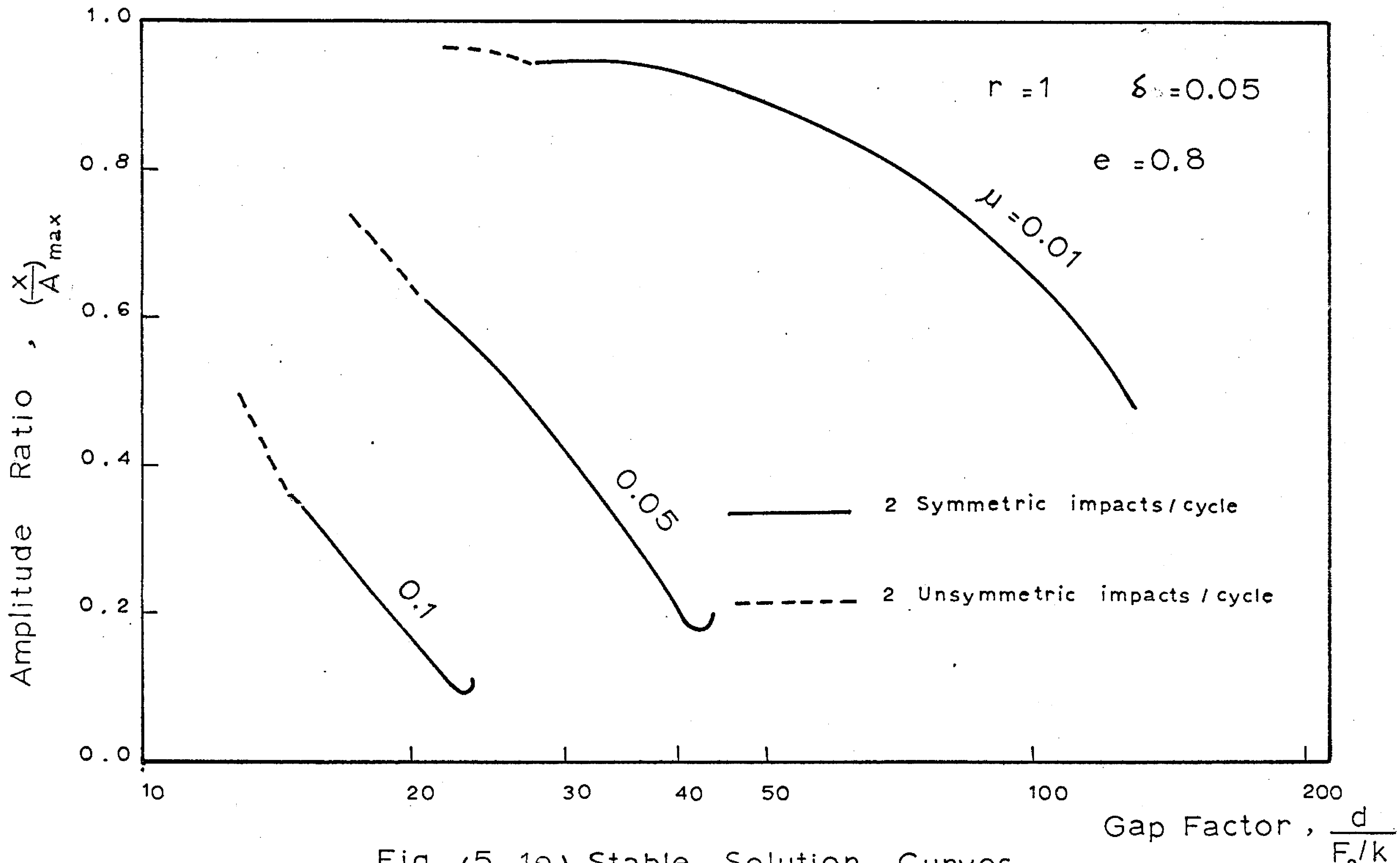


Fig. (5.19) Stable - Solution Curves

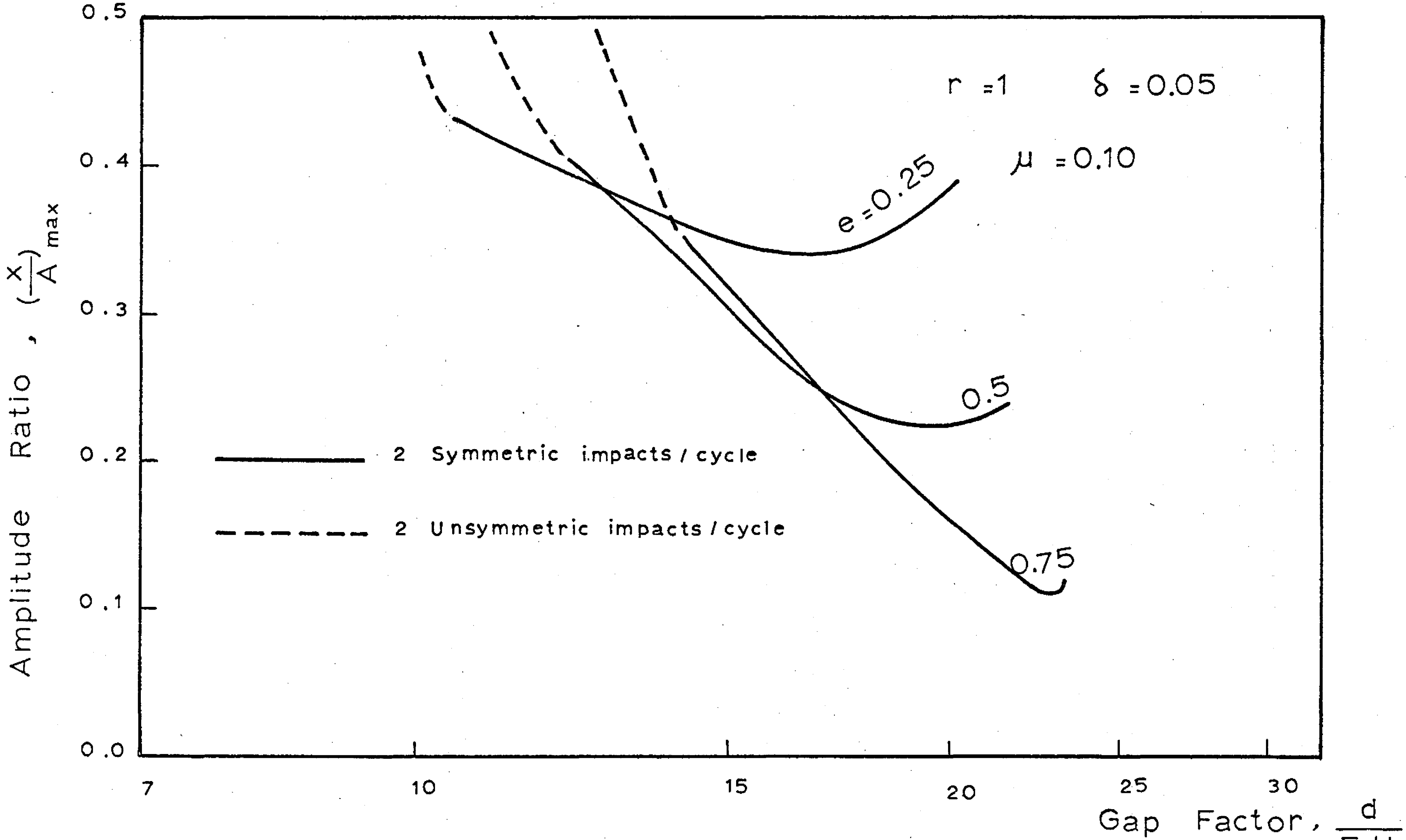


Fig. (5.20) Stable Solution Curves

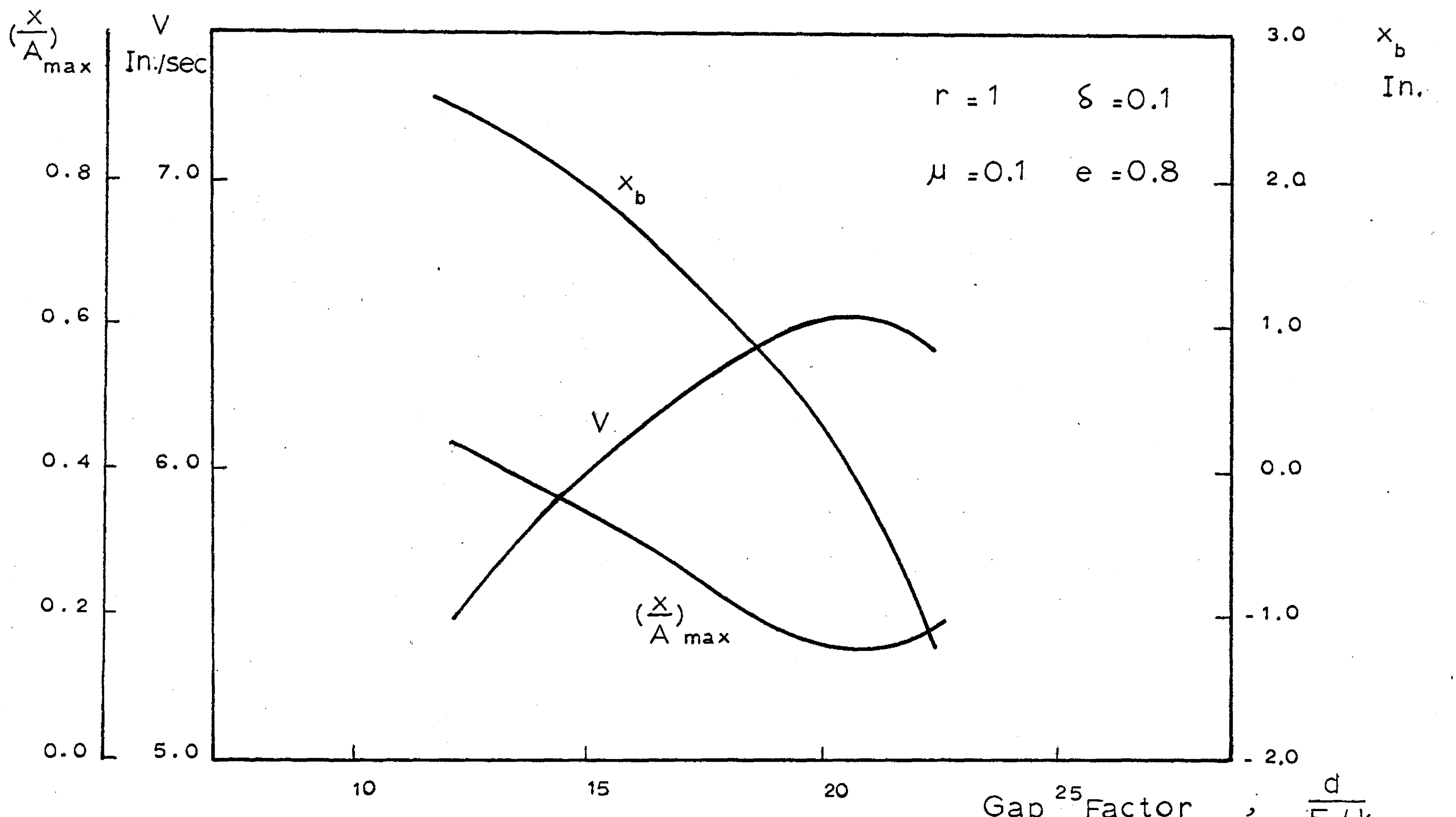


Fig. (5.21) Effect of gap ratio on velocity of the particle V, position of impact x_b & the ratio $(\frac{X}{A})_{max}$

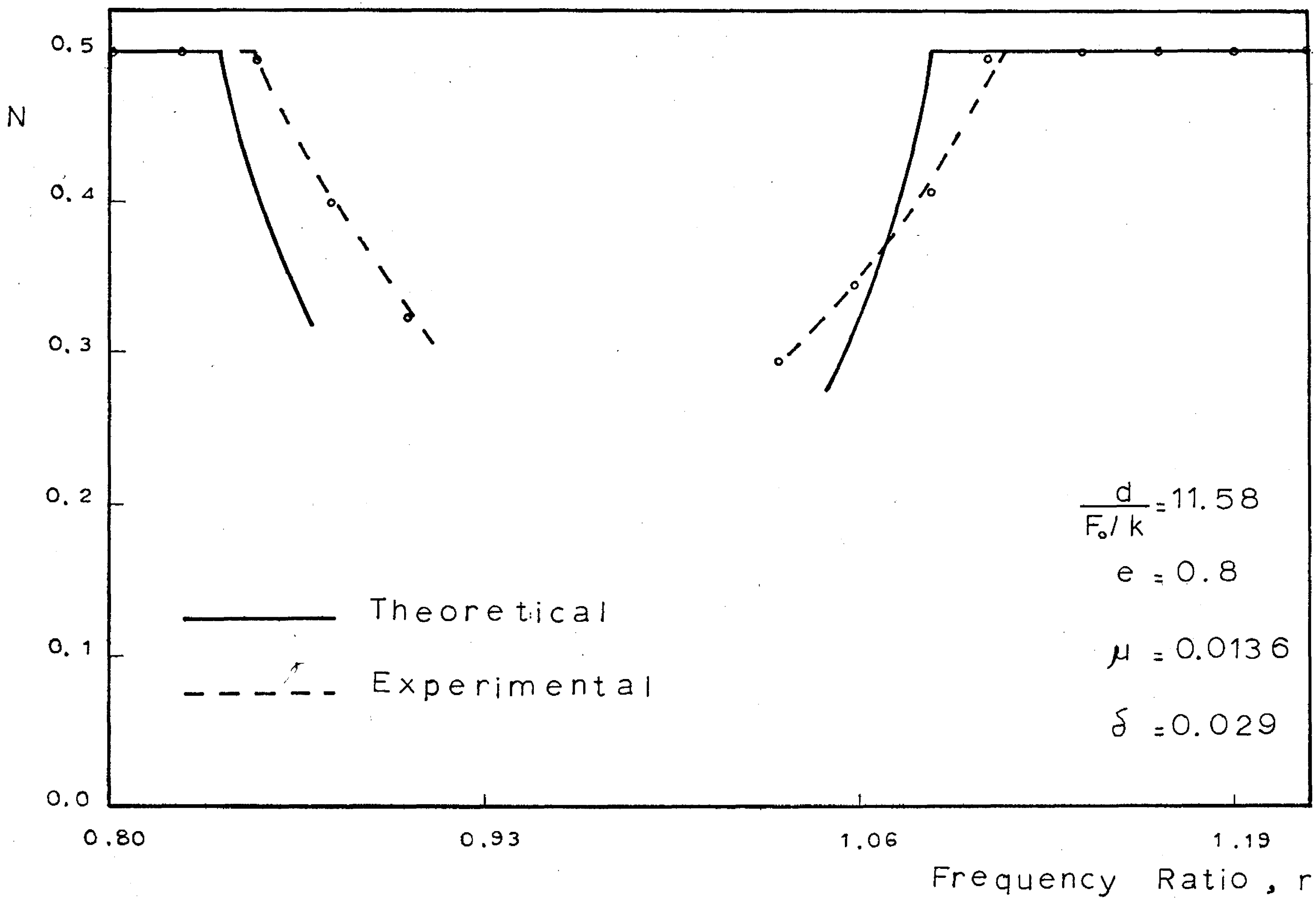


Fig.(5.22) Effect of frequency ratio on the unsymmtry ratio , N

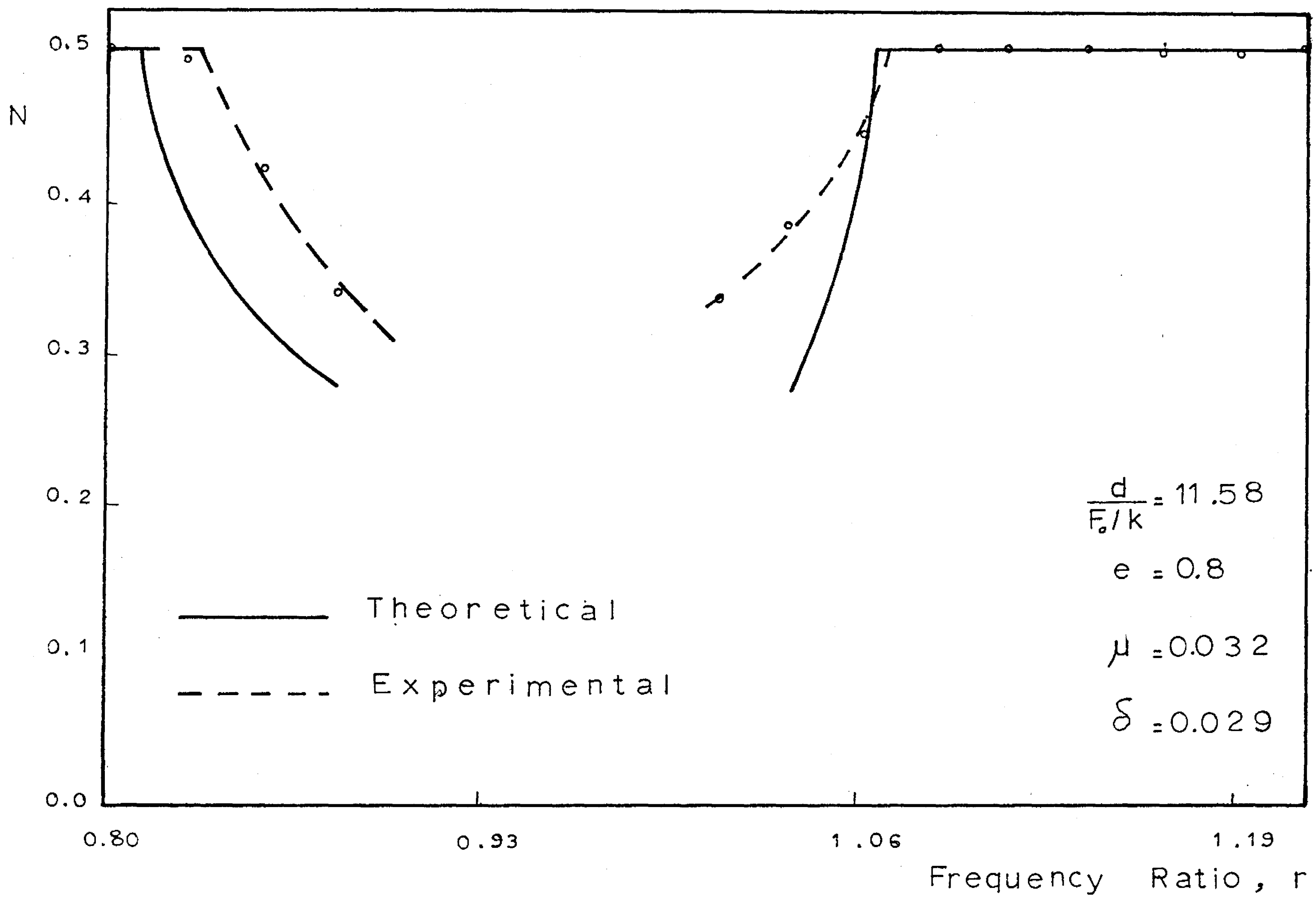


Fig.(5.23) Effect of frequency ratio on the unsymmetry ratio , N

5.2. <u>Discussion of Experimental Work</u>

(a) The excessive (as measured by the human ear) noise level in the vicinity of an operating system resulting from the impacts, especially when the colliding surfaces are hardened, is such an intensity as to require muffling, if the damper is to function over an extended period.

(b) In some cases, even though the two impacts/cycle motion is not stable in the strict mathematical sense, the amplitude of the response is nearly constant and appreciably less than the resulting amplitude when the damper is removed.

Figure (4.4) shows the effect of setting $\mu = 0$ (by removing the particle from its container) while M is vibrating. Obviously, the increased response equals that attained by an equivalent single degree of freedom system subjected to the same excitation.

As soon as μ is returned to its former value, the motion of M resumes its former state.

(c) The actual wave of the response is approximately sinusoidal, and the assumption that the velocity changes discontinuously is justifiable, as shown in Figure (4.4) (a), (b).

(d) Figures (4.6) to (4.14) indicate that the motion of the impact damper is

two symmetric impacts/cycle

or two unsymmetric impacts/cycle

or multiple impacts/cycle

They also indicate that the two unsymmetric impacts/cycle motion exists and is stable.

It is obvious that the efficiency of the impact damper increases as the mass ratio increases. There exists, however, an optimum mass ratio, after which erratic behaviour of the damper starts and the efficiency decreases. This erratic behaviour can be attributed to the fact that energy imparted to the free mass is inadequate to force it to the opposite side of the container. Thus, the free mass starts to oscillate and the amplitude of the vibrating system builds up and subsequently impact occurs between container and the free mass. Due to the impact, the vibrational amplitude decreases, resulting in a vibration wave form that resembles that of the beating phenomena.

It may be also noted that if no compensation is made for the increase in the primary mass due to the addition of the free mass (as in our system), then the natural frequency decreases with impact damper in action.

It is also evident that the impact damper is most efficient at resonance.

From Figures (4.4)(a) to (f), it is clear that the velocity of the primary mass which is under impact, changes at impact discontinuously, while the displacement does not change due to impact. Also, there is a sudden large increase in the acceleration at the moment of impact.

(e) Figures (4.15) to (4.21) indicate that the value of N is not constant. For low frequency ratios N = 0.5, i.e. symmetric two impacts/cycle motion. As the frequency ratio increases, the value of N decreases gradually to about 0.3, i.e. range of unsymmetric two impacts/cycle. As the frequency ratio increases, a state of multiple impacts/cycle exists. Increasing the frequency ratio again, the the unsymmetric two impacts/cycle appears again with N increasing gradually. Finally, a state of symmetric two impacts/cycle starts. It should be noted that sometimes the multiple impacts/cycle motion does not exist.

5.3. Discussion of the Results Obtained by "Sadek".

(a) Fourier series was used to solve the problem, theoretically, which is an approximate method. As a matter of fact, the only other author who used Fourier series, Arnold (4) his experimental results did not agree completely with his theoretical results.

(b) His experimental model is vertical, while his theoretical analysis is for a horizontal model.

(c) No experimental work was done to get the values of N.

(d) Figure 6(a), in his paper, shows the theoretical response curve without impact damper, which is not correct.

(e) He stated that "equally spaced impacts hardly ever occur for reasonably efficient behaviour of the damper" which is not correct.

(f) The value of δ for the experimental model was taken as 0.004 which is very low compared with values of δ for experimental models taken by all other authors, as follow:

Author	δ
Masri	0.1
Grupin	0.1
Shah	0.045

5.4 CONCLUSIONS

(1) The unsymmetric two impacts/cycle motion exists, for a wide range of parameters of the impact damper. and is stable.

(2) The value of unsymmetry ratio N, varies from 0.5 to about 0.3.

(3) Stability boundaries of the steady state solutions are a complicated function of the parameters of the impact damper and the system.

(4) The results obtained by Sadek (7) are not correct.

Some of the main advantages of impact damper would be the relative simplicity of installation, maintenance and facility of variation of damper parameters.

With more investigation and development, the future of the impact damper appears quite promising. For further studies, it would be worth considering the effect of using multiple particles, instead of one or two, in the container and the effect of various soft materials as impacting surfaces and the effectiveness of the impact damper with random and impulse-like excitation.

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APPENDIX I

$$G_1 = \eta\,\omega$$

$$G_2 = \delta\,\omega$$

$$G_3 = e^{-(\delta \omega n \pi / \Omega)}$$

$$G_4 = \eta \omega n \pi / \Omega$$

$$G_5 = \sin(n\pi)$$

$$G_6 = \cos(n\pi)$$

$$G_7 = e^{-\delta \omega (2\pi - n\pi)/\Omega}$$

$$G_8 = \eta\omega(2\pi - n\pi)/\Omega$$

$$G_9 = \frac{(2 - n - en - 2\mu e)}{(2-n)(1+e)}$$

$$G_{10} = \frac{(2\mu + 2 - n - en)}{(2-n)(1+e)}$$

$$G_{11} = \frac{(2e - n - 2\mu - ne)}{(2-n)(1+e)}$$

$$G_{12} = \frac{(2e - n - ne + 2\mu e)}{(2-n)(1+e)}$$

$$G_{13} = n\pi/\Omega$$

$$G_{14} = -n/(2-n)$$

$$G_{15} = G_3 \cdot G_1$$

$$G_{16} = G_3 . G_2$$

$$G_{17} = \Omega . G_6$$

$$G_{18} = \Omega . G_5$$

$$G_{19} = G_7 . G_1$$

$$G_{20} = G_7 . G_2$$

$$G_{21} = G_3 . \sin G_4$$

$$G_{22} = G_3 . \cos G_4$$

$$G_{23} = G_{15} . \cos G_4$$

$$G_{24} = G_{15} . \sin G_4$$

$$G_{25} = G_{16} . \sin G_4$$

$$G_{26} = G_{16} . \cos G_4$$

$$G_{27} = G_7 . \sin G_8$$

$$G_{28} = G_7 . \cos G_8$$

$$G_{29} = G_{19} . \cos G_8$$

$$G_{30} = G_{19} . \sin G_8$$

$$G_{31} = G_{20} . \sin G_8$$

$$G_{32} = G_{20} . \cos G_8$$

$G_{33} = G_{10} / G_9$

$G_{34} = G_{11} / G_9$

$G_{35} = G_{12} / G_9$

$G_{36} = G_{13} / G_9$

$G_{37} = G_{23} / G_{34}$

$G_{38} = G_{24} / G_{34}$

$G_{39} = G_{25} / G_{34}$

$G_{40} = G_{26} / G_{34}$

$G_{41} = G_{17} / G_{34}$

$G_{42} = G_{18} / G_{34}$

$G_{43} = G_1 / G_{35}$

$G_{44} = G_2 / G_{35}$

$G_{45} = G_{17} / G_{35}$

$G_{46} = G_{18} / G_{35}$

$G_{47} = G_{29} / G_{33}$

$G_{48} = G_{30} / G_{33}$

$G_{49} = G_{31} / G_{33}$

$$G_{50} = G_{32} / G_{33}$$

$$G_{51} = \Omega / G_{33}$$

$$G_{52} = G_{37} - G_{39}$$

$$G_{53} = G_{38} + G_{40}$$

$$G_{54} = G_{47} - G_{49}$$

$$G_{55} = G_{48} + G_{50}$$

$$G_{56} = G_{36} . G_{1}$$

$$G_{57} = G_{36} . G_{2}$$

$$G_{58} = G_{36} . \Omega$$

$$G_{59} = G_{1} - G_{52}$$

$$G_{60} = G_{2} - G_{53}$$

$$G_{61} = \Omega - G_{41}$$

$$G_{62} = \Omega - G_{45}$$

$$G_{63} = \Omega - G_{51}$$

$$G_{64} = G_{57} - 1$$

$$G_{65} = G_{58} - G_{5}$$

$$G_{66} = 1 - G_{6}$$

$$G_{67} = G_{27} / (1 - G_{22} . G_{28})$$

$$G_{68} = G_{28} . G_{21} / (1 - G_{22} . G_{28})$$

$$G_{69} = G_{21} + G_{22} . G_{68}$$

$$G_{70} = G_{22} . G_{67}$$

$$G_{71} = G_{59} - G_{60} . G_{68}$$

$$G_{72} = G_{60} . G_{67}$$

$$G_{73} = G_{1} - G_{2} . G_{68} + G_{44} . G_{69}$$

$$G_{74} = G_{70} . G_{44} - G_{2} . G_{67} - G_{43}$$

$$G_{75} = G_{1} - G_{2} . G_{68} + G_{55} . G_{69}$$

$$G_{76} = G_{55} . G_{70} - G_{2} . G_{67} - G_{54}$$

$$G_{77} = G_{56} - G_{64} . G_{68} - G_{69}$$

$$G_{78} = -G_{67} . G_{64} - G_{70}$$

$$G_{79} = G_{71} . G_{74} + G_{72} . G_{73}$$

$$G_{80} = G_{61} . G_{74} + G_{62} . G_{72} / G_{79}$$

$$G_{81} = G_{42} . G_{74} + G_{46} . G_{72} / G_{79}$$

$$G_{82} = G_{62} . G_{71} - G_{61} . G_{73} / G_{79}$$

$$G_{83} = G_{46} . G_{71} - G_{42} . G_{73} / G_{79}$$

$$G_{84} = G_{63} - G_{80} G_{75} - G_{76} G_{82}$$

$$G_{85} = -G_{81} . G_{75} - G_{83} . G_{76}$$

$$G_{86} = G_{65} - G_{80} . G_{77} - G_{82} . G_{78}$$

$$G_{87} = G_{66} - G_{81} . G_{77} - G_{83} . G_{78}$$

APPENDIX II

Derivation of Equation of Motion of the Mass Particle

The equation of motion of the mass particle can be obtained by using Lagrange's equation (18), which states

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} + \frac{\partial V}{\partial q_r} = Q_r \quad (r=1,2,\ldots,n) \qquad (\text{II}.1)$$

where T = kinetic energy of the system

V = potential energy of the system

q_r = generalized co-ordinates

Q_r = generalized forces at q_r which do not have potential

Now, kinetic energy of the particle is given by

$$T = \frac{1}{2} m \dot{y}_1^2$$

$$\frac{\partial T}{\partial \dot{y}_1} = m\dot{y}_1 , \qquad \frac{\partial T}{\partial y_1} = 0 \qquad \frac{\partial V}{\partial y_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}_1}\right) = m\ddot{y}_1 = m(\ddot{y} + \ddot{x})$$

since $y_1 = y + x$ (see Fig. 1).

Since $Q = 0$ for the present case, then substituting proper values into equation (II.1) gives

$$m(\ddot{y} + \ddot{x}) = 0$$

$$\ddot{y} = -\ddot{x}$$

APPENDIX III

A Method of Determining Coefficient of Restitution

The steady state velocity of the mass particle (in the case of symmetric 2 impacts/cycle motion) can be said to be constant and is given by:

$$V = (d_o + 2x_b)\frac{\omega}{\pi} \qquad (III.1)$$

If at $t = 0_-$, the absolute velocity of the mass particle is represented by $v_- = V$ then at $t = 0_+$, $v_+ = -V$.

Now recalling

$$\dot{x}_- = \dot{x}_b \quad , \quad \dot{x}_+ = \dot{x}_a$$

$$v_- = V \quad , \quad v_+ = -V$$

and substituting the appropriate values in equation (2.22) gives:

$$\dot{x}_b = \frac{V(e - 1 - 2\mu)}{(1 + e)} \qquad (III.2)$$

substituting equation (III.1) into (III.2) ultimately gives

$$x_b + \frac{\pi}{2\omega}\left[\frac{1+e}{1-e+2\mu}\right]\dot{x}_b = -\frac{d_o}{2} \qquad (III.3)$$

Similarily from (2.23), $\dot{x}_a$ is given be

$$\dot{x}_a = \frac{V(e - 1 + 2\mu e)}{(1 + e)} \qquad (III.4)$$

and substituting for V from equation (III.1) into equation (III.4) ultimately gives

$$x_b + \frac{\pi}{2\omega}\left[\frac{1+e}{1-e-2\mu e}\right]\dot{x}_a = -\frac{d_o}{2} \qquad (III.5)$$

If equation (III.5) be substracted from (III.3) it would give

$$(1-e-2\mu e)\,\dot{x}_b = (1-e+2\mu)\,\dot{x}_a$$

on simplification, this gives

$$e = \frac{1-(1+2\mu)\dfrac{\dot{x}_a}{\dot{x}_b}}{(1+2\mu)-\dfrac{\dot{x}_a}{\dot{x}_b}} \qquad (III.6)$$

from which e can be evaluated provided $\frac{\dot{x}_a}{\dot{x}_b}$ is known. The velocity ratio $(\frac{\dot{x}_a}{\dot{x}_b})$ in equation (III.6) can be obtained by integrating with respect to time the output of an accelerometer attached to the primary mass M.

But since the value of e for hardened steel to hardened steel is known with quite a good degree of accuracy, and is equal to 0.8, this value of e (0.8) was taken for all theoretical calculations without actually determining it experimentally.

APPENDIX IV

EXPERIMENTAL DETERMINATION OF THE STRUCTURAL DAMPING FACTOR

This was determined by measuring the peak amplitudes of free vibration of the system.

The free vibrations trace of the system is shown in Figure (IV.1). The peak amplitude for each cycle was measured. The value of the damping factor was obtained by using the formula (18)

$$p = \frac{1}{k} \log_e \frac{r_1}{r_2} = \frac{2\pi\delta}{\sqrt{1-\delta^2}}$$

where k = number of oscillations between two points 1 and 2 corresponding to maxima

r_1 = maximum amplitude at point 1

r_2 = maximum amplitude at point 2

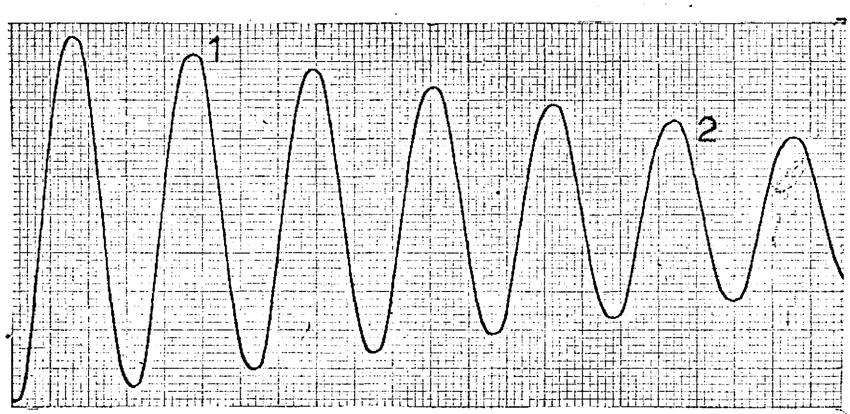

Fig. (IV.1)

The average value of δ was found to be 0.029

APPENDIX V

$$S \equiv \sin \tau$$

$$C \equiv \Omega \cos \tau$$

$$h_1 \equiv e^{-\frac{\delta \Omega \pi}{\omega}} \sin\left(\eta \frac{\pi \Omega}{\omega}\right)$$

$$h_2 \equiv e^{-\frac{\delta \Omega \pi}{\omega}} \cos\left(\eta \frac{\pi \Omega}{\omega}\right)$$

$$\sigma_1 \equiv \frac{\pi}{2\Omega} \frac{1+e}{1-e+2\mu}$$

$$\sigma_2 \equiv \frac{\pi}{2\Omega} \frac{1+e}{1-e-2\mu e}$$

$$\theta_1 \equiv \omega e^{-\frac{\delta \Omega \pi}{\omega}} \left[-\delta \sin\left(\eta \frac{\pi \Omega}{\omega}\right) + \eta \cos\left(\eta \frac{\pi \Omega}{\omega}\right)\right]$$

$$\theta_2 \equiv \omega e^{-\frac{\delta \Omega \pi}{\omega}} \left[-\delta \cos\left(\eta \frac{\pi \Omega}{\omega}\right) - \eta \sin\left(\eta \frac{\pi \Omega}{\omega}\right)\right]$$

APPENDIX VI

$$C_0 \equiv e^{-\frac{\delta\pi}{\Omega}}\left(B_{1_0}\sin\frac{\eta\pi}{\Omega} + B_{2_0}\cos\frac{\eta\pi}{\Omega}\right) - A\sin\tau_0$$

$$C_1 \equiv e^{-\frac{\delta\pi}{\Omega}}\left(\frac{\delta}{\eta}\sin\frac{\eta\pi}{\Omega} + \cos\frac{\eta\pi}{\Omega}\right)$$

$$C_2 \equiv e^{-\frac{\delta\pi}{\Omega}}\left(\frac{1}{\eta}\ \sin\frac{\eta\pi}{\Omega}\right)$$

$$b_1 \equiv \left(\frac{\eta}{\Omega}\cos\frac{\eta\pi}{\Omega}\right)B_{1_0}$$

$$b_2 \equiv -\left(\frac{\eta}{\Omega}\sin\frac{\eta\pi}{\Omega}\right)B_{2_0}$$

$$b_3 \equiv -\left(\frac{\delta}{\Omega}e^{-\frac{\delta\pi}{\Omega}}\right)\left(\left(\sin\frac{\eta\pi}{\Omega}\right)B_{1_0} + \left(\cos\frac{\eta\pi}{\Omega}\right)B_{2_0}\right)$$

$$C_3 \equiv b_3 + e^{-\frac{\delta\pi}{\Omega}}(b_1 + b_2) - A\cos\tau_0$$

$$C_4 \equiv e^{-\frac{\delta\pi}{\Omega}}\left(a\sin\frac{\eta\pi}{\Omega} - (A\cos\tau_0)\cos\frac{\eta\pi}{\Omega}\right) - A\cos\tau_0$$

$$d_1 \equiv \Omega\frac{(1 - C_1)}{V_0 + \Omega C_3}$$

$$d_2 \equiv -\frac{\Omega C_2}{V_0 + \Omega C_3}$$

$$d_3 \equiv - \frac{\pi}{V_o + \Omega C_3}$$

$$d_4 \equiv - \frac{\Omega C_4}{V_o + \Omega C_3}$$

$$d_5 \equiv - \frac{(\Omega C_3 + C_1 V_o)}{V_o + \Omega C_3}$$

$$g_o \equiv - (f_1 B_{1o} + f_2 B_{2o})$$

$$g_1 \equiv - (f_1 \frac{\delta}{\eta} + f_2)$$

$$g_2 \equiv - \frac{f_1}{\eta}$$

$$g_3 \equiv \frac{\eta}{\Omega} (f_1 B_{2o} - f_2 B_{1o})$$

$$g_4 \equiv f_2 A \cos \tau - a f_1$$

$$\rho_o \equiv g_o e^{-\frac{\delta \pi}{\Omega}} - A \Omega \cos \tau$$

$$\rho_1 \equiv g_1 e^{-\frac{\delta \pi}{\Omega}}$$

$$\rho_2 = g_2 e^{-\frac{\delta \pi}{\Omega}}$$

$$p_3 \equiv \left(g_3 - \frac{\delta}{\Omega} g_0\right) e^{-\frac{\delta\pi}{\Omega}} + A\,\Omega \sin \tau_0$$

$$p_4 \equiv g_4\, e^{-\frac{\delta\pi}{\Omega}} + A\,\Omega \sin \tau_0$$

$$f_1 \equiv \delta \sin \frac{\eta\pi}{\Omega} - \eta \cos \frac{\eta\pi}{\Omega}$$

$$f_2 \equiv \delta \cos \frac{\eta\pi}{\Omega} + \eta \sin \frac{\eta\pi}{\Omega}$$

$$a \equiv \frac{A}{\eta}\left(\Omega \sin \tau_0 - \delta \cos \tau_0\right)$$

APPENDIX VII

KEY FOR COMPUTER PROGRAMS SYMBOLS

Fortran Symbols

Program for digital computer method	All other programs	Actual symbol used in mathematical model
A	A ,V	A
D	D	δ
U	AM	μ
–	N,AN	n
FF	FF	F_o/k
R	R	r
E	E	e
WN	W	ω
W	W1	Ω
PI	PI	π
DØ	DØ	d
TIM	TIM	$T\left(=\frac{2\pi}{\Omega}\right)$

Cont.

PSI		ψ
ETA		η
TI		t_{i_+}
XI		x_{i_+}
YI		y_{i_+}
X		x
Y		y
DX		$\dot{x}$
DXI		$\dot{x}_{i_+}$
DYI		$\dot{y}_{i_+}$
EI		E_i
DI		D_i
X1		$(x/A)_{max}$
THET	THET	τ
	B1	B_1
	B2	B_2

Cont.

	B11	$\grave{B}_1$
	B21	$\grave{B}_2$
	XA	x_a
	XG	x_g
	XB	x_b
	XH	x_h
	DXA	$\dot{x}_a$
	DXG	$\dot{x}_g$
	DXB	$\dot{x}_b$
	DXH	$\dot{x}_h$
	V1	V_1
	V2	V_2
	RO	ρ
First Theoretical Solution	XMAXA1 ($0 \leqslant t \leqslant \frac{n\pi}{\Omega_+}$)	$(x/A)_{max}$
	XMAX1 ($0 \leqslant t \leqslant \frac{n\pi}{\Omega_+}$)	$(x)_{max}$
	XMAXA2 ($\frac{n\pi}{\Omega_+} \leqslant t \leqslant \frac{2\pi}{\Omega_-}$)	$(x/A)_{max}$
	XMAX2 ($\frac{n\pi}{\Omega_+} \leqslant t \leqslant \frac{2\pi}{\Omega_-}$)	$(x)_{max}$

Cont.

Solution	Variable	Time range	Quantity
Second Theoretical Solution	XMAXA3	$0 \le t \le \frac{n\pi}{\Omega_-}$	$(x/A)_{max}$
	XMAX3		$(x)_{max}$
	XMAXA4	$\frac{n\pi}{\Omega_+} \le t \le \frac{2\pi}{\Omega_-}$	$(x/A)_{max}$
	XMAX4		$(x)_{max}$

APPENDIX VIII

List of Equipment Used in Experimental Studies

1. 1, amplifier unit, 250 VA Amplifier type 119567, Philips.
2. 1, ammeter
3. 1, vibration generator (exciter), moving coil vibration generator, model 790, Goodmans Industries Ltd., Wimbley, England.
4. 1, capacitance transducer, type 51D05-3 (co-axial) with a tuning plug type 51E03-4, DISA Elektronic, Herlev, Denmark.
5. 1, oscillator, type 51E02-555, DISA Elektronik.
6. 1, reactance converter, type 51E01, DISA Elektronik.
7. 1, cathode ray oscilloscope, type 564 storage oscilloscope, Tektronix Inc., S.W. Millikan Way, Beaverton, Oregon, U.S.A.
8. 1, vibration pick-up pre-amplifier, type 1606, BRUEL and KJAER, Denmark.
9. 1, microphone anplifier, type 2604, BRUEL and KJAER, Denmark.
10. 1, accelerometer, type 4332, BRUEL and KJAER, Denmark.
11. 1, force gauge, model 2103-500, Enderco Corporation, Pasadena, California.
12. 1, frequency generator, model 103, Wavetak, San Diego, California.
13. 1, paper recorder, model 7702, Hewlett, Packard.

14. 1, oscilloscope camera, model C-12, Tektronix, Inc., Portland, Oregon, U.S.A.

APPENDIX IX

```
A4466.                                                    ADEL M.
RUN(P)
SETINDF.
REDUCE.
LGO.
7           6400 END RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C
C     DIGITA COMPUTER METHOD TO GET THE  -EXACT-  MOTION
C     OF A SINGLE DEGREE OF FREEDOM SYSTEM WITH AN
C     DAMPER
C
C
C         TO DETERMINE THE FOLLOWING  .
C         ---------------------------
C                             1)  TIME AT WHICH IMPACT OCCURS
C                             2)  DISPLACEMENT OF  (M)  AT
C                                 IMPACT
C                             3)  RELATIVE DISPLACEMENT OF  (m)
C                                 W.R.T.  (M)  AT IMPACT
C                             4)  VELOCITY OF  (M)  AT IMPACT
C                             5)  RELATIVE VELOCITY OF  (m)
C                                 W.R.T.  (M)  AFTER IMPACT
C                             6)  RATIO  (X/A)MAX
      WRITE (6,470)
      WRITE (6,471)
      WRITE (6,472)
      WRITE (6,473)
C
C     WRITING DATA
C
      PI=4.*ATAN(1.)
      WN=1.
      FF=1.
      E=0.8
      U=0.1
      DO=15.
      W=1.
      D=0.1
      FR=W/(2.*PI)
1000  R=W/WN
      TIM=2.*PI/W
      IF(R.EQ.1.0)GO TO 6
C
C         CALCULATING THE PHASE ANGLE PSI
C
      PSI=ATAN(2.*D*R/(1.-R*R))
      GO TO 8
   6  PSI=1.57
C
```

```
C          CALCULATING THE AMPLITUDE OF MOTION WITHOUT
C          IMPACT DAMPER
C
    8 A=FF/SQRT((1.-R*R)**2+(2.*D*R)**2)
C
      WRITE(6,23)D,FF,E,WN
      WRITE(6,24)W,R,DO,U
      WRITE(6,25)TIM,A
      WRITE(6,27)
      WRITE(6,9)
C
      ETA=SQRT(1.-D*D)
      JKL=0
C
C          INITIAL CONDITIONS
C
      TI=0.0
      XI=0.0
      YI=0.0
      DXI=0.0
      DYI=0.0
      T=TI
      XX=0.0
      MM=0
      YY=DO/2.
      DO 60 I=1,60
      JKJK=0
      AK=0.2
      X1=0.0
C
C          SOLUTION BETWEEN  CONSECUTIVE IMPACTS
C
      EI=XI-A*SIN(W*TI-PSI)
      DI=(D*EI+DXI/WN-A*R*COS(W*TI-PSI))/ETA
      N=0
    5 T=TI+AK
      X=EXP(-D*WN*(T-TI))*(DI*SIN(ETA*WN*(T-TI))+EI*COS
     1 (ETA*WN*(T-TI)))+A*SIN(W*T-PSI)
      Y=-X+XI+YI+(DXI+DYI)*(T-TI)
C
C          CHECKING IF THE NEXT IMPACT IS REACHED
C
      ARG=DO/2.-ABS(Y)
      IF(ABS(X1).GT.ABS(X))GO TO 7
      X1=X
    7 IF(ARG.LT.0.0)GO TO 10
      AK=AK+0.2
      N=N+1
      GO TO 5
   10 IF(Y.GT.0.0)GO TO 11
```

```
      YY=-DO/2.
      GO TO 12
 11   YY=DO/2.
 12   CONTINUE
      K=0
C
C          NEWTON-RAPHSON METHOD FOR SOLVING TRANSCEDENTIAL
C          EQUATIONS
C
 14   T2=T
      M=0
 15   T3=T-TI
      WNT=WN*T3
      DWNT=-D*WNT
      IF(ABS(DWNT).GT.85.0)GO TO 60
      EX=EXP(DWNT)
      EW=ETA*WNT
      SI=SIN(EW)
      CO=COS(EW)
      FT=-YY-EX*(DI*SI+EI*CO)-A*SIN(W*T-PSI)+XI+YI+(DXI+
     1 DYI)*T3
      DX=EX*(ETA*WN*(DI*CO-EI*SI)-D*WN*(DI*SI+EI*CO))+A*
     1 W*COS(W*T-PSI)
      FDFT=-DX+DXI+DYI
      T1=FT/FDFT
      T=T-T1
      IF(ABS(T1).LT.0.002)GO TO 21
      IF(M.EQ.100)GO TO 22
      M=M+1
      GO TO 15
 22   WRITE(6,2)
 51   GO TO 48
 21   YI=YY
      IF(T.LT.TI)GO TO 49
C
C          COMPUTING THE CONDITIONS AT IMPACT
C
 20   XI=EXP(-D*WN*(T-TI))*(DI*SIN(ETA*WN*(T-TI))+EI*COS
     1 (ETA*WN*(T-TI)))+A*SIN(W*T-PSI)
      DX=EXP(-D*WN*(T-TI))*(ETA*WN*(DI*COS(ETA*WN*(T-TI))
     1 -EI*SIN(ETA*WN*(T-TI)))-D*WN*(DI*SIN(ETA*WN*(T-TI))
     1 +EI*COS(ETA*WN*(T-TI))))+A*W*COS(W*T-PSI)
      FDFT=-DX+DXI+DYI
      DXI=DX+U*(1.+E)/(1.+U)*FDFT
      DYI=-E*FDFT
      TI=T
      X1=X1/A
```

```
      IF((ABS(XX)-ABS(X1)).LT.0.00001)GO TO 70
      XX=X1
      GO TO 71
C
 71   WRITE(7,1)I,TI,XI,YI,DXI,DYI,X1
 52   GO TO 60
 49   JKL=JKL+1
      IF(JKL.GT.5)GO TO 60
      WRITE(6,4)
 70   MM=MM+1
 48   T=T2+4.
      K=K+1
      IF(K.GT.15)GO TO 60
      GO TO 14
 60   CONTINUE
 50   CONTINUE
C
  1   FORMAT(20X,I5,F9.2,F9.4,F9.2,3F9.4)
 27   FORMAT(20X,6HIMPACT,4X,1HT,9X,1HX,8X,1HY,6X,1HX,8X,
     1      1HY,10X,1HX//)
  9   FORMAT(20X,6H------,4X,1H-,9X,1H-,8X,1H-,6X,1H-,
     1      8X,1H-,10X,1H-///)
  2   FORMAT(25X,20H   NO  CONVERGENC    )
  4   FORMAT(25X,20HT FOUND LESS THAN TI)
 23   FORMAT(21X,6H    D=,F5.2,6X,5HFO/K=,F5.2,9X,2HE=,
     1      F5.2, 7X,3HWN=,F5.2/)
 24   FORMAT(21X,6H    W=,F5.2,9X,2HR=,F5.2,8X,3HDO=,
     1      F5.2,8X,2HU=,F5.2/)
 25   FORMAT(21X,6H    T=,F5.2,9X,2HA=,F5.2///)
470   FORMAT(48X,10HTABLE 4.2 )
471   FORMAT(48X,10H----------//)
472   FORMAT(41X,23HDIGITAL COMPUTER OUTPUT)
473   FORMAT(41X,23H-----------------------//)
300   STOP
      END
7           6400 END RECORD
8           6400 END FILE
```

```
A4466.                                                          ADEL M.
RUN(P)
SETINDF.
REDUCE.
LGO.
7               6400 END RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C        TO DETERMINE THEORETICALLY    .
C               --------------------------
C                              1) THE VALUE OF       N
C                              2) THE VALUE OF       THET
C                              3) THE VALUE OF       B1
C                              4) THE VALUE OF       B2
C                              5) THE VALUE OF       B11
C                              6) THE VALUE OF       B21
C                              7) THE VALUE OF       XA
C                              8) THE VALUE OF       XG
C                              9) THE VALUE OF       DXA
C                             10) THE VALUE OF       DX
C                             11) THE VALUE OF       DXB
C                             12) THE VALUE OF       DXH
C                             13) THE VALUE OF       V1
C                             14) THE VALUE OF       V2
C               --------------------------
      WRITE(6,470)
      WRITE(6,471)
      WRITE(6,472)
      WRITE(6,473)
C
C          WRITING  DATA
C
      PI=4.*ATAN(1.)
      D=0.1
      FF=1.
      AM=0.1
      E=0.8
      DO=15.
      W=1.
      W1=1.
C
      TIM=2.*PI/W1
      DEL=D
      ET=1.-D**2
      ETA=SQRT(ET)
      R=W1/W
C
C         CALCULATING THE AMPLITUDE OF MOTION WITHOUT
C         IMPACT DAMPER
```

```
C
      A=FF/SQRT((1.-R*R)**2+(2.*D*D)**2)
C
      RO=DO/A
      WRITE(6,23)D,FF,E,W
      WRITE(6,24) W1,R,DO,AM
      WRITE(6,25) TIM,A
      AN=1.
      C=AN/2.
C
      G1=ETA*W
      G2=DEL*W
      G3=1./EXP(DEL*W*AN*PI/W1)
      G4=ETA*W*AN*PI/W1
      G5=SIN(AN*PI)
      G6=COS(AN*PI)
      G99=DEL*W*(2.*PI-AN*PI)/W1
      G7=1.0/EXP(G99)
      G8=ETA*W*PI*(2.-AN)/W1
      G9=(1./((1.+E)*(2.-AN)))*(2.-AN-E*AN-2.*AM*E)
      G10=(1./((1.+E)*(2.-AN)))*(2.*AM+2.-AN-E*AN)
      G11=(1./((1.+E)*(2.-AN)))*(-2.*AM-AN+2.*E-AN*E)
      G12=(1./((1.+E)*(2.-AN)))*(-AN+2.*E-AN*E+2.*AM*E)
      G13=AN*PI/W1
      G14=-AN/(2.-AN)
      G15=G3*G1
      G16=G3*G2
      G17=W1*G6
      G18=W1*G5
      G19=G7*G1
      G20=G7*G2
      G21=G3*SIN(G4)
      G22=G3*COS(G4)
      G23=G15*COS(G4)
      G24=G15*SIN(G4)
      G25=G16*SIN(G4)
      G26=G16*COS(G4)
      G27=G7*SIN(G8)
      G28=G7*COS(G8)
      G29=G19*COS(G8)
      G30=G19*SIN(G8)
      G31=G20*SIN(G8)
      G32=G20*COS(G8)
      G33=G10/G9
      G34=G11/G9
      G35=G12/G9
      G36=G13/G9
      G37=G23/G34
      G38=G24/G34
      G39=G25/G34
```

```
G40=G26/G34
G41=G17/G34
G42=G18/G34
G43=G1 /G35
G44=G2 /G35
G45=G17/G35
G46=G18/G35
G47=G29/G33
G48=G30/G33
G49=G31/G33
G50=G32/G33
G51=W1 /G33
G52=G37-G39
G53=G38+G40
G54=G47-G49
G55=G48+G50
G56=G36*G1
G57=G36*G2
G58=G36*W1
G59=G1-G52
G60=G2-G53
G61=W1-G41
G62=W1-G45
G63=W1-G51
G64=G57-1.
G65=G58-G5
G66=1.-G6
G67=G27/(1.-G22*G28)
G68=(G28*G21)/(1.-G22*G28)
G69=G21+G22*G68
G70=G22*G67
G71=G59-G60*G68
G72=G60*G67
G73=G1-G2*G68+G44*G69
G74=-G2*G67-G43+G70*G44
G75=G1-G2*G68+G55*G69
G76=-G2*G67+G55*G70-G54
G77=G56-G64*G68-G69
G78=-G67*G64-G70
G79=G71*G74+G72*G73
G80=(G61*G74+G62*G72)/G79
G81=(G42*G74+G46*G72)/G79
G82=(G62*G71-G61*G73)/G79
G83=(G46*G71-G42*G73)/G79
G84=-G80*G75-G76*G82+G63
G85=-G81*G75-G76*G83
G86=-G80*G77-G82*G78+G65
G87=-G81*G77-G83*G78+G66
TT=-G84/G85
```

```
C
C         CHECKING THE VALUE OF  (THET )  ,REAL OR
C         IMAGINARY
C
      S1=(RO*G87)**2-(G86**2+G87**2)*(RO**2-G86**2)
      IF(S1.LT.0.0)GO TO 5
      S1=SQRT(S1)
      S2=(RO*G86)**2-(G86**2+G87**2)*(RO**2-G87**2)
      IF(S2.LT.0.0)GO TO 5
      S2=SQRT(S2)
502   T3=(-RO*G87+S1)/(-RO*G86-S2)
C
C              F I R S T   T H E O R E T I C A L
C                         S O L U T I O N
C                         ---------------
C
C
C         CALCULATING THE VALUE OF   (THET )
C
      THET=ATAN(T3)
      P5=-RO*G87+S1
      P6=-RO*G86-S2
      IF(P5.GT.0.0.AND.P6.LT.0.0)THET=THET+PI
      IF(P5.LT.0.0.AND.P6.LT.0.0)THET=THET+PI
      SN=SIN(THET)
      CN=COS(THET)
C
C         CALCULATING    Z1,Z2
C
      Z1=G86*CN+G87*SN+RO
      Z2=G84*CN+G85*SN
      WRITE(6,450)Z1,Z2
      WRITE(6,451)AN,THET
C
C         IF   Z1=0.0   AND   Z2=0.0   ,  THEN THE
C         FOLLOWING VALUES OF   N,THET,B1,B2,B11,B21,XA,
C         XG,DXA,DXB,DXG,DXH,V1,V2,XMAXA,XMAX    ARE A
C         THEORETICAL RESULT
C
C
C         CALCULATING   B1,B2,B11,B21
C
C
      B1=-A*(G80*CN+G81*SN)
      B11=-A*(G82*CN+G83*SN)
      B2=B11*G67+B1*G68
      B21=B1*G69+B11*G70
      WRITE(6,452)B1,B2
      WRITE(6,453)B11,B21
```

```
C
C          CALCULATING   XA,XG
C
C
      XA=B2+A*SIN(THET)
      XG=B21+A*SIN(AN*PI+THET)
      WRITE(6,454)XA,XG
C
C          CALCULATING   DXA,DXB,DXG,GXH
C
      DXA=B1*ETA*W-D*W*B2+A*W1*COS(THET)
      DXB=G33*DXA
      DXG=G34*DXA
      DXH=G35*DXA
      WRITE(6,455)DXA,DXB
      WRITE(6,456)DXG,DXH
C
C          CACULATING   V1,V2
C
      V1=DXA/G9
      V2=G14*V1
      WRITE(6,457)V1,V2
C
C
C          CALCULATING   XMAXA   AND   XMAX
C
      XX=0.0
      T=0.0
505   X=(1./A)*EXP(-D*W*T)*(B1*SIN(ETA*W*T)+B2*COS(ETA*W*
     1  T))+SIN(W1*T+THET)
      IF((ABS(X)).GT.(ABS(XX)))XX=X
      T=T+0.2
      IF(T.GT.(AN*PI/W1))GO TO 506
      GO TO 505
506   XMAXA1=XX
      XMAX1=XX*A
      T=AN*PI/W1
507   S=T-(AN*PI/W1)
      X=(1./A)*EXP(-D*W*S)*(B11*SIN(ETA*W*S)+B21*COS(ETA*
     1  W*S))+SIN(W1*T+THET)
      IF((ABS(X)).GT.(ABS(XX)))XX=X
      T=T+0.2
      IF(T.GT.(2.*PI/W1))GO TO 508
      GO TO 507
508   XMAXA2=XX
      XMAX2=XX*A
      WRITE(6,458)XMAXA1,XMAX1
      WRITE(6,459)XMAXA2,XMAX2
C
C
503   T4=(-RO*G87-S1)/(-RO*G86+S2)
```

```
C
C           S E C O N D   T H E O R E T I C A L
C                      S O L U T I O N
C                      ---------------
C
C
C        CALCULATING THE VALUE OF   (THET )
C
      THET=ATAN(T4)
      P7=-RO*G87-S1
      P8=-RO*G86+S2
      IF(P7.GT.0.0.AND.P8.LT.0.0)THET=THET+PI
      IF(P7.LT.0.0.AND.P8.LT.0.0)THET=THET+PI
      SN=SIN(THET)
      CN=COS(THET)
C
C        CALCULATING    Z1,Z2
C
      Z1=G86*CN+G87*SN+RO
      Z2=G84*CN+G85*SN
      WRITE(6,450)Z1,Z2
      WRITE(6,451)AN,THET
C
C        IF   Z1=0.0   AND   Z2=0.0   ,  THEN THE
C        FOLLOWING VALUES OF   N,THET,B1,B2,B11,B21,XA,
C        XG,DXA,DXB,DXG,DXH,V1,V2,XMAXA,XMAX    ARE A
C        THEORETICAL RESULT
C
C
C      CALCULATING   B1,B2,B11,B21
C
C
      B1=-A*(G80*CN+G81*SN)
      B11=-A*(G82*CN+G83*SN)
      B2=B11*G67+B1*G68
      B21=B1*G69+B11*G70
      WRITE(6,452)B1,B2
      WRITE(6,453)B11,B21
C
C        CALCULATING   XA,XG
C
      XA=B2+A*SIN(THET)
      XG=B21+A*SIN(AN*PI+THET)
      WRITE(6,454)XA,XG
C
C        CALCULATING   DXA,DXB,DXG,GXH
C
      DXA=B1*ETA*W-D*W*B2+A*W1*COS(THET)
      DXB=G33*DXA
      DXG=G34*DXA
      DXH=G35*DXA
```

```
      WRITE(6,455)DXA,DXB
      WRITE(6,456)DXG,DXH
C
C         CALCULATING   V1,V2
C
      V1=DXA/G9
      V2=G14*V1
C
      WRITE(6,457)V1,V2
C
C
C         CALCULATING   XMAXA   AND   XMAX
C
      XX=0.0
      T=0.0
509   X=(1./A)*EXP(-D*W*T)*(B1*SIN(ETA*W*T)+B2*COS(ETA*W*
     1  T))+SIN(W1*T+THET)
      IF((ABS(X)).GT.(ABS(XX)))XX=X
      T=T+0.2
      IF(T.GT.(AN*PI/W1))GO TO 510
      GO TO 509
510   XMAXA3=XX
      XMAX3=XX*A
      T=AN*PI/W1
511   S=T-(AN*PI/W1)
      X=(1./A)*EXP(-D*W*S)*(B11*SIN(ETA*W*S)+B21*COS(ETA*
     1  W*S))+SIN(W1*T+THET)
      IF((ABS(X)).GT.(ABS(XX)))XX=X
      T=T+0.2
      IF(T.GT.(2.*PI/W1))GO TO 512
      GO TO 511
512   XMAXA4=XX
      XMAX4=XX*A
      WRITE(6,461)XMAXA3,XMAX3
      WRITE(6,462)XMAXA4,XMAX4
C
  5   CONTINUE
  6   CONTINUE
C
C
C
C
 23   FORMAT(21X,6H    D=,F5.2,6X,5HFO/K=,F5.2,9X,2HE=,
     1        F5.2, 7X,3HWN=,F5.2/)
 24   FORMAT(21X,6H    W=,F5.2,9X,2HR=,F5.2,8X,3HDO=,
     1        F5.2,8X,2HU=,F5.2/)
 25   FORMAT(21X,6H    T=,F5.2,9X,2HA=,F5.2//)
450   FORMAT(28X,7H    Z1=,E12.5,10X,7H     Z2=,E12.5/)
451   FORMAT(28X,7H     N=,E12.5,10X,7H   THET=,E12.5/)
452   FORMAT(28X,7H    B1=,E12.5,10X,7H     B2=,E12.5/)
453   FORMAT(28X,7H   B11=,E12.5,10X,7H    B21=,E12.5/)
454   FORMAT(28X,7H    XA=,E12.5,10X,7H     XG=,E12.5/)
```

```
455    FORMAT(28X,7H   DXA=,E12.5,10X,7H   DXB=,E12.5/)
456    FORMAT(28X,7H   DXG=,E12.5,/0X,7H   DXH=,E12.5/)
457    FORMAT(28X,7H    V1=,E12.5,10X,7H    V2=,E12.5/)
462    FORMAT(28X,7HXMAXA4=,E12.5,10X,7H XMAX4=,E12.5/)
458    FORMAT(28X,7HXMAXA1=,E12.5,10X,7H XMAX1=,E12.5/)
459    FORMAT(28X,7HXMAXA2=,E12.5,10X,7H XMAX2=,E12.5/)
461    FORMAT(28X,7HXMAXA3=,E12.5,10X,7H XMAX3=,E12.5/)
470    FORMAT(48X,10HTABLE 4.1 )
471    FORMAT(48X,10H----------/)
472    FORMAT(41X,23H THEORETICAL   RESULTS )
473    FORMAT(41X,23H ---------------------- )
       STOP
       END
7            6400 END RECORD
8            6400 END FILE
```

```
A4466.                                                          ADEL M.
RUN(P)
SETINDF.
REDUCE.
LGO.
7            6400 END RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C          STABILITY DETERMINATION FOR THE CASE OF
C          UNSYMMETRIC  2  IMPACTS/CYCLE
C
      DIMENSION  A(4,4),B(4,4),C(4,4),X(4),Y(4)
C
C
C          WRITING DATA
C
      PI=4.*ATAN(1.)
      D=0.1
      ETA=SQRT(1.-D*D)
      E=0.8
      AM=0.1
      W=1.
      W1=1.
      DO=3.
      V=5.
      AN=2.*0.3391615005
      THET=2.94365
      B1=3.18933
      B2=1.16020
      B11=-3.22397
      B21=1.70810
      V1=-7.09090
      V2=3.63926
C
C          CALCULATING  C1,C2,C3,C4
C
      AZ =(V/ETA)*(W1*SIN(THET)-D*COS(THET))
      Z1=SIN(AN*PI/ETA)
      Z2=COS(AN*PI/ETA)
      ZX=1./EXP(D*AN*PI/W1)
      C1=ZX*((D/ETA)*Z1+Z2)
      C2=ZX*Z1*(1./ETA)

      BB2=-B2*Z1*(ETA/W1)
      BB3=-ZX*(D/W1)*(B1*Z1+B2*Z2)
      C3=BB3+(BB1+
      C3=BB3+(BB1+BB2)*ZX+V*COS(AN*PI+THET)


C          CALCULATING  D1,D2,D3,D4,D5
C
      ZZZ=V10+W1*C3
      D1=W1*(1.-C1)/ZZZ
      D2=-W1* C2/ZZZ
```

```
      D3=-PI/ZZZ
      D4=-W1*C4/ZZZ
      D5=-(W1*C3+V10*C1)/ZZZ
C
C         CALCULATING  F1,F2
C
      F1=D*Z1-ETA*Z2
      F2=D*Z2+ETA*Z1
C
C         CALCULATING  G0,G1,G2,G3,G4
C
      G0=-(F1*B1+F2*B2)
      G1=-(F1*(D/ETA)+F2)
      G2=-F1/ETA
      G3=(ETA/W1)*(F1*B2-F2*B1)
      G4=F2*V*COS(THET) -AZ*F1
C
C         CALCULATING  AK1,AK2,AK3,AK4
C
      AK1=(1.-AM*E)/(1.+AM)
      AK2=AM*(1.+E)/(1.+AM)
      AK3=(1.+E)/(1.+AM)
      AK4=(AM-E)/(1.+AM)
C
C         CALCULATING  RO0,RO1,RO2,RO3,RO4
C
      RO0=G0*ZX+V*W1*COS(AN*PI+THET)
      RO1=G1*ZX
      RO2=G2*ZX
      RO3=ZX*(G3-(D/W1)*G0)-V*W1*SIN(AN*PI+THET)
      RO4=G4*ZX-V*W1*SIN (AN*PI+THET)
C
C         CALCULATING THE ELEMENTS OF THE MATRIX  P1
C
      B(1,1)=D5
      B(1,2)=V10*D2/W1
      B(1,3)=-C3*D3
      B(1,4)=V10*D4/W1
      B(2,1)=-AK1*(RO1+RO3*D1)
      B(2,2)=-AK1*(RO2+RO3*D2)
      B(2,3)=AK2-AK1*RO3*D3
      B(2,4)=-AK1*(RO4+RO3*D4)
      B(3,1)=AK3*(RO1+RO3*D1)
      B(3,2)=AK3*(RO2+RO3*D2)
      B(3,3)=AK3*RO3*D3-AK4
      B(3,4)=AK3*(RO4+RO3*D4)
      B(4,1)=D1
      B(4,2)=D2
      B(4,3)=D3
      B(4,4)=1.+D4
C
C         CALCULATING  C11,C21,C31,C41
C
      Z11=SIN (ETA*(2.*PI-AN*PI)/W1)
```

```
      Z21=COS (ETA*(2.*PI-AN*PI)/W1)
      ZX1=1./EXP(D*(2.*PI-AN*PI)/W1)
      AZ1=(V/ETA)*(-D*COS(AN*PI+THET)+W1*SIN(AN*PI+THET))
      C11= ZX1*((D/ETA)*Z11+Z21)
      C21=ZX1*(1./ETA)*Z11
      BB11=B11*(ETA/W1)*Z21
      BB21=-B21*(ETA/W1)*Z11
      BB31=-(D/W1)*ZX1*(B11*Z11+B21*Z21)
      C31=-ZX1*(BB11+BB21)-ZX1*(BB31+V*COS(THET))
      C41=-ZX1*((AZ1*Z11-V*Z21*COS(AN*PI+THET))+V*
     1 COS(THET))
C
C         CALCULATING  D11,D21,D31,D41,D51
C
      ZZZ1=V20+W1*C3
      D11=W1*(1.-C11)/ZZZ1
      D21=-W1*C21/ZZZ1
      D31=-(2.*PI-AN*PI)/ZZZ1
      D41=-W1*C41/ZZZ1
      D51=-(C11*V20+C31*W1)/ZZZ1
C
C         CALCULATING  F11,F21
C
      F11=D*Z11-ETA*Z21
      F21=D*Z21+ETA*Z11
C
C         CALCULATING  G01,G11,G21,G31,G41
C
      G01=-(F11*B11+F21*B21)
      G11=F11*(D/ETA)+F21
      G21=F11/ETA
      G31=-F11*(ETA/W1)*B11+F11*(ETA/W1)*B21
      G41=-AZ1*F11+V*F21*COS(THET)
C
C         CALCULATING    RO11,RO21,RO31,RO41
C
      RO11=ZX1*G11
      RO21=ZX1*G21
      RO31=ZZ1*(-(D/W1)*G01+G31)-V*W1*SIN(THET)
      RO41=ZX1*G41-V*W1*SIN(THET)
C
C         CALCULATING THE ELEMENTS OF THE MATRIX  P1
C
      C(1,1)=D51
      C(1,2)=V20*D21/W1
      C(1,3)=V20*D41/W1
      C(2,1)=AK1*(RO11+RO31*D11)
      C(2,2)=AK1*(RO21+RO31*D21)
      C(2,3)=-(AK2-AK1*RO31*D31)
      C(2,4)=AK1*(RO41+RO3)*D41)
```

```
      C(3,1)=-AK3*(RO1+RO3*D11)
      C(3,2)=-AK3*(RO21+RO31*D21)
      C(3,3)=-AK3*RO31*D31-AK4
      C(3,4)=-AK3*(RO41+RO31*D41)
      C(4,1)=D11
      C(4,2)=D21
      C(4,3)=D31
      C(4,4)=D41+1.
C
C          CALCULATING THE ELEMENTS OF THE STABILITY MATRIX
C
      DO 110 I=1,4
      DO 110 J=1,4
      A(I,J)=0.0
      DO 110 K=1,4
      A(I,J)=A(I,J)+C(I,K)*B(K,J)
110   CONTINUE
C
C          CALCULATING  THE EIGEN-VALUES OF THE STABILITY
C          MATRIX
C
      N=4
      NC=4
      EPS=0.0001
      CALL RUTI(A,N,NC,X,Y,EPS)
C
C          CHECKING THE ABSOLUTE VALUES OF THE EIGEN VALUE
C
      DO 603 I=1,4
      ABSEIG=SQRT(X(I)*X(I)+Y(I)*Y(I))
      IF (ABSEIG.GT.1.)GO TO 604
603   CONTINUE
      WRITE(6,702)
      GO TO 500
604   WRITE(6,701)
C
701   FORMAT(1X,22H  SYSTEM IS NOT STABLE/)
702   FORMAT(1X,18H  SYSTEM IS STABLE/)
500   STOP
      END
```

```
A4466.                                                    ADEL M.
RUN(P)
SETINDF.
REDUCE.
LGO.
7            6400 END RECORD
       PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
      DIMENSION  Z(4,4),Y(4)
C
C          DETERMINATION OF THE STEADY STATE MOTION OF
C          A SINGLE DEGREE OF FREEDOM SYSTEM WITH AN
C          IMPACT DAMPER IN THE SPECIAL CASE OF
C          2 SYMMETRIC IMPACTS/CYCLE MOTION
C
C          WRITING DATA
      PI=4.*ATAN(1.)
      D=0.1
      E=0.2
      AM=0.4
      FF=1.
      ETA=SQRT(1.-D*D)
      DO=3.
      W=1.
      W1=1.25
C
C          CALCULATING S,C,H1,H2,SEG1,SEG2,TH1,TH2
C
1000  S=SIN(THET)
      C=W1*COS(THET)
      H1=(1./EXP(D*PI*W1/W))*SIN(ETA*PI*W1/W)
      H2=(1./EXP(D*PI*W1/W))*COS(ETA*PI*W1/W)
      SEG1=(PI/2.*W1))*((1.+E)/(1.-E+2.*AM))
      SEG2=(PI/2.*W1))*((1.+E)/(1.-E-2.*AM*E))
      TH1=W*(1./EXP(D*PI*W1/W))*(-D*SIN(ETA*PI*W1/W)+
     1 ETA*COS(ETA*PI*W1/W))
      TH2=W*(1./EXP(D*PI*W1/W))*(-D*COS(ETA*PI*W1/W)-
     1 ETA*SIN(ETA*PI*W1/W))
C
C     CALCULATING  H3,H4,H5,H6,H,HH1,HH2,HH3,HH4
C
      H3=(SEG2-SEG1)+SEG1*SEG2*(D*W-TH2)
      H4=SEG1*SEG2*(TH1+ETA*W)
      H5=D*SEG2*W-SEGU*TH2
      H6=SEG1*TH1+ETA*SEG2*W
      H=2.*W*(H3*H1+H4*(1.+H2))/(H5*H1+H6*(1.+H2))
      HH1=(-2.*RO+H*SQRT(H*H+4-RO*RO))/(H*H+4)
      HH2=(-2.*RO-H*SQRT(H*H+4-RO*RO))/(H*H+4)
```

```
      HH3=(-H*RO+2.*SQRT(H*H+4-RO*RO))/(H*H+4)
      HH4=(-H*RO-2.*SQRT(H*H+4-RO*RO))/(H*H+4)
C
C         CALCULATING  THET
C
      THET=ATAN(HH1/HH4)
C
C         CALCULATING THE ELEMENTS OF THE STEADY STATE
C         MOTION MATRIX
C
      N=6
      NA=6
      DO 10 I=1,6
      DO 10 J=1,6
      Z(I,J)=0.0
 10   CONTINUE
      Z(1,1)=1.0
      Z(1,5)=-1.0
      Z(1,6)=-S
      Z(2,3)=1.
      Z(2,4)=-ETA*W
      Z(2,5)=D*W
      Z(2,6)=-C
      Z(3,1)=1.0
      Z(3,4)=H1
      Z(3,5)=H2
      Z(3,6)=-S
      Z(4,2)=1.0
      Z(4,4)=TH1
      Z(4,5)=TH2
      Z(4,6)=-C
      Z(5,1)=1.0
      Z(5,2)=SEG1
      Z(6,2)=1.0
      Z(6,3)=SEG2
      Y(1)=0.0
      Y(2)=0.0
      Y(3)=0.0
      Y(4)=0.0
      Y(5)=-DO/2.0
      Y(6)=-DO/2.0
C
C         SOLVING THE SET OF LINEAR EQUATIONS
C
      CALL  SOLVE(Z,Y,ID,N,NA)
C
```

```
C          CALCULATING  XB,DXB,DXA,B1,B2
C
      XB=Y(1)
      DXB=Y(2)
      DXA=Y(3)
      B1=Y(4)
      B2=Y(5)
C
C
C          WRITING THE  FIRST THEORETICAL SOLTION
C
C
      WRITE(6,320)THET
      WRITE(6,321)XB
      WRITE(6,322)DXB
      WRITE(6,323)DXA
      WRITE(6,324)B1
      WRITE(6,325)B2
C
C          REPEATING THE SAME CALCULATIONS FOR THE OTHER
C          VALUE OF  THET
C
      LZA=LZA+1.
      GO TO (701,40),LZA
701   THET=ATAN(HH2/HH3)
      GO TO 1000
 40   CONTINUE
C
C          WRITING THE SECOND THEORETICAL SOLTION
C
      WRITE(6,320)THET
      WRITE(6,321)XB
      WRITE(6,322)DXB
      WRITE(6,323)DXA
      WRITE(6,324)B1
      WRITE(6,325)B2
C
320   FORMAT(10X,5HTHET=,E20.10/)
321   FORMAT(10X,5HXB   =,E20.10/)
322   FORMAT(10X,5HDXB  =,E20.10/)
323   FORMAT(10X,5HDXA  =,E20.10/)
324   FORMAT(10X,5HB1   =,E20.10/)
325   FORMAT(10X,5HB2   =,E20.10/)
      STOP
      END
```

```
A4466.                                                 ADEL M.
RUN(P)
SETINDF.
REDUCE.
LGO.
7          6400 END RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
      DIMENSION  A(4,4),X(4),Y(4)
C
C         STABILITY DETERMINATION FOR THE SPECIAL CASE OF
C         SYMMETRIC 2 IMPACTS/CYCLE
C
C         WRITING DATA
C
      D=0.1
      ETA=SQRT(1.-D*D)
      W=1.
      W1=1.25
      E=0.2
      PI=4.*ATAN(1.)
      B1=0.91722131767
      B2=-1.0847507256
      V=1.6245538642
      THET=2.4817503658
C
C         CALCULATING  C1,C2,C3,C4
C
      AZ=(V/ETA)*(W1*SIN(THET)-D*COS(THET))
      C1=(1./EXP(D*PI/W1))*((D/ETA)*SIN(ETA*PI*W1/W)
     1 +COS(ETA*PI/W1))
      C2=(1./EXP(D*PI/W1))*((1./ETA)*SIN(ETA*PI/W1))
      BB1=B1*(ETA/W1)*COS(ETA*PI/W1)
      BB2=-B2*(ETA/W1)*SIN(ETA*PI/W1)
      BB3=-(D/W1)*(1./EXP(D*PI/W1)*(SIN(ETA*PI/W1)*B1+
      C3=BB3+(1./EXP(D*PI/W1))*(BB1+BB2)-V*COS(THET)
     1 COS(ETA*PI/W1)*B2)
      C4=1./EXP(D*PI/W1)*(AZ*SIN(ETA*PI/W1)-A*COS(THET)
     1 *COS(ETA*PI/W1))-A*COS(THET)
C
C         CALCULATING  D1,D2,D3,D4,D5
C
      D1=W1*(1.-C1)/(V10+W1*C3)
      D2=-W1*C2/(V10+W1*C3)
      D3=-PI/(V10+W1*C3)
      D4=-W1*C4/(V10+W1*C3)
      D5=-(W1*C3+V10*C1)/(V10+W1*C3)
C
C         CALCULATING  F1,F2
C
      F1=D*SIN(ETA*PI/W1)-ETA*COS(ETA*PI/W1)
      F2=D*COS(ETA*PI/W1)+ETA*SIN(ETA*PI/W1)
```

```
C
C         CALCULATING  G0,G1,G2,G3,G4
C
      G0=-(F1*B1+F2*B2)
      G1=-(F1*(D/ETA)+F2)
      G2=-F1/ETA
      G3=(ETA/W1)*(F1*B2-F2*B1)
      G4=F2*A*COS(THET)-AZ*F1
C
C         CALCULATING  AK1,AK2,AK3,AK4
C
      AK1=(1.-AM*E)/(1.+AM)
      AK2=AM*(1.+E)/(1.+AM)
      AK3=(1.+E)/(1.+AM)
      AK4=(AM-E)/(1.+AM)
C
C         CALCULATING  RO0,RO1,RO2,RO3,RO4
C
      RO0=G0*(1./EXP(D*PI/W1))-A*W1*COS(THET)
      RO1=G1*(1./EXP(D*PI/W1))
      RO2=G2*(1./EXP(D*PI/W1))
      RO3=(1./EXP(D*PI/W1))*(G3-(D/W1)*G0)+A*W1*COS(THET)
      RO4=G4*(1./EXP(D*PI/W1))+A*W1*SIN(THET)
C
C         CALCULATING THE ELEMENTS OF THE STABILITY MATRIX
C
      A(1,1)=D5
      A(1,2)=V10*D2/W1
      A(1,3)=-C3*D3
      A(1,4)=V10*D4/W1
      A(2,1)=-AK1*(RO1+RO3*D1)
      A(2,2)=-AK1*(RO2+RO3*D2)
      A(2,3)=AK2-AK1*RO3*D3
      A(2,4)=-AK1*(RO4+RO3*D4)
      A(3,1)=AK3*(RO1+RO3*D1)
      A(3,2)=AK3*(RO2+RO3*D2)
      A(3,3)=AK3*RO3*D3-AK4
      A(3,4)=AK3*(RO4+RO3*D4)
      A(4,1)=D1
      A(4,2)=D2
      A(4,3)=D3
      A(4,4)=1.+D4
C
C         CALCULATING  THE EIGEN-VALUES OF THE STABILITY
C         MATRIX
C
      N=4
      NC=4
      EPS=0.0001
      CALL RUTI(A,N,NC,X,Y,EPS)
```

```
C
C        CHECKING THE ABSOLUTE VALUES OF THE EIGEN VALUES
C
      DO 603 I=1,4
      ABSEIG=SQRT(X(I)*X(I)+Y(I)*Y(I))
      IF (ABSEIG.GT.1.)GO TO 604
603   CONTINUE
      WRITE(6,702)
      GO TO 500
604   WRITE(6,701)
C
701   FORMAT(1X,22H  SYSTEM IS NOT STABLE/)
702   FORMAT(1X,18H  SYSTEM IS STABLE/)
500   STOP
      END
```