

FILM LOOP IN THE ROLLING LOOP PROJECTOR

A COMPUTER AIDED STUDY OF  
THE FILM LOOP  
IN THE ROLLING LOOP PROJECTOR

By  
HUYNH VAN MINH, B.Eng.

A Thesis  
Submitted to the School of Graduate Studies  
in Partial Fulfilment of the Requirements  
for the Degree  
Master of Engineering

McMaster University

February 1970

MASTER OF ENGINEERING (1970)  
(Mechanical Design)

McMASTER UNIVERSITY  
Hamilton, Ontario.

TITLE: A Computer Aided Study of the Film Loop  
in the Rolling Loop Projector

AUTHOR: Huynh Van Minh, B.Eng.  
(McGill University, Montreal)

SUPERVISOR: Professor W. R. Newcombe

NUMBER OF PAGES: xii, 141

SCOPE AND CONTENTS:

A review of movie projection principles and techniques as applied to conventional projectors and a description of general features, mechanism and film movement of the Rolling Loop Projector are presented.

The characteristics of a film loop under static conditions with coplanar clamping planes are investigated in terms of loop shape, supporting forces and bending moments. An extension of this analysis is applicable to the case when the loop is fixed on a curved surface.

The motion of a loop in the projector is simulated as it grows and rolls on a flat surface and a numerical technique is employed to find the point paths and the velocity and acceleration vectors of these points on the loop at various instants in a projection cycle.

Some design criteria are derived from an examination of the results, and these are presented.

### ACKNOWLEDGEMENTS

The author would like to express his sincere thanks to Professor W. R. Newcombe for his guidance and encouragement throughout the course of the project.

The author is also indebted to Mr. William Shaw, the Director of Development of Multiscreen Corporation for his assistance and suggestions, especially for his kindness in providing information and equipment needed in the project.

Thanks are also extended to Dr. G. Kardos for his useful advice and suggestions.

This work is financially supported by the National Research Council, Ottawa.

## TABLE OF CONTENTS

CHAPTER		<u>PAGE</u>
	LIST OF ILLUSTRATIONS	vi
	LIST OF TABLES	ix
	NOMENCLATURE	x
1	INTRODUCTION	1
2	MOTION PICTURE PROJECTION	4
	2.1 Principles	4
	2.2 Projector Mechanisms	5
	2.3 Mechanics of Intermittent Projectors	11
3	ROLLING LOOP PROJECTOR	13
	3.1 General Concept	13
	3.2 Rolling Loop Projector Mechanism	15
	3.3 Film Movement	17
4	STATIC LOOP ANALYSIS	23
	4.1 General	23
	4.2 Governing Equations	23
	4.3 Other Relations	31
	4.4 Loop Characteristics	36
5	ROLLING LOOP SIMULATION	42
	5.1 General	42
	5.2 Theoretical Model	42
	5.3 Selection of Coordinate Systems	44

CHAPTER		<u>PAGE</u>
	5.4 Method of Solution	46
	5.5 Velocity and Acceleration at Early Stage of Loop Growth	51
6	DISCUSSIONS OF THE RESULTS	55
	6.1 Characteristics of a Film Loop on Flat Surface	55
	6.2 Point Paths	58
	6.3 Velocity and Acceleration in Loop Motion	61
	6.4 Velocity and Acceleration at the Top of the Loop	80
7	CONCLUSIONS	83
8	DESIGN CONSIDERATIONS	87
	REFERENCES	94
APPENDIX A	CHANGE VARIABLE OF INTEGRATION	96
APPENDIX B	STATIC LOOP WITH INCLINED CLAMPED PLANES	98
	B.1 Governing Equations	98
	B.2 Calculation of Parameter K	105
	B.3 Loop Characteristics	108
APPENDIX C	SAMPLE OF COMPUTER PROGRAMMES	114
APPENDIX D	COMPUTER RESULTS	134

## LIST OF ILLUSTRATIONS

FIGURE NUMBER		PAGE
1	Beater mechanism	8
2	"D" roller system	8
3	Cam operated claw mechanism used in the Siemens and Halske projector	8
4	Four-slot Geneva mechanism	10
5	Travel, velocity and acceleration of a typical four-slot Geneva mechanism for 35 mm. film	10
6a,b	Displacement of a point after a loop passing through it	14
7a,b,c	Side view, plan view and front view of the Rolling Loop Projector respectively	16
8a,b,c,d	Film movement at various stages in a frame cycle in the Rolling Loop Projector	20
9	Characteristics of a loop on flat surface with dimensionless parameters	56
10	Characteristics of a loop on flat surface versus gap width ( $E_s = 2.805$ in.)	57
11a,b	Paths of point number 15 and 35 respectively	59
12a,b	Velocity and acceleration diagrams respec- tively at time $t = .2/(24 \times 50)$ sec. with $DS = .187/5$ in.	62

FIGURE NUMBER		PAGE
13a,b	Velocity and acceleration diagrams respectively at time $t = .5/(24 \times 50)$ sec. with $DS = .187/5$ in.	65
14a,b	Velocity and acceleration diagrams respectively at time $t = .8/(24 \times 50)$ sec. with $DS = .187/5$ in.	68
15a,b	Velocity and acceleration diagrams respectively at time $t = 5/(24 \times 50)$ sec. with $DS = 2 \times .187$ in.	70
16a,b	Velocity and acceleration diagrams respectively at time $t = 25/(24 \times 50)$ sec. with $DS = 2. \times .187$ in.	72
17a,b	Velocity and acceleration diagrams respectively at time $t = 45/(24 \times 50)$ sec. with $DS = 2. \times .187$ in.	74
18a,b	Velocity and acceleration diagrams respectively at time $t = 35.2/(24 \times 50)$ sec. with $DS = .187/3$ in.	76
19a,b	Velocity and acceleration diagrams respectively at time $t = 35.2/(24 \times 50)$ sec. with $DS = .187/3.$ in. and $V_F = 0$ .	78
20	Velocity and acceleration amplitudes of the loop top in a frame cycle	81
21	High speed photograph of a film loop after it leaves the projection gate (Projector	86



FIGURE NUMBER		PAGE
21	(Cont'd.) operating speed = 18 frames/sec., camera speed = 1000 frames/sec.)	
22	Different angles of film entry slot	88
23	Proposed film inlet	90

## LIST OF TABLES

TABLE NUMBER	DESCRIPTION
1	Characteristics of a static loop on flat surface with dimensionless parameters
2	Characteristics of a static loop on flat surface versus gap width ( $E_s = 2.805$ in.)
3a,b,c	Sample of numerical results from programme A with $DS = .187/5$ in., $DT = \frac{1}{24} \times \frac{1}{500}$ sec. They are the coordinates of fixed points on the strip, their velocity and acceleration respectively
4	Velocity and acceleration vectors of the loop top in a frame cycle (with $DT = \frac{1}{24 \times 50}$ sec., $DS = 2. \times .187$ in.)
5	Characteristics of a loop on curved surface with $\sin \beta = .0555$

## NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
A	acceleration amplitude
$A_x, A_y$	components of the acceleration vector in the x and y directions respectively
B	flexural rigidity per unit length of the material = $\frac{E'I}{S_T}$
C	parameter = $\sqrt{\frac{P}{E'I}}$
D	diameter of the rotor
DT	time increment
DS	distance between two fixed points on the film strip
E'	modulus of elasticity
E	elliptic integral of the second kind = $E(K, \phi)$
$E_0$	complete elliptic integral of the second kind
$E_1$	elliptic integral of the second kind corresponding to angle $\phi_1$ , = $E(K, \phi_1)$
$E_L$	excess length of film in a loop
F	elliptic integral of the first kind = $F(K, \phi)$
$F_0$	complete elliptic integral of the first kind
$F_1$	elliptic integral of the first kind corresponding to angle $\phi_1$ , = $F(K, \phi_1)$
I	moment of inertia
K	parameter of elliptic integral = $\sin \alpha$

$L_T$	gap width
$L_1$	projected length of $S_1$ on x-axis
$\ell$	half length of the base of an isosceles triangle
$M$	bending moment at any point on the strip
$M_1, M_2$	bending moments at the bottom and at the top of a loop respectively
$P$	supporting force
$P_1$	reaction force at inflection point
$P_C$	critical value of $P$
$R$	ratio of $S_T$ to $L_T$
$s$	length of the equal sides of an isosceles triangle
$S_C$	curvilinear coordinate of the moving origin measured along the film strip
$S_0$	length of the film strip
$S_p$	curvilinear coordinate of selected point
$S_T$	total length of the strip in loop
$V$	velocity amplitude
$V_x, V_y$	components of the velocity vector in x and y directions respectively
$S, x, y$	coordinates of a point wrt. a fixed coordinate system
$S_M, x_M, y_M$	coordinates of a point wrt. a moving coordinate system
$Y_L$	loop height

Greek symbolDescription

$\alpha$	half angle of a tangent to the curve at inflection point
$\beta$	angle of inclination of the clamped planes
$\epsilon$	a small quantity
$\theta$	angle of tangent to the curve
$\psi$	value of $\theta$ after changing variable of integration
$\phi$	limit of $\psi$
$\phi_A$	angle of acceleration vector
$\phi_V$	angle of velocity vector

## 1. INTRODUCTION

The motion picture industry had its beginnings in the late nineteenth century when Thomas Edison invented his peep hole Kinetoscope. Although the motion picture history is a relatively short one, it demonstrates the ingenuity and inventiveness of mankind in overcoming problems in both technical and artistic senses.

Various experimentation and research have been conducted in order to obtain the best mechanism to transport film in movie cameras or movie projectors. So far, specifically in the projection aspect, two different approaches have been attempted. The first approach is to move the film intermittently through the projection gate by intermittent motion mechanisms, and shutter blades are devised to cut off the light when the film is in motion. The second scheme is to run the film continuously at constant speed and employ what one calls an optical intermittent system to freeze the pictures on the screen.

Recently, a totally new kind of film motion has been invented by R. W. Jones in Australia. It is based on the concept that a portion of the film can be transported a certain distance by rolling the film in a series of loops. This type of motion is similar to the first method described in that a portion of the film remains stationary at the

aperture plate for a period of time, but it does not involve intermittent motion of the film or of the mechanism itself. The subject of this thesis is the study of this kind of motion.

Based on the theory of an elastica which is essentially a thin strut deflected beyond the critical load, the characteristics of a strip held between two coplanar clamps are investigated in terms of loop shape, supporting forces and bending moments.

A numerical approach is employed with the aid of the computer to simulate the motion of the film in the projector, in which the loop constrained in a fixed length gap is considered to grow and move together with the gap at constant rates. In order to simplify the theoretical model, this simulation is carried out with the assumption that the loop retains its natural static shape while in motion. The resulting dynamic behaviour of the film in the model - i.e., the motion of various points on the film strip, their velocity and acceleration vectors at various instants - is described in a series of graphs with the help of the Benson Lehner plotter.

The early stage of loop growth is characterized by very high initial transverse velocity and acceleration, and a reversal of the vectors in some part of the velocity and acceleration "profiles" during the transition period. Special attention is paid to the analysis of the loop at this stage.

Under actual conditions the loop is rolled on the inside of a cylindrical surface, the curvature of which may modify all the characteristics of the loop considerably. In order to deal with this situation, Appendix B is set aside for the consideration of the effect of the inclined clamping planes on loop behaviour.

A complete analysis of the loop motion requires an extensive study of various dynamic factors (such as film mass, friction force, irregularities in film material, etc.) which cause the loop shape to deviate from that of the theoretical prediction. However, the results obtained do give a reasonably good approximation to the real loop motion and provide some useful design information.



## 2. MOTION PICTURE PROJECTION

### 2.1 Principles

Motion picture projection, in general, is based on the concept that if an object is moving continuously from one point to another, it must occupy an infinite number of intermediate positions between these two points. Accordingly, a large number of pictures would be required to represent even the simplest bit of movement. When the light coming from an object enters one's eyes an image which has a certain permanency is formed on the retina. This retinal image lasts for a short while before being cancelled by the succeeding images or by the natural function of the eyes. Therefore, human eyes cannot distinguish between the infinite series of positions involved in the actual motion and a set of samples selected from this series at sufficiently short intervals. For example, drops of falling water look like an uninterrupted water thread. Experimenters have found that the eye begins to discern discrete events at an interval greater than about one twentieth of a second. To be able to deceive human eye perception, the replacing of one picture by another must be done in the absence of the projected light. The time for picture interchange is mainly dependent on the brightness of the picture; the brighter the picture, the shorter the period. In movie projection practice the period used is in

the range of one fiftieth of a second.

An alternative solution for this picture interchange is to employ optical compensation, which involves the use of moving lenses, mirrors or prisms which are designed in such a way that the succession of images will melt into one another without dark intervals.

## 2.2 Projector Mechanisms

Based on the principles described above, numerous mechanisms have been devised for the purpose of obtaining a good quality projection picture; however, for various reasons, some of them have fallen into disuse while a few have become widely adapted to projectors for theatrical purposes.

### 2.2.1 Continuous Projectors

In continuous projection the film is moved across the aperture plate at a constant linear velocity. Besides the condensing and objective lenses an oscillating or rotating optical system which consists of lenses, prisms or mirrors or a combination of these is employed to compensate for the continuous motion of the film. The purpose of the compensating elements is to maintain the projected image in the same vertical plane and at exactly the same size throughout the optically effective travel of the compensating elements.

In this kind of system there are always two

superimposed pictures projected on the screen. As soon as the first image begins to fade away because of the reduction of the optical effectiveness of the corresponding compensating element, the next element brings in the following picture which is projected with the same amount of light.

The great advantage of this type of mechanism is that the stationary period of the picture on the screen is prolonged. Consequently, a better tone and a greater depth of perspective of the picture can be obtained. In addition, by moving the film at constant speed film stresses can be reduced to a minimum level, film vibration can be eliminated and thereby it is possible to increase film life and picture steadiness.

One of the draw backs which limit the use of this optical intermittent system for theatrical projection is the great accuracy required in the manufacture and assembly of the compensating optical system. Today the only photographic machines that use the continuous film movement are mostly high speed cameras which can be run at a rate of more than a thousand frames per second.

### 2.2.2 Intermittent Projectors

There are various types of mechanisms which impart an intermittent motion to the film and perform the function of moving one frame at a time across the aperture plate:

#### a. Beater or Dog Movement

This is the first and probably the simplest type of intermittent movement used in projectors. It consists of a revolving eccentric mass (Fig. 1) which moves the film in a series of beats. In every revolution of the mass the beater draws the film down a space of one frame against the drag imposed on the film by a trap at the projection gate. This trap maintains a high tension on the film so that the film back-lash can be prevented and accurate frame registration can be achieved.

Another version of this type is the "D" Roller system (Fig. 2) which includes two rollers, one of which has a "D" shape. These rollers grip the film between them and drive it forward as they rotate. The film drive is not maintained when the flat part of the "D" roller is facing the film.

The main faults of the beater movement are that, in addition to its objectionable noise level, film stresses are quite high because of its inherent jerky movement, and picture registration is not of sufficient quality to meet the standards of motion picture projection.

b. Pin or Claw Movement

This movement is widely used today in cameras and several makes of 16 mm. movie projectors. The film movement is produced by two or more pins which engage in film perforations during frame transfer. The "in" or "out" motion of the pins with respect to the perforations is

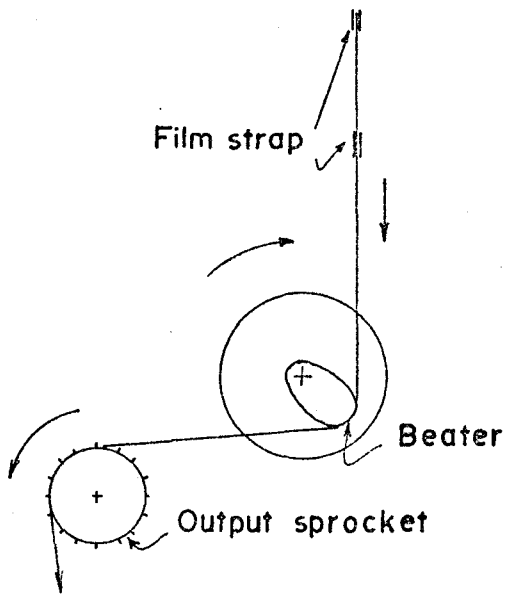


Figure 1 . Beater mechanism

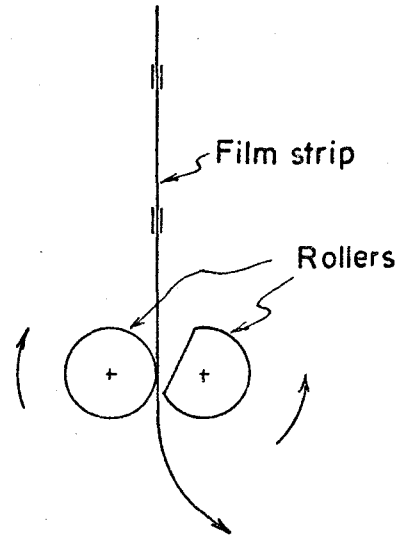


Fig.2 . D roller system

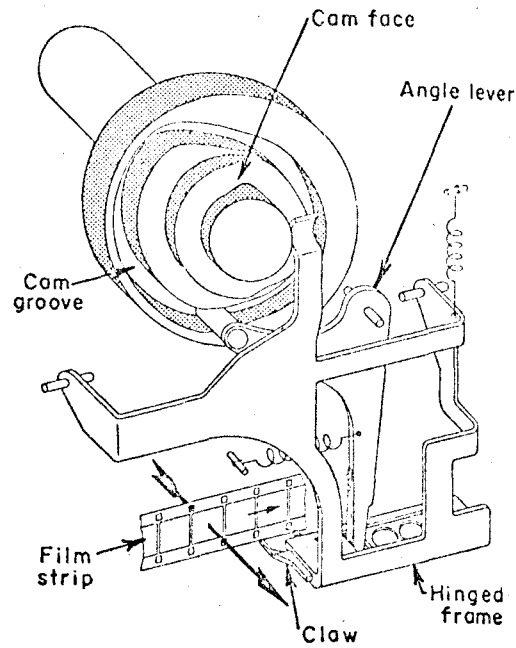


Fig.3. Cam operated claw mechanism used in the Siemens & Halske projector [8].

obtained from the action of a cam, while the "up" and "down" motion is activated by an eccentrically pivoted crank or by a cam-operated equivalent system.

The advantage of having light weight and structural simplicity makes this type of mechanism attractive to the design of compact cameras and portable movie projectors. A version of this type of mechanism used in the Siemens and Halske projector is shown in Figure 3.

c. Geneva Movement

Another source of intermittent film transport is the Geneva or Maltese Cross movement which is now universally used in 35 and 70 mm. motion picture projectors for synchronizing the movement of the film with that of the shutter.

One of the mechanisms in general use is the four slot type Geneva which is shown in Figure 4. The driving member (A) is a disc carrying a pin or a roller which during each revolution of member (A) will engage one of the four radial slots equally spaced on the driven disc (B). Between the radial slots are four concave surfaces to match the circular ring of the driving member. This ring has a cut away surface (D) to provide clearance for the passage of the slotted arms. Member (B) which carries a film sprocket will rotate at an angle of 90 degrees corresponding to a quarter of a revolution of the driving member, and the former will stay stationary during the rest of the cycle by the interlocking of the surfaces (C) and (E).

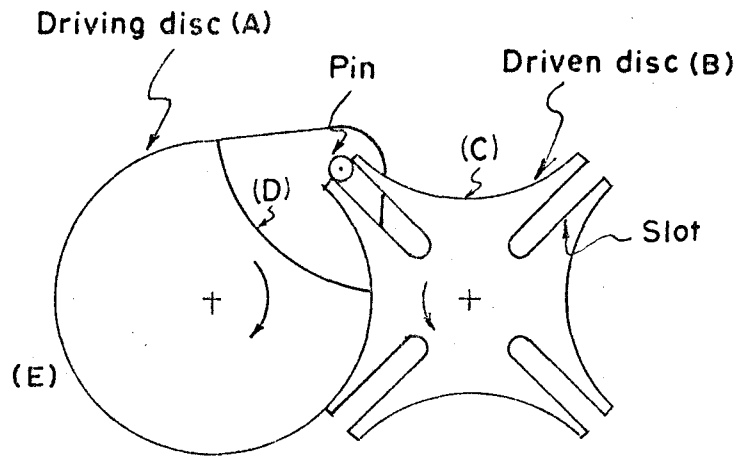


Fig.4. Four slot Geneva mechanism

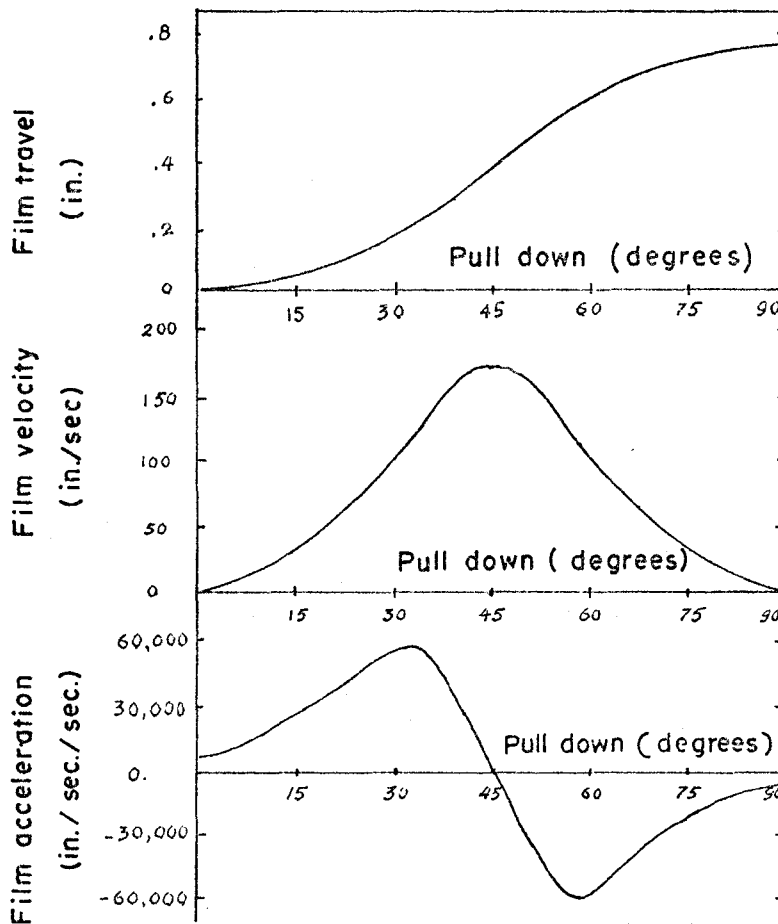


Fig.5. Travel, velocity & acceleration of a four slot Geneva mechanism for 35 mm. film [4].

The Geneva mechanism is simple in construction and it provides accurate and reliable intermittent motion. However, it has the drawback of having points of relatively high angular velocity and acceleration. The characteristics of a typical 4-slots Geneva movement are shown in Figure 5.

### 2.3 Mechanics of Intermittent Projectors

In intermittent projectors the film is pulled across the aperture plate normally at a rate of 24 frames per second and the shutter blade is devised to rotate at the same rate to cover the light when the film is in motion. This results in a flicker frequency of 24 cycles per second. However, a series of flashing pictures will be seen on the screen if the movie is shown at this flicker rate. Therefore, to produce an impression of continuous images to the eyes, an additional shutter blade is employed to increase this frequency to 48 cycles per second, and consequently the actual time for showing one picture frame is reduced to half the time. The illumination efficiency in projection which is mainly dependent on this time interval is theoretically reduced to fifty percent. In wide screen projection where very high intensity arc lamps are used, this factor is of great concern to the projectionists. Therefore, it is desirable to cut down the frame transition period as much as possible; but this reduction is accompanied by an increase in film acceleration by the square of the film



transfer period. Similarly, if the frame size or frame frequency increases, the film stresses will definitely increase proportionally.

For the above reasons the introduction of the Rolling Loop Projector which employs a wave-like film motion is a great break-through in projector design by its reduction of the frame transition period and by its ability to increase the frame size or the frame frequency without exceeding the permissible film stress limits.

### 3. ROLLING LOOP PROJECTOR

#### 3.1 General Concept

Prior to the development of any theory concerning the characteristics of the film loop, it is necessary to have a clear understanding of the Rolling Loop Projector mechanism and the movement of the film through this projector.

The whole philosophy behind the transportation of the film in this mechanism is that a portion of the film can be moved forward in finite steps by rolling a series of loops along the film path through the projector. Consider a strip of film being fixed at the two ends (A) and (B) (Fig. 6a); this strip forms a loop of length  $S_T$  confined in a rectangular slot of a plate (D). The loop originally is at the left hand side of a point P which has a coordinate  $x_1$  with respect to the origin O. After the loop is pushed past P, it is observed that point P is now carried to a new position at a distance  $x_2$  from O (Fig. 6b). If  $L_T$  is the width of the slot, one can write the following relation:

$$x_2 - x_1 = S_T - L_T$$

Therefore, point P is moved a distance equal to the excess length of the film in the slot.

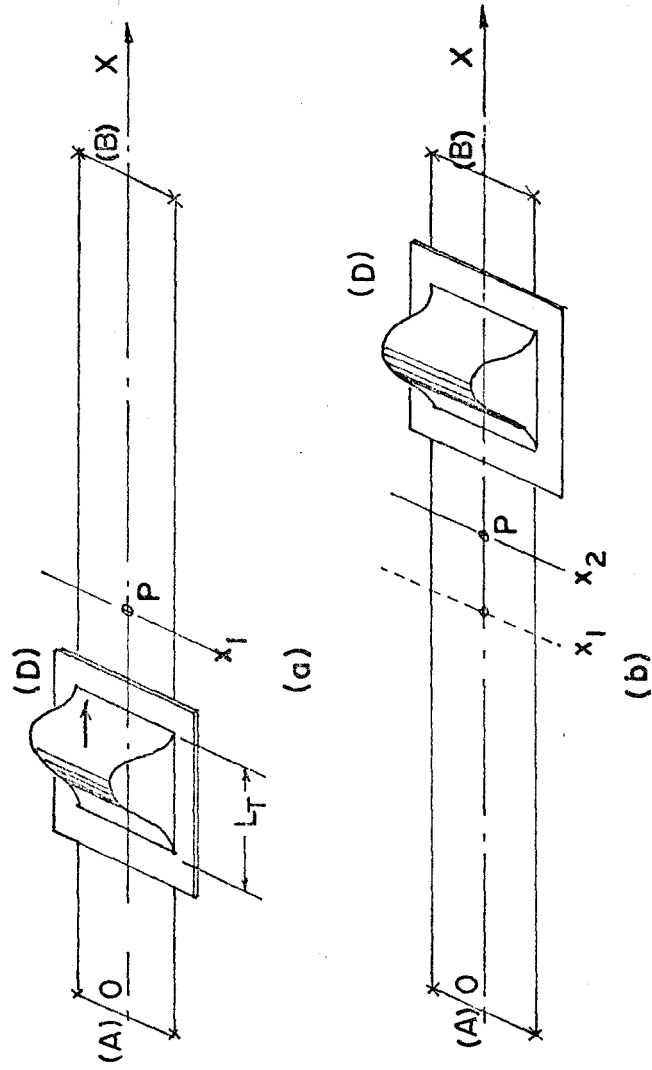


Fig. 6. Displacement of a point after a loop passing through it,  
 (a) Before, (b) After

### 3.2 Rolling Loop Projector Mechanism

The machine described below is a prototype built for testing the ability of the mechanism to project a 70 mm. film<sup>1</sup> with a frame size having an area three times greater than that of the conventional 70 mm. film.

It consists of a rotating drum (A), called the rotor, which has three equally spaced slots (Fig. 7a). Convex shaped shoes (B) of low friction material are fitted to each end of these slots. On each trailing shoe there is a series of air jets whose action is controlled by a cam to reduce the original impact force of the shoe on the film body.

The rotor revolves within a fixed stator (C) (Fig. 7b) with ample clearance between it and the stator so that the film can slide in between without pinching. The inside surface of the stator is provided with a series of air jet holes to produce an air cushion effect between the film and the stator. Film input and output sprockets ( $D_1, D_2$ ) are located on opposite sides of the stator. These are geared to the rotor at a gear ratio which permits the film to be put in and taken out at a constant linear velocity less than that of the rotor periphery.

---

<sup>1</sup> The early model invented in 1966 by P.R. Jones was designed for 35 mm. film and it is reported that this projector was run satisfactorily at a rate of 300 frames per second [10].

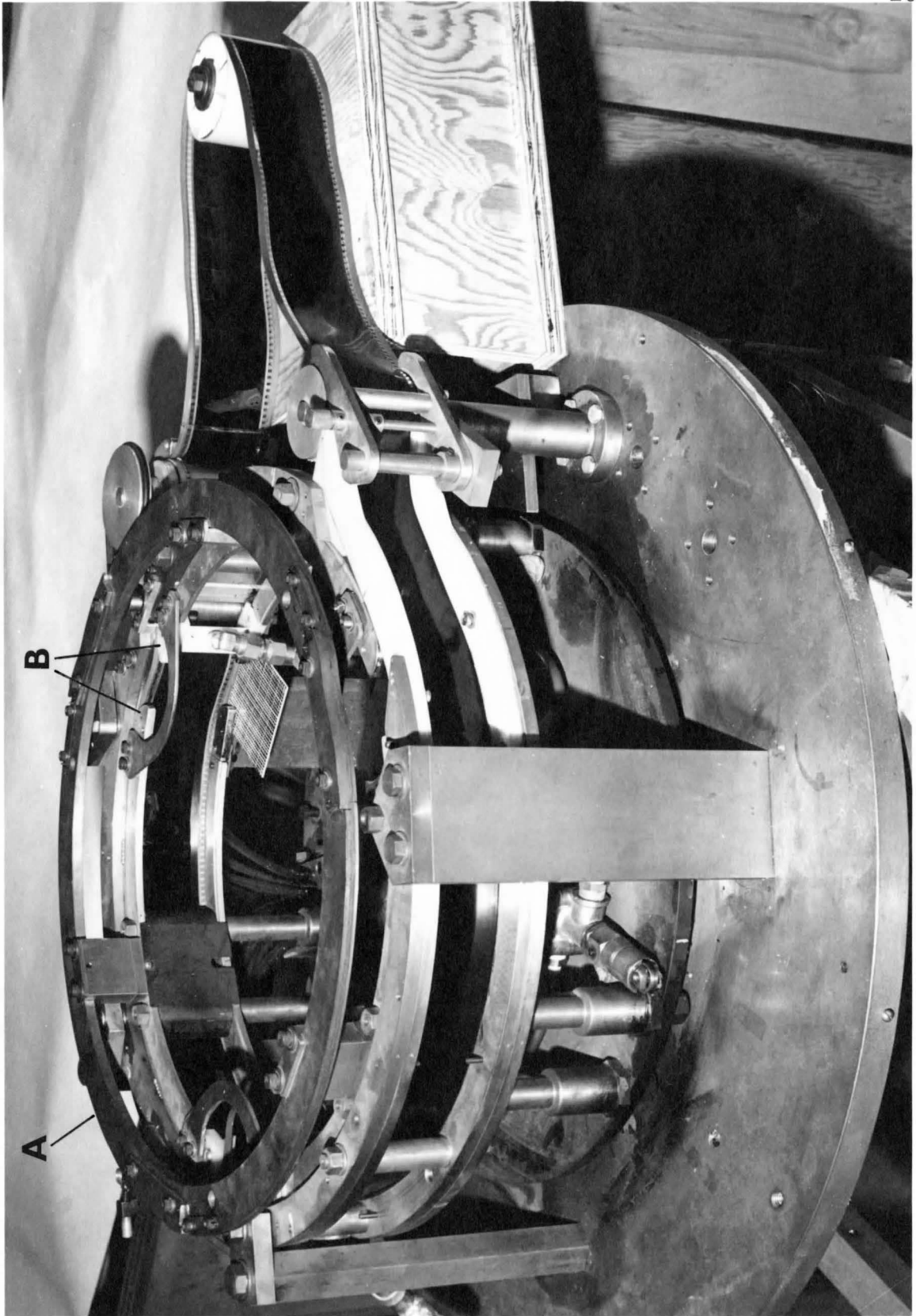


FIGURE 7a. Side view of the Rolling Loop Projector

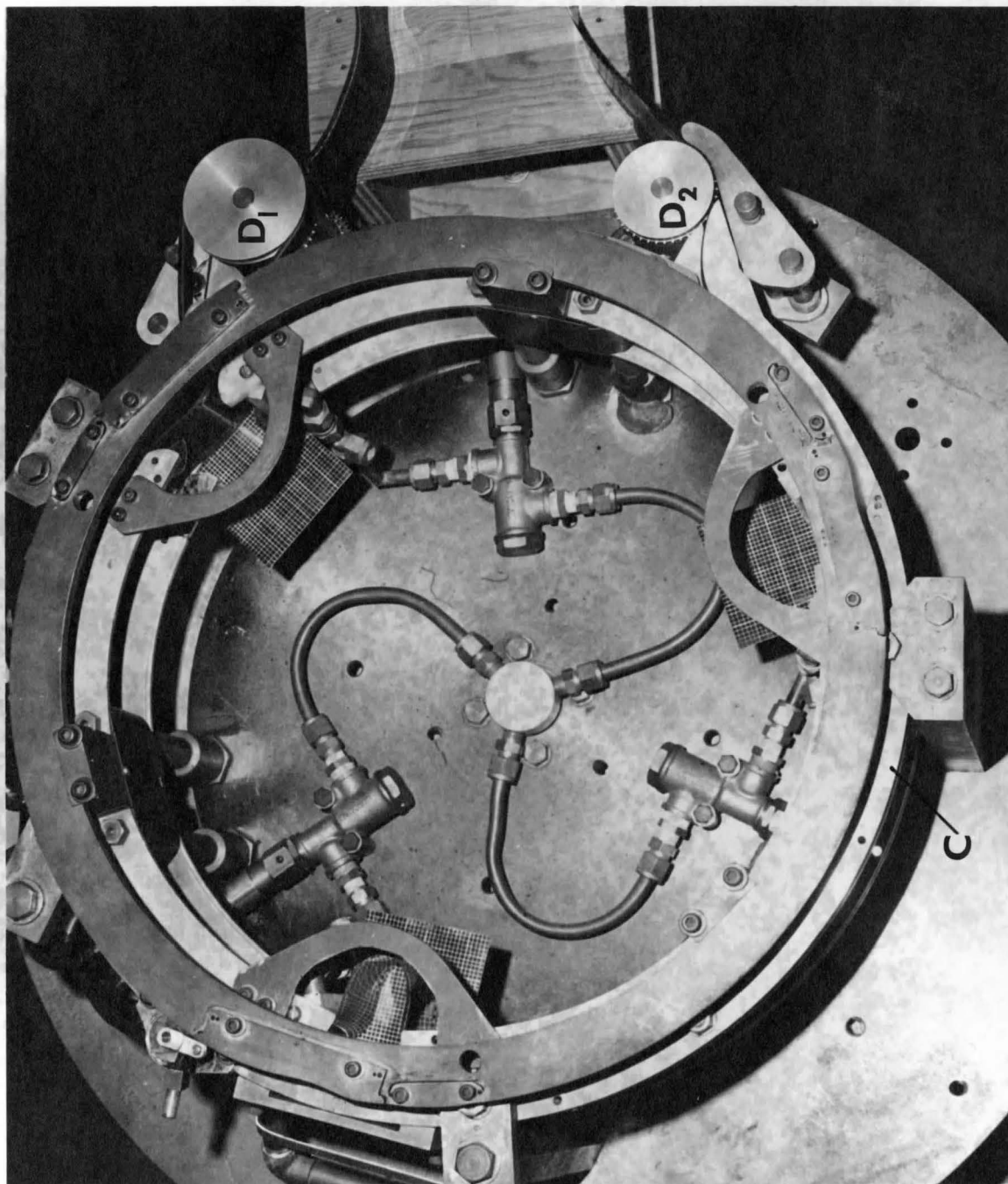


FIGURE 7b. Plan view of the Rolling Loop Projector

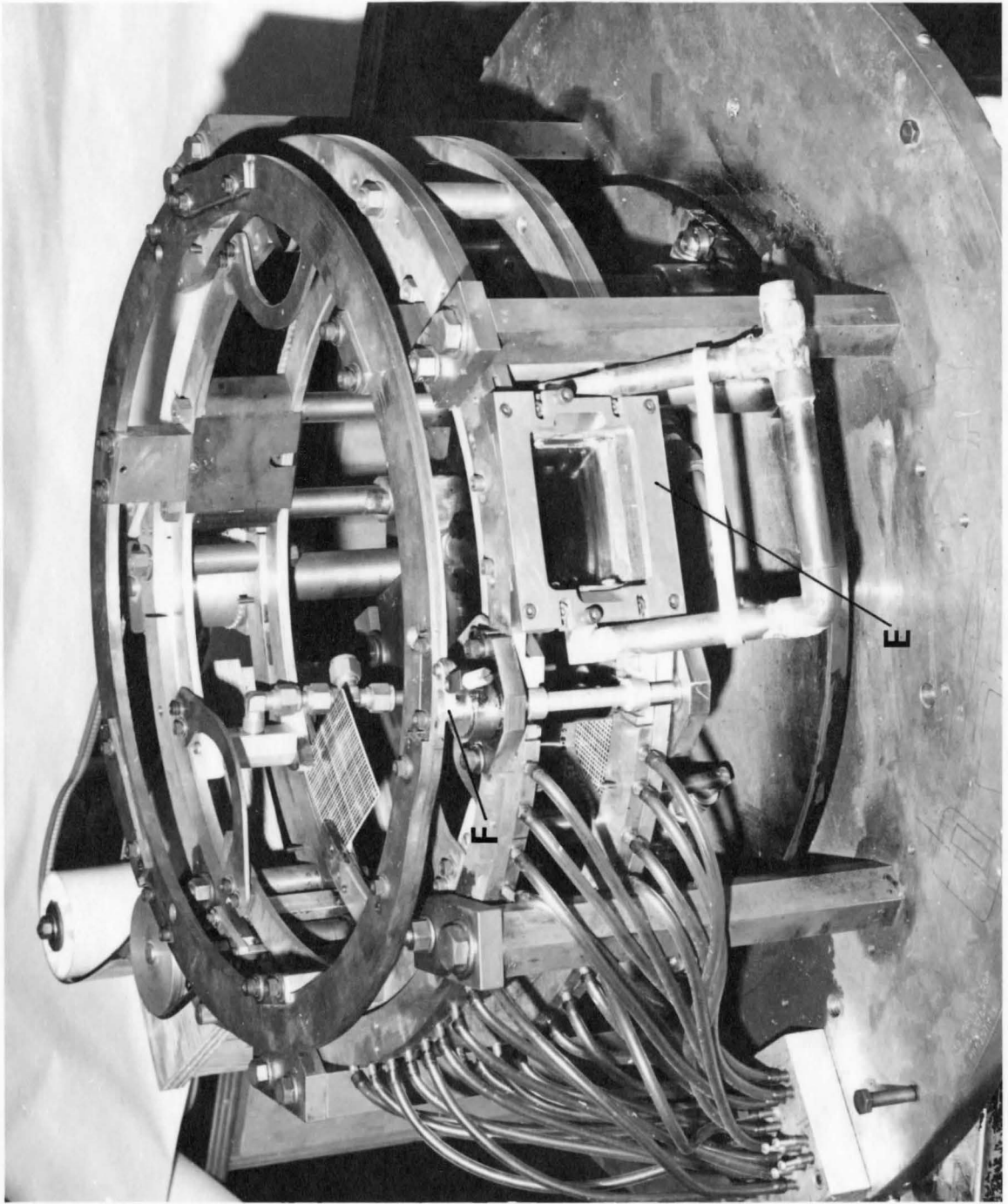


FIGURE 7c. Front view of the Rolling Loop Projector

The stator is equipped with an aperture plate (E) (Fig. 7c) which has two pairs of registration pins at its inside surface to secure the film in place during projection. Close to the aperture plate is a pair of deceleration pins which are activated by a cam unit (F), geared to the motion of the rotor. These deceleration pins are designed to slow the film gradually and to reduce the deceleration force imparted to the film by the register pins as the film is laid on the aperture plate.

### 3.3 Film Movement

The film motion in this projector is unique in the sense that it does not involve acceleration and deceleration in a linear fashion (i.e., along the film body) as in the case of intermittent movement. The process of film movement can be described by observing the consecutive phases in one frame cycle.

In Figure 8a, the film slack is introduced when the projector is threaded and is maintained by constant input - output velocities. This excess film length is accommodated in the rotor gaps. The portion of the film between gap (1) and gap (2) is stationary at the moment considered with respect to the stator, since the film is held fixed by the deceleration and the registration pins. The impact force produced by the initial contact of the rotor shoe and the film is reduced by the action of the air jets provided at the



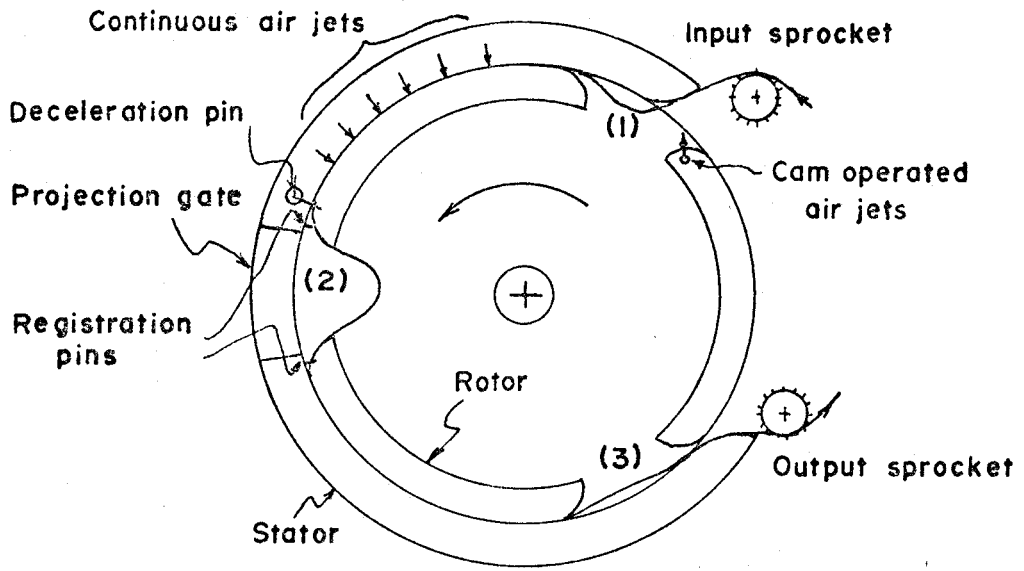


Fig. 8 a.

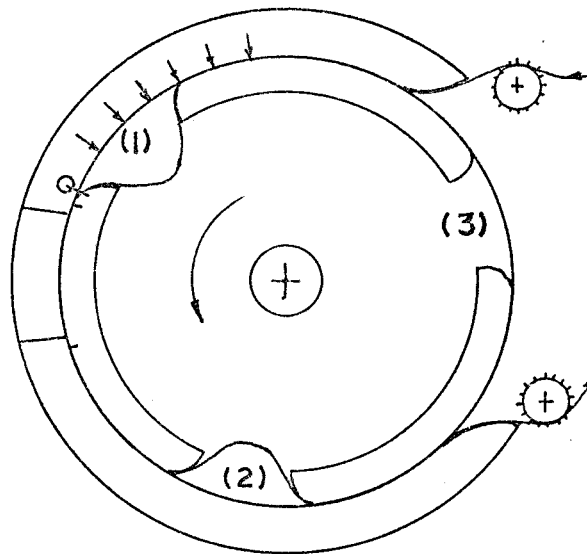


Fig. 8 b.

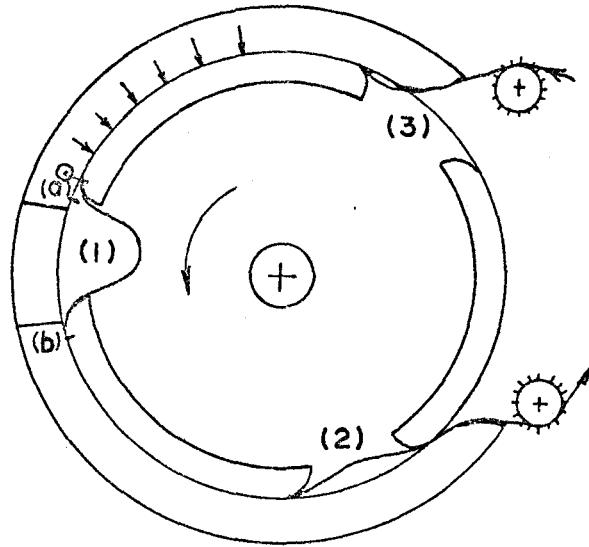


Fig. 8c.

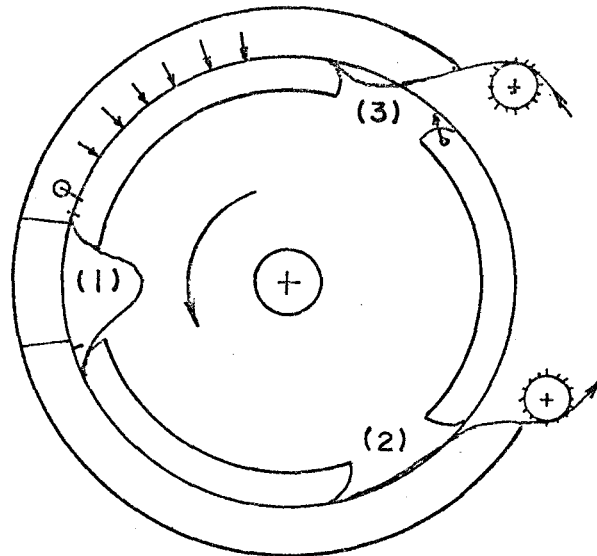


Fig.8d.

Figures 8 a, b, c, d. Film movement at various stages in a frame cycle.

edge of the trailing shoe. As the rotor advances to the position shown in Figure 8b, the loop in gap (1) keeps on increasing in size while the one in gap (2) is decreased. The portion of film in the aperture plate is now stationary and is ready for projection.

When the trailing shoe of gap (1) reaches the deceleration pins (Fig. 8c), these are activated by the cam and slow the film down to a stop. At this moment the loop in gap (1) grows to a full size and an amount of slack film starts building up at the succeeding gap, i.e., gap (3). Thereafter and for a very short period of time, the loop in gap (1) remains at constant height until the leading shoe of this gap leaves the forward registration pin (b).

In the final stage, after the forward portion of the loop in gap (1) has been released from the registration pins (Fig. 8d), the loop in gap (2) "decays" on passage to the exit point and disappears before the film is pulled into the output sprocket. The whole process is then repeated.

In summary, there are three phases of loop movement in the Rolling Loop Projector:

- a) Loop growth with a constant film infeed.
- b) Loop at constant size.
- c) Loop decreasing with a constant film output.

Generally, phase (a) and phase (c) are of the same category and phase (b) is just a particular case of phase (a) where there is no film infeed. Therefore, only the analysis of loop growth will be considered in the following Chapters.

## 4. STATIC LOOP ANALYSIS

Before analysing the film motion in the Rolling Loop Projector, it is necessary to define the loop shape and to investigate its characteristics under static conditions.

### 4.1 General

The following analysis is applied to the case of bending a flexible strip by clamping its two ends on the same plane. Since this problem involves large deflections on the strip, the approximation of the radius of curvature in the formula for the strain, namely:

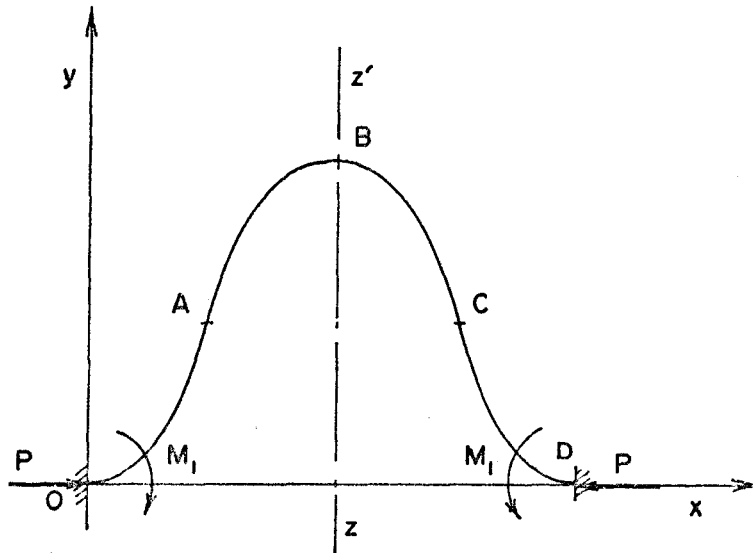
$$\frac{1}{\rho} = \frac{d^2y}{dx^2} / [1 + \left(\frac{dy}{dx}\right)^2]^{3/2} \approx \frac{d^2y}{dx^2}$$

for solving the non-linear differential equations governing the elastic curve of the bending strip is not justified. The approach attempted is based on the theory of an elastica [16] with the application of elliptic integrals and it was found to be quite suitable for numerical computation.

### 4.2 Governing Equations

Considering a flexible strip of a certain material of total length  $S_T$  which is held between two clamps on the same plane at a distance  $L_T$ , where  $S_T$  is greater than or

equal to  $L_T$ . The coordinate system is chosen as shown in the sketch.



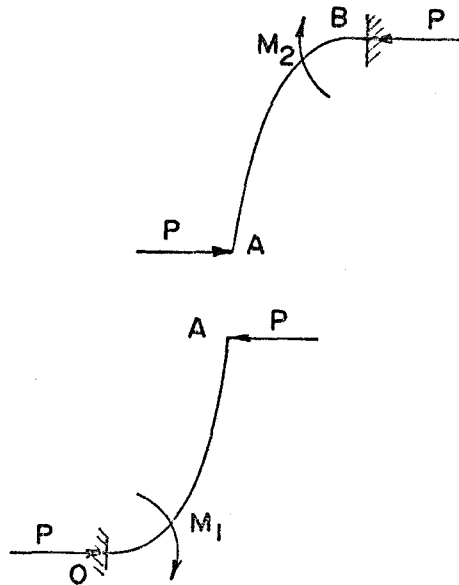
To simplify the problem, the following assumptions are made:

- a) Stress is proportional to strain.
- b) The strip is perfectly flexible and homogeneous with a constant cross-section area.
- c) Deflections are the same across the width of the strip.

Referring to the sketch, the supporting forces  $P$  and the resisting moments  $M_1$  acting at the two ends  $O$  and  $D$  are as shown. Since the end conditions of the strip are the same, the elastic curve of the strip is symmetrical with respect to  $ZZ'$  (parallel to  $oy$  and passing through point  $B$  on the top of the curve). Therefore, it is necessary to

consider only a half of the elastic curve, e.g., portion OB.

Suppose A is a point on portion OB where there is no bending moment but only shearing force exists, then this portion can be broken down to sections OA and AB on which free body diagrams can be drawn as indicated in the sketch.

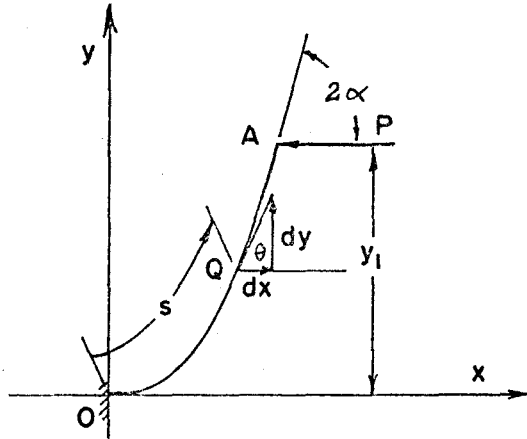


It can be seen that in these diagrams, the action and reaction forces at A and B are of the same magnitude  $P$  and parallel to  $x$ -axis. In addition, it can be shown that the bending moments,  $M_1$  at O and  $M_2$  at B are of the same magnitude but in different directions. Consequently, since all end conditions of the two sections, OA and AB are the same, the curve OAB must be symmetrical with respect to point A.

#### 4.2.1 Curvilinear Coordinates

Consider a point Q on the section OA (shown in the

sketch) with coordinates  $(x,y)$  with respect to coordinate



system  $(oxy)$ . Then, the bending moment at point Q with respect to A is

$$M = P(y_1 - y)$$

where  $y_1$  is the ordinate of point A. But, according to the flexure formula, this bending moment is given by

$$M = E'I \frac{d\theta}{ds}$$

where

$E'$  = Modulus of elasticity of the material

$I$  = Moment of inertia

$\theta$  = Angle of the tangent to the curve at Q with respect to the x-axis.

$s$  = Curvilinear coordinate of Q measured from O.

Then

$$E'I \frac{d\theta}{ds} = P(y_1 - y).$$

Differentiate the above equation with respect to S

$$E'I \frac{d^2\theta}{ds^2} = -P \frac{dy}{ds}.$$

Let

$$c^2 = \frac{P}{E'I} \quad 4-211$$

then the above equation reduces to

$$\frac{d^2\theta}{ds^2} = -c^2 \frac{dy}{ds}$$

or

$$\frac{d^2\theta}{ds^2} = -c^2 \sin \theta \quad 4-212$$

(Since  $\frac{dy}{ds} = \sin \theta$ ).

End conditions require:

At point O  $S=0$ ,  $\theta=0$

At point A  $S=S_1$ ,  $\theta=2\alpha$  and  $\frac{d\theta}{ds} = 0$ , 4-213

where  $2\alpha$  is the maximum angle of  $\theta$  at A.

Multiply the right and left hand sides of 4-212 by  $2d\theta$  and  $2 \frac{d\theta}{ds} ds$ , respectively, and after



reduction

$$d\left(\frac{d\theta}{ds}\right)^2 = -2C^2 \sin \theta d\theta,$$

or

$$d\left(\frac{d\theta}{ds}\right)^2 = 2C^2 d(\cos \theta).$$

Integrating the above equation and noting the end conditions given in 4-213

$$\int_0^{\frac{d\theta}{ds}} d\left(\frac{d\theta}{ds}\right)^2 = 2C^2 \int_{2\alpha}^{\theta} d(\cos \theta)$$

or

$$\frac{d\theta}{ds} = \pm \sqrt{2C} \sqrt{\cos \theta - \cos 2\alpha} \quad 4-214$$

Since  $\theta$  increases with the increasing of  $s$ , a positive sign must be taken. After separating the variables, one gets

$$ds = \frac{d\theta}{\sqrt{2C} \sqrt{\cos \theta - \cos 2\alpha}}. \quad 4-215$$

By changing  $\theta$  to an integratable form (see Appendix A), 4-215 becomes

$$ds = \frac{1}{C} \frac{d\psi}{\sqrt{1 - \sin^2 \alpha \sin^2 \psi}} \quad 4-216$$

where

$$\psi = \sin^{-1} \left( \frac{\sin \frac{\theta}{2}}{\sin \alpha} \right). \quad 4-216a$$

Note that:

$$\left. \begin{array}{l} \text{when } \theta = 0 \quad \psi = 0, \text{ and} \\ \theta = \theta \quad \psi = \sin^{-1} \left( \frac{\sin \frac{\theta}{2}}{\sin \alpha} \right) = \phi \end{array} \right\} 4-217$$

Integrate the two sides of equation 4-216 with a provision for the conditions in 4-217:

$$S = \frac{1}{C} \int_0^{\phi} \frac{d\psi}{\sqrt{1 - \sin^2 \alpha \sin^2 \psi}}$$

Let

$$K = \sin \alpha$$

Then

$$S = \frac{1}{C} \int_0^{\phi} \frac{d\psi}{\sqrt{1 - K^2 \sin^2 \psi}}$$

this equation has the form of an elliptic integral of the first kind and it is denoted by

$$F(K, \phi) = \int_0^{\phi} \frac{d\psi}{\sqrt{1 - K^2 \sin^2 \psi}}$$

Then, the relation for S can be written as

$$S = \frac{1}{C} F(K, \phi) \quad 4-218$$

where  $0 \leq \phi \leq \frac{\pi}{2}$  (by the conditions given in 4-217).

#### 4.2.2 Cartesian Coordinates

For a small element on the curve OA, one can write

$$dy = dS \sin \theta.$$

or

$$dy = dS \cdot 2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}$$

But, according to equation 4-216a

$$\sin \psi = \frac{\sin \frac{\theta}{2}}{\sin \alpha}$$

or

$$\sin \frac{\theta}{2} = K \sin \psi$$

then

$$dy = 2 \cdot K \, dS \sin \psi \cdot \sqrt{1 - K^2 \sin^2 \psi} .$$

Substitute the value of  $dS$  by relation 4-216 in the above, the expression for  $dy$  reduces to

$$dy = \frac{2K}{C} \sin \psi \, d\psi .$$

Integrate both sides:

$$\int_0^y dy = \frac{2K}{C} \int_0^\phi \sin \psi \, d\psi$$

or

$$y = \frac{2K}{C} (1 - \cos \phi) \quad 4-221$$

In a similar way, the expression for  $x$  is developed.

$$dx = dS \cos \theta$$

or

$$dx = dS (1 - 2\sin^2 \frac{\theta}{2})$$

$$(\text{since } \cos \theta = 1 - 2\sin^2 \frac{\theta}{2})$$

On replacing  $\sin \frac{\theta}{2}$  by  $K \sin \psi$ :

$$dx = (1 - 2K^2 \sin^2 \psi) dS$$

or

$$dx = [2(1 - K^2 \sin^2 \psi) - 1] dS .$$

Substituting  $ds$  by the relation 4-216:

$$dx = \frac{1}{C} \left[ 2 \sqrt{1 - K^2 \sin^2 \psi} d\psi - \frac{d\psi}{\sqrt{1 - K^2 \sin^2 \psi}} \right]$$

Integrating:

$$\int_0^x dx = \frac{1}{C} \left[ 2 \int_0^\phi \sqrt{1 - K^2 \sin^2 \psi} d\psi - \int_0^\phi \frac{d\psi}{\sqrt{1 - K^2 \sin^2 \psi}} \right]$$

The elliptic integral of the second kind is defined as

$$E(K, \phi) = \int_0^\phi \sqrt{1 - K^2 \sin^2 \psi} d\psi$$

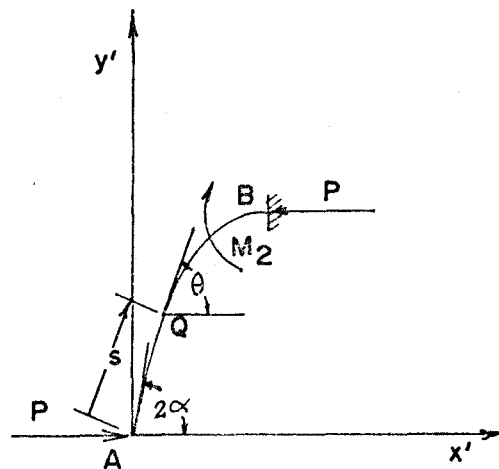
By using the notation for the elliptic integrals, the expression for  $x$  becomes

$$x = \frac{1}{C} [2E(K, \phi) - F(K, \phi)] \quad 4-222$$

### 4.3 Other Relations

In a similar manner, one can derive the equations related to the rest of the elastic curve of the strip.

Consider section AB with a coordinate system  $Ax'y'$  as shown in the sketch.



In referring to equation 4-214, a minus sign must be taken in this case since  $\theta$  decreases with the increasing of  $S$ .

Then, the expression for  $dS$  is

$$dS = - \frac{d\theta}{\sqrt{2} C \sqrt{\cos\theta - \cos 2\alpha}}$$

After changing the variable of integration:

$$dS = - \frac{1}{C} \frac{d\psi}{\sqrt{1-K^2 \sin^2 \psi}}$$

For the limit of integration,  $S$  varies from 0 to  $S$  and  $\theta$  changes from  $2\alpha$  to  $\theta$  (or by 4-216a  $\psi$  changes from  $\frac{\pi}{2}$  to  $\phi$ ), so

$$\int_0^S dS = - \frac{1}{C} \int_{\frac{\pi}{2}}^{\phi} \frac{d\psi}{\sqrt{1-K^2 \sin^2 \psi}}$$

This can be written as

$$S = \frac{1}{C} \left[ \int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1-K^2 \sin^2 \psi}} - \int_0^{\phi} \frac{d\psi}{\sqrt{1-K^2 \sin^2 \psi}} \right]$$

by using notation for the elliptic integrals:

$$S = \frac{1}{C} [F(K, \frac{\pi}{2}) - F(K, \phi)] \quad 4-31$$

where  $\phi$  varies from  $\frac{\pi}{2}$  to 0.

The expression for  $dy$  is now taken a minus sign, i.e.,

$$dy = - dS \sin\theta,$$

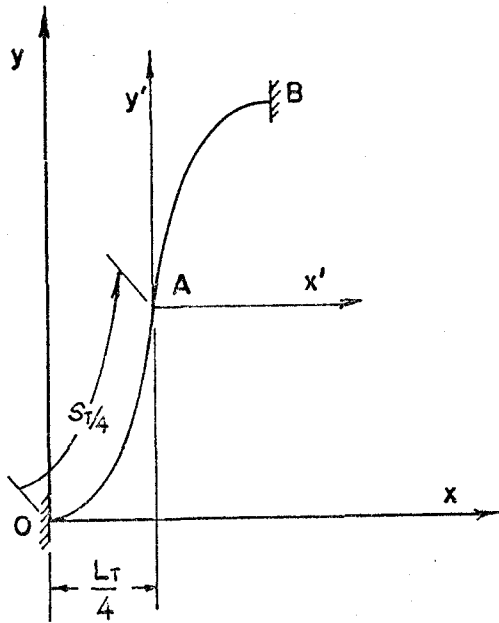
and the limit of integration for  $\psi$  is from  $\frac{\pi}{2}$  to  $\phi$ ; therefore, one can derive the expression for  $y$  as

$$y = \frac{2K}{C} \cos\phi . \quad 4-32$$

In the same fashion and noting the limits of integration, the expression for  $x$  can be obtained:

$$x = \frac{1}{C} [2\{E(K, \frac{\pi}{2}) - E(K, \phi)\} - \{F(K, \frac{\pi}{2}) - F(K, \phi)\}] \quad 4-33$$

After a transformation of coordinate system ( $Ax'y'$ ) to ( $oxy$ ) (see sketch) relations 4-31 to 4-33 become



$$s = \frac{S_T}{4} + \frac{1}{C} [F(K, \frac{\pi}{2}) - F(K, \phi)]$$

or

$$s = \frac{S_T}{2} - \frac{F(K, \phi)}{C},$$

(since  $\frac{F(K, \frac{\pi}{2})}{C} = \frac{S_T}{4}$ , by 4-218);

$$x = \frac{L_T}{4} + \frac{1}{C} [2\{E(K, \frac{\pi}{2}) - E(K, \phi)\} - \{F(K, \frac{\pi}{2}) - F(K, \phi)\}]$$

or

$$x = \frac{L_T}{2} - \frac{1}{C} [2E(K, \phi) - F(K, \phi)]$$

(Since  $\frac{1}{C} \{2E(K, \frac{\pi}{2}) - F(K, \frac{\pi}{2})\} = \frac{L_T}{4}$  ; and

$$y = y_1 + \frac{2K}{C} \cos \phi,$$

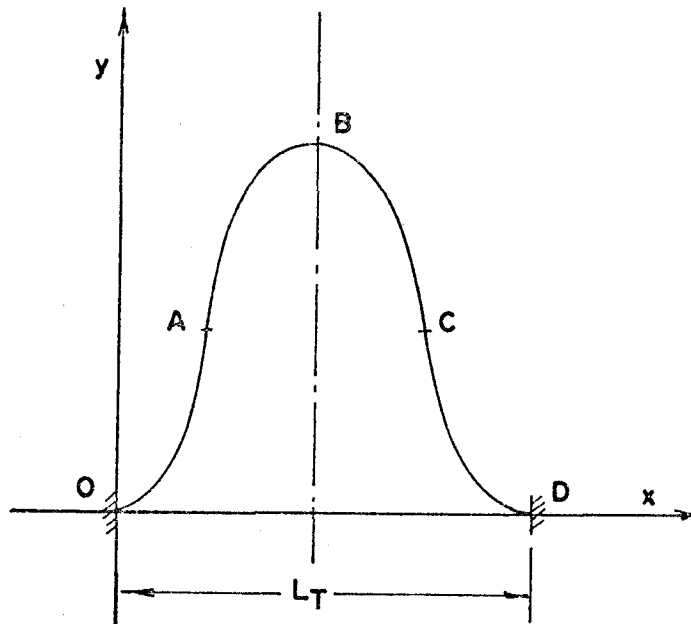
where  $y_1$  is the ordinate of point A and is given by equation 4-221 when  $\phi = \frac{\pi}{2}$ , i.e.,

$$y_1 = \frac{2K}{C}$$

so

$$y = \frac{2K}{C} (1 + \cos \phi).$$

The coordinates of a point on sections BC and CD (see sketch)



can be found in a similar way, and the final results are:

For section BC:

$$S = \frac{S_T}{2} + \frac{1}{C} F(K, \phi),$$

$$x = \frac{L_T}{2} + \frac{1}{C} [2E(K, \phi) - F(K, \phi)], \text{ and,}$$

$$y = \frac{2K}{C} (1 + \cos \phi),$$

where  $\phi$  varies from 0 to  $\frac{\pi}{2}$ .

For section CD:

$$S = S_T - \frac{1}{C} F(K, \phi),$$

$$x = L_T - \frac{1}{C} [2E(K, \phi) - F(K, \phi)],$$

$$y = \frac{2K}{C} (1 - \cos \phi),$$

where  $\phi$  varies from  $\frac{\pi}{2}$  to 0.

### Summary:

Let:

Subscripts A, B, C and D denote segments OA, AB, BC and CD respectively,

$$F = F(K, \phi) \text{ and,}$$

$$E = E(K, \phi).$$

The elastic curve of the bending strip can be described by the following equations:

$$S_A = \frac{F_A}{C} \tag{4-34a}$$

$$x_A = \frac{2E_A - F_A}{C} \tag{4-34b}$$



$$y_A = \frac{2K}{C} [1 - \cos\phi_A] . \quad 4-34c$$

$$S_B = \frac{S_T}{2} - \frac{F_B}{C} \quad 4-35a$$

$$x_B = \frac{L_T}{2} - \frac{1}{C} [2E_B - F_B] \quad 4-35b$$

$$y_B = \frac{2K}{C} [1 + \cos\phi_B] . \quad 4-35c$$

$$S_C = \frac{S_T}{2} + \frac{F_C}{C} \quad 4-36a$$

$$x_C = \frac{L_T}{2} + \frac{1}{C} [2E_C - F_C] \quad 4-36b$$

$$y_C = \frac{2K}{C} [1 + \cos\phi_C] . \quad 4-36c$$

$$S_D = S_T - \frac{F_D}{C} \quad 4-37a$$

$$x_D = L_T - \frac{1}{C} [2E_D - F_D] \quad 4-37b$$

$$y_D = \frac{2K}{C} [1 - \cos\phi_D] . \quad 4-37c$$

where  $\phi_A$  and  $\phi_C$  varies from 0 to  $\frac{\pi}{2}$  and  $\phi_B$  and  $\phi_D$  changes from  $\frac{\pi}{2}$  to 0.

#### 4.4 Loop Characteristics

The characteristics of the strip held between two coplanar clamps will be defined implicitly in terms of loop configuration parameter, i.e., ratio of loop length to distance between two clamped ends. The relations derived below are concerned with the angle of a tangent to the elastic curve at inflection point, the supporting forces, the loop height and the resisting moments at the clamped ends.

#### 4.4.1 Parameter K

If the parameter  $K$  (which is equal to  $\sin \alpha$ ) of the elliptic integral is known, the elastic curve of the bending strip can be defined by using relations from 4-34a to 4-37c.

When  $\phi_A = \frac{\pi}{2}$ , equations 4-34a and 4-34b become

$$S_A = \frac{S_T}{4} = \frac{F(K, \frac{\pi}{2})}{C} \quad 4-411$$

$$x_A = \frac{L_T}{4} = \frac{1}{C} [2E(K, \frac{\pi}{2}) - F(K, \frac{\pi}{2})] \quad 4-412$$

Dividing each side of equation 4-412 by the corresponding side of 4-411 to obtain

$$\frac{L_T}{S_T} = \frac{2E(K, \frac{\pi}{2}) - F(K, \frac{\pi}{2})}{F(K, \frac{\pi}{2})}$$

In rearranging, the above equation becomes

$$\frac{E(K, \frac{\pi}{2})}{F(K, \frac{\pi}{2})} = \frac{1}{2} \left( \frac{L_T}{S_T} + 1 \right)$$

Let

$$R = \frac{S_T}{L_T},$$

$$E_O = E(K, \frac{\pi}{2}),$$

$$F_O = F(K, \frac{\pi}{2}).$$

Then the above equation can be written as

$$\frac{E_O}{F_O} = \frac{1}{2} \left( \frac{1}{R} + 1 \right) \quad 4-413$$

If  $R$  is known,  $K$  can be found by trial and error since the ratio of  $E_o$  to  $F_o$  is a unique function of  $K$ .

#### 4.4.2 Supporting Force

From equation 4-411 the expression for  $C$  is derived as

$$C = \frac{4F(K, \frac{\pi}{2})}{S_T} \quad 4-421$$

But by equation 4-211

$$C^2 = \frac{P}{E'I}$$

or

$$P = C^2 E'I$$

Substitute the value of  $C$  by the expression 4-421 in the above equation, then

$$P = \frac{F^2(K, \frac{\pi}{2}) E'I}{\left(\frac{S_T}{4}\right)^2} \quad 4-422$$

In examining this relation, it is seen that  $P$  is maximum when  $F(K, \frac{\pi}{2})$  is maximum and  $S_T$  is minimum. This situation occurs when  $S_T = L_T$ .

Then

$$K = 0$$

and

$$F(K, \frac{\pi}{2}) = \frac{\pi}{2}$$

So, the maximum supporting force is

$$P_C = \frac{\left(\frac{\pi}{2}\right)^2 E'I}{\left(\frac{L_T}{4}\right)^2}$$

or

$$P_C = \frac{4\pi^2 EI'}{L_T^2},$$

this is the Euler critical load for buckling of a strut of length  $L_T$  and having moment of inertia  $I$  and modulus of elasticity  $E'$ . The force required to hold the strip at the two ends can be written in a dimensionless form as

$$\frac{P}{P_C} = 4 \frac{F^2(K, \frac{\pi}{2})}{\pi^2} \left(\frac{L_T}{S_T}\right)^2$$

or by using short notation:

$$\frac{P}{P_C} = \frac{4F_o^2}{\pi^2 R^2}$$

4-423

#### 4.4.3 Loop Height

When  $\phi_B = 0$ , equation 4-35c becomes

$$y_L = \frac{4K}{C}$$

where  $y_L$  is the loop height.

Replacing the value of  $C$  in this expression by relation 4-421:

$$y_L = \frac{4KS_T}{4F(K, \frac{\pi}{2})}$$

or

$$y_L = \frac{KS_T}{F(K, \frac{\pi}{2})}$$

Dividing the two sides by  $L_T$ :

$$\frac{y_L}{L_T} = \frac{K}{F(K, \frac{\pi}{2})} \left(\frac{S_T}{L_T}\right)$$

or by using short notation:

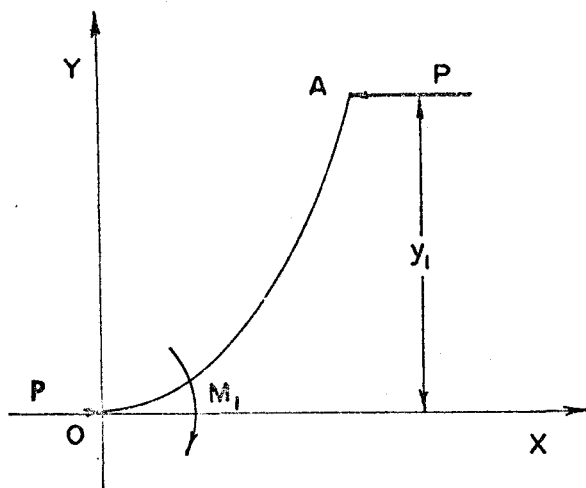
$$\frac{y_L}{L_T} = \frac{KR}{F_O}$$

4-431

#### 4.4.4 Maximum Bending Moments

The bending moment at a point on the strip is proportionally inverse to the radius of curvature and its maxima occur at the two clamped ends and also at the top of the elastic curve where the radius of curvature is the smallest.

In referring to the free body diagram shown in the sketch, the maximum bending moment is



$$M_1 = P \times y_1$$

where

$$y_1 = \frac{2K}{C} \quad (\text{from eq. 4-221 with } \phi = \frac{\pi}{2}).$$

Then

$$M_1 = \frac{2KP}{C}$$

Since from equations 4-421 and 4-422:

$$C = \frac{4 F(K, \frac{\pi}{2})}{S_T}, \text{ and}$$

$$P = \frac{F^2(K, \frac{\pi}{2}) E' I}{\left(\frac{S_T}{4}\right)^2}$$

On substitution and after some reduction, the value of  $M_1$  becomes

$$M_1 = \frac{8K F(K, \frac{\pi}{2}) E' I}{S_T}$$

or in dimensionless form:

$$\frac{M_1 S_T}{E' I} = 8K F(K, \frac{\pi}{2}) \quad 4-441$$

Let  $B$  be the flexural rigidity per unit length of the material, i.e.,

$$B = \frac{E' I}{S_T}$$

Then, 4-441 becomes

$$\frac{M_1}{B} = 8 K F_O \quad 4-442$$

## 5. ROLLING LOOP SIMULATION

### 5.1 General

After the theoretical loop shape has been defined, a numerical technique can be used to simulate the motion of the film in the Rolling Loop Projector from the time the loop starts to grow until it reaches the aperture plate. This type of motion is the combination of a rolling action by virtue of loop displacement and an increase in loop size due to a constant feed at one end.

The objective here is to investigate the paths of certain points on the film loop and to calculate the velocity and acceleration involved in the motion.

### 5.2 Theoretical Model

Consider a strip of film length  $S_0$  which is originally lying on a flat plane. One of its ends is fixed at (O) and the other end (M) is moved toward (O) at a constant velocity  $V_F$  as shown in the sketch. A slider which is essentially a

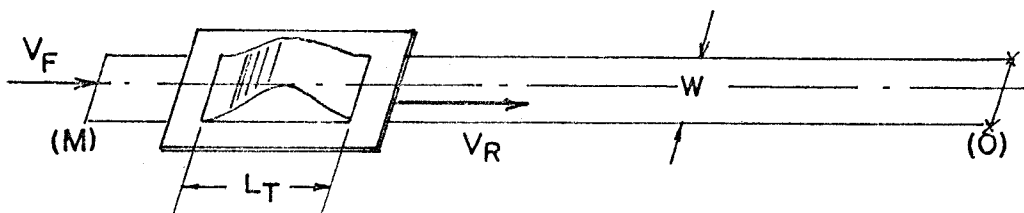


plate with a rectangular gap having dimensions  $L_T$  by  $W$  (where  $W$  is the film width) moves in the same direction as the end (M) with a velocity  $V_R$ . As this slider travels, it traps an amount of film in the gap and thus pushes this section of the film so that the latter moves at the same velocity  $V_R$ . The rest of the film outside the slider gap remains flat.

At any time  $t$ , the rate of film entering the gap is  $V_R$  and the rate of film leaving is  $(V_R - V_F)$ . Thus, the rate of film growth in the gap is

$$V_R - (V_R - V_F) = V_F.$$

Therefore, the amount of excess film in the gap at time  $t$  is

$$E_L = V_F \times t. \quad 5-21$$

In order to simplify the computation the following assumptions are made:

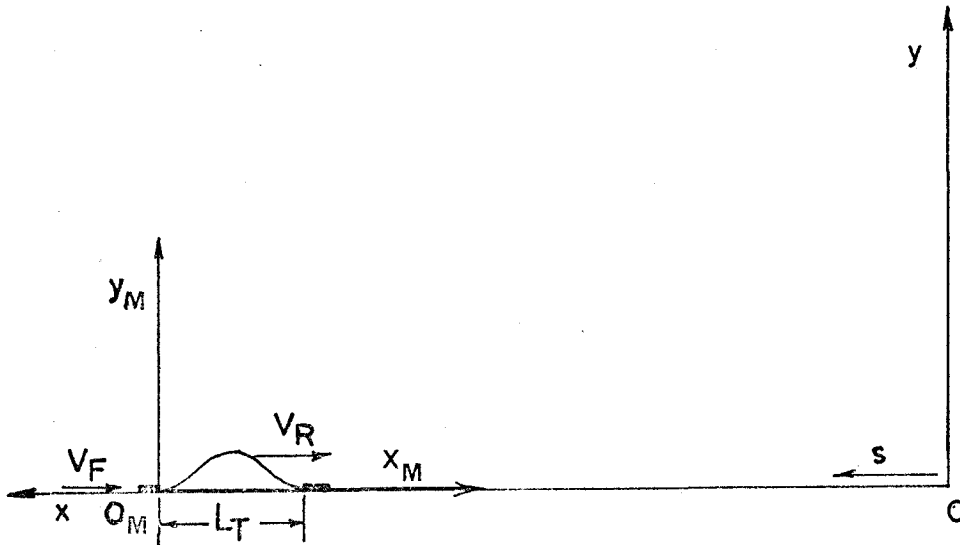
- a) There is no friction involved.
- b) Air resistance is negligible.
- c) Film mass per unit length is small enough that the linear momentum of the loop would not affect the loop shape.

When these conditions are satisfied, the equations defining the loop shape in Chapter 4 can be used in the loop movement simulation.



### 5.3 Selection of Coordinate Systems

A fixed coordinate system  $oxy$  is placed at  $o$  with  $ox$  pointed opposite to the direction of motion of the slider (see sketch).



To be able to apply formulas in Chapter 4 in defining the position of a point on the loop, a moving Cartesian coordinate system  $(o_M x_M y_M)$  is set up attached to the trailing edge of the slider and moved with the slider.

A point on the film strip is identified by a curvilinear coordinate  $S_p$  measured from the origin  $o$ . The position of this point is defined with respect to  $(o_M x_M y_M)$  by the utilization of the formulas derived in Chapter 4 or it is calculated directly with respect to system  $(oxy)$  depending on the location of this point on the film strip. If the coordinates  $(x_M, y_M)$  were sought, they would be transferred to system  $(oxy)$  by a method described below.

Let  $S_M, x_M, y_M$  be the curvilinear and Cartesian coordinates of a point Q on the loop with respect to  $(o_M, x_M, y_M)$  system and  $S_C, x_C, y_C$  be the coordinates of the moving origin  $o_M$  with respect to  $(oxy)$ , then Q is defined in the fixed coordinate system  $(oxy)$  as

$$S = S_C + S_M \quad 5-31a$$

$$x = x_C + y_M \quad 5-31b$$

$$y = y_C + y_M \quad 5-31c$$

where

$$S_C = S_O - (V_R - V_F)t \quad 5-32a$$

$$x_C = X_O - V_R t \quad 5-32b$$

$$y_C = 0 \quad 5-32b$$

( $S_O, X_O$  are values of  $S_C$  and  $X_C$  respectively at time  $t = 0$ ).

#### 5.4 Method of Solution

In order to portray the motion of the loop at various instants, a finite number of points along the film strip is taken and, as mentioned earlier, each point is identified by a curvilinear coordinate  $S_P$  measured from O. Also, the time for the slider to move from one end of the strip to another - that is equivalent to one frame cycle in the Rolling Loop Projector - is divided into convenient time intervals.

At time  $t$ , the coordinates of the moving origin  $O_M$  are

calculated from relations 5-32a and 5-32b, i.e.,

$$S_C = S_O - (V_R - V_F)t$$

$$x_C = x_O - V_R T$$

The length of film inside the gap is deduced from the excess film length given by relation 5-21, i.e.,

$$S_T = E_L + L_T$$

Then

$$S_T = V_F \times t + L_T$$

From this relation the value R which is the ratio of loop length to gap width, can be deduced. Then, by using the expression 4-413, i.e.,

$$\frac{E_O}{F_O} = \frac{1}{2} \left( \frac{1}{R} + 1 \right) ,$$

the parameter K of the elliptic integral can be determined by a trial and error method<sup>1</sup>.

#### 5.4.1 Calculation of Point Paths

The next step is to find the relative position of the point concerned with respect to the location of the loop so that the coordinates of this point can be calculated.

---

<sup>1</sup>The values of the complete or incomplete elliptic integrals are tabulated in Reference [17] or they can be evaluated by calling ELI2, a library subroutine programme available on the Control Data Corp. computer model 6400).

accordingly.

- a. If this point is on the right hand side of the loop, i.e.,  $S_p < (S_c - S_T)$ , then the coordinates of this point are

$$\begin{aligned}x &= S_p \\y &= 0\end{aligned}$$

- b. If it is on the left hand side of the loop, i.e.,  $S_p > S_c$ , then

$$\begin{aligned}x &= x_o = V_F \times t \\y &= 0\end{aligned}$$

- c. If it is on the loop, i.e.,  $S_c - S_p < (S_c - S_T)$ .

Since there are four sets of equations corresponding to four parts of the loop, it is necessary to find out to which part of the loop the point belongs. Then, a trial value of angle  $\phi$  is substituted into the proper equation given in section 4.3 to calculate the coordinate  $S_M$ . The value of  $S_M$  is now compared with the curvilinear coordinate of this point measured from  $O_M$ , i.e.,  $(S_c - S_p)$ . If the difference is too large,  $\phi$  is given a certain increment and the procedure is repeated until the difference is within the preset limits.

When  $\phi$  is found, the coordinates  $x_M$  and  $y_M$  are evaluated with the help of the relations 4-34a to 4-37c. Then they are transformed to  $x, y$  by using relations 5-31b and 5-31c.

#### 5.4.2 Calculation of Velocity and Acceleration

Since the coordinates  $x$  and  $y$  cannot be described analytically as direct functions of time, it is impossible to obtain velocity and acceleration components of a point on the film strip from these relations by differentiating, therefore, a numerical approximation is used instead.

The average velocity components of a point at time  $t$  are calculated as

- Component in the x-direction:

$$V_{x,t} = (x_t - x_{t-1})/DT.$$

- Component in the y-direction:

$$V_{y,t} = (y_t - y_{t-1})/DT$$

where

$V$  denotes velocity,

$x,y$  subscripts for  $x$  and  $y$  directions respectively,

$t,t-1$  subscripts for time  $t$  and  $t-1$  respectively,

$DT$  is time increment.

The amplitude and angle of the average velocity vectors are

$$|V_t| = \sqrt{V_{x,t}^2 + V_{y,t}^2}$$

and

$$\phi_{V,t} = \text{Tan}^{-1} \frac{V_{y,t}}{V_{x,t}}.$$

In a similar way, the average acceleration vector of a point is defined.

Component in the x-direction:

$$A_{x,t} = (V_{x,t+1} - V_{x,t})/DT.$$

Component in the y-direction:

$$A_{y,t} = (V_{y,t+1} - V_{y,t})/DT.$$

Acceleration amplitude and angle are

$$|A_t| = \sqrt{A_{x,t}^2 + A_{y,t}^2}$$

$$\phi_{A,t} = \text{Tan}^{-1} \frac{A_{y,t}}{A_{x,t}}.$$

#### 5.4.3 Data Used in Computer Programmes

Computer programmes were developed to simulate the loop motion using data based on the Rolling Loop Projector prototype described in Chapter 3.

This projector has an 18 inch diameter rotor with three equally spaced gaps and the frame rate for normal projection is 24 frames per second; then, the peripheral velocity of the rotor is

$$V_R = \frac{18}{2} \times 2\pi \times \frac{24}{3} = 452.39 \text{ in./sec.}$$

The equivalent width of the rotor gap is 3 inches, or

$$L_T = 3 \text{ in.}$$

The film type used in this projector has 15 perforations per frame (with perforation pitch of .187 inch) giving a frame length of 2.805 inches and this should be the amount of excess film in the gap when the loop has grown to full size just before it is laid across the aperture. Thus, the film

feeding velocity is

$$V_F = 2.805 / \left(\frac{1}{24}\right) \text{ in./sec.}$$

or

$$V_F = 67.5 \text{ in./sec.}$$

The arc length of the stator along which one rotor gap travels in one frame cycle is

$$\frac{18 \times \pi}{3} \approx 18.8 \text{ inches.}$$

Therefore, the film length required to fill one gap as it proceeds past the film entry point and builds the loop to its full size plus one gap length is:

$$S_o = 18.8 + 3. + 2.805$$

$$S_o = 24.60 \text{ in.}$$

Let MP be the number of fixed points on this film strip with a distance of two perforation pitches apart,<sup>2</sup> then

$$\begin{aligned} \text{MP} &= 24.60 / (.187 \times 2) \\ &= 65.8 \end{aligned}$$

---

<sup>2</sup>Due to the limitation of the memory space in the computer this distance could not be reduced without exceeding the computer capacity. A typical programme with DS= .187×2 in. and  $DT = \frac{1}{24} \times \frac{1}{50}$  sec. requires a memory space of more than 60,000<sub>8</sub> computer words to compile and execute (maximum memory space of the CDC 6400 is 124,000<sub>8</sub> words).

For computational purposes MP was set equal to 66 and  $S_0$  was chosen equal to 25 inches. Also, a time interval of  $(\frac{1}{24} \times \frac{1}{50})$  sec. was arbitrarily selected in the analysis. Shorter time steps and smaller distances between fixed points were taken where greater accuracy was required. For instance, in the starting phase of loop growth, the time interval and the distance selected as

$$DT = \frac{1}{24} \times \frac{1}{500} \text{ sec.}$$

and

$$DS = \frac{.187}{5} \text{ in.} \quad (\text{MP} = 100)$$

And for the intermediate phase (e.g.,  $t = \frac{35}{50} \times \frac{1}{24}$  sec.),

$$DT = \frac{1}{24} \times \frac{1}{500} \text{ sec.}$$

and

$$DS = .187/3 \text{ in.} \quad (\text{MP} = 100)$$

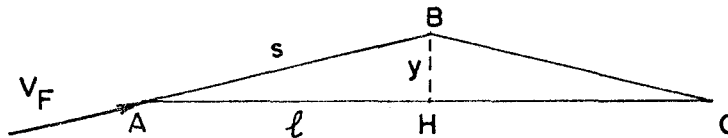
### 5.5 Velocity and Acceleration at the Early Stage of Loop Growth

At the start of loop growth and in a very short time when the film begins to lift off from a flat surface, the velocity and acceleration at the loop top exhibit a very distinctive behaviour that the numerical computation in the simulation fails to indicate. This characteristic can be predicted analytically by an approximation of the loop shape. On the other hand, the effect of film infeed at this stage is more



pronounced than that of the rotor motion as it will be seen later on; therefore, the analysis can be simplified by neglecting the latter.

Let AB and BC be the two sides of an isosceles triangle ABC which represents the approximation of the loop shape at this stage as shown in the sketch. From the geometry



of the loop, the height  $y$  of the top is given by

$$y = \sqrt{s^2 - l^2} \quad 5-51$$

where

$$s = AB$$

$$l = AC/2$$

By differentiating with respect to time and keeping  $l$  constant, relation 5-51 yields

$$\frac{dy}{dt} = (s^2 - l^2)^{-\frac{1}{2}} \frac{ds}{dt} \quad 5-52$$

Since the film is fed in at constant velocity,

$$\frac{ds}{dt} = \text{constant.}$$

Rewrite relation 5-52 in short notation:

$$\dot{y} = \frac{\dot{s}}{[1 - (\frac{l}{s})^2]^{\frac{1}{2}}} \quad 5-52a$$

To find the acceleration, differentiate the expression for velocity in relation 5-52a with respect to time:

$$\ddot{y} = (s^2 - \ell^2)^{-\frac{1}{2}} \frac{d(s\dot{s})}{dt} + s\dot{s} (s^2 - \ell^2)^{-3/2} s \frac{ds}{dt}.$$

but

$$\frac{d(s\dot{s})}{dt} = s\ddot{s} + \dot{s}^2$$

and since

$$\dot{s} = \text{constant}$$

$$\ddot{s} = 0$$

Thus

$$\ddot{y} = \dot{s}^2 (s^2 - \ell^2)^{-\frac{1}{2}} \left(1 - \frac{s^2}{s^2 - \ell^2}\right).$$

After rearranging:

$$\ddot{y} = s\dot{s}^2 \left[1 - \left(\frac{\ell}{s}\right)^2\right]^{-\frac{1}{2}} \left[1 - \frac{1}{1 - \left(\frac{\ell}{s}\right)^2}\right] \quad 5-53$$

When  $s$  is nearly equal to  $\ell$ , i.e.,

$$\ell = s - \epsilon$$

where  $\epsilon$  is a very small quantity.

Then, relation 5-52a becomes

$$\dot{y} = \frac{\dot{s}}{\left[1 - \left(\frac{s - \epsilon}{s}\right)^2\right]^{\frac{1}{2}}} \approx \frac{\dot{s}\sqrt{s}}{\sqrt{2\epsilon}} \quad 5-54$$

and the expression 5-53 yields

$$\ddot{y} = s\dot{s}^2 \left[1 - \left(1 - \frac{\epsilon}{s}\right)^2\right]^{-\frac{1}{2}} \left[1 - \frac{1}{1 - \left(1 - \frac{\epsilon}{s}\right)^2}\right]$$

In rearranging terms and neglecting small quantities:

$$\ddot{y} \doteq \frac{s^{\frac{3}{2}}\dot{s}^2}{\sqrt{2\epsilon}} \left(1 - \frac{\epsilon}{2s}\right)$$

or

$$\ddot{y} \doteq - \frac{\frac{5}{2} \cdot 2}{(2\varepsilon)^{3/2}} \quad 5-55$$

When  $\varepsilon$  approaches zero, 5-54 and 5-55 become

$$\dot{y} \rightarrow + \infty$$

$$\ddot{y} \rightarrow - \infty$$

Therefore, at the beginning of the loop growth, the loop top theoretically moves at infinite upward velocity and is subject to infinitely large deceleration.

## 6. DISCUSSIONS OF THE RESULTS

Based on the equations derived and the simulation technique discussed in the previous Chapters, the computer programmes were developed using data taken from the Rolling Loop Projector prototype. The following are the discussions of some typical results obtained from these programmes.

### 6.1 Characteristics of a Film Loop on Flat Surface

In Figure 9, the loop shape parameters (i.e.,  $K$  and  $y_L$ ), the supporting force and maximum bending moment on the loop in dimensionless form are plotted against the loop configuration parameter, i.e., ratio  $S_T/L_T$ . This ratio is in the range of 1. to 3.8 and this seems to be quite sufficient for the practical application.

In general, the loop shape parameters and the bending moment vary as parabolic functions of the loop configuration parameter while the force curve is hyperbolic. It is noted that when the loop size is increasing, the sine of the angle at the inflection point of the loop and the loop height increases accordingly. When ratio  $\frac{S_T}{L_T}$  reaches a value of 2.14, the tangent to the loop at the inflection point is vertical, and from then on the loop height will increase linearly with respect to the increase of the loop length. On the other hand, the value of bending stress on the film

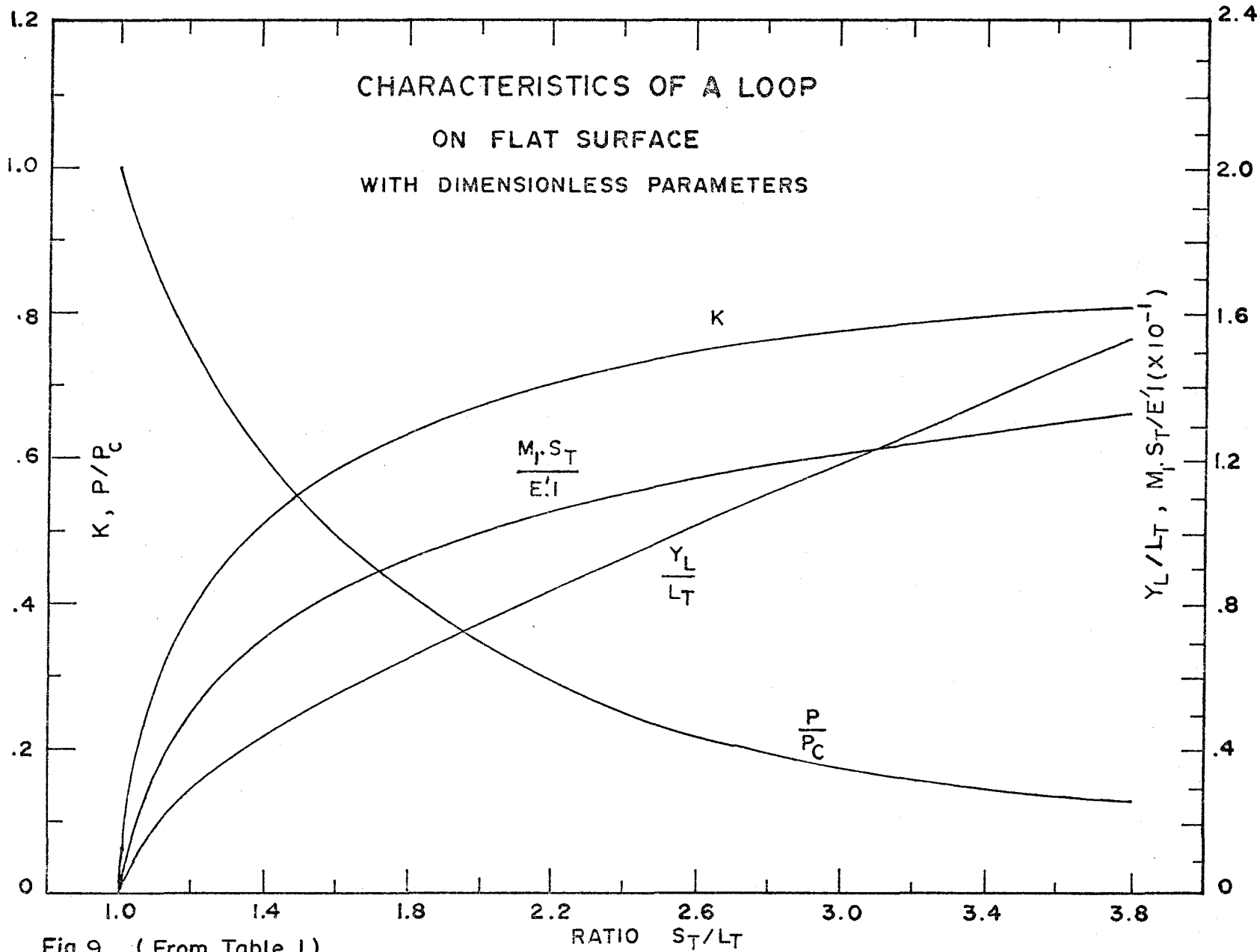


Fig. 9 (From Table I)

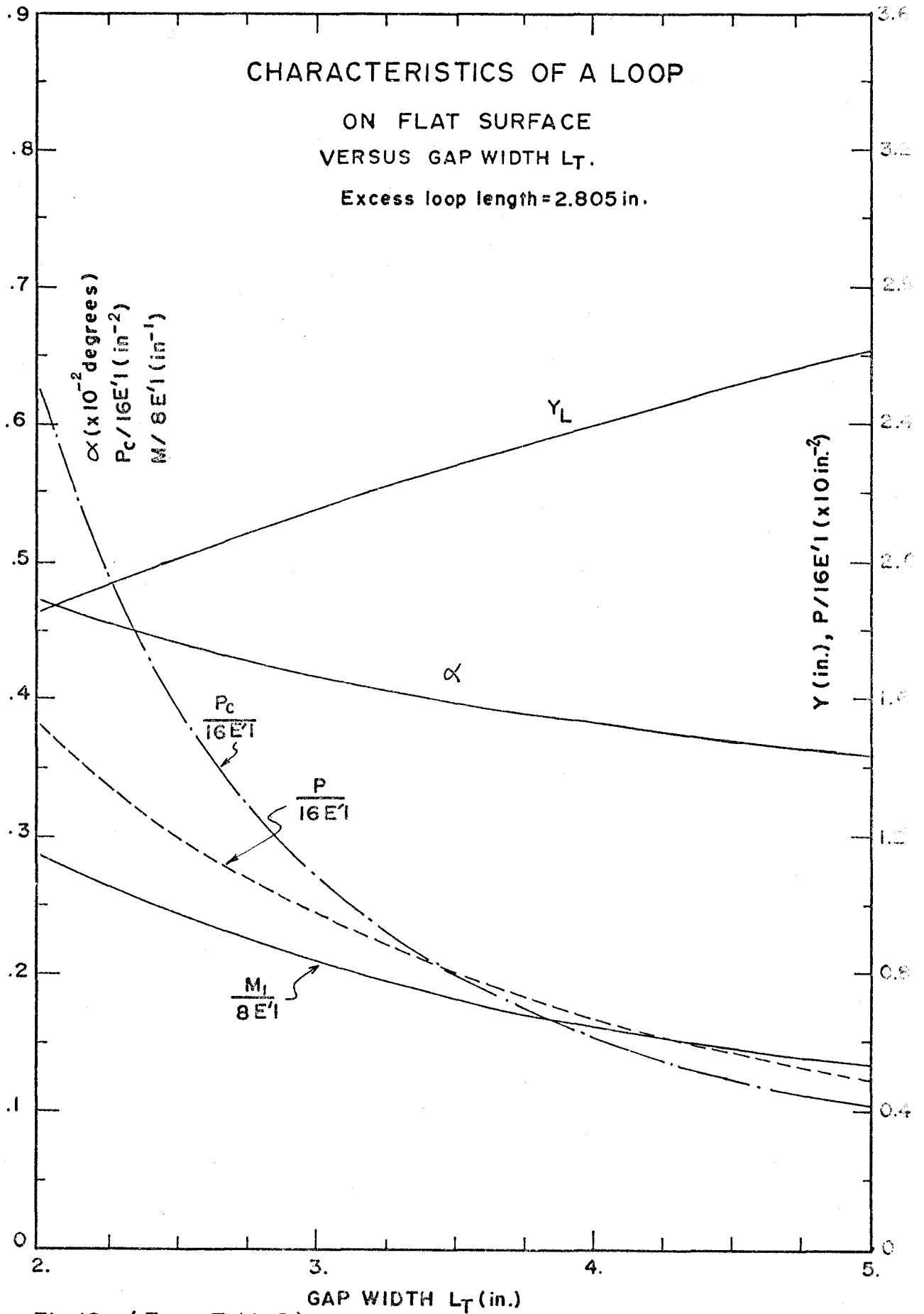


Fig.10 (From Table 2)

material is augmented with the increasing of loop size while the supporting forces at the two ends are reduced.

If those quantities are plotted against the gap width,  $L_T$ , keeping the excess length,  $E_s$ , constant (Fig. 10), it is observed that the angle of the loop at the inflection point, the supporting force and its critical value and bending moment are decreasing when the gap width is increasing. For the loop height, the situation is reversed; it is increased with the increasing of gap width.

## 6.2 Point Paths

Out of 66 fixed points selected on the film strip, two typical points are chosen to show the motion of a particle on the film at various instants (Fig. 11a and 11b)<sup>1</sup>. The coordinates of these points are measured from a fixed coordinate system placed at a point 25 inches from the film entry.

As expected, a point on the film moves along a skewed dome-shaped path with a very sharp top which indicates that a minimum velocity and a maximum deceleration force are expected as this point reaches the top of the loop. As stated earlier, the motion of a point on the loop is a combination of two movements: the rolling action due to the

---

<sup>1</sup> In these figures the number placed close to the x-mark denotes time intervals.

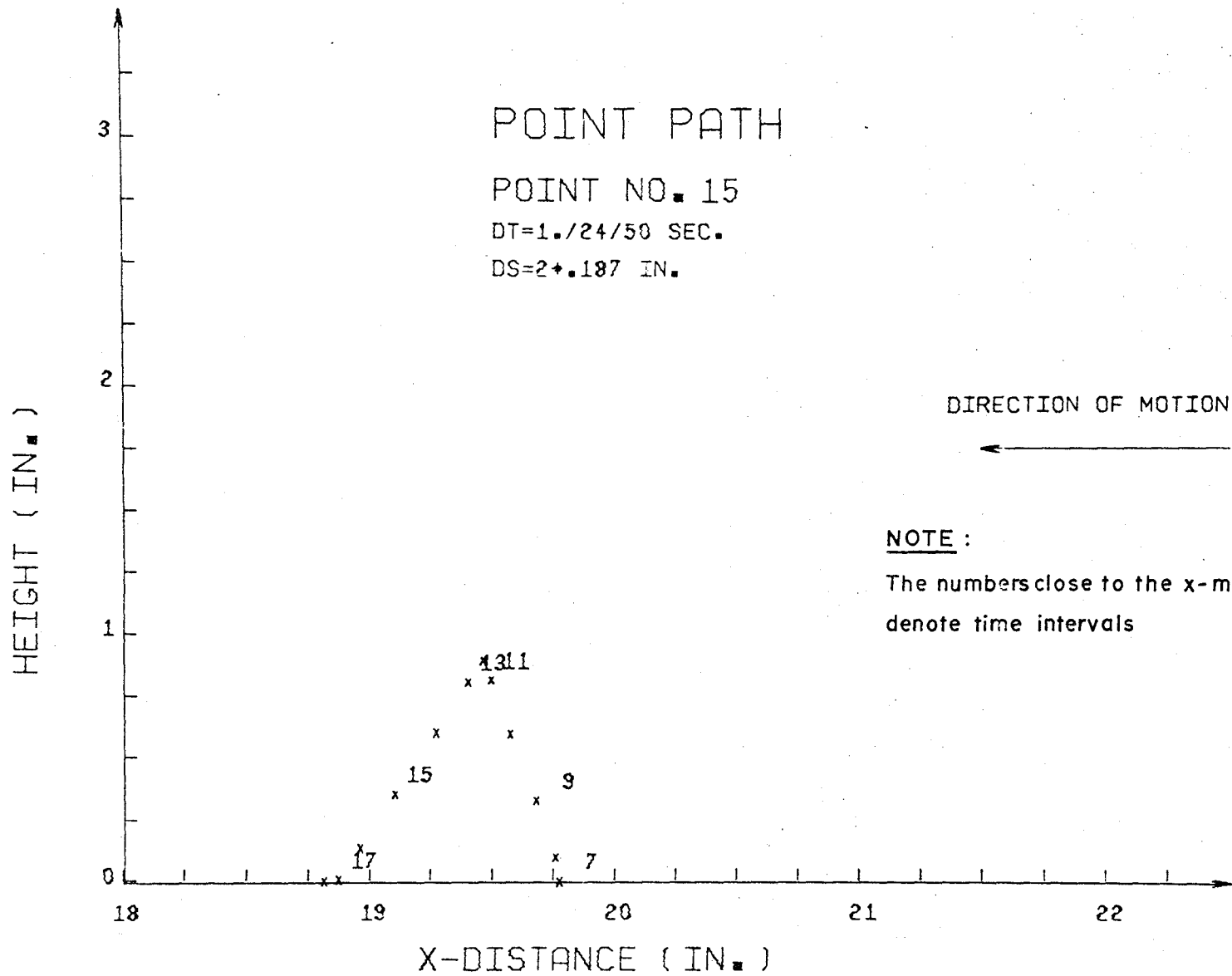


FIGURE 11a.



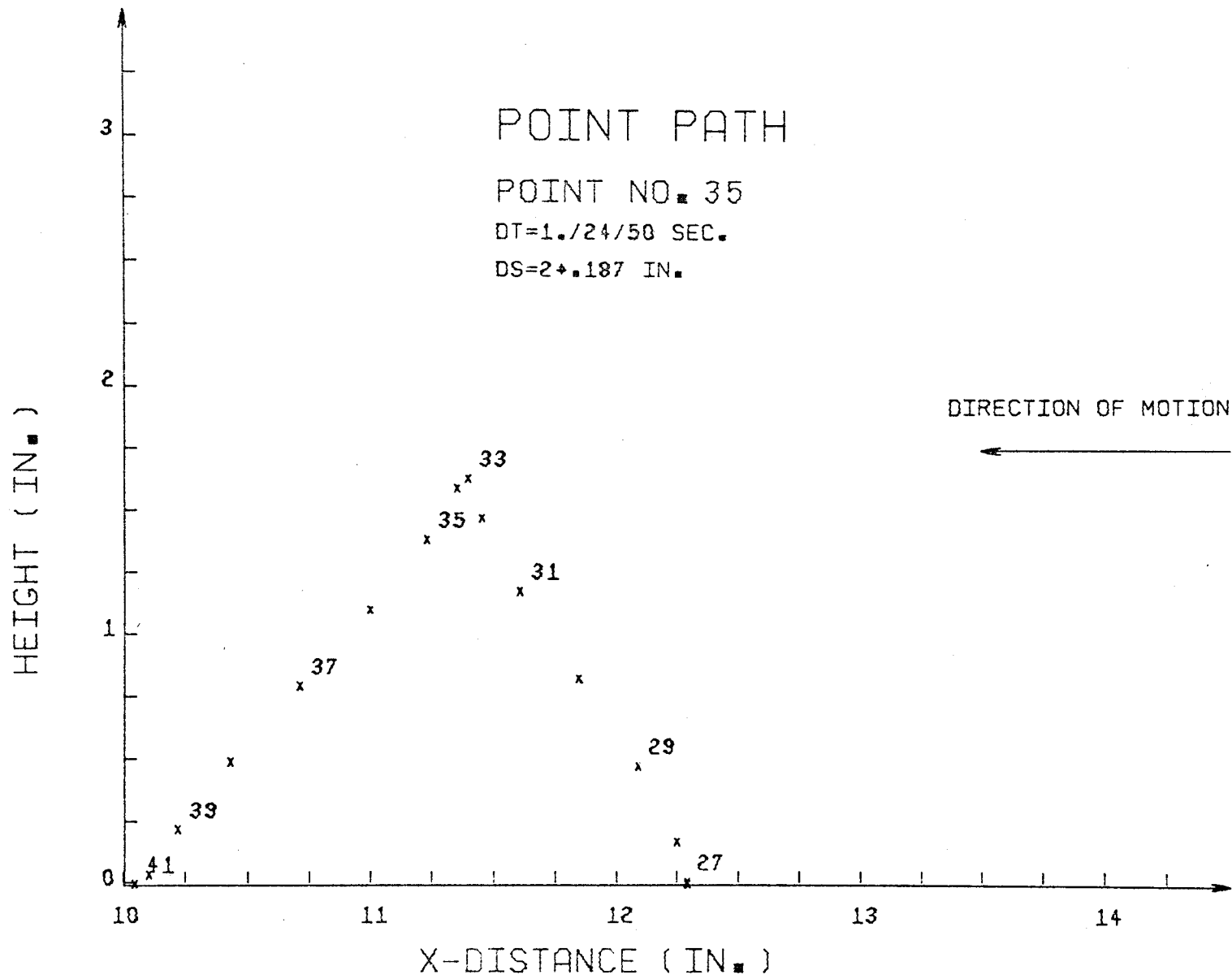


FIGURE 11b.

passing of the loop and the film feeding at one end. The former produces a symmetrical dome-shaped path and the latter destroys the path symmetry by shortening the time for the upward travel and lengthening that of the downward movement of the point.

Also, a comparison of Figures 11a and 11b reveals that in the early stage of film movement, the transversal (i.e., parallel to Y-axis) motion of the film particle in the gap is a predominant factor, while near the end of the movement the lateral motion plays a more important role.

### 6.3 Velocity and Acceleration in Loop Motion

The velocity and acceleration of points on the loop as the film moves along a plane are described by diagrams with vectors proportional to the velocity or acceleration vectors at various points on the loop curve; for simplicity, these diagrams will be called velocity and acceleration profiles or velocity and acceleration distribution diagrams (Figs. 12a to 18b)<sup>2</sup>.

The characteristic of the velocity and acceleration profiles is mainly dependent on the extent of the

---

<sup>2</sup> In these diagrams the loop curve is marked with identifiers to indicate the point numbers which have been assigned to the selected points on the film strip.

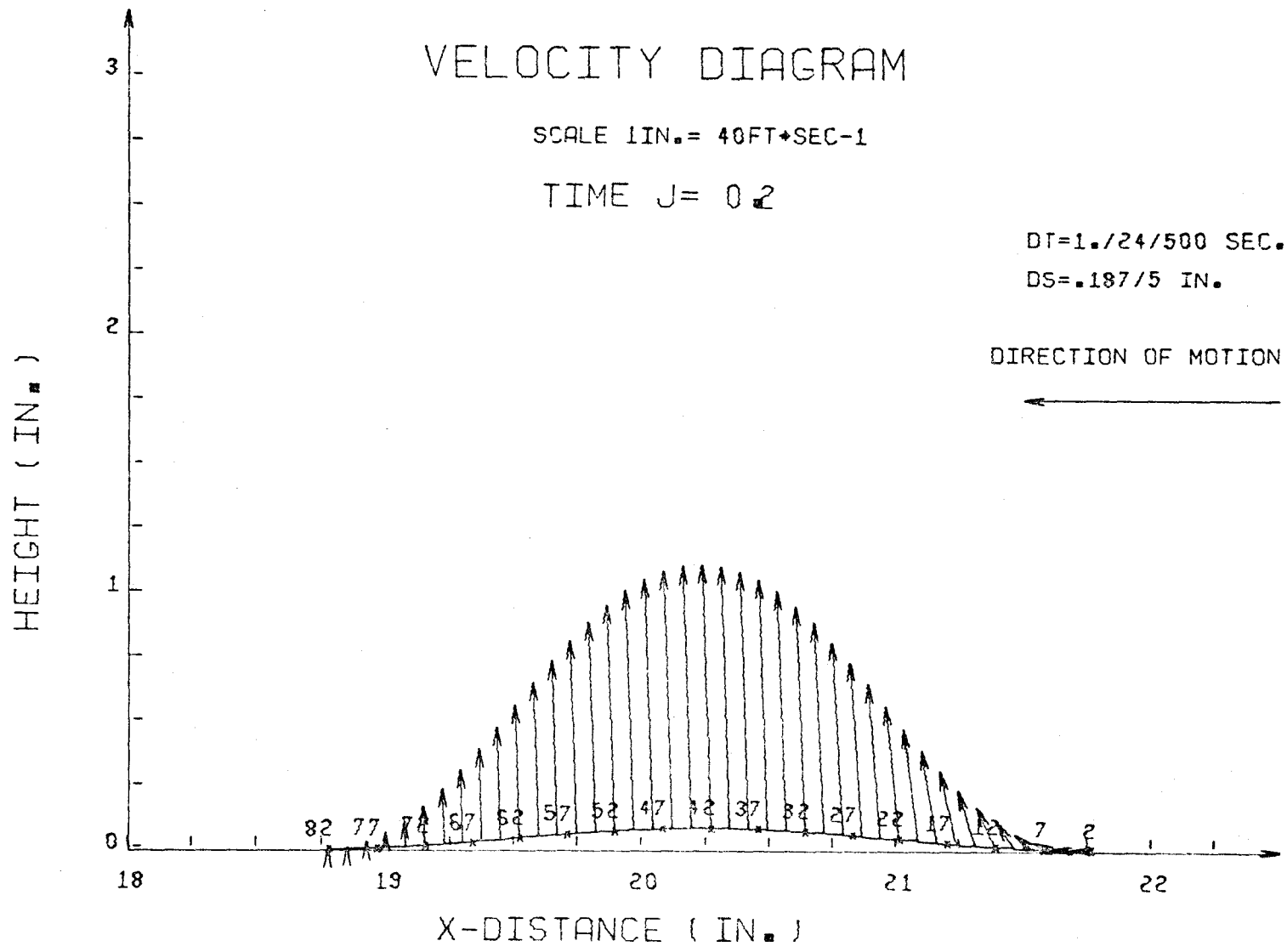


FIGURE 12a. (From Tables 3a and 3b)

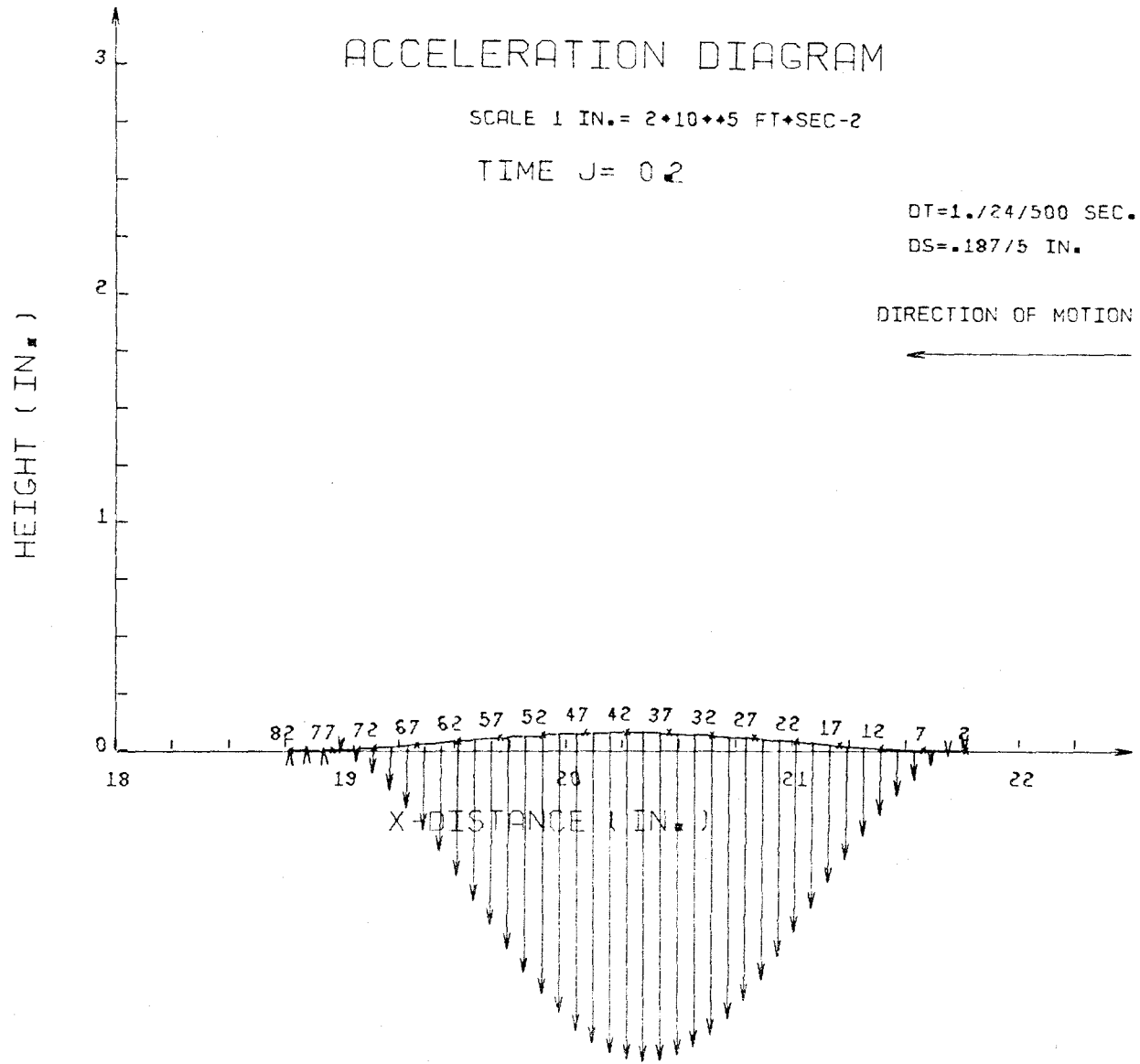


FIGURE 12b. (From Tables 3a and 3c)

contribution of the film feeding and the rolling action. At the start, the film motion is characterized by the sudden uplift movement of the film strip from a flat surface mostly due to the action of film feeding. As indicated in section 5.5, this kind of movement generates an infinitely large velocity and acceleration on the film body and the velocity and acceleration profiles at this stage are dominated by vectors perpendicular to the x-axis. (Figs. 12a and 12b). The peaks of these profiles occur at the center line of the loop.

As the loop grows larger in size, at about  $\frac{1}{500}$  of a frame cycle from the start, the rolling action comes into effect and its contribution to the motion is the addition of the downward components to the velocity vectors near the right hand end of the loop and the upward components to the acceleration vectors on the other side as indicated in Figures 13a and 13b. As for the film infeed, its effect at this stage is the introduction of a horizontal component to the velocity vectors near the film feeding end.

Resulting from the change in magnitude and angle of the vectors on the velocity and acceleration profiles, the so called "knots" appear in these diagrams where the vectors reach their local minima. During the procession of the loop further along the plane the knot in the velocity profile tends

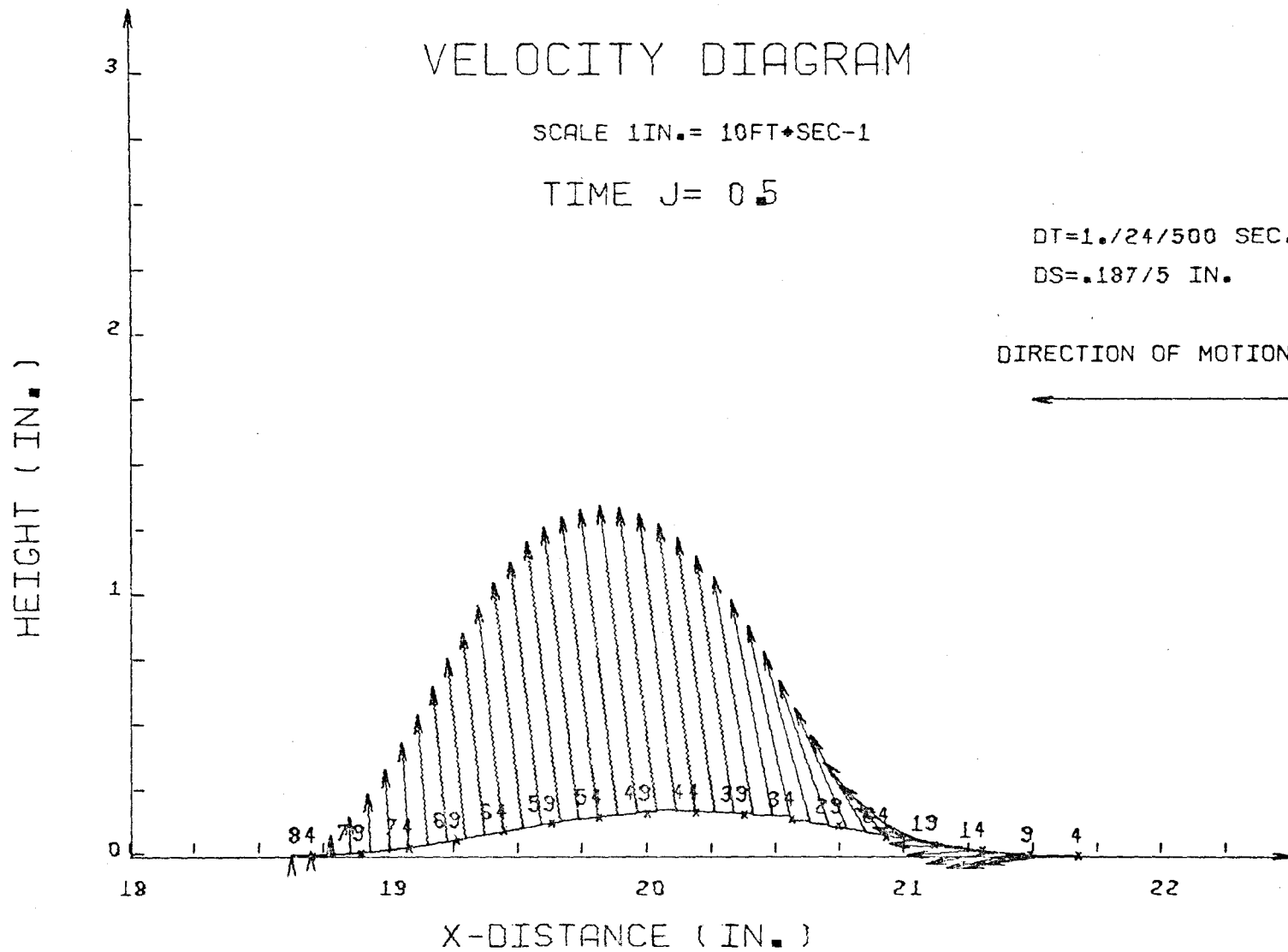


FIGURE 13a.

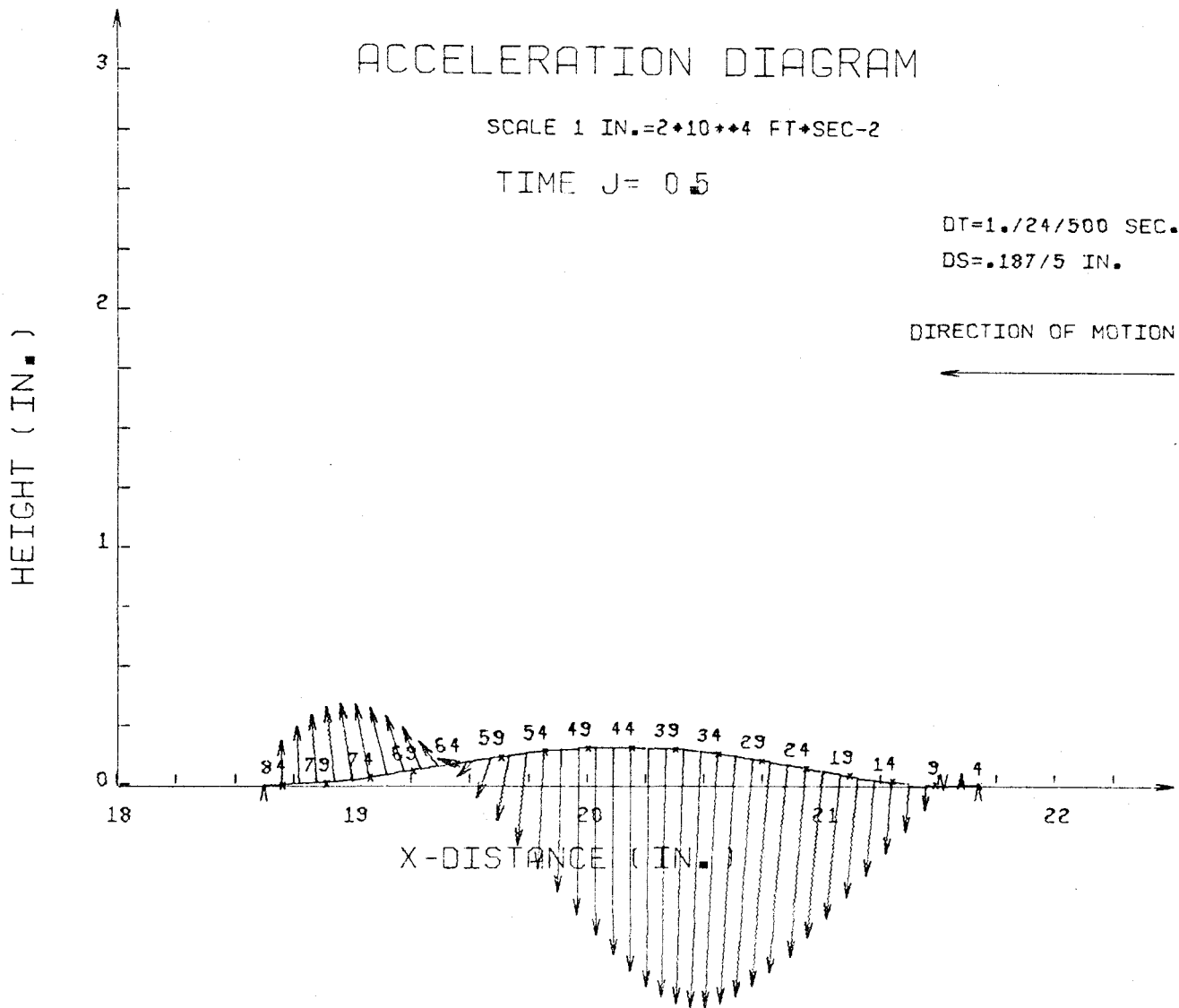


FIGURE 13b.

to move towards the loop top while those of the acceleration diagrams are approaching the inflection points (Figs. 14a to 17b).

At the time of about 1/10th of the frame cycle, the rolling action of the loop is fully developed. The loop motion at this stage is characterized by the appearing of the maximum velocity in the vicinity of the inflection point where the acceleration attained a minimum value and a reverse situation takes place at the loop top and bottom (Figs. 15a to 17b). It is quite interesting to note that in these diagrams, the acceleration vectors are almost perpendicular to the loop curve because of the effect of the rolling motion.

In comparing Figures 18a and 18b which represent the velocity and acceleration profiles of the loop where  $t \approx \frac{1}{35}$  of a frame cycle to the case of pure rolling motion as indicated in Figures 19a and 19b, it is seen that the introduction of the film feeding velocity will create the following effects:

- a) A horizontal component is added to the velocity vector especially close to the right hand side.
- b) Reduce the magnitude of the acceleration vectors.
- c) Shift the point of minimum acceleration towards the opposite direction of the loop motion.

The peculiar behaviour of the film loop acceleration



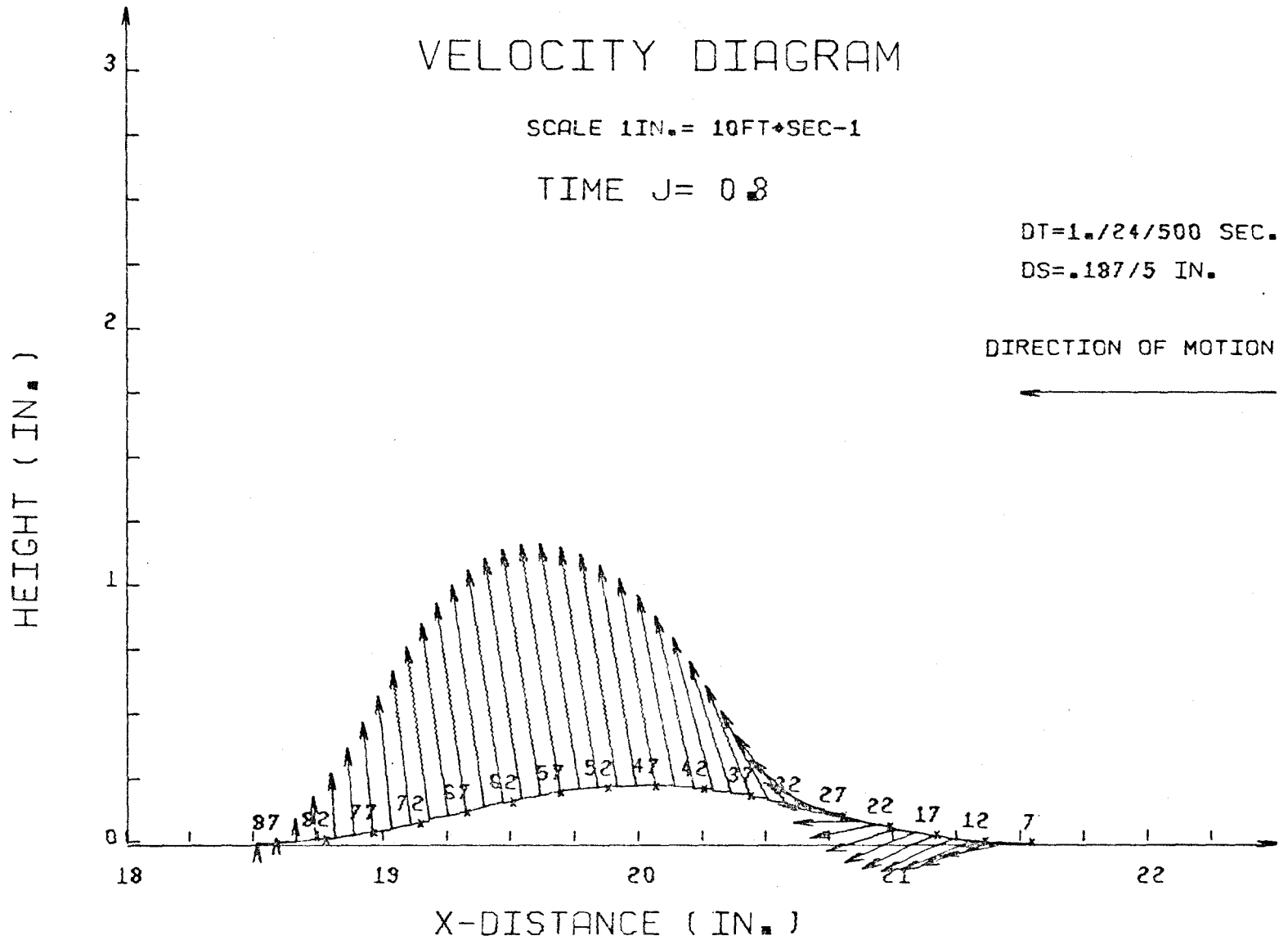


FIGURE 14a.

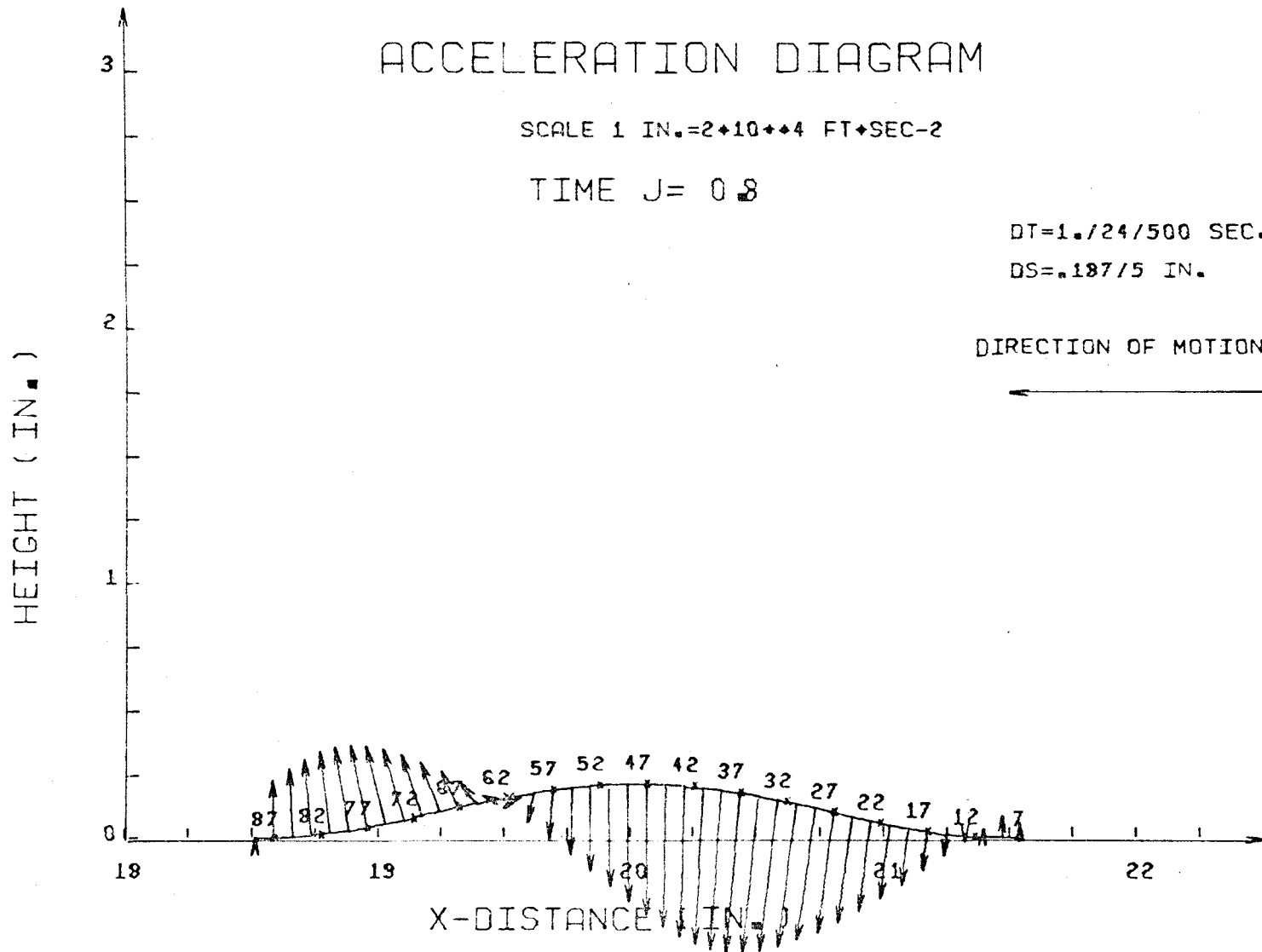


FIGURE 14b.

# ACCELERATION DIAGRAM

SCALE 1 IN. = 10<sup>4</sup> FT·SEC<sup>-2</sup>

TIME J= 5

DT=1./24/50 SEC.

DS=2.♦♦.187 IN.

DIRECTION OF MOTION

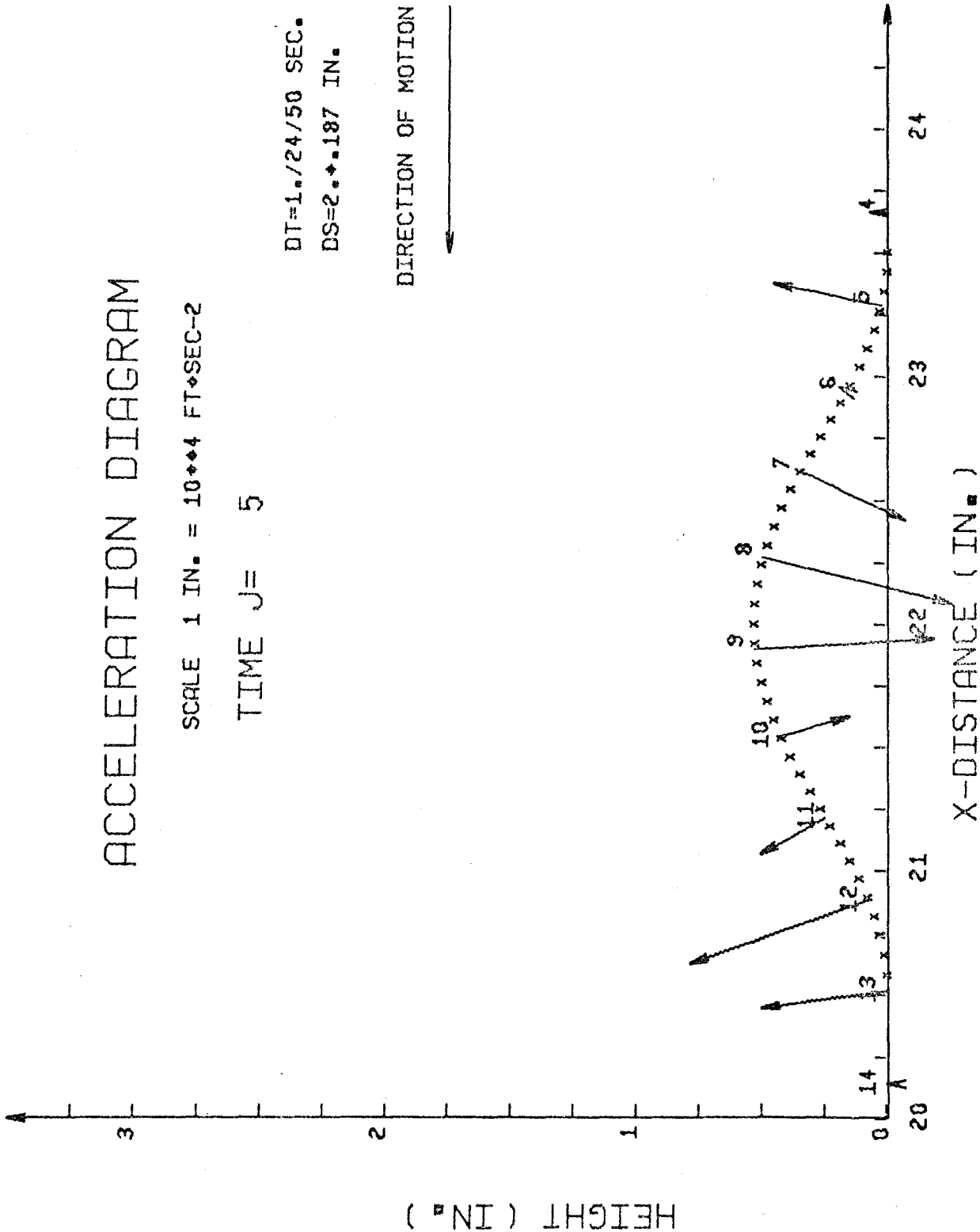


FIGURE 15a.

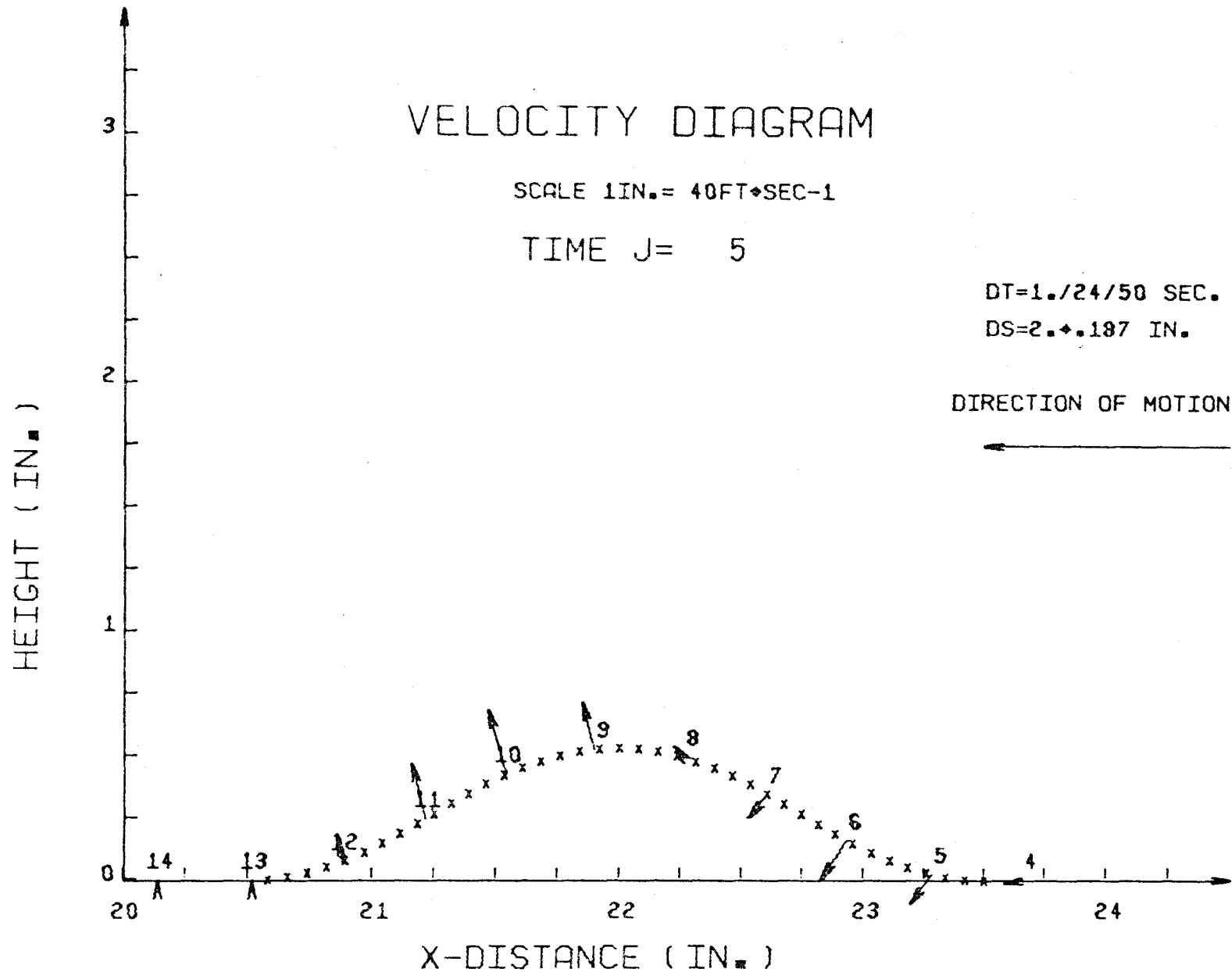


FIGURE 15b.

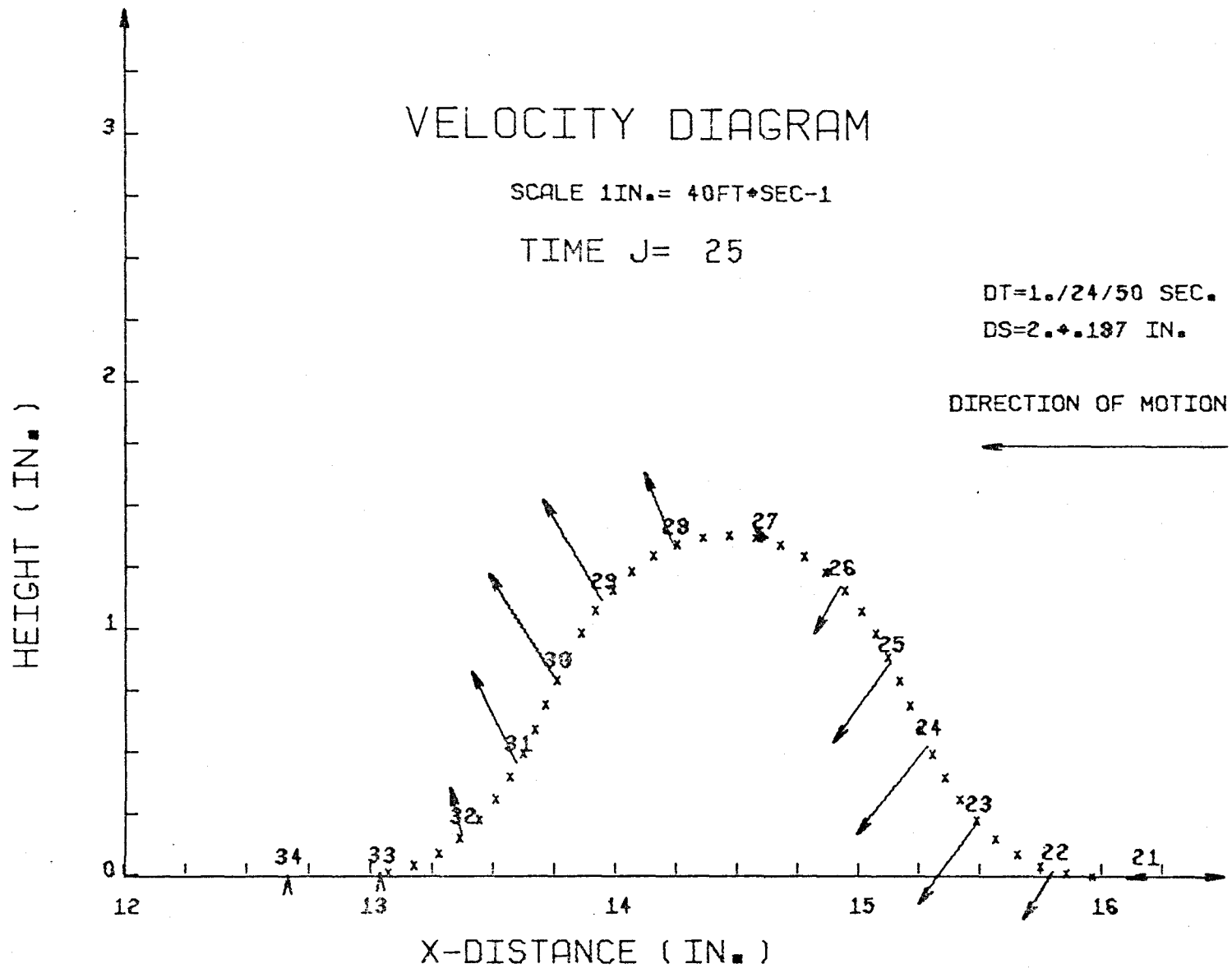


FIGURE 16a.

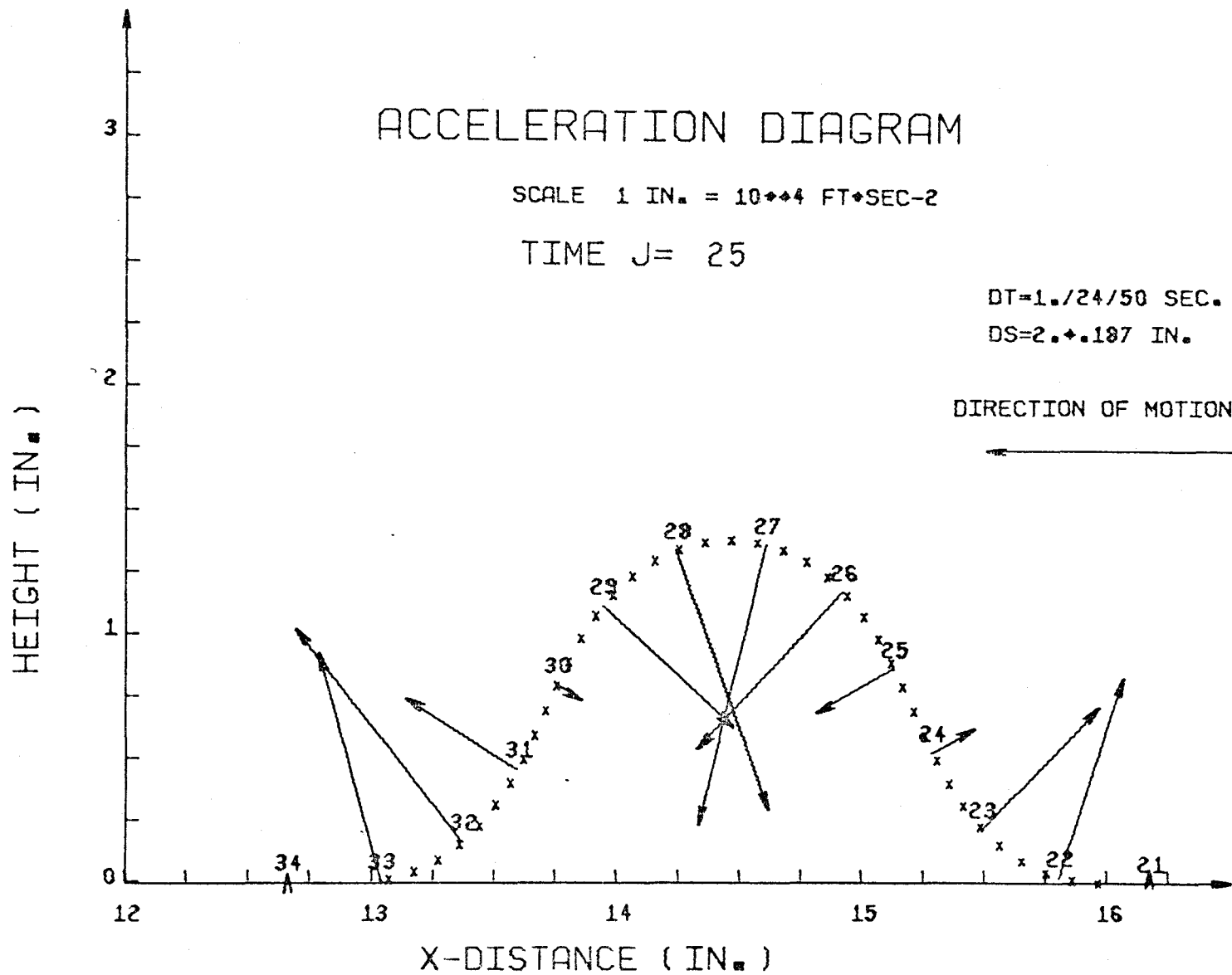


FIGURE 16b.

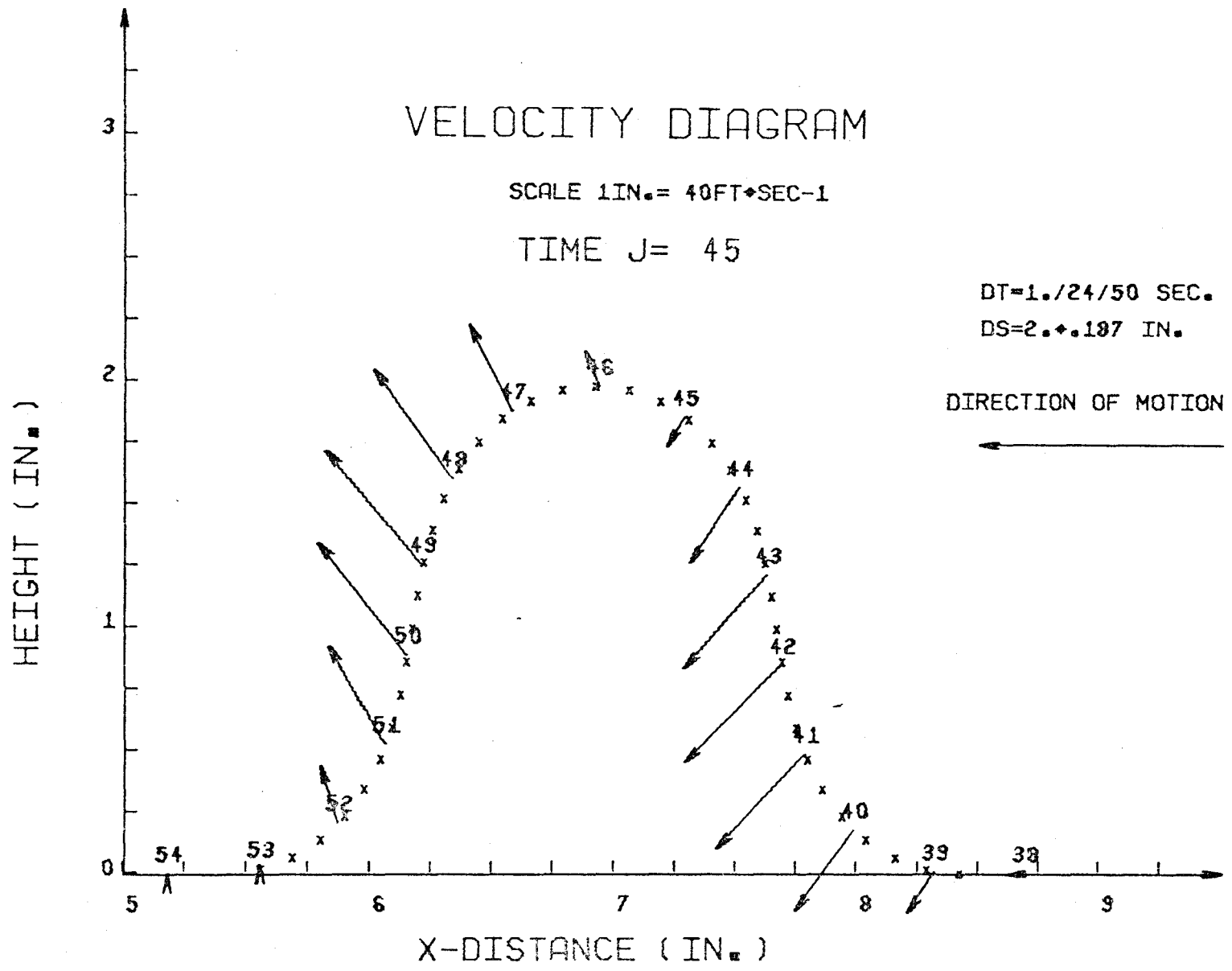


FIGURE 17a.

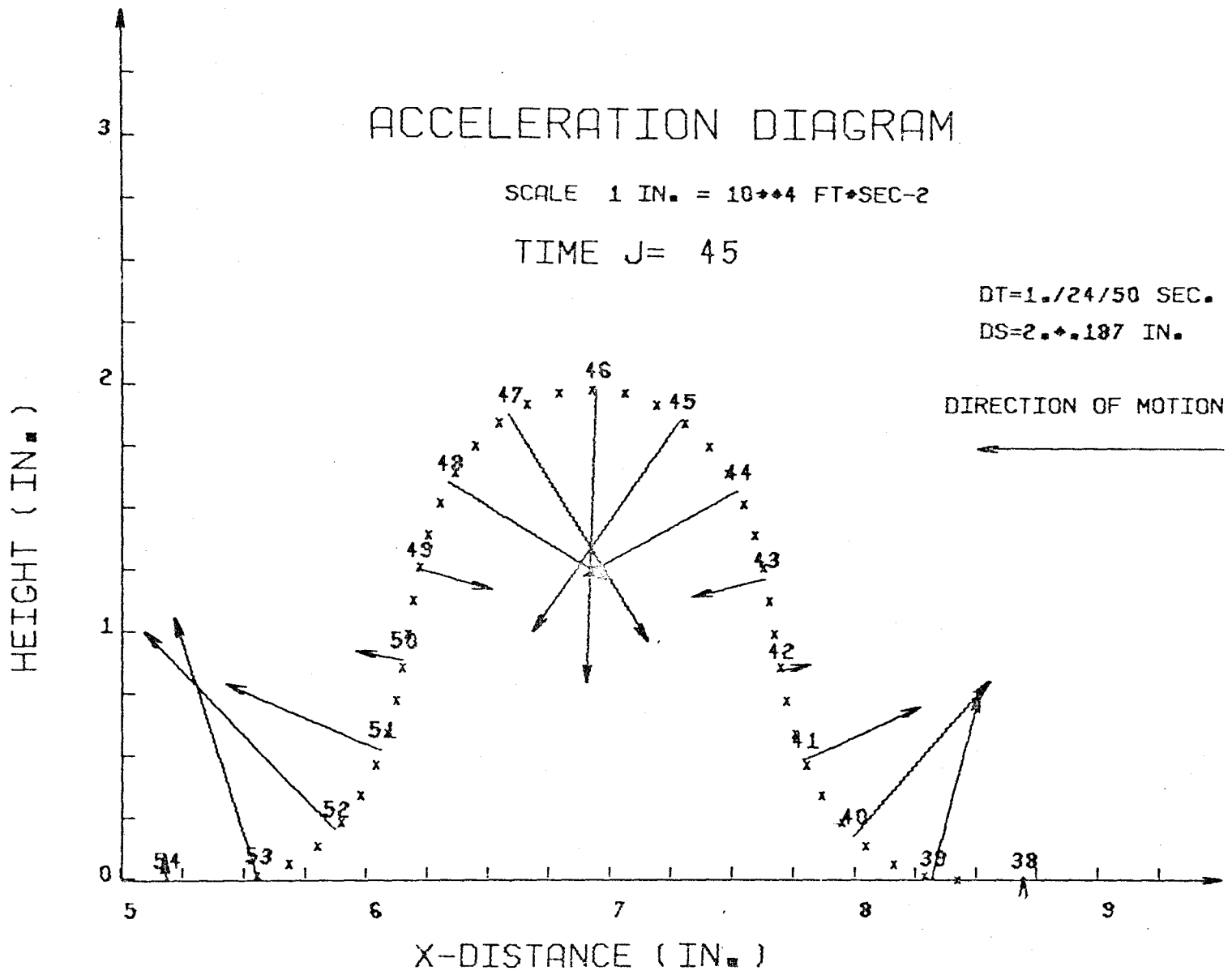


FIGURE 17b.



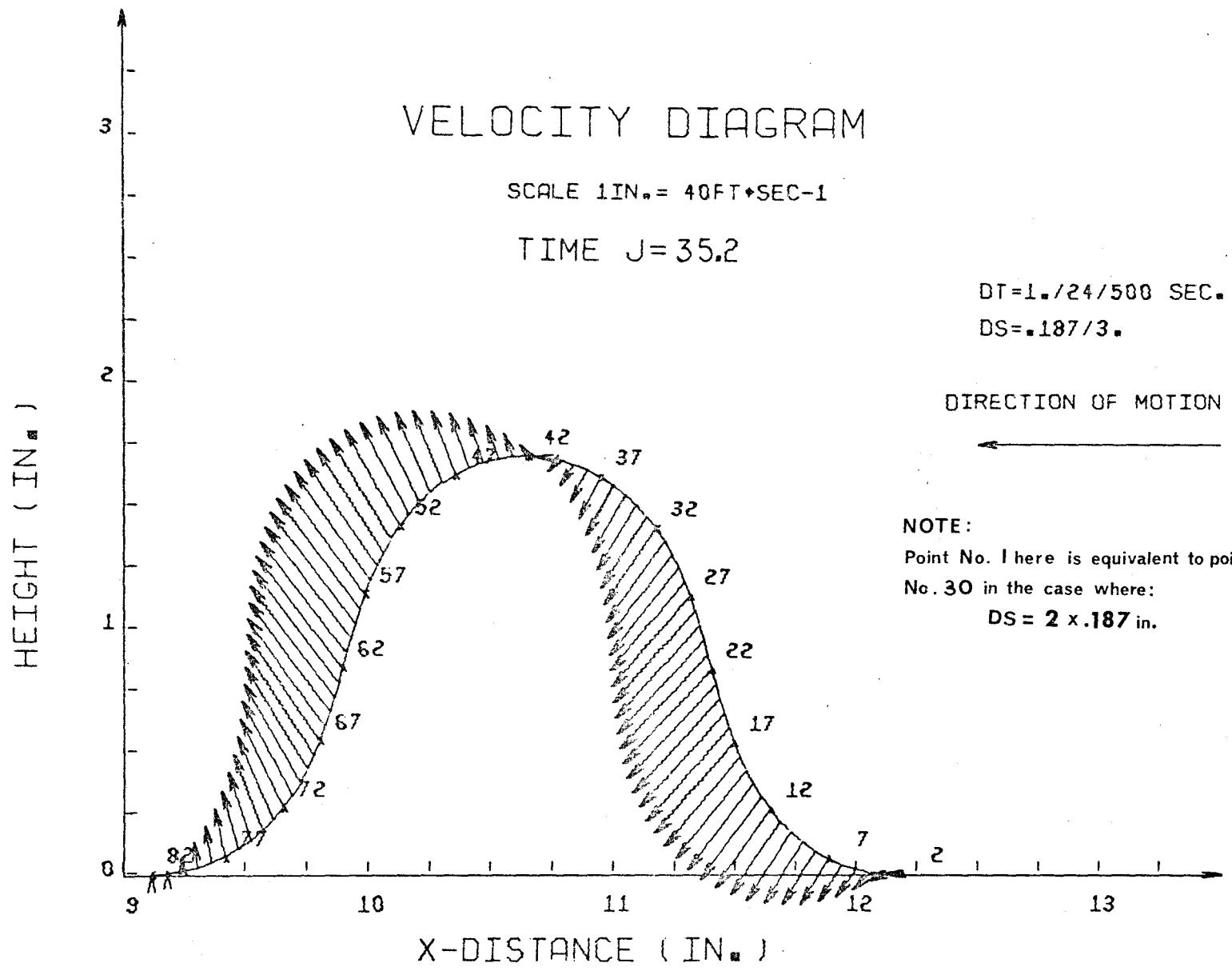


FIGURE 18a.

# ACCELERATION DIAGRAM

SCALE 1 IN. = 10<sup>4</sup> FT\*SEC<sup>-2</sup>

TIME J=35.2

DT=1./24/500 SEC.  
DS=.187/3.

DIRECTION OF MOTION  
←

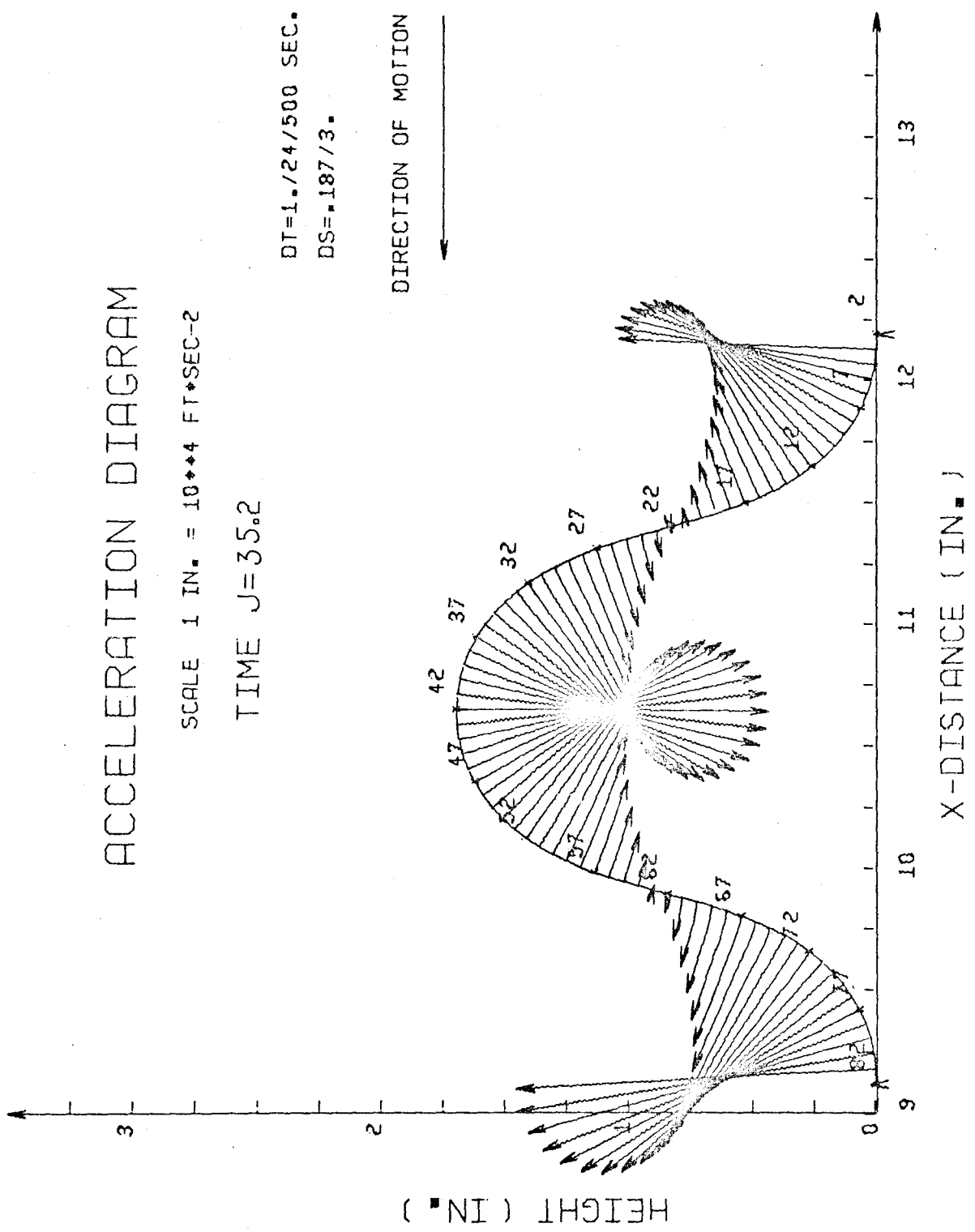


FIGURE 18b.

# VELOCITY DIAGRAM

SCALE 1IN. = 40FT + SEC-1

TIME J = 35.2

$V_F = 0.$

DT = 1./24/500 SEC.  
DS = .187/3.

DIRECTION OF MOTION  $\leftarrow$

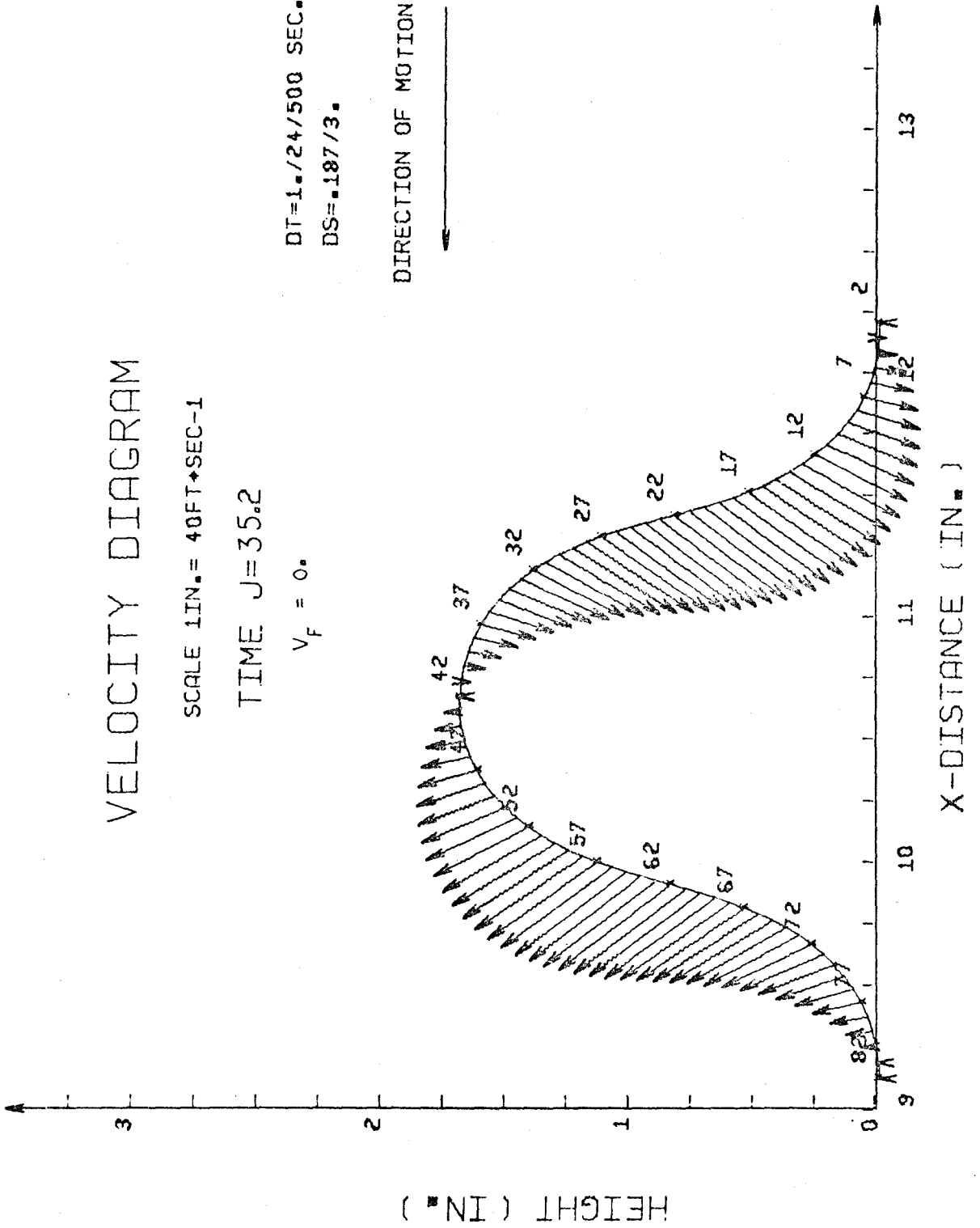


FIGURE 19a.

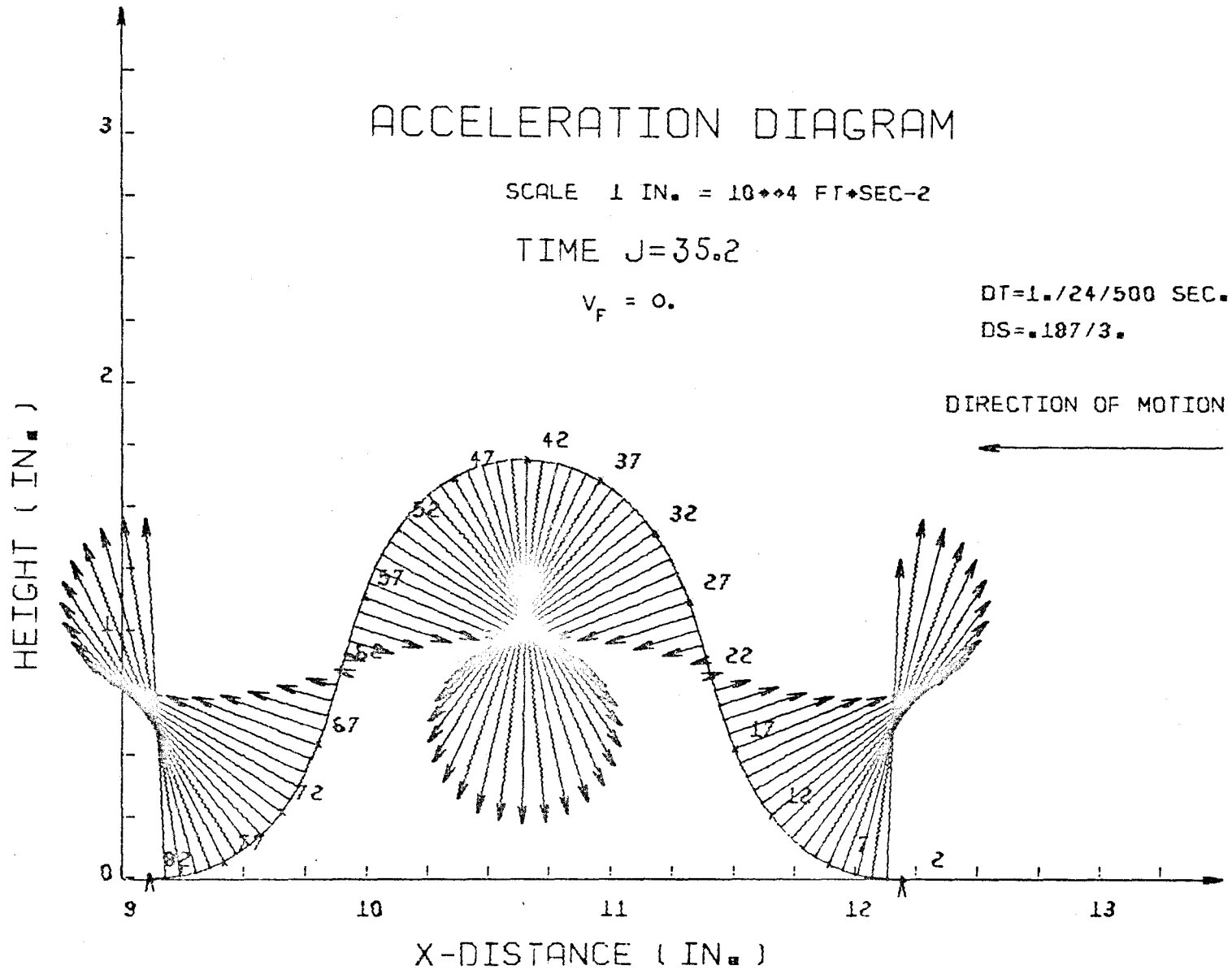


FIGURE 19b.

profile can be best explained by examining the motion of the loop itself. The right hand part of the loop from point 2 to 22 in Figure 18b is seen to be slowing down before it is laid on the flat surface; therefore, the deceleration force must be brought in with an increasing magnitude towards the end of the loop. The top quarter of the loop from point 22 to 42 is accelerated downwards and towards the direction of motion with increasing speed. Consequently, the downward acceleration vectors are attributed to the motion of this part. The same reasoning can be applied to the motion of the rest of the loop.

#### 6.4 Velocity and Acceleration at the Top of the Loop

The variation of the velocity and acceleration amplitudes of a point on the top of the loop in a frame cycle is summarized in Figure 20.

As indicated earlier, the early phase of loop motion shows high velocity and high deceleration in a very short period of time when the effect of film feeding is predominant. As the time is increasing, the acceleration curve drops to a minimum value which corresponds with an inflection point in the velocity curve. This is due to the contribution of the rolling action which results in shifting the point of maximum velocity and acceleration away from the loop center line as indicated in Figure 13a and 13b.

# VELOCITY & ACCELERATION AMPLITUDES OF THE LOOP TOP

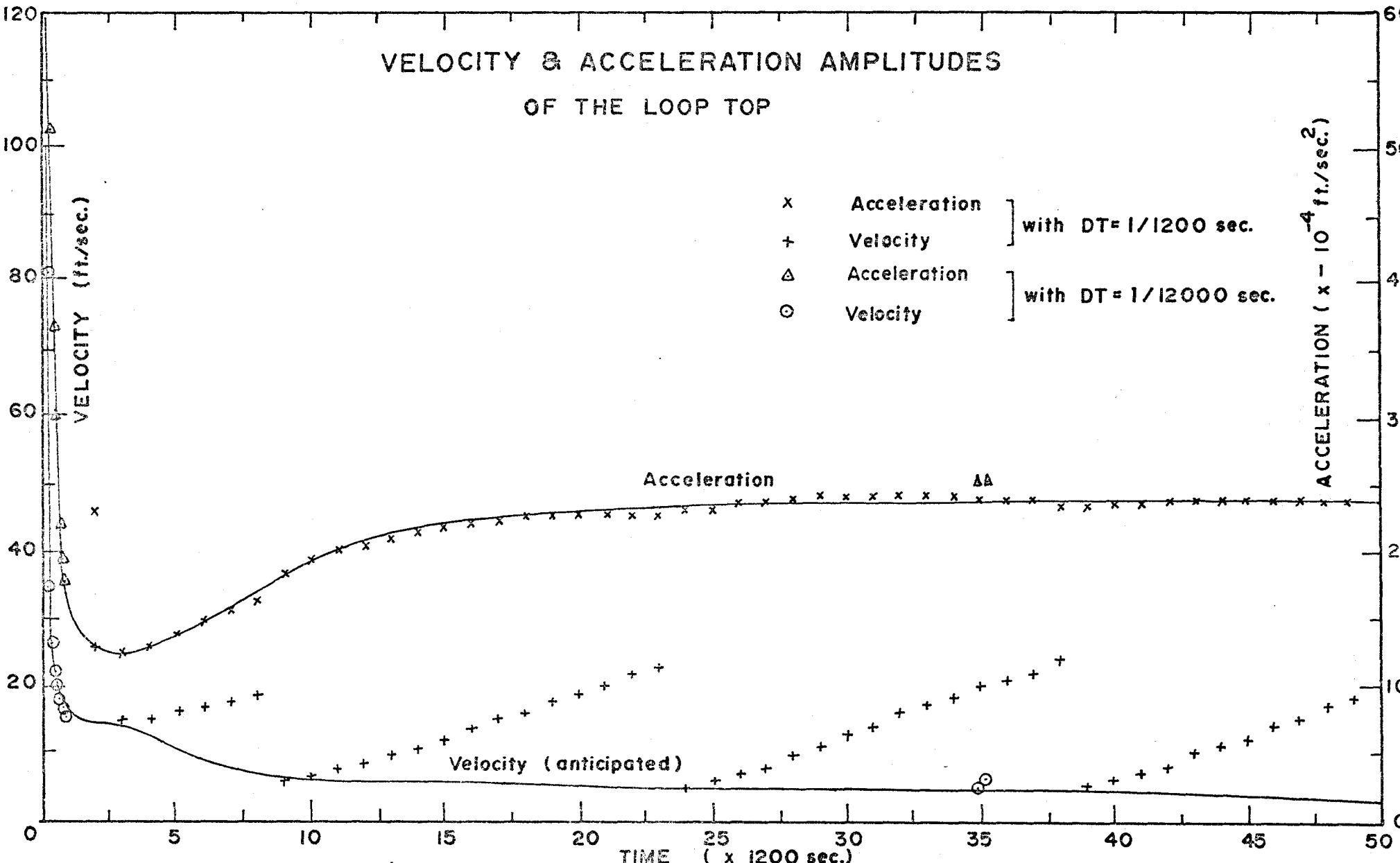


Fig. 20. (From Table 4)

When the knot in the velocity profile proceeds toward the loop top, the velocity decreases in magnitude while the value of the deceleration is increasing. From the time  $t = \frac{1}{15}$  of a frame cycle, when the rolling action dominates the loop motion, the velocity curve drops slowly with a constant rate and in effect a constant deceleration force is expected.

The discrepancy between the results obtained from the computer programmes and the anticipated curve is due to the inherent error in the numerical approximation. In the calculation of these velocity amplitudes, the values are taken at the selected point which is the closest to the top of the loop, and also in this region the velocity is at its minimum value. Accordingly, a large error can be introduced if the chosen point is shifted slightly away from the top.

## 7. CONCLUSIONS

The foregoing study of the loop movement by simulation is restricted to motion on a flat surface with the assumption that the loop shape retains its minimum strain energy form which exists in a static state. The questions arising here are: how close is the approximation, which is based on a theoretical model, to the actual loop motion, and what are the restrictions of the application of these results to the design of the Rolling Loop Projector?

In considering the actual conditions to which the loop is subjected, it is found that there exists various static and dynamic factors which cause the loop shape to deviate from the theoretical shape used as a mathematical model in this thesis. These are the following:

### a. Static Factors

The wave-like motion of the film in the projector is actually produced on the inside of a cylindrical surface. The inclination of the equivalent tangential planes to the film curve at the loop ends creates different end conditions which alter the loop characteristics that had been derived to a certain extent. An analysis for this case is presented in Appendix B.



Furthermore, the loop in the projector is guided and supported by rotor shoes whose configuration is a primary factor in shaping the loop curve not only when the loop is stationary but also when it is in motion as well.

b. Dynamic Factors

The film material has a certain mass which is associated with either linear or angular momentum in the direction of the motion. The film in the rotor gap can be considered equivalent to a certain mass which travels along with the gap at a constant velocity. By the conservation of momentum the change of the amount of film in the gap must be accommodated by a change in the velocity of the film itself. Consequently, in the process of loop growth, where the total amount of film in a rotor gap is increasing, the forward portion of the loop is lagging behind the leading shoe and the rear part is pushed close to the trailing shoe with an end result of tightening the loop curve. This effect is increased at high operating speed by the contribution of air resistance on the projected area of the loop. This situation is further complicated by the existence of the centrifugal force which tends to flatten the loop curve into the stator, and by other factors such as the initial impact of the trailing rotor shoe on the loop before carrying it forward, and the inherent friction force between the shoe and the film.

However, at low operating speed (around 18 frames per second) the loop does approximate the static shape considered. At this speed, the flexural rigidity of the film is large enough to withstand any significant shape modification and, furthermore, the rotor shoes machined to a proper configuration serve as a support to the loop especially when it is full grown. An investigation by high speed photographs of the loop taken at the point where the loop just leaves the projection gate disclosed that the loop shape is quite similar to the theoretical one derived from the static analysis. A typical high speed photograph is shown in Figure 21.

A further study of loop movement, taking into account the factors mentioned above, can be made. However, for the present, the results obtained do provide a basic understanding of the dynamic behaviour of loop movement; and furthermore, this knowledge permits a more precise determination of the design criteria for the Rolling Loop Projector to reduce film stresses and shorten frame transfer period.

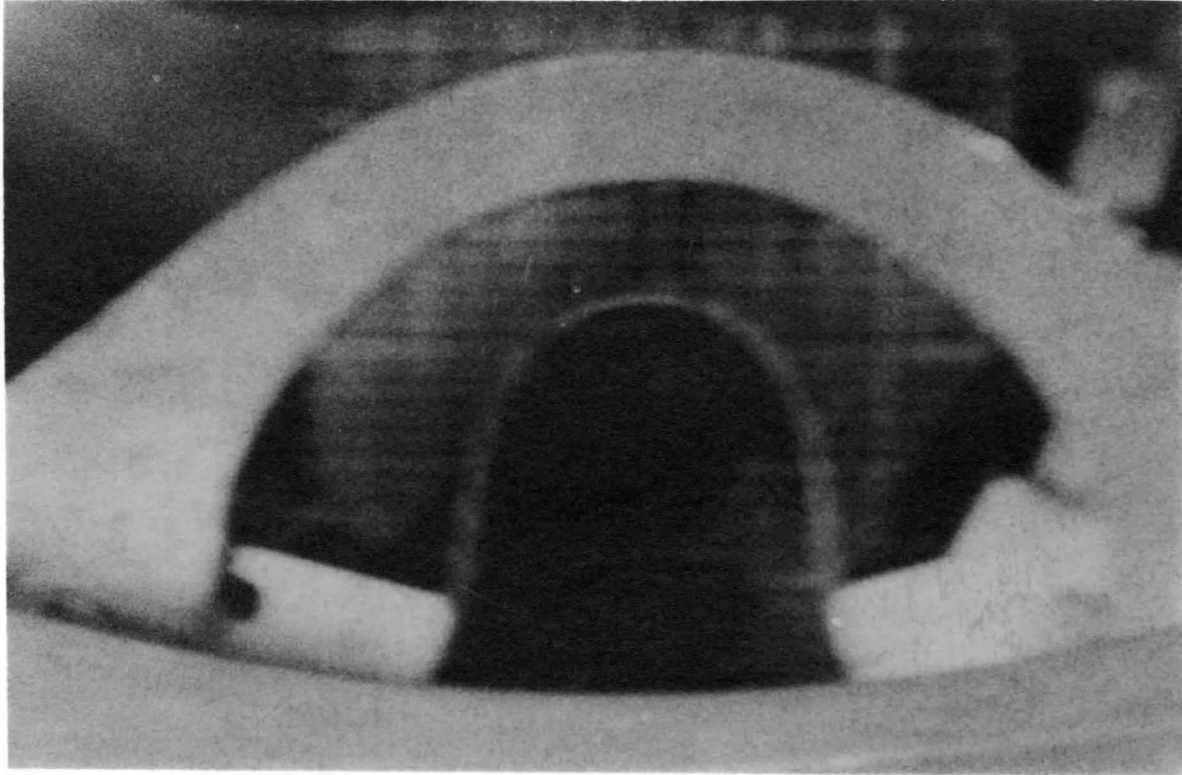


FIGURE 21. High speed photograph of a film loop after it leaves the projection gate (Projector operating speed = 18 frames/sec., camera speed = 1000 frames/sec.)

## 8. DESIGN CONSIDERATIONS

Most of the design work done so far on the Rolling Loop Projector has been based on experimental trial and error techniques. One of the most difficult problems encountered during the development stage of the 70 mm. prototype was the very large stress on the film body generated at the start of the loop growth or at the end of loop decay where the loop height is small. Other design problems arose when an attempt was made to shorten the rotor gap in order to cut down the frame transfer time in a projection cycle. Some ideas and suggestions will be given regarding these topics.

### a. Film Inlet and Outlet

As seen in the acceleration diagrams at the early stage of loop growth, a large acceleration occurs in the film motion when the film loop begins to form from a flat shape. This introduces large compressive loads at the loop ends. If a tangential film entry as shown in Figure 22a were used, the film may buckle before a proper loop was formed because the curvature of the stator is opposite in direction to that of the required loop. An increase in the angle of the film inlet from the tangential position will initiate loop growth in proper direction thus reducing the initial film acceleration

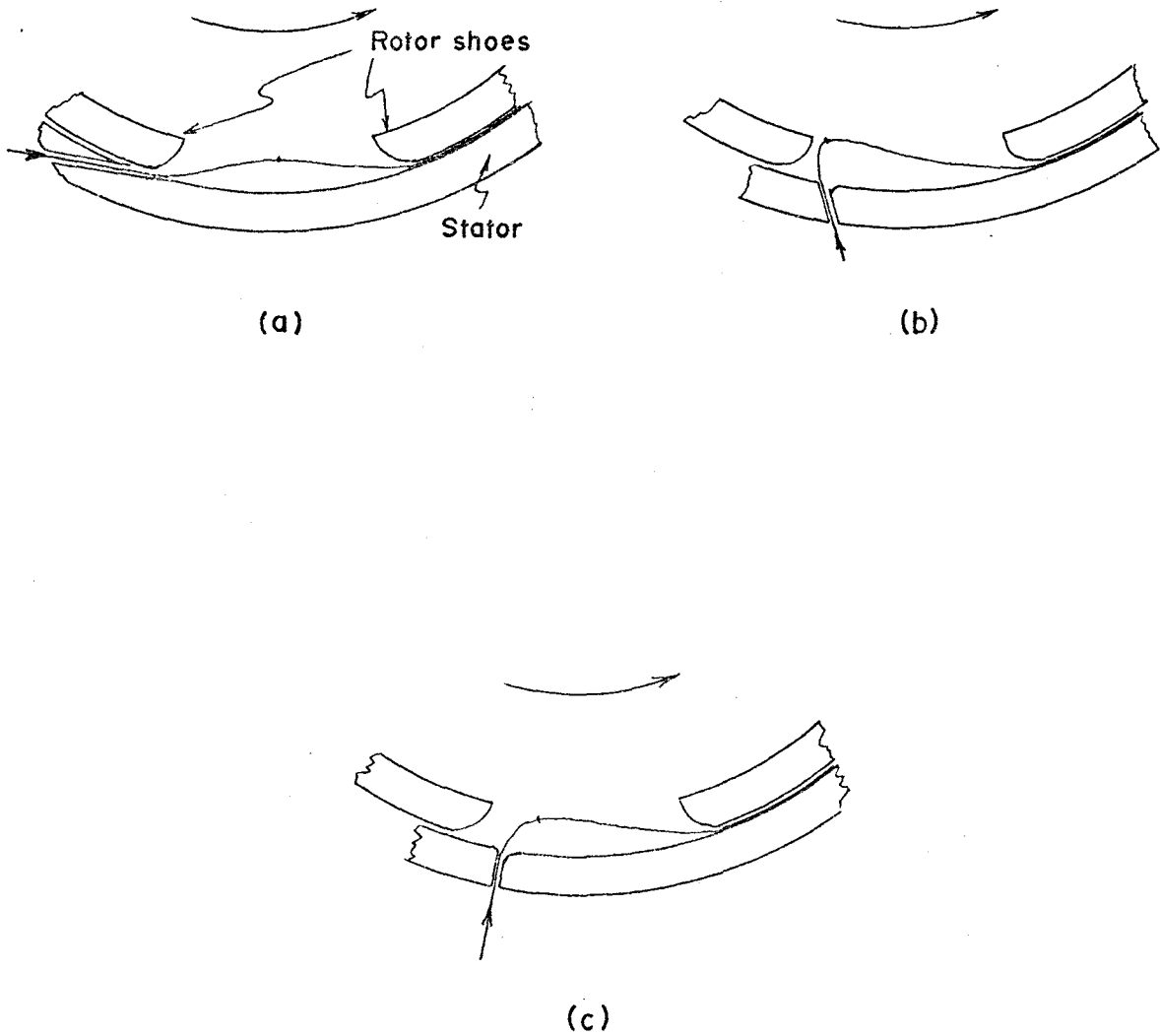


Fig.22. Different angles of film entry slot. (a). Tangential film entry, (b). Large angle film entry, (c) Radial film entry.

and the resultant applied load. However, if very large angle is employed, the top of the loop will be shifted backward closer to the trailing shoe face (Fig. 22b). This will create a situation in which the trailing rotor shoe tends to override the film loop instead of pushing it forward in a rolling action. A 45 degree-film entry was used in the prototype and it seemed to work satisfactorily. However, it is felt that a larger film entry angle might provide a better solution. As shown in Figure 22c, an angle as large as 90 degrees will promote an earlier rolling effect which is favorable for the film motion at this stage. So far, however, the optimum angle for the film entry has not been determined, and this is an area requiring further study.

When the trailing shoe first comes into contact with the rear part of the loop (which is still stationary), an impact force is generated. Although this impact is softened by the air jets on the trailing rotor shoe, the sudden increase in linear momentum of the loop creates a considerable distortion of the loop shape and some loop vibration. The consideration of this effect leads to the proposal of employing a curved stator plate near the film entry with a configuration as shown in Figure 23. With this configuration part of the energy transferred to the loop from the impact is dissipated in bending and the remainder increases the linear momentum of the rear part of the loop. In addition, the

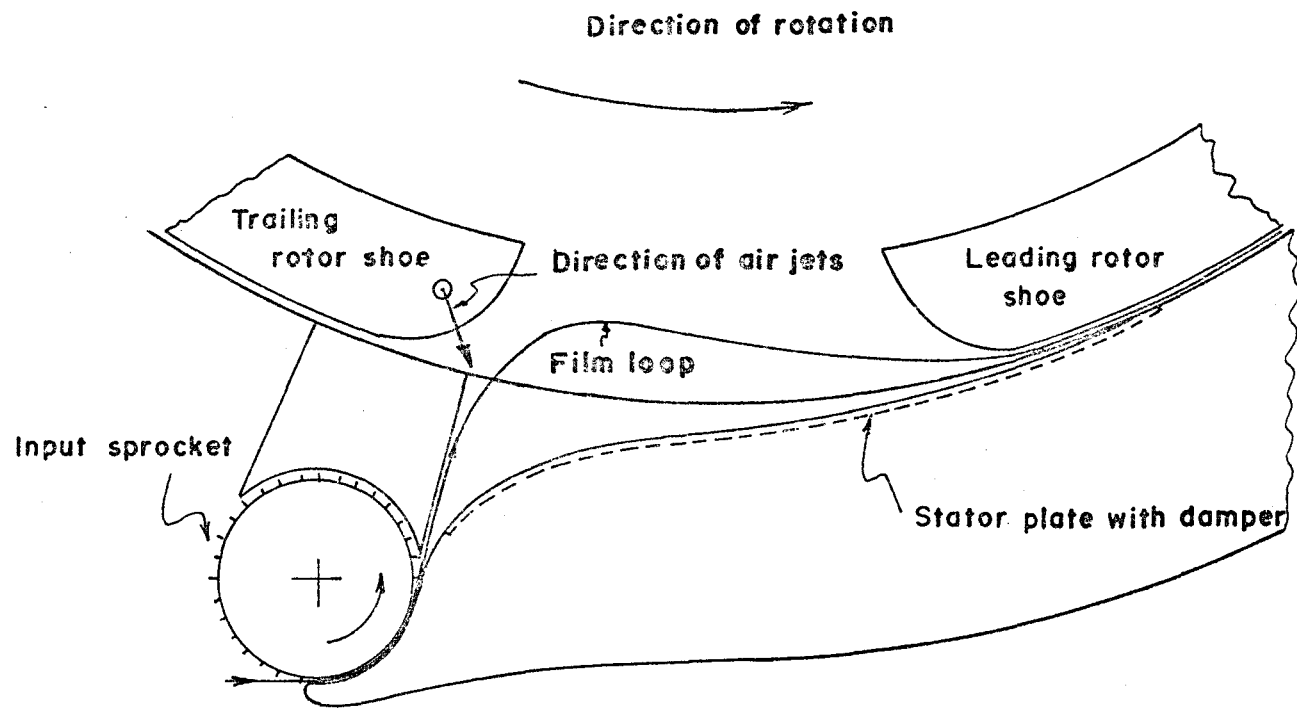


Fig.23. Proposed film inlet

film entry slot is fitted with a plate which provides an air cushion effect to slow the incoming film.

The same concept can be applied to the exit slot. Before the loop has fully disappeared, its linear momentum, which is still carried forward to the exit end, tends to move the film at a speed higher than that of the film output. The combination of a high linear velocity of the film and a large upward deceleration on the film body creates high tension stresses on film perforations with possible resultant film damage. Therefore, an inward (towards rotor center) curved slot with damper plate similar to the inlet slot is recommended. This should permit the film to reduce its size with low deceleration force and to allow some over-travel of the loop.

b. Optimization of the Rotor Gap

The illumination efficiency is a critical factor in projection design, and it is mainly dependent on the frame transfer period. In the Rolling Loop Projector this period is determined by the ratio of the rotor gap width to the circumference of the rotor. For a given rotor diameter a shorter rotor gap width is considered to be highly desirable in that it gives better illumination efficiency.

From the characteristic curves of the static loop (Fig. 10), it is observed that for a given frame length,  $E_s$ , a smaller gap produces a lower loop height and this is an



advantageous condition for a controllable loop shape, especially in high speed operation. On the other hand, a decrease in gap width promotes an increase in the supporting forces at the loop ends and a rise in bending moments on the film material. The bending moment produced, combined with the heating effect on film by projected light, can be detrimental to brittle film. Therefore, the determination of the rotor gap width is a compromise between a shortening of the gap width and a reduction of film stresses. The perforation stress limit is readily obtainable, but the permissible bending stress for the film material is not yet known. Hence, a knowledge of the latter must be sought before an optimization of the rotor gap can be made.

#### c. Other Considerations

Just as the trailing edge of the full grown loop passes over and contacts the fixed registration pins, that portion of the film from the entrance slot around to these pins must be brought suddenly to a stop. This is done by a set of deceleration pins which are actuated by a cam. The design of this cam requires a knowledge of the motion of the film perforation in which the oscillating pins engage. The exact point paths of these perforations can be obtained by using the equations and techniques already developed with some provision for rotor curvature.

Furthermore, in considering the acceleration diagrams at the later stage of loop growth (from the time around half of a frame cycle), it is found that maximum acceleration and deceleration forces are developed near the loop ends and directed towards the rotor shoe curve as shown in Figure 18b. These forces tend to pull the film tightly against the rotor shoes and consequently create large friction force which can cause scratching of the film edges. In the prototype, air is pulsed through jets in the trailing shoe curve as the shoe passes the film entry point. This cushions the contact between the shoe and the trailing portion of the growing loop. It would be advantageous to have a second pulse of air from jets placed close to the bottom of both shoe faces during this later stage of loop growth.

## REFERENCES

- (1) A. J. Holman, "Continuous Non-Intermittent Projectors", Journal of the Society of Motion Picture and Television Engineers, Vol. 16, May 1931.
- (2) J. L. Spence, "Mechanical Advantages of the Optical Intermittent Projectors", Journal of the Society of Motion Picture and Television Engineers, Vol. 18, May 1932.
- (3) F. Tuttle and C. D. Reid, "The Problem of Motion Picture from Continuously Moving Film", Journal of the Society of Motion Picture and Television Engineers, Vol. 20, January 1933.
- (4) A. Hayek, "Design Factors in 35 mm. Intermittent Mechanisms", Journal of the Society of Motion Picture and Television Engineers, Vol. 49, November 1947.
- (5) W. C. Plank, "Non-Intermittent Motion Picture Projection", Journal of the Society of Motion Picture and Television Engineers, Vol. 54, June 1950.
- (6) D. A. Spencer and H. D. Waley, "The Cinema Today", Oxford University Press, 1956.
- (7) J. R. Cameron, "Motion Picture Projection", 14th ed. Cameron, 1959.
- (8) - - - "Canned Fingers Advance Movie Film", Machine Design, Vol. 32, September 1960.

- (9) A. Kingston, "The Development of the Motion Picture Intermittent Mechanism", Photographic Journal, Vol. 108, February 1968.
- (10) P. R. W. Jones, "A New Concept of Film Transport", Society of Motion Picture and Television Engineers, Technical paper presented at the 102nd Technical Conference, September 1967.
- (11) J. Prescott, "Applied Elasticity", New York; Dover Publications, 1961.
- (12) A. E. H. Love, "A Treatise on the Mathematical Theory of Elasticity", New York; Dover Publications, 1964.
- (13) G. Schwesinger, "The Compliance of Film Loops", Journal of the Society of the Motion Picture and Television Engineers, Vol. 57, October 1957.
- (14) S. Timoshenko, "Strength of Materials" Vol. 2, 3rd ed., New York; D. Van Nostrand, 1965.
- (15) M. G. Salvadori and R. J. Schwarz, "Differential Equations in Engineering Problems", Prentice Hall, 1958.
- (16) N. W. McLachlan, "Ordinary Non-Linear Differential Equations in Engineering and Physical Science", Oxford, 1956.
- (17) P. F. Byrd and M. D. Friedman, "Handbook of Elliptic Integrals for Engineers and Physicists", Berlin; Springer, 1954.

APPENDICES

APPENDIX A

CHANGE VARIABLE OF INTEGRATION

Salvadori [15] suggests to transform the equation 4-215, i.e.,

$$dS = \frac{1}{\sqrt{2} C} \frac{d\theta}{\sqrt{\cos\theta - \cos 2\alpha}} \quad \text{A-1}$$

into an intergratable form by changing variable  $\theta$ , in the following manner:

The sine and cosine relation gives

$$\cos\theta = 1 - 2\sin^2 \frac{\theta}{2}$$

then, the denominator of relation A-1 becomes

$$\sqrt{\cos\theta - \cos 2\alpha} = \sqrt{2(\sin^2 \alpha - \sin^2 \frac{\theta}{2})}$$

or

$$\sqrt{\cos\theta - \cos 2\alpha} = \sqrt{2} \sin\alpha \sqrt{1 - \frac{\sin^2 \frac{\theta}{2}}{\sin^2 \alpha}} \quad \text{A-2}$$

Since

$$0 \leq \theta \leq 2\alpha, \quad \text{and}$$

$$2\alpha \leq \pi$$

Therefore

$$0 \leq \frac{\sin \frac{\theta}{2}}{\sin \alpha} \leq 1$$

Let

$$\sin\psi = \frac{\sin \frac{\theta}{2}}{\sin \alpha} \quad \text{A-3}$$

where

$$0 \leq \psi \leq \frac{\pi}{2}$$

Then, equation A-2 becomes

$$\sqrt{\cos\theta - \cos 2\alpha} = \sqrt{2} \sin\alpha \sqrt{1 - \sin^2 \psi}$$

or

$$\sqrt{\cos\theta - \cos 2\alpha} = \sqrt{2} \sin\alpha \cdot \cos\psi \quad \text{A-4}$$

Differentiate both sides of equation A-3:

$$\cos\psi \, d\psi = \frac{1}{2} \frac{\cos \frac{\theta}{2}}{\sin\alpha} \cdot d\theta$$

Separating the variables:

$$d\theta = \frac{2\sin\alpha \cdot \cos\psi \, d\psi}{\cos \frac{\theta}{2}}$$

or

$$d\theta = \frac{2\sin\alpha \cos\psi \, d\psi}{\sqrt{1 - \sin^2 \frac{\theta}{2}}} \quad \text{A-5}$$

$$\left(\text{since } \cos \frac{\theta}{2} = \sqrt{1 - \sin^2 \frac{\theta}{2}}\right) .$$

Substitute  $d\theta$  and  $\sqrt{\cos\theta - \cos 2\alpha}$  by the expressions A-5 and A-4 respectively into relation A-1:

$$ds = \frac{1}{C} \frac{d\psi}{\sqrt{1 - \sin^2 \frac{\theta}{2}}}$$

But, according to A-3

$$\sin \frac{\theta}{2} = \sin\alpha \cdot \sin\psi$$

Then

$$ds = \frac{1}{C} \frac{d\psi}{\sqrt{1 - \sin^2 \alpha \cdot \sin^2 \psi}} \quad \text{A-6}$$

## APPENDIX B

### STATIC LOOP WITH INCLINED CLAMPED PLANES

In the Rolling Loop Projector the film loop is actually rolled on the internal cylindrical surface of the stator. Statically, this loop can be considered as equivalent to a flexible strip held at the two ends - where the loop contacts the curve - by two fixed clamps whose planes are inclined at an angle  $\beta$  to the horizontal plane as shown in Figure B1. Again, in this configuration, the loop shape is symmetrical around  $ZZ'$ , consequently only half of the loop, eg., section OB will be taken into consideration.

#### B.1 Governing Equations

Suppose that A is the inflection point on section OB where there is no bending moment. The reaction force  $P_1$  acting at this point must be parallel to the x-axis as shown in Figure B2. Consider section OA with the coordinate system as shown in Figure B3.

Let

- $S_1$  = length of the curve OA
- $L_1, y_1$  = coordinates of point A
- $M_1$  = bending moment at O
- $P$  = supporting force
- $P_1$  = reaction force at the inflection point.



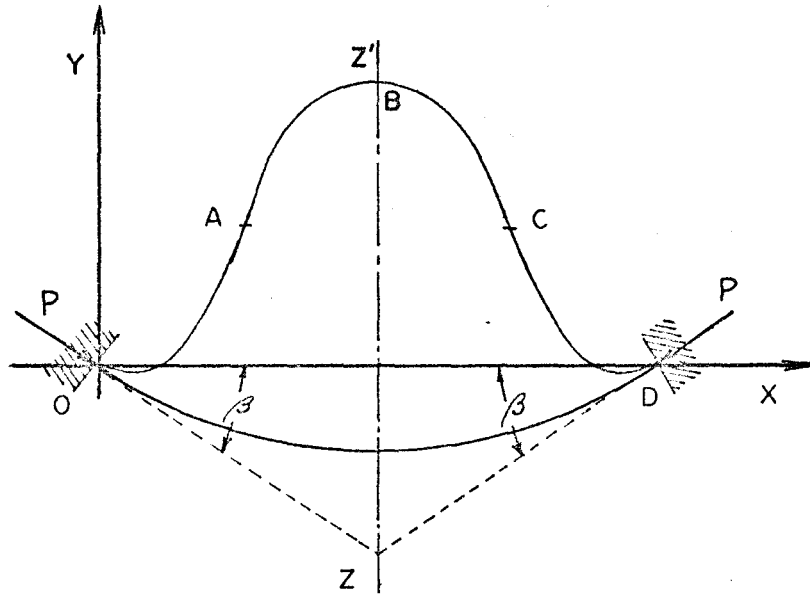


Fig. B1

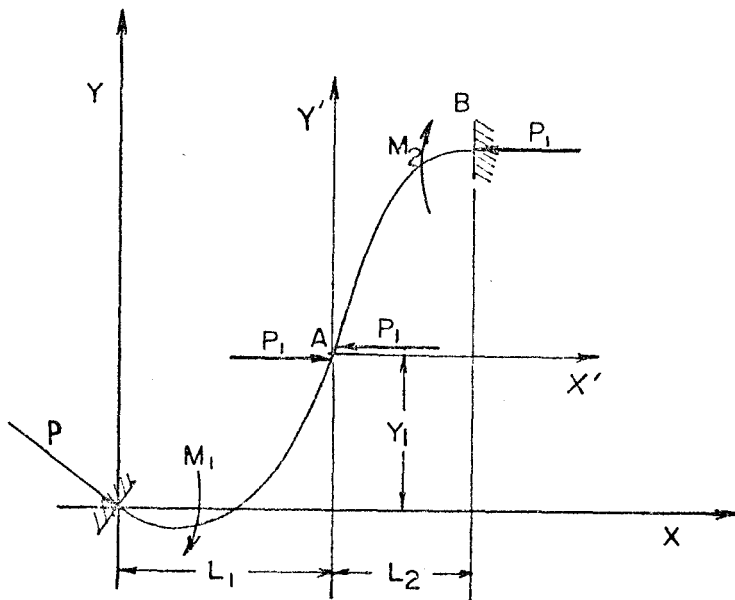


Fig. B2

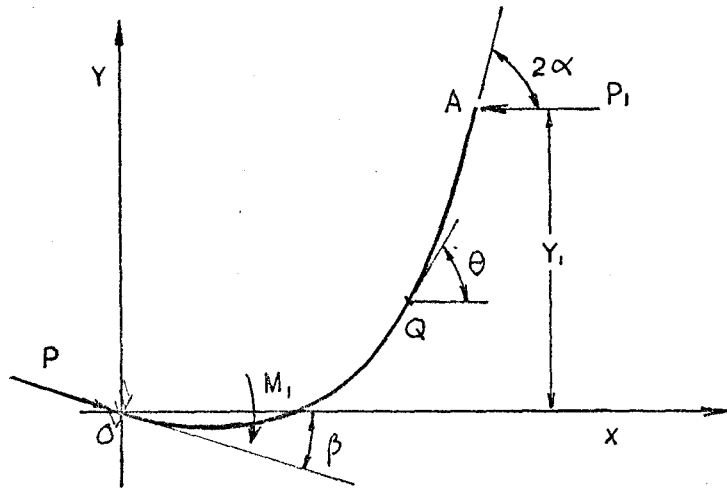


Fig. B3

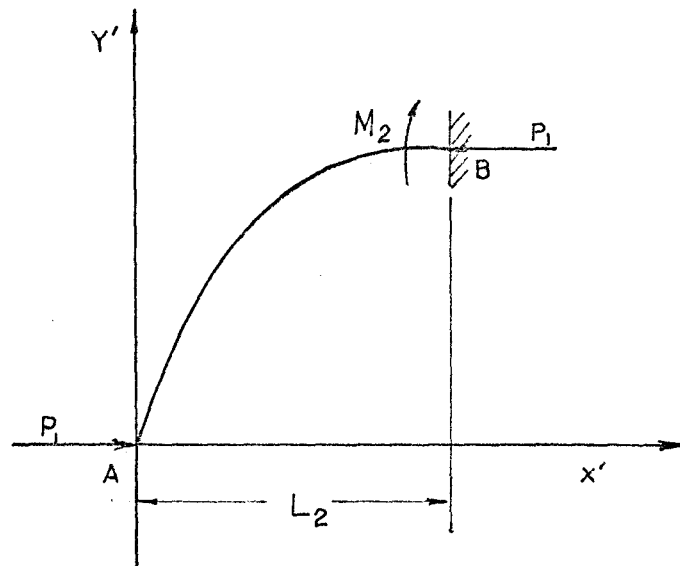


Fig. B4

The first part of the analysis is identical to that of section 4.21 and equations from 4-211 to 4-215 apply. Then

$$ds = \frac{1}{\sqrt{2C}} \frac{d\theta}{\sqrt{\cos \theta - \cos 2\alpha}}.$$

Integrate both sides and note that in this configuration  $\theta$  ranges from  $-\beta$  to  $2\alpha$ , then

$$s = \frac{1}{\sqrt{2C}} \int_{-\beta}^{\theta} \frac{d\theta}{\sqrt{\cos \theta - \cos 2\alpha}}.$$

This integration can be broken down as

$$s = \frac{1}{\sqrt{2C}} \left[ \int_{-\beta}^0 \frac{d\theta}{\sqrt{\cos \theta - \cos 2\alpha}} + \int_0^{\theta} \frac{d\theta}{\sqrt{\cos \theta - \cos 2\alpha}} \right]$$

or

$$s = \frac{1}{\sqrt{2C}} \left[ \int_0^{\beta} \frac{d\theta}{\sqrt{\cos \theta - \cos 2\alpha}} \pm \int_0^{|\theta|} \frac{d\theta}{\sqrt{\cos \theta - \cos 2\alpha}} \right], \text{B-11}$$

where the positive sign is applied for  $\theta > 0$  and vice-versa. After changing the variable of integration by the method described in Appendix A, equation B-11 finally reduces to

$$s = \frac{1}{C} [F(k, \phi_1) \pm F(k, |\phi|)] \quad \text{B-12a}$$

where

$$\phi_1 = \sin^{-1} \frac{\sin \frac{\beta}{2}}{\sin \alpha},$$

$$\phi = \sin^{-1} \frac{\sin \frac{\theta}{2}}{\sin \alpha}$$

and  $\quad +$  sign for  $\phi > 0$   
 $\quad -$  sign for  $\phi < 0$ .

The expression for  $x$  and  $y$  can be deduced in a similar manner, and the results are

$$x = \frac{1}{C} [2\{E(K, \phi_1) \pm E(K, |\phi|)\} - \{F(K, \phi_1) \pm F(K, |\phi|)\}] \quad \text{B-12b}$$

$$y = \frac{2K}{C} (\cos \phi_1 - \cos \phi) \quad \text{B-12c}$$

For section AB shown in Figure B4, the loop has the same end conditions as those analyzed in section 4.3. Applied to coordinate system (Ax'y'), equations from 4-31 to 4-33 give

$$S = \frac{1}{C} [E(K, \frac{\pi}{2}) - F(K, \phi)]$$

$$x = \frac{1}{C} [2\{E(K, \frac{\pi}{2}) - E(K, \phi)\} - \{F(K, \frac{\pi}{2}) - F(K, \phi)\}]$$

$$y = \frac{2K}{C} \cos \phi$$

where  $\phi$  ranges from  $\frac{\pi}{2}$  to 0.

By transferring to coordinate system (oxy) (see Fig. B2), these equations become

$$S = S_1 + \frac{1}{C} [F(K, \frac{\pi}{2}) - F(K, \phi)]$$

but

$$S_1 + \frac{1}{C} F(K, \frac{\pi}{2}) = \frac{S_T}{2}$$

then

$$S = \frac{S_T}{2} - \frac{F(K, \phi)}{C} \quad \text{B-13a}$$

$$x = L_1 + \frac{1}{C} [2\{E(K, \frac{\pi}{2}) - E(K, \phi)\} - \{F(K, \frac{\pi}{2}) - F(K, \phi)\}]$$

but

$$L_1 + \frac{1}{C} [2E(K, \frac{\pi}{2}) - F(K, \frac{\pi}{2})] = \frac{L_T}{2}$$

then

$$x = \frac{L_T}{2} - \frac{1}{C} [2E(K, \phi) - F(K, \phi)]. \quad \text{B-13b}$$

And

$$y = y_1 + \frac{2K}{C} \cos \phi$$

but

$$y_1 = \frac{2K}{C} \cos \phi_1 \quad (\text{from B-12c})$$

thus

$$y = \frac{2K}{C} (\cos \phi_1 + \cos \phi). \quad \text{B-13c}$$

The equations for the rest of the curve can be derived in a similar manner.

For section BC:

$$s = \frac{S_T}{2} + \frac{F(K, \phi)}{C} \quad \text{B-14a}$$

$$x = \frac{L_T}{2} + \frac{1}{C} [2E(K, \phi) - F(K, \phi)] \quad \text{B-14b}$$

$$y = \frac{2K}{C} (\cos \phi_1 + \cos \phi) \quad \text{B-14c}$$

where  $\phi$  increases from 0 to  $\frac{\pi}{2}$ .

For section CD:

$$s = S_T - \frac{1}{C} [F(K, \phi_1) \pm F(K, |\phi|)] \quad \text{B-15a}$$

$$x = L_T - \frac{1}{C} [2\{E(K, \phi_1) \pm E(K, |\phi|)\} - \{F(K, \phi_1) \pm F(K, |\phi|)\}] \quad \text{B-15b}$$

$$y = \frac{2K}{C} (\cos \phi_1 - \cos \phi) \quad \text{B-15c}$$

where  $\phi$  increases from  $-\beta$  to  $\frac{\pi}{2}$ .

Summary:

Let

$$F_1 = F(K, \phi_1)$$

$$E_1 = E(K, \phi_1)$$

By using the same subscripts and notation as in section 4.3, the equations governing the loop shape are

$$S_A = \frac{1}{C} [F_1 \pm F_A] \quad \text{B-16a}$$

$$x_A = \frac{1}{C} [2(E_1 \pm E_A) - (F_1 \pm F_A)] \quad \text{B-16b}$$

$$y_A = \frac{2K}{C} (\cos \phi_1 - \cos \phi_A) \quad \text{B-16c}$$

$$S_B = \frac{S_T}{2} - \frac{F_B}{C} \quad \text{B-17a}$$

$$x_B = \frac{L_T}{2} - \frac{1}{C} [2E_B - F_B] \quad \text{B-17b}$$

$$y_B = \frac{2K}{C} (\cos \phi_1 + \cos \phi_B) \quad \text{B-17c}$$

$$S_C = \frac{S_T}{2} + \frac{F_C}{C} \quad \text{B-18a}$$

$$x_C = \frac{L_T}{2} + \frac{1}{C} (2E_C - F_C) \quad \text{B-18b}$$

$$y_C = \frac{2K}{C} (\cos \phi_1 + \cos \phi_C) \quad \text{B-18c}$$

$$S_D = S_T - \frac{1}{C} (F_1 \pm F_D) \quad \text{B-19a}$$

$$x_D = L_T - \frac{1}{C} [2(E_1 \pm E_D) - (F_1 \pm F_D)] \quad \text{B-19b}$$

$$y_D = \frac{2K}{C} (\cos \phi_1 - \cos \phi_D) \quad \text{B-19c}$$

where

$\phi_A$  and  $\phi_D$  range from  $-\phi_1$  to  $\frac{\pi}{2}$ ,

while  $\phi_B$  and  $\phi_C$  range from 0 to  $\frac{\pi}{2}$ ,

and  $\quad \quad \quad + \text{ sign for } \phi > 0$   
 $\quad \quad \quad - \text{ sign for } \phi < 0.$

Using these equations, the curve of the bending strip was derived and plotted as shown in Figure B5.

## B.2 Calculation of Parameter K

When  $\phi_A = \frac{\pi}{2}$ , relations B-16a and B-16b become

$$S_1 = \frac{1}{C} [F_1 + F_0] \quad \text{B-21}$$

and 
$$L_1 = \frac{1}{C} [2(E_1 + E_0) - (F_1 + F_0)] \quad \text{B-22}$$

Also, when  $\phi_B = \frac{\pi}{2}$ , equations B-17a and B-17b become

$$S_1 = \frac{S_T}{2} - \frac{F_0}{C} \quad \text{B-23}$$

and,

$$L_1 = \frac{L_T}{2} - \frac{1}{C} [2E_0 - F_0]. \quad \text{B-24}$$

Equate B-21 and B-23 to get

$$\frac{1}{C} [F_1 + F_0] = \frac{S_T}{2} - \frac{F_0}{C}$$

or

$$\frac{S_T}{2} = \frac{1}{C} [F_1 + 2F_0]. \quad \text{B-25}$$

Replace  $L_1$  by the expression B-22, relation B-24 becomes

$$\frac{1}{C} [2(E_1 + E_0) - (F_1 + F_0)] = \frac{L_T}{2} - \frac{1}{C} [2E_0 - F_0].$$

LOOP SHAPE ON CURVED SURFACE

$S_T = 5.805$  in.  
 $L_T = 3.0$  in.  
 $\sin(\beta) = 0.166$

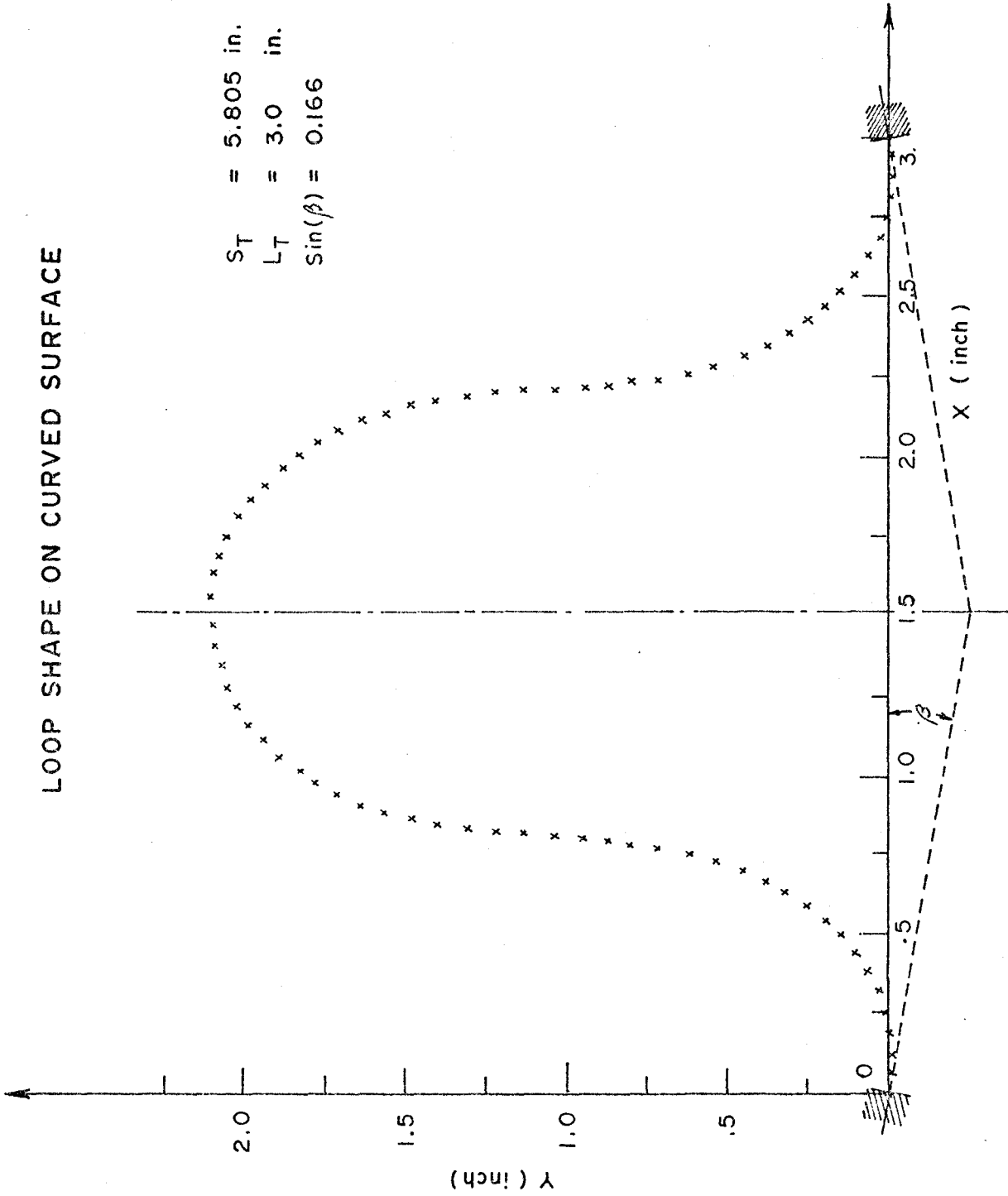


Fig. B5



In rearranging terms:

$$\frac{L_T}{2} = \frac{1}{C} [2(2E_O + E_1) - (2F_O + F_1)] \quad \text{B-26}$$

Dividing each side of relation B-26 by the corresponding side of B-25:

$$\frac{L_T}{S_T} = \frac{2(2E_O + E_1) - (2F_O + F_1)}{2F_O + F_1}$$

or

$$\frac{L_T}{S_T} = 2 \frac{2E_O + E_1}{2F_O + F_1} - 1$$

or

$$\frac{2E_O + E_1}{2F_O + F_1} = \frac{1}{2} \left( \frac{1}{R} + 1 \right) \quad \text{B-27}$$

If the ratio  $R = \frac{L_T}{S_T}$  is known, then the value  $K$  of the

elliptic integrals can be evaluated by relation B-27.

For a given inclination angle  $\beta$ , there is a minimum limit for  $R$ , below which  $K$  does not exist.

From the condition given in equation B-12a

$$\phi_1 = \sin^{-1} \frac{\sin \frac{\beta}{2}}{\sin \alpha}$$

when  $\phi_1 = \frac{\pi}{2}$ , then the value of  $K$  is

$$K = \sin \alpha = \sin \frac{\beta}{2}$$

Then the relation B-27 becomes

$$\frac{E_O}{F_O} = \frac{1}{2} \left( \frac{1}{R} + 1 \right) \quad \text{B-28}$$

where 
$$K = \sin \frac{\beta}{2}$$

(since  $E_1 = E_0$  and  $F_1 = F_0$  when  $\phi_1 = \frac{\pi}{2}$ ).

From relation B-28, the minimum value of  $R$  is calculated for a given angle  $\beta$ . In the case of a strip being fixed on the inner side of a cylindrical surface, if the value of  $R$  is reduced below its minimum for a given angle  $\beta$ , the parameter  $K$  must be reduced accordingly by shifting the points of contact between the strip and the cylindrical surface toward each other so that the relation B-28 is always satisfied. The shape of the strip under this condition is plotted as shown in Figure B6, in which the angle of the curve at the inflection point is equal to  $\beta$ , which is the inclination angle of the clamped planes with respect to the x-axis.

### B.3 Loop Characteristics

#### B.3.1 Parameter $K$ and Supporting Force

The parameter  $K$  of the elliptic integral can be calculated by using the relation B-27 already derived.

In reference to Figure B3, the summation of the forces in the x-direction gives

$$P \cos \beta - P_1 = 0.$$

or

$$P = \frac{P_1}{\cos \beta}$$

LOOP SHAPE ON CURVED SURFACE

WITH MINIMUM LOOP LENGTH

$$L_T = 3.0 \text{ in.}$$

$$\sin(\beta) = 0.166$$

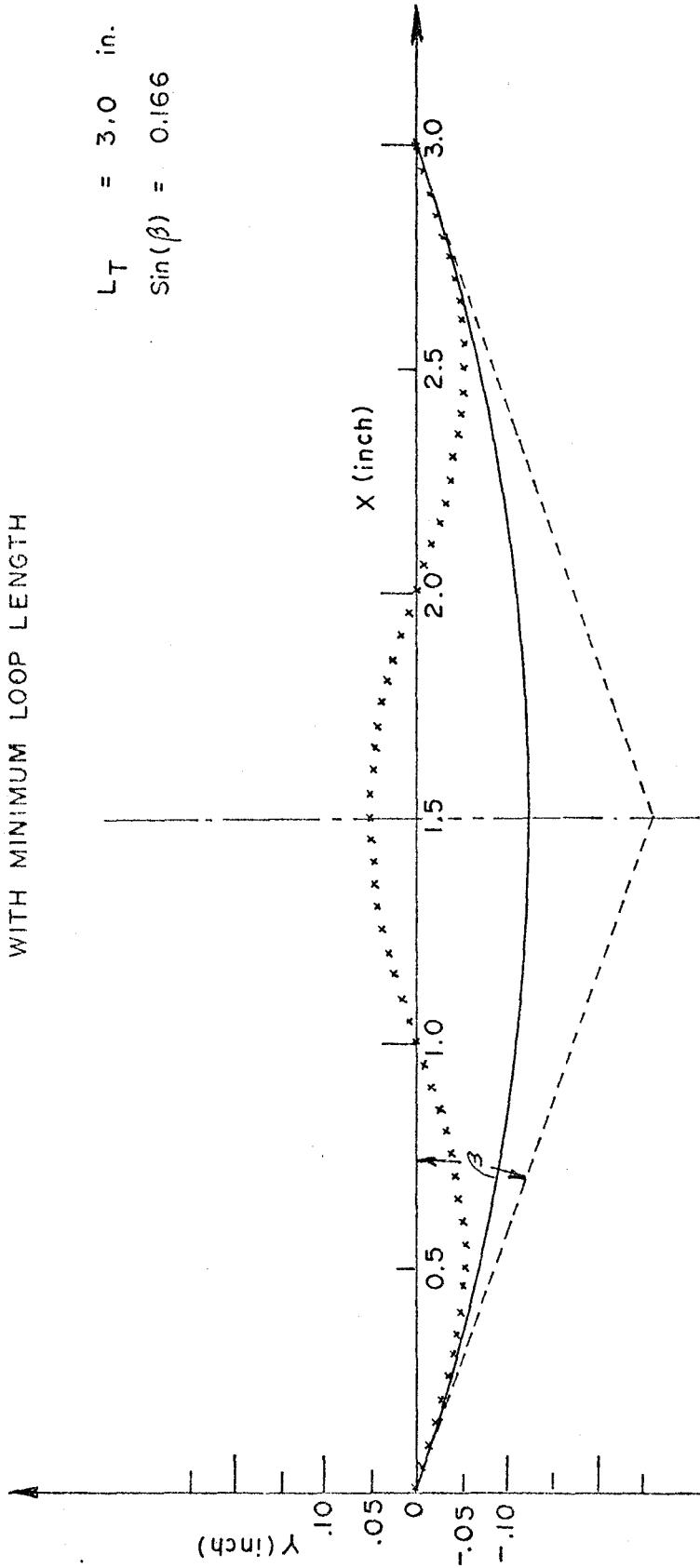


Fig. B6

The parameter C is given by

$$C = \sqrt{\frac{P_1}{E'I}}$$

or

$$P_1 = C^2 E'I$$

Then, the expression for P can be written as

$$P = \frac{C^2 E'I}{\cos \beta} \quad \text{B-311}$$

Furthermore, the value of C is given by equation B-25 as

$$C = \frac{2}{S_T} [F_1 + 2F_0]$$

So, equation B-311 becomes

$$P = \frac{4[F_1 + 2F_0]^2 E'I}{S_T^2 \cos \beta} \quad \text{B-312}$$

The critical supporting force can be found by letting

$K = \sin \frac{\beta}{2}$  and the result<sup>2</sup> is

$$P_C = \frac{36E'I F_0^2}{S_T^2 \cos \beta} \quad \text{B-313}$$

<sup>2</sup>When K approaches zero,  $F_0$  reduces to  $\frac{\pi}{2}$  and B-313 becomes

$$P_C = \frac{9\pi^2 E'I}{S_T^2}$$

This is the Euler critical load for buckling a strut of length  $S_T$  with a buckling mode  $n = 3$ ).

The non-dimensional form of the supporting force is obtained by dividing each side of relation B-312 by the corresponding side of B-313:

$$\frac{P}{P_C} = \frac{1}{9} \left( \frac{F_1 + 2F_0}{F_0} \right)^2 \quad \text{B-314}$$

### B.3.2 Loop Height

When  $\phi_B = 0$ , equation B-17c becomes

$$Y_L = \frac{2K(1 + \cos\phi_1)}{C}$$

but

$$C = \frac{2}{S_T} [F_1 + 2F_0] \quad (\text{from eq. B-25})$$

Thus

$$Y_L = \frac{K(1 + \cos\phi_1) S_T}{F_1 + 2F_0}$$

Dividing both sides by  $L_T$ :

$$\frac{Y_L}{L_T} = \frac{K(1 + \cos\phi_1)}{F_1 + 2F_0} \left( \frac{S_T}{L_T} \right)$$

or

$$\frac{Y_L}{L_T} = \frac{K(1 + \cos\phi_1) R}{F_1 + 2F_0} \quad \text{B-321}$$

### B.3.2 Bending Moments

The bending moments at the clamped ends of the strip are calculated in a similar manner as in section 4.4.4.

$$M_1 = P_1 Y_1$$

but

$$P_1 = P \cdot \cos\beta = \frac{4[F_1 + 2F_0]^2 E'I}{S_T^2} \quad (\text{from eq. B-312}),$$

and

$$y_1 = \frac{2K}{C} \cos\phi_1. \quad (\text{from eq. B-16c})$$

$M_1$  finally reduces to

$$M_1 = \frac{4KE'I}{S_T} (F_1 + 2F_0) \cos\phi_1.$$

On placing in dimensionless form:

$$\frac{M_1 S_T}{EI} = 4K(F_1 + 2F_0) \cos\phi_1$$

or

$$\frac{M_1}{B} = 4K(F_1 + 2F_0) \cos\phi_1 \quad \text{B-331}$$

In a similar fashion, the bending moment at the top of the loop curve is calculated:

$$\frac{M_2}{B} = 4K(F_1 + 2F_0) \quad \text{B-332}$$

The results obtained for the characteristics of the loop are plotted against ratio  $S_T/L_T$  keeping  $\beta$  constant as shown in Figure B7.

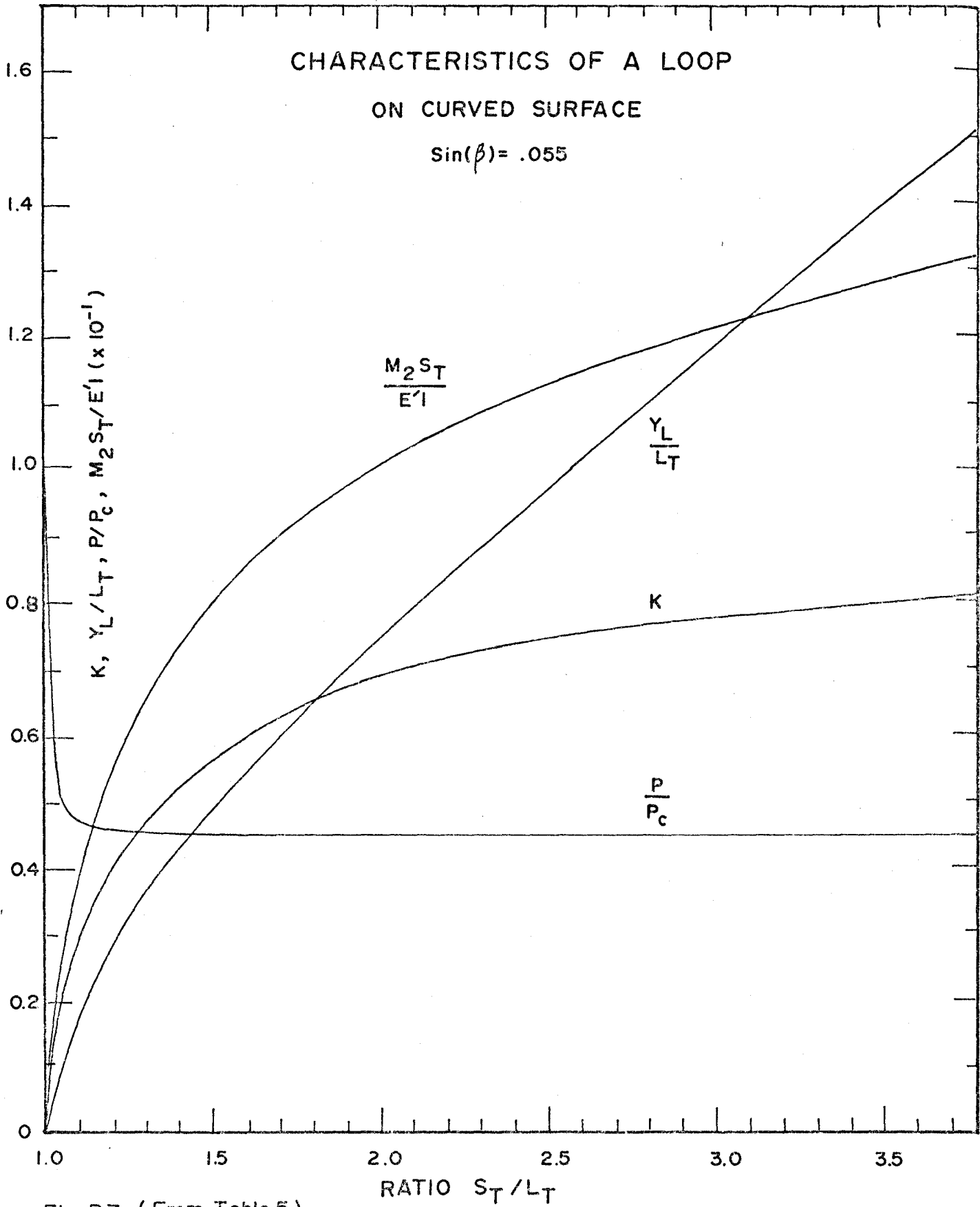


Fig. B7 (From Table 5)

APPENDIC C

SAMPLE OF COMPUTER PROGRAMMES



Note:

The following is a typical set of programmes which were written in Fortran IV to perform the calculation and to plot the velocity and acceleration diagrams of the loop in motion. These programmes were processed on the Control Data Corp. computer (model 6400) in coordination with the Benson Lehner plotter.

```

C     PROGRAMME A
C     *****
C
C     EARLY STAGE OF LOOP GROWTH.
C
C     TO COMPUTE THE COORDINATES OF SELECTED POINTS ON THE FILM STRIP
C     AND TO FIND THE VELOCITY AND ACCELERATION VECTORS OF THESE POINTS
C     DURING THE EARLY STAGE OF LOOP GROWTH.
C
C     DATA USED =
C     -----
C     DISTANCE BETWEEN SELECTED POINTS DS= .1875 INCH.
C     TIME STEPSIZE DT= 1/24/500 SEC.
C     CASES CONSIDERED= TIME J= .2, .3, .....,9
C
C     NOTATION=
C     -----
C     MP           = NUMBER OF FIXED POINTS ON THE FILM STRIP.
C     MT           = NUMBER OF TIME INTERVALS TAKEN.
C     I            = SUBSCRIPT FOR THE ORDER OF THE POINTS ON THE FILM STRIP.
C     J            = SUBSCRIPT FOR TIME INTERVAL.
C     SP(I)        = POSITION OF POINT I ON THE FILM STRIP MEASURED ALONG
C                   THE FILM CURVE FROM ORIGIN O.
C     SC(J)        = CURVILINEAR COORDINATE OF MOVING ORIGIN O(M) WRT.
C                   ORIGIN O.
C     X(I,J)       = ABCISSA OF POINT I AT TIME J WRT. FIXED COORDINATE
C                   SYSTEM OXY.
C     Y(I,J)       = ORDINATE OF POINT I AT TIME J WRT. FIXED COORDINATE
C                   SYSTEM OXY.
C     DT           = TIME INCREMENT.
C     DS           = DISTANCE BETWEEN SELECTED POINTS.
C     VR           = LINEAR VELOCITY OF THE ROTOR.
C     VF           = VELOCITY OF THE FILM INFEEED.
C     AL           = ROTOR GAP WIDTH.
C     ES(J)        = TOTAL LENGTH OF FILM IN ROTOR GAP AT TIME J.
C     SO           = CURVILINEAR COORDINATE OF O(M) AT THE BEGINNING OF THE
C                   PHASE CONSIDERED.
C     XC(J)        = ABCISSA OF O(M) WRT. SYSTEM OXY AT TIME J.
C     XO           = ABCISSA OF O(M) WRT. SYSTEM OXY AT THE BEGINNING OF
C                   THE PHASE CONSIDERED.
C     YO(J)        = HEIGHT OF THE LOOP.
C     SM,XM        = CURVILINEAR COORDINATE AND THE ABCISSA OF A POINT WRT.
C                   MOVING COORDINATE SYSTEM O(M)X(M)Y(M).
C     VA(I,J)      = VELOCITY AMPLITUDE.
C     PHIV(I,J)    = ANGLE OF VELOCITY VECTOR.
C     AA(I,J)      = ACCELERATION AMPLITUDE.
C     PHIA(I,J)    = ANGLE OF ACCELERATION VECTOR.
C
C
C     DIMENSION X(100,10),Y(100,10),SP(100),SC(10),XC(10),ES(10),YO(10)
C     COMMON     VA(100,10),PHIV(100,10),AA(100,10),PHIA(100,10)
C     COMMON X,Y,DT

```

```

C
C   INITIALIZE
C
MP=100
MT=10
NL1=2
NL2=9
DK=0.0
PI=3.1415927
DT=1./24./500.
VR=452.3893
VF=67.5
XO=21.8
SO=XO
AL=3.
VDF=(VR-VF)*DT
VRF=VR*DT
VFF=VF*DT
DS=.187/5.
DX=DS
C
C   AT TIME T=0.
C
J=1
MP1=MP+1
C   ORIGIN OF MOVING COORDINATE SYSTEM.
SC(J)=SO
XC(J)=SO
ES(J)=AL
C   COORDINATES OF A POINT ON THE FILM.
DO 1 I=1,MP
SP(I) =SO-DX*FLOAT(I-1)
X(I,J)=SP(I)
Y(I,J)=0.0
YO(J)=0.0
1  CONTINUE
C
C   AT TIME T, J=2,MT
C
DO 2 J=2,MT
NC1=0
NC2=0
NC3=0
NC4=0
C   CALCULATE COORDINATES OF POINT O(M) AND LENGTH OF FILM IN GAP.
FL=FLOAT(J-1)
SC(J)=SO-VDF*FL
XC(J)=XO-VRF*FL
ES(J)=VFF*FL+AL
C   FIND K AND C.
RA=.5*(AL/ES(J)+1.)
PI2=PI/2.
AK1=0.
RAT1=1.0

```

```

AK=DK
3 CONTINUE
C INTERPOLATION.
CALL ELLIS(RF,AK,PI2,1)
CALL ELLIS(RE,AK,PI2,2)
RAT=RE/RF
IF(RAT.GT.RA) GO TO 4
CALL INTER(AKX,RA,AK1,RAT1,AK,RAT)
C SET K EQUAL TO ITS CLOSEST VALUE FOR THE NEXT INTERPOLATION.
DK=AKX
CALL ELLIS(RF,AKX,PI2,1)
C=4.*RF/ES(J)
STEST=SC(J)-ES(J)
YO(J)=4.*AKX/C
GO TO 5
4 CONTINUE
RAT1=RAT
AK1=AK
AK=AK+.01
GO TO 3
5 CONTINUE
C
C COMPUTE COORDINATES OF POINTS ON THE FILM STRIP.
C
DO 6 I=1,MP
C TESTING TO SEE IF THIS POINT IS ON THE LOOP.
IF(SP(I).GE.SC(J)) GO TO 7
IF(SP(I).GT.STEST) GO TO 8
C POINT IS ON THE FLAT PORTION AT THE RHS. OF THE LOOP.
X(I,J)=SP(I)
Y(1,J)=0.0
GO TO 6
7 CONTINUE
C POINT IS ON THE FLAT PORTION AT THE LHS. OF THE LOOP.
JM=J-1
X(I,J)=X(I,JM)-VFF
Y(I,J)=0.0
GO TO 6
8 CONTINUE
C POINT IS ON THE LOOP.
C FIND ON WHAT PORTION OF THE LOOP THIS POINT IS SITUATED.
SM=SC(J)-SP(I)
IF(SM.LT.(ES(J)/4.)) GO TO 9
IF(SM.LT.(ES(J)/2.)) GO TO 10
IF(SM.LT.(.75*ES(J))) GO TO 11
IF(SM.LT.ES(J)) GO TO 12
STOP 12345
9 CONTINUE
C
C POTION I
C
PHI=0.0
IF(NC1.EQ.1) PHI=PHIX+.01
14 CONTINUE

```

```

C      CALCULATING PHI CORRESPONDING TO SM.
      IF(PHI.GT.PI2) PHI=PI2
      CALL ELLIS(RF,AKX,PHI,1)
      S=RF/C
      IF(S.EQ.SM) GO TO 103
      IF(S.LT.SM) GO TO 13
      CALL INTER(PHIX,SM,PHI1,S1,PHI,S)
      GO TO 102
103   CONTINUE
      PHIX=PHI
102   CONTINUE
      NC1=1
C      CALCULATE COORDINATES X,Y.
      CALL ELLIS(RF,AKX,PHIX,1)
      CALL ELLIS(RF,AKX,PHIX,2)
      XM=1./C*(2.*RE-RF)
      X(I,J)=XC(J)-XM
      Y(I,J)=2.*AKX/C*(1.-COS(PHIX))
      GO TO 6
13    CONTINUE
      S1=S
      PHI1=PHI
      PHI=PHI+.01
      GO TO 14
10    CONTINUE
C
C      PORTION II
C
      PHI=PI2
      IF(NC2.EQ.1) PHI=PHIX-.01
C      CALCULATING PHI CORRESPONDING TO SM.
15    CONTINUE
      IF(PHI.LT.1.0E-09) PHI=0.0
      CALL ELLIS(RF,AKX,PHI,1)
      S=ES(J)/2.-RF/C
      IF (S.EQ.SM) GO TO 105
      IF(S.LT.SM) GO TO 16
      CALL INTER(PHIX,SM,PHI1,S1,PHI,S)
      GO TO 106
105   CONTINUE
      PHIX=PHI
106   CONTINUE
      NC2=1
C      CALCULATE COORDINATES X,Y.
      CALL ELLIS(RF,AKX,PHIX,1)
      CALL ELLIS(RE,AKX,PHIX,2)
      XM= AL/2.-1./C*(2.*RE-RF)
      X(I,J)=XC(J)-XM
      Y(I,J)=2.*AKX/C*(1.+COS(PHIX))
      GO TO 6
16    CONTINUE
      S1=S
      PHI1=PHI
      PHI=PHI-.01

```

```

GO TO 15
11 CONTINUE
C
C PORTION III
C
PHI=0.0
IF(NC3.EQ.1) PHI=PHIX+.01
C CALCULATING PHI CORRESPONDING TO SM.
17 CONTINUE
IF(PHI.GT.PI2) PHI=PI2
CALL ELLIS(RF,AKX,PHI,1)
S=ES(J)/2.+RF/C
IF(S.EQ.SM) GO TO 107
IF(S.LT.SM) GO TO 18
CALL INTER(PHIX,SM,PHI1,S1,PHI,S)
GO TO 108
107 CONTINUE
PHIX=PHI
108 CONTINUE
NC3=1
C CALCULATE COORDINATES X,Y.
CALL ELLIS(RF,AKX,PHIX,1)
CALL ELLIS(RE,AKX,PHIX,2)
XM=AL/2.+1./C*(2.*RE-RF)
X(I,J)=XC(J)-XM
Y(I,J)=2.*AKX/C*(1.+COS(PHIX))
GO TO 6
18 CONTINUE
S1=S
PHI1=PHI
PHI=PHI+.01
GO TO 17
12 CONTINUE
C
C PORTION IV
C
PHI=PI2
IF(NC4.EQ.1) PHI=PHIX-.01
C CALCULATING PHI CORRESPONDING TO SM.
19 CONTINUE
IF(PHI.LT.1.0E-09) PHI=0.0
CALL ELLIS(RF,AKX,PHI,1)
S=ES(J)-RF/C
IF(S.EQ.SM) GO TO 109
IF(S.LT.SM) GO TO 20
CALL INTER(PHIX,SM,PHI1,S1,PHI,S)
GO TO 110
109 CONTINUE
PHIX=PHI
110 CONTINUE
NC4=1
C CALCULATE COORDINATES X,Y.
CALL ELLIS(RF,AKX,PHIX,1)
CALL ELLIS(RE,AKX,PHIX,2)

```

```

XM=AL-1./C*(2.*RF-RF)
X(I,J)=XC(J)-XM
Y(I,J)=2.*AKX/C*(1.-COS(PHIX))
GO TO 6
20 CONTINUE
S1=S
PHI1=PHI
PHI=PHI-.01
GO TO 19
6 CONTINUE
2 CONTINUE
C CALCULATE VELOCITY AND ACCELERATION VECTORS.
CALL VELACE(MP,MT)
C
C PRINT OUT RESULTS.
C
DO 50 J=NL1,NL2
K=J
C PUNCH COORDINATES OF POINTS.
WRITE(8,43)(X(I,J),Y(I,J),I=1,MP)
43 FORMAT(6F12.5)
WRITE(6,72) J
72 FORMAT(1H1,20X,*COORDINATES OF POINTS*,//,21X,*AT TIME J=*,I3)
WRITE(6,33)
33 FORMAT(1H0,2X,* I *,3X,* X *,2X,* Y *,3X,* I *,2X,*
1X *,2X,* Y *,3X,* I *,2X,* X *,2X,* Y *)
DO 36 I=1,33
I1=33+I
I2=66+I
WRITE(6,35) I,X(I,J),Y(I,J),I1,X(I1,J),Y(I1,J),I2,X(I2,J),Y(I2,J)
35 FORMAT(1H ,1X,3(13,2X,F9.4,2X,F9.4,3X))
36 CONTINUE
WRITE(6,37) J
37 FORMAT(1H1,20X,*VELOCITY VECTORS*,//,21X,*TIME J=*,I3)
WRITE(6,39)
39 FORMAT(1H0,2X,* I *,3X,* VA *,2X,* PHIV *,3X,* I *,2X,*
1VA *,2X,* PHIV *,3X,* I *,2X,* VA *,2X,* PHIV *)
DO 38 I=1,33
I1=33+I
I2=66+I
WRITE(6,45) I,VA(I,K),PHIV(I,K),I1,VA(I1,K),PHIV(I1,K),I2,VA(I2,K),
1PHIV(I2,K)
45 FORMAT(1H ,1X,3(13,2X,F9.2,2X,F9.3,3X))
38 CONTINUE
WRITE(6,40) J
40 FORMAT(1H1, 20X,*ACCELERATION VECTORS*,//,21X,*TIME J=*,I3)
WRITE(6,41)
41 FORMAT(1H0,2X,* I *,3X,* AA *,2X,* PHIA *,3X,* I *,2X,*
1AA *,2X,* PHIA *,3X,* I *,2X,* AA *,2X,* PHIA *)
DO 42 I=1,33
I1=33+I
I2=66+I
WRITE(6,45) I,AA(I,K),PHIA(I,K),I1,AA(I1,K),PHIA(I1,K),I2,AA(I2,K),
1PHIA(I2,K)

```

```

42 CONTINUE
50 CONTINUE
C DO 44 J=NL1,NL2
WRITE(8,43)(VA(I,J),PHIV(I,J),I=1,MP)
WRITE(8,43)(AA(I,J),PHIA(I,J),I=1,MP)
44 CONTINUE
STOP
END

C
C
SUBROUTINE VELACE(MP,MT)
-----
C
C SUBROUTINE TO CALCULATE VELOCITY AND ACCELERATION VECTORS FOR
C GIVEN DISPLACEMENTS IN A SPECIFIED TIME.
C
C NOTATION=
C -----
C MP = NUMBER OF FIXED POINTS ON THE FILM STRIP.
C MT = NUMBER OF TIME INTERVALS CONSIDERED.
C X,Y = COORDINATES OF POINT I AT TIME J.
C VA = VELOCITY AMPLITUDE.
C PHIV = ANGLE OF VELOCITY VECTOR.
C AA = ACCELERATION AMPLITUDE.
C PHIA = ANGLE OF ACCELERATION VECTOR.
C VX,VY = COMPONENTS OF VELOCITY VECTOR IN X- AND Y- DIRECTIONS.
C AX,AY = COMPONENTS OF ACCELERATION VECTORS IN X- AND Y-
C DIRECTIONS.
C
C DIMENSION X(100,10),Y(100,10),VA(10),PHIV(10),AA(10),PHIA(10)
COMMON VA1(100,10),PHIV1(100,10),AA1(100,10),PHIA1(100,10)
COMMON X,Y,DT
MT1=MT-1
MT2=MT-2
PI=3.1415927
CON=180./3.1415927
DO 1 I=1,MP
VA(MT)=0.0
PHIV(MT)=0.0
AA(MT1)=0.0
AA(MT)=0.0
PHIA(MT1)=0.0
PHIA(MT)=0.0
DO 2 J=1,MT2
J1=J+1
J2=J+2
C AT TIME J
IF(J.NE.1) GO TO 3
VX1=(X(I,J1)-X(I,J))/DT
VY1=(Y(I,J1)-Y(I,J))/DT
TEST=ABS(VX1)-1.0E-08
IF(TEST.LE.0.0) GO TO 9
PHIV(J)=ATAN(VY1/VX1)
IF(VX1.LT.0.0) PHIV(J)=PHIV(J)+PI

```



```

PHIV(J)=PHIV(J)*CON
GO TO 10
9 CONTINUE
PHIV(J)=90.
IF (ABS(VY1).LE.1.0E-08) PHIV(J)=0.0
10 CONTINUE
VA(J)=SQRT(VX1*VX1+VY1*VY1)
3 CONTINUE
C AT TIME J+1
VX2=(X(I,J2)-X(I,J1))/DT
VY2=(Y(I,J2)-Y(I,J1))/DT
TEST=ABS(VX2)-1.0E-08
IF (TEST.LE.0.0) GO TO 11
PHIV(J1)=ATAN(VY2/VX2)
IF (VX2.LT.0.0) PHIV(J1)=PHIV(J1)+PI
PHIV(J1)=PHIV(J1)*CON
GO TO 12
11 CONTINUE
PHIV(J1)=90.
IF (ABS(VY2).LE.1.0E-08) PHIV(J1)=0.0
12 CONTINUE
VA(J1)=SQRT(VX2*VX2+VY2*VY2)
AX=(VX2-VX1)/DT
AY=(VY2-VY1)/DT
TEST=ABS(AX)-1.0E-08
IF (TEST.LE.0.0) GO TO 13
PHIA(J)=ATAN(AY/AX)
IF (AX.LT.0.0) PHIA(J)=PHIA(J)+PI
PHIA(J)=PHIA(J)*CON
GO TO 14
13 CONTINUE
PHIA(J)=90.
IF (ABS(AY).LE.1.0E-08) PHIA(J)=0.0
14 CONTINUE
AA(J)=SQRT(AX*AX+AY*AY)
VX1=VX2
VY1=VY2
2 CONTINUE
C SHIFT VELOCITY AND ACCELERATION BACK ONE TIME STEP.
DO 42 L=1,MT1
LM=MT-L+1
LM1=LM-1
VA(LM)=VA(LM1)
PHIV(LM)=PHIV(LM1)
AA(LM)=AA(LM1)
PHIA(LM)=PHIA(LM1)
42 CONTINUE
VA(1)=0.0
PHIV(1)=0.0
AA(1)=0.0
PHIA(1)=0.0
DO 40 L=1,MT
VA(L)=VA(L)/12.

```

```

C      AA(L)=AA(L)/12.
C      CONVERT INTO TWO DIMENSIONAL ARRAYS.
      VA1(I,L)=VA(L)
      PHIV1(I,L)=PHIV(L)
      AA1(I,L)=AA(L)
      PHIA1(I,L)=PHIA(L)
40     CONTINUE
1      CONTINUE
      RETURN
      END

C
C
C      SUBROUTINE INTER(AKX,YX,AK1,Y1,AK2,Y2)
C      -----
C
C      SUBROUTINE FOR INTERPOLATION .
C
C      Y=Y1 AT K=AK1,
C      Y=Y2 AT K=AK2,
C      FIND AKX CORRESPONDING TO YX.
C      AKX=AK1+(AK2-AK1)*(YX-Y1)/(Y2-Y1) .
C
C
C      DFLK=AK2-AK1
C      DELY=Y2-Y1
C      DELYX=YX-Y1
C      DELKX=DELK*DELYX/DELY
C      AKX=AK1+DELKX
C      RETURN
C      END

C
C
C      SUBROUTINE ELLIS(R,AK,PHI,INDEX)
C      -----
C
C      SUBROUTINE TO CONVERT THE GENERALIZED ELLIPTIC INTERGRAL INTO
C      COMMONLY USED FORM.
C      LIBRARY SUBROUTINE USED = ELI2.
C
C      AK          = K .
C      PHI         = ANGLE IN RADIAN.
C      R           = VALUE OF THE ELLIPTIC INTERGRAL.
C      INDEX       = INDEX FOR THE TYPE OF ELLIPTIC INTERGRAL,
C                   INDEX = 1 FOR THE FIRST KIND,
C                   INDEX = 2 FOR THE SECOND KIND.
C
C
C      X=TAN(PHI)
C      CK=SQRT(1.-AK*AK)
C      A=1.
C      B=1.
C      CS=CK*CK
C      IF(INDEX.EQ.2) B=CS
C      CALL ELI2(R,X,CK,A,B)
C      R=ABS(R)
C      RETURN
C      END

```

C PROGRAMME B

C \*\*\*\*\*

C TO PLOT VELOCITY AND ACCELERATION VECTORS AT SELECTED POINTS ON  
C THE FILM STRIP AT THE EARLY STAGE OF LOOP GROWTH.  
C DATA ARE READ IN FROM PUNCHED CARDS SUPPLIED BY PROGRAMME A.

C NOTATION=

C -----

C MP = NUMBER OF POINTS SELECTED.  
C MT = TIME INTERVALS CONSIDERED.  
C X,Y = COORDINATES OF POINTS ON THE FILM.  
C VA = VELOCITY AMPLITUDE.  
C PHIV = ANGLE OF VELOCITY VECTOR.  
C AA = ACCELERATION AMPLITUDE.  
C PHIA = ANGLE OF ACCELERATION VECTOR.  
C NPT = SKIP PLOT VECTORS AT EVERY NPT POINTS.  
C NPT2 = SKIP PLOT X-SIGN AND NUMBER AT EVERY NPT POINTS.  
C SCALE1 = SCALE FOR VELOCITY VECTOR.  
C SCALE2 = SCALE FOR ACCELERATION VECTOR.  
C JT = INDEX FOR CHOOSING PROPER SCALES FOR VEL. AND ACCEL.  
C VECTORS.  
C LA = NUMBER OF POINTS TO BE MARKED ON THE FILM CURVE.  
C XT,YT = COORDINATES OF THE ORIGIN OF A SELECTED COORDINATE  
C SYSTEM TO BE PLOTTED .  
C XS,YS = INTERVALS TO BE MARKED ALONG X,Y AXES.  
C XMAX = HALF OF THE MAXIMUM LENGTH ALONG X-AXIS TO BE PLOTTED.  
C YMAX = HALF OF THE MAXIMUM LENGTH ALONG Y-AXIS TO BE PLOTTED.

C NOTE THAT ALL UNITS ARE IN INCHES AND THE SCALE USED FOR PLOTTING  
C IS 2=1.

C DIMENSION X1(100),Y1(100),VA1(100),PH1(100),AA1(100),PH2(100)  
C DIMENSION X(100,10),Y(100,10),VA(100,10),PHIV(100,10),AA(100,10),  
C 1 PHIA(100,10)  
C DIMENSION XX(100),YY(100),VAX(100),PHI1(100),AAX(100),PHI2(100)  
C DIMENSION XP(100),YP(100),B1(3),C1(3),D1(3),B2(3),C2(3),D2(3)  
C DIMENSION E(2),F(2)  
C COMMON X1,Y1,VA1,PH1,AA1,PH2,IB  
C DATAB1/30H SCALE 1IN.= 40FT\*SEC-1 /  
C DATA C1/30H SCALE 1IN.= 20FT\*SEC-1 /  
C DATAD1/30H SCALE 1IN.= 10FT\*SEC-1 /  
C DATAB2/30H SCALE 1 IN.= 2\*10\*\*5 FT\*SEC-2/  
C DATA C2/30H SCALE 1 IN.=5\*10\*\*4 FT\*SEC-2 /  
C DATAD2/30H SCALE 1 IN.=2\*10\*\*4 FT\*SEC-2 /  
C DATA E/20HDT=1./24/500 SEC. /  
C DATA F/20HDS=.187/5 IN. /  
C PI=3.1415927  
C CON=PI/180.  
C NPT=2  
C MP=100  
C MT=8  
C NPT2=5

```

XS=.5
YS=.5
XMAX=5.
YMAX=4.
XT=1.
YT=1.
XO=XT
YO=YT

C
C READ IN DATA
C
DO 2 J=1,MT
READ(5,3)(X(I,J),Y(I,J),I=1,MP)
3 FORMAT(6F12.5)
2 CONTINUE
DO 5 J=1,MT
READ(5,3)(VA(I,J),PHIV(I,J),I=1,MP)
READ(5,3)(AA(I,J),PHIA(I,J),I=1,MP)
5 CONTINUE
C PLOT IDENTIFICATION.
CALL PLOT(12.0,0.0,-3)
CALL IDPLOT
CALL PLOT(12.0,0.0,-3)
CALL PLOT(0.0,-1.6,70)
JJ=1
13 CONTINUE
JT=JJ
C DEFINE SCALE FOR VELOCITY AND ACCELERATION VECTORS.
SCALE1=40.
SCALE2=200000.
IF(JJ.EQ.2) SCALE1=20.
IF(JJ.EQ.2) SCALE2=50000.
IF(JJ.GT.2) SCALE1=10.
IF(JJ.GT.2) SCALE2=20000.
C SET DATA IN ONE DIMENSIONAL ARRAYS.
DO 7 I=1,MP
XX(I)=X(I,JJ)
YY(I)=Y(I,JJ)
VAX(I)=VA(I,JJ)
PHI1(I)=PHIV(I,JJ)
AAX(I)=AA(I,JJ)
PHI2(I)=PHIA(I,JJ)
7 CONTINUE
C SELECT DATA SO THAT Y IS NOT ZERO.
DO 8 I=1,MP
I1=I-1
IF(YY(I).GT.0.0) GO TO 9
8 CONTINUE
9 CONTINUE
I2=I1+2
DO 10 I=I2,MP
I3=I
IF(YY(I).LE.0.0) GO TO 11
10 CONTINUE

```

```

11  CONTINUE
C    CALCULATE THE AMOUNT OF SHIFT FOR Y-AXIS.
    K=INT(XX(I3))
    XC=K
    IL=I1
C    RESET THE ARRAY OF POINTS FOR MARKING X-SIGN.
    DO 25 I=1,MP
    LA=I
    XP(I)=XX(IL)
    XP(I)=XP(I)-XC
    YP(I)=YY(IL)
    IL=IL+NPT2
    IF(IL.GT.I3) GO TO 26
25  CONTINUE
26  CONTINUE
    IA=1
    DO 12 I=I1,I3
    IB=IA
    X1(IA)=XX(I)-XC
    Y1(IA)=YY(I)
    VA1(IA)=VAX(I)
    PH1(IA) = PH1(I)*CON
    AA1(IA)=AAX(I)
    PH2(IA)= PH2(I)*CON
    IA=IA+1
12  CONTINUE
C
C    PLOT VELOCITY DIAGRAM.
C
    XMA=XMAX
    YMA=YMAX
    IM=JJ+1
C    PLOT AXES.
    CALL COORS(XO,YO,XS,YS,XMA,YMA,K,IM)
    CALL PLOT(0.0,-1.6,-3)
C    NUMBERING POINTS.
    NNN=I1
    DO 30 I=1,LA
    XPN=XP(I)*2.+7
    YPN=YP(I)*2.+1.1
    REWIND 4
    WRITE(4,28) NNN
    REWIND 4
    READ(4,29) NNM
    CALL LETTER(3,.12,0.0,XPN,YPN,NNM)
    NNN=NNN+NPT2
30  CONTINUE
    CALL PLTIN(.5,.5,-.5,-.5,0.0,XMAX,0.0,YMAX)
C    PUT X-MARK ON LOOP CURVE.
    CALL PLTMSP(XP,YP,.06,1HX,0.0,LA)
    CALL PLOT(0.0,0.0,3)
C    PLOT VECTORS.
    CALL DRAWVEL(SCALE1,NPT)
C    LABEL THE GRAPH

```

```

IF(JT.EQ.2) GO TO 43
IF(JT.GT.2) GO TO 45
CALL LETTER(30,.12,0.0,4.,6.5,B1)
GO TO 44
43 CONTINUE
CALL LETTER(30,.12,0.0,4.,6.5,C1)
GO TO 44
45 CONTINUE
CALL LETTER(30,.12,0.0,4.,6.5,D1)
44 CONTINUE
CALL LETTER(20,.12,0.0,8.,5.7,E)
CALL LETTER(20,.12,0.0,8.,5.4,F)

C
C PLOT ACCELERATION DIAGRAM
C
XMA=XMAX
YMA=YMAX
CALL PLOT(12.,-1.6,-3)
C PLOT AXES.
CALL COORS(XO,YO,XS,YS,XMA,YMA,K,IM)
CALL PLOT(0.0,-1.6,-3)
C NUMBERING POINTS.
NNN=11
DO 27 I=1,LA
XPN=XP(I)*2.+7
YPN=YP(I)*2.+1.1
REWIND 4
WRITE(4,28) NNN
28 FORMAT(I3)
REWIND 4
READ(4,29) NNM
29 FORMAT(A3)
CALL LETTER(3,.12,0.0,XPN,YPN,NNM)
NNN=NNN+NPT2
27 CONTINUE
CALL PLTIN(.5,.5,-.5,-.5,0.0,XMAX,0.0,YMAX)
C PUT X-MARK ON LOOP CURVE.
CALL PLTMSP(XP,YP,.06,1HX,0.0,LA)
CALL PLOT(0.0,0.0,3)
C PLOT VECTORS.
CALL DRAWAC(SCALE2,NPT)
C LABEL THE GRAPH.
IF(JT.EQ.2) GO TO 53
IF(JT.GT.2) GO TO 55
CALL LETTER(30,.12,0.0,4.,6.5,B2)
GO TO 54
53 CONTINUE
CALL LETTER(30,.12,0.0,4.,6.5,C2)
GO TO 54
55 CONTINUE
CALL LETTER(30,.12,0.0,4.,6.5,D2)
54 CONTINUE
CALL LETTER(20,.12,0.0,8.,5.7,E)
CALL LETTER(20,.12,0.0,8.,5.4,F)

```

```

CALL PLOT(12.,-1.6,-3)
JJ=JJ+1
IF(JJ.GT.MT) GO TO 14
GO TO 13
14 CONTINUE
CALL PLOT(12.0,0.0,-3)
CALL PLOT(0.0,0.0,999)
STOP
END

C
C
SUBROUTINE DRAWVEL(SCALE1,NPT)
-----
C
C
SUBROUTINE TO PLOT VELOCITY VECTORS AND DRAW THE FILM CURVE.
C
C
SCALE1 = SCALE FOR VELOCITY VECTOR.
NPT = SKIP PLOTTING AT EVERY NPT POINTS.
C
C
DIMENSION X1(100),Y1(100),VA1(100),PH1(100),AA1(100),PH2(100)
DIMENSION XA(100),YA(100),XB(100),YB(100),A(2)
COMMON X1,Y1,VA1,PH1,AA1,PH2,IB
DATA A/20H VELOCITY DIAGRAM /
C
SET THE DESIRED POINTS INTO ARRAYS.
II=1
DO 1 I=1,IB
II=I+1
IT=(II/NPT)*NPT
IF(IT.NE.II) GO TO 1
C
XA(II)=2.*X1(I) +1.
YA(II)=2.*Y1(I) +1.
VA=VA1(I)/SCALE1
PHI=PH1(I)
XB(II)=XA(II)+VA*COS(PHI)
YB(II)=YA(II)+VA*SIN(PHI)
M=II
II=II+1
1 CONTINUE
DO 2 I=1,M
XP=XA(I)
YP=YA(I)
XQ=XB(I)
YQ=YB(I)
C
PLOTTING.
C
MOVE WITH PEN UP TO LAST POSITION.
IF (I.LT.2) GO TO 10
CALL PLOT(XP1,YP1,3)
C
PLOT FROM XP1,YP1 TO XP, YP.
CALL PLOT(XP,YP,2)
10 CONTINUE
XP1=XP
YP1=YP
CALL ARROW(XP,YP,XQ,YQ,3)

```

```

2 CONTINUE
CALL LETTER(20,.24,0.0,3.0,7.0,A)
CALL PLOT(0.0,0.0,3)
RETURN
END

C
C
SUBROUTINE DRAWAC(SCALE2,NPT)
-----

C
C SUBROUTINE TO PLOT ACCELERATION VECTORS AND DRAW THE FILM CURVE.
C
C SCALE2 = SCALE FOR ACCELERATION VECTOR.
C NPT = SKIP PLOTTING AT EVERY NPT POINTS.
C
DIMENSION X1(100),Y1(100),VA1(100),PH1(100),AA1(100),PH2(100)
DIMENSION XA(100),YA(100),XB(100),YB(100),A(2)
COMMON X1,Y1,VA1,PH1,AA1,PH2,IB
DATA A/20HACCELERATION DIAGRAM/
C SET DESIRED POINTS INTO ARRAYS.
II=1
DO 1 I=1,IB
II=I+1
IT=(II/NPT)*NPT
IF(IT.NE.II) GO TO 1
XA(II)=2.*X1(I) +1.
YA(II)=2.*Y1(I) +1.
AA=AA1(I)/SCALE2
PHI=PH2(I)
XB(II)=XA(II)+AA*COS(PHI)
YB(II)=YA(II)+AA*SIN(PHI)
M=II
II=II+1
1 CONTINUE
DO 2 I=1,M
XP=XA(I)
YP=YA(I)
XQ=XB(I)
YQ=YB(I)
C PLOTTING
C MOVE WITH PEN UP TO LAST POSITION.
IF (I.LT.2) GO TO 10
CALL PLOT(XP1,YP1,3)
C PLOT FROM XP1,YP1 TO XP, YP.
CALL PLOT(XP,YP,2)
10 CONTINUE
XP1=XP
YP1=YP
CALL ARROW(XP,YP,XQ,YQ,3)
2 CONTINUE
CALL LETTER(20,.24,0.0,3.0,7.0,A)
CALL PLOT(0.0,0.0,3)
RETURN
END

```



```

C
C
SUBROUTINE COORS(XO,YO,XS,YS,XMAX,YMAX,K,JJ)
-----
C
C
SUBROUTINE TO PLOT COORDINATE SYSTEM AND DRAW SCALES.
C
C
NOTATION=
-----
C
XO,YO      = COORDINATES OF THE ORIGIN OF THE COORDINATE SYSTEM TO
C            BE PLOTTED WRT. THE ORIGINAL POSITION OF THE PEN.
C
XS,YS      = SCALES ON X,Y AXES.
C
XMAX,YMAX  = MAXIMUM VALUES OF X,Y.
C
K          = THE AMOUNT OF SHIFT FOR X-VALUES.
C
JJ         = TIME INTERVAL CONSIDERED.
C
C
NOTE THAT SCALE FOR X AND Y IS 2.
C
C
DIMENSION C(2),D(2)
DIMENSION AX(2),BY(2)
DATA AX/20H X-DISTANCE (IN.) /
DATA BY/20H HEIGHT (IN.) /
DATA C/20HPOINT NO. /
DATA D/20H DIRECTION OF MOTION/
XMAX=2.*XMAX
YMAX=2.*YMAX
CALL PLTIN(1.,1.,0.0,0.0,0.0,XMAX,0.0,YMAX)
C
PLOT AXES.
CALL PLOT(0.0,0.0,3)
CALL ARROW(XO,YO,XMAX,YO,3)
CALL PLOT(XO,YO,3)
CALL ARROW(XO,YO,XO,YMAX,3)
C
PLOT SCALES.
I=1
YA=YO-.1
1 X1=XS*FLOAT(I) +.45
X1=X1+.1
IF(X1.GT.XMAX) GO TO 2
CALL LETTER(1,.12,90.,X1,YO,1H-)
I=I+1
GO TO 1
2 I=1
XA=XO-.1
4 Y1=YS*FLOAT(I)+.55
IF(Y1.GT.YMAX) GO TO 3
Y1=Y1-.1
CALL LETTER(1,.12,0.0,XO,Y1,1H-)
I=I+1
GO TO 4
3 CONTINUE
CALL NLFT(K,JJ)
CALL PLOT(0.0,0.0,3)
CALL LETTER(20,.20,0.0,3.2,.3,AX)
CALL LETTER(20,.20,90.,.3,2.5,BY)

```

```

C   PLOT ARROW FOR DIRECTION
C   CALL ARROW(10.,4.5,8.,4.5,3)
C   PLOT LETTER 'DIRECTION OF MOTION'
C   CALL LETTER(20.,12,0.,7.6,4.8,D)
C   PLOT LETTER FOR POINTS.
C   CALL LETTER(20.,18,0.0,4.0,6.5,C)
C   CALL PLOT(0.0,0.0,-3)
C   RETURN
C   END

```

```

C
C   SUBROUTINE NLET(K,JJ)
C   -----

```

```

C   SUBROUTINE FOR NUMBERING SCALES AND LABELLING.

```

```

C   NOTATION =
C   -----

```

```

C   K           = THE AMOUNT OF SHIFT FOR Y AXIS.
C   JJ          = TIME INTERVAL CONSIDERED.

```

```

C
C   J1=JJ
C   REWIND 4
C   WRITE(4,1)J1
C   REWIND 4
C   READ(4,2)J1
C   CALL LETTER(2.,18,0.0,5.7,6.5,J1)
C   DO 3 I=1,5
C   N=K+I-1
C   REWIND4
C   WRITE(4,1)N
1  FORMAT(I2)
C   REWIND4
C   READ(4,2)N
2  FORMAT(A2)
C   SX=FLOAT(I)
C   SX=2.*SX-1.1
C   SY=.7
3  CALL LETTER( 2.,12,0.0,SX,SY,N)
C   CONTINUE
C   DO 4 I=1,4
C   M=I -1
C   REWIND4
C   WRITE(4,1)M
C   REWIND4
C   READ(4,2)M
C   SX=.7
C   SY=FLOAT(I)
C   SY=2.*SY -1.
4  CALL LETTER(2.,12,0.0,SX,SY,M)
C   CONTINUE
C   RETURN
C   END

```

C  
C  
C  
C  
C

SUBROUTINE IDPLOT

-----

PLOTTING NAME AND ACCOUNT NUMBER.

DIMENSION A(2)

DATA A/20HA4365. H V MINH. /

CALL LETTER(20,.30,90.,.4,.4,A)

RETURN

END

-----  
NOTE =

-----

TAPE NUMBER 4 IS USED AS AN INFORMATION TRANSFER UNIT.

APPENDIX D  
COMPUTER RESULTS

$S_T/L_T$	K	$Y_L/L_T$	P/P <sub>c</sub>	$M_1 \cdot S_T/E'I$
1.070	.2549	.1708	.9029	3.257
1.140	.3479	.2446	.8199	4.514
1.210	.4121	.3033	.7484	5.422
1.280	.4612	.3545	.6861	6.147
1.351	.5006	.4010	.6316	6.752
1.421	.5332	.4443	.5835	7.271
1.491	.5607	.4854	.5409	7.727
1.561	.5844	.5246	.5030	8.131
1.631	.6051	.5625	.4690	8.494
1.701	.6232	.5991	.4385	8.822
1.771	.6393	.6349	.4109	9.122
1.841	.6537	.6699	.3859	9.397
1.912	.6666	.7042	.3632	9.650
1.982	.6783	.7379	.3425	9.885
2.052	.6889	.7711	.3235	10.104
2.122	.6987	.8039	.3061	10.307
2.192	.7075	.8363	.2901	10.498
2.262	.7157	.8684	.2753	10.676
2.332	.7233	.9001	.2617	10.844
2.402	.7303	.9316	.2491	11.003
2.473	.7368	.9628	.2373	11.153
2.543	.7428	.9938	.2264	11.295
2.613	.7485	1.0246	.2163	11.429
2.683	.7537	1.0552	.2068	11.557
2.753	.7587	1.0856	.1979	11.678
2.823	.7634	1.1159	.1897	11.794
2.893	.7677	1.1460	.1819	11.905
2.963	.7719	1.1760	.1746	12.010
3.034	.7758	1.2059	.1677	12.111
3.104	.7794	1.2357	.1613	12.208
3.174	.7829	1.2653	.1552	12.300
3.244	.7862	1.2949	.1494	12.389
3.314	.7894	1.3244	.1440	12.474
3.384	.7924	1.3537	.1388	12.556
3.454	.7952	1.3831	.1340	12.635
3.524	.7979	1.4123	.1294	12.711
3.595	.8005	1.4414	.1250	12.785
3.665	.8030	1.4705	.1208	12.855
3.735	.8054	1.4996	.1169	12.923
3.805	.8076	1.5285	.1131	12.989

TABLE 1: Characteristics of a static loop on flat surface with non-dimensional parameters.

$L_T$ (in.)	ALPHA (degrees)	$Y_L$ (in.)	$P_c/16E'I$ (in. <sup>-2</sup> )	$P/16E'I$ (in. <sup>-2</sup> )	$M_1/8E'I$ (in. <sup>-1</sup> )
2.00	46.91	1.863	.6169	.1536	.2863
2.10	46.35	1.893	.5595	.1461	.2766
2.20	45.82	1.923	.5098	.1391	.2674
2.30	45.30	1.952	.4664	.1325	.2588
2.40	44.80	1.981	.4284	.1265	.2506
2.50	44.32	2.010	.3948	.1208	.2429
2.60	43.85	2.038	.3650	.1156	.2355
2.70	43.40	2.066	.3385	.1106	.2285
2.80	42.96	2.093	.3147	.1060	.2219
2.90	42.53	2.120	.2934	.1017	.2156
3.00	42.12	2.146	.2742	.0976	.2095
3.10	41.71	2.173	.2568	.0938	.2038
3.20	41.32	2.199	.2410	.0902	.1983
3.30	40.94	2.224	.2266	.0868	.1930
3.40	40.57	2.250	.2134	.0836	.1880
3.50	40.21	2.275	.2014	.0805	.1832
3.60	39.86	2.300	.1904	.0777	.1786
3.70	39.52	2.324	.1802	.0750	.1742
3.80	39.19	2.349	.1709	.0724	.1700
3.90	38.87	2.373	.1622	.0699	.1659
4.00	38.55	2.397	.1542	.0676	.1621
4.10	38.24	2.420	.1468	.0654	.1583
4.20	37.94	2.444	.1399	.0633	.1547
4.30	37.65	2.467	.1334	.0613	.1512
4.40	37.36	2.490	.1274	.0594	.1479
4.50	37.08	2.512	.1218	.0576	.1447
4.60	36.80	2.535	.1165	.0559	.1416
4.70	36.53	2.557	.1117	.0542	.1386
4.80	36.27	2.579	.1071	.0526	.1357
4.90	36.01	2.601	.1028	.0511	.1329
5.00	35.76	2.623	.0987	.0496	.1302

TABLE 2: Characteristics of a static loop on flat surface (versus gap width  $L_T$ ) with excess loop length,  $E_S = 2.805$  in.

POINT NO.	x(in.)	y(in.)	POINT NO.	x(in.)	y(in.)	POINT NO.	x(in.)	y(in.)
1	21.794	0.000	34	20.563	.074	67	19.331	.026
2	21.757	.000	35	20.525	.076	68	19.294	.023
3	21.720	.000	36	20.488	.078	69	19.256	.020
4	21.682	.001	37	20.451	.079	70	19.219	.017
5	21.645	.001	38	20.413	.080	71	19.182	.015
6	21.607	.002	39	20.376	.081	72	19.144	.012
7	21.570	.003	40	20.339	.082	73	19.107	.010
8	21.533	.005	41	20.301	.082	74	19.070	.008
9	21.495	.006	42	20.264	.082	75	19.032	.006
10	21.458	.008	43	20.226	.082	76	18.995	.005
11	21.421	.010	44	20.189	.082	77	18.958	.003
12	21.383	.012	45	20.152	.081	78	18.920	.002
13	21.346	.015	46	20.114	.080	79	18.883	.001
14	21.309	.017	47	20.077	.079	80	18.845	.001
15	21.271	.020	48	20.039	.078	81	18.808	.000
16	21.234	.023	49	20.002	.076	82	18.771	.000
17	21.197	.026	50	19.965	.075	83	18.733	0.000
18	21.159	.029	51	19.927	.073	84	18.696	0.000
19	21.122	.032	52	19.890	.070	85	18.658	0.000
20	21.085	.035	53	19.853	.068	86	18.621	0.000
21	21.048	.038	54	19.815	.066	87	18.584	0.000
22	21.010	.041	55	19.778	.063	88	18.546	0.000
23	20.973	.045	56	19.741	.060	89	18.509	0.000
24	20.936	.048	57	19.704	.057	90	18.471	0.000
25	20.899	.051	58	19.666	.054	91	18.434	0.000
26	20.861	.054	59	19.629	.051	92	18.397	0.000
27	20.824	.057	60	19.592	.048	93	18.359	0.000
28	20.787	.060	61	19.554	.045	94	18.322	0.000
29	20.749	.063	62	19.517	.042	95	18.284	0.000
30	20.712	.065	63	19.480	.038	96	18.247	0.000
31	20.675	.068	64	19.443	.035	97	18.210	0.000
32	20.638	.070	65	19.405	.032	98	18.172	0.000
33	20.600	.072	66	19.368	.029	99	18.135	0.000

TABLE 3a: Coordinates of selected points on the film strip at time  $t = .2/(24 \times 50)$  sec. with DS = .187/5 in. (from programme A).

POINT NO.	AMPLITUDE (ft./sec.)	ANGLE (degrees)	POINT NO.	AMPLITUDE (ft./sec.)	ANGLE (degrees)	POINT NO.	AMPLITUDE (ft./sec.)	ANGLE (degrees)
1	5.63	180.0	34	74.45	92.3	67	25.87	91.7
2	5.62	180.0	35	76.24	92.2	68	22.93	91.6
3	5.63	178.3	36	77.81	92.1	69	20.10	91.5
4	5.65	174.1	37	79.16	92.1	70	17.40	91.4
5	5.75	167.6	38	80.28	92.0	71	14.85	91.3
6	6.00	159.1	39	81.15	92.0	72	12.46	91.2
7	6.48	149.6	40	81.79	92.0	73	10.24	91.1
8	7.25	140.1	41	82.17	92.0	74	8.22	91.0
9	8.35	131.5	42	82.31	92.0	75	6.39	90.9
10	9.75	124.2	43	82.19	92.0	76	4.78	90.8
11	11.44	118.3	44	81.83	92.0	77	3.38	90.7
12	13.40	113.6	45	81.21	92.0	78	2.22	90.5
13	15.58	109.9	46	80.35	92.0	79	1.30	90.4
14	17.96	106.8	47	79.25	92.0	80	.62	90.3
15	20.52	104.4	48	77.92	92.0	81	.19	90.2
16	23.23	102.4	49	76.36	92.0	82	.01	90.0
17	26.08	100.8	50	74.59	92.1	83	0.00	0.0
18	29.03	99.4	51	72.62	92.1	84	0.00	0.0
19	32.07	98.3	52	70.45	92.1	85	0.00	0.0
20	35.18	97.4	53	68.10	92.1	86	0.00	0.0
21	38.33	96.5	54	65.59	92.1	87	0.00	0.0
22	41.51	95.8	55	62.93	92.1	88	0.00	0.0
23	44.69	95.2	56	60.13	92.1	89	0.00	0.0
24	47.86	94.7	57	57.22	92.1	90	0.00	0.0
25	50.99	94.3	58	54.22	92.1	91	0.00	0.0
26	54.06	93.9	59	51.13	92.1	92	0.00	0.0
27	57.06	93.6	60	47.98	92.0	93	0.00	0.0
28	59.96	93.3	61	44.79	92.0	94	0.00	0.0
29	62.76	93.0	62	41.58	92.0	95	0.00	0.0
30	65.42	92.8	63	38.36	91.9	96	0.00	0.0
31	67.93	92.6	64	35.16	91.8	97	0.00	0.0
32	70.29	92.5	65	32.00	91.8	98	0.00	0.0
33	72.47	92.4	66	28.90	91.7	99	0.00	0.0

TABLE 3b: Velocity vectors at time  $t = .2/(24 \times 50)$  sec. with  $DS = .187/5$  in. (from programme A).



POINT NO.	AMPLITUDE (ft./sec. <sup>2</sup> )	ANGLE (degrees)	POINT NO.	AMPLITUDE (ft./sec. <sup>2</sup> )	ANGLE (degrees)	POINT NO.	AMPLITUDE (ft./sec. <sup>2</sup> )	ANGLE (degrees)
1	0.	0.0	34	549160.	269.9	67	129581.	268.7
2	61.	270.0	35	558481.	269.9	68	110584.	268.5
3	3765.	269.8	36	566125.	269.9	69	92670.	268.4
4	10303.	269.7	37	572045.	269.9	70	75948.	268.2
5	18541.	269.6	38	576206.	270.0	71	60524.	268.0
6	28429.	269.5	39	578582.	270.0	72	46491.	267.7
7	39907.	269.5	40	579158.	270.0	73	33939.	267.3
8	52904.	269.4	41	577932.	270.0	74	22944.	266.6
9	67340.	269.4	42	574910.	270.0	75	13577.	265.4
10	83129.	269.4	43	570112.	270.0	76	5905.	261.7
11	100172.	269.4	44	563568.	270.0	77	643.	169.6
12	118366.	269.4	45	555318.	270.0	78	4332.	95.8
13	137599.	269.4	46	545411.	270.0	79	6718.	92.4
14	157754.	269.4	47	533910.	270.0	80	7311.	91.2
15	178706.	269.4	48	520885.	269.9	81	6097.	90.6
16	200329.	269.4	49	506414.	269.9	82	3082.	90.2
17	222487.	269.4	50	490587.	269.9	83	113.	90.0
18	245046.	269.4	51	473500.	269.9	84	0.	0.0
19	267866.	269.4	52	455258.	269.8	85	0.	0.0
20	290807.	269.5	53	435973.	269.8	86	0.	0.0
21	313727.	269.5	54	415762.	269.7	87	0.	0.0
22	336492.	269.4	55	394750.	269.7	88	0.	0.0
23	358943.	269.5	56	373064.	269.6	89	0.	0.0
24	380955.	269.5	57	350839.	269.6	90	0.	0.0
25	402389.	269.6	58	328211.	269.5	91	0.	0.0
26	423112.	269.6	59	305319.	269.5	92	0.	0.0
27	442994.	269.6	60	282304.	269.4	93	0.	0.0
28	461914.	269.7	61	259307.	269.3	94	0.	0.0
29	479755.	269.7	62	236472.	269.3	95	0.	0.0
30	496404.	269.8	63	213951.	269.0	96	0.	0.0
31	511760.	269.8	64	191855.	269.0	97	0.	0.0
32	525727.	269.8	65	170342.	268.9	98	0.	0.0
33	538219.	269.8	66	149541.	268.8	99	0.	0.0

TABLE 3c: Acceleration vectors at time  $t = .2/(24 \times 50)$  sec. with DS = .187/5 in. (from programme A).

POINT NO.	VEL. AMPL. (ft./sec.)	ANGLE (degrees)	ACCE. AMPL. (ft./sec. <sup>2</sup> )	ANGLE (degrees)	POINT NO.	VEL. AMPL. (ft./sec.)	ANGLE (degrees)	ACCE. AMPL. (ft./sec. <sup>2</sup> )	ANGLE (degrees)
1	0.0	0.0	0.0	0.0	26	7.1	119.0	23566.4	258.7
2	26.4	96.1	23012.5	269.0	27	8.5	115.2	23765.3	260.9
3	15.4	101.6	12595.5	269.5	28	9.8	112.8	23919.2	263.2
4	15.1	103.2	13290.8	271.4	29	11.3	111.4	24030.3	265.5
5	15.7	104.2	14225.1	273.4	30	12.7	110.5	24100.6	267.8
6	16.8	105.0	15116.2	275.5	31	14.1	110.0	24132.1	270.2
7	18.0	105.8	15832.5	277.8	32	15.5	109.9	24126.7	272.5
8	19.2	106.6	16419.7	280.0	33	16.9	110.0	24086.7	274.9
9	5.8	122.2	18750.6	257.5	34	18.3	110.2	24013.8	277.2
10	6.6	118.3	19463.8	258.7	35	19.6	110.6	23910.2	279.6
11	7.6	115.1	20115.3	260.2	36	21.0	110.0	23778.0	281.9
12	8.7	112.7	20690.0	261.8	37	22.3	111.5	23619.0	284.2
13	9.9	111.0	21188.5	263.6	38	23.6	112.1	23435.0	286.5
14	11.1	109.8	21612.9	265.5	39	4.6	137.0	23437.0	254.2
15	12.4	109.1	21965.8	267.5	40	5.7	126.4	23575.5	256.5
16	13.7	108.8	22250.5	269.6	41	6.9	120.0	23681.0	258.8
17	15.1	108.7	22470.4	271.8	42	8.3	116.0	23754.8	261.2
18	16.4	108.8	22629.0	274.0	43	9.7	113.5	23798.2	263.5
19	17.8	109.0	22732.1	276.2	44	11.1	111.9	23812.5	265.8
20	19.1	109.4	22777.5	278.5	45	12.5	111.0	23799.2	268.2
21	20.5	109.9	22773.3	280.8	46	13.8	110.4	23759.5	270.5
22	21.8	110.5	22723.0	283.1	47	15.2	110.2	23694.9	272.8
23	23.1	111.1	22629.5	285.4	48	16.6	110.3	23606.6	275.2
24	4.7	134.4	23026.8	254.3	49	17.9	110.4	23496.0	277.4
25	5.9	124.9	23320.8	256.5	50	19.2	110.7	0.0	0.0

TABLE 4: Velocity and acceleration vectors of the loop top in a frame cycle

(with  $DT = \frac{1}{24} \times \frac{1}{50}$  sec.,  $DS = 2. \times .187$  in.).

$L_T/S_T$	K	$Y_L/L_T$	$P/P_C$	$M_2 \cdot S_T/E'I$
3.735	.8082	1.4876	.4521	13.119
3.665	.8059	1.4588	.4521	13.050
3.595	.8034	1.4300	.4522	12.979
3.525	.8009	1.4011	.4522	12.905
3.455	.7982	1.3721	.4522	12.829
3.385	.7953	1.3430	.4523	12.750
3.315	.7924	1.3139	.4524	12.667
3.245	.7892	1.2846	.4524	12.582
3.174	.7859	1.2553	.4525	12.492
3.104	.7825	1.2259	.4525	12.399
3.034	.7788	1.1964	.4526	12.302
2.964	.7749	1.1667	.4527	12.201
2.894	.7708	1.1369	.4527	12.095
2.824	.7665	1.1071	.4528	11.984
2.754	.7618	1.0770	.4529	11.868
2.684	.7569	1.0468	.4530	11.746
2.613	.7517	1.0164	.4531	11.617
2.543	.7460	.9859	.4532	11.482
2.473	.7400	.9551	.4533	11.340
2.403	.7336	.9242	.4534	11.190
2.333	.7266	.8929	.4535	11.030
2.263	.7191	.8614	.4537	10.862
2.193	.7109	.8296	.4538	10.682
2.123	.7021	.7975	.4540	10.491
2.052	.6924	.7649	.4542	10.287
1.982	.6818	.7319	.4544	10.068
1.912	.6702	.6984	.4547	9.833
1.842	.6573	.6644	.4549	9.578
1.772	.6430	.6296	.4553	9.303
1.702	.6269	.5941	.4556	9.002
1.632	.6089	.5576	.4561	8.673
1.562	.5883	.5200	.4566	8.310
1.491	.5647	.4810	.4572	7.904
1.421	.5372	.4402	.4580	7.449
1.351	.5047	.3971	.4591	6.928
1.281	.4655	.3509	.4605	6.323
1.211	.4166	.2999	.4626	5.598
1.141	.3526	.2415	.4663	4.690
1.071	.2599	.1680	.4747	3.435
1.001	.0278	.0059	1.0000	.524

TABLE 5: Characteristics of a loop on curved surface with  $\sin(\beta) = .0555$ .