MODEL FOR CHARACTERIZING RHEOLOGICAL BEHAVIOUR OF FRESH MORTAR AND CONCRETE

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Abstract

Rheology of fresh mortar and concrete affects the flow of the mixture as well as the corresponding rheological properties. The rheological properties, in turn, are known to affect the placement and workability of the fresh mixture, and therefore can be used to control the quality of the fresh material and the properties of the hardened material. Accordingly, constitutive equations for fresh mortar and fresh concrete were developed to characterize the mixture flow and quantify the rheological properties. Moreover, by incorporating the mixture composition into the constitutive model, one can provide the concrete industry with the necessary tools to design and control the quality of concrete.

This study presents a constitutive material model for mortar and concrete that employs the cell method and builds on the work of Gang et al. It postulates that the shear stress is the sum of three components: static interaction between particles, dynamic interaction between particles, and collision of particles, and that the cell is a representative volume of mixture. For fresh concrete, the effects of particles collision are considered negligible due to high particles concentration, and the resultant constitutive equations become the Bingham model.

Two new models were developed to quantify the rheological properties of fresh concrete, namely plastic viscosity and yield stress. The plastic viscosity model postulates that the cell is a representative volume and that the packing density of the mixture can be used to characterize the aggregate's geometric and physical properties. The model is then extended, to model yield stress following the analogy concept. Evaluation of the proposed models was carried out by comparing their predictive capabilities to experimental data as well as to results of rheological models proposed in the literature. Results revealed that the proposed models provide a higher degree of correlation to the experimental data as well as a more consistent and reliable predictions in comparison to the models currently proposed in the literature for concrete and/or dense suspensions.

The proposed constitutive equations for fresh mortar were also evaluated using experimental data reported in the literature. The results are found to compare very well and that the difference is within the measurement errors.

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Publication List

This thesis consists of the following papers:

Paper I

Mahmoodzadeh F, Chidiac SE. New Models for Predicting Plastic Viscosity and Yield Stress of Fresh Concrete. To be submitted for publication.

Paper II

Chidiac SE, Mahmoodzadeh F. Plastic Viscosity of Fresh Concrete – A critical review of predictions methods. Cement and Concrete Composites 2009;31(8):535–544.

Paper III

Mahmoodzadeh F, Chidiac SE. Yield stress of fresh concrete – A critical review of predictions methods. To be submitted for publication.

Paper IV

Chidiac SE, Mahmoodzadeh F. Constitutive Flow Models for Characterizing the Rheology of Fresh Mortar and Concrete. To be submitted for publication.

Co-Authorship

This thesis has been prepared in accordance with the regulations for a "Sandwich" thesis format or as a compilation of papers stipulated by the Faculty of Graduate Studies at McMaster University and has been co-authored.

September 2005-September 2008:

Paper I: New Models for Predicting Plastic Viscosity and Yield Stress of Fresh Concrete

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The model concept was developed by Dr. S.E. Chidiac. The model was developed by F. Mahmoodzadeh in consultation with Dr. S.E. Chidiac. Paper was written by F. Mahmoodzadeh and edited by Dr. S.E. Chidiac

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Paper II: Plastic Viscosity of Fresh Concrete – A critical review of predictions methods

By: Chidiac S.E. and Mahmoodzadeh F.

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Paper III: Yield Stress of Fresh Concrete – A critical review of predictions methods

By: Mahmoodzadeh F. and Chidiac S.E.,

The experimental program forms part of a broader testing program developed by Dr. S.E. Chidiac. The experimental testing was conducted by F. Mahmoodzadeh, F. Moutassem, and O. Daoud. Data analysis was carried out by F. Mahmoodzadeh in consultation with Dr. S.E. Chidiac. The paper was written by F. Mahmoodzadeh and edited by Dr. S.E. Chidiac

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Paper IV: Constitutive Flow Models for Characterizing the Rheology of Fresh Mortar and Concrete

By: Chidiac S.E. and Mahmoodzadeh F.

The conceptual development of the constitutive equation was developed by Dr. S.E. Chidiac and F. Mahmoodzadeh. The implementation and actual development was carried out by F. Mahmoodzadeh in consultation with Dr. S.E. Chidiac. Data analysis and model calculation was performed by F. Mahmoodzadeh in consultation with Dr. S.E. Chidiac. Paper was written by F. Mahmoodzadeh and edited by Dr. S.E. Chidiac

Contents

Chapter I

1. Thesis Summary

1.1 Introduction

Concrete is composed of aggregate, cementitious materials, water and admixtures. Hydration of cementitious materials bonds the aggregates to create a stone-like material. Owing to its low cost, ease of formation, and durability, concrete continues to be the preferred construction material for civil engineering structures and infrastructure. However, for the concrete industry to remain competitive and sustainable, it is essential to address the quality control issues associated with the industry, and to develop design tools that are compatible with the new concrete, specifically, self-consolidation concrete. Toward that objective, it is postulated that new tools for characterizing the rheological properties and flow of fresh concrete are needed. This stems from the knowledge that workability of fresh concrete, which is controlled by the rheology of fresh concrete, plays a pivotal role in the quality control of fresh concrete and greatly affects the properties of hardened concrete.

In current practice, the rheological behaviour of fresh concrete is characterized using qualitative measures of workability, namely the slump. The slump test was introduced in the 1930's, and continues to be the specified test method for workability of fresh concrete despite the knowledge that it is not a sufficient test method for characterizing the rheological behaviour of fresh concrete [1]. In the 1960's, Tattersall showed that the flow behaviour of concrete obeys Bingham's material model and that two parameters are needed to characterize the flow of fresh concrete, namely yield stress and plastic viscosity. Moreover, the slump measurement was shown to provide a measure of the yield stress. In spite of the deficiency with the slump test, it remains the only standard test method because all other tests that employ concrete rheometers provide an estimate of the rheological properties that are derived from the shear stress and strain rate measurement and a model depicting the flow behaviour inside the rheometer. These estimates of rheological properties were found to correlate statistically but differ in the values for the same concrete mixture [2].

The absence of a standard test method for measuring quantitatively the rheological properties of fresh concrete places additional hurdles towards the development of a fundamental model as the values depend on the type of concrete rheometer. On the other hand, a development of models for characterizing the rheology of concrete mixture based on the composition can form the step necessary to include rheological properties in the design and quality control of fresh concrete. The aim of this study is to develop models based on fundamental principles and understanding of concrete technology.

This thesis consists of five chapters. The first chapter provides a background review of the fundamentals as they pertain to rheology and fresh concrete, followed by the motivation behind this work, the thesis objective and scope. It also provides a summary of the four papers that form part of this thesis, conclusions and recommendations for future research work.

1.2 Rheology of Concrete – a theoretical background

Rheology is defined as "the study of deformation and flow" [3]. It provides a qualitative measure between the shear force and the change of the material shape. The relation between shear stress and deformation is referred to as constitutive equation. Fluids are characterized as Newtonian or non-Newtonian depending on the relation between shear stress, τ, and rate of deformation, $\dot{\gamma}$. For Newtonian fluids, the constitutive equation that governs the flow is given by

$$\tau = \eta \ \dot{\gamma} \tag{1}$$

where η is a material property referred to as plastic viscosity. The general form of the constitutive equation for non-Newtonian fluids is governed by

$$\tau = f(\dot{\gamma}) \tag{2}$$

where f a non-linear spatial and temporal function, represents the material viscous property, namely viscosity. Fluids have been classified depending on the form of the function f. These forms which are shown in Fig. 1 can be described as follows:

- Plastic (Viscoplastic or Bingham plastic) The material behaves like solid and does not flow when the shear stress is less than a threshold value referred to as yield stress, τ_0. When the shear stress is greater than the yield value, the material behaves as fluid with the shear stress linearly proportional to shear strain rate. This model has been used to describe the flow of fresh concrete [4, 5].
- Pseudoplastic (Shear thinning) A fluid possesses shear thinning behaviour when the value of apparent viscosity decreases as the shear strain rate increases. According to Struble [6], the flow of fresh concrete can be classified as "pseudoplastic".
- Dilatant (Shear thickening) Fluids that exhibit an inverse behaviour to those of Pseudoplastic are classified as dilatants. They are said to experience a shear thickening behaviour where the value of apparent viscosity is found to increase with the increase of shear strain rate. Some researchers have reported that under certain conditions, concrete displays a shear thickening behaviour [7].
- Structural Viscosity This material description is an extension of shear thinning and possesses three distinct behaviours: Newtonian behaviour at low and high shear strain rate, and a nonlinear behaviour at intermediate shear strain rate. Concentrated suspensions are reported to follow this model [8].
- Thixotropy, Thixotropic behaviour is described as a reversible decrease at a constant shear rate in the value of apparent viscosity with time. Researchers have reported that the flow of fresh concrete, specifically self-consolidating concrete, is thixotropic [9].

- Rheopexy, Rheopexy is the opposite behaviour of Thixotropy. For these fluids, the value of apparent viscosity increases with time at a constant shear rate. However, the change is usually irreversible.

Concrete and mortar can be characterized as suspended solid particles (aggregates) in viscous medium (cement paste). The corresponding flow is best modeled using steady state non-Newtonian constitutive equations. A comprehensive list of the models developed to predict the rheology of fresh concrete and/or mortar is summarized in Table 1. For fresh concrete, the three models that have received the most attention, namely Bingham, Power Law and Herschel and Bulkley (H-B) models are briefly described.

The Bingham model yields the following constitutive equations,

iff $|\tau| < |\tau_0|$ than $\dot{\gamma} = 0$

iff $|\tau| > |\tau_0|$ than $\tau = \tau_0 + \eta\ \dot{\gamma}$ (3)

The Power Law model provides a nonlinear equation to predict the flow,

$$\tau = m\ \dot{\gamma}^n \tag{4}$$

where n is referred to as the flow index and m the consistency. Depending on the value of n, the behaviour shifts from a Newtonian flow for n=1, to a shear thickening flow for n greater than one, and to shear thinning flow for n less than one.

In the general formulation, the dimension of m depends on the value of n. To standardize the dimension, the term standard shear rate, $\dot{\gamma}_p$, is introduced to Eq. 4 yielding the following equation,

$$\tau = \eta_p \left| \frac{\dot{\gamma}}{\dot{\gamma}_p} \right|^{n-1} \dot{\gamma} \tag{5}$$

η_p is the apparent viscosity at the reference shear rate and can be related to the plastic viscosity by using the following expression,

$$\eta = \eta_p \left| \frac{\dot{\gamma}}{\dot{\gamma}_p} \right|^{n-1} \tag{6}$$

Herschel and Bulkley (H-B) model is a combination of Bingham model and Power Law model. The corresponding constitutive equation is given by

$$\tau = \tau_0 + K \ \dot{\gamma}^n \tag{7}$$

A review of the literature reveals that both Bingham model and H-B model have been used to simulate the flow of fresh concrete [10]. However, closer examination of the reported results strongly suggest that the flow of fresh concrete is governed by the concentration of the suspensions, i.e., for normal slump concrete the flow is best described using Bingham model whereas for high slump concrete such as self-consolidating concrete, the flow is best presented by H-B model [10]. Accordingly, an understanding of the fundamentals governing the flow of concentrated suspension is a pivotal step for characterizing the flow of fresh concrete.

1.3 Concentrated Suspensions

"Dense" or "high" concentration are terms referred to the suspensions, and they imply that the particle size is greater than the average particle separation. The behaviour of concentrated suspension is governed by Navier-Stokes equations, however due to complex boundary conditions namely multi-body interaction, the solution is for most problems not possible. Therefore to derive solution to these flow problems, both the governing partial differential equation, PDE, and boundary conditions are simplified using engineering knowledge and judgment [11].

For the general case, the viscosity of suspensions depends on the particle properties and the fluid properties, where

$$\eta = f(a, \rho_p, n, \eta_0, \rho_0, kT, \dot{\gamma}\, or\, \tau, t) \tag{8}$$

in which a is the radius, ρ_p the density of particle, n the number concentration, η_0 the fluid plastic viscosity, ρ_0 the fluid density, kT the thermal energy, $\dot{\gamma}$ the shear rate, τ the shear stress and t the time. Eq. 8 can be rewritten in dimensionless terms as [11]:

$$\eta_r = f(\varphi, \rho_r, Pe_{\dot{\gamma}}, \mathrm{Re}_{\dot{\gamma}}, t_r) \tag{9}$$

where $\eta_r = \dfrac{\eta}{\eta_0}$; $\varphi = \dfrac{4\pi}{3} na^3$; $\rho_r = \dfrac{\rho_p}{\rho_0}$; $Pe_{\dot{\gamma}} = \dfrac{6\pi\eta_0 a^3 \dot{\gamma}}{kT}$; $\mathrm{Re}_{\dot{\gamma}} = \dfrac{\rho_0 a^2 \dot{\gamma}}{\eta_0}$;

and $t_r = \dfrac{tkT}{\eta_0 a^3}$ (10)

where η_r; φ; ρ_r; $Pe_{\dot{\gamma}}$; $\mathrm{Re}_{\dot{\gamma}}$; t_r are, respectively, the relative viscosity, packing density, relative density, Peclet number, Reynolds number, and relative time. For a steady state neutrally buoyant suspension, Eq. 9 is further reduced to

$$\eta_r = f(\varphi, Pe_{\dot{\gamma}}, \mathrm{Re}_{\dot{\gamma}}) \tag{11}$$

For non-Brownian systems, where the Peclet number is large, i.e., $Pe_{\dot{\gamma}} > 10^3$, with a low Reynolds number, i.e., $\mathrm{Re}_{\dot{\gamma}} < 10^{-3}$, the equation for relative viscosity becomes a function of concentration [11],

$$\eta_r = f(\varphi) \tag{12}$$

Recalling the flow behaviour of suspensions and keeping in mind Eqs. 11 and 12, one can observe that shear thinning occurs for suspension with significant Peclet number, shear thickening occurs for suspensions where the Reynolds number is important, and for suspensions where both terms can be neglected the flow is Newtonian, represented in Eq.

12. In general, the flow of suspensions is non-Newtonian when concentration exceeds 40% of the mixture as is the case for fresh concrete and mortar.

A review of the literature was carried out to assess the adequacy of fundamental models and phenomenological models to estimate the rheological properties of fresh concrete and mortar. The list of models includes those proposed for concrete and for concentrated suspensions proposed for other engineering applications. The assessment included both the development basis of the models as well as the adequacy of the predictions in comparison to measured experimental data. Details of this assessment are presented for plastic and yield stress in Chapters 3 and 4, respectively and will not be duplicated in this chapter.

Of significant relevance to this study that was deduced from the literature review of concentrated suspension, is the cell method. The cell method has been widely used to formulate the viscosity of concentrated suspensions. Accordingly, a brief review of the fundamentals supporting this method is presented in this chapter.

Flow of fluids is governed by the following PDE, referred to as Navier-Stokes equations, which represent momentum balance of fluid element,

$$\overbrace{\rho\left(\underbrace{\frac{\partial u}{\partial t}}_{\substack{\text{Unsteady}\\ \text{acceleration}}} + \underbrace{u.\nabla u}_{\substack{\text{Convective}\\ \text{acceleration}}}\right)}^{\text{Inertia (per volume)}} = \overbrace{\underbrace{-\nabla p}_{\substack{\text{Pressure}\\ \text{gradient}}} + \underbrace{\eta\nabla^2 \mathrm{u}}_{\text{Viscosity}}}^{\text{Divergence of stress}} + \underbrace{f}_{\substack{\text{Other body}\\ \text{forces}}} \tag{13}$$

where **u** represents velocity, and p pressure. When the advective inertia forces are small in comparison to the viscous forces, in steady state becomes, Eq. 13 Stokes equation, i.e.,

$$\eta\nabla^2 \mathrm{u} + f = \nabla p \tag{14}$$

Equation 14 governs the flow for concentrated suspensions. The solution consists of obtaining first the velocity field and then the plastic viscosity of the suspension by applying energy conservation principles.

For the case of incompressible suspensions with rigid spherical particles, which is the simplest case, Einstein [12], by ignoring the body forces, derived an expression for the relative viscosity,

$$\eta_r = 1 + \eta_i \varphi \tag{15}$$

where η_i is the intrinsic viscosity, and equals to 2.5 for rigid spheres [12]. In the derivation, it was assumed that the particle spacing is infinite which corresponds to a low particle concentration. Subsequently, other developments were proposed to address higher concentrations.

Brinkman [13] adopted Einstein's postulation [12] and ignored particle interactions in the development of a mathematical description for plastic viscosity for concentrated suspensions as a function of particle packing. The model postulates that the viscosity of suspension consisting of n particles in a total volume V is a function of n/V, and that Einstein's equation can be used to include the effect of adding solute-particles. This development yields

$$\eta = (1-\varphi)^{-\eta_i} \tag{16}$$

Roscoe [14] followed the same analogy and modified the model using Vand's argument, which states that with increasing the concentration, a certain amount of liquid will freeze between the particles and lead to an increase in the effective concentration. Therefore, at high concentration, the value of effective concentration will be 1.35φ. The corresponding model is

$$\eta = (1-1.35\varphi)^{-2.5} \tag{17}$$

Using crowding theory, Mooney [15] considered the interaction between particles. The theory is based on the concept that the interaction between spheres can be captured by the simple geometric crowding action. The argument is based on the fact that the final viscosity of the suspensions does not depend on the sequence of adding the particles to the suspensions. It assumes that there are two component systems, particles with volume concentration of ϕ_1 and size r_1 and crowd particles r_2 in the remaining free volume $1-\lambda_{12}\phi_1$, where λ_{12} is the crowding factor. By ignoring the higher order interactions, it is assumed that λ_{12} is a function of the ratio r_1/r_2. Toward this development $H(\varphi)$ was defined as the relative viscosity of the suspension,

$$H(\varphi) = \exp\left(\frac{2.5\varphi}{1-k\varphi}\right) \tag{18}$$

where k is self-crowding factor. And, for the multi-size spheres, the following function was proposed,

$$\ln H(\varphi) = 2.5\sum_{i=1}^{n}\frac{\varphi_i}{1-\sum_{j=1}^{n}\lambda_{ji}\varphi_j} \tag{19}$$

Mooney also argued that self-crowding factor, λ_{ii} or k, ranges between 1.35 to 1.91. Comparing the results with the experimental data, it was concluded that the proposed formulation is valid when the concentration ranges from 0 to 0.5.

The models proposed in the literature to predict the plastic viscosity of fresh concrete are for the most cases based on the work of Roscoe [14], Krieger-Dougherty [16] and Mooney [15]. Notably are the models proposed by Murata and Kikukawa [17], Hu and deLarrard [18] and Roshavelov [19]. They also employed the Farris [20] model to account for multi-particle mixture. In brief, these models build on the work of Einstein [12], and do not account for the particles' interaction. The concept of the cell was proposed in the engineering literature as a representative volume of the suspensions. The cell is spherical in shape, filled with fluid and includes a particle at its centre. A schematic view of cell is shown in Fig.2.

Simha [21] and Happel [22] are the pioneers of the cell method. The main difference between the two models is in the modeling of the cell boundary conditions and the cell size. Details of the models are presented in Chapter 2.

Jeffrey and Acrivos [23] criticized the cell method by pointing out that the shape of the cell and the corresponding boundary conditions are arbitrary. They argued that the quantitative significance of the cell method is questionable. They also questioned the postulation that the cells are equidistant. Zholkovskiy et al. [24] proposed a revised formulation for the relative viscosity to overcome the issues of the boundary conditions. The derivation led to the same equation that was developed by Simha. Moreover, they used Happel's definition of the cell because they thought that Simha's definition of the cell contradicts the main assumption of the cell method.

1.4 Motivation

Placement of fresh concrete, which includes transportation, pumping, casting and finishing, depends greatly on the rheology of the mixture. Moreover, recent advances in concrete technology, development of namely self-consolidating concrete, require tighter controls over the rheological properties of the mixture. The importance of concrete rheology was also noted by deLarrard [25]. He stated that delays, lack of quality and cost overruns will become norm if principles and tools of rheology are not implemented in the design and control of fresh concrete. Roshavelov [19] stated that rheology provides a unique approach for characterizing fresh concrete, and that the science of concrete rheology has matured sufficiently to be used for this purpose. In brief, rheology influences the quality of fresh concrete and the properties of hardened concrete specifically the mechanical properties and durability.

1.5 Thesis objective and scope

The objective of this research study is to develop tools for characterizing the rheological properties and behaviour of mortar and fresh concrete. This can be achieved by developing the constitutive equations for simulating flow of the mixture as well as the corresponding equations for predicting the rheological properties. Moreover, it is aimed to use the composition of the mixture as the input information for the model.

The concrete modeled and tested in this study contains no mineral and chemical admixtures, and has a normal slump. The effects of temperatures are not considered in this study.

1.6 Summary of Papers

Paper I: New Models for Predicting Plastic Viscosity and Yield Stress of Fresh Concrete

A model for estimating plastic viscosity and yield stress based on concrete mixture composition can provide the concrete industry with the necessary tool to the design and control the quality of concrete. Although different models have been proposed in the literature, the majority of them are either not comprehensive, not based on fundamental principles, nor do they apply to concrete. This paper presents a new model that is based on the cell method for predicting plastic viscosity. The model is then extended, to model yield stress following the analogy concept. The predictive capabilities of the proposed models are evaluated using experimental results.

Paper II: Plastic Viscosity of Fresh Concrete – A critical review of predictions methods

This paper provides a critical review of the most prevailing models in concrete technology as well as models proposed in the literature for predicting the plastic viscosity of dense suspensions to a total of seven models. New models based on the cell method are proposed in this paper for predicting the plastic viscosity of fresh concrete. Review has revealed that the proposed model provides a higher degree of correlation to the experimental data as well as a more consistent and reliable prediction in comparison to the models currently proposed in the literature for concrete and/or dense suspensions.

Paper III: Yield Stress of Fresh Concrete – A critical review of predictions methods

This paper presents a critical review of the most prevailing models in concrete literature as well as models proposed in the literature for predicting the yield stress of suspensions. Experimental data were used to evaluate the predictive capabilities and adequacy of these models in quantifying the yield stress of fresh normal slump concrete. The results, confirmed that the aggregate characterizing, namely packing density and maximum packing density generally influences the yield stress and also that excessive cement paste through particles interlocking affects yield stress.

The two yield stress models proposed by Mahmoodzadeh and Chidiac, and Ferraris and deLarrard were found to yield good estimates. The yield stress values estimated using BTRHEOM and Slump Rate Machine II (SLRM II) gave similar trends but different values.

Paper IV: Constitutive Flow Models for Characterizing the Rheology of Fresh Mortar and Concrete

Constitutive equation for fresh mortar and fresh concrete provide the characterization of the mixture flow and the quantification of the rheological properties.

This paper presents a constitutive material model for mortar and concrete that builds on the work of Gang et al. and Mahmoodzadeh and Chidiac. It postulates that a) the shear stress is the sum of three components; static interaction between the particles, dynamic interaction between particles, and collision of particles, and b) that the cell is a representative volume of mixture. For fresh concrete the effects of particles collision are assumed negligible due to high concentration, and the equation reduces to the Bingham model. Experimental data reported in the literature was employed to evaluate the predictive capabilities of the constitutive equations. The model results are found to compare very well with the measured experimental data and the difference is within the measurement errors.

1.7 Conclusions

During the course of this study, the rheology of fresh mortar and concrete was studied. The review of the literature revealed that the current models are not complete, i.e., do not account for particles interaction or are phenomenologically based models. It was further revealed that models proposed for concentrated suspensions, especially the ones based on the cell method can be adapted to model mortar and concrete mixtures. The proposed model for simulating rheology of mortar and fresh concrete is based on the composition of the mixtures and consists of three sub models; static interactions of the particle, dynamic interactions of the particles, and particle collisions.

The first submodel provides a description for the yield stress, the second one for the plastic viscosity and the third one for collisions. The corresponding equations are given by

$$\tau = \tau_0 + \tau_{DI} + \tau_{\text{collisions}} \tag{20}$$

$$\tau_0 = \tau_i y(\varphi)^3 \frac{4(1-y(\varphi)^7)}{4(1+y(\varphi)^{10}) - 25y(\varphi)^3(1+y(\varphi)^4) + 42y(\varphi)^5} \tag{21}$$

$$y(\varphi)=(\varphi/\varphi_{\max})^{1/3}\left(1-\mathrm{C}_{\mathrm{Y}}\frac{m_G}{m_W}\right) \tag{22}$$

$$\tau_{DI}=\eta_w\cdot\eta_i\cdot y(\varphi)^3\cdot\frac{4\cdot\left(1-y(\varphi)^7\right)}{4\cdot\left(1+y(\varphi)^{10}\right)-25\cdot y(\varphi)^3\cdot\left(1+y(\varphi)^4\right)+42\cdot y(\varphi)^5}\cdot\dot{\gamma} \tag{23}$$

$$y(\varphi)=(\varphi/\varphi_{\max})^{1/3}\cdot\left(1-C_p\times\frac{m_C}{m_W}\right) \tag{24}$$

$$\tau_{\text{collisions}}=N_{collision}\left(k_p.F_{AP}+\Delta P\right) \tag{25}$$

where τ_i is the intrinsic yield stress and is a function of the shape of the particles, $y(\varphi)$ the ratio of particle size to cell size, φ the volumetric fraction of solid material refers to as packing density, $\varphi_{\max}$ the maximum packing density of the whole mixture, m_G and m_W are respectively the mass of gravel and water of the mix design, and C_{Y} a fitting parameter. η_w represents the viscosity of water, η_i the intrinsic viscosity and is a function of the particle shape, m_C is the mass of cement in the mix design, and C_p a fitting parameter. $N_{collision}$ is the number of collisions, k_p normal stress coefficient, ΔP the average momentum change of the two-particle collision in the mean flow direction, and F_{AP} the force acting on cement paste by a single aggregate particle (equal to drag force). The details are given in the following chapters. Evaluations of the models have revealed the following conclusion:

- **Plastic Viscosity:** Prevailing models used for predicting the plastic viscosity of concrete, with the exception of the Mahmoodzadeh & Chidiac model, are based on theories that were not intended for a medium to high concentration of suspended particles such as concrete. Accordingly, these models do not consider particles interactions.

Examination of the percent error for each prediction has shown that the Mahmoodzadeh & Chidiac model is the only one that is consistent and reliable in predicting the plastic viscosity. Moreover, the errors in the predicted plastic viscosity obtained using the Mahmoodzadeh & Chidiac model when compared to the experimental data are high only for low and high values of plastic viscosities. This is the range where the error in the BTRHEOM measurements is also expected to be high. The same observation cannot be made for other models.

- **Yield stress:** Applying analogy approach, a model based on cell method is formulated for yield stress which incorporates the concrete compositions as the input parameters. Evaluating and comparing with the other models in concrete literature revealed that the proposed model is more representative.
  The results confirm that yield stress is influenced by characterization of mixture particles, namely packing density and maximum packing density, and excessive cement paste through particles interlocking. The proposed models for powder suspension cannot be used to generate the yield stress of fresh concrete. Moreover, models that yielded poor estimates of the yield stress did not adequately account for the particle gradation and/or for the interaction between the aggregate and paste. It should be also noted that examining the results obtained from the BTRHEOM and SLRM II revealed that obtained yield stress have the same trends but not the same values.

- **Constitutive Equation**: A fundamental constitutive equation based on compositions is proposed for mortar and fresh concrete. This model consisted of three parts. The first two are the static and dynamic interactions of the particle which were the yield stress and plastic viscosity. The third one is the collision of the particles which is formulated by modifying Gang et al. [28] based on cell method.

1.8 Future research

The current study has led to the development of a constitutive equation to model the rheology of normal slump concrete. Extension of the model is envisaged for the following cases.

1. Extend the scope to include concrete with mineral and chemical admixtures.
2. Extend the model to account for air entrained.
3. Develop a standard test method for characterizing the aggregate properties.
4. Extend the model to account for the effects of temperature
5. Incorporate the constitutive equation in a numerical model, such as finite element or finite volume, to simulate the actual flow of fresh concrete.
6. Extend the current design of concrete proportioning to include rheological properties.

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Table 1: List of Constitutive equation proposed for modeling flow of fresh concrete and mortar

Referenced model	**Constitutive Equation**
Bingham plastic [1, 10]	$\tau = \tau_0 + \eta_0\ \dot{\gamma}$
Power Law [1,10]	$\tau = m\ \dot{\gamma}^n$
Herschel and Bulkley [1,10]	$\tau = \tau_0 + K\ \dot{\gamma}^n$
Vocadlo [27]	$\tau = (\tau_y^{\frac{1}{A}} + B\ \dot{\gamma})^A$
Dessoff-Kim [27]	$\eta = \eta_0 + \dfrac{A}{\dot{\gamma}}$
Yahia-Khayat [27]	$\tau = \tau_y + 2\sqrt{\tau_y\ \eta\ \dot{\gamma}\ \exp(A\dot{\gamma})}$
Shangraw [27]	$\tau = \tau_y + \eta\ \dot{\gamma} + A\ (1 + \exp(A\dot{\gamma}))$
Sisko model [28]	$\tau = \eta_\infty \dot{\gamma} + K\ \dot{\gamma}^n$
Eyring [10]	$\tau = \eta_0\, B\ \sinh^{-1}(\dot{\gamma}/B)$, $\tau = a\ \dot{\gamma} + B \sinh^{-1}(\dot{\gamma}/C)$
Vom Berg , Ostwald-deWaele [10]	$\tau = \tau_0 + B\ \sinh^{-1}(\dot{\gamma}/C)$
Robertson-Stiff [10]	$\tau = a(\dot{\gamma} + C)^b$
Modified Bingham model [28]:	$\tau = \tau_0 + \mu_P\ \dot{\gamma} + C\dot{\gamma}^2$
Atzeni et al (I). [29]	$\dot{\gamma} = \alpha\ \tau^2 + \beta\ \tau + \delta$

Atzeni et al. (II) [27]	$\tau = \tau_y + \eta_\infty \dot{\gamma} + A \sinh^{-1}\left(\frac{\dot{\gamma}}{B}\right)$
Casson[27, 29]	$\dot{\gamma} = a + b\,\tau^{1/2} + c\tau$ $\sqrt{\tau} = \sqrt{\tau_0} + \sqrt{\eta_p}\,\sqrt{\dot{\gamma}}$
Reiner-Philippoff [30]	$\tau = \left[\eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + (\tau/\tau_s)^2}\right]\dot{\gamma}$
Truncated Power Law[30]	$\tau = \begin{cases} \eta_p\ \dot{\gamma} & if\ \tau\ and\ \lvert\dot{\gamma}\rvert < \lvert\dot{\gamma}_1\rvert \\ \eta_p\ \left\lvert\frac{\dot{\gamma}}{\dot{\gamma}_1}\right\rvert^{n-1}\dot{\gamma} & if\ \tau\ and\ \lvert\dot{\gamma}\rvert > \lvert\dot{\gamma}_1\rvert \end{cases}$
Ellis[29, 30]	$\tau = \left[\frac{\eta_0}{1 + \lvert\tau/\tau_{1/2}\rvert}\right]\dot{\gamma}$ $\dot{\gamma} = a\,\tau + b\ \tau^c$
Meter [27, 30]	$\eta = \left[\frac{1 + \lvert\tau/\tau_m\rvert^{\alpha-1}(\eta_\infty/\eta_0)}{1 + \lvert\tau/\tau_m\rvert^{\alpha-1}}\right]\eta_0$ $\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{1 + \left(\frac{\sigma}{\sigma_2}\right)^A}$
Cross [30]	$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{1 + (\dot{\gamma} t_1)^P}$
Moore [27]	$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{1 + (A\dot{\gamma})}$

Williamson[27, 28]	$\frac{\eta}{\eta_0} = \frac{1}{1+(k\dot{\gamma})^n}$ $\tau = \eta_\infty \dot{\gamma} + \tau_2 \left(\frac{\dot{\gamma}}{\dot{\gamma} + A} \right)$
Williams [30]	$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{(1 + 2t_1{}^2 \dot{\gamma}^2)^P}$
Powell-Eyring [27	$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{\sinh^{-1}(\tau \dot{\gamma})}{\tau \dot{\gamma}}$
Papanastasiou [31]	$\tau = \eta_0 \dot{\gamma} + (1 - e^{(-m\dot{\gamma})}) \tau_0$
Mitsoulis and Abdali [31]	$\tau = k \dot{\gamma}^n + (1 - e^{(-m\dot{\gamma})}) \tau_0$
Dekee and Turcotte [31]	$\tau = \eta_1 \dot{\gamma} e^{(-t_1 \dot{\gamma})} + \tau_0$
Zhu et al. [31]	$\tau = \eta_1 \dot{\gamma} e^{(-t_1 \times \dot{\gamma})} + (1 - e^{(-m\dot{\gamma})}) \tau_0$
A.Yahia, and K.H Khayat [32]	$\tau = (e^{(-\alpha \dot{\gamma})}) \tau_0 + (1 - e^{(-\alpha \dot{\gamma})}) \tau_{max}$
Carreau-Yasuda [27]	$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = (1 + (\lambda \dot{\gamma})^2)^{\frac{n-1}{2}}$

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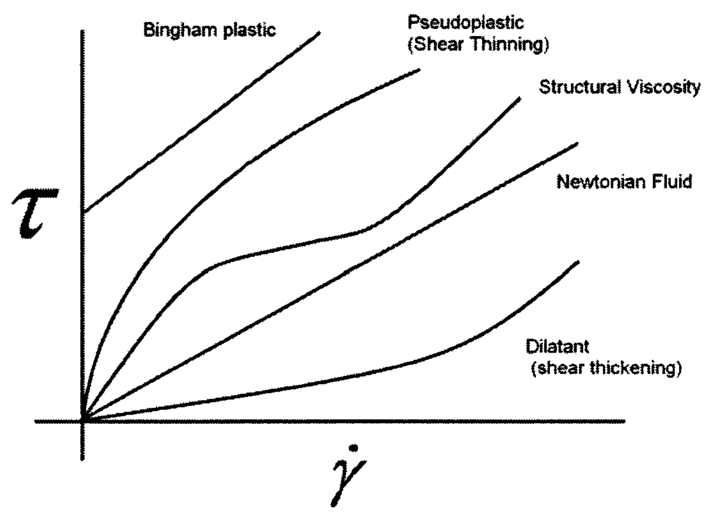

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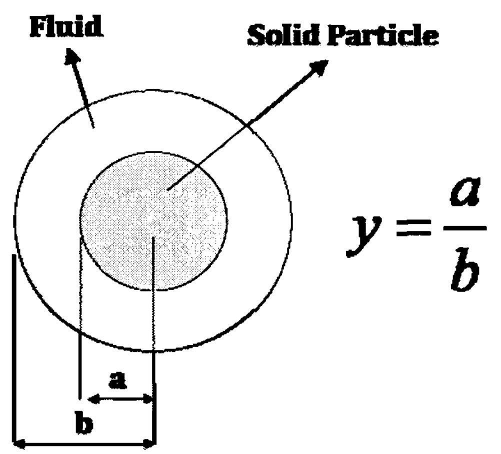

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Chapter II

New Models for Predicting Plastic Viscosity and Yield Stress of Fresh Concrete

Abstract

Plastic viscosity and yield stress are the two rheological properties that affect the placement and workability of fresh concrete and provide a pivotal role in controlling the quality of concrete, including mechanical properties and durability. Therefore, developing a model for estimating these properties based on concrete mixture composition can provide the concrete industry with the necessary tools to design and control the quality of concrete. Although different models have been proposed in the literature, the majority of them are either not comprehensive, not based on fundamental principles, or do not apply to concrete. This paper presents a new model that is based on the cell method for predicting the plastic viscosity. The model is then extended to model yield stress following the analogy concept. The predictive capabilities of the proposed models are evaluated using experimental results.

Keywords: rheology; plastic viscosity; yield stress; Bingham model; concrete mixture; modelling;

1. Introduction

Fresh concrete, which is a mixture of aggregates, cement particles and water, has been modelled as suspended rigid particles (aggregates) in viscous medium (cement paste) [1-5]. Although many constitutive equations have been proposed in the literature to characterize the rheological behaviour of fresh concrete, only the Bingham model and Herschel and Bulkley (H-B) model have received some acceptance [1-5]. For normal slump concrete, the Bingham's material model, which is defined by

$$\tau = \tau_0 + \eta \, \dot{\gamma} \tag{1}$$

has been shown to provide the better fit for the experimental data. In Eq. 1, τ is the shear stress (Pa), τ_0 the yield stress (Pa), η the plastic viscosity (Pa.s), and $\dot{\gamma}$ the shear strain rate (1/s). τ_0 and η, are referred to as Bingham material properties with the first property providing a measure of the shear stress required to initiate flow and the second one a measure of the material resistance to flow after the material begins to flow [1].

Rheological properties of fresh concrete are estimated by means of a concrete rheometer that experimentally measures shear stress versus shear strain rate. Subsequently, an estimate of the yield stress and plastic viscosity is obtained by assuming that the flow of fresh concrete obeys the Bingham model. With the absence of a standard test method for measuring the rheological properties of fresh concrete, any reported experimentally measured properties are specific to the type of concrete rheometer. This stems from previous evaluation of concrete rheometers that revealed that the measurements of rheological properties are different for different types of concrete rheometers [1-5]. For this study, the slump rate machine is used to estimate the rheological properties of fresh concrete [3].

Rheological properties of fresh concrete have been quantified based on the mixture composition. This includes the work reported by Roshavelov [4] and Ferraris and deLarrard [5, 6] to name a few. A brief review of these models and of models proposed in the literature for quantifying rheological properties of concentrated suspensions is provided as background to illustrate their scope and applications. It should, however, be noted that the input parameters for the selected models are volumetric fraction of solid material referred to as packing density, φ_S, and maximum packing density of the whole mixture, $\varphi_{S\max}$. These input parameters are a function of the concrete composition. This is followed by a mathematical description of the proposed model. An experimental program was then developed to evaluate the predictive capabilities of the model.

2. Rheological properties

2.1. Plastic viscosity

For non-Brownian systems with low Reynolds number and large Peclet number, the relative viscosity, η_r, is a function of concentration φ [7], where

$$\eta_r = f(\varphi) \qquad (2)$$

Concrete, which is classified as a dense suspension, is composed of particles of varying sizes and shapes. Maximum packing density, $\varphi_{\max}$, is introduced to account for non mono-sized particles, accordingly,

$$\eta_r = f(\varphi, \varphi_{\max}) \qquad (3)$$

The plastic viscosity models reported in literature can be divided into two groups. The first group includes those models that have been proposed for characterizing the rheological properties of concrete, and the second group consists of models proposed to quantify the plastic viscosity of concentrated suspension. The latter are normally proposed to model other than civil engineering applications. Within each group, these models can be further grouped as phenomenological models or fundamental models.

The model proposed by Ferraris and deLarrard [5, 6] to estimate the plastic viscosity of fresh concrete, which is a phenomenological model, has been shown to provide a good estimate in comparison to others in the same group [1]. It demonstrates that the plastic viscosity is a function of the ratio of volumetric fraction of solid material and the maximum packing density of the whole mixture. The models developed from first principles for estimating the plastic viscosity of fresh concrete tend to combine two theorems, Farris theory [8] to modelling multimodal spherical suspensions and either Roscoe [9], Krieger-Dougherty [10] or Mooney [11], theory to model the rheology. Most notable are the works of Murata and Kikukawa [12], Hu and deLarrard [13] and Roshavelov [4]. These models are based on Einstein's model [14] which is valid for dilute suspensions and do not adequately account for particles interaction.

Murata and Kikukawa [12] implemented Roscoe's [9] equation to quantify the plastic viscosity of concrete. Roscoe [9] adopted Brinkman [15] procedure and Vand's argument, which states that with increasing the concentration, a certain amount of liquid will freeze between particles and lead to an increase in the effective concentration. Brinkman [15] adopted Einstein's argument [14] and has, therefore, developed a mathematical description for plastic viscosity for concentrated suspensions as a function of particle packing. The model does however not include particles interaction. The same argument can be extended to Roshavelov [4] and Hu and deLarrard [13].

There are many models that have been proposed in the literature for modelling the plastic viscosity of polymers and polymers-like materials. The majority of these models are based on fundamental principles and can be represented by four groups, generalized models [16], average method[17-19], analogous approach[20, 21] and cell method[22-26]. Of interest is the cell method which is found to possess the necessary attributes for describing the composition of concrete mixture and the flexibility to incorporate particles interaction, which is a limitation of current models proposed to estimate the plastic viscosity of fresh concrete.

2.2. Yield stress

The phenomenological model proposed by Ferraris and deLarrard [5-6] to estimate the yield stress of fresh concrete has shown to provide good estimates. It postulates that the stress is a function of the volumetric fraction of solid material and the maximum packing density. Toutou and Roussel [27] adopted Coussot's theory [28] to investigate the yield stress. The model is found to yield reasonable results [27]. Szecsy [29], and Noor and Uomoto [30], also proposed phenomenological models to estimate the yield stress of fresh concrete. For a high fluid mixture such as self-compacting concrete, Neilsen [31] developed a model that stems from a geometry function. In addition, there are many models that have been proposed to estimate yield stress for suspensions. Zhou et al. [32] linked the yield stress of suspension to the individual components for powder mixes and Flatt [33-35] developed a model named YODEL, yield stress of multimodal powder suspensions. However, these models are mainly applicable to powder suspensions, and according to Touto and Roussel [27], they cannot be applied to fresh concrete.

3. Proposed model

3.1. Plastic viscosity

Interactions between suspensions in an incompressible viscous fluid are governed by the following partial differential equations, PDEs,

$$\eta_0 \nabla^2 u = \nabla p \tag{4}$$

$$\nabla \cdot u = 0 \tag{5}$$

and subject to boundary conditions. η_0, u and p are plastic viscosity of fluid, the velocity field and pressure, respectively.

The cell method has been widely used in the scientific literature for characterizing plastic viscosity of concentrated suspensions [1, 22-26]. It postulates that the representative volume of the suspension at the microscopic level consists of a spherical cell filled with viscous fluid and containing a solid particle at its centre (Fig. 1), and that the cell interacts at the boundary. To solve the problem Einstein assumed that the boundary of the cell is at infinity, and calculated the relative viscosity by applying energy conservative principles. For concentrated suspensions, it was revealed that a better description of the cell size is merited [23, 24]. Accordingly, Simha [23] and Happel [24], respectively proposed the following cell descriptions,

$$y(\varphi) = \frac{(\varphi/\varphi_{max})^{1/3}}{2-(\varphi/\varphi_{max})^{1/3}} \tag{6}$$

$$y(\varphi) = (\varphi)^{1/3} \tag{7}$$

where the function $y(\varphi)$ is defined as the ratio of the particle radius to the cell radius, see Fig.1. In addition to the cell size and boundary conditions, the shape of the cell was questioned. Frankel and Acrivos [25] proposed a cubic arrangement for the cell, where

$$y(\varphi) = (\varphi/\varphi_{max})^{1/3} \tag{8}$$

Zholkovskiy et al. [26] further develop the model by eliminating the effects of the cell boundary conditions. Their formulation is similar to that of Simha [23], but the cell size follows that of Happel [24]. The formulation led to an expression for the relative viscosity, η_r

$$\eta_r = 1 + \eta_i \lambda \tag{9}$$

and

$$\lambda = y^3 \frac{4(1-y^7)}{4(1+y^{10}) - 25y^3(1+y^4) + 42y^5} \tag{10}$$

η_i is the intrinsic viscosity. Recognizing that the plastic viscosity of concrete is 10^5, Eq. 11 is re-written as

$$\eta_r \cong \eta_i \lambda \tag{11}$$

and, to account for the particle grading in concrete mixtures, it was decided to adopt the mixture maximum packing density concept which is captured in Frankel and Acrivos [25] cell description, but with the following adjustment

$$y(\varphi) = (\varphi/\varphi_{\max})^{1/3}(1-K) \tag{12}$$

where K is a function of the concrete mixture and is defined as follows,

$$K = C_P \frac{m_c}{m_w} \tag{13}$$

where m_C and m_w are the mass of cement and water in the mixture respectively, and C_P a fitting parameter. In summary the model proposed to estimate the plastic viscosity of fresh concrete consists of Eqs. 11, 10 and 12.

3.2. Yield Stress

Theoretically, the yield stress of concentrated suspensions can be derived by following either a hydrodynamic approach [22-26] or an analogous approach [20-21]. The analogous approach has been adopted in this formulation where,

$$\tau = \tau_i \lambda \tag{14}$$

However, the description of the cell was modified to account for the composition of the concrete,

$$y(\varphi) = (\varphi/\varphi_{\max})^{1/3}\left(1 - C_Y \frac{m_G}{m_w}\right) \tag{15}$$

where m_G is the mass of the aggregate, and C_Y a fitting parameter .

4. Experimental program

The concrete mixture was designed using fractional factorial design and proportioned based on CAC guidelines [36]. The corresponding mixture proportions are given in Tables 1 and 2. The variables studied are water to cement ratio (w/c), water content, bulk volume of aggregate and maximum size of aggregate. The experimental program was limited to normal slump concrete.

The concrete mixture was prepared using crushed limestone, siliceous sand, GU-type 10 cement, air entraining admixture and water. The chemical and physical properties of the cement are given in Table 3. Two nominal maximum aggregate sizes including 20 mm and 14 mm were used. The specific gravities, absorption values, and bulk density for the 20 and 14 mm coarse aggregate are 2.75, 0.92%, and 1636 kg/m^3, and 2.74, 0.88%, and 1576 kg/m^3, respectively. The fineness modulus, specific gravities, absorption values, and bulk density for the sand are 2.72, 2.71, 1.58%, and 1812 kg/m^3, respectively. The gradation of the coarse aggregates and sand is shown in Fig. 2. The bulk density, specific gravity, and absorption for CA and sand were measured based on ASTM C127-04 [37] and ASTM C128-04 [38], respectively. The particle size distribution was conducted in accordance with CSA A23.2a [36]. Micro Air, meeting the requirements of ASTM C260-06, was used to achieve 5% air content [39].

Yield stress and plastic viscosity were measured using the slump rate machine II (SLRM II) [3]. These properties were calculated using the experimental measurement and the following equations,

$$\tau_o = 0.0397\left(\frac{\rho}{S_f^{\,2}}\right) \qquad (16)$$

$$\eta = \frac{\rho g H V}{150\pi S_l S_f^{\,2}} t_{slump} \qquad (17)$$

where S_f is the measured slump flow, ρ the density, S_l the measured slump, H the Height of slump cone, *V* the volume of slump cone, *g* the gravitational acceleration, and t_{slump} the measured time of slump. The measured values for mixture non-air-entrained concrete are given in Table 4, and the calculated values are given in Table 5. For the air-entrained concrete, the measurement and properties are given in Table 6, and the rheological properties are given in Table 7.

5. Evaluation of the model

5.1 Non-air-entrained

All of the 8 non-air entrained concrete mixtures were used to validate the model. The non-air-entrained concrete properties and measurements are given in Table 4 whereas the experimental data and model predictions are given in Table 5. For a global statistical assessment of the models, the variance of the error term [1] was calculated. For yield stress and plastic viscosity, the calculated values were 249 Pa and 3 Pa.s, respectively. Fig. 3 illustrates the model correlation with experimental data. The coefficient of correlation for plastic viscosity and yield stress with respect to experimental data are .96 and .83, respectively. These values indicate that the proposed model can be a good representative of rheological properties of non-air concrete. The proposed model is only function of the ratio of the particle radius to the cell radius, y. In Fig. 4, for verification, y was plotted against the results obtained from model and experiment. Good correlations were observed which indicates that these rheological properties can be presented as function of y.

5.2 Air-entrained

The air-entrained concrete properties and measurements are given in Tables 6 and 7. From the 20 mixes given, a set of 10 mixes were used to calibrate the model while the remaining 10 were used to examine the predictive capability of the model. Results of experimental versus model predictions are shown in Fig. 5. The variance of the error term [1] for yield stress, calibration and evaluation, respectively, was 390 Pa and 235 Pa and for plastic viscosity, calibration and evaluation, respectively, was 9 Pa.s and 5 Pa.s. Coefficients of correlations for plastic viscosity are .91 and .89 for calibration and evaluation respectively. However, for yield stress, these values are .71 and .42. By omitting mixture 15, the correlation coefficient will become .71 which is in the acceptable range. In a similar manner to non-air-entrained concrete, the ratio of the particle radius to the cell radius, y was plotted against the results obtained from model and experiment as shown in Fig. 6. This figure confirms that rheological properties can be estimated based on y.

It should be noted that generally the errors in air-entrained concrete is higher than non-air-entrained concrete. This is because the effect of air is ignored both in the proposed model and in the maximum packing density calculation. Consequently, this can be a source of error. Another source of error for yield stress is due to the manual measurement of the slump. Considering these issues, the results are acceptable and these models can be used as additional tools for civil engineers to design concrete and control its quality.

6. Conclusions

This article was prepared to meet the challenge of proposing models to predict plastic viscosity and yield stress of fresh concrete steaming from concrete mixture. Plastic viscosity model of fresh concrete is the result of review of this property for other applications of engineering, i.e. polymer applications. This model is based on cell method

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6. Conclusions

This article was prepared to meet the challenge of proposing models to predict plastic viscosity and yield stress of fresh concrete steaming from concrete mixture. Plastic viscosity model of fresh concrete is the result of review of this property for other applications of engineering, i.e. polymer applications. This model is based on cell method established by Zholkovskiy et al. [34]. The cell size of his model was modified by incorporating the maximum packing density through excessive paste theory.

For yield stress model, analogy approach has been chosen. This approach has been used for plastic viscosity in polymer applications. The plastic viscosity model has been extended to yield stress by modifying the cell size. This idea comes from the fact that the cell size for static status and dynamic is different. To this end, McMaster University data was used to evaluate the proposed model. These models made available tools for engineers to achieve sustainable concrete structures.

Acknowledgments

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Tables:

Table 1: Concrete properties and measurements: non-air-entrained concrete

Mixture #	Density (kg/m^3)	Packing density	Maximum packing density	Slump mm	Slump flow (mm)
1	2385	0.763	0.843	140	280
2	2385	0.774	0.872	240	395
3	2398	0.754	0.835	145	255
4	2376	0.761	0.867	230	360
5	2424	0.779	0.851	115	213
6	2404	0.781	0.878	220	375
7	2403	0.771	0.845	185	290
8	2405	0.770	0.876	200	400

Table 2: Concrete properties and measurements: air-entrained concrete

Mixture #	Density (kg/m3)	Packing density	Maximum packing density	Slump mm	Slump flow mm
1	2298	0.749	0.855	95	227.5
2	2284	0.744	0.873	220	375
3	2359	0.770	0.862	70	205
4	2339	0.755	0.855	100	210
5	2357	0.772	0.882	190	395
6	2278	0.745	0.866	225	400
7	2204	0.739	0.872	180	322.5
8	2298	0.748	0.866	205	325
9	2284	0.751	0.877	210	387.5
10	2304	0.751	0.869	195	315
11	2292	0.754	0.877	190	295
12	2314	0.747	0.848	120	245
13	2287	0.760	0.882	200	320
14	2278	0.751	0.869	200	350
15	2190	0.737	0.863	150	250
16	2221	0.732	0.869	215	350
17	2294	0.756	0.871	195	322.5
18	2211	0.724	0.866	190	340
19	2257	0.735	0.868	200	347.5
20	2279	0.743	0.869	205	337.5

Table 3: Chemical and physical properties of hydraulic cement GU-type 10

	Hydraulic Cement GU-Type 10
SiO_2 (%)	19.7
Al_2O_3 (%)	4.9
Fe_2O_3 (%)	2.4
CaO (%)	62.2
MgO (%)	3.1
SO_3 (%)	3.4
Na_2O (%)	1.3
Loss of Ignition (%)	2.9
Equivalent Alkalies (%)	0.75
Specific Surface Area (Blaine)	4280
% Passing 325 (45um) Mesh (%)	90.7
Time of Setting-Initial (min)	115
Compressive Strength – 28 Day (MPa)	41.9

Table 4: Concrete properties and measurements: non-air-entrained concrete

Mixture #	Density (kg/m^3)	Packing density	Maximum packing density	Slump mm	Slump flow (mm)
1	2385	0.763	0.843	140	280
2	2385	0.774	0.872	240	395
3	2398	0.754	0.835	145	255
4	2376	0.761	0.867	230	360
5	2424	0.779	0.851	115	213
6	2404	0.781	0.878	220	375
7	2403	0.771	0.845	185	290
8	2405	0.770	0.876	200	400

Table 5: Experimental data and model predictions: non-air-entrained concrete

Mixture #	Viscosity (Pa. s)		Error	Yield Stress (Pa)		Error
	Exp.	Model	%	Exp.	Model	%
1	20	25	21	1208	1180	2
2	-	-	-	607	683	11
3	26	23	11	1464	1153	21
4	6	5	14	728	477	34
5	48	47	2	2131	2023	5
6	7	8	14	679	716	5
7	-	-	-	1134	1569	28
8	9	5	43	597	535	10

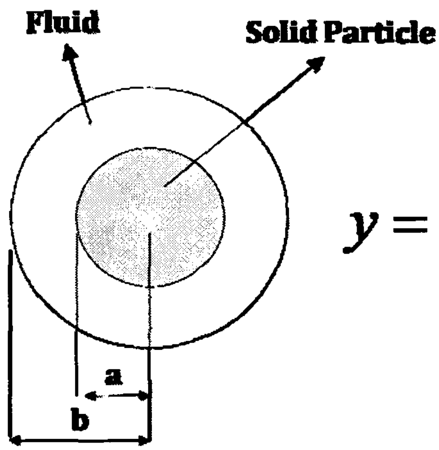
Table 6: Concrete properties and measurements: air-entrained concrete

Mixture #	Density (kg/m3)	Packing density	Maximum packing density	Slump mm	Slump flow mm
1	2298	0.749	0.855	95	227.5
2	2284	0.744	0.873	220	375
3	2359	0.770	0.862	70	205
4	2339	0.755	0.855	100	210
5	2357	0.772	0.882	190	395
6	2278	0.745	0.866	225	400
7	2204	0.739	0.872	180	322.5
8	2298	0.748	0.866	205	325
9	2284	0.751	0.877	210	387.5
10	2304	0.751	0.869	195	315
11	2292	0.754	0.877	190	295
12	2314	0.747	0.848	120	245
13	2287	0.760	0.882	200	320
14	2278	0.751	0.869	200	350
15	2190	0.737	0.863	150	250
16	2221	0.732	0.869	215	350
17	2294	0.756	0.871	195	322.5
18	2211	0.724	0.866	190	340
19	2257	0.735	0.868	200	347.5
20	2279	0.743	0.869	205	337.5

Table 7: Experimental data and model predictions: air-entrained concrete

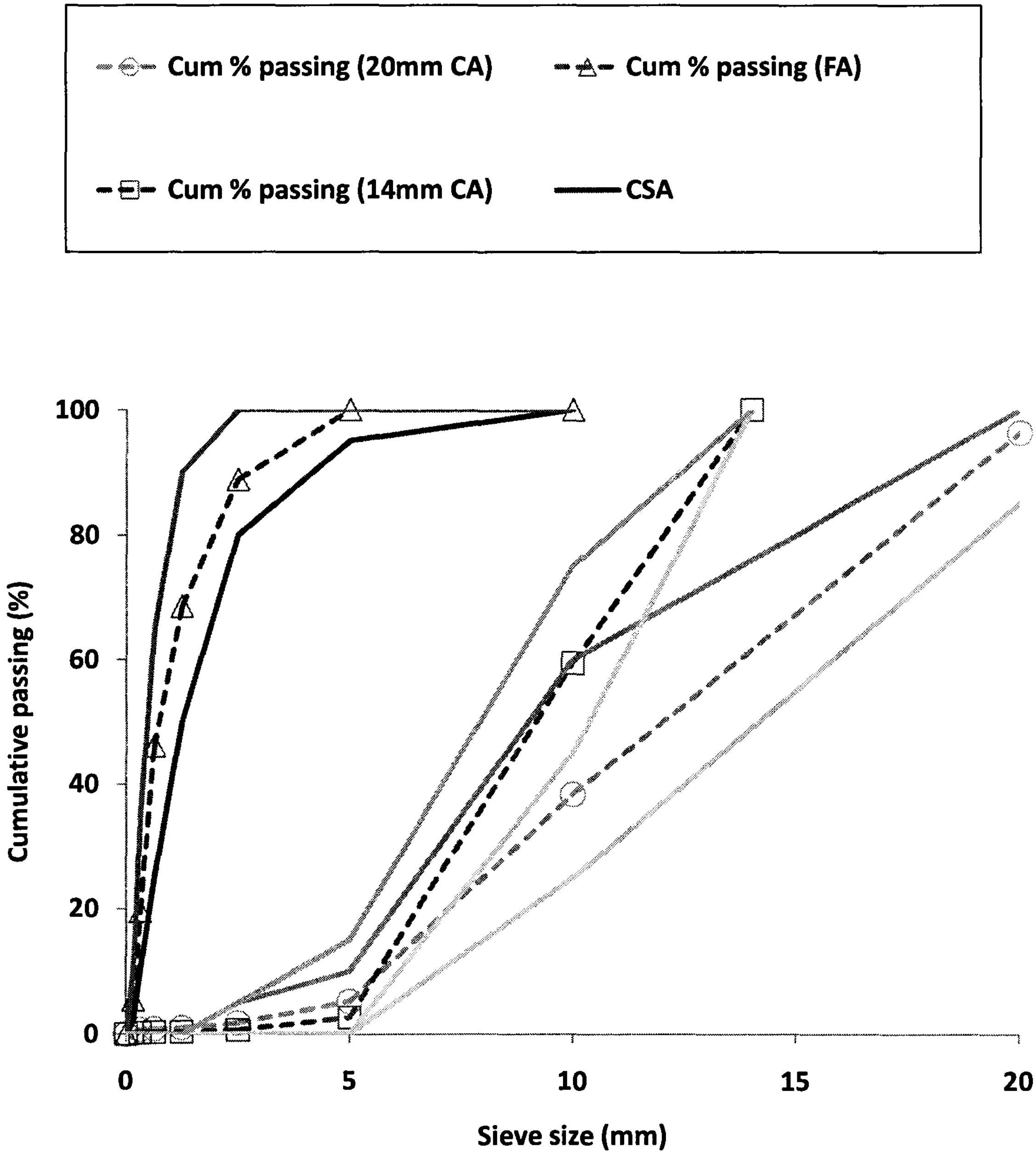
	Mixture	Viscosity (Pa. s)		Error	Yield Stress (Pa)		Error
	#	Exp.	Model	%	Exp.	Model	%
Calibration	1	49	33	33	1763	1310	26
	2	6	8	27	645	768	16
	3	80	87	8	2228	2148	4
	4	56	47	16	2105	1588	25
	5	9	18	49	600	1312	54
	6	5	13	60	565	905	38
	7	9	6	32	841	662	21
	8	10	14	28	864	962	10
	9	8	8	2	604	834	28
	10	9	15	39	922	1004	8
Evaluation	11	12	10	14	1046	904	14
	12	31	42	27	1530	1499	2
	13	11	11	0	887	943	6
	14	8	14	44	738	979	25
	15	-	8	-	1391	796	43
	16	8	5	32	720	613	15
	17	9	17	46	876	1088	20
	18	7	6	14	759	533	30
	19	9	8	8	742	664	11
	20	-	11	-	794	810	2

List of Figures



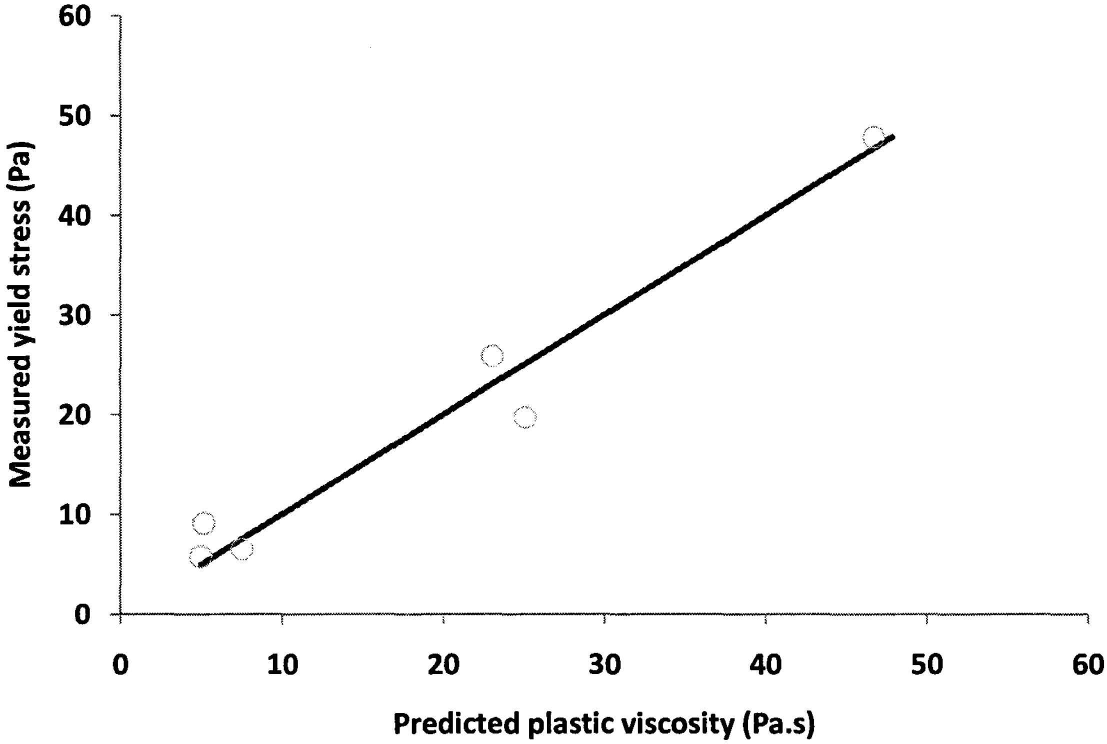
$$y = \frac{a}{b}$$

(1)



Cum % passing (20mm CA)
Cum % passing (FA)
Cum % passing (14mm CA)
CSA
100
80
60
40
20
0
Cumulative passing (%)
0
5
10
15
20
Sieve size (mm)

(2)


60
50
40
30
20
10
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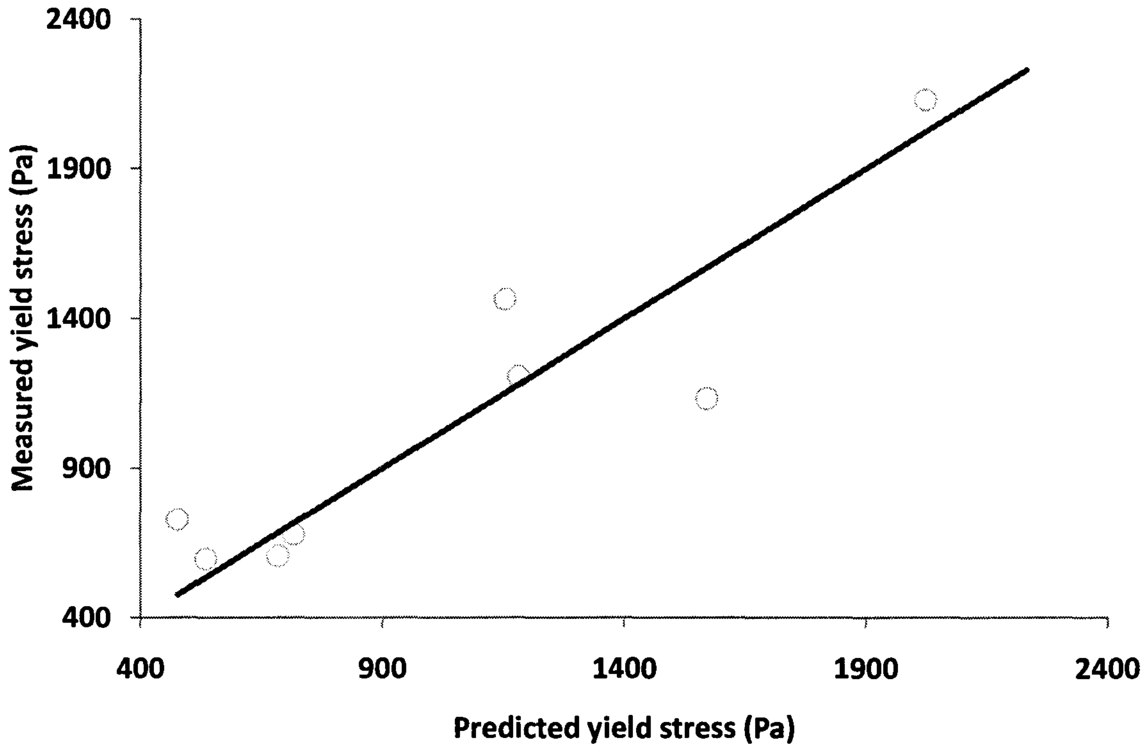
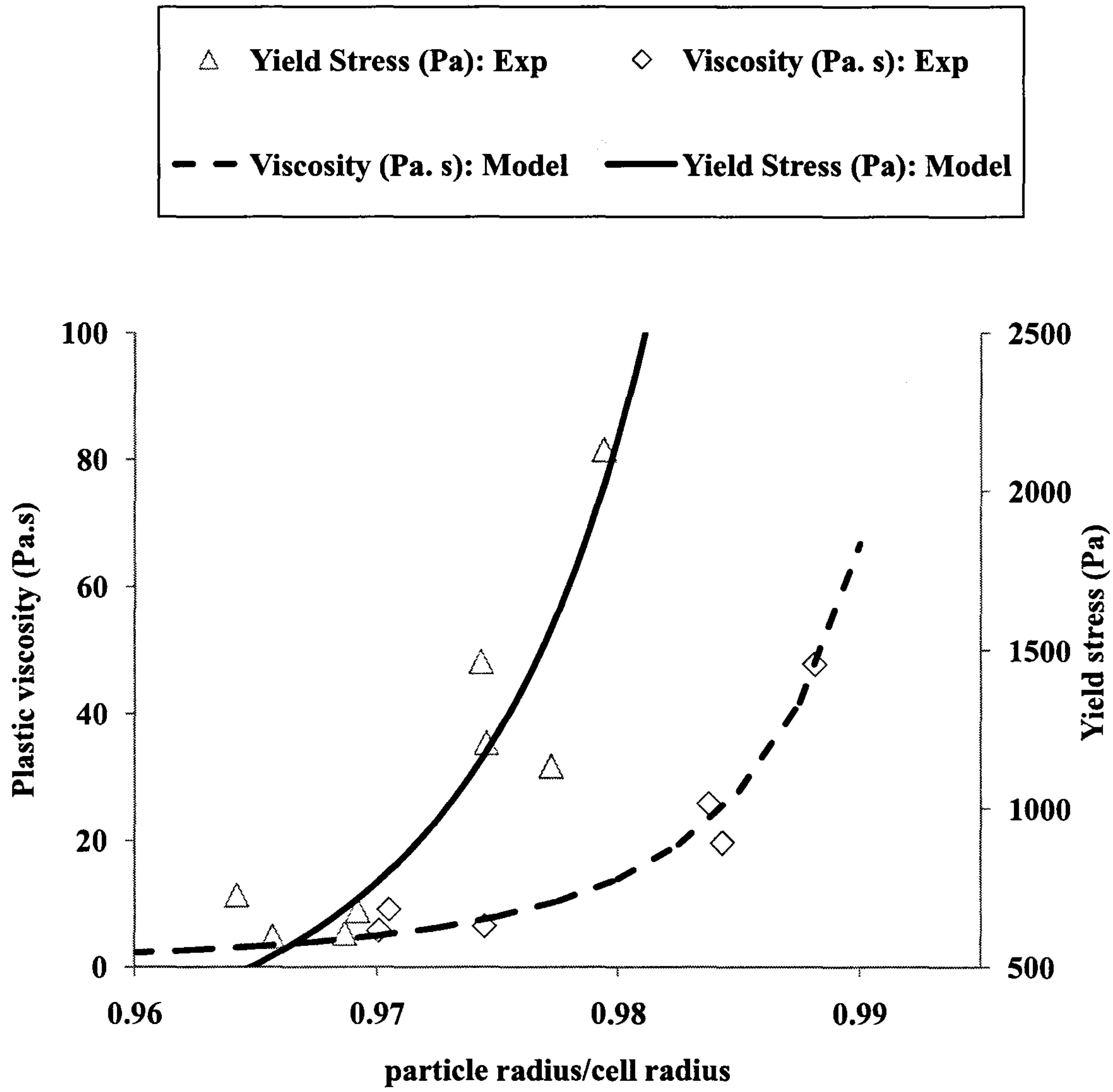
Measured yield stress (Pa)
0
10
20
30
40
50
60
Predicted plastic viscosity (Pa.s)

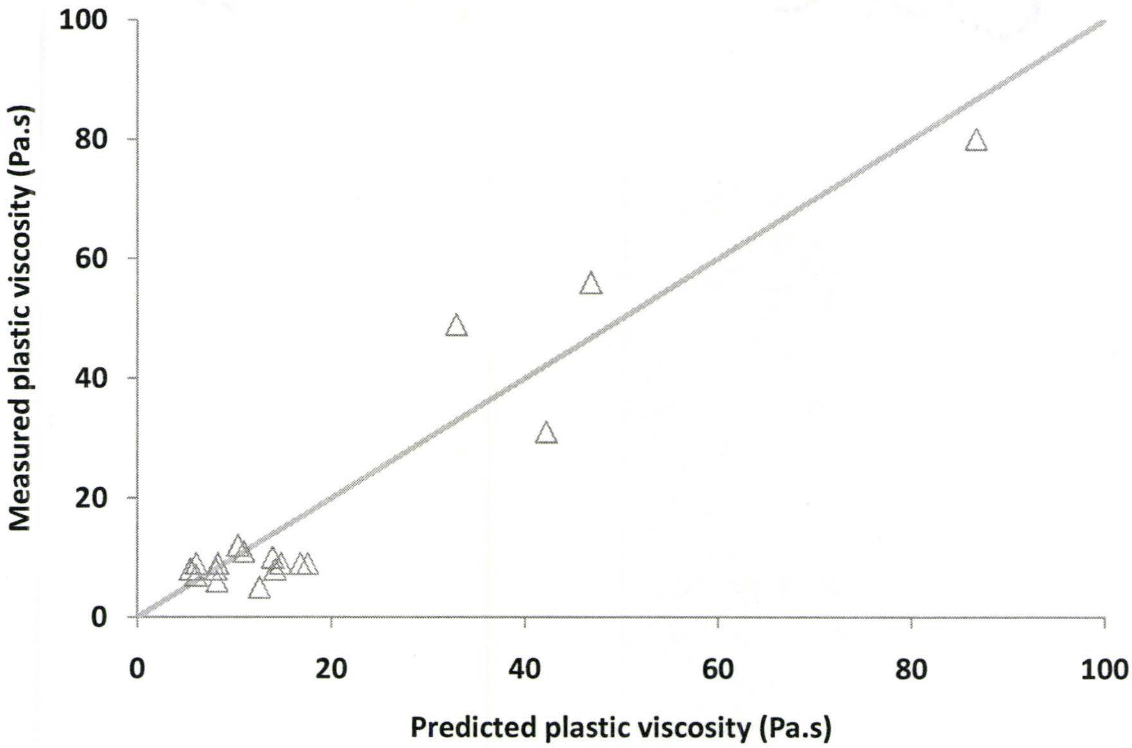
(3a)


2400
1900
1400
900
400
Measured yield stress (Pa)
400
900
1400
1900
2400
Predicted yield stress (Pa)

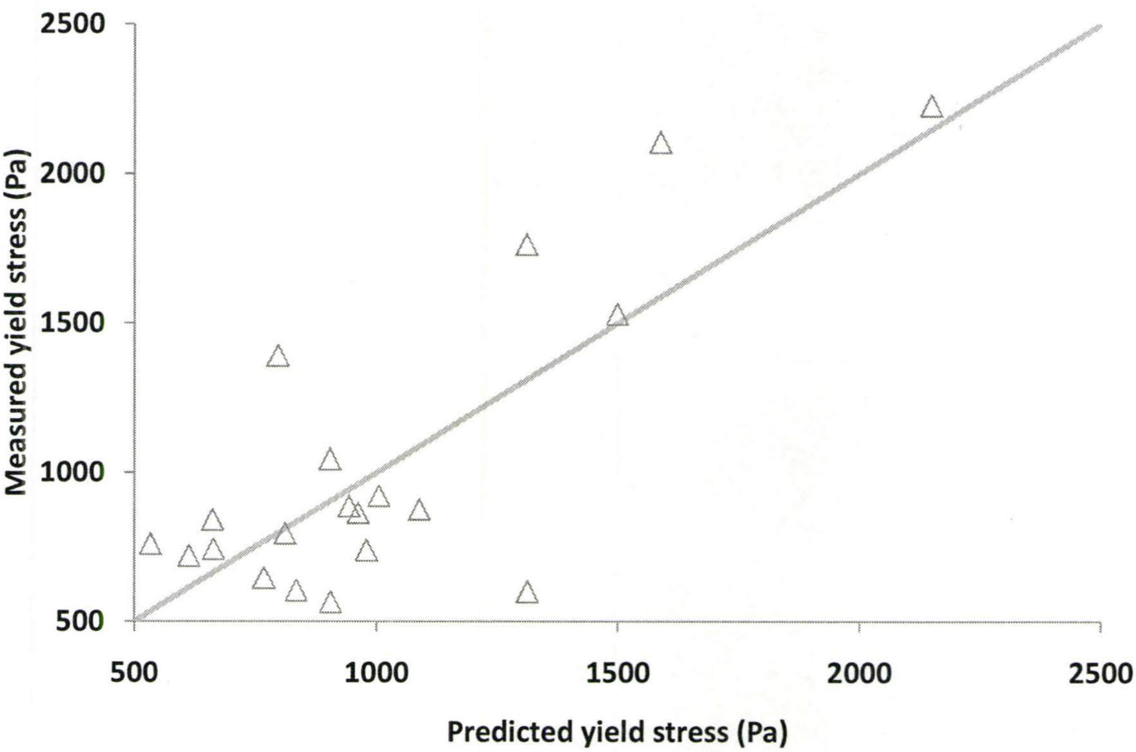
(3b)


Yield Stress (Pa): Exp
Viscosity (Pa. s): Exp
Viscosity (Pa. s): Model
Yield Stress (Pa): Model
100
80
60
40
20
0
2500
2000
1500
1000
500
0.96
0.97
0.98
0.99
Plastic viscosity (Pa.s)
Yield stress (Pa)
particle radius/cell radius

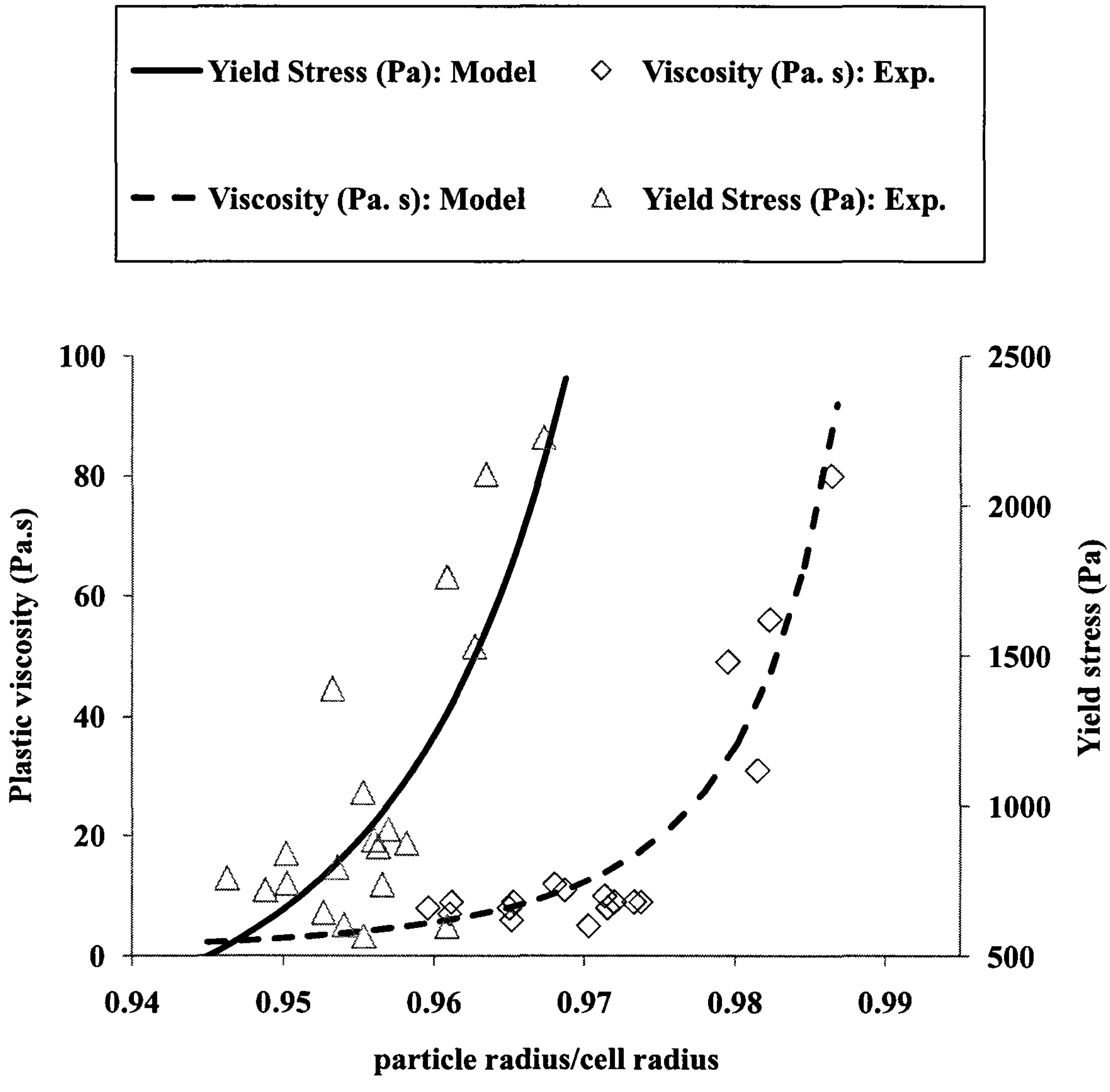
(4)


100
80
60
40
20
0
Measured plastic viscosity (Pa.s)
0
20
40
60
80
100
Predicted plastic viscosity (Pa.s)

(5a)


2500
2000
1500
1000
500
Measured yield stress (Pa)
500
1000
1500
2000
2500
Predicted yield stress (Pa)

(5b)


Yield Stress (Pa): Model
Viscosity (Pa. s): Exp.
Viscosity (Pa. s): Model
Yield Stress (Pa): Exp.
100
80
60
40
20
0
2500
2000
1500
1000
500
0.94
0.95
0.96
0.97
0.98
0.99
Plastic viscosity (Pa.s)
Yield stress (Pa)
particle radius/cell radius

(6)

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Chapter III

Plastic Viscosity of Fresh Concrete - A Critical Review of Predictions Methods

Abstract

Rheological properties of fresh concrete, namely plastic viscosity and yield stress are critical for the concrete industry because they affect placement and workability. Moreover, these rheological properties influence the productivity and quality of concrete, including mechanical properties and durability. Therefore proper characterization of these properties is needed to control the quality of fresh concrete and ensure sustainability of concrete structures.

Fundamental and phenomenological rheological models have been proposed in the literature for characterizing the behaviour of fresh concrete. Establishing a model for predicting the plastic viscosity of concrete based on its composition will be extremely valuable for the concrete industry. This paper provides a critical review of the most prevailing models in concrete technology as well as models proposed in the literature for predicting the plastic viscosity of dense suspensions to a total of seven models. Two new models based on the cell method are proposed in this paper for predicting the plastic viscosity of fresh concrete. Review has revealed that the proposed models provide a higher degree of correlation to the experimental data as well as a more consistent and reliable predictions in comparison to the models currently proposed in the literature for concrete and/or dense suspensions.

Keywords: Concrete mixture; modelling; plastic viscosity; quality control; rheology; sustainability; workability

1. Introduction

Rheology, defined as "the study of deformation and flow", provides a measure between shear stress and rate of deformation. The corresponding constitutive equation can be employed to describe mathematically the flow of fresh concrete. Concrete composed of cement particles, aggregates, water and air, can be characterized as suspended solid particles (aggregate) in viscous media (cement paste) [1-5]. Numerous constitutive equations have been proposed to characterize the rheology of fresh concrete as suspensions, but only Bingham model and Herschel and Bulkley (HB) model have received wide acceptance. For normal concrete, experimental data have confirmed that the flow of fresh concrete follows Bingham's material model, i.e.,

$$\tau = \tau_0 + \eta \, \dot{\gamma} \qquad (1)$$

In which τ is the shear stress (Pa), τ_0 the yield stress (Pa), η the plastic viscosity (Pa.s), and $\dot{\gamma}$ the shear strain rate (1/s). τ_0 and η are referred to as Bingham material properties with the first property providing a measure of the shear stress required to initiate flow and the second one a measure of the material resistance to flow after the material begins to flow. These two rheological properties are therefore needed to quantitatively characterize the flow of fresh concrete [1].

Quantitative characterization of the rheological properties is important to the sustainability of the concrete construction industry for the following reasons: 1) Workability of fresh concrete forms one of the bases of concrete mixture design for quality control purposes – Establishing a quantitative measure for workability will mitigate material waste by properly controlling the quality of fresh concrete; 2) Flow behaviour of fresh concrete impacts the quality of concrete hardened properties [1-3]. Establishing a quantitative measure for workability will mitigate the premature failure of concrete materials and concrete structures; and 3) Concrete placement which includes transportation, pumping, casting and vibration, is affected by the plastic viscosity and yield stress of fresh concrete – Establishing a quantitative measure for workability will provide the tools to design concrete mixture with flow properties suitable for the specificity of the job with the least placement cost. To illustrate the potential of such metric one can review the lessons learned from the newly developed concrete technologies, namely high performance concrete (HPC) and self-consolidating concrete (SCC). Their successful use depends on proper characterization of their rheological properties. SCC needs to flow under its own weight and fill areas that are heavily congested with steel reinforcements without any segregation [5]. HPC, whose mixture possesses a very low water to cement ratio, needs to have adequate flow properties (workability) to fill the various forming system configurations without segregation and without entrapping air voids. These examples demonstrate the need for developing a quantitative characterization of the rheological properties of fresh concrete that can be incorporated in the design and control of concrete mixtures. Towards that need, a study was carried out to evaluate models reported in the literature for predicting the plastic viscosity of fresh concrete. Subsequently, models with acceptable predictions can be incorporated in the design of concrete mixtures to overcome the limitations of current design methods which only consider slump. Slump, which has been correlated to yield stress, is not a sufficient measurement for characterizing the flow properties of fresh concrete.

The most common approach adopted for quantifying the rheological properties of fresh concrete is to measure experimentally shear stress versus shear strain rate using concrete rheometer. And by assuming that the flow of fresh concrete obeys Bingham model, an estimate of the yield strength and plastic viscosity is obtained [1-5]. Other researchers have attempted to quantify the plastic viscosity of fresh concrete based on its composition, specifically the work of Roshavelov [4] and Ferraris and deLarrard [5] to name a few. However, these attempts have received limited success due to the limitations of the models, i.e., the proposed models did not consider particles interaction which is needed for concrete given the high concentration of particles; and due to the need for model validations. This paper provides a critical review of the models proposed in the literature for quantifying the plastic viscosity of fresh concrete as well as the models proposed for quantifying the plastic viscosity of concentrated suspensions intended for chemical and material engineering applications. It should be noted that the input parameters for all the evaluated models are volumetric fraction of solid material refer to as packing density (φ) and maximum packing density of the whole mixture (φ_{max}). These input parameters are a function of concrete composition. Experimental data reported in the literature was used to evaluate the predictive capabilities of these models in quantifying the plastic viscosity of fresh concrete.

2. Rheological models – Plastic viscosity

Two different approaches have been postulated for modeling the plastic viscosity of fresh concrete: phenomenological models and fundamental models. The bases for their development are reviewed briefly. Phenomenological models are founded on observations. The most promising model in this category for fresh concrete is the one proposed by Ferraris and deLarrard [5-6]. They postulated that the plastic viscosity is only a function of the packing density to the maximum packing density of the whole mixture. Using regression analysis, they developed the following model for estimating the plastic viscosity of fresh concrete [5-6],

$$\eta = \exp\left\{26.75 \times \left(\frac{\varphi}{\varphi_{max}} - 0.7448\right)\right\} \tag{2}$$

The predictive capabilities and usefulness of phenomenological models, which includes Ferraris and deLarrard, are limited by the experimental data. Therefore, the use of Ferraris and deLarrard model can lead to erroneous predictions as the bounds of the model, namely $\frac{\varphi}{\varphi_{max}}$, φ and φ_{max} were not defined. Fundamental models proposed to quantify plastic viscosity are based on the science of rheology and fluid mechanics. These models are divided into two groups. The first group includes the models that are prevailing in concrete technology, whereas the second group compiles the models proposed to quantify the plastic viscosity of concentrated suspensions in solvent, typically used for other engineering applications. For the first group, three models were selected for this review as they are considered representative of the recent models put forward in the concrete literature, namely Murata and Kikukawa [7], Hu and deLarrard [8] and Roshavelov [4]. For the second group, the models were classified into four sub-groups: generalized models, analogous approach, cell method, and average method. A complete list of the models evaluated is given in Appendix A. The fundamentals corresponding to these models are briefly discussed and only the models that have provided the best predictions are included in this review.

2.1. Murata and Kikukawa

Murata and Kikukawa [7] implemented Roscoe's [9] equation to quantify the plastic viscosity of concrete, and proposed the following methodology:

1) Calculate the plastic viscosity of cement paste by postulating that cement particles are suspended in water, i.e. there are no physical or chemical interactions between the cement particles and water. Then by recognizing that Roscoe's equation was developed with the premise that the particles are solid, spherical, and identical in shape and size, they proposed an extension to account for the irregularly shaped and non-uniform size of the particles. They proposed the following relation

$$ {}^{i}\eta_r = \frac{{}^{i}\eta}{{}^{i}\eta_0} = \left(1 - \frac{{}^{i}\varphi}{{}^{i}C}\right)^{-{}^{i}k} \tag{3} $$

where superscript "i" is set equal to 1 for cement paste, ${}^{1}\eta_r$ becomes the relative plastic viscosity of cement paste, ${}^{1}\eta$ the plastic viscosity of cement paste, ${}^{1}\eta_0$ the plastic viscosity of water, ${}^{1}\varphi$ the volumetric concentration of cement, ${}^{1}C$ the percentage of absolute volume of cement, and ${}^{1}k$ the coefficient of agglomerated cement particles. Coefficients ${}^{1}k_1$ and ${}^{1}k_2$ are constant and are found through regression.

2) Plastic viscosity of mortar is then established using the same premise stipulated for the cement paste in Eq. (3) with superscript "i" is set equal to 2 for mortar, ${}^{2}\eta_r$ becomes the relative plastic viscosity of mortar, ${}^{2}\eta$ the plastic viscosity of mortar, ${}^{2}\eta_0$ the plastic viscosity of cement paste, ${}^{2}C$ the solid volume ratio of fine aggregate, ${}^{2}\varphi$ the volumetric concentration of fine aggregate, and ${}^{2}k$ a linear function of the fineness modulus.
3) Plastic viscosity of concrete is similarly obtained with superscript "i" is set equal to 3 for concrete, ${}^{3}\eta_r$ becomes the relative plastic viscosity of concrete, ${}^{3}\eta$ the plastic viscosity of concrete, ${}^{3}\eta_0$ the plastic viscosity of mortar, ${}^{3}C$ the solid volume ratio of coarse aggregate and ${}^{3}\varphi$ volumetric concentration of coarse aggregate.

2.2. Hu and deLarrard

For multimodal spherical suspensions, Farris [10] stated that it is possible to ignore the interaction of different classes of particle size in situation where the size ratio of spheres is less than 1/10. Accordingly, he proposed the following functional form:

$$\eta_r = H(\varphi_1)H(\varphi_2)\cdots H(\varphi_N) \tag{4a}$$

where φ_i is the concentration of each size, and $H(\varphi_i)$ the relative plastic viscosity of fluid containing particle size classes from 1 to i to plastic viscosity containing 1 to i-1. Hu and deLarrard [8] incorporated this theory along with Krieger-Dougherty [11] equation to simulate the flow of HPC. They developed the following equations:

$$\eta = \eta_0(1 + k_s p_s)\left(1 - \frac{\varphi_F}{\alpha_F}\right)^{-2.5\alpha_F}\left(1 - \frac{\varphi_C}{\alpha_C}\right)^{-k\alpha_c}\left(1 - \frac{\varphi_G}{\alpha_G}\right)^{-k\alpha_G} \tag{4b}$$

$$\phi_F = \frac{V_F}{V_0 + V_F},\ \phi_C = \frac{V_C}{V_0 + V_F + V_C},\ \phi_G = \frac{V_G}{V_0 + V_F + V_C + V_G}, \tag{4c}$$

$$\varphi_{x\max} = 1 - 0.45\left(\frac{d_x}{D_x}\right)^{0.19} \tag{4d}$$

where k_s and k are found by curve fitting, η_0 the plastic viscosity of water, p_s the proportion of superplasticizer as fraction of its saturating dosage, ϕ_x the volume concentration, V_x the partial volumes, $\varphi_{x\max}$ the maximum packing density and, d_x and D_x the sieve sizes corresponding to 10% and 90%, respectively, of the material concerned. Subscript x in Eq. (4d) can be replaced by 0, F, C and G for water, silica-fume, cement and aggregate, respectively.

2.3. Roshavelov

In 1951, Mooney [12] put forward a model that permits the inclusion of particles interaction by means of crowding theory. It was assumed that the interaction between particles can be captured by a simple geometric crowding factor but did not provide a methodology for developing such interaction functions. By adopting Mooney's theorem, Roshavelov [4] proposed a crowding factor that is based on some geometric argument. The proposed approach was found to have good predictions of the plastic viscosity only at the maximum shear rate.

The second group of fundamental models include those that were developed for dense suspensions. Terms dense or high concentrations refer to the suspensions in which the particle size is greater than the average particle separation. For non-Brownian systems, low Reynolds number and large Peclet number, the relative viscosity is found to be a function of concentration [13], where

$$\eta_r = f(\varphi) \quad (5a)$$

Concrete is classified as dense suspensions with particles of varying sizes and shapes. Accordingly, maximum packing density ($\varphi_{\max}$) was introduced to account for non mono-sized particles, where

$$\eta_r = f(\varphi, \varphi_{\max}) \quad (5b)$$

2.4. Generalized models

Models which have been generalized from other material models, have been proposed to quantify the plastic viscosity of concentrated suspensions [14-15]. In this study, the model put forward by Sudduth [15] is presented,

$$\ln(\eta/\eta_0)=\left(\frac{\eta_i}{k}\right)\left(\frac{1}{\xi-1}\right)\left((1-k\varphi)^{1-\xi}-1\right),\qquad k=1/\varphi_{max}\qquad \text{For: } \xi\neq 1 \tag{6a}$$

$$\eta=\eta_0(1-k\varphi)^{-\eta_i/k},\qquad k=1/\varphi_{max}\qquad \text{For: } \xi=1 \tag{6b}$$

where η_i is the intrinsic viscosity and is a function of the particle shape (for spherical particles, $\eta_i=5/2$), ξ the interaction factor and is found by regression analysis using experimental data. For ξ equals to 0, 1 and 2, respectively, Eq. 6 yields the Arrhenius equation, the Krieger-Dougherty equation, and Mooney equation. Accordingly, Sudduth formulation is the generalized form of Arrhenius, Krieger-Dougherty and Mooney equations.

2.5. Analogous approach

The theoretical treatment to obtain the rheological properties of concentrated suspensions can be divided into two main categories, hydrodynamic approach and analogous approach. For the later one, it is postulated that physical properties such as diffusion coefficient, modulus of elasticity, and thermal conductivity have the same form of constitutive equations, and are therefore treated analogously. In this method, the plastic viscosity is treated as a field property [16]. By taking into account the fact that the plastic viscosity of aggregate is much greater than that of water, the following equation has been developed based on Fan's model [16],

$$\eta_r=C+\varphi^m+\frac{F_s^2}{\varphi-\varphi^m} \tag{7}$$

where C, F_S and m are constant.

2.6. Cell Method

The cell method is widely used for characterizing plastic viscosity in chemical and material engineering applications. It postulates that a spherical cell of fluid containing a particle in the center is the representative volume of the suspension at the microscopic level, and that the cell is subjected to actions at its boundary. The corresponding boundary value problem defined by:

$$\eta_0 \nabla^2 u = \nabla p \tag{8}$$

subjected to $\nabla \cdot u = 0$ (8)yields the solution to the flow problem. The relative viscosity is then obtained by equating the energy dissipation of the cell to the energy dissipated in the fluid of a cell with the same volume. The solution to the partial differential equation leads to Einstein's equation when the boundary of the cell is at infinity. Simha [17] and Happel [18], pioneers of this method, proposed two different cell descriptions,

$$\text{Simha [17]: } y(\varphi) = \frac{(\varphi/\varphi_{max})^{1/3}}{2-(\varphi/\varphi_{max})^{1/3}} \tag{9a}$$

$$\text{Happel [18]: } y(\varphi) = (\varphi)^{1/3} \tag{9b}$$

in which the function y(φ) is defined as the ratio of the radius of the cell to the radius of the particle. Frankel and Acrivos [19] opted to include also the lubrication theory in their model. Moreover, they assumed that cells take cubic arrangement.

$$\text{Frankel and Acrivos [19]: } \quad y(\varphi) = (\varphi/\varphi_{max})^{1/3} \tag{9c}$$

Jeffrey and Acrivos [20] criticized the cell method by pointing out that the selection of the shape of the cell and the boundary conditions are arbitrary, and therefore the quantitative significance is questionable. In addition, the method assumes that all the cells are equidistant which is not always true. In 2006, Zholkovskiy et al. [21] proposed a revised formulation that overcomes the postulated boundary conditions requirement and therefore overcoming the main weakness of the cell method. They also argued that Simha's proposed cell radius contradicts the main assumption of the cell method. They adopted the cell size proposed by Happel in Eq. (9b). Their formulation yielded

$$\eta_r = 1 + y^3 \frac{10(1-y^7)}{4(1+y^{10}) - 25y^3(1+y^4) + 42y^5} \tag{10}$$

Mahmoodzadeh and Chidiac [22] revised Zholkovskiy et al. formulation for quantifying the plastic viscosity of fresh concrete. In the revised formulation, the cell size was modified to account for the maximum packing density. Moreover, it postulated that a) the grading of particles is important, and b) the particle size cannot be equal to that of the cell size even when the packing density is equal to the maximum packing density. Accordingly, only the cell models proposed by Simha and by Frankel and Acrivos can be incorporated in the revised formulation. Mahmoodzadeh and Chidiac's models take the following form:

Using Simha
$$y(\varphi) = \frac{(\varphi/\varphi_{\max})^{1/3}}{2(1+K) - (\varphi/\varphi_{\max})^{1/3}} \tag{11a}$$

Using Frankel & Acrivos
$$y(\varphi) = (\varphi/\varphi_{\max})^{1/3}(1-K) \tag{11b}$$

where K is a function of the concrete mixture and is defined as follows:

$$\begin{cases} K = 0.006\dfrac{Cement}{Water} & Without\ HRWRA \\ K = 3.8\dfrac{HRWRA}{Cement}\dfrac{Water}{Cement + Fine\ Sand + Sand} & With\ HRWRA \end{cases} \tag{12}$$

HRWRA is High Range Water Reducer Admixture. Only the second model (Eq. 11b) is included in the review.

2.7. Average Method

From hydrodynamic point of view, two different approaches have been proposed in the literature to find the effective properties of suspension. The first one is based on equalizing the dissipated energy in the suspension to the dissipated energy in the fluid with the effective properties (Cell Method). This method yields only the effective properties (plastic viscosity), not the full constitutive equation. The second approach relates the average stress tensor of the suspension to the average rate of strain tensor [20]. This method is based on the work of Batchelor [22, 23], in which the bulk stress tensor $(\overline{T})$ is expressed

$$\overline{T} = -\overline{p}I + 2\eta\overline{D} + \overline{T}_P \quad (13)$$

Where $\overline{p}$ is the hydrostatic stress, $\overline{D}$ is the bulk rate of stress tensor that is observed macroscopically, and $\overline{T}_P$ the contribution of the particles to the stress tensor. Equation (13) can be re-worked using Batchelor's work and assuming that particles interaction is negligible $(\overline{T}_P = 0)$ to yield

$$\overline{T} = -\overline{p}I + 2\eta_0 f(\varphi)\overline{D} \quad (14)$$

By comparing Eq. (13) with Eq. (14), the following expression can be derived,

$$\eta_r = \frac{\eta}{\eta_0} = f(\varphi) \quad (15)$$

Martynov et al. [24] applied the average method to model the flow of a viscous liquid with suspended mono-size, rigid spheres. Following Batchelor's work and solving the creep equation in a cell, they developed a closed-form solution to calculate the relative plastic viscosity, where

$$\eta/\eta_0 = 1 + {}^i\eta\,\varphi\left[1 + \frac{15\beta_2}{2\pi}\varphi + \left(\frac{15\beta_2}{2\pi}\varphi\right)^2 + \frac{14}{3}\omega\left(\frac{3}{4\pi}\varphi\right)^{5/3}\right] \quad (16)$$

in which $\beta_2 \approx -0.3594$ and ω is a weight function.

3. Evaluation methods

Experimental data reported in the literature [5] was used to evaluate quantitatively the predictive capabilities of the plastic viscosity models for fresh concrete. The data included 19 concrete mixtures without HRWRA and 17 concrete mixtures with HRWRA. However, the source of the coarse and fine aggregates for all 36 concrete mixtures was the same. Therefore, all 36 mixtures possess the same maximum packing density.

The plastic viscosity measurements were obtained using the BTRHEOM, a parallel plate concrete rheometer. It should be noted that there is no standard test method for measuring the plastic viscosity of concrete and that the reported experimental measurements are only applicable to BTRHEOM given that previous evaluation of concrete rheometers has revealed that the measurements of plastic viscosity are different for different types of concrete rheometer [25]. Based on the authors' experiences with BTRHEOM, the coefficient of variance for stable concrete mixes is in the range of 10 to 15% and that the errors are higher for mixes with low plastic viscosity due to some segregation and for mixes with high plastic viscosity due to added stiffness.

Three tests were used to assess the model predictions of the plastic viscosity. The first test provides a global assessment by calculating the variance of the error term. For the second test, the covariance and correlation were calculated to determine the extent to which the models co-vary. And for the third test, the predictive trends of the models are determined with varying the packing density as an indirect assessment of the particle interaction contribution to the plastic viscosity models.

The variance of the error term (σ) was calculated in accordance with

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(\eta_{i_{\text{Experimental}}} - \eta_{i_{\text{Model}}}\right)^2}{n-q} \tag{17}$$

where the term $(n-q)$ represents the model's degrees of freedom, n the number of data points and q the number of fitting parameters. To assess the models, the experimental data was divided into two sets, the first set is used for determining the coefficient(s) for the model using statistical regressive analyses, and the second set is for testing the predictive capabilities of the models. Ten data points were used for the regression, and 9 points to assess the models predictions for concrete without HRWRA, and 7 points to assess the predictions for concrete with HRWRA. To evaluate the soundness of the proposed number of experimental test points, Ferraris and deLarrard [5-6] phenomenological model, which was fitted to the same data, was first tested. The results from the regression analysis and model predictions for concrete without HRWRA and with HRWRA are shown in Fig. 1. The plots indicate the same distribution of errors for both the predictive and regression analyses, and the errors are found to be similar to the ones reported by Ferraris and deLarrard [5-6]. The variance of the error calculated from the data obtained using the regression analysis and those obtained using the model for concrete without HRWRA is 31 Pa.s and 42 Pa.s, respectively, and 143 Pa.s and 142 Pa.s for concrete containing HRWRA.

4. Comparative analyses

Fig. 1 to 4 provide a visual comparison of the results obtained using regression analysis and models predictions with the experimental data. The models are found to yield different plastic viscosity predictions. Tables 1 and 2 give the number of parameters associated with each model as well as the standard error calculated for the regression analysis and model evaluation corresponding to concrete without HRWRA and with HRWRA, respectively. The results show that the errors for the model evaluation are similar to those calculated for the regression analysis. This indicates that the number of test data used to calibrate the models is adequate for the number of parameters.

Closer examination of the results in Table 1 show that Mahmoodzadeh & Chidiac model has yielded the best results followed by the models developed by Zholkovskiy, Martynov, Sudduth, Fan, Hu & deLarrard, Ferraris & deLarrard, and Murata & Kikukawa. Moreover, the second best model, i.e. Zholkovskiy, has a standard error that is 11% greater than that of Mahmoodzadeh & Chidiac model when predicting the plastic viscosity of concrete without HRWRA. From Table 2, one observes that Mahmoodzadeh & Chidiac model yields the lowest standard error followed by Hu & deLarrard, Ferraris & deLarrard and Zholkovskiy's model. The percent difference in the error between Hu & deLarrard's model and Mahmoodzadeh & Chidiac model is 4%. However, to better assess the correlation between the experimental data and the model predictions, all the experimental data were used for this analysis and the results are given in Tables 3 and 4 for concrete with HRWRA and without HRWRA, respectively. The corresponding 95% confidence bounds are also given in Tables 3 and 4. The results of Table 3 show that Mahmoodzadeh & Chidiac model provides the best correlation, 0.79, with 0.53 and 0.92 as the 95% confidence bounds. Hu & deLarrard is found to be the second best with a correlation value of 0.75 and 95% confidence bounds of 0.44 and 0.90. Although the correlation obtained for the two models is found to be comparable, Hu & deLarrard model requires the calibration of three parameters in comparison to two for Mahmoodzadeh & Chidiac model.

For closer assessment of the errors, the percent difference between the models predictions and the experimental data is shown in Fig. 5. Only the results from models with a correlation factor of 0.7 or higher are shown and for concrete without HRWRA. These results show that the proposed models produce more consistent predictions in comparison to the other three models. The error is found to be less than 25% for all the data with the exception of three data points corresponding to plastic viscosity values of 56, 62 and 146 Pa.s. However, when comparing with the other three models, it was found that Martynov model has five predictions with error greater than 25%, followed by Fan with six predictions and then Hu & deLarrard with nine. Martynov's model predictions with error more than 25% is found to correspond to plastic viscosity values of 56, 62, 146, 89 and 161 Pa.s, Fan's model predictions correspond to plastic viscosity values of 56, 62, 146, 89, 161 and 96 Pa.s, whereas Hu & deLarrard model predictions correspond to plastic viscosity values of 62, 146, 89, 96, 93, 111, 119, 134 and 140 Pa.s. These results indicate that Mahmoodzadeh & Chidiac model, Martynov model and Fan model predictions of plastic viscosity are somewhat consistent, i.e. Martynov model predictions of plastic viscosity that are greater than 25% included the data points from Mahmoodzadeh & Chidiac model and Fan's included those from Martynov. Moreover, the errors in the predictions of Hu & deLarrard model are not consistent with the other three models.

The correlations among the models were calculated to determine the extent to which the models co-vary and the results are given in Table 5. It is found that the model predictions correlate better with each other in comparison to the experimental results with the exception of Mahmoodzadeh & Chidiac and Hu & deLarrard. It is also found that the predictions obtained from Mahmoodzadeh & Chidiac correlate best with those obtained from Hu & deLarrard and vice versa (R^2=0.96). The predictions from Martynov is found to correlate best with those obtained from Sudduth and Zholkovskiy (R^2=0.99). Predictions from Fan's model and Murata & Kikukawa's model are found to correlate best with those of Martynov (R^2=0.94) and Hu & deLarrard (R^2=0.95). These results show that the two models that are found to provide the highest correlation with the experimental data are also found to have best correlation in their predictions. Another important observation is the correlation between Mahmoodzadeh & Chidiac and the predictions obtained using Murata & Kikukawa (R^2=0.92), Sudduth (R^2=0.92), Martynov (R^2=0.91) and Zholkovskiy (R^2=0.92). The same observation is noted for Hu & deLarrard. However, the degree of correlation obtained among models predictions does not appear to have any link to the degree of correlation between the models predictions and the experimental data.

Predicted plastic viscosities for different values of packing density are shown in Fig. 6. Curves obtained for Fan and Martynov are found to differ from the other models. Comparing the results obtained from Hu & deLarrard with those of Mahmoodzadeh & Chidiac, one observes that the two models yield comparable values of plastic viscosity when the packing density is less than or equal to 0.78. The model predictions diverge significantly when the packing density is greater than 0.78. Similar trend is observed for Murata & Kikukawa model and Ferraris & deLarrard model. This difference in model predictions is attributed to particle interactions which become more pronounced as packing density increases.

5. Conclusions

This review indicates that the prevailing models used for predicting the plastic viscosity of concrete, with the exception of Mahmoodzadeh & Chidiac model, are based on theories that were not intended for a medium to high concentration of suspended particles such as concrete. Accordingly, these models do not consider particle interactions and a priori assume a low concentration of spherical solid particles. The impact of these limitations is apparent when the results of Figs. 5 and 6 were examined. The review has also demonstrated that rheological model developed on the basis of fundamental principles is a necessary but not a sufficient requirement for obtaining good results. Understanding of the flow behaviour of fresh concrete is also needed in order to develop a comprehensive model.

Results of Fig. 5 clearly show that the degree of correlation between Mahmoodzadeh & Chidiac model predictions and the experimental data can be misleading. Mahmoodzadeh & Chidiac model and Hu & deLarrard model were found to yield similar degree of correlation. However examination of the percent error for each prediction has shown that the two models are not the same and that Mahmoodzadeh & Chidiac model is the only one that is consistent and reliable in predicting the plastic viscosity. Moreover, the errors in the predicted plastic viscosity obtained using Mahmoodzadeh & Chidiac model when compared to the experimental data are high only for low and high values of plastic viscosities. This is the range where the error in the BTRHEOM measurements is also expected to be high. The same observation cannot be made for Hu & deLarrard model, although a reasonable degree of correlation with the experimental data was obtained. The good fit has been attributed to the higher number of fitting parameters - three in comparison to two for Mahmoodzadeh & Chidiac model.

Although all the models were used to predict the plastic viscosity of fresh concrete using the composition of the mixture, only Mahmoodzadeh & Chidiac model is found to yield results that are consistent and comparable to the experimental ones. This model, although it still requires further testing, can be used by the concrete industry for designing concrete mixture instead of the traditional slump measurement. These predictive methods provide the tools needed to achieve a more consistent and less expensive design of concrete mixtures which is a step in the direction for achieving more sustainable concrete structures.

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Tables

Table 1: Summary of standard error obtained from models calibration and evaluation for concrete without HRWRA and number of fitting parameters.

Model	Number of parameters	Error (Pa.s)	
		Regression	Evaluation
Ferraris & deLarrard	2	31	42
Murata & Kikukawa	4	52	54
Hu and deLarrard	3	34	37
Sudduth	2	31	34
Fan and Boccaccini	3	27	34
Zholkovskiy et al.	1	32	30
Martynov and Syromyasov	2	29	32
Mahmoodzadeh and Chidiac	2	23	27

Table 2: Summary of standard error obtained from models calibration and evaluation for concrete with HRWRA and number of fitting parameters

Model	Number of parameters	Error (Pa.s)	
		Regression	Evaluation
Ferraris & deLarrard	2	143	142
Murata & Kikukawa	-	-	-
Hu and deLarrard	4	102	119
Sudduth	-	-	-
Fan and Boccaccini	-	-	-
Zholkovskiy et al.	1	218	206
Martynov and Syromyasov	-	-	-
Mahmoodzadeh and Chidiac	2	84	114

Table 3: Degree of correlation between models and experimental data for concrete without HRWR

Models	Correlation	Confidence bounds (95%) Lower bound	Upper bounds
Ferraris & deLarrard	0.62	0.24	0.84
Murata & Kikukawa	0.61	0.21	0.83
Hu and deLarrard	0.75	0.44	0.9
Sudduth	0.67	0.31	0.86
Fan and Boccaccini	0.74	0.43	0.89
Zholkovskiy et al.	0.66	0.29	0.86
Martynov and Syromyasov	0.7	0.36	0.88
Mahmoodzadeh and Chidiac	0.79	0.53	0.92

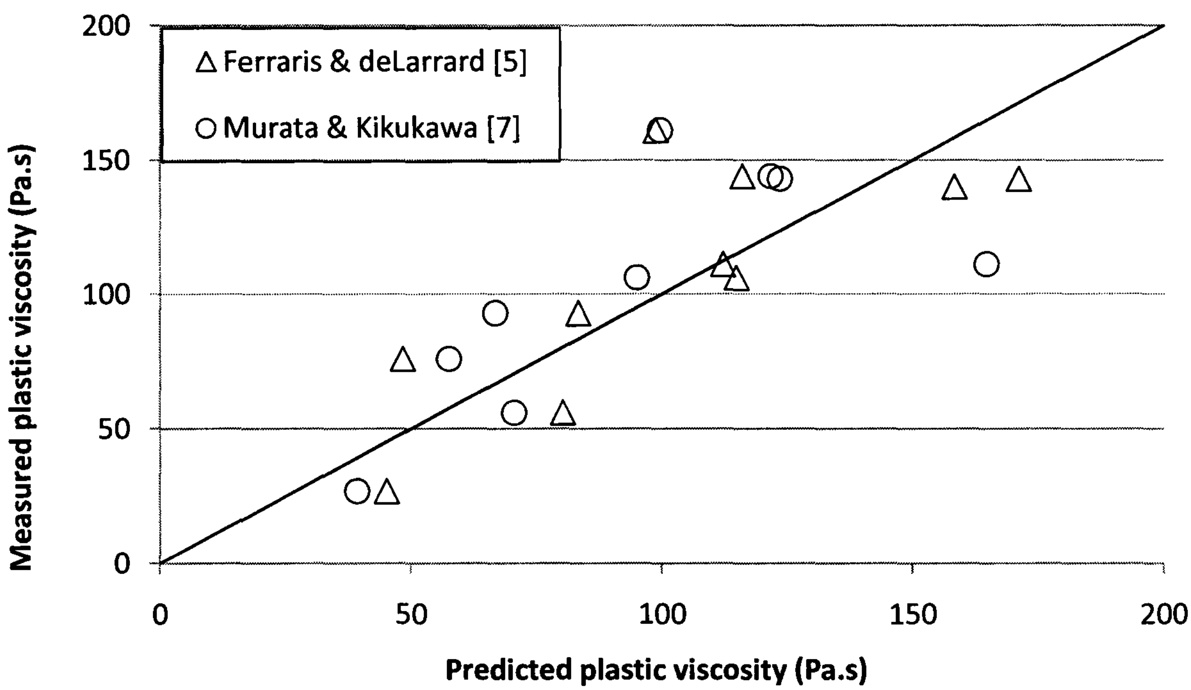
Table 4: Degree of correlation between models and experimental data for concrete with HRWRA

Models	Correlation	Confidence bounds (95%) Lower bound	Upper bounds
Ferraris & deLarrard	0.47	-0.01	0.78
Murata & Kikukawa	-	-	-
Hu and deLarrard	0.83	0.58	0.94
Sudduth	-	-	-
Fan and Boccaccini	-	-	-
Zholkovskiy et al.	-0.05	-0.52	0.44
Martynov and Syromyasov	-	-	-
Mahmoodzadeh and Chidiac	0.82	0.57	0.93

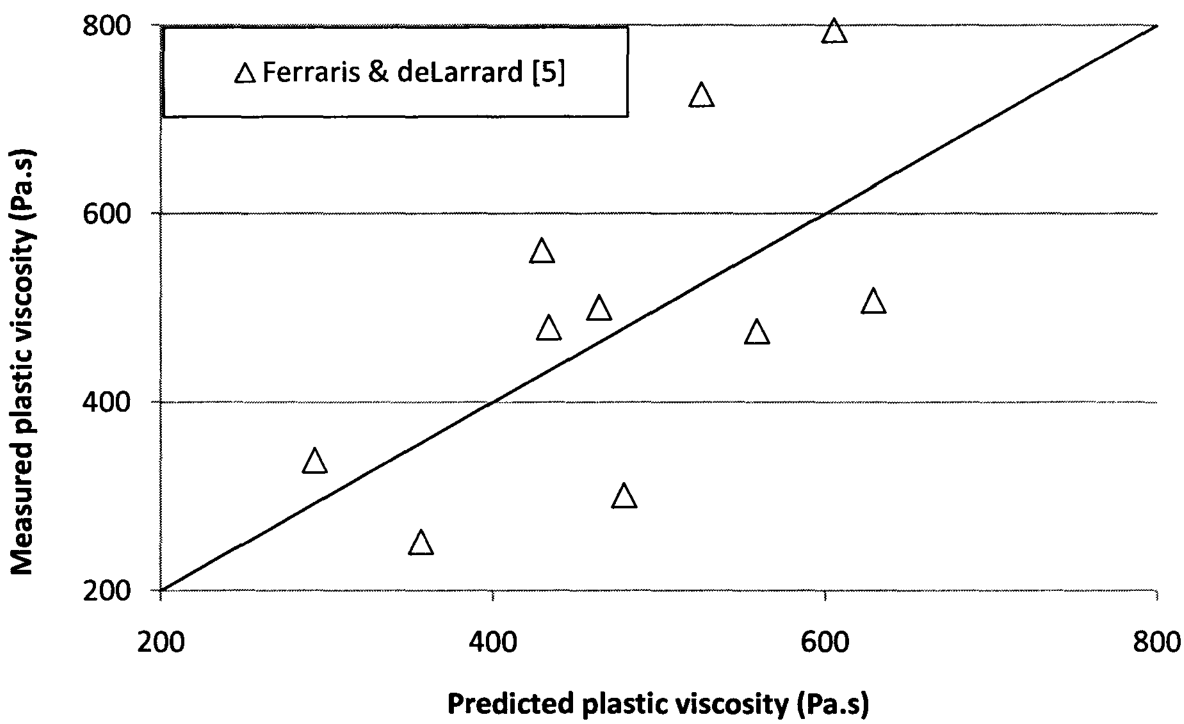
Table 5: Degree of correlation among the models for concrete without HRWRA

	Ferraris and deLarrard	Murata and Kikukawa	Hu and deLarrard	Sudduth	Fan and Boccassini	Martynov and Syromyasov	Zholkovskiy et al.	Mahmoodzadeh and Chidiac
Ferraris and deLarrard	1.00	0.72	0.74	0.85	0.82	0.82	0.79	0.76
Murata and Kikukawa	0.72	1.00	0.95	0.93	0.75	0.90	0.94	0.92
Hu and deLarrard	0.74	0.95	1.00	0.94	0.78	0.91	0.94	0.96
Sudduth	0.85	0.93	0.94	1.00	0.90	0.99	1.00	0.92
Fan and Boccaccini	0.82	0.75	0.78	0.90	1.00	0.94	0.88	0.81
Martynov and Syromyasov	0.82	0.90	0.91	0.99	0.94	1.00	0.99	0.91
Zholkovskiy et al.	0.79	0.94	0.94	1.00	0.88	0.99	1.00	0.92
Mahmoodzadeh and Chidiac	0.76	0.92	0.96	0.92	0.81	0.91	0.92	1.00
Experimental	0.62	0.61	0.75	0.67	0.74	0.70	0.66	0.79

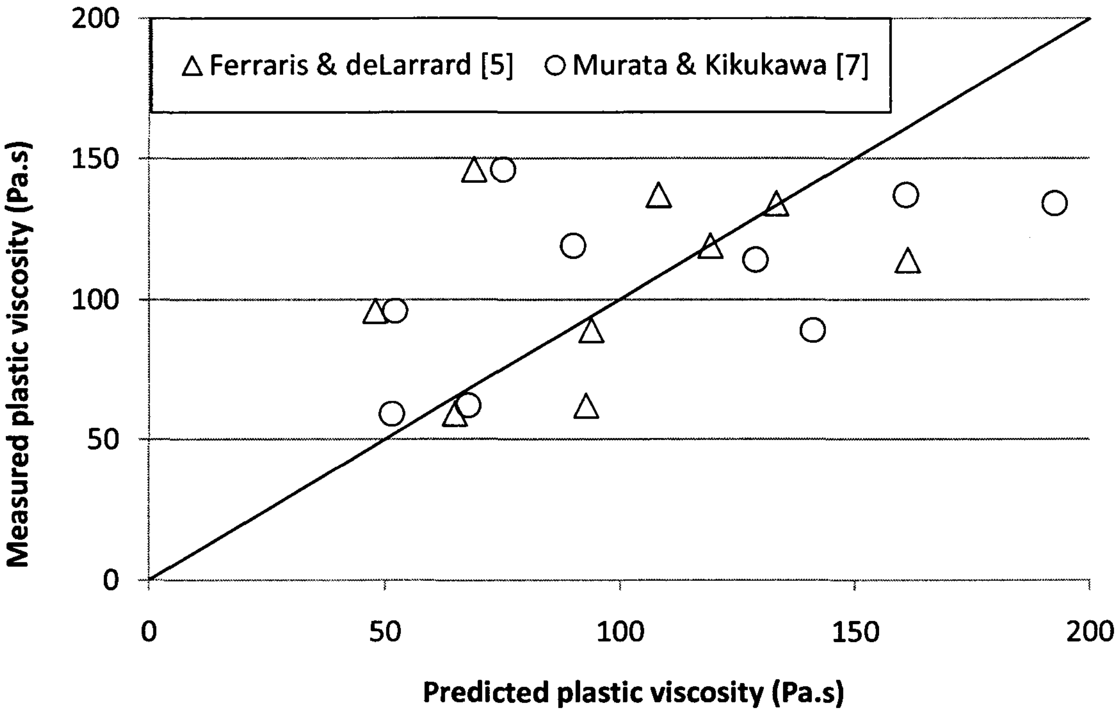
List of Figures


△ Ferraris & deLarrard [5]
○ Murata & Kikukawa [7]
Measured plastic viscosity (Pa.s)
Predicted plastic viscosity (Pa.s)
0
50
100
150
200

(1a)


△ Ferraris & deLarrard [5]
Measured plastic viscosity (Pa.s)
Predicted plastic viscosity (Pa.s)
200
400
600
800

(1b)


200
150
100
50
0
0
50
100
150
200
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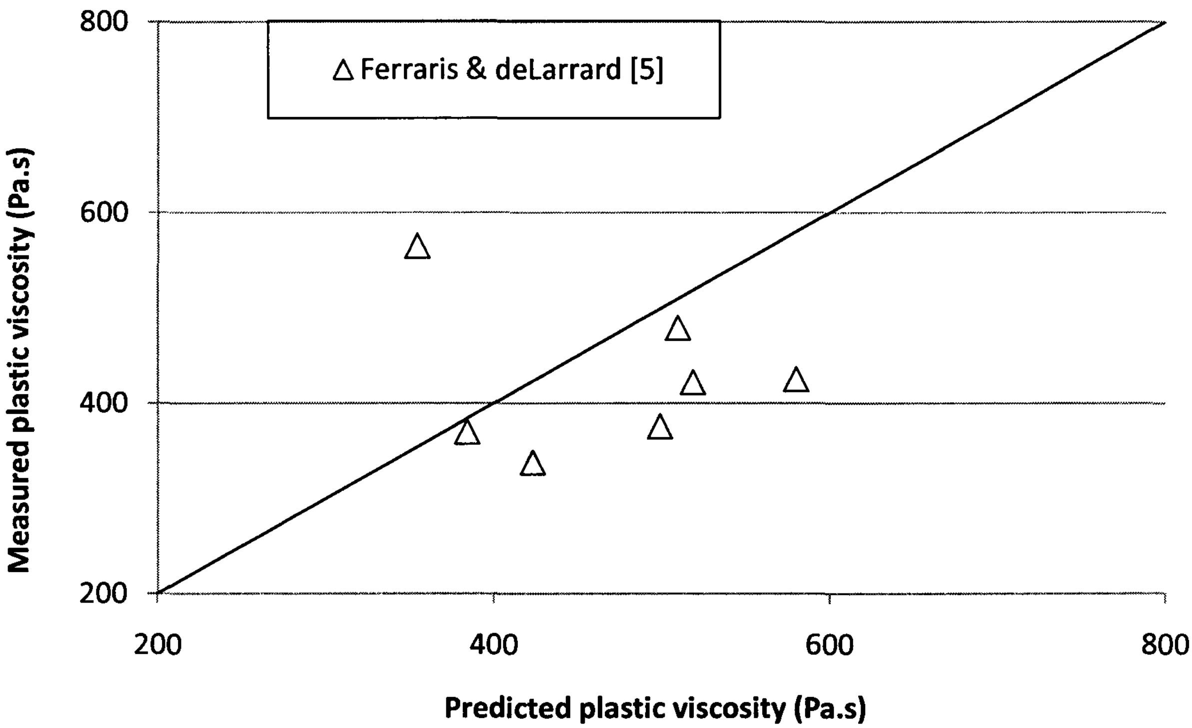
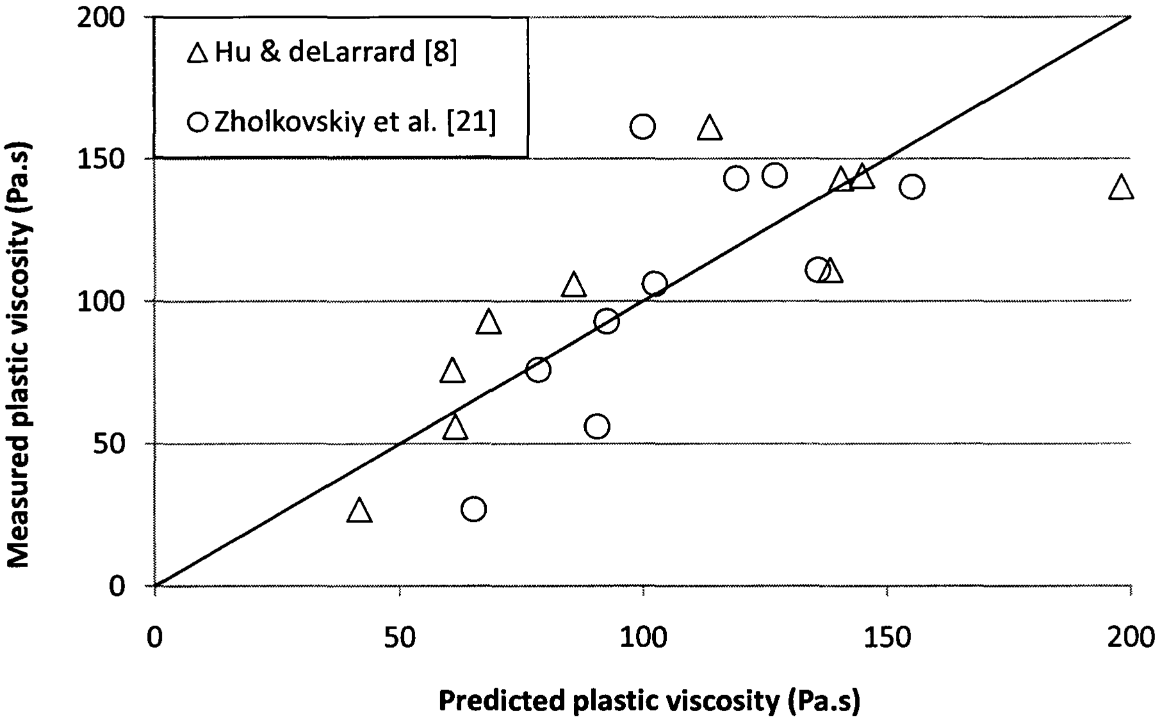
○ Murata & Kikukawa [7]
Measured plastic viscosity (Pa.s)
Predicted plastic viscosity (Pa.s)

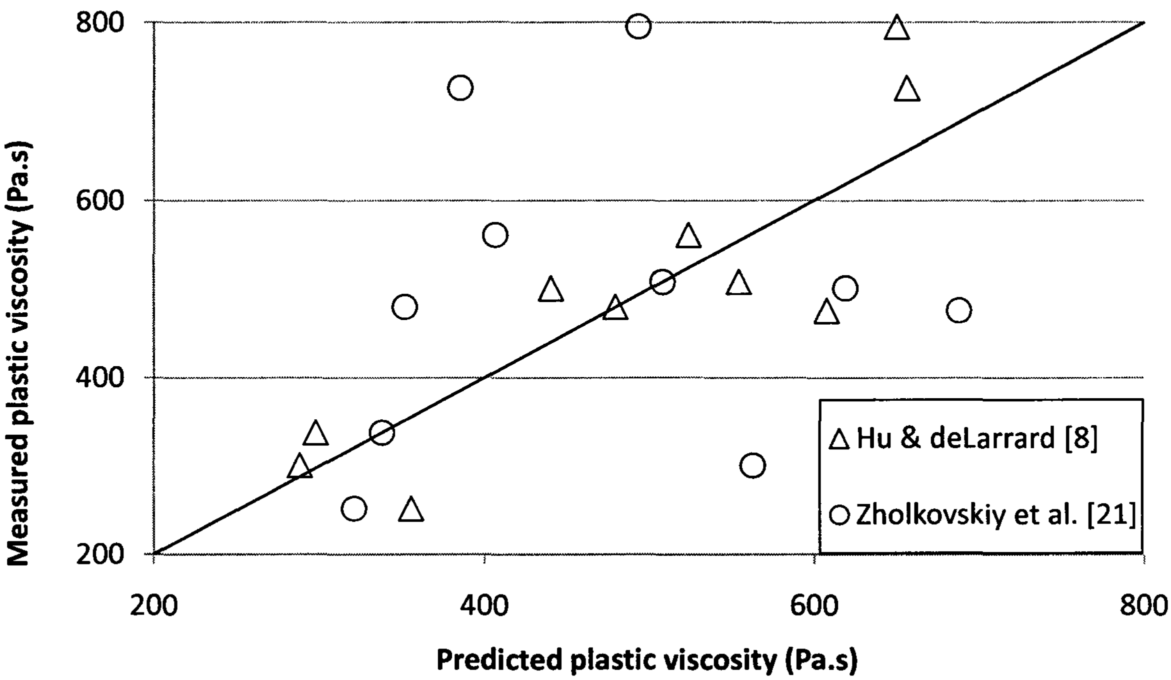
(1c)


800
600
400
200
200
400
600
800
△ Ferraris & deLarrard [5]
Measured plastic viscosity (Pa.s)
Predicted plastic viscosity (Pa.s)

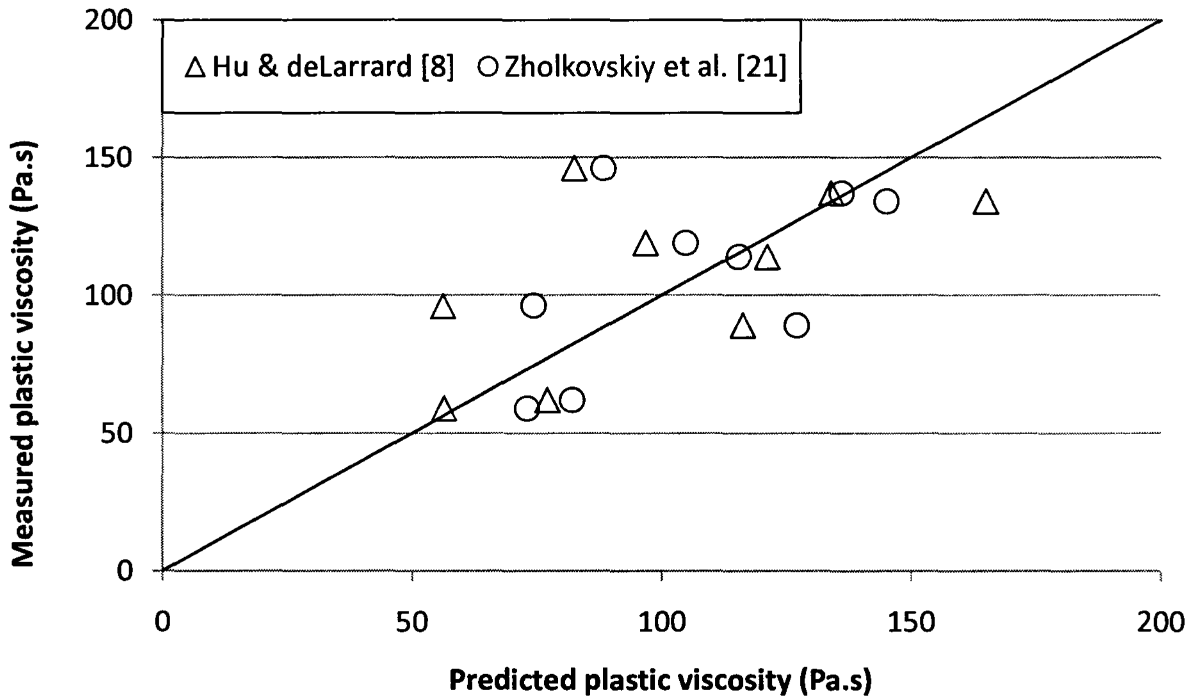
(1-d)


△ Hu & deLarrard [8]
○ Zholkovskiy et al. [21]
Measured plastic viscosity (Pa.s)
Predicted plastic viscosity (Pa.s)
0
50
100
150
200

(2a)


△ Hu & deLarrard [8]
○ Zholkovskiy et al. [21]
Measured plastic viscosity (Pa.s)
Predicted plastic viscosity (Pa.s)
200
400
600
800

(2b)


200
150
100
50
0
0
50
100
150
200
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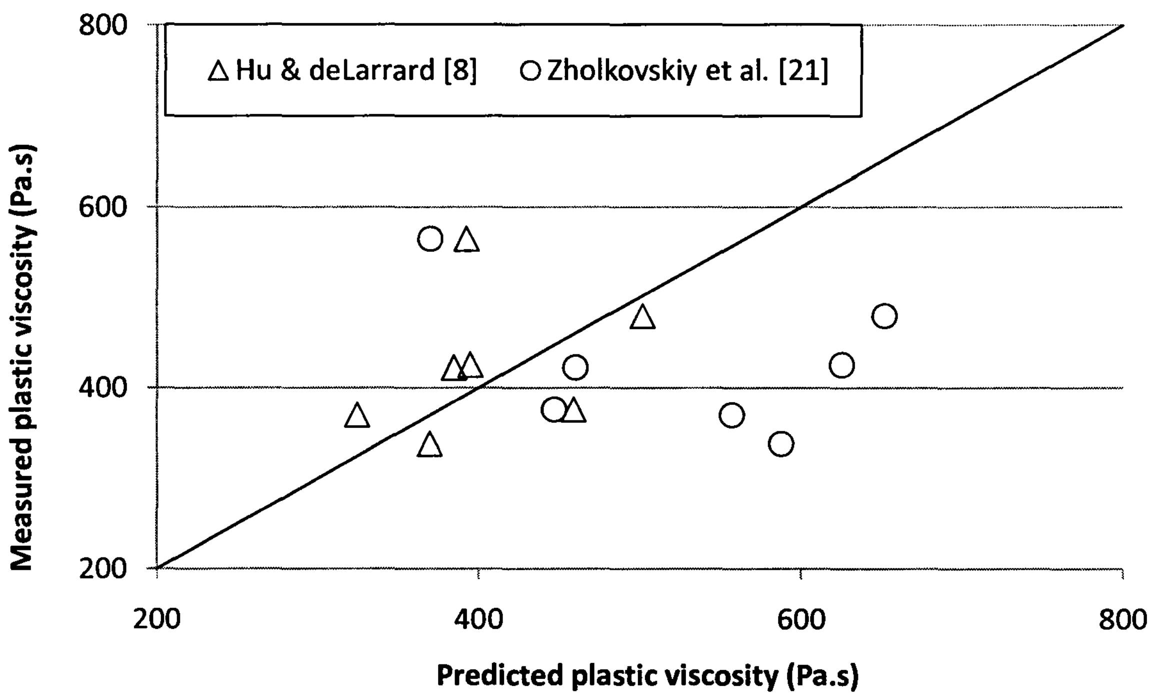
Measured plastic viscosity (Pa.s)
Predicted plastic viscosity (Pa.s)

(2c)


800
600
400
200
200
400
600
800
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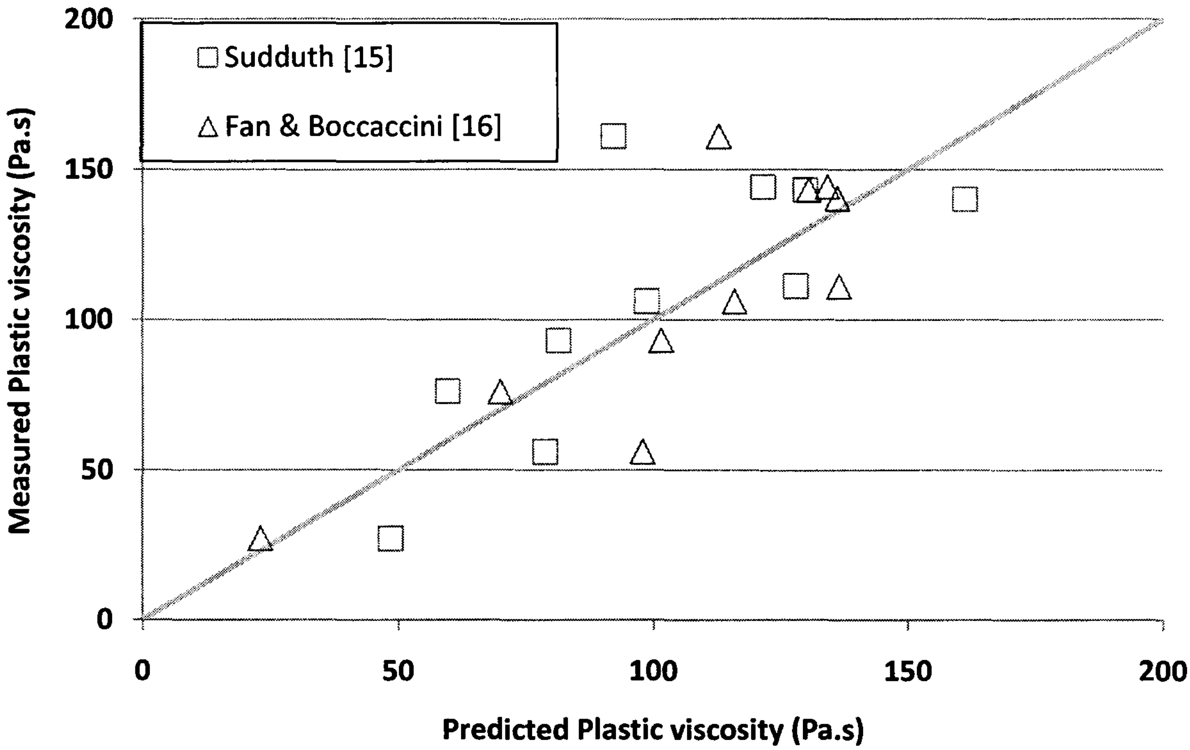
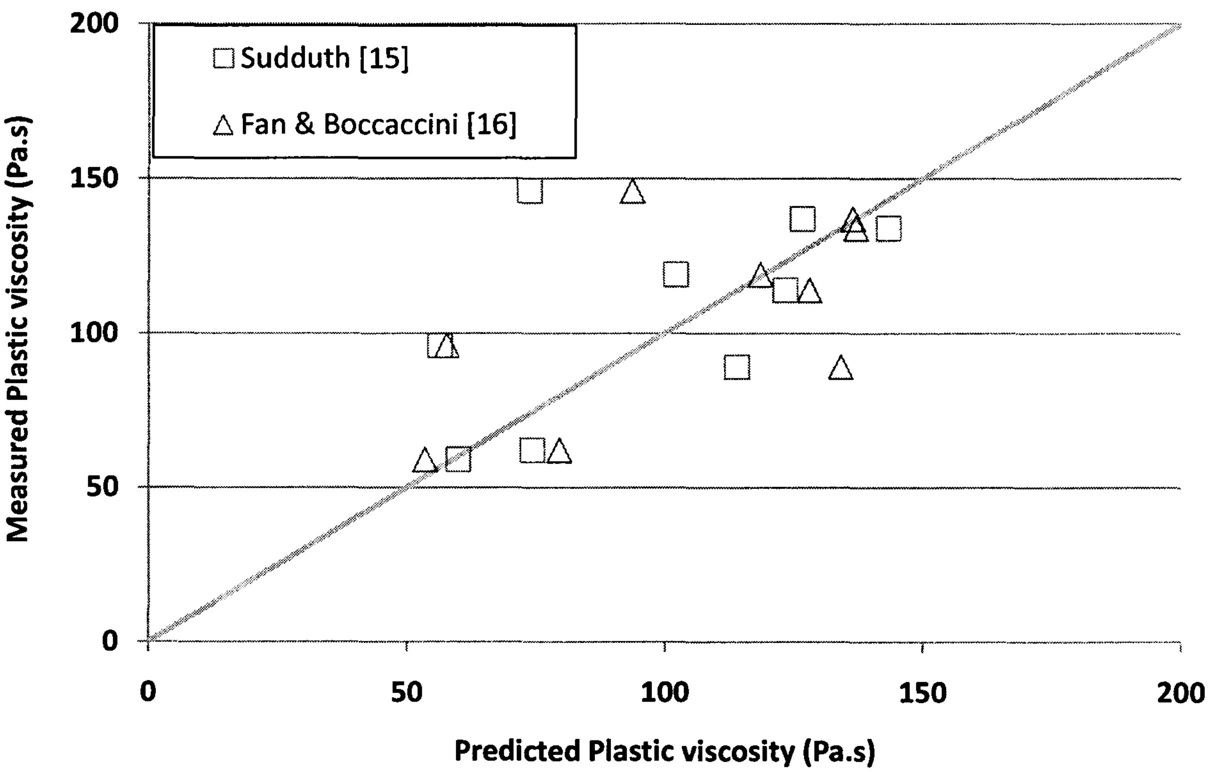
Predicted plastic viscosity (Pa.s)

(2d)



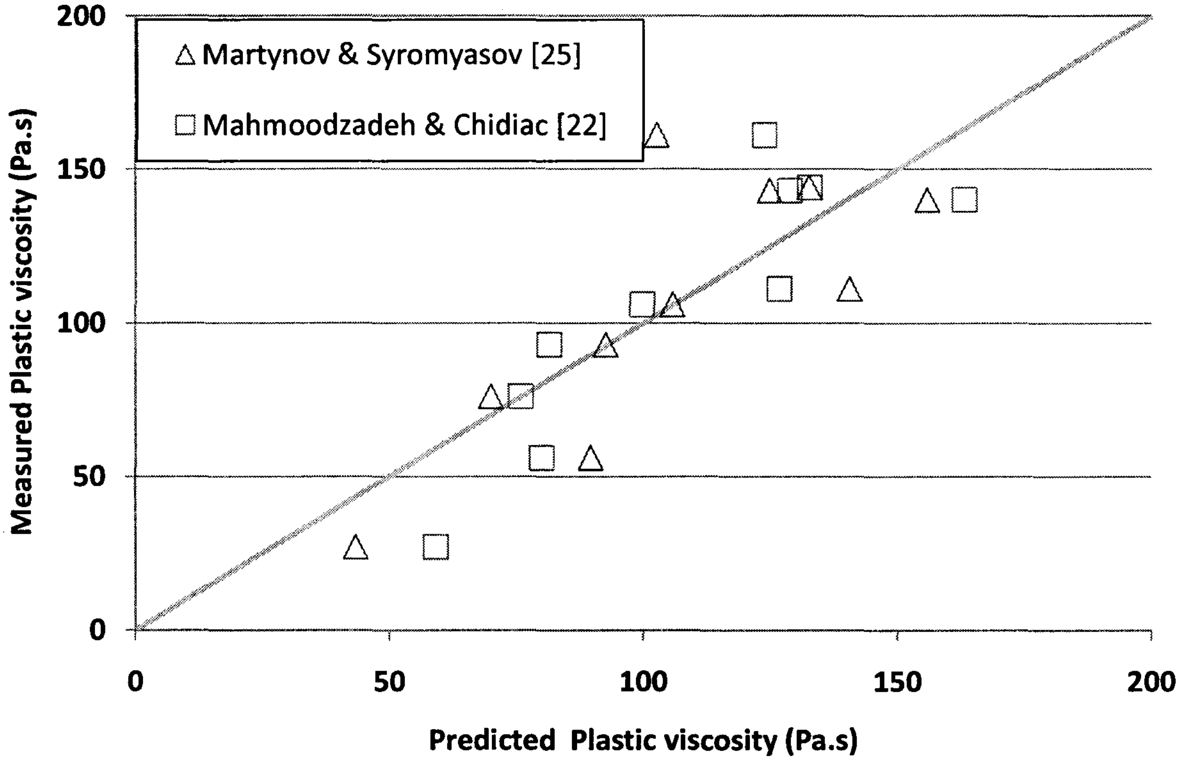
□ Sudduth [15]
△ Fan & Boccaccini [16]
Measured Plastic viscosity (Pa.s)
Predicted Plastic viscosity (Pa.s)
0
50
100
150
200

(3a)

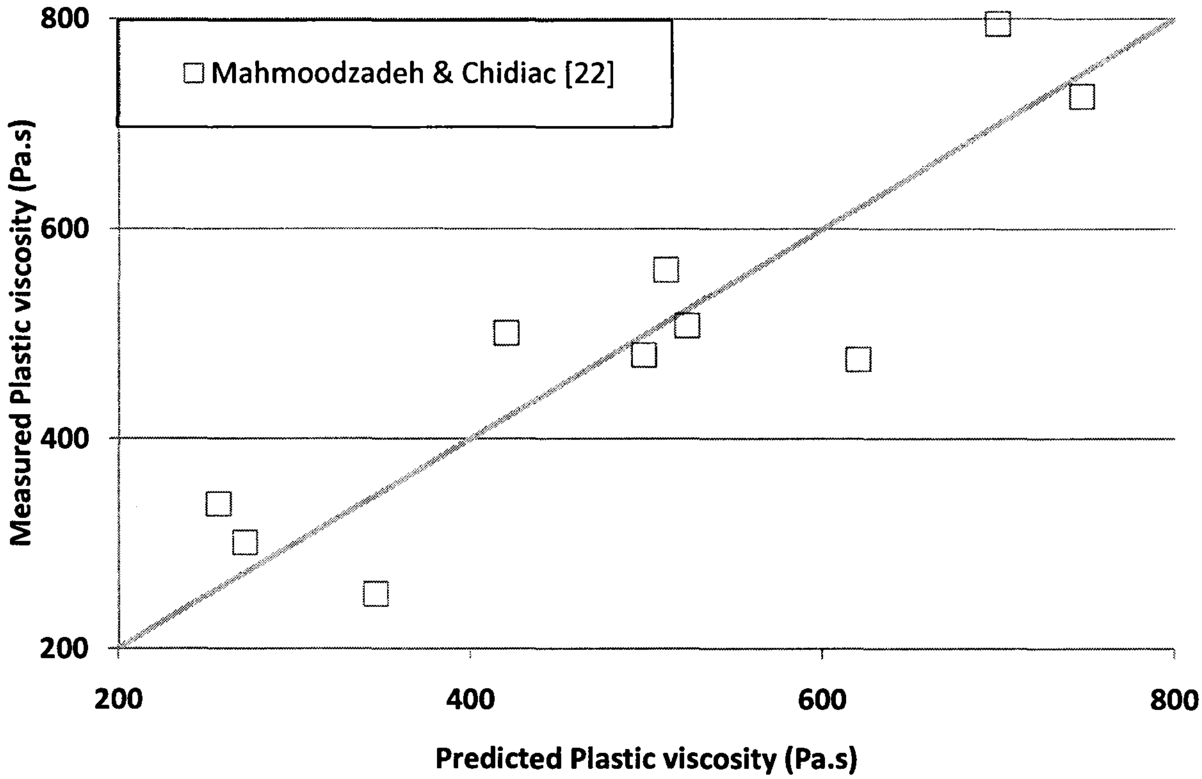


□ Sudduth [15]
△ Fan & Boccaccini [16]
Measured Plastic viscosity (Pa.s)
Predicted Plastic viscosity (Pa.s)
0
50
100
150
200

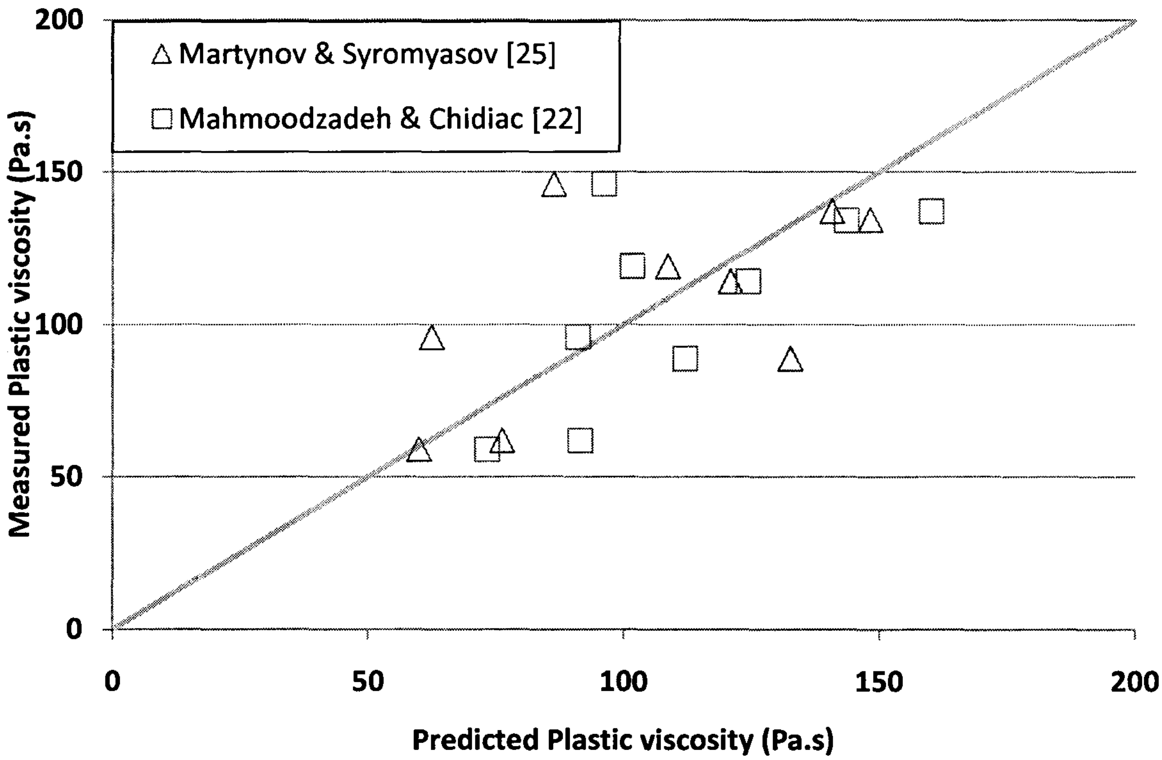
(3b)


△ Martynov & Syromyasov [25]
□ Mahmoodzadeh & Chidiac [22]
Measured Plastic viscosity (Pa.s)
Predicted Plastic viscosity (Pa.s)
0
50
100
150
200

(4a)

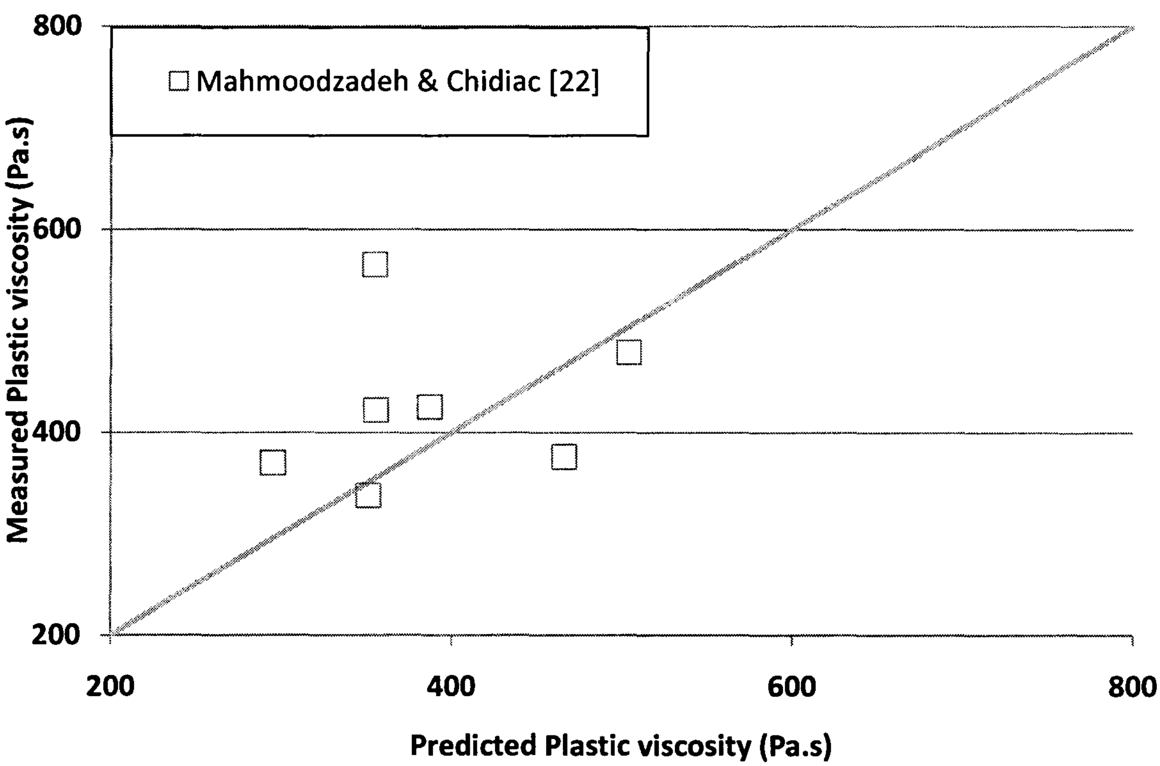

□ Mahmoodzadeh & Chidiac [22]
Measured Plastic viscosity (Pa.s)
Predicted Plastic viscosity (Pa.s)
200
400
600
800

(4b)



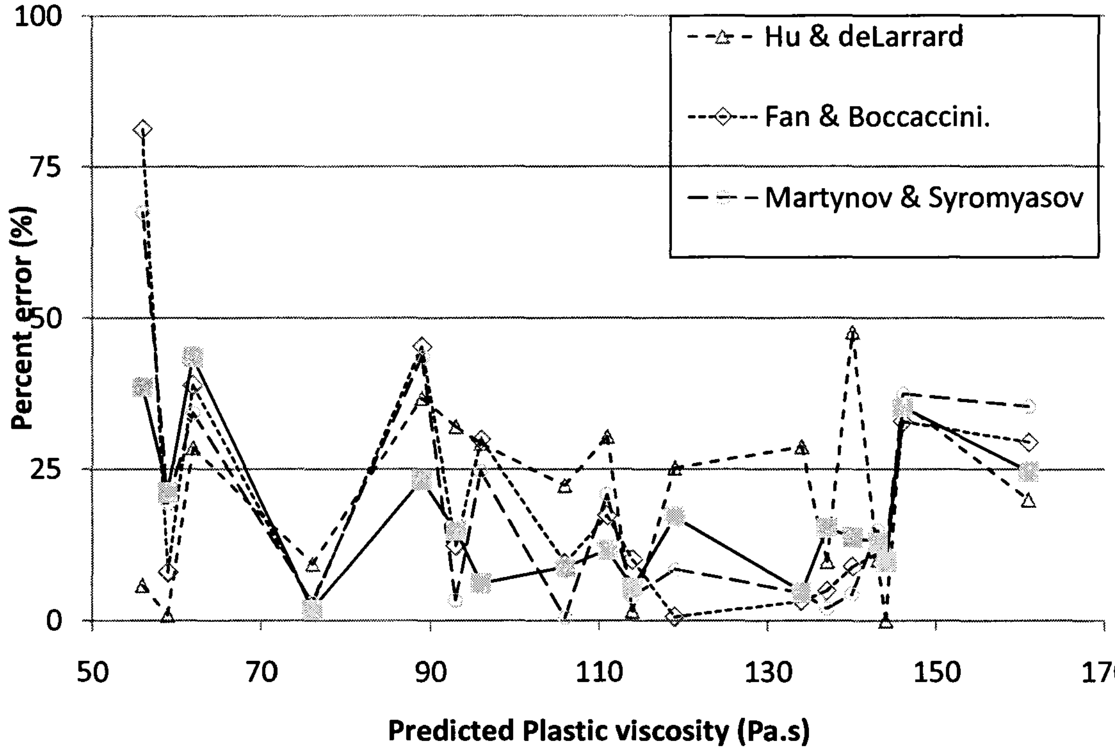
△ Martynov & Syromyasov [25]
□ Mahmoodzadeh & Chidiac [22]
Measured Plastic viscosity (Pa.s)
Predicted Plastic viscosity (Pa.s)
0
50
100
150
200

(4-c)

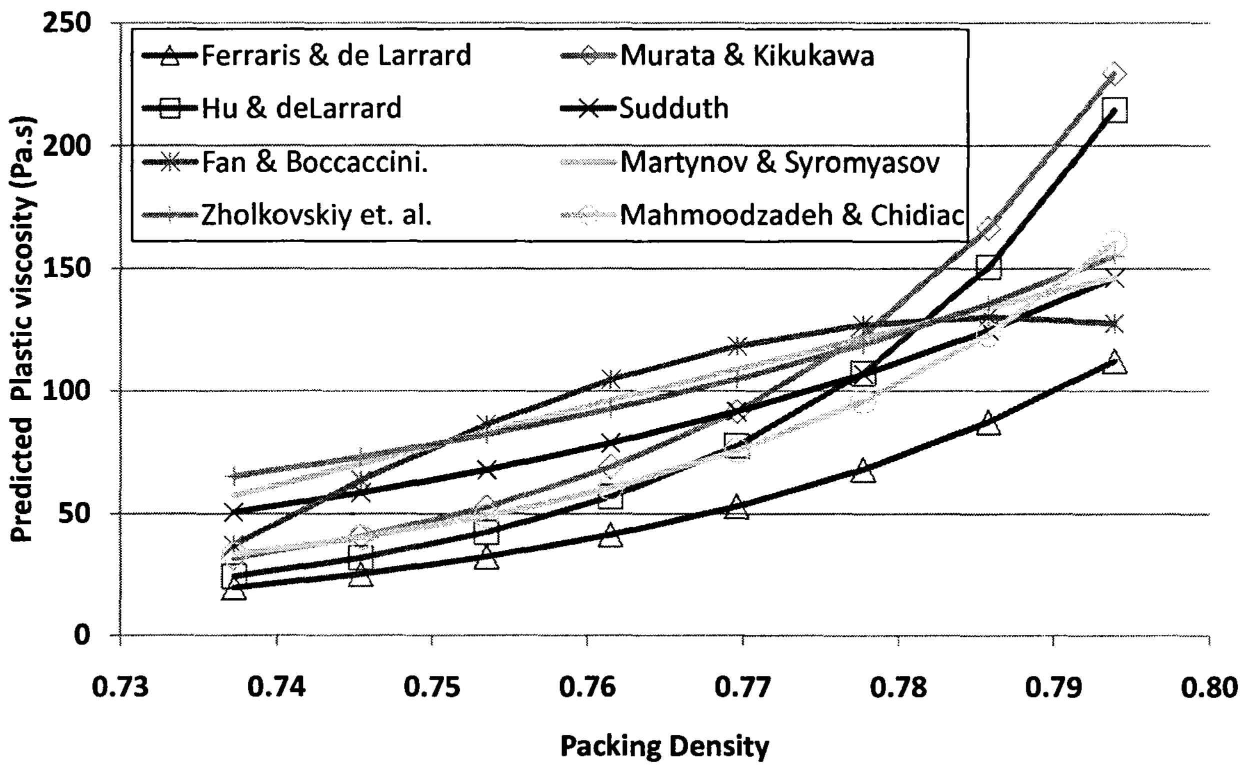


□ Mahmoodzadeh & Chidiac [22]
Measured Plastic viscosity (Pa.s)
Predicted Plastic viscosity (Pa.s)
200
400
600
800

(4-d)


Hu & deLarrard
Fan & Boccaccini.
Martynov & Syromyasov
Percent error (%)
100
75
50
25
0
50
70
90
110
130
150
170
Predicted Plastic viscosity (Pa.s)

(5)


Ferraris & de Larrard
Murata & Kikukawa
Hu & deLarrard
Sudduth
Fan & Boccaccini.
Martynov & Syromyasov
Zholkovskiy et. al.
Mahmoodzadeh & Chidiac
Predicted Plastic viscosity (Pa.s)
250
200
150
100
50
0
0.73
0.74
0.75
0.76
0.77
0.78
0.79
0.80
Packing Density

(6)

Appendix A: Summary of the reviewed models reported in the literature which relate the viscosity of the concentrated suspensions to the concentration of the suspension.

Method	Developed by	Proposed model	Parameters
Generalized	Dabak & Yucel [14]	$\eta_r = \left[1 + \frac{\eta_i \varphi \varphi_{max}}{n(\varphi_{max} - \varphi)})\right]^n$	η_i, n
Generalized	Sudduth [15]	$\ln(\eta / \eta_0) = \left(\frac{\eta_i}{k}\right)\left(\frac{1}{\sigma - 1}\right)\left((1 - k\phi)^{1-\sigma} - 1\right)$ $k = 1 / \varphi_{max}$	η_i, σ
Analogous	Fan & Boccaccini [16]	$\eta^c = \eta_\alpha f_{\alpha c} + \eta_\beta f_{\beta c} + \frac{\eta_\alpha \eta_\beta F_s}{\eta_\beta f_{\alpha III} + \eta_\alpha f_{\beta III}}$ $f_{\alpha c} = f_\alpha^m$, $f_{\alpha c} = f_\beta^n$	Not applicable
		$\eta_r = C + \varphi^m + \frac{F_s^2}{\varphi - \varphi^m}$	C, m, F_S
Analogous	Douglas [27]	$\eta = K\left(1 - \frac{\varphi}{\varphi_{max}}\right)^{-n}$	K, n
Analogous	Bicerano et al. [28]	$\eta_r = \left(1 - \frac{\varphi}{\varphi_{max}}\right)^{-n} \times \lambda$ $\lambda = \left[1 - C_1 \frac{\varphi}{\varphi_{max}} + C_2\left(\frac{\varphi}{\varphi_{max}}\right)^2\right]$	n, C_1, C_2
Cell	Simha [17]	$\eta_r = \eta / \eta_0 = 1 + \eta_i \lambda \varphi$ $\lambda = \frac{4(1 - y^7)}{\left[4(1 + y^{10}) - 25y^3(1 + y^4) + 42y^5\right]}$ $y = a / b = \frac{(\varphi / \varphi_{max})^{1/3}}{\left(2 - (\varphi / \varphi_{max})^{1/3}\right)}$	η_i
		$\eta_r \sim \frac{54}{4f^3}\left(\frac{\varphi^2}{1 - (\varphi / \varphi_{max})^3}\right)$	f

		$\eta / \eta_0 = 1 + \eta_{i2} \varphi \psi$	
Cell	Happel [18]	$\psi = \dfrac{4\gamma^7 + 10 - \frac{84}{11}\gamma^2}{10(1-\gamma^{10}) - 25\gamma^3(1-\gamma^4)}$ $\gamma = a/b = \varphi^{1/3}$	$\eta_{i2} \approx 5.5$
Cell	Frankel & Acrivos [19]	$\eta_r \sim C'\left[\dfrac{(\varphi/\varphi_{max})^{1/3}}{1-(\varphi/\varphi_{max})^{1/3}}\right]$	C'
Cell	Sengun & Probstein [29]	$\eta_r = 1 + C\left(\dfrac{3\pi}{8}\right)\left(\dfrac{\beta}{\beta+1}\right) \times \lambda$ $\lambda = \left[\dfrac{3 + 4.5\beta + \beta^2}{\beta+1} - 3(1+\dfrac{1}{\beta})\ln(\beta+1)\right]$ $\beta = \dfrac{2a}{h_0} = \dfrac{(\phi/\phi_{max})^{1/3}}{1-(\phi/\phi_{max})^{1/3}}$	C
Cell	Malomuzh & Orlov [30]	$\eta_r = \dfrac{\psi(1-\psi)}{\psi(1-\psi) + 1 - \sqrt{1+2\psi^2(1-\psi)}}$ $\psi = R_0^3 / R^3 \approx C\varphi$	C
Cell	Ruiz-Reina et al. [31]	$\eta(\varphi, \lambda) = 1 + \eta_i \varphi \times \lambda$ $\lambda = \dfrac{4(1-\varphi^{7/3})}{4(1+\varphi^{10/3}) - 25\varphi(1+\varphi^{4/3}) + 42\varphi^{5/3}}$	η_i
Cell	Sherwood [32]	$\eta_r = 1 + \eta_i \lambda$ $\lambda = \dfrac{4(y^7 - 1)}{10y^{10} - 25y^7 + 42y^5 - 25y^3 + 4}$ $\eta_r \sim \dfrac{2}{5\varepsilon^3}$, $y = a/b = \varphi^{1/3}$ $\eta_r = 1 + \dfrac{15y^7 + 20}{6y^{10} + 15y^7 - 63y^5 + 50y^3 - 8}$ $\eta_r \sim \dfrac{2}{3\varepsilon^2}$, $y = a/b = \varphi^{1/3}$	η_i

		$$\eta_r = 1 + \frac{25y^7 + 10}{10y^{10} - 10y^7 - 21y^5 + 25y^3 - 4}$$ $$\eta_r \sim \frac{1}{3\varepsilon^2},\ y = a/b = \varphi^{1/3}$$	
Cell	Zholkovskiy et al. [21]	$$\eta_r = 1 + \eta_i y^3 \lambda$$ $$\lambda = \frac{4(1-y^7)}{4(1+y^{10}) - 25y^3(1+y^4) + 42y^5}$$ $$y = a/b = \varphi^{1/3}$$	η_i
Cell	Mahmoodzadeh & Chidiac [22]	$$\eta_r = 1 + \eta_i y^3 \lambda$$ $$\lambda = \frac{4(1-y^7)}{4(1+y^{10}) - 25y^3(1+y^4) + 42y^5}$$ $$y(\varphi) = \frac{(\varphi/\varphi_{\max})^{1/3}}{2(1+K) - (\varphi/\varphi_{\max})^{1/3}}$$ Or $$y(\varphi) = (\varphi/\varphi_{\max})^{1/3}(1-K)$$ $$\begin{cases} K = 0.006 \times \dfrac{Cement}{Water} & \text{Without HRWR} \\ K = 3.8 \times \dfrac{HRWRA}{Cement} & \text{With HRWRA} \\ \times \dfrac{Water}{Cement + Fine\ Sand + Sand} & \end{cases}$$	η_i
Average	Brule & Jongschaap [33]	$$\eta_r = 1 + \frac{3\pi}{16}\left(\frac{1-\varepsilon}{\varepsilon}\right)$$ $$\eta_r = 1 + \frac{\pi}{4}\left(\varepsilon - 1 + \ln(\frac{1}{\varepsilon})\right)$$	$\varepsilon = 1 - \left(\frac{\varphi}{\varphi_m}\right)^1$

Average	Martynov & Syromyasov [25]	$\eta/\eta_0 = 1 + \eta_i \varphi \lambda$ $\lambda = \left[1 + \frac{15\beta_2}{2\pi}\varphi + \left(\frac{15\beta_2}{2\pi}\varphi\right)^2 + \frac{14}{3}\rho\left(\frac{3}{4\pi}\varphi\right)^{5/3}\right]$ $\beta_2 \approx -0.3594$	ρ , η_i

Chapter IV

Yield Stress of Fresh Concrete – A critical review of prediction methods

Abstract

Rheological properties, which include yield stress, affect the workability, placement, and productivity of fresh concrete as well as the quality of hardened concrete. Yield stress also affects the porosity, specifically air voids, which in turn influence the mechanical properties and durability resistance of concrete. Therefore, characterization of these properties is essential to control the quality of fresh and hardened concrete.

This paper presents a critical review of the most prevailing models in concrete literature as well as models proposed in the literature for predicting the yield stress of suspensions. Experimental data were used to evaluate the predictive capabilities and adequacy of these models in quantifying the yield stress of fresh normal slump concrete. The results, confirmed that the aggregate characterizing, namely packing density and maximum packing density generally influences the yield stress and that also excessive cement paste through particles interlocking affects yield stress.

Two yield stress models were found to yield good estimates: Mahmoodzadeh and Chidiac [1, 2] and Ferraris and deLarrard [3, 4]. Yield stress results estimated using BTRHEOM [3] and Slump Rate Machine II (SLRM II) [5, 6], are found to have the same trends but not equal values.

Keywords: concrete mixture; modelling; yield stress, plastic viscosity; rheology; workability

1. Introduction

For normal slump concrete, experimental data has confirmed that the flow of fresh concrete follows Bingham's material model [1-6], i.e.,

$$\tau = \tau_0 + \eta \, \dot{\gamma} \tag{1}$$

in which τ is the shear stress (Pa), τ_0 the yield stress (Pa), η the plastic viscosity (Pa.s) and, $\dot{\gamma}$ the shear strain rate (1/s). τ_0 and η are referred to as Bingham material properties with the first property providing a measure of the stress required to initiate flow and the second one a measure of the material resistance to flow after the material begins to flow [1-6]. These two rheological properties are, therefore, needed to characterize quantitatively the flow of fresh concrete [1-6].

Experimental methods for characterizing yield stress and plastic viscosity of fresh concrete have been reported in the literature [3, 5]. Although these testing methods are very beneficial and can be used to control the quality of fresh concrete, they cannot be incorporated into the design of concrete mixture. To overcome this limitation, researchers have developed models for quantifying the rheological properties from the concrete mixture composition [2-4, 7-11]. Although the proposed models differ in their formulation, they all employ packing density and maximum packing density of the mixture and the aggregate as their primitive variables. This paper provides a brief description of these models and an evaluation of their predictive capabilities by comparing the models results to experimental data reported in the literature.

2. Rheological models – yield stress

Two different approaches for modeling yield stress of fresh concrete have been proposed in the literature. There are models that are based on semi-phenomenological arguments and include the work of Hobbs [7], Ferraris and deLarrard [3], Toutou and Roussel [8], Szecsy [9], Noor and Uomoto [10] and Neilsen [11]; which were developed for concrete. There are models that are based on fundamental principles such as those developed by Zhou et al. [12], Flatt [13], Flatt and Bowen [14-15], Chateau et al. [16] and Mahmoodzadeh and Chidiac [2]. These models address mixtures of suspended concentrated particles. Furthermore, the model proposed by Mahmoodzadeh and Chidiac [2] accounts for concrete mixture by including volumetric fraction of solid material referred to as packing density (φ_S), maximum packing density of the whole mixture $(\varphi_{S\max})$, volume fraction of the aggregate (φ_A), and the maximum volume fraction of aggregates $(\varphi_{A\max})$. A brief description of these models is presented next.

2.1 Hobbs

Using a phenomenological argument and assuming that the Bingham model applies to the flow of both cement paste and concrete, Hobbs [7] developed the following equation,

$$\tau_o = \tau_P\left[1 + \frac{1.5\varphi_A\varphi_{A\max} + \varphi_{A\max}^2}{\varphi_{A\max} - \varphi_A}\right] \tag{2}$$

in which τ_P is the yield stress of cement paste.

2.2 Ferraris & de Larrard

The most promising model for fresh concrete is the semi-empirical model proposed by Ferraris and deLarrard [3-4]. They assumed that yield stress depends only on the volumetric fraction of solid material, and its maximum value. Accordingly, yield stress can be predicted through the following expression:

$$\tau_0 = f(\frac{\varphi_1}{\varphi_{1\max}}, \frac{\varphi_2}{\varphi_{2\max}}, ..., \frac{\varphi_n}{\varphi_{n\max}}) \tag{3}$$

By fitting eq. 3 with experimental data, they developed the following expression [8],

$$\tau_0 = 2.357 + 1.134 K_C^{'} + \sum_i [0.736 - 0.216 \log(d_i)] K_i^{'} \tag{4}$$

where $K_i^{'} = \frac{\varphi_i}{1-\varphi_{i\max}}$, $\varphi_i, \varphi_{i\max}$ and d_i are respectively the volume fraction, the maximum packing volume fraction and the size of particles of the class "i" Subscript "c" refers to cement.

2.3 Toutou and Roussel

Toutou and Roussel [8] investigated the yield stress for cement paste, mortar and concrete. In their formulation, they used the theory proposed by Coussot [17] to account for the gradation of aggregate in concrete. They proposed

$$\tau_0 = \tau_P \left(1 - \frac{\varphi_A}{\varphi_{A\max}}\right)^{-m} \tag{5}$$

where τ_P is the suspending fluid yield stress and can be taken as the yield stress of cement paste. They proposed Legrand formulation [18] to determine τ_P,

$$\tau_P = a \times e^{b(\varphi_C - 0.5)} \tag{6}$$

where a and b are two fitting coefficients.

2.4 Szecsy

Szecsy [9] developed a model that is based on the following conditions:

1. Concrete yield stress should be formulated based on the volume fraction of the aggregates and the paste yield stress.
2. When the aggregate volume fraction is zero, concrete yield stress has to be the same as the paste yield stress.
3. The suspensions can obey non-Newtonian behaviour even when the fluid phase follows Newtonian behaviour. Accordingly, concrete can have a yield stress even when the paste yield stress is zero.

Accordingly, the following model is proposed,

$$\tau_0 = \tau_P + \tau_P^2\ \varphi_A\ \left(10\ \varphi_{A\max}^3 + \varphi_{A\max}^2 + \varphi_{A\max}\right) + D\frac{\varphi_A\ \varphi_{A\max}}{\left(1-\beta_{fa}\right)} \qquad (7)$$

where β_{fa} is the percentage of fine aggregate within the total aggregate, and D a function of the circularity of the coarse aggregate.

2.5 Noor and Uomoto

Noor and Uomoto [10], by arguing that the yield stress of concrete is equal to the yield stress of cement paste if the coarse aggregate content is zero, proposed the following equation,

$$\tau_0 = \tau_P + f(x), \qquad (8)$$

where $f(x)$ accounts for the aggregate content, and x represents the total apparent aggregate volume. x is defined as

$$x = \frac{\varphi_S}{\varphi_{S\max}} + \frac{\varphi_G}{\varphi_{G\max}} \tag{9}$$

in which φ_S is the fine aggregate volume fraction, $\varphi_{S_{\max}}$ the maximum fine aggregate solid volume, φ_G the coarse aggregate volume fraction, and $\varphi_{G_{\max}}$ the maximum coarse aggregate solid volume. Using minimization technique to fit the model with experimental data, they proposed the following functions

$$\tau_P = k_1 \cdot k_2^{\frac{w}{p}} \tag{10}$$

$$f(x) = k_3 \cdot x^{k_4} \tag{11}$$

where k_1, k_2, k_3 and k_4 are fitting parameters, and w/p is water to powder ratio.

2.6 Neilsen

For extreme liquid composites such as self-compacting concrete, Neilsen [11] developed a model for both yield stress and plastic viscosity as a function of particulate phase volume, V_P, suspending medium volume, V_S, the aspect ratio of aggregate (length/ diameter), A, and shape function power, M. The respective model is given by,

$$\tau_0 = \tau_P \left(1 + \psi \varphi_A\right) \tag{12}$$

where ψ is the geometry function and is defined as follows,

$$\psi = \frac{3}{2} \frac{\mu_P + \mu_S - 1}{\mu_S} \tag{13}$$

with μ_P and μ_S are shape functions and are defined by the following expressions,

$$\mu_S = \mu_S^0\left(1-\frac{\varphi_A}{\varphi_s^*}\right)^M \tag{14}$$

$$\mu_P = \mu_P^0\left(1-\frac{\varphi_A}{\varphi_P^*}\right)^M \tag{15}$$

$$\varphi_P^* = -\frac{\mu_P^0}{\mu_S^0}\varphi_s^* \tag{16}$$

$$\mu_P^0 = \begin{cases} \dfrac{3A}{A^2+A+1} & A \le 1 \\ 3\dfrac{A^2-A+1}{4A^2-5A+4} & A > 1 \end{cases} \tag{17}$$

$$\mu_S^0 = \begin{cases} \mu_P^0 & A \le 1 \\ 4\mu_P^0 - 3 & A > 1 \end{cases} \tag{18}$$

2.7 Hu

Hu [19] argued that the parameters that control the yield stress are the excessive paste thickness and the aggregate friction. Using a statistical regression method, these two parameters were combined yielding the following expression for yield stress of fresh concrete,

$$\tau_0 = C_1\left[\tau_m\left(1+\frac{1}{t_m^{'}}\right)\right]^{C_2}\left[Tan\ \theta_{CA}\right]^{C_3} \tag{19}$$

where θ_{CA} represents the friction angle of aggregate, C_1, C_2 and C_3 are fitting parameters, τ_m the yield stress of mortar, and $t_m^{'}$, is the nominal excess mortar thickness,

$$t_m' = \frac{t_m}{r_{CA}} \tag{20}$$

in which t_m is the excess mortar thickness and r_{CA} the average radius of coarse aggregate. It is further assumed that each particle is covered with same mortar thickness, i.e.,

$$t_m = \frac{V_{em}}{S_{CA}} \tag{21}$$

where S_{CA} is the surface area of coarse aggregate, V_{em} the volume of the excess mortar and is given by the following equation,

$$V_{em} = V_m - V_{cm} = \frac{W_w + W_C + W_{FA}}{\gamma_m} - \frac{V_{n-CA}\left(W_{CA} / \gamma_{CA}\right)}{1 - V_{n-CA}} \tag{22}$$

in which V_m and V_{cm} are respectively the volume of mortar and mortar between the voids, and W_w, W_C, W_{FA} and W_{CA} are the weight of water, cement, fine aggregate and coarse aggregate, respectively. γ_m and γ_{CA} are the density of mortar and coarse aggregate, and V_{n-CA} can be calculated according to the specific gravity at SSD condition (ASTM C29). To find τ_m, similar theory is proposed,

$$\tau_m = C_4 \tau_P^{C_5} \left(1 + \frac{1}{t_p'}\right)^{C_6} \tag{23}$$

$$\tau_P = C_7 e^{C_8 w/c} \tag{24}$$

It should be noted that the proposed model accounts for the effect of angularity and shape of the particle through the angle of friction of the aggregate.

2.8 Zhou et al.

Zhou et al. [12] investigated the work of Kapur et al. [20], Scales et al. [21] and Tanaka et al. [22] and proposed a simplified equation for yield stress of powder mixes,

$$\tau_o = \left(\sum \varphi_i \tau_{0i}^{1/2}\right)^2 \tag{25}$$

The proposed formulation links the yield stress of suspension, τ_o, to the yield stress of individual components for powder mixes, τ_{oi}.

2.9 Flatt and Bowen

Flatt [13] and Flatt and Bowen [14-15] developed a yield stress model for multimodal powder suspensions (YODEL). The model postulates that the shear stress is greater than the attractive network forces through the interpretation of the inter-particle forces. The model is given by

$$\tau_0 = m_1 \frac{(\varphi_S - \varphi_0)^2}{\varphi_{S\max}(\varphi_{S\max} - \varphi_S)} \tag{26}$$

where m_1 is a function of inter-particle forces, particle size and particle distribution, and φ_0 is a percolation threshold. This model is developed mainly for powder suspensions where particles interaction is mainly dominant for colloidal suspensions. Toutou & Roussel [8] reviewed Flatt's work and noted that it is only applicable to cement paste.

2.10 Chateau et al.

Chateau et al. [16] studied the yield stress of dense fluids and adopted the homogenization approach. They assumed that the heterogeneities of the secant modulus can be neglected over the fluid phase domain, and that the rheological properties can be estimated from a fictitious linear suspension having the same microstructure. For Bingham materials, their formulation leads to the following equations,

$$\tau_r = \sqrt{(1-\varphi)\eta_r} \tag{27}$$

where τ_r and η_r are, respectively, the relative yield stress, and plastic viscosity of suspensions.

2.11 Mahmoodzadeh and Chidiac

According to Chidiac and Mahmoodzadeh [1] and Mahmoodzadeh and Chidiac [2], plastic viscosity captures the dynamic interaction behaviour of particles and yield stress the static interaction, and that both properties are controlled by excessive paste thickness. They postulated that paste thickness plays a critical role in the interlocking behaviour which affects yield stress while the concrete is at rest, and the degree of lubrication between particles which affects plastic viscosity while the concrete is flowing. Accordingly, they extended the same analogy proposed for plastic viscosity formulation to the yield stress, in which,

$$\tau_0 = \tau_i \, y(\varphi_S)^3 \frac{4\left(1-y(\varphi_S)^7\right)}{4\left(1+y(\varphi_S)^{10}\right)-25y(\varphi_S)^3\left(1+y(\varphi_S)^4\right)+42y(\varphi_S)^5} \tag{28}$$

and

$$y(\varphi_S) = \left(\varphi_S / \varphi_{S\max}\right)^{1/3}\left(1 - C_Y \frac{m_G}{m_W}\right) \tag{29}$$

where τ_i is the "intrinsic" yield stress and is a function of the shape of the particles and C_Y a fitting parameter. $y(\varphi_S)$, is the ratio of particle size to cell size, and the cell consists of a particle at its centre surrounded by fluid. m_G and m_W are, respectively, the weight of coarse aggregate and water of the mixture.

2.12 Remarks

Yield stress models, phenomenological and fundamental based, have employed volume fraction and maximum volume fraction of the aggregates, also known as packing density and maximum packing density respectively. This illustrates that the proposed models postulate that yield stress is mostly influenced by particle interactions and that the latter can be represented by packing density. Closer examination of the models reveals that the model proposed by Hu [19] and that of Mahmoodzadeh and Chidiac [2] are conceptually similar but are fundamentally different. Both yield models account for the paste; Hu [19] employs excessive paste theorem to determine the excess mortar thickness, whereas Mahmoodzadeh and Chidiac [2] adopt the cell method. In the former, the excess mortar thickness, t_m, is constant, Eq. 21, whereas in Mahmoodzadeh and Chidiac [2] the ratio of the mortar thickness to particle size is constant. Another fundamental difference between the two models is the accountability of the shape and aggregate of the particles. Hu [19] proposed the aggregate angle of friction whereas the Mahmoodzadeh and Chidiac [2] have opted to account for the aggregate characteristics through the maximum packing density. Adapting the concept of maximum packing density into Hu's model, one can demonstrate the difference between the two models. For this comparison, V_m and V_{cm} can be represented by

$$V_m = 1 - \varphi_A \tag{30}$$

$$V_{n-CA} = 1 - \varphi_{A\,mac} \tag{31}$$

By substituting the above equation in Eq. 22, 21, and 20, a revised expression of $t_m^{'}$ is derived, where

$$t_m^{'} = \frac{1}{3}\left(\frac{1}{\varphi_A} - \frac{1}{\varphi_{A\max}}\right) \tag{32}$$

In comparison, Mahmoodzadeh and Chidiac [2] yield the following expressions,

$$t_m^{'} = \left(\frac{\varphi_A}{\varphi_{A\max}}\right)^{-\frac{1}{3}} - 1 \tag{33}$$

Fig. 1 shows a plot of Eq. 32, and 33 as a function of packing density and the results show that Hu's model yields larger values for $t_m^{'}$ in comparison to Mahmoodzadeh and Chidiac model. Another significant difference between Hu and Mahmoodzadeh and Chidiac models is the accountability of the different groups of particles, specifically fine and coarse aggregate for concrete. Hu adopts Farris theory [24] whereas Mahmoodzadeh and Chidiac use the maximum packing density to account for the particles gradation. In addition to the differences noted, Hu's model requires a large number of fitting parameters which is found not feasible for the experimental data used to evaluate the model.

3. Experimental program

Experimental data reported in the literature was employed to evaluate the predictive capabilities of the yield stress models. Toward that objective, it was decided to use two experimental programs, with different concrete mixtures, aggregate properties and test methods. The latter was included in the evaluation because there is no standard test method and different test methods have been shown to yield different values [24, 25]. The use of two experimental programs in two different laboratories will permit one to test the models' ability to discriminate among mixtures based on their proportion and aggregate properties. The two experimental programs were carried out at McMaster University (Canada) and the National Institute of Standards and Technology, NIST (USA).

3.1 McMaster University

An experimental program was carried out at McMaster University to evaluate the rheological properties of normal slump concrete mixtures using the slump rate machine II, SLRM II [5, 6]. The mixture proportioning was based on CAC guidelines [26]. The variables studied are water to cement ratio, w/c, water content, bulk volume of aggregate and maximum size of aggregate.

The concrete consisted of crushed limestone coarse aggregate, siliceous sand, GU-type 10 cement and water. The chemical and physical properties of the cement are given in Table 1. Two nominal maximum aggregate sizes were used, 14 mm and 20 mm. The corresponding particle size distribution is given in Fig. 2. The specific gravity, absorption value and bulk density of the 14mm and 20 mm aggregates are, respectively, 2.74 and 2.75, 0.88%, and 0.92%., and 1576 kg/m^3 and 1636 kg/m^3. The gradation of the sand is shown in Fig. 2. The corresponding fineness modulus, specific gravity, absorption and bulk density are 2.71. 2.71. 1.58% and 1812 kg/m^3, respectively. The bulk density, specific gravity and absorption for coarse aggregate and sand were measured in accordance with ASTM C127-04 [27] and ASTM C128-04 [28], respectively. The particle size distribution was in accordance to CSA A23.2a [29].

Eight concrete mixtures were evaluated and the corresponding proportions are given in Table 2. The concrete was mixed using a pan mixer. The dry ingredients were first mixed for 2 minutes; then one third of the water content was added and continued mixing for 2 min. The remaining water was added and the mixing continued for another 2 min. Subsequently, the mixing was stopped for one minute before resuming for one minute. SLRM II was used to measure the slump, slump flow, and time of slump. The yield stress was estimated using the following equation [5]

$$\tau_0 = 0.0397 \frac{\rho}{S_f^{\,2}} \tag{34}$$

where ρ is the density of the mixture and S_f the slump flow. The measured properties and estimated yield stress values are given in Table 2.

3.2 NIST

Ferraris and de Larrard [3] carried out a comprehensive experimental program to test the rheological properties of fresh concrete. Parallel plate concrete rheometer, BTRHEOM, was used to estimate the yield stress and plastic viscosity of the concrete. Eighteen concrete mixtures were evaluated as reproduced in Table 3. The mixture proportions and estimated yield stress values are given in Table 3.

4. Evaluation methods

Two methods are used to assess statistically the predictive capabilities of the yield stress models. A global assessment of the models is first carried out by calculating the variance of the error term [1]. The variance of the error term (σ) was calculated in accordance with

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left(\tau_{i\,Experimental} - \tau_{i\,Model}\right)^2}{n-q} \tag{35}$$

where the term $(n - q)$ represents the model degrees of freedom, n the number of data points and q the number of fitting parameters. The covariance and correlation were also calculated to determine the extent to which these models co-vary [1].

5. Comparative analyses

Two sets of experimental data are needed to evaluate the yield stress models. The first set is used for determining the fitting parameters for the models, using statistical regression analysis, and the second set of experimental data is used for testing the predictions of the models. The number of fitting parameters for the tested models is given in Table 4.

5.1 McMaster University

For the eight concrete mixtures, due to lack of number of data, all measurements were used to calibrate the models. The corresponding yield stress estimations are given in Table 5. Tables 4 and 6 provide a summary of the models standard error and degree of correlation with the experimental data as well as among models. Comparison of the Mahmoodzadeh and Chidiac model, versus Ferraris and deLarrard [3-4] are given as an example in Fig 3.

5.2 NIST

Nine measurements of the 18 concrete mixtures were used to calibrate the models and the remaining nine for evaluating the models. The yield stress estimations according to the models are given in Table 7 and 8. The corresponding standard errors calculated for the regression analysis and model evaluation are given in Table 4. Table 9 provides a summary of the degree of correlation among the models. Results of Table 4 show that the errors are consistent for the regression and evaluation. The error results also reveal that the models proposed by Mahmoodzadeh and Chidiac and Ferraris and deLarrard [3-4] yield the lowest error followed by Chateau et al. [16]. The other models predictions have errors that are large in comparison to Mahmoodzadeh and Chidiac. These results are expected given that Zhou et al. [12] model is intended for powder suspensions, Szecsy [9] and Noor and Uomoto [10] do not adequately account for the interaction between paste and aggregate, and Toutou and Roussel [8] and Hobbs [7] approach does not appear to capture the behaviour of concrete. This phenomenon is further illustrated by plotting the results as shown in Figs. 4 to 6. They show that four models provide acceptable predictions and that the models proposed by Mahmoodzadeh and Chidiac [2] and Ferraris and deLarrard [3-4] provide a better and more consistent prediction in comparison to the experimental data. However, it should be noted that the Ferraris and deLarrard [3-4] model employs four fitting parameters whereas the Mahmoodzadeh and Chidiac model uses 2 parameters.

Fig. 7 presents the results of the Ferraris and deLarrard [3-4] model and Mahmoodzadeh and Chidiac model in terms of percent error. The result revealed that the models predictions, with the exception of one measurement, are less than 20% and with the majority of the predictions below 10%. The results also show that there is no bias in the model prediction and that the error is random for low and high yield stress values. It should also be noted that the reported percent error is within the tolerated experimental measurement errors.

5.3 Standard test method

It was noted that there are no standard test methods. However, a significant statistical correlation was derived among the various methods [24]. This is further demonstrated by plotting the results of the Mahmoodzadeh and Chidiac yield stress model developed for the McMaster University and NIST experimental program, corresponding to SLRM II and BTRHEOM, as a function of the ratio of particle packing density over maximum packing density. The results, plotted in Fig. 8, reveal that both test methods yield the same trend and with SLRM II producing greater yield stress values.

These results are expected, given the difference in the basic formulations adopted to derive these estimates. Specifically, BTRHEOM predictions are derived from shear stress measurements whereas SLRM II predictions are derived from effective shear stress calculations.

6. Conclusions

This paper provides a review of the prevailing models used for predicting the yield stress of fresh concrete and other yield models proposed for concentrated suspensions. The review has led to the following conclusions:

1. Yield stress is mostly affected by the packing density and maximum packing density.
2. Excessive paste thickness is found to affect the yield stress through interlocking of the particles.
3. The proposed models for powder suspensions cannot be used to predict yield stress of fresh concrete.
4. Models that yielded poor estimates of the yield stress did not adequately account for the particle gradation and/or for the interaction between the aggregate and the paste.
5. Two models were found to provide good estimates for the yield stress namely that of Mahmoodzadeh and Chidiac and Ferraris and deLarrard [3-4]. It is worth noting that the Mahmoodzadeh and Chidiac model employs two fitting parameters whereas the Ferraris and deLarrard model [3-4] uses 4.

6. Yield stress results obtained from the BTRHEOM and SLRM II are found to have the same trends but not the same values.

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Tables

Table 1: Chemical and physical properties of hydraulic cement GU-type 10

	Hydraulic Cement GU-Type 10
SiO_2 (%)	19.7
Al_2O_3 (%)	4.9
Fe_2O_3 (%)	2.4
CaO (%)	62.2
MgO (%)	3.1
SO_3 (%)	3.4
Na_2O (%)	1.3
Loss of Ignition (%)	2.9
Equivalent Alkalies (%)	0.75
Specific Surface Area (Blaine)	4280
% Passing 325 (45um) Mesh (%)	90.7
Time of Setting-Initial (min)	115
Compressive Strength – 28 Day (MPa)	41.9

Table 2: McMaster University concrete mixture proportions and properties

Mix#	W/C	Water (kg/m³)	Cement (kg/m³)	CA Size	CA (bulk vol.)	CA (kg/m³)	FA (kg/m³)	Slump flow (mm)	Density (kg/m³)	Yield stress (Pa)
1	0.4	216	540	14	0.50	794	807	280	2385	1208
2	0.6	216	360	14	0.62	971	787	395	2385	607
3	0.4	228	570	14	0.62	971	574	255	2398	1464
4	0.6	228	380	14	0.50	794	912	360	2376	728
5	0.4	205	513	20	0.69	1134	542	213	2424	2131
6	0.6	205	342	20	0.57	928	892	375	2404	679
7	0.4	216	540	20	0.57	928	692	290	2403	1134
8	0.6	216	360	20	0.69	1134	643	400	2405	597

Table 3: NIST concrete mixture proportions and properties

Mix#	Gravel (kg/m³)	Sand (kg/m³)	Fine sand (kg/m³)	Cement (kg/m³)	water (kg/m³)	Packing density	Maximum Packing density	Yield stress (Pa)
1	952	614	190	360	204	0.794	0.853	1599
2	947	611	189	358	208	0.790	0.853	1341
3	943	607	189	356	212	0.786	0.853	983
4	938	604	188	354	216	0.782	0.853	778
5	996	642	199	237	212	0.786	0.854	1162
6	473	972	302	226	251	0.747	0.831	949
7	460	944	293	347	231	0.767	0.835	1234
8	455	934	290	344	239	0.758	0.835	1071
9	450	925	287	340	247	0.750	0.835	906
10	1093	367	114	529	220	0.778	0.834	1665
11	1081	363	113	523	228	0.770	0.834	1281
12	1070	359	111	518	236	0.762	0.834	869
13	851	549	170	527	222	0.776	0.833	1841
14	843	543	169	522	230	0.768	0.833	1115
15	834	537	167	517	238	0.760	0.833	901
16	413	849	264	512	244	0.753	0.823	1496
17	409	840	261	507	252	0.745	0.823	1137
18	405	831	258	501	260	0.737	0.823	771

Table 4: Summary of standard error obtained from models calibration and evaluation for yield stress of fresh concrete and number of fitting parameters.

Model	NIST			McMaster	
	Number of parameters	Error, σ, (Pa)		Number of parameters	Error, σ, (Pa)
		Regression	Evaluation		Regression
Hobbs [7]	2	230	401	2	285
Ferraris and deLarrard [5]	4	138	184	3	373
Toutou and Roussel [8]	3	260	399	3	311
Szecsy [9]	3	364	407	3	320
Noor and Uomoto [10]	4	215	380	4	325
Zhou et al. [12]	5	214	288	4	387
Chateau et al. [16]	2	160	247	2	331
Mahmoodzadeh and Chidiac [2]	2	121	166	2	249

Table 5: Yield stress model prediction of McMaster experimental data (Pa)

Mix#	Experimental	Hobbs	Ferraris and deLarrard	Toutou and Roussel	Szecsy	Noor and Uomoto	Zhou et al.	Chateau et al.	Mahmoodzadeh and Chidiac
1	1208	1413	1234	1429	1319	1443	1277	1474	1180
2	607	684	606	690	659	644	643	674	683
3	1464	1308	1516	1309	1425	1401	1458	1495	1153
4	728	614	504	622	501	517	527	684	477
5	2131	1587	1622	1586	1652	1604	1624	1453	2023
6	679	704	639	706	639	763	679	665	716
7	1134	1458	1500	1440	1553	1486	1460	1468	1569
8	597	604	927	590	789	720	793	672	535

Table 6: Degree of correlation among models for McMaster test data

Experimental	Hobbs	Ferraris and deLarrard	Toutou and Roussel	Szecsy	Noor and Uomoto	Zhou et al.	Chateau et al.	Mahmoodzadeh and Chidiac
0.87	1.00	0.93	1.00	0.97	0.99	0.97	0.98	0.94
0.85	0.93	1.00	0.92	0.99	0.96	0.99	0.93	0.88
0.88	1.00	0.92	1.00	0.97	0.99	0.96	0.98	0.93
0.86	0.97	0.99	0.97	1.00	0.99	1.00	0.96	0.93
0.86	0.99	0.96	0.99	0.99	1.00	0.98	0.97	0.92
0.88	0.97	0.99	0.96	1.00	0.98	1.00	0.96	0.92
0.82	0.98	0.93	0.98	0.96	0.97	0.96	1.00	0.85
0.91	0.94	0.88	0.93	0.93	0.92	0.92	0.85	1.00
1.00	0.87	0.85	0.88	0.86	0.86	0.88	0.82	0.91

Table 7: Yield stress model prediction of NIST experimental data: Evaluation

Mix#	Experimental	Hobbs	Ferraris and deLarrard	Toutou and Roussel	Szecsy	Noor and Uomoto	Zhou et al.	Chateau et al.	Mahmoodzadeh and Chidiac
1	1599	1395	1503	1340	1316	1332	1720	1388	1521
3	983	1287	1152	1255	1286	1294	1060	1158	1121
7	1234	1036	1284	1038	998	1059	1202	1129	1314
9	906	904	791	929	941	994	770	833	711
10	1665	1370	1483	1386	1030	1669	1316	1513	1585
12	869	1178	865	1225	989	1001	952	1048	856
14	1115	994	1199	972	1239	1043	1178	1221	1252
15	901	935	925	926	1214	924	969	1031	921
18	771	781	787	794	960	723	842	830	691

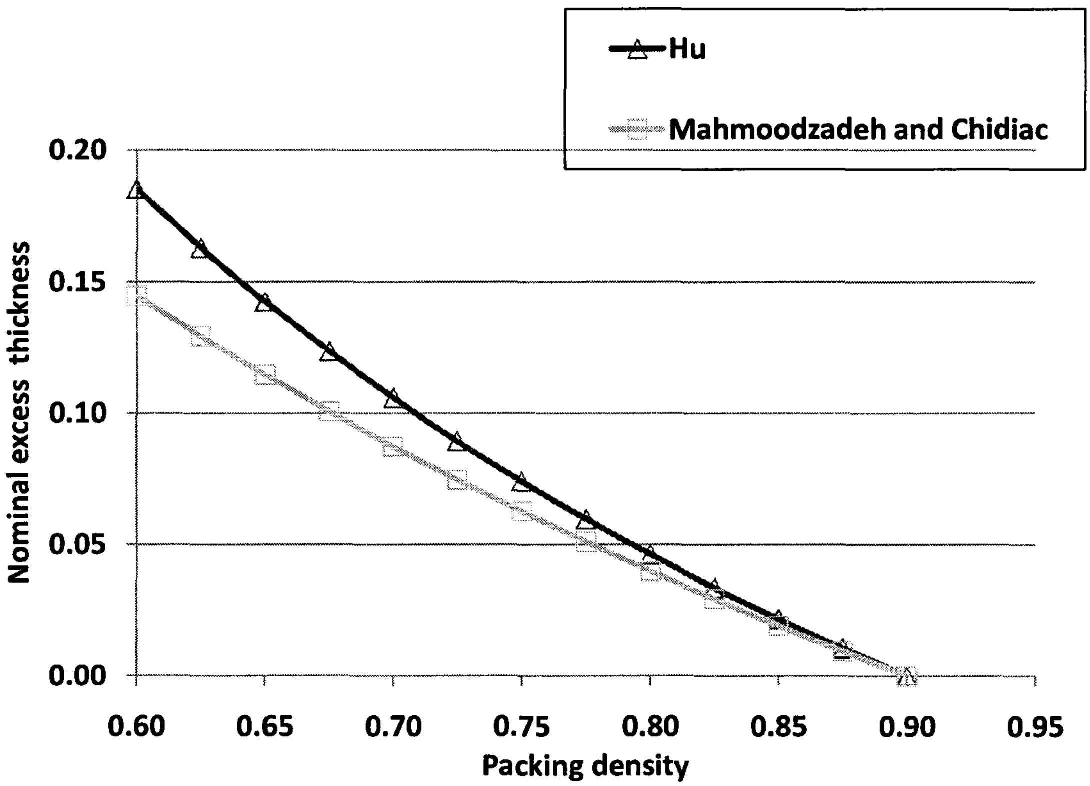
Table 8: Yield stress model prediction of NIST experimental data: Prediction

Mix#	Experimental	Hobbs	Ferraris and deLarrard	Toutou and Roussel	Szecsy	Noor and Uomoto	Zhou et al.	Chateau et al.	Mahmoodzadeh and Chidiac
2	1341	1339	1313	1296	1317	1311	1591	1264	1302
4	778	1236	1018	1214	1287	1274	959	1065	968
5	1162	1344	1035	1425	1277	1524	895	1136	1056
6	949	907	695	965	994	1155	344	836	702
8	1071	1426	997	981	1177	1025	931	962	954
11	1281	1270	1116	1301	988	1151	975	1243	1148
13	1841	1056	1573	1021	1235	1402	1739	1464	1721
16	1496	891	1290	868	1025	779	1212	1125	1283
17	1137	825	997	829	956	748	1025	960	931

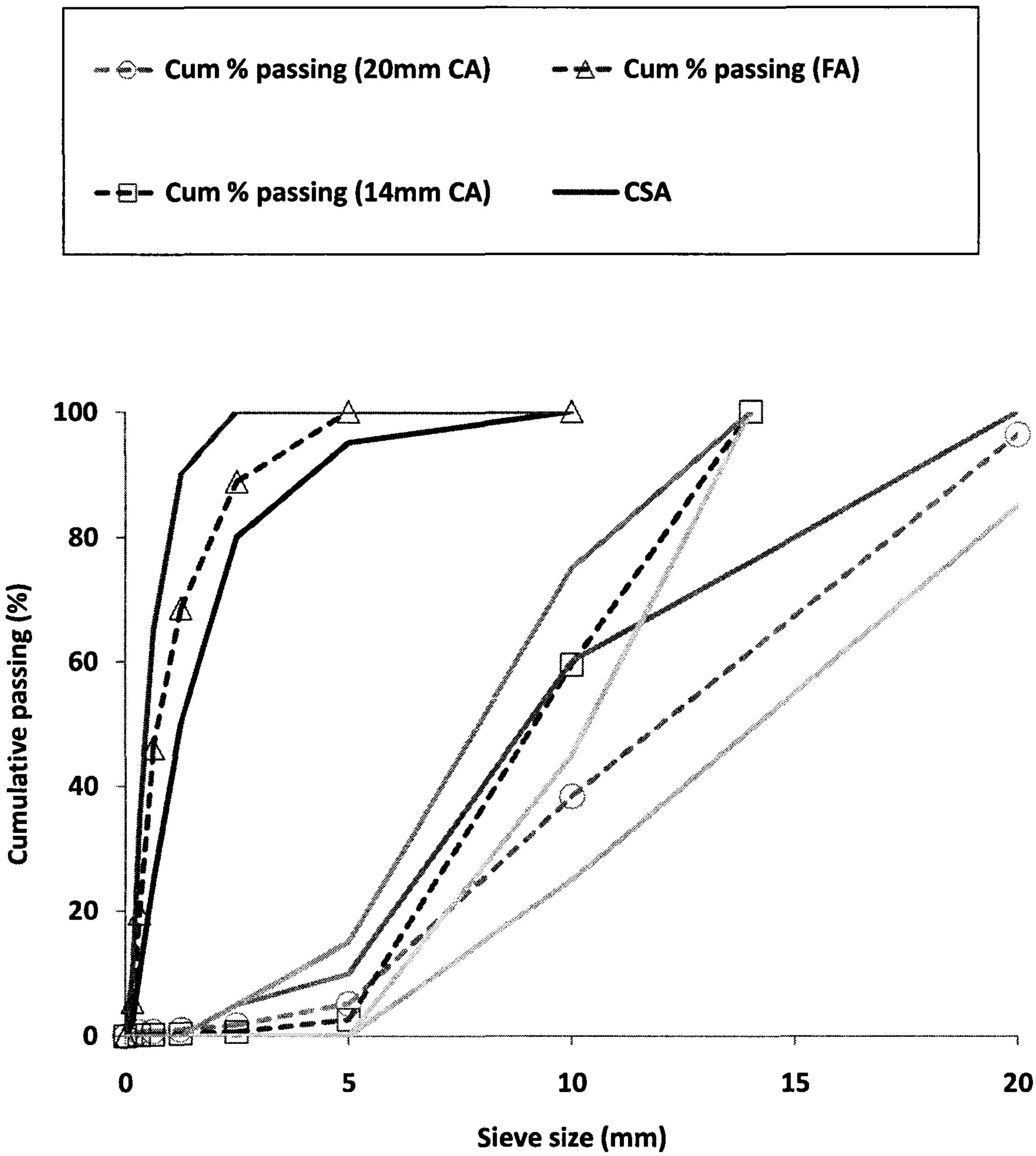
Table 9: Degree of correlation among the models for NIST test data

Experimental	Hobbs	Ferraris and deLarrard	Toutou and Roussel	Szecsy	Noor and Uomoto	Zhou et al.	Chateau et al.	Mahmoodzadeh and Chidiac
1.00	0.51	0.90	0.49	0.35	0.66	0.76	0.84	0.92
0.51	1.00	0.57	0.99	0.48	0.64	0.60	0.85	0.60
0.90	0.57	1.00	0.52	0.62	0.54	0.90	0.87	0.99
0.49	0.99	0.52	1.00	0.42	0.65	0.54	0.82	0.56
0.35	0.48	0.62	0.42	1.00	0.11	0.76	0.50	0.56
0.66	0.64	0.54	0.65	0.11	1.00	0.37	0.76	0.63
0.76	0.60	0.90	0.54	0.76	0.37	1.00	0.78	0.85
0.84	0.85	0.87	0.82	0.50	0.76	0.78	1.00	0.91
0.92	0.60	0.99	0.56	0.56	0.63	0.85	0.91	1.00

List of Figures


Hu
Mahmoodzadeh and Chidiac
0.20
0.15
0.10
0.05
0.00
Nominal excess thickness
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
Packing density

(1)


Cum % passing (20mm CA)
Cum % passing (FA)
Cum % passing (14mm CA)
CSA
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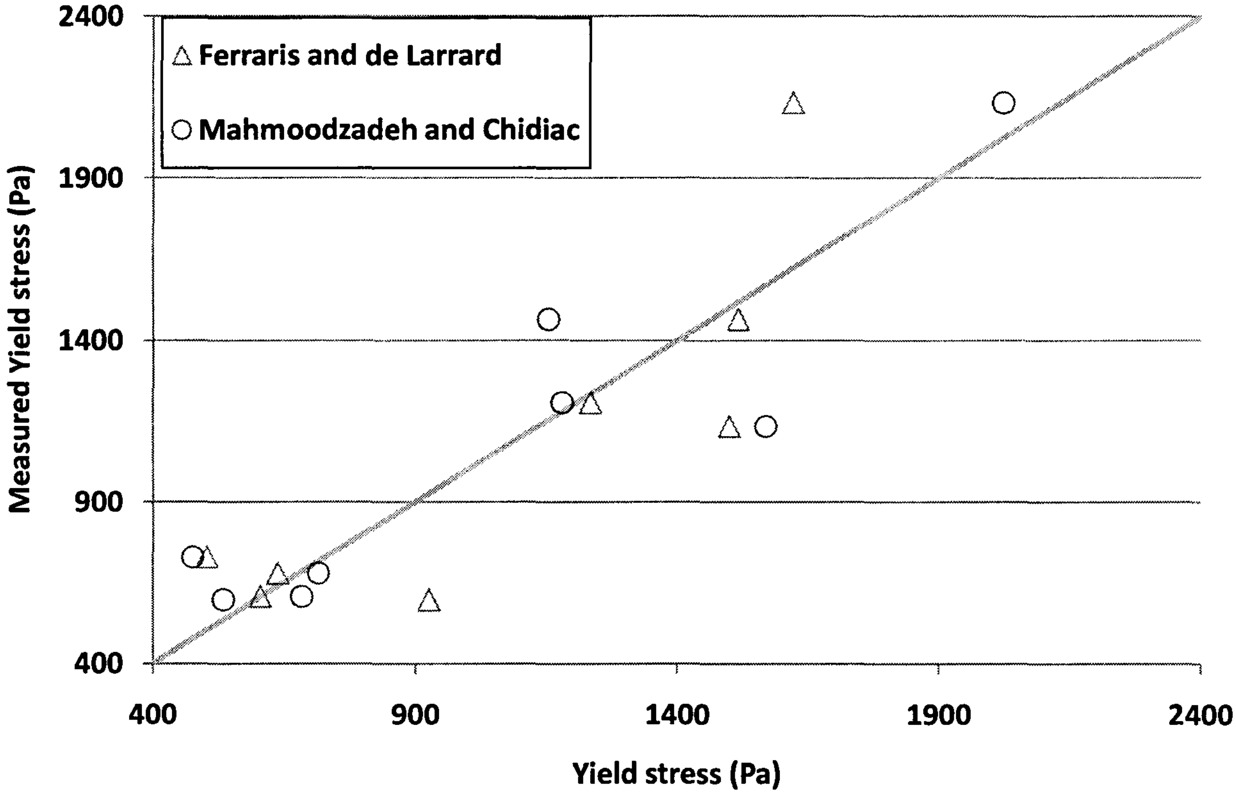
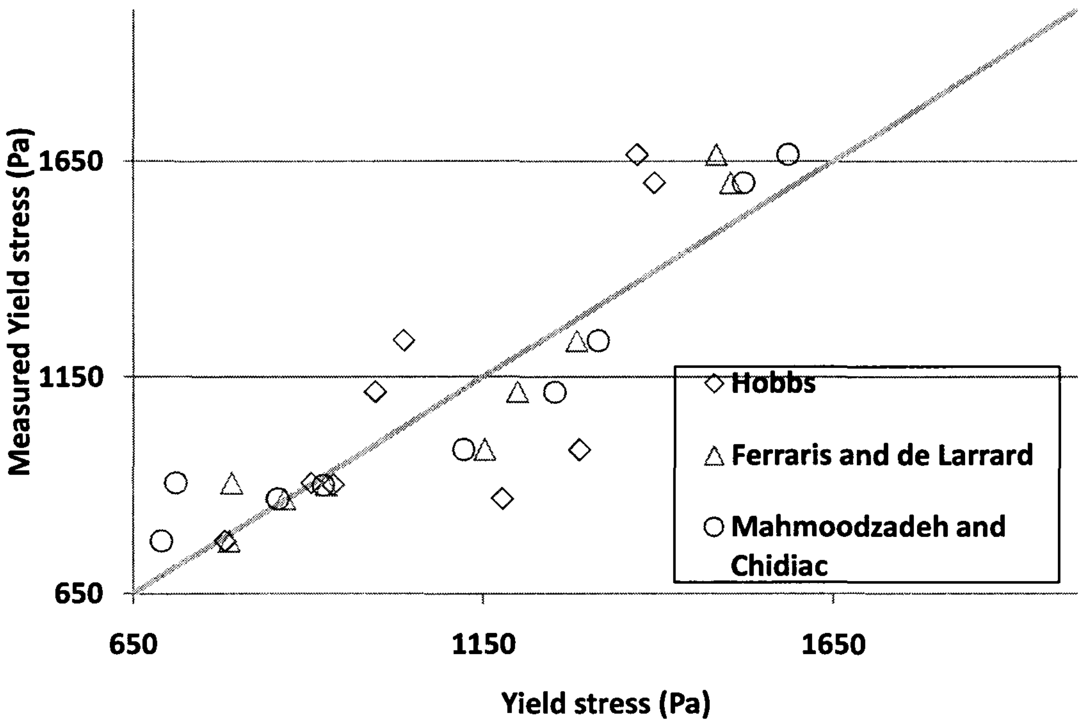
80
60
40
20
0
Cumulative passing (%)
0
5
10
15
20
Sieve size (mm)

(2)

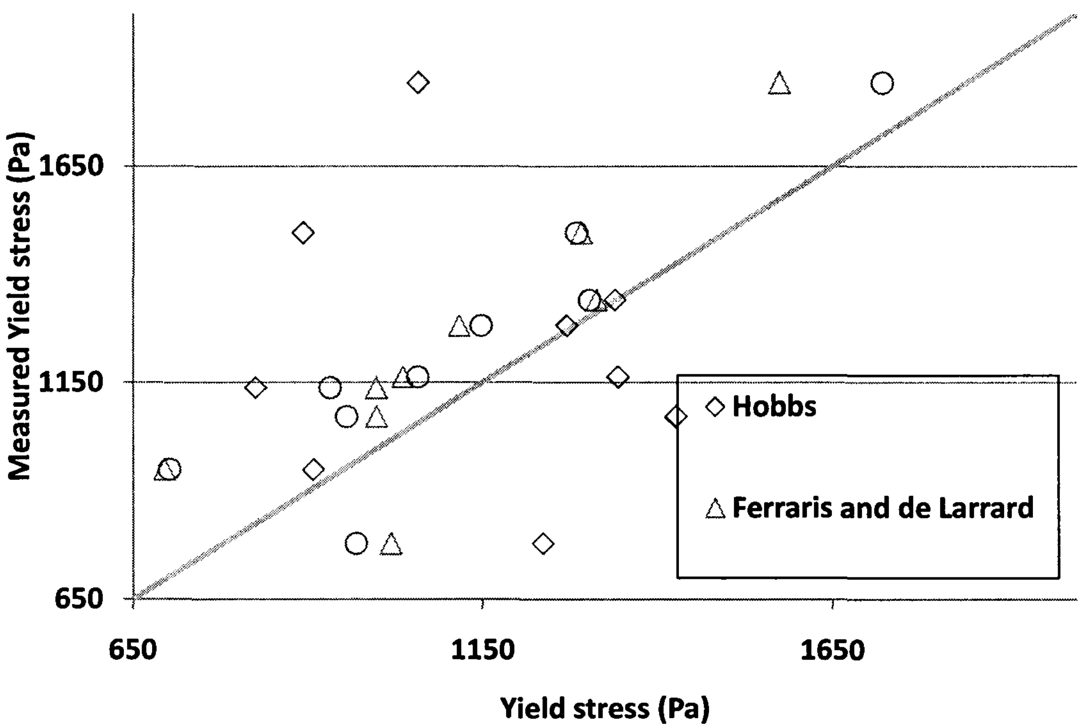


2400
1900
1400
900
400
400
900
1400
1900
2400
Ferraris and de Larrard
Mahmoodzadeh and Chidiac
Measured Yield stress (Pa)
Yield stress (Pa)

(3)


Measured Yield stress (Pa)
1650
1150
650
650
1150
1650
Yield stress (Pa)
Hobbs
Ferraris and de Larrard
Mahmoodzadeh and Chidiac

(4a)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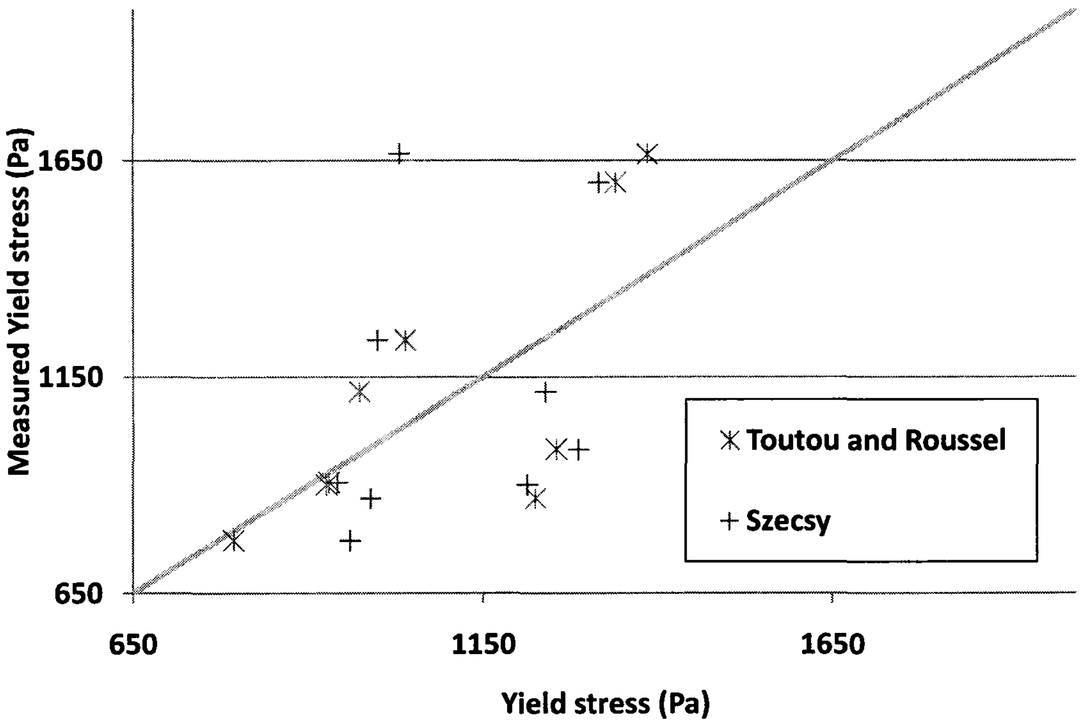
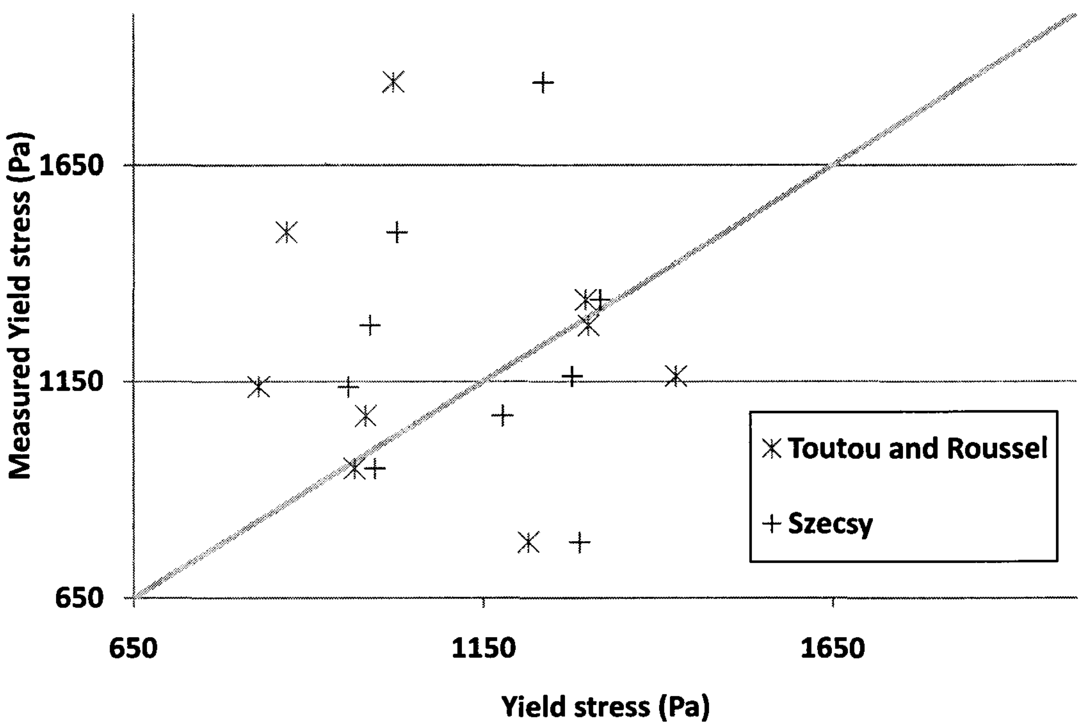
1650
1150
650
650
1150
1650
Yield stress (Pa)
Hobbs
Ferraris and de Larrard

(4b)


Measured Yield stress (Pa)
1650
1150
650
650
1150
1650
Yield stress (Pa)
Toutou and Roussel
Szecsy

(5a)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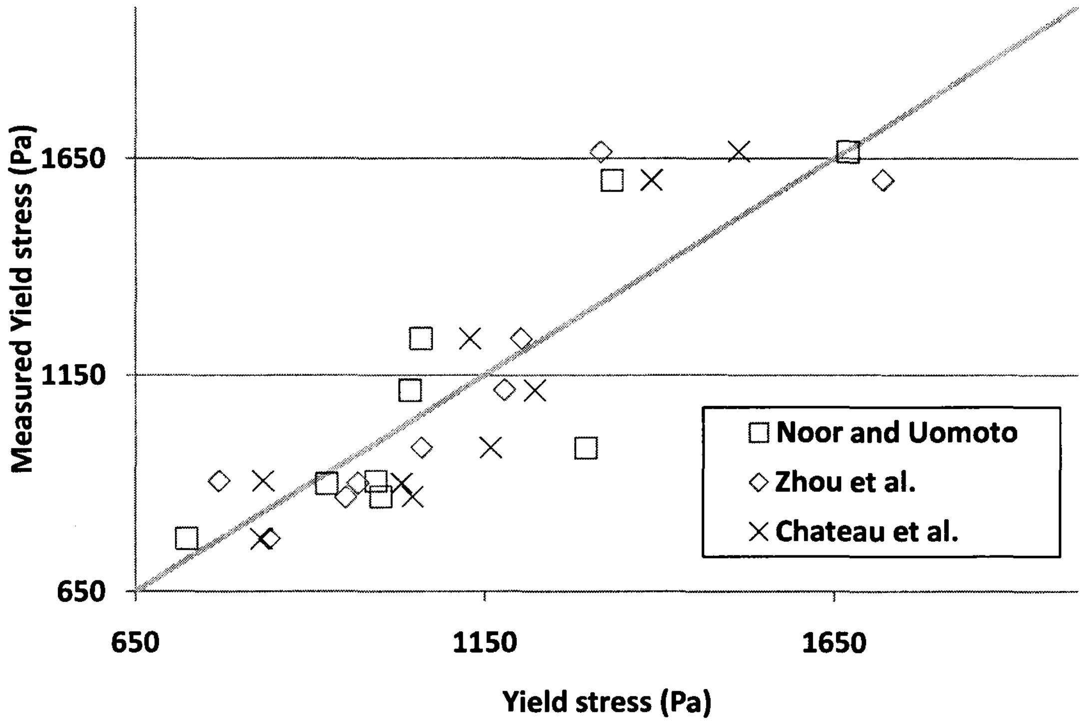
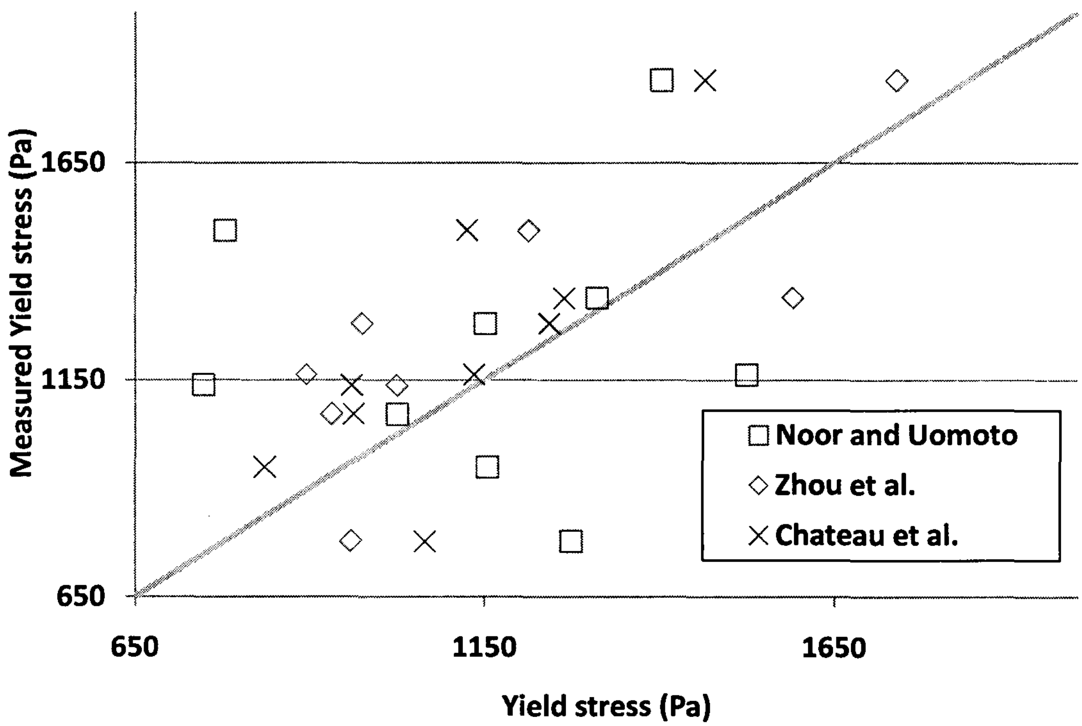
1650
1150
650
650
1150
1650
Yield stress (Pa)
Toutou and Roussel
Szecsy

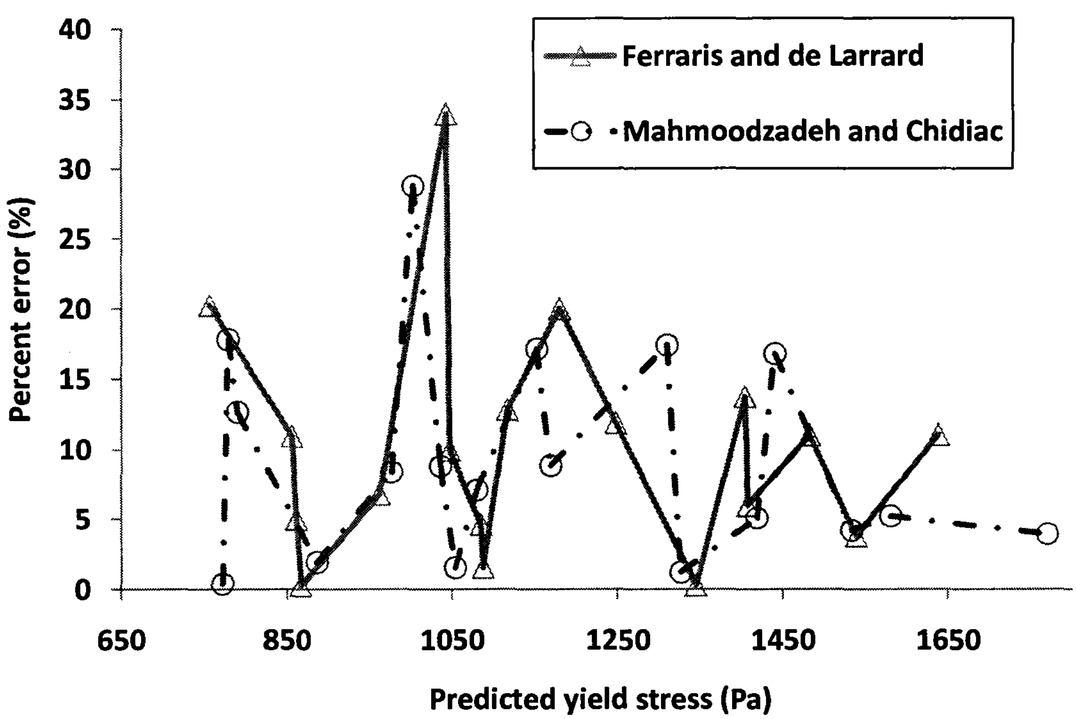
(5b)


Measured Yield stress (Pa)
1650
1150
650
650
1150
1650
Yield stress (Pa)
Noor and Uomoto
Zhou et al.
Chateau et al.

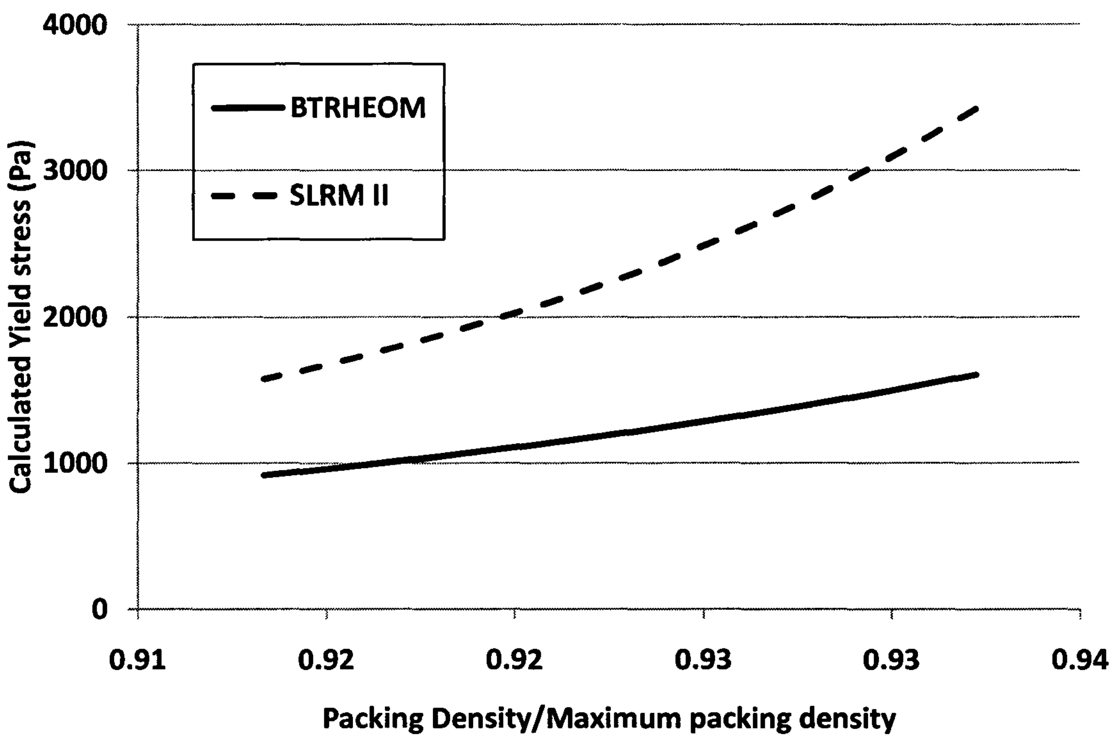
(6a)


Measured Yield stress (Pa)
1650
1150
650
650
1150
1650
Yield stress (Pa)
Noor and Uomoto
Zhou et al.
Chateau et al.

(6b)


Ferraris and de Larrard
Mahmoodzadeh and Chidiac
Percent error (%)
40
35
30
25
20
15
10
5
0
650
850
1050
1250
1450
1650
Predicted yield stress (Pa)

(7)


BTRHEOM
SLRM II
Calculated Yield stress (Pa)
4000
3000
2000
1000
0
0.91
0.92
0.92
0.93
0.93
0.94
Packing Density/Maximum packing density

(8)

Chapter V

Constitutive Flow Models for Characterizing the Rheology of Fresh Mortar and Concrete

Abstract

Constitutive equations for fresh mortar and fresh concrete provide the characterization of the mixture's flow and the quantification of the rheological properties. This paper presents a constitutive material model for mortar and concrete that builds on the work of Gang et al. [1] and Mahmoodzadeh and Chidiac [2]. It postulates that a) the shear stress is the sum of three components; static interaction between particles, dynamic interaction between particles, and collision of particles, and b) that the cell is a representative volume of mixture. For fresh concrete, the effects of particles collision are assumed negligible due to high concentration, and the equation reduces to the Bingham model. Experimental data reported in the literature was employed to evaluate the predictive capabilities of the constitutive equations. The model results are found to compare very well with the measured experimental data and the difference is within the measurement errors.

Keywords: Mortar; Concrete; Constitutive equations; rheology, workability, cell method

1. Introduction

Characterizing the rheological properties and behaviour of fresh concrete is an important step toward controlling the quality of concrete [3-5]. This stems from the fact that placement of fresh concrete, namely transportation, pumping, casting and consolidation, depends on the rheological properties of fresh concrete. Moreover, the significance of characterizing the flowability of fresh concrete is becoming crucial especially for the new concretes such as self-compacting concrete (SCC) where more stringent requirements are needed [5-7].

At present, there is no standard test method for characterizing the rheological properties of fresh concrete, namely yield stress and plastic viscosity. Researchers have, however, developed different types of concrete rheometers and corresponding models for estimating the rheological properties of fresh concrete [7]. A review of these test methods has shown that the proposed methods yield properties that are statistically comparable but the values for the properties are different [8], and although these test methods, once standardized, will provide an excellent tool to control the properties of fresh concrete, they are limited when it comes to the design of concrete mixture. Other researchers have proposed to quantify the rheological properties of concrete on the basis of the composition, specifically the work of Ferraris and deLarrard [6], Roshavelov [9], and Mahmoodzadeh and Chidiac [2]. A critical review of these rheological models has shown that the cell method approach provides a representative description for concrete rheological properties, namely yield stress and plastic viscosity [10-11].

With the exception of the work published by Gang et al. [1], there are no fundamental models that have been proposed in the literature to describe the constitutive behaviour of fresh mortar. For concrete, plasticity and visco-plasticity based models have been proposed in the literature to model the flow behaviour [12]. However, these models assume as a priori that the material will obey Bingham material model and that the corresponding rheological properties are known [12]. These models fail to discriminate between mixtures on the basis of the composition [12].

This paper presents a mathematical description of the proposed constitutive equations for characterizing the flow of fresh mortar and concrete. A brief review of rheology as it pertains to concrete and mortar is first presented. This is followed by the description of the constitutive models for mortar and fresh concrete which build on the work of Gang et al [1] and Mahmoodzadeh and Chidiac [2]. Evaluation of the models is then carried out by comparing the model results with experimental data reported in the literature.

2. Rheology – a theoretical background

Fresh concrete and mortar are composed of cement particles, aggregates, and water. They can be characterized as suspended solid particles (aggregates) in viscous medium (cement paste) [5-6]. The constitutive equations required for simulating the flow of fresh concrete and mortar are difficult to develop because the mixture possesses particles that have varying gradation, shape, surface, texture and angularity. Moreover, the model needs to account for particle interaction. This section provides a brief review of the rheological models for both mortar and concrete that have been proposed in the literature.

2.1 Concrete

Rheology, which is defined as "the study of deformation and flow" [13], is a measure that relates shear force applied to a material to the rate of deformation or change of shape experienced by the material. This relation between shear stress and strain rate is referred to as constitutive equation. This section provides the form of the steady state non-Newtonian constitutive equations proposed in the literature to model the flow of fresh concrete.

Bingham model, represented by Eq. 1, relates the shear stress (τ) and shear strain rate ($\dot{\gamma}$) with a first order polynomial. The corresponding parameters τ_0 and η_0 are referred to, respectively, as yield stress and plastic viscosity.

$$\tau = \tau_0 + \eta_0 \dot{\gamma} \quad (1)$$

Herschel and Bulkley (H-B), which is defined by Eq. 2, is a combination of three parameters, yield stress, plastic viscosity and power index, n. Accordingly, H-B model is expected to provide better predictions over a wider range of shear rates, specifically for the case of strain softening and strain hardening, in comparison to Bingham model [5-6].

$$\tau = \tau_0 + \eta_0 \dot{\gamma}^n \quad (2)$$

Both Bingham model and H-B model have been used primarily to estimate the rheological properties of concrete using experimental measurements. There are other models that have been proposed to provide an estimate of the rheological properties based on the composition of the mixture. Review of these models has revealed that only the models proposed by Mahmoodzadeh and Chidiac [2] and Ferraris and deLarrard [6] do provide good estimates and that the model provided by Mahmoodzadeh and Chidiac [2] yields consistent values for the rheological properties [10-11].

2.2 Mortar

Models proposed in the literature characterizing the flow of fresh mortar are cited in references 1, 14 and 15. Although the majority of these models are phenomenological, they postulate that the flow can be represented by three interactions: static interaction between particles, dynamic interaction between particles and collision between particles and that these three interactions are independent. Accordingly, the model can be represented by

$$\tau = \tau_0 + \tau_{DI} + \tau_{\text{collisions}} \qquad (3)$$

where τ_0 is the shear stress due to static interaction between the particles, τ_{DI} due to dynamic interaction between the particles, and $\tau_{\text{collisions}}$ due to collisions of the particles. Toward the development of a fundamental constitutive rheological model for mortar, Gang et al. [1] assumed that the particles were rigid, non-cohesive and well distributed and that the amount of air was negligible. They have, also, accounted for the high concentration of suspended particles, the different size and shape of the particles, and the interaction and collision of the particles during flow which are necessary requirements to afford a representative description of the flow of fresh mortar. Detailed description of Gang et al. model can be found in reference 1.

3. Constitutive equations for fresh mortar

The proposed model for characterizing the flow of fresh mortar builds on the work of Gang et al. [1]. Specifically, it postulates that shear stress arises due to three interactions between the particles, static interaction, dynamic interaction and collision, and that the three stress components are additive.

3.1 Yield stress

Yield stress, which is one of Bingham rheological properties, is the term that accounts for the static interaction between the particles. Yield stress proposed by Mahmoodzadeh and Chidiac [2] is adopted for this study and is given by

$$\tau_0 = \tau_i y(\varphi)^3 \frac{4(1-y(\varphi)^7)}{4(1+y(\varphi)^{10}) - 25y(\varphi)^3(1+y(\varphi)^4) + 42y(\varphi)^5} \tag{4}$$

and

$$y(\varphi) = (\varphi/\varphi_{\max})^{1/3}\left(1 - \mathrm{C_Y}\frac{m_G}{m_W}\right) \tag{5}$$

where τ_i is the "intrinsic" yield stress and is a function of the shape of the particles, $y(\varphi)$ the ratio of particle size to cell size, φ the volumetric fraction of solid material refers to packing density, $\varphi_{\max}$ the maximum packing density of the whole mixture, m_G and m_W, respectively, the mass of gravel and water of the mixture, and $\mathrm{C_Y}$ a fitting parameter. It should be noted that the proposed model for yield stress differs from that of Gang et al. where the latter assumed that the yield stress for cement paste is equal to that of mortar.

3.2 Particle interactions

Gang et al. [1] postulated that the interaction between two adjacent particles can be mathematically represented by the model shown in Fig. 1. Accordingly, the effect of two particles interaction takes the following form:

$$\tau_{DI} = \eta_p\left[1 + \frac{y}{1-y}\right]\dot{\gamma} \qquad \text{and} \qquad y = \frac{3\varphi_A}{1+1.65\varphi_A} \tag{6}$$

where η_p is the viscosity of cement paste, and φ_A the packing density of aggregate. However, for mortar there are more than two particles that are interacting at one time. To overcome this limitation, and therefore account for multi-particle interaction, the concept of the cell method was proposed by Mahmoodzadeh and Chidiac [2]. The concept is schematically represented in Fig. 2, and it consists of a rigid particle surrounded by fluid.

By accepting the cell as a representative volume, it implies that the particles, which are located at the centre, do not come in contact with each other and that the particles interaction is limited to the interaction of the cells. Accordingly, Mahmoodzadeh and Chidiac [2] have developed an equivalent model that accounts for cells interaction and is given by

$$\tau_{DI} = \eta_w \cdot \eta_i \cdot y(\varphi)^3 \cdot \frac{4 \cdot \left(1 - y(\varphi)^7\right)}{4 \cdot \left(1 + y(\varphi)^{10}\right) - 25 \cdot y(\varphi)^3 \cdot \left(1 + y(\varphi)^4\right) + 42 \cdot y(\varphi)^5} \cdot \dot{\gamma} \tag{7}$$

and

$$y(\varphi) = \left(\varphi / \varphi_{\max}\right)^{1/3} \cdot \left(1 - C_p \frac{m_C}{m_W}\right) \tag{8}$$

where η_w is the viscosity of water, η_i the intrinsic viscosity and is a function of the particle shape, m_C the mass of cement in the mixture, and C_p a fitting parameter.

3.3 Particles collision

The effect of particles collision is included by modifying the particles collision model proposed by Gang et al. [1]. These effects are calculated based on the energy dissipation due to collision of particles moving in parallel horizontal planes. The velocity of the particle is divided into two parts, mean flow velocity (υ_M) and fluctuation velocity (υ_F), and that collision occurs due to fluctuation velocity, where

$$\upsilon = \upsilon_M + \upsilon_F \tag{9}$$

In this formulation, the fluctuation velocity is assumed to be constant for all the particles and is found by solving the partial differential equation, PDE [1],

$$N_{collision}\ \Delta P \frac{\partial u}{\partial Y} = N_{collision}\ F_{AP}\, S + N_{collision}\ \Delta E \tag{10}$$

where:

$$\Delta P = \mu\, m(1+\varepsilon)\left(0.083\, D\frac{\partial u}{\partial Y} + 0.25\, \upsilon_F\right) \tag{11}$$

and

$$\Delta E = \frac{m}{4}\left(1-\varepsilon^2\right)\left\{\left[0.212\, D\frac{\partial u}{\partial Y}\right]^2 + \left[0.6365\, \upsilon_F\right]^2\right\} \tag{12}$$

$$+\frac{m}{4}\left\{\left(0.4244D\frac{\partial u}{\partial Y}\right)^2 - \left(0.4244D\frac{\partial u}{\partial Y} - \mu\left[0.212D\frac{\partial u}{\partial Y}(1+\varepsilon) + 0.6365\upsilon_F(1+\varepsilon)\right]\right)^2\right\}$$

ΔP is the average momentum change of two particles collision in the mean flow direction, $\partial u / \partial Y$ the velocity gradient of mortar, F_{AP} the force acting on cement paste by a single aggregate particle (equals to drag force), S the average distance between the two particles, ΔE the energy loss due to two particles collision, μ the friction coefficient of particles, m the average mass of aggregate particles, ε the coefficient of elastic restitution, and D the average diameter of particles in a horizontal plane.

In Gang et al. model [1], the Reynolds Number, R_e given by

$$R_e = \frac{\rho_P (D/2)^2 \dot{\gamma}}{\eta_P} \tag{13}$$

is assumed to have a high value and consequently the drag coefficient (C_D) is assumed constant. Accordingly, the drag force is represented by

$$F_{AP} = C_D \rho_P \frac{\upsilon_F^2}{2} \cdot \left(\frac{\pi \cdot D^2}{4}\right) \tag{14}$$

where ρ_p is the density of cement paste. Recognizing that the range of mortar strain rate can vary from 0.1 s^{-1} for gravity levelling to 100 s^{-1} during pumping [16] and that the particle size can vary from 0.5 mm to 2 mm, one can deduce that R_e can have a high value of one or greater but it can also have a value as low as 0.001. For the latter case, C_D can no longer be considered constant and the corresponding F_{AP} can be calculated from [17]

$$F_{AP} = 6\pi \cdot \eta_p \cdot \frac{D}{2} \cdot \upsilon_F \tag{15}$$

By adopting Eq. 15, the drag force captures the specificity of the mixture and its placement namely shear strain rate, particle size, density, and viscosity of cement paste. By applying momentum conservation principles, the shear stress due to particles collision becomes

$$\tau_{\text{collisions}} = N_{collision}\left(k_p . F_{AP} + \Delta P\right) \tag{16}$$

where $N_{collision}$ is the number of collisions and k_p the normal stress coefficient. A new approach is proposed to quantify $N_{collision}$ and D.

3.3.1 Average particle diameter

The average diameter of the particles, D_0, shown in Fig. 3, assuming that they are spherical in shape, can be obtained from

$$D_0 = \log^{-1}\left(\frac{\sum_{i=1}^{k} f_i \times \log(D_i)}{\sum_{i=1}^{k} f_i}\right) \tag{17}$$

where D_i is the diameter of the particles in class i, and f_i the corresponding frequency. The average area, A_{AVE}, is then determined by assuming certain geometric distribution. For example, Gang et al. [1] assumed that the angle α has a uniform distribution as shown in Fig. 3. Accordingly, the average area can be obtained from

$$A_{AVE} = \frac{\int_0^{\pi/2} \pi \cdot \left(\frac{D_0}{2} \cdot \cos(\alpha)\right)^2 d\alpha}{\int_0^{\pi/2} d\alpha} = \frac{\pi}{8} D_0^{\,2} \tag{18}$$

and by recalling that the average area is a function of the average particle diameter of the horizontal plane, D, i.e.,

$$A_{AVE} = \frac{\pi}{4} D^2 \tag{19}$$

Gang et al. [1] were then able to develop an expression for calculating the average particle diameter of the horizontal plane,

$$D = \sqrt{0.5}\, D_0 = 0.7071 D_0 \tag{20}$$

However, after analyzing the cutting plane as it moves vertically, it was discovered that it is best to assign the vertical parameter, Y, to have a uniform distribution, refer to Fig. 3c. Accordingly, the average area becomes

$$A_{AVE} = \frac{\int_0^{\frac{D_0}{2}} \pi \times \left(\left(\frac{D_0}{2}\right)^2 - Y^2\right) dY}{\int_0^{\frac{D_0}{2}} dY} = \frac{\pi}{6} D_0^{\,2} \tag{21}$$

Substituting Eq. 19 into Eq. 21, a revised relation for the average diameter is obtained and is given by

$$D = \sqrt{\frac{2}{3}}\, D_0 = 0.82\, D_0 \tag{22}$$

3.3.2 Number of collisions

To calculate the number of collisions, one needs to first calculate the number of particles in a unit material volume, *N*. From Fig. 3, the following formulation can be used,

$$N = \frac{\varphi_A}{A^*_{AVE}} \tag{23}$$

Gang et al. [1] ignored the distance between the particles and substituted Eq. 19 into Eq. 23 to obtain the number of particles. To overcome this simplification, the following average area is proposed for the calculation of N,

$$A^*_{AVE} = \frac{\int_0^{\frac{D_0}{2}} \pi \cdot \left(\left(\frac{D_0}{2} \right)^2 - Y^2 \right) dY}{\int_0^b dY} = \frac{\pi \cdot D_0^{\ 2}}{12 \cdot b} = \frac{\pi \cdot D_0^{\ 2}}{6} \cdot y \tag{24}$$

where b is the cell size as shown in Fig. 2. Subsequently, one can compute the number of particles,

$$N = \frac{\varphi_A}{\frac{\pi D_0^{\ 2}}{6} \cdot y} \tag{25}$$

The number of collisions is obtained using the following expression [1]

$$N_{collision} = N \frac{S_p}{S_p + D} f \tag{26}$$

where f is the frequency of collision and S_p is the distance between particles in the assumed horizontal plane.

3.3.3 Frequency of collisions

Different formulations were proposed in the literature for calculating the frequency of collisions. Marrucci and Denn [18] assumed that the frequency is equal to shear strain rate but Probstein et al. [19] argued that it is much higher. In this study, a similar argument is presented to find the frequency of collisions.

By using the relative velocity of the particles with respect to each other, the only velocity that leads to collisions is the fluctuation velocity (υ_f). For a mixture with high particle concentration, a representative particle is always surrounded in all directions by other particles. The particle moves with the velocity υ_f toward one of the adjacent particles. Recognizing that υ_f has an arbitrary direction, the average of fluctuation velocity (υ_f) of the adjacent particle can be considered equal to zero. Accordingly, the frequency of collisions can be estimated from

$$f = \frac{\upsilon_f}{S} \tag{27}$$

Substituting Eqs. 25 and 27 into Eq. 26 yields:

$$N_{collision} = \frac{\varphi_A}{\pi D_0^2 / 6}\left(\frac{1}{y}\right)\frac{S_p}{S_p + D}\frac{\upsilon_f}{S} \tag{28}$$

By examining Fig. 4, which provides an illustration of two adjacent particles, the following formulation can be extracted,

$$S_p + D = (S + D_0)\cos(\theta) = \frac{D_0}{y}\cos(\theta) \tag{29}$$

Recalling the volume of a spherical particle, $V_{Particle}$, where,

$$V_{Particle} = \frac{\pi}{6} D_0^3 \tag{30}$$

By substituting Eq. 29 and 30 into Eq. 28, and defining the number of particle per unit volume, $\overline{N}$, as

$$\overline{N} = \frac{\varphi_A}{V_{Particle}} \tag{31}$$

the number of particles collision can be calculated from,

$$N_{collision} = \left(\frac{\frac{\cos(\theta)}{y} - \frac{D}{D_0}}{\frac{\cos(\theta)}{y} - \cos(\theta)} \right) \times \overline{N} \times \upsilon_f \cong \overline{N} \times \upsilon_f \tag{32}$$

In summary, the proposed constitutive equations for mortar consist of Eqs. 3, 4, 7, and 16 along with the accompanying equations.

4. Constitutive equation for fresh concrete

The proposed flow model for mortar was extended to model the flow of fresh concrete. For concrete, the concentration of suspended particles in the mixture is high in comparison to mortar. As a consequence, the effect of particles collision becomes negligible. By setting the shear stress term due to particle collision equals to zero in Eq. 3, it becomes the Bingham's model, given in Eq. 1. Using experimental data reported in the literature [6], Mahmoodzadeh and Chidiac [10] and Chidiac and Mahmoodzadeh [11] have shown that the proposed models are adequate and consistent in predicting the rheological properties of fresh concrete namely yield stress, given in Eq. 4, and plastic viscosity, given in Eq. 7.

5. Evaluation of the mortar constitutive equations

Evaluation of the model consists of two parts; the first part is to validate the rheological properties including viscosity and yield stress based on experimental measurements carried out by Ferraris and deLarrard [6], and the second part is to evaluate the ability of the constitutive equation to characterize the mortar flow using experimental work reported by Hu [15].

5.1 Rheological properties

Ferraris and deLarrard carried out an extensive testing program to measure the rheological properties of mortar and concrete [6]. The data corresponding to normal concrete was used to demonstrate the adequacy of the proposed model to predict the rheological properties of concrete [10, 11]. In this study, the data corresponding to normal mortar are used to evaluate rheological properties of mortar. Details of the experimental program are reported in Ferraris and deLarrard [6] and the experimental results are given in Table 1. The model predictions of the four mixtures are also given in Table 1 and shown in Figs. 5 and 6. The results demonstrate that the proposed model predictions are very good.

5.2 Constitutive flow of mortar

An experimental study was conducted by Hu [15] to investigate the flow characteristics of fresh mortar. The mortar mixture was composed of type I Portland cement, river-sand fine aggregate and water. The mixture proportions were 0.4 water to cement ratio, and 2.0 sand to cement ratio. The particle size ranged between 0.6mm and 1.18mm. Brookfield rheometer was used to measure shear stress versus shear strain rate. The loading and unloading sequences which are shown in Fig. 7, indicate an initial preshear cycle prior to the commencement of the test. The test includes a loading cycle, referred to as Up-curve, and an unloading cycle, referred to as Down-curve. The rate is constant for both the up and down curve.

Hu's experimental results are shown in Fig. 8. An understanding of these results is merited prior to the application of the proposed constitutive equations. The results show a jump in the shear stress at the onset of the up-curve before quickly decreasing to shear stress values in the ramp of 500 Pa. Subsequently, the experimental results indicate a small increase in shear stress as the strain rate went from 20 s^{-1} to 100 s^{-1}. Although the pre-shear cycle was intended to break-down the structure, the recorded response indicates a significant resistance to the aggregates movement as they move through the cement paste at low speed. The scientific interpretation of these results suggests that the Reynolds number is low at the onset of the test and thus the drag force is proportional to the velocity of the aggregates and plastic viscosity of the cement paste. Accordingly, the drag coefficient is not constant and the drag force to be determined according to Eq. 15.

As the shear strain continues to increase, the aggregates move at high speed through the cement paste. Accordingly, the drag force becomes proportional to the square of the velocity, and the drag coefficient can be considered constant. Eq. 14 is then used to calculate the corresponding drag force.

For the unloading portion of the test, the same argument can be presented for the high shear strain rate portion of the test. As the test continues the same trend is observed even when the strain rate drops below 20 s^{-1}. The results are attributed to the breakdown of the cement paste structures. Accordingly, drag force is only affected by R_e and not the viscosity of the cement paste. Therefore, Eq. 14 is applicable for the full unloading cycle.

The model results are shown in Fig. 8. Comparing with the experimental data, it can be concluded that the interpretation of the experimental results is sound and that the proposed model is capable of characterizing the behaviour of mortar mixture during the loading and unloading cycle. The model results also show a linear trend once the structure of the paste is broken for both the Up and Down-curve. Moreover, if one accepts Hu's argument [15], then Down-curve can be used to characterize the steady state flow of mortar that is a linear relation between shear stress and strain rate. However, to capture the flow of mortar as a continuum using a finite element framework, the model needs to account for both the loading and unloading. This flow behaviour is captured in the proposed constitutive equation.

6. Conclusion

This paper presents a constitutive model for fresh mortar and concrete consisting of three components, static interaction, dynamic interaction and collision. The first and second components are adopted from the work of Mahmoodzadeh and Chidiac [2] and respectively, represent the yield stress and plastic viscosity. The last component is a modified version of Gang et al. model by introducing the concept of cell method. The evaluation of the model has revealed that

1. The proposed model is capable of predicting the rheological properties of mortar and fresh concrete.
2. For different ranges of shear strain rate, the governing constitutive equation of mortar may be different. The comparison between the model and the provided data by Hu [15] indicates that the proposed model can predict the behaviour of mortar.
3. For concrete, as confirmed with literature, it was proven fundamentally that the fresh concrete obeys Bingham model.

Acknowledgments

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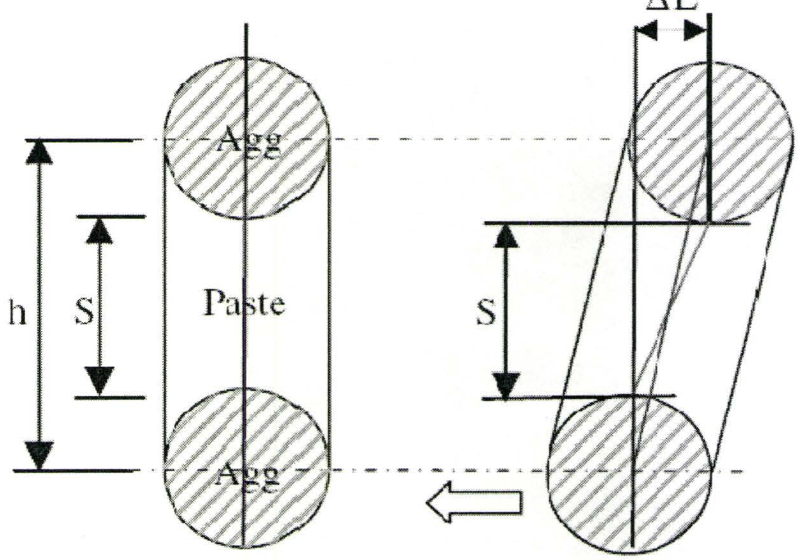
Tables:

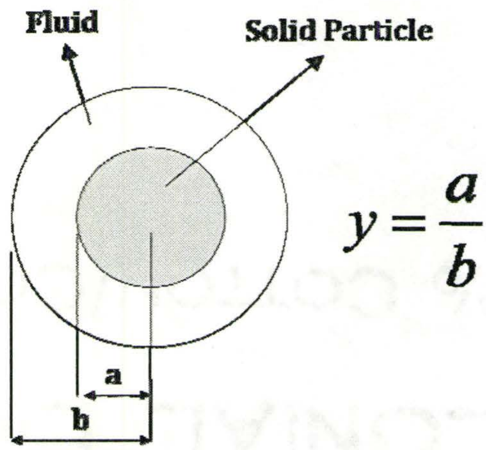
Table 1: NIST mortar proportions and properties and model prediction [6]

Mix#	Sand (kg/m³)	Fine sand (kg/m³)	Cement (kg/m³)	water (kg/m³)	Packing density	Maximum packing density	Yield stress(Pa)			Plastic viscosity (Pa.s)		
							Exp.	Model	Error (%)	Exp.	Model	Error (%)
1	1082	336	475	298	0.699	0.799	1061	829	22	38	35	7
2	984	305	635	296	0.701	0.785	1263	1378	9	34	33	4
3	973	302	628	304	0.693	0.785	1001	1023	2	31	29	6
4	961	298	621	312	0.685	0.785	764	778	2	19	26	38

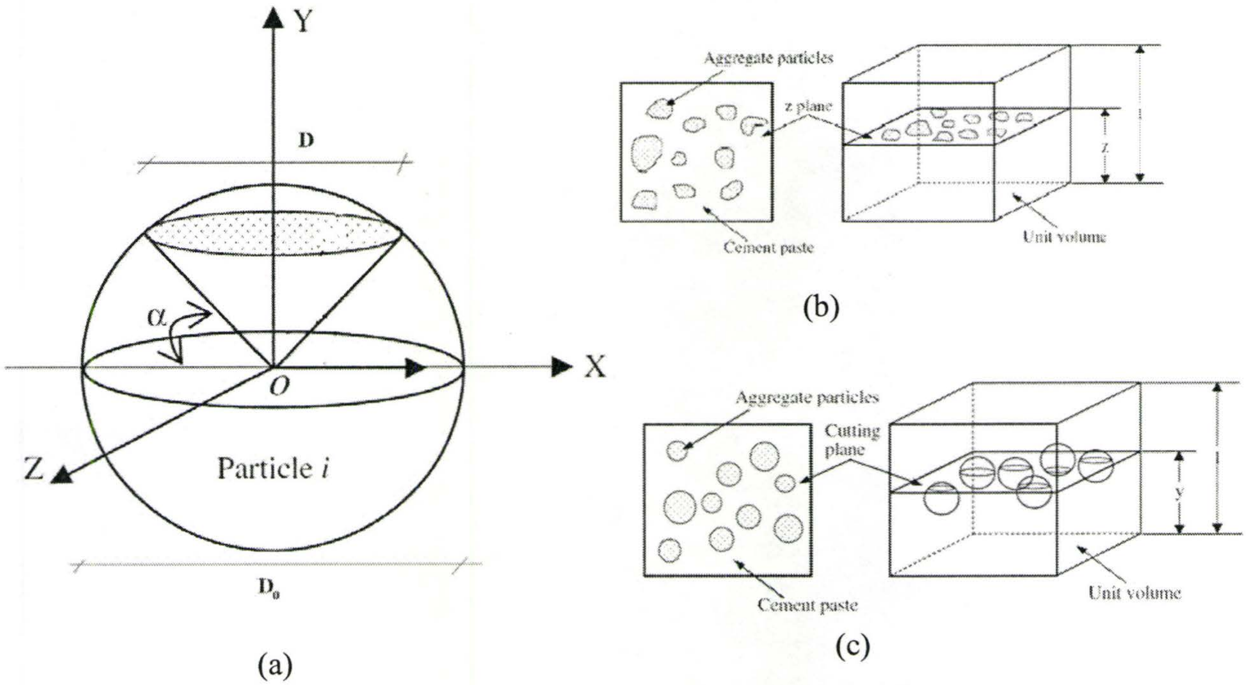
List of Figures



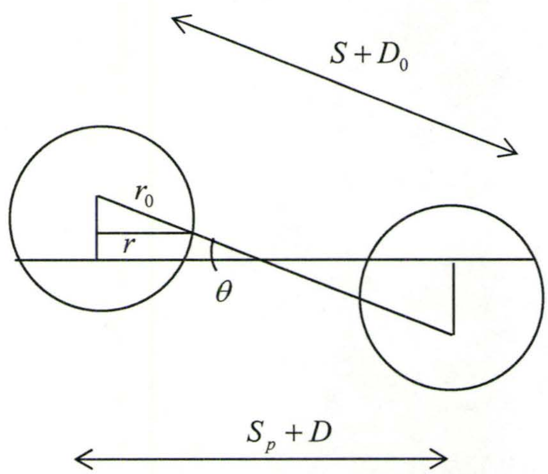
(1)



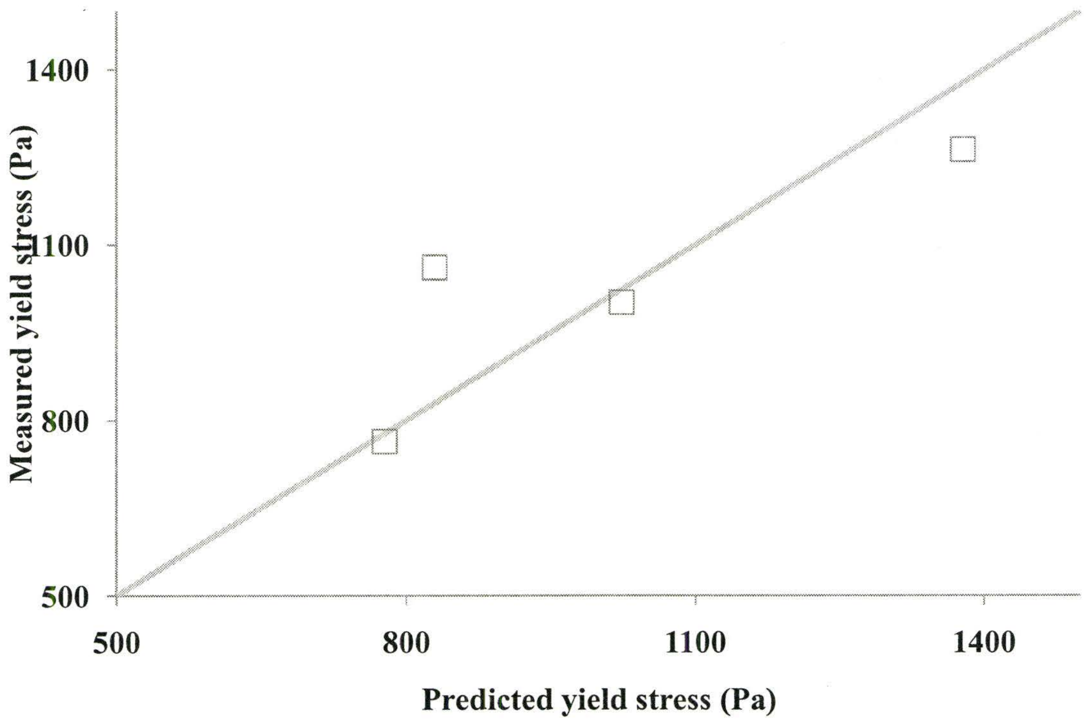
(2)


Y
D
α
X
O
Z
Particle i
D_0
(a)
Aggregate particles
z plane
Cement paste
Unit volume
(b)
Aggregate particles
Cutting plane
Cement paste
Unit volume
(c)

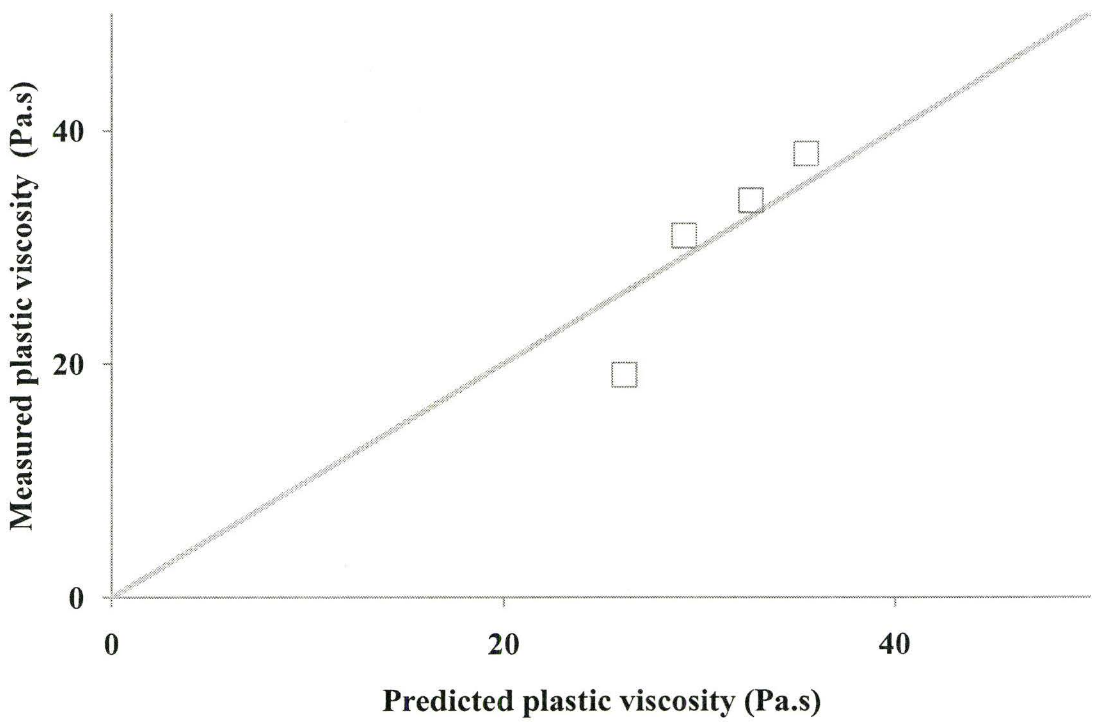
(3)


$S + D_0$
r_0
r
θ
$S_p + D$

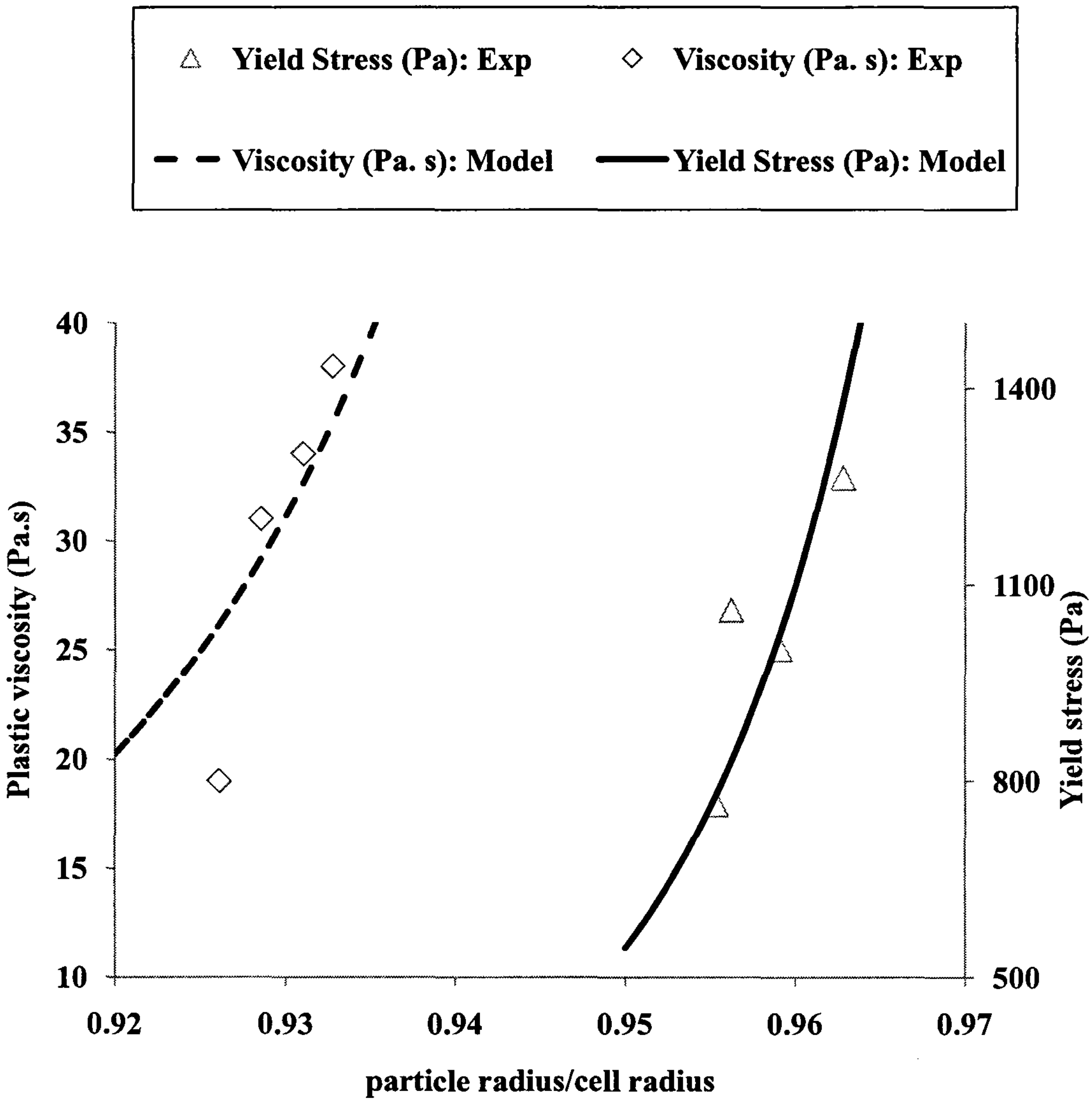
(4)


1400
1100
800
500
Measured yield stress (Pa)
500
800
1100
1400
Predicted yield stress (Pa)

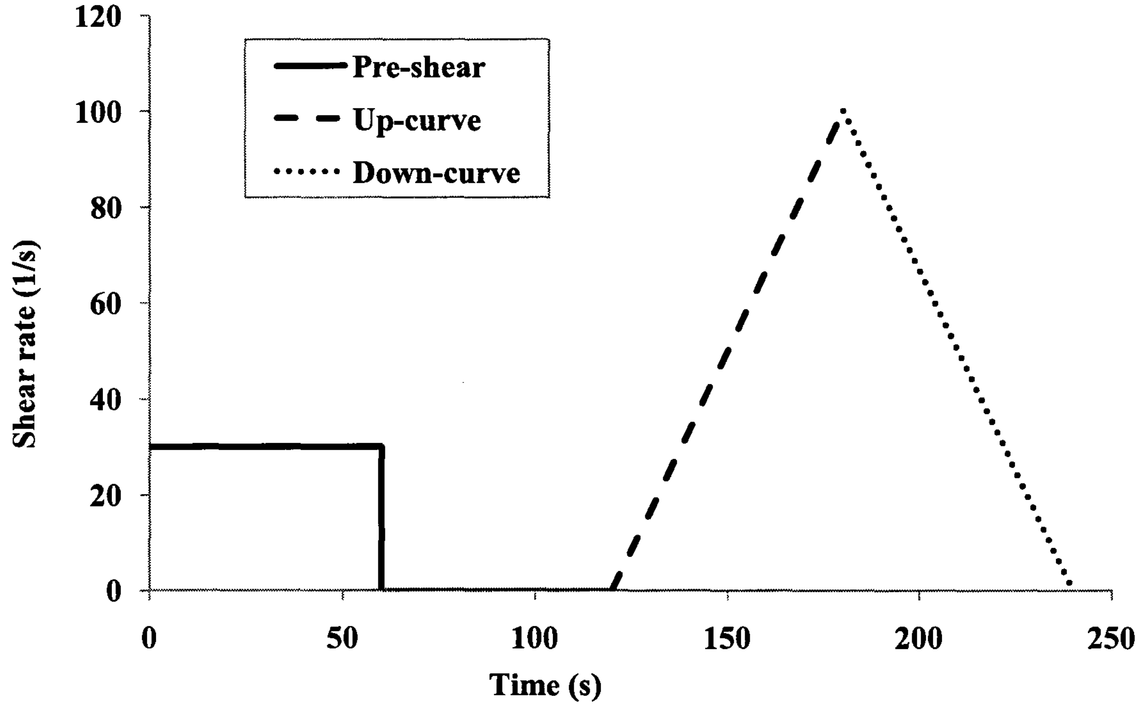
(5a)


40
20
0
Measured plastic viscosity (Pa.s)
0
20
40
Predicted plastic viscosity (Pa.s)

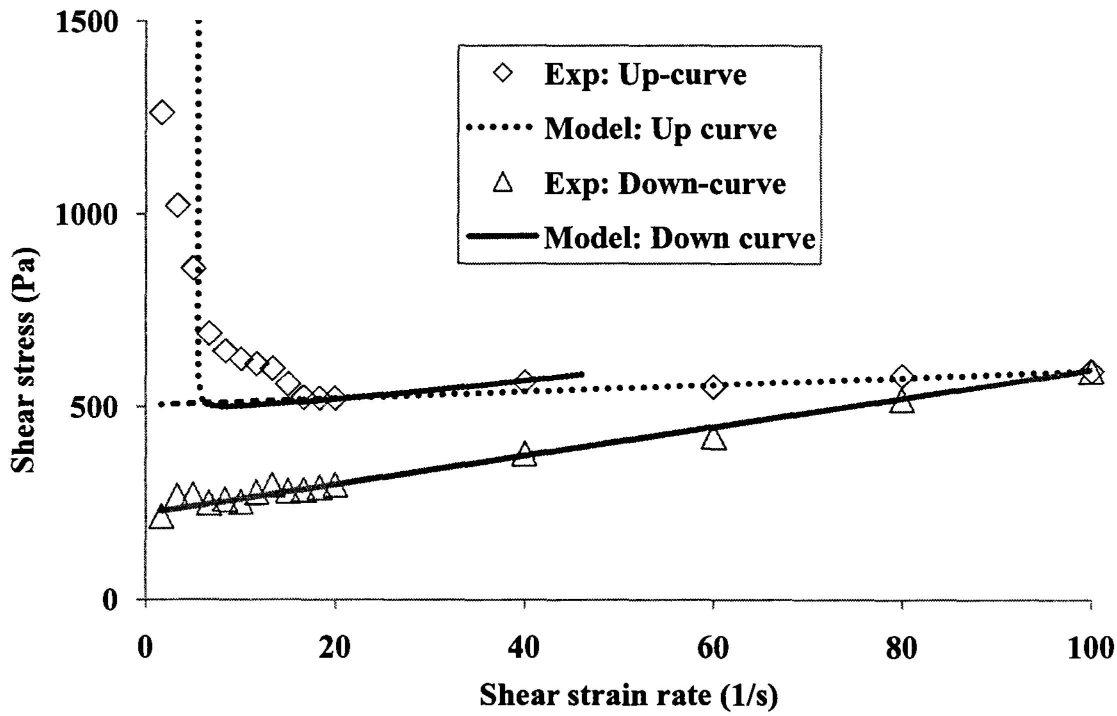
(5b)


Yield Stress (Pa): Exp
Viscosity (Pa. s): Exp
Viscosity (Pa. s): Model
Yield Stress (Pa): Model
Plastic viscosity (Pa.s)
Yield stress (Pa)
40
35
30
25
20
15
10
1400
1100
800
500
0.92
0.93
0.94
0.95
0.96
0.97
particle radius/cell radius

(6)


120
100
80
60
40
20
0
Shear rate (1/s)
Pre-shear
Up-curve
Down-curve
0
50
100
150
200
250
Time (s)

(7)


1500
1000
500
0
Shear stress (Pa)
Exp: Up-curve
Model: Up curve
Exp: Down-curve
Model: Down curve
0
20
40
60
80
100
Shear strain rate (1/s)

(8)