THE FAST ITERATIVE WATER-FILLING POWER CONTROLLER FOR COGNITIVE RADIO NETWORKS

By

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A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Doctor of Philosophy

McMaster University

THE FAST ITERATIVE WATER-FILLING POWER CONTROLLER FOR COGNITIVE RADIO NETWORKS

DOCTOR OF PHILOSOPHY (2010) McMaster University

(McMaster School of Computational Engineering & Science) Hamilton, Ontario

TITLE:	THE FAST ITERATIVE WATER-FILLING POWER CONTROLLER FOR COGNITIVE RADIO NETWORKS
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NUMBER OF PAGES:	xx, 139

To my parents

Abstract

The transmit-power control (TPC) problem is a fundamental problem in cognitive radio design, which aims at determining transmit-power levels for secondary users across available subcarriers. This thesis studies both the theory and the algorithms for the TPC problem for cognitive radio networks, and specifically examines the problem under two different limitations: an interference-power limitation and a low-power limitation. First, the TPC problems are cast into game-theoretic models and the sufficient and necessary optimality conditions for solutions are derived. Sufficient conditions for the existence, uniqueness and stability of a solution are presented as well. Second, the fast iterative water-filling controller (FIWFC) for the TPC problem is developed, which is linearly convergent under certain conditions. The computational complexity is lower than for the iterative water-filling controller (IWFC) for digital subscriber lines. In order to evaluate the FIWFC, simulations are carried out for both stationary and non-stationary radio environments. In addition, the performance of the FIWFC is evaluated, given the presence of measurement errors. The results of these various simulations show that the FIWFC outperforms IWFC in terms of convergence speed in all cases.

Acknowledgements

I am very grateful for the opportunity to have studied and worked with the extraordinary individuals in both the Advanced Optimization Laboratory (AdvOL) and Cognitive Systems Laboratory (CSL) at McMaster University during the course of my Ph.D. First of all, I would like to thank my supervisor, Dr. Simon Haykin, for his guidance and support. I deeply appreciate him for his professional vision, practical insight and profound input on my thesis project. His enthusiasm for research continues to be inspiring. I will no doubt benefit from his positive attitudes towards work throughout my future career. I would like to thank my co-supervisor, Dr. Tamás Terlaky for his advice and support throughout the past five years. The optimization seminars at both the AdvOL and the Fields Institute, as well as the conferences he organized have broaden my vision on both optimization theory and applications. I would also like to thank the other members of the supervisory committee: Dr. Gail Wolkowicz and Nicholas Kevlahan for their thoughtful discussions and suggestions.

I would like to thank the members of the CSL, including Peyman Setoodeh, Farhad Khozeimeh, Adhithya Ravichandran and Yanbo Xue for their collaboration and stimulating discussions, and especially Karl Wiklund who helped me by proofreading my thesis. I would also like to thank the members of the AdvOL for creating a fun and energetic work environment.

Contents

Abbreviations

AVI: Affine Variational Inequality

CDMA: Code Division Multiple Access

DSL: Digital Subscriber Lines

FDMA: Frequency Division Multiple Access

FFT: Fast Fourier Transform

FIWFC: Fast Iterative Water-Filling Controller

GNE: Generalized Nash Equilibrium

GNEP: Generalized Nash Equilibrium Problem

IWFC: Iterative Water-Filling Controller

KKT: Karush-Kuhn-Tucker

MFCQ: Mangasarian-Fromowitz Constraint Qualification

MLCP: Mixed Linear Complementarity Problem

NE: Nash Equilibrium

NEP: Nash Equilibrium Problem

OFDM: Orthogonal Frequency-Division Multiplexing

SNR: Signal-To-Noise Ratio

TPC: Transmit-Power Control

Glossary of the Symbols

K:	number of secondary users
N:	number of subcarriers
S_k^n:	user k's transmit-power over subcarrier n
S_k:	user k's transmit-power vector
S:	transmit-power vector of the network
σ_k^n:	normalized background noise power at the receiver of user k over subcarrier n
σ_k:	normalized background noise power vector at the receiver of user k
σ:	normalized background noise power vector of the network
α_{jk}^n:	normalized interference coefficient from user j's transmitter to user k's receiver over subcarrier n
I_k^n:	interference-plus-noise term experienced by user k over subcarrier n
CAP_k:	battery power budget available to user k
P^n:	interference-power limitation over subcarrier n
UPP^n:	low-power limitation over subcarrier n
Γ:	signal-to-ratio gap
h_{jk}^n:	channel gain from transmitter j to receiver k over subcarrier n

d_{ij}: distance from transmitter j to receiver i

r: path-loss exponent

β^n: attenuation parameter associated with subcarrier n

T_k: index set of the subcarriers utilized by user k

B_k: index set of subcarriers where user k reaches the interference-power limitation

A, C: interference-gain matrices

M: sum of the interference-gain matrix and the identity matrix

$\rho(X)$: the spectral radius of matrix X

$\mathrm{Tr}(X)$: the trace of matrix X

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Chapter 1

Introduction

1.1 Cognitive Radio

The radio spectrum is a natural resource which is used as a medium for data transmission across remotely located transmitters and receivers. However, with increased usage and consequent crowding, underutilization of the radio spectrum has become a serious issue. In [28, 38], it is reported that

- some of the frequency bands of the radio spectrum remain unused most of the time;
- some of the frequency bands are only partially used;
- the rest of the frequency bands are heavily used.

An unused frequency band is called a spectrum hole. It is formally defined in [18, 28]:

Definition 1.1.1 *A spectrum hole is a band of frequencies assigned to a primary user*[1]*, but, at a particular time and specific geographic location, the band is not being utilized by that user.*

Spectrum utilization can be improved by making it possible for secondary users[2] to access these spectrum holes. However, as reported in [16], conventional communication systems are prevented from accessing such frequency bands.

To improve spectrum utilization, cognitive radio has been proposed as a new generation of software-based communication system with the ability to exploit the existence of available spectrum holes [24, 25]. Following [18], cognitive radio is defined as follows:

"Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (i.e. outside world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real-time, with two primary objectives in mind:

- highly reliable communication whenever and wherever needed;
- efficient utilization of the radio spectrum."

This definition implies that a cognitive radio needs to perform the following fundamental tasks: radio-scene analysis, channel identification, transmit-power

[1]Primary users are referred to as those who are licensed to use the frequency spectrum.

[2]Secondary users are referred to as those who attempt to access the frequency spectrum but are not licensed to use it.

control and dynamic spectrum management. A cognitive radio's receiver senses its surrounding environment (i.e. the outside world) to get the necessary information, such as the locations of spectrum holes and the levels of interference. It estimates the channel-state information and predicts the channel capacity for use by the transmitter. This information is passed to the transmitter via a feedback channel. The cognitive radio's transmitter learns from this information, and then appropriately sets certain operating parameters and decision variables (e.g. transmit-power levels) so as to adapt the system's operation to the sensed environment. For the purpose of this thesis, the focus is on transmit-power control.

The most basic form of the cognitive cycle, including the three fundamental tasks outlined previously, is shown below in Figure 1.1 [18].

From the figure, it can be seen that a closed feedback loop is formed by the environment, the receiver, the feedback channel, and the transmitter [18, 19, 36]. The built-in feedback loop provides up-to-date information on the surrounding environment to the transmitter such that it can perform corresponding actions in order to adapt to the current state of the surrounding environment.

Keeping the environmental information up-to-date is critical in cognitive radio design due to the fact that the surrounding environment is nonstationary in time, which is contributed by users and spectrum holes' mobility:

- secondary users in a network may be moving towards different directions with certain speeds;
- secondary users may join or leave a network at any time;

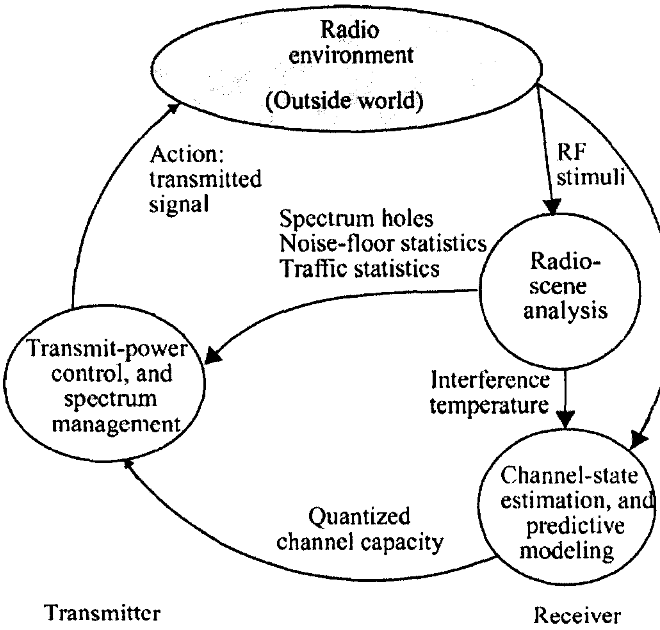


Figure 1.1: Basic cognitive cycle [18]

- spectrum holes may come and go at any time, depending on the communication patterns of primary users.

Therefore, the interference-gain matrix is expected to change. This calls for the consideration of the effect of nonstationary environment in cognitive radio design.

In this thesis, we assume that all the users can share the same frequency bands. Therefore, code division multiple access (CDMA) is applied to cognitive radio networks. CDMA is a channel access method, where multiple users are allowed to send information simultaneously over a single channel. Each transmitter is assigned to a particular code so as to distinguish its information from the other information sent by the other transmitters.

1.2 Transmit Power Control

The transmit-power control (TPC) problem is an important functional block in the feedback loop of a cognitive radio. Using the orthogonal frequency-division multiplexing (OFDM)[3] modulation scheme, each spectrum hole is divided into a number of subcarriers on which parallel data transmissions take place. The TPC problem involves the optimization of a design objective, which aims at allocating those subcarriers to secondary users and determining the transmit powers for all users in the network, based on their current resource capacities and limitations. This problem is well understood for single-user systems comprised of a single transmitter-receiver pair. For such systems, optimality can be achieved by the well-known water-filling algorithm. However, the TPC problem becomes nontrivial when multiple users are involved.

The TPC problem has been formulated as a nonconvex optimization problem which aims at jointly maximizing the total data rates of all the users in a multi-carrier system. Examples of such systems include digital subscriber lines (DSL) [10, 26, 29, 39, 40, 46, 48]. It is difficult to find a global optimal solution to this TPC problem. Theoretically, solving a nonconvex optimization problem is time consuming, since it is not easy to check for global optimality and we lack an efficient algorithm. Some numerical methods have been proposed for solving this problem, such as the convex relaxations [22, 42, 45] and several heuristic methods

[3]In a multi-carrier system using the OFDM modulation scheme, the fast Fourier transform (FFT) algorithm is implemented in the receiver and the inverse FFT algorithm is implemented in the transmitter so as to partition the frequency bands into orthogonal channels. Compared to single-carrier modulation schemes, the OFDM scheme provides higher data rates and greater ability to deal with channel conditions.

[9, 10, 26, 29, 39, 46]. A theoretical treatment of the nonconvex optimization problem for multiuser systems has also been presented in [48].

However, global optimality of a solution is not guaranteed by these algorithms. Practically speaking, finding a globally optimal solution requires some kind of centralized control. To accomplish this, a large amount of information must be exchanged among the different users. This high-level cooperation will consume precious resources, such as spectrum holes and battery power. For mobile wireless devices with a limited battery power budget, battery lifetime is of critical importance. Necessarily, battery lifetime can be preserved by reducing the algorithm's complexity and therefore the computational burden on each user. In fact, due to the highly nonstationary nature of the environment, it is more critical to find an acceptable solution rapidly before the information becomes outdated than to find an optimal solution. Therefore, a suboptimal solution is desirable instead of a globally optimal solution so as to reduce the amount of computational time and resources consumed by the system.

To achieve this goal, the TPC problem for cognitive radios has been formulated using a game-theoretic framework [18, 19, 36]. This thesis takes a similar approach considering the TPC problem for cognitive radios under two different limitations separately, first under an interference-power limitation and then under a low-power limitation. Accordingly, the problems are formulated as a *generalized Nash equilibrium problem* (GNEP) and a *Nash equilibrium problem* (NEP) due to the different power limitations considered.

A solution of the NEP is called a *Nash equilibrium* (NE), while a solution of the GNEP is called a *generalized Nash equilibrium* (GNE). The concept of a mixed-strategy NE for a zero-sum game was first introduced by John von Neumann and Oskar Morgenstern in 1944 [41]. John Forbes Nash first described the concept of NE for any game in his seminal paper [31] in 1950, and shortly later, he gave the formal definition of NE in [32]. The concept of a GNE was formally introduced by Debreu in 1952 [11], which extends the concept of the NE. These solutions are considered as the best responses to the actions of all other "players" (in this case, the users of a cognitive radio network) [11]. For the GNEP and NEP, without knowing any information about the other users' profits and strategy sets, each user acts greedily to optimize its own performance based on local information, without establishing coordination with the other users. This proposition implies that the GNEP and NEP can be solved in a distributed manner.

Both the NEP and GNEP are non-cooperative games. By definition, each player of a non-cooperative game operates greedily. It does not guarantee any fair strategy. Therefore, if we want to include the fairness into cognitive radio networks, we have to implement the coalition among cognitive radio users [35]. In this thesis, we focus on the non-cooperative game for the cognitive radio networks. Coalitional game is a different topic. Hence, it is outside of the scope of this thesis.

The above discussion reveals that a practical method of solving the TPC problem needs to have several key attributes including distributed implementation, low complexity, and fast convergence to a reasonably good solution. The iterative water-filling controller (IWFC)[4] has been proposed as a candidate for

[4]In the literature, the acronym IWFA for iterative water-filling algorithm has been used. In

finding a solution to the TPC problem. The algorithm was proposed for solving the TPC problem for DSL in [44, 47], for a Gaussian multiaccess channel in [49], and later proposed for cognitive radio in [18, 19, 36]. In the IWFC, each user maximizes its own objective individually without specific information about the other users' transmit-power vectors. The essence of the water-filling controller is to decide transmit-power levels by treating the current interference as noise. It can be thought of as pouring an amount of water into a pool the bottom of which has been occupied by a certain amount of water, but the total amount of water cannot exceed a water-filling level threshold. The water-filling level threshold is critical in determining the amount of "water"(i.e. power) for each user. The IWFC iteratively repeats this procedure until a certain prescribed criterion is met. This idea can be found in [44, 47], where a sufficient condition for convergence of the IWFC is given for the two-user case. However, the idea has not been implemented for the TPC problem for cognitive radios, which involves more limiting constraints. As in [36], in order to apply the idea of water-filling to solve the TPC problems for cognitive radios considered herein, a set of nonlinear separable convex optimization subproblems needs to be dealt with at each iteration.

1.3 Contributions to the Literature

This thesis considers the TPC problem for cognitive radios under two different limitations: interference-power limitation and low-power limitation. Progress is

this thesis, the algorithm is used for controlling the transmit power of secondary users. Hence, we call it the iterative water-filling controller (IWFC) hereafter.

made toward theoretical analysis and algorithms regarding the TPC problems for cognitive radios in a multiuser environment. In particular, the characterization of the *Karush-Kuhn-Tucker* (KKT) conditions for the game-theoretical models are derived by studying their fundamental properties. This provides the explicit presentations of the water-filling threshold and then the water-filling solution of the separable convex optimization problem for each user. This result inspires our new algorithms for solving the TPC problem for cognitive radios, which are named "fast iterative water-filling controller (FIWFC)" for the two different limitations. The new algorithms inherit the advantages of the IWFC. Moreover, they can handle the separable convex optimization subproblems efficiently. The computational complexity of the new algorithms are lower than the IWFC for DSL. Under certain conditions, this new algorithm linearly converges to a GNE of the TPC problem independent of initialization. In addition, the sufficient conditions for the uniqueness and stability of solution are given.

1.4 Organization of the Thesis

The rest of the thesis is organized as follows: In Chapter 2, several concepts and theorems relating to Nash game theory and generalized Nash game theory are reviewed. In Chapter 3, the results of the generalized Nash game model of the TPC problem for cognitive radios under interference-power limitation are presented. The theoretical basis of the new algorithm is developed, the new algorithm is introduced, and the solution stability conditions are studied in this chapter. Moreover, a convergence theorem of the new algorithm, sufficient conditions of the uniqueness and stability of solutions are given, and the computational complexity of the new algorithm is analyzed. Chapter 5 describes the same but for the case of low-power limitation. Simulation results comparing the new algorithms for the two different limitations with the IWFC are described in Chapters 4 and 6, respectively. Conclusion and discussion of possible future research are given in Chapter 7. The TPC problem for DSL is studied and the new approach is developed for it in Appendix A. Rigorous proofs of the various theorems described in this thesis are given in the appendices.

Chapter 2

The Nash-Equilibrium and Generalized Nash-Equilibrium Problems

2.1 Definition

The GNEP is defined as below: Suppose that there are K players, with the kth player's strategy depending on the decision variable $S_k \in R^N$. Let

$$S := (S_1, S_2, \cdots, S_K)^\top \in R^{N \times K}$$

be the vector formed by all the players' strategies. For each player k, the utility function $f_k(S_k, S_{-k})$ depends on both its own strategy S_k as well as its opponents' strategies, denoted by S_{-k}. When the opponents' strategy S_{-k} is known, player

k decides its own strategy by solving the following optimization problem:

$$\begin{aligned} \max_{S_k} \quad & f_k(S_k, S_{-k}) \\ \text{s.t.} \quad & S_k \in \Omega_k(S_{-k}) \end{aligned} \tag{2.1}$$

where $\Omega_k(S_{-k})$ is the strategy set of player k depending on its opponents' strategy S_{-k}. The GNEP aims at finding a vector $S^* = (S_1^*, S_2^*, \cdots, S_K^*)^\top$ such that for each k, S_k^* solves problem (2.1). Such a vector S^* is a GNE of the GNEP. If the strategy set $\Omega_k(S_{-k})$ does not depend on its opponents' strategy, i.e. $\Omega_k(S_{-k}) = \Omega_k$ for all the players, then the GNEP is reduced to the standard NEP.

In a NEP, each player's strategy set is independent of its opponents' strategies. while in a GNEP, the strategy set of each player may depend on the other players' strategies. However, GNE still shares an important intrinsic property with NE. It is clear from the definition that when GNE is reached, any individual deviating its own strategy away from the GNE will not increase its own utility. Hence, at GNE, no player has the incentive to change its strategy. Therefore, the GNE is stable in a coordinate-wise sense. Because the GNEP extends the concept of the NEP, the properties of the GNEP described in Sections 2.2 and 2.3 apply to the NEP as well.

2.2 Existence

The existence of GNE has been well-studied in the literature. When Debreu introduced the GNEP in 1952, he also proposed the first existence theorem in the same paper. The main result was subsequently established by Arrow and Debreu in 1954 [6]. A simplified version of the result was given by Ichiishi in 1983 [23]. When the strategy sets $\Omega_k(S_{-k})$ can be represented explicitly by convex inequalities, there is a more direct existence theorem of GNEP [34]. This theorem is based on constraint qualifications, the differentiability of the players' objective functions, and the KKT systems of the players' optimization problems. In 2008, Aussel and Dutta presented a direct existence theorem for GNEP with semistrictly quasi-convex functions[1] [7].

As the maximization of the players' objective functions is considered in this thesis instead of minimization as shown by Aussel and Dutta in [7], their result is modified for a GNEP with semistrictly quasi-concave functions in the following theorem:

Theorem 2.2.1 *Let Ω be the feasible set of a GNEP with $\Omega_k(S_{-k}) = \{S_k : (S_k, S_{-k}) \in \Omega\}$. Suppose that*

- *Ω is nonempty, convex and compact, and*
- *for every player k, the objective function f_k is continuous and semistrictly quasi-concave with respect to S_k.*

Then, the GNEP has at least one GNE.

[1]A function $f : R^n \to R$ is said to be semistrictly quasi-convex if for any x, $y \in R^n$ with $f(x) \neq f(y)$ and $\lambda \in (0,1)$, one has [7]

$$f(\lambda x + (1-\lambda)y) < \max\{f(x), f(y)\}.$$

A function f is called semistrictly quasi-concave if $-f$ is semistrictly quasi-convex.

2.3 KKT Conditions

Suppose that each strategy set $\Omega_k(S_{-k})$ is formed as

$$\Omega_k(S_{-k}) = \{S_k \in R^N : g_k(S_k, S_{-k}) \leq 0\},$$

where g_k is a vector function. Assume that all the functions, g_k and f_k are continuously differentiable. Let $\bar{S}$ be a GNE. If for each player a suitable constraint qualification holds, for example, *Slater's constraint qualification*[2] [8], then there is a Lagrange multiplier vector λ_k such that the KKT conditions for each player's optimization problem are satisfied, which are:

$$\begin{aligned} \nabla_{S_k} L_k(S_k, \bar{S}_{-k}, \lambda_k) &= 0, \\ 0 \leq \lambda_k \perp g_k(S_k, \bar{S}_{-k}) &\leq 0 \end{aligned}$$

where

$$L_k(S_k, \bar{S}_{-k}, \lambda_k) = -f_k(S_k, \bar{S}_{-k}) + \lambda_k^\top g_k(S_k, \bar{S}_{-k}),$$

and $a \perp b$ means that the two vectors a and b are orthogonal. Concatenating all players' KKT conditions as one, we obtain the KKT conditions of the GNEP. The KKT conditions of the GNEP are satisfied by $(\bar{S}, \lambda)$ if, and only if, they

[2]The Slater's constraint qualification is satisfied if $g_k(\cdot, S_{-k})$ is pseudoconvex and continuous, and there is an $S_k \in \Omega_k(S_{-k})$, such that $g_k(S_k, S_{-k}) < 0$.

satisfy the following system:

$$\begin{aligned} \nabla_{\bar{S}} L(\bar{S}, \lambda) &= 0, \\ 0 \leq \lambda \perp g(\bar{S}) &\leq 0, \end{aligned} \tag{2.2}$$

where

$$\lambda := \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_K \end{pmatrix}, \quad g(S) := \begin{pmatrix} g_1(S) \\ g_2(S) \\ \vdots \\ g_K(S) \end{pmatrix},$$

and

$$\nabla_S L(S, \lambda) := \begin{pmatrix} \nabla_{S_1} L_1(S, \lambda_1) \\ \nabla_{S_2} L_2(S, \lambda_2) \\ \vdots \\ \nabla_{S_K} L_K(S, \lambda_K) \end{pmatrix}.$$

On the one hand, this system can be considered as first-order necessary conditions of the GNEP under a suitable constraint qualification. However, its structure is different from that of classical KKT conditions: In actual fact, the system consists of partial differentiation of each function L_k in the Lagrange function vector L with respect to the corresponding part S_k of the variable vector S. On the other hand, system (2.2) is also a sufficient condition for solving the GNEP under the following convexity assumption [12]:

Convexity Assumption For each player, when its opponents' strategy S_{-k} is given, the objective function $f_k(\cdot, S_{-k})$ is concave and the set $\Omega_k(S_{-k})$ is closed and convex.

Let us recall the necessary and sufficient KKT conditions of a GNEP described in [12]:

Theorem 2.3.1 *Suppose that the objective and constraint functions of the GNEP are continuously differentiable:*

- *If there is a GNE $\bar{S}$ of the GNEP at which each player's optimization problem satisfies a suitable constraint qualification, then there is a λ such that $(\bar{S}, \lambda)$ solves system (2.2).*
- *If the Convexity Assumption holds and $(\bar{S}, \lambda)$ solves system (2.2), then $\bar{S}$ is a GNE of the GNEP.*

2.4 Summary

In this chapter, we recalled the concepts of Nash equilibrium and generalized Nash equilibrium problems, which will be used as mathematical models to formulate the TPC problem for cognitive radio networks in Chapters 3 and 5. We also recalled an existence theorem and the KKT conditions of a GNEP. The results for the GNE are also applied to the NE since the concept of a NE is a special case of that of a GNE.

Chapter 3

Transmit-Power Control Problem under Interference-Power Limitation

3.1 Problem Statement

Depending on requirements of a designer, a cognitive radio network can be built in one of two ways:

(i) By using the established communication infrastructure, a cognitive radio network cooperating with existing base stations.

(ii) By working independently from the current communication infrastructure, with secondary users performing transmit-power control in a distributed fashion across the cognitive radio network.

Consequently, a TPC problem can be cast in one of two ways:

(i) a cooperative game, in which all users jointly maximize the total data rate of all the users, subject to some power level constraints; or

(ii) a non-cooperative game, in which each user greedily maximizes its own data rate, subject to its own power level constraints.

In this thesis, the TPC problem is formulated as a non-cooperative game with each user setting its own transmit-power level in a decentralized manner. Obviously, the advantage of choosing a non-cooperative strategy is that a base station is not required and thus cognitive radio networks can be built without being geographically tied to a set of base stations. In this case, instead of jointly maximizing the total data rate, each secondary user in a cognitive radio network maximizes its own data rate in a distributed manner.

3.2 Generalized Nash Game Formulation

Suppose that there are K secondary users and H spectrum holes in a cognitive radio network, and each spectrum hole is divided into 2^L subcarriers. Then, there are $H2^L$ subcarriers available in the network. Let $N = H2^L$. Mathematically, the TPC problem of this network is formulated as a set of optimization subproblems:

For user k,

$$\begin{aligned}
\max_{S_k} \quad & \sum_{n=1}^{N} \log\left(1+\frac{S_k^n}{I_k^n}\right) \\
\text{s.t.} \quad & \sum_{n=1}^{N} S_k^n \leq CAP_k, && (3.1) \\
& S_k^n + I_k^n \leq P^n, \\
& S_k^n \geq 0, && (3.2) \\
& n = 1, \cdots, N,
\end{aligned}$$

where

- $S_k := (S_k^1, S_k^2, \cdots, S_k^N)$, and S_k^n is the decision variable denoting user k's transmit power over subcarrier n,
- $I_k^n := \sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n S_j^n > 0$ denotes the interference-plus-noise term experienced by user k over subcarrier n, $\sigma_k^n > 0$ is the normalized noise power of user k over subcarrier n, and α_{jk}^n is the normalized interference coefficient from user j's transmitter to user k's receiver over subcarrier n,
- $P^n > 0$ denotes the interference-power limitation over subcarrier n, and
- $CAP_k > 0$ is the total battery power budget available to user k.

Here, σ_k^n[1] is defined as the noise power normalized by $\Gamma/|h_{kk}^n|^2$, α_{jk}^n is defined as $\Gamma|h_{jk}^n|^2/|h_{kk}^n|^2$, where Γ is the signal-to-noise ratio (SNR) gap and h_{jk}^n is the channel gain from transmitter j to receiver k over subcarrier n. The term h_{jk}^n

[1] A word of caution: This σ_k^n should not be confused with the symbol σ^2 for noise variance.

can be further calculated from the empirical formula for the path loss in [20],

$$|h_{jk}^n|^2 = \frac{\beta^n}{d_{ij}^r}, \tag{3.3}$$

where d_{ij} is the distance from transmitter j to receiver i, r is the path-loss exponent varing from 2 to 5, and β^n is the frequency-dependent attenuation parameter. Therefore, it can be shown that

$$\alpha_{jk}^n = \Gamma(\frac{d_{ii}}{d_{ij}})^r. \tag{3.4}$$

To preserve the battery power budget constraints, we require that

$$\sum_{k=1}^{K} CAP_k < \sum_{n=1}^{N} (P^n - \sigma_k^n),\ k = 1, \cdots, K. \tag{3.5}$$

This requirement ensures that the power-budget constraints $\sum_{n=1}^{N} S_k^n \leq CAP_k$ are not redundant. Hereafter, the collectivity of these inequalities is referred to as the ***non-triviality requirement***. The TPC problem is a GNEP, which extends the classic Nash equilibrium problem since each user's strategy set depends on the other users' strategies. A point $S := (S_1, S_2, \cdots, S_k)$ is a solution of the TPC problem if, and only if, S_k solves the nonlinear convex optimization subproblem (3.1) for each user k.

For each user, the total interference caused by other users is measured at the receiver and therefore, the objective function in each optimization subproblem (3.1) is strictly concave. In addition, the objective functions are continuous and the constraints are linear. Therefore, a GNE of the TPC exists according to

Theorem 2.2.1. As mentioned in section 2.2, if a GNE is adopted by all the users, then no user has the incentive to deviate from it.

3.3 New Approach for Computing an Equilibrium

We may find a solution of the TPC problem for a cognitive radio network by solving the nonlinear optimization subproblems (3.1) simultaneously by applying the iterative water-filling controller. Some nonlinear optimization solvers have been developed to solve the convex optimization subproblem, which are based on any of the widely used and effective methods that include the Generalized Reduced Gradient method, sequential Quadratic Programming methods, augmented Lagrangian method, and interior-point methods [8]. Interior-point methods solve the primal-dual KKT system. The nonlinear optimization solvers presently in use include, but are not limited to:

- MATLAB Optimization Toolbox using three methods for constrained nonlinear problems: trust region, active set, and interior-point [3],
- MOSEK using an interior-point method [1],
- KNITRO using an interior-point method and active set method [2].

Obviously, performance of the IWFC depends on the optimization solver used.

The new approach described in this thesis is designed specifically for the TPC problem for cognitive radios. It inherits the promising properties of the IWFC.

The key of the new procedure is to find an explicit representation of water-filling level thresholds. Then, the water-filling solution can be updated directly for each user at each iteration using the idea of water-filling. To this end, the following procedure is offered:

(i) write down the KKT conditions of the TPC problem; since the feasible set is a polyhedron, this problem satisfies Slater's constraint qualification. Hence, the KKT conditions are necessary and sufficient.

(ii) transform the KKT conditions into an equivalent **mixed *linear* complementarity problem (MLCP)** without any loss of information; and

(iii) find the necessary and sufficient optimality conditions for the MLCP. In fact, the necessary and sufficient optimality conditions provide a closed-form solution for each nonlinear optimization problem (3.1).

In so doing, the new procedure forms a solid theoretical basis for solving the TPC problem, as depicted in Figure 3.1. The new approach is named the **fast iterative water-filling controller (FIWFC)**, which is also straightforward to implement.

3.4 Theoretical Basis of the New Approach

3.4.1 KKT Conditions

Let λ_k^n be the Lagrange multiplier for the interference-power limitation for user k over subcarrier n, and let μ_k be the Lagrange multiplier for the power budget

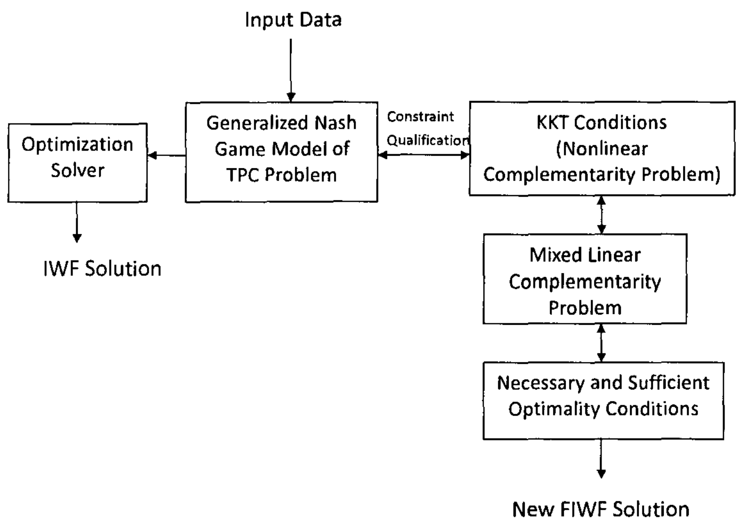


Figure 3.1: Flow graph depicting the theoretical bases for the IWFC and the FIWFC for cognitive radio networks

constraint of user k. Then, the KKT conditions for user k's problem are as shown below:

$$\begin{aligned} 0 \ &\leq \ S_k^n \perp (\lambda_k^n + \mu_k - \frac{1}{S_k^n + I_k^n}) \geq 0 \\ 0 \ &\leq \ \lambda_k^n \perp (P^n - (S_k^n + I_k^n)) \geq 0 \\ 0 \ &\leq \ \mu_k \perp (CAP_k - \sum_{n=1}^{N} S_k^n) \geq 0 \\ & \quad\ n = 1, \cdots, N. \end{aligned} \tag{3.6}$$

By Theorem 2.3.1, the KKT conditions are necessary and sufficient, i.e. there exists a GNE, say S, if and only if, there are Lagrange multipliers α, λ, and μ such that the KKT conditions hold at $(S, \alpha, \lambda, \mu)$. Therefore, by computing a solution of the system (3.6), a solution of the TPC problem can be found. However, the

KKT conditions (3.6) are nonlinear. The following subsection presents a mixed linear complementarity problem, which is an equivalent formulation of the KKT conditions.

3.4.2 Mixed Linear Complementarity Problem

Although the KKT conditions of the TPC problem are nonlinear, the following proposition shows that under the non-triviality requirement, the KKT conditions can be reformulated as a mixed *linear* complementarity problem [36]:

Proposition 3.4.1 *Under the non-triviality requirement, the KKT conditions of the TPC problem are equivalent to the following MLCP:*

$$
\begin{aligned}
0 &\leq S_k^n \perp (S_k^n + I_k^n + \beta_k^n - \nu_k) \geq 0 \\
0 &\leq \beta_k^n \perp (P^n - (S_k^n + I_k^n)) \geq 0 \\
0 &= CAP_k - \sum_{n=1}^{N} S_k^n \\
& \quad \nu_k \text{ is free} \\
n &= 1, \cdots, N,
\end{aligned}
\tag{3.7}
$$

for $k = 1, \cdots, K$.

In turn, the necessary and sufficient conditions of the MLCP are derived, where the representations of the Lagrange multipliers as well as the solutions to each user's optimization subproblem are presented. This idea lends itself readily to computing a GNE of the TPC problem.

3.4.3 Optimality Conditions

After exploring the linear complementarity problem, the necessary and sufficient optimality conditions for the MLCP are presented in the following proposition:

Proposition 3.4.2 *Let*

$$T_k = \{n : S_k^n > 0\}$$

be the index set of subcarriers activated by user k, and

$$B_k = \{n : S_k^n + I_k^n = P^n, S_k^n > 0\}$$

be the index set of subcarriers activated by user k, at which the interference-power limitations are reached. Then, (S, ν, β) solves problem (3.7) if, and only if, (S, ν, β) satisfies the following conditions, referred to as arithmetic simplified optimality conditions:

(1) If $T_k \neq B_k$, then

$$\nu_k = (CAP_k + \sum_{n \in T_k} I_k^n - \sum_{n \in B_k} P^n)/(|T_k| - |B_k|)$$

$$\beta_k^n \begin{cases} = \nu_k - P^n & I_k^n < P^n < \nu_k \\ \geq \nu_k - P^n & I_k^n = P^n < \nu_k \\ = 0 & I_k^n < \nu_k \leq P^n \\ = 0 & \nu_k \leq I_k^n < P^n \\ \geq 0 & \nu_k \leq I_k^n = P^n \end{cases}$$

and

$$S_k^n = \begin{cases} P^n - I_k^n & I_k^n \leq P^n < \nu_k \\ \nu_k - I_k^n & I_k^n < \nu_k \leq P^n \\ 0 & \nu_k \leq I_k^n \leq P^n \end{cases}$$

(2) Otherwise,

- $S_k^n = P^n - I_k^n$, $CAP_k = \sum_{n \in T_k} P^n - \sum_{n \in T_k} I_k^n$ *and* $\nu_k - \beta_k^n = P^n$, *for all* $n \in T_k$*;*

- $\beta_k^n = 0$ *and* $\nu_k \leq I_k^n$ *for all* $n \notin T_k$.

Proof: See APPENDIX B.

According to Theorem 2.3.1, and Propositions 3.4.1 and 3.4.2, we are ready to state the necessary and sufficient optimality theorem in the following corollary.

Corollary 3.4.3 *Suppose that the convexity assumption holds. Then, S is a GNE of the TPC problem under the interference-power limitation* (3.1) *if, and only if, there exist ν and β such that (S, ν, β) satisfies the arithmetic simplified optimality conditions in Proposition 3.4.2.*

3.5 Fast Iterative Water-Filling Controller

3.5.1 Algorithm

This subsection introduces the new algorithm, FIWFC, for finding a GNE of the TPC problem under the interference-power limitation. Enlightened by Proposition 3.4.2, the idea of this new algorithm is to update the transmit-power vector iteratively by using the explicit representation of the water-filling level thresholds. In practice, each cognitive radio can measure the level of interference in its own local environment. Therefore, the measured interference-power level from the previous iteration results can be used to update the current transmit powers. Utilizing the information content of the Lagrange multipliers, the power vector of the TPC problem is updated directly.

To describe the FIWFC in detail, some notation needs to be introduced. Let superscript l indicate the lth iteration of the algorithm, and $\Omega_k(S_{-k}^{(l)})$ be the strategy set of user k at the lth iteration. For $n = 1, \cdots, N$, let

$$I_k^{n(l)} := \sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n S_j^{n(l)}$$

be the interference-plus-noise at the current iteration,

$$T_k^{(l)} := \{n : S_k^{n(l)} > 0\},$$

be the index set of the subcarriers utilized by user k, and

$$B_k^{(l)} := \{n : S_k^{n(l)} + I_k^{n(l)} = P^n, S_k^{n(l)} > 0\},$$

be the index set of subcarriers where user k reaches the interference-power limitation. At each iteration, $\nu_k^{(l)}$ is calculated and the transmit-power vector $S_k^{(l+1)}$ is updated based on information extracted from the current iteration l. In fact, $\nu_k^{(l)}$ plays an important role in the updating scheme of the transmit-power vector by serving as a water-filling level threshold in the updating scheme. The value of $S_k^{(l+1)}$ is set to the difference of the threshold $\nu_k^{(l)}$ and the interference-plus-noise power level $I_k^{n(l)}$, when $\nu_k^{(l)}$ does not exceed the capacity P^n; otherwise, it is set to the difference of P^n and $I_k^{n(l)}$. In addition, the updated transmit-power vector $S_k^{(l+1)}$ as well as the Lagrange multipliers $\nu_k^{(l)}$ and $\beta_k^{n(l)}$ satisfy the KKT conditions (3.7) with $I_k^{n(l)}$ formed by $S_k^{(l)}$.

The FIWFC for cognitive radios under interference-power limitation is described in Algorithm 3.5.1.

Algorithm 3.5.1 FIWFC for Cognitive Radios under Interference-Power Limitation

Let $l = 0$ and $S_k^{(0)}$ be any feasible power vector
repeat
  for $k = 1$ to K **do**
    if $T_k^{(l)} \neq B_k^{(l)}$ **then**
      $\nu_k^{(l)} = \frac{CAP_k + \sum_{n \in T_k^{(l)}} I_k^{n(l)} - \sum_{n \in B_k^{(l)}} P^n}{|T_k^{(l)}| - |B_k^{(l)}|}$
      for $n = 1$ to N **do**

$$S_k^{n(l+1)} = \begin{cases} P^n - I_k^{n(l)} & \text{if} \nu_k^{(l)} \geq P^n > I_k^{n(l)} \\ \nu_k^{(l)} - I_k^{n(l)} & \text{if } P^n \geq \nu_k^{(l)} > I_k^{n(l)} \\ 0 & \text{otherwise} \end{cases}$$

      end for
    else
      Set $\nu_k^{(l)} > \max_n P^n$
      for $n = 1$ to N **do**

$$S_k^{n(l+1)} = \begin{cases} P^n - I_k^{n(l)} & n \in T_k^{(l)} \\ 0 & \text{otherwise} \end{cases}$$

      end for
    end if
  end for
  $l \leftarrow l + 1$
until A stopping criterion is satisfied

The basic idea of this new updating scheme is illustrated in Figure 3.2. In this figure, three cases are considered for the sake of simplicity:

C1. the interference power P^1 is lower than the threshold ν_k; therefore, the transmit-power for user k over this subcarrier is set to $P^1 - I_k^1$;

C2. the interference power P^2 is higher than the threshold ν_k; therefore, the transmit power for user k over this subcarrier is set to $\nu_k - I_k^2$; and

C3. when interference I_k^3 is higher than the threshold, the transmit-power S_k^3 is set to zero.

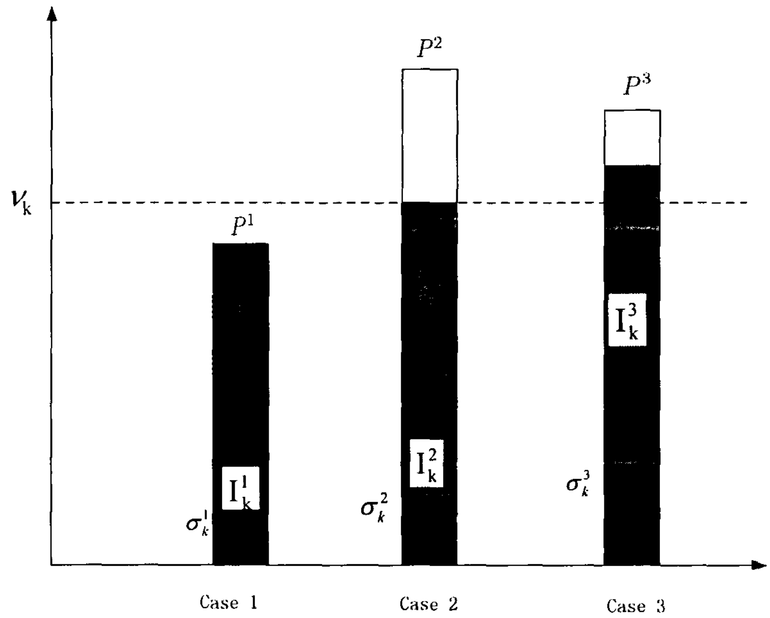


Figure 3.2: Illustrating the basic updating scheme of FIWFC for cognitive radios under interference-power limitation

To compare the FIWFC with the IWFC, the framework of the IWFC is recalled in Algorithm 3.5.2.

Algorithm 3.5.2 Iterative Water-Filling Controller Framework

Let $l = 0$ and $S_k^{(0)}$ be any feasible power vector
repeat
  for $k = 1$ to K **do**

$$\begin{aligned} \bar{S}_k^{(l+1)} = \arg\max \quad & f_k(S_k, S_{-k}^{(l)}) \\ \text{s.t.} \quad & S_k^n + I_k^{n(l)} \le P^n, \\ & \sum_{n=1}^{N} S_k^n \le CAP_k, \\ & S_k^n \ge 0, \\ & n = 1, \cdots, N, \end{aligned}$$

  end for
  $l \leftarrow l + 1$
until A stopping criterion is satisfied

3.5.2 Convergence and Uniqueness

The sufficient conditions for convergence of the FIWFC for cognitive radios under interference-power limitation are presented in this subsection. To show the conditions, some notation needs to be defined, including the interference-gain matrices

$$A = \begin{pmatrix} A_1 \\ A_2 \\ \cdots \\ A_K \end{pmatrix} \in NK \times NK$$

and $C = (C_1, C_2, \cdots, C_N) \in NK \times NK$, where matrix $A_k = (A_{1k}, A_{2k}, \cdots, A_{Kk}) \in N \times KN$ with $A_{kk} = \mathbf{0} \in N \times N$ and

$$A_{ik} = \begin{pmatrix} \alpha_{ik}^1 & 0 & \cdots & 0 \\ 0 & \alpha_{ik}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \alpha_{ik}^N \end{pmatrix} \in N \times N,$$

for $i = 1, \cdots, k-1, k+1, \cdots, K$, and

$$C_n = \begin{pmatrix} C_{1n} \\ C_{2n} \\ \vdots \\ C_{Nn} \end{pmatrix} \in KN \times K$$

with $C_{ni} = 0$ for $i \neq n$, and

$$C_{nn} = \begin{pmatrix} 0 & \alpha_{21}^n & \cdots & \alpha_{K1}^n \\ \alpha_{12}^n & 0 & \cdots & \alpha_{K2}^n \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{1K}^n & \alpha_{2K}^n & \cdots & 0 \end{pmatrix} \in K \times K,$$

for $n = 1, \cdots, N$. Let $\rho(X)$ denote the spectral radius of matrix X. Let $\mathrm{Tr}(X)$ denote the trace of matrix X. The global convergence of the FIWFC is then guaranteed by virtue of the following sufficient conditions.

Theorem 3.5.1 *Suppose that the TPC problem satisfies the non-triviality requirement and the convexity assumption. If one of the following conditions is satisfied,*

- $\rho(A^\top A) < 1$, *or*
- $\rho(C^\top C) < 1$, *or*
- $Tr(A^\top A) < 1$, *or*
- $Tr(C^\top C) < 1$,

then the FIWFC globally and linearly converges to a solution of the TPC problem.

Proof: See APPENDIX C.

Based on Proposition 3.4.2, the conditions that guarantee the uniqueness of the solution of the TPC problem is derived.

Theorem 3.5.2 *With the same conditions in Theorem 3.5.1, the TPC problem has a unique solution.*

Proof: See APPENDIX D.

The conditions in Theorem 3.5.1 are equivalent to the condition:

$$\sum_{\substack{j=1\\j\neq k}}^{K}\sum_{k=1}^{K}\sum_{n=1}^{N} {\alpha_{jk}^{n}}^{2} < 1. \tag{3.8}$$

In particular, if $\alpha_{jk}^n < \frac{1}{K\sqrt{N}}$, then condition (3.8) is satisfied. This condition is restrictive in practice. As mentioned in [36],

$$\alpha_{jk}^n \propto (\frac{d_{ii}}{d_{ij}})^r, \tag{3.9}$$

where d_{ij} is the distance from transmitter j to receiver i and the path-loss exponent r varies from 2 to 5. Therefore, when the distance ratio $\frac{d_{ii}}{d_{ij}}$ is very small, condition (3.8) is satisfied. However, we will see from the simulation results in Chapter 4, the algorithm converges in all the cases considered even when the condition is not satisfied, which implies that the convergence condition may be extended.

3.5.3 Computational Complexity

Although the TPC for cognitive radio networks has a more complicated constraint set than the TPC for DSL contains, the computational complexity of the new algorithm is lower than that of the iterative water-filling controller for DSL as proposed in [47]. It is justified here.

The new algorithm is formulated iteratively. After the initialization, the power allocation is adjusted iteratively for K users in an inner loop. For each user k, T_k and B_k need to be checked once. Theoretically, checking T_k and B_k takes at most N and $N(K+2)$ operations, respectively. More practically, since the interference-plus-noise I_k^n is measured at the receiver based on the previous iteration results, this term does not affect the computational burden on checking B_k. In this sense, checking B_k needs at most $2N$. To evaluate ν_k, the number of additions of the relevant interference-plus-noise terms is at most N, as is that of the interference power limits P^n. Hence, the number of operations of evaluating ν_k is at most $4N + 4$. To evaluate S_k^n needs at most 3 operations, and then the number of operations needed in evaluating user k's power vector $S_k = (S_k^1, S_k^2, \cdots, S_k^N)$ is at most $3N$. Thus, the operations needed for each user is at most $10N + 4$. For K users, the operations needed is at most $K(10N + 4)$. Hence, the total complexity of each iteration is $O(KN)$, which is linear in both the number of users K and the number of subcarriers N, respectively. Indeed, this is the reason why this new algorithm is called the *fast* iterative water-filling controller. Let L_1 be the number of iterations needed in the FIWFC. The total computational complexity of the FIWFC is $O(L_1KN)$. Based on simulation results, L_1 almost stays constant as the size of the data sets changes.

In direct contrast, the total computational complexity of the IWFC for DSL is $O(L_2KN \log N)$ [48, 49], which is higher than the FIWFC by a factor of $\log N$, although the TPC problem for DSL has simpler constraints than the one considered in this thesis. Here, L_2 is the number of iterations needed in the IWFC for DSL.

3.6 Solution Stability

This section studies the stability of a GNE for the TPC problem under perturbation of parameters. In practice, accurate information about the noise and interference may not be obtainable because they are measured by receivers of cognitive radios. The question is, for a given set of measurements close to the accurate values, will the solution of the perturbed TPC problem be close to the correct solution or not? Addressing this question leads to the concept of "solution stability", which has to do with the "continuity" of the perturbed solutions.

3.6.1 MLCP and Affine Variational Inequality (AVI)

In order to take advantage of the well-developed stability theory of AVI, the relationship between MLCP and AVI needs to be studied. First, the definition of AVI is given as follows:

Definition 3.6.1 *Given a subset $\Omega \in R^n$ and an affine function*

$$F = q + Mx, \forall x \in R^n,$$

for some vector $q \in R^n$ and matrix $M \in R^{n\times n}$, the affine variational inequality, denoted as $AVI(\Omega, q, M)$, defines a vector $x \in \Omega$ such that

$$(y - x)^T F(x) \geq 0, \ \forall y \in \Omega. \tag{3.10}$$

The set of solutions to this problem is denoted as $SOL(\Omega, q, M)$.

According to Proposition 1.2.1 in [13], the equivalent AVI formulation for the MLCP (3.7) can be formulated as follows:

$$(Z - S)^T F(S) \geq 0, \forall Z \in \Omega, \tag{3.11}$$

where

$$F := \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_K \end{pmatrix}, \quad F_k := \begin{pmatrix} S_k^1 + I_k^1 \\ S_k^2 + I_k^2 \\ \vdots \\ S_k^N + I_k^N \end{pmatrix},$$

and

$$\Omega = \left\{ S \in R^{KN} : \begin{array}{l} S_k^n + I_k^n \leq P^n, \\ \sum_{n=1}^N S_k^n = CAP_k, \\ S_k^n \geq 0, \\ n = 1, \cdots, N,\ k = 1, \cdots, K \end{array} \right\}.$$

In order to put the function F as well as the functions in the set Ω into matrix form, let us define a matrix M as $M = A + \bar{I}$, where A is defined in Subsection 3.5.2 and $\bar{I}$ is the identity matrix (M can also be set as $M = C + \bar{I}$ depending on how the power vector S is defined, where C is defined in Subsection 3.5.2 as well). Then, the function F and the solution set Ω can be represented as $F = MS + \sigma$ and

$$\Omega = \left\{ S \in R^{KN} : \begin{array}{l} MS + \sigma \leq P, \\ HS = CAP, \\ S \geq 0, \end{array} \right\},$$

respectively, where $H = (H_1, H_2, \cdots, H_K)$, and H_k is a $K \times NK$ matrix where the kth row is the all-ones vector and the other rows are all zero vectors.

3.6.2 Solution Stability of Affine Variational Inequality

Let F be an affine function given by:

$$F(x) = q + Mx, \forall x \in R^n, \tag{3.12}$$

for some vectors $q \in R^n$ and matrix $M \in R^{n \times n}$. Let Ω be represented as:

$$\Omega = \{x \in R^n : g(x) \leq 0, h(x) = 0\}, \tag{3.13}$$

where $g(x) = Ax - b$ and $h(x) = Cx - d$. Following [13], the family of parametric AVIs is defined as follows:

$$\{AVI(\Omega(p), F(\cdot, p)) : p \in P\}. \tag{3.14}$$

Before presenting the stability theorem, two definitions in [13] need to be recalled.

Definition 3.6.2 *The constraints that define the set Ω satisfies the* Mangasarian-Fromowitz Constraint Qualification (MFCQ) *at $x \in \Omega$ if*

(a) the gradients

$$\{\nabla h_j(x) : j = 1, \cdots, l\},$$

are linearly independent, and

(b) there exists a vector $v \in R^n$ such that

$$v^T \nabla h_j(x) = 0, \forall j = 1, \cdots, l$$

$$v^T \nabla g_j(x) \leq 0, \forall i \in \mathrm{I}(x),$$

where $\mathrm{I}(x)$ *is the active index set at* x, *i.e.*

$$\mathrm{I}(x) = \{i : g_i(x) = 0\}.$$

Definition 3.6.3 *Let Ψ be a cone in R^n. A matrix $B \in R^{n \times n}$ is said to be*

(i) copositive *on Ψ if*

$$x^T Bx \geq 0, \forall x \in R^n_+;$$

(ii) strictly copositive *on Ψ if*

$$x^T Bx > 0, \forall x \in R^n_+ \setminus \{0\}.$$

The next result shows the sufficient conditions for parametric stability of a solution x^* in $\mathrm{SOL}(\Omega(p^*), F(\cdot, p^*))$ of a parametric $\mathrm{AVI}(\Omega(p^*), F(\cdot, p^*))$, which is reduced from Theorem 5.4.4 in [13]. The result pertains to the concept of solution stability of the parametric $\mathrm{AVI}(\Omega(p^*), F(\cdot, p^*))$ under the perturbation of the parameter p, which provides useful information including the solvability of the perturbed problems $\mathrm{AVI}(\Omega(p), F(\cdot, p))$ and the quantitative change of the perturbed solution with respect to the solution x^*.

Corollary 3.6.4 *Suppose that the functions $g(\cdot, p)$ and $h(\cdot, p)$ are continuous and convex for each $p \in P$. Let x^* be a solution of the AVI$(\Omega(p^*), F(\cdot, p^*))$. Assume that there exist an open neighborhood Φ of x^*, an open neighborhood Ψ of p^* and positive constants L and L' such that*

$$\sup_{x\in\Omega(P)\cap\Phi} \|F(x,p) - F(x,p^*)\| \leq L\|p - p^*\|, \forall p \in \Psi \cap P, \tag{3.15}$$

$$\|F(x,p^*) - F(x',p^*)\| \leq L'\|x - x'\|, \forall x,\, x' \in \Phi. \tag{3.16}$$

Further assume that the MFCQ holds at $x^* \in \Omega(p^*)$. If M is strictly copositive on R^n, then there exist a neighborhood N of x^* and a neighborhood W of p^* such that, for all $p \in \mathrm{W}$,

$$S_{\mathrm{N}}(p) = SOL(\Omega(p), F(\cdot, p)) \cap \mathrm{N} \neq \emptyset;$$

and there exists a constant $c > 0$ such that, for all p sufficiently close to p^*,

$$\sup\{\|x(p) - x^*\| : x(p) \in S_{\mathrm{N}}(p)\} \leq c\|p - p^*\|. \tag{3.17}$$

The formulation (3.17) indicates that solutions of parametric AVI$(\Omega(p), F(\cdot, p))$ will continuously approach the solution of the original AVI$(\Omega(p^*), F(\cdot, p^*))$ when the parameter p approaches the original p^*.

3.6.3 Solution Stability of the TPC Problem under Interference-Power Limitation

For the TPC in cognitive radio, the measurement errors are encountered in the interference-plus-noise levels. This implies that the interference-gain matrix and normalized noise are the key parameters for stability analysis. This subsection considers the parametric AVI for the TPC problem as follows:

$$(Z-S)^T F(S,p_1,p_2) \geq 0, \, \forall Z \in \Omega(S,p), \tag{3.18}$$
$$p=(p_1,p_2), \, p_1,p_2 \in R^{NK\times NK}$$

where $F(S,p_1,p_2) = (M+p_1M)S+\sigma+\sigma p_2$ and

$$\Omega(p) = \left\{ S \in R^{KN} : \begin{array}{l} (M+p_1M)S+\sigma+\sigma p_2 \leq P, \\ HS = CAP, \\ S \geq 0, \end{array} \right\}.$$

Here, M is not necessarily symmetric.

For the parametric AVI (3.18), the Lipschitz conditions (3.15) and (3.16) in Corollary 3.6.4 are naturally satisfied. In the following, the stability theorem for AVI is applied to our TPC problem:

Corollary 3.6.5 *When the matrix M is strictly copositive and the MFCQ holds at a GNE of the TPC problem (3.1), the GNE will be stable.*

Strict copositiveness of M on R^n is equivalent to the condition that $-M$ is a Hurwitz matrix, which is defined as a matrix where every eigenvalue has a strictly negative real part. If the symmetric part of M, that is $\frac{1}{2}(M + M^T)$, is positive definite, then $-M$ is Hurwitz; if the matrix M is strictly diagonally dominant, then its symmetric part $\frac{1}{2}(M + M^T)$ is positive definite. Therefore, one of the following conditions guarantees that $-M$ will be Hurwitz:

(1)

$$\sum_{j \neq k} \alpha_{jk}^n < 1, \forall n = 1, \cdots, N, \forall k = 1, \cdots, K, \textit{ or} \tag{3.19}$$

(2) in particular,

$$\alpha_{jk}^n < \frac{1}{K-1}, \forall n = 1, \cdots, N, \forall k = 1, \cdots, K. \tag{3.20}$$

Note that the convergence condition developed in Subsection 3.5.2 implies condition (3.20), as does the strict copositiveness. Hence, the solution stability condition is weaker than the convergence condition given in this thesis. We will further discuss the stability conditions in detail in the next subsection.

3.6.4 Discussions on Solution Stability Conditions of the TPC Problem

As mentioned in Section 3.5.2, the matrix M contains the topology information of the cognitive radio networks. The term α_{jk}^n is proportional to the distance ratio $(\frac{d_{ii}}{d_{ij}})^r$ where the path-loss exponent r varies from 2 to 5. When the term α_{jk}^n is smaller than $\frac{1}{K-1}$ for all k and n, the matrix M will be strictly copositive. Then the stability of the solutions will be assured. These conditions require a topology condition to be imposed on a cognitive radio network that each receiver has a relatively shorter distance from its own transmitter, compared to the distances from other activated transmitters in the network.

There may be many good approaches to building a cognitive radio network satisfying this topology requirement. One promising approach is to use an ad hoc network [4, 5]. In cognitive radio ad hoc networks, two or more secondary users are used as wireless hops so as to relay the packets from a source to a destination. Cognitive radio ad hoc networks do not rely on any infrastructure or centralized authority. The relay routes are established and updated among the secondary users according to the topology of the network. Thus, the stability conditions could be satisfied in this ad hoc manner. Figure 3.3 shows an example of a cognitive radio ad hoc network, where the secondary users are placed randomly and the relay routes are chosen according to the shortest Euclidean distances between the secondary users.

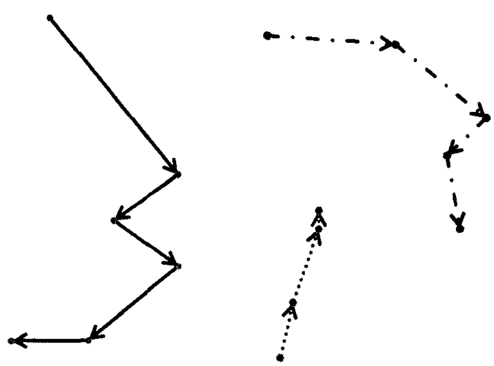


Figure 3.3: An example of a cognitive radio ad hoc network

3.7 Summary

In this chapter, the TPC problem under interference-power limitation was cast as a generalized Nash game model. The solution to this problem is guaranteed by the existence theorem presented in the previous chapter. After establishing the rigorous theoretical basis, we developed the new algorithm, FIWFC for finding a GNE for the this problem. The computational complexity of the FIWFC is lower than that of the IWFC for DSL by a factor of $\log N$, although the TPC problem of DSL has a simpler constraint set. The convergence conditions of the FIWFC and the uniqueness of a solution were given. Although the conditions are restrictive in practice, we will see from the simulation results in the next chapter that the algorithm converges even when the conditions are violated. This implies that the conditions could be extended. We also presented the sufficient conditions of solution stability. For the TPC problem considered in this chapter, a solution is stable when the sum of the interference-gain matrix and the identity matrix is a strictly copositive matrix. However, the stability of a solution cannot be assured when the stability conditions are violated. Using an ad hoc network could help satisfy the stability conditions in practice.

Chapter 4

Simulation Results of the FIWFC for the TPC under Interference-Power Limitation

This chapter presents simulation results that have been performed in order to evaluate the performance of the FIWFC under an interference-power limitation. Different sizes of cognitive radio networks are used as examples and both stationary and non-stationary environments are considered. The convergence behavior of the FIWFC is compared with the IWFC. For the IWFC, the *fmincon* function in MATLAB and the *mskscopt* function in MOSEK were used, to solve the nonlinear separable convex optimization problems (the *fmincon*-based IWFC is denoted as IWFC I and the *mskscopt*-based IWFC as IWFC II, and these designations will be used throughout the rest of thesis.). The latter is designed specifically for separable convex problems using an interior-point method. In the final section of this chapter, the stability of the solutions generated by the FIWFC are evaluated by simulating scenarios under the presence of measurement errors.

The simulation database was generated as follows: the secondary users are randomly located on a $1000 \times 1000m^2$ square area. Similar to the initial set up in [43], the ambient noise power is set as $\sigma^2 = 5 \times 10^{-13}$ and the signal-to-noise ratio gap is $\Gamma = 1/128$. Using the path-loss model (3.3), the path-loss exponent is set to $r = 4$ following the approach of [21, 27, 43] and the attenuation parameter is chosen as $\beta = 0.097$. The battery power budgets CAP_k are generated uniformly from $(100N/2, 100N)$. The interference-power limitations P^n are set to the value $\lceil \sum_{k=1}^{K} CAP_k + N \max_n \sigma_k^n \rceil / N$, where $\lceil x \rceil$ denotes the smallest integer that is not smaller than x. This way of choosing P^n ensures that the non-triviality requirement is satisfied. The low-power limitation values UPP^n are randomly generated from $(CAP_k/2N, 2CAP_k/N)$. The terms K, H and N denote the numbers of users, spectrum holes and subcarriers, respectively.

Two stopping criteria for both the FIWFC and the IWFC are specified:

- The maximum number of iterations to run each data set is set to 100. The algorithms stop when the number of iterations exceeds this value.
- The tolerance is set for the relative change[1] between the power vector in the current iteration and the one in the previous iteration. When it meets a prescribed relatively small tolerance, it indicates that the current solution

[1] The relative change is defined as the Euclidean norm of the difference between the solution of the current iteration and the solution of the previous iteration, divided by the norm of the current solution [14].

is unlikely to improve significantly. For these experiments, the tolerance is set equal to 10^{-8}.

The algorithms stop when either one of these two criteria is satisfied.

4.1 Stationary Environment

This section focuses on a realization of the FIWFC itself and a test of its convergence behavior. To do so, we implement the algorithm in an ideal environment. These kind of environments are called stationary environments, where the initial data including the numbers of secondary users and available subcarriers remain unchanged time-wise, and the secondary users do not move from their starting positions.

In Tables 4.1, 4.2, 4.3, and 4.4, the performance of the IWFC I, IWFC II and FIWFC are compared in stationary environment. Table 4.1 shows the total running time of all users for different data sets. From this table, we see that the FIWFC converges significantly faster than either IWFC I or IWFC II for the data sets considered. In Table 4.2, the average running time is generated by taking the average of the running times over all the secondary users. Hence, it shows the running time needed by each of the secondary users on average. This table shows that the average running time of the FIWFC is affected by the number of subcarriers greater than both IWFC I and IWFC II for the data sets considered. Table 4.3 shows the number of iterations needed by the IWFC I, IWFC II and FIWFC. According to the table, the algorithms converged within 100 iterations in all cases. Actually, all the data sets do not satisfy the convergence conditions.

This implies that the convergence conditions can be extended. Furthermore, the number of iterations needed by the FIWFC is a constant for all the data sets considered. This implies that the number of iterations needed by the FIWFC may not affect the computational complexity in the case of interference-power limitation. The number of iterations needed by the three algorithms are comparable. Table 4.4 shows the sum of the data rates generated by the algorithms. According to this table, the sum rates achieved by the IWFC I, IWFC II and FIWFC are comparable as well.

Table 4.1: Running Time (in Seconds) of the IWFC I, IWFC II and FIWFC under interference-power limitations in stationary environments

(K, H, N)	IWFC I	IWFC II	FIWFC
(4, 1, 8)	3.5371	1.1936	0.0102
(10, 1, 8)	13.628	10.0988	0.0158
(10, 1, 16)	16.247	3.8137	0.02
(20, 2, 16)	44.	28.357	0.0569
(20, 3, 24)	58.7124	21.7084	0.1447
(20, 4, 32)	63.395	27.338	0.2104
(20, 5, 40)	71.1525	21.4440	0.2959
(40, 5, 40)	132.1225	87.7320	1.3658

Table 4.2: Average Running Time (in Seconds) of the IWFC I, IWFC II and FIWFC under interference-power limitations in stationary environments

(K, H, N)	IWFC I	IWFC II	FIWFC
(4, 1, 8)	0.8843	0.2984	0.0025
(10, 1, 8)	1.3628	1.01	0.0016
(10, 2, 16)	1.6247	0.3814	0.002
(20, 2, 16)	2.2	1.4179	0.0028
(20, 2, 24)	2.9356	1.0854	0.0072
(20, 4, 32)	3.1697	1.3669	0.0105
(20, 5, 40)	3.5576	1.0722	0.0148
(40, 5, 40)	3.3031	2.1933	0.0341

Table 4.3: # of Iterations of the IWFC I, IWFC II and FIWFC under interference-power limitations in stationary environments

(K, H, N)	IWFC I	IWFC II	FIWFC
(4, 1, 8)	5	7	7
(10, 1, 8)	6	8	7
(10, 1, 16)	5	7	7
(20, 1, 16)	6	9	7
(20, 2, 24)	5	7	7
(20, 4, 32)	5	7	7
(20, 5, 40)	6	7	7
(40, 5, 40)	6	7	7

Table 4.4: Sum Rate of the IWFC I, IWFC II and FIWFC under interference-power limitations in stationary environments

(K, H, N)	IWFC I	IWFC II	FIWFC
(4, 1, 8)	72.2331	72.234	72.234
(10, 1, 8)	96.9711	97.6054	96.9723
(10, 1, 16)	155.3489	154.9514	154.9516
(20, 1, 16)	575.6113	559.1446	575.614
(20, 2, 24)	81.2864	81.1537	81.1537
(20, 4, 32)	252.9279	249.531	249.5284
(20, 5, 40)	417.464	413.1145	413.1776
(40, 5, 40)	580.5752	545.6939	545.6109

Figures 4.1 and 4.2 show the convergence behaviors of the IWFC I, IWFC II and FIWFC for a data set with 4 users and 8 subcarriers. Figure 4.1 plots the data rates achieved by the three algorithms. Starting with low data rates, all users achieved the higher data rates after the first iteration, and all the data rates dropped at the second iteration. After that, the data rates were adjusted slightly so as to reach an equilibrium. Figure 4.2 shows the sum of the data rates of all users, where the sum rates achieved by the three algorithms are the same as they all reach the same equilibrium.

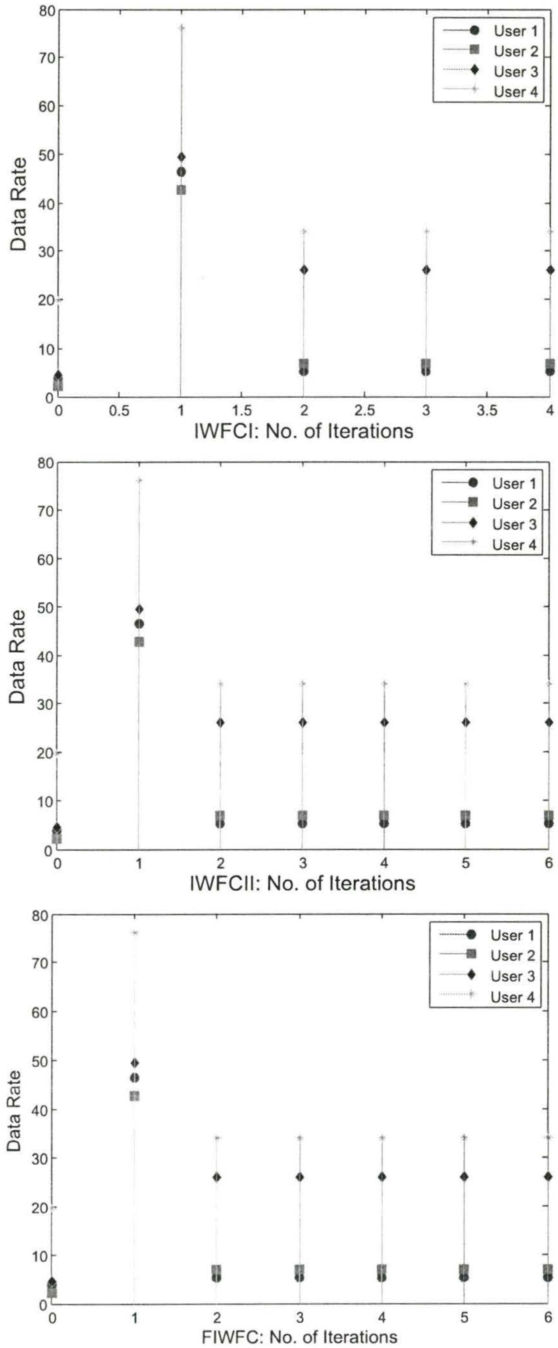


Figure 4.1: The data rates achieved by the IWFC I, IWFC II and FIWFC given 4 users and 8 subcarriers.

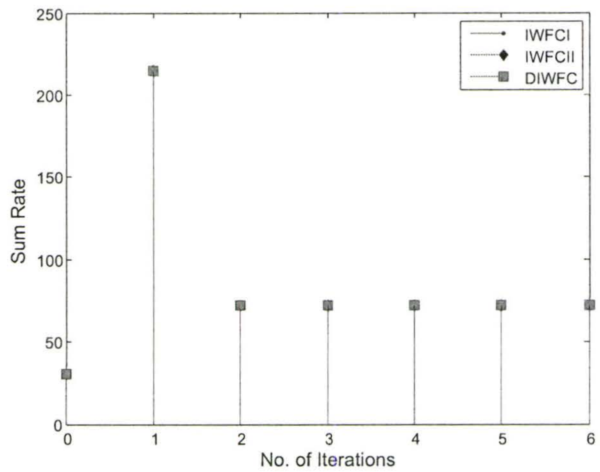


Figure 4.2: The sum of the data rates achieved by the IWFC I, IWFC II and FIWFC given 4 users and 8 subcarriers.

4.2 Nonstationary Environment

In the absence of having access to real-life data (for obvious reasons), a set of realistic scenarios and events is simulated in this section. In this case, all of the secondary users are moving on the grid with different speeds and making turns. Meanwhile, some secondary users come and go, and spectrum holes emerge and disappear with corresponding changes made to the number of subcarriers. This kind of nonstationary environments is more realistic compared to the stationary environments.

As an example, some users are driving along a highway with the speed 108km/h, some are driving on a street with the speed 54km/h, and the rest are walking with the speed 1m/s; On the street, the driving users make a left or right turn every minute, while the walking users may make a random turn every

10 seconds; two new users join at the third iteration, and one spectrum hole having eight subcarriers disappears at the fifth iteration. The changes happen before the algorithms stop. The algorithms start with the same parameter values given in the stationary environments for each data set. Due to these changes with time, the parameters need to be recalculated at each iteration.

Tables 4.5, 4.6, 4.7, and 4.8 show the simulation results generated by the IWFC I, IWFC II and FIWFC in nonstationary environments. Since in nonstatioary environments, secondary users are mobile, the values of the parameters tend to change along the iterations, and they are likely different for the three algorithms at each iteration due to different convergence speeds. Therefore, it is not reasonable to compare the simulation results for the three algorithms in nonstationary environments. Let $\widetilde{K}$, $\widetilde{H}$, and $\widetilde{N}$ denote the ultimate number of users, spectrum holes, and subcarriers after the users join and subcarriers disappear, respectively. Tables 4.5 and 4.6 show the total running time and the average running time for different data sets, respectively. Comparing the running times in stationary environments with those in nonstationary environments, we see that the running time of the FIWFC does not change significantly as a result of the environmental changes, although it is affected by the number of subcarriers, while the running time of both the IWFC I and IWFC II is affected by the environmental change. This is because FIWFC converged so fast that the users did not move too far away from their starting points, and thus the problems are very close to the ones in stationary environments. Table 4.7 shows the number of iterations needed by the IWFC I, IWFC II and the FIWFC in nonstationary environments. From the table, we can see that for the FIWFC, the number of iterations is a constant, while for both the IWFC I and IWFC II, it changes with the size of the data sets. Table 4.8 shows the sum of the data rates. From this table, we see that there are significant differences among the algorithms.

Table 4.5: Running Time (in Seconds) of the IWFC I, IWFC II and FIWFC under interference-power limitations in nonstationary environments

$(\widetilde{K}, \widetilde{H}, \widetilde{N})$	IWFC I	IWFC II	FIWFC
(6, 1, 8)	8.4228	18.2035	0.0305
(12, 1, 8)	24.2668	177.0915	0.0424
(22, 4, 32)	17.3468	30.3175	0.4721
(42, 4, 32)	137.2051	18.0211	1.3488

Table 4.6: Average Running Time (in Seconds) of the IWFC I, IWFC II and FIWFC under interference-power limitations in nonstationary environments

$(\widetilde{K}, \widetilde{H}, \widetilde{N})$	IWFC I	IWFC II	FIWFC
(6, 1, 8)	1.4038	3.0339	0.0051
(12, 1, 8)	2.0222	14.7576	0.0035
(22, 4, 32)	0.7885	1.3781	0.0215
(42, 4, 32)	3.2668	0.4291	0.0321

Table 4.7: # of Iterations of the IWFC I, IWFC II and FIWFC under interference-power limitations in nonstationary environments

$(\widetilde{K}, \widetilde{H}, \widetilde{N})$	IWFC I	IWFC II	FIWFC
(6, 1, 8)	63	23	10
(12, 1, 8)	11	41	10
(22, 4, 32)	25	35	10
(42, 4, 32)	17	32	10

Table 4.8: Sum Rate of the IWFC I, IWFC II and FIWFC under interference-power limitations in nonstationary environments

$(\tilde{K}, \tilde{H}, \tilde{N})$	IWFC I	IWFC II	FIWFC
(6, 1, 8)	108.1487	99.1283	137.7259
(12, 1, 8)	156.7566	308.3914	168.3723
(22, 4, 32)	207	557.5011	389.8124
(42, 4, 32)	859.7974	775.4856	681.4196

Figures 4.3 and 4.4 show the convergence behaviors of the IWFC I, IWFC II and FIWFC in a nonstationary environment given four users and sixteen subcarriers with two new users joining at the third iteration, and eight subcarriers disappearing at the fifth iteration. Figure 4.3 shows the data rates versus the number of iterations. We see that the change in the number of users and available subcarriers affects the distribution of the transmit power among the users. At the third iteration, two new users joined and were allocated resources, which reduced the other users' transmit-power levels. After the fifth iteration, all users' transmit-power levels were brought down significantly as the available resources are reduced. Figure 4.4 shows the sum of the data rates of all users achieved by the IWFC I, IWFC II and FIWFC.

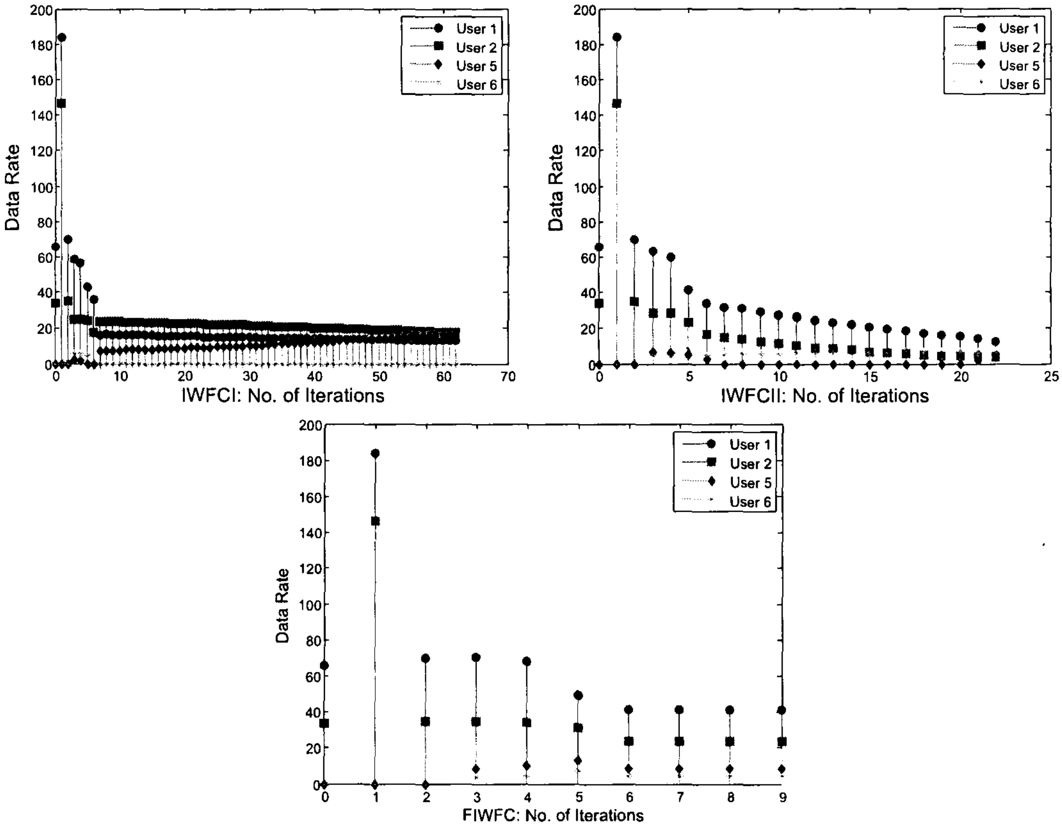


Figure 4.3: The data rates achieved by the IWFC I, IWFC II and FIWFC given an environment of 4 users with 16 subcarriers. Two new users join at the third iteration and eight subcarriers disappear at the fifth iteration.

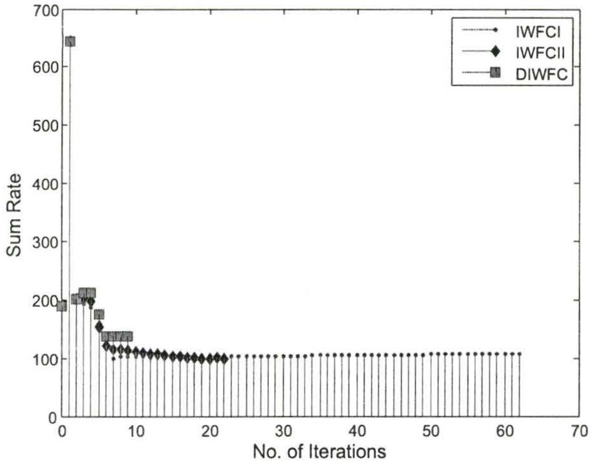


Figure 4.4: The sum rates achieved by the IWFC I, IWFC II FIWFC given an environment of 4 users with 16 subcarriers. Two new users join at the third iteration and eight subcarriers disappear at the fifth iteration.

4.3 Sensitivity and Stability Simulation

This section numerically studies the stability of solutions generated by the FIWFC in the presence of measurement errors. To do so, two key parameters, normalized noise σ and the interference-gain matrix A are perturbed. As an example, a network with 4 users and 8 subcarriers is considered. The movements of the users are simulated in the same way as described in Section 4.2.

The parameters are perturbed within different ranges to examine the behavior of the FIWFC under measurement errors. The average data rates are generated by taking averages of the data rates over all users. The different perturbation ranges are indicated in the legends of the figures. For instance, 20% indicates that the corresponding curves are generated by perturbing the parameters randomly by 20%, i.e., between $(-20\%, 20\%)$. In practice, the interference-plus-noise terms are measured at the receivers at each iteration, and so the measurement errors occur at each iteration as well. Considering this, perturbations to parameters are imposed at each iteration instead of only once at the beginning.

On the one hand, we generate a data set where the sum M of the interference-gain matrix A and the identity matrix $\bar{I}$ is strictly copositive. In this case, the stability conditions are satisfied. Figures (4.5), (4.6), and (4.7) show the stable behaviors of the FIWFC when the interference-gain matrix, the normalized noise, and both of them are perturbed at each iteration, respectively, for this case. The subfigures are plotted as follows:

- Subfigures (a) show the convergence behaviors when the perturbations are imposed within different ranges;
- Subfigures (b) quantify the absolute differences between the different average data rates generated with the perturbation and the average data rate generated with the original parameters.

From subfigures (b), we see that when the perturbation ranges shrink, the average data rates with perturbed parameters approach the average data rate with the original parameters. This indicates that our algorithm achieves a stable GNE for the TPC problem and the stability still holds even when the measurement errors occur at each time instant as the interference-plus-noise term is measured.

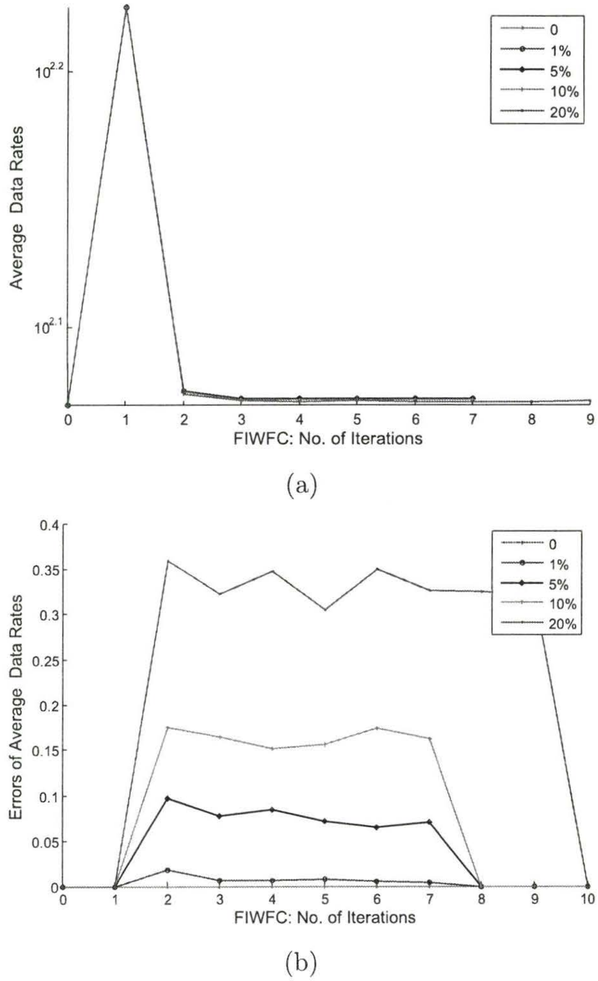
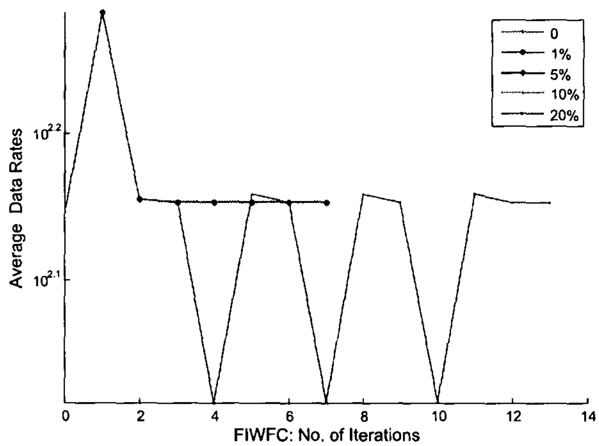
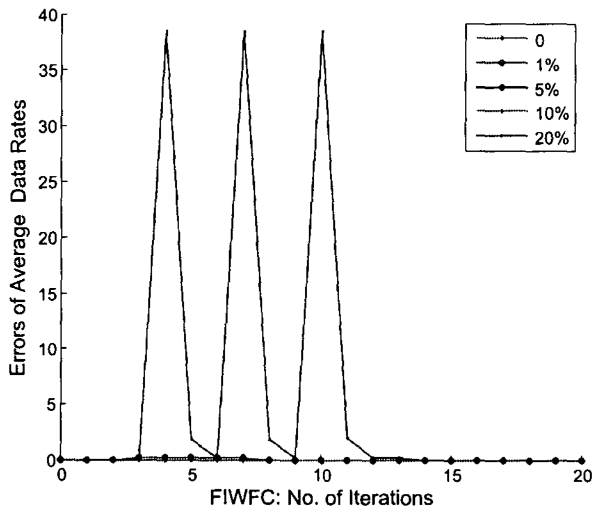


Figure 4.5: (a) The average data rates given perturbed **interference gain matrices** within different ranges. The perturbations are imposed **at each iteration**. (b) The absolute differences between the average data rates for the perturbed and non-perturbed cases versus the number of iterations.

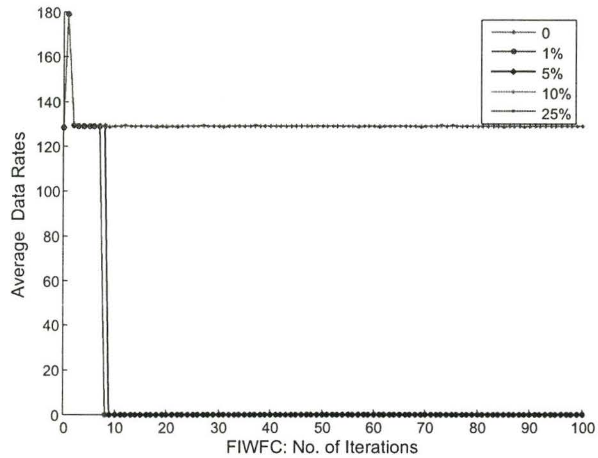


(a)

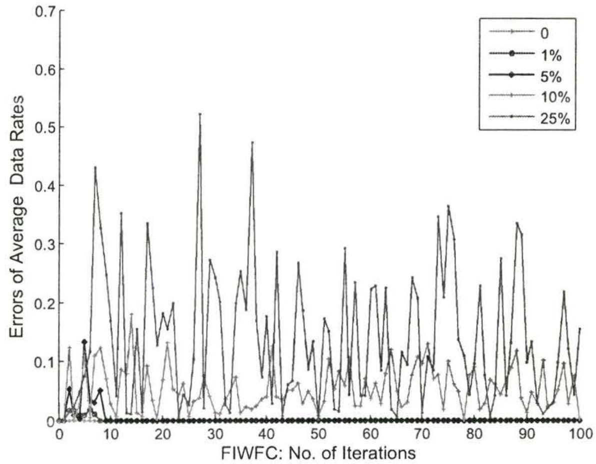


(b)

Figure 4.6: (a) The average data rates with the perturbed **normalized noise** terms within different ranges. The perturbations are imposed **at each iteration**. (b) The absolute differences between the average data rates for the perturbed and non-perturbed cases versus the number of iterations.



(a)



(b)

Figure 4.7: (a) The average data rates given the perturbed **interference-plus-noise** terms within different error ranges. The perturbations are imposed **at each iteration**. (b) The absolute differences between the average data rates for the perturbed and non-perturbed cases versus the number of iterations.

On the other hand, we generate a data set where the matrix M is not strictly copositive. Then the solution stability is not guaranteed. Figure (4.8) shows the error of data rates generated by the FIWFC when the interference-gain matrix is perturbed within different ranges for this case. We see that when the perturbations are imposed, the error curves oscillate along the iterations and could not approach zero, and the algorithms could not converge within 100 iterations, which implies that the solution could be unstable.

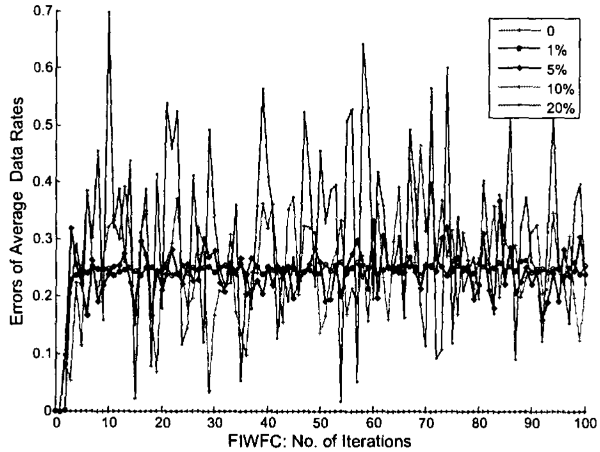


Figure 4.8: The absolute differences between the average data rates for the perturbed and non-perturbed cases versus the number of iterations in the case that the stability conditions are violated.

4.4 Summary

In this chapter, we evaluated the performance of the FIWFC under interference-power limitation from different perspectives:

1. Convergence: The FIWFC converged in both the stationary and nonstationary environments with different sizes of data sets. It also converged when the critical parameters, the interference-gain matrix and normalized noise were perturbed within different ranges. However, when the perturbation range is too large, the algorithm may not converge.

2. Convergence speed: From the simulation results in both the stationary environments, the FIWFC converged significantly faster than the IWFC using the *fmincon* and *mskscopt* functions. The environmental changes did not affect the running time of the FIWFC significantly but they did for the IWFC. The average running time of all the algorithms increased as the number of subcarriers grew. However, the average running time of the FIWFC increased with a faster rate than the IWFC based on the simulation results.

3. Number of iterations: The number of iterations needed by the FIWFC is a constant in both of the stationary and nonstationary environments. However, the number of iterations needed by the IWFC changed significantly with the size of the data sets in the nonstationary environment.

4. Sum rate: The sum rates achieved by the three algorithms are comparable in stationary environments.

5. Sensitivity and stability: In the cases we considered in this chapter, the FIWFC achieved a stable solution when the stability condition was satisfied. However, the solution could be unstable when the condition is violated. In addition, the algorithm may not be able to converge when the stability conditions are violated. As mentioned in Section 3.6.3, this is because the stability condition is weaker than the convergence condition we showed in this thesis. Hence, when the stability condition is not satisfied, the convergence condition is violated.

Chapter 5

Transmit-Power Control Problem for Cognitive Radios under Low-Power Limitation

Chapter 3 focuses on the TPC problem for cognitive radios under the interference-power limitation, in which the interference power at each subcarrier is limited so as to protect the primary users from a prescribed high interference level. This chapter is concerned with protecting the primary users by directly imposing a limitation on the transmit power of each secondary user. The mathematical results developed in the previous chapter are used as the principal tool for developing the fundamental properties and algorithm of this chapter.

5.1 Problem Statement

The TPC problem for cognitive radios operating under a low-power limitation can be formulated as a Nash game, where, instead of jointly maximizing the total data rate, each secondary user maximizes its own data rate individually subject to a given low-power limitation and battery constraint.

5.2 Nash Game Formulation

Suppose that there are K secondary users and N subcarriers. Mathematically, the TPC problem of this network is a Nash game model which consists of K separable nonlinear convex optimization problems. For each user k, the optimization problem is as follows:

$$\begin{aligned} \max_{S_k} \quad & \sum_{n=1}^{N} \log\left(1 + \frac{S_k^n}{I_k^n}\right) \\ \text{s.t.} \quad & \sum_{n=1}^{N} S_k^n \leq CAP_k, \\ & 0 \leq S_k^n \leq UPP^n,\ n = 1, \cdots, N, \end{aligned} \tag{5.1}$$

where UPP^n denotes the low-power limitation over subcarrier n.

For this Nash game model, two situations are considered:

- The first situation is that the battery-power limit for a user is not lower than the total of low-power limitations along the subcarriers, i.e., $CAP_k \geq$

$\sum_{n=1}^{N} UPP^n$. In this case, the battery-power limit constraint

$$\sum_{n=1}^{N} S_k^n \leq CAP_k$$

is redundant. Therefore, for each user k, the model (5.1) reduces to:

$$\begin{array}{ll} \max\limits_{S_k} & \sum\limits_{n=1}^{N} \log\left(1 + \frac{S_k^n}{I_k^n}\right) \\ \text{s.t.} & 0 \leq S_k^n \leq UPP^n,\, n = 1, \cdots, N. \end{array} \tag{5.2}$$

- The second situation is that the prescribed battery-power limit for a user is lower than the total of the low-power limits, i.e., $CAP_k < \sum_{n=1}^{N} UPP^n$. Here, it is necessary to preserve the battery-power limit constraint and the problem is formulated as model (5.1).

In both cases, the problem satisfies Slater's constraint qualification, as the feasible sets are polyhedra. By Theorem 2.3.1, the KKT conditions are necessary and sufficient.

5.3 Theoretical Basis of the Problem

5.3.1 KKT Conditions

For the first situation, the KKT conditions for each user are as follows:

$$\begin{aligned} 0 &\leq S_k^n \perp -\frac{1}{S_k^n + I_k^n} + \lambda_k^n \geq 0 && (5.3) \\ 0 &\leq \lambda_k^n \perp UPP^n - S_k^n \geq 0 && (5.4) \\ & n = 1, \cdots, N. \end{aligned}$$

In fact, it is possible to immediately derive a solution to this model. As $\sigma_k^n > 0$, then the term $\frac{1}{S_k^n + I_k^n}$ must be positive as well. From the inequality on the right-hand side of (5.3), $\lambda_k^n > 0$, for all n. By the complementarity condition in (5.4), we have $S_k^n = UPP^n$, for all n and k. This means that if there is no battery limitation, each user can reach the low-power limitation on each subcarrier.

For the second situation, the KKT conditions for each user in this case are as follows:

$$\begin{aligned} 0 &\leq S_k^n \perp -\frac{1}{S_k^n + I_k^n} + \lambda_k^n + \mu_k \geq 0 && (5.5) \\ 0 &\leq \lambda_k^n \perp UPP^n - S_k^n \geq 0 && (5.6) \\ 0 &\leq \mu_k \perp CAP_k - \sum_{n=1}^{N} S_k^n \geq 0 && (5.7) \\ & n = 1, \cdots, N. \end{aligned}$$

5.3.2 Mixed Linear Complementarity Problem

For the second case, we can also establish an equivalent relation between the KKT conditions and the MLCP. This reduces the nonlinearity of the original problem, which is critical to developing a new, low-complexity algorithm.

Proposition 5.3.1 *Under the condition $CAP_k < \sum_{n=1}^{N} UPP^n$, the set of the KKT conditions of this model is equivalent to the following MLCP:*

$$\begin{aligned} 0 &\leq S_k^n \perp S_k^n + I_k^n + \beta_k^n - \nu_k \geq 0 \\ 0 &\leq \beta_k^n \perp UPP^n - S_k^n \geq 0 \\ 0 &= CAP_k - \sum_{n=1}^{N} S_k^n \\ &\quad \nu_k \textit{ is free} \\ k &= 1, \cdots, K,\ n = 1, \cdots, N. \end{aligned} \tag{5.8}$$

Proof: See APPENDIX E.

5.3.3 Optimality Conditions

Analogous to the result in Subsection 3.4.3, the necessary and sufficient optimality conditions for the MLCP (5.8) are developed in the following proposition:

Proposition 5.3.2 *For each user, if $CAP_k \geq \sum_{n=1}^{N} UPP^n$, then*

$$S_k^n = UPP^n$$

for all n. *Suppose that* $CAP_k < \sum_{n=1}^{N} UPP^n$ *for each* k. *Let*

$$T_k = \{n : S_k^n > 0\}$$

be the index set of subcarriers activated by user k, *and*

$$B_k = \{n : S_k^n = UPP^n > 0, n \in T_k\}$$

be the index set of subcarriers activated by user k, *at which the low-power limitations are reached. Then* (S, ν, β) *solves problem* (5.8) *if, and only if,* (S, ν, β) *satisfies the following conditions, called the arithmetic simplified optimality conditions:*

(1) If $T_k \neq B_k$, *then*

$$\nu_k = (CAP_k + \sum_{n \in T_k/B_k} I_k^n - \sum_{n \in B_k} UPP^n)/(|T_k| - |B_k|)$$

and

$$S_k^n = \begin{cases} 0 & \nu_k - I_k^n \leq 0 \\ \nu_k - I_k^n & 0 < \nu_k - I_k^n \leq UPP^n \\ UPP^n & \nu_k - I_k^n > UPP^n \end{cases}$$

Moreover,

$$\beta_k^n = \begin{cases} 0 & \nu_k - I_k^n \leq 0 \\ 0 & 0 < \nu_k - I_k^n \leq UPP^n \\ \nu_k - I_k^n - UPP^n & \nu_k - I_k^n > UPP^n \end{cases}$$

(2) Otherwise,

$$S_k^n = \begin{cases} UPP^n & n \in B_k \\ 0 & n \notin B_k \end{cases}$$

Proof: See APPENDIX F.

According to Theorem 2.3.1, and Propositions 5.3.1 and 5.3.2, the necessary and sufficient optimality theorem is ready to be stated in the following corollary.

Corollary 5.3.3 *Suppose that the convexity assumption holds. Then, S is a NE of the TPC problem* (5.1) *if, and only if, there exist ν and β such that (S, ν, β) satisfies the arithmetic simplified optimality conditions in Proposition 5.3.2.*

5.4 Fast Iterative Water-Filling Algorithm

5.4.1 Algorithm

This subsection introduces the FIWFC for finding a NE of the TPC problem under the low-power limitation. Enlightened by Proposition 5.3.2, the idea of this new algorithm is to update the transmit-power vector iteratively according to the interference formed by the power vector of the previous iteration. Taking advantage of the information content of the Lagrange multipliers, the power vector of the TPC problem is updated iteratively.

To describe the FIWFC in detail, some notation is needed. Let the superscript l indicate the lth iteration, and Ω_k be the strategy set of user k at the lth iteration.

For $n = 1, \cdots, N$, let

$$I_k^{n(l)} := \sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n S_j^{n(l)}$$

be the interference-plus-noise at the current iteration;

$$T_k^{(l)} := \{n : S_k^{n(l)} > 0\},$$

be the index set of the subcarriers utilized by user k; and

$$B_k^{(l)} := \{n : S_k^{n(l)} = UPP^n, n \in T_k^{(l)}\},$$

be the index set of subcarriers where user k reaches the low-power limit. The FIWFC considers two situations:

1. For those users with $CAP_k \geq \sum_{n=1}^{N} UPP^n$, set $S_k^{n(l)} = UPP^n$ for all n;
2. For those users with $CAP_k < \sum_{n=1}^{N} UPP^n$, the iterative procedure is performed. For such users, form $\nu_k^{(l)}$ and update the transmit-power vector $S_k^{(l+1)}$ based on the information extracted from the current iteration l. In fact, $\nu_k^{(l)}$ plays an important role in the updating scheme of the transmit-power vector, which serves as the water-filling level threshold in the updating scheme. When the interference $I_k^{n(l)}$ is higher than the threshold $\nu_k^{(l)}$, set

$$S_k^{(l+1)} = 0;$$

When the positive difference $\nu_k^{(l)} - I_k^{n(l)}$ is higher than the low-power capacity UPP^n, set

$$S_k^{(l+1)} = UPP^n;$$

otherwise, set it to the positive difference,

$$S_k^{(l+1)} = \nu_k^{(l)} - I_k^{n(l)}.$$

In this way, the updated transmit-power vector $S_k^{(l+1)}$, as well as the Lagrange multipliers $\nu_k^{(l)}$ and $\beta_k^{n(l)}$, satisfy the conditions (5.8) with $I_k^{n(l)}$ formed by $S_k^{(l)}$.

For the sake of illustration, the basic idea of this new updating scheme is shown in Figure 5.1 for the case when $CAP_k < \sum_{n=1}^{N} UPP^n$. This picture considers the case with three subcarriers for user k:

- In subcarrier 1, the low-power limitation UPP^1 is lower than the difference $\nu_k - I_k^1$; therefore, the transmit-power for user k over this subcarrier is set to UPP^1;

- In subcarrier 2, the low-power limitation UPP^2 is higher than the difference $\nu_k - I_k^2$; therefore, the transmit power for user k over this subcarrier is set to $\nu_k - I_k^2$;

- In subcarrier 3, with interference I_k^3 being higher than the threshold, the transmit-power S_k^3 is set to zero.

The FIWFC for cognitive radios under low-power limitation is described in Algorithm 5.4.1.

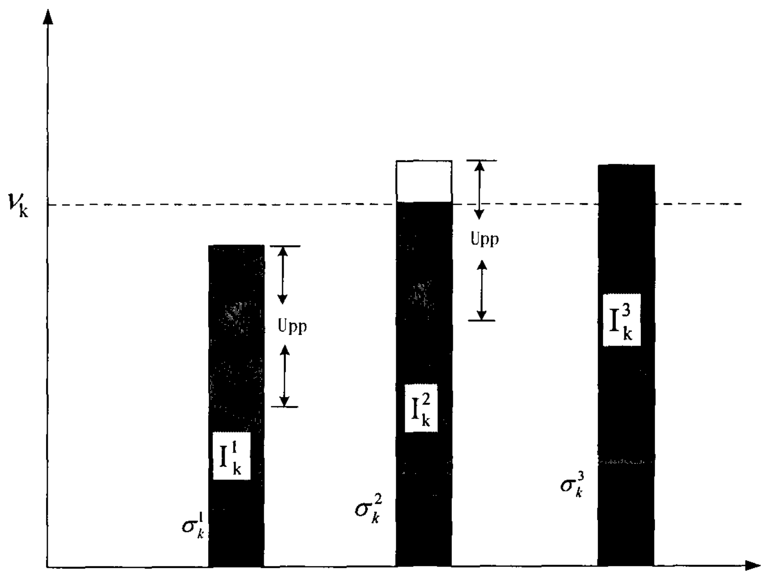


Figure 5.1: Illustrating the basic updating scheme of FIWFC for cognitive radios under low-power Limitation

Algorithm 5.4.1 FIWFC for Cognitive Radios under Low-Power Limitation

Let $l = 0$, $i = 0$, and $S_k^{(0)}$ be any feasible power vector
for $k = 1$ to K **do**
  if $CAP_k \geq \sum_{i=1}^{N} UPP^n$ **then**
    for $n = 1$ to N **do**
      Set $S_k^n = UPP^n$
    end for
  else
    Set $i = i + 1$
    Set $m(i) = k$
  end if
end for
if $i = 0$ **then**
  Stop
end if
repeat
  for $k = m(1)$ to $m(i)$ **do**
    if $T_k^{(l)} \neq B_k^{(l)}$ **then**

$$\nu_k^{(l)} = \frac{CAP_k + \sum_{n \in T_k^{(l)}/B_k^{(l)}} I_k^{n(l)} - \sum_{n \in B_k^{(l)}} UPP^n}{|T_k^{(l)}| - |B_k^{(l)}|}$$

      for $n = 1$ to N **do**

$$S_k^{n(l+1)} = \begin{cases} 0 & \text{if} \nu_k^{(l)} - I_k^{n(l)} \leq 0 \\ \nu_k^{(l)} - I_k^{n(l)} & \text{if } 0 < \nu_k^{(l)} - I_k^{n(l)} \\ & \leq UPP^n \\ UPP^n & \text{otherwise} \end{cases}$$

      end for
    else
      for $n = 1$ to N **do**

$$S_k^{n(l+1)} = \begin{cases} UPP^n & n \in T_k^{(l)} \\ 0 & \text{otherwise} \end{cases}$$

      end for
    end if
  end for
  $l \leftarrow l + 1$
until A stopping criterion is satisfied

5.4.2 Convergence and Uniqueness

Similar to the results developed for the FIWFC under the interference-power limitation as described in Subsection 3.5.2, the convergence and uniqueness results for low-power limitation are derived in the following theorems.

Theorem 5.4.1 *Suppose the TPC problem for cognitive radios under low-power limitation satisfies the convexity assumption. If one of the following conditions is satisfied,*

- $\rho(A^\top A) < 1$, *or*
- $\rho(C^\top C) < 1$, *or*
- $Tr(A^\top A) < 1$, *or*
- $Tr(C^\top C) < 1$,

then the FIWFC under a low-power limitation converges both globally and linearly to a solution of the TPC problem (5.1).

Proof: See APPENDIX G.

Theorem 5.4.2 *With the same conditions in Theorem 5.4.1, TPC problem (5.1) has a unique solution.*

Proof: See APPENDIX H.

5.5 Computational Complexity

The computational complexity of this new algorithm is the same as the FIWFC introduced in Chapter 3. It is justified as follows.

As with the first FIWFC, this new algorithm is formulated iteratively as well. After initialization, the power allocation for the K users is iteratively adjusted within the inner loop. For each user, two situations may be applied:

- When $CAP_k \geq \sum_{n=1}^{N} UPP^n$, the transmit power is set to the low-power limitation. Hence, the complexity of power allocation for this user is $O(N)$.
- When $CAP_k < \sum_{n=1}^{N} UPP^n$, T_k and B_k is checked once. Checking T_k takes $O(N)$ operations. Since, in practice, the interference-plus-noise I_k^n is measured at the receiver, this term does not affect the computational burden. Consequently, checking B_k is an $O(N)$ procedure as well. To evaluate ν_k, the additions of the interference-plus-noise terms need $O(N)$ time, and so do those of the low-power limitations. Hence, the total time devoted to evaluate ν_k is $O(N)$. Furthermore, evaluation of S_k^n is only $O(1)$, and so the total time spent in evaluating $S_k = (S_k^1, S_k^2, \cdots, S_k^N)$ is also $O(N)$.

Thus, in either situation, the operating time for power allocation for each user is $O(N)$. Since K users are involved, the total time for the inner loop is $O(KN)$, which is linear in both the number of users K and the number of subcarriers N, respectively. Let M_1 be the number of iterations needed in the FIWFA. The total computational complexity of the FIWFA is $O(M_1KN)$. Based on the simulation results, M_1 does not change significantly with a change in the size of the data sets. It seems that there is no direct relationship between M_1 and the sizes of data sets.

5.6 Solution Stability of the Nash Game Model under Low-Power Limitation

This section studies stability of a NE for the TPC problem (5.1) under the perturbation of network parameters.

According to Proposition 1.2.1 in [13], the equivalent AVI formulation for the MLCP (5.8) is established as follows:

$$(Z - S)^T F(S) \geq 0, \forall Z \in \Omega, \tag{5.9}$$

where

$$F := \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_K \end{pmatrix}, \; F_k := \begin{pmatrix} S_k^1 + I_k^1 \\ S_k^2 + I_k^2 \\ \vdots \\ S_k^N + I_k^N \end{pmatrix},$$

and

$$\Omega = \left\{ S \in R^{KN} : \begin{array}{l} S_k^n \leq UPP^n, \\ \sum_{n=1}^N S_k^n = CAP_k, \\ S_k^n \geq 0, \\ n = 1, \cdots, N, \; k = 1, \cdots, K \end{array} \right\}.$$

In order to put the function F and the functions in the set Ω into matrix form, define a matrix M as $M = A + \bar{I}$, where A is defined in Subsection 3.5.2 and $\bar{I}$ is the identity matrix (or $M = C + \bar{I}$ depending on the way that the power vector S is defined, where C is also defined in Subsection 3.5.2). Then, the function F and the solution set can be represented as $F = MS + \sigma$ and

$$\Omega = \left\{ S \in R^{KN} : \begin{array}{c} S \leq UPP, \\ HS = CAP, \\ S \geq 0, \end{array} \right\},$$

respectively, where $H = (H_1, H_2, \cdots, H_K)$, and H_k is a $K \times NK$ matrix where the kth row is the all-ones vector and the other rows are all zero vectors.

For the TPC in cognitive radio, the measurement errors occur in the sensed interference-plus-noise terms, which implies that the interference-gain matrix and normalized noise are the key parameters for stability analysis. For those users with a battery-power limit higher than the total low-power limits along the subcarriers, i.e. $CAP_k \geq \sum_{n=1}^{N} UPP^n$, the transmit-power levels are fixed as the low-power limitations, $S_k^n = UPP^n$, and the solutions are obviously stable. Hence, the parametric AVI for the TPC problem are considered for those users with low battery-power limits, i.e. $CAP_k < \sum_{n=1}^{N} UPP^n$ as follows:

$$(Z - S)^T F(S, p_1, p_2) \geq 0, \ \forall Z \in \Omega, \tag{5.10}$$
$$p = (p_1, p_2), \ p_1, p_2 \in R^{NK \times NK}$$

where $F(S, p_1, p_2) = (M + p_1 M)S + \sigma + \sigma p_2$. Here, M is not necessarily symmetric.

For the parametric AVI (5.10), the Lipschitz conditions (3.15) and (3.16) in

Corollary 3.6.4 are naturally satisfied. As a result, the stability theorem for AVI to the TPC problem (5.1) can be applied as follows:

Corollary 5.6.1 *When the matrix M is strictly copositive and the MFCQ holds at a NE of the TPC problem (5.1), the NE will be stable.*

The stability conditions are the same for the TPC problem under interference-power limitation in subsection 3.6.3. As discussed in subsection 3.6.4, the matrix M may be strictly copositive when each receiver has a relatively shorter distance from its own transmitter, compared to the distances from other activated transmitters in the network. This practical requirement could be satisfied using an ad hoc network.

5.7 Summary

In this chapter, the TPC problem under low-power limitation was cast as a Nash game model. The solution to this problem is also guaranteed by the existence theorem presented in Chapter 2. The FIWFC was also introduced for solving the TPC problem under low-power limitation. The computational complexity is the same as that of the FIWFC under the interference-power limitation introduced in Chapter 3. The conditions for the convergence of the FIWFC, the uniqueness of a solution and the solution stability were given as well, and are the same as in the case of interference-power limitation.

Chapter 6

Simulation Results of the FIWFC for the TPC under Low-Power Limitation

In this chapter, simulation results are presented to evaluate the performance of the FIWFC operating under low-power limitation. The convergence behaviors of the FIWFC, the IWFC I and IWFC II are compared for both stationary and nonstationary environments. In the last section, the solution stability is numerically studied by imposing perturbations to the critical parameters including the interference-gain matrix and the normalized noise. In this chapter, the same database and stopping criteria were used as in Chapter 4.

6.1 Stationary Environment

In this section, stationary environments are considered for the purpose of realizing the FIWFC as in Section 4.1.

In Tables 6.1, 6.2, 6.3, and 6.3, the simulation results generated by the IWFC I, IWFC II and FIWFC are compared. Tables 6.1 and 6.2 show the total running times and the average running times in stationary environments, respectively. From these tables, we see that the FIWFC converges significantly faster than either the IWFC I or IWFC II. However, the average running time of the FIWFC is affected greatly by the number of subcarriers for the data sets considered. Table 6.3 shows the numbers of iterations needed for convergence by the IWFC I, IWFC II and the FIWFC. According to the table, the algorithms converged within 100 iterations in all cases. Actually, none of the data sets satisfied the convergence conditions. This also implied that the convergence conditions can be extended. Furthermore, the number of iterations needed by FIWFC is a constant for all the data sets considered. This implies that the number of iterations needed by the FIWFC may not affect the computational complexity in the case of low-power limitation. The numbers of iterations needed by the three algorithms are comparable. Table 6.4 shows the sum data rates achieved by the algorithms. According to the table, the sum rates achieved by the IWFC I, IWFC II and FIWFC are comparable, except for the data set with 10 users and 16 subcarriers, where the IWFC II achieved a data rate about 20% more than that achieved by the IWFC I and FIWFC.

Table 6.1: Running Time (in Seconds) of the IWFC I, IWFC II and FIWFC under low-power limitations in stationary environments

(K, H, N)	IWFC I	IWFC II	FIWFC
(4, 1, 8)	6.0207	6.5434	0.011
(10, 1, 8)	17.0673	9.8206	0.0137
(10, 2, 16)	19.9753	4.6417	0.0241
(20, 2, 16)	42.7408	17.5715	0.0572
(20, 5, 40)	65.2201	18.337	0.3017
(40, 5, 40)	133.3974	38.943	1.3343

Table 6.2: Average Running Time (in Seconds) of the IWFC I, IWFC II and FIWFC under low-power limitations in stationary environments

(K, H, N)	IWFC I	IWFC II	FIWFC
(4, 1, 8)	1.5052	1.6360	0.0027
(10, 1, 8)	1.7067	0.981	0.0014
(10, 2, 16)	1.998	0.4642	0.0024
(20, 2, 16)	2.1370	0.8786	0.0029
(20, 5, 40)	3.2610	0.9168	0.0151
(40, 5, 40)	3.3349	0.9736	0.0334

Table 6.3: # of Iterations of the IWFC I, IWFC II and FIWFC under low-power limitations in stationary environments

(K, H, N)	IWFC I	IWFC II	FIWFC
(4, 1, 8)	5	7	7
(10, 1, 8)	5	7	7
(10, 2, 16)	5	8	7
(20, 2, 16)	5	7	7
(20, 5, 40)	6	7	7
(40, 5, 40)	6	7	7

Table 6.4: Sum Rate of the IWFC I, IWFC II and FIWFC under low-power limitations in stationary environments

(K, H, N)	IWFC I	IWFC II	FIWFC
(4, 1, 8)	37.0586	37.0587	37.0587
(10, 1, 8)	135.0205	135.0205	135.0205
(10, 2, 16)	202.4697	239.3432	202.4704
(20, 2, 16)	257.9637	257.9642	257.9642
(20, 5, 40)	417.464	413.1145	413.1776
(40, 5, 40)	580.5752	545.6109	545.6939

Figures 6.1 and 6.2 show the convergence behaviors of the IWFC I, IWFC II and FIWFC for a data set with 4 users and 8 subcarriers. Figure 6.1 plots the data rates achieved by the IWFC I, IWFC II and FIWFC. From the figure, we see that the data rates only changed slightly after the second iteration. Figure 6.2 shows the sum of the data rates of all users achieved by the IWFC I, IWFC II and FIWFC. In this case, the sum rates achieved by three algorithms are the same as they reached the same equilibrium.

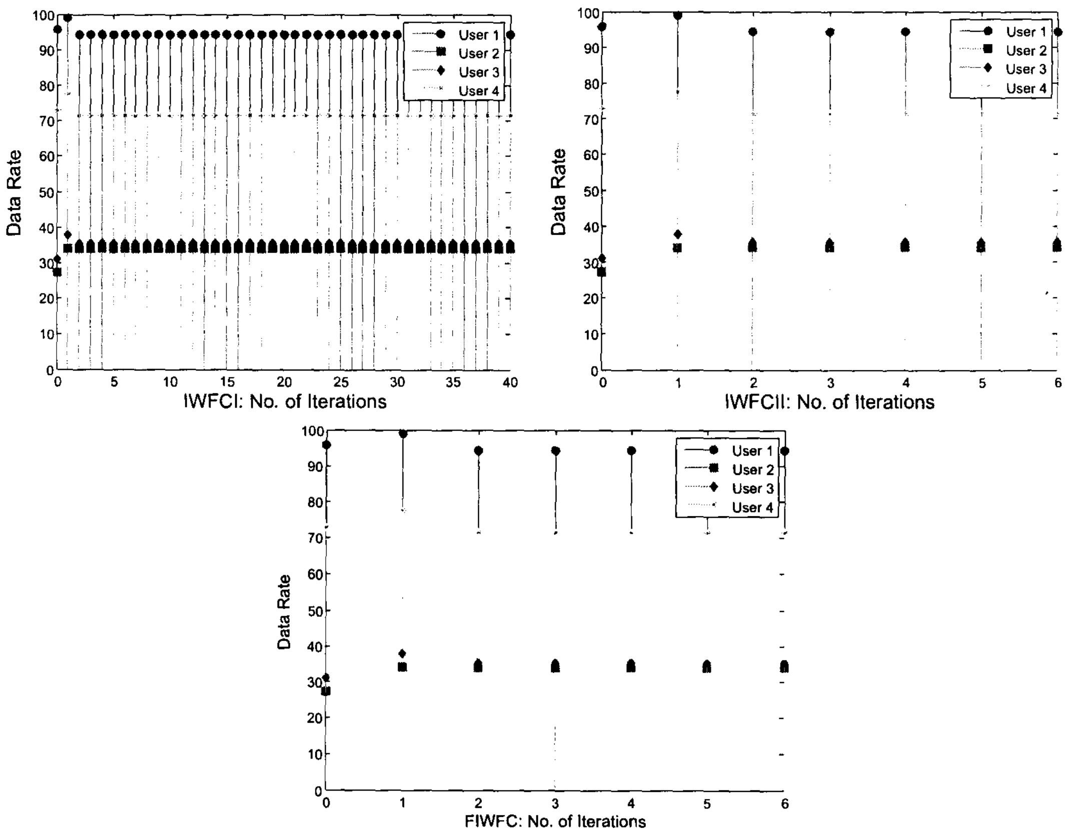


Figure 6.1: The data rates achieved by the IWFC I, IWFC II and FIWFC given 4 users and 8 subcarriers.

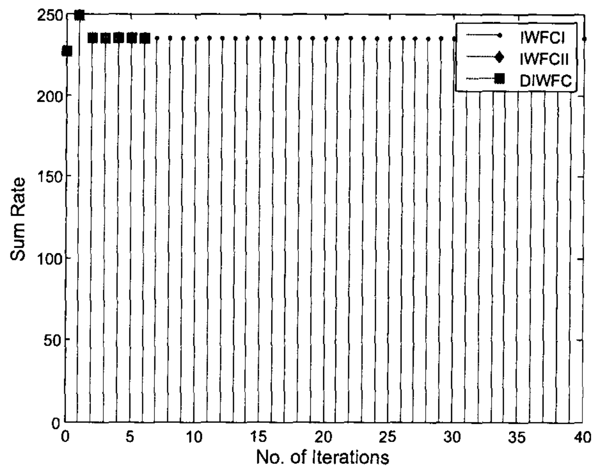


Figure 6.2: The sum of the data rates achieved by the IWFC I, IWFC II and FIWFC given 4 users and 8 subcarriers.

6.2 Nonstationary Environment

This section reports the simulation results obtained for the case of nonstationary environments. The same database of nonstationary environments is considered as in Section 4.2.

Tables 6.5, 6.6, 6.7, and 6.8 show the simulation results generated by the IWFC I, IWFC II and FIWFC in nonstationary environments. Since in nonstatioary environments, secondary users are mobile, the values of the parameters tend to change along the iterations, and they are likely different for the three algorithms at each iteration due to different convergence speeds. Therefore, it is not reasonable to compare the simulation results for the three algorithms in nonstationary environments. Tables 6.5 and 6.6 show the total running times and the average running times for different data sets, respectively. As stated in

Section 4.2, we also see that by comparing the running times of these algorithms in both stationary environments and nonstationary environments, the running time of the FIWFC is not significantly affected by the environmental changes, although it does now change with the size of cognitive radio networks, while the running time of both the IWFC I and IWFC II is affected by the environmental change. Table 6.7 shows the number of iterations needed by the IWFC and the FIWFC in various nonstationary environments. From this table, it is clear that the number of iterations needed by the FIWFC is not affected by the environmental changes, and the IWFC I could not converge within 100 iterations. Table 6.8 shows that the sum of data rates generated by the IWFC I, IWFC II and FIWFC. There are also significant differences among the algorithms.

Table 6.5: Running Time (in Seconds) of the IWFC I, IWFC II and FIWFC under low-power limitations in nonstationary environments

$(\widetilde{K}, \widetilde{H}, \widetilde{N})$	IWFC I	IWFC II	FIWFC
(6, 1, 8)	10.191	34.3336	0.0258
(12, 1, 8)	9.724	1.427837	0.0339
(22, 4, 32)	26.7573	23.7074	0.425392
(42, 4, 32)	52.7802	41.269522	1.2975

Table 6.6: Average Running Time (in Seconds) of the IWFC I, IWFC II and FIWFC under low-power limitations in nonstationary environments

$(\tilde{K}, \tilde{H}, \tilde{N})$	IWFC I	IWFC II	FIWFC
(6, 1, 8)	1.6985	5.7223	0.0043
(12, 1, 8)	0.8103	0.1190	0.0028
(22, 4, 32)	1.2162	1.0776	0.0193
(42, 4, 32)	1.2567	0.9826	0.0309

Table 6.7: # of Iterations of the IWFC I, IWFC II and FIWFC under low-power limitations in nonstationary environments

$(\tilde{K}, \tilde{H}, \tilde{N})$	IWFC I	IWFC II	FIWFC
(6, 1, 8)	100	17	10
(12, 1, 8)	100	9	10
(22, 4, 32)	100	65	10
(42, 4, 32)	100	51	10

Table 6.8: Sum Rate of the IWFC I, IWFC II and FIWFC under low-power limitations in nonstationary environments

$(\tilde{K}, \tilde{H}, \tilde{N})$	IWFC I	IWFC II	FIWFC
(6, 1, 8)	91.0691	92.4289	110.2813
(12, 1, 8)	32.3824	94.5366	102.9132
(22, 4, 32)	520.6649	432.0761	311.6943
(42, 4, 32)	526.7673	757.1632	742.6092

Figures 6.3 and 6.4 show the convergence behaviors of the IWFC I, IWFC II and FIWFC in a nonstationary environment given four users and sixteen subcarriers, with two new users joining at the third iteration, and eight subcarriers

disappearing at the fifth iteration. Figure 6.3 shows the data rates versus the number of iterations. We see that the change in the number of users and available subcarriers affects the distribution of the transmit power among the users. At the third iteration, two new users joined the network and they were allocated resources, which reduced some other users' transmit-power levels. After the fifth iteration, all users' transmit-power levels were brought down significantly as some subcarriers disappeared. Figure 6.4 shows the sum of the data rates of all users achieved by the IWFC I, IWFC II and FIWFC in the case. From Figures 6.3 and 6.4, we see that the algorithms can still converge to an equilibrium at the end, although the environment changes happen before the algorithms stop.

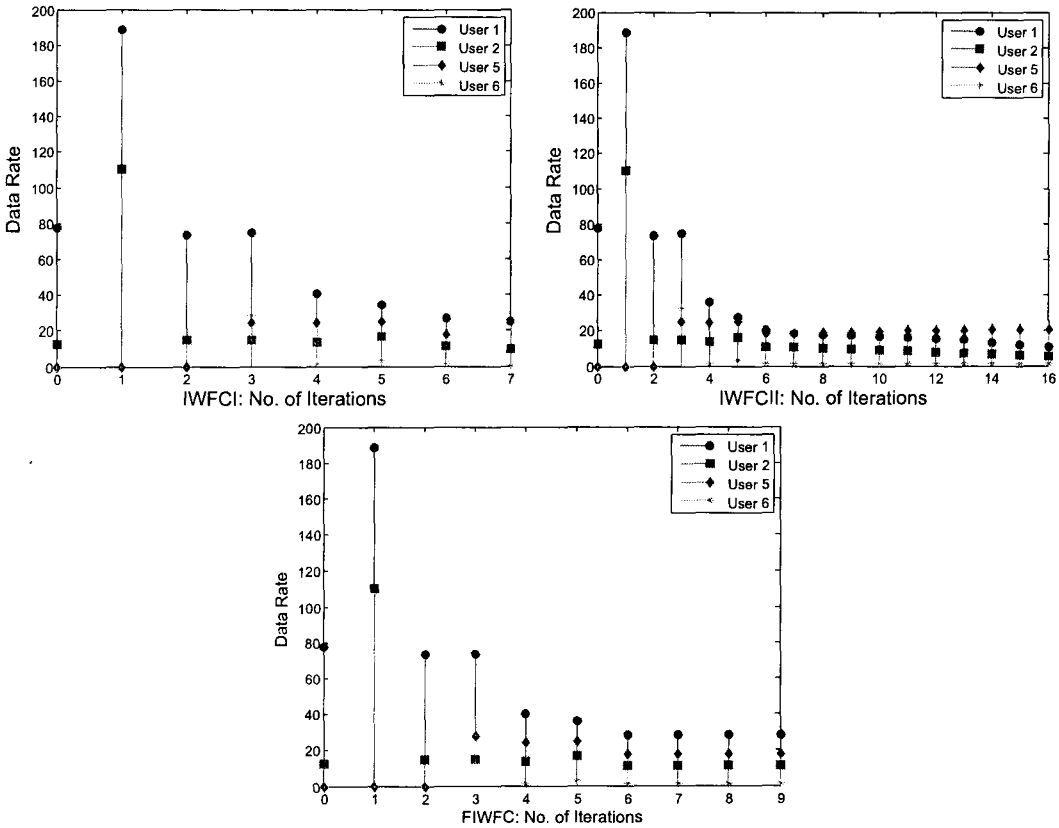


Figure 6.3: The data rates achieved by the IWFC I, IWFC II and FIWFC given an environment of 4 users with 16 subcarriers. Two new users join at the third iteration and eight subcarriers disappear at the fifth iteration.

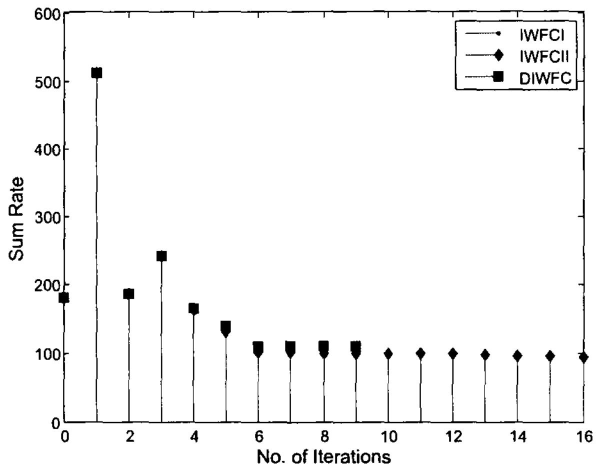


Figure 6.4: The sum rates achieved by the IWFC I, IWFC II and FIWFC given an environment of 4 users with 16 subcarriers. Two new users join at the third iteration and eight subcarriers disappear at the fifth iteration.

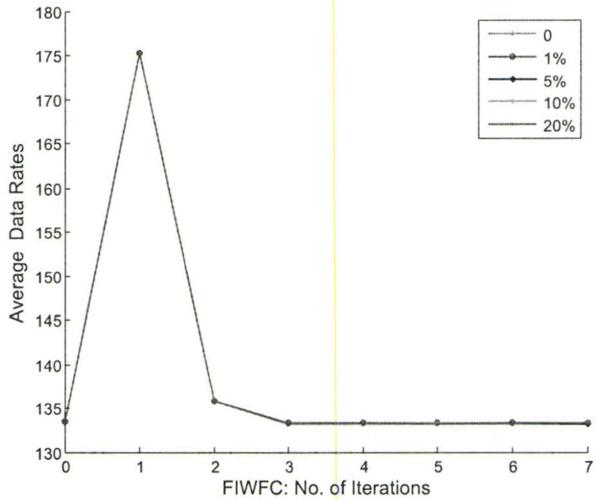
6.3 Sensitivity and Stability Simulation

This section demonstrates the convergence behavior of the FIWFC in the presence of measurement errors. For this investigation, two key parameters, the normalized noise σ, and the interference-gain matrix A, are perturbed randomly. As in Section 4.3, a cognitive radio network with 4 users and 8 subcarriers is considered. Instead of only once at the beginning, perturbations to parameters are imposed at each iteration.

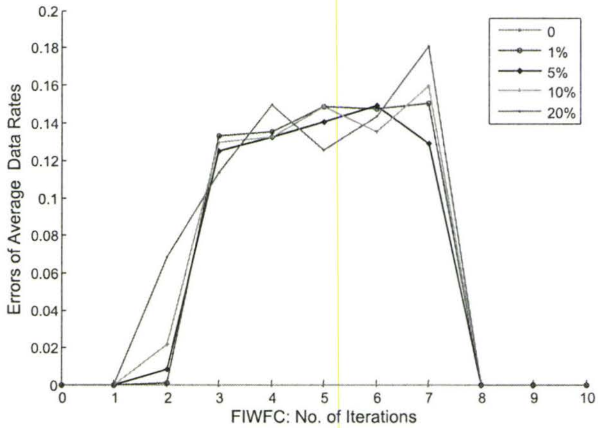
On the one hand, we generate a data set where the sum M of the interference-gain matrix A and the identity matrix $\bar{I}$ is strictly copositive. In this case, the stability conditions are satisfied. Figures 6.5, 6.6, and 6.7 show the stable behaviors of the FIWFC for the TPC, when the interference-gain matrix, the normalized noise, or both are perturbed at each iteration, respectively. The subfigures of each plot are presented as follows:

- Subfigures (a) show the average data rates generated by the FIWFC when the perturbations are imposed within different ranges;
- Subfigures (b) quantify the absolute differences between the different average data rates generated under the perturbations and the average data rate generated with the original parameters.

In the latter set of subfigures, we see that as the range of perturbation decreases, the average data rates under the perturbations approach the average data rates for the case of perturbation-free. This indicates that although the algorithm may not converge when the perturbation ranges are relatively large, it achieves a stable NE for the TPC problem under the low-power limitation and that the stability still holds even when measurement errors occur at each time instant as the interference-plus-noise terms are measured.



(a)



(b)

Figure 6.5: (a) The average data rates given perturbed **interference gain matrices** within different ranges. As before, the perturbations are imposed **at each iteration**. (b) The absolute differences between the average data rates for the perturbed and non-perturbed cases versus the number of iterations.

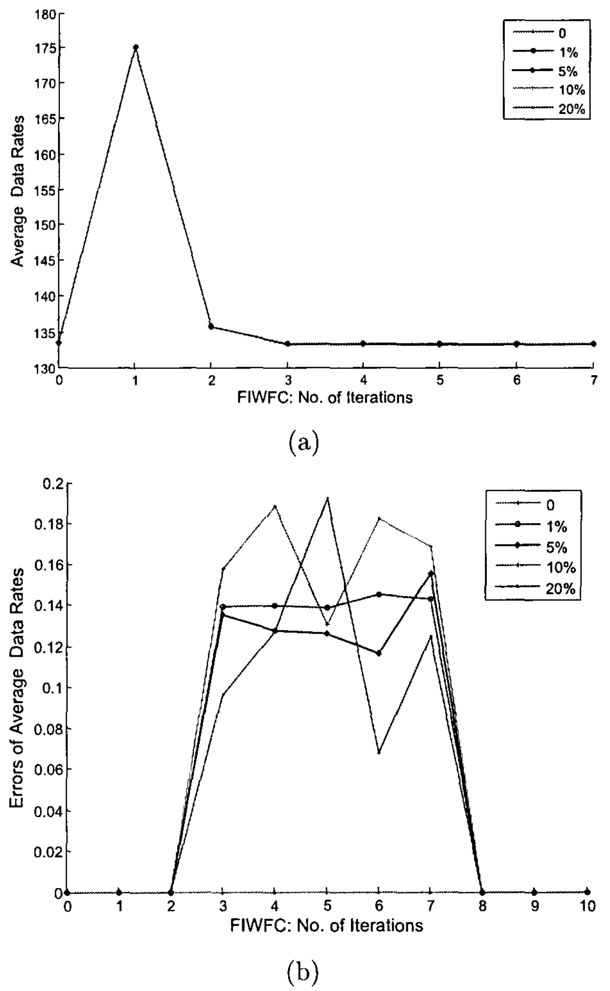
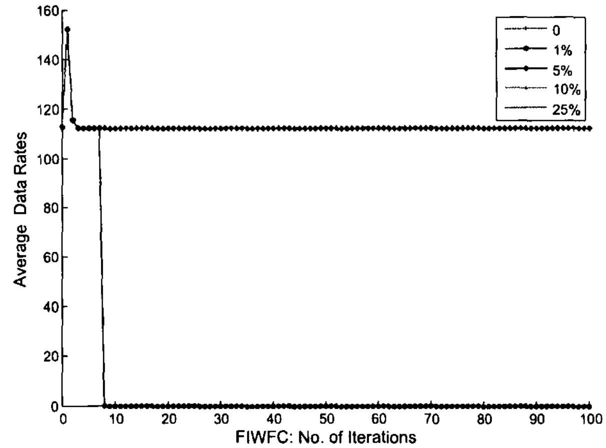
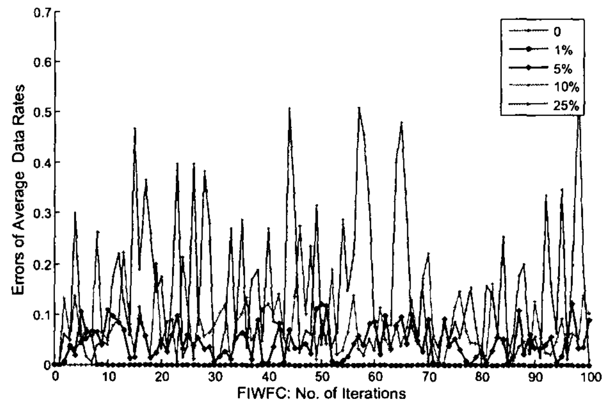


Figure 6.6: (a) The average data rates with the perturbed **normalized noise** terms within different ranges. The perturbations are imposed **at each iteration.** (b) The absolute differences between the average data rates for the perturbed and non-perturbed cases versus the number of iterations.



(a)



(b)

Figure 6.7: (a) The average data rates with the perturbed **interference-plus-noise** terms within different ranges. The perturbations are imposed **at each iteration**. (b) The absolute differences between the average data rates for the perturbed and non-perturbed cases versus the number of iterations.

On the other hand, we generate a data set where the matrix M is not strictly copositive. Then the solution stability is not guaranteed. Figure (6.8) shows the error of data rates generated by the FIWFC when the interference-gain matrix are perturbed within different ranges for this case. We see that when the perturbations are imposed, the algorithms could not converge within 100 iterations. Moreover, the error curves are oscillating along the iterations and could not approach zero, which implies that the solution is unstable.

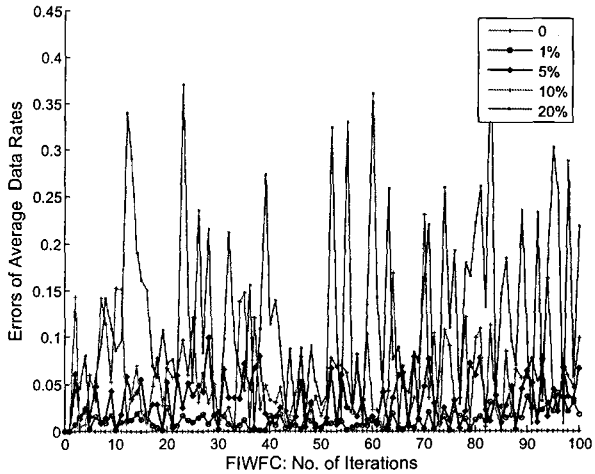


Figure 6.8: The absolute differences between the average data rates for the perturbed and non-perturbed cases versus the number of iterations in the case that the stability conditions are violated.

6.4 Summary

In this chapter, we evaluated the performance of the FIWFC under low-power limitation from different perspectives. We came to the same conclusions as we made for the simulation results for the case of interference-power limitation in Chapter 4:

1. Convergence: The FIWFC converged in both the stationary and nonstationary environments with different sizes of data sets. It also converged when the critical parameters, the interference-gain matrix and normalized

noise were perturbed within different ranges. However, when the perturbation range is too large, the algorithm may not converge.

2. Convergence speed: From the simulation results in the stationary environments, the FIWFC converged significantly faster than the IWFC using the *fmincon* and *mskscopt* functions. The environmental changes did not affect the running time of the FIWFC significantly but they did for the IWFC. The average running times of all the algorithms increased as the number of subcarriers grew. However, the average running time of the FIWFC increased with a faster rate than the IWFC.

3. Number of iterations: The number of iterations needed by the FIWFC is a constant in both of the stationary and nonstationary environments. However, the number of iterations needed by the IWFC changed significantly with the size of the data sets.

4. Sum rate: The sum rates achieved by the three algorithms are comparable in stationary environments except for one data set with 10 users and 16 subcarriers.

5. Sensitivity and Stability: In the cases we considered in this chapter, the FIWFC achieved a stable solution when the stability conditions were satisfied. However, the solution could be unstable when the condition is violated. In addition, the algorithm may not be able to converge when the stability conditions are violated.

Chapter 7

Contribution to the Literature on Cognitive Radio

7.1 Concluding Remarks

In order to improve utilization of the radio spectrum, this thesis has studied both the theory and algorithms for the transmit-power control (TPC) problem on cognitive radio networks. Following other work in this field, and owing to the distributed nature of the problem, the approach taken was based on game-theoretic principles. The TPC problem was considered under two different limitations.

First, the TPC problem for cognitive radios under interference-power limitations was formulated as a generalized Nash game model, which aims at maximizing the data rate of each user, subject to the constraints of the battery power budgets of each user and interference-power limitation at each subcarrier.

Second, the TPC problem for cognitive radios under low-power limitations was formulated as a Nash game model, which aims at maximizing the data rate of each user, subject to the constraints of the battery power budgets of each user and low-power limitations at each subcarrier.

The solid mathematical results presented herein in order to characterize the right threshold are critical in the development of the fast iterative water-filling scheme. Sufficient conditions for the uniqueness and stability of a solution of the TPC problem were derived for both cases. Novel, simple and fast algorithms, called the FIWFCs, for finding a solution of the TPC problem were derived for the two cases. With the FIWFCs in place, we may now summarize its attributes:

(1) The sufficient condition of linear and global convergence is derived for the FIWFC. Based on the simulation results, the algorithm converges in all the cases considered even when the condition is not satisfied, which implies that the convergence condition may be extended.

(2) The FIWFC accommodates operations in a nonstationary radio environment. Although the convergence theorem only gives the conditions for the convergence of the FIWFC in stationary environments, through simulation, we found that the algorithm converges globally to an equilibrium in non-stationary environments as well. Moreover, based on the experiments, the number of iterations needed by the FIWFC is affected neither by the size of the data set, nor by environmental changes, which is remarkable. The running time is also not affected by the environmental changes, although it does change with the size of the data set.

(3) Both the classical IWFC and the FIWFC provide mechanisms for improving the utilization of the radio spectrum. However, the FIWFC attains this practical goal much faster than the IWFC. Theoretical analysis shows that the computational complexity of the algorithm under both the interference-power and low-power limitations are lower than the IWFC for DSL, although the TPC problem for DSL contains a simpler constraint set. This improvement is attributed to the fact that the water-filling threshold for the new FIWFC can be found directly by taking the advantage of the necessary and sufficient conditions of the solutions to the TPC problem.

(4) Inherited from the IWFC, the FIWFC can be implemented in a distributed and decentralized manner, which is a property that is desirable from a practical perspective.

7.2 Future Work

In the future, the following issues will be considered:

- **Robustness**:In practice, the perturbations are uncertain to the users since each of them is operating distributively with a lack of full information of the network. Robust optimization is a useful approach to modeling uncertainty-affected problems, and to operating under a lack of full information on uncertainty. Therefore, we plan to apply this approach to model our problem in a form that can be solved efficiently and to guarantee the robust performance of a solution. Currently, there are lack of theoretical results and algorithms for robust generalized Nash games in the literature.

Robust optimization has been applied only to Nash games [17, 33]. Our new algorithm is designed specifically for the game-theoretical model but not the robust game-theoretical model. Extending the theoretical results and algorithms on game-theoretical models to robust game-theoretical models requires further research.

- **Frequency division multiple access (FDMA)**: FDMA is a channel access method, where users are given an individual allocation of one or more frequency bands. If FDMA is applied to cognitive radio networks, secondary users can not share a frequency band, in contrast to CDMA. To mathematically model this, we need to impose an orthogonal condition on every user pair over each frequency band. As a result, the feasible set of this problem is not convex, which make the problem difficult to solve. We plan to extend the theoretical results established in this thesis, so as to develop a low-complexity algorithm for solving the TPC problem for cognitive radios applying FDMA.

- **Relaxation**: As simulation results showed, the FIWFC converged even when the convergence conditions were violated. Hence, we plan to relax the sufficient conditions of convergence. It is expected that some relaxed sufficient conditions can be derived from the theory of non-negative matrices. A non-negative matrix is a matrix in which all the elements are not less than zero. It has many nice properties. In linear algebra, the Perron–Frobenius theorem, proved by Oskar Perron (1907) for positive matrices and Georg

Frobenius (1912) for certain classes of non-negative matrices, has been extended to non-negative matrices [15, 30]. It is asserted that the largest eigenvalue of a non-negative matrix will be non-negative and it is bounded by the largest value of the sum of each row of the matrix. It may well be that the use of non-negative matrices provides a new approach for relaxing the conditions of convergence.

- **Evaluation**: The TPC for DSL was studied in Appendix A following the same steps for the TPC for cognitive radios in this thesis. However, the implementation has not been done yet. We plan to evaluate the performance of the FIWFC for DSL only in stationary environments, as the users and available frequency bands are static in a DSL environment.

Appendix A

The TPC Problem for DSL

A.1 Introduction

Digital Subscriber Lines (DSL) system is referred to as the technology with which high-speed data transmission can happen via a wired network. Specifically, DSL is a multicarrier communication system where the frequency spectrum is divided into a number of frequency carriers to make parallel data transmission possible. In a DSL environment, with twisted copper wires from different homes bundled together, mutual interference exists among different wires, which is called crosstalk. Therefore, a multiuser model is more suitable than a single-user one to describe DSL systems. Figure A.1 illustrates a typical DSL environment.

The crosstalk dominates the total noise and interference and it is the main factor affecting the performance of a DSL system. To control the crosstalk from one line to its neighboring lines, the current approach enforces a fixed power for each line. This "static" approach restricts the speed and coverage of DSL

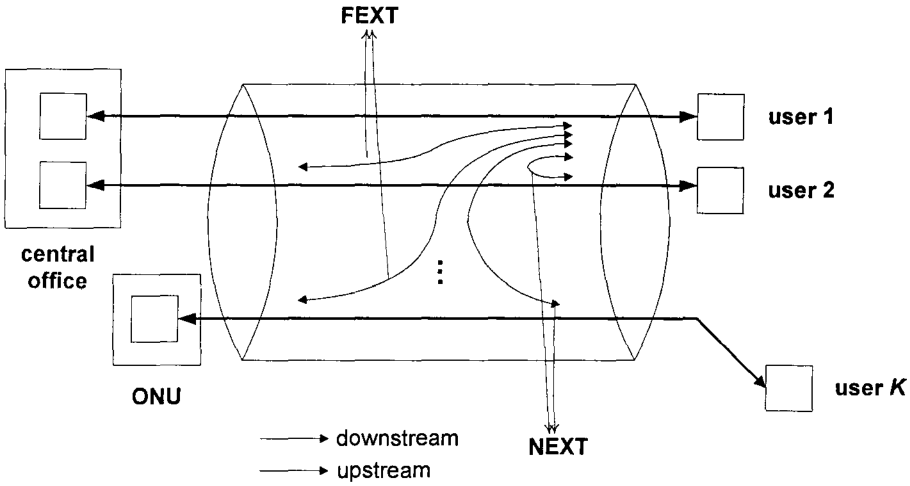


Figure A.1: Multiuser DSL

service [37]. With increasing demands for higher speed service, the improvement of DSL systems is evidently necessary. The TPC for DSL addresses this need, which allows an adaptive allocation of power over multiple lines regarding power capacities and mutual interference across the lines, such that the total achievable data rate is optimized.

A.2 Problem Statement

The TPC problem for DSL is formulated as a Nash game, where, instead of jointly maximizing the total data rate, each user maximizes its own data rate individually subject to a battery constraint. Then, the TPC problem can be implemented distributively with minimal central control. In this way, the service providers in the competitive market can share the same bundles.

A.3 Nash Game Formulation of the DSL Problem

Suppose that there are K users and N subcarriers. Mathematically, the TPC problem for DSL is a Nash game model which consists of K separable nonlinear convex optimization problems. For each user k, the optimization problem is as follows:

$$\begin{aligned} \max_{S_k} \quad & \sum_{n=1}^{N} \log\left(1 + \frac{S_k^n}{I_k^n}\right) \\ \text{s.t.} \quad & \sum_{n=1}^{N} S_k^n \le CAP_k, \\ & S_k^n \ge 0, \\ & n = 1, \cdots, N, \end{aligned} \tag{A.1}$$

where

$$I_k^n = \sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n S_j^n > 0.$$

This problem satisfies Slater's constraint qualification, as the feasible set is a polyhedron. Therefore, by Theorem 2.3.1, the KKT conditions are necessary and sufficient.

A.4 Theoretical Basis of the Problem

The KKT conditions for problem A.1 are:

$$\begin{aligned} 0 &\le S_k^n \perp (-\frac{1}{S_k^n + I_k^n} + \mu_k) \ge 0 \\ 0 &\le \mu_k \perp (CAP_k - \sum_{n=1}^{N}(S_k^n + I_k^n)) \ge 0 \\ &n = 1, \cdots, N,\ k = 1, \cdots, K. \end{aligned} \tag{A.2}$$

As the KKT conditions are necessary and sufficient, the TPC problem for DSL can be solved by computing a KKT solution of system (A.2). However, the KKT conditions (A.2) are nonlinear. We further transform them equivalently to a mixed *linear* complementarity problem (MLCP).

Proposition A.4.1 *The KKT conditions of the TPC problem for DSL are equivalent to the following MLCP:*

$$\begin{aligned} 0 &\le S_k^n \perp S_k^n + I_k^n - \nu_k \ge 0 \\ 0 &= CAP_k - \sum_{n=1}^{N} S_k^n \\ &\nu_k \textit{ is free} \\ n &= 1, \cdots, N,\ k = 1, \cdots, K. \end{aligned} \tag{A.3}$$

The proof is similar to that for Proposition 5.3.1 in Appendix E.

Analogous to the results in Subsections 3.4.3 and 5.3.3, the sufficient and necessary optimality conditions for the MLCP (A.3) are developed in the following proposition:

Proposition A.4.2 *S is a solution to the MLCP (A.3) if, and only if, there is a vector ν such that (S, ν) satisfies the following optimality conditions:*

$$S_k^n = \begin{cases} \nu_k - I_k^n & I_k^n < \nu_k \\ 0 & \nu_k \le I_k^n \end{cases}$$

where

$$\nu_k = (CAP_k + \sum_{n \in T_k} I_k^n)/|T_k|$$

and

$$T_k = \{n : S_k^n > 0\}.$$

Proof: First, we show that if S is a solution to the MLCP (A.3), then there is a vector ν such that (S, ν) satisfies the arithmetic simplified optimality conditions. If $S_k^n > 0$, by (A.3), we have that

$$S_k^n + I_k^n - \nu_k = 0 \tag{A.4}$$

Then,

$$S_k^n = \nu_k - I_k^n \tag{A.5}$$

Summing (A.4) for all $n \in T_k$, we have

$$\sum_{n \in T_k} S_k^n + \sum_{n \in T_k} I_k^n - \sum_{n \in T_k} \nu_k = 0 \tag{A.6}$$

Substituting $\sum_{n=1}^{N} S_k^n = CAP_k$ in (A.6), we have that

$$\nu_k = \frac{CAP_k + \sum_{n \in T_k} I_k^n}{|T_k|} \tag{A.7}$$

It is straightforward to check that if (S, ν) satisfies the arithmetic simplified optimality conditions, then S is a solution to MLCP (A.3). This completes the proof.

From the above discussion, we see that the TPC problem for DSL can be solved by finding a solution satisfying the conditions described in Proposition A.4.2.

Corollary A.4.3 *Suppose that the convexity assumption holds. Then, S is a NE of the TPC problem for DSL (A.1) if, and only if, there exist ν and β such that the triplet (S, ν, β) satisfies the arithmetic simplified optimality conditions in Proposition A.4.2.*

A.5 Algorithm

This section introduces the FIWFC for finding a NE of the TPC problem for DSL. Enlightened by Proposition A.4.2, the idea of this new algorithm is to iteratively update the transmit-power vector according to the water-filling threshold ν_k.

For the sake of illustration, the basic idea of this new updating scheme is shown in Figure A.2. This picture considers the two possible cases for user k:

- In subcarrier 1, the interference level I_k^1 is lower than the water-filling threshold ν_k; hence the transmit-power for user k over this subcarrier is set to the difference $\nu_k - I_k^1$;

- In subcarrier 2, the interference level I_k^2 is not lower than the water-filling threshold ν_k; hence the transmit-power for user k over this subcarrier is set to zero.

The FIWFC for DSL is described in Algorithm A.5.1.

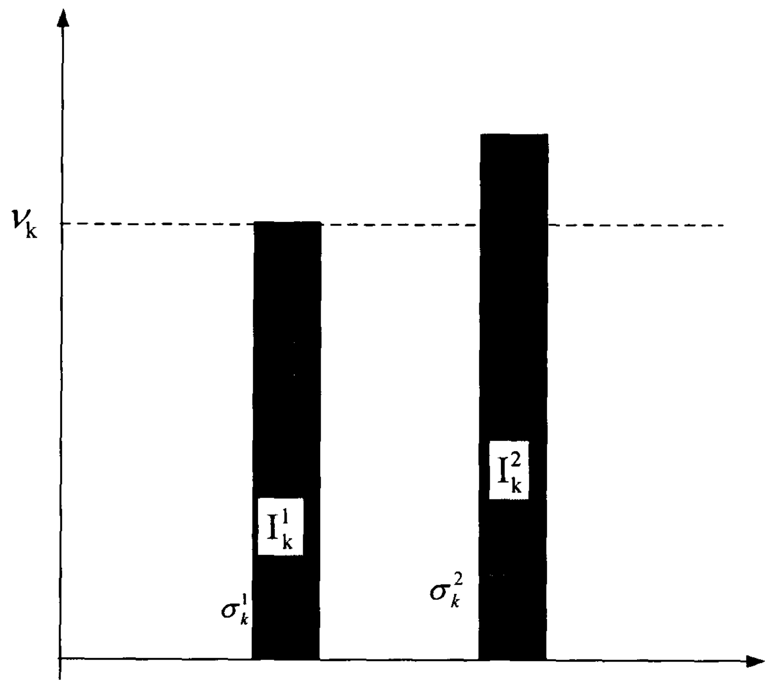


Figure A.2: Illustrating the basic updating scheme of FIWFC for DSL

Algorithm A.5.1 FIWFC for DSL

Let $l = 0$ and $S_k^{(0)}$ be any feasible power vector
repeat
  for $k = 1$ to K **do**
    $\nu_k^{(l)} = (CAP_k + \sum_{n \in T_k^{(l)}} I_k^{n(l)})/|T_k^{(l)}|$
    for $n = 1$ to N **do**

$$S_k^{n(l+1)} = \begin{cases} \nu_k^{(l)} - I_k^{n(l)} & \nu_k^{(l)} > I_k^{n(l)} \\ 0 & \text{otherwise} \end{cases}$$

    end for
  end for
  $l \leftarrow l + 1$
until A stopping criterion is satisfied

A.6 Convergence and Uniqueness

Similar to the results developed for the FIWFC for cognitive radios under both the interference-power and low-power limitations as described in Subsection 3.5.2 and 5.4.2, the convergence and uniqueness results for the FIWFC for DSL are derived in the following theorem.

Theorem A.6.1 *Suppose the TPC problem for DSL satisfies the convexity assumption. If one of the following conditions is satisfied,*

- $\rho(A^\top A) < 1$, *or*
- $\rho(C^\top C) < 1$, *or*
- $Tr(A^\top A) < 1$, *or*
- $Tr(C^\top C) < 1$,

then the FIWFC for DSL converges both globally and linearly to a unique solution of the TPC problem (A.1).

A.7 Computational Complexity

The computational complexity of this new algorithm is the same as the FIWFC for cognitive radio networks introduced in Chapters 3 and 5. It is justified in the following.

After initialization, the power allocation is adjusted iteratively for the K users. For each user k, T_k needs to be checked once. Theoretically, checking T_k takes $O(N)$ operations. To evaluate ν_k, the number of additions of the interference-plus-noise terms is less than N. Hence, the total complexity of evaluating ν_k is $O(N)$. To evaluate S_k^n takes only $O(1)$ time, and therefore the total time spent in evaluating user k's power vector $S_k = (S_k^1, S_k^2, \cdots, S_k^N)$ is $O(N)$. Thus, the total operating time for each user is linear in N. For K users, the total complexity of each iteration is $O(KN)$, which is linear in both the number of users K and the number of subcarriers N, respectively. Let L be the number of iterations needed in the FIWFC for DSL. The total computational complexity is $O(LKN)$.

A.8 Summary

The TPC problem for DSL was cast as a Nash game model and the solution also exists according to the existence theorem presented in Chapter 2. The FIWFC was introduced to solve the TPC problem for DSL, with the same computational complexity as under the interference-power limitation introduced in Chapter 3. The conditions for algorithm convergence and solution uniqueness were also presented.

Appendix B

Proof of Proposition 3.4.2

The proof proceeds as follows:

- **Case 1:** $T_k \neq B_k$.

  First, we show that if (S, ν, β) solves the mixed LCP (3.7), then (S, ν, β) satisfies the arithmetic simplified optimality conditions. If $I_k^n < P^n < \nu_k$, then we show that $\beta_k^n > 0$. If $\beta_k^n = 0$, then

$$S_k^n + I_k^n + \beta_k^n - \nu_k \;\leq\; P^n - \nu_k < 0.$$

  This contradicts the first complementarity condition. Hence, $\beta_k^n > 0$. By the second complementarity condition, we have $S_k^n = P^n - I_k^n$. Since $I_k^n < P^n$, then $S_k^n > 0$. By the first complementarity condition, we have

$$\begin{aligned} 0 &= S_k^n + I_k^n + \beta_k^n - \nu_k \\ &= P^n + \beta_k^n - \nu_k. \end{aligned}$$

Immediately, we find that $\beta_k^n = \nu_k - P^n$.

If $I_k^n = P^n < \nu_k$, by the second complementarity condition, we have $S_k^n = 0$. By the first complementarity condition, we have $\beta_k^n \geq \nu_k - I_k^n = \nu_k - P^n$.

If $I_k^n < \nu_k \leq P^n$, we show that $\beta_k^n = 0$. If $\beta_k^n > 0$, then $S_k^n + I_k^n = P^n$ by the second complementarity condition. Since $I_k^n < P^n$, then $S_k^n > 0$. Hence, by the first complementarity condition, we have $P^n + \beta_k^n - \nu_k = 0$. Hence, $P^n < \nu_k$, which contradicts the inequality $\nu_k \leq P^n$. Therefore, we have $\beta_k^n = 0$. By the first complementarity condition, we have $S_k^n + I_k^n \geq \nu_k$. Since $I_k^n < \nu_k$, we have $S_k^n > 0$. Then, $S_k^n + I_k^n = \nu_k$. Moreover, it implies that $\beta_k^n = 0$ for $n \in T_k$ but $n \notin B_k$.

If $\nu_k \leq I_k^n \leq P^n$, we show that $S_k^n = 0$. Assuming $S_k^n > 0$, we have

$$\begin{aligned} S_k^n + I_k^n + \beta_k^n - \nu_k &\geq S_k^n + \beta_k^n \\ &> \beta_k^n \\ &\geq 0, \end{aligned}$$

which contradicts the first complementarity condition. Therefore, we have $S_k^n = 0$. Moreover, if $P^n > I_k^n$, then $\beta_k^n = 0$.

After summing up all the S_k^n and performing some linear algebra, we find that

$$\nu_k = (CAP_k + \sum_{n \in T_k} I_k^n - \sum_{n \in B_k} P^n)/(|T_k| - |B_k|).$$

Second, we show that if (S, ν, β) satisfies the arithmetic simplified optimality conditions, then (S, ν, β) solves the mixed LCP (3.7). Suppose next that (S, ν, β) satisfies the conditions:

$$\nu_k = (CAP_k + \sum_{n \in T_k} I_k^n - \sum_{n \in B_k} P^n)/(|T_k| - |B_k|)$$

$$\beta_k^n \begin{cases} = \nu_k - P^n & I_k^n < P^n < \nu_k \\ \geq \nu_k - P^n & I_k^n = P^n < \nu_k \\ = 0 & I_k^n < \nu_k \leq P^n \\ = 0 & \nu_k \leq I_k^n < P^n \\ \geq 0 & \nu_k \leq I_k^n = P^n \end{cases}$$

and

$$S_k^n = \begin{cases} P^n - I_k^n & I_k^n \leq P^n < \nu_k \\ \nu_k - I_k^n & I_k^n < \nu_k \leq P^n \\ 0 & \nu_k \leq I_k^n \leq P^n. \end{cases}$$

If $I_k^n < P^n < \nu_k$, then $\beta_k^n = \nu_k - P^n > 0$ and $S_k^n = P^n - I_k^n$, which implies that the second complementarity condition is satisfied, and $S_k^n + I_k^n + \beta_k^n - \nu_k = 0$ implies that the first complementarity condition is satisfied. If $I_k^n = P^n < \nu_k$, then $S_k^n = P^n - I_k^n = 0$ and $S_k^n + I_k^n + \beta_k^n - \nu_k > 0$. If $I_k^n < \nu_k \leq P^n$, $S_k^n + I_k^n + \beta_k^n - \nu_k = 0$ and $\beta_k^n = 0$. If $\nu_k \leq I_k^n < P^n$, $S_k^n = 0$, $S_k^n + I_k^n + \beta_k^n - \nu_k > 0$ and $\beta_k^n = 0$. If $\nu_k \leq I_k^n = P^n$, $S_k^n = 0$,

$S_k^n + I_k^n + \beta_k^n - \nu_k \geq 0$ and $I_k^n = P^n$. Since

$$(|T_k| - |B_k|)\nu_k = CAP_k + \sum_{n\in T_k} I_k^n - \sum_{n\in B_k} P^n,$$

we have

$$\begin{aligned}
\sum_{n\in T_k} S_k^n &= \sum_{n\in B_k} S_k^n + \sum_{n\in T_k, n\notin B_k} S_k^n \\
&= |B_k|(P^n - I_k^n) + (|T_k| - |B_k|)(\nu_k - I_k^n) \\
&= (|T_k| - |B_k|)\nu_k + \sum_{n\in B_k} P^n - \sum_{n\in T_k} I_k^n \\
&= CAP_k.
\end{aligned}$$

- **Case 2:** $T_k = B_k$.

  Suppose next that (S, ν, β) solves the mixed LCP (3.7). If $n \in T_k$, then $S_k^n = P^n - I_k^n$ since $T_k = B_k$. Summing up for all $n \in T_k$, we have $CAP_k = \sum_{n\in T_k} P^n - \sum_{n\in T_k} I_k^n$ by the equation in (3.7). We also have $\nu_k - \beta_k^n = P^n$ since $S_k^n + I_k^n + \beta_k^n - \nu_k$ has to be zero from the first complementarity condition in (3.7). If $n \notin T_k$, then $n \notin B_k$. Consequently, $\beta_k^n = 0$ from the second complementarity condition, and $\nu_k \leq I_k^n$ from the first complementarity condition.

  Suppose that (S, ν, β) satisfies the conditions:

  - $S_k^n = P^n - I_k^n$, $CAP_k = \sum_{n\in T_k} P^n - \sum_{n\in T_k} I_k^n$ and $\nu_k - \beta_k^n = P^n$, for all $n \in T_k$;
  - $\beta_k^n = 0$, $P^n \geq I_k^n$ and $\nu_k \leq I_k^n$ for all $n \notin T_k$.

If $n \in T_k$, then $S_k^n + I_k^n + \beta_k^n - \nu_k = P^n - P^n = 0$. Therefore, the first complementarity condition is satisfied. The second complementarity condition and the equation are satisfied naturally since $S_k^n = P^n - I_k^n$. If $n \notin T_k$, then the second complementarity condition is satisfied since $\beta_k^n = 0$ and $P^n \geq I_k^n$, and the first complementarity condition is satisfied since $\nu_k \leq I_k^n$. Therefore, (S, ν, β) solves the mixed LCP (3.7).

Appendix C

Proof of Theorem 3.5.1

The proof proceeds as follows: Let $S^{(l)} = \{S_k^{(l)}\}_{k=1}^K$, with $S_k^{(l)} = \{S_k^{n(l)}\}_{n=1}^N$, $\beta_k^{n(l)}$ and $\nu_k^{(l)}$ be the lth iteration solution of the FIWFC. Define $I_k^{n(l)} = \sigma_k^n + \sum_{j\neq k} \alpha_{jk}^n S_j^{n(l)}$. Therefore, the triplet $(S^{(l+1)}, \beta^{(l+1)}, \nu^{(l+1)})$ satisfy the following mixed linear complementarity system

$$
\begin{aligned}
0 &\leq S_k^{n(l+1)} \perp S_k^{n(l+1)} + I_k^{n(l)} + \beta_k^{n(l+1)} - \nu_k^{(l+1)} \geq 0 \\
0 &\leq \beta_k^{n(l+1)} \perp P^n - (S_k^{n(l+1)} + I_k^{n(l)}) \geq 0 \\
0 &= CAP_k - \sum_{n=1}^{N} S_k^{n(l+1)}
\end{aligned}
\tag{C.8}
$$

for $k = 1, \cdots, K$, and $n = 1, \cdots, N$. Let χ be the Cartesian product of the feasible sets defined as

$$\chi = \Pi_{k=1}^K \chi_k.$$

Since $S^{(l+1)}$ is a solution of the KKT conditions described in (C.8), it is a solution of the following variational inequality (VI):

$$\sum_{n=1}^{N}(S_k^n - S_k^{n(l+1)})(S_k^{n(l+1)} + I_k^{n(l)}) \geq 0 \tag{C.9}$$

for all $S = \{S_k\}_{k=1}^{K} \in \chi$, with $S_k = \{S_k^n\}_{n=1}^{N}$. Suppose that $\bar{S}$ is a solution of the TPC problem. By Theorem 2.3.1 and Proposition 3.4.2, there are $\bar{\nu}$ and $\bar{\beta}$, such that the triplet $(\bar{S}, \bar{\nu}, \bar{\beta})$ solves the mixed LCP (3.7). Therefore, it is a solution of the VI:

$$\sum_{n=1}^{N}(S_k^n - \bar{S}_k^n)(\bar{S}_k^n + \bar{I}_k^n) \geq 0, \tag{C.10}$$

for all $S \in \chi$. Hence, from VIs (C.9) and (C.10), we have

$$\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})(S_k^{n(l+1)} + I_k^{n(l)}) \geq 0, \tag{C.11}$$

and

$$\sum_{n=1}^{N}(S_k^{n(l+1)} - \bar{S}_k^n)(\bar{S}_k^n + \bar{I}_k^n) \geq 0. \tag{C.12}$$

After adding (C.11) to (C.12), and rearranging the terms, we have

$$\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})^2 \leq \sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)(\bar{S}_k^n - S_k^{n(l+1)}). \tag{C.13}$$

By the Cauchy-Schwarz inequality, we have

$$\sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)(\bar{S}_k^n - S_k^{n(l+1)}) \leq \sqrt{\sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)^2}\sqrt{\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})^2}. \quad (C.14)$$

By inequalities (C.13) and (C.14), we find

$$\sqrt{\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})^2} \leq \sqrt{\sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)^2}.$$

This is true for $k = 1, \cdots, K$. Therefore,

$$\sum_{k=1}^{K}\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})^2 \leq \sum_{k=1}^{K}\sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)^2. \quad (C.15)$$

That is,

$$\begin{aligned} \|\bar{S} - S^{(l+1)}\| &\leq \|A(\bar{S} - S^{(l)})\| \\ &\leq \|A\|\|\bar{S} - S^{(l)}\|. \end{aligned}$$

Since $\|A\| = \sqrt{\rho(A^\top A)}$, then $\rho(A^\top A) < 1$ implies that the sequence $\{S^{(l)}\}$ linearly converges. Since for the 2-norm, $\|A\| = \sqrt{\mathrm{Tr}(A^\top A)}$, then $\mathrm{Tr}(A^\top A) < 1$ implies the sequence $\{S^{(l)}\}$ linearly converges as well.

Similarly, if we arrange $S^{(l)}$ in this way: $S^{(l)} = \{S^{n(l)}\}_{n=1}^N$, with $S^{n(l)} = \{S_k^{n(l)}\}_{k=1}^K$, we have

$$\|\bar{S} - S^{(l+1)}\| \leq \|C\|\|\bar{S} - S^{(l)}\|.$$

Then, if $\rho(C^\top C) < 1$, the sequence $\{S^{(l)}\}$ converges linearly.

Therefore, if $\rho(A^\top A) < 1$, or $\rho(C^\top C) < 1$, or $\mathrm{Tr}(A^\top A) < 1$, or $\mathrm{Tr}(C^\top C) < 1$, the FIWFC linearly converges.

Appendix D

Proof of Theorem 3.5.2

The proof proceeds as follows: Suppose that $\bar{S}$ and $\hat{S}$ are two solutions of the GNEP. Similar to the derivation of (C.9), we have

$$\sum_{n=1}^{N}(\bar{S}_k^n - \hat{S}_k^n)(\hat{S}_k^n + \hat{I}_k^n) \geq 0, \tag{D.16}$$

and

$$\sum_{n=1}^{N}(\hat{S}_k^n - \bar{S}_k^n)(\bar{S}_k^n + \bar{I}_k^n) \geq 0. \tag{D.17}$$

Therefore, in a manner similar to the derivation of inequality (C.15), from inequalities (C.11) and (C.12), we have

$$\sum_{k=1}^{K}\sum_{n=1}^{N}(\bar{S}_k^n - \hat{S}_k^n)^2 \leq \sum_{k=1}^{K}\sum_{n=1}^{N}(\hat{I}_k^n - \bar{I}_k^n)^2,$$

which, in turn, implies that,

$$\begin{aligned} |\bar{S}-\hat{S}\| &\leq \|A(\bar{S}-\hat{S})\| \\ &\leq \|A\|\|\bar{S}-\hat{S}\|, \end{aligned}$$

or, equivalently,

$$\begin{aligned} \|\bar{S}-\hat{S}\| &\leq \|C(\bar{S}-\hat{S})\| \\ &\leq \|C\|\|\bar{S}-\hat{S}\| \end{aligned}$$

depending on how the vector S is arranged. Hence, if $\rho(A^\top A) < 1$, or $\rho(C^\top C) < 1$, or $\mathrm{Tr}(A^\top A) < 1$, or $\mathrm{Tr}(C^\top C) < 1$, we have $\|\bar{S} - \hat{S}\| = 0$, which implies that the solution of the TPC problem is unique.

Appendix E

Proof of Theorem 5.3.1

The proof proceeds as follows: First, we show that if $(S_k^n, \lambda_k^n, \mu_k)$, for all n, solves the KKT conditions for all users k, then there are $(S_k^n, \beta_k^n, \nu_k)$ for all n and k, solving the MLCP. Now we prove that $\mu_k > 0$ for all k. If $\mu_k = 0$, then $\lambda_k^n \geq \frac{1}{S_k^n + I_k^n} > 0$ by the inequality on the right-hand side of (5.5). Hence, according to the complementarity condition in (5.6), we have $S_k^n = UPP^n > 0$. It implies that

$$CAP_k - \sum_{n=1}^{N} S_k^n = CAP_k - \sum_{n=1}^{N} UPP^n.$$

By (5.7), we have $CAP_k - \sum_{n=1}^{N} UPP^n \geq 0$, which contradicts the condition $CAP_k < \sum_{n=1}^{N} UPP^n$. Therefore, $\mu_k > 0$ for all k.

Dividing the inequality on the right hand side of (5.5) by μ_k and $S_k^n + I_k^n$. we have:

$$S_k^n + I_k^n + \lambda_k^n(S_k^n + I_k^n)/\mu_k - \frac{1}{\mu_k} \geq 0. \tag{E.18}$$

Let $\nu_k = \frac{1}{\mu_k}$ and $\beta_k^n = \lambda_k^n(S_k^n + I_k^n)/\mu_k$. Then, $\nu_k > 0$ and the sign of β_k^n is the same as that of λ_k^n. Hence, by substituting ν_k and β_k^n, we can rewrite the KKT conditions (5.5), (5.6), and (5.7) equivalently as the linear complementarity problem (5.8).

Second, we show that if the triplet (S, β, ν) solves the MLCP, then there is the triplet (S, λ, μ) that solves the KKT conditions. Now we must have that $\nu_k > 0$. Otherwise,

$$S_k^n + I_k^n + \beta_k^n - \nu_k > 0,$$

and from the first complementarity condition in MLCP (5.8), we have that

$$S_k^n = 0, \ \ \forall n = 1, \cdots, N,$$

which contradicts the equality conditions in (5.8). Therefore, the KKT conditions (5.5), (5.6), and (5.7) hold by letting $\mu_k = \frac{1}{\nu_k}$ and $\lambda_k^n = (S_k^n + I_k^n)/(\beta_k^n \nu_k)$. This completes the proof.

Appendix F

Proof of Proposition 5.3.2

The proof proceeds as follows: First, we show that if (S, ν, β) solves the mixed LCP (5.8), then (S, ν, β) satisfies the arithmetic simplified optimality conditions.

- **Case 1:** $T_k \neq B_k$.

  If $\nu_k - I_k^n \leq 0$, then the complementarity condition in the first condition in (5.8) does not hold for any $S_k^n > 0$. Hence,

  $$S_k^n = 0.$$

  From the second condition in (5.8), we have

  $$\beta_k^n = 0.$$

  If $\nu_k - I_k^n > 0$, we have $S_k^n > 0$. Otherwise, if $S_k^n = 0$, then $I_k^n + \beta_k^n - \nu_k \geq 0$ by the first condition in (5.8). Hence, $\beta_k^n \geq \nu_k - I_k^n > 0$. By the complementarity condition in the second condition in (5.8), we have that $S_k^n = UPP^n > 0$, which contradicts the assumption that $S_k^n = 0$. Therefore, $S_k^n > 0$ when $\nu_k - I_k^n > 0$. By the complementarity condition in the first condition in (5.8), we have

$$S_k^n + I_k^n + \beta_k^n - \nu_k = 0,$$

which is

$$S_k^n + \beta_k^n = \nu_k - I_k^n.$$

If $UPP^n \geq \nu_k - I_k^n$, then $UPP^n - S_k^n \geq \beta_k^n$. By the complementarity condition in the second condition of (5.8), the smaller value between $UPP^n - S_k^n$ and β_k^n has to be zero, which yields $\beta_k^n = 0$. Therefore,

$$S_k^n = \nu_k - I_k^n. \tag{F.19}$$

If $UPP^n \leq \nu_k - I_k^n$, then

$$UPP^n - S_k^n \leq \beta_k^n.$$

By the complementarity condition in the second condition of (5.8), the smaller of $UPP^n - S_k^n$ and β_k^n has to be zero, which yields

$$UPP^n - S_k^n = 0,$$

and

$$\beta_k^n = \nu_k - I_k^n - UPP^n.$$

Therefore,

$$S_k^n = UPP^n. \tag{F.20}$$

By the third condition in (5.8), (F.19) and (F.20), we have

$$\begin{aligned} 0 &= CAP_k - \sum_{n \in T_k/B_k} S_k^n - \sum_{n \in B_k} S_k^n \\ &= CAP_k - \sum_{n \in T_k/B_k} (\nu_k - I_k^n) - \sum_{n \in B_k} UPP^n. \end{aligned}$$

Finally, we have that

$$\nu_k = \frac{CAP_k + \sum_{n \in T_k/B_k} I_k^n - \sum_{n \in B_k} UPP^n}{|T_k| - |B_k|}.$$

- **Case 2:** $T_k = B_k$.

  If $n \in T_k$, then $S_k^n = UPP^n$ since $T_k = B_k$. If $n \notin T_k$, then $n \notin B_k$. Consequently, $S_k^n = 0$.

It is straightforward to check if (S, ν, β) satisfies the arithmetic simplified optimality conditions, then (S, ν, β) solves the mixed LCP (5.8).

This completes the proof.

Appendix G

Proof of Theorem 5.4.1

The proof proceeds as follows: Let $S^{(l)} = \{S_k^{(l)}\}_{k=1}^K$, with $S_k^{(l)} = \{S_k^{n(l)}\}_{n=1}^N$, $\beta_k^{n(l)}$ and $\nu_k^{(l)}$ be the lth iteration solution of the FIWFC. Define $I_k^{n(l)} = \sigma_k^n + \sum_{j\neq k} \alpha_{jk}^n S_j^{n(l)}$. If $CAP_k \geq \sum_{n=1}^{n} UPP^n$, then $S_k^{n(l+1)} = UPP^n$, for all n. Otherwise, the triplet $(S^{(l+1)}, \beta^{(l+1)}, \nu^{(l+1)})$ satisfies the following mixed linear complementarity system

$$\begin{aligned}
0 &\leq S_k^{n(l+1)} \perp S_k^{n(l+1)} + I_k^{n(l)} + \beta_k^{n(l+1)} - \nu_k^{(l+1)} \geq 0, \; n = 1, \cdots, N \\
0 &\leq \beta_k^{n(l+1)} \perp UPP^n - S_k^{n(l+1)} \geq 0, \; n = 1, \cdots, N \\
0 &= CAP_k - \sum_{n=1}^{N} S_k^{n(l+1)}
\end{aligned} \tag{G.21}$$

for $n = 1, \cdots, N, k = 1, \cdots, K$. Next, let Ω_k be user k's feasible set, defined as

$$\Omega_k = \left\{ S \in R^{KN} : \begin{array}{l} S_k^n \leq UPP^n, \\ \sum_{n=1}^{N} S_k^n \leq CAP_k, \\ S_k^n \geq 0, \\ n = 1, \cdots, N. \end{array} \right\}.$$

Let Ω be the Cartesian product of all users' feasible sets, defined as

$$\Omega = \prod_{k=1}^{K} \Omega_k.$$

Since $S^{(l+1)}$ is a solution of the KKT conditions (G.21), it is a solution of the following VI:

$$\sum_{n=1}^{N} (S_k^n - S_k^{n(l+1)})(S_k^{n(l+1)} + I_k^{n(l)}) \geq 0, \ \forall S \in \Omega. \tag{G.22}$$

Suppose that $\bar{S}$ is a solution of the TPC problem. By Theorem 2.3.1 and Proposition 5.3.2, there are $\bar{\nu_k}$ and $\bar{\beta_k}$ such that the triple $(\bar{S}_k, \bar{\nu_k}, \bar{\beta_k})$ solves MLCP (5.8). Therefore, it is a solution of the VI:

$$\sum_{n=1}^{N} (S_k^n - \bar{S}_k^n)(\bar{S}_k^n + \bar{I}_k^n) \geq 0, \ \forall S \in \Omega. \tag{G.23}$$

Hence, from the VIs (G.22) and (G.23), we have

$$\sum_{n=1}^{N} (\bar{S}_k^n - S_k^{n(l+1)})(S_k^{n(l+1)} + I_k^{n(l)}) \geq 0, \tag{G.24}$$

and

$$\sum_{n=1}^{N}(S_k^{n(l+1)} - \bar{S}_k^n)(\bar{S}_k^n + \bar{I}_k^n) \geq 0. \tag{G.25}$$

After summing (G.24) and (G.25) and rearranging terms, we have

$$\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})^2 \leq \sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)(\bar{S}_k^n - S_k^{n(l+1)}). \tag{G.26}$$

By the Cauchy-Schwarz inequality, we have

$$\sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)(\bar{S}_k^n - S_k^{n(l+1)}) \leq \sqrt{\sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)^2}\sqrt{\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})^2}. \tag{G.27}$$

By the inequalities (G.26) and (G.27), we have that

$$\sqrt{\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})^2} \leq \sqrt{\sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)^2}.$$

This is true for $k = 1, \cdots, K$, as when $CAP_k \geq \sum_{n=1}^{n} UPP^n$, we have

$$\bar{S}_k^n - S_k^{n(l+1)} = UPP^n - UPP^n = 0.$$

Therefore,

$$\sum_{k=1}^{K}\sum_{n=1}^{N}(\bar{S}_k^n - S_k^{n(l+1)})^2 \leq \sum_{k=1}^{K}\sum_{n=1}^{N}(I_k^{n(l)} - \bar{I}_k^n)^2. \tag{G.28}$$

That is

$$\begin{aligned}\|\bar{S} - S^{(l+1)}\| &\leq \|A(\bar{S} - S^{(l)})\| \\ &\leq \|A\|\|\bar{S} - S^{(l)}\|.\end{aligned}$$

Since $\|A\| = \sqrt{\rho(A^\top A)}$, then the inequality $\rho(A^\top A) < 1$ implies that the sequence $\{S^{(l)}\}$ linearly converges. Since for the 2-norm, $\|A\| = \sqrt{\mathrm{Tr}(A^\top A)}$, then $\mathrm{Tr}(A^\top A) < 1$ implies that the sequence $\{S^{(l)}\}$ linearly converges as well.

Similarly, if we arrange $S^{(l)}$ in this way: $S^{(l)} = \{S^{n(l)}\}_{n=1}^N$ with $S^{n(l)} = \{S_k^{n(l)}\}_{k=1}^K$, we have

$$\|\bar{S} - S^{(l+1)}\| \leq \|C\|\|\bar{S} - S^{(l)}\|.$$

Then, if $\rho(C^\top C) < 1$, then the sequence $\{S^{(l)}\}$ linearly converges.

Therefore, if $\rho(A^\top A) < 1$, or $\rho(C^\top C) < 1$, or $\mathrm{Tr}(A^\top A) < 1$, or $\mathrm{Tr}(C^\top C) < 1$, the FIWFC linearly converges.

Appendix H

Proof of Theorem 5.4.2

The proof proceeds as follows: Suppose that $\bar{S}$ and $\hat{S}$ are two solutions of the GNEP. Similar to deriving (G.22), we have

$$\sum_{n=1}^{N}(\bar{S}_k^n - \hat{S}_k^n)(\hat{S}_k^n + \hat{I}_k^n) \geq 0, \tag{H.29}$$

and

$$\sum_{n=1}^{N}(\hat{S}_k^n - \bar{S}_k^n)(\bar{S}_k^n + \bar{I}_k^n) \geq 0. \tag{H.30}$$

Therefore, similar to deriving the inequality (G.28) from the inequalities (G.24) and (G.25), we have

$$\sum_{k=1}^{K}\sum_{n=1}^{N}(\bar{S}_k^n - \hat{S}_k^n)^2 \leq \sum_{k=1}^{K}\sum_{n=1}^{N}(\hat{I}_k^n - \bar{I}_k^n)^2,$$

which implies that,

$$
\begin{aligned}
|\bar{S} - \hat{S}\| &\leq \|A(\bar{S} - \hat{S})\| \\
&\leq \|A\|\|\bar{S} - \hat{S}\|,
\end{aligned}
$$

or, equivalently,

$$
\begin{aligned}
\|\bar{S} - \hat{S}\| &\leq \|C(\bar{S} - \hat{S})\| \\
&\leq \|C\|\|\bar{S} - \hat{S}\|
\end{aligned}
$$

depending on how the vector S is arranged. Hence, if $\rho(A^\top A) < 1$, or $\rho(C^\top C) < 1$, or $\mathrm{Tr}(A^\top A) < 1$, or $\mathrm{Tr}(C^\top C) < 1$, we have $\|\bar{S} - \hat{S}\| = 0$, which implies that the solution of the TPC problem is unique.

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