Dynamic Models of Cognitive Radio Networks

DYNAMIC MODELS OF COGNITIVE RADIO NETWORKS

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'Would you tell me, please, which way I ought to go from here?'

'That depends a good deal on where you want to get to,' said the Cat.

'I don't much care where—' said Alice.

'Then it doesn't matter which way you go,' said the Cat.

'—so long as I get SOMEWHERE,' Alice added as an explanation.

'Oh, you're sure to do that,' said the Cat, 'if you only walk long enough.'

— Lewis Carroll, Alice's Adventures in Wonderland.

To my mother and the loving memory of my father

Abstract

A cognitive radio network is a multi-user system, in which different users compete for limited resources in an opportunistic manner, interacting with each other for access to the available resources. The fact that both users and spectrum holes (i.e., under-utilized spectrum subbands) can come and go in a stochastic manner, makes a cognitive radio network a highly dynamic and challenging wireless environment. Finding robust decentralized resource-allocation algorithms, which are capable of achieving reasonably good solutions fast enough in order to guarantee an acceptable level of performance even under worst-case interference conditions, is crucial in such an environment.

Considering a non-cooperative framework, the iterative waterfilling algorithm (IWFA) is a potentially good candidate for transmit-power control in cognitive radio networks for achieving a Nash-equilibrium point. IWFA is appealing because of its low complexity, fast convergence, distributed nature, and convexity. It can be reformulated as an affine variational inequality (AVI) problem. Employing the theory of projected dynamic (PD) systems, an affine dynamic model is obtained for the evolution of the network's state. This dynamic model allows us to study both equilibrium and disequilibrium behaviour of the network. The proposed dynamic framework also facilitates sensitivity and stability analysis of the system.

The fact that changes happen in a cognitive radio network because of continuous dynamics as well as discrete events, makes it a hybrid dynamic (HD) system. Decision making is then a multiple-time-scale process. Modeling the system using the theory of PD lends itself to describing the cognitive radio network as a constrained piecewise affine (PWA) system and therefore, benefiting from various mathematical tools, which have been well demonstrated in control theory.

Usually users use asynchronous update schemes and they update their transmit powers at different rates. The feedback channel introduces a time-varying delay in the control loop of a cognitive radio, which means sometimes users update their transmit powers using out-dated information. Therefore, the network is practically speaking a multiple-time-varying-delay system with uncertainty. Robust exponential stability of the network is studied in this framework.

Theories of evolutionary variational inequalities and projected dynamic systems on Hilbert spaces were used to extend the developed framework further in order to address the multiple-time-scale nature of the cognitive radio network.

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Finally, I would like to thank my family. Their endless support, encouragement, and love have always been a source of energy for me to pursue higher education.

Abbreviations

ADC	Analog-to-Digital Converter
AVI	Affine Variational Inequality
DAC	Digital-to-Analog Converter
DLD	Double-Layer Dynamic
DSL	Digital Subscriber Lines
EVI	Evolutionary Variational Inequality
FCC	Federal Communications Commission
FDE	Functional Differential Equation
FFT	Fast Fourier Transform
HD	Hybrid Dynamic
ICI	Inter-Carrier Interference
IEEE	Institute of Electrical and Electronics Engineers
IFFT	Inverse Fast Fourier Transform
ISI	Inter-Symbol Interference
IWFA	Iterative Waterfilling Algorithm
KKT	Karush-Kuhn-Tucker
LCP	Linear Complementarity Problem
MIMO	Multiple Input Multiple Output

MLCP	Mixed Linear Complementarity Problem
MTM	Multitaper Method
NCP	Nonlinear Complementarity Problem
ODE	Ordinary Differential Equation
OFDM	Orthogonal Frequency Division Multiplexing
PD	Projected Dynamic
PDSD	Projected Dynamic System with Delay
PWA	Piecewise Affine
QoS	Quality of Service
SINR	Signal-to-Interference plus Noise Ratio
SNR	Signal-to-Noise Ratio
TD	Temporal Difference
VI	Variational Inequality

Glossary of Symbols

α_k^{ij} Normalized interference gain from transmitter j to receiver i on subcarrier k

β_k Frequency-dependent attenuation parameter associated with subcarrier k

γ_k^i Lagrange multiplier associated with the constraint imposed by the permissible interference power level

λ_k^i Lagrange multiplier associated with the constraint that prevents cognitive radios to transmit on non-idle subcarriers

λ Decay rate

Γ Signal-to-noise ratio gap

Π_K Projection operator onto the feasible set K

$\boldsymbol{\varrho}$ The combined effect of the background noise in both forward and feedback channels in the network

σ_k^i Normalized background noise power at the receiver input of user i on subcarrier k

$\sigma_k^{\max}$ Maximum normalized background noise power on subcarrier k

$\boldsymbol{\sigma}^i$ Normalized background noise power vector at the receiver input of user i

$\boldsymbol{\sigma}$ Normalized background noise power vector of the network

τ Time

$\tau^i(t)$ Time-varying delay introduced by user i's feedback channel

$\tau^{ij}(t)$	Time-varying delay with which user i receives update information from user j
ϕ	Initial function
Ψ^i_{t0}	Initial set associated with user i
Ψ_{t0}	Initial set for the network
$a(t)$	Step-size
CAP_k	The permissible interference power level on subcarrier k
d_{ij}	Distance from transmitter j to receiver i
f^i	Objective function of user i
Δf	Frequency offset
h^{ij}_k	Channel gain from transmitter j to receiver i over the flat-fading subchannel associated with subcarrier k
I^i_k	Noise plus interference experienced by user i on subcarrier k
$\bar{I}^i_k$	Nominal noise plus interference experienced by user i on subcarrier k
ΔI^i_k	Perturbation term in noise plus interference experienced by user i on subcarrier k
$\mathbf{I}^i$	Noise plus interference vector experienced by user i
$\mathbf{\Delta I}^i$	Perturbation term in noise plus interference vector experienced by user i
K^i	Feasible set of user i
K	Feasible set of the network
$\text{int}K$	Interior of the feasible set K
∂K	Boundary of the feasible set K
L^i	Lagrangian of the optimization problem for user i
$\mathbf{M}$	Matrix of normalized interference gains

$\mathbf{M}_k$	Tone matrix associated with subcarrier k
$\mathbf{M}^{ij}$	Diagonal matrix, whose diagonal elements are normalized interference gains from transmitter j to receiver i at different subcarriers
m	Number of active cognitive radio transceivers in the region of interest
M_1	Set of users that are able to transmit with their maximum powers
M_2	Set of users that are not able to transmit with their maximum powers
n	Total number of subcarriers in an OFDM framework that can be potentially available for communications
p_k^i	User i's transmit power on subcarrier k
$\mathbf{p}^i$	User i's power vector
$\mathbf{p}^{-i}$	Joint power vectors of users other than user i
$\mathbf{p}$	Network power vector
$\mathbf{p}^*$	Nash-equilibrium point
$p_{\max}^i$	User i's maximum power
PS	Subset of subcarriers that cannot be assigned to cognitive radios
r	Path-loss exponent
$S(\mathbf{p})$	Set of inward normals at $\mathbf{p} \in X$
$\mathcal{S}_t$	Adjustment scheme at time t
t	Time
u^i	Lagrange multiplier associated with maximum power constraint

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Chapter 1

Introduction

"Spectrum is like air; we need to keep it clean, open, and green for our environment" [2].

1.1 Motivation

Mobile communications and broadband internet access have been playing key roles in the development of our society in recent years. The increasing number of users of internet-enabled wireless devices, illustrates the shift from traditional application-specific radio technology to service-oriented information delivery systems. Regarding the ever-increasing demand for more advanced applications that require the exchange of higher volumes of data, communication technologies are progressing toward providing secure and seamless connectivity of mobile devices to any network, anytime, and anywhere [3].

Although the future of telecommunication industries looks very promising, there are concerning issues regarding spectrum management that should be addressed immediately. The electromagnetic spectrum is a natural resource, the use of which for radio and television broadcasting, mobile wireless communications, and radar applications is regulated by government agencies. Unfortunately, several measurement studies conducted in North America [4–8] and elsewhere [9–11] have revealed that this precious resource is very much underutilized by the primary users. According to predictions made by the International Telecommunications Union and the Organization for Economic Cooperation and Development, unless serious actions are taken towards smart, efficient, and dynamic management of the electromagnetic spectrum, the worldwide mobile communication network will collapse by the year 2050. In order to allocate the spectrum dynamically and openly, future wireless devices should be service-oriented terminals, which are more compatible with computer systems and support unlocked and multiple wireless standards [2].

Cognitive science provides the tool for building a new generation of devices with dynamic applications. These cognitive machines will be able to build up their rules of behaviour over time through learning from experiential interactions with the environment. They should be able to deal with environmental uncertainties and properly perform tasks of different kinds in a wide range of environmental conditions. In other words, robustness must be a major design criterion. Although intelligence is considered as a computational problem, accurate study of the biological systems in general and especially the structure of the brain will provide a reliable guide for building cognitive machines. Therefore, computer science, biology and other related disciplines will play key roles in the newly emerged field of *cognitive dynamic systems* (CDS) [12]. Regarding the fact that cognitive science has its roots in *cybernetics* [13], it is critical for the success of this field of study to pay attention to the history and learn from it. This way, mistakes that led to the failure of cybernetics to some extent can be avoided [14].

Cognitive radio is a special class of cognitive machines. It offers a novel way of solving the spectrum utilization problem [15,16]. It solves the problem by, first, sensing the radio environment to identify those subbands of the electromagnetic spectrum that are underutilized and, second, providing the means for making those subbands available for employment by secondary users. Typically, the subbands allocated for wireless communications are the property of legally licensed owners, which, in turn, make them available to their own customers: the primary users. From the perspective of cognitive radio, underutilized subbands are referred to as *spectrum holes.* A spectrum hole is a band or subband of frequencies assigned to a primary user, but at a particular time and specific geographic location, it is not being utilized by that user, partially or fully.

Naturally, the entire operation of cognitive radio hinges on the availability of spectrum holes. The identification and exploitation of spectrum holes poses technical challenges rooted in computer software and hardware, signal processing, communication theory, control, optimization, and game theory, just to name a few disciplines. Moreover, the operation of cognitive radio is compounded further by the fact that the spectrum holes come and go in a rather stochastic manner.

The large number of heterogeneous elements in a cognitive radio network that interact with each other indirectly through the limited resources makes the cognitive radio network a *complex dynamic system* [1,16] or a *system of systems* [17]. In such an environment, each element is a decision-maker. Different degrees of coupling between different decision-makers of one tier or between decision-makers from different tiers influence their chosen policies. Change of policies affects the interaction between the decision makers and alters the degrees of coupling between them. In other words, both *upward and downward causations* [18] play key roles in a cognitive radio network and lead to positive or negative emergent behaviour, which is not explicitly programmed in different elements. Since the global behaviour of the network cannot be reduced to the local behaviour of different elements, taking an approach to build a dynamic model, which provides a global description of the network behaviour, is of critical importance. The fact that it is impractical to perform experiments with large decentralized wireless networks with hundreds or thousands of nodes in order to understand their global behaviour, highlights the importance of analytical approaches even more [19]. Such models enable us to predict the future and based on the obtained knowledge engineer it to improve network robustness against potential disruptions.

This research focuses on resource allocation in cognitive radio networks in which users access the available spectrum in an opportunistic manner. The goal of this research is to identify dominant sources of uncertainty in practical cognitive radio networks as thoroughly as possible and build analytical models that describe the behaviour of the network from a global perspective. Having identified the sources of uncertainty and established the underlying theory to predict the behaviour of the network, proper control policies will be proposed for risk management and improvement of network robustness. A theoretical framework is developed to address the multiple-time-scale decision making in cognitive radio networks based on the theory of double-layer dynamic systems.

1.2 Mathematical Toolbox

A principled basis for the dynamic allocation and management of resources in a cognitive radio network is developed based on the fusion of ideas from game theory, control theory, and optimization.

1.2.1 Game Theory

Game theory provides an analytical toolbox for modeling and analyzing situations in which multiple decision-makers (players) with possibly conflicting interests interact. *Rationality* and *strategically reasoning* are two basic assumptions in game theory. These assumptions reflect that each decision-maker has a well-defined objective and acts based on its knowledge or expectation of other decision-makers' behaviors [20].

In engineering, in many cases that deal with decentralized control systems, controllers are designed in a centralized manner and then implemented in a decentralized way [21, 22]. This method is not truly decentralized and may cause some problems in practice. Game theory provides a natural framework for analysis and design of truly decentralized control systems. John Nash's paper on "Parallel Control" is perhaps the pioneering work in this area [23]. Influenced by his earlier work on equilibria in non-cooperative games [24, 25], Nash proposed to build computers in which components work in a more autonomous way. Basar and Olsder's book on dynamic non-cooperative games [26] focuses more on control theoretic aspects and interprets optimal control problems as one-player games. Also, in [27] the robust control problem was interpreted as a zero-sum game in which the controller tries to maximize the system's utility while the environment is trying to minimize the system's utility.

In wireless networks the radio communication channel is usually shared between different transmitter-receiver (transceiver) pairs. In such environment, multiple users compete for limited resources and the behaviour of each user affects the performance of neighboring users. It is therefore not surprising that game theory has attracted the attention of many researchers in the field of communication networks especially those who are working on cognitive radio.

1.2.2 Control Theory

Control engineering is an exciting and challenging field with a multi-disciplinary nature and strong mathematical foundation. A control engineer's systematic insight can be easily extended to be utilized in other fields. The present challenge to control engineers is the modeling and control of modern, complex, and interrelated systems. To face this challenge, we need something dramatically different from traditional control techniques possibly new control structures coming out of the neuroscience world.

Control systems are found throughout nature at the levels of genes, proteins, cells, and entire systems [28]. Some of the natural control systems have unequaled degrees of sophistication [29]. Increased understanding of the scientific and engineering principles behind the living organisms as well as the way they interact with the world and learn from it will lead to fantastic breakthroughs in the design and application of intelligent machines that are truly cognitive.

A living organism interacts with nature through observation and action. Inspired by the perception-action cycle in the brain, a cognitive radio transceiver is built as a closed loop feedback system, which embodies the radio environment, radio-scene analyzer, feedback channel, and radio-environment actuator. Moreover, a cognitive radio network is a hybrid dynamic system with both continuous and discrete dynamics.

Therefore, cognitive radio networks have the potential for presenting a rich spectrum of dynamic behaviours.

1.2.3 Optimization

In a complex system such as a cognitive radio network, every decision-making process will be a multi-criteria optimization problem with possibly conflicting objectives [30]. In order to make certain rational decisions, a user needs to gather information and process it. Data acquisition and computation capabilities of users are limited and they can only make the best decisions regarding their knowledge and resources. Also, real life cognitive radios are subject to uncertainties that cannot necessarily be dealt by statistical analysis. In this environment, robust optimization provides an essential tool for making decisions based on worst-case conditions.

1.3 Vision for the Thesis

The thesis is organized as follows:

- Chapter 2 discusses the primary communication resources and the spectrum underutilization problem in the current communication networks. After mentioning the advantages of the OFDM scheme, a review of cognitive radio with emphasis on the cognitive-information-processing cycle, is followed by a discussion on the constraints imposed by the cognitive-radio environment.

- Chapter 3 studies different sources of uncertainty in cognitive radio networks and the two approaches that can be taken to deal with uncertainty in the context of transmit-power control; stochastic optimization and robust optimization.

The concept of robustness is reviewed and its importance for designs concerning complex and large-scale systems such as cognitive radio networks is emphasized.

- Chapter 4 studies the cognitive radio network dynamics with emphasis on the equilibrium behaviour of the network. Tools from information theory and optimization are employed to formulate the transmit-power-control problem in a cognitive radio network as a robust game. The equilibrium solution is found using the robust version of the iterative waterfilling algorithm (IWFA). IWFA is formulated as a variational inequality (VI) problem, which facilitates studying the existence and uniqueness of the equilibrium solution. Also, it paves the way for investigating the network behaviour in a dynamic framework.

- Chapter 6 studies the cognitive radio network dynamics with emphasis on the disequilibrium (transient) behaviour of the network. Tools from control theory are employed to find a differential equation, which governs the evolution of the network's state trajectory before reaching the equilibrium. The stationary points of this dynamic model coincide with the equilibrium points of the corresponding VI model, developed in Chapter 4. Stability of the network in the presence of perturbation and time delay is addressed. Also, the network is modeled as a hybrid dynamic system.

- Chapter 8 extends the theoretical framework developed in Chapter 4 and Chapter 6 to capture the multiple-time-scale nature of the cognitive radio network.

- Simulation results are presented in Chapters 5, 7, and 9. The testbed used for simulations is explained in Chapter 5. Chapters 5 and 7 present computer experiments for small-scale networks, which are carefully designed to highlight and clarify the key points of the theoretical frameworks developed in Chapters 4 and 6, respectively. Chapter 9 presents the computer experiment for a large-scale network with emphasis on the double-layer dynamics of cognitive radio networks.

- The thesis concludes in Chapter 10 by reviewing the contributions of the thesis to the literature.
- The Appendix provides the proofs of theorems and propositions.

Chapter 2

Cognitive Radio

2.1 Spectrum Utilization

The poor utilization of the spectrum is a result of current inefficient spectrum management policies. In November 2002, the Federal Communications Commission (FCC) published a report, aimed at improving the way in which this limited and precious resource is managed in the United States [4]. Since then spectrum occupancy measurement campaigns have been conducted in different countries (Table 2.1). However, the results highly depend on the sensing locations, the spectrum sensing method, and the chosen threshold to distinguish idle bands from occupied bands.

In the United States, measurements have shown that from January 2004 to August 2005, on average, only 5.2% of the radio spectrum was actually in use [5]. Measurements over a period of 2 days in November 2005 showed that the average spectrum occupancy in the band 30-3000 MHz was 13.1% and 17.4% for New York and Chicago, respectively [6]. In [7], the spectrum occupancy in the band 400-7200 MHz was compared for an urban area (Atlanta, Georgia) and a rural area (North Carolina). The

Table 2.1: Spectrum utilization in different countries

Country	Region	Frequency Range (MHz)	Usage (%)
USA	-	-	5.2
	New York	30-3000	13.1
	Chicago	30-3000	17.4
	Limestone	30-3000	1.7
	Atlanta	400-7200	6.5
	North Carolina (A Rural Aea)	400-7200	0.8
New Zealand	Auckland	806-2750	6.2
Singapore	-	80-5850	4.54
Qatar	Doha	700-3000	15.3

respective measurements were 6.5% and 0.8%. At the Loring Commerce Centre, Limestone, Maine, USA, measurements over a period of 3 days in the band 30-3000 MHz, showed that the average spectrum usage was 1.7%. Occupancy varied from less than 1% to 24.65% in different subbands. The maximum occupancy of 24.65% was reported for the band 470-512 MHz [8].

In Auckland, New Zealand, the spectrum occupancy was reported to be 6.2% over the frequency range 806-2750 MHz [9].

In Singapore, the average spectrum occupancy in the band 80-5850 MHz, based on measurements over a period of 12 days, was reported to be 4.54% [10].

In Doha, Qatar, measurements performed over a period of 3 days in the 700-3000 MHz frequency band showed that the spectrum utilization was 1% for the indoor environment and 15.3% for the outdoor environment [11].

In Aachen, Germany, measurements over a period of 7 days next to the main railway station in the band 20-3000 MHz, showed that the spectrum utilization was 32% for the indoor environment and about 100% for the outdoor environment. However, in such a place, the sensors were exposed to high-level ambient noise and the inability of the energy detectors to distinguish man-made noise from primary users' signals led to this unexpectedly high occupancy measurement [31]. We should be cautious about the spectrum sensing method that we adopt in order to avoid such misleading results.

The employed spectrum sensors should be able to detect spectrum holes, provide high spectral-resolution capability, estimate the average power in each subband of the spectrum, and identify the unknown directions of interfering signals. Cyclostationarity is another desirable property that could be used for signal detection and classification. Therefore, the multitaper method (MTM) for nonparametric spectral estimation was proposed in [32] as the method of choice for spectrum sensing in cognitive radio because it accomplishes these tasks accurately, effectively, robustly, and in a computationally feasible manner.

2.2 Primary Communication Resources

There are two primary resources in a cognitive radio network; channel bandwidth and transmit power. The operation of the transmit-power controller is complicated by a phenomenon that is peculiar to cognitive radio communication, namely, the fact that spectrum holes come and go, depending on the availability of subbands as permitted by licensed users. To deal with this phenomenon and thereby provide the means for improved utilization of the radio spectrum, a cognitive radio system must have the

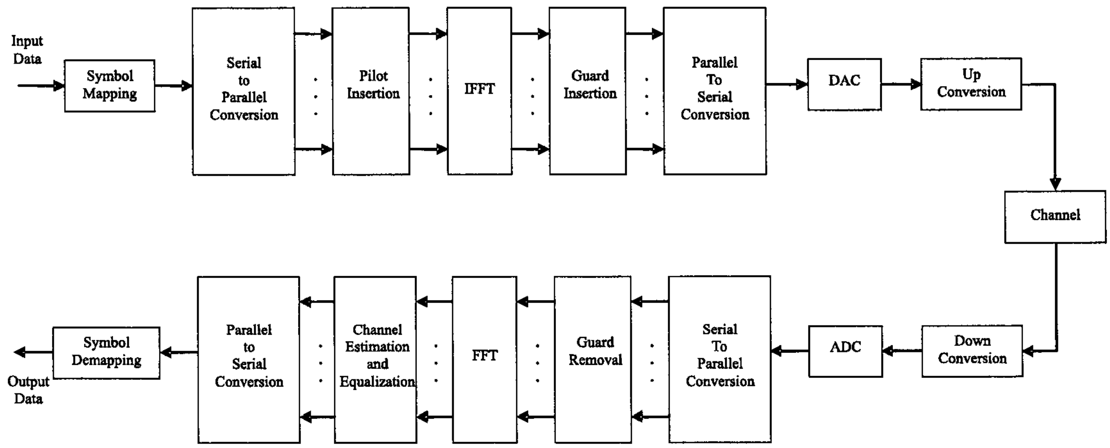


Figure 2.1: Block diagram of an OFDM transceiver.

ability to fill the spectrum holes rapidly and efficiently. In other words, cognitive radios have to be frequency-agile radios with flexible spectrum shaping abilities. The orthogonal frequency-division multiplexing (OFDM) scheme can provide the required flexibility, and is therefore a good candidate for cognitive radio [1,16,33–35]. OFDM can be employed in a cognitive radio network by dividing the primary user's unused bandwidth into a number of subbands available for use by the cognitive radio systems. In order to achieve low mutual interference between primary and secondary users, an adaptive transmit filter can be used to prevent usage of a set of subcarriers, which are being used by the primary users. Moreover, the fast Fourier transform (FFT) block in the OFDM demodulator (Figure 2.1) can be used for spectral analysis [33].

OFDM is a multi-carrier scheme in which a wideband signal is converted to a number of narrowband signals. Then closely-spaced orthogonal subcarriers are used to transmit these narrowband data segments simultaneously. In effect, a frequency selective fading channel is divided into a number of narrowband flat fading subchannels. OFDM has many advantages over single-carrier transmission [36–40]:

- It improves the efficiency of spectrum utilization by the simultaneous use of multiple orthogonal subcarriers, which are densely packed.

- The OFDM waveform is first built in the frequency domain and then it is transformed into the time domain, thereby providing flexible bandwidth allocation.

- *Interleaving* the information over different OFDM symbols provides robustness against loss of information caused by flat-fading and noise effects.

- Although the spectrum tails of subcarriers overlap with each other, at the center frequency of each subcarrier all other subcarriers are zero. Theoretically this prevents *inter-carrier interference* (ICI). However, time and frequency synchronization is critical for ICI prevention as well as correct demodulation, and is a major challenge in the physical layer design [41].

- Since a narrowband signal has a longer symbol duration than a wideband signal, OFDM takes care of *inter-symbol interference* (ISI) caused by multipath delay of wireless channels. However, guard time intervals, which are longer than the channel impulse response, are introduced between OFDM symbols to eliminate the ISI by giving enough time for each transmitted OFDM symbol to dissipate considerably [38].

- Due to the low ISI, less complex equalization is required at the receiver, which leads to a simpler receiver structure.

In summary, frequency diversity enables OFDM to provide higher data rates, more flexibility in controlling the waveform characteristics, and greater robustness against channel noise and fading compared to single-carrier transmission schemes.

Using the OFDM-based modulation scheme, the bandwidth allocation can be considered as a subcarrier assignment problem [38]. The resource management problem may then consist of subcarrier assignment and power control. While the availability of channel bandwidth depends on the communication patterns of primary users, a cognitive radio has complete control over its own transmit power. In other words, among the two primary resources, power is the only variable that can be manipulated by cognitive radio users. As mentioned previously, a subcarrier will not be assigned to a cognitive radio if its transmit power on that subcarrier is zero. Therefore, the resource-allocation problem can be reduced to the transmit-power control and can be considered as a distributed control problem. Scalable decentralized algorithms with reasonable computational complexity are naturally preferred.

2.3 Cognitive-Information-Processing Cycle

In signal-processing terms, a feature that distinguishes cognitive radio from conventional wireless communication, is the cognitive-information-processing cycle [1, 16]. This cycle applies to a secondary (unserviced) user, where a transmitter at one location communicates with a receiver at some other location via a spectrum hole, that is, a licensed subband of the radio spectrum that is underutilized at a particular point in time and at a particular location. The cognitive cycle encompasses two basic operations; radio-scene analysis of the surrounding wireless environment at the receiver, and dynamic spectrum management/transmit-power control at the transmitter. Information on spectrum holes and the forward channel's condition, extracted by the scene-analyzer at the receiver, is sent to the transmitter via a feedback channel. The feedback channel is a physical channel available to all cognitive radio users. It can be established in three ways [42]:

- A specific spectrum band is licensed and reserved as a dedicated universal channel for cognitive radios.
- Available spectrum holes are used by cognitive radios both for data transmission and feedback channel.
- Cognitive radio units establish their feedback channels using unlicensed bands.

The feedback channel can always be established using the unlicensed bands independent of the availability of spectrum holes. Also, unlike the universal feedback channel, it does not have the problem of spectrum licensing. Therefore, using the unlicensed bands for establishing the feedback channel is the best choice [42].

Dynamic spectrum manager solves a limited-resource distribution problem and is designed to dynamically assign available spectrum holes to cognitive radio units in a fair and efficient manner [42]. The information that transmitter receives through the feedback channel enables it to adaptively adjust the transmitted signal and update its transmit power over desired channels. Using a predictive model, the cognitive radio is enabled to predict the availability duration of spectrum holes, which, in turn, determines the horizon of the transmit-power control. The combination of the radio-scene analyzer, the feedback channel, the dynamic spectrum manager/transmit-power controller, and the wireless link constitutes a closed-loop feedback system as depicted in Figure 2.2 [1, 16].

The detailed-information-processing cycle of Figure 2.2 can be summarized as Figure 2.3. The cognitive-information-processing loop resembles the perception-action

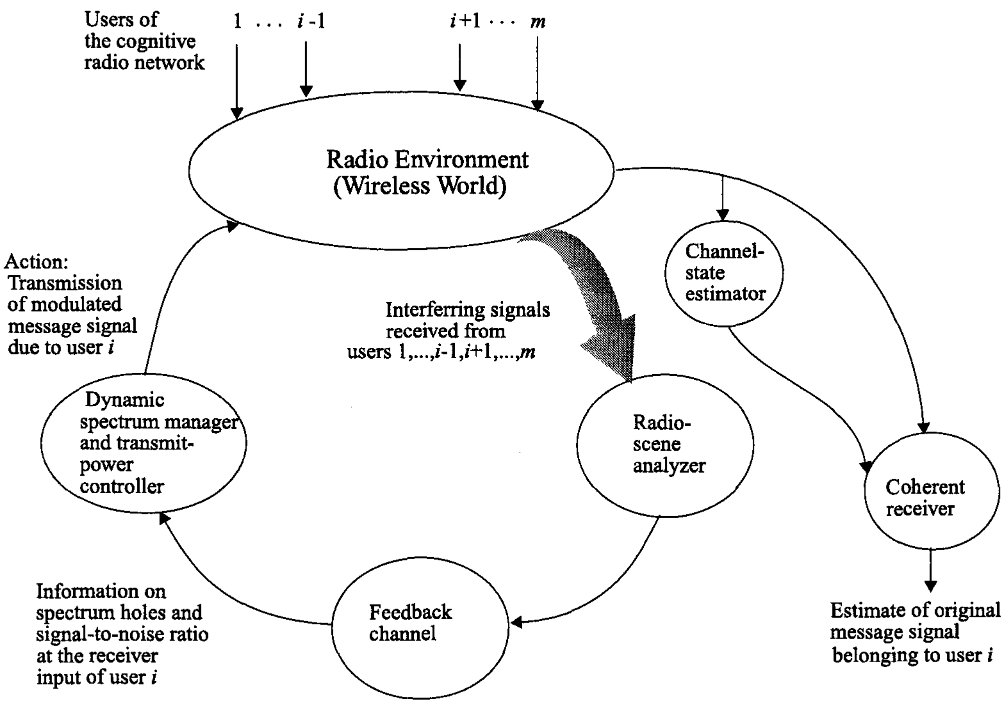


Figure 2.2: Basic information-processing cycle for user i in a cognitive radio network [1].

cycle in the brain. The radio-environment actuator performs dynamic spectrum management and transmit-power control.

2.4 Network of Cognitive Radios

In a cognitive radio network, the radio communication channel is shared between different transceivers and each user's action affects the performance of neighboring users while they compete for limited resources. At any instant of time, new users may join the network or old users may leave the network. Also, primary users may start or stop communication and therefore, they may occupy or release some frequency bands in a stochastic manner. All of these occurrences can be considered as discrete

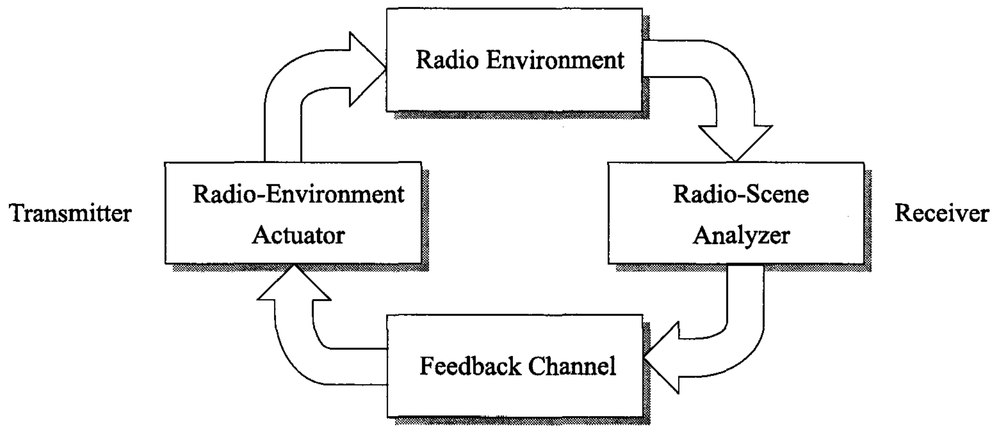


Figure 2.3: The cognitive-information-processing cycle in cognitive radio.

events compared to the real-time evolution of each user's power vector, which can be considered as evolving in continuous time. It follows therefore that the cognitive-radio problem is a mixture of continuous dynamics and discrete events. In other words, a cognitive radio network is a hybrid dynamic system of the sort described in [43,44].

The feedback channel will naturally introduce some delay in the control loop, and some of the users may use inaccurate or out-dated interference measurements to update their transmit powers. Also, they may update their transmit powers with different frequencies. Therefore, in a real-life situation, the resource-allocation algorithm would have to be implemented in a distributed asynchronous manner [45–48].

In a competitive multi-agent environment with limited resources such as a cognitive radio network, where the actions of all agents (users) are coupled via available resources, finding a global optimum for the resource-allocation problem can be computationally intractable and time consuming. Moreover, such optimization would require huge amounts of information exchange between different users that will consume precious resources. In a highly dynamic environment, where both users and resources can freely come and go, finding a reasonably good or "just right" solution (i.e. a suboptimal solution) that can be obtained fast enough is the only practical goal. Otherwise, spectrum holes may disappear before they can be utilized for communication. In such a situation, the concept of equilibrium is very important [49]. It is therefore not surprising that *game theory* has attracted the attention of many researchers in the field of communication networks.

Recently, several tutorials on game theory have been published for communication engineers. A nice survey on applications of game theory in wired communication systems is presented in [50]. The monograph [19] covers the non-cooperative game theory and in the final chapter mentions some research areas in wireless communications and networking that can benefit from game theoretic approaches. The technical report [51] explains the terminology of non-cooperative game theory using four simple examples from wireless communications. The concept of equilibria and the related theorems are presented in [52]. The tutorial paper [53] explains the cooperative game theory. In September 2008, IEEE Journal on Selected Areas in Communications published a special issue on game theory in communication systems and John Nash wrote a foreword for that issue. Also, in September 2009, IEEE Signal Processing Magazine published another special issue on game theory in signal processing and communications. The latter includes the mentioned tutorial papers on equilibria and cooperative games. Also, the references [54–58] are worth mentioning among the others for application of game theory in wireless communication systems and cognitive radio networks.

In game theory, the *Nash equilibrium* is considered to be a concept of fundamental importance. This equilibrium point is a solution such that none of the agents has an incentive to deviate from it unilaterally. In other words, in a Nash-equilibrium point, each user's chosen strategy is the "best response" to the other users' strategies [20,24, 25]. Regarding the highly time-varying nature of communication networks in general and especially cognitive radio networks, a Nash-equilibrium solution is a reasonable candidate, eventhough it may not always be the best solution in terms of spectral efficiency [59].

The above discussion reveals that several key attributes such as distributed implementation, low complexity, and fast convergence to a reasonably good solution, provide an intuitively satisfying framework for choosing and designing resource-allocation algorithms for cognitive radio. It is with this kind of framework in mind that in [1, 16, 60], the IWFA has been proposed as a good candidate for finding a Nash equilibrium solution for resource allocation in cognitive radio networks.

2.5 Problem Constraints

Regarding the coexistence of both primary and secondary users in certain subbands, there are two spectrum sharing schemes [61]:

- *Protective spectrum sharing* in which primary users do not allow coexistence of secondary users in their non-idle subbands. In OFDM scheme, secondary users should not transmit over non-idle subbands and perhaps some other contiguous subcarriers, which are used as guard bands.

- *Aggressive spectrum sharing* in which coexistence of primary and secondary users in the same subbands is allowed on the condition that interference power experienced by the primary user's receiver remain below a specified threshold.

A set of constraints must be imposed on each user's transmit power in each subcarrier to maintain a limit on the interference produced. In [62], a fixed limit on each user's transmit power in each subcarrier is considered in order to guarantee that all users transmit at low powers and do not cause high interference. However, this approach may be too conservative from spectral efficiency point of view especially when a subband is not crowded. In [63, 64], global and flexible constraints were proposed instead of individual and rigid constraints. The peak average interference tolerable by the primary user's receiver is used to put a limit on cognitive radios' transmit powers. The measurements are performed at the primary user's receiver and the results are sent to secondary users' transmitters. This approach requires information exchange between primary users and secondary users and can be used in a market-model spectrum-sharing regime that involves pricing. In [60], the *interference temperature* limit, which was proposed by FCC, was used as a local and flexible constraint. In the proposed approach, each user's receiver measures the interference power level on each subcarrier and sends the results to its corresponding transmitter through the feedback channel. The transmitter adjusts its transmit power vector in a way that it does not violate the *permissible interference power level* limit (interference temperature limit).

2.6 Summary

Reported experiments that show the poor utilization of the spectrum in different parts of the world were reviewed. OFDM scheme was mentioned as the method of choice for cognitive radio because of its flexible spectrum shaping abilities. The building blocks of the cognitive-information-processing cycle, which distinguishes cognitive radio from conventional wireless communications, were explained. Characteristics of a cognitive radio network, and the constraints that are imposed on different cognitive radios were presented.

Chapter 3

Robustness

"To be uncertain is to be uncomfortable, but to be certain is to be ridiculous."

Chinese proverb

According to the Institute of Electrical and Electronics Engineers (IEEE), "*the robustness of a system is the degree to which a system or component can function correctly in the presence of invalid inputs or stressful conditions*" [65].

3.1 The Concept of Robustness

Much too often in the literature, optimality is considered as the driving force for obtaining the best performance possible. Such an objective may well work satisfactorily when considering small-scale applications or toy problems. However, when the application of interest is of a complex or large-scale kind, exemplified by a cognitive radio network, we find ourselves confronted with a much more pressing system requirement: robustness.

Most, if not all, control design strategies exemplified by transmit-power control, are based on the selection of a model for the plant. Selection of the model is influenced by mathematical tractability and prior knowledge that we may have about the plant, a generic term used to describe part of a dynamic system that is supposed to be controlled. Unfortunately, no matter how hard we try and irrespective of all the prior knowledge we may have about the system, there will always be some discrepancy between the actual physical behaviour of the plant and the corresponding behaviour of the hypothetical model. The response produced at the output of the plant due to a prescribed input signal is determined by the underlying physics of the plant. On the other hand, when the corresponding behaviour of the plant is considered, the response of the model due to the same input signal deviates invariably from the actual response of the plant due to unavoidable model uncertainty. The challenge in designing the controller is to make sure that the errors are kept small enough to be acceptable from an operational viewpoint, regardless of all operating conditions that are likely to arise in practice.

3.2 Transmit-Power Control

In spectrum sensing that constitutes a basic cognitive function in the receiver, the issue of prime interest is that of variance versus bias of estimation [32]. When we go on to consider the associated cognitive function of transmit-power control in the transmitter, the issue of prime interest is robustness versus optimality [60].

In the context of cognitive radio, the physical plant represents the communication channel between the transmitter and receiver, the radio-scene analyzer plays the role of the sensor, and the radio-environment actuator is the controller. Since the sensor and the actuator are not collocated, they have to be connected by a physical feedback channel and the controller receives the sensor measurements via the feedback channel. Due to the different uncertainty sources in a cognitive radio network, adjusting the transmit power of a cognitive radio requires solving an optimization problem under uncertainty.

3.3 Dominant Sources of Uncertainty

The dominant sources of uncertainty in a cognitive radio network are:

- *Primary Users:* In a cognitive radio network, spectrum holes come and go, depending on the availability of idle subbands. Therefore, primary users' activities are the cause of *supply-side risk.* Communication patterns of primary users determine the availability and the duration of availability of resources. The availability of the spectrum holes determines the joint feasible set of the resource-allocation optimization problems that are solved by individual secondary users. In other words, it determines the joint set of the action spaces of all secondary users in the corresponding game. As mentioned before, the availability duration of spectrum holes determines the control horizon for the radio-environment actuators of secondary users. Depending on the subbands of interest and the dynamics of activities of primary users in those subbands, two different cases are observed:

    a) The activities of the primary users and therefore, their occupancy of the corresponding subbands are well-defined. A good example for this case would be the use of TV bands for cognitive radios.

b) The activities of the primary users and therefore, the appearance and disappearance of spectrum holes are more dynamic and far less predictable than the former case. A good example for this case would be the use of cellular bands for cognitive radios.

- *Secondary Users:* Anytime users can leave the network and new users can join the network in a stochastic manner. This is the cause of *demand-side risk* in the network.

- *Mobility:* Users move all the time. Because of the mobility, the interference that a user causes on other users and mutually the interference that other users cause on that particular user in the network are time-varying.

- *Multiple Time-Varying Delays:* The feedback channel plays a fundamental role in the design and operation of cognitive radio. Feedback may naturally introduce delay in the control loop and different transmitters may receive statistics of noise and interference with different time delays. Moreover, the sporadic feedback causes users to use out-dated statistics to update their power vectors. The time-varying delay in the control loop of each cognitive radio is another source of uncertainty that degrades the performance and may cause stability problems.

- *Noise:* The ambient noise depends on different activities in the environment and is caused by both natural and man-made phenomena.

3.4 Dealing with Uncertainty

During the time intervals that the activity of primary users does not change and the available spectrum holes are fixed, two approaches can be taken to deal with the uncertainty caused by joining and leaving of other cognitive radios as well as their mobility; *stochastic optimization* and *robust optimization* [66]. The pros and cons of these two approaches are discussed here.

If there is good knowledge about the probability distribution of the uncertainty sources, then the uncertainty can be dealt with by means of probability and related concepts. In this case, calculation of the expected value will not be an obstacle and therefore, transmit-power control can be formulated as a stochastic optimization problem.

However, since in practice, little may be known about the probability distribution, the stochastic optimization approach that utilizes the expected value is not a suitable approach. In this case, robust optimization techniques that are based on worst-case analysis, without involving probability theory, are more appropriate, although such techniques may well be overly conservative in practice. Suboptimality in performance is, in effect, traded in favor of robustness.

Stochastic optimization guarantees some level of performance on average, and sometimes the desired quality of service may not be achieved, which means a lack of reliable communication. On the other hand, robust optimization guarantees an acceptable level of performance under worst-case conditions. It is a conservative approach because real-life systems are not always in their worst behaviour, but it can provide seamless communication even in the worst situations. Regarding the dynamic nature of the cognitive radio network and the delay introduced by the feedback channel, the statistics of interference that is used by the transmitter to adjust its power may not represent the current situation of the network. In these cases, robust optimization is equipped to prevent permissible interference power level violation by taking into account the worst-case uncertainty in the interference and noise. Therefore, sacrificing optimality for robustness seems to be a reasonable proposition. However, the use of a predictive model may make it possible for the user to choose the uncertainty set adaptively according to environmental conditions and therefore, may lead to less conservative designs.

3.5 Summary

The concept of robustness and its formal definition were reviewed. The dominant sources of uncertainty in a cognitive radio network were identified. Since the resource-allocation problem in a cognitive radio network is an optimization problem under uncertainty, stochastic and robust optimization can be used to address the uncertainty issue. It was mentioned that robust optimization would be a better choice for the problem at hand.

Chapter 4

Network Dynamics Viewed from Information-Theoretic and Optimization Perspectives

There are two ways to build a cognitive radio network, one being evolutionary and the other revolutionary. In the evolutionary viewpoint, the currently established communication infrastructures can be utilized and cognitive radio networks are built around existing base stations. In this framework, the base stations or spectrum brokers [67] are responsible for assigning channels to cognitive radios; well-known algorithms proposed in the multi-cellular network literature for distributed optimal data rate and power control can be employed [46, 68–73]. On the other hand, in the revolutionary viewpoint, which is the focus of this research, there are no base stations or communication infrastructures; hence, channel assignment and power control would have to be performed jointly. As mentioned in Chapter 2, the IWFA is a potentially good candidate for resource-allocation in cognitive radio networks because of its low complexity, fast convergence, distributed nature, and convexity.

4.1 Waterfilling Interpretation of Information Capacity Theorem

Capacity is interpreted as the ability of a channel to convey information and is related to the noise characteristic of the channel. Shannon's capacity theorem [74] defines the fundamental limit on the rate of error-free transmission over a noisy communication channel. The information capacity of a channel is defined as the maximum of the *mutual information* between the channel input and the channel output over all distributions on the input that satisfy the power constraint [75, 76].

However, capacity is a theoretical ultimate transmission rate for reliable communication over a noisy channel. In practice, depending on the acceptable probability of error, there is a gap between the channel capacity and what is achievable by a practical coding and modulation scheme, called signal-to-noise ratio (SNR) gap, Γ, which is zero at theoretical capacity [77].

The information capacity of a continuous channel of bandwidth B Hz, perturbed by additive white Gaussian noise of power spectral density $N_0/2$ and limited in bandwidth to B, is given by

$$C = B \log_2\left(1 + \frac{P}{N_0 B}\right) \tag{4.1}$$

where P is the average transmitted power. The above formula reveals the interplay among three key parameters; channel bandwidth, average transmitted power, and noise power spectral density. While the dependence of the information capacity, C, on channel bandwidth, B, is linear, its dependence on SNR, P/N_0B, is logarithmic. Therefore, it will be easier to increase the information capacity of a communication channel by expanding its bandwidth rather than increasing the transmit power for a prescribed noise variance [76].

In a cognitive radio network the communication channel is often shared between several transmitter-receiver pairs and information exchange between each pair interferes with the communication between the others. Such a channel is called an *interference channel* [78]. The capacity of interference channels is poorly understood even for simple cases. The set of all possible data rates achievable by all users, is called the *rate region.* The sum-rate expression is a non-convex function and finding the optimal power allocations for different users that guarantees the global maximum sum-rate is in general an NP-hard problem [79, 80].

Instead of solving the optimization problem globally, we settle for a suboptimal solution by viewing the problem as a non-cooperative game [77]. The competing users try to maximize their data rates greedily by distributing their powers in a channel above the noise level but below a constant level determined by the permissible interference level (Figure 4.1). It is called the *waterfilling (pouring)* interpretation in the sense that the process by which power is distributed is identical to the way in which water distributes itself in a vessel [76].

4.2 Iterative Waterfilling Algorithm (IWFA)

The IWFA was originally developed for digital subscriber lines (DSL) [81–83]. In this algorithm, users sequentially update their transmit power vectors over the available

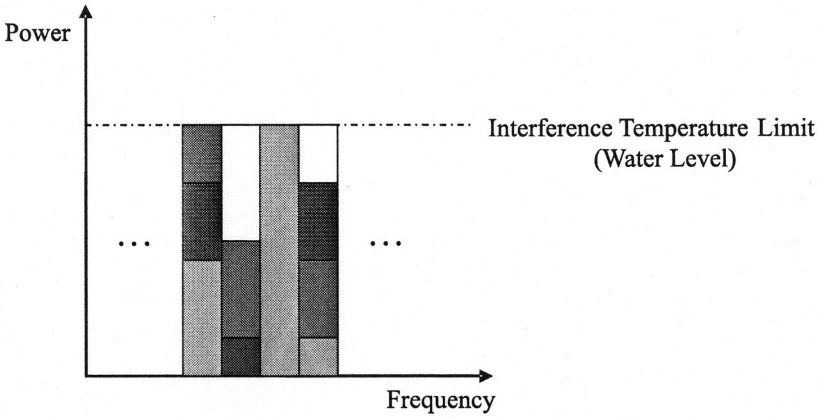


Figure 4.1: Waterfilling interpretation of the information-capacity theorem.

frequency tones in a fixed updating order, considering the transmit power of other users as interference. The sequential nature of the algorithm requires some form of central scheduling to determine the order in which users update their transmit powers [47,48]. In a cognitive radio network, which is an infrastructure-less network, such a central scheduling does not exist and also synchronization between the nodes is difficult. Therefore, users update their transmit powers in a totally asynchronous manner. The IWFA is well-suited for cognitive radio networks. In particular, the practical virtues of the algorithm are:

- The transmit power control problem is formulated as a *game* or a distributed *convex optimization* problem;
- It is implemented in a *decentralized* manner;
- The algorithm converges fast; it has a *linear convergence* property under certain conditions [84];

- Each user acts greedily to optimize its own performance based on local information, and the users do not need to communicate with each other to establish coordination between themselves. This tends to *reduce the complexity* of the cognitive radio network.

Finding a Nash equilibrium for the DSL game was reformulated as a *nonlinear complementarity problem* (NCP) in [85]. In an NCP, the vector $\mathbf{x} \in \mathbb{R}^n$, should be found such that

$$\mathbf{x} \geq 0, \quad \mathbf{F}(\mathbf{x}) \geq 0, \quad \mathbf{x}^T\mathbf{F}(\mathbf{x}) \geq 0 \tag{4.2}$$

where $\mathbf{F}$ is a nonlinear mapping from $\mathbb{R}^n$ to $\mathbb{R}^n$. The problem will be a *linear complementarity problem* (LCP) if $\mathbf{F} = \mathbf{M}\mathbf{x} + \mathbf{q}$ for a matrix $\mathbf{M}$ and a vector $\mathbf{q}$ with appropriate dimensions [86]. In [84], the DSL game problem was reformulated as an LCP. Reformulation of the IWFA as an NCP and an LCP provides very interesting insights into this problem such as establishing the linear convergence under certain conditions on interference gains. Also, conditions on interference gains are obtained to guarantee convergence of the algorithm to a unique Nash-equilibrium point [47,48,82,84]. However, the algorithm has some drawbacks:

- It is suboptimal;
- It is defenseless against clever selfish users that try to exploit dynamic changes or limited resources.

Moreover, regarding the dynamic nature of the cognitive radio environment and the speed of changes, the current transmit power values may not provide a good initial point for the next iteration. In this case, it may be better to start the iterative procedure from a randomly picked initial point in the new feasible set.

In what follows, the resource-allocation problem in cognitive radio networks is presented in the IWFA framework. While the predictive model can help for dealing with the appearance and disappearance of spectrum holes, robustification of the algorithm is proposed to address the issues related to unavoidable changes in the number of users and their mobility.

Assume that there are m active cognitive radio transmitter-receiver pairs in the region of interest, and n subcarriers in an OFDM framework could potentially be available for communication. Let PS denote the subset of subcarriers that are being used by primary users and cannot be assigned to cognitive radios. Since spectral efficiency is the main goal of cognitive radio, the utility function chosen by each user to be maximized is the data rate. Thus, the IWFA lets user i solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{p}^i} \quad & f^i\left(\mathbf{p}^1, \ldots, \mathbf{p}^m\right) = \sum_{k=1}^{n} \log_2\left(1 + \frac{p_k^i}{I_k^i}\right) \\ \text{subject to} \quad : \quad & \sum_{k=1}^{n} p_k^i \leq p_{\max}^i \\ & p_k^i + I_k^i \leq CAP_k, \quad \forall k \notin PS \\ & p_k^i = 0, \quad \forall k \in PS \\ & p_k^i \geq 0 \end{aligned} \tag{4.3}$$

Sometimes, this formulation is called rate-adaptive waterfilling. p_k^i denotes user i's transmit power on subcarrier k. The noise plus interference experienced by user i on subcarrier k because of the transmission of other users is:

$$I_k^i = \sigma_k^i + \sum_{j \neq i} \alpha_k^{ij} p_k^j \tag{4.4}$$

Since cognitive radio is *receiver centric*, I_k^i is measured at receiver i.

The positive parameter σ_k^i is the normalized background noise power at user i's receiver input on the kth subcarrier. The non-negative parameter α_k^{ij} is the normalized interference gain from transmitter j to receiver i on subcarrier k and we have $\alpha_k^{ii} = 1$. The term α_k^{ij} is the combined effect of two factors:

- Propagation path-loss from transmitter j to receiver i on subcarrier k.
- Subcarrier amplitude reduction due to the frequency offset Δf.

Mathematically α_k^{ij} is defined as

$$\alpha_k^{ij} = \frac{\Gamma |h_k^{ij}|^2}{|h_k^{ii}|^2} \tag{4.5}$$

where Γ is the SNR gap and h_k^{ij} is the channel gain from transmitter j to receiver i over the flat-fading subchannel associated with subcarrier k. Regarding the empirical formula for the path loss [87], we have

$$|h_k^{ij}|^2 = \frac{\beta_k}{(d_{ij})^r} \tag{4.6}$$

where d_{ij} is the distance from transmitter j to receiver i. The *path-loss exponent*, r, varies from 2 to 5, depending on the environment, and the *attenuation parameter*, β_k, is frequency dependent. Therefore,

$$\alpha_k^{ij} \propto \left(\frac{d_{ii}}{d_{ij}}\right)^r \tag{4.7}$$

and in general $\alpha_k^{ij} \neq \alpha_k^{ji}$. If user i's receiver is closer to its transmitter compared to other active transmitters in the network, we will have $\alpha_k^{ij} \leq 1$.

Also, $p^i_{\max}$ is user i's maximum power and CAP_k is the maximum allowable interference on subcarrier k. CAP_k is determined in a way to make sure that the permissible interference power level limit will not be violated at the primary users' receivers [1, 16]. The previously mentioned properties of IWFA are more elaborated in what follows based on the mathematical formulation of (4.3).

In IWFA, user i assumes that p_k^j is fixed for $j \neq i$. Therefore, the optimization problem in (4.3) is a concave maximization problem in $\mathbf{p}^i = [p_1^i, \ldots, p_n^i]^T$, which can be converted to a convex minimization problem by considering $-f^i$ as the objective. The first constraint states that the total transmit power of user i on all subcarriers should not exceed its maximum power (power budget). The second constraint set guarantees that the interference caused by all users on each subcarrier will be less than the maximum allowed interference on that subcarrier. If primary users do not let the secondary users use the non-idle subcarriers in their frequency bands, then cognitive radios should not use those subcarriers for transmission. The third constraint set guarantees this by forcing the related components of the secondary user i's power vector to be zero. If primary users let the coexistence of secondary users on non-idle subcarriers in condition that they do not violate the permissible interference power level, then the third constraint set can be relaxed and the second constraint set suffices.

As mentioned previously, IWFA is implemented in a decentralized manner. In order to solve the optimization problem (4.3), it is not necessary for user i to know the value of p_k^j for $\forall j \neq i$. The I_k^i defined in (4.4) is measured by user i's receiver rather than calculated, and therefore users do not need to exchange information. It is not even necessary for user i to know the number of other users in the network. Therefore, changing the number of users in the network does not affect the complexity of the optimization problem that should be solved by each user. Hence, there is not any scaling problem.

While the action of user i is denoted by its power vector $\mathbf{p}^i$, following the notation in the game theory literature, the joint actions of the other $m-1$ users are denoted by $\mathbf{p}^{-i}$. Three major types of adjustment schemes, $\mathcal{S}$, can be used by the users to update their actions [26]:

(i) Iterative waterfilling [81–83]: users update their actions in a predetermined order [26]:

$$\mathbf{p}^{-i}(\mathcal{S}_t) = [\mathbf{p}^1(t+1), \cdots, \mathbf{p}^{i-1}(t+1), \mathbf{p}^{i+1}(t), \cdots, \mathbf{p}^m(t)] \tag{4.8}$$

(ii) Simultaneous iterative-waterfilling [47]: users update their actions simultaneously regarding the most recent actions of the others [26]:

$$\mathbf{p}^{-i}(\mathcal{S}_t) = \mathbf{p}^{-i}(t) \tag{4.9}$$

(*iii*) Asynchronous iterative-waterfilling [48] is an instance of an adjustment scheme that user i receives update information from user j at random times with delay [26]:

$$\mathbf{p}^{-i}(\mathcal{S}_t) = [\mathbf{p}^1(\tau_t^{i,1}), \cdots, \mathbf{p}^{i-1}(\tau_t^{i,i-1}), \mathbf{p}^{i+1}(\tau_t^{i,i+1}), \cdots, \mathbf{p}^m(\tau_t^{i,m})] \tag{4.10}$$

where $\tau_t^{i,j}$ is an integer valued random variable satisfying

$$\max(0, t-d) \leq \tau_t^{i,j} \leq t+1 \quad j \neq i \quad i,j \in \mathbb{N} \tag{4.11}$$

which means that the delay does not exceed d time units.

Due to lack of central scheduling and difficulty of synchronization between different users in a cognitive radio network, the asynchronous adjustment scheme is more realistic than the other two.

4.3 IWFA as a Multi-Stage Optimization Problem in light of System Uncertainties

Since a cognitive radio network is a hybrid dynamic system, policies are defined on the event space as well as on the state space and therefore, each user needs to solve the corresponding optimization problem in two stages based on events and states.

4.3.1 Event-Based Optimization

A set of state transitions is called an event. Events determine the dimension of the state space. When primary users stop communication, they release subbands, which can be used by cognitive radios. This event increases the dimension of the optimization problems that are solved by secondary users. On the other hand, when primary users start communication, they occupy subbands. This event decreases the dimension of the optimization problems that are solved by secondary users. Each user's dynamic spectrum manager chooses a set of appropriate channels for communication. Finding the optimal set of channels for each user is equivalent to the well-known graph colouring problem in graph theory [42]. In [88] a novel self-organizing spectrum management scheme is proposed, which uses *Hebbian learning* [89, 90] and solves the problem in a decentralized manner. This way, cognitive radios will be able to learn communication patterns of the primary users and build a predictive model, which determines the control horizon for the transmit-power controller. In the time intervals between such events, the state dimension of each user remains unchanged and state-based optimization is performed to find the optimal transmit power vector.

4.3.2 State-Based Optimization

In the time intervals in which the available spectrum holes are fixed, the cognitive radio environment still has a dynamic nature, secondary users move all the time, they can leave the network and new users can join the network in a stochastic manner. Because of these activities, the interference plus noise term (4.4) in the objective function and the second constraint set of the optimization problem (4.3) are both time-varying; the IWFA therefore assumes the form of an optimization problem under uncertainty. As mentioned in Chapter 3, stochastic and robust optimization can be employed to deal with the uncertainty caused by joining and leaving of other cognitive radios as well as their mobility. After discussing the pros and cons of these two approaches, it was concluded that the robust optimization is a more reasonable approach, hence the material that follows.

4.4 Robust IWFA

Because of different sources of uncertainty, the noise plus interference term is the summation of two components: a nominal term, $\bar{I}$, and a perturbation term, ΔI, as

$$I_k^i = \bar{I}_k^i + \Delta I_k^i \tag{4.12}$$

In the following, the objective functions for both stochastic and robust versions of the optimization problem (4.3) are presented.

If there is good knowledge about the probability distribution of the uncertainty term, ΔI, the IWFA problem (4.3) can be formulated as a stochastic optimization problem with the following objective function.

$$\max_{\mathbf{p}^i} \left[\mathbb{E}_{\mathbf{\Delta I}^i} \sum_{k=1}^{n} \log_2 \left(1 + \frac{p_k^i}{\bar{I}_k^i + \Delta I_k^i} \right) \right] \tag{4.13}$$

where $\mathbb{E}$ denotes the statistical expectation operator and

$$\mathbf{\Delta I}^i = \left[\Delta I_1^i, \ldots, \Delta I_n^i \right]^T \tag{4.14}$$

The formulation of IWFA as a robust game in the sense described in [91] is basically a *max-min* problem in which each user tries to maximize its own utility while the environment and the other users are trying to minimize that user's utility [27,92]. Worst-case interference scenarios have been studied for DSL in [93]. Considering an ellipsoidal uncertainty set, the IWFA problem (4.3) can be formulated as the following robust optimization problem.

$$\begin{aligned}
\max_{\mathbf{p}^i} \quad & \left[\min_{\|\mathbf{\Delta I}^i\| \leq \varepsilon} \sum_{k=1}^{n} \log_2 \left(1 + \frac{p_k^i}{\bar{I}_k^i + \Delta I_k^i} \right) \right] \\
\text{subject to} \; : \quad & \sum_{k=1}^{n} p_k^i \leq p_{\max}^i \\
& \max_{\|\mathbf{\Delta I}^i\| \leq \varepsilon} \left(p_k^i + \bar{I}_k^i + \Delta I_k^i \right) \leq CAP_k, \quad \forall k \notin PS \\
& p_k^i = 0, \quad \forall k \in PS \\
& p_k^i \geq 0
\end{aligned} \tag{4.15}$$

A larger ε accounts for larger perturbations, and the second set of constraints guarantee that the permissible interference power level will not be violated for any perturbation from the considered uncertainty set.

4.4.1 The Cost of Robustness

In addition to conservativism, there is yet another price to be paid for achieving robustness. Although the IWFA problem (4.3) is a convex optimization problem, appearance of the perturbation term, ΔI, in the denominator of signal-to-interference plus noise ratio (SINR) in the objective function of the robust IWFA problem (4.15), makes it a non-convex optimization problem. A robust optimization technique is proposed in [94] for solving non-convex and simulation-based problems. The proposed method is based on the assumption that the cost and constraints as well as their gradient values are available. The required values can even be provided by numerical simulation subroutines. It operates directly on the surface of the objective function, and therefore does not assume any specific structure for the problem. In this method, the robust optimization problem is solved in two steps, which are applied repeatedly in order to achieve better robust designs.

- *Neighborhood search*: The algorithm evaluates the worst outcomes of a decision by obtaining knowledge of the cost surface in a neighborhood of that specific design.
- *Robust local move*: The algorithm excludes neighbors with high costs and picks an updated design with lower estimated worst-case cost. Therefore, the decision is adjusted in order to counterbalance the undesirable outcomes.

Linearity of constraints of the robust optimization problem (4.15), especially the second set of constraints that involves the perturbation terms, improves the efficiency of the algorithm.

4.5 Reformulation of IWFA as a Variational Inequality (VI) Problem

A Nash equilibrium game can be reformulated as a VI problem [95–97]. To be specific, Denoting the feasible set of (4.3) by K^i, we may rewrite the optimization problem (4.3) as

$$\begin{aligned} \min_{\mathbf{p}^i} \quad & -f^i\left(\mathbf{p}^1, \ldots, \mathbf{p}^m\right) \\ \text{subject to} \quad : \quad & \mathbf{p}^i \in K^i \end{aligned} \tag{4.16}$$

We recall the following theorem from [96, 97].

Theorem 4.1: Let K^i be a closed convex subset of $\mathbb{R}^n$ and $-f^i$ be a convex and continuously differentiable function in $\mathbf{p}^i$ for $i = 1, \cdots, m$. $\mathbf{p}^* = \left[\mathbf{p}^{*1^T}, \cdots, \mathbf{p}^{*m^T}\right]^T$ is a Nash equilibrium of the game if, and only if, it is a solution of the following VI problem VI$(K, \mathbf{F})$:

$$(\mathbf{p} - \mathbf{p}^*)^T \mathbf{F}(\mathbf{p}^*) \geq 0 \tag{4.17}$$

where

$$\mathbf{F}(\mathbf{p}) = -\left[\nabla_{\mathbf{p}^i} f^i\right]_{i=1}^m \tag{4.18}$$

and

$$\begin{aligned} K = \{\mathbf{p} \in \mathbb{R}^{m \times n} |\ & p_k^i = 0, \ \ \forall k \in PS, \forall i = 1, \ldots, m; \\ & 0 \leq p_k^i + I_k^i \leq CAP_k, \ \ \forall k \notin PS, \forall i = 1, \ldots, m; \\ & \sum_{k=1}^n p_k^i \leq p_{\max}^i, \ \ \forall i = 1, \ldots, m\} \end{aligned} \tag{4.19}$$

Calculating the gradients in (4.18) leads to fractional terms with the sum of the power and interference plus noise in the denominators:

$$\begin{aligned} \nabla_{\mathbf{p}^i} f^i &= \left[\frac{1}{p_1^i + I_1^i}, \cdots, \frac{1}{p_n^i + I_n^i}\right]^T \\ &= \left[\frac{1}{\sigma_1^i + \sum_{j=1}^m \alpha_1^{ij} p_1^j}, \cdots, \frac{1}{\sigma_n^i + \sum_{j=1}^m \alpha_n^{ij} p_n^j}\right]^T \end{aligned} \tag{4.20}$$

Alternatively, following the approach of [84], a nice formulation of the IWFA as a VI problem is obtained that facilitates study of the network in a dynamic framework.

The discussion presented in this section is built on [84], and extends the proposed reformulation of IWFA to the cognitive-radio problem. In particular, we are allowed to utilize some existing mathematical tools and also benefit from ongoing research in other fields. The *Lagrangian* of the optimization problem in (4.3) for the user i is now written as

$$L^i\left(\mathbf{p}^1,\ldots,\mathbf{p}^m\right) = -f^i + u^i\left(\sum_{k=1}^{n} p_k^i - p_{\max}^i\right) + \sum_{k\notin PS}\gamma_k^i\left(\sigma_k^i + \sum_{j=1}^{m}\alpha_k^{ij}p_k^j - CAP_k\right) + \sum_{k\in PS}\lambda_k^i p_k^i \tag{4.21}$$

Therefore, we have

$$\begin{cases} \gamma_k^i = 0, \lambda_k^i > 0 & k \in PS \\ \lambda_k^i = 0, \gamma_k^i > 0 & k \notin PS \end{cases} \tag{4.22}$$

The Karush-Kuhn-Tucker (KKT) conditions [98–100] for user i and $\forall k = 1,\ldots,n$ are as follows:

$$\begin{aligned} & 0 \le p_k^i \;\perp\; -\frac{1}{\sigma_k^i + \sum_{j=1}^{m}\alpha_k^{ij}p_k^j} + u^i + \gamma_k^i + \lambda_k^i \ge 0 \\ & 0 \le u^i \;\perp\; p_{\max}^i - \sum_{k=1}^{n} p_k^i \ge 0 \\ & 0 \le \gamma_k^i \;\perp\; CAP_k - \sigma_k^i - \sum_{j=1}^{m}\alpha_k^{ij}p_k^j \ge 0, \;\; \forall k \notin PS \\ & p_k^i = 0, \;\; \forall k \in PS \end{aligned} \tag{4.23}$$

where "⊥" signifies orthogonality of the corresponding variables.

Regarding the availability of spectrum for secondary usage, two cases may happen. If the network faces spectrum scarcity, some of the users may not be able to transmit with their maximum powers. Then, the first constraint in (4.3) will be redundant for those particular users. On the other hand, if the available spectrum is enough for all of the users to transmit with their maximum powers, the following inequality will be satisfied.

$$\sum_{j=1}^{m} p^j_{\max} < \sum_{k \notin PS} (CAP_k - \sigma_k^{\max}) \tag{4.24}$$

where $\sigma_k^{\max}$ is the maximum normalized background noise power on subcarrier k. In this case, similar to Proposition 1 of [84], which was proved for DSL, it can be shown that the system described in (4.23) is equivalent to a *mixed linear complementarity system* (mixture of a linear complementarity problem with a system of linear equations) [101].

Proposition 4.1: Suppose that (4.24) holds, then the system (4.23) is equivalent to the following mixed linear complementarity system:

$$\begin{aligned}
&0 \le p^i_k \perp \sigma^i_k + \sum_{j=1}^{m} \alpha^{ij}_k p^j_k + \nu^i + \varphi^i_k + \varsigma^i_k \ge 0 \\
&0 \le \varphi^i_k \perp CAP_k - \sigma^i_k - \sum_{j=1}^{m} \alpha^{ij}_k p^j_k \ge 0, \ \ \forall k \notin PS \\
&p^i_{\max} - \sum_{k=1}^{n} p^i_k = 0 \\
&p^i_k = 0, \ \ \forall k \in PS
\end{aligned} \tag{4.25}$$

where

$$\begin{aligned} \nu^i &= -\frac{1}{u^i} \\ \varphi_k^i &= \frac{\gamma_k^i \left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)}{u^i} \\ \varsigma_k^i &= \frac{\lambda_k^i \left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)}{u^i} \end{aligned} \tag{4.26}$$

and

$$\begin{aligned} u^i &= -\frac{1}{\nu^i} \\ \gamma_k^i &= -\frac{\varphi_k^i}{\nu^i \left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)} \\ \lambda_k^i &= -\frac{\varsigma_k^i}{\nu^i \left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)} \end{aligned} \tag{4.27}$$

While each user solves the above *mixed linear complementarity problem* (MLCP) with time-varying constraints, they should finally reach an equilibrium. The linear equation in (4.25) dictates that each user transmits with its maximum power, which leads to the worst-case interference condition. Intuitively it makes sense that each user transmits with its maximum power in order to achieve maximum data rate.

In the most general case, where (4.24) is not valid, some of the users in the network will be able to transmit with their maximum powers and the others will not. We define two sets, M_1 and M_2, which include these two groups of users, respectively. Intuitively speaking, when users adjust their power vectors based on rate-adaptive waterfilling (4.3) in which they try to maximize their data rates subject to power constraints, they either transmit with their maximum power or with the highest power permitted by the interference limits. In the case of spectrum scarcity, where (4.24) is not valid, for user $i \in M_1$, which is able to transmit with its maximum power, $u^i > 0$ and we have:

Proposition 4.2: Suppose that (4.24) is not valid and user i is able to transmit with its maximum power, then the system (4.23) is equivalent to the mixed linear complementarity system (4.25).

On the other hand, when user i cannot transmit with its maximum power, the first constraint in (4.3) will be redundant and $u^i = 0$. The KKT conditions in (4.23) are reduced to:

$$\begin{aligned}
&0 \leq p_k^i \ \perp \ -\frac{1}{\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j} + \gamma_k^i + \lambda_k^i \geq 0 \\
&0 \leq \gamma_k^i \ \perp \ CAP_k - \sigma_k^i - \sum_{j=1}^m \alpha_k^{ij} p_k^j \geq 0, \quad \forall k \notin PS \\
&p_k^i = 0, \quad \forall k \in PS
\end{aligned} \tag{4.28}$$

In this case, we have:

Proposition 4.3: Suppose that (4.24) is not valid and the first constraint in (4.3) can be relaxed for user i, then the system (4.28) is equivalent to the following mixed linear complementarity system:

$$\begin{aligned}
&0 \leq p_k^i \ \perp \ \sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j + \varphi_k^i + \varsigma_k^i \geq 0 \\
&\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j = CAP_k, \quad \forall k \notin PS \\
&p_k^i = 0, \quad \forall k \in PS
\end{aligned} \tag{4.29}$$

where

$$\begin{aligned}\varphi_k^i &= -\frac{1}{\gamma_k^i} \\ \varsigma_k^i &= \frac{\lambda_k^i\left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)}{\gamma_k^i}\end{aligned} \tag{4.30}$$

and

$$\begin{aligned}\gamma_k^i &= -\frac{1}{\varphi_k^i} \\ \lambda_k^i &= -\frac{\varsigma_k^i}{\varphi_k^i\left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)}\end{aligned} \tag{4.31}$$

The linear equation in (4.29) suggests that, when user i cannot transmit with its maximum power, it transmits with the highest permissible power, dictated by the interference temperature limit. Again, intuitively it makes sense.

Users that belong to M_1 solve the MLCP (4.25) and users that belong to M_2 solve the MLCP (4.29). Let us concatenate the corresponding variables for all users as follows:

$$\mathbf{p} = \left[\mathbf{p}^i\right]_{i=1}^m = \begin{bmatrix} \begin{bmatrix} p_1^1 \\ \vdots \\ p_n^1 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} p_1^m \\ \vdots \\ p_n^m \end{bmatrix} \end{bmatrix} \tag{4.32}$$

$$\boldsymbol{\sigma} = \left[\boldsymbol{\sigma}^i\right]_{i=1}^{m} = \begin{bmatrix} \begin{bmatrix} \sigma_1^1 \\ \vdots \\ \sigma_n^1 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \sigma_1^m \\ \vdots \\ \sigma_n^m \end{bmatrix} \end{bmatrix} \tag{4.33}$$

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}^{11} & \cdots & \mathbf{M}^{1m} \\ \vdots & \cdots & \vdots \\ \mathbf{M}^{m1} & \cdots & \mathbf{M}^{mm} \end{bmatrix} \tag{4.34}$$

where $\mathbf{M}^{ij}$s are diagonal matrices

$$\mathbf{M}^{ij} = \begin{bmatrix} \alpha_1^{ij} & \cdots & 0 \\ \vdots & \cdots & \vdots \\ 0 & \cdots & \alpha_n^{ij} \end{bmatrix} \tag{4.35}$$

The MLCPs (4.25) and (4.29) are the KKT conditions for an *affine variational inequality* (AVI) problem [96], defined by the affine mapping

$$\mathbf{F}(\mathbf{p}) = \boldsymbol{\sigma} + \mathbf{M}\mathbf{p} \tag{4.36}$$

and the polyhedron [84]:

$$K = \{\mathbf{p} \in \mathbb{R}^{m\times n} |\ p_k^i = 0,\ \ \forall k \in PS, \forall i = 1,\ldots,m;$$
$$p_k^i + I_k^i \leq CAP_k,\ \ \sum_{k=1}^{n} p_k^i = p_{\max}^i,\ \ \forall k \notin PS,\ \ \forall i \in M_1;$$
$$p_k^i + I_k^i = CAP_k,\ \ \forall k \notin PS,\ \ \forall i \in M_2\} \quad (4.37)$$

Hence, the IWFA can be formulated as an AVI problem VI(K, $\boldsymbol{\sigma}$+$\mathbf{Mp}$) or AVI(K, $\boldsymbol{\sigma}$, $\mathbf{M}$). The vector $\mathbf{p}^*$ is a Nash equilibrium point of the IWFA if, and only if, $\mathbf{p}^* \in K$ and $\forall \mathbf{p} \in K$ [84, 96]:

$$(\mathbf{p} - \mathbf{p}^*)^T (\boldsymbol{\sigma} + \mathbf{Mp}^*) \geq 0 \quad (4.38)$$

The above AVI problem can be interpreted as a robust optimization problem in which $\mathbf{p}$ is subject to uncertainty and known only to belong to K [102]. In the following, it is shown that the AVI reformulation of IWFA facilitates the study of the disequilibrium behaviour and stability analysis of the cognitive radio network.

4.6 Solution Characteristics

Monotonicity establishes the essential conditions for existence and uniqueness of the solution of the VI problem. The following definition and theorem are recalled from [96].

Definition 4.1: A mapping $F : K \subseteq \mathbb{R}^n \to \mathbb{R}^n$ is said to be

(a) *monotone* on K if

$$(\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y}))^T (\mathbf{x} - \mathbf{y}) \geq 0, \quad \forall \mathbf{x}, \mathbf{y} \in K; \tag{4.39}$$

(b) *strictly monotone* on K if

$$(\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y}))^T (\mathbf{x} - \mathbf{y}) > 0, \quad \forall \mathbf{x}, \mathbf{y} \in K, \mathbf{x} \neq \mathbf{y}; \tag{4.40}$$

(c) *ξ-monotone* on K for some $\xi > 1$ if there exists a constant $c > 0$ such that

$$(\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y}))^T (\mathbf{x} - \mathbf{y}) \geq c\|\mathbf{x} - \mathbf{y}\|^{\xi}, \quad \forall \mathbf{x}, \mathbf{y} \in K; \tag{4.41}$$

(d) *strongly monotone* on K if there exists a constant $c > 0$ such that

$$(\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y}))^T (\mathbf{x} - \mathbf{y}) \geq c\|\mathbf{x} - \mathbf{y}\|^{2}, \quad \forall \mathbf{x}, \mathbf{y} \in K, \tag{4.42}$$

i.e. if F is 2-monotone on K.

Strong monotonicity implies ξ-monotonicity, ξ-monotonicity implies strict monotonicity, and strict monotonicity implies monotonicity but the reverse is not true.

Theorem 4.2: Let $K \subseteq \mathbb{R}^n$ be closed convex and $F : K \subseteq \mathbb{R}^n \to \mathbb{R}^n$ be continuous.
(a) If F is strictly monotone on K, then VI(K, F) has at most one solution.
(b) If F is ξ-monotone on K for some $\xi > 1$, then VI(K, F) has a unique solution.

Therefore, the VI$(K, \boldsymbol{\sigma} + \mathbf{Mp})$ has at most one solution if $\boldsymbol{\sigma} + \mathbf{Mp}$ is strictly monotone and it has a unique solution, $\mathbf{p}^*$, if $\boldsymbol{\sigma} + \mathbf{Mp}$ is ξ-monotone for some $\xi > 1$.

Monotonicity of the affine map $\mathbf{Mp} + \boldsymbol{\sigma}$, where $\mathbf{M}$ is not necessarily symmetric, is equivalent to the condition that all of the eigenvalues of $\mathbf{M}$ have non-negative real parts. Also, strict monotonicity, ξ-monotonicity, and strong monotonicity of $\mathbf{M}\mathbf{p}+\boldsymbol{\sigma}$, as well as the condition that all of the eigenvalues of $\mathbf{M}$ have positive real parts are all equivalent [96]. The latter condition is equivalent to the statement that $-\mathbf{M}$ is a *Hurwitz* matrix. This statement follows from the definition: A Hurwitz matrix is a matrix, which all of its eigenvalues have negative real parts [103]. Since $\mathbf{M}$ is a non-negative real matrix, in this case the symmetric part of $\mathbf{M}$, $\frac{1}{2}\left(\mathbf{M}+\mathbf{M}^T\right)$, will be positive definite. Therefore, if matrix $-\mathbf{M}$ is Hurwitz, the existence of a unique equilibrium solution for the IWFA game will be guaranteed.

In order to get an idea about the conditions under which matrix $-\mathbf{M}$ is Hurwitz in a practical cognitive radio network, let us regroup the elements of the power vector in (4.32) based on subcarriers instead of users.

$$\mathbf{q} = [\mathbf{p}_k]_{k=1}^{n} = \begin{bmatrix} \begin{bmatrix} p_1^1 \\ \vdots \\ p_1^m \end{bmatrix} \\ \vdots \\ \begin{bmatrix} p_n^1 \\ \vdots \\ p_n^m \end{bmatrix} \end{bmatrix} \tag{4.43}$$

Accordingly, by rearranging rows and columns of matrix $\mathbf{M}$, the following block diagonal matrix is obtained.

$$\mathbf{N} = \begin{bmatrix} \mathbf{M}_1 & \cdots & \mathbf{0} \\ \vdots & \cdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{M}_n \end{bmatrix} \tag{4.44}$$

where $\mathbf{M}_k$s are *tone matrices* [84]

$$\mathbf{M}_k = \begin{bmatrix} 1 & \cdots & \alpha_k^{1m} \\ \vdots & \cdots & \vdots \\ \alpha_k^{m1} & \cdots & 1 \end{bmatrix} \tag{4.45}$$

Matrices $\mathbf{M}$ and $\mathbf{N}$ have the same set of eigenvalues. Regarding the block diagonal structure of matrix $\mathbf{N}$, if all of the eigenvalues of every tone matrix, $\mathbf{M}_k$, have positive real values or if the symmetric part of every tone matrix, $\frac{1}{2}\left(\mathbf{M}_k + \mathbf{M}_k^T\right)$, is positive definite, then $-\mathbf{M}$ will be Hurwitz. If tone matrices are strictly diagonally dominant, then their symmetric parts will be positive definite. Therefore, the following condition guarantees that $-\mathbf{M}$ will be Hurwitz.

$$\sum_{j=1,\neq i}^{m} \alpha_k^{ij} < 1, \;\; \forall i = 1, \ldots, m, \; \forall k = 1, \ldots, n \tag{4.46}$$

For instance, if

$$\alpha_k^{ij} < \frac{1}{m-1}, \;\; \forall i, j = 1, \ldots, m, \; \forall k = 1, \ldots, n \tag{4.47}$$

the Hurwitz condition will be guaranteed [81–84].

As shown in (4.7), the interference gains, α_k^{ij}, depend on the distance between a receiver and its own transmitter relative to its distance from other active transmitters in the network. Therefore, the Hurwitz condition of matrix $-\mathbf{M}$ depends on the topology of the network. Roughly speaking, if each user's receiver has the proper distance from its own transmitter, which is short compared to its distance from other active transmitters in the network, then it can be guaranteed that the network will reach a stable unique equilibrium.

The existence and uniqueness results for IWFA are extended in [48,84] and broader conditions are obtained compared to those presented in [81–83]. However, as it will be clear in Chapter 6, the condition (4.46) provides insight on the stability of real-life cognitive radio networks.

In general, cognitive radio networks are infrastructure-less networks and connection between source and destination nodes are established through self-organization and ad hoc networking [104–106]. In a self-organized ad hoc cognitive radio network, when a source node wants to communicate with a destination node, a multi-hop path is established between them, which is called a *communication tube*. In general, the communication tube is dynamic and nodes can enter the tube or leave it due to their mobility. It can also bend and change its shape in order to preserve connectivity [107]. The multi-hop (relay) communication for cognitive radio has been the focus of extensive research. In the context of *spectrum-aware routing* or *opportunistic-spectrum routing*, a transceiver explores and utilizes the cooperative diversity in the network to build a multi-hop communication path in which the intermediate nodes that are willing to cooperate relay the message [108–110]. Moreover, the dynamic spectrum manager guarantees that the neighboring transmitting nodes will not use the same set of channels in order to reduce the interference [42, 88]. Therefore, the above mentioned condition for existence of a unique equilibrium solution is practically achievable through dynamic spectrum management, opportunistic-spectrum routing, and flexibility of the communication tube between source and destination nodes.

4.7 Summary

The resource-allocation problem in cognitive radio networks was formulated as a non-cooperative game. Users solve a two-stage optimization problem to select a set of proper channels for communication and adjust their transmit powers in those channels. The robust version of IWFA was suggested as a proper candidate for finding the equilibrium solution using local and flexible constraints based on the permissible interference level in the network. IWFA was formulated as an affine variational inequality problem and sufficient conditions for existing of a unique equilibrium solution were presented. The AVI formulation paves the way for investigating the network behaviour in a dynamic framework. Regarding the relationship between variational inequalities and temporal difference (TD) methods [111], extending the developed framework to equip the cognitive radios with learning capability is suggested as a direction for future research.

Chapter 5

Computer Experiments I

"Something is always discarded when the results of experiment are trimmed down to fit formulas and equations. That something, much or little, which is thrown away has frequently been of scientific importance equal to what is retained in the mathematics." E. T. Bell (1883-1960)

IEEE 802.22 standard for wireless regional area networks provides fixed wireless broadband services for cognitive radios in TV broadcast bands on a non-interfering basis. Spectrum sensing is performed on the operating channel as well as the adjacent channels to make sure that cognitive radios will not cause harmful interference to TV signals and wireless microphones. The sensing receiver sensitivity for digital TV, analog TV, and wireless microphone is -116 dBm, -94 dBm, and -107 dBm, respectively. Channel detection time is 2 s. Probability of detection is 0.9 and probability of false alarm is 0.1. The standard supports superframes of groups of 16 frames with a frame size of 10 ms. Also, excess delay of up to 37 μs can be absorbed by the cyclic prefix [112, 113].

Simulation results are now presented to support theoretical underpinnings of the previous chapter. It is assumed that the cognitive-radio transceivers are distributed randomly in the region of interest with uniform distribution (Figure 5.1); this assumption is intuitively satisfying. Similar to [84,85] the normalized interference gains α_k^{ij} are chosen randomly from the interval $(0, 1/(m-1))$ with uniform distribution, which are less than $1/(m-1)$, in order to guarantee that the tone matrices will be strictly diagonally dominant. Thus, the corresponding matrix $-\mathbf{M}$ will be Hurwitz. The ambient noise is assumed to be zero-mean Gaussian and the variance of the noise experienced by each user in each subcarrier is chosen from the interval $(0, 0.1/(m-1))$ with uniform distribution. The power budgets $p_{\max}^i$ are chosen randomly from the interval $(n/2, n)$ with uniform distribution too. For scenarios that consider the time-varying delay in the control loops, delays are chosen randomly. As shown in Figure 5.2, user mobility changes the communication path partially or completely, which, in turn, changes the interference gains and matrix $\mathbf{M}$. The same thing happens when new users join the network or old users leave the network.

5.1 Robust IWFA vs Classic IWFA

5.1.1 Stochastic Variability in the Network Configuration

The transmit power control problem in a cognitive radio network using the classic IWFA and its robust version were presented in Section 4.2 and Section 4.4, respectively. In a cognitive radio network, when a spectrum hole disappears, users may have to seek or else increase their transmit powers in other spectrum holes and this increases the interference. Also, when new users join the network, current users in

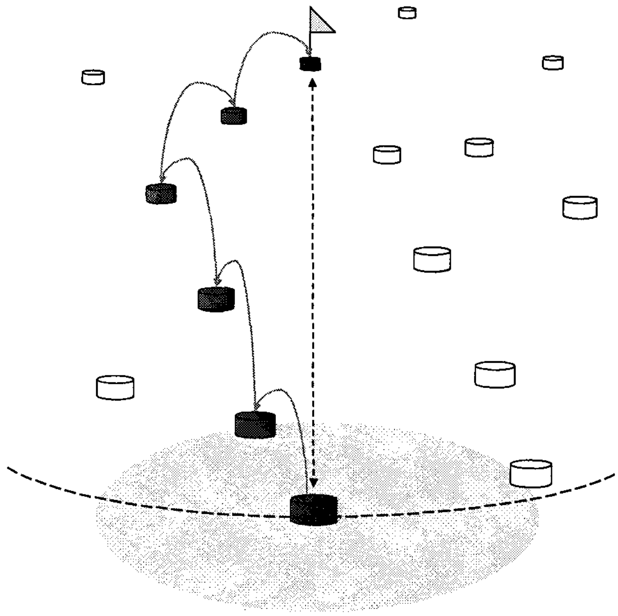
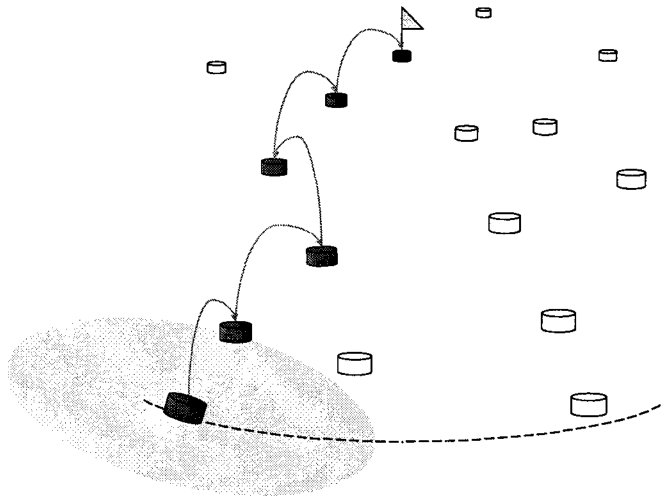


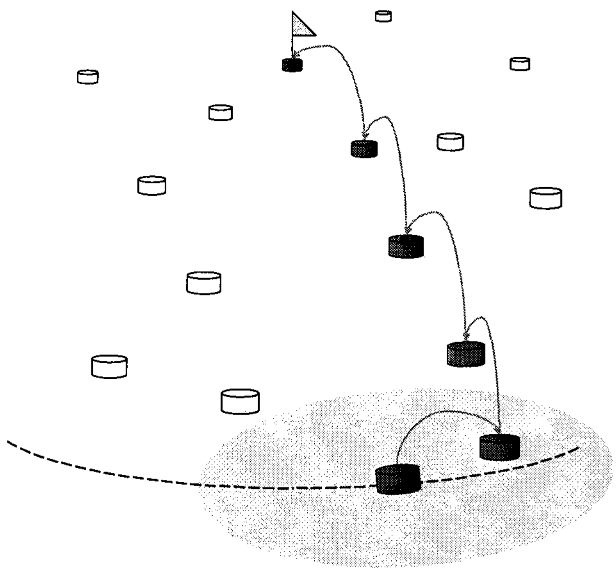
Figure 5.1: Multi-hop communication path between a source node and a destination node.

the network, experience more interference. Therefore, the joining of new users or the disappearance of spectrum holes makes the interference condition worse. Also, the cross-interference between users is time-varying because of mobility of the users. Results related to two typical but extreme scenarios are presented here to show superiority of the robust IWFA (4.15) over the classic IWFA (4.3) in dealing with the above issues.

The first scenario addresses a network with $m = 5$ nodes and $n = 2$ available subcarriers, and all of the users simultaneously update their transmit powers using the interference measurements from the previous time-step. As mentioned previously, the asynchronous adjustment scheme is the most realistic one when network simplicity is at a premium. However, here simultaneous adjustment was employed to implement extreme cases, which emphasize the practical effectiveness of robust IWFA and its



(a)

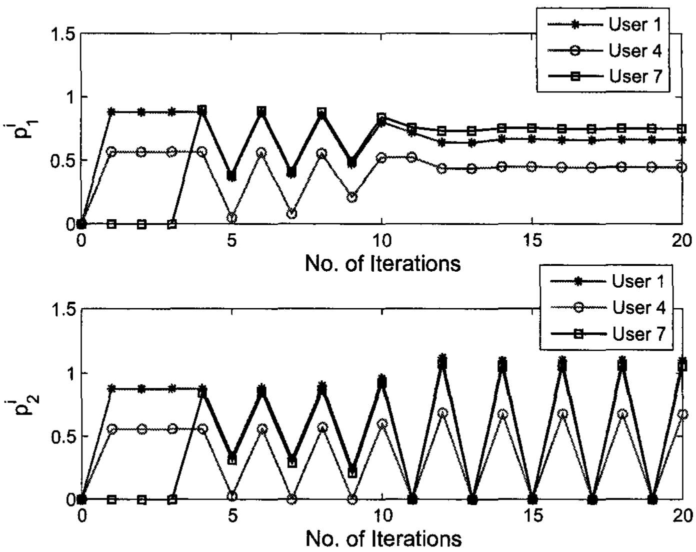


(b)

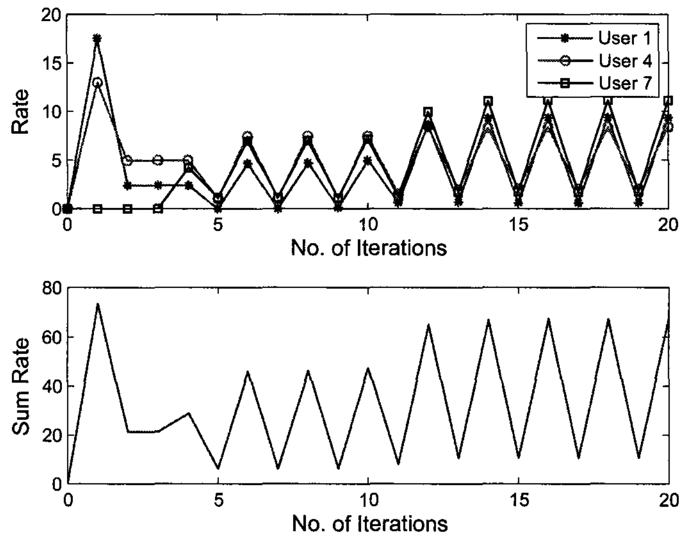
Figure 5.2: Effect of user mobility on the communication path: (a) partially changed, (b) completely changed.

superiority over the classic IWFA. At the fourth time-step, two new users join the network, which increases the power level of interference. The interference gains are also changed randomly at different time instants to consider mobility of the users. Figure 5.3 and Figure 5.4 show the transmit power of three users (users one, four, and seven) on two different subcarriers for the classic IWFA and robust IWFA, respectively. At the second subcarrier, the classic IWFA is not able to reach an equilibrium. Data rates achieved by the chosen users are shown too. Also, the total data rate in the network is plotted against time, which is a measure of spectral efficiency. Although the average sum rate achieved by the classic IWFA is close to the average sum rate of the robust IWFA, it fluctuates and in some time instants the data rate is very low, which indicates lack of spectrally efficient communication. Although, the oscillation occurs mainly because of using simultaneous update scheme, it also highlights practical effectiveness of the robust IWFA.

In the second scenario, a network with $m = 5$ nodes and $n = 4$ available subcarriers is considered. Again at the fourth time-step two new users join the network but at the eighth time-step the third subcarrier is not available anymore (i.e. a spectrum hole disappears). Results are shown in Figure 5.5 and Figure 5.6, which, again show superiority of the robust IWFA. For classic IWFA, immediately after the disappearance of the third subcarrier, power in the fourth subcarrier starts to oscillate and after changing the interference gains randomly, we observe the same behaviour in other subcarriers. In contrast to the robust IWFA, the classic IWFA fails again to achieve an equilibrium.

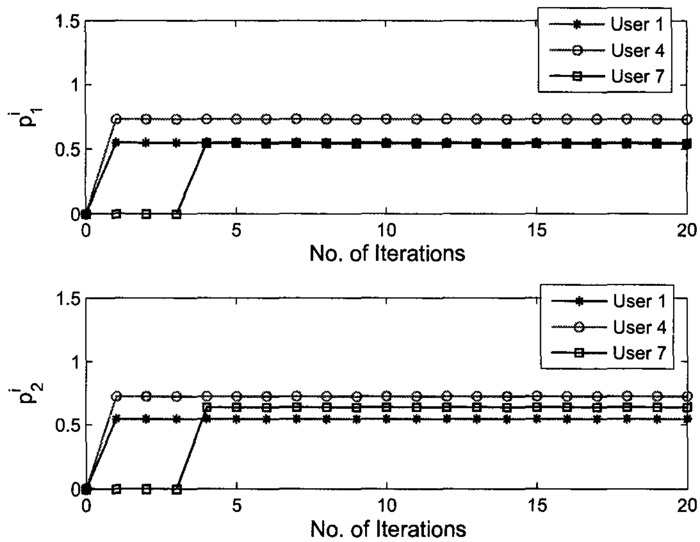


(a)

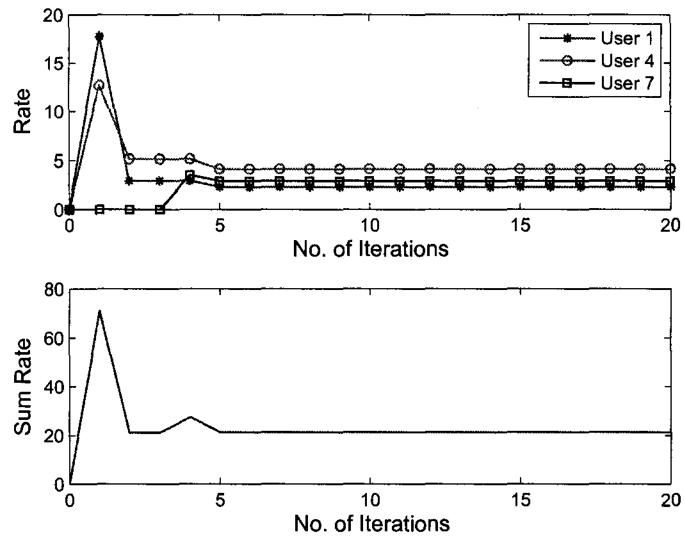


(b)

Figure 5.3: Resource allocation results of simultaneous IWFA, when 2 new users join a network of 5 users and interference gains are changed randomly to address the mobility of the users: (a) transmit powers of 3 users on 2 subcarriers, (b) data rates of 3 users and the total data rate in the network.

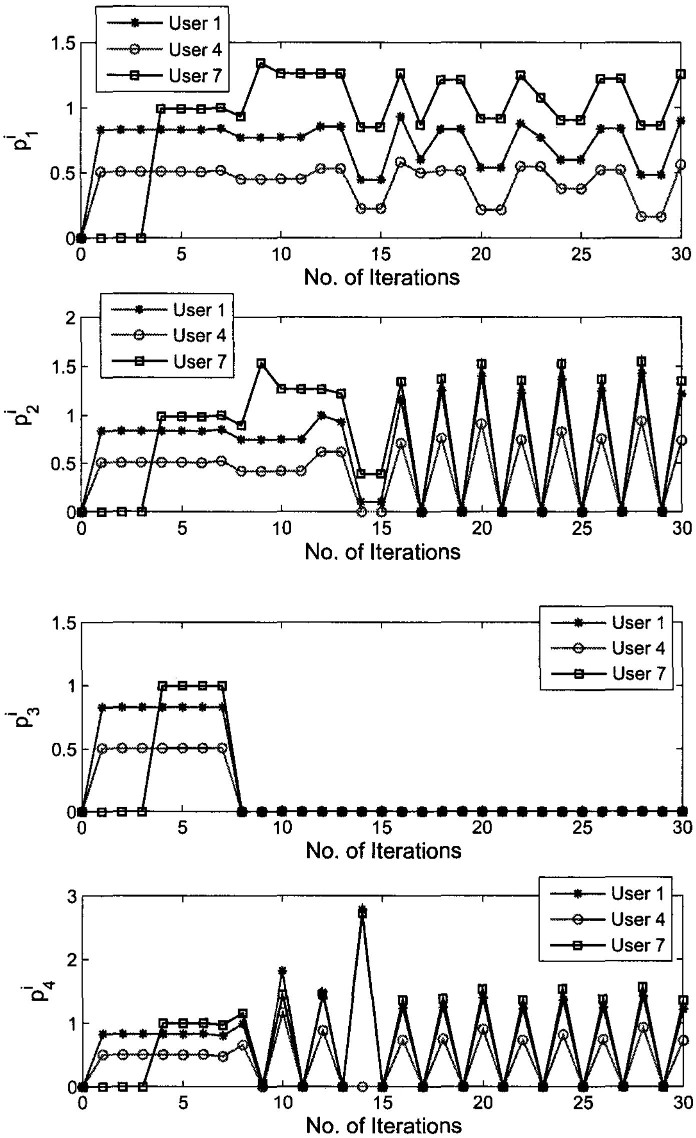


(a)

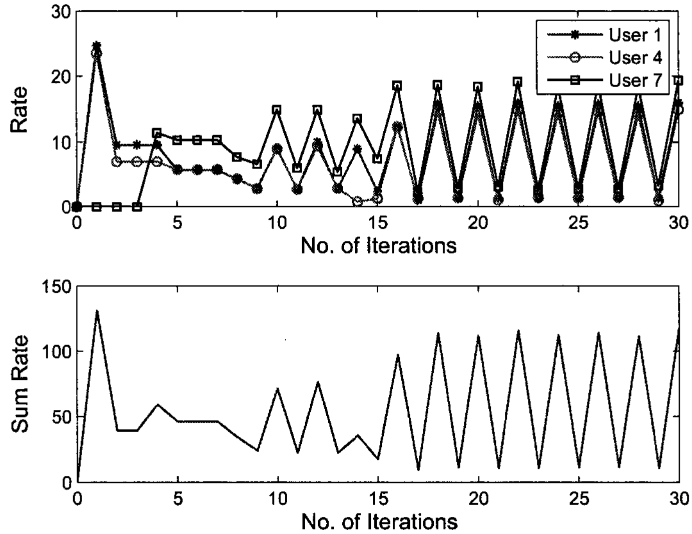


(b)

Figure 5.4: Resource allocation results of simultaneous robust IWFA, when 2 new users join a network of 5 users and interference gains are changed randomly to address the mobility of the users: (a) transmit powers of 3 users on 2 subcarriers, (b) data rates of 3 users and the total data rate in the network.

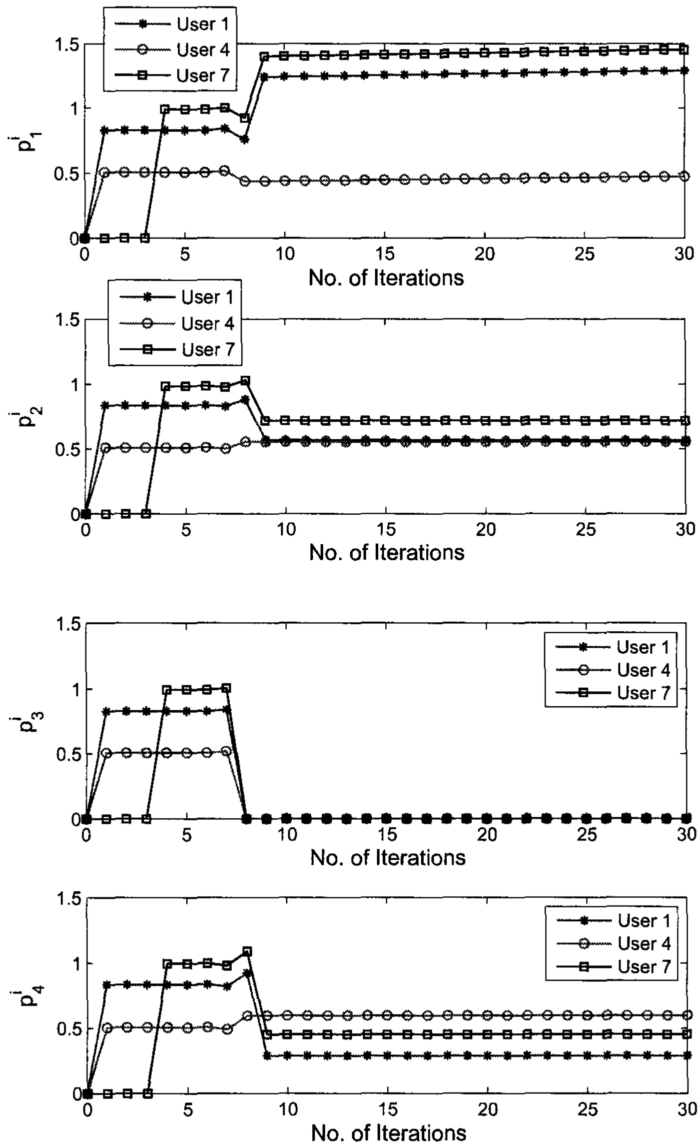

User 1
User 4
User 7
p^i_1
No. of Iterations
p^i_2
No. of Iterations
p^i_3
No. of Iterations
p^i_4
No. of Iterations

(a)



(b)

Figure 5.5: Resource allocation results of simultaneous IWFA, when 2 new users join a network of 5 users, a subcarrier disappears, and interference gains are changed randomly to address the mobility of the users: (a) transmit powers of 3 users on 4 subcarriers, (b) data rates of 3 users and the total data rate in the network.

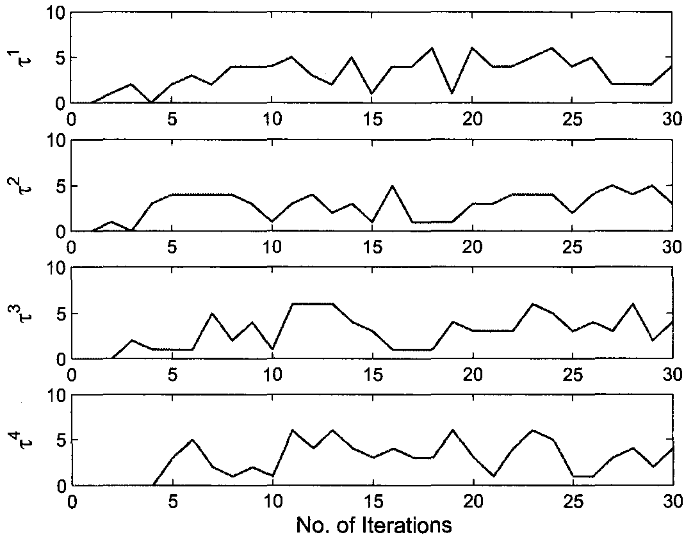

User 1
User 4
User 7
p_1^i
p_2^i
p_3^i
p_4^i
No. of Iterations
0
5
10
15
20
25
30
0
0.5
1
1.5

(a)

5.1.2 Delay

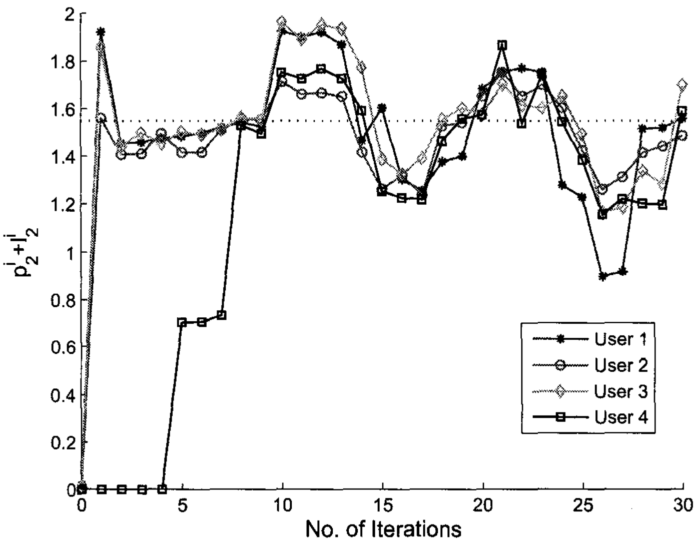
As mentioned previously, sporadic feedback introduces a time-varying delay in the transmit power control loop, which causes different users to update their transmit powers based on out-dated statistics. For instance, when the network configuration and therefore interference pattern changes, some users receive the related information after a delay. If the interference on a subcarrier increases and the transmitter is not informed immediately, it will not reduce its transmit power and it may violate the permissible interference power level for a while until it receives updated statistics of the interference in the forward channel. Similarly, this may happen to some users that update their transmit powers at lower rates compared to others. In the third scenario, a new user joins a network of three users, who are competing for utilizing two subcarriers. Each user's transmitter receives statistics of the interference plus noise with a time-varying delay. Figure 5.7(a) shows the randomly chosen time-varying delays introduced by each user's feedback channel. Sum of transmit power and interference plus noise at the second subcarrier at the receiver of each user is plotted in Figure 5.7(b) and Figure 5.7(c) for classic IWFA and robust IWFA, respectively. Dashed lines show the limit imposed by the permissible interference power level. Although the classic IWFA is less conservative, it is not as successful as the robust IWFA at preventing violations of the permissible interference power level. Similar results are obtained when users update their transmit powers with different frequencies.

These small-scale problems were designed and typical results were presented to compare the performance of classic IWFA and robust IWFA. In some extreme cases because of occurrence of discrete events such as the appearance and disappearance of spectrum holes and users, the IWFA cannot achieve an equilibrium solution and

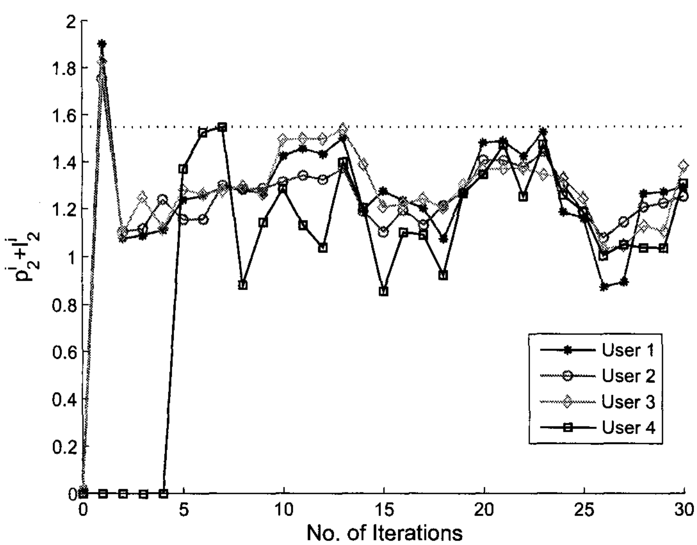


(a)

calculated results oscillate in subsequent time-steps especially if we use the simultaneous update scheme. This confirms the point that in a hybrid dynamic system such as a cognitive radio network in which switching happens between different subsystems even if all of them are stable, the whole system may become unstable because of switching between the subsystems. In the presented cases, the robust IWFA was able to achieve an equilibrium solution. Also, when some users update their transmit powers with lower frequencies or use out-dated information, the robust IWFA can prevent violating the permissible interference power level. Classic IWFA lacks this ability although it achieves superior data rates.



(b)



(c)

Figure 5.7: Resource allocation results of IWFA, when interference gains change randomly with time and users use out-dated information to update their transmit powers: (a) time-varying delays introduced by each user's feedback channel. Sum of transmit power and interference plus noise for 4 users achieved by (b) classic IWFA. (c) robust IWFA. Dashed lines show the limit imposed by the permissible interference power level.

5.2 Summary

Simulation results were presented to compare the performance of classic IWFA vs robust IWFA. Toy scenarios were considered in order to develop insight. Results show superiority of the robust IWFA over the classic IWFA in dealing with different practical issues in a cognitive radio environment, which is achieved by putting up with a reduction in achievable data rate for reliable performance. As mentioned previously, IWFA is defenseless against malicious users that do not follow the rules and do not decrease their transmit power, when the interference level is high. Such users can exploit the limited resources and achieve higher data rates compared to well-behaved users. In effect, therefore, a malicious user may act like a jammer and have the same effect on the network that disappearance of a spectrum hole has. Hence, other users' power vectors may fluctuate and the network may not be able to reach an equilibrium. Similar results on oscillation of transmit powers and therefore, data rates in the presence of a jammer were reported in [114]. In this situation, robust IWFA enables the well-behaved users to reach an equilibrium with possibly lower data rates.

Chapter 6

Network Dynamics Viewed from Control-Theoretic Perspectives

Although the components of the network may remain unchanged in complex and large-scale networks, the general behaviour of the network can change drastically over time. If the SINR of a communication link drops below a specified threshold for a relatively long time, the connection between the transmitter and receiver will be lost. For this reason, in addition to the equilibrium resource allocation that was discussed in Chapter 4, the transient behaviour of the network deserves attention too [115]. Therefore, studying the equilibrium states in a dynamic framework by methods that provide information about the disequilibrium behaviour of the system is critical, which is the focus of this chapter.

6.1 Projected Dynamic (PD) System

In the previous chapters, IWFA was proposed as an approach to find an equilibrium solution for the resource-allocation problem in cognitive radio networks. Also, the IWFA was reformulated as a VI problem. The *projected dynamic* (PD) systems theory [95] can be utilized to associate an ordinary differential equation (ODE) to the obtained VI. A projection operator, which is discontinuous, appears in the right-hand side of the ODE to incorporate the feasibility constraints of the VI problem into the dynamics. This ODE provides a dynamic model for the competitive system whose equilibrium behaviour is described by the VI. Also, the stationary points of the ODE coincide with the set of solutions of the VI, which are the equilibrium points. Thus, the equilibrium problem can be studied in a dynamic framework. This dynamic model enables us not only to study the transient behaviour of the network, but also to predict it.

Before we proceed, we need to recall some mathematical definitions from [95]. The set of inward normals at $\mathbf{p} \in K$ is defined as

$$S(\mathbf{p}) = \{\boldsymbol{\gamma} : \ \|\gamma\| = 1 \ , \ \langle \gamma, \mathbf{p} - \mathbf{y} \rangle \leq 0, \ \ \forall \mathbf{y} \in K\} \tag{6.1}$$

Then, the projection of $\mathbf{b} \in \mathbb{R}^n$ onto K at $\mathbf{p} \in K$ can be written as

$$\Pi_K(\mathbf{p}, \mathbf{b}) = \mathbf{b} + \max\left(0, \langle \mathbf{b}, -\mathbf{s}^* \rangle\right) . \ \mathbf{s}^* \tag{6.2}$$

where $\mathbf{s}^*$ is a vector in $S(\mathbf{p})$ that satisfies the condition

$$\langle \mathbf{b}, -\mathbf{s}^* \rangle = \max_{\mathbf{s} \in S(\mathbf{p})} \langle \mathbf{b}, -\mathbf{s} \rangle \tag{6.3}$$

By this projection operator, a point in the interior of K is projected onto itself, and a point outside of K is projected onto the closest point on the boundary of K. The following ODE

$$\dot{\mathbf{p}} = \Pi_K(\mathbf{p}, \mathbf{b}(\mathbf{p})) \tag{6.4}$$

with the initial condition

$$\mathbf{p}(t_0) = \mathbf{p}_0 \in K \tag{6.5}$$

is called a projected dynamic system.

Theorem 6.1: Assume that K is a convex polyhedron. Then, the equilibrium points of the PDS(K,F) coincide with the solutions of VI(K,F) [95].

Let us replace $\mathbf{b}(\mathbf{p})$ with $-\mathbf{F}(\mathbf{p}) = -(\boldsymbol{\sigma} + \mathbf{M}\mathbf{p})$ in (6.4). Then, the *stationary points* of the following PDS

$$\dot{\mathbf{p}} = \Pi_K(\mathbf{p}, -\mathbf{F}(\mathbf{p})) \tag{6.6}$$

coincide with the solutions of the VI problem of (4.38).

The associated dynamic model to the equilibrium problem will be realistic only if there is a unique solution path from a given initial point. The following theorem addresses the existence and uniqueness of the solution path for the above ODE [95].

Definition 6.1: A mapping $F : K \subseteq \mathbb{R}^n \to \mathbb{R}^n$ is said to be Lipschitz continuous if there is an $L > 0$, such that

$$\|\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})\| \leq L\|\mathbf{x} - \mathbf{y}\|, \ \ \forall \mathbf{x}, \mathbf{y} \in K; \tag{6.7}$$

Theorem 6.2: If $\mathbf{F}$ in the initial value problem (6.4) and (6.5) is *Lipschitz* continuous, then for any $\mathbf{p}_0 \in K$, there exists a unique solution $\mathbf{p}(t)$ to the initial value problem (6.4) and (6.5).

For the affine mapping $\mathbf{F}(\mathbf{p}) = (\boldsymbol{\sigma} + \mathbf{M}\mathbf{p})$, we have

$$\|\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})\| = \|\mathbf{M}(\mathbf{x} - \mathbf{y})\| \tag{6.8}$$

According to the multiplicative property of the matrix norm [116]:

$$\|\mathbf{M}(\mathbf{x} - \mathbf{y})\| \leq \|\mathbf{M}\|.\|\mathbf{x} - \mathbf{y}\| \tag{6.9}$$

For the Euclidean norm, we have [116]:

$$\bar{\sigma}(\mathbf{M}) \leq \|\mathbf{M}\| \leq \sqrt{mn}\,\bar{\sigma}(\mathbf{M}) \tag{6.10}$$

where $\bar{\sigma}(\mathbf{M})$ is the maximum singular value of $\mathbf{M}$. From (6.8) to (6.10) we have:

$$\|\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})\| \leq \sqrt{mn}\,\bar{\sigma}(\mathbf{M})\|\mathbf{x} - \mathbf{y}\| \tag{6.11}$$

The interference channel is a multiple-input-multiple-output (MIMO) dynamic system with the state-transition matrix $\mathbf{M}$ in which the transmitted signal by each transmitter in each subcarrier is an input and the received signal by each receiver in each subcarrier is an output. In a MIMO system, the largest gain (amplification) for any input direction is equal to the maximum singular value of the state-transition matrix [116]. The communication channel attenuates the transmitted signals in all directions and therefore, the Lipschitz continuity is a valid assumption.

When $\mathbf{p}(t)$ is in the interior of the feasible set, $\mathbf{p}(t) \in \text{int}K$, the projection operator in (6.6) is

$$\Pi_K(\mathbf{p}, -\mathbf{F}(\mathbf{p})) = -\mathbf{F}(\mathbf{p}) \tag{6.12}$$

If $\mathbf{p}(t)$ reaches the boundary of the feasible set, $\mathbf{p}(t) \in \partial K$, we have

$$\Pi_K(\mathbf{p}, -\mathbf{F}(\mathbf{p})) = -\mathbf{F}(\mathbf{p}) + z(\mathbf{p})\mathbf{s}^*(\mathbf{p}) \tag{6.13}$$

where

$$\mathbf{s}^*(\mathbf{p}) = \underset{\mathbf{s}\in S(\mathbf{p})}{\text{argmax}}\langle -\mathbf{F}(\mathbf{p}), -\mathbf{s}\rangle \tag{6.14}$$

and

$$z(\mathbf{p}) = \max\left(0, \langle -\mathbf{F}(\mathbf{p}), -\mathbf{s}^*(\mathbf{p})\rangle\right) \tag{6.15}$$

From (6.12) and (6.13), it follows that [95]:

$$\|\Pi_K(\mathbf{p}, -\mathbf{F}(\mathbf{p}))\| \leq \| - \mathbf{F}(\mathbf{p})\| \tag{6.16}$$

Therefore, because of the projection operator, the right-hand side of the differential equation (6.6) is discontinuous on the boundary of K. If at some t, $\mathbf{p}(t)$ reaches the boundary of K and $-\mathbf{F}(\mathbf{p}(t))$ points out of the boundary, then the right-hand side becomes the projection of $-\mathbf{F}$ onto the boundary. The state trajectory then evolves on the boundary. In summary, the projection operator keeps the state trajectory in the feasible set. At some later time, the state trajectory may enter a lower dimensional part of the boundary or even go to the interior of K [95], where the evolution of the state trajectory is governed by the differential equation

$$\dot{\mathbf{p}} = -\mathbf{F}(\mathbf{p}) = -(\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}), \quad \forall \mathbf{p}(t) \in K \tag{6.17}$$

Since $-\mathbf{F}(\mathbf{p})$ is an affine mapping, the differential equation (6.17) represents an *affine system*. Moreover, the state trajectory of this affine system must remain in the feasible set K. Therefore, the system described by (6.17) is a constrained affine system [117].

Iterative algorithms based on time discretization of the PD system (6.6) are proposed in [95] for computation of the system state trajectory. At each time-step t, the proposed algorithms solve the minimum-norm problem:

$$\min_{\mathbf{p}(t+1)\in K} \quad \|\mathbf{p}(t+1) - [\mathbf{p}(t) - a(t)\mathbf{F}(\mathbf{p}(t))]\,\| \tag{6.18}$$

or equivalently, solve the following quadratic programming problem:

$$\begin{aligned} \min_{\mathbf{p}(t+1)\in K} \quad & \frac{1}{2}\mathbf{p}^T(t+1)\mathbf{p}(t+1) \\ & - [\mathbf{p}(t) - a(t)\mathbf{F}(\mathbf{p}(t))] \cdot \mathbf{p}(t+1) \end{aligned} \tag{6.19}$$

where "·" signifies the dot product. A good approximation of the state trajectory may be achieved by choosing a small value for the step-size $a(t)$. It should be noted that although quadratic programming is indeed computationally demanding, it does not feature in operation of the robust IWFA; rather, it is a burden incurred in carrying out simulation experiments to study the behaviour of the whole network.

6.2 Stability of the Perturbed PD System

The stability of a system is an important issue in the study of feedback control systems and therefore, deserves special attention. It can be interpreted as the ability of the system to maintain or restore its equilibrium state against external perturbations. In other words, system stability is linked to system sensitivity to perturbations.

The formulation of the IWFA as AVI$(K, \boldsymbol{\sigma}, \mathbf{M})$, is helpful to study the sensitivity of a solution $\mathbf{p}^*$ as the pair $(K, \boldsymbol{\sigma} + \mathbf{Mp})$ is perturbed. It would be interesting to know if the perturbed system has a solution close to $\mathbf{p}^*$; and in case that such a solution exists, if it will converge to $\mathbf{p}^*$ as the perturbed AVI approaches the original one. This way of thinking leads to the concept of *solution stability* [96]. The monotonicity conditions play a key role in the analysis of both local and global stability [95,96].

The local uniqueness of $\mathbf{p}^*$, which was studied in Chapter 4, is not sufficient to guarantee the solvability of the perturbed AVI, but it is important for sensitivity analysis because every unique solution of a VI problem is an *attractor* of all solutions of nearby VIs [96]. Alternatively, the following theorems are recalled from [95] about stability of the corresponding PD system in order to answer the following questions:

- If the initial state of the network is close to an equilibrium, will the state trajectory remain in a neighborhood of the equilibrium?

- Starting from an arbitrary initial state, will the state trajectory asymptotically approach an equilibrium and at what rate?

Theorem 6.3: Suppose that $\mathbf{p}^*$ solves VI$(K, \boldsymbol{\sigma} + \mathbf{Mp})$. If the mapping $\boldsymbol{\sigma} + \mathbf{Mp}$ is strictly monotone at $\mathbf{p}^*$, then $\mathbf{p}^*$ is a strict monotone attractor for the PDS$(K, -(\boldsymbol{\sigma} + \mathbf{Mp}))$.

Theorem 6.4: Suppose that $\mathbf{p}^*$ solves VI$(K, \boldsymbol{\sigma} + \mathbf{M}\mathbf{p})$. If the mapping $\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}$ is ξ-monotone at $\mathbf{p}^*$ with $\xi < 2$, then $\mathbf{p}^*$ is a finite-time attractor.

Theorem 6.5: Suppose that $\mathbf{p}^*$ solves VI$(K, \boldsymbol{\sigma} + \mathbf{M}\mathbf{p})$. If the mapping $\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}$ is strongly monotone at $\mathbf{p}^*$, then $\mathbf{p}^*$ is exponentially stable.

Following the discussion in Section 4.6, if matrix $-\mathbf{M}$ is Hurwitz, the existence of a unique equilibrium solution for the IWFA game, which is exponentially stable, will be guaranteed. As will be clear later, the Hurwitz property of matrix $-\mathbf{M}$ is also needed to guarantee the robust exponential stability of the system in the presence of multiple-time-varying delays. As mentioned before, this condition is practically achievable through dynamic spectrum management [42, 88] and spectrum-aware routing [108–110].

6.3 The PD System Viewed as a Constrained Piece-wise Affine (PWA) System

As mentioned before, a cognitive radio network is a hybrid dynamic system with both continuous and discrete dynamics. Changes occur in the network due to discrete events such as the appearance and disappearance of users and spectrum holes, as well as continuous dynamics described by differential equations that govern the evolution of transmit power vectors of users over time. When conditions change due to these kinds of discrete events, each user will have to solve a new optimization problem similar to the one described in (4.3) and the network deviates from the achieved equilibrium point and it is desirable to converge to a new one reasonably fast. Also, the occurrence of an event such as a change in the number of users or available subcarriers will change the parameters $\boldsymbol{\sigma}$ and $\mathbf{M}$ in (6.17). Accordingly, the problem is formulated in terms of an ensemble of subsystems and the global state space is:

- partitioned into polyhedral regions described in (4.37) that follow the varying realizations of the network at different time intervals, and
- an affine state equation, similar to that described in (6.17), is associated with each polyhedral region that governs the evolution of state trajectory in that region.

It follows therefore, that the whole network can be modeled as a constrained *piecewise affine* (PWA) system [117]:

$$\dot{\mathbf{p}} = -\mathbf{M}(\boldsymbol{v})\mathbf{p} - \boldsymbol{\sigma}(\boldsymbol{v}), \quad \forall \mathbf{p}(t) \in K(\boldsymbol{v}) \tag{6.20}$$

where $\boldsymbol{v}$ is a key vector, which is a function of time and discrete events, and describes which affine subsystem is currently a valid representation of the network [118].

The stationary points of each one of these dynamic subsystems coincide with the equilibrium points of the corresponding game resulting from solving the related optimization problems. In summary, the occurrence of discrete events changes the equilibrium point and causes the state trajectory to deviate from an equilibrium point and therefore, converge to another equilibrium point. Each one of these equilibrium points may have a *region of attraction* around it such that if the system is perturbed, the solution remains in that region close to the solution of the unperturbed system. This issue was addressed to some extent in the previous section and it will be studied more in the next section.

6.4 Stability of the Perturbed PD System in the Presence of Time Delay

The feedback channel plays a fundamental role in the design and operation of cognitive radio. Indeed, feedback is the *facilitator of computational intelligence*, without which the radio loses its cognitive capability. The discovery of spectrum holes may prompt the need to establish the feedback channel from the receiver to the transmitter of a cognitive radio. In this case, a fraction of the available spectrum holes will be used for the feedback channel to send relevant information from a user's receiver to its transmitter to take the appropriate action. In effect, therefore, feedback channels are not fixed and instead of having permanent feedback, we have sporadic feedback. However, this problem can be avoided by using the unlicensed bands for establishing a common feedback channel [42]. Also, in order to be conservative in consuming the precious bandwidth that can be used for data transmission, the feedback should be low-rate and quantized. Therefore, rather than sending the actual values of the required parameters identified by the radio scene-analyzer, the practical approach is to feed their respective quantized values back to the transmitter [1].

Feedback may naturally introduce delay in the control loop and different transmitters may receive statistics of noise and interference with different time delays. Moreover, the sporadic feedback causes the users to adopt out-dated statistics to update their power vectors. The time-varying delay in the control loop degrades the performance and may cause stability problems. Analysis of stability and control of time-delay systems is a topic of practical interest and has attracted the attention of many researchers [119–122]. Robust stability of the system under time-varying delays is the focus of this section. The dynamic model of the previous sections can be used to find out if the network is able to achieve a *retarded equilibrium*, which is stable. If an equilibrium point is not stable, the system may not be able to maintain that state long enough because of perturbations, and there is the potential possibility that an equilibrium can not even be established.

The dynamic model of (6.6) will be a PD system with delay (PDSD) [123] in the form of the following *functional differential equation* (FDE) [119,124,125]:

$$\dot{\mathbf{p}}(t) = \Pi_K\left(\mathbf{p}(t), -\mathbf{F}_d\left(\mathbf{p}\right)\right) \tag{6.21}$$

$\mathbf{F}_d$ can be written as

$$\mathbf{F}_d\left(\mathbf{p}\right) = \begin{bmatrix} \mathbf{F}^1\left(\mathbf{p}^1(t), \mathbf{p}^{-1}(\mathcal{S}_t)\right) \\ \vdots \\ \mathbf{F}^i\left(\mathbf{p}^i(t), \mathbf{p}^{-i}(\mathcal{S}_t)\right) \\ \vdots \\ \mathbf{F}^m\left(\mathbf{p}^m(t), \mathbf{p}^{-m}(\mathcal{S}_t)\right) \end{bmatrix} \tag{6.22}$$

where $\mathbf{p}^{-i}(\mathcal{S}_t)$ denotes a continuous-time asynchronous adjustment scheme similar to (4.10).

Let the given initial time be t_0. In order to determine the continuous solution, $\mathbf{p}(t)$ of (6.21) for $t > t_0$, we need to know a continuous *initial function*, $\phi(t)$, where $\mathbf{p}(t) = \phi(t)$ for $t_0 - \tau^{i,j} \leq t \leq t_0$, $\forall i, j = 1, \ldots, m$. The initial function may be obtained from measurements. Since the system described in (6.21) and (6.22) is a multiple-delay system, each deviation defines an initial set $\Psi_{t_0}^{i,j}$, consisting of the point t_0 and those values $t - \tau^{i,j}(t)$ for which $t - \tau^{i,j}(t) < t_0$ when $t \geq t_0$ [126].

Therefore, the initial condition for the system described in equation (6.21) is

$$\mathbf{p}(\theta) = \boldsymbol{\phi}(\theta), \quad \forall \theta \in \Psi_{t_0} \tag{6.23}$$

where $\boldsymbol{\phi} : \Psi_{t_0} \mapsto \mathbb{R}^{m \times n}$ is a continuous norm-bounded initial function [126, 127] and

$$\begin{aligned} \Psi_{t_0} &= \bigcup_{i,j=1, i \neq j}^{m} \Psi_{t_0}^{i,j} \\ &= \bigcup_{i,j=1, i \neq j}^{m} \left\{ t \in \mathbb{R} : t = \kappa - \tau^{i,j}(\kappa) \leq 0, \ \kappa \geq t_0 \right\} \end{aligned} \tag{6.24}$$

The $\mathbf{F}_d(\mathbf{p})$ in (6.22) can be written as the following summation:

$$\begin{aligned} \mathbf{F}_d(\mathbf{p}) = \quad & \mathbf{p}(t) + \sum_{i=1}^{m} \sum_{j=1, \neq i}^{m} \mathbf{M}_d^{ij} \mathbf{p}(t - \tau^{i,j}(t)) \\ & + \sum_{i=1}^{m} \sum_{j=1, \neq i}^{m} \Delta \mathbf{M}_d^{ij} \mathbf{p}(t - \tau^{i,j}(t)) + \boldsymbol{\varrho}(t) \end{aligned} \tag{6.25}$$

where $\mathbf{M}_d^{ij}$ is obtained by replacing all the blocks in $\mathbf{M}$ except $\mathbf{M}^{ij}$ by $n \times n$ zero matrices, and $\Delta \mathbf{M}_d^{ij}$ is a perturbation in $\mathbf{M}_d^{ij}$. The term $\boldsymbol{\varrho}$ is the combined effect of the background noise in both forward and feedback channels.

Therefore, the associated constrained affine system that governs the network's dynamics is described by the differential equation

$$\begin{aligned} \dot{\mathbf{p}}(t) = \quad & -\mathbf{p}(t) - \sum_{i=1}^{m} \sum_{j=1, \neq i}^{m} \mathbf{M}_d^{ij} \mathbf{p}(t - \tau^{i,j}(t)) \\ & - \sum_{i=1}^{m} \sum_{j=1, \neq i}^{m} \Delta \mathbf{M}_d^{ij} \mathbf{p}(t - \tau^{i,j}(t)) - \boldsymbol{\varrho}(t) \end{aligned} \tag{6.26}$$

$\forall \mathbf{p}(t) \in K$, which is a multiple-time-varying-delay system with uncertainty. It can be written as

$$\begin{aligned}\dot{\mathbf{p}}(t) = \quad & -\mathbf{p}(t) - \sum_{\ell=1}^{m(m-1)} \mathbf{M}_d^\ell \mathbf{p}(t - \tau^\ell(t)) \\ & - \sum_{\ell=1}^{m(m-1)} \Delta\mathbf{M}_d^\ell \mathbf{p}(t - \tau^\ell(t)) - \boldsymbol{\varrho}(t)\end{aligned} \tag{6.27}$$

This reformulation is an instance of the general systems that were studied in [127]. Following the approach of [127], we assume that $\forall t \geq t_0$, the time-varying delays $\tau^\ell(t)$ satisfy

$$\tau^\ell(t) \leq \tau(t) \leq \bar{\tau} \tag{6.28}$$

$$\dot{\tau} \leq \delta < 1 \tag{6.29}$$

where $\bar{\tau} > 0$, $\delta \geq 0$, and $\tau(t)$ is a strictly positive continuous differentiable function. Also, the uncertainties are assumed to be bounded for all $\mathbf{p}$ and at all times, such that the following pair of conditions holds:

$$\|\boldsymbol{\varrho}(t)\| \leq b_d \|\mathbf{p}(t)\| \tag{6.30}$$

and

$$\|\Delta\mathbf{M}_d^\ell(t)\mathbf{p}(t))\| \leq b_d^\ell \|\mathbf{p}(t)\| \tag{6.31}$$

where $b_d \geq 0$ and $b_d^\ell \geq 0$. If there exist $\zeta \geq 1$ and $\lambda > 0$ such that

$$\|\mathbf{p}(t)\| \leq \zeta \sup_{\theta \in \Psi_{t_0}} \{\|\mathbf{p}(\theta)\|\} e^{-\lambda(t-t_0)} \tag{6.32}$$

then the uncertain time-delay system of (6.27), is said to be robustly exponentially stable with a decay rate of λ. In other words, the trivial solution, $\mathbf{p} = 0$, of the system, is exponentially stable with a decay rate of λ for all admissible uncertainties [127].

Recognizing that

$$\mathbf{I} + \sum_{i=1}^{m(m-1)} \mathbf{M}_d^\ell = \mathbf{M} \tag{6.33}$$

we conclude the robust exponential stability of the network from Theorem 4 of [127], which is repeated here with some modification to conform to our problem.

Theorem 6.6: Consider the system (6.27) with initial condition (6.23), and assume that $-\mathbf{M}$ is a Hurwitz stable matrix satisfying

$$\|e^{\mathbf{M}t}\| \leq ce^{-\eta t} \tag{6.34}$$

for some real numbers $c \geq 1$ and $\eta > 0$. In the left hand side of the above equation, e denotes a "matrix" exponential operator. If the inequality

$$\frac{c}{\eta}\left[\bar{\tau} \sum_{\ell=1}^{m(m-1)} \left(\mu_1^\ell + \mu_2^\ell\right) + b_d + \sum_{\ell=1}^{m(m-1)} b_d^\ell\right] < 1 \tag{6.35}$$

holds, then the transient response of $\mathbf{p}(t)$ satisfies

$$\|\mathbf{p}(t)\| \leq \zeta \sup_{\theta \in \Psi_{t_0}} \{\|\phi(\theta)\|\} e^{-\rho \int_{t_0}^{t} \frac{d\theta}{\tau(\theta)}}, \quad \forall t \geq t_0, \; \zeta \geq 1 \tag{6.36}$$

where

$$\mu_1^\ell = \|\mathbf{M}_d^\ell\| + \|\mathbf{M}_d^\ell\| b_d \tag{6.37}$$

$$\mu_2^\ell = \sum_{j=1}^{m(m-1)} \|\mathbf{M}_d^\ell \mathbf{M}_d^j\| + \|\mathbf{M}_d^\ell\| \sum_{j=1}^{m(m-1)} b_d^j \tag{6.38}$$

and $\rho > 0$ is the unique positive solution of the transcendental equation

$$1 - \frac{c}{\eta} b_d - \frac{\rho}{\eta \tau(0)} = \mu_3 \, \frac{c}{\eta} \, e^{\frac{\rho}{1-\delta}} \tag{6.39}$$

where

$$\mu_3 = \bar{\tau} \sum_{\ell=1}^{m(m-1)} \mu_1^\ell + \bar{\tau} e^{\frac{\rho}{1-\delta}} \sum_{\ell=1}^{m(m-1)} \mu_2^\ell + \sum_{\ell=1}^{m(m-1)} b_d^\ell \tag{6.40}$$

Furthermore, the system described by (6.27) and (6.23) is robustly exponentially stable with a decay rate $\rho/\bar{\tau}$.

The left-hand side of the transcendental equation (6.39), is a continuous decreasing function of ρ and its right-hand side is a continuous increasing function, and by virtue of (6.35) at $\rho = 0$, the right-hand side is less than the left-hand side. Therefore (6.39) has a unique positive solution, as desired.

6.5 Summary

The cognitive radio network dynamics were studied with emphasis on the disequilibrium (transient) behaviour of the network. Theory of projected dynamic systems was used to develop a dynamic model, which governs the evolution of the network's state trajectory before reaching the equilibrium state. The stationary points of this dynamic model coincide with the equilibrium points of the corresponding VI model, developed in Chapter 4. The network was modeled as a constrained piecewise affine system and its stability in the presence of perturbation and multiple-time-varying delays was studied.

Chapter 7

Computer Experiments II

Using the testbed described in Chapter 5, simulation results are now presented to support theoretical underpinnings of the previous chapter. Network dynamics are simulated for both delay-free and multiple-time-varying-delay cases. Also, the solution stability is studied under system perturbation. Numerical values for parameters are chosen in the same way that was described in Chapter 5.

7.1 Projected Dynamic System

To study the transient behaviour of a cognitive radio network, a scenario is considered for three users and three subcarriers, so that we may arrive at insightful conclusions. Moreover, three is chosen merely for the sake of visualization. It is assumed that all the users update their power vectors simultaneously under the assumption that the network experiences worst-case interference conditions.

Figure 7.1 depicts state trajectories for three users obtained from a discrete-time approximation of the PD system by solving the quadratic programming described in

(6.19), when the following sequence of events happens. First, all three subcarriers are idle and can be used by the secondary users. Therefore, the state trajectories evolve in the three-dimensional space (i.e. $p_1^i p_2^i p_3^i$ space). Then, the second subcarrier is no longer available and state trajectories enter the two-dimensional space and evolve in $p_1^i p_3^i$ plane. After that the same thing happens to the third subcarrier and state trajectories evolve in one-dimensional space (i.e. p_1^i line). After a while, subcarrier three becomes idle and therefore available again. Thus, the state trajectories enter from p_1^i line to $p_1^i p_3^i$ plane. When subcarrier two becomes available again, state trajectories enter from $p_1^i p_3^i$ plane to $p_1^i p_2^i p_3^i$ space.

It is obvious that the power trajectories enter from higher-dimensional spaces to lower-dimensional spaces according to the number of available subcarriers, and again they go back to higher-dimensional spaces when users have access to more subcarriers, which is what should happen during a successful operation. The achieved equilibrium points for different users as they exist between occurrences of the mentioned events, are shown by stars on their state trajectories. Also, arrows in Figure 7.1 show the direction of evolution of states for different users.

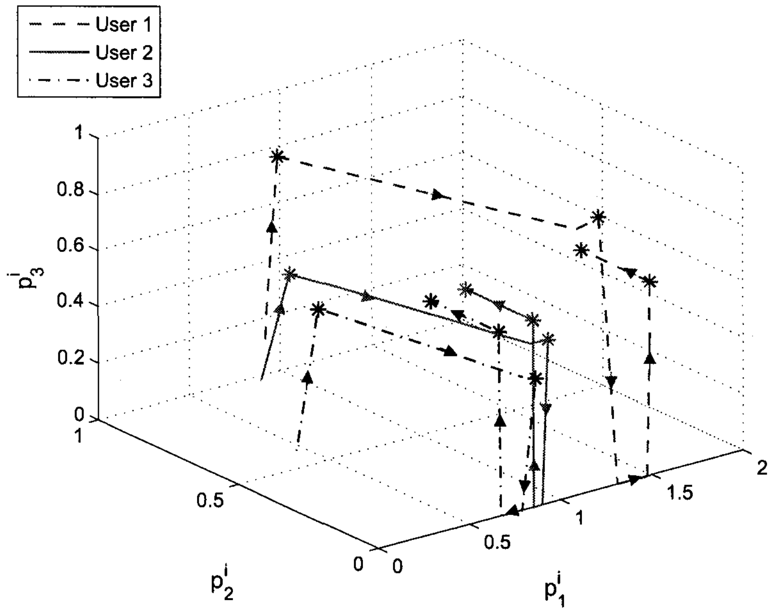


Figure 7.1: Power trajectories for a network of 3 users with 3 available subcarriers obtained from the associated PD system, when both the interference gains and the number of subcarriers change by time. Direction of evolution of states and the achieved equilibrium points are shown by arrows and stars, respectively. Trajectories enter lower dimensional spaces when spectrum holes disappear and then again go back to higher dimensional spaces when new spectrum holes are available. When the second subcarrier is not idle, trajectories enter $p_1^i p_3^i$ plane and when the third subcarrier is not also idle anymore, trajectories enter p_1^i line. After a while when third and then second subcarriers become available again, state trajectories go back to $p_1^i p_3^i$ plane and then $p_1^i p_2^i p_3^i$ space.

7.2 Sensitivity Analysis

To study the solution stability via simulation, the system is perturbed and the equilibrium point of the perturbed system is calculated. The interference-gain matrix and the noise vector are respectively perturbed as $\mathbf{M} + w_{\mathbf{M}}\Delta\mathbf{M}$ and $\boldsymbol{\sigma} + w_{\boldsymbol{\sigma}}\Delta\boldsymbol{\sigma}$, where $w_{\mathbf{M}}$ are $w_{\boldsymbol{\sigma}}$ are weights. The perturbation terms $\Delta\mathbf{M}$ and $\Delta\boldsymbol{\sigma}$ are chosen in the same way that $\mathbf{M}$ and $\boldsymbol{\sigma}$ were chosen, respectively as described in Chapter 5. Results at three different subcarriers are shown separately in Figure 7.2. As the perturbation terms decay (i.e. the weights $w_{\mathbf{M}}$ are $w_{\boldsymbol{\sigma}}$ move toward zero) and the perturbed system approaches the original one, behaviour of the perturbed system converges to the solution of the original system, which is shown by stars in Figure 7.2. The arrows show the direction in which the solution of the perturbed system converges to the solution of the original system. This experiment validates the notion of solution stability that was discussed previously.

When delays introduced by the feedback channels are considered, it may take longer for both the original system and the perturbed systems to achieve an equilibrium. Under the conditions mentioned in Chapter 6, the robust exponential stability of the system is guaranteed and similar results are obtained in simulations for the time-delay cases with constraints on delays.

7.3 Multiple-Time-Varying Delays

Simulation results for the above network of three users and three potentially available subcarriers, with a similar sequence of events, mentioned in Section 7.1, are repeated with asynchronous adjustment scheme. In the beginning, all three subcarriers are idle and can be used by secondary users. Then, the second subcarrier is no longer available, and after that the same thing happens to the third subcarrier. After a while, subcarriers two and then three become idle and therefore available again. Power trajectories and achieved equilibrium points are shown in Figure 7.3. Figure 7.4 depicts the random delays in adjustment schemes, $\tau^i(t)$, used by different users, which shows that most of the time users have used out-dated information to update their power vectors. Results confirm the ability of the system to achieve retarded equilibria under the conditions given in *Theorem 6.6.* By increasing the delay the performance of the system will degrade and eventually the system becomes unstable.

7.4 Summary

Simulations were conducted to demonstrate the concept of solution stability. The system was perturbed and its equilibrium solution was calculated. By decaying the perturbation terms, the equilibrium solution of the perturbed system converged to the equilibrium solution of the original system. The ability of the dynamic model, obtained using PD system theory, was validated by simulations for both delay-free and multiple-time-varying-delay cases. The results presented here show that by appearance and disappearance of spectrum holes, the state trajectory of the network enters higher and lower dimensional subspaces in the global state space, respectively.

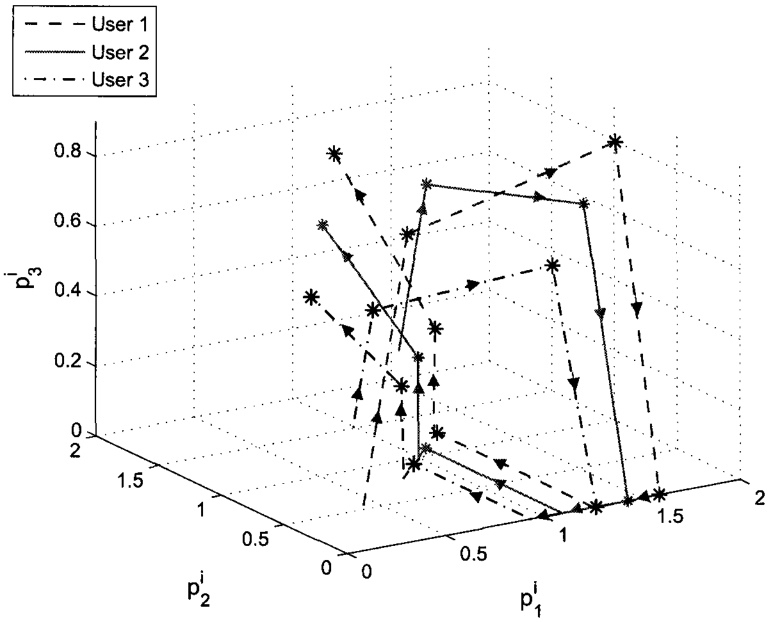


Figure 7.3: Power trajectories for a network of 3 users with 3 available subcarriers obtained from the associated multiple-time-varying-delay PD system with uncertainty, when both the interference gains and the number of subcarriers change by time. Direction of evolution of states and the achieved equilibrium points are shown by arrows and stars, respectively. Trajectories enter lower dimensional spaces when spectrum holes disappear and then again go back to higher dimensional spaces when new spectrum holes are available. When the second subcarrier is not idle, trajectories enter $p_1^i p_3^i$ plane and when the third subcarrier is not also idle anymore, trajectories enter p_1^i line. After a while when second and then third subcarriers become available again, state trajectories go back to $p_1^i p_2^i$ plane and then $p_1^i p_2^i p_3^i$ space.

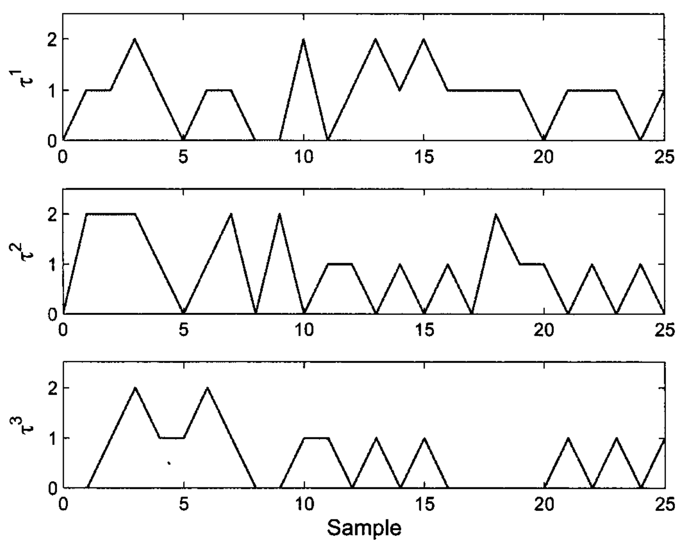


Figure 7.4: Time-varying delays introduced by feedback channels in transmit power control loops for a network of 3 users.

Chapter 8

Double-Layer Network Dynamics

There are two worlds of wireless communications: the legacy (old) wireless world and the cognitive (new) wireless world. The previous chapters were focused on the new world with spectrum holes being the medium through which the two worlds interact. Releasing subbands by primary users allows the cognitive radio users to perform their normal tasks and therefore, survive. In other words, the old world affects the new world through appearance and disappearance of the spectrum holes and there is a *master-slave* relationship between them. This chapter addresses the fact that the two worlds of wireless communications are going on side by side. This makes a cognitive radio network a *multiple-time-scale* dynamic system; a large-scale time in which the activities of primary users change, and a small-scale time in which the activities of secondary users change accordingly. Such systems are called *double-layer dynamic* (DLD) systems [128]. This chapter extends the developed theoretical framework of the previous chapters to capture the multiple-time-scale nature of cognitive radio networks and lays the groundwork for further research. This topic is similar to an uncharted territory and has a great deal of potential both in theoretical and practical

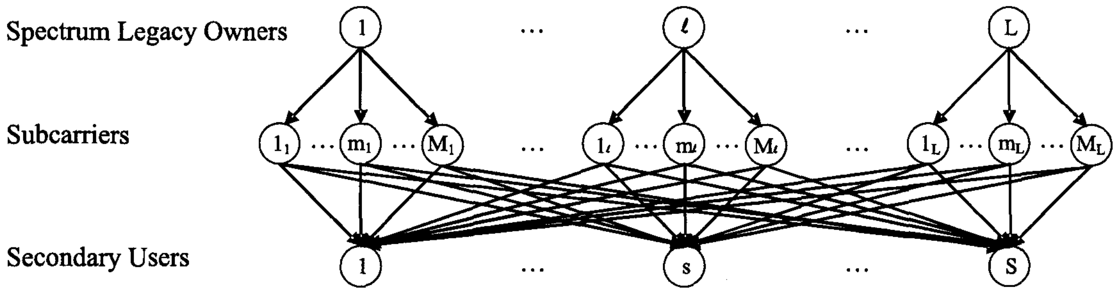


Figure 8.1: The spectrum supply chain network.

terms.

8.1 Two-Time-Scale Behaviour

A cognitive radio network, which is a system of systems, is a *goal-seeking* system in the sense described in [129]. The following classes of problems are involved in developing a cognitive radio network:

- Specifying the goal that the system is pursuing (i.e. efficient spectrum utilization and ubiquitous network connectivity).
- Discriminating between the available alternatives based on the meaning of a desirable decision.
- Choosing a desirable action based on a decision-making process.

By the same token, every subsystem in the network (i.e. every cognitive radio) is a goal-seeking system too.

Due to the master-slave relationship between the legacy and the cognitive wireless worlds, the spectrum supply chain network has a hierarchical structure [130].

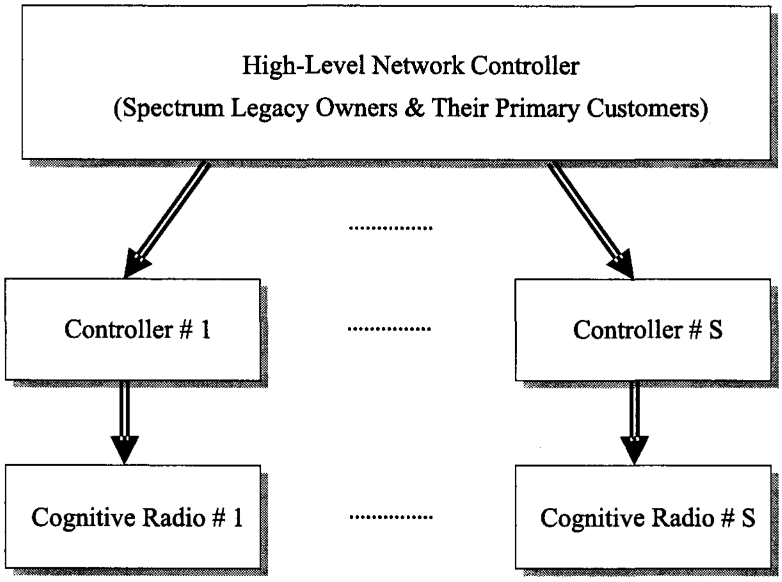


Figure 8.2: Decentralized hierarchical control structure in a cognitive radio network.

As mentioned in Chapter 4, the resource-allocation problem should be solved in two stages regarding discrete events and continuous states. Therefore, the local controllers in Figure 8.2 are two-level controllers [132,133]. The corresponding two-level control scheme is shown in Figure 8.3. The supervisory-level (i.e. the higher-level) controller is, in effect, an event-based controller that deals with appearance and disappearance of spectrum holes. The radio-scene analyzer will inform the supervisory-level controller, if it detects a change in the status of the available spectrum holes. In that case, the supervisory-level controller calls for reconfiguration of the transmitter in order to adapt the transmitting parameters to the new set of available channels. The field-level (i.e. the lower-level) controller is a state-based controller that adjusts the transmit power over the set of available channels chosen by the supervisory-level controller according to the interference level in the radio environment. A cognitive radio may build an internal model for the external world. This model is used to

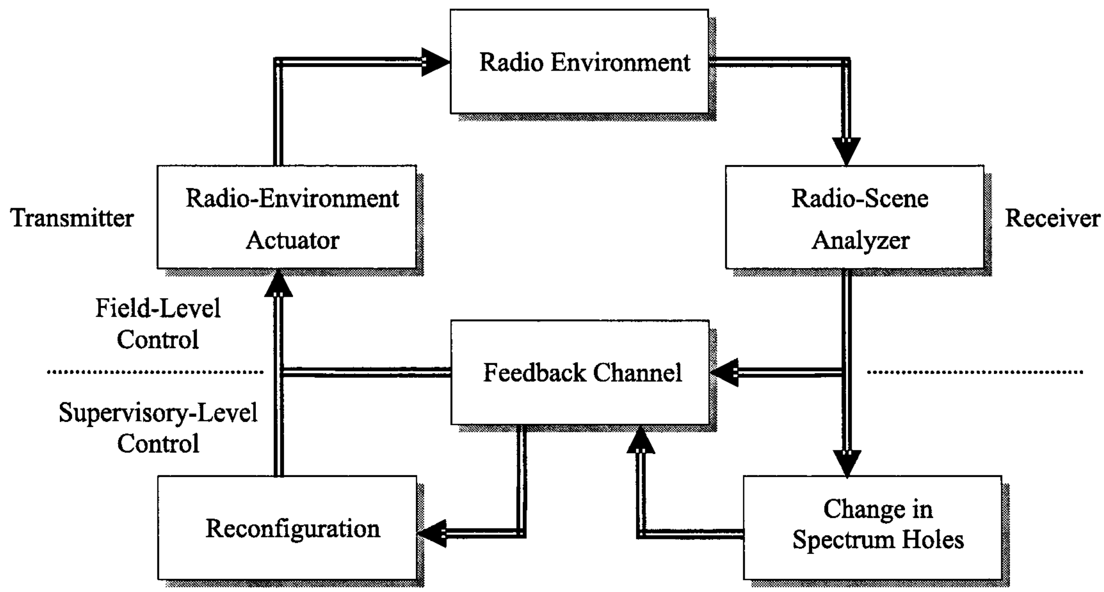


Figure 8.3: Two-level control scheme for cognitive radio.

predict the availability of certain subbands, the duration of their availability, and the approximate interference level in those subbands. These information will be critical for providing seamless communication in the dynamic wireless environment. Both the supervisory-level and the field-level controllers will benefit from a predictive model, which determines the control horizon, to plan ahead. The subsequent sections are focused on developing models that describe both equilibrium and transient behaviours of cognitive radio networks, emphasizing on the two-time-scale nature of the network.

8.2 Evolutionary Variational Inequalities (EVI)

It has been emphasized throughout the thesis that dynamics play a central role in cognitive radio networks. Therefore, the joint feasible set of the active users in the network is time-varying in nature and the finite-dimensional feasible set (4.37) captures a snapshot of the dynamic network with a time-varying feasible set. Regarding the continuous nature of time, we need to deal with infinite-dimensional feasible sets, if we consider time explicitly in the structure of the feasible set. Hence, the results should be extended to *Hilbert spaces.*

Definition 8.1: A *Hilbert space* is a generalization of *Euclidean space*, which is complete, separable, and possibly infinite-dimensional [134].
Hilbert space extends the results of vector algebra and calculus to spaces with any finite or infinite number of dimensions. The real space L^2 is a Hilbert space. This section extends the results of Section 4.5 by explicitly considering time in the obtained AVI-based model. Since the time-varying nature of the network's feasible set is explicitly considered in the formulation, theory of time-dependent VI or evolutionary VI (EVI) should be employed to obtain an equilibrium model for the network. The EVI-based model gives a curve of equilibria over a time interval of interest $[0, T]$. The predictive model will provide a reasonable estimate for T.

By considering time as an additional scalar parameter, the joint feasible set will be the following subset of the Hilbert space $L^2([0,T], \mathbb{R}^{m \times n})$.

$$K = \bigcup_{t \in [0,T]} K_t \tag{8.1}$$

where K_t was described in (4.37). The network, whose feasible set is described by K_t at a specific time instant $t \in [0, T]$, is a snapshot of the dynamic network with the time-varying feasible set (8.1) at that particular time instant. The EVI-based equilibrium model of the network may therefore, be stated as follows: find the $\mathbf{p}^* \in K$

such that the condition

$$\int_0^T (\mathbf{p} - \mathbf{p}^*)^T (\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}^*)\, dt \geq 0, \quad \forall \mathbf{p} \in K \tag{8.2}$$

holds [135]. In the next section, the results of Chapter 6 are extended to Hilbert spaces in order to study the equilibrium states of the network obtained from the above EVI in a dynamic framework.

8.3 Projected Dynamic Systems on Hilbert Spaces

A generalization of the theory of PD systems on Hilbert spaces is used to model the transient behaviour of the network, whose equilibrium behaviour is described by the EVI. Two distinct time-frames are considered: large-scale time-frame t and small-scale time-frame τ. There is a PD system corresponding to each $t \in [0, T]$, which is denoted by PDS_t. However, the evolution time variable for PDS_t is denoted by τ, which is different from time t. PDS_t describes the time evolution of the state trajectory of the system towards an equilibrium point on the curve of equilibria corresponding to the moment t. The following PD system

$$\frac{d\mathbf{p}(.,\tau)}{d\tau} = \Pi_K\left(\mathbf{p}(.,\tau), -\mathbf{F}(\mathbf{p}(.,\tau))\right) \tag{8.3}$$

with the initial condition

$$\mathbf{p}(.,0) = \mathbf{p}_0(.) \in K \tag{8.4}$$

is established as a dynamic model for the network that governs the transient behaviour of the network preceding the attainment of an equilibrium. The above PD system's stationary points coincide with the equilibrium points of the corresponding EVI problem. The associated dynamic model to the equilibrium problem will be realistic only if there is a unique solution path from a given initial point. The following theorem addresses the existence and uniqueness of the solution path for the above PD system [136, 137].

Theorem 8.1: Let H be a Hilbert space and K be a nonempty, closed, convex subset. Let $F : K \to H$ be a Lipschitz-continuous vector field and $\mathbf{p}_0 \in K$. Then the initial value problem

$$\frac{d\mathbf{p}(\tau)}{d\tau} = \Pi_K\left(\mathbf{p}(\tau), -\mathbf{F}(\mathbf{p}(\tau))\right), \quad \mathbf{p}(0) = \mathbf{p}_0 \in K \tag{8.5}$$

has a unique absolutely continuous solution on the interval $[0, \infty)$.

The next section answers the question that if the competitive behaviour of the users will lead to an equilibrium state in the network and if that equilibrium state is unique.

8.4 Solution Characteristics

Monotonicity properties of the underlying vector field of EVI/PDS play a key role. The following theorem states the conditions under which there exists a unique equilibrium solution.

Theorem 8.2: If $\mathbf{F}(\mathbf{p}) = \boldsymbol{\sigma} + \mathbf{M}\mathbf{p}$ is strictly monotone and Lipschitz continuous on K, then there exists $\mathbf{p}^* \in K$ such that

- $\mathbf{p}^*$ uniquely solves the EVI problem
- $\mathbf{p}^*$ uniquely solves $\Pi_K(\mathbf{p}(.,\tau), -\mathbf{F}(\mathbf{p}(.,\tau))) = 0$

It was discussed in Chapters 4 and 6 that in a real-life network, if the distance between receivers and their corresponding transmitters are short enough compared to their distances from other active transmitters in the network, then the strict monotonicity condition is satisfied and therefore, the network will have a unique equilibrium. The unique equilibrium state is the solution of the EVI problem, which coincides with the stationary point of the corresponding PD system. The next section answers the following two questions:

- If the initial state of the network is close to an equilibrium (i.e. if the competitive game starts near an equilibrium), will the state trajectory remain in a neighborhood of the equilibrium?
- Starting from an initial state, will the state trajectory asymptotically approach an equilibrium and at what rate?

8.5 Sensitivity and Stability Analyses

In EVI, monotonicity establishes the essential conditions for the existence and uniqueness of the solutions. In PD system, monotonicity is used to study stability of the perturbed system. The following definitions are recalled from [128].

Definition 8.2: A mapping F is called

(a) *pseudo-monotone* on K if

$$\langle \mathbf{F}(\mathbf{x}), \mathbf{y} - \mathbf{x} \rangle \geq 0 \Longrightarrow \langle \mathbf{F}(\mathbf{y}), \mathbf{y} - \mathbf{x} \rangle \geq 0, \quad \forall \mathbf{x}, \mathbf{y} \in K; \tag{8.6}$$

(b) *strictly pseudo-monotone* on X if

$$\langle \mathbf{F}(\mathbf{x}), \mathbf{y}-\mathbf{x}\rangle \geq 0 \Longrightarrow \langle \mathbf{F}(\mathbf{y}), \mathbf{y}-\mathbf{x}\rangle > 0, \quad \forall \mathbf{x}, \mathbf{y} \in K, \mathbf{x} \neq \mathbf{y}; \tag{8.7}$$

(c) *strongly pseudo-monotone* on X if there exists a constant $c > 0$ such that

$$\langle \mathbf{F}(\mathbf{x}), \mathbf{y}-\mathbf{x}\rangle \geq 0 \Longrightarrow \langle \mathbf{F}(\mathbf{y}), \mathbf{y}-\mathbf{x}\rangle \geq c\|\mathbf{x}-\mathbf{y}\|^2, \quad \forall \mathbf{x}, \mathbf{y} \in K, \mathbf{x} \neq \mathbf{y} \tag{8.8}$$

Definition 8.3: Let K be a closed, convex subset of a Hilbert space.

(a) A point $\mathbf{x}^* \in K$ is a monotone attractor for the PD system, if there exists a neighborhood V of $\mathbf{x}^*$ such that the distance $d(t) = \|\mathbf{x}(t) - \mathbf{x}^*(t)\|$ is a non-increasing function of t, for any solution $\mathbf{x}(t)$ starting in the neighborhood V.

(b) A point $\mathbf{x}^* \in K$ is a strict monotone attractor, if the distance $d(t)$ is decreasing. The following theorem addresses the stability of the network [136].

Theorem 8.3: Assume $\mathbf{F} : K \to L^2([0,T], \mathbb{R}^{m\times n})$ is Lipschitz continuous on K

- If $\mathbf{F}$ is strictly pseudo-monotone on K, then the unique curve of equilibria is a strict monotone attractor.
- If $\mathbf{F}$ is strongly pseudo-monotone on K, then the unique curve of equilibria is exponentially stable and an attractor.

Due to the properties of L^2-norm, the system is expected to evolve uniformly towards its equilibrium on the curve of equilibria for almost all $t \in [0,T]$. The above theorem provides the stability properties of the curve of equilibria as a whole in the sense that the curve attracts the trajectories of almost all PDS_t and therefore, it is possible for the curve to be reached for some instants $t \in [0,T]$ [136].

The implications between the different monotonicity notions are as follows [138]:

strong pseudo-monotonicity $\Rightarrow$ strict pseudo-monotonicity $\Rightarrow$ pseudo-monotonicity

strong monotonicity $\Rightarrow$ strong pseudo-monotonicity

strict monotonicity $\Rightarrow$ strict pseudo-monotonicity

monotonicity $\Rightarrow$ pseudo-monotonicity

It was discussed in Chapter 4 that Hurwitz condition of matrix $-\mathbf{M}$ guarantees strong monotonicity and therefore, guarantees the exponential stability of the unique curve of equilibria. In practice, the Hurwitz condition of matrix $-\mathbf{M}$ is achieved by establishing a low-interference regime through dynamic spectrum management and ad hoc routing. While dynamic spectrum manager makes sure that the neighboring transmitters will not use the same set of channels [42,88], opportunistic-spectrum ad hoc routing [108–110], which was previously described in Section 4.6, can guarantee that the distance between receivers and their corresponding transmitters are short enough compared to their distances from other active transmitters in the network.

8.6 Summary

This chapter addressed the two-time-scale behaviour of the cognitive radio network due to the coexistence of legacy and cognitive wireless worlds. Such a system is called a double-layer dynamic system. By extending the developed theoretical framework of Chapters 4 and 6 to explicitly include time as a parameter, both equilibrium and transient behaviors of the network were studied using the theories of evolutionary variational inequalities and projected dynamic systems on Hilbert spaces, respectively. Sufficient conditions for existence of a stable unique curve of equilibria and hints on how these conditions can be established in a real-life network were presented. This chapter proposed a new way of thinking, which requires further investigation in future.

Chapter 9

Computer Experiment III

"Science is made of mistakes, which are useful to make, because they lead, little by little, to the truth." Jules Verne (1828-1905)

A large-scale computer experiment is presented in this chapter to support theoretical underpinnings of the previous chapter. According to IEEE 802.11a standard for wireless local area networks, 48 out of 64 subcarriers are dedicated to data transmission. A network of 120 users is considered and it is assumed that 48 subcarriers can be potentially available for data transmission. Numerical values for parameters are chosen in the same way that was described in Chapter 5.

9.1 Curve of Equilibria

Initially, the network faces spectrum scarcity and users are not able to transmit with their maximum powers. The following sequence of events happens in the network:

- New users join the network.

- Some of the subcarriers are not available anymore for secondary usage.
- Network is perturbed close to its equilibrium state by randomly changing the interference gains, which occurs due to the mobility of users.
- More subcarriers are available for secondary usage.
- Some of the subcarriers are not available anymore for secondary usage.

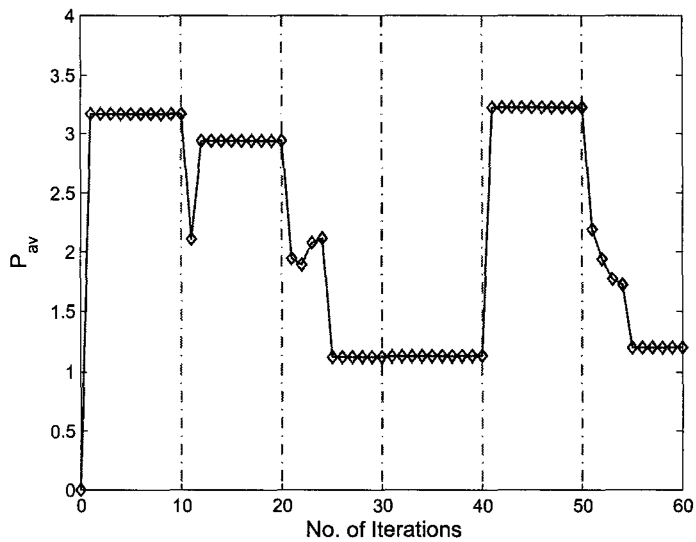
The interference gains were changed randomly due to user mobility as well as appearance and disappearance of users.

The average transmit power and the average data rate achieved by users after occurrence of each event are depicted in Figure 9.1. Power and data rate are plotted vs the number of iterations. Occurrences of events are shown by dashed lines. As shown in the figure, the network deviates from the equilibrium point, when an event occurs. Starting from an initial state dictated by the event, network moves toward a new equilibrium. In the diagram, 10 iterations were shown between two consequent events but the convergence is fast and in practice less iterations are required to reach a new point on the curve of equilibria from an arbitrary initial state, provided that the conditions for existence of a stable unique curve of equilibria are satisfied. Also, when the initial state dictated by a discrete event is not far from the achieved equilibrium (i.e. the network is perturbed around its equilibrium state), the state trajectory remains close to the equilibrium, which is the case for event 3.

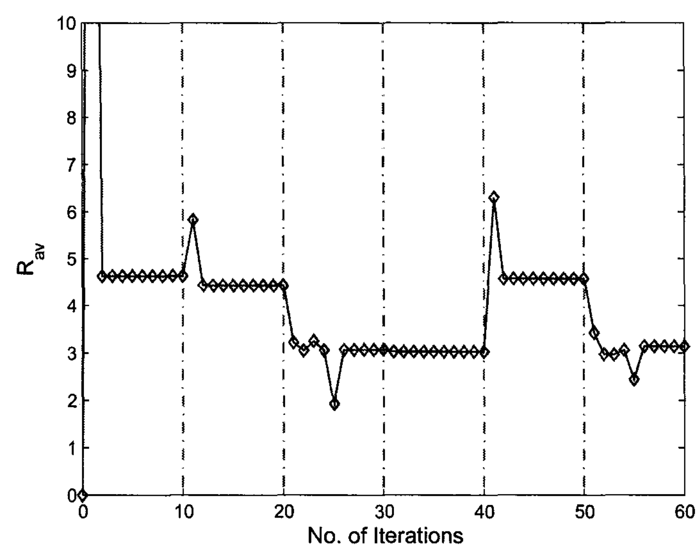
9.2 Summary

Simulations were conducted to demonstrate the double-layer dynamics of a cognitive radio network. Network deviates from its equilibrium state due to discrete events.

After the occurrence of each event, the network state trajectory starts from a new initial state and moves toward a new equilibrium. Provided that a stable unique curve of equilibria exists, if the initial state is close to the established equilibrium, the state trajectory will remain in a neighborhood of the equilibrium and if the initial state is relatively far from the established equilibrium, the state trajectory will approach a new equilibrium fast.



(a)



(b)

Figure 9.1: Dynamic behaviour of a network of 120 users and 48 potentially available subcarriers. Dashed lines show the occurrence of events. When an event occurs, network deviates from the established equilibrium. Starting from the initial state dictated by the event, network moves toward a new equilibrium: (a) average power and (b) average data rate are plotted vs the number of iterations.

Chapter 10

Contribution to the Literature

This research was focused on choosing an appropriate algorithm for resource allocation in cognitive radio networks and finding dynamic models that describe the global behaviour of the network, when different users employ the proposed algorithm. Ideas from information theory, optimization, game theory, and control theory were fused to develop such models.

Different formulations of IWFA have been proposed in the literature for resource allocation in wireless networks based on fixed local constraints [62] and flexible global constraints [63, 64] on transmit power per subcarrier. While the former may be way too conservative, the latter requires information exchange between primary and secondary users, which is more suitable for a market-model spectrum-sharing regime. This thesis provided a receiver-centric design based on flexible local constraints on transmit power per subcarrier dictated by interference-temperature limit [60]. There is no need for information exchange between different users in the proposed approach and it is well suited for an open spectrum-sharing regime. Also, the thesis highlighted the uncertainty issue in cognitive radio networks and proposed a robust version of the transmit-power controller, which improves the network robustness against malicious users as well as changes in the number of users, network topology, and available spectrum holes [60].

Along with [84] and [64], the thesis studied a new line of analysis of resource-allocation games in communication networks based on theory of VIs in order to provide conditions that guarantee existence of a unique equilibrium solution. Also, the VI-based reformulation of the resource-allocation game facilitates study of the network in a dynamic framework [60].

Transient behavior of communication networks, when iterative resource-allocation algorithms are employed, is generally under-explored. Analysis of transient behavior of a cellular network in which multiple users use Foschini-Miljanic distributed power control algorithm and share a single channel was studied in [115]. This thesis introduced a new line of analysis of transient behaviors in communication networks based on theory of PD systems. It facilitates sensitivity analysis and provides conditions that guarantee stability of networks in which multiple users use IWFA and share multiple channels [60].

In cognitive radio networks, the asynchronous adjustment scheme for resource allocation is the most realistic one among different options. In [139], convergence of asynchronous IWFA was proved by providing a set of conditions, which guarantee that the conditions of *asynchronous convergence theorem* in [45] are satisfied. Using the network dynamic model developed based on theory of PD systems, this thesis provided a new approach based on theory of dynamic systems to extend the available convergence results by proving convergence of asynchronous IWFA under uncertainty [60].

The thesis also introduced a new line of analysis of the multiple-time-scale dynamic behavior of cognitive radio networks in which large-scale time of operation applies to activities of the primary users and small-scale time of operation applies to secondary users.

The thesis contributions to the literature are summarized as follows.

Network Dynamics Viewed from Information-Theoretic and Optimization Perspectives

- The resource-allocation problem in a cognitive radio network was formulated as a non-cooperative game.
- Iterative waterfilling algorithm was used to find the Nash equilibrium solution of the game.
- Local and flexible power constraints based on the maximum allowable interference level in each channel were used in the formulation of the corresponding optimization problems that are solved by different users.
- Dominant sources of uncertainty in cognitive radio networks were identified.
- A robust version of the iterative waterfilling algorithm was presented to deal with uncertainty.
- The corresponding game was reformulated as an affine variational inequality problem and existence of a unique equilibrium state was addressed.
- Based on the theoretical results, conditions under which the stability of real-life cognitive radio networks are guaranteed were discussed.

Network Dynamics Viewed from Control-Theoretic Perspectives

- Theory of projected dynamic systems was employed to find a dynamic model that describes both equilibrium and transient behaviours of the network.
- Sensitivity analysis for the equilibrium states was presented.
- Hybrid systems theory was used to build a tracking method for the disequilibrium behaviour of the network.
- In this framework, the network was viewed as an ensemble of constrained piecewise affine systems.
- A novel approach based on the theory of dynamic systems was presented to address the convergence of the asynchronous IWFA under uncertainty.

Double-Layer Network Dynamics

- A model was built that
  - can be used as a testing tool for policy forecast, and
  - incorporates time evolution as the life span (control horizon) of a given policy.
- Two types of time dependency were studied
  - Time-dependent equilibria
  - Time-dependent behaviour away from the predicted curve of equilibria

Theories of evolutionary variational inequalities and projected dynamic systems on Hilbert spaces were used to study these two types of time dependency, respectively.

Appendix: Proofs

Proof of Theorem 4.1:

Proof of this theorem can be found in [96], Chapter 1, Proposition 1.4.2 and the discussion that follows it. Essential outline of the proof is as follows.

The VI formulation of the game is obtained by writing down the KKT conditions for each player's optimization problem and concatenating the KKT systems of all players in the form of a mixed complementarity problem.

Due to convexity and minimum principle, $\mathbf{p}^*$ is a Nash equilibrium if, and only if, for each $i = 1, \cdots, m$

$$-\left(\mathbf{p}^i - \mathbf{p}^{i*}\right)^T \nabla_{\mathbf{p}^i} f^i(\mathbf{p}^*) \geq 0, \quad \forall \mathbf{p}^i \in K^i \tag{A.1}$$

Thus, if $\mathbf{p}^*$ is a Nash equilibrium, then by concatenating these individual VIs, it follows easily that $\mathbf{p}^*$ must solve the prescribed VI.

Conversely, if $\mathbf{p}^*$ solves the VI problem, then

$$(\mathbf{p} - \mathbf{p}^*)^T \mathbf{F}(\mathbf{p}^*) \geq 0, \quad \forall \mathbf{p} \in K \tag{A.2}$$

In particular, for each $i = 1, \cdots, m$, let $\mathbf{p}$ be the vector whose jth subvector is equal to $\mathbf{p}^{*j}$ for $j \neq i$ and ith subvector is equal to $\mathbf{p}^i$, where $\mathbf{p}^i$ is an arbitrary element of the set K^i. The above inequality then becomes (A.1).

□

Proof of Proposition 4.1:

Let $(p_k^i, u^i, \gamma_k^i, \lambda_k^i)$ satisfy (4.23) and assume that the complement set of PS is nonempty. Since power is non-negative and $\sigma_k^i > 0$, $0 \leq \alpha_k^{ij} \leq 1$, it is known that

$$\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j > 0 \quad \forall k = 1, \dots, n \tag{A.3}$$

It can be proved by contradiction that $u^i > 0$. To show this, we first note that if $u^i = 0$, then

$$\gamma_k^i + \lambda_k^i \geq \frac{1}{\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j} > 0 \quad \forall k = 1, \dots, n \tag{A.4}$$

If $k \notin PS$, then $\lambda_k^i = 0$ and from (A.4) we must have $\gamma_k^i > 0$. Regarding the third complementarity condition in (4.23), $\gamma_k^i > 0$ leads to

$$CAP_k - \sigma_k^i - \sum_{j=1}^{m} \alpha_k^{ij} p_k^j = 0 \tag{A.5}$$

Therefore, we have

$$CAP_k - \sigma_k^{\max} \leq CAP_k - \sigma_k^i = \sum_{j=1}^{m} \alpha_k^{ij} p_k^j \tag{A.6}$$

Taking the summation over $k \notin PS$ from both sides of this equation leads to

$$\sum_{k \notin PS} (CAP_k - \sigma_k^{\max}) \leq \sum_{k \notin PS} \sum_{j=1}^{m} \alpha_k^{ij} p_k^j \tag{A.7}$$

$p_k^i = 0$, $\forall k \in PS$ and $\forall i = 1, \ldots, m$, so we have

$$\sum_{k \in PS} \sum_{j=1}^{m} \alpha_k^{ij} p_k^j = 0 \tag{A.8}$$

Therefore, we can rewrite (A.7) as

$$\begin{aligned} \sum_{k \notin PS} (CAP_k - \sigma_k^{\max}) \quad & \leq \sum_{k \notin PS} \sum_{j=1}^{m} \alpha_k^{ij} p_k^j + \sum_{k \in PS} \sum_{j=1}^{m} \alpha_k^{ij} p_k^j \\ & = \sum_{k=1}^{n} \sum_{j=1}^{m} \alpha_k^{ij} p_k^j \end{aligned} \tag{A.9}$$

Since $0 \leq \alpha_k^{ij} \leq 1$, we have

$$\sum_{j=1}^{m} \alpha_k^{ij} p_k^j \leq \sum_{j=1}^{m} p_k^j \tag{A.10}$$

and therefore

$$\sum_{k \notin PS} (CAP_k - \sigma_k^{\max}) \leq \sum_{k=1}^{n} \sum_{j=1}^{m} p_k^j \tag{A.11}$$

Changing the order of the two summations in the right-hand side of (A.11), we get

$$\sum_{k \notin PS} (CAP_k - \sigma_k^{\max}) \leq \sum_{j=1}^{m} \sum_{k=1}^{n} p_k^j \tag{A.12}$$

From the first inequality constraint of (4.3), we know that

$$\sum_{k=1}^{n} p_k^j \leq p_{\max}^j \tag{A.13}$$

Thus,

$$\sum_{k \notin PS} (CAP_k - \sigma_k^{\max}) \leq \sum_{j=1}^{m} p_{\max}^j \tag{A.14}$$

which contradicts (4.24). Thus $\forall k \notin PS$ and $\forall i = 1, \ldots, m$, in addition to λ_k^i, γ_k^i must be zero too and we must therefore have $u^i > 0$ in order to satisfy the first complementary condition in (4.23). Defining the following variables:

$$\begin{aligned} \nu^i &= -\frac{1}{u^i} \\ \varphi_k^i &= \frac{\gamma_k^i \left(\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j \right)}{u^i} \\ \varsigma_k^i &= \frac{\lambda_k^i \left(\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j \right)}{u^i} \end{aligned} \tag{A.15}$$

we do get a solution to (4.25).

Conversely, assume that $(p_k^i, \nu^i, \varphi_k^i, \varsigma_k^i)$ satisfies (4.25). This time, we must have $\nu^i < 0$. Otherwise,

$$\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j + \nu^i + \varphi_k^i + \varsigma_k^i > 0 \tag{A.16}$$

and then the first complementarity condition in (4.25) yields

$$p_k^i = 0, \quad \forall k = 1, \ldots, n \tag{A.17}$$

which contradicts the equality constraint in (4.25). Therefore, (4.23) holds by having

$$
\begin{aligned}
u^i &= -\frac{1}{\nu^i} \\
\gamma_k^i &= -\frac{\varphi_k^i}{\nu^i\left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)} \\
\lambda_k^i &= -\frac{\varsigma_k^i}{\nu^i\left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)}
\end{aligned}
\tag{A.18}
$$

This completes the proof. □

Proof of Proposition 4.2:

The proof is straightforward. The same steps in the proof of *Proposition 4.1* after showing that $u^i > 0$ should be followed. The relation between the corresponding variables defined in (A.15) and (A.18). □

Proof of Proposition 4.3:

Let $(p_k^i, \gamma_k^i, \lambda_k^i)$ satisfy (4.28) and assume that the complement set of PS is nonempty. Since power is non-negative and $\sigma_k^i > 0$, $0 \leq \alpha_k^{ij} \leq 1$, it is known that

$$\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j > 0 \quad \forall k = 1, \ldots, n \tag{A.19}$$

If $k \notin PS$, then $\lambda_k^i = 0$ and from the first complementary condition in (4.28) we have

$$\gamma_k^i \geq \frac{1}{\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j} > 0 \quad \forall k = 1, \ldots, n \tag{A.20}$$

Regarding the second complementarity condition in (4.28), $\gamma_k^i > 0$ leads to

$$\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j = CAP_k \tag{A.21}$$

Defining the following variables:

$$\begin{aligned} \varphi_k^i &= -\frac{1}{\gamma_k^i} \\ \varsigma_k^i &= \frac{\lambda_k^i \left(\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j\right)}{\gamma_k^i} \end{aligned} \tag{A.22}$$

we do get a solution to (4.29).

Conversely, assume that $(p_k^i, \varphi_k^i, \varsigma_k^i)$ satisfies (4.29). This time, we must have $\varphi_k^i < 0$. Otherwise,

$$\sigma_k^i + \sum_{j=1}^{m} \alpha_k^{ij} p_k^j + \varphi_k^i + \varsigma_k^i > 0 \tag{A.23}$$

and then the first complementarity condition in (4.29) yields

$$p_k^i = 0, \quad \forall k = 1, \dots, n \tag{A.24}$$

which contradicts the equality constraint in (4.29). Therefore, (4.28) holds by having

$$\begin{aligned} \gamma_k^i &= -\frac{1}{\varphi_k^i} \\ \lambda_k^i &= -\frac{\varsigma_k^i}{\varphi_k^i \left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)} \end{aligned} \tag{A.25}$$

This completes the proof. □

Proof of Theorem 4.2:

Proof of this theorem can be found in [96], Chapter 2, Theorem 2.3.3. Essential outline of the proof is as follows.

(a) Assume that F is strictly monotone on K. If $x \neq x'$ are two distinct solutions of the VI(K, F), $\forall y \in K$, we have

$$(y - x)^T F(x) \geq 0 \quad \text{and} \quad (y - x')^T F(x') \geq 0 \tag{A.26}$$

Substitute $y = x'$ into the first inequality and $y = x$ into the second inequality:

$$(x' - x)^T F(x) \geq 0 \quad \text{and} \quad (x - x')^T F(x') \geq 0 \tag{A.27}$$

Add these two inequalities:

$$(x' - x)^T \left(F(x') - F(x)\right) \leq 0 \tag{A.28}$$

This inequality contradicts the strict monotonicity property of F, thus establishing statement (a).

(b) Let $\bar{F} : \mathbb{R}^n \to \mathbb{R}^n$ denote a continuous extension of F, then SOL(K, F) = SOL$(K, \bar{F})$. If F is ξ-monotone on K for some $\xi > 1$, then $\exists x^{ref} \in K$ such that the set

$$L_< = \{x \in K |\; F(x)^T (x - x^{ref}) < 0\} \tag{A.29}$$

is bounded (possibly empty). This implies that there exists a bounded open set Ω

and a vector $x^{ref} \in K \cap \Omega$ such that

$$F(x)^T(x - x^{ref}) \geq 0, \ \ \forall x \in K \cap \partial\Omega \tag{A.30}$$

where $\partial\Omega$ denotes the topological boundary of Ω. This implies that the VI(K, F) has a solution. Moreover, if the set

$$L_{\leq} = \{x \in K |\ F(x)^T(x - x^{ref}) \leq 0\} \tag{A.31}$$

which is nonempty and larger than $L_{<}$, is bounded, then SOL(K, F) is nonempty and compact. The uniqueness of the solution follows from part (a). □

Proof of Theorem 6.1:

Proof of this theorem can be found in [95], Chapter 2, Theorem 2.4. Essential outline of the proof is as follows.

$$\Pi_K\left(\mathbf{p}^*, -\mathbf{F}(\mathbf{p}^*)\right) = 0 \Leftrightarrow \begin{cases} \text{either } \ \mathbf{F}(\mathbf{p}^*) = 0, \ \text{ or} \\ \mathbf{p}^* \in \partial K; \ \ \mathbf{F}(\mathbf{p}^*) = \alpha\mathbf{s}, \ \ \alpha > 0, \ \ \mathbf{s} \in S(\mathbf{p}^*) \end{cases} \tag{A.32}$$

which is equivalent to VI(K, F). □

Proof of Theorem 6.2:

Proof of this theorem for solutions in Euclidean space can be found in [95], Chapter 2, Theorem 2.5. The proof is based on the assumption that F is Lipschitz continuous with linear growth. In [140], Chapter 6, Theorem 6.1 and [141], Theorem 3.1, results were generalized from Euclidean space to Hilbert spaces of arbitrary dimensions. Also, the linear growth condition was relaxed. Essential outline of the proof for Hilbert spaces is presented in the Proof of Theorem 8.1. Essential outline of the proof for Euclidean space is as follows.

The associated ODE with discontinuous right-hand side is written as a pair of two equations. The first one is the ODE without the projection operator and the second one is a mapping that restricts the solution of the first equation to K. This approach benefits from the results of the *Skorokhod problem* [142] for finding such a mapping. The Skorokhod problem defines a mapping from the space of paths to itself [95].

Definition A.1: Let $\psi \in D([0,\infty), \mathbb{R}^{m\times n})$ with $\psi(0) \in K$ be given. Then (ϕ, η) solves the Skorokhod problem with respect to K if $\forall t \in [0,\infty)$

(i) $\phi(t) = \psi(t) + \eta(t), \quad \phi(0) = \psi(0)$

(ii) $\phi(t) \in K$

(iii) $|\eta(t)| < \infty$

(iv) $|\eta(t)| = \int_{(0,t]} I_{\partial K}(\phi(\tau))d|\eta(\tau)|$, where I is an indicator function.

(v) There exists measurable $\gamma : [0,\infty) \rightarrow \mathbb{R}^{m\times n}$ such that $\gamma(\tau) \in s(\phi(\tau))$ and $\eta(t) = \int_{(0,t]} \gamma(\tau)d|\eta(\tau)|$, where s is the inward normal. □

Proof of Theorem 6.3:

Proof of this theorem can be found in [95], Chapter 3, Theorem 3.6. Essential outline of the proof is as follows.

Consider the Lyapunov function

$$V(t) = \frac{1}{2}\|\mathbf{p}(t) - \mathbf{p}^*\|^2 \tag{A.33}$$

Then

$$\dot{V}(t) = \langle(\mathbf{p}(t) - \mathbf{p}^*), \Pi_K(\mathbf{p}(t), \boldsymbol{\sigma} + \mathbf{M}\mathbf{p}(t))\rangle \tag{A.34}$$

Regarding (6.16), it can be shown that

$$\dot{V}(t) \leq \langle(\mathbf{p}(t) - \mathbf{p}^*), -(\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}(t))\rangle \tag{A.35}$$

Due to strict monotonicity, we have

$$\dot{V}(t) < 0 \tag{A.36}$$

when $\mathbf{p}(t) \neq \mathbf{p}^*$, and

$$\lim_{t\to\infty} V(t) = 0 \tag{A.37}$$

Therefore, $\mathbf{p}^*$ is a is a strict monotone attractor. □

Proof of Theorem 6.4:

Proof of this theorem can be found in [95], Chapter 3, Theorem 3.8. Essential outline of the proof is as follows.

Consider the Lyapunov function $V(t)$ as (A.33). Since ξ-monotonicity implies strict monotonicity, $V(t)$ is strictly decreasing. It can be shown that due to ξ-monotonicity, $V(t)$ reaches zero and then it remains zero. Hence, there exists a T such that

$$\begin{cases} V(t) > 0, t \leq T \\ V(t) = 0, t > T \end{cases} \tag{A.38}$$

Therefore, $\mathbf{p}^*$ is a is a finite-time attractor. □

Proof of Theorem 6.5:

Proof of this theorem can be found in [95], Chapter 3, Theorem 3.7. Essential outline of the proof is as follows.

Consider the Lyapunov function $V(t)$ as (A.33). Regarding (6.16), it can be shown that

$$\dot{V}(t) \leq -\|\mathbf{M}\|.\|\mathbf{p}(t) - \mathbf{p}^*\|^2 \tag{A.39}$$

If there exists a $t_0 \geq 0$ for which $\|\mathbf{p}(t_0) - \mathbf{p}^*\| = 0$, we have

$$\|\mathbf{p}(t) - \mathbf{p}^*\| = 0, \ \ \forall t \geq t_0 \tag{A.40}$$

Since strong monotonicity implies monotonicity, we have

$$\|\mathbf{p}(t) - \mathbf{p}^*\| \leq \|\mathbf{p}_0 - \mathbf{p}^*\| \leq c\|\mathbf{p}_0 - \mathbf{p}^*\|e^{-\eta t} \tag{A.41}$$

where $c = e^{\eta t_0}$. Assume that

$$\|\mathbf{p}(t) - \mathbf{p}^*\| \neq 0, \ \ \forall t \geq 0 \tag{A.42}$$

Dividing both sides of (A.39) by $V(t)$ and taking the integral, we obtain

$$\|\mathbf{p}(t) - \mathbf{p}^*\| \leq c'\|\mathbf{p}_0 - \mathbf{p}^*\|e^{-\eta t} \tag{A.43}$$

Therefore, $\mathbf{p}^*$ is exponentially stable. □

Proof of Theorem 6.6:

The proof uses ideas given in [127, 143]. Let us consider the following differential equation:

$$\dot{\mathbf{y}}(t) = -(\eta - cb_d)\,\mathbf{y}(t) + q(t)\mathbf{y}\,(t - \tau(t)) \tag{A.44}$$

where

$$q(t) = \left(\eta - cb_d - \frac{\rho}{\tau(t)}\right) e^{-\rho \int_{t-\tau(t)}^{t} \frac{d\theta}{\tau(\theta)}} \tag{A.45}$$

It can be verified that

$$\mathbf{y}(t) = C_0 e^{-\rho \int_{t_0}^{t} \frac{d\theta}{\tau(\theta)}} \tag{A.46}$$

is a solution of (A.44), where C_0 is a constant. The mean-value theorem is applied to $\int_{t-\tau(t)}^{t} \frac{d\theta}{\tau(\theta)}$ twice . It follows that $\exists\ \theta_1, \theta_2 \in \mathbb{R}$ satisfying $0 < \theta_1 < \theta_2 < 1$ such that

$$\begin{aligned} \int_{t-\tau(t)}^{t} \frac{d\theta}{\tau(\theta)} &= \frac{\tau(t)}{\tau(t) - \theta_1 \tau(t)\dot{\tau}(t - \theta_2\tau(t))} \\ &= \frac{1}{1 - \theta_1 \dot{\tau}(t - \theta_2 \tau(t))} \leq \frac{1}{1-\delta} \end{aligned} \tag{A.47}$$

For $\rho > 0$ satisfying (6.39), we have

$$q(t) \geq \left(\eta - cb_d - \frac{\rho}{\tau(t_0)}\right) e^{-\frac{\rho}{1-\delta}} = c\mu_3 \tag{A.48}$$

Now we show that for a proper choice of C_0, the solution of (A.46) is an upper bound for the solution of (6.27) and (6.23).

Let us choose C_0 such that the following inequalities are satisfied simultaneously:

$$\mathbf{y}(t) \geq \|\phi(\theta)\|, \quad \forall\theta \in \psi_{t_0} \tag{A.49}$$

$$C_0 \geq c \sup_{\theta\in\Psi_{t_0}} \|\phi(\theta)\| \tag{A.50}$$

Solution of (6.27) can be written as

$$\begin{aligned}\mathbf{p}(t) = \mathbf{p}(t_0)e^{-\mathbf{I}t} - \int_{t_0}^{t} e^{-\mathbf{I}(t-\theta)} \sum_{\ell=1}^{m(m-1)} \mathbf{M}_d^\ell \mathbf{p}(\theta - \tau^\ell(\theta))d\theta \\ - \int_{t_0}^{t} e^{-\mathbf{I}(t-\theta)}[\varrho(\theta) + \Delta\mathbf{M}_d^\ell(\theta)\mathbf{p}(\theta - \tau^\ell(\theta))]d\theta\end{aligned} \tag{A.51}$$

Regarding (6.30), (6.31), and (6.34), $\forall t \geq t_0$ we have

$$\begin{aligned}\|\mathbf{p}(t)\| \leq ce^{-\eta t}\sup_{\theta\in\Psi_{t_0}} \|\phi(\theta)\| + \int_{t_0}^{t} ce^{-\eta(t-\theta)} \sum_{\ell=1}^{m(m-1)} \left(\|\mathbf{M}_d^\ell\| + b_d^\ell\right) \|\mathbf{p}(\theta - \tau^\ell(\theta))\|d\theta \\ + \int_{t_0}^{t} ce^{-\eta(t-\theta)} b_d \|\mathbf{p}(\theta)\|d\theta\end{aligned} \tag{A.52}$$

Considering the term $cb_d\mathbf{y}(t) + q(t)\mathbf{y}\left(t - \tau(t)\right)$ in (A.44) as an inhomogeneous term, The solution of this equation can be written as

$$\mathbf{y}(t) = C_0 e^{-\eta t} + \int_{t_0}^{t} cb_d e^{-\eta(t-\theta)}\mathbf{y}(\theta)d\theta + \int_{t_0}^{t} e^{-\eta(t-\theta)} q(\theta)\mathbf{y}(\theta - \tau(\theta))d\theta \tag{A.53}$$

In order to compare $\|\mathbf{p}(t)\|$ with $\mathbf{y}(t)$, we define $\mathbf{z}(t) = \|\mathbf{p}(t)\| - \mathbf{y}(t)$. From (A.52) and (A.53), $\forall t \geq t_0$ we have

$$\mathbf{z}(t) \leq \left(c \sup_{\theta \in \Psi_{t_0}} \|\phi(\theta)\| - C_0 \right) e^{-\eta t}$$
$$+ c \int_{t_0}^{t} e^{-\eta(t-\theta)} \left(\sum_{\ell=1}^{m(m-1)} \left(\|\mathbf{M}_d^\ell\| + b_d^\ell \right) \mathbf{z}(\theta - \tau(\theta)) + b_d \mathbf{z}(\theta) \right) d\theta$$
$$+ \int_{t_0}^{t} e^{-\eta(t-\theta)} \left(c \sum_{\ell=1}^{m(m-1)} \left(\|\mathbf{M}_d^\ell\| + b_d^\ell \right) - q(\theta) \right) \mathbf{y}(\theta - \tau(\theta)) d\theta \quad \text{(A.54)}$$

Inequalities (A.48) and (A.49) lead to:

$$\mathbf{z}(t) \leq c \int_{t_0}^{t} e^{-\eta(t-\theta)} \left(\sum_{\ell=1}^{m(m-1)} \left(\|\mathbf{M}_d^\ell\| + b_d^\ell \right) \mathbf{z}(\theta - \tau(\theta)) + b_d \mathbf{z}(\theta) \right) d\theta \quad \text{(A.55)}$$

Also, (A.49) implies that

$$\mathbf{z}(t) \leq 0, \quad \forall t \in \Psi_{t_0} \quad \text{(A.56)}$$

Since $\mathbf{z}(t)$ is continuous, the above inequality holds in some neighborhood of t_0. Assume that t^* is the smallest t for which $\mathbf{z}(t^*) > 0$. Due to (A.56) and the fact that $\mathbf{z}(\theta) \leq 0$ for any $0 \leq \theta \leq t^*$, it follows from (A.55) that $\mathbf{z}(t^*) \leq 0$, which contradicts the assumption. Hence, $\mathbf{z}(t) \leq 0$ for all $t \geq t_0$, which leads to

$$\|\mathbf{p}(t)\| \leq C_0 e^{-\rho \int_{t_0}^{t} \frac{d\theta}{\tau(\theta)}} = \zeta \sup_{\theta \in \Psi_{t_0}} \{\|\phi(\theta)\|\} e^{-\rho \int_{t_0}^{t} \frac{d\theta}{\tau(\theta)}}, \quad \forall t \geq t_0 \quad \text{(A.57)}$$

where

$$\zeta = \frac{C_0}{\sup\limits_{\theta \in \Psi_{t_0}} \|\phi(\theta)\|} \geq 1 \tag{A.58}$$

Finally, from the boundedness of $\tau(t)$, we have

$$\|\mathbf{p}(t)\| \leq \zeta \sup_{\theta \in \Psi_{t_0}} \{\|\phi(\theta)\|\} e^{-\frac{\rho}{\bar{\tau}}}, \quad \forall t \geq t_0 \tag{A.59}$$

Therefore, the system (6.27) and (6.23) is robustly exponentially stable with a decay rate $\frac{\rho}{\bar{\tau}}$. □

Proof of Theorem 8.1:

Proof of this theorem can be found in [140], Chapter 6, Theorem 6.1 and [141], Theorem 3.1. Essential outline of the proof is as follows.

Let L be the Lipschitz constant and $\|\mathbf{p}\| \leq b$ for $b > 0$. Consider the interval $[0, l]$ where $l = \frac{b}{\|\mathbf{F}(\mathbf{p}_0)\|+bL}$.

(i) Construct the sequence $\{\mathbf{p}_k(.)\}$ of absolutely continuous functions defined on $[0, l]$ with values in K such that $\forall k \geq 0$ $\mathbf{p}_k(0) = \mathbf{p}_0$ and for almost all $t \in [0, l]$ and every neighborhood $\mathcal{M} \in K \times K$ of 0, $\{\mathbf{p}_k(.)\}$ and the sequence of its derivatives $\{\mathbf{p}'_k(.)\}$ have the following property:

$$(\mathbf{p}_k(t), \mathbf{p}'_k(t)) \in \text{graph}(\mathbf{F} - \tilde{N}_k) + \mathcal{M}, \quad \forall k \geq k_0(t, \mathcal{M}) \tag{A.60}$$

where

$$\tilde{N}_k(\mathbf{p}) = \{\mathbf{n} \in N_k(\mathbf{p}) | \; \|\mathbf{n}\| \leq \|\mathbf{F}(\mathbf{p})\|\} \subseteq N_k(\mathbf{p}) \tag{A.61}$$

and $N_k(\mathbf{p})$ is the normal cone to the set K at the point $\mathbf{p}$:

$$N_k(\mathbf{p}) = \{\mathbf{n} \in K | \; \langle \mathbf{n}, \mathbf{p} - \mathbf{x} \rangle \geq 0, \quad \forall \mathbf{x} \in K\} \tag{A.62}$$

The uniform convergence of the sequence $\{\mathbf{p}_k(.)\}$ can be proved.

(ii) After proving the uniform convergence of the sequence of approximate solutions $\{\mathbf{p}_k(.)\}$ to a limit $\mathbf{p}(.)$, select a subsequence for which the sequence of derivatives $\{\mathbf{p}'_k(.)\}$ in $L^\infty([0, l], K)$ converges weakly to the derivative of $\mathbf{p}(.)$.

(iii) It is shown that $\mathbf{p}(.)$ is a solution to the initial value problem (6.4) and (6.5).

(iv) From(i)-(iii), we know that the problem has solutions on the interval $[0, l]$. Assume that $\mathbf{p}_1(.)$ and $\mathbf{p}_2(.)$ are two solutions starting at the point $\mathbf{p}_0$. It is shown that

$$\|\mathbf{p}_1(t) - \mathbf{p}_2(t)\|^2 \leq 0 \tag{A.63}$$

Therefore, $\mathbf{p}_1(t) = \mathbf{p}_2(t), \ \ \forall t \in [0, l]$, which proves the uniqueness of the solutions.

(v) Having the unique solution for the interval $[0, l]$, we consider $t_0 = l$ and apply the theorem again. Hence, we obtain a solution for an extended time interval. By continuing this process, a solution can be obtained for $t \in [0, \infty)$. □

Proof of Theorem 8.2:

The proof uses ideas given in [96, 128, 144].

(a) Uniqueness of the solution is concluded from Theorem 4.2, and the fact that F is an affine mapping.

(b) In [128], Proposition 3.1, it is proved that the PD system has at most one equilibrium point. Essential outline of the proof is as follows.

Assume that the PD system has at least two solutions $\mathbf{p}_1 \neq \mathbf{p}_2 \in K$. Then,

$$\Pi_K\left(\mathbf{p}_1, -\mathbf{F}(\mathbf{p}_1)\right) = 0 \text{ and } \Pi_K\left(\mathbf{p}_2, -\mathbf{F}(\mathbf{p}_2)\right) = 0 \tag{A.64}$$

Equivalently, this means that $-\mathbf{F}(\mathbf{p}_1) \in N_K(\mathbf{p}_1)$ and $-\mathbf{F}(\mathbf{p}_2) \in N_K(\mathbf{p}_2)$, where N_K is the normal cone. Since the set-valued mapping $\mathbf{p} \rightarrow N_K(\mathbf{p})$ is a monotone mapping, we have

$$(-\mathbf{F}(\mathbf{p}_1) + \mathbf{F}(\mathbf{p}_2))^T(\mathbf{p}_1 - \mathbf{p}_2) \geq 0 \tag{A.65}$$

or equivalently

$$(\mathbf{F}(\mathbf{p}_2) - \mathbf{F}(\mathbf{p}_1))^T(\mathbf{p}_2 - \mathbf{p}_1) \leq 0 \tag{A.66}$$

On the other hand, from strict monotonicity property of F, we have

$$(\mathbf{F}(\mathbf{p}_2) - \mathbf{F}(\mathbf{p}_1))^T(\mathbf{p}_2 - \mathbf{p}_1) > 0 \tag{A.67}$$

The last two equations lead to a contradiction. Therefore, the PD system has at most one equilibrium point. Solutions of the EVI problem are the same as the stationary points of the PD system and vice versa [128]. From (a) we know that the EVI has a unique solution. Therefore, the PD system has a unique equilibrium as well. □

Proof of Theorem 8.3:

Proof of this theorem can be found in [140], Chapter 7, Theorem 7.2 and Theorem 7.6. Essential outline of the proof is as follows.

(a) Consider the Lyapunov function

$$V(t) = \frac{1}{2}\|\mathbf{p}(t) - \mathbf{p}^*\|^2 \tag{A.68}$$

Then

$$\dot{V}(t) = \langle(\mathbf{p}(t) - \mathbf{p}^*), \Pi_K\left(\mathbf{p}(t), -(\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}(t))\right)\rangle \tag{A.69}$$

Regarding (6.16), we have

$$\dot{V}(t) \leq \langle(\mathbf{p}(t) - \mathbf{p}^*), -(\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}(t))\rangle \tag{A.70}$$

Strict pseudo-monotonicity leads to

$$\langle(\mathbf{p}(t) - \mathbf{p}^*), -(\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}(t))\rangle < 0 \tag{A.71}$$

and

$$\dot{V}(t) < 0 \tag{A.72}$$

Therefore, $\mathbf{p}^*$ is a is a strict monotone attractor.

(b) Since F is strongly pseudo-monotone, there exists $c > 0$ such that

$$\langle \mathbf{F}(\mathbf{p}^*), \mathbf{p} - \mathbf{p}^* \rangle \geq 0 \Longrightarrow \langle \mathbf{F}(\mathbf{p}), \mathbf{p} - \mathbf{p}^* \rangle \geq c\|\mathbf{p} - \mathbf{p}^*\|^2, \ \ \forall \mathbf{p} \in K \tag{A.73}$$

which implies that

$$\langle \mathbf{p}(t) - \mathbf{p}^*, -(\boldsymbol{\sigma} + \mathbf{M}\mathbf{p}(t)) \rangle \leq -c\|\mathbf{p} - \mathbf{p}^*\|^2 \tag{A.74}$$

From (A.69), we have

$$\dot{V}(t) \leq -c\|\mathbf{p} - \mathbf{p}^*\|^2 \tag{A.75}$$

Integration of the above inequality leads to

$$\frac{1}{2}\|\mathbf{p}(t) - \mathbf{p}^*\|^2 \leq \|\mathbf{p}_0 - \mathbf{p}^*\|^2 e^{-ct} \Rightarrow \|\mathbf{p}(t) - \mathbf{p}^*\| \leq \sqrt{2}\|\mathbf{p}_0 - \mathbf{p}^*\| e^{-\frac{c}{2}t} \tag{A.76}$$

which shows that $\mathbf{p}^*$ is exponentially stable. As $t \to \infty$, we obtain $\mathbf{p}(t) \to \mathbf{p}^*$ and therefore, $\mathbf{p}^*$ is an attractor. □

Bibliography

[1] S. Haykin, "Fundamental issues in cognitive radio," *in Cognitive Wireless Communication Networks, E. Hossain and V. K. Bhargava, Eds., Springer*, pp. 1–43, 2007.

[2] W. W. Lu, "Point-of-view article: Opening the U.S. mobile communications market," *Proceedings of IEEE*, vol. 96, no. 9, pp. 1450–1452, 2008.

[3] P. Stuckmann and R. Zimmermann, "Toward ubiquitous and unlimited-capacity communication networks: European research in framework programme 7," *IEEE Communications Magazine*, vol. 45, no. 5, pp. 148–157, 2007.

[4] FCC, *Spectrum policy task force report.* ET Docket No. 02-135, 2002.

[5] M. McHenry, "Report on spectrum occupancy measurements," *Shared Spectrum Company, http://www.sharedspectrum.com/*, 2005.

[6] M. A. McHenry, P. A. Tenhula, D. McCloskey, D. A. Roberson, and C. S. Hood, "Chicago spectrum occupancy measurements and analysis and a long-term studies proposal," in *Proc. Workshop on Technology and Policy for Accessing Spectrum (TAPAS)*, pp. 1–12, 2006.

[7] A. Petrin and P. G. Steffes, "Analysis and comparison of spectrum measurements performed in urban and rural areas to determine the total amount of spectrum usages," in *Proc. International Symposium on Advanced Radio Technologies*, pp. 9–12, 2005.

[8] T. Erpek, M. Lofquist, and K. Patton, "Spectrum occupancy measurements: Loring Commerce Centre, Limestone, Maine, Sep. 18-20, 2007," *Technical Report, Shared Spectrum Company*, pp. 1–35, 2007.

[9] R. I. Chiang, G. B. Rowe, and K. W. Sowerby, "A quantitative analysis of spectral occupancy measurements for cognitive radio," in *Proc. IEEE Vehicular Technology Conference*, pp. 3016–3020, 2007.

[10] M. H. Islam, C. L. Koh, S. W. Oh, X. Qing, Y. Y. Lai, C. Wang, Y. C. Liang, B. E. Toh, F. Chin, G. L. Tan, and W. Toh, "Spectrum survey in Singapore: Occupancy measurements and analysis," in *Proc. 3rd International Conference of Cognitive Radio Oriented Wireless Network and Communications (CROWNCOM08)*, pp. 1–7, 2008.

[11] K. A. Qaraqe, H. Celebi, M. S. Alouini, A. El-Saigh, L. Abuhantash, M. Al-Mulla, O. Al-Mulla, A. Jolo, and A. Ahmed, "Measurement and analysis of wideband spectrum utilization in indoor and outdoor environments," in *Proc. IEEE 18th Signal Processing and Communications Applications Conference (SIU-2010)*, pp. 1–4, 2010.

[12] S. Haykin, "Point-of-view article: Cognitive dynamic systems," *Proceedings of IEEE*, vol. 94, no. 11, pp. 1910–1911, 2006.

[13] N. Wiener, *Cybernetics: or Control and Communication in the Animal and the Machine.* MIT Press, 1965.

[14] J. P. Dupuy, *On the Origins of Cognitive Science: The Mechanization of the Mind.* MIT Press, 2009.

[15] J. Mitola, *Cognitive radio: An integrated agent architecture for software defined radio.* Ph.D. Dissertation, Royal Institute Technolology (KTH), Stockholm, Sweden, 2000.

[16] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, 2005.

[17] M. Jamshidi, "System of systems engineering - New challenges for the 21st century," *IEEE Aerospace and Electronic Systems Magazine*, vol. 23, no. 5, pp. 4–19, 2008.

[18] J. Pearl, *Causality: Models, Reasoning and Inference.* Cambridge University Press, 2nd ed., 2009.

[19] A. MacKenzie and L. DaSilva, *Game Theory for Wireless Engineers.* Morgan and Claypool Publishers, 2006.

[20] M. J. Osborne and A. Rubenstein, *A Course in Game Theory.* The MIT Press, 1994.

[21] D. D. Siljak, *Decentralized Control of Complex Systems.* Academic Press, 1991.

[22] A. G. O. Mutambara, *Decentralized Estimation and Control for Multisensor Systems.* CRC Press, 1998.

[23] H. W. Kuhn and S. Nasar, *The Essential John Nash.* Princeton University Press, 2002.

[24] J. F. Nash, "Equilibrium points in n-person games," *in Proceedings of the National Academy of Science*, vol. 36, pp. 48–49, 1950.

[25] J. F. Nash, "Non-cooperative games," *Annals of Mathematics*, vol. 54, pp. 286–295, 1951.

[26] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory.* SIAM, 1999.

[27] T. Basar and P. Bernhard, *H_∞ Optimal Control and Related Minimax Design Problems: A Dynamic Game Approach.* 2nd ed., Birkhauser, 1995.

[28] F. J. Doyle, "Robust control in biology: From genes to cells to systems," in *Proc. The International Federation of Automatic Control (IFAC)*, pp. 3470–3479, 2008.

[29] J. B. Burl, *Linear Optimal Control: H_2 and H_∞ Methods.* Addison-Wesley, 1999.

[30] V. I. Zhukovskiy and M. E. Salukvadze, *The Vectror-Valued Maximin.* Academic Press, 1994.

[31] M. Wellens, J. Wu, and P. Mahonen, "Evaluation of spectrum occupancy in indoor and outdoor scenario in the context of cognitive radio," in *Proc. International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*, pp. 1–8, 2007.

[32] S. Haykin, D. J. Thomson, and J. H. Reed, "Spectrum Sensing for cognitive radio," *Proceedings of IEEE*, vol. 97, no. 5, pp. 849–877, 2009.

[33] T. A. Weiss and F. K. Jondral, "Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency," *IEEE Communications Magazine*, vol. 42, no. 3, pp. S8–S14, 2004.

[34] U. Berthold, F. K. Jondral, S. Brandes, and M. Schnell, "OFDM-based overlay systems: a promising approach for enhancing spectral efficiency," *IEEE Communications Magazine*, vol. 45, no. 12, pp. 52–58, 2007.

[35] H. Arslan, H. A. Mahmoud, and T. Yucek, "OFDM for cognitive radio: merits and challenges," *in H. Arslan (Editor), Cognitive Radio, Software Defined Radio, and Adaptive Wireless Systems, Springer*, 2007.

[36] T. Keller and L. Hanzo, "Adaptive multicarrier modulation: A convenient framework for time-frequency processing in wireless communications," *Proceedings of the IEEE*, vol. 88, no. 5, pp. 611–640, 2000.

[37] A. R. S. Bahai, B. R. Saltzberg, and M. Ergen, *Multi-Carrier Digital Communications: Theory and Applications of OFDM.* 2nd Ed, Springer, 2004.

[38] K. E. Nolan, *Reconfigurable OFDM systems.* Ph.D. Dissertation, University of Dublin, Trinity College, 2005.

[39] Y. Li and G. Stuber, *Orthogonal Frequency Division Multiplexing for Wireless Communications.* Springer, 2006.

[40] M. Sterand, T. Svensson, T. Ottosson, A. Ahlen, A. Svensson, and A. Brunstrom, "Towards systems beyond 3G based on adaptive OFDMA transmission," *Proceedings of IEEE*, vol. 95, no. 12, pp. 2432–2455, 2007.

[41] M. Morelli, C. C. J. Kuo, and M. O. Pun, "Synchronization techniques for orthogonal frequency division multiple access (OFDMA): A tutorial review," *Proceedings of IEEE*, vol. 95, no. 7, pp. 1394–1427, 2007.

[42] F. Khozeimeh and S. Haykin, "Dynamic spectrum management for cognitive radio: An overview," *Wireless Communications and Mobile Computing*, pp. 1–13, 2009.

[43] A. van der Schaft and H. Schumacher, *An Introduction to Hybrid Dynamical Systems*. Springer, 2000.

[44] R. Goebel, R. G. Sanfelice, and A. R. Teel, "Hybrid Dynamical Systems," *IEEE Control Systems Magazine*, vol. 29, no. 2, pp. 28–93, 2009.

[45] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Prentice-Hall, 1989.

[46] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 7, pp. 1341–1347, 1995.

[47] G. Scutari, D. P. Palomar, and S. Barbarossa, "Simultaneous iterative water-filling for Gaussian frequency-selective interference channels," in *Proc. IEEE International Symposium on Information Theory*, pp. 600–604, 2006.

[48] G. Scutari, D. P. Palomar, and S. Barbarossa, "Asynchronous iterative water-filling for Gaussian frequency-selective interference channels: A unified framework," in *Proc. Information Theory and Applications Workshop*, pp. 349–358, 2007.

[49] I. Konnov, *Equilibrium Models and Variational Inequalities.* Springer, 2007.

[50] E. Altman, T. Boulogne, R. El-Azouzi, T. Jimenez, and L. Wynter, "A survey on networking games in telecommunication," *Computers and Operations Research*, vol. 24, no. 3, pp. 58–68, 2007.

[51] M. Felegyhazi and J. P. Hubaux, "Game theory in wireless networks: A tutorial," *Technical Report, EPFL*, pp. 1–15, 2007.

[52] S. Lasaulce, M. Debbah, and E. Altman, "Methodologies for analyzing equilibria in wireless games," *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 41–52, 2009.

[53] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Basar, "Coalitional game theory for communication networks," *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 77–97, 2009.

[54] M. Biagi, *Cross-layer optimization of multi-antenna 4G-WLANs.* Ph.D. Dissertation, University of Rome La Sapienza, Rome, Italy, 2005.

[55] N. Nei and C. Comaniciu, "Adaptive channel allocation spectrum ettiquette for cognitive radio networks," in *Proc. First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, pp. 269–278, 2005.

[56] J. O. Neel, *Analysis and design of cognitive radio networks and distributed radio resource management algorithms.* Ph.D. Dissertation, Virginia Polytechnic Institute and State University, 2006.

[57] C. Liang, *A game-theoretic approach to power management in MIMO-OFDM ad hoc networks.* Ph.D. Dissertation, Drexel University, 2006.

[58] M. Felegyhazi, *Non-cooperative behavior in wireless networks.* Ph.D. Dissertation, Ecole Polytechnique Federale De Lausanne (EPFL), Switzerland, 2007.

[59] M. Felegyhazi, M. Cagalj, S. S. Bidokhti, and J. P. Hubaux, "Non-cooperative multi-radio channel allocation in wireless networks," in *Proc. IEEE International Conference on Computer Communications (INFOCOM)*, pp. 1442–1450, 2007.

[60] P. Setoodeh and S. Haykin, "Robust transmit power control for cognitive radio," *Proceedings of IEEE*, vol. 97, no. 5, pp. 915–939, 2009.

[61] Y. Zhang, *Resource Allocation for OFDM-Based Cognitive Radio Systems.* Ph.D. Dissertation, University of British Columbia, Canada, 2008.

[62] F. Wang, M. Krunz, and S. Cui, "Price-based spectrum management in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 74–87, 2008.

[63] G. Scutari, D. P. Palomar, J. S. Pang, , and F. Facchinei, "Flexible design of cognitive radio wireless systems," *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 107–123, 2009.

[64] J. S. Pang, G. Scutari, D. P. Palomar, and F. Facchinei, "Design of cognitive radio systems under temperature-interference constraints: A variationall inequality approach," *IEEE Transactions on Signal Processing*, vol. 58, no. 6, pp. 3251–3271, 2010.

[65] A. Nagurney and Q. Qiang, *Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World.* John Wiley and Sons, 2009.

[66] M. Fukushima, "Stochastic and robust approaches to optimization problems under uncertainty ," in *Proc. of International Conference on Informatics Research for Development of Knowledge Society Infrastructure (ICKS)*, pp. 87–94, 2007.

[67] I. F. Akyildiz, W. Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks*, vol. 50, pp. 2127–2159, 2006.

[68] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 641–646, 1993.

[69] N. Bambos, S. C. Chen, and G. J. Pottie, "Channel access algorithms with active link protection for wireless communication networks with power control," *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp. 583–597, 2000.

[70] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 291–303, 2002.

[71] D. O'Neill, D. Julian, and S. Boyd, "Seeking Foschinis genie: optimal rates and powers in wireless networks," *Submitted to IEEE Transactions on Vehicular Technology*, 2003.

[72] S. Stánczak, M. Wiczanowski, and H. Boche, *Resource Allocation in Wireless Networks: Theory and algorithms.* Springer, 2006.

[73] C. W. Tan, D. Palomar, and M. Chiang, "Robustness-energy tradeoff in cellular network power control," *IEEE/ACM Transactions on Networking*, vol. 17, no. 3, pp. 912–925, 2009.

[74] C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication.* University of Illinois Press, 1998.

[75] T. M. Cover and J. A. Thomas, *Elements of Information Theory.* 2nd Ed, Wiley-Interscience, 2006.

[76] S. Haykin, *Communication Systems.* Wiley, 4th ed., 2001.

[77] W. Yu, *Competetion and Cooperation in Multi-user Communication Environments.* Ph.D. Dissertation, Stanford University, USA, 2002.

[78] A. B. Carleial, "Interference Channels," *IEEE Transactions on Information Theory*, vol. 24, no. 1, pp. 60–70, 1978.

[79] Z. Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 57–73, 2008.

[80] S. Hayashi and Z. Q. Luo, "Spectrum management for interference-limited multiuser communication systems," *IEEE Transactions on Information Theory*, vol. 55, no. 3, pp. 1153–1175, 2009.

[81] K. B. Song, S. T. Chung, G. Ginis, and J. M. Cioffi, "Dynamic spectrum management for next-generation DSL systems," *IEEE Communications Magazine*, vol. 40, no. 10, pp. 101–109, 2002.

[82] W. Yu, G. Ginis, and J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1105–1115, 2002.

[83] S. T. Chung, S. J. Kim, J. Lee, and J. M. Cioffi, "A game theoretic approach to power allocation in frequency-selective Gaussian interference channels," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, pp. 316–316, 2003.

[84] Z. Q. Luo and J. S. Pang, "Analysis of iterative waterfilling algorithm for multiuser power control in digital subscriber lines," *EURASIP Journal on Applied Signal Processing*, vol. 2006, pp. 1–10, 2006.

[85] N. Yamashita and Z. Q. Luo, "A nonlinear complementarity approach to multiuser power control for digital subscriber lines," *Optimization Methods and Software*, vol. 19, no. 5, pp. 633–652, 2004.

[86] Z. Q. Luo, J. S. Pang, and D. Ralph, *Mathematical Programs with Equilibrium Constraints.* Cambridge University Press, 1996.

[87] S. Haykin and M. Moher, *Modern Wireless Communications.* Prentice-Hall, 2005.

[88] F. Khozeimeh and S. Haykin, "Self-organizing dynamic spectrum management for cognitive radio networks," in *Proc. The 8th Conference on Communication Networks and Services Research (CNSR)*, pp. 1–7, 2010.

[89] D. Hebb, *The Organization of Behavior: A Neuropsychologocal Theory.* Wiley, 1949.

[90] S. Haykin, *Neural Networks and Learning Machines.* Prentice Hall, 3rd ed., 2009.

[91] M. Aghassi and D. Bertsimas, "Robust Game Theory," *Mathematical Programming Series B*, vol. 107, pp. 231–273, 2006.

[92] J. M. Danskin, *The Theory of Max-Min.* Springer-Verlag, 1967.

[93] M. K. Brady, *Techniques for Interference Analysis and Spectrum Management of Digital Subscriber Lines.* Ph.D. Dissertation, University of California, Berkeley, USA, 2006.

[94] K. M. Teo, *Nonconvex Robust Optimization.* Ph.D. Dissertation, Massachusets Institute of Technology, USA, 2007.

[95] A. Nagurney and D. Zhang, *Projected Dynamical Systems and Variational Inequalities with Applications.* Springer, 1996.

[96] F. Facchinei and J. S. Pang, *Finite-Dimensional Variational Inequalities and Complementarity Problems.* Springer, 2003.

[97] F. Facchinei and C. Kanzow, "Generalized Nash equilibrium problems," *4OR: A Quarterly Journal of Operations Research*, vol. 5, no. 3, pp. 173–210, 2007.

[98] S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004.

[99] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms.* Wiley-Interscience, 3rd ed., 2006.

[100] D. P. Bertsekas, A. Nedic, and A. Ozdaglar, *Convex Analysis and Optimization.* Athena Scientific, 2003.

[101] R. W. Cottle, J. S. Pang, and R. E. Stone, *The Linear Complementarity Problem.* Academic Press, 1992.

[102] M. L. Aghassi, *Robust Optimization, game theory, and variational inequalities.* Ph.D. Dissertation, Massachusetts Institute of Technology (MIT), 2005.

[103] H. K. Khalil, *Nonlinear Systems.* New York: Prentice-Hall Inc., 3rd ed., 2001.

[104] R. W. Thomas, D. H. Friend, L. A. DaSilva, and A. B. MacKenzie, "Cognitive networks: Adaptation and learning to achieve end-to-end performance objectives," *IEEE Communications Magazine*, pp. 51–57, 2006.

[105] P. Demestichas, G. Dimitrakopoulos, J. Strassner, and D. Bourse, "Introducing reconfigurability and cognitive networks concepts in the wireless world," *IEEE Vehicular Technology Magazine*, pp. 32–39, 2006.

[106] P. Mahonen, "Cognitive wireless networks: Your network just became a teenager," in *Proc. IEEE International Conference on Computer Communications (INFOCOM)*, 2006.

[107] O. K. Tonguz and G. Ferrari, *Ad Hoc Wireless Networks: A Communication-Theoretic Perspective.* John Wiley and Sons, 2006.

[108] K. B. Lataief and W. Zhang, "Cooperative communications for cognitive radio networks," *Proceedings of IEEE*, vol. 97, no. 5, pp. 878–893, 2009.

[109] Q. Zhang, J. Jia, and J. Zhang, "Cooperative relay to improve diversity in cognitive radio networks," *IEEE Communications Magazine*, vol. 47, no. 2, pp. 111–117, 2009.

[110] K. C. Chen, B. K. Cetin, Y. C. Peng, N. Prasad, J. Wang, and S. Lee, "Routing for cognitive radio networks consisting of opportunistic links," *Wireless Communications and Mobile Computing*, vol. 10, no. 4, pp. 451–466, 2010.

[111] D. P. Bertsekas, "Projected equations, variational inequalities, and temporal difference methods," *Technical Report LIDS-P-2808, MIT*, pp. 1–26, 2009.

[112] G. L. Stuber, S. M. Almalfouh, and D. Sale, "Interference analysis of TV-band whitespace," *Proceedings of IEEE*, vol. 97, no. 4, pp. 741–754, 2009.

[113] C. R. Stevenson, G. Chouinard, Z. Lei, W. Hu, S. J. Shellhammer, and W. Caldwell, "IEEE 802.22: The first cognitive radio wireless regional area network standard," *IEEE Communications Magazine*, vol. 47, no. 1, pp. 130–138, 2009.

[114] R. H. Gohary, Y. Huang, Z. Q. Luo, and J. S. Pang, "A generalized iterative water-filling algorithm for distributed power control in the presence of a jammer," *IEEE Transactions on Signal Processing*, vol. 57, no. 7, pp. 2660–2674, 2009.

[115] M. Fazel, D. M. Gayme, and M. Chiang, "Transient Analysis for Wireless Power Control," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, pp. 1–6, 2006.

[116] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design.* John Wiley and Sons, 2nd ed., 2006.

[117] F. J. Christophersen, *Optimal Control of Constrained Piecewise Affine Systems.* Springer, 2007.

[118] J. Roll, *Local and Piecewise Affine Approaches to System Identification.* Ph.D. Dissertation, Linköping University, Linköping, Sweden, 2003.

[119] V. B. Kolmanovskii and V. R. Nosov, *Stability of Functional Differential Equations.* Academic Press, 1986.

[120] V. B. Kolmanovskii and L. E. Shauikhet, *Control of Systems with Aftereffect.* Oxford University Press, 1997.

[121] K. Gu and S. Niculescu, "Survey on recent results in the stability and control of time-delay systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 125, pp. 158–165, 2003.

[122] J. P. Richard, "Time-delay systems: An overview of some recent advances and open problems," *Automatica*, vol. 39, pp. 1667–1694, 2003.

[123] L. Engelson, "On dynamics of traffic queues in a road network with route choice based on real time traffic information," *Transportation Research Part C*, vol. 11, pp. 161–183, 2003.

[124] J. K. Hale and S. M. V. Lunel, *Introduction to Functional Differential Equations.* Springer-Verlag, 1993.

[125] V. B. Kolmanovskii and A. D. Myshkis, *Introduction to the Theory and Applications of Functional Differential Equations.* Kluwer Academic Publishers, 1999.

[126] L. E. El'sgol'ts and S. B. Norkin, *Introduction to the Theory and Application of Differential Equations with Deviating Arguments.* Academic Press, 1973.

[127] S. I. Niculescu, C. E. de Souza, L. Dugard, and J. M. Dion, "Robust exponential stability of uncertain systems with time-varying delays," *IEEE Transactions on Automatic Control*, vol. 43, no. 5, pp. 743–748, 1998.

[128] M. G. Cojocaru, P. Daniele, and A. Nagurnay, "Double-layered dynamics: A unified theory of projected dynamical systems and evolutionary variational inequalities," *European Journal of Operational Research*, vol. 175, pp. 494–507, 2006.

[129] M. D. Mesarovic and Y. Takahara, *Abstract Systems Theory.* Springer-Verlag, 1989.

[130] A. Nagurney, *Supply Chain Network Economics: Dynamics of Prices, Flows, and Profits.* Edward Elgar Publishing, 2006.

[131] D. C. Tarraf and H. H. Asada, "Decentralized hierarchical control of multiple time scale systems," in *Proc. American Control Conference*, pp. 1121–1122, 2002.

[132] D. Mignone, *Control and Estimation of Hybrid Systems with Mathematical Optimization.* Ph.D. Dissertation, Swiss Federal Institute of Technology (ETH), Switzerland, 2002.

[133] T. Steffen, *Control Reconfiguration of Dynamical Systems.* Springer, 2005.

[134] A. N. Kolmogorov and S. V. Fomin, *Introductory Real Analysis.* Dover, 1970.

[135] P. Daniele, *Dynamic Networks and Evolutionary Variational Inequalities.* Edward Elgar Publishing, 2006.

[136] M. Cojocaru, S. Hawkins, H. Thille, and E. Thommes, "Double-layer and hybrid dynamics of equilibrium problems: Applications to markets of environmental products," *in Nonlinear Analysis and Variational Problems, P. M. Pardalos, T. M. Rassias, and A. A. Khan, Eds., Springer*, pp. 213–234, 2010.

[137] P. Daniele, S. Giuffre, A. Maugeri, and S. Pia, "A panoramic view on projected dynamical systems," *in Nonlinear Analysis and Variational Problems, P. M. Pardalos, T. M. Rassias, and A. A. Khan, Eds., Springer*, pp. 235–258, 2010.

[138] N. Hadjisavvas and S. Schaible, "Generalized Monotone Maps," *in Handbook of Generalized Convexity and Generalized Monotonicity, N. Hadjisavvas, S. Kmlosi, and S. Schaible, Eds., Springer*, pp. 389–420, 2005.