SHEAR STRENGTHENING OF RC BEAMS
SHEAR STRENGTHENING OF RC BEAMS USING EXTERNALLY BONDED AND ANCHORED FRP U-WRAPSS

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TITLE: Shear Strengthening of RC Beams Using Externally Bonded and Anchored FRP U-wraps

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Lay Abstract

Damaged or older reinforced concrete structures can be rehabilitated by using externally bonded fibre-reinforced polymer (FRP) sheets, which are bonded to the concrete surface using an epoxy adhesive. For the case of shear strengthening of beams, it is common for FRP sheets to be wrapped around the sides and bottom of the beam, resembling a U-shape. The problem with this configuration is that under high levels of load the FRP sheets tend to peel off the concrete surface (debonding). This limits the effectiveness of the rehabilitation and results in the inefficient use of the FRP. A new method for anchoring the FRP sheets to the concrete surface is investigated in this research study. The use of a new in-situ π-shape anchor shows promising results, as it delays debonding and provides a large increase in strength with less FRP needed.
Abstract

Externally bonded FRP U-wraps are a common shear strengthening configuration for RC beams, however premature debonding of the wraps is a major problem, which limits the effectiveness and efficiency of the FRP strengthening. In this investigation a new π-shape carbon anchor was used to fasten the FRP U-wraps to the concrete in an attempt to prevent/delay debonding of the wraps and increase their effectiveness. Fourteen large scale rectangular beams with a 1900 mm span, 400 mm height, and 170 mm width were tested in three-point bending with various configurations of FRP shear strengthening. Shear pre-cracks were introduced in the beams at angles of 30 and 45 degrees in an attempt to control the inclination angle of the shear crack and determine its effect on the FRP shear resistance. The FRP shear strengthening configurations included un-anchored U-wraps, U-wraps with anchors, U-wraps with horizontal strips, and full wraps. The results showed that the use of a variable shear crack inclination angle in the CSA S806-12 (2012) standard led to overestimated shear resistance predictions for beams with a single shear crack, therefore a conservative 45 degree shear crack inclination is recommended for design. The use of the proposed carbon anchors resulted in a 74% increase in shear strength over the un-anchored U-wrapped beams, while only using half the amount of FRP. The use of the anchors also resulted in a 286% increase in the ultimate FRP strain over the un-anchored U-wraps, and allowed the FRP wraps to achieve 58% of their rupture strain. The use of horizontal strips provided similar results to the anchors and may be used as a less labour intensive alternative, but this issue needs further investigation.
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List of Abbreviations and Symbols

In this thesis all lengths are in $mm$, areas in $mm^2$, forces in $N$, and stresses in $MPa$ unless stated otherwise.

$A_f$ = Cross-sectional area of FRP wrap

$A_s$ = Cross-sectional area of longitudinal tensile steel bars

$a_g$ = Maximum aggregate size

$b_w$ = Width of beam web

$b_f = w_f = w_{FRP}$ = Width of FRP wrap

$c$ = Neutral axis depth from top of beam cross-section

$d$ = Distance from extreme compression fibre to the centroid of the longitudinal steel reinforcement

$d_v$ = Effective shear depth of the beam

$d_{fv} = d_f$ = Effective FRP depth

$D_{FRP}$ = Stress distribution factor

$E_{FRP} = E_f$ = Elastic Modulus of FRP

$E_s$ = Elastic Modulus of Steel

$f'_{c}$ = Cylindrical compressive strength of concrete

$f_{cu}$ = Cube compressive strength of concrete

$f_{ctm}$ = Average tensile strength of concrete

$f_{ck}$ = Characteristic strength of concrete

$f_t = f_{ct}$ = Tensile strength of concrete

$f_{FRP}$ = Tensile strength of FRP

$f_{FRP,e} = f_{fed}$ = Effective stress in FRP
\( f_{fd} \) = Design debonding strength of the FRP
\( f_{du} \) = Ultimate design strength of the FRP
\( f_1 \) = Principal tensile stress
\( f_2 \) = Principal compressive stress
\( G_f \) = Interfacial fracture energy
\( G_{fk} \) = Specific fracture energy of the FRP-concrete interface
\( h = h_w \) = Height of beam
\( h_{FRP,e} = h_{f, e} \) = Effective height of FRP
\( h_{df} \) = Vertical distance from the crack tip to the point of the intersection between the debonding front and the critical shear crack
\( h_t \) = Vertical distance from the top of the FRP wrap to the crack tip
\( h_b \) = Thickness of the concrete cover
\( k_1 \) = Modification factor which accounts for concrete strength
\( k_2 \) = Modification factor which accounts for the wrapping configuration of the FRP
\( k_v \) = Strain reduction factor
\( k_b \) = Geometric coefficient
\( L_{max} \) = The maximum value among the bond lengths of all the FRP strips intersected by the critical shear crack
\( L_e \) = The effective bond length, defined as the length of bond that gives the maximum bond strength
\( M_f \) = Factored moment
\( n_f \) = The number of layers of FRP used
\( N_f \) = Factored axial load at the section
\( r_c \) = Corner radius for the beam
\[ s_f = S_{FRP} = S_F = \text{Center to center spacing of the FRP wraps} \]

\[ s_{ze} = \text{Effective crack spacing factor} \]

\[ S_z = \text{Crack spacing factor} \]

\[ s_{m\theta} = \text{Spacing of inclined cracks} \]

\[ s_{mx} = \text{Crack spacing factor in the longitudinal direction} \]

\[ s_{mv} = \text{Crack spacing factor in the transverse direction} \]

\[ t_f = \text{Thickness of the FRP wrap} \]

\[ \nu = \text{Shear stress on beam} \]

\[ V_r = \text{Shear resistance of beam} \]

\[ V_c = \text{Concrete shear resistance} \]

\[ V_{cl} = \text{Concrete shear resistance due to aggregate interlock} \]

\[ V_{FRP} = \text{FRP shear resistance} \]

\[ V_f = \text{Factored shear force at section} \]

\[ w = \text{Shear crack width} \]

\[ w_{e,p} = \text{Shear crack end width when the FRP shear contribution reaches its peak value} \]

\[ \rho_f = \text{FRP reinforcement ratio} \]

\[ \rho_s = \text{Traverse-steel reinforcement ratio} \]

\[ \alpha = \alpha_F = \text{Angle that the FRP fibres make with the longitudinal axis of the beam} \]

\[ \alpha_1 = \text{Concrete stress block parameter} \]

\[ \beta = \text{A factor accounting for the extent of concrete contribution to shear resistance} \]

\[ \beta_1 = \text{Concrete stress block parameter} \]

\[ \beta_c = \text{Concrete-cracking coefficient} \]

\[ \beta_L = \text{Coefficient to compensate for insufficient anchorage length} \]
\[ \beta_w = \text{FRP width-to-spacing coefficient} \]
\[ \theta = \text{Shear crack inclination angle} \]
\[ \tau_f = \text{Maximum interfacial shear stress} \]
\[ \delta_f = \text{Interfacial slip at the shear crack} \]
\[ \varepsilon_{fu} = \varepsilon_{max} = \text{The ultimate FRP strain} \]
\[ \varepsilon_{fe} = \text{Effective FRP strain} \]
\[ \varepsilon_x = \text{Longitudinal strain at the mid-depth of the cross-section} \]
\[ \varepsilon_t = \text{Transverse strain at the mid-depth of the cross-section} \]
\[ \varepsilon_1 = \text{Principal tensile strain} \]
\[ \varepsilon_2 = \text{Principal compressive strain} \]
\[ \varepsilon'_c = \text{Concrete strain corresponding to the maximum concrete stress} \]
\[ \varepsilon_c = \text{Concrete strain at extreme compression fibre} \]
\[ \sigma_{FRP,max} = \text{The maximum FRP stress at debonding or rupture} \]
\[ \varphi_R = \text{Factor regarding the curvature of the corners of the beam} \]
\[ \gamma_{fd} = \text{Partial resistance factor for FRP} \]
\[ \lambda = \text{The normalised maximum bond length} \]
Declaration of Academic Achievement

I have designed, built, and tested all of the specimens used for this research study and have analyzed all of the test results. I have received assistance from the technicians in the Applied Dynamics Laboratory at McMaster University for the building and testing phases of my research. I have also received assistance and guidance from my thesis supervisor in all aspects and stages of my research.
Chapter 1: Introduction

1.1 General

In ordinary reinforced concrete beams, shear loading is resisted by a combination of the inherent concrete shear resistance and the shear resistance provided by internal steel stirrups. This resistance mechanism is ideal for normal reinforced concrete structures, however, when structures are damaged or found to be deficient they must be repaired and restored. This is evident around the world with a growing number of reinforced concrete bridges and structures nearing the end of their service life that must be rehabilitated due to damage from fatigue or corrosion of the internal steel reinforcement.

Fibre-reinforced polymer (FRP) composites have been implemented into construction as an external strengthening solution to reinforced concrete structures in need of repair. Their high resistance to corrosion, high strength to weight ratio, and minimal labour requirements make them an attractive material to use for strengthening and a cost effective alternative to replacing damaged structural members. For external strengthening of reinforced concrete beams, FRP composites in the form of sheets are bonded to the concrete surface using an adhesive. For shear strengthening, due to the top surface of beams usually being inaccessible, FRP sheets are often bonded to the sides of the beam (side bonded) or bonded to the sides and bottom of the beam (U-wrapped).

Despite the advantageous material properties of FRP, a major problem with its use in external strengthening is that the FRP wraps can fail prematurely due to debonding i.e., prior to reaching their ultimate tensile strength. Therefore, the full utilization and efficient use of the FRP along with its accompanying strength benefits are not achieved.

Researchers have developed several anchorage systems in order to prevent/delay the debonding of FRP sheets and promote their efficient use (Koutas and Triantafillou, 2013; Bae and Belarbi, 2013; Mofidi et al., 2012; Ortega et al., 2009; Grelle and Sneed, 2013; Jinno et al., 2001; Al-Mahaidi and Kalfat, 2011; Khalifa and Nanni, 2000; Deifalla and Ghobarah, 2010). Some anchorage systems applicable to shear strengthening include horizontal FRP strips, FRP spike anchors, and mechanical anchorage devices. There are some disadvantages to using current anchorage systems, which include disagreement in the literature about the effectiveness of horizontal FRP strips, FRP spike anchors and mechanical anchorage systems having to penetrate through the FRP wrap, which can cause damage to the wrap, and mechanical anchorage devices fabricated with steel, which is corrosive. Although the use of
these anchorage devices for shear strengthened beams has demonstrated improvement in delaying the debonding process of FRP wraps, the problem has not been adequately resolved yet.

1.2 Problem and Study Motivation

Premature debonding of external FRP strengthening is a major problem for retrofitted RC beams, limiting the effectiveness of the FRP and leading to its inefficient use. A new anchorage system is proposed in this research study to delay/eliminate the onset of debonding and increase the efficiency of the FRP strengthening. The anchor is fabricated from carbon-fibre, which makes it non-corrosive, it does not involve any penetration through the FRP wrap, preventing high stress concentrations and damage to the wrap, and it is designed to be fabricated in-situ. Although designed to be effective at delaying debonding, the effects of these anchors on shear strengthening are unknown and will be investigated in this research study.

Theoretically, among the several factors which influence the contribution of FRP wraps to the shear resistance of a retrofitted beam is the angle of inclination of the diagonal shear cracks. For this reason, several existing models used to predict the resistance provided by FRP wraps are based on a variable truss angle analogy (CSA S806-12, 2012; CNR-DT 200/2004, 2004; Chen et al., 2013; Chen and Teng, 2003a,b; Mofidi and Chaallal, 2011). This assumes that the shear resistance provided by the FRP strengthening is dependent on the shear crack inclination angle and that all FRP wraps crossing the shear crack are equally active at resisting the load. This might not be the case for intermittent FRP wraps, due to their linear elastic behaviour and inability to redistribute stresses. The amount of load an FRP wrap will resist depends on the location of the wrap along the beam span, the height at which the shear crack crosses the wrap, and the shape of the shear crack. Therefore, not all wraps may be equally active at resisting the load and the shear crack angle might have little effect on the resistance contributed by the FRP strengthening. Little research has been done to investigate the influence of the shear crack inclination on the effectiveness of the FRP strengthening. This is due to the difficulty of being able to control the inclination angle of the shear crack. To address this issue, in this research study shear pre-cracks will be introduced at inclination angles of 30 and 45 degrees to the longitudinal axis of the beams in an attempt to control the inclination of the shear crack and investigate its influence on the effectiveness of the FRP shear strengthening.
1.3 Objectives and Scope of Research

The purpose of this study is to investigate the effectiveness of a new anchorage system developed to delay/prevent the premature debonding of externally bonded FRP U-wraps and to investigate the influence of the shear crack inclination angle on the effectiveness of FRP shear strengthening. The specific objectives are:

- To compare the FRP shear resistance predictions provided by available models to the experimental FRP shear resistance in order to determine the veracity of the models.
- To determine the behaviour of shear-deficient reinforced concrete beams strengthened with various FRP shear strengthening configurations.
- To compare the behaviour of anchored and un-anchored U-wrapped beams, in order to determine the effectiveness of the proposed anchorage device.
- To compare the behaviour of FRP shear strengthened beams with different shear crack inclination angles, in order to determine the influence of the shear crack inclination angle on the FRP shear strengthening.
- To compare different modifications and configurations of the proposed anchorage device, in order to gain better understanding of the anchoring mechanisms and the effectiveness of the individual components of the anchorage device.
- To develop a step-by-step procedure for the installation of the anchors in the field.
- To quantify the strain distribution in the FRP wraps for various shear strengthening configurations.

This research program consisted of an experimental phase. Fourteen tests were performed on large-scale reinforced concrete beams with various shear strengthening configurations and shear pre-crack inclination angles of 30 and 45 degrees. The test variables included the shear crack inclination angle, the shear strengthening configuration, the number of layers of the FRP laminate used for each wrap, and the presence of anchors.
1.4 Thesis Layout

This thesis is organized into six chapters. The chapters are as follows:

Chapter 1: Introduction – This chapter discusses the purpose and motivation behind this research study, the objectives and goals of this research study, the scope of this research program, and the organization of this thesis.

Chapter 2: Background and Literature Review – This chapter provides an extensive literature review and background on FRP shear strengthening configurations, their common failure modes, factors influencing the FRP-concrete bond strength, models and guidelines to predict the shear resistance contributed by the FRP strengthening, and the available anchorage devices used for shear strengthening.

Chapter 3: Experimental Program – This chapter discusses the testing program and set-up, the test specimens, their fabrication, shear strengthening, material properties, and the instrumentation used for their testing.

Chapter 4: Experimental Results – This chapter presents the observed behaviour of the beams, their failure mechanisms, load-deflection responses, strain in their internal tensile reinforcement, and strain in their FRP wraps.

Chapter 5: Analysis of Experimental Results – This chapter discusses the failure modes of the beams, their load-deflection responses and failure loads, the ultimate strains and strain distribution in their FRP wraps, and the influence of the shear crack inclination angle, the use of anchors and horizontal strips, and the influence of the concrete surface preparation on the effectiveness of the FRP strengthening.

Chapter 6: Summary, Conclusions, and Recommendations for Future Work – This chapter provides a summary of this research study, the main conclusions that can be drawn from this study, and recommendations for future research in this area.
Chapter 2: Background and Literature Review

2.1 General

In modern construction, reinforced concrete members are designed to fail in flexure rather than shear. This is due to the fact that flexural failure is ductile and predictable whereas shear failure is brittle and sudden (Wight and MacGregor, 2012). A concrete member could become shear critical due to several factors. A prime factor is due to the damage of internal shear reinforcement which could be caused by corrosion, fatigue, or chemical attack. Another factor is the design of reinforced concrete structures using out-dated design guidelines and standards, which could render the structure shear deficient based on updates in current codes and standards. Human errors involved with the design and construction of reinforced concrete members, and a change of design loads over time could also result in a shear critical member.

It is evident that shear deficiency can be a problem for reinforced concrete structures. Therefore, a shear strengthening technique is an economical solution to increase shear capacity and avoid the possibility of brittle shear failure.

2.2 Shear Failure Modes of Non-strengthened Reinforced Concrete Beams

When reinforced concrete beams are subject to shear, it results in the formation of diagonal cracks called shear cracks. Shear failure results along these cracks in a brittle manner. There are five modes of shear failure which can take place, depending on various elements of the beam, such as its amount of steel reinforcement, geometry, properties of concrete, and the load applied on the beam (Raju, 2014).

The first mode of shear failure is called a Diagonal Tension Failure, and is caused by a diagonal crack propagating through the depth of the beam, and ultimately causing collapse. For beams having sufficient longitudinal reinforcement to form a compression zone, diagonal cracks propagate through the web but do not spread to the compression zone. This results in the concrete in compression zone above the tip of the shear crack being crushed, which is the
second mode of shear failure, called *Shear Compression Failure*. The third mode is called *Shear Tension Failure* and is caused by horizontal cracks forming along the tension reinforcement in a reinforced concrete beam. For concrete beams with thin web sections, the concrete in the web section between diagonal cracks can become crushed, which is another potential failure mode, called *Web Crushing Failure* or *Diagonal Compression Failure*. Finally, the fifth failure mode due to shear is called *Arch-Rib Failure*, which is caused by a diagonal crack from the loading area to the support and is only relevant for deep or short span beams (Pillai and Menon, 2003).

FRP shear strengthening can prevent diagonal tension failure, which is the prevailing shear failure mode in the majority of slender shear deficient beams. Some of the other failure modes can also be mitigated by the appropriate retrofit method, but such methods are outside the scope of the current study.

### 2.3 Strengthening of Reinforced Concrete Beams

There are several methods used to strengthen reinforced concrete beams, consisting of external post-tensioning, externally bonded steel elements, the use of fibre-reinforced polymer (FRP) composites, textile reinforced concrete (TRC), and near-surface mounted (NSM) reinforcement. In addition, a combination of these strengthening techniques can also be used as an effective type of strengthening. These strengthening techniques are called composite strengthening systems because they modify and reinforce the original concrete member (Heiza, 2014).

One must analyze several factors to determine which method of strengthening is appropriate for a specific strengthening project. Some of the factors include the architectural requirements for the structure, its environmental conditions, the budget for the strengthening project, the required useful life of the structure, and access to the structure (Paul, 2002).

This literature review will be focused on the use of externally bonded FRP as a strengthening technique for shear deficient reinforced concrete beams, with particular focus on preventing diagonal tension failure.
2.4 Fibre-reinforced Polymers (FRP)

The use of fibre-reinforced polymers (FRP) for rehabilitation and strengthening of structures has grown significantly over the past two decades. This can be attributed to its advantageous physical, mechanical, and chemical properties and long term behaviour over various strengthening construction materials, such as steel. FRP materials are non-corrosive and demonstrate good durability in harsh environments. They also exhibit very high strength to weight ratios due to their low density and high tensile strength. Another advantage of using FRP materials is its quick installation and minimal labor requirements. FRP materials are also electromagnetically inert, allowing their use for structures that house sensitive equipment. With regards to the stress-strain relationship of FRP materials, they do not yield before rupture and exhibit a linear elastic stress-strain relationship until failure (ISIS-EC-M4, 2004).

FRP materials are composites, meaning they are made up of two or more materials of different chemical, mechanical, and physical properties. FRP materials are made from the embedment of high strength fibres in a polymeric matrix. The fibres provide the FRP strength and stiffness, whereas the matrix separates the fibres and allows load transfer among them. There are three main types of fibres commonly used; namely, glass (GFRP), carbon (CFRP), and aramid (AFRP), with carbon fibres having the highest tensile strength and elastic modulus. The most commonly used types of polymers for the matrix are epoxy, vinyl esters, and polyesters.

The three defining characteristics of all FRP materials are its geometry, fibre orientation, and volumetric ratio (concentration of fibres). FRP is an anisotropic material, principally strong along the direction of the fibres. Therefore, the FRP materials should not be heavily loaded transverse to their fibre orientation. If the strengthening case involves loading in both directions, bi-directional FRP materials can be used. For reinforced concrete shear strengthening applications, FRP sheets are used with an adhesive in order to bond the FRP to the surface of the concrete (ACI 440.2R-08, 2008; CNR-DT 200/2004, 2004). This will be discussed in more detail in the subsequent sections.

2.4.1 FRP Sheets

The use of FRP sheets or laminates is common for strengthening of reinforced concrete beams, where the strength of the sheets is dependent on their fibre orientation and concentration. FRP sheets can be uni-directional, where fibres are orientated in the direction
of the length of the sheets, *bi-directional*, where the same ratio of fibres are oriented in two directions, and *multi-axial* or *multi-directional*, where fibres are orientated in more than two directions. In regards to geometry, FRP sheets are very thin (<0.2 mm), allowing them to be bent and rolled up, or relatively thick and stiff (>1.6 mm) (CNR-DT 200/2004, 2004).

### 2.4.2 Adhesives

The bonding of the FRP sheets to the concrete surface for strengthening of structures and the impregnation of dry FRP sheets requires the use of adhesives. It is common for FRP sheets to be distributed as a dry product and to be impregnated with resin adhesive on site for installation on the concrete surface. In regards to installation, the adhesive can either be applied directly to the dry FRP sheets to impregnate them first and then fix them on the concrete surface, or directly on the concrete surface to which the dry FRP sheets are applied and the adhesive permeates through them by applying pressure to the bonded FRP surface.

The effectiveness of adhesion between the FRP sheets and the concrete surface depends on the application technique and surface treatment of the concrete before application. The concrete surface must be treated prior to application of the adhesive by increasing the roughness of the surface, which improves the bond between the adhesive and concrete. The most practical adhesive for retrofit with FRP materials is epoxy resin, because it exhibits good resistance to moisture and chemical agents and has excellent adhesive properties (CNR-DT 200/2004, 2004; ACI 440.2R-08, 2008).

### 2.5 FRP Shear Strengthening of Reinforced Concrete Beams

Extensive research has been undertaken in the shear strengthening of reinforced concrete beams with fiber reinforced polymers (Chen and Teng 2003a,b; Bousselham and Chaallal 2004, 2006a, 2008; Triantafillou, 1998; Carolin and Taijsten 2005; Khalifa and Nanni 2000, 2002; Bukhari et al., 2010; Zhang and Hsu, 2005; Jayaprakash et al., 2007; Grande et al. 2007). Unlike flexural strengthening, which involves the application of a longitudinal FRP sheet along the tension face of a reinforced concrete beam, shear strengthening uses FRP sheets along the sides of the beam. The sheets can be applied in three different configurations, including full wrapping, U-wrapping, and side bonding, as illustrated in Figure 2-1. The FRP sheets can be applied as discrete strips with some spacing between them, or as a continuous sheet along the length of the member. It is common for the fiber
orientation in the sheets to be at 90 degrees to the longitudinal axis of the beam; however, fibers can also be orientated at 45 or 135 degrees to the longitudinal axis of the beam. Therefore, the combination of different bonding configurations, fiber distributions, and fiber orientations can result in several different shear strengthening schemes.

![Diagram of FRP Wraps](image)

(a)

![Diagram of FRP shear strengthening configurations](image)

(b) (c) (d)

Figure 2-1: FRP shear strengthening configurations (a) side view of FRP shear strengthened beam with discrete strips (b) cross-sectional view of FRP full wrapped beam (c) cross-sectional view of FRP U-wrapped beam (d) cross-sectional view of FRP side-bonded beam

2.5.1 Full Wrap Configuration

For a full wrap configuration, the FRP sheets are fully bonded around the reinforced concrete beam, as illustrated in Figure 2-1b. The predominant failure mode for the full wrapping
configuration is FRP rupture (Chen and Teng, 2003b; Teng et al., 2009; Cao et al., 2005). Debonding of the FRP along the sides of the beam has been observed prior to rupture of the full wrap (Cao et al, 2005; Teng et al., 2009). Since strengthening projects are done on existing structures, the full wrapping configuration is not always an available option because a slab usually rests on the top surface of most reinforced concrete beams, restricting access to the top surface of the beam.

Cao et al. (2005) conducted an experimental study on the failure process of reinforced concrete beams strengthened with FRP full wraps. The study consisted of 18 rectangular, shear critical reinforced concrete beams which were strengthened with unidirectional carbon and glass FRP full wraps spaced at 40 to 150 mm intervals. The strengthened beams were loaded to failure and the failure modes were examined. For most specimens, complete debonding of the FRP sheets on the sides of the beam which intersected the critical shear crack occurred before failure. For all specimens, the ultimate failure was caused by the rupture of the FRP sheets along the shear crack.

Teng et al. (2009) also conducted an experimental study on the strengthening of reinforced concrete beams with FRP full wraps. The study was aimed at comparing the effectiveness of strengthening between full wrapped CFRP sheets with either the sides bonded or unbonded to the beam. Six rectangular, shear strengthened reinforced concrete beams were tested in order to compare the side bonded full wrap configuration to the full wrap configuration with the sides unbonded. The results showed that for the full wrap configuration with the sides unbonded, the average maximum strain in the FRP was 11% less than the ultimate strain found from tensile tests, whereas for bonded sides, the average maximum strain in the FRP was 29% less. Therefore, the debonding process of a full wrap with bonded sides can significantly affect the rupture strain of the FRP.

2.5.2 U-wrapped and Side Bonded Configurations

For U-wrapped configurations, the FRP sheets are attached to the two sides and bottom of a reinforced concrete beam (Figure 2-1c), whereas for side bonded configurations, the FRP sheets are bonded only to the two sides of the beam (Figure 2-1d). These configurations are both an option when the top surface of the beam is not accessible for a full wrapping configuration. The predominant failure mode for these configurations is FRP debonding because the U-wraps or side bonded FRP are not as securely bonded to the beam as the full wrap configuration (Belarbi and Acun, 2013). In addition, the U-wrapped configuration is more effective than the side bonded configuration and therefore, can withstand a higher strain before debonding (Grande et al., 2007; Belarbi and Acun, 2013).
Grande et al (2007) conducted an experimental study which compared the shear contribution of fully wrapped, U-wrapped, and side bonded strengthening configurations. Rectangular reinforced concrete beams were strengthened using different configurations of evenly spaced CFRP sheets with an internal shear reinforcement spacing of 300 mm. From the results, it was found that the shear contribution of the FRP strengthening for the fully wrapped beam was 70% greater than the FRP shear contribution of the U-wrapped beam, and 325% greater than the FRP shear contribution for the side bonded beam. This shows that the full wrap configuration is the most effective for strengthening, followed by the U-wrapped configuration, and least effective, the side bonded configuration.

Belarbi and Acun (2013) present an extensive review of the failure modes for a diverse set of experimental data provided by NCHRP Report 678 (2011) on shear strengthening. The results showed that no debonding failures were reported for fully wrapped beams, 83% of failures were due to debonding for U-wrapped beams, and 92% of failures were due to debonding for side bonded beams.

2.5.3 Parameters Influencing the Shear Capacity of Strengthened Beams

There are several parameters that have been investigated which can influence the effectiveness of a shear strengthening configuration. Some parameters are the use of bi-directional sheets, the orientation of the fibres, the spacing of the FRP sheets, the thickness or amount of layers of the FRP sheet, the size of the strengthened beam, and the spacing of the internal shear reinforcement (Bukhari et al., 2010; Zhang and Hsu, 2005; Jayaprakash et al., 2007; Bousselham and Chaallel, 2008; Mofidi and Chaallal, 2011).

Bukhari et al. (2010) conducted an experimental study analyzing the effectiveness of shear strengthening for various configurations. Seven two-span rectangular reinforced concrete beams were tested, with one being the control beam and having no external strengthening. No internal shear reinforcement was provided within the interior shear spans of all beams in order to ensure shear failure in this area and because the goal of the research was simply to compare shear strengthening configurations. Based on the results, a beam strengthened by a 304.8 mm wide continuous CFRP side strip with its fibres orientated at 90 degrees to the longitudinal axis of the beam, increased the shear strength of the beam by 54% compared to the control beam. However, a beam strengthened with the same CFRP sheet but with fibres orientated at 45 degrees to the longitudinal axis of the beam increased the shear strength of the beam by 92% compared to the control beam. Therefore, orientating the fibres of the FRP at 45 degrees to the longitudinal axis of the beam provides superior strengthening over
conventional vertical FRP sheets. The only disadvantage of inclined FRP wraps is that they cannot accommodate substantial load reversal.

Zhang and Hsu (2005) also conducted an experimental study on the effect of fibre orientation for FRP external shear strengthening. Four foot long rectangular reinforced concrete beams were tested with several different fiber orientations for external strengthening with evenly spaced CFRP strips, including one control beam which was not strengthened. The reinforced concrete beams had sufficient internal flexural strengthening but did not contain any internal shear reinforcement, as the objective was only to compare fibre orientations. The results showed that with fibres orientated at 90 degrees to the longitudinal axis of the beam there was a 60% increase in shear capacity over the control beam, however with fibres orientated at 45 degrees to the longitudinal axis of the beam there was an 80% increase in shear capacity over the control beam. Therefore, this shows that orienting the fibers at 45 degrees to the longitudinal axis of the beam is beneficial to the effectiveness of the strengthening.

Jayaprakash et al. (2007) have conducted an experimental study on the use of bi-directional FRP sheets, and the effects of internal shear reinforcement spacing and external FRP spacing on the strengthening of reinforced concrete beams. A series of reinforced concrete T-beams externally strengthened with bi-directional CFRP sheets, were tested with an internal shear reinforcement spacing of 120 mm or 210 mm and an external strengthening spacing of 150 mm or 200 mm, for each of the internal shear reinforcement spacings. The results firstly showed that all strengthened beams failed in flexure with a ductile failure, therefore bi-directional sheets helps to control debonding of the CFRP from the concrete surface. For the case of the 210 mm internal stirrup spacing, the external CFRP spacing of 150 mm increased the shear capacity of the beam by 17% compared to the external CFRP spacing of 200 mm. However, for the case of the 120 mm internal stirrup spacing, the shear capacity for the external CFRP spacing of 150 mm was 11% less than the shear capacity with the external CFRP spacing of 200 mm. Therefore, this shows that an increase of both the external and internal shear reinforcement does not increase the shear capacity of the beam.

Bousselham and Chaallal (2008) reviewed an experimental study on the effects of the FRP thickness, the use of bi-directional FRP sheets, the size of the reinforced concrete beams being strengthened, and the internal shear spacing, on the shear capacity of strengthened beams. The study consisted of 17 full size reinforced concrete T-beams of depth 350 mm and 175 mm. For each depth, beams were reinforced with no internal stirrups, internal stirrups spaced at half the depth of the beam, and internal stirrups spaced at a quarter of the depth of the beam. Also, for each amount of internal reinforcement, beams were either not strengthened, strengthened with 1 layer of external CFRP sheets, or strengthened with 2 layers of external CFRP sheets. The CFRP sheets used for strengthening were bi-directional.
CFRP sheets applied continuously in a U-wrapped configuration around the web. The results firstly showed that none of the specimens failed due to debonding of the FRP sheets, which could be attributed to the use of bi-directional CFRP sheets. Secondly, it was found that doubling the thickness of the CFRP sheet did not lead to a proportional increase in shear resistance. This is seen in the case of no internal shear reinforcement for the 350 mm depth beams. With a single layer of CFRP the increase in shear capacity over the un-strengthened control beam was 48%, however for a double layer of CFRP the increase was only 50%. Thirdly, it was found that increasing the amount of internal shear reinforcement by decreasing the spacing of the stirrups leads to a lower contribution of the CFRP to the shear resistance of the beam. This can be seen from the 350 mm depth beam with no internal shear reinforcement and strengthened with 2 layers of external CFRP having an increase in shear capacity due to the CFRP of 50% over the un-strengthened control beam, whereas the same beam with stirrups spaced at 175 mm and strengthened with 2 layers of external CFRP had an increase in shear capacity due to the CFRP of 2% over the un-strengthened beam. Finally, it was also seen that the beam with a depth of 175 mm, no stirrups, and 1 layer of CFRP strengthening had an increase in shear strength due to the CFRP of 65% over the un-strengthened control beam, whereas the beam with the same properties but with a depth of 350 mm had an increase of only 48%. Therefore, it is evident that the beam depth can influence the contribution of externally applied FRP to the shear strength of the beam.

2.6 Failure Mechanics of FRP Shear Strengthened Reinforced Concrete Beams

The two main failure mechanisms for reinforced concrete beams strengthened in shear with FRP sheets are FRP rupture or FRP debonding. FRP rupture is the predominant failure mechanism for full wrap configurations, whereas FRP debonding is common for side bonded and U-wrap configurations (Chen and Teng, 2003b; Chen and Teng, 2003a; Teng et al., 2009; Belarbi and Acun, 2013). Both failure mechanisms will be discussed in detail in the subsequent sections.
2.6.1 FRP Rupture

FRP rupture is the failure of the FRP sheet caused by tearing of the fibres when the tensile load exceeds the ultimate tensile strength of the FRP and is a predominant failure mode for the full wrap configuration. Although it is rare, FRP rupture can also be a failure mechanism for the U-wrap configuration (Belarbi and Acun, 2013; Chen and Teng, 2003a; Grande et al, 2007). The use of mechanical anchorage devices at the ends of side bonded or U-wrapped configurations can delay the debonding process and could result in the FRP reaching its ultimate tensile strength and failing due to rupture (Ceroni and Pecce, 2010).

Ceroni and Pecce (2010) conducted an experimental study on the bond strength between CFRP sheets and concrete with the use of an anchorage device. Bond tests were performed by bonding one end of a CFRP sheet to the side of a compressed concrete block and gripping the other end of the sheet in a universal machine to apply tensile load to it. Three types of anchorage devices, a lateral CFRP sheet, a near surface mounted CFRP bar, and a CFRP spike anchor were used at the ends of the bonded side of the CFRP sheets and the results were compared to the unanchored case. From the results, it was found that in a few cases, the use of anchorage devices resulted in the elimination of debonding and failure due to rupture.

Cao et al. (2005) and Teng et al. (2009) provide a description of the failure process for strengthened beams that fail due to FRP rupture. FRP rupture occurs from the development of a shear crack along the concrete beam. With the formation of a shear crack, the concrete becomes ineffective in resisting the tensile stresses along the crack and all tensile stresses are resisted by the FRP sheets intersecting the crack. As the width of the shear crack increases, the strain and tensile load in the FRP sheet also increase until the tensile strength of the sheet is reached, resulting in rupture. In most cases, the FRP ultimate tensile strength is reached first at the lower end of a shear crack. Therefore, once the ultimate tensile strength is reached, the rupture process begins with the failure of the highest stressed FRP sheet which intersects the lower end of a shear crack. The stresses then redistribute themselves among the remaining wraps crossing the shear crack and the FRP rupture process is repeated along the crack, ultimately resulting in failure of the beam. Once rupture starts, it propagates rapidly along the shear crack, therefore minimal warning is given before the failure of the beam. In several cases of full wrap configurations, partial debonding of the sides of the wrap occurs before failure due to rupture of the FRP.

When the FRP sheets are applied in a full wrap or U-wrap configuration, the ultimate tensile strength of the sheets are reduced compared to their strength using tension tests with flat coupons (Chen and Teng, 2003b; Teng et al., 2009). This is due to factors related to the surface of the concrete beams and their geometry. A major factor deals with the corners and
edges of beams. These can create localized stress concentrations in the FRP and could lead
to tearing of the FRP if they are sharp. The rounding of sharp corners is needed to reduce
these localized stress concentrations, but regardless of the rounded corner radius, the change
in the stress direction at corners introduces radial stresses perpendicular to the fibres in the
FRP, which tend to weaken the FRP. Nevertheless, corners of a beam cross-section must be
rounded when using an FRP full wrap or U-wrap configuration. The ACI 440.2R-08 (2008)
guidelines state that a minimum radius of 13 mm must be provided when the FRP sheet is
wrapped around outside corners, whereas the CNR-DT 200/2004 (2004) guidelines require a
minimum radius of 20 mm (Teng et al., 2009).

Cao et al. (2005) conducted an experimental study, as previously discussed, on the failure
process of rectangular reinforced concrete beams strengthened with fully wrapped
unidirectional carbon and glass FRP sheets. The results showed that two beams failed by
premature FRP rupture immediately after the fully wrapped sheets partially debonded along
the sides of the beam. The lower ultimate load in the FRP sheets was attributed to the
corners of the beams not being well rounded.

2.6.2 FRP Debonding

FRP debonding is a premature failure mechanism of shear strengthened beams. It occurs in
U-wrapped and side bonded configurations, due to a limited anchorage length. This is in
contrast to fully wrapped beams, where sufficient anchorage is provided by the FRP sheet
being wrapped around the top and bottom surfaces of the beam. FRP debonding is the
process of an FRP sheet peeling off the concrete surface to which it is bonded, prior to
reaching its ultimate tensile strength (Chen and Teng, 2003a; Sas et al., 2008; Teng and
Chen, 2009; Lu et al., 2009; Colalillo and Sheikh, 2014; Mofidi and Challal, 2011; Chen et
al., 2012).

Teng and Chen (2009) and Colalillo and Sheikh (2014) discuss the debonding process for U-
wrapped and side bonded beams. Debonding failures occur due to the formation of shear
creacks in the strengthened beam. Once a shear crack forms, the concrete surface along the
crack becomes inactive in resisting the tensile loads and a vertical separation of the rigid
cement sections on either side of the crack occurs. This results in localized debonding of
the FRP sheets at the crack surface and high tensile stresses throughout the surrounding area
of the sheets bridging the crack. The high tensile stresses must be transferred from the FRP
to the concrete sections by interfacial stresses between the FRP sheet and concrete surface.
These high interfacial shear stresses cause further localized debonding towards the free edge
of the sheet and also cause the effective bond length to migrate. The localized debonding
will spread towards the free end until the bond length becomes insufficient to transfer the required interfacial stresses, resulting in debonding of the free end of the sheets. This debonding process will continue along the crack until enough of the FRP wraps have deboned from the concrete surface, leading to the failure of the reinforced concrete beam prior to the FRP reaching its ultimate tensile strength. Generally, once debonding initiates, due to the inability of concrete and FRP to redistribute stresses and lower stress peaks, the debonding quickly spreads towards the free end of the FRP wrap, unless it is arrested by the presence of an anchorage device.

The FRP wraps are inactive before a shear crack develops in the beam and localized debonding of the FRP wrap occurs. This is due to the load being resisted by the concrete prior to cracking. For U-wrapped configurations, the FRP debonding will occur at the free ends of the sheet, as there is only one free end on either side. However, with side bonded configurations, the debonding process can either occur at the top or bottom free ends of the sheet on either side of the beam. Therefore, the use of anchorage devices at these locations of the U-wrapped or side bonded sheets can delay or possibly eliminate the onset of complete debonding. Anchorage systems will be discussed in detail in the subsequent section.

There are two locations where the FRP can debond, which is at the FRP/epoxy interface or at the concrete/epoxy interface. Debonding failures commonly occur in the concrete at a small distance from the FRP/epoxy interface (Chen and Teng, 2003a; Lopez-Gonzales et al. 2012). This results in a thin layer of concrete attached to the FRP sheet when debonding occurs and can be attributed to the bond strength of the concrete layer being much lower than the bond strength of the epoxy, meaning that the concrete is the weaker link. Although almost all debonding failures occur in the concrete, some debonding failures can occur at the epoxy/FRP interface. This can be attributed to several factors, including inadequate concrete surface conditions for bonding, inadequate amount of epoxy used for bonding, or the use of a high strength concrete that has greater bond strength than the bond strength of epoxy.

Lopez-Gonzales et al. (2012) conducted an experimental study on the effect of concrete strength and adhesive thickness on the strength of FRP/concrete bonds. Beam tests were used to test 6 specimens with concrete strengths of 20, 40, and 60 MPa, where 2 and 3 adhesive layers were tested for each of the concrete strengths. Based on the results, they concluded that for low strength concrete members, failure occurred in the concrete and was independent of the adhesive thickness, however for higher strength concretes, the greater adhesive layer resulted in delaying the debonding failure.

There are several factors which can influence the bond strength of FRP to concrete, including the FRP axial rigidity and concrete strength, the bond length, the concrete and epoxy strengths, the concrete surface preparation before bonding, the interfacial fracture energy,
and the use of anchorage devices. These factors will be discussed in more detail in the subsequent sections.

2.6.2.1 FRP Axial Rigidity and Concrete Strength

The axial rigidity and axial stiffness of FRP sheets are a major influencing factor on the strength of an FRP/concrete bond.

Triantafillou (1998) first proposed a polynomial equation that related the strain in the FRP at shear failure to the axial rigidity of the FRP sheets. This equation was derived by curve fitting of 40 experimental data produced by several researchers. The problem with this equation was that debonding failures were not considered individually, and the concrete strength was not included in the equation.

The equation was later modified by Khalifa et al. (1998) and Triantafillou and Antonopoulos (2000) by introducing the concrete compressive strength as a governing parameter. Other parameters also introduced were the effective bond length, which is defined as the length beyond which the bond strength does not increase with further increase in bond length, and a reduction factor for the relation between the stiffness of internal shear reinforcement to FRP shear reinforcement. These modifications were made by empirical methods.

The axial rigidity can be represented as:

\[
\text{axial rigidity} = E_f \rho_f
\]

(2.1)

where \( E_f \) is the elastic modulus of the FRP in the principal fiber orientation, and \( \rho_f \) is the FRP reinforcement ratio which can be represented by equation 2.2:

\[
\rho_f = \frac{2t_f b_f}{b_w s_f}
\]

(2.2)

where \( t_f \) is the thickness of the FRP wrap, \( b_f \) is the width of the FRP wrap, \( b_w \) is the width of the concrete cross-section, and \( s_f \) is the spacing of the FRP wraps.

The effective strain model proposed by Triantafillou and Antonopoulos (2000) is used in the \textit{fib} Task Group 9.3 Bulletin 14 Technical Report (2001) and shows that an increase in the axial rigidity of the FRP will lead to a decrease in effective strain, however, an increase in the concrete strength will produce an increase in effective strain and delay debonding.
Godat et al. (2012) developed a finite element model in order to investigate the influence of several parameters on the effectiveness of FRP shear strengthened beams. A three-dimensional finite element model was produced using ADINA software and appropriate material models were employed, as well as interface elements able to represent the bond-slip characteristics and failure criterion, used at the concrete/FRP interface. Concrete strengths of 25, 30, 35, 40, and 45 MPa and FRP elastic moduli values of 102 to 233 GPa were used to see the effect of concrete strength and FRP elastic modulus on the bond strength. As expected, it was found that the increase in the elastic modulus and in turn the axial rigidity of the FRP resulted in a decrease in the effective axial strain, whereas an increase in the concrete compressive strength resulted in an increase in the effective axial strain.

2.6.2.2 Bond Length and Configuration of the FRP Sheets

The bond length of FRP sheets have a direct influence on whether the sheets will fail due to debonding or rupture. An increase in bond length would result in an increase in the resistance of the FRP. This relationship is true until the effective bond length is reached, which is the length of the bond at which the maximum bond strength is achieved. If the length of the bond is increased past this effective length, there is no increase in bond strength. Therefore, when a long bond length is used, only part of the bond length is active in resisting the load, and the average interfacial stresses based on the entire length of the bond is inappropriate (Maeda et al, 1997; Teng and Chen, 2009; Mofidi and Challal, 2011).

The idea of the effective bond length was developed from a research study by Maeda et al (1997) in which it was demonstrated that increasing the bond length beyond a certain point would not result in an increase in bond strength. Several bond strength models have adopted this idea including Chen and Teng (2003b), Mofidi and Challal (2011), Chen et al. (2013), Colalillo and Sheikh (2014) and several others. Several expressions for the effective bond length have shown a direct relation between the square root of the axial rigidity of the FRP sheet and the inverse of the forth root of the compressive strength of concrete or the inverse of the square root of the tensile strength of the concrete. In recent models, such as Colalillo and Sheikh (2014) the influence of interfacial stress and slip has been implemented in the calculation of the effective bond length.

The most effective location for a side bonded FRP strip would be in the middle of a shear crack, in order to obtain the maximum bond length for the sheet sections on either side of the crack. For an FRP U-wrap, the most effective location would be at the lower end of the crack in order to provide the maximum bond length for the critical sheet section above the
crack. For U-wrapped configurations, the critical region of the wrap is bonded above the shear crack, since it contains the free end of the wrap.

2.6.2.3 Concrete Surface Preparation

The behaviour of concrete members externally strengthened with FRP is greatly dependant on a sound concrete substrate and proper preparation of the concrete surface. An improperly prepared surface can result in debonding of the FRP reinforcement layer before achieving the design load transfer (ACI 440.2R-08, 2008).

The CNR-DT 200/2004 (2004) guidelines recommend four steps for concrete surface preparation before the application of FRP. A summary of these steps are presented below:

1. Sandblasting of the concrete surface should be performed in order to provide a roughness degree of 0.3 mm. This level of roughness should be measured by suitable instruments.
2. Poor concrete surfaces should be treated with a consolidating agent before primer application takes place.
3. The concrete surface should be cleaned to remove any dust, laitance, oil, surface lubricants, foreign particles, or any other bond-inhibiting material.
4. All inside and outside corners and sharp edges should be rounded to a minimum radius of 20 mm.

2.6.2.4 Interfacial Fracture Energy

The interfacial fracture energy is the work done by the interfacial shear stress until the debonding of the FRP sheet from the concrete surface. It is equal to the area under the interfacial stress-slip curve which can be found by using experimental tests on the FRP/concrete interface. Lopez-Gonzalez et al. (2012) discusses the two most common experimental tests to evaluate the shear stress in the FRP/concrete interface, which are single and double shear tests. A single shear test is performed by bonding an FRP strip to a concrete prism. The concrete prism is held in place and a tensile load is applied on the other end of the FRP strip. A double shear test is similar to the single shear test, except two FRP strips are applied on opposite sides of the concrete prism. Both single and double shear tests have been performed by various researchers in experimental studies, such as Mazzotti et al. (2009), Cao et al. (2007), Toutanji et al (2012), Nigro et al. (2011), and several others.
Lu et al. (2005) conducted an extensive review of available bond models as well as using meso-scale finite element results with appropriate numerical smoothing and concluded that typical bond slip curves should contain an ascending branch with a continuous stiffness degradation up to the maximum bond stress, and then a curved descending branch to zero bond stress at a finite value of slip. Three models were proposed to approximate the bond slip curve, which were the precise model, the simplified model, and the bilinear model. The precise model provides the best approximation of the bond slip curve but is also the most complex, whereas the simplified model and bilinear models provide a simplified approach without significant loss of accuracy.

Several empirical models have been proposed for the interfacial fracture energy based on the concrete tensile strength, and in turn, the concrete compressive strength, which can be related to its tensile strength (Toutanji et al, 2012). Chen et al. (2013) recently presented a new shear strength model to take into account the interaction between the internal shear reinforcement and external FRP reinforcement in which the interfacial fracture energy is used to predict the bond strength and is shown to be empirically related to the square root of the concrete tensile strength. Sas et al. (2008) also presented a shear strength model in which the interfacial fracture energy was used to predict the bond strength and was shown to be related to the concrete compressive strength.

### 2.7 Models to Predict Resistance of FRP Shear Strengthening

#### 2.7.1 Shear Resistance Concept

The concept of shear resistance for reinforced concrete beams is based on the understanding that the total shear resistance for a beam is equal to the combination of shear resistance contributed by the concrete, the internal steel shear reinforcement, and the FRP shear strengthening. The total shear resistance of beams strengthened with external FRP can be represented by the following:

\[
V_{\text{Total}} = V_c + V_s + V_{\text{FRP}}
\]  

(2.3)

Since the beams in this experimental program have a strengthened region which is devoid of internal shear reinforcement, the shear resistance due to internal steel stirrups will not be
considered. The calculation of the individual concrete and FRP shear resistances will be discussed in the subsequent sections.

2.7.2 Concrete Shear Resistance ($V_c$)

The concrete shear resistance can be calculated using the appropriate standard or guidelines corresponding to the standard or guideline used to calculate the FRP shear resistance.

The concrete shear resistance outlined in the CSA A23.3-04 (2004) concrete design standard is based on the modified compression field theory (Vecchio and Collins, 1986). Two methods are available to calculate the concrete shear resistance, which are the simplified method and general method. The difference between these two methods for the concrete shear resistance is in the calculation of the term $\beta$, which accounts for the so-called concrete contribution to shear resistance. The simplified method allows one to calculate $\beta$, or assume a value based on design standard recommendation. The general method’s process to determine $\beta$ is somewhat more elaborate and may require an iterative procedure. The concrete shear resistance equation and the two methods to determine $\beta$ are presented below:

$$V_c = \lambda \beta \sqrt{f'c} b_w d_v$$

(2.4)

where $\lambda$ is a factor to account for the density of concrete, $\beta$ is a factor accounting for the extent of concrete contribution to shear resistance, $f'c$ is the compressive strength of the concrete, $b_w$ is the width of the beam, and $d_v$ is the effective shear depth of the beam.

The effective shear depth, $d_v$, can be calculated from the equation 2.5:

$$d_v = max\left\{ \frac{0.9 d}{0.72h} \right\}$$

(2.5)

where $d$ is the distance from the extreme compression fibre to the centroid of the longitudinal tensile reinforcement and $h$ is the beam height.

Using the simplified method, the term $\beta$ can either be assumed as 0.18 or can be calculated using the equation below:

$$\beta = \frac{230}{1000+d_v}$$

(2.6)
The latter equation accounts for beam size effect on its shear strength. Using the general method, the term $\beta$ can be calculated using the following equations:

$$\beta = \frac{0.4}{1 + 1500\varepsilon_x} \cdot \frac{1300}{1000+s_{ze}}$$  \hspace{1cm} (2.7)

$$\varepsilon_x = \frac{M_f + V_f + 0.5N_f}{2(E_s A_s)}$$  \hspace{1cm} (2.8)

$$s_{ze} = \frac{35S_z}{15+a_g}$$  \hspace{1cm} (2.9)

where $\varepsilon_x$ is the longitudinal strain at the mid-depth of the cross-section, $s_{ze}$ is the effective crack spacing factor, $M_f$ is the factored moment, $V_f$ is the factored shear force, $N_f$ is the factored axial load at the section, $E_s$ is the elastic modulus of the longitudinal reinforcement, $A_s$ is the area of the longitudinal reinforcement, $S_z$ is the crack spacing factor, and $a_g$ is the nominal maximum course aggregate size used in the concrete mix.

2.7.3 Shear Resistance Due to FRP ($V_{FRP}$)

There are several models available to calculate the shear resistance contributed by externally applied FRP laminates used in shear strengthening. The FRP shear contribution for most models is based on the truss model with a variable or 45 degree shear crack angle. The main difference among the models presented below is in the calculation or limits of the effective strain in the FRP for various strengthening configurations and the inclination angle of the shear crack. For simplicity, the CSA S806-12 (2012), ACI 440.2R-08 (2008), and CNR-DT 200/2004 (2004) will be addressed as standards.

2.7.3.1 Chen and Teng (2003a, b) Model

Chen and Teng provided a model to calculate the shear resistance contributed by FRP U-wraps, side strips, and full wraps.

This model is based on several assumptions. Firstly, a difference in stress is assumed to be experienced by the FRP wraps surrounding the shear crack and a stress distribution factor is used in this model to account for this. Secondly, the shear resistance of the FRP is assumed
to be governed by a single critical shear crack that dominates the debonding failure process (Chen and Teng, 2003a,b).

The FRP shear resistance, $V_{FRP}$, is based on a variable shear crack inclination angle and can be calculated as:

$$V_{FRP} = 2f_{FRP,e}t_{FRP}w_{FRP} \frac{h_{FRP,e}(\cot \theta + \cot \alpha) \sin \alpha}{s_{FRP}}$$

where, $f_{FRP,e}$ is the effective stress in the FRP at the ultimate state, $t_{FRP}$ is the thickness of FRP wraps, $w_{FRP}$ is the width of the individual FRP wraps, $s_{FRP}$ is the center-to-center spacing of the FRP wraps along the longitudinal axis of the beam, $h_{FRP,e}$ is the effective height of FRP which is taken as 0.9$d$, $\theta$ is the inclination angle of the shear crack, which is assumed as 45 degrees for design, and $\alpha$ is the angle between the FRP fibres and longitudinal axis of the beam.

The effective stress in the FRP, $f_{FRP,e}$ for FRP U-wraps can be calculated as follows:

$$f_{FRP,e} = D_{FRP} \sigma_{FRP,max}$$

$$D_{FRP} = \begin{cases} \frac{2(1-\cos \frac{\pi \lambda}{\pi \lambda})}{\pi \lambda \sin \frac{2\lambda}{\pi}}, & \text{when } \lambda \leq 1 \\ 1 - \frac{\pi - 2}{\pi \lambda}, & \text{when } \lambda > 1 \end{cases}$$

$$\sigma_{FRP,max} = 0.427 \beta_w \beta_L \frac{E_{FRP} \sqrt{f_c}}{t_{FRP}}$$

$$\beta_w = \frac{2-w_{FRP}/(s_{FRP} \sin \beta)}{\sqrt{1+w_{FRP}/(s_{FRP} \sin \beta)}}$$

$$\beta_L = \begin{cases} \sin \frac{\pi \lambda}{2}, & \text{if } \lambda < 1 \\ 1, & \text{if } \lambda \geq 1 \end{cases}$$

$$\lambda = \frac{L_{max}}{L_e}$$

$$L_{max} = \frac{h_{FRP,e}}{\sin \beta}$$
\[ L_e = \sqrt{\frac{E_{FRP} f_{FRP}}{\sqrt{f'c}}} \]  

(2.18)

where \( D_{FRP} \) is the stress distribution factor, \( \sigma_{FRP,max} \) is the maximum stress at debonding, \( f'c \) is the compressive strength of the concrete, \( \beta_w \) is the strip width ratio factor, \( \beta_l \) is the bond length factor, \( \lambda \) is the normalised maximum bond length, \( L_{max} \) is the maximum value among the bond lengths of all the FRP strips intersected by the critical shear crack, and \( L_e \) is the effective bond length, defined as the length of bond that gives the maximum bond strength.

For fully wrapped beams, the stress distribution factor and maximum FRP stress at rupture are given by:

\[ D_{FRP} = 0.5 \]  

(2.19)

\[ \sigma_{FRP,max} = \begin{cases} 0.8f_{FRP}, & \text{if } \frac{f_{FRP}}{E_{FRP}} \leq \epsilon_{max} \\ 0.8\epsilon_{max}E_{FRP}, & \text{if } \frac{f_{FRP}}{E_{FRP}} > \epsilon_{max} \end{cases} \]  

(2.20)

where \( f_{FRP} \) is the tensile strength of the FRP, and \( \epsilon_{max} \) is the ultimate strain for the FRP.


The FRP shear strengthening provisions in the Italian CNR-DT 200/2004 (2004) standard are based on fracture mechanics in order to determine the stress in the FRP at debonding. The standard provides equations to determine the effective stress in the FRP for side bonded, U-wrapped, and fully wrapped strengthening configurations. All FRP wraps are assumed to achieve their full capacity at ultimate state. Also, the shear resistance contributed by the FRP is based on the shear crack inclination angle.

The FRP shear resistance is expressed as:

\[ V_{FRP} = \frac{1}{\gamma_{rd}} \cdot 0.9d \cdot f_{feq} \cdot 2t_f \cdot (\cot \theta + \cot \alpha) \cdot \frac{w_f}{s_f} \]  

(2.21)

where \( \gamma_{rd} \) is a safety factor, \( d \) is the distance from the extreme compression fibre to the centroid of the longitudinal tensile reinforcement, \( f_{feq} \) is the effective stress in the FRP, \( t_f \) is the thickness of the FRP wrap, \( \theta \) is the angle of inclination of the shear crack, which is recommended as 45 degrees unless a more detailed calculation is made, and \( \alpha \) is the angle the
FRP fibres make with the longitudinal axis, \( w_f \) is the width of the FRP wraps, and \( s_f \) is the center to center spacing between the FRP wraps.

The effective stress for U-wraps can be calculated as:

\[
f_{fed} = f_{fdd} \left[ 1 - \frac{1}{3} \frac{L_e \sin \alpha}{\min (0.9d; h_w)} \right]
\]  
(2.22)

The effective stress for full wraps can be calculated as:

\[
f_{fed} = f_{fdd} \left[ 1 - \frac{1}{6} \frac{L_e \sin \alpha}{\min (0.9d; h_w)} \right] + \frac{1}{2} (\varphi_R f_{fd} - f_{fdd}) \left[ 1 - \frac{L_e \sin \alpha}{\min (0.9d; h_w)} \right]
\]  
(2.23)

The terms used to calculate the effective stress can be calculated using the following equations:

\[
\varphi_R = 0.2 + 1.6 \frac{r_c}{b_w}, \quad 0 < \frac{r_c}{b_w} \leq 0.5
\]  
(2.24)

\[
f_{fdd} = \sqrt{\frac{2E_f G_{fk}}{t_f}}
\]  
(2.25)

\[
L_e = \sqrt{\frac{E_f t_f}{2\sigma_{ctm}}}
\]  
(2.26)

\[
G_{fk} = 0.03k_b \sqrt{f_{ck} f_{ctm}}
\]  
(2.27)

\[
k_b = \sqrt{\frac{2-w_f/s_f}{1+w_f/400}} \geq 1
\]  
(2.28)

where \( \varphi_R \) is a factor regarding the curvature of the corners of the beam, \( r_c \) is the corner radius for the beam, \( f_{fdd} \) is the design debonding strength of the FRP reinforcement, \( f_{fd} \) is the ultimate design strength of the FRP, \( b_w \) is the beam width, \( h_w \) is the beam depth, \( L_e \) is the effective bond length, \( f_{ctm} \) is the average tensile strength of the concrete, \( f_{ck} \) is the characteristic strength of concrete, \( G_{fk} \) is the specific fracture energy of the FRP-concrete interface, \( k_b \) is a geometric coefficient, and \( E_f \) is the elastic modulus of the FRP.
2.7.3.3 ACI 440.2R-08 (2008) Standard

The ACI standard provides a simplified model to determine the shear resistance contributed by FRP strengthening. The model is based on a research study by Khalifa et al. (1998). The standard provides calculations for the debonding strain of the FRP for U-wrapped and side bonded configurations in the form of a strain reduction factor applied to the ultimate FRP strain. The standard recommends the use of mechanical anchorage devices at the ends of FRP sheets in order to increase the amount of strain the FRP sheets can withstand before debonding. The standard ultimately limits the strain at 0.004 in order to ensure no loss of aggregate interlock of the concrete. For fully wrapped beams, the standard assumes an effective FRP strain of 0.004. It is important to note that the FRP shear resistance equation provided by the standard does not include a variable shear crack inclination angle and assumes a shear crack inclination angle of 45 degrees for all beams. Also, all FRP wraps are assumed to achieve their full capacity at the ultimate state.

The FRP shear resistance is calculated using:

\[
V_{FRP} = \frac{A_f \varepsilon_{fe} E_f (\sin \alpha + \cos \alpha) d_{fv}}{s_f} \tag{2.29}
\]

where \( A_f \) is the cross sectional area of the two legs of the FRP, \( \varepsilon_{fe} \) is the effective strain in the FRP, \( E_f \) is the elastic modulus of the FRP, \( \alpha \) is the angle between the FRP fibres and the longitudinal axis, \( d_{fv} \) is the effective FRP depth, taken as the distance from the extreme compression fibre to the centroid of the tensile reinforcement, and \( s_f \) is the center to center spacing of the FRP wraps.

For U-wrapped or side bonded configurations, the effective strain of the FRP sheets before debonding can be calculated using the following equations:

\[
\varepsilon_{fe} = k_v \varepsilon_{fu} \leq 0.004 \tag{2.30}
\]

\[
k_v = \frac{k_1 k_2 L_e}{11900 \varepsilon_{fu}} \leq 0.75 \tag{2.31}
\]

\[
L_e = \frac{23300}{(f_{te} E_f)^{0.58}} \tag{2.32}
\]

\[
k_1 = \left( \frac{f_{te}}{27} \right)^{2/3} \tag{2.33}
\]
where, $k_1$ is a modification factor which accounts for concrete strength, $k_2$ is a modification factor which accounts for the wrapping configuration of the FRP, $L_e$ is the active bond length of the FRP sheet, $\varepsilon_{fu}$ is the ultimate FRP strain, $n_f$ is the number of layers of FRP, $t_f$ is the thickness for one layer of the FRP reinforcement, and $f'c$ is the compressive strength of the concrete.

### 2.7.3.4 CSA S806-12 (2012) Standard

The Canadian Standards Association S806-12 (2012) standard presents similar recommendations as the ACI 440 2R-08 standard. One of the differences between the two are that the CSA standard uses the variable truss-angle analogy, whereas the ACI uses the 45 degree truss. Therefore, similar to the Italian CNR-DT 200/2004 (2004) standard, the CSA standard makes the FRP shear strength a function of the angle of inclination of the shear crack. Also, for the CSA standard, the bond reduction factor $k_2$ is calculated using a single equation for both U-wrapped and side bonded FRP configurations. For anchors, the CSA standard assumes an effective FRP strain of 0.005 and for full wraps an effective FRP strain of 0.006 is used.

The FRP shear resistance is calculated using:

$$V_{FPR} = \frac{A_F\varepsilon_{Fe}E_Fd_p(cot\theta+cot\alpha_F)sin\alpha_F}{s_F}$$

where $A_F$ is the cross sectional area of the two legs of the FRP wrap, $\varepsilon_{Fe}$ is the effective FRP strain, $E_F$ is the FRP elastic modulus, $d_p$ is the effective shear depth, $\theta$ is the shear crack inclination angle which can be calculated using the general method for shear resistance provided by the CSA A23.3-04 (2004) standard, $\alpha_F$ is the angle between the FRP fibres and longitudinal axis, and $s_F$ is the center to center spacing of the FRP.

The effective FRP strain, $\varepsilon_{Fe}$ for U-wrapped and side bonded beams can be calculated using equations 2.30-2.33. The term $k_2$ can be calculated using the equation presented below:

$$k_2 = \begin{cases} \frac{d_{fu}-L_e}{d_{fu}}, \text{U-wrapped} \\ \frac{d_{fu}-2L_e}{d_{fu}}, \text{Side bonded} \end{cases}$$

(2.34)
2.7.3.5 Chen et al. (2013) Model

Chen et al. (2013) is the most up-to-date model used to predict the FRP shear resistance for U-wrapped or side bonded beams. The model incorporates the interaction between internal steel stirrups and external FRP strengthening. Some assumptions made in the derivation of this model are that the shear failure is governed by a single critical shear crack with a linear crack shape. The width of the critical shear crack is assumed to vary linearly from the crack tip to the crack end. A variable truss-angle analogy was also used for this model, resulting in the FRP shear resistance being based on the inclination angle of the shear crack. A stress distribution factor is used in this model to account for the differences in stress experienced by the FRP wraps surrounding the shear crack. This model also incorporates fracture mechanics to determine the effective FRP stress at debonding.

The FRP shear resistance can be calculated using Eq. 2.10 and the effective FRP stress can be calculated using Eq. 2.11 in Section 2.7.3.1. The terms needed to determine the effective FRP stress for debonding failure of U-wrapped beams can be calculated as follows:

\[
D_{FRP} = 1 - \left(1 - \frac{\pi}{4}\right) \cdot \frac{h_{df}}{h_{f,e}}
\]  
\[
\sigma_{f,\text{max}} = \left\{ \begin{array}{ll}
\sin \left(\frac{\pi}{2} \cdot \frac{L}{L_e}\right) \sqrt{\frac{2E_fG_f}{t_f}}, & L_{\text{max}} < L_e \\
\sqrt{\frac{2E_fG_f}{t_f}}, & L_{\text{max}} \geq L_e
\end{array} \right.
\]  
\[
h_{df} = 2\delta_f \frac{h_{f,e}}{w_{e,p}\sin(\theta+\beta)}
\]  
\[
w_{e,p} = \delta_f \frac{1 + \frac{\pi}{2} \left(\frac{h_{f,e}}{L_e\sin\beta} - 1\right)}{\sin(\theta+\beta)}
\]  
\[
L_{\text{max}} = \frac{h_{f,e} + h_c + h_p}{\sin\beta}
\]  
\[
L_e = \sqrt{\frac{E_f\tau_f}{\sqrt{F_c}}}
\]
\[
\delta_f = \frac{2G_f}{\tau_f}
\] (2.43)

\[
G_f = 0.308\beta_w\sqrt{f_t}
\] (2.44)

\[
\tau_f = 1.5\beta_w f_t
\] (2.45)

\[
\beta_w = \frac{2-w_f/(s_f\sin\beta)}{1+w_f/(s_f\sin\beta)}
\] (2.46)

\[
f_t = 0.395f_{cu}^{0.55}, \quad f_{cu} = \frac{f_{tc}}{0.8}
\] (2.47)

where, \(f_{cu}\) is the cube compressive strength of the concrete, \(E_f\) is the elastic modulus of the FRP, \(t_f\) is the thickness of FRP strips, \(\beta_w\) is the strip width ratio factor, \(w_f\) is the width of the individual FRP strips, \(s_f\) is the center-to-center spacing of the FRP strips along the longitudinal axis of the beam, \(h_{f,e}\) is the effective height of FRP, which is taken as 0.9d, \(w_{e,p}\) is the crack end width when the FRP shear contribution reaches its peak value, \(h_{df}\) is the vertical distance from the crack tip to the point of the intersection between the debonding front and the critical shear crack, \(h_t\) is the vertical distance from the top of the FRP strip to the crack tip, \(h_b\) is the vertical distance from the beam bottom to the crack end, \(L_{max}\) is the maximum value among the bond lengths of all the FRP strips intersected by the critical shear crack, \(L_e\) is the effective bond length, \(\beta\) is the angle between the fiber direction and the beam longitudinal axis, \(\tau_f\) is the maximum interfacial shear stress, \(\delta_f\) is the interfacial slip at the shear crack, \(G_f\) is the interfacial fracture energy, and \(f_t\) is the tensile strength of the concrete.

### 2.7.3.6 Mofidi and Chaallal (2011) model

Mofidi and Chaallal (2011) also presented an up-to-date model to predict the FRP shear resistance in U-wrapped and side bonded strengthening configurations. The model includes the interaction effects between internal steel stirrups and external FRP strengthening. Also a more complex, multi crack pattern is incorporated into the model, whereas other models assume a single critical shear crack to govern the shear failure of the beam. A variable truss-angle analogy was also used for this model, resulting in the FRP shear resistance being based on the shear crack inclination angle.

According to this model, the FRP shear resistance can be calculated using:
where \( t_f \) is the thickness of the FRP wrap, \( w_f \) is the width of the FRP wrap, \( \varepsilon_{fe} \) is the effective strain in the FRP wrap, \( E_f \) is the elastic modulus of the FRP, \( \theta \) is the shear crack inclination angle and is assumed as 45 degrees for design purposes, \( \alpha \) is the angle that the FRP fibres make with the longitudinal axis, \( d_f \) is the FRP depth, which is defined as the distance from the extreme compression fibre to the centroid of the tensile reinforcement, and \( s_f \) is the center to center spacing of the FRP.

The effective strain in the FRP for debonding failure of U-Wrapped beams can be calculated using the following equations:

\[
\varepsilon_{fe} = 0.31 \cdot \beta_C \cdot \beta_L \cdot \beta_w \cdot \sqrt{\frac{f_{ec}}{f'_{fc}}} \quad (2.49)
\]

\[
\beta_C = \frac{0.6}{\sqrt{\rho_f \cdot E_f + \rho_s \cdot E_s}} \quad (2.50)
\]

\[
\beta_w = \sqrt{\frac{2 - w_f / s_f}{1 + w_f / s_f}} \quad (2.51)
\]

\[
\beta_L = \begin{cases} 1, & \text{if } \lambda \geq 1 \\ (2 - \lambda) \cdot \lambda, & \text{if } \lambda < 1 \end{cases} \quad (2.52)
\]

\[
\lambda = \frac{L_{\text{max}}}{L_e} \quad (2.53)
\]

\[
L_{\text{max}} = \frac{d_f}{\sin \alpha} \quad (2.54)
\]

\[
L_e = \frac{E_f \cdot t_f}{2 \cdot f_{ct}} \quad (2.55)
\]

\[
f_{ct} = 0.53 \sqrt{f_{ec}} \quad (2.56)
\]

where \( \beta_C \) is the concrete-cracking coefficient, \( \beta_L \) is the coefficient to compensate for insufficient anchorage length, \( \beta_w \) is the FRP width-to-spacing coefficient, \( f'_{ec} \) is the concrete compressive strength, \( \rho_f \) and \( \rho_s \) are the FRP reinforcement ratio and traverse-steel reinforcement ratio, \( L_{\text{max}} \) is the maximum available bond length, \( L_e \) is the effective bond length, and \( f_{ct} \) is the concrete tensile strength. For the calculation of \( \beta_C \), \( E_f \) is in gigapascals.
2.7.4 Accuracy of FRP Shear Resistance Models

Several researchers have reviewed and compared existing FRP shear guidelines to a large database of experimental data, including Chen et al. (2013), Colalillo and Sheikh (2014), Sas et al. (2009), and Mofidi and Chaallal (2011).

Chen et al. (2013) compared shear strength predictions for FRP debonding failures using the ACI 440.2R-08 (2008), CNR-DT 200/2004 (2004), and their own model with a large database of experimental tests on shear strengthened reinforced concrete beams that failed due to debonding. Based on the results, it was found that the coefficient of determination for a plot of the predicted shear strength versus the experimental shear strength for the ACI standard was 0.563, for the CNR standard was 0.267, and for Chen et al. (2013) was 0.798.

Colalillo and Sheikh (2014) also compared the shear strength predictions using the ACI 440.2R-08 (2008) standard, Chen and Teng (2003a), and the CNR-DT 200/2004 (2004) standard to the experimental shear strength from a database of 119 shear strengthened specimens that failed due to deboning. The results show that the average experimental to predicted shear strength ratio for the ACI standard was 1.33, for Chen and Teng (2003a) was 1.23, and for CNR standard was 1.13.

Sas et al. (2009) have also compared the shear strength predictions for shear strengthened beams that fail due to debonding using the Chen and Teng (2003a) model to a large database of experimental shear strengthened beams that have failed due to debonding. The experimental database consisted of over 200 tests on T-beams and rectangular beams. The prediction of the FRP shear contribution using the Chen and Teng (2003a) model showed a large scatter with the experimental results and drastically underestimated or overestimated the shear capacity for several cases of rectangular beams, however for T-beams the model provided a safe prediction.

Mofidi and Chaallal (2011) also conducted a comparison of the theoretical FRP shear contribution to the shear contribution obtained from experimental results based on a database of 75 shear strengthened reinforced concrete beams that failed due to debonding. The models used to obtain the theoretical FRP shear contributions were the ACI 440.2R-08 (2008) standard, the CNR-DT-200/2004 (2004) standard, and their own model. The results showed that the coefficient of determination between the theoretical FRP shear contribution and the experimental results for the ACI predictions was 0.37, for the CNR predictions was 0.42, and for the Mofidi and Chaallal (2011) predictions was 0.61.

Therefore, one can see that to date, none of the available methods can predict the actual strength of FRP shear strengthened beams with a high degree of precision. This may partly
be due to the quality of workmanship, as in most cases lab specimens are prepared by individuals with insufficient experience in FRP retrofit.

2.8 Anchorage Devices for FRP Shear Strengthening

From previous sections, it is evident that the use of U-wrapped and side bonded configurations for shear strengthening of reinforced concrete beams can lead to premature failure due to debonding of the FRP sheets intersecting a shear crack. In order to solve this problem, several research studies have been conducted on the use of anchorage devices at the ends of FRP sheets for U-wrapped and side bonded configurations (Koutas and Triantafillou, 2013; Bae and Belarbi, 2013; Mofidi et al., 2012; Ortega et al., 2009; Grelle and Sneed, 2013; Jinno et al., 2001; Al-Mahaidi and Kalfat, 2011; Khalifa and Nanni, 2000; Deifalla and Ghobarah, 2010). It has been proven that the capacity of shear strengthened beams with U-wrapped and side bonded configurations can be greatly increased with the implementation of anchorage devices, as they delay or even eliminate premature debonding failure (Koutas and Triantafillou, 2013; Bae and Belarbi, 2013; Mofidi et al., 2012).

There are several different types of anchorage devices for shear strengthened reinforced concrete beams, which include horizontal FRP strips, FRP spike anchors, mechanical anchor systems, and π-anchors. These systems are described in the following sections.

2.8.1 Horizontal FRP Strips

Horizontal FRP strips can be used as an anchorage device for shear strengthened beams with U-wrapped or side bonded configurations of FRP sheets, as seen in Figure 2-2. The horizontal FRP strip can be continuous or discrete and is applied along the length of a beam, perpendicular to the vertical FRP strips used for shear strengthening. The horizontal FRP strip is usually bonded to the top end of the FRP sheet for U-wrapped configurations and can be bonded to the top and bottom ends for side bonded configurations. This type of anchorage system requires the least amount of labour compared to others, and therefore has an easy installation process. In addition, the fibre orientation for these sheets can be at 90 degrees or 45 degrees to the longitudinal axis of the beam and are usually bi-directional (Bae and Belarbi, 2013).
There has been some contradiction among the results from the use of horizontal FRP strips as an anchorage device for shear strengthened beams. Schnerch (2001) used horizontal FRP strips for a study on the shear strengthening of full scale prestressed concrete I-girders and reinforced concrete T-beams and reported that the horizontal FRP anchorage strip did not delay debonding of the vertical FRP sheets and did not increase the shear capacity of the strengthened beam. On the other hand, an experimental study by Al-Mahaidi and Kalfat (2011) on the use of horizontal FRP strips for U-wrapped and side bonded configurations showed that they delayed the debonding process and increased the shear capacity of the strengthened beams. Bae and Belarbi (2013) also conducted an experimental study on the use of horizontal FRP strips as an anchorage device for strengthened reinforced concrete T-beams. Twelve reinforced concrete T beams were tested with internal shear and flexural reinforcement. Beams were strengthened with unidirectional CFRP strips in a U-wrapped configuration with the use of horizontal strips. Based on the results, it was found that the failure of strengthened beams with horizontal strips as an anchorage device was still caused by debonding, but the debonding failure was delayed and the shear capacity was increased over the control beam. The strengthened beam with the use of horizontal strips resulted in a 54.5% strength gain over the un-strengthened control beam, whereas the strengthened beam without horizontal strips only had a strength gain of 26.3%.

Figure 2-2: U-wrapped beam with horizontal strips (a) side view (b) cross-sectional view
2.8.2 FRP Spike Anchor

The FRP spike anchor is a very practical anchorage device consisting of resin impregnated fibre roving’s. As shown in Figure 2-3, the anchor is produced by bundling up a portion of fibres and tying a section of the fibre length with plastic ties. Immediately after the application of the FRP shear strengthening, while the epoxy is still wet, the tied portion of the fibres are inserted into predrilled holes filled with saturant in order to secure the fibres. After this, the dry section of the fibres is splayed and bonded on top of the FRP wraps in a fan arrangement using epoxy resin. Spike anchors are applied to the free ends of the FRP sheet for shear strengthened beams. For the case of T-beams, since the FRP sheets have free ends at the connection between the beam flange and web, the spike anchors can either be embedded horizontally into the web or vertically into the flange (Smith and Kim, 2008; Bae and Belarbi, 2013). Koutas and Triantafillou (2013) have determined that the embedment of the anchors vertically into the flange of the beam is much more effective than horizontally into the web. A disadvantage of spike anchors is that they must be inserted through the FRP wrap, therefore reducing the effective area of the FRP wrap which might cause damage to the wrap under loading.
Figure 2-3: U-wrapped beam with spike anchors (a) side view (b) cross-sectional view (c) image of fibres that make up spike anchor

Spike anchors have been used widely with the flexural strengthening of reinforced concrete beams, however there has not been much research in its application to shear strengthening configurations.

Orton (2007) conducted an experimental study on the use of spike anchors for shear strengthening applications and reported that they allowed the FRP wraps to reach their ultimate tensile strength and avoid premature debonding failure.

Jinno et al. (2001) also conducted an experimental study on T-beams shear strengthened with U-wrapped FRP sheets reinforced with spike anchors embedded vertically inside the flange of the beam. The results of the study showed that these anchors greatly increased the shear capacity of the strengthened beams and delayed the debonding process.

Baggio et al. (2014) also conducted an experimental study on the use of spike anchors for the FRP shear strengthening of rectangular reinforced concrete beams. The beams were 150 mm wide by 350 mm deep by 2440 mm long and contained shear and flexural internal reinforcement. Partial depth, 100 mm wide GFRP U-wraps spaced at 200 mm were used to strengthen the beams with and without the use of spike anchors. The results showed that the use of spike anchors provided a 56% increase in shear strength over the control beam, whereas the strengthened beam without anchors only provided a 36% increase in shear strength over the control beam.

Kim et al. (2014) conducted an experimental study on the strengthening of reinforced concrete T-beams using CFRP U-wraps with spike anchors. The beams were 610 mm deep and were tested with three different shear span to depth ratios of 1.5, 2.1, and 3. The beams
also had internal shear and flexural reinforcement. The beams were strengthened with U-wraps with and without spike anchors. One spike anchor was used at the top of each U-wrap and was embedded through the wrap into the concrete. The CFRP laminate thickness was 0.011 mm and had a tensile elongation at rupture of 10,050 με. Comparing anchored and un-anchored U-wrapped beams with a shear span to depth ratio of 3, it was found that the anchored U-wrapped beam resulted in an increase in strength of 40% over the un-anchored U-wrapped beam.

2.8.3 Mechanical Anchor Systems

Mechanical anchors have been investigated for its application in FRP shear strengthened beams by several researchers, including Bae and Belarbi (2013), Ortega et al. (2009), and Mofidi et al (2012). They can be used to reinforce both U-wraps and side bonded FRP sheets against debonding. As seen in Figure 2-4, the mechanical anchorage system consists of a steel or FRP composite plate at the ends of the FRP wrap. The steel or composite FRP plates are then bolted to the concrete surface using steel anchor bolts. The steel anchor bolts also penetrate the FRP sheets bonded to the plates (Grelle and Sneed, 2013; Bae and Belarbi, 2013).

An extension of this concept can also be used to anchor FRP U-wrapped and side bonded sheets, called sandwiched mechanical anchors. The sandwiched mechanical anchors consist of the end of the FRP wrap being looped around the steel or FRP composite plate, as seen in Figure 2-4c. An additional plate is then placed on top of the first plate and anchor bolts are bolted through the two plates and FRP sheet into the concrete surface, in order to lock the FRP sheet between the plates and transfer load to the concrete. The sandwiched mechanical anchors are superior to conventional mechanical anchors as they provide a layered connection and greater resistance to slippage of the FRP sheets underneath the plates. In addition, both conventional and sandwiched mechanical anchors can be applied either as discontinuous or continuous plates along the beam length (Grelle and Sneed, 2013; Bae and Belarbi, 2013)
Bae and Berarbi (2013) conducted an experimental study comparing the behaviour of full scale shear strengthened T-beams using CFRP U-wrapped sheets reinforced with discontinuous mechanical and sandwiched mechanical anchors. The plates used were hybrid FRP plates consisting of glass and carbon hybrid pultruded strips embedded in a vinyl ester resin. Also, three evenly spaced steel anchor bolts were used per anchorage device to bolt the plates to the concrete. The results showed that the use of the discontinuous mechanical anchor did not allow the FRP sheet to reach its ultimate strength due to a bearing failure of the FRP sheets around the anchor bolts. On the other hand, the use of the sandwiched...
mechanical anchor did not result in bearing failure of the FRP sheets around the anchor bolt and in turn allowed the FRP sheet to reach its ultimate strength. The use of the discontinuous mechanical anchor resulted in a 48.1% increase in shear strength and the use of the sandwiched discontinuous mechanical anchor resulted in an increase in shear strength of 74.6% over the un-strengthened beam, whereas the un-anchored strengthened beam had an increase in strength of only 26.3% over the un-strengthened beam.

Ortega et al. (2009) conducted another experimental study on the use of sandwiched mechanical anchors for shear strengthened T-beams with a continuous steel plate and steel anchor bolts. The results showed that the continuous steel plate exhibited a buckling failure mode due to the large spacing between the anchors. They recommended the use of discontinuous plates for each FRP strip.

Mofidi et al. (2012) also conducted research on the effect of sandwiched mechanical anchors for FRP U-wrapped T-beams. The anchor plates were made of aluminum and steel anchor bolts were used. The results showed that the shear strengthened T-beams with sandwiched mechanical anchors exhibited an average increase in shear capacity as high as 41% over un-strengthened beams, whereas the FRP strengthened beam without anchors exhibited an increase in shear capacity of 25% over un-strengthened beams.

2.8.4 $\pi$-Anchor

The $\pi$-anchor was developed by Mostafa and Razaqpur (2013) and combines a horizontal strip type anchorage device with spike anchors. It consists of an anchor head plate which is bonded to the FRP laminate and concrete surface. The anchor head plate is also connected to the concrete through anchor legs which are embedded into the concrete, as illustrated in Figure 2-5. This system provides a greater bonded surface area for the FRP sheets and the transfer of shear stresses deep into the concrete member through the embedded anchor legs. One advantage of this system compared to conventional mechanical anchors is that there is no embedment through the FRP wrap, therefore high stresses and damage in the FRP wrap is prevented.
Mostafa and Razaqpur (2013) tested the \( \pi \)-anchors on 21 T-beams strengthened in flexure. The test variables comprised of the amount of CFRP reinforcement, use of the anchors, number of anchors used, and the locations of the anchors. The anchors were placed along the CFRP sheet bonded to the tension face of the beams. For one of the beams, with four layers of CFRP reinforcement, the anchors allowed the FRP sheet to reach 94% of its ultimate strain before failure due to debonding.

Cameron (2012) also performed experimental tests on the use of the \( \pi \)-anchor for flexural strengthened T-beams. Six T-beams were tested with no flexural strengthening, flexural strengthening without anchors, and flexural strengthening with anchors. Based on the results, for the beam strengthened with the use of the anchors, the FRP laminate achieved a maximum strain equal to 80% of its ultimate strain, which was a 94% increase over the strengthened beam without anchors.

This new anchor has been proven effective for the flexural strengthening of reinforced concrete beams, however, its effectiveness in shear strengthened beams has not yet been investigated. Hence a variant of the \( \pi \)-anchor will be used in this study to check its effectiveness in shear strengthening.

### 2.9 Summary

This chapter has discussed the causes of shear deficiency in reinforced concrete beams, the mechanics of shear failures for these beams, and strengthening techniques to reinforce them. FRP materials were introduced as a strengthening method and the configurations used for the
shear strengthening of reinforced concrete beams were discussed. The failure modes of the FRP shear strengthened configurations were examined with an emphasis on debonding failure for side bonded and U-wrapped configurations, and several models were presented to shear resistance provided by the FRP strengthening. Finally, several anchorage devices were discussed as reinforcement against debonding failure for side bonded and U-wrapped FRP shear strengthened reinforced concrete beams.

The following conclusions can be drawn from this literature review:

1) The main parameter for all FRP shear resistance models is the effective stress/strain in the FRP at failure, and is a distinguishing feature of most models.

2) The FRP shear resistance models presented may not be acceptable for all conditions, as influencing factors such as the interaction between the internal and external shear reinforcement are not considered in most models.

3) Spike anchors are an effective anchorage device for continuous or wide intermittent FRP wraps, however they are ineffective for thin wraps due to the formation of high stress concentrations and damage of the fibers in the FRP. An effective anchorage device for thin intermittent wraps needs further research.

4) The ACI 440.2R-08 (2008) guidelines assume a shear crack inclination angle of 45°, whereas all other guidelines and models include a variable shear crack inclination angle when calculating the FRP shear resistance. The influence of the shear crack inclination angle on the shear resistance contributed by the FRP and the veracity of the FRP shear resistance models discussed should also be investigated.

5) The π-anchor developed by Mostafa and Razaqpur (2013) has potential for effective use in shear strengthened beams, however, experiments must be conducted to demonstrate its effectiveness in FRP shear strengthened beams.
Chapter 3: Experimental Program

3.1 General

The objective of this experimental program is to determine the effectiveness of a new anchor designed to delay/prevent the premature debonding of FRP U-wraps that are used to strengthen shear deficient reinforced concrete beams, and to determine the influence of the shear crack inclination angle on the effectiveness of the external shear strengthening of reinforced concrete beams with FRP. The following sections will describe the testing program and set-up, the test specimens, their fabrication, shear strengthening, material properties, and the instrumentation used for their testing.

3.2 Testing Program and Set-up

A total of fourteen tests were performed on shear-deficient reinforced concrete beams. Although twelve beams were fabricated, two of the beams were re-strengthened and re-tested. Inclined pre-cracks, simulating shear cracks were created in the beams during fabrication. Half of the beams were fabricated with a crack angle of 30 degrees, and the other half, with a crack angle of 45 degrees to the longitudinal axis of the beams. Some of the beams were externally strengthened in shear along their pre-cracked section using either FRP U-wraps, FRP U-wraps with one anchor per side, FRP U-wraps with one horizontal strip per side, or FRP full wraps, as described in more detail later in Section 3.3.3.

All beams were tested in the Applied Dynamics Laboratory at McMaster University. The beams were 2200 mm long and were tested in three-point loading with a shear span of 950 mm. The beams were simply supported by a roller at one end and a pin at the other end. A jack supported by a steel frame attached to the strong floor of the laboratory was used to apply load to the beams. The jack had a stroke of 500 mm and a load capacity of 890 kN. All beams were loaded using displacement control at a rate of 0.6 mm/min. The load was transferred to the beam via a plate placed on the top surface of the beam at mid-span, directly below the jack. A layer of Hydrostone was used between the supports and the beam and between the loading plate and the beam to eliminate any stress concentrations due to irregularities on the concrete surface. A schematic of the loading and support dimensions is...
presented in Figure 3-1 and an image of a beam in the test set-up prior to testing is presented in Figure 3-2.

Figure 3-1: Loading and support dimensions

Figure 3-2: Image of a beam in the test setup prior to testing
3.3 Test Specimens

All beams tested in this experimental program had identical size and shape. Figure 3-3 shows the typical cross-section and flexural reinforcement of the beams. The cross section was 170 mm wide, 400 mm deep, with an effective depth of 309 mm. The top, sides, and bottom concrete covers were 25, 30, and 35 mm. Each beam was 2200 mm long with a shear span length of 950 mm. The shear span to depth ratio of the beams was 3.07, which is considered a slender beam and promotes the formation of diagonal shear cracks.

Inclined pre-cracks at angles of 30 and 45 degrees to the longitudinal axis of the beam were created by inserting a 0.8 mm thick polycarbonate sheet into the formwork at the appropriate angle before casting of the concrete. The polycarbonate sheets started at the tension face in the beams and reached to a height of 80% of the beam depth, as illustrated in Figure 3-4. This was done in order to promote the formation of a shear crack along the sheet during testing.
All beams were identically reinforced in flexure with four 20M steel bars in tension and two 20M steel bars in compression. The internal shear reinforcement consisted of 10M stirrups with standard 90 degree hooks spaced at 200 mm center to center, except when an isolated stirrup was placed near the beam ends. The beams were designed using the CSA A23.3-04 (2004) standard, as well as the CSA S806-12 (2012) standard. All beams with the exception of select control beams with full internal reinforcement, contained a section which was devoid of internal shear reinforcement in the region spanning the inclined pre-crack. This was done in order to induce shear failure in the pre-cracked region and to mobilize the FRP reinforcement to resist the applied shear. The internal reinforcement details for the beams are shown in Figure 3-5a, b, and c. Figure 3-5a shows the internal reinforcement details for the control beams with full internal reinforcement. Figure 3-5b shows the internal reinforcement details for the 30 degree pre-cracked beams and Figure 3-5c shows the internal reinforcement details for the 45 degree pre-cracked beams.
3.4 Fabrication of Test Specimens

The twelve beams in this experimental study were cast in wooden formwork made from 2X4’s and ¾” plywood. Steel reinforcement cages were assembled using tie wire, as seen in Figure 3-6a, and were placed inside the forms and centered, resting on plastic chairs to ensure the proper concrete cover in the beams. The steel reinforcement cages were placed into the formwork inverted, to allow for the polycarbonate sheets used to form inclined pre-cracks in the beams, to be inserted through the top of the formwork. Due to the flexibility of the polycarbonate sheets, 1 mm diameter steel rods were attached to the edges and center of the sheets, in order to stiffen them. The steel rods were covered with tape to prevent them from bonding with the wet concrete during casting. Angled clamps were fabricated and screwed to the top of the formwork in order to secure the polycarbonate sheets during concrete casting.
and ensure the appropriate inclination angle of the sheets. Figure 3-6b shows the polycarbonate sheet prior to being inserted into the formwork, and Figure 3-6c shows the steel cage and polycarbonate sheet inserted into the formwork before casting of the concrete.

Figure 3-6: Images of beam components prior to casting of the concrete (a) steel cages during assembly (b) polycarbonate sheet prior to being inserted into the formwork (c) steel cage and polycarbonate sheet inserted into the formwork before casting of the concrete
All forms and polycarbonate sheets were well lubricated with formwork oil to allow for removal of the plates and easy stripping of the forms. In order to prevent the polycarbonate plates from bending under the weight of the wet concrete during casting, the formwork was filled half way with concrete before the plates were inserted. After casting of the concrete, the beams were kept hydrated to prevent any loss of moisture. The beams were covered with wet burlap and were left in the formwork for three days. After this time, the beams were stripped of their formwork and were stored inside the laboratory to cure until they reached their 28 day strength. Straps were used to transport the beams by crane in the laboratory.

Figure 3-7 shows images of the beams after casting of the concrete. Figure 3-7a shows an image of the beams in their formwork after casting of the concrete and Figure 3-7b shows an image of the beams after being stripped of their formwork.

Figure 3-7: Images of beams after casting of the concrete (a) beams in their formwork after casting of the concrete (b) beams after being stripped of their formwork
3.5 Shear Strengthening of Test Specimens

Several of the beams tested in this experimental program were externally strengthened in shear using FRP. Different configurations of shear strengthening were used in order to compare the effectiveness of each. Select beams were shear strengthened in their pre-cracked regions using un-anchored U-wraps, U-wraps with the use of anchors, U-wraps with the use of horizontal strips, or full wraps.

Table 3.1 presents a summary of the beams tested in this experimental program and their shear strengthening configurations. The beams are designated in the following manner: WW-XX-YY-ZZ, where WW indicates the shear strengthening configuration, XX indicates the pre-crack inclination angle, YY indicates if anchors were used and the type of anchors used, and ZZ indicates the number of layers in each FRP wrap.
Table 3-1: Summary of Beams Tested

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Description</th>
<th>Number of Layers of FRP</th>
<th>U-wrapped</th>
<th>Fully Wrapped</th>
<th>Use of Anchor</th>
<th>Use of Horizontal Strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-30-NA-NA</td>
<td>-Un-strengthened</td>
<td>0</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>-No internal stirrups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-30-NA-2L</td>
<td>-Strengthened with U-wraps</td>
<td>2</td>
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<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>-Strengthened with U-wraps</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>-Anchor configuration 1 used</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-30-A2-1L</td>
<td>-Strengthened with U-wraps</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>-Anchor configuration 2 used</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>-Strengthened with U-wraps</td>
<td>1</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>-Horizontal strips used</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-30-NA-1L</td>
<td>-Strengthened with full wraps</td>
<td>1</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
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<td>C1-30-NA-NA</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>-Full internal stirrups</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>-Strengthened with U-wraps</td>
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<td>No</td>
<td>No</td>
</tr>
<tr>
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<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>-Strengthened with U-wraps</td>
<td>1</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>-Anchor configuration 1 used</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>-Strengthened with U-wraps</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>-Anchor configuration 2 used</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>-Strengthened with U-wraps</td>
<td>1</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>-Horizontal strips used</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-45-NA-1L</td>
<td>-Strengthened with full wraps</td>
<td>1</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>C1-45-NA-NA</td>
<td>-Un-strengthened control beam</td>
<td>0</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>-Full internal stirrups</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
3.5.1 Un-anchored Strengthened Beams

Beams strengthened without the use of anchors consisted of un-anchored U-wrapped and full wrapped beams. SikaWrap 1400 C carbon-fiber wraps were used for the U-wrapped and fully wrapped strengthening configurations. The wraps were 1.3 mm thick, 30 mm wide, and were spaced at 200 mm center to center along the pre-cracked region of the beams. Hence, the cross-sectional area for each leg of the U-wrap or full wrap per layer of FRP was 39 mm$^2$. Two layers of the FRP wraps were used for the un-anchored U-wrapped beams to provide a thickness of 2.6 mm for the U-wraps. For the full wrapped beams, only one layer of the FRP wraps were used, as higher FRP strains were anticipated. A schematic of the cross section of beams strengthened with un-anchored U-wraps or full wraps is presented in Figure 3-8a. The side view of the 30 degree pre-cracked beams strengthened with un-anchored U-wraps or full wraps is shown in Figure 3-8b, and the side view of the 45 degree pre-cracked beams strengthened with un-anchored U-wraps or full wraps is presented in Figure 3-8c.
Figure 3-8: FRP strengthening details (a) cross section of beam strengthened with an un-anchored U-wrap or full wrap (b) side view of 30 degree pre-cracked beams strengthened with un-anchored U-wraps or full wraps (c) side view of 45 degree pre-cracked beams strengthened with un-anchored U-wraps or full wraps
3.5.2 FRP Installation for Un-anchored Strengthened Beams

Before the FRP wraps could be applied to the beams, the concrete surface was prepared using the following procedure:

1) The bottom edges of the U-wrapped beams and the top and bottom edges of the fully wrapped beams were rounded to a radius of 20 mm. This was done to reduce stress concentration in the FRP at the edges of the beam. An image of one of the beams rounded edges can be seen in Figure 3-9a.

2) The concrete surface was roughened using a needle scaler. This was done to provide an open roughened surface for improved bonding. Figure 3-9b shows an image of the concrete surface being roughened. For the 45 degree pre-cracked beams strengthened with un-anchored U-wraps, the concrete surface was grinded down to the aggregate level before being roughened.

![Concrete surface preparation](image_url)

(a) (b)

Figure 3-9: Concrete surface preparation (a) rounded edges (b) roughened surface

The FRP wraps used in this experimental program were installed using the procedure recommend for the SikaWrap 1400C by Sika® Canada (https://can.sika.com/). The FRP installation procedure was as follows:
1) U-wrapped beams were placed inverted on saw horses prior to strengthening to allow for easier installation of the FRP U-wrap. Fully wrapped beams were placed in their normal orientation on saw horses prior to strengthening.

2) The locations of the wraps were marked on the beams prior to strengthening to ensure their correct spacing. Figure 3-10a shows an image of the marks placed on a beam prior to strengthening.

3) All FRP wraps were cut to the appropriate length and width prior to strengthening using heavy duty scissors. Figure 3-10b shows an image of an FRP wrap being cut to size.

4) Sikadur 300 epoxy, designed to bond the FRP to the concrete surface was prepared according to the instructions provided by Sika® Canada (https://can.sika.com/). The two parts of the epoxy were weighed to achieve the appropriate ratio as provided by the manufacturer. Part B was added to Part A and the two parts were mixed for 5 minutes with a low speed mixing drill until the epoxy was uniformly blended. Figure 3-10c shows an image of the epoxy parts being weighed prior to mixing them. Figure 3-10d shows an image of the two parts of the epoxy being mixed together with a mixing drill.

5) Sikadur 300 epoxy was applied to the concrete surface by brush before the application of the FRP wraps. The FRP wraps were then saturated with the epoxy and were applied by hand onto the concrete surface and pressed down.

6) The FRP wraps were rolled with a fluted roller in order to remove any air pockets and excess epoxy, and to ensure all areas of the FRP wrap were pressed on to the concrete surface. Figure 3-10e shows an image of the FRP wraps being rolled with the roller after application on the concrete surface.
Figure 3-10: FRP installation (a) markings showing the location of the FRP wraps (b) wraps being cut to their appropriate length and width (c) Sikadur 300 epoxy components being weighed prior to mixing (d) epoxy being mixed using a low speed mixing drill (e) FRP wraps being rolled using a fluted roller
This completes the FRP installation process for un-anchored strengthened beams. After installation of the FRP, the epoxy needs to cure before any load can be applied to the beam. In this experimental study, the strengthened beams were not tested until at least one week after the FRP installation.

3.5.3 Strengthened Beams with Anchors or Horizontal Strips

Select beams in this experimental program were strengthened with FRP U-wraps with the use of the proposed anchors or horizontal strips. The proposed anchor is conceptually based on the π-anchor developed by a research team at McMaster University (Mostafa and Razaqpur, 2013). Although based on the π-anchor concept, the manufacturing of the anchor used in this experimental program differed from the manufacturing process used by Mostafa and Razaqpur (2013). The proposed anchor is designed to be manufactured in-situ, whereas the π-anchor was designed to be pre-fabricated.

The manufacturing process of the proposed anchor was as follows:

1) Holes were first drilled in the beams at the appropriate locations of the anchor legs prior to the strengthening of the beams. The holes had a diameter of 14 mm and were drilled to a depth of 65 mm. The outer edges of the holes were rounded to minimize stress concentration in the carbon-fibre rope to be inserted into the holes in a later step. An image of the holes drilled in one of the beams with anchors can be seen in Figure 3-11a. Figure 3-11b shows the outer edges of the holes being rounded.

2) Two rope sections of SikaWrap Anchor-C carbon-fibre rope were cut to a length of 98 mm prior to strengthening of the beams to create the anchor legs. The selected length allowed for full insertion of the carbon-fibre rope into the anchor holes and 30 mm of length to splay onto the anchor head plates. Plastic ties were tightened on the ends of the carbon-fibre rope sections that were to be inserted into the anchor holes. This was done to keep the fibers intact and together when being inserting into the anchor holes. Figure 3-11c shows a carbon-fibre rope section that was cut to size with a plastic tie tightened on one of the ends.

3) Two layers of 80 x 200 mm strips of the SikaWrap 1400 C carbon-fibre fabric were cut to create the anchor head plate. The first strip had fibers running in the traverse direction, whereas the second strip had fibers running in the longitudinal direction. Two holes were created in the strips at the appropriate locations by spreading apart their fibers and inserting wooden dowels to ensure the appropriate hole diameters, as seen in Figure 3-11d.
4) The FRP U-wrap was first applied to the beam using the installation process for un-anchored strengthened beams described in section 3.5.2. This can be seen in step (i) of Figure 3-12.

5) Sikadur 300 epoxy was applied to the concrete surface by brush before the application of the carbon-fibre strips, which were cut to size in step 3. The carbon-fibre strips were then saturated with the epoxy and were applied by hand onto the concrete surface and the free end of the U-wrap bonded in step 4, as seen in part (ii) and (iii) of Figure 3-12. The first layer of the strips had its fibers running in the
transverse direction as seen in part (ii) of Figure 3-12, whereas the second layer had its fibers running in the longitudinal direction as seen in part (iii) of Figure 3-12. Care was taken to ensure that the holes made in the strips in step 3, coincided with the pre-drilled anchor holes in the beams from step 1.

6) Sikadur 300 epoxy was then injected into the predrilled anchor holes using a plastic syringe. Figure 3-13a shows an image of the epoxy being injected into the predrilled anchor holes.

7) The previously cut sections of the SikaWrap Anchor C carbon-fibre rope in step 3 were saturated with Sikadur 300 epoxy and inserted into the predrilled anchor holes, as seen in step (iv) of Figure 3-12.

8) The protruding length of the carbon-fibre rope sections were splayed onto the anchor head plate in a 360 degree fan and brushed with additional Sikadur 300 epoxy. Figure 3-13b shows an image of the protruding section of the carbon-fibre rope splayed onto the anchor head plate in a 360 degree fan.

![Image of anchor installation steps](image)
This completes the anchor installation. As with the un-anchored strengthened beams, the strengthened beams with anchors were given at least one week of curing time for the epoxy before being tested. A schematic of the top view and 3-D view of the assembled anchor is presented in Figure 3-14.
After testing the beams with the above anchor installation, it was noticed that the free end of the U-wrap eventually debonded from the concrete while still being bonded to the anchor head plate, leading to slip between the U-wrap and anchor head plate at failure. Therefore, the bond between the U-wrap and concrete surface, at the free end of the wrap was found to be the weak link. To improve the situation, the proposed anchor was modified. The modification involved applying the first layer of the carbon fibre strips to the concrete prior to the application of the U-wrap, as seen in part (i) of Figure 3-15. The U-wrap was then applied to the concrete surface, on top of the first carbon-fibre strip, as seen in part (ii), followed by the application of second carbon-fibre strip, as seen in part (iii), and finally the anchor legs as seen in step (iv) of Figure 3-15. It is important to note that for the modified anchor, both layers of the carbon-fibre strips had their fibers running in the transverse direction.

![Figure 3-15: Illustration of modified anchor installation steps](image)

After testing the beams with anchors, it was surmised that the anchor legs embedded into the concrete may have played a minimal role in the anchor mechanism. In order to investigate the influence of the anchor legs on the effectiveness of the anchors, select beams were strengthened with U-wraps with the use of only the horizontal carbon-fibre strips. The installation process of the horizontal strips was identical to the installation process for anchor configuration 1, with the exclusion of pre-drilled anchor holes, holes made in the carbon-fibre strips, and the anchor legs which were embedded into the concrete and splayed onto the anchor head plate. Figure 3-16 shows an illustration of the horizontal strip installation procedure. Refer to the installation procedure of anchor configuration 1 for steps (i), (ii), and (iii) in Figure 3-16.
3.6 Material Properties of Test Specimens

3.6.1 Concrete

The twelve beams in this experimental study were constructed from concrete supplied by a ready mix concrete supplier. A single mix was used for making the twelve beams and the twelve 150 mm x 300 mm concrete cylinders used to determine the compressive and tensile strength of the specified mix. The specified design strength of the concrete was 35 MPa, with a maximum aggregate size of 12.5 mm and a slump of 140 mm. A plasticiser as well as extra water was added to the mix during casting to maintain the 140mm slump.

Table 3-2 shows the concrete average 28 day compressive strength as 45 MPa, Table 3-3 shows its average tensile strength as 3.7 MPa, and Table 3-4 shows its average end of testing compressive strength as 52 MPa. Nine cylinders were tested in compression using the compressive test outlined in the CSA A23.2 (2014) standard. The remaining three cylinders were tested in tension using a split cylinder test as per the CSA A23.2 (2014) standard.
Table 3-2: Concrete 28-Day Compressive Strength  
(150 x 300 mm Cylinders)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Compressive Strength ($f'_c$) (MPa)</th>
<th>Average Compressive Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>49</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-3: Concrete 28-Day Split Cylinder Tensile Strength  
(150 x 300 mm Cylinders)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Tensile Strength ($f'_t$) (MPa)</th>
<th>Average Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2</td>
<td>3.7</td>
</tr>
<tr>
<td>2</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.9</td>
<td></td>
</tr>
</tbody>
</table>
Table 3-4: End of Testing Compressive Strength of Concrete (Approximate Age = 101 Days)

(150 x 300 mm Cylinders)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Compressive Strength ($f'_c$) (MPa)</th>
<th>Average Compressive Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>2</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>54</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-17 shows images of the concrete cylinders after casting and compressive and tensile testing. An image of the concrete cylinders after casting is shown in Figure 3-17a. An image of a concrete cylinder during and after its compressive test is shown in Figure 3-17b, and an image of a concrete cylinder during and after its split cylinder test is shown in Figure 3-17c.

(a)
Figure 3-17: Concrete cylinder after casting and compressive and split cylinder tensile testing (a) concrete cylinders after casting (b) concrete cylinder during and after compressive testing (c) concrete cylinder during and after split cylinder tensile test
3.6.2 Internal Steel Reinforcement

Grade 400 steel was used for the longitudinal tensile and compressive reinforcement, as well as for the internal shear reinforcement. The tensile and compressive longitudinal reinforcement consisted of 20M bars. Tensile tests in accordance with the ASTM A370 (2016) guidelines were performed on three 500 mm length steel specimens from the same batch as the steel used for the longitudinal reinforcement in the beams. A Tinius Olsen Universal Testing machine with a maximum capacity of 600 kN was used to carry out the tensile tests, and a standard electronic extensometer with a gauge length of 100 mm was used for strain measurements. Figure 3-18 shows the tensile stress-strain relationship of the three specimens. It is important to note that in order to avoid damage to the extensometer, it was removed prior to failure of the specimens, therefore the complete stress-strain relationship until failure was not captured. Table 3-5 shows the yield strength, yield strain, ultimate strength, and modulus of elasticity, and average yield strength for samples.

![Tensile stress-strain relationship for steel specimens tested](image_url)
Table 3-5: Properties of Steel Specimens Tested

<table>
<thead>
<tr>
<th>Samples</th>
<th>Yield Strength (MPa)</th>
<th>Yield Strain (με)</th>
<th>Ultimate Strength (MPa)</th>
<th>Modulus of Elasticity (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>420</td>
<td>0.00212</td>
<td>592</td>
<td>198,113</td>
</tr>
<tr>
<td>2</td>
<td>424</td>
<td>0.00213</td>
<td>593</td>
<td>199,061</td>
</tr>
<tr>
<td>3</td>
<td>415</td>
<td>0.00207</td>
<td>589</td>
<td>200,483</td>
</tr>
<tr>
<td>Average</td>
<td>420</td>
<td>0.00211</td>
<td>591</td>
<td>199,219</td>
</tr>
</tbody>
</table>

3.6.3 Carbon-fibre Sheet

The SikaWrap 1400C carbon-fibre fabric was used in this experimental study to produce the U-wraps, full wraps, and anchor head plates used to strengthen the test specimens. The SikaWrap 1400C is a unidirectional, high strength carbon fibre fabric that is flexible and can be wrapped around complex geometries. FRP is an elastic material up to failure, therefore the stress in the FRP can be determined at any value of strain using its modulus of elasticity. Table 3-6 provides the physical and mechanical properties of the SikaWrap 1400C carbon-fibre sheet as reported by the manufacturer, Sika Canada Inc ® (https://can.sika.com/).

Table 3-6: SikaWrap 1400C Physical and Mechanical Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength</td>
<td>1355 MPa</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>115 700 MPa</td>
</tr>
<tr>
<td>Elongation at Break</td>
<td>2.15%</td>
</tr>
<tr>
<td>Thickness</td>
<td>1.3 mm</td>
</tr>
<tr>
<td>Colour</td>
<td>Black</td>
</tr>
</tbody>
</table>
3.6.4 Epoxy

Sikadur 300 epoxy was used for all strengthening configurations for this experimental program. It is a clear, two component, high strength epoxy specifically designed for use as a primer and impregnating resin for the SikaWrap 1400 C carbon fibre fabric which was used for strengthening. The mix ratio of the two components is 2.38:1 by volume and 2.9:1 by weight. Table 3-7 provides the physical and mechanical properties of this epoxy as reported by the manufacturer, Sika Canada Inc ® (https://can.sika.com/).

Table 3-7: Sikadur 300 Physical and Mechanical Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colour</td>
<td>Clear, Amber</td>
</tr>
<tr>
<td>Mix Ratio (A:B) by Volume</td>
<td>2.38:1</td>
</tr>
<tr>
<td>Mix Ratio (A:B) by Weight</td>
<td>2.9:1</td>
</tr>
<tr>
<td>Viscosity</td>
<td>500 cps</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>55 MPa</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>1724 MPa</td>
</tr>
<tr>
<td>Elongation at Break</td>
<td>3%</td>
</tr>
</tbody>
</table>

3.6.5 Carbon-fibre Rope

SikaWrap Anchor C carbon fibre rope was used to create the legs for the proposed anchorage device. The SikaWrap Anchor C is a unidirectional, carbon-fibre rope specifically designed to anchor CFRP fabrics on concrete and masonry. Table 3-8 presents the physical and mechanical properties for the SikeWrap Anchor C carbon-fibre rope as reported by the manufacturer, Sika Canada Inc ® (https://can.sika.com/).
Table 3-8: SikaWrap Anchor C Physical and Mechanical Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colour</td>
<td>Black</td>
</tr>
<tr>
<td>Diameter</td>
<td>10 mm</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>1590 MPa</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>215 000 MPa</td>
</tr>
<tr>
<td>Elongation at Break</td>
<td>0.74%</td>
</tr>
</tbody>
</table>

3.7 Instrumentation Used During Testing of Specimens

The instrumentation used in this experimental program consisted of electrical resistance strain gauges, string potentiometers, and a load cell. Two 5 mm, 120 Ω electrical resistance strain gauges were used to measure the strain in the internal tensile steel reinforcement at the mid span and pre-cracked section of the beams. Figure 3-19a shows a schematic of the locations of the internal strain gauges along the tensile steel reinforcement. The sections of the longitudinal tensile steel bars where the strain gauges were to be installed were grinded to provide a smooth surface prior to installation. An image of one of the strain gauges installed on a longitudinal tensile steel bar prior to the assemblage of the steel cages is presented in Figure 3-19b.

![Diagram showing strain gauge installation](image-url)
Figure 3-19: Internal strain gauges placed on longitudinal tensile reinforcement (a) schematic of internal strain gauge locations on tensile steel reinforcement in the beam (b) an image of a strain gauge installed on a tensile steel bar

Six 5 mm, 120 Ω electrical resistance strain gauges were also used to measure the strain in the FRP wraps of strengthened beams. This allowed the strain in the FRP wraps surrounding the shear pre-crack to be monitored during testing. Figure 3-20a shows a schematic of the locations of the external strain gauges installed on the FRP wraps surrounding the shear pre-crack. Sections of the FRP wraps where strain gauges were to be installed were sanded down to provide a smooth surface prior to installation. Figure 3-20b shows an image of strain gauges installed on the FRP wraps prior to testing of the strengthened beam.
Figure 3-20: External strain gauges installed on FRP wraps (a) A schematic of the locations of the strain gauges installed on the FRP wraps of strengthened beams (b) an image of strain gauges installed on the FRP wraps of a strengthened beam
A load cell attached to a hydraulic actuator was used to measure load during testing. The load cell had a capacity of 890 kN.

Three string potentiometers were also used to determine the displacement of the beam at midspan and quarter length points. Screws were drilled into the beam at the locations of the string potentiometers and the strings were fastened to the screws. Figure 3-21a shows the locations of the string potentiometers and Figure 3-21b shows an image of a string potentiometer installed prior to testing.

Figure 3-21: String potentiometers (a) a schematic of the locations of the string potentiometers (b) an image of string potentiometers installed on a beam prior to testing
3.8 Summary

This chapter has described the testing program and set-up, the test specimens, their fabrication, shear strengthening, material properties, and the instrumentation used for their testing. Fourteen beams were tested with half of the beams having a shear pre-crack inclination of 30 degrees and the other half, 45 degrees to the longitudinal axis of the beam. All beams were shear strengthened using FRP at the pre-cracked regions of the beams, which were left devoid of internal shear reinforcement. The strengthening configurations included un-anchored U-wraps, anchored U-wraps, U-wraps with horizontal strips, and full wraps. The test variables included the shear pre-crack inclination angle, the shear strengthening configuration, the presence of anchors or horizontal strips, and the number of layers of FRP laminate used for the wraps. One can refer to Table 3-1 in Section 3.5 for a detailed description of the beams tested. The results of the experimental tests performed on the beams, including their observed behaviour, failure modes, load-deflection response, internal tensile reinforcement strain, and FRP strain development will be discussed in the next chapter.
Chapter 4: Experimental Results

4.1 General

This chapter presents the experimental results obtained for the beams tested in this investigation. The following sections will discuss the observed behaviour, failure modes, load-displacement behaviour, internal tensile reinforcement strain, and FRP strain development for all beams. The analysis and discussion of the results will be presented in the following chapter.

4.2 Summary of Experimental Results

Several failure modes were observed among the 14 beams tested in this experimental program. The failure modes observed consisted of diagonal tension failure, FRP debonding, FRP rupture, slip between the FRP wrap and anchor, flexural failure due to concrete crushing, and flexural failure due to buckling of the compression reinforcement. Table 4-1 provides a summary of the test results and failure modes observed for the beams tested in this experimental program. The provided results include ultimate load, shear strength, and corresponding mid-span deflection for each of the beams tested.
Table 4-1: Summary of Test Results and Failure Modes

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Ultimate Load (kN)</th>
<th>Mid-span Deflection at Ultimate Load (mm)</th>
<th>Ultimate Shear Force (kN)</th>
<th>Failure Mode</th>
<th>Max Strain in FRP Wrap (με)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-30-NA-NA</td>
<td>155</td>
<td>4.5</td>
<td>77.5</td>
<td>Shear-D</td>
<td>NA</td>
</tr>
<tr>
<td>U-30-NA-2L</td>
<td>197</td>
<td>4.30</td>
<td>98.5</td>
<td>FRP-D</td>
<td>1759</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>290</td>
<td>8.5</td>
<td>145</td>
<td>FRP-D FRP-S Shear-DT</td>
<td>6357</td>
</tr>
<tr>
<td>U-30-A2-1L</td>
<td>342</td>
<td>19.3</td>
<td>171</td>
<td>FRP-D FRP-S Shear-DT Flexure-B</td>
<td>6790</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>298</td>
<td>7.25</td>
<td>149</td>
<td>Shear-D</td>
<td>3420</td>
</tr>
<tr>
<td>F-30-NA-1L</td>
<td>346</td>
<td>43.2</td>
<td>173</td>
<td>FRP-R Flexure-CC Flexure-B Shear-DT</td>
<td>11800</td>
</tr>
<tr>
<td>C1-30-NA-NA</td>
<td>331</td>
<td>25.4</td>
<td>165.5</td>
<td>Flexure-CC</td>
<td>NA</td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>331</td>
<td>41.7</td>
<td>165.5</td>
<td>Flexure-CC</td>
<td>3060</td>
</tr>
<tr>
<td>(Grinded Surface Preparation)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-45-NA-1L</td>
<td>286</td>
<td>7.8</td>
<td>143</td>
<td>FRP-D</td>
<td>2310</td>
</tr>
<tr>
<td>(Grinded Surface Preparation)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>353</td>
<td>54.7</td>
<td>176.5</td>
<td>Flexure-CC</td>
<td>6030</td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>338</td>
<td>42.9</td>
<td>169</td>
<td>Flexure-CC</td>
<td>4583</td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>338</td>
<td>47.1</td>
<td>169</td>
<td>Flexure-CC</td>
<td>3300</td>
</tr>
<tr>
<td>F-45-NA-1L</td>
<td>338</td>
<td>47.0</td>
<td>169</td>
<td>Flexure-CC</td>
<td>2870</td>
</tr>
<tr>
<td>C1-45-NA-NA</td>
<td>334</td>
<td>36.4</td>
<td>167</td>
<td>Flexure-CC</td>
<td>NA</td>
</tr>
</tbody>
</table>
D = debonding, DT = diagonal tension failure, S = slip between U-wrap and anchor head plate, B = buckling of compression reinforcement, CC = concrete crushing in compression zone

It is important to note that the shear crack inclination angle was measured at the mid-depth of each beam.

4.3 Results for 30 Degree Pre-cracked Beams

4.3.1 Control Beam (C-30-NA-NA)

Beam C-30-NA-NA was a control beam, devoid of internal shear reinforcement in its pre-cracked region and un-strengthened. The beam failed by diagonal tension failure, with a shear crack inclination angle of approximately 32 degrees to the longitudinal axis of the beam, as seen in Figure 4-1. During testing the first signs of cracking began with flexural cracks which appeared at a load of 40 kN around the mid-span of the beam. As the test progressed, a shear crack began to propagate along the fabricated pre-crack at a load of 120 kN, where a slight drop in load and change in slope can be seen on the load-deflection curve, presented in Figure 4-2. The shear cracks continued to widen as the load increased until the beam failed suddenly in diagonal tension failure along the fabricated shear crack path at a load 155 kN and mid-span deflection of 4.43 mm. After the maximum load was achieved, a steep drop in load occurred with increase in deflection. The brittle nature of a shear failure was clearly exemplified by the failure of this beam.

Figure 4-1: Diagonal tension shear failure of beam C-30-NA-NA
The internal tensile reinforcement strain is presented in Figure 4-3. A minute level of strain was recorded in the longitudinal tensile reinforcement at the pre-cracked region of the beam prior to the formation of shear cracks in that region at a load of 120 kN. After this, the strain in the longitudinal reinforcement increased as the crack width increased. The strain gauge began to give erroneous readings at a load of 140 kN and therefore those readings are not included in Figure 4-3. The strain response of the longitudinal tensile reinforcement at the mid-span of the beam is also not included in Figure 4-3 due to the internal strain gauge malfunctioning at this location.

It is interesting to note that despite the relatively large strain experienced by the longitudinal reinforcement, the failure mode was brittle, which confirms the prevailing belief that shear failure is brittle, irrespective of the plastic response of the steel reinforcement.
4.3.2 U-wrapped Beam without Anchors (U-30-NA-2L)

Beam U-30-NA-2L was strengthened using two layers of FRP U-wraps without the use of anchors. The beam was only loaded until the FRP U-wraps debonded, therefore full failure of the beam did not occur during testing. This was done in order to retest the beam after retrofitting it with U-wraps with the use of horizontal strips. In this beam, the middle FRP wrap, which was the highest stressed wrap, debonded at a load of 197 kN after reaching a recorded strain of 1759 με. The mid-span deflection of the beam at debonding was 4.3 mm and the shear crack crossing the middle FRP wrap had an inclination angle of approximately 33 degrees to the longitudinal axis of the beam. An image of the debonded middle wrap can be seen in Figure 4-4.

![Image of debonded middle FRP wrap](image)

Figure 4-4: Middle FRP U-wrap of beam U-30-NA-2L after debonding

The load deflection response of this beam is presented in Figure 4-5. The load-deflection curves show an approximately bi-linear response. During testing, the first signs of cracking began with flexural cracks at mid-span, at a load of 28 kN. As the load increased, several more flexural cracks began to occur. At 50 kN, a large flexural crack occurred at mid-span and caused a loud cracking noise and a slight drop in load. The first signs of shear cracking began at 110 kN with the formation of a major shear crack propagating along the artificial shear crack path. This is seen in the load-deflection response of the pre-cracked region of the beam with a sudden increase in deflection at 110 kN. As the load increased, cracking noises could be heard in the FRP until the middle FRP strip suddenly debonded along the north end of the beam, followed by the south end. A loud noise was heard during debonding.
Figure 4-5 Load-deflection response of beam U-30-NA-2L

Figure 4-6, shows the longitudinal steel strain variation with load at the mid-span of the beam. The initial formation of flexural cracks at mid-span is represented by a change of slope of the longitudinal tensile steel strain response at a load of 28 kN. The longitudinal reinforcement at mid-span reached a maximum strain of 2051 με at the debonding load and did not yield during testing. The strain response of the longitudinal reinforcement at the pre-cracked region of the beam is not included in Figure 4-6 due to the internal strain gauge malfunctioning at this location.

Figure 4-6: Strain in longitudinal tensile reinforcement of beam U-30-NA-2L

The variation of the strain in the FRP wraps with applied load is presented in Figures 4-7 and 4-8. The initial shear crack that formed at a load of 110 kN is clearly represented by a sudden increase of 500 με in the middle FRP wrap at this load. As the load increased, the crack width increased which resulted in the strain in the middle FRP wrap increasing with load until debonding of the wrap at a load of 197 kN. The strains recorded in the left and right FRP wraps were both close to zero during testing due to the shear crack not being in the
vicinity of the strain gauges placed on these wraps or the wraps being inactive at resisting the shear load.

With reference to the strain distribution in the middle FRP wrap in Figure 4-8, the strain recorded at the top and bottom of the wrap were close to zero throughout loading, as the shear crack crossed the middle wrap near its center. The strain in the bottom of the FRP wrap increased slowly with load after the formation of the initial shear crack at a load of 110 kN, however the strain was low due to the shear crack being relatively far from the bottom strain gauge location. Before the wrap debonded at a load of 197 kN, the strain instantaneously increased in the top and middle of the FRP wrap by 200 με. This indicates that the FRP in the vicinity of the crack debonded and the interfacial shear stresses had to be resisted by the remaining bonded regions. However, in the absence of anchors, the resistance of the remaining bonded length was easily overcome and full debonding occurred.

Figure 4-7: Strain in FRP wraps surrounding the shear crack for beam U-30-NA-2L

Figure 4-8: Strain distribution of middle FRP wrap for beam U-30-NA-2L
4.3.3 U-wrapped Beams with Anchors (U-30-A1-1L and U-30-A2-1L)

Beams U-30-A1-1L and U-30-A2-1L were strengthened using one layer of FRP U-wraps with the use of anchors. Beam U-30-A1-1L was strengthened with the use of the first anchor configuration and it failed in diagonal tension shear failure attributed to slip between the FRP U-wrap and anchor at a load of 290 kN and a mid-span deflection of 8.6 mm, as seen in Figure 4-9a. The shear crack was at an angle of inclination of approximately 33 degrees to the longitudinal axis of the beam, and the middle wrap which was the highest strained, achieved a recorded strain of 6357 micro-strain before failure.

Beam U-30-A2-1L was strengthened with the second anchor configuration. The beam failed in combined diagonal tension failure due to slip between the anchor head plate and the U-wrap and flexural failure due to concrete crushing in the compression zone under the loading point, as seen in Figure 4-9b. Also, buckling of the longitudinal compression reinforcement occurred at the pre-cracked region of the beam. The beam achieved an ultimate load of 342 kN and an associated mid-span deflection of 19.3 mm. Multiple shear cracks occurred in this beam, however the main shear crack had an inclination angle of approximately 31 degrees to the longitudinal axis of the beam and the middle wrap which was the highest strained, reached 6790 με.
The first signs of cracking for beam U-30-A1-1L began with flexural cracks at mid-span at a load of 37 kN. The first shear crack occurred within the pre-cracked region of the beam at a load of 109 kN, forming along the pre-existing shear crack. This can be seen in the load-deflection response of the beam, presented in Figure 4-10a, as a sudden increase in deflection at the pre-cracked region of the beam occurred at a load of 109 kN. As the load increased, the shear crack width increased and loud cracking was heard in the FRP until shear failure occurred at a load of 290 kN. At the failure load, the shear crack opened and the beam failed.
in diagonal tension failure. Slip between the U-wrap and anchor head plate was observed in the critical wrap intersecting the shear crack.

The load-deflection response of beam U-30-A2-1L is presented in Figure 4-10b and is representative of a ductile flexural failure. The first signs of cracking occurred at a load of 48 kN with flexural cracks initiating at mid-span. At a load of 140 kN, the first signs of shear cracking began with the formation of a major shear crack along the pre-existing shear crack path. As the load increased, the shear crack width also increased and loud cracking noises could be heard in the FRP. Also, additional shear cracks began to form parallel to the initial shear crack, intersecting the center of the left wrap. At a load of 297 kN the longitudinal tensile reinforcement began to yield, and the mid-span deflection began to increase rapidly, with a slowly increasing load. Loud cracking continued in the FRP as the load slowly increased. After reaching a maximum load of 342 kN, concrete crushing was observed under the loading plate and the load began to drop slowly. After the load dropped to 282 kN the beam failed in shear/flexural failure due to diagonal tension failure and buckling of the longitudinal compression reinforcement in the unreinforced section of the beam. At failure, a section of the concrete cover surrounding the buckled compression reinforcement broke off and the critical shear crack along the manufactured shear crack path opened up simultaneously. Slip between the U-wrap and anchor head plate and breakage of the anchor head plate were observed in the critical wrap crossing the shear crack. Although eventually compression steel buckling and combined flexure and shear failure occurred, the use of the anchors enabled the beam to achieve a large plastic deformation and maximum deflection of over 25 mm.
For beam U-30-A1-1L, the longitudinal tensile steel at mid-span yielded at a load of 263 kN with a corresponding strain of 2914 με, as seen in Figure 4-11a. Also, the maximum strain in this steel reached nearly 5000 με. The longitudinal tensile steel strain response at the pre-cracked region of the beam was not included due to the internal strain gauge at this location malfunctioning. The longitudinal tensile steel strain variation with load for beam U-30-A2-1L is presented in Figure 4-11b. The tensile reinforcement yielded at a load of 275 kN corresponding to a strain of 2900 με at mid-span. A sudden increase in strain can also be seen in the strain response at the pre-cracked region of the beam at a load of 140 kN when a major shear crack formed along the shear crack path. The maximum strain in this case reached over 8000 με, indicating noticeably ductile behaviour.
Figures 4-12 and 4-13 show the variation of strain in the FRP wraps with applied load for the anchored beams. For both anchored beams, one can see that there was zero strain in the FRP wraps until the formation of a shear crack crossing them. Thereafter, the strain in the FRP suddenly increased and continued to increase as the shear crack width increased with load. For both beams, the middle FRP wrap was the highest strained wrap, with a maximum recorded strain of 6357 με for beam U-30-A1-1L and 6790 με for beam U-30-A2-1L. In both beams, the strain readings increased significantly before failure, which is attributed to the critical shear crack opening prior to failure. It is worthy to observe the relatively high value of the maximum FRP strain in these beams due to the use of anchors.

Considering the strain distribution in the middle FRP wrap presented in Figure 4-13, it is evident that the distribution is similar in the two anchored beams. The strain gauge at the top of the wraps, above the anchors is seen to have zero strain throughout testing. The strain at the bottom of the wraps followed the same response shape as the center of the wraps, with lower values of strain due to the shear crack not being in the vicinity of the strain gauge placed at the bottom of the wrap.
Figure 4-12: Strain in FRP wraps surrounding the shear crack for anchored U-wrapped beams (a) U-30-A1-1L (b) U-30-A2-1L
Figure 4-13: Strain distribution in middle FRP wrap for anchored U-wrapped beams  
(a) U-30-A1-1L (b) U-30-A2-1L

4.3.4 U-wrapped Beam with Horizontal Strip (U-30-H-1L)

Beam U-30-H-1L was manufactured by re-strengthening the already tested U-30-NA-2L beam, which was tested until the debonding of its U-wraps. The beam was re-strengthened by removing the debonded U-wraps, grinding any remaining epoxy from the surface, roughening the surface using a needle scaler, and applying the new strengthening configuration. Beam U-30-H-1L was strengthened using one layer of U-wraps with the use of horizontal strips. The beam failed due to shear failure near the support. This may have been caused by movement of the steel cage during concrete pour, which would have resulted in the longitudinal reinforcement failing to extend past the support.

Before failure, the beam reached a maximum load of 298 kN, corresponding to a mid-span deflection of 7.25 mm and debonding of the highest strained left FRP wrap occurred, as seen in Figure 4-14. After the left wrap debonded, a sudden decrease in load was observed until failure of the beam at a load of 259 kN. A maximum strain of 3420 με was recorded in the left wrap before debonding, however the shear crack intersected the wrap 50 mm away from the strain gauge, therefore the true strain in the wrap at the shear crack would be greater. The main shear crack for this beam developed parallel to the pre-existing shear crack path, crossing the bottom of the middle wrap and the two wraps closest to mid-span at an inclination angle of 31 degrees to the longitudinal axis of the beam.
The load-deflection response of the beam is presented in Figure 4-15. One can see that the beam displays basically a linear response. Since the beam was previously tested before being re-strengthened, shear and flexural cracks were already present along the beam at the start of testing. The previously developed shear and flexural cracks began to open up at a load of 10 kN and is represented by a change of slope in the load-deflection response at this load. As the load was increased the crack widths increased as well. At a load of 201 kN a new shear crack began to develop on the left side of the pre-cracked region, parallel to the pre-existing shear crack, and the pre-existing shear crack began to close. The new shear crack crossed the bottom of the middle wrap and in between the bottom and center of the left wrap. As the load increased, the width of the new shear crack increased, while the other crack remained closed and inactive. Loud cracking noises could also be heard in the FRP. After reaching a maximum load of 298 kN the highest strained, left FRP wrap debonded from the concrete surface and the load dropped to 278 kN. The load continued to drop after this point, until a load of 259 kN when the beam suddenly failed in shear failure near the support.
The longitudinal tensile reinforcement reached a maximum strain of 2450 με, corresponding to a load of 250 kN at the mid-span of the beam, as seen in Figure 4-16. The strain variation with applied load for the longitudinal tensile reinforcement at the pre-cracked region of the beam was not included in Figure 4-16 due to the malfunctioning of the internal strain gauge at this location.

![Figure 4-16: Strain in longitudinal tensile reinforcement of beam U-30-H-1L](image)

The strain variation in the FRP wraps with applied load is presented in Figures 4-17 and 4-18. The strain in the middle wrap began to increase at a load of 10 kN. This was due to the existing shear crack along the pre-existing shear crack path re-opening and causing an increase in strain. At 150 kN, the pre-existing shear crack began to close and the strain in the middle wrap began to decrease. At a load of 201 kN, the new shear crack formed parallel to the pre-existing shear crack path, causing a sudden increase in strain in the left FRP wrap intersecting the crack. As the load increased, this new shear crack became the critical crack and the left wrap became the highest strained wrap. At a load of 298 kN the left wrap debonded from the concrete surface reaching a maximum strain of 3420 με. The strain in the FRP is anticipated to be higher than this value because the shear crack did not cross the wrap at the location of the strain gauge. The strain at the center of the middle wrap was not shown after a load of 190 kN due to the strain gauge malfunctioning at this location.

With reference to Figure 4-18, it is important to note that the highest strained part of the wrap was at its bottom, due to the critical shear crack forming parallel to the pre-existing shear crack path and crossing the bottom of the middle wrap and the center of the left wrap. Prior to the formation of the new shear crack at a load of 201 kN, the center of the middle wrap was the highest strained, with zero strain recorded at the top and bottom of the wrap.
4.3.5 Fully Wrapped Beam (F-30-NA-1L)

Beam F-30-NA-1L was strengthened with one layer of FRP, which was fully wrapped around the beam cross-section. The beam reached a maximum load of 346 kN, corresponding to a mid-span deflection of 43.2 mm, before failing in combined shear and flexural failure due to concrete crushing under the loading plate and rupture of the FRP wraps, as seen in Figure 4-19. Two shear cracks formed along the pre-cracked region of the beam. One of the shear cracks formed along the pre-existing shear crack path at an angle of inclination of approximately 29 degrees to the longitudinal axis of the beam. The other shear crack formed parallel to the pre-existing shear crack path, passing through the two wraps closest to mid-span at an inclination angle of approximately 30 degrees to the longitudinal axis of the beam. For this beam, the left wrap was the highest strained wrap and achieved a maximum strain of 11800 με.
Figure 4-19: Image of beam F-30-NA-1L at failure

Figure 4-20 and 4-21 present the load-deflection response of the beam and the strain variation in the longitudinal tensile reinforcement with applied load. The load-deflection response clearly shows that the beam failed in a ductile manner due to combined shear and flexural failure. The first signs of cracking were flexural cracks initiating at mid-span at a load of 70 kN. This can be seen in the load-deflection response and strain in the longitudinal tensile reinforcement, with a change in slope at mid-span at this load. The first shear crack occurred at a load of 121 kN, forming along the pre-existing shear crack path. As the load increased, cracking noises could be heard in the FRP and the shear crack width increased. Also, another shear crack formed parallel to the original shear crack, crossing the two wraps closest to mid-span. At a load of 299 kN the longitudinal tensile reinforcement began to yield, resulting in a rapidly increasing deflection with a slowly increasing load. This can be seen in the load-deflection response, with a change of slope at the load of 299 kN. The strain variation in the longitudinal tensile reinforcement with applied load shows that the bottom layer of the longitudinal tensile steel at mid-span yielded at a load of 286 kN, corresponding to a strain of 2900 με. Loud cracking noises continued during yielding and the beam eventually reached a maximum load of 346 kN. Thereafter, concrete crushing began to occur in the compression zone under the loading plate and the load began to drop gradually. Despite the dropping load, cracking noises could still be heard from the FRP due to the concrete weakening and the width of the shear cracks increasing. After the load decreased to a value of 206 kN, the two highest strained FRP wraps which were closest to mid-span, ruptured and resulted in beam failure. The strain in the longitudinal tensile reinforcement at the pre-cracked region of the beam was not included in Figure 4-21 due to the strain gauge malfunctioning at this location.
The strain in all FRP wraps was close to zero prior to the development of first shear crack at a load of 121 kN. Thereafter, the strains in the FRP increased with an increase in the crack width, as seen in Figure 4-22. It is important to note that even after the maximum load of 346 kN was reached and the load began to drop, the strain in the FRP wraps still increased due to the continued opening of the shear cracks as the concrete weakened. The left wrap was the highest strained wrap and achieved a maximum strain of 11800 με before its rupture.
Figure 4-22: Strain in FRP wraps surrounding the shear crack for beam F-30-NA-1L

The strain distribution in the middle FRP wrap is presented in Figure 4-23. One can see that initially, the center of the wrap was the highest strained section, followed by the bottom of the wrap with a lower strain, and the top of the wrap with zero strain. As the load began to decrease after reaching its maximum value, the strain increased rapidly near the top of the wrap and eventually the top and bottom areas of the wrap became the highest strained.

Figure 4-23: Strain distribution in middle FRP wrap for beam F-30-NA-1L
4.3.6 Internally Reinforced Control Beam (C1-30-NA-NA)

Beam C1-30-NA-NA was a control beam with full internal shear and flexural reinforcement. The objective of testing this beam was to determine for reference, the capacity of the 30° pre-cracked beams with full internal reinforcement and the effect of the shear crack inclination angle on the shear strength of a conventionally reinforced beam. The beam failed in flexure, due to concrete crushing in the compression zone under the loading plate, as seen in Figure 4-24. Although this beam was designed to have a shear capacity lower than its flexural capacity, it ultimately failed in flexure due to the conservative nature of the shear design equations and the incorrect assumption that the concrete contribution to shear strength would be reduced by the presence of the polycarbonate plate inserted into the beam which would cause a relatively smooth surface profile for the pre-existing shear crack. The beam reached a maximum load of 331 kN, corresponding to a mid-span deflection of 25 mm. The main shear crack formed along the pre-existing shear crack path at an inclination angle of approximately 36 degrees to the longitudinal axis of the beam.

![Image of beam C1-30-NA-NA at failure](image)

Figure 4-24: Image of beam C1-30-NA-NA at failure

The first cracks observed in the beam were flexural cracks at mid-span initiating at a load of 31 kN. This can be seen in the load-deflection response and strain in the longitudinal tensile reinforcement, presented in Figures 4-25 and 4-26 with a change in slope at the load of 31 kN. The first shear cracking occurred at a load of 122 kN with a shear crack initiating along the pre-existing shear crack path. At a load of 282 kN the longitudinal tensile reinforcement began to yield and the deflection began to increase rapidly with a slowly increasing load. As seen in Figure 4-26, the bottom layer of the longitudinal tensile reinforcement yielded at a load of 260 kN, corresponding to a strain of 2930 με at the mid-span of the beam. After reaching a maximum load of 331 kN, concrete crushing began to occur in the compression zone.
zone under the loading plate and the load began to drop rapidly. The strain in the longitudinal tensile reinforcement at the pre-cracked region of the beam is not included in Figure 4-26, due to the strain gauge malfunctioning at this location.

Figure 4-25: Load-deflection response of beam C1-30-NA-NA

Figure 4-26: Strain in longitudinal tensile reinforcement of beam C1-30-NA-NA
4.4 Results of 45 Degree Pre-cracked Beams

4.4.1 U-wrapped Beams without Anchors (U-45-NA-2L and U-45-NA-1L)

Beams U-45-NA-1L and U-45-NA-2L were strengthened with one and two layers of FRP U-wraps. The surface preparation prior to strengthening for these beams differed from all other beams in this experimental program. Prior to strengthening, the concrete surface was grind down to the aggregate level and was then roughened using a needle scalar, whereas for all other beams the concrete surface was not grinded. The reason for the change of surface preparation was due to suspicion of inadequate bonding using the regular surface preparation procedure in the beams tested earlier. Although not the focus of this experimental procedure, a major increase in bond strength was observed from grinding the concrete surface down to the aggregate level.

Beam U-45-NA-2L was strengthened using two layers of FRP U-wraps and failed in flexure due to crushing of the concrete in the compression zone under the loading plate, as seen in Figure 4-27a. The beam reached a maximum load of 331 kN, corresponding to a mid-span deflection of 41.7 mm. The middle FRP wrap was the highest strained wrap and achieved a maximum strain of 3060 µε. Several shear cracks formed along the pre-cracked region of the beam. The first shear crack formed at an inclination angle of approximately 51 degrees to the longitudinal axis of the beam, intersecting the bottom of the right wrap and the top of the middle wrap. The second shear crack formed parallel to the first, crossing the center of the middle wrap and the top of the left wrap at an inclination angle of approximately 41 degrees to the longitudinal axis of the beam. A third shear crack also formed parallel to the other two, intersecting the center of the left wrap and bottom of the middle wrap at an inclination angle of approximately 41 degrees to the longitudinal axis of the beam.

Beam U-45-NA-1L was strengthened using one layer of FRP U-wraps and resulted in debonding of the left FRP wrap, as seen in Figure 4-27b. The beam was not tested to failure, as testing was stopped after the left wrap debonded. This was done in order to retest the beam after retrofitting it with U-wraps with the use of horizontal strips. The beam reached a maximum load of 286 kN at a mid-span deflection of 7.8 mm. The left wrap was the highest strained and reached a maximum strain of 2310 µε, however the closest shear crack to the strain gauge on the left wrap was 70 mm, therefore, the maximum strain value at the crack section would have been larger. Two shear cracks formed in the pre-cracked region of the beam. The first shear crack formed along the pre-existing shear crack path at an inclination angle of approximately 45 degrees to the longitudinal axis of the beam. The other shear crack formed parallel to the manufactured shear crack path, crossing the bottom of the middle
wrap and the top of the left wrap at an inclination angle of approximately 43 degrees to the longitudinal axis of the beam.
For beam U-45-NA-1L, the first signs of cracking were flexural cracks at mid-span, initiating at a load of 22 kN. This can be seen by a change of slope at this load in the load-deflection response and strain variation in the longitudinal tensile reinforcement presented in Figures 4-28a and 4-29a. The first signs of shear cracking occurred at a load of 132 kN, with the formation of shear cracks along the pre-cracked region of the beam. As the load increased, the width of the shear cracks also increased, and cracking noises could be heard in the FRP. At a load of 285 kN, the longitudinal tensile reinforcement began to yield, represented by a flattening slope in the load-deflection curve of the beam. Looking at Figure 4-29a, it can be seen that the bottom layer of the longitudinal tensile reinforcement yielded at a load of 262 kN with an associated strain of 3090 με at mid-span. The beam reached a maximum load of 331 kN, corresponding to a mid-span deflection of 41.7 mm. Thereafter, concrete crushing occurred in the compression zone under the loading plate and the load began to drop. The strain in the longitudinal tensile reinforcement at the pre-cracked region of the beam was not included in Figure 4-29a due to the strain gauge malfunctioning at this location.

The load-deflection response and strain variation in the longitudinal tensile reinforcement with applied load for beam U-45-NA-1L is presented in Figures 4-28b and 4-29b. The first signs of cracking were flexural cracks at mid-span, initiating at a load of 39 kN, and can be seen in Figures 4-28b and 4-29b with the associated change in slope at this load. The first signs of shear cracking occurred at a load of 126 kN with the formation of shear cracks in the
pre-cracked region of the beam, along and parallel to the pre-existing shear crack path. As the load increased, cracking noises could be heard in the FRP and the shear cracks widened. After reaching a load of 286 kN, the left wrap, which was the highest strained, debonded and the load dropped to 271 kN. The test was stopped after this point, as the beam would be re-tested in another strengthening configuration. Looking at Figure 4-29b, one can see that the longitudinal tensile reinforcement at mid-span yielded at a load of 268 kN, corresponding to a strain of 2900 με. The strain in the longitudinal tensile reinforcement at the pre-cracked region of the beam was not included in Figure 4-29b due to the strain gauge malfunctioning at this location.

Figure 4-28: Load-deflection response of un-anchored U-wrapped beams (a) U-45-NA-2L (b) U-45-NA-1L
Figure 4-29: Strain in longitudinal tensile reinforcement of un-anchored U-wrapped beams (a) U-45-NA-2L (b) U-45-NA-1L

Figure 4-30 presents the strain variation in the FRP wraps with applied load for the un-anchored U-wrapped beams. One can see that for beam U-45-NA-2L the middle wrap and left wrap were highly strained as several shear cracks intersected both wraps. The middle wrap was the highest strained wrap and reached a maximum strain of 3060 $\mu$ε. For beam U-45-NA-1L, the highest strained wrap was the left wrap, as the main shear crack developed parallel to the pre-existing shear crack path, crossing the bottom of the middle wrap and in-between the top and center of the left wrap. The left wrap debonded at a load of 286 kN, associated with a strain of 2310 $\mu$ε, however, this strain would have been higher due to the shear crack not crossing the strain gauge placed on this wrap.
The strain distribution in the middle FRP wrap for beam U-45-NA-2L is provided in Figure 4-31a and shows that the highest strained region of the wrap was at its center, which is where the main shear crack intersected it. Figure 4-31b provides the strain distribution in the middle FRP wrap for beam U-45-NA-1L. The highest strained region of this wrap was at its bottom, as the main shear crack developed parallel to the pre-existing shear crack path, crossing the bottom of the middle wrap and between the top middle of the left wrap.

Figure 4-30: Strain in FRP wraps surrounding the shear crack for un-anchored U-wrapped beams (a) U-45-NA-2L (b) U-45-NA-1L
4.4.2 U-wrapped Beams with Anchors (U-45-A1-1L and U-45-A2-1L)

Beams U-45-A1-1L and U-45-A2-1L were strengthened using one layer of FRP U-wraps with the use of anchors. Beam U-45-A1-1L was strengthened using the first anchor configuration and failed in flexure due to concrete crushing in the compression zone under the loading plate at a load of 353 kN and a corresponding mid-span deflection of 54.7 mm, as seen in Figure 4-32a. Two shear cracks formed in the pre-cracked region of the beam. A steep shear crack crossed the bottom of the right wrap and top of the middle wrap at an angle of inclination of approximately 60 degrees to the longitudinal axis of the beam. Another
shear crack intersected the middle wrap in-between its bottom and center and the center of the left wrap at an inclination angle of approximately 40 degrees to the longitudinal axis of the beam. The left wrap was the highest strained wrap and achieved a maximum strain of 6030 με before flexural failure.

Beam U-45-A2-1L was strengthened using the second anchor configuration and also failed in flexural failure due to concrete crushing in the compression zone under the loading plate, as seen in Figure 4-32b. The beam reached an ultimate load of 338 kN, corresponding to a mid-span deflection of 42.9 mm. Two shear cracks formed in the pre-cracked region of the beam. A shear crack crossed the bottom of the right wrap and between the top and center of the middle wrap at an inclination angle of approximately 42 degrees to the longitudinal axis of the beam. Another shear crack intersected the middle wrap in-between its bottom and center and the left wrap in-between its top and center at an inclination angle of approximately 43 degrees to the longitudinal axis of the beam. The left wrap was the highest strained wrap and achieved a maximum recorded strain of 4583 με.
Figure 4-33 presents the load-deflection response of beams U-45-A1-1L and U-45-A2-1L. One can see that both beams exhibit a ductile flexural failure. The first signs of cracking in both beams were flexural cracks at mid-span. The first flexural cracks for beam U-45-A1-1L initiated at a load of 29 kN and for beam U-45-A2-1L, at 23 kN. This can be seen in Figure 4-33 with a change in the slope of the load-deflection response at mid-span of both beams at those loads. The first signs of shear cracking for beam U-45-A1-1L occurred at a load of 129 kN and for beam U-45-A2-1L, at 130 kN, with the formation of shear cracks along the pre-cracked region of the beams. As the load increased, the shear cracks widened and cracking noises could be heard in the FRP. For beam U-45-A1-1L, its longitudinal tensile reinforcement began to yield at a load of 310 kN and the slope of the load-deflection curve began to flatten. For beam U-45-A2-1L this yield point began at a load of 281 kN. Beam U-45-A1-1L reached a maximum load of 353 kN, corresponding to a mid-span deflection of 54.7 mm and beam U-45-A2-1L reached a maximum load of 338 kN, with an associated mid-span deflection of 42.9 mm. In both beams, after this maximum load was reached, concrete crushing began to occur in the compression zone under the loading plate and the load began to fall.
For beam U-45-A1-1L the bottom layer of the longitudinal tensile reinforcement at mid-span yielded at a load of 280 kN, corresponding to a strain of 3000 με, as seen in Figure 4-34a. For beam U-45-A2-1L the bottom layer of the longitudinal tensile reinforcement at mid-span yielded at a load of 260 kN, associated with a strain of 2935 με, as seen in Figure 4-34b. For both beams, the maximum strain reached in the longitudinal reinforcement at mid-span was between 9000 to 10000 με, and was significantly larger than the strain in the reinforcement at the pre-cracked region of the beam, which did not experience yielding.
Figure 4-34: Strain in longitudinal tensile reinforcement of anchored U-wrapped beams
(a) U-45-A1-1L (b) U-45-A2-1L

Figures 4-35 and 4-36 present the strain variation in the FRP wraps with applied load. For both beams, the left wrap was the highest strained wrap achieving a maximum strain of 6030 \( \mu \varepsilon \) for beam U-45-A1-1L and 4583 \( \mu \varepsilon \) for beam U-45-A2-1L. Since the shear crack for beam U-45-A2-1L did not cross the left wrap at the strain gauge location, the actual strain in this wrap is expected to be higher than recorded. It is important to note that for both beams, the left wrap was the first wrap to be strained, followed by the middle wrap at a much higher load due to the natural progression of the shear crack development.
In regards to Figure 4-36, the strain distribution in the middle FRP wrap for both beams showed that the highest strain was recorded at the center of the wrap, followed by the bottom of the wrap with a lower strain. The strain in the top of the wrap, above the anchor head plate was close to zero throughout loading.

Figure 4-35: Strain in FRP wraps surrounding the shear crack for anchored U-wrapped beams (a) U-45-A1-1L (b) U-45-A2-1L
4.4.3 *U-wrapped Beam with Horizontal Strips (U-45-H-1L)*

Beam U-45-H-1L was tested after re-strengthening the U-45-NA-1L beam which failed due to debonding of its left U-wrap. The beam was re-strengthened by removing the debonded left U-wrap, grinding any remaining epoxy from the surface, roughening the surface using a needle scaler, and applying the new strengthening configuration. Beam U-45-H-1L was strengthened using one layer of U-wraps with horizontal strips near the top of the wraps. It is...
important to note that only the left wrap was re-strengthened to produce beam U-45-H-1L, as this was the only wrap that debonded during the testing of beam U-45-NA-1L.

Beam U-45-H-1L failed in flexure due to the crushing of the concrete in the compression zone under the loading plate, as seen in Figure 4-37. The beam reached a maximum load of 339 kN, corresponding to a mid-span deflection of 46.5 mm. The major shear cracks from the previously tested U-45-NA-1L beam further developed during the testing of this beam. It is important to note that the horizontal strip was effective in eliminating debonding of the left wrap, as it was the highest strained wrap and reached a maximum recorded strain of 3300 micro-strain. Also, the actual maximum strain in the left FRP wrap would have been higher than the recorded value, as the closest shear crack to the strain gauge on the left wrap was 70 mm away.

Figure 4-37: Image of beam U-45-H-1L at failure

Figure 4-38 presents the load-deflection response for beam U-45-H-1L, which exhibits a ductile flexural failure mode. The cracking pattern remained the same as the previously tested U-45-NA-1L beam. At a load of 10 kN, the already existing cracks began to open up gradually with increasing load. The tensile reinforcement began to yield at a load 290 kN, as the load-deflection curve for mid-span began to flatten at this load. The bottom layer of the longitudinal tensile reinforcement at mid-span yielded at a load of 268 kN, corresponding to a strain of 2730 με, as seen in Figure 4-39. The strain in the longitudinal tensile reinforcement at mid-span in Figure 4-39 is not shown after yielding, as the strain readings became erratic. After reaching a maximum load of 339 kN, concrete crushing began to occur in the compression zone under the loading plate and the load began to drop rapidly. Although not the focus of this experimental test, the middle wrap which was not strengthened with a horizontal strip, debonded before failure of the beam.
The left FRP wrap was the highest strained wrap and reached a maximum strain of 3300 \( \mu \varepsilon \) before the beam failed in flexure, as seen in Figure 4-40. It is important to note that the debonding of the left wrap, similar to the previously tested U-45-NA-1L beam, was avoided with the use of the horizontal strip. The full strain response of the middle wrap was not shown, as its strain readings became erratic after a certain point. For the opposite side of the middle wrap, the strain increased at a load greater than that which initiated the strain increase in the left wrap. Since the left wrap did not debond similar to the previous U-45-NA-1L test, the strain in the middle wrap increased as the failure path began to change, and this ultimately resulted in debonding of the middle wrap before beam failure. The strain distribution in the middle FRP wrap is not shown for this beam, as this wrap was not strengthened with a horizontal strip and was not the focus of the experimental test.
4.4.4 Fully Wrapped Beam (F-45-NA-1L)

Beam F-30-NA-1L was strengthened with one layer of FRP, which was completely wrapped around the beam cross-section. The beam failed due to shear failure near the support, which may have been caused by movement of the steel cage during casting of the concrete, resulting in the longitudinal reinforcement failing to extend past the support. An image of the beam at failure can be seen in Figure 4-41a. Before failure, the beam reached a maximum load of 290 kN. Since the full potential of the FRP wraps was not seen, due to the premature failure of the beam, it was re-tested by repositioning it so that the failed portion of the beam extended past the support. This resulted in the support being positioned in-between the right and middle wrap, rendering the right wrap ineffective at resisting the load. The shear span for the side of the beam with FRP strengthening was kept the same as before, which resulted in the side of the beam without FRP having a shorter shear span. Since the two shear span lengths were not the same for the beam, a difference in shear load would be experienced on the two sides of the beam.

After re-testing the beam, it reached a maximum load of 361 kN, corresponding to a mid-span deflection of 43.2 mm before failing in flexure due to concrete crushing in the compression zone under the loading plate, as seen in Figure 4-41b. Due to the two shear spans being different lengths under the new loading, the maximum shear load acting on the retrofitted side was 169 kN and it would correspond to a maximum load of 338 kN if the load had been applied symmetrically as in the case of the other beams. The main shear crack
formed parallel to the pre-existing shear crack path, intersecting the bottom of the middle wrap and center of the left wrap at an inclination angle of approximately 35 degrees to the longitudinal axis of the beam. The left wrap was the highest strained wrap for both tests. It reached a maximum strain of 3580 με before failure for the first test. However, when re-testing the beam, the maximum strain in the left wrap was only 2870 με, as the failure path changed when repositioning the beam for re-testing. This resulted in another shear crack forming closer to mid-span, which did not cross the FRP wraps.

Figure 4-41: Images of beam F-45-NA-1L at failure (a) original test (b) re-test
Figures 4-42 and 4-43 present the load-deflection response and strain in the longitudinal tensile reinforcement for the two tests performed on beam F-45-NA-1L. One can see that the load-deflection curves for the beam showed an essentially bi-linear response in the original test and a characteristic non-linear response associated with ductile flexural failure in the re-test. For the original test, the first cracks observed were flexural cracks at mid-span initiating at a load of 33 kN. This can be seen in Figure 4-42a and 4-43 with a change in the slope in the load-deflection curve and longitudinal tensile strain at this load. The first shear cracking occurred at a load of 154 kN with the formation of a major shear crack, parallel to the pre-existing shear crack path. At a load of 290 kN, the beam suddenly failed in shear at the support. Looking at Figure 4-43, the longitudinal tensile steel at mid-span yielded at a load of 205 kN, corresponding to a strain of 2860 micro-strain, however, these values might have been due to malfunctioning of the strain gauge at this location.

When re-testing the beam, the cracks from the previous test began to open at a load of 12 kN. Based on Figure 4-42b, during re-testing the longitudinal tensile reinforcement began to yield at a load of 323 kN as the load-deflection curve at mid-span began to flatten. After reaching a maximum load of 361 kN, concrete crushing began to occur in the compression zone under the loading plate and the load began to drop rapidly. The strain variation in the longitudinal tensile reinforcement with applied load was not shown for the re-test due to both internal strain gauges malfunctioning after the original test.
Figure 4-42: Load-deflection response of beam F-45-NA-1L (a) original test (b) re-test

Figure 4-43: Strain in the longitudinal tensile reinforcement of beam F-45-NA-1L for the original test

The strain variation in the FRP wraps with applied load is presented in Figures 4-44 and 4-45. One can see that the left wrap was the highest strained wrap during both tests. The strains recorded in the original test are much greater in all FRP wraps than for the re-test, as the failure path changed during re-testing due to the re-positioning of the beam. The strain response of the right wrap is not included in Figure 4-44b due to the new placement of the beam rendering the right wrap ineffective in resisting the shear.

With reference to Figure 4-45, it is evident that in both tests the bottom of the middle wrap was the highest strained. This was due to the main shear crack forming parallel to the pre-existing shear crack path and intersecting the bottom of the middle wrap and the center of the left wrap.
Figure 4-44: Strain in the FRP wraps surrounding the shear crack for beam F-45-NA-1L
(a) original test (b) re-test
4.4.5 Internally Reinforced Control Beam (C1-45-NA-NA)

Beam C1-45-NA-NA was a control beam with full internal shear and flexural reinforcement. The objective of testing this beam was to determine for reference, the capacity of the 45 degrees pre-cracked beams with full internal reinforcement and the effect of the shear crack inclination angle on the shear strength of a conventionally reinforced beam. The beam failed in flexural failure due to concrete crushing in the compression zone under the loading plate, as seen in Figure 4-46. Although the beam was designed to have a shear capacity lower than its flexural capacity, the beam ultimately failed in flexure due to the conservative nature of the shear design equations and the apparently incorrect assumption that the concrete shear resistance along the pre-existing, smooth shear crack would be less than along the rough cracks normally formed in diagonally cracked beams. The beam reached a maximum load of 334 kN, corresponding to a mid-span deflection of 36.4 mm. The main shear crack formed along the pre-existing shear crack path at an inclination angle of approximately 39 degrees to the longitudinal axis of the beam.
Figure 4-47 presents the load-deflection response of the beam, which clearly shows that the beam failed in a flexural ductile mode. The first cracks observed were flexural cracks at mid-span initiating at a load of 32 kN. This can be seen in Figure 4-47 with a change in the slope of the load-deflection curve at this load. The first shear cracking occurred at a load of 125 kN with a shear crack initiating along the pre-existing shear crack path. At a load of 284 kN the longitudinal tensile reinforcement began to yield as the deflection began to increase rapidly while the load increased slowly. After reaching a maximum load of 334 kN, concrete crushing began to occur in the compression zone under the loading plate and the load began to drop rapidly.

**Figure 4-46: Image of beam C1-45-NA-NA at failure**

**Figure 4-47: Load-deflection response of beam C1-45-NA-NA**
4.5 Summary

The results presented in this chapter showed that the external FRP U-wraps with anchors and the full wraps both allowed the test beams to reach their full capacity. On the other hand, the U-wraps without anchors, even when doubled in area were not able to allow the beam to reach its full capacity, due to pre-mature debonding of the wraps.

In the following chapter, the expected strength of the beams based on some existing design standards will be examined and the role of the anchors in allowing the beams to reach their design capacity will be investigated.
Chapter 5: Analysis of Experimental Results

5.1 General:

In this chapter the results of the fourteen tests in this research program will be analyzed. The analysis will be carried out using prevailing semi-analytical relationships developed by some researchers or recommended by national standards. Most of the standards do not deal with anchored FRP wraps, so the computed strengths are for similar wraps without anchors. The anchor configurations used in the current study have not been previously used, so there is no available method that can predict their performance.

The following sections will discuss the failure mechanisms of the beams, their load-deflection responses and failure loads, the ultimate strains and strain distribution in their FRP wraps, and the influence of the shear crack inclination angle, the use of anchors and horizontal strips, and the influence of the concrete surface preparation on the effectiveness of the FRP strengthening.

5.2 Observed Failure Mechanisms of Beams

Several failure mechanisms were observed during this experimental program. Seven beams failed in shear, while the rest of the beams failed in flexure. Certain laminate configurations eliminated shear failure and resulted in flexural failure. The failure mechanisms observed during testing were diagonal tension shear failure, FRP debonding, slip between the FRP U-wrap and anchor head plate, breakage of the anchor head plate, FRP rupture, buckling of the longitudinal compression reinforcement, and concrete crushing in the compression zone under the loading. In beams experiencing flexural failures, the failure mode was ductile, while in those experiencing shear failures, it was brittle. Table 5-1 presents a summary of the failure mechanisms for the tested beams. Figure 5-1 shows typical images of the different failure mechanisms observed during this investigation.
Table 5-1: Summary of Failure Mechanisms Observed

<table>
<thead>
<tr>
<th>Beams</th>
<th>Failure Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-30-NA-NA</td>
<td>Diagonal tension shear failure</td>
</tr>
<tr>
<td>U-30-NA-2L</td>
<td>FRP debonding</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>Slip between FRP U-Wrap and anchor head plate</td>
</tr>
<tr>
<td>U-30-A2-1L</td>
<td>Slip between FRP U-wrap and anchor head plate, breakage of anchor head plate, and buckling of longitudinal compression reinforcement</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>FRP debonding</td>
</tr>
<tr>
<td>F-30-NA-1L</td>
<td>FRP rupture and concrete crushing in the compression zone under the loading point</td>
</tr>
<tr>
<td>C1-30-NA-NA</td>
<td>Concrete crushing in the compression zone under the loading point</td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>Concrete crushing in the compression zone under the loading point</td>
</tr>
<tr>
<td>U-45-NA-1L</td>
<td>FRP debonding</td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>Concrete crushing in the compression zone under the loading point</td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>Concrete crushing in the compression zone under the loading point</td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>Concrete crushing in the compression zone under the loading point</td>
</tr>
<tr>
<td>F-45-NA-1L</td>
<td>Concrete crushing in the compression zone under the loading point</td>
</tr>
<tr>
<td>C1-45-NA-NA</td>
<td>Concrete crushing in the compression zone under the loading point</td>
</tr>
</tbody>
</table>
Based on the failure mechanisms listed in Table 5-1, several conclusions can be drawn. Firstly, one can see that the majority of the 45 degree pre-cracked beams failed due to flexure.
and exhibited a higher strength compared to their companion 30 degree beams. Secondly, buckling of the longitudinal compression reinforcement only occurred in the 30 degree pre-cracked beams after the maximum load was reached. This can be attributed to these beams having a longer pre-cracked region which was closer to mid-span than the 45 degree pre-cracked beams. The longer crack reduced the effective concrete area resisting compression and led to crushing of the concrete and buckling of the compression bars because they were not surrounded by closed stirrups, which tend to prevent such buckling. The difference in the strengths of the two sets of beams will be discussed later.

It is important to note that failure due to FRP debonding began from the top of the U-wrap and proceeded down the depth of the beam. At failure, the U-wrap did not completely debond from the sides of the beams and a bonded region along the lower depth of the beams still existed. The failure plane for all debonding failures occurred at the concrete-adhesive interface, a few millimeters into the concrete.

For strengthened beams with anchors, slip between the U-wrap and anchor head plate and breakage of the anchor head plate was observed. For strengthened beams with horizontal strips, debonding of the horizontal strip and U-wrap was observed, however the debonding process was delayed when compared to the un-anchored U-wrapped beam.

Since brittle shear failure is an undesirable failure mode, it is reassuring to know that this mode of failure can be avoided in shear retrofitting beams with anchors. It is also worth noticing that the achievement of ductile failure does not require both ductile flexural and shear reinforcement. As long as the flexural reinforcement is ductile and the beams shear capacity exceeds its flexural capacity, ductility can be maintained despite the brittle nature of FRP.

### 5.3 Load-deflection Curves

The load-deflection curves for the beams tested in this experimental program differed based on their strengthening configuration and pre-crack inclination angle. A practically bi-linear load-deflection response was exhibited by the beams that failed in shear, whereas the beams that failed in flexure, or a combination of shear and flexure, showed a non-linear response. The load- mid-span deflection curves of the tested beams are shown in Figure 5-2. The 30 degree pre-cracked beams that failed in shear are presented in Figure 5-2a, and the remaining 30 degree pre-cracked beams that failed in flexure are presented in Figure 5-2b. The 45
degree pre-cracked beams that failed in shear are presented in Figure 5-2c, and the remaining 45 degree pre-cracked beams that failed in flexure are presented in Figure 5-2d.
Several observations can be made regarding the load-deflection curves presented in Figure 5-2. Firstly, the stiffness for all beams changed during the onset of cracking in the concrete and at yielding of the longitudinal tensile reinforcement. Secondly, the majority of the FRP retrofitted beams that failed in flexure displayed larger mid-span deflections over the un-retrofitted beams with full internal shear reinforcement. This suggests that the shear retrofit does not adversely affect the ductility of flexural beam members or their mode of failure, as long as the shear strength of the beam is higher than its flexural strength. Thirdly, it is important to notice that full wraps are highly effective because as Figure 5.2b shows, the 30
degree pre-cracked beam with one layer of full wraps achieved the highest strength and mid-span deflection for the 30 degree pre-cracked beams. On the contrary, the 30 degree pre-cracked beam with two layers of un-anchored U-wraps was the least effective retrofit and failed in shear at a relatively small load and mid-span deflection.

5.4 Ultimate Load of Beams

The ultimate loads, the shear resistance, and the observed shear crack inclination angle of the beams tested in this experimental program are summarized in Table 5-2.

Table 5-2: Ultimate Load of Test Beams

<table>
<thead>
<tr>
<th>Beam</th>
<th>Pre-crack Angle (Degrees)</th>
<th>Ultimate Load (kN)</th>
<th>Shear Resistance (kN)</th>
<th>Observed Shear Crack Inclination (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-30-NA-NA</td>
<td>30</td>
<td>155</td>
<td>77.5</td>
<td>32</td>
</tr>
<tr>
<td>U-30-NA-2L</td>
<td>30</td>
<td>197</td>
<td>98.5</td>
<td>33</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>30</td>
<td>290</td>
<td>145</td>
<td>33</td>
</tr>
<tr>
<td>U-30-A2-1L</td>
<td>30</td>
<td>342</td>
<td>171</td>
<td>31</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>30</td>
<td>298</td>
<td>149</td>
<td>31</td>
</tr>
<tr>
<td>F-30-NA-1L</td>
<td>30</td>
<td>346</td>
<td>173</td>
<td>29</td>
</tr>
<tr>
<td>C1-30-NA-NA</td>
<td>30</td>
<td>331</td>
<td>165.5</td>
<td>36</td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>45</td>
<td>331</td>
<td>165.5</td>
<td>41</td>
</tr>
<tr>
<td>U-45-NA-1L</td>
<td>45</td>
<td>286</td>
<td>143</td>
<td>43</td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>45</td>
<td>353</td>
<td>176.5</td>
<td>40</td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>45</td>
<td>338</td>
<td>169</td>
<td>43</td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>45</td>
<td>338</td>
<td>169</td>
<td>43</td>
</tr>
<tr>
<td>F-45-NA-1L</td>
<td>45</td>
<td>338</td>
<td>169</td>
<td>35</td>
</tr>
<tr>
<td>C1-45-NA-NA</td>
<td>45</td>
<td>334</td>
<td>167</td>
<td>39</td>
</tr>
</tbody>
</table>

Since the beams were symmetric and symmetrically loaded (except beam F-45-NA-1L), the maximum shear resisted by each beam was equal to half of its ultimate load. The ultimate load for beam F-45-NA-1L, in Table 5-2 is the converted value, equivalent to the ultimate load if the beam was loaded symmetrically. It is important to note that all shear crack inclination angles reported in the last column of Table 5-2 were measured at the mid-depth of each beam. In general, the measured shear crack inclination angles are reasonably close to
the pre-crack inclination, however the beams with internal steel stirrups tend to deviate more from the expected value.

In order to further assess the observed ultimate strength of the current test beams, their expected theoretical shear resistance will be calculated using available methods in design standards and other reliable references. The FRP shear resistance will be calculated using the CSA S806-12 (2012) standard, the ACI 440.2R-08 (2008) guidelines, the CNR-DT 200/2004 (2004) guidelines, the Mofidi and Chaallal (2011) model, the Chen and Teng (2003a,b) model, and the Chen et al. (2013) model. The concrete shear resistance will be calculated using the simplified method provided in the CSA A23.3-04 (2004) standard and will be used in conjunction with the CSA S806-12 (2012), Chen and Teng (2003a,b), Mofidi and Chaallal (2011) and Chen et al. (2013) Models. The ACI-318-05 (2005) simplified concrete shear resistance will be used in conjunction with the ACI 440.2R-08 (2008) guidelines and the Eurocode-2 (2004) concrete shear resistance will be used together with the CNR-DT 200/2004 (2004) guidelines. For a detailed overview of the theoretical shear resistance of strengthened beams, one can refer to Section 2.7 in Chapter 2 of this thesis.

All FRP strain values provided in the shear resistance models will be used in the calculations. For example, for the CSA S806-12 (2012) design standard the FRP strain is limited to 5000 με for U-wraps with a proven anchorage system and 6000 με for full wraps. For the ACI-440.2R-08 (2008) guidelines, the FRP strain is limited to 4000 με for full wraps. U-Wraps with a proven anchorage system are not considered in the ACI guidelines, therefore a strain of 4000 με will also be used for the beams strengthened with anchors and horizontal strips. The CNR-DT 200/2004 (2004) guidelines and the Chen and Teng (2003a, b) model provide design equations to calculate the effective strain for U-wraps and full wraps, however the use of mechanical anchors is not considered. The FRP shear resistance models by Mofidi and Chaallal (2011) and Chen et al. (2013) only consider the case of failure due to debonding of un-anchored U-wraps.

The measured shear crack inclination angles presented in Table 5-2 will be used in the calculation of the FRP shear resistance. It is important to note that the FRP shear resistance is independent of the shear crack inclination angle for the ACI 440.2R-08 (2008) guidelines, as an angle of 45 degrees is assumed for all beams. In all other methods, the FRP shear resistance is dependent on the shear crack inclination angle.

Due to changes in the concrete compressive strength and the incorrect assumption that the concrete shear resistance would be significantly decreased by the pre-existing shear crack, the computed shear resistance predictions are much higher than those computed during initial design. Due to the difference in concrete compressive strength during the testing period, the average of the measured concrete compressive strength at 28 days and at the end of the
testing period was used for calculations. This average strength was 48.5 MPa. Table 5-3 presents a comparison of the experimental and theoretical shear resistance values by different methods. The shear resistance calculated using the Chen et al. (2013) model is denoted as $V_{r, Chen1}$ and the Chen and Teng (2003a,b) model is denoted as $V_{r, Chen2}$.

Table 5-3: Comparison of Theoretical and Experimental Shear Resistance

<table>
<thead>
<tr>
<th>Beam</th>
<th>$V_{r,exp}$ (kN)</th>
<th>$V_{r, exp}^{CSA}$</th>
<th>$V_{r, exp}^{ACI}$</th>
<th>$V_{r, exp}^{CNR}$</th>
<th>$V_{r, exp}^{Mofidi}$</th>
<th>$V_{r, exp}^{Chen1}$</th>
<th>$V_{r, exp}^{Chen2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-30-NA-2L</td>
<td>98.5</td>
<td>1.35</td>
<td>1.13</td>
<td>1.34</td>
<td>1.29</td>
<td>1.32</td>
<td>1.35</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>145</td>
<td>1.11</td>
<td>0.80</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>U-30-A2-1L</td>
<td>171</td>
<td>0.99</td>
<td>0.68</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>149</td>
<td>1.13</td>
<td>0.78</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>F-30-NA-1L</td>
<td>173</td>
<td>1.15</td>
<td>0.67</td>
<td>0.82</td>
<td>N/A</td>
<td>N/A</td>
<td>0.96</td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>165.5</td>
<td>0.69</td>
<td>0.67</td>
<td>0.72</td>
<td>0.67</td>
<td>0.68</td>
<td>0.70</td>
</tr>
<tr>
<td>U-45-NA-1L</td>
<td>143</td>
<td>0.68</td>
<td>0.68</td>
<td>0.75</td>
<td>0.75</td>
<td>0.70</td>
<td>0.71</td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>176.5</td>
<td>0.78</td>
<td>0.66</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>169</td>
<td>0.77</td>
<td>0.69</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>169</td>
<td>0.77</td>
<td>0.69</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>F-45-NA-1L</td>
<td>169</td>
<td>1.02</td>
<td>0.69</td>
<td>0.77</td>
<td>N/A</td>
<td>N/A</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Based on Table 5-3, several observations can be made regarding the shear strength values. With reference to the 30 degree pre-cracked beams, one can see that the CSA S806-12 (2012) standard provided the closest predictions to the experimental results, with the exception of the un-anchored U-wrapped beam. Despite having the closest predictions, the standard overestimated the shear strength of most of the 30 degree pre-cracked beams. Values of FRP strain higher than the limits imposed by the standard were also recorded in the beams. The ACI 440.2R-08 (2008) guidelines provided conservative predictions for all beams except the un-anchored U-wrapped beam. This is due to the guide’s conservative assumption of a 45 degree shear crack inclination angle and the limiting of the effective FRP strain to 0.004 for fully wrapped beams, which was also used for anchored beams in the calculations. The CNR-DT 200/2004 (2004) guidelines only provided FRP shear predictions for the U-wrapped and fully wrapped beams. It overestimated the shear resistance of the un-anchored U-wrapped beam, but provided good predictions for the shear resistance of the fully wrapped beam. The models of Mofidi and Chaallal (2011), Chen et al. (2013), and Chen and Teng (2003a) also overestimated the shear resistance of the un-anchored U-wrapped beam.
A possible reason for the overestimated shear resistance is that all models, with the exception of Chen et al. (2013) and Chen and Teng (2003a,b) assume that all FRP wraps crossing the shear crack are equally effective at resisting the load and achieve the same ultimate strain at the beam failure, which was not observed for beams with a single shear crack. For these beams, a critical wrap provided the majority of the shear resistance. Lu et al. (2009) conducted a study on the strain distribution in the FRP wraps crossing a shear crack. A finite element analysis was used to model four different shear crack shapes and the FRP wraps crossing them. The FRP wraps crossing the shear cracks were shown to experience large differences in strain depending on their location along the crack and the shape of the shear crack. These differences in strain experienced by the FRP U-wraps crossing the shear crack could result in a lower FRP contribution to the total shear resistance than expected. The models of Chen et al (2013) and Chen and Teng (2003a,b) account for this phenomenon by introducing a strain distribution factor to reduce the effective strain in the FRP, however, due to a higher predicted debonding strain, the shear resistance is still overestimated.

Another reason for the overestimated shear resistance could be due to an incorrect concrete contribution to the shear resistance. The majority of the concrete shear resistance in a cracked section can be attributed to aggregate interlock due to the rough concrete surface along the shear crack, however, the shear contribution from aggregate interlock directly depends on the width of the shear crack. The larger the width of the shear crack, the less the shear that can be resisted by aggregate interlock. The concrete shear strength given by the CSA A23.3-04 (2004) standard assumes a shear crack width associated with internal steel reinforcement, however when FRP wraps are used instead of internal steel stirrups, the width of the shear crack can be much larger than with internal steel, and this should be reflected in the concrete shear resistance. This will be discussed in more detail in the following section.

All shear resistance predictions for the 45 degree pre-cracked beams were conservative. This is due to the assumption that around 1.4 wraps were active at resisting the load with angles close to 45 degrees, however due to the multiple shear crack pattern observed for all 45 degree beams, the wraps were further activated by the additional shear cracks and therefore two wraps or close to two wraps were fully activated. Higher levels of strain experienced in the FRP than the strain values assumed in the above methods also resulted in the conservative predictions. The higher levels of strain achieved in the un-anchored U-wrapped beams might have been partially due to the superior surface preparation applied to these beams, resulting in an increased bond strength. Due to the fact that in several cases the use of a shear crack inclination lower than 45 degrees resulted in un-conservative estimation of the shear strength of these beams, it is recommended that in all cases a 45 degree shear crack inclination angle be used for design.
5.5 Alternative Methods for Predicting the Shear Resistance of Beams

Based on Section 5.4, several of the models and guidelines that were used overestimated the shear resistance of the 30 degree pre-cracked beams and were conservative in their predictions for the 45 degree pre-cracked beams. Several alternatives will be explored in this section in order to determine the reason for the inaccurate predictions. This section will focus on the predictions based on the CSA S806-12 (2012) and CSA A23.3-04 (2004) standards. The following alternatives that will be examined are a modified concrete shear resistance for beams without shear reinforcement, given by the CSA A23.3-04 (2004) standard, a concrete shear resistance based on the modified compression field theory, shear resistance predictions using the CSA S806-12 (2008) with a 45 degree shear crack inclination angle for design, and a way to implement anchors for models that do not consider them.

5.5.1 Concrete Shear Resistance for Beams without Shear Reinforcement

For a detailed overview of the concrete shear resistance calculations given by the CSA A23.3-04 (2004) standard, one can refer to Section 2.7.2 in Chapter 2 of this thesis. The CSA standard provides an alternative method for calculating the concrete shear contribution factor, \( \beta \), for concrete sections with no transverse reinforcement. This method was not used previously as it was assumed that the FRP shear strengthening provided the minimum transverse reinforcement required, however it is unclear as to whether this applies to FRP. For sections with no transverse reinforcement, \( \beta \) can be calculated by:

\[
\beta = \frac{230}{1000 + \frac{355Z}{15+a_g}}
\] (5.1)

The value of \( \beta \) calculated using this method was 0.168, whereas the previous method assumed value of \( \beta \) of 0.18. This new value of \( \beta \) results in a 7% decrease in the concrete shear resistance, however this makes a small change to the total shear resistance of the beams, and results in the shear resistance still being over-estimated.
5.5.2 Concrete Shear Resistance Using the Modified Compression Field Theory

The majority of the concrete shear resistance in a cracked section can be attributed to aggregate interlock due to the rough concrete surface along the shear crack, which directly depends on the width of the shear crack. Figure 5-3 shows a schematic of the local forces at the shear cracked section of a beam.

![Internal forces at shear cracked section of a beam](image)

Based on mechanics, the nominal or average shear stress in a beam can be computed by:

\[ \tau = \frac{V}{b_w d_v} \]  (5.2)

where \( V \) is the shear force acting on the section, \( b_w \) is the width of the section, and \( d_v \) is its shear depth.

If the shear crack inclination angle is known, as in the case of the current test beams, the principal compressive stress in the concrete can be found based on equilibrium using the following:

\[ f_2 = \frac{V}{b_w d_v} (\tan \theta + \frac{1}{\tan \theta}) \]  (5.3)

where \( \theta \) is the shear crack inclination angle.

The longitudinal strain in the beam at mid-height can be determined using the following equation given in the CSA A23.3-04 standard, which considers the longitudinal strain caused by the shear and moment at a section:
where $M_f$ is the moment at a section of the beam, $V_f$ is the shear at a section of the beam, $E_s$ is the modulus of elasticity of steel, and $A_s$ is the area of the longitudinal tensile steel reinforcement.

Based on compatibility, and knowing $\varepsilon_x$ and $\theta$ one can determine the principal tensile strain, $\varepsilon_1$ in terms of $\varepsilon_2$ using the following:

$$\varepsilon_1 = \frac{\varepsilon_x (1 + \tan^2 \theta) - \varepsilon_2}{\tan^2 \theta}$$ (5.5)

Using the stress-strain relationship of diagonally cracked concrete as given by Vecchio and Collins (1982), one can determine the maximum principal compressive stress in the concrete, $f_{2,\text{max}}$ in terms of $\varepsilon_2$ by substituting $\varepsilon_1$ from Eq 5.5 into the following:

$$f_{2,\text{max}} = \frac{f'_{c}}{0.8 + 170 \varepsilon_1}$$ (5.6)

One can determine the principal compressive strain in the concrete, $\varepsilon_2$, by solving the following quadratic equation, and substituting $f_{2,\text{max}}$ in terms of $\varepsilon_2$ from Eq 5.5 and 5.6.

$$0 = f_{2,\text{max}} \left[ 2 \left( \frac{\varepsilon_2}{e'_{c}} \right) - \left( \frac{\varepsilon_2}{e'_{c}} \right)^2 \right] - f_2$$ (5.7)

The value of $\varepsilon_1$ can then be solved by substituting $\varepsilon_2$ back into Eq. 5.5. Knowing $\varepsilon_1$, the width of the shear cracks can be determined using:

$$W = \varepsilon_1 S_{m\theta}$$ (5.8)

where $S_{m\theta}$ is the spacing of the inclined cracks, which can be calculated using:

$$S_{m\theta} = \frac{1}{\left( \frac{\sin\theta}{s_{mx}} + \frac{\cos\theta}{s_{mv}} \right)}$$ (5.9)

where $s_{mx}$ and $s_{mv}$ are the crack spacings in the longitudinal and transverse directions. Vecchio and Collins (1986) state that $s_{mx}$ can be taken as 1.5 times the maximum spacing of
the stirrups, and $S_{mv}$ can be taken as 1.5 times the spacing of the distributed longitudinal reinforcement.

Once the width of the shear crack is calculated, the concrete shear resistance along the shear crack can be calculated. Collins and Mitchell (1991) present an equation for the concrete shear resistance along the shear crack, $V_{ci}$, in terms of the width of the shear crack. The equation has been simplified from the expressions developed by Vecchio and Collins (1986) using experimental data from a study by Walraven (1981). The equation is presented below:

$$V_{ci} = \left( \frac{0.18 \sqrt{f'_{c}}}{0.3 + \frac{24w}{a+16}} \right) b_w d_v$$  \hspace{1cm} (5.10)

where $a$ is the maximum size of aggregates in the concrete. If the crack widths are not wide, the concrete shear resistance may be governed by the tensile stresses in the uncracked concrete, which is given by:

$$V_c = \frac{0.33 \sqrt{f'_{c}} \cot \theta}{1 + \sqrt{500 \varepsilon_s}} b_w d_v$$ \hspace{1cm} (5.11)

Hence, if $V_{ci}$ is less than $V_c$ than the concrete shear resistance shall be taken as $V_{ci}$. If this is not the case, than the concrete shear resistance shall be taken as $V_c$.

Table 5-4 presents the predicted shear resistance of the beams using the modified compression field theory to compute the concrete shear resistance and the CSA S806-12 (2012) standard to calculate the FRP shear resistance. An average concrete compressive strength of 48.5 MPa was used for calculations. The shear force along the crack is constant for all beams, however, the moment is constantly changing, therefore the maximum moment along the pre-cracked region was used in Eq. 5.5. Also, beam F-45-NA-1L was not included in the analysis due to the problem during its testing as described earlier.
Based on Table 5-4, one can see that for several of the 30 degree pre-cracked beams, the concrete shear resistance was decreased due to the large width of some of the shear cracks, compared to the value given by the CSA A23.3-04 (2004) simplified method. Due to this, the shear resistance predictions are closer to the experimental shear resistance, however, the un-anchored U-wrapped beam is still overestimated.

For the 45 degree beams, the concrete shear resistance was close to the predicted values by the CSA A23.3-04 (2004) simplified method. Therefore, the shear resistance predictions are still conservative and can be attributed to the factors discussed previously.

Based on the analysis of the concrete shear resistance, one can conclude that the over-predictions for the 30 degree pre-cracked beams are mainly due to the over-estimated FRP shear resistance. The CSA S806-12 (2012) standard assumes that all FRP wraps crossing the shear crack equally resist the load and reach their maximum capacity at failure, however, this was not observed during testing. For all beams, the right wrap closest to the support was inactive at resisting the load, until failure occurred in the critical wrap. This is due to the close proximity of the wrap to the support and the material properties of FRP. Therefore, the assumption that all FRP wraps crossing the shear crack are active and their shear resistance is additive can lead to major over-predictions in this circumstance. This would affect the prediction of the 30 degree beams with a single shear crack because the right wrap would be one of the main wraps, as in the case of beams U-30-NA-2L, U-30-A1-1L, and U-30-H-1L. Since beam U-30-H-1L was re-tested and its pre-existing crack closed with the opening of a new crack during testing, it can be assumed as a single shear crack. For beams with multiple

---

Table 5-4: Predicted Shear Resistance Using the Modified Compression Field Theory

<table>
<thead>
<tr>
<th>Beam</th>
<th>$V_c$ (kN)</th>
<th>$V_{FRP}$ (kN)</th>
<th>$V_{r, Predicted}$ (kN)</th>
<th>$V_{r, Experimental}$ (kN)</th>
<th>$\frac{V_{r, Predicted}}{V_{r, Experimental}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-30-NA-2L</td>
<td>64.62</td>
<td>72.04</td>
<td>136.66</td>
<td>98.50</td>
<td>1.39</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>48.15</td>
<td>100.06</td>
<td>148.20</td>
<td>145.00</td>
<td>1.02</td>
</tr>
<tr>
<td>U-30-A2-1L</td>
<td>37.09</td>
<td>108.14</td>
<td>145.79</td>
<td>171.00</td>
<td>0.85</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>41.94</td>
<td>108.14</td>
<td>150.08</td>
<td>149.00</td>
<td>1.01</td>
</tr>
<tr>
<td>F-30-NA-1L</td>
<td>31.85</td>
<td>138.95</td>
<td>170.80</td>
<td>173.00</td>
<td>0.99</td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>60.50</td>
<td>53.81</td>
<td>114.31</td>
<td>165.50</td>
<td>0.69</td>
</tr>
<tr>
<td>U-45-NA-1L</td>
<td>59.99</td>
<td>36.83</td>
<td>96.82</td>
<td>143.00</td>
<td>0.68</td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>60.77</td>
<td>77.44</td>
<td>138.21</td>
<td>176.50</td>
<td>0.78</td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>57.38</td>
<td>69.68</td>
<td>127.06</td>
<td>169.00</td>
<td>0.75</td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>57.38</td>
<td>69.68</td>
<td>127.06</td>
<td>169.00</td>
<td>0.75</td>
</tr>
</tbody>
</table>
shear cracks, like beam U-30-A2-1L and the 45 degree pre-cracked beams, this would not have as much of an impact because the additional shear cracks can further activate other wraps in order to make up for the inactive right wrap, and would result in safe predictions despite the right wrap being inactive. Beam F-30-NA-1L also had multiple shear cracks in the pre-cracked region of the beam and should have had safe predictions, however, its maximum load was restricted by its flexural capacity. It is important to note that the shear resistance predictions of the 45 degree un-anchored U-wrapped beams are highly conservative due to a higher number of wraps activated than assumed because of the formation of multiple shear cracks and a higher FRP strain than the assumed debonding strain, due to an enhanced surface preparation.

The shear resistance that a wrap will contribute directly depends on its location along the beam and the height at which the shear crack crosses the wrap. Following the same procedure previously discussed, one can find the longitudinal strain, $\varepsilon_x$, for individual wraps, corresponding to the location of the wrap along the beam and the beam height at which the shear crack crosses the wrap. The component of the longitudinal strain due to shear is constant along the beam height and also along the shear span for the test beams. The longitudinal strain due to shear can be calculated by:

$$\varepsilon_{x,\text{Shear}} = \frac{V}{2E_s A_s}$$  \hfill (5.12)

The component of the longitudinal strain due to flexure can be calculated individually for each wrap by using strain compatibility to determine the strain at the appropriate depth that the shear crack crosses the wrap, corresponding to the moment along the span where the wrap is located, as seen in Figure 5-4.

![Figure 5-4: Schematic of longitudinal strain due to flexure for an individual wrap](image)

The transverse strain in the direction of the FRP fibers can be calculated using the following:
\[ \varepsilon_t = \frac{\varepsilon_1 + \varepsilon_2 \tan^2 \theta}{1 + \tan^2 \theta} \]  \hspace{1cm} (5-13)

Figure 5-5 shows the transverse strains corresponding to the FRP wraps for beam U-30-NA-2L calculated using this method.

![Normalized Transverse Strain Distribution](image)

**Figure 5-5:** Normalized transverse strain distribution along shear crack

Based on Figure 5-5, one can see that the location of the wrap along the beam span and the height at which the shear crack crosses the wrap can inherently cause one wrap to be strained more than others and can limit the amount of strain on wraps. This shows that the full usage of all wraps crossing the shear crack may not be utilized and may lead to un-conservative predictions if assumed so.

Another cause for overestimated shear resistance prediction by the CSA S806-12 (2012) standard could be due to some wraps having a limited capacity attributed to an insufficient anchorage length, in the case of un-anchored U-wraps. This would affect wraps intersecting the top of the shear crack. One can analyze the effect of this by multiplying the equation for debonding stress given by Chen and Teng (2003a) by the area of the FRP wrap in order to obtain the shear resistance per wrap and rearranging it in terms of the effective length. This is given by:

\[ V_{FRP \text{ per wrap}} = 0.427 \beta_w \beta_L \sqrt{f'_{c}} w_{FRP} L_e \]  \hspace{1cm} (5.14)

One can refer to Section 2.7.3.1 in Chapter 2 for a detailed overview of the Chen and Teng (2003a) model. For beam U-30-NA-2L, the effective length was calculated to be 207 mm. Two of the three wraps crossing the shear crack had bond lengths greater than this value,
however the wrap intersecting the top of the shear crack only had a bond length of 80 mm, therefore its maximum capacity would be limited. This would result in a 78% decrease in its ultimate capacity and could cause over-predictions if not taken into account. In order to account for these reductions in the shear capacity, the CSA S806-12 (2012) FRP shear resistance equation can be modified to represent the shear resistance per FRP wrap, given as:

\[ V_{FRP\,per\,wrap} = A_F \varepsilon_F E_f \]  \hspace{1cm} (5.15)

The modified shear resistance for the 30 degree beams with a single shear crack, using Eq 5.15, considering the reductions discussed, and neglecting the shear contribution of the extreme right wrap is presented in Table 5-5. The concrete shear resistance given by the modified compression field theory was used, and the FRP strain values were calculated in accordance with the CSA standard.

Table 5-5: Modified Shear Resistance Predictions for Beams with a Single Shear Crack

<table>
<thead>
<tr>
<th>Beam</th>
<th>( V_c ) (kN)</th>
<th>( V_{FRP} ) (kN)</th>
<th>( V_{r,\text{Predicted}} ) (kN)</th>
<th>( V_{r,\text{Experimental}} ) (kN)</th>
<th>( \frac{V_{r,\text{Predicted}}}{V_{r,\text{Experimental}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-30-NA-1L</td>
<td>64.62</td>
<td>35.09</td>
<td>99.71</td>
<td>98.5</td>
<td>1.01</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>48.15</td>
<td>76.71</td>
<td>124.85</td>
<td>145</td>
<td>0.86</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>41.94</td>
<td>81.22</td>
<td>123.17</td>
<td>149</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Based on Table 5-5 one can see that the modified prediction for beam U-30-NA-2L is much closer to the experimental results, as the debonding strain assumed in the CSA standard was slightly higher than the experimental debonding strain. Good predictions are given for the other 30 degree beams, however they are not as close to the experimental results as beam U-30-NA-1L due to a higher level of strain achieved in the wraps than assumed in the CSA predictions.

5.5.3 CSA S806-12 (2012) Shear Resistance Prediction Using a 45 Degree Shear Crack Inclination Angle

As seen from Section 5.4, the CSA S806-12 (2012) standard overestimated the shear resistance of the 30 degree pre-cracked beams. It was recommended that a 45 degree shear crack inclination should be used in the FRP shear resistance equations given by the standard. Table 5-6 presents the shear resistance predictions using the CSA S806-12 (2012) standard
with a fixed 45 degree shear crack inclination. The concrete shear resistance was calculated in accordance with the CSA A23.3-04 (2004) simplified method. An average concrete compressive strength of 48.5 MPa was used for calculations.

Table 5-6: Shear Resistance Predictions Using a Fixed 45 Degree Shear Crack

<table>
<thead>
<tr>
<th>Inclination Angle</th>
<th>Beam</th>
<th>$V_{FRP}$ (kN)</th>
<th>$V_{r,Predicted}$ (kN)</th>
<th>$\frac{V_{r,Predicted}}{V_{r,Experimental}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 degree</td>
<td>U-30-NA-2L</td>
<td>46.78</td>
<td>107.78</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>U-30-A1-1L</td>
<td>64.98</td>
<td>125.98</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>U-30-A2-1L</td>
<td>64.98</td>
<td>125.98</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>U-30-H-1L</td>
<td>64.98</td>
<td>125.98</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>F-30-NA-1L</td>
<td>77.97</td>
<td>138.97</td>
<td>0.80</td>
</tr>
<tr>
<td>30 degree</td>
<td>U-45-NA-2L</td>
<td>46.78</td>
<td>107.78</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>U-45-NA-1L</td>
<td>34.35</td>
<td>95.35</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>U-45-A1-1L</td>
<td>64.98</td>
<td>125.98</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>U-45-A2-1L</td>
<td>64.98</td>
<td>125.98</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>U-45-H-1L</td>
<td>64.98</td>
<td>125.98</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Based on Table 5-6, one can see that when using the CSA A23.3-04 (2004) simplified concrete shear resistance with the CSA S806-12 (2012) FRP shear resistance, the use of a 45 degree shear crack inclination gives safe predictions for the 30 degree beams with a single shear crack, excluding the un-anchored beam. Since the CSA standard provides debonding strain predictions close to the experimental debonding strain and does not account for reductions due to some wraps having insufficient anchorage lengths, it can still provide unsafe predictions for U-wrapped beams without anchors. In the case of multiple shear cracks forming, and a higher strain in the FRP reached than anticipated, the predictions can be conservative, as in the case of the 45 degree beams and beam U-30-A2-1L, however, it is difficult to predict the shear crack pattern and therefore, one must design for a single shear crack.

5.5.4 Implementation of Anchors

Several models used to predict the shear resistance contributed by FRP strengthening fail to implement the effects of anchors. As intermediate debonding of a U-wrap occurs, it spreads
along the wrap until reaching the anchors at its top, therefore the greater FRP shear resistance from the use of the anchors is attributed to the increase in interface area between the FRP wraps and concrete. One can implement the effects of anchors in models that do not account for them by using the interface area between the FRP wraps and concrete due to the anchors, instead of the effective length of the wrap times its width. Ceroni and Pecce (2010) stated that the effect of an end transverse strip can be estimated as a spread of the bond shear stresses along an angle of 45 degrees, as seen for the case of the proposed anchors in Figure 5-6.

![Diagram of bond shear stresses with proposed anchor](image)

**Figure 5-6:** Schematic of bond shear stresses with the proposed anchor

One can implement the effect of anchors for the Chen and Teng (2003a) model by multiplying the debonding stress equation by the area of the FRP wrap to give the shear resistance per wrap and rearranging the equation in terms of the effective length, as follows:

\[
V_{FRP \text{ per wrap}} = 0.427 \beta W \beta L \sqrt{f'_c} (w_{FRP} L_e) \tag{5.16}
\]

One can refer to Section 2.7.3.1 in Chapter 2 for a detailed overview of the Chen and Teng (2003a) model. Since the debonding stress predicted by Chen and Teng (2003a) was greater than the experimental debonding stress, the recommended coefficient for design of 0.315 will be used instead of 0.427. The term with the width of the wrap and effective length can be replaced by the trapezoidal interface area of the anchor, shown in Figure 5-6, which was
calculated to be 8800 \(mm^2\). This value should be multiplied by 2 when considering both sides of the wrap. Table 5-7 presents the shear predictions of the anchored beams using Eq. 5.16. An average concrete compressive strength of 48.5 MPa was used for calculations. The right wrap was neglected for all beams, and the reductions associated with the position of the wrap along the beam and the depth at which the shear crack crosses the wrap for beams with a single shear crack was considered. For beams with multiple shear cracks, it was assumed that the wraps intersecting the shear cracks, excluding the right wrap, were fully activated. Also the term \(\beta_L\) to account for insufficient anchorage length was assumed as 1 for all wraps, as anchorage length is not an issue for anchored wraps. The modified concrete shear resistances calculated in Section 5.5.2 were used to calculate the total shear resistance of the beams.

Table 5-7: Shear Resistance Predictions Using the Chen and Teng (2003a) Model with Consideration for Anchors

<table>
<thead>
<tr>
<th>Beam</th>
<th>(V_c) (kN)</th>
<th>(V_f) (kN)</th>
<th>(V_{r,\text{Predicted}}) (kN)</th>
<th>(\frac{V_{r,\text{Predicted}}}{V_{r,\text{Experimental}}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-30-A1-1L</td>
<td>48.15</td>
<td>83.25</td>
<td>131.39</td>
<td>0.90</td>
</tr>
<tr>
<td>U-30-A2-1L</td>
<td>37.65</td>
<td>97.94</td>
<td>135.59</td>
<td>0.79</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>41.94</td>
<td>88.15</td>
<td>130.08</td>
<td>0.87</td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>60.77</td>
<td>97.94</td>
<td>158.71</td>
<td>0.90</td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>57.38</td>
<td>97.94</td>
<td>155.32</td>
<td>0.92</td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>57.38</td>
<td>73.50</td>
<td>130.88</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Based on Table 5-7, one can see that the assumption of the spread of bond shear stresses along a 45 degree angle by Ceroni and Pecce (2010) gives good predictions when implemented in the Chen and Teng (2003a) model. For beam U-45-H-1L, only one of the wraps had horizontal strips and this was taken into account in the calculations, however, since an enhanced surface preparation was applied to this beam, the un-anchored U-wrap had a much higher debonding load than assumed and resisted more shear. Overall, the above proposed method for computing the shear strength of retrofitted beams with anchors seems to give reasonable results.
5.6 Flexural Analysis of Beams

Although not the focus of this experimental program, due to the fact that many of the beams in the current investigation failed in flexure, it is important that their theoretical flexural resistance be determined so that the experimental results can be validated. Since all the beams have the same cross-sectional dimensions and concrete and steel properties, they are expected to have the same flexural strength. The theoretical ultimate strength of the beams was determined based on strain compatibility analysis in conjunction with the constitutive laws of the concrete and steel reinforcement. The equivalent rectangular stress block was used to compute the concrete force and the location of its point of application. The stress block parameters were calculated in accordance with the CSA A23.3-04 (2004) design standard. The average concrete strength at 28 days and at the end of the testing period was used for calculations. The average concrete strength was 48.5 MPa. The steel yield stress and modulus of elasticity used for calculations was 420 MPa and 199,219 MPa, as determined from the steel tensile tests performed in this experimental program. The effect of strain hardening was also considered in the analysis by using the steel stress-strain behaviour, as obtained from the steel tensile tests. Figure 5-7 shows the assumed strain and stress profiles of the section at failure.

Figure 5-7: Concept of strain compatibility method (a) strain profile of beam at flexural failure (b) concrete stresses corresponding to strain profile (c) equivalent concrete stress block corresponding to strain profile
For beams that failed in flexure, the strain in the extreme compression fibre was assumed to be 0.0035, due to the failure of the concrete at that fibre. Using the strain compatibility method the location of the neutral axis, the strain in the longitudinal compressive and tensile steel reinforcement, and the ultimate moment were found. Since the flexural reinforcement and cross-sectional properties were the same for all beams, theoretically all beams that failed in flexure should have the same neutral axis location, strain in the longitudinal reinforcement, and ultimate moment. The location of the neutral axis \( c \) was calculated to be 71.3 mm from the top of the beam, the strain in the bottom most tensile reinforcement was calculated to be 13,373 με, and the ultimate load corresponding to the flexural resistance was calculated to be 310.4 kN.

Table 5-8 compares the predicted ultimate load, corresponding to the maximum moment resistance, to the experimental ultimate load for the beams that failed in flexure or a combination of shear and flexure. All internal strain gauges malfunctioned prior to flexural failure, therefore comparisons of predicted strains to experimental strains in the longitudinal tensile reinforcement was not possible.

Table 5-8: Comparison of Predicted/Experimental Values for Beams that Failed in Flexure

<table>
<thead>
<tr>
<th>Beam</th>
<th>Experimental Ultimate Load for Beams that Failed in Flexure ( P_{U,\text{experimental}} ) (kN)</th>
<th>( \frac{P_{U,\text{predicted}}}{P_{U,\text{experimental}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-30-A2-1L</td>
<td>342</td>
<td>0.91</td>
</tr>
<tr>
<td>F-30-NA-1L</td>
<td>346</td>
<td>0.90</td>
</tr>
<tr>
<td>C1-30-NA-NA</td>
<td>331</td>
<td>0.94</td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>331</td>
<td>0.94</td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>353</td>
<td>0.89</td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>338</td>
<td>0.92</td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>338</td>
<td>0.92</td>
</tr>
<tr>
<td>F-45-NA-1L</td>
<td>338</td>
<td>0.92</td>
</tr>
<tr>
<td>C1-45-NA-NA</td>
<td>334</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Based on Table 5-8, the predicted ultimate loads corresponding to the maximum flexural resistance of the beams are in reasonable agreement with the experimental ultimate loads for the beams that failed in flexure. Deviations in the experimental ultimate loads might be due to variability in the concrete and steel properties among the beams.

Strain compatibility was also used to determine the location of the neutral axis, strain in the longitudinal steel reinforcement, and strain in the concrete at the extreme compression fibre,
corresponding to the equivalent moment at the ultimate load, for beams that failed in shear. Since failure of the concrete was not reached, the strain at the extreme compression fibre would have been less than the ultimate strain of concrete and the typical stress block parameters corresponding to the failure of concrete could not be used. Using the recommendations of Collins and Mitchell (1999), the stress block parameters related to the maximum strain within the concrete were found. The concrete stress block parameters depend on the concrete compressive strength and the strain in the concrete corresponding to the maximum compressive stress. Assuming a parabolic stress variation in the concrete and a compressive strength of 48.5 MPa, the strain in the concrete at the maximum stress was calculated to be 0.00213. Using this information, the strain in the concrete at the extreme compression fibre, the strain in the longitudinal tensile steel reinforcement, and the location of the neutral axis of the beams that failed in shear were calculated. Table 5-9 presents these predicted values for the beams that failed in shear using strain compatibility. Figure 5-8 presents a comparison of the predicted to experimental strain response in the longitudinal tensile steel for beams that failed in shear.

Table 5-9: Comparison of Predicted/Experimental Values for Beams that Failed in Shear

<table>
<thead>
<tr>
<th>Beams</th>
<th>Strain in Concrete at the Extreme Compression Fibre (με)</th>
<th>Location of the Neutral Axis (mm)</th>
<th>Predicted Strain in Longitudinal Tensile Reinforcement (εS1Predicted) (με)</th>
<th>Recorded Strain in Longitudinal Tensile Reinforcement (εS1Experimental) (με)</th>
<th>εS1Predicted/εS1Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-30-NA-NA</td>
<td>600</td>
<td>106.19</td>
<td>1300</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>U-30-NA-2L</td>
<td>790</td>
<td>107.36</td>
<td>1704</td>
<td>2009</td>
<td>0.85</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>1600</td>
<td>87.68</td>
<td>4585</td>
<td>4597</td>
<td>0.99</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>2100</td>
<td>74.32</td>
<td>7477</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>U-45-NA-1L</td>
<td>1450</td>
<td>94.13</td>
<td>3771</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>
With reference to Table 5-9, some strain gauge readings could not be used due to malfunctioning or erratic strain recordings at the failure of the beams. However, the appropriate strain readings recorded at failure for beams U-30-NA-2L and U-30-A1-1L were in relatively good agreement with the predicted strain in the longitudinal tensile reinforcement.

Based on Figure 5-8, the predicted longitudinal tensile steel strain response using strain compatibility is in good agreement with the experimental strain responses for the beams that failed in shear. Beam C-30-NA-NA was not included in this comparison because the strain gauge on the longitudinal tensile steel at mid-span of this beam malfunctioned before testing. A slight offset in the elastic region between the predicted and experimental strain response is apparent for some of the beams. This can be attributed to a sudden jump in strain in the longitudinal tensile steel at mid-span upon the onset of concrete cracking, which was not considered in the predicted response. Beam U-30-H-1L, which was a re-strengthened beam, showed the exact predicted response in the elastic region for the strain in the longitudinal tensile steel at mid-span. Flexural cracks had already formed during the previous test and therefore the jump in strain due to the onset of cracking was non-existent during re-testing.
5.7 Ultimate Strain in FRP

It is important to analyze the maximum strain achieved in the FRP wraps in order to determine the effectiveness of the FRP strengthening. The strains that should be experienced by the FRP to provide the required shear resistance can be calculated by rearranging the CSA S806-12 (2012) FRP shear resistance equation. The FRP strain can be calculated by the rearranged CSA standard FRP shear resistance equation presented below:

\[ \varepsilon_{Fe} = \frac{\sigma_{Vf}}{A_{F}E_{F}d_{b}(\cot \theta + \cot \alpha_{F})\sin \alpha_{F}} \] (5.17)

For a detailed overview of the FRP shear strengthening design procedure in this standard, one can refer to Section 2.7 in Chapter 2 of this thesis.

It is assumed that the total shear resistance found for the beam minus the concrete shear resistance is the FRP shear resistance. The concrete shear resistance was calculated by using the modified compression field theory, as in Section 5.5.2. Once the FRP shear resistance is known, the predicted FRP strains can be calculated using Eq. 5.17. For the beams strengthened with un-anchored U-wraps, the predicted debonding strain will be compared to the experimental debonding strain. Due to the difference in concrete compressive strength during the testing period, an average compressive strength of 48.5 MPa will be used in the calculations. The shear crack inclination angles observed during testing were used in the calculations.

Table 5-10 presents a comparison of the ultimate strains recorded in the FRP wraps and the computed ultimate strains based on the CSA S806-12 (2012) standard.
Table 5-10: Comparison of Predicted/Experimental Ultimate Strains in the FRP Wraps

<table>
<thead>
<tr>
<th>Beam</th>
<th>Recorded Ultimate Strain (με)</th>
<th>Predicted Ultimate Strain (CSA) (με)</th>
<th>Predicted Strain Recorded Strain (CSA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-30-NA-2L</td>
<td>1759</td>
<td>1800 (Debonding Strain)</td>
<td>1.02</td>
</tr>
<tr>
<td>U-30-A1-1L</td>
<td>6357</td>
<td>4840</td>
<td>0.76</td>
</tr>
<tr>
<td>U-30-A2-1L</td>
<td>6790</td>
<td>6165</td>
<td>0.91</td>
</tr>
<tr>
<td>U-30-H-1L</td>
<td>3420</td>
<td>4949</td>
<td>1.45</td>
</tr>
<tr>
<td>F-30-NA-1L</td>
<td>11800</td>
<td>6020</td>
<td>0.51</td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>3060</td>
<td>1800 (Debonding Strain)</td>
<td>0.56</td>
</tr>
<tr>
<td>U-45-NA-1L</td>
<td>2310</td>
<td>2643 (Debonding Strain)</td>
<td>1.14</td>
</tr>
<tr>
<td>U-45-A1-1L</td>
<td>6030</td>
<td>7472</td>
<td>1.24</td>
</tr>
<tr>
<td>U-45-A2-1L</td>
<td>4583</td>
<td>8009</td>
<td>1.75</td>
</tr>
<tr>
<td>U-45-H-1L</td>
<td>3300</td>
<td>8009</td>
<td>2.43</td>
</tr>
</tbody>
</table>

Based on Table 5-10, one can see that for the 30 degree pre-cracked beams, many of the predicted strain values are less than the experimental strains, as a higher number of effective wraps is assumed than that observed during testing. Since the CSA FRP shear strength equation includes a variable shear crack angle, it assumes that at least 2 FRP wraps will be equally effective at resisting the load and will share the same ultimate strain. For the beams with a single shear crack, after the reductions due to the location of the wraps along the beam span and the depth at which the shear crack crosses the wrap, the number of equivalent effective wraps was between 1.4 and 2 wraps. For beam U-30-A2-1L, multiple shear cracks occurred and resulted in further activation of the left wrap and close to two equivalent effective wraps, hence the CSA standard provides a good prediction for this beam. It is important to note that for beam U-30-H-1L, since the shear crack did not cross the strain gauge on the critical wrap, a higher strain than recorded would have been experienced in the FRP. Also for beam F-30-NA-1L, the strain in the FRP continued to increase as the load dropped, which is attributed to the weakening of the concrete and the opening of the shear crack, therefore an accurate comparison of predicted to experimental strain cannot be made.
With reference to the 45 degree pre-cracked beams, the predicted strain values were greater than the experimental, as more wraps were activated during testing than assumed. This is due to the formation of multiple shear cracks in the pre-cracked region of all 45 degree pre-cracked beams. This caused further activation of the wraps and close to 2 equivalent effective wraps. The CSA standard assumes around 1.4 wraps to be active and therefore predicts higher strains. Also, the strain recordings for the 45 degree beams are not completely accurate, due to the shear crack not crossing the wraps at the location of the strain gauges for several of the beams.


Table 5-11: Comparison of Predicted/Experimental Debonding Strains in Un-anchored FRP U-wraps

<table>
<thead>
<tr>
<th>Beam</th>
<th>Recorded Debonding or Ultimate Strain (με)</th>
<th>Pred $\varepsilon_{FRP}^{exp}$ (CSA)</th>
<th>Pred $\varepsilon_{FRP}^{exp}$ (ACI)</th>
<th>Pred $\varepsilon_{FRP}^{exp}$ (Mofidi)</th>
<th>Pred $\varepsilon_{FRP}^{exp}$ (Chen, 2013)</th>
<th>Pred $\varepsilon_{FRP}^{exp}$ (Chen, 2003a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-30-NA-2L</td>
<td>1759</td>
<td>1.02</td>
<td>0.80</td>
<td>0.89</td>
<td>1.02</td>
<td>1.06</td>
</tr>
<tr>
<td>U-45-NA-2L</td>
<td>3060</td>
<td>0.59</td>
<td>0.46</td>
<td>0.51</td>
<td>0.59</td>
<td>0.61</td>
</tr>
<tr>
<td>U-45-NA-1L</td>
<td>2310</td>
<td>1.14</td>
<td>0.95</td>
<td>1.35</td>
<td>1.25</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Based on Table 5-11, the predicted debonding strains using all models are in good agreement with the experimental debonding strain for beam U-30-NA-2L. The CSA and ACI standards provided the closest predictions. For beam U-45-NA-2L, the predicted debonding strain using all models differed greatly from the recorded ultimate strain. This can be attributed to the superior surface preparation procedure used for this beam. For beam U-45-NA-1L, an accurate comparison between the predicted and recorded debonding strain cannot be made as the shear crack did not cross the strain gauge on the critical wrap, therefore, the actual strain in the wrap is expected to be higher than recorded.
Table 5-12 presents a comparison of the predicted and experimental rupture strain for beam F-30-NA-1L. The CNR-DT 200/2004 (2004) guidelines and the Chen and Teng (2003b) model are used to predict the rupture strain for fully wrapped beams.

Table 5-12: Comparison of Predicted/Experimental Rupture Strain in FRP Full Wrap

<table>
<thead>
<tr>
<th>Beam</th>
<th>Recorded Rupture Strain (με)</th>
<th>Predicted Strain Recorded Strain (CNR)</th>
<th>Predicted Strain Recorded Strain (Chen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-30-NA-1L</td>
<td>11800</td>
<td>0.26</td>
<td>0.79</td>
</tr>
</tbody>
</table>

The CNR guidelines provide an equation to calculate the rupture strain for full wrapped beams, dependent on the debonding strain and the radius of the curved edges of the beams. The rupture strain predicted using this model is inaccurate with respect to the recorded rupture strain. Chen and Teng (2003b) recommended that the rupture strain for full wraps should be equal to 80 percent of the maximum tensile strain of the wrap. This is to account for the bending of the FRP wraps around the corners of the beams. The predicted rupture strain using this model is in better agreement with the recorded rupture strain. It is important to note that the stress distribution factor was neglected in the calculation of the effective rupture strain.

Furthermore, the use of anchors greatly increased the maximum strain in the FRP wrap before debonding, from 14% of the rupture strain for un-anchored U-wraps to 58% of the rupture strain for U-wraps with the use of anchors. Based on the CSA-S806-12 (2012) standard, the new anchors can be considered a proven anchorage system, as they allowed the FRP wraps to achieve a strain greater than 5000 micro-strain.

**5.8 Strain Distribution in FRP Wraps**

It is important to investigate the distribution of strain in the FRP wraps so that one may gain a better understanding of the mechanism which enables the FRP wraps to resist shear. Three strain gauges were placed along the middle wrap of all strengthened beams, positioned at the top, center, and bottom of the wraps to capture the FRP strain profile during loading. The
strain gauges along the middle FRP wrap were placed at 20, 200, and 380 mm from the bottom of the beam.

This section will focus on the strain profile in the un-anchored U-wrapped, anchored U-wrapped, and fully wrapped beams tested in this experimental program. Only 30 degree pre-cracked beams will be used for this section, as FRP failure was not observed for the majority of the 45 degree pre-cracked beams and the critical wrap was not the middle wrap. The strain profile was identical for the U-wrapped beams with the two anchor configurations, therefore only one of the anchored beams will be considered in this section. The strain profiles in the FRP at different stages of loading for the FRP configurations tested in this experimental program are presented in Figure 5-9.

![Diagram showing strain profiles in FRP wrap at different stages of loading for un-anchored and anchored U-wrapped beams.](image-url)
Figure 5-9: Strain profile in FRP wrap at various stages of loading (a) un-anchored U-wrapped beam (b) anchored U-wrapped beam (c) fully wrapped beam

An important issue governing the strain profile in the FRP wraps is the location where the main shear crack intersects the wrap. For all strain profiles presented, the shear crack intersected the wrap near its center region.

Several observations can be made based on the FRP strain profiles for the un-anchored U-wrap presented in Figure 5-9a. Firstly, the zero strain profile at 0.5Pu clearly shows that the FRP is not utilized until the onset of shear cracking, which renders the concrete ineffective at resisting the tensile stresses. At 0.75Pu, shear cracking occurred, resulting in localized debonding and high tensile stresses in the region of the wrap crossing the crack. This is seen with the sudden increase in strain at the center of the wrap, whereas the top and bottom regions of the wrap were unstrained. At Pu, which was the debonding load of the wrap, the top and bottom regions were now strained, as high interfacial stresses caused localized debonding to spread along the wrap. As the localized debonding spread towards the top of the wrap, it ultimately resulted in the debonded of the free end, due to the remaining bond length being insufficient to transfer the interfacial stresses (Teng and Chen, 2009; Colalillo and Sheikh, 2014).

Several observations can also be made regarding the FRP strain profiles for the anchored U-wrapped beam presented in Figure 5-9b. It is important to note that the top strain gauge was placed above the anchor head plate and was damaged at failure due to slip between the anchor head plate and U-wrap. At a load of 0.5Pu, the center of the wrap was strained, whereas the top and bottom regions remained unaffected. This is attributed to the formation
of a shear crack crossing the center of the FRP wrap and resulting in localized debonding. As the load was increased, the localized debonding spread towards the top and bottom regions of the wrap. This is clearly seen in the bottom region of the wrap, as it becomes strained at loads of 0.75 $P_u$ and $P_u$. There is no strain observed in the top region of the wrap throughout loading, due to the strain gauge positioned above the anchor head plate, however, high stresses would have been present at the anchor location due to the spread of localized debonding towards the top of the wrap. The anchor was effective at transferring high interfacial stresses to the concrete in the top region of the wrap, resulting in the delay of debonding at the free end and the ability of the wrap to achieve a high level of strain.

With reference to Figure 5-9c, it is important to note that the FRP strain for the fully wrapped beam continued to increase as the load dropped after the beam reached its maximum load, therefore, the strain profiles at maximum load and at failure were both analyzed. The initial FRP strain profiles for the full wrap at loads of 0.5$P_u$, 0.75$P_u$, and $P_u$ were similar to the strain profiles for the anchored U-wrap, however at the failure of the beam, the strain became highest at the top and bottom regions of the wrap. This could be attributed to the high stress in the FRP at the corners of the beam due to the bending of the FRP and the expansion of concrete at failure. Concrete undergoes volumetric expansion when subjected to axial stress equal to around 90% of its compressive strength. The full wrap acts as a confining reinforcement and exerts stress on the concrete in order to restrain its expansion. The stress that the concrete exerts on the FRP near the corners of the beam may lead to localized debonding at the corners and is expected to increase the stress in the FRP near the corners of the beam. Since the top and bottom strain gauges were positioned quite close to the corners of the beam, these high stresses would have been recorded.

Cao et al. (2005) conducted an experimental study on 18 rectangular reinforced concrete beams strengthened with full CFRP wraps. The beams were 150 mm wide, 250 mm deep, and had a length of 2 m. The beams had a shear span to shear depth ratio of 2.7 and were tested in four-point bending with the strengthened span containing internal shear reinforcement of 6M stirrups spaced at 200 mm. Four strain gauges were placed along the depth of one of the CFRP wraps that intersected a main shear crack. The strain profile along the wrap was presented at various stages of loading, as seen in Figure 5-10, and is similar to the strain profile for the full wrap plotted in Figure 5-9c. Cao et al. (2005) also attributed the larger strains at the top and bottom of the wrap at higher stages of loading to the bending effect of the FRP around the corners of the beam.
5.9 Effect of the Shear Crack Inclination Angle

One focus of this experimental program was to determine the influence of the shear crack inclination angle on the effectiveness of FRP strengthening. Inclined pre-cracks were introduced in the beams at angles of 30 and 45 degrees, although during testing, the actual shear cracks deviated slightly from these angles. In this section, the effect of the shear crack inclination angle with reference to the current test results will be discussed.

It is inappropriate to compare the un-anchored U-wrapped beams and the U-wrapped beams with horizontal strips, due to differences in the concrete surface preparation between the 30 and 45 degree pre-cracked beams. It is also inappropriate to compare the fully wrapped beams, due to a problem encountered during testing of the fully wrapped 45 degree pre-cracked beam. Therefore, this comparison will be limited to 30 and 45 degree pre-cracked beams with anchors.

Figure 5-11 presents a comparison of the ultimate load of 30 and 45 degree pre-cracked beams strengthened with U-wraps with the use of anchor configuration 1 and 2.
Figure 5-11: Comparison of ultimate load for 30 and 45 degree pre-cracked beams with anchors

Based on Figure 5-11, one can see that for U-wrapped beams with anchor configuration 1, the 45 degree pre-cracked beam had a higher strength than the 30 degree pre-cracked beam. For U-wrapped beams with anchor configuration 2, both 30 and 45 degree pre-cracked beams had around the same strength. Theoretically, one expects the opposite result because the flatter, 30 degree shear crack is expected to engage more U-wraps and mobilize them, leading to a higher strength. A flat angle also increases the bond length from the free end of the U-wrap to its intersection with the shear crack, which should lead to a higher strength.

Figure 5-12 presents a comparison of the maximum FRP strain measured in the two sets of beams with different shear crack inclination angles. For both anchor configurations, the 45 degree pre-cracked beams experienced a lower strain in the FRP than the 30 degree pre-cracked beams. However, the actual strain achieved in the 45 degree pre-cracked beam with anchor configuration 2 might have been higher than recorded, since the shear crack did not cross the critical wrap at the location of the strain gauge. Reliance on the recorded value of strains may not be appropriate because the measured strain is a function of the location of the strain gauge from the shear crack. As one moves away from the crack, the strain drops rapidly along the FRP wrap, except when full debonding of the wrap occurs.
A comparison of the load-deflection curves of the two sets of beams with different shear crack inclination angles is presented in Figure 5-13. For both anchor configurations, the 45 degree pre-cracked beams experienced a much higher mid-span deflection than the 30 degree pre-cracked beams. This is due to the 30 degree pre-cracked beams failing in shear or combined shear and flexure, whereas the 45 degree pre-cracked beams failed in flexure. Based on Table 5-1 in Section 5.2, both 30 degree pre-cracked beams with anchors failed due to damage incurred by the FRP wrap, while in the 45 degree pre-cracked beams, damage to the FRP wraps was not observed and the beams were able to reach their full flexural capacity.
In order to determine whether these experimental results agree with existing shear theories, the current theory regarding the effect of the shear crack inclination angle on the shear strength is revised. Based on the CSA S806-12 (2012) design standard, which is based in part on the modified compression field theory (Vecchio and Collins, 1986), the influence of the shear crack inclination angle, \( \theta \), on the effectiveness of the FRP strengthening is reflected by the following equation:

\[
V_{FRP} = \frac{A_f \cdot E_f \cdot \epsilon_f \cdot d_y \cdot \cot \theta}{S_f} \tag{5.18}
\]

According to Eq. 5.18, a shallow shear crack inclination angle results in an increased FRP contribution to the shear resistance.

It is clear that the observed anchored beams did not follow the expected trend based on the CSA S806-12 (2012) standard. The 45 degree pre-cracked beams observed had an equal or greater shear resistance to the 30 degree beams. This is due to the formation of multiple shear cracks in the 45 degree beams, which is not considered in Eq. 5.18. The multiple shear cracks further engage the FRP wraps and result in a larger shear resistance than if only a single shear crack were present. Also, as seen in Section 5.5.2, the concrete shear resistance was greater in the 45 degree anchored beams than in the 30 degree beams when calculated using the modified compression field theory.
Based on this, one can conclude that the shear crack inclination has little effect on the FRP shear resistance. Therefore, for beams with a single shear crack, the CSA standard can overestimate the FRP shear resistance since it does not consider reductions in the shear capacity or inactivity of some wraps due to their location along the beam span, the depth at which the shear crack crosses them, and the possibility of an insufficient anchorage length. Due to this it is recommended that a conservative 45 degree shear crack inclination angle be used in the CSA FRP shear resistance equation with a safety factor for un-anchored U-wraps for design.

5.10 Effect of Anchors

Another focus of the current experimental program is to determine the effectiveness of using the proposed anchors to delay/eliminate debonding of U-wraps and increase the shear capacity of the strengthened beams. Two configurations of the proposed anchor were tested in this experimental program. Only the anchor configuration with the most effective results will be discussed in this section. A comparison of the anchor configurations will be discussed in a later section.

For the 30 degree pre-cracked beams, the second anchor configuration provided the most effective results and will be used in the current comparison. For the 45 degree pre-cracked beams, both beams with anchors failed in flexure independent of the anchor configuration. The beam with anchor configuration one failed in flexure at a higher load than the other anchored beam, therefore the 45 degree pre-cracked beam with anchor configuration one will also be discussed in this section.

The anchored U-wrapped beams will be compared to the companion un-anchored U-wrapped beams. Beam U-30-NA-2L is the un-anchored U-wrapped beam for the 30 degree pre-cracked beams and beam U-45-NA-1L is the un-anchored U-wrapped beam for the 45 degree pre-cracked beams.

In Figure 5-14 the ultimate strength of the anchored and un-anchored U-wrapped beams are compared. As seen, the use of anchors provides a large increase in strength over the un-anchored U-wrapped beams. For the 30 degree pre-cracked beams, a strength increase of 74% with a 50% decrease in the thickness of the FRP wraps was observed when using anchors over the un-anchored case. The anchored beam contained half the amount of FRP because a higher strain was anticipated in the anchored FRP wraps. For the 45 degree pre-cracked beams, a strength increase of at least 23% percent was observed when using anchors
over the un-anchored case. The anchored beam failed due to flexure with no damage done to the FRP strengthening, therefore a higher shear resistance could have been achieved, had the beam not failed in flexure. It is important to note that a superior concrete surface preparation was used for the 45 degree pre-cracked U-wrapped beam without anchors.

![Comparison of ultimate load for anchored and un-anchored U-wrapped beams](image)

**Figure 5-14:** Comparison of ultimate load for anchored and un-anchored U-wrapped beams

Figure 5-15 shows a comparison of the maximum recorded FRP strain in the anchored and un-anchored companion beams. The use of anchors delayed debonding and allowed the FRP U-wraps to achieve much greater strain than when un-anchored. For the 30 degree pre-cracked beams, the anchored U-wraps were able to achieve a maximum strain of 6790 µε, corresponding to an increase in strain of roughly 286% over the un-anchored wraps. The comparison of strain might not be completely accurate for the 30 degree pre-cracked beams due to the difference in thickness of the U-wraps for the anchored and un-anchored cases. For the 45 degree pre-cracked beams, the anchored U-wraps were able to achieve a recorded strain of 6030 µε, corresponding to an increase in strain of roughly 161% over its companion un-anchored beam. Since the anchored U-wrapped beam failed in flexure, without damage to the FRP wrap, the anchored U-wraps could have achieved a higher strain if the beam failed in shear. It must be noted that the recorded strain in the 45 degree pre-cracked U-wrapped beam without anchors may not be the true maximum strain reached in the FRP because the shear crack did not intersect the critical U-wrap at the location of the strain gauge.
Figure 5-15: Comparison of ultimate strain recorded in the FRP for anchored and un-anchored U-wrapped beams

A comparison of the load-deflection curves of the anchored and un-anchored beams is presented in Figure 5-16. It can be observed that the use of anchors resulted in increased strength and deflection compared to the un-anchored case. The use of anchors allowed the beams to achieve yielding before failure which resulted in a large increase in deflection compared to the un-anchored U-wrapped beams that failed prematurely due to debonding of the U-wraps. An increased deflection is greatly beneficial, as it provides sufficient warning before failure of the beam.
To gauge the veracity of the current results, one could compare the current results with similar results from previous experimental studies involving anchors.

Kim et al. (2014) conducted an experimental study on the strengthening of reinforced concrete T-beams using CFRP U-wraps with spike anchors. The beams were 610 mm deep and were tested with three different shear span to depth ratios of 1.5, 2.1, and 3. The beams also had internal shear and flexural reinforcement. The beams were strengthened with U-wraps with and without spike anchors. One spike anchor was used at the top of each U-wrap and was embedded through the U-wrap into the concrete. The CFRP laminate thickness was 0.011 mm and had a tensile elongation at rupture of 10,050 με. Comparing anchored and un-anchored U-wrapped beams with a shear span to depth ratio of 3, it was found that the anchored U-wrapped beam resulted in an increase in strength of 40% over the un-anchored U-wrapped beam.

Mofidi et al. (2012) conducted an experimental study on the effectiveness of several different anchorage systems used in reinforced concrete T-beams strengthened with FRP U-wraps. Nine tests were performed on full scale reinforced concrete T-beams with a depth of 254 mm, a length of 4520 mm, and a shear span to depth ratio of 3. The beams were reinforced internally in flexure and shear with four 25M steel bars in flexure and 8mm diameter steel stirrups spaced at 350 mm in shear. The beams were strengthened with a continuous un-
anchored CFRP U-wrap, and a continuous CFRP U-wrap with various end anchorage systems, including a surface bonded, flat CFRP bar, a double-aluminum-plate mechanical anchorage, an embedded, round CFRP bar end anchorage, and an embedded flat CFRP laminate end anchorage. The most effective anchorage system was the embedded flat CFRP laminate end anchorage, which provided the modest shear strength gain of 17% over the un-anchored U-wrapped beam.

It could be argued that the preceding comparisons may not be entirely appropriate because the specimens in both studies had internal shear reinforcement and the CFRP used in these experiments had a much smaller thickness. Despite the above caveat, the proposed anchor used in this experimental program has yielded promising results when compared to similar experimental studies involving other anchor systems.

5.11 Comparison of Anchor Configurations

In this experimental program, two different anchor configurations were investigated. It is important to examine their relative performance. The first configuration consisted of the two layers of the anchor head plate being installed on top of the FRP U-wrap. The second anchor configuration consisted of a sandwiched approach, where one layer of the anchor head plate was installed under the U-wrap, and the other layer on top. For a detailed overview of the anchor configurations one can refer to Section 3.5.3 in Chapter 3. Since both anchored 45 degree pre-cracked beams failed in flexure, independent of the shear strengthening, only the 30 degree pre-cracked beams will be used for comparison in this section.

Figure 5-17 compares the ultimate strength of U-wrapped beams with anchor configuration 1 and 2. One can see that the U-wrapped beam with anchor configuration 2 had higher strength over the beam with anchor configuration 1. The U-wrapped beam with anchor configuration 2 provided an 18% increase in strength over the beam with anchor configuration 1.
Figure 5-17: Comparison of ultimate load for anchored U-wrapped beams

Figure 5-18 compares the ultimate strain achieved in the FRP U-wrap with anchor configuration 1 and 2. It is clear that the U-wrapped beam with anchor configuration 2 achieved a higher ultimate strain in the FRP than the beam with anchor configuration 1. The U-wrapped beam with anchor configuration 2 achieved a 7% higher ultimate strain over the beam with anchor configuration 1. It is clear that the increase in strain is not the same as the increase in strength, which is due to the U-wrapped beam with anchor configuration 2 having multiple shear cracks, whereas the U-wrapped beam with anchor configuration 1 only had a single shear crack. Multiple shear cracks can further engage some FRP wraps and increase the shear capacity due to this.

Figure 5-18: Comparison of ultimate strain recorded in the FRP for anchored U-wrapped beams
Based on the comparison of the load-deflection curves for the anchored beams presented in Figure 5-19, it is evident that the use of anchor configuration 2 delayed beam failure compared to the beam strengthened using anchor configuration 1. Configuration 2 allowed the beam to reach its full flexural capacity and experience large deformation before failure, while configuration 1 did not prevent premature failure and led to a brittle failure of the beam. It is important to note that the failure modes for the anchors differed slightly. The beam with anchor configuration 2 failed due to slip between the anchor head plate and U-wrap and breakage of the anchor head plate, whereas for the beam with anchor configuration 1, only slip between the anchor head plate and the U-wrap was observed at failure.

![Comparison of load-deflection responses at mid-span of U-wrapped beams with anchors](image)

**Figure 5-19**: Comparison of load-deflection responses at mid-span of U-wrapped beams with anchors

### 5.12 Comparison of Horizontal Strips with Anchors

To investigate the effect of anchor legs versus its head plate on shear transfer, the U-wraps of select beams were retrofitted with horizontal strips of laminate applied transverse to the U-wrap ends, forming a T-shape. The horizontal strips were identical to the anchor head plates in anchor configuration 1. It was speculated that the anchor legs may play a minimal role in the anchoring mechanism of resisting shear and preventing debonding of the U-wraps. For a more detailed description of the horizontal strips, reference can be made to Section 3.5.3 in Chapter 3.

A comparison of the U-wrapped beams with horizontal strips to anchored and un-anchored U-wrapped beams will be conducted in this section. The un-anchored U-wrapped beams used for comparison are beams U-30-NA-2L and U-45-NA-1L. Since the 45 degree pre-
cracked beams with anchors and horizontal strips failed in flexure, only the 30 degree pre-cracked beams will be used for comparison of the horizontal strip and anchor. The anchored U-wrapped beam used for comparison is beam U-30-A1-1L.

Figure 5-20 presents a comparison of the ultimate strength of the beams concerned. One can see that the U-wrapped beams with horizontal strips showed an increase in strength compared to the un-anchored U-wrapped beams. For the 30 degree pre-cracked beams, the use of horizontal strips provided a 51% increase in strength with a 50% decrease in the area of the FRP wraps over the un-anchored beam. A similar increase in strength was observed for the beam with anchors. For the 45 degree pre-cracked beams, the use of horizontal strips provided an increase in strength of at least 18% over the un-anchored U-wrapped beam. The U-wrapped beam with horizontal strips failed in flexure, without damage to the FRP strengthening, therefore a higher strength could have been achieved if the beam had failed in shear.

Figure 5-20: Comparison of ultimate load for U-wrapped beams with horizontal strips, with anchors, and without anchors

A comparison of the ultimate strain in the FRP wraps for the selected beams is presented in Figure 5-21. The horizontal strips allowed the FRP wraps to achieve an increased strain by delaying debonding. For the 30 degree pre-cracked beams, the use of horizontal strips allowed the FRP U-wrap to achieve an increase in ultimate strain of at least 101%. The recorded ultimate strain in the FRP U-wrap with anchors was greater than the ultimate FRP strain recorded in the U-wrap with horizontal strips. This can be attributed to the shear crack failing to intersect the critical FRP wrap at the location of the strain gauge for the beam with horizontal strips. The comparison of the ultimate strain in the FRP wraps for the 30 degree
U-wrapped beam with horizontal strips and the un-anchored U-wrapped beam may not be accurate because of the difference in thickness of the U-wraps for these beams. For the 45 degree pre-cracked beams, the beam with horizontal strips provided an increase in ultimate strain in the FRP of at least 43% over the un-anchored U-wrapped beam. The U-wrapped beam with horizontal strips failed in flexure, independent of the FRP strengthening, therefore, a higher strain could have been achieved in the FRP if the beam failed in flexure.

![Figure 5-21: Comparison of the ultimate strain recorded in the FRP for U-wrapped beams with horizontal strips, with anchors, and without anchors](image)

Figure 5-21: Comparison of the ultimate strain recorded in the FRP for U-wrapped beams with horizontal strips, with anchors, and without anchors

Figure 5-22 presents a comparison of the load-deflection curves for the selected beams. The use of horizontal strips allowed the beam to experience an increased mid-span deflection compared to the un-anchored U-wrapped beam. For the 30 degree pre-cracked beams, the use of horizontal strips resulted in roughly double the mid-span deflection of the un-anchored case, however both beams still had a brittle failure. Also, for the 30 degree pre-cracked beams, the use of horizontal strips resulted in roughly the same deflection as the anchored beam. For the 45 degree pre-cracked beams, the use of horizontal strips allowed the beam to reach its full flexural capacity, resulting in a large increase in the mid-span deflection over the un-anchored case and a ductile failure.
Figure 5-22: Comparison of load-deflection response at mid-span for U-wrapped beams with horizontal strips, anchors, and without anchors (a) 30 degree pre-cracked beams (b) 45 degree pre-cracked beams

It is important to note that the failure modes of U-wraps with horizontal strips and anchors differed greatly, as seen in Figure 5-23. The U-wrap with horizontal strips failed due to debonding of the wrap along with the attached horizontal strip. This differed from the failure mode of the U-wrap with anchors, where slip between the wrap and anchor head plate was observed at failure. This shows that the anchor head plate is effective at resisting shear, however without the use of the anchor legs embedded through the head plate into the concrete surface, debonding of the head plate will occur. The use of the carbon-fibre legs
effectively prevents the anchor head plate from debonding, resulting in slip between the anchor head plate and U-wrap.

(a) U-wrapped beam with anchors (b) U-wrapped beam with horizontal strips

Figure 5-23: Images of U-wrapped beams with horizontal strips and anchors at failure

The results of these tests are similar to the experimental results found by Bae and Belarbi (2013), where reinforced concrete T-beams with internal shear and flexural reinforcement were strengthened with U-wraps with the use of a continuous horizontal strip as an anchorage system. It was found that the failure of strengthened beams with horizontal strips as anchorage devices was still caused by debonding, however, the debonding failure was delayed and the shear capacity was increased over the un-anchored U-wrapped beam. For Bae and Belarbi (2013), the use of FRP U-wraps with horizontal strips resulted in a 54% increase in shear strength over the un-strengthened control beam, whereas the un-anchored U-wrapped beam resulted in an increase in shear strength of only 26% over the control beam.

In this study, the use of horizontal strips provided an increased ultimate strength, strain and deflection over the un-anchored U-wrapped beam. In terms of comparison of the horizontal
strips to anchors, the ultimate strength comparison and load-deflection comparisons show similar results. It was difficult to compare the ultimate strain values in the FRP U-wrap, as the shear crack did not intersect the strain gauge for the beam with horizontal strips. The difference between anchors and horizontal strips was observed in their failure modes, which demonstrated that the anchor legs are effective at preventing debonding of the anchor head plate.

5.13 Effect of Concrete Surface Preparation

Although not the focus of this experimental program, the influence of the concrete surface preparation on the effectiveness of FRP strengthening was observed during testing. An enhanced concrete surface preparation procedure was used for the 45 degree un-anchored U-wrapped beams. The enhanced surface preparation procedure involved the concrete surface being grinded down to the aggregate level and roughened using a needle scalar prior to strengthening, whereas for the normal surface preparation for all other beams, the concrete surface was not grinded down to the aggregate level prior to strengthening. The reason for the change of surface preparation was due to the suspicion that the regular surface preparation may not have been adequate. Based on the CSA S806-12 (2012) design standard, as well as the manufacturer’s installation instructions (Sika Canada Inc ®, https://can.sika.com), the basic requirement for surface preparation of the concrete is to provide a roughened surface.

The 30 and 45 degree pre-cracked U-wrapped beams without anchors will be compared in this section to determine the influence of the concrete surface preparation on the effectiveness of the FRP strengthening. Beams U-45-NA-2L and U-30-NA-2L will be used for comparison. The comparison will be based on the debonding/ultimate strain in the FRP U-wraps.

Figure 5-24 presents a comparison of the debonding/ultimate strain recorded in the FRP U-wraps of the un-anchored beams being considered.
Figure 5-24: Comparison of the ultimate strain recorded in the FRP for un-anchored the U-wrapped beams considered

Based on Figure 5-24, the enhanced concrete surface preparation resulted in at least an 80% increase in the ultimate strain that can be achieved in the FRP U-wrap over the beam with a normal concrete surface preparation. It is important to note that the U-wrap for the beam with normal concrete surface preparation debonded at the ultimate strain recorded in the FRP, however, the beam with the grinded surface preparation failed in flexure, absent of damage to the FRP strengthening. Therefore, a higher strain could have been achieved in the FRP U-wrap of the beam with the enhanced concrete surface preparation if it failed in shear.

With respect to Table 5-11 in Section 5.7, it is evident that the predicted debonding strains are in good agreement with the U-wrapped beam which had a normal concrete surface preparation, however, the recorded ultimate strain for the U-wrapped beam with the enhanced concrete surface preparation was much greater than the predicted values.

Furthermore, the enhanced surface preparation procedure of grinding the concrete surface to the aggregate level and then roughening it can create a much stronger bond between the FRP and concrete and result in FRP U-wraps being able to withstand a higher level of strain before debonding occurs.
5.14 Summary

The results of this chapter showed that the shear predictions using several standards and models with a variable shear crack inclination angle overestimated the shear resistance of the 30 degree pre-cracked beams with a single shear crack. For the CSA S806-12 (2012) standard this was attributed to the assumption that all wraps crossing the shear crack equally resist the load and reach their maximum capacity concurrently. This assumption can lead to overestimation because, as observed during testing, not all wraps crossing the shear crack are active at the same time. Also, the effectiveness of a wrap depends on its location along the beam span and the depth at which the shear crack crosses it. For un-anchored U-wraps an insufficient anchorage length can also greatly affect the capacity of the wrap. For beams with multiple shear cracks, the activation of the wraps is increased and conservative predictions were provided by the CSA S806-12 (2012) standard. The use of the proposed anchors in this study provided a large increase in the shear resistance over the un-anchored U-wrapped beams and has yielded promising results when compared to other anchor systems previously tested by others. Anchor configuration 2 proved to be the most effective configuration and the use of horizontal strips provided similar results to the anchors, with the exception of its failure mechanism. Also, although not the focus of this experimental program, the superior surface preparation applied to select beams provided an increased bond strength and shear resistance for un-anchored U-wraps.

In the following chapter a summary of this research study will be presented along with conclusions that can be drawn from its results and analysis. Also, several recommendations for future work in this area will be presented.
Chapter 6: Summary, Conclusions, and Recommendations for Future Work

6.1 Summary

This research study was undertaken in order to investigate the influence of a new anchorage system on the effectiveness of FRP U-wrapped beams and the influence of the shear crack inclination angle on the effectiveness of external FRP shear strengthening.

Twelve beams were tested in three-point bending with a shear span to depth ratio of 3.07. Two of the beams were re-tested after a new retrofit was applied to them. The beams had a length of 2200 mm and a shear span of 950 mm. Inclined shear pre-cracks were fabricated in the beams, with half of the beams having a pre-crack inclination angle of 30 degrees and the other half, 45 degrees to the longitudinal axis of the beam. The pre-cracked regions of the beams were left devoid of internal shear reinforcement, and instead were externally strengthened in shear with FRP.

Select beams were strengthened with un-anchored FRP U-wraps, FRP U-wraps with the use of the proposed anchorage system, FRP U-wraps with the use of horizontal strips, and FRP full wraps. All FRP wraps were 30 mm wide with center to center spacing of 200 mm. The proposed anchor was designed to be fabricated in-situ and consisted of a carbon-fibre head plate, bonded to the FRP U-wrap and concrete surface, and two carbon-fibre legs which were embedded into predrilled holes in the concrete and splayed on to the head plate. The head plate was made from two layers of carbon-fibre strips which were 1.3 mm thick, 80 mm deep, and 200 mm wide, and the two legs were made from 10 mm diameter carbon-fiber rope embedded 65 mm into pre-drilled holes in the concrete.

Two configurations of the anchor were used in this experimental program. The original configuration consisted of the U-wrap applied to the beam first, followed by two layers of the horizontal carbon-fibre strip applied on top of the U-wrap, forming the head plate, and then the application of the anchor legs. After testing, it was found that the bond between the free end of the U-wrap and the concrete surface was the weak link in the anchorage system and leads to slip between the anchor head plate and the U-wrap. To improve the situation, the anchor was modified by applying the first layer of the horizontal carbon-fibre strip to the concrete prior to the application of the U-wrap. The U-wrap was then applied to the concrete surface, on top of the first carbon-fibre strip, followed by the application of second carbon-
fibre strip, and finally the anchor legs. In order to investigate the role of the anchor legs in the anchor resistance mechanism, select U-wrapped beams were also tested with the use of only the horizontal carbon-fibre strips, identical to the original anchor configuration, but excluding the anchor legs.

Although not the purpose of this experimental program, select strengthened beams also had a different concrete surface preparation applied to them prior to strengthening, which involved the concrete surface being grinded down to the aggregate level.

Seven of the beams failed in shear or a combination of shear and flexure, whereas the rest of the beams failed in flexure. It was found that the CSA S806-12 (2012) standard overestimated the FRP shear resistance for the 30 degree pre-cracked beams with a single shear crack. For beams with multiple shear cracks, greater activation of the FRP wraps was achieved and conservative predictions were provided by the CSA S806-12 (2012) standard. It was also found that the use of un-anchored U-wraps increased the shear strength of the beam by 27% over the un-strengthened control beam, whereas the use of U-wraps with anchor configuration 1 increased the shear strength of the beam by 87% and the use of U-wraps with anchor configuration 2 increased the shear strength by 120% over the control beam. The U-wrapped beams with the use of horizontal strips showed similar strength increases as the strengthened beams with anchor configuration 1, however a difference in their failure mode was observed. Finally, the un-anchored U-wrapped beam with an enhanced surface preparation showed an 80% increase in the ultimate strain over the U-wrapped beam with a normal surface preparation procedure.

### 6.2 Conclusions

The focus of this research study was to investigate the influence of a new anchorage system on the effectiveness of FRP U-wrapped beams and the influence of the shear crack inclination angle on the effectiveness of external FRP shear strengthening. Based on this, the following conclusions have been reached:

1) The CSA S806-12 (2012) standard assumes that the shear resistance provided by FRP strengthening is dependant on the shear crack inclination angle and all wraps crossing the shear crack equally resist the load and reach their maximum capacity at failure. This led to overestimation of the shear resistance by the CSA standard for the beams with a single shear crack. During testing, it was observed that the shear crack inclination angle had little effect on the shear resistance contributed by the FRP, as
not all wraps crossing the shear crack were active at the same time, the effectiveness of the wrap depended on its location along the beam span and the depth at which the shear crack crossed it, and for un-anchored U-wraps an insufficient anchorage length greatly decreased the capacity of a wrap.


3) Both configurations of the new anchorage system proved to be highly effective at delaying the debonding process of U-wraps and allowed the wraps to achieve high strain and in turn resulted in a higher shear resistance. Anchor configuration 2 proved to be most effective and resulted in a 74% increase in shear strength with a 50% decrease in the area of the FRP wraps over the un-anchored U-wrapped beam. The use of anchor configuration 2 also resulted in a 286% increase in strain in the wraps over the un-anchored U-wraps.

4) In the current tests, the anchor legs seem to play a minimal role in the anchor resistance mechanism, as seen by the similar shear strength results between the beam with horizontal strips and anchors. The anchor legs were only observed to be effective at eliminating debonding of the head plate at failure. However this issue requires further study. If the validity of the above statements can be demonstrated in other tests, the elimination of the anchor legs will provide a much less labour intensive installation process.

5) Although not the focus of this experimental program, an enhanced surface preparation procedure, involving grinding of the concrete surface down to the aggregate level resulted in a stronger bond between the FRP and concrete, and delayed the debonding process of un-anchored U-wraps.

6.3 Recommendations for Further Work

In order to gain a better understanding of the effects of the new anchorage system and shear crack inclination angle on the shear strengthening of beams, the following are recommendations for future studies:
1) The assumption that all wraps crossing a given diagonal shear crack contribute equally to the ultimate shear resistance of a retrofitted beam needs further investigation. One approach to addressing this issue may be to compute the strain in each FRP wrap at the point where it crosses the diagonal crack. The strain may be calculated using the procedures of the modified compression field theory, but it must be validated by detailed non-linear finite element analysis and appropriate experimental data.

2) The effects of modifying the anchor dimensions, such as the head plate dimensions, embedment length of the legs, splayed length of the legs onto the head plate, and the angle of the splayed section should be investigated.

3) The effects of the anchors on FRP U-wraps with a smaller thickness and larger width should be investigated.

4) Further research using finite element modeling should be conducted to gain a better understanding of the stress transfer mechanism between the FRP U-wrap and the anchor, and the strain profile in the FRP wrap with and without the use of anchors.

5) Due to a limited number of beams, only the horizontal strip configuration identical to anchor configuration 1 was tested. Therefore, further tests should be done to compare the effects of horizontal strips and anchors and to investigate the use of the horizontal strip configuration identical to anchor configuration 2.

6) The externally strengthened regions of the beams in this research study were left devoid of internal shear reinforcement for simplicity. Further testing should be conducted with both internal and external shear reinforcement to investigate the effects of strengthening with more realistic conditions.

7) Only shear pre-crack inclination angles of 30 and 45 degrees were considered in this research study, however further testing with more shear crack inclination angles should be conducted to gain a better understanding of the effect of the shear crack inclination angle on the FRP strengthening of beams.

8) Further testing involving the concrete surface preparation with more test specimens should be conducted to gain a better understanding of the effect of the concrete surface preparation on the bond strength between FRP and concrete.


