Vibration of an inflatable self-rigidizing toroidal satellite component

VIBRATION OF AN INFLATABLE

SELF-RIGIDIZING TOROIDAL SATELLITE COMPONENT

BY

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ABSTRACT

Inflatable structures have attracted much interest in space applications. The three main components of inflatable satellites are inflatable struts, an inflatable torus as the structural support component, and some sort of lens, aperture, or array housed inside the boundary of the torus. This project is devoted towards understanding the dynamic characteristics of an inflated torus with a focus on the self-rigidizing torus, SRT, developed by United Applied Technologies.

The self-rigidizing torus is manufactured from flat sheets of Kapton[®] that are formed into curved films with the regular pattern of hexagonal domes. The inflated torus can support its structural shape even when there is no internal pressure.

Modal testing is used to determine the dynamic properties of the structure for comparison with the numerical model. The feasibility of using a non-contact in-house fabricated electromagnetic excitation is investigated. The first four, in-plane and out-of-plane, damped natural frequencies and their corresponding damping ratios and modes shapes are extracted and compared with prior experimental studies. A preliminary finite element modal analysis is carried out for a torus made of flat film and the results are compared with prior studies. Kapton 300JP[®]'s frequency-dependent modulus of elasticity is determined.

Owing to the large number of hexagonal domes in the self-rigidizing torus, a simplified sub-structuring technique is used. Each hexagonal dome is replaced with a statically equivalent flat hexagon with the same mass and stiffness as the hexagonal dome. Then the finite element modal analysis of the self-rigidizing torus is carried out for an equivalent torus made of flat film. The geometric nonlinearity and the effect of the follower load on the stiffness are included in this analysis. The methodology is verified through the correlation between the analytical and modal test results of the self-rigidizing torus.

To my lovely daughters, Romina and Armita, and my husband, Ata

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NOMENCLATURE

- *E* Modulus of elasticity, complex modulus in viscoelastic materials
- *E'* Storage modulus
- *E*" Loss modulus
- F Force
- *h* Wall thickness
- H₁ Frequency response function estimator
- *I* Moment of inertia
- p Internal pressure
- r Tube radius
- R Ring radius
- t_k Thickness of the film of Kapton[®]
- t_e Thickness of epoxy
- t_p Spacing between the interdigitated electrode
- u_0 Zero-order uncertainty
- *u_c* Instrument uncertainty
- *u_d* Design-stage uncertainty
- U_i Translational motion in "i" direction
- d_{ij} Piezoelectric strain constant
- Σ_i Rotational motion about the "i" direction
- δ Phase difference (Viscoelastic materials)
- ε Strain
- σ Stress

CHAPTER 1 INTRODUCTION

The antenna design requirements for satellites drove their size to dimensions that could not be deployed into space using solid parts. This problem introduced a revolutionary change in satellite design. It was suggested that inflatable portions be used instead of large rigid antennas.

Inflatable structures, also known as Gossamer or membrane structures have attracted much interest in space applications such as large telescopes, antennas, solar sails, sun shields, and solar thermal propulsion. These structures possess many advantageous properties such as ultra-lightweight, small stowage volume (packing efficiency), high strength-to-mass ratio, low overall space program cost, design flexibility, and on-orbit deployability (Jenkins, 2001 and Cadogan et al.2006). Inflatable structures can also reduce the total system mass. Since they have very few mechanical components, they are of lower deployment complexity, and thus higher reliability (Peng et al., 2005, Cadogan et al. 2006).

The three main components of several gossamer communication or imaging satellites are inflatable struts, an inflatable torus as the structural support components and some sort of lens, aperture, or array housed inside the torus boundary. The circular or elliptical torus provides the boundary for any housed membrane or lens, and it ties together the satellite or craft supporting struts (Griffith and Main, 2002 and Lewis and Inman, 2001).

While having many advantageous properties, the inflatable structures have additional challenges when compared to the rigid ones. Many traditional space structures have vibration problems. As the Hubble Space telescope moved in and out of the earth's shadow, it was subjected to vibrations induced by thermal shock, thus making it useless for 15 min at a time (Nurre et al., 1995). Normally, space structures have vibration problems due to low structural stiffness and a lack of air damping in the vacuum of space. These problems are worse for the inflatable structures because of their lower structural stiffness and material damping.

The inflated antenna must maintain a desired surface accuracy in order to focus all the incoming light and microwave on a receiver. The vibration of satellite distorts the antenna surface, thus making it dysfunctional for a certain amount of time. Satellites can be excited mainly under shock disturbances and harmonic excitations. Shock sources can be meteoroid impact, thermal shock, and satellite repositioning. In general, harmonic excitations come from rotating unbalance on the satellite itself, e.g., rotating imbalance in a reaction wheel (Lewis and Inman, 2001) and unbalance of onboard rotating gyroscope (Jha, 2002). Attenuating these disturbances is crucial to maintaining the satellite performance. Besides using mechanisms to maintain the antenna's surface accuracy, one possible way to attenuate the inflatable antenna overall vibration is to control the vibration of its main support system, i.e., the inflated torus.

It is anticipated that the harsh environment of space, intense temperature differentials and micrometeorite bombardment, will most likely threaten the pressurized satellite components. As the structure moves from orbital eclipse into orbital day, the temperature of the gas inside the structure can rise drastically, and eventually the pressure increases. Conversely, as the structure moves from the orbital day into the darkness of orbital eclipse, the pressure can drop significantly.

Addressing this issue, United Applied Technologies (Huntsville, AL, USA) has designed and manufactured a new thin film casting process for the toroidal satellite component. The membrane of the so called self-rigidizing torus, SRT, has a regular pattern of hexagonal domes. The inflated SRT, due to the additional stiffness provided by the pushed out hexagonal domes, can support its own structural shape even when there is no internal pressure.

In recent inflatable space applications, Kapton[®], which is a polyimide film, has been used as a membrane material. It is stable at both high and low temperature extremes and is tough and abrasion resistant. It was reported that the elastic modulus of Kapton[®] varies with the excitation frequency, temperature, and the level of excitation (Tinker, 1998, and Leigh et al., 2001).

Accurate knowledge of the structural dynamics of inflatable structure is crucial to the structural design and control of the spacecraft structure. Therefore, this study is devoted towards understanding the dynamic characteristics of the main support of the inflatable system, the torus, with the focus on the SRT.

1.1 Scope of the Work

Owing to the importance of the inflatable assemblies, characterizing their dynamic behavior both experimentally and analytically is of great interest. In this regard,

taking the building block approach to model and understand the behavior of each of their components seems very helpful. One main component that needs to be investigated in detail is the structural support: the inflatable toroidal structure.

This study is devoted towards understanding the dynamic characteristics of the self-rigidizing torus. This project is divided into two main tasks: experiments and analytical simulation. In the experimental task, the variation of the modulus of elasticity of Kapton 300JP[®] versus frequency is determined first and then modal testing of the SRT is carried out. The analytical simulation involves finite element modal analysis of an inflatable complete smooth-surfaced torus first and thereafter the analysis of the SRT.

As the first step in the experimental process, the dependency of the modulus of elasticity of Kapton $300JP^{\mbox{\sc B}}$ on frequency is examined by performing the experiments with dynamic mechanical analyzer, DMA. This is necessary because there is no source of data in the open literature showing this relationship. If the modulus of elasticity exhibits significant material nonlinearity, this would have to be included in the finite element model.

As the next step in the experimental process, the modal testing of the SRT is carried out to determine the dynamic properties of the structure for comparison to the numerical model. The feasibility of employing a non-contact in-house fabricated electromagnetic excitation is investigated. While the nonlinearity of the test structure is accentuated, the first four in-plane and out of plane damped natural frequencies within the frequency bandwidth of 1-25 Hz and their corresponding damping ratios and mode shapes are extracted and compared with prior studies. It should be mentioned that the out of plane mode shapes resemble the bending modes of a free-free beam, while in the in-plane modes, the structure symmetrically bends in the plane of the torus.

As the first step in the analytical simulation, using $ANSYS^{TM}$, some preliminary analyses for a complete smoothed surface inflatable torus are carried out and the results are compared with the published analytical (closed-form) data. This is necessary to evaluate the capability of commercial finite element software in modal analysis of inflatable toroidal shells. This analysis consists of a parametric study of the effect of the aspect ratio (tube radius / ring radius), pressure and thickness on the natural frequencies and their corresponding mode shapes.

The next step of the analytical simulation deals with the numerical simulation of the SRT. Owing to the large number of hexagonal domes in the SRT shell, a simplified sub-structuring technique is used. In this approach, each hexagonal dome is replaced with a statically equivalent flat hexagon with the same mass, width and stiffness as the hexagonal dome. Then the finite element modal analysis of the SRT is carried out for an equivalent torus made of flat film. The geometric nonlinearity and the effect of the follower load on the stiffness are included in this analysis. The bonding regions are included in the model. Natural frequencies and mode shapes determined by the finite element modal analysis are compared with those obtained from the modal testing.

1.2 Outline of Dissertation

Chapter 1 introduced the inflatable satellite technology, its advantages and disadvantages, and the scope of this study. The descriptions of the remaining chapters are summarized below. In Chapter 2, the state-of-the-art on different aspects of this research is surveyed. In this context, the contributions of this research are discussed. In Chapter 3, the description of the SRT is presented. Thereafter, the experimental results regarding the modulus of elasticity of Kapton 300JP[®] and the epoxy used for bonding are presented. Chapter 4 deals with the experimental modal analysis of SRT. The experimental setup, excitation and sensing setup, experimental procedure, nonlinearity and modal parameter identification are presented. In Chapter 5, the finite element modal analysis of the specific case of an inflated pressurized toroidal shell with a smooth surface is given. The effects of aspect ratio, pressure, and thickness on the natural frequencies and mode shapes of the inflated torus are studied. In Chapter 6, the sub-structuring approach for the finite element modal analysis of the SRT along with its results is provided. Chapter 7 ends this dissertation with the summary and conclusions of this research. It also provides recommendations for further investigation.

CHAPTER 2 LITERATURE REVIEW

As discussed in Chapter 1, an inflated torus serves as an important structural component for an inflatable system. It was also pointed out that the vibration of these structures must be suppressed for their proper functioning. The goal of this research is to understand the dynamic characteristics of the main support of the inflatable system, the torus, with the emphasis on the self-rigidizing torus.

This chapter starts with a review of the concept of gossamer spacecraft, section 2.1, followed by a review on the membranes used for inflatable structures, subsection 2.1.1. In subsection 2.1.2, rigidizable structures are introduced. Thereafter, a critical review of the literature on the experimental and numerical modal analysis of the inflatable tori is presented. This includes a review of the smart material, subsection 2.2.1, followed by survey on the experimental modal analysis of gossamer spacecraft and inflated torus, subsection 2.2.2. In Section 2.3, previous major projects which involved the analysis and modeling of gossamer structures are discussed. Subsection 2.3.1 discusses the pre-stress and the follower action of the pressure load, with subsection 2.3.2 reviewing the prior analytical and numerical studies of an inflated torus. This chapter ends with the proposed research.

2.1 Gossamer Spacecraft

1

A gossamer spacecraft is any such space-inflated craft that exhibit the two distinct characters of ultra-low mass and minimal stowage volume. With gossamer technology, huge structures can be deployed into space at a significantly reduced cost.

Many inflatable structures have been successfully used in space including re-entry ballutes (IRDT), space suits, communications satellites such as ECHO, missile decoys, impact attenuation airbags such as the Luna, Pathfinder and MER missions to Mars, and even in the Russian Voskhod 2 airlocks (Cadogan et al., 2006).

Fig. 2.1 shows the In-Space Technology Experiments Program (IN-STEP) of Inflatable Antenna Experiment (IAE) which was performed in 1996 by L'Garde and NASA JPL together (Lewis and Inman, 2001 or Freeland et al., 1993). IAE was a technology demonstrator flight for inflatable structures. The total weight of the 14 meter diameter inflatable deployable reflector antenna structure was 60 kg (Freeland and Bilyeu, 1992). The system's basic elements included the (a) inflatable torus as the support structure for the reflector, (b) the inflatable parabolic membrane reflector structure, (c) the inflatable struts, which had the capability for supporting a feed or subreflector structure, but, for this experiment, terminated at the canister, (d) the canister, which interfaced the antenna experiment to the Spartan, supported the stowed antenna and other experiment equipment, incorporated deployable doors to access the antenna to the space environment, and provided mounting for the surface measurement system, (e) the instrumentation system that consisted of high-resolution television cameras and a digital imaging radiometer, that were mounted in the canister, and (f) the Spartan, which was the carrier for the experiment and gave initiation commands to the experiment controller (Freeland et al. 1993). Antennas of this type could be used for space and mobile



Figure 2.1 Experimental orbital configuration of inflatable antenna used in IAE (Freeland et al., 1993).

communications, earth observations radiometry, active microwave sensing, astronomical observations, microspacecraft and space-based radar. The IN-STEP experiment objective was to (a) validate the successful deployment of a 14 meter diameter, inflatable deployable, offset parabolic reflector antenna structure in a zero gravity environment, (b) measure the reflector surface precision, which was on the order of 2 mm rms, for several different sun angles and inflation pressures in a realistic gravity and thermal environment, and (c) investigate the fundamental modes and structural damping characteristics of this unique type of space structure (Freeland and Bilyeu, 1992, and Freeland and Veal, 1998). Despite difficulty encountered in the inflation/deployment phase, the flight was successful (Tinker, 1998).

As mentioned earlier, an inflatable torus serves as one of the main components of a gossamer structure. As shown in Fig. 2.2, a toroidal shell has double curvature, i.e., ring radius, R, and tube radius, r. This causes coupling between bending and stretching actions of a load and results in smaller deflections when compared to those of a beam, plate, or cylindrical structure. Owing to this coupling and the internal pressure, an inflatable toroidal shell can carry a significant load. Moreover, despite their quite thin walls, the internal pressure and large surface area provide moderately high strength in inflated tori. Owing to its axisymmetric configuration, an inflated torus is considered as a stable structure (Ruggiero et al., 2003).

The aforementioned properties make an inflatable torus useful for providing structural support to a gossamer structure. Analysis of an inflatable torus has its unique problems usually not encountered in the analysis of conventional metal or composite structures, which makes it a practical and interesting research topic.





2.1.1 Membranes for gossamer structures

Two common characteristics of all membrane/inflatable structures are flexibility and low packaging volume, which are achieved due to the combination of a material's low modulus and thinness.

Currently membrane/inflatable materials are either thin polymer sheets or films, or thin walled polymer tubes. In recent inflatable space applications, Kapton[®] which is a polyimide film made by DuPont has been used as a membrane material. Among different types of polyimide films available for space application, Kapton[®] has minimum degradation in tensile properties and solar absorbance under exposure to a simulated five year low earth orbit and geosynchronous orbit (Slade, 2001). It is stable at both high and low temperature extremes and is tough and abrasion resistant. It is reported that Kapton[®] exhibits significant non-linear properties. Its elastic modulus varies with the excitation frequency, temperature, and the level of excitation (Tinker, 1998, and Leigh et al., 2001). Jenkins (2001) reported that typical membrane materials have time-dependent and nonlinear behavior.

2.1.2 **Rigidizable structures**

Typically, the gossamer structures need to maintain their internal pressure to keep their structural stiffness. Though, chances that meteoroid or other objects impact the structure are very low, it is desirable that the structure become rigidized after deployment. Hence, there would be no need to maintain the internal pressure to keep the structural shape.

Curing the material by the ultraviolet light in space, use of thermoset materials or water-based materials are three different rigidization techniques that were mentioned by Lewis and Inman (2001).

Addressing the rigidization issue, United Applied Technologies (Huntsville, AL, USA) designed and manufactured a new thin film casting process for toroidal satellite component, which once inflated could support its own structural shape even when its internal pressure was released. The flat thin sheets of Kapton 300JP[®] were formed into curved thin films with regular pattern of hexagonal domes. The inflated self-rigidizing torus, SRT, could support its own structural shape even when there was no internal pressure, due to the additional stiffness provided by the pushed out hexagonal domes.

2.2 Experimental Modal Analysis

2.2.1 Smart materials

Intelligent or smart structures are a subset of active structures. They have highly distributed actuator and sensor systems, which are part of the load bearing system and have structural functionality, and distributed control functions and computing architectures as well (Crawley, 1994). Piezoelectric materials, shape memory alloys, electrostrictive materials, magnetostrictive materials, electrorheological, magnetorheological fluids and fiber optics have all been integrated with structures to make smart structures (Sethi and Song, 2005). Among these materials, piezoelectric and magnetostrictive materials have the capability to serve as both sensors and actuators (Pradhan and Reddy, 2004). Ceramics and polymers are two general class of piezoelectric materials used in vibration control. The piezopolymers require extremely high voltages and have limited control authority; hence they are used typically as sensors. The well-known piezopolymer is the polyvinylidene fluoride (PVDF or PVF2). Piezoceramics are used as both actuators and sensors for a wide range of frequency. The well-known piezoceramic is the Lead Zirconate Titanate (PZT) (Preumont, 2002).

A piezoelectric film actuator's weakness is that it provides small forces. Jha (2002) mentioned that the traditional piezoelectric materials, e.g., PZT and PVDF, induces strain coefficients in the vertical direction (d_{33}) higher (almost double) than the induced strain coefficients in the longitudinal direction (d_{31} , d_{32}), Fig. 2.3. While the actuation effect is due to longitudinal coefficients, for the in-plane actuation, it is more effective to use the d_{33} coefficient.



Figure 2.3 A monolithic piezoelectric wafer with coordinate system (Jha, 2002).

Monolithic PZT imposes certain limitations for its use in practical applications. The PZT material's brittle nature requires additional attention during the handling and bonding procedures. Besides, their poor conformability to curved surfaces necessitates extra surface treatment and further manufacturing capabilities (Sodano et al., 2004).

Motivated by these facts, two enhanced performance piezoelectric actuators have been developed. They are the Macro-Fiber Composite actuator (MFC) developed by NASA Langley Research Center (Wilkie et al., 2000) and the Active Fiber Composites actuator (AFC) developed by MIT (Hagood and Bent, 1993, Bent and Hagood, 1995, Bent, 1997 and 1999). The new high performance MFC/AFC actuators are typically directional or anisotropic and offer much higher flexibility, conformability, induced strain energy density and durability than its monolithic piezoceramic predecessors (Azzouz et al., 2001, Sodano et al., 2004, and Ruggiero et al., 2005). They employ the interdigitated electrodes and utilize the high d_{11} piezoelectric charge constant, which is normally two times more than d_{31} , Fig. 2.4. In a composite actuator, the piezoceramic fibers and electrodes are placed in such a way that the electric field coincides with the direction of induced strain along a preferred direction. This leads to a higher directional dependent electro-mechanical coupling. Since the MFC is a piezoelectric device; its linear constitutive behavior is governed by the strain-stress-field equation for such materials.

MFC[®] comes in a variety of sizes and abilities, and has low mass, fast time response, and negligible added stiffness, which makes it an excellent candidate for the control and dynamics application of inflatable structures (Sodano et al., 2004).





2.2.2 Experimental modal analysis of gossamer spacecraft and inflated torus

Main et al. (1995) performed modal tests of an inflated beam in a ground laboratory as well as a near weightlessness low gravity simulator aircraft. They quoted the possible sources of damping of the inflated beam as viscous damping due to moving through the outside air, viscous damping from motion and compression in the enclosed gas, and damping resulted from the beam fabric stretching. The team showed that the pressurization stress levels in the beam's fabric affect the strain-rate damping. It was also shown that the viscous damping was independent of pressure. Owing to the inflated structures relative flexibility, these structures are prone to wrinkles in their fabric skin. Hence environmental factors can have a dramatic effect on the structure behavior. It was shown that in the ground test, due to the changes in the stress distribution in the fabric, the damping estimates were too high or too low.

Griffith and Main (2000) used impact hammer and accelerometer to identify natural frequencies and structural and viscous damping of the first three in-plane and out of plane bending modes of a torus. Their test structure was an inflated Kapton HN^{\otimes} torus with a 1.98 m major diameter and a 0.15 m tube diameter. They studied the shell pre-stress effect on modal parameters for two inflation pressures of 5.52 kPa (0.8 psig) and 6.89 kPa (1.0 psig). The existence of shell modes made the determination of higher order bending modes from frequency response function difficult. In order to minimize the accelerometer's mass effect, the measurements were taken in a direction perpendicular to gravity. By modifying the hammer tip, they distributed the impact energy to a large area and were able to excite the inflatable torus global modes. For the in-plane measurement, the torus was suspended at three points by monofilament line, while a single strand was used for the out-of-plane testing. The pendulum rigid body mode frequency for the inplane and out-of-plane configuration was 0.3 Hz and 1 Hz, respectively. At 5.52 kPa, the first in-plane and out-of-plane frequencies were 16 Hz and 13.30 Hz, respectively. They found that a possible acoustic resonance of the enclosed gas (air) is present. Natural frequencies increased with increasing internal pressure and modal damping ratio followed the opposite trend. They showed that structural damping increases with increasing internal pressure while the viscous damping follows the opposite trend. Modal damping for the out-of-plane modes was generally higher than those for the in plane modes. Their study of damping was based on only two pressurization levels. For a better understanding of damping behavior, they suggested performing more tests in both ambient and vacuum condition. A significant drop in the coherence broadband at a lower pressurization level of 5.52 kPa was observed; the maximum coherence was below 0.8. They explained that the drop in the coherence could be due to the structure nonlinear behavior for the whole frequency range of study or an additional input from the suspension. Since the broadband drop in coherence was not found at the higher pressure, 6.89 kPa, the possibility of additional input from the suspension was ruled out. They concluded that since at lower pressure levels, the structure is less stiff and more apt to wrinkle, more nonlinearity is expected and it is why the coherence dropped.

Slade et al. (2000) and Slade (2001) conducted modal testing on the Pathfinder 3, prototype inflatable structure for the Shooting Star Experiment (SSE). The prototype inflatable solar concentrator consisted of a torus/lens assembly supported by three struts. This was attached to a circular aluminum plate which simulated the engine interface attachment and provided connections for inflation hardware. Tests were conducted in the atmospheric pressure and room temperature conditions as well as in a thermal vacuum chamber. Owing to the Kapton[®] film high flexibility, instead of exciting the structure directly on the inflatable surface, a single point random shaker excitation was used at a location on the interface plate. As for the noncontact measurement, a scanning laser vibrometer was employed. They took data at 70°F for three inflation pressures of 0.25, 0.5, and 0.75 psig, and thereafter, the test was conducted at cryogenic environment, -55° F and pressure of 0.5 psig. Owing to a number of factors, including the limitation of the shaker, high flexibility and high damping of the inflatable components, local shell modes, increased brittleness of the film and loss of shaker performance at low temperature and the choice of structural suspension during testing, they could not get consistent results. They found out that there are considerable differences between the structure performance in vacuum and atmospheric environments. Only a few mode shapes occurred for both the vacuum and ambient conditions. While all modes had high damping (>>1%), the first few modes were very heavily damped. The cryogenic test case showed lower damping for the lower modes than the ambient test one, which was expected due to the presence of viscous damping (air resistance) in the ambient test. The damping level in the cryogenic test was even lower than that of the other vacuum tests. This indicated that decreasing the temperature increased the brittleness of the structure and decreased the damping level. Their study showed an increasing trend of natural frequencies with increasing internal pressure.

The modal analysis techniques used by Griffith and Main (2000), Slade et al. (2000) and Slade (2001) highlighted some of the disadvantages associated with traditional techniques in modal testing of extremely lightweight inflatable structures.

Several other methods of modal testing are available, and one method that shows increasing promise is the use of smart materials particularly piezoelectric patches.

Agnes and Rogers (2000) tried to obtain a frequency response function of an inflated torus, a children's swimming pool with the floor panel removed. The torus ring diameter and tube diameter were 152 cm (60 in) and 40.5 cm (12 in), respectively. The torus was excited using a conventional electrodynamic shaker and separately, with a PVDF patch. The structure vibratory response was measured using a laser vibrometer around the perimeter of the torus face. Both the shaker and the PVDF patch excited the torus with a chirp signal from 0.5–50 Hz. They identified the resonant frequencies using the multivariate mode indicator function. Further analysis deemed impossible, due to "the presence of significant nonlinear effects, noise attributable to unmeasured disturbances,

and the low level of signal in both tests" (Agnes and Rogers, 2000). The results demonstrated that the PVDF excitation produced less interference with the suspension.

Park et al. (2002a) performed a modal analysis of an inflated rubber tire inner tube. They used an electromagnetic shaker as a point input to the torus, and compared the frequency response functions generated using an accelerometer and a PVDF patch as sensors. The work also included an investigation of MFC actuators. The MFC actuators and the PVDF sensors hardly interfered with the suspension modes of the torus. With rubber wire suspension, the rigid body modes occurred at 1-2 Hz, which was negligible compared with the frequency range of interest, 10-200Hz. Finally, they attempted to control the vibration of the fourth out-of-plane mode using a positive position feedback controller. Both the four-layer bimorph PVDF actuator and MFC actuator were able to reduce the vibration of the rubber torus by approximately 50%. Fig. 2.5 shows both unimorph and bimorph patch configuration on a shell.



Figure 2.5 A shell with unimorph and bimorph patches (Jha, 2002).

Park et al. (2002b and 2003) performed a similar set of experiments on the same Kapton[®] torus originally tested by Griffith and Main (2000). They excited the Kapton[®] torus using an electromagnetic shaker as well as an MFC actuator and they measured the frequency response function using an accelerometer and PVDF sensor. The team demonstrated that MFC actuators and the PVDF sensors hardly interfere with the suspension modes of a free-free torus. While the torus was suspended with a single rubber wire, the rigid body modes occurred at 1-2 Hz. The first identified damped natural

frequency was 12.81 Hz. Since the MFC actuators were exciting both planes of motion; the resulting mode shapes demonstrated considerable contributions from both planes of excitation. They mentioned that the MFC actuator's mass loading is negligible, which is a great advantage compared to conventional sources of excitation. Since the in-plane measurement points were located along the flaps, the structure's stiffest part, getting the in-plane modes was difficult. In contrast, the out-of-plane modes were much easier to collect. Once again, as they found in their earlier research (Park et al., 2002a), the sensors clearly picked up the signal 180° apart from the actuator (on both sides). It was concluded that the MFC actuator was able to excite the torus global modes and the PVDF sensor was able to obtain frequency response functions with high coherence. In all experimental studies, the frequency response functions were exhibiting a noticeable amount of noise, which could be due to the shell modes being present in the bandwidth.

Single input single output (SISO) modal testing techniques are unable to reliably distinguish between pairs of modes that are inherent to axi-symmetric structures like an inflated torus. Furthermore, it is questionable as to whether a single actuator could reliably excite the global modes of a true gossamer craft, such as a 25 m diameter torus. Ruggiero et al. (2002, 2004a) demonstrated the feasibility of using multiple MFC patches and sensors to excite an inflated torus with high coherence, multiple input multiple output (MIMO). Their test structure was an inflated Kapton[®] torus with a 1.8 m ring diameter and a 0.15 m tube diameter and internal pressure of 0.5 psi. The torus was suspended with a single rubber wire with a rigid body mode occurred between 1 and 2Hz; the first identified damped natural frequency was 13.26 Hz. Since the measurement points for the in-plane modes were located along the joined region, torus' stiff part, it was difficult to get the in-plane motions. The authors used two half patches of MFC as sensors and two full patches as actuators. The sensors clearly picked up the signal within 180° of the actuator (on both sides), which once again indicated that the MFC actuator was able to excite the torus global modes. Using multiple actuators, they properly excited the structure global modes and distinguished between pairs of modes at nearly identical resonant frequencies. They showed that MFC excitation produces less interference with suspension modes of the free-free torus than excitations from a conventional shaker. The MIMO technique was able to decipher the modal asymmetry. This phenomenon happens when the mass imbalance on the symmetric structure splits the resonant peak in two. They concluded that MIMO testing provides a better energy distribution and even actuation force and requires less power than the SISO testing.

Sodano et al. (2004) showed that an MFC could be used as a sensor and actuator to find modal parameters of an inflatable structure and to attenuate its vibration. They developed a self-sensing (collocated sensor and actuator) circuit for an MFC. Their experimental results indicated that this strategy could suppress structural vibration, while reducing the number of system components. For an inflatable torus (the same torus as in Ruggiero et al., 2004a), they implemented a positive feedback control system that consisted of two pairs of sensors and actuators and reduced the vibration of the first mode by approximately 70%.

Ruggiero et al. (2004b) performed experimental modal analysis of a self-rigidizing torus, SRT, using a 5lb electromagnetic shaker and two sets of accelerometer, single input multiple output (SIMO). The torus was suspended vertically using two rubber wires causing a pendulum frequency of approximately 3.5 Hz. The SRT was excited with sine sweep, 1-12 Hz, once in the out-of-plane direction and then in the in-plane direction. The shaker's stinger was attached to the torus by beeswax. The measured out-of-plane frequencies were 2.0, 4.9 and 5.5 (split mode pair), and 11.2 Hz. The only identified inplane frequency was 3Hz. Their in-plane measurement was taken along the glue-stiffened peel seam (flaps) and owing to inertial loading on the structure, using the accelerometer caused frequency shift problems. In their study, the pendulum frequency was higher than the first natural frequency. The rule of thumb is that the suspension frequency should be less than ten percent of the first natural frequency (Allemang, 1999).

Later, Ruggiero and Inman (2005) compared the SISO and MIMO modal analyses techniques for the nonlinear dynamic behavior of the inflated Kapton® torus and membrane mirror satellites. Their test torus had a 1.8 m ring diameter and a 0.15 m tube diameter. An aluminized thin film of Mylar was cut and bonded to the interior of the torus ring. The internal pressure was maintained at 2758 Pa. The test structure was suspended at one point with rubber wire. The rigid body mode occurred at frequency of below 2 Hz. The first damped natural frequency was identified at 11.18 Hz. MFC patches were employed as sensors actuators. Two half patches were used as sensors and two full patches as actuators. It was observed that during a MIMO test, there was less interference or disturbance of the test setup as compared to the SISO methodology. However, the MFC sensor was sensitive to the electromagnetic effect of the surrounding equipment. The MIMO methodology required fewer numbers of tests and testing time than the SISO one. The identified damping ratios at each mode were substantially different between the two methodologies. They concluded that, while the SISO methodology has the weak point of assuming a linear behavior for the test's structure; it provides quite accurate mode shapes. The MIMO methodology not only provided fairly accurate mode shapes, but also, even in the presence of the nonlinearities, gave a better estimate of the test structure's modal parameter. To protect the prototype satellite component from air currents generated by the laboratory's air conditioning, a localized isolation chamber was built around it.

In 2006, Song et al. used a 30.5 cm (12 inch) subwoofer to acoustically excite the SRT structure by a low frequency white acoustic noise, 1-200 Hz. The torus' displacement response was acquired by laser displacement sensor, SISO. The SRT was suspended horizontally at three points with 3.2 meter long monofilament lines causing a pendulum frequency of 0.28 Hz. The torus was excited once in the out-of-plane and then in the in-plane direction. The damped natural frequencies and modal damping were

evaluated. The out-of-plane resonant frequencies were 2.88, 9.51 and 14.61 Hz (split mode pair). In the in-plane direction the frequencies were 5.07, 13.02 and 22.08 Hz. Only the first two in-plane modes exhibited the expected classical ring mode shape. The third in-plane mode had the oval shape of the first mode. In the frequency response function calculation, instead of the actual acoustic signal, the white noise signal from the signal generator was used as the input signal. Achieving frequencies of below 18 Hz with a 30.5 cm subwoofer seems almost unlikely. This was tested by the author with a 25.4 cm (10 inch) subwoofer.

In both previous studies on the SRT, the inflation nozzle was left open to the atmospheric pressure which made the SRT opt to wrinkle and sag in the gravity force presence.

From this survey, we can conclude that modal testing of inflatable structures presents a number of challenges, e.g., maintaining the desired constant pressure, pump noise, airflow into the structure while maintaining the pressure, pneumatic resonance, possible inadequate excitation, limited number of measurement points, coupling between the suspension system and the structure.

2.3 Analysis and Modeling of Gossamer Structures

Modeling of membrane response may be done by using one or more of the following procedures:

- Classical mathematical analysis,
- Numerical analysis.

The classical mathematical analysis yields closed-form solutions for only a few special simple cases of membrane problems. Hence, for general cases of membrane problems the numerical analysis must be employed. Experimental analysis may be applied to produce results for a specific case, or to verify and tune more general mathematical or numerical models.

Membranes are intrinsically nonlinear. Their degree of nonlinearity, which is integrated into the field equation, depends on the formulation philosophy. "Thus, theories may be developed ranging from linear to small strain-finite rotation (geometric nonlinearity only) and finite strain formulations that are fully nonlinear both in geometry and materials (Jenkins, 2001)."

In the field of structural design of inflated structures a great amount of work has been done, while, the work performed in the structural analysis area is limited (Griffith and Main, 2002).

2.3.1 The internal pressure effect

The internal pressure produces a pre-stress (initial stress) field in the inflatable structure which affects the load bearing capability, i.e., the structure's effective stiffness. The pre-stresses can be calculated in the static analysis and then incorporated in the dynamic equations.

In addition to the pre-stress effects, the internal pressure also induces the follower load, which arises because the pressure force tends to act normal to the deformed surface during the vibration. Thus, as the shell vibrates, the force direction changes, producing a displacement-dependent force. The stiffness generated by this deflection-dependent force also alters the structure's effective stiffness. This effect is called the follower action of the pressure load (Jha and Inman, 2004a).

2.3.2 Analytical and numerical studies of an inflated torus

Timoshenko (1940) used linear membrane theory to perform a static analysis of an inflated torus. Though his classical solution gave acceptable stress distribution, the displacement solution had singularities. Since then, many researchers proposed alternative solutions in order to find a more accurate analysis and remove the displacement singularities. Ruggiero et al. (2003) performed an extensive literature survey of the analytical study of an inflated torus. They named Jordan (1962), Reissner (1963), Sanders and Leipins (1963) as the researchers who used nonlinear membrane theories. They also named Colbourne and Flügge (1967) and Leonard (1988) as researchers who discussed the effect of pre-stress on the membrane structure while the latter addressed the nonlinear behavior as well. More on the literature survey on pressurized arches, which resemble incomplete inflated tori, and buckling and stability of inflated torus can be found in Ruggiero et al. (2003) and references therein.

Since the satellite structures are exposed to a variety of time-varying loads, studying their dynamic behavior is very important. The free vibration analysis is necessary in order to get the natural frequencies and mode shapes as basis for forced vibration analysis and control design. Free vibration analysis of toroidal shells without pressure have been studied by many researchers (Jha et al., 2002 and references therein); there have been few studies on toroidal shell subjected to pressure. Liepins (1965a and 1965b) performed

a linear free vibration analysis of a toroidal membrane under internal pressure. He solved the governing equations using a finite difference method and derived the natural frequencies and mode shapes by a trial and error method in the Holzer fashion. He found that when the torus tube radius is very small compared to the torus ring radius, low aspect ratio, and the pre-stress is large, the torus vibrates in ring modes. He also observed that only the lower frequencies are affected by the pre-stresses induced by the internal pressure.

Jordan predicted the vibratory frequencies of a pressurized torus shell using the Rayleigh quotient, (1966), and performed vibration testing on a 1.4 m diameter complete circular aluminum torus shell for both free and rigidly held boundary conditions (1967). He realized that during the test, structural-pneumatic interaction has occurred. Jordan proved the existence of the acoustic resonance of the air inside the shell by replacing part of the air by helium. Liepins (1967) extended his previous work, on the free vibrations of pressure pre-stresses toroidal shells with constant thickness and various boundary conditions based on the linear membrane theory. His results agreed well with the experimental results of Jordan (1967). Saigal et al. (1986) obtained a closed-form solution for the natural frequencies and mode shapes of a pre-stressed toroidal membrane with fixed boundary conditions. They narrowed down their study to a torus with very small aspect ratio and assumed that the in-plane displacement components of their torus are zero and that the torus ring radius are very large compared to the tube radius (very small aspect ratio). The results were valid only for special cases due to the very limiting assumptions made to simplify the governing equations. None of the mentioned researchers dealt with the inflatable toroidal shell.

Slade and Tinker (1999) and Slade (2000) studied the dynamics of an inflatable cylinder using both closed-form and finite element beam model. They used beam theory to calculate the bending stiffness of struts, EI, at each modal frequency. A nonlinearly decreasing relationship between stiffness and frequency was obtained. Then, they used MSC NASTRANTM and MSC PATRANTM to simulate the behavior of the SSE Pathfinder 3 article. Both frequency-dependent beam models and shell models were constructed and compared for their efficiency. Owing to the limitation of NASTRAN, moment of inertia, I, was held constant and E varied with frequency. It was pointed out that the idea of calculating an effective E for each mode was based on the assumption that the structure behaves as a beam (constant I). Thus, this assumption had a limited validity for a shell model. Since, the material properties were not constant; it was not possible to use eigensolution method to find the modal properties of the structure. As an alternative, a forcing function at the same RMS level as in the vacuum test (Slade et al. 2000) was applied to the model. The frequency response solution was generated, and from the frequency response function the resonant peaks were identified. While the shell models were more complex and computationally intensive, they provided more insight in to the dynamics of the structure than the beam models. In the shell models, several modes were found that were not detected in the modal tests of the structure.

Smalley et al. (2001) included the pressure loading effect on the structural stiffness in the modeling and modal analysis of thin film inflatable strut. The stiffness matrix was generated in a nonlinear static analysis in MSC/NASTRANTM software and thereafter the generated stiffness matrix was imported into an eigenvalue analysis. The methodology was verified by comparing the analytical model and modal test results. Free-free and cantilever cases were analyzed at various pressures. For the free-free cases the results fell within an acceptable margin while for the cantilever cases large errors occurred. The paper recommended an investigation in the use of nonlinear material properties.

Williams et al. (2001) modeled the interaction of PVDF patches attached to an inflated torus using pre-stressed flat membrane theory; membranes cannot resist bending moments. The study was performed both analytically and numerically. The Rayleigh-Ritz method and the ANSYSTM finite element software were used to approximate the natural frequencies and mode shapes of the layered system. Finite element analysis was performed with both membrane and plate elements and the results were compared with the analytically obtained results. They investigated the feasibility of using membrane theory to account for the effects of surface-mounted piezopolymer patches used as sensors or actuators. They found that the membrane theory is able to account for only the patch's added mass, not the additional stiffness. Besides, the excitation of transverse vibration was impossible using membrane theory. However, the actuation was simulated as an applied in-plane force at the base layer's neutral axis, which gave the PVDF patch the ability to suppress out-of-plane disturbances by changing the tension in the base layer as a function of the applied voltage. While they were able to model the piezoelectric patches actuation behavior, the sensing behavior modeling seemed unfeasible with the membrane theory. Moreover, due to the fact that membranes do not have bending stiffness, it was not possible to predict the excitation of transverse vibration and the amount of control authority was highly restricted. In the end, they concluded that considering the membrane theory limitation for modeling the layering of thin films, thin plate theory which allows the tension to change in the film as it undergoes transverse vibration, must be used. They demonstrated that the addition of PVDF sensors slightly decreases the torus natural frequencies, as expected from the additional mass, but has little effect on the mode shapes.

Lewis (2000) and Lewis and Inman (2001) analyzed a pre-stressed inflated torus using the linear thin shell elements, Shell93, of the commercial finite element software package ANSYSTM. Shell93 elements have eight nodes, four corner and four mid-side nodes. Each node has six degrees of freedom, translation in and rotation about the x, y, and z axis. The torus under the study had circular cross-section and was made out of a film of Kapton[®] with PVDF piezoelectric patches on it. The patches were placed on the torus by collocating additional Shell93 elements on top of the torus elements. The boundary condition was taken as free-free. They investigated the effect of torus aspect ratio and internal pressure on the vibratory responses, natural frequencies and mode

shapes. The study showed that at small aspect ratios, all the lower modes were orthogonal pairs of ring modes. At the higher order modes, other types of modes showed up, e.g., sloshing, breathing and axisymmetric modes. As the aspect ratio increased, the sequence of modes changed dramatically and ring modes moved to higher order modes and the other type of modes appeared in the lower order modes. It was demonstrated that increasing the pressure resulted in increased natural frequencies. While it was mentioned that for large aspect ratios, increasing the pressure, resulted in the order of mode shapes changing significantly, they did not address this change specifically. The mass and the sloshing effects of the enclosed gas on the torus dynamics were ignored. In their finite element model, Kapton[®] and PVDF were assumed to lie in the same plane instead of being layered, hence only part of the PVDF stiffness was considered and the model had a slightly lower bending stiffness than the actual structure. The study showed that adding the patches had slightly lowered the torus frequencies, but at least for the first fourteen modes, the mode shape sequences did not change. At the end, the torus' vibration was attenuated using two different control tools, observer-based full state feedback and direct output velocity feedback.

Leigh et al. (2001) used both linear beam and shell elements, of the finite element code MSC/NASTRANTM, to model an inflated torus along with three struts and compared the results with experiments. The beam element was generally unable to predict the modes. The lack of correlation between the model and test was blamed on the limited number of measurement points and the probability of beam and shell modes coupling. Varying thickness and the glued joints modulus, as well as the flaps and accelerometer added mass were included in their model. In order to account for the weight of the pressurized air inside the structure, they included nonstructural mass in their model. For the torus alone (without the joining flaps), the model's results were compared with the experimental results of Griffith and Main (2000). They qualified their results as "encouraging", although both in-plane and out-of-plane measurements were high. They also ran into some difficulties because the flaps' numerous local modes had obscured the torus modes of interest.

Both the finite element study by Lewis (2000) and Leigh et al. (2001) predicted that the shell mode activities are less significant for small aspect ratios (less than 0.1), i.e., the structure demonstrated mainly ring mode shapes.

Lewis and Inman (2001), Leigh et al. (2001) and Park et al. (2003) included only the pre-stress effects due to the internal pressure and ignored the follower action of pressure force and they used linear shell elements with linear material properties.

Owing to the wall's small thickness and relatively high internal pressure, the initial deflection in an inflated structure becomes quite high. This necessitates the consideration of geometric nonlinearity.

Jha (2002), Jha et al. (2002), Jha and Inman (2004a) used Sanders' shell theory and derived the static and dynamic equations for a thin shell under pressure using Hamilton's principle. To take into account the pre-stress effects of internal pressure, the geometric nonlinearity was used, and to model the follower action of pressure force, the work done by internal pressure during the shell vibration was considered.

Jha et al. (2002) performed free vibration analysis using Galerkin's method and Fourier series. They studied the effect of aspect ratio, pressure and thickness on the natural frequencies of a free-free inflated torus of circular cross-section with linear material properties. It was demonstrated that increasing the aspect ratio and thickness, decreased the natural frequencies while increasing the pressure increased the natural frequencies, Jha (2002) and Jha et al. (2002).

Jha and Inman (2002) modeled an actuator and sensor made of piezoelectric material patches attached to a pressurized inflated torus using Sander's linear shell theory. The torus had circular cross section with a free-free boundary condition. They considered the effect of the patches' stiffness and mass on the actuation and sensing capabilities. Both unimorph and bimorph piezoelectric patches arrangement were studied. They found out that a bimorph actuator configuration does not provide any better actuation than a unimorph one. As a special case of inflated torus, a circular cylinder was studied. The actuators' mass and stiffness effects on the shell response were found to be negligible. They also studied the effect of the piezoelectric patches' locations and sizes on the modal controllability and modal observability. They showed that a higher patch area may not provide better actuating and sensing capabilities. It was also shown that the considered sensor structure could not detect torsional and rigid-body modes. Later, employing a genetic algorithm, these models were used to find optimal sizes and locations of actuators and sensors so that the actuators and sensors provided good control and sensing capabilities (Jha and Inman, 2003). Their study showed that the size of optimal actuators were bigger than the one of the optimal sensors. Using the optimal actuators and sensors, they applied linear-quadratic optimal controller to demonstrate the inflated torus vibration attenuation. Jha (2002) and Jha and Inman (2004b) used a sliding mode controller and observer for the vibration control of an inflated torus using MFC actuators and PVDF sensors. It was found that, due to the close natural frequencies and complicated mode shapes, vibration control of an inflated torus is relatively difficult yet feasible with piezoelectric patches. A large number of actuators and sensors were needed for good controllability and observability and a large number of controlled modes were needed to reduce the spillover effect. They also studied the controller and observer robustness properties against the model uncertainty, unknown disturbances, and spillover effects.

Park at al. (2003) developed a predictive model for a pre-stressed Kapton[®] inflated torus with its joining flaps, using the linear shell elements of the commercial finite element software package ANSYSTM. They did not provide more details about the
type of element used in the model. In their study, the mass and sloshing effects from internal gas on the torus overall dynamics were not considered. Since the enclosed air mass would be on the same order as the torus mass, the exclusion of these effects was quoted as a source of deviation between the modeling and the experiment results. For the first three ring modes, the difference in the natural frequencies between their finite element analysis (FEA) and shell theory, Jha 2002, was as high as 17%. For the first seven natural frequencies associated with ring modes, the difference between the FEA and their experimental results varied between 3.6-19%; the in-plane and out-of-plane modes sequence was adequately predicted. They mentioned that the presence of flaps in finite element analysis (FEA) lowers the natural frequencies and affects the mode shapes. In the presence of flaps, the out-of-plane modes came before the in-plane modes while by excluding them in the FEA model, the in-plane modes came before the out of plane ones.

In the vibration analysis of an inflated structure, the direct action of the internal pressure force, which is in addition to the pre-stress' effects, and accurate geometric nonlinearities, must be considered. Failure to do so results in inaccurate rigid-body and non-rigid-body natural frequencies and mode shapes as shown by Jha and Inman (2004a). They presented the importance of geometric nonlinearity and a comparison of the results (natural frequencies and mode shapes) obtained by employing different approximate shell theories.

Jha (2002) pointed out the shortcomings in the results obtained by using commercial finite element codes for an inflatable torus. An inflated torus with a free boundary condition should have six rigid-body modes with six zero natural frequencies. The commercial finite element codes used by Lewis and Inman (2001), Leigh et al. (2001) and Park et al. (2003) did not take into account the direct effect of pressure force as well as the geometric nonlinearity, hence they gave inaccurate natural frequencies and mode shapes for non-rigid-body modes and produced nonzero rigid-body natural frequencies.

The literature survey for the dynamic analysis of inflatable toroidal shells made of Kapton[®] is summarized in the Table 2.1.

Table 2.1Summarized literature survey on dynamic analysis of Kapton[®] inflatable torus

Author	Year	Structure	Analytical	Finite element	Geometric nonlinearity	Follower load	Mass of the enclosed gas	Material nonlinearity	Actuator in Modal experiment	Sensor in Modal experiment	Vacuum condition
Griffith and Main	2000	Torus							Impact hammer	Accelerometer	
Griffith and Main	2002	Torus							Impact hammer	Accelerometer	
Jha	2002	Torus	X		X	X					
Jha et al.	2002	Torus	X		X	X					
Jha and Inman	2002	Torus / Piezoelectric	X			X					
Jha and Inman	2003	Torus / Piezoelectric	X		X	X					
Jha and Inman	2004a	Torus	X		X	X					
Leigh et al.	2001	Torus / flaps / lens / struts		X	X		X				
Lewis and Inman	2001	Torus / PVDF		x					,		
Park et al.	2002a	Torus / flaps							Shaker / MFC	Accelerometer / PVDF	
Park et al.	2002b	Torus / flaps							Shaker / MFC	Accelerometer / PVDF	
Park et al.	2003	Torus / flaps		x					Shaker / MFC	Accelerometer / PVDF	

Author	Year	Structure	Analytical	Finite element	Geometric nonlinearity	Follower load	Mass of the enclosed gas	Material nonlinearity	Actuator in Modal experiment	Sensor in Modal experiment	Vacuum condition
Ruggiero and Inman	2005	Torus / mirror							MFC	MFC	
Ruggiero et al.	2002	Torus		_					MFC	MFC	
Ruggiero et al.	2004a	Torus							MFC	MFC	
Ruggiero et al.	2004b	SRT				i			Shaker	Accelerometer	
Slade	2000	Torus / lens / struts		X				x	Shaker	Laser vibrometer	X
Slade et al.	2000	Torus / lens / struts		 					Shaker	Laser vibrometer	X
Slade	2001	Torus / lens / struts							Shaker	Laser vibrometer	X
Sodano et al.	2004	Torus							MFC	MFC	
Song et al.	2006	SRT							Loud speaker	Laser displacement	

2.4 Summary and Proposed Research

As pointed out earlier, the goal of this research is to understand the dynamic characteristics of the self-rigidizing torus, both experimentally and analytically.

There are a number of unresolved issues in the existing experimental modal analysis techniques related to the excitation and sensing methodologies. Previous modal experiments of the SRT did not detect the in-plane modes. Ruggiero et al. (2004b) derived only the first in-plane mode, and in the study by Song et al. (2006) only the first two in-plane modes exhibited the expected classical ring mode shape. The third in-plane mode had the oval shape of the first mode (Song et al., 2006). Considerable improvements in the modal experiment can be achieved by employing non-contacting excitation and sensing transducers, which is precisely the case in the present research. For the verification of the finite element analysis, a more precise modal experiment of the SRT is required. The first contribution of this research is:

The development of a more accurate experimental modal analysis of the SRT, using non-contact excitation and sensing method.

The second contribution is:

The design and manufacturing of a non-contact electromagnetic excitation system and using it in the modal experiments.

While some experimental studies are available for the SRT, as noted in Table 2.1, analytical structural dynamic studies of these types of structures lag behind the experimental one even for the inflatable pressurized smoothed-surface torus. Analytical effort for the inflatable torus has mostly focused on quasi-static effects, such as shape, configuration, strength, deployment, and wrinkling (Fang and Lou, 1999). The finite element analysis performed by Lewis and Inman (2001), Leigh et al. (2001) and Park et al. (2003) was for a pressurized smoothed-surface torus. These studies did not take into account the direct effect of pressure force as well as the geometric nonlinearity. Therefore, apart from giving inaccurate natural frequencies and mode shapes for nonrigid-body modes, they produced nonzero rigid-body natural frequencies. Up to date, the finite element modal analysis of the torus with hexagonal domes has not reported. Prior to any attempt to model the SRT, a preliminary finite element analysis of a complete pressurized torus with smoothed-surface must be performed and validated. The results must be compared with the published analytical results. For a more accurate finite element analysis, both the geometric nonlinearity and the effect of the follower load on the stiffness must be included in the analysis. The third contribution is:

The accurate finite element modal analysis of a complete torus using the commercial finite element software while including geometric nonlinearities and the effect of the follower load on the stiffness for more accurate analyses.

The fourth contribution is:

The sub-structuring approach for finite element modeling of the hexagonal domes of the SRT and the finite element modal analysis of the self-rigidizing torus using the proposed sub-structuring technique.

Since the bonded regions have higher stiffness than that of membrane, they have to be accounted for in the modeling of the SRT. The fifth contribution is:

The inclusion of the bonded regions in the modeling of the SRT.

In order to include the bondings, the material properties of the epoxy used in those regions are required. Owing to the lack of manufacturer data, the sixth contribution is:

The determination of the modulus of elasticity of the 3M scotch-weld epoxy adhesive 2216 B/A translucent[®].

Owing to lack of data in open literature showing the relation between the modulus of elasticity of Kapton $300JP^{\text{(R)}}$ and the frequency, this dependency needs, first, to be investigated. If the modulus of elasticity exhibits significant material nonlinearity, this should be included in the finite element model. The seventh contribution is:

The determination of the variation of the modulus of elasticity of Kapton $300 JP^{$ ® versus frequency.

It is believed that the finding of the current research will improve the state of knowledge of the dynamic characteristics of the SRT. This will help in the development of better techniques for its vibration attenuation.

CHAPTER 3 TEST STRUCTURE AND MATERIAL PROPERTIES

The physical properties of the self-rigidizing torus, SRT, which is the structure under focus of this research, are described in Section 3.1. While the modulus of elasticity of Kapton $300JP^{\text{(B)}}$ is derived using tensile test, in subsection 3.2.1., with the dependency of this property on frequency being examined in subsection 3.2.2. The modulus of elasticity of 3M scotch-weld epoxy adhesive, which is used in joining regions of the SRT, is obtained in Section 3.3.

3.1 Self-rigidizing Torus, SRT

Three SRT prototypes were designed and manufactured at the United Applied Technologies (Huntsville, Alabama). The first two SRT were used by Ruggiero et al. (2004b) and Song et al. (2006), respectively. The structure which is the focus of modal testing and modeling in the present thesis, is the third SRT prototype.

The tested structure was an extremely flexible self-rigidizing inflatable torus. The 181 cm ring diameter and 22 cm tube diameter torus was constructed of thin films of Kapton 300JP[®]. The flat 76 micrometer thick Kapton[®] sheets were formed into curved 46 micrometer thin films with regular pattern of hexagonal domes of 32.5 micrometer thickness, Fig. 3.1.a. The top and bottom of the torus, which each were constructed from seven joined segments at shear seams, were joined together at the inner and outer peel seams (flaps) to form the complete torus, Fig. 3.1.b.

The "3M scotch-weld epoxy adhesive 2216 B/A translucent[®]" was used to join these fourteen segments. The width of the circumferential peel seams and the average width of shear seam were 5.1cm (2 inches) and 2.1cm (0.82 inch), respectively.

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(a)



(b)

Figure 3.1 The self-rigidizing inflatable torus, (a) Complete torus; (b) Peel and shear seams.

Ideally, in the absence of the gravity force, the inflated torus, due to the additional stiffness provided by the pushed out hexagonal domes, could support its own structural shape even when there was no internal pressure. The geometric properties of the SRT are provided in Table 3.1.

Property	Value
Film thickness	46×10 ⁻⁶ m
Ring radius	0.907 m
Tube radius	0.111 m
Average shear seam width	0.021 m
Average peel seam width	0.051 m
Average shear and peel seam thickness	$2.88 \times 10^{-4} \text{ m}$
Average overlapped shear and peel seam thickness	$4.48 \times 10^{-4} \mathrm{m}$
Hexadome width (major radius)	7×10 ⁻³ m
Hexadome height	3×10 ⁻³ m
Hexadome thickness	$32.5 \times 10^{-6} \mathrm{m}$
Gap between the adjacent hexadomes	1×10^{-3} m
Mass	0.560 kg

Table 3.1.Geometric properties of the SRT.

3.2 Kapton 300JP[®] Material Properties

3.2.1 Tensile test

The elastic modulus of "Kapton 300JP[®]" was measured using the INSTRON 4411 tensile testing machine. The test was performed according to the "ASTM D882-02 Standard Test Method for Tensile Properties of Thin Plastic 0% relative humidity Sheeting". The specimens were cut according to "ASTM D 6287-05 Standard Practice for Cutting Film and Sheeting Test Specimens", by a dual blade cutter by KUTRIMMER, model IDEAL 1038.

Five samples were conditioned at least 48 hours prior to test at 25 degrees Celsius and 50% humidity. The tensile test performed with initial grip separation of 250 mm and

rate of grip separation of 25 mm/min and the initial strain rate of 0.1 mm/mm.min. The samples' width and thickness was 10.50 mm and 84 μ m.

The true stress and true stain were calculated using the following relations:

$$\varepsilon_{true} = \ln(1 + \varepsilon_{eng}), \qquad (3.1)$$

$$\sigma_{true} = \sigma_{eng} (1 + \varepsilon_{eng}).$$

The true stress versus true strain curves are presented in Fig. 3.2. The material exhibits Hookean behaviour at low strain. By compensating the toe region on the stress-strain curve, for the strain of 0.002 to 0.02, the modulus of elasticity was found to be 2.976 GPa, which compares well with the modulus provided by DuPont for Kapton300JP[®] (2.79 GPa).



Figure 3.2 Kapton 300JP[®], true stress versus true strain.

3.2.2 Dynamic material properties

Inflatable components were constructed from polyimide film materials that are highly nonlinear, with the modulus of elasticity varying as a function of frequency, temperature and the level of excitation (Tinker, 1998, and Smalley et al., 2001).

Since there was no source of data in the open literature that shows the relation between the modulus of elasticity of Kapton 300JP[®] and the frequency, this dependency was examined by performing the experiments with dynamic mechanical analyzer, DMA.

Using the dynamic mechanical analyzer, Q800 DMA system by TA Instruments, for the frequency range of 0.1 to 55 Hz, the loss and storage moduli of Kapton 300JP[®] were determined for the tension mode. In this mode, the sample was placed in tension between a fixed and a moveable clamp. The test was performed according to the "ASTM D4065-01 Standard Practice for Plastic: Dynamic Mechanical Properties: Determination and Report of Procedures".

In this dynamic oscillatory test, a sinusoidal strain was applied to the film and the resultant sinusoidal stress was measured. Then the complex modulus, stress / strain, was calculated. DMA measures the phase difference, δ , between the two sine waves (stress and strain) as well. For pure elastic materials this phase lag is zero and for pure viscous materials is 90 degrees. Viscoelastic materials exhibit an intermediate phase difference. The complex modulus, *E*, is formed by two quantities, storage modulus, *E'*, and loss modulus, *E''*. The storage modulus is a measure of the energy stored in a cycle and it is related to the sample's stiffness. On the other hand, the loss modulus is a measure of the energy lost in a cycle by the ability of the sample in dissipating the mechanical energy through molecular motion. This is the viscous component of the complex modulus (ASTM D 4092-01 Standard terminology for Plastics: Dynamic Mechanical Properties). The absolute value of the complex modulus is the modulus of elasticity. The relation between moduli and δ is as follows:

$$E = E' + iE''$$

$$(3.2)$$

$$\tan \delta = E'' / E'$$

The DMA test performed for six samples; the sizes are provided in Table 3.2. The control variable was set to 0.2% strain and for the frequency range of 0.1 to 55 Hz, the loss and storage moduli were measured at the temperature of 23 degrees Celcius. The post-experiment analysis on the raw data was performed using the Thermal Advantage Software from TA Instruments. For each sample, the variation of the loss modulus, storage modulus and the calculated modulus of elasticity (Eq. 3.2) with frequency are shown in Figs. 3.3, 3.4, and 3.5, respectively.

The mean tensile modulus of elasticity for the frequency range of 0.1 to 25 Hz and 0.1 to 55 Hz is 3.50 GPa and 3.53 GPa, respectively. It is observed that contrary to the storage modulus and the modulus of elasticity, which are only slightly frequency-dependent, the loss modulus is more frequency-dependent over the measurement range (Figs. 3.3. to 3.5.). Up to 5Hz, the increase in the storage modulus and the modulus of elasticity with increasing frequency is dramatic and above the frequency of 5Hz, this trend slows down. The loss modulus is two orders of magnitude smaller than the storage modulus. Hence, Kapton 300JP[®] behaves more like an elastic material than a typical viscoelastic one. Henceforth we can use the measured elastic modulus for the desired frequency range of 1 to 25 Hz.

Kapton[®] is a form of plastic in which its molecules are oriented randomly. The torus was made of a 76 micrometer (3mil) film which was heated, stretched and formed. Owing to this process, its thickness had changed significantly, to approximately 32.5 micrometer. Since the process of heating, stretching and particularly cooling was a slow process (not in the order of fraction of minute), the molecules were still oriented randomly. Thus, it was possible to still use the properties obtained for a 3 mil Kapton[®] sheet.

Sample number	Length (mm)	Width (mm)	Thickness (mm)
1	12.476	8.22	0.09
2	12.526	7.40	0.09
3	12.526	7.40	0.09
4	12.492	7.30	0.08
5	12.527	7.96	0.08
6	12.526	7.96	0.08

Table 3.2 DMA's sample size.



Figure 3.3 Loss moduli versus frequency plotted with two scales; top plot is in full scale and bottom one is in a smaller scale.



Figure 3.4 Storage moduli versus frequency; top plot is in full scale and bottom one is in a smaller scale.



Figure 3.5 Moduli of elasticity versus frequency; top plot is in full scale and bottom one is in a smaller scale.

The mechanical properties of the Kapton 300JP[®] are summarized in Table 3.3. In the absence of first hand experimental data and nominal material properties provided by DuPont for Poisson's ratio, the value used by all other researchers (e.g., Lewis and Inman, 2001) was adopted, i.e., 0.34. The measured mass density was 1405 kg/m³, which compares well with the density provided by the manufacturer, 1399.7 kg/m³.

Property	Value
Mass density	1399.7 kg/m ³
Poisson's ratio	0.34
DuPont elastic modulus	2.79 GPa
DMA elastic modulus	3.53 GPa
INSTRON elastic modulus	2.98 GPa

Table 3.3Mechanical properties of Kapton 300JP[®].

3.3 Epoxy's Modulus of Elasticity

The modulus elasticity of the "3M scotch-weld epoxy adhesive 2216 B/A translucent[®]," was measured with the tensile test of five samples of a layered film of Kapton 300JP[®]/epoxy/Kapton 300JP[®]. Sample sizes are listed in Table 3.4. As with the Kapton[®] tensile test, the samples were conditioned at least 48 hours prior to test at 25 degrees Celsius and 50% humidity. The tensile test performed with initial grip separation of 250 mm and rate of grip separation of 25 mm/min and the initial strain rate of 0.1mm/mm.min.

Sample number	Width (mm)	Thickness (µm)	
1	10.00	200	
2	10.00	200	
3	10.50	200	
4	10.50	220	
5	10.50	220	

Table 3.4Composite sample size

Fig. 3.6, illustrates two film of Kapton[®] (k) glued to each other by epoxy (e) and pulled by the force F at one end while being constrained on the other end. The width is unity and the thickness of Kapton[®] and epoxy are denoted by t_k and t_e , respectively.



Figure 3.6 Two films of Kapton[®] glued by epoxy.

Knowing the modulus of elasticity of Kapton 300JP[®] and the layered film, the modulus of elasticity of the epoxy could be evaluated. Considering the fact that each layer undergoes the same strain and the total force is the sum of each layer's force, the equivalent modulus of elasticity was calculated as follows:

$$\varepsilon_{total} = \varepsilon_{Kapton} = \varepsilon_{epoxy},\tag{3.3}$$

or

$$\frac{\sigma_{total}}{E_{total}} = \frac{\sigma_{Kapton}}{E_{Kapton}} = \frac{\sigma_{epoxy}}{E_{epoxy}}$$
(3.4)

or

$$\frac{F_{total}}{(t_e + 2t_k)E_{total}} = \frac{F_{Kapton}}{2t_k E_{Kapton}} = \frac{F_{epoxy}}{t_e E_{epoxy}}$$
(3.5)

and, the force governing equation is,

$$F_{total} = F_{Kapton} + F_{epoxy} \tag{3.6}$$

Substituting Eq. 3.5 into Eq. 3.6 yields:

$$(t_e + 2t_k)E_{total} = 2t_k E_{Kapton} + t_e E_{epoxy}$$
(3.7)

The true stress versus true strain curves for the layered (composite) film is illustrated in Fig. 3.7. The layered film demonstrates Hookean behaviour at low strain. By compensating the toe region, its average modulus of elasticity was calculated, 2.30 GPa. Thereafter, using the Eq. 3.7, the average modulus of elasticity of the epoxy was found to be 0.363 GPa.

The mass density was adopted from the technical sheets provided by 3M Inc. Owing to the lack of first hand experimental data as well as manufacturer data, a Poisson's ratio of 0.34 was selected; the same value as the Kapton $300JP^{\textcircled{R}}$ Poisson's ratio. The mechanical properties of the epoxy are summarized in Table 3.5.



Figure 3.7 Composite Kapton[®]/Epoxy/Kapton[®] true stress versus true strain.

Property	Value
Mass density	1042 kg/m ³
Poisson's ratio	0.34
Elastic modulus	0.363 GPa

Table 3.5Mechanical properties of the 3M scotch-weld epoxy adhesive 2216B/A translucent[®].

CHAPTER 4 EXPERIMENTAL MODAL ANALYSIS OF SRT

This chapter details the experimental modal analysis of the SRT. Section 4.1 describes the experimental setup used in this study. In Section 4.2, the description of the non-contacting electromagnetic exciter and laser sensing setup are presented. The modal testing procedure is discussed in Section 4.3. In Section 4.4, the nonlinearity of the structure is examined. The derived natural frequencies, mode shapes and modal damping are presented in Section 4.5 and compared with prior studies. The summary and conclusion drawn from the modal testing results, presented in Section 4.6, ends this chapter.

4.1 Experimental Setup

The test structure was suspended horizontally from the laboratory's ceiling at six equally spaced points using 3.3 meter long suspension systems, as shown in Fig. 4.1. A very soft spring suspension system was used to provide the lowest possible rigid body frequencies while having the strength to suspend the test article above the floor. Each suspension, which consisted of a series of one slinky, two soft springs and two rubber bands, was attached to the torus through holes located in the outer circumferential peel seams (flaps). The measurements were performed after the rubber bands were allowed to creep for more than two weeks.

Once suspended, the torus was inflated by a small aquarium pump. Using a pressure regulator, the pressure was maintained below 1379 Pa (0.2 psig), shown in Fig. 4.2.a. The amount of pressure was monitored by a monometer, Fig. 4.2.b. Thereafter, the supply pump was removed and the inflation nozzle was left open to the atmospheric pressure to allow the internal pressure to equal that of the surrounding air. Leaving the air nozzle open to the atmospheric pressure had challenges. The inflatable torus hardly could maintain its shape long enough for one continuous set of measurement, i.e., due to the earth's gravity, it immediately wrinkled and sagged. Hence, using the regulator, a very

small internal pressure of 149.3 Pa (0.022 psi or 0.6 inch-water) could be maintained during the experiment to prevent the sag and wrinkle of the torus. The leakage flow rate was slow enough to allow useful measurement.



Figure 4.1 SRT modal experiment setup.

The pendulum and plunge (bouncing up and down) frequencies of the test structure were found to be 0.27Hz and 0.29Hz, respectively. It was expected that the first natural frequency would be between 2.0 to 3.0 Hz, (Ruggiero et al., 2004b and Song et al., 2006). Hence, the suspension frequencies were less than ten percent of the first natural frequency and therefore in an acceptable range. To reduce the mass loading effect of the air tube, the tube was suspended freely from a stand. In order to minimize the effect of the air current disturbance over the SRT, all the air ventilation ducts of the lab were blocked, and the air conditioner was turned off during the measurements.

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(a)



(b)

Figure 4.2 (a) Pressure regulator, (b) Monometer.

4.2 Excitation and Sensing Setup

Considering the low stiffness and high flexibility of inflatable structures, the choice of suitable sensing and actuation systems in the dynamic testing was limited. The extremely flexible nature of the SRT caused point excitation to result in only local deformation; hence the excitation method should be cautiously selected.

Using the contact excitation systems would have had drawbacks. For instance, had an electromagnetic shaker as the excitation system been used and the input energy is not properly distributed throughout the structure, the shaker could have pierced or ripped the membrane (Ruggiero and Inman, 2006). Considering the very low mass of the SRT, employing smart materials as actuators or sensors would have caused frequency shift. The same reasoning applies with the application of accelerometer as a sensor. Therefore, a noncontact actuation and sensing system was employed in the present study. For the instrument specification refer to the Appendix A.

4.2.1 Excitation Setup

The test structure was excited via a non-contacting electromagnetic exciter. An inhouse fabricated electromagnet was made out of four layers of 28 gauge copper wire on an iron core. The core had a 4.5 mm (0.18 in) diameter and was 47 mm (1.85 in) long. The electromagnet was placed close to the test structure where a strong permanent Rare Earth magnet was attached to the torus. In order to reduce the effect of local deformation of the very flexible membrane of the SRT, the 9.54 mm (3/8 in) in diameter magnet was attached to the torus by a 2.5cm (1 in) by 3.5 cm (1.4 in) double sided exterior mounting tape. By this modification, the excitation energy was distributed to a larger area of the SRT surface and the global bending modes of the structure were excited without causing local excitation (deformation.) The magnet had 5 mm (1/5 in) thickness, and weighed 2.69 grams (0.095 ounce), which was approximately 0.5% of the torus weight.

A time-changing current applied to the electromagnetic exciter resulted in a timechanging magnetic field. Hence the excitation system, without even coming into contact with the SRT, applied significant forces to it. Since the deflected modes were in a different orthogonal direction, the magnet was attached at the outer surface of the torus at an approximate 45 degree angle with respect to the peel seam (Fig. 3.1) to excite all the modes. The configuration of the electro-magnetic excitation system is illustrated in Fig. 4.3.

While it was impossible to keep the distance between the solenoid and magnet constant at all time, an effort was done to maintain this gap at approximately 3 cm.



Figure 4.3 Electromagnetic excitation setup.

4.2.2 Sensors

As noncontact measurement transducers, laser displacement sensors were used to avoid the stiffness of instrumentation wiring as well as frequency shift problem associated with the inertial loading and damping on the structure. The laser measurement system was very versatile, which allowed easy changing of the location and number of the measurement points.

Two Keyence laser triangulation displacement sensor head LK-G82 and a common controller LK-G3001 were used. The laser displacement sensors were calibrated by comparison method with a Kistler 8638B50 accelerometer, described in Appendix B. Since measurement of the force applied to the torus was impossible, the reaction of the force on the core of electromagnet was measured using an Interface MB-5 Miniature Beam load cell. The strain gage based load cell was isolated from the electromagnetic field of the solenoid by an 18 cm (7in) aluminium bar, Fig. 4.4. The load cell was placed between the isolation aluminium bar and an aluminium bracket. The bracket was clamped to a rigid stand.

In order to capture the scaled mode shapes, drive point measurement was necessary. The modes of interest were ring modes, i.e., in-plane and out-of-plane motions

of the torus. The directions of the in-plane and out of plane measurements are shown in Fig. 4.3.

For each of the in-plane and out-of-plane measurements, displacement was measured by roving the laser displacement sensors at fourteen evenly spaced points along the inner surface and top of the structure, respectively. The measurement points coincide with the fourteen shear seam sections. The sensor locations around the perimeter of the SRT are shown in Fig. 4.5. The drive point measurement was taken at location 1.

Response measurements on the surface of the SRT were complicated by the fact that the Kapton[®] film is optically transparent, which allowed much of the laser light to pass through the test article. Preliminary test proved that by attaching reflective tape at the measurement points, a better coherence could be achieved. Henceforth, reflective tapes were used at all measurement points which added a total of approximately 8 grams (0.282 ounce) to the mass of the torus, which was 1.4% of the torus mass.



Figure 4.4 Electromagnetic exciter and load cell ensemble.



Figure 4.5 A sketch of the SRT, showing the fourteen locations of the sensors as well as the suspension points around the perimeter. The solid and dashed radial lines are the connection shear seams in the top half and bottom half of the SRT, respectively.

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4.3 Experimental Procedure

Careful selection of excitation signal (specific length and bandwidth) was found to be critical for a flexible inflatable torus when trying to obtain reliable vibratory responses with reasonable accuracy.

Since by narrowing down the bandwidth, sufficient input energy could be provided to the entire structure and the global modes would be excited, a periodic chirp signal was used for the input signal. The periodic chirp signal could characterize the nonlinearity and it is leakage free (self-windowing) and had the best signal to noise ratio characteristic when compared to other type of excitation signals.

The analog chirp signal of 1 to 25 Hz was generated using Matlab Signal Processing Toolbox for periods of 10 seconds. The 24 Hz frequency bandwidth was chosen to focus on the area of rich satellite dynamics. The chirp signal was fed to a National Instrument card PCI-6024E. Thereafter, the signal was fed to a power amplifier, Panasonic RAMSA WP-1200, and the voltage-adjusted signal, 3.4 V, was sent to the electromagnetic exciter. This voltage was selected based on the experimental study that a voltage of above 3.4 V would have burned the solenoid. The data collected from the laser displacement sensors and load cell were sent back into the National Instrument card, and signal processing was performed with Matlab Signal Processing Toolbox. The cut-off frequency for the low pass filters of displacement sensors was 30 Hz. To remove the effect of the rigid body modes, a fourth order digital high pass Butterworth filter with the cut off frequency of 1 Hz was used. Butterworth filter is an electronic filter designed to have a frequency response, which is as flat as mathematically possible in the passband.

The internal pressure was maintained at approximately 149.3 Pa (0.0216 psig or 0.6 inch water), with the room temperature being kept at approximately 26 degrees Celsius during the experiment. There was no control on the humidity.

The frequency of the pump diaphragm was measured at 60 Hz, described in Appendix C. Any effect of the pump on the frequency response functions would therefore have appeared at 60Hz, which was above the desired frequency range of 1 to 25 Hz. In a preliminary test, no noticeable effect of pump noise and the air flow into the structure was found on the measurement of frequency response function, described in Appendix C.

Owing to the current-squared nature of the generated magnetic force, even a single frequency sinusoidal input signal to the electromagnet, results in a complex periodic force signal (Ewins, 2000).

All frequency response function measurements were estimated using the H_1 estimation algorithm, which cancels noise at the output. The measurement sampling frequency was 2000 Hz and no windowing was used.

In the signal processing phase, both cyclic (linear) and power spectra averaging was used. In each measurement, one block of excitation took place without acquiring the associated input and output data, i.e., one delay block. This caused the transient response to any start or change in the periodic excitation to decay out of the response signals, so both the input and outputs signals would be periodic with respect to the observation period (Allemang, 1999). Each delay block was equal to the observation period, 10 seconds. To reduce the possible leakage error, cyclic averaging of the time domain data was employed. After the delay block, four continuous blocks of time data, i.e., excitation and responses, were captured and linearly averaged. Hence, the total test time for each cyclic average was five continuous blocks of data. Fig. 4.6.a shows a typical excitation for a single measurement; one delay block followed by four continuous blocks of chirp excitation. To provide a clear view of the chirp excitation, one block (10 s) is splitted into two parts of 0 to 6 seconds and 6 to 10 seconds, which are illustrated in Figs. 4.6.b and 4.6.c.

In the estimation of the frequency response function, linear (stable) spectral averaging was performed to remove the variance error due to extraneous random noise and randomly excited nonlinearities. Twelve ensemble of delay and captured data blocks were used in the spectral averaging of the cross and auto power spectra.



Figure 4.6 Excitation signal from the signal generator for (a) single measurement consisting of five blocks of 10 seconds, (b) 0 to 6 seconds, and (c) 6 to 10 seconds.

Coherence function, which is a measure of the input-output relationship, was used to determine the effectiveness of the excitation in these tests. The coherence was examined for each data block. Any broadband drop in coherence denoted poor excitation. When this occurred, the measurement was discarded and repeated.

Fig. 4.7 shows a typical plot of coherence and frequency response function for the drive point measurement. The coherence and frequency response function plot for point 8, for the in-plane and out of plane motion, are given in Figs. 4.8 and 4.9, respectively. Point 8 is located at 180 degree from the drive point about the torus ring section, as shown in Fig. 4.5.

The coherence is below one; the drive point is of no exception, as shown in Fig. 4.7. The drop in the coherence broadband could be attributed to nonlinear behavior in the structure, unmeasured inputs acting on the system, or bias error such as noise or leakage in the measurements. The SRT would inherently wrinkle and sag in the presence of the gravity force, which caused nonlinear behavior in the structure. The drop in the coherence was most likely due to the nonlinear behavior of the structure. The modal testing performed by Griffith and Main (2002) for an inflated Kapton HN[®] torus, showed the same trend. Additional (unwanted) input was less likely, because while the input was shut down the coherence was almost one. The effects of leakage could not be eliminated; they could have been reduced only by proper averaging, increasing frequency resolution, use of periodic excitation and use of window functions (Van Karsen, 2007). In this regard, cyclic averaging of the time domain data was employed and the periodic chirp signal which was leakage free (self-windowing) was used.

The frequency response functions have a significant amount of noise, closely spaced and low amplitude peaks (Figs. 4.7 to 4.9). Local high flexibilities and high damping might have affected the propagation of energy through the structure and eventually could have caused poor input power spectra.



Figure 4.7 (a) Coherence and (b) frequency response function for the drive point.



(b)

Figure 4.8 (a) Coherence and (b) frequency response function for point 8 in-plane motion.



(b)

Figure 4.9 (a) Coherence and (b) frequency response function for point 8 out of plane motion.

4.4 Nonlinearity Detection

The inflatable torus was assumed to be nonlinear. While keeping the internal pressure at approximately 149.3 Pa (0.0216 psig or 0.6 in water), the lightly pressurized torus was excited at two different levels of excitation of 1.8 and 3.4 V. The excitation level of 1.8 V was the lowest voltage that could excite the torus globally, i.e. 180 degree from the drive point about the torus ring section. As mentioned in the previous section, the 3.4 V excitation was the highest voltage could be used without burning the solenoid.

Fig. 4.10 shows the frequency response functions for the in-plane and out-ofplane measurements of sensor location 1. Any deviation between the two curves indicates some sort of nonlinear behaviour. For a linear system, increasing the input by a factor should increase the output by the same factor. For nonlinear modes, the frequency and width of the modal peak vary by the level of excitation. By overlaying the two frequency response functions (Fig.4.10), for each in-plane and out of plane measurements, it is observed that the response of the SRT displays weak nonlinear system behavior. This means not only the estimated frequency but also the corresponding damping of some modes was valid only at the level of excitation (3.4 V) used in the test.

4.5 Modal Parameter Identification

After completing the data measurements, the collected frequency response functions were analyzed using the STAR Modal software from Spectral Dynamics, Inc.

The curve fitting was done by Global Curve Fitting method. This method splits the curve fitting process up into two steps. In the first step, using all the measurement data, the frequency and damping are estimated. In the second step, by a second estimation process, it obtains the residues (mode shapes) from the known and fixed values for the frequency and damping. Since the frequency and damping estimates can be obtained by processing all the frequency response functions and not a single measurement, these modal parameter estimates are more accurate than the estimates by curve fitters that fit frequency response functions one at a time. The second advantage of the Global Curve Fitting is that since the damping is known from the first step, the mode shapes are generally more precisely estimated in the second step (Richardson and Formenti, 1985).



Figure 4.10 Nonlinearity detection (a) in-plane, (b) out-of-plane frequency response functions of location 1; left plots are in logarithmic scale and the right ones are in the linear scale.

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The identified resonant frequencies and modal dampings of both in-plane and out of plane modes of this study and prior work done by Ruggiero et al. (2004b) and Song et al. (2006) are summarized in Tables 4.1 and 4.2, respectively. The method of fabrication, as well as the extensive repair that was done on the flaps, resulted in thickness variations along the joined regions. This contributed to non-uniform stiffness and mass distributions in both the torus and the flaps, affecting the modal parameters of the SRT. Besides, the flaps were not completely flat and had wrinkles at various locations. The mass imbalance and boundary conditions created modal asymmetry in this axisymmetric structure and consequently divided some resonant frequencies in two well separated frequencies. In Tables 4.1 and 4.2 the split mode pairs are indicated by asterisk signs.

The measured frequencies by Ruggiero et al. (2004b) are lower than those obtained in the present study, Table 4.1, indicating the higher stiffness of the current structure. In the present study, the SRT was pressurized at a very low pressure, while in the Ruggiero et al. study the inflation nozzle was left open to the atmospheric air, which allowed the SRT to wrinkle. Though in their study, the SRT was suspended vertically, lack of internal pressure, though small, could have reduced the stiffness and consequently reduced the identified frequencies of the SRT. As discussed in chapter 2, many researchers have shown that increasing the pressure increases the stiffness of the structure and consequently the identified frequencies. The second possible cause for this trend is the use of accelerometer in their modal experiment. The contact measurement transducers,

Table 4.1 Comparison between non-rigid body frequencies determined by the present study and prior work done by Ruggiero et al. (2004b) and Song et al. (2006). The asterisk indicates a split mode pair.

	Non-rigid body frequencies (Hz)							
Mada		In-plane		Out of plane				
WIGGe	Present study	Ruggiero et al.	Song et al.	Present study	Ruggiero et al.	Song et al.		
1	4.68	3.0	5.07	3.21	2.0	2.88		
1*	6.07							
2	8.43		13.02	8.12	4.9	9.51		
2*	9.46				5.5			
3	13.32		22.08	12.79	11.2	14.61		
4	17.21			17.55				

	Damping (%)							
Mode	In-pla	ine	Out of plane					
	Present study	Song et al.	Present study	Song et al.				
1	2.38	3.60	9.27	16.12				
1*	5.55							
2	3.87	6.97	2.63	9.61				
2*	2.34							
3	1.65	12.14	2.39	7.73				
4	2.58		1.82					

Table 4.2Comparison between modal damping determined by the present study and
prior work done by Song et al. (2006). The asterisk indicates a split mode pair.

due to inertial loading on the structure, might have caused frequency shift toward lower frequencies. In the Ruggiero et al. study, the SRT was suspended vertically from the ceiling using two rubber wires causing a pendulum frequency of approximately 3.5 Hz, which was greater than the first natural frequency of 2 Hz extracted by this team, (Table 4.1). It is known that suspension frequencies greater than 10% of the first natural frequencies lead to inaccurate frequency identification. There was no information available regarding the plunge frequency of the SRT in that study.

In the study by Song et al. (2006), only the first two in-plane and out of plane modes exhibited the expected classical mode shape. The third in-plane mode had the oval shape of the first mode. Except for the first out of plane mode, their measured frequencies are higher than those obtained in the present study. Table 4.1, indicating the high stiffness of their structure. In their study, the SRT was suspended at three points by 3.2 meter long monofilament lines with pendulum frequency of 0.28 Hz. These suspension lines could have presented stiffness in the out of plane direction and depending on the location of the suspension points and the vibration nodes could have affected the modal parameter identification. The monofilament lines generally should not cause a significant stiffness in the in-plane direction. Though, in the in-plane deformation, the structure might have deformed in the out of plane direction as well, and the added stiffness by the support might have increased the identified frequencies and could have changed the mode shapes of the structure. In the Song et al. study, similar to the Ruggiero et al. tests, the inflation nozzle was also left open to the atmospheric air which made the SRT wrinkle and sag in the presence of the gravity force. This could have affected the stiffness of the SRT as well. But the most important difference between the results of their study and the present work lies in the method of excitation and collection of data. They had used the acoustic
excitation. Besides, in the calculation of the frequency response function instead of the actual acoustic signal, the white noise signal from the signal generator was used as the input signal.

The variations of the in-plane and out of plane modal damping ratios with frequencies are shown in Figs. 4.11 and 4.12, respectively. The modal damping ratios identified by Song et al. (2006), except for the first in-plane one, are significantly greater than the ones from the current study. Generally, accurate estimation of damping and mode shapes is more difficult than the accurate estimation of frequency from the frequency response function measurements (Richardson and Formenti, 1985). Damping is the most difficult parameter to estimate precisely and the mode shape is coupled to it. Even if the curve fitting function matches the measured frequency response function very well, when there is a large error in damping calculation, the residue (mode shape) would be in large error too (Richardson and Formenti, 1985). The difference between modal damping ratios estimated by the present study and by Song et al. (2006) could be as a result of different modal parameter extraction technique, different localized mass loading and stiffness of each of the SRTs, different testing condition such as suspensions, pressures and even ambient temperature. The high level of damping might be due to the effect of the air resistance being present in the ambient laboratory condition.



Figure 4.11 Comparison between in-plane modal damping determined by the present study and prior work done by Song et al. (2006).



Figure 4.12 Comparison between out of plane modal damping determined by the present study and prior work done by Song et al. (2006).

The identified mode shapes of the first four in-plane and out of plane modes are shown in Figs. 4.13 to 4.17. The blue and red shapes represent the un-deformed and deformed shape of the SRT, respectively. The numbers on the red shapes stand for the measurement points. The in-plane modes have the oval, triangle, square and pentagon shape for the first, second, third and fourth modes, respectively. The out of plane modes resemble the bending mode of a free-free beam.



First mode – split (6.07 Hz) (b)



Experimentally identified first in-plane mode shapes.



(b) Second mode – split (9.46 Hz)



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(b) Fourth mode (17.21 Hz)





- (b) Second mode (8.12Hz)
- Figure 4.16 Experimentally identified (a) first, and (b) second out of plane mode shapes.



Figure 4.17 Experimentally identified (a) third and (b) fourth out of plane mode shapes.

4.6 Summary

In this chapter experimental modal analysis of the SRT was presented. An inhouse fabricated non-contacting electromagnetic exciter and two laser displacement sensors were employed. The sensors clearly picked up the signal within 180° of the actuator, which indicated that the excitation system was able to excite the torus' global modes.

First, it was shown that the SRT had a weak nonlinear behaviour. Hence, the identified natural frequencies and modal damping were valid at the level of excitation used in the testing, 3.4V. Then, the first four in-plane and out of plane damped natural frequencies and their corresponding modal damping and mode shapes were identified and compared with prior studies.

As discussed in the literature review, Ruggiero et al. (2004b) could identify the first three out of plane modes' natural frequencies and only the first in-plane one. Their second out of plane mode was a split mode. They did not provide the modal damping ratios. Song et al. (2006) identified the first three in-plane and out of plane modes' natural frequencies and their corresponding modal damping. In their study, only the first two identified in-plane modes exhibited the expected classical ring mode shape. The third in-plane mode had the oval shape of the first mode. In the present study, the first and second in-plane modes, particularly the in-plane ones, as well as decipher the modal asymmetry proved that the present experimental modal analysis was more accurate than prior studies. The modal damping ratios identified by Song et al. (2006), except for the first in-plane mode, were significantly greater than the ones from the current study.

The identified frequencies of the present study were higher than the measured frequencies by Ruggiero et al. (2004b) and lower than those obtained by Song et al. (2006), as presented in Table 4.1. This indicated the higher stiffness of the current structure than the former study and its lower stiffness than the latter one.

The modal experiment of the SRT proved that dynamic testing of this structure was very challenging. It was impossible to keep the electromagnet and the permanent magnet exactly at the same distance even in one set of measurements. The drive point measurement was very challenging due to spatial limitation. The width of the shear seam (Fig. 3.1) was 2.1 cm, and it was not possible to accommodate the laser sensor head and the electromagnet support in this small area, in such a way that both displacement measurement and excitation perform exactly at the same point and direction.

Also, since maintaining the internal pressure was done manually, it was less likely to achieve the same pressure at all times. Variable pressure contributed to variable stiffness which might have altered the modal parameters of the structure. Owing to the presence of very small punctures in the SRT, the air leakage made the distribution of pressure non-uniform in the torus.

The torus was suspended from the ceiling by very soft and long springs. The suspension frequencies were less than ten percent of the first natural frequency, 3.21 Hz, and it was not expected to affect the modal parameters directly. While during the experiments all the air ventilation ducts of the lab were blocked and the air conditioner was turned off, still the effect of the air current disturbance over the SRT was observable.

CHAPTER 5 FINITE ELEMENT MODAL ANALYSIS OF A COMPLETE INFLATED TORUS

The previous finite element analyses of inflatable tori, Lewis and Inman (2001), Leigh et al. (2001) and Park et al. (2003) were carried out using commercial finite element softwares. None of the mentioned studies take into account the direct effect of pressure force and the geometric nonlinearity. Hence, apart from giving inaccurate natural frequencies and mode shapes for the non-rigid-body modes, they produced nonzero rigid-body natural frequencies.

Prior to any attempt to model the SRT, a preliminary finite element analysis of a complete pressurized torus with smooth surface must be performed and validated. The present chapter deals with the finite element modal analysis of a complete pressurized torus. For a more accurate analysis, both the geometric nonlinearity and the effect of the follower load on the stiffness are included in the study.

The assumptions and the modeling procedure are provided in Section 5.1. Section 5.2 deals with the model verification. In subsection 5.2.1, the model convergence is demonstrated by solving for the modal frequencies with varying mesh densities. The rigid body frequencies are examined in subsection 5.2.2, with the mode shapes being presented in subsection 5.2.3. Thereafter in Section 5.3, the modal results of the developed model are compared and verified with the analytical study's results obtained by Jha (2002). To better understand the effect of different parameters on the vibratory characteristics of the inflated torus, parametric studies are performed and presented in Section 5.4. Subsections 5.4.1 to 5.4.3 detail the study of the effect of aspect ratio, pressure, and the membrane thickness on the modal parameters of the torus, respectively. The results are compared with the published analytical results. This chapter ends with a summary and conclusions drawn from the finite element analysis of the complete torus, Section 5.5.

5.1 Finite Element Model

In the standard modal analysis, the structure is free from active loads. Since the stiffness of the inflatable structures are directly related to the internal load, a standard modal analysis that does not take into account the internal pressure gives incorrect results (Smalley et al., 2001). In the present study, to address the aforementioned problem, the stiffness matrix was generated in a nonlinear static analysis. The finite element analysis was carried out using the commercial software ANSYSTM.

Throughout the nonlinear static analysis procedure, which takes into account the large deformation, the stiffness matrix was updated over a nonlinear curve. Upon the completion of the nonlinear static analysis and the convergence of the solution, the modal analysis restart was performed. This procedure imported the original geometry from the bulk data section and the stiffness matrix from the last iteration of the nonlinear static analysis. The SHELL181 element was used in the analysis. This element has 4-nodes with six degrees of freedom at each node: translations in and rotations about the x, y, and z-axes. SHELL181 is well suited for linear, large rotation, and/or large strain nonlinear applications and is also suitable for analyzing thin to moderately-thick layered shell structures. It accounts for follower (load stiffness) effects of distributed pressures, and employs the first order shear deformation's theory, known as Mindlin-Reissner.

The preliminary study was carried out based on the following assumptions: cross sections remained circular after the static deformation; the wall thickness was uniform; and membrane material was isotropic and had linear elastic property.

It was assumed that the vibration amplitude was very small, thus pressure was constant both after achieving the static deformation and during the torus vibration. Geometric nonlinearity (large strain) was considered only within the pre-stress, and the effect of the follower load on the stiffness was included in the analysis. The boundary condition was free-free. Except for the internal pressure load, the other effects of the enclosed gas were ignored; i.e., mass and structural acoustic interaction. The inflatable torus was non-rigidized and had no flaps (complete torus). Damping effects were neglected.

Since the wall thickness was very small, the pre-stresses were of membrane type, i.e., uniform throughout the thickness, and the pre-stresses related to the transverse direction were assumed negligible.

Since damping effects were neglected, the system matrices were symmetrical. Therefore, the Block Lanczos algorithm, which is a fast and robust algorithm for eigensolution computation was used. The Block Lanczos Method is a very efficient algorithm to perform a modal analysis for large models and recommended for most applications as the default solver.

5.2 Finite Element Model Verification

The convergence of the model of the torus was studied comparing the non-rigid body modes' natural frequencies of different mesh densities. Thereafter, the rigid body modes and their frequencies were studied. The natural frequencies obtained from the converged model were then compared with the results obtained from the analytical solution by Jha (2002), presented in Section 5.3. The geometric and mechanical properties adopted by Jha for the torus are listed in Tables 5.1 and 5.2, respectively. Internal pressure of 3447.38 Pa (0.5 psi) was applied to the torus.

5.2.1 Model convergence

To confirm the convergence of the model, the inflated torus was meshed with 360, 720, 1440 and 2880 elements. Table 5.3 shows the first ten natural frequencies for each model. The heading "N×M" refers to the mesh density of the model, i.e., a "24×60" is a torus meshed with 24 elements around the tube and 60 elements around the ring for a total of 1440 elements. The "24×120" model is selected as the reference model. Differences between natural frequencies of each "N×M" model and the "24×120" model are listed in Table 5.4.

The " 24×60 " model predicted the modes very well with a maximum difference of 1.02% at the ninth mode. Hence, the " 24×60 " model appeared to be a reasonable compromise between accuracy and computation time. This mesh size was therefore used for the remainder of this study.

Table 5.1	Geometric	properties of
	d opr	

ine SK1.		
Property	Value	
Film thickness	46×10 ⁻⁶ m	
Ring radius	7.62 m	
Tube radius	1.22 m	

Table 5.2	Mechanical	properties	of
	Vantan®		

Kapioi	1.
Property	Value
Mass density	1418 kg/m ³
Poisson's ratio	0.34
Elastic modulus	2.55 GPa

	Frequency by the number of elements (Hz)					
Mode number	12×30	24×30	12×60	24×60	12×120	24×120
1	6.82	6.96	6.75	6.88	6.73	6.86
2	7.14	7.29	7.04	7.18	7.01	7.15
3	17.50	17.57	17.38	17.52	17.28	17.52
4	17.82	18.10	17.45	17.67	17.44	17.56
5	18.64	18.94	18.14	18.44	18.01	18.31
6	24.33	24.48	24.15	24.34	24.15	24.30
7	28.16	28.15	28.05	28.04	28.02	28.01
8	31.20	31.53	29.99	30.35	29.69	30.05
9	31.42	32.75	31.15	31.52	30.82	31.20
10	38.15	38.27	37.78	37.94	37.68	37.85

Table 5.3Torus model convergence.

Table 5.4 Co

Comparison of "N×M" and "24×120" model.

	Difference in frequencies (%)				
Mode number	12×30	24×30	12×60	24×60	12×120
1	0.58	1.41	1.62	0.29	1.93
2	0.14	1.86	1.56	0.43	2.00
3	0.11	0.28	0.80	0.04	1.39
4	1.46	3.06	0.64	0.63	0.69
5	1.79	3.41	0.93	0.71	1.67
6	0.12	0.73	0.62	0.16	0.63
7	0.54	0.50	0.14	0.11	0.03
8	3.80	4.91	0.20	0.99	1.22
9	0.71	4.96	0.16	1.02	1.23
10	0.78	1.11	0.19	0.24	0.44

5.2.2 Rigid body motions

No boundary condition was assigned to the finite element model of the inflated torus. Therefore, the free-free vibration analysis results in six rigid-body modes. Rigid body modes are defined if the relative positions (distance) of any two particles of the structure do not change with time. A non-trivial solution for these deflections is when the whole structure moves, while the relative motion of any two point of the structure is zero. For the case of a free-free boundary condition, such motion can be divided into three pure translations along and three rotations about three perpendicular axes, Fig. 5.1.

Zero relative displacement between two points results in zero strain and eventually no re-storing force in the structure. This causes non-oscillatory deflection, and hence zeros frequencies for the rigid body motion. The first six natural frequencies corresponding to rigid body modes obtained from the "24×60" finite element model are listed in Table 5.5. The first three modes correspond to the translational motions (U_x, U_y, U_z) and the rest are the rotational ones (Σ_x , Σ_y , Σ_z). The rigid body frequencies were at least two orders of magnitude smaller than the first non-rigid body frequencies and smaller than 0.1 Hz. Hence, the present finite element analysis gave six approximately zero frequencies corresponding to six rigid body modes.



Figure 5.1 Directions of the displacements and rotations of the rigid body modes of inflated torus.

Mode number	Frequency (Hz)
1	0.0000
2	0.0000
3	0.67E-06
4	0.77E-01
5	0.89E-01
6	0.89E-01

m 11 c c	- ·	1 • .		1 1 1
lable 1 1	Erection of the second seco	orresponding to	SIX RIGIC	body modes
14010 5.5	r requencies e	on esponding to	SIATIEIG	oouy moues.

5.2.3 Mode shapes

The modes can be categorized into two main groups, symmetric and antisymmetric. In symmetric modes, the meridional cross section is symmetric with respect to mid-plane of the torus. In the antisymmetric modes, the meridional cross section has an antisymmetric pattern with respect to the torus mid-plane.

The first eighteen non-rigid body mode shapes and their corresponding natural frequencies are shown in Fig. 5.2. Modes 1, 3, 4, 6, 8, 10, 12, 13, and 18 are anti-symmetric modes. The other modes are the symmetric one. Out of the anti-symmetric modes, modes 1, 4, 8 and 13 are out of plane (bending) mode. They resemble the bending modes of a free-free beam. Out of the symmetric ones, modes 2, 5, 9 and 16 bend in plane of the torus and vibrate by making oval, triangular, square and pentagon shapes, respectively. These modes are known as the in-plane modes. Modes 6, 10, 12 and 18 are the twisting modes. Except for the axisymmetric modes, 3 and 7, all modes are in orthogonal pairs. Only one mode of each pair is shown in Fig. 5.2. The other mode of the pairs can be obtained by rotation. In axisymmetric modes, the meridional cross section does not change along the longitudinal direction of the torus, but the cross section deforms either antisymmetrically, mode 3, or symmetrically, mode 7. Mode 3 resembles a cone. At the breathing mode, mode 7, the tube expands and contracts uniformly and at the sloshing mode, 11, the tube expands and contracts from side to side. Figs. 5.3 and 5.4 depict the side view of the cone mode and the sloshing mode, respectively, at three instants of time.

By reviewing the natural frequencies, one can conclude that the natural frequencies of the torus come in clusters, e.g. modes 3, 4 and 5, and then modes 7, 8 and 9. Considering the fact that some modes are in a pair, in each of the mentioned clusters, there are five frequencies in a very small range of frequencies. This necessitates consideration of a high number of modes in the vibration testing and control. When natural frequencies of the structure are close, imperfects can end up exciting some modes over others.

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Mode 1 (6.88 Hz) 1st antisymmetric (out of plane)



Mode 2 (7.18 Hz) 1st symmetric (in-plane)



Mode 4 (17.67 Hz) 2nd antisymmetric (out of plane)



Mode 5 (18.44 Hz) 2nd symmetric (in-plane)



Mode 3 (17.52 Hz) 1st axisymmetric antisymmetric (cone)



Mode 6 (24.34 Hz) 3rd antisymmetric (twisting)

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Mode 7 (28.04 Hz) 2nd axisymmetric symmetric (breath)



Mode 8 (30.53 Hz) 4th antisymmetric (out of plane)



Mode 10 (37.94 Hz) 5th antisymmetric (twisting)



Mode 11 (38.31 Hz) 4th symmetric (sloshing)



Mode 9 (31.52 Hz) 3rd symmetric (in-plane)



Mode 12 (43.09 Hz) 6th antisymmetric (twisting)

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Mode 13 (43.10 Hz) 7th antisymmetric (out of plane)



THE

Mode 16 (44.12 Hz) 7th symmetric (in-plane)



Mode 14 (43.37 Hz) 5th symmetric (fish)



Mode 15 (43.53 Hz) 6th symmetric (hourglass)

Mode 17 (44.65 Hz) 8th symmetric



Mode 18 (44.71 Hz) 8th antisymmetric (twisting)



Mode shapes of the inflated torus



Figure 5.3 Side view of the cone mode at three instant of time.



Figure 5.4 Side view of the sloshing mode at three instant of time.

5.3 External Verification

For the first ten non-rigid body modes, the finite element results were compared with the analytical work done by Jha (2002). The natural frequencies and mode shapes are listed in Table 5.6 and Table 5.7, respectively.

Table 5.6

frequencies	frequencies of the FEA and Jha model			
Mode number	FEA (Hz)	Jha (Hz)	Error (%)	
1	6.88	6.90	0.29	
2	7.18	7.24	0.84	
3	17.52	17.71	1.08	
4	17.67	17.85	1.02	
5	18.44	18.55	0.60	
6	24.34	24.85	2.09	
7	28.04	28.45	1.46	
8	30.35	30.33	0.07	
9	31.52	31.61	0.28	
10	37.94	38.53	1.54	

Comparison of

shapes of the	FEA and J	na model
Mode number	FEA	Jha
1	1 Bend	1 Bend
2	Oval	Oval
3	Cone	2 Bend
4	2 Bend	Cone
5	Triangle	Triangle

Twisting

3 Bend

Square

Twisting

Breathing Breathing

Twisting

3 Bend

Square

Twisting

6

7

8

9

10

Table 5.7	Comparison of mode
ahamaa af tha	EEA and the model

The finite element third and fourth mode shapes are, cone and out of plane modes, respectively; the corresponding mode shapes for the third and fourth mode of the Jha results are out of plane and cone, respectively. Considering the fact that no change in the order of the other mode shapes has been observed, and the frequencies of the third and fourth mode are very close (17.52 Hz and 17.67 Hz), we can conclude that the finite element model compares very well with the analytical solution. The maximum difference between the frequencies is for the 6th mode, which is 2.09%.

Since the present finite element model gave six approximately zero frequencies corresponding to six rigid body modes and for the non-rigid-body modes compares very well with the analytical solution, it can be concluded that the element type is properly selected.

5.4 Parametric Study

In this section, the effect of the aspect ratio (r/R), internal pressure (p) and the wall thickness (h) on mode shapes and natural frequencies of the torus are presented. Each model was meshed with 60 elements around the ring diameter and 24 elements around the

tube diameter. The torus pressure and geometric properties, shown in Table 5.1, will be changed one at a time.

5.4.1 Aspect ratio

The ring radius was held constant at 7.62 m. For ten different aspect ratios, the first ten natural frequencies and their corresponding mode shapes are listed in Table 5.8 and Table 5.9, respectively. By thoroughly examining the results one can conclude that the aspect ratio has a strong impact on both the natural frequencies and the sequence of mode shapes. This is due to the fact that, any increase in the aspect ratio increases the stiffness of the torus and consequently increases the natural frequency.

Table 5.8	Frequencies for the torus with different aspect ratio, values in the shaded
	areas represent the ring modes (in-plane/out of plane).

	Frequency (Hz)									
Mode number	Aspect Ratio 0.06	Aspect Ratio 0.16 (Ref.)	Aspect Ratio 0.26	Aspect Ratio 0.36	Aspect Ratio 0.46					
1	1 2.94		9.52	11.22	12.32					
2	3.03	7.17	10.25	12.46	13.85					
3	8.27	17.53	16.34	15.38	14.65					
4	8.44	17.66	22.56	24.00	24.01					
5	15.62	18.43	23.80	25.04	24.92					
6	15.87	24.34	24.28	25.88	25.82					
7	18.89	28.04	27.42	27.09	25.91					
8	24.72	30.34	33.99	28.83	25.95					
9	25.05	31.51	34.15	29.12	26.76					
10	25.41	37.94	35.08	30.74	28.78					

Mode number	de number Aspect Ratio 0.06		Aspect Ratio 0.26	Aspect Ratio 0.36	Aspect Ratio 0.46	
1	1 Bend	1 Bend	1 Bend	1 Bend	1 Bend	
2	Oval	Oval	Oval	Oval	Oval	
3	2 Bend	Cone	Cone	Cone	Cone	
4	Triangle	2 Bend	2 Bend	Twisting	Breathing	
5	3 Bend	Triangle	Twisting	2 Bend	Twisting	
6	Square	Twisting	Triangle	Breathing	Twisting	
7	Breathing	Breathing	Breathing	Triangle	2 Bend	
8	4 Bend	3 Bend	3 Bend	Hourglass	Hourglass	
9	Pentagon	Square	Hourglass	Twisting	Twisting	
10	Hourglass	Twisting	Fish	Sloshing	Triangle	

Table 5.9Mode shape for the torus with different aspect ratio, mode shapes in the
shaded areas represent the ring modes (in-plane/out of plane).

By increasing the aspect ratio, the sequence of mode shape changes significantly. For the lowest aspect ratio of 0.06, skinny tube, the first six modes are orthogonal pair of ring modes alternating between in-plane and out of plane modes of the torus. For the highest aspect ratio of 0.46, fat tube, the axisymmetric and twisting modes dominate even in the lower modes, i.e., the local deformation of the meridional curve (cross-section). On the other hand, the frequencies at high aspect ratios are much closer to each other than the ones at the low aspect ratios.

Fig. 5.5 shows the effect of aspect ratio on the first ten natural frequencies of the torus, disregarding their corresponding mode shapes. Except for the first, second and fourth natural frequencies, which increase by the increase of the aspect ratio, the majority of frequencies first increase then decrease as the aspect ratio is increased. On the other hand, examining the natural frequency corresponding to each specific mode shape, one can conclude that by increasing the aspect ratio the natural frequencies increase. Fig. 5.6 shows the effect of the aspect ratio on the first four ring modes of the inflated torus.



Figure 5.5 Effect of the aspect ratio on the first ten natural frequencies of the inflated torus.

Jha (2002) provided the first ten natural frequencies and mode shapes for the aspect ratios of 0.06 and 0.26. The present finite element results compare very well with his analytical solution, the maximum difference between the frequencies is 2.4% and 3.2% for the aspect ratios of 0.06 and 0.26, respectively. Figs. 5.7 and 5.8 show the aforementioned comparison.



Figure 5.6 Effect of the aspect ratio on the first four ring modes of the inflated torus .





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Figure 5.8 Comparison of the FEA and Jha frequencies for the aspect ratio of 0.26.

5.4.2 Internal pressure

As the inflatable satellite moves from orbital eclipse to orbital day and vice versa, the temperature of the enclosed gas changes, which consequently results in changes in the internal pressure (Tinker, 1998). Therefore, it is important to know the impact of the change in pressure on the frequencies and mode shapes. Table 5.10 and 5.11 summarize the effect of the internal pressure on the first ten natural frequencies and mode shapes, respectively. The variation of frequency versus pressure is illustrated in Fig. 5.9. As the pressure increases, the natural frequencies increase. At lower pressures, the rate of the increase in frequencies is high, but further increases in the pressure slow down the rate of increase in the natural frequencies. The pressure affects directly the stiffness of the inflatable torus, and thereby increasing the internal pressure increases the effective stiffness of the inflated torus, which cause an increase in the natural frequencies. The change in internal pressure affects the sequence of mode shapes as well. For example, at higher pressure, the breathing mode moves to lower mode shape number, i.e., from the 10th place at 1000Pa to the 7th place at 2500Pa. Besides, at high pressures, above 4500 Pa, there is only one twisting mode in the first ten modes, whereas there are two twisting modes at low pressures. At higher pressure, e.g., 4500Pa, the antisymmetric axisymmteric (cone) mode has moved from its third place at 500Pa to fourth place.

	Frequency (Hz)										
number	Pressure 500 (Pa)	Pressure 1000 (Pa)	Pressure 1500 (Pa)	Pressure 2500 (Pa)	Pressure 3500 (Pa)	Pressure 4500 (Pa)	Pressure 5500 (Pa)	Pressure 6500 (Pa)			
1	5.39	6.09	6.42	6.73	6.89	6.99	7.07	7.14			
2	5.40	6.22	6.61	6.99	7.19	7.31	7.4	7.46			
3	10.79	13.62	15.20	16.8	17.55	17.91	18.06	18.18			
4	12.98	15.38	16.42	17.29	17.68	17.96	18.20	18.34			
5	13.21	15.83	17.01	18.01	18.46	18.71	18.88	19.01			
6	17.10	20.20	22.00	23.69	24.36	24.67	24.81	24.88			
7	18.69	23.74	26.78	27.91	28.04	28.09	28.10	28.09			
8	18.84	24.05	27.35	29.49	30.38	30.78	31.02	31.18			
9	21.59	26.10	27.40	30.51	31.55	32.07	32.26	32.44			
10	23.14	26.38	29.85	35.42	38.01	38.34	38.31	38.26			

Table 5.10Frequencies for the torus with different internal pressure, values in the shaded areas represent the ring modes
(in-plane/out of plane).

		Frequency (Hz)										
number	Pressure 500 (Pa)	Pressure 1000 (Pa)	Pressure 1500 (Pa)	Pressure 2500 (Pa)	Pressure 3500 (Pa)	Pressure 4500 (Pa)	Pressure 5500 (Pa)	Pressure 6500 (Pa)				
1	1 Bend	1 Bend	1 Bend	end 1 Bend 1 Ben		1 Bend	1 Bend	1 Bend				
2	Oval	Oval	Oval	Oval	Oval	Oval	Oval	Oval				
3	Cone	Cone	Cone	Cone	Cone	2 Bend	2 Bend	2 Bend				
4	2 Bend	2 Bend	2 Bend	2 Bend	2 Bend	Cone	Cone	Cone				
5	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle				
6	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting				
7	3 Bend	3 Bend	3 Bend	Breathing	Breathing	Breathing	Breathing	Breathing				
8	Square	Square	Square	3 Bend								
9	Twisting	Twisting	Breathing	Square	Square	Square	Square	Square				
10	4 Bend	Breathing	Twisting	Twisting	Twisting	Sloshing	Sloshing	Sloshing				

 Table 5.11
 Mode shapes for the torus with different internal pressure, mode shapes in the shaded areas represent the ring modes (in-plane/out of plane).

As mentioned earlier, by increasing the internal pressure, the inflatable torus becomes stiffer, which causes less deformation on the meridional cross section. Fig. 5.10 demonstrates the effect of the pressure on the second in-plane mode for pressures of 1500Pa and 5500Pa. At lower pressure, the torus has lower stiffness hence there is a significant bulge in its deformation.

Closed form solution, Jha (2002), finite element modeling, Lewis and Inman (2001), and experimental studies, Slade et al. (2000) and Griffith and Main (2002), all validate the increasing trend of frequency by the increase of pressure.



Figure 5.9 Effect of the internal pressure on the first ten natural frequencies of the inflated torus.



2nd in-plane bending mode, p=1500 Pa



2nd in-plane bending mode, p=5500 Pa

Figure 5.10 Effect of the internal pressure on the mode shapes

5.4.3 Thickness

For various tube wall thicknesses, the first ten natural frequencies and mode shapes are listed in Tables 5.12 and 5.13, respectively. Fig. 5.11 demonstrates the effect of the wall thickness on the first six ring mode frequencies. In general, increasing the tube's thickness, decrease the frequencies. The analytical study by Jha (2002) validates this trend. The rate of change for small values of thickness is high and as the tube become thicker, the change in frequencies becomes smaller. Increasing the thickness increases both the stiffness and mass of the torus. While generally, the former increases the frequency, the latter has an adverse effect on frequency. For thin walls, the main source of stiffness is the effect of the internal pressure and the effect of mass is more dominant. Therefore, a small change in thickness on stiffness becomes more important and

	Frequency (Hz)											
number	25 (μm)	37.5 (μm)	50 (μm)	75 (μm)	100 (μm)	150 (μm)	200 (μm)	250 (μm)	375 (μm)	500 (μm)	750 (μm)	1000 (μm)
1	7.30	7.15	7.05	6.89	6.75	6.52	6.33	6.14	5.75	5.44	4.94	4.55
2	7.64	7.49	7.38	7.19	7.02	6.74	6.49	6.27	5.82	5.47	4.97	4.62
3	18.43	18.23	18.03	17.55	16.93	15.74	14.70	13.82	12.15	10.99	9.44	8.46
4	18.48	18.38	18.15	17.68	17.36	16.73	16.11	15.52	14.22	13.17	11.63	10.56
5	19.35	19.06	18.84	18.46	18.09	17.36	16.66	15.99	14.55	13.41	11.79	10.68
6	24.75	24.89	24.79	24.36	23.81	22.60	21.44	20.43	18.53	17.30	15.78	14.74
7	27.99	28.08	28.10	28.04	27.94	27.62	25.82	24.12	21.06	19.03	16.42	14.84
8	31.57	31.25	30.97	30.38	29.66	27.79	26.28	24.45	21.25	19.18	16.53	14.88
9	32.88	32.51	32.21	31.56	30.71	28.50	27.14	26.51	23.51	21.84	20.02	18.21
10	38.00	38.23	38.32	38.01	35.92	31.49	28.53	26.55	24.98	23.51	20.32	18.27

Table 5.12	Frequencies for the torus with different wall thickness, values in the shaded areas represent the ring modes (in-
	plane/out of plane).

Mode	Frequency (Hz)											
number	25 (μm)	37.5 (μm)	50 (μm)	75 (μm)	100 (μm)	150 (μm)	200 (μm)	250 (μm)	375 (μm)	500 (μm)	750 (μm)	1000 (μm)
1	1 Bend	1 Bend	1 Bend	1 Bend	1 Bend	1 Bend	1 Bend	1 Bend	1 Bend	1 Bend	Oval	Oval
2	Oval	Oval	Oval	Oval	Oval	Oval	Oval	Oval	Oval	Oval	1 Bend	1 Bend
3	Cone	Cone	2 Bend	2 Bend	Cone							
4	2 Bend	2 Bend	Cone	Cone	2 Bend							
5	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle	Triangle
6	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting	Twisting	3 Bend
7	Breath	Breath	Breath	Breath	Breath	Breath	3 Bend	Square				
8	3 Bend	3 Bend	3 Bend	3 Bend	3 Bend	3 Bend	Square	Square	Square	Square	Square	Twisting
9	Square	Square	Square	Square	Square	Square	Breath	Breath	Twisting	Twisting	Twisting	4 Bend
10	Sloshing	Sloshing	Sloshing	Twisting	Twisting	Twisting	Twisting	Breathing	Breathing	Hourglass	4 Bend	Pentagon

 Table 5.13
 Mode shapes for the torus with different wall thickness, mode shapes in the shaded areas represent the ring modes (in-plane/out of plane).



Figure 5.11 Effect of the tube thickness on the first ring mode frequencies.

therefore, the effects of mass and stiffness on frequencies counterbalance each other. This is why at high thicknesses the rate of change in frequencies becomes low.

At high thicknesses, the natural frequencies are closely spaced which makes the vibration testing and control difficult. The other effect of thickness is on the mode shapes. By increasing the thickness, the ring modes move to lower mode orders and the twisting modes move to higher modes. Fig. 5.12 illustrates the second out of plane mode shapes of the torus with the thickness of 25 μ m and 1000 μ m. The local shape change of the meridional section in thinner tubes is less than the one in the thicker tubes due to the dominant effect of pressure on the effective stiffness of thin wall torus.



 2^{nd} out of plane bending mode, $h = 25 \mu m$



 2^{nd} out of plane bending mode, $h = 1000 \mu m$

Figure 5.12 Effect of wall thickness on the mode shapes.

5.5 Summary

In this chapter finite element modal analysis of a complete torus was presented. The finite element model compares very well with the analytical solution. Since the finite element analysis also predicted six approximately zero frequencies corresponding to six rigid body modes, we concluded that the element type was properly selected. A parametric study on the effect of aspect ratio, pressure and thickness on natural frequencies and mode shapes was carried out.

It was observed that increasing the aspect ratio or the pressure increases the natural frequencies, while increasing the thickness had the opposite effect. While change in any of the aforementioned parameters changed the sequence of modes, the aspect ratio

had the most impact on this variation. Hence, the relative importance of the type of modes to be controlled depends mainly on the aspect ratio of the inflated torus. Considering the fact that the ring modes are easier to be controlled, proper selection of the aspect ratio and thickness can maximize the number of ring modes at the lower frequencies.

CHAPTER 6 The SRT finite element modal analysis

Dynamic analysis of large flexible spacecraft structures may require the analysis of finely detailed models. As a result of the size of these structures and the inclusion of the fine details of the stiffening feature, the conventional finite element model would produce a model with millions of degrees of freedom, which would necessitate large computational effort. In the aerospace industry two typical approaches have been used to address this issue. One approach is to develop simplified models with equivalent structures. And the second approach is the use of physical substructures for model reduction. The modal model is computed for each component and then reassembled to compute the global modal model of the original structure (Lore et al., 2005).

In the present chapter, due to the large number of hexagonal domes in the SRT shell, a simplified sub-structuring technique is proposed and used. Each hexagonal dome is replaced with a statically equivalent flat hexagon of the same width, mass and stiffness. The SRT is then modeled by an equivalent torus with smooth surface.

Section 6.1 details the sub-structuring approach. In Section 6.2 the procedure of building the hexadome is described. The method of finding the equivalent flat hexagon is presented in Section 6.3. The finite element modal analysis of the SRT is carried out in Section 6.4. This chapter ends with a summary and conclusions drawn from the finite element modal analysis of the SRT, Section 6.5.

6.1 Sub-structuring Approach

The following steps were taken in the sub-structuring process. First, the geometry of one single hexagonal dome was built. This was done by pressurizing the central region of a hexagonal plate to match the actual dome shape. This step ended by updating the geometry of the model to form the hexagonal dome shape. Second, by modifying the thickness or density, a flat hexagon with the same width, mass and modulus of elasticity (Kapton $300JP^{\text{(B)}}$) of the hexagonal dome was selected. Third, through a tensile test and

under a uniform displacement applied on the boundaries of models, the reaction forces of both hexagonal dome and hexagonal flat plate were calculated. The reaction forces were compared and the relative stiffness of the two models was calculated. Then, the stiffness of the SRT model was modified (changed) by the inverse ratio of the reaction forces of the hexadome to that of the flat plate. Thereafter, the SRT was modeled by a torus with smooth surface.

Such an approach combined accuracy and acceptable computational cost and made it possible to perform modal analysis of the structure using multipurpose commercial software. This approach also allowed updating the stiffness matrix and taking into account the large deformation under nonlinear static pressurization. Furthermore, the effect of the follower load on the stiffness was included in the analysis.

6.2 Building a Hexagonal Dome

In order to build the geometry of the dome (geometry given in Table 3.1), a hexagonal plate with the major radius of 7.5 mm and thickness of 32.5 μ m was constrained at a distant of 0.5 mm from each side. Hexagonal domes in the SRT were separated from each other by 1mm flat film.

Linear elastic and isotropic properties were assigned to the membrane. The mechanical properties of Kapton 300JP[®] were adopted from Table 3.2. For the modulus of elasticity, the tensile test result was used.

By applying sufficient normal pressure load on the plate, the un-constrained part of the hexagonal plate deformed to a dome shape with the desired height of 3 mm, as shown in Fig. 6.1. Then the geometry of the finite element model was updated to the deformed configuration. The model had 1941 nodes and 1872 SHELL181 elements.


Equivalent Hexagonal Plate

6.3

In the sub-structuring process, each hexagonal dome was replaced with a statically equivalent flat hexagon of the same width, mass and stiffness. Three different flat hexagons with different thicknesses and densities were selected.

First, a thickness of 32.5 μ m (same as dome thickness) and then 46 μ m (same as the SRT film thickness) was selected. In both cases, in order to keep the mass constant, the density was modified. For the third case, the density was kept the same as of Kapton 300JP[®], provided in Table 3.3, and the thickness was modified. The thickness and density of the three cases are listed in Table 6.1.

Table 6.1

Thickness (µm)	Density (kg/m ³)
32.5	1707
39.65	1399
46	1206

Equivalent flat hexagon thickness and density.



Figure 6.2 Schematic of tensile test of hexadome and equivalent hexagon.

Thereafter, the tensile test modeling of the flat hexagon and hexadome was undertaken. The sides of both hexagons were stretched for an arbitrary uniform displacement of 0.01 mm, as illustrated in Fig. 6.2.

For both the hexadome and the three flat hexagons, the reaction forces in the x and y directions on the boundary lines are listed in Table 6.2. Considering the fact that the displacement boundary conditions were equal, the ratio of reaction force to modulus of elasticity of both cases should have been equal:

$$\frac{F_{flat_hexagon}}{F_{hexadome}} = \frac{E_{flat_hexagon}}{E_{hexadome}}$$
(6.1)

This ratio for the flat hexagon with the thickness of 32.5 μ m in the x and y direction was found to be 9.362 and 9.315, respectively, with the average of 9.34. This means that the flat hexagon was 9.34 times stiffer than the hexadome. Henceforth, the torus with hexagonal dome could be simulated with a torus having a flat surface pending that the modulus of elasticity of the Kapton 300JP[®] film was reduced 9.34 times. This ratio for a 39.65 μ m and 46 μ m thick flat hexagons was 11.39 and 13.22, respectively. These ratios are listed in Table 6.3.

Table 6.2Reaction forces.

Flat hexagon thickness (µm)	Flat hexagon density (kg/m ³)	$F_{flat_hexagon}\Big _x$ (N)	F _{hexadome_x} (N)	$F_{flat_hexagor} _{y}$ (N)	$F_{hexadom \epsilon_y}$ (N)
32.5	1707	1.4670	0.1567	0.84492	0.0907
39.65	1399.7	1.7897	0.1567	1.0308	0.0907
46	1206	2.0764	0.1567	1.1959	0.0907

Table 6.3Stiffness ratios.

Flat hexagon thickness (µm)	Flat hexagon density (kg/m ³)	<u>E _{flat _}hexagon</u> E _{hexadome}
32.5	1707	9.34
39.65	1399.7	11.39
46	1206	13.22

6.4 Finite Element Modal Analysis of Self-rigidizing Torus

The finite element analysis was the continuation of the FEA provided in Chapter 5. The modal analysis was carried out based on the following assumptions:

Linear elastic and isotropic properties were assigned to Kapton 300JP[®]. Internal pressure of 149.3 Pa (0.022 psig) equal to that used in the modal experiment study was applied to the torus and assumed to be constant during the static and modal finite element analyses. Geometric nonlinearity (large strain) was also considered during the pre-stress analysis. Furthermore, the effect of the follower load on the stiffness was included in the model. Except for the internal pressure load, the other effects of the enclosed gas were ignored. The effect of air damping was disregarded. Free-free boundary conditions were considered in the analysis. Shell wrinkling effects were not taken into account within the finite element model. Both peel seams and shear seams of the SRT were included in the model.

Using the commercial finite element software package ANSYSTM, and SHELL181 element, a two-step approach was undertaken. First, a nonlinear static pressurization resulted in an updated stiffness matrix that took into account the large deformation. Thereafter, through a restart procedure, the updated stiffness matrix was imported to a linear modal analysis section.

The geometry of the SRT, material properties of Kapton $300JP^{\text{(B)}}$ and 3M scotchweld epoxy adhesive 2216 B/A translucent^(B) were adopted from Table 3.1, Table 3.3 and Table 3.5, respectively. For Kapton $300JP^{\text{(B)}}$ the tensile elastic modulus was used. The mass of the inner and outer peel seams were 0.0877 kg and 0.1114 kg, respectively. Therefore, the flaps constituted 35.5% of the SRT's total mass. The modulus of elasticity of the shear seam and peel seam was 1.35 GPa. For the section with the overlapped shear and peel seams, the modulus of elasticity was 0.966 GPa.

Once the modal analysis was carried out, it was observed that bonded regions added locally distributed masses to the structure and had higher stiffness than that of the toroidal membrane, which had to be accounted in the modeling of the torus, as well. When the peel seams, as shown in Fig. 3.1, were added to the model, it was observed that numerous low frequency local modes of the flaps obscured the torus' modes of interest. Hence, for comparison purposes, two different models were developed. First, the peel seams (flaps) were added to the model and modal analysis was carried out. Second, instead of adding the peel seams to the model, structural mass elements MASS21 with the mass equal to the mass of the flaps were used at the nodes shared by the peel seams and the torus. That is, the mass of each peel seam, inner and outer, was distributed equally on the corresponding seam's nodes by the structural mass element. The model of the SRT with flaps had 23296 nodes, 23296 elements, and 139776 degree of freedom; consisting of 2 elements in flap width, 48 elements around the tube, and 448 elements along the ring. The lumped mass model had 24755 nodes, 25704 elements, and 148512 degree of freedom; consisting of 476 mass elements along each of the inner and outer ring, 52 elements around the tube, and 476 elements along the ring.

Modal analysis was carried out for the three different combinations of thickness and density listed in Table 6.1. The mode shapes were mainly in two categories of inplane and out of plane modes. The in-plane frequencies, for the cases of lumped mass and peel seem, are listed in Tables 6.4 and 6.5, respectively.

The frequencies of the out of plane modes for the cases of lumped mass and peel seem are listed in Tables 6.6 and 6.7, respectively. For the case of lumped mass, the mode shapes for the first four in-plane and out of plane modes are shown in Figs. 6.3 and 6.4, respectively.

Mode number	Frequency (Hz)			
	Thickness 32.5µm Thickness 39.65µm Thickness 46µm			
1	3.54	3.54	3.54	
2	7.66	7.67	7.67	
3	9.90	9.91	9.92	
4	11.23	11.25	11.26	

Table 6.4Frequencies for the in-plane modes for lumped mass application.

Table 6.5Frequencies for the in-plane modes for peel seam application.

Mode number	Frequency (Hz)			
	Thickness 32.5µm	Thickness 39.65µm	Thickness 46µm	
1	4.00	4.00	4.00	
2	9.76	9.76	9.77	
3	15.98	15.99	16.00	
4	24.26	24.27	24.27	

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Mode number	Frequency (Hz)		
	Thickness 32.5µm	Thickness 39.65µm	Thickness 46µm
1	3.91	3.91	3.91
2	9.36	9.36	9.36
3	13.73	13.73	13.73
4	17.19	17.20	17.20

Table 6.6Frequencies for the out of plane modes for lumped mass
application.

Table 6.7Frequencies for the out of plane modes for peel seam application.

Mode number	Frequency (Hz)			
	Thickness 32.5µm	Thickness 39.65µm	Thickness 46µm	
1	3.60	3.63	3.65	
2	11.26	11.36	11.46	
3	15.77	15.79	15.82	
4	19.06	19.09	19.11	

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1st in-plane



2nd in-plane

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3rd in-plane







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1st out of plane



2nd out of plane

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3rd out of plane





Figure 6.4 The first four out of plane mode shapes.

6.5 Summary

In the present chapter, a simplified sub-structuring technique for the finite element modal analysis of the SRT was proposed. Each hexadome was substituted by a flat hexagon of equal mass and density. The SRT was then modeled as a torus with smooth surface. Three different flat hexagons with different thicknesses and densities but equal masses were selected. All three combinations of density and thickness resulted in the same frequencies and sequence of mode shapes, as denoted in Tables 6.4 to 6.7.

Two models of the SRT were developed. In the first one, the flaps were added to the SRT model. While in the second approach, instead of adding the flaps, structural mass elements with the mass equal to the mass of the peel seams were uniformly applied at the inner and outer ring of the SRT model. By comparing the two cases of lumped mass and peel seams, we conclude that except for the first out of plane mode, the frequencies of the model with flaps are higher than the corresponding model with lumped mass, as shown in Figs. 6.5 and 6.6. This means that the model with the flaps is stiffer than the model with lumped mass. Adding the flaps increased the stiffness of the torus mainly in the in-plane direction, affecting the in-plane mode frequencies more than the out of plane ones.

By reviewing the natural frequencies, one can conclude that the natural frequencies of the torus came in clusters.



Figure 6.5 Comparison of in-plane frequencies.

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CHAPTER 7 SUMMARY AND CONCLUSIONS

The goal of this research was to understand the dynamic characteristics of the self-rigidizing torus. In the following paragraphs, we briefly review what was done and give an overall concluding comparison of the experimental and the analytical modal results.

The modulus of elasticity of Kapton 300JP[®] was determined using the tensile testing. Then the variation of this property versus frequency was examined with DMA. It was observed that among the modulus of elasticity, the storage modulus and the loss modulus, the latter was more frequency-dependent over the measurement range (0 to 55 Hz). The loss modulus was two orders of magnitude smaller than either the storage modulus or the modulus of elasticity. Hence, Kapton 300JP[®] behaves more like an elastic material than a typical viscoelastic one. Hence, for the desired frequency range of 1 to 25 Hz, we used the elastic modulus obtained from the tensile test. As part of the material property testing, the modulus elasticity of the "3M scotch-weld epoxy adhesive 2216 B/A translucent[®]" was measured with the tensile test. This property was necessary in the modeling of the SRT.

The experimental modal analysis of the SRT was carried out using the in-house fabricated non-contacting electromagnetic exciter and two laser displacement sensors. The excitation system was able to excite the torus' global modes. The SRT demonstrated weak nonlinear system behavior: the estimated frequency and the corresponding damping of some modes were valid only at the level of excitation used in the test, 3.4V. The first four in-plane and out of plane damped natural frequencies and their corresponding modal damping and mode shapes were identified and compared with prior studies.

As presented in Section 2.2.2, Ruggiero et al. (2004b) could identify the first inplane mode's natural frequency as well as the first three out of plane ones. They did not provide the modal damping ratios. Song et al. (2006) identified the first three in-plane and out of plane modes' natural frequencies and their corresponding modal damping. In the latter study, only the first two identified in-plane modes exhibited the expected classical ring mode shape. The third in-plane mode had the oval shape of the first mode. The present study was able to identify the first four in-plane and out of plane modes as well as decipher the modal asymmetry, which proved that the present experimental modal analysis was more accurate than prior studies.

Thereafter, finite element modal analysis of a complete pressurized torus was performed. For a more accurate analysis, both the geometric nonlinearity and the effect of the follower load on the stiffness were included in the study. The finite element model compared very well with the analytical solution; it also gave six approximately zero frequencies corresponding to the six rigid body modes. Therefore, it was concluded that the element type was properly selected. A parametric study on the effect of aspect ratio, pressure and thickness on the natural frequencies and mode shapes was carried out. It was observed that increasing the aspect ratio (tube radius / ring radius) or the pressure increased the natural frequencies, while increasing the thickness had the opposite effect.

Owing to the large number of hexagonal domes in the SRT shell, a simplified sub-structuring technique was proposed and used. Each hexagonal dome was replaced with a statically equivalent flat hexagon of the same width, mass and stiffness. The SRT was then modeled by an equivalent torus with smooth surface. Since peel seams and shear seams together constituted more than one third of the SRT's total mass, they were included in the model. Two different models were developed. First, the flaps (peel seams) were added to the model and modal analysis was carried out. Second, instead of adding the peel seams to the model, structural mass elements with the mass equal to the mass of the flaps were used at the inner and outer ring nodes. Particularly in the in-plane direction, the model with the flaps was found to be stiffer than the model with the lumped mass.

Overall, there was a satisfactory agreement between the experimentally measured frequencies and those predicted in the finite element approach, as shown in Tables 7.1 and 7.2. The results indicated that in the out of plane direction, the finite element models were slightly stiffer than the experimental one. While in the in-plane direction, except for the first mode, the experimental model was stiffer than the finite element model with the lumped mass and was softer than the model with the flaps. A few sources of discrepancies were indentified in this study, explaining the aforementioned differences between the experimental and the predicted frequencies.

In the finite element model the pressure was assumed to be constant in the entire torus, which, due to leakage of air, this assumption was not completely correct.

Owing to the extremely small values of the film thickness, any increase in the thickness significantly affects the stiffness and mass of the structure and eventually the eigenvalues. For example, the average seams' thickness was above six times the shell film thickness. Furthermore, the method of fabrication, as well as the extensive repair that was done on the flaps, resulted in variations of the thickness of the joined regions. This

contributed to non-uniform stiffness and mass distribution in both the torus and the flaps, affecting the modal parameters of the SRT. The flaps were not completely flat and had wrinkles at various locations. In the finite element study, the flaps were assumed to be flat with a uniform thickness equal to the average thickness of the inner and outer flaps.

	Frequencies (Hz)			
number	Experiment	Lumped mass	Flaps	
1	4.68 / 6.07	3.54	4.00	
2	8.4 / 9.4	7.66	9.76	
3	13.3	9.90	15.98	
4	17.2	11.23	24.26	

Table 7.1	Frequencies for the in-
	plane modes

Table 7.2	Frequencies for the out of
	plane modes

26.1	Frequencies (Hz)		
number	Experiment	Lumped mass	Flaps
1	3.21	3.91	3.60
2	8.1	9.36	11.26
3	12.8	13.73	15.77
4	17.5	17.19	19.06

Though attempts were made to use a very soft suspension and simulate a free-free boundary condition, the boundary condition was still not completely free-free and the suspension system had constrained the outer flaps. This might have affected the response of the torus. At the six suspension points, the outer flaps were inverted upward and they were not staying flat as it was assumed in the finite element analysis, Fig. 7.1. These local deformations of flaps altered the stiffness of the structure and could have affected the modal parameters.

To reduce the effect of the air tube's mass (to inflate the torus) on the SRT, the tube was suspended from a stand. However, since the plastic air tube was not soft enough, it might have constrained the torus which was not modeled in the finite element analysis.

The air current disturbance as well as the effect of the air damping might be another cause of discrepancy between the experimental and predicted results, which were also omitted in the model.



Figure 7.1 Inverted flaps.

7.1 Research Contributions

As pointed out earlier, the goal of this research was to understand the dynamic characteristics of the self-rigidizing torus, both experimentally and analytically. The main contributions of the work in this dissertation are:

1) The development of a more accurate experimental modal analysis of the SRT using non-contact excitation and sensing method.

2) The design and manufacturing of a non-contact electromagnetic excitation system and using it in the modal experiments.

3) The accurate finite element modal analysis of a complete torus using the commercial finite element software, while including geometric nonlinearities and the effect of the follower load on the stiffness for more accurate analyses.

4) The sub-structuring approach for finite element modeling of the hexagonal domes of the SRT and the finite element modal analysis of the self-rigidizing torus using the proposed sub-structuring technique.

5) The inclusion of the bonded regions in the modeling of the SRT.

6) The determination of the modulus of elasticity of the 3M scotch-weld epoxy adhesive 2216 B/A translucent[®].

7) The determination of the variation of the modulus of elasticity of Kapton $300JP^{$ [®] versus frequency.

7.2 **Recommendations for Future Work**

There are several ways in which the present study can be advanced. A few of such possibilities are summarized below.

Bales (1982) observed that change in the ambient temperature during the modal test and the amount of moisture absorbed by the membrane material had significant effect on the dynamic response of the membrane structures. Owing to nonlinear material property and the light weight of the structure, the test environment has a considerable impact on the modal test results. Increasing the atmospheric pressure, from near vacuum to almost one atmosphere, results in the rearrangement of the order of the modes, decreased resonant frequencies, mode shape differences and increased structural damping coefficients (Agnes et al., 2000). One way to improve the state of knowledge is to perform the experiments in a more controlled environment.

Since the air resistance was not incorporated in the modeling, the finite element results should have been compared with the vacuum test results. Performing the test in a vacuum would provide a better benchmark for the model verification.

One complexity in the analysis of an inflatable structure arises from wrinkling. Analysis can be carried out to understand the phenomenon of wrinkling and ways to avoid it. Furthermore, since a single actuator might not perfectly excite the global modes, a MIMO modal testing is suggested as the next step of the present study.

Since there was no information available about the process of heating, stretching and forming of the film of Kapton $300JP^{\textcircled{R}}$, the modulus of elasticity of 32.5 µm thick hexadome was adopted from the property of the 76µm sheet. As verified by Dr. J. Vlachopoulos, the manufacturing process could not be in the order of a fraction of a minute. Henceforth the molecules were still oriented randomly and the properties of the film would not change "significantly". Since the modal frequencies are dependent on the modulus of elasticity, an accurate value for this property is necessary for a more precise analysis.

Vibration and shape control of the SRT is a very critical issue in ensuring the optimal performance of this ultra-flexible structure. Design of spatially distributed controllers is recommended for future work. A typical characteristic of gossamer structures is that the membrane resonant frequencies come in high densities over small

bandwidths (Ruggiero et al., 2006). This dense modal behavior can cause significant complications in the control methodologies.

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Appendix A Specification of the instruments

1. Accelerometer

Type: 8638B50

Maker: Kistler

Sensitivity $\pm 5\%$: 100mV/g

Acceleration range: $\pm 50g$

Resonant frequency: 22kHz

Linearity: ±1%

2. Laser displacement sensor

Type: LK-G82

Maker: Keyence

Reference distance: 80mm

Measuring range: ±15mm

Spot diameter: Approx. Ø70µm

Resolution: 0.2µm

Repeatability: 0.2µm

Linearity: ±0.05% of full scale

Temperature characteristics: 0.01% of full scale /°C

3. Load cell

Type: MB-5

Maker: Interface Miniature Beam

Capacity: 5lbf

Nominal output: 3 lbf

Recommended excitation: 10 VDC

Resonant frequency: 950Hz

Nonlinearity: $\pm 0.03\%$ of full scale

Hysteresis: ±0.02% of full scale

Nonrepeatability: ±0.01% Rated output

Temperature effect on zero: ±0.15 % Rated output/55.6°C

Zero balance: $< \pm 1\%$ Rated output

Compensated temperature range: -15°C to 65°C

Creep, in 20 min: ±0.025%

4. Power amplifier

Type: RAMSA WP-1200

Maker: Panasonic

Output maximum load: $200W / 4\Omega$ per channel

Number of channels: 2

5. Accelerometer calibrator

Type: 4291

Maker: Bruel and Kjaer

6. Air pump

Maker: Optima

Capacity: Aquarium 30+ US gallons

7. NI PCI-6024E DAQ Card

Number of channels: 2 voltage outputs / 16 voltage inputs

Nominal range at full scale: $\pm 10V$

Output / Input coupling: DC

8. Dual blade cutter

Model: IDEAL 1038

Maker: KUTRIMMER

Appendix B Laser displacement sensor calibration

Using an accelerometer calibrator, Bruel and Kjaer Type 4291, both the KEYENCE LK-G82 laser displacement sensors calibrated by comparison method. A Kistler 8638B50 accelerometer mounted on the calibrator and both the laser displacement sensors beam simultaneously aimed at accelerometer during the measurement. As illustrated in Fig. B.1. the amplitude of acceleration for the three transducers is the same. The measured frequency of the shaker is 80.1938 Hz which was consistent in all measurements, Fig. B.2.

As expected there is a 180 degree phase difference between the accelerometer measurement and the displacement measurements, Fig. B.3. At the frequency of 80.1938 Hz the measured phase change of laser A, laser B, and accelerometer is -180.0199, -180.0672° and 179.8510° respectively. As shown in Fig. B.4, the coherence between the transducers is 1.0 at the frequency of 80Hz for a span of 5Hz, which proves the accuracy of the laser displacement sensors.



Figure B.1. Time history



Figure B.2. FFT.

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Figure B.3. Phase.



Figure B.4. Coherence.

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Appendix C Aquarium pump frequency

The frequency of the air pump's diaphragm evaluated using both the Kistler 8638B accelerometer and the Keyence LK-G82 laser displacement sensor. From the auto power spectrum density graphs, Fig. C.1, the diaphragm frequency is found to be 60Hz. Then while the pump was running, the effect of the pump's noise and the airflow through the soft hoses, on the torus, is studied, Fig. C.2. No noticeable effect on the auto power spectrum density observed.







Figure C.1. Pump auto power spectrum density measured by (a) accelerometer (b) laser displacement sensor



Figure C.2. Torus auto power spectrum density measured by laser displacement sensor

Appendix D Uncertainty analysis

The ANSI/ASME International' PTC 19.1 Test Uncertainty is used for the uncertainty analysis. This method refers to random (zero-order) and systematic (instrument) errors.

The zero-order uncertainty, u_0 , of each instrument is an estimate of the expected random uncertainty. A numerical value of one-half of the instrument resolution or the least digital count of the instrument was assigned to u_0 .

The instrument uncertainty, u_c , is an estimate of the systematic error for the instrument. The u_c was estimated by combining all known elemental errors using the root-sum-squares (RSS) method.

$$u_c = \pm \sqrt{\sum_{i=1}^{N} u_i^2}$$

The design-stage uncertainty, u_d , for each instrument was calculated by combining the instrument uncertainty with the zero-order uncertainty,

$$u_d = \sqrt{u_0^2 + u_c^2}$$

Based on the instruments specifications, presented in Appendix A, the uncertainty for the load cell and the laser displacement sensor was calculated as 0.03 N and 0.0002 mm, respectively.