ADVANCED ELECTROMAGNETIC MODELING OF THE INTERACTION OF THE MICROWAVE FIELD WITH HUMAN TISSUES
ADVANCED ELECTROMAGNETIC MODELING OF THE INTERACTION OF THE MICROWAVE FIELD WITH HUMAN TISSUES

By

Li Liu, M.Sc., B.Sc.

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AUTHOR: Li Liu
M. Sc.
Department of Electrical and Computer Engineering (Wuhan University)

SUPERVISOR: N. K. Nikolova, Professor
Ph. D. (University of Electro-Communications)
P. Eng. (Ontario)
Canada Research Chair (in High Frequency Electromagnetics)

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ABSTRACT

This thesis contributes significantly to the advanced electromagnetic (EM) modeling of the interaction of the microwave field with human tissues. The proposed EM models achieve unprecedented computational efficiency, accuracy, and reliability.

The EM modeling is crucial in (a) multiphysics and (b) EM analysis in support of optimization procedures with applications in design optimization and inverse problem solutions. The challenge in such applications stems from the fact that EM modeling requires extensive computational resources. Therefore, the reduction of these computational requirements is necessary in order to handle the complexity of multiphysics modeling and microwave imaging.

In this thesis, an efficient EM/thermal analysis of the interaction of the radio-field (RF) fields of mobile phones with the human eyes is presented. Another advanced application developed here is the solution of inverse problem in microwave imaging and detection by making use of response sensitivity analysis.

Two methods are proposed for the evaluation of the maximum specific absorption rate (SAR) in the human eyes due to RF exposure from handheld devices. They account for the existence of resonance in the eye and are applied to the case of
near-field exposure. The first method is semi-analytical. As an input, it requires the measured or simulated open-space near field of the device under test in the absence of the eye. As an output, depending on the mutual position and orientation of the eye and the device, it produces the maximum SAR value in the eye averaged over 1 and 10 grams of tissue. The second method is experimental. It requires the fabrication of a simple eye phantom and relies on a measurement with an SAR robot. The proposed methods allow for the fast and reliable SAR evaluation of newly developed handheld devices in an industrial environment. Results concerning the temperature rise in the eyes are also presented. They are based on detailed simulation eye models.

A conceptually new detection algorithm is proposed in this thesis for the localization of electrically small scatterers in a known background medium. The algorithm requires the knowledge of the electric field distribution inside the known background medium where no scatterers are present. It is based on a self-adjoint response sensitivity computation which can be performed in real time. Using the E-field distribution in the background medium, it provides three-dimensional maps of the Fréchet derivative within the imaged volume. The peaks and dips in these maps identify the locations where the permittivity and conductivity of the measured medium differ from those in the background medium. The background medium can be heterogeneous. The performance of the detection algorithm is studied in terms of the number of transmission/reception points, the dielectric contrast of the scatterer compared to the background medium, and the size of the scatterer. Its resolution is
also addressed. The proposed detection algorithm is successfully applied in breast cancer detection.
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List of Abbreviations

2D Two Dimensional
3D Three Dimensional
ABC Absorbing Boundary Condition
EM Electromagnetic
FDTD Finite-Difference Time-Domain
PIFA Printed Inverted-F Antenna
RF Radio Frequency
RHCP patch Right-hand Circularly Polarized Patch
SAR Specific Absorption Rate
SASA Self-Adjoint Sensitivity Analysis
TD Time Domain
TLM Transmission Line Modeling
Chapter 1

INTRODUCTION

1.1 MOTIVATION

Many applications in science and technology rely increasingly on electromagnetic (EM) field computations in either man-made or natural problems. The solution approaches in EM may be roughly divided into two: analytical modeling and numerical modeling. Analytical models are solutions of Maxwell’s equations in a given environment (defined by its medium parameters) and for given boundary conditions (the geometry). Analytical models based on field theory typically express the solutions for the chosen electric and magnetic field components in terms of chosen analytical basis functions (such as sine or cosine functions, Bessel and/or Hankel series, etc.).

The need to solve realistic problems of arbitrary geometry in a reliable way with high accuracy is one of the major driving factors in developing numerical modeling. EM problems have become extremely complex and can not be handled via analytical approaches. Major areas where complex EM problems arise and where numerical approaches are used, include but are not limited to, antenna/tissue
interaction and microwave imaging (these two will be addressed in the thesis), as well as propagation, antenna analysis and design, biomedical engineering, electromagnetic compatibility (EMC), electromagnetic interference (EMI), microwave networks, microwave tomography, sensors, and integrated systems. Numerical models are characterized by their accuracy, reliability, efficiency, versatility, and applicability.

Modern research in the area of numerical modeling focuses on two major types of problems: (a) multiphysics and (b) optimization. All the systems important for scientific and industrial applications are inherently multiphysics involving mechanical, fluid, thermal, electromagnetic, biological, etc., processes. Numerical simulation of these multiphysics problems requires the development of sophisticated models and methods for their integration, as well as efficient numerical algorithms and advanced computational techniques.

In recent decades, the growing need for the quality improvement of industrial and practical processes leads to the rapid development of optimization. The rapid development of optimum-seeking algorithms was brought about by the progress in computer science and the applied mathematics of optimization. Numerical optimization is a growing research area which responds to the growing interest in optimization in engineering, science, and business. EM numerical modeling is coupled with optimization processes for applications in design optimization and inverse problem solutions.
This thesis addresses research in the above areas of modern EM numerical methods through: (a) an efficient EM/thermal analysis of the interaction of the radio-frequency (RF) fields of handheld devices with the human eyes; (b) response sensitivity analysis for the solution of inverse problem in microwave imaging and detection.

The widespread use of mobile phones is a relatively recent phenomenon. Their use has escalated over the last decade, and the number of worldwide subscribers was more than 1.6 billion by the year 2005 [1]. The extensive use of mobile phones has been accompanied by: (i) strict regulations for the industry and (ii) public debate about possible adverse health effects.

The non-governmental organization accepted as an independent expert body by the United Nations (UN), the World Health Organization (WHO), as well as the International Labor Organization (ILO), the European Union (EU), and most of the developed countries, is the International Commission on Non-Ionizing Radiation Protection (ICNIRP). The Commission is established for the purpose of advancing non-ionizing radiation (NIR) protection for the benefit of people and the environment and in particular to provide guidance and recommendation on protection from NIR exposure. In 1998, the ICNIRP published its latest guidelines for exposure limits, which were based on the evaluation of all available and known analytical/numerical/experimental results [2]. In these guidelines as well as elsewhere, in the frequency range of 10 MHz to a few GHz (including 900 MHz and
1800 MHz cellular phone frequencies). the specific absorption rate (SAR) is accepted as a main physical parameter. The most recognized EM exposure standards, including ICNIRP [2]-[4], adopt the SAR averaged over the whole body (SAR$_{wB}$) as the basic parameter to evaluate the safety of an exposure. Sometimes, although SAR$_{wB}$ is below the basic limit, the local SAR values can be rather high in a confined body region, where the power absorption takes place, such as a human eye near an antenna of a mobile phone. In such conditions, limits on the local SAR averaged over tissue masses of 1 g or 10 g have been introduced in the standards, as 1.6 W/kg over 1 g or 2 W/kg over 10 g.

The evaluation of the actual amount of heating (temperature rise) due to RF exposure is crucial when possible tissue damage is investigated [5]-[8] although it is rarely the subject of regular testing in an industrial environment. Exposure guidelines [2] recommend that RF induced tissue temperature rise does not exceed 1 °C. Adherence to the exposure SAR limits [2]-[4] should ensure that the tissue temperature rise is well below the 1 °C limit.

The coupling of EM energy to the complex human tissues (e.g. eye tissues), described by SAR, is difficult if not impossible to approach analytically. Hence, SAR and thermal analysis rely mostly on simulations. The full-wave simulations, which include a complicated eye model and the device under test (e.g. a mobile phone) are time-consuming and case-specific. Thus, they are not suitable for regular industrial testing and evaluation. Therefore, it is important to have a fast and reliable
numerical modeling to evaluate the SAR and the temperature rise in the tissues for newly designed mobile phones. This is where one of the contributions of this thesis lies.

EM waves are widely used in subsurface imaging. The applications include, but are not limited to, tumor detection, landmines and unexploded ordnance, archeological discoveries, crack detection etc. One crucial application of subsurface imaging is tumor detection. Microwave imaging for breast cancer detection has been a widely investigated topic. In the U.S., breast cancer is the most common nonskin malignancy in women and the second leading cause of female cancer mortality. Approximately 180,000 new cases of breast cancer were diagnosed and approximately 44,000 deaths occurred in 1994 alone [9]. Breast cancer will occur in approximately one in eight women by the age of 90 years. Detection of breast cancer at an early stage increases the likelihood of successful treatment and long-term survival. Screen film mammography is currently the most effective method of detecting asymptomatic breast cancer [10].

The important limitations of current diagnostic tools motivate the quest for alternatives. The rationale for pursuing electromagnetic methods is strong. First and foremost, there is data in the literature [11][12], which show that the electromagnetic properties of breast malignancy are significantly different than normal in the high megahertz to low gigahertz spectral range. Second, microwave illumination can effectively penetrate the breast at these frequencies. Third, the breast is a small
readily accessible tissue volume, making it an ideal site for deploying advanced near-field imaging concepts that exploit model-based image reconstruction methodology.

The imaging and detection problem is extremely complex. The main difficulty in producing high-quality microwave images is due to the complex diffraction mechanism of the wave propagation in the strongly inhomogeneous and dispersive biological tissues. Another challenging problem is due to the required forward models, which are extremely expensive full-wave EM simulations. A rough estimate shows that such simulations would require in excess of 6 GB of RAM (random-access memory) for the most common techniques, e.g., finite elements or finite differences. Provided that sufficient memory is allocated, the computational time for a single system analysis would be in excess of 6 hours on a modern personal computer even if hardware acceleration is implemented. Therefore, efficient numerical modeling is necessary in order to handle the complexity of microwave imaging. It is an essential tool in research toward reliable (ensuring repeatability and accuracy of data acquisition and image reconstruction), efficient (providing answers in a reasonable amount of time), practical (microwave measurement apparatus is readily available), relatively inexpensive (e.g., compared to X-ray systems), and safe imaging and detection methodologies. One of the contributions of this thesis aims at taking this research one step further toward practical clinical systems.

This thesis presents advanced electromagnetic modeling addressing: (a) an efficient EM/thermal analysis of the interaction of the RF fields of mobile phones
with the human eyes: (b) the solution of inverse problem in microwave imaging and
detection. Firstly, two feasible methods are presented for the evaluation of the SAR
in the human eyes due to exposure to RF fields. The methods are applied to the case
of near-field exposure to handheld devices; however, they are applicable to the case
of far-field illumination as well. The first method is based on a semi-analytical
model of the eye. As an input, it requires the measured or simulated open-space field
of the device under test. As an output, depending on the mutual position and
orientation of the eye and the device, it produces the maximum SAR value in the eye
averaged over 1 and 10 grams of tissue. The second method is based on
measurements. It requires the fabrication of a simple eye phantom and relies on a
standard measurement with an SAR robot. The measurement method is
complementary to the semi-analytical model and can be used to verify its output
through one additional field measurement in open space. Moreover, a detailed
simulation eye model is developed, which allows for the calculation of the
temperature rise in the eyes for a given device under test. The proposed methods
allow for the fast and reliable SAR and thermal evaluation of newly developed
handheld devices in an industrial environment.

Secondly, a novel general formulation of the response sensitivity analysis is
proposed and implemented in a computationally efficient algorithm for the detection
of electrically small scatterers in a known background medium. The responses of the
background medium where no scatterers are present are modeled via ultra-wideband
time-domain simulation. Using these modeled responses and the measured responses of the examined object, 3D derivative maps are obtained within the object's volume. The minima and the maxima in these maps indicate the locations where the voxel permittivities and conductivities differ significantly between the measured and modeled media. Localization of the scatterers in a complex heterogeneous example is successfully conducted. The limitations of the detection algorithm and its resolution are studied using a homogeneous background example in terms of the number of transmission/reception points, the dielectric contrast and the size of the scatterer.

1.2 CONTRIBUTIONS

The author has contributed substantially to a number of original developments presented in this thesis. These are briefly described next.

(1) A fast and accurate semi-analytical model to evaluate the maximum SAR in the human eyes due to RF exposure from handheld devices, published in [13]-[16].

(2) An experimental method for the evaluation of the maximum SAR in the human eyes due to RF exposure from handheld devices, published in [14]-[16].

(3) A detailed simulation eye model used to analyze the temperature rise due to RF exposure from handheld devices, published in [15][16].
(4) A conceptually new detection algorithm for the localization of electrically small scatterers in a known background medium, published in [17]-[19].

(5) Implementation of the novel near-field detection based on self-adjoint sensitivity computation for scatterers embedded in lossy homogeneous and heterogeneous background media, published in [19].


(7) Development of systematic assessment criteria for the performance of ultra wide band sensors [21].

1.3 OUTLINE OF THE THESIS

This thesis presents novel approaches to (1) evaluate the maximum SAR in the human eye due to RF exposure from handheld devices, and corresponding temperature rise; (2) detect electrically small scatterers in the known background medium, and its application in breast cancer detection.

Chapter 2 starts with a comprehensive study on the resonant effects in the human eye at near-field exposure with account for the type and orientation of the antennas. Then a fast semi-analytical model is introduced, which can predict reliably the maximum SAR value in the eye averaged over 1 or 10 grams of tissue. The model is verified through simulations and through measurements in a phantom which captures the resonance in the eye. The direct measurement of the maximum SAR
value for a given device under test through the phantom is addressed in the end of Chapter.

Chapter 3 addresses the results on the temperature rise in the eye under conditions of maximum allowed induced SAR and maximum allowed transmitted power. It starts with an introduction to the high-fidelity detailed eye model. The details of the eye include the retina and the choroid. These are important from a thermal point of view since they have significant blood flow. The modes of operation of the handheld devices are taken into account in the end of Chapter. At the different operation modes, the transmitted power may vary significantly, which brings large differences in the estimated maximum SAR and temperature rise. The focus is on the worst-case scenario when the transmitted power from handheld device is the largest.

Chapter 4 addresses a conceptually new detection algorithm for the localization of electrically small scatterers in a known background medium. It starts with introducing the formulation of the response sensitivity analysis. Then, it is discussed from the viewpoint of the linear Born approximation. It shows that the formulation of the objective functions and summarizes the detection algorithm. Verification through an example with a homogeneous background is also given. Furthermore, the performance of the algorithm is also studied versus the number of the transmission/reception points, the contrast between the scatterer and the background medium, and the size of the scatterer, and as well as the resolution demonstrating the algorithm's ability to discern two electrically small scatterers. The
algorithm is applied in an example with a heterogeneous background in the end of Chapter.

Chapter 5 presents the breast cancer detection using microwave frequencies and self-adjoint sensitivity analysis. The technique can be applied into lossy heterogeneous dielectric structures arising in biomedical applications of microwave imaging, where the dielectric losses are usually significant. The detection algorithm based on the self-adjoint response sensitivity analysis is tested on a realistic breast-tumor model through ultra-wideband time-domain simulation. The detection relies on images obtained from the response derivatives with respect to the voxel permittivities and conductivities in the known background medium, which is a close approximation of the healthy (or normal) breast tissue. Abnormal regions in the object under test are successfully detected and localized. The Chapter further describes the performance of the detection algorithm in the realistic breast-tumor model versus the number of the transmission/reception points, the contrast between the scatterer and the background medium, and the size of the scatterer, and as well as the resolution demonstrating the algorithm’s ability to discern two electrically small scatterers.

The thesis concludes with Chapter 6 where suggestions for future research are outlined.
REFERENCES


Chapter 2

EVALUATION OF THE SPECIFIC ABSORPTION RATE IN THE HUMAN EYES

2.1 INTRODUCTION

The human eye comprises some of the most sensitive body tissues as far as radio-frequency (RF) exposure is concerned [1][2]. At the same time, modern mobile communication services offer data formats such as text messaging, email, video and internet, where the handheld is placed directly in front of the eyes, sometimes in close proximity. Thus, it is important to have a fast and reliable means of evaluating the specific absorption rate (SAR) in the eyes for newly designed handheld devices. The industry follows strict guidelines, which regulate the exposure limits [3]-[7]. In the case of the eye, the limit for the SAR averaged over 10 g of tissue is 2 W/kg in the frequency range from 0.5 GHz to 3.5 GHz in the IEEE standard [3] and the International Commission on Non-Ionizing Radiation Protection (ICNIRP) guidelines [4]. In addition, according to the Federal Communications Commission (FCC) guidelines [6], the limit for near-field exposure in public environment is 1.6 W/kg for any tissue averaged over 1 g.
The evaluation of the actual amount of heating (temperature rise) due to RF exposure is crucial when possible tissue damage is investigated [1][8]-[10] although it is rarely the subject of testing in an industrial environment. Exposure guidelines [4] recommend that RF induced tissue temperature rise does not exceed 1 °C. Adherence to the SAR exposure limits [3]-[6] should ensure that the above temperature limit is observed.

Recently, a variety of computational methods for quantifying the SAR have been developed [11]. Analytical methods are suitable for simplified geometries. However, the coupling of electromagnetic (EM) energy to the complex eye tissues is difficult if not impossible to approach analytically. Thus, SAR and thermal models rely mostly on numerical simulations [12][13]. The full-wave simulations, which include the eye model and the device under test are time-consuming and case-specific. Thus, they are not suitable for regular industrial testing and evaluation.

Possible resonances in the eye may lead to locally enhanced SAR values. This has been addressed through numerical methods [13]. However, no rigorous theoretical model of the resonant effects was provided in order to relate the geometrical and material parameters of the eye to the value and the location of the maximum SAR. In the work [14], we outlined a fast semi-analytical model which exploits the resonant properties of the human eye. Here, we discuss in detail the resonant properties of the human eye and specify the resonance-based fast semi-analytical model. It predicts the maximum SAR in the eye (averaged over 1 g and 10
g of tissue) for radiating devices whose open-space field is known at the eye location. The open-space field of the device under test may be measured or simulated in the absence of the eye. Near-field exposure is studied in order to address the now common use of handhelds in internet and data formats. The model takes into account the mutual position/orientation of the eye and the device. It is verified through EM simulations as well as measurements.

Regardless of how accurate the SAR models may be, SAR measurements are necessary. Here, the phantom technology is essential. Much work has been done on head and whole-body phantoms [7][15], which are made of plastic containers shaped as a human body and/or head filled with homogenous gels or liquids representing the averaged electrical properties of tissues. The existing homogenous head and torso phantoms do not represent properly the eyes. The eyes have distinct electrical properties compared to the surrounding tissues and the air. Thus, they exhibit resonant behavior in the frequency range of interest. Inhomogeneous head phantoms exist [16][17] but they are relatively difficult to construct and have not addressed the SAR measurement inside the eye. In [14], we proposed a method to measure the SAR in an eye phantom, which is described here in greater detail. It requires the fabrication of a simple inhomogeneous eye liquid phantom and a standard measurement with an SAR robot such as DASY4 (Dosimetric Assessment Systems) [18]. The measurement technique exploits the same physical principles as the semi-analytical model and is complementary to it in the sense that: (1) it can provide the
In summary, here we present the results of a comprehensive study on the resonant effects in the human eye at near-field exposure with account for the type and orientation of the antennas. A fast semi-analytical model is proposed which can predict reliably the maximum SAR value in the eye averaged over 1 or 10 grams of tissue. The model is verified through simulations and through measurements in a phantom which captures the resonance in the eye. The phantom can also be used to measure directly the maximum SAR value for a given device under test.

2.2 ANALYTICAL MODEL OF THE RESONANT EFFECTS

The resonant effects in the eye can be approximated with the simplified geometry shown in Fig. 2.1. The eyeball is modeled by a lossy dielectric sphere immersed in an infinite head-equivalent medium referred to as the background medium. The uniform sphere representing the eyeball uses only the relative permittivity and the conductivity of the vitreous body, which constitutes more than 90% of the eye volume. Its radius is set at 1.3 cm, an average value for an adult. The properties of
the background medium are a weighted average of the properties of the major tissues surrounding the eyeball: skin, fat, brain and bone. It extends about $\lambda_{\text{max}} / 2$ from the surface of the eyeball where $\lambda_{\text{max}}$ is the maximum wavelength in the eyeball medium. The total considered volume is $3.3 \times 3.3 \times 3.3 \text{ cm}^3$ with the eyeball centered in it. The weighting coefficient for each tissue depends on the volume it occupies as a proportion of the background volume [see Table 2.1] [29]. The permittivities and conductivities of the eye tissues, such as vitreous body, skin, fat, etc., can be obtained from [30][31]. Note that the biological tissues are dispersive. The properties of the tissues shown in Table 2.1 are taken at 1.8 GHz.

Fig. 2.1  The geometry of the simplified analytical model. The human eyeball is approximated by a lossy dielectric sphere immersed in an infinite head-equivalent medium referred to as the background medium. The constitutive parameters are taken at 1.8 GHz.
Table 2.1
Properties of the Constituents of the Background Medium at 1.8 GHz

<table>
<thead>
<tr>
<th>Tissue</th>
<th>( \varepsilon_r )</th>
<th>( \sigma ) (S/m)</th>
<th>Weighting Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>5.35</td>
<td>0.078</td>
<td>0.94</td>
</tr>
<tr>
<td>Skin</td>
<td>38.87</td>
<td>1.18</td>
<td>0.02</td>
</tr>
<tr>
<td>Bone</td>
<td>11.78</td>
<td>0.29</td>
<td>0.02</td>
</tr>
<tr>
<td>Brain</td>
<td>42.24</td>
<td>1.10</td>
<td>0.02</td>
</tr>
<tr>
<td>Background medium</td>
<td>6.0</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

The simple two-medium structure allows for analytical modeling. Due to the high permittivity contrast between the eyeball and the background medium, the sphere has distinct resonant modes [32]. The work in [32] assumes loss-free media. Here, we extend the model to the case of a lossy sphere immersed in a lossy medium. We derive the resonant modes by solving the Helmholtz equations for the \( E \) and \( H \) field in terms of spherical functions [33]-[35]. They represent the waves in terms of transverse electric (TE\(_r\)) and transverse magnetic (TM\(_r\)) modes. Inside the eyeball, a TE\(_r\) mode is described by

\[
E_r = 0 \\
E_\theta = -m(\varepsilon_e r \sin \theta)^{-1} \cdot \hat{J}_n(\gamma_1 r) \cdot P_n^m(\cos \theta) \cdot \Phi_\phi^r \phi \\
E_\phi = (\varepsilon_e r)^{-1} \cdot \hat{J}_n(\gamma_1 r) \cdot P_n^m(\cos \theta)^{\prime} \cdot \Phi_\phi^r \phi 
\]

\[
H_r = (j \omega \mu_e e)^{-1} [\hat{J}_n(\gamma_1 r)^{\prime} - \gamma \cdot \hat{J}_n(\gamma_1 r)] P_n^m(\cos \theta) \cdot \Phi_\phi^r \phi \\
H_\theta = (j \omega \mu_e e)^{-1} \cdot \hat{J}_n(\gamma_1 r)^{\prime} \cdot P_n^m(\cos \theta)^{\prime} \cdot \Phi_\phi^r \phi \\
H_\phi = m(j \omega \mu_e e r \sin \theta)^{-1} \cdot \hat{J}_n(\gamma_1 r)^{\prime} \cdot P_n^m(\cos \theta) \cdot \Phi_\phi^r \phi 
\]

20
where \( \Phi_z(\phi) = \pm C \sin(m\phi) + D \cos(m\phi) \), \( P_n^m(\cos \theta) \) is an associated Legendre function of the first kind, and \( \hat{J}_n(\gamma_1 r) \) is the spherical Bessel function of the first kind defined by Schelkunoff [35]. The complex propagation constant \( \gamma_1 = j\omega \left[ \mu(\varepsilon_0\varepsilon_r - j\sigma_1 / \omega) \right]^{1/2} \) is that of the eyeball. In the background medium, the expressions are similar and obtained from (1)-(2) by replacing \( \hat{J}_n(\gamma_1 r) \) with the spherical Hankel function of the second kind \( \hat{H}_n^{(2)}(\gamma_2 r) \) where \( \gamma_2 = j\omega \left[ \mu(\varepsilon_0\varepsilon_r - j\sigma_2 / \omega) \right]^{1/2} \). For the TM\(_r\) modes, the expressions are dual to those in (2.1)-(2.2).

The continuity conditions at the interface between the eyeball and the background medium lead to the eigenvalue equations:

a) TE\(_{r}^{(n,m,p)}\) modes

\[
\frac{J_{n+1/2}(\gamma_1 a)}{\gamma_1 J_{n-1/2}(\gamma_1 a)} = \frac{H_{n+1/2}^{(2)}(\gamma_2 a)}{\gamma_2 H_{n-1/2}^{(2)}(\gamma_2 a)}
\]  
(2.3)

b) TM\(_{r}^{(n,m,p)}\) modes

\[
\frac{\gamma_1}{j\omega \varepsilon_1} J_{n+1/2}(\gamma_1 a) - \frac{n}{j\omega \varepsilon_1 a} = \frac{\gamma_2}{j\omega \varepsilon_2} H_{n+1/2}^{(2)}(\gamma_2 a) - \frac{n}{j\omega \varepsilon_2 a}
\]  
(2.4)

Here, \( a \) is the radius of the eyeball and \( \varepsilon_{1,2} = \varepsilon_\| \varepsilon_r,2 - j\sigma_{1,2} / \omega \). The equations are solved for \( \omega \), which yields an infinite discrete spectrum of resonant frequencies. The order of the obtained roots is labeled as \( p \).
Table 2.2 shows the resonant frequencies of the dominant modes of the eyeball resonator [as shown in Fig. 2.1]. These frequencies are close to those used in cellular services. They are also in good agreement with the FDTD model in [13]. Note that the resonant frequencies obtained by solving (2.3) and (2.4) are complex [36]. Their real parts are the actual oscillation frequencies [shown in Table 2.2 in GHz]. The imaginary parts are the damping coefficients. For example, the TM\(_{r}^{(1,0,2)}\) and TE\(_{r}^{(1,0,1)}\) modes have the same oscillation frequency of 1.8 GHz. However, their respective damping coefficients are different, \(1.5 \times 10^8\) s\(^{-1}\) and \(3.3 \times 10^8\) s\(^{-1}\), respectively.

The field distribution in the eyeball is obtained for each mode from (2.1)-(2.2) and the respective dual TM\(_{r}\) expressions. Note that the field distribution depends on \(n\) and \(m\) only. For example, the TM\(_{r}^{(1,0,p)}\) and TE\(_{r}^{(1,0,p)}\) modes are:

a) TM\(_{r}^{(1,0,p)}\)

\[
E_r = 2\sqrt{2}r^{-2}\hat{J}_1(\gamma r) \cdot \cos \theta \\
E_\theta = r^{-1}[\gamma \hat{J}_0(\gamma r) - r^{-1}\hat{J}_1(\gamma r)]\sin \theta \\
E_\phi = 0
\]  

(2.5)

b) TE\(_{r}^{(1,0,p)}\)
\[ E_r = 0, \ E_\theta = 0 \]
\[ E_\phi = r^{-1} j_1(\gamma_1 r) \sin \theta \]  \hspace{1cm} (2.6)

where \( \gamma_1 \) is taken at the desired frequency.

To build the SAR analytical model, we need the normalized field distribution at a given frequency. If this frequency is resonant, a linear combination of all resonant modes is taken. For example, at 1.8 GHz, the \( \text{TM}_{1.0.p} \) and \( \text{TE}_{1.0.p} \) modal distributions in (2.5) and (2.6) are used. When the resonant modes are more than one, the linear combination requires the proper coefficients multiplying the modal terms. Since we are interested in a normalized distribution only, one of the coefficients is always set to one. The remaining coefficients are obtained by minimizing the difference between the field distribution predicted by the linear combination of modes and that of a reference model. The reference model used here is a simulation (with HFSS ver. 10.1.2 [37]) of the structure shown in Fig. 2.1 at the desired frequency with a plane-wave excitation.

It suffices to match only the field-magnitude value at the eye periphery normalized with respect to the maximum field-magnitude value, which, for the dominant modes, is in the eyeball center [see the value \( \bar{E}_h \) in Fig. 2.2]. At 1.8 GHz, for example, with a coefficient of 1 for the \( \text{TM}_{1.0.p} \) mode, we determine the coefficient of the \( \text{TE}_{1.0.p} \) term as \((0.21 + j0.12\times10^{-4})\). The minimization was performed with the function \textit{fmincon} in MATLAB [38]. The matched normalized field-magnitude distributions from the simulation and the two-mode analytical model
at 1.8 GHz are shown in Fig. 2.2. Thus, we reduce the description of the normalized field distribution in the eye model to a single complex number, the coefficient of the TE_{r}^{(1,0,p)} term.

Using the above procedure, analytical approximate models for the field distribution in the eye can be built at any desired frequency. We refer to these models as semi-analytical because, in general, their modal coefficients are obtained via a matching procedure to a reference model.

If the frequency of interest is different from a resonant frequency [see Table 2.2], one may need to include more modes in the analytical model. As an example, in Fig. 2.3, we show a comparison at 1.2 GHz. The field distribution predicted by the two-mode combination does not match well the simulated field distribution. Note that the optimized coefficient for the TE_{r}^{(1,0,p)} mode at 1.2 GHz is 0.20 + j0.32×10^{-4}. To achieve better match, one additional mode is included, the TM_{r}^{(2,0,p)} mode. The model is then re-optimized to yield the coefficients for the TE_{r}^{(1,0,p)} and TM_{r}^{(2,0,p)} modes as (0.20 + j0.22×10^{-4}) and (0.07 + j0.01×10^{-4}), respectively. The three-mode analytical curve is also plotted in Fig. 2.3. It matches better the reference curve.

The modeling results, both analytical and simulation, confirm the existence of resonance in the eye. Note that the field has its maximum in the center of the eyeball where it is more than twice stronger than its value at the eye periphery. The resonance has a low quality factor due to the significant loss.
Fig. 2.2  The field magnitude distribution along the eye’s axis at 1.8 GHz obtained from the two-mode analytical model and the HFSS simulation. The eyeball radius is $a = 13$ mm.

Fig. 2.3.  The field magnitude distribution along the eye’s axis at 1.2 GHz obtained with: (1) the two-mode analytical model optimized at 1.2 GHz, (2) the three-mode analytical model optimized at 1.2 GHz, and (3) the HFSS simulation.
2.3 ANTENNA ORIENTATION: NEAR FIELD PARAMETRIC STUDY

Here, we present a parametric study on the impact of the antenna orientation on the maximum SAR value and the SAR distribution in the eye. In a simulation example, we use a half-wave dipole as the excitation antenna. Its length is 80 mm. The frequency is 1.8 GHz and the radiated power is $P_{\text{rad}} = 10$ W. We take into consideration several head tissues: skin, fat, bone and brain whose constitutive parameters are found in [30][31]. This simplified model represents the eyeball as a sphere whose constitutive parameters are those of the vitreous body. The dimensions of each shape component are shown in Fig. 2.4. The distance $D$ from the surface of the eye to the center of the dipole is 50 mm. We rotate the dipole along its center from the vertical position $0^\circ$ to the horizontal position $90^\circ$ to obtain the SAR distribution along the axis defined by the points $A$ and $B$ versus the dipole orientation. The SAR is averaged over 1 g of tissue in order to capture local resonance effects. Averaging the eye SAR over 10 g of tissue is not suitable here bearing in mind that the mass of the entire eye is between 11 and 12 g for an adult [29]. In all following results, the 10-g SAR is calculated as the SAR averaged over the entire eye.
Fig. 2.4. The structure and dimensions of each component in the simulation example. A half-wave dipole of length 80 mm is used as the excitation antenna at 1.8 GHz. The radiated power is $P_{\text{rad}} = 10$ W. The eyeball is represented as a sphere whose constitutive parameters correspond to the vitreous body [see Fig. 1]. Around the eyeball, several head tissue details such as skin, fat, bone and brain are taken into account. The distance $D$ from the surface of the eye to the center of the dipole is 50 mm. The dipole is rotated along its center from the vertical position $0^\circ$ to the horizontal position $90^\circ$ to obtain the SAR distribution along the axis defined by the points $A$ and $B$ versus the dipole orientation.

The two plots in Fig. 2.5 show the dependence of the 1-g SAR distribution on the dipole orientation. As the antenna angle increases, the maximum SAR increases. The obtained SAR distributions are similar to those in [39], where the source is a plane wave, despite the fact that the eye here is extremely close to the antenna and the illumination is very different from a plane wave. This indicates the decisive role of resonance modes in the SAR distribution. In the worst-case scenario, the
maximum 1-g SAR occurs in the center of the eyeball and this corresponds to an orientation angle of 90°. The major reason for this angle to yield the worst maximum SAR is the decreasing distance between the eye and the tip of the dipole where the field intensity is very high. When the dipole is placed horizontally (at 90°), the distance from the dipole tip to the eye surface is only 10 mm and the 1-g maximum SAR is 3.4 W/kg.

Similarly, we investigate the worst cases for several other antennas at different distances from the surface of the eyeball. Considered antennas include [29] an electrically small dipole, an electrically small loop, a right-hand circularly polarized (RHCP) patch [40], and a printed inverted-F antenna (PIFA) [41]. The antenna sizes are listed in Table 2.3. With all these antennas, the 1-g SAR distributions are very similar to those in Fig. 2.5. Their maximum values (worst-case orientations) are between 3.0 W/kg for the small dipole and 7.0 W/kg for the PIFA at 1.8 GHz. In all cases, the distance from the center of the antenna to the surface of the eyeball is 50 mm and the radiated power is 10 W as in the case of the half-wavelength dipole.

The SAR distributions obtained through this study are similar to the normalized distributions predicted by the semi-analytical model. In particular, the normalized field-magnitude values at the eye surface facing the antenna, especially in the worst-case scenario, match closely those of the semi-analytical model. Thus, the analytical model can be used to quickly estimate the maximum 1-g SAR in the eye interior provided the field magnitude at the eye periphery is known.
Fig. 2.5. Simulated results of 1-g SAR dependence on the half-wave dipole orientation ($f = 1.8 \text{ GHz}$, $P_{\text{rad}} = 10 \text{ W}$): (a) from $0^\circ$ to $40^\circ$; (b) from $40^\circ$ to $90^\circ$. At $90^\circ$, the half-wave dipole is oriented horizontally with its tip toward the eye.
TABLE 2.3
SIZES OF THE ANTENNAS USED IN THE SIMULATIONS

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Dipole</td>
<td>Length: 26.5 mm</td>
</tr>
<tr>
<td>Small Loop</td>
<td>Radius: 4.5 mm</td>
</tr>
<tr>
<td>RHCP Patch</td>
<td>Length of patch: 8 mm</td>
</tr>
<tr>
<td></td>
<td>Length of substrate: 11.25 mm</td>
</tr>
<tr>
<td></td>
<td>Width of patch: 4 mm</td>
</tr>
<tr>
<td></td>
<td>Width of substrate: 11.25 mm</td>
</tr>
<tr>
<td>PIFA</td>
<td>Length: 2 mm</td>
</tr>
<tr>
<td></td>
<td>Width: 1.8 mm</td>
</tr>
</tbody>
</table>

2.4 PREDICTING THE MAXIMUM SAR WITH A SEMIANALYTICAL MODEL

The prediction of the maximum SAR is based on the ratio

\[ R_{mb} = \frac{|E_{max}|}{|E_{h}|} = \overline{E}_h^{-1} \]  \hspace{1cm} (2.7)

of the maximum field-magnitude value \( |E_{max}| \) occurring in the center of the eye and its boundary value \( |E_{h}| \) at the surface. The normalized field-magnitude value at the eye periphery \( \overline{E}_h \) is the inverse of \( R_{mb} \) and is illustrated in Fig. 2.2 for the frequency of 1.8 GHz. There, \( R_{mb} \) is approximately 2.13 for the analytical curve and 2.01 for the simulation curve.
The ratio $R_{mh}$ is provided by the semi-analytical model at the frequency of interest. It allows for the estimation of the maximum field magnitude $\tilde{E}_{\text{max}}$ from an approximate field-magnitude boundary value $|\tilde{E}_b|$ as

$$\tilde{E}_{\text{max}} = R_{mh} |\tilde{E}_b|.$$  

(2.8)

Here, $|\tilde{E}_b|$ is approximated by the near field of the antenna in open space (i.e., in the absence of the eye and the head) at the location corresponding to the eye surface. The open-space near field of the device can be easily obtained either by measurement with an SAR robot or a simulation in open space. Note that only one simulation in open space is required in order to obtain the needed near-field values corresponding to all possible positions and orientations of the device relative to the eye.

The value of $|\tilde{E}_b|$ is taken at the boundary point $A$ [see Fig. 2.4 or Fig. 2.6]. It is not immediately clear whether the total field at the eye periphery can be approximated by the incident field alone since this implies that effects of backscatter and mutual coupling are neglected. We demonstrate that such an approximation is reasonable by providing the percentage difference between the incident and total field-magnitude values at point $A$, both obtained via full-wave simulations. In the first simulation, the antenna radiates in open space while in the second simulation it radiates in the presence of the simple eye model as illustrated in Fig. 2.6. The results are summarized in Table 2.4.
Fig. 2.6. The eye model in the HFSS simulation. The point A shows where the near-field of the antenna is recorded in an open-space scenario. The orientation angle of the shown antenna corresponds to a worst-case SAR scenario for the mutual orientation of the eye and the antenna.

| TABLE 2.4 |
| DIFFERENCE BETWEEN MAGNITUDE VALUES AT POINT A FOR INCIDENT AND TOTAL E-FIELD (IN %) |

<table>
<thead>
<tr>
<th>Freq.: 1.8 GHz</th>
<th>Distances (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{rad}$: 1.0 W</td>
<td>20</td>
</tr>
<tr>
<td>Small Dipole</td>
<td>0.90</td>
</tr>
<tr>
<td>Small Loop</td>
<td>0.71</td>
</tr>
<tr>
<td>RHCP Patch</td>
<td>0.89</td>
</tr>
<tr>
<td>PIFA</td>
<td>0.53</td>
</tr>
</tbody>
</table>
From $\vec{E}_{\text{max}}$, the maximum SAR value is estimated as

$$\text{SAR}_{\text{max}} = \sigma \vec{E}_{\text{max}}^2 \rho_m^{-1}. \quad (2.9)$$

where $\sigma$ is the conductivity of the vitreous body and $\rho_m$ is its mass density ($\rho_m = 1.050 \, \text{kg/m}^3$).

In summary, the combined use of the semi-analytical model and the measured or simulated open-space near-field of the device under test yields an efficient procedure for the maximum SAR evaluation in the eye. It involves the following steps.

1) Obtain the ratio $R_{\text{mb}}$ of the maximum-to-boundary field-magnitude value using the semi-analytical model.

2) Measure or simulate the open-space field of the device under test and obtain its magnitude $|\vec{E}_b|$ at the point $A$, which corresponds to the center of the eye opening.

3) Calculate $\vec{E}_{\text{max}}$ from $R_{\text{mb}}$ and $|\vec{E}_b|$ using (2.8).

4) Calculate the maximum SAR from $\vec{E}_{\text{max}}$ using (2.9).
2.5 VALIDATION OF THE SEMI-ANALYTICAL MODEL THROUGH SIMULATIONS

To verify the ability of our semi-analytical model to predict the maximum SAR averaged over 1 g and 10 g of tissue, we compare its output with a simulation model based on Fig. 2.4. A 3-D view of the simulated structure with a dipole is shown in Fig. 2.6. We investigate the antennas listed in Table 2.3 when they are located at distances of 20, 40, 80 and 160 mm from the eye. The distances are taken from the antenna center to the surface of the eyeball. Each antenna is oriented so that, for the given distance, the maximum SAR occurs. For each antenna, two simulations are carried out: one in open-space without the eye and another in the presence of the eye as shown in Fig. 2.6. The first simulation provides the open-space near-field magnitude at point A at the eye periphery [see Fig. 2.6], which is needed in the semi-analytical model. The second simulation is used to obtain the maximum SAR in the eye directly, which is then compared to that predicted by the semi-analytical model. All the simulations are carried out with Ansoft HFSS [37]. Mesh refinement is applied with 0.5 % convergence error.

The comparisons between the semi-analytical model and the simulations in terms of the maximum 1-g SAR are summarized in Table 2.5. Overall, the difference between the semi-analytical and the simulation models is less than 6 % [see Difference I]. Note that the semi-analytical model overestimates the maximum SAR, which makes it adequate for worst-case estimations. This is due to the fact that the
tissue heterogeneity in the simulation model distorts the resonant field distribution and decreases the maximum SAR value in comparison with the semi-analytical model. As for the maximum 10-g SAR at 1.8 GHz [summarized in Table 2.6], the difference between the semi-analytical model and the simulations [see Difference I] is less than 4.5%.

We also present comparisons with a very detailed simulation model shown in Fig. 2.7. It includes additional tissues — cornea, camera anterior, lens, sclera, choroid, retina, and nervus opticus — whose electrical properties are available in [30]. We observe [see Tables 2.5 and 2.6, Difference II] that the additional tissue heterogeneity reduces further the maximum SAR value in comparison with the simplified simulation and the semi-analytical model. The maximum observed difference between the semi-analytical model and the detailed simulation is 18% for the case of the maximum SAR averaged over 1 g of tissue and 10% for the SAR averaged over 10 g of tissue.

In summary, the semi-analytical model can adequately predict the maximum SAR value in the eye from the open-space near field of the device under test. There is no need for full-wave simulations of the device in the proximity of complex head and eye models. We emphasize that the semi-analytical model has negligible computational resources while each full-wave simulation requires about 6 hours.\(^1\) Moreover, the near field of the device under test can be easily acquired with an SAR

\(^1\)The simulations are performed on an Intel® Pentium® 4; CPU is at 3.2 GHz, RAM is 1 GB.
robot at an industrial testing facility. The measured near field can be used in the semi-analytical model, thus completely avoiding the need for time-consuming simulations. The related measurement technique is addressed next.

### TABLE 2.5

**Comparison of Maximum 1-g SAR (W/kg)**

<table>
<thead>
<tr>
<th>Freq.: 1.8 GHz</th>
<th>Distances (mm)</th>
<th>20</th>
<th>40</th>
<th>80</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{rad}}$: 1.0 W</td>
<td>Small Dipole</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-analytical</td>
<td>2.30</td>
<td>0.514</td>
<td>0.108</td>
<td>0.0216</td>
<td></td>
</tr>
<tr>
<td>Simplified Simulation</td>
<td>2.18</td>
<td>0.486</td>
<td>0.104</td>
<td>0.0205</td>
<td></td>
</tr>
<tr>
<td>$^a$ Difference I in %</td>
<td>5.5</td>
<td>5.8</td>
<td>3.8</td>
<td>5.4</td>
<td></td>
</tr>
<tr>
<td>Detailed Simulation</td>
<td>2.00</td>
<td>0.435</td>
<td>0.096</td>
<td>0.0188</td>
<td></td>
</tr>
<tr>
<td>$^b$ Difference II in %</td>
<td>15</td>
<td>18</td>
<td>13</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Small Loop</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-analytical</td>
<td>4.10</td>
<td>0.824</td>
<td>0.241</td>
<td>0.0438</td>
<td></td>
</tr>
<tr>
<td>Simplified Simulation</td>
<td>3.87</td>
<td>0.780</td>
<td>0.228</td>
<td>0.0418</td>
<td></td>
</tr>
<tr>
<td>Difference I in %</td>
<td>5.9</td>
<td>5.6</td>
<td>5.7</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>Detailed Simulation</td>
<td>3.53</td>
<td>0.708</td>
<td>0.208</td>
<td>0.0380</td>
<td></td>
</tr>
<tr>
<td>Difference II in %</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>RHCP Patch</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-analytical</td>
<td>2.60</td>
<td>0.640</td>
<td>0.160</td>
<td>0.0430</td>
<td></td>
</tr>
<tr>
<td>Simplified Simulation</td>
<td>2.46</td>
<td>0.608</td>
<td>0.152</td>
<td>0.0410</td>
<td></td>
</tr>
<tr>
<td>Difference I in %</td>
<td>5.7</td>
<td>5.3</td>
<td>5.3</td>
<td>4.9</td>
<td></td>
</tr>
<tr>
<td>Detailed Simulation</td>
<td>2.28</td>
<td>0.580</td>
<td>0.145</td>
<td>0.0393</td>
<td></td>
</tr>
<tr>
<td>Difference II in %</td>
<td>14</td>
<td>10</td>
<td>10</td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td>PIFA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-analytical</td>
<td>5.00</td>
<td>1.18</td>
<td>0.350</td>
<td>0.0690</td>
<td></td>
</tr>
<tr>
<td>Simplified Simulation</td>
<td>4.80</td>
<td>1.13</td>
<td>0.340</td>
<td>0.0670</td>
<td></td>
</tr>
<tr>
<td>Difference I in %</td>
<td>4.2</td>
<td>4.4</td>
<td>3.0</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>Detailed Simulation</td>
<td>4.50</td>
<td>1.06</td>
<td>0.320</td>
<td>0.0633</td>
<td></td>
</tr>
<tr>
<td>Difference II in %</td>
<td>11</td>
<td>11</td>
<td>9</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Difference I is the difference between the semi-analytical model and the simplified simulation model as shown in Fig. 6.

$^b$ Difference II is the difference between the semi-analytical model and the detailed simulation model as shown in Fig. 7.
<table>
<thead>
<tr>
<th>Freq.: 1.8 GHz</th>
<th>Distances (mm)</th>
<th>20</th>
<th>40</th>
<th>80</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{rad}}$: 1.0 W</td>
<td>Semi-analytical</td>
<td>0.820</td>
<td>0.180</td>
<td>0.0380</td>
<td>0.0077</td>
</tr>
<tr>
<td></td>
<td>Simplified</td>
<td>0.790</td>
<td>0.175</td>
<td>0.0370</td>
<td>0.0074</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Difference I in %</td>
<td>3.7</td>
<td>2.9</td>
<td>2.7</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>Detailed Simulation</td>
<td>0.759</td>
<td>0.168</td>
<td>0.0360</td>
<td>0.0070</td>
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<tr>
<td></td>
<td>Difference II in %</td>
<td>8.0</td>
<td>7.1</td>
<td>5.6</td>
<td>10</td>
</tr>
<tr>
<td>Small Dipole</td>
<td>Semi-analytical</td>
<td>1.44</td>
<td>0.290</td>
<td>0.0860</td>
<td>0.0150</td>
</tr>
<tr>
<td></td>
<td>Simplified</td>
<td>1.39</td>
<td>0.280</td>
<td>0.0823</td>
<td>0.0145</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Difference I in %</td>
<td>3.6</td>
<td>3.6</td>
<td>4.5</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Detailed Simulation</td>
<td>1.33</td>
<td>0.270</td>
<td>0.0780</td>
<td>0.0140</td>
</tr>
<tr>
<td></td>
<td>Difference II in %</td>
<td>8.3</td>
<td>7.4</td>
<td>10</td>
<td>7.1</td>
</tr>
<tr>
<td>Small Loop</td>
<td>Semi-analytical</td>
<td>0.903</td>
<td>0.229</td>
<td>0.0570</td>
<td>0.0154</td>
</tr>
<tr>
<td></td>
<td>Simplified</td>
<td>0.875</td>
<td>0.222</td>
<td>0.0550</td>
<td>0.0149</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Difference I in %</td>
<td>3.2</td>
<td>3.2</td>
<td>3.6</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>Detailed Simulation</td>
<td>0.835</td>
<td>0.212</td>
<td>0.0530</td>
<td>0.0143</td>
</tr>
<tr>
<td></td>
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<td>8.1</td>
<td>8.0</td>
<td>7.5</td>
<td>7.7</td>
</tr>
<tr>
<td>RHCP Patch</td>
<td>Semi-analytical</td>
<td>1.79</td>
<td>0.420</td>
<td>0.125</td>
<td>0.0246</td>
</tr>
<tr>
<td></td>
<td>Simplified</td>
<td>1.73</td>
<td>0.410</td>
<td>0.123</td>
<td>0.0241</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Difference I in %</td>
<td>3.5</td>
<td>2.4</td>
<td>1.6</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Detailed Simulation</td>
<td>1.65</td>
<td>0.390</td>
<td>0.117</td>
<td>0.0230</td>
</tr>
<tr>
<td></td>
<td>Difference II in %</td>
<td>8.5</td>
<td>7.7</td>
<td>6.8</td>
<td>7.0</td>
</tr>
</tbody>
</table>
Fig. 2.7. The geometry of the detailed simulation model.

2.6 SAR ESTIMATION WITH EYE PHANTOM MEASUREMENT

To confirm the results of the above resonance-based models through SAR measurements, we propose a simple heterogeneous eye phantom shown in Fig. 2.8. It emulates the simple two-media model in Fig. 2.1. Fig. 2.8a is a cross-sectional view of the eye phantom and Fig. 2.8b shows the physical phantom with its corresponding dimensions. The glass flask standing for the eyeball contains the liquid whose dielectric properties are similar to those of the vitreous body. The flask is fitted and glued to a circular opening at the bottom of a plastic box. The plastic box contains another liquid whose dielectric properties are similar to the background medium of Fig. 2.1. The recipes for the two liquids for measurements at 1.8 GHz are listed in Table 2.7. The relative permittivity and conductivity are measured by a dielectric
probe [42]. Note that the bottom of the glass flask is exposed to air thus emulating the eye opening. The handheld device is positioned in front of this opening (under the plastic box) at the desired distances.

![Diagram of eye phantom](image)

**Fig. 2.8.** The eye phantom: (a) 2-D cut containing the measurement axis and the device under test; (b) the physical phantom and its dimensions.
### TABLE 2.7
**RECIPE FOR THE LIQUID PHANTOM (FREQ. 1.8 GHz)**

<table>
<thead>
<tr>
<th>Section</th>
<th>Material (%)</th>
<th>( \varepsilon_r )</th>
<th>( \sigma ) (S/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eyeball</td>
<td>Water (89)</td>
<td>Sugar (10)</td>
<td>Salt (1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>69</td>
<td>1.9</td>
</tr>
<tr>
<td>Background</td>
<td>2-propanol</td>
<td></td>
<td>5.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.2</td>
</tr>
</tbody>
</table>

Using a DASY4 SAR measurement robot [18], the SAR distribution in the eye can be directly measured along the vertical axis of the flask from its bottom up to the base of the flask neck as indicated by the blue dash arrow line in Fig. 2.8a. Fig. 2.9 shows the robot, which positions the probe inside the phantom through the flask opening and takes multiple measurements along the vertical axis.

In order to validate the phantom measurement, we compare the measured SAR distribution along the flask axis with that obtained from a simulation with a PIFA. This validation is necessary since the near-field measurements may be affected by coupling between the SAR probe and the device under test. In this simulation, the PIFA [41] excites a structure representing our phantom as shown in Fig. 2.8 where the SAR probe is not present. In the phantom measurement and the simulation, the distance between the antenna and the bottom of the flask is 20 mm. The comparison is shown in Fig. 2.10. The SAR distribution curves are normalized. Two simulation curves are plotted: one shows the SAR distribution when the SAR is averaged over 1 g of tissue; the second one shows the SAR averaged over 10 g of tissue. It is obvious
that the measurement is in agreement with the simulation when the SAR is averaged over 1 g of tissue. Besides, the small size of the eye phantom prevents the probe from obtaining accurate 10-g SAR values.

Fig. 2.9. The SAR measurement robot and the eye phantom.
To confirm experimentally the maximum SAR prediction of the semi-analytical model, we compare its value with the measured maximum SAR in the phantom. The open-space near field of the device under test is acquired with DASY4 by simply removing the eye phantom and measuring the field magnitude value at point A [see Fig. 2.8a]. This value is used in the semi-analytical model as $|\mathbf{E}_b|$. We compare the maximum 1-g SAR obtained with the semi-analytical model with the one which is directly measured inside the phantom [see Table 2.8]. The distances from each antenna to the surface of the eyeball are 20, 40, 80 and 160 mm. The difference between the semi-analytical model and the eye phantom is less than 6%.
TABLE 2.8
COMPARISON OF MAXIMUM 1-G SAR (W/KG)

<table>
<thead>
<tr>
<th>Freq.: 1.8 GHz</th>
<th>Distances (mm)</th>
<th>Models</th>
<th>20</th>
<th>40</th>
<th>80</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Semi-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>analytical</td>
<td>0.77</td>
<td>0.17</td>
<td>0.035</td>
<td>0.0097</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Phantom</td>
<td>0.75</td>
<td>0.18</td>
<td>0.045</td>
<td>0.01</td>
</tr>
<tr>
<td>Difference %</td>
<td></td>
<td>Measured</td>
<td>2.7</td>
<td>5.6</td>
<td>2.2</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The measurement technique is complementary to the semi-analytical model in the sense that: (1) it can provide to the semi-analytical model the open-space near field at the eye opening for the desired position and orientation of the device; (2) the measured SAR values in the liquid eye phantom match closely those predicted by the semi-analytical model when $|\vec{E}_b|$ is measured with the SAR robot.

In summary, the eye-phantom measurement can be used to directly evaluate the SAR in the eye due to the device under test. If, however, an eye phantom is not available, the near field of the device can be measured with the SAR robot and then it can be used in the semi-analytical model to obtain the maximum SAR estimate.

Alternatively, if an SAR robot is not available, the near field of the device can be simulated in open space and the boundary field-magnitude values at the eye opening can be obtained at all desired distances, orientations and frequencies of interest. Using these values, the semi-analytical model can predict reliably the maximum SAR value in the eye interior.
2.7 CONCLUSION

We have presented the results of a comprehensive study of the SAR in the human eyes due to near-field exposure to handheld communication devices. The study focuses on the resonance effects in the eyes. Both simulations and measurements confirm the decisive role of the resonance in the field and SAR distributions in the eye at the frequencies used for mobile communication. It is also found that the antenna orientation has a strong effect on the maximum SAR values. This is attributed to the near-zone reactive field of the antenna which varies rapidly as a function of both angular position and distance.

We have proposed two resonance-based methods for the fast and reliable SAR evaluation of handheld devices regarding their interaction with the eyes—a semi-analytical model and a phantom measurement with an SAR robot. Both methods are easy to implement in an industrial SAR testing facility. The experimental method requires the fabrication of a simple eye phantom and relies on a measurement with an SAR robot.

As an input, the semi-analytical method requires the measured or simulated open-space near field of the device under test in the absence of the eye. As an output, depending on the mutual position and orientation of the eye and the device, it produces the maximum SAR value in the eye averaged over 1 and 10 grams of tissue. It is fast and versatile. This is because the near-field acquisition of the device under test can be obtained through a single simulation or measurement, which
provides the near-field values at all possible relative positions and orientations with respect to the device under test. The measurement is preferred because it is impractical to simulate accurately the whole handheld device.

The semi-analytical model requires the near-field data of the device, which can be obtained either by simulation or by measurement in open space. The agreement of the semi-analytical model with the eye-phantom measurements is better than 6% at several distances and frequencies of interest when the SAR is averaged over 1 g of tissue. It is also in good agreement with the time-consuming simulation models. Here, we have summarized our findings at 1.8 GHz; however, we have confirmed the validity of the model at all frequencies shown in Table 2.1.

To validate the semi-analytical model, a simple heterogeneous eye phantom has been fabricated. It is easy to make and it approximates the eye well. It allows for quick and convenient measurements with an SAR robot. It also allows for the measurement of the maximum SAR (averaged over 1 g of tissue) as well as the SAR distribution along the axis of the eyeball. The measured SAR values and those provided by the semi-analytical model are in close agreement.
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Chapter 3

TEMPERATURE RISE IN THE HUMAN EYES IN THE NEAR FIELD OF HANDHELD DEVICES

3.1 INTRODUCTION

In recent years, there has been increased public concern about the influence of radio-frequency (RF) and microwave radiation on the human body. It is well known that some significant thermal damage can occur in sensitive tissue under conditions of partial-body exposure to intense electromagnetic (EM) field. Among them, the eye is more susceptible to microwave radiation. Thus, research on the temperature increase in the eye for RF exposure is of great interest.

Despite of the strict safety regulations, the public concern continues in view of the advent of new mobile services that offer data formats such as text messaging, email, video and internet, where the wireless handheld is placed directly in front the eyes, sometimes in close proximity. Thus, the evaluation of the actual amount of heating (temperature rise) due to RF exposure is crucial when possible tissue damage is investigated. Exposure guidelines specified by the International Commission on Non-Ionizing Radiation Protection (ICNIRP) [1] recommend that RF induced tissue
temperature rise does not exceed 1 °C. Adherence to the exposure specific absorption rate (SAR) limits [1]-[5] should ensure that the tissue temperature rise is well below the 1 °C limit.

In Chapter 2, two feasible methods have been proposed for the evaluation of the SAR in the human eyes due to exposure to RF fields. The methods are applied to the case of near-field exposure to handheld devices; however, they are applicable to the case of far-field illumination as well. The first method is based on a semi-analytical model of the eye. As an input, it requires the measured or simulated open-space field of the device under test. As an output, depending on the mutual position and orientation of the eye and the device, it produces the maximum SAR value in the eye averaged over 1 and 10 grams of tissue. The second method is based on measurements. It requires the fabrication of a simple eye phantom and relies on a standard measurement with an SAR robot. The measurement method is complementary to the semi-analytical model and can be used to verify its output through one additional field measurement in open space. The proposed methods allow for the fast and reliable SAR evaluation of newly developed handheld devices in an industrial environment.

The SAR alone is not sufficient to evaluate possible thermal effects. The tissue heating is strongly influenced not only by the power dissipated in the local volume but also by the way in which absorption is distributed in surrounding areas, by the thermal characteristics of the tissues, and finally, by the heat exchange with
the external environment [6]. Therefore, a thermal analysis is necessary, in addition to the SAR analysis, in order to assess the safety of the exposure [1]-[5].

In early research on the temperature rise in the eye, the eye was simplified as an object thermally isolated from the head where the effect of blood flow was not taken into account [6][7]. Here, we study the temperature rise in the eye due to RF exposure, considering: (1) the thermal flow at the boundary between the eyeball and the surrounding head tissues; and (2) the effect of blood flow in the choroid and other thin eye tissues. Such study requires a detailed eye model with high resolution. Anatomically based models derived from the Visible Human Project (VHP) [9] or from magnetic resonance imaging (MRI) are usually employed. The eye has been modeled in many different ways including: (i) 3D models with low resolution (1-2 mm) [6][10]-[12]; (ii) 2D models with high resolution (0.25 mm) [13][14]; (iii) 3D models with high resolution (0.25 mm) [15]-[17]. To study the temperature rise in the eye, we have adopted a high-resolution 3D model similar to those in [15]-[17]. Here, we present the maximum temperature rise in relation to the SAR limits prescribed in the safety standards. We also take into account the modes of operation of the handheld, especially, in the Global System for Mobile communications (GSM).

In summary, here we present results on the temperature rise in the eye under conditions of maximum allowed induced SAR and maximum allowed transmitted power.
3.2 THERMAL EYE MODEL AND THERMAL ANALYSIS

Unlike the estimation of the maximum SAR value discussed in Chapter 2, the estimation of the temperature rise in the eye requires a high-fidelity detailed model of the eye such as the one in Fig. 3.1. It includes tissues such as the retina and the choroid. These are important from a thermal point of view since they have significant blood flow.

Fig. 3.1  A 2D cut showing the geometry of the detailed simulation eye model. The 3D eye model is rotationally symmetric.
The complete simulation-based eye model is a full-wave EM simulation coupled with a thermal simulation. We use Ansoft ePhysics ver. 2 [18] whose static thermal solver is based on the diffusion equation [19]:

\[ \nabla \cdot (K \nabla T) + \rho \text{SAR} + A - B(T - T_b) = C \rho \frac{\partial T}{\partial t}. \]  

(3.1)

Here, \(T\) is the unknown temperature (°C), \(C\) is the specific heat (J/(kg °C)), \(K\) is the thermal conductivity (W/(m °C)), \(B\) is the blood flow (W/(m³ °C)), \(T_b\) is the blood temperature (in °C), and \(A\) is the basal metabolic rate (W/m³). The metabolic mechanisms have been neglected because the induced temperature rise is expected to be small. The thermal properties of the eye tissues are listed in Table 3.1 [10][15].

The convective boundary conditions are applied on the skin-air and cornea-air interfaces [19]:

\[ -K \frac{\partial T}{\partial n} = H(T_s - T_e) \]  

(3.2)

where \(H\) is the convection coefficient (W/(m² °C)), \(T_s\) is the unknown surface temperature, and \(T_e\) is the fluid temperature (corresponding to the air temperature). The convection coefficient between the skin and the air is assumed to be 10.5 W/(m² °C) [20], while the convection coefficient between the cornea surface and the air is 20 W/(m² °C) [21]. The thermal flow at the boundary between the eyeball and the surrounding head tissues is represented with a convection coefficient of 65 W/(m² °C) [22]. In addition, the room temperature is assumed to be 25 °C and the body temperature is assumed to be 37 °C.
<table>
<thead>
<tr>
<th>Tissues</th>
<th>Specific Heat $C$ (J/(kg°C))</th>
<th>Heat Cond. $K$ (W/(m°C))</th>
<th>Blood Flow Rate $B$ (W/(m³°C))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vitreous body</td>
<td>4178</td>
<td>0.58</td>
<td>0</td>
</tr>
<tr>
<td>Camera anterior</td>
<td>3997</td>
<td>0.58</td>
<td>0</td>
</tr>
<tr>
<td>Lens</td>
<td>3000</td>
<td>0.40</td>
<td>0</td>
</tr>
<tr>
<td>Cornea</td>
<td>4200</td>
<td>0.58</td>
<td>0</td>
</tr>
<tr>
<td>Choroid, Retina, Sclera</td>
<td>4200</td>
<td>0.58</td>
<td>13500</td>
</tr>
<tr>
<td>Nerve</td>
<td>3600</td>
<td>0.503</td>
<td>35000</td>
</tr>
<tr>
<td>Skin</td>
<td>3500</td>
<td>0.42</td>
<td>9100</td>
</tr>
<tr>
<td>Fat</td>
<td>2500</td>
<td>0.25</td>
<td>520</td>
</tr>
<tr>
<td>Bone</td>
<td>1300</td>
<td>0.40</td>
<td>1000</td>
</tr>
<tr>
<td>Brain</td>
<td>3600</td>
<td>0.50</td>
<td>35000</td>
</tr>
</tbody>
</table>

The static thermal model represents a worst-case scenario where the radiating unit operates in a continuous regime. Since the power levels are low, the tissue properties are assumed constant and independent of the temperature. Thus, the EM and thermal simulations can be performed sequentially.

At the initial state, the temperature distribution in the eye without any RF field exposure is obtained from the steady-state bioheat equation [21]:

$$ \nabla \cdot (K \nabla T) - B(T - T_b) = 0. \quad (3.3) $$

subject to the boundary conditions (3.2).

We examine the temperature distribution along the axis defined by the points $A$ and $B$ in Fig. 3.1 before and after exposure to the radiation from different antennas.
where the maximum SAR value is set to be equal to the limits prescribed in the safety standards. Fig. 3.2 shows the temperature distribution due to the PIFA placed 20 mm in front of the eye. The output power of the PIFA is 1 W at 1.8 GHz, which results in the maximum allowable SAR (averaged over 10 g) of 2 W/kg, as per the ICNIRP guidelines [1]. We observe that the highest temperature rise of less than 0.5 °C occurs in the vitreous body.

Fig. 3.3 shows the temperature distribution in the same configuration, except that the PIFA now has output power of 0.2 W, which leads to a maximum SAR (averaged over 1 g) of 1.6 W/kg, as per the FCC guidelines [4]. The observed highest temperature rise is now less than 0.3 °C.

Both scenarios are well below the limit of 1 °C [1]. Note that the temperature rise is not directly proportional to the radiated power due to the blood flow and the thermal diffusion. The temperature rise in other critical tissues such as the lens and the cornea is negligible. The results agree with observations made in [10] and [15] if one takes into account the differences in power levels and antenna orientation, as well as the assumption in [10] that the eye is thermally isolated from the head.

We note that the scenario above is exceedingly pessimistic in the assumed eye-antenna distance as well as the transmitted power (see section below). It confirms that as far as the temperature rise in the eye is concerned, wireless handhelds operate within safe limits by a very wide margin.
Fig. 3.2. Axial temperature distribution in the thermal eye model before and after exposure to the field of a PIFA at 1.8 GHz when the 10-g SAR in the eye is 2 W/kg. The ambient temperature at the cornea is 25 °C. The distance between the PIFA and the eye surface is \( D = 20 \) mm and the radiated power is 1 W.

Fig. 3.3. Axial temperature distribution in the thermal eye model before and after exposure to the field of a PIFA at 1.8 GHz when the 1-g maximum SAR in the eye is 1.6 W/kg. The distance between the PIFA and the eye surface is \( D = 20 \) mm and the radiated power is 0.2 W.
3.3 WORST CASE SCENARIO WITH GSM HANDHELD DEVICES

GSM is the most popular standard for mobile communications. By the present standards, the GSM handhelds are permitted to transmit maximum power of 2 W for GSM 900 and 1 W for GSM 1800. However, as in all time division multiple access systems, the average power transmitted by a mobile phone is not continuous. In GSM, it is one-eighth of the maximum values, which are 0.25 W and 0.125 W, for the respective frequency bands. Further, it can be reduced by a significant amount due to the effects of adaptive power control and discontinuous transmission [23]. The power control of a GSM phone automatically reduces the emitted power by up to a factor of 1000 if the intensity is not needed for stable transmission. Mobile phones operating in the standby mode lead to much lower exposure compared to mobile phones operating at maximum power. On average, in the conversation mode, the transmission power could be reduced to 1/100 full power, that is 20 mW. An illustration of the GSM signal for each operation mode is shown in Fig. 3.4 [24].

![GSM Signal](image_url)

**Fig. 3.4.** The amplitude of the GSM mobile phone transmission on reception of an incoming call [24].
SAR testing of handhelds is usually done for the worst case of continuous maximum-power transmission. These are the conditions assumed in our thermal model in the previous section. In the case of a transmitted power of 1 W at 1.8 GHz, which is the maximum allowed in GSM, the tested PIFA must be placed as close to the eye as 20 mm in order to induce the maximum allowable SAR (averaged over 10 g) of 2 W/kg. This SAR in turn leads to a maximum local temperature increase of less than 0.5 °C as shown in Fig. 3.2.

3.4 UNCERTAINTY STUDY OF THE EYE

A literature survey resulted in no certain information regarding the dielectric and thermal properties of each component of the human eye. To account for the uncertain properties of the eye tissues, EM and thermal analyses here are carried out by varying the dielectric properties, i.e., permittivity and conductivity, of the vitreous body of the eye, over a range of values which are reported in the literatures [25]-[27].

The maximum temperature rise in the eye when varying the dielectric properties of the eye is summarized in Table 3.2. The eye is exposed to the radiation from the PIFA placed at 20 mm in front of the eye and its output power is 1 W at 1.8 GHz, which results in the maximum allowable SAR (averaged over 10 g) of 2 W/kg. The permittivity and conductivity of the vitreous body have the variance of 13.5 % and 18.5 %, respectively.
TABLE 3.2
MAXIMUM TEMPERATURE RISE IN THE EYE WITH THE VARYING DIELECTRIC PROPERTIES OF THE VITREOUS BODY

<table>
<thead>
<tr>
<th>Properties of Vitreous Body</th>
<th>Variance</th>
<th>Max. Temperature Rise</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_r$</td>
<td>+ 13.5%</td>
<td>0.51 °C</td>
</tr>
<tr>
<td></td>
<td>- 13.5%</td>
<td>0.49 °C</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>+ 18.5%</td>
<td>0.83 °C</td>
</tr>
<tr>
<td></td>
<td>- 18.5%</td>
<td>0.20 °C</td>
</tr>
</tbody>
</table>

Fig. 3.5. Axial temperature distribution in the thermal eye model before and after exposure to the field of a PIFA at 1.8 GHz when the 10-g SAR in the eye is 2 W/kg. The ambient temperature at the cornea is 37 °C. The distance between the PIFA and the eye surface is $D = 20$ mm and the radiated power is 1 W.

Another uncertain factor is the ambient environment temperature. Here, the thermal analysis is carried out assuming hot summer outdoor temperature (37 °C).

Fig. 3.5 shows the temperature distribution along the pupillary axis before and after
exposure to the radiation leading to the maximum allowable SAR prescribed by ICNIRP and IEEE. The observed highest temperature rise in the eye is less than 0.6 °C and occurs in the vitreous body. This temperature rise is slightly higher than the one observed for an ambient temperature of 25 °C, but is still less than the maximum allowable temperature rise of 1 °C.

3.5 CONCLUSION

We have considered a detailed EM/thermal simulation model of the eye interacting with a variety of antennas. The maximum temperature rise in the eye is investigated at distances and transmitted power levels such that prescribed SAR limits are approached. The results show that the maximum temperature rise in the eye is well below the recommended limit of 1 °C and that it occurs in the vitreous body.

We have also considered the worst-case scenario of a GSM phone transmitting continuously at full power at very close proximity to the eyes. The results show that, even in the most pessimistic scenario, the temperature rise is well below the recommended limit of 1 °C and that it occurs in the vitreous body.
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Chapter 4

NEAR-FIELD DETECTION AT MICROWAVE FREQUENCIES BASED ON SELF-ADJOINT RESPONSE SENSITIVITY ANALYSIS

4.1 INTRODUCTION

Microwave imaging is a powerful method for noninvasive evaluation, testing, and diagnostics in medicine [1]-[6], nondestructive testing [7], [8], geophysical prospecting, remote sensing and underground surveillance [9], [10]. Microwave security screening and surveillance systems are also being developed [11], [12].

Imaging via microwave measurements aims at a complete reconstruction of the medium complex permittivity. In contrast, detection aims only at partial information about the interior of the object under test. This interior is often assumed known in its normal state and the goal of detection is to uncover possible abnormalities (e.g., structural flaws, hidden weapons, cancerous tissue).

Permittivity reconstruction represents a complex inverse scattering problem, which is usually strongly nonlinear and is solved iteratively by updating a forward
model of the profile estimate. The forward models often rely on electromagnetic (EM) simulations because analytical models are not available. The use of time-intensive simulations as forward models in iterative optimization procedures precludes real-time performance. Moreover, the solution suffers from non-uniqueness [13], [14]. Thus, it is beneficial to exploit all a priori knowledge [15]-[17] toward the acceleration of the inverse solution. In detection, the a priori information about the normal state of the object under test is crucial, especially for real-time performance.

When solving an inverse problem iteratively, an objective function $\Phi$ is formulated as a measure of the difference between the measured (or target) responses and the responses produced by the forward model of the current profile estimate. The goal is to minimize $\Phi$. In detection, there is no objective function per se; however, the algorithms inherently exploit comparisons between the known “normal” responses and the measured responses.

Overall, we can view the detection procedure as an inverse-problem solution, which is sufficiently linearized by a priori knowledge to allow for a direct single-step solution. In this work, we investigate a new detection approach which relies on the knowledge of the incident field distributions $E^\text{inc}_{ik} \ (k=1,\ldots,K)$ within the imaged volume.\(^1\) Here, the index $k$ corresponds to a given transmitter (or excitation

---

\(^1\) In this work, the physical fields such as the electric field $E$ are denoted with upper-case bold letters, while matrices and vectors are in bold italics.
source). By definition, the incident field is the one arising under known excitation and boundary conditions in the known background medium. Note that the background medium need not be homogeneous. The incident field distributions can be derived by simulations or by measurements. Acquiring three-dimensional (3D) vector-field distributions through field-probe measurements is in principle possible unless the background medium is a mechanically impenetrable solid object. Such measurement system is expensive as it requires precision positioning control. In the presented preliminary studies, full-wave electromagnetic simulations were used to acquire $\mathbf{E}_d^{inc}(k = 1, \ldots, K)$. Note that the incident field acquisition is a calibration step performed before the test measurements as it is independent of the particular object under test. The detection algorithm itself performs the calculations fast in real time.

The proposed detection method exploits a self-adjoint response-sensitivity formula. Response sensitivity analysis has been widely used in computer-aided design [19]-[24] and inverse-problem solutions [17], [25]-[28]. It provides the Jacobian matrix (response gradients, response sensitivities) from which the gradient of the objective function $\nabla_p \Phi$ is computed. Here, $p = [p_1, \ldots, p_N]^T$ is the vector of system parameters. When $p$ consists of the voxel permittivities and conductivities in the imaged region, $\nabla_p \Phi$ is a form of the Fréchet derivative [18]. The objective function $\Phi$ developed here for the purposes of sensitivity-based detection treats the signals' magnitudes and phases as separate responses. This allows for
implementations in various scenarios where phase information may or may not be available.

Computing the Jacobian of a simulated EM structure is not straightforward. This is a well-known problem in the numerical modeling of complex distributed systems [29]. The problem stems from the non-analytical (often intractable) dependence of the response on the system parameters $p$. A simple but inefficient and inaccurate solution is provided by the response-level finite-difference approximations of the Jacobian or its response surface approximations in the $N$-dimensional parameter space. These techniques are prohibitively slow with EM simulations because they require at least one additional system analysis per system parameter. In imaging, the number of parameters may be on the order of thousands. Another prohibitive constraint is due to the limited accuracy of the simulated responses, which causes catastrophic cancellations in the finite-difference estimates of the response derivatives [29]-[31].

The adjoint-variable method for response-sensitivity computations is significantly more efficient than the response-level approximations — Jacobians are computed with two system analyses at the most, regardless of the number of the system parameters [32], [33]. More importantly, when numerical field solutions are used, the adjoint-based sensitivity estimates are accurate, i.e., they are of the same order of accuracy as the response estimates and do not suffer from catastrophic cancellation [30]. Note that the adjoint-variable sensitivity formula is exact when the
derivatives of the system matrices with respect to the system parameters are known analytically. However, since the formula uses numerical field solutions, its output is not exact and some degradation of accuracy can be expected due to numerical errors in the field solutions.

Self-adjoint sensitivity formulas have been proposed where the second (adjoint) system analysis is eliminated [17], [27], [28], [31], [34]-[37]. This is achieved by formulating an adjoint-problem solution which can be obtained from the solution of the original problem by simple mathematical transformations. This is the major advantage of the self-adjoint formulations. It is the reciprocity of the EM problem which allows for the self-adjoint sensitivity analysis (SASA). The time required by the SASA is negligible in comparison with the simulation time. The memory requirements are also usually modest — they cannot exceed those of the simulation itself [38].

In this chapter, we develop a new general methodology for response sensitivity analysis. It is independent of the forward model, i.e., the method of simulation. The methodology is applied as a self-adjoint formula whose only restriction is that the problem must be reciprocal, i.e., the medium is linear with symmetric constitutive tensors, \( \mathbf{c} = \mathbf{c}^T \) and \( \mathbf{\mu} = \mathbf{\mu}^T \).

The detection algorithm generates sensitivity maps, i.e., plots of the Fréchet derivatives (the derivatives of the objective function \( \Phi \) with respect to the voxel permittivities and conductivities) versus the 3D coordinates of the voxels. The
sensitivity maps can be interpreted as images showing areas in the background medium where the voxel permittivities differ significantly from those in the measured object. The algorithm exploits these maps to arrive at a diagnostic conclusion of whether scatterers (e.g., defects in materials, abnormal tissues in organs, etc.) are likely to be present or not in the examined object.

The formulation of the response sensitivity analysis is derived in section 2 and discussed in section 3 from the viewpoint of the linear Born approximation. In section 4, the formulation of the objective functions is addressed. The algorithm is summarized in section 5. Verification through an example with a homogeneous background is presented in section 6. There, the performance of the algorithm is also studied versus the number of the transmission/reception points, the contrast between the scatterer and the background medium, and the size of the scatterer. The resolution is also addressed demonstrating the algorithm's ability to discern two electrically small scatterers. In section 7, the algorithm is applied in an example with a heterogeneous background. In all numerical examples, we employ a commercial EM simulator [39].
4.2 BACKGROUND

4.2.1. Adjoint-variable method in electromagnetics

The generic response in the frequency-domain EM sensitivity analysis is the functional

\[ F(\mathbf{E}, \mathbf{p}) = \iiint \mathcal{f}(\mathbf{E}, \mathbf{p}) d\Omega \]  

(4.1)

where \( \Omega \) is the computational volume, \( \mathbf{E} \) is the field solution, \( \mathbf{p} \) represents the parameters of interest, and \( \mathcal{f}(\mathbf{E}, \mathbf{p}) \) is the local response, which depends on the field solution \( \mathbf{E} \). The dependence of \( \mathcal{f} \) on \( \mathbf{p} \) is implicit through \( \mathbf{E} \) but there may be explicit dependence as well.

The goal of sensitivity analysis is to compute the response gradient (response sensitivity)

\[ \nabla_{\mathbf{p}} F = \begin{bmatrix} \frac{\partial F}{\partial p_1} & \frac{\partial F}{\partial p_2} & \cdots & \frac{\partial F}{\partial p_N} \end{bmatrix}. \]  

(4.2)

The adjoint-variable method offers an efficient way to obtain all sensitivity information with only two simulations: original and adjoint.

A time-harmonic EM problem involving linear materials can be cast in a linear operator form:

\[ \mathbb{L}\mathbf{E} = \mathbf{g}. \]  

(4.3)

Here, \( \mathbb{L} \) is the linear EM operator and \( \mathbf{g} \) results from the excitation and/or the
by the derivative of the local response with respect to the field solution $\frac{\partial f}{\partial E_\xi}$, \( \xi = x, y, z \), as per (4.12). The adjoint field $\hat{E}$ is not uniquely defined until the boundary conditions complementing (4.13) are set. They are usually chosen to be the same as those of the original problem [41], [42]. This offers several advantages including self-adjoint sensitivity formulations.

4.2.2. Self-adjoint formulation

In self-adjoint formulations, the adjoint system analysis is eliminated since the adjoint-problem solution $\hat{E}$ is derived from the solution $\bar{E}$ of the original problem by simple mathematical transformations without solving (4.13). Self-adjoint approaches have been developed for both time- and frequency-domain problems; see, for example, [31], [34], [35], [37], [43], [44].

The self-adjoint formulation becomes possible in problems where: (i) the constitutive tensors are symmetric (reciprocity holds), i.e., $\varepsilon = \varepsilon^T$ and $\mu = \mu^T$. (ii) the distribution of the adjoint excitation $\hat{g}$ in 3D space is the same as that of the original excitation $g$, and (iii) the boundary conditions of the adjoint problem are the same as those in the original problem. Condition (i) renders the linear operator $\mathcal{L}$ symmetric:

$$\mathcal{L}^T = \mathcal{L}.$$  \hspace{1cm} (4.16)

The symmetry of the reciprocal EM problem often reflects in the symmetry of the
system matrix after discretization is applied. For example, in a reciprocal medium, the finite-element method (FEM) features a symmetric real-valued matrix [45].

Further, if condition (ii) is fulfilled, i.e.,
\[ \hat{g} = \kappa g \quad (4.17) \]
then, from (4.3), (4.13) and (4.16)-(4.17), it follows that
\[ \hat{E} = \kappa \hat{E} \quad (4.18) \]
Here, \( \kappa \) is a complex constant. Another case in which self-adjoint formulation is possible is the case where
\[ L' = L^* \quad (4.19) \]
i.e., \( L \) is Hermitian,\(^2\) and
\[ \hat{g} = \kappa g^* \quad (4.20) \]
Then
\[ \hat{E} = \kappa \hat{E}^* \quad (4.21) \]

The self-adjoint formulation in time-domain EM analysis has been addressed in [35], [36]. In [41], it is shown that the time-domain adjoint problem is equivalent to the original problem if: (i) the tensors \( \varepsilon \) and \( \mu \) are symmetric, (ii) the boundary conditions are the same. (iii) the time runs backward along the reversed time variable \( \tau = T_{\text{max}} - t \), where \( T_{\text{max}} \) is the time at which the original simulation ends. and (iv) the adjoint problem is excited by \( -\hat{g} \) such that \( -\hat{g}(\tau) \) and \( g(t) \) have identical

\(^2\) The EM operator is Hermitian in the loss-free case [40].
waveforms in their respective times and identical distributions in 3D space; i.e.,
\( g(r,t) = -\kappa \hat{g}(r,\tau) \) where \( \kappa \) is a real constant and \( r \) is the position in space.

In an active microwave imaging scenario, the measurement data are often acquired through sensors (antennas) which serve as both transmitters and receivers. In such cases, the distribution and the polarization of the excitations coincide with those of the responses. As a result, \( \hat{g} \) has the same distribution and polarization as \( g \) and a self-adjoint formulation is possible.

4.2.3. Application on discrete grids with point sources and point responses

Current-density point sources have been used as means of excitation in both time-domain [36] and frequency-domain [38] analyses. Here, we apply the spectral self-adjoint sensitivity formula [38] due to its memory efficiency. This formula produces the response sensitivity with respect to the permittivity and conductivity of each voxel within the region of interest.

We assume that the excitation and observation points reside at the same \( K \) locations \( r_k \) \( (k = 1,...,K) \). The spectral self-adjoint sensitivity formula operates on the field phasors at the frequencies of interest. Thus, the current-density sources \( J_{k}^{(i)} = J_{k}^{(i)} \hat{p}_k \) are characterized by their location \( r_k \) and their frequency \( \omega_i \) \( (i = 1,\ldots,N_f) \). Here, \( \hat{p}_k \) is the \( k \)th source polarization unit vector (assumed to be the
same at all frequencies of interest) and \( J_k^{(i)} = |J_k^{(i)}| \exp(j \phi_k^{(i)}) \) is the source phasor.

The sources are excited one by one and the scattered field is recorded at all \( K \) locations. Each complex response \( F_{jk}^{(i)} \) is thus characterized by the position of the observation point \( \mathbf{r}_j \) \((j = 1, \ldots, K)\), the position of the source \( \mathbf{r}_k \) \((k = 1, \ldots, K)\), and the frequency \( \omega_i \) \((i = 1, \ldots, N_f)\). \( F_{jk}^{(i)} \) samples the desired field component of polarization \( \mathbf{p}_j \) (assumed frequency independent):

\[
F_{jk}^{(i)} = \hat{\mathbf{p}}_j \cdot \mathbf{E}_k^{(i)}(\mathbf{r}_j)
\]

where \( \mathbf{E}_k^{(i)} \) is the vector field phasor due to the \( k \)th source at the frequency \( \omega_i \). In the current application, it is assumed that only one polarization vector \( \hat{\mathbf{p}}_j \) is associated with the observation location \( \mathbf{r}_j \). This is because the same antennas are used for both transmission and reception. Thus, at \( \mathbf{r}_j \), \( \hat{\mathbf{p}}_j \) is the polarization vector of the \( j \)th transmitting/receiving antenna. Thus, \( K \times K \) complex-valued responses are acquired at each frequency of interest. This scenario is relevant in microwave measurements of the scattering parameters (S-parameters) [46].

The analyses of the forward problem (4.3) at all frequencies of interest and for all excitations produce the complex-valued vector field solutions \( \mathbf{E}_k^{(i)}(\mathbf{r}) \) \((k = 1, \ldots, K; i = 1, \ldots, N_f)\). The field solutions are typically available at discrete points. e.g.,

---

3 For simplicity, it is assumed here that the polarization vectors \( \hat{\mathbf{p}}_k \) describe linear polarization, i.e., that they are real-valued unit vectors.

4 The complex scattering parameter \( S_{kj} \) is the ratio of the signal received at the \( k \)th location and the signal transmitted from the \( j \)th location.
on a finite-element or a finite-difference grid, where the analyzed volume is
discretized into mesh elements or voxels whose position is given by \( \mathbf{r} \). Note that in
principle \( \mathbf{E}^{(i)}(\mathbf{r}) \) can be measured directly, provided the medium is mechanically
penetrable, but an adequate EM simulation model is a cheaper and faster option.

Applying the method of [38] to the case of the responses in (4.22), the self-
adjoint relationship (4.18) is obtained as

\[
\mathbf{E}^{(i)}(\mathbf{r}) = \kappa^{(i)} \mathbf{E}^{(i)}(\mathbf{r}) \quad k = 1, \ldots, K \quad i = 1, \ldots, N_f
\]  

(4.23)

where

\[
\kappa^{(i)} = \frac{1}{j\omega\mu_0 J_{\text{ik}} \Delta \Omega_k \Delta h^2} 
\]  

(4.24)

Here, \( J_{\text{ik}} = |J_{\text{ki}}| \exp(j\varphi_{k}) \) is the \( k \)th current-density source phasor in A/m\(^2\). \( \Delta \Omega_k \)
is the volume of the voxel where the \( k \)th source resides, \( \mu_0 \) is the permeability of
vacuum, and \( \Delta h \) is the smallest mesh edge. Note that the approach of [38] uses a
finite-difference discretization grid: therefore, \( \Delta \Omega_k = \Delta x_k \Delta y_k \Delta z_k \), and on a uniform
grid \( \Delta h = \min(\Delta x, \Delta y, \Delta z) \).

The sensitivity formula (4.15) applies to all \( E_{jk}^{(i)} \ (j, k = 1, \ldots, K; i = 1, \ldots, N_f) \),

\[
\frac{\partial E_{jk}^{(i)}}{\partial p_n} = -\kappa_j^{(i)} \int_{\Omega} \int_{\Omega} \mathbf{E}_{jk}^{(i)} \cdot \frac{\partial \mathbf{R}(\mathbf{E}_{ki}^{(i)})}{\partial p_n} d\Omega. \quad n = 1, \ldots, N
\]  

(4.25)

assuming that the respective explicit derivatives are zero. As long as the responses
are recorded in an observation region outside the examined volume, this assumption
is valid. When the analyzed volume is discretized into finite-volume voxels, the sensitivity integral (4.25) can be written as

\[
\frac{\partial F_r^{(i)}}{\partial p_n} = -\kappa_j^{(i)} \sum_{r' \in \Omega} \left[ E_r^{(i)} \cdot \frac{\partial R(\vec{E}_k^{(i)})}{\partial p_n} \right] \Delta \Omega', \quad n = 1, \ldots, N \tag{4.26}
\]

where \( r' \) is a location in the examined region \( \Omega \) and \( \Delta \Omega' \) is the voxel’s volume at this location. The numerical implementation of the residual derivative \( \frac{\partial R(\vec{E}_k^{(i)})}{\partial p_n} \) depends on: (i) the type of parameters (shape or constitutive parameters), and (ii) the assumed discretization model.\(^5\) Here, we are interested in the voxel constitutive parameters. On a FDTD discretization grid and in an isotropic medium, \( \frac{\partial R(\vec{E}_k^{(i)})}{\partial p_n} \) is [36], [38],

\[
\begin{bmatrix}
\frac{\partial R(\vec{E}_k^{(i)})}{\partial p_n}
\end{bmatrix}_{r'} = -\frac{\partial \alpha(r')}{\partial p_n} \vec{E}_k^{(i)}(r') - \frac{\partial s(r')}{\partial p_n} \vec{E}_k^{(i)}(r') \tag{4.27}
\]

where

\[
\alpha(r') = \left( \frac{\Delta h}{c \Delta t} \right)^2 \varepsilon_r(r'), \quad s(r') = \left( \frac{\mu_0 \Delta h^2}{2 \Delta t} \right) \sigma(r') \tag{4.28}
\]

\[
\vec{E}_k^{(i)}(r') = -\alpha r' \Delta t^2 \vec{E}_k^{(i)}(r') \tag{4.29}
\]

\[
\vec{E}_k^{(i)}(r') = 2 j \alpha r' \Delta t \vec{E}_k^{(i)}(r') \tag{4.30}
\]

In (4.27)-(4.30), \( c \) is the speed of light in vacuum, \( \Delta t \) is the discrete step in time, \( \varepsilon_r(r') \) is the relative permittivity and \( \sigma(r') \) is the specific conductivity in the

\(^5\) Note that the discretization model assumed by the sensitivity formula need not be the same as that of the simulation producing the field solution.
respective voxel. Note that $\alpha$ and $s$ are linear functions of the voxel’s permittivity and conductivity, respectively. Thus, when the parameter of interest $p_n$ is this voxel’s permittivity or conductivity, the derivatives $\partial \alpha / \partial p_n$ and $\partial s / \partial p_n$ are constants. For example, if $p_n = \varepsilon$, at a given voxel, then $\partial \alpha / \partial p_n = \Delta h^2 / (c \Delta t)^2$ and $\partial s / \partial p_n = 0$ at this voxel. At all other voxels, these derivatives are zero and the summation in (4.26) has only one term.

It is instructive to rewrite the residual derivative (4.27) in a more general form:

$$\left[ \frac{\partial R(\vec{E}_k^{(i)})}{\partial p_n} \right]_r = k_i^2 \Delta h^2 \vec{E}_k^{(i)}(\vec{r}') \cdot \frac{\partial}{\partial p_n} \left[ \varepsilon_r(\vec{r}') - j \frac{\sigma(\vec{r}')}{\omega \varepsilon_0} \right].$$

(4.31)

Here, $k_i = \omega \sqrt{\mu_0 \varepsilon_0}$. If $p_n$ is a constitutive parameter of the $n$th voxel alone, after substitution of (4.31) in (4.26), the response derivative is obtained as

$$\left[ \frac{\partial R^{(i)}_{jk}}{\partial p_n} \right]_r = \frac{k_i^2 \Delta \Omega_n}{j \omega \mu_0 J_k^{(i)}} \left( \vec{E}_j^{(i)} \cdot \vec{E}_k^{(i)} \right)_{r=r_n} \frac{\partial}{\partial p_n} \left( \varepsilon_{r,n} - j \frac{\sigma_n}{\omega \varepsilon_0} \right).$$

(4.32)

Here, the subscript $n$ describes a quantity associated with the $n$th voxel only ($p_n = \varepsilon_{r,n}$ or $p_n = \sigma_n$). Note that the differentiated term in (4.32) is the complex voxel permittivity

$$\varepsilon^{(i)}_{r,n} = \varepsilon_{r,n} - j \sigma_n / (\omega \varepsilon_0).$$

(4.33)

In a dispersive medium, both the real and imaginary parts of $\varepsilon^{(i)}_{r,n}$ are frequency dependent although their superscripts $(i)$ are omitted hereafter for easier notation.
The result in (4.32) helps to understand the proposed detection algorithm from the viewpoint of scattering theory as explained next.

4.3 RELATION BETWEEN THE SELF-ADJOINT SENSITIVITY FORMULA AND THE LINEAR BORN APPROXIMATION

In the case of weak scatterers, the linear Born approximation assumes that the total field in the examined region can be replaced by the incident field; see. e.g., [47]. Using the adopted notations, the scattered field vectors in a nonmagnetic isotropic medium are approximated as

\[ \mathbf{E}_s^{(i)sc}(\mathbf{r}_j) = k_i^2 \iiint_{\Omega} (\bar{\varepsilon}_{r,j}^{(i)} - \bar{\varepsilon}_{r,b}^{(i)}) \mathbf{G}(r_j, r') \mathbf{E}_r^{(j)inc}(r') d\Omega' , \quad (j, k = 1, \ldots, K; i = 1, \ldots, N_j) \]

(4.34)

where \( \bar{\varepsilon}_{r,b}^{(i)} \) and \( \bar{\varepsilon}_{r,j}^{(i)} \) are the complex relative permittivities of the known background medium and the unknown scatterer, respectively. By definition, the incident field \( \mathbf{E}_r^{(i)inc} \) is the field due to the \( k \)th source in the background medium. \( \mathbf{G}(r, r') \) is the Green function tensor for the vector Helmholtz \( \mathbf{E} \)-field equation in this medium:

\[ \nabla \times \nabla \times \bar{\mathbf{G}}(r, r') - k_i^2 \varepsilon_r \bar{\mathbf{G}}(r, r') = \bar{\mathbf{1}} \delta(r - r') . \]

(4.35)

Thus, the incident field at \( \mathbf{r}_j \) due to a current-density point source at \( \mathbf{r}' \) is determined through \( \bar{\mathbf{G}}(r_j, r') \) as [47]
\[ E^{(i)\text{inc}}(\mathbf{r}_j) = j\omega \mu_0 \Delta \Omega_{r'} \mathbf{G}(\mathbf{r}_j, \mathbf{r}'), \mathbf{J}^{(i)}(\mathbf{r}'), \quad i = 1, \ldots, N_f \]  

(4.36)

where \( \Delta \Omega_{r'} \) is the volume of the voxel at the source location \( \mathbf{r}' \). In an isotropic medium, \( \mathbf{G}(\mathbf{r}_j, \mathbf{r}') = \mathbf{G}^T(\mathbf{r}', \mathbf{r}_j) \) [47]; therefore,

\[ E^{(i)\text{inc}}(\mathbf{r}') \equiv E^{(i)\text{inc}}(\mathbf{r}') = j\omega \mu_0 \Delta \Omega_j \mathbf{G}(\mathbf{r}_j, \mathbf{r}') \mathbf{J}^{(i)}(\mathbf{r}_j) \]  

(4.37)

where \( \mathbf{J}^{(i)}(\mathbf{r}_j) = \hat{\mathbf{p}}_j J^{(i)}_j, \quad J^{(i)}_j = |\exp(j\varphi^{(i)}_j)|. \)

A perturbation in the permittivity or conductivity of a single voxel, the \( n \)th voxel, in the background medium constitutes a weak-scattering problem. The respective scattering field can be viewed as the change in the incident field due to this perturbation:

\[ E_{jk}^{(i)\text{inc}} = \Delta_n E_{jk}^{(i)\text{inc}} = k_j^2 \Delta_n \mathbf{e}_{jk}^{(i)\text{inc}} \mathbf{G}(\mathbf{r}_j, \mathbf{r}_n) E_j^{(i)\text{inc}}(\mathbf{r}_n) \Delta \Omega_n \]  

(4.38)

so that the field \( E_j^{(i)} = E_j^{(i)\text{inc}} + \Delta_n E_{jk}^{(i)\text{inc}} \) is the total field in the perturbed problem.

Dividing both sides of (4.38) by the parameter perturbation \( \Delta \rho_n \) and taking the dot product with the polarization vector \( \hat{\mathbf{p}}_j \) of the \( j \)th source produces an expression for the response \( F_{jk}^{(i)} \):

\[ \frac{\partial E_{jk}^{(i)}}{\partial \rho_n} = \lim_{\Delta \rho_n \to 0} \frac{\hat{\mathbf{p}}_j \cdot \Delta_n E_{jk}^{(i)}}{\Delta \rho_n} = k_j^2 \hat{\mathbf{p}}_j^T \mathbf{G}(\mathbf{r}_j, \mathbf{r}_n) E_k^{(i)\text{inc}}(\mathbf{r}_n) \Delta \Omega_n \frac{\partial \mathbf{e}_{jk}^{(i)\text{inc}}}{\partial \rho_n}. \]  

(4.39)

Using (4.37) with \( \mathbf{r}' = \mathbf{r}_n \), the response derivative is written as

\[ \frac{\partial E_{jk}^{(i)}}{\partial \rho_n} = \frac{k_j^2 \Delta \Omega_n}{j\omega \mu_0 \Delta \Omega_j} \left( E_{jk}^{(i)\text{inc}} \cdot E_k^{(i)\text{inc}} \right)_{\mathbf{r}' = \mathbf{r}_n} \frac{\partial \mathbf{e}_{jk}^{(i)\text{inc}}}{\partial \rho_n}. \]  

(4.40)
which is identical with the sensitivity formula in (4.32).

It is now clear that the response derivatives (4.32) in the problem defined by the background medium describe the rate of increase of the scattered field with respect to the voxel constitutive parameters. Electromagnetic simulations of this background-medium problem can supply the respective incident-field solutions. While the simulations can be rather time-consuming, the sensitivity calculations using (4.40) are fast and can be performed in real time. The practicality of the proposed detection algorithm stems from the fact that the simulations model the background medium only. They are independent of the object under test and can be performed before the test measurements commence.

The above theory also shows that the limitations of the detection algorithm relate to those of the linear Born approximation. The algorithm relies substantially on the knowledge of the incident field and performs best with electrically small targets. Thus the intended applications are in near-field microwave imaging and the detection of electrically small abnormalities. As explained next, the algorithm exploits a large number of responses by employing multiple transmission/reception points as well as multiple frequencies in a very wide frequency band. This improves its detection sensitivity and robustness to errors in the incident field distribution as well as measurement errors.
4.4 OBJECTIVE FUNCTION FORMULATION

In imaging, the objective function is customarily defined as [18]

$$\Phi(\tilde{\epsilon}) = \| F(\tilde{\epsilon}) - \tilde{F} \| + \delta \cdot \| \tilde{\epsilon} - \tilde{\epsilon}_b \|$$  \hspace{1cm} (4.41)

where $\tilde{F}$ is the vector of target responses, $F$ is the vector of responses obtained in the forward model, and $\| \cdot \|$ represents a suitable, e.g., $l_2$, norm. The second term in (4.41) is the regularization term with $\delta$ usually set between 0 and 0.5. The vector $\tilde{\epsilon}$ represents the unknown complex permittivity profile of the reconstructed object while $\tilde{\epsilon}_b$ is the known background permittivity profile.

For the purposes of detection, here, the regularization term in the objective function is dropped and a quadratic norm is adopted. Also, the signal magnitude and its phase are considered as separate responses. This allows for implementations in various scenarios where phase information may or may not be available. The objective functions based on the magnitudes of the responses are defined as

$$F_M^{(i)}(\tilde{\epsilon}) = 0.5 \sum_{j,k=1}^{K} \left( |F^{(i)}_{jk}| - |\tilde{F}^{(i)}_{jk}| \right)^2, \; i = 1,\ldots,N_f.$$  \hspace{1cm} (4.42)

Their derivatives are

$$\frac{\partial F_M^{(i)}}{\partial p_n} \bigg|_{p_n = \epsilon_{t,n}, \epsilon_{e}} = \sum_{j,k=1}^{K} \left( |F^{(i)}_{jk}| - |\tilde{F}^{(i)}_{jk}| \right) \frac{\partial |F^{(i)}_{jk}|}{\partial p_n}, \; n = 1,\ldots,N,$$  \hspace{1cm} (4.43)

where for any $F^{\pm}_{jk} = F$. 

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The objective functions based on the phase of the responses are defined as

$$ F_{ij}^{(i)}(\vec{\xi}) = 0.5 \sum_{j,k=1}^{K} \left[ \exp \left( j \angle F_{jk}^{(i)} \right) - \exp \left( j \angle \bar{F}_{jk}^{(i)} \right) \right], \quad i = 1, \ldots, N_f. \tag{4.45} $$

Their respective derivatives are

$$ \frac{\partial F_{ij}^{(i)}}{\partial p_n} \bigg|_{p_n = r_{1n}, \sigma_n} = \sum_{j,k=1}^{K} \sin \left( \angle F_{jk}^{(i)} - \angle \bar{F}_{jk}^{(i)} \right) \frac{\partial \angle F_{jk}^{(i)}}{\partial p_n}, \quad n = 1, \ldots, N, \quad i = 1, \ldots, N_f \tag{4.46} $$

where for any $ F_{jk}^{(i)} = F $,

$$ \frac{\partial F}{\partial p_n} = |F|^{-2} \left[ \text{Re} F \cdot \text{Im} \left( \frac{\partial F}{\partial p_n} \right) - \text{Im} F \cdot \text{Re} \left( \frac{\partial F}{\partial p_n} \right) \right]. \tag{4.47} $$

The complex response derivatives $ \partial F_{jk}^{(i)}/\partial p_n $ are computed using the sensitivity formula (4.32).

Each parameter $ p_n $ corresponds to a location $ r_n $ in the region of interest. Thus, each of the four types of objective-function sensitivities described above ($ \partial F_{M}^{(i)}/\partial \epsilon_{n}, \partial F_{M}^{(i)}/\partial \sigma_n, \partial F_{V}^{(i)}/\partial \epsilon_{n}, \partial F_{V}^{(i)}/\partial \sigma_n, n = 1, \ldots, N, i = 1, \ldots, N_f $) can be plotted as functions of $ r_n, n = 1, \ldots, N $, at each frequency of interest. These plots are referred to as sensitivity maps.

It is clear that sensitivity maps obtained from a single response ($ j, k = 1 $) cannot identify the scatterers because, after normalization, the distributions in space are
identical to those of the background: \[ \left[ \partial |F_{i}^{(i)}| / \partial p_n \right]_{r=r_e} \text{ or } \left[ \partial \angle F_{i}^{(i)} / \partial p_n \right]_{r=r_e}, \]
\[ p_n = \varepsilon_{r,n} \sigma_n, \quad n = 1, \ldots, N, \quad i = 1, \ldots, N_j. \]
However, with multitude of responses, the derivatives at points, whose \( \varepsilon_r \) and/or \( \sigma \) in the object under test differ from those in the background, will add up coherently to produce a distinct maximum or minimum in the sensitivity map. On the contrary, where \( \varepsilon_r \) and \( \sigma \) in the object under test are the same as those in the background, the derivatives of the responses will add incoherently. Note that the differences between “background” responses and “target” responses, \( (|F_{i}^{(i)}| - |\overline{F}_{i}^{(i)}|) \) in (4.43) and \( \sin \left( \angle F_{i}^{(i)} - \angle \overline{F}_{i}^{(i)} \right) \) in (4.46), play an important role as weighting coefficients for the respective derivative terms although by themselves they are not functions of position.

For example, if \( |F_{i}^{(i)}| = |\overline{F}_{i}^{(i)}| \), the respective sensitivity distribution \( \partial |F_{i}^{(i)}| / \partial p_n \), \( n = 1, \ldots, N \), plays no role in the formation of the magnitude sensitivity map \( \partial F_M / \partial p_n \), \( n = 1, \ldots, N \); see (4.43). If the sign of \( (|F_{i}^{(i)}| - |\overline{F}_{i}^{(i)}|) \) is the same as the sign of the derivative it multiplies, the contribution of this derivative at this location is positive. The interpretation is that this voxel parameter will have to decrease in order to reduce the objective function \( F_M \) in (4.42). Conversely, if the sign of \( (|F_{i}^{(i)}| - |\overline{F}_{i}^{(i)}|) \) is opposite to that of the derivative it multiplies, the contribution of this derivative at this location is negative, i.e., this voxel parameter will have to increase in order to reduce \( F_M \).
The sensitivity maps are usually plotted as slice images intersecting the 3D object under test at planes of interest. Regions of pronounced peaks (positive maxima) or dips (negative minima) indicate locations where the parameter of interest in the background medium ($\varepsilon_{r,b}$ or $\sigma_b$) is significantly higher or significantly lower, respectively, than that in the object under test. The peaks and the dips in the 2D slice plots of the sensitivity maps are identified by a zero gradient,

$$\frac{\partial M}{\partial u} = \frac{\partial M}{\partial v} = 0$$  \hspace{1cm} (4.48)

where $u$ and $v$ are the two mutually orthogonal plot axes and $M$ can be any of the four types of sensitivity maps or a linear combination thereof.

A particularly large number of responses is acquired in holography techniques for microwave imaging [48]. The detection algorithm developed here can make use of this information to locate small scatterers which the holography imaging algorithm is likely to miss.

4.5 DETECTION ALGORITHM

The procedure to build the sensitivity maps and their suitable linear combinations is summarized next. Tomographic-style (slice) images are generated at each plane of interest by following the steps below.

**Step 1 Map Generation:** At each frequency of interest ($i = 1, \ldots, N_f$), the
reverse the sign of all combination maps in order to obtain maxima instead of minima depending on the desired color schemes.

4.6 HOMOGENEOUS EXAMPLE AND RESULTS

In all examples, the sources employ the same band-limited pulse waveform, which covers the frequency band from 3.0 GHz to 10.0 GHz. The magnitude spectrum at 3.0 GHz and 10.0 GHz is approximately 0.33 of the maximum spectral component which is at 6.5 GHz. We employ a circular array of transmission/reception points. An example configuration of the array is shown in Fig. 4.1.

![Diagram of the array configuration](image)

**Fig. 4.1.** Configuration of the circular array of transmission/reception points.

4.6.1 Model set-up

In order to verify the proposed detection algorithm, we set up the background and
target models for simulation with QuickWave [39]. As a time-domain simulator, QuickWave can provide the phasors of the electric field over the ultra wide frequency band. Also, it has a friendly user interface to export the electric field phasors at grid points of interest. Fig. 4.2(a) shows a 2D cut of the target model which serves to obtain the target (or “measured”) responses. It consists of a homogeneous background and a lossy spherical scatterer. The spherical scatterer has its center at (20, 20, 20) mm and its diameter is 2 mm. Its constitutive parameters are $\varepsilon_r = 30$ and $\sigma = 3$ S/m and are assumed frequency-independent. The constitutive parameters of the background are $\varepsilon_b = 6$ and $\sigma_b = 0.2$ S/m (also frequency-independent).

The six excitation points are uniformly distributed on a circle of diameter 14 mm (see points $P_{1, 2, ..., 6}$ in Fig. 4.2). These are also the points where the responses are recorded.

The overall computational domain is a box with a corner at (0, 0, 0) mm, which extends 40 mm along the $x$, $y$ and $z$ axes. It is terminated with absorbing boundaries. The finite-difference mesh is uniform with $\Delta h = 0.5$ mm. Fig. 4.2(b) shows the 2D cut of the background model. It is identical with the target model except for the absence of the scatterer.
mm, with an x-axis error of 0.4 % and a y-axis error of 2.2 %. Finally, three combination maps are plotted as slice images in 3D in Fig. 4.4. The combination maps lie in three orthogonal planes (x-, y- and z-planes) which intersect at the center of the sphere. They indicate the significant difference in the dielectric properties of the background medium and the target problem at the center of the Jacobian computational domain.

Fig. 4.3. Four types of sensitivity maps in the plane $z = 20$ mm at 10 GHz: (a) $M_{\varepsilon}$; (b) $M_{\sigma}$; (c) $P_{\varepsilon}$; (d) $P_{\sigma}$.  

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TABLE 4.1
MINIMA OF THE SENSITIVITY MAPS FROM 3 GHz TO 10 GHz
WHEN THE TRUE TARGET LOCATION IS (20.0, 20.0) MM IN THE Z = 20.0 MM PLANE

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Frequency (GHz) $\frac{\partial}{\partial \varepsilon_r}$</th>
<th>$M$ Location (x, y) (in mm)</th>
<th>$P$ Location (x, y) (in mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>a None</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>None</td>
<td>(19.0, 21.5)</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>None</td>
<td>(22.0, 22.5)</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>(20.0, 20.0)</td>
<td>(23.0, 22.5)</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>(20.0, 20.0)</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>(20.0, 20.0)</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>(20.0, 20.0)</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>(20.0, 20.0)</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>(20, 20)</td>
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<tr>
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<td>4</td>
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<td>None</td>
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<tr>
<td></td>
<td>5</td>
<td>(18.5, 20.5)</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>(22.0, 23.0)</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>None</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>None</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>None</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>None</td>
<td>(20.0, 20.0)</td>
</tr>
</tbody>
</table>

$^a$ None refers to a map which does not have peaks or dips.

TABLE 4.2
MINIMA OF THE FREQUENCY-AVERAGED SENSITIVITY MAPS
WHEN THE TRUE TARGET LOCATION IS (20.0, 20.0) MM IN THE Z = 20.0 MM PLANE

<table>
<thead>
<tr>
<th>Derivative</th>
<th>$M$ Location (x, y) (in mm) $\frac{\partial}{\partial \varepsilon_r}$</th>
<th>$P$ Location (x, y) (in mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(20.0, 20.0)</td>
<td>(20.362, 21.1293)</td>
</tr>
<tr>
<td></td>
<td>(19.9697, 20.6212)</td>
<td>(20.0, 20.0)</td>
</tr>
</tbody>
</table>
Fig. 4.4. 3D set of three combination maps with the target's center at (20, 20, 20) mm.

The same example is repeated with various locations of the spherical target. In all cases, the sensitivity maps exhibit minima, which identify correctly the target's location. For example, when the target's center is at the true location (23.5, 20.0, 20.0) mm, the predicted location is obtained as (22.50, 19.99, 20.0), which has an x-axis error of 4.3 % and a y-axis error of 0.05 %. The locations predicted by the minima of the four types of sensitivity maps from 3 GHz to 10 GHz in this case are listed in Table 4.3. The minima of the four frequency-averaged sensitivity maps are listed in Table 4.4. Fig. 4.5 shows the combination maps in a 3D plot where the slice images intersect at the true location of the target.
TABLE 4.3
MINIMA OF THE SENSITIVITY MAPS FROM 3 GHz TO 10 GHz
WHEN THE TRUE TARGET LOCATION IS (23.5, 20.0) MM IN THE Z = 20.0 MM PLANE

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Frequency (GHz)</th>
<th>Minimum in M Location (x, y) (in mm)</th>
<th>Minimum in P Location (x, y) (in mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>None</td>
<td>(20.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>None</td>
<td>(22.5, 20.0)</td>
</tr>
<tr>
<td>$\frac{\partial}{\partial \epsilon}$</td>
<td>5</td>
<td>None</td>
<td>(23.5, 20.0)</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>None</td>
<td>(24.0, 19.0)</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>(23.5, 20.0)</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>(23.0, 20.0)</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>(23.0, 20.0)</td>
<td>(16.5, 20.0)</td>
</tr>
<tr>
<td>$\frac{\partial}{\partial \sigma}$</td>
<td>3</td>
<td>(21.0, 20.0)</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>(22.0, 20.0)</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>(23.0, 20.0)</td>
<td>(20.5, 20.0)</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>(24.0, 20.0)</td>
<td>(21.0, 20.0)</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>None</td>
<td>(21.5, 20.0)</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>None</td>
<td>(22.0, 20.0)</td>
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<td></td>
<td>9</td>
<td>None</td>
<td>(22.5, 20.0)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>None</td>
<td>(22.5, 20.0)</td>
</tr>
</tbody>
</table>

TABLE 4.4
MINIMA OF THE FREQUENCY-AVERAGED SENSITIVITY MAPS
WHEN THE TRUE TARGET LOCATION IS (23.5, 20.0) MM IN THE Z = 20.0 MM PLANE

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Minimum in M Location (x, y) (in mm)</th>
<th>Minimum in P Location (x, y) (in mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial}{\partial \epsilon}$</td>
<td>(23.1250, 20.0)</td>
<td>(22.4857, 19.8571)</td>
</tr>
<tr>
<td>$\frac{\partial}{\partial \sigma}$</td>
<td>(22.8684, 20.1053)</td>
<td>(21.508, 20.0)</td>
</tr>
</tbody>
</table>
Fig. 4.5. 3D set of three combination maps with the target’s center at (23.5, 20.0, 20.0) mm.

4.6.3 The effect of the number of transmission/reception points

Here, the goal is to examine the capability of the sensitivity-based detection algorithm depending on the number of transmission/reception points. The performance of the algorithm is quantified through the localization error,

\[ e = \frac{1}{2} \left( \frac{|x - \bar{x}|}{\bar{x}} + \frac{|y - \bar{y}|}{\bar{y}} \right) \times 100 \]  \hspace{1cm} (4.56)

where \((x, y)\) are the coordinates of the predicted location and \((\bar{x}, \bar{y})\) are the coordinates of the true location in a given combination map, which contains the target’s location.
For this study, we use the target model where the spherical scatterer is located at (23.5, 20.0, 20.0) mm. Five scenarios are considered where the number of transmission/reception points is set as two, three, four, six, and eight, respectively. These points are always symmetrically distributed in a circular array as shown in Fig. 4.2 where the array is in the plane $z = 20$ mm containing the target's center. The localization error versus the number of transmission/reception points is shown in Fig. 4.6. The error decreases fast as the number of transmission/reception points is increased. When six points are used, the localization error is already 1.8%. Adding more points does not bring any significant improvement in this simple homogeneous example. A more complex example where the background is heterogeneous or the radius of the circular array is larger will likely require more points in order to achieve adequate accuracy.

Fig. 4.6. The localization error versus the number of transmission/reception points.
4.6.4 Conductivity contrast

The permittivity and conductivity contrasts between the scatterer and the background are crucial. In medical diagnostics, such as breast cancer, it is reported that the conductivity contrast between malignant and healthy tissues is prominent [49]-[51]. Also, in concealed weapon or minerals detection, the conductivity contrast is very large. Here, we study the capability of the proposed detection algorithm depending on the conductivity contrast. Again, the case is considered where the true location of the spherical target is at (23.5, 20.0, 20.0) mm. Six symmetric transmission/reception points are used in this example. The permittivity of the scatterer is set equal to that of the background (\(\varepsilon_s = \varepsilon_{rb} = 6\)). The following conductivity contrast ratios, \(\sigma_s / \sigma_{rb}\), are considered: five, twelve, twenty-five, fifty, seventy-five, and a hundred. The localization errors versus the conductivity contrast, for different diameters of the spherical scatterer, are shown in Fig. 4.7(a). A study regarding the contrast in the relative permittivity shows very similar behavior of the localization error versus the number of transmission/reception points and versus the size of the target.
4.6.5 Target size

The smallest target size that can be detected by the algorithm quantifies its sensitivity. We use again the case where the spherical target is at (23.5, 20.0, 20.0) mm with six transmission/reception points. The diameter of the spherical scatterer varies from one cell size ($\Delta h = 0.5$ mm), to two, four, six and eight cell sizes. Fig. 4.7(b) shows the localization error versus the diameters of the spherical targets with conductivity contrasts of five, fifty and a hundred. The errors decrease drastically with higher conductivity contrasts as well as with larger sizes of the target. In the case of infinite contrast, which describes a perfectly conducting scatterer, even a single-cell scatterer is accurately located with the localization error of 1.76%. 


Fig. 4.7. The localization error versus: (a) the conductivity contrast with different diameters of the scatterer; (b) the diameter of the scatterer with the different conductivity contrasts. The number of transmission/reception points is six.
4.6.6 Resolution

The resolution of the detection algorithm can be defined as the smallest distance between two electrically small scatterers at which they can be distinguished. Two targets are considered in the same homogeneous background ($\varepsilon_{rb} = 6$ and $\sigma_b = 0.2$ S/m). The diameter of both scatterers is 2 mm (or $4\Delta h$). The permittivity of the scatterers is the same with that of the background, while their conductivity is fifty times that of the background. The distance between the two scatterers is varied as: $\lambda_{\min}/2$, $\lambda_{\min}/3$, $\lambda_{\min}/4$, and $\lambda_{\min}/6$. Here, $\lambda_{\min} = 14$ mm is the shortest wavelength in the background medium corresponding to $10$ GHz. The combination maps are shown in Fig. 4.8 for the four distances between the scatterers. The two scatterers can be distinguished until the distance between them is about $\lambda_{\min}/4$. Below this separation distance, the minima corresponding to the two scatterers appear to fuse together in the combination map [see Fig. 4.8(d)], i.e., the two targets cannot be distinguished.
Fig. 4.8. Combination maps for the case of two scatterers in the plane $z = 20$ mm at distances: (a) $\lambda_{\text{min}} / 2$; (b) $\lambda_{\text{min}} / 3$; (c) $\lambda_{\text{min}} / 4$; (d) $\lambda_{\text{min}} / 6$. 
4.7 HETEROGENEOUS EXAMPLE AND RESULTS

The proposed detection algorithm has been also tested on problems where the background is heterogeneous. The cross-section of the heterogeneous background model is shown in Fig. 4.9(a) together with a circular array of six transmission/reception points. The background includes three different regions: the region outside the array (medium 1, \( \varepsilon_r = 6 \) and \( \sigma = 0.2 \) S/m), inside the array (medium 2, \( \varepsilon_r = 7 \) and \( \sigma = 0.3 \) S/m), and an object of a complex shape inside the array (medium 3, \( \varepsilon_r = 8 \) and \( \sigma = 0.4 \) S/m). The overall computational domain is a cube 40 mm on a side and it is terminated with absorbing boundaries. The FDTD mesh is uniform with a cell size \( \Delta h = 0.5 \) mm.

Fig. 4.9(b) shows the target model, which is identical to the background except for the presence of a spherical scatterer whose center is at (19.0, 19.0 20.0) mm and its radius is 0.5 mm. Its constitutive parameters are \( \varepsilon_r = 30 \) and \( \sigma = 3 \) S/m.

For complete 3D sensitivity analysis, the circular array scans the height of the imaged cylindrical region (height is 40 mm) in steps of 2 mm. In each acquisition plane, the transmission points are excited one at a time and the scattered field is recorded at all reception points. The excitation waveform is the same as in the previous examples.
Fig. 4.9. Cross-section in the $xy$ plane of the heterogeneous example: (a) forward model; (b) target model.
It has been established empirically that the highest quality sensitivity maps in a given plane of interest are obtained from the responses acquired at reception points belonging to this plane. Note that, in principle, the sensitivity maps can use all responses including those acquired in planes different from the mapped plane. We attribute this to the fact that the scattered field contributions due to small scatterers in planes different from those of the transmitters/receivers are weak and they are often at the error level of the numerical solution.

The vertical scanning by a cylindrical array allows for obtaining sensitivity maps in two types of planes: horizontal planes, which are the planes of the circular array itself, and vertical planes, which are defined by the vertical positions of each array element. In Fig. 4.10(a), five horizontal planes are shown with red dash circles. One such plane is also shown in Fig. 4.10(b) with the six symmetric transmission/reception points. In Fig. 4.10(c), a vertical cut in the 3D domain is shown, where these planes appear as red dash-lines. In each horizontal plane, \(6 \times 6 = 36\) responses are acquired at each frequency of interest. With eight sampled frequencies from 3 GHz to 10 GHz (same as in the homogeneous-background example), this makes for 288 complex responses. The respective \(6 \times 8 = 48\) complex-valued vector field solutions in this plane are recorded and used to calculate the sensitivity maps as per (4.32).
One vertical plane defined by the positions $P_3$ and $P_6$ is shown with black dash-line in Fig. 4.10(a); this is also the plane of the 2D cut shown in Fig. 4.10(c). Five vertical planes are shown with black dash-line in Fig. 4.10(b), all containing the line of the vertical positions of $P_1$. To build the sensitivity maps in a vertical plane, the same measurements are used as for the maps in the horizontal planes. Take as an example the plane shown in Fig. 4.10(c). Here, we have at our disposal the responses $F_{jk}^{(i)}$, $j, k = 3, 6$, $i = 1, \ldots, 8$, acquired in five measurements (for the five vertical positions of the array). This makes for a total of 160 complex responses. The respective $2 \times 5 \times 8 = 80$ complex-valued vector field solutions in this plane are used to calculate the sensitivity maps as per (4.32).

In this example, the vertical positions of the array are $z = 16, 18, 20, 22$ and 24 mm where the $z = 20$ mm plane contains the center of the spherical scatterer.
Fig. 4.10. Planes of field acquisition in the heterogeneous example: (a) 3D configuration showing five horizontal planes (red dash-line circles) and one vertical plane (black dash-line rectangle); (b) horizontal plane 1 where the intersections with five vertical planes are shown with black dash-lines; (c) one vertical plane where the intersections with five horizontal planes are shown with red dash-lines.
The set of three combination maps intersecting at the target's center at (19.0, 19.0 20.0) mm is shown in Fig. 4.11. The corresponding minima are listed in Table 4.5. It is evident that the heterogeneous background medium is not a significant obstacle for the detection algorithm. It should be pointed out, however, that here exact knowledge of the background is assumed. It is a matter of further thorough studies to establish how inaccuracies in the knowledge of the background would affect the performance of the algorithm and to quantify the respective limitations.

Fig. 4.11. 3D plot of combination maps for the heterogeneous example. The maps intersect at the target's center whose true location is (19.0, 19.0, 20.0) mm.

<table>
<thead>
<tr>
<th>Location (x, y) (in mm) in the plane z = 20 mm</th>
<th>True Center</th>
<th>Map Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(19.00, 19.00)</td>
<td>(19.65, 19.73)</td>
<td></td>
</tr>
</tbody>
</table>
4.8 CONCLUSIONS

A new general formulation of the response sensitivity analysis of electromagnetic problems has been presented. This formulation is independent of the method of field analysis. Its only limitation stems from the assumption for the reciprocity of the electromagnetic problem.

The self-adjoint sensitivity analysis is the basis of a novel detection algorithm. The Fréchet derivatives (the Jacobian) of specifically formulated objective functions are computed with respect to the voxel permittivities and conductivities in a model of the known background medium. The objective functions are a measure of the difference between this model's responses and the target responses, which are acquired with the object under test. The responses are formulated in terms of transmitted and reflected signals. This conforms to microwave measurements where the scattering parameters are acquired. The algorithm benefits from the availability of both the magnitudes and the phases of the scattering parameters but its objective functions are so formulated that the algorithm can still function with magnitude information only.

The detection is realized by examining the 3D plots of the objective-function sensitivities versus the position of the voxels inside the object under test whose constitutive parameters are examined. The obtained sensitivity maps are not images of the object's dielectric profile but rather images of the difference between its
dielectric profile and that of the background medium. Significant negative-value minima (or positive-value maxima) are indicative of the locations at which the constitutive parameters of the background medium are smaller (or greater) than those in the examined object.

Validation examples have been presented for the case of a homogeneous background medium where both the background model and the target model are realized through time-domain simulations. The locations of electrically small scatterer are determined with a maximum $x$-axis error of 4.3% and a $y$-axis error of 2.2% when a six-element array of transmission/reception points is employed. The error consistently decreases as the number of transmission/reception points increases.

The detection algorithm is also studied in an effort to determine the effect of various factors on its performance. These factors include: (i) the number of transmission/reception points, (ii) the conductivity and permittivity contrasts between the scatterer and the background medium, and (iii) the size of the scatterer in terms of the discretization cell size. The resolution of the algorithm is also assessed in an example involving two electrically small scatterers, the distance between which as varied.

Finally, the case of heterogeneous background is also studied. It is found that the detection capability of the algorithm is not influence by the fact that the background is heterogeneous provided that the background complex-permittivity profile is known.
The aim of the current study is to examine the feasibility of the proposed sensitivity-based detection algorithm. The results are encouraging and warrant further studies and experimentation with applications of microwave imaging and diagnostics in cancer detection, concealed-object detection and structural integrity. It is yet to be determined how inaccuracies of the knowledge of the background medium would influence the algorithm performance in various scenarios including homogeneous and strongly heterogeneous media.

Finally, it should be pointed out that the sensitivity-analysis approach proposed here is a powerful technique from the point of view of optimization-based image reconstruction. It can drive efficiently gradient-based optimizers toward a full reconstruction of the dielectric profile of a target. While the detection algorithm proposed here is a single-step diagnostic approach, a complete image reconstruction would obviously require a lengthy iterative-update procedure. In this context, it is worth emphasizing that the proposed algorithm can achieve detection in real time since the calculation of the sensitivity maps is very fast. The field distributions in the background medium, which are required by the sensitivity formula, can be obtained beforehand as part of the system calibration since they are not dependent on the particular object under test.
REFERENCES


Chapter 5

BREAST CANCER DETECTION USING MICROWAVE AND SELF-ADJOINT SENSITIVITY ANALYSIS

5.1 INTRODUCTION

Breast cancer affects many women and early detection aids in fast and effective treatment. X-ray mammography is currently the most effective imaging method for detecting clinically occult breast cancer. However, despite significant progress in improving mammographic techniques for detecting and characterizing breast lesions, mammography still has high false-negative rates [1] and high false-positive rates [2]. These difficulties are attributed to insufficient contrast between normal and malignant tissues at X-ray frequencies. In X-ray tomography, tissue is differentiated based on density. However in most cases, the tissue density does not depend on its physiological state. Important tissue characteristics such as temperature, blood content, blood oxygenation and ischemia cannot be differentiated by X-ray tomography. For soft tissues like human breast, X-ray cannot image the breast anomalies at an early stage, as there is no significant variation in density between normal and malignant breast tissues [3].
Microwave imaging is a new technology which has potential applications in the field of diagnostic medicine [4][5]. The basic motivation for this is improved physiologic and pathophysiologic correlation, especially in soft tissue. This expectation is based on the molecular (dielectric) rather than atomic (density) based interactions of the microwave radiation with the target when compared with X-ray imagery. When exposed to microwaves, the high water content of malignant breast tissues causes more significant microwave scattering than normal fatty breast tissues that have low water content.

Microwaves could be used effectively for the detection of biological anomalies like tumors at an early curable stage. At microwave frequencies, the sensitivity, specificity and the ability to detect small tumors are due to the dielectric contrast between normal and malignant breast tissues [6]. Malignant breast tissues exhibit considerable increase in bound water content compared to the normal tissues and hence a high value of permittivity. It is reported that dielectric permittivity and conductivity increase for cancerous breast tissue could be three or more times greater than the host tissue [7]. Due to the improved dielectric contrast, better tissue characterization too is possible.

Chapter 4 has introduced a novel detection algorithm that uses a self-adjoint response-sensitivity formulation. The work presented in Chapter 4 addresses problems where the background medium is uniform. Here, the focus is on the far more challenging problem of detecting abnormalities in a strongly heterogeneous
dissipative medium such as that presented by living tissue. To derive the background medium for human breast, we make use of images from magnetic resonance imaging (MRI). The goal of this preliminary investigation is to determine the feasibility of the proposed detection algorithm in breast cancer detection. Moreover, the limitations of the detection algorithm in terms of the number of transmission/reception points, the dielectric contrast and the size of the scatterer are studied in the heterogeneous background examples.

5.2 MODEL SET-UP

A MRI-based numerical model of the breast is set up in QWED [8]. The CAD model of the breast is an ‘.UDO’ file that is created by using segmented T1 weighted MRI images. This is done using an in-house built algorithm developed in MATLAB [9], whose details are given in [10].

The MRI-based breast model with a tumor (a spherical scatterer) is shown in Fig. 5.1. This simulation provides the target responses. The scatterer is absent in the forward model. The generated breast model is a 22 mm thick transversal section using 10 images from the whole MRI set. The latter usually consists of more than 90 images, which is simply to limit the computational time due to the overwhelming requirement with such detailed models. The breast model spans 22 mm along the z-axis, approximately 45 mm along the x- and y-axis. It is surrounded by a background
cubic air box, with a side of 80 mm. The spherical scatterer has its center at (40.25, 40.25, 10.25) mm and its diameter is 10 mm. The background is terminated with absorbing boundaries. The FDTD mesh is uniform with cell size $\Delta h = 0.5$ mm.

As shown in Fig. 5.1(b), the interior of the breast model consists of four types of normal tissues – muscle, fibro-glandular tissue (fibroglandular adjoining the muscle), transitional tissue (trans) and adipose fat (fat). The assigned tissue dielectric properties are also shown in Fig. 5.1(b). The scatterer (tumor stimulant) is embedded in the transitional tissue. The tissue classification is based on the latest studies [10][11]. However, there are two major approximations taken in this numerical model to keep the computational time and resources at a reasonable level: (i) the skin is not included in the CAD model (due to the complications arising in the construction of a 2 mm thin layer surrounding the breast model boundaries); (ii) the tissue dielectric properties are not exactly taken from the study. It is because the permittivity values are very high for the dense tissues such as muscle and fibro-glandular tissue, which brings down the effective wavelength in the medium. This in turn requires a very small cell-size that consequently increases the computational time. However, note that the dielectric contrast between the various tissue types is kept similar to those estimated in the study [11].

The breast model requires twenty-four transmission/reception points for effective localization. The effect of the number of points is discussed later. These points exploit multi-frequency responses in the ultra-wide band (UWB) from 3 GHz
to 10 GHz. The wide bandwidth of the response set provides optimal use of the significant tissue penetration at low frequencies and the improved resolution at high frequencies. They are evenly distributed on a rectangular array, along the coronal cut [black rectangle in Fig. 5.1(a)] through the 3D breast model. The transmission/reception points lying on this rectangular plane are uniformly distributed along the z-axis with an interval of 2 mm and along the y-axis with an interval of 5 mm. This yz-plane is shown in Fig. 5.1(c). It passes through the embedded tumor stimulant in the target model. The yz-plane is employed in recording $\mathbf{E}^{\text{inc}}(\mathbf{r})$ for the Jacobian computation described in Chapter 4.
Fig. 5.1 Set-up in QWED of the MRI-based breast model with a tumor stimulant: (a) A 2.2 cm thick 3D model obtained from 10 breast slices from a MRI set. (b) The transverse cut through the 3D model showing the various normal tissues comprising the breast model and surrounding the tumor. (c) The coronal cut through the tumor center in the 3D breast model, as shown by the black rectangle in (a). The $yz$ cross-section also shows the twenty-four transmission/reception points placed around the breast model.
5.3 DETECTION AND LOCATION OF A SPHERICAL TARGET

The Jacobian maps at various stages of the detection algorithm are shown in Fig. 5.2. Fig. 5.2(a) shows the frequency-averaged magnitude maps $M_{\nu}^M$ and $M_{\sigma}^M$ added together, obtained at Step 4 (map averaging), as explained in Chapter 4. The predicted location of the scatterer from this map is (40.25, 41.725, 10.96) mm. Fig. 5.2(b) shows the frequency-averaged phase maps $M_{\nu}^P$ and $M_{\sigma}^P$ added together. The predicted location of the scatterer from this map is (40.25, 41.25, 10.645) mm. It is observed that the phase maps localize the tumor stimulant with fewer artifacts as compared to the magnitude maps.

The combination map that was generated at Step 5 (map combination) is shown in Fig. 5.2(c). The corresponding predicted location is at (40.25, 40.5, 10.18) mm. It is very close to the true location of the tumor's center (40.25, 40.25, 10.25) mm.
Fig. 5.2. Jacobian maps (in the $yz$-plane at $x = 40.25$ mm): (a) frequency-averaged magnitude map ($M_{\sigma}^M + M_{\sigma}^M$) with predicted location (40.25, 41.725, 10.96) mm; (b) frequency-averaged phase maps ($M_{\phi}^P + M_{\phi}^P$) with predicted location (40.25, 41.25, 10.645) mm; (c) combination map with predicted location (40.25, 40.5, 10.18) mm.

The true location of the scatterer is (40.25, 40.25, 10.25) mm.
5.4 EFFECT OF THE NUMBER OF TRANSMISSION/RECEPTION POINTS

According to the performance study in the homogeneous-background examples (see Section 4.6 of Chapter 4), the number of acquired responses has a significant effect on the accuracy of the detection algorithm. Here, the goal is to examine its capability depending on the number of transmission/reception points when it is applied to a problem with complex heterogeneous background. The performance of the algorithm is quantified through the localization error in the yz-plane, as defined in (4.56), i.e.,

\[ e = \frac{1}{2} \left( \left| \frac{y - \bar{y}}{y} \right| + \left| \frac{z - \bar{z}}{z} \right| \right) \times 100\% \quad (5.1) \]

where \((y, z)\) are the coordinates of the predicted location and \((\bar{y}, \bar{z})\) are the coordinates of the true location in a given combination map, which contains the target's location.

For this study, we use the target model of Fig. 5.1 where the spherical scatterer is located at \((40.25, 40.25, 10.25)\) mm. Four scenarios are considered where the number of transmission/reception points is set as six, eight, twelve and twenty-four respectively. These points are always symmetrically distributed in a rectangular linear array as shown in Fig. 5.1 where the array is in the plane \(x = 40.25\) mm containing the target's center. The localization error versus the number of transmission/reception points is shown in Fig. 5.3. The error decreases fast and dramatically as the number of transmission/reception points is increased. When
twenty-four points are used, the localization error is 3.03 %. Compared with the simple homogeneous example in Section 4.6 of Chapter 4, the more complex example with the heterogeneous background requires more points in order to achieve adequate accuracy.

Fig. 5.3. The localization error versus the number of transmission/reception points.
5.5 TARGET SIZE

The smallest target size that can be detected by the algorithm quantifies its sensitivity. We use again the case where the spherical target is at (40.25, 40.25, 10.25) mm with twenty-four transmission/reception points. The diameter of the spherical scatterer varies from 2 mm (equivalently $4\Delta h$), to 4 mm, 6 mm, 8 mm and 10 mm. Fig. 5.4 shows the localization error versus the diameters of the spherical targets. The errors decrease drastically with larger sizes of the target. Note that the result becomes unreliable when the diameter of the target is too small, e.g. 2 mm.

![Graph showing localization error versus diameter of scatterer](image)

Fig. 5.4. The localization error versus the diameter of the scatterer. The number of transmission/reception points is twenty-four.
5.6 RESOLUTION

The resolution of the detection algorithm, as defined in Chapter 4, is the smallest distance between two electrically small scatterers at which they can be distinguished. Multiple targets (scatterers, or tumors) are considered with the same constitutive properties ($\varepsilon_r = 18$ and $\sigma = 7.5 \text{ S/m}$). They are immersed in the region of the transitional tissue of the breast model, as shown in Fig. 5.5. Both the forward model and the target model are surrounded by twenty-four transmission/reception points for effective localization, as shown in Fig. 5.5(b). The diameters and central locations of the multiple spherical targets are listed in Table 5.1.

According to the detection algorithm explained in Chapter 4, the combination map is plotted in Fig. 5.6. Note that each peak in Fig. 5.6 represents the predicted location of each spherical target. The predicted locations of multi-targets through the detection algorithm and their respective localization errors are also listed in Table 5.1. It is necessary to emphasize that the shortest distance between the centers of two distinguished spherical targets is 5 mm, equivalently, $\lambda_{\text{min}} / 3$. Here, $\lambda_{\text{min}} = 15$ mm is the shortest wavelength in the breast transitional tissue at 10 GHz. When the separation distance is below $\lambda_{\text{min}} / 3$, the multi-targets start to fuse together, i.e., they cannot be distinguished.
Fig. 5.5. The localization error versus the diameter of the scatterer. The number of transmission/reception points is twenty-four.

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>True Location (y, z) in mm</th>
<th>Predicted Location (y, z) in mm</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tumor 1</td>
<td>10</td>
<td>(40.25, 10.25)</td>
<td>(40.49, 10.50)</td>
</tr>
<tr>
<td>Tumor 2</td>
<td>8</td>
<td>(45.0, 14.5)</td>
<td>(45.51, 16.14)</td>
</tr>
<tr>
<td>Tumor 3</td>
<td>8</td>
<td>(40.0, 3.5)</td>
<td>(40.49, 3.13)</td>
</tr>
<tr>
<td>Tumor 4</td>
<td>6</td>
<td>(40.0, 20.0)</td>
<td>(40.49, 17.16)</td>
</tr>
<tr>
<td>Tumor 5</td>
<td>4</td>
<td>(45.0, 5.0)</td>
<td>(44.01, 6.91)</td>
</tr>
</tbody>
</table>
Fig. 5.6. The localization error versus the diameter of the scatterer. The number of transmission/reception points is twenty-four.

5.7 CONCLUSION

The newly proposed detection algorithm provides efficient localization of tumor stimulants, about one cm in diameter, in a realistic breast-tumor detection problem. The example is novel in terms of the detailed tissue classification and the realistic dielectric property contrast represented in the numerical model using MRI images. Moreover, limitations of the approach in terms of number of transmission/reception
points, distance between these points (scanning distance), have been investigated.

This is still a preliminary investigation and further studies are required. More realistic tissue properties and model sizes must be considered.

REFERENCES


Chapter 6

CONCLUSIONS

This thesis has presented advanced electromagnetic (EM) modeling of the interaction of the microwave field with human tissues. The proposed EM modeling methodologies are reliable, efficient and practical. The first advanced model targets the evaluation of the maximum specific absorption rate (SAR) and the maximum temperature rise in the human eyes due to exposure to radio-frequency (RF) fields generated by wireless handheld devices. The second advanced model aids the detection of electrically small scatterers in a known background medium.

In Chapter 2, we proposed two feasible methods for the evaluation of the maximum SAR in the human eyes due to exposure to RF fields. The methods are applied to the case of near-field exposure to handheld devices; however, they are applicable to the case of far-field illumination as well. While the first method is based on a semi-analytical model, the second is based on measurements. The two methods are complementary to each other and can be used to verify each other's output. They allow for the fast and reliable SAR evaluation of newly developed handheld devices in an industrial environment.
In Chapter 3, we developed a detailed simulation eye model, which allows for the calculation of the temperature rise in the eyes for a given device under test. The thermal study was carried out using the detailed eye model, which includes all anatomical details of size equal or greater than 0.25 mm. It suggests insignificant temperature rise in the human eyes even in the case of exposure to continuous full-power radiation from a typical wireless handheld device. It also confirms that our previously proposed semi-analytical model predicts adequately the worst-case scenario SAR value and its location.

In Chapter 4, a conceptually new detection algorithm was proposed for the localization of electrically small scatterers in a known background medium. The algorithm requires the knowledge of the electric field distribution inside the known background medium where no scatterers are present. It is based on a self-adjoint response sensitivity computation which can be performed in real time. Using the E-field distribution in the background medium, it provides three-dimensional maps of the Fréchet derivative within the imaged volume. The peaks and dips in these maps identify the locations where the permittivity and conductivity of the measured medium differ from those in the background medium. The background medium can be heterogeneous. In a homogeneous-medium example, the performance of the detection algorithm is studied in terms of the number of transmission/reception points, the dielectric contrast of the scatterer compared to the background medium,
and the size of the scatterer. Its resolution is also addressed. Detection of a small scatterer in a heterogeneous background is demonstrated.

In Chapter 5, the performance of the proposed algorithm was tested on a realistic breast-tumor detection problem through ultra-wideband time-domain simulations. The detection relies on images obtained from the response derivatives with respect to the voxel permittivities and conductivities in the modeled “background” medium, which is a close approximation of the healthy (or normal) breast tissue. Abnormal regions in the object under test are successfully detected and localized with a sufficient number of transmission/reception points. The limitations of the approach in terms of number of transmission/reception points, size of the tumor stimulant and the distance between two distinguishable tumors were investigated.

From the experience gained during the course of this work, the author suggests the following research topics to be addressed in future developments.

(1) Applying sensitivity analysis to practical problems of special interest such as antenna design, microwave imaging, etc.

(2) In the area of microwave imaging, exploiting the self-adjoint response sensitivity-based detection algorithm to produce enhanced 3D real-time imaging.

(3) In the area of breast cancer detection, investigating the improved breast model with dispersive tissue properties.


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