

**LEARNING BY DOING AND OPTIMAL
FISCAL AND MONETARY POLICY**

LEARNING BY DOING AND OPTIMAL FISCAL AND MONETARY
POLICY

by

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ABSTRACT

This thesis studies a number of issues in optimal fiscal and monetary policy using the Ramsey framework. Specifically, it focuses on the effects of learning-by-doing and organizational capital on optimal policy responses. The first essay investigates the optimal capital income taxation in presence of learning-by-doing effects. The main result is that the optimal tax rate on capital income is significantly positive in the long run even though the product market is imperfectly competitive. This finding contrasts with results obtained in the literature that the capital income tax should be zero if the product market is perfectly competitive and negative if the product market is imperfectly competitive. The second essay studies the effects of learning-by-doing, and price rigidities on the dynamic properties of optimal fiscal and monetary policy variables. The main result is that, contrary to the findings of other papers in this literature, optimal Ramsey inflation is very stable and persistent over the business cycle. A second important result is that optimal tax policy is counter-cyclical - tax rates fall during recession and rise during boom. This finding contrasts with pro-cyclical tax results obtained in standard sticky price Ramsey models. Finally, the third essay studies welfare maximizing fiscal and monetary policy rules in a model with sticky prices, learning-by-doing in the technology, and distortionary taxation. Specifically, it considers monetary feedback rules whereby

the nominal interest rate is set as a function of output and inflation. The main finding is that the optimal interest-rate rules call for a very strong response to inflation and a very weak response to output. Also, the optimal interest-rate rules are forward looking. This result contrasts with the backward looking optimal interest rate rules obtained in the existing optimal policy literature. The optimized fiscal rule is passive in the sense that tax revenues increase only mildly in response to increases in government liabilities. The optimized regime yields a level of welfare that is very close to that implied by the Ramsey optimal policy.

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PREFACE

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Chapter 1

Introduction

The basic idea of learning-by-doing and knowledge accumulation is that firms learn from production experience and accumulate this firm-specific knowledge that increases their productivity. There is a vast empirical literature (e.g. McGrattan and Prescott (2005), Atkeson and Kehoe (2005), Thornton and Thompson (2001), Benkard (2000), Irwin and Klenow (1994), Bahk and Gort (1993), Prescott and Visscher (1980)) which document the pervasive presence and the importance of learning effects in virtually every area of the economy. Also, some recent papers (e.g. Johri and Lahiri (2008), Cooper and Johri (2002), Chang, Gomes, and Schorfheide (2002)) show that a learning-by-doing (LBD) mechanism magnifies the propagation of shocks and improves the moment matching performance of Dynamic Stochastic General Equilibrium (DSGE) models. However, as far as we are aware of, there are

no previous studies of optimal fiscal and monetary policy that incorporate a learning-by-doing mechanism into DSGE models. In this thesis, we introduce learning-by-doing mechanism in previously studied DSGE models and study optimal fiscal and monetary policy in a Ramsey framework following the tradition of Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991).

In his seminal paper (Ramsey (1927)), Ramsey studied a static, one consumer economy with many goods. A government requires fixed amounts of each of these goods, which are purchased at market prices, financed through the levy of flat-rate excise taxes on the consumption goods. In this setting, Ramsey sought to characterize the excise tax pattern(s) that would maximize the utility of the consumer (or minimize the ‘welfare cost’ of taxation). Pigou (1947) and later Kydland and Prescott (1977), Barro (1979), Turnovsky and Brock (1980), and others noted that Ramseys formulation could be applied to the study of fiscal policy over time if the many goods being taxed were interpreted as dated deliveries of a single, aggregate consumption good. In this reinterpretation, the excise tax on ‘good t ’ is interpreted as the general level of taxes in period t . Several other authors like Bailey (1956), Friedman (1969), Phelps (1973), and Calvo (1978) developed the observation that one could apply the Ramsey formulation to the study of optimal monetary policy as well as fiscal policy by interpreting cash holdings as a second “good” and with the ‘inflation tax’ induced by a positive nominal interest rate playing the role as a tax

on cash holdings. In this thesis, we study both real and monetary Ramsey models and find that learning-by-doing matters for both optimal fiscal and monetary policy.

The real Ramsey model of chapter 2 reconsiders the optimal Ramsey taxation in light of the notion that firms accumulate organizational capital. The level of organizational capital in period $(t+1)$ depends on the levels of organizational capital and labor input employed in period t . An important policy recommendation emerges from this chapter: the optimal long-run (steady-state) capital income tax rate should be significantly positive even though firms operate in monopolistically competitive product markets. This finding is opposite to the finding of earlier standard Ramsey monetary models. Standard Ramsey models of optimal fiscal policy predict that in the long run capital income tax should be zero in a perfectly competitive economy and negative if the product market is imperfectly competitive (see Judd (2002), Atkeson, Chari, and Kehoe (1999), Chamley (1986), and Judd (1985)). The zero or negative capital income tax is based on the fact that capital is a stock while labor is a pure flow. A tax on labor income distorts only the static trade-off between consumption and leisure. However, a tax on capital income distorts the intertemporal trade-off between current and future consumption. In other words, taxes on stocks causes cumulative distortions over an infinite time period while taxes on a pure flow cause distortions only for a single period. Therefore, it is not optimal to tax a stock variable. The classic stock-flow distinction is not so obvious in our

model. Labor is not a stock per se, but it contributes to generating organizational capital - a pure stock- every period. Taxes on both labor income and capital income distort the intertemporal trade-off between current consumption and future consumptions. In our model, there is nothing very special about physical capital and it is optimal for the Ramsey planner to tax both sources of income to finance the exogenous spending.

In chapter 3 we study optimal fiscal and monetary policy in a standard monetary Ramsey model augmented with price stickiness and organizational learning-by-doing in the production technology. The main result is that contrary to the findings of other papers in this literature optimal Ramsey inflation is very stable and persistent over the business cycle. While a dynamic link between current production and future productivity is the key for the inflation persistence, the real cost of price adjustment is the key for the very low volatility in optimal inflation. Both of these mechanisms work through the firms' optimal pricing condition – namely the New Keynesian Philips Curve. Learning-by-doing influences inflation persistence by introducing a dynamic consideration in the firms' price setting decision. A current price change not only affects revenue and production today, it also affects knowledge accumulation, productivity, costs and hence profits in all future periods. This dynamic link between current production and future productivity induces the Ramsey planner to use the inflation in a more persistent manner. Although learning-by-

doing generates persistence in optimal inflation it can not reduce inflation volatility by itself. If prices are flexible, there is no real resource cost of price adjustment and the Ramsey planner still finds it optimal to use inflation as a lump-sum tax on households' financial wealth. But, if there is a price adjustment cost, as is the case in our model, the Ramsey planner faces another tradeoff. On one hand, the planner would like to use surprise inflation as a state contingent lump-sum tax or transfer on nominal wealth. In this way the planner avoids the need to use distortionary taxes over the business cycle. On the other hand, the planner has a strong incentive to stabilize inflation in order to minimize the price adjustment costs. In line with the literature, this tradeoff is overwhelmingly resolved in favor of inflation stability. A second important result is that optimal tax policy is counter-cyclical - tax rates fall during recession and rise during boom. This finding contrasts with pro-cyclical tax results obtained in standard sticky price Ramsey models. The basic intuition for the result is that in the presence of learning-by-doing, the Ramsey planner finds it relatively costly to raise taxes in response to a negative technology shock. Higher taxes would reduce hours, output, and hence organizational capital which will magnify the effects of the shock further by reducing future productivity. Therefore, the planner would optimally lower taxes to raise the after tax return to work and minimize the effects of the shock. Finally, inflation, nominal interest rate, and labor income tax rates are relatively lower in our model as compared to models without

learning-by-doing. This result is a direct consequence of a relatively lower markup generated by the presence of learning-by-doing in our model.

Ramsey outcomes are mute on the issue of what policy regimes can implement them. The information on policy one can extract from the solution to the Ramsey problem is limited to the equilibrium behavior of policy variables such as tax rates, the nominal interest rate, etc. as a function of the state of the economy. Therefore, in chapter 4 we modify the monetary Ramsey model of chapter 3 to address the issue of implementation of optimal policy by limiting attention to simple monetary and fiscal rules. These rules are defined over a small set of readily available macro indicators and are designed to ensure local uniqueness of the rational expectations equilibrium. We characterize welfare maximizing monetary and fiscal feedback rules and find that such policy rules can attain very similar levels of welfare as does the Ramsey optimal policy. The interest rate rule features a very strong response to inflation and a very weak response to output. The optimized fiscal rule is passive in the sense that tax revenues increase only mildly in response to increases in government liabilities. Also, the optimal interest rate rule is superinertial means that the monetary authority is forward looking. Finally, inflation is very stable under the optimal policy rule, a feature also characterized by the Ramsey policy.

Chapter 2

Learning-by-doing and Optimal Capital Income Taxation

2.1 Introduction

Optimal factor income taxation has been an important and interesting topic in the macro and public finance literature. Standard economic theory argues against any sort of capital income taxation. Using neoclassical growth models, Chamley (1985) and Judd (1986) established that capital income should not be taxed in the long run. Atkeson, Chari and Kehoe (1999) show that the Chamley-Judd result holds even after relaxing a number of crucial assumptions made by Chamley. Judd (2002) augments the standard growth model to allow for imperfectly competitive product

markets while Chugh (2007), Schmitt-Grohe and Uribe (2005) augment the standard model with a rich array of real and nominal rigidities in addition to a imperfectly competitive product market. All of these studies find that the optimal steady-state tax on capital income is negative.

This chapter reconsiders the optimal Ramsey taxation literature in light of the notion that firms accumulate organizational capital. The level of organizational capital in period $(t+1)$ depends on the levels of organizational capital and labor input employed in period t . Even though firms operate in monopolistically competitive product markets, we find that the optimal steady-state capital income tax rate is significantly positive¹.

Standard neoclassical models suggest a zero tax on capital income and a positive tax on labor income because of the fact that capital is a stock while labor is a pure flow². A tax on labor income distorts only the static trade-off between consumption and leisure. However, a tax on capital income distorts the intertemporal trade-off between current and future consumption. Atkeson et. al (1999) show that a

¹There are some other modelling choices that might invalidate the zero or negative capital income tax result in the long run. In life cycle models the optimal capital income tax can be different from zero if the tax code cannot explicitly be conditioned on the age of the household. See Alvarez, Burbidge, Farrell, and Palmer (1992), Erosa and Gervais (2002) for detail. Using Bewley (1986) class of models, S. Rao Aiyagari (1995) shows that if households face tight borrowing constraints and are subject to uninsurable idiosyncratic income risk, then the optimal tax system will in general include a positive capital income tax. As Aiyagari (1995) shows, Bewley-type models resemble an overlapping generations model with finite-lived agents.

²See Jones, Manuelli, and Rossi (1997) for details on this point.

constant capital income tax is equivalent to an increasing sequence of consumption taxes. In other words, taxes on stocks causes cumulative distortions over an infinite time period while taxes on a pure flow cause distortions only for a single period. Therefore, it is not optimal to tax a stock variable. Market power adds an additional distortion in the economy- a tendency towards an under-accumulation of capital. With monopoly power, as Judd (2002) shows, an optimal policy should promote efficiency along the capital accumulation margin. Providing a capital subsidy can boost capital accumulation and achieve this optimality goal.

The classic stock-flow distinction is not so obvious in our model. Labor is not a stock per se, but it is generating organizational capital - a pure stock- every period. Taxes on both labor income and capital income distort the intertemporal trade-off between current consumption and future consumptions. Taxes on labor income affect labor supply in the current period. Current hours, however, are contributing to the accumulation of organizational capital next period which affects the marginal productivity and labor income in all future periods. With a learning-by-doing mechanism, market power not only induces under-accumulation of physical capital, it also induces under-accumulation of organizational capital. Therefore, there is nothing very special about physical capital in our model.

Using a representative agent dynamic general equilibrium model Jones, Manuelli, and Rossi (1997) show that the optimality of zero capital income also applies to labor

income in a model with human capital. This result holds so long as the technology for accumulating human capital displays constant returns to scale in the stock of human capital and goods used (not including raw labor). This chapter complements their work in a number of ways. First, we introduce imperfect competition which is a key feature of modern dynamic economies. Second, we model organizational capital which consists of the human capital of the firm's employees. As Atkeson et. al (2005) find, this is an important factor of production and the payments to owners of organizational capital are 37 percent of the net payments to owners of physical capital in the US economy. Third, we solve the Ramsey problem using a timeless perspective solution algorithm. The usual Ramsey equilibrium concept suffers from the time inconsistency problem but Woodford's (2003) timeless perspective solution does not. The difference between the usual Ramsey equilibrium concept and the timeless perspective concept is that the structure of the optimality conditions associated with the Ramsey equilibrium is time invariant. In choosing optimal policy the government is assumed to honor commitments made in the past. By contrast, under standard Ramsey equilibrium definition, the equilibrium conditions in the initial periods are different from those applying to later periods.

The remainder of the chapter is organized as follows. Section 2.2 describes the model while section 2.3 discusses about parameterizations and computation technique. Section 2.4 presents numerical solution results and section 2.5 concludes.

2.2 The model

The model economy involves households, firms, and the government. The structure of the economy is a standard growth model augmented with three frictions - monopolistic competition in the product market, learning-by-doing in the technological environment, and distortionary taxation. The firms possess a degree of monopoly power and hence, can earn positive economic profits. As owners of all the firms, households receive profits as dividends. However, the crucial feature of the model economy that serves as the basis of our results is the introduction of firm-level learning-by-doing effects in the production technology.

2.2.1 Households

We suppose that the economy is populated by a continuum of identical, infinitely lived households. The households' preferences are defined over consumption, c_t , and labor effort, n_t and are described by the standard time separable utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t), \quad (2.1)$$

where $\beta \in (0, 1)$ represents a subjective discount factor, E_t denotes the mathematical expectations operator conditional on information available at the beginning of period t , c_t is consumption and n_t is hours worked in period t .

The representative household faces the following period-by-period budget constraint:

$$c_t + i_t + b_t \leq (1 - \tau_t^n)w_t n_t + (1 - \tau_t^k)r_t k_t + b_{t-1}R_{t-1} + \pi_t, \quad (2.2)$$

where i_t denotes investment, b_t represents one-period real government bonds carried into period $t + 1$, k_t denotes capital. Households derive income by supplying labor and capital services to firms at rates w_t and r_t , earning interest on their government bond holdings, and, as owners of the firms, receive profits π_t in the form of dividends. τ_t^n and τ_t^k are the tax rates imposed on labor and capital income, respectively. The capital stock depreciates at the rate δ , so it evolves according to

$$k_{t+1} = (1 - \delta)k_t + i_t, \quad (2.3)$$

We normalize the number of total hours available to households to 1. That is,

$$n_t + l_t \leq 1, \quad (2.4)$$

where, l_t denotes leisure.

Households are also constrained by the transversality conditions that prevent them from engaging in Ponzi schemes. A representative household's problem is to maximize the utility function (2.1) subject to (2.2), (2.3), (2.4) and the no-Ponzi-game borrowing limit. The household first-order conditions and the associated transversality conditions are:

$$c_t : \quad U_{ct} = \lambda_t, \quad (2.5)$$

$$n_t : \quad -U_{nt} = \lambda_t(1 - \tau_t^n)w_t, \quad (2.6)$$

$$k_{t+1} : \quad \lambda_t = \beta E_t \lambda_{t+1} [(1 - \tau_{t+1}^k)r_{t+1} + 1 - \delta], \quad (2.7)$$

$$b_t : \quad \lambda_t = \beta E_t \lambda_{t+1} R_t, \quad (2.8)$$

$$tvc : \quad \lim_{t \rightarrow \infty} \beta^t \lambda_t k_{t+1} = 0, \quad (2.9)$$

$$tvc : \quad \lim_{t \rightarrow \infty} \beta^t \lambda_t b_{t+1} = 0, \quad (2.10)$$

where λ_t is the Lagrange multiplier associated with the household budget constraint (2.2). Here U_{ct} and U_{nt} are the partial derivatives of $U(c_t, n_t)$ with respect to c_t and n_t . The interpretation of these first order conditions is quite standard. Equation (2.7) is the consumption-savings optimality condition. It states that marginal rates of substitution between present and future consumption equals after-tax return on savings. Equation (2.7) implies that capital income tax creates a dynamic distortion in the consumption-savings margin. Equation (2.8) determines optimal bond holdings. Equations (2.7) and (2.8) imply that after-tax returns on capital and bonds to be equalized each period. Combining (2.5) and (2.6) gives

$$\frac{U_{lt}}{U_c(t)} = (1 - \tau_t^n)w_t \quad (2.11)$$

Eq. (2.11) gives the optimal labor-leisure choice. It states that marginal rate of substitution between consumption and leisure equals the after-tax wage. Clearly,

the labor income tax rate distorts the consumption-leisure margin. Given the wage rate, households will tend to work less and consume less the higher is τ^n . This distortion is purely static in a standard monopolistically competitive model. But, as will be clear in the next section, in our model the labor income tax also creates a dynamic distortion.

2.2.2 The Government

The government faces an exogenous stream of real expenditures that it must finance through the labor income tax, the capital income tax, and the issuance of real risk-free one-period debt. Its period-by-period budget constraint is given by

$$g_t + R_{t-1}b_{t-1} = b_t + \tau_t^n w_t n_t + \tau_t^k r_t k_t \quad (2.12)$$

R_t denotes the gross one-period, risk-free, real interest rate in period t . g_t denotes per capita government spending on the final good.

2.2.3 Production

The production side of the economy features two sectors: an intermediate goods sector that produces differentiated goods using labor, physical capital and organizational capital, and a final goods sector that uses intermediate goods to produce a unique final good.

Final Goods Producers

Government consumption goods, private consumption goods and investments are physically indistinguishable. There are a large number of producers who produce this unique final good in a perfectly-competitive environment. Final goods producers require only the differentiated intermediate goods as inputs and use the following CES technology for converting intermediate goods into final goods.

$$y_t = \left[\int_0^1 y_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \quad (2.13)$$

where $\eta > 1$ denotes the intratemporal elasticity of substitution across different varieties of intermediate goods, and differentiated intermediate goods are indexed by $i \in [0, 1]$.

Each period final goods firms choose inputs y_{it} for all $i \in [0, 1]$ and output y_t to maximize profits given by

$$y_t - \int_0^1 p_{it} y_{it} di \quad (2.14)$$

subject to (2.13) where p_{it} is the relative price of the i th intermediate good³. The solution to this problem gives us the input demand functions:

$$y_{it} = p_{it}^{-\eta} (y_t). \quad (2.15)$$

³We normalize the final good's price, p_t , to 1

The zero profit condition can be used to infer the relationship between the final good price and the intermediate goods prices:

$$p_t (= 1) \equiv \left[\int_0^1 p_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \quad (2.16)$$

Intermediate Goods Producers

There are a large number of intermediate goods producers, indexed by the letter i who operate in a Dixit-Stiglitz style imperfectly competitive economy. Each of these firms produces a single variety i using three factor inputs - physical capital, k_{it} , organizational capital, h_{it} , and labor services, n_{it} . The production technology facing each firm is given by

$$y_{it} = F(k_{it}, h_{it}, n_{it}),$$

where y_{it} is the intermediate good variety produced by firm i . The function F is assumed to be concave, and strictly increasing in all three arguments. Following Cooper and Johri (2002), we shall assume the following specific functional form for the production technology.

$$y_{it} = z_t k_{it}^\alpha n_{it}^{1-\alpha} h_{it}^\theta \quad (2.17)$$

The technology differs from a standard neo-classical production function because the firm carries a stock of organizational capital which is an input in the production technology. Organizational capital refers to the information accumulated by the

firm, through the process of past production, regarding how best to organize its production activities and deploy its inputs. As a result, the higher the level of organizational capital, the more productive the firm is.⁴ Learning-by-doing leads to the accumulation of organizational capital and as in Chang, Gomes and Schorfheide (2002), learning depends upon the labor input used by the firm⁵.

We assume that organizational capital in the current period depends on the level of labor employment and the stock of organizational capital in the previous period:

$$h_{i,t+1} = (1 - \delta^h)h_{it} + h_{it}^\gamma n_{it}^{1-\gamma}, \quad (2.18)$$

where δ^h is the depreciation rate of organizational capital and $0 < \delta^h, \gamma < 1$. All producers begin life with a positive and identical endowment of organizational capital. The restriction $0 < \delta^h < 1$ is consistent with the empirical evidence supporting the hypothesis of organizational forgetting. Argote, Beckman, and Epple (1990) provide empirical evidence for this hypothesis of organizational forgetting associated with the construction of Liberty Ships during World War II. Similarly, Darr, Argote, and Epple (1995) provide evidence for this hypothesis for pizza franchises

⁴Atkeson and Kehoe (2005) model and estimate the size of organizational capital for the US manufacturing sector and find that it has a value of roughly 66 percent of physical capital.

⁵Prescott et. al (1980) also define firm-specific human capital as Organization capital - "The capacity of the organization to function effectively as a production unit is determined largely by the level and meshing of the skills of the employees. Employee skills are our final example of organization capital. The case for the human capital of employees being a part of the capital stock of the firm is well established. Productivity in the future depends on levels of human capital in the future.".

and Benkard (2000) provides evidence for organizational forgetting associated with the production of commercial aircraft.

While learning-by-doing is often associated with workers and modeled as the accumulation of human capital, a number of economists have argued that firms are also store-houses of knowledge. Atkeson & Kehoe (2005) note “At least as far back as Marshall (1930, bk.iv, chap. 13.I), economists have argued that organizations store and accumulate knowledge that affects their technology of production. This accumulated knowledge is a type of unmeasured capital distinct from the concepts of physical or human capital in the standard growth model”. Similarly Lev and Radhakrishnan (2005) write, “Organization capital is thus an agglomeration of technologies, business practices, processes and designs, including incentive and compensation systems that enable some firms to consistently extract out of a given level of resources a higher level of product and at lower cost than other firms”. There are at least two ways to think about what constitutes organizational capital. Some, like Rosen (1972), think of it as a firm specific capital good while others focus on specific knowledge embodied in the matches between workers and tasks within the firm. In modeling organizational capital we follow the second line of thinking.

We assume that the firm must satisfy demand at the posted price. That is, we impose

$$y_{it} \geq (p_{it})^{-\eta}(y_t) \tag{2.19}$$

The decision problem of the firm is to choose the plans for n_{it} , k_{it} , h_{it+1} , and p_{it} so as to maximize discounted life-time profits⁶:

$$\sum_{t=0}^{\infty} Q_t \{p_{it}y_{it} - w_t n_{it} - r_t k_{it}\}$$

subject to (2.17), (2.18), and (2.19), where Q_t is the appropriate discount factor to use to price revenue and costs in adjoining periods which is determined in the household problem⁷.

The first-order conditions associated with the firm's problem are then:

$$n_{it} : \quad w_t = mc_{it}(1 - \alpha) \frac{y_{it}}{n_{it}} + \Psi_{it}(1 - \gamma)n_{it}^{-\gamma}h_{it}^{\gamma} \quad (2.20)$$

$$k_{it} : \quad r_t = mc_{it}\alpha \frac{y_{it}}{k_{it}} \quad (2.21)$$

$$h_{i,t+1} : \quad \Psi_{it} = Q_{t+1}E_t \left[mc_{i,t+1}\theta \frac{y_{i,t+1}}{h_{i,t+1}} + \Psi_{i,t+1} \left\{ (1 - \delta^h) + \gamma h_{i,t+1}^{\gamma-1} n_{i,t+1}^{1-\gamma} \right\} \right] \quad (2.22)$$

$$p_{it} : \quad mc_{it} = \frac{\eta - 1}{\eta} p_{it}, \quad (2.23)$$

where Ψ_{it} and mc_{it} are the Lagrange multipliers associated with the organizational

⁶All input payments are assumed to be made in units of the final good.

⁷Combining (2.5) and (2.8) we get the pricing formula for a one-period risk-free real bond $1 = R_t \frac{\beta u_{c,t+1}}{u_{ct}}$, which implies the following real pricing kernel between period t and $t + 1$:

$$Q_{t+1} = \frac{\beta u_{c,t+1}}{u_{ct}}$$

Consumers discount factor is appropriate to discount period $t + 1$ profit because they own all intermediate firms and thus receive all the profits.

capital accumulation equation and production function respectively. Equations (2.21) and (2.23) are standard. Equation (2.22) determines the optimal use of organizational capital by the firm. One additional unit of organizational capital has a (marginal) value, in terms of profits, of Ψ_{it} to the producer in the current period. The right hand side of (2.22) measures the value of having available an additional unit of organizational capital for use by the firm in the following period. First, the additional organizational capital directly contributes to the production in the following period as captured by the first term on the right hand side. Second, the additional organizational capital today has a positive effect on the future stock of organizational capital which is captured by the two terms inside the curly bracket. First term is the un-depreciated additional stock and the second term is the new organizational capital stock generated by this additional stock. This higher stock of organizational capital has a value of $\Psi_{i,t+1}$ to the producer. All this must be discounted by the factor Q_{t+1} . The condition (2.22) implies that organizational capital will be accumulated up to the point where the value of an additional unit of organizational capital today is equal to the discounted value of this organizational capital next period. Firm's labor demand function is quite different in our model.

Combining (2.20) and (2.22) we get:

$$w_t = mc_{it}(1 - \alpha) \frac{y_{it}}{n_{it}} + Q_{t+1} \left[mc_{i,t+1} \theta \frac{y_{i,t+1}}{h_{i,t+1}} + \Psi_{i,t+1} \left\{ (1 - \delta^h) + \gamma h_{i,t+1}^{\gamma-1} n_{i,t+1}^{1-\gamma} \right\} \right] \times (1 - \gamma) n_{it}^{-\gamma} h_{it}^{\gamma} \quad (2.24)$$

The second term on the right hand side of (2.24) does not appear in the standard model of monopolistic competition. In the standard model, a firm's labor hiring decision is solely based on the marginal product of labor in the current period. But in our model, in addition to that basic contribution firms also take into account the positive effect of an additional unit of labor in accumulating organizational capital in the following period. One additional unit of labor can generate $(1 - \gamma) n_{it}^{-\gamma} h_{it}^{\gamma}$ units of organizational capital in the following period. Each of these additional units of organizational capital has a value of Ψ_{it} to the firm. So, the right hand side of (2.24) gives the total marginal benefit of having available an additional unit of labor input.

We restrict our attention to a symmetric equilibrium in which all firms make the same decisions. We thus drop all the subscripts i . That is, in equilibrium $y_{it} = y_t$, $c_{it} = c_t$, $p_{it} = p_t = 1$, $k_{it} = k_t$, $n_{it} = n_t$, $h_{it} = h_t$ and the aggregate production technology and organizational capital accumulation are given by

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} h_t^{\theta} \quad (2.25)$$

$$h_{t+1} = (1 - \delta^h) h_t + h_t^{\gamma} n_t^{1-\gamma} \quad (2.26)$$

We can also aggregate the firm's optimality conditions, equations (2.20)- (2.23), as

$$w_t = mc_t(1 - \alpha)\frac{y_t}{n_t} + \Psi_t(1 - \gamma)n_t^{-\gamma}h_t^\gamma \quad (2.27)$$

$$r_t = mc_t\alpha\frac{y_t}{k_t} \quad (2.28)$$

$$\Psi_t = Q_{t+1} \left[mc_{t+1}\theta\frac{y_{t+1}}{h_{t+1}} + \Psi_{t+1} \{(1 - \delta^h) + \gamma h_{t+1}^{\gamma-1} n_{t+1}^{1-\gamma}\} \right] \quad (2.29)$$

$$mc_t = \frac{\eta - 1}{\eta} \quad (2.30)$$

2.2.4 Equilibrium

In the presence of government policy there are many competitive symmetric equilibria, indexed by different government policies. This multiplicity motivates the Ramsey problem. In our model competitive and Ramsey equilibria are defined as follows:

Competitive Equilibrium

A competitive equilibrium is a set of plans $\{ c_t, n_t, k_{t+1}, h_{t+1}, i_t, w_t, r_t, b_t, mc_t, \lambda_t, \Psi_t, \text{ and } R_t \}$, such that the household maximizes expected lifetime utility taking as given prices and policies; the firms maximize profit taking as given the wage rate, capital rental rate, and the demand function; the labor market clears, the capital market clears, the bond market clears, the government budget constraint and the aggregate resource constraint are satisfied. In other words, all the processes above

satisfy conditions (2.3), (2.5)-(2.10), (2.12), (2.25)-(2.30) and the aggregate resource constraint

$$c_t + g_t + i_t = z_t k_t^\alpha n_t^{1-\alpha} h_t^0 \quad (2.31)$$

given policies $\{\tau_t^n, \tau_t^k\}$, exogenous processes $\{z_t, g_t\}$, and the initial conditions k_{-1}, h_{-1}, z_0, g_0 .

The Ramsey Equilibrium

The Ramsey equilibrium is the unique competitive equilibrium that maximizes the household's expected lifetime utility. Following Schmitt-Grohé and Uribe (2007), we assume that the benevolent Ramsey planner has been operating for an infinite number of periods and it honors the commitments made in the past. This form of policy commitment is known as 'optimal from the timeless perspective' (Woodford, 2003). In more technical terms, the Ramsey Equilibrium is defined as a set of stationary processes $c_t, n_t, k_{t+1}, h_{t+1}, i_t, w_t, r_t, \tau_t^n, \tau_t^k, b_t, mc_t, \lambda_t, \Psi_t$ for $t \geq 0$ that maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$$

subject to the conditions (2.3), (2.5)-(2.10), (2.12), (2.25)-(2.30) and (2.31), for $t > -\infty$, given exogenous processes g_t and z_t , values of all the variables dated

$t < 0$, the values of the Lagrange multipliers associated with the constraints listed above dated $t < 0$. Under traditional Ramsey equilibrium concept, the equilibrium conditions in the initial periods are different from those applied to later periods. But under Woodford's timeless definition, the optimality conditions associated with Ramsey equilibrium are time invariant.

2.3 Parameterization and Solution Method

The time unit in our model is one quarter. We set $\beta = .9902$ so that the discount rate is 4 percent (Prescott (1986)) per year. We assume that the period utility function takes the following GHH⁸ specification

$$U(c_t, n_t) = \frac{[c_t - \rho n_t^v]^{1-\sigma}}{1-\sigma}$$

The value of the coefficient of relative risk aversion parameter, σ , ranges from 1 to 2 in the literature. We used a value of 1.2 for σ . Given this preference the labor supply elasticity is $\frac{1}{v-1}$. We set $v = 1.4$ so that the labor supply elasticity is 2.5. The value for ρ is set so that the steady state labor supply is 0.2. Table 2.1 presents the structural parameters used in the baseline model.

We assign a value of 0.3 to the cost share of capital, α . This is consistent with the empirical regularity that in the developed countries wages represent about 70

⁸These preferences have been introduced in the macro literature by Greenwood, Hercowitz, and Huffman (1988) and widely used by many authors thereafter.

TABLE 2.1

BASELINE PARAMETER VALUES

Parameters	Value	Description
β	.9902	Subjective discount rate
σ	1.2	Coefficient of relative risk aversion
ν	1.4	Labor supply elasticity parameter
ρ	4.71	Preference parameter- ties the steady state labor supply to 0.2
α	0.3	Share of capital in the production technology
δ	0.02	Depreciation of physical capital
θ	0.16	Elasticity of output with respect to organizational capital
δ^h	0.02	Depreciation rate of organizational capital
γ	0.55	Org. capital production parameter, $h_{t+1} = (1 - \delta^h)h_t + h_t^\gamma n_t^{1-\gamma}$

percent of total cost. Following Johri (2009), we set $\theta = 0.16$ and $\delta = .02$. This value of θ corresponds to a “learning rate” of just under twelve percent and is taken from production function estimates for US manufacturing industries provided in Cooper and Johri (2002). To maintain symmetry with the physical capital we set δ^h equal to .02⁹. Following Johri (2009) we set γ equal to 0.55. We conduct a detailed sensitivity analysis with respect to the values of all learning-by-doing parameters.

We characterize the Ramsey steady-state numerically using the methodology outlined in Schmitt-Grohé, and Uribe (2005). Their publicly available numerical tools allow the computation of Ramsey policy in a general class of stochastic dynamic general equilibrium models.

⁹McGrattan & Prescott (2005) use an estimate of 11% (annual rate) for the depreciation rate of intangible capital stocks which is approximately equivalent to our quarterly value.

2.4 Results

We consider the long run Ramsey equilibrium without any uncertainty. After obtaining the dynamic first-order conditions of the Ramsey problem, we impose the steady state and numerically solve the resulting non-linear system using the Schmitt-Grohé, and Uribe (2005) algorithm. This gives us the exact numerical solution of the Ramsey problem.

2.4.1 Optimal Taxes in Learning-by-doing model

In a standard neoclassical model taxing capital income is bad because it distorts the intertemporal trade-off between current and future consumption while labor income tax distorts the static trade-off between consumption and leisure. A tax on capital income reduces the return to saving and thus affects future consumption. But, labor income taxation does not have any effect on future consumption. If the labor income tax is high households tend to work less and consume less in the current period. There is no effect of this distortion in the future periods.

In our model, however, labor income taxes create a wedge in the labor-leisure margin which is no longer static. By working less, households not only sacrifice current consumption they also sacrifice future consumption. Less work this period means less organizational capital accumulated next period. This in turn implies less

labor productivity and less consumption in all future periods. Since both the labor income tax and the capital income tax distort the intertemporal trade-off between current and future consumption, optimal rates for both are positive in our model. The relative magnitude of the two tax rates depends on how strongly the labor supply affects the accumulation of organizational capital. The long run relationship between labor supply and the stock of organizational capital is controlled by the value of three learning-by-doing parameters- δ^h , γ and θ . The following three tables present optimal tax rates with various values of learning-by-doing parameters.

Table 2.2 presents the optimal tax rates for various degrees of depreciation rate of organizational capital. Column 2 shows the optimal tax rates under the baseline parameter values. The next three columns display the optimal taxes with three different values of δ^h . Although the optimal capital income tax rate is positive in all cases it is decreasing in the value of δ^h . The reason this occurs is that the

TABLE 2.2

OPTIMAL TAX RATES FOR VARIOUS VALUES OF δ^h

Optimal Tax rates	$\delta^h = .02$	$\delta^h = .05$	$\delta^h = .1$	$\delta^h = .15$
τ^k	0.2309	0.1695	0.0834	0.0521
τ^n	0.2607	0.2722	0.2967	0.3050

higher the δ^h the less is the effect of a change in labor supply on the long run stock of organizational capital. Using (2.18) we obtain the steady state stock of

organizational capital as

$$h^{ss} = \frac{n^{ss}}{\delta h^{\frac{1}{1-\gamma}}} \quad (2.32)$$

where h^{ss} and n^{ss} are steady state levels of organizational capital stock and labor input respectively. Equation (2.32) gives

$$\frac{dh^{ss}}{dn^{ss}} = \frac{1}{\delta h^{\frac{1}{1-\gamma}}} \quad (2.33)$$

Equation (2.33) implies that the higher the value of δ^h the lower is the effect of a given change in labor input on the steady state stock of organizational capital. Consequently, a tax on labor income generates a relatively lower dynamic distortion. As a result, the optimal tax scheme calls for a relatively higher tax rate on labor income and lower tax rate on capital income. Table 2.3 presents the optimal tax rates for various values of γ . The capital income tax rate is increasing in the value

TABLE 2.3

OPTIMAL TAX RATES FOR VARIOUS VALUES OF γ

Optimal Tax rates	$\gamma = .45$	$\gamma = .5$	$\gamma = .55$	$\gamma = .6$	$\gamma = .65$
τ^k	0.2037	0.2200	0.2309	0.2376	0.2449
τ^n	0.2684	0.2635	0.2607	0.2597	0.2586

of γ . Again, equation (2.33) provides the intuition. The bigger the value of γ , the larger is the effect of a given change of labor input on the steady state stock of organizational capital. Consequently, a given labor income tax generates relatively

higher distortion of future consumption, making it optimal to lower the labor income tax a bit and raise the capital income tax a bit.

Finally, table 2.4 displays the optimal tax rates for various values of θ . The optimal tax rate on capital income is decreasing in the value of θ . As equation (2.32) shows, the steady state stock of organizational capital does not depend on θ . Therefore, the dynamic distortion caused by the labor income tax is not affected by the value of θ . However, the steady state stock of physical capital depends positively

TABLE 2.4
OPTIMAL TAX RATES FOR VARIOUS VALUES OF θ

Optimal Tax rates	$\theta = .14$	$\theta = .15$	$\theta = .16$	$\theta = .17$	$\theta = .18$
τ^k	0.2482	0.2396	0.2309	0.2219	0.2122
τ^n	0.2572	0.2590	0.2607	0.2626	0.2647

on the value of θ . Combining (2.7), (2.28), and (2.32) we can derive the following expression for the steady state stock of physical capital

$$k^{ss} = \left[\frac{\eta - 1}{\eta} \frac{\alpha(1 - \tau^k)}{\beta^{-1} - 1 + \delta} \delta h^{\frac{1-\alpha}{1-\gamma}} h^{ss(1-\alpha+\theta)} \right]^{\frac{1}{1-\alpha}} \quad (2.34)$$

Note that given the value of h^{ss} the higher the value of θ the higher the intertemporal distortions generated by a given amount of capital income tax. Hence, it is optimal for the Ramsey planner to lower the capital income tax when the value of θ is relatively high.

2.4.2 Optimal taxation without Learning-by-doing

We claim that the positive capital income tax result is solely generated by the learning-by-doing mechanism in our model. To defend our claim we also check whether the increasing returns in the production technology or the inability to tax only one form of capital stock can generate the same results. For this purpose we solve the Ramsey problem for two related models. In the first model we retain the assumptions of increasing returns to scale in the production technology and imperfect competition in the product market but firms are no longer allowed benefit from accumulating organizational capital. In the second model we have two capital stocks, but only one of which can be taxed. There are increasing returns to scale and imperfect competition but the learning-by-doing mechanism also does not exist.

An Economy with Increasing Returns but without Learning-by-doing

In this economy the representative household, the representative final good firm, and the government have exactly the same problems as described in section (2). However, the representative intermediate good firm's problem is different. The Learning-by-doing mechanism does not exist and hence the firm faces a static decision making problem. The production technology is assumed to be

$$y_{it} = z_t k_{it}^\alpha n_{it}^{1.16-\alpha}. \quad (2.35)$$

We use $(1.16-\alpha)$ as the labor share so that this model and the Learning-by-doing model have the same increasing returns in production technology. The representative firm's problem is to maximize profit given by

$$p_{it}y_{it} - w_t n_{it} - r_t k_{it} \quad (2.36)$$

subject to (2.35) and (2.19). The first order conditions associated with this problem are then:

$$n_{it} : \quad w_t = mc_{it}(1.16 - \alpha) \frac{y_{it}}{n_{it}} \quad (2.37)$$

$$k_{it} : \quad r_t = mc_{it} \alpha \frac{y_{it}}{k_{it}} \quad (2.38)$$

$$p_{it} : \quad mc_{it} = \frac{\eta - 1}{\eta} p_{it}, \quad (2.39)$$

We impose symmetry in the production sector and solve the Ramsey problem for this economy using the same baseline parameter values described in section 2.3. The resulting solution gives us the following optimal tax rates:

$$\tau^k = -0.1260, \quad \tau^n = 0.3332$$

This is the standard capital subsidy result in presence of imperfectly competitive product market. Capital income tax distorts the intertemporal trade-off between present consumption and future consumption but the labor income tax does not. In addition, with market power, there is a tendency towards an under-accumulation of capital, so providing a subsidy boosts the capital stock.

An Economy with Two Physical Capital Stocks

There are two physical capitals in this economy. The government, however, can tax only the incomes from one capital, and labor input. The income from the other capital service can't be taxed. The production technology is described by

$$y_{it} = z_t k_{1it}^\alpha n_{it}^{1-\alpha} k_{2it}^\theta \quad (2.40)$$

where k_{jit} is the physical capital of type j . The representative intermediate-goods firm's problem is to choose the input levels and the price of its product to maximize profit

$$p_{it}y_{it} - w_t n_{it} - r_{1t}k_{1it} - r_{2t}k_{2it} \quad (2.41)$$

subject to (2.19) and (2.40) where r_{jt} is the rental rate for type j capital. The first order conditions associated with the firms problem are:

$$n_{it} : \quad w_t = mc_{it}(1 - \alpha) \frac{y_{it}}{n_{it}} \quad (2.42)$$

$$k_{1it} : \quad r_{1t} = mc_{it} \alpha \frac{y_{it}}{k_{1it}} \quad (2.43)$$

$$k_{2it} : \quad r_{2t} = mc_{it} \theta \frac{y_{it}}{k_{2it}} \quad (2.44)$$

$$p_{it} : \quad mc_{it} = \frac{\eta - 1}{\eta} p_{it}, \quad (2.45)$$

The representative household's budget constraint is now given by

$$c_t + i_{1t} + i_{2t} + b_t \leq (1 - \tau_t^n)w_t n_t + (1 - \tau_t^k)r_{1t}k_{1t} + r_{2t}k_{2t} + b_{t-1}R_{t-1} + \pi_t, \quad (2.46)$$

where i_{jt} is the investment on type j capital. The two capital stocks evolve according to

$$k_{1,t+1} = (1 - \delta_1)k_{1t} + i_{1t} \quad (2.47)$$

$$k_{2,t+1} = (1 - \delta_2)k_{2t} + i_{2t}, \quad (2.48)$$

where δ_j is the depreciation rate of type j capital.

The representative household's problem is to maximize (2.1), subject to (2.46), (2.47), and (2.48). The first order conditions associated with the household's problem are

$$c_t : \quad U_{ct} = \lambda_t, \quad (2.49)$$

$$n_t : \quad -U_{nt} = \lambda_t(1 - \tau_t^n)w_t, \quad (2.50)$$

$$k_{1,t+1} : \quad \lambda_t = \beta\lambda_{t+1} [(1 - \tau_{t+1}^k)r_{1,t+1} + 1 - \delta_1], \quad (2.51)$$

$$k_{2,t+1} : \quad \lambda_t = \beta\lambda_{t+1} [r_{2,t+1} + 1 - \delta_2], \quad (2.52)$$

$$b_t : \quad \lambda_t = \beta\lambda_{t+1}R_t, \quad (2.53)$$

$$tvc : \quad \lim_{t \rightarrow \infty} \beta^t \lambda_t k_{1,t+1} = 0, \quad (2.54)$$

$$tvc : \quad \lim_{t \rightarrow \infty} \beta^t \lambda_t k_{2,t+1} = 0, \quad (2.55)$$

$$tvc : \quad \lim_{t \rightarrow \infty} \beta^t \lambda_t b_{t+1} = 0, \quad (2.56)$$

The aggregate resource constraint of the economy is given by

$$c_t + g_t + i_{1t} + i_{2t} = z_t k_t^\alpha n_t^{1-\alpha} k_{2t}^\theta. \quad (2.57)$$

We solve the Ramsey problem numerically using the same set of baseline parameter values as we did in section (3). From the resulting solution we obtain the following optimal tax rates:

$$\tau^k = -0.1284, \quad \tau^n = 0.4432$$

The two capital model is, therefore, not able to deliver the positive capital income tax result. Again, the main reason lies in the fact that the labor income tax does not distort the intertemporal trade-off between current and future consumptions while capital income tax does.

2.5 Conclusion

We introduce Learning-by-doing and imperfect competition in the product market into an otherwise standard infinite horizon dynamic general equilibrium model. The numerical solution of the associated Ramsey problem characterizes the optimal capital and labor income tax. While the introduction of only monopoly power calls for a capital income subsidy, the Learning-by-doing model generates a positive tax on capital income. In a standard model without learning-by-doing the labor income tax distorts only the static trade-off between consumption and leisure while the capital income tax distorts the intertemporal trade-off between current consumption and future consumption. Therefore, the labor income tax is clearly a better choice as far

as optimality is concerned. In our model, however, both capital and labor income tax distort the dynamic trade-off between current consumption and future consumption. Consequently, it is optimal for the Ramsey planner to tax both capital income and labor income. The relative magnitudes of the tax rates depend crucially on the values of the learning-by-doing parameters. In the next chapter we introduce price rigidities and organizational learning mechanism in a monetary Ramsey model and primarily study the dynamic properties of key fiscal and monetary policy variables.

Chapter 3

Organizational Learning and Optimal Fiscal and Monetary Policy

3.1 Introduction

Ramsey models featuring flexible-price environments find that optimal inflation is highly volatile and serially uncorrelated (see Chari, Christiano, and Kehoe (1991); Calvo and Guidotti (1993); Chari and Kehoe (1999)). The government has nominal, non-state-contingent liabilities outstanding and, under the Ramsey plan, it uses surprise inflation as a lump-sum tax on financial wealth. Essentially, inflation plays

the role of a shock absorber of unexpected innovations in the fiscal deficit. Similarly, in Ramsey models with nominal rigidities optimal inflation is still characterized by very little persistence but is very stable in such environments (see Schmitt-Grohé and Uribe, 2004b; Siu, 2004). The fact that very little or no inflation persistence emerges with optimal Ramsey inflation in the literature motivated Chugh (2007) to answer the question originally raised by Chari and Kehoe (1999)- whether there are any general equilibrium settings which can rationalize inflation persistence as part of the Ramsey policy. Chugh (2007) introduces capital and habit persistence in preferences in a otherwise standard flexible-price Ramsey model and finds that optimal inflation is substantially persistent and highly volatile - even more volatile than the standard flexible-price Ramsey models would suggest.

As the above discussion demonstrates it has proven difficult to find Ramsey models where optimal inflation is both persistent and stable. The main contribution of this chapter is to address this issue by proposing a Ramsey model where optimal inflation has these two properties. In particular, we extend a standard cash-credit good monetary Ramsey model by adding price stickiness and organizational learning-by-doing (LBD) mechanism in the production technology. By delivering a crucial result – optimal inflation is characterized by substantial persistence and very low volatility – our model fills an important gap in the Ramsey literature.

The basic mechanism regarding organizational learning and knowledge accumu-

lation is that organizations learn from the production process and accumulate this firm-specific knowledge — known as organizational capital — that raises productivity^{1,2}. One critical feature of this knowledge is that it is produced jointly with output and embodied in the organization itself. To model organizational learning and knowledge accumulation we follow Cooper and Johri (2002) and introduce a firm-level learning-by-doing effect into the production technology³. In particular, production in any period by a firm leads to the accumulation of organizational capital by the firm. This causes increases not only in productivity in the next period but also in the stock of organizational capital in all future periods. To introduce price stickiness we follow Rotemberg (1982) and assume that firms incur quadratic costs in adjusting their nominal prices.

Our result of stable and persistence Ramsey inflation depends on both learning-by-doing and the price stickiness. While learning-by-doing mainly generates the persistence in optimal inflation, price rigidity generates the stability in it. Both of

¹Atkeson and Kehoe (2005) note "... At least as far back as Marshall (1930, bk. iv, chap. 13.I), economists have argued that organizations store and accumulate knowledge that affects their technology of production. This accumulated knowledge is a type of unmeasured capital distinct from the concepts of physical or human capital in the standard growth model...."

²Organizational learning and knowledge accumulation has long been considered significant too (see Atkeson and Kehoe, 2005; Cooper and Johri, 2002; Prescott and Visscher, 1980; Rosen, 1972 and many others). In particular, Atkeson and Kehoe (2005) model and estimate the size of organizational capital for the US manufacturing sector and find that it has a value of roughly 66 percent of physical capital.

³This particular theme of modeling organizational capital has a long tradition. Rosen (1972), Ericson and Pakes (1995), Atkeson and Kehoe (2005) and many others have developed models in which organization capital is acquired by endogenous learning by doing.

these mechanisms work through the intermediate firms' optimal pricing condition – namely the New Keynesian Philips Curve. Learning-by-doing influences inflation persistence by introducing a dynamic consideration in the firms' price setting decision. A current price change not only affects revenue and production today, it also affects knowledge accumulation, productivity, costs and hence profits in all future periods. This dynamic link between price changes and future productivity induces the Ramsey planner to use the inflation in a more persistent manner. To make it more intuitive, suppose there is an inflation defined by a price increase this period. Firms now have to cut production to match the lower demands. Lower output production this period causes lower accumulation of production knowledge which raises tomorrow's costs by lowering productivity. Facing higher marginal costs in the next period, the firms set higher prices (which causes inflation again) in the next period as compared to environments without learning-by-doing. By parallel arguments, lower prices (deflation) this period will induce firms to set relatively lower prices in the next period as well. In Chugh (2007) persistence in optimal inflation is generated through a very different mechanism. His result depends on consumption-smoothing. With capital and habit, the ability to and the preference for consumption-smoothing is enhanced significantly. This generates a persistent real interest rate which implies persistent inflation through the Fisher relationship.

Although, learning-by-doing generates persistence in optimal inflation it cannot

reduce inflation volatility by itself. If prices are flexible, there is no real resource cost of price adjustment and the Ramsey planner still finds it optimal to use inflation to synthesize state-contingent returns from nominal risk-free government bonds. When price adjustment costs are introduced in the model, the Ramsey planner faces a tradeoff. On the one hand, the Ramsey planner would like to use surprise inflation because it serves as a non-distortionary instrument to finance innovations in the government budget and this is preferred to changes in distorting proportional labor income tax. On the other hand, the Ramsey planner has strong incentives to stabilize inflation to minimize the costs associated with inflation changes. As Schmitt-Grohé and Uribe (2004b) and Siu (2004) find, even with a very small degree of price stickiness, this tradeoff is overwhelmingly resolved in favor of inflation stability. When price stickiness is introduced into a LBD model the inflation persistence increases further as compared to a LBD model with flexible prices. The main reason for this is that in a model with both LBD and price stickiness, the inflation directly depends on past, present, and future values of some variables through the New Keynesian Philips Curve. This generates some extra smoothness in the optimal inflation path.

Another interesting and important result in our work is that optimal tax policy is counter-cyclical - tax rates fall during recessions. This finding contrasts with procyclical tax results obtained in standard sticky price Ramsey models (see Chugh,

2006; Schmitt-Grohé and Uribe, 2004b). The basic intuition for the result is that in the presence of learning-by-doing, the Ramsey planner finds it relatively costly to raise taxes in response to a negative technology shock. Higher taxes would reduce hour, output, and hence organizational capital which will magnify the shock further. Therefore, the planner would optimally lower taxes to raise the after tax return to work and minimize the effects of the shock. In a standard model without LBD, the planner does not face this dynamic shock amplifying effect of a higher tax and optimally increases the tax rate in a recession to finance exogenous government spending.

Finally, average inflation, nominal interest rate, and labor income tax rates are relatively lower in our model as compared to models without learning-by-doing. This is consistent with Schmitt-Grohé and Uribe (2004a), and Chugh (2006) results that inflation, nominal interest rate, and labor income tax rates increase with market power. Schmitt-Grohé and Uribe (2004a) explains that monopoly profits represent pure rents for the owners of the monopoly power. The Ramsey planner would like to tax these rents at 100 percent rate because it would be non-distortionary. If profit taxes are unavailable or restricted to be less than 100%, the Ramsey planner uses the nominal interest rate as an indirect tax on profits. As the markup (market power) increases, the profit share increases and the Ramsey planner needs a higher nominal interest rate to tax these larger profits. Inflation increases with markup because

on average inflation has a direct relationship with nominal interest rate through the Fisher relation. The labor income tax base falls as the economy becomes less competitive and the Ramsey planner needs to increase labor income tax rate when the markup goes up. The presence of learning-by-doing decreases the markup and hence the monopoly profit which calls for relatively lower inflation, nominal interest and labor income tax.

The remainder of the chapter is organized as follows. The next section presents and describes the model while section 3.3 discusses about parameterizations and functional forms. Section 3.4 analyzes both steady-state and dynamic properties of Ramsey allocations and section 3.5 concludes.

3.2 The model

The model economy involves a large number of households and final good firms, a continuum of intermediate good producing firms, and the government. The structure of the economy is a standard growth model augmented with some new features and frictions - monopolistic competition in the product market, learning-by-doing in the technological environment, sticky prices, a money demand by households, and distortionary labor income taxation. The intermediate firms possess a degree of monopoly power and hence, can earn positive economic profits. As owners of

all the firms, households receive profits as dividends. However, the crucial features of the model economy that serve as the basis of our results are the firm-level learning-by-doing mechanism in the production technology and a quadratic cost of price-adjustment. The uncertainty in the economy is generated from two sources - stochastic productivity and government spending. We characterize, in turn, the economic environments faced by the households, the firms, and the government.

3.2.1 Households

The economy is populated by a large number of identical, infinitely lived households. Household's preferences are defined over processes of consumption and leisure. Money demand is motivated by a standard cash-credit goods environment. Household has to spend cash to purchase a subset of consumption goods. The representative household's objective function is given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t), \quad (3.1)$$

where, c_{1t} denotes consumption of cash goods, c_{2t} denotes consumption of credit goods, n_t denotes fraction of household's unit time endowment devoted to labor, $\beta \in (0, 1)$ denotes the subjective discount factor, and E_0 denotes the mathematical expectation operator conditional on information available in period 0.

The household faces two sequences of constraints. The flow budget constraint in

period t is given by

$$\frac{M_t}{P_{t-1}} + \frac{B_t}{P_{t-1}} = (1 - \tau_{t-1}^n)w_{t-1}n_{t-1} + R_{t-1}\frac{B_{t-1}}{P_{t-1}} + \frac{M_{t-1}}{P_{t-1}} - c_{1t-1} - c_{2t-1} + pr_{t-1}, \quad (3.2)$$

where M_t is the nominal money held at the end of securities-market trading in period t , B_t is the nominal, risk-free one-period bond held at the end of securities-market trading in period t , R_t is the gross nominal interest rate on these bonds, and P_t is the nominal price. w_t is the real wage rate and subject to a proportional tax rate τ_t^n . As the owner of the firms the household receives profit, pr_t , on a lump-sum basis with a one-period lag. We follow the same timing convention used in standard cash-credit goods environments. At the start of period t , after observing the shocks, households trade money and assets in a centralized securities market. This trading is followed by simultaneous trading in the goods-markets and the factor market. The household sells labor n_t and buys cash and credit goods. Purchases of the cash good are subject to a cash-in-advance constraint

$$c_{1t} \leq \frac{M_t}{P_t}. \quad (3.3)$$

Purchases of the cash good are settled at the end of period t , while purchases of the credit goods and selling of the labor service are settled at the beginning of period $t + 1$.

Let λ_t and ϕ_t denote the Lagrange multipliers on the flow budget constraint and the cash-in-advance constraint respectively. Then the first-order conditions of the

household's maximization problem are (3.2)-(3.3) holding with equality and

$$c_{1t} : \quad u_{1t} - \phi_t - \beta E_t \lambda_{t+1} = 0, \quad (3.4)$$

$$c_{2t} : \quad u_{2t} - \beta E_t \lambda_{t+1} = 0, \quad (3.5)$$

$$n_t : \quad -u_{3t} + \beta E_t [\lambda_{t+1} (1 - \tau_t^n) w_t] = 0, \quad (3.6)$$

$$M_t : \quad -\frac{\lambda_t}{P_{t-1}} + \frac{\phi_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{P_t} = 0, \quad (3.7)$$

$$B_t : \quad -\frac{\lambda_t}{P_{t-1}} + \beta E_t \frac{R_t \lambda_{t+1}}{P_t} = 0, \quad (3.8)$$

where u_{1t} denotes the value of marginal utility of cash good in period t (similarly for u_{2t}), and u_{3t} denotes the value of marginal utility of labor in period t . Equation (3.8) gives rise to a standard Fisher equation,

$$1 = R_t E_t \left[\frac{\beta \lambda_{t+1}}{\lambda_t} \frac{1}{\pi_t} \right], \quad (3.9)$$

where $\pi_t = P_t/P_{t-1}$, is the gross inflation rate between period $t - 1$ and period t . Combining (3.9) with (3.4) and (3.7) we can express the Fisher relation in terms of marginal utilities,

$$1 = R_t E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right], \quad (3.10)$$

which gives us the pricing formula for a one-period risk-free nominal bond. Denoting the nominal pricing kernel between period t and $t + 1$ as Q_{t+1} , we can write

$$Q_{t+1} = \left(\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right), \quad (3.11)$$

which implies the real pricing kernel as

$$q_{t+1} = Q_{t+1}\pi_{t+1}. \quad (3.12)$$

Combination of (3.4), (3.5), (3.7) and (3.8) implies a relationship between the gross nominal interest rate and the marginal rate of substitution between cash and credit goods

$$R_t = \frac{u_{1t}}{u_{2t}}. \quad (3.13)$$

Finally, combining equations (3.5) and (3.6), we obtain

$$\frac{u_{3t}}{u_{2t}} = (1 - \tau_t^n)w_t. \quad (3.14)$$

Equation (3.14) gives the optimal labor-leisure choice. It states that the presence of a non-zero labor income tax rate drives a wedge between the marginal rate of substitution between leisure-consumption and the real wage. Equation (3.13) states that a non-zero nominal interest rate drives a wedge between the marginal rate of substitution between cash-credit good consumption and the marginal rate of transformation between them, which is unity.

3.2.2 Production

The production environment consists of two sectors: an intermediate goods sector that produces differentiated goods using labor and organizational capital, and a

final goods sector that uses intermediate goods to produce a unique final good. The two sector feature of the production environment is a standard convention in New Keynesian models. However, a critical feature of our model is the presence of a learning-by-doing mechanism in the production technology of the intermediate goods firms.

Final Goods Producers

Government consumption goods, cash consumption goods, and credit consumption goods are physically indistinguishable. There are a large number of producers who produce this unique final good in a perfectly-competitive environment. Final goods producers require only the differentiated intermediate goods as inputs and use the following CES technology for converting intermediate goods into final goods.

$$y_t = \left[\int_0^1 y_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \quad (3.15)$$

where $\eta > 1$ denotes the intratemporal elasticity of substitution across different varieties of consumption goods, and differentiated intermediate goods are indexed by $i \in [0, 1]$.

Each period final goods firms choose inputs y_{it} for all $i \in [0, 1]$ and output y_t to

maximize profits given by

$$P_t y_t - \int_0^1 P_{it} y_{it} di \quad (3.16)$$

subject to (3.15). Here P_t denotes the nominal price of the final good and P_{it} denotes the nominal price of the intermediate good i . The solution to this problem yields the input demand functions

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} (y_t). \quad (3.17)$$

Intermediate Goods Producers

There is a continuum of intermediate goods producers, indexed by the letter i , who operate in a Dixit-Stiglitz style imperfectly competitive economy. Each of these firms produces a single variety i using two factor inputs - organizational capital, h_{it} , and labor services, n_{it} . The production technology of each firm i is given by

$$y_{it} = z_t F(h_{it}, n_{it}),$$

where y_{it} is the intermediate good variety produced by firm i . The variable z_t denotes an aggregate, exogenous, and stochastic productivity shock. The function F is assumed to be concave, and strictly increasing in two arguments. The stock of organizational capital is predetermined in the sense that h_{it} reflects the stock of organizational capital chosen at time $t - 1$. As in Cooper and Johri (2002), we

assume that the production technology has the following specific functional form:

$$y_{it} = z_t n_{it}^\alpha h_{it}^\theta \quad (3.18)$$

A key innovation in this chapter is the presence of the organizational capital in the production technology of intermediate goods firms. Organizational capital refers to the stock of firm-specific knowledge which is jointly produced with output and embodied in the organization itself. Organizational capital is acquired by endogenous learning by doing. In other words, firms accumulate the stock of organizational capital through the process of past productions regarding how best to organize its production activities and deploy the optimal mix of inputs. In this model we assume that organizational is accumulated according to:

$$h_{i,t+1} = (1 - \delta^h)h_{it} + h_{it}^\gamma y_{it}^\epsilon, \quad (3.19)$$

where δ^h is the depreciation rate of organizational capital and $0 < \delta^h < 1, \gamma < 1$. This accumulation equation might be viewed as a technology that uses the existing stock of organizational capital and current plant output as productive inputs for the production of future organizational capital. All producers begin life with a positive and identical endowment of organizational capital. The restriction $0 < \delta^h < 1$ is consistent with the empirical evidence supporting the hypothesis of organizational forgetting⁴. This Cooper and Johri (2002) framework of how learning-by-doing leads

⁴Organizational forgetting is the hypothesis that a firm's stock of production experience depreciates over time. Argote, Beckman, and Epplé (1990) provide empirical evidence for the hypothesis

to productivity increases is not particularly new in the literature. Rosen (1972), Prescott & Visscher (1980), Bahk & Gort (1993), Irwin & Klenow (1994), Jarmin (1994), Ericson & Pakes (1995), Benkard (2000), Thompson (2001), Thornton & Thompson (2001), Atkeson & Kehoe (2005) and many others have developed and tested models in which organization capital is acquired by endogenous learning-by-doing.

Prices are assumed to be sticky à la Rotemberg (1982). Specifically, in changing their prices intermediate goods firms face a real resource cost which is quadratic in the inflation rate of the good it produces.

$$\frac{\varphi}{2} \left(\frac{P_{it}}{P_{it-1}} - \pi \right)^2 \quad (3.20)$$

The parameter φ measures the degree of price stickiness. The higher is φ , the more sluggish is the adjustment of nominal prices. Price are fully flexible if φ equals zero.

The parameter π denotes the steady state inflation rate.

We assume that the firm must satisfy demand at the posted price. That is, every firm i faces the following constraint:

$$y_{it} \geq \left(\frac{P_{it}}{P_t} \right)^{-\eta} y_t \quad (3.21)$$

of organizational forgetting associated with the construction of Liberty Ships during World War II. Similarly, Darr, Argote, and Epple (1995) provide evidence for this hypothesis for pizza franchises and Benkard (2000) provides evidence for organizational forgetting associated with the production of commercial aircraft.

The intermediate firm takes aggregate demand y_t and the aggregate price level P_t as given. Therefore, the decision problem of the representative firm i is to choose the plans for n_{it} , h_{it+1} , and P_{it} so as to maximize the present discounted value of life-time profits:

$$\sum_{t=0}^{\infty} Q_t P_t \left\{ \frac{P_{it}}{P_t} y_{it} - w_t n_{it} - \frac{\varphi}{2} \left(\frac{P_{it}}{P_{it-1}} - \pi \right)^2 \right\}$$

subject to (3.18), (3.19), and (3.21). Here Q_t is the consumer's stochastic discount factor which is given by equation (3.11). As households own all the intermediate firms and thus receive their profits, it is appropriate to use their nominal discount factor in pricing revenue and costs in adjoining periods.

Let $P_t \Psi_{it}$ and $P_t mc_{it}$ be the Lagrange multipliers associated with the constraints (3.19) and (3.21) respectively. Then the first-order conditions of the firm's maximization problem with respect to labor and organizational capital are, respectively,

$$n_{it} : \quad w_t = mc_{it} \alpha \frac{y_{it}}{n_{it}} \quad (3.22)$$

$$h_{i,t+1} : \quad \Psi_{it} = E_t q_{t+1} \left[mc_{i,t+1} \theta \frac{y_{i,t+1}}{h_{i,t+1}} + \Psi_{i,t+1} \left\{ (1 - \delta^h) + \gamma h_{i,t+1}^{\gamma-1} y_{i,t+1}^\varepsilon \right\} \right] \quad (3.23)$$

Lagrange multiplier mc_{it} has the interpretation of marginal costs. This can be seen more clearly if we rearrange (3.22) as,

$$mc_{it} = \frac{w_t}{z_t F_n(h_t, n_t)}. \quad (3.24)$$

Given all else the same, a larger stock of organizational capital, h_t , implies a lower marginal cost, mc_t . The first order condition with respect to P_{it} yields a New Keynesian Phillips Curve,

$$(1 - \eta) \left(\frac{P_{it}}{P_t} \right)^{-\eta} y_t + mc_{it} \eta \left(\frac{P_{it}}{P_t} \right)^{-\eta} y_t \left(\frac{P_t}{P_{it}} \right) = \eta \varepsilon \Psi_{it} h_{it}^\gamma y_{it}^\varepsilon \left(\frac{P_t}{P_{it}} \right) + \varphi \left(\frac{P_{it}}{P_{it-1}} - \pi \right) \left(\frac{P_t}{P_{it-1}} \right) - E_t Q_{t+1} \varphi \left(\frac{P_{it}}{P_{it-1}} - \pi \right) \left(\frac{P_{t+1}}{P_{it}} \right) \left(\frac{P_{t+1}}{P_{it}} \right) \quad (3.25)$$

Since all intermediate firms face the same wage rate, face the same downward sloping demand curves, have access to the same production technology, marginal costs mc_{it} , are identical across all firms. Consequently, they hire the same amount of labor and produce the same amount of output. Therefore, we can restrict our attention to a symmetric equilibrium in which all firms make the same decisions. We thus drop all the subscripts i . That is, in equilibrium $y_{it} = y_t$, $p_{it} = p_t$, $mc_{it} = mc_t$, $\Psi_{it} = \Psi_t$, $n_{it} = n_t$, $h_{it} = h_t$. Equations (3.22), (3.23), and (3.25) can be simplified as:

$$w_t = mc_t \alpha \frac{y_t}{n_t} \quad (3.26)$$

$$\Psi_t = Q_{t+1} \pi_{t+1} \left[mc_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} + \Psi_{t+1} \{ (1 - \delta^h) + \gamma h_{t+1}^{\gamma-1} y_{t+1}^\varepsilon \} \right] \quad (3.27)$$

$$[1 - \eta + \eta mc_t] y_t = \varphi (\pi_t - \pi) \pi_t - \varphi E_t [q_{t+1} (\pi_{t+1} - \pi) \pi_{t+1}] + \Psi_t \eta \varepsilon h_t^\gamma y_t^\varepsilon. \quad (3.28)$$

Equation (3.26) is standard. When $mc_t < 1$, labor price w_t is less than the corresponding social marginal product $\alpha \frac{y_t}{n_t}$. Equation (3.27) determines the optimal use of organizational capital by the firm. One additional unit of organizational capital has a (marginal) value, in terms of profits, of Ψ_t to the producer in the current period. The right hand side of (3.27) measures the value of having available an additional unit of organizational capital for use by the firm in the following period. First, the additional organizational capital directly contributes to the intermediate good production in the following period as captured by the first term on the right hand side. Second, the additional organizational capital today has a positive effect on the future stock of organizational capital which is captured by the two terms inside the curly bracket. First term is the un-depreciated additional stock and the second term is the new organizational capital stock generated by this additional stock. This higher stock of organizational capital has a value of Ψ_{t+1} to the producer. Finally, all of these next period values must be discounted by the factor $Q_{t+1}\pi_{t+1}$. The condition (3.27) implies that organizational capital will be accumulated up to the point where the value of an additional unit of organizational capital today is equal to the discounted value of this organizational capital next period.

Finally, condition (3.28) represents the New Keynesian Phillips Curve which can

be rearranged as,

$$\left[mc_t - \frac{\eta - 1}{\eta} \right] \eta y_t = \varphi (\pi_t - \pi) \pi_t - \varphi E_t [q_{t+1} (\pi_{t+1} - \pi) \pi_{t+1}] + E_t q_{t+1} \left[mc_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} + \Psi_{t+1} \{ (1 - \delta^h) + \gamma h_{t+1}^{\gamma-1} y_{t+1}^\varepsilon \} \right] \eta \varepsilon h_t^\gamma y_t^\varepsilon. \quad (3.29)$$

Price setting condition (3.29) describes an equilibrium relationship between the current deviation of marginal cost, mc_t , from marginal revenue, $(\eta - 1)/\eta$, current inflation, π_t , expected future inflation, and expected change in future organizational capital. Under full price flexibility and without learning-by-doing effect in the technology, the firm would always set marginal revenue equal to marginal cost (the firm does not have any term on the right hand side of equation (3.29)). However, in the presence of either learning-by-doing effect in the production technology or the price adjustment costs, this practice is not optimal. Pricing decision in the current period has consequences for future costs and hence profits. Therefore, firms set prices to equate an average of current and future expected marginal costs to an average of current and future expected marginal revenues.

Quadratic price adjustment costs impose some additional restrictions on firm's price setting behavior which are captured by the first two terms on the right hand side of equation (3.29). By choosing a particular price in period t the firm incurs a direct cost in the current period which is captured by the first term. In addition, this price change has consequences for the menu costs the firm will incur in period

$t + 1$ which is reflected in the second term. Finally, the last term reflects the fact that the firm takes into account that its pricing decision today affects organizational capital tomorrow through the effect on demand and hence output. The expression $\eta \varepsilon h_t^\gamma y_t^\varepsilon$ ($= \frac{\partial h_{t+1}}{\partial y_t} \frac{\partial y_t}{\partial P_t}$) represents the marginal change in organizational capital in period $t + 1$ due to a change in price in period t . The expression $q_{t+1}[\dots]$ represents the present value of a period $t + 1$ additional unit of organizational capital. For making a dynamically optimal decision the firm must consider this future costs incurred by the current pricing decision⁵.

3.2.3 The Government

The government faces an exogenous, stochastic and unproductive stream of real expenditures denoted by g_t . These expenditures are financed through labor income taxation, money creation, and issuance of one-period, risk-free, nominal debt. The government's period-by-period budget constraint is then given by

$$M_t + B_t + P_{t-1} \tau_{t-1}^n w_{t-1} n_{t-1} = M_{t-1} + R_{t-1} B_{t-1} + P_{t-1} g_{t-1}. \quad (3.30)$$

As in Chari et. al (1991), government consumption is a credit good and thus g_{t-1} is not paid until period t . The government does not have the ability to directly

⁵In a non-Ramsey DSGE model, Johri (2009) discusses how LBD introduces a dynamic link between current production and future productivity and generates endogenous inertia in prices and output.

tax profits of the intermediate goods firms which is one of the reasons for the non-optimality of the Friedman Rule. Using the cash in advance constraint (3.3), we can eliminate the M terms and rewrite the government budget constraint as

$$c_{1t}\pi_t + b_t\pi_t + \tau_{t-1}^n w_{t-1} n_{t-1} = c_{1t-1} + R_{t-1} b_{t-1} + g_{t-1}, \quad (3.31)$$

where $b_t = \frac{B_t}{P_t}$ denotes the real value of the nominal government debt in period t .

3.2.4 Resource Constraint

Aggregating the time- t household budget constraint and the time- t government budget constraint yields the following resource constraint for the economy,

$$c_{1t-1} + c_{2t-1} + g_{t-1} + \frac{\varphi}{2}(\pi_{t-1} - \pi)^2 = y_{t-1}. \quad (3.32)$$

The price adjustment cost appears in the resource constraint due to the fact that it represents an identical real resource cost incurred by the all intermediate goods firms. As discussed in Chugh (2006), the economy-wide resource frontier describes production possibilities for period $t-1$ because of the timing convention of the model – particularly, because all goods are paid for with a lag of one period, summing the time- t household and government budget constraints gives rise to the time $t-1$ resource constraint.

3.2.5 Equilibrium

In the presence of government policy there are many competitive equilibria, indexed by different government policies. This multiplicity motivates the Ramsey problem.

In our model competitive and Ramsey equilibria are defined as follows:

Competitive Equilibrium

A competitive monetary equilibrium is a set of endogenous plans $\{c_{1t}, c_{2t}, n_t, w_t, h_{t+1}, M_t, B_t, mc_t, \Psi_t, \pi_t\}$, such that the household maximizes utility taking as given prices and policies; the firms maximizes profit taking as given the wage rate, and the demand function; the labor market clears, the bond market clears, the money-market clears, the government budget constraint and the aggregate resource constraint are satisfied. In other words, all the processes above satisfy conditions (3.10), (3.14), (3.19), (3.26)- (3.28), (3.40)- (3.32) given policies $\{\tau_t^n, R_t\}$, and the exogenous processes $\{z_t, g_t\}$.

The Ramsey Equilibrium

The Ramsey equilibrium is the unique competitive equilibrium that maximizes the household's expected lifetime utility. And the optimal fiscal and monetary policy is the process $\{R_t, \tau_t^n\}$ associated with this Ramsey equilibrium. Following Schmitt-Grohé and Uribe (2007), we assume that the benevolent Ramsey Government has

been operating for an infinite number of periods and it honors the commitments made in the past. This form of policy commitment is known as ‘optimal from the timeless perspective’ (Woodford (2003)). Under this concept of Ramsey equilibrium, the structure of the optimality conditions associated with the equilibrium is time invariant. On the other hand, under the conventional concept of Ramsey equilibrium, the equilibrium conditions in the initial periods are different from those applying to later periods. However, the timeless approach to analyzing dynamic properties of Ramsey allocation is comparable to the conventional approach because existing studies using conventional approach limit attention to the properties of equilibrium time series excluding the initial transition. Formally, we can define the Ramsey Equilibrium as a set of stationary processes $\{c_{1t}, c_{2t}, n_t, h_{t+1}, M_t, B_t, mc_t, \Psi_t, \pi_t, \tau_t^n, R_t\}$ that maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, n_t)$$

subject to the resource constraint

$$c_{1t} + c_{2t} + g_t + \frac{\varphi}{2}(\pi_t - \pi)^2 - z_t n_t^\alpha h_t^\theta = 0, \quad (3.33)$$

the household’s first-order condition on bond accumulation

$$1 - R_t E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right] = 0, \quad (3.34)$$

the cash-credit goods consumption optimality condition

$$R_t - \frac{u_{1t}}{u_{2t}} = 0, \quad (3.35)$$

the optimal consumption-leisure condition

$$\frac{u_{3t}}{u_{2t}} - (1 - \tau_t^n) m c_t \alpha \frac{y_t}{n_t} = 0, \quad (3.36)$$

the organizational capital accumulation technology

$$h_{t+1} - (1 - \delta^h) h_t + h_t^\gamma y_t^\varepsilon = 0, \quad (3.37)$$

the intermediate firms first-order condition on organizational capital accumulation

$$\Psi_t = E_t Q_{t+1} \pi_{t+1} \left[m c_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} + \Psi_{t+1} \left\{ (1 - \delta^h) + \gamma h_{t+1}^{\gamma-1} y_{t+1}^\varepsilon \right\} \right] \quad (3.38)$$

the New Keynesian Phillips Curve

$$[1 - \eta + \eta m c_t] y_t = \varphi (\pi_t - \pi) \pi_t - \varphi E_t [q_{t+1} (\pi_{t+1} - \pi) \pi_{t+1}] + \Psi_t \eta \varepsilon h_t^\gamma y_t^\varepsilon \quad (3.39)$$

and the time $t + 1$ government budget constraint

$$c_{1t+1} \pi_{t+1} + b_{t+1} \pi_{t+1} + \tau_t^n m c_t \alpha \frac{y_t}{n_t} - c_{1t} - R_t b_t - g_t = 0, \quad (3.40)$$

given exogenous process g_t , and z_t , values of all the variables dated $t < 0$, the values of the Lagrange multipliers associated with the constraints listed above dated $t < 0$. Under traditional Ramsey equilibrium concept, the equilibrium conditions

in the initial periods are different from those applied to later periods. But under Woodford's timeless definition, the optimality conditions associated with Ramsey equilibrium are time invariant.

3.3 Parameterization and Functional Forms

The time unit in our model is one quarter. We set $\beta = .9902$ so that the discount rate is 4 percent (Prescott, 1986) per year. We follow Chugh (2007) in choosing the utility function and assume that the period utility function takes the following specification

$$\ln c_t - \frac{\zeta}{1 + \mu} n_t^{1+\mu}, \quad (3.41)$$

where,

$$c_t = [(1 - \sigma)c_{1t}^\nu + \sigma c_{2t}^\nu]^{\frac{1}{\nu}} \quad (3.42)$$

Chugh (2007) use the parameter values for σ and ν from Siu (2004) who estimates them using the household optimality condition (3.10). We also use the same estimates $\sigma = 0.62$ and $\nu = 0.79$ as our base line. The parameter μ governs disutility of work. We choose $\mu = 1.7$ which is consistent with Hall (1997) estimates of the elasticity of marginal disutility of work. The preference parameter ζ was calibrated so that in the steady-state of the model without learning-by-doing and without nominal rigidities the consumer spends about one-third of his time working. We hold

the corresponding value of ζ (9.73) constant in all the environments considered in the chapter. We choose $\theta = 0.15$, $\gamma = 0.6$, and $\varepsilon = 0.4$ in line with Cooper & Johri (2002). We set $\delta^h = .1$ which is equivalent to a yearly depreciation rate of 40%. This value is in line with Benkard's (2000) estimate which suggests that the stock of experience depreciates by 39% yearly.

The exogenous processes for government spending, g_t , and productivity, z_t , are assumed to follow independent AR(1) in their logarithms,

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g$$

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z$$

with $\epsilon_t^z \sim iidN(0, \sigma_z^2)$ and $\epsilon_t^g \sim iidN(0, \sigma_g^2)$. \bar{g} is the steady-state level of government spending and we calibrate this value so that government spending constitutes 17 percent of steady-state output. We choose the first-order autocorrelation parameters $\rho_z = 0.95$ and $\rho_g = 0.97$, the standard deviation parameters $\sigma_z = 0.007$ and $\sigma_g = 0.02$ in line with Chugh (2007) and the RBC literature. Following Schmitt-Grohé and Uribe (2006) we set i) the degree of imperfect competition parameter $\eta = 6$, and ii) the initial liabilities to government B_1/P_0 so that in the nonstochastic steady-state the government debt-to-GDP ratio is 44 percent per year. Finally, in line with Chugh (2006)⁶ we set the price-rigidity parameter $\varphi = 5.88$ which implies an

⁶Chugh (2006) derives a detailed mapping between Calvo price-rigidity parameter and the Rotemberg price-rigidity parameter.

average price stickiness of three quarters. Table-3.1 presents the baseline values of the structural parameters we use to obtain our main results.

TABLE 3.1

BASELINE PARAMETER VALUES

Parameters	Value	Description
β	.9902	subjective discount rate
η	6	price elasticity of demand
σ	0.62	credit good share parameter in consumption
ν	0.79	elasticity parameter in consumption
ζ	calibrated	preference parameter
μ	1.7	parameter governing disutility of work
α	0.85	share of labor in the production technology
θ	0.15	share of organizational capital in production technology
δ^h	0.1	depreciation rate of organizational capital
γ	0.4	OC accumulation parameter, $h_{t+1} = (1 - \delta^h)h_t + h_t^\gamma y_t^\varepsilon$
ε	0.6	OC accumulation parameter
φ	5.88	price adjustment cost parameter
\bar{g}	calibrated	steady-state level of govt. spending
ρ_g	0.97	persistence in log govt. spending
σ^g	0.02	standard deviation of log govt. spending
ρ_z	0.95	persistence in log productivity
σ^z	0.007	standard deviation of log productivity

3.4 Quantitative Results

We characterize and solve the Ramsey equilibrium numerically using the methodology outlined in Schmitt-Grohé and Uribe (2006). They develop a set of numerical tools that allow the computation of Ramsey policy in a general class of dynamic

stochastic general equilibrium models. We first describe the optimal policy in the Ramsey steady-state and then present the simulation based dynamic results.

3.4.1 Ramsey Steady-States

To characterize the long-run state of the Ramsey equilibrium, first we derive the dynamic first-order conditions of the Ramsey problem. Then we impose the steady state and numerically solve the resulting non-linear system. This gives rise to the exact numerical solution of the long-run Ramsey problem.

Table 3.2 presents the Ramsey steady-state values of net inflation, the net nominal interest rate⁷, and labor income tax rate under four different environments of interests. All the environments we consider are characterized by imperfectly competitive product market and hence the Friedman rule ceases to be optimal. Optimal nominal interest rates are positive in all four cases because of the presence of monopoly profits. As explained by Schmitt-Grohé and Uribe (2004a), monopoly profits represent pure rents for the owners of the monopoly power, which the Ramsey planner would like to tax at 100 percent rate because it would be non-distortionary.

If profit taxes are unavailable, which is the case in our environments, or restricted to

⁷Note that both the net inflation rate, $\bar{\pi}$, and the net nominal interest rate, \bar{R} , are expressed in percent per year. For example, in the last row of table 3.2, yearly net inflation rate $\bar{R} = 1.7832\%$, implies yearly gross interest rates $= (1 + 0.017832)$. This implies a quarterly gross interest rate of $R = 1.017832^{1/4}$. Now, if we use the steady state relation $\pi = \beta R$, we can recover the quarterly gross inflation rate $\pi = 0.9902 \times 1.017832^{1/4} = 0.9946$. This implies a yearly net inflation rate $\bar{\pi} = (0.9946)^4 - 1 = -0.021315$, which is shown in the last row of table 3.2.

be less than 100%, the Ramsey planner uses inflation/nominal interest rate⁸ as an indirect tax on profits. Thus, the Friedman rule of a zero net nominal interest rate is no longer optimal. And consequently, the optimal inflation is also higher than the Friedman deflation (equal to the negative of the real interest rate).

The presence of learning-by-doing (LBD) reduces the optimal rate of inflation

TABLE 3.2

OPTIMAL STEADY-STATE POLICY IN DIFFERENT ENVIRONMENTS

	$\bar{\pi}$	\bar{R}	τ^n
Flexible price, no LBD	-0.1367	3.6002	0.2342
Flexible price with LBD	-1.4248	2.5143	0.2331
Sticky price, no LBD	-0.6108	3.4409	0.2340
Sticky price with LBD	-2.1315	1.7832	0.2328

Note: The inflation rate, $\bar{\pi}$, and the net nominal interest rate, \bar{R} , are expressed in percent per year.

and nominal interest rate in both flexible and sticky price environments. This finding is consistent with the market power intuition just discussed and Schmitt-Grohé and Uribe (2004a), and Chugh (2006) finding that steady-state nominal interest rate/inflation increases with market power. For a given price elasticity of demand intermediate firms' markup and hence market power falls due to the presence of learning-by-doing. This can be easily seen by rearranging the steady-state version

⁸In the steady state inflation and nominal interest rate have a direct relationship through the Fisher relation (3.10): $\pi = \beta R$.

of the New Keynesian Phillips Curve (3.29),

$$mc = \frac{\eta - 1}{\eta} + \Psi \epsilon h^\gamma y^{\epsilon-1}. \quad (3.43)$$

The real marginal cost, mc , increases from $\frac{\eta-1}{\eta}$ toward 1 because of the presence of learning-by-doing effect (the second term on the right hand side of equation (3.43)). The higher the mc , the lower the markup⁹ and hence monopoly profit. Thus, the steady state net nominal interest rate and inflation are lower than the rate suggested by a otherwise similar model without learning-by-doing. Finally, the labor income tax rate is also falling with learning-by-doing which is consistent with Schmitt-Grohé and Uribe (2004a). The labor tax base shrinks with monopoly power because the higher the monopoly power, the higher the wedge between wages and marginal product of labor. And a lower tax base calls for a higher labor income tax rate. In our model, learning-by-doing decreases the market power, increases the labor tax base and hence calls for a lower rate of the labor income tax.

We can also analyze how the steady-state policy responds to different values of learning parameters. Table 3.3 displays the steady-state Ramsey policy for different values of ϵ , γ , and δ^h . In this exercise while we change the value of one of the parameters we keep the other parameter constant at the baseline value. As the table shows, nominal interest rate, and consequently inflation rates, decline as either ϵ or

⁹Note that mc is the real marginal cost in our model and hence $\frac{1}{mc}$ represents the gross markup.

γ rises and δ^h falls. Again, the intuition draws from equation (3.43). A higher value

TABLE 3.3

STEADY-STATE POLICY FOR VARIOUS VALUES OF ε , γ AND δ^h

		$\bar{\pi}$	\bar{R}	τ^n	mc	n
$\varepsilon =$	0.35	-0.44645	3.5357	0.23667	0.9314	0.3658
	0.40	-2.1315	1.7832	0.23282	0.9473	0.3688
	0.45	-3.0239	0.85511	0.22892	0.9638	0.3719
$\gamma =$	0.55	-0.25756	3.6425	0.2339	0.9420	0.3678
	0.60	-2.1315	1.7832	0.23282	0.9473	0.3688
	0.65	-3.7673	0.081981	0.22942	0.9620	0.3715
$\delta^h =$	0.11	-0.73511	3.2355	0.23359	0.9446	0.3683
	0.10	-2.1315	1.7832	0.23282	0.9473	0.3688
	0.09	-3.0847	0.79189	0.23213	0.9497	0.3692

Note: The inflation rate, $\bar{\pi}$, and the net nominal interest rate, \bar{R} , are expressed in percent per year.

for either ε or γ or a lower value for δ^h (= higher value of Ψ) imply higher rate of learning and a higher value for mc (the table clearly shows these expected changes in the value of mc). And a higher value of mc implies a lower markup and hence lower monopoly profit. Finally, the labor income tax rates falls as ε or γ increases or δ^h falls. As the last column of the table confirms, this result is due to an increasing labor tax base.

3.4.2 Ramsey Dynamics

We compute the numerical solutions to the Ramsey problem based on a second-order approximation of the Ramsey planner's decision rules. We approximate the model in levels around the non-stochastic steady-state based on the perturbation algorithm described in Schmitt-Grohé and Uribe (2004a). As in Schmitt-Grohé and Uribe (2004b), we first generate simulated time series of length 100 for the variables of interest and then compute the first and second moments. We repeat the procedure 500 times and report the averages of the moments. Table 3.4 presents the simulation based moments for key real and policy variables generated from different model environments. As the table shows, the central result of the chapter – stable and persistent inflation – is generated only when both learning-by-doing and price rigidity are introduced in the model. While learning-by-doing generates the persistence, price rigidity generates the stability in the Ramsey inflation.

The top panel of Table 3.4 displays results for the model without any price rigidities or learning-by-doing effects in the production technology. As in Schmitt-Grohé and Uribe (2004a), inflation is characterized by high volatility and low persistence in this environment. The reason is that the Ramsey planner uses surprise inflation as a lump-sum tax on households' financial wealth. Inflation does not impose any real resource cost to the economy and hence it is optimal to use it in response

to unanticipated changes in the state of the economy. By varying the price level in response to shocks the Ramsey planner actually makes the riskless nominal debt state-contingent in real terms. In this flexible price environment, debt serves as a shock absorber which allows the Ramsey planner to maintain very smooth paths for the distortionary labor income taxes and interest rates over the business cycle. This intuition is supported by the very low standard deviation and high persistence of the labor income tax τ^n .

The second panel of Table 3.4 shows results for the model with flexible prices and learning-by-doing effect in the production technology. As price is still fully flexible and inflation does not incur any resource costs, LBD itself can't reduce the volatility of optimal inflation. The main contribution of learning is the generation of substantial persistence in optimal inflation and in a few other variables. We can draw intuition for the higher inflation persistence from the New Keynesian Phillips curve (3.29). Without the price stickiness this pricing equation becomes:

$$\left[mc_t - \frac{\eta - 1}{\eta} \right] = q_{t+1} \left[mc_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} + \Psi_{t+1} \{ (1 - \delta^h) + \gamma h_{t+1}^{\gamma-1} y_{t+1}^\epsilon \} \right] \varepsilon h_t^\gamma y_t^{\epsilon-1} \quad (3.44)$$

Although the quadratic price adjustment costs are absent in this environment the presence of LBD makes the pricing decision of the firm dynamic. Intermediate firms realize that a current price change affects organizational capital, productivity, cost,

and hence profits in all future periods. Therefore, they no longer follow a static pricing rule of equating time t marginal cost, mc_t , and marginal revenue, $\frac{\eta-1}{\eta}$. For maximizing life time profits they now take into account the future effects (which is captured by the terms on the right hand side of equation (3.44)) on cost and profits of a current pricing decision. This dynamic feature on the part of firms' price setting behavior significantly influences inflation persistence. More intuitively, if there is deflation (due to price reduction) this period, firms have to increase output to meet the additional demands. More output this period causes larger accumulation of production knowledge which lowers future costs. As the firms face relatively lower marginal costs in the next period, they set lower prices (which causes further deflation) in the next period as compared to environments without learning-by-doing. By similar arguments, higher prices (inflation) this period will induce firms to set relatively higher prices in the next period as well. As inflation becomes more persistent, so does labor input, n (given that output y_t , and the other input organizational capital h_t -a stock, are very persistent). Finally, through the Fisher relation (3.10), higher persistence in both consumption and inflation implies higher persistence in the nominal interest rate, R_t .

The third panel of Table 3.4 presents results for the model with sticky prices but without any learning-by-doing effect in the production technology. This model is comparable to other standard sticky price Ramsey models - e.g. Schmitt-Grohé and

Uribe (2004b) and Siu (2004). In line with their findings, the volatility of optimal inflation decreases substantially as compared to the baseline model of the top panel - the standard deviation of inflation falls from over three to near zero - but the autocorrelation coefficient still has a value near zero. The reason for this inflation stability is that when price adjustment is costly, the Ramsey planner balances the shock absorbing benefits of state-contingent inflation against the associated resource misallocation costs. In particular, he/she keeps the price changes to a minimal level because the associated resource misallocation costs largely dominate the value of state-contingent lump-sum levies on nominal wealth.

The bottom panel of Table 3.4 shows results for the model with both sticky prices and learning-by-doing effect. Optimal inflation is now characterized by very low volatility and very high persistence - exactly opposite to the inflation dynamics found in the baseline model of the top panel. After going through the results of different models it is now somewhat clear that while learning-by-doing generates the high persistence the price rigidities generates the low volatility in optimal inflation. The magnitude of inflation volatility is almost unchanged between the model of panel 3 (sticky prices without LBD) and the full model of the bottom panel (sticky prices with LBD). However, the inflation persistence increased significantly in the full model (Sticky prices with LBD) as compared to the model of panel 3 (sticky prices without LBD). As equation (3.27) indicates, a very stable path of optimal inflation

implies a more stable path for the value of organizational capital, Ψ_t . This extra stability in the value of organizational capital has contributed to the persistence of optimal inflation through the New Keynesian Phillips curve (3.29). Another way to think about it is that in a model with both LBD and price stickiness, the inflation directly depends on past, present, and future values of variables through the New Keynesian Philips Curve. More specifically with LBD, inflation π_t depends on h_{t-1} , y_{t-1} , h_t , y_t , mc_t , h_{t+1} , y_{t+1} , mc_{t+1} , Φ_{t+1} , π_{t+1} through (3.29). This generates some extra smoothness in the optimal inflation path.

Finally, as Table 3.4 clearly shows, tax policy is pro-cyclical - tax rates fall during a boom and rise during a recession - in the models without LBD. To finance an exogenous stream of spending, the Ramsey planner increases the tax rates when output is relatively low. However, with LBD the tax policy becomes counter-cyclical. The basic intuition is that the planner does not want hours to fall when negative technology shock hits as this lowers organizational capital and magnifies/propagates the shock further. Instead, the planner will lower taxes to raise the after tax return to work. In other words, he/she leans against the wind. How can the planner do this? He must be increasing the other tax at the same time to pay for the reduction in labor tax revenues. This is indeed the case as the correlation of inflation with the technology shock becomes more strongly negative. Also, notice that average inflation and nominal interest rates fall in presence of LBD. Again, the reason for

this is that learning-by-doing reduce markup and hence monopoly profits. Average labor tax rates falls in LBD models mostly due to the increase of the tax base.

3.5 Conclusion

This chapter characterizes optimal fiscal and monetary policy with price rigidity and organizational learning-by-doing in the production technology. The economic environment considered features a government that finances an exogenous stream of spending by levying distortionary income taxes, printing money, and issuing nominal risk-free debts. Our central finding is that, the inflation associated with the Ramsey allocation is very stable and persistence over the business cycle. The key for our results is some new features in the New Keynesian Philips Curve – the firms' optimal price setting condition. Inflation is optimally persistence because there is a dynamic link between current production and future productivity. And inflation is optimally stable because changes in inflation come at a resource cost. Another important result is that optimal tax policy is counter-cyclical in our model which contrasts with pro-cyclical tax results obtained in standard sticky price Ramsey models. Finally, the presence of organizational learning increases the competitiveness of the product market and hence reduces the nominal interest rates, inflation, and the labor income tax rates. The next chapter focuses on the implementation of

optimal Ramsey policies characterized in this chapter. In particular, it studies simple welfare maximizing fiscal and monetary feedback rules that can attain virtually the same level of welfare as under the Ramsey optimal policy.

TABLE 3.4

DYNAMIC PROPERTIES OF RAMSEY ALLOCATION

Variable	Mean	Std. Dev.	Auto. corr.	Corr(x,y)	Corr(x,g)	Corr(x,z)
Flexible prices without LBD						
τ^n	0.2360	.06169	0.8457	-0.3064	0.8728	-0.1332
$\bar{\pi}$	-0.4335	3.5436	-0.0141	-0.1167	0.1455	-0.0933
\bar{R}	3.5614	0.6243	0.1135	0.1043	0.0480	-0.0944
y	0.3351	0.0052	0.9014	1.0000	-0.1030	0.9692
n	0.3352	0.0012	0.3277	0.2179	-0.4278	-0.0147
c	0.2763	0.0040	0.9430	0.7935	-0.6456	0.7114
Flexible prices with LBD						
τ^n	0.2335	0.3649	0.8709	-0.6642	0.6140	0.2729
$\bar{\pi}$	-1.4248	3.0655	0.6626	-0.4391	0.4963	-0.2411
\bar{R}	2.5143	0.8588	0.9114	-0.5297	0.7652	-0.5790
y	0.11573	0.0163	0.8995	1.0000	0.0487	0.9957
n	0.3467	0.0005	0.8931	-0.4102	0.8600	-0.4665
c	0.9449	0.0094	0.9010	0.9769	-0.1400	0.9877
Sticky prices without LBD						
τ^n	0.2389	1.1572	0.8232	-0.2360	0.9559	-0.2576
$\bar{\pi}$	-0.6108	0.0400	0.0337	-0.1348	0.1552	-0.1243
\bar{R}	3.2409	0.8531	-0.1591	-0.3501	0.3623	-0.3416
y	0.3819	0.0065	0.8981	1.0000	-0.0519	0.9977
n	0.3320	0.0004	0.9319	0.0514	-0.7189	-0.0044
c	0.2872	0.0046	0.9054	0.8743	-0.4868	0.8515
Sticky prices with LBD						
τ^n	0.2323	0.7321	0.9497	0.5851	0.6373	0.4592
$\bar{\pi}$	-2.1234	0.0397	-0.9059	-0.2282	0.7621	-0.4452
\bar{R}	1.7974	0.1514	0.9171	-0.0871	-0.8523	0.1260
y	1.0189	0.0158	0.8991	1.0000	0.2405	0.9552
n	0.3690	0.0021	0.9113	-0.2725	0.8121	-0.5139
c	0.8449	0.0093	0.9024	0.8087	-0.3167	0.9373

Note: The inflation rate, $\bar{\pi}$, and the net nominal interest rate, \bar{R} , are expressed in percent per year.

Chapter 4

Organizational Learning and Optimal Fiscal and Monetary Policy Rules

4.1 Introduction

There has been much recent work studying optimal monetary policy rules (see the survey by McCallum (1999)). Most studies on simple feedback policy rules are not fully micro-based. Ad hoc criteria such as the implied volatilities of output and inflation are employed to evaluate policy. Also, it is standard practice in this literature to completely ignore fiscal policy. The implicit assumption in these models is that the

fiscal budget is balanced at all times by means of lump-sum taxation. However, empirical studies (e.g. Favero and Monacelli (2003)) show that characterizing postwar U.S. fiscal policy as passive at all times is at odds with the facts. In addition, most often these optimal policy models are conducted in cashless environments. This assumption introduces an inflation-stabilization bias into optimal monetary policy. For the presence of a cash-in-advance constraint creates a motive to stabilize the nominal interest rate rather than inflation. In a recent paper, Schmitt-Grohé and Uribe (2007) address these issues by studying welfare maximizing simple monetary and fiscal policy rules in a medium scale stochastic dynamic general equilibrium model where nominal rigidities induce inefficiencies, and where there is a nontrivial demand for money. As opposed to other papers in the literature (e.g. Kollmann (2008)), they use the Ramsey-optimal policy as a point of comparison.

In this chapter we compute welfare maximizing monetary and fiscal feedback rules in a stochastic dynamic general equilibrium economy which has monopolistic competition and tax distortions. More notably, the monopolistic firms in this economy learn from their production experiences which raises their future productivity. Nominal rigidities are introduced by assuming that firms incur quadratic costs in adjusting their prices. The central focus of this chapter is to investigate whether the policy conclusions arrived at in the existing literature regarding the optimal conduct of monetary policy are robust in an economy where a learning-by-doing mechanism

exists in the production technology. Following Schmitt-Grohé and Uribe (2007), this chapter characterizes monetary and fiscal policy rules that are optimal within a family of implementable and simple rules. Simple rules are ones where policy variables such as the nominal interest rate, and taxes are set as a function of a few observable aggregates such as output, inflation, and government debt. Policy rules are implementable if they are associated with a unique rational expectations equilibrium. The optimal rule is the rule that maximizes welfare of the representative household. The numerical evaluation of welfare is conducted using a second-order accurate solution to the equilibrium behavior of endogenous variables.¹

The results in this chapter are consistent with the findings in the literature. First, we show that the optimal interest-rate rules call for an very strong response to inflation and a very weak response to output. Our model strongly suggests that the monetary authority must not respond to output. In the literature (see Schmitt-Grohé and Uribe (2007), and Kollmann (2008)), the output coefficient in the interest-rate rule is very small and positive but not exactly zero. In our model, it is zero in all different monetary policies we consider. This result is consistent with the explanation often offered for (see Schmitt-Grohé and Uribe (2007)) why a policy of “leaning against the wind” is not appropriate in response to a technology shock.

¹Computationally, we employ the numerical solution algorithms used in Schmitt-Grohé and Uribe (2004c), and Schmitt-Grohé and Uribe (2007).

Under a policy of leaning against the wind the nominal interest rate rises whenever output rises. This increase in the nominal interest rate in turn hinders prices falling by as much as marginal costs causing markups to increase. With an increase in markups, output does not increase as much as it would have otherwise, preventing the efficient rise in output (see, for example, Rotemberg and Woodford (1997)). In our model, leaning against the wind policy not only prevents the efficient rise in output, it also prevents an efficient rise in organizational capital. Consequently, not responding to output is even more important in our model as compared to in a model without learning-by-doing in production. Second, the interest rule is superinertial which means that the monetary authority is forward looking. This result contrasts with the backward looking optimal policy obtained in the related literature. Third, The optimal monetary and fiscal rule combination yields a level of welfare that is very close to that implied by the Ramsey optimal policy. Finally, the optimized rules induce a stable rate of inflation, a feature also characterized by the Ramsey policy.

The remainder of the chapter is organized as follows. The next section presents and describes the model while section 4.3 define the equilibrium concepts. section 4.4 discusses about computation, welfare measure, functional forms and parameterizations. Section 4.5 presents the quantitative results and section 4.6 concludes.

4.2 The model

The model economy comprises of a large number of households and final good firms, a continuum of intermediate good producing firms, and the government. The structure of the economy is a standard growth model augmented with some new features and frictions - monopolistic competition in the product market, learning-by-doing in the technological environment, sticky prices, a money demand by households, and distortionary labor income taxation. The firms possess a degree of monopoly power and hence, can earn positive economic profits. As owners of all the firms, households receive profits as dividends. Two important features of the model economy are the firm-level learning-by-doing mechanism in the production technology and quadratic cost of price-adjustment. The uncertainty in the economy is generated from two sources - stochastic productivity and government spending. We characterize, in turn, the economic environments faced by the households, the firms, and the government.

4.2.1 Households and Firms

The problem of final goods producers and intermediate goods producers are exactly the same as described in Chapter 3. Consequently, we have the same first order conditions from the production sector as in Chapter 3. The problem of the repre-

sentative household is as described in Chapter 3 except the flow budget constraint is now

$$\frac{M_t}{P_{t-1}} + \frac{B_t}{P_{t-1}} = (1 - \tau_{t-1}^D)w_{t-1}n_{t-1} + R_{t-1} \frac{B_{t-1}}{P_{t-1}} + \frac{M_{t-1}}{P_{t-1}} - c_{1t-1} - c_{2t-1} + (1 - \tau_{t-1}^D)pr_{t-1} - \tau_{t-1}^L, \quad (4.1)$$

where τ_t^L is the lump sum tax. M_t is the nominal money held at the end of securities-market trading in period t , B_t is the nominal, risk-free one-period bond held at the end of securities-market trading in period t , R_t is the gross nominal interest rate on these bonds, and P_t is the nominal price. w_t is the real wage rate and subject to a proportional income tax rate τ_t^D . As the owner of the firms the household receives profit, pr_t , which is also subject to the income tax rate τ_t^D . We follow the same timing convention used in standard cash-credit goods environments (see Chari, Christiano, and Kehoe (1991) and Chugh (2007)). At the start of period t , after observing the shocks, households trade money and assets in a centralized securities market. This trading is followed by simultaneous trading in the goods-markets and the factor market. The household sells labor n_t and buys cash and credit goods. Purchases of the cash good are subject to a cash-in-advance constraint

4.2.2 The Government

The government faces an exogenous, stochastic and unproductive stream of real expenditures denoted by g_t . These expenditures are financed through labor income,

and profit taxation, money creation, and issuance of one-period, risk-free, nominal debt. The government's period-by-period budget constraint is then given by

$$M_t + B_t + P_{t-1}\tau_{t-1} = M_{t-1} + R_{t-1}B_{t-1} + P_{t-1}g_{t-1}. \quad (4.2)$$

Here, τ_t is total tax revenues from labour income and profit taxes. As in Chari et. al (1991), government consumption is a credit good and thus g_{t-1} is not paid until period t . The government does not have the ability to directly tax profits of the intermediate goods firms which is one of the reasons for the non-optimality of the Friedman Rule. Let $l_{t-1} \equiv \frac{R_{t-1}B_{t-1}}{P_{t-1}}$ denote real government debt liabilities outstanding at the end of period $t - 1$. Now, using the cash in advance constraint (3.3), we can rewrite the government budget constraint as

$$c_{1t}\pi_t + \frac{l_t}{R_t}\pi_t + \tau_{t-1} = c_{1t-1} + l_{t-1} + g_{t-1}, \quad (4.3)$$

We consider various alternative fiscal policy specifications that involve either lump sum or distortionary income taxation. Total tax revenues, τ_t , consist of revenue from lump-sum taxation, τ_t^L , and revenue from income taxation, $\tau_t^D(w_t n_t + pr_t) = \tau_t^D y_t$. That is,

$$\tau_t = \tau_t^L + \tau_t^D y_t \quad (4.4)$$

Following Schmitt-Grohé and Uribe (2007), the fiscal regime is defined by the following rule:

$$\tau_t - \tau^* = \bar{\theta}(l_{t-1} - l^*), \quad (4.5)$$

where $\bar{\theta}$ is a parameter, and τ^* and l^* denote the deterministic Ramsey steady-state values of τ_t and l_t , respectively. Fiscal rule (4.5) implies, the fiscal authority sets tax revenues in period t , τ_t , as a linear function of the real value of total debt liabilities. We focus on two alternative fiscal regimes. In one all taxes are lump sum ($\tau^D = 0$), and in the other all taxes are distortionary.

As in Schmitt-Grohé and Uribe (2007), we assume that the monetary authority sets the short-term nominal interest rate according to a simple feedback rule belonging to the following class of Taylor (1993)-type rules

$$\ln(R_t/R^*) = \alpha_R \ln(R_{t-1}/R^*) + \alpha_\pi E_t \ln(\pi_{t-i}/\pi^*) + \alpha_y E_t \ln(y_{t-i}/y^*); i = -1, 0, \text{ or } 1, \quad (4.6)$$

where y^* , R^* , and π^* denote the nonstochastic Ramsey steady-state levels of aggregate demand, nominal interest rate, and inflation, respectively and α_R , α_π , and α_y are parameters. The index i can take three values 1, 0, or -1. We refer to the interest rate rule as backward looking when $i = 1$, as contemporaneous when $i = 0$, and as forward looking when $i = -1$. As Schmitt-Grohé and Uribe (2007) note, the focus is restricted to this class of interest rate feedback rules, primarily because these rules are defined in terms of readily available macroeconomic indicators.

4.2.3 Resource Constraint

Aggregating the time- t household budget constraint and the time- t government budget constraint yields the following resource constraint for the economy,

$$c_{1t-1} + c_{2t-1} + g_{t-1} + \frac{\varphi}{2}(\pi_{t-1} - \pi)^2 = y_{t-1}. \quad (4.7)$$

The price adjustment cost appears in the resource constraint due to the fact that it represents an identical real resource cost incurred by the all intermediate goods firms. As discussed in Chugh (2006), the economy-wide resource frontier describes production possibilities for period $t-1$ because of the timing convention of the model – particularly, because (all) goods are paid for with a lag of one period, summing the time- t household and government budget constraints gives rise to the time $t-1$ resource constraint.

4.3 Equilibrium

In the presence of government policy there are many competitive equilibria, indexed by different government policies. This multiplicity motivates the Ramsey problem.

In our model Ramsey and competitive equilibria are defined as follows:

4.3.1 The Ramsey Equilibrium

The Ramsey equilibrium is the unique competitive equilibrium that maximizes the household's expected lifetime utility. And the optimal fiscal and monetary policy is the process $\{R_t, \tau_t^D\}$ associated with this Ramsey equilibrium. Following Schmitt-Grohé and Uribe (2007), we assume that the benevolent Ramsey Government has been operating for an infinite number of periods and it honors the commitments made in the past. This form of policy commitment is known as 'optimal from the timeless perspective' (Woodford, 2003). Under this concept of Ramsey equilibrium, the structure of the optimality conditions associated with the equilibrium is time invariant. On the other hand, under the conventional concept of Ramsey equilibrium, the equilibrium conditions in the initial periods are different from those applying to later periods. However, the timeless approach to analyzing dynamic properties of Ramsey allocation is comparable to the conventional approach because existing studies using the conventional approach limit attention to the properties of equilibrium time series excluding the initial transition. Formally, we can define the Ramsey Equilibrium as a set of stationary processes $\{c_{1t}, c_{2t}, n_t, h_{t+1}, M_t, l_t, mc_t, \Psi_t, \pi_t, \tau_t^L, \tau_t^D, R_t\}$ that maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, n_t)$$

subject to the resource constraint

$$c_{1t} + c_{2t} + g_t + \frac{\varphi}{2}(\pi_t - \pi)^2 - z_t n_t^\alpha h_t^\theta = 0, \quad (4.8)$$

the household's first-order condition on bond accumulation

$$1 - R_t E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right] = 0, \quad (4.9)$$

the optimal consumption-leisure condition

$$\frac{u_{3t}}{u_{2t}} - (1 - \tau_t^D) m c_t \alpha \frac{y_t}{n_t} = 0, \quad (4.10)$$

the organizational capital accumulation technology

$$h_{t+1} - (1 - \delta^h) h_t + h_t^\gamma y_t^\varepsilon = 0, \quad (4.11)$$

the intermediate firms first-order condition on organizational capital accumulation

$$\Psi_t = E_t Q_{t+1} \pi_{t+1} \left[m c_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} + \Psi_{t+1} \{ (1 - \delta^h) + \gamma h_{t+1}^{\gamma-1} y_{t+1}^\varepsilon \} \right] \quad (4.12)$$

the New Keynesian Phillips Curve

$$\begin{aligned} [1 - \eta + \eta m c_t] y_t &= \varphi (\pi_t - \pi) \pi_t - \varphi E_t [q_{t+1} (\pi_{t+1} - \pi) \pi_{t+1}] \\ &\quad + \Psi_t \eta \varepsilon h_t^\gamma y_t^\varepsilon, \end{aligned} \quad (4.13)$$

the equation representing the total tax revenue

$$\tau_t = \tau_t^L + \tau_t^D y_t \quad (4.14)$$

and the time $t + 1$ government budget constraint

$$c_{1t}\pi_t + \frac{l_t}{R_t}\pi_t + \tau_{t-1} - c_{1t-1} - l_{t-1} - g_{t-1} = 0, \quad (4.15)$$

given exogenous process g_t , and z_t , values of all the variables dated $t < 0$, the values of the Lagrange multipliers associated with the constraints listed above dated $t < 0$.

4.3.2 Competitive Equilibrium

A stationary competitive monetary equilibrium is a set of processes c_{1t} , c_{2t} , n_t , w_t , τ_t^D , h_{t+1} , M_t , l_t , τ_t^L , τ_t , mc_t , Ψ_t , π_t , for $t = 0, 1, \dots$ that remain bounded in some neighborhood around the deterministic steady-state and satisfy conditions (3.10), (3.14), (3.19), (3.26)- (3.28), (4.3)- (4.7) and either $\tau_t^L = 0$ (in the case of distortionary taxation) or $\tau_t^D = 0$ (in the case of lump sum taxation), given initial values for h_0 , l_{-1} , and exogenous processes z_t , and g_t .

4.4 Computation, Welfare Measure, Functional forms and Parameterization

Following Schmitt-Grohé and Uribe (2007) we wish to find the monetary and fiscal policy rule combination (a set of values for α_{pi} , α_y , α_R , and $\bar{\theta}$) that is optimal and implementable within the family defined by equations (4.5) and (4.6). The require-

ments for a implementable policy are i) the rule must ensure local uniqueness of the rational expectations equilibrium, ii) the rule must induce nonnegative equilibrium dynamics for the nominal interest rate. For an implementable policy to be optimal, the contingent plans for consumption and hours of work associated with that policy must yield the highest level of unconditional lifetime utility. In other words, we look for policy parameters that maximize $E[V_t]$, where

$$V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, n_{t+j}),$$

and E denotes the unconditional expectation operator. The point of reference for policy evaluation is the time-invariant equilibrium process of the Ramsey optimal allocation. We compute conditional and unconditional welfare costs of following the optimized simple policy rule relative to the Ramsey policy.

4.4.1 Parameterization and Functional Forms

The time unit in our model is one quarter. We set $\beta = .9902$ so that the discount rate is 4 percent (Prescott, 1986) per year. We follow Chugh (2007) in choosing the utility function and assume that the period utility function takes the following specification

$$\ln c_t - \frac{\zeta}{1 + \mu} n_t^{1+\mu}, \quad (4.16)$$

where,

$$c_t = [(1 - \sigma)c_{1t}^v + \sigma c_{2t}^v]^{\frac{1}{v}} \quad (4.17)$$

Chugh (2007) use the parameter values for σ and v from Siu (2004) who estimates them using the household optimality condition (3.10). We also use the same estimates $\sigma = 0.62$ and $v = 0.79$ as our base line. The parameter μ governs disutility of work. We choose $\mu = 1.7$ which is consistent with Hall's (1997) estimates of the elasticity of marginal disutility of work. Parameter ζ was calibrated so that in the steady-state of the model without learning-by-doing and without nominal rigidities the consumer spends about one-third of his time working. We hold the corresponding value of ζ (9.73) constant in all the environments considered in the chapter. We choose $\theta = 0.15$, $\gamma = 0.6$, and $\varepsilon = 0.4$ in line with Cooper & Johri (2002). We set $\delta^h = .1$ which is equivalent to a yearly depreciation rate of 40%. This value is in line with Benkard's (2000) estimate which suggests that the stock of experience depreciates by 39% yearly.

The exogenous processes for government spending, g_t , and productivity, z_t , are assumed to follow independent AR(1) in their logarithms,

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g$$

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z$$

with $\epsilon_t^z \sim Nid(0, \sigma_z^2)$ and $\epsilon_t^g \sim Nid(0, \sigma_g^2)$. \bar{g} is the steady-state level of government spending and we calibrate this value so that government spending constitutes 17 percent of steady-state output. We choose the first-order autocorrelation parameters $\rho_z = 0.95$ and $\rho_g = 0.97$, the standard deviation parameters $\sigma_z = 0.007$ and $\sigma_g = 0.02$ in line with Chugh (2007) and the RBC literature. Following Schmitt-Grohé and Uribe (2006) we set i) the degree of imperfect competition parameter $\eta = 6$, and ii) the initial liabilities to government B_1/P_0 so that in the nonstochastic steady-state the government debt-to-GDP ratio is 44 percent per year. Finally, in line with Chugh (2006)² we set the price-rigidity parameter $\varphi = 5.88$ which implies an average price stickiness of three quarters. Table 4.1 presents the baseline values of the structural parameters we use to obtain our main results.

4.4.2 Measuring Welfare Costs

Following Schmitt-Grohé and Uribe (2007), we conduct policy evaluations by computing the welfare cost of a particular monetary and fiscal regime relative to the time-invariant equilibrium process associated with the Ramsey policy. Let us denote the Ramsey policy by r , and an alternative policy regime by a . Then we can define the welfare associated with the time-invariant equilibrium implied by the Ramsey

²Chugh (2006) derives a detailed mapping between Calvo price-rigidity parameter and the Rotemberg price-rigidity parameter.

TABLE 4.1

BASELINE PARAMETER VALUES

Parameters	Value	Description
β	.9902	subjective discount rate
η	6	price elasticity of demand
σ	0.62	credit good share parameter in consumption
ν	0.79	elasticity parameter in consumption
ζ	calibrated	preference parameter
μ	1.7	parameter governing disutility of work
α	0.85	share of labor in the production technology
θ	0.15	share of organizational capital in production technology
δ^h	0.1	depreciation rate of organizational capital
γ	0.4	OC accumulation parameter, $h_{t+1} = (1 - \delta^h)h_t + h_t^\gamma y_t^\varepsilon$
ε	0.6	OC accumulation parameter
φ	5.88	price adjustment cost parameter
\bar{g}	calibrated	steady-state level of govt. spending
ρ_g	0.97	persistence in log govt. spending
$\sigma^{\varepsilon_t^g}$	0.02	standard deviation of log govt. spending
ρ_z	0.95	persistence in log productivity
$\sigma^{\varepsilon_t^z}$	0.007	standard deviation of log productivity

policy conditional on a particular state of the economy in period 0 as

$$V_0^r = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^r, n_t^r),$$

where c_t^r and n_t^r denote the contingent plans for consumption and hours under the Ramsey policy. Similarly, we can define the conditional welfare associated with policy regime a as

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^a, n_t^a).$$

We assume that at time zero all state variables of the economy equal their respective Ramsey steady-state value. Consequently, computing expected welfare conditional on the initial state being the nonstochastic steady state ensures that the economy begins from the same initial point under all possible policies.

We denote λ^c as the welfare cost of adopting policy regime a instead of the Ramsey policy conditional on a particular state in period zero. λ^c is defined as the fraction of regime r 's consumption process that a household would be willing to give up to be as well off under regime a as under regime r . More formally, λ^c is implicitly defined by

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t u((1 - \lambda^c)c_t^r, n_t^r).$$

Given our particular functional form for the period utility function, equation (4.16), the above expression can be written as

$$\begin{aligned} V_0^a &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log((1 - \lambda^c)c_t^r) - \frac{\zeta}{1+\mu} n_t^{r1+\mu} \right\} \\ &= \frac{\log(1-\lambda^c)}{1-\beta} + V_0^r \end{aligned}$$

Now, solving for λ^c we obtain

$$\lambda^c = 1 - e^{[(V_0^a - V_0^r)(1-\beta)]}$$

In equilibrium, V_0^a and V_0^r are functions of the initial state vector x_0 and the parameter σ_ϵ scaling the standard deviation of the exogenous shocks (see Schmitt-Grohé

and Uribe, 2004a). Therefore, we can write $V_0^a = V^{ac}(x_0, \sigma_\epsilon)$ and $V_0^r = V^{rc}(x_0, \sigma_\epsilon)$.

Consequently, the conditional welfare cost can be expressed as

$$\lambda^c = 1 - e^{[(V^{ac}(x_0, \sigma_\epsilon) - V^{rc}(x_0, \sigma_\epsilon))(1 - \beta)]} \quad (4.18)$$

Clearly, λ^c is a function of x_0 and σ_ϵ , which we write as

$$\lambda^c = \Lambda^c(x_0, \sigma_\epsilon).$$

We restrict attention to an approximation that is accurate up to second order and omit all higher order terms. Now, consider a second-order approximation of the function Λ^c around the point $x_0 = x$ and $\sigma_\epsilon = 0$, where x denotes the deterministic Ramsey steady state of the state vector. This gives³,

$$\lambda^c \approx \Lambda^c(x, 0) + \Lambda_{\sigma_\epsilon}^c(x, 0)\sigma_\epsilon + \frac{\Lambda_{\sigma_\epsilon \sigma_\epsilon}^c(x, 0)}{2}\sigma_\epsilon^2. \quad (4.19)$$

Similarly, the unconditional welfare cost is given by

$$\lambda^u \approx \Lambda^u(x, 0) + \Lambda_{\sigma_\epsilon}^u(x, 0)\sigma_\epsilon + \frac{\Lambda_{\sigma_\epsilon \sigma_\epsilon}^u(x, 0)}{2}\sigma_\epsilon^2. \quad (4.20)$$

4.5 Results

We consider two broad policy environments in this chapter. In one environment there is only a monetary authority which follows the monetary policy rule but there

³ Since we wish to characterize welfare conditional upon the initial state being the deterministic Ramsey steady state, in performing the second-order expansion of Λ^c only its first and second derivatives with respect to σ_ϵ have to be considered.

is no fiscal authority. Analytically, the absence of a fiscal authority is equivalent to modeling a government that operates under passive fiscal policy and collects all of its revenue via lump-sum taxation (see Schmitt-Grohé and Uribe, 2007). In the other environment there are both fiscal and monetary authorities and they follow their respective policy feedback rules.

4.5.1 An Economy Without any Fiscal Feedback Rule

Table 4.2 reports policy evaluations for the economy where the government operates under passive fiscal policy and collects all the revenues via lump-sum taxation. The point of comparison is the time-invariant stochastic real allocation associated with the Ramsey policy. The table reports conditional and unconditional welfare costs, λ^c and λ^u , as defined in equations (4.19) and (4.20).

In table 4.2 we consider five different monetary policies: Three constrained optimal interest-rate feedback rules and two non-optimized rules. We find that all the optimal interest-rate rules call for an aggressive response to inflation and a mute response to output. The inflation coefficient of the optimized rules take the largest value allowed in the search, namely 3.⁴ The optimized rules are very effective as

⁴In setting the largest value of the inflation coefficient we follow Schmitt-Grohé and Uribe, 2007. Removing the upper bound on policy parameters optimal policy calls for a larger inflation coefficient but yields only a negligible improvement in welfare.

TABLE 4.2

OPTIMAL MONETARY POLICY

Interest-Rate Rule	$\hat{R} = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R \hat{R}_{t-1}$							
	α_π	α_y	α_R	$\bar{\theta}$	Conditional Welfare Cost ($\lambda^c \times 100$)	Unconditional Welfare Cost ($\lambda^u \times 100$)	σ_π	σ_R
Ramsey Policy	-	-	-	-	0	0	0.002	0.19
<u>Optimized Rules</u>								
Contemporaneous($i = 0$)	3	0.00	0.51	-	0.000	0.000	0.065	0.62
Backward($i = 1$)	3	0.00	0.32	-	0.000	0.001	0.078	0.66
Forward($i = -1$)	3	0.00	0.43	-	0.001	0.001	0.352	0.36
<u>Non-Optimized Rules</u>								
Taylor Rule	1.5	0.5	-	-	0.488	0.535	10.1	10.6
Simple Taylor Rule	1.5	-	-	-	0.052	0.057	2.83	3.33

Note: (1) In the optimized rules, the policy parameters α_π , α_y , and α_R are restricted to lie in the interval [0,3]. (2) Conditional and unconditional welfare costs, $\lambda^c \times 100$ and $\lambda^u \times 100$, are defined as the percentage decrease in the Ramsey optimal consumption process necessary to make the level of welfare under the Ramsey policy identical to that under the evaluated policy. (3) The standard deviation of inflation and the nominal interest rate is measured in percent per year.

they deliver welfare levels remarkably close to those achieved under the Ramsey policy.

Optimality requires that the interest-rate rules do not respond to output. In the related literature (see Schmitt-Grohé and Uribe (2007), and Kollmann (2008)), the output coefficient in the interest-rate rule is very small and positive but not exactly zero. In our model, it is zero in all different monetary policies we consider. An explanation often offered (see Schmitt-Grohé and Uribe (2007)) for why a policy of

“leaning against the wind” is not appropriate in response to supply shock such as a technology shock, is that under such policy the nominal interest rate rises whenever output rises. This rise in the nominal interest rate in turn hinders prices falling by as much as marginal costs causing markups to increase. With an increase in markups, output does not increase as much as it would have otherwise, preventing the efficient rise in output. In our model, leaning against the wind policy not only prevent the efficient rise in output, it also prevent efficient rise in organizational capital. Consequently, not responding to output is even more important in our model as compared to in a model without learning-by-doing in production. This point is also conveyed with simplicity by comparing the welfare consequences of a simple interest-rate rule that responds only to inflation with a coefficient of 1.5 to those of a standard Taylor rule that responds to inflation as well as output with coefficients 1.5 and 0.5, respectively. From table 4.2 we see that the Taylor rule that responds to output is significantly welfare inferior to the simple interest-rate rule that responds only to inflation.

The optimized rules also induce a stable rate of inflation, a feature also characterized by the Ramsey policy. Another fact about the optimal interest rules is that they feature interest-rate inertia, which means that the monetary authority reacts to inflation much more aggressively in the long run than in the short run. The fact that the interest rule is not superinertial (i.e., α_R does not exceed unity) means that

the monetary authority is backward looking.

Another important policy issue is what measure of inflation and aggregate activity the central bank should respond to. In particular, should the monetary authority respond to past, current, or expected future values of output and inflation. We address this issue by computing optimal backward-looking and forward-looking interest-rate rules. Table 4.2 shows that there are no welfare gains from targeting past or future values of inflation and output as opposed to current values of these macroeconomic indicators. Essentially, standard deviations of inflation and interest rate somewhat rise if the monetary authority switches from current to forward-looking or backward-looking interest-rate rules.

4.5.2 An Economy with Both Fiscal and Interest Rate Feedback Rules

We now consider the more realistic case in which lump sum taxes are unavailable and the fiscal authority has to levy distortionary income taxes to finance public expenditures. In particular, total tax receipts are given by $\tau_t = \tau_t^D y_t$. The point of reference to perform policy evaluation is again the Ramsey optimal policy. As in Schmitt-Grohé and Uribe (2007), we impose that in the steady state of the Ramsey equilibrium the debt-to-GDP ratio be 44 percent annually. Given this restriction,

the Ramsey steady state implies an income tax rate, τ^D , equal to 21.7 percent. And, the Ramsey steady state rate of inflation is -0.02 percent per year.

Over the business cycle the government commits to the fiscal and interest-rate rules given in equations (4.6) and (4.5), respectively. In this case we find the following optimal policy rule combination

$$\ln(R_t/R^*) = 3 \ln(\pi_t/\pi^*) + 0.0009 \ln(y_t/y^*) + 1.21 \ln(R_{t-1}/R^*) \quad (4.21)$$

and

$$\tau_t - \tau^* = 0.22(l_{t-1} - l^*), \quad (4.22)$$

The main characteristics of optimized policy in this economy are very much similar to those obtained in the economy with lump-sum taxes and no fiscal feedback rule. Similar to Schmitt-Grohé and Uribe 2007, the optimized interest-rate rule features a strong response to inflation and a very very weak (almost zero) response to output. Also, the optimized fiscal rule calls for a very weak response to increases in government liabilities. The optimized regime yields a level of welfare that is very close to that implied by the Ramsey optimal policy. However, the response to the last period's interest rate is quite different in our model. In Schmitt-Grohé and Uribe 2007, the value of the parameter α_R is less than one which implies that the optimized rule is backward looking. In our model, $\alpha_R = 1.21 > 1$ implies that the optimal rule is forward looking. The welfare cost of the optimized policy relative to

the Ramsey policy conditional on the initial state being the deterministic Ramsey steady state is only 0.0100 percent of consumption per period. Finally, inflation is very stable under the optimal policy rule. The standard deviation of inflation is only 6 basis points per year.

4.6 Conclusion

In this chapter we compute welfare maximizing monetary and fiscal feedback rules, in a dynamic general equilibrium model with learning-by-doing and sticky prices. The government makes exogenous final good purchases, levies a distortionary income tax, and issues nominal one non-state contingent one period bonds. The main criteria for choosing the policy rules are simplicity and implementability. Simplicity implies that the rules must be ones where policy variables such as the nominal interest rates, and taxes are set as a function of a few number of observable aggregates such as output, inflation, and government debt. Policy rules are implementable if they are associated with a unique rational expectations equilibrium. The optimal rule is the rule that maximizes welfare of the individual agent. Within the class of simple and implementable rules we find that: first, the optimal interest-rate rules call for a very strong response to inflation and a very weak response to output. Second, the optimized interest rate rule is superinertial (i.e., α_R exceeds unity) means that the

monetary authority is forward looking. Third, the optimal monetary and fiscal rule combination attains a level of welfare which is very close to level of welfare generated by the Ramsey-optimal policy. Finally, the optimized rules induce a stable rate of inflation, a feature also characterized by the Ramsey policy. In the next chapter, we summarize the main findings of this research project and conclude the thesis.

Chapter 5

Conclusion

In this thesis we augment standard imperfectly competitive Ramsey models by incorporating learning by doing mechanism, and nominal price rigidities and show that learning-by-doing mechanism matters for both optimal fiscal and monetary policy making. Some of our results turn conventional wisdom from traditional Ramsey models on its head¹.

In chapter 2, we characterize the optimal capital and labor income taxation in a real imperfectly competitive Ramsey model in which the organization capital is embodied in the firms workers. We find that optimal capital income tax is significantly positive in the long run. This is a very important and remarkable result as

¹We must admit that more sensitivity testing would have been nice but was beyond the thesis's scope.

conventional wisdom suggests a subsidy on capital income if the product market is characterized by imperfect competition. We have argued that the key driving force behind the capital income tax result is the implicit stock nature of the labor input.

In chapter 3, we study optimal fiscal and monetary policy in an economy in which firms learn from their production experience and face price adjustment costs. The benevolent planner has access to labor income taxes, nominally risk-free debt, and money creation. Money demand is motivated through a cash-in-advance constraint on a subset of goods purchased by consumers. Two main results emerge. First, optimal Ramsey inflation is very stable and persistent over the business cycle. second, optimal tax policy is counter-cyclical - tax rates fall during recession and rise during boom.

Finally, in chapter 4, we employ the model of chapter 3 to characterize optimal interest-rate and fiscal feedback rules that best implement Ramsey-optimal stabilization policy. We find that the optimal interest-rate rule is active inflation, mute in output, and super inertial. The optimized fiscal rule is not so active as tax revenues increase only mildly in response to increases in government liabilities. These rules achieve virtually the same level of welfare as the Ramsey optimal policy.

Appendix

1 Computation of Ramsey Equilibria

This appendix describes the procedure for solving for the steady state and the dynamics implied by the Ramsey equilibrium of a DSGE macroeconomic model as applied in Schmitt-Grohé and Uribe (2006), Schmitt-Grohé and Uribe (2007) and described in Schmitt-Grohé and Uribe (2009). The procedure yields an exact numerical solution for the steady state and second-order accurate dynamics.

Given a policy regime, described by the process $\{\tau_t\}$, the competitive equilibrium conditions of a model, such as the one in chapter 3, can be written as

$$E_t \mathbf{C}(x_t, y_t, \tau_t, s_t, x_{t+1}, y_{t+1}, \tau_{t+1}, s_{t+1}) = 0, \quad (\text{A.1})$$

$$s_{t+1} - s_t = \rho(s_t - s) + \eta\sigma\epsilon_{t+1} \quad (\text{A.2})$$

where x_t is an $n_x \times 1$ vector of endogenous predetermined variables, y_t is an $n_y \times 1$ vector of endogenous nonpredetermined variables, τ_t is an $n_\tau \times 1$ vector of policy instruments chosen by the Ramsey Planner/government, s_t is an $n_s \times 1$ vector of exogenous predetermined variables, and ϵ_t is an $n_\epsilon \times 1$ vector of exogenous i.i.d. innovations with mean zero and unit standard deviations. The matrix of parameters ρ is of order $n_s \times n_s$, the vector of parameters s is of order $n_s \times 1$, the matrix of parameters η is of order $n_s \times n_\epsilon$, and $\sigma > 0$ is a scalar scaling the amount of uncertainty in the economy. The function \mathbf{C} maps $\mathbb{R}^{n_y+n_x+n_\tau+n_s}$ into $\mathbb{R}^{n_y+n_x}$.

The period- t objective function of the Ramsey planner is the utility function of the representative household which is given by $U(y_t)$ for all t .²

The Ramsey planner discounts time at the rate $\beta \in (0, 1)$. The portion of the Lagrangian associated with the Ramsey problem that is relevant for the purpose of computing optimal policy from the timeless perspective is given by

$$\begin{aligned} \mathcal{L} = & \dots + U(y_t) + \beta E_t U(y_{t+1}) \\ & + \beta^{-1} \Lambda'_{t-1} \mathbf{C}(x_{t-1}, y_{t-1}, \tau_{t-1}, s_{t-1}, x_t, y_t, \tau_t, s_t) \\ & + \Lambda'_t E_t \mathbf{C}(x_t, y_t, \tau_t, s_t, x_{t+1}, y_{t+1}, \tau_{t+1}, s_{t+1}) \\ & + \beta E_t \Lambda'_{t+1} \mathbf{C}(x_{t+1}, y_{t+1}, \tau_{t+1}, s_{t+1}, x_{t+2}, y_{t+2}, \tau_{t+2}, s_{t+2}) + \dots \end{aligned}$$

Let Θ_t denote the vector of variables that the Ramsey planner chooses and are

²Depending on the type of preference, x_t, τ_t , and s_t can also enter in the utility function.

realized in period t . The vector Θ_t is given by

$$\Theta_t = \begin{bmatrix} x_{t+1} \\ y_t \\ \tau_t \end{bmatrix}$$

The first-order condition of the Ramsey planner with respect to Θ_t is given by

$$\frac{\partial \mathcal{L}}{\partial \Theta_t} = 0, \text{ or}$$

$$\frac{\partial U(t)}{\partial \Theta_t} + \beta \frac{\partial U(t+1)}{\partial \Theta_t} + \beta^{-1} \Lambda'_{t-1} \frac{\partial C(t-1)}{\partial \Theta_t} + \Lambda'_t E_t \frac{\partial C(t)}{\partial \Theta_t} + \beta E_t \Lambda'_{t+1} \frac{\partial C(t+1)}{\partial \Theta_t} = 0 \quad (\text{A.3})$$

The first-order condition with respect to the vector of Lagrange multipliers, Λ_t , is $\frac{\partial \mathcal{L}}{\partial \Lambda_t} = 0$, or equation (A.1). Obtaining the set of Ramsey equilibrium conditions given in equation (A.3) can be extremely tedious if done manually. Therefore, as in Schmitt-Grohé and Uribe (2007), we derive those equilibrium conditions analytically using Matlab's symbolic math toolbox.

1.1 OLS Approach To Computing the Ramsey Steady State

In a deterministic steady state $\sigma = 0$ and all endogenous and exogenous variables are constant. The Ramsey equilibrium conditions, then simplify to:

$$A(x, y, \tau; s) + B(x, y, \tau; s)\Lambda = 0 \quad (\text{A.4})$$

and

$$C(x, y, \tau; s) = 0 \tag{A.5}$$

where $A(x, y, \tau; s)'$ is the steady state of $\frac{\partial U(t)}{\partial \Theta_t} + \beta \frac{\partial U(t+1)}{\partial \Theta_t}$, $B(x, y, \tau; s)'$ is the steady state of $\beta^{-1} \Lambda'_{t-1} \frac{\partial \mathbf{C}(t-1)}{\partial \Theta_t} + \Lambda'_t E_t \frac{\partial \mathbf{C}(t)}{\partial \Theta_t} + \beta E_t \Lambda'_{t+1} \frac{\partial \mathbf{C}(t+1)}{\partial \Theta_t}$, and $C(x, y, \tau; s)$ is the steady state of $\mathbf{C}(t)$. The goal is to obtain the steady-state values of x_t , y_t , and τ_t . The algorithm Schmitt-Grohé and Uribe (2004b) and Schmitt-Grohé and Uribe (2007) propose consists in first constructing a non-negative function $\Omega(\tau)$ mapping \mathbb{R}^{n_τ} into \mathbb{R}^+ . Once the function $\Omega(\tau)$ has been constructed, a numerical minimization package is used to find that value of τ that minimizes $\Omega(\tau)$. Essentially, the function $\Omega(\tau)$ measures the distance between the steady state value of $\frac{\partial \mathcal{L}}{\partial \Theta_t}$ and zero, given τ and given x and y such that the steady state value of $\frac{\partial \mathcal{L}}{\partial \Lambda_t}$ is zero.

1.2 Computing Ramsey Dynamics

The complete set of Ramsey equilibrium conditions is given by (A.1), (A.39), and (A.3), which is a system of $2n_y + 2n_x + n_\tau + n_s$ stochastic difference equations in the $2n_y + 2n_x + n_\tau + n_s$ variables x_{t+1} , y_t , τ_t , s_{t+1} , and Λ_t . We compute first- and second-order accurate approximations to the set of Ramsey equilibrium conditions using the perturbation package described in Schmitt-Grohé and Uribe (2004c). The equilibrium dynamics are approximated around the Ramsey steady state obtained

in the previous section. The procedure involves expanding the system given in equations (A.1) and (A.3).

2 Equilibrium Conditions and the Steady State:

Chapter-2

This appendix presents the complete set of equilibrium conditions and derives the steady state values for all endogenous variables, given values for policy variables, τ^k and τ^n , for the model in chapter 2.

2.1 Complete Set of Equilibrium Conditions

A competitive equilibrium is a set of plans $\{ c_t, n_t, k_{t+1}, h_{t+1}, i_t, \tau_t^n, \tau_t^k, b_t, mc_t, \lambda_t, \Psi_t, \text{ and } R_t \}$ that remain bounded in some neighborhood around the deterministic

steady-state and satisfy the conditions

$$y_t - z_t k_t^\alpha n_t^{1-\alpha} h_t^\theta = 0 \quad (\text{A.6})$$

$$h_{t+1} - (1 - \delta^h) h_t - h_t^\gamma n_t^\varepsilon = 0 \quad (\text{A.7})$$

$$k_{t+1} - (1 - \delta) k_t - i_t = 0, \quad (\text{A.8})$$

$$\Psi_t - \frac{\beta u_{c,t+1}}{u_{ct}} \left[m c_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} + \Psi_{t+1} \{ (1 - \delta^h) + \gamma h_{t+1}^{\gamma-1} n_{t+1}^{1-\gamma} \} \right] = 0 \quad (\text{A.9})$$

$$m c_t - \frac{\eta - 1}{\eta} = 0 \quad (\text{A.10})$$

$$U_{ct} - \lambda_t = 0 \quad (\text{A.11})$$

$$-U_{nt} - \lambda_t (1 - \tau_t^n) \left[m c_t (1 - \alpha) \frac{y_t}{n_t} + \Psi_t \varepsilon n_t^{\varepsilon-1} h_t^\gamma \right] = 0 \quad (\text{A.12})$$

$$\lambda_t - \beta \lambda_{t+1} \left[(1 - \tau_{t+1}^k) m c_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta^k \right] = 0 \quad (\text{A.13})$$

$$\lambda_t - \beta \lambda_{t+1} R_t = 0 \quad (\text{A.14})$$

$$g_t + R_{t-1} b_{t-1} - b_t - \tau_t^n w_t n_t - \tau_t^k r_t k_t = 0 \quad (\text{A.15})$$

$$c_t + g_t + i_t - z_t k_t^\alpha n_t^{1-\alpha} h_t^\theta = 0 \quad (\text{A.16})$$

given initial values for h_0 , k_0 , and b_{-1} and exogenous processes g_t and z_t . Here u_{ct} , and u_{nt} denote the marginal utilities for c_t , n_t , respectively. Note that in chapter 2 we impose CRTS in the production technology for organizational capital accumulation and assume $\varepsilon = 1 - \gamma$. Given our utility function these marginal utilities are given

by

$$U_{ct} = [c_t - \rho n_t^v]^{-\sigma}$$

$$U_{nt} = [c_t - \rho n_t^v]^{-\sigma} (v \rho n_t^{v-1})$$

Combination of (A.11) and (A.17) gives

$$v \rho n_t^{v-1} = (1 - \tau_t^n) \left[m c_t (1 - \alpha) \frac{y_t}{n_t} + \Psi_t \varepsilon n_t^{\varepsilon-1} h_t^\gamma \right] \quad (\text{A.17})$$

2.2 Steady State Given τ^k and τ^n

We impose the steady state by omitting all the time subscripts from the above system of equations. Then, we compute the steady state values for all endogenous variables given values for R and τ^n . From (A.62) From (A.14) and (A.10)

$$R^{ss} = \frac{1}{\beta} \quad (\text{A.18})$$

$$m c^{ss} = \frac{\eta - 1}{\eta} \quad (\text{A.19})$$

Solving the steady state equivalent of equation (A.13) we obtain

$$(k/y)^{ss} = \frac{\alpha(1 - \tau^k) m c^{ss}}{1/\beta - 1 + \delta^k} \quad (\text{A.20})$$

From (A.7) and (A.16)

$$h = n^{\frac{\varepsilon}{1-\gamma}} \delta^{h \frac{1}{\gamma-1}} \quad (\text{A.21})$$

$$(c/y)^{ss} = 1 - \delta^k (k/y)^{ss} - s_g,$$

where s_g is the state (g/y) ratio and we assume that $s_g = 0.2$. Solving the steady state equivalent of equation (A.9) and combining with (A.21) we get

$$\Psi = \frac{\beta m c^{ss\theta}}{1 - \beta(1 - \delta^h) - \beta\gamma\delta^h} \frac{y}{h} \tag{A.22}$$

And combining (A.6) and (A.21) we have

$$y = (k/y)^{ss \frac{\alpha}{1-\alpha}} n^{1 + \frac{\theta\varepsilon}{(1-\gamma)(1-\alpha)}} \delta^{h \frac{\theta}{(\gamma-1)(1-\alpha)}} \tag{A.23}$$

Now substituting (A.22) and (A.23) in (A.17) and solving for n yields

$$n^{ss} = \left[\frac{1}{v\rho} (1 - \tau^n) \left\{ m c^{ss} (1 - \alpha) + \varepsilon \delta^h \frac{\beta m c^{ss\theta}}{1 - \beta(1 - \delta^h) - \beta\gamma\delta^h} \right\} (k/y)^{ss \frac{\alpha}{1-\alpha}} \delta^{h \frac{\theta}{(\gamma-1)(1-\alpha)}} \right]^{\frac{1}{v-1-\theta}} \tag{A.24}$$

Now the steady state values of all other endogenous variables follow directly

$$h^{ss} = n^{ss} \frac{\epsilon}{1-\gamma} \delta^h \frac{1}{\gamma-1} \quad (\text{A.25})$$

$$y^{ss} = (k/y)^{ss} \frac{\alpha}{1-\alpha} n^{ss} + \frac{\theta \epsilon}{(1-\gamma)(1-\alpha)} \delta^h \frac{\theta}{(\gamma-1)(1-\alpha)} \quad (\text{A.26})$$

$$\Psi^{ss} = \frac{\beta m c^{ss} \theta}{1 - \beta(1 - \delta^h) - \beta \gamma \delta^h} \frac{y^{ss}}{h^{ss}} \quad (\text{A.27})$$

$$k^{ss} = (k/y)^{ss} y^{ss} \quad (\text{A.28})$$

$$i^{ss} = \delta^k k/y^{ss} \quad (\text{A.29})$$

$$c^{ss} = (c/y)^{ss} y^{ss} \quad (\text{A.30})$$

$$g^{ss} = s_g y^{ss} \quad (\text{A.31})$$

$$\lambda^{ss} = [c^{ss} - \rho n^{ss}]^{-\sigma} \quad (\text{A.32})$$

$$b^{ss} = [\tau^n w^{ss} n^{ss} + \tau^k r^{ss} k^{ss} - g^{ss}] / (R^{ss} - 1) \quad (\text{A.33})$$

where, $w^{ss} = m c^{ss} (1 - \alpha) \frac{y^{ss}}{n^{ss}} + \Psi^{ss} \epsilon n^{ss \epsilon - 1} h^{ss \gamma}$ and $r^{ss} = m c^{ss} \alpha \frac{y^{ss}}{k^{ss}}$.

3 Equilibrium Conditions and the Steady State:

Chapter-3

This appendix presents the complete set of equilibrium conditions and derives the steady state values for all endogenous variables, given values for policy variables, R and τ^n , for the model in chapter 3.

3.1 Complete Set of Equilibrium Conditions

A stationary competitive equilibrium is a set of processes c_{1t} , c_{2t} , n_t , h_{t+1} , y_t , Ψ_t , mc_t , π_t , λ_t , b_t , ϕ_t , τ_t^n , and R_t for $t = 0, 1, \dots$ that remain bounded in some neighborhood around the deterministic steady-state and satisfy the conditions

$$y_t - z_t n_t^\alpha h_t^\theta = 0 \quad (\text{A.34})$$

$$h_{t+1} - (1 - \delta^h)h_t - h_t^\gamma y_t^\varepsilon = 0 \quad (\text{A.35})$$

$$\Psi_t - \frac{\beta u_{1t+1}}{u_{1t}} \left[mc_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} - \Psi_{t+1} \{ (1 - \delta^h) - \gamma h_{t+1}^{\gamma-1} y_{t+1}^\varepsilon \} \right] = 0 \quad (\text{A.36})$$

$$\left[mc_t - \frac{\eta - 1}{\eta} \right] \eta y_t - \varphi (\pi_t - \pi) \pi_t + \varphi E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} (\pi_{t+1} - \pi) \pi_{t+1} \right] - \Psi_t \eta \varepsilon h_t^\gamma y_t^\varepsilon = 0 \quad (\text{A.37})$$

$$u_{1t} - \phi_t - \beta E_t \lambda_{t+1} = 0 \quad (\text{A.38})$$

$$u_{2t} - \beta E_t \lambda_{t+1} = 0 \quad (\text{A.39})$$

$$u_{3t} + \beta E_t \left[\lambda_{t+1} (1 - \tau_t^n) mc_t \alpha \frac{y_t}{n_t} \right] = 0 \quad (\text{A.40})$$

$$\lambda_t \pi_t - \phi_t + \beta E_t \lambda_{t+1} = 0 \quad (\text{A.41})$$

$$1 - R_t E_t \left[\frac{\beta \lambda_{t+1}}{\lambda_t} \frac{1}{\pi_t} \right] = 0 \quad (\text{A.42})$$

$$c_{1t+1} \pi_{t+1} + b_{t+1} \pi_{t+1} + \tau_t^n mc_t \alpha y_t - c_{1t} - R_t b_t - g_t = 0 \quad (\text{A.43})$$

$$c_{1t} + c_{2t} + g_t + \frac{\varphi}{2} (\pi_t - \pi)^2 - y_t = 0 \quad (\text{A.44})$$

given initial values for h_0 and b_{-1} and exogenous processes g_t and z_t . Here u_{1t} , u_{2t} , and u_{3t} denote the marginal utilities for c_{1t} , c_{2t} , and n_t , respectively. For the

functional form of the utility function in our model ($u(c_t, n_t) = \ln c_t - \frac{\zeta}{1+\mu} n_t^{1+\mu}$

where, $c_t = [(1 - \sigma)c_{1t}^v + \sigma c_{2t}^v]^{\frac{1}{v}}$, these marginal utilities are given by

$$u_{1t} = \frac{(1 - \sigma)c_{1t}^{v-1}}{[(1 - \sigma)c_{1t}^v + \sigma c_{2t}^v]} \quad (\text{A.45})$$

$$u_{2t} = \frac{\sigma c_{2t}^{v-1}}{[(1 - \sigma)c_{1t}^v + \sigma c_{2t}^v]} \quad (\text{A.46})$$

$$u_{3t} = -\zeta n_t^\mu \quad (\text{A.47})$$

Solving (A.41) for ϕ and substituting in (A.38) yields,

$$u_{1t} = \lambda_t \pi_t \quad (\text{A.48})$$

Now, combining (A.48) with (A.42) gives,

$$1 = R_t E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right] \quad (\text{A.49})$$

Also, combination of (A.39), (A.48), and (A.42) yields,

$$R_t = \frac{u_{1t}}{u_{2t}} \quad (\text{A.50})$$

Finally, combination of (A.39) and (A.40) implies,

$$\frac{u_{3t}}{u_{2t}} = (1 - \tau_t^n) m c_t \alpha \frac{y_t}{n_t} \quad (\text{A.51})$$

Substituting (A.45) and (A.46) in equation (A.50), we have

$$\frac{(1 - \sigma)c_{1t}^{v-1}}{\sigma c_{2t}^{v-1}} = R_t \quad (\text{A.52})$$

$$\Rightarrow c_{1t} = \left(\frac{\sigma}{1 - \sigma} \right)^{\frac{1}{v-1}} R_t^{\frac{1}{v-1}} c_{2t} \quad (\text{A.53})$$

Using this relationship between c_{1t} and c_{2t} , we can rewrite (A.45) and (A.46) as

$$u_{1t} = \frac{1}{c_{1t} \left[1 + \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1}{1-\nu}} R_t^{\frac{\nu}{1-\nu}} \right]} \quad (\text{A.54})$$

$$u_{2t} = \frac{1}{c_{2t} \left[1 + \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1}{\nu-1}} R_t^{\frac{\nu}{\nu-1}} \right]} \quad (\text{A.55})$$

After all these substitutions, the competitive equilibrium can be redefined by the following reduced system of equations

$$y_t - z_t n_t^\alpha h_t^\theta = 0 \quad (\text{A.56})$$

$$h_{t+1} - (1 - \delta^h) h_t - h_t^\gamma y_t^\varepsilon = 0 \quad (\text{A.57})$$

$$\Psi_t - \frac{\beta u_{1t+1}}{u_{1t}} \left[m c_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} - \Psi_{t+1} \{ (1 - \delta^h) - \gamma h_{t+1}^{\gamma-1} y_{t+1}^\varepsilon \} \right] = 0 \quad (\text{A.58})$$

$$\left[m c_t - \frac{\eta - 1}{\eta} \right] \eta y_t - \varphi (\pi_t - \pi) \pi_t + \varphi E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} (\pi_{t+1} - \pi) \pi_{t+1} \right] - \Psi_t \eta \varepsilon h_t^\gamma y_t^\varepsilon = 0 \quad (\text{A.59})$$

$$c_{1t} - \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1}{\nu-1}} R_t^{\frac{1}{\nu-1}} c_{2t} = 0 \quad (\text{A.60})$$

$$-\frac{u_{3t}}{u_{2t}} - (1 - \tau_t^n) m c_t \alpha \frac{y_t}{n_t} = 0 \quad (\text{A.61})$$

$$1 - R_t E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right] = 0 \quad (\text{A.62})$$

$$c_{1t+1} \pi_{t+1} + b_{t+1} \pi_{t+1} + \tau_t^n m c_t \alpha y_t - c_{1t} - R_t b_t - g_t = 0 \quad (\text{A.63})$$

$$c_{1t} + c_{2t} + g_t + \frac{\varphi}{2} (\pi_t - \pi)^2 - y_t = 0 \quad (\text{A.64})$$

In a competitive equilibrium, these conditions are satisfied by the processes c_{1t} , c_{2t} , n_t , h_{t+1} , y_t , Ψ_t , $m c_t$, π_t , b_t , τ_t^n , and R_t .

3.2 Steady State Given R and τ^n

We impose the steady state by omitting all the time subscripts from the above system of equations. Then, we compute the steady state values for all endogenous variables given values for R and τ^n . From (A.62)

$$\pi^{ss} = \beta R \quad (\text{A.65})$$

From equation (A.58), we get

$$h = y^{\frac{\epsilon}{1-\gamma}} \delta^h \frac{1}{\gamma-1} \quad (\text{A.66})$$

Combining (A.66) with (A.59) yields

$$\Psi = \frac{\beta mc\theta}{1 - \beta(1 - \delta^h) - \beta\gamma\delta^h} \frac{y}{h} \quad (\text{A.67})$$

Now, substituting the value of Ψ in (A.59) and solving for mc we obtain

$$mc^{ss} = \frac{\eta - 1}{\eta} \frac{1}{\left[1 - \frac{\beta\theta\epsilon\delta^h}{1 - \beta(1 - \delta^h) - \beta\gamma\delta^h}\right]} \quad (\text{A.68})$$

Combination of (A.60) and (A.64) gives

$$\left(\frac{c_2}{y}\right)^{ss} = \frac{1 - s_g}{\left[1 + \left(\frac{\sigma}{1-\sigma}\right)^{\frac{1}{v-1}} R_t^{\frac{1}{v-1}}\right]} \quad (\text{A.69})$$

where s_g is the steady state g/y ratio. We impose the restriction that $s_g = 0.2$.

Now, from equation (A.61) we have

$$n^{ss} = \left\{ \frac{\alpha(1 - \tau)mc^{ss}}{\zeta \left[1 + \left(\frac{\sigma}{1-\sigma}\right)^{\frac{1}{v-1}} R_t^{\frac{1}{v-1}}\right] \left(\frac{c_2}{y}\right)^{ss}} \right\}^{\frac{1}{1+\mu}} \quad (\text{A.70})$$

Substituting the value of h from (A.66) in the production function and solving for y we obtain

$$y^{ss} = \left[zn^\alpha \delta^h \frac{\theta}{\gamma-1} \right]^{\frac{1}{1-\frac{\epsilon\theta}{\gamma}}} \quad (\text{A.71})$$

Therefore, we have

$$h^{ss} = y^{ss \frac{\epsilon}{1-\gamma}} \delta^h \frac{1}{\gamma-1} \quad (\text{A.72})$$

$$\Psi^{ss} = \frac{\beta mc^{ss\theta}}{1 - \beta(1 - \delta^h) - \beta\gamma\delta^h} \frac{y^{ss}}{h^{ss}} \quad (\text{A.73})$$

$$c_2^{ss} = \left(\frac{c_2}{y} \right)^{ss} y^{ss} \quad (\text{A.74})$$

$$c_1^{ss} = \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1}{v-1}} R^{\frac{1}{v-1}} c_2^{ss} \quad (\text{A.75})$$

$$g^{ss} = s_g y^{ss} \quad (\text{A.76})$$

And finally, from equation (A.63) we have

$$b^{ss} = [c_1^{ss}(\pi - 1) + \alpha\tau^n mc^{ss}y^{ss} - g^{ss}] / (R - \pi) \quad (\text{A.77})$$

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