

**EMPIRICAL DYNAMIC STOCHASTIC GENERAL
EQUILIBRIUM MODELS**

**ESSAYS ON EMPIRICAL DYNAMIC STOCHASTIC GENERAL
EQUILIBRIUM MODELS**

By

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ABSTRACT

The overall goal of this thesis is to provide a deeper understanding of the role of dynamic stochastic general equilibrium (DSGE) models as foundations upon which empirical work is conducted. This is a very broad topic with a large existing literature. For this purpose, my dissertation focuses on applying the tools and rich structure of DSGE models to answer questions that have hitherto been studied only by using a reduced-form characterization. I have chosen to look at two specific macroeconomic questions of interest: the economic consequences of oil price shocks in Canada and the role of intangible capital (IC) in explaining cyclical dynamics of S&P500 earnings. Chapter 2 look at the economic consequences of oil price shocks in a structural vector autoregressions (VAR) framework. Chapter 3 builds on this by developing an open economy DSGE model to investigate the impact of oil price shocks on the aggregate Canadian economy and to quantify the relative contribution of U.S. and Canadian monetary policy in transmitting oil price shocks. Chapter 4 studies another interesting macroeconomic phenomenon: the excess volatility of aggregate profits. We embed intangible capital into an otherwise standard real business cycle (RBC) model to examine the role of intangible capital in driving cyclical dynamics of S&P500 earnings. A common feature of my papers is the application of Bayesian time series techniques to macroeconomic data to pursue new insights on “the impact of oil price shocks on economic

activities”, “the role of monetary policy in transmitting oil price shocks” in new open economic macroeconomics (NOEM) literature and “intangible capital and corporate earnings” in U.S. business cycle literature.

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PREFACE

The essay in Chapter 4 is co-authored with Professor Alok Johri, McMaster University. Both authors played an equal role in developing all aspects of the paper.

TABLE OF CONTENTS

Abstract	iii
Acknowledgements	v
Preface	vii
List of Figures	xii
List of Tables	xv
1 Introduction	1
2 Oil Price Shocks and Their Transmission Mechanism in Canada:	
a VAR Analysis	12
2.1 Introduction	12
2.2 Stylized Facts of Oil Prices	18
2.3 The Empirical Model	23
2.3.1 Identifications of Structural Shocks	24

2.3.2	Effects of an oil price shock	28
2.3.3	Effects of domestic monetary policy shocks	30
2.3.4	Effects of U.S. monetary policy shocks	33
2.3.5	Sources of Fluctuations in Canadian Macroeconomic Vari- ables	34
2.3.6	The Importance of International Transmission Channels	36
2.3.7	Application: Forecasting Performance	40
2.4	Robustness of the VAR results	42
2.4.1	Traditional Identification Strategy	42
2.5	Conclusion	44

3 The Impact of Oil Price Shocks on Canadian Economy: A

	Structural Investigation	47
3.1	Introduction	47
3.2	The Model Economy	54
3.2.1	Household Sector	54
3.2.2	Inflation, the Real Price of Oil, the Real Exchange Rate and the Terms of Trade	59
3.2.3	International Risk Sharing and Uncovered Interest Parity	61
3.2.4	Domestic Production	62
3.2.5	Domestic Importers	65
3.2.6	Oil Producing Firms	67

3.2.7	Monetary Policy	68
3.2.8	The Foreign Economy	68
3.2.9	Exogenous Processes	69
3.2.10	Market Clearing and Equilibrium	70
3.3	Empirical Implementation	71
3.3.1	Calibration and Prior Specifications	72
3.3.2	Posterior Estimates	77
3.3.3	Effects of Oil Price Shocks	78
3.4	Sensitivity Analysis	94
3.4.1	Sensitivity Analysis with Different Degree of Nominal Rigidities	94
3.4.2	Persistence of the Oil Price Shock	100
3.5	Conclusion	101
4	Intangible Capital, Corporate Earnings and the Business Cy- cle	103
4.1	Introduction	103
4.1.1	Related literature	109
4.2	The Model Economy	112
4.2.1	The Household's problem	112
4.2.2	The Firm's Problem	114
4.2.3	Equilibrium	119

4.2.4	Intangible Capital in Steady State	120
4.3	Empirical Method and Results	123
4.3.1	Data and Specification of Priors	124
4.3.2	Posterior Estimates	128
4.3.3	Model Fit and Marginal Data Densities	130
4.3.4	Explaining key features of business cycles	131
4.3.5	Impulse-Response Dynamics	139
4.4	Sensitivity Analysis	145
4.5	Conclusion	148
5	Conclusion	152
	Appendix	155
1	Data Description	155
2	Implementation of Sign Restrictions	157
3	Calculating Posterior Distribution and Moments	161
	References	177

List of Figures

2.1	Oil Price vs Real/Nominal Exchange Rate	18
2.2	Oil Price vs Real/Nominal Exchange Rate: Filtered	19
2.3	Dynamic Correlations: Oil Price vs. Key Macroeconomic Variables	20
2.4	Impulse Response to a one S.D. in Oil Prices	29
2.5	Impulse Response to a one S.D. in Domestic Interest Rates . .	31
2.6	Impulse Response to a one S.D. in U.S. Interest Rates	32
2.7	Impulse Responses to an Oil Price Shock: the Close Economy	38
2.8	In-Sample Forecasting	40
2.9	Out-of-Sample Forecasting	41
2.10	Impulse Responses to an Oil Price Shock: Choleski Decomposition	44
3.1	Impulse Response to a 5% in Oil Prices: VAR vs. Model (1985:1-2008:3)	80
3.2	Historical Contributions of Oil Price Shocks, 1998:3 - 2008:3 .	82

3.3	Counterfactual Experiment: Keep R constant	90
3.4	Sensitivity Analysis: Cumulative Output Changes under Alternative Policy Rules	96
3.5	Sensitivity Analysis: Price Level Changes under Alternative Policy Rules	97
3.6	Sensitivity Analysis: Cumulative Output Changes under Alternative Foreign Policy Rules	98
3.7	Sensitivity Analysis: Price Level Changes under Alternative Foreign Policy Rule	99
4.1	Earning Dynamics: Simulation Vs Data.	104
4.2	Ratio: I^z/Y	120
4.3	Investment Ratios at the steady state	121
4.4	Capital-output ratio at steady state.	125
4.5	Labor share at steady state.	126
4.6	Cross-correlation between Earning and Output.	134
4.7	Earning Dynamics: Simulation Vs Data.	135
4.8	Sensitivity Analysis: relative volatility of earnings with respect to output.	136
4.9	Sensitivity Analysis: contemporaneous correlation of earnings with respect to output.	136

4.10 Sensitivity Analysis: relative volatility of investment in IC with respect to output.	138
4.11 Lead-Lag Cross Correlation: IC model vs Data.	139
4.12 Impulse Response Function (posterior mean).	140
4.13 Impulse responses: earnings, output and investments in intan- gible capital.	142
4.14 Impulse Response (posterior mean): Solow Residual vs. IC. . .	144
4.15 Sensitivity Analysis: Impulse responses of output and hours with ϵ varying	146
4.16 Sensitivity Analysis: Impulse responses of output and hours with γ varying	147
4.17 Sensitivity Analysis: Impulse responses of earnings with ϵ varying	149
4.18 Sensitivity Analysis: Impulse responses of earnings with γ varying	150

List of Tables

2.1	Dynamic correlations	21
2.2	Sign restrictions	26
2.3	Forecast Error Variance Decomposition: the VAR model	34
2.4	Forecast Error Variance Decomposition for Oil Shocks	37
3.1	The Calibrated Parameters	73
3.2	Prior and Posterior Distribution	75
3.3	The Contribution of Oil Shocks to the Canadian Economic Fluctuations	84
3.4	Sixteen Quarters Cumulative Output Changes	91
3.5	Sixteen Quarters Cumulative Output Changes under Different Monetary Policy Rules	93
3.6	Sensitivity Analysis: Persistence of Oil Price Shocks	100
4.1	Prior Distribution for the Structural Parameters	128
4.2	Posterior Estimates for the Structural Parameters	129

4.3	Goodness of Fit	130
4.4	Autocorrelation Statistics	132
4.5	Second-Order Unconditional Moments	132

Chapter 1

Introduction

Macroeconomists have for a long time tried to explore which mechanisms are important to explain business cycles. The empirical and theoretical analysis of cyclical fluctuations, however, is one of the more debated fields in economics. There are many competing methods for applying quantitative analysis and theories on the sources of business cycle fluctuations.

On the empirical side, the vector autoregression (VAR) methodology developed by Sims (1980), extended by Blanchard and Quah (1989), Bernanke (1986), Sims and Zha (2006) and many others, has become a popular tool in assessing closed-economy/open-economy business cycle fluctuations. A typical VAR model starts with the estimation of its reduced-form, where each variable is expressed as a linear function of its own lagged values, the past values of all other variables in the VAR system and a serially uncorrelated error term.

The error terms in these reduced form regressions are usually correlated across equations, which can be interpreted as the linear combination of underlying orthogonal innovations in the variables. Thus structural interpretations of these surprise innovations require further assumptions about the causal relationship among the variables in the system. Traditionally, structural disturbances are identified via imposing short-run or/and long-run restrictions.

The VAR methodology, however, has been criticized on various grounds. First, criticisms to the nature of traditional identification strategy have repeatedly appeared in the modeling literature. Recent studies include Cooley and LeRoy (1985), Cooley and Dwyer (1998), Cushman and Zha (1997), Canova and Pina (1999) and Chari, McGrattan, and Kehoe (2005).¹ The problems found in this strand of the literature have motivated the development of alternative identification schemes that do not resort to such zero-type identifying restrictions. Faust (1998), Canova and Nicoló (2002), Uhlig (2005) and Peersman (2005) develop an identification strategy by restricting the sign and/or shape of impulse response functions to shocks to overcome the shortcomings of traditional zero restrictions documented in the VAR literature. Specifically, this strategy employs a set of theoretically coherent information to select im-

¹Cooley and LeRoy (1985) criticize the Cholesky decompositions because the recursive structures are hard to obtain in dynamic general equilibrium models. Cooley and Dwyer (1998) and Chari, McGrattan, and Kehoe (2005) question the robustness of the long-run VAR model in identifying structural shocks in answering business cycle questions. Canova and Pina (1999) show that a large class of DSGE models nearly never provide the zero restrictions employed in identifying monetary shocks for SVAR models. Cushman and Zha (1997) show that the recursive Cholesky approach produce empirical anomalies in a small open economy setup when financial variables are included.

pulse response functions from the complete set of orthogonal alternatives obtained from rotations of the contemporaneous impact matrices, rather than relying solely on reduced-form equations and traditional zero restrictions. In Chapter 2, I show that as opposed to the recursive Cholesky approach, a VAR with sign restrictions can jointly identify oil price, domestic supply and foreign and domestic monetary policy shocks without producing empirical anomalies that are observed in past empirical literature on the effects of monetary policy in closed and open economies. Second, the weak theoretical foundation makes VAR models vulnerable to the Lucas Critique for policy simulation because the reduced-form estimates are inconsistent with rational expectations hypothesis. Thus the atheoretical nature of even structural VARs makes this methodology ill-suited to testing one structural interpretation against another.

On the theoretical side, two recent developments in the literature – namely real business cycle models and New Keynesian theory – have led to a large amount of research on DSGE models of rational, forward-looking economic behavior along with well-specified micro-foundations. The main advantage of DSGE models is that they provide a theoretically coherent framework for policy analysis. Such a modeling approach, as opposed to atheoretical VAR models, typically takes an explicit view on the causal relationships between macroeconomic variables. Log-linearized DSGE models produce a state-space form solution, which can be written in a restricted VAR setup (see, for ex-

ample, Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007)). However, as opposed to more flexible VAR methods, the cross-equation restrictions implied by DSGE models are often considered as too “stringent” when taken to the data (see, for example, Chang, Gomes, and Schorfheide (2002)).

Since King, Plosser, and Rebelo (1988), early RBC studies have extensively used the calibration method. This involves first documenting empirical regularities (namely, stylized facts that the model is built to account for), then calibrating the structural parameters (e.g., labor share, depreciation rate, the discount factor and so on) and specifying the dynamic process of the exogenous shocks (e.g., by estimating Solow residuals from a production function to approximate the technology shock process) to match the unconditional moments generated by the model to the unconditional moments observed in the data. Calibrated models are useful in gaining empirical insights while this approach does not provide a full evaluation of model performance. Instead of the calibration method, many researchers subscribe to the generalized method of moments (GMM) to estimate a set of structural parameters that is best chosen to match a subset of empirical moments in U.S. data. (Christiano and Eichenbaum (1992), Burnside, Eichenbaum, and Rebelo (1993) and Johri and Letendre (2007)) Similarly, Ambler, Dib, and Rebei (2004) use the simulated moment method (SMM) to estimate deep parameters in a DSGE model.

More recently, the focus has shifted slightly toward models with a better

empirical fit to improve existing tools for policy evaluation and forecasting analysis. This thesis makes use of Bayesian methods, which have become popular in the DSGE literature, to confront the model directly with the data. Recent work includes Smets and Wouters (2004), Rabanal and Rubio-Ramírez (2005), and Lubik and Schorfheide (2005, 2007). An and Schorfheide (2007) provide a detailed overview of the literature of Bayesian estimation. The basic idea behind Bayesian analysis is to combine information contained in data (likelihood function) with prior information (prior distribution) to form posterior beliefs on the parameters (posterior distribution). For instance, let Θ_i define a vector of deep parameters in a particular model, $\mathcal{M}_i$. In the Bayesian context, the posterior distribution of Θ_i can be thought of as a way of weighting the likelihood information contained in the observed data by the prior density $p(\Theta_i|\mathcal{M}_i)$. Given a prior, the posterior density kernel² of Θ_i can be written as:

$$p(\Theta_i|Y^t, \mathcal{M}_i) \propto \mathcal{L}(Y^t|\Theta_i, \mathcal{M}_i)p(\Theta_i|\mathcal{M}_i)$$

where $\mathcal{L}(Y^t|\Theta_i, \mathcal{M}_i)$ is the likelihood conditional on the observed data, $Y^t = \{y_1, \dots, y_T\}_{t=1}^T$. The sequence of posterior draws can be obtained using Markov

²Note that Bayes' Theorem states that

$$p(\Theta_i|Y^t, \mathcal{M}_i) = \frac{\mathcal{L}(\Theta_i|Y^t, \mathcal{M}_i)p(\Theta_i|\mathcal{M}_i)}{\int \mathcal{L}(\Theta_i|Y^t, \mathcal{M}_i)p(\Theta_i|\mathcal{M}_i)d\Theta_i}$$

But recognizing $\int \mathcal{L}(\Theta_i|Y^t, \mathcal{M}_i)p(\Theta_i|\mathcal{M}_i)d\Theta_i$ is constant for $\mathcal{M}_i$, I only need to be able to evaluate the posterior density up to a proportionate constant using

$$p(\Theta_i|Y^t, \mathcal{M}_i) \propto \mathcal{L}(\Theta_i|Y^t, \mathcal{M}_i)p(\Theta_i|\mathcal{M}_i)$$

Chain Monte Carlo (MCMC) methods. Point estimates of Θ_i can be obtained from calculating the sample mean or median from the simulated Markov chains. Similarly, inferences of Θ_i are derived from computing the percentiles of these posterior draws.

There are several oft-cited advantages of using Bayesian methods. First, in contrast to the estimation of reduced-form equations that suffer from identification problems (see, for example, Leeper and Zha (2000)), this full-information likelihood-based method takes advantages of dynamic general equilibrium models that posit primitive structural parameters. Second, macroeconomic time series are often quite short and the Bayesian approach outperforms GMM and maximum likelihood in small samples in that all available non-sample information can be introduced in the form of prior distributions (Rabanal and Rubio-Ramírez (2005)). Third, numerical maximization of likelihood is often difficult in practice due to identification problems inherent in DSGE models³ and non-linearity in parameters. Thus adding priors introduces “curvature” into the objective function, which allows for easier maximization of the posterior likelihood compared to only the likelihood function (see An and Schorfheide (2007)). Last, Bayesian estimation and model comparison are consistent even when the model is misspecified (see Fernandez-Villaverde and Rubio-Ramirez (2007)). Bayesian methods, however, also suffer from a few drawbacks. First, it is of-

³For example, two structural parameters that enter the model only proportionally will not be recoverable simultaneously. The likelihood function will be flat in some dimensions of the parameter space.

ten difficult to obtain good information to form prior beliefs. Second, prior distributions might substantially influence posterior estimates. For instance, a tightly specified prior can produce a well behaved posterior distribution, even if the likelihood function has little information on the parameters of interest, such as overly small standard errors for the estimated parameters.

My dissertation focuses on applying the tools and rich structure of DSGE models to answer questions that have hitherto been studied only by using a reduced-form characterization. For this purpose, I have chosen to look at two specific macroeconomic questions of interest: the economic consequences of oil price shocks in Canada and the role of intangible capital in explaining cyclical dynamics of S&P500 earnings. A common feature of my work is the application of Bayesian time series techniques to macroeconomic data to pursue new insights on “the impact of oil price shocks on economic activities”, “the role of monetary policy in transmitting oil price shocks” in NOEM literature and “intangible capital and corporate earnings” in U.S. business cycle literature. In addition to DSGE models, part of this analysis is based on more flexible econometric techniques such as VAR models to uncover sources of business cycle fluctuations and the transmission mechanism of structural shocks.

In Chapters 2 and 3, I concentrate on a discussion of the economic consequences of world oil price shocks on an oil-exporting economy. Stunning run-ups and precipitous falls in the dollar price of oil over the past few years

has stimulated renewed interests in the macroeconomic consequences of oil price shocks. From the point of view of an oil-exporting economy, the task of identifying the international channels through which oil price shocks affect its economic performance becomes even more perplexing if one considers the fact that U.S. monetary policy itself influences oil prices and also influences the macroeconomic consequences of these changes in oil prices because the U.S. dollar has served as the sole pricing and settlement currency in oil transaction since 1975. Hence the fluctuating value of the dollar, which is affected by U.S. monetary policy, plays an important role in exacerbating the run-ups and precipitous fall in the world oil prices. On the other hand, systematic reaction of U.S. monetary policy blurs the distinction between the direct impact of oil price shocks on economic activities and the indirect effects through U.S. monetary policy.⁴ Although there is a large volume of literature that addresses the role of U.S. monetary policy in interacting with oil price shocks in a closed-economy context in which exchange rate movement, integrated trade and financial markets are absent,⁵ the literature is largely silent on the issue of whether there is a significant channel of transmission of oil price shocks on an oil-exporting country through the systematic responses of U.S. monetary policy.

⁴Blinder and Rudd (2008) provide an excellent literature review on the nature of the apparent changes in the macroeconomic effects of oil shocks in the late 1990s compared to the oil shocks in 1970s, as well as on some of the possible causes.

⁵Recent studies include Bernanke, Gertler, and Watson (1997), Leduc and Sill (2004), Carlstrom and Fuerst (2004) and others.

In Chapter 2, I examine the macroeconomic effects of oil price shocks and the oil shock transmission mechanism in Canada within a structural VAR characterization. Chapter 2 is a contribution to the growing literature of VAR models with sign restrictions. Specifically, I use a structural VAR with sign restrictions to jointly identify oil price, domestic supply and foreign and domestic monetary policy shocks. This identification strategy is successful in explaining some long standing empirical puzzles of open economics, including the price puzzles and the exchange rate puzzles. Several insights follow from this effort. First, the effects of oil price and monetary policy shocks on macroeconomic variables are consistent with the predictions of a broad set of theoretical models. Second, I find that oil shocks have a stimulative effect on Canadian aggregate demand, appreciate the Canadian dollar, improve the terms of trade and reduce real wages. Third, foreign disturbances, including innovations in oil prices and the U.S. interest rate, have a significant influence on Canadian economic activities. Finally, our empirical analysis indicates that the U.S. interest rate and the bilateral exchange rate both play an important role as channels of transmission of oil price shocks. Chapter 2, however, does not fully single out the contribution of endogenous monetary policy from the impact of oil shocks because many reduced-form estimates are inconsistent with rational expectations hypothesis. That is, the atheoretical nature of even structural VAR make this methodology ill-suited to testing one structural in-

terpretation versus another. In what follows, to make our policy analysis overcome the Lucas Critique and to better understand the mechanism through which oil price shocks affect Canadian economic activities, I push the analysis one step forward by developing an open-economy DSGE model while treating the VAR-based results as the empirical facts that need to be explained.

In Chapter 3, I investigate the economic consequences of oil price shocks using an open-economy DSGE model that incorporates demand for and supply of oil while allowing for interaction between home and foreign monetary policy. I apply Bayesian techniques to estimate this model using Canadian and U.S. data. The use of this estimated model allows us to quantify the relative importance of oil price shocks, domestic policy and U.S. monetary policy as contributing factors to the changes in domestic macroeconomic variables. I show that domestic monetary policy is a key channel that accounts for over 40% of discounted variation in domestic output across a four year horizon after an oil shock. In contrast, the U.S. monetary policy turns out to be of lesser importance in propagating the oil price shocks in a small oil-exporting economy through the international channel.

In Chapter 4 (joint with Alok Johri), I use an estimated DSGE model to look at another macroeconomic question of interests: intangible capital and corporate earnings. Specifically, aggregate corporate profits are highly volatile and procyclical. Most dynamic general equilibrium models of the business

cycle cannot deliver these basic features of the data. In this chapter, I develop a model of the U.S. economy in which firms expend resources to create IC, which serves as an additional input in their production technology. In keeping with the data, the model delivers profits that are many times more volatile than output. An estimated version of the model implies that IC investments are large and pro-cyclical. IC acts as a propagation mechanism, generating inertial responses to shocks. Overall, the model fits the aggregate data much better than a model without IC.

The thesis is organized as follows. In chapter 2, I document stylized facts of oil prices and Canadian macroeconomic variables and examine the macroeconomic effects of oil price shocks and the oil shock transmission mechanism in Canada within a VAR framework. Chapter 3 estimates an open-economy DSGE model and provides the model-based estimates of the relative contribution of each channel in the transmission of oil price shocks to the Canadian economy. Chapter 4 focuses on intangible capital and the dynamic features of aggregate corporate earnings. The last chapter offers conclusions, hinting to potential extensions of the models and summarizing main findings.

Chapter 2

Oil Price Shocks and Their Transmission Mechanism in Canada: a VAR Analysis

2.1 Introduction

One important empirical characteristic of oil prices is that increases in oil prices have been associated with a contemporaneous increase in the federal funds rate. There is a large volume of literature that addresses the role of U.S. monetary policy in interacting with oil price shocks. In light of the contributions of Bernanke, Gertler, and Watson (1997), Leduc and Sill (2004), Carlstrom and Fuerst (2004), Hooker (1996), Hamilton (1996), Hamilton and

Herrera (2004) and others, it seems safe to speak of a consensus on the pattern of the U.S. economy's response to an oil price shock. Nonetheless, empirical evidence on the economic consequences of oil price shocks and their transmission mechanism in small open economies is subject to numerous debates.¹ One of the difficulties encountered is the inability to correctly identify underlying shocks and single out the contributions of transmission channels in an open economy setup.

The most common approach to identify oil price shocks is the Cholesky factorization, where oil prices are allowed to influence all the other macroeconomic variables contemporaneously but where shocks to other variables have no immediate impact on the prices (see Burbidge and Harrison (1984), Blanchard and Galí (2007) and Leduc and Sill (2004)). Kormilitsina (2009) and Pieschacón (2009) identify oil price shocks by imposing strict exogeneity of oil prices. These identification assumptions will clearly be incorrect if economic developments in the country of consideration affect the world oil price contemporaneously. As Blanchard and Galí (2007) point out, it is inappropriate

¹Burbidge and Harrison (1984) estimate a closed-economy VAR for Canada. They find that oil price shocks have negative impact on Canadian industrial production. Mork and Mysen (1994) report a statistically significant negative correlation between oil price increases and GDP growth for the U.S., Canada, France, Germany and Japan over the period 1967 through 1994. Based on more recent data for 1980:1 - 2003:3, however, Cologni and Manera (2008) find that oil price shocks have an insignificant stimulative impact on the Canadian GDP. The relationship between oil prices and the value of the Canadian dollar is also a topic of debate. Amano and van Norden (1995) report a negative relationship between energy prices and the Canadian dollar. That is, higher real energy prices lead to a depreciation of the Canadian dollar. In later work, however, Issa, Lafrance, and Murray (2006) find that such a relationship broke down in the early 1990s.

to consider the real price of oil as an exogenous variable because it equals the dollar price of oil dividing by an endogenous variable, the GDP deflator. Intuitively, because the U.S. dollar is the sole pricing and settlement currency in oil transactions, the fluctuating value of the dollar, which is affected by U.S. monetary policy, plays an important role in exacerbating run-ups and precipitous falls in world oil prices. Therefore, an identification approach that allows for simultaneity between prices of oil and the US funds rate has the potential to generate more credible results.

As documented in many empirical studies in a small open-economy setup, the recursive Cholesky approach also produce empirical anomalies (e.g. “price” and “exchange rate” puzzles) when financial variables are included.² As opposed to a large economy, like the U.S., the monetary authority in a small open economy is likely to respond quickly to the foreign variables while the Cholesky approach that constrains the contemporaneous impact matrix to be triangular fails to allow for these features. Cushman and Zha (1997) provide a solution to these empirical puzzles for Canada by using a non-recursive identification approach that allows monetary policy to react contemporaneously

²Previous empirical research on the effects of monetary policy in closed and open economies found evidence of a number of anomalies, such as the “price” and “exchange rate” puzzles. For example, Sims (1992) analysis five major industrial countries (not including Canada) in VARs. For several countries, the response of the price level are wrong as positive home interest rate innovations are associated with an increase in the price level rather than a decrease (the “price” puzzle). Sims (1992) also observes depreciation of home currency in response to positive home interest rate innovations (the “exchange rate” puzzle). Grilli and Roubini (1995) document that monetary contraction in other G-7 countries is associated with an depreciation of their currency value relative to the U.S. dollar.

to a variety of domestic and foreign variables. Kim and Roubini (2000) follow a similar identification strategy in allowing contemporaneous interaction between domestic monetary policy variables and the exchange rate as well as U.S. interest rates for the non-US G-7 countries to study the effects of monetary policy in an open economy context.

To allow better identification of contemporaneous interactions in a small open economy context, the present chapter uses a structural VAR with sign restrictions to evaluate the effects of oil prices on major Canadian macroeconomic variables, including output, interest rate, real wage rate and CPI inflation. I also include the U.S. interest rate, the bilateral real exchange rate between U.S. and Canada and U.S. output to capture the most important transmission channels through which oil prices may affect Canadian economic activities indirectly. Similar to Canova and Nicoló (2002), Uhlig (2005) and Peersman (2005), my identification scheme is to use a set of theoretically coherent information to select impulse response functions from the complete set of orthogonal alternatives that can be obtained from rotations of the contemporaneous impact matrices. In this way, I jointly identify the oil price, domestic supply and U.S. and domestic monetary policy shocks with mutually exclusive sign restrictions on each shock. As opposed to traditional zero restrictions, my identification scheme appears to be successful in identifying oil price and monetary policy shocks and in resolving the empirical anomalies

associated with the effects of monetary policy shocks found in the literature.

The empirical analysis yields several insights. First, I find that oil price shocks have a stimulative impact on Canadian aggregate demand. Canada's output rises following a positive oil price shock, while U.S. output falls. The oil price shock generates significant hump-shaped responses of inflation and of domestic and U.S. interest rates. The comovement of domestic inflation and output after an oil price shock exhibits the key features of an aggregate demand shock. As would be expected, a positive oil price shock leads to a significant appreciation of Canadian dollars and a fall in the real wage. Second, oil price shocks make an important contribution to the forecast variance of domestic variables. Similar findings can be found in Dib (2008) for Canada, Blanchard and Galí (2007) for the U.S. and Pieschacón (2009) for Mexico and Norway. Third, I find that by 12 quarters after an oil shock, U.S. interest rate disturbances account, respectively, for 35%, 35% and 22% of the forecast variation in exchange rates, the domestic interest rate and inflation, but that they only play a limiting role (on average, less than 10% by 12 quarters) on the forecast variation in Canadian output. This finding is in line with quantitative evidence from reduced-form VAR and the calibrated DSGE models for Canada. (see, for example, Cushman and Zha (1997)³; Mendoza (1991); Schmitt-Grohé

³Cushman and Zha (1997) propose a block exogenous assumption for Canada and therefore treat the foreign block simply to follow a recursive ordering structure. Without identifying each of foreign shock, they conclude that foreign factors, including exports, imports, foreign output, prices, interest rate and commodity prices, are the most important driver of the Canadian business cycle. More than 74% of output fluctuation is accounted for by the

(1998)).

The most important lesson from our empirical analysis is that there is a significant channel of oil shock transmission through the systematic responses of U.S. monetary policy and real exchange rates. To illustrate this point, I estimate another closed-economy Canadian VAR model that ignores the effects of exchange rate and American variables. In terms of the percentage of forecast variance, the comparison between the alternative VAR models reveals that the addition of U.S. variables to the closed-economy version of the Canadian model reduces the effects of oil shocks on the Canadian variables. The difference between the two corresponding percentages is interpreted as the contribution of an oil shock picked up by American variables. I show that a substantial part (roughly 50%) of an oil shock is indirectly transmitted to the Canadian economy through the international channel.

The rest of the chapter is organized as follows. Section 1.2 reviews the stylized facts of oil prices and Canadian economic activities for the period 1980:1 – 2008:3. Section 1.3 discusses the empirical models and results. Section 1.4 checks the robustness of the empirical results. Section 1.5 concludes.

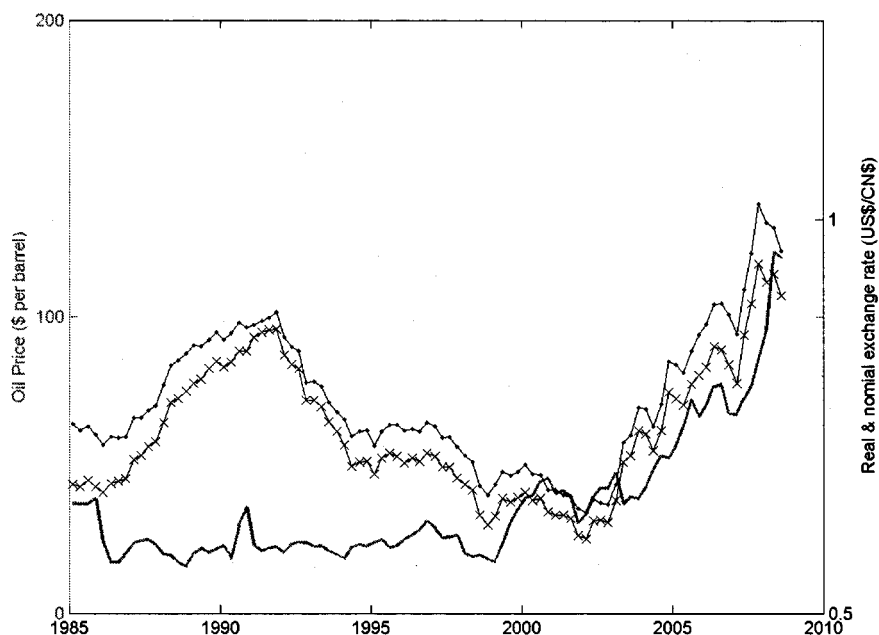


FIG. 2.1 - *The solid line refers to the historical oil prices (\$ per barrel); the crossed line to the nominal exchange rate (US\$/CA\$); the dotted line to the real exchange rates.*

2.2 Stylized Facts of Oil Prices

In this section, I present some stylized facts regarding the nature of the relationship between the oil prices and major U.S. and Canada macroeconomic variables. It is widely believed that oil price fluctuations play an important role in driving the business cycles of small oil-exporting countries. The fluctuations in world oil price are enormously volatile. For instance, the standard deviation of Hodrick-Prescott (HP) filtered (logged) quarterly real oil prices is 15 times larger than Canadian GDP during the post-Bretton Woods era while the relative standard deviation of the real bilateral (US\$/CA\$) exchange rate is roughly 3 times as volatile. Figure 2.1 shows that the relevance of oil prices

for the bilateral Canada-U.S. exchange rate movements appears to be quite weak between the 1980s and the early 1990s, but that since the late 1990s, the appreciation of the Canadian dollar has largely coincided with rising real oil prices. Figure 2.2 displays the same set of variables, which are here detrended by the HP filter. This transformation gives us a better sense of the magnitude of the comovement between oil prices and exchange rates at business-cycle frequencies. Clearly, there is a strong correspondence between oil price and the bilateral exchange rate movements at business-cycle frequencies.

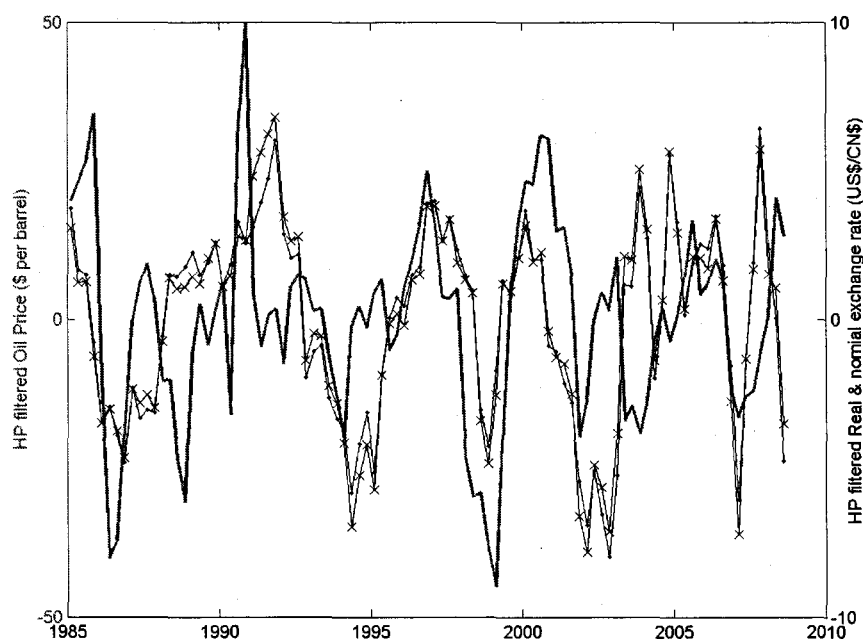


FIG. 2.2 - The solid line refers to the historical oil prices (\$ per barrel); the crossed line to the nominal exchange rate (US\$/CA\$); the dotted line to the real exchange rates.

I estimate the dynamic correlations between the real price of oil and the aggregate variables of interest. I measure the degree of comovement of world oil

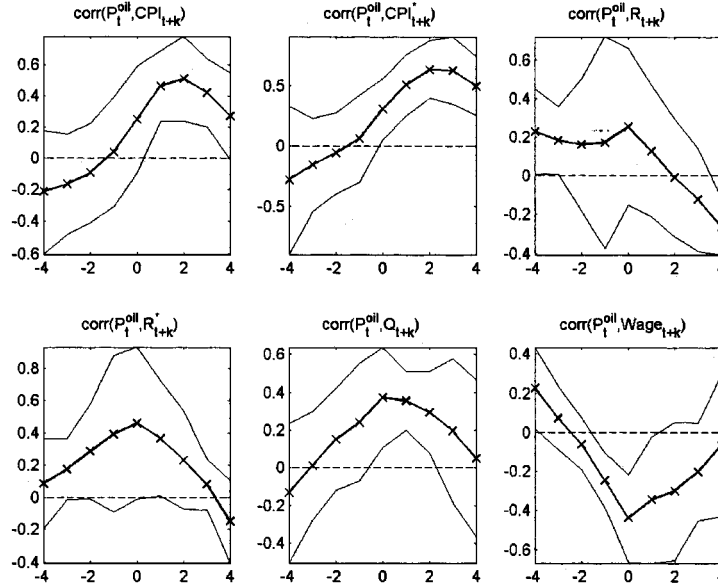


FIG. 2.3 - The line with "x" markers represents the cross-correlation coefficients, while the intervals between the two solid lines contain two standard errors.

prices with the cycle by the correlation coefficient $\rho(j)$, where $j \in \{0, \pm 1, \pm 2, \pm 3, \pm 4\}$.

Information regarding whether the contemporaneous correlation coefficient, $\rho(0)$, is positive, zero or negative can indicate whether oil prices are procyclical, acyclical or countercyclical, respectively.⁴ Moreover, the cross-correlation coefficient $\rho(j)$ provides information about whether the cycle of oil prices is leading, is synchronous, or is lagging the cycle of the reference variables as $|\rho(j)|$ reaches a maximum for a negative, zero or positive j , respectively.

⁴Following Fiorito and Kollintzas (1994) and Serletis and Shahmoradi (2005), I say that the series is strongly contemporaneously correlated, weakly contemporaneously correlated, and contemporaneously uncorrelated with the cycle based on $0.23 \leq |\rho(0)| < 1$, $0.1 \leq |\rho(0)| < 0.23$, $0 \leq |\rho(0)| < 0.1$, respectively. Note that Fiorito and Kollintzas (1994) provide more details on how to determine the cutoff points of 0.1 and 0.23

TABLE 2.1

DYNAMIC CORRELATIONS									
	$j = -4$	$j = -3$	$j = -2$	$j = -1$	$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
<i>(A) $\rho(p_t^{oil}, x_{t+j})$</i>									
CPI	-0.21	-0.16	-0.09	0.04	0.25	0.46	0.51	0.42	0.27
CPI_{us}	-0.28	-0.16	-0.06	0.06	0.31	0.50	0.63	0.62	0.49
Output	0.24	0.31	0.37	0.39	0.23	0.09	-0.05	-0.17	-0.27
GDP_{us}	0.18	0.19	0.24	0.23	0.09	-0.17	-0.31	-0.41	-0.42
R	0.23	0.18	0.16	0.17	0.25	0.12	-0.01	-0.13	-0.26
R_{us}	0.08	0.17	0.28	0.39	0.46	0.36	0.23	0.08	-0.15
US\$/CA\$	-0.13	0.01	0.15	0.24	0.37	0.36	0.29	0.20	0.05
Real Wages	0.23	0.07	-0.06	-0.25	-0.44	-0.34	-0.30	-0.20	-0.07
<i>(B) $corr(r_t, x_{t+j})$</i>									
R^{us}	0.13	0.38	0.58	0.70	0.81	0.72	0.56	0.37	0.17
Inflation(π)	-0.11	0.00	0.14	0.23	0.32	0.31	0.27	0.26	0.11

Note: the cross correlations based on the series detrended by HP filters.

Figure 2.3 and Table 2.1 show the degree of cyclical correlations between the oil price and a comprehensive set of quarterly Canadian aggregate time series by the correlation coefficient $\rho(j)$. Figure 2.3 provides a visual illustration of the magnitude of the dynamic correlations with 95% error bands. Table 2.1 reports the cross correlations based on the HP filter, at lags and leads of one, two, three and four quarters.⁵

Several key features are evident in Table 2.1. First, the inflationary nature of oil prices is apparent. Oil prices are positively contemporaneously correlated with CPI, and lead CPI by two quarters. Second, the oil prices are procyclical and lag the Canadian GDP by one quarter, as indicated by the HP filter, while

⁵The cross-correlations based on the Baxter-King filter display a similar dynamic pattern.

they lag U.S. cycles by two quarters. Third, there are systematic movements of nominal interest rates in relation to oil prices. Oil prices are positively contemporaneously correlated with both the Canadian and U.S. short-term interest rates, suggesting that the central bank may simultaneously react to oil price shocks due to their inflationary nature. Moreover, the contemporaneous correlation of oil prices is strongly positive with the bilateral Canada-U.S. exchange rate (US\$/CA\$), suggesting that a sharp increase in oil prices appreciates the value of the Canadian dollar against its U.S. counterpart. Finally, oil prices are negatively and strongly contemporaneously correlated with the Canadian real wages as indicated in the last row of Panel A in Table 2.1.

Aside from the relationship between oil and macro-variables, Panel B reports the lead-lag relationship between the Canadian short-run interest rate and the U.S. interest rate, as well as Canadian inflation. First, the Canadian interest rates are highly synchronized with the cycle of US interest rates, which suggests, to some extent, the convergency of Canadian monetary policy with U.S. macroeconomic policy. Second, the short-run interest rate has a significant contemporaneous comovement with CPI inflation, which indicates that as an explicit inflation targeter, the Canadian monetary authority typically raises/lowers the short-term interest rate when inflation appears to be above/below target. However, the correlations between inflation and future interest rates are negative ($\text{corr}(\pi_t, r_{t+4}) = -0.11$), which is in line with the

conventional wisdom that interest rates and inflation tend to be inversely related.

2.3 The Empirical Model

In the following section, I use a VAR model to provide more structural evidence on the macroeconomic effects of underlying shocks for Canada. In the VAR specification, I use the following variables: Canadian output, real world oil prices, Canadian CPI inflation, the bilateral Canada-U.S. real exchange rate, real hourly wages, Canadian and U.S. short-term nominal interest rates and U.S. output.⁶ All Canadian data are obtained from CANSIM, Statistics Canada. For the U.S., all data are taken from the database at the Federal Reserve Bank of St. Louis. The oil prices come from the IMF Primary Commodity Prices database. I take the natural logarithm of each variable with the exception of the short-term interest rates and inflations. The VAR model is estimated for the sample period 1980:Q1 - 2008:Q3 with four lags. The real oil prices, domestic output and inflation are included as the focus of our interest. The remaining variables in the model are added to capture the most important transmission channels through which oil prices may affect economic activity indirectly, in part by incorporating changes in domestic monetary policies and U.S. monetary policies. These channels include a variety of demand

⁶See Appendix 1 for a more detailed description of the data.

and supply-side effects of oil prices operating via exchange rates, financial variables and the international trade in goods.

2.3.1 Identifications of Structural Shocks

Consider a structural VAR of the form

$$A_0 Y_t = A(L) Y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, I_m), \quad (2.1)$$

where Y_t is an $m \times 1$ vector of endogenous variables, A_0 is an invertible contemporaneous impact matrix, $A(L)$ denotes a polynomial coefficient matrices of the form $A(L) = \sum_{s=1}^p A_s L^s$, L is the lag operator and ε_t is a vector of structural shocks, which are mutually independent.

In identifying the shocks in Canada, I employ a structural VAR approach with sign restrictions as in Canova and Nicoló (2002), Uhlig (2005) and Peersman (2005).⁷ I take this identification strategy for two reasons. First, this strategy employs a set of theoretically coherent information to select impulse response functions from the complete set of orthogonal alternatives obtained from rotations of the contemporaneous impact matrices, rather than relying solely on reduced-form equations and traditional zero restrictions. The most common approach to identifying oil price shocks is the Cholesky factorization, where oil prices are allowed to influence all the other macroeconomic variables

⁷A discussion of the technicalities of the identification procedure can be found in Appendix 2. For a detailed description of the implementation of sign restrictions, I refer to Canova and Nicoló (2002).

contemporaneously but where shocks to other variables have no immediate impact on the prices (see Burbidge and Harrison (1984), Blanchard and Galí (2007) and Leduc and Sill (2004)). Without imposing such stringent restrictions, Faust (1998), Canova and Nicoló (2002), Uhlig (2005) and Peersman (2005) investigate alternative identification strategies that overcome the shortcomings of traditional zero restrictions. Instead, they use sign restrictions on the impulse response functions or cross-correlation to identify the underlying structural shocks. Second, non-recursive identification structures outperform the Cholesky approach in addressing the empirical anomalies in an open economy setup, such as the “exchange rate puzzle” and the “price puzzle”, for the small open economy when financial variables are included. Cushman and Zha (1997) show that as the monetary authority in a small open economy is likely to respond quickly to the foreign variables, the Cholesky approach fails to allow for these features. In light of this, I impose sign restrictions on the VAR model to allow better identification of contemporaneous interactions, in that there is no need to assume that certain variables affect others only with a lag and that monetary policy shocks in a small open economy can be correctly identified in the absence of empirical puzzles.

Despite the fact that our VAR model has eight variables, I will not have a complete decomposition of a reduced-form shock into all of its structural components, instead identifying only four shocks: oil price shocks, domestic

supply shocks, domestic monetary shocks and U.S. monetary shocks because the effects of oil price, U.S. monetary policy and domestic monetary policy shocks on Canadian economy are the most important ingredients of this chapter.⁸ Therefore, I jointly identify these three shocks with mutually exclusive sign restrictions on each shock. In addition, I also identify the domestic supply shock jointly. Because a positive oil price shock is likely to be confused with a negative domestic supply shock (see, for example, Peersman (2005), and Leduc and Sill (2004)), the inclusion of the domestic supply shock should help to more effectively isolate the oil price shock. Following Peersman (2005), I assume that an oil price shock has the largest contemporaneous impact on oil prices so as to discriminate a supply shock from an oil price shock. Since identifying these shocks of interest is sufficient for the purpose of this paper, I do not impose restrictions on other shocks.

TABLE 2.2
SIGN RESTRICTIONS

	r_{us}	P_{oil}	Y	π	$Wage$	r	$Q_{CA\$/US\$}$	Y_{us}
Oil price shock	≥ 0	≥ 0		≥ 0		≥ 0		≤ 0
U.S. monetary shock	≥ 0					≥ 0	≥ 0	≤ 0
Canada monetary shock			≤ 0	≤ 0		≥ 0	≤ 0	
Canada supply shock			≥ 0	≤ 0		≤ 0	≥ 0	

Note: A " $\geq$ " (or " $\leq$ ") indicates that the impulse response of the variable in VAR is restricted to be non-negative (non-positive) for 4 quarters after a shock. A blank entry indicates that no restriction is imposed on the response.

⁸Similarly, Uhlig (2005) and Canova, Gambetti, and Pappa (2007) employ sign restrictions to identify only a single shock or a subset of shocks and leave the rest of the shocks unidentified.

Table 2.2 summarizes our identification restrictions. The sign conditions are derived from a standard aggregate demand and supply diagram.⁹ In addition, the restrictions are generally consistent with empirical cross-correlations between variables calculated in the empirical section. Consider a positive innovation of the oil price. On the one hand, for Canada, a sharp rise in oil prices appreciates the bilateral exchange rate and real income gains, while on the other hand, the strong Canadian dollar dampens exports, and the higher interest rates deter the aggregate demand and tend to reduce output. As the two forces counter each other, I do not impose any sign restriction on the output responses to an oil price shock, instead letting the data determine the sign of this response. I assume that the oil price shocks must not decrease CPI inflation or the nominal interest rate. As the signs on CPI inflation, interest rates and exchange rates are the same as those for a negative domestic supply shock, oil price shocks are likely to be confused with domestic technology shocks. However, in contrast to an oil price shock, the sign of the response of oil prices to a supply shock is ambiguous. Hence, I do not impose a restriction on the response of oil prices after a supply shock.

The domestic and foreign monetary policy shocks have distinct effects on real domestic activities. I assume that after a contractionary domestic mon-

⁹Similarly, Peersman (2005) and Baumeister and Peersman (2008) derive the sign conditions from a standard aggregate demand and supply scheme. Canova and Nicoló (2002) and Canova, Gambetti, and Pappa (2007) derive sign restrictions from DSGE models to identify structural shocks in the data.

etary policy shock, the response of output and inflation are not positive, and the tightening monetary policy appreciates the domestic currency against its U.S. counterpart. I do not impose sign restrictions on the world oil prices, U.S. interest rates or U.S. output because these variables are assumed to be exogenous to a monetary policy shock originating from the Canadian economy. After an unexpected rise in the U.S. interest rate, I assume that the response of U.S. output is not positive and that the Canadian dollar depreciates against U.S. dollar on impact. According to the uncovered interest rate parity condition and the empirical cross-correlation between the Canadian and U.S. interest rates, I expect a non-negative impact on the domestic interest rate. Furthermore, given the fact that world crude oil prices are denominated in U.S. dollars, there is no immediate increase in the oil price. Note also that I choose the time period over which the prior sign restrictions are binding equal to four quarters and that reducing or increasing this value does not significantly alter the results.

2.3.2 Effects of an oil price shock

Figure 2.4 displays the impulse responses for a horizon of up to 40 quarters after a one-standard-deviation shock to the real oil prices. The solid line denotes the posterior mean impulse response and the shaded area defines the

16th and 84th percentile confidence bands.¹⁰

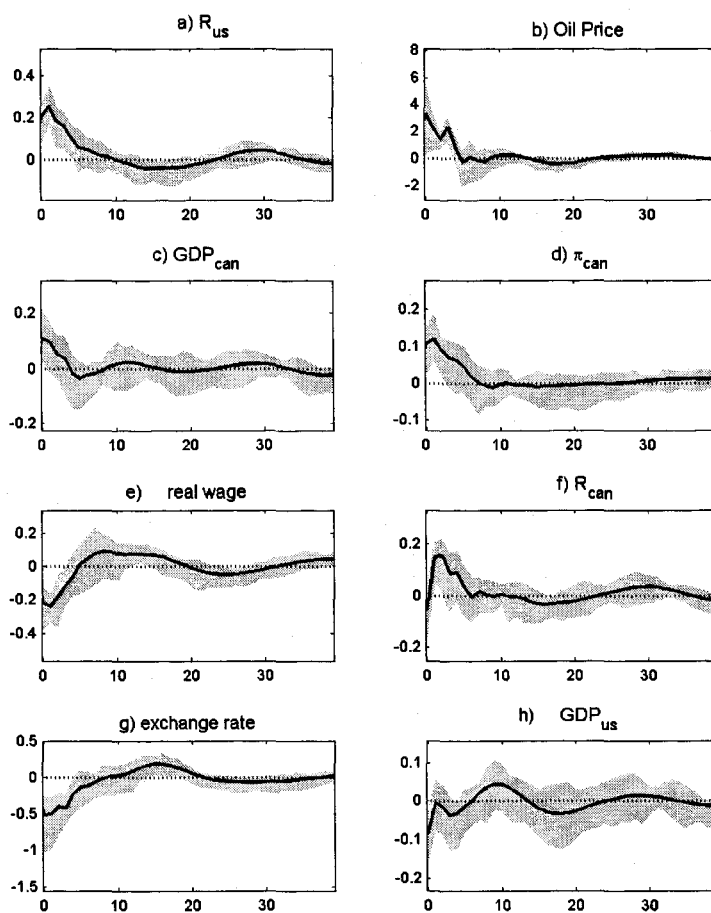


FIG. 2.4 - *Dynamic responses to a positive oil price shock. The shaded area represents the 70% error band.*

As Canada is a net oil exporter, an oil price shock can be thought of as a positive shock to the terms of trade, which would in general lead to an expansion of the domestic economy and generate real income gains. It is

¹⁰The confidence bands are computed by a Bayesian method. As sample distribution of impulse responses may not be normal, the bands is not necessarily centered around the mean of response.

not surprising that the domestic output runs up following an oil shock, while the U.S. output responds negatively to the same shock. A sharp rise in the real oil price is associated with a statistically significant appreciation of the Canadian dollar against its U.S. counterpart and a significant increase in CPI inflation. The monetary authority in Canada and the U.S. endogenously react to a positive oil price shock due to its inflationary effect. Additionally, the real hourly wage rate falls significantly and gradually returns to its pre-shock level after four quarters. Similar finding about real wages can be found in Rotemberg and Woodford (1996) for U.S.

2.3.3 Effects of domestic monetary policy shocks

To check on whether our identification strategy is successful in jointly identifying underlying structural shocks, I now examine the identified Canadian and U.S. monetary policy shocks. Figure 2.5 displays the impulse responses to a domestic tightening monetary policy shock. The unexpected contractionary monetary policy shock is accompanied by an immediate and significant appreciation of the Canadian dollar that lasts about two years - the exchange rate effect. The inflation rate first drops by a small (but statistically significant) amount and remains below the steady state for about three quarters - the price effect. Output shows a statistically significant but relative small decline for about four quarters. The real hourly wage responds gradually and

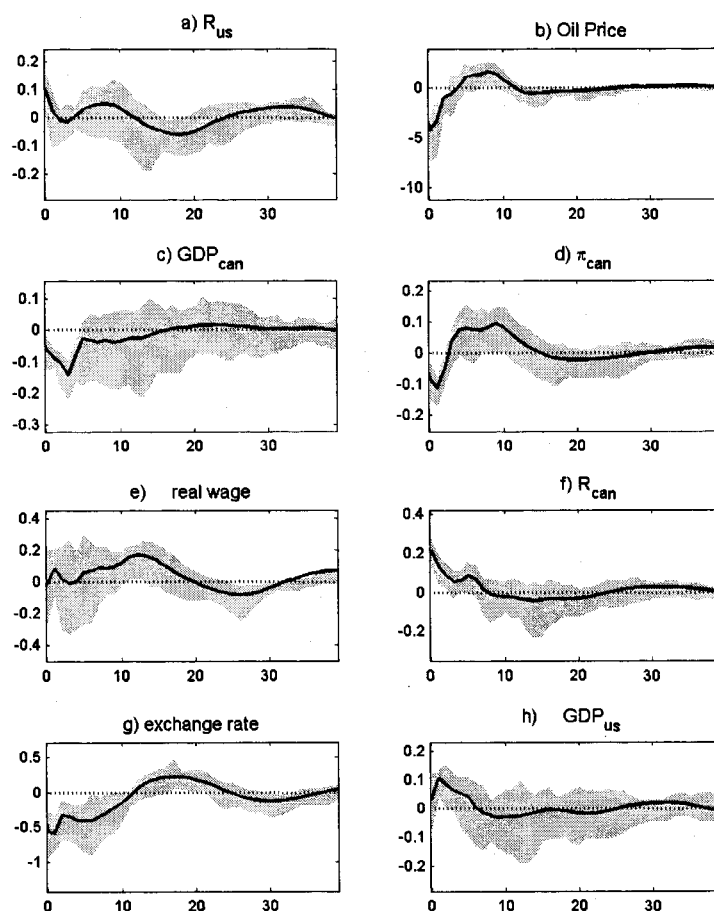


FIG. 2.5 - *Dynamic responses to a contractionary domestic monetary policy shock. The shaded area represents the 70% error band.*

positively, though the response is insignificant. It is not surprising that in case of U.S. fund rates and U.S. output, the reactions of these variables to Canadian monetary disturbances are neither statistically significant nor large. In addition, the monetary policy shock is quite persistent, generating pronounced hump-shaped responses from all of the domestic variables in the system. Our

results are consistent with traditional theoretical predictions that a tightening monetary policy shock has recessionary consequences.

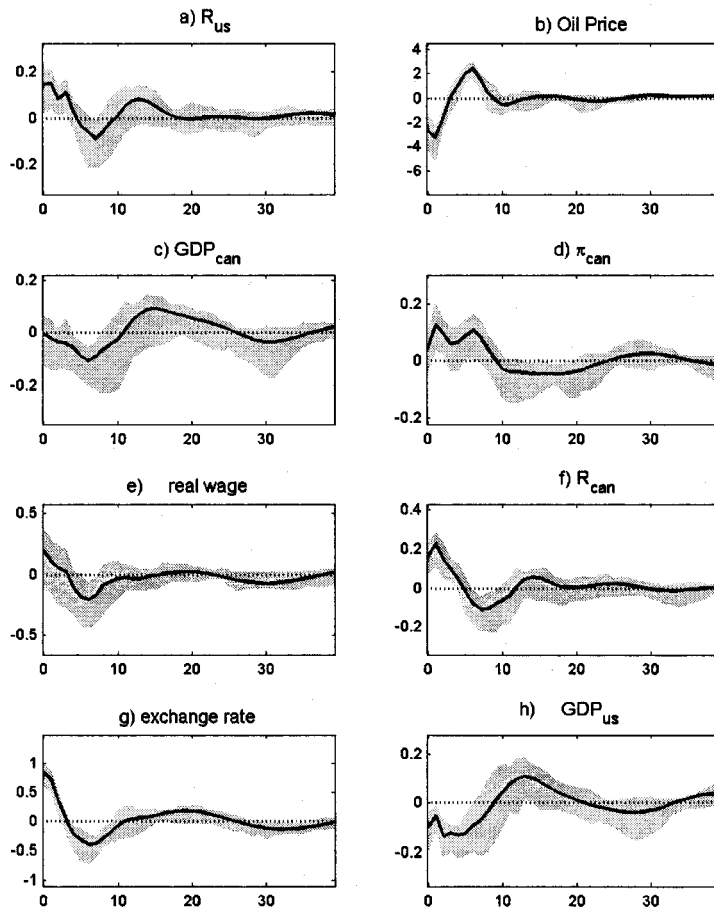


FIG. 2.6 - Dynamic responses to a contractionary U.S. monetary policy shock. The shaded area represents the 70% error band.

2.3.4 Effects of U.S. monetary policy shocks

Finally, I consider the effects of U.S. contractionary monetary policy shocks on the Canadian economy in Figure 2.6. As the U.S. funds rate increases, the Canadian dollar depreciates strongly and significantly relative to the U.S. dollar on impact, but the depreciation is short-lived, as the real exchange rate shows evidence of overshooting. This finding agrees with Cushman and Zha (1997) for Canada and with Kim and Roubini (2000) for G7 countries including Canada. The devaluation of the Canadian dollar is associated with an inflationary effect as the domestic inflation rates tend to increase significantly after the domestic depreciation. However, I also observe that the domestic monetary authority responds strongly to the U.S. rate increases with an interest rate hike, which exerts downward pressure on domestic inflation. As these two forces counter each other, the responses of the CPI inflation rates are statistically insignificant (except for a significant positive response in the second quarter). As the higher interest rate dampens aggregate demand, domestic output falls persistently and significantly before returning to its pre-shock level. A positive innovation in U.S. fund rates has a significant negative impact on real oil prices.

TABLE 2.3
FORECAST ERROR VARIANCE DECOMPOSITION: THE VAR MODEL

Quarter Ahead	P_{oil} shock	R shock	R_{us} shock	Other shocks
<i>(A) Real Exchange Rate</i>				
1	18.19	20.96	47.40	13.46
4	24.47	24.49	36.86	14.18
8	19.14	29.41	35.08	16.37
12	17.90	30.27	35.02	16.82
20	17.15	28.80	30.42	23.63
40	15.87	28.01	30.18	25.93
<i>(B) Domestic Interest Rate</i>				
1	2.59	42.05	24.06	31.29
4	19.62	25.58	38.59	16.21
8	16.46	22.19	33.00	28.35
12	14.69	20.13	34.67	30.50
20	14.45	20.48	33.53	31.53
40	14.98	20.37	31.45	33.20
<i>(C) CPI Inflation</i>				
1	25.77	14.07	4.83	55.33
4	23.26	13.84	18.27	44.63
8	18.29	18.37	23.95	39.39
12	15.90	24.88	22.22	37.00
20	12.89	21.02	22.07	44.02
40	12.26	20.39	22.02	45.33
<i>(D) Real GDP</i>				
1	21.07	6.33	0.06	72.53
4	9.96	16.25	1.45	72.34
8	7.03	12.98	8.86	71.14
12	6.61	12.83	9.69	70.87
20	5.41	10.39	16.15	68.04
40	5.41	9.48	16.91	68.20

2.3.5 Sources of Fluctuations in Canadian Macroeconomic Variables

To assess the role of underlying structural shocks in fluctuations of macroeconomic variables, I also calculate the forecast error variance decomposition

(FEVD) based on the VAR. Table 2.3 shows the contribution of the structural shocks to the variation in real exchange rates, the domestic interest rate, CPI inflation and domestic output, at several horizons $t = 1, 4, 8, 12, 20, 40$. Since I leave some individual shocks unidentified, The label “other shocks” in Table 2.3 refers to the decomposition from a subset of unidentified individual shocks as well as the identified Canadian supply shock. The results of the variance decomposition exercise show several features. First, the oil price shock has an important contribution to the forecast variance of domestic variables. For instance, the oil shock has the highest contribution to the variance of real output and CPI inflation on impact, accounting for 21.07 and 25.77 percent of the total variance, respectively. In the case of the domestic interest rate and the bilateral Canada-U.S. real exchange, the contribution peaks during the fourth quarter after an oil price shock, account for 19.62 and 24.47 percent, respectively. Interestingly, oil shock contributes to a large share of the domestic interest rate fluctuation in longer horizons rather than on impact. This fact might indicate that the changes in oil prices gradually pass-through to the core inflation, which is the target of the Canadian monetary authority. Second, the impact of fluctuation in U.S. fund rates on the Canadian economy is significant. In all cases, by the 20th quarter, U.S. monetary policy shock still explains a large amount of the variance of the Canadian macroeconomic variables. A similar finding can be found in Cushman and Zha (1997). Third,

domestic monetary policy disturbances is an important source of fluctuations in the real exchange rate and CPI inflation. In the case of Canadian output, the contribution of a domestic monetary policy shock peaks after a year, accounting for around 16.3 percent of the total output variance, while the impact on output decreases over longer horizons. This finding agrees with the results in Bernanke and Mihov (1995) and Sims and Zha (2006) for the U.S. and in Cushman and Zha (1997) for Canada. Finally, the oil shock together with U.S. and Canadian monetary policy shocks is an important source of fluctuations in the Canadian economy, especially in the short run.

2.3.6 The Importance of International Transmission Channels

International transmission channels are likely to play an important role in explaining the effects of oil shocks on Canada's economy, but how significant are they, for instance, relative to the direct effects of oil shocks? To single out the indirect effects of oil shock transmission through the international financial and trade channel on the Canadian variables, I estimate another closed-economy VAR model that ignores the effects of U.S. variables and the real exchange rate.¹¹ I apply the same identification strategy with similar sign

¹¹Sims and Zha (2006) and Bernanke, Gertler, and Watson (1997) use the VAR model to uncover the endogenous effects of foreign monetary policy. However, the estimates obtained from counterfactual experiments in a VAR context are highly controversial because many reduced form estimates are inconsistent with rational expectations.

TABLE 2.4
FORECAST ERROR VARIANCE DECOMPOSITION FOR OIL SHOCKS

Quarter Ahead	Real GDP	Interest Rate	CPI Inflation	Real Exchange Rate
<i>(A) Closed-economy: 5-variable system</i>				
1	45.71	46.09	15.52	-
4	15.05	68.28	17.75	-
8	47.76	65.84	19.11	-
12	55.45	69.44	21.15	-
20	52.07	68.75	21.32	-
40	52.38	68.28	21.44	-
<i>(B) Open-economy: 8-variable system</i>				
1	21.07	2.59	25.77	18.19
4	9.96	19.62	23.26	24.47
8	7.03	16.46	18.29	19.14
12	6.61	14.69	15.90	17.90
20	5.41	14.45	12.89	17.15
40	5.41	14.98	12.26	15.87

restrictions. Table 2.4 reports, at several horizons, the contribution of the oil shocks to the forecast error variance of the Canadian variables.

In terms of the percentage of forecast variance, the comparison of VAR systems reveals that the addition of U.S. variables and the bilateral real exchange rate to the closed-economy version of the Canadian model reduces the effects of oil shocks on the Canadian variables. Panel A in Table 2.4 shows that under the closed-economy scenario, oil price shocks account for 52%, 69% and 21% of the forecast variance of the Canadian GDP, interest rates and CPI inflation by 20 quarters, respectively. I interpret the differences between the FEVD results in Panels A and B as the proportion of oil shocks picked up by the U.S. interest rate and real exchange rates. Apparently, the variance decomposi-

tion of Canadian GDP indicates that a substantial part of an oil shock (nearly 50%) is indirectly transmitted to the Canadian economy through international channels.

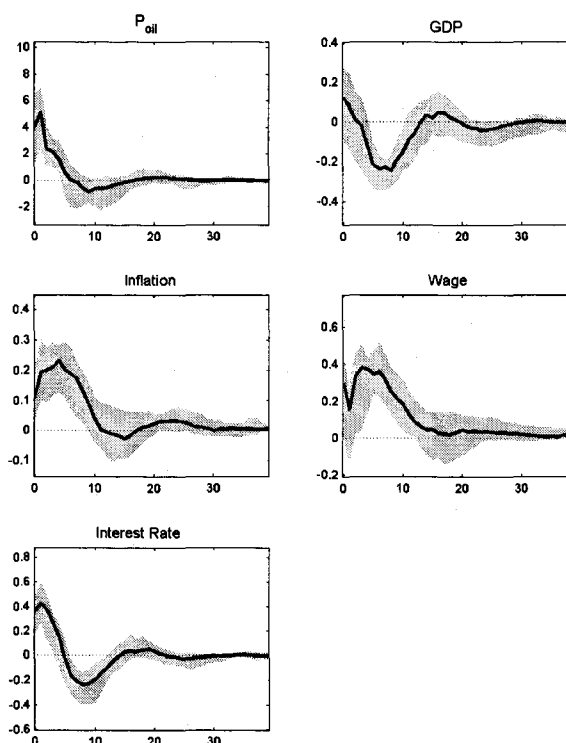


FIG. 2.7 - The solid line refers to the mean responses; the shade area to 95% error band.

Figure 2.7 displays the impulse responses to an oil price shock in the closed-economy system. The most controversial response to an oil shock is the one of real wage which is quite different across the two model systems of the economy. In the closed-economy system, real wage displays statistically significant and hump-shaped responses. Similar findings can be found in Burbidge and Harrison (1984). On the contrary, the responses of real wage derived from the

VAR with eight-variable system fall on impact, which is consistent with results reported in Rotemberg and Woodford (1996), Blanchard and Galí (2007) and others. From a theoretical point of view, this falling reflects that persistent increases in oil prices raise production costs, which requires domestic producers to adjust their prices and wages to stabilize the production cost and to restore external competitiveness.

As shown in Figure 2.7, the responses of output fall after an oil price shock, which suggests the feature of “Dutch disease” occurring in the Canadian economy. That is, soaring oil prices, combined with the exchange rate appreciation and real income gains owing to its oil component, would reduce Canada’s cost competitiveness in manufactured goods and trigger domestic adjustments by reallocating resources, which would be potentially harmful to the Canadian economy. However, the view of Dutch disease has been questioned by many empirical studies for Canada (see, for example, Macdonald (2007)). As shown in the variance decomposition analysis, the oil price shock identified in the closed-economy system is entangled with other latent foreign shocks. When including U.S. variables and the real exchange rate in the model, I find that oil price shocks have a stimulative effect on the Canadian aggregate demand. As opposed to the responses in the closed-economy system, responses of output increase after an oil shock in the open-economy model.

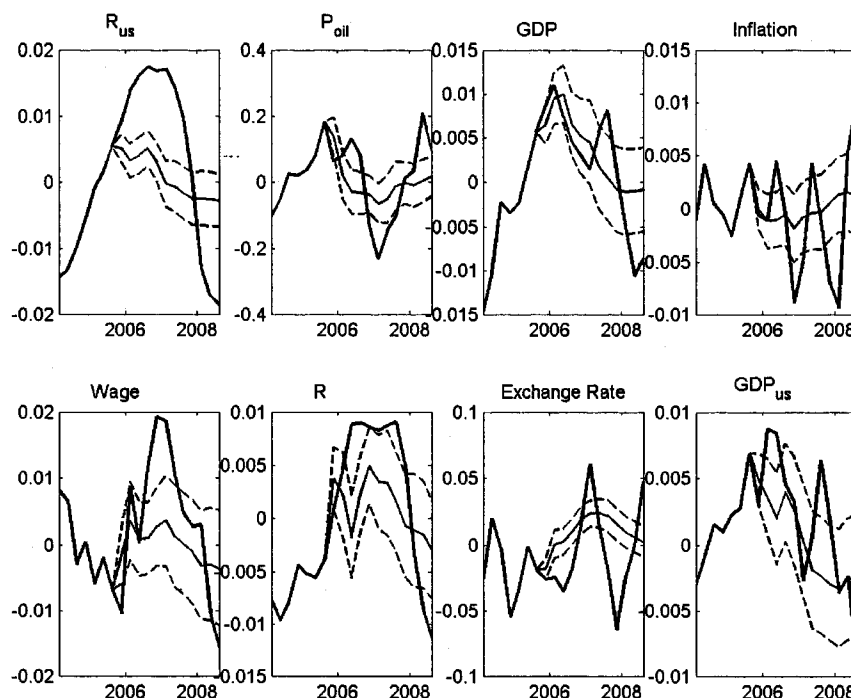


FIG. 2.8 - Note: The thick line refers to the data; the thin line to the posterior mean forecasting; the dashed lines to 15th and 85th percentile of posterior forecasting.

2.3.7 Application: Forecasting Performance

I assess the prediction performance of our VAR model by conducting a traditional in-sample forecasting exercise over 2005:3 - 2008:3 in Figure 2.8. The bounds appear to be relatively good measures of the forecast uncertainty in the sense that there are only a few exceptions where the actual data go beyond the 70% error bands. For instance, the predictions on the U.S. fund rates are more moderate than their actual development was during 2006 - 2007. The movement in bilateral exchange rate is also quite volatile, which largely falls in the 15% tails of the forecast distribution during 2007. Figure

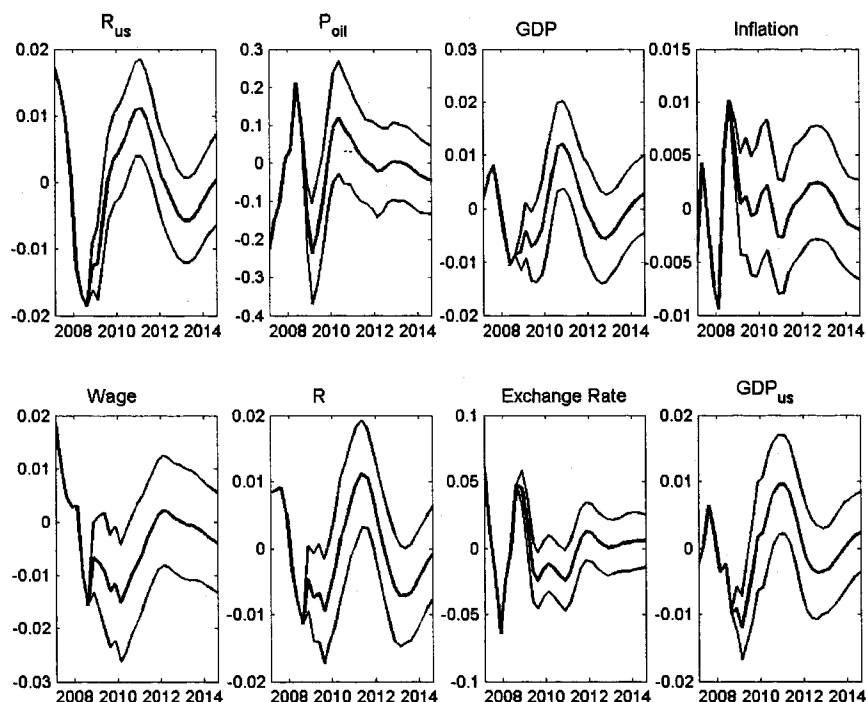


FIG. 2.9 - Note: The solid line refers to the posterior mean forecasting; the dashed lines to 15th and 85th percentile of posterior forecasting.

2.9 extends the series for the eight macro-economic variables with forecasts running out through 2008:4 - 2014:3. Apparently, the VAR projections for 2008:4 - 2009:3 coincide with the most recent episode of recession in the U.S. and Canada. The model also predicts the precipitous fall in oil prices in 2008 and their recovery in the middle of 2009. Note that the forecast uncertainty increases significantly as the horizon lengthens.

2.4 Robustness of the VAR results

In this section, the robustness of the baseline results is examined against a number of alternative specifications. I first examine whether the identification strategy matters for our results by deriving the impulse responses from a Cholesky factorization. I then investigate the sensitivity of our results to the alternative specifications of macro variable systems.

2.4.1 Traditional Identification Strategy

To check for the robustness of our empirical results, I compare them with the impulse responses obtained from the VAR with the traditional zero restrictions. In this recursive identification procedure, I place the oil prices at the top of the ordering so as to allow oil prices to influence all other macroeconomic variables contemporaneously but allow other structural shocks to affect the oil prices only with a lag. Figure 2.10 displays the impulse responses to an oil price shock. The shaded area represents the two-standard-deviation confidence intervals, obtained using a Monte Carlo procedure.

The most controversial response to an oil shock is those of interest rates because the impulse responses are quite different across the two identification strategies. Under the traditional Cholesky decomposition, both U.S. and Canadian interest rates fall persistently in the wake of oil shocks. This finding indicates that the contemporaneous correlations between the oil price and in-

terest rates are negative while a host of empirical studies suggest otherwise. As the U.S. dollar is the sole pricing and settlement currency for oil according to OPEC's 1975 decision, the fluctuating value of the dollar, which is determined by the U.S. monetary policy, plays an important role in exacerbating the run-up and precipitous fall in world oil prices. Therefore, the traditional zero restrictions that other variables have no immediate impact on the oil prices may be misspecified.

It is worth noting that a potential problem associated with the conventional approach is that the responses of oil prices to an oil shock are much higher than those derived from the VAR with sign restrictions. This finding might indicate that part of the oil shock identified by the conventional approach is picked up by other shocks when sign restrictions are applied.

Another dimension worth examining for robustness is the choice of variables included. I check three alternative specifications of VAR models. In the alternative eight-variable system, the real wage rates are replaced by the terms of trade. In the seven-variable system, the U.S. output is removed from the VAR model. In the nine-variable system, the U.S. CPI inflation is included in the data. All the estimated results from the alternative models confirm the findings in our baseline eight-variable VAR model and are not reported to conserve space.

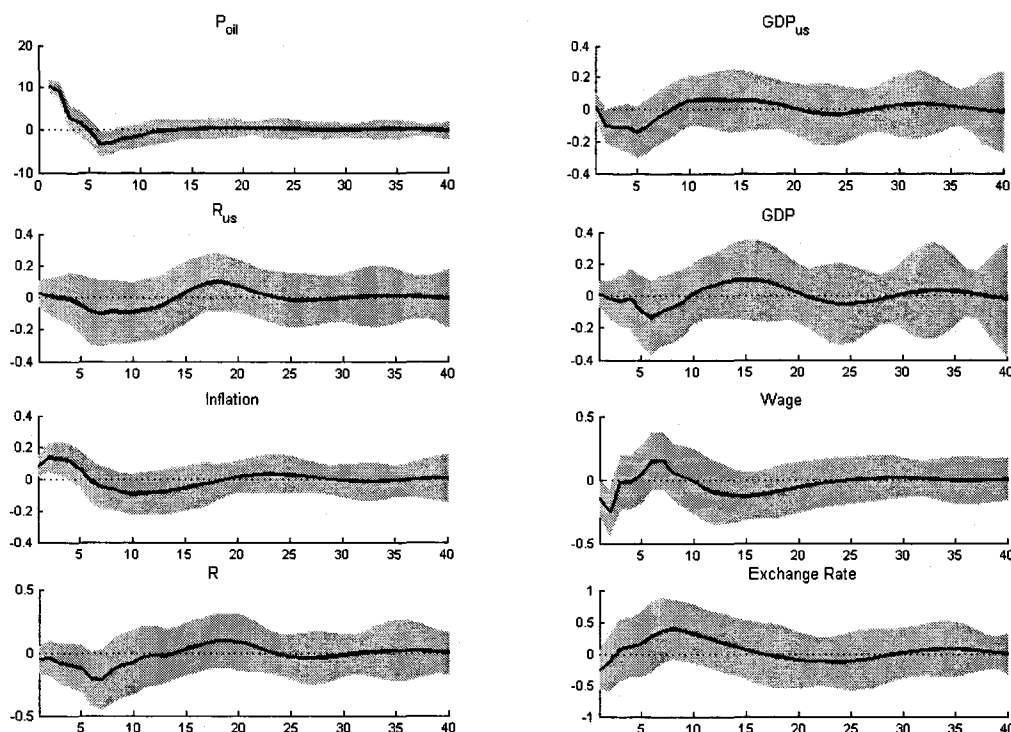


FIG. 2.10 - Note: The solid line refers to the mean responses to an oil price shock; the shade area to 95% error band.

2.5 Conclusion

Empirical analysis of the impact of oil prices and their transmission mechanism in small open economies has been subject to numerous debates. One of the difficulties encountered is the inability to correctly identify and single out the contributions of structural shocks in a small economy. The present chapter tries to contribute to this research agenda by using a structural VAR with sign restrictions to jointly identify oil price, domestic supply and foreign and domestic monetary policy shocks and examine the roles of transmission channels of oil shocks in an open-economy setup.

Four major conclusions can be drawn from this empirical analysis. First, our identification scheme appears to be successful in identifying oil price and monetary policy shocks by imposing theoretically coherent information to select impulse response functions. The effects of each structural shock on macroeconomic variables are consistent with the predictions of a broad set of theoretical models. In particular, as for identifying the effects of monetary policy in an open economy context, our structural VAR approach provides a solution to the price puzzle and exchange rate puzzles. Second, based on data for 1980:1 - 2008:3, I find that oil shocks have a stimulative effect on Canadian aggregate demand, appreciate the Canadian dollar, improve the terms of trade and reduce real wages. Third, foreign disturbances, including innovations in oil prices and the U.S. interest rate, have a significant influence on Canadian economic activities.

Finally, our empirical analysis indicates that the U.S. interest rate and the bilateral exchange rate both play important roles as channels of transmission of oil price shocks. To illustrate this point, I estimate another closed-economy Canadian VAR model. In terms of the percentage of the forecast variance, the comparison between the VAR systems reveals that the additional international transmission channels, in comparison to the closed-economy model, reduce about 50% of the contribution of oil shocks to the Canadian variables. This finding suggests that as the U.S. dollar is the sole pricing and settlement

currency in oil transaction, the systematic responses of U.S. monetary policy may act as a significant transmission channel of oil price shocks in determining macroeconomic fluctuations in oil-producing countries.

Chapter 2, however, does not fully single out the contribution of endogenous monetary policy from the impact of oil shocks because many reduced-form estimates are inconsistent with rational expectations. That is, the a-theoretical nature of even structural VAR make this methodology ill-suited to testing one structural interpretation versus another. In what follows, to make our policy analysis overcome the Lucas Critique and to better understand the mechanism through which oil price shocks affect Canadian economic activities, I push the analysis one step forward by developing an open-economy DSGE model while treat the VAR-based results as the empirical facts that need to be explained.

Chapter 3

The Impact of Oil Price Shocks on Canadian Economy: A Structural Investigation

3.1 Introduction

Dramatic changes in world oil prices have important consequences for global economic activities. While much attention has focused on the implication of oil price shocks for domestic output and inflation in oil-importing economies, the situation is vastly different for those countries that both consume and export oil. As would be expected, soaring oil prices have expansionary effects on aggregate demand in oil-exporting economies due to the boost

in oil incomes, which generates substantial shifts in purchasing power between oil-exporting and oil-importing countries.¹ Such shifts in wealth position have largely coincided with appreciation of the currencies of oil-exporting countries and improvement in their terms of trade, as documented in many empirical studies. (see, for example, Issa, Lafrance, and Murray (2006); Dupuis and Marcil (2008); Francis (2008)) Furthermore, there are also consequences due to the appreciation of oil-exporting country's currency, namely, the "Dutch disease". That is, changes in relative prices may lead to the reallocation of demand between domestic- and foreign-produced non-oil products, which dampens the positive impacts of oil shocks on an oil-exporting economy. The nature and extent of these relative price effects for an oil-exporting country remain topics of debate.

Moreover, from the point of view of an oil-exporting economy, the task of identifying the international channels through which oil price shocks affect its economic performance becomes even more perplexing if one considers the fact that U.S. monetary policy itself influences oil prices and also influences macroeconomic consequences of these changes in oil prices. While there is a large volume of literature that addresses the role of U.S. monetary policy in interacting with oil price shocks in a closed-economy context in which ex-

¹In a recent working paper, Bodenstein, Erceg, and Guerrieri (2007) use a calibrated two country DSGE model to show that the magnitude of such wealth effects depends on financial market structures.

change rate movement, integrated trade and financial markets are absent,² the literature is largely silent on the issue of whether there is a significant channel of transmission of oil price shocks on an oil-exporting country through the systematic responses of U.S. monetary policy. From an open economy perspective, these international channels are important because, with rapid financial deregulation around the world, the subsequent tightening of monetary policy in U.S. after an oil shock would contemporaneously affect both U.S. terms of trade and exchange rate, and therefore play a more important role than ever before in explaining cross-country linkages through a relative price channel, especially for an oil-exporting country like Canada.

This chapter studies Canada as an oil-exporting country and concentrate the discussion on the role of monetary policy in transmitting oil price shocks within a Canada-U.S. two-country DSGE framework. Several papers have partially addressed some of these issues. Dib (2008) develops a small open economy DSGE model for Canada in which a commodity production sector is embedded and the commodities are invoiced in U.S. dollars. Like many other papers using the small open economy structure, Dib (2008) assumes away any causal relationship between commodity prices and foreign variables, which are merely captured by an AR(1) process individually. This assumption clearly impedes a causal interpretation between oil prices and U.S. monetary policy

²Recent studies include Bernanke, Gertler, and Watson (1997), Leduc and Sill (2004), Carlstrom and Fuerst (2004) and others.

and does not allow one to examine the importance of the resulting relative price channel in transmitting oil shocks to the home country. By controlling for the exchange rate, U.S. interest rate and output, Chapter 2 finds that roughly 50% of the total impact of oil price shocks on the Canadian economy are picked up by these international channels. Chapter 2, however, does not fully single out the contribution of endogenous monetary policy from the impact of oil shocks because the weak theoretical foundation makes structural VAR models vulnerable to the Lucas Critique for policy simulations.

In the present chapter, I contribute to this research agenda by using a fully specified DSGE model to provide model-based estimates of the direct impact of oil price shocks on an oil-exporting economy and the indirect effects through U.S. and domestic monetary policy. Specifically, I assume that the model is comprised of two countries, an oil-exporting home country and an oil-importing foreign country and that oil is used as an input in production and also as part of the consumption bundle.³ Oil transactions are assumed to be invoiced in the foreign currency, which generates an asymmetry, whereby domestic firms bear more exchange rate risk than foreign firms when facing oil price shocks. That is, the production cost of a domestic firm depends not only on world oil prices but also its bilateral exchange rate. Moreover, the foreign economy is unaffected by economic activities in the home country, while it

³Note that I introduce oil usage directly in the production function as in Carlstrom and Fuerst (2004) and Dhawan and Jeske (2007) but unlike Leduc and Sill (2004), who tie oil use to capacity utilization.

does influence the home country via trade (including oil and non-oil trade) and financial markets. To allow the model to deliver realistic responses to monetary policy, I include three types of nominal rigidity, as wages, domestic goods prices and imported goods prices are all set using a Calvo-style structure. Taylor rules capture the fact that home and foreign monetary authorities endogenously react to oil price shocks due to their inflationary nature. I estimate this model for Canadian and U.S. data by using Bayesian techniques. The use of the estimated model allows us, by means of a counterfactual experiment, to establish the relative contribution of each channel in transmitting oil shocks.

Our main findings are as follows. First, I find that the predictions of the DSGE model are successful in replicating the VAR-based impulse response functions. Oil price increases tend to stimulate the Canadian economy. Oil shocks are mainly responsible for the increase in domestic aggregate demand due to the boost in oil income, appreciation of the Canadian dollar and improvements in the terms of trade in Canada. Second, domestic monetary policy is an important transmission channel because it largely determines the effects of oil shocks on domestic aggregate demand. This occurs because the monetary authority, under Taylor-type rules, aggressively responds to the inflation pressure that accompanies an oil shock increase, which leads to an increase in the real interest rate and dampens the expansionary impact of oil shocks.

According to our estimated results, systematic monetary policy accounts for roughly 41% of the discounted variation in domestic output over a four-year horizon after an oil price shock. Third, the results indicate that U.S. monetary policy is of lesser importance than Canadian as responses in U.S. policy rates only account for 8% of the discounted variation in Canadian output. This finding is in line with the results in Schmitt-Grohé (1998).⁴

Fourth, the above conclusions raise a further question regarding whether the economic consequences of an oil price shock will change under different monetary policy regimes. Should the central bank target core inflation or headline inflation (with oil price changes included)? This chapter contributes to this debate by teasing out the implications of alternative policy rules. That is, I estimate the effect of oil shocks in terms of the discounted variation in domestic output and price level under different monetary policy rules. I find that the core inflation-targeting rule is superior to headline inflation targeting because the expansionary effect of an oil shock on the domestic economy is more substantial, though it achieves a roughly comparable degree of headline inflation stabilization. This occurs because the nominal rigidities in prices and wages restrain the pass-through of oil price shocks to core inflation, which lead to a more moderate spike in policy rate if the central bank uses core inflation

⁴Schmitt-Grohé (1998) treats both the world interest rates and the terms of trade as transmission mechanism instead of as root disturbances and shows that the computed impulse responses from both of these channels failed to match the estimated responses of output, investment and employment simultaneously.

as a relevant inflation measure.

Finally, I use sensitivity analysis to establish that the small contribution of the U.S. interest rate to the Canadian economy is robust, which is in line with much of the open-economy DSGE literature. (see, for example, Mendoza (1991)⁵ and Schmitt-Grohé (1998))

Our model is closely related to new open economic macroeconomics (NOEM) literature. Recent studies include Galí and Monacelli (2005), Monacelli (2005), Lubik and Schorfheide (1992), Lubik and Schorfheide (2007) and Adolfson, Laséen, Lindé, and Villani (2007) and many others.⁶ The unifying features of the open economy models in this literature include the integration of nominal rigidities and market imperfections into DSGE models along with well-specified microfoundations. The present study extends the NOEM model developed by Galí and Monacelli (2005) by embedding a domestic oil-producing sector in which demand for and supply of oil are incorporated. I also introduce a number of nominal and real frictions that have proven to be important for the empirical fit of DSGE models.

⁵Mendoza (1991) extended a standard RBC model to the case of a small open economy and showed that the world interest rate disturbances do not play a significant role in driving the Canadian business cycles. Note that the parameters of Mendoza (1991) model is calibrated to match stylized facts of the Canadian economy.

⁶Galí and Monacelli (2005) derive the aggregate relationships from an optimizing dynamic model to analyze the macroeconomic implication of different monetary policy regimes for the small open economy. Building on this study, Monacelli (2005) introduces incomplete path-through of exchange rate changes to import prices and shows that a slow pass-through has important consequences in the analysis of optimal monetary policy. Lubik and Schorfheide (1992) apply Bayesian techniques to estimate a small-scale two-country model using data from the U.S. and Euro Area. Lubik and Schorfheide (2007) investigate whether exchange rate movements are an important concern for four small open economies in determining their monetary policy.

The rest of the chapter is organized as follows. Section 2.2 describes a multi-sector open economy DSGE model. Section 2.3 presents the empirical results and discusses the effects of oil price shocks. Section 2.4 examines the robustness of the results. Section 2.5 concludes.

3.2 The Model Economy

3.2.1 Household Sector

The economy is populated by a continuum of households indexed by $j \in [0, 1]$. The preference of the household j is defined over consumption of final goods and leisure:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t(j) - hC_{t-1}) - \frac{N_t(j)^{1+\sigma_L}}{1+\sigma_L} \right\} \quad (3.1)$$

where E_0 denotes the mathematical expectation condition on date $t = 0$ information, $C_t(j)$ is total consumption, and $N_t(j)$ is the amount of type j labor provided by the household j . Parameter σ_L is the inverse elasticity of work effort with respect to the real wage. Preferences display the external habit formation in consumption governed by $h \in [0, 1]$ and C_{t-1} is the past aggregate consumption. The final good consumption, $C_t(j)$ is given by,

$$C_t(j) = \left[\gamma^{\frac{1}{\epsilon}} (O_{c,t}(j))^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)^{\frac{1}{\epsilon}} (Z_t(j))^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (3.2)$$

where parameter ϵ is the elasticity of substitution between oil and non-oil consumption, and γ denotes the oil share in the total consumption bundle. $O_{c,t}(j)$

represents oil consumption, and $Z_t(j)$ is a composite non-oil consumption index of imported and domestically produced goods defined as:

$$Z_t(j) = \omega^{-\omega}(1 - \omega)^{(\omega-1)} C_{F,t}(j)^\omega C_{H,t}(j)^{1-\omega} \quad (3.3)$$

where $\omega \in [0, 1]$ is the import ratio measuring the degree of openness. The demand for oil and non-oil consumption are given by

$$Z_t(j) = (1 - \gamma) \left(\frac{P_{Z,t}}{P_t} \right)^{-\epsilon} C_t(j), \quad O_{c,t}(j) = \gamma \left(\frac{P_{o,t}}{P_t} \right)^{-\epsilon} C_t(j) \quad (3.4)$$

where $P_{o,t}$ and $P_{Z,t}$ are the price of oil and the core consumption deflator, respectively. Analogously, the demand functions for home and foreign non-oil goods are given by,

$$C_{H,t}(j) = (1 - \omega) \left(\frac{P_{H,t}}{P_{Z,t}} \right)^{-1} Z_t(j), \quad C_{F,t}(j) = \omega \left(\frac{P_{F,t}}{P_{Z,t}} \right)^{-1} Z_t(j) \quad (3.5)$$

where $P_{H,t}$ and $P_{F,t}$ are the prices of the home produced and imported goods, respectively. Thus, the consumption-based price index, P_t , and the non-fuel consumption price index is given by

$$P_t = \left[\gamma P_{o,t}^{1-\epsilon} + (1 - \gamma) P_{Z,t}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \quad P_{Z,t} = P_{F,t}^\omega P_{H,t}^{1-\omega} \quad (3.6)$$

The household j holds one-period internationally traded contingent claims at the end of period t , receives payments of his labor income and dividends from the domestic firms, importers and oil producers. Hence, his budget constraint at time t is given by,

$$E_t(Q_{t,t+1} B_{t+1}(j)) + P_t C_t(j) = B_t(j) + W_t(j) N_t(j) + \Pi_t(j) \quad (3.7)$$

where $W_t(j)$ is the nominal wage set by household j , $\Pi_t(j)$ is the profits that household j is entitled to. $Q_{t,t+1}$ is the stochastic discount rate on nominal payoffs, and $B_{t+1}(j)$ is the nominal payoff on a portfolio held at the end of period t . The complete international financial markets implies that there exists a complete set of state-contingent claims. When utility is separable between consumption and leisure, complete financial markets ensures consumption is equaled across households.

It is convenient to break down the problem of a household into two steps. First, the household j either re-optimizes its wage or keeps it fixed, depending on whether it is allowed to re-adjust its nominal wages at the beginning of period t . Second, given the level of his nominal wage determined in the first step, the household chooses a strategy $\{C_t(j), B_{t+1}(j)\}_{t=0}^{\infty}$ to maximize its expected lifetime utility given by (3.1), subject to (3.7) and a borrowing constraint $B_t(j) \geq -\bar{B}$ for a large positive number $\bar{B}$.

Consumption Decisions

I first consider the first order conditions for choosing consumption and bonds holding.

$$\Lambda_t(j) = \frac{1}{C_t(j) - hC_{t-1}} \quad (3.8)$$

$$E_t(Q_{t,t+1}) = E_t \left[\beta \frac{P_t}{P_{t+1}} \frac{\Lambda_{t+1}(j)}{\Lambda_t(j)} \right] \quad (3.9)$$

where $\Lambda_t(j)$ is the Lagrange multiplier associated with the household j 's budget constraint and $E_t(Q_{t,t+1}) = (1 + i_t)^{-1}$ is the nominal return on a state-contingent security maturing in $t+1$ and i_t denotes the nominal interest rate. Since I am assuming the existence of a complete set of state-contingent claims, consumption is equalized across households. Therefore, in what follows, I can omit index j from consumption.

$$E_t \left[\beta \frac{P_t}{P_{t+1}} \left(\frac{C_t - hC_{t-1}}{C_{t+1} - hC_t} \right) \right] = \frac{1}{1 + i_t} \quad (3.10)$$

Wage Setting

Each household j is a monopolistic supplier of a differentiated labor service. Assuming there is a set of perfectly competitive labor service assemblers that hire labor from each household and bundle it into an aggregate labor service unit, N_t that is then used by the domestic goods producer and oil producer. The labor service unit is defined as:

$$N_t = \left(\int_0^1 N_t(j)^{\frac{\epsilon_L - 1}{\epsilon_L}} dj \right)^{\frac{\epsilon_L}{\epsilon_L - 1}} \quad (3.11)$$

where ϵ_L measures the wage elasticity of demand among differentiated labor services. The labor service assembler maximizes its profits in the labor market by solving

$$\max W_t N_t - \int W_t(j) N_t(j) dj \quad (3.12)$$

where $N_t(j)$ is the labor supply of household j , $W_t(j)$ is the wage rate set by the household, and W_t is the aggregate wage index defined below. From the

above maximization problems, individual households face a downward sloping demand function:

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_L} N_t \quad (3.13)$$

where the aggregate wage index, W_t , can be defined as,

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_L} dj \right)^{\frac{1}{1-\epsilon_L}} \quad (3.14)$$

Following Erceg, Henderson, and Levin (2008) where the wage setting is subject to a nominal rigidity in a discrete time version of Calvo (1983), I assume that each household can reoptimize its wage in a given period with a constant probability $(1 - \phi_w)$, which is independent of other households and of the time elapsed since last adjustment. Thus, the law of large number implies that only a $(1 - \phi_w)$ fraction of households are able to re-optimize their nominal wage each period while the other fraction ϕ_w cannot. For those who are not allowed to re-optimize their nominal wage in the current period, I assume that their nominal wages remain constant during the interval between re-optimizations, and that they must supply any quantity of labor service demanded at the wage that they have decided on. Thus, a constant elasticity of substitution (CES) aggregate wage index evolves according to the difference equation

$$W_t = \left(\phi_w W_{t-1}^{1-\epsilon_L} + (1 - \phi_w) W_t^{T^{1-\epsilon_L}} \right)^{\frac{1}{1-\epsilon_L}} \quad (3.15)$$

If instead re-optimization is possible in the current period, a household j

will set the target wage, $W_t^T(j)$ to maximize

$$\max E_{t-1} \left\{ \sum_{k=0}^{\infty} (\beta \phi_w)^k \left[\Lambda_{t+k} N_{t,t+k}(j) \frac{W_t^T(j)}{P_{t+k}} - \frac{1}{1 + \sigma_L} N_{t,t+k}(j)^{1+\sigma_L} \right] \right\} \quad (3.16)$$

subject to the labor demand (3.13). Note that E_{t-1} denotes the mathematical expectation condition on date $t - 1$ information, which implies that wage setting occurs prior to the realization of any aggregate disturbance at time t , (see Rotemberg and Woodford (1996) and $N_{t,t+k}(j)$ stands for the labor supply of household j in period $t + k$ if the last wage re-optimization occurs in t . The variable Λ_{t+k} denotes the household's marginal utility of consumption at date $t + k$. The solution of the above problem satisfies the following first-order condition

$$E_{t-1} \left\{ \sum_{k=0}^{\infty} (\beta \phi_w)^k \left[\Lambda_{t+k} N_{t,t+k}(j) \frac{W_t^T(j)}{P_{t+k}} - \frac{\epsilon_L}{\epsilon_L - 1} N_{t,t+k}(j)^{1+\sigma_L} \right] \right\} = 0 \quad (3.17)$$

3.2.2 Inflation, the Real Price of Oil, the Real Exchange Rate and the Terms of Trade

This section describes the key relationships between headline inflation, core inflation, the real price of oil, the real exchange rate, and the term of trade. I define and derive a number of identities that are extensively used thereafter.

I start by defining the real price of oil expressed in terms of final consumption goods as $P_{o,t}^r \equiv \frac{P_{o,t}}{P_t}$. Using equation (3.6), the relative price of non-oil

goods can be written as

$$\frac{P_{Z,t}}{P_t} = \left(\frac{1 - \gamma P_{o,t}^{r, 1-\epsilon}}{1 - \gamma} \right)^{\frac{1}{1-\epsilon}} \quad (3.18)$$

Taking logs and the first difference, I arrive at an identity linking headline inflation, core inflation and the change in the real price of oil,

$$\pi_t = \pi_{z,t} + \frac{\gamma}{1 - \gamma} (p_{o,t}^r - p_{o,t-1}^r) \quad (3.19)$$

where $p_{o,t}^r$ is the log of the real price of oil.

Next, I define the terms of trade (TOT) of non-oil goods as $S_{H,t} \equiv \frac{P_{F,t}}{P_{H,t}}$, i.e. the price of foreign goods per unit of home good. Note that an increase in $S_{H,t}$ is equivalent to an increase in competitiveness for the domestic economy. Equation (3.6) can be rewritten as

$$\frac{P_{H,t}}{P_{Z,t}} = S_{H,t}^{-\omega} \quad (3.20)$$

Taking logs and the first difference, I derive a relationship between the core inflation, domestic inflation and the terms of trade as follows,

$$\pi_{Z,t} = \pi_{H,t} + \omega(s_{H,t} - s_{H,t-1}) \quad (3.21)$$

I derive the relationships between the real exchange rate, real prices of oil and the terms of trade. Firstly, I define the bilateral real exchange rate as $Q_t \equiv \frac{e_t P_t^*}{P_t}$, the ratio of the two countries' CPIs, both expressed in the domestic currency. Additionally, I assume that the law of one price (LOP) holds for oil, but incomplete pass-through for non-oil imports is allowed. The

motivation behind this assumption is that Canada, as a small open economy, is a price taker with little bargaining power in the international energy market. For its oil exports, oil prices are determined exogenously in the rest of the world. Therefore, the real exchange rate can also be written as the ratio of the two countries' real prices of oil, $Q_t \equiv \frac{P_{o,t}^r}{P_{o,t}^{*r}}$. On the import side, following Monacelli (2005), I assume monopolistic importers purchase foreign non-oil goods at world market prices and sell these goods to domestic consumers at a price $P_{F,t}(j)$ equal to a markup over the cost. Thus, LOP holds at the wholesale level but rigidities arising from the domestic importers allow the domestic import prices to deviate from the world price in the short run.

3.2.3 International Risk Sharing and Uncovered Interest Parity

Under the assumption of complete international financial markets, the expected nominal return from state contingent claims in the domestic currency must be equal to the expected domestic-currency return from the foreign state contingent claims. A first order condition analogous to (3.9) must also hold for the household j in the foreign economy.

$$E_t(Q_{t,t+1}) = E_t \left[\beta \frac{P_t^*}{P_{t+1}^*} \frac{e_t}{e_{t+1}} \frac{\Lambda_{t+1}^*(j)}{\Lambda_t^*(j)} \right] \quad (3.22)$$

where $\Lambda_t^*(j)$ denotes the foreign household j 's marginal utility of consumption at date t .

Combining (3.9) and (3.22), together with the real exchange rate definition,

it follows that

$$\Lambda_t = \vartheta \Lambda_t^* Q_t \quad (3.23)$$

for all t and where ϑ is a constant which depends on initial conditions about relative net asset positions.

The assumption of a complete international financial market recovers another relationship, the uncovered interest parity (UIP) condition. If I equate the nominal returns (in terms of domestic currency) of a state contingent security in a foreign currency $e_t(1 + i_t^*)^{-1} = E_t(Q_{t,t+1}e_{t+1})$ to that of the same state contingent security in domestic currency, $(1 + i_t)^{-1} = E_t Q_{t,t+1}$, I obtain

$$E_t \left[Q_{t,t+1} \left((1 + i_t) - (1 + i_t^*) \frac{e_{t+1}}{e_t} \right) \right] = 0 \quad (3.24)$$

3.2.4 Domestic Production

Aggregation of Domestic Goods

The domestic good aggregator, Y_t is the bundle of differentiated goods which is supplied to domestic and foreign consumers. The Dixit-Stiglitz aggregator is

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_H - 1}{\epsilon_H}} di \right)^{\frac{\epsilon_H}{\epsilon_H - 1}} \quad (3.25)$$

where $Y_t(i)$ represents the quantity of a differentiated good i , The optimal allocation of expenditure across differentiated goods implies a downward-sloping

demand function for a good i ,

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_H} Y_t \quad (3.26)$$

where $P_{H,t}(i)$ denotes the price of good $Y_t(i)$ and ϵ_H measures the price elasticity of demand among differentiated goods. $P_{H,t}$ denote the price index of composite domestic goods given by

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\epsilon_H} di \right)^{\frac{1}{1-\epsilon_H}} \quad (3.27)$$

Intermediate Producers

In the domestic goods market, there is a continuum of monopolistic competitive firms producing differentiated goods indexed by $i \in [0, 1]$ with a CES production function

$$Y_t(i) = A_t \left[(1 - \alpha)^{\frac{1}{\epsilon_m}} N_{m,t}(i)^{\frac{\epsilon_m-1}{\epsilon_m}} + \alpha O_{m,t}(i)^{\frac{\epsilon_m-1}{\epsilon_m}} \right]^{\frac{\epsilon_m}{\epsilon_m-1}} \quad (3.28)$$

where $N_{m,t}(i)$ is the labor input, and $O_{m,t}(i)$ is oil used in production. A_t is a domestic technology shock. The level of technology is common to all firms. Parameter ϵ_m defines the elasticity of substitution between labor and oil in production. Note that for the sake of simplicity, this elasticity is constrained to be the same as for households.

A producer i solves its maximization problem in two stages. In the first stage, taking the price of both inputs as given, the producer chooses the cost

minimizing quantities of labor and oil to solve the following static cost minimization problem:

$$\min_{N_{m,t}(i), O_{m,t}(i)} W_t N_{m,t}(i) + P_{o,t} O_{m,t}(i)$$

subject to (3.28). This cost minimization implies that firm i 's nominal marginal cost $MC_t(i)$ is given by:

$$MC_t(i)^{\epsilon_m} = \frac{W_t^{\epsilon_m}}{(1-\alpha)(Y_t(i)/N_{m,t}(i))} = \frac{P_{o,t}^{\epsilon_m}}{\alpha(Y_t(i)/O_{m,t}(i))} \quad (3.29)$$

Letting $MC_t^r(i) \equiv (MC_t(i)/P_{H,t})$ denotes firm i 's real marginal cost, I have

$$MC_t^r(i) = A_t^{-1} \left[(1-\alpha)W_t^{r^{1-\epsilon_m}} + \alpha P_{o,t}^{r^{1-\epsilon_m}} \right]^{\frac{1}{1-\epsilon_m}} S_{H,t}^{\omega} \left(\frac{1-\gamma P_{o,t}^{r^{1-\epsilon}}}{1-\gamma} \right)^{\frac{-1}{1-\epsilon}} \quad (3.30)$$

where $W_t^r \equiv \frac{W_t}{P_t}$. Notice that the real marginal cost depends not only on the real prices of inputs but also on the terms of trade of non-oil goods and that all these factors are common to all firms.

These domestic firms are assumed to set prices in a Calvo-staggered fashion. Analogously to the case of households, a fraction $(1-\phi_h)$ of firms selected randomly, set new prices optimally each period, while the remaining fraction ϕ_h of firms keep their price unchanged. The CES aggregate price index consequently evolves according to

$$P_{H,t} = \left[(1-\phi_h)P_{H,t}^{T^{1-\epsilon_H}} + \phi_h P_{H,t-1}^{1-\epsilon_H} \right]^{\frac{1}{1-\epsilon_H}} \quad (3.31)$$

Since firms are not allowed to re-optimize on their prices every period, the optimal price determination becomes an inherently dynamic problem. In the

second stage, the domestic good producer i chooses its target price $P_{H,t}^T(i)$ so as to maximize the present value of its real profits:

$$\max E_{t-1} \left\{ \sum_{k=0}^{\infty} (\beta \phi_h)^k \frac{\Lambda_{t+k}}{\Lambda_t} Y_{t,t+k}(i) \left[\frac{P_{H,t}^T(i)}{P_{H,t+k}} - MC_{t,t+k}^r \right] \right\} \quad (3.32)$$

subject to the demand function (3.26). $Y_{t,t+k}(i)$ denotes the sales of good i in period $t+k$, if the most recent price adjustment came into effect in period t . The term $\beta^k \frac{\Lambda_{t+k}}{\Lambda_t}$ represents the appropriate discount factor for future profits between date t and $t+k$, and ϕ_h^k stands for the probability that prices will not be re-optimized for k period. The solution to above maximization problem is given by

$$E_{t-1} \left\{ \sum_{k=0}^{\infty} (\beta \phi_h)^k \frac{\Lambda_{t+k}}{\Lambda_t} Y_{t,t+k}(i) \left[\frac{P_{H,t}^T(i)}{P_{H,t+k}} - \frac{\epsilon_H}{\epsilon_H - 1} MC_{t,t+k}^r \right] \right\} = 0 \quad (3.33)$$

Note that if domestic good prices are flexible, i.e. if $\phi_h = 0$, a representative firm chooses the price for its differentiated product as a constant markup over the nominal marginal cost $P_{H,t}^T = \frac{\epsilon_H}{\epsilon_H - 1} P_{H,t} MC_t^r$.

3.2.5 Domestic Importers

Following Monacelli (2005) I assume that endogenous deviation from purchasing power parity (PPP) in the short term arises due to the existence of monopolistically competitive importers. Although the law of one price holds at the wholesale level for the differentiated foreign goods, importers sell these goods to domestic consumers and charge a mark-up over their cost, which re-

sults in a wedge between domestic and import prices of foreign goods in the short run. I define the law of one price gap as

$$\Psi_t \equiv \frac{e_t P_{F,t}^*}{P_{F,t}} \quad (3.34)$$

Combining (3.34), (3.18) and (3.20) yields the following equation linking LOP gap, the real price of oil, the terms of trade and the real exchange rate:

$$\Psi_t = Q_t \left(\frac{1 - \gamma^* P_{o,t}^{r*1-\epsilon}}{1 - \gamma^*} \right)^{\frac{1}{1-\epsilon}} \left(\frac{1 - \gamma P_{o,t}^{r1-\epsilon}}{1 - \gamma} \right)^{\frac{-1}{1-\epsilon}} S_{H,t}^{\omega-1} \quad (3.35)$$

Similar to domestic producers, importers operate under Calvo-style pricing structure, with $1 - \phi_f$ importers setting the target prices, $P_{F,t}^T$ optimally each period. The CES aggregate price index consequently evolves according to the relation

$$P_{F,t} = \left[(1 - \phi_f) P_{F,t}^{T1-\epsilon_F} + \phi_f P_{F,t-1}^{1-\epsilon_F} \right]^{\frac{1}{1-\epsilon_F}} \quad (3.36)$$

and retailers setting prices in period t face a demand curve

$$C_{F,t}(i) = \left(\frac{P_{F,t}^T(i)}{P_{F,t}} \right)^{-\epsilon_F} C_{F,t} \quad (3.37)$$

Thus, a particular importer i maximizes the present value of its retail profits:

$$\max E_t \left\{ \sum_{k=0}^{\infty} (\beta \phi_f)^k \frac{\Lambda_{t+k}}{\Lambda_t} C_{F,t+k}(i) \left[\frac{P_{F,t+k}^T(i)}{P_{F,t+k}} - \Psi_{t+k} \right] \right\} \quad (3.38)$$

subject to the demand function (3.37). The firm's optimization problem implies the first order condition

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta \phi_f)^k \frac{\Lambda_{t+k}}{\Lambda_t} C_{F,t+k}(i) \left[\frac{P_{F,t+k}^T(i)}{P_{F,t+k}} - \frac{\epsilon_F}{\epsilon_F - 1} \Psi_{t+k} \right] \right\} = 0 \quad (3.39)$$

3.2.6 Oil Producing Firms

I now discuss the features of oil production in the model economy. Since a small oil exporting country is a price taker with little bargaining power in the international oil market, I assume world oil prices are determined exogenously. In addition, I assume that the domestic economy is populated by identical local oil producers who, taking the world oil price as given, extract oil to satisfy the domestic and foreign demand. A representative oil producer pays a fix cost to maintain its oil well and produce oil according to the following decreasing return to scale technology that uses labor as input

$$O_t = A_t N_{o,t}^{\alpha_o} \bar{X}^{1-\alpha_o} \quad (3.40)$$

where $N_{o,t}$ is the labor used in the oil sector, and $\bar{X}$ denotes the other factor input (i.e. oil well) which is constant in the short run. Parameter $\alpha_o \in (0, 1)$ is the share of labor in oil production.

Taking oil prices and wages as given, the firm chooses labor demand in order to maximize profits:

$$\max_{N_{o,t}} P_{o,t} O_t - W_t N_{o,t} - \Xi_t \quad (3.41)$$

subject to (3.40). Variable Ξ_t denotes the fixed cost for oil extraction. The first order condition with respect to $N_{o,t}$ requires equating the marginal product of labor to the real wage in terms of oil.

$$\frac{W_t^r}{P_{o,t}^r} = \alpha_o A_t N_{o,t}^{\alpha_o-1} \bar{X}^{1-\alpha_o} \quad (3.42)$$

Oil market clearing condition requires that

$$O_t = O_{c,t} + O_{m,t} + O_{x,t} \quad (3.43)$$

where $O_{x,t}$ is the oil export to the foreign economy. Thus, the sum of the home oil production equals the sum of home and foreign oil consumption.

3.2.7 Monetary Policy

I assume the monetary policy follows a Taylor-type rule in the following form

$$\log\left(\frac{r_t}{r}\right) = \rho_r \log\left(\frac{r_{t-1}}{r}\right) + (1 - \rho_r) \left(\phi_1 \log\left(\frac{\pi_{Z,t}}{\pi_Z}\right) + \phi_2 \log\left(\frac{y_t}{y}\right) \right) + \varepsilon_{R,t} \quad (3.44)$$

where r , π_Z , y are the steady-state values of r_t , $\pi_{Z,t}$, y_t . The parameter ρ_r is the degree of interest rate smoothing, which reflects the central bank's preference for interest rate stability;⁷ ϕ_1 and ϕ_2 are the relative weights measuring central bank's response to log-deviation of core inflation and output from their steady-state values, respectively; and $\varepsilon_{R,t}$ denotes an iid monetary policy shock.

3.2.8 The Foreign Economy

The foreign economy is assumed to be an oil importer whose oil demand is satisfied by the oil-exporting economy. I assume that the foreign economy is unaffected by economic activities in the small open economy, but influences the

⁷See, for example, Clarida and Gertler (2001) and Sack and Wieland (2000)

latter via international trade (including fuel and non-fuel trade). For simplicity, behavioral similarity is assumed between domestic and foreign household and producers. Domestic and foreign economies, however, can differ in price-setting and monetary policy. I arrive at a similar set of optimality conditions describing the dynamic behaviors of the foreign economy with all variables taking a superscript ($\star$) and parameters properly substituted. Note that with the domestic economy being small relative to the foreign economy, foreign non-oil consumption approximately comprises only foreign-produced goods such that $Z^\star = C_F^\star$ and $P_Z^\star = P_F^\star$. I assume that the monetary authority in the foreign economy adjusts its nominal interest rate in response to deviations of the foreign inflation and output gap, analogously to (3.44).

3.2.9 Exogenous Processes

There are eight structural shock processes in the model economy, including an oil price shock, domestic technology shock, world technology shock, domestic and foreign wage markup shocks, a UIP shock, domestic and foreign monetary policy shocks. I can describe the stochastic process of these shocks with log-linearized form by the univariate representation

$$\xi_t = \rho_{\xi,t} \xi_{t-1} + \varepsilon_{\xi,t}, \quad \varepsilon_{\xi,t} \sim N(0, \sigma_\xi^2) \quad (3.45)$$

where $\xi_t = \left\{ P_{ot}^{r\star}, \ln(A_t), \ln(A_t^\star), \xi_{Rt}, \xi_{Rt}^\star, \xi_{UIPt}, \xi_{Wt}, \xi_{Wt}^\star \right\}$.

3.2.10 Market Clearing and Equilibrium

Since all households, intermediate good-producing firms and importers make identical decisions, in any symmetric equilibrium I must have $P_{Ht}^T = P_{Ht}^T(i)$, $P_{Ft}^T = P_{Ft}^T(i)$, $W_t^T = W_t^T(i)$, $N_{mt} = N_{mt}(i)$, $Y_t = Y_t(i)$, $C_{Ht} = C_{Ht}(i)$, $C_{Ft} = C_{Ft}(i)$. Symmetric conditions must hold in the foreign economy. An equilibrium requires that all markets clear. For the home country, the following balanced trade relation must hold:

$$P_{H,t}CA_t = P_{H,t}C_{H,t}^* + P_{O,t}O_{x,t} - P_{F,t}C_{F,t} = 0 \quad (3.46)$$

where CA_t denotes the current account. Labor and energy market clearing at home require that

$$N_t = N_{m,t} + N_{o,t}, \quad \text{and} \quad O_t = O_{c,t} + O_{m,t} + O_{x,t} \quad (3.47)$$

the domestic and foreign good market clearing require that

$$Y_t = C_{H,t} + C_{H,t}^*, \quad \text{and} \quad Y_t^* = C_t^* \quad (3.48)$$

For the foreign economy, balanced trade and energy market clearing require

$$P_t^*C_t^* = P_{Z,t}^*Y_t^* - P_{O,t}^*O_{M,t}^*, \quad \text{and} \quad O_{x,t} = O_{c,t}^* + O_{m,t}^* \quad (3.49)$$

Lastly, let $P_{y,t}$ denote the value added deflator. The value added satisfies the following relations:

$$P_{y,t}GDP_t = P_{H,t}C_{H,t} + P_{O,t}O_{c,t} + P_{H,t}CA_t \quad (3.50)$$

Note that the characterization of the equilibrium did not require introducing either value added or the value added deflator, but these are needed to compare the implications of the model to the data.

3.3 Empirical Implementation

In this section I discuss our methodology for estimating and evaluating the empirical performance of the DSGE model. I make use of Bayesian methods that have become popular in the DSGE literature.⁸ Note that the equilibrium system of a DSGE model can be linearly approximated around its stationary steady-state in the form of

$$AE_t(\hat{\mathbf{x}}_{t+1}|\mathbf{I}_t) = \mathbf{B}\hat{\mathbf{x}}_t + \mathbf{C}(F)E(\varepsilon_{\xi,t}|\mathbf{I}_t) \quad (3.51)$$

where $\hat{\mathbf{x}}_t$ is a vector of endogenous variables⁹, $E_t(\hat{\mathbf{x}}_{t+1}|\mathbf{I}_t)$ is the expectation of $\hat{\mathbf{x}}_{t+1}$ given period t information, ε_t is a vector of exogenous stochastic processes underlying the system, and $\mathbf{C}(F)$ is a matrix polynomial of the forward operator F . The solution of the log-linearized system (4.24) can be written in the following state-space form:

$$\hat{s}_t = \mathbf{P}\hat{s}_{t-1} + \mathbf{C}_1\varepsilon_{\xi,t} \quad (3.52)$$

$$\hat{y}_t = \mathbf{Q}\hat{s}_{t-1} \quad (3.53)$$

⁸See, for example, Rabanal and Rubio-Ramírez (2005), Lubik and Schorfheide (2007), and Del Negro and Schorfheide (2001).

⁹For any stationary variable x_t , I define $\hat{x}_t = \left(\frac{x_t - \bar{x}}{\bar{x}}\right)$ as the percentage deviation from its steady-state value, $\bar{x}$.

where the vector $\varepsilon_{\xi,t}$ contains 8 orthogonal structural innovations in the model. I then update the state-space form solution by adding a set of measurement equations that link the observed time series to the vector of unobserved state variables. I further use the Kalman filter to evaluate the likelihood function of the state-space form solution and combine this likelihood function with our specified prior knowledge about these deep parameters to form the posterior distribution function. The sequence of posterior draws can be obtained using Markov chain Monte Carlo (MCMC) methods. I use the random-walk Metropolis-Hasting algorithm as described in Schorfheide (2000) to numerically generate the Markov chains for the structural parameters. Point estimates of the structural parameters can be obtained by calculating the sample mean or median from the simulated Markov chains. Similarly, inferences of these parameters are derived from computing the percentiles of these posterior draws. Thus, given the sequence of posterior draws of parameters, I compute posterior statistics of interest that are often used to validate model performance, such as impulse response function and historical decomposition.

3.3.1 Calibration and Prior Specifications

Not all the model parameters are estimated. Because some parameters are weakly identified or the data under analysis contain only limited information about them, these model parameters in the model are calibrated using the

corresponding data statistics. Table 3.1 summarizes the calibrated parameters. Parameters specific to the oil sector are set as follows. The share of labor in the production of oil α_o is assigned a value of 8% so that the labor input in the oil sector is about 2% of the Canadian total labor force. Following Blanchard and Galí (2007), I calibrate γ^* and α^* as the share of oil in U.S. consumption and the share of oil in U.S. output to 1.5% and 3%, respectively. The parameter determining the share of oil in Canadian output is fixed at 5%, consistent with the calibration in Macklem, Osakwe, Pioro, and Schembri (2001) who calculate the share parameters in production from Canadian 1996 input-output tables. Assuming economic structure of Canada is similar to that of the U.S., I calibrate the share of oil in the Canadian consumption bundle to 3%.

TABLE 3.1
THE CALIBRATED PARAMETERS

Parameters	Description	Values
β	the discount factor	0.99
α_o	the share of labor in the production of oil	0.08
γ	the share of oil in Canada consumption	0.03
γ^*	the share of oil in US consumption	0.015
α	the share of oil in Canada non-oil output	0.05
α^*	the share of oil in US non-oil output	0.03
ω	the share of imported goods in Canada non-oil consumption	0.40
ϵ	the elasticity of substitution between oil and non-oil consumption	0.18
ϵ_m	the elasticity of substitution between oil and labor input	0.18

I calibrate the model so that the share of foreign goods in the Canadian consumption basket, ω is fixed at 40% to match the average import-to-GDP

ratio observed in the data, while I set the share of foreign goods in the U.S. consumption basket ω^* to ensure that the real exchange rate at steady state is equal to unity, which implies that the law of one price holds in the long-term. I assume a low elasticity of substitution of oil in the consumption bundles as well as in the production function. The prior means of ϵ and ϵ_m are both equal to 0.18, which is in line with the estimates obtained by Rotemberg and Woodford (1996) who use data on twenty two-digit U.S. manufacturing sectors. Kim and Loungani (1992) and Kilian (2008) also use a similar degree of substitutability between oil and other factors in production. Finally, the discount factor β is set to 0.99, which implies an annual real interest rate of 4%.

I estimate a vector of 32 structural parameters using Bayesian techniques. Table 3.2 summarizes information about the priors. I use fairly loose priors for the parameters describing the monetary policy rule. The priors for ϕ_1 and ϕ_2 , which are both described by a Gamma distribution, are centered on the values commonly associated with the Taylor rule. The interest rate smoothing parameter ρ_R is assumed to follow a beta distribution with a mean of 0.5 and a standard error of 0.2. I choose identical priors for parameters of the foreign policy rule. The priors for the price and wage stickiness parameters are chosen based on evidence on the average frequency of price changes. For Canada, I set the prior mean for the Calvo-adjustment parameter for price setting to 0.65, which corresponds to an approximate three-quarter contract length

TABLE 3.2
PRIOR AND POSTERIOR DISTRIBUTION

Parameter	Prior Mean	Posterior Mean	90% Confidence Interval	Density	Prior S.D.
h	0.5	0.566	[0.514, 0.621]	Beta	0.1
σ_L	1.6	1.662	[0.871, 2.415]	Gamma	0.5
σ_L^*	2	1.323	[0.794, 1.845]	Gamma	0.5
ϵ_H	8	8.008	[6.332, 9.560]	Gamma	1
ϵ_F	8	7.235	[5.633, 8.826]	Gamma	1
ϵ_L	8	5.938	[4.210, 7.633]	Gamma	1
ϵ_F^*	8	7.312	[5.777, 8.817]	Gamma	1
ϕ_H	0.6	0.617	[0.561, 0.674]	Beta	0.05
ϕ_F	0.6	0.561	[0.522, 0.604]	Beta	0.025
ϕ_W	0.85	0.886	[0.817, 0.953]	Beta	0.05
ϕ_H^*	0.5	0.561	[0.522, 0.604]	Beta	0.05
ϕ_W^*	0.5	0.886	[0.817, 0.953]	Beta	0.05
ϕ_1	2.5	1.766	[2.014, 2.658]	Gamma	0.25
ϕ_2	0.65	0.507	[0.396, 0.607]	Gamma	0.1
ϕ_1^*	2	1.937	[2.144, 2.865]	Gamma	0.25
ϕ_2^*	0.6	0.265	[0.603, 0.999]	Gamma	0.15
ρ_A	0.5	0.781	[0.697, 0.865]	Beta	0.2
ρ_A^*	0.5	0.941	[0.909, 0.972]	Beta	0.2
ρ_O	0.5	0.683	[0.594, 0.783]	Beta	0.2
ρ_R	0.5	0.659	[0.595, 0.721]	Beta	0.2
ρ_R^*	0.5	0.658	[0.601, 0.711]	Beta	0.2
ρ_{UIP}	0.5	0.602	[0.517, 0.684]	Beta	0.2
ρ_{PIH}	0.5	0.512	[0.417, 0.61]	Beta	0.2
ρ_W	0.5	0.713	[0.611, 0.802]	Beta	0.2
σ_a	0.05	0.025	[0.012, 0.039]	Inv-Gamma	∞
σ_a^*	0.05	0.024	[0.020, 0.029]	Inv-Gamma	∞
σ_{mkw}	0.05	0.011	[0.008, 0.014]	Inv-Gamma	∞
σ_o^*	0.05	0.12	[0.106, 0.133]	Inv-Gamma	∞
σ_{mkh}	0.05	0.017	[0.014, 0.020]	Inv-Gamma	∞
σ_r	0.05	0.017	[0.014, 0.020]	Inv-Gamma	∞
σ_r^*	0.05	0.008	[0.007, 0.009]	Inv-Gamma	∞
σ_{uip}	0.05	0.015	[0.013, 0.017]	Inv-Gamma	∞

as reported in Ambler, Dib, and Rebei (2004) and Lubik and Schorfheide (2007). The stickiness of wages is set at the same level. For the U.S., I use

information reported by Bils and Klenow (2004)¹⁰ to set the prior mean for the Calvo probability parameter θ_H^* to 0.5. The stickiness of U.S. wages is set at the same level. The priors for the preference parameters are chosen as follows. The prior for the habit parameter h is assumed to fluctuate around 0.5 with a standard error of 0.2, and the inverse elasticity of labor supply σ_L and σ_L^* are both assumed to be centered at 1 with a standard error of 0.4. I set the elasticity of substitution between different types of labor ϵ_L and ϵ_L^* to be around 8, which translates into a wage markup equal to 15%, which is consistent with the findings in the micro-econometric studies by Griffin (1996) as based on U.S. data. A number of studies for the U.S. have reported quite a large range for the value of the elasticity of substitution between differentiated goods, ϵ_F .¹¹ The prior mean of ϵ_F^* is set to 8, which implies a steady state markup of price over marginal cost equal to 15%. I assume identical priors for ϵ_H and ϵ_F .

The persistence parameters of the exogenous AR(1) processes are assumed to follow a beta distribution with a mean of 0.5 and a standard deviation of 0.2. The standard errors of various innovations are assumed to follow the same inverse gamma distribution, with a mean of 0.05 and a standard error of infinity.

¹⁰Bils and Klenow (2004) document that an average of 26% of U.S. sectoral prices change every 3.3 months which implies a Calvo probability parameter of $\theta_H^* = 0.5$

¹¹Chari, Kehoe, and McGrattan (2000) set this parameter at 10, while Rabanal and Rubio-Ramírez (2005) set it at 6, while Christiano, Eichenbaum, and Evans (2005) estimate it to be around 3.

3.3.2 Posterior Estimates

...Given the data and the prior specification, I simulate two independent Markov chains of size 500,000 using the method described above. Based on the two independent Markov chains, in Table 3.2 I report mean estimates as well as 95% error band for the posterior estimates in the third and fourth column. The results are largely in line with the previous literature.

The estimation results for the degree of external habit persistence $h = 0.57$ are relatively low when compared with Justiniano and Preston (2008) estimates for Canada, but are close to the estimates obtained from other studies for the U.S. and the Euro area (see, for example, Lubik and Schorfheide (1992); Smets and Wouters (2004)). The posterior mean for the coefficient on the lagged interest rate in the Canadian interest rate reaction function is estimated at 0.66 and the weight on inflation and changes in output are 1.77 and 0.51, respectively. These estimates are well consistent with the values obtained from recent studies on Canada (Justiniano and Preston (2008); Lubik and Schorfheide (2007)). The estimates for the policy parameters in the U.S. interest rate reaction function is 0.65 for the coefficient on the lagged interest rate and the weight on inflation and changes in output are 1.94 and 0.27, respectively. Similar findings can be found in Taylor (1993) and Smets and Wouters (2004). Note that given the small standard deviation of monetary policy shocks, the interest rate reaction functions provide a fairly good

description of monetary policy during the post-Bretton Woods era.

The mean estimate of the proportion of firms that do not re-optimize their prices in a given quarter is $\phi_H = 0.62$ for domestic firms and $\phi_F = 0.56$ for import retailers. The Calvo-style pricing coefficients imply that the average duration of price contracts is around 2 quarters for domestic firms and 2.5 quarters for import retailers. The mean estimate of the probability that households do not adjust their nominal wages in a given quarter is around 0.58, which implies that nominal wages are changed almost every 2 quarters. These aggregate degrees of nominal price and wage rigidities are lower than those reported for the Euro area in Smets and Wouters (2004) but are comparable with estimates for the U.S. in Christiano, Eichenbaum, and Evans (2005).

3.3.3 Effects of Oil Price Shocks

In this section, I examine the implications of the estimated model in terms of the effects of oil price shocks and evaluate the role of monetary policies in interacting with oil shocks.

Impulse Responses: a Brief Comparison with a VAR

To validate the DSGE model, I compare the impulse-response functions implied by our estimated DSGE model with those obtained from an a-theoretical

VAR in Figure 3.1.¹² The circled line refers to impulse responses generated by the theoretical model, shown as percentage deviations from the steady state; the solid line refers to VAR-based impulse responses and the shaded area represents the 70% error band.

Overall, the estimated model is successful in replicating the impulse responses obtained from the VAR, with all variables exhibiting their expected qualitative behaviors. In particular, the estimated model performs quite well in matching the responses of output. As Canada is a net oil exporter, positive oil price shocks lead to an expansion of aggregate demand due to the boost in oil incomes. The model generates a positive response of domestic output after a positive innovation in the price of oil, at approximately 0.15 percent above the steady state, as predicted by the VAR. With oil price changes being passed through to its oil component, the CPI inflation rises on impact, which exerts an upward pressure on the policy rate. Thus, CPI inflation and interest rate rise after an oil shock. In both cases, the model-based responses are qualitatively similar to their VAR-based counterpart in terms of the sign of the responses and dynamic persistence. However, the model underpredicts the magnitude of the response of inflation.

¹²To enable a comparison between the DSGE model and the a-theoretical VAR, I employ a procedure proposed in Chapter 2 to jointly identify the structural shocks in oil prices, domestic supply, and U.S. and Canadian interest rates in the VAR system. This identification strategy use theoretically coherent information to select impulse response functions from the complete set of orthogonal alternatives obtained from rotations of the contemporaneous impact matrices.

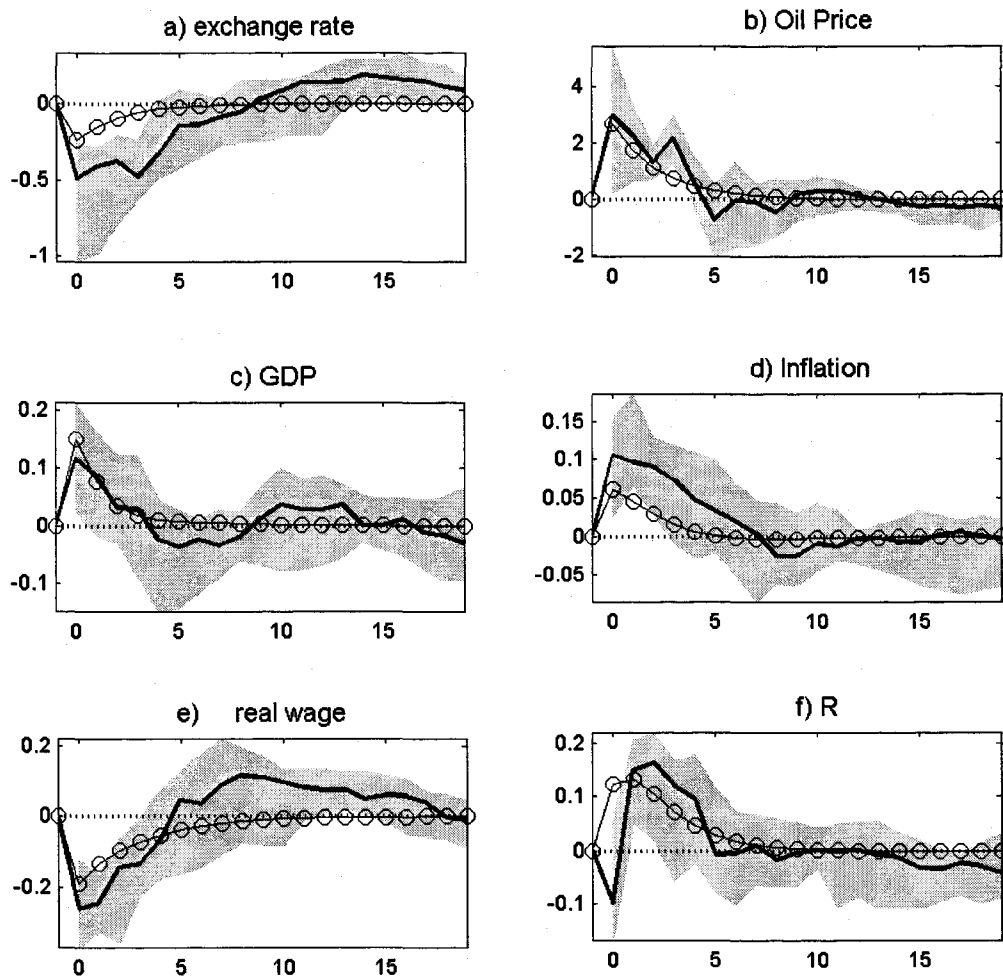


FIG. 3.1 - *Dynamic responses to a positive oil price shock. The shaded area represents the 70% error band. The circled line refers to the responses from the model.*

The real wage rate falls in response to the oil price shock. The model predicts that the negative response of the wage rate to an oil price shock is 0.2 percent, which fit the empirical impulse responses quite well. Intuitively, this occurs as a result of the sluggish adjustment of nominal wage and the

boosted price level owing to oil price shocks. Although in the medium term the difference becomes more substantial, the response of the real wage in the theoretical model is still within the 70% error band of the empirical impulse response. The model generates a negative and persistent response of the real exchange rate to an increase in oil prices, which implies that increases in oil prices appreciate Canadian dollars. The response predicted by the model, however, is smaller in magnitude than its VAR-based counterpart.

Variance and Historical Decompositions

The posterior impulse responses show the dynamic behavior of the macroeconomic variables in response to an oil shock. Of equal importance is the role that oil price shocks have played in historical episodes. To determine the role of oil price shocks in the fluctuations of macroeconomic variables, I calculate the contribution of the oil price shocks from the estimated model. Specifically, to isolate the influence of oil price shocks on a given variable, I turn off all the other exogenous shocks and simulate the model. I then obtain the paths for the variable of interest that would have emerged if only the oil price shocks had been present. To analyze in detail the effects of oil price shocks, I focus on the two recent episodes that began in 1999 and 2002 that involved large oil price shocks.¹³ Figure 3.2 and Table 3.3 display the decomposition results

¹³Following Blanchard and Gali (2007), a large oil price shock is defined as an episode

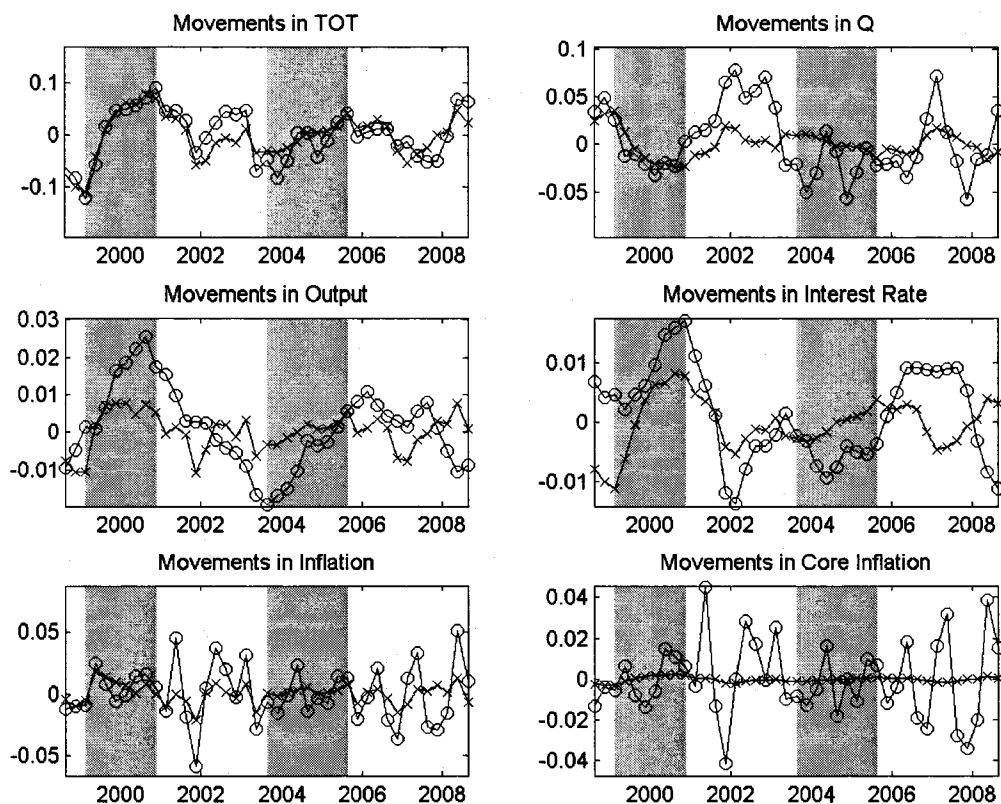


FIG. 3.2 - Note: Movements in key macroeconomic variables due to oil price shocks. The circled line refers to data and the line with marker "x" to the simulated series with oil shocks only

implied by the estimated model.

Figure 3.2 displays the behavior of the oil price shocks recovered from Bayesian estimation for the period from 1998:3 to 2008:3. The shaded vertical areas correspond to the oil shock episodes. Several observations can be made from this figure. First, the oil price shock is the primary source of variations in the terms of trade. As shown in the top left panel in Figure 3.2, the oil price shocks explain most of the dynamics in the terms of trade. Second, there involving a cumulative change in the (log) price of oil above 50%, which is subsequently sustained for more than a year.

is the relative importance of oil price shocks in explaining the variation in the real exchange rate changes over time. Note that the first oil shock episode leads to a sharp appreciation in Canadian dollars, while no clear relationship is observed between oil price shocks and the real exchange rate during the second episode. Third, the oil price shocks explain a significant amount of variation in CPI inflation; note that the timing of the first oil shock episode coincides with a sharp increase in CPI inflation. The pattern of CPI inflation during the more recent oil shock episode, however, is different. The magnitude of the changes owing to oil price shocks is much smaller than that observed in the first episode. In contrast, oil price shocks have negligible effects on core inflation. This is consistent with Hooker (2002) for the U.S., who indicates that oil price shocks have hardly passed into core inflation since the 1980's. Finally, the oil shocks partly explain the run-ups in Canadian output during the two oil shock episodes. Over the entire period, the oil price shocks account for a fair amount of fluctuations in the domestic output. As for the domestic interest rates, oil price shocks are mainly responsible for the increase in the domestic interest rate and tend to precede the latter variable's rises during the two highlighted episodes.

Table 3.3 provides statistics regarding the role of oil price shocks as a source of fluctuation, including their percentage contribution to the volatility of each variable in both absolute and relative terms. In addition, I calculate the statis-

TABLE 3.3 - The Contribution of Oil Shocks to the Canadian Economic Fluctuations

	<i>Conditional Standard Deviation</i>			<i>ConditionalSD UnconditionalSD</i>		
	1980:1-2008:3	Episode 1	Episode 2	1980:1-2008:3	Episode 1	Episode 2
Terms of Trade	3.89	6.59	2.48	0.76	0.91	0.63
Real exchange Rate	1.27	2.13	0.81	0.36	1.18	0.37
Output	0.52	0.62	0.29	0.38	0.65	0.34
Interest rate	0.39	0.72	0.25	0.25	1.24	1.04
CPI inflation	0.88	0.79	0.28	0.44	0.68	0.21
Core inflation	0.11	0.19	0.07	0.05	0.18	0.06

tics for the two oil episodes of interest. The estimated standard deviations of the oil-driven components in the major macroeconomic variables are given in the first three columns of Table 3.3. The last three columns of Table 3.3 show the relative contribution of oil price shocks to variations in these variables, measured as the ratio of the conditional to unconditional standard deviation. The results for conditional variance are consistent with the observations from the historical decompositions. For domestic output, the oil shocks account for approximately 38% of unconditional variation over the whole sample period, while during the turmoil in the first episode, the contribution of oil shocks increases to 65%. Oil price shocks contribute around 76% to the variation in the terms of trade over the entire period. This fraction reaches as high as 91% in the first episode but declines to 63% in the second episode. The oil price shocks on average account for about 36% of real exchange rate variations, while the contribution of oil shocks changes dramatically during the first episode. According to our estimates, the variation in the oil-driven component is 1.18

times larger than that of the unconditional variation of the bilateral exchange rate, which implies that there must be some other opposing forces that simultaneously affect the Canadian dollar. The oil-driven component of interest rates displays similar features. Most of the variation in interest rates during the two oil episodes can be attributed to the oil price shocks. In the cases of domestic CPI and core inflation, the fractions of overall variance explained by oil price shocks are about 44% and 5%, respectively. Note that in the absence of energy components, core inflation is much less affected by changes in oil prices. This finding is consistent with the observations in the recent literature. (Blanchard and Galí (2007); Kilian (2008))

The Transmission Mechanism of an Oil Price Shock

Since our theoretical model seems to perform reasonably well in response to structural shocks¹⁴, I now turn to an exercise of obvious interest, namely, determining the direct and indirect effect of an oil price shock on Canadian economic activity and delineating how an oil price shock is propagated in an oil-exporting economy. In this section, I explain some of the intuition behind the transmission mechanism of an oil price shock in an oil-exporting economy. In contrast to the closed-economy model, the transmission mechanism here emphasizes the influence of the world oil price shocks on the domestic economy

¹⁴The estimated model is also able to match the VAR-based impulse responses to the other identified shocks, such as responses to the domestic monetary shock, U.S. monetary policy shock and domestic supply shock.

not only through the standard intertemporal risk-sharing channel but also through international trade in good markets.

First, persistent increases in the oil prices have an immediate and positive income effect on the household's budget constraint because of boosted oil income. Increases in the price of oil generate real income gains for the household in the oil exporting country, which lead to increases in total consumption. According to equation (3.2), total consumption is allocated between oil and non-oil goods depending on their relative prices. Increases in the relative price of oil result in a substitution of oil goods for non-oil goods. The substitution effect is determined by the elasticity of substitution between oil and non-oil goods.

Second, as opposed to the direct wealth effect of oil price shocks, the fact that the domestic monetary authority endogenously reacts to oil price shocks due to inflation pressure blurs the distinction between the direct impact of oil price shocks on economic activity and their indirect effect through domestic monetary policy. This transmission mechanism has been emphasized by Bernanke, Gertler, and Watson (1997), Leduc and Sill (2004) and Carlstrom and Fuerst (2004). Note that oil price shocks have direct impacts on CPI inflation, with oil price changes being passed through to the oil component of the CPI. In addition, there is a second-round effect of oil price shocks on inflation through boosted production cost as shown in 3.30. However, the pres-

ence of sticky nominal wages and prices impedes the adjustment of real wages and delivers a sluggish adjustment in real marginal cost, which restrains the second-round effects of an oil price shock. Consequently, the presence of the oil component in headline CPI inflation opens a question whether monetary policy should react to the headline CPI inflation or the core CPI inflation.

Third, the open economy aspect of the model allows the household to smooth away shocks through international trade when the price of oil is higher. A fair amount of empirical work has documented that oil shocks induce domestic adjustments beyond the domestic channel. Issa, Lafrance, and Murray (2006) find that rising oil prices improve Canada's terms of trade and appreciate Canadian dollar against the U.S. dollar. Korhonen and Juurikkala (2007), Jahan-Parvar and Mohammadi (2009) and many others have shown that soaring oil prices lead to real exchange rate appreciation in oil-producing countries. Nonetheless, there are also consequences due to the appreciation of the Canadian dollar, which results in changes in the relative prices for domestic goods and the reallocation of demand between domestic- and foreign-produced products. Hence, the exchange rate movements have their own impact on aggregate demand, which dampens the direct effects of oil shocks.

Last, our model implies that oil price shocks also cause instantaneous reactions of U.S. monetary policy, which may influence aggregate demand in Canada through a movement in the relative prices of goods and services. For

instance, tightening U.S. monetary policy that follows oil price shocks causes a depreciation of the Canadian dollar against the U.S. dollar, which counteracts the appreciation of the Canadian dollar caused by rising oil prices. These two opposing forces contemporaneously act on domestic aggregate demand, and thus, the central bank would have to determine the relative importance of each force. Note that Ragan (2005) uses a narrative approach to discuss the consequences of the exchange rate movements and monetary policy rules.

Consequences of an Oil Price Shock When Monetary Policy Variables Remain Fixed

In this section, I decompose the effects of an oil price shock into direct and indirect components and provide rough estimates of the contribution of endogenous domestic and foreign policy response. In the first experiment, I simply fix the domestic fund rate at its steady state value for the whole simulation, in the manner of Carlstrom and Fuerst (2004).¹⁵ That is, I shut off the domestic monetary policy response that would otherwise be implied by the theoretical model after an oil shock. For convenience, I refer to this experiment as Counterfactual 1. Note that in this case, U.S. monetary policy is allowed to respond to an oil price shock that affects the domestic economy

¹⁵Sims and Zha (2006) and Bernanke, Gertler, and Watson (1997) employ similar approaches to investigate the contribution of endogenous monetary policy changes in a VAR context. They shut down the policy response by setting the policy instrument as its baseline value throughout the simulation.

through international transmission channels. The difference between the total effect of an oil price shock on the domestic variables and the effect when the domestic policy response is shut down is then interpreted as a measure of the contribution of the endogenous domestic policy response.

In the second experiment, labeled Counterfactual 2, I keep both domestic and foreign monetary policy responses constant throughout the counterfactual simulation. This scenario is intended to eliminate the resulting domestic and foreign policy component of an oil price shock, leaving only the direct effect of the oil price shocks on the economy. Similar to Counterfactual 1, the difference between the effect when only the domestic policy response is turned off and the effect when both domestic and foreign policies are turned off is then interpreted as a measure of the contribution of the systematic foreign policy response to an oil price shock.

I first compare our baseline results to those obtained in Counterfactual 1. Figure 3.3 presents the impulse response functions of domestic value added and foreign value added for an increase of one standard deviation in the price of oil. The circled lines indicate that the absence of an endogenous reaction of domestic monetary policy leads to higher domestic output and nearly unaffected U.S. output, as one would anticipate. The cross line provides estimates of the indirect contribution of endogenous monetary policy due to domestic

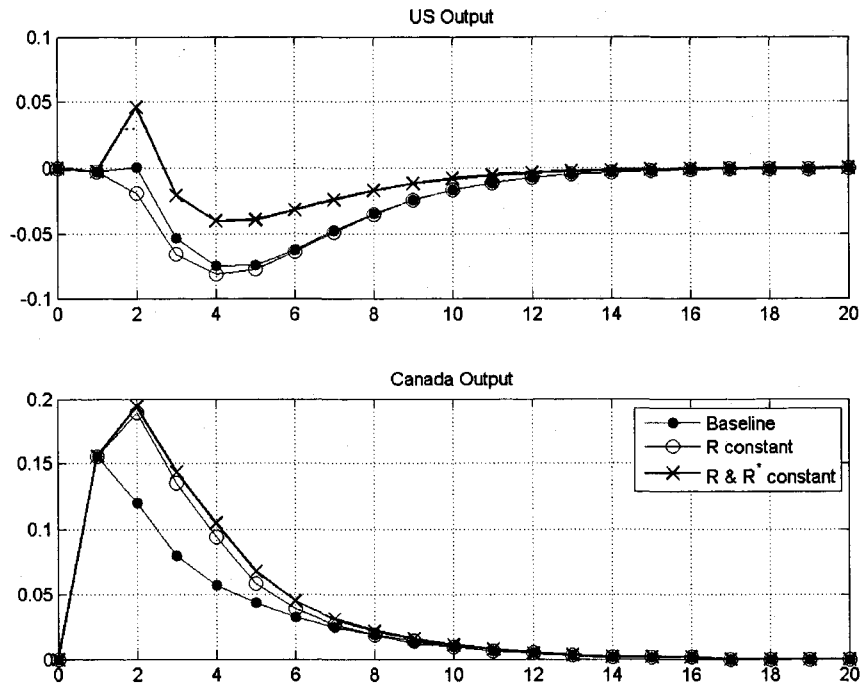


FIG. 3.3 - Note: Dynamic responses to a positive oil price shock under counterfactual scenarios. The dotted line refers to the benchmark model; the circled line to the scenario where R is kept constant for four quarters; the line with marker "x" to the scenario where R and R^* are kept constant for the whole simulation.

monetary policy and foreign monetary policy in aggregate. The results indicate that if the responses of foreign monetary policy were kept constant after an oil price shock, foreign output would have increased by 0.05 percent. As compared to Counterfactual 1, Counterfactual 2 indicates that the absence of an endogenous reaction of foreign policy is much more stimulating for foreign output because the unresponsive fund rates in the U.S. lead to a larger effect on inflation and thus a larger decline in the real rate of interest. Counterfactual 2, however, implies that an endogenous reaction of foreign policy accounts

for little of the variation in domestic economic activity.

TABLE 3.4
SIXTEEN QUARTERS CUMULATIVE OUTPUT CHANGES

	Monetary Policy			
	Baseline	Taylor Rule	Counterfactual 1	Counterfactual 2
(A) sixteen-quarter cumulative Canada output changes:				
Statistics L^y	3.60		5.10	4.80
% changes relative to Baseline	—		41.67%	33.33%
(B) sixteen-quarter cumulative U.S. output changes:				
Statistics L^{y*}	−0.39		−0.39	−0.26
% changes relative to Baseline	—		0%	33.33%

In the spirit of Dhawan and Jeske (2007), I use the following measure of output changes to estimate the contribution of endogenous monetary policies

$$L^y = \sum_{t=1}^T \beta^{t-1} (\exp(\hat{y}_t) - 1) \quad (3.54)$$

where $\hat{y}_t$ is the impulse response function of output. L^y is a measure of the present discounted stream of the output changes in period t . Table 3.4 summarizes the results presented in this section. In Panels A and B, I report the 16-quarter cumulative Canada and U.S. output changes based on an oil price shock under counterfactual experiments in Panel A and B, respectively. The second row of each panel measures the changes under each scenario relative to the baseline Taylor rule. As discussed earlier, the difference between the two values represents the indirect effect of oil price shocks on output through endogenous monetary policies. For instance, in Counterfactual 1, output would have increased by 5.1% instead of 3.6%. The difference of 1.5% is attributed to

endogenous responses of domestic monetary policy, which accounts for 41.67% of the total impact of an oil price shock. In Counterfactual 2, both domestic and foreign interest rates are stable; in this case, domestic output would have increased by 4.8%. This finding indicates that endogenous response of foreign monetary policy to oil price shocks has relatively small effects (i.e., 6%) on the cumulative changes in domestic output. Panel B in Table 3.4 shows the contribution of monetary policies to U.S. output decline. As would be expected, in Counterfactual 1, Canadian monetary policy does not affect the U.S. output at all. In Counterfactual 2, if the U.S. interest rate is kept constant, 0.26% of the 0.39% decline in U.S. output can be attributed to oil (i.e., 66.67%), and the remainder to the systematic monetary policy (that is, 33.33%). This is essentially Carlstrom and Fuerst (2004) conclusion for the U.S.

Consequences of an Oil Price Shock under Alternative Monetary Policy Rules

While the previous experiments provide rough estimates of the contribution of a standard Taylor-type rule in the wake of an oil price shock, these experiments do not address the following counterfactual question regarding whether the economic consequences of an oil price shock would change under different monetary policy regimes. If monetary policy plays an important role in explaining the transmission mechanism of oil price shocks, I should observe

different responses when policy rules change. In the rest of the section, I investigate if the central bank should use the headline CPI inflation, as opposed to core inflation as a relevant inflation measure. I address this issue by explicitly measuring the effect of an oil shock in terms of discounted variation in domestic output and price level. In what follows, I consider two alternative inflation-targeting regimes.

Rule 1 - Baseline: Our baseline Taylor rule refers to equation (3.44).

Rule 2 - Headline inflation: In the second scenario, the central bank uses the headline CPI inflation (including fuels) as a relevant inflation measure.

The monetary policy rule is specified as follows¹⁶

$$\log\left(\frac{r_t}{r}\right) = \rho_r \log\left(\frac{r_{t-1}}{r}\right) + (1 - \rho_r) \left(\phi_1 \log\left(\frac{\pi_t}{\pi}\right) + \phi_2 \log\left(\frac{y_t}{y}\right) \right) + \varepsilon_{R,t} \quad (3.55)$$

TABLE 3.5 - Sixteen Quarters Cumulative Output Changes under Different Monetary Policy Rules

	Domestic Monetary Policy Rules	
	Core Inflation Targeting	Headline Inflation Targeting
(A) sixteen-quarter cumulative Canada output changes:		
Statistics L^y	3.60	3.14
% changes relative to Baseline	—	−12.8%
(B) sixteen-quarter cumulative changes in the price level:		
Impact on CPI	111.53	110.42
% changes relative to Baseline	—	−1%

¹⁶The parameter values are kept the same as in the baseline version of the model for the alternative specification of the monetary policy rule.

Table 3.5 summarizes the results of the two alternative monetary rules over a medium-term horizon. Because the main focus will be how different alternative monetary policy rules affect the macroeconomic outcomes of oil price shocks, I report the cumulative output gains and permanent changes in price level under alternative policy rules as well as their relative contribution to the baseline Taylor rule. As shown in Table 3.5, a comparison between the alternative policy rules reveals that there is a tradeoff between output and inflation. In terms of the cumulative output gain and the impact on permanent changes in price level in the medium run, I find that for a central bank with explicit inflation targets, the core inflation-targeting rule outperforms headline inflation-targeting because the expansionary effect of an oil shock on the domestic economy is more substantial while still achieving a roughly comparable degree of headline inflation stabilization.

3.4 Sensitivity Analysis

3.4.1 Sensitivity Analysis with Different Degree of Nominal Rigidities

The persistent increases in the price of oil raise the price of factor inputs and production costs, thus requiring a domestic adjustment. However, as the rising oil prices appreciates the Canadian dollar, domestic wages and prices

would decline to stabilize the production costs and restore external competitiveness. Because neither prices nor wages are flexible enough to adjust quickly, the wage and price rigidities delay the adjustment process and therefore, play an important role in determining the real effect of monetary policy as well as the second-round pass-through of the oil price shocks. To quantify how important the degree of price and wage rigidity is to our results, I conduct a set of sensitivity exercises that vary the values of ϕ_H and ϕ_w .¹⁷ I report the 16-quarter cumulative changes in domestic output and CPI inflation owing to an oil price shock of one standard deviation under two specifications of domestic monetary policy rules as shown in Figures 3.4 and 3.5, respectively. The bottom layer on the first row in each chart plots the results obtained from the baseline model, while the top layer in each chart shows the counterfactual results with domestic interest rates kept constant for the whole simulation. The charts in the second row in each figure plot the contribution of the endogenous reaction of domestic monetary policy. Note that the contribution of monetary policy responses is measured as the vertical gap between the two layers. Several observations can be made from Figures 3.4 and 3.5.

First, other things equal, adding more stickiness (i.e., increasing ϕ_H and ϕ_w) limits the second-round effect of oil price shocks. The more stickiness

¹⁷I also perform sensitivity analysis with respect to ϕ_F , which is the degree of rigidity of imported good prices. To conserve space, the results are provided upon request.

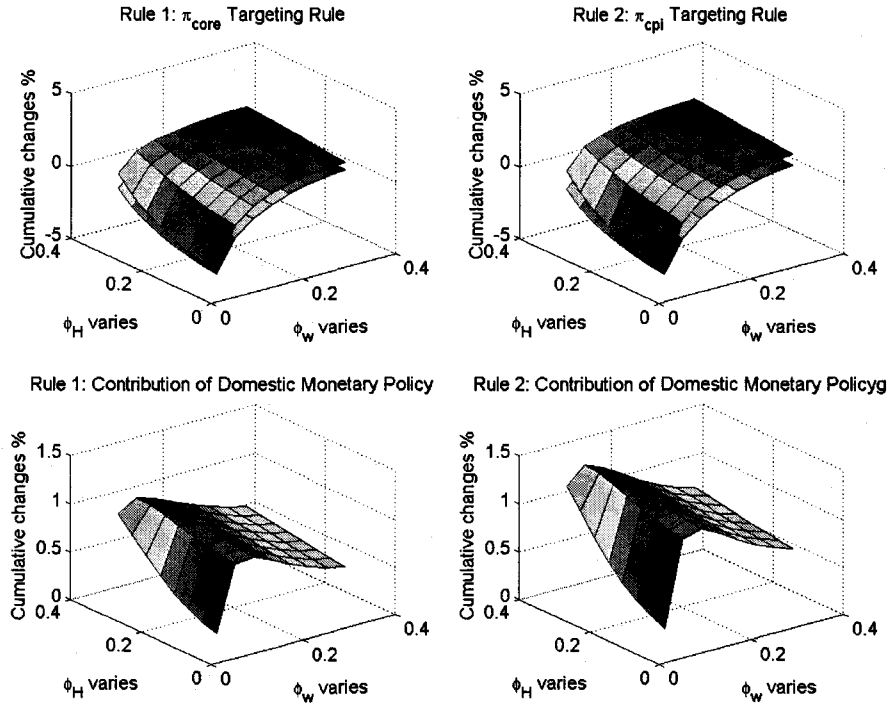


FIG. 3.4 - Note: Sixteen-quarter cumulative Canadian output changes with ϕ_H and ϕ_W varying

there is in the prices and wages, the less contractionary is the domestic monetary policy and the less inflationary the effect of oil shocks becomes. This observation can be applied to both of the monetary policy regimes. As shown in Figure 3.5, the degree of wage rigidities plays a more important role in restraining the pass-through of oil price shocks on to production costs and inflations. Inflation declines more sharply along the ϕ_W axis than along the ϕ_H axis.

Second, different monetary regimes display similar pattern in terms of cumulative output and inflation changes with respect to the degree of nominal

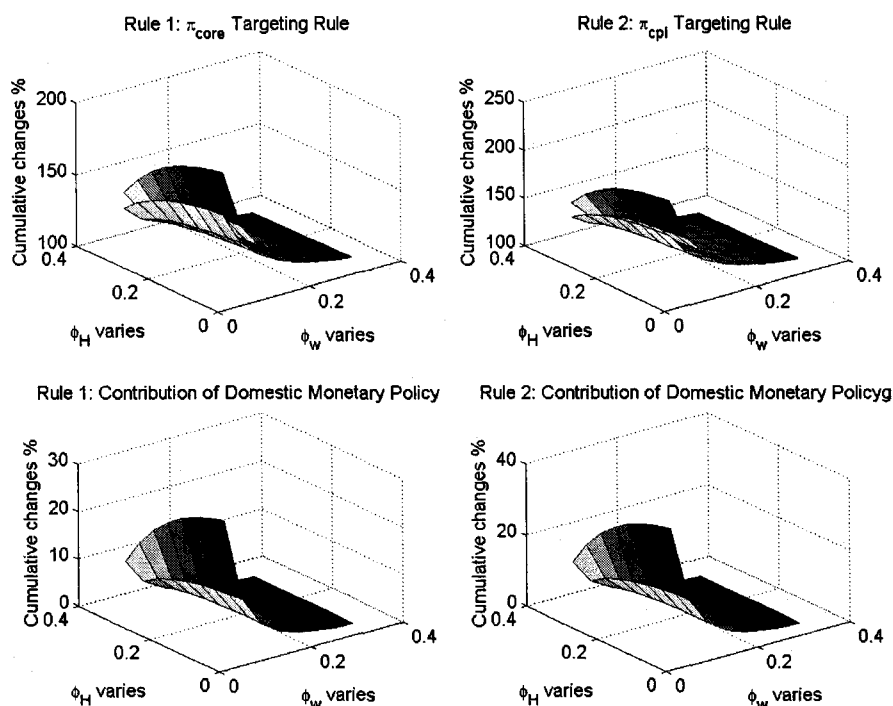


FIG. 3.5 - *Note: Dynamic responses to a positive oil price shock with ϕ_w varying*

rigidities. In a low-rigidity scenario, the quick adjustment in prices and wages following an oil price shock would raise core and headline inflation as well as their expectancies. The policy rate is instantaneously raised by quite a significant amount in response to the boosted inflations. Consequently, as Figure 3.4 illustrates, there is a noticeable drop in domestic output owing to the aggressive tightening monetary policy. As the degree of nominal rigidity increases, the degree to which oil price shocks are passed onto core inflation is weakened and the expansionary effect of oil price shocks on domestic economic activity is more evident.

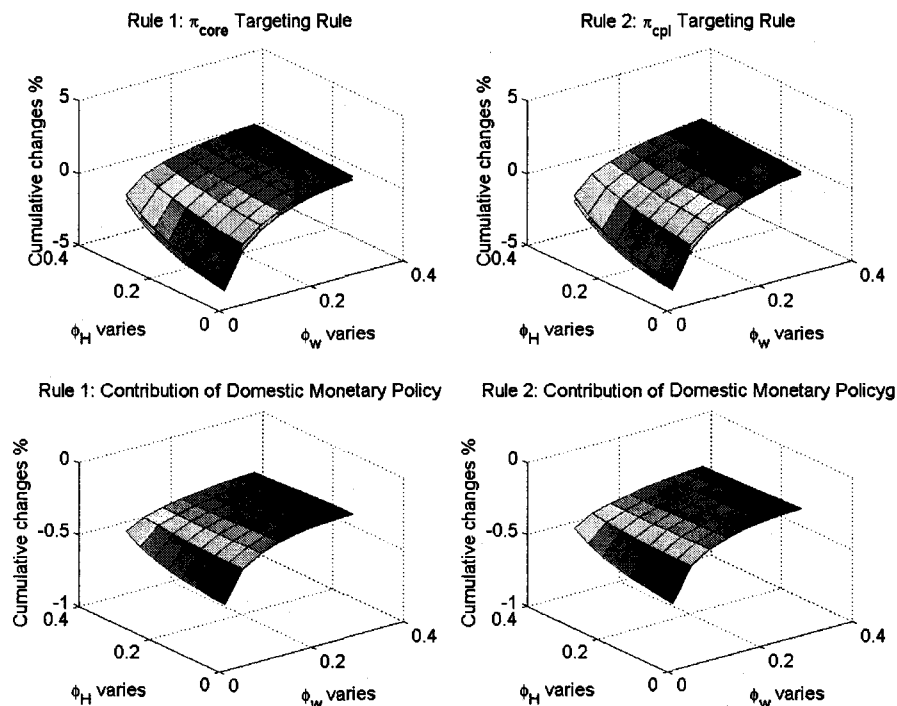


FIG. 3.6 - Note: *Dynamic responses to a positive oil price shock with ϕ_H varying*

Third, as shown in the charts in the second rows in Figures 3.4 and 3.5, the contribution of the alternative monetary policy rules to the economic consequences of oil price shocks shows a similar pattern but with different magnitudes. Under both policy rules, the central bank would exacerbate the consequences of oil price shocks on variations in inflation and output with low levels of nominal rigidities. For instance, Figure 3.5 shows that in a low-rigidity scenario, a one-standard deviation increase in the oil price would raise the CPI price level on average by 30% after 16 quarters if the monetary authority targeted either inflation measure, and while it would increase by over 50% if the

domestic interest rate were kept constant throughout the simulation.

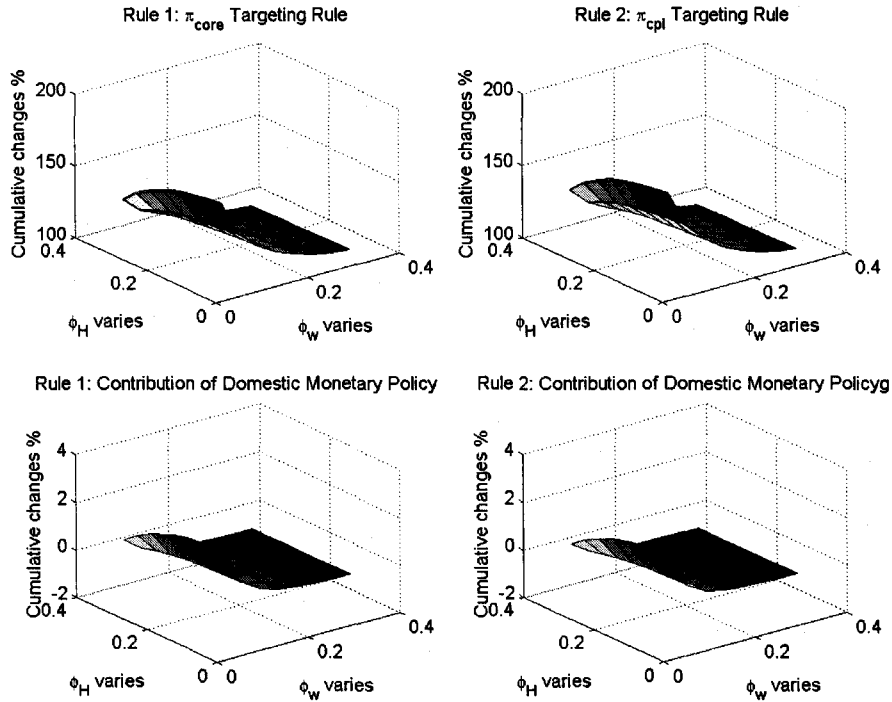


FIG. 3.7 - Note: Dynamic responses to a positive oil price shock with ϕ_w varying

Another question of interest is the contribution of U.S. monetary policy rules to domestic economic activity with respect to the degree of nominal rigidities. I display the results in Figures 3.6 and 3.7. The simulation results in Figures 3.6 and 3.7 indicate that there is little contribution of the U.S. monetary policy to the domestic output and price level 16 quarters ahead, with a wide range of nominal stickiness and irrespective of the policy regimes applied by the U.S. monetary authority. The small contribution of U.S. interest rates tends to be align with the DSGE literature (Mendoza (1991) and Schmitt-

Grohé (1998)).

3.4.2 Persistence of the Oil Price Shock

I now assess the sensitivity of our results to the degree of persistence in the oil shock process by increasing its AR(1) coefficient from 0.68 to 0.99. The results are reported in Table 3.6, which shows the cumulative output changes due to an increase of one standard deviation in oil prices, under two alternative domestic monetary policy rules. As shown in Table 3.6, the results under alternative Taylor rules are essentially indistinguishable in the case of a very persistent shock with respect to the level of oil prices. Note as well that with a more persistent shock, the degree to which the oil shock stimulates the domestic economy tends to be more substantial (24.21% and 23.85%). In contrast, the recessionary effects of oil price shocks on U.S. economic activity become more severe (-4.43% and -4.38%).

TABLE 3.6
SENSITIVITY ANALYSIS: PERSISTENCE OF OIL PRICE SHOCKS

	Core Inflation Targeting Rule	Headline Inflation Targeting Rule
(A) Sixteen-Quarter Cumulative Domestic Output Changes		
Baseline model	3.60	3.14
high persistence, $\rho_O = 0.99$	24.21	23.85
(B) Sixteen-Quarter Cumulative U.S. Output Changes		
Baseline model	-0.47	-0.54
high persistence, $\rho_O = 0.99$	-4.43	-4.38

3.5 Conclusion

This paper analyzes the direct impact of oil price shocks on economic activity in an oil-exporting country and their indirect effect through the endogenous responses of domestic and U.S. monetary policy. Such questions have seemed particularly relevant in recent years due to the fact that the enormous volatility in world oil prices has challenged the role of the U.S. dollar in pricing oil transactions and thus the dollar's international dominance.

To assess the relative importance of oil price shocks and endogenous domestic and U.S. monetary policies, I consider an open economy DSGE model with an embedded oil sector. To confront the model with the data, I use Bayesian methods to combine prior knowledge with information contained in the historical data. The estimated model is successful in replicating the features of the empirical impulse response functions obtained from the VAR. The use of the estimated model allows us to decompose the economic consequences of oil shocks via counterfactual simulations into three components, including the direct impact of oil shocks as well as the indirect effects through endogenous responses of U.S. monetary policy and the indirect effects through endogenous responses of domestic monetary policy. I find that endogenous domestic monetary policy rules are important transmission mechanisms for oil price shocks in determining their effects on aggregate demand. According to our estimated results, the reaction of U.S. monetary policy to oil shocks

through the international transmission channels has no major influence on the Canadian macroeconomic variables. According to our counterfactuals, I find that endogenous responses of U.S. monetary policy to oil price shocks only account for roughly 6% of the sixteen-quarter cumulative changes in Canadian output.

Furthermore, the presence of the oil component in headline CPI inflation opens a question whether monetary authority should react to the headline inflation or core inflation. Our analysis suggests that the core inflation-targeting rule outperforms the headline-inflation targeting Taylor rule in terms of discounted variation in domestic output and price level. I conclude by conducting some robustness analysis. I find that, with low levels of nominal rigidities, the central bank in a small economy will exacerbate the economic consequence of oil price shocks on inflation and output changes. In terms of how the U.S. monetary policy rules contribute to the domestic economic performance after an oil shock, I find that the result indicating the small contribution of the U.S. interest rate is robust.

In the following chapter, we documents another interesting macroeconomic phenomena: the excess volatility of aggregate profits. We embed intangible capital into an otherwise standard RBC model to examine the role of intangible capital in driving cyclical dynamics of S&P500 earnings.

Chapter 4

Intangible Capital, Corporate Earnings and the Business Cycle

Keqiang Hou & Alok Johri

4.1 Introduction

Large swings in the profitability of U.S. corporations is an important aspect of the business cycle. As shown in Figure 4.1, profits, as measured by aggregate real earnings of S&P 500 corporations, are about seven times as volatile as output and have a contemporaneous correlation of about 0.5. While investors fret over reported earnings and the financial media devote endless energy to

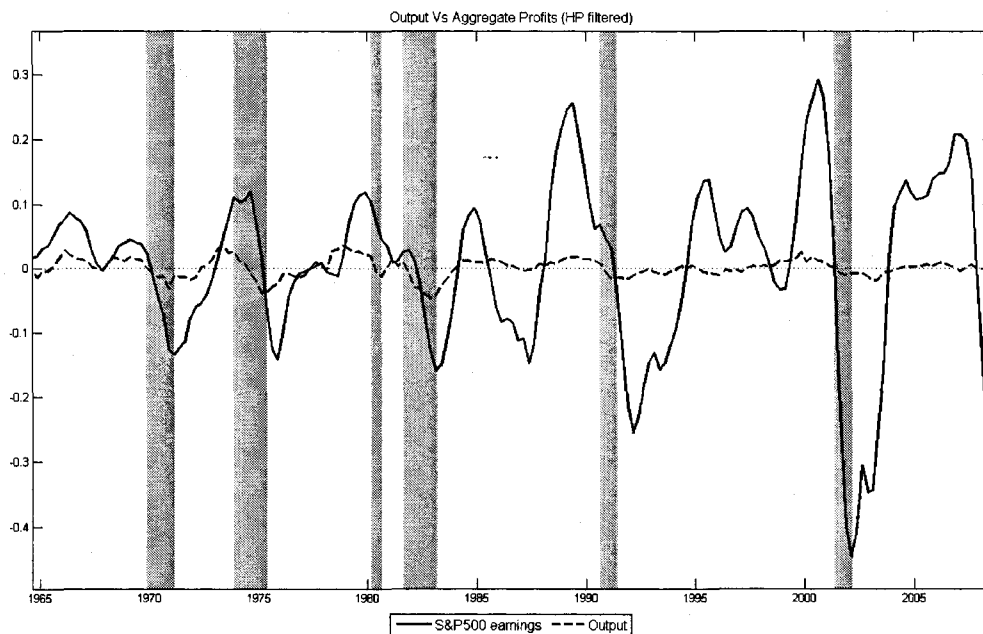


FIG. 4.1 - U.S. output Vs aggregate profits. The solid line refers to the corporate earning data and the dashed line to the U.S. real output. Both series are detrended by the HP filter. The shaded vertical areas correspond to the NBER recession periods.

anticipating corporate results during “earnings season”, the macroeconomics literature on economic fluctuations is largely silent on the phenomenon.

A little reflection reveals that most popular models of business cycles are not consistent with the earnings data. The typical specification of the production technology calls for constant returns to scale in labour and physical capital services. With competitive markets this implies zero profits. One obvious way to generate profits is to assume decreasing returns to scale. This implies that profits move in proportion to output, yielding both a relative volatility number as well as a contemporaneous correlation coefficient equal to unity. Another obvious possibility is to assume that firms have some market

power. Once again the typical formulation of imperfect competition implies a constant markup of price over marginal cost which implies the model cannot generate profits that are more volatile than output.¹

In this chapter we explore the idea that competitive firms may still generate profits because they produce intangible capital. Intangible capital is modeled as a third input in the technology for producing final goods in addition to labour and physical capital. If firm level investments in intangible capital are pro-cyclical, then the extra productivity unleashed by these investments can lead to profits rising more than output in periods of high activity. We embed this feature into an otherwise standard dynamic general equilibrium model and estimate the model using aggregate U.S. data. We find that the model generates aggregate profits that are much more volatile than output and are positively correlated with output as well. Simulations based on our estimates of model parameters reveal that earnings are roughly eight times as volatile as output and the correlation coefficient between the two series is 0.49. Investments in IC in the model are in fact procyclical and this cyclicalit helps to explain the observed variation in real corporate earnings over the cycle.²

¹There do exist a few models with time varying markups which could potentially do the job, but a model with counter-cyclical markups (which is often the case) is unlikely to work. An intriguing exception is the new study by Edmond and Veldkamp (2009) which posits a model with counter-cyclical markups but procyclical profit shares. However their model lacks capital accumulation so it is hard to assess the quantitative importance of the proposed mechanism in a full blown quantitative DGE model. The focus of the authors is also somewhat different. They discuss the relative volatility of the profit share as opposed to that of profits.

²The procyclicalit of IC is reminiscent of the evidence that R&D expenditure is procyclical. While R&D may be viewed as investment in intangible capital, most observers believe

When firms raise spending on IC, this comes at the expense of reduced current profits but results in much higher profits in the quarters that follow.

Investments in intangible capital can be thought of as any expenditures by the firm (that are not included in physical capital investment) that raise it's ability to produce or that lower it's costs of production. Corrado, Hulten, and Sichel (2006) mention spending on "innovative property (eg., R&D) and economic competencies as well as software and other computerized information..." as some types of intangible capital. This includes spending on strategic planning, spending on redesigning or reconfiguring existing products in existing markets, investments to retain or gain market share, and investments in brand names. They further divide R&D expenditures into a "scientific and non-scientific" category where the latter includes new product development in the services sector.

Recent evidence from the U.S. economy also suggests that these investments are a large and growing part of the economy. For example, Corrado, Hulten, and Sichel (2006) report that the ratio of intangible to tangible investment increased from roughly 1.10 in the previous decade to over 1.3 since the turn of the century. This aggregate work is backed up by microeconomic studies such as Brynjolfsson, Hitt, and Yang (2002) which discuss the importance of firm specific investments in (what they refer to as) organizational capital in

it to be a small part of the total expenditure on IC. Barlevy (2007) offers an alternate explanation for the procyclicality of R&D.

determining the success of firms. A number of recent studies have also argued that investments in intangible capital by firms are important for understanding medium run observations on productivity and asset returns (McGrattan and Prescott (2007); Hall (2001)).

Despite the increased interest of economists in understanding the role played by, and magnitude of, intangible capital investments, little effort has been devoted to understanding the business cycle implications of an economy which devotes a significant amount of resources to the accumulation of intangible capital. We hope to fill this void. The goal is to go beyond a study of aggregate earnings dynamics and to understand how investments in intangible capital alter the dynamic response of the typical business cycle model to aggregate shocks as well as to understand how the business cycle influences the creation of intangible capital.

We estimate this model using Bayesian techniques and compare the performance of the model with intangible capital to one without IC. A number of interesting results emerge from this effort. We find that the ability of the *IC* model to explain aggregate output and hours data is many orders of magnitude higher than the model without IC. The estimation results indicate that in order to choose the *no-IC* model over the *IC* model, we need to assign a prior probability 7.11×10^{11} times larger to the *no-IC* model than to the *IC* model. The model is an effective propagation mechanism, with output

displaying considerably more inertia in response to shocks than the standard model (the AR(1) coefficient of output growth equals 0.3 vs. 0.003). This occurs because an increase in IC leads to future increases in productivity thus propagating shocks over time.

Our estimates imply that the steady state ratio of investments in intangible relative to tangible capital is about 0.75 which is less than the estimates reported by Corrado, Hulten, and Sichel (2006) and slightly more than those reported by McGrattan and Prescott (2007). Firm investments in intangible capital allow it to earn a small profit in steady state of less than one percent of aggregate output which is in keeping with the earnings data.

As mentioned before, the model generates aggregate earnings behaviour that closely mimics the behaviour of the data.³ In contrast, the *no-IC* model predicts a correlation of unity between earnings and output as well as a relative standard deviation which is unity as well.⁴ A variant of the *no-IC* model where firms accumulate physical capital performs even worse, predicting a negative correlation with output. We think the ability of the *IC* model to explain earnings is important because it allows the model to distinguish itself from a host of other mechanisms to improve the fit of DGE models to observed

³In this study, we use S&P500 earnings as a proxy for real profits of firms. The earning data is taken from Shiller (2000). We also examined the corporate net cash flow (CNCF) series as an alternative proxy. The standard deviation of Hodrick-Prescott filtered (logged) quarterly CNCF relative to output is roughly 3.1 and the correlation with output is positive (0.55).

⁴Earnings in the *no-IC* model are calculated by allowing decreasing returns in the production technology for a fair comparison with the *IC* model. Strictly speaking our estimates of the *no-IC* model imply constant returns to scale in production and therefore zero earnings.

output and hours series. Examples of the mechanisms we have in mind are based on modifications to the household side of the model such as changes in preferences (habit formation in leisure) or other ways to introduce dynamics in labour supply (learning effects, human capital etc.).⁵

4.1.1 Related literature

The idea that firms accumulate unobserved inputs that raise productivity has also been explored in the organizational capital literature. Here the accumulation of the unobserved input typically involves a learning-by-doing process by which firms accumulate knowledge as a by-product of production. Examples of dynamic general equilibrium models with this feature have been explored in closed and open economy settings in Cooper and Johri (2002), Johri and Lahiri (2008) and Johri (2009). A key difference between those models and the IC model is that firms must expend valuable resources to produce intangible capital while the learning-by-doing process creates organizational capital as a by-product of production. We do borrow from that literature the idea that the contribution of past knowledge diminishes with time.⁶

Our IC model is related to, but different from, a number of recent ideas ex-

⁵Examples of these include Perli and Sakellaris (1998), Chang, Gomes, and Schorfheide (2002), Bouakez and Kano (2006), Jones, Manuelli, and Siu (2005). While these may be important mechanisms in their own right, they cannot help explain the cyclical behaviour of earnings.

⁶There is considerable evidence that this is indeed the case. See Benkard (2000) for examples as well as Cooper and Johri (2002) as well as Johri and Letendre (2007) for aggregate evidence.

plored in the business cycle context using dynamic general equilibrium models. The first set of models study the implications of human capital for economic fluctuations. Early work on these ideas was published in Perli and Sakellaris (1998). These models typically model human capital as being accumulated by workers. Workers supply a joint input of human capital and raw labour to the production technology and get paid a wage commensurate with the return of the composite labour input. For example, Jones, Manuelli, and Siu (2005) study the business cycle properties of an endogenous growth model with human capital accumulation. The planner can accumulate human capital by setting aside consumption goods as he would for investments in physical capital. Similarly De Jong and Ingram (2009) estimate a DGE model of skill acquisition using techniques similar to those used here. Unlike the previous study, skill or human capital is acquired purely as a function of time but not of goods. The modeling of human capital also differs in that it enters the production technology with a lag. In contrast with these models where agents expend resources on human capital, Chang, Gomes, and Schorfheide (2002) invoke the notion of by-product learning-by-doing to accumulate human capital, again purely as a function of hours. Similarly Kim and Lee (2007) combine both aspects into their human capital accumulation equation. As mentioned earlier, our IC model focuses on decisions made at the firm level which lead to IC being a state variable in the firms problem allowing the firm to earn profits

even though it operates in a competitive industry. Our model also differs from most of the above in that firms use both labour and physical capital to create new intangible capital.

Our model of intangible capital is also related to the endogenous R&D models of medium term fluctuations such as Comin and Gertler (2006). While the details of the model are very different, certain aspects of the R&D process have similarities to our intangible capital technology. In the Comin and Gertler model, agents invest consumption goods in a process that yields innovations, which if adopted, become new varieties of intermediate goods. As in Romer (2006), this leads to an increase in productivity in the final good technology.

Finally our *IC* model is similar to McGrattan and Prescott (2007). The notion of unmeasured capital used in that paper is much broader than our notion of intangible capital but shares many similarities in how the technology for the creation of this capital is modeled. Nonetheless, the models differ in crucial respects. All expenses on IC in our model are accounted for and there is no “sweat equity” type notions used here. Moreover, we focus on explaining US business cycles generally while their focus is on the 1990’s. Finally we provide estimates of our model while they focus on calibration.

The rest of the chapter is organized as follows. Section 3.2 lays out the basic structure of our model economy. Section 3.3 discusses the econometric methodology and the data and then presents the empirical results. Sensitivity

analysis can be found in section 3.4. Section 3.5 concludes.

4.2 The Model Economy

In this section, we specify a decentralized dynamic general equilibrium model in which firms accumulate intangible capital. For convenience we refer to this model as the *IC* model which we will contrast with the standard model referred to as the *no-IC* model. While all agents in the economy operate in competitive markets, we will assume a single representative household and firm for convenience.

4.2.1 The Household's problem

The representative household maximizes its expected discounted utility over an infinite time horizon:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, \mathcal{B}_t). \quad (4.1)$$

where

$$U(C_t, N_t, \mathcal{B}_t) = \ln(C_t) - \phi \mathcal{B}_t N_t$$

Here β is the discount factor and the utility function in period t depends positively on contemporaneous consumption, C_t and labor supply, N_t . The variable $\mathcal{B}_t$ represents a shock to preferences and follows a first-order autore-

gressive process with an iid error term:

$$\ln \mathcal{B}_t = \rho_b \ln \mathcal{B}_{t-1} + \epsilon_{bt} \quad (4.2)$$

In each period, the representative household supplies labor and physical capital to the firm, taking as given the wage rate w_t and the rental rate on capital, r_t^k . In addition, as the owner of the firm, the household receives any real profits earned by the firm, π_t . The sequence of budget constraints is given by

$$C_t + I_t = w_t N_t + r_t^k K_t + \pi_t. \quad (4.3)$$

The right-hand side of the budget constraint represents the sources of wealth: labor income $w_t N_t$; the return on the real capital stock, $r_t^k K_t$ and the profits earned by the firm. The left-hand side shows the uses of wealth: consumption spending and investment (I_t) in physical capital. Investment augments the physical capital stock over time according to

$$K_{t+1} = I_t + (1 - \delta)K_t \quad (4.4)$$

where $\delta \in (0, 1)$ is a constant depreciation rate for physical capital.

Given initial values, the household chooses $\{C_t, N_t, I_t, K_{t+1}\}, t = 0, 1, 2, \dots$, to maximize the objective function (4.1) subject to the budget constraint (4.3) and the capital accumulation equation (4.4). The first-order conditions asso-

ciated with this problem are:

$$w_t = B_t \frac{U_{n,t}}{U_{c,t}} \quad (4.5)$$

$$1 = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} (r_{t+1}^k + 1 - \delta) \right] \quad (4.6)$$

where $U_{c,t}$ and $U_{n,t}$ are, respectively, the marginal utility of consumption and marginal utility of leisure. Since the household problem is standard and well understood, we will omit any discussion of the optimality conditions and move on to the firm's problem.

4.2.2 The Firm's Problem

We assume that all production occurs at a single firm that behaves competitively and takes factor prices as given. The firm produces the final good according to the following constant returns to scale technology which uses labour, physical capital and intangible capital as inputs:

$$y_t = (A_t u_t^n N_t)^\alpha (u_t^k K_t)^{1-\alpha-\varepsilon} Z_t^\varepsilon. \quad (4.7)$$

The presence of a third input, intangible capital, Z_t , in addition to the usual labor N_t and physical capital K_t is what distinguishes the model from the typical business cycle structure. Not all of the labor and capital hired by the firm goes to the production of the final good, y_t . The variables, u_t^n and u_t^k denote, respectively, the fraction of labor and physical capital which the firm allocates to output production. The remainder of labor and capital are used

to produce new intangible capital. The technology shock, A_t , is assumed to follow a random walk with drift process:

$$\ln A_t = \gamma_a + \ln A_{t-1} + \epsilon_{at} \quad (4.8)$$

where ϵ_{At} are *iid* shocks.

The stock of intangible capital for the next period, requires labor, physical capital and intangible capital. The IC technology is given by

$$Z_{t+1} = \left[(A_t(1 - u_t^n)N_t)^{\alpha_1} ((1 - u_t^k)K_t)^{1-\alpha_1} \right]^{(1-\gamma)} Z_t^\gamma \quad (4.9)$$

where $\alpha_1(1 - \gamma)$ represents the elasticity of hours spent on creating IC in the current period with respect to IC in the next period. When $\alpha_1 = 0$, the firm allocates all of it's labour to the production of the final good and when $\alpha_1 = 1$, physical capital is no longer used in intangible capital creation. The other parameter $\gamma \in (0, 1)$, indicates that the contribution of past intangible capital decays the further back in time that it was created. This captures the idea that the relevance of knowledge falls with time as the economic environment undergoes change. This is consistent with the notion of organizational forgetting explored in Benkard (2000) and the depreciation of organizational capital discussed in the learning literature.⁷ We would expect the performance of the *IC* model to approach that of the *no-IC* model for values of γ close to unity.

⁷Note the IC technology may be viewed as a log-linear accumulation equation for IC with γ governing the depreciation rate of IC. Clarke (2006) suggests that the dynamics of organizational capital models with linear and log-linear accumulation equations is very similar. We expect the same to be true here.

To see this, note that $\gamma = 1$ implies that intangible capital is constant over time. Note also that $\gamma = 0$ implies that intangible capital available to the firm at present makes no contribution to future levels of intangible capital. The productivity shock appears in equation (4.9) in order to ensure a balanced growth path and can be understood to imply that increases in the productivity of labour over time apply to both activities of the firm. This appears to be a reasonable assumption.⁸

In each period, the firm maximizes the present value of real profits:

$$\mathcal{X}_t = \max E_t \sum_{s=0}^{\infty} \Xi_{t+s} (y_{t+s} - w_{t+s}N_{t+s} - r_{t+s}^k K_{t+s})$$

subject to (4.7), and (4.9). where the variable $\Xi_{t+s} = \beta \frac{U_{c,t+s}}{U_{c,t}}$ is the appropriate endogenous discount factor for the firm. The first order conditions are given by:

$$w_t = \alpha \frac{y_t}{N_t} + \alpha_1(1 - \gamma)\lambda_t^Z \frac{Z_{t+1}}{N_t} \quad (4.10)$$

$$r_t^k = (1 - \alpha - \varepsilon) \frac{y_t}{K_t} + (1 - \gamma)(1 - \alpha_1)\lambda_t^Z \frac{Z_{t+1}}{K_t} \quad (4.11)$$

$$\alpha \frac{y_t}{u_t^n} = \alpha_1(1 - \gamma)\lambda_t^Z \frac{Z_{t+1}}{1 - u_t^n} \quad (4.12)$$

$$(1 - \alpha - \varepsilon) \frac{y_t}{u_t^k} = (1 - \gamma)(1 - \alpha_1)\lambda_t^Z \frac{Z_{t+1}}{1 - u_t^k} \quad (4.13)$$

$$\lambda_t^Z = E_t \left[\frac{\Xi_{t+1}}{\Xi_t} \left(\varepsilon \frac{y_{t+1}}{Z_{t+1}} + \gamma \lambda_{t+1}^Z \frac{Z_{t+2}}{Z_{t+1}} \right) \right] \quad (4.14)$$

where λ_t^Z is the Lagrange multiplier associated with equation (4.9). Equations

⁸One might expect that there should be some randomness associated with the ability of the firm to create intangible capital. In practice, however, an additional shock in the IC equation gets confounded with the preference shock and cannot be separately identified when estimating the model. As a result we chose to leave it out.

(4.10) and (4.11) differ from the typical conditions in that the firm will not equate the marginal product of labor and physical capital, respectively, to their factor prices. Rather, the prices will be higher than the marginal products. This occurs because only a part of labour and capital is used in production, the rest is used to produce intangible capital which in turn raises production and hence profits in the future. This dynamic consideration facing the firm when it decides how much labour and capital to hire shows up in the additional term involving Z_{t+1} that appears on the right hand side of both conditions.

Equations (4.12) and (4.13) state that the firm should allocate inputs from the production of final good to intangible capital in such a way that the marginal decrease in output is exactly equal to the value of the marginal increase in intangible capital made available to the firm as a result of the switch. Replacing (4.12) and (4.13) in (4.10) and (4.11) respectively yields (4.15) and (4.16) below. Since u^n and u^k are positive fractions, it is clear that factor prices exceed their marginal products in final output production. Note also that the u^i 's act as time varying wedges between factor prices and marginal products.⁹

$$w_t = \frac{1}{u_t^n} \cdot \alpha \frac{y_t}{N_t} \quad (4.15)$$

$$r_t^k = \frac{1}{u_t^k} \cdot (1 - \alpha - \varepsilon) \frac{y_t}{K_t} \quad (4.16)$$

Equation (4.14) establishes the marginal value of an extra unit of intangible

⁹The wedge between marginal product of labour and wages in the data is often interpreted as evidence of monopoly power. In steady state, our model estimates would imply a markup of $\frac{1}{0.79} = 1.27$ even though the firm behaves competitively.

capital to the firm. The benefit comes not only from the extra production of final good made possible but also from the additional intangible capital that can be produced in the future.

Recall that firm earnings or profit each period is given by

$$\pi_t = y_t - w_t N_t - r_t^k K_t. \quad (4.17)$$

Substituting (4.15) and (4.16) for factor prices yields

$$\pi_t = y_t \left(1 - \frac{1}{u_t^n} \cdot \alpha - \frac{1}{u_t^k} \cdot (1 - \alpha - \varepsilon) \right). \quad (4.18)$$

It is clear from (4.18) that the time varying nature of the factor proportions (u^i 's) are crucial for breaking the tight link between earnings and output implied by the model without intangible capital.

Defining the firm's investment in intangible capital as I_t^z where

$$I_t^z = w_t N_t (1 - u_t^n) + r_t^k K_t (1 - u_t^k), \quad (4.19)$$

and rearranging this using (4.15) and (4.16) we can write the relationship between output and investment in IC as

$$\frac{I_t^z}{y_t} = \left(\frac{1}{u_t^n} - 1 \right) \cdot \alpha + \left(\frac{1}{u_t^k} - 1 \right) \cdot (1 - \alpha - \varepsilon). \quad (4.20)$$

Clearly investments in IC are increasing in output and decreasing in the share of factors allocated to output, u 's. If these u 's are procyclical then they work towards generating counter-cyclical investment in IC, though we would expect the positive relationship between output and investment in IC to dominate.

The above equation (4.19) also allows us to tease out the relationship between the firm's earnings and investments in its IC:

$$\pi_t = \varepsilon y_t - I_t^z. \quad (4.21)$$

Equation (4.21) brings out the trade-off facing the firm nicely. Investments in intangible capital allow the firm to become more efficient in the future, raising future profits, however, this comes at the cost of reducing current period profits by diverting resources away from the production of the final good.

The *no-IC* model is just a special case of the *IC* model so we do not discuss it in detail. The efficiency conditions would be the same as here as long as u_t^i 's were equal to unity for all t . In addition (4.9) would be eliminated.

4.2.3 Equilibrium

A competitive equilibrium consists of sequences of allocations $\{C_t, n_t, I_t, K_{t+1}, \pi_t, u_t^n, u_t^k, Z_{t+1}, y_t\}_{t=0}^\infty$, and prices $\{w_t, r_t^k\}_{t=0}^\infty$ such that, taking as given K_0, Z_0 and exogenous processes $\{A_t, B_t\}_{t=0}^\infty$:

- $\{C_t, n_t, I_t, \pi_t, K_{t+1}\}_{t=0}^\infty$ solve the household problem
- $\{u_t^n, u_t^k, Z_{t+1}, y_t\}_{t=0}^\infty$ solve the firm's problem
- Market clearing conditions for goods, labor and physical capital are satisfied.

4.2.4 Intangible Capital in Steady State

In this section we discuss some properties of intangible capital in steady state and the impact of varying key parameters associated with the intangible capital process.

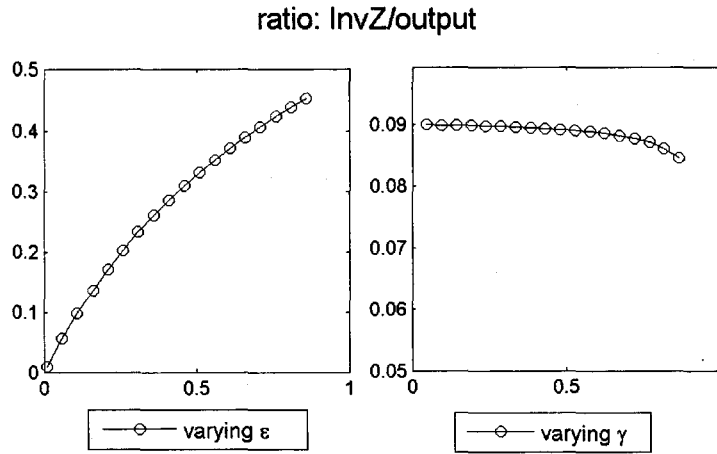


FIG. 4.2
RATIO AT STEADY STATE: I^z/Y

The ratio of investment in intangible capital to output $\frac{\bar{I}^z}{\bar{y}}$ and the ratio of the investment in intangible relative to tangible capital $\frac{\bar{I}^z}{\bar{I}}$ in steady state are now, respectively, given by

$$\frac{\bar{I}^z}{\bar{y}} = \beta\epsilon \frac{1-\gamma}{1-\beta\gamma} \quad (4.22)$$

$$\frac{\bar{I}^z}{\bar{I}} = \frac{\bar{r}\beta\epsilon \frac{1-\gamma}{1-\beta\gamma}}{\bar{z} \left(1 - \alpha - \epsilon + (1 - \alpha_1)\beta\epsilon \frac{1-\gamma}{1-\beta\gamma} \right)} \quad (4.23)$$

where $\bar{z}$ denotes the constant growth rate of output and $\bar{r} = \frac{\bar{z}}{\beta} + \delta - 1$ denotes the steady state interest rate.

According to equation (4.22), the ratio of the investment in IC to output, I^z/Y depends only on the parameters ε and γ but not on α_1 . Figure 4.2 shows how this ratio varies with respect to these two parameters. The results indicates that ε is the key parameter in determining the value of I^z/Y while γ only has a marginal effect. I^z/Y increases strongly with ε because this parameter controls the contribution of intangible capital to output.

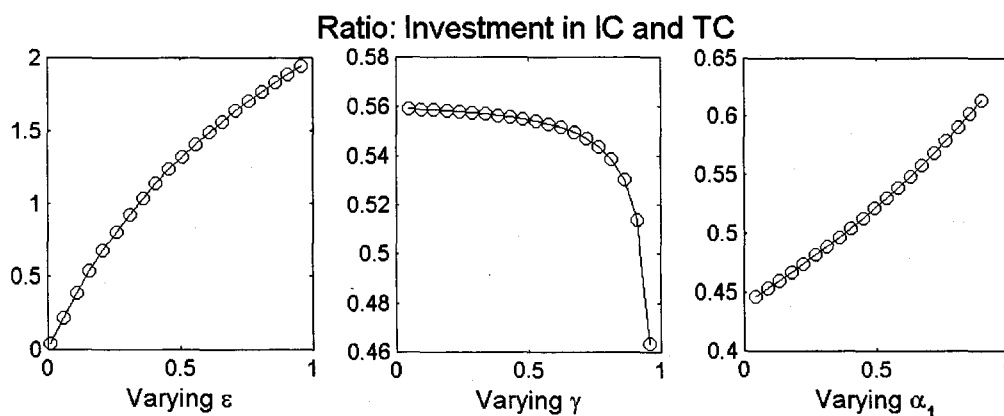


FIG. 4.3
INVESTMENT RATIOS AT THE STEADY STATE

A number of studies for the U.S. have reported quite a large range for the ratio of investment in intangible relative to tangible capital (TC) (see, for example, Corrado, Hulten, and Sichel (2006) and McGrattan and Prescott (2007)). We find that our model can imply a wide range of values for this ratio as key parameters of the IC process are varied. Figure 4.3 displays these relationships. The panels plot, respectively, the values of the steady

state ratio as ε , α_1 and γ vary. We find that the ratio of the two investments is increasing strongly in both ε and α_1 and is decreasing in γ . To see the impact of ε on the results, note first that physical capital contributes to output both directly via the production function and indirectly as an input in the creation of intangible capital. Next note that as the share of intangible capital in output increases, the share of physical capital in output becomes smaller due to the restriction of constant return to scale. Thus, as ε increases, the direct effect of intangible capital in goods production becomes more important while the direct effect of tangible capital in output is weakened. This fact makes investment in intangible capital more favourable over investment in physical capital, which leads to a larger $\bar{I}^z/\bar{I}$. By contrast, as γ increases, the current stock of intangible capital contributes more to future intangible capital. Therefore, the firm has less incentive to invest in intangible capital, which leads to a fall in $\bar{I}^z/\bar{I}$. Finally, as α_1 gets larger, physical capital contributes less to the production of Z_{t+1} relative to Z_t , which dampens the indirect effect of physical capital on output through future intangible capital. So it makes sense to invest more in intangible relative to physical capital. Figure 4.3 suggests that for plausible values of parameters, our model predicts that this ratio will be less than unity.

4.3 Empirical Method and Results

In this section we discuss our methodology for estimating and evaluating the empirical performance of two competing models. We make use of Bayesian methods which have become popular in the DSGE literature.¹⁰ Note that the equilibrium system of a DSGE model can be linearly approximated around its stationary steady-state in the form of

$$\mathbf{A}E_t(\hat{\mathbf{x}}_{t+1}|\mathbf{I}_t) = \mathbf{B}\hat{\mathbf{x}}_t + \mathbf{C}(F)E(\xi_t|\mathbf{I}_t) \quad (4.24)$$

where $\hat{\mathbf{x}}_t$ is a vector of endogenous variables¹¹, $E_t(\hat{\mathbf{x}}_{t+1}|\mathbf{I}_t)$ is the expectation of $\hat{\mathbf{x}}_{t+1}$ given period t information, ξ_t is a vector of exogenous stochastic processes underlying the system, and $\mathbf{C}(F)$ is a matrix polynomial of the forward operator F . The solution of the log-linearized system (4.24) can be written in the following state-space form:

$$\hat{s}_{t+1} = \mathbf{P}\hat{s}_t + \mathbf{C}_1\xi_{t+1} \quad (4.25)$$

$$\hat{y}_t = \mathbf{Q}\hat{s}_t \quad (4.26)$$

where the vector $\xi = \begin{bmatrix} \hat{\xi}_{At} \\ \hat{\xi}_{pt} \end{bmatrix}$ contains technology and preference innovations.

Then we update the state-form solution by adding a set of measurement equations which link the observed time series to the vector of unobserved state

¹⁰See, for example, Rabanal and Rubio-Ramírez (2005); Lubik and Schorfheide (2007); Del Negro and Schorfheide (2001)

¹¹For any stationary variable x_t , we define $\hat{x}_t = \left(\frac{x_t - \bar{x}}{\bar{x}}\right)$ as the percentage deviation from its steady-state value, $\bar{x}$.

variables. We further use the Kalman filter to evaluate the likelihood function of the state-space form solution and combine the likelihood function with our specified prior knowledge about these deep parameters to form the posterior distribution function. The sequence of posterior draws can be obtained using Markov Chain Monte Carlo (MCMC) methods. We use the random-walk Metropolis-Hasting algorithm as described in Schorfheide (2000) to numerically generate the Markov chains for the structural parameters.

4.3.1 Data and Specification of Priors

We use quarterly U.S. data taken from the Federal Reserve Bank of St. Louis' Fed database. The data sample consists of seasonally adjusted quarterly time series, from 1959:1 to 2008:3, on total hours for non-agricultural industries and growth rate of real GDP in chained 2000 dollars. Both series are expressed in per capita terms by dividing by the civilian non-institutional population, ages 16 and over.

To specify our priors, we use information about key ratios in steady state to pin down prior means. This is particularly important for ε , α_1 and γ as there is little guidance in the literature about reasonable values to use. These ratios are the capital output ratio and the labour share in steady state which

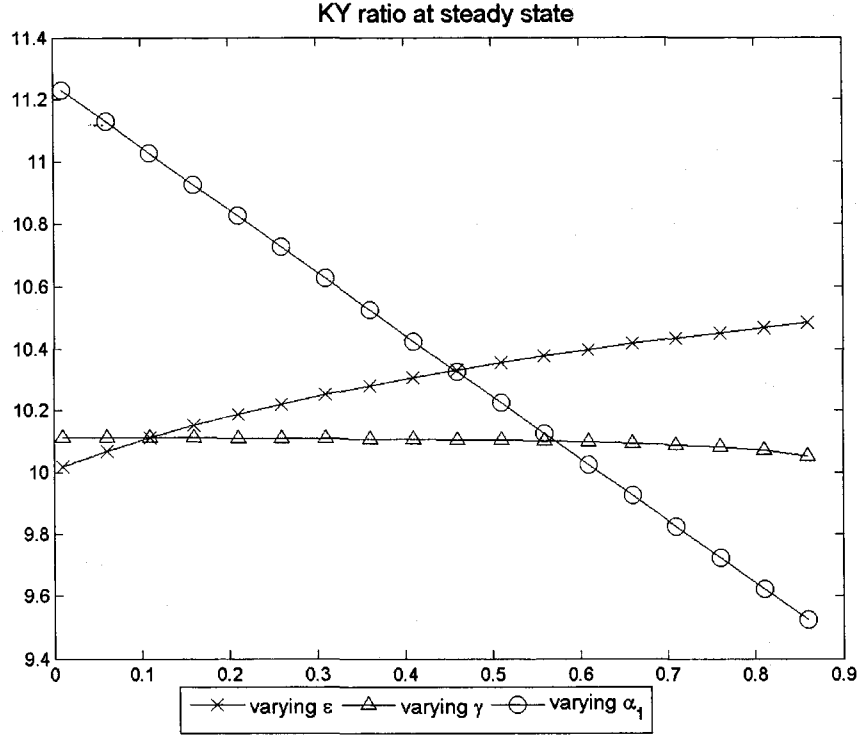


FIG. 4.4
CAPITAL-OUTPUT RATIO AT STEADY STATE.

are given by the following equations

$$\frac{\bar{K}}{\bar{y}} = \frac{\bar{z}}{\bar{r}} \left(1 - \alpha - \varepsilon + (1 - \alpha_1)\beta\varepsilon \frac{1 - \gamma}{1 - \beta\gamma} \right) \quad (4.27)$$

$$L\bar{S}H = \alpha + \alpha_1\beta\varepsilon \frac{1 - \gamma}{1 - \beta\gamma} \quad (4.28)$$

Figure (4.4) and (4.5) displays the sensitivity of the capital-output ratio and the labour share with respect to these intangible capital parameters. As we can see, γ does not have a big influence on these ratios. Therefore, we assign a common uniform prior distribution to γ , with a lower bound of 0 and an

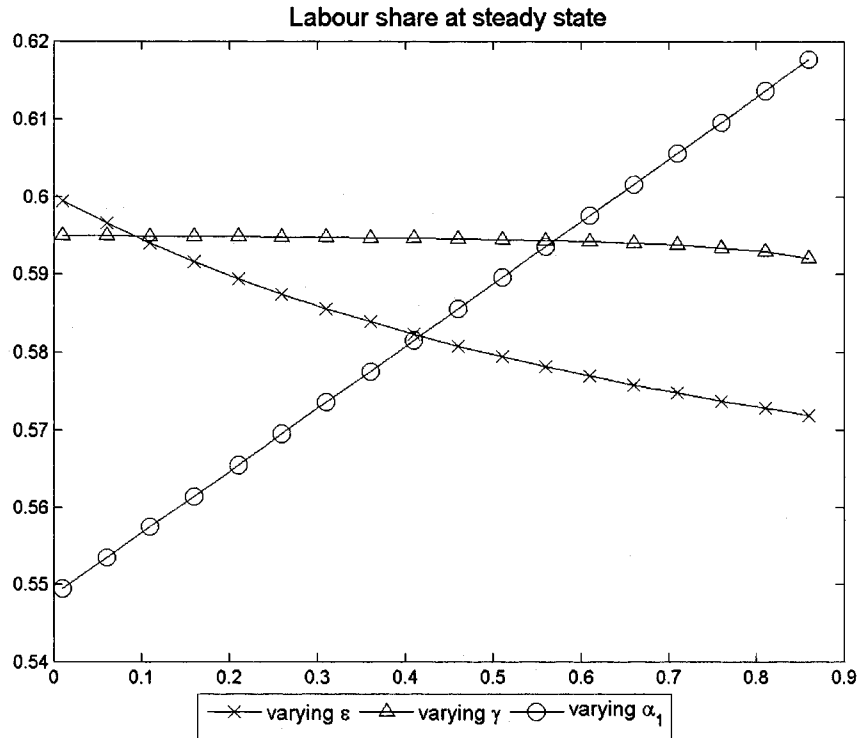


FIG. 4.5
LABOR SHARE AT STEADY STATE.

upper bound of 1. In contrast, to assign prior means to ε and α_1 , we choose parameter values such that the capital-output ratio and the labour share are calibrated to 10.1 and 0.6, respectively. Note that the steady state ratios are fairly sensitive to the values of these two parameters. Based on this fact, we choose relatively tight priors. Specifically, we choose a beta distribution on ε with a mean of 0.09 and standard error of 0.05. The 95 percent confidence interval for ε extends from 0 to 0.71. We also give a beta distribution to α_1 with a mean of 0.55 and standard error of 0.2. The 95 percent confidence

interval for α_1 extends from 0 to 0.99.

We assign a Beta distribution with mean of 0.8 and standard deviation of 0.1 to ρ_p the autoregressive coefficients of the stationary exogenous process. We choose uninformative inverse gamma distributions for the precision of the structural shocks, $\{\sigma_A, \sigma_p\}$.

Table 4.1 presents the marginal prior distributions for the structural parameters. The choice of prior distributions for parameters reflect restrictions on their natural domain, such as non-negativity or interval restrictions. In addition, the priors on the structural parameters are assumed to be independent of each other, which allows for easier construction of the joint prior density used in the MCMC algorithm.

The depreciation rate of capital δ is assumed to follow a Beta distribution with a mean of 0.025 and standard error of 0.003. The prior for α is described by a Beta distribution with a mean of 0.55 (which implies a labor share of 0.6) and standard error of 0.05. Regarding the labor supply elasticity, we assume ϕ follows a Gamma distribution with a mean of 2 with a standard error of 0.5. As these deep parameters are largely in line with the literature, we use tight priors¹² to make the estimated model a-priori comparable to those in the literature. In all models, we calibrate the discount factor β equal to 0.99, which implies a steady-state annually real interest rate of 4 per cent.

¹²The prior variance were chosen to reflect a reasonable degree of uncertainty over the calibrated values of parameters.

TABLE 4.1

PRIOR DISTRIBUTION FOR THE STRUCTURAL PARAMETERS

Parameter	Range	Density	Mean	S.D.
IC Parameters, Prior:				
ε	$[0, 1]$	Beta	0.09	0.05
γ	$[0, 1]$	Uniform	-	-
α_1	$[0, 1]$	Beta	0.55	0.2
Additional Parameters:				
α	$[0, 1]$	Beta	0.55	0.05
γ_a	$\mathcal{R}$	Normal	0.005	0.005
δ	$[0, 1]$	Beta	0.025	0.003
ϕ	$\mathcal{R}^+$	Gamma	2	0.5
ρ_p	$[0, 1]$	Beta	0.8	0.1
σ_A	$\mathcal{R}^+$	Inverse Gamma	0.02	∞
σ_p	$\mathcal{R}^+$	Inverse Gamma	0.02	∞

4.3.2 Posterior Estimates

Table 4.2 reports the posterior distribution of parameters based on 250,000 draws from two independent Markov chains. Of special interest here are the intangible capital parameters. The estimate of ε , in the *IC* model is approximately 0.173 while that of γ is 0.6. Our estimate of α_1 equal to 0.85 suggests that labour is much more important in creating intangible capital than physical capital. The other structural parameters defining preferences and technology are estimated to be roughly of the same magnitude in both models, and the posterior means are consistent with a number of other calibrated and estimated DSGE models. The estimated posterior mean of α is 0.54, which implies the labor share in output equals 0.67. Our estimate of the quarterly depreciation rate of tangible capital δ and the deterministic growth rate γ_a are

2.2% and 0.34%, respectively. The estimated process for the stationary preference shock, $\mathcal{B}_t$, is highly persistent with the standard deviation of innovations equal to 0.5%. The posterior standard deviation of a permanent technology shock is 1.2. These estimates imply a capital output ratio of 9.11. In addition, the implied ratio of intangible to tangible capital investment is 0.748. As a comparison, McGrattan and Prescott (2007) report this ratio equals 0.42.

The posterior means of the structural parameters in the *no-IC* model are consistent with estimates reported in previous studies (see, for example, Chang, Gomes, and Schorfheide (2002); Ireland (2004)).

TABLE 4.2

POSTERIOR ESTIMATES FOR THE STRUCTURAL PARAMETERS

Parameter	<i>no-IC</i> model		<i>IC</i> model	
	Post. Mean	S.D.	Post. Mean	S.D.
ε	-	-	0.173	0.034
γ	-	-	0.592	0.075
α_1	-	-	0.848	0.086
α	0.614	0.019	0.535	0.020
γ_a	0.0037	0.001	0.0034	0.001
δ	0.024	0.003	0.022	0.003
ϕ	0.863	0.018	0.887	0.018
ρ_p	0.976	0.010	0.978	0.010
σ_A	0.010	0.001	0.012	0.001
σ_p	0.005	0.000	0.005	0.000

Notes: The posterior means are calculated from the output of the Metropolis-Hastings algorithm and *S.D.* denotes the standard deviation.

4.3.3 Model Fit and Marginal Data Densities

In this section, we compare the *no-IC* model to the *IC* model in terms of how well they fit the aggregate data. Given the estimates of the two competing models, we conduct a comparison of the overall time series fit between the DSGE models and a comparable a-theoretical VAR model. Table 4.3 reports the marginal data densities¹³ and posterior odds ratios. The results indicate that in order to choose the *no-IC* model over the *IC* model, we need to assign a prior probability 7.11×10^{11} times larger to the *no-IC* model than to the *IC* model. This result indicates that introducing intangible capital into our standard DSGE models leads to a significant improvement in the ability of these models to fit the aggregate data. The time-series fit of *IC* model remains inferior to that of the VAR(4) as is typically found in the literature.

TABLE 4.3

GOODNESS OF FIT

Statistic	<i>no-IC</i> model	<i>IC</i> model	VAR(4)
Prior probability, $\pi_{i,0}$	1/3	1/3	1/3
Log marginal data density	1215.07	1242.36	1265.94
Posterior odds ratio	1.00	7.11×10^{11}	1.23×10^{22}
Posterior probability, $\pi_{i,T}$	8.08×10^{-23}	3.89×10^{-11}	1.00

Notes: Marginal data densities for the DSGE models are computed by Geweke's (1999) modified harmonic-mean estimator. The marginal data density of the VAR is computed via Monte Carlo approximation of one-step-ahead predictive densities.

¹³The marginal data densities are computed using Geweke's modified harmonic mean estimator and a Markov chain of 150,000 draws for each specification of DSGE models.

4.3.4 Explaining key features of business cycles

Propagation

As emphasized in numerous studies (e.g. Cogley and Nason (1995) and King, Plosser, and Rebelo (1988)), successful business cycle models must contain effective mechanisms to propagate shocks over time. It is well known that the *no-IC* model falls short on this account. For example, output growth is positively autocorrelated over short horizons and weakly autocorrelated over longer horizons (Cogley and Nason (1995) and Chang, Gomes, and Schorfheide (2002)). The *IC* model is able to propagate shocks since firms respond to an increase in productivity by acquiring more intangible capital thus raising future productivity.

In Table 4.4, we compare the autocorrelations of output growth predicted by the estimated *no-IC* and *IC* models to those of US data for the period 1960:1 to 2008:3. The results clearly show that the *no-IC* model predicts autocorrelations of output growth to be essentially zero, while the *IC* model is capable of generating positive autocorrelations of output growth, which match the autocorrelations of the data quite well over short horizons. In order to formally evaluate the models using model-based and observed autocorrelations, we specify a posterior expected loss function, L_q .¹⁴ The measure of loss reported in Table 4.4 confirms that the *IC* model ($L_q = 0.019$) does much better

¹⁴See Schorfheide (2000) for a detailed discussion of these loss functions and their interpretations

than the *no-IC* model ($L_q = 0.149$) in explaining output growth autocorrelations.

TABLE 4.4

AUTOCORRELATION STATISTICS

Statistic	Lag	<i>no-IC</i> model	<i>IC</i> model	VAR(4)
(A) Output Growth, $\text{corr}(\Delta \ln Y_t, \Delta \ln Y_{t-j})$:				
	1	0.003	0.301	[0.217, 0.322]
	2	0.003	0.191	[0.189, 0.207]
	3	0.003	0.119	[-0.026, 0.067]
	4	0.002	0.073	[-0.036, -0.026]
L_q risk	1 – 4	0.149	0.013	

Corporate Earning Dynamics and Intangible Capital

TABLE 4.5

SECOND-ORDER UNCONDITIONAL MOMENTS

Moments	US Data	<i>no-IC</i> model	<i>IC</i> model
σ_y	1.62	1.61	1.61
σ_c/σ_y	0.78	0.43	0.51
σ_i/σ_y	4.49	2.52	2.93
σ_n/σ_y	1.13	1.16	1.16
σ_w/σ_y	0.55	0.55	0.58
$\sigma_{\text{earn}}/\sigma_y$	7.36	1.00	8.50
$\text{corr}(c, y)$	0.87	0.97	0.94
$\text{corr}(n, y)$	0.88	0.88	0.88
$\text{corr}(i, y)$	0.89	0.99	0.98
$\text{corr}(\text{earn}, y)$	0.49	1.00(-0.97)	0.15

Notes: The statistics are based on Hodrick-Prescott filtered quarterly U.S. data for the period 1965:1-2008:3. All variables are in logarithms.

As discussed in the introduction to this chapter, a key feature of the *IC*

model is its ability to explain the behaviour of real corporate earnings over the business cycle. Earnings in S&P 500 corporations are much more volatile than output and positively correlated with it. As shown in Table 4.5, the *IC* model is able to deliver both features of the earning's data.¹⁵ In the *no-IC* model, firms generate no profits due to the assumption of constant returns to scale in labour and capital. One way to allow a comparison is to impose decreasing returns to scale on the production technology. We do this by keeping the return on labour and capital equal to that used in the *IC* model.¹⁶ Not surprisingly, earnings generated by the *no-IC* model using this method are perfectly correlated with output and have the same volatility as output.

In order to give the model without intangible capital a better chance to account for the behaviour of earnings we consider a modified version of the *no-IC* model where the firm, rather than households, accumulate physical capital. As a result of this change, the firm's earnings are now given by

$$E_t = y_t - w_t n_t - I_t \quad (4.29)$$

where the notation is the same as before. Since the firm will not pay all it's output to the factor of production, there will be some earnings generated.

In Figure 4.6 we plot the cross-correlation of earnings in period t with both

¹⁵Table 4.5 reports the Hodrick-Prescott filtered moments. Note that the moments of corporate earning are sensitive to the filtering method. The corresponding linearly detrended correlation coefficient between earning and output is 0.50 in the model and 0.52 in the data. The relative standard deviation numbers are, respectively, 5.51 in the model and 6.78 in the data. The dynamic correlations shows similar lead-lag pattern.

¹⁶In decreasing returns to scale case, we choose $\alpha = 0.59$ and $\theta = 0.84 - \alpha$.

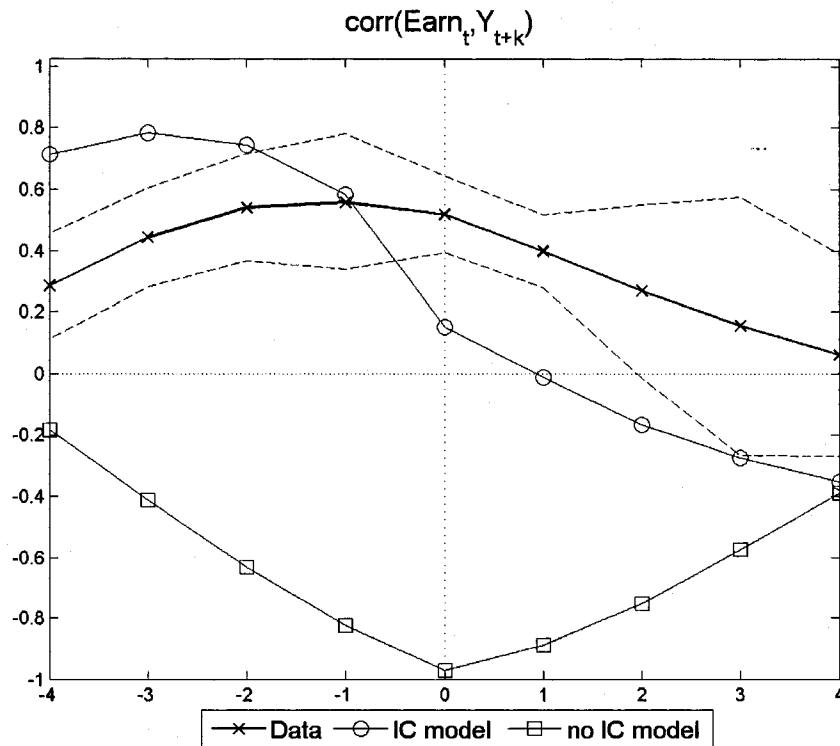


FIG. 4.6 - The line with "x" markers refers to the cross-correlation coefficients from data; the circled line to those from the *IC* model; the triangled line to those from *no-IC* model (II).

leads and lags of output for the data, the *IC* model as well as the version of *no-IC* model described above. While the *IC* model does a fair job of capturing the lead lag relationship, the alternative model predicts a strongly negative relationship which is completely counter-factual. In Figure 4.7, we plot simulated earnings from the *IC* model against the data. The figure shows that the model predictions are quite good. The overall correlation between the two series is 0.53.

To further explore the role of intangible capital in explaining earning dy-

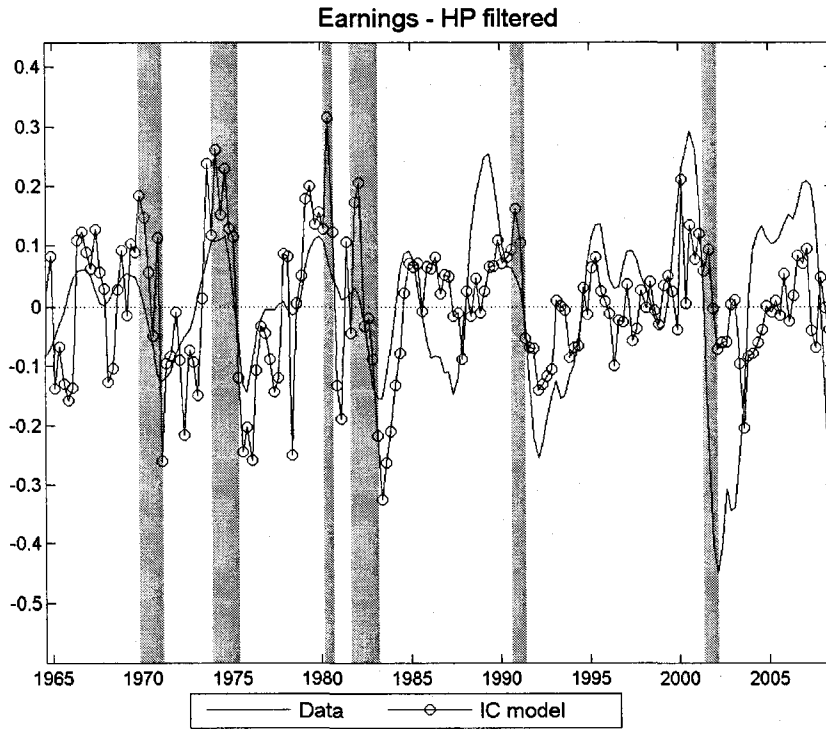


FIG. 4.7 - The solid blue line refers to the corporate earning data and the red circled line to the simulated series from *IC* model. Both series are detrended by the HP filter. The shaded vertical areas refer to the NBER recession periods.

namics, we display the relationship between the relative standard deviation and the contemporaneous correlation of earnings with respect to output and key intangible capital parameters in Figure 4.8 and 4.9, respectively. Figure 4.8 indicates that the volatility of earnings is increasing in ε and α_1 but decreasing in γ . The relationship between ε and earnings is clear from

$$E_t = \varepsilon y_t - I_t^z. \quad (4.30)$$

Note also that given the volatility of output, the volatility of earnings depends on how strongly investment in intangible capital responds to shocks.

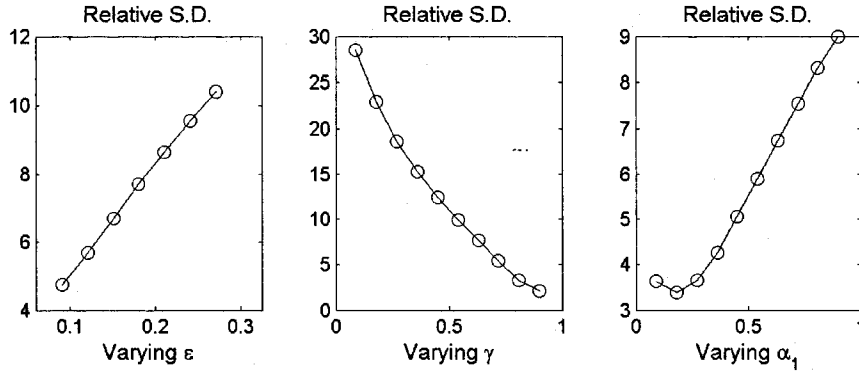


FIG. 4.8 - Sensitivity Analysis: relative volatility of earnings with respect to output.

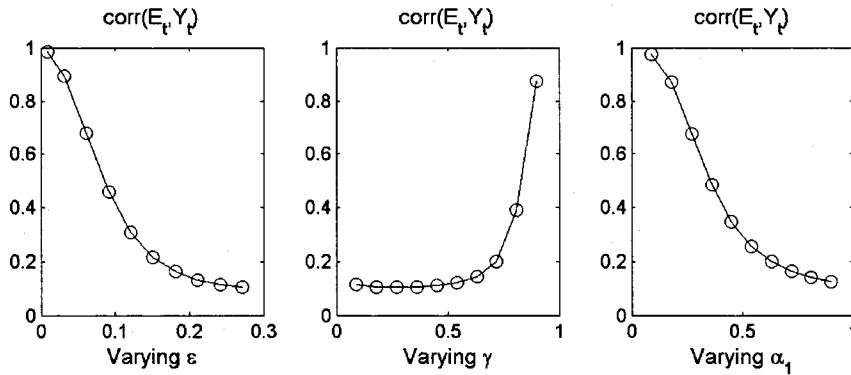


FIG. 4.9 - Sensitivity Analysis: contemporaneous correlation of earnings with respect to output.

Figure 4.10 shows how the standard deviation of I_t^z is influenced by α_1 and γ . Recall that as γ goes to one, the model approaches one in which intangible capital becomes constant. This occurs because of the constant returns to scale assumption imposed on the intangible capital equation. As γ rises, the contribution of labour and capital to intangible capital accumulation falls towards zero. With no change in intangible capital, earnings can only respond to movements in output, leading to a fall in the volatility of earnings. Turning

to α_1 , we note that as α_1 rises, the contribution of labour to the creation of new intangible capital increases while the contribution of physical capital falls. Since the productivity shock is labour augmenting, an increase in α_1 makes the shock more potent in creating intangible capital. This induces the firm to transfer a bigger proportion of an already rising total hours over to the creation of intangible capital. Since hours respond more strongly to shocks than physical capital, the rising contribution of hours on I_t^z overwhelms the falling contribution of capital leading to a larger response of I_t^z . As before, bigger movements in investment in intangible capital translate into more volatility of earnings.¹⁷ Figure 4.9 shows the contemporaneous correlation between earnings and output as the same parameters are varied. This moment, however, is decreasing in ε and α_1 but increasing in γ . Comparing Figure 4.8 and 4.9, a trade-off between volatility and correlation is clearly visible as any one parameter changes. This is to be expected given that any attempt to increase investment in intangible capital will result in lower contemporaneous earnings (but higher future profits).

¹⁷Detailed sensitivity analysis with impulse responses (not shown here) confirm that I_t^z responds more strongly as α_1 is increased. This increase in volatility occurs mostly in response to productivity shocks and to a lesser extent with preference shocks. It also reveals that u^n falls more if α_1 is raised.

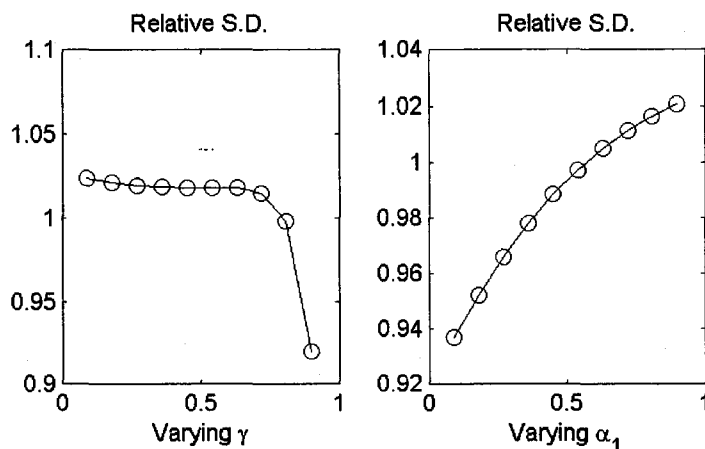


FIG. 4.10 - Sensitivity Analysis: relative volatility of investment in IC with respect to output.

Standard Business Cycle Moments

In this section, we discuss the performance of the model with respect to the typical second moments reported in the business cycle literature. Table 4.5, reports these moments for the *IC* and *no-IC* models and contrasts them with their unconditional data counterparts. All the business cycle statistics reported in Table 4.5 are calculated using stationary cyclical deviations based on the Hodrick-Prescott filter and are calculated using the estimated shocks for each model.

Both models underpredict the volatility in consumption and investment relative to output but the *IC* model does better on both accounts. Both models get the relative volatility of hours and wages roughly right. The contemporaneous correlations of the above macroeconomic variables with output are also similar to each other and to the data.

Figure 4.11 reports the lead lag pattern in the cross-correlation of hours, consumption and investment with output. The *IC* model clearly follows the patterns seen in the data.¹⁸

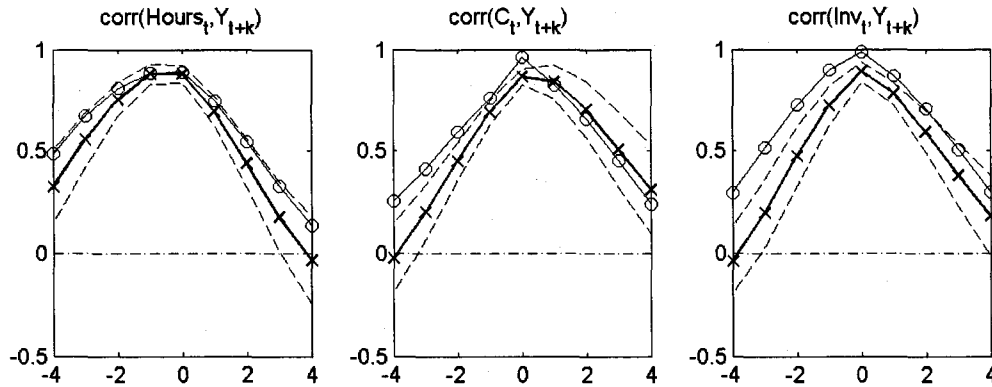


FIG. 4.11 - Lead-Lag Cross Correlation: IC model vs Data. Note that the crossed line refers to the correlations obtained from the data; the circled line to those implied by the model; the dashed lines to the 95% error bands

4.3.5 Impulse-Response Dynamics

In this section we display and discuss the impulse response of key variables of the model to the two shocks. Our goals are two-fold. First, we wish to compare the model responses to the data. Second, we wish to further explore the role played by the presence of intangible capital on the dynamics of the model.

¹⁸The correlation patterns of the *no-IC* model are similar and not reported for this reason.

A brief comparison with a VAR

To shed more light on how well the *IC* model captures the dynamics of output and hours worked, we compare the impulse-responses from both structural models with the counterparts from the estimated a-theoretical VAR.¹⁹

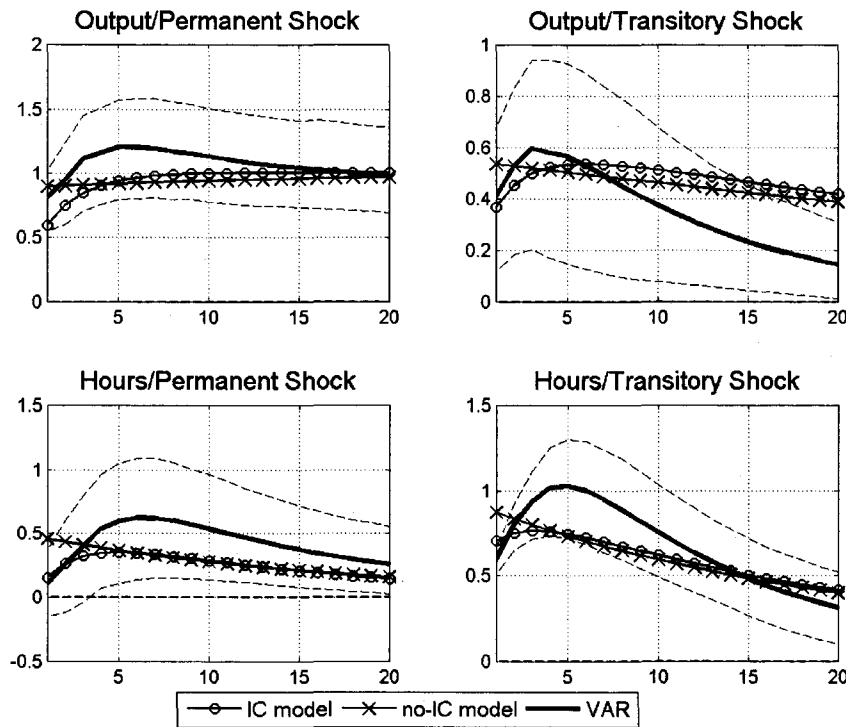


FIG. 4.12
IMPULSE RESPONSE FUNCTION (POSTERIOR MEAN).

The first column of Figure 4.12 presents the posterior means of the impulse-response of output and hours worked to a one-standard deviation increase in

¹⁹To enable a comparison between the DSGE models and the a-theoretical VAR model, we employ Blanchard and Quah (1989) method to identify the permanent and transitory shocks in the VAR.

labour augmenting productivity generated by the *no-IC*, *IC* and VAR models, respectively. The two models generate completely different initial dynamics in response to the technology shock. In the *IC* model, both output and hours worked display an inertial response which tracks the shape of the VAR-based counterpart more closely than the *no-IC* model. In particular, the response of output and hours rises for the first few quarters before peaking which is similar to the VAR based response. This feature is missing in the absence of intangible capital.

The second column of Figure 4.12 reports the posterior means of the impulse response to a one-standard deviation shock to the transitory process in each model. As documented by the literature, (see, for example, Blanchard and Quah (1989); Cogley and Nason (1995)) the VAR response of output to the transitory shock exhibits a pronounced hump-shape and trend reverting path. The *no-IC* model lacks this hump in output, while the *IC* model produces a pronounced hump-shaped output response, which matches the VAR response fairly well in the first few quarters. The response of hours worked to a transitory shock in the *IC* model also generates a small hump-shaped response while the *no-IC* model displays monotonic convergence of hours worked towards its steady state.

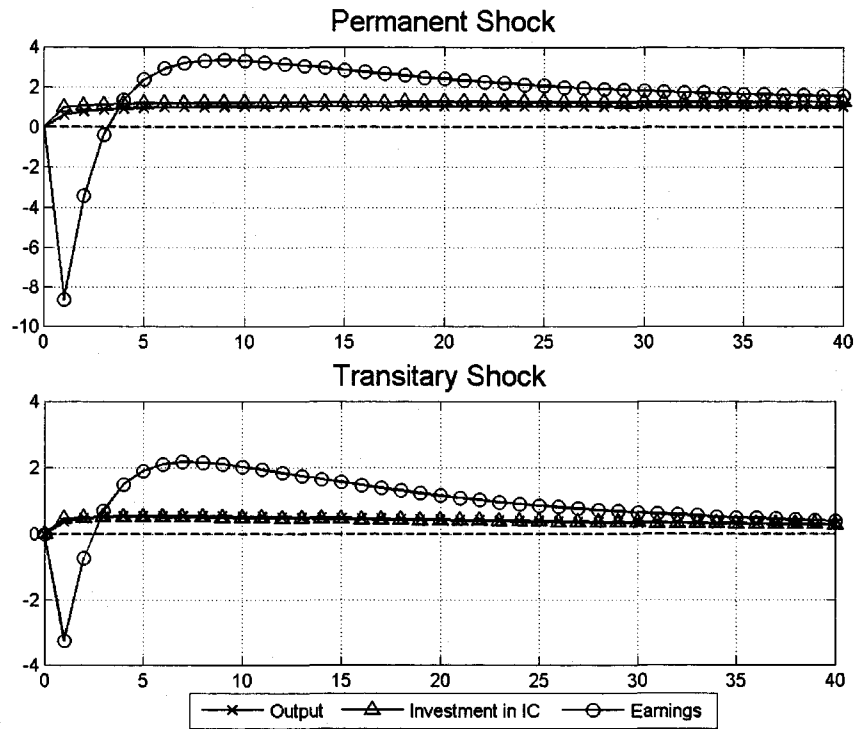


FIG. 4.13 - Impulse responses: earnings, output and investments in intangible capital.

The Role of Intangible Capital in Propagating Shocks

In order to understand how the presence of intangible capital changes the response of the basic DSGE model to shocks, we plot the response of investment in intangible capital in Figure 4.13 along with output and earnings. The top panel plots the response to a permanent technology shock while the bottom panel plots the response to a transitory shock.

The advent of a positive technology shock leads to a permanent rise in the productivity of labour. This induces the firm to hire more hours, as we saw in the previous section. Since productivity has increased in both the creation of

goods as well as intangible capital, there is an incentive to expand both output as well as invest more in intangible capital. Given our estimates of $\alpha_1 > \alpha$, the shock has a bigger impact on the IC equation than the production function so the firm also chooses to substitute some labour towards the creation of new intangible capital. This is achieved by reducing u_t^n below steady state levels (not shown). As a result, the firm ramps up investment in intangible capital slightly more than output. This investment in future productivity occurs even though earnings temporarily fall below steady state. Eventually, the ensuing rise in intangible capital becomes sufficiently large that the extra productivity unleashed is sufficient to pay for the extra investment in intangible capital and yet allow earnings to rise above steady state.

A very similar pattern is evident in the bottom panel where the firm is induced to hire more labour because of a fall in wages (not shown) driven by the preference shock. The large increase in hours worked at the firm allows an expansion of both output and investment in intangible capital. One difference relative to the top panel is that both variables rise by about the same amount. The firm raises u_t^n thus diverting resources towards production. Once again earnings fall below steady state for a couple of periods but this is offset by many future quarters of above steady state earnings. Figure 4.13 also illustrates why the *IC* model can lower the correlation of earnings and output below unity.

The impact of investment in intangible capital on endogenous productivity

dynamics of the economy becomes even more clear by studying the impulse response of the Solow Residual in the *IC* model. Defining the Solow Residual in the usual way it can be easily shown that it is composed of both endogenous and exogenous variables:

$$SR_t = \frac{y_t}{N_t^\alpha K_t^{1-\alpha-\varepsilon}} = (A_t u_t^n)^\alpha (u_t^k)^{1-\alpha-\varepsilon} Z_t^\varepsilon. \quad (4.31)$$

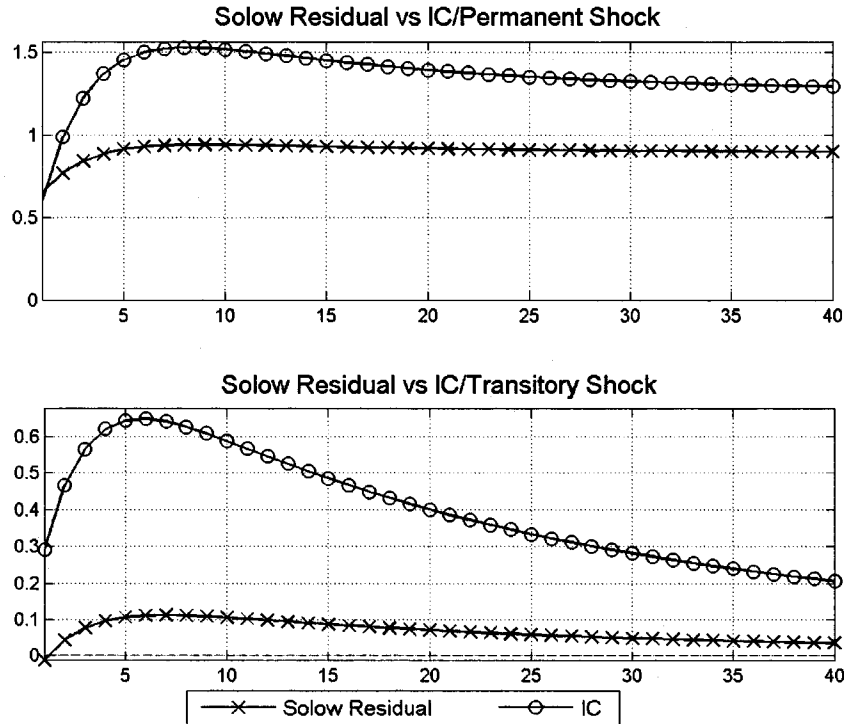


FIG. 4.14
IMPULSE RESPONSE (POSTERIOR MEAN): SOLOW RESIDUAL VS. IC.

The Solow residual now varies with movement in either intangible capital or the share of factors used in goods production even in the absence of any

productivity shock. Figure 4.14 shows the response of the Solow residual to a preference shock in the lower panel and to a productivity shock in the upper panel. The lower panel shows the Solow residual dips on impact as the firm diverts resources towards accumulating intangible capital and away from production of the final good (a fall in u_t^n). This leads to a persistent rise in productivity as the extra intangible capital goes to work producing more final goods. It is clear that the model generates a highly persistent endogenous productivity response. This extra productivity is the payoff for giving up some earnings in the initial periods. According to equation (4.31), three components contribute to the impulse dynamics of Solow Residual. It is worth noting that the shares of factors used in good production do not move much and thus movement in intangible capital is the primary ingredient in the dynamic behaviour of Solow Residual after the initial period. The response of the Solow Residual to productivity shocks is similar in that SR_t rises above A_t and only slowly returns to its new steady state level.

4.4 Sensitivity Analysis

In this section, we discuss how key *IC* model parameters influence the impulse responses from the *IC* model. Figure 4.15 captures the role of the parameter ϵ as it varies while the rest of the parameters in the *IC* model remain the same as before. In all cases explored below, impulse responses

from the *IC* model to both technology and preference shocks are shown.

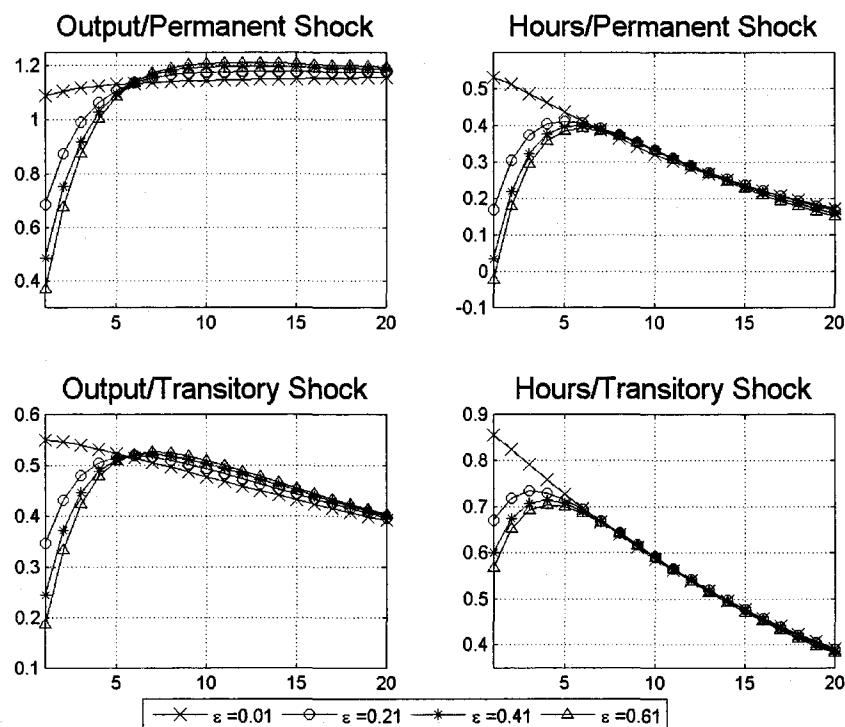


FIG. 4.15 - Sensitivity Analysis: Impulse responses of output and hours with ϵ varying

The results indicate that as intangible capital becomes more important in production of the final good, the responses of output and hours worked display more inertia. This occurs because the desire of the firm to acquire intangible capital increases as the parameter rises. This leads to a larger diversion of resources away from production when the shocks hit the economy. As a result, productivity falls more on impact, the higher is ϵ . Since productivity will rise in the near future, firms shift the hiring of labour forward in time leading to a

smaller increase in hours worked and output on impact. Figure 4.16 shows the

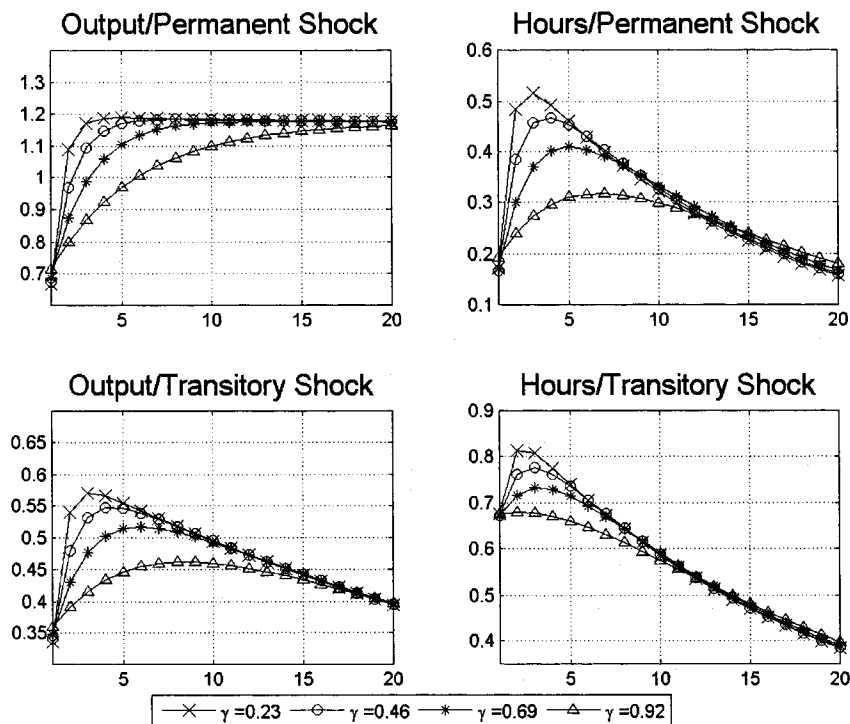


FIG. 4.16 - Sensitivity Analysis: Impulse responses of output and hours with γ varying

impulse response functions as the value of γ progressively increases towards unity. As might be expected, based on our earlier discussion, the higher is γ , the closer are the responses to the *no-IC* model. Figure 4.17 and 4.18 show the impulse response functions of earnings and investment in IC when the values of ε and γ increase towards unity, respectively. The results regarding earnings in Figure 4.17 and 4.18 reinforce the features of earning dynamics shown in Figure 4.8. The higher is ε , the more pronounced are the responses of earnings. Note as well that the pattern of impulse response of earning changes as ε varies. As

discussed earlier, the larger is ε , the greater importance of the intertemporal trade-off for firms' earnings. A firm is more willing to give up current profits to invest in intangible capital when ε gets bigger. On the contrary, the higher is γ , the less valuable is the current investment in intangible capital to future intangible capital stock and therefore the more dampened are the responses of earnings. As shown in the lower panels of Figure 4.17 and 4.18, there is strong correspondence between earnings and investment in IC responses. Apparently, when firms raise investment in IC, this comes at the expense of reduced current profits but results in much higher profits in the quarters that follow. We end our sensitivity analysis by noting that the sensitivity of relative volatility and comovement of earnings were discussed earlier in Figure 4.9.

4.5 Conclusion

An important feature of business cycles is that profits increase in booms and fall in recessions. Using real earnings data for S&P 500 corporations as a proxy for aggregate profits we find that earnings are roughly seven times more volatile than aggregate output. Most business cycle models cannot deliver this feature of the data. In this paper we explore the idea that firms generate profits because they produce intangible capital. Intangible capital is modeled as a third input in the technology for producing final goods in addition to labour and physical capital. Firms can invest in creating intangible capital by

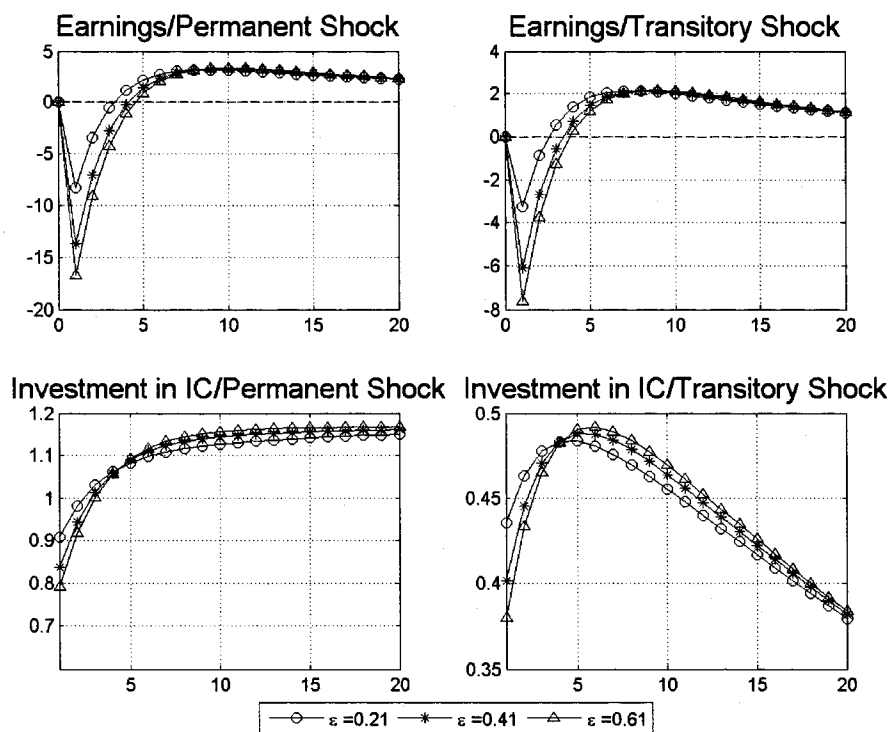


FIG. 4.17

SENSITIVITY ANALYSIS: IMPULSE RESPONSES OF EARNINGS WITH ε VARYING

diverting resources away from the production of final goods. We embed this model of intangible capital into an otherwise standard DSGE model and ask if it can deliver reasonable predictions about aggregate profits without sacrifices on the usual metrics used to evaluate business cycle models.

The model is estimated using bayesian techniques and provides a significantly better fit with aggregate U.S. time series data on hours and output than the model without intangible capital. Simulation results from an esti-

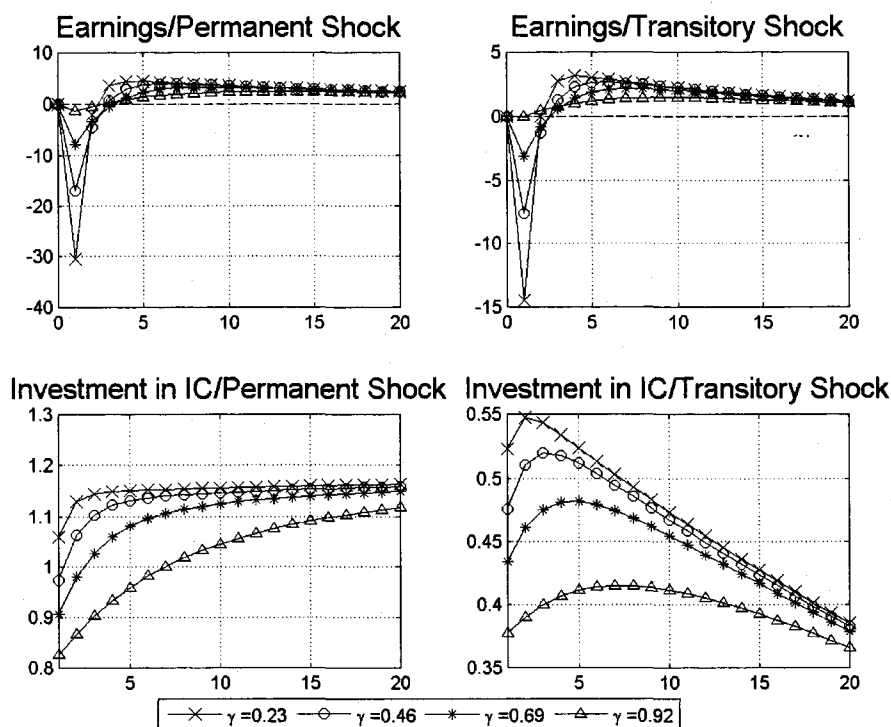


FIG. 4.18 - Sensitivity Analysis: Impulse responses of earnings with γ varying

mated version of our model are similar in many respects to a number of recent models which also show an improvement in fit as well as an improved ability to propagate shocks and generate inertial responses which mimic a-theoretical VAR-based impulse response functions. What distinguishes our model from many of these exercises is the ability of our model to explain the dynamics of aggregate real corporate earnings. The intangible capital model can generate earnings volatility which is roughly the same magnitude as the data. It can also deliver less than perfect co-movement between earnings and output, which is another feature of the data as well as broadly capture the lead-lag

patterns of these variables. In the absence of intangible capital, the model fails to replicate these features of the data.

To our knowledge, this paper is the first to provide aggregate estimates for a model with intangible capital, a phenomenon which is inherently hard to measure directly. We find that investments in intangible capital are procyclical and substantial in magnitude and play a major role in generating endogenous movements in productivity over the cycle. Our estimates imply that investments in intangible capital are roughly three-fourths the size of investments in physical capital.

Chapter 5

Conclusion

The overall goal of this thesis is to provide a deeper understanding of the role of DSGE models as foundations upon which empirical work is conducted. This is a very broad topic with a large existing literature. Therefore, I have chosen to look at two specific macroeconomic questions of interest. Chapter 2 look at the economic consequences of oil price shocks in a structural vector autoregressions (VAR) framework. Chapter 3 builds on this by developing an open economy DSGE model to pursue new insights on “the impact of oil price shocks on Canadian economic activities”, “the role of U.S. and Canadian monetary policy in transmitting oil price shocks”. Chapter 4 documents another interesting macroeconomic phenomenon: the excess volatility of aggregate profits. We embed this model of intangible capital into an otherwise standard RBC model to examine the role of intangible capital in explaining

cyclical dynamics of S&P500 earnings.

- Future Research Directions

We treat the structural VAR in Chapter 2 as a reference model and its impulse response functions as the empirical facts that need to be explained by a DSGE model. Further work is needed to bring these two strands of macro-modeling methodologies closer together. In light of Del Negro and Schorfheide (2001), future research could therefore be directed toward using quantitative restrictions implied by a DSGE model for statistical VAR identification.

Another potential area of research is to examine the problem related to policy analysis under misspecified models. For instance, the findings and conclusions in Chapter 3 rest on an important modeling assumption: balance of trade for each country is zero. However, it is well known that the U.S. experiences a large and persistent trade deficit. Consequently, imposing that the U.S. trade balance be zero at each period of the time is counterfactual and might assume away the indirect effects of U.S. monetary policy through the international financial channel. Therefore, it is important to assess implications of using alternative assumptions on DSGE models. Del Negro and Schorfheide (2001) propose a DSGE-VAR methodology to assess the degree of potential mis-specification of underlying structural models.

Finally, the analysis in Chapter 4 reveals an interesting prediction of the *IC* model on the dynamic behavior (short-run vs long-run) of earnings. In

response to persistent shocks (permanent or highly persistent) the *IC* model predicts that corporate earnings are negative in the short-run because firms would like to substitute away resources from production labor and capital for investment in intangible capital. Profits increase over time as intangible capital builds up. This prediction might be testable by the data. That is, in response to a persistent shock, the “measured” current profit might decrease in the short run but will increase in the long run as the firm builds up intangible capital. It might be interesting to look at firm-level data to test this hypothesis. I can also connect these predictions with the stock value of firms. For example, in response to a permanent or persistent productivity shocks, there might be a weak contemporaneous correlation between the profits and stock value. The stock value should increase (despite a negative profit in the short run) for the firms with intensive investment on intangible capital (e.g. Google and Apple in their early stages).

Appendix

1 Data Description

The common period covered by all series is 1980:Q1 - 2008:Q3. Some series at monthly frequencies will be converted to quarterly by simple averaging. The Canadian data for this project are obtained from CANSIM. Series numbers are indicated in brackets and correspond to CANSIM database numbers.

- output is measured by real GDP per capita [V1992067]
- The CPI inflation rate is measured by changes in consumer price index [V41690973]
- The average nominal wage is measured by average hourly labor earnings (wage and salary [V500266] / total working hours [V2348296])
- Short-term Interest Rate is measured by the rate on Canadian three-

month treasury bills [V122484]

- The terms of trade is measured by the ratio of exports deflator [V1997750] to the imports deflator [V1997753]
- The series in per capita terms are obtained by dividing by the Canadian civilian population aged 15 and over [V2062810]

The world oil prices are measured by simple average of three spot prices of crude oil (Dated Brent, West Texas Intermediate, and the Dubai Fateh), US\$ per barrel [POILAPSP], which come from IMF Primary Commodity Prices database available at

<http://www.imf.org.external/np/res/commod/externaldata.csv>.

The U.S. data are quarterly from 1959Q1 to 2008Q3 and obtained from the Federal Reserve Bank of St. Louis with series numbers in brackets. The aggregate profits data are taken from U.S. Bureau of Economic Analysis, National Income and Product Accounts as well as Robert Shiller's home page.

- U.S. output is measured by real U.S. GDP of chained 2000 dollars [GDPC1]
- Consumption is measured by real personal consumption expenditures of chained 2000 dollars [PCECC96]
- Investment is measured by real gross private domestic investment of chained 2000 dollars [GPDIC96]

- Hours worked is measured by hours of all persons in nonfarm business sector [HOANBS]
- Wage is measured by compensation of employees: wages & salary accruals [WASCUR]
- Aggregate profit is measured by the real S&P500 composite earnings obtained by Shiller (2000), which is available at <http://www.econ.yale.edu/shiller/data.htm>.
- All of the above series are expressed in per capita terms by dividing by the U.S. civilian non-institutional population, ages 16 and over [CNP16OV]
- U.S. inflation rate is measured by changes in consumer price index for all urban consumers [CPIAUCSL]
- U.S. interest rate is measured by the rate on U.S. three-month treasury bills [TB3MS]
- The real exchange rate is measured by average Canadian dollars per unit of U.S. dollar [EXCAUS]

2 Implementation of Sign Restrictions

In the following, we briefly describe the technical features of VAR identification with sign restrictions based on Canova and De Nicolò (2002).

Consider a reduced form VAR representation, which is associated with the structural model in equation (2.1), is given by

$$Y_t = B(L)Y_{t-1} + e_t, \quad e_t \sim N(0, \Sigma), \quad (\text{A.1})$$

where $B(L) = A_0^{-1}A(L)$, and $e_t = A_0^{-1}\varepsilon_t = V\varepsilon_t$ is the vector of one-step ahead prediction error with variance-covariance matrix $\Sigma = E[e_t e_t'] = VE[\varepsilon_t \varepsilon_t']V' = VV'$. Without imposing further restrictions the decomposition of Σ is not unique. The multiplicity of these decompositions comes from the fact that for any orthogonal matrix Q satisfying $QQ' = I$, $\Sigma = VQQ'V'$ produces another admissible decomposition.

In order to tracing out all possible impulse response functions to the orthogonal structural shocks, we transform equation (A.1) into a vector moving average representation (see Canova and De Nicolo, 2002 and Uhlig, 2005)

$$Y_t = C(L)e_t = C(L)V\varepsilon_t \quad (\text{A.2})$$

$$\begin{aligned} C(L) &= I_m + C_1L + C_2L^2 + \dots \\ &= \left[I_m - B_1L - B_2L^2 - \dots - B_pL^p \right]^{-1} \end{aligned} \quad (\text{A.3})$$

where $C_i = J'M^iJ$, $i = 0, 1, 2, \dots, \infty$ and J and M are given by

$$J_{mp,m} = \begin{bmatrix} I_m \\ 0_m \\ 0_m \\ \vdots \\ 0_m \end{bmatrix}, \quad M_{mp,mp} = \begin{bmatrix} B_1 & B_2 & \dots & B_{p-1} & B_p \\ I_m & 0_m & \dots & 0_m & 0_m \\ 0_m & I_m & \dots & 0_m & 0_m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_m & 0_m & \dots & I_m & 0_m \end{bmatrix}$$

The term $C_i V$ denotes the corresponding VAR impulse responses to a structural shock in ε_t .¹ Now consider a $m \times m$ Givens matrix $Q_{pq}(\theta)$ of the form

$$Q_{pq}(\theta) = \begin{bmatrix} 1 & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & \ddots & \dots & \dots & \dots & \dots & 0 \\ \vdots & \vdots & \cos \theta & \dots & \sin \theta & \vdots & \vdots \\ \vdots & \vdots & \vdots & 1 & \vdots & \vdots & \vdots \\ \vdots & \vdots & \sin \theta & \dots & \cos \theta & \vdots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \ddots & 0 \\ 0 & \dots & \dots & \dots & \dots & 0 & 1 \end{bmatrix}$$

where $\theta \in [0, \pi]$, the subscript (p, q) indicates the rows p and q which are

¹Consider a temporary shock to the k^{th} component of ε_t such that ε_t^k equals unity. Then the response of $Y_{t+i} = C_i V \varepsilon_t^k$, for $i = 1, 2, 3, \dots$, which is just the k^{th} column of C_i . Thus, the columns of C_i give us the impulse responses of each endogenous variable, at time $t + j$, to shocks administered at time t .

rotated by the angle θ and $I_k = Q_{pq}(\theta)Q_{pq}(\theta)'$ for all θ . We then have that

$$Q(\theta) = Q_{12}(\theta_{12})Q_{13}(\theta_{13}) \dots Q_{1m}(\theta_{1m})Q_{23}(\theta_{23}) \dots Q_{2m}(\theta_{2m}) \dots Q_{m-1,m}(\theta_{m-1,m}) \quad (\text{A.4})$$

is also an orthogonal matrix such that $\Sigma = VQ(\theta)Q(\theta)'V'$ and $e_t = VQ(\theta)Q(\theta)'\varepsilon_t = \tilde{V}\tilde{\varepsilon}_t$.² The vector $\tilde{\varepsilon}_t$ is the new set of rotated orthogonal shocks, which has the same covariance matrix as ε_t but which has a different impact upon Y_t through its impact matrix $C_i A_0^{-1}Q(\theta)$. The empirical distribution for the impulse responses are derived in a Bayesian framework. Bayesian estimation proceeds by taking many draws from the posterior distribution of the VAR coefficients $B(L)$ which belongs to the Normal-Wishart family as well as many draws on Givens matrix $Q(\theta)$. Specifically, in a m variable system, according to equation (A.4), $Q(\theta)$ depends upon $\frac{m(m-1)}{2}$ bivariate rotation matrices $Q_{pq}(\theta)$ with $\theta = \theta_1, \dots, \theta_{\frac{m(m-1)}{2}}$. As shown in Uhlig (2005) and Canova and De Nicolò (2002), all possible rotation can be produced by drawing θ from a uniform distribution on $[0, \pi]$. For each draw, we calculate the corresponding impulse responses and check whether the sign restrictions are satisfied. In this way, all possible impulse response functions can be traced out by varying the angle $\theta \in [0, \pi]$ and only those $Q(\theta)$ s that generate impulse responses comply with the sign restrictions will be retained.

²Simply define $\tilde{V} = VQ(\theta)$ and $\tilde{\varepsilon}_t = \varepsilon_t'Q(\theta)$.

3 Calculating Posterior Distribution and Moments

We wish to estimate a DSGE model $\mathcal{M}_i$ and its associated vector of structural parameters Θ_i . We update the state-form solution (4.25)-(4.26) by adding a set of measurement equation which links the observed time series to the vector of unobserved state variables:

$$S_{t+1} = AS_t + B\varepsilon_{t+1} \quad (\text{A.5})$$

$$Y_t = CS_t \quad (\text{A.6})$$

where the matrices A , B and C are functions of the models' structural parameters, and C represents the relationship between the observed data Y_t and variables in state equation S_t . $S_t = \{\hat{x}_t\}$ from equations (4.25) and Y_t contains only two observed control variables in $\{\hat{y}_t\}$ from equation (4.26). Specifically, Y_t is a 2×1 vector of observable variables, including GDP growth and hours worked; ε_t is the vector containing technology and preference innovations.³. Given the state-space form defined by (A.5) - (A.6), the likelihood function of the model $\mathcal{M}_i$, can be constructed by applying the Kalman filter as outlined by Hamilton (1994):

$$\ln \mathcal{L}(\Theta|Y^T, \mathcal{M}_i) = -\frac{nT}{2} \ln 2\pi - \sum_{t=1}^T \left[\frac{1}{2} \ln |\Omega_{t|t-1}| + \frac{1}{2} \omega_t' \Omega_{t|t-1}^{-1} \omega_t \right] \quad (\text{A.7})$$

³Note that in contrast to Ireland (2004), we do not specify the measurement errors in measurement equations.

where the vector Θ_i contains the parameters to be estimated; $\{\omega_t\}_{t=1}^T$ ⁴ is a series of innovations that are used to evaluate the likelihood function $\mathcal{M}_i$ for the data sample, Y^T , and $\Omega_{t|t-1} = E\omega_t\omega_t'$ is the variance-covariance matrix that depends on the structural parameters, Θ_i .

We further combine the likelihood function with our specified prior knowledge about these deep parameters to form the posterior distribution function. In the Bayesian context, the posterior distribution of Θ_i can be thought of as a way of weighting the likelihood information contained in the observed data by the prior density $p(\Theta_i|\mathcal{M}_i)$. Given a prior, the posterior density kernel⁵ of Θ_i can be written as:

$$p(\Theta|Y^T, \mathcal{M}_i) \propto \mathcal{L}(Y^T|\Theta, \mathcal{M}_i)p(\Theta|\mathcal{M}_i) \quad (\text{A.8})$$

where $\mathcal{L}(Y^T|\Theta, \mathcal{M}_i)$ is the likelihood conditional on the observed data, $Y^T = \{y_1, \dots, y_T\}_{t=1}^T$. The sequence of posterior draws can be obtained using Markov Chain Monte Carlo (MCMC) methods. We use the random-walk Metropolis-Hasting algorithm as described in Schorfheide (2000) to numerically generate the Markov chains for the structural parameters. Point estimates of Θ_i can

⁴ ω_t is defined as $\omega_t = y_t - \hat{y}_{t|t-1}$ and $\omega_t \sim N(0, \Omega_{t|t-1})$ is assumed normally distributed

⁵Note that Bayes' Theorem states that

$$p(\Theta|Y^T, \mathcal{M}_i) = \frac{\mathcal{L}(\Theta|Y^T, \mathcal{M}_i)p(\Theta|\mathcal{M}_i)}{\int \mathcal{L}(\Theta|Y^T, \mathcal{M}_i)p(\Theta|\mathcal{M}_i)d\Theta}$$

But recognizing $\int \mathcal{L}(\Theta|Y^T, \mathcal{M}_i)p(\Theta|\mathcal{M}_i)d\Theta$ is constant for $\mathcal{M}_i$, we only need to be able to evaluate the posterior density up to a proportionate constant using

$$p(\Theta|Y^T, \mathcal{M}_i) \propto \mathcal{L}(\Theta|Y^T, \mathcal{M}_i)p(\Theta|\mathcal{M}_i)$$

be obtained from calculating the sample mean or median from the simulated Markov chains. Similarly, inference of Θ_i are derived from computing the percentiles of these posterior draws.

Furthermore, given the sequence of posterior draws, $\{\Theta_i^j\}_{j=1}^N \sim p(\Theta_i|Y^T, \mathcal{M}_i)$, by the law of large numbers:

$$E(g(\Theta_i)|Y^T) = \frac{1}{N} \sum_{j=1}^N g(\Theta_i^j) \quad (\text{A.9})$$

where $g(\cdot)$ is some function of interest, such as impulse response functions and moments. We can employ Markov chain Monte Carlo (MCMC) methods to evaluate equation (A.9) with $\{\Theta_i^j\}_{j=1}^N$.

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