### DIGITAL SEPARATION TECHNIQUES

### APPLIED TO A

### TEMPERATE KARST TERRAIN

**BY :** 

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#### Abstract

An important aspect in spatial analysis is a digital separation technique. There are several techniques possible, but trend-residual techniques and filtering techniques are emphasized in this report. The areas studied were a 9.6 km<sup>2</sup> gridded area in the Mammoth Cave region in Kentucky and the Lewisburg region in West Virginia. The basic geology and surface patterns are Although they are similar in respect to described. geology, the doline distribution is much smaller and more dense in Kentucky than in West Virginia. Both areas appear stable and subsidence dolines appear dominantly. The two doline development models are presented and the evidence cited tends to support the MDCP model. The predicted clustering of daughters about uniform/randomly distributed parents is found in Kentucky, no study was found for West Virginia. Both areas show that doline long axis are oriented parallel to regional joint sets.

A uniformly dense grid proves accurate and unbiased when the elevation data are contoured. The Surface II Graphics package proves more than adequate in producing all the maps needed for this study despite the lack of flexibility in some areas.

A trend-residual analysis was conducted to the fourth order for both the Kentucky and West Virginia areas. Despite minor technical problems, the results are positive. The method clearly separates the local doline variance from the regional trend, but there does appear to be a consistent bias towards nearby ridges. It also appears that there is an enhancement-suppression effect from the residual analysis such that certain doline forms are enhanced while others are suppressed as the trend order progresses from the first to the fourth order.

The filtering technique also shows some excellent results from the digital separation method. Several filter types are discussed and the theory of their design is also presented. Very successful results were achieved by a zero summed filter as well as a 3 x 3 moving average filter. Several maps are produced from the analyses and are computer generated and these are also presented. There are several conclusions given at the end.

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#### CHAPTER ONE: INTRODUCTION

Digital separation techniques may be applied to all forms of geographical data that can be expressed as a surface or a set of numerical values. These are techniques which are often used to define the spatial patterns that exist within a surface or set of numerical data, by separating the features into distinct components. This technique is often required because there is usually a fair amount of white noise (variance) within the spatial data which masks the true spatial patterns. By identifying some spatial patterns it may be possible to describe some relationships between processes and form.

#### Objectives

The purpose of this thesis is to present some digital separation techniques commonly used, and to describe how these may be applied to a karst environment. More specifically, digital separation techniques were applied to doline plains in an attempt to separate two components: the regional component (regional trend), and the local variances (dolines). In so doing, it may be possible to examine and draw some conclusions about the spatial patterns of dolines in a plain.

This report will explain how dolines form and how

processes affect their distributions in the plain. There are presently two models which attempt to explain spatial distributions of solutional dolines. Both the MCDP and MIRP are presented and discussed. Evidence from the Kentucky area suggests that the MDCP model is the more applicable model of the two. Much of this report is based extensively on map analyses which first requires consideration of gridding techniques. A review of some techniques is given and the method that was actually used in this study is described. Once the data is obtained from the topographic maps, it is necessary to generate contour maps which are representative of the true surface. A computer graphics package was chosen to produce the contour maps. Reasons for selecting the Surface II Graphics system are given and the potential of the package is also detailed. The accuracy of the computer generated maps is discussed and compared to the actual surfaces.

There are several techniques available to separate the regional trends from the local variances, including trend-residual surface analysis, and filtering analysis. Trend surface analysis is a statistical technique which attempts to describe the regional trends by means of regression equations. The residual analysis is the variance represented by the difference between the calculated regional trend and the actual surface. When

considering the two analyses, it is possible to determine the regional component and local variance and to draw some conclusions from the patterns that exist. Spatial filters are a different form of separation technique. These are numerical matrices which are passed over the gridded data and the design of the filter determines which features are separated from others. The theory behind spatial filters, their design and some results are presented. Because much of the analysis deals with comparisons of maps, many of the conclusions lack quantitative support, but the conclusions are significant and do not require that they be substantiated quantitatively.

#### Study Area

The original proposal was to analyze and compare the spatial distribution patterns of surface karst formations in temperate and tropical latitudes, but time constraints restricted the work to a comparison of doline distributions in temperate latitudes. Dr. D.C. Ford (McMaster University) suggested two typical temperate doline plains for the comparison; the Kentucky and the West Virginia regions. The two areas are geographically close: Kentucky (Mammoth Cave Nat. Park) is situated at N37° 05' - W85° 57' 30" and West Virginia (Lewisburg) is N37° 45' - W80° 22' 30". The Kentucky area shows a very

dense doline pattern and there does not appear to be any spatial patterns existing in the doline distributions. West Virginia shows a less dense doline plain, but many more uvala forms than does Kentucky.

#### Geology

Kemmerly (1976, 1982, 1985) and Kemmerly and Towe (1978) have conducted extensive studies in the southern Kentucky and northern Tennessee regions. From their studies, it is reported that the Kentucky area is underlain dominantly by the St. Louis and Ste. Genevieve Limestones. The units are of Mississippian age. The St. Louis Limestone is 40 to 80 m thick and is a coarsegrained, thin-bedded to massive limestone with some siltstone and dolomite. The limestone is relatively pure with 91% to 97% calcite. The Ste. Genevieve Limestone is 15 to 50 m thick and consists of white to blue-grey, fine-to coarse-grained, thin- to thick-bedded limestones. The Ste. Genevieve is also quite pure with 95% to 97% calcite. The units typically dip to the NW by approximately 6m/km. Kemmerly (1982, 1985) reports the existence of three dominant joint sets in the directions of N20°E to N40°E, N70°E to N80°E and N20°W to N30°W. The surface soils consist largely of clayey, cherty, residuum which is overlain by silty colluvium and local loess

deposits. The residuum from the St. Louis limestone has greater permeability than does the Ste. Genevieve. The residuum layer ranges in thickness from 0 to 20m, the colluvium ranges between 0 to 5m and the loess may range from 0 to 2m (Kemmerly, 1982).

Much of the geology in the West Virginia region is described by Medville (1977). The Greenbrier group is divided into several distinct facies. The lower group is the Patton Limestone which is a dark grey, partly oolitic, sandy and impure limestone. Ascending upwards is the Taggard shales intermittent with a 5 ft thick limestone The Pickaway limestone overlies the Taggard units unit. and is a hard limestone ranging in thickness from 30 to 40 The major unit in this area is the Union Limestone ft. ranging in thickness from 135 ft to 150 ft. It is basically a light grey, slightly sandy, oolitic unit. Overlying the Union Limestone is an 18 to 20 ft thick Greenville shale which is overlain by the Alderson Limestone, a 40 ft thick dark grey shaley limestone. According to Medville (1977) the regional dip is 3 to 4 degrees to the northwest and the strike is approximately N65 E and plays a significant role in the hydrology of the Similarly, Lowman Jr. and Webster Jr. and Allenby area. (1980) found from air photo studies that one of the longest transverse fault systems strike in a N80 E

direction. The two areas are slightly different in their geologic setting. The limestones of the Kentucky area are thicker, more massive units where those in the West Virginia area consist of many more thinner limestone units. They differ with respect to regional joint sets where there is only one dominant joint set orientation in West Virginia and there are three dominant set orientations in the Kentucky area. The significance of this difference in joint sets is explained later in this report.

#### Specific Site Description

The actual map analysis was conducted on a much smaller area within the regions described previously. For a proper comparison, an area of 9.66km<sup>2</sup> (4.2km x 2.3km) was used in both areas as the basis for the map analyses. With respect to the doline densities and the nature of their distribution, both the Kentucky and West Virginia areas have certain similarities and differences which are explained following.

The Kentucky area is found from  $W85^{\circ}75'$  30" to  $W86^{\circ}$  00' 57", and from N37° 05' 00" to N37° 75' 30". The area is just south of the Mammoth Cave Plateau in a fairly level plain. At first glance it appears that there are hundreds of individual dolines (see Figure 1, all figures,

referred to in this report, appear in the appendix), but under closer inspection it is seen that over half of the study area consists of one large uvala form. The uvala forms a very large depression form within which are found hundreds of dolines. The smaller dolines tend not to be deep as does the uvala. Whether the large uvala formation has developed from a coalescence of many other dolines, or developed as a single doline is unknown, although it is more likely the former. Kemmerly (1982) has reported that typical doline lengths range from 3 to 1500m and depths range from 3 to 30m. Uvalas typically range from 50 to 5000m and 5 to 50m in depth. These dimensions reported by Kemmerly (1982) are very similar to those found in the present Kentucky study area. Although there are approximately 50 ponded dolines shown on the topographic map of the study area, there is no evidence of surface rivers. This suggests that the area has a high rate of permeability and probably a well developed subsurface drainage system. The ponding of some dolines may either be due to natural clogging by clays, or they may be manmade (ie. farmer blocks drainage conduit to develop a watering hole.)

The elevation of the Kentucky study area ranges from near 200m (600ft) in the west to 255m (750ft) in the north-west. All contour elevations, on maps provided in

this report, are given in feet. The general slope of the area is downward in an easterly direction with a drop of 12m/km. Although this is twice the dip slope recorded by Kemmerly (1982), the 12m/km is representative of a much more localized area than reported by Kemmerly. Finally one might assume from the number of roads, railways and pipelines that dissect the area that this region is relatively stable. These may also suggest that the doline formations are solutional in origin, as opposed to a more catastrophic or collapse origin.

The West Virginia study area is situated in what appears to be a large, dry, flat bottomed valley. Whether the valley form has a structural origin (eg. graben or rift fault valley), or a fluvial origin (eg. outwash channel, paleoriver valley) is unclear. This is important since the origin of the valley indicates the type of subsurface sediment units that may exist. The area is found from  $W80^{\circ} 24'$  16" to  $W80^{\circ} 26'$  58" and from  $N37^{\circ} 49'$  41" to  $N37^{\circ} 52'$  30", approximately 7km north of Lewisburg, W.Va. (Figure 2). The first and obvious difference, between the two areas, is that the number of smaller dolines is much less than in Kentucky. However, there are similar frequencies of the larger uvala forms. From the topographic map, of West Virginia, it was determined that the lengths of the dolines range from 20 m to 350 m and

from less than 6 m to 13 m in depth. The depths are much less accurate since there is some interpolation of elevations that are necessary, and the scale of the map is such that very small dolines are not registered. The larger uvala forms range in length from 625 to over 3100m and from 6 to 40m in depth. Like those found in Kentucky, the West Virginia dolines and uvalas tend to be elliptical, but unlike those in Kentucky the West Virginia features tend to be unidirectional. The dolines and moreso the uvalas in West Virginia tend to have a systematic north to south orientation of their long axis. It is impossible to determine any systematic orientation of the dolines in Kentucky, at least not from the topographic maps.

There are approximately 30 ponded doline forms in the present West Virginia region and like the Kentucky area, there are no streams flowing on the surface. This fact, like that found in Kentucky, suggests that surface runoff is quickly drawn down to the subsurface drainage system, facilitated by the high permeability of the carbonate rocks. The presence of many roads, residential areas, and the Greenbrier Valley Airport and runways just off the north east corner of the map suggests that this is a relatively stable environment with respect to doline development. It too suggests that the dolines may have a

solutional rather than catastrophic origin. The elevation of the area drops from 816 m (2400 ft) in the north to about 748 m (2200 ft) in the south, which represents a southwardly dip of 16m/km, comparable to that recognized in the Kentucky study area.

It is apparent then, that there are a great number of similarities between the Kentucky and West Virginia regions. The areas have similar doline formations which compare in size and depth, as do the uvala forms. There is a lack of surface streams in both areas and both appear to be stable with a predominance of solutional forms. The main differences are in reference to the frequency of small doline forms and the fact that there is an apparent north - south orientation of dolines in West Virginia. It will be shown later that there is actually a pattern of doline orientation in the Kentucky area as reported by Kemmerly (1976, 1982, 1985).

#### CHAPTER 2: DOLINES; THEIR DEVELOPMENT AND DISTRIBUTIONS

Dolines are a basin or funnel shaped hollow in limestone, which range in diameter from a few meters to a kilometer and from a few meters to several hundred meters in depth (Monroe, 1970, in Ritter, 1978). These features are typically found in carbonate terrain, but vary in form due to differences in the limestones and geologic setting. Although the most frequently acknowledged doline forms are the collapse dolines, due to their catastrophic nature, the more spatially common dolines are the solutional forms. Kemmerly (1982) reports that the most common doline in the Kentucky - Tennessee region is the solutional and subsidence doline, therefore much of the following report will deal with the development and spatial distributions of these solutional forms.

#### Doline Development

As their name suggests, solutional dolines are formed by the dissolution of carbonate rocks by the action of aggressive or acidic waters (Jennings, 1985). The aggressive water action is most pronounced at sites of structural weakness. Continual action by aggressive waters may eventually cause the zone of weakness to enlarge and thereby allow for larger sediments to be carried downward. In time, the dissolving process and

sediment removal will form a conical depression which has an internal drainage system, linked to a subsurface drainage system.

The distinction between subsidence dolines and solutional dolines is related to their forms. Where the solutional dolines are formed directly on carbonate rocks, the subsidence dolines have a carbonate base with an overlying residuum. Despite the fact that both forms are generated by the same dissolution process, there is a significant distinction between the two forms. Considering the two study areas, and the reports on their carbonate and soil lithologies, it is safe to say that both areas are dominated by subsidence rather that solutional dolines.

The following processes are believed to be the main factors affecting the initiation and effecting the growth of dolines. These include secondary (vertical) permeability, ground water recharge, hydraulic gradient, and regolith thickness and shearstrength which also effects the lateral growth of dolines (Williams, 1972a, 1972b; Drake and Ford, 1972; Sweeting, 1972; and Palmquist, 1979, in Kemmerly, 1982).

Secondary permeability is most efficient along existing joint sets. As a result, there is a strong tendency for the long axis of a doline to be aligned along

joint sets, and/or parallel to them. This tendency has been reported by many authors and has been studied in many different areas (eg. Kemmerly, 1976, 1982, 1985; Day, 1976; Dicken, 1935; La Valle, 1968; Melton, 1934; Jennings, 1985; and Williams, 1972). It is quite possible that the north - south orientation of dolines in the West Virginia study area is indicative of a parallel joint set orientation. Medville (1977) reports a regional strike orientation of N65°E in this area of West Virginia and an orientation of N80°E is reported by Lowman Jr. and Webster Jr. and Allenby (1980). Although the evidence is not conclusive, it does appear that the doline long axis orientation in the West Virginia area is structurally controlled. In his study of the Kentucky region, Kemmerly (1976, 1982) has reported that the dolines are aligned along the regional joint sets, these being  $N20^{\circ}E$  to  $N40^{\circ}E$ , N70°E to N80°E and N20°W to N30°W. The fact that there are three dominant doline orientation classes existing in the same plain may be the reason why it is so difficult to determine any doline alignment patterns from the topographic maps.

The significance of the regolith thickness concerns the production of  $CO_2$ . The aggressive waters derive from the presence of  $CO_2$  in solution, which combines with water to form an acid ( $HCO_3^{-}$ ) solution. The

presence of plants and micro-organisms in the regolith are conducive to the production of biogenic carbon dioxide (Monroe, 1968 in Day, 1976). Therefore, the efficiency of dissolution is a function of the availability of plants and micro-organisms and the availability of water as a transporting agent. Kemmerly (1982) found however, that this factor was not significant in the Kentucky region. Its effect in West Virginia is indeterminable.

Ground water recharge and hydraulic gradient are two factors which go hand in hand. Obviously, the rate at which water can reach the carbonates determines, to some extent, the rate of dissolution. In both study areas, it was seen that there were no surface river systems visible and they must therefore have well developed groundwater recharge systems.

Regolith shearstrength affects the enlargement process effecting the doline development process. The enlargement of dolines are a result of slope processes, which themselves are a function of the regolith shearstrength. Although significant as an enlargement process, it may not be as significant as a factor of initiation.

It is evident that solutional dolines initiate and align preferentially to regional joint sets. The probability of initiation is also greater at multiple

joint intersections. This is evident in Kentucky as reported by Kemmerly (1976, 1982). The fact that the West Virginia dolines appear to be unidirectional may be due to a lack of competing joint sets, therefore, they align parallel to the only joint system evident (N80°E or N65°E). It is clear that there are certain processes which help to initiate doline development.

#### **Doline Distribution Patterns**

In knowing the factors affecting doline initiation and growth, it is easier to explain their spatial distribution patterns. There are two models which attempt to explain distribution patterns. The first is the Mutually Independent Random Process Model (MIRP) developed by McConnell and Horn (1972, in Kemmerly 1982, 1985). Briefly, it states that since doline forming processes are spatially intermixed and mutually independent, dolines are randomly distributed. They also suggest that there is no distinction between large and small dolines except possibly as a function of age. The second model is the Multigenerational Diffusion and Competition Process Model (MDCP) developed collectively by Ford (1972), Williams (1972a, 1972b) and Palmquist (1976, 1979) in Kemmerly (1982). It agrees with the random processes concept of the MIRP model and that they are spatially independent,

however, it proposes that there is a distinction between larger and smaller dolines. According to the model, the presence and growth of a large doline causes the sapping of sediment clogged joints in the vicinity of it. This allows for accelerated dissolution along the newly opened joints and the development of smaller dolines initially. Therefore, the development of secondary dolines is due to the presence of primary dolines and the smaller dolines should be clustered about the larger, randomly distributed dolines.

Kemmerly has produced several studies of doline distributions in Kentucky, the most significant was in 1982. Through statistical analysis, he was able to show several significant trends. From a Nearest Neighbor Analysis of over 25,000 dolines (42 quadrangles), Kemmerly (1982) showed that there are actually two distinct doline populations. The first are larger and deeper and have a well developed swallet system, unlike the smaller and shallower second population. This suggests that there are different processes acting to form the two populations. He was also able to show that 26 of the 42 quadrangles had a random distribution of dolines, as expected by both the MIRP and MDCP models. From a Nearest Neighbor Analysis of dolines with respect to swallet order, he was able to show that 22 quadrangles had a uniform primary doline

distribution with clustered secondary dolines. He also showed that 6 other quadrangles had a random primary doline distribution with a clustered secondary distribution. These secondaries and primaries explain the multigenerational aspect and the uniform primaries, the competition.

It is apparent that the initiation of dolines, although random, has a greater probability of occurring along joints and moreso at multiple joint intersections. There is also the fact that most processes that are significant in initiating and developing dolines, are also randomly distributed. This suggests that the presence of a doline is more likely where these processes are combined and enhanced by each other. Finally, it is also shown that there may be a structural control component to doline orientation as evidence in both areas.

An understanding of doline initiation and development is important since, it helps to understand and explain distribution patterns existing in other regions. The next section deals with the generation and manipulation of computer contoured maps, which is an attempt to describe and separate trends that may exist in either or both study areas.

#### CHAPTER 3: DATA ACQUISITION

As it is with most types of map analyses, when considering spatial distributions and patterns, it is necessary to acquire the maximum amount of information with the least amount of time or effort. Several decisions which must be considered are, method of extracting data, representation of the data, if the population is represented by the sample and the ease in which further information can be gathered from the sample. Considering the contouring of the two study areas, decisions that were made included the type of gridding method to employ, how representative of the actual topography were the contour maps and how useful were the generated maps in obtaining additional information on doline distributions. One other consideration was the computer contouring package and its flexibility and performance.

#### Gridding Techniques

To extract elevation values from surfaces (topographic maps), it is common to use a gridded overlay from which all elevations are extracted from all grid intersections. There are several types of grid overlay methods available. Makarovic (1973) proposes that the progressive sampling method is the most accurate. The

method is progressive since it represents rougher surfaces with a denser grid system. A plain surface, is overlain by a wide spaced grid network. By describing the rougher areas with a dense grid, the results are more accurate since variations are described more accurately. Ackerman (1978) also proposed that an uneven grid spacing would give more accurate results. To eliminate concerns of what grid spacings in an uneven filter would be most appropriated, it was decided to use a uniformly dense grid. Since it is the distribution of dolines that is of interest in this study, the grid size selected was based on the smallest group of dolines present on the topographic maps. The final result was a grid with a cell size of 0.5cm x 0.5cm.

The actual size of the study area and total grid size was determined by the most suitable area on the West Virginia topographic map. The actual site in West Virginia was selected to cover the greatest amount of the doline plain and yet miss the ridge formations on the east and west. The area was bounded in the south by the town of Lewisburg and the map border in the north. The final area coverage was 9.6km<sup>2</sup> (4.2km x 2.3km). The selected area allowed a grid overlay with 43 columns and 24 rows comprising of 0.5cm grid cells. From this grid size it was possible to extract 1032 elevation points, which

allowed an area coverage of 107.5 elevation data/km<sup>2</sup>. For a proper comparison, the same grid size overlay was used for both Kentucky and West Virginia. The grid network is dense enough to represent the doline roughness as well as providing accurate results when it comes time to contour the sample surfaces.

#### **Computer Contouring Method**

Once the topographic map has been digitized, the grid sample must then be contoured. The contouring may be done either by hand or by computer. For obvious reasons the grid samples from Kentucky and West Virginia were contoured by computer, using the Surface II Graphics System (Sampson, 1984). It must be determined however, if the graphics package and grid size are adequate in describing the original surfaces.

Slootweg (1978) and Engelen and Huybrechts (1981) have determined that manually contoured maps tend to show several inaccuracies in surface representation due to the bias introduced by the person that is contouring. Slootweg (1978) compared the automated and manually produced contour maps of the Atlantic ridge bathymetry. Engelen and Huybrechts (1981) conducted a similar type of study in the Dry Hesbaye region of eastern Belgium, and Ackerman (1978) studied the Soehnstetten region in

Germany. All three studies reported that manually derived maps showed contours that were skewed toward large features, especially those that were believed to be significant.

All maps appearing in this report, except figures 1 and 2, are generated by the Surface II Graphics System. The Surface II package is flexible in the way it reads data. The package also allows for a great deal of control by the operator, allowing control over contour intervals, map size, printing, line smoothing, closed depression notation, line bolding and contour line interpolations. It also has an extensive variety of possible outputs including trend surface maps, residual surface maps, isopach maps, profiles, three dimensional representations any many others.

The accuracy of the maps is an important feature. In general, areas with large differences in relief and outlined with small contour intervals, tended to have many contours which crossed. Believing this as a functional problem with Surface II to handle minute details, the problem was eliminated by using larger contour intervals. The problem with using larger contour intervals is that some of the doline forms are not as detailed as they could be, therefore contour maps of several contour intervals, for both Kentucky and West Virginia, meant that some

detail will then be lost. For this reason, computer generated maps for both areas and for several contour intervals are given (see Figures 3-7). Another problem with the computer maps is the loss of some depression forms since some contours would extend beyond the map border, and the computer cannot interpolate beyond the map edges. If more data was added so the contours would be completed within the map, then they would definitely appear as closed depressions. However, if more data was added, it would simple cause other contours to continue off the page. Finally, the contour interpolation method, as it is based on gridded data, tends to generalize forms. In areas where there is a valley or doline form on the original maps, if a data point does not represent the valley then it will not appear on the computer generated map.

Although the graphics package has some limitations, the overall quality of the computerized maps is fairly good. All major depression and uvala forms in West Virginia are shown in similar form and depth on the computer generated map (see Figure 2 and 4). The generated maps of Kentucky shows the very large uvala form as it appears on the original map (see Figures 1 and 6). Figure 5 is the same area of Kentucky but with a 5 ft contour interval, which was an attempt to detail more of

the smaller dolines that are present in Figure 1. Since so many of the dolines in Kentucky are small and shallow, the contour interpolation processed the features as one large uvala form. This fact is an indication that many of the doline features are not significantly different from each other, at least not enough to describe them as separate features. Despite the minor inadequacies of the map generation procedures, the contour maps are good representations of the actual surfaces, and with the data stored in memory there are many other possible maps that can be generated by simple manipulations of the data (eg. trend surface analysis, residual analysis, and filtering analysis).

# CHAPTER 4: STATISTICAL ANALYSIS AND DIGITAL SEPARATION TECHNIQUES

Any surface may consist of both positive and negative relief features. Therefore, to describe the general spatial pattern it is necessary to separate the surface into its individual components (eg. separate the positive relief features from the negative relief feature. By separating the components it is possible to separate the desired from the undesired features. Remembering that dolines are negative relief features, the objective of any separation technique is to isolate the doline (local variances) from the general topography (regional trend). There are generally two methods of separating digital information: (1) trend surface analysis which isolates the local variances (residual surface) from the regional surface, and (2) filtering techniques, which are more a mechanical separating method than statistical. Both methods are presented in this report representing a good contrast of methods and since both methods are utilized by the Surface II Graphics System. Following, is a discussion of the methods and their respective results.

#### **Trend Analysis**

Ayeni (1979) neatly describes the purpose of using a stepwise trend analysis; " to select the 'best' trend surface equation (or best deterministic function) which is most effective in describing both the regional trends and local deviations". A trend surface analysis is a higher order regression analysis. The basic approach in defining the trend is to estimate the surface expressions by the least squares method. The purpose of least squares is to minimize the square of the deviations from the trend. Each successive order of the trend analysis attempts to model an additional component of the actual surface which the previous order neglected to do. With each successive order the actual surface expression is modelled more accurately, and the number of orders permitted is equal to one less than the total number of data points in the sample population. However, it is hardly feasible to conduct a trend analysis to an order of 1031 in this study. Usually, the limit of successive orders required is defined by the significance of the variations that the trend can explain. If a higher order function explains significantly more of the variation than the next lower order, it is required to continue to the next higher This process stops when the higher order fails to order. explain significantly more of the surface variations than

the previous order. For the purpose of this study it was not necessary to determine the highest order required statistically since it is only intended to present a comparative analysis to the filtering technique. As a result, it was sufficient to only compute up to the fourth order trend for both Kentucky and West Virginia.

The first order trend is the polynomial trend which is represented by the following equation:

$$Z = a + bX + cY + \mathcal{E}i \tag{1}$$

Z	:	estimated surface elevation	
Χ,Υ	:	independent variables	
${\mathcal E}$ i	:	standard error of estimate	
(residual variance)			
a,b,	c:	population parameter estimates	

The second or quadratic order trend equation is given by

 $Z = a + bX + cY + dX^2 + eXY + fY^2 + \hat{e}i$  (2) where e,f are the population parameter estimates. The third and fourth order trends are similar in form as equation 1 and 2 except that they are higher polynomials (Ayeni, 1979; Chorley and Haggett, 1968; Thomas, 1968; Krumbein, 1959; Krumbein, 1963; and Allen and Krumbein, 1962).

As mentioned, the objective of trend surface is to minimize the deviations from the trend. With each trend order, more of the variation is explained and the amount of the total variation that is explained is expressed as the Pearson's r or  $\mathbb{R}^2$ . As part of the computer generated output,  $R^2$  values for each trend order are presented in the following table.

Table 1: Comparative R<sup>2</sup> values for 4 trend orders forKentucky and West Virginia

Trend Order	Kentucky	West Virginia
1st	0.078	0.235
2nd	0.536	0.345
3rd	0.588	0.367
4th	0.632	0.416

From the table it is clear that the trend orders for Kentucky explain more of the total variations (63%) than do those of West Virginia (41%). The main reason for this difference relates to the magnitude of the surface topography ranges. In West Virginia, the fairly level plain is frequently interrupted by positive and negative relief features and these range in elevations. In Kentucky however, the magnitude of the north ridges creates a bias such that the magnitude of the negative relief features are insignificant in comparison. As a result, the trend analysis attempts to model the ridges and a comparatively flat plain, which results in a higher Pearson r value. A factor which may contribute to the  $R^2$ values is the absolute volume of sample data. It is possible that because there are so many elevations in such a small area (1031 points) and since the trend surface has

only been computed to the fourth order, the degree of freedom is very low comparatively. As a result, the higher  $R^2$  values may be due to the low degree of freedom rather than a function of the explained variations.

The trend surface results for West Virginia are seen in figures 8 - 11, and those for Kentucky are seen in figures 16 - 19. With each successive order surface in West Virginia, it is evident that the trend begins to double over on itself. The trend progresses from a constant, southward dip in figure 8 to an 'S' shaped surface in figure 11. It is also noticed that none of the West Virginia trends distinguishes any doline forms, which is very different than the Kentucky trends.

The trend in Kentucky progresses from a constant, easterly dip in figure 16 to a distinctive bowl shape in figures 17 - 19. As mentioned previously, it is believed that the trend in Kentucky is more a function of the north ridges than it is of the central dolines and uvala. It is possible that it is a combination of the two but their relative magnitudes are so different. Since it appears that the trend surface maps do no adequately describe the local variations, but only the regional trends, than it is quite possible that the residual surfaces will define the doline plains.

#### **Residual Analysis**

Because of the nature of this study and the nature of trend surface analysis, it is quite possible that a residual analysis reveals more information of the doline distributions in the study areas. The residual component is a random normal variate, whose variance is extracted as the variance of the observed values from predicted. The residual magnitudes are represented in the trend equation by the i term (see eq. 1,2). By mapping the residuals it will be possible to distinguish how the dolines differ from the regional trend since it was evident from the previous section that the dolines are not a function of the regional trend. Residual mapping may take many forms. Thomas (1968) defines the following ways i/n which residuals are commonly presented; the basic residual (predicted - observed = residual), ration (predicted/observed = residual), the relative residual ((predicted - observed)/ observed = residual), and the standardized residual ((predicted - observed)/constant = residual). The Surface II Graphics package presents residuals only in the basic residual form. The production of a residual map is a by-product of the trend analysis and requires that only one parameter be changed in the The graphics package provides virtually no program. flexibility in terms of residual map types. To produce

the other residual map types it is necessary to compute the values separately and then to reenter them into the graphics package. Therefore, only the basic residual type have been mapped.

The basic residuals from the West Virginia trend analysis are seen in figures 12 -15. The dolines appearing in these figures are strikingly similar to those in the computer generated contour map and the actual topographic The main uvala form in the map center is very map. similar to the actual surface. The residual map betters the generated contour map in that some of the forms that should appear as depressions in the contour maps (eg. those along the map edges), do appear as depressions in the residual maps. However, there seems to be an enhancement - suppression effect with some of the doline forms as the residual order increases. For instance. doline forms in the northwest and north areas are enhanced as the residual order increase, but are then suppressed in the fourth order residual surface. Contrary, those dolines in the southeast and south tend to have a cyclic enhancement - suppression pattern. The large features in the first order are suppressed in the second order, only to be enhanced and the depressed in the third and fourth orders respectively. But overall, the dolines that appear in the residual maps are a good representation of those in

the actual surface.

The residual maps of the Kentucky study area are seen in figures 20 -23. There are some similarities and differences apparent between these residual maps and the generated, and actual, contour maps. Like the generated contour maps, the residual maps do not describe the smaller doline forms that appear on the original surface. However, the residual maps do identify the larger uvala The uvala forms are most evident in figure 20, forms. where it distinguishes four large uvala forms. Comparing the residual maps to the generated contour maps, figure 20 is most similar since the others begin to show the enhancement - suppression effect that was evident with the West Virginia residual surfaces. Unlike that found in West Virginia, the enhancement - suppression effect severely alters the main (centrally located) depression The depression forms in the northern area of the form. map are suppressed with increasing residual order, while they are enhanced in the southern region. The fact that they are suppressed in the northern ridge area may confirm the fact that the trend modelling process is biased by the ridges, therefore minimizing the deviations in the north rather than the south.

Therefore, both sets of residual surfaces detail the depression forms indicating that they are indeed local

variances and not a feature of the regional topography. There are some minor distinctions between the results from both aresa but the dolines are generally well represented. Both areas have noticeable enhancement = suppression effects, and it appears to enhance preferentially in one direction. Exactly why this effect is present is unclear, but it may be caused by a biassed error towards ridge formations. Despite minor technical drawbacks, a trendresidual analysis is a viable method for separating doline landscapes. The combinations of the procedures allows the local doline variations to be separated from the regional component and at the same time, maintain an accurate representation of the original doline forms. An added bonus of this method is that statistical methods can be applied in certain cases, and the significance of relationships, explained variations, and data normallicy are possible to determine. The next section deals with a second method which digitally separates elevation data.

#### Filtering Methods

Filtering is a method often used to separate regional components from local ones. The filters themselves range in complexity but the simplest method separates the extreme forms from the average plain, thereby producing two separate surfaces which can be used

to describe the original surface (Velleman, 1980). It is apparent from the literature, that there are numerous types of digital filters, each designed differently and designed to produce different results.

Velleman (1980) identifies two basic types of data filter categories: (1) The linear filters replace an outlier with an average value of the nearby values, (2) The non-linear filters replace outliers with a median value which is also extracted from the values nearby. The non-linear filter type seems more appropriate since the averaging process of the linear filters may tend to skew the results if their is an extreme value present in the averaging window. One specific type of linear filter is the box filter. This filter is designed to replace a `corrupted' value by the average of the values within a box formation surrounding it. It seems unlikely, but McDonnell (1981) claims that the box technique is independent of the box size. Lee (1980) also proposes a variation of the linear filters. The technique is based not only on the average of the surrounding values, but includes a threshold as defined by the local variance. Therefore, the corrupted value is only replaced if the local variance threshold is exceeded. Although expected to produce a biassed output, the linear filter techniques have provided suitable results (Lee, 1980).

An example of a non linear filter is a rank filter (Heygster, 1981). The ranking technique is based on the grey scale and it operates by ranking a box by its median Once ranked, the boxes can be kept or eliminated. value. Although this technique is applied to digital image processing with the grey scale, which ranges from 0 - 256, it may be possible to apply this technique to surface features. A simple filtering technique is presented by Nakagawa and Rosenfeld (1978). Their method is based on the substitutions of values (either one, or zero) if the surrounding area is dominated by the opposing number. Therefore, if a value of one is recorded in an area dominated by zeros, then it will be substituted by a zero, and vice-versa. To apply this technique to geographical landforms one would either have to create a larger classifying scheme based on more than two numbers, or the other option is to consider only surfaces with extremes such as large graben fault systems and the like.

The Surface II Graphics package has a filtering subroutine within it, one which is based on the works by Robinson (1969), Robinson and Charlesworth and Ellis (1969), and Robinson and Merrian (1972). The filters developed by Robinson are spatial filters intended for filtering geologic data. These filters are designed to pass the desired features while eliminating the

undulations which are not desired (Robinson, 1979; Robinson and Charlesworth and Ellis, 1979). The filter itself is a matrix comprised of weights which when summed, may either equal unity or zero. If the sum is unity, then the average elevation of the surface being passed is kept unchanged. If the sum is zero, the average elevation range will be eliminated and all else will remain unchanged. The matrix weights are defined by the width of the desired features as well as their frequency of appearance. The widths and frequencies are described by means of Fourier transformations; a collection of several sine and cosine curves of different frequencies and amplitudes which when summed define the desired surface topography being filtered.

One advantage of this method is that the operator has complete control over the features to be passed and eliminated. This is an important feature since it is this option which is lacking in the trend-residual surface analysis. One disadvantage of this method is that there is a loss of data along the edges of the map. The loss is due to the physical nature of the filtering process since values cannot be filtered if only part of the filter is covering the area, therefore the loss of data is equal to half the width of the filter matrix. Another problem is that each filter is site specific so that a new filter must be designed for each new surface and feature being filtered. One final problem is that the data to be filtered must be uniformly distributed, but this is not a problem with the data in this report because of the grid technique that was used.

There are several filtered diagrams presented. Α lack of knowledge in designing filters was an obstacle. Filters were designed to equal zero since unity meant that desired features were eliminated, and these were applied on a trial and error basis. A moving average filter, suggested by Robinson (1969) was also applied. When viewing figures 24 - 31, it is clear that there are very positive results. Figures 24 and 25 are of West Virginia and Kentucky respectively. A zero-sum filter was passed over the surfaces and the results are very dramatic. Figure 24 shows many of the doline forms found on the original map of West Virginia, as is should, and it shows many more than the trend surface method does. Figure 25 shows that a good proportion of the area is covered by dolines as is the case with the actual surface. This is the first map of the Kentucky area that does not

coalesce all depression forms into one large uvala.

Figures 26 and 27 represent the result from a 5 x 5 moving average filter. The detail on each is minimal though there are indications of the main doline forms.

The interesting feature of spatial filters derives from figures 28 and 29. These are exactly the same as figures 26 and 27 except that the moving average filter is a 3 x 3 matrix rather than a 5 x 5 matrix. It is amazing the magnitude of the increase in detail that results from a reduction in the filter matrix size. The figures show much more of the dolines that are present, as well as showing the general regional component. The bowl-like regional trend shown by trend surface in figure 17, is also evident in figure 29. The same `S' regional shape shown in figure 11 is also evident in figure 28.

Finally, figures 30 and 31 represent filtered residual surfaces of the fourth order for West Virginia and Kentucky respectively. Although these figures show the main dolines, the filter used was designed for a different surface by Robinson (Sampson, 1984). Theoretically, the filtered maps should not appear as good as they do, but exactly why they worked is unclear. The one feature that is absent from the Surface II Graphics system is the ability to save the data that is not passed by the filters since these would also reveal some information on the surface trends.

It is clear that filtering is a method that can be used to separate land surface features. It is shown that the results from this method are extremely good. It just

so happened that the filters that were used also happened to work since it was by trial and error. Despite the lack of statistical information to support relationships that may be found through filtering, it is clear that this method is certainly a viable alternative to the commonly used trend-residual analysis techniques.

#### CHAPTER 5: SUMMARY AND CONCLUSIONS

#### Summary

It was seen in chapter 1 that the geology and lithology of the Kentucky and West Virginia areas are very similar. The basic difference is that the limestone units were thicker, but fewer in number in Kentucky than in West Virginia. It was shown in chapter 2 that doline initiation and development is dependent on the structural and erosional processes. It was shown that since the structural and erosional processes are distributed randomly. Then doline distributions are also randomly distributed. There are two models presented, the MDCP model being the more favorable of the two. The model proposes that daughter dolines are distributed in a cluster about parent dolines, which themselves are either distributed uniformly due to competition, or randomly if in the immature state. The model was substantiated by Kemmerly (1982,1985) who conducted volumes of research in the Kentucky region. It was shown that in both study areas, dolines tend to align according to the regional joints. The complex pattern found in Kentucky is a result of three major joint sets, whereas there is only one in West Virginia.

Several methods of gridding and acquiring data was shown in chapter 3. It was decided that a uniformly dense grid would best suit the purposes of this thesis, as well as eliminating the possibility of biassed errors. It was also shown that the combination of the dense grid (1032 points) and the Surface II Graphics Package contouring ability, that the contour maps are an accurate representation of the actual surfaces. The fact that the smaller dolines are coalesced into larger uvala forms or are non existent is mainly due to the contour interpolation procedures.

Chapter 4 presented the theories behind digital separation techniques as well as two methods presently used in separating digital values. The trend-residual analysis is a method that has been in use for many years and its purpose is to model a surface statistically by the least squares method. The results from the analysis were very positive, showing that it clearly separates the regional component from the local variances (dolines). The maps also showed that the statistical method is accurate in describing the true surface. The filtering technique is a more mechanical technique. It requires that a spatial filter (zero-summed) be design to suite the needs of the operator and to fit the particular data. Although the contoured results were obtained by trial and error, it was shown that the technique is quite suitable to separating the dolines from the regional surface. In some instances it was evident that the technique actually described the doline patterns better than the trendresidual technique did.

#### Conclusions

The objective of this report was to present and discuss digital separation techniques as they are applied to a karst, doline landscape. It was evident from this report that,

1) Dolines are distributed randomly due to the random nature of the affecting and effecting processes. Uniform distributions are indicative of a competitive environment.

2) There is a structural alignment of doline long axis. In Kentucky there are three distinct orientations, and only one in West Virginia.

3) The nature of the grid is an important consideration and a dense, uniform grid is accurate in extracting and allows for representative results.

4) The Surface II Graphics package allows for several types of maps to be produced and with good accuracy.

5) Trend-residual analysis is a good digital separation technique. It provides accurate results and allows for further statistical studies. It can adequately separate the local doline variances from the regional trend and with similar accuracy in both Kentucky and West Virginia.

6) Filtering techniques are a viable option to the trendresidual method. It allows for more operator control over results and the results accurately describe the true surface. Although they are more specific in their use than trend analysis they are an excellent alternative.

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APPENDIX



Topographic surface map of West Virginia. ci = 20 ft

FIGURE 1



Topographic surface map of Kentucky. ci = 10 ft

FIGURE 2





CONTOUR MAP OF W.V. DOLINES Fig. 3













Fig 15



Fig 12 FIRST ORDER RESIDUAL ANALYSIS OF W.V. DOLINES





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Fig 22 THIRD ORDER RESIDUAL ANALYSIS OF KY. DOLINES



Fig 23 FOURTH ORDER RESIDUAL ANALYSIS OF KY. DOLINES



Fig 20 FIRST ORDER RESIDUAL ANALYSIS OF KY. DOLINES







FILTER OPERATION ON W.V.

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FILTERED FOURTH ORDER TREND RESIDUAL ANALYSIS OF KY. DOLINES

