OPTIMIZATION OF SHUTOFF RODS IN A CANDU REACTOR

by

Joseph KOTLARZ, BEng.

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AUTHOR: Joseph Kotlarz, BEng. (RMC)
SUPERVISOR: Dr. O.A. Trojan
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In CANDU reactors, mechanical devices called shutoff rods are used to shutdown the reactor if required. These rods are made of high thermal neutron absorbing material such as cadmium. The number and the locations of the shutoff rods are optimized for a given reactor configuration. Optimization here means minimizing the number of rods and maximizing their reactivity depth or effectiveness.

Optimization may be studied in various ways but the method selected is both simple and basic. It is apparent that if the interaction effects between the individual shutoff rods are reduced, their worths will increase. The optimum distance between two rods was determined to be 130 cm. Also, the best location of a third rod with respect to two already placed at an optimum separation was studied. Finally, these results were used in order to determine the optimum distance between banks of shutoff rods. These banks of rods were arranged in such a way as to achieve maximum flux flattening with all the rods inserted in the core. A 22 shutoff rod configuration for an adjuster flattened CANDÚ reactor gave a total change of 5.6% in $k_{eff}$. 

ACKNOWLEDGEMENTS

I wish to thank the staff of Atomic Energy of Canada Limited, Power Projects, especially the Physics and Analysis Division for their benevolent assistance in the research for this project. Special thanks to Dr. V.K. Mohindra, D.A. Jenkins and Dr. O.A. Trojan for their help and guidance and Mr. M.Y. Mamourian for his enthusiasm and helpful comments.
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1. INTRODUCTION

The shutdown of a nuclear reactor due to some malfunction must be anticipated at all times. This is achieved generally by controlling or limiting the fission process. Most reactors can be shut down in seconds by introducing a neutron absorber which renders the system subcritical. These absorbers may be termed as "black" (impenetrable) to neutrons or "grey". To ensure a shutdown, a conservative safety philosophy calls for a backup system such that if the first should fail, the other is available. Presently, there are three types of shutdown systems (SOS) used in CANDU reactors. These include: 1) mechanical shutoff rods (SOR's); 2) moderator dump; and 3) poison injection. Liquid shutoff rods are in the conceptual design stage.

Moderator dump is no longer designed as the backup to a primary SOS as in Pickering A, due to long times associated with its use. The poison injection system (used in Gentilly-1) is termed as very effective, however, a complex ion-exchange process is needed for subsequent cleanup. The primary shutdown system employs mechanical shutoff rods, each of which has a permanently fixed guide tube in the reactor core. Closely associated are the liquid shutoff rods which introduce, in Euler-type tubes, a liquid poison (gadolinium nitrate or boric acid and lithium hydroxide mixture). This poison can be quickly removed to restart the reactor. This is the newest system which has been adopted also by the Italians in the CIRENE reactor and the British in the SGHWR at Winfrith. (1,2,3)

Considering the importance of the shutoff rods, the effectiveness of the mechanical shutdown system is usually analyzed assuming that two of the most important rods are unavailable. A conservative safety policy requires a total reactivity load much higher than that normally required to safely shut the reactor down. Such a policy requires a large number of SOR's and hence a significant expenditure. To cut capital costs, optimization of the number and location of shutoff rods is required.
This report investigates the optimization of cadmium SOR's in a CANDU reactor (Figures 1 and 2). Optimal configurations were studied by applying model restraints and using flux distributions obtained from a two-group, three-dimensional neutron diffusion code.

2. THEORY AND MODEL SIMULATION

Mechanical shutoff rods simulated in this study are made of a strong, absorbing, cylindrical stainless steel-cadmium sandwich (see Figure 3 for details). There are numerous drive mechanisms, however, an electromagnetic clutch assembly is common. When the clutch is de-energized, the SOR falls into the guide tube under the influence of gravity and is sometimes aided by an accelerating spring. A typical insertion time would be of the order of one to two seconds. To restart the reactor, the SOR assembly is winched out requiring a time of about sixty seconds.

Accurate predictions regarding the effect of a SOR on the system requires extensive calculations. In general, neutron behaviour in a critical reactor may be summarized numerically as:

\[ \text{PRODUCTION} = \text{LOSS} = \text{LEAKAGE} + \text{ABSORPTION} \]

Using diffusion theory, the fast and thermal energy neutron groups are balanced in differential equation form. (The fast energy group refers to all neutrons above 0.5 ev.) (See Table 1 for the notation used.)

\[ \text{Fast-Gp}^{(5)} \quad \lambda \chi_1 (\nu \Sigma_{f1} \phi_1 + \nu \Sigma_{f2} \phi_2) = D_1 \nabla^2 \phi_1 + \Sigma_{a1} \phi_1 + \Sigma_{R1} \phi_1 \] (1)

\[ \text{Thermal-Gp}^{(5)} \quad \lambda \chi_2 (\nu \Sigma_{f1} \phi_1 + \nu \Sigma_{f2} \phi_2) + \Sigma_{R2} \phi_2 = -D_2 \nabla^2 \phi_2 + \Sigma_{a2} \phi_2 \] (2)

Over 95% of all fissions occur due to thermal neutrons, therefore, \( \nu \Sigma_{f1} \) is combined with \( \nu \Sigma_{f2} \). Further, \( \chi_1 \) is approximately equal to 1.0 and \( \chi_2 \) equals 0.0. In the study of absorption in SOR's in a CANDU reactor fast fluxes are
not usually considered since the cross-sections are small in the fast energy range compared to the thermal range. Insertion of a shutoff rod is associated with a significant local increase in the thermal absorption cross-section in equation (2):

In order to solve equations (1) and (2) for a given reactor state, a model is created by superimposing a grid system on the reactor. The separation of the grid line depends upon reactivity device locations and will be discussed within the simulation section. Such a model is shown in Figure 4. Using established three-dimensional neutron diffusion codes the flux at the centre of each grid cell was calculated. The leakage term in equations (1) and (2) at a point \((1,J,K)\) is formulated by the seven point difference equation:

\[
D_i \nabla^2 \phi_i(1,J,K) = D_i(\Delta X)^{-2} [\phi_i(1+1,J,K) - 2\phi_i(1,J,K) + \phi_i(1-1,J,K)]
\]

\[
+ D_i(\Delta Y)^{-2} [\phi_i(1,J+1,K) - 2\phi_i(1,J,K) + \phi_i(1,J-1,K)]
\]

\[
+ D_i(\Delta Z)^{-2} [\phi_i(1,J,K+1) - 2\phi_i(1,J,K) + \phi_i(1,J,K-1)]
\]

Assuming complete symmetry in the three directions, the code was used with an octant model of the core. This assumption is not entirely correct, however, since absolute symmetry cannot be achieved in a real reactor. For purposes of this analysis the results are acceptable.
The basic numerical methods used in superimposing a grid on the
octant model are as described. Primarily, a flux guess is made and a
convergence criteria is specified. Iterations based on an original flux
\[ \phi_i^n \] where \( n \) equals 0, solve for a flux \( \phi_i^{n+1} \) by an updating procedure until
the flux convergence criterion is met:

\[
\frac{\phi_i^{n+1} - \phi_i^n}{\phi_i^{n+1}} \leq \varepsilon .
\] (4)

A reasonable criterion is for \( \varepsilon \) to be \( 10^{-4} \). By iterative
processes the eigenvalue, \( \lambda \), converges more rapidly than flux such that for

\[
\frac{\lambda^{n+1} - \lambda^n}{\lambda^{n+1}} \leq \varepsilon ,
\] (5)

the flux convergence is often an order of magnitude less.

3. MODELLING PROCEDURE

A basic model has been established using a typical CANDU reactor
gometry (see Figure 5). As mentioned above, complete symmetry was assumed
and an octant was simulated. In the model there are 120 fuel channels, 52
of which are in a specified inner core. The outer core and the reflector
make up the remaining two basic materials. The basis by which these materials
were outlined was by an equal volume representation of the model to the
physical shape and size of the reactor. The reactor was further divided into
cells and the above diffusion equations were applied to each of these cells.
In other words, it was assumed that the properties of various fuel regions
and reactivity devices within the reactor could be smeared over an effective
cell. Cell dimensions were normally chosen in increments of \( (\Delta X \Delta Y \Delta Z) \)
corresponding to \( (28.575 \times 28.575 \times 49.53 \text{ cm}^3) \) unless restricted by reactivity
devices. X and Y increments are based on a typical lattice cell pitch featuring one channel per cell. The 49.53 cm in the axial direction corresponds to the length of a fuel bundle. Next and most importantly, material properties were assigned to each cell, depicting diffusion coefficients, absorption, removal and yield cross-sections.

Simulation commenced with a reference case model which consisted of three banks of eight adjusters and six zone controllers (one-half full). Material properties for the reference case were obtained from lattice codes. Furthermore, these codes calculated the incremental properties characteristic of shutoff rods. From the obtained flux distribution, further simulations were anticipated by "strategically" introducing SOR's.

The simple introduction of a SOR into the reactor complicates the modelling by creating more mesh points. In addition, incremental properties are smeared over a cell representing the reactivity device. An example of a cell representing an adjuster rod would correspond to a cross-sectional area of 57.15 x 49.53 cm$^2$. The adjuster rod lengths vary from 5 to 8 lattice pitches (1 lattice pitch = 28.575 cm), the longer ones being in the centre of the core. A zone controller represents the same AXAZ cell, however, it is typically 12 lattices in length. It must be remembered that these aforementioned device lengths must be doubled when talking about the entire reactor. In addition, any devices lying on the transverse or vertical axial midplanes represent a "half-effect", with the centrally located SOR having only a "quarter-effect".

At this stage, a logical procedure of SOR positioning is required in order to study model effects, in particular, the interaction between SOR's.

4. PROCEDURE

For the reference model flux distributions were plotted in the axial and radial directions (Figure 6). The local depressions in the radial direction

* This refers to the SOR placement in a region of high flux in order to achieve a maximum worth per rod.
are due to the adjuster rods resulting in an overall flattened distribution. A shutoff rod has the effect of depressing the thermal flux. Its range of effectiveness depends primarily on where it is located. For example, a high flux region such as the core center will make the shutoff rod worth much more than if it was placed in the outer core region. In addition, the worth of a shutoff rod is dependent upon its interaction with other control devices, namely, other shutoff rods. The following procedure was used to study possible locations for SOR's taking into account rod interactions. Prior to describing this procedure, optimization of shutoff rods will be defined as applicable to this project.

Optimization here has two aspects. Firstly, a minimum number of SOR's is determined for a given reactor shape and size. In this case, the interaction effects between the shutoff rods are minimal. Such an array of SOR's will be referred to as a "configuration". Secondly, given the total reactivity depth required, more SOR's may have to be added to the above minimum "configuration". In this case, the interaction between SOR's will no longer be minimal, however, the number of SOR's required can still be optimized for the given situation. This latter case requires that the additional SOR's be arranged in such a way as to achieve maximum flux flattening with all SOR's inserted. Such an array will be termed as an optimum "arrangement".

The process of determining the minimum number of SOR's required for a given reactivity load was restricted at first to a quadrant. To begin, a central and then an eccentric SOR was placed axially in order to study the interaction between any two shutoff rods. A spatial restriction is that the SOR must physically be at least one lattice pitch (28.575 cm) away from any other reactivity device.

Next, the interaction between three SOR's was studied with two of the rods along the axial direction being optimally spaced and under the above mentioned physical restraint. The third rod was moved in an arc about the centre of the reactor. Using flux distributions and physical limitations,
the optimum location of the third SOR was obtained. Before locating a fourth optimum SOR, a crucial reorganization of the optimal three-rod configuration was made, considering totally the geometrical limitations imposed by the reactor. To this stage, the major geometrical constraint was that reactivity devices be at least one lattice pitch from one another with the exception of the central shutoff rod. Considering now that the central SOR is in the immediate vicinity of an adjuster, and by applying previously obtained results from two and three rod interactions, a rearrangement of the optimal three-rod configuration was made (see Figure 8, rods (1), (2), and (3)). In conjunction a fourth SOR was optimally located using the previously attained optimum shutoff rod separations. The optimum "configuration" was obtained whereby the interaction between all shutoff rods was minimal and the total reactivity worth was maximum. This optimal configuration of four ((1), (2), (3), (4)) SOR's per quadrant (Figure 8) was adopted as the basis to determine an optimum arrangement.

It is clear from the above that additional SOR's no longer provide an optimum solution. However, using the four optimally placed SOR's and tracing the flux pattern throughout the reactor grid, high flux regions are easily located. The next optimally placed SOR would be in this higher flux region. Now, optimization will mean minimizing the number of SOR's to achieve a required reactivity depth since their geometrical placements are no longer optimum.

In summary of optimization criteria, a maximum worth per rod was obtained by putting as many rods into the core as proximity or minimum interaction would allow. If after having exhausted these locations and still not having met a required reactivity load, more SOR's will be required. Then a flux distribution is obtained from the previous optimum case, the region of maximum flux is found, the constraints are determined and finally the "best" location for the next SOR is selected in order to achieve a maximum flux flattening. This method of obtaining an optimum flux distribution and then placing a SOR in an area of high flux can be continued to the point of
achieving a reactivity depth specified by the reactor design. In a broad sense, the SOR arrangements obtained by this process describe physically the phrase, "optimization of shutoff rods".

5. RESULTS

Table 2 illustrates the average thermal flux due to the interaction between the central and one eccentric shutoff rod per quadrant. By plotting the worth of this second rod against their separation, an optimum of 130 cm separation was found (Figure 7). At this distance, the fluxes are most nearly equalized and highest in average value.

Table 3 shows the interaction and optimal placement of three shutoff rods. The two rods already optimized in position lie along the axial axis. The optimal separation is 161 cm rather than 130 cm due to physical constraints. In this particular case, a zone controller was positioned at 123 cm and at least one lattice pitch is required between reactivity devices. An optimum configuration (Case 3) gave an average worth of 2.11 mk per rod.

Based on the previous results, a final optimum configuration of four SOR's per quadrant was established. Note that there is no central SOR. This configuration (rod, (1) to (4)) is shown in Figure 8 as a "rhombic" geometry. Table 4 correlates $k_{\text{eff}}$, the total worth of the SOR's and their individual average thermal fluxes. In addition, it shows the optimal effects of twenty and twenty-two shutoff rods. Twenty-two SOR's (optimized) have a total worth of 56.2 mk.

6. CONCLUSIONS

For the given CANDU reactor, an optimization of SOR's may be simply approached by a study of rod interaction - in particular, two and three shutoff rods. Geometrical constraints play a crucial role in the optimization
but in addition, the criterion for an optimum case is to achieve maximum flux flattening with all the rods inserted in the core.

The optimum configuration of SOR's for a typical CANDU system takes the geometrical form of a rhombic solution. This solution gave a total reactivity worth of 36.73 mk or 2.3 mk per rod. An extension of this study showed how its basic solution or optimum SOR configuration could be used in the optimization for a specified reactivity load greater than that attained by the four optimally placed rods. The continued optimization utilizing high flux distributions located a fifth and sixth rod as shown in previous Figure 8. Such an arrangement gave a total change of 5.6% in $k_{\text{eff}}$ over the reference case.
APPENDIX A

CENTRAL SOR MATHEMATICAL ANALYSIS

Analytical calculations of $\Delta k_{\text{eff}}$ for a central shutoff rod are generally performed using one-group and two-group treatment. Equations describing these two methods are contained in Table 5.\(^{(6)}\)

Most theories dealing with control rods assume them to be parallel to the fuel channels. This is not the case with the CANDU reactor where SOR's are perpendicular. To accommodate this situation, the CANDU reactor geometry was simply altered on an equal volume basis (Figure 10) giving a radius equal to 351 cm. Figures 11, 12 and 13 describe physically the symbols used in the above referenced criticality equations and Table 6 gives their values as obtained from a lattice parameter code. Using the values of Table 6 in the equations presented in Table 5 the results (Table 7) showed one-group overestimates and two-group underestimates SOR worth proportionally by 9.9%. An average set of data showed differences of about 16% from the calculated values. This simple approach used to obtain preliminary analytical results should be considered in a possible future project for developing analytical methods for analyzing the worth of central and eccentric shutoff rods.
## TABLE I

### SYMBOLS DEFINED

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>Neutron group. For i = 1, fast neutrons and i = 2 denotes thermal neutrons</td>
</tr>
<tr>
<td>$k_{\text{eff}}$</td>
<td>Effective Multiplication Factor</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Eigenvalue corresponding to $1/k_{\text{eff}}$</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Fraction of neutrons born fast or thermal</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Mean number of secondary neutrons per fission</td>
</tr>
<tr>
<td>$\nu\Sigma_{f_i}$</td>
<td>Neutron yield cross-section</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Diffusion coefficient</td>
</tr>
<tr>
<td>$\Sigma_{a_i}$</td>
<td>Neutron absorption cross-section</td>
</tr>
<tr>
<td>$\Sigma_{r_i}$</td>
<td>Removal cross-section</td>
</tr>
<tr>
<td>$\Phi_i^n$</td>
<td>Neutron flux corresponding to group &quot;i&quot; and iteration number n used in numerical methods</td>
</tr>
<tr>
<td>Case</td>
<td>Distance of Second Rod from Central (cm)</td>
</tr>
<tr>
<td>------</td>
<td>-----------------------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Reference Case$^+$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>130</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
</tr>
</tbody>
</table>

$^+$ All adjusters in and zone controllers one-half full

$^*$ \[
\left| \frac{(k_{\text{eff}})\text{rod(s)}}{(k_{\text{eff}})\text{ref}} - 1 \right|
\]

$^{++}$ \[
\left| \frac{(k_{\text{eff}})2 \text{ rods}}{(k_{\text{eff}})\text{control rod}} - 1 \right|
\]
TABLE 3
THREE ROD CONFIGURATIONS

<table>
<thead>
<tr>
<th>Case</th>
<th>((Z_i, X_i)^+) (cm, cm)</th>
<th>(k_{\text{eff}}) (mk)</th>
<th>Total Worth of Rods (mk)*</th>
<th>Average Thermal Flux in each SOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>((Z_1, X_1))</td>
</tr>
<tr>
<td>1</td>
<td>(i=1 (0,0))</td>
<td>0.989578</td>
<td>10.8869</td>
<td>0.3735032</td>
</tr>
<tr>
<td></td>
<td>(i=2 (161,0))</td>
<td></td>
<td></td>
<td>0.3078395</td>
</tr>
<tr>
<td></td>
<td>(i=3 (0,114.3))</td>
<td></td>
<td></td>
<td>0.4106712</td>
</tr>
<tr>
<td>2</td>
<td>(i=1 (0,0))</td>
<td>0.985728</td>
<td>14.7351</td>
<td>0.3735032</td>
</tr>
<tr>
<td></td>
<td>(i=2 (161,0))</td>
<td></td>
<td></td>
<td>0.3078395</td>
</tr>
<tr>
<td></td>
<td>(i=3 (30,114.3))</td>
<td></td>
<td></td>
<td>0.4106712</td>
</tr>
<tr>
<td>3</td>
<td>(i=1 (0,0))</td>
<td>0.985684</td>
<td>14.7791</td>
<td>0.3735032</td>
</tr>
<tr>
<td></td>
<td>(i=2 (161,0))</td>
<td></td>
<td></td>
<td>0.3078395</td>
</tr>
<tr>
<td></td>
<td>(i=3 (123.825, 114.3))</td>
<td></td>
<td></td>
<td>0.4106712</td>
</tr>
</tbody>
</table>

\* \(\frac{(k_{\text{eff}})\text{rods}}{1.00047} - 1\)

\( (Z_i, X_i) \) refers to SOR location where \(i\) stands for the SOR number
TABLE 4

OPTIMUM CONFIGURATIONS (MORE THAN 4 SORS)

<table>
<thead>
<tr>
<th>Case</th>
<th>No. and Placement ( (Z_i, X_i) )</th>
<th>( I &lt; \text{eff} ) (mk)</th>
<th>Total Worth of Rods (mk)</th>
<th>Average Thermal Flux in Each SOR</th>
<th>Worth per rod (mk)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( i=1 ) (50,2) &lt;br&gt; ( i=2 ) (50,6) &lt;br&gt; ( i=3 ) (180,4) &lt;br&gt; ( i=4 ) (180,8) (4)</td>
<td>0.967318</td>
<td>36.7348</td>
<td>(( Z_1, X_1 )) 0.3999723</td>
<td>0.5124329</td>
</tr>
<tr>
<td></td>
<td>( i=1 ) (50,2) &lt;br&gt; ( i=2 ) (50,6) &lt;br&gt; ( i=3 ) (180,4) &lt;br&gt; ( i=4 ) (180,8) &lt;br&gt; ( i=5 ) (50,8) (5)</td>
<td>0.955523</td>
<td>44.9259</td>
<td>(( Z_2, X_2 )) 0.5370126</td>
<td>0.3237817</td>
</tr>
<tr>
<td></td>
<td>( i=1 ) (50,2) &lt;br&gt; ( i=2 ) (50,6) &lt;br&gt; ( i=3 ) (180,4) &lt;br&gt; ( i=4 ) (180,8) &lt;br&gt; ( i=5 ) (50,8) &lt;br&gt; ( i=6 ) (161,0) (5-1/2)</td>
<td>0.944251</td>
<td>56.1926</td>
<td>(( Z_3, X_3 )) 0.4737825</td>
<td>0.3782369</td>
</tr>
</tbody>
</table>

+ (cm, lattices) where 1 lattice = 28.575 cm
## TABLE 5

**CENTRAL SOR CRITICALITY, FLUX AND $\Delta k_{\text{eff}}$ EQUATIONS**

<table>
<thead>
<tr>
<th>THEORY</th>
<th>EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One-group</strong></td>
<td></td>
</tr>
<tr>
<td>$Y_o(\lambda_1 r)$</td>
<td>$Y_o(\lambda_1 R_O) + \varepsilon \lambda_1 Y_1(\lambda_1 R_O)$</td>
</tr>
<tr>
<td>$J_o(\lambda_1 r)$</td>
<td>$J_o(\lambda_1 R_O) + \varepsilon \lambda_1 J_1(\lambda_1 R_O)$</td>
</tr>
<tr>
<td>$\phi(r)$</td>
<td>$A \left[ 1 - \frac{2.44 \delta k}{\pi M^2 \alpha_o^2} \left( 0.116 + \frac{1}{\alpha r} \right) \right]$</td>
</tr>
<tr>
<td>$\Delta k_{\text{eff}}$</td>
<td>$7.5 \frac{M^2}{R^2} \left( 0.116 + \frac{1}{2.4 \frac{R}{R'_l}} \right)$</td>
</tr>
<tr>
<td><strong>Two-group</strong></td>
<td></td>
</tr>
<tr>
<td>$\Delta k_{\text{eff}}$</td>
<td>$7.5 \frac{M^2}{R^2} \left[ 0.116 \left( 1 + \frac{L_s^2}{L^2} \right) + \frac{L_s^2}{L^2} \ln \frac{L L_s}{M R'_l} + \frac{\ln R}{2.4 \frac{R}{R'_l}} \right]^{-1}$</td>
</tr>
</tbody>
</table>
### TABLE 6

**CENTRAL SOR PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L^2$</td>
<td>232.32 cm²</td>
</tr>
<tr>
<td>$L_s^2$</td>
<td>156.29 cm²</td>
</tr>
<tr>
<td>$M^2$</td>
<td>388.61 cm²</td>
</tr>
<tr>
<td>$D_1$</td>
<td>1.28 cm</td>
</tr>
<tr>
<td>$D_2$</td>
<td>0.95 cm</td>
</tr>
<tr>
<td>$R_o$</td>
<td>5.636 cm</td>
</tr>
<tr>
<td>$R$</td>
<td>350.7 cm</td>
</tr>
</tbody>
</table>

$R_1 = R_o - d$ where $d = \frac{0.71 \lambda_t + \frac{4}{3} \lambda_t}{2}$

$\lambda_t = 3D_i$, where $i = 1, 2$

---

### TABLE 7

**CENTRAL SOR MATHEMATICAL ANALYSIS**

<table>
<thead>
<tr>
<th>$R'$ (cm)</th>
<th>$d$ (cm)</th>
<th>One-group $\Delta k_{eff}$ (mk)</th>
<th>Two-group $\Delta k_{eff}$ (mk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.72826</td>
<td>2.908</td>
<td>5.7844</td>
<td>4.7151</td>
</tr>
<tr>
<td>2.22226</td>
<td>3.414</td>
<td>6.1193</td>
<td>4.4137</td>
</tr>
<tr>
<td>1.71626</td>
<td>3.920</td>
<td>6.3450</td>
<td>4.0849</td>
</tr>
</tbody>
</table>

* Obtained from lattice parameters code

** Obtained on an equal volume basis

*** From Appendix A
FIGURE 1  ELEVATION VIEW OF REACTOR

FIGURE 2  PLAN VIEW OF REACTOR
OFFSET DRIVE ARRANGEMENT

FIGURE 3
FIGURE 4  GRID REPRESENTATION OF REACTOR
FIGURE 5  REACTOR MODEL - OCTANT
FIGURE 6  NORMALIZED FLUX VS RADIAL/AXIAL DISTANCE

RADIAL DISTRIBUTION

AXIAL DISTRIBUTION
FIGURE 7  WORTH (ECCENTRIC SOR) VS DISTANCE FROM CENTRAL SOR

MARCH 1976
FIGURE 8  PLAN VIEW

ADJUSTER

ZONE CONTROLLER

SHUTOFF ROD
FLUX AVERAGING EXAMPLE SHOWING 2 SORs, ADJUSTER (1a) AND ZONE CONTROLLER OVERLAP

FIGURE 9 FLUX AVERAGING
REFERENCES

1. E.M. Yaremy, "Reactor and Station Control", Nuclear Energy Symposium, 1974 Conference CNA/AECL, Montreal, Canada.


5. ORNL Report ORNL-3199.
