Models for Managing Supply and Demand Uncertainties in Supply Chains

MODELS FOR MANAGING SUPPLY AND DEMAND UNCERTAINTIES IN SUPPLY CHAINS

BY

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Abstract

We propose a classification framework for the operations management literature that has looked at pricing and ordering in supply chains when supply and/or demand are uncertain. We then focus on developing three new models for managing supply and demand uncertainties in supply chains.

In the first model, we study a two period sourcing problem of a firm under two sets of contracts. The contracts differ in terms of acquisition costs and the level of risk that they impose on the firm. We provide the conditions where the optimum solution is unique and also explore the behaviour of the optimum solution analytically and numerically. One application of our model is in the agribusiness supply chain and we provide numerical examples based on data from the almond industry in California.

In the second model we look at a joint ordering, pricing and capacity planning problem. We characterize the optimum policy both in single and multi-period cases. In addition, we study the impact of fixed production costs on the optimum policy.

The third model is devoted to coordination between a buyer and a supplier where there is a possibility of improving the supplier by both players. We analyze the problem under a Stackelberg game setting where the buyer is the leader. We show that the buyer either tries to amplify the investment of the supplier by order inflation or assumes all the investment costs. We investigate the behaviour of the optimum

solution under different strategies.

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Contents

Abstract **iii**

Acknowledgements **v**

1 Introduction **1**

1.1 Overview and Motivation 1

1.2 Contributions and Organization of Thesis 3

2 Literature Review and a Classification Framework **7**

2.1 Related Review Papers 8

2.2 A Classification Framework 9

2.3 Lot-Sizing Problem with Uncertain Supply and Demand 11

2.3.1 Single Supplier ($U|U|.|.|1|q$) 12

2.3.2 Two Suppliers ($U|U|.|.|2|q$) 28

2.3.3 Multiple Suppliers ($U|U|.|.|M|q$) 38

2.4 Joint Lot-Sizing and Pricing with Uncertain Supply and Demand . . 41

2.5 Classifying the Papers Based on Their Solution Procedures 45

3 A Two-Period Sourcing Model with Demand and Supply Risks **51**

3.1 Introduction 51
3.1.1 Literature Review of Production Planning Problem in Agricultural Environment 54
3.2 Assumptions and Notations 55
3.3 Production Planning with No Carry-over 57
3.4 Production Planning with Carry-over 62
3.4.1 Empirical Test of Price Ratios 69
3.5 Numerical Case Analysis: California Almond Industry 70
3.6 Conclusions 76

4 Optimal Capacity, Pricing and Production Policies under Supply and Demand Uncertainties 82

4.1 Introduction 82
4.1.1 Literature Review of Joint Lot-Sizing and Pricing Problems Under Uncertain Supply and Demand 84
4.2 Problem Description and Modelling 85
4.2.1 Expected demand fill rate elasticity 90
4.3 Single Period Model 93
4.4 Multi-Period Model with no Fixed Costs 95
4.4.1 Case 1: No penalty cost 95
4.4.2 Model with penalty cost 100
4.5 Multi-Period Model with Production Fixed Cost 102
4.6 Numerical Analysis 104
4.6.1 Effect of dynamic pricing 105
4.6.2 Effect of renting option 107

4.6.3 Effect of uncertainty of supply 108

4.7 Conclusion 110

5 Buyer-Supplier Coordination through Capacity Investment under Demand and Supply Risks 137

5.1 Introduction 137

5.1.1 Literature Review of Coordination in Supplier Development Programs 140

5.2 Problem Description and Formulation 143

5.3 Centralized Case 151

5.4 Investment Sharing Contract 152

5.5 Numerical Analysis 154

5.5.1 Comparison of Investment Options 154

5.5.2 Behaviour of Optimal Solution under Different Settings of the Contract 158

5.6 Conclusions 161

6 Concluding Remarks 176

6.1 Thesis Summary 176

6.2 Future Work 179

List of Tables

2.1 Existing notation systems in Operations Management 10
2.2 Proposed $a|b|c|d|e|f$ notation system for lot-sizing and pricing problems 11
2.3 Lot-sizing papers with a single supplier and single product in single period and continuous review cases 13
2.4 Lot-sizing papers with a single supplier and single product in finite horizon case 14
2.5 Lot-sizing papers with a single supplier and single product in infinite horizon case 15
2.6 Lot-sizing papers with a single supplier and multi-products 16
2.7 Lot-sizing papers with two suppliers in single period case 29
2.8 Lot-sizing papers with two suppliers in multi-period case 30
2.9 Lot-sizing papers with more than two supplier 39
2.10 Joint lot-sizing and pricing papers 42
2.11 Proposed solution procedure of reviewed studies in lot-sizing 46
2.12 Proposed solution procedure of reviewed studies in lot-sizing 47
2.13 Proposed solution procedure of reviewed studies in joint lot-sizing and pricing 49
3.1 Summary of notation 56

3.2 Price ratios (based on data from the Almond Board of California (2012)) 70

4.1 Summary of notation 92

4.2 Description of the Figure 4.8 135

5.1 Summary of notation 145

5.2 The supply chain profit for different production costs ($p = 30$) 156

List of Figures

3.1 Sensitivity analysis with respect to first period's yield standard deviation. 71

3.2 Sensitivity analysis of production quantity with respect to first period's mean demand. 73

3.3 Sensitivity analysis of production quantity with respect to first period's demand standard deviation. 74

3.4 Sensitivity analysis of production quantity with respect to the ratio of the production cost over purchasing cost. 75

4.1 Difference between dynamic and static pricing with respect to initial asset 105

4.2 profit increase with respect to renting rate 107

4.3 Behaviour of average optimal prices with respect to renting rate . . . 108

4.4 Behaviour of production land with respect to standard deviation of supply 109

4.5 Behaviour of average land change with respect to standard deviation of supply 109

4.6 Behaviour of average price with respect to standard deviation of supply 110

4.7 Behaviour of expected profit with respect to standard deviation of supply 110

4.8 Behaviour of EDFR elasticity under different settings 136

5.1 Optimality of different options for the buyer 155
5.2 Optimality of different options for the supply chain 156
5.3 The behaviour of optimal order quantity 157
5.4 The behaviour of optimal investment 158
5.5 Variation of supply chain's profit with respect to β 159
5.6 Optimal order quantity with respect to β 159
5.7 Variation of total investment with respect to β 160
5.8 Variation of supplier's investment with respect to β 160

Chapter 1

Introduction

1.1 Overview and Motivation

The process of determining the timing and the quantity of production and procurement is referred to as the *lot-sizing* problem (Yano and Lee (1995)). It has a wide application in different areas such as health care systems (Cho (2010), Arifoğlu *et al.* (2012)), remanufacturing systems (Inderfurth (1997), Mukhopadhyay and Ma (2009)) and agribusiness supply chains (Jones *et al.* (2001), Kazaz and Webster (2011)). As a result, in different studies the structure of this problem has been customized based on its application. Despite these differences, a common trait in the lot-sizing problem is finding a balance between demand and supply. A mismatch between demand and supply can result in downgrading the firm's corporate performance (Hendricks and Singhal (2009)).

The constant market changes and sophisticated nature of human behaviour makes it difficult for companies to forecast future demand accurately. This uncertainty of demand can significantly impact the performance of a company. There is a vast body of literature that considers uncertainty of demand (e.g., Mula *et al.* (2006)).

The existence of uncertainty in the supply chain is not limited to demand uncertainty. Many firms experience inconsistency in their production and procurement processes which is resulting in supply uncertainty. Electronic fabrication and assembly systems, closed-loop supply chains and agricultural production systems are some of sectors of that suffer from uncertainty of supply (Yano and Lee (1995)). This uncertainty can be caused by many factors such as machine break-downs, weather conditions and natural disasters. The main forms of supply uncertainty that have been discussed in the literature are as follows:

- **Yield uncertainty:** Common in agricultural production and manufacturing systems where there is a possibility of producing defective items (Yano and Lee (1995)).

- **Capacity uncertainty:** Occurs in production systems that suffer from unexpected machine break-downs and maintenances (Ciarallo *et al.* (1994)).

- **Disruption:** Infrequent risky events such as natural disasters and terrorist attacks, that may lead to system shut-downs and temporary closures (Snyder *et al.* (2010)).

- **Lead-time uncertainty:** Results from inconsistency and randomness in the production and procurement delivery time (Dolgui *et al.* (2013)).

- **Supply information uncertainty:** Most of the firms use computerized systems to keep the records of their inventory. However the physical amount of inventory can be affected by some unforeseen factors such as shrinkage, theft and misplacement. These factors cause a gap between the inventory records and the actual number of products. This is referred to as supply information uncertainty (Chen and Mersereau (2013)).

Most of the demand driven strategies, such as just-in-time, can be incapacitated by uncertainty of supply (Simchi-Levi *et al.* (2002)). Boonyathan and Power (2007) demonstrated that the uncertainty of supply has a greater impact on the performance of a company in comparison to the uncertainty of demand. Thus, it is important to consider supply uncertainty, in addition to demand uncertainty, when solving lot-sizing problems, even though it makes the analysis more complex. In this thesis we look at lot-sizing problems under different settings while considering the uncertainty of supply and demand. We focus on the structure of the optimum decisions and how they behave when the conditions of the problem change.

In some literature the term *uncertainty* is understood in the context of Knightian uncertainty: case where the distribution of the possible outcomes of an event are unknown (Knight (1921)). In these sources the case where the distribution of possible outcomes is known, the terms *risk* or *random* are used instead. However, in many other sources the terms uncertainty and random are used interchangeably. In this thesis, unless it is confusing to the author, we adopt the latter approach.

1.2 Contributions and Organization of Thesis

In this section we describe the scope and structure of the thesis as well as some of the main assumptions that are considered in each chapter. In addition we mention some of the main contributions in each chapter.

In Chapter 2 we conduct a state of the art review of the literature on lot-sizing when supply and demand are random. We propose a classification scheme for problems in this area and use it to classify the reviewed literature. We also report on the different methodologies employed in the reviewed literature.

In Chapter 3 we study the problem of sourcing a product when the demand and yield may be uncertain. We consider a two-period model where the supplier's production quantity in the second period is a function of the amount produced in the first period. This is a common situation in industries where the production capacity cannot be changed in a short period of time such as in the almond industry. In this industry usually a two year contract between the supplier (farmer) and the buyer (handler) is preferred. The buyer can sign two sets of contracts: a production contract where she is responsible for the uncertainty of yield or a purchasing contract where the provided quantity is guaranteed by the supplier at a higher cost. The buyer has to decide about the the quantities to buy through the production and purchasing contracts. The buyer has the option to carry excess inventory from the first period to the second. We establish some analytical properties of our proposed model and perform comparative static analysis to study the buyers decisions. In particular, we show under which conditions the buyer may benefit from purchasing contracts. To gain some practical insights we also apply our model to some real data from the California almond industry.

In Chapter 4 we consider a multi-period model for determining optimal capacity, production quantities and prices when supply and a price-sensitive demand are uncertain. According to the forecast of supply and demand, the decision maker may increase her capacity in each period. In cases where there is excess capacity, she has the option to rent or sell out her capacity. We introduce the concept of expected demand fill rate elasticity and characterize the conditions in which one-sided production, pricing and capacity acquisition policy is optimal. We extend the results to the case when there is a fixed production cost and show that a two-sided production, pricing and acquisition policy is optimal. We also investigate the conditions when it is optimal for the decision maker to rent out her capacity. In the special case of a single period newsvendor problem we characterize the conditions in which the problem is unimodal.

In Chapter 5 we develop a model for coordination between a buyer and a supplier in the context of a supplier development program. We look at simultaneous investment of a buyer and a supplier in capacity improvement of the supplier. We apply a Stackelberg game setting where the buyer is the leader and investigate the problem under uncertainty of supply and demand. Both players can invest directly on capacity improvement to make sure that their desired production volume is achieved. We show that the players have an opportunistic behaviour toward investment. When the buyer finds that the supplier is motivated enough to invest, he avoids any direct contribution on capacity improvement. In this situation the buyer follows an order quantity inflation strategy to increase the investment of the supplier. However when the supplier does not show the desire to make enough investment, the buyer will engage in direct investment in the suppliers capacity. As a result the supplier will not have any contribution to the investment. Our analysis demonstrates that the order inflation strategy can overshadow the effect of double marginalization. In addition, we find that a coordination contract works best when the supplier and buyer have similar profit margin profiles.

The conclusion of thesis is provided in Chapter 6. In this chapter some of the main findings of each chapter are discussed. Also we provide some ideas for further research extensions.

Note that in each chapter similar notations maybe used with different definitions. This choice is made in order to make our notation as concise as possible.

Chapter 2

Literature Review and a Classification Framework

In this chapter we survey the studies that have considered the lot-sizing problem under uncertain supply and demand. Some of the reviewed papers are in the context of production and supply chain planning with an emphasis on inventory replenishment problems. In addition, due to the importance of pricing problems and its close relation with the lot-sizing problem we have devoted a separate section to studies of joint lot-sizing and pricing problems. Furthermore, we introduce a framework to classify the literature in these areas. Our motivation for this framework came from the overwhelming task of tracing the different assumptions of the studies in this area and trying to relate them to each other. Borrowing from the fields of scheduling and queueing theory, we thought it is time that a similar classification process be developed for this field. We apply the proposed framework to categorize the reviewed papers according to their major assumptions and use it to highlight the existing gaps in the literature. Finally we analyze the different methodologies employed in these areas.

2.1 Related Review Papers

There is an extensive body of literature in the field of lot-sizing. Correspondingly the number of review papers in this context is significant and each have focused on a particular setting of the problem. Jans and Degraeve (2008) review single level lot-sizing papers. Gupta and Keung (1990) and Simpson and Erenguc (1996) focus on multi-stage settings. Single item lot-sizing studies are reviewed by Brahimi *et al.* (2006). Karimi *et al.* (2003), Quadt and Kuhn (2007) and Buschkühl *et al.* (2008) survey the related literature to capacitated lot-sizing. Gelders and Van Wassenhove (1981) tried to bridge the gap between academia and practice by reviewing some of the major contributions that have the potential to be implemented in real world problems. The joint lot-sizing and scheduling problem literature is reviewed by Elmaghraby (1978), Potts and Wassenhove (1992), Drexl and Kimms (1997), Zhu and Wilhelm (2006) and Ben-Daya *et al.* (2008). De Bodt *et al.* (1984) concentrate on the papers that have considered dynamic demand. Supply chain coordination is considered by Cachon (2003) and Robinson *et al.* (2009). Maes and Wassenhove (1988), Jans and Degraeve (2007) and Guner Goren *et al.* (2008) narrow the scope of their review based on the type of the solution algorithm. The literature of supplier selection in lot-sizing is reviewed by Aissaoui *et al.* (2007) and Tajbakhsh *et al.* (2007). The studies that emphasize on the role of quality in inventory models are reviewed by Wright and Mehrez (1998). Grosfeld-Nir and Gerchak (2004) review the context of multiple lot-sizing with random yield.

Some of the review papers focused on a specific form of uncertainty. The studies with uncertainty in the context of production and supply chain planning are reviewed by Murthy and Ma (1991), Mula *et al.* (2006) and Peidro *et al.* (2008). Due to the overlap between lot-sizing and production planning, a small portion of the lot-sizing with uncertainty literature is reviewed in these studies, but it is not their main focus.

Yano and Lee (1995), Snyder *et al.* (2010), Dolgui *et al.* (2013) and Chen and Mersereau (2013) concentrated on different forms of supply uncertainty. Yano and Lee (1995) proposed a comprehensive review of the lot-sizing literature with uncertain yield. Snyder *et al.* (2010) looked at the studies that have considered disruption in their modelling. Dolgui *et al.* (2013) reviewed the context of supply planning under lead time uncertainty. Chen and Mersereau (2013) proposed a review on the studies that have considered inaccuracy in supply and demand information.

2.2 A Classification Framework

The vast applications of inventory and pricing models creates an appropriate groundwork for researchers to investigate these problems under different settings and assumptions. Consequently we are facing a massive number of studies that each has enlightened a diminutive portion of the field of lot-sizing and pricing. This makes it challenging to track a particular class of studies that have commonality in some particular aspects.

The lot-sizing and pricing is not the only domain that suffers from this issue. A similar pattern exists in some of the conventional subjects of Operations Management. Therefore in some areas, researchers have developed a standard that concisely and conveniently identifies the major assumptions of the models being studied. In Table 2.1 we include two of the most well known standardization systems. Kendall (1953)

Table 2.1: Existing notation systems in Operations Management

Notation system	Explanation	Area
$A/S/c/K/N/D$	A : time between arrivals S : processing time of jobs c : number of servers K : capacity of queue N : size of the population of jobs D : queue discipline	Queueing
$\alpha\|\beta\|\gamma$	α : machine environment β : job characteristics γ : objective function	Scheduling

introduced a three field notation for queueing models that was later extended to six fields. Graham and Lawler (1977) proposed the three-field notation for scheduling problems.

This has motivated us to follow a similar approach and introduce a six-field $a|b|c|d|e|f$ notation system for the lot-sizing and pricing models. Table 2.2 clarifies the notation. For instance $U|U|P_F|2|1|q$ represents a two products, single supplier, periodic review with finite horizon lot-sizing problem in which the demand and supply are uncertain. In the rest of the paper we use this notation to describe the reviewed models from the literature.

Although this notation can be used to classify all the papers in the context of lot-sizing, due to the scope of this thesis, here we only use it to classify the studies that consider both uncertainties of demand and supply.

Table 2.2: Proposed $a|b|c|d|e|f$ notation system for lot-sizing and pricing problems

Field	Description	Example
a	Nature of demand	Certain (C) Uncertain (U)
b	Nature of supply	Certain (C) Uncertain (U)
c	Inventory review policy	Single period (P_1) Periodic review-finite horizon (P_F) Periodic review-infinite horizon (P_I) Continuous review (C)
d	Number of products	Single product (1) Multi-products (M)
e	Number of suppliers	Single supplier (1) Two suppliers (2) Multi-suppliers (M)
f	Decision Variables	Lot-sizing (q) Pricing (p) Both lot-sizing and pricing (q, p)

2.3 Lot-Sizing Problem with Uncertain Supply and Demand

In this section we review the studies that consider the lot-sizing problem under uncertain supply and demand conditions with no pricing decision. In Section 2.4 we review the studies that consider joint lot-sizing and pricing under the same conditions. The papers in this section are classified based on their assumptions about the number of suppliers.

2.3.1 Single Supplier ($U|U|.|.|1|q$)

In this section we review the papers that consider only one supplier. In these studies it is assumed that the firm either procures supply form an unreliable supplier or has an imperfect production system. Tables 2.3, 2.4, 2.5 and 2.6 categorize the studies in this context based on the proposed standard notation. Table 2.3 shows the studies with a single product under a single period or a continuous review cases. The studies that consider multi-periods with a finite horizon are listed in Table 2.4. papers that deal with the infinite horizon case are included in Table 2.5. Table 2.6 shows the studies that consider the multi-products. Note that some of the studies are mentioned in more than one category since they consider more than one form of a problem.

Based on the characteristics of the problem, the researchers have implemented different ways to represent the uncertainty of supply in their models. This representation can change the structure of the model and its analysis significantly. Below we discuss the different form that were used to model supply yield uncertainty.

Stochastically Proportional Yield

Assume that the firm's order quantity is Q. Under this setting the firm receives a portion of this quantity represented by YQ, where Y is a positive random variable. Yano and Lee (1995) noted that this type of representation is appropriate for systems that have a poor adaptability to random environmental changes.

Shih (1980) incorporates the concept of uncertain yield in the economic order quantity (EOQ) model and a newsvendor problem. In the first case the holding and

Table 2.3: Lot-sizing papers with a single supplier and single product in single period and continuous review cases

Problem Class	List of papers	
$U\|U\|P_1\|1\|1\|q$	• Shih (1980) • Noori and Keller (1986) • Gerchak *et al.* (1986) • Basu (1987) • Ehrhardt and Taube (1987) • Gerchak *et al.* (1988) • Henig and Gerchak (1990) • Ciarallo *et al.* (1994) • Wang and Gerchak (1996) • Hwang and Singh (1998) • Erdem and Özekici (2002) • Weng and McClurg (2003) • Inderfurth (2004a) • Gallego and Hu (2004) • Gupta and Cooper (2005) • Rekik *et al.* (2006)	• Grasman *et al.* (2007) • Chiang and Feng (2007) • Kök and Shang (2007) • He and Zhang (2008) • Wang (2009) • Xu (2010) • Arifoglu and Özekici (2010) • Schmitt *et al.* (2010) • Teunter and Flapper (2011) • Arifoglu and Özekici (2011) • Tan and Çömden (2012) • Arifoğlu *et al.* (2012) • Kaki *et al.* (2013) • Wu *et al.* (2013) • Okyay *et al.* (2014)
$U\|U\|C\|1\|1\|q$	• Noori and Keller (1986) • Moinzadeh and Lee (1987) • Moinzadeh and Lee (1989)	• Chang (2004) • Uthayakumar and Parvathi (2009)

ordering costs are minimized while the overage and shortage costs are minimized in the second case. It is demonstrated that both problems are convex with respect to order quantity. Noori and Keller (1986) consider a special case of the newsvendor problem where the uncertainty of supply is either uniform or exponential. This assumption has enabled them to find the explicit form of the optimum solution. Inderfurth (2004a) also considers a special case where both yield and demand are uniform and characterize the optimum solution based on this assumption. Rekik *et al.* (2006) extend

Table 2.4: Lot-sizing papers with a single supplier and single product in finite horizon case

Framework	List of papers
$U\|U\|P_F\|1\|1\|q$	• Gerchak *et al.* (1988) • Parlar and Gerchak (1989) • Henig and Gerchak (1990) • Ciarallo *et al.* (1994) • Parlar *et al.* (1995) • Wang and Gerchak (1996) • Erdem and Özekici (2002) • Iida (2002) • Sloan (2004) • Gallego and Hu (2004) • Gupta and Cooper (2005) • Babai and Dallery (2006) • Bollapragada and Rao (2006) • Kök and Shang (2007) • Bensoussan *et al.* (2009) • Wang and Tomlin (2009) • Arifoglu and Özekici (2010) • Cho (2010) • Arifoglu and Özekici (2011)

the work of Inderfurth (2004a) by considering a normal distribution for demand in addition to a Uniform distribution. They also model the uncertainty of the yield in both additive and multiplicative forms. Under these assumptions, they propose closed form optimum solutions. Kaki *et al.* (2013) assume that demand and yield rates are stochastically dependent. In order to show that, they use bi-variant normal distributions for both demand and yield rates. The main focus of the study is on how the the correlation between supply and demand can affect the optimal solution.

Teunter and Flapper (2011) consider a remanufacturing system that buys used products, repairs them and then sells them in a spot market. The acquired products are segmented into different classes based on their quality with a specific probability. The remanufacturing cost of each type of products increases as the quality of the product decreases. Under this situation Teunter and Flapper (2011) propose the optimum used product acquisition policy.

Table 2.5: Lot-sizing papers with a single supplier and single product in infinite horizon case

Framework	List of papers	
$U\|U\|P_I\|1\|1\|q$	• Henig and Gerchak (1990) • Tang (1990) • Ciarallo *et al.* (1994) • Ciarallo *et al.* (1994) • Wang and Gerchak (1996) • Song and Zipkin (1996) • Denardo and Lee (1996) • Denardo and Tang (1997) • Nurani *et al.* (1997) • Güllü (1998) • Özekici and Parlar (1999) • Bollapragada and Morton (1999) • Webster and Weng (2001) • Erdem and Özekici (2002)	• Iida (2002) • Bollapragada *et al.* (2004a) • Bollapragada *et al.* (2004b) • Li *et al.* (2004) • Gallego and Hu (2004) • Inderfurth and Transchel (2007) • Li *et al.* (2008) • Huh and Nagarajan (2009) • Inderfurth (2009) • Arifoglu and Özekici (2010) • Schmitt *et al.* (2010) • Arifoglu and Özekici (2011) • Inderfurth and Vogelgesang (2013) • Kutzner and Kiesmüller (2013)

Gerchak *et al.* (1988) extend the previous results to the multi-period case. They demonstrate that in both single and multi-period cases there exist a reorder point that defines the time of ordering in terms of the level of inventory. The reorder point is not affected by the variability of the yield. They also show that unlike the deterministic yield case, a base-stock policy is not necessarily optimal.

In the absence of analytical results, some studies focused on developing efficient solution algorithms. Parlar and Gerchak (1989) consider the case where the variable and holding costs are quadratic. They note that myopic algorithms are not applicable in solving the problem in a multi-period case and develop a heuristic solution. Another attempt to develop efficient heuristic solution is the study of Bollapragada

Table 2.6: Lot-sizing papers with a single supplier and multi-products

Framework	List of papers	
$U\|U\|P_1\|M\|1\|q$	• Hsu and Bassok (1999) • Abdel-Malek *et al.* (2008)	• Maddah *et al.* (2009)
$U\|U\|C\|M\|1\|q$	• Kogan (2009)	
$U\|U\|P_F\|M\|1\|q$	• Proth *et al.* (1997)	• Qinghe *et al.* (2001)
$U\|U\|P_I\|M\|1\|q$	• Gong and Matsuo (1997) • Duenyas and Tsai (2000)	• Grasman *et al.* (2008)

and Morton (1999). By using different approximations, they develop three heuristic algorithms based on the first order optimality conditions to solve the infinite horizon case. They report that their algorithm performs well when compared to the optimal solution. In a technical note, Inderfurth and Transchel (2007) highlight that the optimality conditions that are used by Bollapragada and Morton (1999) are not correct. Moreover they explore the effect of this mistake on the results by some numerical examples. Li *et al.* (2008) also consider the same problem and found some upper and lower bounds for the optimum solution. Based on these bounds, they develop a heuristic algorithm that is able to outperform the proposed algorithm by Bollapragada and Morton (1999) in many cases. Huh and Nagarajan (2009) implement the concept of linear inflation rules (LIR), introduced by Zipkin (2000), to develop a heuristic algorithm for the problem. Their algorithm is based on picking the best candidate from LIR policies. Abdel-Malek *et al.* (2008) investigate the capacitated newsvendor problem in which a common budget is used for the production of M products. They develop a Newton based algorithm to find the optimum solution.

Instead of minimizing costs, Gerchak *et al.* (1986) model the problem as the maximization of the profit in a single period case. Based on the dependency of the variable cost on the yield rate, they develop two models. They demonstrate that under both cases the objective functions are concave, but a base stock policy is not optimal. Ehrhardt and Taube (1987) show more explicit results by characterizing the demand with uniform and beta distribution functions. Hsu and Bassok (1999) extend the previous works by considering the multi-product case. They demonstrate that the model can be decomposed into a network flow problem and propose three different algorithms for solving the problem. Maddah *et al.* (2009) assume that based on the quality of the outputs, the products are segmented into two groups with independent stochastic demand. They prove that the objective function is concave and find a unique optimum solution.

Gupta and Cooper (2005) question the common perception that a higher yield rate results in higher expected profit. They indicate that under some conditions, a stochastically larger yield results in lower expected profit. This result is proved for both single period and multi-period cases. They also show conditions under which a better yield rate results in higher profits.

Chiang and Feng (2007) consider a manufacturer who provides a single product for a set of independent retailers. Using a simulation based approach, they show how information sharing can improve the coordination in this two echelon supply chain. The concept of coordination in a two echelon system when the uncertainty of supply is stochastically proportional is also investigated by He and Zhang (2008), He and Zhang (2010), Xu (2010) and Wang (2009). They all first assume that each party decides independently, under Stackelberg game setting, and then considered a centralized form of the problem. Both the supplier and the retailer have to decide about the order/production quantity considering the randomness of yield and demand. He and Zhang (2008) demonstrate the unimodality of the objective functions under different settings. Xu (2010) extends the previous work by considering a secondary market for the supplier. Wang (2009) adds the concept of vendor managed inventory into the model and demonstrates how this type of contract enhances the performance of the supply chain. Cho (2010) focuses on an influenza vaccine supply chain. He assumes that the government is responsible for providing sufficient influenza vaccines considering the supply uncertainty of the manufacturer and the uncertain behaviour of the customers. The problem is a three echelon case where the manufacturer decides about the production quantity, government decides about the order quantity and the supply policy and the target population decides whether to get the vaccine shot. The problem for the government is a finite horizon lot-sizing problem in which decisions are made according to information updates in each period. Arifoğlu *et al.* (2012) consider a same problem under different settings. They assumed that the population are divided into two groups based on their infection dis-utility. This segmentation shows the priority of getting the influenza shot. The main focus of this study is on how the uncertainty in supply and the behaviour of the people can affect the efficiency of the vaccine supply chain.

Some of the studies have incorporated the concept of yield uncertainty into more complicated production systems. Inderfurth (2009) incorporates the concept of stochastically proportional yield in the materials requirement process (MRP). He focuses on policies for mitigating the effects of uncertainty. He shows that linear order rules keep the state of the system steady despite yield and demand variations. A capacitated multi-product lot-sizing problem in a multi-stage manufacturing system is explored by Gong and Matsuo (1997). The goal is to satisfy the demand for each product considering the limited capacity of each stage in the production system.

A small number of studies has been devoted to models with a continuous review policy. Kogan (2009) considers a manufacturer that has to decide about the production rate while the yield and demand rates are functions of time. Chang (2004) investigates an EOQ model with a fuzzy yield rate and demand. By implementing the fuzzy estimate of the objective function he develops the optimum policy and determines when the proposed fuzzy model can be reduced into the basic EOQ model.

Uncertainty in Capacity

Under this setting it is assumed that the production/ordering process has a random capacity. Let C represent the random capacity of the system. If the ordering quantity is Q, then the received quantity is equal to $\min\{C, Q\}$.

Ciarallo *et al.* (1994) consider such a system in single period, finite horizon and infinite horizon cases. In their problem, a machine is responsible for production of a certain product and it may be down for a portion of a period. This unavailability results in an uncertain and limited capacity. They prove that unlike in the stochastically proportional yield case, a base stock policy is optimal for such a problem. Iida (2002) defines an upper bound and a lower bound for the base stock level in this problem. It is indicated that these bounds converge as the number of periods increases. Bollapragada *et al.* (2004b) assume that if a portion of the order is not delivered in its corresponding period, due to a lack of (uncertain) capacity, then it would be delivered in the next period. They also consider a customer service level constraint. Bollapragada *et al.* (2004b) explore the problem under the assumption that the firm follows a base stock policy. Assuming quasi-concavity of the demand distribution,they show that the problem results in a convex programming. Instead of minimizing the total discounted cost, Güllü (1998) minimizes the average cost under the base stock inventory replenishment policy. The novel formulation approach allowed for mapping the problem to a G/G/1 queueing problem and subsequently establishing several results related to the performance of the system.

A newsvendor problem with uncertain capacity and a risk averse decision maker is investigated by Wu *et al.* (2013). They propose two models based on the concepts of Value at Risk (VaR) and Conditional Value at Risk (CVaR). It is shown that although in a risk neutral case the optimum quantity is not affected by the uncertainty of capacity, this value may significantly change by incorporating VaR and CVaR measures in the model.

Bollapragada *et al.* (2004a) consider a multi-echelon assembly system with a customer service level constraint that follows a base stock policy for all the raw materials. They are able to develop a decomposition based heuristic algorithm to find a near optimal solution for the problem. The problem of a multi-stage production system with uncertain capacity in every stage is considered by Hwang and Singh (1998). They assume that running the production on each stage includes a fixed setup cost, in addition to the variable cost. Hwang and Singh (1998) prove that in each stage n the optimal production policy is characterized by two critical values (s_n, S_n). If the input is less that s_n the production process is halted at that stage. If it is more than S_n we only produce S_n products. Otherwise all the received materials will be produced.

Qinghe *et al.* (2001) consider a supply chain planning problem. The firm has to decide about the production quantities of different products as well as the volume of transportation to each customer. Instead of limiting the production capacity, they assume that the storage capacity is limited. They assumed that the uncertainty of capacity is normally distributed while the uncertainty of demand is fuzzy. A genetic algorithm is developed to solve the problem.

Bernoulli Production Process

In this situation the probability of producing a good unit is p. So generally in Q produced units, the probability that n products are good follows a binomial distribution and is equal to $\binom{Q}{n}p^n(1-p)^{Q-n}$.

Tang (1990) considers a serial production system with a binomial yield uncertainty in each stage. He introduces the concept of a restoration rule in order to find an efficient manufacturing policy. Denardo and Lee (1996) extend the previous work by considering the possibility of rework and uncertain processing time. Denardo and Tang (1997) consider the same problem and show that restoration rules keep the system stable even under highly volatile demand and yield.

Grasman *et al.* (2008) consider a multi-product system where products are produced on a single machine. Instead of discarding the scrap products, they are returned to the production line for rework. The objective of the study is to find the optimum base stock level that minimizes the costs of production.

Disruption Risk

Under some circumstances there is a possibility that the whole supply system get disrupted. In this situation usually the supplier is not able to provide products until the system got repaired. The optimization models in this context aim to find the optimal replenishment policy by considering the probability of disruption.

Most of the studies in this line of research usually consider the possibility of having more than one supplier to mitigate the disruption risk. These studies are reviewed in Section 2.3.2. One of the few studies that consider this setting with only one supplier is that of Li *et al.* (2004). They consider an infinite horizon case of the problem and implement the concept of renewal theory to minimize the total cost. It is demonstrated that a failure rate dependent base-stock policy is optimum for the problem.

State Dependent Uncertainty

In all the aforementioned studies it is assumed that the uncertainties of supply in different periods are independent. However in many situations the state of the system dictates the distribution of the supply uncertainty. It is common to consider that the state of the system follows a Markov chain. Any of the aforementioned structure can be used to model the uncertainty of yield. Nevertheless the state dependency of the yield uncertainty affects the analytical and numerical investigation of the problems significantly.

Parlar *et al.* (1995) consider disruption risk with a Markovian property. They assume that the probability of the disruption depends on the state of the system. By considering a fixed ordering cost, they prove that the optimal inventory replenishment policy has a (s, S) structure. Özekici and Parlar (1999) extend the previous research by assuming that not only the supply, but also demand uncertainty depends on the state of the system. When there is no fixed ordering cost, they show that a base-stock policy is optimal. They also present the conditions that guarantee the optimality of the (s, S) policy under the presence of a fixed ordering cost. A variant of the previous model with uncertain capacity is discussed in the study of Erdem and Özekici (2002). The optimality of the base-stock policy for both finite and infinite horizon is proved.

Gallego and Hu (2004) consider two Markov chains; one defines the distribution of demand and the other defines the distribution of the yield uncertainty. The uncertainty of yield is assumed to be stochastically proportional. They assume that the capacity of the supplier is limited and determine that in the finite horizon case a state dependent inflated base stock policy is optimal. they also find conditions that ensure the optimality of this policy the infinite horizon case. Arifoglu and Özekici (2010) extend the previous work by assuming that the state of the system is partially observable. They prove that a state dependent inflated base-stock policy is still optimal. Arifoglu and Özekici (2011) consider a variant of the Arifoglu and Özekici (2010) model by assuming the uncertainty of supply exists in the capacity of the supplier. The optimality of a state dependent base-stock policy is established under such asituation. They also show that a state dependent (s, S) policy is optimal in the case that there is a fixed ordering cost and the system may be disrupted.

Song and Zipkin (1996) model the uncertainty in replenishment lead time when it is state dependent. They prove the optimality of the base-stock policy in their model.

Instead of an unreliable supply system, some of the studies assume that the inventory level information is inaccurate. Kök and Shang (2007) consider a manufacturing system that needs to conduct inspection to assess the inventory level, but as time passes the information looses its accuracy. The reliability of the information is state dependent and is defined by the time that has passed from the last inspection. The firm has to decide about the order quantity and the frequency of inspection. They introduce an inspection-adjusted base-stock policy and proved its optimality. Bensoussan *et al.* (2009) consider a situation where that the most recent information available to the inventory manager is dated. They assume that the information delay time is stochastic. Bensoussan *et al.* (2009) prove that in the absence of a fixed ordering cost, a base-stock policy is optimal while in its presence, the optimal inventory policy has an (s, S) form.

Webster and Weng (2001) explore the situation where there may be inaccuracy in forecasting the demand and production yield. The forcasted quantity in each period is a function of the last period data, so it can be considered as a state dependent system. The performance of the system is analyzed under such situation.

Sloan (2004) consider a manufacturing system with deterioration in its performance. The probability of producing good products is based on the state of the system. Considering the maintenance cost, the firm decides about the maintenance schedule and production quantity. Sloan (2004) determines the conditions that ensure the existence of a threshold for both production and maintenance in the optimal policy.

Other Forms of Supply Uncertainty

In this section we review the papers that implemented a rather distinguished form of uncertainty into their models or applied a combination of the aforementioned structures.

The paper of Henig and Gerchak (1990) is an extension of Gerchak *et al.* (1986) study with a general form of yield uncertainty. They also assume that holding, shortage and variable costs have a general form. They proved that in the optimal inventory policy there exists a re-order point that determines the order time based on the level of inventory. Grasman *et al.* (2007) consider a single period case of the problem and extend the results of Henig and Gerchak (1990). A general form of yield uncertainty is also considered by Bollapragada and Rao (2006). They assume that the supplier may split an order and deliver it in different periods. The demand is assumed to be normally distributed and has to be satisfied with a specific probability in each period. Without a service level constraint, the problem is proved to be a convex programming. In addition they develop a heuristic algorithm to solve the problem in the general case. The application of a general yield uncertainty in a continuous review system is investigated in the study of Uthayakumar and Parvathi (2009). They assume that the firm has the opportunity to reduce the setup cost, leadtime and variability of yield by investment. By proving the convexity of the objective function, they develop an algorithm to find the optimum solution.

Wang and Gerchak (1996) consider a stochastically proportional uncertainty in both the capacity and the output rate. They conclude that the objective function is quasi-convex in both the finite and infinite horizon cases and as a result the optimal policy is of a re-order point type. Okyay *et al.* (2014) also examine the coexistence of uncertainty in both the capacity and the output rate in a single period case. They assume that the demand and yield uncertainty are related and find the conditions under which the optimum solution is unique. It is indicated that with only stochastically proportional yield the objective function is concave. Considering only uncertainty in capacity, the unimodality of the objective function depends on the conditional distribution of yield with respect to demand.

Noori and Keller (1986), Moinzadeh and Lee (1987), Moinzadeh and Lee (1989), Babai and Dallery (2006) investigate the (r, Q) inventory system with uncertain supply. Noori and Keller (1986) use the concept of conditional expected value and variance to model the uncertainty in the problem. They propose an iterative approach to find the optimum solution. Moinzadeh and Lee (1987) assume that the demand arrivals follow Poisson distribution. Considering the inventory system as a markov chain, they develop an approximation for the objective function in order to simplify the solving procedure of the problem. Moinzadeh and Lee (1989) extend the previous study by considering a more general form of demand. They also assume that the order would be split randomly into two shipments with known lead time. Same as the previous study, they propose and approximation to simplify the analysis of the problem. Babai and Dallery (2006) categorize the optimum solution when the lead time is stochastic. They also consider an additive noise for delivered quantity that is normally distributed.

Wang and Tomlin (2009) consider a firm which provides her supply from an overseas manufacturer with uncertain delivery time. They develop a multi-period model in which the products would be sold in the last period. Wang and Tomlin (2009) assume that if the products are not delivered before the final period, the demand would be lost. The firm has to decide about the order quantity as well as the time of ordering. As the time passes the firm gets more accurate information about the demand however ordering late increases the probability of not receiving any product. Weng and McClurg (2003) consider a relation of a supplier and a retailer when the delivery time of the supplier is uncertain. The structure of the proposed problem is of a Stakelberg game in which the retailer is the leader. Initially the retailer decides about the order quantity and the supplier has a chance to whether accept or reject the offer. They look at both coordinated and uncoordinated cases. The effect of uncertain lead time in an assembly system is examined in the paper of Proth *et al.* (1997). They consider an assembly system that produces M product from N components and the delivery time of each component is stochastic. The firm has to decide about the order quantity of each component as well as the production quantities of the products. Due to the complexity of the problem, they propose a heuristic algorithm to solve the problem.

Schmitt *et al.* (2010) explore a retailer inventory policy with an unreliable supplier and normally distributed demand. They assume that the received quantity by the retailer follows a normal distribution with the mean value of her order quantity and a fixed standard deviation. Also they assume that the supplier may be disrupted. Under such situation the closed-form optimum order quantity can not be found so they use approximation to get a better understanding of the solution.

Nurani *et al.* (1997) model a single stage manufacturing system that may go out of control based on a geometric distribution. The customers enter the system according to a Poisson distribution and the production time follows an exponential distribution. The firm does not realize that the process is not under control until a customer is served with a defected item. When the customers' demand is satisfied based on a FIFO policy, the production strategy is of a re-order point type. Under a LIFO policy the re-order point is characterized based on the number of grey items (the products that may be faulty). Kutzner and Kiesmüller (2013) also assume the system may go out of control, but unlike Nurani *et al.* (1997) it can be realized only when an inspection is conducted. They consider a binomial distribution for the number of faulty item where the probability of producing defective products increases when the system is out of control. They model the problem using renewal theory and use an approximation to simplify the analysis. The numerical analysis shows the promising performance of the approximation.

Duenyas and Tsai (2000) consider a production system where the output is graded into different qualities with known and constant probabilities. Demand for quality levels are uncertain and independent. They assume that the products have downward substitutability. Duenyas and Tsai (2000) propose a heuristic algorithms to solve the problem and demonstrated the efficiency of the algorithm numerically.

The focus of Inderfurth (2009) study is on determining a level of safety stock that guarantees the proper service level. They consider stochastically proportional, binomial and interrupted geometric yields and introduced different approaches to calculate the safety stock under each case.

2.3.2 Two Suppliers ($U|U|.|.|2|q$)

In this section we review the studies that consider two sources of supply for the firm. Under such situations, the firm has to decide about the order quantity from each source. Table 2.7 classifies the studied papers in the context of dual-sourcing in

Table 2.7: Lot-sizing papers with two suppliers in single period case

Framework	List of papers	
$U\|U\|P_1\|1\|2\|q$	• Anupindi and Akella (1993) • Parlar and Wang (1993) • Gurnani *et al.* (1996) • Swaminathan and Shanthikumar (1999) • Gurnani *et al.* (2000) • Jones *et al.* (2001) • Kouvelis and Milner (2002) • Kazaz (2004) • Zikopoulos and Tagaras (2007) • Ketzenberg *et al.* (2009) • Mukhopadhyay and Ma (2009)	• He and Zhang (2010) • Iakovou *et al.* (2010) • Tiwari *et al.* (2011) • Sting and Huchzermeier (2012) • Xanthopoulos *et al.* (2012) • Hong *et al.* (2013) • Hu *et al.* (2013a) • Hu *et al.* (2013b) • Ma *et al.* (2013) • Cho and Tang (2013)
$U\|U\|P_1\|M\|2\|q$	• Saghafian and Van Oyen (2012)	• Inderfurth (2004b)

single period case. Also Table 2.8 shows the papers with dual supplier in multi-period case. We classify the papers based on the characteristics of the suppliers and timing of the decisions.

Simultaneous ordering from two suppliers

In such studies it is assumed that the firm is dealing with two suppliers, usually both unreliable, and sends the orders to suppliers simultaneously.

Parlar and Wang (1993) address a single period problem with stochastically proportional yield under EOQ (deterministic demand) and newsvendor (stochastic demand) settings. They prove the concavity of the objective function under both settings. Anupindi and Akella (1993) analyze the problem under three settings. In the first model the whole order quantity is either delivered in the period of study (with

Table 2.8: Lot-sizing papers with two suppliers in multi-period case

Framework	List of papers
$U\|U\|P_F\|1\|2\|q$	• Simpson (1978) • Anupindi and Akella (1993) • Gurnani *et al.* (1996) • Inderfurth (1997) • Swaminathan and Shanthikumar (1999) • Wang and Fang (2001) • Kiesmüller and Minner (2003) • Karabuk and Wu (2003) • Hu *et al.* (2008) • Pac *et al.* (2009) • Chen *et al.* (2012) • Kouvelis and Li (2013)
$U\|U\|P_I\|1\|2\|q$	• Tomlin (2006) • Ketzenberg *et al.* (2009) • Ahiska *et al.* (2013)
$U\|U\|P_F\|M\|2\|q$	• Meybodi and Foote (1995)

probability of β) or in the next period (with the probability of $1-\beta$). Such situation may happen when there is an uncertainty in the timing of delivery. In the second model a portion of the order quantity is delivered and the rest is discarded. In the third model, a portion of the order is delivered in the period of study and the rest is delivered in the next period. In each model they found the optimal ordering policy for both single and multi-period cases. They proved that the optimal policy in all the situations is of a re-order type. Swaminathan and Shanthikumar (1999) consider the first model presented by Anupindi and Akella (1993) and analyze it when the distribution of the yield is discrete. They demonstrated that the same re-order policy is optimum for inventory replenishment.

Iakovou *et al.* (2010) consider a dual-sourced problem when there is a risk of disruption for both suppliers. Based on the severity of the disruption, the suppliers may still be able to provide a portion of the demand. It is proved that the objective function is concave and optimality conditions are necessary and sufficient for the optimum solution. In a capacitated variant of the problem, they demonstrate the sufficiency of KKT conditions for the optimum solution. Xanthopoulos *et al.* (2012) extend the previous study by considering a service level constraint. They demonstrate that the resulting model is a convex programming.

Zikopoulos and Tagaras (2007) consider the problem in a reverse supply chain. They assumed that the uncertainty of the suppliers are stochastically proportional and may be related. Initially the firm decides about the order quantity from each supplier and after realization of the yield, determines the refurbishing quantity. It is demonstrated that the optimum solution of the problem is unique.

The problem of supplier selection in an assembly system is addressed by Gurnani *et al.* (1996). They assume that the final product requires two components that can be supplied by two separate suppliers or by one joint supplier. The structure of yield uncertainty is defined in a similar way to that of the first model of Anupindi and Akella (1993). They noted that the firm would order from individual suppliers only when the inventory level is less than a specific threshold. A variant of the problem is considered by Gurnani *et al.* (2000) in which the uncertainty of the suppliers are stochastically proportional. They show that the objective function is concave.

In some cases, in addition to an unreliable supplier, the firm has the opportunity to satisfy some of the demand from a reliable supplier. Usually the variable cost of the reliable supplier is higher. Hu *et al.* (2013b) assume that the uncertainty of the unreliable supplier is stochastically proportional as well as it may be disrupted. In addition to a centralized system, they explored the decentralized form of the problem under a Stackelberg game setting. They establish the uniqueness of the optimum solution in both cases. Hong *et al.* (2013) assume that in addition to the unreliable supplier (stochastically proportional), the firm has the option of sourcing from spot market with an uncertain acquisition price. They assumed that the firm is risk averse, so that in addition to maximizing the profit it is also desired to minimize its variance. They assume that yield, demand and the spot market price are normally distributed and correlated.

The finite horizon version of the problem is explored by Meybodi and Foote (1995), Wang and Fang (2001), Pac *et al.* (2009) and Chen *et al.* (2012). Pac *et al.* (2009) assume that the uncertainty of supply is stems from the uncertainty of the workforce. The firm has both permanent workforce that may be temporarily unavailable and contracted workforce that are always available. They consider inventory and work force planning simultaneously in order to minimize the cost. Chen *et al.* (2012) assume that one of the suppliers may be disrupted while the capacity of the reliable supplier is limited. The focus of the study is on minimizing the holding, shortage, variable and ordering costs. They implement the concept of CK-convexity to characterize the optimal inventory policy. Meybodi and Foote (1995) considers the problem of production planning and scheduling simultaneously. A goal programming approach is implemented to minimize the total production cost, workforce instability and customer dissatisfaction. The coexistence of in-house production and subcontracting gives a dual-sourcing structure to the problem. A heuristic algorithm is proposed to solve the problem. Wang and Fang (2001) focus on finding the optimum aggregate production planning by minimizing the production costs and workforce instability. Their model determines the regular time production quantity, overtime production quantity, outsourcing quantity, inventory and backorder levels and hiring and firing policies. They assume that production price, subcontracting cost, work force level, production capacity and market demand are fuzzy in nature.

For the infinite horizon case, Ahiska *et al.* (2013) assume that one of the suppliers may be disrupted while the other one is perfectly reliable. The uncertainty of the unreliable supplier is modelled with a Markov process with two states, up and down. The only time that the unreliable supplier accepts the order is when she is in the up state. They conduct an extensive numerical analysis of the problem and demonstrate how the sourcing strategies may change. Due to the presence of fixed ordering cost, the (s, S) policy is the optimal ordering strategy for the problem.

Presence of a backup supplier

Under this setting, based on a forecasted demand, the firm orders from an unreliable supplier. After a certain amount of time and observing the state of the uncertain parameters (at least partially), the firm has a chance to make another order from the same or a backup supplier.

Tiwari *et al.* (2011) consider a two-stage newsvendor problem with stochastically proportional yield. At time zero the firm decides about the order quantity, considering the uncertainty of supply and demand. After realization of the yield and demand, the firm has a chance to order again. The second order also suffers from yield uncertainty. They explore the optimum ordering policy with an emphasis on uniformly distributed yield.

Karabuk and Wu (2003) consider capacity planning in the semi-conductor industry. In their model, in each period the firm has to decide about capacity expansion based on the forecasted demand and unreliability of the implemented technology. Capacity expansion is a time consuming process, so the firm may use outsourcing for sudden fluctuations of demand. In order to solve the model, the authors propose three different ways to approximate the optimum strategy and analyze them numerically. Another configuration of the capacity planning problem is examined by Sting and Huchzermeier (2012). At the beginning of the first stage the buyer decides about the order quantity from the unreliable supplier. It is assumed that the capacity of the supplier is uncertain. Meanwhile she has to decide about the capacity that she wants to reserve on the backup supplier. After realization of yield and demand, she decides about the order quantity from the backup supplier which is limited to the predefined capacity. They demonstrate the relation of the optimal sourcing strategy with the marginal cost of ordering. They also extend the model to the case that demand and supply uncertainty are correlated. Tomlin (2006) considers an unreliable supplier with production lead time and the possibility of disruption that can be modelled with a Markov process. In addition a reliable backup supplier, with an instantaneous delivery time and the possibility of capacity expansion, is considered. The backup supplier requires a certain amount of time to increase her capacity. Tomlin (2006) addresses carrying excessive inventory, ordering from the reliable supplier and passive acceptance as the three strategies that the firm may use to alleviate the risk of the system. The main focus of the paper is on the conditions that each strategy seems to be more attractive for the firm. Saghafian and Van Oyen (2012) consider a variant of Sting and Huchzermeier (2012) model. The firm is dealing with a dedicated and a backup supplier who are susceptible to disruption. At the beginning of the first period, the firm decides about the capacity reservation on the backup supplier according to her prediction of the dedicated supplier state. After realization of the state of the dedicated supplier (which determines the probability of his disruption) the firm puts an order for both suppliers. The authors explore the optimum solution for both single and two products cases.

Kouvelis and Li (2013) look at the problem of supplying from an off-shore supplier. The production process of the supplier is unreliable and is modelled in a stochastically proportional form. After realization of the yield, the firm can send an emergency order to compensate the shortfall. In addition, after the production process, the firm has to choose between two transportation modes. One of the modes is slow but cheap and the other one is fast but expensive. After characterizing the optimum solution of the problem, the main focus of the paper is on analyzing the effect of uncertainty on the optimum solution.

Jones *et al.* (2001) and Kazaz (2004) study the problem in an agricultural environment. Jones *et al.* (2001) develop a two period model for production of hybrid seed corn. The vendor decides about the amount of production in the first period and after realization of yield, they have a chance to reproduce to satisfy the unmet demand. They demonstrate that the objective function in each period is concave and the optimum solution is unique. Kazaz (2004) consider the production of olive oil. The producer has the chance to either produce the fruits herself or buy them from other farmers. They assumed that the procurement happens after realization of the yield in the first stage and acquisition price is a function of yield. They present a closed-form solution for the second stage problem and show that the objective function of the first stage is concave.

Hu *et al.* (2008) consider a firm with two production plants that are producing the same product facing an uncertain demand. The plants have limited uncertain capacity and are following a base stock policy. At the beginning of the period the firm decides about the production quantity in each plant. At the end of the period, after realization of capacity and demand, the firm has to decide about the transshipment of inventory between facilities. They prove that even if there is no immediate need, it is sometimes profitable to ship the materials to the facility with a lower holding cost.

Some studies consider two echelon models in which the supplier has more than one way of providing the products. The work of He and Zhang (2010) is an extension that of Xu (2010) by considering the dependency of a second market selling price on the yield rate and an emergency order for the supplier. Hu *et al.* (2013a) consider a special contract between the supplier and retailer called a flexible order quantity contract. Instead of an order quantity, the retailer gives an upper bound and a lower bound to the supplier. If the produced quantity of the supplier is less than the lower bound, he has to compensate the shortage by an emergency order to raise the quantity to the lower bound. On the other hand if he produces more than the upper bound, only the amount of products equal to the upper bound is send to the retailer. On the other cases the exact produced amount is shipped to the retailer. They analyze both centralized and decentralized cases of the model and demonstrate that such policy can improve the performance of the supply chain. Cho and Tang (2013) investigate the relation of a manufacturer and a retailer where the manufacturer sets a wholesale price and the retailer specifies her order quantity accordingly. After realization of yield and demand, if there is any excess inventory on manufacturer side, the retailer may order for the second time with the new set price by the manufacturer. Cho and

Tang (2013) also analyze two other cases that retailer sets her ordering quantity either before realization of the yield or after. They conclude that delaying the ordering does not necessarily benefits the retailer. In addition they show that in the centralized case the volatility of demand may be desirable although this is not the case for yield volatility. Ma *et al.* (2013) focus on the relation of a manufacturer and a distributor. The manufacturer produces a product by two components where the yield of one of which is uncertain with a stochastically proportional form. If there is a shortage in the manufacturer production, the rest of the distributor order would be satisfied from the spot market with an uncertain price. They analyzed the problem under a VMI arrangement with an emphasis on the role of the spot market on the optimum solution.

Manufacturing/remanufacturing systems

The dual-sourcing structure is very common in manufacturing/remanufacturing systems. In such systems, in addition to producing new products, the firm satisfies a part of demand by gathering and refurbishing the used materials. Usually there is a huge uncertainty in the quality of the used products, so most of the studies in this line of research lies in the context of uncertain supply lot-sizing. The differences between manufacturing new products and remanufacturing used materials gives the structure of dual-sourced problems to such studies.

Simpson and Erenguc (1996) explore the problem under a finite-horizon periodic review setting. They assume that the number of returned products is random, however there is no flaw in producing new products. In each period the firm decides about the production quantity of new products, remanufacturing quantity and the number of used material to be scrapped. They prove that the optimum policies for both manufacturing and remanufacturing quantity is in the form of base-stock. Inderfurth (1997) extends the previous research by considering lead time. He proves that when the lead times of remanufacturing and production processes are identical, a same order-up-to policy is optimal. Kiesmüller and Minner (2003) consider the same problem as Inderfurth (1997) and found more explicit results. In another study Inderfurth (2004b) considers a situation where new products cannot be substituted by remanufactured items while there is an opportunity to satisfy the demand of remanufactured items by new ones.

Mukhopadhyay and Ma (2009) assume that the uncertainty in remanufacturing products is stochastically proportional. They consider two cases where the production quantity of the new materials can be determined either after realization of the yield or before. They explore the optimum solution in each case and propose its closed-form when yield and demand are normally distributed. Ketzenberg *et al.* (2009) look at the value of information in such a problem and how investing information sharing can improve the performance of the system.

2.3.3 Multiple Suppliers ($U|U|.|.|M|q$)

A small portion of the literature has been devoted to the study of multiple suppliers lot-sizing problem with uncertain supply. This problem is also referred to as supplier selection in the lot sizing literature. Table 2.9 lists the studies in this context.

Dada *et al.* (2007) analyze the problem when the capacities of the suppliers are uncertain and dependent on the order quantity. They concluded that , generally , the cost of production dominates the reliability when it comes to indexing suppliers for

Table 2.9: Lot-sizing papers with more than two supplier

Framework	List of papers
$U\|U\|P_1\|1\|M\|q$	• Chen *et al.* (2001) • Lin and Chen (2003) • Yang *et al.* (2007) • Dada *et al.* (2007) • Federgruen and Yang (2008) • Federgruen and Yang (2009) • Burke *et al.* (2009) • Tan and Çömden (2012) • Merzifonluoglu and Feng (2014)
$U\|U\|P_F\|1\|M\|q$	• Petrovic *et al.* (1998) • Petrovic *et al.* (1999) • Federgruen and Yang (2011) • Tan and Çömden (2012)
$U\|U\|P_I\|1\|M\|q$	• Ferrer and Whybark (2009)
$U\|U\|P_1\|M\|M\|q$	• Sakawa *et al.* (2001)

selection. Federgruen and Yang (2008) assume that the uncertainty of the suppliers are stochastically proportional and there is a fixed ordering cost. They characterize the optimum solution in both identical and non-identical suppliers cases. Federgruen and Yang (2009) extend their previous research by including a chance constrained approach for characterizing the risk of demand unsatisfaction. They also demonstrate the more important role of cost when comparing the reliability of suppliers. In another extension, Federgruen and Yang (2011) consider the problem in a multi-period case. They show that the replenishment of inventory follows a reorder point policy. In term of supplier selection, the results are same as the study of Dada *et al.* (2007). In addition they show that the number of selected suppliers in each period is a decreasing function of the initial inventory. Burke *et al.* (2009) consider the case where the demand is uniformly distributed. They show the concavity of the objective function and emphasize on the role of cost as the main driver of supplier selection.

Yang *et al.* (2007) focus on developing an efficient algorithm to solve the problem when uncertainty of the suppliers' yields is stochastically proportional. They selected a Newton-based search procedure as the basis of their proposed algorithm. Merzifonluoglu and Feng (2014) consider the case when the suppliers are capacitated and the yield rates normally distributed. A newsvendor setting is proposed to model the single period case of the problem. Three different approaches are proposed to solve the problem: a linear relaxation, a branch an bound algorithm (to solve the problem optimally) and a heuristic algorithm (to solve the problem efficiently).

Chen *et al.* (2001) consider a manufacturer that decides about order quantity and the inspection quantity received from each supplier. If any defective item is observed, it can be repaired and sold in the spot market. They characterize the optimum ordering and inspection policy.

Tan and Çömden (2012) focus on the application of the problem in an agricultural environment. They consider a firm working under a production contract and has to decide about the time of seeding and the acreage they want to put under cultivation. They assume that the uncertainty of yield has a normal distribution. The model has been extended to the multi-period and multi-supplier case.

Petrovic *et al.* (1998), Petrovic *et al.* (1998), Sakawa *et al.* (2001) and Lin and Chen (2003) consider supply chain planning with uncertain supply. Petrovic *et al.* (1998) and Petruzzi and Dada (1999) propose a simulation based model to analyze the performance of a supply chain with multiple suppliers that use a base-stock policy. They assume fuzzy demand and supply. Sakawa *et al.* (2001) consider a manufacturer with a set of plants that needs to satisfy the demands in different regions. The firm has to decide about the production and transshipment quantities. They assume fuzzy capacity and demand. Lin and Chen (2003) consider a supply chain including a set of manufacturers, a single distributor and a set of retailers. They propose a model which minimizes the total production and ordering costs and develop a genetic algorithm to solve it.

Ferrer and Whybark (2009) attempt to develop a general structure for the inventory control of ramanufacturing systems that highly suffer from uncertainty of supply. They consider a firm that buys used products, disassemble them, finds the functional components, reassembles and sells them in the spot market. They use a similar structure to the materials requirement planning (MRP) method, so the proposed approach is simple to understand and implement.

2.4 Joint Lot-Sizing and Pricing with Uncertain Supply and Demand

Successful businesses need to integrate their internal cross-functional operations before they can coordinate their external supply chain operations (Braunscheidel and Suresh (2009) and Stevens and Graham (1989)). One such important area of cross-functional integration is sales and operations planning where there is a need for going beyond simple data flow integration to systematic joint optimization (Oliva and Watson (2011) and Van Landeghem and Vanmaele (2002)). Thus, one would expect an interest studying joint lot-sizing and pricing. Nevertheless , the studies that consider uncertain supply and demand simultaneously are almost rare. In this section we review the literature on joint lot-sizing and pricing optimization. Table 2.10 shows the reviewed studies.

Table 2.10: Joint lot-sizing and pricing papers

Framework	List of papers	
$U\|U\|P_1\|1\|1\|p,q$	• Li *et al.* (2009) • Cai *et al.* (2009) • Pan and So (2010)	• He (2013) • Surti *et al.* (2013)
$U\|U\|P_F\|1\|1\|p,q$	• Li and Zheng (2006) • Chao *et al.* (2008)	• Feng (2010) • Zhu (2013)
$U\|U\|P_I\|1\|1\|p,q$	• Li and Zheng (2006) • Chao *et al.* (2008)	• Feng (2010)
$U\|U\|P_1\|1\|2\|p,q$	• Xu and Lu (2013)	
$U\|U\|P_F\|1\|2\|p,q$	• Yan and Liu (2009)	• Zhou and Yu (2011)
$U\|U\|P_1\|M\|2\|p,q$	• Shi *et al.* (2011)	• Kazaz and Webster (2011)
$U\|U\|P_F\|1\|M\|p,q$	• Feng and Shi (2012)	

He (2013) considers a single period newsvendor problem with pricing and uncertain supply. He assumes that the order quantity and pricing decisions are taken sequentially and developed two models based on the order of decision making. He (2013) assumes that the uncertainty of supply is stochastically proportional and the demand function has a linear form with additive uncertainty. He proposes an enumerative procedure to determine the optimal solution and demonstrates the conditions that guarantee the uniqueness of the optimum solution. Surti *et al.* (2013) propose a similar model to He (2013) and provide results for both the simultaneous and the sequential decisions cases. They also allow the error in the demand function to be additive as well as multiplicative. Xu and Lu (2013) model a variant of the He (2013) problem where the decisions are made simultaneously and the firm compensates the unsatisfied demand by an emergency order. They also concentrate on the conditions that the objective function is unimodal.

Under a multi-period setting, Li and Zheng (2006) propose a model to maximize the profit when the unsatisfied demand is backordered and the yield is stochastically proportional. They indicate that unlike the certain yield case (Federgruen and Heching, 1999), base-stock list-price policy is not optimum. Chao *et al.* (2008) explore the problem for the case that the capacity is uncertain, stochastically increasing and concave. They also consider a linear price sensitive demand with additive uncertainty. They prove the optimality of base-stock list-price policy under the mentioned assumptions. Feng (2010) extends the previous work by considering a general demand response function, a more general penalty and salvage cost and unrestricted capacity uncertainty. They prove that the optimal replenishment policy is in the form of re-order point. Feng and Shi (2012) consider the same problem in the presence of multi-suppliers. They show that although a base-stock list price policy is optimal when the suppliers are reliable, this result does not hold for the unreliable supplier case. However, similar to the previous work, the optimum policy has the form of a re-order point. Zhu (2013) explore the problem under the presence of a disruption risk. They assume that the firm first orders the raw material and after realization of the supply, decides about the production quantity and pricing. The optimal ordering policy follows a base-stock policy. However, the production policy is in the form of a re-order point. In addition, the pricing policy is of a list-price policy that depends on the level of raw material and finished products inventory. The problem of dual-sourcing with one reliable supplier is examined by Yan and Liu (2009). They assume that the unreliable supplier may be disrupted and there is a fixed ordering cost for both suppliers. The optimal ordering policy for both suppliers follows a modified version of an (s, S) policy and list-pricing is the optimal pricing strategy.

Implication of joint lot-sizing and pricing in remanufacturing systems can be found in the studies of Li *et al.* (2009), Zhou and Yu (2011) and Shi *et al.* (2011). Li *et al.* (2009) propose a two stage model in which in the first stage the vendor sets a collection price that defines the production quantity and in the second stage they set a selling price to maximize the revenue. They prove the uniqueness of the solution in each stage. Zhou and Yu (2011) consider a firm that has to decide about the requisition effort in order to collect the used products, new production amount and the price of products. They assume that the number of products that are collected is random and the demand has a linear additive form. They introduce some proper structural characteristics of the problem and optimum solution. Shi *et al.* (2011) extend the previous work by considering multi-products and including a capacity constraint in a single period case. They propose a Lagrangian relaxation based algorithm and show that it has a promising performance.

Cai *et al.* (2009) explore the relation of a producer and a distributor in the fresh product industry. At the beginning of the period the distributor decides about the order quantity and the amount of effort that she wants to put in order to keep the material fresh during the transportation. The quality and the amount of non-defective materials during the transportation are increasing functions of the amount of effort and they are stochastic. After realization of supply, the distributor sets the selling price by considering an iso-elastic demand function. On the other side of the supply chain, the producer decides about the wholesale price. They analyze the problem under centralized and decentralized cases and determine the behaviour of the optimum solution in each case.

Pan and So (2010) consider an assembly system where the supply of the one of the components is uncertain. The uncertainty of the supply is assumed to be in form of stochastically proportional and the problem is examined under both deterministic and uncertain demand cases. Pan and So (2010) show that threshold on the price that makes the production profitable. Interestingly this threshold is equal for deterministic and uncertain demand cases.

2.5 Classifying the Papers Based on Their Solution Procedures

Several methodologies have been used in the reviewed literature. In this section we take a closer look at these methodologies. We have classified these methodologies in five categories:

1. *Exact Algorithms*: Papers that develop and show exact solution algorithms such as branch and bound and Bender's decomposition based approaches. These papers are often related to models that employ mathematical programs that involves discrete variables for the selection of suppliers.

2. *Heuristics*: Studies that propose procedures for finding solutions that have provable quality, theoretically or numerically, but not exact. An example would be an EOQ lot sizing used in an uncertain environment or a high-low pricing used in a multi-period problem.

3. *Structural analysis*: Papers that study the convexity of the objective functions

and perform comparative statics.

4. *Approximation*: Papers that approximate the objective function with more tractable functions, such as linear or quadratic functions.

5. *Simulation*: Papers that explicitly use a simulation approach where they address more complex setting that are not easy to represent with stylized mathematical models.

We note that some of the studies used more than one methodology

Table 2.11 shows the frequency of use of these different methodologies. Tables 2.12 and Table 2.13, in Appendix of this chapter, match the different methodologies with those used in the reviewed papers.

Table 2.11: Proposed solution procedure of reviewed studies in lot-sizing

Exact algorithm	Heuristic	Structural analysis	Approximation	Simulation
9 (8%)	17 (15%)	73 (65%)	11 (10%)	3 (3%)

Appendix

Table 2.12: Proposed solution procedure of reviewed studies in lot-sizing

Study	Exact algorithm	Heuristic	Structural analysis	Approximation	Simulation
Simpson (1978)			*		
Shih (1980)			*		
Noori and Keller (1986)			*		
Gerchak *et al.* (1986)			*		
Basu (1987)			*	*	
Ehrhardt and Taube (1987)			*		
Moinzadeh and Lee (1987)				*	
Gerchak *et al.* (1988)			*		
Moinzadeh and Lee (1989)				*	
Parlar and Gerchak (1989)		*			
Henig and Gerchak (1990)			*		
Tang (1990)			*		
Anupindi and Akella (1993)			*		
Parlar and Wang (1993)			*		
Ciarallo *et al.* (1994)			*		
Meybodi and Foote (1995)		*			
Parlar *et al.* (1995)			*		
Denardo and Lee (1996)			*		
Gurnani *et al.* (1996)			*		
Song and Zipkin (1996)			*		
Wang and Gerchak (1996)			*		
Denardo and Tang (1997)			*		
Gong and Matsuo (1997)			*		
Inderfurth (1997)			*		
Nurani *et al.* (1997)			*		
Proth *et al.* (1997)				*	
Hwang and Singh (1998)			*		
Güllü (1998)			*		
Petrovic *et al.* (1998)[a]					*
Bollapragada and Morton (1999)		*			
Hsu and Bassok (1999)	*				
Özekici and Parlar (1999)			*		
Petrovic *et al.* (1999) [a]					*
Swaminathan and Shanthikumar (1999)			*		
Duenyas and Tsai (2000)		*	*		
Gurnani *et al.* (2000)			*		
Chen *et al.* (2001)			*		
Jones *et al.* (2001)			*		
Qinghe *et al.* (2001)[a]	*				
Wang and Fang (2001)[a]				*	
Webster and Weng (2001)			*		
Erdem and Özekici (2002)			*		

Continued on next page

Table 2.12 – continued from previous page

Study	Exact algorithm	Heuristic	Structural analysis	Approximation	Simulation
Iida (2002)		*	*		
Kouvelis and Milner (2002)			*		
Kiesmüller and Minner (2003)				*	
Karabuk and Wu (2003)				*	
Lin and Chen (2003)		*			
Weng and McClurg (2003)			*		
Inderfurth (2004a)			*		
Bollapragada *et al.* (2004a)			*		
Bollapragada *et al.* (2004b)	*	*	*		
Li *et al.* (2004)		*	*		
Gallego and Hu (2004)			*		
Inderfurth (2004b)			*		
Kazaz (2004)			*		
Chang (2004)[a]				*	
Sloan (2004)			*		
Gallego and Hu (2004)			*		
Gupta and Cooper (2005)			*		
Babai and Dallery (2006)			*		
Bollapragada and Rao (2006)		*	*		
Rekik *et al.* (2006)			*		
Tomlin (2006)			*		
Dada *et al.* (2007)			*		
Grasman *et al.* (2007)			*		
Chiang and Feng (2007)					*
Inderfurth and Transchel (2007)		*			
Kök and Shang (2007)		*	*		
Yang *et al.* (2007)	*				
Zikopoulos and Tagaras (2007)			*		
Abdel-Malek *et al.* (2008)	*				
Federgruen and Yang (2008)			*		
Grasman *et al.* (2008)		*	*		
Li *et al.* (2008)		*	*		
He and Zhang (2008)			*		
Hu *et al.* (2008)			*		
Bensoussan *et al.* (2009)			*		
Burke *et al.* (2009)			*		
Federgruen and Yang (2009)			*		
Ferrer and Whybark (2009)		*			
Huh and Nagarajan (2009)		*			
Inderfurth (2009)		*			
Ketzenberg *et al.* (2009)			*		
Kogan (2009)		*			
Maddah *et al.* (2009)	*	*			
Mukhopadhyay and Ma (2009)			*		
Pac *et al.* (2009)			*		
Uthayakumar and Parvathi (2009)	*				

Continued on next page

Table 2.12 – continued from previous page

Study	Exact algorithm	Heuristic	Structural analysis	Approximation	Simulation
Wang (2009)			*		
Wang and Tomlin (2009)			*		
He and Zhang (2010)			*		
Iakovou *et al.* (2010)			*		
Xu (2010)			*		
Arifoglu and Özekici (2010)			*		
Cho (2010)			*		
Schmitt *et al.* (2010)			*	*	
Teunter and Flapper (2011)				*	
Tiwari *et al.* (2011)			*		
Arifoglu and Özekici (2011)			*		
Federgruen and Yang (2011)			*		
Arifoğlu *et al.* (2012)			*		
Chen *et al.* (2012)			*		
Saghafian and Van Oyen (2012)			*		
Sting and Huchzermeier (2012)			*		
Tan and Çömden (2012)			*		
Xanthopoulos *et al.* (2012)			*		
Ahiska *et al.* (2013)	*				
Kaki *et al.* (2013)			*		
Cho and Tang (2013)			*		
Hong *et al.* (2013)			*		
Hu *et al.* (2013a)			*		
Hu *et al.* (2013b)			*		
Ma *et al.* (2013)			*		
Wu *et al.* (2013)			*		
Okyay *et al.* (2014)			*		
Inderfurth and Vogelgesang (2013)		*			
Kouvelis and Li (2013)			*		
Kutzner and Kiesmüller (2013)				*	
Merzifonluoglu and Feng (2014)	*	*			

a. used fuzzy logic for modelling the problem

Table 2.13: Proposed solution procedure of reviewed studies in joint lot-sizing and pricing

Study	Exact algorithm	Heuristic	Structural analysis	Approximation	Simulation
Li and Zheng (2006)			*		
Chao *et al.* (2008)			*		
Cai *et al.* (2009)			*		
Li *et al.* (2009)			*		

Continued on next page

Table 2.13 – continued from previous page

Study	Exact algorithm	Heuristic	Structural analysis	Approximation	Simulation
Yan and Liu (2009)			*		
Feng (2010)			*		
Pan and So (2010)			*		
Kazaz and Webster (2011)			*		
Shi *et al.* (2011)		*			
Zhou and Yu (2011)			*		
Feng and Shi (2012)			*		
He (2013)			*		
Surti *et al.* (2013)			*		
Xu and Lu (2013)			*		
Zhu (2013)			*		

Chapter 3

A Two-Period Sourcing Model with Demand and Supply Risks

3.1 Introduction

In this chapter we study a two-period sourcing problem when both demand and yield are uncertain and the production quantities in the second period is a function of that in the first period. These setting are common in closed loop manufacturing-remanufacturing systems and agricultural production environments. In particular our study is motivated by the production of almond and the problems this industry has been facing during the last decade. Almonds are versatile tree nuts and widely available, compared to other competing nuts such as cashews and walnuts. The state of California in the United States of America produces about 80% of the world almonds. In 2011, there were about 6500 almond farms in California that produced 2.02 billion pounds with total sales of more than 2.8 billion dollars (Almond Board of

California (2012)). While this industry is an important player in the overall agricultural economy it also has it own problems. According to Ezell (2010) the return on an acre of almond trees has been decreasing at an alarming rate from 2004, when it was at $2382/acre, to 2009, when it reached $475/acre, a drop of 80%. Accordingly Ezell estimated that the total loss in revenue in 2008/09 amounts to $1.6 billion. This loss is attributed mostly to the large increase in production in 2008 of 1.614 billion pounds, an increase of about 300 million pounds of saleable almonds. This has caused analysts in this industry to question the farmers decision on production and the handlers decisions on inventory carry overs. Thus our interest in studying the problem of sourcing in this industry to hopefully avoid the revenue losses that the industry experienced in the 2008/09 years. In order to do that we first need to understand the characteristics of this industry and agricultural production.

One of the major concerns in any agricultural environment is high uncertainty in yield (Roberts *et al.* (2004)), which increases the risk of production significantly. One of the ways that farmers can alleviate this source of risk is by signing *production contracts*. In this sector production contracts refer to an arrangement in which the farmer does not guarantee a specific amount of products and instead the buyer agrees to purchase all the production of a particular acreage (Zawada (2004)). This type of contracts has become more popular during the last decade. According to an estimate by USDA, 41 percent of the value of U.S. agricultural production in 2005 were sold under production contracts, compared to about 39 percent in 2003 and 36 percent in 2001 (Kunkel *et al.* (2009)). A firm, often called handler in the almond industry, signs a set of production contracts with the farmers and at the end of the season harvests the crops and sells them in the spot market with uncertain demand. In our model we assumed that the handler has the opportunity to provide her supply from both production and purchasing contracts. In a production contract the handler is the one who bears the uncertainty of supply, however in a *purchasing contract* the farmer guarantees to provide a certain amount of products.

The terms of the contracts may differ due to the characteristics of the crop. The alternate bearing behaviour of almond trees leads many farmers to prefer two-year contracts. Under such a contract the handler can not change the size of the rented farm in the second year. In other words, the production of the second year is a function of acreage of the farm that is rented in the beginning of the first year. But this value may differ from that of the first period due to changes in the farm and weather conditions. For purchasing contracts the two parties agree at the beginning of the first period on the amount of product that the farmer would supply for both periods.

Based on the special setting of contracts we developed a two period sourcing problem with random supply and demand. We consider two cases of the problem. In the first case we assume that due to storage and quality issues the handler does not carry any inventory to the second period and sell all the excess production with a lower price in a secondary market, such as a charity organization that buys the excess inventory with lower price. In the second case we assume that the handler has the ability to store and preserve the quality of the excess production and carry the overproduction to the next period. We study the problem in both cases analytically and numerically in order to provide some insights about the characteristics of the models and behaviour of the optimum solutions with regard to variation of parameters.

3.1.1 Literature Review of Production Planning Problem in Agricultural Environment

In Chapter 2 we provided a review on lot-sizing problem under uncertain supply and demand. Here we highlight some of the studies that have focused on production planning in agricultural environment in order to clarify our contribution in this field.

The studies that consider production planning in agricultural environments are almost rare. Jones *et al.* (2001) developed a two period model for production of hybrid seed corn. The vendor decides about the amount of production in the first period and after the realization of yield they have a chance to produce more to satisfy the unmet demand. According to their model assumptions their objective function in each period is concave and the optimum solution is unique. Kazaz (2004) consider the production of olive oil. In their model the producer has the chance to cultivate her own supply or buy the fruits from other farmers. The procurement amount is decided after the realization of the yield. Kazaz and Webster (2011) extend that model by incorporating a pricing decision. Tan and Çömden (2012) considered a firm working under a production contract that has to decide about the time of seeding and the acreage they want to put under cultivation. They assumed that the uncertainty of yield has a normal distribution. The model has been extended for the case where there are N farms with different yields and for the multi-period case. Huh and Lall (2013) considered the problem of land and irrigation allocations under forward contract farming from the farmer's point of view. The farmer has to decide about the size of the allocated land to each crop and how to irrigate the land in a single season. They assumed the yield of each crop is a function of water input and is affected by the amount of rainfall, which is uncertain. Our research differs from the above studies as we consider time dependency as well as the case where there are two options for providing supply before the realization of existing uncertainties.

3.2 Assumptions and Notations

In this section we present a description of the problem, assumptions and notations. The almond trees bear fruit every other year so the farmers in this industry prefer to sign a two year contract in order to reduce their risk of production. The handler can provide her required supply from two sets of contracts. The first is a production contract where the handler decides about the amount of almond trees that she wants to rent. In this situation the output of production is uncertain due to the nature of the agricultural environment. The potential amounts of production in both periods are determined based on the rented acreage. But these values may be different because of alternate bearing behaviour of almond trees and some other factors such as availability of the facilities in different periods. We consider a general function for the second period potential production in order to preserve the applicability of the model in different cases. The second type of contract is a purchasing contract where the farmer guarantees to provide a specified quantity. In this situation the handler does not experience any uncertainty in supply, but the cost of acquiring the supply may be higher in comparison with the production contract.

The production decisions are made by considering the uncertainty of yield and demand. To model the uncertainty of the yields we use a stochastically proportional yield for both periods. According to Yano and Lee (1995) stochastically proportionality is suitable for modelling uncertainties that are caused by environmental factors such as weather conditions.

Alston *et al.* (1995) noted that for almonds, due to the size of the market, the producers usually do not set the price of their products and it is determined by the market. Therefore in our study we assume that the price is an exogenous factor. In addition, due to the large size of the market and uncertainties due to environmental factors, usually the firm does not incur any penalty cost for unsatisfied demand.

The notation that is used in this are presented in Table 3.1:

Table 3.1: Summary of notation

Parameters:	
p_i :	Unit price in period $i = 1, 2$
m_i :	Unit price of secondary market in period $i = 1, 2$, $m_i \leq p_i$
c_i :	Unit cost of production in period $i = 1, 2$, $c_i \leq p_i$
h_i :	Unit inventory cost at the end of period $i = 1, 2$
s_i :	Unit cost of acquiring products via a purchasing contract in period $i = 1, 2$, $s_i \geq c_i$
Y_i :	Random variable representing the proportional yield in period i with expected value μ_{Y_i}, standard deviation σ_{Y_i}, p.d.f. g_i and c.d.f. G_i. We assume Y_i is defined on the range $[\underline{k}_i, \overline{k}_i]$ $i = 1, 2$
D_i :	Random variable representing the demand in period i with expected value μ_{D_i}, p.d.f. f_i and c.d.f. F_i. We assume D_i is defined on the range $[A_i, B_i]$ $i = 1, 2$
Decision variables:	
I_i :	Amount of product in period i that is acquired through a purchasing contract, $i = 1, 2$
q :	Amount of trees that is rented in the first period.
$l(q)$:	The amount of trees that is rented in the second period. Due to the two year production contract, this value is a function of q.

3.3 Production Planning with No Carry-over

Some studies have shown that the outbreak of salmonellosis during the period 2001 to 2006 can be associated to the consumption of raw almonds (Keady *et al.* (2004), Isaacs *et al.* (2005), Ledet Müller *et al.* (2007)). Thus regulators in the USA have introduced laws in 2007 imposing strict controls on handlers' processing and storage of almonds so as to reduce the risk of such outbreaks (Federal Register (2007)). Handlers faced high investment costs and difficulties of providing storage facilities that meet these new standards. This has forced some of the handlers to sell all of their supply of almonds in the given year, even if at lower prices, to avoid the exorbitant compliance costs associated with carrying it for another year. Another driver for selling all the produce in the same period is the difficulty in preserving the quality of the product if it is carried over for an additional year. This has motivated us to focus on the case where the handler does not carry any inventory and sell all the excess production at a lower price. We assume that the handler's goal is to maximize the profit.

Noting that in an agricultural environment the variable cost of production is independent of the yields random variables Y_1 and Y_2, the profit function of the handler can be formulated as

$$\begin{aligned}\Pi(q, I_1, I_2) = & p_1 E[\min\{Y_1 q + I_1, D_1\}] - c_1 q - s_1 I_1 + m_1 E[Y_1 q + I_1 - D_1]^+ \\ & + p_2 E[\min\{Y_2 l(q) + I_2, D_2\}] + m_2 E[Y_2 l(q) + I_2 - D_2]^+ \\ & - c_2 l(q) - s_2 I_2. \end{aligned} \tag{3.1}$$

The explicit form of objective function is as follows:

$$
\begin{aligned}
\Pi(q, I_1, I_2) =&(p_1 - m_1) \int_{\underline{k}_1}^{\overline{k}_1} \int_{A_1}^{y_1 q + I_1} z_1 f_1(z_1) g_1(y_1) dz_1 dy_1 \\
&+ (p_1 - m_1) \int_{\underline{k}_1}^{\overline{k}_1} \int_{y_1 q + I_1}^{B_1} (y_1 q + I_1) f_1(z_1) g_1(y_1) dz_1 dy_1 \\
&+ m_1 \mu_{Y_1} q + m_1 I_1 - c_1 q - s_1 I_1 + m_2 \mu_{Y_2} l(q) + m_2 I_2 \\
&+ (p_2 - m_2) \int_{\underline{k}_2}^{\overline{k}_2} \int_{A_2}^{y_2 l(q) + I_2} z_2 f_2(z_2) g_2(y_2) dz_2 dy_2 \\
&+ (p_2 - m_2) \int_{\underline{k}_2}^{\overline{k}_2} \int_{y_2 l(q) + I_2}^{B_2} (y_2 l(q) + I_2) f_2(z_2) g_2(y_2) dz_2 dy_2 \\
&- c_2 l(q) - s_2 I_2. \qquad (3.2)
\end{aligned}
$$

The above objective function is formulated as a two period newsvendor problem. In each period the selling amount is the minimum of the on hand inventory and the demand.

The presence of $l(q)$ complicates the analysis of the problem. Let $I_2^*(q)$ denote the optimum value of I_2 with respect to q. In the next two propositions we show some properties of the objective function and optimal solutions.

Proposition 3.1.

a) The objective function is submodular with respect to q and I_1. If $l(q)$ is increasing (decreasing) it is also submodular (supermodular) with respect to q and I_2.

b) If $l(q)$ is concave and $s_2 \mu_{Y_2} \geq c_2$ then $\Pi(q, I_1, I_2^*(q))$ is concave.

c) If $l(q)$ is linear then $\Pi(q, I_1, I_2)$ is concave.

The proof of Proposition 3.1 requires the proof of two lemmas and is included in the Appendix of this chapter.

Part (a) of Proposition 3.1 implies that as we increase the size of the production contract, the size of the purchasing contract decreases. Parts (b) and (c) of the above proposition guarantee that (under the stated condition) the optimum solution of the problem is unique and first order optimality conditions are sufficient to find it. The condition $s_2\mu_{Y_2} \geq c_2$ implies that long term supply contracts come with a premium to account for the yield uncertainty as is typically the case in the almond industry (Micke (1996), Chapter 2). As of the condition on $l(q)$, in practice the handler would either rent the same land or a factor of that rented in the revious year (i.e., a linear from). Thus, we believe the conditions we have in Proposition 3.1 are reasonable and cover most practical settings.

Proposition 3.1 can also aid us in analyzing the multi-period problem. Consider

$$\mathbf{I} = (I_1, I_2, ..., I_n)$$

and

$$\begin{aligned}\tilde{\Pi}(q, \mathbf{I}) =& p_1 E[\min\{Y_1 q + I_1, D_1\}] - c_1 q - s_1 I_1 + m_1 E[Y_1 q + I_1 - D_1]^+ \\ & \sum_{t=2}^{n} \{p_t E[\min\{Y_t l_t(q) + I_t, D_t\}] - c_t l_t(q) - s_t I_t \\ & + p_{bt} E[Y_t l_t(q) + I_t - D_t]^+ \} . \end{aligned} \tag{3.3}$$

The objective function (3.3) is decomposable as

$$\tilde{\Pi}(q, \mathbf{I}) = \sum_{t=1}^{n} J_t(q, I_t)$$

where

$$J_1(q, I_1) = p_1 E[\min\{Y_1 q + I_1, D_1\}] - c_1 q - s_1 I_1 + m_1 E[Y_1 q + I_1 - D_1]^+$$

and

$$J_t(q, I_t) = p_t E[\min\{Y_t l_t(q) + I_t, D_t\}] - c_t l_t(q) - s_t I_t + p_{at} E[Y_t l_t(q) + I_t - D_t]^+.$$

According to Proposition 3.1 we can show that $J_t(q, I_t^*(q))$ is concave with respect to q. Therefore $\sum_{t=1}^{n} J_t(q, I_t^*(q))$ is concave with respect to q (since it is a summation of a set of concave functions) and as a result $\tilde{\Pi}(q, \mathbf{I})$ is unimodal.

This result is formalized in Corollary 3.1.

Corollary 3.1 (**Multi-Period Extension**). *If $l_t(q)$ is concave and $s_t \mu_{Y_t} \geq c_t$, $\forall t$, then $\tilde{\Pi}(q, \boldsymbol{I})$ is unimodal.*

According to Corollary 3.1 we can extend the results to the case that the handler signs contracts that last from more than two years. Corollary 3.1 is also important since it projects the problem of producing n products from a common raw material. An example of such industry is the oil refinery where a set of products such as gasoline, petroleum naphtha and asphalt base are extracted from crude oil.

Proposition 3.2. *If $s_1 \mu_{Y_1} - c_1 + (s_2 \mu_{Y_2} - c_2) l_q(q) \leq 0$ for all the values of q then $q^* = 0$.*

Proof. According to first order conditions we have:

$$\Pi_{I_1}(q, I_1^*(q), I_2) = (p_1 - m_1)E\left[\bar{F}_1(Y_1 q + I_1^*(q))\right] - s_1 + m_1 = 0 \tag{3.4}$$

$$\Pi_{I_2}(q, I_1, I_2^*(q)) = (p_2 - m_2)E\left[\bar{F}_2(Y_2 l(q) + I_2^*(q))\right] - s_2 + m_2 = 0 \tag{3.5}$$

The first derivative of the objective function with respect to q is:

$$\begin{aligned}\Pi_q(q, I_1^*(q), I_2^*(q)) =& (p_1 - m_1)E\left[Y_1\bar{F}_1(Y_1 q + I_1^*(q))\right] - c_1 + m_1\mu_{Y_1}\\ &+ (p_2 - m_2)E\left[Y_2 l_q(q)\bar{F}_2(Y_2 l(q) + I_2^*(q))\right]\\ &- c_2 l_q(q) + m_2\mu_{Y_2} l_q(q)\end{aligned}$$

$\bar{F}_1(Y_1 q + I_1^*(q))$ and $\bar{F}_2(Y_2 l(q) + I_2^*(q))$ are decreasing in Y_1 and Y_2 respectively. So according to Chebyshev's inequality we get

$$\begin{aligned}\Pi_q(q, I_1^*(q), I_2^*(q)) \leq& (p_1 - m_1)\mu_{Y_1}E\left[\bar{F}_1(Y_1 q + I_1^*(q))\right] - c_1 + m_1\mu_{Y_1}\\ &+ (p_2 - m_2)\mu_{Y_2}E\left[l_q(q)\bar{F}_2(Y_2 l(q) + I_2^*(q))\right]\\ &- c_2 l_q(q) + m_2\mu_{Y_2} l_q(q)\\ =& s_1\mu_{Y_1} - c_1 + (s_2\mu_{Y_2} - c_2) l_q(q)\end{aligned}$$

The last equality can be found from Equations (3.4) and the first derivative of the objective function with respect to I_2. Now if $s_1\mu_{Y_1} - c_1 + (s_2\mu_{Y_2} - c_2)l(q) \leq 0$ then $\Pi_q(q, I_1^*(q), I_2^*(q)) \leq 0$ for all the values of q and as a result $q^* = 0$.

□

Proposition 3.2 provides the condition under which signing a production contract is not optimal for the producer. In that event it is better to source through a purchasing contract. A milder sufficient condition for the result of Proposition 3.2 is $s_1 \leq \frac{c_1}{\mu_{Y_1}}$ and $s_2 \leq \frac{c_2}{\mu_{Y_2}}$ when $l(q)$ is increasing. Note that $\frac{c_1}{\mu_{Y_1}}$ and $\frac{c_2}{\mu_{Y_2}}$ are the expected costs of producing one unit in periods 1 and 2, respectively. Thus, the conditions guarantee that we can have the same expected output from a production contract as when sourcing with a purchasing contract at a lower cost. Therefore, it is more beneficial and reliable to use the purchasing contract instead of a production contract. Also, from the above proposition it can be concluded that having a large profit in one period by sourcing under a production contract justifies some losses in the other period.

The proof of Proposition 3.2 also reveals that similar to the classic one-period newsvendor problem if $I_1^* > 0$ and $I_2^* > 0$ then there is a certain level of unsatisfied demand in each period when it is optimal to source using a production and purchasing contracts. More specifically we have $Pr\left\{Y_1 q^* + I_1^* \leq D_1\right\} = \frac{s_1 - m_1}{p_1 - m_1}$, in the first period, and $Pr\left\{Y_2 l(q^*) + I_2^* \leq D_2\right\} = \frac{s_2 - m_2}{p_2 - m_2}$, in the second period. It is interesting to note that the level of lost sales is only dependent on the difference of the unit purchase cost and the unit price relative to that of the price of the secondary market and not the cost of production.

3.4 Production Planning with Carry-over

In this section we consider the case where the handler has proper facilities to carry the excess production to the next period. Given the existence of the appropriate storage facilities, the carried inventory does not lose its quality and can be sold with the same price as the fresh product. As is common in practise, the handler carries all the excess inventory to the second period and incurs the associated holding cost.

Our objective is to maximize the expected profit

$$\begin{aligned}\Pi(q, I_1, I_2) =& p_1 E[\min\{Y_1 q + I_1, D_1\}] - c_1 q - s_1 I_1 - h_1 E[R(q, I_1)] \\ &+ p_2 E[\min\{Y_2 l(q) + I + R(q, I_1), D_2\}] \\ &- h_2 E[Y_2 l(q) + I_2 + R(q, I_1) - D_2]^+ - c_2 l(q) - s_2 I_2 \end{aligned} \tag{3.6}$$

where $R(q, I_1)$ is a random variable equal to $\max\{Y_1 q + I_1 - D_1, 0\}$ and represents the amount of inventory at the end of the first period. If $h_2 < 0$ it accounts for the unit salvage value of the inventory available at the end of the second period. We can rewrite objective function (3.6) as

$$\begin{aligned}\Pi(q, I_1, I_2) =& -(p_1 + h_1 + h_2) E\left[\max\{Y_2 l(q) + I_1 - D_1, 0\}\right] \\ &+ (p_2 + h_2) E\left[\min\{Y_2 l(q) + I_2 + \max\{Y_1 q + I_1 - D_1, 0\}, D_2\}\right] \\ &- h_2 \mu_{Y_2} l(q) - h_2 I_2 - s_2 I_2 - c_2 l(q) + p_1 \mu_{Y_1} q + p_1 I_1 - s_1 I_1 - c_1 q.\end{aligned}$$

Based on conditional probabilities we can explicitly write the objective function as

$$\begin{aligned}
\Pi(q, I_1, I_2) =& -(p_1+h_1+h_2)\int_{\underline{k}_1}^{\overline{k}_2}\int_0^{y_1q+I_1}(y_1q+I_1-z_1)f_1(z_1)g_1(y_1)dz_1dy_1 \\
&+(p_2+h_2) \\
&\cdot\Bigg\{\int_{\underline{k}_1}^{\overline{k}_1}\int_{y_1q+I_1}^{B_1}\int_{\underline{k}_2}^{\overline{k}_2}\int_{A_1}^{L_1(q,I_2)} z_2f_2(z_2)g_2(y_2)f_1(z_1)g_1(y_1)dz_2dy_1dz_1dy_1 \\
&+\int_{\underline{k}_1}^{\overline{k}_1}\int_{y_1q+I_1}^{B_1}\int_{\underline{k}_2}^{\overline{k}_2}\int_{L_1(q,I_2)}^{B_2} L_1(q,I_2)f_2(z_2)g_2(y_2)f_1(z_1)g_1(y_1)dz_2dy_1dz_1dy_1 \\
&+\int_{\underline{k}_1}^{\overline{k}_1}\int_{A_1}^{y_1q+I_1}\int_{\underline{k}_2}^{\overline{k}_2}\int_{A_2}^{L_2(q,I_2)} z_2f_2(z_2)g_2(y_2)f_1(z_1)g_1(y_1)dz_2dy_1dz_1dy_1 \\
&+\int_{\underline{k}_1}^{\overline{k}_1}\int_{A_1}^{y_1q+I_1}\int_{\underline{k}_2}^{\overline{k}_2}\int_{L_2(q,I_2)}^{B_2} L_2(q,I_2)f_2(z_2)g_2(y_2)f_1(z_1)g_1(y_1)dz_2dy_1dz_1dy_1\Bigg\} \\
&-h_2\mu_{Y_2}l(q)-h_2I_2-s_2I_2-c_2l(q)+p_1\mu_{Y_1}q+p_1I_1 \\
&-s_1I_1-c_1q \qquad (3.7)
\end{aligned}$$

in which $L_1(q, I_2) = y_2l(q) + I_2$ and $L_2(q, I_2) = L_1(q, I_2) + y_1q + I_1 - z_1$. The derivatives of the objective function can be found based on the above function.

Let $I_2^*(q, I_1)$ denote the optimum value of I_2 with respect to q and I_1. In Proposition 3.3 we establish some properties for the objective function (3.6).

Proposition 3.3.

a) If $l(q)$ is concave and $s_2\mu_{Y_2} \geq c_2$ and $p_1 + h_1 \geq s_2$ then $\Pi(q, I_1, I_2^*(q, I_1))$ is concave with respect to q.

b) If $p_1 + h_1 \geq p_2$ then $\Pi(q, I_1, I_2^*(q, I_1))$ is concave with respect to I_1.

Proof of Proposition 3.3. The proof of part (a) is similar to that of Proposition 3.1.

The objective function is clearly concave with respect to I_2, so we can find the optimum value of I_2 with respect to I_1 and q and then optimize the problem with respect to these two variables.

Let

$$\Theta = \Pi_{qq}(q, I_1, I_2^*(q, I_1))\Pi_{I_2I_2}(q, I_1, I_2^*(q, I_1)) - \Pi^2_{qI_2}(q, I_1, I_2^*(q, I_1))$$

then if $\Theta \geq 0$ the objective function is concave with respect to q. We can show that

$$\begin{aligned}\Theta \geq &\Pi_{I_2I_2}(q, I_1, I_2^*(q, I_1)).\\ &\big\{(p_2 + h_2)E\left[(Y_2 l_{qq}(q) + R_{qq}(q, I_1))\bar{F}_2(Y_2 l(q) + I_2^*(q) + R(q, I_1))\right] - c_2 l_{qq}(q)\\ &- h_2 E\left[Y_2 l_{qq}(q) + R_{qq}(q, I_1)\right] - (p_1 + h_1)E\left[R_{qq}(q, I_1)\right]\big\}.\end{aligned}$$

If $l(q)$ is concave and $c_b\mu_{Y_2} \geq c_2$ and $p_1 + c_h a \geq c_b$ then $\Theta \geq 0$ implying the objective function is concave with respect to q.

b) The proof of part (b) follows a similar logic. We define $\hat{\Theta}$ as

$$\hat{\Theta} = \Pi_{I_1I_1}(q, I_1, I_2^*(q, I_1))\Pi_{I_2I_2}(q, I_1, I_2^*(q, I_1)) - \Pi^2_{I_1I_2}(q, I_1, I_2^*(q, I_1))$$

.

If $\hat{\Theta} \geq 0$ then the objective function is concave with respect to I_1. We can show that

$$\begin{aligned}\hat{\Theta} \geq & \Pi_{I_2I_2}(q, I_1, I_2^*(q, I_1)). \{-(p_1 + h_1 + h_2)E\left[R_{I_1I_1}(q, I_1)\right] \\ & + (p_2 + h_2)E\left[R_{I_1I_1}(q, I_1)\bar{F}_2(Y_2 l(q) + I_2^*(q, I_1) + R(q, I_1))\right]\}.\end{aligned}$$

Therefore if $p_1 + h_1 \geq p_2$ then $\Pi(q, I_1, I_2^*(q, I_1))$ is concave with respect to I_1. □

In Proposition 3.3 we have reduced the problem to a two-variable problem and provide conditions under which the reduced objective function is concave with respect to each of the variables. Although these conditions do not guarantee the unimodality of the objective function, they can be useful in solving the problem more efficiently. For instance according to part (b) of this proposition, if $p_1 + h_1 \geq p_2$ the optimum values of I_1 and I_2 can be found by using the first order conditions as a function of q and then solving the resulting single variable problem.

We note that the condition in part b) would imply the condition in part a) knowing that it is reasonable to assume that $p_2 \geq s_2$. In Section 3.4.1 we provide emprical evidence that support the condition $p_1 + h_1 \geq p_2$.

The major difference between the two models proposed in Section 3.3 and Section 3.4 is the relation between I_1 and I_2 through carrying inventory. The next proposition clarifies the relation of the optimum value of I_2 with respect to I_1.

Proposition 3.4. *For any value of q, $-1 \leq \frac{\partial I_2^*(q,I_1)}{\partial I_1} \leq 0$.*

Proof. According to first order optimality conditions and implicit function theorem we have:

$$\begin{aligned}\frac{\partial I_2^*(q,I_1)}{\partial I_1} &= -\frac{\Pi_{I_1 I_2}}{\Pi_{I_2 I_2}} \\ &= -\frac{-E\left[F_1(Y_1 q + I_1) f_2(Y_2 l(q) + I_2 + R(q,I_1))\right]}{-E\left[f_2(Y_2 l(q) + I_2 + R(q,I_1))\right]}.\end{aligned}$$

Since $F_1(.)$ is between 0 and 1, we conclude that $-1 \leq \frac{\partial I^*(q,I_1)}{\partial I_1} \leq 0$. □

According to Proposition 3.4 by increasing I_1, or the amount that will be carried from period 1 to period 2, would decrease the optimum value of I_2. Furthermore, for every unit increase in the value of I_1, the decrease of I_2 is bounded by 1. This suggests that carry overs would have an impact on sourcing decisions in the second period, but they do not necessarily replace new and additional purchase. This finding supports the recommendations from the almond industry analysts (Ezell (2010)).

In some cases a firm has the chance to eliminate the uncertainty of demand by signing contracts with the customers. In this situation the demand would be deterministic and known. The handler would then be interested in knowing if if signing such a contract can increase the expected profit. We address this question in Theorem 3.1.

We assume that the deterministic demand takes values μ_{D_1} and μ_{D_2} in periods 1 and 2, respectively. The expected profit is

$$\begin{aligned}\Pi(q,I_1,I_2) =& p_1 E[\min\{Y_1 q + I_1, \mu_{D_1}\}] - c_1 q - s_1 I_1 - h_1 E[\bar{R}(q,I_1)] \\ &+ p_2 E[\min\left\{Y_2 l(q) + I + \bar{R}(q,I_1), \mu_{D_2}\right\}] \\ &- h_2 E[Y_2 l(q) + I_2 + \bar{R}(q,I_1) - \mu_{D_2}]^+ - c_2 l(q) - s_2 I_2 \qquad (3.8)\end{aligned}$$

where $\bar{R}(q, I_1) = \max\{Y_1 q + I_1 - \mu_{D_1}, 0\}$. Let (q^*, I_1^*, I_2^*) denote the optimum solution of the problem with uncertain demand and $(\bar{q}^*, \bar{I}_1^*, \bar{I}_2^*)$ the optimum solution of the problem with deterministic demand.

Theorem 3.1. *If* $p_1 + c_h \geq p_2$*, then we have* $\Pi(q^*, I_1^*, I_2^*) \leq \bar{\Pi}(\bar{q}^*, \bar{I}_1^*, \bar{I}_2^*)$.

Proof. We rewrite Equation (3.6) as a function of $\hat{D}_1$ and $\hat{D}_2$:

$$\begin{aligned} J(\hat{D}_1, \hat{D}_2) =& -(p_1 + h_1 + h_2)E\left[\max\left\{Y_1 q + I_1 - \hat{D}_1, 0\right\}\right] - c_1 q - c_a I_1 \\ &+ (p_2 + h_2)E\left[\min\left\{Y_2 l(q) + I_2 + \max\left\{Y_1 q + I_1 - \hat{D}_1, 0\right\}, \hat{D}_2\right\}\right] \\ &- h_2 E[Y_2 l(q) + I_2] - c_2 l(q) - c_b I_2. \end{aligned}$$

$J(\hat{D}_1, \hat{D}_2)$ is clearly concave with respect to $\hat{D}_1$ (since minimization preserves concavity). The second derivative of $J(\hat{D}_1, \hat{D}_2)$ with respect to $\hat{D}_1$ is

$$\begin{aligned} J_{\hat{D}_1,\hat{D}_1}(\hat{D}_1, \hat{D}_2) =& -\frac{(p_1 + h_1 + h_2) g_1\left(\frac{\hat{D}_1 - I_1}{q}\right)}{q} \\ &- (p_2 + h_2)\int_{\frac{\hat{D}_1 - I_1}{q}}^{K_1} \left[\frac{g_2\left(\frac{\hat{D}_2 - y_1 q - I_1 + \hat{D}_1 - I_2}{l(q)}\right)}{l(q)}\right] g_1(y_1) dy_1 \\ &+ (p_2 + h_2)\int_0^{\frac{\hat{D}_2 - I_2}{l(q)}} \left[\frac{g_1\left(\frac{\hat{D}_1 - I_1}{q}\right)}{q}\right] g_2(y_2) dy_2. \end{aligned} \tag{3.9}$$

If $p_1 + h_1 \geq p_2$ then Equation (3.9) is negative and consequently $J(\hat{D}_1, \hat{D}_2)$ is concave with respect to $\hat{D}_1$. So based on Jensen's inequality we have:

$$\Pi(q^*, I_1^*, I_2^*) \leq \bar{\Pi}(\bar{q}^*, \bar{I}_1^*, \bar{I}_2^*)$$

and that completes the proof. □

As an extension to the above theorem, it is easy to show that if the firm is able to sell more than the mean demand then the optimum expected profit in the deterministic case is larger than that of the uncertain case. As a managerial insight, the firm can expect a higher profit by signing a long term contract with customers when the total supplied quantity is higher than the average forecasted demand.

3.4.1 Empirical Test of Price Ratios

The condition $p_1 + h_1 \geq p_2$ implies that $\frac{p_2}{p_1} \leq 1 + \gamma$, where we assume $h_1 = \gamma p_1$, a common assumption in the inventory literatures, see for example, Nahmias (2005). For almonds a reasonable value for γ is 11% (Klonsky (2010)). This condition implies that the effective unit revenue in the second period is lower than that of the first period. This situation can happen where the second period is expected to have higher yields and/or a significant amount was carried from period one. As per Table 3.2 we can see that in practice such a situation seems to be common, occurring about 56% of the time assuming a holding cost rate of 11%.

Table 3.2: Price ratios (based on data from the Almond Board of California (2012))

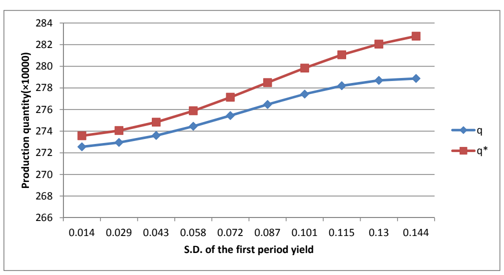
Year	Price	$\frac{p_2}{p_1}$
2002/03	1.11	-
2003/04	1.57	1.41
2004/05	2.21	1.41
2005/06	2.81	1.27
2006/07	2.06	0.73
2007/08	1.75	0.85
2008/09	1.45	0.83
2009/10	1.65	1.14
2010/11	1.79	1.08
2011/12	1.92	1.07

3.5 Numerical Case Analysis: California Almond Industry

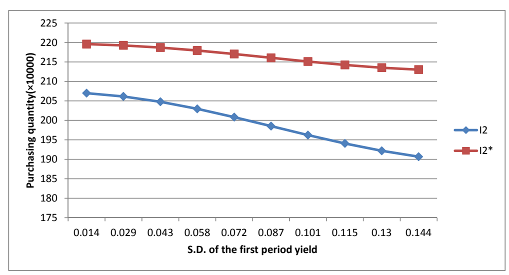
In this section we investigate the behaviour of the optimum solution with regard to parameters of the problem. Our main objective is to understand the impact of demand and yield uncertainty on optimal supply quantities and profits. Thus, our parametric analysis focuses on the standard deviations and mean values of the yield and demand parameters.

We chose our parameters based on actual data from the Almond Board of California 2012 report. Period 1 stands for the 2011/12 season and period 2 for the 2012/13 season. From Table 3.2 we take p_1 = \$1.92/lb and according to the historical trend assume p_2 =\$1.95/lb. Given Klonsky (2010) cost estimation and the bearing acre yield information (p. 20, Almond Board of California (2012)) we estimate the cost at c_1 =\$1.05/lb and c_2 =\$1.04/lb. Based on Klonsky (2010) estimate of the cash overhead we estimate h_1 = \$0.21/lb. Based on equipment, harvest and trees costs we estimate $s_1 = s_2$ = \$1.82/lb. Also based on the costs and marginal profit, we set $m_1 = m_2 =$ \$1.2/lb. For the sake of consistency in both cases, we assumed that h_2 =\$-1.2/lb. From a linear regression of production in the last ten years we find that $l(q)$ can be estimated by a linear relationship ($R^2 \approx 0.8$) with $\alpha = 125198.4$ and $\beta = 0.99$. According to the bearing acre yield available data (p. 20, Almond Board of California (2012)) we estimate that $g_1, g_2 \sim U(0.51, 1)$ with p-values of 0.97 and 0.79 for the Kolmogorov-Smirnov and χ^2 tests respectively, suggesting that the test for uniformity holds. We assume that we are dealing with a small size producer with $f_1 \sim \mathcal{N}(2000000, 200000)$ and $f_2 \sim \mathcal{N}(3000000, 300000)$.

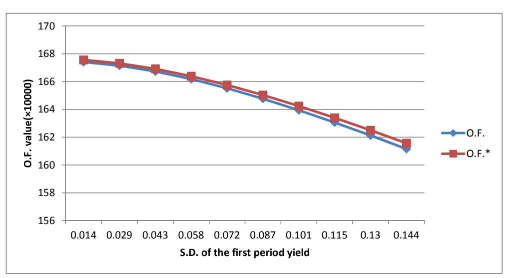
In the following figures, q, I and $O.F.$ indicate the optimum quantity, inventory and objective function of the case where we have carry over and q^*, I^* and $O.F.^*$ show the same values for the case when there is no inventory carrying.



(a) Production Quantity.



(b) Purchasing Quantity.



(c) Expected profit.

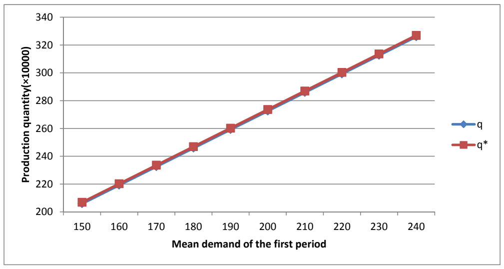
Figure 3.1: Sensitivity analysis with respect to first period's yield standard deviation.

The increase of the first period's yield standard deviation increases the risk of production. Because of existence of the secondary market, the cost of overproduction is less than cost of underproduction. Therefore the handler tends to alleviate the risk by increasing the production quantity. This phenomenon is demonstrated in Figure 3.1a. For both cases the increase of the optimum quantity with respect to standard deviation of the yield shows a S-shape pattern. As expected, the production quantity in the case where we have carry-over is less than the case we do not. We also note that the no carry-over case is more sensitive to the variation of the yield standard deviation. This again confirms the industry analysts call for encouraging handles to carry over produce from season to the next, in contrast to hastily getting rid of it, of at low prices, in one season (Ezell (2010)).

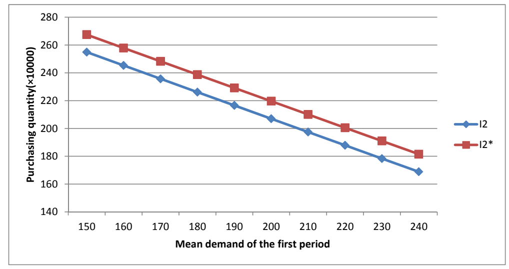
Figure 3.1b represents the variation of the purchasing quantity with respect to the standard deviation of the yield in the second period. Due to the submodularity of the objective function, as the production quantity increases with respect to standard deviation of the yield, the purchasing quantity demonstrates an opposite behaviour. In the case where we have carry-over, some of the required products in the second period are provided through the carry over inventory. Therefore the amount of the purchased quantity is less in the case with carry-over.

Expected profit decreases as the standard deviation of the yield increases. This behaviour is demonstrated in Figure 3.1c. We also note that the expected profits are higher in the case where inventory is carried over. This benefit is more significant when the yield uncertainty is higher.

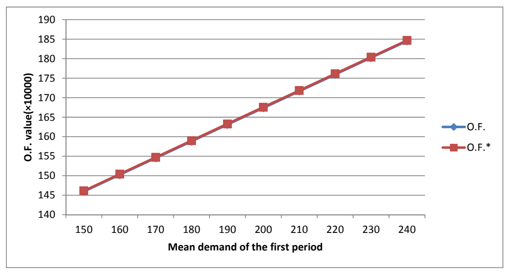
Figures 3.2a and 3.2b demonstrate the behaviour of the production and purchasing quantity of the second period with respect to the mean value of the demand in the



(a) Production Quantity.



(b) Purchasing Quantity.



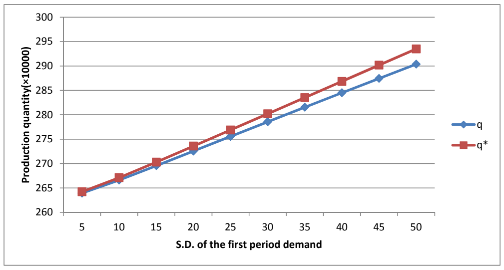
(c) Expected profit.

Figure 3.2: Sensitivity analysis of production quantity with respect to first period's mean demand.

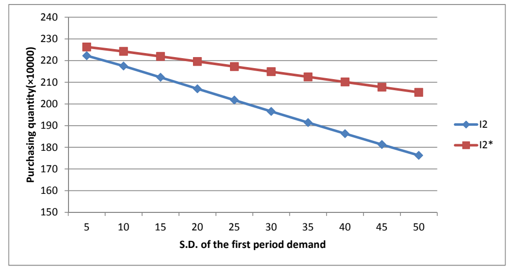
first period. The production quantity shows a linear increase as the mean value of the demand increases. However, based on the submodularity of the objective function, the purchasing quantity in the second period decreases as q increases.

Increase of the mean demand implies an increase of sale and as a result an increase of the expected profit as shown in Figure 3.2c. We note that while the purchasing quantity is higher when there are no carry overs, the production and expected profits are less sensitive to the carry over decision under variations of the first period expected demand.

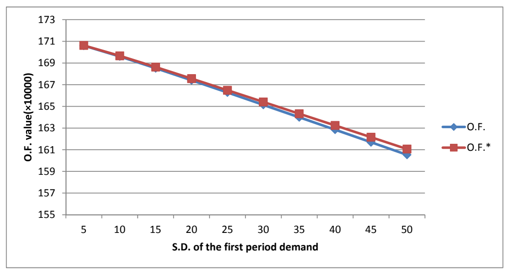
Figures 3.3a and 3.3b illustrate behaviour of the production and purchasing quantities with respect to the standard deviation of demand in the first period. Similar to the pattern we observed under the yield uncertainty, the production (purchasing)



(a) Production Quantity.



(b) Purchasing Quantity.

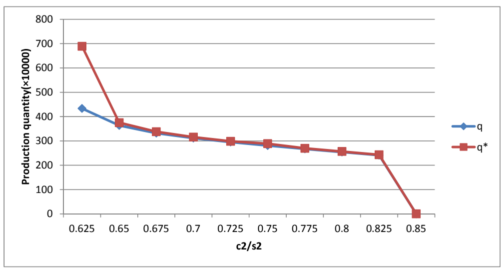


(c) Expected profit.

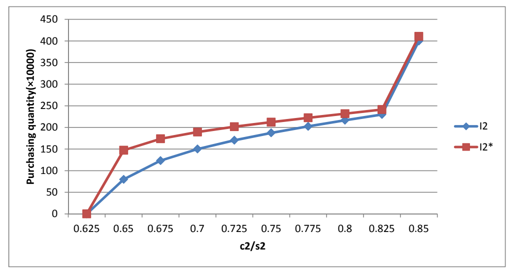
Figure 3.3: Sensitivity analysis of production quantity with respect to first period's demand standard deviation.

quantity increases (decreases) as the first period demand uncertainty increases. As the variability of the first period's demand increases, the risk of under production increases and so the handler tends to increase the supply quantity when using a production contract. Consequently, in the second period purchasing contract amount, I_2, decreases to balance the increase in the first periods production contract amount, q. In the case where we have carry-overs, increases of q in the first period contributes to the increase in the available quantity in period 2 through the effects of an increase in $l(q)$ and inventory. As a result, and given that demand variability in the second period is kept constant, increase of q in the first period (due to variation of demand) causes more over production in the second period. Therefore it is better for the handler to decrease I_2 at a faster rate than that of the increase in q.

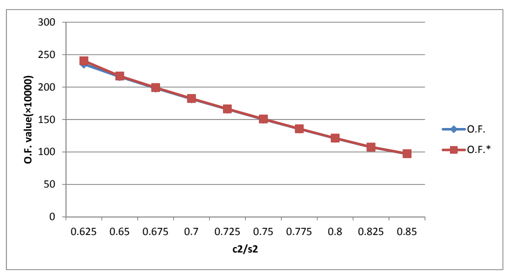
As expected, due to the increase of the risk of demand, the expected value of the optimum profit decreases as shown in Figure 3.3c. We also observe that in this case carry overs are also more relevant with purchase contracts, especially when the first period's demand uncertainty is high.



(a) Production Quantity.



(b) Purchasing Quantity.



(c) Expected profit.

Figure 3.4: Sensitivity analysis of production quantity with respect to the ratio of the production cost over purchasing cost.

Figures 3.4a, 3.4b and 3.4c demonstrate the behaviours of the optimum production quantity, purchasing quantity and objective function with respect to the ratio of the production cost over purchasing cost, respectively. Clearly as we increase this ratio, the purchasing contract becomes more attractive. Figures 3.4a and 3.4b show two break points where only implementing one of the contracts is optimal. When the ratio is small (below 0.625) only the production contract is justified. When the ratio is high (above 0.825) only the purchasing contract is optimal. In order to increase the ratio $\frac{c_2}{s_2}$ we fixed the value of s_2 and increased the value of c_2. Therefore by increasing the ratio, the optimum value of the profit decreases in both cases, as illustrated in 3.4c.

3.6 Conclusions

We have presented models for studying sourcing decisions in multi-period newsvendor settings that have risky demand and supply. The sourcing can happen through two types of contracts: a production contract that is typically priced lower, but comes with more supply uncertainty, and a purchasing contract that charges a premium, but guarantees the agreed upon delivery quantities. Given the presence of both demand and supply uncertainties as well as the possibility for dual supply contracts we had to deal with complex optimization problems for which we were able to establish concavity results under some reasonably practical, and empirically verifiable, conditions. As an application of our models we used data from the Almond Board of California. After collecting parameter data we run some numerical sensitivity analysis to understand the role of demand and supply uncertainty as well as the dual supply contract options. We found that as the demand and supply uncertainty increase it is recommended to have carry overs from one season to the next. This is consistent with recent ad hoc recommendations from industry practitioners when they have observed that most almond handles seem to irrationally try to clear all their produce in one season.

Appendix

In order to prove Proposition 3.1 we need the following lemmas:

Lemma 3.1.

a) $\Pi(q, I_1, I_2)$ *is concave over* I_2.

b) Let $I_2^*(q)$ *be the optimum value of* I_2 *with respect to* q. *It follows that:*

$$(p_2 - m_2)E\left[\bar{F}_2(Y_2 l(q) + I_2^*(q))\right] = c_b - m_2$$

c) $\Pi_{I_2,I_2}(q, I_1, I_2^*(q)) \leq 0$.

Proof. a) The second derivative of the $\Pi(q, I_1, I_2)$ with respect to I_2 is:

$$\Pi_{I_2,I_2}(q, I_1, I_2) = -(p_2 - m_2)E\left[f_2(Y_2 l(q) + I_2)\right]$$

Clearly since $p_2 \geq m_2$ then the objective function is concave with respect to I_2.

b) Since the objective function is concave with respect to I_2, for any values of q and I_1, there is a unique I_2 that maximizes the objective function and it can be found by setting the first derivative equal to zero. So we have:

$$(p_2 - m_2)E\left[\bar{F}_2(Y_2 l(q) + I_2^*(q))\right] = c_b - m_2 \tag{3.10}$$

Note that the optimum value of I_2 is independent of the value of I_1.

c) We can use a same logic as part (a) to prove this part. □

Lemma 3.2. *Define* $\Delta(q, I_2^*(q))$ *as follows:*

$$\Delta(q, I_2^*(q)) = -(p_2 - m_2)\int_0^{K_2} y_2^2(l_q(q))^2 f_2(y_2 l(q) + I_2^*(q))g_2(y_2)dy_2.$$

Then we have $\Delta(q, I_2^*(q))\Pi_{I_2,I_2}(q, I_1, I_2^*(q)) \geq (\Pi_{q,I_2}(q, I_1, I_2^*(q))^2$.

Proof. The second derivative of the objective function with respect to q and I_2 is:

$$\Pi_{q,I_2}(q, I_1, I_2) = -(p_2 - m_2)E\left[Y_2 l_q(q) f_2(Y_2 l(q) + I_2)\right] \tag{3.11}$$

We define $w(y_2) = f_2(y_2 l(q) + I_2^*(q))g_2(y_2)$ and the inner product as:

$$< a, b >_w = \int_0^{K_2} a(y_2)b(y_2)w(y_2)dy_2$$

The norms can be defined as follows:

$$\|a\|_w^2 = \int_0^{K_2} a^2(y_2)w(y_2)dy_2$$

and

$$\|b\|_w^2 = \int_0^{K_2} b^2(y_2)w(y_2)dy_2$$

By setting $a(y_2) = y_2 l_q(q)$ and $b(y_2) = 1$ and based on Cauchy-Schwartz inequality we have:

$$\Delta(q, I_2^*(q))\Pi_{I_2,I_2}(q, I_1, I_2^*(q)) - (\Pi_{q,I_2}(q, I_1, I_2^*(q))^2$$
$$= (p_2 - m_2)^2 \|a\|_w^2 \|b\|_w^2 - (p_2 - m_2)^2(< a, b >_w)^2 \geq 0$$

□

Proof of Proposition 3.1. a) The second derivative of objective function with respect to q and I_1 is:

$$\Pi_{q,I_1}(q, I_1, I_2) = -(p_2 - m_1)E\left[Y_2 f_1(Y_1 q + I_1)\right] \tag{3.12}$$

$\Pi_{q,I_1}(q, I_1, I_2)$ is negative for all the value of q and I_1. $\Pi_{q,I_2}(q, I_1, I_2)$ is also negative for all the values of q and I_2 in the case that $l(q)$ is increasing and and it is positive otherwise (look at Equation (3.11)). These are sufficient for submodularity and supermodularity of $\Pi(q, I_1, I_2)$ with respect to q and I_1 and also q and I_2 (Topkis, 1998).

b) The objective function can be segmented as follows:

$$\Pi(q, I_1, I_2^*(q)) = J(q, I_1) + \hat{J}(q, I_2^*(q))$$

where

$$J(q, I_1) = (p_1 - m_1)E\left[\min\left\{Y_1 q + I_1, D_1\right\}\right] - s_1 I_1 - c_1 q + m_1 E\left[Y_1 q + I_1\right]$$

and

$$\begin{aligned}\hat{J}(q, I_2^*(q)) =& (p_2 - m_2)E\left[\min\left\{Y_2 l(q) + I_2^*(q), D_2\right\}\right] - c_b I_2^*(q) - c_2 l(q) \\ &+ m_2 E\left[Y_2 l(q) + I_2^*(q)\right]\end{aligned}$$

$J(q, I_1)$ is clearly concave with respect to q and I_1. Now we need to prove that $\hat{J}(q, I_2^*(q))$ is concave with respect to q.

since $\hat{J}_{I_2}(q, I_2^*(q)) = 0$, based on implicit function theorem and chain rule we can show:

$$\frac{\partial^2 \hat{J}(q, I_2^*(q))}{\partial q^2} = \hat{J}_{qq}(q, I_2^*(q)) - \frac{\hat{J}_{qI_2}^2(q, I_2^*(q))}{\hat{J}_{I_2 I_2}(q, I_2^*(q))}$$

So according to part (c) of Lemma 3.1, if $\Theta = \hat{J}_{qq}(q, I_2^*(q))\hat{J}_{I_2 I_2}(q, I_2^*(q)) - \hat{J}_{qI_2}^2(q, I_2^*(q)) \geq 0$, then $\hat{J}(q, I_2^*(q))$ is concave with respect to q. We can show

$$\begin{aligned}\hat{J}_{qq}(q, I_2^*(q)) =& (p_2 - m_2)E\left[Y_2 l_{qq}(q)\bar{F}_2(Y_2 l(q) + I_2^*(q))\right] - c_2 l_{qq}(q) + m_2 \mu_{Y_2} l_{qq}(q) \\ &- (p_2 - m_2)E\left[Y_2^2 l_q^2(q) f_2(Y_2 l(q) + I_2^*(q))\right]\end{aligned}$$

$$\begin{aligned}\Theta = &\left\{(p_2 - m_2)E\left[Y_2 l_{qq}(q)\bar{F}_2(Y_2 l(q) + I_2^*(q))\right] - c_2 l_{qq}(q) + m_2\mu_{Y_2} l_{qq}(q)\right.\\ &\left. - (p_2 - m_2)E\left[Y_2^2 l_q^2(q) f_2(Y_2 l(q) + I_2^*(q))\right]\right\}\hat{J}_{I_2 I_2}(q, I_2^*(q)) - \hat{J}_{qI_2}^2(q, I_2^*(q))\end{aligned}$$

According to Lemma 3.1 and Lemma 3.2 we have:

$$\Theta \geq \left\{(p_2 - m_2)E\left[Y_2 l_{qq}(q)\bar{F}_2(Y_2 l(q) + I_2^*(q))\right] - c_2 l_{qq}(q) + m_2\mu_{Y_2} l_{qq}(q)\right\}\hat{J}_{I_2 I_2}(q, I_2^*(q))$$

Since $Y_2 l_{qq}(q)$ is increasing with respect to Y_2 and $\bar{F}_2(Y_2 l(q)+I_2^*(q))$ is a decreasing function of Y_2, based on Chebyshev's inequality and part (b) of Lemma 3.2 we have:

$$\begin{aligned}\Theta &\geq \left\{(p_2 - m_2)\mu_{Y_2} l_{qq}(q)E\left[\bar{F}_2(Y_2 l(q) + I_2^*(q))\right] - c_2 l_{qq}(q) + m_2\mu_{Y_2} l_{qq}(q)\right\}\hat{J}_{I_2 I_2}(q, I_2^*(q))\\ &= \left\{(s_2\mu_{Y_2} - c_2) l_{qq}(q)\right\}\hat{J}_{I_2 I_2}(q, I_2^*(q))\end{aligned}$$

So if $l(q)$ is concave and $s_2\mu_{Y_2} \geq c_2$ then $\Pi(q, I_1, I_2^*(q))$ is concave.

c) Same as part (b) we can decompose the objective function into $J(q, I_1)$ and $\hat{J}(q, I_2)$. Since minimization preserves concavity both functions are concave and as a result the objective function is concave. □

Chapter 4

Optimal Capacity, Pricing and Production Policies under Supply and Demand Uncertainties

4.1 Introduction

The models in this chapter can be applied in both manufacturing and agricultural settings. We chose to adopt an agricultural context for two reasons. First, despite the importance of the agricultural sector in reducing poverty (Ligon and Sadoulet (2008) and Cervantes-Godoy and Dewbre (2010)) and its contribution to economic development (Diao *et al.* (2007) and Vogel (1994)), the management science and operations research community has spent far less attention to this field than that of manufacturing (Lowe and Preckel (2004)). Second, despite support from policy makers, such as the Canadian government's "Business Risk Management Programs," Martin and Stiefelmeyer (2011)), the agribusiness industry still has a lot to learn and apply from the fields of demand and production management (Taylor (2006)).

In all businesses, integration of the internal cross-functional operations is a prerequisite for coordination of the external supply chain operations (Braunscheidel and Suresh (2009) and Stevens and Graham (1989)) and agribusiness supply chain is no exception to that. Integration of production planning and sales is one of the important areas that requires systematic joint optimization. Agricultural supply chains used to suffer from excess stock as the overall global production often surpasses the demand. However, during the last few years the rate of demand growth is accelerating and quickly closing the the gap between supply and demand (Trostle (2008)). This phenomenon creates a competitive market further stressing the importance of sales management.

The main characteristic of agricultural production is the uncertainty of supply. The uncertainty in the demand side further exacerbates uncertainty in the agribusiness supply chain. Even staple food such as potatoes have uncertain demand (Taylor (2006)). To mitigate the effects of these uncertainties farmers often sign production contracts with produce handlers. In such a contract the farmer simply rents out the farm to the handler to avoid dealing with the demand and supply uncertainties. Handlers are the firms that purchase the agricultural products from the farmers and sell them to the wholesalers and/or retailers. In a production contract the handler bears the uncertainty of supply and the spot market demand. Although in production contract the farmer benefits from reducing the production risks (especially that of demand), they may lose the opportunity of making higher profits by selling later in the season at high prices. Thus, the farmer faces a trade-off between gaining control over prices and assuming the costs of mismatch between demand and supply due to their uncertainties. For more detailed discussion on production contracts the reader is referred to Kunkel *et al.* (2009)).

The aim of this study is therefore to develop efficient production and pricing policies for a farmer to optimally manage uncertainty in both supply and demand. The farmer may choose to rent out the farm under a production contract. She may also decide to change her production capacity by acquiring new land or selling a portion of the land. To capture these features we develop a multi-period model and extend it to the both cases where there is a fixed production cost or penalty cost. In addition, we examine the special case of a single-period newsvendor pricing under both supply and demand uncertainties and offer a unifying analysis framework. In particular, we introduce the concept of *Expected Demand Fill Rate (EDFR) elasticity* as an extension of the *Lost Sales Rate (LSR) elasticity*, developed by Kocabiyikoğlu and Popescu (2011) for the uncertain demand case. Previously it has been demonstrated that in the presence of supply uncertainty, it is difficult to find a proper structure for the optimum price (Li and Zheng (2006), Feng (2010)). In this study we show that by conditioning over EDFR elasticity a proper structure for the optimum price can be found.

4.1.1 Literature Review of Joint Lot-Sizing and Pricing Problems Under Uncertain Supply and Demand

In Section 2.4 of Chapter 2 we reviewed the studies in the field of joint lot-sizing and pricing under uncertain supply and demand. In this section we present some of the main differences between the current study and the existing literature.

He (2013), Surti *et al.* (2013) and Xu and Lu (2013) investigated the single period joint ordering and pricing under uncertain demand and supply. He (2013) and Xu and Lu (2013) assumed that the decisions are made sequentially and considered specific demand and production functions. Surti *et al.* (2013) considered both the simultaneous and sequential decisions. In the single period case of our study, we look at the case where the decisions are made simultaneously and consider generalized demand and yield functions.

Li and Zheng (2006), Chao *et al.* (2008), Yan and Liu (2009), Feng (2010), Feng and Shi (2012) and Zhu (2013) investigated this problem in multi-period case. They all assumed that the unsatisfied demand is backlogged which simplifies the analysis of the problem. Each of these studies considered a special form of supply uncertainty, nevertheless they all showed that it is cumbersome to characterize the behaviour of optimum price. In this study we assume that the unsatisfied demand is lost considering very general production and demand functions. Also we demonstrate that by conditioning over EDFR elasticity we can find a proper behaviour for optimal price.

4.2 Problem Description and Modelling

A farmer has the option to sign one of two types of contract with a handler. A *production contract* in which the farmer rents out the farm to the handler. In such a contract the handler is responsible for uncertainty of supply and the production cost. The second option is a *selling contract* where the farmer takes responsibility of the production and its related supply uncertainty. However, in this contract the farmer has the power of setting the product price and consequently managing the demand. The contract is signed before the cultivation season so that the pricing decision has to be made before realization of the yield.

We consider a multi-period finite horizon of length T periods. In each period the farmer has to make three main decisions. The first decision is finding the size of the land that the farmer wants to cultivate in period t, denoted q^t. The unit cost of production in period t is c^t. The total size of farmland to be cultivated or rented in period t is z^t. Thus, $z^t - q^t$ is the land that is available for renting. The demand for leasing the farm is random and represented by X^t in period t with ψ^t and Ψ^t its p.d.f. and c.d.f., respectively. We assume the demand varies in the interval $[\underline{k}^t, \overline{k}^t]$. The farmer revenue from renting out each unit of the farm is r^t in period t.

The production yield is highly volatile in agricultural environments. Let Y^t with p.d.f. g^t and c.d.f. G^t represent the uncertain factor of the yield in period t. We assume it varies in the interval $[\underline{l}^t, \overline{l}^t]$. Given this uncertainty, we model the production quantity in each period as the random variable $U^t(q^t, Y^t)$.

The farmer's second decision is to decide about the price of its produce during the planning horizon, $p^t \in [\underline{p}^t, \overline{p}^t]$. We consider a price sensitive and uncertain demand represented by $D^t(p^t, V^t)$. V^t is the random factor of demand with p.d.f. ϕ^t and c.d.f. Φ^t that varies in the interval $[\underline{v}^t, \overline{v}^t]$. Due to long lead times and large market sizes in the agribusiness supply chain, it is reasonable to assume that unsatisfied demand is lost. We assume that there is a penalty cost, b^t, for each unit of unsatisfied demand in period t. Due to inventory issues and the perishable nature of agricultural products, we assume that the farmer does not keep any inventory and would sell the excess production in a secondary market. We consider h^t as the unit salvage value where $h^t \leq c^t \leq \underline{p}^t$.

The third decision of the farmer is with regard to the size of the farm land.

According to the forecast of the farmer with regard to future demand, she might want to increase her production capacity or sell a portion of the land in order to maximize her profit. We consider a^t as the amount of land that is either acquired or sold in period t and we assume that the land is cultivable in the next period. The positive value of a^t represents acquiring new land while the negative value shows selling decision. δ^t represents the unit cost (price) of acquiring new farm land (selling farm land) in period t. If the available land at the beginning of the period t is z^t then $m^t = z^t + a^t$ is the cultivable land in the next period. Table 4.1 provides a summary of notation.

To simplify the exposition, in the sequel we will omit the time index whenever their omission does not create confusion to the reader. Considering α as the discount factor, a dynamic programming formulation of our problem is

$$f^t(z) = \delta^t z + \max_{0 \leq q \leq z, 0 \leq m, p} \left\{ J^t(z, p, q, m) \right\}$$

in which

$$\begin{aligned} J^t(z, p, q, m) =& pE\left[\min\left\{D^t(p, V^t), U^t(q, Y^t)\right\}\right] + h^t E\left[U^t(q, Y^t) - D^t(p, V^t)\right]^+ \\ & - b^t E\left[D^t(p, V^t) - U^t(q, Y^t)\right]^+ - c^t q - \delta^t m + r^t E\left[\min\left\{z - q, X^t\right\}\right] \\ & + \alpha f^{t+1}(m). \end{aligned} \tag{4.1}$$

At the end of the planning horizon, we assume that the unit value of the available land is r^T so that

$$f^T(z) = r^T z. \tag{4.2}$$

For analytical tractability of the problem we consider the following assumptions.

Assumption 4.1. *We assume that for all the values of t, $u^t(q, y^t)$ is an increasing function of y^t and q. In addition we assume it is a continuous, twice differentiable and concave function of q.*

A special case that satisfies Assumption 1 is the stochastically proportional yield where $U^t(q, Y^t) = Y^t q$. Another example that satisfies Assumption 1 is $U^t(q, Y^t) = \min\{Y^t, q\}$.

Assumption 4.2. *Consider $d^t(p, v^t)$ as the deterministic demand in period t and $\tau^t(p, v^t) = pd^t(p, v^t)$ as the revenue function when demand is deterministic. We assume $d^t(p, v^t)$ is a decreasing function of p and an increasing function of v^t. Also we assume $\tau^t(p, v^t)$ is a concave function of p for all the values of t.*

The above assumptions are common in the revenue management literature (e.g., Petruzzi and Dada (1999), Kocabiyikoğlu and Popescu (2011) and Pang (2011)).

Consider $u^t(q, y^t)$ as the deterministic form of the yield function. In order to facilitate the theoretical analysis of the problem, we introduce $v^t(p, u^t(q, y^t))$ and $y^t(q, d^t(p, v^t))$ as follows:

$$d^t(p, v^t(p, u^t(q, y^t))) = u^t(q, y^t) \tag{4.3}$$

$$u^t(q, y^t(q, d^t(p, v^t))) = d^t(p, v^t) \tag{4.4}$$

So if the observation of the uncertain part of demand takes the value of $v^t(p, u^t(q, y^t))$

then the supply matches the demand and the farmer will not experience any overproduction or underproduction. This is a generalized form of the stocking factor introduced by Petruzzi and Dada (1999). Similarly, if the observation of the uncertain part of yield takes the value of $y^t(q, d^t(p, v^t))$, supply and demand will be equal.

One of the goals of this study is to find the optimum production and pricing policies with regard to the current available land. To this end we introduce two classes of policies.

Definition 4.1 (One-Sided Production, Pricing and Capacity Planning Policy)**.** *A production, pricing and capacity planning policy,* (q^{*t}, p^{*t}, m^{*t}) *is one-sided if given* z*, the available acreage, there exist* $z^{\circ t}$ *such that* $q^{*t} = \min\{z, z^{\circ t}\}$, $p^{*t} = \max\{P^t(z), P^t(z^{\circ t})\}$, *where* $P^t(z)$ *is a non-increasing function of* z*, and* $m^{*t} = \hat{m}^t$ *where* $\hat{m}^t$ *is a constant value in each period.*

Definition 4.2 (Two-Sided Production, Pricing and Capacity Planning Policy)**.** *A production, pricing and capacity planning policy,* (q^{*t}, p^{*t}, m^{*t})*, is two-sided if given* z*, the available acreage, then*

$$q^{*t} = \begin{cases} 0 & \text{if } z \leq n^t \\ z & \text{if } n^t < z < N^t \\ N^t & \text{if } N^t \leq z \end{cases}, \; p^{*t} = \begin{cases} P^t(n^t) & \text{if } z \leq n^t \\ P^t(z) & \text{if } n^t < z < N^t \\ P^t(N^t) & \text{if } N^t \leq z \end{cases} \text{ and } m^{*t} = \bar{m}^t$$

where (n^t, N^t)*,* $(\underline{z}^t, \bar{z}^t)$ *and* $\bar{m}^t$ *are parameters and* $P^t(z)$ *is non-increasing functions of* z*.*

We will provide comments on the intuition behind these policies in the context of the models that we discuss later in the paper.

4.2.1 Expected demand fill rate elasticity

The concept of LSR elasticity was first introduced by Kocabiyikoğlu and Popescu (2011) in the context of a newsvondor problem with pricing under deterministic yield. Some other studies, such as de Vericourt and Lobo (2009), Chen *et al.* (2011) and Pang (2011), have also used conditioning over LSR elasticity to obtain some properties for their objective functions. Here we generalize this concept by incorporating the uncertainty of yield.

For any values of p and q, we define $s^t(p,q)$ as the expected amount of demand that is filled by slightly changing the values of p and q in period t. In another word, $s^t(p,q)$ represents the slope of the demand satisfaction function in terms of p and q. We call $s^t(p,q)$ the expected demand fill rate (EDFR) and it is equal to

$$s^t(p,q) = E\left[U_q^t(q,Y^t)\bar{\Phi}^t(v^t(p,U^t(q,Y^t)))\right], \tag{4.5}$$

in which $U_q^t(q,Y^t)$ is the derivative of $U^t(q,Y^t)$ with respect to q and $\bar{\Phi}^t(.) = 1-\Phi^t(.)$. The term $\bar{\Phi}^t(v^t(p,U^t(q,Y^t)))$ defines the probability of demand exceeding supply. Let $w^t = p^t - h^t + b^t$ be the unit lost sale cost. EDFR elasticity determines the variation of the EDFR with respect to variation of w^t for a given quantity and it can be defined as

$$\Theta^t(p,q) = \frac{-w^t s_p^t(p,q)}{s^t(p,q)}, \tag{4.6}$$

where $s_p^t(p,q)$ is the first derivative of $s^t(p,q)$ with respect to p. EDFR elasticity can be considered as the generalization of the lost sale rate elasticity introduced by Kocabiyikoğ̃lu and Popescu (2011). For the case that $U^t(q,Y^t) = q$ the EDFR elasticity and LSR elasticity are equal.

Table 4.1: Summary of notation

Parameters:	
T	Length of the planning horizon
c^t	Unit production cost in period t
h^t	Unit salvage value in period t
b^t	Unit penalty cost in period t
r^t	Revenue from renting out each unit of farm in period t
δ^t	Unit cost(price) of acquiring new farm land (selling farm land) in period t
α	Discount factor
K	Fixed production cost
Y^t	Random factor of the yield in period t with p.d.f g^t and c.d.f G^t that varies in the interval $[\underline{l}^t, \overline{l}^t]$
V^t	Random factor of the demand in period t with p.d.f ϕ^t and c.d.f Φ^t that varies in the interval $[\underline{v}^t, \overline{v}^t]$
X^t	Random demand for leasing the farm in period t with p.d.f ψ^t and c.d.f Ψ^t that varies in the interval $[\underline{k}^t, \overline{k}^t]$
Decision variables:	
q^t	The size of the land that is put under cultivation in period t
p^t	The price for each unit of production in period t
a^t	The amount of land that is either acquired or sold in period t
m^t	The amount of land at the end of period t
q^t	The size of the available land at the end of period t
z^t	The size of the available land at the beginning of period t
$U(q^t, Y^t)^t$	The yield function in period t
$D(p^t, Y^t)^t$	The demand function in period t

4.3 Single Period Model

In this section we consider the single period case. The structure of the problem in the single period case is same as that of the newsvendor problem with pricing under uncertain supply and demand. By analyzing this problem we are perusing two goals. First create the groundwork for the multi-period case, and second develop a better understanding on the newsvendor problem with pricing which is the foundation of many related problems in this field. In this section we assume that there are no penalty costs and no limitation on the production quantity (as a result the renting option is also neglected). The objective function can be formulated as follows:

$$f(p,q) = R(p,q) + hE\left[U(q,Y)\right] - cq \tag{4.7}$$

$$R(p,q) = (p-h)E\left[\min\left\{D(p,V), U(q,Y)\right\}\right] \tag{4.8}$$

As it was mentioned earlier, de Vericourt and Lobo (2009), Chen *et al.* (2011) and Pang (2011) applied conditioning over LSR elasticity to find some nice structural results for the newsvendor problem with pricing when yield is certain. Uncertainty of yield limits the application of LSR elasticity in our problem. However conditioning over EDFR elasticity enables us to find some proper structural results in our problem.

Proposition 4.1. *Consider $p^*(q)$ as the optimum price for a given q and $q^*(p)$ as the optimum quantity with respect to p, then:*

a) if $\Theta(p,q) \geq 1/2$ the objective function is concave.

b) if $\Theta(p^*(q),q) \geq 1/2$ the objective function is unimodal.

c) if $\Theta(p, q^*(p)) \geq 1/2$ *the objective function is unimodal.*

The proof of Proposition 4.1 and subsequent proofs are provided in Appendix A.

Proposition 4.1 provides the conditions that guarantee the uniqueness of the optimal solution. In this situation we can use first order condition to find the optimal p and q. In the following proposition we offer a different condition that guarantees the unimodality of the objective function.

Proposition 4.2. *If* $\Theta(p, q)$ *is increasing in* q*, then* $\Theta(p^*(q), q) \geq 1$*.*

Proposition 4.2 simplifies the analysis of the problem. In the case that we have a more specific structure for the problem, more detailed results can be found based on this proposition. For example in Corollary 4.1 we consider the case where uncertainty of yield is additive and demonstrate that when V and Y have special distributions, the objective function is unimodal.

Corollary 4.1. *Let* $U(q, Y) = u(q) + Y$*. Then:*

a) If V *is IFR (has an increasing failure rate) and* d_p/d_v *is decreasing in* v *then* $f(p, q)$ *is unimodal.*

b) If Y *is DFR (has a decreasing failure rate) then* $f(p, q)$ *is unimodal.*

The uniform and Erlang distributions are examples of distributions that have an increasing failure rate. The Weibull distribution with a scale parameter β satisfying $0 < 1/\beta < 1$ is DFR. The exponential distribution is both IFR and DFR.

4.4 Multi-Period Model with no Fixed Costs

In this section we investigate the multi-period case of the problem. We assume there are no fixed costs associated to production decisions. First we look at some of structural properties of the objective function.

Proposition 4.3.

a) $f^t(z)$ *is non-decreasing for all the periods.*

b) $J^t(z, p, q, m)$ *is supermodular in* (z, q).

According to Proposition 4.3, the farmer benefits from having a larger land at the beginning of periods. This is understandable when we see that in our context the size of land plays the role of capacity of production. As a result more available land means more opportunity for production and renting out the farm.

Supermodularity of $J^t(z, p, q, m)$ implies that the optimum production quantity is non-decreasing as the available land increases. In this section we consider two versions of the problem. First we assume that the firm does not incur any penalty cost for the unsatisfied demand. This assumption is reasonable in agricultural environment since usually the size of the market is much bigger than the size of the individual producers. In the second case we consider the problem with penalty cost.

4.4.1 Case 1: No penalty cost

In this situation $J^t(z, p, q, a)$ can be rewritten as follows:

$$J^t(z, p, q, m) = R^t(p, q) + L^t(z, q, a) + \alpha f^{t+1}(m) \tag{4.9}$$

in which

$$R^t(p,q) = (p - h^t)E\left[\min\left\{D^t(p,V^t), U^t(q,Y^t)\right\}\right] \tag{4.10}$$

$$L^t(z,q,m) = h^t E\left[U^t(q,Y^t)\right] - c^t q - \delta^t m + r^t E\left[\min\left\{z-q, X^t\right\}\right] \tag{4.11}$$

In the next proposition we extend the results that we have in Proposition 4.1 to the multi-period case.

Proposition 4.4. *If $\Theta^t(p,q) \geq 1/2$, $\forall t$ then $f^t(z)$ is concave for all the values of t.*

Proposition 4.4 provides conditions that guarantee the uniqueness of the solution in each period. As a result, in each period there is a unique p, q and m that maximize the profit. Consider $\tilde{q}(z)$ as the optimum production quantity which is a function of available land. Then based on Proposition 4.3, $\tilde{q}(z)$ is a non-decreasing function of z. Finding similar structural properties for the optimal price would require more limiting conditions. The next Theorem provides conditions in which one-sided production, pricing and capacity planning policy is optimal.

Theorem 4.1. *If $\Theta^t(p,q) \geq 1$, $\forall t$, then one-sided production, pricing and capacity planning policy is optimal.*

According to Theorem 4.1, if the mentioned condition is satisfied, we put all the available land under cultivation unless $z > z^{\circ t}$. In this case $q^* = z^{\circ t}$ and the rest of the land is available for renting. The requisition policy is fairly simple and intuitive. If the available land is more than $\hat{m}^t$, the farmer sells a portion of the land and keep $\hat{m}^t$ unit of the farm land. If $z < \hat{m}^t$, the farmer buys $\hat{m}^t - z$ unit of land. When $z \leq z^{\circ t}$, the optimum price decreases as the available land increases. When z passes $z^{\circ t}$, the optimum price remains constant and it is equal to $P^t(z^{\circ t})$.

Produce or rent?

The first decision that the farmer has to make is to whether start production or rent out all the available farm. The farmer can avoid any risk that is related to production or demand by renting out all the available farm. In this situation the farmer does not have to worry about the optimum price and quantity. However starting her own production may provide a situation where the farmer can increase her profit by setting a proper price and putting enough land under cultivation. In the next proposition we provide the conditions in which it is not optimal for the farmer to start her own production.

Proposition 4.5. *It is not optimal to produce in period t, if and only if for all the values of p the following inequality holds:*

$$(p - h^t)s^t(p, 0) + h^t E\left[U_q^t(0, Y^t)\right] \leq c^t + r^t\bar{\Psi}^t(z) \tag{4.12}$$

If $u^t(0, y^t) = 0$, $\forall y^t$, then (4.12) can be simplified as

$$\overline{p}^t E\left[U_q^t(0, Y^t)\right] \leq c^t + r^t\bar{\Psi}^t(z) \tag{4.13}$$

Condition (4.12) is intuitive; it states that the marginal increase in revenue when $q = 0$ should not exceed the sum of the marginal production cost and the marginal decrease in revenue with regard to renting the farm. It is interesting to note that when $u^t(0, y^t) = 0$, $\forall y^t$, condition (4.13) is not dependent on the salvage value. Although the price in the secondary market affects the production quantity, it has no effect on the decision of whether or not to start production.

Practitioners and academics sometimes avoid explicitly modelling uncertainties by using expected values of the random variables. This approach can simplify and facilitate the analysis of the problem, however, it lead to suboptimal outcomes where the shortage and overage costs may be underestimated. It is therefore interesting to analytically study how ignoring uncertainty can affect the farmer's decision on production. Consider $\bar{f}^t(z)$ as the value function of the problem where μ_{Y^t} is used instead of Y^t (certain yield case) and $\bar{\bar{f}}^t(z)$ as the value function of the case where V^t is replaced by its mean value, μ_{V^t} (certain demand case). Also consider $\xi^t(p,q,v^t) = Pr\left\{U(q,Y^t) \geq d^t(p,v^t)\right\}$ as the overproduction probability under the certain demand case. Then we have the following lemma.

Lemma 4.1.

a) If $u^t(q,y^t)$ is concave with respect to y_t, then $\bar{f}^t(z) \geq f^t(z)$.

b) If $d^t(p,v^t)$ is concave with respect to v^t, then $\bar{\bar{f}}^t(z) \geq f^t(z)$.

c) If

$$\frac{d^t_{v^t}(p,v^t)\xi^t_{v^t}(p,q,v^t)}{d^t_{v^tv^t}(p,v^t)\xi^t(p,q,v^t)} \leq -1 \quad \forall p,q,v^t, \tag{4.14}$$

then $\bar{\bar{f}}^t(z) \geq f^t(z)$.

d) If $d^t(p,v^t)$ is convex with respect to v^t and

$$\frac{d^t_{v^t}(p,v^t)\xi^t_{v^t}(p,q,v^t)}{d^t_{v^tv^t}(p,v^t)\xi^t(p,q,v^t)} \geq -1 \quad \forall p,q,v^t, \tag{4.15}$$

then $\bar{\bar{f}}^t(z) \leq f^t(z)$.

Lemma 4.1 shows that using a certain yield model, when $u^t(q,y^t)$ is concave, will result in overestimation of the profit. It also shows that, under some condition, the certain demand case may result in either overestimation or underestimation of the profit. Misleading values of the expected profit can also cause the farmer to deviate from the optimal decision. In the next proposition we investigate how removing the uncertainty from the model can affect the production decision. To that end we let $\bar{J}^t(z,p,q,a)$ and $\bar{\bar{J}}^t(z,p,q,a)$ be the functions that correspond to $\bar{f}^t(z)$ and $\bar{\bar{f}}^t(z)$, respectively.

Proposition 4.6. *Assuming that $u^t(0,y^t)=0$, $\forall y^t$ we have*

a) If $u^t(q,y^t)$ is concave with respect to y_t, then $\bar{J}^t_q(z,p,0,a) \geq J^t_q(z,p,0,a)$.

b) If $d^t(p,v^t)$ is concave with respect to v^t or,

$$\frac{d^t_{v^t}(p,v^t)\xi^t_{v^t}(p,q,v^t)}{d^t_{v^tv^t}(p,v^t)\xi^t(p,q,v^t)} \leq -1 \quad \forall p,q,v^t,$$

then $\bar{\bar{J}}^t_q(z,p,0,a) \geq J^t_q(z,p,0,a)$.

c) If $d^t(p,v^t)$ is convex with respect to v^t and

$$\frac{d^t_{v^t}(p,v^t)\xi^t_{v^t}(p,q,v^t)}{d^t_{v^tv^t}(p,v^t)\xi^t(p,q,v^t)} \geq -1 \quad ,\forall p,q,v^t,$$

then $\bar{\bar{J}}^t_q(z,p,0,a) \leq J^t_q(z,p,0,a)$.

According to Proposition 4.6 part (a), under some situations, by using certain yield model, the farmer may start production while it is optimal to just rent out all the available farm. The same situation may happen when the farmer is using a certain demand model (Proposition 4.6 part (b)). From part (c), we deduce that the farmer may rent out all the available farm while it is better to have some production (by using the certain demand case).

4.4.2 Model with penalty cost

Here we assume that every unit of unsatisfied demand would be penalized. Adding penalty cost would complicate the analysis of the problem. For the sake of simplicity, in this section we assume that the uncertainty of demand has an additive form so that $D^t(p, V^t) = d^t(p) + V^t$. The random variable V^t is the random factor of demand and without loss of generality we assume $E[V^t] = 0$. Consider $p^t(d)$ as the reverse function of $d^t(p)$. Since $d^t(p)$ is a decreasing function of p, then there is a one to one relation between demand and price. Based on the upper and lower bounds of the price, we can define the upper and lower bounds for d accordingly. Assume that $d \in [\underline{d}^t, \overline{d}^t]$. Therefore, instead of the demand function, we can use price as a function of demand in the form of $P^t(d, V^t) = p^t(d) + V^t$. Using the price function, the recursive form of the problem is:

$$f^t(z) = \delta^t z + \max_{0 \le q \le z, 0 \le m, d} \left\{ J^t(z, d, q, m) \right\}$$

in which

$$\begin{aligned} J^t(z, d, q, m) = & p(d) E\left[\min\left\{d + V^t, U^t(q, Y^t)\right\}\right] + h^t E\left[U^t(q, Y^t) - d - V^t\right]^+ \\ & - b^t E\left[d + V^t - U^t(q, Y^t)\right]^+ - c^t q - \delta^t m \\ & + r^t E\left[\min\left\{z - q, X^t\right\}\right] + \alpha f^{t+1}(m) \end{aligned} \tag{4.16}$$

To study the value function we define the lost sale rate elasticity in the deterministic yield case as follows:

$$\Gamma^t(d,q) = -\frac{(p^t(d) - h^t + b^t)\phi(q-d)}{p^t_d(d)\bar{\Phi}(q-d)}.$$

In the following proposition we establish conditions under which the value function is concave.

Proposition 4.7. *If* $\Gamma^t(d,q) \geq 1$ *and* $p^t_{dd}(d) + p^t_d(d)d \leq 0$, $\forall d,q,t$, *then* $f^t(z)$ *is concave for all the values of* t*.*

In Proposition 4.7 we used the same conditions that are used in the study of Pang (2011) to prove the optimality of the (s,S) policy. Proposition 4.7 bridges the results of the certain yield case to the uncertain yield case. Note that in Proposition 4.4 we assumed that $d(p)$ has a general structure. It is easy to show that for some more restricted demand functions, such as the linear case, the conditions mentioned in Proposition 4.4 are sufficient to prove the concavity of $f^t(z)$ in the presence of a penalty cost. We also note that there are some situations in which EDFR elasticity is more than 0.5 while LSR elasticity is less (An example of this situation is provided in Appendix B). Therefore we cannot claim that EDFR elasticity conditions imply those of the LSR elasticity. In the next theorem we show when a one-sided policy is optimal.

Theorem 4.2. *If the conditions of Proposition 4.7 are satisfied and* $\Theta^t(d,q) \geq 1$ *for all the values of* d,q,t *then a one-sided production, pricing and capacity planning policy is optimal.*

In the presence of a penalty cost we require more restricted conditions to prove the optimality of the one-sided production, pricing and capacity planning policy. The structure of the result is the same as the one that is described for Theorem 4.1. In term of the production decisions, the model with a penalty cost shows almost the same behaviour as that with no penalty costs.

Remark 4.1. *It is not optimal to produce in period t, if and only if for all the values of p we have the following inequality:*

$$(p(d) - h^t + b^t)s^t(p,0) + h^t E\left[U_q^t(0,Y^t)\right] \leq c^t + r^t\bar{\Psi}^t(z) \tag{4.17}$$

If $u^t(0,y^t) = 0$, $\forall y^t$, then (4.17) can be simplified to

$$(p(\overline{d})^t + b^t)E\left[U_q^t(0,Y^t)\right] \leq c^t + r^t\bar{\Psi}^t(z) \tag{4.18}$$

The proof is provided analogous to Proposition 4.5. Similar to the case of no penalty cost model, we note that when $u^t(0,y^t) = 0$, $\forall y^t$, the salvage value has no effect on the decision of starting production. However, we see that the penalty cost plays a role. Having a larger penalty cost motivates the farmer to start production to avoid the costs of unsatisfied demand.

4.5 Multi-Period Model with Production Fixed Cost

In some situations, the farmer requires to provide some facilities and equipments as a prerequisite of the production. This can impose a fixed production cost to the farmer.

We consder M^t as the production fixed cost in period t. We define $\lambda(q)$:

$$\lambda(q) = \begin{cases} 0 & \text{if } q = 0 \\ 1 & \text{if } q > 0 \end{cases},$$

then we define $\hat{J}^t(z, p, q, m)$ as follows:

$$\hat{J}^t(z, p, q, m) = R^t(p, q) + L^t(z, q, a) + M^t\lambda(q) + \alpha f^{t+1}(m). \tag{4.19}$$

In the presence of fixed production costs, production may become less profitable. As a result the farmer may need to follow a different policy.

Theorem 4.3. *Let $b^t = 0$, $\forall t$. If $U^t(0, Y^t) = 0$ and $\Theta^t(p, q) \geq 1$, $\forall t$, then the two-sided production, pricing and capacity planning policy is optimal.*

According to Theorem 4.3, in period t, when we have a fixed production cost it is optimal to not start production unless the available land has a certain level, n^t. Also if z is greater than n^t but less than N^t all the available land will go under cultivation. The corresponding price will decrease as z increases. If $z > N^t$, then the optimal size of the land for cultivation is N^t and the rest will be rented out. As a result we keep the optimal price constant for all the values of z greater than N^t.

In the presence of penalty cost, the conditions for optimality of two-sided production, pricing and capacity planning policy become more restrictive. The following remark addresses these conditions.

Remark 4.2. *Let $b^t > 0$, $\forall t$. If the conditions of Theorem 4.2 are satisfied, then a two-sided production, pricing and capacity planning policy is optimal.*

The proof is analogous to Theorem 4.3.

Remark 4.2 extends Theorem 4.3 when there is a penalty cost. In this situation the farmer will no start production until she is certain that the expected profit from production can compensate for the fixed cost.

4.6 Numerical Analysis

In this section we explore some of the characteristics of the model using numerical analysis. In order to implement a reasonable range of parameters in our example, we use the California almond industry's data. Given the availability of data, we consider the model with no penalty and fixed cost. We assume that the planning horizon is 6 years ($T = 6$) with a discount factor of 0.95. According to Freeman *et al.* (2012), the almond yield varies between 1600 lb to 2800 lb per acre in California. We assume that the uncertainty of yield has a stochastically proportional form and it is uniformly distributed between 1600 lb and 2800 lb. Also Freeman *et al.* (2012) estimated the overall cost of production, land renting rate and land purchasing rate to be \$3675/acre, \$349/acre and \$7000/acre respectively. We assume that the salvage value is \$1.09/lb which is the production cost per pound without considering cash overhead costs and non-cash overhead costs (Freeman *et al.*, 2012). We consider a small size farmer with average potential demand of 600000 lb that is varying normally with a standard deviation of 1000 lb. These values have been chosen to make the numerical computation tractable. The demand is considered to be linear in which \$1 increase in price reduces the potential demand by 16% (Russo *et al.* (2008)). According to an increasing trend in almond consumption during the last decade (Russo *et al.* (2008)), we assume that the average potential demand of the farmer increases 5% each year.

4.6.1 Effect of dynamic pricing

In the study of de Vericourt and Lobo (2009) it was demonstrated that the initial asset of the firm has a reverse effect on the effectiveness of dynamic pricing. Here we investigate the same question where the initial asset is determined by the acreage of the land that the farmer owns in the beginning of the first period. We compare two policies: a dynamic pricing policy in which the farmer can change the price in each period and a static pricing policy in which the farmer finds a single optimal price for all the periods and is not allowed to change it.

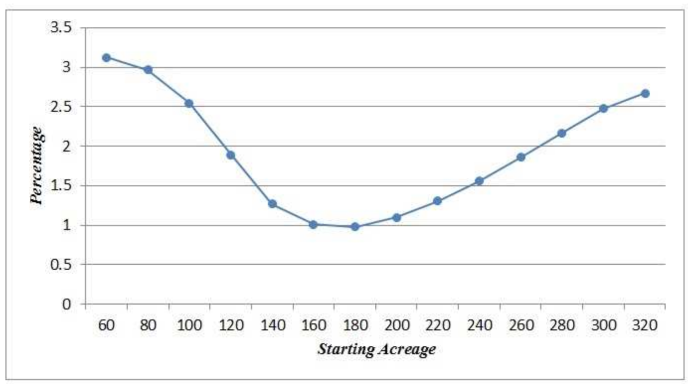


Figure 4.1: Difference between dynamic and static pricing with respect to initial asset

Figure 4.1 demonstrates the value of dynamic pricing. In contrast to the finding of de Vericourt and Lobo (2009), the difference between dynamic and static pricing is not monotone. The major reason for this difference is the shape of the demand and the nature of the production. In our analysis we assumed that demand is increasing however de Vericourt and Lobo (2009) considered no variation in the average of demand. Also in our study the production capacity is determined by the farmland. If the farmer wants to increase the production capacity she has to endure the acquisition cost. In the study of de Vericourt and Lobo (2009) the asset is money which increases when the firm makes profit.

When the demand is increasing, in a static pricing strategy, the farmer faces a dilemma. Low prices may be suitable for the beginning periods (since the demand is low), but as the time goes on, the farmer can take the advantage of high demand in the last periods by charging high prices. When the initial acreage is low the farmer buys a large amount of farm in the first few periods to be able to satisfy the demand in the latter periods. In this way, the farmer can match the supply and demand from the second period but the first period remains a problem (the size of the farm is too small to satisfy the demand). In dynamic pricing the farmer sets a high price for the first period to match the production and demand. However in static pricing the price is almost low in the first period because it can not be changed through time and he needs to compromise (high price is not optimum for the second and third periods). The difference between profits in the first period is the major reason for the high difference between the dynamic and static pricing strategies. As the size of the farm increases the variation of the price decreases for the dynamic case and it gets closer to the static case. In both cases the cost of acquisition prevents the farmer from buying more farm to satisfy the demand completely and the farmer relies more on high prices. When z_1 is high, again the variation of the price goes up for the dynamic case. The farmer wants to take advantage of low demand in the beginning period and high demand in final periods when he has enough capacity for production. However in the static case, the price should stay constant and the farmer has to compromise between low demand in the beginning and high demand at final stages. We believe this is the main reason for the higher difference between profits for high values of z_1.

Despite the difference between the profits, still the percentage of the difference stays below 3.5% which needs to be considered by the farmer in implementation of dynamic pricing as it may be costly.

4.6.2 Effect of renting option

In this part we investigate the role of the renting option. Figure 4.2 shows the gain for the farmer when there is a renting option with respect to the renting rate.

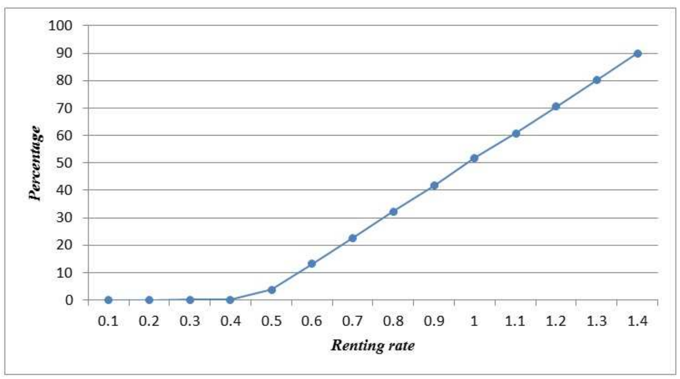


Figure 4.2: profit increase with respect to renting rate

As the renting rate goes up, the farmer's profit increases. This is intuitive as the share of renting revenue in the farmer's profit increases as the renting rate goes up.

Figure 4.3 shows the behaviour of the average optimal prices with respect to the renting rate.

When the renting rate is low, the farmer prefers to allocate all the available land to production. As a result a slight increase in the renting rate does not change the decision of the production quantity as well as the price. However when r increases, renting out the farm becomes more appealing. So the farmer assigns less farmland

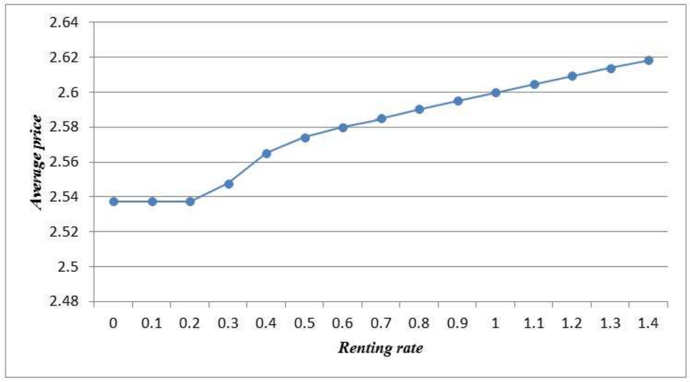


Figure 4.3: Behaviour of average optimal prices with respect to renting rate

to production and increases the price to match the production and demand. This phenomena is interesting since the growth of renting fees does not only affects the price of producers which use renting farm lands (by increasing the cost of production), it also affects the price of the farmers which are using their own land for production, even when their cost of production is constant.

4.6.3 Effect of uncertainty of supply

Here we investigate the behaviour of the optimal policies as the uncertainty of yield increases. We assume that the average yield stays constant (2200 lb/acre) while the standard deviation increases. Figure 4.4 shows the behaviour of the average amount of land that is assigned to production with respect to the uncertainty of yield.

Since in the farmer enjoys the benefit of the salvage value, the cost of under-production is more than overproduction. As a result when the uncertainty of yield increases the farmer tends to increase q to alleviate the risk of underproduction. In the same manner, the farmer tends to acquire more land during different periods to increase the production capacity. This behaviour is demonstrated in Figure 4.5.

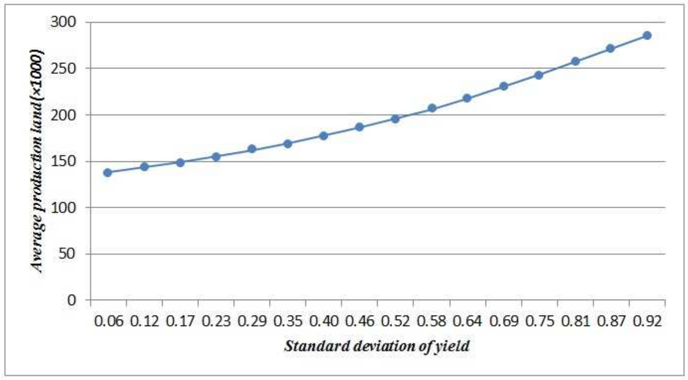


Figure 4.4: Behaviour of production land with respect to standard deviation of supply

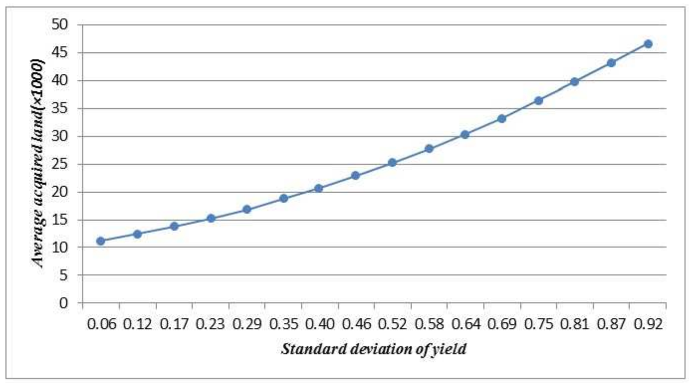


Figure 4.5: Behaviour of average land change with respect to standard deviation of supply

Another policy that decreases the chance of underproduction is increasing the price so that the demand declines. Figure 4.6 shows the increasing trend of price as a mitigation strategy for the increase of yield uncertainty.

As we increase the uncertainty of yield the expected profit decreases. This behaviour is demonstrated in Figure 4.7. According to Lemma 4.1 part (a) the highest profit happens when the yield is certain. In this situation the farmer's risk of underproduction is minimized.

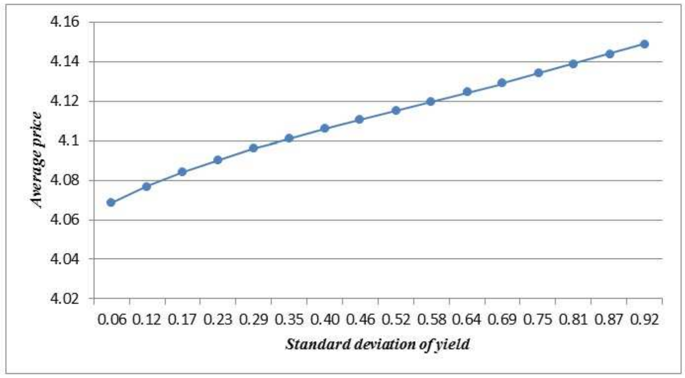


Figure 4.6: Behaviour of average price with respect to standard deviation of supply

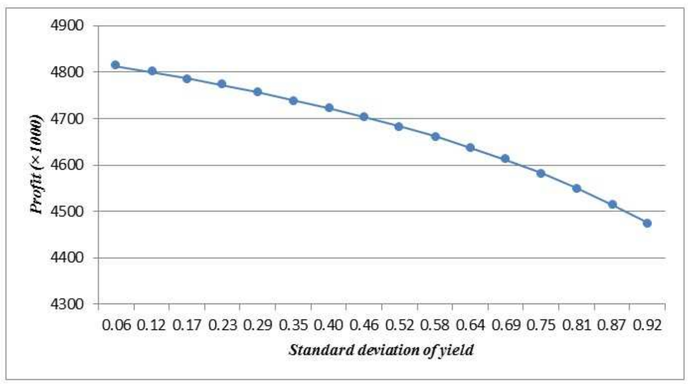


Figure 4.7: Behaviour of expected profit with respect to standard deviation of supply

4.7 Conclusion

In this section we looked at the problem of production planning, pricing and capacity planning of a farmer in a single and multi-period cases. We introduced the concept of EDFR elasticity as an extension to LSR elasticity developed by Kocabiyikoğlu and Popescu (2011). This elasticity facilitates the analysis of the newsvendor problem with pricing under uncertain supply and demand where the application of LSR elasticity seems to be cumbersome.

In the multi-period case we provided the condition where the optimal policy of the farmer is one-sided production, pricing and capacity planning. This is an interesting policy because it provides a structural form for pricing, despite the fact that some of the previous researchers such as Li and Zheng (2006) and Feng (2010) demonstrated that under uncertainty of supply, pricing decision usually does not show a well formed behaviour.

In the case that there is a fixed cost for production, the farmer switches to a two-sided production, pricing and capacity planning policy. In this situation the farmer has to make sure that the benefit from production surpasses the fixed cost. Similar as the one-sided policy, optimal prices have a proper structural form in this situation too.

We looked at the conditions where it is not optimal for the farmer to start production and interestingly we find that this decision is not affected by the salvage value. This result is not intuitive since an increase in the salvage value, decreases the risk of overproduction which seems to be a good motivation for starting the production. In contrast, a penalty cost has a direct affect on this decision. Increasing the penalty cost motivates the farmer to start production in order to reduce the unmet demand. Removing the uncertainty from the model can result in overestimation or underestimation of the profit. This can be influential on the production decision. The farmer may start or stop production while it is optimal to do the opposite action.

In our numerical analysis we found that the importance of dynamic pricing does not necessarily diminish as the initial asset goes up which is in contrast to the finding of de Vericourt and Lobo (2009). When increasing the production capacity is costly and the demand is increasing, in some cases the role of dynamic pricing becomes more important for the firm that has a larger initial assets. We also show that in the case where the cost of underproduction is relatively higher than that of overproduction, the farmer may choose to increase the production quantity as well as the price in order to alleviate the risks associated with a volatile yield.

Appendix A

We need the following lemma to proof Proposition 4.1.

Lemma 4.2. *We have the following equations:*

a) $v_u(p, u(q, y)) = \frac{1}{d_v(p,v(p,u(q,y)))}$

b) $v_p(p, u(q, y)) = -\frac{d_p(p,v(p,u(q,y)))}{d_v(p,v(p,u(q,y)))}$

c) $s_p(p, q) = E\left[\frac{U_q(q,Y)d_p(p,v(U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right]$

Proof. a) By taking the derivative with respect to q from both sides of Equation (4.3) and based on the chain rule we have:

$$\begin{aligned}
&d_v(p, v(p, u(q, y)))v_u(p, u(q, y))u_q(q, y) = u_q(q, y) \\
\Rightarrow &v_u(p, u(q, y)) = \frac{1}{d_v(p, v(p, u(q, y)))}
\end{aligned}$$

b) By taking derivatives with respect to p from both sides of Equation (4.3) we have:

$$\begin{aligned}
&d_v(p, v(p, u(q, y)))v_p(p, u(q, y)) + d_p(p, v(p, u(q, y))) = 0 \\
\Rightarrow &v_p(p, u(q, y)) = -\frac{d_p(p, v(p, u(q, y)))}{d_v(p, v(p, u(q, y)))}
\end{aligned}$$

c) By taking derivative with respect to p from both sides of Equation (4.5), we have:

$$s_p(p,q) = -E\left[U_q(q,Y)\phi(v(p,U(q,Y)))v_p(p,U(q,Y))\right]$$

and according to part (b) we have:

$$s_p(p,q) = E\left[\frac{U_q(q,Y)d_p(p,v(p,U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right]$$

□

Proof of Proposition 4.1.

a) Consider $\hat{\tau}(p,v) = (p-h)d(p,v)$. First we show that if $\tau(p,v)$ is concave, then $\hat{\tau}(p,v)$ is concave. We have:

$$\hat{\tau}_{pp}(p,v) = 2d_p(p,v) + (p-h)d_{pp}(p,v)$$

If $d_{pp}(p,v) \leq 0$, since $d(p,v)$ is decreasing in p and $p \geq h$, then $\hat{\tau}_{pp}(p,v) \leq 0$. Otherwise we have:

$$\begin{aligned}\hat{\tau}_{pp}(p,v) &= 2d_p(p,v) + (p-h)d_{pp}(p,v)\\ &\leq 2d_p(p,v) + pd_{pp}(p,v)\\ &= \tau_{pp}(p,v) \leq 0\end{aligned}$$

so $\hat{\tau}(p,v)$ is concave. The conditional form of $R(p,q)$ is:

$$R(p,q) = E\left[\int_{A}^{v(p,U(q,Y))} \hat{\tau}(p,\epsilon)\phi(\epsilon)d\epsilon\right] + (p-h)E\left[\int_{v(p,U(q,Y))}^{B} U(q,Y)\phi(\epsilon)d\epsilon\right] \tag{4.20}$$

According to Equation (4.20) and by using Lemma 4.2 part (a) and (b), the second derivatives of $R(p,q)$ with respect to p and q are:

$$R_{qq} = (p-h)E\left[U_{qq}(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right] - (p-h)E\left[\frac{U_q^2(q,Y)\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right] \tag{4.21}$$

$$R_{pq} = E\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right] + (p-h)E\left[\frac{U_q(q,Y)d_p(p,v(p,U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right] \tag{4.22}$$

$$R_{pp} = \Omega(p,q) - (p-h)E\left[\frac{d_p^2(p,v(p,U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right] \tag{4.23}$$

in which

$$\Omega(p,q) = E\left[\int_{A}^{v(p,U(q,Y))} \hat{\tau}_{pp}(p,\epsilon)\phi(\epsilon)d\epsilon\right]$$

Since $\hat{\tau}(p,\epsilon)$ is concave with respect to p, $\Omega(p,q) \leq 0$. According to Assumptions 1 and 2, $R(p,q)$ is also concave with respect to p and q separately. The determinant of the Hessian is:

$$\begin{aligned}
\Delta(p,q) =& (p-h)E\left[U_{qq}(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right].R_{pp}(p,q) \\
& -(p-h)\Omega(p,q).E\left[\frac{U_q^2(q,Y)\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right] \\
& +(p-h)^2E\left[\frac{U_q^2(q,Y)\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right] \\
& .E\left[\frac{d_p^2(p,v(p,U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,z(p,U(q,Y)))}\right] \\
& -E\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right]^2 \\
& -(p-h)^2E\left[\frac{U_q(q,Y)d_p(p,v(p,U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right]^2 \\
& -2(p-h)E\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right] \\
& .E\left[\frac{U_q(q,Y)d_p(p,v(p,U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right] \\
\geq & -E\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right]^2 \\
& -2(p-h)E\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right] \\
& .E\left[\frac{U_q(q,Y)d_p(p,v(p,U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right]
\end{aligned} \tag{4.24}$$

where in the last inequality we made use of the Cauchy-Schwartz inequality. Based on Lemma 4.2 part (c) we can rewrite inequality (4.24) as follows:

$$\begin{aligned}\Delta(p,q) \geq & -s^2(p,q) - 2ps(p,q)s_p(p,q) \\ \geq & s^2(p,q)\left[-\frac{2ps_p(p,q)}{s(p,q)} - 1\right] \\ = & s^2(p,q)\left[2\Theta(p,q) - 1\right]\end{aligned} \tag{4.25}$$

So if $\Theta(p,q) \geq 1/2$ then $\Delta(p,q) \geq 0$ and $R(p,q)$ is concave.

b)It is easy to show that:

$$\frac{\partial^2 f(p^*(q),q)}{\partial q^2} = f_{qq}(p^*(q),q) - \frac{f_{pq}^2(p^*(q),q)}{f_{pp}(p^*(q),q)}.$$

Similar to the proof of part (a) we have:

$$\begin{aligned}\Delta(p^*(q),q) \geq & s^2(p^*(q),q)\left[-\frac{2p^*(q)s_p(p^*(q),q)}{s(p^*(q),q)} - 1\right] \\ = & s^2(p^*(q),q)\left[2\Theta(p^*(q),q) - 1\right]\end{aligned} \tag{4.26}$$

So if $\Theta(p^*(q),q) \geq 1/2$ then $f(p^*(q),q)$ is concave and as a result $f(p,q)$ is unimodal. The proof of part (c) is analogous to part (b).

□

In order to prove Proposition 4.2 we will use the following lemma.

Lemma 4.3. *For any given quantity q, the optimal price $p^*(q)$ is unique and solves*

$$\int_0^q s(p^*(q),\epsilon)(1-\Theta(p^*(q),\epsilon))d\epsilon = 0$$

Proof. By taking the first derivative of $f(p,q)$ with respect to q we can show that:

$$\begin{aligned} f_q(p,q) &= pE\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right] - c \\ &= ps(p,q) - c \end{aligned} \tag{4.27}$$

Integrating from both sides of Equation (4.27) we have:

$$f(p,q) = p\int_0^q s(p,\epsilon)d\epsilon - cq \tag{4.28}$$

Since the objective function is concave with respect to p then for any value of q there is a unique $p^*(q)$ that maximize the objective function and satisfies the first order condition. So by taking the first derivative from Equation (4.28) with respect to p and replacing p with $p^*(q)$ we have:

$$\begin{aligned} f_p(p^*(q),q) &= \int_0^q (s(p^*(q),\epsilon) + p^*(q)s_p(s(p^*(q),\epsilon))d\epsilon \\ &= \int_0^q s(p^*(q),\epsilon)(1-\Theta(p^*(q),\epsilon))d\epsilon = 0 \end{aligned}$$

□

Proof of Proposition 4.2. Our proof is similar to that of Theorem 1 in Kocabiyikoğlu and Popescu (2011). We know that

$$f_p(p,q) = \int_0^q W(p,\epsilon)d\epsilon$$

where

$$W(p,\epsilon) = s(p,\epsilon)(1-\Theta(p,\epsilon))$$

According to Lemma 4.3 we have

$$\int_0^q s(p^*(q),\epsilon)(1-\Theta(p^*(q),\epsilon))d\epsilon = 0$$

Since $s(p,\epsilon)$ is positive and $\Theta(p,q)$ is increasing in q, then $W(p,\epsilon)$ only intersects zero once and at the intersection $W_\epsilon(p,\epsilon) \leq 0$. Therefore $W_\epsilon(p^*(q),q)$ should be less than zero to satisfy Lemma 4.3. The positivity of $s(p^*(q),q)$ implies that $\Theta(p^*(q),q) \geq 1$. □

Proof of Corollary 4.1 .

a) Under the assumption that $U(q,Y) = u(q) + Y$ then $U_q(q,Y) = u_q(q)$. According to Lemma 4.3 we have:

$$\int_0^q (s(p^*(q),\epsilon) + p^*(q)s_p(p^*(q),\epsilon))d\epsilon = 0$$

$$\Rightarrow \int_0^q E\left[u_q(\epsilon)\bar{\Phi}(v(p^*(q),U(\epsilon,Y)))\left\{1 - \right.\right. \\ \left.\left. \frac{-p^*(q)d_p(p^*(q),v(p^*(q),U(\epsilon,Y)))}{d_v(p,v(p^*(q),U(\epsilon,Y)))}h^V(v(p^*(q),U(\epsilon,Y)))\right\}\right] d\epsilon = 0 \tag{4.29}$$

where $h^V(.)$ is the hazard rate function of V. Note that

$$\frac{\partial v(p^*(q), U(\epsilon, Y))}{\partial Y} = v_U(p^*(q), U(\epsilon, Y))U_Y(\epsilon, Y).$$

According to Assumption 4.1 and Lemma 4.2 we have $\frac{\partial v(p^*(q),U(\epsilon,Y))}{\partial Y} \geq 0$. As a result if V is IFR and d_p/d_v is decreasing in v, the second part of Equation (4.29) is decreasing in Y. Since $\bar{\Phi}(v(p^*(q), U(\epsilon, Y)))$ is also decreasing in Y, according to Chebyshev's algebraic inequality we have:

$$\Rightarrow \int_0^q E\left[u_q(\epsilon)\bar{\Phi}(v(p^*(q), U(\epsilon, Y)))\right] .E\left[1 - \frac{-p^*(q)d_p(p^*(q), v(p^*(q), U(\epsilon, Y)))}{d_v(p, v(p^*(q), U(\epsilon, Y)))} h^V(v(p^*(q), U(\epsilon, Y)))\right] d\epsilon \leq 0. \tag{4.30}$$

Consider $\mathcal{X}(\epsilon)$ as follows:

$$\mathcal{X}(\epsilon) = E\left[1 - \frac{-p^*(q)d_p(p^*(q), v(p^*(q), U(\epsilon, Y)))}{d_v(p, v(p^*(q), U(\epsilon, Y)))} h^V(v(p^*(q), U(\epsilon, Y)))\right].$$

$E\left[u_q(\epsilon)\bar{\Phi}(v(p^*(q), U(\epsilon, Y)))\right]$ is positive for all the values of ϵ. In addition if V is IFR and d_p/d_v is decreasing in v, then $\mathcal{X}(\epsilon)$ is decreasing in ϵ. So in order to satisfy

Inequality (4.30) we must have:

$$\begin{aligned}
&\mathcal{X}(q) \leq 0 \\
\Rightarrow &E\left[1 - \frac{-p^*(q)d_p(p^*(q), v(p^*(q), U(\epsilon, Y)))}{d_v(p, v(p^*(q), U(\epsilon, Y)))} h^V(v(p^*(q), U(\epsilon, Y)))\right] \leq 0 \\
\Rightarrow &E\left[-\frac{-p^*(q)d_p(p^*(q), v(p^*(q), U(\epsilon, Y)))}{d_v(p, v(p^*(q), U(\epsilon, Y)))} h^V(v(p^*(q), U(\epsilon, Y)))\right] \leq -1. \quad (4.31)
\end{aligned}$$

According to Proposition 4.1, inequality (4.26) can be rewritten as:

$$\begin{aligned}
\Delta(p^*(q), q) &\geq s(p^*(q), q)\left[-s(p^*(q), q) - p^*(q)s_p(p^*(q), q)\right] \\
&\geq E\Bigg[u_q(q)\bar{\Phi}(v(p^*(q), U(q, Y)))\Bigg\{1 - \\
&\frac{-p^*(q)d_p(p^*(q), v(p^*(q), U(q, Y)))}{d_v(p, v(p^*(q), U(q, Y)))} h^V(v(p^*(q), U(q, Y)))\Bigg\}\Bigg].
\end{aligned}$$

Applying Chebyshev's algebraic inequality we obtain

$$\begin{aligned}
\Delta(p*(q), q) &\geq E\left[u_q(q)\bar{\Phi}(v(p^*(q), U(q, Y)))\right] \\
&.E\left[\frac{-2p^*(q)d_p(p^*(q), v(p^*(q), U(q, Y)))}{d_v(p, v(p^*(q), U(q, Y)))} h^V(v(p^*(q), U(q, Y))) - 1\right] \\
&\geq 0.
\end{aligned}$$

The last inequality follows from inequality (4.31).

b) By changing the bounds of the integral we have:

$$\begin{aligned} s(p,q) &= E_Y\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right] \\ &= E_Y\left[\int_{v(p,U(q,Y))}^{\bar{v}} U_q(q,Y)\phi(\epsilon)d\epsilon\right] \\ &= E_V\left[\int_{\underline{l}}^{y(q,D(p,V))} u_q(q,x)g(x)dx\right]. \end{aligned}$$

So we have

$$s_p(p,q) = E_V\left[u_q(q,y(q,D(p,V)))y_D(q,D(p,V))D_p(p,V)g(y(q,D(p,V)))\right].$$

Note that when $U(q,Y) = u(q) + Y$, then we have $y_D(q,D(p,V)) = 1$ and

$$s(p,q) = E_V\left[u_q(q)G(y(q,D(p,V)))\right].$$

So we can rewrite the EDFR elasticity as

$$\begin{aligned} \Theta(p,q) &= \frac{-pE_V\left[u_q(q)D_p(p,V)g(y(q,D(p,V)))\right]}{E_V\left[u_q(q)G(y(q,D(p,V)))\right]} \\ &= \frac{-pE_V\left[D_p(p,V)\bar{G}(y(q,D(p,V)))h^Y(y(q,D(p,V)))\right]}{E_V\left[G(y(q,D(p,V)))\right]} \end{aligned}$$

in which $h^Y(.)$ is the hazard rate function of Y. We also have

$$\begin{aligned} & u(q,y(q,D(p,V))) = D(p,V) \\ \Rightarrow & u_q(q,y(q,D(p,V))) + u_y(q,y(q,D(p,V)))y_q(q,D(p,V)) = 0 \\ \Rightarrow & y_q(q,D(p,V)) = -\frac{u_q(q,y(q,D(p,V)))}{u_y(q,y(q,D(p,V)))} \end{aligned} \quad (4.32)$$

Based on Assumption 4.1 and equation (4.32) it follows that $y_q(q, D(p,V)) \leq 0$. Accordingly $G(y(q, D(p,V)))$ is decreasing in q and $\bar{G}(y(q, D(p,V)))$ is increasing in q. Now if Y is DFR then $\Theta(p,q)$ is increasing in q and based on Proposition 4.2, $f(p,q)$ is unimodal. □

Proof of Proposition 4.3.

a) Consider $\mathfrak{J}^t(z,q) = r^t E[\min\{z-q, X^t\}]$. Clearly $\mathfrak{J}^t_z(z,q) = \bar{\Psi}(z-q)$ is positive. Also $\delta^t z$ is increasing with respect to z and based on the linear constraint $q \leq z$, the solution is non-decreasing with respect to z. So if $f^{t+1}(.)$ is a non-decreasing function, then $f^t(.)$ is non-decreasing. Since $f^T(z) = r^T z$ is a non-decreasing function, by induction $f^t(.)$ is non-decreasing in each period.

b) The second derivative of $\mathfrak{J}^t(z,q)$ with respect to z and q is $\mathfrak{J}^t_{zq}(z,q) = \psi^t(z-q) \geq 0$ which is the necessary and sufficient condition for supermodularity of $\mathfrak{J}^t(z,q,a)$ and $J^t(z,p,q,a)$ (Topkis, 1998). □

Proof of Proposition 4.4. According to part (a) of Proposition 4.1, $R^t(p,q)$ is jointly concave with respect to p and q when $\Theta^t(p,q) \geq 1/2$. According to Assumption 1 and the fact that minimization preserves concavity, $L^t(z,q,m)$ is also concave. Since the constraint $0 \leq q \leq z$ is linear, it is a lattice. Also $\delta^t z$ is linear and as a result concave. Therefore under the assumption that $\Theta^t(p,q) \geq 1/2$, if $f^{t+1}(.)$ is concave then $f^t(z)$ is concave. Since $f^T(z)$ is a linear function of z, it is concave, and by induction if $\Theta^t(p,q) \geq 1/2$, $f^t(z)$ is concave for each time period t. □

Proof of Theorem 4.1. For ease of exposition, we drop index t. According to Proposition 4.4 if $\Theta(p,q) \geq 1$ then $J(z,p,q,m)$ is concave and as a result there is a unique solution for p, q and m. According to Proposition 4.3 part (b), $J(z,p,q,m)$ is supermodular in (z,q). So if $J_q(z,p^*,z,m^*) \geq 0$ then $q^* = z$. Also if $J_q(z,p^*,z,m^*) < 0$, there exist a $z^\circ < z$ such that $J_q(z,p^*,z^\circ,m^*) = 0$ and $q^* = z^\circ$.

We know that:

$$\begin{aligned}
R_{pq} &= E\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right] \\
&\quad + (p-h)E\left[\frac{U_q(q,Y)d_p(p,v(p,U(q,Y))\phi(v(p,U(q,Y)))}{d_v(p,v(p,U(q,Y)))}\right] \\
&= s(p,q) + ps(p,q) \\
&= -s(p,q)(\Theta(p,q)-1)
\end{aligned}$$

So if $\Theta(p,q) \geq 1$ then $R_{pq}(p,q) \leq 0$ which is a necessary and sufficient condition for submodularity of $R(p,q)$ (and consequently $J(z,p,q,m)$) in (p,q) (Topkis (1998)). So if $q^* = z$ then $p^* = P(z)$, which is a decreasing function (since q^* is an increasing function of z). Also, when $q^* = z^\circ$, then $p^* = P(z^\circ)$. Also based on concavity of $J(z,p,q,m)$, we can see that the one-sided capacity planning policy is optimal and this completes the proof. □

Proof of Proposition 4.5. For ease of exposition we drop index t. According to Equation (4.20), the first derivative of $R(p,q)$ with respect to q is:

$$R_q(p,q) = (p-h)E\left[U_q(q,Y)\bar{\Phi}(v(p,U(q,Y)))\right] = (p-h)s(p,q) \tag{4.33}$$

Also the explicit form of $L(z,q,m)$ is

$$L(z,q,m) = hE\left[U(q,Y)\right] - cq - \delta m + r\int_{\underline{k}}^{z-q} x\psi(x)dx + r(z-q)\bar{\Psi}(z-q)$$

so its first derivative with respect to q is

$$L_q(z,q,m) = hE\left[U_q(q,Y)\right] - c - r\bar{\Psi}(z-q) \tag{4.34}$$

Since $J(z,p,q,m)$ is concave with respect to q (based on Proposition 4.4), the highest value of $J_q(z,p,q,m)$ is at $q=0$. So if $J_q(z,p,0,m) \leq 0$ for all the values of p, then the optimum value of q is equal to zero. Therefore, if

$$(p-h)s(p,0) + hE\left[U_q(0,Y)\right] \leq c + r\bar{\Psi}(z)$$

then $q^* = 0$.

In order to prove the other way of this statement, assume that there is a $\tilde{p} \in [\underline{p}, \overline{p}]$ such that:

$$(\tilde{p}-h)s(\tilde{p},0) + hE\left[U_q(0,Y)\right] > c + r\bar{\Psi}(z)$$

then $J_q(z,\tilde{p},q,m)$ is decreasing in q and there exist a $q^* > 0$ such that $J_q(z,\tilde{p},q^*,m) = 0$, and based on the first order conditions, it is optimum.

In the case that $U(q,0) = 0$, then $\bar{\Phi}(v(p,U(0,Y))) = 1$. In addition, since $U(q,Y)$ is increasing, then (4.13) reduces to

$$\overline{p}E\left[U_q(0,Y)\right] \leq c + r\bar{\Psi}(z)$$

□

Proof of Lemma 4.1.

a) If $u^t(q, y^t)$ is concave with respect to y^t, then $J^t(y^t)$ (as a function of y^t) is concave with respect to y^t (since minimization preserves concavity). As a result, according to Jensen's inequality $J^t(\mu_{Y^t}) \geq E\left[J^t(Y^t)\right]$ for the same values of z, p, q and a, and $\bar{f}^t(z) \geq f^t(z)$.

b) The proof is analogous to part (a). Since minimization preserves concavity, if $d^t(p, v^t)$ is concave with respect to v^t, then $\bar{\bar{f}}^t(z) \geq f^t(z)$.

c) We drop the index t here to simplify the demonstration of the proof. Consider $Q(p, q, v)$ as follows:

$$Q(p, q, v) = (p - h)E\left[\min\left\{d(p, v), U(q, Y)\right\}\right]$$

We can write $Q(p, q, v)$ with respect to $y(q, d(p, v))$ as follows:

$$Q(p, q, v) = (p - h)\int_{\underline{l}}^{y(q,d(p,v))} U(q, \epsilon)g(\epsilon)d\epsilon + (p - h)\int_{y(q,d(p,v))}^{\bar{l}} d(p, v)g(\epsilon)d\epsilon$$

Note that $\xi(p, q, v) = \bar{G}(y(q, d(p, v))) \geq 0$. So we have:

$$\xi_v(p, q, v) = -d_v(p, v)y_d(q, d(p, v))g(y(q, d(p, v)))$$

The second derivative of $Q(p,q,v)$ with respect to v is:

$$\begin{aligned} Q_{vv}(p,q,v) &= (p-h)d_{vv}(p,v)\bar{G}(y(q,d(p,v))) \\ &\quad - (p-h)d_v^2(p,v)y_d(q,d(p,v))g(y(q,d(p,v))) \\ &= (p-h)d_{vv}(p,v)\bar{G}(y(q,d(p,v)))\left[1+\frac{d_v(p,v)\xi_v(p,q,v)}{d_{vv}(p,v)\xi(p,q,v)}\right] \end{aligned} \tag{4.35}$$

We have:

$$u(q,y(q,d(p,v))) = d(p,v)$$

By taking the derivative with respect to v from both side we have

$$y_d(q,d(p,v)) = \frac{1}{u_y(q,y(q,d(p,v)))}.$$

According to Assumption 4.1 $u_y(.) \geq 0$ and as a result $y_d(q,d(p,v)) \geq 0$. So according to Assumption 4.2, $\xi_v(p,q,v) \leq 0$.

Now if $d(p,v)$ is concave with respect to v then $d_{vv}(.) \leq 0$ and the right hand side of (4.35) is clearly less than zero. Also if $d_{vv}(.) > 0$ and

$$\frac{d_v(p,v)\xi_v(p,q,v)}{d_{vv}(p,v)\xi(p,q,v)} \leq -1$$

then $Q_{vv}(p,q,v) \leq 0$. The rest of the proof follows from Jensen's inequality.

d) According to Equation (4.35) if $d_v v(.) \geq 0$ and

$$\frac{d_v(p,v)\xi_v(p,q,v)}{d_{vv}(p,v)\xi(p,q,v)} \geq -1$$

then $Q_{vv}(p,q,v) \geq 0$. The rest of the proof follows from Jensen's inequality. □

Proof of Proposition 4.6.

a) Since $u^t(0,y^t) = 0$, then when $q = 0$ we have: $\bar{J}^t(z,p,0,m) = J^t(z,p,0,m)$. Assume $u^t(q,y^t)$ is concave. According to Lemma 4.1 part (a), we have $\bar{J}^t(z,p,\epsilon,m) \geq J^t(z,p,\epsilon,m)$. and as result:

$$\lim_{\epsilon\to 0} \bar{J}^t(z,p,\epsilon,m) \geq \lim_{\epsilon\to 0} J^t(z,p,\epsilon,m)$$
$$\Rightarrow \bar{J}^t_q(z,p,0,m) \geq J^t_q(z,p,0,m)$$

The proof of part (b) and (c) are analogous. □

In order to prove Proposition 4.7, we need the following lemmas.

Lemma 4.4. *Consider $\bar{R}(d,q)$ as the deterministic yield revenue function. Then we have:*

$$\bar{R}(d,q) = p(d)E_V\left[\min\{d+V,q\}\right].$$

Assume $\bar{R}(d,q)$ is concave. Then $\hat{R}(d,q,y)$ is concave in (d,q), where

$$\hat{R}(d,q,y) = p(d)E_V\left[\min\{d+V,U(q,y)\}\right].$$

Proof. Assume $q_1, q_2 \geq 0$ and $d_1, d_2 \in [\underline{d}, \overline{d}]$. Also let β be an arbitrary number such that $0 \leq \beta \leq 1$. We have:

$$\begin{aligned}&\hat{R}(\beta d_1 + (1-\beta)d_2, \beta q_1 + (1-\beta)d_2, y)\\&= (\beta d_1 + (1-\beta)d_2)E_V[\min\{\beta d_1 + (1-\beta)d_2 + V, U(\beta q_1 + (1-\beta)q_2, y)\}].\end{aligned}$$

Since $U(q, y)$ is concave and $\bar{R}$ is increasing in q, we have

$$\begin{aligned}&\hat{R}(\beta d_1 + (1-\beta)d_2, \beta d_1 + (1-\beta)q_2, y)\\&\geq (\beta d_1 + (1-\beta)d_2)E_V[\min\{\beta d_1 + (1-\beta)d_2 + V, \beta U(q_1, y) + (1-\beta)U(q_2, y)\}]\\&\geq \beta d_1 E_V[\min\{d_1 + V, U(q_1, y)\}] + (1-\beta)d_2 E_V[\min\{d_2 + V, U(q_2, y)\}]\\&= \beta\hat{R}(d_1, q_1, y) + (1-\beta)\hat{R}(d_2, q_2, y)\end{aligned}$$

which completes the proof. □

Lemma 4.5. *If $\hat{R}(d, q, y)$ is concave then $E_Y[\hat{R}(d, q, Y)]$ is concave.*

Proof. Consider $q_1, q_2 \geq 0$ and $d_1, d_2 \in [\underline{d}, \overline{d}]$. Also let β be an arbitrary number satisfying $0 \leq \beta \leq 1$. We have

$$
\begin{aligned}
E\left[\hat{R}(\beta d_1 + (1-\beta)d_2, \beta q_1 + (1-\beta)q_2, Y)\right] \geq E\Big[&\beta \hat{R}(d_1, q_1, Y) \\
&+(1-\beta)\hat{R}(d_2, q_2, Y)\Big] \\
= \beta E&\left[\hat{R}(d_1, q_1, Y)\right] \\
&+ (1-\beta)E\left[\hat{R}(d_2, q_2, Y)\right]
\end{aligned}
$$

which completes the proof. □

Lemma 4.6. *If* $\Gamma(d,q) \geq 1$ *and* $p_{dd}(d) + p_d(d)d \leq 0$, $\forall d, q$, *then* $R(d,q)$ *is concave.*

Proof. According to Pang (2011) (Lemma 2) if $\Gamma(d,q) \geq 1$ and $p_{dd}(d) + p_d(d)d \leq 0$, $\forall d, q$, then $\bar{R}(d,q)$ is concave.

According to Lemma 4.4 if $\bar{R}(d,q)$ is concave then $\hat{R}(d,q,Y)$ is concave in q and d. From Lemma 4.5 it follows that $R(d,q)$ is concave

□

Proof of Proposition 4.7. We can rewrite Equation (4.16) as follows:

$$
\begin{aligned}
J^t(z,d,q,m) =& (p(d) - h^t + b^t)E\left[\min\left\{d + V^t, U^t(q, Y^t)\right\}\right] + h^t E\left[U^t(q, Y^t)\right] \\
&- b^t d - c^t q - \delta^t m + r^t E\left[\min\left\{z - q, K^t\right\}\right] + \alpha E f^{t+1}(m).
\end{aligned}
$$

The first part of the equation is concave according to Lemma 4.6. The rest of the function is also concave since minimization preserves concavity. The rest of the prove follows in a similar way to that of Proposition 4.4 □

Proof of Theorem 4.2. Consider $\bar{R}(d,q)$ as follows:

$$\bar{R}(d,q) = (p(d) - h + b)E\left[\min\left\{d + V, U(q,Y)\right\}\right].$$

The second derivative of $\bar{R}(d,q)$ with respect to d and q is:

$$\begin{aligned}\bar{R}_{dq}(d,q) = & \; p^t_d(d)E\left[U^t_q(q,Y^t)\bar{\Phi}(U^t_q(q,Y^t) - d)\right] \\ & + (p^t(d) - h^t + b^t)E\left[U^t_q(q,Y^t)\phi(U^t_q(q,Y^t) - d)\right] \\ = & \; p^t_d(d)E\left[U^t_q(q,Y^t)\bar{\Phi}(U^t_q(q,Y^t) - d)\right](1 - \Theta(d,q)).\end{aligned}$$

Therefore if $\Theta(d,q) \geq 1$ then $\bar{R}_{dq}(d,q) \geq 0$ which is sufficient for supermodularity of $\bar{R}(d,q)$ with respect to d and q. The rest of the proof is is similar to that of Theorem 4.1. □

Proof of Theorem 4.3. The more explicit form of objective value in the presence of fixed cost is as follows:

$$\begin{aligned}f^t(z) = \max \Big\{ & \underset{a \geq 0}{max} \left\{r^t E\left[\min\left\{z, X^t\right\}\right] - \delta^t a + \alpha f^{t+1}(z+a)\right\}, \\ & \underset{0 \leq q \leq z, 0 \leq a, p}{max} \left\{J^t(z,p,q,a) - M\right\} \Big\}\end{aligned}$$

Since $f^{t+1}(z+a)$ is independent from production and pricing decisions in period t, we can rewrite $f^t(z)$ as follows:

$$f^t(z) = \underset{a \geq 0}{max} \left\{-\delta^t a + \alpha f^{t+1}(z+a)\right\} + \max \left\{r^t E\left[\min\left\{z, X^t\right\}\right],\right.$$
$$\left. \underset{0 \leq q \leq z, 0 \leq p}{max} \left\{R^t(p,q) + h^t E\left[U(q, Y^t)\right] - c^t q + r^t E\left[\min\left\{z-q, X^t\right\}\right] - M\right\} \right\}$$

According to Proposition 4.1 the objective value is concave with respect to a and submodular with respect to (z, a) in all the periods. So a^{*t} follows the mentioned optimal policy. Consider $\Upsilon^t(z)$ and $\Pi^t(z)$ as follows:

$$\Upsilon^t(z) = r^t E\left[\min\left\{z, X^t\right\}\right]$$
$$\Pi^t(z) = \underset{0 \leq q \leq z, 0 \leq p}{max} \left\{J^\circ(p,q,z)\right\}$$

in which

$$J^\circ(p,q,z) = R^t(p,q) + h^t E\left[U(q, Y^t)\right] - c^t q + r^t E\left[\min\left\{z-q, X^t\right\}\right] - M.$$

Clearly, $\max\left\{\Upsilon^t(z), \Pi^t(z)\right\}$ is continuous. If $\Upsilon^t(z) \geq \Pi^t(z)$ for all the values of z, then $q^{*t} = 0$ for all the values of z and $n = +\infty$. Also p^{*t} can take any value since $R^t(p, 0) = 0$ which can be considered as a constant for all the values of z.

Now assume that there exists an n such that $\Pi^t(n) = \Upsilon^t(n)$. We need to show that $\Pi^t(z) \leq \Upsilon^t(z),\ \forall z < n$ and $\Pi^t(z) \geq \Upsilon^t(z),\ \forall z > n$. Consider the q and p which maximizes $\Pi^t(n)$ as q_n and p_n. By taking the first derivatives from $J^\circ(p,q,z)$ and $\Upsilon^t(z)$ with respect to z we have:

$$\Upsilon_z(z) = r^t \bar{\Psi}(z) \leq J_z^\circ(p,q,z) = r^t \bar{\Psi}(z-q) \tag{4.36}$$

As a result based on Inequality (4.36) and the fact that $\Pi^t(n) = \Upsilon^t(n)$, for any value of $z \geq n$ we have:

$$J_z^\circ(p_n, q_n, n) = \Upsilon^t(n) \Rightarrow J_z^\circ(p_n, q_n, z) \geq \Upsilon^t(z) \Rightarrow \Pi(z) \geq \Upsilon^t(z)$$

Now lets consider the case where $z < n$. Assume q^* and p^* are the corresponding solutions for $\Pi^t(z)$. Two cases may happen:

Case 1: Assume that $q^* < z$. In this situation as we decrease z, there will be no change in the values of q^* and p^* and as a result the part of $J^\circ(p, q, z)$ that changes is $r^t E\left[\min\left\{z - q, K^t\right\}\right]$. From (4.36), $\Upsilon_z^t(z) \leq J^\circ(p, q, z)$ and so $\Pi(z) \leq \Upsilon^t(z)$ for $z < n$.

Case 2: Assume that $q^* = z$. In this situation the optimal solution is binding. So we can rewrite $\Pi^t(z)$ as a function of z as follows:

$$\Pi^t(z) = R^t(p^*(z), z) + h^t E\left[U(z, Y^t)\right] - c^t z - M.$$

Assume $\Pi_z^t(z) < r^t \bar{\Psi}(z)$. Then if we pick ϵ small enough, we have

$$\begin{aligned} R^t(p^*(z), z) + h^t E\left[U(z, Y^t)\right] - c^t z - M <& R^t(p^*(z - \epsilon), z - \epsilon) + h^t E\left[U(z - \epsilon, Y^t)\right] \\ & - c^t(z - \epsilon) - M + r^t E\left[\min\left\{\epsilon, X^t\right\}\right] \end{aligned}$$

which contradicts the fact that $\Pi^t(z)$ is the maximization of J°. Consequently $\Pi_z^t(z) \geq r^t \bar{\Psi}(z)$, and then $\Pi(z) \leq \Upsilon^t(z)$ for $z < n$.

Based on the above discussion for $z < n$ we have:

$$f^t(z) = \max_{a \geq 0} \left\{ -\delta^t a + \alpha f^{t+1}(z+a) \right\} + r^t E \left[\min \left\{ z, X^t \right\} \right]$$

and the optimum value of q is equal to zero and we set the optimum price as $P(n)$ (since it can take any value). For $z \geq n$ we have:

$$f^t(z) = \max_{a \geq 0} \left\{ -\delta^t a + \alpha f^{t+1}(z+a) \right\} + \Pi^t(z)$$

and based on concavity of $J^{\circ}(p, q, z)$ and its submodularity over (p, q) and its supermodularity over (z, q) (Theorem 4.1), then a two-sided production, pricing and capacity planning policy is optimal. □

Appendix B

Examples of EDFR elasticity's behaviour with respect to production quantity

In the Figure 4.8 EDFR elasticity is plotted with different demand and yield uncertainty distributions. Table 4.2 shows the parameters that are used in plotting each of the figures. We tried different combination of distributions for yield and demand uncertainty and in all cases the EDFR elasticity demonstrates an increasing behaviour with respect to q.

Table 4.2: Description of the Figure 4.8

Figure number	Yield form	Demand form	Demand parameters	Distribution of V	Distribution of Y
Figure 4.8a	$U(q,Y)=Yq$	$D(p,V)=a-bp+V$	$a=400$, $b=3$	Exp(1/50)	$U(0,1)$
Figure 4.8b				$\mathcal{N}(50,5)$	$\mathcal{N}(0.5,0.1)$
Figure 4.8c				$\mathcal{N}(50,5)$	$U(0,1)$
Figure 4.8d				$U(50,100)$	$\mathcal{N}(0.5,0.1)$
Figure 4.8e				Weibull(200,5)	Beta(2,2)
Figure 4.8f		$D(p,V)=ap^{-b}V$	$a=1000$, $b=1.5$	Exp(1)	$U(0,1)$
Figure 4.8g				$\mathcal{N}(1,0.15)$	$\mathcal{N}(0.5,0.1)$
Figure 4.8h				$U(0.5,1.5)$	$\mathcal{N}(0.5,0.1)$
Figure 4.8i				Weibull(1,5)	Beta(2,2)

Example where the EDFR elasticity is greater than lost sale rate elasticity

Assume that $D(p,V)=200-3p$ where $V\sim\mathcal{N}(100,5)$. Also assume that $U(q,Y)=q+Y$ where $Y\sim U(-15,15)$. Similar to Kocabiyikoğlu and Popescu (2011), we define lost sale rate elasticity, $\mathscr{E}(p,q)$, as

$$\mathscr{E}(p,q)=\frac{-pw_p(p,q)}{w(p,q)}$$

where $w(p,q)=Pr\{D(p,V)\geq q\}$. Under such a condition the EDFR elasticity for $q\geq 255$ and $p\geq 10$ is greater than 1/2, however this is not the case for lost sale rate elasticity. For instance $\Theta(10,255)=0.536$ but $\mathscr{E}(10,255)=0.027$.

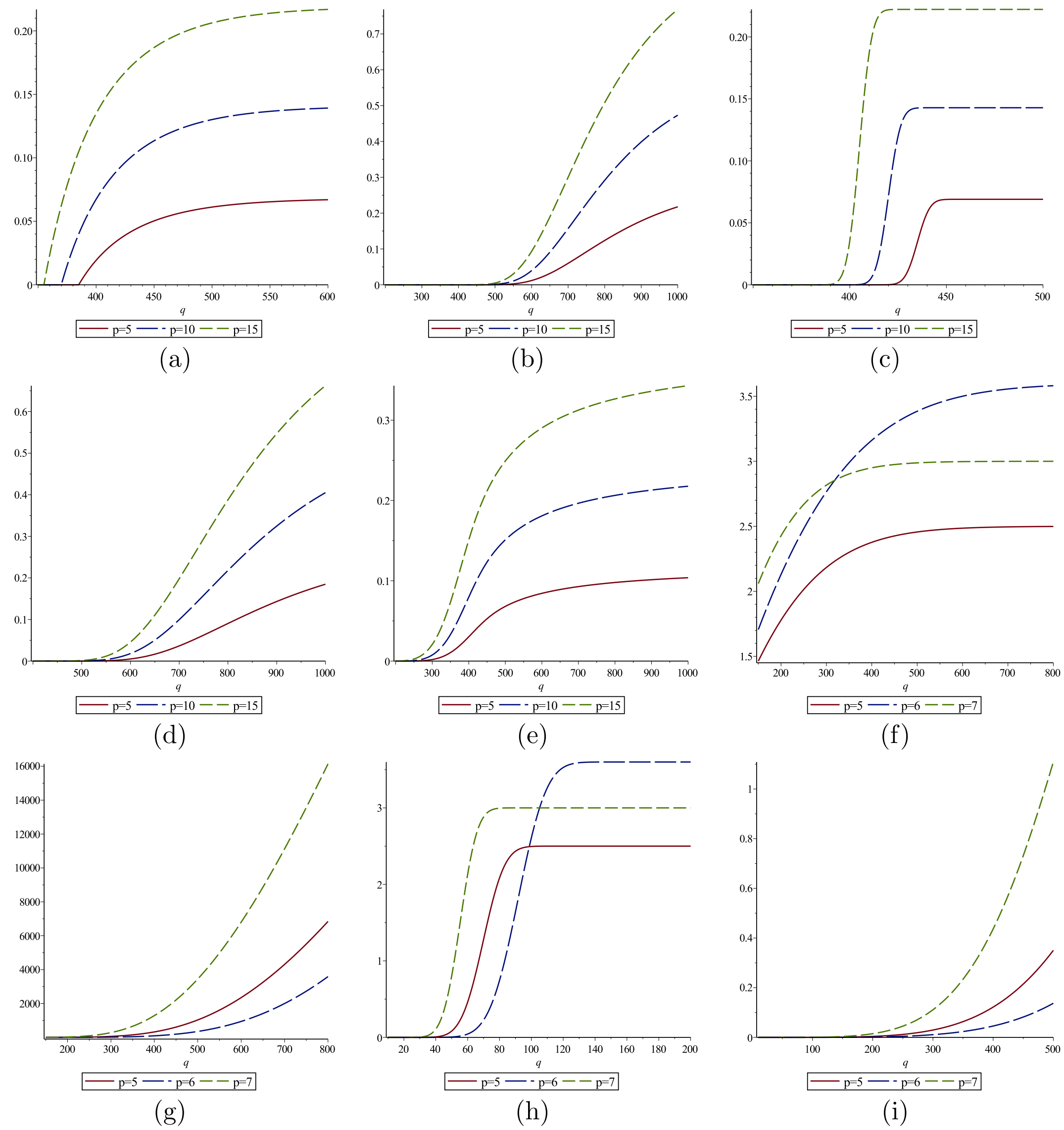


Figure 4.8: Behaviour of EDFR elasticity under different settings

Chapter 5

Buyer-Supplier Coordination through Capacity Investment under Demand and Supply Risks

5.1 Introduction

A vast majority of companies are trying to find an edge in the current intense market by focusing on their core competencies and outsourcing their operations to other firms (Krause *et al.* (2007), Krause *et al.* (2000)). In addition many companies, such as Toyota and Ford, prefer to use a single supplier for most of their outsourced parts in order to keep the consistency of quality (Sako (2004), Simchi-Levi *et al.* (2002)). As a result the firms have become more dependent and vulnerable to the performance of their suppliers. This issue highlights the strategic role of supplier development programs as an inevitable part of an efficient supply chain. The current studies have revealed that the performance of the buyers can be significantly improved by practising this type of programs (Krause (1997), Wagner and M. (2006)). This is the reason that many major companies, such as Toyota and Samsung have invested in supplier development programs (Sako (2004), Jeon (2012)).

These programs can vary from informal assessment of the supplier to direct investment of the buyer in supplier's operations (Krause and Ellram (1997)). In the cases that the supplier suffers from insufficiency of budget or she is skeptical about the outcome of the investment, the buyer can directly provide the funding to enhance the system. Otherwise the supply chain may experience deficiency in fulfilling the customers' demand. For instance, Boeing and Airbus have reported a huge demand backlog which is mainly because of incapability and unwillingness of their suppliers to increase their production capacity (Parker and Shotter (2012), Broderick (2015)). Some of the firms in the auto-industry have reported a similar problem with their component manufacturers (Bansal and Mathur (2012)).

In the direct involvement of the buyer in improving the supplier's operations, there is always a chance that the supplier will try to play games to increase her own profits at the expense of that the buyer. Although the buyer's commitment can provide a tool to increase the profitability of the whole system, the supplier can choose to strategically decrease her efforts to increase her share from the profit. Yan and Kull (2014) reported on how this behaviour may happen in collaboration of the buyer and supplier in new product development. To avoid such supplier opportunism in capacity expansion projects, the buyer has to decide about how to induce supplier's development without demotivating the supplier.

In this chapter we look at the lot-sizing problem of a buyer and a supplier where both parties have the opportunity to invest on the capacity of the supplier. We investigate the problem under a Stackelberg game setting where the buyer is the leader. In order to capture the essence of real world problems, we assume that the capacity of the supplier, as well as the demand of the buyer, is uncertain. Although considering the uncertainties of supply and demand complicates the analysis of the problem, neglecting it may limit the applicability of the model.

We demonstrate that both the buyer and supplier try to take advantage of the other player's investment in order to maximize their profit. This opportunistic behaviour results in not sharing the investment and consequently only one of the parties would invest on improving capacity. As the leader, the buyer has the initiative to determine which party carries the investment. In the case that the buyer sees the supplier motivated enough to invest on the capacity, he tries to induce her by inflating the order quantity. On the other hand if there is a lack of motivation by the supplier, the buyer decreases the order quantity and invests directly in the capacity.

The order inflation strategy of the buyer weakens the effect of double marginalization on the order quantity. There are cases where the order quantity in decentralized case surpasses the optimal order quantity in the centralized case. However when the buyer starts investing in the capacity, the system experiences a drop in order quantity that goes below the centralized case. We also look at the case where the supplier decides about the amount of investment where a portion of the investment is paid by the buyer. We study whether such an approach can increase the total investment and how the investment of the supplier gets affected.

5.1.1 Literature Review of Coordination in Supplier Development Programs

In this section we look at the studies that consider the relation of a buyer and his suppliers under a supplier development program. In this context different aspects of supplier development, such as cost reduction (Li (2013) and Kim and Netessine (2013)) and quality improvement (Yang and Pan (2004)) have been investigated. Here we look at the studies that consider the problem of a buyer that can improve the reliability of his suppliers directly or indirectly. This improvement can be done by direct investment of the buyer or by inducing the supplier to increase her investment on her capacity reliability.

The first line of research is the studies that assume that each party in the supply chain invests only on its own production system. Cachon and Lariviere (2001), Tomlin (2003), Erkoc and Wu (2005) and Ozer and Wei (2006) considered buyer's capacity reservation problem. The first two studies considered the buyer-lead models under an option contract. The focus of Cachon and Lariviere (2001) is on the effect of demand information sharing in an environment where the buyer may not be truthful about the information. Tomlin (2003) assumed that both the buyer and the supplier have to invest on their own capacities and the wholesale price is determined based on the order quantity of the buyer. They investigated how the buyer can motivate the supplier to increase her capacity by sharing the profit. Erkoc and Wu (2005) assumed that the supplier, as the leader, announces the capacity reservation fee. The buyer decides about the amount of capacity that she wants to reserve accordingly and afterwards the supplier starts to build his capacity. Similar to Cachon and Lariviere (2001), Ozer and Wei (2006) investigated the notion of demand information sharing between a supplier and a manufacturer. They showed that the supplier can develop a menu of contracts for capacity reservation and the corresponding prices in order to maximize his profit and induce the manufacturer to share the demand information.

Taylor and Plambeck (2007) considered the case where the capacity development is a long process and should be done before realization of demand. In this situation the buyer motivates the supplier to expand her capacity by some off-contract negotiations. They showed how the buyer can lead these informal negotiations on price and quantity to maximize his profit. Li and Debo (2009a) looked at a two period model where the buyer has the opportunity buy from a second supplier in the second period. They showed how the capacity development cost and the demand distribution can affect the advantages of dual-sourcing over sole-sourcing. Plambeck and Taylor (2007) considered the case where two buyers share a common supplier. Each buyer exerts some market effort to expand his sale, while the supplier invests on her capacity. After realization of demand, the firms can renegotiate on sharing the excess capacity. Yang and Murthy (2014) considered investment in the context of recovery after disruption. They assumed that the supplier determines his recovery time by the effort that he puts in action, while the buyer has to decide whether to stick to the same supplier or switch to a backup supplier.

All the aforementioned studies assumed that there is no uncertainty in the supply. Hwang and Demiguel (2013) assumed that the supplier can improve the capacity by investment, however some of the capacity may not be available due to unforeseen circumstances. They demonstrated that the buyer can induce the reliability improvement effort of the supplier by inflating his order quantity. They expanded their results to a random yield case and studied coordination policies under each case. Hu *et al.*

(2013b) explored the relation of a buyer and a supplier before and after disruption. They assumed that if the supplier develops the restoration capability before disruption, she can recover some of her production capability by investing after disruption, although the recovered amount is uncertain. In order to increase the effort of the supplier, the buyer provides incentives by increasing the order quantity or price.

In another stream of literature, the buyer is involved directly in the reliability of the supplier. Li *et al.* (2009) looked at the problem of a buyer that is investing on the capacity of two competing suppliers while the suppliers exerting some effort to reduce their own production costs. In the study of Li and Debo (2009b), a buyer invests on a transferable capacity which can be specified to any supplier. The buyer selects a supplier using an auction and decides whether to sign a long-term or a short-term contract. They found that the chance of building a long-term relationship motivates the suppliers to bid more aggressively. The studies of Li *et al.* (2009) and Li and Debo (2009b) also considered the case when supply is deterministic. Qi *et al.* (2015) focused on the relation of two competing buyers that are investing in a common supplier. Competing under a Cournot setting, the buyers have to be aware of how their investment may affect the performance of their competitor.

There are few studies that considered the chance of investment for both the buyer and the supplier on the production capability of the supplier. Bai and Sarkis (2014) investigated the relation of a buyer and a set of suppliers where the production process of suppliers follows a Cobb-Douglas production function. They considered the problem under a new product development setting where the buyer can invest on the knowledge sharing while all players can invest on tangible matters such as human resources.

Our study is more aligned with the paper of Tang *et al.* (2014). They looked at a supplier development setting where the supplier may be disrupted and the buyer is facing a deterministic demand. The supplier can invest on his operations in order to decrease the probability of disruption. In order to motivate the supplier, the buyer may cover a portion of the investment cost. The buyer may also use order inflation to induce the supplier to invest. They demonstrated that based on the disruption severity, the buyer may use an investment sharing contract or the combination of both strategies. The results were also extended to the case where the demand is uncertain and the buyer can use a supply diversification strategy to mitigate the supply risk. Our study differs from the study of Tang *et al.* (2014) from three different angles. First of all, we explore the problem under uncertainty of capacity with a general function. The all-or-nothing disruption, as considered in Tang *et al.* (2014), is a special case of our model. Second, although in Tang *et al.* (2014) the buyer is involved in the investment, the final decision is made by the supplier. However in our model, the buyer may invest on improving the capacity, without considering the supplier's action on capacity investment. Finally, a large portion of the study of Tang *et al.* (2014) has been devoted to the deterministic demand case, while we assume that both supply and demand are uncertain.

5.2 Problem Description and Formulation

Consider a buyer that is facing an uncertain demand D with a p.d.f $f(.)$ and a c.d.f $F(.)$. Each unit sold to the customers has a revenue p. The buyer procures the products from a single supplier. The supplier charges the buyer $w \leq p$ for each unit of product sold. The supplier's unit production cost is $c \leq w$. The supplier capacity is uncertain. This uncertainty can be caused by any unexpected interruption during the production process due to, for example, its old age and lack of reliability. One way to improve the reliability of the production process is through investments by the supplier and/or the buyer. One way to model reliability improvement is to consider reliablity as a decion variable and define the improvment cost as a fucntion of reliability (e.g., Tang *et al.* (2014)). Another approach is to consider investment as a decision variable, and define the reliability or the capacity as a fucntion of investment (e.g., Liu *et al.* (2010) and Sting and Huchzermeier (2010)). We have incorporated the second approach in our modeling in order to analyze the investment as a decision directly. Consider r^s and r^b as the amount of investment by the supplier and the buyer, respectively. The available capacity is $U(r^s + r^b, Y)$ in which Y is a random factor that represents the uncertainty of capacity. Let $g(.)$ and $G(.)$ be the p.d.f and c.d.f of $Y \in [\underline{l}, \bar{l}]$. We consider y as the observation of Y and $u(x, y)$ as the observation of $U(x, Y)$ accordingly. We assume that $u(x, y)$ is concave with respect to x and strictly increasing with respect to x and y. The assumptions with respect to x are consistent with the fact that the investment improves the capacity and also its marginal effect decreases as the investment increases. As for the assumption with respect to y, we note that it covers the commonly used special cases of additive and multiplicative capacity random errors. We also assume that for any value of $x > 0$, $\lim_{y\to+\infty} u(x, y) = +\infty$.

To analyze the interplay between the buyer and supplier, we model the investment problem as a Stackelberg game where the buyer is the leader. The chronological order of events are as follows: The buyer decides about his order quantity q^b based on his perception about the distributions of demand and supply uncertainty. In addition, the buyer determines the amount of budget that he wants to invest on the supplier's capacity. After realization of the actions of the buyer, the supplier decides about her production quantity, q^s, and investment contribution for capacity improvement. Since the supplier faces the uncertainty of capacity, the delivered quantity, $\tilde{q}^s$ will always be less than or equal to q^s. Similar to Tang *et al.* (2014) and Ozer and Wei (2006), we assume that both players endure no penalty and salvage costs. Table 5.1 provides the summary of notation.

Table 5.1: Summary of notation

Parameters:	
c	Supplier's unit production cost
w	Supplier's unit selling cost
p	Buyer's unit selling cost
Y	Random factor of supplier's capacity with p.d.f g and c.d.f G that varies in interval $[\underline{l}, \bar{l}]$
y	Realization of random variable Y
D	Random demand with p.d.f f and c.d.f F
Decision variables:	
q^b	Buyer's order quantity
q^s	Supplier's targeted production quantity
q^c	Centralized case targeted production quantity
r^b	Buyer's investment in supplier's capacity
r^s	Supplier's investment in her capacity
r^c	Centralized case investment in capacity
$U(r^s + r^b, Y)$	Supplier's capacity function
$u(r^s + r^b, y)$	Realizarion of Supplier's capacity function
$\tilde{q}^s$	Supplier's actual production quantity

Based on this description the supplier's problem can be modelled as follows:

$$\max_{q^s,r^s \geq 0} \Pi(q^s, r^s|q^b, r^b) = wE\left[\min\left\{q^b, \min\left\{q^s, U(r^b + r^s, Y)\right\}\right\}\right] - cE\left[\min\left\{q^s, U(r^b + r^s, Y)\right\}\right] - r^s \tag{5.1}$$

In order to analyze the problem, we let $y(r^b + r^s, q^b)$ be the value of uncertain part of capacity for which capacity perfectly equals q^b (i.e., $u(r^b + r^s, y(r^b + r^s, q^b) = q^b)$. The first decision of the supplier is to determine her optimal production quantity, considering the order quantity of the buyer. The next proposition addresses this issue.

Proposition 5.1. *The optimal value of the targeted production quantity of the supplier is* $q^{s^*} = q^b$.

All the proofs are provided in Appendix A of this chapter.

According to Proposition 5.1, the supplier optimal targeted production quantity, q^{s^*} is to exactly match the order quantity of the buyer. However due to the uncertainty of capacity, the delivered quantity, $\tilde{q}^s$, will be less than q^b. Based on this proposition, we can rewrite the problem of the supplier as follows:

$$\max_{r^s \geq 0} \Gamma(r^s|q^b, r^b) = (w - c)E\left[\min\left\{q^b, U(r^b + r^s, Y)\right\}\right] - r^s \tag{5.2}$$

In order to be able to satisfy the demand, the supplier has to decide about the amount of money that she wants to invest on capacity. Lack of investment may result in low production and huge lost sales. On the other hand the over-investment may create excess capacity and impose extra cost on the supplier. In the next proposition we analyze the behaviour of the optimum investment of the supplier.

Proposition 5.2.

a) $\Gamma(r^s|q^b, r^b)$ *is concave with respect to* r^s.

b) $\frac{\partial r^{s^*}(r^b)}{\partial r^b} = -1,$ *if* $r^{s^*}(r^b) > 0$.

c) $\frac{\partial r^{s^*}(q^b)}{\partial q^b} \geq 0$.

Part (a) of Proposition 5.2 demonstrates that the optimum investment of the supplier is unique and can be found by setting the first derivative of the objective function equal to zero. Part (a) and (b) help us understand how the supplier reacts to the buyers order and investment actions. Part (b) reveals that the investment of the buyer demotivates the supplier from investing on her capacity. This behaviour may nullify the investment of the buyer when looked at from an overall buyer-supplier system perspective. Consequently the buyer should be very cautious when he selects direct involvement in developing the supplier production capabilities.

Part (c) of Proposition 5.2 concentrates on the effect of order quantity of the buyer on the supplier's investment. Increase in the buyer's order creates an opportunity for the supplier to increase her profitability. This situation induces the supplier to increase her investment in order to reduce the chance of lost sales. As a result the buyer may see order inflation as a strategy to increase the supplier's production capacity. Note that the variation of the optimum supplier's investment with respect to buyer's order quantity is not affected by the production cost of the supplier nor the wholesale price.

Predicting the optimum response of the supplier, the buyer has to decide about his order quantity, as well as his direct commitment in the capacity of the supplier.

The problem of the buyer can be defined as follows:

$$\max_{q^b, r^b \geq 0} \Theta(q^b, r^b) = pE\left[\min\left\{D, \min\left\{q^b, U(r^b + r^{s^*}(q^b, r^b), Y)\right\}\right\}\right] - wE\left[\min\left\{q^b, U(r^b + r^{s^*}(q^b, r^b), Y)\right\}\right] - r^b \tag{5.3}$$

It is important that the buyer manages his direct investment, knowing that increasing the investment induces the supplier to shrink her commitment on the investment. In the next proposition we look at the dynamics of the optimum investment of the buyer and supplier.

Proposition 5.3. *In the equilibrium,* $r^{b^*} r^{s^*}(q^b, r^{b^*}) = 0$.

Proposition 5.3 suggests that the buyer avoids direct involvement when he finds the supplier is motivated enough to improve capacity. When the investment of the supplier is high enough, the buyer avoids any payment to shift the investment cost to the supplier. However in some situations the investment of the supplier does not meet the expectations of the buyer. As a result the buyer may experience a high level of lost sale. This condition forces the buyer to get directly involved in the improvement of capacity. As a consequence the supplier takes advantage of the situation and cancels all her effort for improving her capacity.

Now the question is how the optimum order quantity of the buyer differs according to the opportunistic behaviour of the buyer and supplier in investment. Consider $q_1^{b^*}$ and $q_2^{b^*}$ as the optimum order quantity of the buyer when $r^{s^*}(q^b, r^{b^*}) = 0$ and $r^{b^*} = 0$, respectively. First we look at the situation where the buyer takes control of the investment and as a result $r^{s^*}(q^b, r^{b^*}) = 0$.

Proposition 5.4. *If* $r^{s^*}(q_1^b, r^{b^*}) = 0$*, the optimum value of the order quantity of the buyer is* $q_1^{b^*} = \bar{F}^{-1}(\frac{w}{p})$.

According to Proposition 5.4, the optimum value of the order quantity is unique. However this does not mean that the objective function is concave with respect to q_1^b (see Ciarallo *et al.* (1994)). When there is no investment by the supplier, the buyer sets his order quantity equal to critical fractile solution. Note that this value is not affected by neither uncertainty of supply nor the amount of investment of the buyer. In the next proposition we look at the optimum investment of the buyer when the supplier has no share.

Proposition 5.5. *If* $r^{s^*}(q_1^b, r^{b^*}) = 0$*, the optimum value of* r^b *is unique and solves the first order condition.*

According to Proposition 5.5, the optimum value of the buyer's investment can be found by setting the first derivative of the objective function equal to zero. When there is no contribution by the supplier, the buyer ignores the production cost of the supplier in his decisions.

In the case where the supplier invests on the improvement of capacity, the ordering behaviour of the buyer is different. The next proposition compares the ordering quantity of the buyer when the supplier makes the investment with the case where the buyer makes the investment.

Proposition 5.6. *The ordering quantity of the buyer increases when the supplier makes the investment (*$q_2^{b^*} \geq q_1^{b^*}$*).*

The leader role of the buyer gives him the power to decide whether he wants to invest directly on the capacity, or leaves the investment to the supplier. In the case that the buyer does not find direct investment profitable enough, he induces the supplier to increase her investment by an order inflation strategy. So the buyer has to find a balance between the cost of over-ordering and the benefit of having more capacity. In this case it can be demonstrated that there is a unique solution for the optimum order quantity.

Proposition 5.7. *Consider $v(x, y)$ as the value of investment where for an observation of Y we have $u(v(x, y), y) = x$. Let $E\left[u_{r^s}(v(\bar{F}^{-1}(\frac{w}{p}), \underline{l}), Y)\right] \geq 1/(w-c)$. If $r^{b^*} = 0$, the optimum value of the order quantity can be found by solving for the optimality conditions.*

The conditions in Proposition 5.7 provide sufficient conditions for the optimum order quantity to be bounded. According to this proposition, although the presence of investment by the supplier complicates the analysis of the problem, we can still find the optimum solution by using the first order conditions.

The question that the buyer is facing is to whether invest on improvement of the capacity or let the supplier make the investment. The general form of the capacity improvement function and the presence of uncertainty in both supply and demand makes the analysis of these cases complex. However, we can find some special conditions in which the buyer benefits from transferring the investment to the supplier.

Proposition 5.8. *If $w - c \geq p - w$, then it is optimal for the buyer to not invest on the improvement of the capacity.*

Proposition 5.8 demonstrates how the marginal profit of both parties plays a role in the behaviour of the buyer. When the profit margin of the supplier is higher, the potential investment by the supplier surpasses the investment of the buyer. Consequently the delivered quantity to the buyer will be increased. Also the buyer does not need to tolerate any investment related costs. Note that since we do not consider any penalty or salvage cost the profit margins are equal to the unit lost sales cost. So according to Proposition 5.8, the higher unit lost sales cost of the supplier is a signal to the buyer that there will be a proper level of investment by the supplier.

This situation imposes an extra cost on the supplier which decreases her profit. However, based on Proposition 5.6, when the investment is vested to the supplier, the buyer increases his order quantity to maintain a certain level of investment. This phenomenon inflates the sales of the supplier and results in a higher profit. In some cases the effect of order inflation even dominates the investment costs. Therefore transferring the investment from the buyer to the supplier is more desirable by the supplier too. We provide an example of such situation in Appendix B of this chapter.

5.3 Centralized Case

Here we look at the centralized case of the problem where the profit of the supply chain is maximized. In this situation the problem is reduced to a newsvendor problem with uncertain capacity and investment opportunity. Considering q^c and r^c as the order quantity and the investment, the problem can be formulated as follows:

$$\begin{aligned} \underset{q^c,r^c\geq 0}{max}\ \Omega(q^c,r^c) =& pE\left[\min\left\{D,\min\left\{q^c,U(r^c,Y)\right\}\right\}\right] \\ & - cE\left[\min\left\{q^c,U(r^c,Y)\right\}\right] - r^c. \end{aligned} \tag{5.4}$$

Analyzing the centralized case, provides a proper perspective about the factors that distance the supply chain from coordination in a decentralized case. The structure of the problem in the centralized case is similar to the case where the buyer invests on capacity improvement.

Proposition 5.9. $\Omega(q^c, r^c)$ *is unimodal and* $q^{c^*} = \bar{F}^{-1}(\frac{c}{p}) \geq q_1^{b^*}$

Proposition 5.9 suggests that the optimum solution of the centralized case can easily be found by setting the first derivatives equal to zero. Also the order quantity in the centralized case is greater than the case where the buyer invests on the capacity. This behaviour is aligned with the concept of double marginalization. However this may not be the case when the supplier makes the investment. The order inflation in some situations can dominate the effect of double marginalization and as a result the order quantity in the decentralized case exceeds that of the centralized case. An example of such a situation is provided in Appendix B of this chapter.

5.4 Investment Sharing Contract

The opportunistic behaviour of the buyer and the supplier prevents any coordination on capacity improvement. The buyer either invests directly or motivates the supplier to increase her investment by order inflation. Either way, the investment may not be enough to coordinate the supply chain. Under such conditions, the performance of the supply chain can be improved by some contract that obliges both parties to deviate from the equilibrium. There are not many studies that consider the coordination of a buyer and a supplier on reliability improvement. The study of Tang *et al.* (2014) is one of the few that investigated such situation. They assumed that the investment decision is made by the supplier and the buyer accepts to pay a certain portion of the investment. In addition, they assumed that the buyer decides about the portion that he wants to pay. We consider a similar setting for the contract and assume that the commitment of the buyer is pre-negotiated and known at the beginning of the period. Let $0 \leq \beta \leq 1$ represent the commitment of the buyer in the investment. Also consider q^m as the order quantity of the buyer and r^m as the total amount of investment under the contract (similar to Proposition 5.4, it is easy to show that the optimum targeted production quantity of the supplier is equal to q^m). So the buyer invests βr^m on the capacity and the rest is paid by the supplier. It is clear that under such a contract the optimum order quantity is smaller in comparison to the case that the supplier makes all the investment. However it can be either higher or lower than the case where the buyer makes all the investment. The examples of such situations are provided in Section 5.5. In the Proposition 5.10 we explore the behaviour of the optimum investment.

Proposition 5.10.

a) For any value of q^m, $\frac{\partial i^{m^*}(\alpha)}{\partial \alpha} \geq 0$

b) If $q^m = q^b$ *then* $r^{m^*} \geq r^{s^*}$.

According to the above proposition, the total amount of investment grows as the portion of the investment by the buyer increases. This is a promising fact for the coordination of the buyer and the supplier. Although the total investment increases (for the same order quantity), one may expect that under a contract, the amount of investment by the supplier is lower than the case where the supplier is the sole-investor. However we can show that there are situations that not only the contract increases the total investment, but also the individual investment by the supplier increases. An example of such a situation is provided in Appendix B of this chapter.

In Section 5.5 we provide a more comprehensive numerical analysis of the optimum solution under a contract.

5.5 Numerical Analysis

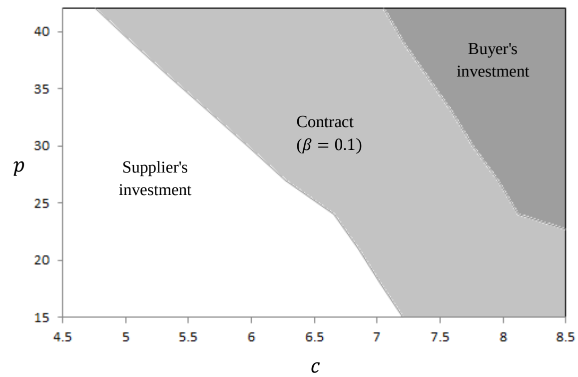
In this section we carry a numerical analysis in order to better understand the investment behaviours of the buyer and supplier and the role of contracts. We used a Cobb-Doglas function to represents the effect of investment on capacity. The parameters are set as follows: $Y \sim \exp(100)$, $D \sim \exp(300)$, $c = 6$, $w = 10$, $p = 30$, $\beta = 0.1$ and $U(r^b + r^s, Y) = A.(K + r^s + r^b)^{\alpha} + Y$ where $A = 10$, $K = 50$ and $\alpha = 0.5$. Using the numerical analysis, we want to answer two main questions. First, how does the buyer decide between his complete investment, contract (if he has the option) and complete investment of the supplier. Second how does the optimum solution behaves under different settings of the contract.

5.5.1 Comparison of Investment Options

The first decision that the buyer has to make is to find the optimum investment strategy. As the leader, the buyer has to decide whether he wants to take charge in the investment, vest it to the buyer or sign a contract that enables the players to share the investment (if there is such an opportunity). One of the main factors that affect such a decision is the profit margin of the buyer and the supplier. In Proposition 5.8 we provided one of the conditions in which transferring the investment to the supplier is more profitable for the buyer. Here we analyze this decision numerically.

Figure 5.1 shows the optimal decision of the buyer, under different profit margins.

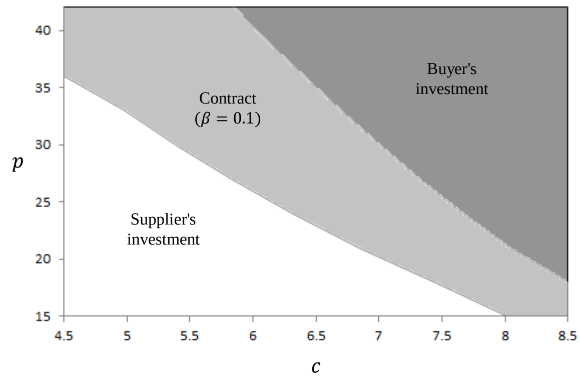
Figure 5.1: Optimality of different options for the buyer



The buyer selects a sole-investment strategy when there are more extreme differences between the profit margins. When the fraction of the profit margin of the supplier over the profit margin of the buyer is high, the buyer selects the supplier as the sole investor. In this situation the investment of the supplier is high enough to prevent the buyer from having any contribution in capacity improvement. As this fraction decreases, it is better for the buyer to boost the investment by taking a share in the investment. In this situation the buyer does not have to tolerate all the investment related costs, while the capacity is enhanced enough to satisfy the demand. When the market position of the buyer in terms of profit margin is much superior to the supplier, it is optimal for the buyer to take charge of the investment completely. In this situation due to the higher lost sale cost of the buyer, he becomes more sensitive to underproduction. Consequently the buyer takes the complete responsibility over investment.

Figure 5.2 demonstrates which option yields the highest profit for the supply chain under different values of players' profit margins. The profit margin of the buyer over supplier plays a key role in the superiority of different options. When this value is low, the supply chain performs the best when the supplier is the only investor. As the fraction increases, contracting dominates the sole-investment of the supplier. When

Figure 5.2: Optimality of different options for the supply chain



the buyer faces a great profit margin, while this value is low for the supplier, the best performance of the supply chain is achieved under the buyer's sole-investment. This behaviour is roughly compatible with the optimal investment decision of the buyer. Consequently the self-oriented behaviour of the buyer does not necessarily result in poor performance of the supply chain. In spite of this similarity, in comparison to the optimal decision of the buyer, the supply chain's profit seems to be more sensitive to the variation of the buyer's profit margin.

Table 5.2: The supply chain profit for different production costs ($p = 30$)

c	No coordination		Contract ($\beta = 0.1$)		Centralized
	Profit	D.C	Profit	D.C	Profit
4.5	3876.48	%1.2	3854.91	%1.7	3922.10
5	3715.96	%0.9	3705.58	%1.2	3748.73
5.5	3549.36	%0.9	3552.27	%0.8	3580.45
6	3373.26	%1.2	3391.76	%0.7	3417.20
6.5	3183.88	%2.3	3219.15	%1.2	3258.88
7	3025.70	%2.6	3028.51	%2.5	3105.42
Average		%1.5		%1.3	

Table 5.2 shows the profit of the supply chain in decentralized, contract and centralized cases for different values of production costs. Columns D.C represents the difference between each case and the centralized case. On average both decentralized

and contract cases show a little deviation from the centralized case. One of the reasons for this behaviour is the effect of order inflation that reduces the influence of double marginalization. The effect of contract on improving the performance of the system is very marginal. When the production cost is very low or very high, the contract actually has a negative effect on the system. Note that we only investigated one setting of the contract ($\beta = 0.1$). Clearly a better performance is expected if the profit of the supply chain is optimized over different values of β.

Figure 5.3: The behaviour of optimal order quantity

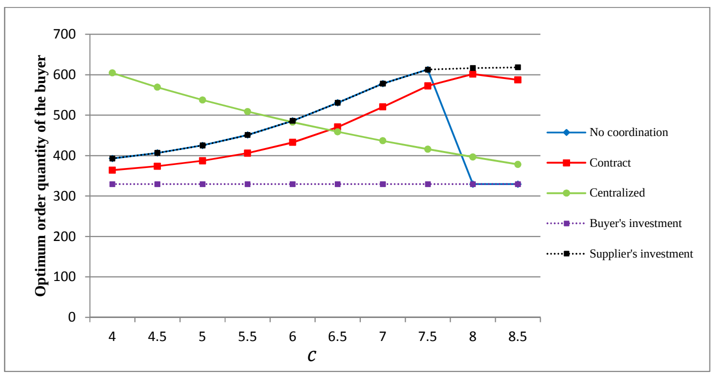
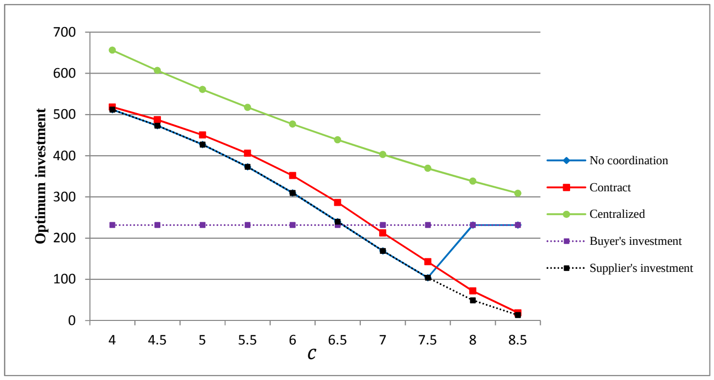


Figure 5.3 shows the behaviour of the optimum order quantity of the buyer for different supplier's production cost. As the production cost increases, the supplier becomes less motivated to invest in the capacity. This situation is not desirable for the buyer, so he increases the order quantity to induce the supplier to have more investment. However when the production costs reach a certain point, the cost of order inflation becomes so high that the buyer prefers to do the investment himself and reduce the order quantity to the critical fractile solution.

This behaviour is observable in Figure 5.4 too. This figure pictures the optimum investment with regard to different values of production cost.

The increase of the production cost, decreases the investment of the supplier. The order inflation has a positive effect on the supplier's investment, but it is not enough

Figure 5.4: The behaviour of optimal investment



to stop the downward trend of the optimal investment. The jump in the optimal investment represents the point where the buyer decides to shift the investment from the supplier to himself. From this point on, the buyer keeps the order quantity and the investment constant.

5.5.2 Behaviour of Optimal Solution under Different Settings of the Contract

In this section we investigate how the behaviour of the optimum solution changes with respect to different values of β. The constant β defines the share of the buyer in total investment where the supplier is the decision maker about the amount of investment.

Figure 5.5 shows the supply chain's profit with respect to different values of β. Figure 5.6 indicates the variation of the optimum order quantity for different sharing contracts. Three levels of production cost are considered to provide a better understanding about the behaviour of the optimum solution. When the production cost is low, the supply chain performs the best when β is set as low as possible. In this situation the optimum order quantity is at its highest and the supplier is motivated

Figure 5.5: Variation of supply chain's profit with respect to β

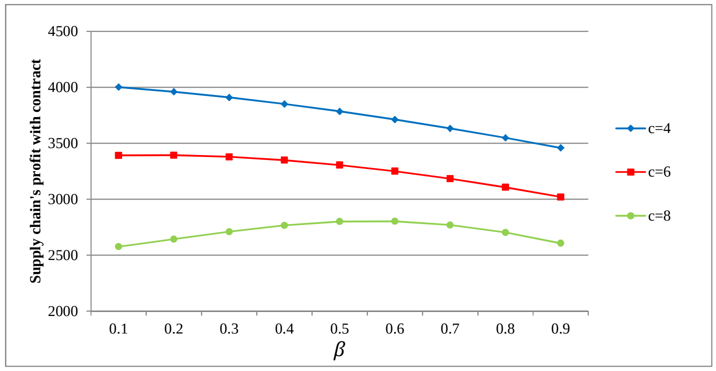
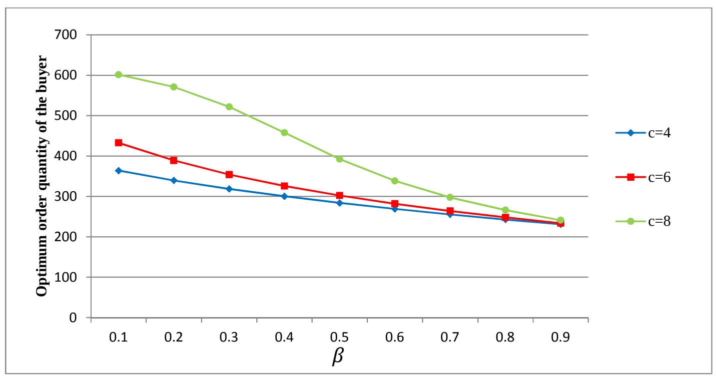


Figure 5.6: Optimal order quantity with respect to β



enough to make a sufficient investment on the capacity. When the production cost is higher, the investment of the supplier may not be enough. In this situation a greater share from the buyer can improve the system significantly. For instance in the case when the production cost is equal to 8, increasing β from 0.1 to 0.6 can improve the profit of supply chain by over 8 percent. This increase is enough to make contracting preferable in comparison to the no coordination case.

The behaviours of the optimum total investment and supplier's individual investment are demonstrated in Figures 5.7 and 5.8, respectively. An increase of β can affect the investment from two different aspects. First by reducing the cost of investment of the supplier, it motivates her to increase the investment which increases the total investment. On the other hand transfering the costs to the buyer will result in a reduction of the optimal order quantity which has a negative effect on the supplier's investment. When the production cost is low, the negative effect of order quantity deflation dominates the effect of lower investment cost and as a result the total investment decreases. However for high production costs, increasing the share of buyer's investment can be significantly effective on increasing the total investment. This behaviour suggests that when the supplier is less motivated to invest in comparison to the buyer, it is better to increase the share of the buyer in order to have a better performance.

Figure 5.7: Variation of total investment with respect to β

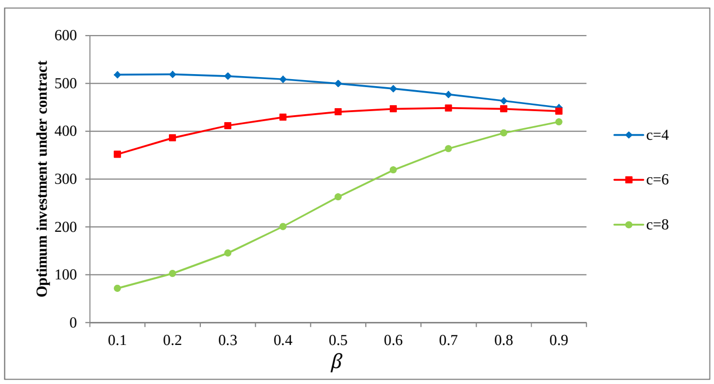
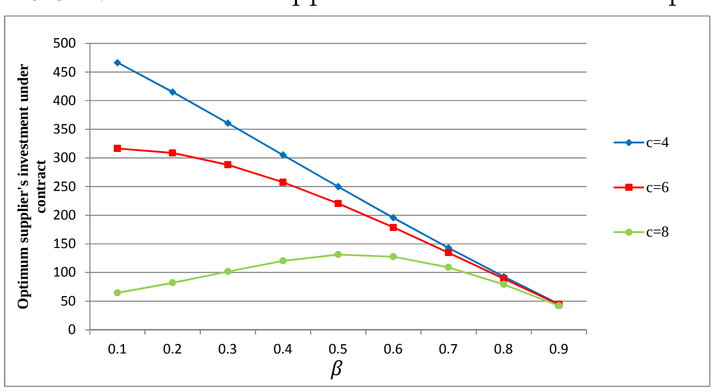


Figure 5.8: Variation of supplier's investment with respect to β



Increasing the share of the buyer does not necessarily mean that the supplier will decrease her investment. When the production cost is high, as β goes up the supplier increases her investment to boost the investment of the buyer. As a result the increase of buyer's share, can increase the total investment significantly. However in lower production costs, since the individual investment of the supplier is high enough, by increasing the buyer's share, the supplier keeps the total investment almost constant and just enjoys the transfer of investment cost from her side to the buyer's side.

5.6 Conclusions

Both theoretical and experimental analyses have demonstrated the positive role of supplier development programs in supply chains. In this study we looked at the relation of a buyer and a supplier on the supplier's capacity improvement. We provide insight for the buyers in order to determine their involvement in supplier's improvement. The problem is investigated under a Stackelberg game setting where the buyer is the leader and the players are facing uncertain demand and capacity.

We first showed that the problems of the buyer and the supplier are concave and the first order conditions can be used to find the optimum solution. Also it can be shown that at the equilibrium, the buyer and the supplier will not invest in the capacity simultaneously. When the supplier is motivated enough, the buyer does not invest in the capacity in order to avoid any additional cost. For instance when the profit margin of the supplier is higher than the buyer, it is optimal for the buyer to avoid any investment on the capacity. In this situation the buyer uses order inflation to induce the supplier to have more investment. This inflation can be so high that it dominates the effect of double marginalization. As a result the order quantity in the decentralized case surpasses the order quantity in the centralized case. There are cases where the supplier welcomes transferring the investment to her side since the benefits of order inflation overshadow the cost of investment. When the supplier shows a lack of motivation in the investment, the buyer invests directly in the capacity of the supplier. In this situation the supplier sets her investment equal to zero and the buyer drops his order quantity to the critical fractile solution. When the investment is completely done by the buyer, the optimal order quantity is not affected by neither uncertainty of supply nor the amount of investment.

One of the ways that the investment can be increased is by sharing the investment under some contract. We look at the case where the investment is determined by the supplier and a portion of the investment is paid by the buyer. Numerical analysis of the problem suggests that this type of contracts performs best when the profit margin of both players are similar. By increasing the share of the player that has a better situation in market (in term of profit margin) the performance of the contract can be improved. It has been indicated that decreasing the share of the supplier in the total investment does not necessary mean that the supplier reduces her investment. In some cases both the investment of the supplier and the buyer increases when the share of the buyer increases proportionally.

Appendix A

Proof of Proposition 5.1. In order to proof this proposition, we have to show that for any value of r^s, $\Pi(q^b, r^s|q^b, r^b) \geq \Pi(\hat{q}, r^s|q^b, r^b)$, for all the values of $\hat{q} \neq q^b$.

Case 1: Consider $\hat{q} < q^b$. Then we have:

$$\begin{aligned}
\Pi(\hat{q}, r^s|q^b, r^b) &= wE\left[\min\left\{q^b, \min\left\{\hat{q}, U(r^b + r^s, Y)\right\}\right\}\right] \\
&\quad - cE\left[\min\left\{\hat{q}, U(r^b + r^s, Y)\right\}\right] - r^s \\
&= wE\left[\min\left\{\hat{q}, U(r^b + r^s, Y)\right\}\right] \\
&\quad - cE\left[\min\left\{\hat{q}, U(r^b + r^s, Y)\right\}\right] - r^s \\
&= (w - c)E\left[\min\left\{\hat{q}, U(r^b + r^s, Y)\right\}\right] - r^s \\
&\leq (w - c)E\left[\min\left\{q^b, U(r^b + r^s, Y)\right\}\right] - r^s = \Pi(q^b, r^s|q^b, r^b)
\end{aligned}$$

Case 2: Consider $\hat{q} > q^b$. Then we have:

$$\begin{aligned}
\Pi(\hat{q}, r^s|q^b, r^b) &= wE\left[\min\left\{q^b, \min\left\{\hat{q}, U(r^b + r^s, Y)\right\}\right\}\right] \\
&\quad - cE\left[\min\left\{\hat{q}, U(r^b + r^s, Y)\right\}\right] - r^s \\
&= wE\left[\min\left\{q^b, U(r^b + r^s, Y)\right\}\right] \\
&\quad - cE\left[\min\left\{\hat{q}, U(r^b + r^s, Y)\right\}\right] - r^s \\
&\leq (w - c)E\left[\min\left\{q^b, U(r^b + r^s, Y)\right\}\right] - r^s = \Pi(q^b, r^s|q^b, r^b)
\end{aligned}$$

and this completes the proof. □

Proof of Proposition 5.2. a) Since $r^b + r^s$ is a linear function, $u(x, y)$ is concave with respect to x and minimization preserves concavity, $\Gamma(r^s|q^b, r^b)$ is concave with respect to r^s.

b) The first and the second derivatives of $\Gamma(.)$ with respect to r^s are:

$$\Gamma_{r^s}(.) = (w-c)\int_{\underline{l}}^{y(r^b+r^s,q^b)} u_{r^b+r^s}(r^b+r^s,\epsilon)g(\epsilon)d\epsilon - 1 \tag{5.5}$$

$$\begin{aligned}\Gamma_{r^sr^s}(.) =&(w-c)\Bigg\{\int_{\underline{l}}^{y(r^b+r^s,Q)} u_{r^b+r^s,r^b+r^s}(r^b+r^s,\epsilon)g(\epsilon)d\epsilon \\ &+ y_{r^b+r^s}(r^b+r^s,q^b)u_{r^b+r^s}(r^b+r^s,y(r^b+r^s,q^b))g(y(r^b+r^s,q^b))\ \Bigg\}\end{aligned}$$

Also the second derivative of $\Gamma(.)$ with respect to r^b and r^s is:

$$\begin{aligned}\Gamma_{r^br^s}(.) =&(w-c)\Bigg\{\int_{\underline{l}}^{y(r^b+r^s,Q)} u_{r^b+r^s,r^b+r^s}(r^b+r^s,\epsilon)g(\epsilon)d\epsilon \\ &+ y_{r^b+r^s}(r^b+r^s,q^b)u_{r^b+r^s}(r^b+r^s,y(r^b+r^s,q^b))g(y(r^b+r^s,q^b))\ \Bigg\}\end{aligned}$$

According to implicit function theorem we have:

$$\frac{\partial r^{s^*}(r^b)}{\partial r^b} = -\frac{\Gamma_{r^br^s}(.)}{\Gamma_{r^sr^s}(.)} = -1$$

c) We have:

$$u(r^b+r^s, y(r^b+r^s,q^b)) = q^b$$

By taking the derivative with respect to r^s from both sides we have:

$$\begin{aligned} & u_{r^b+r^s}(r^b + r^s, y(r^b + r^s, q^b)) + u_y(r^b + r^s, y(r^b + r^s, q^b)).y_{r^b+r^s}(r^b + r^s, q^b) = 0 \\ \Rightarrow & y_{r^b+r^s}(r^b + r^s, q^b) = -\frac{u_{r^b+r^s}(r^b + r^s, y(r^b + r^s, q^b))}{u_y(r^b + r^s, y(r^b + r^s, q^b))} \end{aligned} \tag{5.6}$$

Since $u(x, y)$ is increasing in both x and y, then $y_{r^b+r^s}(r^b+r^s, q^b) \leq 0$. The second derivative of $\Gamma(.)$ with respect to q^b and r^s is:

$$\Gamma_{q^b r^s}(.) = -(w - c)y_{r^b+r^s}(r^b + r^s, q^b)g(y(r^b + r^s, q^b)) \geq 0 \tag{5.7}$$

According to the implicit function theorem we have:

$$\frac{\partial r^{s^*}(q^b)}{\partial q^b} = -\frac{\Gamma_{q^b r^s}(.)}{\Gamma_{r^s r^s}(.)} \geq 0$$

Note that the optimum value of the supplier's investment is bounded below by zero. So $\frac{\partial r^{s^*}(q^b)}{\partial q^b} = -1$ when $r^{s^*} > 0$. □

Proof of Proposition 5.3. Consider $\Delta(q^b, r^b, r^s)$ as follows:

$$\begin{aligned} \Delta(q^b, r^b, r^s) =& pE\left[\min\left\{D, \min\left\{q^b, U(r^b + r^s, Y)\right\}\right\}\right] \\ & - wE\left[\min\left\{q^b, U(r^b + r^s, Y)\right\}\right] \end{aligned}$$

Clearly $\Delta_{r^s}(q^b, r^b, r^s) = \Delta_{r^b}(q^b, r^b, r^s)$. The first derivative of $\Theta(q^b, r^b)$ with respect to r^b is:

$$\Theta_{r^b}(q^b, r^b) = \Delta_{r^b}(q^b, r^b, r^{s^*}(q^b, r^b)) + \Delta_{r^s}(q^b, r^b, r^{s^*}(q^b, r^b)).\frac{\partial r^{s^*}(q^b, r^b)}{\partial r^b} - 1$$

According to Proposition 5.2 part (b), when $r^{s^*}(q^b, r^b) > 0$ we have $\frac{\partial r^{s^*}(q^b,r^b)}{\partial r^b} = -1$. It follows that $\Theta_{r^b}(q^b, r^b) = -1$ when $r^{s^*}(q^b, r^b) > 0$ and as a result the optimum value of r^b is equal to 0. This completes the proof. □

Proof of Proposition 5.4. If $r^{s^*}(q_1^b, r^{b^*}) = 0$ the objective function of the buyer will be reduced to $\hat{\Theta}(q_1^b, r^b)$ which is as follows:

$$\hat{\Theta}(q_1^b, r^b) = pE\left[\min\left\{D, \min\left\{q_1^b, U(r^b, Y)\right\}\right\}\right] - wE\left[\min\left\{q_1^b, U(r^b, Y)\right\}\right] - r^b$$

The first derivative of $\hat{\Theta}(q_1^b, r^b)$ with respect to q_1^b is:

$$\hat{\Theta}_{q_1^b}(q_1^b, r^b) = \bar{G}(y(r^b, q_1^b))\left[p\bar{F}(q_1^b) - w\right]$$

By using the first order condition we have $q_1^{b^*} = \bar{F}^{-1}(\frac{w}{p})$. Now if we check the second order condition we have:

$$\begin{aligned}\hat{\Theta}_{q_1^b q_1^b}(q_1^b, r^b) = & pg(y(r^b, q_1^b))y_{q_1^b}(r^b, q_1^b)\bar{F}(q_1^b) \\ & - p\bar{G}(y(r^b, q_1^b))f(q_1^b) + wg(y(r^b, q_1^b))y_{q_1^b}(r^b, q_1^b).\end{aligned}$$

By replacing q_1^b with $q_1^{b^*}$ we get:

$$\hat{\Theta}_{q_1^b q_1^b}(q_1^{b^*}, r^b) = -p\bar{G}(y(r^b, q_1^{b^*}))f(q_1^{b^*}) \leq 0$$

As a result $q_1^{b^*} = \bar{F}^{-1}(\frac{w}{p})$ is the optimum value of the order quantity. □

Proof of Proposition 5.5. The first and second derivatives of $\hat{\Theta}(q_1^b, r^b)$ with respect to

r^b is:

$$\hat{\Theta}_{r^b}(q_1^b, r^b) = \int_{\underline{l}}^{y(r^b,q_1^b)} u_{r^b}(r^b,\epsilon)\left[p\bar{F}(u(r^b,\epsilon)) - w\right] g(\epsilon)d\epsilon - 1 \tag{5.8}$$

and

$$\begin{aligned}\hat{\Theta}_{r^b r^b}(q_1^b, r^b) = &\int_{\underline{l}}^{y(r^b,q_1^b)} u_{r^b r^b}(r^b,\epsilon)\left[p\bar{F}(u(r^b,\epsilon)) - w\right] g(\epsilon)d\epsilon \\ &- p\int_{\underline{l}}^{y(r^b,q_1^b)} u_{r^b}^2(r^b,\epsilon) f(u(r^b,\epsilon)) g(\epsilon)d\epsilon \\ &+ y_{r^b}(r^b,q_1^b) u_{r^b}(r^b, y(r^b,q_1^b)) g(y(r^b,q_1^b))\left[p\bar{F}(q_1^b) - w\right]\end{aligned} \tag{5.9}$$

By replacing q_1^b with $q_1^{b^*}$ in Equation (5.9) we have:

$$\begin{aligned}\hat{\Theta}_{r^b r^b}(q_1^{b^*}, r^b) = &\int_{\underline{l}}^{y(r^b,q_1^{b^*})} u_{r^b r^b}(r^b,\epsilon)\left[p\bar{F}(u(r^b,\epsilon)) - w\right] g(\epsilon)d\epsilon \\ &- p\int_{\underline{l}}^{y(r^b,q_1^{b^*})} u_{r^b}^2(r^b,\epsilon) f(u(r^b,\epsilon)) g(\epsilon)d\epsilon\end{aligned} \tag{5.10}$$

Clearly the second term of the Equation (5.10) is negative. Note that $T(r^b,\epsilon) = p\bar{F}(u(r^b,\epsilon)) - w$ is a decreasing function of ϵ (since $u(r^b,\epsilon)$ is increasing in ϵ and $\bar{F}(.)$ is decreasing). Also $T(r^b, y(r^b, q_1^{b^*})) = 0$. Consequently $T(r^b,\epsilon)$ is positive for any value of ϵ that is less than $y(r^b, q_1^{b^*})$ and this is sufficient for negativity of the first term of Equation (5.10). Therefore $\hat{\Theta}(q_1^{b^*}, r^b)$ is concave with respect to r^b. □

Proof of Proposition 5.6. When the supplier invests on the capacity, according to Proposition 5.3 the buyer sets his investment equal to zero. In this situation the objective function of the buyer will be reduced to $\bar{\Theta}(q_2^b, r^b)$ which is as follows:

$$\begin{aligned}\bar{\Theta}(q_2^b, r^{s^*}(q_2^b)) =& pE\left[\min\left\{D, \min\left\{q_2^b, U(r^{s^*}(q_2^b), Y)\right\}\right\}\right] \\ & - wE\left[\min\left\{q_2^b, U(r^{s^*}(q_2^b), Y)\right\}\right]\end{aligned}$$

The first derivative $\bar{\Theta}(q_2^b, r^{s^*}(q_2^b))$ with respect to q_2^b is:

$$\begin{aligned}\frac{\partial\bar{\Theta}(q_2^b, r^{s^*}(q_2^b))}{\partial q_2^b} &= \bar{\Theta}_{q_2^b}(q_2^b, r^{s^*}(q_2^b)) + \bar{\Theta}_{r^s}(q_2^b, r^{s^*}(q_2^b))\frac{\partial r^{s^*}(q_2^b)}{\partial q_2^b} \\ &= \bar{G}(y(r^{s^*}(q_2^b), q_2^b))\left[p\bar{F}(q_2^b) - w\right] \\ &\quad + \left[\int_{\underline{l}}^{y(r^{s^*}(q_2^b), q_2^b)} u_{r^s}(r^{s^*}(q_2^b), \epsilon)\left\{p\bar{F}(u(r^{s^*}(q_2^b), \epsilon)) - w\right\} g(\epsilon)d\epsilon\right] \\ &\quad \cdot\frac{\partial r^{s^*}(q_2^b)}{\partial q_2^b}.\end{aligned} \tag{5.11}$$

Clearly for any value $q_2^b \leq \bar{F}^{-1}(\frac{w}{p})$, the first part of Equation (5.11) is positive. According to Proposition 5.2, part (c), $\frac{\partial r^{s^*}(q_2^b)}{\partial q_2^b}$ is positive. Also since $u(x, y)$ is increasing with respect to x, then $u_{r^s}(r^{s^*}(q_2^b), \epsilon)$ is positive for any values of ϵ. Now considering $T(r^s, \epsilon) = p\bar{F}(u(r^s, \epsilon)) - w$, according to Proposition 5.5, $T(r^s, \epsilon)$ is positive for any value of $\epsilon \leq y(r^s, \bar{F}^{-1}(\frac{w}{p}))$. As a result $\frac{\partial\bar{\Theta}(q_2^b, r^{s^*}(q_2^b))}{\partial q_2^b} \geq 0$ for any value of $q_2^b \leq \bar{F}^{-1}(\frac{w}{p})$ which completes the proof. □

In order to prove Proposition 5.7, first we need to prove the following lemma.

Lemma 5.1. *Assume that $\rho(x)$ is unimodal (with a maximum) and x^* is its optimum value. Let $\delta(x)$ be an increasing function of x. Also assume that $\rho(x)$ and $\delta(x)$ are both continuous and we have $\lim_{x\to+\infty}\delta(x) \geq x^*$ and $\lim_{x\to 0}\delta(x) \leq x^*$. Then $\rho(\delta(x))$ is unimodal.*

Proof. Since $\delta(x)$ is increasing with respect to x, there is a one on one correspondence between x and δ. Since $\lim_{x\to+\infty}\delta(x) \geq x^*$ and $\lim_{x\to 0}\delta(x) \leq x^*$, then there exists a $\hat{x}$ where $\delta(\hat{x}) = x^*$. First we show that $\rho(\delta(x))$ is increasing for any value of $x < \hat{x}$.

Consider ϵ as a sufficiently small positive number. Then:

$$x \leq x + \epsilon \leq \hat{x} \Rightarrow \delta(x) \leq \delta(x+\epsilon) \leq \delta(\hat{x}) = x^*$$

Since $\rho(x)$ is unimodal then for any value of $x \leq x^*$, $\rho(x)$ is increasing. As a result for any value of $x < \hat{x}$, $\rho(\delta(x)) \leq \rho(\delta(x+\epsilon))$. To prove that $\rho(\delta(x))$ is decreasing for any value $x \geq \hat{x}$ can be done in an analogous way. As a result $\rho(\delta(x))$ is unimodal. □

Proof of Proposition 5.7. We consider

$$\rho(q_2^b) = pE\left[\min\left\{D, q_2^b\right\}\right] - wq_2^b$$

and

$$\delta(q_2^b) = \min\left\{q_2^b, u(r^{s^*}(q_2^b), y)\right\}$$

Since $r^{s^*}(q_2^b)$ is increasing (according to Proposition 5.2, part (b)), $\delta(q_2^b)$ is increasing in q_2^b, for any value of y. Considering $q_2^{b^*}$ as the optimum value of $\rho(q_2^b)$ then $q_2^{b^*} = \bar{F}^{-1}(\frac{w}{p})$. We know that:

$$\lim_{q_2^b\to 0}\delta(q_2^b) = 0 \leq q_2^{b^*}.$$

Now we need to prove:

$$\lim_{q_2^b \to +\infty} u(r^{s^*}(q_2^b), y) \geq q_2^{b^*} = \bar{F}^{-1}(\frac{w}{p})$$

We know that $\lim_{q_2^b \to +\infty} u(r^s, y(r^s, q_2^b)) = q_2^b = +\infty$, so $\lim_{q_2^b \to +\infty} y(r^s, q_2^b) = +\infty$. Based on the initial assumption we have:

$$\begin{aligned}
& E\left[u_{r^s}(v(\bar{F}^{-1}(\frac{w}{p}), \underline{l}), Y)\right] \geq 1/(w-c) \\
\Rightarrow & \int_{\underline{l}}^{\bar{l}} u_{r^s}(v(\bar{F}^{-1}(\frac{w}{p}), \underline{l}), \epsilon) g(\epsilon) d\epsilon \geq 1/(w-c) \\
\Rightarrow & \lim_{q_2^b \to +\infty} \int_{\underline{l}}^{y(r^s, q_2^b)} u_{r^s}(v(\bar{F}^{-1}(\frac{w}{p}), \underline{l}), \epsilon) g(\epsilon) d\epsilon \geq 1/(w-c).
\end{aligned} \tag{5.12}$$

Based on the fact that $u_{r^s}(r^s, y)$ is decreasing with respect to r^s and Equations (5.5) and (5.12) we have:

$$\begin{aligned}
r^{s^*}(q_2^b) \geq v(\bar{F}^{-1}(\frac{w}{p}), \underline{l}) & \Rightarrow r^{s^*}(q_2^b) \geq v(\bar{F}^{-1}(\frac{w}{p}), y), \ \forall y \\
& \Rightarrow u(r^{s^*}(q_2^b), y) \geq \bar{F}^{-1}(\frac{w}{p}), \ \forall y.
\end{aligned}$$

Note that:

$$\begin{aligned}
& u(v(r^s, y), y) = r^s \\
\Rightarrow & v_y(r^s, y) = -\frac{u_y(v(r^s, y), y)}{u_v(v(r^s, y), y)} \leq 0.
\end{aligned}$$

□

In order to prove Proposition 5.8, we need to prove the following lemma.

Lemma 5.2. *Let $\rho(x)$ be a continuous and unimodal function. Also consider $\gamma(x)$ and $\delta(x)$ as two increasing and continuous functions where $\gamma(x) \geq \delta(x)$. Also let $\gamma(0) = \delta(0)$. Then $\underset{x}{max}\rho(\gamma(x)) \geq \underset{x}{max}\rho(\delta(x))$.*

Proof. Consider x^* as the optimum value of x that maximizes $\rho(\delta(x))$. Since $\gamma(0) = \delta(0)$ and $\gamma(x) \geq \delta(x)$, there exists $\hat{x}$ such that $\gamma(\hat{x}) = \delta(x^*)$. So we have:

$$\underset{x}{max}\rho(\gamma(x)) \geq \rho(\gamma(\hat{x})) = \rho(\delta(x^*)) = \underset{x}{max}\rho(\delta(x))$$

and that completes the proof. □

Proof of Proposition 5.8. Consider $r^{b^*}(q)$ $(r^{s^*}(q))$ as the optimum value of buyer(supplier) investment when the supplier(buyer) investment is zero. Based on Equation (5.8) we have:

$$\begin{aligned}
&\int_{\underline{l}}^{y(r^{b^*}(q),q)} u_{r^b}(r^{b^*}(q),\epsilon)\left[p\bar{F}(u(r^{b^*}(q),\epsilon)) - w\right] g(\epsilon)d\epsilon - 1 = 0 \\
\Rightarrow &\int_{\underline{l}}^{y(r^{b^*}(q),q)} u_{r^b}(r^{b^*}(q),\epsilon)\left[p - w\right] g(\epsilon)d\epsilon \geq 1 \\
\Rightarrow &\int_{\underline{l}}^{y(r^{b^*}(q),q)} u_{r^b}(r^{b^*}(q),\epsilon) g(\epsilon)d\epsilon \geq \frac{1}{p-w}
\end{aligned} \tag{5.13}$$

Also according to Equation (5.5) we have:

$$\int_{\underline{l}}^{y(r^{s^*}(q),q)} u_{r^s}(r^{s^*}(q),\epsilon) g(\epsilon)d\epsilon = \frac{1}{w-c} \tag{5.14}$$

Based on the assumption we have:

$$\begin{aligned}
&w - c \geq p - w \\
\Rightarrow &\frac{1}{w-c} \leq \frac{1}{p-w} \\
\Rightarrow &\int_{\underline{l}}^{y(r^{s^*}(q),q)} u_{r^s}(r^{s^*}(q), \epsilon) g(\epsilon) d\epsilon \leq \int_{\underline{l}}^{y(r^{b^*}(q),q)} u_{r^b}(r^{b^*}(q), \epsilon) g(\epsilon) d\epsilon
\end{aligned}$$

Consider $\tau(x,q) = \int_{\underline{l}}^{y(x,q)} u_x(x,\epsilon)g(\epsilon)d\epsilon$. Since $u_x(x,\epsilon)$ and $y(x,q)$ are decreasing with respect to x, then from the previous inequality we can conclude that $r^{s^*}(q) \geq r^{b^*}(q)$.

We now define

$$\rho(q) = pE\left[\min\left\{D, q\right\}\right] - wq$$

and

$$\gamma(q) = \min\left\{q, u(r^{s^*}(q), y)\right\}, \quad \delta(q) = \min\left\{q, u(r^{b^*}(q), y)\right\}$$

Since $r^{s^*}(q) \geq r^{b^*}(q)$, then $\gamma(q) \geq \delta(q)$ for all the values of y. Also $\gamma(0) \geq \delta(0) = 0$. Then based on Lemma 5.2 and by taking the expected value over y we have:

$$\bar{\Theta}(q, r^{s^*}(q)) = E\left[\rho(\gamma(q))\right] \geq E\left[\rho(\delta(q))\right] \geq E\left[\rho(\delta(Q))\right] - r^{b^*}(q) = \hat{\Theta}(q, r^{b^*}(q)), \quad \forall q.$$

□

Proof of Proposition 5.9. The proof is analogous to the proofs of Proposition 5.4 and 5.5 □

Proof of Proposition 5.10. a) Consider $\tilde{\Gamma}(r^m|q^m)$ as the objective function of the supplier under a contract.

$$\tilde{\Gamma}(r^m|q^m) = (w-c)E\left[\min\left\{q^m, U(r^m, Y)\right\}\right] - (1-\beta)r^m$$

The second derivatives of $\tilde{\Gamma}(r^m|q^m)$ with respect to r^m and β are:

$$\begin{aligned}\tilde{\Gamma}_{r^m r^m}(.) =&(w-c)\,\{ \int_{\underline{l}}^{y(r^m,q^m)} u_{r^m,r^m}(r^m,\epsilon)g(\epsilon)d\epsilon \\ &+ y_{r^m}(r^m,q^m)u_{r^m}(r^m,y(r^m,q^m))g(y(r^m,q^m))\ \} \\ \tilde{\Gamma}_{r^m\beta}(.) =&1.\end{aligned}$$

So based on the implicit function theorem we have:

$$\frac{\partial r^{m^*}(\beta)}{\partial\beta} = -\frac{1}{\tilde{\Gamma}_{r^m r^m}(.)} \geq 0.$$

b) Clearly in the case that $\beta = 0$, $r^{m^*} = r^{s^*}$ (for the case that the supplier is the investor). According to part (a), r^{m^*} increases as the value of β increases, so $r^{m^*} \geq r^{s^*}$. □

Appendix B

Example where the supplier's investment is more profitable for both parties

Assume $Y \sim \exp(100)$, $D \sim \exp(300)$, $c = 1$, $w = 10$, $p = 30$ and $U(r^b + r^s, Y) = A.(K + r^s + r^b)^\alpha + Y$ where $A = 10$, $K = 50$ and $\alpha = 0.5$. Under this situation, when the buyer makes the investment, the optimum objective value of the buyer is 2281 and this value for the supplier is 2231 ($q_1^{b^*} = 329$, $r^{b^*} = 231.6$). However when the investment is done by the supplier, the profits of the buyer and supplier increase to 2686 and 2271 respectively ($q_2^{b^*} = 356.18$, $r^{b^*} = 660.15$).

Example where the order quantity in the decentralized case surpasses the centralized case

Assume $Y \sim \exp(100)$, $D \sim \exp(300)$, $c = 6$, $w = 10$, $p = 30$ and $U(r^b + r^s, Y) = A.(K + r^s + r^b)^\alpha + Y$ where $A = 10$, $K = 50$ and $\alpha = 0.5$. In this situation the optimal order quantity in the centralized (q^{c^*}) case is equal to 486 however this value in the decentralized case is 483.

Example where the investment of the supplier under a contract is higher than the decentralized case

Assume $Y \sim \exp(100)$, $D \sim \exp(300)$, $c = 6$, $w = 10$, $p = 30$, $\beta = 0.1$ and $U(r^b + r^s, Y) = A.(K + r^s + r^b)^\alpha + Y$ where $A = 10$, $K = 50$ and $\alpha = 0.5$. In this situation the optimum amount of investment by the supplier is $r^{s^*} = 309.78$. Under a contract the optimum amount of investment by the supplier is $(1-\beta)r^{m^*} = 316.55$.

Chapter 6

Concluding Remarks

6.1 Thesis Summary

In this thesis we concentrated on the problem of lot-sizing under uncertain supply and demand. We looked at this problem under three different settings: purchasing and production planning, joint lot-sizing and pricing and joint lot-sizing and capacity improvement.

In Chapter 2 we presented a framework for classifying models in the area of lot-sizing. We believe this framework will help in communicating results in this growing area as has been the practice in the areas of queueing and scheduling. Using this framework we reviewed the literature on lot-sizing and pricing when both demand and supply are uncertain. We have used our proposed framework to classify 139 papers. It is clear that there is an increasing interest in studying these problems. Furthermore, although the focus has earlier been on single product and supplier problems, which is hardly realistic for today's supply chains, we have found an increasing interest in the study of multiple products and/or suppliers in the last 15 years or so.

In Chapter 3 we focused on the sourcing problem of a handler in the almond industry. There are two ways for the handler to supply her required products. First through production contracts where the handler basically rent the farm and start her own production. The second way is to buy the products directly from the farmers by paying a premium. Based on the special characteristic of almond trees, a two year contract is preferable in this industry. As a result we developed a two year sourcing model under uncertainty of supply and demand. Considering both uncertainty of supply and demand complicates the analysis of the problem but we were able to prove the concavity of the objective functions under some conditions. We also provided the conditions in which purchasing contract is superior to a production contract. We numerically analyzed the problem using real data from the California almond industry. In particular we explored the behaviour of optimal decisions with regard to uncertainty of supply and demand. The analysis demonstrated that when the uncertainty of supply is high, the handler tends to move from a purchasing contract to a production contract. The handler shows a similar behaviour with regard to the uncertainty of demand. This is an interesting behaviour since one would expect that under high uncertainty the handler would avoid the risk of over and under production by preferring a purchasing contract.

In Chapter 4 we incorporated the pricing and capacity planning decisions with the lot-sizing problem. A multi-period model was developed where in each period the farmer has to decide about the amount of the land that he wants to put under cultivation, the price of each unit of product, and the amount of land that he wants to purchase at the end of the season to increase his production capacity. We also considered the option of renting out the farm. In the single period case, the structure of the model is similar to a newsvendor problem with pricing under uncertain supply and demand. In order to be able to analyze the problem, we developed the concept of expected demand fill rate elasticity as an extension to the lost sales rate elasticity introduced by Kocabiyikoğlu and Popescu (2011). By implementing this concept, we found the sufficient conditions for unimodality of the problem. In multi-period case we found the conditions where a one-sided production, pricing and capacity planning policy is optimal. We extend our results to the case where there is a fixed production cost. In this situation, the optimal policy of the farmer becomes a two-sided production, pricing and capacity planning policy. We also investigated the effects of dynamic pricing, renting and uncertainty of supply on optimal decisions numerically. It has been revealed that the effect of dynamic pricing does not necessarily decrease as the initial asset of the farmer increases. Also the farmer tends to put more land under cultivation and set higher prices for his products when the uncertainty of supply increases.

In Chapter 5 we studied the relation of a buyer and a supplier in a supplier development context under a Stackelberg game setting. We assumed that the capacity of the supplier is uncertain but it can be improved by investment. Both the supplier and the buyer have the opportunity to invest on the supplier's capacity. Also the buyer is facing a random demand. As the leader, the buyer first decides about the order quantity and the amount of investment and then the supplier decides about the production quantity and her own part of the investment. We proved that the buyer and the supplier show an opportunistic behaviour toward investment. Consequently the players will not invest on the capacity simultaneously. If the buyer finds the supplier motivated enough to invest sufficiently on capacity, he cancels all his direct involvement on improving the capacity. Instead, he picks an order inflation strategy to induce the investment of the supplier. In this situation, the effect of the order inflation strategy may dominate the effect of double marginalization. However in the case that the investment of the supplier will not be enough, the buyer gets directly involved in investment. Furthermore the supplier avoids any investment to maximize her profit. Contracts can be used as a tool to make the simultaneous investment of the players possible. We looked at the situation where the investment decision is made by the supplier, however a portion of the investment is paid by the buyer. This type of contract seems to be most effective when both players experience similar profit margins.

6.2 Future Work

In this section we look at the opportunities that exist in order to extend the proposed studies.

In the context of agricultural production, in some markets, producers may reduce the risk of productions by insurance policies on their crops. A premium will be paid to cover for instances where yields go below a present target. An interesting extensions can investigate optimal insurance policies and their impact on pricing and supply contracts. Another common practical variation is for the handler to jointly share the production cost with the producer (e.g., by investing in equipment) and in turn have priority access to products at reduced prices. An important question to look at is what the optimal investment levels are and how these relate to supply contracts, especially when some investment, such as in fertilizers, may have a direct positive impact on yields.

One of the advantages of the agricultural production is the diversity of products. Different types of crops can be produced in a single farmland. Considering multiple products as an extension to our works can be interesting. They can include complexities such as products substitutability and demand dependency. In addition, in an agricultural environment using the same farmland for different crops would usually imply that their yields are dependent and may require the rotation of the crops thorough consecutive periods.

In the problem of buyer and supplier coordination on supplier's capacity, we found that players will not invest simultaneously. One line of research that can be interesting is the use of contracts to facilitate the coordination. We looked at a very special case in our study, but further elaboration is required. Different types of contract and their effect on the performance of supply chain can be considered. Also the application mechanism design, and how the buyer can develop a menu of contracts to ensure a certain level of collaboration is interesting. Another avenue of research that is worth looking at is considering information asymmetry in the model. Usually the supplier has a better understanding about the uncertainty of supply and how the investment can improve it. Considering the effect of this asymmetry on the optimal decisions of the buyer and the supplier can provide a better understanding of the relation of buyers and suppliers.

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