PSEUDOPLASTIC ENTRY FLOW IN STRAIGHT AND CONVERGENT CHANNELS
PSEUDOPLASTIC ENTRY FLOW IN STRAIGHT
AND CONVERGENT CHANNELS

by

JACQUES LAROCQUE, B.A., B.Sc.A.

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Master of Engineering

McMaster University
April 1973
TITLE: Pseudoplastic entry flow in straight and convergent channels

AUTHOR: Jacques Larocque, B.A. (Université de Montréal)
        B.Sc.A. (Ecole Polytechnique de Montréal)

SUPERVISOR: Dr. John Vlachopoulos

NUMBER OF PAGES: xi, 188
ABSTRACT

Power law fluids are analysed for entry flow in straight and converging channels in their pseudoplastic region \(0.0 \leq n \leq 1.0\). The motion and energy equations simplified by the boundary layer assumptions were solved by an implicit finite difference scheme with a marching procedure. To circumvent the difficulties arising from an infinite viscosity at zero shear rate, a minimum value of shear rate was used making the fluid newtonian at low shear rates.

Entrance flows between parallel plates of infinite width (Slit) for uniform entry profile are discussed in Part I and converging flows for non-parallel flat plates is the subject of Part II of this work. Results are compared with their equivalent in the current literature for the newtonian case; new results are presented for non-newtonian fluids. These results include velocity and temperature profiles, pressure drops, Nusselt number, and entry lengths as a function of the flow behavior index \((n)\) and the taper angle.
RESUME

Des fluides obéissant à la loi de puissance sont étudiés pour des écoulements d'entrée en conduites droites et convergentes, dans leur région pseudoplastique (0<n<1.). Les équations de mouvement d'abord simplifiées par la théorie de la couche limite, sont ensuite résolues par une méthode de différences finies implicites, incluant l'équation d'énergie. Une valeur minimum de taux de cisaillement est utilisée de manière à ne pas rendre la viscosité infinie à bas taux de cisaillement.

En partie I, on traite d'écoulements d'entrée entre plaques planes parallèles pour profil d'entrée plat; ces écoulements sont étudiés pour plaques non parallèles en partie II. Pour les fluides newtoniens, on compare avec des résultats précédents et quelques nouveaux sont obtenus pour des fluides non newtoniens. Les résultats présentés comprennent des profils de vitesse et de température, des chutes de pression, des nombres de Nusselt et des longueurs d'entrée en fonction de l'indice de puissance (n) et des conditions géométriques.
ACKNOWLEDGEMENTS

The author would like to thank his supervisor Dr. Vlachopoulos for his help and guidance during the course of this work. Sincere thanks are also extended to Doug Buchanan for the use of his plotting routines.

Gratitude is expressed to the National Research Council for their financial support in form of a postgraduate scholarship and bursary, and to the Chemical Engineering Department for their assistantship.

Finally the author is very grateful to his wife, Carole, for her encouragements, her sacrifices and her typing of this thesis.
# TABLE OF CONTENTS

| FIGURE INDEX                              | ix      |
| TABLE INDEX                               | xi      |

**PART I: CHANNEL ENTRY FLOWS FOR PSEUDOPLASTIC FLUIDS** 1-49

I- 1 INTRODUCTION 1

I- 2 THEORETICAL BACKGROUND AND LITERATURE SURVEY 3

I-2.1 Methods of solution 3

I-2.2 Solution of boundary layer equations by series expansions 4

I-2.3 Numerical solutions 6

I- 3 SOLUTION OF THE PROBLEM 8

I-3.1 The constitutive equation 8

I-3.2 The hydrodynamic problem 9

I-3.3 Energy equation 19

I-3.4 Numerical procedure 23

I-3.5 Convergence, stability and step size 25

I- 4 RESULTS AND DISCUSSION 28

I-4.1 The hydrodynamic problem 28

a) Newtonian case 28

b) Non-Newtonian fluids 32

I-4.2 Heat transfer problem 37

I- 5 CONCLUSIONS AND RECOMMENDATIONS 46

NOTATION 47
II-A: Derivation of momentum equation for converging flow 123
II-B: Mesh sizes used for converging flows 125
II-C: Additional results for converging entry flow 126
III : Finite difference solution of channel entry flow with an iterative scheme 144
IV : Algorithm and computer programs 155
## FIGURE INDEX

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-2.1</td>
<td>Channel entry flow</td>
<td>5</td>
</tr>
<tr>
<td>I-3.1</td>
<td>Straight channel physical system</td>
<td>10</td>
</tr>
<tr>
<td>I-3.2</td>
<td>Finite difference mesh</td>
<td>14</td>
</tr>
<tr>
<td>I-3.3</td>
<td>Auxiliary mesh</td>
<td>16</td>
</tr>
<tr>
<td>I-4.1</td>
<td>$U_{\text{max}}$ vs X</td>
<td>29</td>
</tr>
<tr>
<td>I-4.2</td>
<td>Velocity development</td>
<td>30</td>
</tr>
<tr>
<td>I-4.3</td>
<td>Entry length vs flow behavior index</td>
<td>34</td>
</tr>
<tr>
<td>I-4.4</td>
<td>Entry pressure drop vs flow behavior index</td>
<td>36</td>
</tr>
<tr>
<td>I-4.5</td>
<td>Bulk temperature vs X</td>
<td>39</td>
</tr>
<tr>
<td>I-4.6</td>
<td>Bulk temperature vs X</td>
<td>40</td>
</tr>
<tr>
<td>I-4.7</td>
<td>Bulk temperature vs X</td>
<td>42</td>
</tr>
<tr>
<td>I-4.8</td>
<td>Bulk temperature vs X</td>
<td>43</td>
</tr>
<tr>
<td>I-4.9</td>
<td>Bulk temperature vs X</td>
<td>44</td>
</tr>
<tr>
<td>I-4.10</td>
<td>Local Nusselt Number vs X</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>II-3.1</td>
<td>Physical system</td>
<td>55</td>
</tr>
<tr>
<td>II-3.2</td>
<td>Finite difference network</td>
<td>60</td>
</tr>
<tr>
<td>II-3.3</td>
<td>$U_{\text{max}}/\overline{U}$ vs R for various grid spacing</td>
<td>68</td>
</tr>
<tr>
<td>II-4.1</td>
<td>Developed velocity Profile for $\beta=5^\circ$</td>
<td>71</td>
</tr>
<tr>
<td>II-4.2</td>
<td>Velocity development</td>
<td>73</td>
</tr>
<tr>
<td>II-4.3</td>
<td>Newtonian fluid Pressure curves</td>
<td>74</td>
</tr>
<tr>
<td>II-4.4</td>
<td>$U_{\text{max}}/\overline{U}$ vs R</td>
<td>76</td>
</tr>
<tr>
<td>II-4.5</td>
<td>$U_{\text{max}}/\overline{U}$ vs R</td>
<td>77</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>II-4.6</td>
<td>$U_{\text{max}}/\bar{U}$ vs $R$</td>
<td>78</td>
</tr>
<tr>
<td>II-4.7</td>
<td>Entry Lengths</td>
<td>79</td>
</tr>
<tr>
<td>II-4.8</td>
<td>Effect of different Peclet numbers on Nusselt number for $\beta=5.0^\circ$</td>
<td>81</td>
</tr>
<tr>
<td>II-4.9</td>
<td>Effect of different Peclet numbers on bulk temperature for $\beta=5.0^\circ$</td>
<td>82</td>
</tr>
<tr>
<td>II-4.10</td>
<td>Effect of different Peclet numbers on Nusselt number for $\beta=2.5^\circ$</td>
<td>83</td>
</tr>
<tr>
<td>II-4.11</td>
<td>Effect of different Peclet numbers on bulk temperature for $\beta=2.5^\circ$</td>
<td>84</td>
</tr>
<tr>
<td>II-4.12</td>
<td>$U_{\text{max}}/\bar{U}$ vs $R$</td>
<td>85</td>
</tr>
<tr>
<td>II-4.13</td>
<td>$U_{\text{max}}/\bar{U}$ vs $R$</td>
<td>86</td>
</tr>
<tr>
<td>II-4.14</td>
<td>Pressure vs $R$</td>
<td>88</td>
</tr>
<tr>
<td>II-4.15</td>
<td>$U_{\text{max}}/\bar{U}$ vs $R$</td>
<td>89</td>
</tr>
<tr>
<td>II-4.16</td>
<td>Nusselt number vs $R$</td>
<td>90</td>
</tr>
<tr>
<td>II-4.17</td>
<td>Bulk temperature vs $R$</td>
<td>91</td>
</tr>
<tr>
<td>II-4.18</td>
<td>Effect of Peclet number on Nusselt number for $n=.75$</td>
<td>93</td>
</tr>
<tr>
<td>II-4.19</td>
<td>Effect of Peclet number on bulk temperature for $n=.75$</td>
<td>94</td>
</tr>
<tr>
<td>II-4.20</td>
<td>Effect of Peclet number on Nusselt number for $n=.75$</td>
<td>95</td>
</tr>
<tr>
<td>II-4.21</td>
<td>Effect of Peclet number on bulk temperature for $n=.75$</td>
<td>96</td>
</tr>
<tr>
<td>II-4.22</td>
<td>Average Nusselt number</td>
<td>97</td>
</tr>
</tbody>
</table>
### TABLE INDEX

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-3.1</td>
<td>Mesh sizes of this work</td>
<td>26</td>
</tr>
<tr>
<td>I-4.1</td>
<td>Entry length for 99% of final centerline velocity</td>
<td>31</td>
</tr>
<tr>
<td>I-4.2</td>
<td>Excess pressure drop</td>
<td>31</td>
</tr>
<tr>
<td>I-4.3</td>
<td>Pressure gradient</td>
<td>37</td>
</tr>
<tr>
<td>I-4.4</td>
<td>Asymptotic local Nusselt number</td>
<td>38</td>
</tr>
<tr>
<td>II-3.1</td>
<td>Roughness of mesh size</td>
<td>67</td>
</tr>
</tbody>
</table>
PART I

CHANNEL ENTRY FLOWS FOR PSEUDOPLASTIC FLUIDS

I-1 INTRODUCTION

In the first part of this thesis, the problem of entrance flows of pseudoplastic fluids in a channel formed by two parallel plates is analysed; the two plates are considered semi-infinite i.e. infinitely wide with negligible end effects. The fluid to be studied is a power law fluid. Heat transfer effects are also discussed at different Prandtl numbers.

This problem has drawn a lot of attention in the current literature although most of it has gone to newtonian fluids for tube flow. Since that problem deals with partial differential equations, there are three major methods of solution: (1) exact analytical methods, (2) approximate methods, (3) numerical methods. The solution we are proposing here is a numerical finite difference procedure of the implicit type.

First, the theoretical background is presented with a literature survey followed by the solution of the problem itself. The three conservation equations (Continuity,
Momentum and Energy) are derived according to the assumptions used and transformed through finite difference approximations. Then, in a third section, the results are described along with a discussion on their significance. Conclusions are finally drawn from the results, which include the possible extension of this work.
I-2. METHODS OF SOLUTION

This problem has been solved by many methods in the 50 or so papers related to it, nearly all of them about Newtonian fluids either with or without boundary layer assumptions. Methods using the boundary layer assumptions are of the following types: (1) linearization of inertia terms, (2) integral methods, (3) series expansions, (4) numerical finite difference of the boundary layer equations. Methods without boundary layer assumptions: (5) finite difference solution of the full Navier-Stokes equations.

The first two methods will only be mentioned here without any explanation because they were not really extended to more general application after their original publication; the method of linearization was originated by Boussinesq (1891) and Langhaar (1942); Schiller (1922) did some work with the integral method (this type of solution was retaken by McKillop (1964) and McKillop et al. (1970) for the entry flow in the immediate entry). The last three methods, because of their wider use, will be explained in more detail. The method used here, is of the boundary layer finite difference type.
I-2.2 Solution of boundary layer equations by series expansions

The boundary layer concept is based on the fact that in viscous flow problems e.g. flows of polymers, the viscous effects have more importance than the inertial effects in the fluid; naturally this will force us to remain in an acceptable range for the Reynolds number; we can say that for most flows with Reynolds number over 50, the above concept is valid as demonstrated by Christiansen and Kelsey (1973) for tube flow at a contraction.

Schlichting (1934) applied the boundary layer concept in solving entry flow problems for straight channels (uniform flat entry). His method consisted of two different solutions applied to each of the upstream and downstream parts of the problem. In the upstream part, the boundary layer is formed in a small region near the wall, where viscous effects and velocity gradients are important, and the central core is uniformly accelerated as pictured in Figure I-2.1, for this part, Schlichting obtained an expression for the axial velocity in form of a series expansion with respect to the two independant variables (2-dimensional flow). This expansion, if inserted in the axial momentum equation gives a differential equation with an infinite number of terms, of which, as a first approximation, we have the Blasius
differential equation for a flat plate (depending on how many terms are taken in the expansion). Now for the downstream part, Schlichting assumes a velocity profile which is a perturbation of the true final parabolic one. The final step of the method is to join the two solutions at a suitable point where they are compatible to each other. In a recent paper where the upstream solution of Schlichting's method is questioned, Van Dyke (1970) is presenting a solution using a slightly different upstream expansion.

Figure I-2.1: Channel entry flow

Entry flow for non-newtonian fluids is more recent. The first calculations (in channels) were done by Collins and Schowalter (1963); they worked out the solution for channel entry flow of pseudoplastic fluids represented by the power law equation. Their method is about the same as Schlichting's method except that they took more terms in their series in order to have a better precision. Their results are used extensively for comparison purposes with the present work.
I- 2. 3 Numerical solutions

The boundary layer concept is also used with finite difference methods. Bodoia and Osterle (1961) have initiated work in this area followed by Hwang and Fan (1963, 1964). The method consists in starting with the Navier-Stokes equations which are simplified through the boundary layer assumptions; this means the y-direction of the equation of momentum (for 2-dimensional problem) is neglected implying that the pressure gradient is zero in the y-direction and that the motion in this direction is not important to the total problem. This evidently is not correct in the first region of the flow; but it becomes valid as soon as the thickness of the boundary layer is relatively large. In the present work this method is followed in solving the channel entry flow problem and the solution is extended to power-law fluids.

Three more publications are using the finite difference methods for the full Navier-Stokes equations; Wang and Longwell (1964), Brandt & Gillis (1966), and McDonald et al. (1972). They are solving those equations for newtonian fluids by relaxation with a two-dimensional grid at different Reynolds numbers. Their method, although quite complex, should be considered as one of the more precise because of the absence of boundary layer approximations;
we must say that for non-newtonian fluids, this method becomes extremely complicated particularly the search for convergence criteria as defined in Brandt and Gillis (1966).
I- 3 SOLUTION OF THE PROBLEM

I- 3. 1 The constitutive equation

The fluid to be used is represented by a power law constitutive equation (Ostwald-de Waele equation) from Bird et al. (1960) p. 101.

\[ \tau = -K \left| I_2 \right|^{n-1} \Delta \]

where \( \tau \): stress tensor
\( I_2 \): second invariant \( = \Delta : \Delta \) of \( \Delta \)
\( K \): power-law consistency index
\( n \): flow behavior index
\( \Delta \): rate of deformation tensor (symmetrical)

\[ \Delta = \begin{bmatrix} 2 \frac{\partial u}{\partial x} & 2 \frac{\partial u}{\partial y} \\ 2 \frac{\partial u}{\partial y} & 2 \frac{\partial v}{\partial y} \end{bmatrix} \]

For an incompressible fluid in cartesian coordinates, we have:

\[ \frac{1}{2} I_2 = 2 (\frac{\partial u}{\partial x})^2 + 2 (\frac{\partial v}{\partial y})^2 + (\frac{\partial u}{\partial y})^2 \]
the first two terms are neglected because, although they are important at the immediate entry, their effect diminishes very quickly. For more explanations, the reader should consult Schowalter (1960).

So equation I-3.1 becomes:

\[ T = -K \left( \frac{\partial \bar{u}}{\partial y} \right)^{n-1} \Delta \]  
(I-3.3)

This constitutive equation can be used directly in the momentum equation if the usual drawback, namely the infinite viscosity at low shear rates, is eliminated; this is done by setting a minimum value for \( \frac{\partial \bar{u}}{\partial y} \) making the fluid newtonian at lower shear rates. It has been recognized experimentally that all non-newtonian fluids are exhibiting a newtonian behavior at low shear rates so this working procedure is very much in line with reality.

I-3.2 The hydrodynamic problem

The physical system as drawn in Figure I-3.1 is made of two semi-infinite parallel flat plates spaced by a length of "2a". The velocity profiles are sought along x for a uniform entry profile with the use of the continuity and momentum equation in a manner quite similar to the analysis
of Bodoia and Osterle (1961) for a newtonian fluid; Hwang and Fan (1961) have solved the same problem for magneto-hydrodynamic flow of a newtonian fluid.

![Figure I-3.1: Straight channel physical system](image)

The following assumptions are made in order to solve the problem:

- The flow is 2-dimensional (Motion in the z-direction is negligible and no end effect).
- All fluid properties are constant: C, C_p, k, K.
- The effect of gravitational force is negligible.
- The flow is laminar.
- Prandtl's boundary layer assumptions apply.

With these assumptions, we obtain:

- Continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (I-3.4)
\]
- Momentum equation in the x-direction:

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{dp}{dx} + \frac{\partial \tau_{xy}}{\partial y} \]  (I-3.5)

where \( \tau_{xy} = -K \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \)

- Integral form of the continuity equation:

\[ \int_{0}^{a} u dy = u_{o} a \]  (I-3.6)

The following variable transformations are used

\[ X = \frac{(X)}{a} / Re \]  (I-3.7)

where \( Re = \frac{\rho u_{o}^{2-n}}{K} \) (2a)

\[ Y = y/a \]  (I-3.8)

\[ U = u/u_{o} \]  (I-3.9)
\[ V = \frac{(2a)^n u_o}{K} (1-n) \rho v \]  \hspace{1cm} (I-3.10)

\[ p = \frac{p-p_o}{\rho u_o^2} \]  \hspace{1cm} (I-3.11)

Then the continuity equation is

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  \hspace{1cm} (I-3.12)

the integral form becomes

\[ \int_{0}^{1} U dY = 1 \]  \hspace{1cm} (I-3.13)

and the momentum equation is

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{dp}{dx} + 2^n \frac{\partial}{\partial X} \left( \frac{U^{n-1}}{\frac{\partial U}{\partial Y}} \right) \]  \hspace{1cm} (I-3.14)

And the boundary conditions are

at \( Y = 0 \) and \( X > 0 \) (centerline): \( \frac{\partial U}{\partial Y} = 0 \) and \( V = 0 \)  \hspace{1cm} (I-3.15)

at \( Y = 1 \) and \( X > 0 \) (wall): \( U = 0 \) and \( V = 0 \)
and the initial condition is

\[
\text{at } X = 0 \text{ and } 0 \leq Y \leq 1 : U = 1 \text{ and } V = 0
\]

\[
P = P_0 = 0
\]

(I-3.16)

There are two ways of solving this problem since the momentum is a nonlinear equation. First we can solve it as a linear approximation by an appropriate choice of difference approximations using a normal implicit scheme similar to the one of Bodoia & Osterle (1961) and Hwang & Fan (1963); another way is to use an iterative scheme in which the values that have just been calculated are reintroduced into the equations to compute again a set of new values. (A solution with an iterative scheme is described in Appendix III).

The implicit type method without iteration starts with the original assumptions for the velocity profile (initial condition) and proceeds column by column with linear equations by solving the coefficient matrix for the hydrodynamic problem in order to compute the velocities; with the velocities, we are then able to compute the temperatures directly. Figure I-3.2 is giving the mesh configuration:
Finite difference representations are chosen in order to get linear equations. So for the continuity equation I-3.12, the following difference approximations are used.

\[
\frac{\partial U}{\partial x} = \frac{U_{j+1,k+1} - U_{j,k+1} + U_{j+1,k} - U_{j,k}}{2\Delta x}
\]

(I-3.17)

\[
\frac{\partial V}{\partial y} = \frac{V_{j+1,k+1} - V_{j+1,k}}{\Delta y}
\]
In the integral form of the continuity equation, we use the Simpson integration formula which gives:

\[ U_{j+1,1} + 4 \sum_{n} U_{j+1,2n} + 2 \sum_{n} U_{j+1,2n+1} = 3m \quad 1 \leq n \leq m/2 \]

(I-3.18)

where \( m \) = total number of points across less 1

For the momentum equation, we have the following finite difference approximations:

\[ \frac{\partial U}{\partial x} = \frac{U_{j+1,k} - U_{j,k}}{\Delta x} \]

\[ \frac{\partial U}{\partial y} = \frac{1}{2} \frac{U_{j,k+1} - U_{j,k-1}}{2 \Delta y} + \frac{1}{2} \frac{U_{j+1,k+1} - U_{j+1,k-1}}{2 \Delta y} \]

(I-3.19)

\[ \frac{dP}{dx} = \frac{P_{j+1} - P_{j}}{\Delta x} \]

\[ U = U_{j,k} \quad \text{and} \quad V = V_{j,k} \]
For the last term of the momentum equation, a secondary grid superimposed over the previous one is used along Y (See Figure I-3.3 below).

![Diagram of auxiliary mesh]

Figure I-3.3: Auxiliary mesh

Now the difference representation for the last term of the momentum equation is taken at \((j+1/2, k)\) which is the central point of that grid, so we have:

\[
\frac{\partial \tau}{\partial Y} = \frac{1}{2} \frac{\tau_{j,k+1/2} - \tau_{j,k-1/2}}{\Delta Y} + \frac{1}{2} \frac{\tau_{j+1,k+1/2} - \tau_{j+1,k-1/2}}{\Delta Y}
\]

\[(I-3.20)\]
where

\[ \tau_{j,k+1/2} = \begin{vmatrix} U_{j+1,k} - U_{j,k} \\ \Delta Y \end{vmatrix}^{n-1} \frac{U_{j,k} - U_{j,k-1}}{\Delta Y} \]

\[ \tau_{j,k-1/2} = \begin{vmatrix} U_{j,k} - U_{j,k-1} \\ \Delta Y \end{vmatrix}^{n-1} \frac{U_{j,k} - U_{j,k-1}}{\Delta Y} \]

(I-3.21)

\[ \tau_{j+1,k+1/2} = \begin{vmatrix} U_{j+1,k+1} - U_{j,k} \\ \Delta Y \end{vmatrix}^{n-1} \frac{U_{j+1,k+1} - U_{j+1,k}}{\Delta Y} \]

\[ \tau_{j+1,k-1/2} = \begin{vmatrix} U_{j,k} - U_{j,k-1} \\ \Delta Y \end{vmatrix}^{n-1} \frac{U_{j+1,k} - U_{j+1,k-1}}{\Delta Y} \]

So the momentum equation after simplification gives (details shown in appendix I-A):

\[ A_k \cdot U_{j+1,k-1} + B_k U_{j+1,k} + C_k U_{j+1,k+1} + D_k P_{j+1} = E_k \]

(I-3.22)

where \( A_k = -\frac{V_{j,k}}{4\Delta Y} - \frac{|Gm|^{n-1}}{\Delta Y^2} \)
\[ B_k = \frac{U_{j,k}}{\Delta X} + \frac{|Gp|^n - 1}{\Delta Y^2} |Gm|^n - 1 \]

\[ C_k = \frac{V_{j,k}}{4\Delta Y} - \frac{|Gp|^n - 1}{\Delta Y^2} \]

\[ D_k = \frac{1}{\Delta X} \]

\[ E_k = \frac{P_j}{\Delta X} + \frac{U_{j,k}^2}{\Delta X} - \frac{V_{j,k}}{4\Delta Y} \left( U_{j,k+1} - U_{j,k-1} \right) + \frac{|Gp|^n - 1}{\Delta Y^2} \]

\[ (U_{j,n+1} - U_{j,n}) - \frac{|Gm|^n - 1}{\Delta Y^2} (U_{j,n} - U_{j,n-1}) \]

where \( Gp = 2 \frac{U_{j,k+1} - U_{j,k}}{\Delta Y} \)

\( Gm = 2 \frac{U_{j,k} - U_{j,k-1}}{\Delta Y} \)

By combining the last equation with equation I-3.18, we obtain a set of "m" linear equations with "m" velocities plus "1" pressure which permits us to solve for those values as demonstrated in section I-3.4.
For the continuity equation, we have an explicit expression with respect to velocities along \( y \) if the axial velocities are known. Replacing I-3.17 in I-3.12, the following equation is obtained:

\[
V_{j+1,k+1} = V_{j+1,k} - \frac{\Delta y}{2\Delta x} \left( U_{j+1,k+1} + U_{j+1,k} - U_{j,k+1} - U_{j,k} \right)
\]

(I-3.23)

Since \( V_{j+1,1} = 0.0 \) (Centerline boundary condition) then all the "\( V \)" can be computed directly.

I- 3.3 Energy equation

If we make the same assumptions for the energy equation as we did previously for the hydrodynamic problem, we have the following energy equation

\[
u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \alpha \frac{\partial^2 t}{\partial y^2}
\]

(I-3.24)

where \( \alpha = \frac{k}{\rho C_p} \) (thermal diffusivity)
The previous variable transformations (equation I-3.7 to I-3.11) are retained and the following one is added:

\[ T = \frac{t - t_w}{t_o - t_w} \quad (I-3.25) \]

where \( t_o \) : Entry temperature \( (T_o = 1) \)

\( t_w \) : Wall temperature \( (T_w = 0) \)

After simplification the following equation is obtained

\[ U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{2}{Pr} \frac{\partial^2 T}{\partial Y^2} \quad (I-3.26) \]

where \( Pr = \frac{C_P K}{\left(\frac{2a}{u_o}\right)} 1-n \)

(Non-Newtonian Prandtl Number)

It should be noted that when \( n=1 \) for a newtonian fluid the Prandtl number takes the usual form.

The boundary and initial conditions are as follows for the constant wall temperature problem:

B.C.: at \( Y = 0 \) and \( X > 0 \) : \( \frac{\partial T}{\partial Y} = 0 \) \quad (I-3.27)

at \( Y = 1 \) and \( X > 0 \) : \( T = 0 \)

I.C.: at \( X = 0 \) and \( 0 < Y < 1 \) : \( T = 1 \) \quad (I-3.28)
Now the following finite difference approximations are used:

\[
U = \frac{U_{j,k} + U_{j+1,k}}{2} \quad V = \frac{V_{j,k} + V_{j+1,k}}{2}
\]

\[
\frac{\partial T}{\partial x} = \frac{T_{j+1,k} - T_{j,k}}{\Delta x}
\]

(I-3.29)

\[
\frac{\partial T}{\partial y} = \frac{1}{2} \frac{T_{j,k+1} - T_{j,k-1}}{2\Delta y} + \frac{1}{2} \frac{T_{j+1,k+1} - T_{j+1,k-1}}{2\Delta y}
\]

\[
\frac{\partial^2 T}{\partial y^2} = \frac{(T_{j+1,k+1} - 2T_{j+1,k} + T_{j+1,k-1}) + (T_{j+1,k+1} - 2T_{j+1,k} + T_{j+1,k-1})}{2\Delta y^2}
\]

By substitution into equation I-3.26, we obtain a series of "m" linear equations with "m" temperature points to be solved; the resulting matrix is found in Appendix I-B. In the next section, the numerical procedure is described.

Other heat transfer quantities are used in this part I; they are the Bulk temperature \( T_b \) (mixing cup temperature) and the local Nusselt number. The Bulk temperature is defined by the following equation

\[
T_b = \frac{\int_0^1 T_{j+1,k} U_{j+1,k} \, dy}{\bar{U}}
\]

where \( \bar{U} \) = average velocity = 1
The local Nusselt number is defined by the following:

\[ \text{Nu} = \frac{hL}{k} \]  \hspace{1cm} (I-3.30)

where: 
- \( h \) = heat transfer coefficient
- \( L \) = characteristic length = \( 2a \)
- \( k \) = thermal conductivity

From the heat flux at the wall, we have that

\[ q_w = h (t_w - t_b) = \left( -k \frac{\partial t}{\partial y} \right)_{y=b} \]  \hspace{1cm} (I-3.31)

which, if we adopt the previous variable transformations, gives

\[ \text{Nu} = \frac{\left( \frac{\partial T}{\partial Y} \right)_{Y=1}^2}{T_b} \]  \hspace{1cm} (I-3.32)

The value of the gradient at the wall is calculated by the same finite difference approximation as used in Katotakis (1969), Vlachopoulos and Keung (1972) and Katotakis and Vlachopoulos (in press) at column "j+l".

\[ \left( \frac{\partial T}{\partial Y} \right)_{Y=1} = \frac{1}{6\Delta Y} \left( -11T_{j+1,m+1} + 18T_{j+1,m} - 9T_{j+1,m-1} + 2T_{j+1,m-2} \right) \]  \hspace{1cm} (I-3.33)

where \( m+1 \) is the grid point on the wall.
To solve the hydrodynamic problem, the momentum (I-3.22) and integral continuity (I-3.18) equations are used to form a coefficient matrix; the unknowns are the "m" axial velocities and one pressure so we have the following set of linear equations:

\[
\begin{bmatrix}
1 & 4 & 2 & 4 & \cdots & 0 \\
B_1 & (C_1+A_1) & \cdots & D_1 \\
A_2 & B_2 & C_2 & \cdots & D_2 \\
A_3 & B_3 & C_3 & \cdots & D_3 \\
& \ddots & \ddots & \ddots & \ddots \\
A_k & B_k & C_k & \cdots & D_k \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
A_{m-1} & B_{m-1} & C_{m-1} & D_{m-1} & \vdots \\
A_m & B_m & C_m & D_m & \vdots \\
\end{bmatrix}
\begin{bmatrix}
U_{j+1,1} \\
U_{j+1,2} \\
U_{j+1,3} \\
\vdots \\
U_{j+1,k} \\
\vdots \\
U_{j+1,m} \\
p_{j+1} \\
\end{bmatrix}
= 
\begin{bmatrix}
3m \\
E_1 \\
E_2 \\
E_3 \\
\vdots \\
E_k \\
\vdots \\
E_{m-1} \\
E_m \\
\end{bmatrix}
\]

This set of equations was solved by a program using the Gauss elimination method written especially for the above coefficient matrix in order to minimize computing time (See Gauss subroutine in Appendix IV for details). Once the axial velocities are obtained, one can find the perpendicular velocities explicitly by using the normal form of the continuity equation (I-3.22); the y-velocity at the centerline is zero so the remaining ones can be computed from there on. This gives a fully implicit set of equations which means that once
the starting profile is known, that is at column 0 (from the initial condition), one can compute directly column 1 and keep this "marching procedure" until enough information is generated (in our case, all calculations were stopped at X = 1.0).

For the energy equation, a slightly different coefficient matrix is obtained where there is no additional unknown apart from the "m" temperatures in a column: so we have the following system:

\[
\begin{bmatrix}
G_1 & (F_1 + H_1) \\
F_2 & G_2 & H_2 \\
& F_k & G_k & H_k \\
& & F_{m-1} & G_{m-1} & H_{m-1} \\
& & & F_m & G_m
\end{bmatrix}
\begin{bmatrix}
T_{j+1,1} \\
T_{j+1,2} \\
\vdots \\
T_{j+1,k} \\
\vdots \\
T_{j+1,m-1} \\
T_{j+1,m}
\end{bmatrix}
= \begin{bmatrix}
J_1 \\
J_2 \\
\vdots \\
J_k \\
\vdots \\
J_{m-1} \\
J_m
\end{bmatrix}
\]

(I-3.35)

The coefficient matrix is known as a tridiagonal matrix. A recursion formula, Carnahan et al. (1969) and Hwang & Fan (1963), was used to solve this set; this formula which is based on Gaussian elimination, is also called Thomas's method.
Problems with stability and convergence arise from the substitution in a differential equation of finite difference approximations; they are both discussed very well in Richtmyer (1957) for linear differential equations. The finite difference solutions of non-linear equations has yet to reach the same level of understanding as the linear ones so we must still rely on linear equation type of analysis to appreciate convergence and stability.

Convergence is the problem of getting the same exact solution from the differential equation and the difference equation; it arises from the fact that the difference equation is solved for a mesh with finite grid spaces. It is normally solved by reducing the mesh size up to a point where there is convergence of the finite difference solution; it simply means that the truncation error caused for example by taking a gradient between two points, will be less if those two points are closer. Evidently in doing so, computing time and experimental error must be taken in consideration in order to get a reasonable mesh size; with this in mind the network size of Table I-3.1 was used. This mesh gave solutions which would not vary more than .1% by reducing its size for normal calculations.
including calculations at very low flow behavior index); since between 1 and 2% is the normal accepted value of the experimental error, this was judged acceptable.

<table>
<thead>
<tr>
<th>up to X</th>
<th>m</th>
<th>(ΔY)</th>
<th>ΔX</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0004</td>
<td>80</td>
<td>(.0125)</td>
<td>0.00002</td>
</tr>
<tr>
<td>.002</td>
<td>80</td>
<td>(.0125)</td>
<td>0.0001</td>
</tr>
<tr>
<td>.01</td>
<td>40</td>
<td>(.025)</td>
<td>0.0005</td>
</tr>
<tr>
<td>.03</td>
<td>20</td>
<td>(.05)</td>
<td>0.001</td>
</tr>
<tr>
<td>.1</td>
<td>20</td>
<td>(.05)</td>
<td>0.005</td>
</tr>
<tr>
<td>1.0</td>
<td>20</td>
<td>(.05)</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table I-3.1: Mesh sizes of this work

A look at mesh sizes of Hwang and Fan (1965) is showing that they are slightly rougher than the ones in Table I-3.1; it was found that for flow-behavior index < .25 a finer mesh was necessary to stay within the .1% value.

Stability is the second problem, one must face when dealing with finite difference solutions; instability problems are coming mainly from round-off errors in the computations which are giving discrepancies between the exact solution of the difference equation and its numerical solution. Stability is normally ensured by checking that an error introduced in the computation will remain bounded. Katotakis (1969) has done a stability analysis on the boundary layer momentum equation using Von Newman's theory (See Richtmeyer 1957).
However this method of analysis of the growth of a general error term, must be used with linear equations so the momentum equations were linearized by assuming that coefficients of inertia terms are constant; furthermore in our work, the viscosity should be considered constant to get linear equations; Katotakis (1969) is doing the stability analysis of the resulting linear equations; however it must be stated that, since viscosity can vary by as much as $10^4$, this analysis should be regarded only as a trend indicator.

The energy equation was analysed by Hwang & Fan (1965) and by Richtmeyer (1957). It is found to be always stable when the equation is considered linear. Although Hwang and Fan are recommending a finer mesh for the heat transfer part than the one used in table I-3.1, the present work has demonstrated that using a finer mesh size did not change the results of the temperature profile noticeably and the mesh used gave an excellent agreement for the bulk temperature with their result for newtonian fluid. Furthermore, it was found that using an auxiliary mesh for the heat transfer problem by computing, at the intermediate grid points linear approximations of the velocities, was causing instabilities in the form of harmonics; this phenomenon was detected by an oscillating Nusselt number. So the same mesh size was used for the hydrodynamic and the heat transfer set of equations.
I-4. RESULTS AND DISCUSSION

I-4.1 The hydrodynamic problem

A) Newtonian case

As seen in the literature survey, the parallel plates entry flow problem for the Newtonian case has been solved by numerous methods. Van Dyke (1970) and McDonald et al. (1972) in their respective paper are offering excellent comparisons of the development of the centerline velocity. Figure I-4.1 is showing the good agreement between Wang & Longwell, Bodoia & Osterle (1961) and the present work; however the full numerical solutions of Brandt & Gillis (1964) and McDonald et al. (1972) are showing discrepancy with the previous three curves. This discrepancy with the full numerical solution of Wang & Longwell seems to be due to different boundary condition (vorticity is zero for W. & L. and "v" is zero for B. & G. and M. et al.).

Whereas the agreement of boundary layer numerical solutions is good at Re = 75 for the no vorticity case, Brandt and Gillis (1964) are showing that a similar agreement exist at Re>300 for zero perpendicular velocity case; however since it is very difficult to say which solution is best corresponding to reality, our results should apply whenever the Reynolds number is over 200. In figure I-4.2, the velocity development is shown which is almost undistinguishable from the one of Bodoia&Osterle.
Figure I-4.1: $U_{\text{max}}$ vs $X$

$U_{\text{max}}$(max vs X)

$X = (x/a)/Re$

$Re = 75$

- Bodoia & Osterle (1961) and Present work
- Wang & Longwell (1964) and McDonald et al. (1972) at $x=0$
- Brandt & Gillis (1964) and McDonald at al. (1972) at $x=0$
Figure I-4.2: Velocity development
The entry length is defined as the length of the channel taken by the centerline axial velocity to reach 99% of its fully developed value. We can see that the results of the present work are close to Hwang and Fan results and in the same range as other workers.

\[
X_{e99} = \left( \frac{x}{a} \right) Re
\]

Table I-4.1: Entry length for 99% of final centerline velocity

<table>
<thead>
<tr>
<th>Author</th>
<th>( Re )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schlichting (1934)</td>
<td>0.0800</td>
</tr>
<tr>
<td>Present work</td>
<td>0.0838</td>
</tr>
<tr>
<td>Hwang and Fan (1961)</td>
<td>0.0844</td>
</tr>
<tr>
<td>Bodoia and Osterle (1961)</td>
<td>0.0880</td>
</tr>
<tr>
<td>Brandt and Gillis (1966)</td>
<td>0.0884</td>
</tr>
<tr>
<td>Roidt and Cess (1962)</td>
<td>0.0908</td>
</tr>
</tbody>
</table>

The excess pressure drop \( P_d \) is calculated the following way for a newtonian fluid.

\[
P_d = \lim_{X \to \infty} [P(0) - P(X) - 6X]
\]  

\( P_d \) is due to departure from parabolic flow in the inlet region of the entrance flow. Table I-4.2 shows results of other workers.

<table>
<thead>
<tr>
<th>Author</th>
<th>( P_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic-energy end correction</td>
<td>0.271</td>
</tr>
<tr>
<td>Schlichting (1934)</td>
<td>0.300</td>
</tr>
<tr>
<td>Hwang and Fan (1963)</td>
<td>0.312</td>
</tr>
<tr>
<td>Roidt and Cess (1962)</td>
<td>0.315</td>
</tr>
<tr>
<td>Brandt and Gillis (1966)</td>
<td>0.331</td>
</tr>
<tr>
<td>Bodoia and Osterle (1961)</td>
<td>0.338</td>
</tr>
<tr>
<td>Present work</td>
<td>0.343</td>
</tr>
</tbody>
</table>

Table I-4.2: Excess pressure drop
For Brandt and Gillis, the value is given for Re = ∞ since the boundary layer assumptions are valid for high Reynolds number. We can see that the results of the present work are pretty well in same range as the ones obtained by other workers. The final value of the pressure gradient calculated from this work is exactly 6.000 which is the value that can be calculated for the Poiseuille flow analytically.

B) Non-Newtonian Fluids

There are not many solutions on power law fluids for channels so we will limit the comparison to the work done by Collins and Schowalter (1963). For n=1 (newtonian fluid) their entry length is .069 compared .067 for this work (this entry length is 98% of the fully developed center-line velocity) and the excess pressure lost is .34 compared to .343 here.

From this, we can conclude that our results are compatible with those of Collins and Schowalter, so a comparison for non-newtonian fluids can be undertaken. Collins and Schowalter are solving the problem with an extension of the Schlichting's analytical method so the fact that for pseudoplastic fluid (n<1.0), the viscosity becomes infinite when the axial velocity gradient is zero, did not give them any problem. In a numerical solution like
the one presented here, this limitation arises as we saw earlier, in the calculations because of the finite difference form of the constitutive equation. The minimum value of the gradient, in solving the problem, was taken in dimensionless form as:

\[ \left| \frac{\partial U}{\partial Y} \right| = \dot{\gamma}_m \]  \hspace{1cm} (I-4.2)

This procedure was used by Ozoe and Churchill (1972) for free convection problems. A range of minimum gradient from 0.001 to 0.5 was used in this work. \( \gamma_m^{n-1} \) is then the value of the factor by which we must multiply the viscosity at unity shear rate (the viscosity at unity shear rate is the consistency index of the power law equation: \( K \)) to get the true viscosity. In the figure I-4.3, we can see the effect of the value of the minimum gradient on the entry length vs flow behavior index curves as well as a comparison with Collins and Schowalter results. This figure shows that as the value of \( \gamma_m \) increases, the maximum of the curves are decreasing and leaning a bit toward the left; values for \( n > .75 \) are not changing very much; these effects are easily explained by the fact that for flow behavior index the profiles are flatter so the minimum gradient value is used, in the numerical solution, not only at the centerline but also in the flat profile region around the centerline; as soon as the value of the minimum
Figure I-4.3: Entry length vs flow behavior index

Collins & Schowalter (1963)

\( \dot{\gamma}_m = 0.001 \) and \( \gamma_m = 0.01 \)
\( \dot{\gamma}_m = 0.1 \)
\( \dot{\gamma}_m = 0.5 \)

0 point where the entry length is calculated
gradient goes below .01, there is very little change in the entry length values. The discrepancy at low flow behavior index with Collins and Schowalter results, can partially be attributed to the fact that only one point (n=.2) was calculated in that region and a full curve from 0.4 to 0.0 was drawn.

The entry pressure drop coefficient was found relatively insensitive to the value of the minimum gradient except when this value was quite high e.g. $\gamma_m = 0.5$. In figure I-4.4, we can see that a noticeable difference in the end pressure drop correction can be seen only at low flow behavior index. In this figure, we are also comparing with results of Collins and Schowalter (1963) and Tiu et al. (1972) whose method is based on a previous paper written by Lungren et al. (1964), using a linearization method of momentum and machanical energy equations. We can see that our results are in very close agreement with Tiu's paper but shows some differences with Collins and Schowalter. In table I-4.3, we can see that for minimum gradient of .01 (and less) the pressure gradient (which is constant after the immediate entry) is very close to the exact analytical one where as for 0.5 the value begins to differ appreciably. The implication of this, is that, for pressure calculation, the value of the minimum gradient at which the fluid becomes newtonian at low shear stress does not have much effect.
Kinetic energy correction
Collins & Schowalter (1963)
Tiu et al. (1972) and Present work ($\gamma_m = 0.01$)
Present work ($\gamma_m = 0.5$)

Figure I-4.4: Entry pressure drop vs flow behavior index
Table I-4.3: Pressure gradient

In appendix I-C the reader will find some of the results not discussed in this section. Axial velocity developments are presented with respect to the axial distance for various flow behavior index at 5 different distances from the centerline (Y = 0 is the centerline). Then pressures are plotted also along the axial variable "x".

I- 4.2 Heat transfer problem

For heat transfer we need to look at three variables. The first two are $\gamma_m$ and $n$ which are related to the constitutive equation used here and the third is the Prandtl number (Pr). In figure I-4.5, we can see what influence the flow behavior index has on the bulk temperature. Change of the minimum gradient has no noticeable effect on bulk temperature as we can see by comparing figure I-4.5 to I-4.6 (in fact we have not more than 3\% variation in bulk temperature at a flow behavior index of .25). The same is not true for the
Nusselt number as we can see in Table I-4.4 where we compare asymptotic local Nusselt number for different minimum gradients and different flow behavior index with values given by Vlachopoulos and Keung (1972) whose calculations are for fully developed velocity profiles. The different values of the Nusselt number for different minimum gradient is probably caused by the fact that

<table>
<thead>
<tr>
<th>$\hat{y}_m$</th>
<th>0.5</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>Vlachopoulos and Keung (1972)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.25</td>
<td>4.162</td>
<td>4.212</td>
<td>4.218</td>
<td>4.218</td>
<td>4.22</td>
</tr>
<tr>
<td>.50</td>
<td>3.954</td>
<td>3.965</td>
<td>3.966</td>
<td>3.966</td>
<td>3.97</td>
</tr>
<tr>
<td>.75</td>
<td>3.839</td>
<td>3.841</td>
<td>3.841</td>
<td>3.841</td>
<td>----</td>
</tr>
<tr>
<td>1.00</td>
<td>3.763</td>
<td>3.763</td>
<td>3.763</td>
<td>3.763</td>
<td>3.767</td>
</tr>
</tbody>
</table>

Table I-4.4: Asymptotic local Nusselt number

A more parabolic like velocity profile is encountered at high minimum gradient causing lower velocities near the wall (to maintain a constant flow rate). Those lower velocities near the wall are in turn affecting the value of the wall temperature gradient which is part of the Nusselt number.
FIGURE I-4.5  BULK TEMPERATURE VS X
BULK TEMPERATURE VS X

\[ \gamma_m = 0.5 \quad Pr = 1.0 \]

- \( n = 0.15 \)
- \( n = 0.25 \)
- \( n = 0.50 \)
- \( n = 0.75 \)
- \( n = 1.00 \)

FIGURE I-4.6  BULK TEMPERATURE VS X
The bulk temperature values are similar to results of Hwang and Fan (1965); a high Prandtl number has the effect of lowering the heat transfer rate; this can be seen in figure I-4.7 to I-4.9 where the bulk temperature decrease is slower as the Prandtl number is increased. The Nusselt numbers have the same patterns except that they take longer to attain their asymptotic values at high Prandtl number; when the Prandtl is low the bulk temperature falls very fast to zero making the calculation of Nusselt number inexact because we are dealing with very small temperatures when we are calculating the temperature gradient at the wall.

Some results plotted in a log-log form can be seen in figure I-4.10. More results are available in Appendix I-C concerning bulk temperature and Nusselt Number for various values of the parameters.
FIGURE I-4.7: BULK TEMPERATURE VS X

Pr = 0.5 \quad \dot{m} = 0.01

\begin{align*}
&\text{--- ---} \quad n = 0.15 \\
&\text{--- ---} \quad n = 0.25 \\
&\text{--- ---} \quad n = 0.50 \\
&\text{--- ---} \quad n = 0.75 \\
&\text{--- --- ---} \quad n = 1.00
\end{align*}
BULK TEMPERATURE VS X

Pr = 1.0

\( \dot{\gamma}_m = 0.01 \)

- --- n = 0.15
- --- n = 0.25
- --- n = 0.50
- --- n = 0.75
- --- n = 1.00

FIGURE I-4.8: BULK TEMPERATURE VS X
FIGURE I-4.9 BULK TEMPERATURE VS X

Pr = 5. \( \dot{\gamma}_m = 0.01 \)

- \( n = 0.15 \)
- \( n = 0.25 \)
- \( n = 0.50 \)
- \( n = 0.75 \)
- \( n = 1.00 \)
Figure I-4.10: Local Nusselt Number vs X
In this part I pseudoplastic fluids have been studied for entry flow with a finite difference method; for Newtonian fluids, good agreement was found to exist with previous results in the hydrodynamic and heat transfer problem. In the non-newtonian case, comparisons with Collins & Schowalter (1963) and Tiu et al. (1972) gave excellent agreement with the present work for entry length and excess pressure drops except in low flow behavior index for the entry length. Heat transfer was found to decrease by lower flow behavior index in accordance with other results.

It must be noted that the present method can be used with practically any type of entry profile. In fact a sixth order polynomial entry profile was tried with satisfactory results.

Improving the present work could be done by including viscoelastic fluids; it is suggested that this be done employing something similar to a deviatoric stress tensor as used in Balmer and Kauzlarich (1972). Another improvement would be to extend this work to channel flow where side effects are important; this case would probably help the understanding of single screw extruders particularly if heat transfer is included. Further viscous dissipation effects should be included.
**NOTATION**

a : Half space between plates

$A_k$ : Series of constants defined in equation I-3.20

$B_k$ : Series of constants defined in equation I-3.20

$C_k$ : Series of constants defined in equation I-3.20

$C_p$ : Specific heat

$D_k$ : Series of constants defined in equation I-3.20

$E_k$ : Series of constants defined in equation I-3.20

$F_k$ : Series of constants defined in equation I-B.1

$G_k$ : Series of constants defined in equation I-B.1

$G_m$ : Constant defined in equation I-3.20

$G_p$ : Constant defined in equation I-3.20

h : Heat transfer coefficient

$H_k$ : Series of constants defined in equation I-B.1

$I_2$ : Second invariant of $\Delta$

$J_k$ : Series of constants defined in equation I-B.1

k : Thermal conductivity

K : Power law consistency index

L : Characteristic length

m : Number of grid points across flow less one

n : Flow behavior index

$Nu$ : Nusselt number

P : Pressure

$P_o$ : Pressure at the entry

P : Dimensionless pressure ($=p-p_o/\rho u_o^2$)
P_d : Excess pressure drop
Pr : Prandtl number
q_w : Heat flux at the wall
Re : Reynolds number
t : Temperature
t_o : Temperature at the entry
t_w : Temperature at the wall
T : Dimensionless temperature \((= (t - t_w)/(t_o - t_w))\)
T_b : Bulk temperature
u : Axial velocity (in x-direction)
u_o : Axial velocity at entry (uniform)
U : Dimensionless axial velocity \((= u/u_o)\)
\bar{U} : Average axial velocity (dimensionless)
v : Velocity in y-direction
V : Dimensionless velocity in y-direction \(\left(\frac{2a}{(2a)n u_o \frac{1-n \rho v}{k}}\right)\)
x : Coordinate along the channel
X : Dimensionless coordinate along the channel \((= (y/a/Re))\)
X_e : Entry length at 98% of the fully developed centerline velocity
X_{e99} : Entry length at 99% of the fully developed centerline velocity
y : Coordinate across channel
Y : Dimensionless coordinate across channel \((= y/a)\)
z : Coordinate along the width of plates
Greek letters

\( \alpha \) : Thermal diffusibility \((=k/\rho C_p)\)

\( \gamma_m \) : Minimum gradient

\( \dot{\Lambda} \) : Rate of deformation tensor

\( \Delta X \) : Increment along x

\( \Delta Y \) : Increment along y

\( \tau \) : Stress point (in the grid)

\( \tau^{\infty} \) : Stress tensor

\( \rho \) : Fluid density

\( \omega \) : Vorticity

Subscript

\( j \) : For mesh point in \( x \)-direction

\( k \) : For mesh point in \( y \)-direction

\( m \) : Number of grid points across flow less one
PART II

CONVERGING FLOWS FOR PSEUDOPLASTIC FLUIDS

II- 1 INTRODUCTION

Converging flows of newtonian fluids have been studied for a while since the Navier-Stokes equations in this case can be solved analytically either with, or without the boundary layer approximations, and because analytical solutions were the main method of solution before the advent of computers in the late forties. Since then some numerical solutions were developed and compared to the exact solutions.

In this second part, converging flows for pseudo-plastic (power law fluid) and newtonian fluids are analysed by a finite difference integration method. Solutions for newtonian fluids will be presented and compared to the few available results found in the literature. They are produced in order to ascertain the attainability of exact fully developed velocity profile at various flow conditions; this aspect of the problem seems to have been neglected by preceding authors.

In the first section of this part, a survey of the current literature is presented, followed by a second section where the solution of the problem is presented. Finally the results of this work are outlined and discussed in a last section.
II- 2 LITERATURE SURVEY AND METHODS OF SOLUTION

II- 2.1 Analytical solutions

One of the few exact analytical solutions of the Navier-Stokes equations is about flow through converging and diverging channels. The solution was found independently by two workers, G. Hamel (1916) and G. Jeffery (1915); they reduced the Navier-Stokes equations to an ordinary differential equation which in turn was solved through the use of elliptic functions. Millsaps and Pohlhausen (1953) extended the calculations of Jeffery and Hamel for the hydrodynamic part and solved the energy equation transformed into an ordinary differential equation with a finite difference method.

Another method of solving the problem is to make the boundary layer assumptions and therefore simplify the motion equations. Again they are solved exactly; that is what Pohlhausen (1921) did in an early paper. Reeves and Kippenhan (1962) in a subsequent paper are presenting a very good comparison of the two methods; their results show that the boundary layer velocity distribution is very close to the exact solution for a Reynolds number of 50; so we can conclude that for Reynolds number higher than that value the boundary layer assumptions will give solutions very close to the exact value.
There are a few more approximate solutions to converging flow problem. Williams (1963) is presenting one in which, as he points out, "the equations of motion for flow in slender channels at moderate or high Reynolds number are identical in form to the boundary layer equations", although he goes on to say that those two solutions are not the same because of different boundary conditions, his results (At $Re = 600, 1384$ and $5000$) are the same as Jeffery-Hamel's exact solution. In a more recent paper, Balmer and Kauzlarich (1971) are presenting results for an elastic fluid sheared in its newtonian region; even through they are using a power law constitutive equation, their exponent must be one as a condition for transforming a partial differential equation describing the flow, into an ordinary one by similarity analysis; this of course, limits their results.

All the solutions discussed up to this point, are for fully developed flow; the next ones including those in the next section using numerical methods, are solved with entry profile which means that we are able to check if, at certain flow conditions, we are effectively reaching the fully developed profiles; furthermore velocity distributions can be followed as the fluid proceeds in the converging channel.

Atabek (1972) linearized the two momentum equations in order to solve for arbitrary entry profile; the weak
points of his analytical solution seem to be the assumptions that the fully developed profile is obtainable at the apex of the converging plates and that the velocity distribution is taken as a perturbation of the fully developed profile throughout.

II- 2. 2 Numerical methods

Sutterby (1965) was one of the first workers to develop a numerical method for converging flows. He simplified the Navier-Stokes equations with approximations and then solved those equations by finite differences; unfortunately his results are for converging tubes and for initial parabolic profile making comparisons almost impossible to make with our results.

Another paper was published by Yang and Price (1972) on converging plane-walled channels for newtonian fluids. The numerical solution presented here is similar to their solution; as a matter of fact our equations for the newtonian case are identical and the results are almost the same. Since they seem to have employed a rougher mesh than the one used here, a comparison is presented with their results to see the effect of the grid size on the convergence of the solution.
II-3 SOLUTION OF THE PROBLEM

II-3.1 Constitutive equation

The constitutive equation to be used will be the power law equation. Since we are dealing with polar coordinates we can expect that this constitutive equation will be quite complex; approximations will be made in order to simplify this equation, in a way similar to Sutterby (1965); first the assumption of purely radial flow is made (that assumption was made also by Hamel and Jeffery to solve analytically for newtonian fluid).

So we have that:

\[ u = \frac{F(\theta)}{r} \quad \text{and} \quad v = 0 \]  \hspace{1cm} (II-3.1)

where \( F(\theta) \) is a function of \( \theta \) only

The physical system is represented in figure II-3.1:
Figure II-3.1: Physical system

We know that for power law fluids we have the following stress-shear relation:

\[ \tau = K \left| \sqrt{\frac{I_2}{2}} \right|^{n-1} \Delta \]

where \( \tau \) = stress tensor

\( I_2 \) = second invariant of \( \Delta \)

\( K \) = power law consistency index

\( n \) = flow behavior index

\( \Delta \) = rate of deformation tensor (symmetrical)

\[ \Delta = \begin{bmatrix}
\frac{\partial u}{\partial r} & \frac{1}{r} \frac{\partial u}{\partial \theta} \\
\frac{1}{r} \frac{\partial (v/r)}{\partial r} + \frac{1}{r} \frac{\partial u}{\partial \theta} & \frac{2}{r^2} \frac{\partial v}{\partial \theta} + \frac{u}{r}
\end{bmatrix} \]
If we expand the second invariant we have

\[ I_2 = \Delta \Delta = 2 \left( \frac{\partial u}{\partial r} \right)^2 + 2 \left( \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{v}{r} \right)^2 + \left( r \frac{\partial (v/r)}{\partial r} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right)^2 \]

(II-3.3)

Then with the assumption of radial flow we will have

\[ I_2 = 2 \left( \frac{F'}{r^2} \right)^2 \]

(II-3.4)

where \( F' = \frac{dF}{d\theta} \)

So

\[ r = K \left| \frac{F'}{r^2} \right|^\frac{n-1}{\Delta} \]

(II-3.5)

II- 3.2 Hydrodynamic problem

The continuity and momentum equations are developed in order to solve the flow problem; polar coordinates as shown earlier in figure II-3.1 are used. At the inlet, the entry profile will be either uniform or parabolic to approximate the change from a reservoir to converging plates or the change from parallel plates to converging ones. The following assumptions will be made to simplify the equations:
- Only small angles are used (θ-dir. motion is neglected)
- Incompressible flow
- No end effects (semi-infinite plates i.e. motion in z-direction is negligible)
- Steady flow
- No body forces
- Constant fluid properties
- No exit effects (from the sink at the apex)

With these assumptions in polar coordinates, the equations of motion are from Bird et al. (1960) (p. 83):

- Continuity

\[
\frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial \theta} = 0 \quad (II-3.6)
\]

- Momentum equation in r-direction

\[
u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{dp}{dr} - \frac{1}{\rho} \left( \frac{\partial}{\partial r} \left( \frac{\partial (r^2\tau_{rr})}{\partial r} \right) + \frac{\partial \tau_{r\theta}}{\partial \theta} + \tau_{\theta \theta} \right) \]

\[
(II-3.7)
\]

- Continuity in the integral form

\[
\bar{u}_o \beta r_o = \int_0^\beta ur \, d\theta \quad (II-3.8)
\]

Note: \( \bar{u}_o \) is the average velocity at the entry which means that for uniform entry, \( u_o \) represents the actual flat velocity profile.
We did not include the momentum equation in the θ-direction because only small angles of convergence are used (<15°) so the motion in the θ-direction will be considered negligible; this approximation is similar to the one used by Sutterby (1965) and Yang & Price (1972). In the equation of momentum, we are replacing all the stresses by the equivalent constitutive equations as defined in the previous section. After several manipulations described in appendix II-A, we get the following momentum equation:

\[ u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{dp}{dr} + \frac{K}{\rho} \frac{\gamma^{n-1}}{r} \left( \frac{3}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \left( \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial u}{\partial \theta} \right) \right) \]

\[ + \frac{K}{r} \left( 2r \frac{\partial u}{\partial r} \frac{\partial (\gamma^{n-1})}{\partial r} + \left( \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right) \frac{\partial (\gamma^{n-1})}{\partial \theta} \right) \]

where \( \dot{\gamma} = \frac{1}{r} \frac{\partial u}{\partial \theta} \)

The term \( \frac{\partial^2 u}{\partial r^2} \) is neglected because it represents only a smaller part of the axial diffusion of momentum \( \left( \frac{3}{r} \frac{\partial u}{\partial r} \text{ and } \frac{u}{r^2} \text{ are other terms} \right) \). In the last term, only \( \frac{1}{r} \frac{\partial u}{\partial \theta} \frac{\partial (\gamma^{n-1})}{\partial \theta} \) will be retained because it does not contain any θ-direction velocity or derivative with respect to r which after the initial flow development are rapidly becoming negligible. So we finally are left with the following equation:

\[ u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{dp}{dr} + \frac{K}{\rho} \frac{\gamma^{n-1}}{r} \left( \frac{3}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \left( \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial u}{\partial \theta} \right) \right) + \frac{K}{r^2} \frac{\partial u}{\partial \theta} \frac{\partial (\gamma^{n-1})}{\partial \theta} \]

(II-3.10)
With the use of the following variable transformation

\[
U = u/\bar{u}_o \quad R = 1-r/r_o
\]
\[
V = v/\bar{u}_o \quad \phi = \theta/\beta
\]
\[
P = (p-p_0)/\rho\bar{u}_o^2
\]  \hspace{1cm} (II-3.11)

we obtain

- Continuity equation

\[
\frac{\partial ((1-R)U)}{\partial R} + \frac{1}{\beta} \frac{\partial V}{\partial \phi} = 0
\]  \hspace{1cm} (II-3.12)

- Momentum equation

\[
U \frac{\partial U}{\partial R} + \frac{1}{\beta (1-R)} \frac{\partial U}{\partial \phi} + \frac{V^2}{(1-R)} = - \frac{dP}{dR} + \frac{1}{Re} \left[ \frac{1}{\beta (1-R)} \frac{\partial U}{\partial \phi} \right]^{n-1}
\]
\[
+ \frac{1}{Re} \frac{1}{\beta} \frac{\partial U}{\partial \phi} \frac{1}{\beta (1-R)} \frac{\partial U}{\partial \phi} \]  \hspace{1cm} (II-3.13)

where \( \text{Re} = \frac{\bar{u}_o 2-n}{\rho \tau_0^n} \) = Reynolds number

- Integral form of continuity

\[
\left[ \int_{0}^{1} U(1-R) \, d\phi \right]_{R=R}^{R=R+\Delta R} = \left[ \int_{0}^{1} U(1-R) \, d\phi \right]_{R=R+\Delta R}
\]  \hspace{1cm} (II-3.14)
One point to be noted is the fact that in the physical system as it stands in figure I-3.1, the velocity $u$ is in the opposite direction of $r$ the radial length so, before transforming with the use of the above variables, this must be taken into account by replacing $u$ by $-u$.

In terms of dimensionless variables, the boundary conditions and initial condition are as follows:

- **Boundary conditions**

  at the wall $\phi = 1$  
  \[
  U = 0 \text{ and } V = 0 
  \]

  at the centerline $\phi = 0$  
  \[
  \frac{\partial U}{\partial \phi} = 0 \text{ and } V = 0
  \]

- **Initial condition (for uniform entry)**

  at the entry $R = 0$  
  \[
  U = 1 \text{ and } V = 0
  \]

In order to use the finite difference scheme, the grid in figure II-3.2 is employed:

Figure II-3.2: Finite difference network
The finite difference approximations used are:

\[
\frac{\partial U}{\partial R} = \frac{U_{j+1,k} - U_{j,k}}{\Delta R}
\]

\[
\frac{\partial U}{\partial \phi} = \frac{U_{j+1,k+1} - U_{j+1,k-1}}{2\Delta \phi}
\]

\[
\frac{\partial V}{\partial \phi} = \frac{V_{j+1,k+1} - V_{j+1,k}}{\Delta \phi}
\]

\[
\frac{\partial^2 U}{\partial \phi^2} = \frac{U_{j+1,k+1} - 2U_{j+1,k} + U_{j+1,k-1}}{\Delta \phi^2}
\]

\[
\frac{\partial P}{\partial R} = \frac{P_{j+1} - P_j}{\Delta R}
\]

The continuity equation becomes:

\[
V_{j+1,k+1} = V_{j+1,k} - \frac{\beta \Delta \phi}{2\Delta R} \left( (1-R)_{j+1} U_{j+1,k} + U_{j+1,k+1} \right) - (1-R)_j \left( U_{j,k} + U_{j,k+1} \right)
\]

\[
(II-3.18)
\]

Similarly the momentum equation becomes:

\[
A_k U_{j+1,k-1} + B_k U_{j+1,k} + C_k U_{j+1,k+1} + D_k P_{j+1} = E_k
\]

\[
(II-3.19)
\]

where

\[
A_k = - \frac{V_{j,k} - \frac{G_{j,k+1} - G_{j,k-1}}{\beta(1-R)_j}}{2\Delta \phi} - \frac{G_{j,k}}{((1-R)_j \beta \Delta \phi)^2}
\]
\[ B_k = \frac{U_{j,k}}{\Delta R} + \frac{G_{j,k}^2}{(1-R)_{j,\beta} \Delta \phi} + \frac{3G_{j,k}}{(1-R)_{j,\Delta R}} \]

\[ C_k = \frac{V_{j,k} - G_{j,k+1} - G_{j,k-1}}{\beta(1-R)_{j,2\Delta \phi}} + \frac{G_{j,k}}{(1-R)_{j,\Delta \phi}^2} \]

\[ D_k = \frac{1}{\Delta R} \]

\[ E_k = \frac{U_{j,k}^2}{\Delta R} - \frac{V_{j,k}^2}{(1-R)_{j,\Delta R}} + \frac{P_i}{(1-R)_{j,\Delta R}} + \frac{3G_{j,k}U_{j,k}}{(1-R)_{j,\Delta R}^2} + \frac{P}{(1-R)_{j,\Delta R}^2} \]

where \[ G_{j,k} = \frac{1}{\text{Re}} \left| \frac{1}{\beta(1-R)_{j,2\Delta \phi}} \right|^{n-1} \]

and the integral form of the continuity equation is

\[ (1-R) \sum_{k=1}^{m} U_{j+1,k} = (1-R) \sum_{k=1}^{m} U_{j,k} \]

(II-3.20)
For the energy equation, viscous dissipation is considered negligible and all the assumptions made for the hydrodynamic problem are retained.

The energy equation then becomes:

\[
\frac{u \frac{\partial t}{\partial r} + v \frac{\partial t}{\partial r}}{r \frac{\partial \theta}{\partial r}} = \frac{k}{\rho C_p} \left( \frac{1}{r} \frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \theta^2} \right) \quad (II-3.21)
\]

Following Yang & Price (1972) the term \( \frac{\partial^2 t}{\partial r^2} \) will be neglected since \( \frac{1}{r} \frac{\partial t}{\partial r} \) which is the other term for axial heat diffusion is more important.

To put that equation in dimensionless form, we will use the variable transformations already mentioned in the previous section (equations I-3.11) and

\[
T = \frac{t - t_w}{t_e - t_w} \quad (II-3.22)
\]

where \( t_e \) and \( t_w \) are the temperature at the entry and the wall respectively. So the energy equation becomes

\[
\frac{U \frac{\partial T}{\partial \phi} + \frac{V}{\beta (1-R)} \frac{\partial T}{\partial \phi}}{\beta (1-R)} = \frac{1}{Pe} \left( \frac{1}{(1-R)} \frac{\partial T}{\partial \phi} + \frac{1}{(\beta (1-R))^2} \frac{\partial^2 \phi}{\partial \phi^2} \right) \quad (II-3.23)
\]

where \( Pe = \frac{\bar{u} \rho C_p r_o}{k} = \text{Peclet number} \)
It should be noted that here, as in the hydrodynamic problem, "u" must be replaced by "-u" to account for the opposite direction of "u" and "r" in the physical system of figure II-3.1.

The same mesh network as for the hydrodynamic problem is used along with the following temperature finite difference representations:

\[
\frac{\partial T}{\partial R} = \frac{T_{j+1,k} - T_{j,k}}{\Delta R}
\]

\[
\frac{\partial T}{\partial \phi} = \frac{T_{j+1,k+1} - T_{j+1,k-1}}{2\Delta \phi}
\]

\[
\frac{\partial^2 T}{\partial \phi^2} = \frac{T_{j+1,k-1} - 2T_{j+1,k} + T_{j+1,k+1}}{\Delta \phi^2}
\]

After substitutions, the energy equation becomes

\[
H_k T_{j+1,k-1} + J_k T_{j+1,k} + L_k T_{j+1,k+1} = M_k \quad \text{for } 1 \leq k \leq m
\]

where

\[
H_k = \frac{V_{j,k}}{\beta(1-R)} - \frac{1}{\beta \left(1-R\right) 2\Delta \phi} \left(\frac{2}{\text{Pe} \left(\beta \left(1-R\right) \Delta \phi\right)}\right)^2
\]

\[
J_k = \frac{1}{\Delta R} \left(U_{j,k} - \frac{1}{\text{Re} \left(1-R\right)}\right) + \frac{2}{\beta(1-R) 2\Delta \phi} \left(\frac{2}{\text{Pe} \left(\beta \left(1-R\right) \Delta \phi\right)}\right)^2
\]

\[
L_k = \frac{V_{j,k}}{\beta(1-R)} - \frac{1}{\beta \left(1-R\right) 2\Delta \phi} \left(\frac{2}{\text{Pe} \left(\beta \left(1-R\right) \Delta \phi\right)}\right)^2
\]

\[
M_k = \left(\frac{T_{j,k}}{\Delta R}\right) U_{j,k} + \frac{1}{\text{Pe} \left(1-R\right)}
\]
The bulk temperature or mixing cup temperature is defined by the following equation

\[ T_b = \frac{\int_0^1 U_{j+1,k} T_{j+1,k} d\phi}{\bar{U}} \]

where \( \bar{U} \) = average velocity

The local Nusselt number is obtained as previously:

\[ Nu(\text{local}) = \frac{hL}{K} = \left(\frac{\partial T}{\partial \phi}\right)_w \frac{L}{r_o \beta \Delta T} \]

If "\( L \)" is taken as the total length of the arc at the entry and "\( \Delta T \)" as \( (T_b - T_w) \), we end up with the following expression

\[ Nu_R = \left(\frac{\partial T}{\partial \phi}\right)_w \frac{2}{(1-R) T_b} \]

Now the average Nusselt number is calculated using the expression below where the integration of the local Nusselt number will be performed using the trapezoidal rule

\[ \bar{Nu} = \frac{1}{R} \int_0^R Nu_R \, dR \]

In order to calculate the temperature gradient at the wall, the expression below is used as described in Katotakis (1969) and Vlachopoulos & Keung (1972); this four point formula can be easily derived from a Taylor series expansion.

\[ \frac{\partial T}{\partial \phi}_w = \frac{1}{6\Delta \phi} \left( -11 T_{j,m+1} + 18 T_{j,m} - 9 T_{j,m-1} + 2 T_{j,m-2} \right) \]

where \( m+1 \) is the grid point at the wall.
II- 3.4 Numerical procedure

The numerical procedure to compute the velocities, pressures and temperatures, is the same as the one used in Part I so it will not be repeated here and the reader should see Section I-3.4 for the details.

II- 3.5 Stability and convergence

Similarly to what was said in Section I-3.5, when someone is dealing with highly nonlinear equations like the motion equations, there are no absolute stability criteria that one can obtain as in the case of linear equations.

As previously, convergence is ascertained by decreasing the mesh sizes; one tries to get results that do not change significantly with the use of finer grid sizes. Computer time requirements must be taken into account at this point. To achieve convergence in our case, nine different mesh sizes which are shown in Table II-3.1 in order of decreasing roughness, were used to make a graph of the ratio of maximum velocity over the average velocity along "R". (These mesh sizes are described in detail in appendix II-B).
Table II-3.1: Roughness of mesh size

<table>
<thead>
<tr>
<th>Mesh size symbol</th>
<th>Roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-21</td>
<td>Increasing roughness</td>
</tr>
<tr>
<td>C-22</td>
<td>Decreasing roughness</td>
</tr>
<tr>
<td>C-23</td>
<td></td>
</tr>
<tr>
<td>C-41</td>
<td></td>
</tr>
<tr>
<td>C-42</td>
<td></td>
</tr>
<tr>
<td>C-43</td>
<td></td>
</tr>
<tr>
<td>C-82</td>
<td></td>
</tr>
<tr>
<td>V-83</td>
<td></td>
</tr>
<tr>
<td>V-164</td>
<td></td>
</tr>
</tbody>
</table>

In the mesh symbols, the first digit is a letter where, C stands for constant number of grid spaces across the flow and V means variable number of grid spaces across as the marching procedure goes on; the first number multiplied by 10 gives the number of grid spaces across (exception: for V-164, we use 16); the last digit is a number describing the kind of spacing axially, where a higher number means a finer mesh size. In figure II-3.3, we can see that as the mesh size becomes finer the curves are going down with the best one being C-43 or V-83 or V-164. There can be a difference as high as 3% for example at R=.1 between the roughest and finest mesh; however this difference is almost unnoticeable for R>.35. In this graph, the work of Yang & Price (1972) has been included, of which the present work is an extension; it was found that C-22 had the closest fit with their results; as we can see from Table II-3.1, this is in our estimation, one of the rougher mesh sizes. In a private communication with one of the authors, Price (1972), it was
Figure II-3.3: $\frac{U_{\text{max}}}{U}$ vs $R$ for various grid spacing
found that at the entry they were using 39 grid spaces perpendicular to the flow and going down to 19 after a while; since their physical system included the full taper angle and not half of it as in the present work, we effectively should have a similarity between their 39 point analysis and our 20 in C-22. So our conclusion is that they should have used a finer mesh for their work since their results can be improved. The mesh size V-83 was used throughout the rest of this work.
II- 4  RESULTS AND DISCUSSION

II- 4. 1  Comparison for the Newtonian case

The results of the present work for a newtonian fluid will be compared principally with data of two recent articles namely Yang & Price (1972) and Atabek (1972) whenever possible and with Jeffery-Hamel's exact analytical solution of fully developed flow as presented by Millsaps and Pohlhausen (1953).

The Jeffery-Hamel solution requires the use of Jacobian elliptic function making the calculations tedious; so Millsaps and Pohlhausen are giving results for only three Reynolds numbers ranging from 684 to 5000. An excellent agreement is found to exist with the present work; the velocity profiles were taken here at \( R = 0.7 \) when they were evidently fully developed, are undistinguishable from the true profile. Atabek, for developed velocity profiles, is admittedly reporting profiles in excess of 7% to the two values at intermediate Reynolds numbers. Similarly, our profiles were found to be flatter than Atabek's with differences of up to 9% at \( R = 4400 \). The equation of Atabek is also valid for low Reynolds number but not less than 50 was used in this work as recommended by Reeves and Kippenhan (1962) for a good agreement between velocity distributions of exact boundary layer solution and Navier-Stokes exact solution. In figure II-4.1, those results are shown:
Figure II-4.1: Developed velocity Profile for $\beta=5^\circ$
This figure also includes cases where the axial velocity is normalized with the average velocity in order to show that Atabek's work and the present one are giving similar results at very high Reynolds numbers. In figure II-4.2 the development of the axial velocity with respect to R (the axial dimension) is shown; it is apparent that velocities are increasing faster according to our calculations than in the solution of Atabek.

Pressure curves are found in figure II-4.3. Most of the results are within 10% from the results of other workers and the curve shapes are the same. Unfortunately, Atabek is solving at very high Reynolds numbers where the flow is almost potential. We presume that results would differ somewhat at lower Reynolds number because of the differences in velocity profiles. Pressure results of Yang and Price (1972) are also shown; we can see that they agree reasonably well with those of the present work. The biggest differences are occurring at the entry where, as it has been pointed out, the grid size has an influence on the results.
Figure II-4.2: Velocity development
Figure II-4.3: Newtonian fluid Pressure curves

\[ P = \frac{p - p_0}{\rho \left( \frac{u}{U} \right)^2} \]

- Present work
- Yang & Price (1972)
- Atabek (1972)

\( \beta = 5^\circ \) and \( \text{Re} = 1000 \)

\( \beta = 5^\circ \) and \( \text{Re} = 5730 \)

\( \beta = 5^\circ \) and \( \text{Re} = 17190 \)
II- 4.2 Other results for Newtonian case

In this section, results obtained by the present method, which are very difficult to compare with others, are discussed. Furthermore the rapidity with which a fluid is developing will be studied.

As it was seen above, two parameters are affecting the hydrodynamic problem, the Reynolds number and the taper angle. In the next three figures, II-4.4 to II-4.6, the center-line velocity against the axial distance at different Reynolds numbers (Fig. II-4.4 for $\beta=2.5$ and Fig. II-4.5 for $\beta=5.0$) and at different angle values (Fig. II-4.6 for $Re=1000$) is shown. It appears that the flow is slower to develop as the Reynolds number or the taper angle are increased. The following expression can be thought of as a definition of an entry length:

$$\lim_{R \to 1} \left[ 0.99 \left( \frac{U_{\text{max}}}{U} \right) \right] = \left( \frac{U_{\text{max}}}{U} \right)_{R=R_{\text{ent}}}$$

where $R_{\text{ent}}$ is the entry length

Figure II-4.7 is an example of the kind of curves obtained if $R_{\text{ent}}$ is plotted with respect to the Reynolds number. Curves are similar to those obtained by Atabek (1972) i.e. they are reaching a maximum at a certain value of the Reynolds number. The value of that maximum is less than the one obtained by Atabek: this confirms what we saw earlier about the slower development of the velocity profiles.
Figure II-4.4: $U_{\text{max}/U}$ vs $R$
$\beta = 5^\circ$ (tot. angle = 10°)

FIGURE II-4.5: $\frac{U_{\text{max}}}{U}$ vs $R$
FIGURE II-4.6: \( \frac{U_{\text{max}}}{U} \) vs \( R \)
Figure II-4.7: Entry Lengths
The heat transfer problem can be analysed by varying three parameters (for a newtonian fluid) namely the Reynolds number, the Peclet number and $\beta$. In figure II-4.8 to II-4.11, one can see the effect of different Peclet numbers on the bulk temperature and on the local Nusselt number. Since the Peclet number is the product of the Reynolds number and the Prandtl number, one can expect the same effect when we are lowering the value of the Peclet number as we had in the first part when lowering the Prandtl number; the bulk temperature is effectively decreasing much faster at low Peclet number. The same is also true for the local Nusselt number as shown in those figures.

II- 4.3 Pseudoplastic case

The value of flow behavior index was limited almost exclusively to values over .5 since, in most cases, potential flow was obtained throughout for lower values. At a flow behavior index of .6, the fluid was found to exhibit potential flow behavior for moderately high Reynolds number as is shown in figure II-4.12 where the ratio of centerline velocity over the average velocity is plotted with respect to $R$. The same kind of graph is presented in the next figure (II-4.13) for $n=.75$ with a varying Reynolds number. The trends are the same as reported for newtonian fluids; a new characteristic is

\[ (1) \text{ Since we use high value of F.B.I., the minimum gradient value is not important.} \]
Figure II-4.8: Effect of different Peclet numbers on Nusselt number for $\beta=5^\circ$. 

**Graph Description:**
- The graph illustrates the variation of Nusselt number with respect to the dimensionless radius $R(1 - R/RO)$.
- Different Peclet numbers ($Pe$) are represented by various lines on the graph:
  - $Pe = 100$
  - $Pe = 500$
  - $Pe = 1000$
  - $Pe = 3000$
  - $Pe = 10000$
- The conditions for the figure are: $n=1.0$, $Re=1000$, $\beta=5^\circ$ (total angle $10^\circ$).

**Equation and Conditions:**
- The equation for the Nusselt number is not explicitly stated in the graph, but it is implied to be used in the context of the Peclet numbers and Reynolds number to calculate the Nusselt number for the given parameters.
Figure II-4.9: Effect of different Peclet numbers on bulk temperature for $\beta=5.0^\circ$
Figure II-4.10: Effect of different Peclet numbers on Nusselt number for $\beta=2.5^\circ$
Figure II-4.11: Effect of different Peclet numbers on bulk temperature for $\beta = 2.5^\circ$
$U_{\text{MAX} = \text{CENTER-LINE}} / U(\text{AVERAGE})$ vs $R$

$n = 0.6$, $\beta = 2.5^\circ$ (tot. angle $= 5^\circ$)

- $Re = 34,380$
- $Re = 15,000$
- $Re = 1,000$
- $Re = 300$
- $Re = 50$

Figure II-4.12: $U_{\text{max}} / U$ vs $R$
Figure II-4.13: $U_{\text{max}}/U$ vs $R$
that the curves are reaching a maximum and then decreasing. This can be explained by the fact that, at lower flow behavior index, a fluid takes more time to develop as we saw for parallel plates entry; this fact will mean that at a certain value of $R$, the rapidity of development will not be fast enough to compensate for the acceleration due to converging effect of the walls.

From figure II-4.14, we can see that pressure drops are decreasing as the flow behavior index is lowered. In their first part, the curves are presenting an almost constant pressure gradient but as soon as the fluid reaches a region where the flow becomes noticeably accelerated, the gradient increases sharply. We can see also that pressure curves have a minimum which is caused by potential flow. In figure II-4.15, one can see the effect of the flow behavior index on curves of centerline velocity normalized by the average velocity plotted with respect to $R$.

For the heat transfer part, we are producing in figure II-4.16 curves of the bulk temperature at different flow behavior indices. The curves are lower as the flow index is decreased up to a point where potential flow again is attained; we can see that heat transfer is decreased by the non-newtonian characteristic. The next figure II-4.17 is showing the value of the local Nusselt number along $R$ again, at different values of the flow behavior index; as a matter of fact, the lower bulk temperature seems to be the principal
Figure II-4.14: Pressure vs R

Re=300  $\beta=2.5^\circ$ (tot. angle=5°)

- $n=0.65$
- $n=0.7$
- $n=0.8$
- $n=0.9$
- $n=1.0$
Figure II-4.15: $U_{\text{max}}/U$ vs $R$
Figure II-4.16: Bulk temperature vs R
Figure II-4.17: Nusselt number vs R

NU (LOCAL)

β=2.5° (tot. angle=5°)  Re=300

n= .65
n= .7
n= .8
n= .9
n=1.0

R(1-R/RO)

0.00  0.10  0.20  0.30  0.40  0.50  0.60  0.70

0.0  5.0  10.0  15.0  20.0  25.0
cause for the higher Nusselt number at low index since the temperature gradients were quite similar from one behavior index to the other.

Varying the Peclet number has a similar effect on the heat transfer as described for newtonian fluids. This can be seen in figure II-4.18 to II-4.21. In figure II-4.22, the average Nusselt number is plotted against R for n=1.00 and n=.75.
Figure II-4.18: Effect of Peclet number on Nusselt number for $n=0.75$
Figure II-4.19: Effect of Peclet number on bulk temperature for $n = 0.75$
Figure II-4.20: Effect of Peclet number on Nusselt number for $n=.75$
Figure II-4.21: Effect of Peclet number on bulk temperature for $n=0.75$
Figure II-4.22: Average Nusselt number
In this second part, pseudoplastic fluids are analysed in converging channels. Excellent agreement was found to exist between this work and the exact fully developed solution of Jeffery-Hamel for newtonian fluids. Some discrepancy (Newtonian Fluids) with the analytical solution of Atabek (1972) was encountered, the result of the present work giving faster development of profiles, and agreement was good with the work of Yang and Price (1972) except in some cases where the finer mesh size used here gave some differences. New results were obtained for pseudoplastic fluids in converging flow in which a recession of the centerline velocity is predicted after the initial development. The heat transfer problem is also analysed with results expressed in terms of the bulk temperature and the Nusselt number where this last quantity was found to increase after the initial decrease contrarily to the parallel plates where the Nusselt number reaches an asymptotic value.

Recommendations for this part are in two directions, experimental and theoretical. Since the author has found no experimental work in this field, it is recommended that the experimental side of this work be done. On the theoretical aspect, we recommend that the extension of this thesis include viscoelastic fluids again along the line of the devialoric stress tensor as used in Balmer and Kauzlarich (1972).
\textbf{NOTATION}

\begin{itemize}
  \item $A_k$: Series of constants defined in equation II-3.19
  \item $B_k$: Series of constants defined in equation II-3.19
  \item $C_k$: Series of constants defined in equation II-3.19
  \item $C_p$: Specific heat
  \item $D_k$: Series of constants defined in equation II-3.19
  \item $E_k$: Series of constants defined in equation II-3.19
  \item $F$: Function of $\theta$ defining the axial velocity in equation II-3.1
  \item $G_{j,k}$: Expression defined in equation II-3.19
  \item $h$: Heat transfer coefficient
  \item $H_k$: Series of constants defined in equation II-3.25
  \item $I_2$: Second invariant of $\nabla$
  \item $J_k$: Series of constants defined in equation II-3.25
  \item $k$: Thermal conductivity
  \item $K$: Power law consistency index
  \item $L$: Characteristic length
  \item $L_k$: Series of constants defined in equation II-3.25
  \item $m$: Total number of points less one in radial direction
  \item $M_k$: Series of constants defined in equation II-3.25
  \item $n$: Flow behavior index of power law fluid
  \item $\overline{Nu}$: Average Nusselt number
  \item $\text{Nu}_R$: Nusselt number (local)
  \item $p$: Pressure
\end{itemize}
\( p_0 \) : Pressure at entry
\( P \) : Dimensionless pressure \( \left( \frac{p-p_0}{\rho \bar{u}_0^2} \right) \)
\( \text{Pe} \) : Peclet number
\( r \) : Radial coordinate
\( r_0 \) : Radial length
\( R \) : Dimensionless radial coordinate \( \left( =1-r/r_0 \right) \)
\( \text{Re} \) : Reynolds number
\( R_{\text{ent}} \) : Radial entry length
\( t \) : Temperature
\( t_e \) : Temperature at the entry
\( t_w \) : Temperature at the wall
\( T \) : Dimensionless temperature \( \left( =\frac{t-t_w}{t_e-t_w} \right) \)
\( T_b \) : Bulk temperature
\( u \) : Radial velocity
\( \bar{u} \) : Average radial velocity
\( \bar{u}_0 \) : Average radial velocity at the entry
\( U \) : Dimensionless radial velocity \( \left( =u/\bar{u}_0 \right) \)
\( \bar{U} \) : Average dimensionless radial velocity
\( \nu \) : Angular velocity
\( \nu \) : Dimensionless angular velocity \( \left( =v/\bar{u}_0 \right) \)

**Greek letters**

\( \beta \) : Half the taper angle
\( \dot{\gamma} \) : Variable defined in equation II-3.9
\( \Delta \): Tensor of rate of deformation

\( \theta \): Angular coordinate

\( T \): Stress tensor

\( \rho \): Density

\( \phi \): Dimensionless angular coordinate \((=\theta/\beta)\)

**Subscript**

\( j \): Point number in radial direction

\( k \): Point number in angular direction

\( m \): Total number of points less one in radial direction
REFERENCES


BALMER, R.T., and J.J. Kauzlarich; "Similarity solutions for converging or diverging...", A.I.Ch.E. Jl, 17, 1181 (1971).


BOUSSINESQ, J.; Comptes rendus, 113, 9 (1891).


BUCHANAN, D.; Private communication (Plotting routines), McMaster University (1973).


JEFFERY, G.; "Steady motion of a viscous fluid", Phil. Mag., 29, 455 (1915).


KATOTAKIS, S., and J. VLACHOPOULOS; "Application of a General Finite Difference Method to Boundary Layer flow" CSME Trans. (in press)


OKA, S.; "Pressure development in a non-newtonian flow through a tapered tube", Private communication (author address: Depart. of Physics, Kero University, Hiyoski, Y. Bohama, Japan).

OZOE, H. and S.W. CHURCHILL; "Hydrodynamic stability and natural convection in Oswald-de Waele and Ellis fluids: the development of a numerical solution", A.I.Ch.E. J1, 18, 1196 (1972).


SCHILLER, L.; "Untersuchungen uber laminare und turbulente strömung", ZAMM, 2, 96-106 (1922).


SCHLICHTING, H.; "Laminare Kanaleinlaufstromung", ZAMM, 14, 368 (1934).

SCHOWALTER, W.R.; "The application of boundary-layer theory to power law pseudoplastic fluids: similar solutions", A.I.Ch.E. Jl, 6, 24 (1960).


WANG, Y.L. and P.A. LONGWELL; "Laminar flow in the inlet section of parallel plates", A.I.Ch.E. Jl, 10, 323 (1964).


YANG, J.W. and G.M. PRICE; Private communication, (1972).
APPENDIX I-A

Derivation of momentum equation (equation I-3.23) from finite difference approximations (equation I-3.19 and I-3.20).

We start from equation I-3.14

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + 2n \partial^2 \left( \frac{u}{\partial y} \right)^{n-1} \left( \frac{u}{\partial y} \right) \]

(I-A.1)

by replacing directly the finite difference approximations of equations I-3.19 and I-3.20

\[
\begin{align*}
U_{j+1,k} - U_{j,k} & = \frac{1}{\Delta x} \left( \frac{U_{j,k+1} + U_{j,k-1} + U_{j+1,k+1} + U_{j+1,k-1}}{2} - P_{j+1} - P_j \right) \\
+ & 2n \left( \frac{U_{j,k+1} - U_{j,k}}{\Delta y} \right)^{n-1} \left( \frac{U_{j,k+1} - U_{j,k}}{\Delta y} \right)
\end{align*}
\]

(I-A.2)
Rearranging and putting all the known terms on the right side and all the unknown on the left side

\[ U_{j+1,k-1} \left( -\frac{V_{j,k}}{4\Delta Y} - \frac{|Gm|^{n-1}}{\Delta Y^2} \right) \]

\[ + \quad U_{j+1,k} \left( \frac{U_{j,k}}{\Delta X} + \frac{|Gp|^{n-1}}{\Delta Y^2} + \frac{|Gm|^{n-1}}{\Delta Y^2} \right) \]

\[ + \quad U_{j+1,k+1} \left( \frac{V_{j,k}}{4\Delta Y} - \frac{|Gp|^{n-1}}{\Delta Y^2} \right) + P_{j+1} \left( \frac{1}{\Delta X} \right) = \]

\[ + \frac{P_{j+1} U_{j,k}^2}{\Delta X^2} - V_{j,k} \left( \frac{U_{j,k+1} - U_{j,k-1}}{4\Delta Y} \right) \]

\[ + Gp |n-1| \left( \frac{U_{j,k+1} - U_{j,k}}{\Delta Y^2} \right) - Gm |n-1| \left( \frac{U_{j,k} - U_{j,k-1}}{\Delta Y^2} \right) \]

where \( Gp = 2 \frac{U_{j,k+1} - U_{j,k}}{\Delta Y} \)

\( Gm = 2 \frac{U_{j,k} - U_{j,k-1}}{\Delta Y} \)
APPENDIX I-B

Transformation of energy equation

The energy equation is (equation I-3.26)

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{2}{Pr} \frac{\partial^2 T}{\partial Y^2} \quad \text{(I-B.1)}$$

Using the finite difference equation I-3.29, we get

$$F_k T_{j+1,k-1} + G_k T_{j+1,k} + H_k T_{j+1,k+1} = J_k$$

$$F_k = - \frac{V_{j,k} + V_{j+1,k}}{2} \frac{2}{4\Delta Y} \frac{2}{Pr \Delta Y^2}$$

$$G_k = \frac{U_{j,k} + U_{j+1,k}}{2\Delta X} + \frac{2}{Pr \Delta Y^2}$$

$$H_k = \frac{V_{j,k} + V_{j+1,k}}{2} \frac{2}{4\Delta Y} \frac{2}{Pr \Delta Y^2}$$

$$J_k = \frac{U_{j,k} - U_{j+1,k}}{2\Delta X} \frac{T_{j,k} - \frac{V_{j,k} + V_{j+1,k}}{2} \frac{T_{j+1,k} - T_{j-1,k}}{4\Delta Y}}{2} \frac{T_{j,k+1} - 2T_{j,k} + T_{j,k-1}}{Pr \Delta Y^2}$$

108
APPENDIX I-C

Additional results for straight channel entry flow

I-C.1 : Velocity vs Y for n=0.05
I-C.2 : Velocity vs Y for n=0.15
I-C.3 : Velocity vs Y for n=0.30
I-C.4 : Velocity vs Y for n=0.60
I-C.5 : Velocity vs Y for n=1.00
I-C.6 : Velocity vs X for n=0.15
I-C.7 : Velocity vs X for n=0.15
I-C.8 : Velocity vs X for n=0.50
I-C.9 : Velocity vs X for n=1.00
I-C.10: Pressure vs X
I-C.11: Bulk temperature vs X
I-C.12: Bulk temperature vs X
I-C.13: Nusselt number vs X
Figure I-C.1: Velocity vs Y for n=.05

VELOCITY VS Y FOR N= .05

\( \gamma_m = 0.01 \)

\( X=1.0 \)
\( X=0.05 \)
\( X=0.002 \)
VELOCITY VS Y FOR N= 0.15

Figure I-C.2: Velocity vs Y for n=0.15
Figure I-C.3: Velocity vs Y for n=.30
VELOCITY VS Y FOR N= 0.60

Figure I-C.4: Velocity vs Y for n=.60
Figure I-C.5: Velocity vs Y for \( n=1.00 \)
Figure I-C.6: Velocity vs X for n= .15
Figure I-C.7: Velocity vs X for n=0.15
Figure I-C.8: Velocity vs X for n = .50
Figure I-C.9: Velocity vs X for n=1.00
Figure I-C.10: Pressure vs X
Figure I-C.11: Bulk temperature vs X
Figure I-C.12: Bulk temperature vs X

- Pr = .01
- \( \gamma_m = .01 \)
- n = .15
- n = .25
- n = .50
- n = .75
- n = 1.00
Figure I-C.13: Nusselt number vs X
APPENDIX II-A

Derivation of momentum equation for converging flow

\[
U \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{\partial v}{\partial r} = -\frac{1}{\rho} \frac{dp}{dr} - \frac{1}{\rho r} \left( \frac{\partial \left(\frac{r \tau_{rr}}{r}\right)}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} - \tau_{\theta\theta} \right)
\]

\[\text{(II-A.1)}\]

If we expand the last term in accordance with the constitutive equation, we get

\[
+ \frac{K}{\rho r} \left( \frac{\partial (G) 2 \frac{\partial u}{\partial r}}{\partial r} + \frac{\partial (G) \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r} \frac{\partial u}{\partial \theta}}{\partial \theta} \right) - 2(G) \left( \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{u}{r} \right)
\]

\[\text{(II-A.2)}\]

where \(G = \left| \frac{1}{r} \frac{\partial u}{\partial \theta} \right|^{n-1} \)

By using the following two identities taken from the continuity equation

\[
\frac{1}{r} \frac{\partial v}{\partial \theta} = -\frac{1}{r} \frac{\partial (ru)}{\partial r} = -\frac{\partial u}{\partial r} - \frac{u}{r}
\]

\[\text{(II-A.3)}\]
we have with the following last term

\[ + \frac{K(G)}{\rho} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2} \right) u + \frac{\partial^2 u}{\partial \theta^2} + \frac{3}{r} \frac{\partial u}{\partial r} + K \frac{\partial^3 u}{\partial \rho^3} \frac{\partial^3 v}{\partial \rho^3} + \frac{1}{\partial \rho} \frac{\partial u}{\partial \rho} \frac{\partial v}{\partial \rho} + \frac{1}{r} \frac{\partial u}{\partial r} \frac{\partial v}{\partial r} \]

\[ + \frac{\partial (G)}{\partial r} 2 \frac{\partial}{\partial r} \frac{\partial u}{\partial r} \frac{\partial v}{\partial r} \]

(II-A.4)
### APPENDIX II-B

Mesh sizes used for converging flows

<table>
<thead>
<tr>
<th>Number of grid points across</th>
<th>Value of $\Delta R$</th>
<th>Grid size valid up to $R$=</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A)</strong> C-21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>20</td>
<td>0.05</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>B)</strong> C-41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>40</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>40</td>
<td>0.05</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>C)</strong> C-22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.0001</td>
<td>0.001</td>
</tr>
<tr>
<td>20</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>20</td>
<td>0.05</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>D)</strong> C-42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.0001</td>
<td>0.001</td>
</tr>
<tr>
<td>40</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>40</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>40</td>
<td>0.05</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>E)</strong> C-82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>0.0001</td>
<td>0.001</td>
</tr>
<tr>
<td>80</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>80</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>80</td>
<td>0.05</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>F)</strong> C-43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.0001</td>
<td>0.001</td>
</tr>
<tr>
<td>40</td>
<td>0.0005</td>
<td>0.01</td>
</tr>
<tr>
<td>40</td>
<td>0.0025</td>
<td>0.1</td>
</tr>
<tr>
<td>40</td>
<td>0.01</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>G)</strong> C-23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.0001</td>
<td>0.001</td>
</tr>
<tr>
<td>20</td>
<td>0.0005</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>0.0025</td>
<td>0.1</td>
</tr>
<tr>
<td>20</td>
<td>0.01</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>H)</strong> V-83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>0.0001</td>
<td>0.001</td>
</tr>
<tr>
<td>40</td>
<td>0.0005</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>0.0025</td>
<td>0.1</td>
</tr>
<tr>
<td>20</td>
<td>0.01</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>I)</strong> V-164</td>
<td></td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>0.00002</td>
<td>0.0004</td>
</tr>
<tr>
<td>160</td>
<td>0.0001</td>
<td>0.002</td>
</tr>
<tr>
<td>80</td>
<td>0.0005</td>
<td>0.01</td>
</tr>
<tr>
<td>40</td>
<td>0.001</td>
<td>0.03</td>
</tr>
<tr>
<td>20</td>
<td>0.0025</td>
<td>0.1</td>
</tr>
<tr>
<td>20</td>
<td>0.01</td>
<td>1.0</td>
</tr>
</tbody>
</table>
APPENDIX II-C

Additional results for converging entry flow

II-C.1 : Velocity vs R for n=1.00 at $\beta=0.5^\circ$
II-C.2 : Velocity vs R for n=1.00 at $\beta=2.5^\circ$
II-C.3 : Velocity vs R for n=1.00 at $\beta=5^\circ$
II-C.4 : Velocity vs R for n=1.00 at $\beta=7.5^\circ$
II-C.5 : Velocity vs R for n=0.75 at Re=5000
II-C.6 : Velocity vs R for n=0.75 at Re=1000
II-C.7 : Velocity vs R for n=0.75 at Re=300
II-C.8 : Velocity vs R for n=0.75 at Re=50
II-C.9 : Pressure vs R for different $\beta$
II-C.10: Pressure vs R for different Re
II-C.11: Pressure vs R for different Re
II-C.12: Bulk temperature vs R for different $\beta$
II-C.13: Bulk temperature vs R for different Re
II-C.14: Nusselt number vs R for different Re
II-C.15: Bulk temperature vs R for different Re
II-C.16: Nusselt number vs R for different Re
II-C.17: Bulk temperature vs R for different Re
VELOCITY VS R FOR N=1.00

\[ \beta = 0.5^\circ \text{ (tot. angle}=1^\circ) \]

\[ \text{Re} = 1000 \]

Figure II-C.1: Velocity vs R for n=1.00
Figure II-C.2: Velocity vs R for n=1.00

VELOCITY VS R FOR N=1.00

$\beta = 2.5^\circ$ (tot. angle=5\(^\circ\))

Re = 1000
Figure II-C.3: Velocity vs R for n=1.00
Figure II-C.4: Velocity vs R for n=1.00

VELOCITY vs R FOR N=1.00

$\beta=7.5^\circ$ (tot. angle=15$^\circ$)

Re = 1000
VELOCITY VS R FOR N= .75

$\beta=2.5^\circ$ (tot. angle=$5^\circ$)

Re = 5000

Figure II-C.5: Velocity vs R for $n=.75$
Figure II-C.6: Velocity vs R for n= .75

\[ \beta = 2.5^\circ \text{ (tot. angle}=5^\circ) \]

Re = 1000
VELOCITY VS R FOR N = 0.75

$\beta = 2.5^\circ$ (tot. angle=$5^\circ$)

$Re = 300$

Figure II-C.7: Velocity vs R for n=0.75
Figure II-C.8: Velocity vs R for n= .75

VELOCITY VS R FOR N= .75

β=2.5° (tot. angle=5°)
Re = 50
Figure II-C.9: Pressure vs R

**PRESS VS R**

Re = 1000

- \( \beta = 0.5 \)
- \( \beta = 1.5 \)
- \( \beta = 2.5 \)
- \( \beta = 5.0 \)
- \( \beta = 7.5 \)
\( \beta = 2.5^\circ \) (tot. angle=5\(^\circ\)) \quad n = 0.75

- \( \text{Re} = 10000 \)
- \( \text{Re} = 5000 \)
- \( \text{Re} = 1000 \)
- \( \text{Re} = 300 \)
- \( \text{Re} = 50 \)

Figure II-C.10: Pressure vs R
$\beta = 2.5^\circ$ (tot. angle = $5^\circ$)  \hspace{1mm} n=1.0$

Re = 10000

Re = 5000

Re = 1000

Re = 300

Re = 50

Figure II-C.11: Pressure vs R
Figure II-C.12: Bulk temperature vs R
Figure II-C.13: Bulk temperature vs R
Figure II-C.14: Nusselt number vs R

Nusselt Number vs $R$

$n = 0.75 \quad \beta = 2.5^\circ$

- $Re = 10000$
- $Re = 5000$
- $Re = 1000$
- $Re = 300$
- $Re = 50$

Figure II-C.14: Nusselt number vs $R$
Figure II-C.15: Bulk temperature vs $R$
Figure II-C.16: Nusselt number vs $R$
Figure II-C.17: Bulk temperature vs R

- $n=1.0$, $\beta=2.5^\circ$ (tot. angle=$5^\circ$)
- Re = 10000
- Re = 5000
- Re = 1000
- Re = 300
- Re = 50
APPENDIX III

Finite difference solution of channel entry flow with an iterative scheme

III- 1 Theory

As mentioned in the first part of this work, we can use an iterative scheme if when introducing finite difference representations, we still have non-linear equations. Whereas the fully implicit scheme can proceed from one column to the other, the iterative one uses approximations of the values to be calculated. In the present work, the first approximation used, is the values of the preceding column; after the first calculation, the approximation is taken the sum of 85% of the previously calculated values and 15% of the old values. (convergence being generally faster with this ratio). The word "values" here applies only to the axial velocities and pressure but not to y-direction velocities since these are obtained explicitly from the axial velocities. After each iteration, the new calculated values are compared with the approximations and the iterations are stopped if convergence is obtained (less than .5% difference between approximations and calculated values).
For the iterative scheme, all the equations and the difference transformations are the same except for the following four finite difference approximations:

\[ V = \frac{V_j + V_{j+1}}{2} \quad U = \frac{U_j + U_{j+1}}{2} \]

\[ \tau_{j+1,k+1/2} = \left| \frac{U_{j+1,k+1} - U_{j+1,k}}{\Delta Y} \right|^{n-1} \frac{U_{j+1,k+1} - U_{j+1,k}}{\Delta Y} \]

\[ \tau_{j+1,k-1/2} = \left| \frac{U_{j+1,k} - U_{j+1,k-1}}{\Delta Y} \right|^{n-1} \frac{U_{j+1,k} - U_{j+1,k-1}}{\Delta Y} \]

(III-1)

Since each difference equation is similar to the ones already used with the implicit scheme except for the momentum equation, only the transformation of this last one will be retaken here.

Starting from equation I-3.14

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial P}{\partial X} + 2n \frac{\partial}{\partial Y} \left( \frac{\partial U}{\partial Y} \right)^{n-1} \frac{\partial U}{\partial Y} \]

(III-2)
we replace directly the finite difference approximations

I-3.19, I-3.20 and III-1

\[
\begin{align*}
U_{j+1, k+1, k+1} + U_{j+1, k, k+1} + U_{j, k+1, k+1} + U_{j, k+1, k} + U_{j, k, k+1} + U_{j, k, k} = & \\
U_{j, k+1, k+1} - U_{j, k, k+1} - U_{j, k, k} - U_{j, k, k-1} - U_{j, k-1, k} - U_{j, k-1, k-1} = & \\
\end{align*}
\]

\[
\begin{align*}
\frac{\Delta x}{\Delta y} \left[ U_{j, k+1, k+1} - U_{j, k, k+1} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k+1, k} - U_{j, k, k} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k+1, k-1} - U_{j, k, k-1} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k, k+1} - U_{j, k, k} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k, k-1} - U_{j, k, k} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k, k} - U_{j, k, k-1} \right] = & \\
\frac{\Delta y}{\Delta x} \left[ U_{j, k+1, k+1} - U_{j, k, k+1} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k+1, k} - U_{j, k, k} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k+1, k-1} - U_{j, k, k-1} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k, k+1} - U_{j, k, k} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k, k-1} - U_{j, k, k} \right] + \frac{\Delta y}{\Delta x} \left[ U_{j, k, k} - U_{j, k, k-1} \right] = & \\
\end{align*}
\]

Rearranging, we have

\[
A_k U_{j+1, k-1} + B_k U_{j+1, k} + C_k U_{j+1, k+1} + D_k P_{j+1, k} = E_k
\]

(III-4)

where \( A_k = - \frac{V_{j, k} + V_{j+1, k}}{8\Delta y} + \frac{Gmn_{n-1}}{\Delta y^2} \)

\( B_k = \frac{U_{j, k} + U_{j+1, k}}{2\Delta x} + \frac{Gmn_{n-1} + Gpn_{n-1}}{\Delta y^2} \)
\[
C_k = \frac{V_{j,k} - V_{j+1,k}}{8\Delta Y} - \frac{|Gpn|^{n-1}}{\Delta Y^2}
\]

\[
D_k = \frac{1}{\Delta x}
\]

\[
E_k = \frac{p_j}{\Delta x} + \frac{U_{j,k} + U_{j+1,k}}{2} \frac{U_{j,k}}{\Delta x} - \frac{V_{j,k} + V_{j+1,k}}{2} \frac{V_{j,k} + V_{j+1,k}}{\Delta Y} + \frac{U_{j,k+1} - U_{j,k-1}}{4\Delta Y}
\]

\[
+ |Gp|^{n-1} \frac{Gp}{\Delta Y} + |Gm|^{n-1} \frac{Gm}{\Delta Y}
\]

where \( Gpn = 2 \frac{U_{j+1,k+1} - U_{j+1,k}}{\Delta Y} \)

\( Gmn = 2 \frac{U_{j+1,k} - U_{j+1,k-1}}{\Delta Y} \)

\( Gp = 2 \frac{U_{j,k+1} - U_{j,k}}{\Delta Y} \)

\( Gm = 2 \frac{U_{j,k} - U_{j,k-1}}{\Delta Y} \)

We can see here that \( (U_{j,k} + U_{j+1,k}), (V_{j,k} + V_{j+1,k}) \), \( Gpn \) and \( Gmn \) are making this equation nonlinear; so we must set a value for those variables before computing the \( j+1 \) column.

The solving procedure remains exactly the same as used for the implicit scheme.
III- 2 Results and discussion for the iterative scheme

This kind of solution was also investigated to obtain results for dilatant fluids. The overall results from this method are giving slower developing profile than non-iterative solutions. In figure III-1, we are plotting the entry length (98% of fully developed profile) and comparing with similar non-iterative results of this work and Collins and Schowalter values. We can see that the entry lengths are longer than the usual ones; this is probably due to the fact that the effect of the wall is somewhat damped by the iterative scheme; the first reaction (on the first calculation) being more "explosive" than the following ones which are lowering this forward thrust. This should be especially true with the first few columns. We can see that for this kind of solution, results were obtainable for n>1.0 i.e. for dilatant fluids; it must be stated though that when n is higher than 1.75 there are some instabilities particularly near the wall: those instabilities were not strong enough to disturb the flow as a whole and were taking the form of an oscillating perpendicular (Y-direction) velocity. This kind of oscillation was also noted for the values of the temperatures which made calculations of the Nusselt number difficult because we must have accurate temperature values near the wall to calculate the proper temperature gradient. So at n=2.00 our statement that values do not change appreciably with changes of mesh sizes does not hold at times.
ENTRY LENGTH VS FLOW BEHAVIOR INDEX

\[ \gamma_m = 0.01 \]

Collins & Schowalter (1963)

Non-iterative scheme

Iterative scheme

FIGURE III-1: ENTRY LENGTH VS FLOW BEHAVIOR INDEX
The values of the excess pressure drops correction are lower than the previously reported values; this is shown in figure III-2 where excess pressure drops are plotted against the flow behavior index. Varying the minimum gradient on those curves has not any effect on lower flow behavior index as in the non-iterative but has some at $n=2.0$. Figure III-3 shows a fluid developing along $X$ at five different values of $Y$ at $n=1.0$; we can see the differences between the two schemes as the non-iterative one is developing faster than the other one.

For the heat transfer, the results of the bulk temperature are similar in forms with values just a shade higher than the ones with no iterations. Fig.III-4 shows the Nusselt number. For dilatant fluids, the values of the asymptotic local Nusselt numbers are in accordance with the ones given by Vlachopoulos and Keung (1972) as we can see in table III-1 below:

<table>
<thead>
<tr>
<th>$\gamma_m$</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4.162</td>
<td>3.954</td>
<td>3.767</td>
<td>3.683</td>
<td>3.636</td>
</tr>
<tr>
<td>0.1</td>
<td>4.212</td>
<td>3.959</td>
<td>3.767</td>
<td>3.682</td>
<td>3.635</td>
</tr>
<tr>
<td>(Pr = 1.0)</td>
<td>0.01</td>
<td>4.212</td>
<td>3.966</td>
<td>3.767</td>
<td>3.682</td>
</tr>
<tr>
<td>(Pr = 5.0)</td>
<td>0.01</td>
<td>4.218</td>
<td>3.967</td>
<td>3.768</td>
<td>3.683</td>
</tr>
<tr>
<td>0.005</td>
<td>4.218</td>
<td>3.966</td>
<td>3.767</td>
<td>3.682</td>
<td>3.635</td>
</tr>
</tbody>
</table>

Vlachopoulos & Keung (1972)  

<table>
<thead>
<tr>
<th></th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.22</td>
<td>3.97</td>
<td>3.767</td>
<td>-</td>
<td>3.64</td>
<td></td>
</tr>
</tbody>
</table>

Table III-1: Asymptotic Nusselt number for iterative system (at Pr = 1.0)
Figure III-2: Excess pressure drop vs flow behavior index
Figure III-3: Comparison of iterative and non-iterative scheme
Figure III-4: Local Nusselt number vs X (Iterative scheme)
Those two systems as we saw, are giving different results for the hydrodynamic problem but are giving similar ones for the heat transfer part (this was predictable since we are using exactly the same energy equation for the two cases). In the present literature, there is no iterative scheme with integration procedure (solutions of the full Navier-Stokes equations are solutions of elliptic equations in which all the grid points in the 2-dimensional mesh are solved simultaneously). It could be that solutions of the iterative type should be done without the boundary layer assumptions because they are too precise for this kind of problem making it a must to solve the entire equations. However we must say that this method can be improved sensibly by an appropriate choice of step sizes and finite difference approximations so it should not be discarded as an inferior method.
APPENDIX IV

Algorithm and computer programs

1: Basic algorithm
2: Main program for straight channel with non iterative implicit scheme
3: Main program for straight channel with iterative scheme
4: Subroutine "Const" for above programs
5: Subroutine "Gauss"
6: Subroutine "Dessin" for straight channel programs
7: Main program for converging channel
8: Subroutine "Const" for converging channel program
9: Subroutine "Dessin" for converging channel program

155
ALGORITHM

Note:- Apply to all three main programs

-Broken lines apply to iterative scheme

1. Read mesh sizes
2. Yes
   Read other parameters
6. No
5. Establish starting profile and output
3. Initialisation Call "Const"

- (Iterative scheme)
  1st approximation UCON=VOLD
  Establish coefficient matrix
  Solve it to get UNEW, PNEW

Yes

Stop

No

Is program terminating

156
Compute explicit VNEW

Do we have convergence? No

Yes

For the present column
Do we do heat transfer calculation

Yes

- Establish coefficient matrix (For heat transfer)
- Solve it

Calculate TBULK, NUS...

Output the results? No

Yes

Output results (Listing form)

- Prepare values for plot
- Plot (If all results are in)

Is this fluid analysis finished?

Yes

No

Do we change grid spacing in axial direction

Change grid size in axial direction check if same needed in cross flow direction

Change new values to old values and prepare for next column
NON-ITERATIVE IMP\CIT METHOD

PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10)

THIS IS A FINITE DIFFERENCE METHOD TO SOLVE THE MOTION
EQUATIONS FOR THE ENTRANCE REGION OF A CHANNEL

THIS METHOD IS USING A SERIE OF VALUES KNOWN AS THE OLD VALUES TO
COMPUTE ANOTHER SET OF VALUES KNOWN AS THE NEW VALUES CORRESPONDING
TO THE NEXT COLUMN (AXIALLY) SO THAT FROM THE ENTRY SET OF VALUES WE
CAN GO DOWNSTREAM AS FAR AS WE WANT PROVIDING WE HAVE THE BOUNDARY
CONDITIONS

LIST OF VARIABLES
AN= EXPONENT OF THE POWER LAW FLUID (FLOW BEHAVIOR INDEX')
UOLD(-),VOLD(-),TOLD(-),POLD= VECTORS OF VELOCITY (X-DIR), VELOCITY
(Y-DIR), TEMPERATURE AND PRESSURE ALREADY KNOWN
UNEW(-),VNEW(-),TNEW(-),PNEW= VECTORS OF VELOCITY (X-DIR), VELOCITY
(Y-DIR), TEMPERATURE AND PRESSURE TO BE COMPUTED (NEW)
UO(-),VO(-),UN(-),VN(-)= VECTORS OF VELOCITY ATTACHED TO HEAT TRANSFER
GRID
UCON(-),VCON(-)= VECTORS OF VELOCITY USED IN ITERATIVE TYPE MESH
TO CHECK THE CONVERGENCE
UUMAXN(-)= VECTOR OF VELOCITY DIVIDED BY MAXIMUM VELOCITY (CENTERLINE)
(OUTPUT PURPOSE)
GRA(-),GRAMIN,GRAM1= GRADIENTS - VECTOR OF GRADIENTS (FOR OUTPUT),
MINIMUM GRADIENT AS DEFINED IN READ STAT., MINIMUM GRADIENT AS
DEFINED IN PROGRAM
FACT = (GRAM1) **(AN-1)
X= VALUE OF X AT WHICH THE PROGRAM IS NOW
XT= NEXT VALUE OF X AT WHICH TEMPERATURE PROFILE WILL BE CALCULATED
TBULK= BULK TEMPERATURE OF FLOW
NUS= NUSSELT NUMBER
PR= PRANDLT NUMBER
LCON= NUMBER OF TIME WE TRIED TO ACHIEVE CONVERGENCE
TOL= TOLERANCE BETWEEN OLD AND NEW VALUES (ITERATIVE TYPE MESH)
PAR= PORTION OF NEW VELOCITY TO ADD TO THE OLD ONE TO ACHIEVE CONVERGENCE
VARIABLES DESCRIBING THE MESH SIZE (MESH SIZES ARE VARYING AS WE PROCEED)
INCY(-)= VECTOR CORRESPONDING TO NUMBER OF GRID SPACE ACROSS FLOW
INC(-)= VECTOR CORRESPONDING TO GRID SPACE IN X-DIR
ULIX(-)= VECTOR CORRESPONDING TO LIMIT UP TO WHICH THE PRECEDING
X-DIR GRID SPACE IS VALID
NOTE= THE INDICES FROM 1 TO 10 ARE FOR MOMENTUM MESH SIZE AND 1 TO 20
ARE FOR THAT TRANSFER MESH SIZE
NDX,NDXT= VARIABLES CORRESPONDING TO INDICES OF PREVIOUS VARIABLES
TO KEEP TRACK OF THE NEXT MESH SIZE (FOR MOMENTUM MESH AND
HEAT TRANSFER MESH)
CONSTANTS RELATED MOMENTUM MESH
A(-,-)= MATRIX OF COEFFICIENTS FOR MOMENTUM MESH
CP,ABSCP,CPA,ABSCP1,CPANEW,CM,ABSCM,CMA,ABSCMN,CMANEW= CONSTANT USE
IN THE SETTING OF THE COEFFICIENT MATRIX (MOM. MESH)
NMAX1= NUMBER OF SPACE (DY) IN Y-DIR
NMAX= NUMBER OF POINT IN Y-DIR (NMAX = NMAX1 + 1)
DY= GRID SIZE IN Y-DIR
DX= GRID SIZE IN X-DIR
NMAX4,DDY2,DDY2,DX1,DX2X,D2Y= SERIES OF CONSTANTS DEFINED IN SUBROU.
CONST CALCULATED FROM 4 PREVIOUS
CONSTANTS RELATED TO HEAT TRANSFER MESH
A1(-,-)= MATRIX OF COEFFICIENTS FOR HEAT TRANSFER MESH
NMAX1T,DXT,DXT,NMAXT= SIMILAR AS MOMENTUM MESH COUNTERPART BUT FOR
HEAT TRANSFER
NMAX4T,PRDY= DEFINED IN CONST FROM 4 PREVIOUS
VOLD0(-),VOLD0(-),TOLD0(-),UUMAX0(-'),LCON0,POLD0,TBULKO,XOLD= ALL
THOSE VARIABLES ARE THERE JUST FOR 'OUTPUTING' CONVENIENCE
**Variables and Constants Definition**

- **YD**: Used in the output to situate a point in Y-dir (at centerline YD=0.0, at wall YD=1.0).
- **LL**: Flag to output every second time.
- **NMAXO**: NMAXO=0 stands for old values otherwise same as before.
- **U98**: 98% of last UCL (fully developed).
- **NPOS**: Counter for insertion of velocity profile.
- **LN**: Counter for different fluids.
- **L1, L2**: Counter for pressure drop, Nusselt number.
- **VX(-)**: Values of Y (-1.0 to +1.0).
- **VY(-)**: Velocity in X-dir for those previous VX(-).
- **XPRESS(-)**: Pressure drop at those X.
- **XNUS(-)**: Values of X (0.0 to 2.0).
- **FU(-)**: Flow behavior index (similar to AN).
- **EL(-)**: Entry length for those previous AN.

**Common Variables**

- **NMAX1**: NMAX1,PNEW(5,165),UNEW(165),VNEW(165),TNEW(165),UOLD(165),
  VOLD(165),TOLD(165),VOLDO(165),TOLDO(165),UU(165)
- **1**: UUMAXO(165),UN(165),V(165),VO(165),VA(165),GR(165)
- **DIMENSION**: VY(21,3),VX(21),PRESS(100,5),XPRESS(100),UCL(100),EL(5)
- **1**: FBI(5),NU(100,5),XNU(100),TBU(100,5),UVEL(70,5),VECTOR(11)
- **REAL**: NU

**Parameters Reading**

- **READ IN ALL THE GRID SIZE**
  - **N** = 0
  - **N** = **N** + 1
  - **READ**: 5,1105) INCY(N),XINC(N),ULIX(N)
  - **WRITE**: 6,1105) INCY(N),XINC(N),ULIX(N)
  - **IF**: (INCY(N) NE. 0) GO TO 7
  - **IF** (N GT. 10) GO TO 3
  - **N** = 10
  - GO TO 7

**Parameter Reading**

- **READ THE PARAMETERS**
  - **READ**: 5,1109) MAX,(FBI(N),N=1,MAX)
  - **1109 FORMAT**: 15,10F5.2
  - **4 READ**: 5,1100) PR,GRAMIN,TOL
  - **LN** = 0
  - **IF**: (PR EQ. 0.0) STOP
  - **5 IF**: (LN GE. MAX) GO TO 850
  - **LN** = **LN** + 1
  - **AN** = FBI(LN)

**Minimum Gradient Definition**

- **GRAMI** = GRAMIN
- **FACT** = (GRAMI)**(AN-1.)

**Output Parameters**

- **WRITE**: 6,1110) PR,GRAMIN,FACT,TOL

**Initialisation**

- **TGRAO** = 0.0
- **SUM** = 0.0
- **X** = 0.0
- **NPOS** = 1
- **L1** = 1
- **L2** = 1
- **GRA(1)** = GRAMIN
- **LL** = 0
TBULK = 0.0
NDX = 1
POLD = 0.0
XT = X + XINC(11)

**COMPUTE ALL THE CONSTANTS NEEDED**

NDXT = 11
CALL CONST(INCY(NDX), XINC(NDX), INCY(NDXT), XINC(NDXT), NMAX1, NMAX,
10X, DY, NMAX4, DDY2, DY2, DX1, DY2X, D2Y, NMAX1T, NMAXT, NMAX4T, DYT, DXT, PK,
2PRDY)

DO 102 N=1,NMAXT
UO(N) = 1.0
VO(N) = 0.0
TOLD(N) = 1.0
TNEW(N) = 0.0
102 CONTINUE
UO(NMAXT) = 0.0
TOLD(NMAXT) = 0.0
DO 1 N=1,NMAX
UOLD(N) = 1.0
VOLD(N) = 0.0
1 CONTINUE

**OUTPUT THE STARTING VALUES**

WRITE(6,1001) AN, X, X, POLD, POLD

DO 2 N=1,NMAX, NMAX4
WRITE(6,1002) N, UOLD(N), VOLD(N)
2 CONTINUE

**INTEGRATION ITERATION BEGINS**

11 X = X + DX

**SET THE COEFFICIENT MATRIX**

DO 10 N=1,NMAX1
CP = (UOLD(N+1) - UOLD(N))/DY
ABSCP = ABS(CP)
IF(ABSCP .LT. GRAMI) ABSCP = GRAMI
GRA(N) = ABSCP
CPA = ((2.0*ABSCP)**(AN-1.0))/(DY**2)
IF(N .NE. 1) GO TO 12
CM = -CP
CMA = CPA
A(5,N) = -(CPA*CP - CMA*CM)/DY - POLD/DX - UOLD(1)*UOLD(1)/DX
GO TO 13
12 CONTINUE
CM = (UOLD(N) - UOLD(N-1))/DY
ABSCM = ABS(CM)
IF(ABSCM .LT. GRAMI) ABSCM = GRAMI
CMA = ((2.0*ABSCM)**(AN-1.0))/(DY**2)
A(5,N) = -(CPA*CP - CMA*CM)/DY - POLD/DX + VOLD(N)*(UOLD(N+1) - UOLD(N-1)) + 0.5/D2Y - (UOLD(N)*UOLD(N))/DX
GO TO 13
13 CONTINUE
A(1,N) = +CMA + VOLD(N)/ (4.0 * DY)
A(2,N) = -CMA - CPA - (UOLD(N)/DX)
A(3,N) = +CPA - VOLD(N)/ (4.0*DY)
A(4,N) = -DX
10 CONTINUE
A(3,1) = A(3,1) + A(1,1)

**SOLVE THE MATRIX (GAUSS ELIMINATION ALGORITHM)**

CALL GAUSS
UNEW(NMAX) = 0.0
C...COMPUTE VELOCITIES IN Y-DIR
VNEW(1) = 0.0
DO 30 J=2,NMAX1
  30 VNEW(J) = VNEW(J-1) - DY2X*(UNEW(J)+UNEW(J-1)-UOLD(J)-UOLD(J-1))
VNEW(NMAX) = 0.0
C...HEAT TRANSFER PART
C...SEE HEAT TRANSFER COLUMN SHOULD BE DONE
  IF(X .LT. 0.999*XT) GO TO 130
C...SET UP VELOCITY FIELD FOR ENERGY EQUATION
  NTN = NMAX/T/NMAX1
  NN = 0
  DO 111 N=1,NMAX1
  DO 111 L=L/NTN
  NN = NN+1
  VN(NN) = VNEW(N) - (VNEW(N)-VNEW(N+1))*FLOAT(L-1)/FLOAT(NTN)
  111 UN(NN) = UNEW(N) - (UNEW(N)-UNEW(N+1))*FLOAT(L-1)/FLOAT(NTN)
VNEW(NMAX) = 0.0
C...SET THE COEFFICIENT MATRIX (ENERGY EQUATION)
  DO 125 N=1,NMAX1T
  VCO = (VO(N)+VN(N))/8.*DYT
  A1(1,N) = -PRDY -VCO
  A1(2,N) = (UO(N)+UN(N))/DX +2.*PRUY
  A1(3,N) = -PRDY +VCO
  IF(N .EQ. 1) GO TO 121
  TNEW(N) = (UO(N)+UN(N))*TOLD(N)/DX
  1 -VCO*(TOLD(N+1)-TOLD(N-1))
  2 +(TOLD(N+1) - 2.*TOLD(N) +TOLD(N-1))*PRDY
  GO TO 120
  121 TNEW(N) = (UO(1)+UN(1))*TOLD(1)/DX +2.*(TOLD(2)-TOLD(1))*PRDY
  120 CONTINUE
  125 CONTINUE
A1(3,1) = A1(3,1) + A1(1,1)
C...SOLVE MATRIX
  CALL DIAG3(A1,TNEW,NMAX1T)
  TNEW(NMAXT) = 0.0
C...COMPUTE BULK TEMPERATURE AND NUSSELT NUMBER
  TBUKL = (UN(1)*TNEW(1) + TNEW(NMAXT)*UN(NMAXT))/2.
  DO 122 N=1,NMAXT
  TNEW(N) = TNEW(N)+UN(N)*TNEW(N)
  122 TNEWL = TBUKL + (UN(N)*TNEW(N))
  TBUKL = TBUKL/FLOAT(NMAX1T)
  TGRA = (-11.0*TNEW(NMAXT)+18.0*TNEW(NMAXT-1)-9.0*TNEW(NMAXT-2)
  +2.0*TNEW(NMAXT-3))/6.*DYT
  NUS = (2.0*TGRA)/(TBULK)
  SUM = SUM+ (.25*DXT*(TGRA+TGRAO))
  10000 CONTINUE
  AVNUS = SUM/X
  TGRAO = TGRA
  IF(X .LT. 0.01 .OR. L2 .GT. 100) GO TO 9998
  TBUKL = TBULK
  NU(L2,LN) = NUS
  XNU(L2) = X
  L2 = L2 +1
  9998 CONTINUE
  DO 123 N=1,NMAXT
  UO(N) = UN(N)
  VO(N) = VN(N)
123 TOLD(N) = TNEW(N)
C...SET XT FOR NEXT TIME
   XT = X + XINC(NDXT)
   GO TO 39
130 IF(X * GT* 0.001) GO TO 45
   DO 131 N=1,NMAXT
131 TNEW(N) = 0.0
   TBULK = 0.0
C
C...OUTPUT THE RESULTS (PRINT OUT FORM)
39 LL = LL +1
   ANTN = FLOAT(NMAX1T)/FLOAT(NMAX1)
   NTN = NMAX1T/NMAX1
   IF(LL * NE* 2) GO TO 44
   LL = 0
   WRITE(6,1001) AN,AVNUS,XOLD,POLD,X,PNEW,TBULK,NUS
   MM = 1
   IF(NMAX * NE* NMAX0) MM = 2
   NMAX4D = NMAX4*2
   DO 40 N=1,NMAX,NMAX4D
      M = MM*(N-1) +1
   NN = IFIX(ANTN*(FLOAT(N)-1.)) +1
   NNN = IFIX(ANTN*(FLOAT(M)-1.)) +1
   YD = FLOAT(N-1)/FLOAT(NMAX1)
   UUMAXN(N) = UNEW(N)/UNEW(1)
   WRITE(6,1002)N,YD,L,JOLD0(M),LJUMAXO(M),VOLDO(M),TOLD(NNN),
   1 UNEW(N),UUMAXN(N),VNEW(N),TNEW(NN),NN,GRA(N)
40 CONTINUE
   GO TO 45
C...PUT RESULT IN RESERVE BECAUSE OF NON PUBLICATION
44 DO 53 N=1,NMAX,NMAX4
   NN = NTN*(N-1) +1
   UOLDO(N) = UNEW(N)
   VOLDO(N) = VNEW(N)
   UUMAXO(N) = UOLDO(N)/UOLDO(1)
53 TOLDO(N) = TOLD(NN)
   XOLD = X
   TBULKO = TBULK
   POLDO = PNEW
C
C...PART TO SET VALUES FOR PLOTTING SUBROUTINE
45 GO TO (801,802,803),NPOS
803 IF(X * LT* 1.0) GO TO 701
   GO TO 702
802 IF(X * LT* 0.05) GO TO 701
   GO TO 702
801 IF(X * LT* 0.002) GO TO 46
C
C...PREPARE VELOCITY FIELD
702 L = 0
    DO 700 N=1,NMAX,NMAX4
       L = L+1
       LI = 22-L
       NN = NMAX+1-N
       VY(L,NPOS) = UNEW(NN)
       VY(LI,NPOS) = UNEW(NN)
       VX(L) = (FLOAT(L-1)/10.0)-1.0
700 VX(LI) = FLOAT(LI-1)/10.0
NPOS = NPOS + 1

C...PREPARE PRESSURE DROPS
701 IF(L1 .GT. 100) GO TO 710
PRESS(L1,LN) = -PNEW
XPRESS(L1) = X
UCL(L1) = UNEW(L1)
IF(L1 .GT. 70) GO TO 709
J = 0
DO 705 N=1,NMAX,NMAX4
J = J+1
705 VECTOR(J) = UNEW(N)
UVEL(L1,1) = VECTOR(1)
UVEL(L1,2) = VECTOR(4)
UVEL(L1,3) = VECTOR(6)
UVEL(L1,4) = VECTOR(8)
UVEL(L1,5) = VECTOR(10)

709 L1 = L1 + 1
710 CONTINUE
IF(NPOS .NE. 4) GO TO 46
NPOS = NPOS - 1
DO 900 N=1,21
900 WRITE(6,901)VX(N),VY(N,1),VY(N,2),VY(N,3)
901 FORMAT(12F11.5)

C...COMPUTING ENTRY LENGTH VS FLOW BEHAVIOR INDEX
U98 = 0.98*UNEW(1)
L = 0
220 L = L+1
IF(UCL(L) .LT. U98) GO TO 220
L = L-1
EL(LN) = XPRESS(L) + ((XPRESS(L+1)-XPRESS(L))/UCL(L+1)-UCL(L))
1*(U98-UCL(L))
CALL DESSIN(1,VX,VY,XPRESS,PRESS,UVEL,XNU,NU,TBU,FBI,EL,LN)
GO TO 5

C...TO CHANGE GRID SPACING IN X-DIR (IF NECESSARY)
46 NMAXTO = NMAXT
NMAXO = NMAX
IF(XT .LT. (1.01*ULIX(NDXT))) GO TO 49
IF(X .LT. (0.99*ULIX(NDX))) GO TO 48
NDX = NDXT + 1
48 NDXT = NDXT + 1
XT = X +XINC(NDXT)
47 CALL CONST(INCY(NDX),XINC(NDX),INCY(NDXT),XINC(NDXT),NMAX1,NMAX,
1DX,DY,NMAX4,DDY2,DY2,DX1,DY2X,D2Y,NMAX4T,NMAXT,NMAX4T,DYT,DXT,PK,
2PRDY)
GO TO 60
49 IF(X .LT. (0.99*ULIX(NDX))) GO TO 51
NDX = NDXT + 1
GO TO 47

60 IF(NMAXT .EQ. NMAXTO) GO TO 61

C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / MOMENTUM EQUATION
DO 129 N=1,NMAXT
NN = 2*N - 1
UO(N) = UO(NN)
VO(N) = VO(NN)
TNEW(N) = TNEW(NN)
129 TOLD(N) = TOLD(NN)
61 IF(NMAX .EQ. NMAXO) GO TO 51
C... TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / ENERGY EQUATION
DO 22 N=1,NMAX
   NN = 2*N-1
   UNEW(N) = UNEW(NN)
   VNEW(N) = VNEW(NN)
22 CONTINUE
C... ONE COLUMN COMPLETE, PREPARE FOR NEXT COLUMN
51 POLD = PNEW
   DO 52 N=1,NMAX
      UOLD(N) = UNEW(N)
52 VOLD(N) = VNEW(N)
   GO TO 11
C... PLOT AND OUTPUT COMPARABLE VALUES (PRESSURE, NUSSELT) OF ALL FLUIDS
850 CONTINUE
   CALL DESSIN(3,VX,VY,XPRESS,PRESS,UVEL,XNU,NU,TBU,FBI,EL,LN)
   GO TO 4
997 DO 998 N=LN,MAX
998 FBI(N) = FBI(N+1)
   GO TO 10000
   LN = LN - 1
   MAX = MAX - 1
   GO TO 5
1001 FORMAT(///,8X,* N =*F4.2,80X*AVNUS=*F9.4/3X,*X=*F7.5,10X*PRESS=*F7.4,4X*X*UVEL=*F7.5,
1   //4X*N*,4X*X*Y/D*,6X*U(N)*,9X*U/UMAX*,7X*V(N)*,9X*T(L)*,T(L)**F8.5)
1002 FORMAT (1X,14,3X,F5.3,14F13.6,4X,4F13.6,14,F6.3)
1100 FORMAT(3F10.4)
1105 FORMAT(15,F10.6)
1110 FORMAT(2X,*PRANDTL NUMBER=* F12.4/2X,*MINIMUM GRADIENT=*F6.3,12X
   1*(MIN. GRAD.)*EXP(N-1)/*F10.4/2X,*TOLERANCE FOR CONVERGING TYPE M/*F8.5)
END
PROGRAM TST (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, TAPE10)
THIs IS A FINITE DIFFERENCE METHOD TO SOLVE THE MOTION
EQUATIONS FOR THE ENTRANCE REGION OF A CHANNEL
THIS METHOD IS USING A SERIES OF VALUES KNOWN AS THE OLD VALUES TO
COMPUTE ANOTHER SET OF VALUES KNOWN AS THE NEW VALUES CORRESPONDING
TO THE NEXT COLUMN (AXIALLY) SO THAT FROM THE ENTRY SET OF VALUES W
CAN GO DOWNSTREAM AS FAR AS WE WANT PROVIDING WE HAVE THE BOUNDARY
CONDITIONS
LIST OF VARIABLES
AN= EXPONENT OF THE POWER LAW FLUID (FLOW BEHAVIOR INDEX)
UOLD(-), VOLD(-), TOLD(-), POLD= VECTORS OF VELOCITY (X-DIR), VELOCITY
(Y-DIR), TEMPERATURE AND PRESSURE ALREADY KNOWN
UNEW(-), VNEW(-), TNEW(-), PNEW= VECTORS OF VELOCITY (X-DIR), VELOCITY
(Y-DIR), TEMPERATURE AND PRESSURE TO BE COMPUTED (NEW)
UO(-), VO(-), UN(-), VN(-)= VECTORS OF VELOCITY ATTACHED TO HEAT TRANSFER
GRID
UCON(-), VCON(-)= VECTORS OF VELOCITY USED IN ITERATIVE TYPE MESH
TO CHECK THE CONVERGENCE
UUMAXN(-)= VECTOR OF VELOCITY DIVIDED BY MAXIMUM VELOCITY (CENTERLINE)
(OUTPUT PURPOSE)
GRA(-), GRAMIN, GRAMI= GRADIENTS - VECTOR OF GRADIENTS FOR OUTPUT),
MINIMUM GRADIENT AS DEFINE IN READ STAT., MINIMUM GRADIENT AS
DEFINE IN PROGRAM
FACT = (GRAMI) **(AN-1)
X= VALUE OF X AT WHICH THE PROGRAM IS NOW
XT= NEXT VALUE OF X AT WHICH TEMPERATURE PROFILE WILL BE CALCULATED
TBULK= BULK TEMPERATURE OF FLOW
NUS= NUSSELT NUMBER
PR= PRANDLT NUMBER
LCON= NUMBER OF TIME WE TRIED TO ACHIEVE CONVERGENCE
TOL= TOLERANCE BETWEEN OLD AND NEW VALUES (ITERATIVE TYPE MESH)
PAR= PORTION OF NEW VELOCITY TO ADD TO THE OLD ONE TO ACHIEVE CONVERGENCE
VARIABLES DESCRIBING THE MESH SIZE (MESH SIZES ARE VARYING AS WE PROCEED)
INCY(-)= VECTOR CORRESPONDING TO NUMBER OF GRID SPACE ACROSS FLOW
XINC(-)= VECTOR CORRESPONDING TO GRID SPACE IN X-DIR
ULIX(-)= VECTOR CORRESPONDING TO LIMIT UP TO WHICH THE PRECEEDING
X-DIR GRID SPACE IS VALID
NOTE= THE INDICES FROM 1 TO 10 ARE FOR MOMENTUM MESH SIZE AND 1 TO 20
ARE FOR THAT TRANSFER MESH SIZE
NDX, NDX2= VARIABLES CORRESPONDING TO INDICES OF PREVIOUS VARIABLES
TO KEEP TRACK OF THE NEXT MESH SIZE (FOR MOMENTUM MESH AND
HEAT TRANSFER MESH)
CONSTANTS RELATED MOMENTUM MESH
A1(-,-)= MATRIX OF COEFFICIENTS FOR MOMENTUM MESH
CP, ABSCP, CPA, ABSPPN, CPANEW, CMS, ABDM, CMS, CMAP, ABDMG, CPANEW= CONSTANT USE
IN THE SETTING OF THE COEFFICIENT MATRIX (MOM. MESH)
NMAX1= NUMBER OF SPACE (DY) IN Y-DIR
NMAX= NUMBER OF POINT IN Y-DIR (NMAX = NMAX1 + 1)
DY= GRID SIZE IN Y-DIR
DX= GRID SIZE IN X-DIR
NMAX4, DXY, DX1, DY2, XDY, D2Y= SERIES OF CONSTANTS DEFINED IN SUBROU.
CONST CALUCULATED FROM 4 PREVIOUS
CONSTANTS RELATED TO HEAT TRANSFER MESH
A1(-,-)= MATRIX OF COEFFICIENTS FOR HEAT TRANSFER MESH
NMAX1=DXT, DXT, NMAXT= SIMILAR AS MOMENTUM MESH COUNTERPART BUT FOR
HEAT TRANSFER
NMAX4, PRDY= DEFINED IN CONST FROM 4 PREVIOUS
VOLD(-), VOLD1(-), TOLD(-), UUMAXO(-), LCONO, POLDO, TBULK0, XOLD= ALL
THOSE VARIABLES ARE THERE JUST FOR 'OUTPUTING' CONVENIENCE
c yd = used in the output to situate a point in y-dir (at centerline yd=0.0, at wall yd=1.0)
c ll = flag to output every second time
nmax0,nmax0=0 stands for old values otherwise same as before
constants and variables related to plotting subroutines
c ucl(-) = velocity (x-dir) at center line
c u98 = 98 per cent of last ucl (fully developed)
np = counter for insertion of velocity profile
c ln = counter for different fluids
c l1, l2 = counter for pressure drop, nusselt number
vx(-) = values of y (-1.0 to +1.0)
c vy(-,-) = velocity in x-dir for those previous vx(-)
c xpress(-) = values of x (0.0 to 1.0)
c press(-,-) = pressure drop at those x
xnus(-) = values of x (0.0 to 2.0)
c nu(-,-) = nusselt number at those x
fbi(-) = flow behavior index (similar to an)
c el(-) = entry length for those previous an
common nmax0,pnew1,a(5,165),unew(165),vnew(165),tnew(165),vold(165)
1,vold(165),told(165),vold0(165),vold0(165),told0(165),umaxn(165)
2,umax0(165),un(165),vnew(165),u0(165),vo(165),a1(3,165),gra(165)
3,uncon(165),vcon(165)
dimension vy(21,3),vx(21),press(100,5),xpress(100),ucl(100),el(5)
1,fbi(5),nu(100,5),xnu(100),tbu(100,5),uvel(70,5),vector(11)
dimension incy(20),ulix(20),xinc(20)
real nus,nu
**read in all the grid size
n = 0
7 n = n +1
read(5,1105) incy(n),xinc(n),ulix(n)
write(6,1105) incy(n),xinc(n),ulix(n)
if (incy(n) .ne. 0) go to 7
if (n .gt. 10) go to 3
n = 10
go to 7
**read the parameters
3 read(5,1109) max,(fbi(n),n=1,max)
1109 format(15,10f5.2)
4 read(5,1100) pr,gramin,tol
ln = 0
if(pr .eq. 0.0) stop
5 if(ln .ge. 850) go to 850
ln = ln +1
an = fbi(ln)
**define minimum gradient
gram = gramin
fact = (gramin)**(an-1.)
**output the parameters
write(6,1110) pr,gramin,fact,tol
**initialisation of counters and others
x = 0.0
np = 1
l1 = 1
l2 = 1
gra(1) = gramin
ll = 0
 tbulk = 0.0
NDX = 1
POLD = 0.0
XT = X +XINC(11)

C...COMPUTE ALL THE CONSTANTS NEEDED
NDXT = 11
CALL CONST(ICY(NDX),XINC(NDX'),ICY(NDXT),XINC(NDXT'),NMAX1,NMAX,
IDX,DY,NMAX4,DDY2,dy2,DX1,dy2X,D2Y,NMAX1T,NMAXT,NMAX4T,DYT,DXT,PK,
2PRDY)

C...PUT IN THE STARTING VELOCITIES AND TEMPERATURE PROFILE
DO 102 N=1,NMAXT
UO(N) = 1.0
V0(N) = 0.0
TOLD(N) = 1.
TNEW(N) = 0.0
102 CONTINUE
DO 1 N=1,NMAX
UOLD(N) = 1.0
V0LD(N) = 0.0
1 CONTINUE

C...OUTPUT THE STARTING VALUES
WRITE(6,1001)AN,X,X,POLD,POLD
DO 2 N=1,NMAX,NMAX4
2 WRITE(6,1002)N,UOLD(N),V0LD(N)

C...INTEGRATION ITERATION BEGINS
11 X = X +DX

C...SET KNOWN VALUES WITH OLD VALUES
DO 100 N=1,NMAX
UCON(N) = UOLD(N)
100 CONTINUE

C...SET THE COEFFICIENT MATRIX
DO 10 N=1,NMAX1
UNEW(N) = (UCON(N)+UOLD(N))/2.
VNEW(N) = (VCON(N)+V0LD(N))/2.
CP = (UOLD(N+1)-UOLD(N))/DY
ABSCP = ABS(CP)
IF(ABSCP .LT. GRAMI) ABSCP = GRAMI
GRA(N) = ABSCP
CPA = ((2.*ABSCP)**(AN-1.))/(DY**2)
ABSCPN = ABS((UCON(N+1)-UCON(N))/DY)
IF(ABSCPN .LT. GRAMI) ABSCPN = GRAMI
CPANEW = ((2.*ABSCPN)**(AN-1.))/(DY**2)
IF(N .NE. 1)GO TO 12
CM = -CP
CMA = CPA
CMANEW = CPANEW
A(5,1) = (CPA*CP-CMA*CM)*DY
-POLD/DX-UOLD(I)*UOLD(I)/DX
GO TO 13
12 CONTINUE
CM = (UOLD(N)-UOLD(N-1))/DY
ABSCM = ABS(CM)
IF(ABSCM .LT. GRAMI) ABSCM = GRAMI
CMA = ((2.*ABSCM)**(AN-1.))/(DY**2)
ABSCMN = ABS((UCON(N)-UCON(N-1))/DY)
IF(ABSCMN .LT. GRAMI) ABSCMN = GRAMI
CMANEW = ((2.*ABSCMN)**(AN-1.))/(DY**2)
\[ A(5,N) = -\left( C_P A \cdot C_M A \cdot C_M \right) D \] 
\[ -P_{\text{old}} \frac{D}{\text{y}} \] 
\[ 1 - \frac{V_{\text{new}}(N) \cdot (U_{\text{old}}(N+1) - U_{\text{old}}(N-1))}{2 \cdot D_2 D_y} - \frac{U_{\text{old}}(N) \cdot (U_{\text{old}}(N) / D_x)}{2} \] 

13 CONTINUE

\[ A(1,N) = \frac{V_{\text{new}}(N)}{2 \cdot D_2 D_y} + C_{\text{manew}} \] 
\[ A(2,N) = -\frac{U_{\text{new}}(N)}{D_x} - C_{\text{manew}} - C_{\text{pnew}} \] 
\[ A(3,N) = -\frac{V_{\text{new}}(N)}{2 \cdot D_2 D_y} + C_{\text{pnew}} \] 
\[ A(4,N) = -D_x \] 

10 CONTINUE

\[ A(3,1) = A(3,1) + A(1,1) \] 

C. SOLVE THE MATRIX (GAUSS ELIMINATION ALGORITHM)

CALL GAUSS

UNEW(NMAX) = 0.0

C. COMPUTE VELOCITIES IN Y-DIR

VNEW(1) = 0.0

DO 30 J = 2, NMAX1

30 VNEW(J) = \frac{V_{\text{new}}(J-1) - D Y_2 \cdot (U_{\text{new}}(J) + U_{\text{new}}(J-1) - U_{\text{old}}(J) - U_{\text{old}}(J-1))}{V_{\text{new}}(NMAX)} = 0.0

LCON = LCON + 1

C. COMPARE OBTAINED VALUES WITH OLD VALUES

N = 0

IF (LCON.GT. 25) GO TO 112

110 N = N + 1

IF (ABS(UNEW(N) - UCON(N)).GT. (TOL*UNEW(N))) GO TO 105

IF (N.LT. NMAX) GO TO 110

C. HEAT TRANSFER PART

C. SEE HEAT TRANSFER COLUMN SHOULD BE DONE

112 IF (X.LT. (0.999*XT)) GO TO 130

NN = 0

C. SET UP VELOCITY FIELD FOR ENERGY EQUATION

NTN = NMAX1/NMAX1T

DO 111 N = 1, NMAX1, NTN

111 VN(NN) = VNEW(N)

UN(NMAX1T) = 0.0

VN(NMAX1T) = 0.0

C. SET THE COEFFICIENT MATRIX (ENERGY EQUATION)

DO 125 N = 1, NMAX1T

121 TNEW(N) = (UO(N) + UN(N)) / DXT + 2 \cdot PRDY

A(3,1) = A(3,1) + A(1,1)

C. SOLVE MATRIX

CALL DIAG3CA1, TNEW, NMAX1T

TNEW(NMAX1T) = 0.0

C. COMPUTE BULK TEMPERATURE AND NUSSELT NUMBER

TBULK = \frac{T_{\text{new}}(1) \cdot U(1) + T_{\text{new}}(NMAX1T) \cdot U(NMAX1T)}{2}

DO 122 N = 2, NMAX1T

122 TBULK = TBULK + CUN(N) \cdot TNEW(N)
TBULK = TBULK/FLAT(NMAX1T)
NUS = (-11.*TNEW(NMAXT)+18.*TNEW(NMAXT-1)-9.*TNEW(NMAXT-2)
+2.*TNEW(NMAXT-3))/(3.*DYT*TBULK)
IF(X .LT. 0.01 .OR. L2 .GT. 100) GO TO 9998
TB(L2,LN) = TBULK
NU(L2,LN) = NUS
XNU(L2) = X
L2 = L2 +1
9998 CONTINUE
DO 123 N=1,NMAXT
UO(N) = UN(N)
VO(N) = VN(N)
123 TOLD(N) = TNEW(N)
C...SET XT FOR NEXT TIME
XT = X +XINC(NDXT)
GO TO 39
130 IF(X .GT. 0.001) GO TO 45
DO 131 N=1,NMAXT
131 TNEW(N) = 0.0
TBULK = 0.0
C...OUTPUT THE RESULTS (PRINT OUT FORM)
39 LL = LL +1
ANTN = FLOAT(NMAX1)/FLOAT(NMAX1T)
NTN = NMAX1T/NMAX1
IF(LL .NE. 2) GO TO 44
LL = 0
WRITE(6,1001) AN,LCONO,LCON,XOLD,POLD,TBULK,X,PNEW,TBULK,NUS
MM = 1
IF(NMAX .NE. NMAXO) MM = 2
NMAX4D = NMAX4*2
DO 40 N=1,NMAX,NMAX4
M = MM*(N-1) +1
NN = IFIX(ANTN*(FLOAT(N)-1.)+1
NNN = IFIX(ANTN*(FLOAT(M)-1.)+1
YD = FLOAT(N-1)/FLOAT(NMAX1)
UUMAXN(N) = UNEW(N)/UNEW(1)
WRITE(6,1002) N,YD,ULDO(N),UUAMXO(M),VLODO(M),TLD(NNN),
1 UNEW(N),UUMAX(N),VNEW(N),TNEW(NN),GRA(N)
40 CONTINUE
GO TO 45
C...PUT RESULT IN RESERVE BECAUSE OF NON PUBLICATION
44 CONTINUE
DO 53 N=1,NMAX,NMAX4
NN = NTN*(N-1) +1
UO(N) = UNEW(N)
VLODO(N) = VNEW(N)
UUMAXO(N) = UOLODO(N)/UOLODO(1)
53 TLODO(NN) = TNEW(NN)
XOLD = X
LCONO = LCON
TBULK = TBULK
POLD = PNEW
C...PART TO SET VALUES FOR PLOTTING SUBROUTINE
45 GO TO (801,802,803),NPOS
803 IF(X .LT. 1.0) GO TO 701
GO TO 702
802 IF(X LT 0.05) GO TO 701
GO TO 702
801 IF(X LT 0.002) GO TO 46
C...PREPARE VELOCITY FIELD
702 L = 0
DO 700 N=1,NMAX,NMAX4
L = L+1
LI = 22-L
NN = NMAX+1-N
VY(L,NPOS) = UNEW(N)
VY(LI,NPOS) = UNEW(N)
VX(L) = (FLOAT(L-1)/10.0)-1.0
700 VX(LI) = FLOAT(LI-1)/10.0
NPOS = NPOS +1
C...PREPARE PRESSURE DROPS
701 IF(LI GT 100) GO TO 710
PRESS(LI,LN) = PNEW
XPRESS(L1) = X
UCL(LI) = UNEW(I)
IF(LI GT 70) GO TO 709
J = 0
DO 705 N=1,NMAX,NMAX4
J = J+1
705 VECTOR(J) = UNEW(N)
UVEL(LI,1) = VECTOR(1)
UVEL(LI,2) = VECTOR(4)
UVEL(LI,3) = VECTOR(6)
UVEL(LI,4) = VECTOR(8)
UVEL(LI,5) = VECTOR(10)
709 L1 = L1 +1
710 CONTINUE
IF(NPOS NE 4) GO TO 46
NPOS = NPOS -1
DO 900 N=1,21
900 WRITE(6,901)VX(N),VY(N,1),VY(N,2),VY(N,3)
DO 906 N=1,70
906 WRITE(6,901)XPRESS(N),UVEL(N,J),J=1,5
901 FORMAT(12F11.5)
C...COMPUTING ENTRY LENGTH VS FLOW BEHAVIOR INDEX
U98 = 0.98*UNEW(I)
L=0
220 L = L+1
IF(UCL(L) LT U98) GO TO 220
L = L-1
EL(LN) = XPRESS(L) +((XPRESS(L+1)-XPRESS(L))/(UCL(L+1)-UCL(L)))*
1 *(U98-UCL(L))
FBII(LN) = AN
GO TO 5
C...TO CHANGE GRID SPACING IN X-DIR (IF NECESSARY)
46 NMAXTO = NMAX
NMAXO = NMAX
IF(XT LT (1.01*ULIX(NDX))) GO TO 49
IF (X LT (0.99*ULIX(NDX)) ) GO TO 48
NDX = NDX +1
48 NDXT = NDXT +1
XT = X +XINC(NDXT)
47 CALL CONST(INCY(NDX),XINC(NDX),INCY(NDXT),XINC(NDXT),NMAX1,NMAX, 
1DX,DY,NMAX4,DDY2,DY2,DXI,DY2X,D2Y,NMAX1T,NMAX1T,NMAX4T,DYT,DXT,PK, 
2PRDY) 
GO TO 60 
49 IF (X .LT. (0.99*ULIX(NDX)) ) GO TO 51 
NDX = NDX +1 
GO TO 47 
60 IF(NMAXX .EQ. NMAXTO) GO TO 61 
C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / MOMENTUM EQUATION 
DO 129 N=1,NMAX 
NN = 2*N -1 
UO(N) = UO(NN) 
VO(N) = VO(NN) 
TNEW(N) = TNEW(NN) 
129 TOLD(N) = TOLD(NN) 
61 IF(NMAX .EQ. NMAXO)GO TO 51 
C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / ENERGY EQUATION 
DO 22 N=1,NMAX 
NN = 2*N-1 
UNEW(N) = UNEW(NN) 
VNEW(N) = VNEW(NN) 
22 CONTINUE 
C...ONE COLUMN COMPLETE, PREPARE FOR NEXT COLUMN 
51 POLD = PNEW 
DO 52 N=1,NMAX 
UOLD(N) = UNEW(N) 
52 VOLD(N) = VNEW(N) 
GO TO 11 
C...PLOT AND OUTPUT COMPARABLE VALUES (PRESSURE, NUSSELT) OF ALL FLUIDS 
850 CONTINUE 
CALL DESSIN(3,VX,VY,XPRESS,PRESS,UVEL,XNU,NU,TBUL,FBI,EL,GN) 
GO TO 4 
999 DO 998 N=LN,MAX 
998 FBI(N) = FBI(N+1) 
WRITE(6,1500)CON 
1500 FORMAT( * NUMBER OF ITERATIONS TOO HIGH =*,14) 
LN = LN -1 
MAX = MAX -1 
GO TO 5 
C...PART OF RESET OF ITERATIONS FOR CONVERGING TYPE MESH 
105 CONTINUE 
PAR = 0.85 
DO 103 N=1,NMAX 
UCON(N) = PAR*UNEW(N) + (1-PAR)*UCON(N) 
103 VCON(N) = PAR*VNEW(N) + (1-PAR)*VCON(N) 
IF(UCON(N) .LT. -0.01 .OR. UCON(N) .GT. 4.)GO TO 999 
GO TO 101 
1001 FORMAT (//,16X,* N =*,F4.2,12X*CON=*,I3,50X*CON=*,I3, 
1 /,3X,*X=*,F8.6,5X*PRES=*,F8.6*6X*TBUlk=*,F8.6,4X*PREa=*, 
1F8.4*4X *TBULk=*,F8.4,4X*NUS=*,F8.4, 
2 /,4X*N=*,4X*Y/D=*,6X*U(N)*,9X*U/UMAX*,7X*V(N)*,9X*T(L)*, 
3 13X*U(N)*,9X*U/UMAX*,7X*V(N)*,9X*T(L)*,7X*L*)) 
1002 FORMAT (1X*I4,3X*F5.3,4F13.6,4X*4F13.6,I4,F6.3) 
1100 FORMAT ( 3F10.5) 
1105 FORMAT (I5,2F10.6) 
1110 FORMAT(1HL1,2X,*PRANDTL NUMBER=*,F12.4,2X,*MINIMUM GRADIENT=*,F6.4 
1,12X*(MIN. GRAD.)*EXP(N-1)=*,F10.4,2X,*TOLERANCE FOR CONVERGING TYP
2E MESH=*F8.5)  
END

CD TOT 0403
SUBROUTINE CONST(INCY,XINC,INCYT,XINCT,NMAX1,NMAX,X,DY,NMAX4,DDY2,1,DY2,DXI,DY2X,D2Y,NMAX1T,NMAXT,NMAX4T,DYT,DXT,PR,PRDY)

C
C...SUBROUTINE COMPUTING CONSTANTS OF THE MAIN PROGRAM
C

DX = XINC
NMAX1 = INCY
DY = 1./FLOAT(NMAX1)
NMAX = NMAX1 +1
NMAX4 = NMAX1/10
DDY2 = 0.5/(DY*DY)
DY2 = 1./(DY*DY)
DXI = 1./DX
DY2X = DY/(2.*DX)
D2Y = 2.*DY
NMAX1T = INCYT
NMAXT = NMAX1T +1
NMAX4T = NMAX1T/10
DYT = 1.0/FLOAT(NMAX1T)
DXT = 2.0*XINCT
PRDY = 1.0/(PR*2.0*DYT*DYT)
PRDY = 2.*PRDY
RETURN
END.
SUBROUTINE GAUSS

C

C...SUBROUTINE TO SOLVE THE MOMENTUM EQUATION COEFFICIENT MATRIX
C...THIS IS DONE USING THE GAUSS ELIMINATION METHOD
C

COMMON NMAX1,PNEW,A(5,165),UNEW(165)
DIMENSION B(5,165)
NMAX = NMAX1 + 1
DO 1 I=1,5
DO 1 J=1,NMAX1
1 B(I,J) = A(I,J)
   J = 1
3 DO 2 I=2,5
2 B(I,J) = B(I,J)*(-A(1,J+1)/A(2,J))
   A(1,J+1) = A(1,J+1)+B(2,J)
   A(2,J+1) = A(2,J+1)+B(3,J)
   A(4,J+1) = A(4,J+1)+B(4,J)
   A(5,J+1) = A(5,J+1)+B(5,J)
   J = J+1
4 DO 4 I=2,5
4 B(I,J) = A(I,J)
   IF(J .LT. NMAX1)GO TO 3
   J = NMAX1
5 F = -A(3,J-1)/A(2,J)
   B(2,J) = B(2,J)*F
   B(4,J) = B(4,J)*F
   B(5,J) = B(5,J)*F
   A(3,J-1) = A(3,J-1) + B(2,J)
   A(4,J-1) = A(4,J-1) + B(4,J)
   A(5,J-1) = A(5,J-1) + B(5,J)
   J = J-1
   IF(J .GT. 1) GO TO 5
   A(1,1) = 1.0
   J = 0
13 J = J+2
   A(1,J) = 4.0
   A(1,J+1) = 2.0
   IF(NMAX1-J+1,11,12,13
12 A(1,J+1) = 0.0
   A(1,J+2) = FLOAT(3*NMAX1)
   GO TO 15
11 A(1,J) = 0.0
   A(1,J+1) = FLOAT(3*NMAX1)
15 DO 14 J=1,NMAX1
   F = -A(1,J)/A(2,J)
   A(1,J) = A(1,J) + A(2,J)*F
   A(1,NMAX) = A(1,NMAX) + A(4,J)*F
14 A(1,NMAX+1) = A(1,NMAX+1) + A(5,J)*F
   PNEW = A(1,NMAX+1)/A(1,NMAX)
   DO 20 J=1,NMAX1
20 UNEW(J) = (A(5,J)-A(4,J)*PNEW)/A(2,J)
RETURN
END
SUBROUTINE DESSIN(IFLAG,VX,VY,XPRES,PRES,UVEL,XNU,NU,TBU,FBI,EL,NF)
1)

C.... SUBROUTINE USING THE PLOTTING SUBROUTINES OF DOUG BUCHANAN
C
DIMENSION VY(21,NF),VX(21),PRES(100,NF),XPRES(100),X(70),
1EL(NF),FBI(NF),XNU(100),NU(100,NF),UVEL(70,5),TBU(100,NF)
REAL NU
C.... COMMON BLOCK
COMMON DUMMY(3),XX(5000)
COMMON /INFOPLT/LINEFX(30),LPTS(30)
COMMON /LABL/ LABEL,LETS(4)
COMMON /LBL/ YLVERT(7),YRVERT(7),XLHOR(7),XUHOR(7)
COMMON /TITLES/MNYPLT,TITLE1(2),TITLE2(2),TITLE3(2),TITLE4(2)
COMMON /Scales/ ISIZE
COMMON /BORDER/ XMIN,XMAX,YMIN,YMAX
C.... CONSTANTS
NREAD = 0
FMTE = 0
NPT = 1
NORD = 3
XSIZE = 15.0
YSIZE = 15.0
LABEL = 3
MNYPLT = 1
LINES = 2
C.... MAIN TITLES
TITLE1(1) = 1OH HPVL
TITLE1(2) = 1OH LAROCQUE
TITLE2(1) = 1OH ENTRY FLOW
TITLE2(2) = 1OH
TITLE3(1) = 1OH FOR POWER
TITLE3(2) = 1OH LAW FLUID
NT = 21
IPOINT = 0
IPOINT = 0
PLOT = 200
DO 10 N=1,10
LINEFX(N) = 3
10 LPTS(N) = N
GO TO (100,200,300),IFLAG
C.... PART FOR THE VELOCITY FIELD PLOT(U VS Y)
100 LETS(1) = 20
LETS(2) = 5
LETS(3) = 24
YLVERT(1) =1OH VELOCITY=
YLVERT(2) =1OH U(VENT)
XLHOR(1) = 5H Y/A
XUHOR(1) =1OHVELOCITY V
XUHOR(2) = 1OH S Y FOR N=
ENCODER(10,15,XUHOR(3),FBI(NF)
15 FORMAT(F4.2,6X)
ISIZE = 2
YMAX = 2.0
YMIN = 0.0
NF1 = 3
J6 = 1+NT*NF1+NT
CALL NPLOTS(NF1,NT,PLOT,IPOINT,NREAD,VY,VX,J6,LINES,1,FMTE,NPT,NORD,XSIZE,YSIZE)
RETURN
C...PART FOR THE VELOCITY FIELD PLOT(U VS X)
200 NF1 = 5
NT = 70
LETS(1) = 20
LETS(2) = 10
LETS(3) = 24
YLVERT(1) = 10H VELOCITY=
YLVERT(2) = 10H U/VELOCITY(ENT)
XLHOR(1) = 10H X/A/RE
XUHOR(1) = 10H VELOCITY V
XUHOR(2) = 10H X FOR N=
ENCOD(10,15,XUHOR(3),FBI(NF))
ISIZE = 3
YMAX = 2.0
YMIN = 0.0
XMAX = 0.3
XMIN = 0.0
J6 = 1+NT*NF1+NT
DO 20 N=1,70
20 X(N) = XPRES(N)
CALL NPLOTS(NF1,NT,IPLOT,IPPOINT,NREAD,UVEL,X
1 FMTE,NPT,NORD,XSIZE,YSIZE)
RETURN
300 CONTINUE
C...PART FOR PRESSURE DROPS PLOT (P VS X)
DO 11 N=1,10
11 LINEFX(N) = N
XSIZE = 20.0
NT = 100
IPOINT = 0
IPLOT = 200
J6 = 1+NT*NF +NT
ISIZE = 1
XMIN = 0.0
XMAX = 0.9
LETS(1) = 10
LETS(2) = 10
LETS(3) = 10
YLVERT(1) = 10HPRESSURE
XLHOR(1) = 10H X(X/A/RE)
XUHOR(1) = 10 HPRESS VS X
901 FORMAT(11F11.5)
CALL NPLOTS(NF,NT,IPLOT,IPPOINT,NREAD,PRES,XPRES,J6,LINES,
1 FMTE,NPT,NORD,XSIZE,YSIZE)
C...PART FOR NUSSELT NUMBER PLOT(NU VS X)
LETS(1) = 10
LETS(2) = 10
LETS(3) = 20
YLVERT(1) = 10HNU (LOCAL)
XLHOR(1) = 10H X/A/RE
XUHOR(1) = 10H NUSSELT NU
XUHOR(2) = 10H NUSSELT VS X
CALL NPLOTS(NF,NT,IPLOT,IPPOINT,NREAD,NU,XNU
1 FMTE,NPT,NORD,XSIZE,YSIZE)
C...PART FOR BULK TEMPERATURE PLOT (TBULK VS X)
LETS(1) = 10
LETS(2) = 10
LETS(3) = 25
YLVERT(1) = 10HBULK TEMP.
XLHOR(1) = 10H X/A/RE
XUHOR(1) = 10HBULK TEMPE
DO 902 N=1,NT
902 WRITE(6,901)XPRES(N),(PRES(N,I),I=1,NF),(NU(N,J),J=1,NF)
XUHOR(2) = 10HRATURE VS
XUHOR(3) = 5HX
CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,TFBUXN,NU,J6,LINES,
1 FMTE,NPT,NORD,XSIZE,YSIZE)
C..PART FOR ENTRY LENGTH VS FLOW BEHAVIOR INDEX PLOT
NT = NF
NF = 1
IPOINT = 1
IPLOT = 0
J6 = 1+NT*NF +NT
LETS(1) = 26
LETS(2) = 24
LETS(3) = 37
YLVERT(1) = 10H ENTRY LEN
YLVERT(2) = 10HGTH (VALUE
YLVERT(3) = 6H OF X)
XLHOR(1) = 10H FLOW BEHA
XLHOR(2) = 10HVIOR INDEX
XLHOR(3) = 4H N
XUHOR(1) = 10HENTRY LENG
XUHOR(2) = 10HTH VS FL
XUHOR(3) = 10HOW BEHAVIO
XUHOR(4) = 7HR INDEX
ISIZE = 3
XMIN = 0.0
XMAX = 2.0
YMIN = 0.0
YMAX=0.30
DO 903 N=1,NT
903 WRITE(6,901)FBI(N),EL(N)
CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,EL,FBI ,J6,LINES,
1 FMTE,NPT,NORD,XSIZE,YSIZE)
RETURN
END
PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10)

THIS IS A FINITE DIFFERENCE METHOD TO SOLVE THE MOTION
EQUATIONS FOR THE ENTRANCE REGION OF A CONVERGING CHANNEL

THIS METHOD IS USING A SERIES OF VALUES KNOWN AS THE OLD VALUES TO
COMPUTE ANOTHER SET OF VALUES KNOWN AS THE NEW VALUES CORRESPONDING
TO THE NEXT COLUMN (AXIALLY) SO THAT FROM THE ENTRY SET OF VALUES WE
CAN GO DOWNSTREAM AS FAR AS WE WANT PROVIDING WE HAVE THE
BOUNDARY CONDITIONS

LIST OF VARIABLES
AN= EXPONENT OF THE POWER LAW FLUID (FLOW BEHAVIOR INDEX)
ANGLE=TOTAL TAPER ANGLE
BETA=HALF THE TOTAL TAPER ANGLE
UOLD(1-),VOLD(1-),TOLD(1-),POLD= VECTORS OF VELOCITY (X-DIR), VELOCITY
(Y-DIR), TEMPERATURE AND PRESSURE ALREADY KNOWN
UNEW(1-),VNEW(1-),TNEW(1-),PNEW= VECTORS OF VELOCITY (X-DIR), VELOCITY
(Y-DIR), TEMPERATURE AND PRESSURE TO BE COMPUTED (NEW)
UAVE=AVERAGE VELOCITY IN X-DIRECTION
UMAUAV= CENTERLINE VELOCITY (MAX)/AVERAGE VELOCITY
U99=99 PERCENT OF UMAUAV
UUMAXN(1-)= VECTOR OF VELOCITY DIVIDED BY MAXIMUM VELOCITY (CENTERLINE)
(OUTPUT PURPOSE)
UUMAXO(1-)= SAME AS PRECEEDING ONE
GRA(1-),GRAMIN,GRAMI= GRADIENTS - VECTOR OF GRADIENTS (FOR OUTPUT),
MINIMUM GRADIENT AS DEFINE IN READ STAT., MINIMUM GRADIENT AS
DEFINED IN PROGRAM
FACT = (GRAMI) **(AN-1)
R= VALUE OF R AT WHICH THE PROGRAM IS NOW (RADIAL LENGTH)
ROLD= OLD VALUE OF R
TBULK= BULK TEMPERATURE OF FLOW
NUS= NUSSLETT NUMBER
AVNUS= AVERAGE NUSSLETT NUMBER
PE= PECLET NUMBER
PGRAO= PRESSURE GRADIENT
RE= REYNOLDS NUMBER
RC= REYNOLDS NUMBER WHERE THE CENTERLINE VELOCITY IS TAKEN AS
CHARACTERISTIC VELOCITY
SUM= RUNNING SUM FOR CALCULATION OF AVNUS
TGRA= TEMPERATURE GRADIENT

VARIABLES DESCRIBING THE MESH SIZE (MESH SIZES ARE VARYING AS WE PROCEED)
INCY(1-)= VECTOR CORRESPONDING TO NUMBER OF GRID SPACE ACROSS FLOW
XINC(1-)= VECTOR CORRESPONDING TO GRID SPACE IN X-DIR
ULIX(1-)= VECTOR CORRESPONDING TO LIMIT UP TO WHICH THE PRECEDING
X-DIR GRID SPACE IS VALID
NDR= VARIABLE CORRESPONDING TO INDICES OF PREVIOUS VARIABLES
TO KEEP TRACK OF THE NEXT MESH SIZE (MOMENTUM MESH AND
HEAT TRANSFER MESH)

CONSTANTS RELATED MOMENTUM MESH
A(-,)= MATRIX OF COEFFICIENTS FOR MOMENTUM MESH
CONS,RUPJ,= CONSTANTS USED IN SETTING OF COEFFICIENT MATRIX
NMAX1= NUMBER OF SPACE IN TETA-DIRECTION (ANGULAR DIRECTION)
NMAX= NUMBER OF POINT IN Y-DIR (NMAX = NMAX1 + 1)
DTETA= GRID-SIZE IN TETA-DIRECTION (ANGULAR DIRECTION)
DR= GRID-SIZE IN R-DIRECTION (RADIAL DIRECTION)
DRT2,DR1,DTR= SERIE OF CONSTANTS DEFINED IN SUBROUTINE
CONST CALCULATED FROM 4 PREVIOUS
CONSTANTS AND VARIABLES RELATED TO HEAT TRANSFER MESH
B(-,-) = MATRIX OF COEFFICIENTS FOR HEAT TRANSFER MESH
TBULKO, NUSO = VARIABLE USED FOR OUTPUTING PURPOSES (SAME MEANING
AS WITHOUT *O*)
YD = USED IN THE OUTPUT TO SITUATE A POINT IN TLTA-DIRECTION (AT CENTERLINE
YD= 0.0, AT WALL YD=1.0)
LL = FLAG TO OUTPUT EVERY SECOND TIME

CONSTANTS AND VARIABLES RELATED TO PLOTTING SUBROUTINES
U98 = 98 PER CENT OF LAST UMAUAV (FULLY DEVELOPED)
NPOS = COUNTER FOR INSERTION OF VELOCITY PROFILE
LN = COUNTER FOR DIFFERENT FLUIDS
L1, L2 = COUNTER FOR PRESSURE DROP, NUSSELT NUMBER
VX(-,-) = VALUES OF Y (-1.0 TO +1.0)
VY(-,-) = VELOCITY IN R-DIRECTION FOR THOSE PREVIOUS VX(-)
XPRESS(-,-) = VALUES OF R (0.0 TO 1.0)
PRESS(-,-) = PRESSURE DROP AT THOSE R
TBU(-,-) = BULK TEMPERATURE AT THOSE R
NU(-,-) = NUSSELT NUMBER AT THOSE R
VECTOR(-,-) = USED IN PREPARING UVEL(-,-)
FBI(-) = FLOW BEHAVIOR INDEX (SIMILAR TO AN)
UVEL(-,-) = VELOCITY IN R-DIRECTION

COMMON NMAX1,RIJI,PNEW,A(5,165),UNEW(165),UOLD(165),VNEW(165),
VOLD(165),TNEW(165),TOLD(165),UMAXN(165),UMAXO(165),B(3,165)
DIMENSION INCY(N),XINC(N),ULIX(N)
DIMENSION VY(21,3),VX(21),PRESS(110,5),XPRESS(110),UMAXA(110,5)
1,FB1(5),NU(110,5),TBU(110,5),VECTOR(12),UVEL(70,5)
DIMENSION C(165),VGRAD(165)
REAL NUS,NUSC,NU
N = 0

READ IN ALL THE GRID SIZE
7 N = N +1
READ(5,1105)INCY(N),XINC(N),ULIX(N)
WRITE(6,1105)INCY(N),XINC(N),ULIX(N)
IF (INCY(N) .NE. 0) GO TO 7

READ THE PARAMETERS
6 LN = 0
LM = 0
READ(5,1109)MAX,(FBI(N),N=1,MAX)
IF(EOF(5))10000,9
9 IF(MAX .EQ. 1) GO TO 200
1009 FORMAT (15,10F5.2)
4 READ(5,1106)RE,PE,ANGLE,GRAMIN,TOL
IF(RE .EQ. 0.0) STOP
5 IF(LN .GE. MAX) GO TO 850
LN = LN +1
LM = LM +1
AN = FBI(LN)

DEFINE MINIMUM GRADIENT
8 GRAM = GRAMIN

INITIALISATION OF COUNTERS AND OTHERS
R = 0.0
NUSO = 0.0
TGRAO = 0.0
SUM = 0.0
ROLD = 0.0
NPOS = 1
L1 = 1
LL = -1
L = 0
NDR = 1

C... COMPUTE ALL THE CONSTANTS NEEDED
CALL CONStENCY(NDR),XINC(NDR),ANGLE,BETA,NMAX1,DR,DTETA,NMAX,NMAX
14,DRDT2,DRI,BT2R)

C... DEFINE THE STARTING PROFILES
DO 1 K=1,NMAX
   TOLD(K) = 1.0
   UOLD(K) = 1.0
   VOLD(K) = 0.0
1 CONTINUE

POLD = 0.0
PNEW = 0.0

C... OUTPUT THE STARTING VALUES
WRITE(6,1003)AN,R,POLD
DO 2 N=1,NMAX,NMAX4
   WRITE(6,1U02) N ,UOLD( N J,VUL0(NJ

C... OUTPUT THE PARAMETERS
FACT = (GRAM1)**(AN-1.)
WRITE(6,101 U)RE,PE,ANGLE,GRAM1,FACT,TOL

C... INTEGRATION ITERATION BEGINS
11 R = R +DR
RIJ = 1.-ROLD
RIJ1 = 1.-R
AJ = 1./(RIJ*BETA*DTETA)
AJ2RE =((AJ*AJ)/RE)
BJRE =((1./(RIJ*DR))/RE)

C... COMPUTE VARIABLES NECESSARY IN SETTING UP OF COEFFICIENT MATRIX
DO 13 N=1,NMAX1
   IF(N .NE. 1)GO TO 12
   C(N) = (GRAM1/(RIJ*BETA)**(AN-1.))
   GO TO 13
12 GRA = (ABS(0.5*(UOLD(N+1)-UOLD(N-1)))/DTETA
IF (GRA .LT. GRAMI) GRA = GRAMI
C(N) = (GRA/(RIJ*BETA)**(AN-1.))
13 CONTINUE
GRA = (ABS(UOLD(NMAX)-UOLD(NMAX-1))/DTETA)
IF (GRA .LT. GRAMI) GRA = GRAMI
C(NMAX) = (GRA/(RIJ*BETA)**(AN-1.))
CONS = RE*BETA*RIJ*DTETA
DO 14 N=2,NMAX1
14 VGRAD(N) = (C(N+1)-C(N-1))/CONS
VGRAD(1) = 0.0

C... SET THE COEFFICIENT MATRIX
DO 10 N=1,NMAX1
   AJ2REC = AJ2RE*C(N)
   BJREC = BJRE*C(N)
   A(3,N) = +5*AJ*(VOLD(N)-VGRAD(N)) -AJ2REC
   A(1,N) = -5*AJ*(VOLD(N)-VGRAD(N)) -AJ2REC
   A(2,N) = UOLD(N)/DR +3.*BJREC+2.*AJ2REC
   A(4,N) = +DRI
   A(5,N) = POLD/DR -VOLD(N)*VOLD(N)/RIJ +UOLD(N)*(UOLD(N)/DR
   1+3.*BJREC+( (1./RIJ)**2/RE)*C(N))
10 CONTINUE
A(3,1) = A(3,1) + A(1,1)

C... SOLVE THE MATRIX (GAUSS ELIMINATION ALGORITHM)
CALL GAUSS
UNEWS(NMAX) = 0.0

C... COMPUTE VELOCITIES IN Y-Dir
VNEW(1) = 0.0
DO 30 J = 2, NMAX
30 VNEW(J) = VNEW(J-1) - BT2K*(RIJ1*(UNEWS(J) + UNEW(J-1))
            + RIJ*(UOLD(J) + UOLD(J-1)))
VNEW(NMAX) = 0.0

C... COMPUTE AVERAGE VELOCITY
UAVE = UNEW(1)/2.

DO 47 N = 2, NMAX
47 UAVE = UAVE + UNEW(N)
UAVE = UAVE/FLOAT(NMAX)

C... HEAT TRANSFER PART
BJT = 1.0/(BETA*RIJ*DTETA)
BJT2P = BJT*BJT/PE

C... SET UP VELOCITY FIELD FOR ENERGY EQUATION
C... SET THE COEFFICIENT MATRIX (ENERGY EQUATION)
DO 60 N = 1, NMAX
RUPJ = (UOLD(N) + 1.0/(PE*RIJ))/DR
B(1,N) = -VOLD(N)*BJT/2.0 - BJT2P
B(2,N) = RUPJ + 2.0*BJT2P
B(3,N) = +VOLD(N)*BJT/2.0 - BJT2P
TNEW(N) = TOLD(N)*RUPJ
60 CONTINUE
R(3,1) = B(3,1) + B(1,1)

C... SOLVE MATRIX
CALL DIAG3(B, TNEW, NMAX)
TNEW(NMAX) = 0.0

C... COMPUTE BULK TEMPERATURE AND NUSSELT NUMBER
TBULK = (TNEW(1)*UNEW(1) + TNEW(NMAX)*UNEW(NMAX))/2.
DO 122 N = 2, NMAX
122 TBULK = TBULK + (UNEW(N)*TNEW(N))
TBULK = (TBULK/FLOAT(NMAX))/UAVE
TGRA = (-11.0*TNEW(NMAX) + 18.0*TNEW(NMAX-1) - 9.0*TNEW(NMAX-2) + 2.0*TNEW(1))
NUS = (2.0*TGRA)/(1.0 - TBULK)
SUMO = SUM
SUM = (5.0*DR*(NUS+NUSO)) + SUM
AVNUS = SUM/R

C... OUTPUT THE RESULTS (PRINT OUT FORM)
L = L+1
IF(L .NE. 2) GO TO 45
L = 0
LL = LL+1
IF(LL .EQ. 3) GO TO 99
98 CONTINUE
RC = RE*(1.0-R)*UNEW(1)
WRITE(6,1001) AN, AVNUS, ROLD, POLD, TBULK, NUSO, R, RC, PNEW, TBULK, NUS
NMAX4D = NMAX4*D
DO 40 N = 1, NMAX, NMAX4D
YD = FLOAT(N-1)/FLOAT(NMAX)
UUMAXO(N) = UOLD(N)/UOLD(1)
UUMAXN(N) = UNEW(N) / UNEW(1)

40 WRITE (6, 1002) N, YD, UOLD(N), UUMAXO(N), VOLD(N), TOLD(N),
 1 UNEW(N), UUMAXN(N), VNEW(N), TNEW(N)

C

C***PART TO SET VALUES FOR PLOTTING SUBROUTINE

45 GO TO (801, 802, 803), NPOS

803 IF(R .LT. 0.76) GO TO 701
  GO TO 702

802 IF(R .LT. 0.05) GO TO 701
  GO TO 702

801 IF(R .LT. 0.005) GO TO 701

702 LJ = 0

C***PREPARE VELOCITY FIELD

DO 700 N=1, NMAX, NMAX4
  LJ = LJ+1
  LI = 22-LJ
  NN = NMAX+1-N
  VY(LJ, NPOS) = UNEW(NN) / UAVE
  VY(LI, NPOS) = UNEW(NN) / UAVE
  VX(LJ) = (FLOAT(LJ-1))/10.0 - 1.0

700 VX(LI) = (FLOAT(LI-1))/10.0
  NPOS = NPOS +1

701 IF(L1 .GT. 110 .OR. R .LT. 0.05) GO TO 710

C***PREPARE PRESSURE DROPS, U(MAX)/UAVE, BULK TEMPERATURE AND NUSSLEIT NUMBE

XPRESS(L1) = R
  PRESS(L1, LM) = -PNEW
  TBU(L1, LM) = TBUK
  NU(L1, LM) = NUS
  UMAUAV(L1, LM) = UNEW(1) / UAVE
  PGRA = (POLD-PNEW)/DR
  IF(L1 .EQ. 0) WRITE(6, 1007) UAVE, UMAUAV(L1, LM), PGRA
  IF(L1 .GT. 70) GO TO 709
  J = 0
  DO 705 N=1, NMAX, NMAX4
    J = J+1
    VECTOR(J) = UNEW(N) / UAVE

705 CONTINUE
  UVEL(L1, 1) = VECTOR(1)
  UVEL(L1, 2) = VECTOR(4)
  UVEL(L1, 3) = VECTOR(6)
  UVEL(L1, 4) = VECTOR(8)
  UVEL(L1, 5) = VECTOR(10)

709 L1 = L1 +1

710 CONTINUE
  IF(NPOS .NE. 4) GO TO 46
  NPOS = NPOS -1
  U99 = .99*UMAUAV(110, LM)
  N = 0

C***COMPUTE ENTRY LENGTH

220 N = N +1
  IF(N .GT. 110) GO TO 899
  IF(UMAUAV(N, LM) .LT. U99) GO TO 220
  N = N -1

  IF(N .LT. 1) GO TO 899
  EL = XPRESS(N) * ((XPRESS(N+1) - XPRESS(N)) / (UMAUAV(N+1, LM) 1 - UMAUAV(N, LM))) * (U99 - UMAUAV(N, LM))
  WRITE (6, 1104) EL
1104 FORMAT(2X,*ENTRY LENGTH =* F10.4)  
899 DO 900 N=1,21  
900 WRITE(6,901) VX(N),VY(N,1),VY(N,2),VY(N,3)  
899 DO 906 N=1,70  
906 WRITE(6,901) XPRESS(N), (UVEL(N,J),J=1,5)  
901 FORMAT (1IF11.5)  
IF(MAX. EQ. 1)GO TO 200  
CALL DESSIN(2,VX,VY,XPRESS,PRESS,UVEL,UMAUAV,NU,TBU,FBI,LM)  
GO TO 5  
C  
C...TO CHANGE GRID SPACING IN X-DIR (IF NECESSARY)  
46 IF(R.LT.(0.99*ULIX(NDR))) GO TO 51  
NDR = NDR +1  
CALL CONST(INCY(NDR),XINC(NDR),ANGLE,BETA,NMAX1,DR,DTEA,NMAX,NMAX  
14,DRDT2,DR1,DR2R)  
IF(INCY(NDR). EQ. INCY(NDR-1)) GO TO 51  
C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / MOMENTUM EQUATION  
DO 22 N=1,NMAX  
NN = 2*N-1  
UOLD(N) = UOLD(NN)  
VOLD(N) = VOLD(NN)  
UNEW(N) = UNEW(NN)  
VNEW(N) = VNEW(NN)  
TNEW(N) = TNEW(NN)  
TOLD(N) = TOLD(NN)  
22 CONTINUE  
C...ONE COLUMN COMPLETE, PREPARE FOR NEXT COLUMN  
51 POLD = PNEW  
ROLD = R  
NUSO = NUS  
TBULK0 = TBULK  
TGRAO = TGRA  
DO 52 N=1,NMAX  
TOLD(N) = TNEW(N)  
UOLD(N) = UNEW(N)  
52 VOLD(N) = VNEW(N)  
C...GO BACK FOR NEXT COLUMN  
GO TO 11  
99 CONTINUE  
LL = 0  
GO TO 98  
850 CONTINUE  
C...PLOT AND OUTPUT COMPARABLE VALUES (PRESSURE, NUSSEL) OF ALL FLUIDS  
CALL DESSIN(3,VX,VY,XPRESS,PRESS,UVEL,UMAUAV,NU,TBU,FBI,LM)  
IF(MAX. EQ. 1)GO TO 6  
LN = 0  
GO TO 4  
200 READ(5,1106) RE, PE, ANGLE, GRAMIN, TLC  
IF(RE. EQ. 0.0) GO TO 850  
AN = FBI(1)  
LN = 1  
LM = LM +1  
GO TO 8  
10000 STOP  
1001 FORMAT (18X,N = *,F4.2,80X*AVNUS=*F8.4,12X,*R=*F7.5,3X*PRESS=*
1F8.4,3X*TBULK=*F7.4,3X*NUS=*F8.4,14X,*K=*F7.5,*(F6.0*)*,3X*PRESS=*
1F8.4,3X*TBULK=*F7.4, 3X*NUS=*F8.4, / 5X*N*,4X*Y/D*,7X
2*U(N)*, 9X,*U/UMAX*, 7X,*V(N)*, 9X,*T(N)*, 19X,*U(N)*9X,*U/UMAX*7X,
3 *V(N)*9X,*T(N)*
1002 FORMAT (1X,I5,3X,F5.3,4F13.6,10X,4F13.6)
1003 FORMAT (1H1,10X,* N =*, F4.2, //, 3X, * K =*F6.4,
14X, * PRESSURE =*, F8.5, //, 9X,*N*, 4X,
2*U(N)*, 9X, *V(N)*)
1007 FORMAT (80X,*UAVE =* F7.4,8X,*UMAX/UAVE =* F7.4,6X*PGRA=*F7.3)
1010 FORMAT (15X*REYNOLDS NUMBER=* F10.3,15X*PECLET NUMBER=*F10.3,
1 15X*ANGLE (TOTAL) =*F5.1, 2X,*MINIMUM GRADIENT=*F6.4,12X
1*(MIN. GRAD.)EXP(N-1)=*F10.4, /2X,*TOLERANCE FOR CONVERGING TYPE ME
2SH=*F8.5)
1105 FORMAT(I5,2F10.5)
1106 FORMAT(12F10.4)
END
SUBROUTINE CONSTINC,Y,XINC,ANGLE,BETA,NMAX1,DR,DTETA,NMAX,NMAX4
1,DRDT2,DRI,BT2R)

C..SUBROUTINE COMPUTING CONSTANTS OF THE MAIN PROGRAM (FOR CONVERGING FLOW)
C
BETA = (ANGLE/2.)*(3.14159/180.)
NMAX1 = INCY
DR = XINC
DTETA = 1./FLOAT(NMAX1)
NMAX = NMAX1 +1
NMAX4 = NMAX1/10
DRDT2 = DR/(DTETA*DTETA)
WRITE(6,1006) DRDT2
1006 FORMAT(2X,* DR/DTETAXP2= *,E12.5)
DRI = 1./DR
BT2R = BETA*DTETA/(2.*DR)
RETURN
END
SUBROUTINE DESSIN(IFLAG, VX, VY, XPRES, PRES, UVEL, UMAUAV, NU, TSU, FBU, NF)

C
C...SUBROUTINE USING THE PLOTTING SUBROUTINES OF D. BUCHANAN
C
DIMENSION VY(21, NF), VX(21), PRES(110, NF), XPRES(110), UMAUAV(110, NF)

DIMENSION X(70)
REAL NU

C...COMMON BLOCK
COMMON DUMMY(3), XX(5000)
COMMON /INFOPLT/LINEFX(30), LPTS(30)
COMMON /LABL/ LABEL, LETS(4)
COMMON /LBL/ YLVERT(7), YRVERT(7), XLHOR(7), XUHOR(7)
COMMON /TITLES/MNYPLT, TITLE1(2), TITLE2(2), TITLE3(2), TITLE4(2)
COMMON /SCALES/ ISIZE
COMMON /BORDER/ XMIN, XMAX, YMIN, YMAX

C...CONSTANTS
NREAD = 0
FMTE = 0
NPT = 1
NORD = 3
XSIZE = 15.0
YSIZE = 15.0
LABEL = 3
MNYPLT = 1
LINES = 2
TITLE1(1) = '10H HPVL J.
TITLE1(2) = '10H LAROCQUE
TITLE2(1) = '10HCONVERGING
TITLE2(2) = '10H FLOW
TITLE3(1) = '10H FOR POWER
TITLE3(2) = '10H LAW FLUID
NT = 21
IPOINT = 0
IPLOT = 200
DO 10 N=1, 10
LINEFX(N) = 3
10 LPTS(N) = N
GO TO (100, 200, 300), IFLAG

C...PART FOR THE VELOCITY FIELD PLOT (U VS Y)
100 LETS(1) = 20
LETs(2) = 5
LETs(3) = 24
YLVERT(1) = '10H VELOCITY=
YLVERT(2) = '10HU/U(ENT',XLHOR(1) = '5H Y/A
XUHOR(1) = '10HVELOCITY V
XUHOR(2) = '10HS Y FOR N=
ENCOD(10, 15, XUHOR(3), FBU(NF)
15 FORMAT(F4, 2, 6X)
ISIZE = 2
YMAX = 2.0
YMIN = 0.0
NF1 = 3
J6 = 1+NT*NF1 +NT
CALL NPLOTS(NF1, NT, IPLOT, IPOINT, NREAD, VY, VX, J6, LINES,
1 FMTE, NPT, NORD, XSIZE, YSIZE)
RETURN
**PART FOR THE VELOCITY FIELD PLOT (U VS X)**

200 NF1 = 5

NT = 70

LETS(1) = 20

LETS(2) = 10

LETS(3) = 24

YLVERT(1) = 10H VELOCITY=

YLVERT(2) = 10HU/U(AVE)

XLHOR(1) = 10HR(1. -R/RO)

XUHOR(1) = 10HVELOCI\y

XUHOR(2) = 10HS R FOR N=

ENCOD(10,15,XUHOR(3))FBI(NF)

ISIZE = 3

YMAX = 2.0

YMIN = 0.0

XMAX = 0.4

XMIN = 0.0

J6 = 1+NT*NF1+NT

DO 210 N=1,NT

210 XCN) = XPRESCN)

CALL NPLOTS(NF1,NT,IPLOT,IP0INT,NREAD,UVEL,X

FMTE,NPT,NORD,XSIZE,Y SIZE)

RETURN

**PART FOR THE PRESSURE DROPS PLOT**

DO 11 N=1,10

11 LINEFX(N) = N

XSIZE = 20.0

C PRESSURE VS X

NT = 110

IPOINT = 0

IPLOT = 200

J6 = 1+NT*NF +NT

ISIZE = 1

XMIN = 0.0

XMAX = 0.8

LETS(1) = 10

LETS(2) = 10

LETS(3) = 10

YLVERT(1) = 10H PRESSURE

XLHOR(1) = 10HR(1. -R/RO)

XUHOR(1) = 10H PRESS VS R

DO 902 N=1,NT

902 WRITE(6,901) XPRES(N), (PRES(N,I),I=1,NF),(UMAUAV(N,J),J=1,NF)

CALL NPLOTS(NF,NT,IPLOT,IP0INT,NREAD,PRES,XPRES,J6,LINES,

FMTE,NPT,NORD,XSIZE,Y SIZE)

**PART FOR U(MAX)/U(AVERAGE) PLOT**

LETS(3) = 35

YLVERT(1) = 10H UMA/UAV

XUHOR(1) = 10HU(MAX=CENT

XUHOR(2) = 10HER LINE)/U

XUHOR(3) = 10H(AVERAGE)

XUHOR(4) = 5HVS R

ISIZE = 3

YMAX = 2.0

YMIN = 1.0

CALL NPLOTS(NF,NT,IPLOT,IP0INT,NREAD,UMAUAV,XPRES,J6,LINES,
1 FMTE,NPT,NORD,XSIZE,YSIZE)

C...PART FOR THE NUSSELT NUMBER PLOT
LETS(1) = 10
LETS(2) = 10
LETS(3) = 20
YLVERT(1) = 10HNU (LOCAL)
XLHOR(1) = 10HR(1.*R/RO)
XUHOR(1) = 10HNUSELT NU
XUHOR(2) = 10HMBER VS R
DO 903 N=1,NT
903 WRITE(6,901) XPRES(N),(TBU(N,I),I=1,NF),(NU(N,J),J=1,NF)
901 FORMAT(11F11.5)
YMAX = 5.
YMIN = 0.0
CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,NU,XPRES,J6,LINES,1 FMTE,NPT,NORD,XSIZE,YSIZE)

C...PART FOR THE BULK TEMPERATURE PLOT
LETS(1) = 10
LETS(2) = 10
LETS(3) = 25
YLVERT(1) = 10HBUK TEMP.
XLHOR(1) = 10HR(1.*R/RO)
XUHOR(1) = 10HBUK TEMPE.
XUHOR(2) = 10HRATURE VS
XUHOR(3) = 5HR
YMAX = 1.0
YMIN = 0.0
CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,TBU,XPRES,J6,LINES,1 FMTE,NPT,NORD,XSIZE,YSIZE)
RETURN
END