PSEUDOPLASTIC ENTRY FLOW IN STRAIGHT AND CONVERGENT CHANNELS

PSEUDOPLASTIC ENTRY FLOW IN STRAIGHT AND CONVERGENT CHANNELS

by

JACQUES LAROCQUE, B.A., B.Sc.A.

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Master of Engineering

McMaster University

April 1973

MASTER OF ENGINEERING (1973)
(Chemical Engineering)

McMaster University
Hamilton, Ontario

TITLE: Pseudoplastic entry flow in straight and convergent channels

AUTHOR: Jacques Larocque, B.A. (Université de Montréal)
B.Sc.A. (Ecole Polytechnique de Montréal)

SUPERVISOR: Dr. John Vlachopoulos

NUMBER OF PAGES: xi, 188

ABSTRACT

Power law fluids are analysed for entry flow in straight and converging channels in their pseudoplastic region ($0.0 \leqslant n \leqslant 1.0$). The motion and energy equations simplified by the boundary layer assumptions were solved by an implicit finite difference scheme with a marching procedure. To circumvent the difficulties arising from an infinite viscosity at zero shear rate, a minimum value of shear rate was used making the fluid newtonian at low shear rates.

Entrance flows between parallel plates of infinite width (Slit) for uniform entry profile are discussed in Part I and converging flows for non-parallel flat plates is the subject of Part II of this work. Results are compared with their equivalent in the current literature for the newtonian case; new results are presented for non-newtonian fluids. These results include velocity and temperature profiles, pressure drops, Nusselt number, and entry lengths as a function of the flow behavior index (n) and the taper angle.

RESUME

Des fluides obéissant à la loi de puissance sont étudiés pour des écoulements d'entrée en conduites droites et convergentes, dans leur région pseudoplastique ($0<n<1.$). Les équations de mouvement d'abord simplifiées par la théorie de la couche limite, sont ensuite résolues par une méthode de différences finies implicites, incluant l'équation d'énergie. Une valeur minimum de taux de cisaillement est utilisée de manière à ne pas rendre la viscosité infinie à bas taux de cisaillement.

En partie I, on traite d'écoulements d'entrée entre plaques planes parallèlles pour profil d'entrée plat; ces écoulements sont étudiés pour plaques non parallèlles en partie II. Pour les fluides newtoniens, on compare avec des résultats précédents et quelques nouveaux sont obtenus pour des fluides non newtoniens. Les résultats présentés comprennent des profils de vitesse et de température, des chutes de pression, des nombres de Nusselt et des longueurs d'entrée en fonction de l'indice de puissance (n) et des conditions géométriques.

ACKNOWLEDGEMENTS

The author would like to thank his supervisor Dr. Vlachopoulos for his help and guidance during the course of this work. Sincere thanks are also extended to Doug Buchanan for the use of his plotting routines.

Gratitude is expressed to the National Research Council for their financial support in form of a postgraduate scholarship and bursary, and to the Chemical Engineering Department for their assistantship.

Finally the author is very grateful to his wife, Carole, for her encouragements, her sacrifices and her typing of this thesis.

TABLE OF CONTENTS

FIGURE INDEX

Page

TABLE INDEX

PART I

CHANNEL ENTRY FLOWS FOR PSEUDOPLASTIC FLUIDS

I- 1 INTRODUCTION

In the first part of this thesis, the problem of entrance flows of pseudoplastic fluids in a channel formed by two parallel plates is analysed; the two plates are considered semi-infinite i.e. infinitely wide with negligible end effects. The fluid to be studied is a power law fluid. Heat transfer effects are also discussed at different Prandtl numbers.

This problem has drawn a lot of attention in the current literature although most of it has gone to newtonian fluids for tube flow. Since that problem deals with partial differential equations, there are three major methods of solution: (1) exact analytical methods, (2) approximate methods, (3) numerical methods. The solution we are proposing here is a numerical finite difference procedure of the implicit type.

First, the theoretical background is presented with a literature survey followed by the solution of the problem itself. The three conservation equations (Continuity, Momentum and Energy) are derived according to the assumptions used and transformed through finite difference approximations. Then, in a third section, the results are described along with a discussion on their significance. Conclusions are finally drawn from the results, which include the possible extension of this work.

I- 2. THEORETICAL BACKGROUND AND LITERATURE SURVEY

I- 2. 1 Methods of solution

This problem has been solved by many methods in the 50 or so papers related to it, nearly all of them about newtonian fluids either with or without boundary layer assumptions. Methods using the boundary layer assumptions are of the following types: (1) linearization of inertia terms, (2) integral methods, (3) series expansions, (4) numerical finite difference of the boundary layer equations. Methods without boundary layer assumptions: (5) finite difference solution of the full Navier-Stokes equations.

The first two methods will only be mentioned here without any explanation because they were not really extended to more general application after their original publication; the method of linearization was originated by Boussinesq (1891) and Langhaar (1942); Schiller (1922) did some work with the integral method (this type of solution was retaken by McKillop (1964) and McKillop et al. (1970) for the entry flow in the immediate entry). The last three methods, because of their wider use, will be explained in more detail. The method used here, is of the boundary layer finite difference type.

I- 2. 2 Solution of boundary layer equations by series expansions

The boundary layer concept is based on the fact that in viscous flow problems e.g. flows of polymers, the viscous effects have more importance than the inertial effects in the fluid; naturally this will force us to remain in an acceptable range for the Reynolds number; we can say that for most flows with Reynolds number over 50, the above concept is valid as demonstrated by Christiansen and Kelsey (1973) for tube flow at a contraction.

Schlichting (1934) applied the boundary layer concept in solving entry flow problems for straight channels (uniform flat entry). His method consisted of two different solutions applied to each of the upstream and downstream parts of the problem. In the upstream part, the boundary layer is formed in a small region near the wall, where viscous effects and velocity gradients are important, and the central core is uniformly accelerated as pictured in Figure I-2.1, for this part, Schlichting obtained an expression for the axial velocity in form of a series expansion with respect to the two independant variables (2-dimensional flow). This expansion, if inserted in the axial momentum equation gives a differential equation with an infinite number of terms, of which, as a first approximation, we have the Blasius differential equation for a flat plate (depending on how many terms are taken in the expansion). Now for the downstream part, Schlichting assumes a velocity profile which is a perturbation of the true final parabolic one. The final step of the method is to join the two solutions at a suitable point where they are compatible to each other. In a recent paper where the upstream solution of Schlichting's method is questionned, Van Dyke (1970) is presenting a solution using a slightly different upstream expansion.

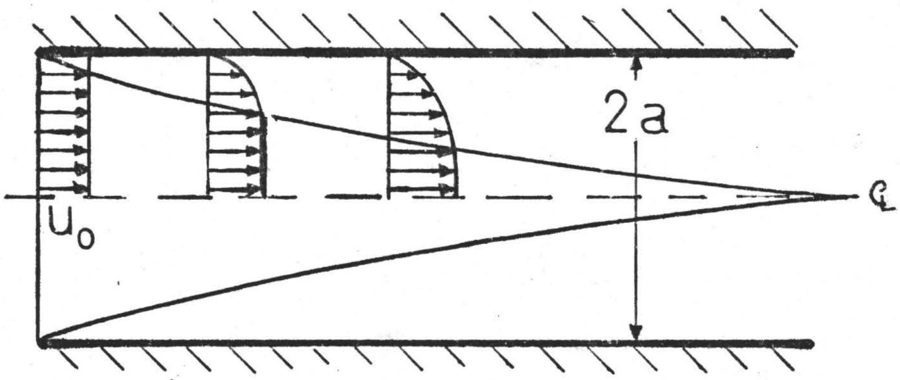


Figure I-2.1: Channel entry flow

Entry flow for non-newtonian fluids is more recent. The first calculations (in channels) were done by Collins and Schowalter (1963); they worked out the solution for channel entry flow of pseudoplastic fluids represented by the power law equation. Their method is about the same as Schlichting's method except that they took more terms in their series in order to have a better precision. Their results are used extensively for comparison purposes with the present work.

I- 2. 3 Numerical solutions

The boundary layer concept is also used with finite difference methods. Bodoia and Osterle (1961) have initiated work in this area followed by Hwang and Fan (1963, 1964). The method consists in starting with the Navier-Stokes equations which are simplified through the boundary layer assumptions; this means the y-direction of the equation of momentum (for 2-dimensional problem) is neglected implying that the pressure gradient is zero in the y-direction and that the motion in this direction is not important to the total problem. This evidently is not correct in the first region of the flow; but it becomes valid as soon as the thickness of the boundary layer is relatively large. In the present work this method is followed in solving the channel entry flow problem and the solution is extended to power-law fluids.

Three more publications are using the finite difference methods for the full Navier-Stokes equations; Wang and Longwell (1964), Brandt & Gillis (1966), and McDonald et al. (1972). They are solving those equations for newtonian fluids by relaxation with a two-dimensional grid at different Reynolds numbers. Their method, although quite complex, should be considered as one of the more precise because of the absence of boundary layer approximations;

we must say that for non-newtonian fluids, this method becomes extremely complicated particularly the search for convergence criteria as defined in Brandt and Gillis (1966).

I- 3 SOLUTION OF THE PROBLEM

I- 3. 1 The constitutive equation

The fluid to be used is represented by a power law constitutive equation (Ostwald-de Waele equation) from Bird et al. (1960) p. 101.

$$\underline{\underline{\tau}} = -K\left|\sqrt{\frac{I_2}{2}}\right|^{n-1} \underline{\underline{\Delta}} \qquad \text{(I-3.1)}$$

where $\underline{\underline{\tau}}$: stress tensor

I_2 : second invariant $(=\underline{\underline{\Delta}}:\underline{\underline{\Delta}})$ of $\underline{\underline{\Delta}}$

K : power law consistency index

n : flow behavior index

$\underline{\underline{\Delta}}$: rate of deformation tensor (symmetrical)

$$\text{where } \underline{\underline{\Delta}} = \begin{bmatrix} 2\frac{\partial u}{\partial x} & 2\frac{\partial u}{\partial y} \\ 2\frac{\partial u}{\partial y} & 2\frac{\partial v}{\partial y} \end{bmatrix}$$

For an incompressible fluid in cartesian coordinates, we have:

$$\frac{1}{2} I_2 = 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(2\frac{\partial u}{\partial y}\right)^2 \qquad \text{(I-3.2)}$$

the first two terms are neglected because, although they are important at the immediate entry, their effect diminishes very quickly. For more explanations, the reader should consult Schowalter (1960).

So equation I-3.1 becomes:

$$\underline{\underline{\tau}} = -K\left|\frac{\partial u}{\partial y}\right|^{n-1} \underline{\underline{\Delta}} \qquad \text{(I-3.3)}$$

This constitutive equation can be used directly in the momentum equation if the usual drawback, namely the infinite viscosity at low shear rates, is eliminated; this is done by setting a minimum value for $\left|\frac{\partial u}{\partial y}\right|$ making the fluid newtonian at lower shear rates. It has been recognized experimentaly that all non-newtonian fluids are exhibiting a newtonian behavior at low shear rates so this working procedure is very much in line with reality.

I- 3. 2 The hydrodynamic problem

The physical system as drawn in Figure I-3.1 is made of two semi-infinite parallel flat plates spaced by a length of "2a". The velocity profiles are sought along x for a uniform entry profile with the use of the continuity and momentum equation in a manner quite similar to the analysis of Bodoia and Osterle (1961) for a newtonian fluid; Hwang and Fan (1961) have solved the same problem for magneto-hydrodynamic flow of a newtonian fluid.

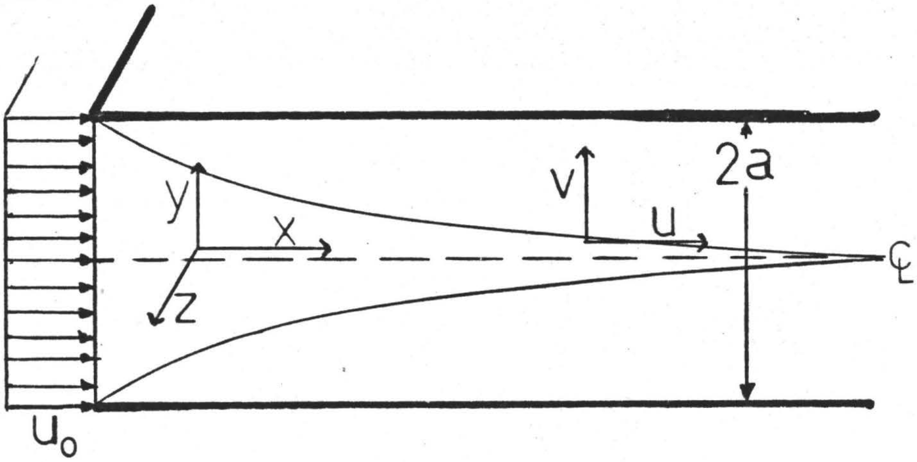


Figure I-3.1: Straight channel physical system

The following assumptions are made in order to solve the problem:

- The flow is 2-dimensional (Motion in the z-direction is negligible and no end effect).
- All fluid properties are constant: C, C_p, k, K.
- The effect of gravitational force is negligible.
- The flow is laminar.
- Prandtl's boundary layer assumptions apply.

With these assumptions, we obtain:

- Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(I-3.4)}$$

- Momentum equation in the x-direction:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{dp}{dx} + \frac{\partial \tau_{xy}}{\partial y} \qquad \text{(I-3.5)}$$

$$\text{where } \tau_{xy} = -K \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y}$$

- Integral form of the continuity equation:

$$\int_o^a u dy = u_o a \qquad \text{(I-3.6)}$$

The following variable transformations are used

$$X = \left(\frac{x}{a}\right) / Re \qquad \text{(I-3.7)}$$

$$\text{where } Re = \frac{\rho u_o^{2-n} (2a)^n}{K}$$

$$Y = y/a \qquad \text{(I-3.8)}$$

$$U = u/u_o \qquad \text{(I-3.9)}$$

$$V = \frac{(2a)^n u_o^{1-n} \rho v}{K} \qquad \text{(I-3.10)}$$

$$P = \frac{p-p_o}{\rho u_o^2} \qquad \text{(I-3.11)}$$

Then the continuity equation is

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad \text{(I-3.12)}$$

the integral form becomes

$$\int_0^1 U dY = 1 \qquad \text{(I-3.13)}$$

and the momentum equation is

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{dP}{dX} + 2^n \frac{\partial \left(\left| \frac{\partial U}{\partial Y} \right|^{n-1} \frac{\partial U}{\partial Y} \right)}{\partial Y}$$

(I-3.14)

And the boundary conditions are

at $Y = 0$ and $X>0$ (centerline): $\frac{\partial U}{\partial Y} = 0$ and $V = 0$

(I-3.15)

at $Y = 1$ and $X>0$ (wall) : $U = 0$ and $V = 0$

and the initial condition is

$$\text{at } X = 0 \text{ and } 0 \leqslant Y \leqslant 1 \quad : U = 1 \text{ and } V = 0$$

$$P = P_o = 0$$

$$\text{(I-3.16)}$$

There are two ways of solving this problem since the momentum is a nonlinear equation. First we can solve it as a linear approximation by an appropriate choice of difference approximations using a normal implicit scheme similar to the one of Bodoia & Osterle (1961) and Hwang & Fan (1963); another way is to use an iterative scheme in which the values that have just been calculated are reintroduced into the equations to compute again a set of new values. (A solution with an iterative scheme is described in Appendix III).

The implicit type method without iteration starts with the original assumptions for the velocity profile (initial condition) and proceeds column by column with linear equations by solving the coefficient matrix for the hydrodynamic problem in order to compute the velocities; with the velocities, we are then able to compute the temperatures directly. Figure I-3.2 is giving the mesh configuration:

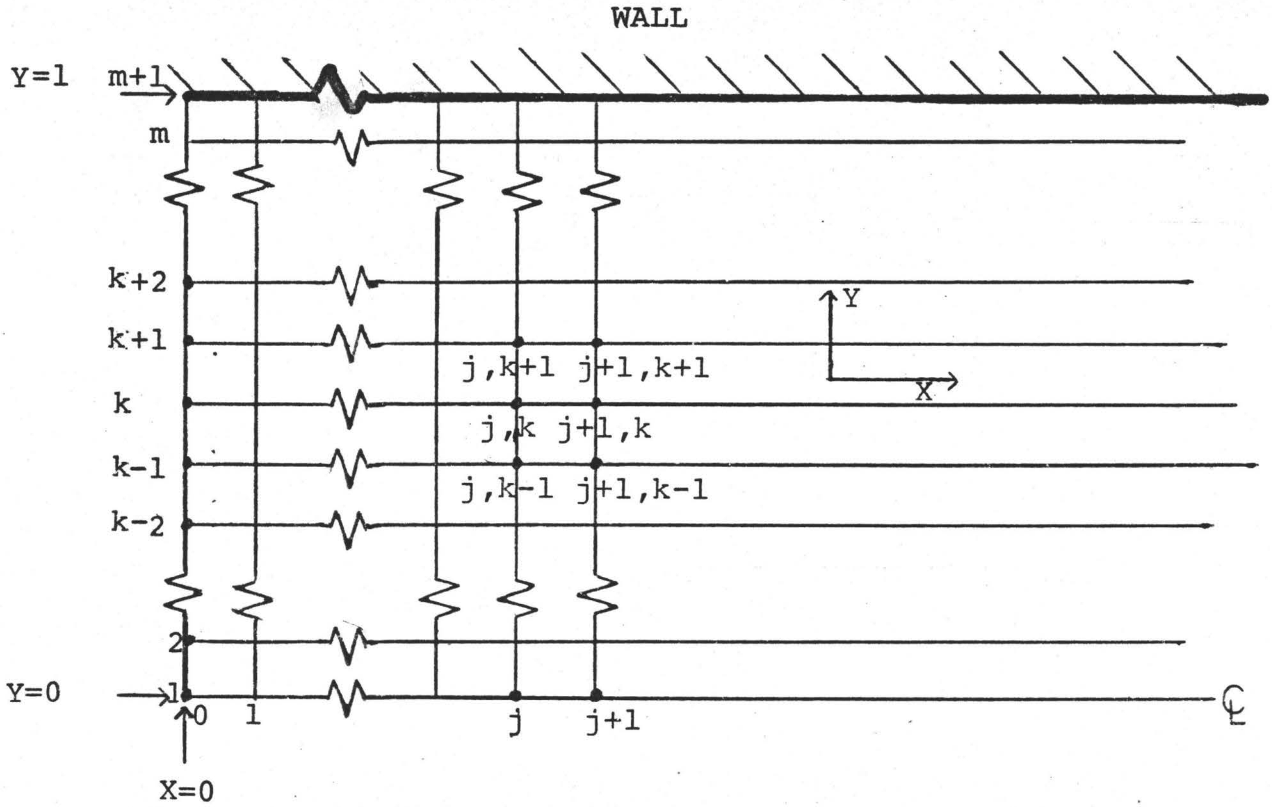


Figure I-3.2: Finite difference mesh

Finite difference representations are chosen in order to get linear equations. So for the continuity equation I-3.12, the following difference approximations are used.

$$\frac{\partial U}{\partial X} = \frac{U_{j+1,k+1} - U_{j,k+1} + U_{j+1,k} - U_{j,k}}{2\Delta X}$$

$$\frac{\partial V}{\partial Y} = \frac{V_{j+1,k+1} - V_{j+1,k}}{\Delta Y} \qquad \text{(I-3.17)}$$

In the integral form of the continuity equation, we use the Simpson integration formula which gives:

$$U_{j+1,1} + 4\sum_n U_{j+1,2n} + 2\sum_n U_{j+1,2n+1} = 3m \qquad 1 \leq n \leq m/2 \tag{I-3.18}$$

where m = total number of points across less 1

For the momentum equation, we have the following finite difference approximations:

$$\frac{\partial U}{\partial X} = \frac{U_{j+1,k} - U_{j,k}}{\Delta X}$$

$$\frac{\partial U}{\partial Y} = \frac{1}{2}\,\frac{U_{j,k+1} - U_{j,k-1}}{2\Delta Y} + \frac{1}{2}\,\frac{U_{j+1,k+1} - U_{j+1,k-1}}{2\Delta Y}$$

(I-3.19)

$$\frac{dP}{dX} = \frac{P_{j+1} - P_j}{\Delta X}$$

$$U = U_{j,k} \qquad \text{and} \qquad V = V_{j,k}$$

For the last term of the momentum equation, a secondary grid superimposed over the previous one is used along Y (See Figure I-3.3 below).

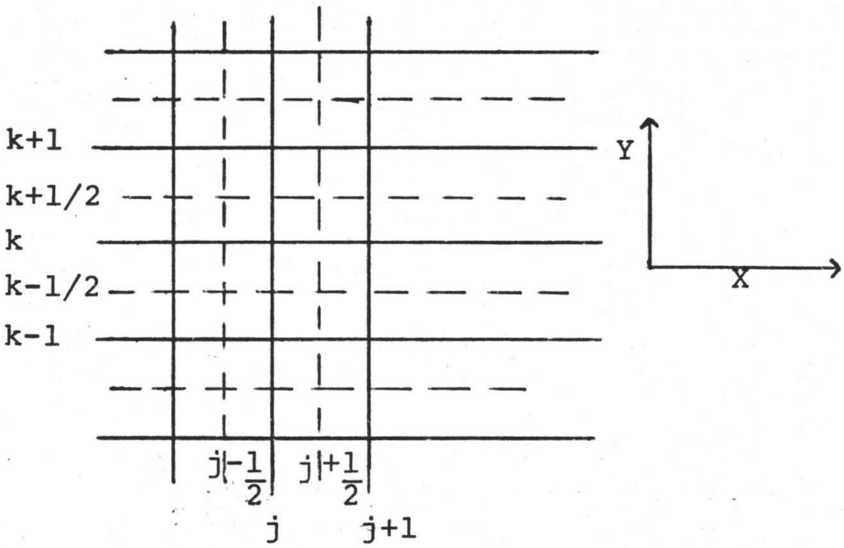


Figure I-3.3: Auxiliary mesh

Now the difference representation for the last term of the momentum equation is taken at (j+1/2, k) which is the central point of that grid, so we have:

$$\frac{\partial \tau}{\partial Y} = \frac{1}{2}\,\frac{\tau_{j,k+1/2} - \tau_{j,k-1/2}}{\Delta Y} + \frac{1}{2}\,\frac{\tau_{j+1,k+1/2} - \tau_{j+1,k-1/2}}{\Delta Y}$$

(I-3.20)

where

$$\tau_{j,k+1/2} = \left|\frac{U_{j,k+1} - U_{j,k}}{\Delta Y}\right|^{n-1} \frac{U_{j,k} - U_{j,k-1}}{\Delta Y}$$

$$\tau_{j,k-1/2} = \left|\frac{U_{j,k} - U_{j,k-1}}{\Delta Y}\right|^{n-1} \frac{U_{j,k} - U_{j,k-1}}{\Delta Y}$$

(I-3.21)

$$\tau_{j+1,k+1/2} = \left|\frac{U_{j,k+1} - U_{j,k}}{\Delta Y}\right|^{n-1} \frac{U_{j+1,k+1} - U_{j+1,k}}{\Delta Y}$$

$$\tau_{j+1,k-1/2} = \left|\frac{U_{j,k} - U_{j,k-1}}{\Delta Y}\right|^{n-1} \frac{U_{j+1,k} - U_{j+1,k-1}}{\Delta Y}$$

So the momentum equation after simplification gives (details shown in appendix I-A):

$$A_k \quad U_{j+1,k-1} + B_k \, U_{j+1,k} + C_k \, U_{j+1,k+1} + D_k \, P_{j+1} = E_k$$

(I-3.22)

$$\text{where } A_k = - \frac{V_{j,k}}{4\Delta Y} - \frac{|Gm|^{n-1}}{\Delta Y^2}$$

$$B_k = \frac{U_{j,k}}{\Delta X} + \frac{|Gp|^{n-1} + |Gm|^{n-1}}{\Delta Y^2}$$

$$C_k = \frac{V_{j,k}}{4\Delta Y} - \frac{|Gp|^{n-1}}{\Delta Y^2}$$

$$D_k = \frac{1}{\Delta X}$$

$$E_k = \frac{P_j}{\Delta X} + \frac{U_{j,k}^2}{\Delta X} - \frac{V_{j,k}}{4\Delta Y}\left(U_{j,k+1} - U_{j,k-1}\right) + \frac{|Gp|^{n-1}}{\Delta Y^2}$$

$$\left(+U_{j,n+1} - U_{j,n}\right) - \frac{|Gm|^{n-1}}{\Delta Y^2}\left(U_{j,n} - U_{j,n-1}\right)$$

$$\text{where } Gp = 2\,\frac{U_{j,k+1} - U_{j,k}}{\Delta Y}$$

$$Gm = 2\,\frac{U_{j,k} - U_{j,k-1}}{\Delta Y}$$

By combining the last equation with equation I-3.18, we obtain a set of "m" linear equations with "m" velocities plus "1" pressure which permits us to solve for those values as demonstrated in section I-3.4.

For the continuity equation, we have an explicit expression with respect to velocities along y if the axial velocities are known. Replacing I-3.17 in I-3.12, the following equation is obtained:

$$V_{j+1,k+1} = V_{j+1,k} - \frac{\Delta Y}{2\Delta X}\left(U_{j+1,k+1} + U_{j+1,k} - U_{j,k+1} - U_{j,k}\right)$$

(I-3.23)

Since $V_{j+1,1} = 0.0$ (Centerline boundary condition) then all the "V" can be computed directly.

I- 3. 3 Energy equation

If we make the same assumptions for the energy equation as we did previously for the hydrodynamic problem, we have the following energy equation

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \alpha \frac{\partial^2 t}{\partial y^2} \qquad \text{(I-3.24)}$$

where $\alpha = \frac{k}{\rho C_p}$ (thermal diffusivity)

The previous variable transformations (equation I-3.7 to I-3.11) are retained and the following one is added:

$$T = \frac{t-t_w}{t_o-t_w} \qquad \text{(I-3.25)}$$

where t_o : Entry temperature (T_o=1)

t_w : Wall temperature (T_w=0)

After simplification the following equation is obtained

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{2}{Pr} \frac{\partial^2 T}{\partial Y^2} \qquad \text{(I-3.26)}$$

$$\text{where } Pr = \frac{C_p\ K\left(\frac{2a}{u_o}\right)^{1-n}}{k}$$

(Non-Newtonian Prandtl Number)

It should be noted that when n=1 for a newtonian fluid the Prandtl number takes the usual form.

The boundary and initial conditions are as follows for the constant wall temperature problem:

B.C.: at $Y = 0$ and $X>0$: $\frac{\partial T}{\partial Y} = 0$

at $Y = 1$ and $X>0$: $T = 0$ (I-3.27)

I.C.: at $X = 0$ and $0<Y<1$: $T = 1$ (I-3.28)

Now the following finite difference approximations are used:

$$U = \frac{U_{j,k} + U_{j+1,k}}{2} \qquad V = \frac{V_{j,k} + V_{j+1,k}}{2}$$

$$\frac{\partial T}{\partial X} = \frac{T_{j+1,k} - T_{j,k}}{\Delta X}$$

(I-3.29)

$$\frac{\partial T}{\partial Y} = \frac{1}{2}\,\frac{T_{j,k+1} - T_{j,k-1}}{2\Delta Y} + \frac{1}{2}\,\frac{T_{j+1,k+1} - T_{j+1,k-1}}{2\Delta Y}$$

$$\frac{\partial^2 T}{\partial Y^2} = \frac{\left(T_{j+1,k+1} - 2T_{j+1,k} + T_{j+1,k-1}\right) + \left(T_{j,k+1} - 2T_{j,k} + T_{j,k-1}\right)}{2\Delta Y^2}$$

By substitution into equation I-3.26, we obtain a series of "m" linear equations with "m" temperature points to be solved; the resulting matrix is found in Appendix I-B. In the next section, the numerical procedure is described.

Other heat transfer quantities are used in this part I; they are the Bulk temperature T_b (mixing cup temperature) and the local Nusselt number. The Bulk temperature is defined by the following equation

$$T_b = \frac{\int_0^1 T_{j+1,k}\, U_{j+1,k}\, dY}{\bar{U}}$$

where $\bar{U}$ = average velocity = 1

The local Nusselt number is defined by the following:

$$Nu = \frac{hL}{k} \qquad (I-3.30)$$

where: h = heat transfer coefficient

L = characteristic length = 2a

k = thermal conductivity

From the heat flux at the wall, we have that

$$q_w = h\,(t_w - t_b) = \left(-k\,\frac{\partial t}{\partial y}\right)_{y=b} \qquad (I-3.31)$$

which, if we adopt the previous variable transformations, gives

$$Nu = \frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=1} 2}{T_b} \qquad (I-3.32)$$

The value of the gradient at the wall is calculated by the same finite difference approximation as used in Katotakis (1969), Vlachopoulos and Keung (1972) and Katotakis and Vlachopoulos (in press) at column "j+1".

$$\left(\frac{\partial T}{\partial Y}\right)_{Y=1} = \frac{1}{6\Delta Y}\left(-11T_{j+1,m+1} + 18T_{j+1,m} - 9T_{j+1,m-1} + 2T_{j+1,m-2}\right)$$

$$(I-3.33)$$

where m+1 is the grid point on the wall

I- 3. 4 Numerical procedure

To solve the hydrodynamic problem, the momentum (I-3.22) and integral continuity (I-3.18) equations are used to form a coefficient matrix; the unknows are the "m" axial velocities and one pressure so we have the following set of linear equations:

$$\begin{bmatrix} 1 & 4 & 2 & 4 & \cdots & & & 0 \\ B_1 & (C_1+A_1) & & & & & & D_1 \\ A_2 & B_2 & C_2 & & & & & D_2 \\ & A_3 & B_3 & C_3 & & & & D_3 \\ & & \ddots & \ddots & \ddots & & & \vdots \\ & & & A_k & B_k & C_k & & D_k \\ & & & & \ddots & \ddots & \ddots & \vdots \\ & & & & A_{m-1} & B_{m-1} & C_{m-1} & D_{m-1} \\ & & & & & A_m & B_m & D_m \end{bmatrix} \times \begin{bmatrix} U_{j+1,1} \\ U_{j+1,2} \\ U_{j+1,3} \\ \vdots \\ \\ U_{j+1,k} \\ \vdots \\ U_{j+1,m} \\ P_{j+1} \end{bmatrix} = \begin{bmatrix} 3m \\ E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_k \\ \vdots \\ E_{m-1} \\ E_m \end{bmatrix} \tag{I-3.34}$$

This set of equations was solved by a program using the Gauss elimination method written especially for the above coefficient matrix in order to minimize computing time (See Gauss subroutine in Appendix IV for details). Once the axial velocities are obtained, one can find the perpendicular velocities explicitly by using the normal form of the continuity equation (I-3.22); the y-velocity at the centerline is zero so the remaining ones can be computed from there on. This gives a fully implicit set of equations which means that once the starting profile is known, that is at column 0 (from the initial condition), one can compute directly column 1 and keep this "marching procedure" until enough information is generated (in our case, all calculations were stopped at X = 1.0).

For the energy equation, a slighty different coefficient matrix is obtained where there is no additional unknown apart from the "m" temperatures in a column : so we have the following system:

$$\begin{bmatrix} G_1 & (F_1+H_1) & & & & \\ F_2 & G_2 & H_2 & & & \\ & \ddots & \ddots & \ddots & & \\ & & F_k & G_k & H_k & \\ & & & \ddots & \ddots & \ddots \\ & & & F_{m-1} & G_{m-1} & H_{m-1} \\ & & & & F_m & G_m \end{bmatrix} \times \begin{bmatrix} T_{j+1,1} \\ T_{j+1,2} \\ \vdots \\ T_{j+1,k} \\ \vdots \\ T_{j+1,m-1} \\ T_{j+1,m} \end{bmatrix} = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_k \\ \vdots \\ J_{m-1} \\ J_m \end{bmatrix}$$

(I-3.35)

The coefficient matrix is known as a tridiagonal matrix. A recursion formula, Carnahan et al. (1969) and Hwang & Fan (1963), was used to solve this set; this formula which is based on Gaussian elimination, is also called Thomas's method.

I-3.5 Convergence, stability and step size

Problems with stability and convergence arise from the substitution in a differential equation of finite difference approximations; they are both discussed very well in Richtmyer (1957) for linear differential equations. The finite difference solutions of non-linear equations has yet to reach the same level of understanding as the linear ones so we must still rely on linear equation type of analysis to appreciate convergence and stability.

Convergence is the problem of getting the same exact solution from the differential equation and the difference equation; it arises from the fact that the difference equation is solved for a mesh with finite grid spaces. It is normally solved by reducing the mesh size up to a point where there is convergence of the finite difference solution; it simply means that the truncation error caused for example by taking a gradient between two points, will be less if those two points are closer. Evidently in doing so, computing time and experimental error must be taken in consideration in order to get a reasonable mesh size; with this in mind the network size of Table I-3.1 was used. This mesh gave solutions which would not vary more than .1% by reducing its size for normal calculations (not including calculations at very low flow behavior index); since between 1 and 2% is the normal accepted value of the experimental error, this was judged acceptable.

up to X	m	(ΔY)	ΔX
0.0004	80	(.0125)	0.00002
.002	80	(.0125)	0.0001
.01	40	(.025)	0.0005
.03	20	(.05)	0.001
.1	20	(.05)	0.005
1.0	20	(.05)	0.01

Table I-3.1: Mesh sizes of this work

A look at mesh sizes of Hwang and Fan (1965) is showing that they are slightly rougher than the ones in Table I-3.1; it was found that for flow-behavior index <.25 a finer mesh was necessary to stay within the .1% value.

Stability is the second problem, one must face when dealing with finite difference solutions; instability problems are coming mainly from round-off errors in the computations which are giving discrepancies between the exact solution of the difference equation and its numerical solution. Stability is normally ensured by checking that an error introduced in the computation will remain bounded. Katotakis (1969) has done a stability analysis on the boundary layer momentum equation using Von Newman's theory (See Richtmeyer 1957).

However this method of analysis of the growth of a general error term, must be used with linear equations so the momentum equations were linearized by assuming that coefficients of inertia terms are constant; furthermore in our work, the viscosity should be considered constant to get linear equations; Katotakis (1969) is doing the stability analysis of the resulting linear equations; however it must be stated that, since viscosity can vary by as much as 10^4, this analysis should be regarded only as a trend indicator.

The energy equation was analysed by Hwang & Fan (1965) and by Richtmeyer (1957). It is found to be always stable when the equation is considered linear. Although Hwang and Fan are recommending a finer mesh for the heat transfer part than the one used in table I-3.1, the present work has demonstrated that using a finer mesh size did not change the results of the temperature profile noticeably and the mesh used gave an excellent agreement for the bulk temperature with their result for newtonian fluid. Furthermore, it was found that using an auxiliary mesh for the heat transfer problem by computing, at the intermediate grid points linear approximations of the velocities, was causing instabilities in the form of harmonics; this phenomenon was detected by an oscillating Nusselt number. So the same mesh size was used for the hydrodynamic and the heat transfer set of equations.

I- 4. RESULTS AND DISCUSSION

I-4.1 The hydrodynamic problem

A) Newtonian case

As seen in the literature survey, the parallel plates entry flow problem for the newtonian case has been solved by numerous methods. Van Dyke (1970) and McDonald et al. (1972) in their respective paper are offering excellent comparisons of the development of the center-line velocity. Figure I-4.1 is showing the good agreement between Wang & Longwell, Bodoia & Osterle (1961) and the present work; however the full numerical solutions of Brandt & Gillis (1964) and McDonald et al. (1972) are showing discrepancy with the previous three curves. This discrepancy with the full numerical solution of Wang & Longwell seems to be due to different boundary condition (vorticity is zero for W. & L. and "v" is zero for B. & G. and M. et al.). Whereas the agreement of boundary layer numerical solutions is good at Re = 75 for the no vorticity case, Brandt and Gillis (1964) are showing that a similar agreement exist at Re>300 for zero perpendicular velocity case; however since it is very difficult to say which solution is best corresponding to reality, our results should apply whenever the Reynolds number is over 200. In figure I-4.2, the velocity development is shown which is almost undistinguishable from the one of Bodoia&Osterle

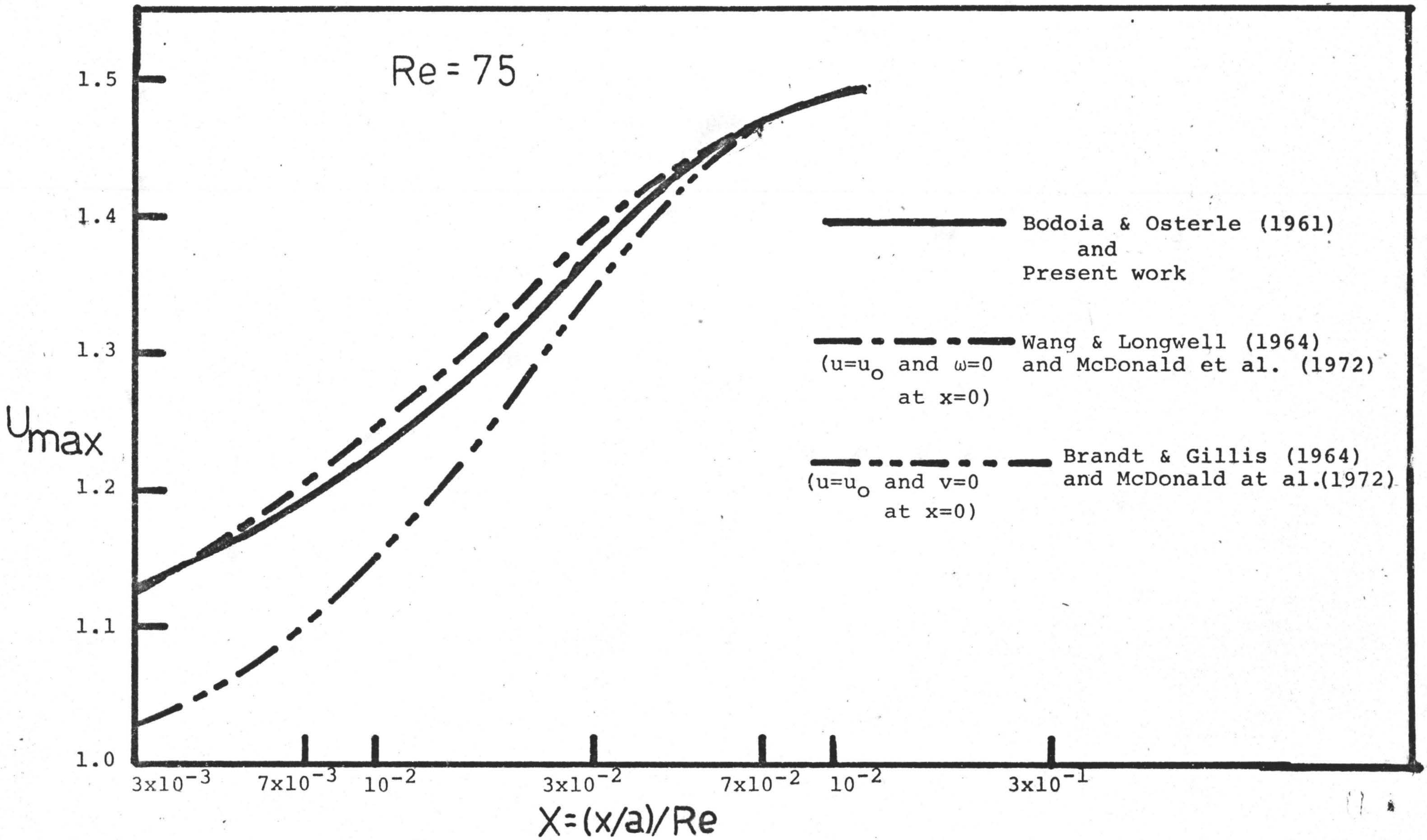


Figure I-4.1: U_{max} vs X

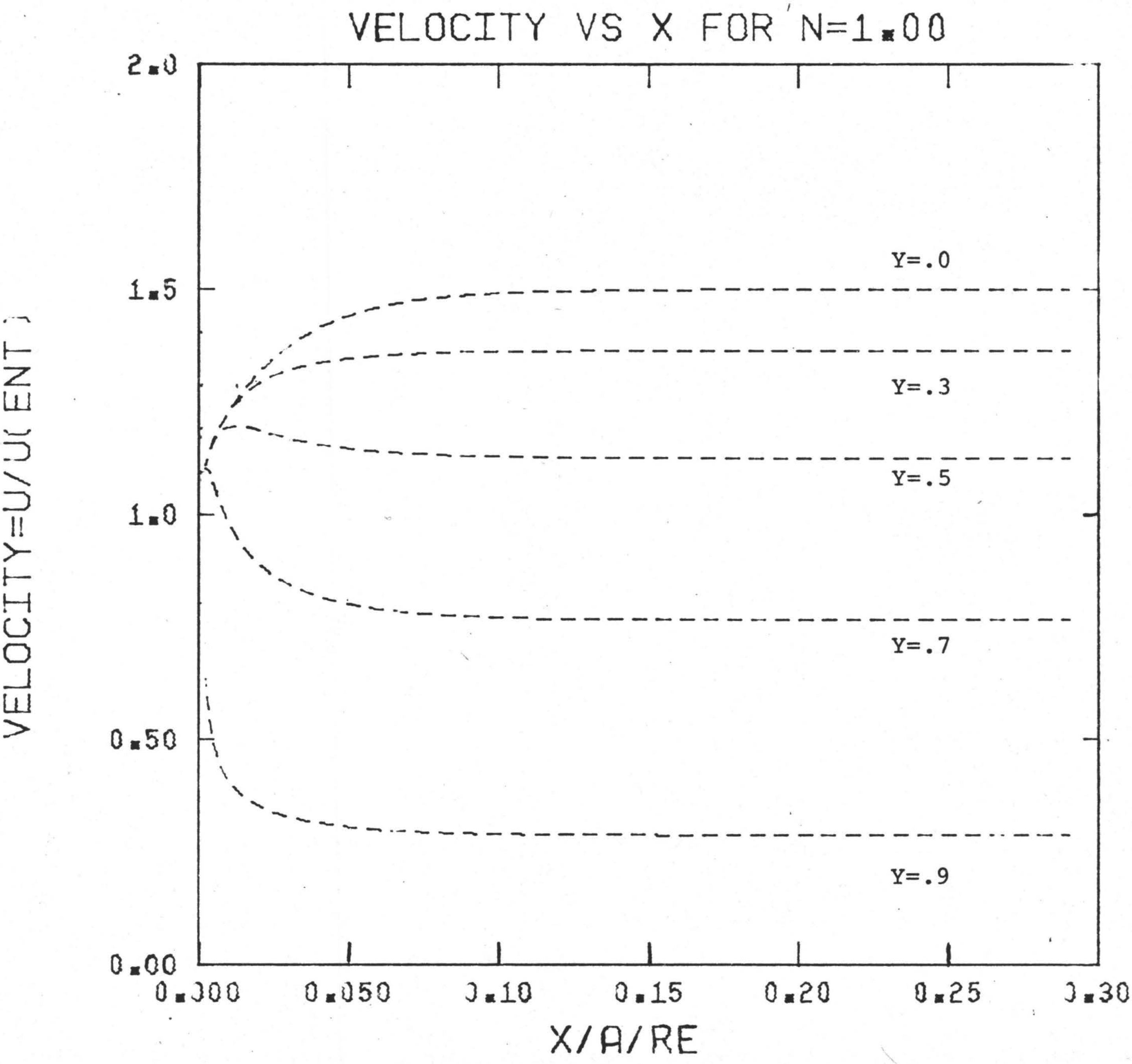


Figure I-4.2: Velocity development

The entry length is defined as the length of the channel taken by the centerline axial velocity to reach 99% of its fully developed value. We can see that the results of the present work are close to Hwang and Fan results and in the same range as other workers.

$X_{e99} = (x/a)$ Re	Author
.0800	Schlichting (1934)
.0838	Present work
.0844	Hwang and Fan (1961)
.0880	Bodoia and Osterle (1961)
.0884	Brandt and Gillis (1966)
.0908	Roidt and Cess (1962)

Table I-4.1: Entry length for 99% of final centerline velocity

The excess pressure drop "P_d" is calculated the following way for a newtonian fluid.

$$P_d = \lim_{X\to\infty} [P(0) - P(X) - 6X] \qquad \text{(I-4.1)}$$

P_d is due to departure from parabolic flow in the inlet region of the entrance flow. Table I-4.2 shows results of other workers

P_d	
.271	Kinetic-energy end correction
.300	Schlichting (1934)
.312	Hwang and Fan (1961)
.315	Roidt and Cess (1962)
.331	Brandt and Gillis (1966)
.338	Bodoia and Osterle (1961)
.343	Present work

Table I-4.2: Excess pressure drop

For Brandt and Gillis, the value is given for Re = ∞ since the boundary layer assumptions are valid for high Reynolds number. We can see that the results of the present work are pretty well in same range as the ones obtained by other workers. The final value of the pressure gradient calculated from this work is exactly 6.000 which is the value that can be calculated for the Poiseuille flow analytically.

B) Non-Newtonian Fluids

There are not many solutions on power law fluids for channels so we will limit the comparison to the work done by Collins and Schowalter (1963). For n=1 (newtonian fluid) their entry length is .069 compared .067 for this work (this entry length is 98% of the fully developed center-line velocity) and the excess pressure lost is .34 compared to .343 here.

From this, we can conclude that our results are compatible with those of Collins and Schowalter, so a comparison for non-newtonian fluids can be undertaken. Collins and Schowalter are solving the problem with an extension of the Schlichting's analytical method so the fact that for pseudoplastic fluid (n<1.0), the viscosity becomes infinite when the axial velocity gradient is zero, did not give them any problem. In a numerical solution like the one presented here, this limitation arises as we saw earlier, in the calculations because of the finite difference form of the constitutive equation. The minimum value of the gradient, in solving the problem, was taken in dimensionless form as:

$$\left|\frac{\partial U}{\partial Y}\right| = \dot{\gamma}_m \qquad \text{(I-4.2)}$$

This procedure was used by Ozoe and Churchill (1972) for free convection problems. A range of minimum gradient from .001 to 0.5 was used in this work. $\gamma_m{}^{n-1}$ is then

the value of the factor by which we must multiply the viscosity at unity shear rate (the viscosity at unity shear rate is the consistency index of the power law equation:K) to get the true viscosity. In the figure I-4.3, we can see the effect of the value of the minimum gradient on the entry length vs flow behavior index curves as well as a comparison with Collins and Schowalter results. This figure shows that as the value of γ_m increases, the maximum of the curves are decreasing and leaning a bit toward the left; values for $n > .75$ are not changing very much; these effects are easily explained by the fact that for flow behavior index the profiles are flatter so the minimum gradient value is used, in the numerical solution, not only at the centerline but also in the flat profile region around the centerline ; as soon as the value of the minimum

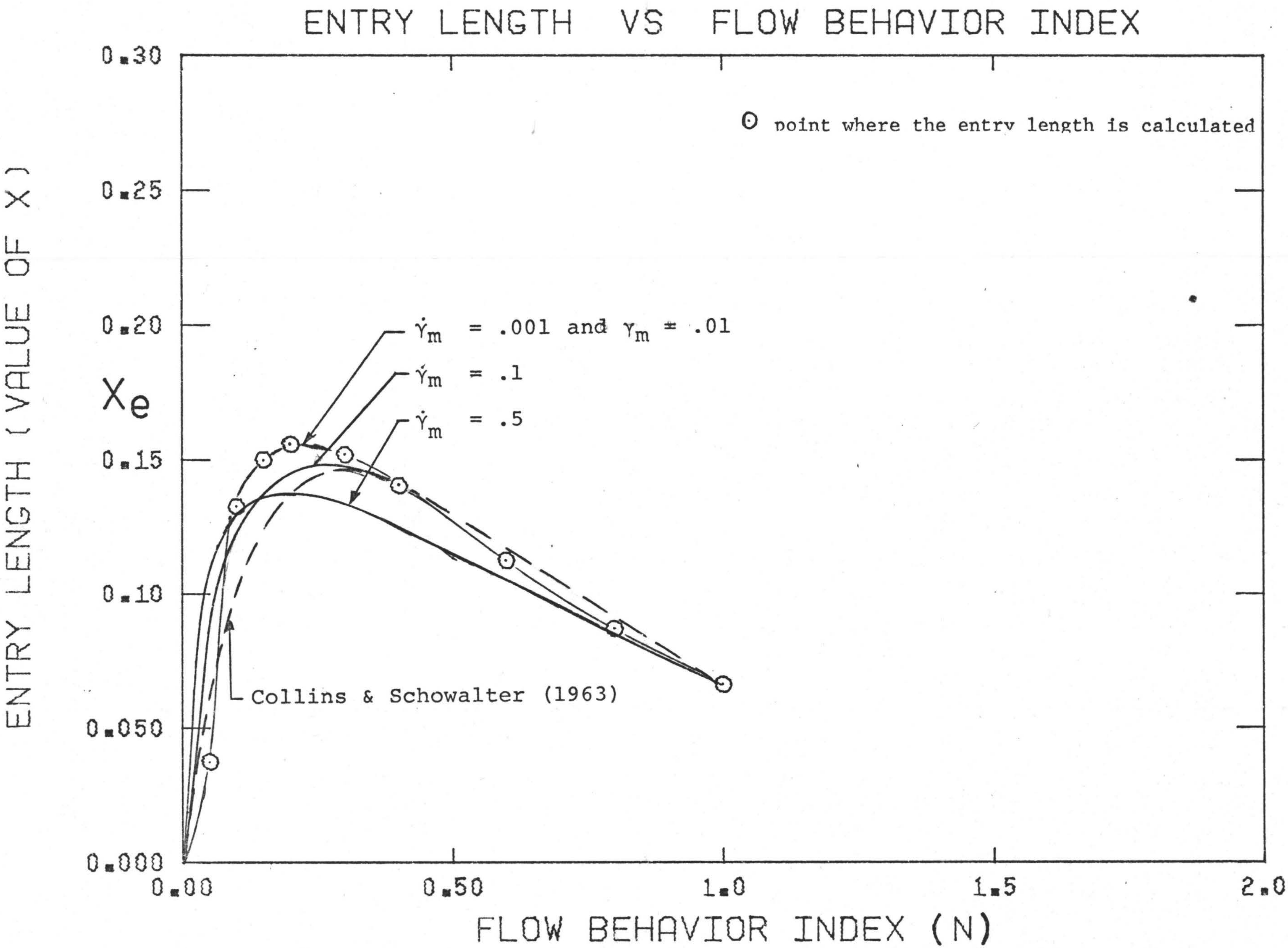


Figure I-4.3: Entry length vs flow behavior index gradient goes below .01, there is very little change in the entry length values. The discrepancy at low flow behavior index with Collins and Schowalter results, can partially be attributed to the fact that only one point (n=.2) was calculated in that region and a full curve from 0.4 to 0.0 was drawn.

The entry pressure drop coefficient was found relatively insensitive to the value of the minimum gradient except when this value was quite high e.g. $\gamma_m = 0.5$. In figure I-4.4, we can see that a noticeable difference in the end pressure drop correction can be seen only at low flow behavior index. In this figure, we are also comparing with results of Collins and Schowalter (1963) and Tiu et al. (1972) whose method is based on a previous paper written by Lungren et al. (1964), using a linearization method of momentum and machanical energy equations. We can see that our results are in very close agreement with Tiu's paper but shows some differences with Collins and Schowalter. In table I-4.3, we can see that for minimum gradient of .01 (and less) the pressure gradient (which is constant after the immediate entry) is very close to the exact analytical one where as for 0.5 the value begins to differ appreciably. The implication of this, is that, for pressure calculation, the value of the minimum gradient at which the fluid becomes newtonian at low shear stress does not have much effect.

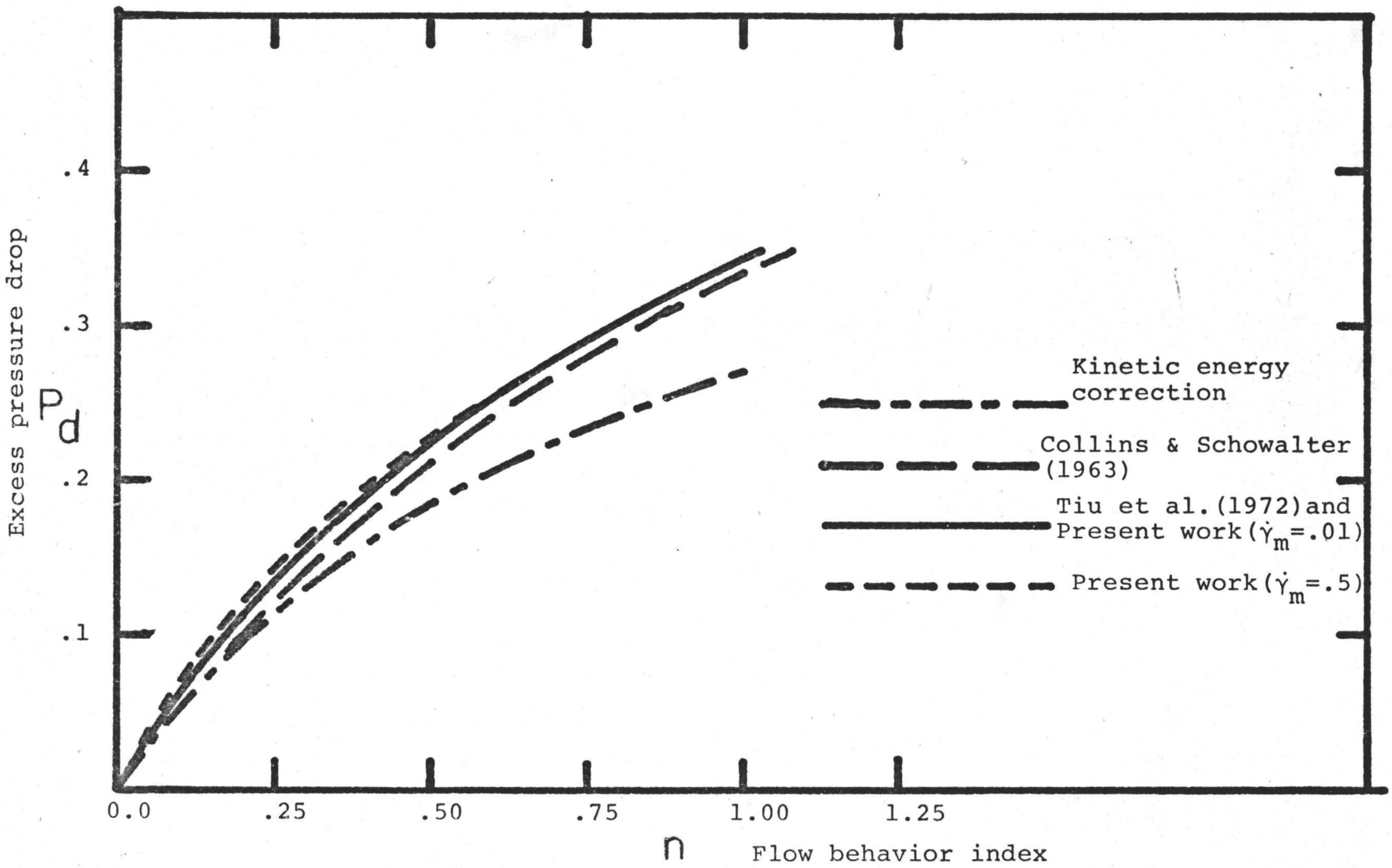


Figure I-4.4: Entry pressure drop vs flow behavior index

n / $\dot{\gamma}_m$	.25	.50	.75	1.0
.5	1.8510	2.8216	4.1460	6.000
.01	1.8620	2.8280	4.1490	6.000
Exact analytical value	1.8612	2.8284	4.1489	6.000

Table I-4.3: Pressure gradient

In appendix I-C the reader will find some of the results not discussed in this section. Axial velocity developments are presented with respect to the axial distance for various flow behavior index at 5 different distances from the centerline (Y = 0 is the centerline). Then pressures are plotted also along the axial variable "X".

I- 4. 2 Heat transfer problem

For heat transfer we need to look at three variables. The first two are γ_m and n which are related to the constitutive equation used here and the third is the Prandtl number (Pr). In figure I-4.5, we can see what influence the flow behavior index has on the bulk temperature. Change of the minimum gradient has no noticeable effect on bulk temperature as we can see by comparing figure I-4.5 to I-4.6 (in fact we have not more than 3% variation in bulk temperature at a flow behavior index of .25). The same is not true for the

Nusselt number as we can see in Table I-4.4 where we compare asymptotic local Nusselt number for different minimum gradients and different flow behavior index with values given by Vlachopoulos and Keung (1972) whose calculations are for fully developed velocity profiles. The different values of the Nusselt number for different minimum gradient is probably caused by the fact that

n \ $\dot{\gamma}_m$	0.5	0.1	0.01	0.001	Vlachopoulos and Keung (1972)
.25	4.162	4.212	4.218	4.218	4.22
.50	3.954	3.965	3.966	3.966	3.97
.75	3.839	3.841	3.841	3.841	----
1.00	3.763	3.763	3.763	3.763	3.767

Table I-4.4: Asymptotic local Nusselt number

a more parabolic like velocity profile is encountered at high minimum gradient causing lower velocities near the wall (to maintain a constant flow rate). Those lower velocities near the wall are in turn affecting the value of the wall temperature gradient which is part of the Nusselt number.

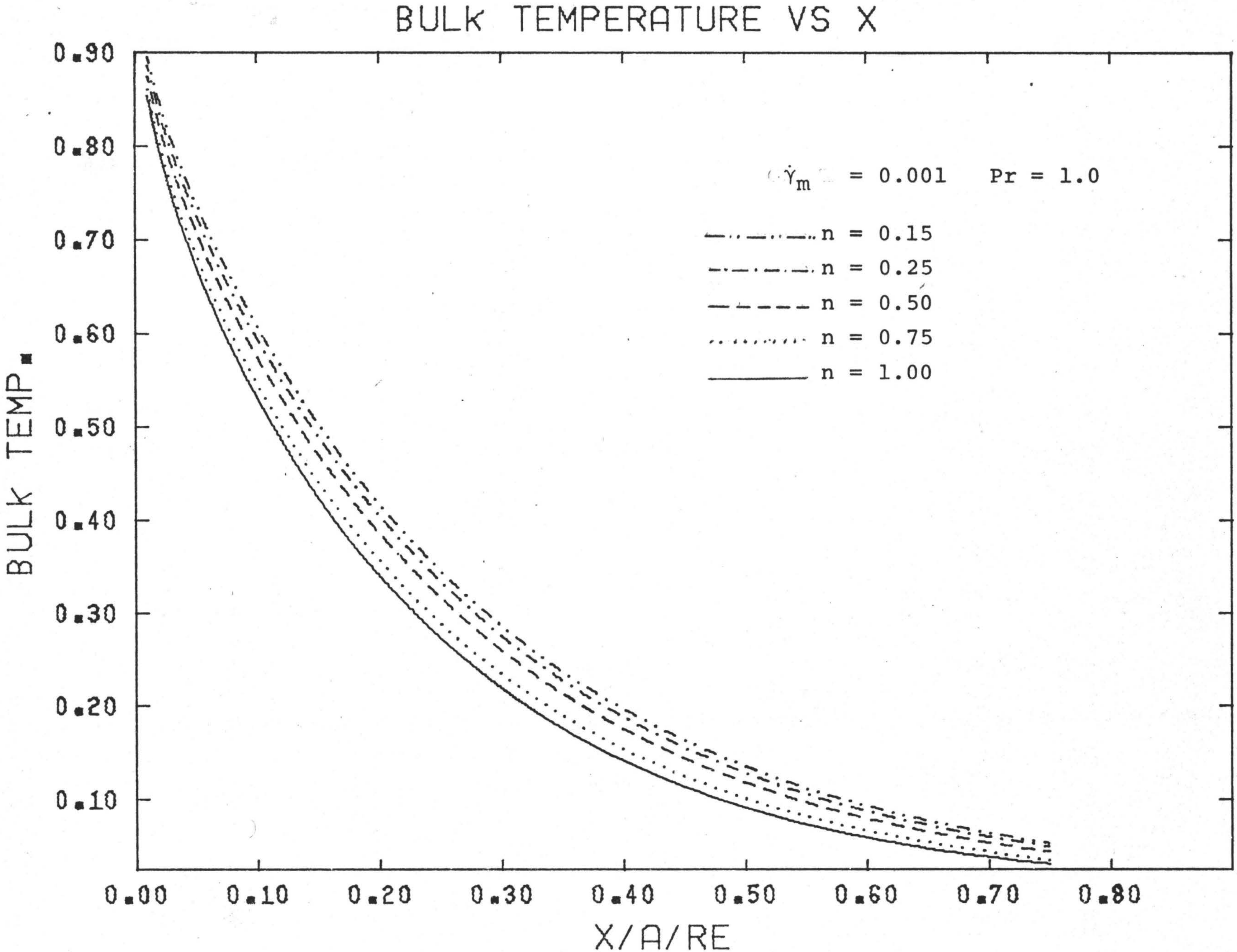


FIGURE I-4.5 BULK TEMPERATURE VS X

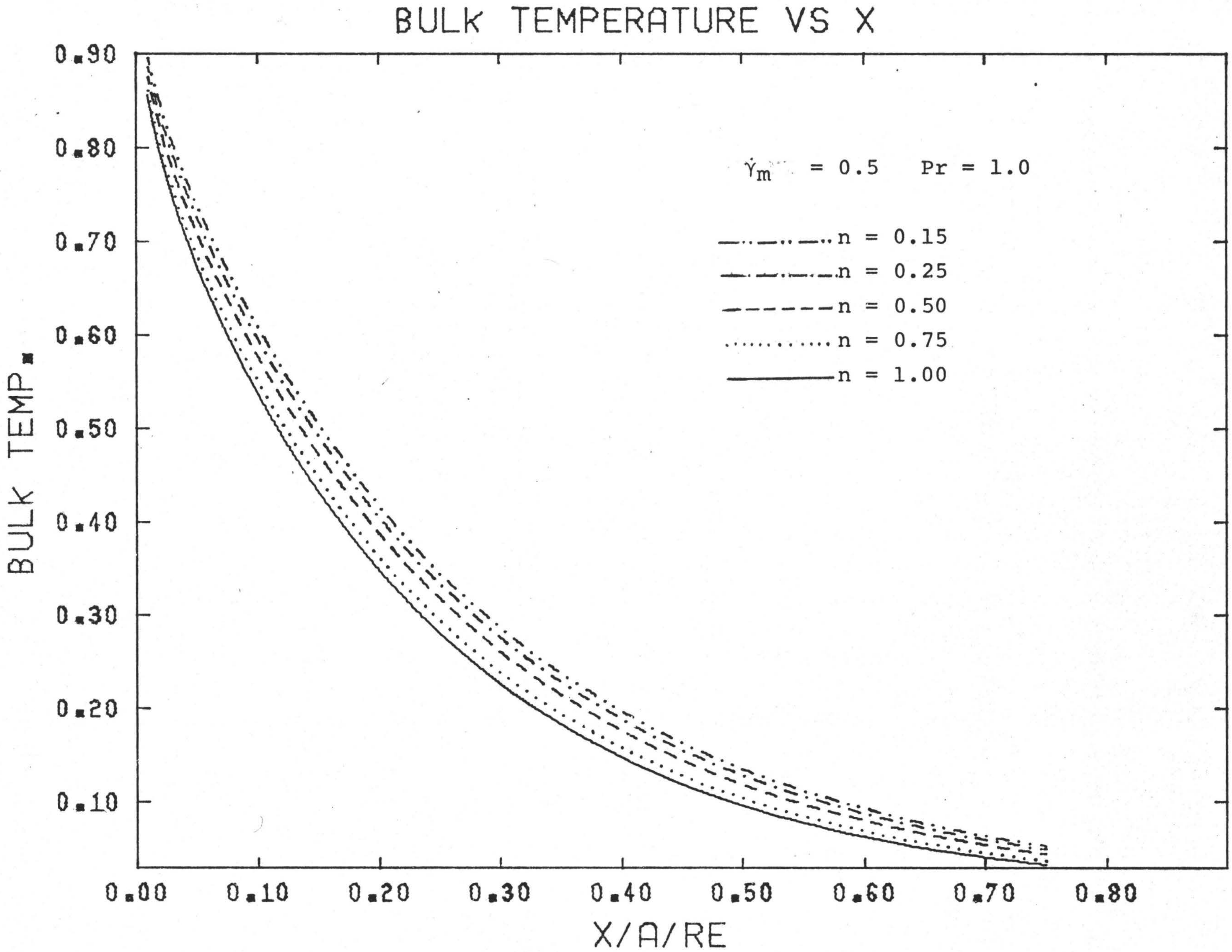


FIGURE I-4.6 BULK TEMPERATURE VS X

The bulk temperature values are similar to results of Hwang and Fan (1965); a high Prandtl number has the effect of lowering the heat transfer rate; this can be seen in figure I-4.7 to I-4.9 where the bulk temperature decrease is slower as the Prandtl number is increased. The Nusselt numbers have the same patterns except that they take longer to attain their asymptotic values at high Prandtl number; when the Prandtl is low the bulk temperature falls very fast to zero making the calculation of Nusselt number inexact because we are dealing with very small temperatures when we are calculating the temperature gradient at the wall. Some results plotted in a log-log form can be seen in figure I-4.10. More results are available in Appendix I-C concerning bulk temperature and Nusselt Number for various values of the parameters.

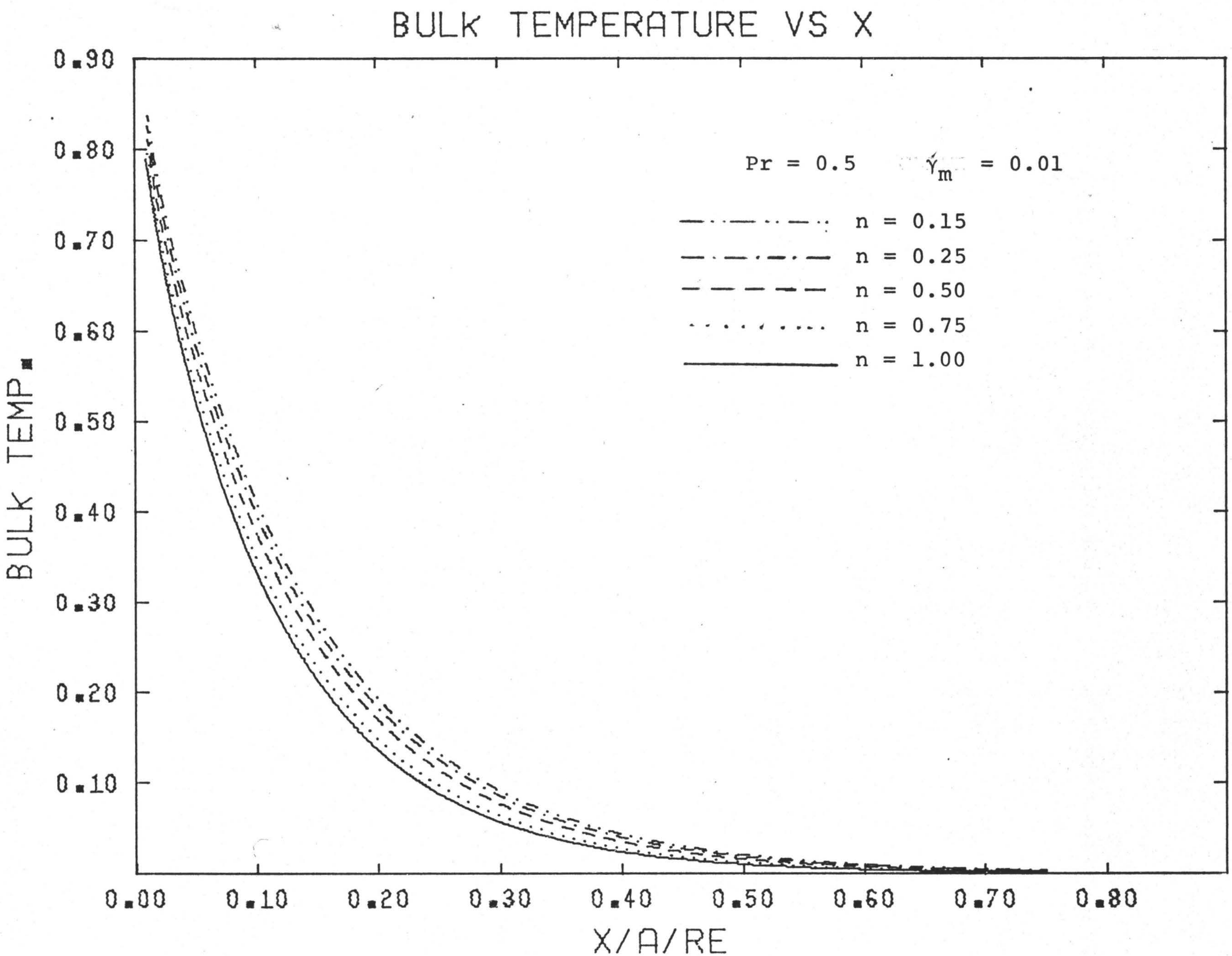


FIGURE I-4.7: BULK TEMPERATURE VS X

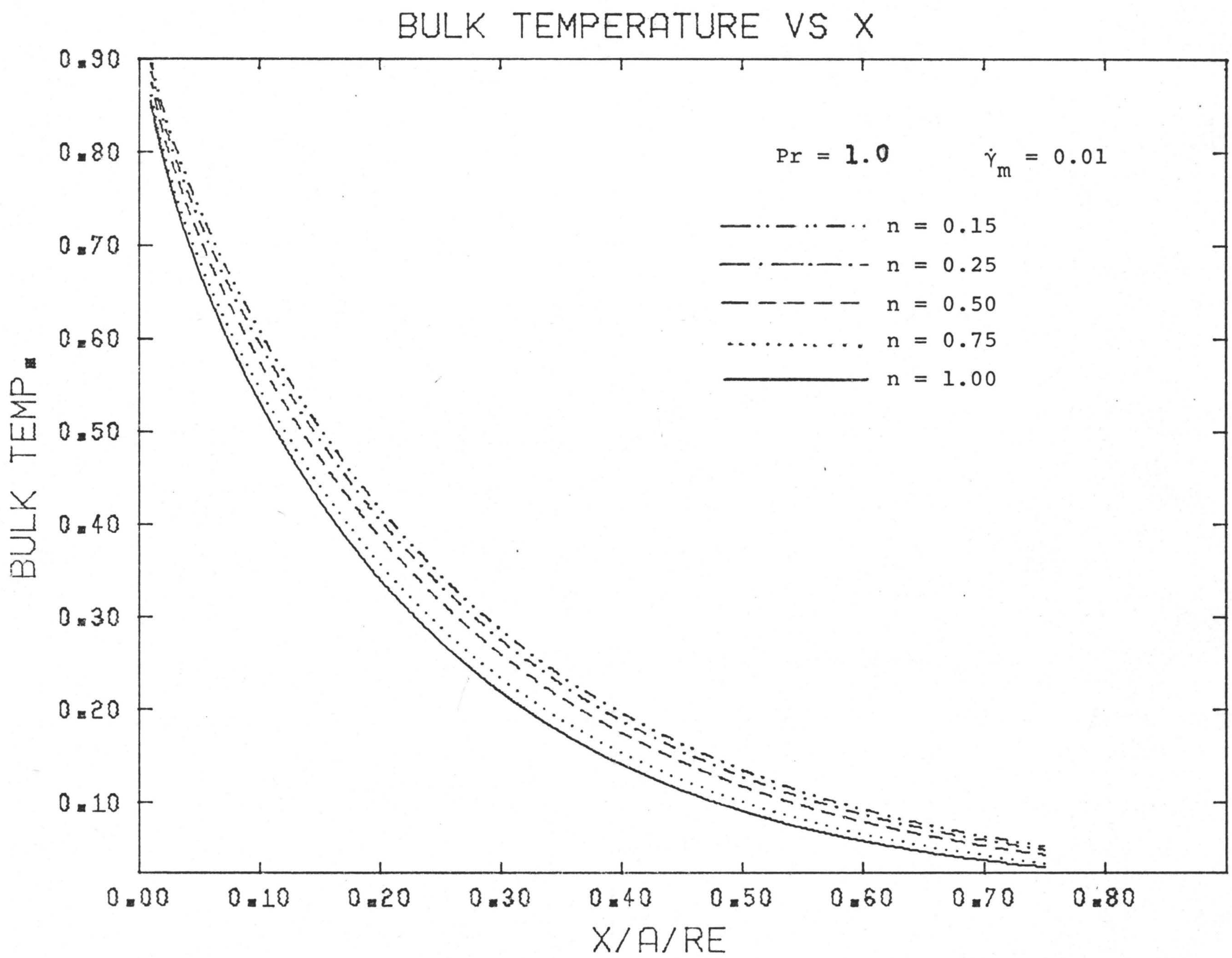


FIGURE I-4.8: BULK TEMPERATURE VS X

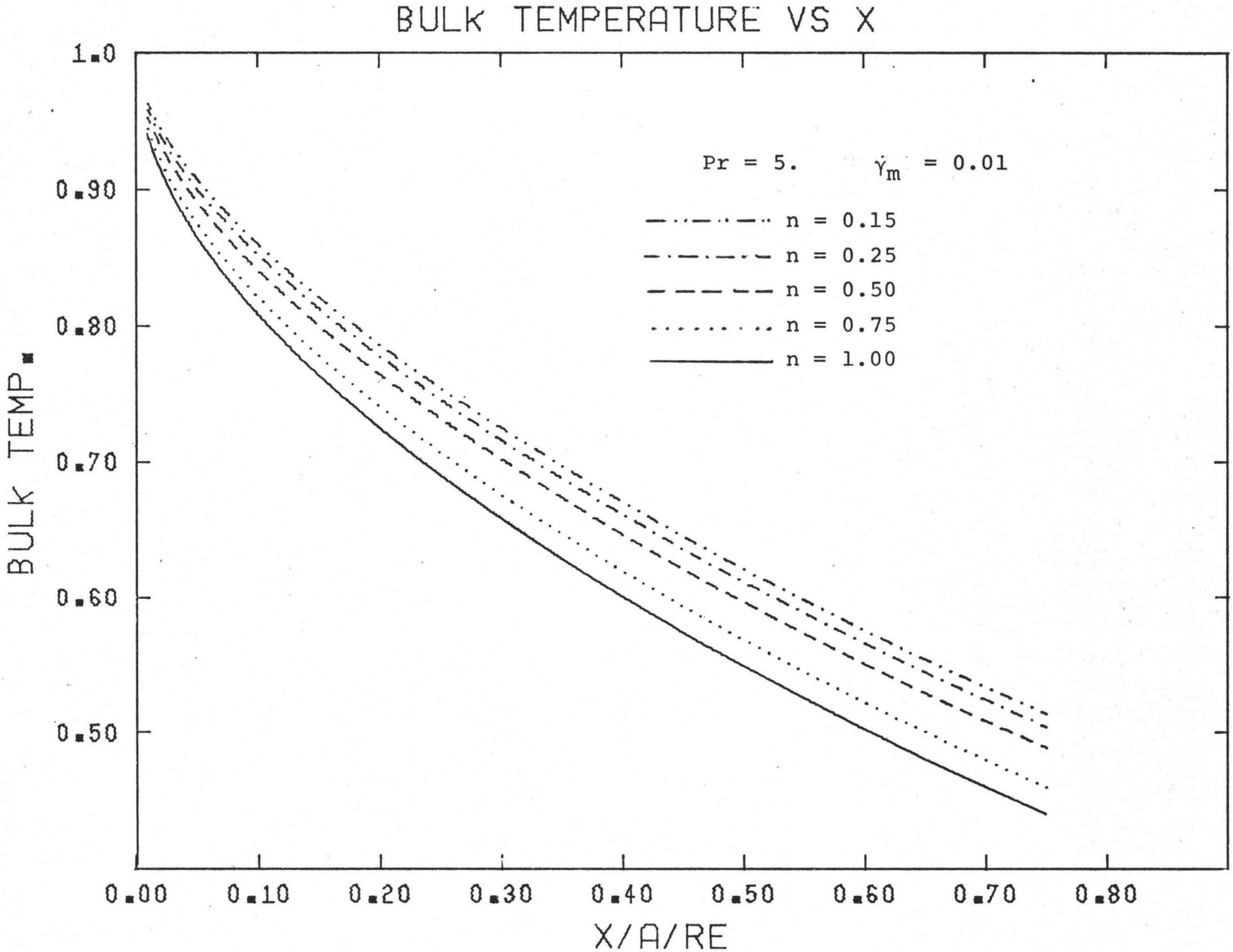


FIGURE I-4.9 BULK TEMPERATURE VS X

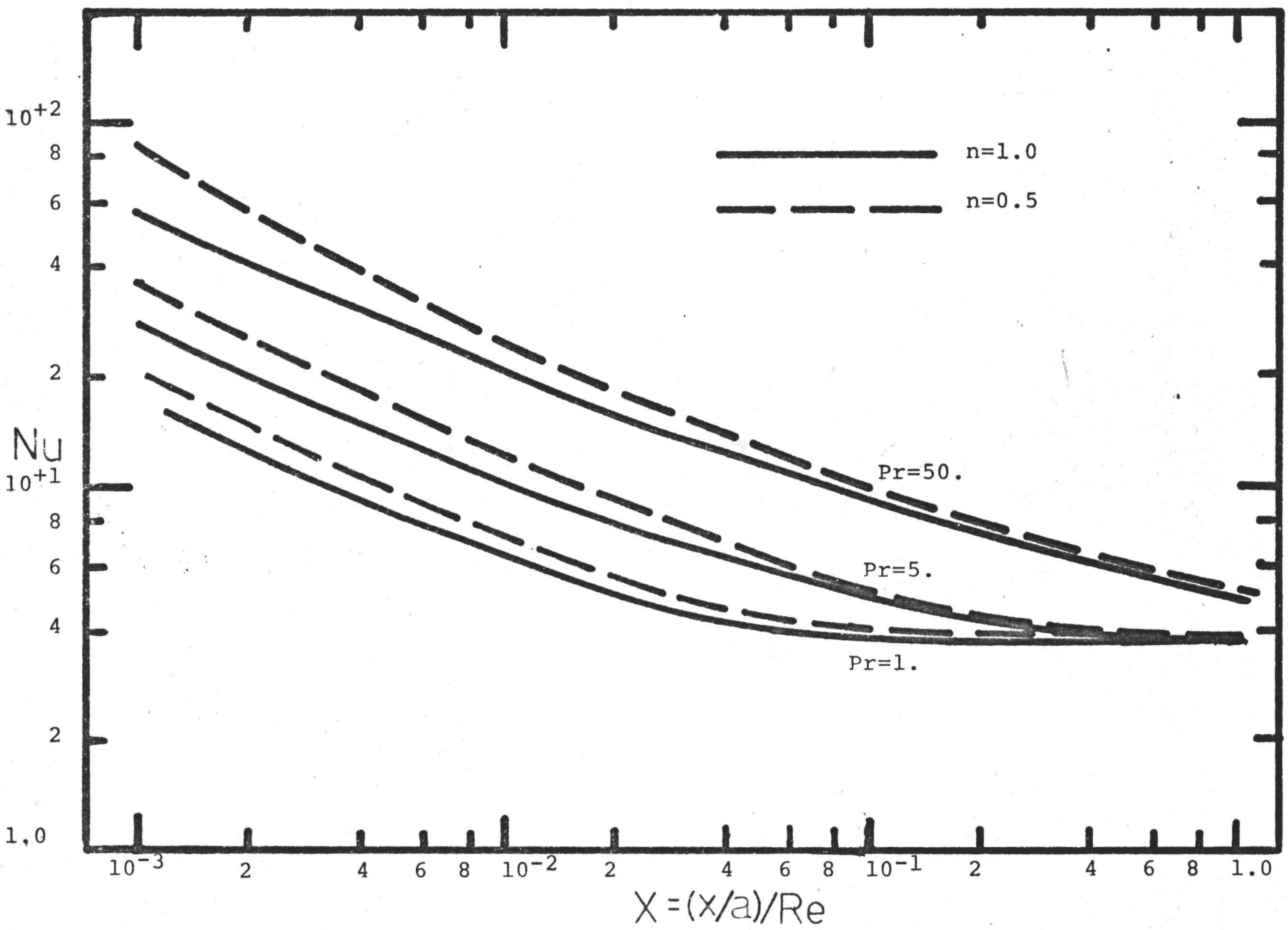


Figure I-4.10: Local Nusselt Number vs X

I- 5 CONCLUSIONS AND RECOMMENDATIONS

In this part I pseudoplastic fluids have been studied for entry flow with a finite difference method; for Newtonian fluids, good agreement was found to exist with previous results in the hydrodynamic and heat transfer problem. In the non-newtonian case, comparisons with Collins & Schowalter (1963) and Tiu et al. (1972) gave excellent agreement with the present work for entry length and excess pressure drops except in low flow behavior index for the entry length. Heat transfer was found to decrease by lower flow behavior index in accordance with other results.

It must be noted that the present method can be used with practically any type of entry profile. In fact a sixth order polynomial entry profile was tried with satisfactory results.

Improving the present work could be done by including viscoelastic fluids; it is suggested that this be done employing something similar to a deviatoric stress tensor as used in Balmer and Kauzlarich (1972). Another improvement would be to extend this work to channel flow where side effects are important; this case would probably help the understanding of single screw extruders particularly if heat transfer is included. Further viscous dissipation effects should be included.

NOTATION

a	:	Half space between plates
A_k	:	Series of constants defined in equation I-3.20
B_k	:	Series of constants defined in equation I-3.20
C_k	:	Series of constants defined in equation I-3.20
C_p	:	Specific heat
D_k	:	Series of constants defined in equation I-3.20
E_k	:	Series of constants defined in equation I-3.20
F_k	:	Series of constants defined in equation I-B.1
G_k	:	Series of constants defined in equation I-B.1
Gm	:	Constant defined in equation I-3.20
Gp	:	Constant defined in equation I-3.20
h	:	Heat transfer coefficient
H_k	:	Series of constants defined in equation I-B.1
I_2	:	Second invariant of $\underline{\underline{\Delta}}$
J_k	:	Series of constants defined in equation I-B.1
k	:	Thermal conductivity
K	:	Power law consistency index
L	:	Characteristic length
m	:	Number of grid points across flow less one
n	:	Flow behavior index
Nu	:	Nusselt number
p	:	Pressure
p_o	:	Pressure at the entry
P	:	Dimensionless pressure $(=p-p_o/\rho u_o{}^2)$

P_d : Excess pressure drop

Pr : Prandtl number

q_w : Heat flux at the wall

Re : Reynolds number

t : Temperature

t_o : Temperature at the entry

t_w : Temperature at the wall

T : Dimensionless temperature $(=(t-t_w)/(t_o-t_w))$

T_b : Bulk temperature

u : Axial velocity (in x-direction)

u_o : Axial velocity at entry (uniform)

U : Dimensionless axial velocity $(=u/u_o)$

$\bar{U}$: Average axial velocity (dimensionless)

v : Velocity in y-direction

V : Dimensionless velocity in y-direction $\left(=\frac{(2a)^n \, u_o{}^{1-n} \, \rho \, v}{k}\right)$

x : Coordinate along the channel

X : Dimensionless coordinate along the channel (=(y/a/Re))

X_e : Entry length at 98% of the fully developed centerline velocity

X_{e99} : Entry length at 99% of the fully developed centerline velocity

y : Coordinate across channel

Y : Dimensionless coordinate across channel (=y/a)

z : Coordinate along the width of plates

Greek letters

α : Thermal diffusibity $(=k/\rho C_p)$

γ_m : Minimum gradient

$\underline{\underline{\Delta}}$: Rate of deformation tensor

ΔX : Increment along x

ΔY : Increment along y

τ : Stress point (in the grid)

$\underline{\underline{\tau}}$: Stress tensor

ρ : Fluid density

ω : Vorticity

Subscript

j : For mesh point in x-direction

k : For mesh point in y-direction

m : Number of grid points across flow less one

PART II

CONVERGING FLOWS FOR PSEUDOPLASTIC FLUIDS

II- 1 INTRODUCTION

Converging flows of newtonian fluids have been studied for a while since the Navier-Stokes equations in this case can be solved analytically either with, or without the boundary layer approximations, and because analytical solutions were the main method of solution before the advent of computers in the late forties. Since then some numerical solutions were developed and compared to the exact solutions.

In this second part, converging flows for pseudoplastic (power law fluid) and newtonian fluids are analysed by a finite difference integration method. Solutions for newtonian fluids will be presented and compared to the few available results found in the literature. They are produced in order to ascertain the attainability of exact fully developed velocity profile at various flow conditions; this aspect of the problem seems to have been neglected by preceeding authors.

In the first section of this part, a survey of the current literature is presented, followed by a second section where the solution of the problem is presented. Finally the results of this work are outlined and discussed in a last section.

II- 2 LITERATURE SURVEY AND METHODS OF SOLUTION

II- 2. 1 Analytical solutions

One of the few exact analytical solutions of the Navier-Stokes equations is about flow through converging and diverging channels. The solution was found independently by two workers, G. Hamel (1916) and G. Jeffery (1915); they reduced the Navier-Stokes equations to an ordinary differential equation which in turn was solved through the use of elliptic functions. Millsaps and Pohlhausen (1953) extended the calculations of Jeffery and Hamel for the hydrodynamic part and solved the energy equation transformed into an ordinary differential equation with a finite difference method.

Another method of solving the problem is to make the boundary layer assumptions and therefore simplify the motion equations. Again they are solved exactly; that is what Pohlhausen (1921) did in an early paper. Reeves and Kippenhan (1962) in a subsequent paper are presenting a very good comparison of the two methods; their results show that the boundary layer velocity distribution is very close to the exact solution for a Reynolds number of 50; so we can conclude that for Reynolds number higher than that value the boundary layer assumptions will give solutions very close to the exact value.

There are a few more approximate solutions to converging flow problem. Williams (1963) is presenting one in which, as he points out, "the equations of motion for flow in slender channels at moderate or high Reynolds number are identical in form to the boundary layer equations", although he goes on to say that those two solutions are not the same because of different boundary conditions, his results (At Re = 600, 1384 and 5000) are the same as Jeffery-Hamel's exact solution. In a more recent paper, Balmer and Kauzlarich (1971) are presenting results for an elastic fluid sheared in its newtonian region; even through they are using a power law constitutive equation, their exponent must be one as a condition for transforming a partial differential equation describing the flow, into an ordinary one by similarity analysis; this of course, limits their results.

All the solutions discussed up to this point, are for fully developed flow; the next ones including those in the next section using numerical methods, are solved with entry profile which means that we are able to check if, at certain flow conditions, we are effectively reaching the fully developed profiles; furthermore velocity distributions can be followed as the fluid proceeds in the converging channel.

Atabek (1972) linearized the two momentum equations in order to solve for arbitrary entry profile; the weak points of his analytical solution seem to be the assumptions that the fully developed profile is obtainable at the apex of the converging plates and that the velocity distribution is taken as a perturbation of the fully developed profile throughout.

II- 2. 2 Numerical methods

Sutterby (1965) was one of the first workers to develop a numerical method for converging flows. He simplified the Navier-Stokes equations with approximations and then solved those equations by finite differences; unfortunately his results are for converging tubes and for initial parabolic profile making comparisons almost impossible to make with our results.

Another paper was published by Yang and Price (1972) on converging plane-walled channels for newtonian fluids. The numerical solution presented here is similar to their solution; as a matter of fact our equations for the newtonian case are identical and the results are almost the same. Since they seem to have employed a rougher mesh than the one used here, a comparison is presented with their results to see the effect of the grid size on the convergence of the solution.

II- 3 SOLUTION OF THE PROBLEM

II- 3. 1 Constitutive equation

The constitutive equation to be used will be the power law equation. Since we are dealing with polar coordinates we can expect that this constitutive equation will be quite complex; approximations will be made in order to simplify this equation, in a way similar to Sutterby (1965); first the assumption of purely radial flow is made (that assumption was made also by Hamel and Jeffery to solve analytically for newtonian fluid).

So we have that:

$$u = \frac{F(\theta)}{r} \qquad \text{and } v = 0 \qquad \text{(II-3.1)}$$

where $F(\theta)$ is a function of θ only

The physical system is represented in figure II-3.1:

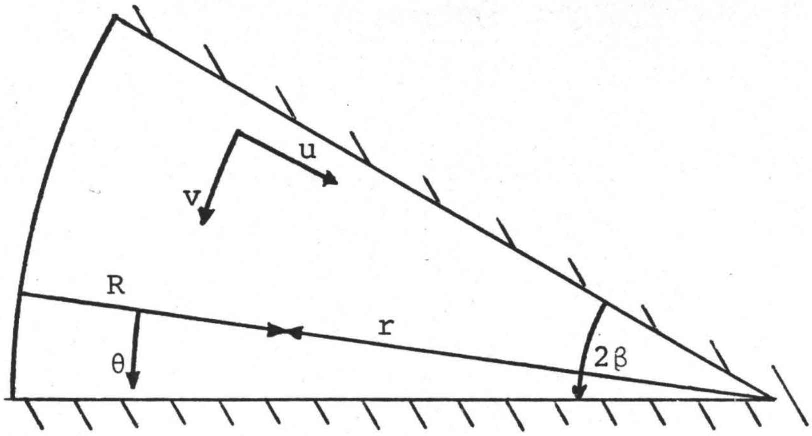


Figure II-3.1: Physical system

We know that for power law fluids we have the following stress-shear relation:

$$\underline{\underline{\tau}} = K \left| \sqrt{\frac{I_2}{2}} \right|^{n-1} \underline{\underline{\Delta}} \qquad \text{(II-3.2)}$$

where $\underline{\underline{\tau}}$ = stress tensor

I_2 = second invariant of $\underline{\underline{\Delta}}$

K = power law consistency index

n = flow behavior index

$\underline{\underline{\Delta}}$ = rate of deformation tensor (symmetrical)

$$\text{where } \underline{\underline{\Delta}} = \begin{bmatrix} 2\frac{\partial u}{\partial r} & r\frac{\partial (v/r)}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} \\ r\frac{\partial (v/r)}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} & 2\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r} \end{bmatrix}$$

If we expand the second invariant we have

$$I_2 = \underline{\underline{\Delta}}:\underline{\underline{\Delta}} = 2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{v}{r}\right)^2 + \left(r\frac{\partial (v/r)}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta}\right)^2$$

(II-3.3)

Then with the assumption of radial flow we will have

$$I_2 = 2\left(\frac{F'}{r^2}\right)^2 \qquad \text{(II-3.4)}$$

$$\text{where } F' = \frac{dF}{d\theta}$$

So

$$\underline{\underline{\tau}} = K\left|\frac{F'}{r^2}\right|^{n-1}\underline{\underline{\Delta}} \qquad = K\left|\frac{1}{r}\frac{\partial u}{\partial \theta}\right|^{n-1}\underline{\underline{\Delta}} \qquad \text{(II-3.5)}$$

II- 3. 2 Hydrodynamic problem

The continuity and momentum equations are developed in order to solve the flow problem; polar coordinates as shown earlier in figure II-3.1 are used. At the inlet, the entry profile will be either uniform or parabolic to approximate the change from a reservoir to converging plates or the change from parallel plates to converging ones. The following assumptions will be made to simplify the equations:

- Only small angles are used(θ-dir. motion is neglected)
- Incompressible flow
- No end effects (semi-infinite plates i.e. motion in z-direction is negligible)
- Steady flow
- No body forces
- Constant fluid properties
- No exit effects (from the sink at the apex)

With these assumptions in polar coordinates, the equations of motion are from Bird et al. (1960) (p. 83):

- Continuity

$$\frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial \theta} = 0 \qquad \text{(II-3.6)}$$

- Momentum equation in r-direction

$$u\,\frac{\partial u}{\partial r} + \frac{v}{r}\,\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\,\frac{dp}{dr} - \frac{1}{\rho}\left(\frac{1}{r}\left(\frac{\partial (r\tau_{rr})}{\partial r}\right) + \frac{\partial \tau_{r\theta}}{\partial \theta} + \tau_{\theta\theta}\right)$$

$$\text{(II-3.7)}$$

- Continuity in the integral form

$$\bar{u}_o \beta r_o = \int_o^{\beta} ur\, d\theta \qquad \text{(II-3.8)}$$

Note: $\bar{u}_o$ is the average velocity at the entry which means that for uniform entry, u_o represents the actual flat velocity profile.

We did not include the momentum equation in the θ-direction because only small angles of convergence are used ($<15^o$) so the motion in the θ-direction will be considered negligible; this approximation is similar to the one used by Sutterby (1965) and Yang & Price (1972). In the equation of momentum, we are replacing all the stresses by the equivalent constitutive equations as defined in the previous section. After several manipulations described in appendix II-A, we get the following momentum equation:

$$u \frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{dp}{dr} + \frac{K|\dot{\gamma}|^{n-1}}{\rho}\left(\frac{3}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\left(\frac{\partial^2 u}{\partial \theta^2} + u\right) + \frac{\partial^2 u}{\partial r^2}\right)$$

$$+ \frac{K}{r}\left(2r \frac{\partial u}{\partial r}\frac{\partial\left(|\dot{\gamma}|^{n-1}\right)}{\partial r} + \left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}\right)\frac{\partial\left(|\dot{\gamma}|^{n-1}\right)}{\partial \theta}\right)$$

(II-3.9)

where $\dot{\gamma} = \frac{1}{r}\frac{\partial u}{\partial \theta}$

The term $\frac{\partial^2 u}{\partial r^2}$ is neglected because it represents only a smaller part of the axial diffusion of momentum ($\frac{3}{r}\frac{\partial u}{\partial r}$ and $\frac{u}{r^2}$ are other terms). In the last term, only $\frac{1}{r}\frac{\partial u}{\partial \theta}\frac{\partial\left(|\dot{\gamma}|^{n-1}\right)}{\partial \theta}$ will be retained because it does not contain any θ-direction velocity or derivative with respect to r which after the initial flow development are rapidly becoming negligible. So we finally are left with the following equation:

$$u \frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{dp}{dr} + \frac{K|\dot{\gamma}|^{n-1}}{\rho}\left(\frac{3}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\left(\frac{\partial^2 u}{\partial \theta^2} + u\right)\right) + \frac{K}{r^2}\frac{\partial u}{\partial \theta}\frac{\partial\left(|\dot{\gamma}|^{n-1}\right)}{\partial \theta}$$

(II-3.10)

With the use of the following variable transformation

$$U = u/\bar{u}_o \qquad R = 1-r/r_o$$

$$V = v/\bar{u}_o \qquad \phi = \theta/\beta$$

$$P = (p-p_o)/\rho\bar{u}_o^{\,2} \tag{II-3.11}$$

we obtain

- Continuity equation

$$\frac{\partial((1-R)U)}{\partial R} + \frac{1}{\beta}\frac{\partial V}{\partial \phi} = 0 \tag{II-3.12}$$

- Momentum equation

$$U\frac{\partial U}{\partial R} + \frac{1}{\beta}\frac{V}{(1-R)}\frac{\partial U}{\partial \phi} + \frac{V^2}{(1-R)} = -\frac{dP}{dR} + \frac{1}{Re}\left|\frac{1}{\beta(1-R)}\frac{\partial U}{\partial \phi}\right|^{n-1}$$

$$\left[-\frac{3}{(1-R)}\frac{\partial U}{\partial R} + \frac{1}{(1-R)^2}\left(\frac{1}{\beta^2}\frac{\partial^2 U}{\partial \phi^2}+U\right)\right] + \frac{1}{Re}\,\frac{1}{\beta}\,\frac{\partial U}{\partial \phi}\,\frac{\partial\left|\frac{1}{\beta(1-R)}\frac{\partial U}{\partial \phi}\right|^{n-1}}{\partial \phi}$$

(II-3.13)

where $Re = \dfrac{\bar{u}_o^{\,2-n}\, r_o^{\,n}\,\rho}{K}$ = Reynolds number

- Integral form of continuity

$$\left[\int_0^1 U(1-R)\, d\phi\right]_{R=R} = \left[\int_0^1 U(1-R)\, d\phi\right]_{R=R+\Delta R} \tag{II-3.14}$$

One point to be noted is the fact that in the physical system as it stands in figure I-3.1, the velocity "u" is in the opposite direction of "r" the radial length so, before transforming with the use of the above variables, this must be taken into account by replacing "u" by "-u".

In terms of dimensionless variables, the boundary conditions and initial condition are as follows

- Boundary conditions

$$\begin{aligned} &\text{at the wall} && \phi = 1 && U = 0 \text{ and } V = 0 \\ &\text{at the centerline} && \phi = 0 && \frac{\partial U}{\partial \phi} = 0 \text{ and } V = 0 \end{aligned} \qquad \text{(II-3.15)}$$

- Initial condition (for uniform entry)

$$\text{at the entry } R = 0 \qquad U = 1 \quad \text{and } V = 0 \qquad \text{(II-3.16)}$$

In order to use the finite difference scheme, the grid in figure II-3.2 is employed:

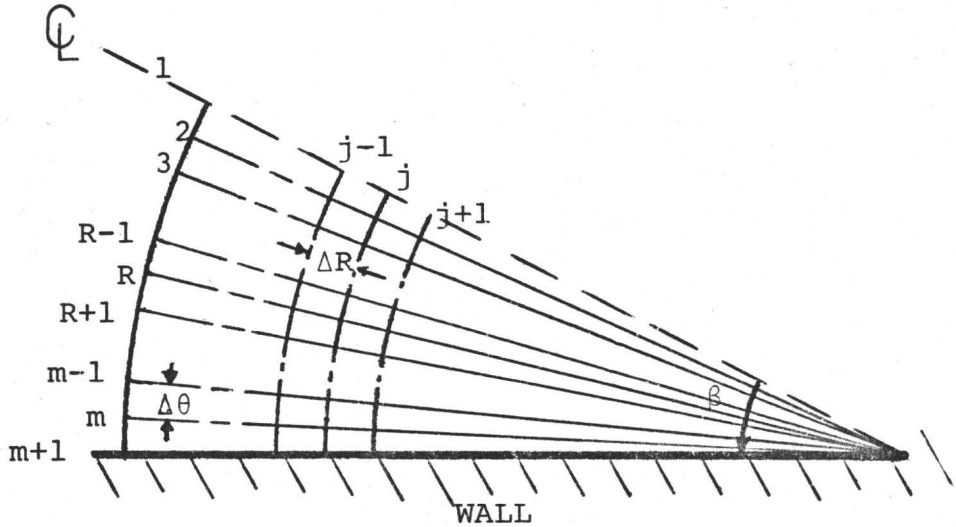


Figure II-3.2: Finite difference network

The finite difference approximations used are:

$$\frac{\partial U}{\partial R} = \frac{U_{j+1,k} - U_{j,k}}{\Delta R}$$

$$\frac{\partial U}{\partial \phi} = \frac{U_{j+1,k+1} - U_{j+1,k-1}}{2\Delta\phi}$$

$$\frac{\partial V}{\partial \phi} = \frac{V_{j+1,k+1} - V_{j+1,k}}{\Delta\phi} \qquad \text{(II-3.17)}$$

$$\frac{\partial^2 U}{\partial \phi^2} = \frac{U_{j+1,k+1} - 2U_{j+1,k} + U_{j+1,k-1}}{\Delta\phi^2}$$

$$\frac{dP}{dR} = \frac{P_{j+1} - P_j}{\Delta R}$$

The continuity equation becomes:

$$V_{j+1,k+1} = V_{j+1,k} - \frac{\beta\Delta\phi}{2\Delta R}\left((1-R)_{j+1}\left(U_{j+1,k} + U_{j+1,k+1}\right) - (1-R)_j\left(U_{j,k} + U_{j,k+1}\right)\right) \qquad \text{(II-3.18)}$$

Similarly the momentum equation becomes:

$$A_k\, U_{j+1,k-1} + B_k\, U_{j+1,k} + C_k\, U_{j+1,k+1} + D_k\, P_{j+1} = E_k \qquad \text{(II-3.19)}$$

$$\text{where } A_k = -\frac{V_{j,k} - \dfrac{G_{j,k+1} - G_{j,k-1}}{\beta(1-R)_j\ 2\Delta\phi}}{\beta(1-R)_j\ 2\Delta\phi} - \frac{G_{j,k}}{((1-R)_j\beta\Delta\phi)^2}$$

$$B_k = \frac{U_{j,k}}{\Delta R} + \frac{G_{j,k}^{2}}{((1-R)_j \ \beta \ \Delta\phi)^2} + \frac{3 \ G_{j,k}}{(1-R)_j \ \Delta R}$$

$$C_k = + \frac{V_{j,k} - \frac{G_{j,k+1} - G_{j,k-1}}{\beta(1-R)_j \quad 2\Delta\phi}}{\beta(1-R)_j \quad 2\Delta\phi} - \frac{G_{j,k}}{((1-R) \ \Delta\phi)^2}$$

$$D_k = \frac{1}{\Delta R}$$

$$E_k = \frac{U^2_{j,k}}{\Delta R} - \frac{V^2_{j,k}}{(1-R)_j} + \frac{Pi}{\Delta R} + \frac{3G_{j,k} \ U_{j,k}}{(1-R)_j \ \Delta R} + \frac{P}{(1-R)^2_j}$$

$$\text{where } G_{j,k} = \frac{1}{Re} \left| \frac{1}{\beta(1-R)_j} \frac{\left(U_{j,k+1} - U_{j,k-1}\right)}{2\Delta\phi} \right|^{n-1}$$

and the integral form of the continuity equation is

$$(1-R)_{j+1} \sum_{k=1}^{m} U_{j+1,k} = (1-R)_j \sum_{k=1}^{m} U_{j,k}$$

(II-3.20)

II- 3. 3 Energy equation

For the energy equation, viscous dissipation is considered negligible and all the assumptions made for the hydrodynamic problem are retained.

The energy equation then becomes:

$$u \frac{\partial t}{\partial r} + \frac{v}{r} \frac{\partial t}{\partial \theta} = \frac{k}{\rho C_p} \left(\frac{1}{r} \frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \theta^2} \right) \qquad \text{(II-3.21)}$$

Following Yang & Price(1972) the term $\frac{\partial^2 t}{\partial r^2}$ will be neglected since $\frac{1}{r} \frac{\partial t}{\partial r}$ which is the other term for axial heat diffusion is more important:
To put that equation in dimensionless form, we will use the variable transformations already mentionned in the previous section (equations I-3.11) and

$$T = \frac{t - t_w}{t_e - t_w} \qquad \text{(II-3.22)}$$

where t_e and t_w are the temperature at the entry and the wall respectively. So the energy equation becomes

$$U \frac{\partial T}{\partial R} + \frac{1}{\beta} \frac{V}{(1-R)} \frac{\partial T}{\partial \phi} = \frac{1}{Pe} \left(- \frac{1}{(1-R)} \frac{\partial T}{\partial R} + \frac{1}{(\beta(1-R))^2} \frac{\partial^2 T}{\partial \phi^2} \right)$$

(II-3.23)

where $Pe = \frac{\bar{u}_o \, \rho \, C_p \, r_o}{k}$ = Peclet number

It should be noted that here, as in the hydrodynamic problem, "u" must be replaced by "-u" to account for the opposite direction of "u" and "r" in the physical system of figure II-3.1.

The same mesh network as for the hydrodynamic problem is used along with the following temperature finite difference representations:

$$\frac{\partial T}{\partial R} = \frac{T_{j+1,k} - T_{j,k}}{\Delta R}$$

$$\frac{\partial T}{\partial \phi} = \frac{T_{j+1,k+1} - T_{j+1,k-1}}{2\Delta\phi} \tag{II-3.24}$$

$$\frac{\partial^2 T}{\partial \phi^2} = \frac{T_{j+1,k-1} - 2T_{j+1,k} + T_{j+1,k+1}}{\Delta\phi^2}$$

After substitutions, the energy equation becomes

$$H_k\, T_{j+1,k-1} + J_k\, T_{j+1,k} + L_k\, T_{j+1,k+1} = M_k \qquad \text{for } 1 \leq k \leq m \tag{II-3.25}$$

where
$$H_k = \frac{V_{j,k}}{\beta(1-R)\ 2\Delta\phi} - \frac{1}{Pe(\beta(1-R)\Delta\phi)^2}$$

$$J_k = \frac{1}{\Delta R}\left(U_{j,k} - \frac{1}{Re(1-R)}\right) + \frac{2}{Pe(\beta(1-R)\Delta\phi)^2}$$

$$L_k = \frac{V_{j,k}}{\beta(1-R)\ 2\Delta\phi} - \frac{1}{Pe(\beta(1-R)\Delta\phi)^2}$$

$$M_k = \left(\frac{T_{j,k}}{\Delta R}\right) U_{j,k} + \frac{1}{Pe(1-R)}$$

The bulk temperature or mixing cup temperature is defined by the following equation

$$T_b = \frac{\int_0^1 U_{j+1,k}\, T_{j+1,k}\, d\phi}{\overline{U}}$$

where $\overline{U}$ = average velocity

The local Nusselt number is obtained as previously:

$$Nu(\text{local}) = \frac{hL}{K} = \frac{\left(\frac{\partial t}{\partial \phi}\right)_w L}{r_o\ \beta\ \Delta T}$$

If "L" is taken as the total length of the arc at the entry and "ΔT" as $(T_b - T_w)$, we end up with the following expression

$$Nu_R = \frac{\left(\frac{\partial T}{\partial \phi}\right)_w 2}{(1-R)\ T_b}$$

Now the average Nusselt number is calculated using the expression below where the integration of the local Nusselt number will be performed using the trapezoidal rule

$$\overline{Nu} = \frac{1}{R}\int_0^R Nu_R\, dR$$

In order to calculate the temperature gradient at the wall, the expression below is used as described in Katotakis (1969) and Vlachopoulos & Keung (1972); this four point formula can be easily derived from a Taylor series expansion.

$$\frac{\partial T}{\partial \phi_w} = \frac{1}{6\Delta\phi}\left(-11\ T_{j,m+1} + 18\ T_{j,m} - 9\ T_{j,m-1} + 2\ T_{j,m-2}\right)$$

where m+1 is the grid point at the wall

II- 3. 4 Numerical procedure

The numerical procedure to compute the velocities, pressures and temperatures, is the same as the one used in Part I so it will not be repeated here and the reader should see Section I-3.4 for the details.

II- 3. 5 Stability and convergence

Similarly to what was said in Section I-3.5, when someone is dealing with highly nonlinear equations like the motion equations, there are no absolute stability criteria that one can obtain as in the case of linear equations.

As previously, convergence is ascertained by decreasing the mesh sizes; one tries to get results that do not change significantly with the use of finer grid sizes. Computer time requirements must be taken into account at this point. To achieve convergence in our case, nine different mesh sizes which are shown in Table II-3.1 in order of decreasing roughness, were used to make a graph of the ratio of maximum velocity over the average velocity along "R". (These mesh sizes are described in detail in appendix II-B).

Mesh size symbol	Roughness
C-21 C-22 C-23 C-41 C-42 C-43 C-82 V-83 V-164	↓ Decreasing roughness

Table II-3.1: Roughness of mesh size

In the mesh symbols, the first digit is a letter where, C stands for constant number of grid spaces across the flow and V means variable number of grid spaces across as the marching procedure goes on; the first number multiplied by 10 gives the number of grid spaces across (exception: for V-164, we use 16); the last digit is a number describing the kind of spacing axially, where a higher number means a finer mesh size. In figure II-3.3, we can see that as the mesh size becomes finer the curves are going down with the best one being C-43 or V-83 or V-164. There can be a difference as high as 3% for example at R=.1 between the roughest and finest mesh; however this difference is almost unnoticeable for R>.35. In this graph, the work of Yang & Price (1972) has been included, of which the present work is an extension; it was found that C-22 had the closest fit with their results; as we can see from Table II-3.1, this is in our estimation, one of the rougher mesh sizes. In a private communication with one of the authors, Price (1972), it was

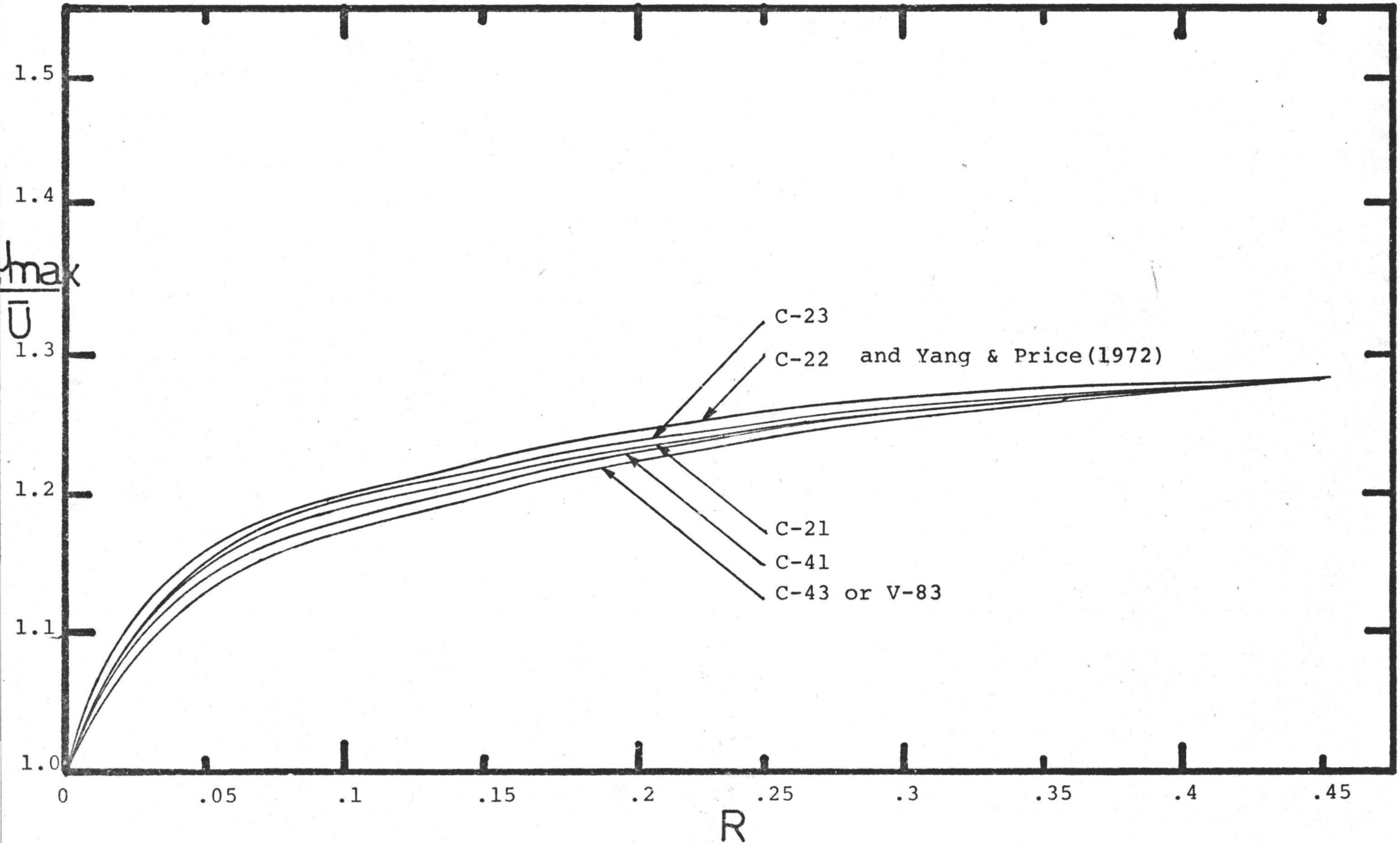


Figure II-3.3: $U_{max}/\overline{U}$ vs R for various grid spacing found that at the entry they were using 39 grid spaces perpendicular to the flow and going down to 19 after a while; since their physical system included the full taper angle and not half of it as in the present work, we effectively should have a similarity between their 39 point analysis and our 20 in C-22. So our conclusion is that they should have used a finer mesh for their work since their results can be improved. The mesh size V-83 was used throughout the rest of this work.

II- 4 RESULTS AND DISCUSSION

II- 4. 1 Comparison for the Newtonian case

The results of the present work for a newtonian fluid will be compared principally with data of two recent articles namely Yang & Price (1972) and Atabek (1972) whenever possible and with Jeffery-Hamel's exact analytical solution of fully developed flow as presented by Millsaps and Pohlhausen(1953).

The Jeffery-Hamel solution requires the use of Jacobian elliptic function making the calculations tedious; so Millsaps and Pohlhausen are giving results for only three Reynolds numbers ranging from 684 to 5000. An excellent agreement is found to exist with the present work; the velocity profiles were taken here at R=.7 when they were evidently fully developed, are undistinguishable from the true profile. Atabek, for developed velocity profiles, is admittedly reporting profiles in excess of 7% to the two values at intermediate Reynolds numbers. Similarly, our profiles were found to be flatter than Atabek's with differences of up to 9% at R=4400. The equation of Atabek is also valid for low Reynolds number but not less than 50 was used in this work as recommended by Reeves and Kippenhan (1962) for a good agreement between velocity distributions of exact boundary layer solution and Navier-Stokes exact solution. In figure II-4.1, those results are shown:

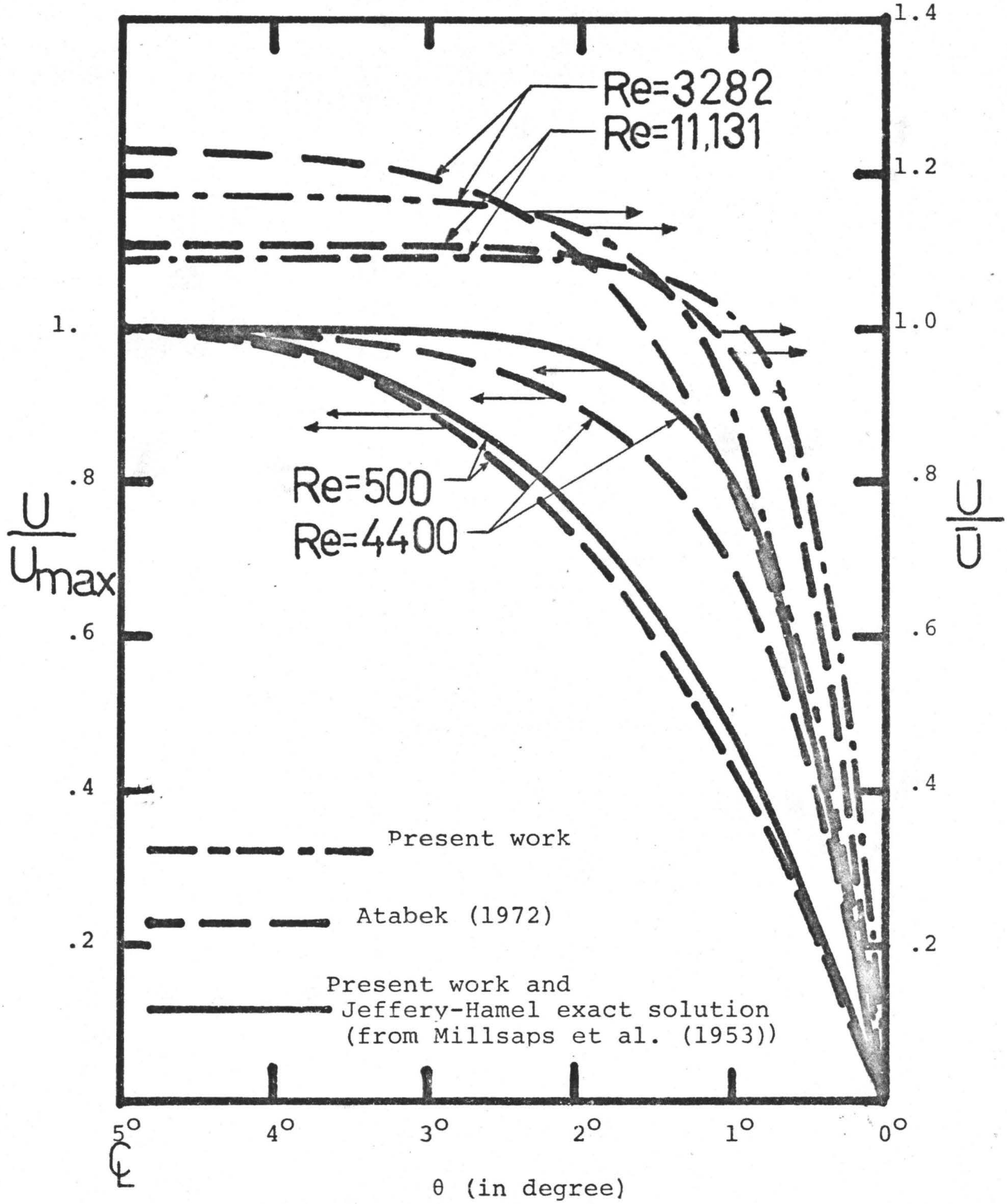


Figure II-4.1: Developed velocity Profile for $\beta=5^{\circ}$

This figure also includes cases where the axial velocity is normalized with the average velocity in order to show that Atabek's work and the present one are giving similar results at very high Reynolds numbers. In figure II-4.2 the development of the axial velocity with respect to R (the axial dimension) is shown; it is apparent that velocities are increasing faster according to our calculations than in the solution of Atabek.

Pressure curves are found in figure II-4.3. Most of the results are within 10% from the results of other workers and the curve shapes are the same. Unfortunately, Atabek is solving at very high Reynolds numbers where the flow is almost potential. We presume that results would differ somewhat at lower Reynolds number because of the differences in velocity profiles. Pressure results of Yang and Price (1972) are also shown; we can see that they agree reasonably well with those of the present work. The biggest differences are occuring at the entry where, as it has been pointed out, the grid size has an influence on the results.

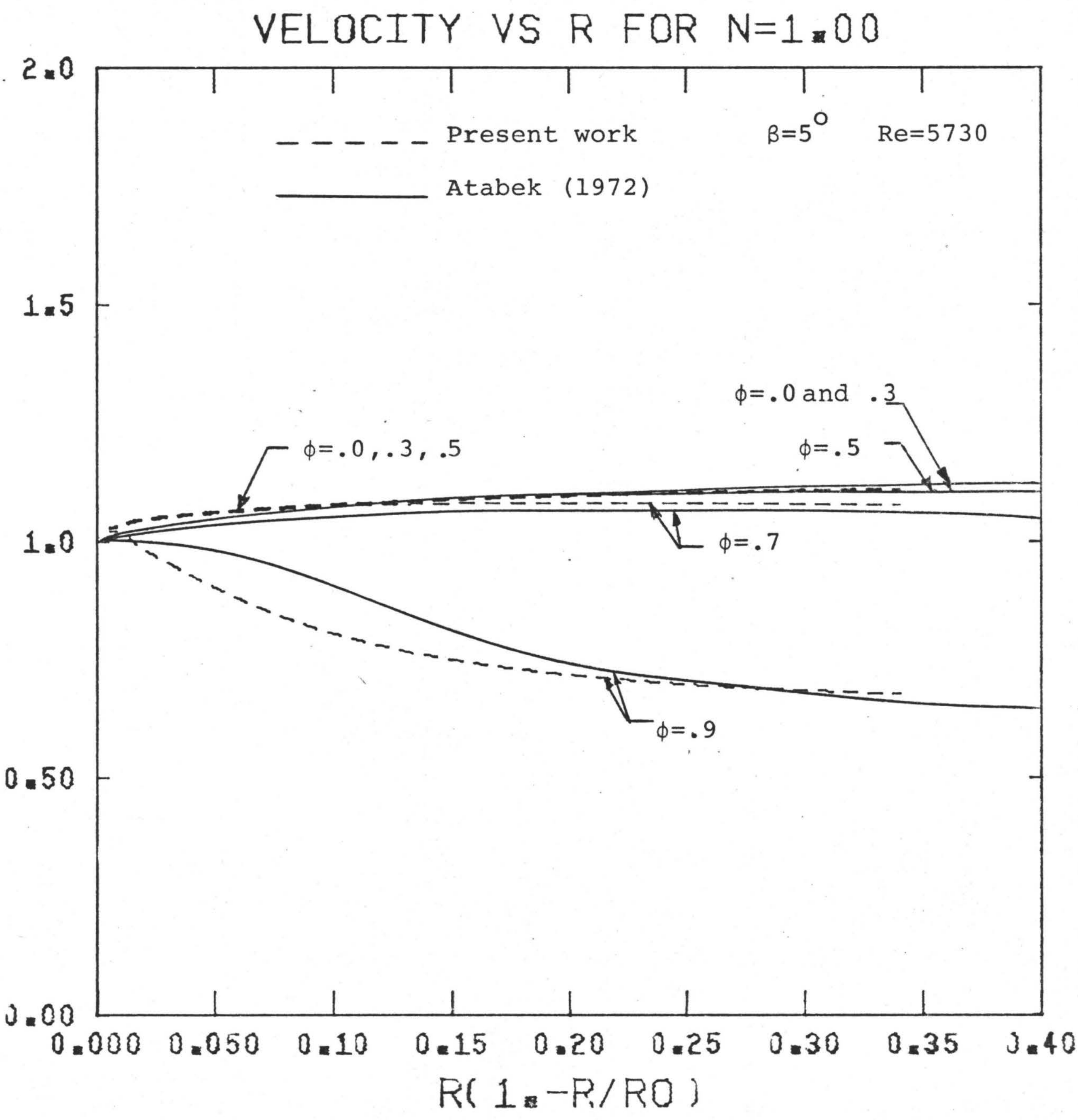


Figure II-4.2: Velocity development

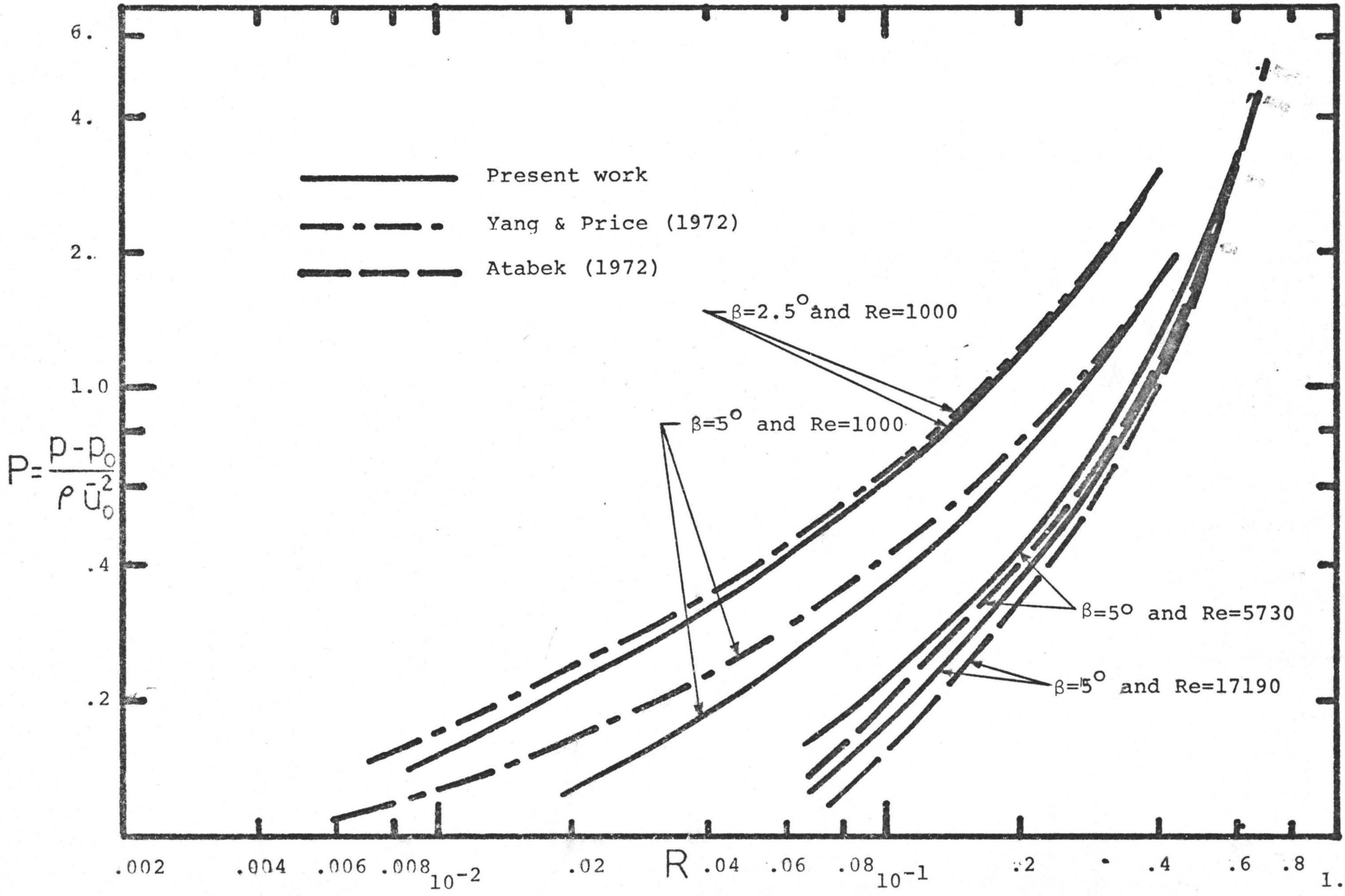


Figure II-4.3: Newtonian fluid Pressure curves

II- 4. 2 Other results for newtonian case

In this section, results obtained by the present method, which are very difficult to compare with others, are discussed. Furthermore the rapidity with which a fluid is developing will be studied.

As it was seen above, two parameters are affecting the hydrodynamic problem, the Reynolds number and the taper angle. In the next three figures, II-4.4 to II-4.6, the center-line velocity against the axial distance at different Reynolds numbers (Fig. II-4.4 for β=2.5 and Fig. II-4.5 for β=5.0) and at different angle values (Fig. II-4.6 for Re=1000) is shown. It appears that the flow is slower to develop as the Reynolds number or the taper angle are increased. The following expression can be thought of as a definition of an entry length:

$$\lim_{R\to 1}\left[0.99\left(U_{max}/\overline{U}\right)\right] = \left(U_{max}/\overline{U}\right)_{R=R_{ent}}$$

where R_{ent} is the entry length

Figure II-4.7 is an example of the kind of curves obtained if R_{ent} is plotted with respect to the Reynolds number. Curves are similar to those obtained by Atabek (1972) i.e. they are reaching a maximum at a certain value of the Reynolds number. The value of that maximum is less than the one obtained by Atabek: this confirms what we saw earlier about the slower development of the velocity profiles.

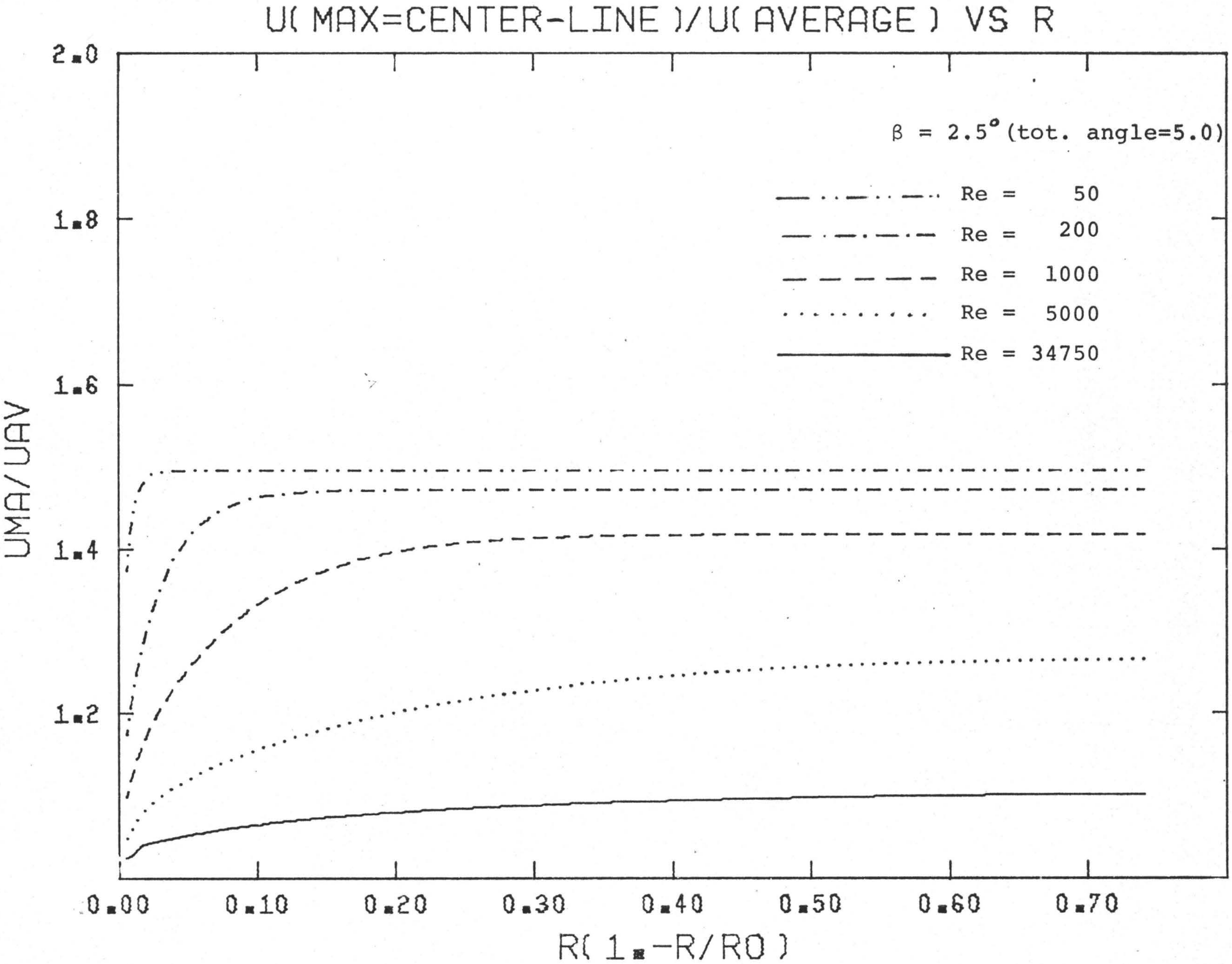


FIGURE II-4.4: $U_{max}/\overline{U}$ vs R

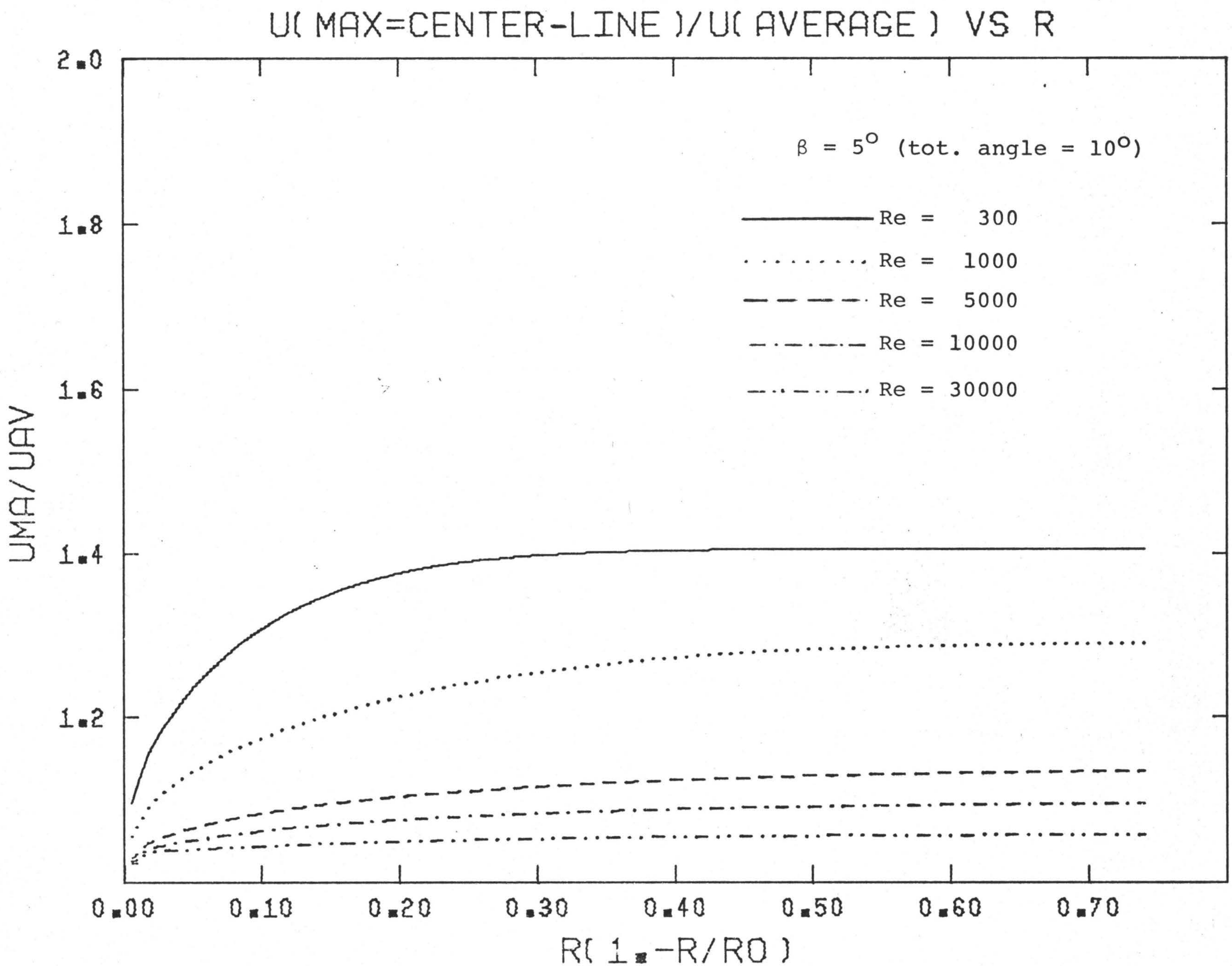


FIGURE II-4.5: $U_{max}/\overline{U}$ vs R

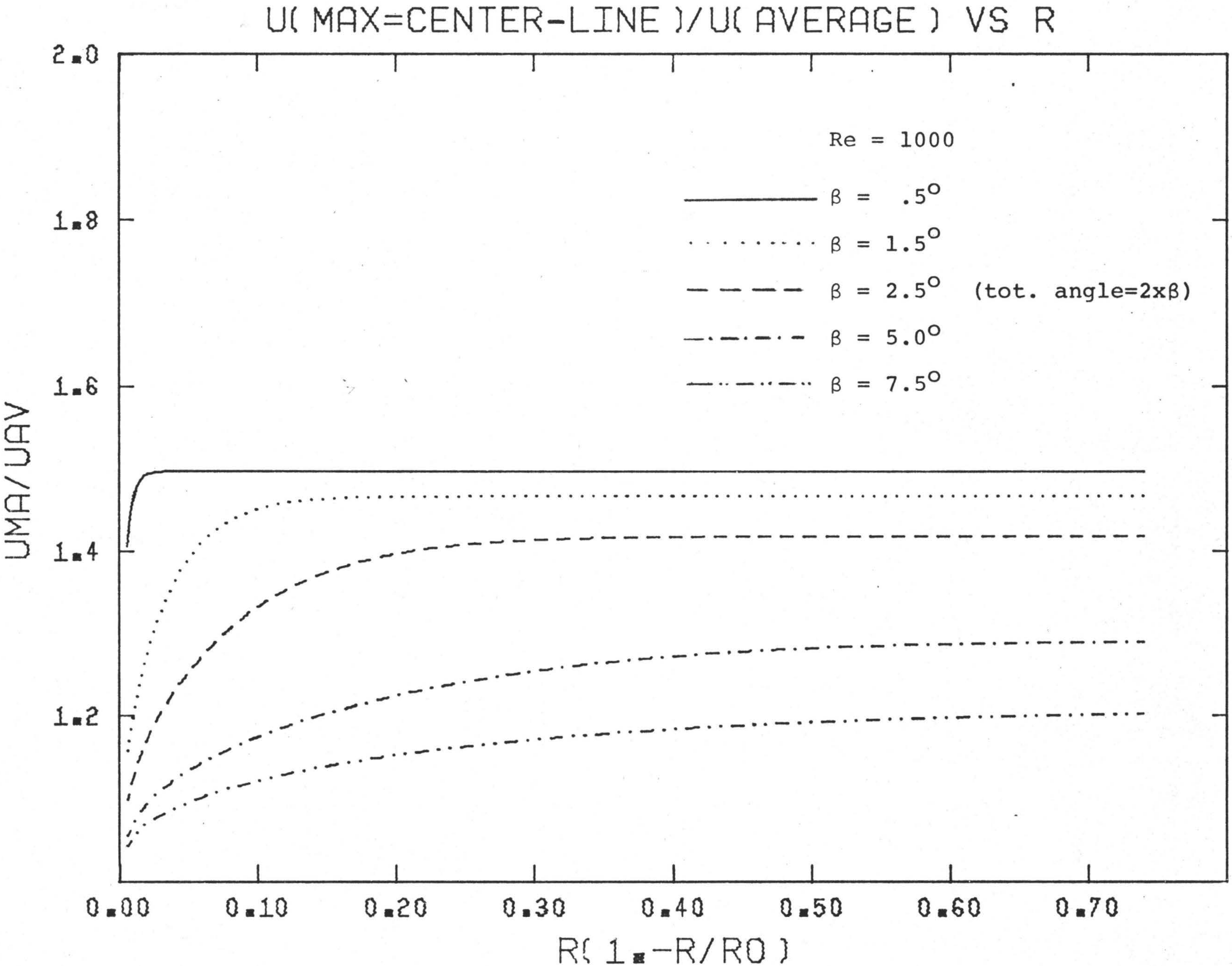


FIGURE II-4.6: $U_{max}/\bar{U}$ vs R

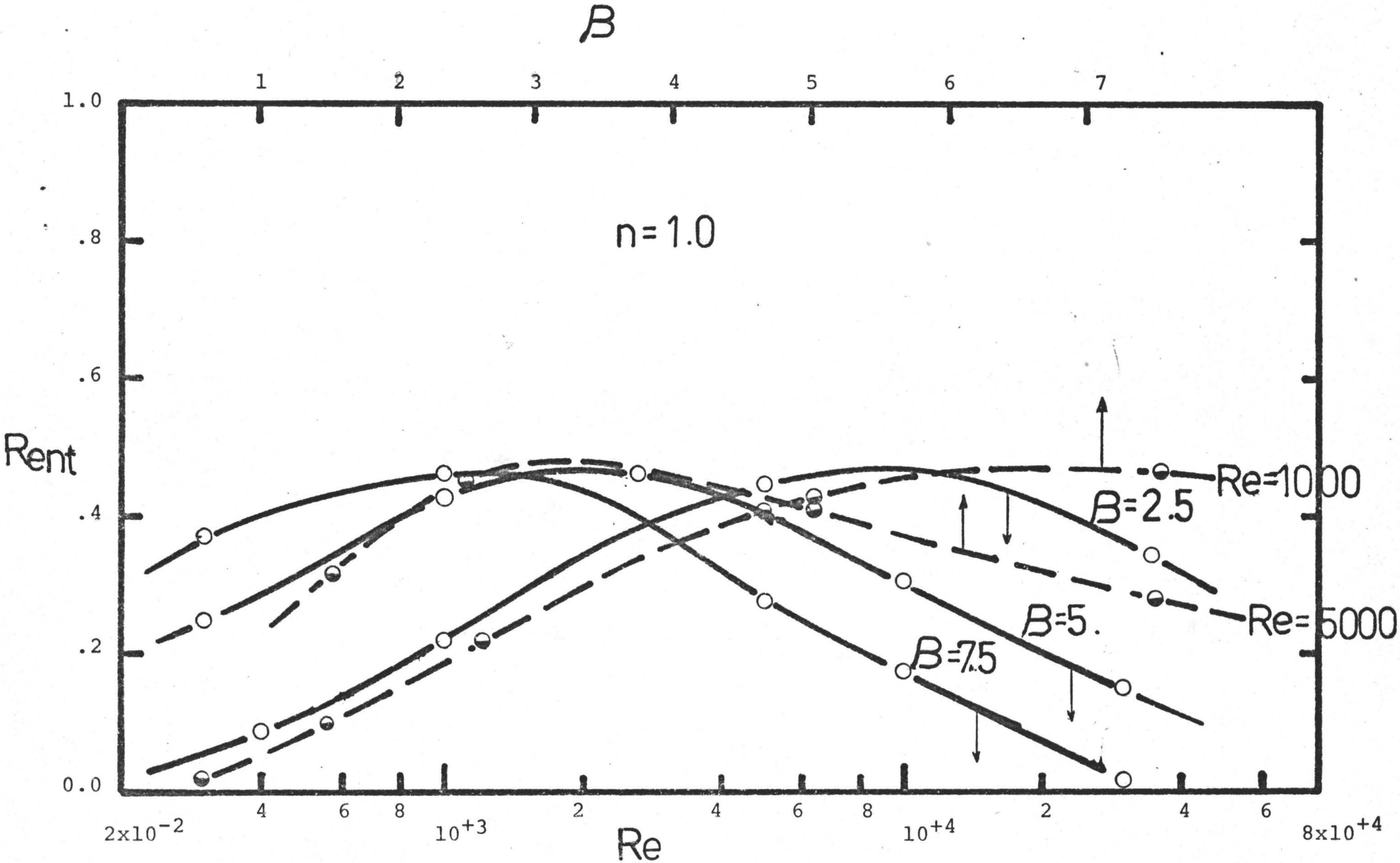


Figure II-4.7: Entry Lengths

The heat transfer problem can be analysed by varying three parameters (for a newtonian fluid) namely the Reynolds number, the Peclet number and β. In figure II-4.8 to II-4.11, one can see the effect of different Peclet numbers on the bulk temperature and on the local Nusselt number. Since the Peclet number is the product of the Reynolds number and the Prandtl number, one can expect the same effect when we are lowering the value of the Peclet number as we had in the first part when lowering the Prandtl number; the bulk temperature is effectively decreasing much faster at low Peclet number. The same is also true for the local Nusselt number as shown in those figures.

II- 4. 3 Pseudoplastic case

The value of flow behavior index was limited almost exclusively to values over .5 since, in most cases, potential flow was obtained throughout for lower values.[1] At a flow behavior index of .6, the fluid was found to exhibit potential flow behavior for moderately high Reynolds number as is shown in figure II-4.12 where the ratio of centerline velocity over the average velocity is plotted with respect to R. The same kind of graph is presented in the next figure (II-4.13) for n=.75 with a varying Reynolds number. The trends are the same as reported for newtonian fluids; a new characteristic is

(1): Since we use high value of F.B.I., the minimum gradient value is not important.

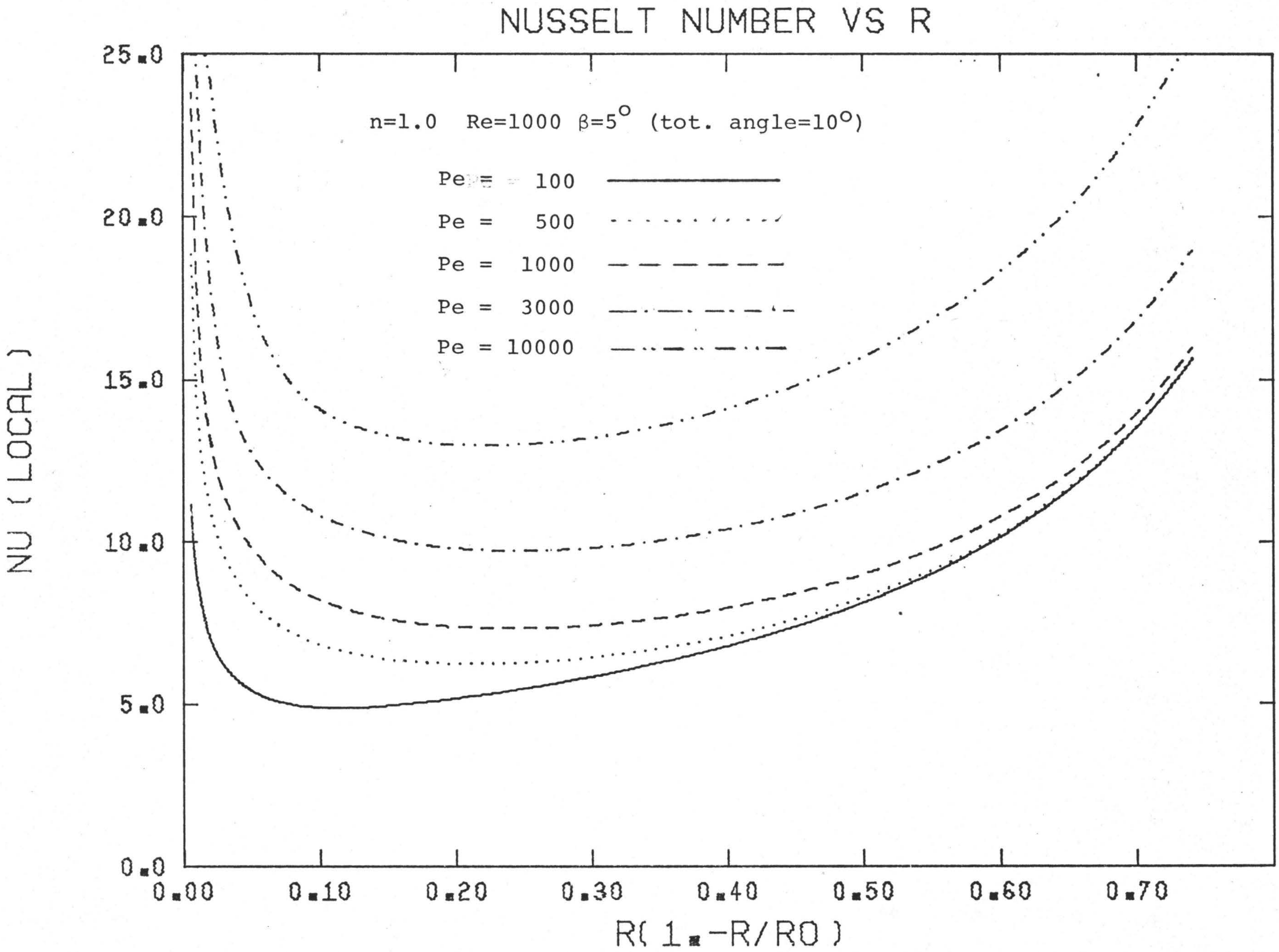


Figure II-4.8: Effect of different Peclet numbers on Nusselt number for $\beta=5.^{\circ}$

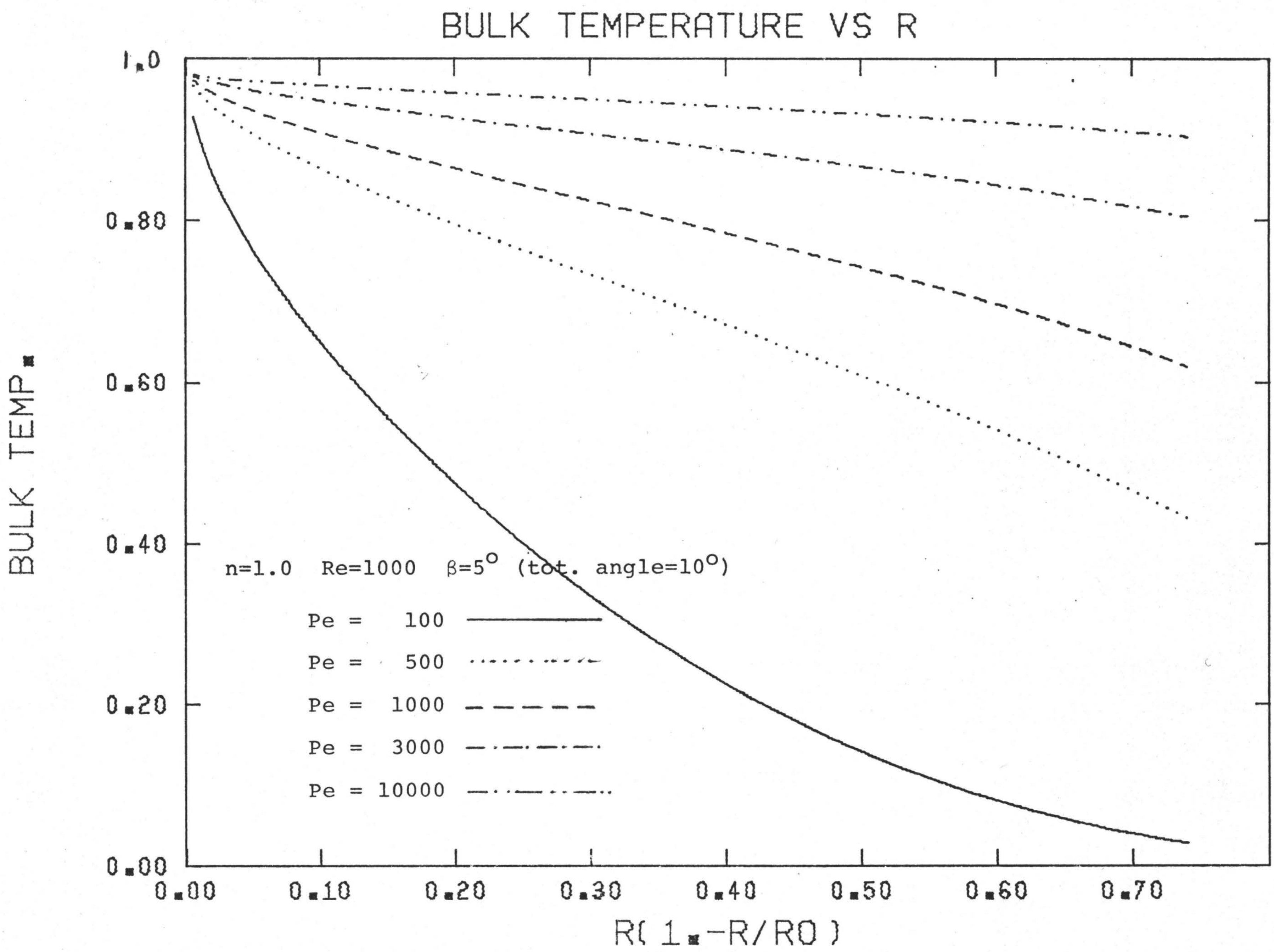
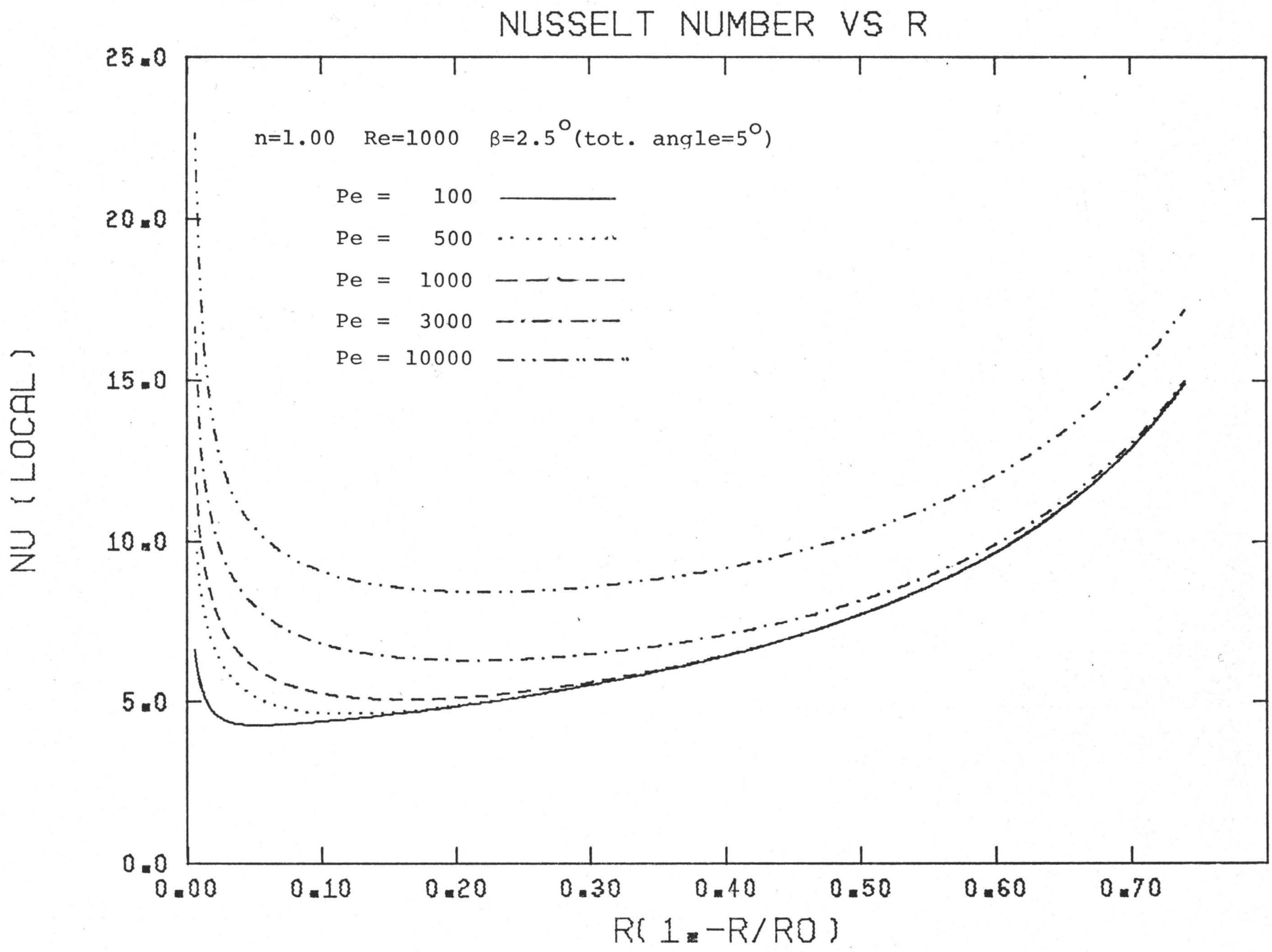


Figure II-4.9: Effect of different Peclet numbers on bulk temperature for $\beta=5.0^{\circ}$



FigureII-4.10: Effect of different Peclet numbers on Nusselt number for $\beta=2.5^{o}$

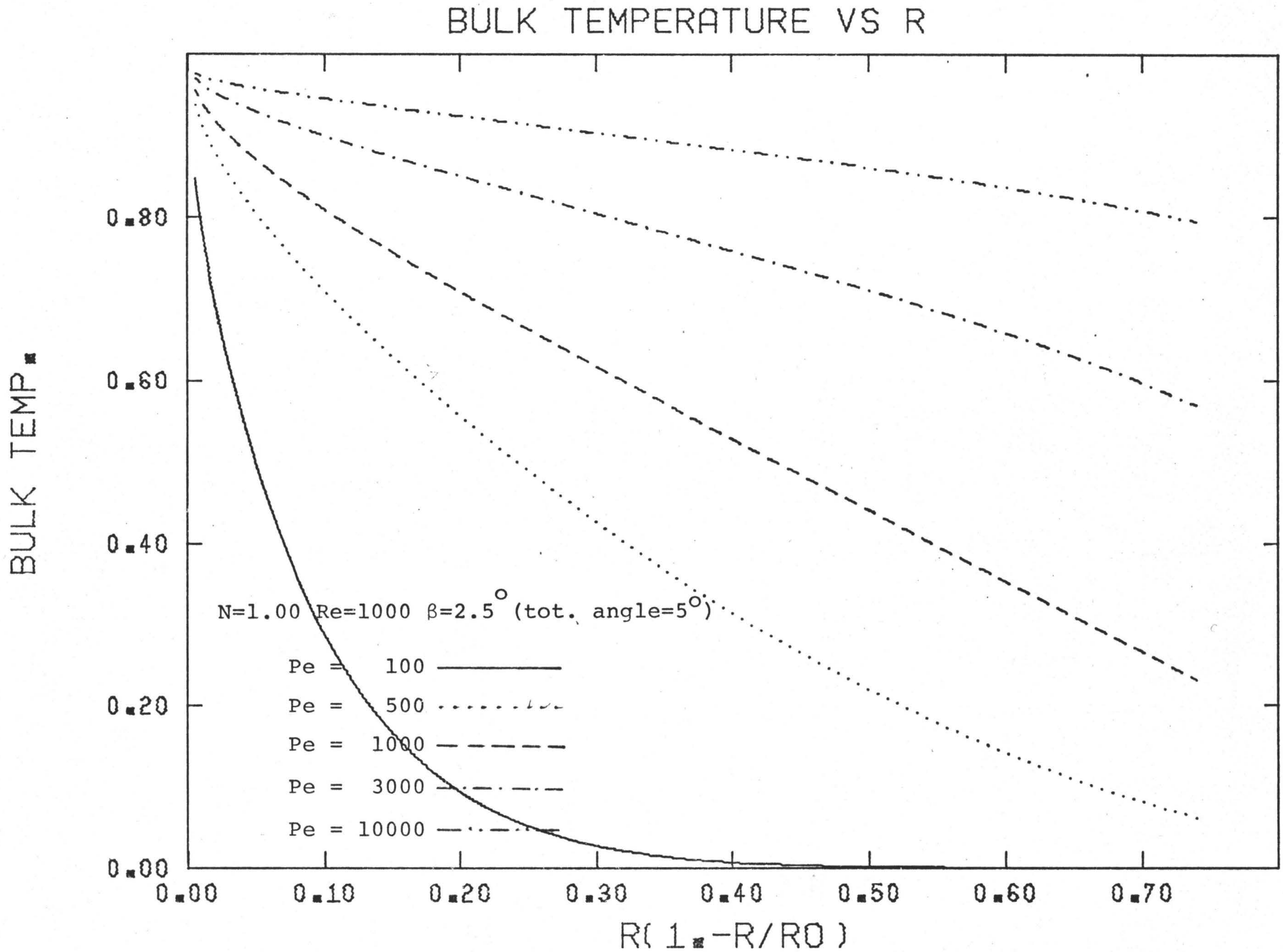


Figure II-4.11: Effect of different Peclet numbers on bulk temperature for $\beta=2.5^{\circ}$

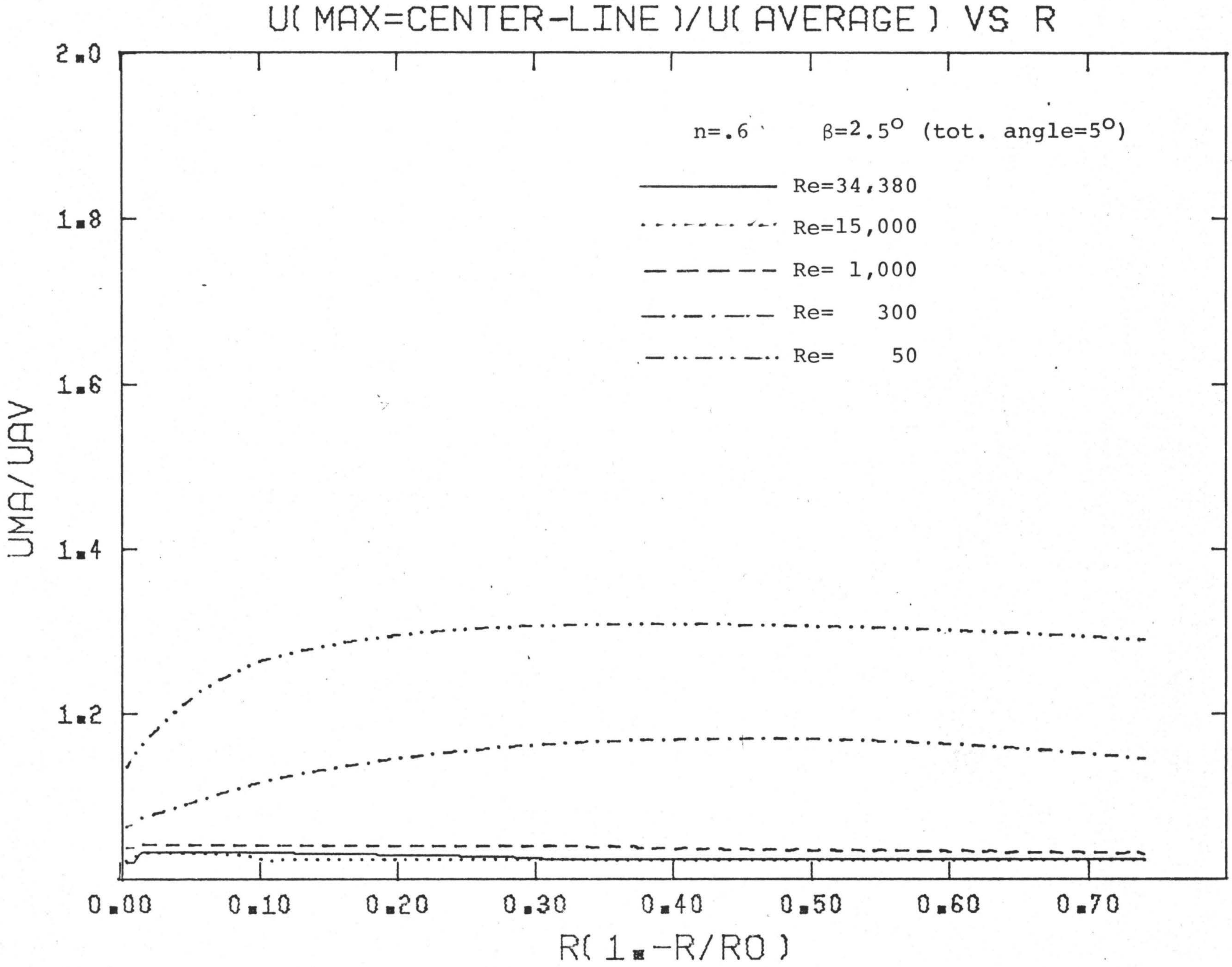


Figure II-4.12: $U_{max}/\overline{U}$ vs R

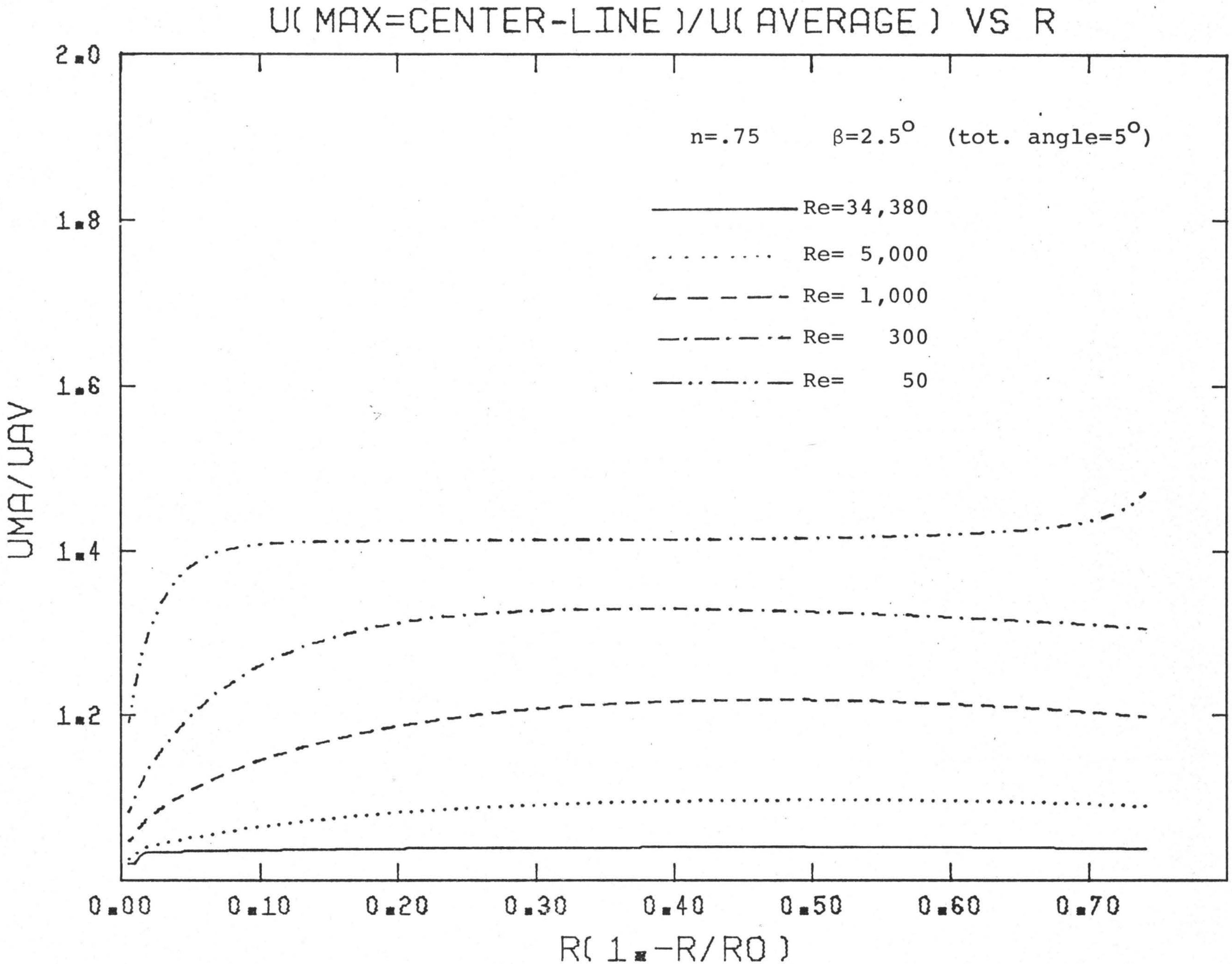


Figure II-4.13: $U_{max}/\overline{U}$ vs R

that the curves are reaching a maximum and then decreasing. This can be explained by the fact that, at lower flow behavior index, a fluid takes more time to develop as we saw for parallel plates entry; this fact will mean that at a certain value of R, the rapidity of development will not be fast enough to compensate for the acceleration due to converging effect of the walls.

From figure II-4.14, we can see that pressure drops are decreasing as the flow behavior index is lowered. In their first part, the curves are presenting an almost constant pressure gradient but as soon as the fluid reaches a region where the flow becomes noticeably accelerated, the gradient increases sharply. We can see also that pressure curves have a minimum which is caused by potential flow. In figure II-4.15, one can see the effect of the flow behavior index on curves of centerline velocity normalized by the average velocity plotted with respect to R.

For the heat transfer part, we are producing in figure II-4.16 curves of the bulk temperature at different flow behavior indices. The curves are lower as the flow index is decreased up to a point where potential flow again is attained; we can see that heat transfer is decreased by the non-newtonian characteristic. The next figure II-4.17 is showing the value of the local Nusselt number along R again, at different values of the flow behavior index; as a matter of fact, the lower bulk temperature seems to be the principal

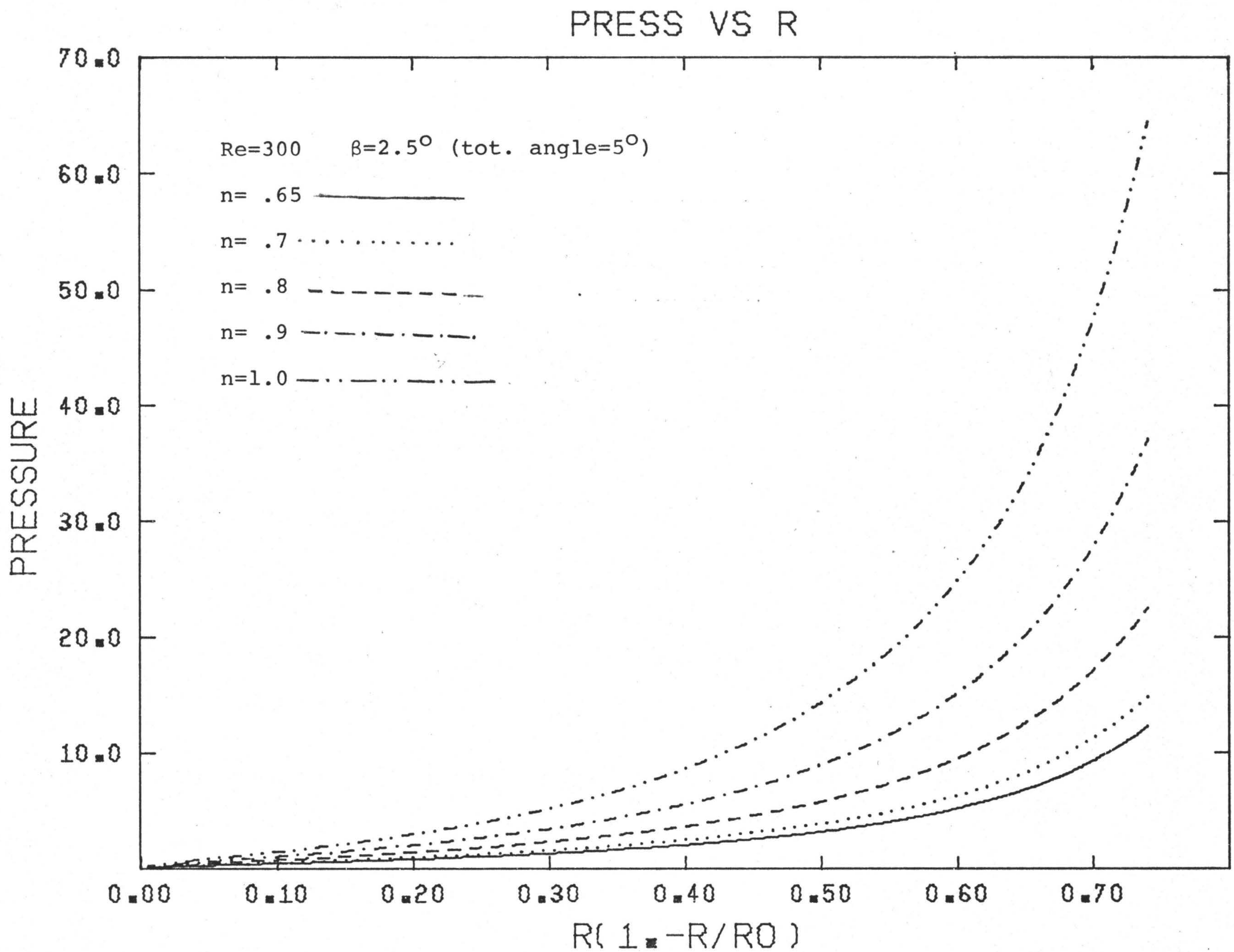


Figure II-4.14: Pressure vs R

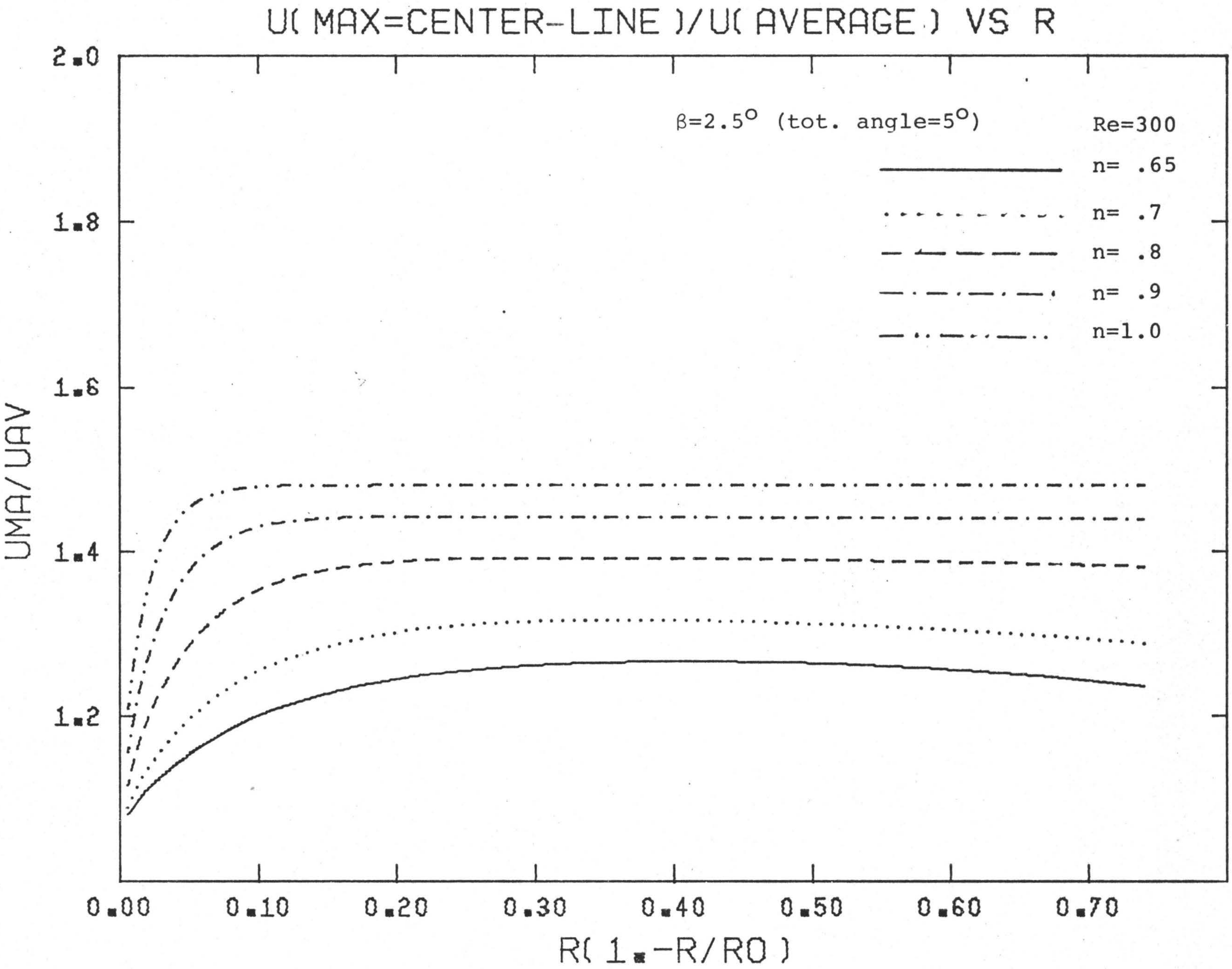


Figure II-4.15: $U_{max}/\overline{U}$ vs R

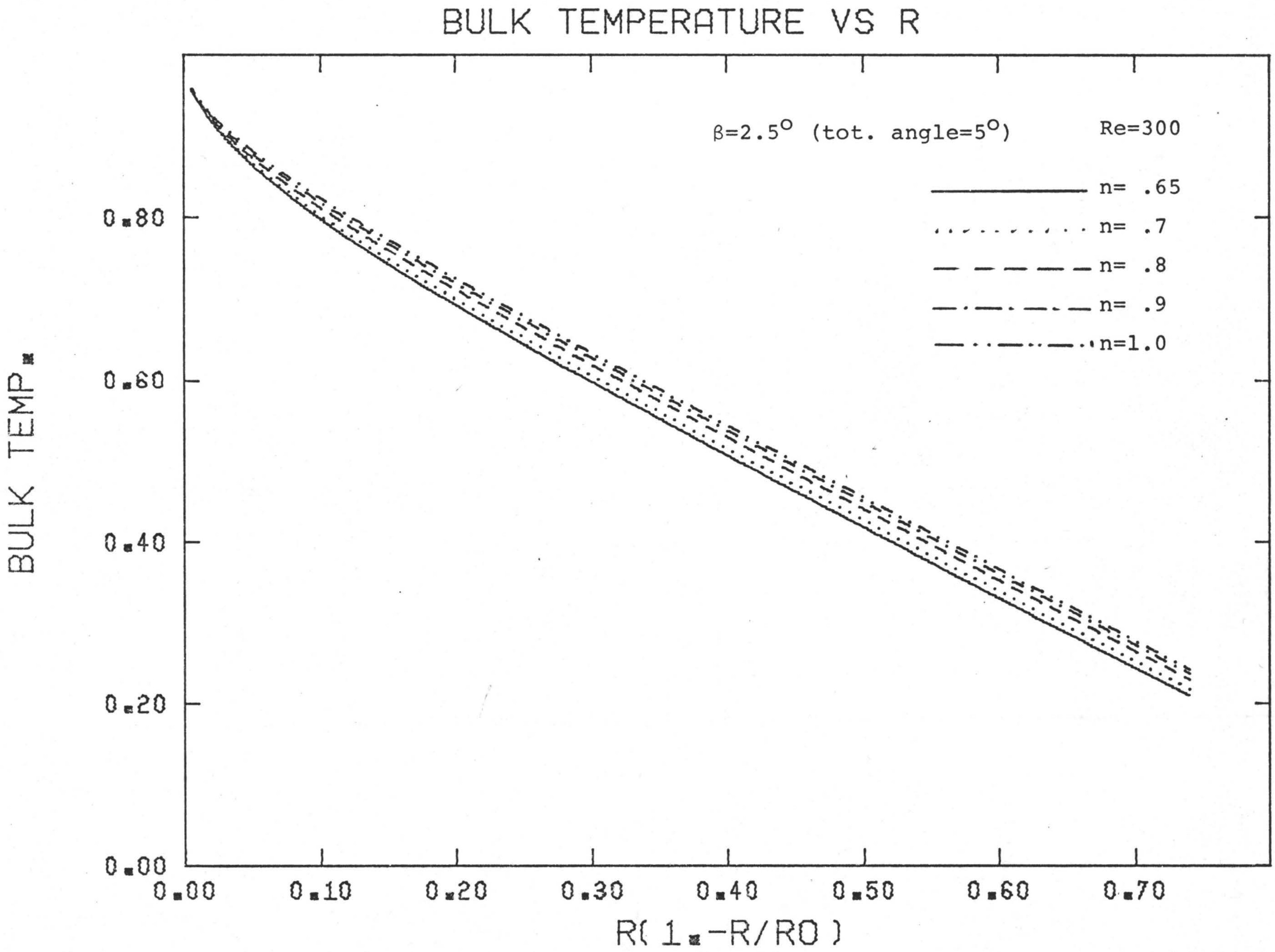


Figure II-4.16: Bulk temperature vs R

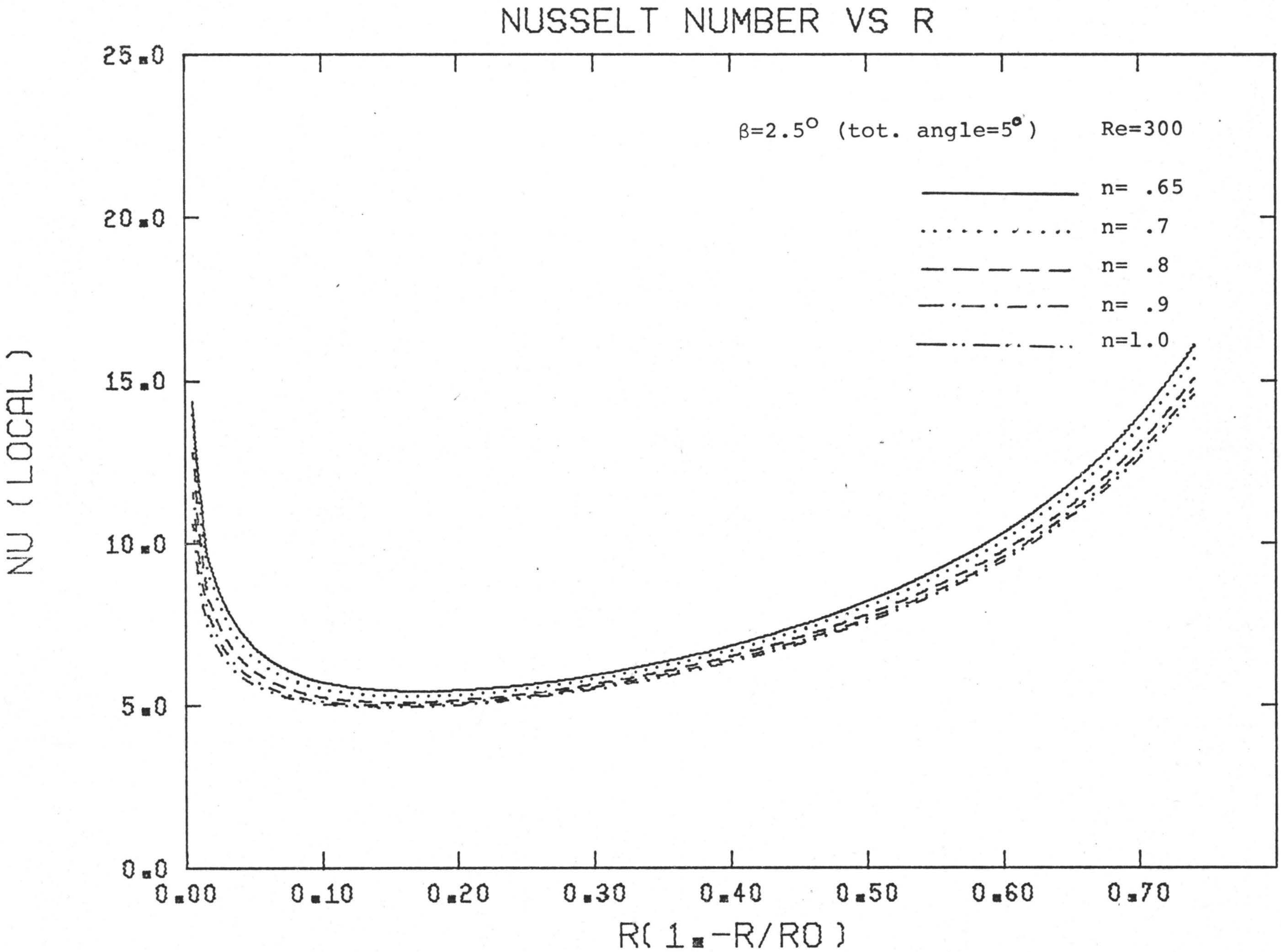


Figure II-4.17: Nusselt number vs R

cause for the higher Nusselt number at low index since the temperature gradients were quite similar from one behavior index to the other.

Varying the Peclet number has a similar effect on the heat transfer as described for newtonian fluids. This can be seen in figure II-4.18 to II-4.21. In figure II-4.22, the average Nusselt number is plotted against R for n=1.00 and n=.75.

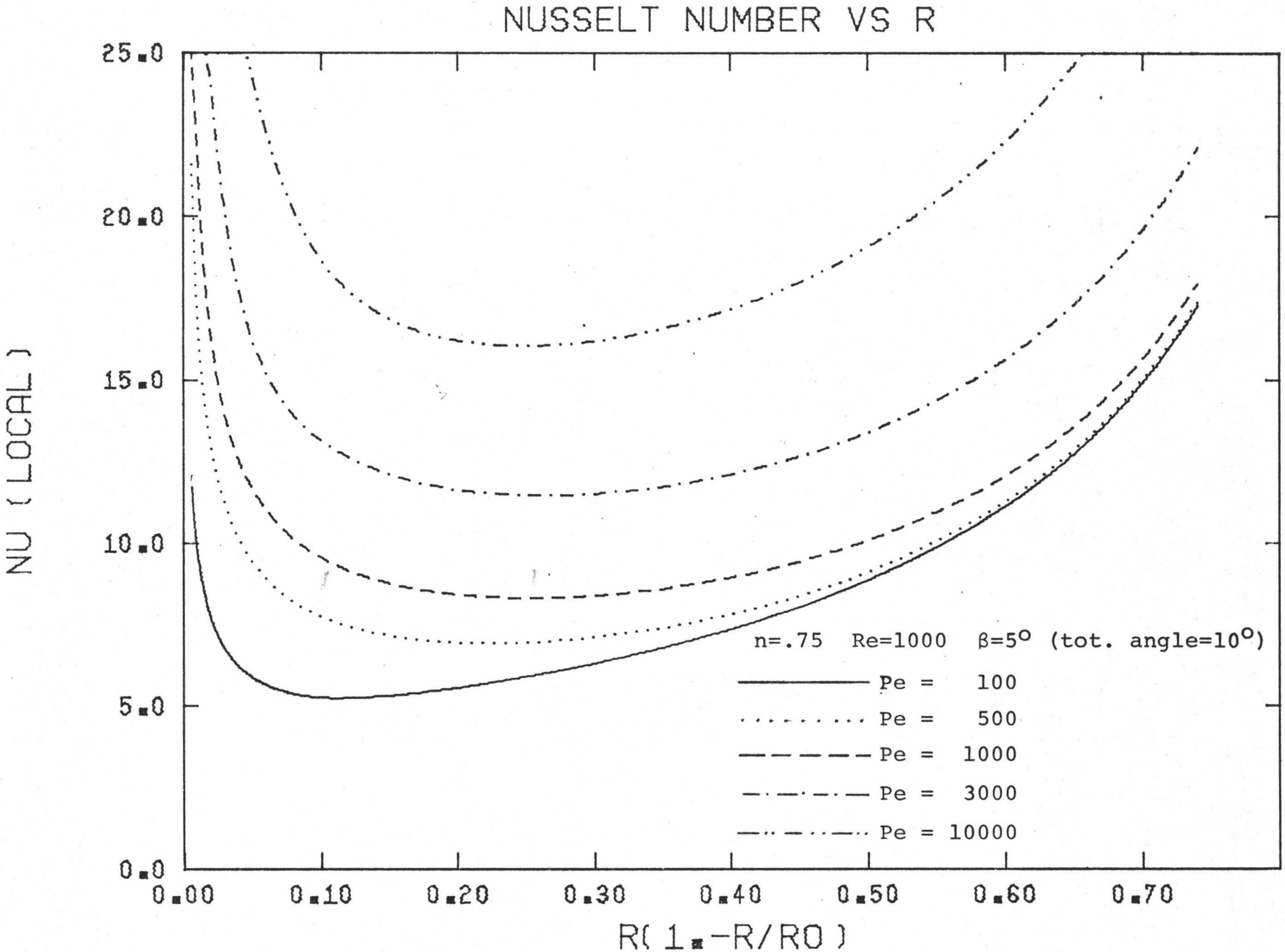


Figure II-4.18: Effect of Peclet number on Nusselt number for n=.75

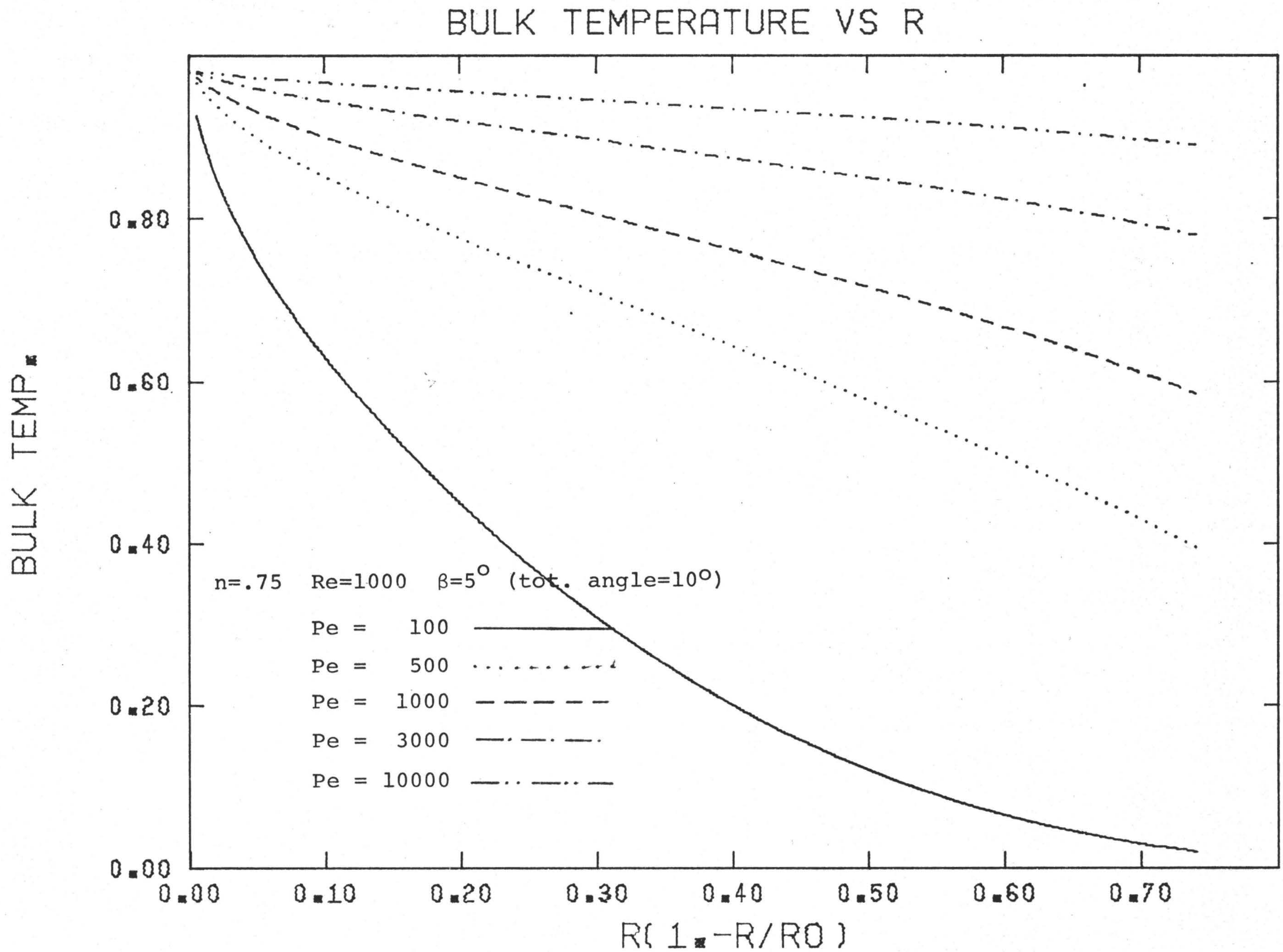


Figure II-4.19: Effect of Peclet number on bulk temperature for n=.75

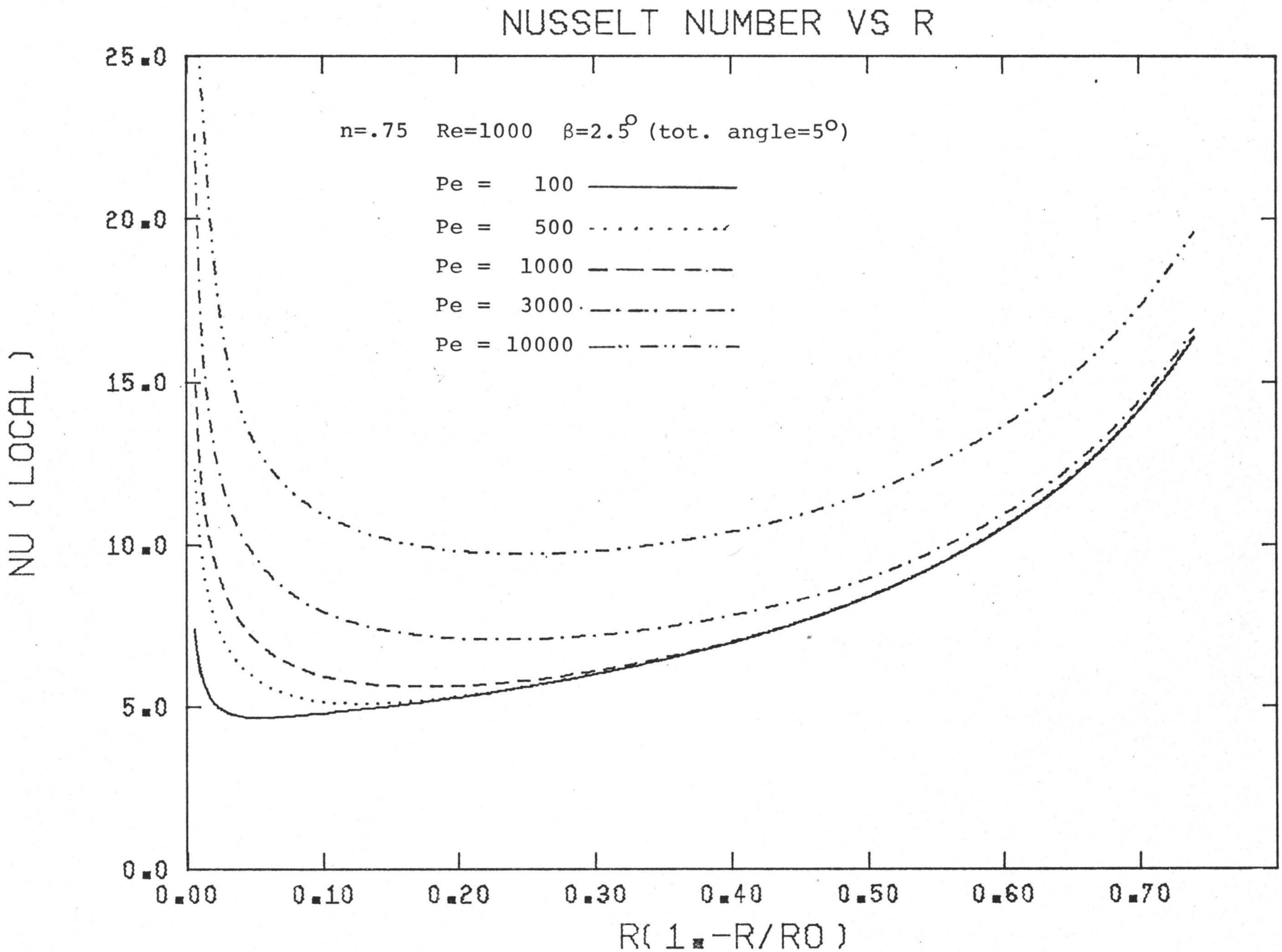


Figure II-4.20: Effect of Peclet number on Nusselt number for n=.75

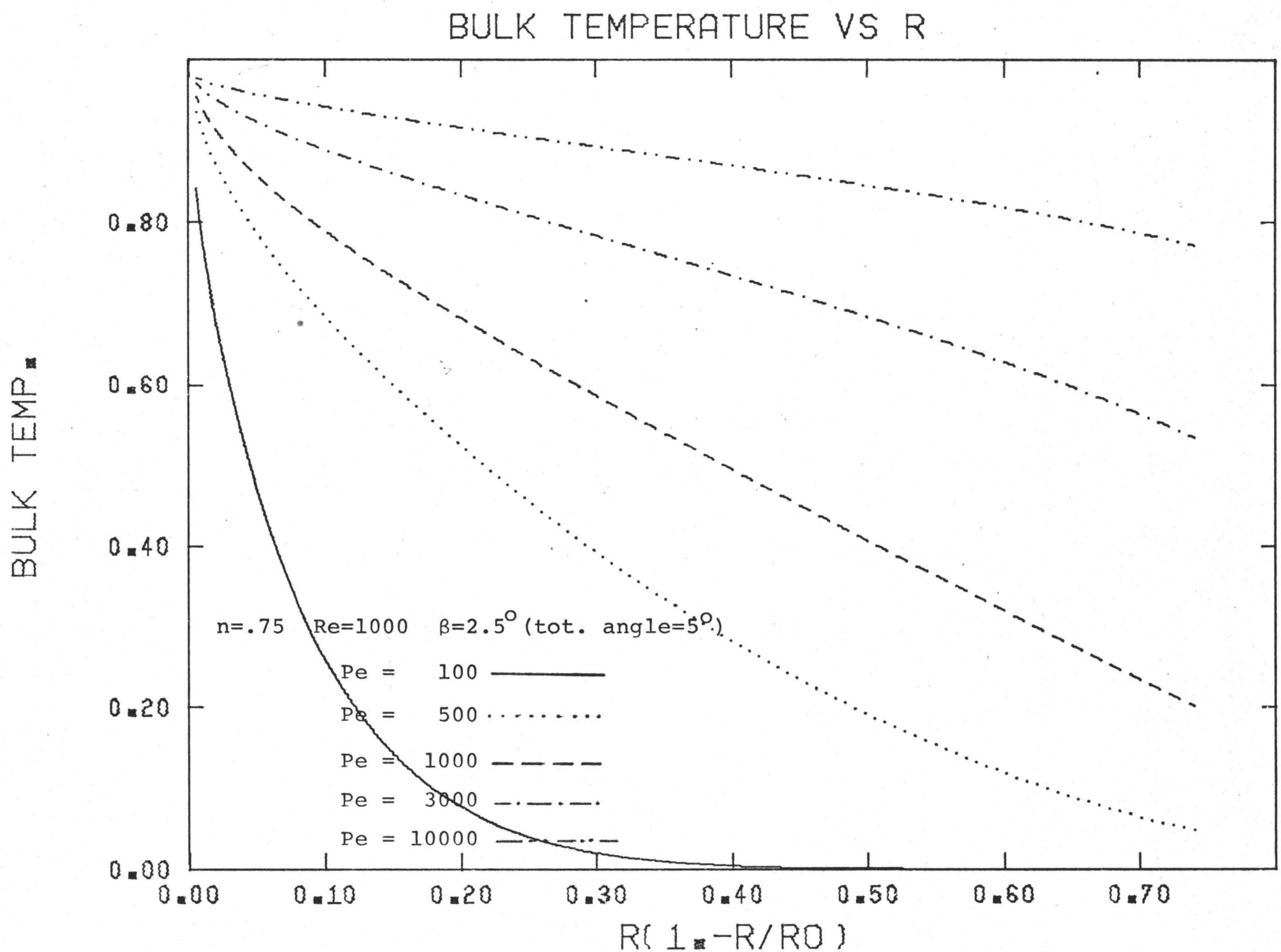


Figure II-4.21: Effect of Peclet number on bulk temperature for n=.75

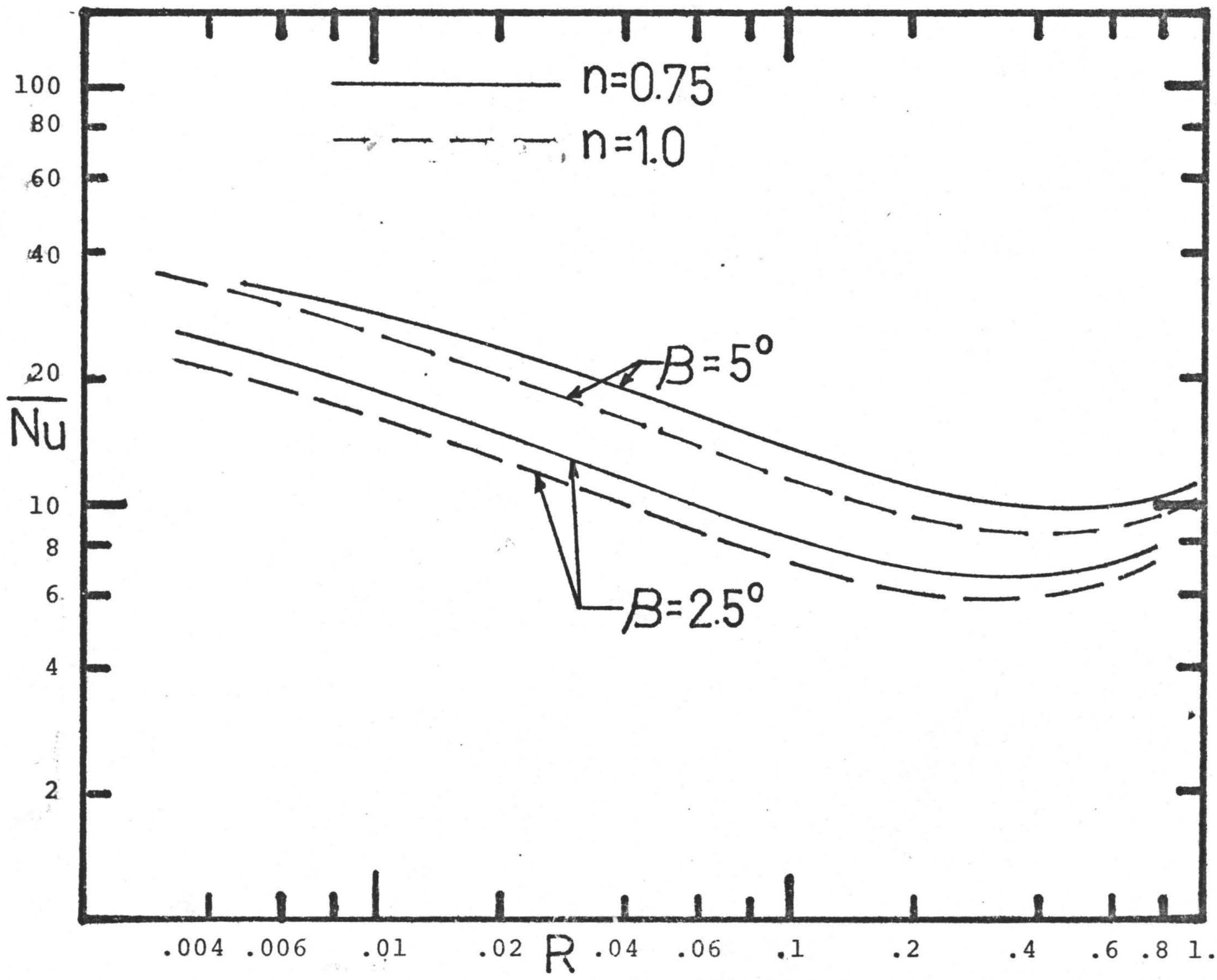


Figure II-4.22: Average Nusselt number

II- 5 CONCLUSIONS AND RECOMMENDATIONS

In this second part, pseudoplastic fluids are analysed in converging channels. Excellent agreement was found to exist between this work and the exact fully developed solution of Jeffery-Hamel for newtonian fluids. Some discrepancy (Newtonian Fluids) with the analytical solution of Atabek (1972) was encountered, the result of the present work giving faster development of profiles, and agreement was good with the work of Yang and Price (1972) except in some cases where the finer mesh size used here gave some differences. New results were obtained for pseudoplastic fluids in converging flow in which a recession of the centerline velocity is predicted after the initial development. The heat transfer problem is also analysed with results expressed in terms of the bulk temperature and the Nusselt number where this last quantity was found to increase after the initial decrease contrarily to the parallel plates where the Nusselt number reaches an asymptotic value.

Recommendations for this part are in two directions, experimental and theoretical. Since the author has found no experimental work in this field, it is recommended that the experimental side of this work be done. On the theorical aspect, we recommend that the extension of this thesis include visco-elastic fluids again along the line of the devialoric stress tensor as used in Balmer and Kauzlarich (1972).

NOTATION

A_k : Series of constants defined in equation II-3.19

B_k : Series of constants defined in equation II-3.19

C_k : Series of constants defined in equation II-3.19

C_p : Specific heat

D_k : Series of constants defined in equation II-3.19

E_k : Series of constants defined in equation II-3.19

F : Function of θ defining the axial velocity in equation II-3.1

$G_{j,k}$: Expression defined in equation II-3.19

h : Heat transfer coefficient

H_k : Series of constants defined in equation II-3.25

I_2 : Second invariant of $\underline{\underline{\Delta}}$

J_k : Series of constants defined in equation II-3.25

k : Thermal conductivity

K : Power law consistency index

L : Characteristic length

L_k : Series of constants defined in equation II-3.25

m : Total number of points less one in radial direction

M_k : Series of constants defined in equation II-3.25

n : Flow behavior index of power law fluid

$\overline{Nu}$: Average Nusselt number

Nu_R : Nusselt number (local)

p : Pressure

p_o : Pressure at entry

P : Dimensionless pressure $(=\frac{p-p_o}{\rho \bar{u}_o^2})$

Pe : Peclet number

r : Radial coordinate

r_o : Radial length

R : Dimensionless radial coordinate $(=1-r/r_o)$

Re : Reynolds number

R_{ent} : Radial entry length

t : Temperature

t_e : Temperature at the entry

t_w : Temperature at the wall

T : Dimensionless temperature $(=(t-t_w)/(t_e-t_w))$

T_b : Bulk temperature

u : Radial velocity

$\bar{u}$: Average radial velocity

$\overline{u_o}$: Average radial velocity at the entry

U : Dimensionless radial velocity $(=u/\bar{u}_o)$

$\bar{U}$: Average dimensionless radial velocity

v : Angular velocity

V : Dimensionless angular velocity $(=v/\bar{u}_o)$

<u>Greek letters</u>

β : Half the taper angle

$\dot{\gamma}$: Variable defined in equation II-3.9

$\underline{\underline{\Delta}}$: Tensor of rate of deformation

θ : Angular coordinate

$\underline{\underline{\tau}}$: Stress tensor

ρ : Density

ϕ : Dimensionless angular coordinate $(=\theta/\beta)$

<u>Subscript</u>

j : Point number in radial direction

k : Point number in angular direction

m : Total number of points less one in radial direction

REFERENCES

ATABEK, H.B.; "Development of flow in the entrance region of a converging channel", Appl. Sci. Res., 27, 103 (1972).

BALMER, R.T., and J.J. Kauzlarich; "Similarity solutions for converging or diverging...", A.I.Ch.E. Jl, 17, 1181 (1971).

BODOIA, J.R. and J.F. OSTERLE; "Finite difference analysis of plante Poiseuille and Couette flow developments" Appl. Sci. Res., A10, 265 (1961).

BOUSSINESQ, J.; Comptes rendus, 113, 9 (1891).

BRANDT, A. and J. GILLIS; "Magneto-hydrodynamic flow in the inlet region of a straight channel", Physics of Fluids, 9, 690 (1966).

BUCHANAN, D.; Private communication (Plotting routines), McMaster University (1973).

CARNAHAN, B., H.A. LUTHER and J.P. WILKER; Applied Numerical methods, John Wiley & Sons, New York, (1969).

CHRISTIANSEN, E.B. and S.J. KELSEY; "Isothermal and non isothermal, laminar, inelastic non-newtonian tube-entrance flow following a contraction", Chem. Eng. Sci., 28, 1099 (1973).

COLLINS, M. and W.R. SCHOWALTER; "Laminar flow in the inlet region of a straight channel", Physics of Fluids, 5, 1122 (1962).

COLLINS, M. and W.R. SCHOWALTER; "Behavior of non-newtonian fluids in the inlet region of a channel", A.I.Ch.E. Jl., 9, 98 (1963).

COLLINS, M. and W.R. SCHOWALTER; "Behavior of non-newtonian fluids in the entry region of a pipe", A.I.Ch.E. Jl, 9, 804 (1963).

HAMEL, G.; "Spiralformige Bewegung zaher Flussigkeiten", Jahresber d. Dt. Methematikes Vereinigung, 25, 34 (1916).

HWANG, C.L. and L.T. FAN; "A finite difference analysis of laminar MHD flow in the entrance region of a flat rectangular duct", Appl. Sci. Res., B10, 329 (1963).

HWANG, C.L. and L.T. FAN; "Finite difference analysis of forced convection heat transfer in entrance region of a flat rectangular duct", Appl. Sci. Res., A13, 401 (1964).

JEFFERY, G.; "Steady motion of a viscous fluid", Phil. Mag., 29, 455 (1915).

KATOTAKIS, S.; "Numerical solution of the laminar boundary layer equations", M. Eng. thesis, McMaster University (1969).

KATOTAKIS, S. and J. VLACHOPOULOS; "Application of a General Finite Difference Method to Boundary Layer flow"CSME Trans. (in press)

LANGHAAR, H.L.; "Steady flow in transition length of a straight tube", Jl Appl. Mech., 9, A55 (1942).

McDONALD, J.W., V.E. DENNY and A.F. MILLS; "Numerical solutions of the Navier-Stokes equations in inlet region", Jl Appl. Mech., (ASME trans), 39, 873 (1972).

McKILLOP, A.A.; "Heat transfer for laminar flow on non-newtonian fluids in entrance region of a tube", Int. Jl Heat Mass transfer, 7, 853 (1964).

McKILLOP, A.A., J.C. HARPER, N.J. BADER and A.Y. KORAYEM; "Variable viscosity entrance-region flow of non-newtonian liquids", Int. Jl Heat Mass transfer, 13, 901 (1970).

MILLSAPS, K. and K. POHLHAUSEN; "Thermal distribution in Jeffery-Hamel flow between non parallel plane walls", Jl of Aeronautical Sc., 20, 187 (1953).

OKA, S.; "Pressure development in a non-newtonian flow through a tapered tube", Private communication (author address: Depart. of Physics, Kero University, Hiyoski, Y. Bohama, Japan).

OZOE, H. and S.W. CHURCHILL; "Hydrodynamic stability and natural convection in Oswald-de Waele and Ellis fluids: the development of a numerical solution", A.I.Ch.E. Jl, 18, 1196 (1972).

POHLHAUSEN, K.; "Zur nahesungsweisen integration der differential gleichung der Grenzschicht", ZAMM, 1, 252 (1921).

RAMA MURTHY, A.V. and D.V. BOGER; "Developing velocity profiles on the downstream side of a contraction for inelastic polymer solutions", Trans. Soc. Rheology, 15, 709 (1971).

REEVES, B.L. and C.J. KIPPENHAN; "A particular class of similar solutions of the equations of momentum and energy of a viscous fluids", Jl of Aerospace Sci., 29, 38 (1962).

RICHTMEYER, R.D.; Difference methods for initial-value problems, Interscience publishers Inc., New York, (1957).

ROIDT, M. and R.D. CESS; "An approximate analysis of laminar MHD flow in the entrance region of a flat duct", Jl Appl. Mech., (ASME trans.), 29, 171 (1962).

SCHILLER, L.; "Untersuchungen uber laminare und turbulente stromung", ZAMM, 2, 96-106 (1922).

SCHLICHTING, H.; Boundary-layer theory, McGraw-Hill, New York, (1968).

SCHLICHTING, H.; "Laminare Kanaleinlaufstromung", ZAMM, 14, 368 (1934).

SCHOWALTER, W.R.; "The application of boundary-layer theory to power law pseudoplastic fluids: similar solutions", A.I.Ch.E. Jl, 6, 24 (1960).

SUTTERBY, J.L.; "Finite difference analysis of viscous laminar converging flow in conical tubes", Appl. Sci. Res., A15, 241 (1965).

TIU, C., D.V. BOGER and A.L. HAMOS; "Generalized method for redicting loss coefficients in entrance region flow for elastic fluids", Chem. Eng. Jl, 4, 113 (1972).

VAN DYKE, M.; "Entry flow in a channel", Jl Fluid Mech., 44, 813 (1970).

WANG, Y.L. and P.A. LONGWELL; "Laminar flow in the inlet section of parallel plates", A.I.Ch.E. Jl, 10, 323 (1964).

WILLIAMS, J.C.; "Viscous compressible and incompressible flow in slender channels", A.I.A.A. Jl, 1, 186 (1963).

YANG, J.W. and G.M. PRICE; "Laminar flow development and heat transfer in converging plane-walled channels", Appl. Sci. Res., 25, 361 (1972).

YANG, J.W. and G.M. PRICE; Private communication, (1972).

APPENDIX I-A

Derivation of momentum equation (equation I-3.23) from finite difference approximations (equation I-3.19 and I-3.20).

We start from equation I-3.14

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{dP}{dX} + 2^n \frac{\partial\left(\left|\frac{\partial U}{\partial Y}\right|^{n-1} \frac{\partial U}{\partial Y}\right)}{\partial Y} \tag{I-A.1}$$

by replacing directly the finite difference approximations of equations I-3.19 and I-3.20

$$U_{j,k} \frac{U_{j+1,k}-U_{j,k}}{\Delta X} + V_{j,k} \; \frac{1}{2} \frac{U_{j,k+1}-U_{j,k-1}}{2\Delta Y} + \frac{1}{2} \frac{U_{j+1,k+1}-U_{j+1,k-1}}{2\Delta Y} = -\frac{P_{j+1}-P_{j}}{\Delta X}$$

$$+2^n \frac{\left|\frac{U_{j,k+1}-U_{j,k}}{\Delta Y}\right|^{n-1} \frac{U_{j,k+1}-U_{j,k}}{\Delta Y} - \left|\frac{U_{j,k}-U_{j,k-1}}{\Delta Y}\right|^{n-1} \frac{U_{j,k}-U_{j,k-1}}{\Delta Y}}{2\Delta Y}$$

$$+ \frac{\left|\frac{U_{j,k+1}-U_{j,k}}{\Delta Y}\right|^{n-1} \frac{U_{j+1,k+1}-U_{j+1,k}}{\Delta Y} - \left|\frac{U_{j,k}-U_{j,k-1}}{\Delta Y}\right|^{n-1} \frac{U_{j+1,k}-U_{j+1,k-1}}{\Delta Y}}{2\Delta Y} \tag{I-A.2}$$

Rearranging and putting all the known terms on the right side and all the unknown on the left side

$$U_{j+1,k-1}\left(-\frac{V_{j,k}}{4\Delta Y}-\frac{|Gm|^{n-1}}{\Delta Y^2}\right)$$

$$+\ U_{j+1,k}\left(\frac{U_{j,k}}{\Delta X}+\frac{|Gp|^{n-1}}{\Delta Y^2}+\frac{|Gm|^{n-1}}{\Delta Y^2}\right)$$

$$+\ U_{j+1,k+1}\left(\frac{V_{j,k}}{4\Delta Y}-\frac{|Gp|^{n-1}}{\Delta Y^2}\right)+P_{j+1}\left(+\frac{1}{\Delta X}\right)=$$

$$+\frac{P_j}{\Delta X}+\frac{U_{j,k}^{2}}{\Delta X}-V_{j,k}\left(\frac{U_{j,k+1}-U_{j,k-1}}{4\Delta Y}\right)+|Gp|^{n-1}\left(\frac{U_{j,k+1}-U_{j,k}}{\Delta Y^2}\right)-|Gm|^{n-1}\left(\frac{U_{j,k}-U_{j,k-1}}{\Delta Y^2}\right)$$

$$\text{where } Gp = 2\,\frac{U_{j,k+1}-U_{j,k}}{\Delta Y}$$

$$Gm = 2\,\frac{U_{j,k}-U_{j,k-1}}{\Delta Y}$$

APPENDIX I-B

Transformation of energy equation

The energy equation is (equation I-3.26)

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{2}{Pr} \frac{\partial^2 T}{\partial Y^2} \tag{I-B.1}$$

Using the finite difference equation I-3.29, we get

$$F_k T_{j+1,k-1} + G_k T_{j+1,k} + H_k T_{j+1,k+1} = J_k$$

$$F_k = -\frac{V_{j,k} + V_{j+1,k}}{2 \quad 4\Delta Y} - \frac{2}{Pr \; 2\Delta Y^2}$$

$$G_k = \frac{U_{j,k} + U_{j+1,k}}{2\Delta X} + \frac{2}{Pr \; \Delta Y^2}$$

$$H_k = \frac{V_{j,k} + V_{j+1,k}}{2 \quad 4\Delta Y} - \frac{2}{Pr \; 2\Delta Y^2}$$

$$J_k = \frac{U_{j,k} - U_{j+1,k}}{2} \frac{T_{j,k}}{\Delta X} - \frac{V_{j,k} + V_{j+1,k}}{2} \frac{T_{j,k+1} - T_{j,k-1}}{4\Delta Y} + \frac{T_{j,k+1} - 2T_{j,k} + T_{j,k-1}}{Pr \; \Delta Y^2}$$

APPENDIX I-C

Additional results for straight channel entry flow

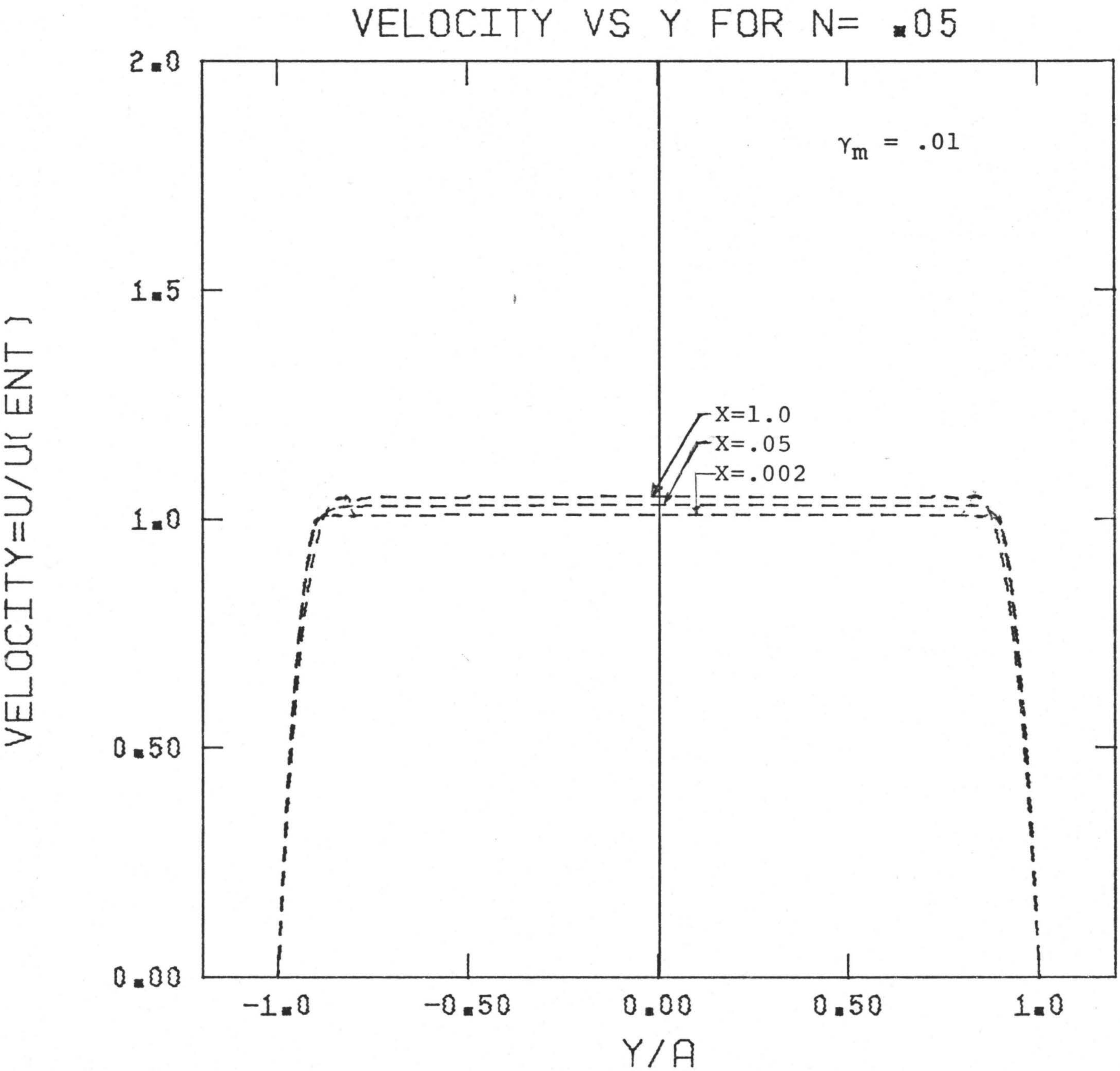


Figure I-C.1: Velocity vs Y for n=.05

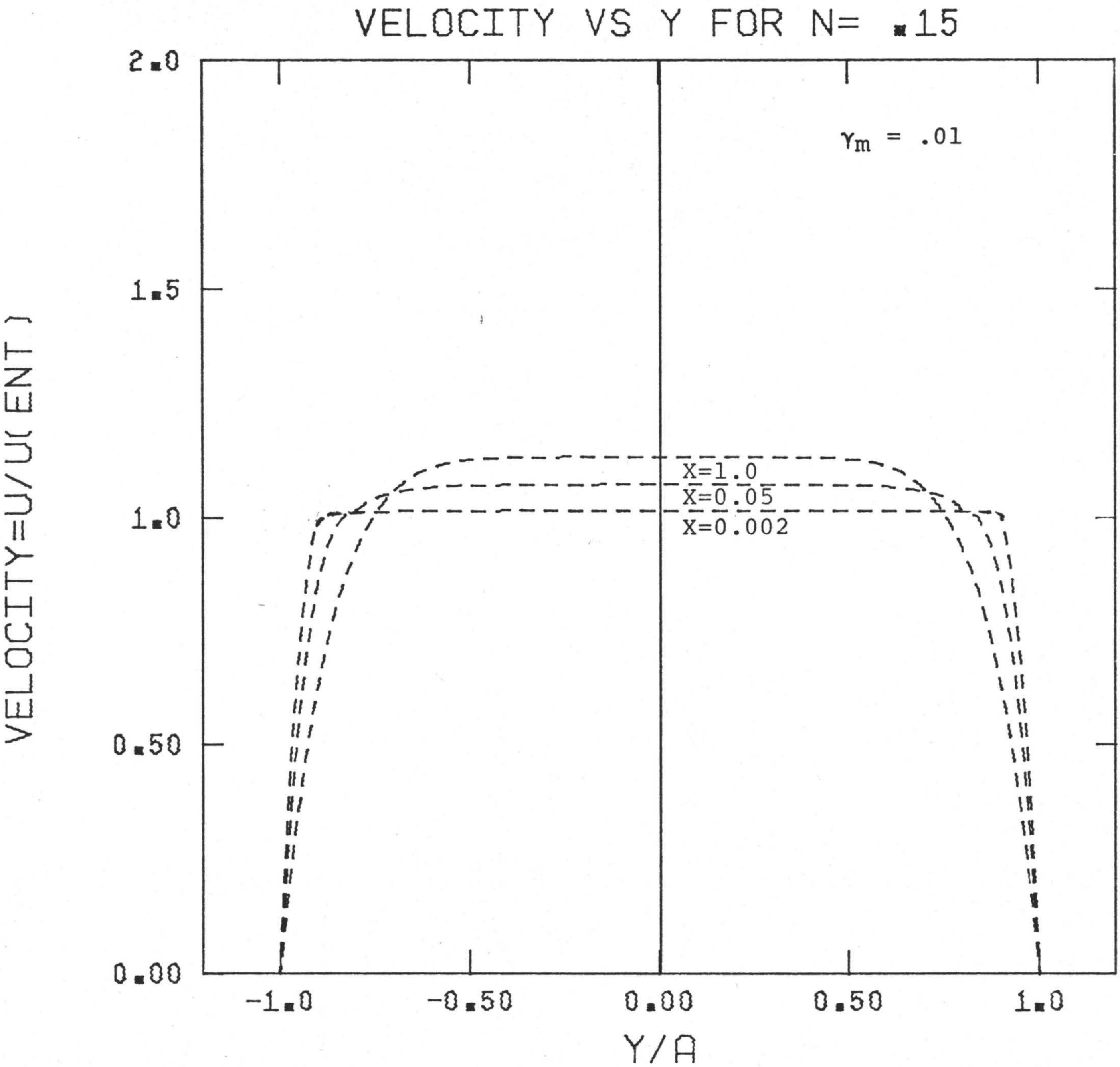


Figure I-C.2: Velocity vs Y for n=.15

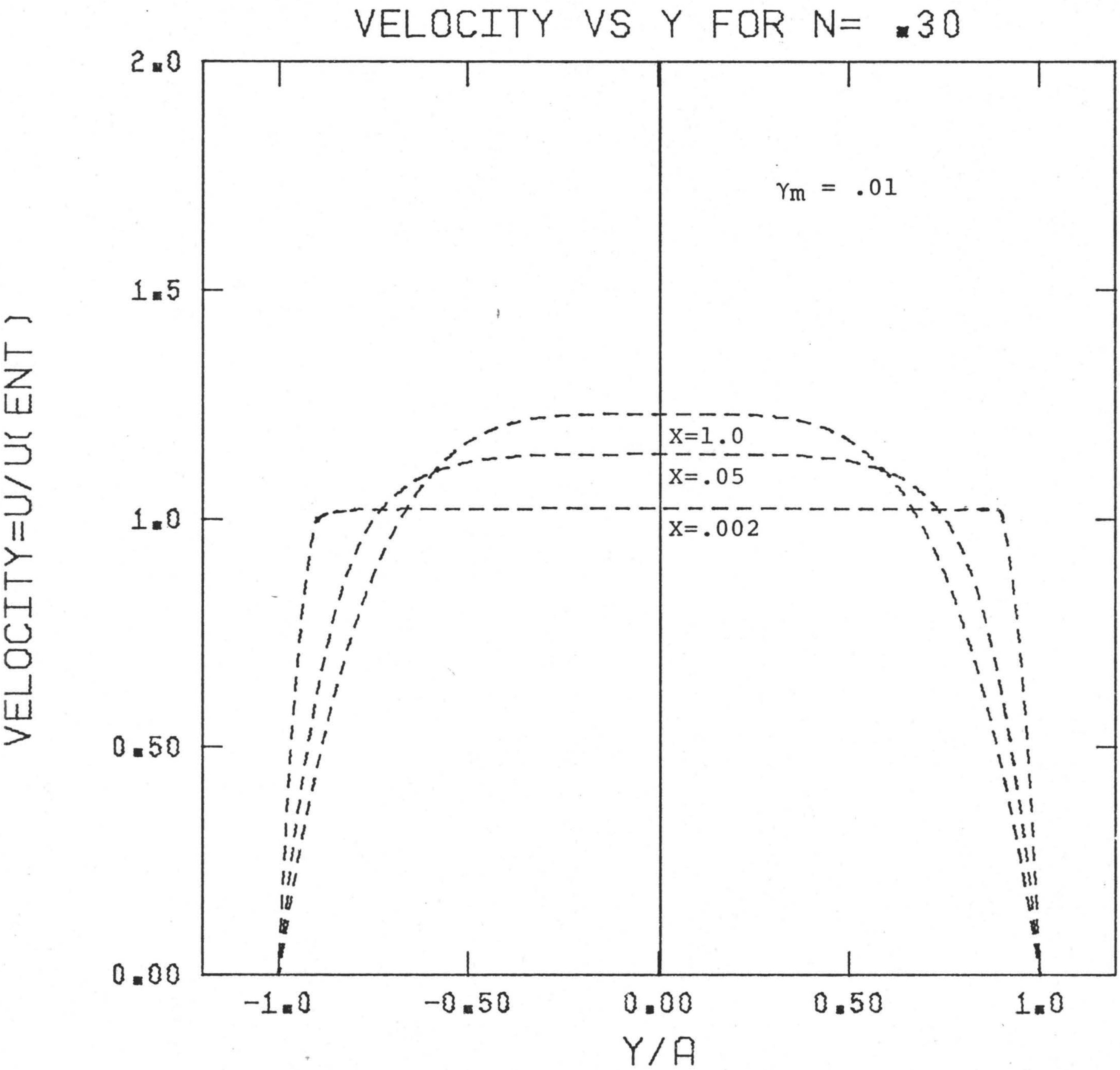


Figure I-C.3: Velocity vs Y for n=.30

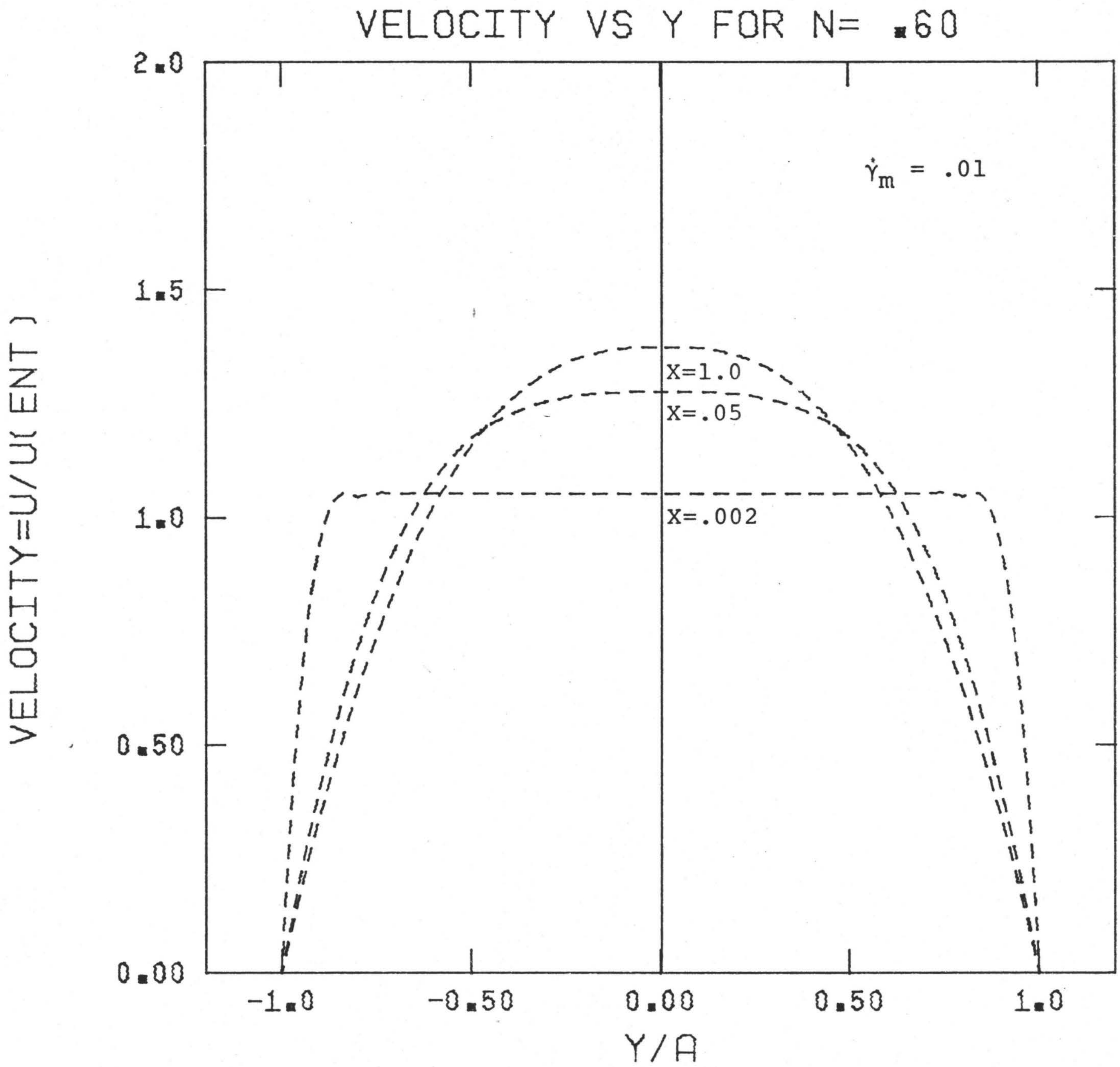


Figure I-C.4: Velocity vs Y for n=.60

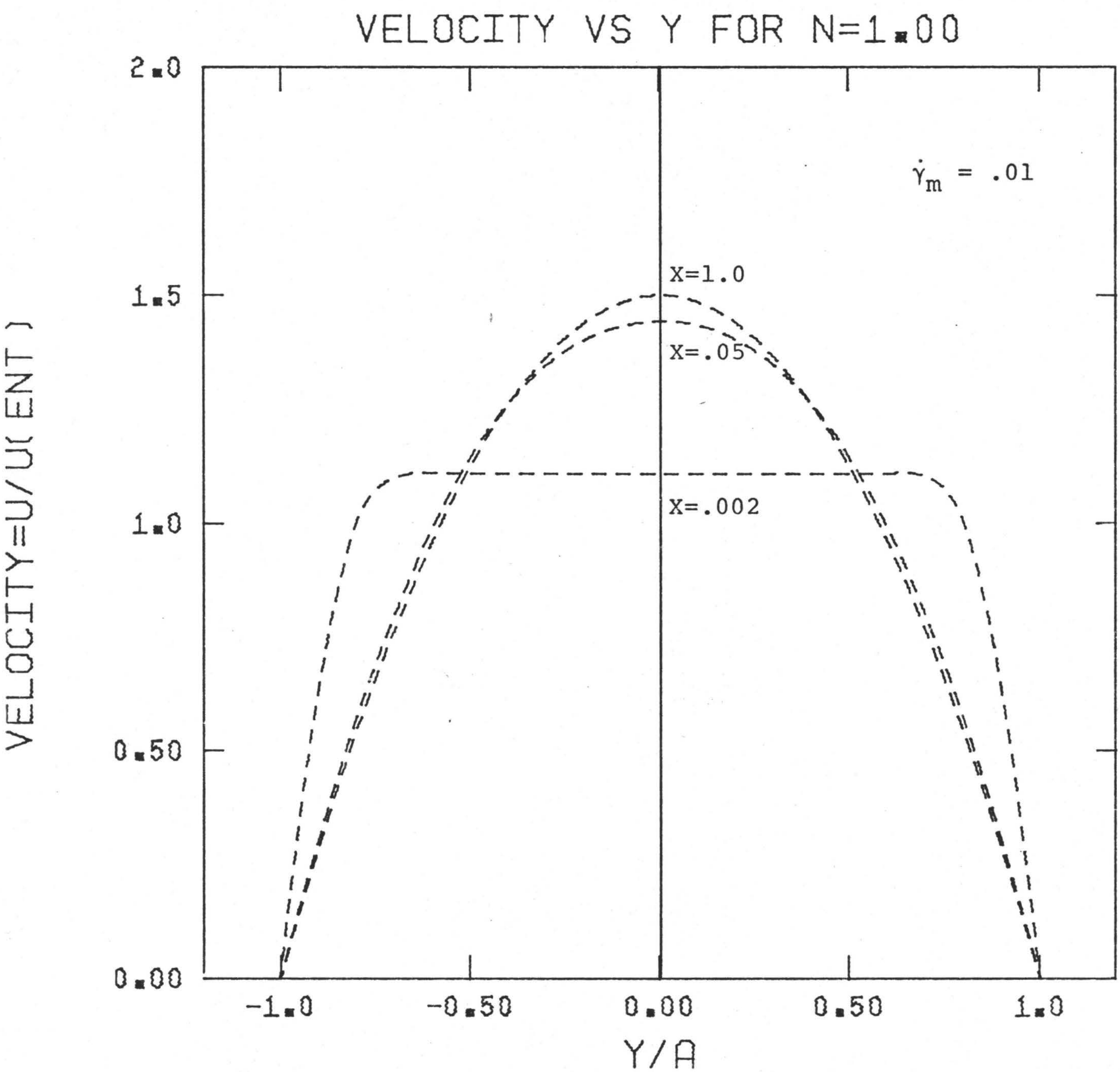


Figure I-C.5: Velocity vs Y for n=1.00

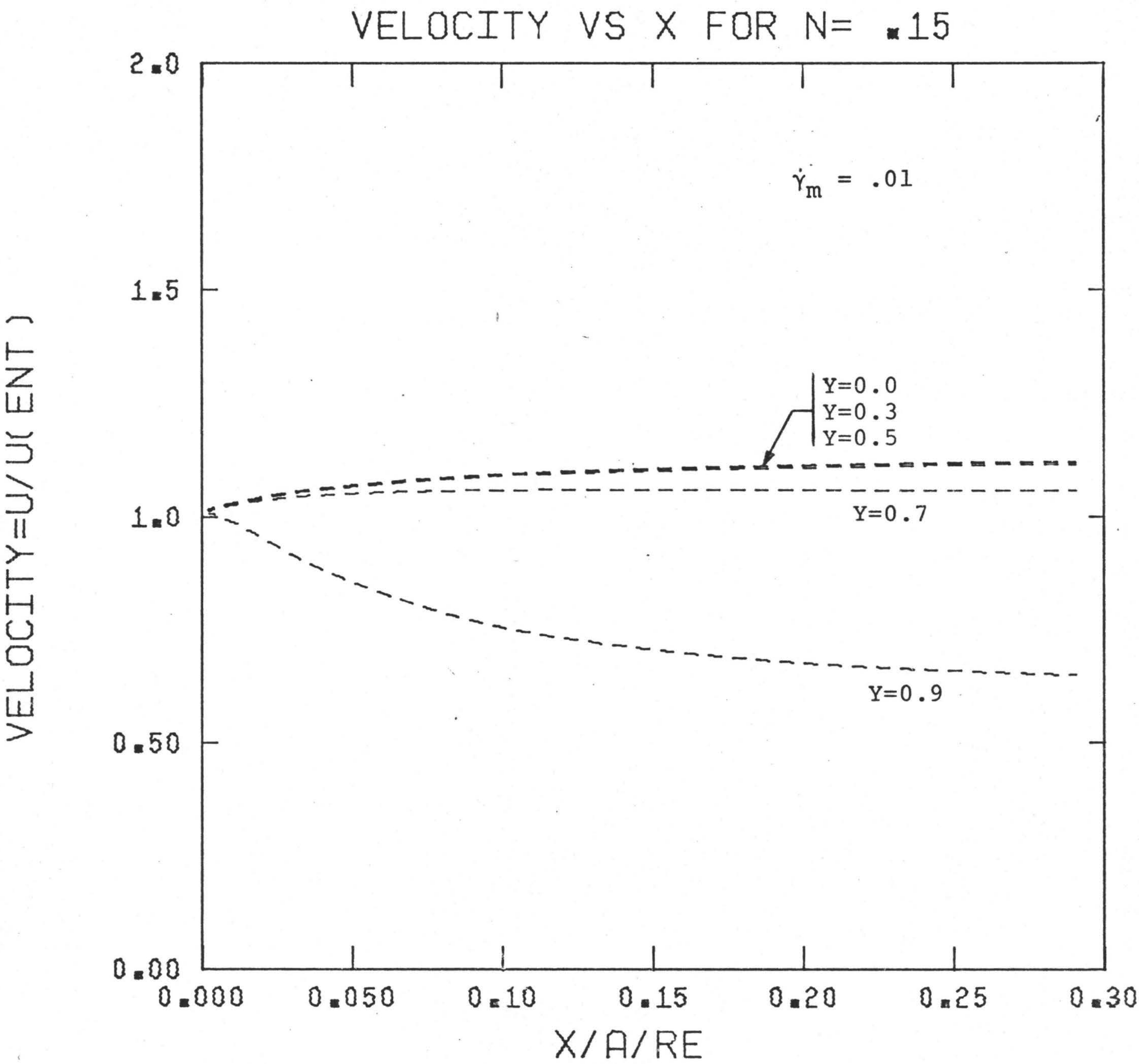


Figure I-C.6: Velocity vs X for n=.15

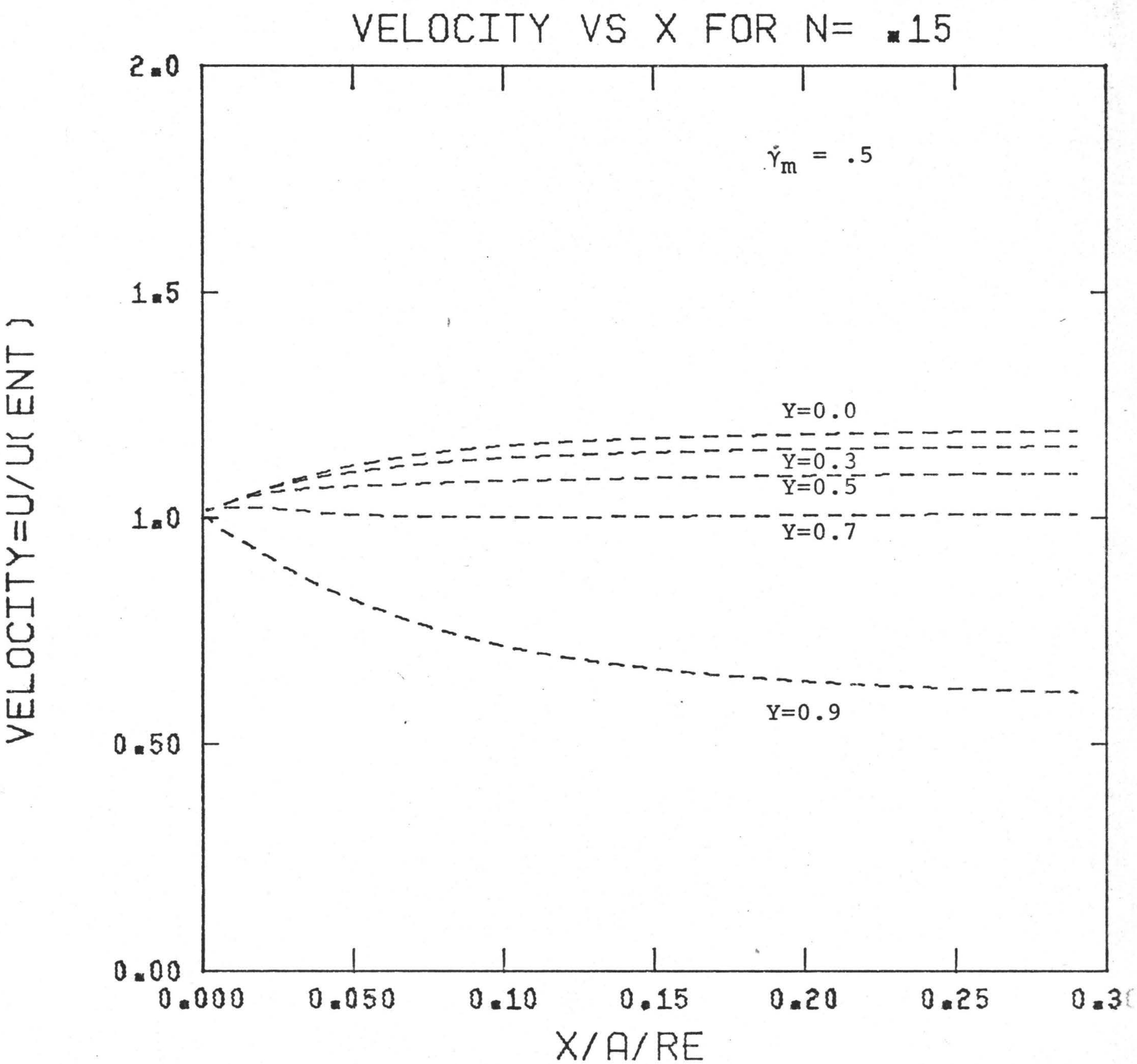


Figure I-C.7: Velocity vs X for n=.15

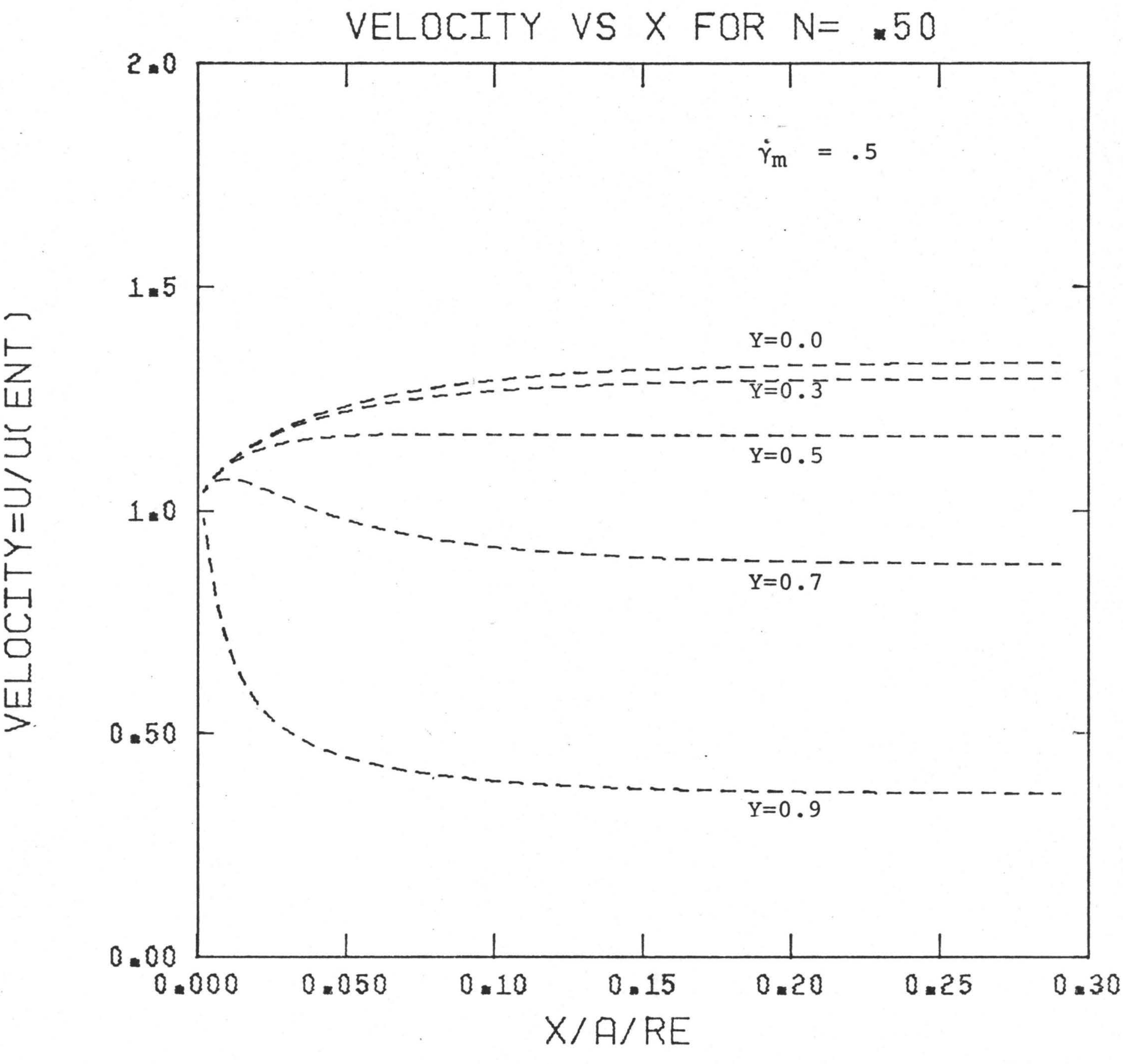


Figure I-C.8: Velocity vs X for n=.50

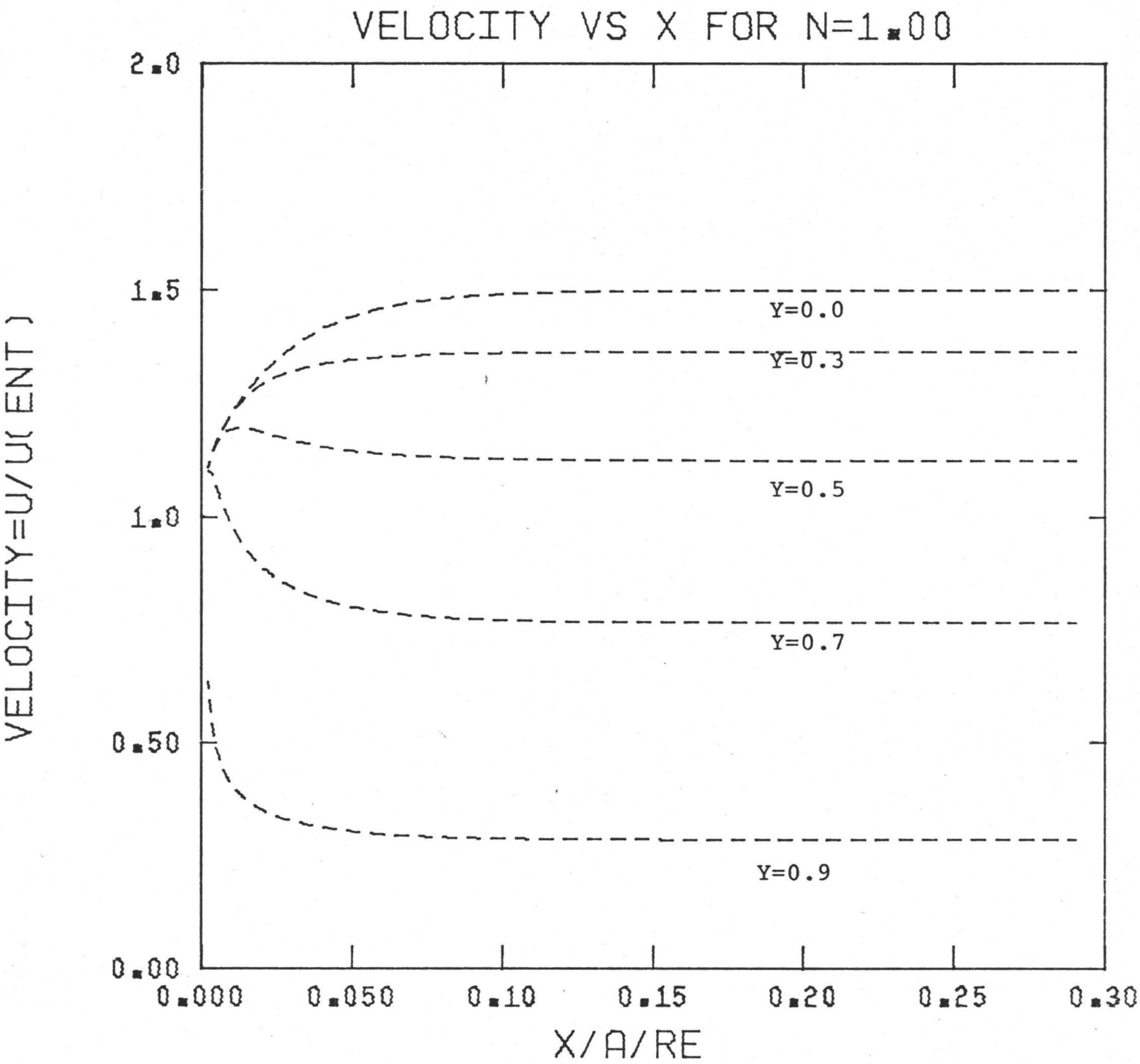


Figure I-C.9: Velocity vs X for n=1.00

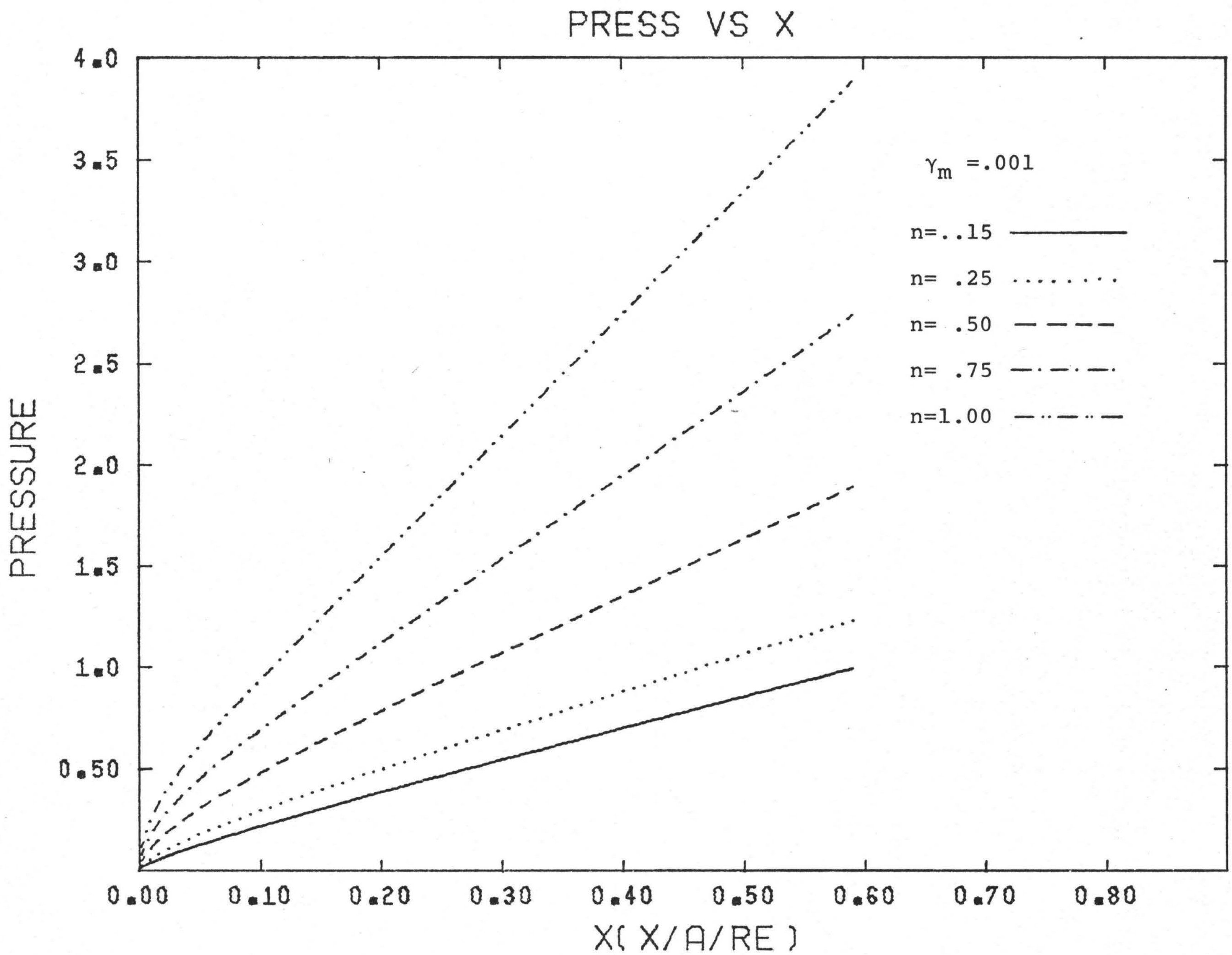


Figure I-C.10: Pressure vs X

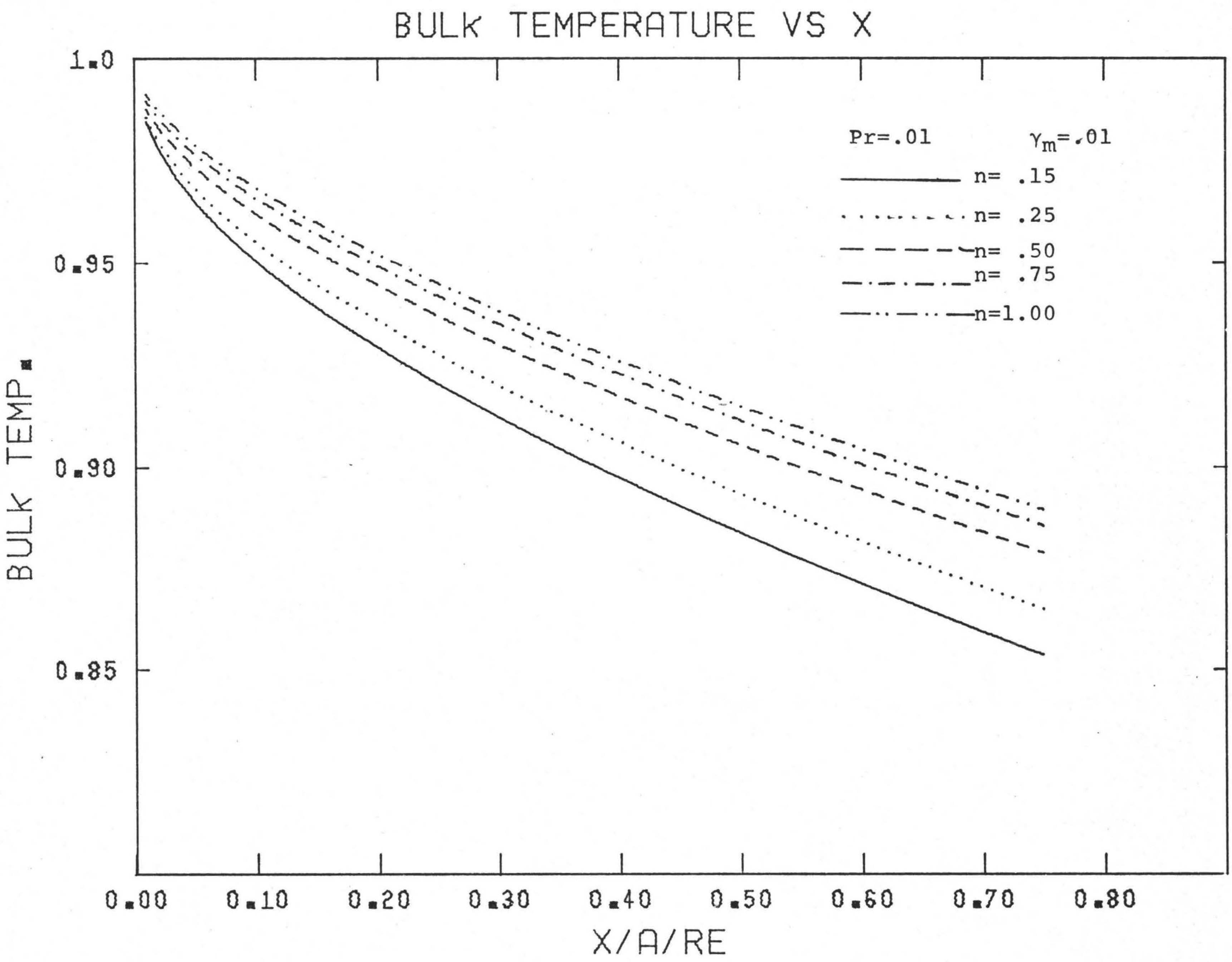


Figure I-C.11: Bulk temperature vs X

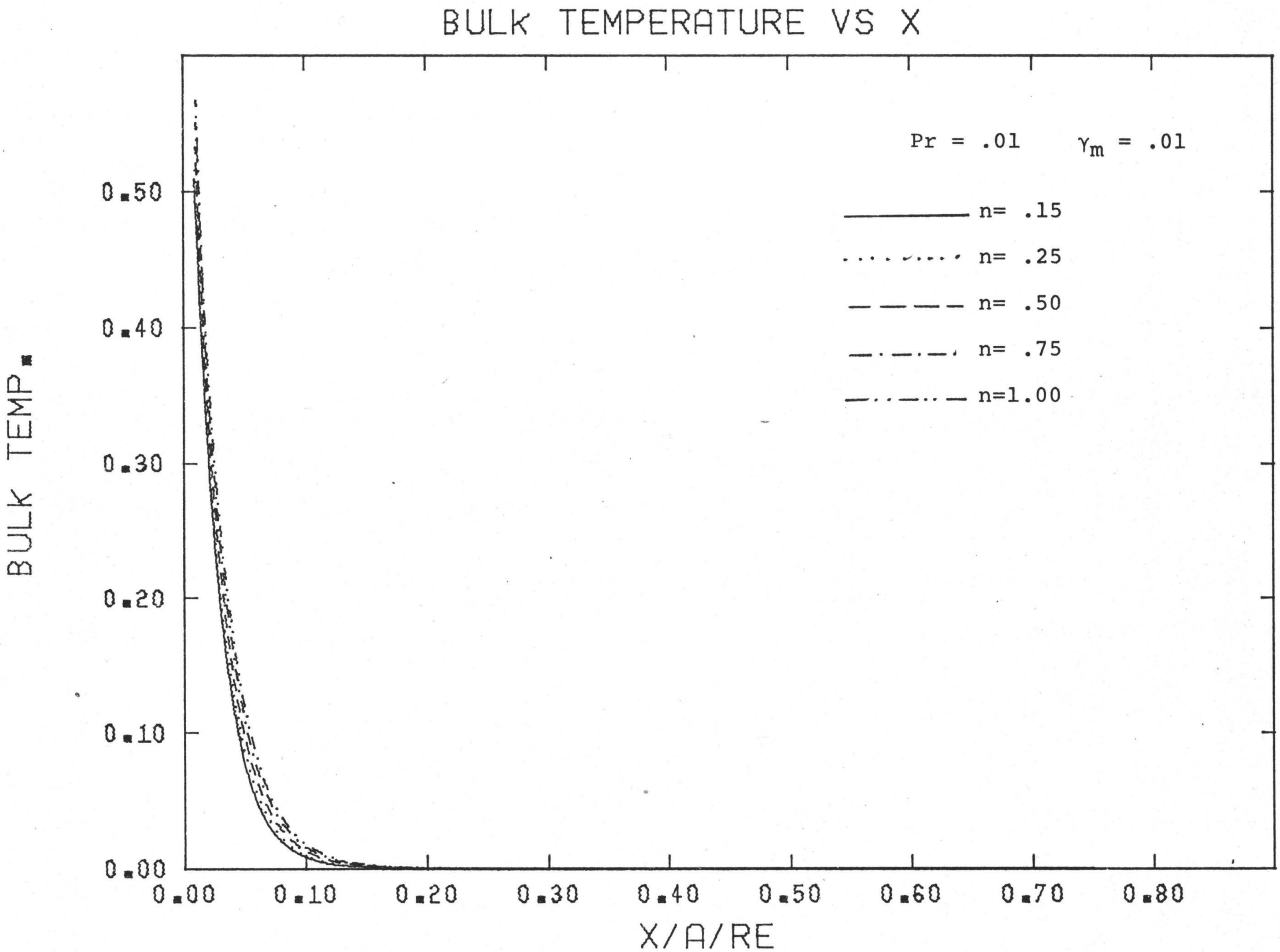


Figure I-C.12: Bulk temperature vs X

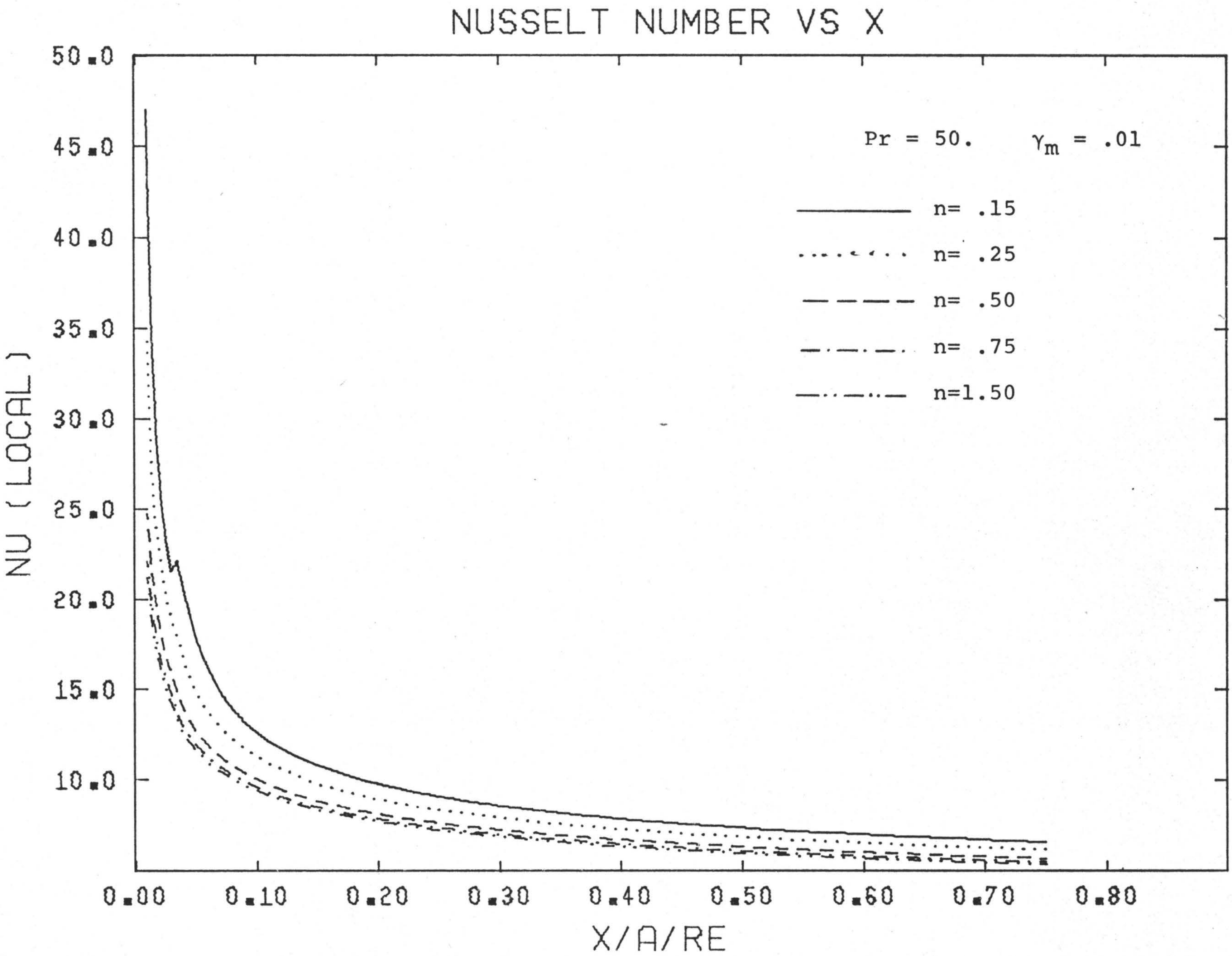


Figure I-C.13: Nusselt number vs X

APPENDIX II-A

Derivation of momentum equation for converging flow

$$U \frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{1}{\rho r}\left(\frac{\partial\left(r\tau_{rr}\right)}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} - \tau_{\theta\theta}\right)$$

(II-A.1)

If we expand the last term in accordance with the constitutive equation, we get

$$+\frac{K}{\rho r}\quad \frac{\partial\left(r(G)2\frac{\partial u}{\partial r}\right)}{\partial r} + \frac{\partial\left((G)\ \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}\right)}{\partial \theta} - 2(G)\ \left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right)$$

(II-A.2)

$$\text{where } (G) = \left|\frac{1}{r}\frac{\partial u}{\partial \theta}\right|^{n-1}$$

By using the following two identities taken from the continuity equation

$$\frac{1}{r}\frac{\partial v}{\partial \theta} = -\frac{1}{r}\frac{\partial (ru)}{\partial r} = -\frac{\partial u}{\partial r} - \frac{u}{r}$$

(II-A.3)

$$\frac{\partial v}{\partial r \partial \theta} = \frac{\partial\left(-\frac{\partial (ru)}{\partial r}\right)}{\partial \theta} = -2\ \frac{\partial u}{\partial r} - \frac{r\partial^2 u}{\partial r^2}$$

we have with the following last term

$$+ \frac{K(G)}{\rho} \quad \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2} \quad U + \frac{\partial^2 u}{\partial \theta^2} \quad + \frac{3}{r}\frac{\partial u}{\partial r} \quad + \frac{K}{\rho} \quad \frac{\partial (G)}{\partial \theta} \quad \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}$$

$$+ \frac{\partial (G)}{\partial r} \quad 2 \; \frac{\partial \; r\frac{\partial u}{\partial r}}{\partial r}$$

(II-A.4)

APPENDIX II-B

Mesh sizes used for converging flows

	Number of grid points across	Value of ΔR	Grid size valid up to R=
A) C-21	20	.001	.01
	20	.01	.1
	20	.05	1.0
B) C-41	40	.001	.01
	40	.01	.1
	40	.05	1.0
C) C-22	20	.0001	.001
	20	.001	.01
	20	.01	.1
	20	.05	1.0
D) C-42	40	.0001	.001
	40	.001	.01
	40	.01	.1
	40	.05	1.0
E) C-82	80	.0001	.001
	80	.001	.01
	80	.01	.1
	80	.05	1.0
F) C-43	40	.0001	.001
	40	.0005	.01
	40	.0025	.1
	40	.01	1.0
G) C-23	20	.0001	.001
	20	.0005	.01
	20	.0025	.10
	20	.01	1.0
H) V-83	80	.0001	.001
	40	.0005	.01
	20	.0025	.1
	20	.01	1.0
I) V-164	160	.00002	.0004
	160	.0001	.002
	80	.0005	.01
	40	.001	.03
	20	.0025	.1
	20	.01	1.0

APPENDIX II-C

Additional results for converging entry flow

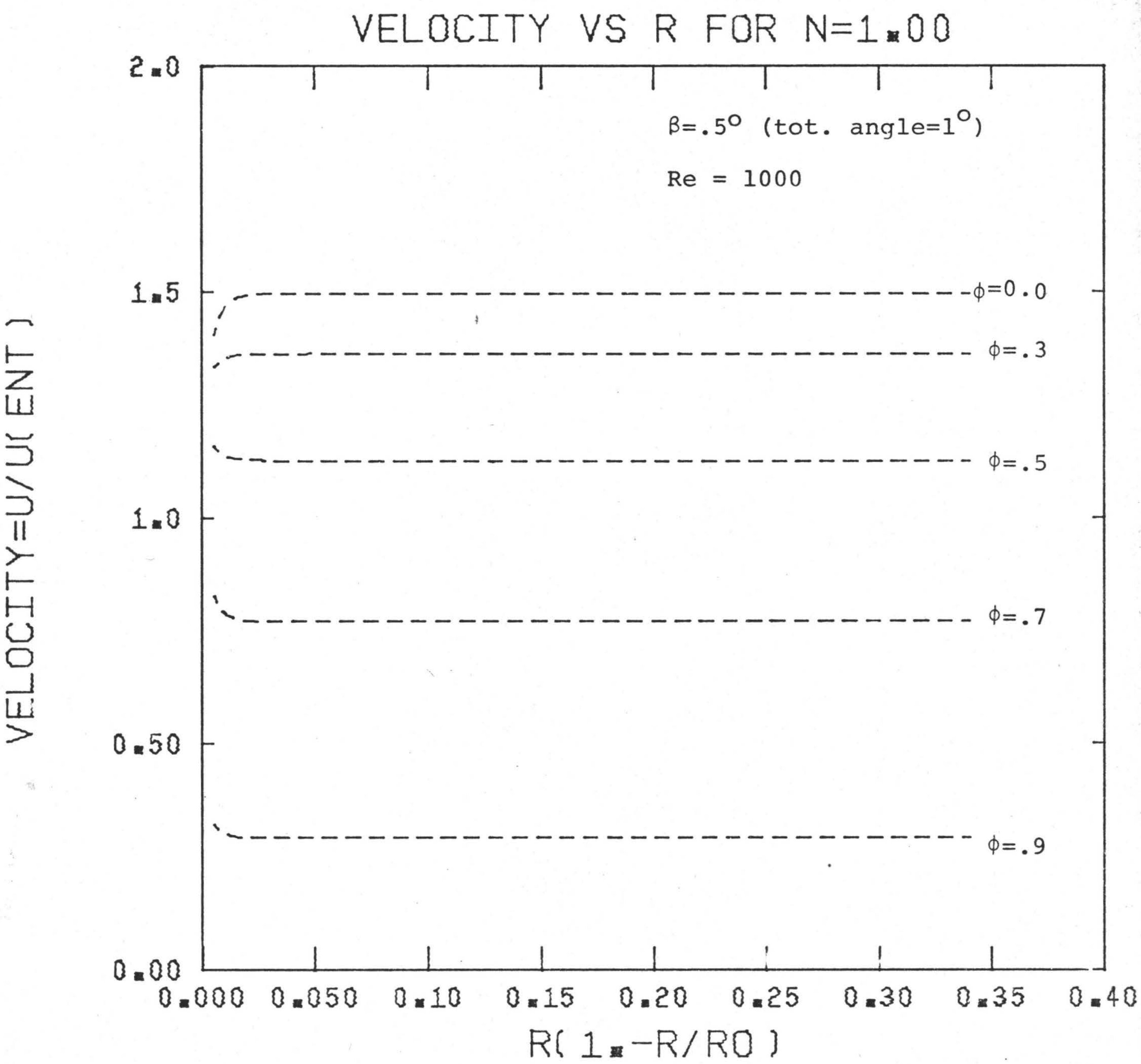


Figure II-C.1: Velocity vs R for n=1.00

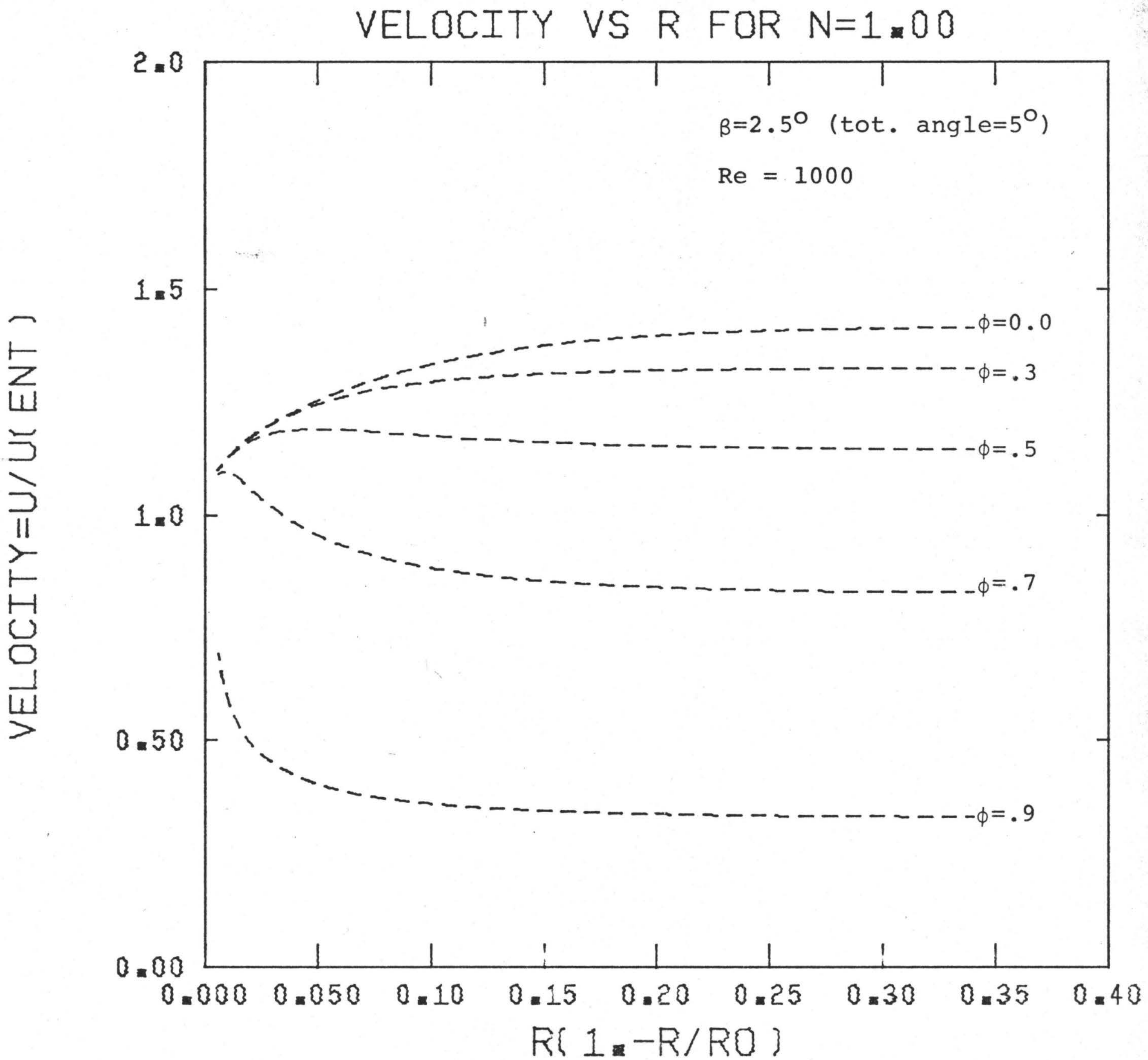


Figure II-C.2: Velocity vs R for n=1.00

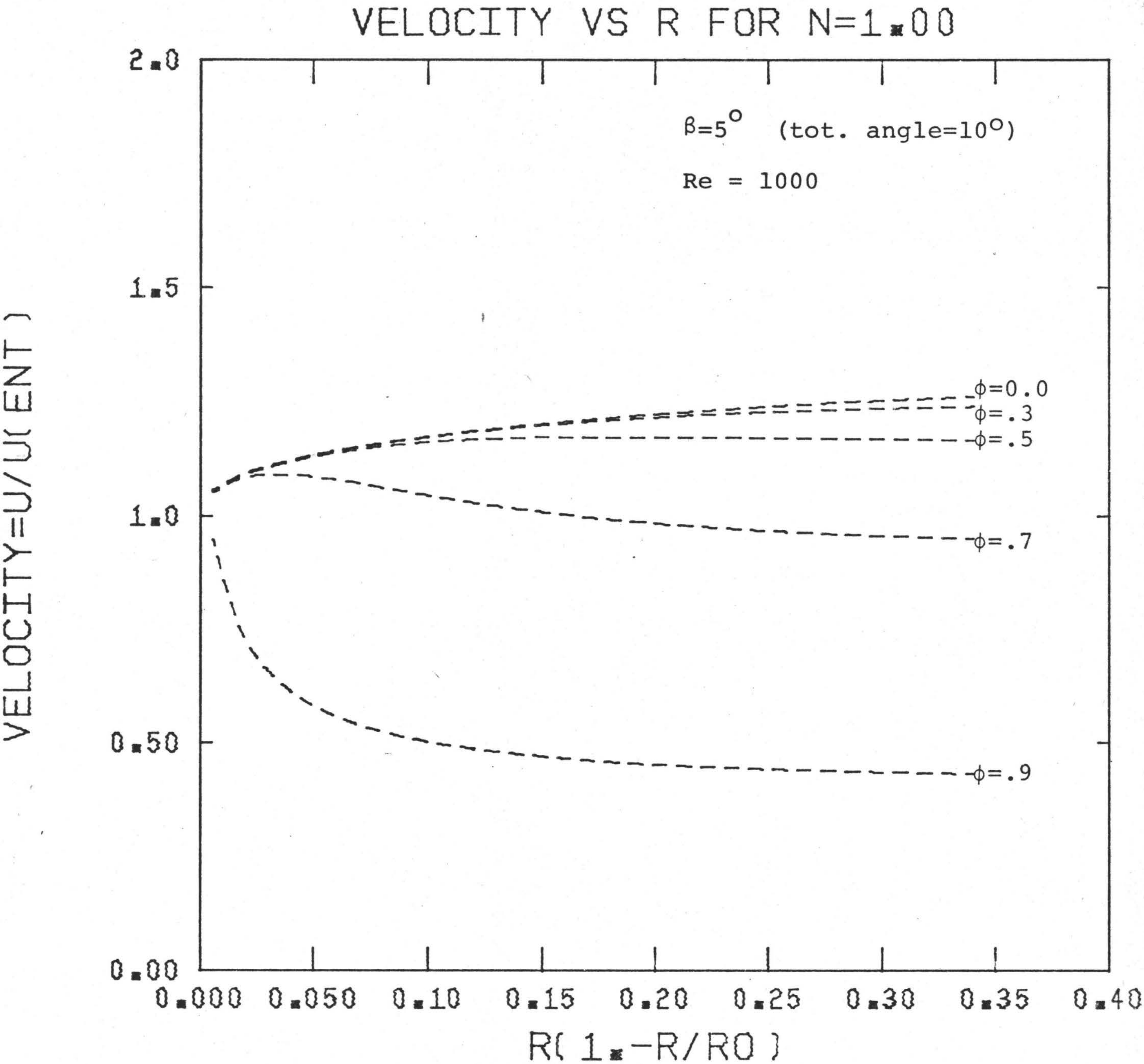


Figure II-C.3: Velocity vs R for n=1.00

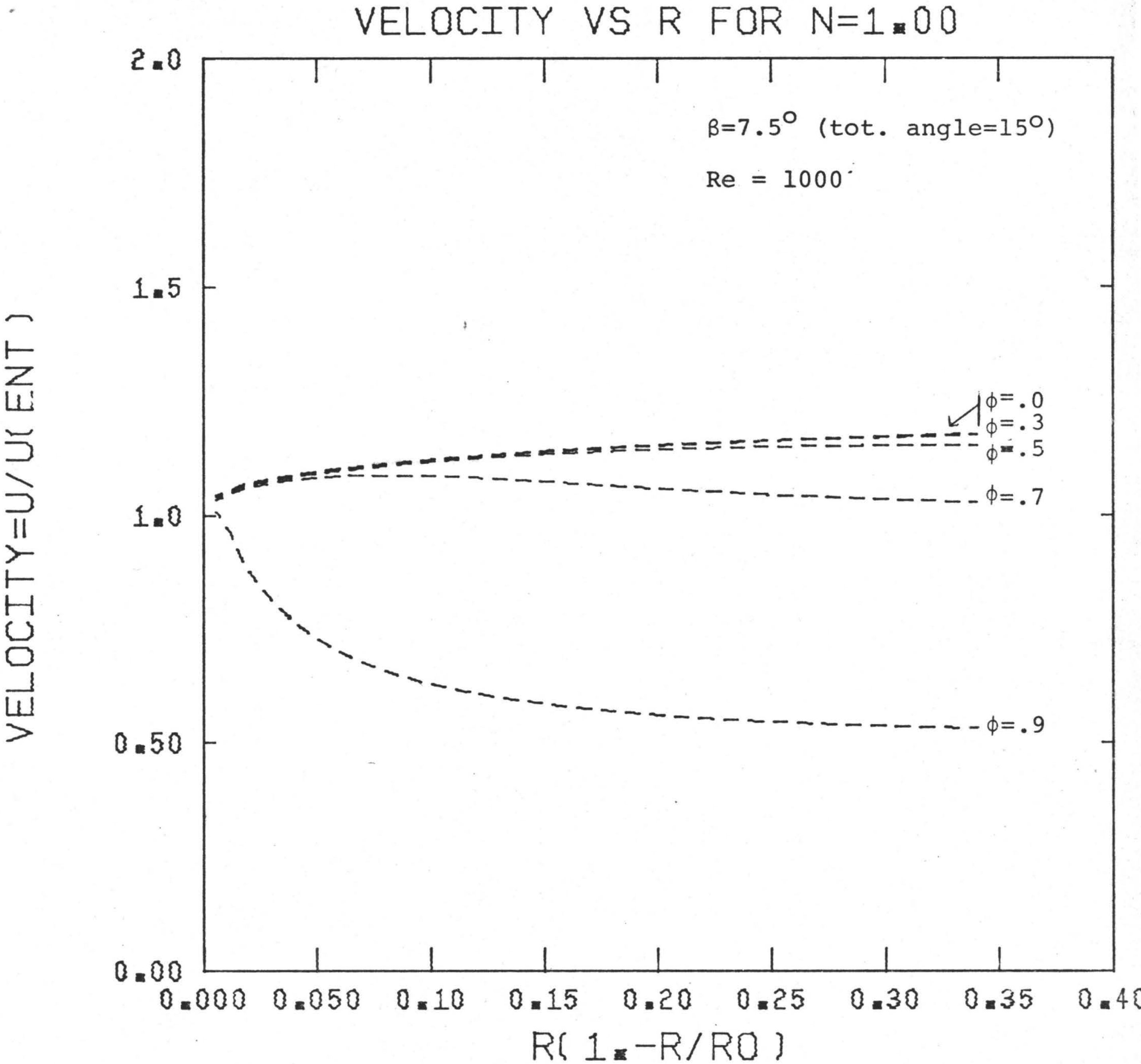


Figure II-C.4: Velocity vs R for n=1.00

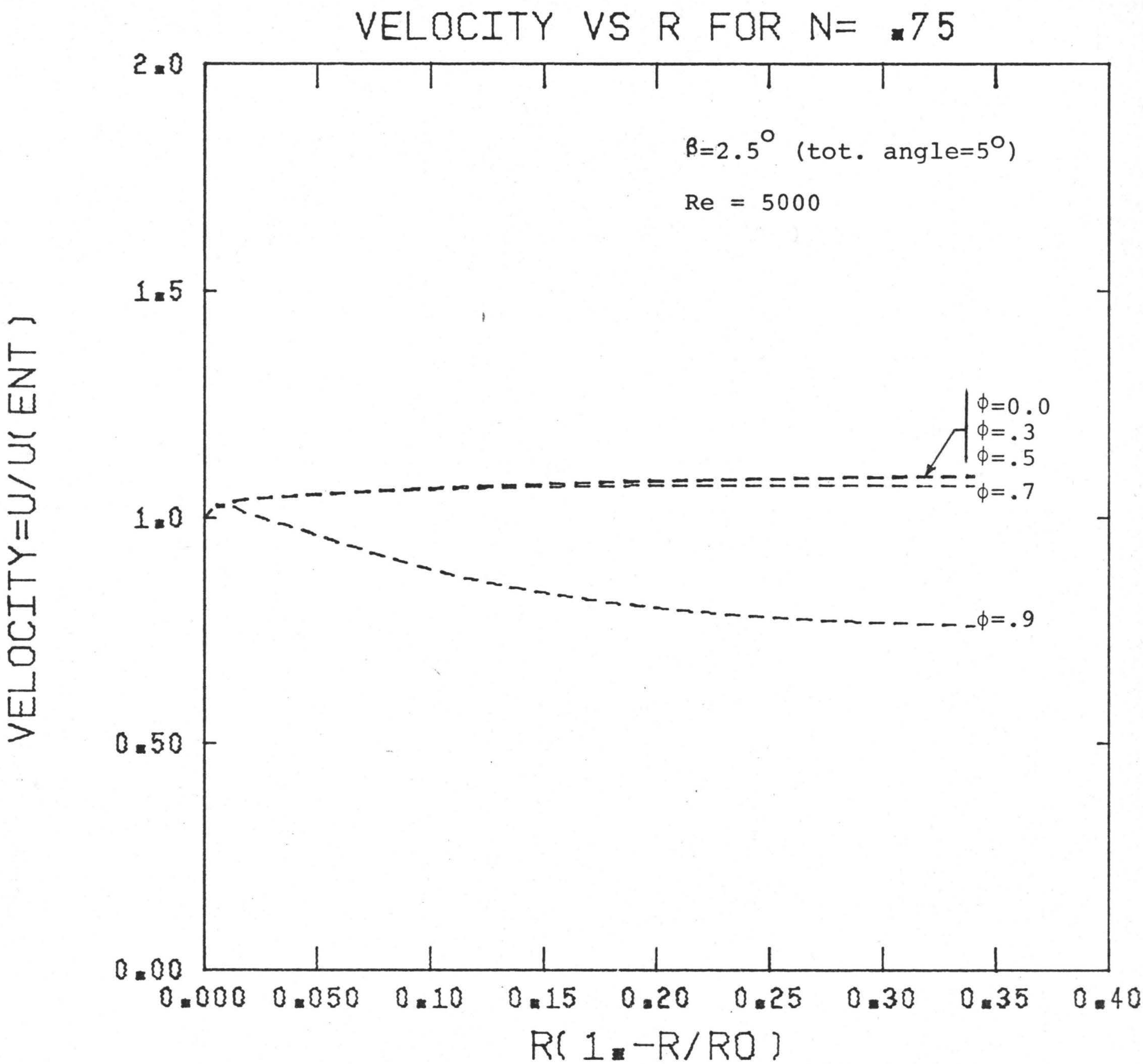


Figure II-C.5: Velocity vs R for n=.75

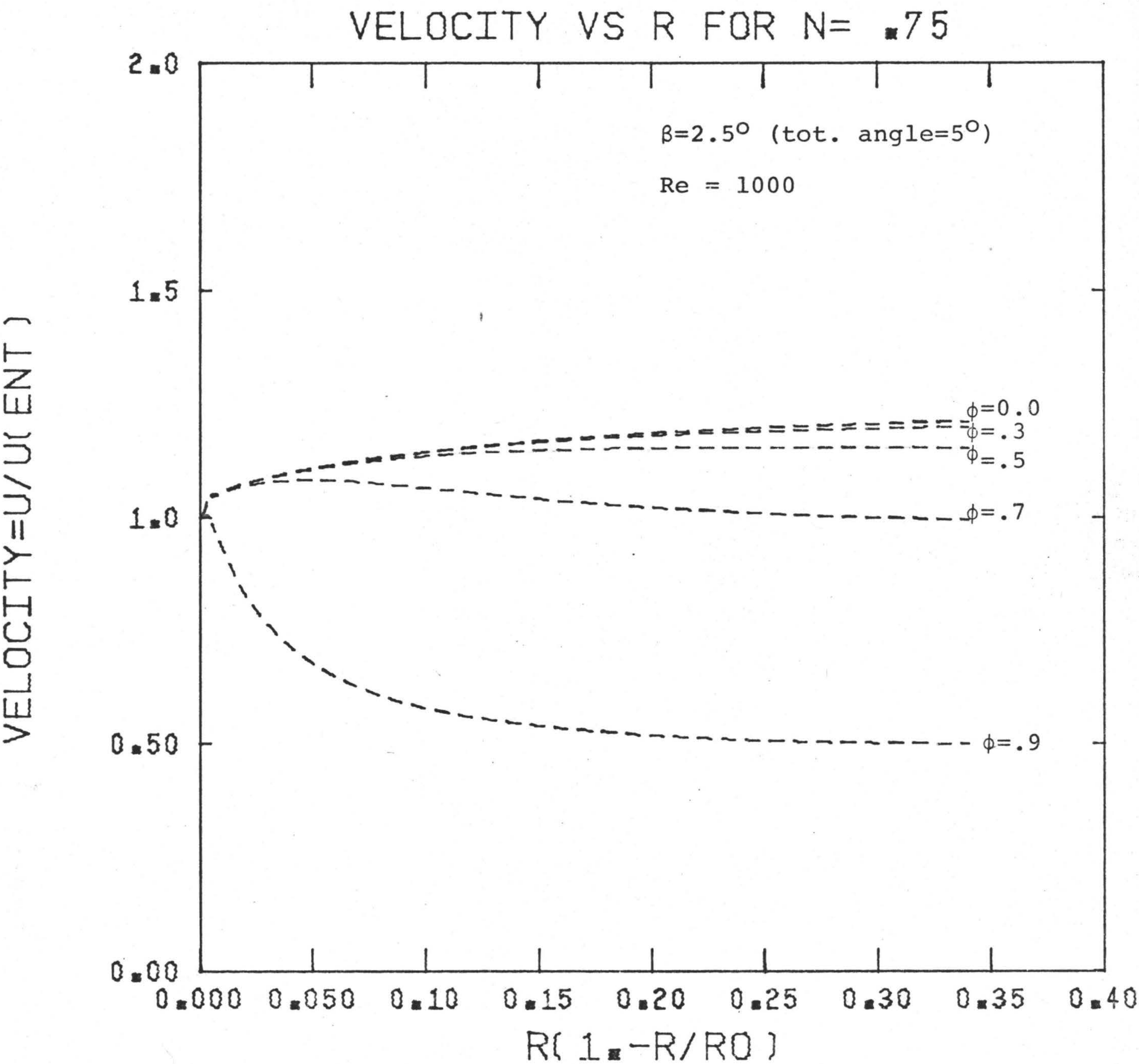


Figure II-C.6: Velocity vs R for n=.75

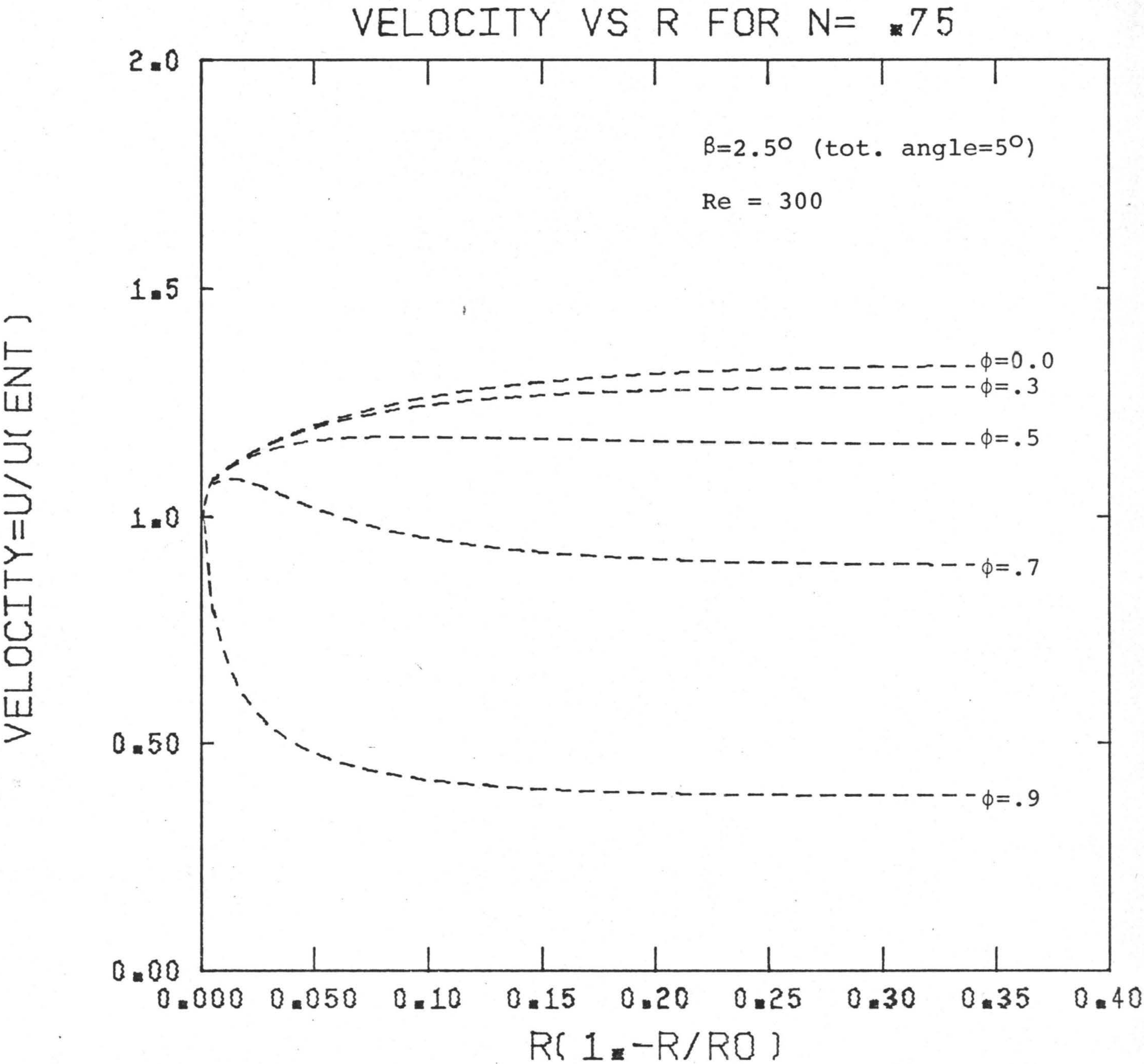


Figure II-C.7: Velocity vs R for n=.75

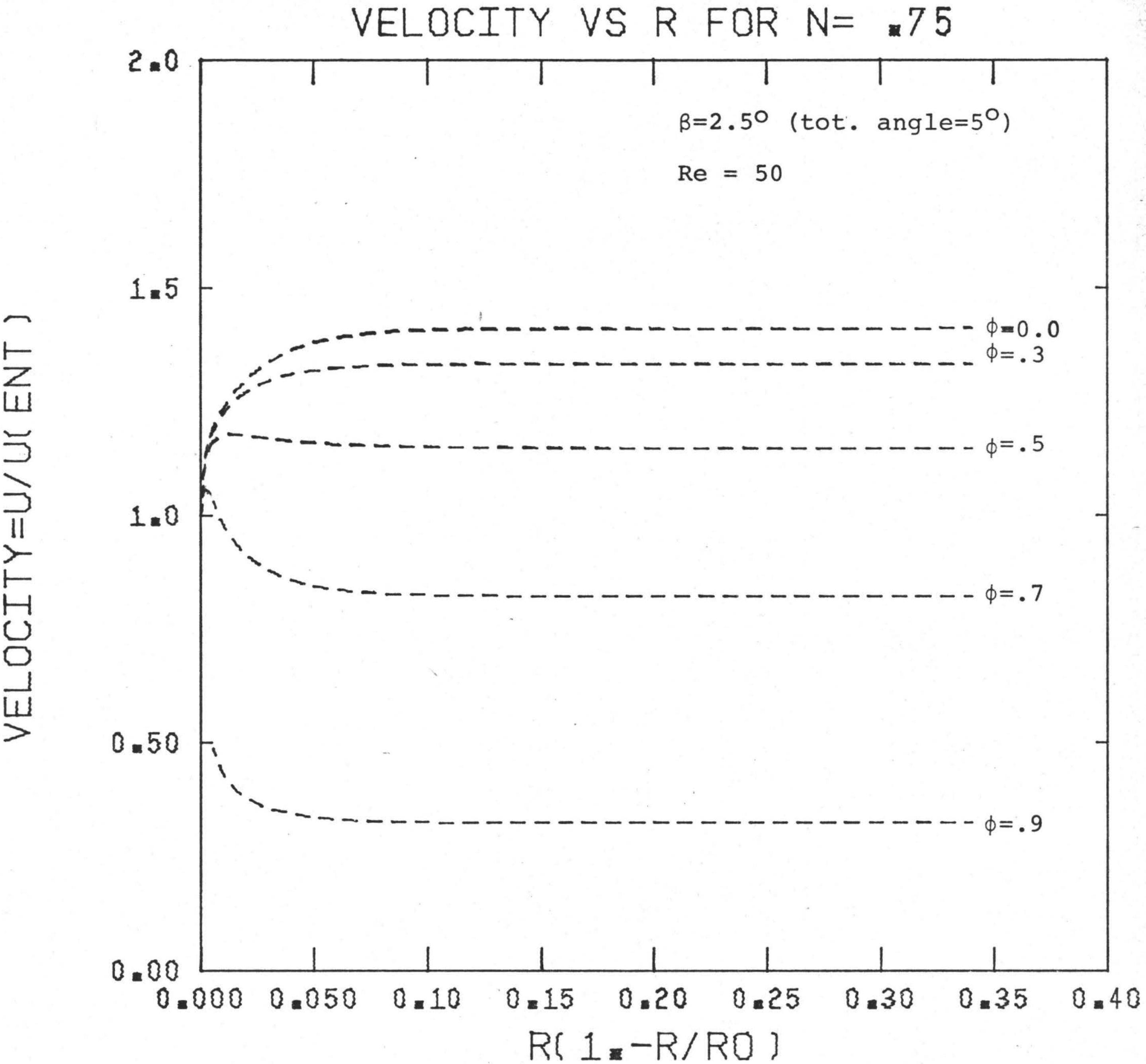


Figure II-C.8: Velocity vs R for n=.75

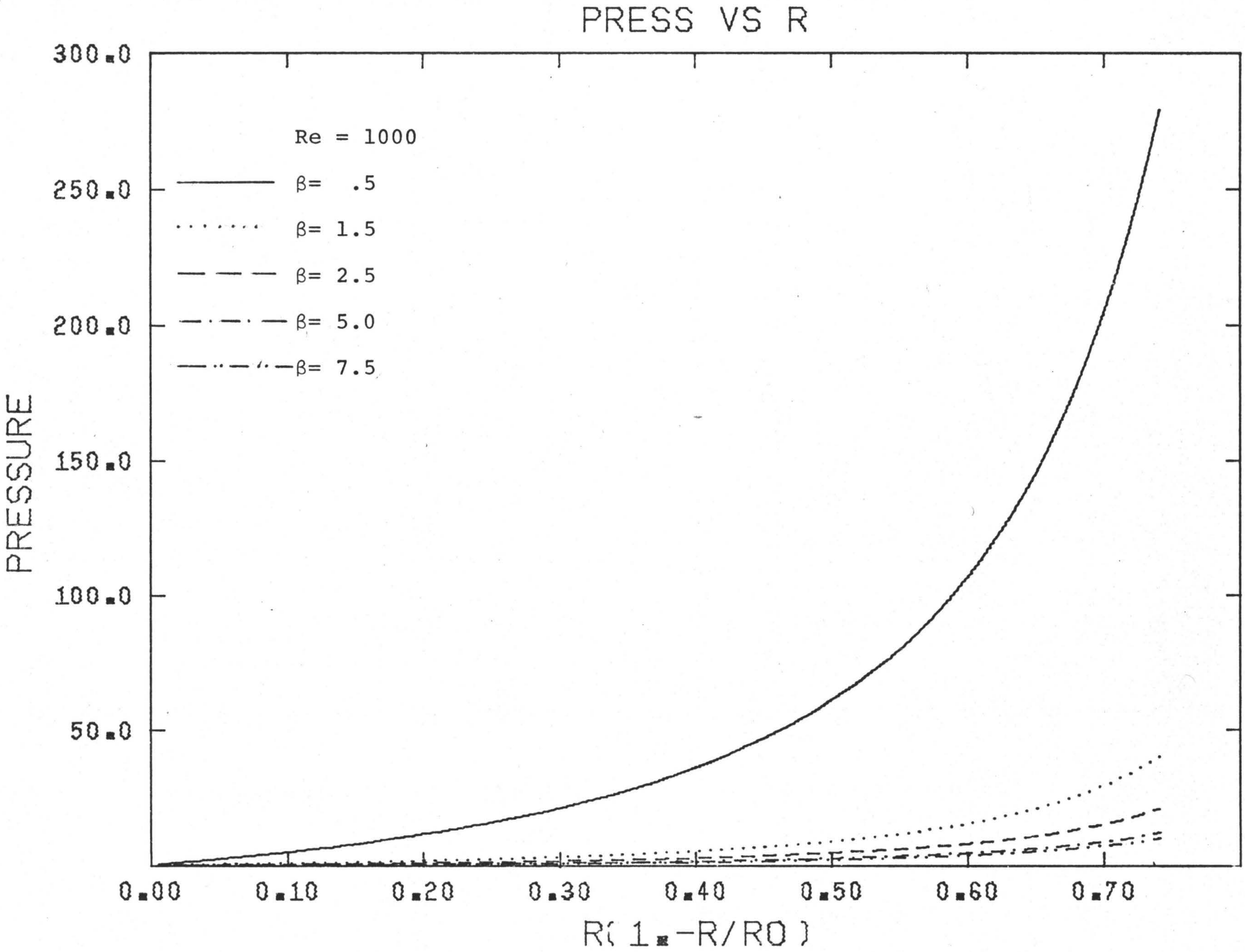


Figure II-C.9: Pressure vs R

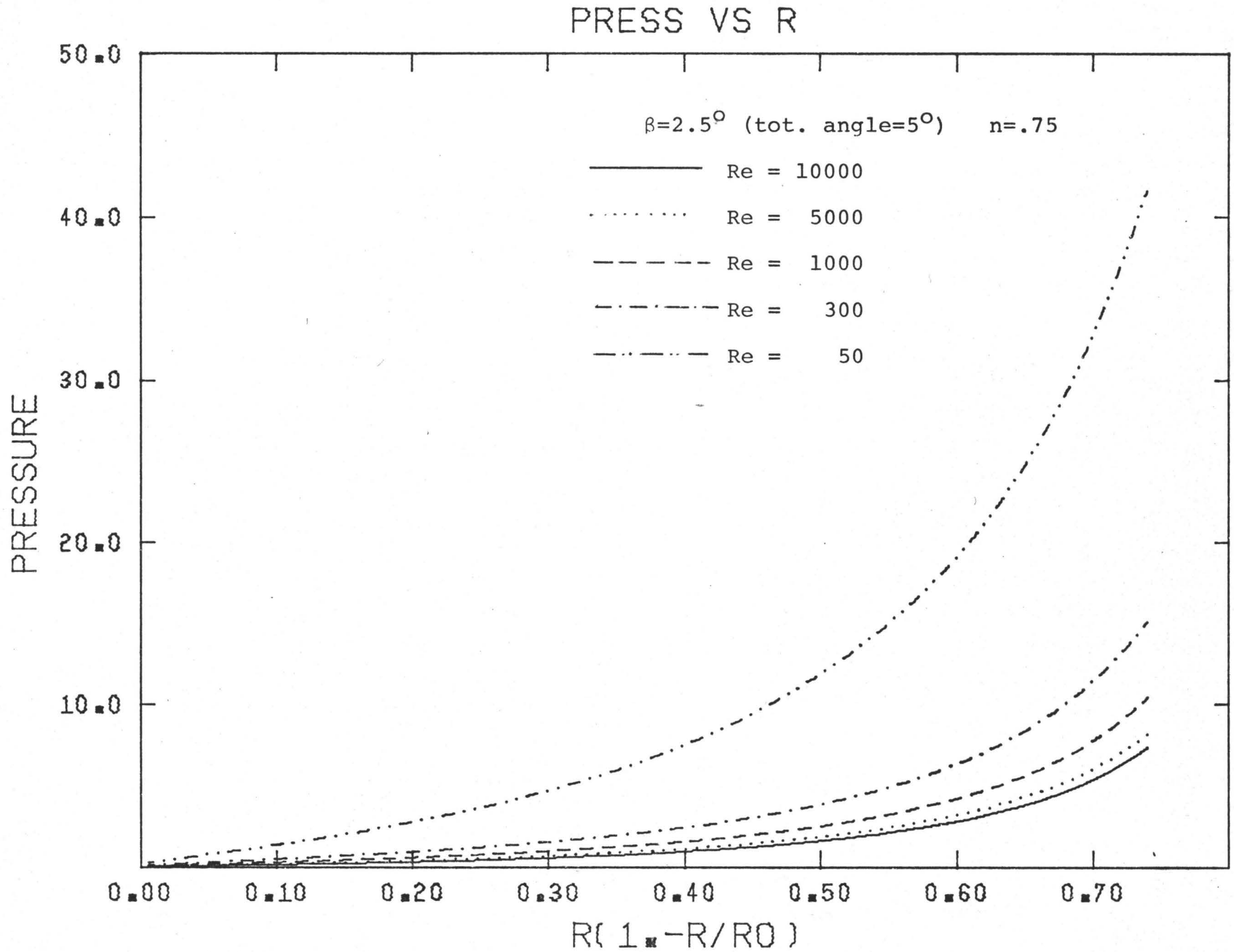


Figure II-C.10: Pressure vs R

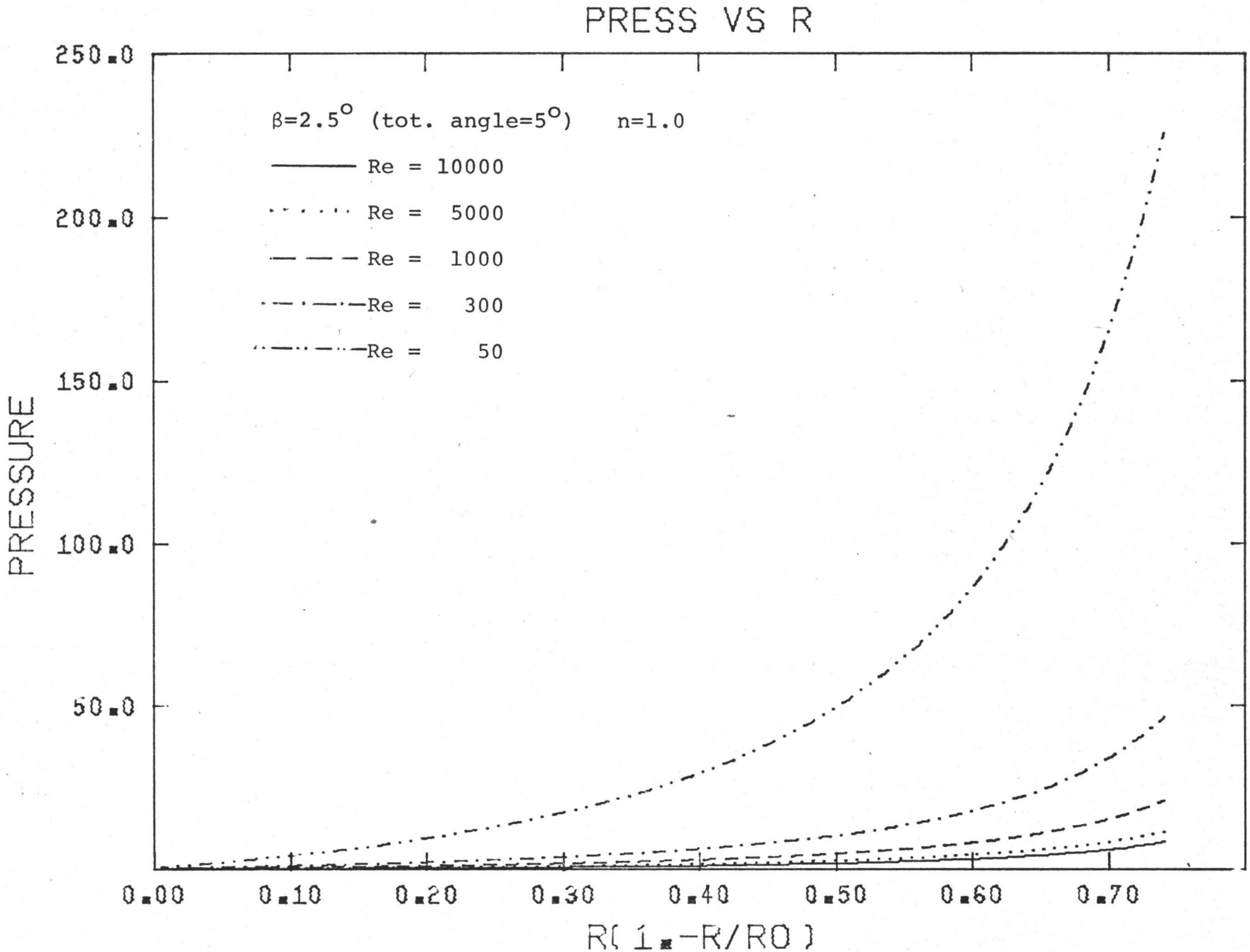


Figure II-C.11: Pressure vs R

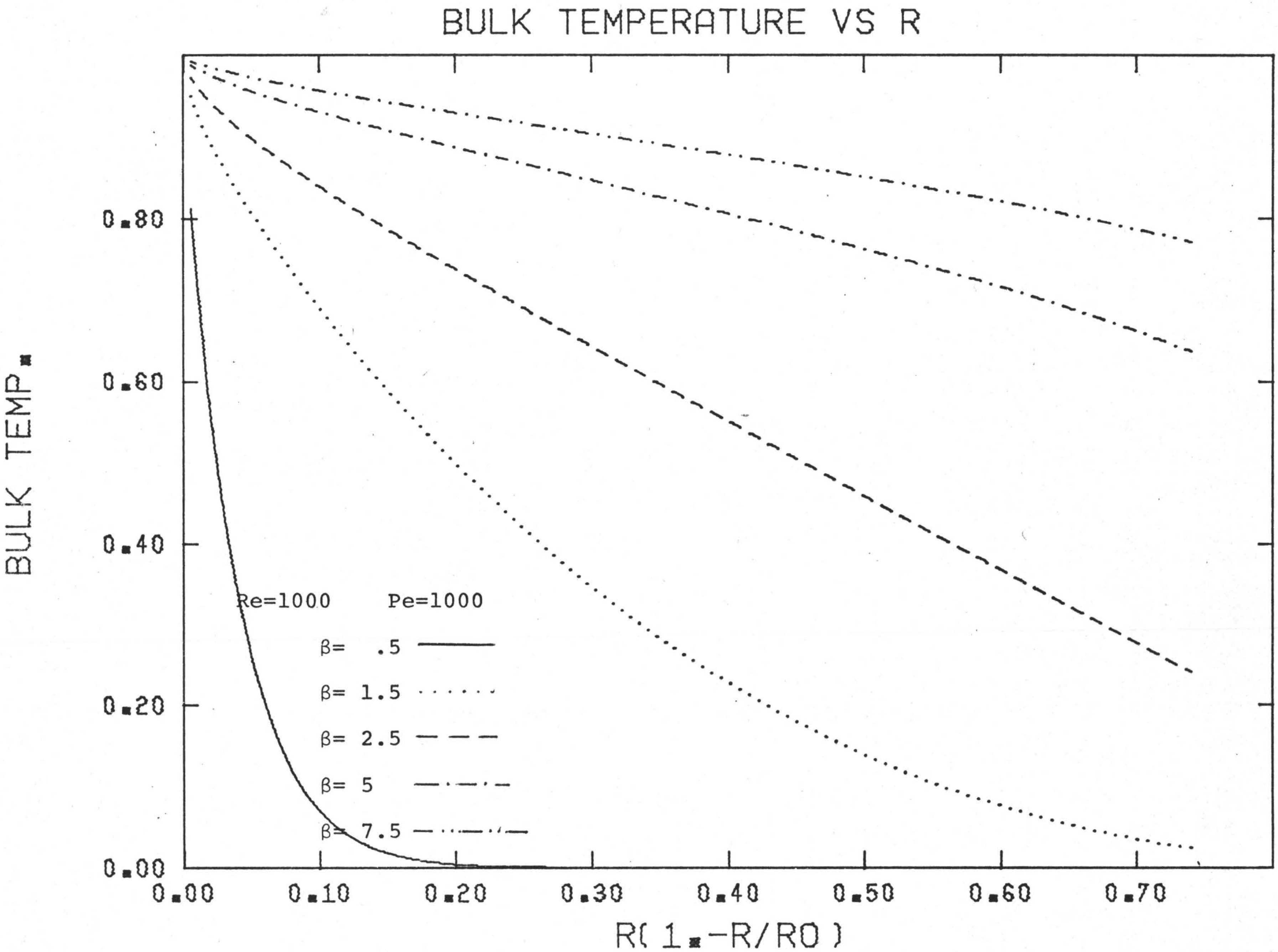


Figure II-C.12: Bulk temperature vs R

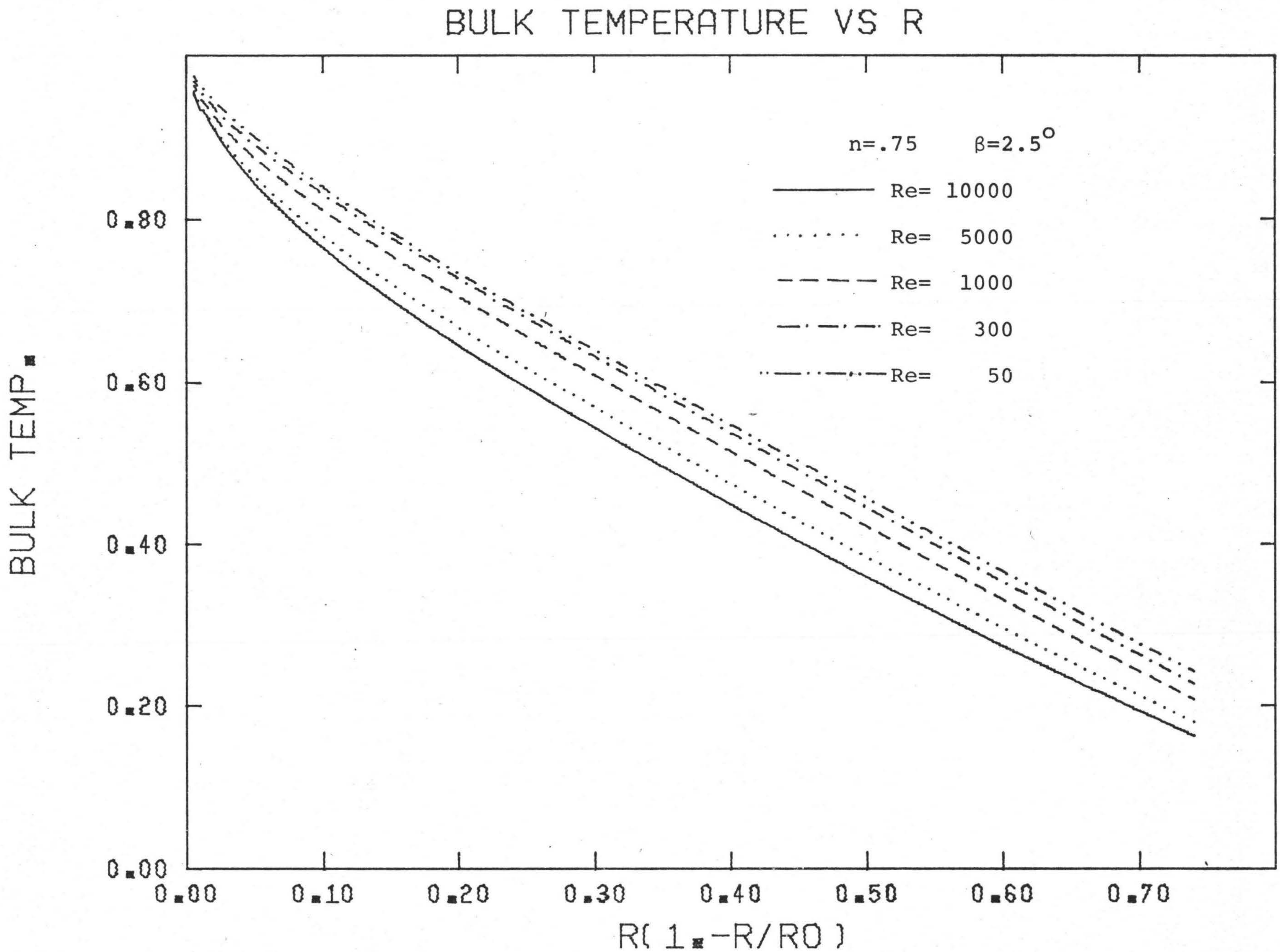


Figure II-C.13: Bulk temperature vs R

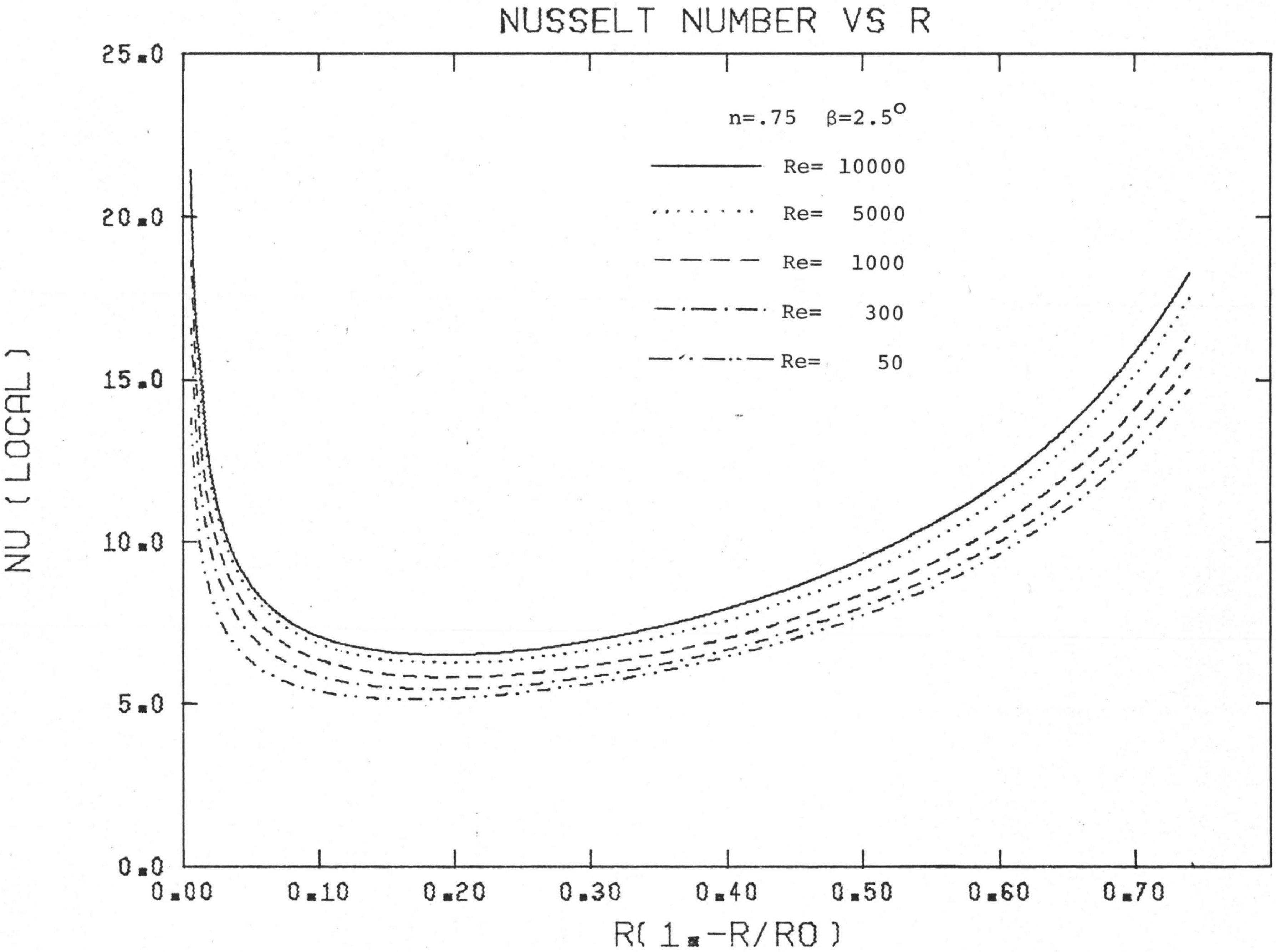


Figure II-C.14: Nusselt number vs R

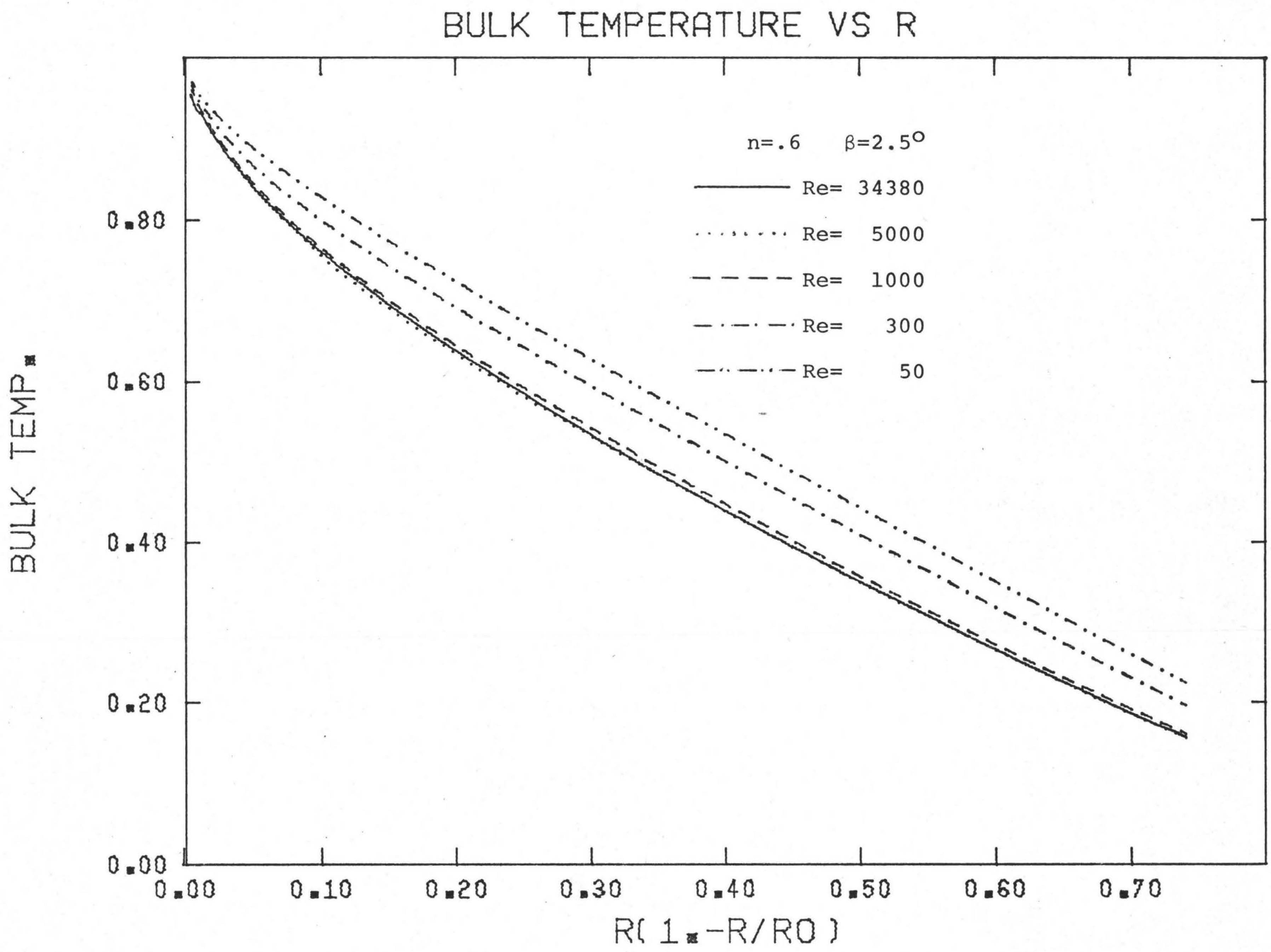


Figure II-C.15: Bulk temperature vs R

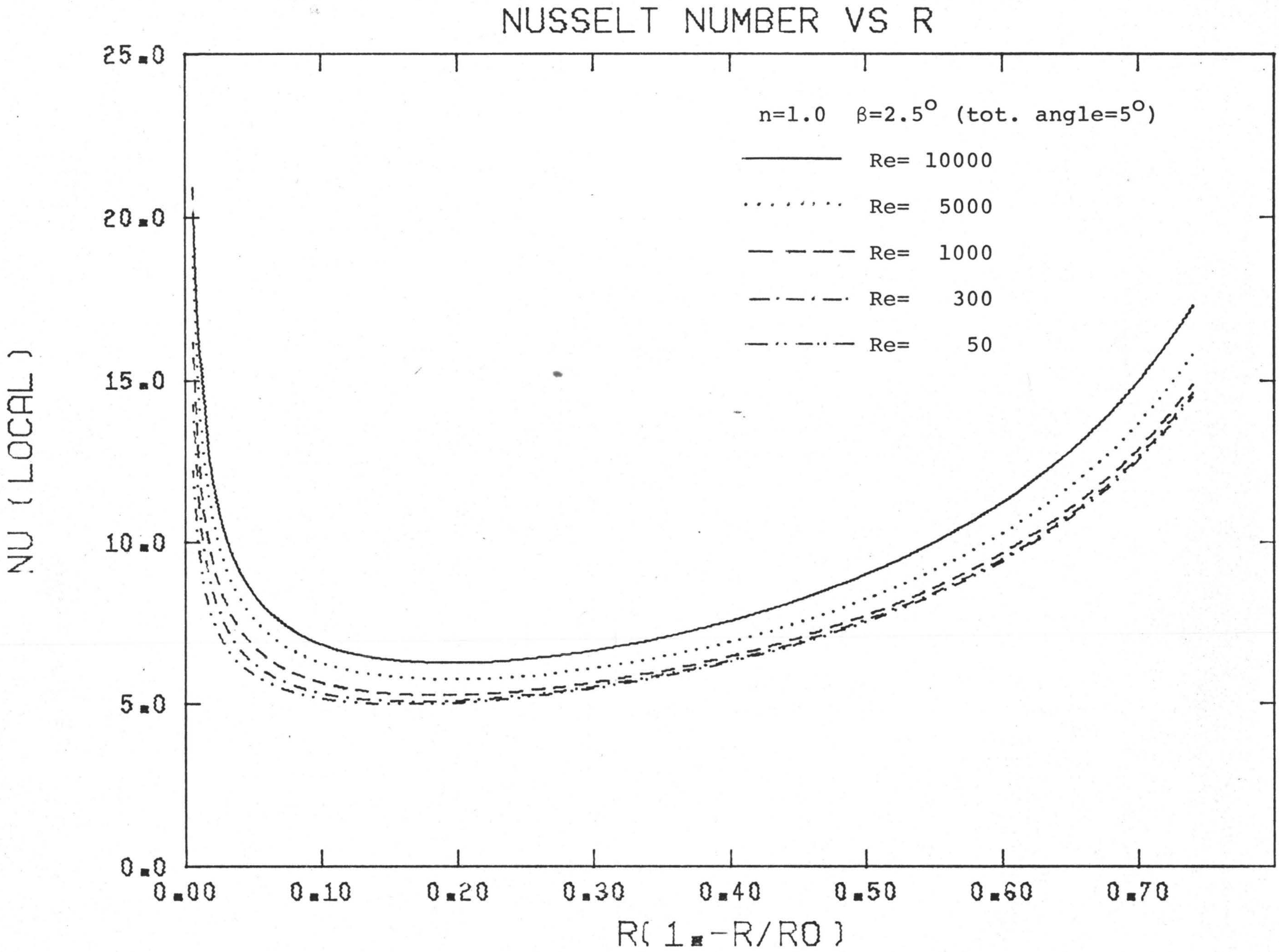


Figure II-C.16: Nusselt number vs R

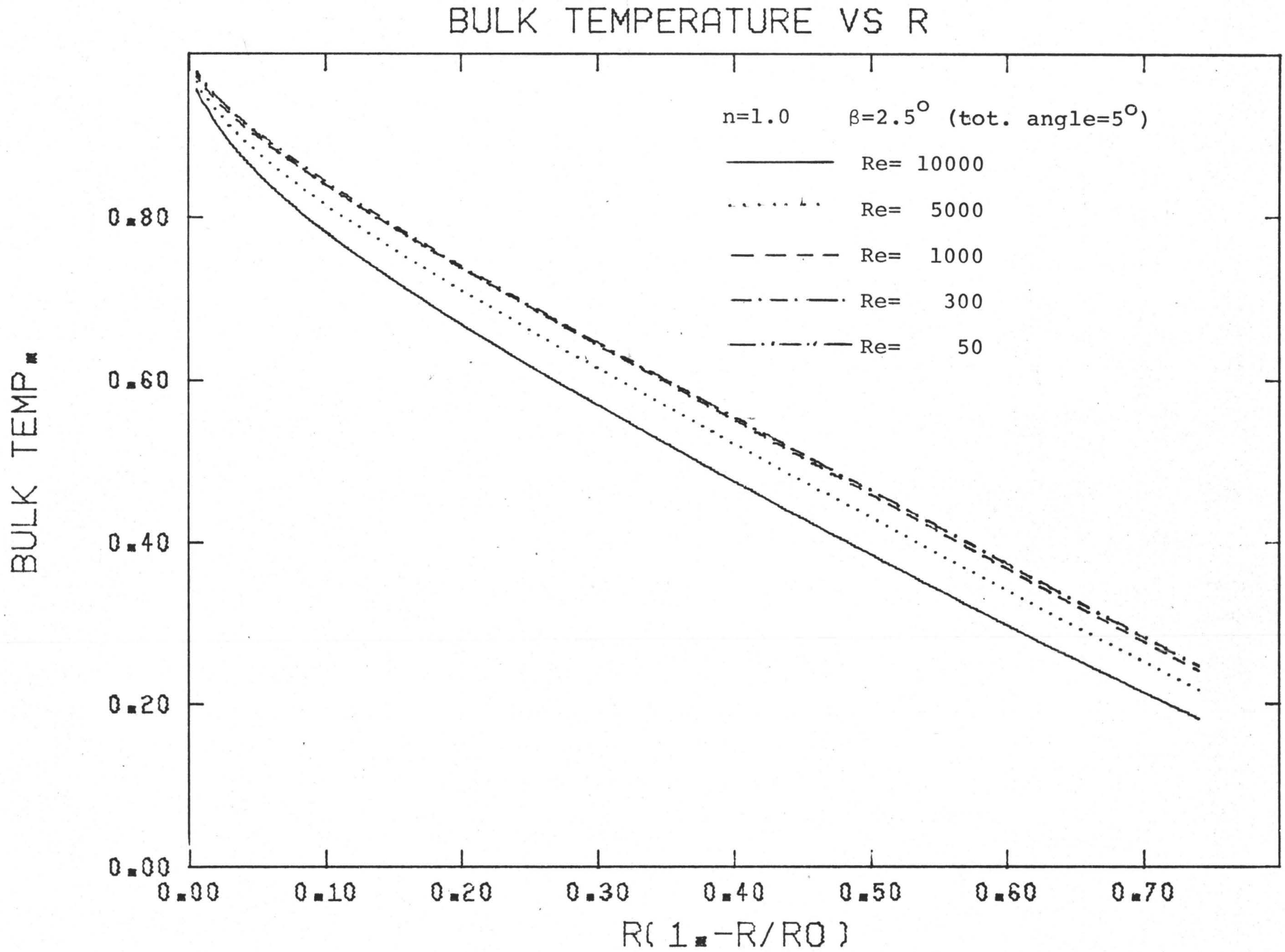


Figure II-C.17: Bulk temperature vs R

APPENDIX III

Finite difference solution of channel entry flow with an iterative scheme

III- 1 Theory

As mentionned in the first part of this work, we can use an iterative scheme if when introducing finite difference representations, we still have non-linear equations. Whereas the fully implicit scheme can proceed from one column to the other, the iterative one uses approximations of the values to be calculated. In the present work, the first approximation used, is the values of the preceeding column ; after the first calculation, the approximation is taken the sum of 85% of the previously calculated values and 15% of the old values. (convergence being generally faster with this ratio). The word "values" here applies only to the axial velocities and pressure but not to y-direction velocities since these are obtained explicitly from the axial velocities. After each iteration, the new calculated values are compared with the approximations and the iterations are stopped if convergence is obtained (less than .5% difference between approximations and calculated values).

For the iterative scheme, all the equations and the difference transformations are the same except for the following four finite difference approximations:

$$V = \frac{V_j + V_{j+1}}{2} \qquad U = \frac{U_j + U_{j+1}}{2}$$

$$\tau_{j+1,k+1/2} = \left| \frac{U_{j+1,k+1} - U_{j+1,k}}{\Delta Y} \right|^{n-1} \frac{U_{j+1,k+1} - U_{j+1,k}}{\Delta Y}$$

(III-1)

$$\tau_{j+1,k-1/2} = \left| \frac{U_{j+1,k} - U_{j+1,k-1}}{\Delta Y} \right|^{n-1} \frac{U_{j+1,k} - U_{j+1,k-1}}{\Delta Y}$$

Since each difference equation is similar to the ones already used with the implicit scheme except for the momentum equation, only the transformation of this last one will be retaken here.

Starting from equation I-3.14

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{dP}{dX} + 2^n \frac{\partial \left(\left| \frac{\partial U}{\partial Y} \right|^{n-1} \frac{\partial U}{\partial Y} \right)}{\partial Y}$$

(III-2)

we replace directly the finite difference approximations I-3.19, I-3.20 and III-1

$$\frac{U_{j,k}+U_{j+1,k}}{2}\,\frac{U_{j+1,k}-U_{j,k}}{\Delta X}+\frac{V_{j,k}+V_{j+1,k}}{2}\,\frac{1}{2}\,\frac{U_{j,k+1}-U_{j,k-1}}{2\Delta Y}+\frac{1}{2}\,\frac{U_{j+1,k+1}-U_{j+1,k-1}}{2\Delta Y}$$

$$=-\frac{P_{j+1}-P_{j}}{\Delta X}+2^{n}\,\frac{\left|\frac{U_{j,k+1}-U_{j,k}}{\Delta Y}\right|^{n-1}\frac{U_{j,k+1}-U_{j,k}}{\Delta Y}-\left|\frac{U_{j,k}-U_{j,k-1}}{\Delta Y}\right|^{n-1}\frac{U_{j,k}-U_{j,k-1}}{\Delta Y}}{2\,\Delta Y}$$

$$+\frac{\left|\frac{U_{j+1,k+1}-U_{j+1,k}}{\Delta Y}\right|^{n-1}\frac{U_{j+1,k+1}-U_{j+1,k}}{\Delta Y}-\left|\frac{U_{j+1,k}-U_{j+1,k-1}}{\Delta Y}\right|^{n-1}\frac{U_{j+1,k}-U_{j+1,k-}}{\Delta Y}}{2\,\Delta Y}$$

(III-3)

Rearranging, we have

$$A_{k}\,U_{j+1,k-1}+B_{k}\,U_{j+1,k}+C_{k}\,U_{j+1,k+1}+D_{k}\,P_{j+1}=E_{k}$$

(III-4)

$$\text{where } A_{k}=-\frac{V_{j,k}+V_{j+1,k}}{8\Delta Y}-\frac{\left|Gmn\right|^{n-1}}{\Delta Y^{2}}$$

$$B_{k}=+\frac{U_{j,k}+U_{j+1,k}}{2\Delta X}+\frac{\left|Gmn\right|^{n-1}+\left|Gpn\right|^{n-1}}{\Delta Y^{2}}$$

$$C_k = \frac{V_{j,k} - V_{j+1,k}}{8\Delta Y} - \frac{\left|Gpn\right|^{n-1}}{\Delta Y^2}$$

$$D_k = \frac{1}{\Delta X}$$

$$E_k = \frac{P_j}{\Delta X} + \frac{U_{j,k} + U_{j+1,k}}{2} \frac{U_{j,k}}{\Delta X} - \frac{V_{j,k} + V_{j+1,k}}{2} \frac{U_{j,k+1} - U_{j,k-1}}{4\Delta Y}$$

$$+ \left|Gp\right|^{n-1} \frac{Gp}{\Delta Y} + \left|Gm\right|^{n-1} \frac{Gm}{\Delta Y}$$

$$\text{where } Gpn = 2 \frac{U_{j+1,k+1} - U_{j+1,k}}{\Delta Y}$$

$$Gmn = 2 \frac{U_{j+1,k} - U_{j+1,k-1}}{\Delta Y}$$

$$Gp = 2 \frac{U_{j,k+1} - U_{j,k}}{\Delta Y}$$

$$Gm = 2 \frac{U_{j,k} - U_{j,k-1}}{\Delta Y}$$

We can see here that $\left(U_{j,k} + U_{j+1,k}\right)$, $\left(V_{j,k} + V_{j+1,k}\right)$, Gpn and Gmn are making this equation nonlinear; so we must set a value for those variables before computing the j+1 column.

The solving procedure remains exactly the same as used for the implicit scheme.

III- 2 Results and discussion for the iterative scheme

This kind of solution was also investigated to obtain results for dilatant fluids. The overall results from this method are giving slower developing profile than non-iterative solutions. In figure III-1, we are plotting the entry length (98% of fully developed profile) and comparing with similar non iterative results of this work and Collins and Schowalter values. We can see that the entry lengths are longer than the usual ones; this is probably due to the fact that the effect of the wall is somewhat damped by the iterative scheme; the first reaction (on the first calculation) being more "explosive" than the following ones which are lowering this forward thrust. This should be especially true with the first few columns. We can see that for this kind of solution, results were obtainable for n>1.0 i.e. for dilatant fluids; it must be stated though that when n is higher than 1.75 there are some instabilities particularly near the wall: those instabilities were not strong enough to disturb the flow as a whole and were taking the form of an oscillating perpendicular (Y-direction) velocity. This kind of oscillation was also noted for the values of the temperatures which made calculations of the Nusselt number difficult because we must have accurate temperature values near the wall to calculate the proper temperature gradient. So at n=2.00 our statement that values do not change appreciably with changes of mesh sizes does not hold at times.

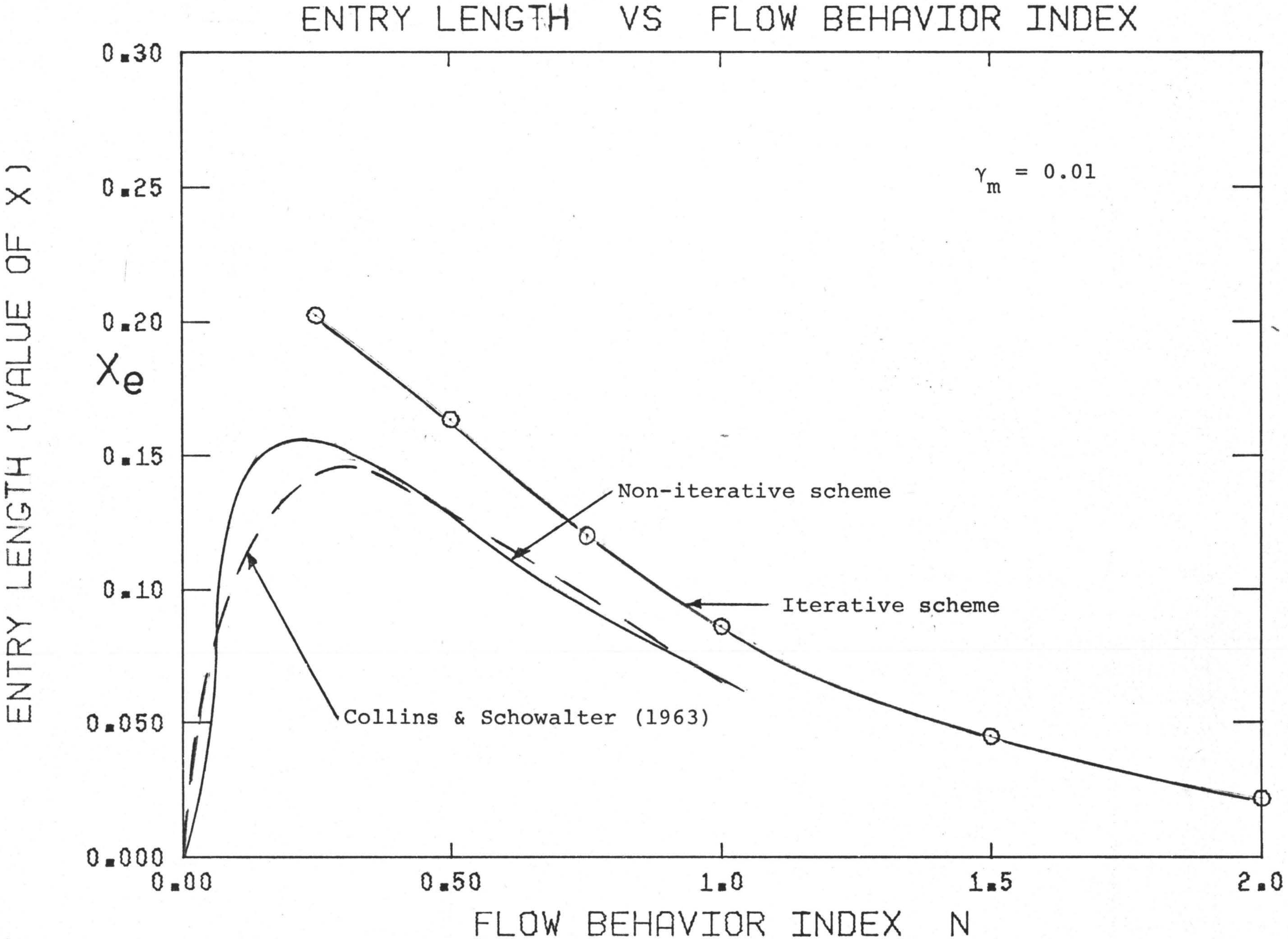


FIGURE III-1: ENTRY LENGTH VS FLOW BEHAVIOR INDEX

The values of the excess pressure drops correction are lower than the previously reported values; this is shown in figure III-2 where excess pressure drops are plotted against the flow behavior index. Varying the minimum gradient on those curves has not any effect on lower flow behavior index as in the non-iterative but has some at n=2.0. Figure III-3 shows a fluid developing along X at five different values of Y at n=1.0; we can see the differences between the two schemes as the non-iterative one is developing faster than the other one.

For the heat transfer, the results of the bulk temperature are similar in forms with values just a shade higher than the ones with no iterations. Fig.III-4 shows the Nusselt number. For dilatant fluids, the values of the asymptotic local Nusselt numbers are in accordance with the ones given by Vlachopoulos and Keung (1972) as we can see in table III-1 below:

	γ_m	.25	.5	1.0	1.5	2.0
	0.5	4.162	3.954	3.767	3.683	3.636
	0.1	4.212	3.959	3.767	3.682	3.635
(Pr = 1.0)	0.01	4.212	3.966	3.767	3.682	
(Pr = 5.0)	0.01	4.218	3.967	3.768	3.683	3.637
	0.005	4.218	3.966	3.767	3.682	3.635
Vlachopoulos & Keung (1972)		4.22	3.97	3.767	-	3.64

Table III-1: Asymptotic Nusselt number for iterative system (at Pr = 1.0)

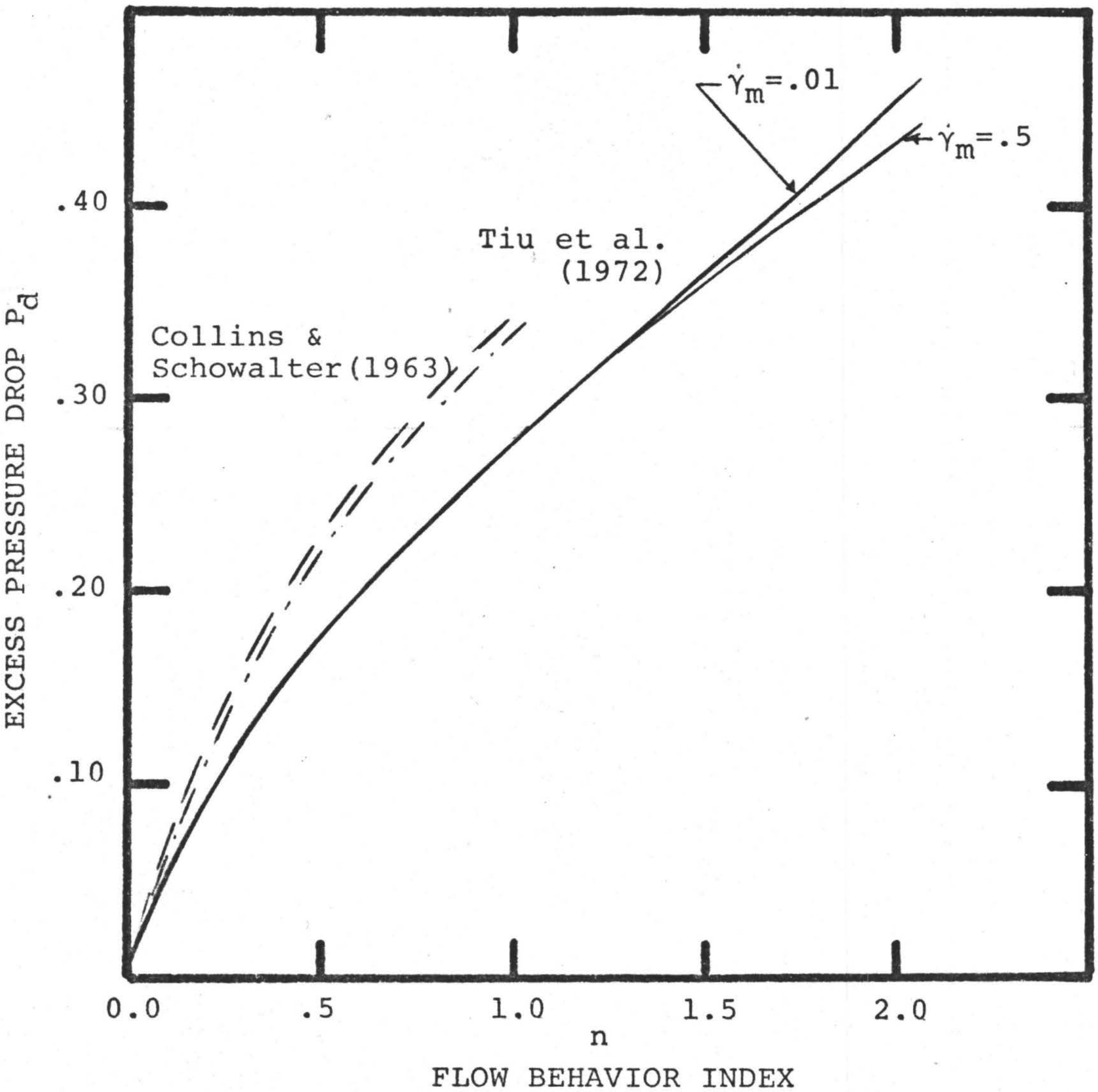


Figure III-2: Excess pressure drop vs flow behavior index

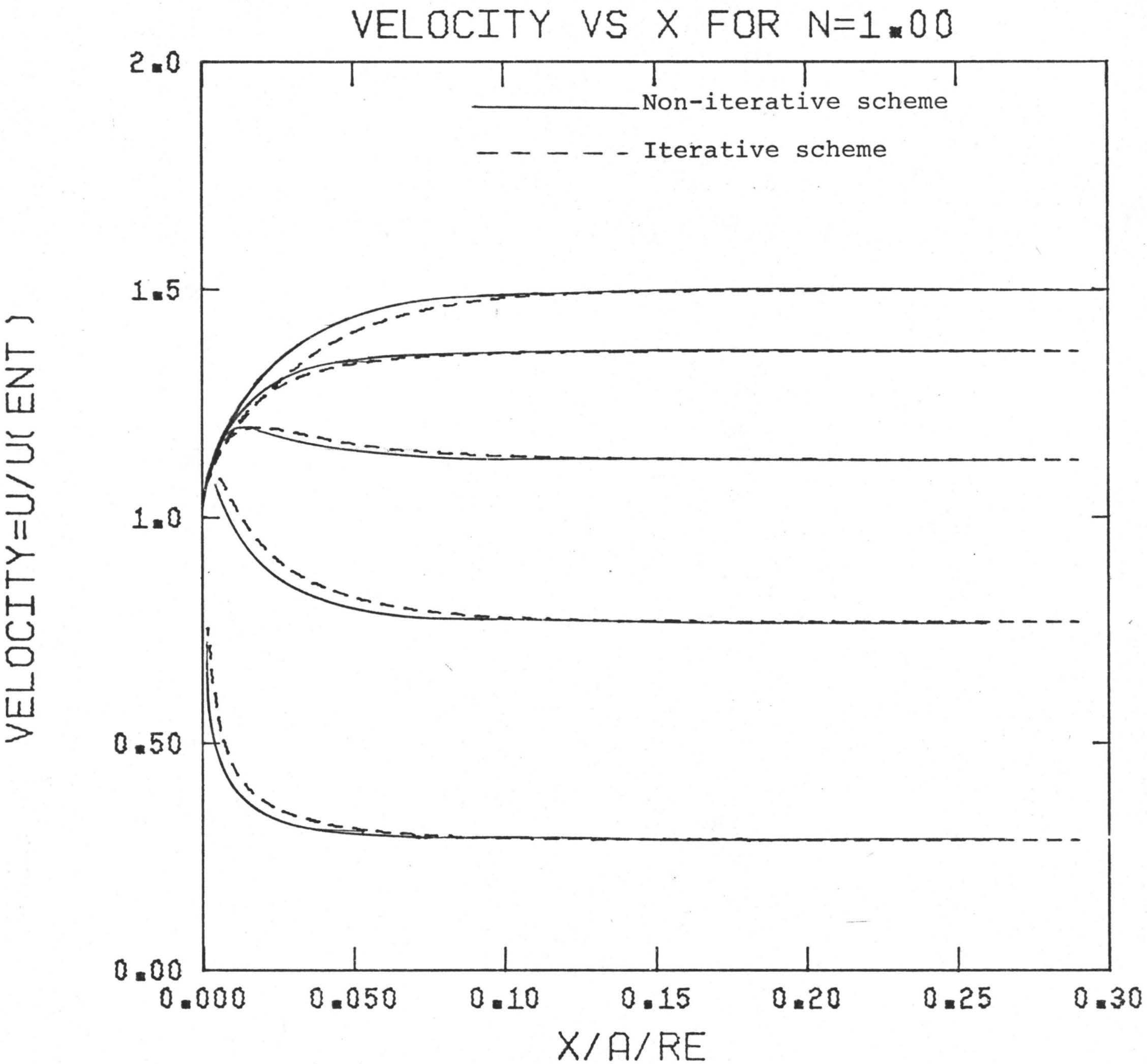


Figure III-3: Comparison of iterative and non-iterative scheme

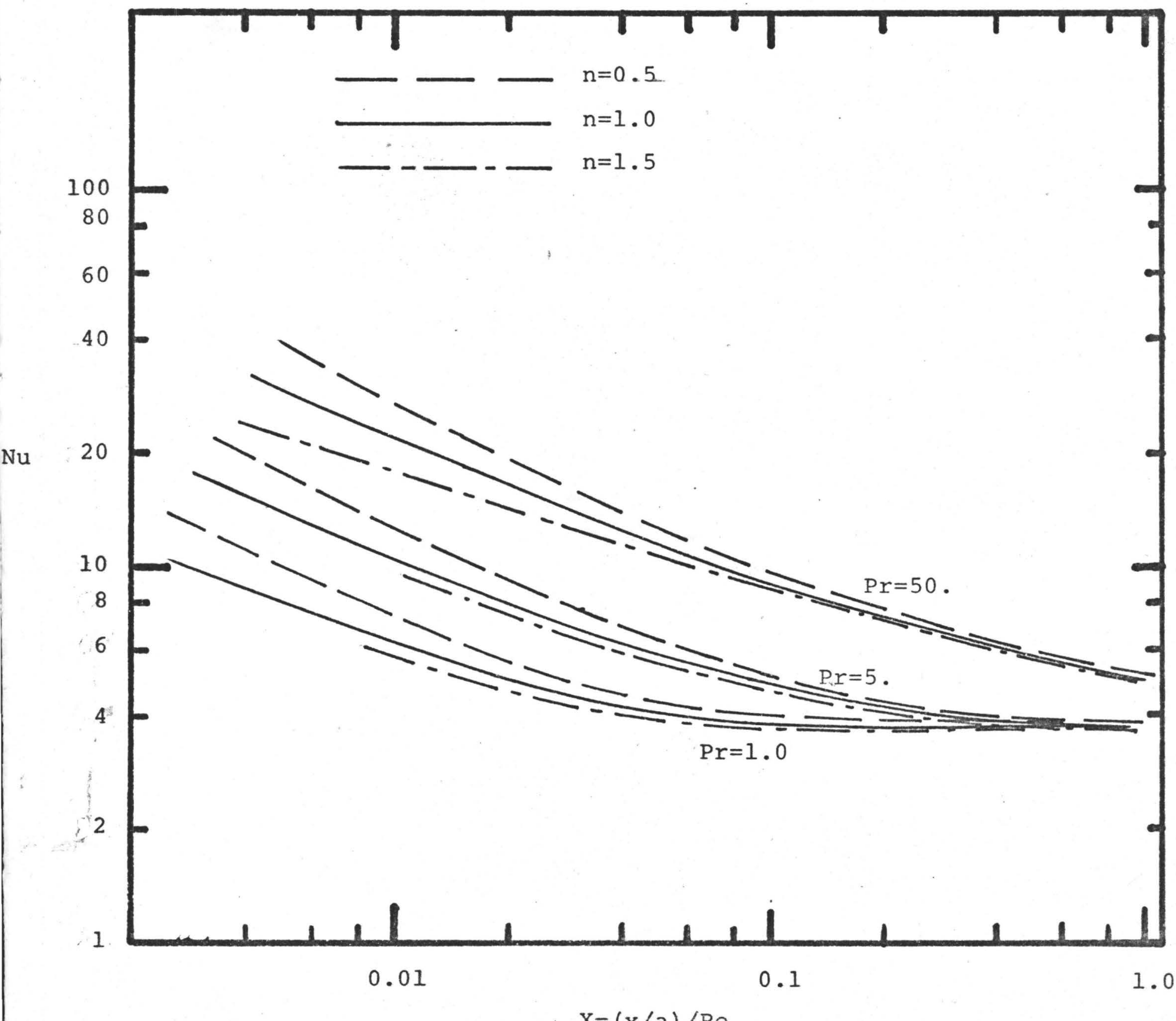


Figure III-4: Local Nusselt number vs X (Iterative scheme)

Those two systems as we saw, are giving different results for the hydrodynamic problem but are giving similar ones for the heat transfer part (this was predictable since we are using exactly the same energy equation for the two cases). In the present literature, there is no iterative scheme with integration procedure (solutions of the full Navier-Stokes equations are solutions of elliptic equations in which all the grid points in the 2-dimensional mesh are solved simultaneously). It could be that solutions of the iterative type should be done without the boundary layer assumptions because they are too precise for this kind of problem making it a must to solve the entire equations. However we must say that this method can be improved sensibly by an appropriate choice of step sizes and finite difference approximations so it should not be discarded as an inferior method.

APPENDIX IV

Algorithm and computer programs

1: Basic algorithm

2: Main program for straight channel with non iterative implicit scheme

3: Main program for straight channel with iterative scheme

4: Subroutine " Const" for above programs

5: Subroutine "Gauss"

6: Subroutine "Dessin" for straight channel programs

7: Main program for converging channel

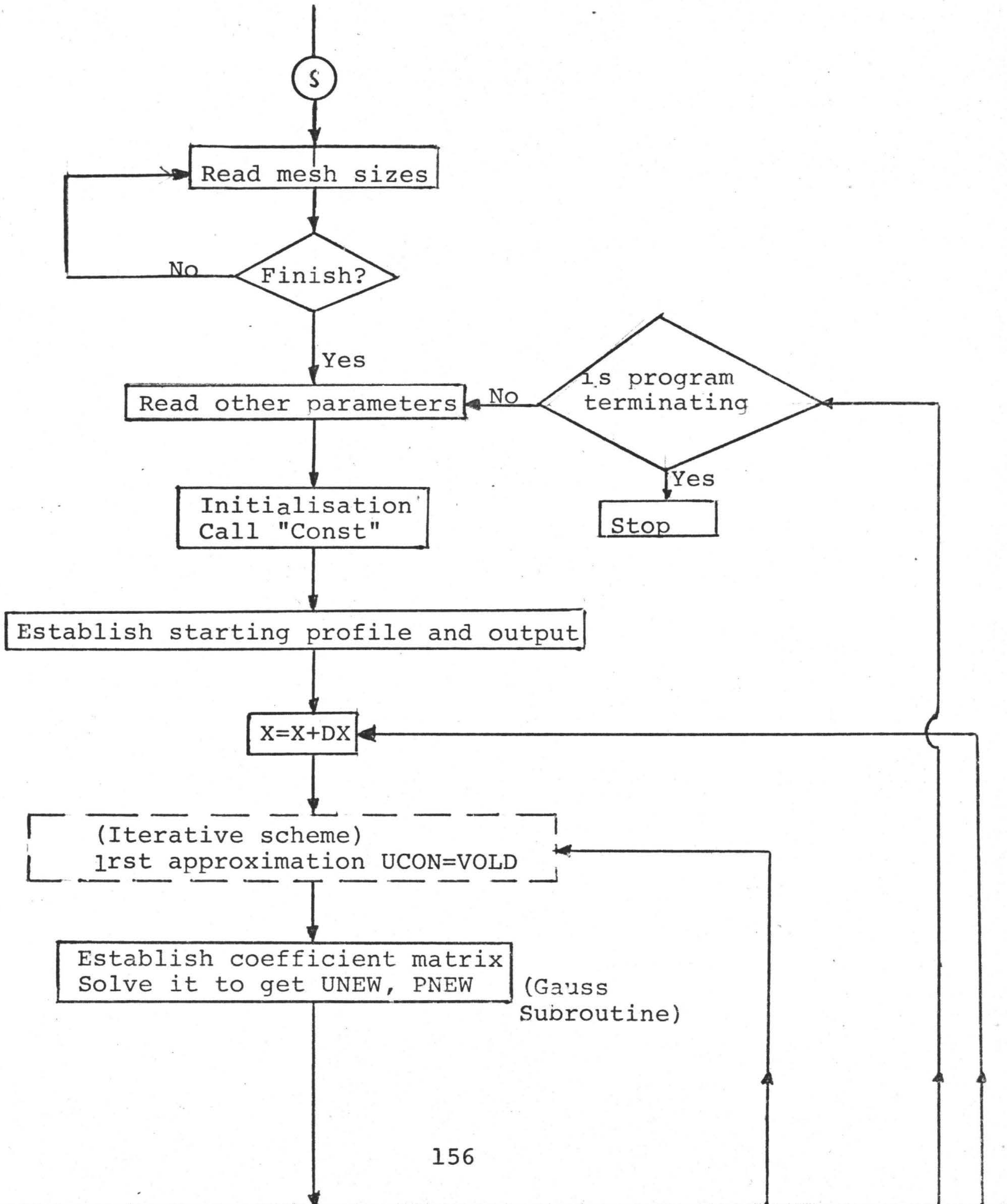
8: Subroutine "Const" for converging channel program

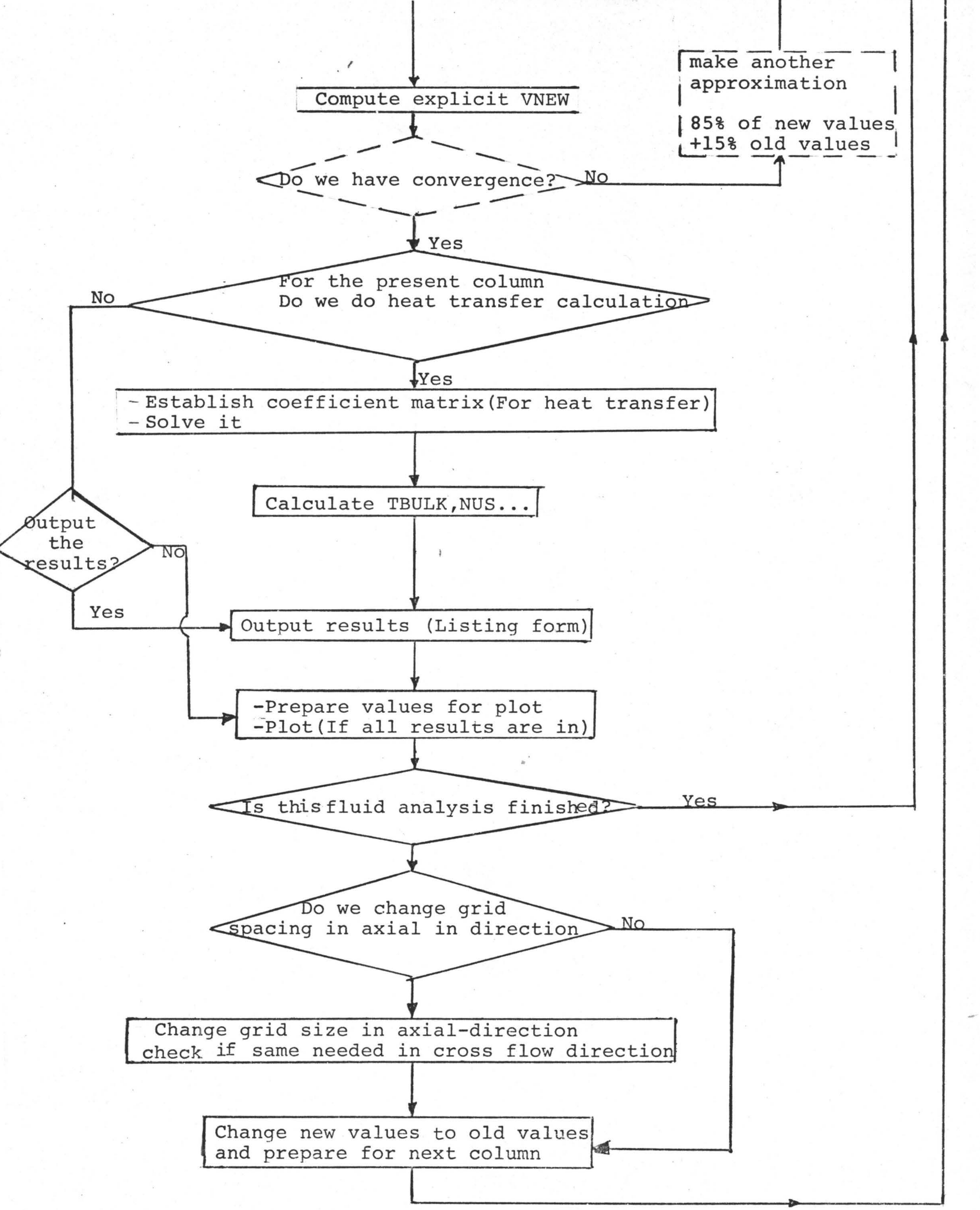
9: Subroutine "Dessin" for converging channel program

ALGORITHM

Note:-Apply to all three main programs

-Broken lines apply to iterative scheme




make another approximation
85% of new values +15% old values
Compute explicit VNEW
Do we have convergence?
No
Yes
For the present column Do we do heat transfer calculation
No
Yes
-Establish coefficient matrix(For heat transfer)
-Solve it
Calculate TBULK,NUS...
Output the results?
No
Yes
Output results (Listing form)
-Prepare values for plot
-Plot(If all results are in)
Is this fluid analysis finished?
Yes
Do we change grid spacing in axial in direction
No
Change grid size in axial-direction check if same needed in cross flow direction
Change new values to old values and prepare for next column

```
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10)
C     THIS IS A FINITE DIFFERENCE METHOD TO SOLVE THE MOTION
C           EQUATIONS FOR THE ENTRANCE REGION OF A CHANNEL
C     THIS METHOD IS USING A SERIE OF VALUES KNOWN AS THE OLD VALUES TO
C           COMPUTE ANOTHER SET OF VALUES KNOWN AS THE NEW VALUES CORRESPONDING
C           TO THE NEXT COLLUMN (AXIALLY) SO THAT FROM THE ENTRY SET OF VALUES WE
C        CAN GO DOWNSTREAM AS FAR AS WE WANT PROVIDING WE HAVE THE BOUNDARY
C           CONDITIONS
C LIST OF VARIABLES
C     AN= EXPONENT OF THE POWER LAW FLUID (FLOW BEHAVIOR INDEX)
C     UOLD(-),VOLD(-),TOLD(-),POLD= VECTORS OF VELOCITY (X-DIR), VELOCITY
C           (Y-DIR), TEMPERATURE AND PRESSURE ALREADY KNOWN
C     UNEW(-),VNEW(-),TNEW(-),PNEW= VECTORS OF VELOCITY (X-DIR), VELOCITY
C           (Y-DIR), TEMPERATURE AND PRESSURE TO BE COMPUTED (NEW)
C     UO(-),VO(-),UN(-),VN(-)=  VECTORS OF VELOCITY ATTACHED TO HEAT TRANS+ER
C           GRID
C     UCON(-),VCON(-)= VECTORS OF VELOCITY USED IN ITERATIVE TYPE MESH
C           TO CHECK THE CONVERGENCE
C     UUMAXN(-)= VECTOR OF VELOCITY DIVIDED BY MAXIMUM VELOCITY (CENTERLINE)
C           (OUTPUT PURPOSE)
C     GRA(-),GRAMIN,GRAMI= GRADIENTS - VECTOR OF GRADIENTS (FOR OUTPUT),
C           MINIMUM GRADIENT AS DEFINEDIN READ STAT., MINIMUM GRADIENT AS
C           DEFINEDIN PROGRAM
C     FACT = (GRAMI) **(AN-1)
C     X= VALUE OF X AT WHICH THE PROGRAM IS NOW
C     XT= NEXT VALUE OF X AT WHICH TEMPERATURE PROFILE WILL BE CALCULATED
C     TBULK= BULK TEMPERATURE OF FLOW
C     NUS= NUSSELT NUMBER
C     PR= PRANDLT NUMBER
C     LCON= NUMBER OF TIME WE TRIED TO ACHIEVE COMVERGENCE
C     TOL= TOLERANCE BETWEEN OLD AND NEW VALUES (ITERATIVE TYPE MESH)
C     PAR= PORTION OF NEW VELOCITY TO ADD TO THE OLD ONE TO ACHIEVE CONVERGENCE
C VARIABLES DESCRIBING THE MESH SIZE (MESH SIZES ARE VARYING AS WE PROCEED)
C     INCY(-)= VECTOR CORRESPONDING TO NUMBER OF GRID SPACE ACROSS FLOW
C     XINC(-)= VECTOR CORRESPONDING TO GRID SPACE IN X-DIR
C     ULIX(-)= VECTOR CORRESPONDING TO LIMIT UP TO WHICH THE PRECEDING
C          X-DIR GRID SPACE IS VALID
C NOTE= THE INDICES FROM 1 TO 10 ARE FOR MOMENTUM MESH SIZE AND 1 TO 20
C           ARE FOR THAT TRANSFER MESH SIZE
C     NDX,NDXT= VARIABLES CORRESPONDING TO INDICES OF PREVIOUS VARIABLES
C           TO KEEP TRACK OF THE NEXT MESH SIZE (FOR MOMENTUM MESH AND
C           HEAT TRANSFER MESH)
C CONSTANTS RELATED MOMENTUM MESH
C     A(-,-)= MATRIX OF COEFFICIENTS FOR MOMENTUM MESH
C     CP,ABSCP,CPA,ABSCPN,CPANEW,CM,ABSCM,CMA,ABSCMN,CMANEW= CONSTANT USE
C           IN THE SETTING OF THE COEFFICIENT MATRIX (MOM. MESH)
C     NMAX1= NUMBER OF SPACE (DY) IN Y-DIR
C     NMAX= NUMBER OF POINT IN Y-DIR (NMAX = NMAX1 + 1)
C     DY= GRID SIZE IN Y-DIR
C     DX= GRID SIZE IN X-DIR
C     NMAX4,DDY2,DY2,DXI,DY2X,D2Y= SERIES OF CONSTANTS DEFINED IN SUBROU.
C           CONST CALCULATED FROM 4 PREVIOUS
C CONSTANTS RELATED TO HEAT TRANSFER MESH
C     A1(-,-)= MATRIX OF COEFFICIENTS FOR HEAT TRANSFER MESH
C     NMAX1T,DXT,DXT,NMAXT= SIMILAR AS MOMENTUM MESH COUNTEPART BUT FOR
C           HEAT TRANSFER
C     NMAX4T,PRDY= DEFINED IN CONST FROM 4 PREVIOUS
C     VOLDO(-),VOLDO(-),TOLDO(-),UUMAXO(-),LCONO,POLDO,TBULKO,XOLD= ALL
C           THOSE VARIABLES ARE THERE JUST FOR 'OUTPUTING' CONVENIANCE
```

```
C     YD= USED IN THE OUTPUT TO SITUATE A POINT IN Y-DIR (AT CENTERLINE YD=0.0,
C           AT WALL YD=1.0)
C     LL= FLAG TO OUTPUT EVERY SECOND TIME.
C     NMAXO,NMAXTO= O STANDS FOR OLD VALUES OTHERWISE SAME AS BEFORE.
C CONSTANTS AND VARIABLES RELATED TO PLOTTING SUBROUTINES
C     UCL(-)= VELOCITY (X-DIR) AT CENTER LINE
C     U98= 98 PER CENT OF LAST UCL (FULLY DEVELOPPED)
C     NPOS= COUNTER FOR INSERTION OF VELOCITY PROFILE
C     LN= COUNTER FOR DIFFERENT FLUIDS
C     L1, L2= COUNTER FOR PRESSURE DROP, NUSSELT NUMBER
C     VX(-)= VALUES OF Y (-1.0 TO + 1.0)
C     VY(-,-)= VELOCITY IN X-DIR FOR THOSE PREVIOUS VX(-)
C     XPRESS(-)= VALUES OF X (0.0 TO 1.0)
C     PRESS(-,-)= PRESSURE DROP AT THOSE X
C     XNUS(-)= VALUES OF X (0.0 TO 2.0)
C     NU(-,-)= NUSSELT NUMBER AT THOSE X
C     FBI(-)= FLOW BEHAVIOR INDEX (SIMILAR TO AN)
C     EL(-)= ENTRY LENGHT FOR THOSE PREVIOUS AN
      COMMON NMAX1,PNEW,A(5,165),UNEW(165),VNEW(165),TNEW(165),UOLD(165)
     1,VOLD(165),TOLD(165),UOLDO(165),VOLDO(165),TOLDO(165),UUMAXN(165)
     2,UUMAXO(165),UN(165),VN(165),UO(165),VO(165),A1(3,165),GRA(165)
      DIMENSION VY(21,3),VX(21),PRESS(100,5),XPRESS(100),UCL(100),EL(5)
     1,FBI(5),NU(100,5),XNU(100),TBU(100,5),UVEL(70,5),VECTOR(11)
      DIMENSION INCY(20),ULIX(20),XINC(20)
      REAL NUS,NU
C...READ IN ALL THE GRID SIZE
      N = 0
    7 N = N +1
      READ(5,1105) INCY(N),XINC(N),ULIX(N)
      WRITE(6,1105)INCY(N),XINC(N),ULIX(N)
      IF (INCY(N) .NE. 0) GO TO 7
      IF (N .GT. 10)GO TO 3
      N = 10
      GO TO 7
C...READ THE PARAMETERS
    3 READ(5,1109)MAX,(FBI(N),N=1,MAX)
 1109 FORMAT(I5,10F5.2)
    4 READ(5,1100)PR,GRAMIN,TOL
      LN = 0
      IF(PR .EQ. 0.0) STOP
    5 IF(LN .GE. MAX)GO TO 850
      LN = LN +1
      AN = FBI(LN)
C...DEFINE MINIMUM GRADIENT
      GRAMI = GRAMIN
      FACT = (GRAMI)**(AN-1.)
C...OUTPUT THE PARAMETERS
      WRITE(6,1110)PR,GRAMI,FACT,TOL
C...INITIALISATION OF COUNTERS AND OTHERS
       TGRAO = 0.0
      SUM = 0.0
      X = 0.0
      NPOS = 1
      L1 = 1
      L2 = 1
      GRA(1) = GRAMI
      LL = 0
```

```
      TBULK = 0.0
      NDX = 1
      POLD = 0.0
      XT = X +XINC(11)
C...COMPUTE ALL THE CONSTANTS NEEDED
      NDXT = 11
      CALL CONST(INCY(NDX),XINC(NDX),INCY(NDXT),XINC(NDXT),NMAX1,NMAX,
     1DX,DY,NMAX4,DDY2,DY2,DXI,DY2X,D2Y,NMAX1T,NMAXT,NMAX4T,DYT,DXT,PR,
     2PRDY)
      DO 102 N=1,NMAXT
      UO(N) = 1.0
      VO(N) = 0.0
      TOLD(N) = 1.
      TNEW(N) = 0.0
  102 CONTINUE
       UO(NMAXT) = 0.0
      TOLD(NMAXT) = 0.0
      DO 1 N=1,NMAX
      UOLD(N) = 1.0
      VOLD(N) = 0.0
    1 CONTINUE
      UOLD(NMAX)  = 0.0
C...OUTPUT THE STARTING VALUES
      WRITE(6,1001)AN,X,X,POLD,POLD
      DO  2 N=1,NMAX,NMAX4
    2 WRITE(6,1002)N,UOLD(N),VOLD(N)
C...INTEGRATION ITERATION BEGINS
   11 X = X +DX
C...SET THE COEFFICIENT MATRIX
      DO 10 N=1,NMAX1
      CP = (UOLD(N+1)-UOLD(N))/DY
      ABSCP = ABS(CP)
      IF(ABSCP .LT. GRAMI) ABSCP = GRAMI
      GRA(N) = ABSCP
      CPA = ((2.*ABSCP)**(AN-1.))/(DY**2)
      IF(N .NE. 1)GO TO 12
      CM = -CP
      CMA = CPA
      A(5,1)=-(CPA*CP-CMA*CM)*DY             -POLD/DX-UOLD(1)*UOLD(1)/DX
      GO TO 13
   12 CONTINUE
      CM = (UOLD(N)-UOLD(N-1))/DY
      ABSCM = ABS(CM)
      IF(ABSCM .LT. GRAMI) ABSCM = GRAMI
      CMA = ((2.*ABSCM)**(AN-1.))/(DY**2)
      A(5,N)= -(CPA*CP-CMA*CM)*DY                           -POLD/DX
     1 +VOLD(N)*((UOLD(N+1)-UOLD(N-1))*0.5/D2Y)-(UOLD(N)*UOLD(N))/DX
   13 CONTINUE
      A(1,N)= +CMA    +VOLD(N)/(4.*DY)
      A(2,N)=-CMA-CPA-(UOLD(N)/DX)
      A(3,N)= +CPA  -VOLD(N)/(4.*DY)
      A(4,N) = -DXI
   10 CONTINUE
      A(3,1) = A(3,1) +A(1,1)
C...SOLVE THE MATRIX (GAUSS ELIMINATION ALGORITHM)
      CALL GAUSS
      UNEW(NMAX) = 0.0
```

```
C...COMPUTE VELOCITIES IN Y-DIR
      VNEW(1) = 0.0
      DO 30 J=2,NMAX1
   30 VNEW(J) = VNEW(J-1) -DY2X*(UNEW(J)+UNEW(J-1)-UOLD(J)-UOLD(J-1))
      VNEW(NMAX) = 0.0
C...HEAT TRANSFER PART
C...SEE HEAT TRANSFER COLLUMN SHOULD BE DONE
      IF(X .LT. (0.999*XT))GO TO 130
C...SET UP VELOCITY FIELD FOR ENERGY EQUATION
      NTN = NMAX1T/NMAX1
      NN = 0
      DO 111 N=1,NMAX1
      DO 111 L=1,NTN
      NN = NN+1
      VN(NN) = VNEW(N) -(VNEW(N)-VNEW(N+1))*(FLOAT(L-1)/FLOAT(NTN))
  111 UN(NN) = UNEW(N) -(UNEW(N)-UNEW(N+1))*(FLOAT(L-1)/FLOAT(NTN))
      UN(NMAXT) = 0.0
      VN(NMAXT) = 0.0
C...SET THE COEFFICIENT MATRIX (ENERGY EQUATION)
      DO 125 N=1,NMAX1T
      VCO = (VO(N)+VN(N))/(8.*DYT)
      A1(1,N) = -PRDY -VCO
      A1(2,N) = (UO(N)+UN(N))/DXT +2.*PRDY
      A1(3,N) = -PRDY +VCO
      IF(N .EQ. 1)GO TO 121
      TNEW(N)=(UO(N)+UN(N))*TOLD(N)/DXT
     1-VCO*(TOLD(N+1)-TOLD(N-1))
     2 +(TOLD(N+1) -2.*TOLD(N) +TOLD(N-1))*PRDY
      GO TO 120
  121 TNEW(N)=(UO(1)+UN(1))*TOLD(1)/DXT +2.*(TOLD(2)-TOLD(1))*PRDY
  120 CONTINUE
  125 CONTINUE
      A1(3,1) = A1(3,1) +A1(1,1)
C...SOLVE MATRIX
      CALL DIAG3(A1,TNEW,NMAX1T)
      TNEW(NMAXT)= 0.0
C...COMPUTE BULK TEMPERATURE AND NUSSELT NUMBER
      TBULK = (TNEW(1)*UN(1) +TNEW(NMAXT)*UN(NMAXT))/2.
      DO 122 N=2,NMAX1T
  122 TBULK = TBULK +(UN(N)*TNEW(N))
      TBULK = TBULK/FLOAT(NMAX1T)
      TGRA     =  (-11.*TNEW(NMAXT)+18.*TNEW(NMAXT-1)-9.*TNEW(NMAXT-2)
     1 +2.*TNEW(NMAXT-3))/(6.*DYT)
      NUS = (2.*TGRA)/(TBULK)
      SUM = SUM+(.25*DXT*(TGRA+TGRAO))
10000 CONTINUE
      AVNUS = SUM/X
      TGRAO = TGRA
      IF(X .LT. 0.01 .OR. L2 .GT. 100) GO TO 9998
      TBU(L2,LN) = TBULK
      NU(L2,LN)=  NUS
      XNU(L2) = X
      L2 = L2 +1
 9998 CONTINUE
      DO 123 N=1,NMAXT
      UO(N) = UN(N)
      VO(N) = VN(N)
```

```
  123 TOLD(N) = TNEW(N)
C...SET XT FOR NEXT TIME
      XT = X +XINC(NDXT)
      GO TO 39
  130 IF(X .GT. 0.001) GO TO 45
      DO 131 N=1,NMAXT
  131 TNEW(N) = 0.0
      TBULK = 0.0
C
C...OUTPUT THE RESULTS (PRINT OUT FORM)
   39 LL = LL +1
      ANTN = FLOAT(NMAX1T)/FLOAT(NMAX1)
      NTN = NMAX1T/NMAX1
      IF(LL .NE. 2) GO TO 44
      LL = 0
      WRITE(6,1001) AN,AVNUS,XOLD,POLDO,X,PNEW,TBULK,NUS
      MM = 1
      IF(NMAX .NE. NMAXO) MM = 2
      NMAX4D = NMAX4*2
      DO 40 N=1,NMAX,NMAX4D
      M = MM*(N-1) +1
      NN = IFIX(ANTN*(FLOAT(N)-1.)) +1
      NNN= IFIX(ANTN*(FLOAT(M)-1.)) +1
      YD = FLOAT(N-1)/FLOAT(NMAX1)
      UUMAXN(N) = UNEW(N)/UNEW(1)
      WRITE(6,1002)N,YD,UOLDO(M),UUMAXO(M),VOLDO(M),TOLDO(NNN),
     1 UNEW(N),UUMAXN(N),VNEW(N),TNEW(NN),NN,GRA(N)
   40 CONTINUE
      GO TO 45
C...PUT RESULT IN RESERVE BECAUSE OF NON PUBLICATION
   44 DO 53 N=1,NMAX,NMAX4
      NN = NTN*(N-1) +1
      UOLDO(N) = UNEW(N)
      VOLDO(N) = VNEW(N)
      UUMAXO(N) = UOLDO(N)/UOLDO(1)
   53 TOLDO(NN) = TNEW(NN)
      XOLD = X
      TBULKO = TBULK
      POLDO = PNEW
C
C...PART TO SET VALUES FOR PLOTTING SUBROUTINE
   45 GO TO (801,802,803),NPOS
  803 IF(X .LT. 1.0)GO TO 701
      GO TO 702
  802 IF(X .LT. 0.05) GO TO 701
      GO TO 702
  801 IF(X .LT. 0.002) GO TO 46
C...PREPARE VELOCITY FIELD
  702 L = 0
      DO 700 N=1,NMAX,NMAX4
      L = L+1
      LI = 22-L
      NN = NMAX+1-N
      VY(L,NPOS ) = UNEW(NN)
      VY(LI,NPOS) = UNEW(NN)
      VX(L) = (FLOAT(L-1)/10.)-1.0
  700 VX(LI) = FLOAT(LI-11)/10.0
```

```
      NPOS = NPOS +1
C...PREPARE PRESSURE DROPS
  701 IF(L1 .GT. 100) GO TO 710
      PRESS(L1,LN) =-PNEW
      XPRESS(L1) = X
      UCL(L1) = UNEW(1)
      IF(L1 .GT. 70) GO TO 709
      J = 0
      DO 705 N=1,NMAX,NMAX4
      J = J+1
  705 VECTOR(J) = UNEW(N)
      UVEL(L1,1) = VECTOR(1)
      UVEL(L1,2) = VECTOR(4)
      UVEL(L1,3) = VECTOR(6)
      UVEL(L1,4) = VECTOR(8)
      UVEL(L1,5) = VECTOR(10)
  709 L1 = L1 +1
  710 CONTINUE
      IF(NPOS .NE. 4) GO TO 46
      NPOS = NPOS -1
      DO 900 N=1,21
  900 WRITE(6,901)VX(N),VY(N,1),VY(N,2),VY(N,3)
  901 FORMAT(12F11.5)
C...COMPUTING ENTRY LENGHT VS FLOW BEHAVIOR INDEX
      U98 = 0.98*UNEW(1)
      L=0
  220 L = L+1
      IF(UCL(L) .LT. U98) GO TO 220
      L = L-1
      EL(LN) = XPRESS(L) +((XPRESS(L+1)-XPRESS(L))/(UCL(L+1)-UCL(L)))
     1 *(U98-UCL(L))
      CALL DESSIN(1,VX,VY,XPRESS,PRESS,UVEL,XNU,NU,TBU,FBI,EL,LN)
      GO TO 5
C...TO CHANGE GRID SPACING IN X-DIR (IF NECESSARY)
   46 NMAXTO = NMAXT
      NMAXO = NMAX
      IF(XT .LT. (1.01*ULIX(NDXT))) GO TO 49
      IF (X .LT. (0.99*ULIX(NDX)          )) GO TO 48
      NDX = NDX +1
   48 NDXT = NDXT +1
      XT = X +XINC(NDXT)
   47 CALL CONST(INCY(NDX),XINC(NDX),INCY(NDXT),XINC(NDXT),NMAX1,NMAX,
     1DX,DY,NMAX4,DDY2,DY2,DXI,DY2X,D2Y,NMAX1T,NMAXT,NMAX4T,DYT,DXT,PR,
     2PRDY)
      GO TO 60
   49 IF (X .LT. (0.99*ULIX(NDX)          )) GO TO 51
      NDX = NDX +1
      GO TO 47
   60 IF(NMAXT .EQ. NMAXTO) GO TO 61
C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / MOMENTUM EQUATION
      DO 129 N=1,NMAXT
      NN = 2 *N -1
      UO(N) = UO(NN)
      VO(N) = VO(NN)
      TNEW(N) = TNEW(NN)
  129 TOLD(N) = TOLD(NN)
   61 IF(NMAX .EQ. NMAXO)GO TO 51
```

```
C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / ENERGY EQUATION
      DO 22 N=1,NMAX
      NN = 2*N-1
      UNEW(N) = UNEW(NN)
      VNEW(N) = VNEW(NN)
   22 CONTINUE
C...ONE COLLUMN COMPLETE, PREPARE FOR NEXT COLLUMN
   51 POLD = PNEW
      DO 52 N=1,NMAX
      UOLD(N) = UNEW(N)
   52 VOLD(N) = VNEW(N)
      GO TO 11
C...PLOT AND OUTPUT COMPARABLE VALUES (PRESSURE, NUSSELT) OF ALL FLUIDS
  850 CONTINUE
      CALL DESSIN(3,VX,VY,XPRESS,PRESS,UVEL,XNU,NU,TBU,FBI,EL,LN)
      GO TO 4
  999 DO 998 N=LN,MAX
  998 FBI(N) = FBI(N+1)
      GO TO 10000
      LN = LN -1
      MAX = MAX-1
      GO TO 5
 1001 FORMAT(///,16X,* N =*F4.2,80X*AVNUS=*F9.4,/3X,*X=*F7.5,10X*PRES=*
     1 F8.4,36X*X=*F7.5,4X*PRES=*F8.4,4X*TBULK=*F6.4,4X*NUSS=*F7.4,
     2      //4X*N*,4X*Y/D*,6X*U(N)*,9X*U/UMAX*,7X*V(N)*,9X*T(L)*,
     3 13X*U(N)*,9X*U/UMAX*,7X*V(N)*,9X*T(L)*,7X*L* )
 1002 FORMAT (1X,I4,3X,F5.3,4F13.6,4X,4F13.6,I4,F6.3)
 1100 FORMAT(3F10.4)
 1105 FORMAT (I5,2F10.6)
 1110 FORMAT(2X,*PRANDTL NUMBER=* F12.4,/2X,,*MINIMUM GRADIENT=*F6.4,12X
     1*(MIN. GRAD.)EXP(N-1)=*F10.4,/2X,*TOLERANCE FOR CONVERGING TYPE ME
     2SH=*F8.5)
      END
```

CD TOT   0377

```
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10)
C     THIS IS A FINITE DIFFERENCE METHOD TO SOLVE THE MOTION
C           EQUATIONS FOR THE ENTRANCE REGION OF A CHANNEL
C     THIS METHOD IS USING A SERIE OF VALUES KNOWN AS THE OLD VALUES TO
C           COMPUTE ANOTHER SET OF VALUES KNOWN AS THE NEW VALUES CORRESPONDING
C           TO THE NEXT COLLUMN (AXIALLY) SO THAT FROM THE ENTRY SET OF VALUES W
C        CAN GO DOWNSTREAM AS FAR AS WE WANT PROVIDING WE HAVE THE BOUNDARY
C           CONDITIONS
C LIST OF VARIABLES
C     AN= EXPONENT OF THE POWER LAW FLUID (FLOW BEHAVIOR INDEX)
C     UOLD(-),VOLD(-),TOLD(-),POLD= VECTORS OF VELOCITY (X-DIR), VELOCITY
C           (Y-DIR), TEMPERATURE AND PRESSURE ALREADY KNOWN
C     UNEW(-),VNEW(-),TNEW(-),PNEW= VECTORS OF VELOCITY (X-DIR), VELOCITY
C           (Y-DIR), TEMPERATURE AND PRESSURE TO BE COMPUTED (NEW)
C     UO(-),VO(-),UN(-),VN(-)=  VECTORS OF VELOCITY ATTACHED TO HEAT TRANSFER
C           GRID
C     UCON(-),VCON(-)= VECTORS OF VELOCITY USED IN ITERATIVE TYPE MESH
C           TO CHECK THE CONVERGENCE
C     UUMAXN(-)= VECTOR OF VELOCITY DIVIDED BY MAXIMUM VELOCITY (CENTERLINE)
C           (OUTPUT PURPOSE)
C     GRA(-),GRAMIN,GRAMI= GRADIENTS - VECTOR OF GRADIENTS (FOR OUTPUT),
C           MINIMUM GRADIENT AS DEFINE IN READ STAT., MINIMUM GRADIENT AS
C           DEFINE IN PROGRAM
C     FACT = (GRAMI) **(AN-1)
C     X= VALUE OF X AT WHICH THE PROGRAM IS NOW
C     XT= NEXT VALUE OF X AT WHICH TEMPERATURE PROFILE WILL BE CALCULATED
C     TBULK= BULK TEMPERATURE OF FLOW
C     NUS= NUSSELT NUMBER
C     PR= PRANDLT NUMBER
C     LCON= NUMBER OF TIME WE TRIED TO ACHIEVE COMVERGENCE
C     TOL= TOLERANCE BETWEEN OLD AND NEW VALUES (ITERATIVE TYPE MESH)
C     PAR= PORTION OF NEW VELOCITY TO ADD TO THE OLD ONE TO ACHIEVE CONVERGENCE
C VARIABLES DESCRIBING THE MESH SIZE (MESH SIZES ARE VARYING AS WE PROCEED)
C     INCY(-)= VECTOR CORRESPONDING TO NUMBER OF GRID SPACE ACROSS FLOW
C     XINC(-)= VECTOR CORRESPONDING TO GRID SPACE IN X-DIR
C     ULIX(-)= VECTOR CORRESPONDING TO LIMIT UP TO WHICH THE PRECEDING
C          X-DIR GRID SPACE IS VALID
C NOTE= THE INDICES FROM 1 TO 10 ARE FOR MOMENTUM MESH SIZE AND 1 TO 20
C           ARE FOR THAT TRANSFER MESH SIZE
C     NDX,NDXT= VARIABLES CORRESPONDING TO INDICES OF PREVIOUS VARIABLES
C           TO KEEP TRACK OF THE NEXT MESH SIZE (FOR MOMENTUM MESH AND
C           HEAT TRANSFER MESH)
C CONSTANTS RELATED MOMENTUM MESH
C     A(-,-)= MATRIX OF COEFFICIENTS FOR MOMENTUM MESH
C     CP,ABSCP,CPA,ABSCPN,CPANEW,CM,ABSCM,CMA,ABSCMN,CMANEW= CONSTANT USE
C           IN THE SETTING OF THE COEFFICIENT MATRIX (MOM. MESH)
C     NMAX1= NUMBER OF SPACE (DY) IN Y-DIR
C     NMAX= NUMBER OF POINT IN Y-DIR (NMAX = NMAX1 + 1)
C     DY= GRID SIZE IN Y-DIR
C     DX= GRID SIZE IN X-DIR
C     NMAX4,DDY2,DY2,DXI,DY2X,D2Y= SERIES OF CONSTANTS DEFINED IN SUBROU.
C           CONST CALCULATED FROM 4 PREVIOUS
C CONSTANTS RELATED TO HEAT TRANSFER MESH
C     A1(-,-)= MATRIX OF COEFFICIENTS FOR HEAT TRANSFER MESH
C     NMAX1T,DXT,DXT,NMAXT= SIMILAR AS MOMENTUM MESH COUNTEPART BUT FOR
C           HEAT TRANSFER
C     NMAX4T,PRDY= DEFINED IN CONST FROM 4 PREVIOUS
C     VOLDO(-),VOLDO(-),TOLDO(-),UUMAXO(-),LCONO,POLDO,TBULKO,XOLD= ALL
C           THOSE VARIABLES ARE THERE JUST FOR 'OUTPUTING' CONVENIANCE
```

```
C     YD= USED IN THE OUTPUT TO SITUATE A POINT IN Y-DIR (AT CENTERLINE YD=0.0,
C           AT WALL YD=1.0)
C     LL= FLAG TO OUTPUT EVERY SECOND TIME.
C     NMAXO,NMAXTO= O STANDS FOR OLD VALUES OTHERWISE SAME AS BEFORE.
C CONSTANTS AND VARIABLES RELATED TO PLOTTING SUBROUTINES
C     UCL(-)= VELOCITY (X-DIR) AT CENTER LINE
C     U98= 98 PER CENT OF LAST UCL (FULLY DEVELOPPED)
C     NPOS= COUNTER FOR INSERTION OF VELOCITY PROFILE
C     LN= COUNTER FOR DIFFERENT FLUIDS
C     L1, L2= COUNTER FOR PRESSURE DROP, NUSSELT NUMBER
C     VX(-)= VALUES OF Y (-1.0 TO + 1.0)
C     VY(-,-)= VELOCITY IN X-DIR FOR THOSE PREVIOUS VX(-)
C     XPRESS(-)= VALUES OF X (0.0 TO 1.0)
C     PRESS(-,-)= PRESSURE DROP AT THOSE X
C     XNUS(-)= VALUES OF X (0.0 TO 2.0)
C     NU(-,-)= NUSSELT NUMBER AT THOSE X
C     FBI(-)= FLOW BEHAVIOR INDEX (SIMILAR TO AN)
C     EL(-)= ENTRY LENGHT FOR THOSE PREVIOUS AN
      COMMON NMAX1,PNEW,A(5,165),UNEW(165),VNEW(165),TNEW(165),UOLD(165)
     1,VOLD(165),TOLD(165),UOLDO(165),VOLDO(165),TOLDO(165),UUMAXN(165)
     2,UUMAXO(165),UN(165),VN(165),UO(165),VO(165),A1(3,165),GRA(165)
     3,UCON(165),VCON(165)
      DIMENSION VY(21,3),VX(21),PRESS(100,5),XPRESS(100),UCL(100),EL(5)
     1,FBI(5),NU(100,5),XNU(100),TBU(100,5),UVEL(70,5),VECTOR(11)
      DIMENSION INCY(20),ULIX(20),XINC(20)
      REAL NUS,NU
C...READ IN ALL THE GRID SIZE
      N = 0
    7 N = N +1
      READ(5,1105) INCY(N),XINC(N),ULIX(N)
      WRITE(6,1105)INCY(N),XINC(N),ULIX(N)
      IF (INCY(N) .NE. 0) GO TO 7
      IF (N .GT. 10)GO TO 3
      N = 10
      GO TO 7
C...READ THE PARAMETERS
    3 READ(5,1109)MAX,(FBI(N),N=1,MAX)
 1109 FORMAT(I5,10F5.2)
    4 READ(5,1100)PR,GRAMIN,TOL
      LN = 0
      IF(PR .EQ. 0.0) STOP
    5 IF(LN .GE. MAX)GO TO 850
      LN = LN +1
      AN = FBI(LN)
C...DEFINE MINIMUM GRADIENT
      GRAMI = GRAMIN
      FACT = (GRAMI)**(AN-1.)
C...OUTPUT THE PARAMETERS
      WRITE(6,1110)PR,GRAMI,FACT,TOL
C...INITIALISATION OF COUNTERS AND OTHERS
      X = 0.0
      NPOS = 1
      L1 = 1
      L2 = 1
      GRA(1) = GRAMI
      LL = 0
      TBULK = 0.0
```

```
      NDX = 1
      POLD = 0.0
      XT = X +XINC(11)
C...COMPUTE ALL THE CONSTANTS NEEDED
      NDXT = 11
      CALL CONST(INCY(NDX),XINC(NDX),INCY(NDXT),XINC(NDXT),NMAX1,NMAX,
     1DX,DY,NMAX4,DDY2,DY2,DXI,DY2X,D2Y,NMAX1T,NMAXT,NMAX4T,DYT,DXT,PR,
     2PRDY)
C...PUT IN THE STARTING VELOCITIES AND TEMPERATURE PROFILE
      DO 102 N=1,NMAXT
      UO(N) = 1.0
      VO(N) = 0.0
      TOLD(N) = 1.
      TNEW(N) = 0.0
  102 CONTINUE
      DO 1 N=1,NMAX
      UOLD(N) = 1.0
      VOLD(N) = 0.0
    1 CONTINUE
C...OUTPUT THE STARTING VALUES
      WRITE(6,1001)AN,X,X,POLD,POLD
      DO  2 N=1,NMAX,NMAX4
    2 WRITE(6,1002)N,UOLD(N),VOLD(N)
C...INTEGRATION ITERATION BEGINS
   11 X = X +DX
C...SET KNOWN VALUES WITH OLD VALUES
      DO 100 N=1,NMAX
      UCON(N) = UOLD(N)
  100 VCON(N) = VOLD(N)
      LCON = 0
  101 CONTINUE
C...SET THE COEFFICIENT MATRIX
      DO 10 N=1,NMAX1
      UNEW(N) = (UCON(N)+UOLD(N))/2.
      VNEW(N) = (VCON(N)+VOLD(N))/2.
      CP = (UOLD(N+1)-UOLD(N))/DY
      ABSCP = ABS(CP)
      IF(ABSCP .LT. GRAMI) ABSCP = GRAMI
      GRA(N) = ABSCP
      CPA = ((2.*ABSCP)**(AN-1.))/(DY**2)
      ABSCPN = ABS((UCON(N+1)-UCON(N))/DY)
      IF(ABSCPN .LT. GRAMI) ABSCPN = GRAMI
      CPANEW = ((2.*ABSCPN)**(AN-1.0))/(DY**2)
      IF(N .NE. 1)GO TO 12
      CM = -CP
      CMA = CPA
      CMANEW = CPANEW
      A(5,1)=-(CPA*CP-CMA*CM)*DY          -POLD/DX-UOLD(1)*UOLD(1)/DX
      GO TO 13
   12 CONTINUE
      CM = (UOLD(N)-UOLD(N-1))/DY
      ABSCM = ABS(CM)
      IF(ABSCM .LT. GRAMI) ABSCM = GRAMI
      CMA = ((2.*ABSCM)**(AN-1.))/(DY**2)
      ABSCMN = ABS((UCON(N)-UCON(N-1))/DY)
      IF(ABSCMN .LT. GRAMI) ABSCMN = GRAMI
      CMANEW = ((2.*ABSCMN)**(AN-1.0))/(DY**2)
```

```
      A(5,N)= -(CPA*CP-CMA*CM)*DY                              -POLD/DX
     1 -VNEW(N)*(UOLD(N+1)-UOLD(N-1))/(2.*D2Y) -UNEW(N)*(UOLD(N)/DX)
   13 CONTINUE
      A(1,N) = VNEW(N)/(2.*D2Y) +CMANEW
      A(2,N) =-UNEW(N)/DX   -CMANEW-CPANEW
      A(3,N) =-VNEW(N)/(2.*D2Y) +CPANEW
      A(4,N) =-DXI
   10 CONTINUE
      A(3,1) = A(3,1) +A(1,1)
C...SOLVE THE MATRIX (GAUSS ELIMINATION ALGORITHM)
      CALL GAUSS
      UNEW(NMAX) = 0.0
C...COMPUTE VELOCITIES IN Y-DIR
      VNEW(1) = 0.0
      DO 30 J=2,NMAX1
   30 VNEW(J) = VNEW(J-1) -DY2X*(UNEW(J)+UNEW(J-1)-UOLD(J)-UOLD(J-1))
      VNEW(NMAX) = 0.0
      LCON = LCON +1
C...COMPARE OBTAINED VALUES WITH OLD VALUES
      N = 0
      IF(LCON .GT. 25)GO TO 112
  110 N = N +1
      IF(ABS(UNEW(N)-UCON(N)) .GT. (TOL*UNEW(N))) GO TO 105
      IF(N .LT. NMAX)GO TO 110
C...HEAT TRANSFER PART
C...SEE HEAT TRANSFER COLLUMN SHOULD BE DONE
  112 IF(X .LT. (0.999*XT))GO TO 130
      NN = 0
C...SET UP VELOCITY FIELD FOR ENERGY EQUATION
      NTN = NMAX1/NMAX1T
      DO 111 N=1,NMAX1,NTN
      NN = NN +1
      UN(NN)= UNEW(N)
  111 VN(NN)= VNEW(N)
      UN(NMAXT) = 0.0
      VN(NMAXT) = 0.0
C...SET THE COEFFICIENT MATRIX (ENERGY EQUATION)
      DO 125 N=1,NMAX1T
      A1(1,N) = -PRDY
      A1(2,N) = (UO(N)+UN(N))/DXT +2.*PRDY
      A1(3,N) = -PRDY
      IF(N .EQ. 1)GO TO 121
      TNEW(N)=(UO(N)+UN(N))*TOLD(N)/DXT
     1 -(VO(N)+VN(N))*(TOLD(N+1)-TOLD(N-1))/(4.0*DYT)
     2 +(TOLD(N+1) -2.*TOLD(N) +TOLD(N-1))*PRDY
      GO TO 120
  121 TNEW(N)=(UO(1)+UN(1))*TOLD(1)/DXT +2.*(TOLD(2)-TOLD(1))*PRDY
  120 CONTINUE
  125 CONTINUE
      A1(3,1) = A1(3,1) +A1(1,1)
C...SOLVE MATRIX
      CALL DIAG3(A1,TNEW,NMAX1T)
      TNEW(NMAXT) = 0.0
C...COMPUTE BULK TEMPERATURE AND NUSSELT NUMBER
      TBULK = (TNEW(1)*UN(1) +TNEW(NMAXT)*UN(NMAXT))/2.
      DO 122 N=2,NMAX1T
  122 TBULK = TBULK +(UN(N)*TNEW(N))
```

```
      TBULK = TBULK/FLQAT(NMAX1T)
      NUS      =  (-11.*TNEW(NMAXT)+18.*TNEW(NMAXT-1)-9.*TNEW(NMAXT-2)
     1 +2.*TNEW(NMAXT-3))/(3.*DYT*TBULK)
      IF(X .LT. 0.01 .OR. L2 .GT. 100) GO TO 9998
      TBU(L2,LN) = TBULK
      NU(L2,LN)=  NUS
      XNU(L2) = X
      L2 = L2 +1
 9998 CONTINUE
      DO 123 N=1,NMAXT
      UO(N) = UN(N)
      VO(N) = VN(N)
  123 TOLD(N) = TNEW(N)
C...SET XT FOR NEXT TIME
      XT = X +XINC(NDXT)
      GO TO 39
  130 IF(X .GT. 0.001) GO TO 45
      DO 131 N=1,NMAXT
  131 TNEW(N) = 0.0
      TBULK = 0.0
C
C...OUTPUT THE RESULTS (PRINT OUT FORM)
   39 LL = LL +1
      ANTN = FLOAT(NMAX1)/FLOAT(NMAX1T)
      NTN = NMAX1T/NMAX1
      IF(LL .NE. 2) GO TO 44
      LL = 0
      WRITE(6,1001) AN,LCONO,LCON,XOLD,POLD,TBULKO,X,PNEW,TBULK,NUS
      MM = 1
      IF(NMAX .NE. NMAXO) MM = 2
      NMAX4D = NMAX4*2
      DO 40 N=1,NMAX,NMAX4D
      M = MM*(N-1) +1
      NN = IFIX(ANTN*(FLOAT(N)-1.)) +1
      NNN= IFIX(ANTN*(FLOAT(M)-1.)) +1
      YD = FLOAT(N-1)/FLOAT(NMAX1)
      UUMAXN(N) = UNEW(N)/UNEW(1)
      WRITE(6,1002)N,YD,UOLDO(M),UUMAXO(M),VOLDO(M),TOLDO(NNN),
     1 UNEW(N),UUMAXN(N),VNEW(N),TNEW(NN),NN,GRA(N)
   40 CONTINUE
      GO TO 45
C...PUT RESULT IN RESERVE BECAUSE OF NON PUBLICATION
   44 CONTINUE
      DO 53 N=1,NMAX,NMAX4
      NN = NTN*(N-1) +1
      UOLDO(N) = UNEW(N)
      VOLDO(N) = VNEW(N)
      UUMAXO(N) = UOLDO(N)/UOLDO(1)
   53 TOLDO(NN) = TNEW(NN)
      XOLD = X
      LCONO = LCON
      TBULKO = TBULK
      POLDO = PNEW
C
C...PART TO SET VALUES FOR PLOTTING SUBROUTINE
   45 GO TO (801,802,803),NPOS
  803 IF(X .LT. 1.0)GO TO 701
```

```
      GO TO 702
  802 IF(X .LT. 0.05) GO TO 701
      GO TO 702
  801 IF(X .LT. 0.002) GO TO 46
C...PREPARE VELOCITY FIELD
  702 L = 0
      DO 700 N=1,NMAX,NMAX4
      L = L+1
      LI = 22-L
      NN = NMAX+1-N
      VY(L,NPOS ) = UNEW(NN)
      VY(LI,NPOS) = UNEW(NN)
      VX(L) = (FLOAT(L-1)/10.)-1.0
  700 VX(LI) = FLOAT(LI-11)/10.0
      NPOS = NPOS +1
C...PREPARE PRESSURE DROPS
  701 IF(L1 .GT. 100) GO TO 710
      PRESS(L1,LN) =-PNEW
      XPRESS(L1) = X
      UCL(L1) = UNEW(1)
      IF(L1 .GT. 70) GO TO 709
      J = 0
      DO 705 N=1,NMAX,NMAX4
      J = J+1
  705 VECTOR(J) = UNEW(N)
      UVEL(L1,1) = VECTOR(1)
      UVEL(L1,2) = VECTOR(4)
      UVEL(L1,3) = VECTOR(6)
      UVEL(L1,4) = VECTOR(8)
      UVEL(L1,5) = VECTOR(10)
  709 L1 = L1 +1
  710 CONTINUE
      IF(NPOS .NE. 4) GO TO 46
      NPOS = NPOS -1
      DO 900 N=1,21
  900 WRITE(6,901)VX(N),VY(N,1),VY(N,2),VY(N,3)
      DO 906 N=1,70
  906 WRITE(6,901)XPRESS(N),(UVEL(N,J),J=1,5)
  901 FORMAT(12F11.5)
C...COMPUTING ENTRY LENGHT VS FLOW BEHAVIOR INDEX
      U98 = 0.98*UNEW(1)
      L=0
  220 L = L+1
      IF(UCL(L) .LT. U98) GO TO 220
      L = L-1
      EL(LN) = XPRESS(L) +((XPRESS(L+1)-XPRESS(L))/(UCL(L+1)-UCL(L)))
     1 *(U98-UCL(L))
      FBI(LN) = AN
      GO TO 5
C...TO CHANGE GRID SPACING IN X-DIR (IF NECESSARY)
   46 NMAXTO = NMAXT
      NMAXO = NMAX
      IF(XT .LT. (1.01*ULIX(NDXT))) GO TO 49
      IF (X .LT. (0.99*ULIX(NDX)          )) GO TO 48
      NDX = NDX +1
   48 NDXT = NDXT +1
      XT = X +XINC(NDXT)
```

```
   47 CALL CONST(INCY(NDX),XINC(NDX),INCY(NDXT),XINC(NDXT),NMAX1,NMAX,
     1DX,DY,NMAX4,DDY2,DY2,DXI,DY2X,D2Y,NMAX1T,NMAXT,NMAX4T,DYT,DXT,PR,
     2PRDY)
      GO TO 60
   49 IF (X .LT. (0.99*ULIX(NDX)           )) GO TO 51
      NDX = NDX +1
      GO TO 47
   60 IF(NMAXT .EQ. NMAXTO) GO TO 61
C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / MOMENTUM EQUATION
      DO 129 N=1,NMAXT
      NN = 2 *N -1
      UO(N) = UO(NN)
      VO(N) = VO(NN)
      TNEW(N) = TNEW(NN)
  129 TOLD(N) = TOLD(NN)
   61 IF(NMAX .EQ. NMAXO)GO TO 51
C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / ENERGY EQUATION
      DO 22 N=1,NMAX
      NN = 2*N-1
      UNEW(N) = UNEW(NN)
      VNEW(N) = VNEW(NN)
   22 CONTINUE
C...ONE COLLUMN COMPLETE, PREPARE FOR NEXT COLLUMN
   51 POLD = PNEW
      DO 52 N=1,NMAX
      UOLD(N) = UNEW(N)
   52 VOLD(N) = VNEW(N)
      GO TO 11
C...PLOT AND OUTPUT COMPARABLE VALUES (PRESSURE, NUSSELT) OF ALL FLUIDS
  850 CONTINUE
      CALL DESSIN(3,VX,VY,XPRESS,PRESS,UVEL,XNU,NU,TBU,FBI,EL,LN)
      GO TO 4
  999 DO 998 N=LN,MAX
  998 FBI(N) = FBI(N+1)
      WRITE(6,1500)LCON
 1500 FORMAT(    * NUMBER OF ITERATIONS TOO HIGH =*,I4)
      LN = LN -1
      MAX = MAX-1
      GO TO 5
C...PART OF RESET OF ITERATIONS FOR CONVERGING TYPE MESH
  105 CONTINUE
      PAR = 0.85
      DO 103 N=1,NMAX
      UCON(N) = PAR*UNEW(N) +(1.-PAR)*UCON(N)
  103 VCON(N) = PAR*VNEW(N) +(1.-PAR)*VCON(N)
      IF(UCON(1) .LT. -.01 .OR. UCON(1) .GT. 4.)GO TO 999
      GO TO 101
 1001 FORMAT (///,16X,* N =*,F4.2,12X*CON=*I3,50X*CON=*I3,
     1 /,3X,*X=*F8.6,5X*PRES=*F8.4,6X*TBULK=*F8.4,20X,*X=*F8.6,4X*PRES=*
     1F8.4,4X *TBULK=*F8.4,4X*NUS=*F8.4,
     2      //4X*N*,4X*Y/D*,6X*U(N)*,9X*U/UMAX*,7X*V(N)*,9X*T(L)*,
     3 13X*U(N)*,9X*U/UMAX*,7X*V(N)*,9X*T(L)*,7X*L* )
 1002 FORMAT (1X,I4,3X,F5.3,4F13.6,4X,4F13.6,I4,F6.3)
 1100 FORMAT(    3F10.5)
 1105 FORMAT (I5,2F10.6)
 1110 FORMAT(1H1,2X,*PRANDTL NUMBER=* F12.4,/2X,,*MINIMUM GRADIENT=*F6.4
     1,12X*(MIN. GRAD.)EXP(N-1)=*F10.4,/2X,*TOLERANCE FOR CONVERGING TYP
```

```
2E MESH=*F8.5)
 END
```

CD TOT 0403

```
      SUBROUTINE CONST(INCY,XINC,INCYT,XINCT,NMAX1,NMAX,DX,DY,NMAX4,DDY2
     1 ,DY2,DXI,DY2X,D2Y,NMAX1T,NMAXT,NMAX4T,DYT,DXT,PR,PRDY)
C
C...SUBROUTINE COMPUTING CONSTANTS OF THE MAIN PROGRAM
C
      DX = XINC
      NMAX1 = INCY
       DY = 1./FLOAT(NMAX1)
       NMAX = NMAX1 +1
       NMAX4 = NMAX1/10
      DDY2 = 0.5/(DY*DY)
      DY2 = 1./(DY*DY)
      DXI = 1./DX
      DY2X = DY/(2.*DX)
      D2Y = 2.*DY
      NMAX1T = INCYT
      NMAXT = NMAX1T +1
      NMAX4T = NMAX1T/10
      DYT = 1.0/FLOAT(NMAX1T)
      DXT = 2.0*XINCT
      PRDY = 1.0/(PR*2.0*DYT*DYT)
      PRDY = 2.*PRDY
      RETURN
      END
```

```
      SUBROUTINE GAUSS
C
C...SUBROUTINE TO SOLVE THE MOMENTUM EQUATION  COEFFICIENT MATRIX
C...THIS IS DONE USING THE GAUSS ELIMINATION METHOD
C
      COMMON NMAX1,PNEW,A(5,165),UNEW(165)
      DIMENSION B(5,165)
      NMAX = NMAX1 + 1
      DO 1 I=1,5
      DO 1 J=1,NMAX1
    1 B(I,J) = A(I,J)
      J = 1
    3 DO 2 I=2,5
    2 B(I,J) = B(I,J)*(-A(1,J+1)/A(2,J))
      A(1,J+1) = A(1,J+1)+B(2,J)
      A(2,J+1) = A(2,J+1)+B(3,J)
      A(4,J+1) = A(4,J+1)+B(4,J)
      A(5,J+1) = A(5,J+1)+B(5,J)
      J = J+1
      DO 4 I=2,5
    4 B(I,J) = A(I,J)
      IF(J .LT. NMAX1)GO TO 3
      J = NMAX1
    5 F = -A(3,J-1)/A(2,J)
      B(2,J) = A(2,J)*F
      B(4,J) = A(4,J)*F
      B(5,J) = A(5,J)*F
      A(3,J-1) = A(3,J-1) +B(2,J)
      A(4,J-1) = A(4,J-1) +B(4,J)
      A(5,J-1) = A(5,J-1) +B(5,J)
      J = J-1
      IF(J .GT. 1) GO TO 5
      A(1,1) = 1.0
      J = 0
   13 J = J+2
      A(1,J) = 4.0
      A(1,J+1) = 2.0
      IF(NMAX1-J)11,12,13
   12 A(1,J+1) = 0.0
      A(1,J+2) = FLOAT(3*NMAX1)
      GO TO 15
   11 A(1,J) = 0.0
      A(1,J+1) = FLOAT(3*NMAX1)
   15 DO 14 J=1,NMAX1
      F= -A(1,J)/A(2,J)
      A(1,J) = A(1,J) +A(2,J)*F
      A(1,NMAX) = A(1,NMAX) +A(4,J)*F
   14 A(1,NMAX+1) = A(1,NMAX+1) +A(5,J)*F
      PNEW = A(1,NMAX+1)/A(1,NMAX)
      DO 20 J=1,NMAX1
   20 UNEW(J) = (A(5,J)-A(4,J)*PNEW)/A(2,J)
      RETURN
      END
```

```
      SUBROUTINE DESSIN(IFLAG,VX,VY,XPRES,PRES,UVEL,XNU,NU,TBU,FBI,EL,NF)
     1)
C
C...SUBROUTINE USING THE PLOTTING SUBROUTINES OF DOUG BUCHANAN
C
      DIMENSION VY(21,NF),VX(21),PRES(100,NF),XPRES(100),X(70),
     1EL(NF),FBI(NF),XNU(100),NU(100,NF),UVEL(70,5),TBU(100,NF)
      REAL NU
C...COMMON BLOCK
      COMMON DUMMY(3),XX( 5000)
      COMMON /INFOPLT/LINEFX(30),LPTS(30)
      COMMON /LABL/ LABEL,LETS(4)
      COMMON /LBL/ YLVERT(7),YRVERT(7),XLHOR(7),XUHOR(7)
      COMMON /TITLES/MNYPLT,TITLE1(2),TITLE2(2),TITLE3(2),TITLE4(2)
      COMMON /SCALES/ ISIZE
      COMMON /BORDER/ XMIN,XMAX,YMIN,YMAX
C ...CONSTANTS
      NREAD = 0
      FMTE = 0
      NPT = 1
      NORD = 3
      XSIZE = 15.0
      YSIZE = 15.0
      LABEL = 3
      MNYPLT = 1
      LINES = 2
C...MAIN TITLES
      TITLE1(1) = 10H HPVL    J.
      TITLE1(2) = 10H LAROCQUE
      TITLE2(1) = 10HENTRY FLOW
      TITLE2(2) = 10H
      TITLE3(1) = 10H FOR POWER
      TITLE3(2) = 10H LAW FLUID
      NT = 21
      IPOINT = 0
      IPLOT = 200
      DO 10 N=1,10
      LINEFX(N) = 3
   10 LPTS(N) = N
      GO TO (100,200,300),IFLAG
C...PART FOR THE VELOCITY FIELD PLOT(U VS Y)
  100 LETS(1) = 20
      LETS(2) = 5
      LETS(3) = 24
      YLVERT(1) =10H VELOCITY=
      YLVERT(2) =10HU/U(ENT)
      XLHOR(1) = 5H Y/A
      XUHOR(1) = 10HVELOCITY V
      XUHOR(2) = 10HS Y FOR N=
      ENCODE(10,15,XUHOR(3))FBI(NF)
   15 FORMAT(F4.2,6X)
      ISIZE = 2
      YMAX = 2.0
      YMIN = 0.0
      NF1 = 3
      J6 = 1+NT*NF1 +NT
      CALL NPLOTS(NF1,NT,IPLOT,IPOINT,NREAD,VY,VX    ,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
      RETURN
```

```
C...PART FOR THE VELOCITY FIELD PLOT(U VS X)
  200 NF1 = 5
      NT = 70
      LETS(1) = 20
      LETS(2) = 10
      LETS(3) = 24
      YLVERT(1) = 10H VELOCITY=
      YLVERT(2) = 10HU/U(ENT)
      XLHOR(1) = 10H X/A/RE
      XUHOR(1) = 10HVELOCITY V
      XUHOR(2) = 10HS X FOR N=
      ENCODE(10,15,XUHOR(3))FBI(NF)
      ISIZE = 3
      YMAX = 2.0
      YMIN = 0.0
      XMAX = 0.3
      XMIN = 0.0
      J6 = 1+NT*NF1+NT
      DO 20 N=1,70
   20 X(N) = XPRES(N)
      CALL NPLOTS(NF1,NT,IPLOT,IPOINT,NREAD,UVEL,X      ,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
      RETURN
  300 CONTINUE
C...PART FOR PRESSURE DROPS PLOT (P VS X)
      DO 11 N=1,10
   11 LINEFX(N) = N
      XSIZE = 20.0
       NT = 100
       IPOINT = 0
      IPLOT = 200
      J6 = 1+NT*NF +NT
      ISIZE = 1
      XMIN = 0.0
      XMAX = 0.9
      LETS(1) = 10
      LETS(2) = 10
      LETS(3) = 10
      YLVERT(1) = 10HPRESSURE
      XLHOR(1) = 10H X(X/A/RE)
      XUHOR(1) = 10HPRESS VS X
  901 FORMAT(11F11.5)
      CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,PRES,XPRES,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
C... PART FOR NUSSELT NUMBER PLOT(NU VS X)
      LETS(1) = 10
      LETS(2) = 10
      LETS(3) = 20
      YLVERT(1) = 10HNU (LOCAL)
      XLHOR(1) = 10H X/A/RE
      XUHOR(1) = 10HNUSSELT NU
      XUHOR(2) = 10HMBER VS X
      CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,NU,XNU      ,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
C...PART FOR BULK TEMPERATURE PLOT (TBULK VS X)
      LETS(1) = 10
      LETS(2) = 10
```

```
      LETS(3) = 25
      YLVERT(1) = 10HBULK TEMP.
      XLHOR(1) = 10H X/A/RE
      XUHOR(1) = 10HBULK TEMPE
      DO 902 N=1,NT
  902 WRITE(6,901)XPRES(N),(PRES(N,I),I=1,NF),(NU(N,J),J=1,NF)
      XUHOR(2) = 10HRATURE VS
      XUHOR(3) = 5HX
      CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,TBU,XNU,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
C...PART FOR ENTRY LENGTH VS FLOW BEHAVIOR INDEX PLOT
      NT = NF
      NF = 1
      IPOINT = 1
       IPLOT = 0
      J6 = 1+NT*NF +NT
      LETS(1) = 26
      LETS(2) = 24
      LETS(3) = 37
      YLVERT(1) = 10H ENTRY LEN
      YLVERT(2) = 10HGTH (VALUE
      YLVERT(3) = 6H OF X)
      XLHOR(1) = 10H FLOW BEHA
      XLHOR(2) = 10HVIOR INDEX
      XLHOR(3) = 4H  N
      XUHOR(1) = 10HENTRY LENG
      XUHOR(2) = 10HTH  VS  FL
      XUHOR(3) = 10HOW BEHAVIO
      XUHOR(4) = 7HR INDEX
      ISIZE = 3
      XMIN = 0.0
      XMAX = 2.0
      YMIN = 0.0
      YMAX=0.30
      DO 903 N=1,NT
  903 WRITE(6,901)FBI(N),EL(N)
      CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,EL,FBI   ,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
      RETURN
       END
```

```
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10)
C     THIS IS A FINITE DIFFERENCE METHOD TO SOLVE THE MOTION
C             EQUATIONS FOR THE ENTRANCE REGION OF A CONVERGING CHANNEL
C     THIS METHOD IS USING A SERIE OF VALUES KNOWN AS THE OLD VALUES TO
C           COMPUTE ANOTHER SET OF VALUES KNOWN AS THE NEW VALUES CORRESPONDING
C           TO THE NEXT COLLUMN (AXIALLY) SO THAT FROM THE ENTRY SET OF VALUES WE
C           CAN GO DOWNSTREAM AS FAR AS WE WANT PROVIDING WE HAVE THE
C           BOUNDARY CONDITIONS
C
C LIST OF VARIABLES
C     AN= EXPONENT OF THE POWER LAW FLUID (FLOW BEHAVIOR INDEX)
C     ANGLE=TOTAL TAPER ANGLE
C     BETA=HALF THE TOTAL TAPER ANGLE
C     UOLD(-),VOLD(-),TOLD(-),POLD= VECTORS OF VELOCITY (X-DIR), VELOCITY
C           (Y-DIR), TEMPERATURE AND PRESSURE ALREADY KNOWN
C     UNEW(-),VNEW(-),TNEW(-),PNEW= VECTORS OF VELOCITY (X-DIR), VELOCITY
C           (Y-DIR), TEMPERATURE AND PRESSURE TO BE COMPUTED (NEW)
C     UAVE=AVERAGE VELOCITY IN X-DIRECTION
C     UMAUAV=CENTERLINE VELOCITY (MAX)/AVERAGE VELOCITY
C     U99=99 PERCENT OF UMAUAV
C     UUMAXN(-)= VECTOR OF VELOCITY DIVIDED BY MAXIMUM VELOCITY (CENTERLINE)
C           (OUTPUT PURPOSE)
C     UUMAXO(-)= SAME AS PRECEEDING ONE
C     GRA(-),GRAMIN,GRAMI= GRADIENTS - VECTOR OF GRADIENTS (FOR OUTPUT),
C           MINIMUM GRADIENT AS DEFINE IN READ STAT., MINIMUM GRADIENT AS
C             DEFINED IN PROGRAM
C     FACT = (GRAMI) **(AN-1)
C     R= VALUE OF R AT WHICH THE PROGRAM IS NOW (RADIAL LENGTH)
C     ROLD= OLD VALUE OF R
C     TBULK= BULK TEMPERATURE OF FLOW
C     NUS= NUSSELT NUMBER
C     AVNUS= AVERAGE NUSSELT NUMBER
C     PE= PECLET NUMBER
C     PGRAO= PRESSURE GRADIENT
C     RE= REYNOLDS NUMBER
C     RC= REYNOLDS NUMBER WHERE THE CENTERLINE VELOCITY IS TAKEN AS
C             CHARACTERISTIC VELOCITY
C     SUM= RUNNING SUM FOR CALCULATION OF AVNUS
C     TGRA= TEMPERATURE GRADIENT
C
C VARIABLES DESCRIBING THE MESH SIZE (MESH SIZES ARE VARYING AS WE PROCEED)
C     INCY(-)= VECTOR CORRESPONDING TO NUMBER OF GRID SPACE ACROSS FLOW
C     XINC(-)= VECTOR CORRESPONDING TO GRID SPACE IN X-DIR
C     ULIX(-)= VECTOR CORRESPONDING TO LIMIT UP TO WHICH THE PRECEDING
C          X-DIR GRID SPACE IS VALID
C     NDR= VARIABLE CORRESPONDING TO INDICES OF PREVIOUS VARIABLES
C             TO KEEP TRACK OF THE NEXT MESH SIZE (MOMENTUM MESH AND
C           HEAT TRANSFER MESH)
C
C CONSTANTS RELATED MOMENTUM MESH
C     A(-,-)= MATRIX OF COEFFICIENTS FOR MOMENTUM MESH
C     C(-),RIJ,RIJ1,AJ,AV2RE,AJ2REC,BJRE,BJREC,BJT,BJT2P,VGAAD(-),
C             CONS,RUPJ,= CONSTANTS USED IN SETTING OF COEFFICIENT MATRIX
C     NMAX1= NUMBER OF SPACE IN TETA-DIRECTION (ANGULAR DIRECTION)
C     NMAX= NUMBER OF POINT IN Y-DIR (NMAX = NMAX1 + 1)
C     DTETA= GRID SIZE IN TETA-DIRECTION (ANGULAR DIRECTION)
C     DR= GRID SIZE IN R-DIRECTION (RADIAL DIRECTION)
C     DRDT2,DRI,BT2R= SERIE OF CONSTANTS DEFINED IN SUBROUTINE
C           CONST CALCULATED FROM 4 PREVIOUS
```

```
C
C  CONSTANTS AND VARIABLES RELATED TO HEAT TRANSFER MESH
C     B(-,-)= MATRIX OF COEFFICIENTS FOR HEAT TRANSFER MESH
C     TBULKO,NUSO= VARIABLE USED FOR OUTPUTING PURPOSES (SAME MEANING
C               AS WITHOUT *O*)
C     YD= USED IN THE OUTPUT TO SITUATE A POINT IN TETA-DIRECTION (AT CENTERLINE
C               YD= 0.0, AT WALL YD=1.0)
C     LL= FLAG TO OUTPUT EVERY SECOND TIME.
C
C CONSTANTS AND VARIABLES RELATED TO PLOTTING SUBROUTINES
C     U98= 98 PER CENT OF LAST UMAUAV (FULLY DEVELOPED)
C     NPOS= COUNTER FOR INSERTION OF VELOCITY PROFILE
C     LN= COUNTER FOR DIFFERENT FLUIDS
C     L1, L2= COUNTER FOR PRESSURE DROP, NUSSELT NUMBER
C     VX(-)= VALUES OF Y (-1.0 TO + 1.0)
C     VY(-,-)= VELOCITY IN R-DIRECTION FOR THOSE PREVIOUS VX(-)
C     XPRESS(-)= VALUES OF R (0.0 TO 1.0)
C     PRESS(-,-)= PRESSURE DROP AT THOSE R
C     TBU(-,-)= BULK TEMPERATURE AT THOSE R
C     NU(-,-)= NUSSELT NUMBER AT THOSE R
C     VECTOR(-)= USED IN PREPARING UVEL(-,-)
C     FBI(-)= FLOW BEHAVIOR INDEX (SIMILAR TO AN)
C     UVEL(-,-)= VELOCITY IN R-DIRECTION
C
      COMMON NMAX1,RIJ1,PNEW,A(5,165),UNEW(165),UOLD(165),VNEW(165),
     1VOLD(165),TNEW(165),TOLD(165),UUMAXN(165),UUMAXO(165),B(3,165)
      DIMENSION INCY(20),XINC(20),ULIX(20)
      DIMENSION VY(21,3),VX(21),PRESS(110,5),XPRESS(110),UMAUAV(110,5)
     1 ,FBI(5),NU(110,5),TBU(110,5),VECTOR(12),UVEL(70,5)
      DIMENSION C(165),VGRAD(165)
      REAL NUS,NUSO,NU
      N = 0
C...READ IN ALL THE GRID SIZE
    7 N = N +1
      READ(5,1105)INCY(N),XINC(N) ,ULIX(N)
      WRITE(6,1105)INCY(N),XINC(N),ULIX(N)
      IF (INCY(N) .NE. 0)GO TO 7
C...READ THE PARAMETERS
    6 LN = 0
      LM = 0
      READ(5,1109)MAX,(FBI(N),N=1,MAX)
      IF(EOF(5))10000,9
    9 IF(MAX .EQ. 1)GO TO 200
 1109 FORMAT (I5,10F5.2)
    4 READ(5,1106)RE,PE,ANGLE,GRAMIN,TOL
      IF(RE .EQ. 0.0)STOP
    5 IF(LN .GE. MAX) GO TO 850
      LN = LN +1
      LM = LN
      AN = FBI(LN)
C...DEFINE MINIMUM GRADIENT
    8 GRAMI = GRAMIN
C...INITIALISATION OF COUNTERS AND OTHERS
      R = 0.0
      NUSO = 0.0
       TGRAO = 0.0
      SUM = 0.0
```

```
      ROLD = 0.0
      NPOS = 1
      L1 = 1
      LL = -1
      L = 0
      NDR = 1
C...COMPUTE ALL THE CONSTANTS NEEDED
      CALL CONST(INCY(NDR),XINC(NDR),ANGLE,BETA,NMAX1,DR,DTETA,NMAX,NMAX
     14,DRDT2,DRI,BT2R)
C...DEFINE THE STARTING PROFILES
      DO 1 K=1,NMAX
      TOLD(K) = 1.0
      UOLD(K) = 1.0
      VOLD(K) = 0.0
    1 CONTINUE
      POLD = 0.0
      PNEW = 0.0
C...OUTPUT THE STARTING VALUES
      WRITE(6,1003)AN,R,POLD
      DO  2 N=1,NMAX,NMAX4
    2 WRITE(6,1002)N,UOLD(N),VOLD(N)
C...OUTPUT THE PARAMETERS
      FACT = (GRAMI)**(AN-1.)
      WRITE(6,1010)RE,PE,ANGLE,GRAMI,FACT,TOL
C...INTEGRATION ITERATION BEGINS
   11 R = R +DR
      RIJ = 1.-ROLD
      RIJ1 = 1.-R
      AJ = 1./(RIJ*BETA*DTETA)
      AJ2RE =((AJ*AJ)/RE)
      BJRE =((1./(RIJ*DR))/RE)
C...COMPUTE VARIABLES NECESSARY IN SETTING UP OF COEFFICIENT MATRIX
      DO 13 N=1,NMAX1
      IF(N .NE. 1)GO TO 12
      C(N)=(GRAMI/(RIJ*BETA))**(AN-1.0)
      GO TO 13
   12 GRA =(ABS(0.5*(UOLD(N+1)-UOLD(N-1))))/DTETA
      IF(GRA .LT. GRAMI)GRA = GRAMI
       C(N)= (GRA/(RIJ*BETA))**(AN-1.0)
   13 CONTINUE
      GRA      =   (ABS(UOLD(NMAX)-UOLD(NMAX-1))/DTETA)
      IF (GRA .LT. GRAMI)GRA = GRAMI
      C(NMAX) = (GRA/(RIJ*BETA))**(AN-1.)
      CONS = RE*BETA*RIJ*DTETA
      DO 14 N=2,NMAX1
   14 VGRAD(N) = (C(N+1)-C(N-1))/CONS
      VGRAD(1) = 0.0
C...SET THE COEFFICIENT MATRIX
      DO 10 N=1,NMAX1
      AJ2REC = AJ2RE*C(N)
      BJREC = BJRE*C(N)
       A(3,N) = +.5*AJ*(VOLD(N)-VGRAD(N)) -AJ2REC
       A(1,N) = -.5*AJ*(VOLD(N)-VGRAD(N)) -AJ2REC
      A(2,N) = UOLD(N)/DR +3.*BJREC+2.*AJ2REC
      A(4,N) = +DRI
       A(5,N) = POLD/DR -VOLD(N)*VOLD(N)/RIJ +UOLD(N)*(UOLD(N)/DR
     1+3.*BJREC+(((1./RIJ)**2)/RE)*C(N))
```

```
   10 CONTINUE
      A(3,1) = A(3,1) +A(1,1)
C...SOLVE THE MATRIX (GAUSS ELIMINATION ALGORITHM)
      CALL GAUSS
      UNEW(NMAX) = 0.0
C...COMPUTE VELOCITIES IN Y-DIR
      VNEW(1) = 0.0
      DO 30 J=2,NMAX1
   30 VNEW(J) = VNEW(J-1) -BT2R*(RIJ1*(UNEW(J)+UNEW(J-1))
     1 -RIJ*(UOLD(J)+UOLD(J-1)))
      VNEW(NMAX) = 0.0
C...COMPUTE AVERAGE VELOCITY
      UAVE = UNEW(1)/2.
      DO 47 N=2,NMAX1
   47 UAVE = UAVE +UNEW(N)
      UAVE = UAVE/FLOAT(NMAX1)
C...HEAT TRANSFER PART
      BJT = 1./(BETA*RIJ*DTETA)
      BJT2P = BJT*BJT/PE
C...SET UP VELOCITY FIELD FOR ENERGY EQUATION
C...SET THE COEFFICIENT MATRIX (ENERGY EQUATION)
      DO 60 N=1,NMAX1
      RUPJ = (UOLD(N) +1./(PE*RIJ))/DR
      B(1,N) = -VOLD(N)*BJT/2.0 -BJT2P
      B(2,N) = RUPJ + 2.*BJT2P
      B(3,N) = +VOLD(N)*BJT/2.0 -BJT2P
      TNEW(N) = TOLD(N)*RUPJ
   60 CONTINUE
      B(3,1) = B(3,1) +B(1,1)
C...SOLVE MATRIX
      CALL DIAG3(B,TNEW,NMAX1)
      TNEW(NMAX) = 0.0
C...COMPUTE BULK TEMPERATURE AND NUSSELT NUMBER
      TBULK = (TNEW(1)*UNEW(1) +TNEW(NMAX)*UNEW(NMAX))/2.
      DO 122 N=2,NMAX1
  122 TBULK = TBULK +(UNEW(N)*TNEW(N))
      TBULK =(TBULK/FLOAT(NMAX1))/UAVE
      TGRA = (-11.*TNEW(NMAX)+18.*TNEW(NMAX-1)-9.*TNEW(NMAX-2)+2.*TNEW(
     1NMAX-3))/(6.*DTETA)
      NUS = (2.*TGRA)/((1.-R)*TBULK)
      SUMO = SUM
      SUM = (.5*DR*(NUS+NUSO)) +SUM
      AVNUS = SUM/R
C
C...OUTPUT THE RESULTS (PRINT OUT FORM)
      L = L+1
      IF(L .NE. 2)GO TO 45
      L = 0
      LL = LL+1
      IF(LL .EQ. 3)GO TO 99
   98 CONTINUE
      RC = RE*(1.-R)*UNEW(1)
      WRITE(6,1001)AN,AVNUS,ROLD,POLD,TBULKO,NUSO,R,RC,PNEW,TBULK,NUS
      NMAX4D = NMAX4*2
      DO 40 N=1,NMAX,NMAX4D
      YD = FLOAT(N-1)/FLOAT(NMAX1)
      UUMAXO(N) = UOLD(N)/UOLD(1)
```

```
      UUMAXN(N) = UNEW(N)/UNEW(1)
   40 WRITE (6,1002)N,YD,UOLD(N),UUMAXO(N),VOLD(N),TOLD(N),
     1UNEW(N),UUMAXN(N),VNEW(N),TNEW(N)
C
C...PART TO SET VALUES FOR PLOTTING SUBROUTINE
   45 GO TO (801,802,803),NPOS
  803 IF(R .LT. 0.76)GO TO 701
      GO TO 702
  802 IF(R .LT. 0=05) GO TO 701
      GO TO 702
  801 IF(R .LT. 0.005) GO TO 701
  702 LJ= 0
C...PREPARE VELOCITY FIELD
      DO 700 N=1,NMAX,NMAX4
      LJ = LJ+1
      LI = 22-LJ
      NN = NMAX+1-N
      VY(LJ,NPOS ) = UNEW(NN)/UAVE
      VY(LI,NPOS) = UNEW(NN)/UAVE
      VX(LJ) = (FLOAT(LJ-1)/10.)-1.0
  700 VX(LI) = FLOAT(LI-11)/10.0
      NPOS = NPOS +1
  701 IF(L1 .GT. 110 .OR. R .LT. 0.005)GO TO 710
C...PREPARE PRESSURE DROPS, U(MAX)/UAVE, BULK TEMPERATURE AND NUSSELT NUMBE
      XPRESS(L1) = R
      PRESS(L1,LM) =-PNEW
      TBU(L1,LM) = TBULK
      NU(L1,LM) = NUS
      UMAUAV(L1,LM) = UNEW(1)/UAVE
       PGRA =(POLD-PNEW)/DR
      IF(L .EQ. 0)WRITE(6,1007)UAVE,UMAUAV(L1,LM),PGRA
      IF(L1.GT.70)GO TO 709
      J = 0
      DO 705 N=1,NMAX,NMAX4
      J = J+1
      VECTOR(J) = UNEW(N)/UAVE
  705 CONTINUE
      UVEL(L1,1) = VECTOR(1)
      UVEL(L1,2) = VECTOR(4)
      UVEL(L1,3) = VECTOR(6)
      UVEL(L1,4) = VECTOR(8)
      UVEL(L1,5) = VECTOR(10)
  709 L1 = L1 +1
  710 CONTINUE
      IF(NPOS .NE. 4) GO TO 46
      NPOS = NPOS -1
      U99 = .99*UMAUAV(110,LM)
      N = 0
C...COMPUTE ENTRY LENGTH
  220 N = N +1
      IF(N .GT. 110)GO TO 899
      IF(UMAUAV(N,LM) .LT. U99)GO TO 220
      N = N -1
      IF(N .LT. 1)GO TO 899
       EL = XPRESS(N) +((XPRESS(N+1)-XPRESS(N))/(UMAUAV(N+1,LM)
     1 -UMAUAV(N,LM)))*(U99-UMAUAV(N,LM))
      WRITE (6,1104)EL
```

```
 1104 FORMAT(2X,*ENTRY LENGTH =* F10.4)
  899 DO 900 N=1,21
  900 WRITE(6,901)VX(N),VY(N,1),VY(N,2),VY(N,3)
      DO 906 N=1,70
  906 WRITE(6,901)XPRESS(N),(UVEL(N,J),J=1,5)
  901 FORMAT (11F11.5)
      IF(MAX .EQ. 1)GO TO 200
      CALL DESSIN(2,VX,VY,XPRESS,PRESS,UVEL,UMAUAV,NU,TBU,FBI,LM)
      GO TO 5
C
C...TO CHANGE GRID SPACING IN X-DIR (IF NECESSARY)
   46 IF(R .LT.(0.99*ULIX(NDR))) GO TO 51
      NDR = NDR +1
      CALL CONST(INCY(NDR),XINC(NDR),ANGLE,BETA,NMAX1,DR,DTETA,NMAX,NMAX
     14,DRDT2,DRI,BT2R)
      IF(INCY(NDR) .EQ. INCY(NDR-1)) GO TO 51
C...TO CHANGE GRID SPACING IN Y-DIR (IF NECESSARY) / MOMENTUM EQUATION
      DO 22 N=1,NMAX
      NN = 2*N-1
      UOLD(N) = UOLD(NN)
      VOLD(N) = VOLD(NN)
      UNEW(N) = UNEW(NN)
      VNEW(N) = VNEW(NN)
      TNEW(N) = TNEW(NN)
      TOLD(N) = TOLD(NN)
   22 CONTINUE
C...ONE COLLUMN COMPLETE, PREPARE FOR NEXT COLLUMN
   51 POLD = PNEW
      ROLD = R
      NUSO = NUS
      TBULKO = TBULK
      TGRAO = TGRA
      DO 52 N=1,NMAX
      TOLD(N) = TNEW(N)
      UOLD(N) = UNEW(N)
   52 VOLD(N) = VNEW(N)
C...GO BACK FOR NEXT COLLUMN
      GO TO 11
   99 CONTINUE
      LL = 0
      GO TO 98
  850 CONTINUE
C...PLOT AND OUTPUT COMPARABLE VALUES (PRESSURE, NUSSELT) OF ALL FLUIDS
      CALL DESSIN(3,VX,VY,XPRESS,PRESS,UVEL,UMAUAV,NU,TBU,FBI,LM)
      IF(MAX .EQ. 1)GO TO 6
      LN = 0
      GO TO 4
  200 READ(5,1106)RE,PE,ANGLE,GRAMIN,TOL
      IF(RE .EQ. 0.0)GO TO 850
      AN = FBI(1)
      LN = 1
      LM = LM +1
      GO TO 8
10000 STOP
 1001 FORMAT (///,16X,* N =*,F4.2,80X*AVNUS=*F8.4,/2X,*R=*F7.5,3X*PRES=*
     1F8.4,3X*TBULK=*F7.4,3X*NUS=*F8.4,14X,*R=*F7.5,*(*F6.0*)*,3X*PRES=*
     1F8.4,3X*TBULK=*F7.4 ,   3X*NUS=*F8.4,/ 5X*N*,4X*Y/D*,7X
```

```
    2*U(N)*, 9X,*U/UMAX*, 7X,*V(N)*,9X,*T(N)*,19X,*U(N)*9X,*U/UMAX*7X,
    3 *V(N)*9X,*T(N)* )
1002 FORMAT (1X,I5,3X,F5.3,4F13.6,10X,4F13.6)
1003 FORMAT (1H1,10X,* N =*,F4.2,//,3X,*  R =*F6.4,
    14X, * PRESSURE =*,F8.5,                       //,9X,*N*,          4X,
    2*U(N)*, 9X,              *V(N)*)
1007 FORMAT (80X,*UAVE =* F7.4,8X,*UMAX/UAVE =* F7.4,6X*PGRA=*F7.3)
1010 FORMAT(///,5X*REYNOLDS NUMBER=* F10.3,15X*PECLET NUMBER=*F10.3,
    1 15X*ANGLE (TOTAL)=*F5.1,          /2X,,*MINIMUM GRADIENT=*F6.4,12X
    1*(MIN. GRAD.)EXP(N-1)=*F10.4,/2X,*TOLERANCE FOR CONVERGING TYPE ME
    2SH=*F8.5)
1105 FORMAT(I5,2F10.5)
1106 FORMAT(12F10.4)
     END
```

```
      SUBROUTINE CONST(INCY,XINC,ANGLE,BETA,NMAX1,DR,DTETA,NMAX,NMAX4
     1 ,DRDT2,DRI,BT2R)
C
C...SUBROUTINE COMPUTING CONSTANTS OF THE MAIN PROGRAM(FOR CONVERGING FLOW)
C
      BETA = (ANGLE/2.)*(3.14159/180.)
      NMAX1 = INCY
      DR = XINC
      DTETA = 1./FLOAT(NMAX1)
      NMAX = NMAX1 +1
      NMAX4 = NMAX1/10
      DRDT2 = DR/(DTETA*DTETA)
      WRITE(6,1006) DRDT2
 1006 FORMAT(2X,* DR/DTETAEXP2= *,E12.5)
      DRI = 1./DR
      BT2R = BETA*DTETA/(2.*DR)
      RETURN
      END
```

```
      SUBROUTINE DESSIN(IFLAG,VX,VY,XPRES,PRES,UVEL,UMAUAV,NU,TBU,FBI,NF
     1)
C
C...SUBROUTINE USING THE PLOTTING SUBROUTINES OF D. BUCHANAN
C
      DIMENSION VY(21,NF),VX(21),PRES(110,NF),XPRES(110),UMAUAV(110,NF)
     1       ,FBI(NF)             ,NU(110,NF),UVEL(70,5),TBU(110,NF)
      DIMENSION X(70)
      REAL NU
C...COMMON BLOCK
      COMMON DUMMY(3),XX( 5000)
      COMMON /INFOPLT/LINEFX(30),LPTS(30)
      COMMON /LABL/ LABEL,LETS(4)
      COMMON /LBL/ YLVERT(7),YRVERT(7),XLHOR(7),XUHOR(7)
      COMMON /TITLES/MNYPLT,TITLE1(2),TITLE2(2),TITLE3(2),TITLE4(2)
      COMMON /SCALES/ ISIZE
      COMMON /BORDER/ XMIN,XMAX,YMIN,YMAX
C...CONSTANTS
      NREAD = 0
      FMTE = 0
      NPT = 1
      NORD = 3
      XSIZE = 15.0
      YSIZE = 15.0
      LABEL = 3
      MNYPLT = 1
      LINES = 2
      TITLE1(1) = 10H HPVL    J.
      TITLE1(2) = 10H LAROCQUE
      TITLE2(1) = 10HCONVERGING
      TITLE2(2) = 10H FLOW
      TITLE3(1) = 10H FOR POWER
      TITLE3(2) = 10H LAW FLUID
      NT = 21
      IPOINT = 0
      IPLOT = 200
      DO 10 N=1,10
      LINEFX(N) = 3
   10 LPTS(N) = N
      GO TO (100,200,300),IFLAG
C...PART FOR THE VELOCITY FIELD PLOT (U VS Y)
  100 LETS(1) = 20
      LETS(2) = 5
      LETS(3) = 24
      YLVERT(1) =10H VELOCITY=
      YLVERT(2) =10HU/U(ENT)
      XLHOR(1) = 5H Y/A
      XUHOR(1) = 10HVELOCITY V
      XUHOR(2) = 10HS Y FOR N=
      ENCODE(10,15,XUHOR(3))FBI(NF)
   15 FORMAT(F4.2,6X)
      ISIZE = 2
      YMAX = 2.0
      YMIN = 0.0
      NF1 = 3
      J6 = 1+NT*NF1 +NT
      CALL NPLOTS(NF1,NT,IPLOT,IPOINT,NREAD,VY,VX    ,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
      RETURN
```

```
C...PART FOR THE VELOCITY FIELD PLOT (U VS X)
  200 NF1 = 5
      NT = 70
      LETS(1) = 20
      LETS(2) = 10
      LETS(3) = 24
      YLVERT(1) = 10H VELOCITY=
      YLVERT(2) = 10HU/U(AVE)
      XLHOR(1) = 10HR(1.-R/RO)
      XUHOR(1) = 10HVELOCITY V
      XUHOR(2) = 10HS R FOR N=
      ENCODE(10,15,XUHOR(3))FBI(NF)
      ISIZE = 3
      YMAX = 2.0
      YMIN = 0.0
      XMAX = 0.4
      XMIN = 0.0
      J6 = 1+NT*NF1+NT
      DO 210 N=1,NT
  210 X(N) = XPRES(N)
      CALL NPLOTS(NF1,NT,IPLOT,IPOINT,NREAD,UVEL,X      ,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
      RETURN
  300 CONTINUE
C...PART FOR THE PRESSURE DROPS PLOT
      DO 11 N=1,10
   11 LINEFX(N) = N
      XSIZE = 20.0
C     PRESSURE VS X
      NT = 110
       IPOINT = 0
      IPLOT = 200
      J6 = 1+NT*NF +NT
      ISIZE = 1
      XMIN = 0.0
      XMAX = 0.8
      LETS(1) = 10
      LETS(2) = 10
      LETS(3) = 10
      YLVERT(1) = 10HPRESSURE
      XLHOR(1) = 10HR(1.-R/RO)
      XUHOR(1) = 10HPRESS VS R
      DO 902 N=1,NT
  902 WRITE(6,901) XPRES(N),(PRES(N,I),I=1,NF),(UMAUAV(N,J),J=1,NF)
      CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,PRES,XPRES,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
C...PART FOR U(MAX)/U(AVERAGE) PLOT
      LETS(3) = 35
      YLVERT(1) = 10H UMA/UAV
      XUHOR(1) = 10HU(MAX=CENT
      XUHOR(2) = 10HER LINE)/U
      XUHOR(3) = 10H(AVERAGE)
      XUHOR(4) = 5HVS R
      ISIZE = 3
      YMAX = 2.0
      YMIN = 1.0
      CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,UMAUAV,XPRES,J6,LINES,
```

```
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
C...PART FOR THE NUSSELT NUMBER PLOT
      LETS(1) = 10
      LETS(2) = 10
      LETS(3) = 20
      YLVERT(1) = 10HNU (LOCAL)
      XLHOR(1) = 10HR(1.-R/RO)
      XUHOR(1) = 10HNUSSELT NU
      XUHOR(2) = 10HMBER VS R
      DO 903 N=1,NT
  903 WRITE(6,901) XPRES(N),(TBU(N,I),I=1,NF),(NU(N,J),J=1,NF)
  901 FORMAT(11F11.5)
      YMAX = 25.
      YMIN = 0.0
      CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,NU,XPRES  ,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
C...PART FOR THE BULK TEMPERATURE PLOT
      LETS(1) = 10
      LETS(2) = 10
      LETS(3) = 25
      YLVERT(1) = 10HBULK TEMP.
      XLHOR(1) = 10HR(1.-R/RO)
      XUHOR(1) = 10HBULK TEMPE
      XUHOR(2) = 10HRATURE VS
      XUHOR(3) = 5HR
      YMAX = 1.0
      YMIN = 0.0
      CALL NPLOTS(NF,NT,IPLOT,IPOINT,NREAD,TBU,XPRES,J6,LINES,
     1 FMTE,NPT,NORD,XSIZE,YSIZE)
      RETURN
       END
```