STUDY OF THE THREE-BODY NUCLEAR FORCE

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by

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SCOPE AND CONTENT:

Until now, the nuclear many body problem has been restricted almost entirely to considerations of the two-body force. However the meson theory of nuclear forces predicts that the exchange of mesons between three or more particles will give rise to a three-body or many-body force. The meson theory which has succesfully explained the main features of the phenomenological nucleon-nucleon potential, is expected to provide a good basis for the study of three-body nuclear forces. Three-body nuclear forces can occur among baryons such as N, Λ, Σ and Ξ. So far, however only bound states of nucleons (nuclei) and of nucleons and Λ (hypernuclei) have been observed experimentally. Hence only the three-nucleon force and the ΛNN force are considered. It will be seen that the ΛNN force plays a more important role than the three-nucleon force. Thus in the present work the ΛNN force will be studied in greater detail than the three-nucleon force. First the long and intermediate range parts of the ΛNN force are derived from meson theory. Their effects on the binding energies of ${}^{3}_{\Lambda}H$, ${}^{5}_{\Lambda}He$ and nuclear matter are then estimated.

Since the short range part of the force is not known, no definite conclusion can be drawn. However it is found that the three-body ΛNN force can play an important role in the nuclear structure problem. The effects of the three-nucleon force in ${}^{3}_{\Lambda}H$ and in nuclear matter are also briefly discussed.

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CONTENTS

Chapter		Page
1.	INTRODUCTION - THREE - BODY NUCLEAR FORCE	1
2.	DERIVATION OF THE LONG - RANGE TWO - PION - EXCHANGE (TPE) ΛNN FORCE	9
	2.1 Evaluation in the static approximation using Miyazawa formalism	9
	2.2 Comparison with the previously obtained ΛNN forces	16
3.	EFFECTS OF THE TPE ΛNN FORCE	19
	3.1 Effect in $^{3}_{\Lambda}H$	19
	3.2 Effect in $^{5}_{\Lambda}He$	24
	3.3 Effect in nuclear matter	28
4.	EVALUATION OF THE INTERMEDIATE RANGE THREE - PION - EXCHANGE ΛNN FORCE	38
5.	EFFECTS OF THE THREE - PION -EXCHANGE ΛNN FORCE	55
	5.1 Effect in $^{3}_{\Lambda}H$	55
	5.2 Effect in $^{5}_{\Lambda}He$	57
	5.3 Effect in nuclear matter	60
6.	DERIVATION AND EFFECTS OF THE LONG RANGE TPE THREE - NUCLEON FORCE	68
	6.1 Derivation of the three-nucleon (3N) force	68
	6.2 Effects of the TPE 3N force in triton	73
	6.3 Effect in nuclear matter	79
7.	DISCUSSION AND CONCLUSION	84

CONTENTS (Continued)

		Page
APPENDIX 1	DETAILS OF THE CALCULATION OF THE EFFECT OF THE TPE ΛNN FORCE ON $^5_\Lambda$He	90
APPENDIX 2	EXPRESSIONS NEEDED IN THE CALCULATION OF THE THREE - PION - EXCHANGE ΛNN FORCE	94
APPENDIX 3	CALCULATIONAL DETAILS ON THE EFFECT OF THE THREE - PION - EXCHANGE ΛNN FORCE ON $^5_\Lambda$He	96
APPENDIX 4	NUMERICAL CONSTANTS	99

LIST OF FIGURES

Figure		Page
2.1	The two-pion-exchange diagram for ΛNN force. The blob on the Λ-line contains all the possible processes.	10
2.2	The configuration of the three particles, ΛNN.	13
2.3	Typical time-ordered diagrams for the two-pion-exchange ΛNN force. The diagram of Fig.2.1 contains all of these.	17
3.1	The Λ-α potentials due to the TPE ΛNN force. $U_p^{(3)}(I)$, $U_p^{(3)}(II)$, and $U_s^{(3)}$ are defined by eqs.(3.19),(3.20) and (3.21),respectively. The cutoff radius is taken to be $d_{\Lambda N} = 1$ fm.	34
3.2	The Λ-α potential $U = U^{(2)} + U^{(3)}$, where $U^{(2)}$ and $U^{(3)}$ are the contributions from the two-body Λ-N force and the three-body TPE ΛNN force, respectively. $U^{(2)}$ is defined by eq.(3.14),and $U^{(3)} = U_p^{(3)} + U_s^{(3)}$. Here U and $U^{(2)}$ are shown . For $U^{(3)}$, the cutoff radius is taken to be $d_{\Lambda N} = 1$ fm.	35
3.3(a)	Effect of the TPE ΛNN potentials in nuclear matter for a Λ-N cutoff of 0.6 fm. The effect is plotted against d_{NN}, the cutoff for a step-function type N-N correlation. The full line gives the total contribution arising from $U_\Lambda(s)$,$U_\Lambda(I)$ and $U_\Lambda(II)$. The notation is explained in the text.	36
3.3(b)	Same as in Fig.3.3(a), but for Λ-N cutoff of 1.0 fm.	37

Figures		Page
4.1	The different three-pion-exchange diagrams for the ΛNN force (Fig.4.1(c) and 4.1(d)). The contribution of 4.1(d) is approximated by the sum of the contribution of 4.1(a) and 4.1(b). α, β and γ are the indices of the third component of isotopic spin and p, q, and k are the four-momenta of the pions.	53
4.2	Diagram which has to be subtracted once from the total contribution of 4.1(a) and 4.1(b).	54
4.3	Configuration of the three particles Λ, N_1 and N_2.	54
4.4	Diagram which is subtracted from diagrams 4.1(a) and 4.1(b) since it involves only two-body forces.	54
5.1	The Λ- N distance dependence of the central part of the three-pion-exchange ΛNN potential for ${}^{3}_{\Lambda}H$ (C_3) and for ${}^{5}_{\Lambda}He$ and nuclear matter (C_3'), is compared with that of the central part of the TPE ΛNN potential (curve C_2).	62
5.2	The Λ- N distance dependence of the tensor part of the three-pion-exchange ΛNN potential for ${}^{3}_{\Lambda}H$ (T_3) and for ${}^{5}_{\Lambda}He$ and nuclear matter (T_3'), is compared with that of the tensor part of the TPE ΛNN potential (curve T_2).	63
5.3	The Λ- α potentials arising from the three-pion-exchange ΛNN force (central part $P_c(r_\Lambda)$ and tensor part $P_T(r_\Lambda)$ are compared with those arising from the TPE ΛNN force (central p-wave part $U_p^3(I)$ and tensor p-wave part	

LIST OF FIGURES (Continued)

Figures		Page
	$U_p^3(II)$, for the cutoff distances $d_{\Lambda N} = 1$ fm and $d_{NN} = 0.6$ fm.	66
6.1	The two-pion-exchange diagram for the three-body 3-nucleon force.	68
6.2	Diagram for the three-body force due to the π-N scattering via the σ - meson.	69
6.3	The diagrams for the s-wave π-N scattering. The diagram (a) represents the ''indirect interaction'' via the σ- meson, and the diagram (b) stands for all other ''direct interaction''.	70

CHAPTER 1

INTRODUCTION - THREE-BODY NUCLEAR FORCES

One of the major trends in nuclear physics is an attempt to explain nuclear structure in terms of the two-body nuclear forces which are determined from nucleon-nucleon scattering data. This program relies on the hope that three-body and many-body forces are unimportant. On the other hand, from the meson theory of nuclear forces, where it is believed that forces are due to the exchange of mesons, it is expected that the exchange of mesons between three particles or more will give rise to a three-body or many-body force.

The meson theory of the two-body nucleon-nucleon (N-N) force has been extensively studied by many authors. The one-pion-exchange potential, OPEP, well established theoretically, gives a good description of the N-N interaction at large distances, distances greater than about 2 fm (Iwadare et al, 1956; Cziffra et al, 1959). For interaction distances less than about 2 fm, the potential due to the exchange of more mesons or heavier mesons has to be considered. Thus the two-pion-exchange potential, TPEP, has been calculated and is fairly well understood theoretically (Konuma et al, 1957; Cottingham and Vinh Mau, 1963). The potential due to the exchange of heavier mesons than π has also been considered, giving rise to the one-boson-exchange potentials, OBEP, (Arndt et al, 1966; Bryan and Scott, 1967). The OBEP, TPEP together with the OPEP reproduce the main features of the long and intermediate range part of the phenomenological N-N potential. The meson theory of nuclear forces has been extended to the interaction between various baryons N-Λ, N-Σ, Λ-Λ etc... However experimental data are as yet too meager to provide a proper comparison with theoretical predictions. Since meson theory gives a good account for the two-body nuclear interaction, in particular for the N-N force, it is expected that meson theory will provide a good basis for the study of the three-body nuclear forces. Accordingly the exchange of mesons can take place between three particles and can give rise to a ''meson theoretical'' three-body force.

Three-body nuclear forces can occur among baryons such as N, Λ, Σ and Ξ. So far, however, only bound states of nucleons (nuclei) and of nucleons and Λ (hypernuclei) have been observed experimentally. Thus the three-body nuclear forces which are expected to play a role in nuclear structure, are the three-nucleon force and the ΛNN force. As will be seen the three-body ΛNN force is stronger than the three-nucleon force. On the other hand the charge symmetry of the strong interaction forbids the exchange of one pion in the Λ-N interaction, thus the longest ranged two-body Λ-N force arises due to TPE and has a shorter range than the N-N force. Therefore the ΛNN force will be relatively more important than the three-nucleon force and then it will be studied in greater detail.

In principle, if two-body forces are completely known and if many-body problems can be solved exactly with the two-body interaction, then any discrepancy between the theoretical predictions and experimental data, can be claimed to be due to many-body forces. In practice, the two-body interaction is not very well known and even if it is well known, it is practically impossible to solve the many-body problems exactly.

Nevertheless there seems to be some evidence for such discrepancy, in particular in the case of the Λ-N interaction.

The best source of information about the Λ-N interaction has come so far from the phenomenological analyses of the binding energies for light hypernuclei, mainly s-shell hypernuclei. Recently Λ-N scattering experiments have provided more direct information. If there is no many-body force, these two approaches should lead to the same results. Extensive analyses, in terms of the binding energies of s-shell hypernuclei, lead to an s-state Λ-N potential with strong spin dependence: it is much more attractive in the spin singlet state than in the spin triplet state. These results have been well described by Dalitz (1966). The most direct information, derived from Λ-p scattering experiments, is now becoming available (Alexander et al,1966, 1968). The s-wave scattering lengths and effective ranges, determined from Alexander et al's experiments, are not very different for the singlet and triplet states, if the intrinsic range of the force is chosen to be around 2 fm or slightly greater (Alexander et al, 1966; Ali et al, 1967). This is contrary to the results of the previous analyses which utilised the binding energy calculations for light hypernuclei. This suggests a possible importance of the three-body ΛNN force and that the Λ-N force determined from the Λ-p scattering will not give the right binding energies.

It has been shown that a Λ-N potential, reproducing the data of Alexander et al, ''overbinds'' light hypernuclei, in particular ${}^{5}_{\Lambda}$He (Bhaduri et al, 1967; Herdon and Tang, 1967). Thus there is the possibility that a three-body repulsive ΛNN force could play an important role in these hypernuclei. It was first pointed out by Weitzner(1958) that for strong repulsive ΛNN forces, the binding energies of light hypernuclei could be accounted for with a two-body Λ-N force without strong spin dependence. Recently, the effects of ΛNN forces in s-shell hypernuclei, in connection with the new scattering data, have been discussed by Gal (1966). If such ΛNN forces exist, then the low-energy scattering parameters previously extracted from the binding energy data of s-shell hypernuclei, assuming only a two-body Λ-N force, are the result of some ''effective Λ-N interaction''. ''This effective Λ-N force'' cannot be directly compared with the free two-body Λ-N force.

On the other hand it is interesting to have an accurate estimate of the binding energy of a Λ in nuclear matter, since it is the same as the depth of the average one-body potential, U_Λ, in which it moves and this depth gives a measure of the strength of the ''effective Λ-N interaction''. It was shown by Ali et al (1967) that the Λ-N potential that fits the Λ-p scattering data overbinds the Λ in nuclear matter. Here again, one of the effects which may reduce the depth, U_Λ, in nuclear matter, may arise from the presence of a strong three-body ΛNN force.

At the present time, the three-body ΛNN force can only be calculated using a meson field theoretical approach. This method has already been applied to the determination of a ''theoretical'' Λ-N force by several authors (Nogami et al, 1964; Deloff and Wrezecinko, 1964; Rimpault and Vinh Mau, 1965). In such a calculation, the longest range part of the three-body ΛNN force, arising from TPE, has the same range as that of the TPE Λ-N force which has been shown to be important. The definition or the interpretation of the three-body force is not free from ambiguity. In fact it is still a matter of serious contraversy (Brown et al, 1968;

Mc Kellar and Rajaraman,1968). However in the present work the static meson theory is taken throughout.

The TPE ΛNN force is derived in the static approximation in chapter 2 and is compared with the previously obtained ΛNN forces. Only the p-wave and s-wave π- Λ interaction are considered here, higher partial waves being ignored. Then the ΛNN force, W, consists of two parts, arising from the p- and s-wave π-Λ interaction:

$$W = W_p + W_s \qquad \ldots(1.1)$$

For W_s, a ''suppression factor'' of the s-wave π-Λ interaction is introduced in analogy to the corresponding situation of the s-wave π-N interaction. It will be shown that for a reasonable suppression factor, W_s is unimportant. W_p consists of a central and a tensor term. The tensor term appears as a product of two tensor operators and depends on the angle between the two ΛN vectors. This term contributes significantly to the potential energy of the system.

Unfortunately, the ΛNN force, that is derived , is of a very singular nature at short distances and cannot be taken literally. In any case, in the short range region ($r_{\Lambda N} \lesssim 0.7$ fm) processes other than TPE will become important so that the contribution of TPE should not be taken alone. Therefore only the tail of the TPE potential is considered here and this potential is set equal to zero when the Λ-N distance is less than a cut-off distance, $d_{\Lambda N}$. For $d_{\Lambda N}$ greater than 1 fm only the TPE potential is expected to contribute significantly to the ΛNN force.

The contribution of this TPE ΛNN force to the binding energy of a Λ, in ${}^{3}_{\Lambda}H$, ${}^{5}_{\Lambda}He$ and in nuclear matter, is examined in chapter 3. The effect in ${}^{3}_{\Lambda}H$ is found to be relatively small and can be compensated by slightly changing the two-body Λ-N force. The Λ- α potential in $^{5}_{\Lambda}$He due to Wp is repulsive but its srength is sensitive to d $_{\Lambda N}$. The contribution from the tensor part of W_p is large but finite in the limit of d $_{\Lambda N} \to 0$. For a reasonable d $_{\Lambda N}$, the contribution of W_p is still quite substantial and used in conjunction with a Λ-N force that fits the scattering data of Alexander <u>et al</u>, gives a B_{Λ} for $^{5}_{\Lambda}$He much closer to the experimental value than that obtained by Bhaduri <u>et al</u> (1967). The effect on B_{Λ} in nuclear matter is considered using perturbation theory. It is found that the contribution from W_p, mainly repulsive, is dominant and is extremely sensitive to N-N correlation. However, as pointed out above, processes other than TPE, contributing to the intermediate range part of the ΛNN force, may modify these results appreciably.

The three-pion-exchange three-body ΛNN force may be important in the intermediate range part since it is the next lowest order contribution after TPE and since it has been shown that the TPE Λ-N force is important (Nogami <u>et al</u>, 1964; Deloff and Wrezecinko, 1964; Rimpault and Vinh Mau, 1965). In the case of the meson theory of the N-N interaction, it has been shown that the OPEP gives a good description of the N-N interaction at distances greater than 2 fm (Cziffra <u>et al</u>, 1959; Breit and Hull, 1960). For distances smaller than this, the OPEP is dominated by the TPEP. It is of interest therefore to investigate at what distance the TPE ΛNN force becomes dominated by the three-pion-exchange ΛNN force. It will be also possible to say to what extent the previous results, on the effect of the TPE three-body ΛNN force in light hypernuclei and in nuclear matter, are reliable.

The three-pion-exchange ΛNN force, P, is derived in the static approximation in chapter 4. In chapter 5 the effects of P on B_Λ in ${}^3_\Lambda H$, in ${}^5_\Lambda He$ and in nuclear matter are examined. In ${}^3_\Lambda H$ it is found that P depends strongly on the Λ-N distance and that the contribution of P to B_Λ in ${}^3_\Lambda H$ is attractive and larger than that of the TPE ΛNN force, W_p. For $d_{\Lambda N} \approx 1$ fm, the overall effect (P and W_p) is found to be attractive and relatively small. The Λ-α potential in ${}^5_\Lambda He$ due to P, $P(r_\Lambda)$, is evaluated. The potential $P(r_\Lambda)$ is found to be repulsive (except for $d_{\Lambda N} = 1$ fm and $r_\Lambda \approx 0$ fm) and its magnitude is sensitive to $d_{\Lambda N}$. It further reduces the binding energy of ${}^5_\Lambda He$ and for $d_{\Lambda N} \approx 1$ fm the effect of P is smaller than that of W_p. A first order perturbation calculation is done to estimate the contribution of P to B_Λ in nuclear matter. This contribution is found to be repulsive and its magnitude depends strongly on $d_{\Lambda N}$ and is larger than that of W_p. Even for $d_{\Lambda N} \approx 1$ fm, the overall effect (P and W_p) is found to be large and repulsive.

In the case of the N-N interaction, there are also discrepancies between theoretical predictions and experimental data when many-body problems are solved with the two-body interaction. Such a disparity occurs in triton, ${}^3H$, where many authors have evaluated the binding energy due to a two-body N-N potential that fits N-N scattering data. So far the value obtained is always smaller than the experimental value by an amount of the order of 1 Mev or more (Delves and Blatt, 1967; Davies, 1967). There is a similar situation in nuclear matter where the binding energy obtained using a two-body N-N force is smaller than the ''experimental value'' extrapolated from heavy nuclei by an amount of the order of 3 Mev (Brueckner and Masterson 1962; Bhargava and Sprung, 1967). So it is possible that the three nucleon force may also play an important role in the study of nuclear structure.

The long range part of the three nucleon force, arising from two-pion-exchange, is derived in chapter 6 and its effects on the binding energies of triton and nuclear matter are examined. The effect of the (controversial) I = J=0 dipion resonance (the σ-meson) is considered. The three-body potential consists of three parts :

$$F = F_p + F_s + F_\sigma \qquad \ldots(1.2)$$

where F_σ is due to the (virtual) π-N scattering via the σ-meson, while F_p and F_s are due respectively to the p-wave and s-wave π-N scattering via ''direct interactions''. The direct s-wave π-N interaction is set up so that, together with the π-N interaction via the σ-meson, it reproduces the observed s-wave π-N scattering length. Since the short range part of the three-nucleon force is not known, the three-body potential is taken to be zero for N-N distances less than a cut-off distance d. The triton wave function is taken from a variational calculation for a hard-core two-nucleon potential (hard-core radius D). The result is sensitive to d and also to the hard-core radius D. For instance, at D = 0.4 fm, the contribution to the binding energy can be as big as about 1 Mev (attractive). The effect in nuclear matter is evaluated by a first order perturbation theory. The contribution is found to be order of a few Mev, between -0.7 and 4.0 Mev, depending on the value of the cut-off d and on the N-N correlation function.

A discussion of the results is given in chapter 7.

Throughout the whole calculation, the units $\hbar$ = c = 1 are used.

CHAPTER 2

DERIVATION OF THE LONG-RANGE TWO-PION-EXCHANGE (TPE) ΛNN FORCE

2.1 Evaluation in the static approximation using Miyazawa formalism

There is an arbitrariness in the definition of the Λ-N and ΛNN potentials which is connected with the number of channels considered in the solution of the Shrödinger equation. For the two-body interaction if two channels are taken, Λ-N and Σ-N, then correspondingly the wave function has two components. The potential is then a 2×2 matrix :

$$V = \begin{bmatrix} V(\Lambda N \rightarrow \Lambda N) & V(\Sigma N \rightarrow \Lambda N) \\ V(\Lambda N \rightarrow \Sigma N) & V(\Sigma N \rightarrow \Sigma N) \end{bmatrix} \qquad \ldots(2.1)$$

As shown by Uehara (1960), the Σ-N component of the wave function can be eliminated and the one-channel formalism can be used. Then the Λ-N potential in the one-channel formalism which is of course different from $U(\Lambda N \rightarrow \Lambda N)$ in (2.1), is energy-dependent, but this energy-dependence is negligible if the energy of the system is well below the Σ-N threshold. The Λ-N and ΛNN potentials, in the two-channel formalism, have been discussed by Uehara (1960). Since only the bound state is considered it is sufficiently accurate to use the one-channel formalism. The two-pion-exchange ΛNN force,† arising from the diagram shown in Fig. 2.1, is derived in the static approximation where the nucleons and Λ are considered to be at rest.

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† Here the effect of the pion-pion intraction (π-Λ interaction via the σ-meson) has not been considered. It will be done for the three-nucleon force (chapter 6).

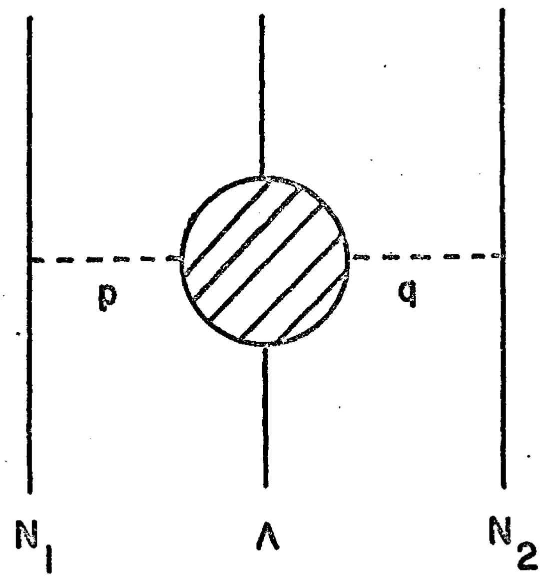


Fig. 2.I

Following Miyazawa (1956, 1957), the S matrix element corresponding to the diagram 2.1 can be written ($c = \hbar = 1$) :

$$S = - \frac{4\pi f_N^2 (\boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2)}{(2\pi)^6 \mu^2} \int d^3p \; d^3q \; \frac{\boldsymbol{\sigma}^1 \cdot \mathbf{p} \; \boldsymbol{\sigma}^2 \cdot \mathbf{q} \langle \mathbf{p} | S_\Lambda | \mathbf{q} \rangle}{(\mathbf{p}^2 + \mu^2)\;(\mathbf{q}^2 + \mu^2)} e^{-i(\mathbf{p}\cdot\mathbf{r}_1 - \mathbf{q}\cdot\mathbf{r}_2)} \qquad \ldots(2.2)$$

Here f_N is the pseudo-vector πNN coupling constant ($f_N^2 = 0.08$), $\mathbf{p}$ and $\mathbf{q}$ are the momenta of the exchanged pions, $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of the two nucleons and μ is the pion mass. The scattering matrix, $\langle \mathbf{p} | S_\Lambda | \mathbf{q} \rangle$, for the zero energy pion scattering by Λ , is given by

$$\langle \mathbf{p} | S_\Lambda | \mathbf{q} \rangle = 2\pi i \; \delta(0) \; (A_\Lambda(0) \; \boldsymbol{\sigma}^\Lambda \cdot \mathbf{p} \; \boldsymbol{\sigma}^\Lambda \cdot \mathbf{q} + C_\Lambda(0) \; \boldsymbol{\sigma}^\Lambda \cdot \mathbf{q} \; \boldsymbol{\sigma}^\Lambda \cdot \mathbf{p}$$

$$+ 2 D_{\Lambda}(0) \,) \, e^{i(\underset{\sim}{p} - \underset{\sim}{q}) \cdot \underset{\sim}{r}_{\Lambda}} \qquad \ldots(2.3)$$

where $\underset{\sim}{r}_{\Lambda}$ is the coordinate of Λ. The functions A_{Λ}, C_{Λ} and D_{Λ} are obtained by setting p_0 equal to zero in the dispersion relations for the p-wave and s-wave π- Λ scattering. From Nogami and Bloore (1964), they are given by :

$$A_{\Lambda}(p_0) = C_{\Lambda}(-p_0) = \frac{4\pi\,(f_{\Lambda}/\mu)^2}{\Delta + p_0 - i\epsilon} + \frac{1}{2\pi}\int_0^{\infty} \frac{dk \;\; \sigma_3(k)}{\omega_k\,(\omega_k - p_0 - i\epsilon)}$$

$$+ \frac{1}{6\pi}\int_0^{\infty} \frac{dk}{\omega_k} \; \frac{2\sigma_1(k) + \sigma_3(k)}{\omega_k + p_0 - i\epsilon} \;, \qquad \ldots(2.4)$$

$$D_{\Lambda}(p_0) = 2\pi\, a_{\Lambda} + \frac{p_0^2 - \mu^2}{2\pi}\int_0^{\infty} \frac{dk}{\omega_k}\,\sigma_s(k)\left(\frac{1}{\omega_k - p_0 - i\epsilon} + \frac{1}{\omega_k + p_0 - i\epsilon}\right) \qquad \ldots(2.5)$$

Here f_{Λ} is the $\pi\,\Lambda\,\Sigma$ pseudo-vector coupling constant, $\omega_k = (k^2 + \mu^2)^{1/2}$, $\Delta = m_{\Sigma} - m_{\Lambda}$ and σ_1 and σ_3 are the p-wave π- Λ scattering cross sections in the states with $J = 1/2$ and $3/2$ respectively. In the following, the p-wave π- Λ scattering is assumed to be dominated by the Y_1^* (1385) resonance in σ_3, and σ_1 is ignored. In (2.5), a_{Λ} is the scattering length and σ_s the cross section of the s-wave π- Λ scattering. If expression (2.3) is substituted in eq. (2.2) S is obtained in the form $S = -2\pi i\, \delta(0)\, W$. The quantity W, which is interpreted as the ΛNN potential arising from diagram 2.1, is given by :

$$W = W_p + W_s \;, \qquad \ldots(2.6)$$

$$W_p = -{}^1/_6 \; C_{p\Lambda} \overset{1}{\boldsymbol{\tau}} \cdot \overset{2}{\boldsymbol{\tau}} \left\{ \overset{1}{\boldsymbol{\sigma}} \cdot \overset{\Lambda}{\boldsymbol{\sigma}} + S_{1\Lambda}(x)\, T(x) \;, \right.$$

$$\left. \overset{2}{\boldsymbol{\sigma}} \cdot \overset{\Lambda}{\boldsymbol{\sigma}} + S_{2\Lambda}(y)\, T(y) \right\}_+ Y(x)\, Y(y), \qquad \ldots(2.7)$$

$$W_s = C_{s\Lambda} \overset{1}{\boldsymbol{\tau}} \cdot \overset{2}{\boldsymbol{\tau}} \; \overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{x} \; \overset{2}{\boldsymbol{\sigma}} \cdot \boldsymbol{y} \; (\mu x + 1) \; (\mu \; y + 1) \; Y(x) \; Y(y)/(\mu x y)^2 .$$

$$\ldots(2.8)$$

Here, $\{A, B\}_+ = AB + BA$, $\boldsymbol{x} = \boldsymbol{r}_1 - \boldsymbol{r}_\Lambda$, $\boldsymbol{y} = \boldsymbol{r}_2 - \boldsymbol{r}_\Lambda$ (see Fig. 2.2) and

$$C_{p\Lambda} = \frac{2(\mu f_N)^2}{3} \left(\frac{f_\Lambda^2}{\Delta} + \frac{\mu^2}{6\pi^2} \int_0^\infty dk \; \frac{\sigma_3(k)}{\omega_k^2} \right), \qquad \ldots(2.9)$$

$$C_{s\Lambda} = - \; (\mu f_N)^2 \; D_\Lambda(0) \; / \; (2\pi), \qquad \ldots(2.10)$$

$$S_{1\Lambda}(x) = 3 \; \frac{\overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{x} \; \overset{\Lambda}{\boldsymbol{\sigma}} \cdot \boldsymbol{x}}{x^2} - \overset{1}{\boldsymbol{\sigma}} \cdot \overset{\Lambda}{\boldsymbol{\sigma}} \; , \qquad \ldots(2.11)$$

$$T(x) = 1 + \frac{3}{\mu x} + \frac{3}{(\mu x)^2} \; , \quad Y(x) = \frac{e^{-\mu x}}{\mu x} \; . \qquad \ldots(2.12)$$

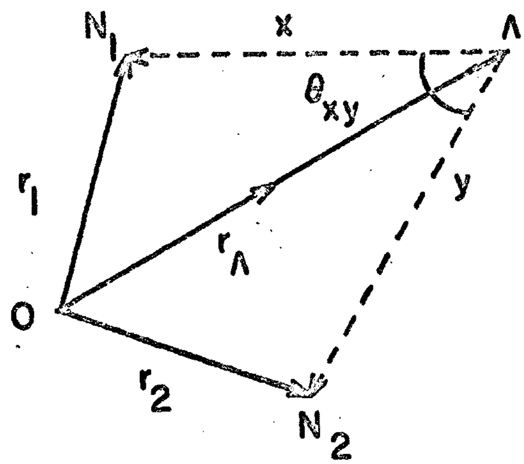


Fig. 2.2

In the following chapter, the potentials W_p and W_s will be considered in the case where the nucleons and Λ are in the s-shell. The expectation values of the τ 's and σ 's are denoted by brackets $\langle \quad \rangle$. The expectation value $\langle \boldsymbol{\sigma}^1 \cdot \mathbf{x} \; \boldsymbol{\sigma}^2 \cdot \mathbf{y} \rangle$ should be proportional to $\mathbf{x}\cdot\mathbf{y}$ for any two vectors $\mathbf{x}$ and $\mathbf{y}$. The constant of proportionality can be found by putting x = y and doing the angular integration. Therefore :

$$\langle \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2 \; \boldsymbol{\sigma}^1 \cdot \mathbf{x} \; \boldsymbol{\sigma}^2 \cdot \mathbf{y} \rangle = \frac{1}{3} \, \mathbf{x}\cdot \mathbf{y} \langle \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2 \; \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2 \rangle \qquad \ldots(2.13)$$

and using (2.13)

$$\langle \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2 \left\{ \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^\Lambda \, , \, S_{2\Lambda}(y) \right\}_+ \rangle = 0,$$

$$\langle \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2 \quad \left\{ S_{1\Lambda}(x),\ \underset{\sim}{\sigma}^2 \cdot \underset{\sim}{\sigma}^\Lambda \right\}_+ \rangle = 0 \qquad \ldots(2.14)$$

and

$$\langle \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2 \quad \left\{ S_{1\Lambda}(x),\ S_{2\Lambda}(y) \right\}_+ \rangle = 2\,(3\cos^2\theta_{xy} - 1)$$

$$\times \langle \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2 \quad \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\sigma}^2 \rangle \quad , \qquad \ldots(2.15)$$

where $\cos\theta_{xy} = \underset{\sim}{x}\cdot\underset{\sim}{y} / (xy)$. Thus, for the nucleons and Λ in the s-shell, W_p and W_s, are reduced to :

$$W_p = -\frac{C_{p\Lambda}}{3}\left[1 + (3\cos^2\theta_{xy} - 1)\,T(x)\,T(y)\right] \quad Y(x)\,Y(y) \langle \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2 \quad \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\sigma}^2 \rangle \qquad \ldots(2.16)$$

$$W_s = \frac{1}{3}\,C_{s\Lambda} \cos\theta_{xy} \quad (\mu x + 1)\,(\mu y + 1) \quad Y(x)\,Y(y) \,/\, (\mu^2 xy) \quad \langle \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2\ \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\sigma}^2 \rangle . \qquad \ldots(2.17)$$

When Λ is far from the nucleons, the angle θ_{xy} is small, hence W_p is repulsive and W_s is attractive (it will be seen that $\langle \quad \rangle = -3$).

The coefficient $C_{p\Lambda}$ and $C_{s\Lambda}$ are now evaluated. For the first term in $C_{p\Lambda}$ (2.9) various estimates of f_Λ^2 have been done indicating that f_Λ^2 is slightly smaller than f_N^2 (Martin and Wali, 1963; Raman, 1966; Kwan Kim, 1967; Cham and Meire, 1968). It is assumed here that $f_\Lambda^2 = f_N^2 = 0.08$, although this may be an overestimate of f_Λ^2 by a factor $\lesssim 2$.

As in Nogami and Bloore (1964), the second term in $C_{p\Lambda}$ has been evaluated assuming a Breit Wigner resonance formula for Y_1^* (1385). Thus it is found that :

$$C_{p\Lambda} = 1.43 \text{ Mev} \qquad \ldots(2.18)$$

The first term in (2.9) constitutes 73% of $C_{p\Lambda}$, while the rest comes from Y_1^*. If the value $f_\Lambda^2 = 0.04$ is taken, $C_{p\Lambda}$ is found to be equal to 0.89 Mev. The true value of $C_{p\Lambda}$ will be between 1.43 and 0.89 Mev.

If the two-channel formalism is used, the first term in (2.9) should be dropped, because it is interpreted as a repetition of the OPE two-body potential (Nogami and Bloore, 1964). In other words, the ΛNN force in the one-channel formalism contains effects of two-body forces in the two-channel formalism. The potential W_p is much stronger than the TPE three-nucleon force, derived, for instance, by Fujita and Miyazawa (1957), which corresponds to the ΛNN force in the two-channel rather than the one channel formalism.

For $C_{s\Lambda}$, since a_Λ and σ_s are not known experimentally, theoretical estimates are necessary. The lowest order perturbation theory gives $D_\Lambda(0) \approx 2\pi a_\Lambda$, $a_\Lambda = -2 (f_\Lambda/\mu)^2 \times (m_\Sigma + m_\Lambda)$, which results in $C_{s\Lambda} \approx -(\mu f_N)^2 a_\Lambda = 29.5$ Mev. However, such a simple perturbation calculation of a_Λ is very poor approximation. There is a corresponding problem in the three-nucleon force (Fujita and Miyazawa, 1957; Quang-Ho-Kim, 1966). There , a_Λ is replace by $(a_1 + 2a_3)/3$, where a_1 and a_3 are the s-wave π-N scattering lengths in the I = 1/2 and 3/2 states, respectively. According to Hamilton and Woolcock (1963) experimental values are $a_1 = (0.171 + 0.005)/\mu$ and $a_3 = (-0.088 \pm 0.004)/\mu$. Hence

$$0 \leqslant -(a_1 + 2a_3)/3 \leqslant 0.0\,1/\mu.$$

On the other hand, it has been shown that for the soft pion (pion of zero four momentum) $a_1 + 2a_3 = 0$ (Weinberg 1966). However, the lowest-order perturbation calculation gives $-(a_1 + 2a_3)/3 = 4 m_N (f_N/\mu)^2 = 2.12/\mu$. This is an overestimate by a factor of 200 or more. Various mechanisms have been considered for this ''pair suppression'' (Amati and Fubini, 1962). These mechanisms all seem to be applicable also to the s-wave π-Λ interaction. Thus a suppression factor of the same order of magnitude is expected there. This factor is tentatively assumed to be 100, namely $a_\Lambda = -2(f_\Lambda/\mu)^2 (m_\Sigma + m_\Lambda)/100$. It has been also shown that for the soft pion $a_\Lambda = 0$ (Weinberg, 1966; Tomozawa, 1966). In analogy with the π-N case the integral containing σ_s in (2.5) is assumed to be negligible. Then, it is found that :

$$C_{s\Lambda} = 0.30 \text{ Mev} \qquad \ldots(2.19)$$

As will be seen in the following chapter, results will not be altered even when $C_{s\Lambda}$ is as large as 3 Mev.

2.2 Comparison with the previously obtained ΛNN forces.

The ΛNN potential, obtained in the previous section, will be compared with those so far derived. The TPE ΛNN force has been evaluated, in various approximation, by Weitzner (1958), Spitzer (1958), Bach (1959), Uehara (1960), Chalk and Downs (1963). Weitzner' s and Spitzer's results differ from all the later works and it is not easy to retrace their calculation.

In Bach's work, the ΛNN force is calculated in the static approximation by perturbation theory using time-ordered diagrams. There are three types of diagrams, as shown in Fig. 2.3, which are called NB, B and SB.

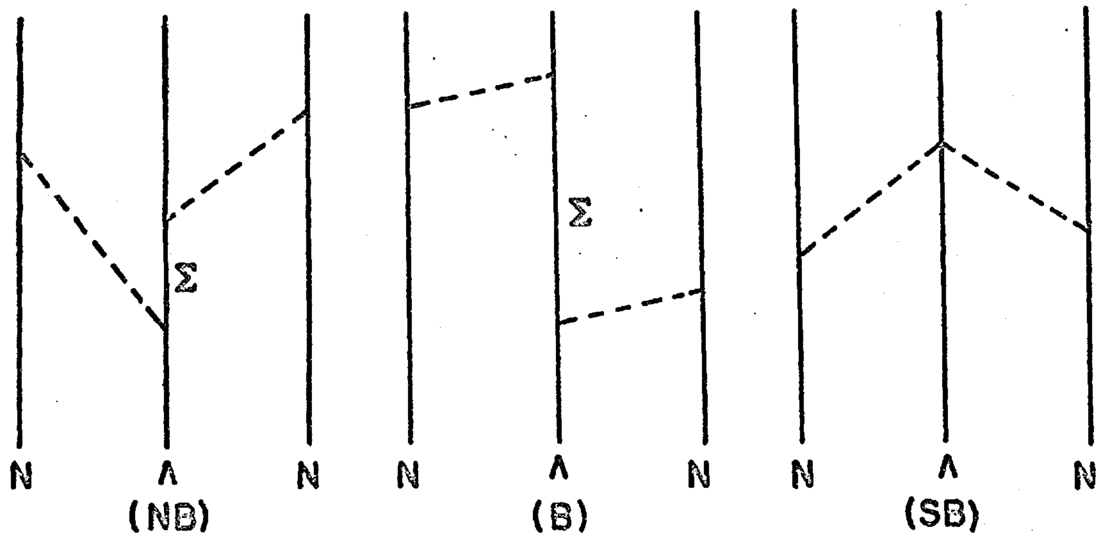


Fig.2.3

The diagrams NB and B are due to the p-wave and SB is due to the s-wave π- Λ interaction. The ΛNN potentials, arising from these diagrams, are denoted by V_{NB}, V_B and V_{SB}, respectively. The mass difference, $\Delta = m_\Sigma - m_\Lambda$, is neglected in the energy denominators except in the second intermediate state of B. However, if Δ is not neglected and if the sum of the contributions from all the 16 NB diagrams and 8 B - diagrams is done, the potential that results is the same as W_p but without the contribution from Y_1^*. This result was not obtained in Bach's work, perhaps because V_{NB} and V_B were calculated separately. The diagram of Fig. 2.1 is not time-ordered and contains both NB and B. The same calculation was repeated by Chalk and Downs and Bach's V_{NB} and V_B were confirmed. The value of the coupling constant f_Λ^2, taken by Bach, is slightly smaller than that of f_Λ^2

used here. Incidentally Bach's results have been misquoted by Dalitz (1965) and by Gal (1966). The central part of Bach's V_B is quoted as ~ 2 Mev $\times$ Y(x) Y(y), which is actually 2 times Bach's original V_B.

Gal's strength parameter C_G, corresponding to $C_{p\Lambda}$, is taken equal to 17 Mev which is much stronger than the estimate of $C_{p\Lambda}$ done here. It is argued that the contribution from the Y_1^* - intermediate process may considerably modify the value of C_G. However, $C_{p\Lambda}$ includes contribution from Y_1^*. Also in Gal's work only the central part of W_p is considered and it will be shown, in the next chapter, that the effect of the tensor part of W_p dominates over that of the central part. Uehara's ΛNN potential is for the two-channel formalism and is obtained from W_p by dropping the first term in $C_{p\Lambda}$ (2.9).

For the s-wave diagram SB, as was noted by Chalk and Downs, Bach's V_{SB} has a wrong factor. The diagram SB was calculated by Chalk and Downs by perturbation theory, firstly in static approximation and then in a relativistic way. Their result in the static approximation agrees with W_s, without the suppression factor. Their relativistic calculation shows that the recoil effect is unimportant for the long range part of the ΛNN potential. Since the suppression factor was not introduced their potential is much stronger than W_s.

CHAPTER 3

EFFECTS OF THE TPE ΛNN FORCE

3.1 Effect in ${}^3_\Lambda H$

The effects of the potentials W_p (2.16) and W_s (2.17) on B_Λ in ${}^3_\Lambda H$ are estimated by perturbation theory. The unperturbed wave function is taken to be the wave function obtained by Downs, Smith and Truong (1963) from a variational calculation of the binding energy of ${}^3_\Lambda H$.

It has the form :

$$\Psi = N^{-1/2} f(x)\, f(y)\, g(z)\, \xi \chi \qquad \ldots(3.1)$$

where $\underset{\sim}{x} = \underset{\sim}{r}_1 - \underset{\sim}{r}_\Lambda$, $\underset{\sim}{y} = \underset{\sim}{r}_2 - \underset{\sim}{r}_\Lambda$, $\underset{\sim}{z} = \underset{\sim}{r}_1 - \underset{\sim}{r}_2$ and

$$\begin{aligned} f(r) &= 0 && \text{for } r < D \\ &= \exp\left[-\alpha(r-D)\right] - \exp\left[-\beta(r-D)\right] && \text{for } r > D, \end{aligned} \qquad \ldots(3.2)$$

$$\begin{aligned} g(r) &= 0 && \text{for } r < D, \\ &= \exp\left[-\gamma(r-D)\right] - \exp\left[-\delta(r-D)\right] && \text{for } r > D. \end{aligned} \qquad \ldots(3.3)$$

Here D is the hard-core radius of the Λ-N and N-N forces and the factor $N^{-1/2}$ normalizes Ψ to unity. The function ξ is the isospin singlet wave function for the two nucleons and χ is the spin wavefunction of ${}^3_\Lambda H$. The optimum variational parameters, obtained by Downs et al (1963), are listed in table 3.1 for the hard-core radii D = 0.2, 0.4 and 0.6 fm together with the corresponding normalization factors .†

TABLE 3.1

Parameters for the wavefunction, with the corresponding normalization factors N, for three different hard-core radii D. (Downs et al, 1963)

D (fm)	α (fm^{-1})	β (fm^{-1})	γ (fm^{-1})	δ (fm^{-1})	N (fm^{6})
0.2	0.297	7.36	0.547	6.52	3.825
0.4	0.327	6.94	0.578	4.55	3.658
0.6	0.389	11.28	0.606	4.79	2.928

The expectation values of W_p (2.16) and W_s (2.17) with respect to Ψ (3.1) are denoted by $< W_p >$ and $< W_s >$ respectively. The expectation value of the τ's and σ's is :

$$<\underset{\sim}{\tau}^{1} \cdot \underset{\sim}{\tau}^{2} \quad \underset{\sim}{\sigma}^{1} \cdot \underset{\sim}{\sigma}^{2}> \quad = -3 \qquad \ldots(3.4)$$

† The normalization factor N listed in table 3.1 differs from Chalk and Downs'N (listed in table 1 in Chalk and Downs (1963)) by a factor $8\pi^2$. Their wavefunction is normalized as :

$$8\pi^2 \int dx\, dy\, dz\, xyz\, \Psi(x,y,z,)^2 = 1$$

whereas here the factor $8\pi^2$ of the volume element is dropped throughout.

Then $\langle W_p \rangle$ and $\langle W_s \rangle$ can be written :

$$\langle W_p \rangle = \langle W_p (I) \rangle + \langle W_p (II) \rangle \qquad ...(3.5)$$

with

$$\langle W_p (I) \rangle = C_{p\Lambda} \; I \left[Y(x)\, Y(y) \right], \qquad ...(3.6)$$

$$\langle W_p (II) \rangle = C_{p\Lambda} \; I \left[(3 \cos^2 \theta_{xy} - 1)\, T(x)\, T(y)\, Y(x)\, Y(y) \right] \qquad ...(3.7)$$

and

$$\langle W_s \rangle = - C_{s\Lambda} \; I \left[\cos \theta_{xy} \, (\mu x + 1)\, (\mu y + 1)\, Y(x)\, Y(y) / (\mu xy)^2 \right] \qquad ...(3.8)$$

where

$$I [\cdot \cdot \cdot] = N^{-1} \int dx\, dy\, dz\, xyz \, \left\{ f(x)\, f(y)\, g(z) \right\}^2 [\cdot \cdot \cdot] \qquad ...(3.9)$$

The integration domain of eq.(3.9) is such that x, y vary from $d_{\Lambda N}$ (cutoff of the ΛNN potential) to infinity and z takes values from D to infinity (assuming $d_{\Lambda N} \geqslant D$), subject to the triangular inequalities :

$$x + y \geqslant z, \; y + z \geqslant x, \text{ and } z + x \geqslant y.$$

The integral (3.6) has been done analytically and for the integrals (3.7) and (3.8) first the z integration was done analytically and then the xy-integration numerically.

The results are shown in table 3.2 for five different values of the cutoff $d_{\Lambda N}$ and three different values of the hard-core radius D. It is found that $\langle W_s \rangle$ is always negative (attractive) and smaller than $\langle W_p (I) \rangle$ and $\langle W_p (II) \rangle$. The ''central part'' $\langle W_p (I) \rangle$ is always positive (repulsive). The tensor part $\langle W_p (II) \rangle$ is predominant and changes sign depending on $d_{\Lambda N}$ and D.

TABLE 3.2

EXPECTATION VALUES OF THE DIFFERENT PARTS OF THE ΛNN POTENTIAL IN MEV FOR DIFFERENT CUTOFF $d_{\Lambda N}$ AND HARD-CORE RADIUS D

	$d_{\Lambda N}$ (fm)	0.2	0.4	0.6	1.	1.4
$\langle W_s \rangle$ in Mev	D = 0.2 fm	- 0.023	- 0.022	- 0.018	- 0.009	- 0.004
	D = 0.4 fm		- 0.015	- 0.014	- 0.009	- 0.004
	D = 0.6 fm			- 0.014	- 0.010	- 0.004
$\langle W_p \rangle$ in Mev	D = 0.2 fm $\langle W_p(I) \rangle$	0.091	0.084	0.064	0.030	0.011
	D = 0.2 fm $\langle W_p(II) \rangle$	- 0.726	- 0.231	0.354	0.297	0.118
	D = 0.4 fm $\langle W_p(I) \rangle$		0.074	0.068	0.034	0.012
	D = 0.4 fm $\langle W_p(II) \rangle$		- 0.700	- 0.432	0.130	0.090
	D = 0.6 fm $\langle W_p(I) \rangle$			0.084	0.044	0.015
	D = 0.6 fm $\langle W_p(II) \rangle$			- 1.010	- 0.075	0.074

According to Chalk and Downs (1963) the contribution of the two-body Λ-N force to the binding energy of ${}^{3}_{\Lambda}H$ is given by :

$$2 \langle V_{\Lambda N} \rangle = \begin{cases} 24.2 \text{ Mev} \\ 70.8 \text{ Mev} \end{cases} \text{ for } D = \begin{cases} 0.4 \text{ fm} \\ 0.6 \text{ fm} \end{cases} \qquad \ldots(3.10)$$

The relative importance of the ΛNN force will be given by the ratio :

$$R = \frac{\langle W_p + W_s \rangle}{2 \langle V_{\Lambda N} \rangle} = \begin{cases} 2.6\% \\ 1.3\% \end{cases} \text{ for } D = d_{\Lambda N} = \begin{cases} 0.4 \text{ fm} \\ 0.6 \text{ fm} \end{cases} \qquad \ldots(3.11)$$

This small ratio justifies the use of a perturbation calculation to estimate the effect of the ΛNN force.

In view of the small binding energy of Λ in ${}^{3}_{\Lambda}H$ ($B = 0.21 \pm 0.20$ Mev) (Gajeroski et al,1967) the absolute value of $(W_p + W_s)$ can be quite substantial depending on $d_{\Lambda N}$ and D. As was discussed in the introduction, however, a reasonable value of the cutoff radius $d_{\Lambda N}$ may be taken as 1 fm. This follows from the fact that the TPE ΛNN potential, W, is not very meaningful at short distances where processes other than TPE are likely to be important. On the other hand, if the same core radii for Λ-N and N -N are assumed, it will be reasonable to take $D \approx 0.4$ fm. Then it is found that $\langle W_p + W_s \rangle = 0.155$ Mev for $d_{\Lambda N} = 1$ fm and $D = 0.4$ fm. Thus the effect is small, and can probably be compensated for by a slight change in the Λ-N force.

Finally, the results are compared with those of previous works. The expectation value of V_B(§ 2.2) which approximately corresponds to W_p, has been evaluated by Bach (1959) who found that the ratio $\langle V_B \rangle / (2 \langle V_{\Lambda N} \rangle) = 2\,\%$ for a cutoff radius of 0.55 fm. However the wavefunction which was used in Bach's work does not satisfy proper boundary conditions. Bach's calculation has been refined in the work of Abou-Hadid (1962) where a proper hard-core wavefunction was used. Abou-Hadid's conclusion essentially agrees with Bach's. It was concluded, as in the present work, that the effect of the ΛNN force in ${}^{3}_{\Lambda}H$ is small. The expectation value $\langle W_s \rangle$ has been evaluated by Chalk and Downs (1963). Since no suppression factor was introduced, their $\langle W_s \rangle$ is about 100 times as large as the $\langle W_s \rangle$ obtained here. Still their $\langle W_s \rangle$ is at most 5 % of the expectation value of the two-body Λ-N force. The expectation value $\langle W_p \rangle$ has not been evaluated.

3.2 Effect in $^5_\Lambda He$

The calculation of B_Λ in $^5_\Lambda He$, using only a two-body s-state Λ-N potential that fits the low-energy data of Alexander et al (1966, 1968), is first briefly described. The details of the calculation can be found in the work of Bhaduri et al (1967). The low-energy parameters of the Λ-N potential from Λ-p scattering data of Alexander et al are :

$$a_s = -2.46 \text{ fm}, \; a_t = -2.07 \text{ fm}, \; r_s = 3.87 \text{ fm}, \; r_t = 4.50 \text{ fm}. \qquad \ldots(3.12)$$

These are the most probable values. In order to bypass the construction of a complete hard-core (or soft-core) potential which makes binding-energy calculations rather cumbersome, the following form of the Λ-N potential is chosen :

$$\begin{aligned} V_{s,t}(r) &= 0 && \text{for } r < d_{s,t} . \\ &= -A_{s,t}\, e^{-\nu r}/\nu r && \text{for } r > d_{s,t} . \end{aligned} \qquad \ldots(3.13a)$$

Here the subscripts s,t stand for singlet and triplet spin states respectively. The range parameter is taken as $\nu = 1.3992 \text{ fm}^{-1}$, corresponding to TPE, while

$$d_s = 1.017 \text{ fm}, \; A_s = 204.1 \text{ Mev}, \; d_t = 1.180 \text{ fm}, \; A_t = 223.3 \text{ Mev} \qquad \ldots(3.13b)$$

are determined by fitting parameters (3.12). Justification for choosing such a form for the Λ-N potential is given in the work of Bhaduri et al (1967). Since this Λ-N potential consists of a fairly weak attractive tail, the one-body average field that the Λ experiences can be obtained by folding in the nucleon density distribution with this potential :

$$U^{(2)}(r_\Lambda) = \int \rho(r_i) \quad V(|\underset{\sim}{r}_i - \underset{\sim}{r}_\Lambda|)\, d^3 r_i \qquad \ldots(3.14)$$

where $V = V_s + 3\, V_t$, $\rho\,(r_i)$ is the density distribution of the nucleons in

α and all the vectors are measured from the center of α. The superscript (2) on U refers to the fact that this part of the average field originates from a two-body Λ-N force. The complete U should also contain contributions from three-body ΛNN forces. The normalized density distribution for the nucleons is taken as :

$$\rho(r_i) = (\beta/\pi^{1/2})^3 \exp(-\beta^2 r_i^2) \qquad \ldots(3.15)$$

where $\beta = 0.85056\ \mathrm{fm}^{-1}$. In this calculation the α is assumed to be undistorted in the presence of the Λ. The binding, B_Λ, due to $U^2(r_\Lambda)$ was found to be 6.45 Mev (see Table 3.3), whereas the experimental value is 3.08 Mev (Gajeroski et al, 1967).

In the presence of ΛNN forces W_p(2.16) and W_s(2.17), the average field $U(r_\Lambda)$ will be modified. The average fields generated by W_p and W_s are denoted by $U_p^{(3)}(r_\Lambda)$ and $U_s^{(3)}(r_\Lambda)$, respectively. Then

$$U_p^{(3)}(r_\Lambda) = 6\, C_{p\Lambda} \int d^3r_1\, d^3r_2\, \rho(r_1)\, \rho(r_2)\, W_p(\underset{\sim}{r}_1 - \underset{\sim}{r}_\Lambda, \underset{\sim}{r}_2 - \underset{\sim}{r}_\Lambda), \qquad \ldots(3.16)$$

where the factor 6 comes from the six possible ΛNN bonds and all the other quantities have been defined in chapter 2. The expectation value of the expression in τ's and σ's in W_p(3.16) can be shown to be identical for ${}^5_\Lambda$He and ${}^3_\Lambda$H. Thus the value given in (3.4) is used in W_p(3.16). Putting $\underset{\sim}{x} = \underset{\sim}{r}_1 - \underset{\sim}{r}_\Lambda$, $\underset{\sim}{y} = \underset{\sim}{r}_2 - \underset{\sim}{r}_\Lambda$ (see Fig. 2.2), eq. (3.16) can be written :

$$U_p^{(3)}(r_\Lambda) = 6\, C_{p\Lambda} \int d^3x\, d^3y\, \rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x}|)\rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{y}|)\ W_p(x, y) \qquad \ldots(3.17)$$

It is shown, in Appendix 1, that the $\underset{\sim}{x}$ and $\underset{\sim}{y}$ integration can be separated to yield :

$$U_p^{(3)} = U_p^{(3)}(I) + U_p^{(3)}(II) \qquad \ldots(3.18)$$

where :

$$U_p^{(3)}(I) = 6\, C_{p\Lambda} \left\{ \int d^3x\, \rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x}|)\ \ Y(x) \right\}^2 , \qquad \ldots(3.19)$$

$$U_p^{(3)}(II) = 3\, C_{p\Lambda} \left\{ \int d^3x\, \rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x}|)\ \ (3\cos^2\theta_x - 1)\, Y(x)\ T(x) \right\}^2 \qquad \ldots(3.20)$$

with $\cos\theta_x = (\underset{\sim}{x}.\underset{\sim}{r}_\Lambda)/(xr_\Lambda)$. Only $U_p^{(3)}(I)$ has been considered previously by other authors (Dalitz, 1965; Gal, 1966). However $U_p^{(3)}(II)$ is the dominant term in eq. (3.18), as can be seen from Fig 3.1. It is also seen from (3.19) and (3.20) that $U_p^{(3)}$ is always repulsive in character. A similar analysis for W_s, the ΛNN force arising from the s-wave π-interaction, yields

$$U_s^{(3)}(r_\Lambda) = -\,6\, C_{s\Lambda} \left\{ \int d^3x\, \rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x}|)\ \cos\theta_x\, (1 + \mu x)\ Y(x)/(\mu x) \right\}^2$$

$$\ldots(3.21)$$

which is always attractive. In the Appendix 1, it is shown that the angular interactions in $\underset{\sim}{x}$ can easily be done in (3.20) and (3.21) and the problem reduces to the numerical integration of one-dimensional integrals to give $U_p^{(3)}$ and $U_s^{(3)}$. With $C_{s\Lambda} = 0.30$ Mev, $U_s^{(3)}$ turns out to be completely negligible compared to $U_p^{(3)}$. The situation would not change appreciably even if $C_{s\Lambda}$ were ten times larger. In Fig. 3.1, $U_p^{(3)}(I)$, $U_p^{(3)}(II)$ and $U_s^{(3)}$ are plotted for a cutoff $d_{\Lambda N} = 1$ fm. In Fig. 3.2, $U^{(2)}$ and $U = U^{(2)} + U_p^{(3)} + U_s^{(3)}$ are plotted for the same scale to show how the average field is modified by ΛNN forces.

TABLE 3.3

CALCULATED VALUES OF B_Λ IN ${}^5_\Lambda He$ FOR VARIOUS CUTOFF DISTANCES $d_{\Lambda N}$ OF THE ΛNN POTENTIAL. THE NOTATION IS EXPLAINED IN SECTION 3.2. THE EXPERIMENTAL VALUE OF B_Λ IS 3.08 MEV. (GAJEROSKI ET AL, 1967)

B_Λ in ${}^5_\Lambda He$ with $U^{(2)}$ only = 6.45 Mev			
B_Λ (in Mev) when three-body forces are included			
Cutoff $d_{\Lambda N}$ (fm)	0.6	1.0	1.4
$U^{(2)} + U_p^{(3)}(I)$	6.01	6.23	6.36
$U^{(2)} + U_p^{(3)}(I) + U_p^{(3)}(II)$	2.99	4.35	5.46
$U^{(2)} + U_p^{(2)} + U_s^{(3)}$	3.09	4.42	5.50

In table 3.3, the binding energy B_Λ in ${}^5_\Lambda He$ is tabulated with and without ΛNN forces for various cutoffs. $U^{(2)}$ is calculated with the parameters given in (3.13 b) and shows considerable overbinding. It is seen that this overbinding is drastically reduced when $U_p^{(3)}(II)$ is included in the calculation. This repulsive effect is probably overestimated since N-N correlations in the α-particle are neglected. It is quite clear from table 3.3 that three-body forces cannot be ignored in the calculation of B_Λ in ${}^5_\Lambda He$. An effective Λ-N force that is extracted purely from an analysis of ${}^3_\Lambda H$ and ${}^5_\Lambda He$ bindings, would therefore be much less attractive in the triplet s-state than the free Λ-N interaction .

3.3 Effect in nuclear matter

The binding energy $U_\Lambda^{(3)}$ of a Λ in nuclear matter due to the ΛNN force W is calculated by first-order perturbation theory in a way similar to that formulated by Bodmer and Sampanthar (1962). The wavefunction of nuclear matter in the Fermi-gas model is :

$$\Psi_{kA} = \Psi_k(\Lambda)\,\phi_A(1,2,\ldots,A) \qquad \ldots(3.22)$$

where A is the number of nucleons ($A \rightarrow \infty$ for nuclear matter) and

$$\Psi_k(\Lambda) = \frac{1}{\sqrt{\Omega}}\, e^{i\,\underset{\sim}{k}\cdot\underset{\sim}{r}_\Lambda}\, \chi_\Lambda \qquad \ldots(3.23)$$

$$\phi_A = \frac{1}{\sqrt{A!}}\ \mathrm{Det}\left[\ \phi_p(i)\ \right] , \qquad \ldots(3.24)$$

$$\phi_p(i) = \frac{1}{\sqrt{\Omega}}\, e^{i\,\underset{\sim}{p}\cdot\underset{\sim}{r}_i} \times \text{spin function} \times \text{isospin function} \qquad \ldots(3.25)$$

The function χ_Λ represents the spin state of Λ and $\phi_p(i)$ is the wave function of the i-th nucleon in the state p where p represents the momentum, the spin and isospin state of the nucleon. The quantity Ω is the volume of nuclear matter, $\Omega \rightarrow \infty$ with A and $A/\Omega = \rho$ where ρ is the density of nuclear matter. The expectation value of the potential $\sum_{i<j} W(i,j,\Lambda)$ is

$$U_{\Lambda}^{(3)} = \frac{1}{2} \sum_{i,j}^{A} \int \Psi_k(\Lambda)\, \phi_{p_i}(1)\, \phi_{p_j}(2)\, W(1, 2, \Lambda)$$

$$\times (\phi_{p_i}(1)\, \phi_{p_j}(2) - \phi_{p_i}(2)\, \phi_{p_j}(1))\; d^3r_1\, d^3r_2\, d^3r_{\Lambda} \;. \qquad \ldots(3.26)$$

The binding energy $U_{\Lambda}^{(3)}$ consists of two contributions : a direct term and an exchange term. The direct term vanishes identically because of the spin-isospin saturation. The expectation value of the τ's and σ's, for the exchange term, is :

$$\langle \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2\; \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\sigma}^2 \rangle = 36 \qquad \ldots(3.27)$$

The evaluation of the spatial part of $U_{\Lambda}^{(3)}$ will now be considered. It can be shown that the function $K(z)$ defined by :

$$K(z) = K(|\underset{\sim}{r}_1 - \underset{\sim}{r}_2|) = \frac{1}{2\Omega^2} \sum_{i,j}^{A} e^{-i\underset{\sim}{p}_i \cdot \underset{\sim}{r}_1}\, e^{-i\underset{\sim}{p} \cdot \underset{\sim}{r}_2}\, e^{i\underset{\sim}{p}_i \cdot \underset{\sim}{r}_2}\, e^{i\underset{\sim}{p} \cdot \underset{\sim}{r}_1} \qquad \ldots(3.28)$$

gives when $A \rightarrow \infty$

$$K(z) = \frac{\rho^2}{32}\, D^2(k_f z). \qquad \ldots(3.29)$$

Here $\rho = 0.170\ \mathrm{fm}^{-3}$ (equilibrium density of nucleon matter), k_f is the fermi momentum and

$$D(k_f z) = \frac{3j_1(k_f z)}{k_f z}, j_1(r) = \frac{\sin r}{r^2} - \frac{\cos r}{r}.$$

Then $U_\Lambda^{(3)}$ can be written

$$U_\Lambda^{(3)} = U_\Lambda(I) + U_\Lambda(II) + U_\Lambda(s)$$

where $U_\Lambda(I)$ is the contribution from $W_p(I)$, $U_\Lambda(II)$ originates from $W_p(II)$ and $U_\Lambda(s)$ from W_s. From eqs. (3.26) to (3.29) it can be seen that :

$$U_\Lambda(I) = \frac{3}{8} C_{p\Lambda} \rho^2 \int D^2(k_f |\underset{\sim}{x} - \underset{\sim}{y}|)\, Y(x)\, Y(y)\, d^3x\, d^3y, \qquad \ldots(3.30)$$

$$U_\Lambda(II) = \frac{3}{8} C_{p\Lambda} \rho^2 \int D^2(k_f |\underset{\sim}{x} - \underset{\sim}{y}|)\, (3 \cos^2\theta_{xy} - 1)\, Y(x)\, Y(y)\, T(x)\, T(y)\, d^3x\, d^3y$$

$$\ldots(3.31)$$

$$U_\Lambda(s) = -\frac{3}{8} C_{s\Lambda} \rho^2 \int D^2(k_f |\underset{\sim}{x} - \underset{\sim}{y}|) \cos\theta_{xy} \left(1 + \frac{1}{\mu x}\right)\left(1 + \frac{1}{\mu y}\right) Y(x)\, Y(y)\, d^3x\, d^3y.$$

$$\ldots(3.32)$$

Here $\cos\theta_{xy} = \underset{\sim}{x}\cdot\underset{\sim}{y}/(xy)$ which is shown in Fig. 2.2, the variables x and y are the Λ-N distances, and $|\underset{\sim}{x} - \underset{\sim}{y}|$ is the N-N distance. All the integrands, including that in (3.31), are well-behaved even for the limit $x, y \to 0$. An inspection of $U_\Lambda(II)$ in eq. (3.31) now reveals why it is so sensitive to the N-N correlation. The function $(3 \cos^2\theta_{xy} - 1)$ is positive for $\theta_{xy} = 0$ to 55° and then turns negative. Small values of θ_{xy} correspond to the case when the two nucleons are relatively close to each other, and this is the part most affected by the N-N correlation function. The potential $U_\Lambda(II)$

is not positive definite, unlike U_{Λ} (I), and tends to turn negative if the N-N correlation is strongly repulsive, thereby excluding smaller values of θ_{xy}. In order to study the effect of N-N correlation, a step-function has been used :

$$\theta(|\underset{\sim}{x} - \underset{\sim}{y}| - d_{NN}) = 0 \quad \text{for } |\underset{\sim}{x} - \underset{\sim}{y}| < d_{NN}$$

$$= 1 \quad \text{for } |\underset{\sim}{x} - \underset{\sim}{y}| > d_{NN} \qquad \ldots(3.33)$$

Here it is not worthwhile to use a more sophisticated correlation function because :

(a) It is not known sufficiently well. In particular, it depends sensitively on the form of the N-N potential chosen, whether it contains a hard-core or a soft-core, and if it is state-dependent.

(b) There is already considerable uncertainty in W due to the Λ-N cutoff $d_{\Lambda N}$.

The integrals (3.30), (3.31) and (3.32) are evaluated numerically after putting the N-N correlation function $\theta(|\underset{\sim}{x} - \underset{\sim}{y}| - d_{NN})$ in the integrand. The results are shown in table 3.4 and graphically in Fig 3.3. Roughly speaking, d_{NN} should be chosen halfway between the N-N hard-core radius (assuming hard-core N-N potential) and the healing distance, which is state-dependent. A reasonable value of d_{NN}, on this basis, is about 0.7 fm. From Fig. 3.3 it can be seen that corresponding to this value of d_{NN}, $U_{\Lambda}^{(3)} = 1.8$ Mev for $d_{\Lambda N} = 0.6$ fm and is about 8.7 Mev for $d_{\Lambda N} = 1.0$ fm.

TABLE 3.4

EFFECT OF THE TPE THREE-BODY ΛNN POTENTIALS IN NUCLEAR MATTER. THE NOTATIONS ARE EXPLAINED IN THE TEXT. $d_{\Lambda N}$ IS THE CUTOFF FOR THE ΛNN POTENTIAL, WHILE d_{NN} IS THE CUTOFF FOR A STEP-FUNCTION TYPE N-N CORRELATION.

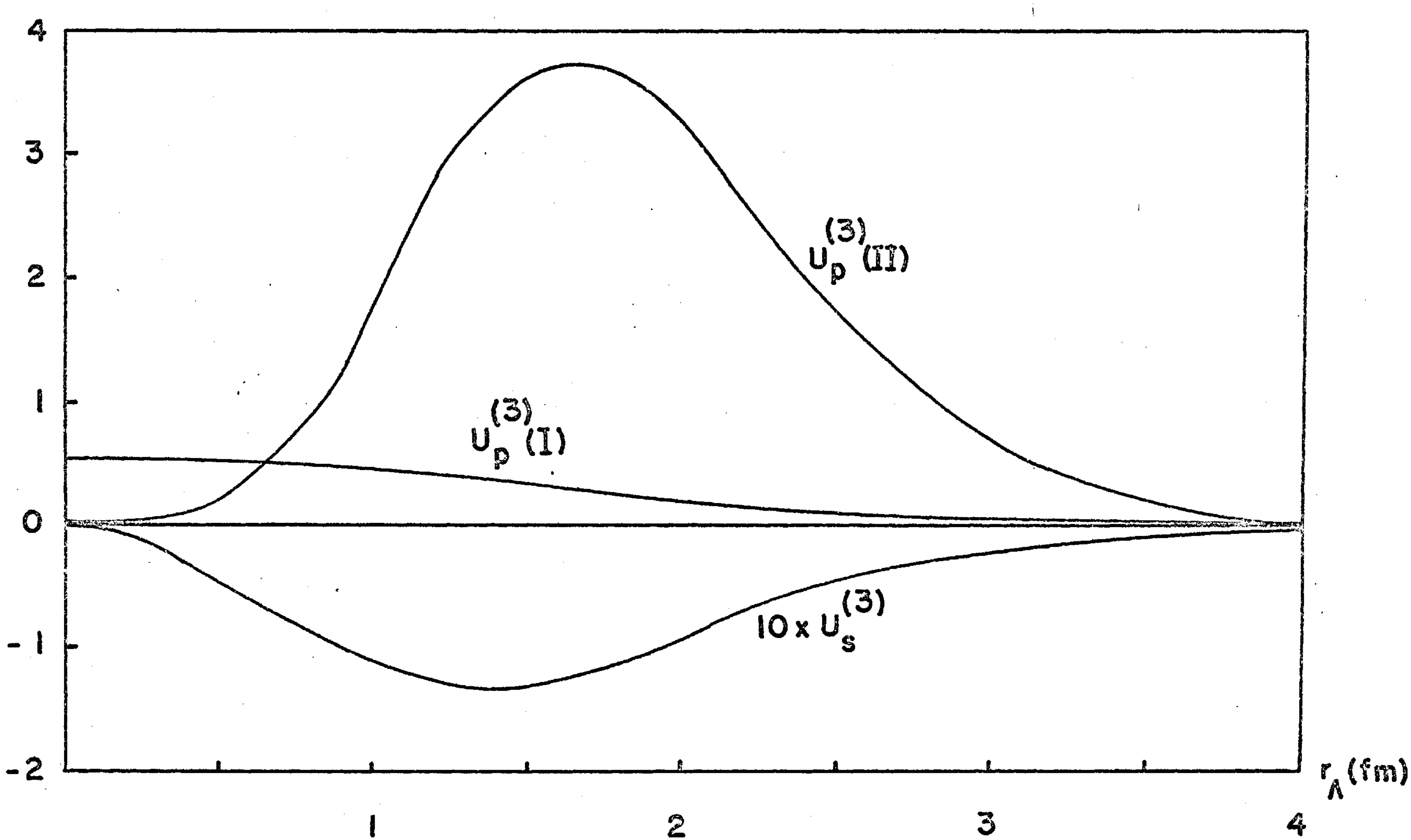
$d_{\Lambda N}$	0.6 fm			1. fm			
d_{NN}	0.4 fm	0.6 fm	1 fm	0.4 fm	0.6 fm	1 fm	1.4 fm
$U_\Lambda^{(s)}$ (Mev)	- 0.534	- 0.474	- 0.290	- 0.358	- 0.333	- 0.242	- 0.131
$U_\Lambda^{(I)}$ (Mev)	1.362	1.299	1.057	0.843	0.807	0.668	0.465
$U_\Lambda^{(II)}$ (Mev)	13.950	4.687	- 6.849	12.533	9.948	3.109	- 1.354
U_Λ^{3} (Mev)	14.777	5.512	- 6.082	13.017	10.422	3.535	- 1.076

However, with a slightly larger value of d_{NN}, $U_\Lambda^{(3)}$ may well turn negative for the case where $d_{\Lambda N}$ is 0.6 fm. A repulsion of 3.535 Mev is obtained for $d_{\Lambda N} = d_{NN} = 1$ fm, which is close to what Nyman (1967) obtained. The main point to emerge, however, is that $U_\Lambda^{(3)}$ is extremely sensitive to $d_{\Lambda N}$ as well as the N-N correlation function. If the very long-range part of the ΛNN potential (for $d_{\Lambda N} > 1$ fm) were being considered then its effect is definitely repulsive and can be as large as 10 Mev. However, it would be misleading to quote any definite number and therefore, the point of view adopted here, disagrees in spirit to Nyman's work.

Recently Gal (1966, 1967) introduced in his phenomenological analysis of light hypernuclei, a ΛNN force of the type $W_p(I)$, which is completely central. Only Gal's $C_{p\Lambda}$ was much larger than the $C_{p\Lambda}$ used here and was taken to be 17 Mev as compared to 1.43 Mev used here. The contribution to U_Λ^3 of such a central potential is rather insensitive to N-N correlation, as was already clear from the work of Bodmer and Sampanthar (1962). It behaves very differently from the dominant non-central term $W_p(II)$. For example, with $d_{\Lambda N} = 0.6$ fm and $d_{NN} = 0.7$ fm, its contribution to $U_\Lambda^{(3)}$ is 16 Mev, compared to a value of 2 Mev from $W_p(II)$ but for $d_{\Lambda N} = 1$ fm and $d_{NN} = 0.7$ fm it is about 10 Mev, nearly the same as the value found here. By taking a completely central phenomenological ΛNN force like Gal's, there is the danger that the repulsive effect may be overestimated in calculating B_Λ .

Before concluding this chapter, it would be in order to mention that an extensive analysis of the effects of the TPE ΛNN force, W, in ${}^{13}_{\Lambda}C$, ${}^{17}_{\Lambda}O$, ${}^{29}_{\Lambda}Si$ and ${}^{41}_{\Lambda}Ca$ has been done by Friesen and Tomusiak (1968). Their conclusion is very similar to that of the present work in ${}^{5}_{\Lambda}He$. Namely, in all cases considered, they found that the ΛNN force results in an appreciable repulsion between the Λ and the core nucleus. Of course their conclusion is subject to the ambiguituies which have been discussed in the case of ${}^{5}_{\Lambda}He$.

Fig.3.1



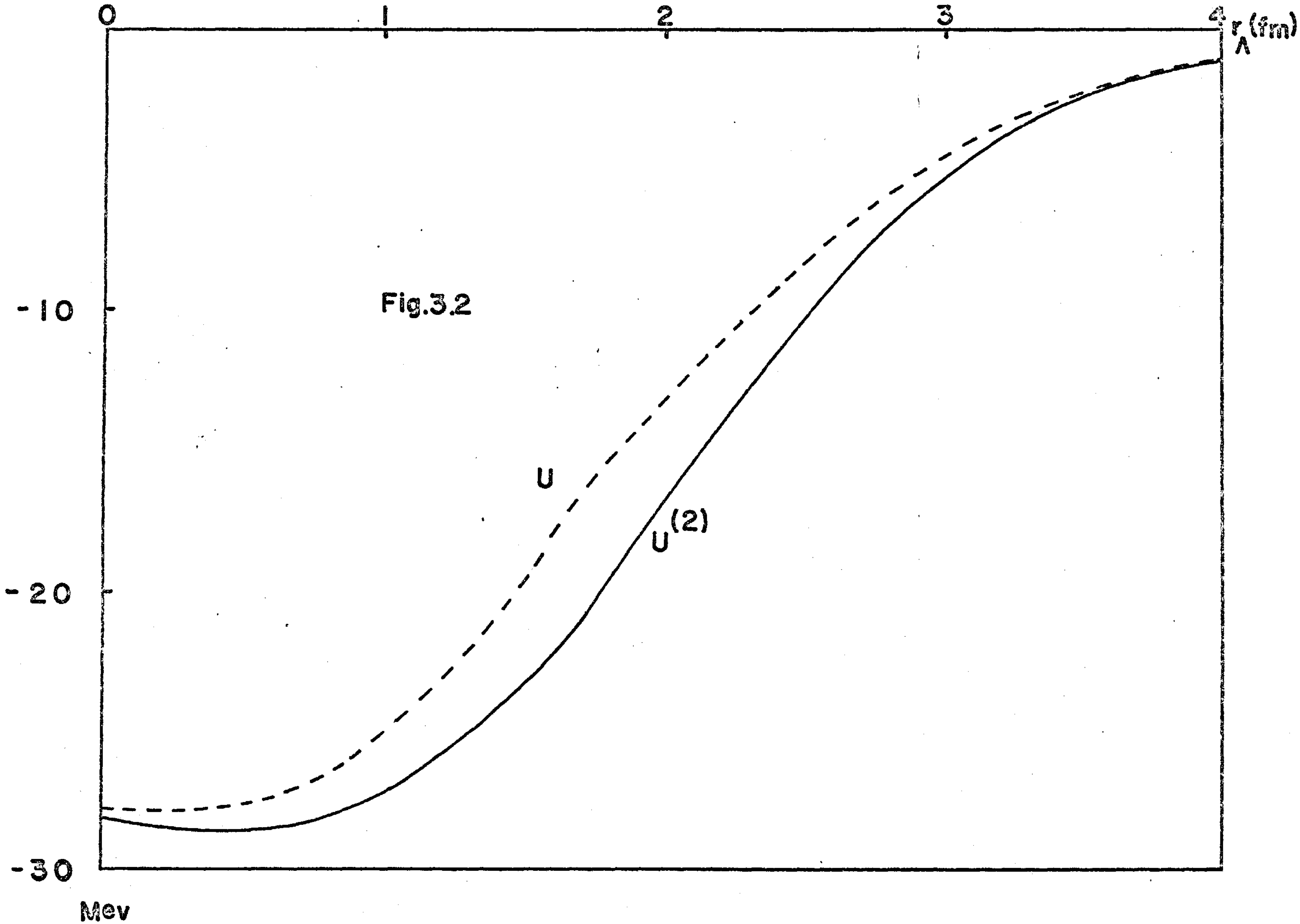


Fig.3.2

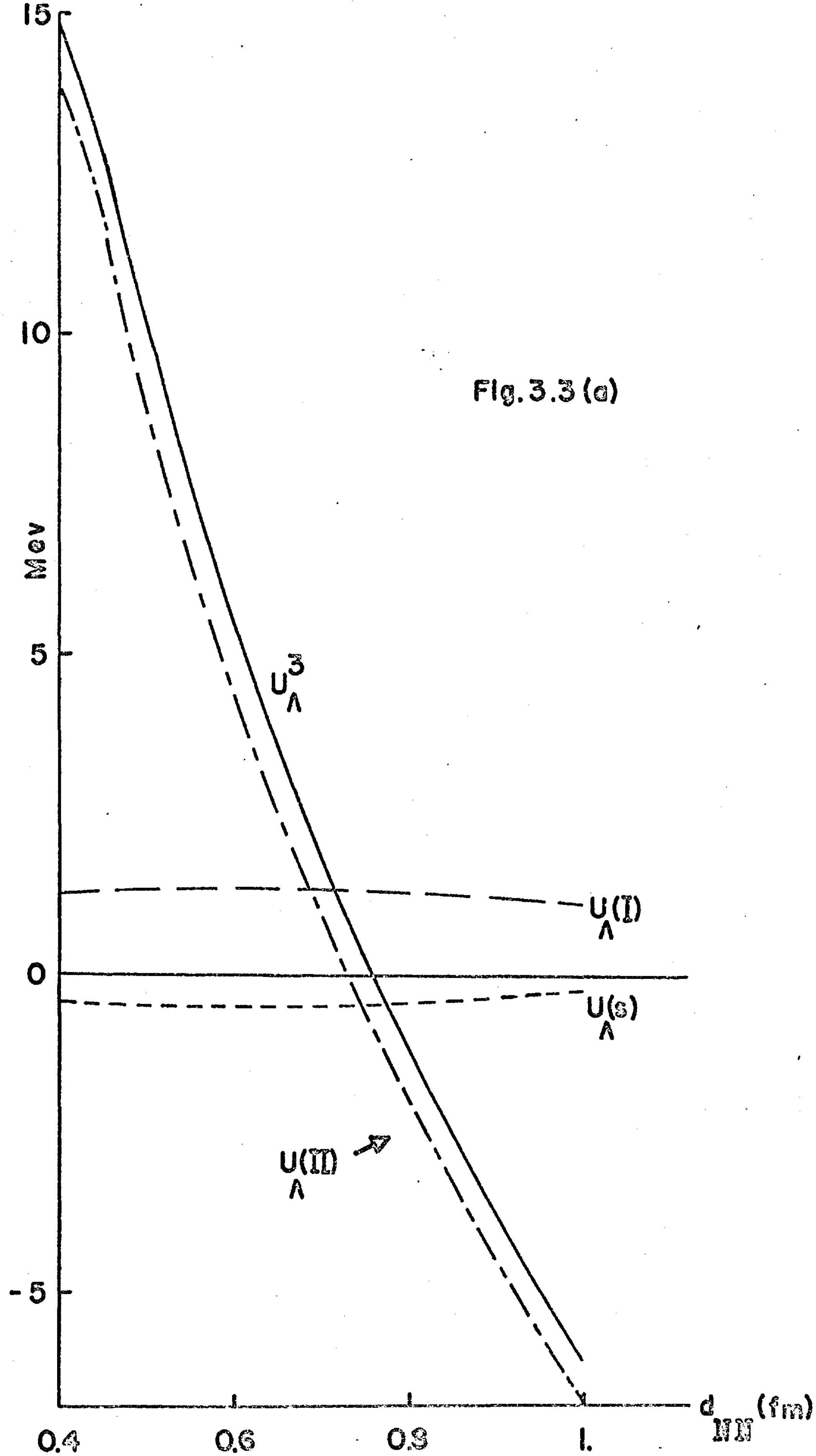


Fig. 3.3 (a)
Mev
15
10
5
0
-5
U^3_Λ
$U_\Lambda(I)$
$U_\Lambda(s)$
$U_\Lambda(II)$
0.4
0.6
0.8
1.
d_{NN} (fm)

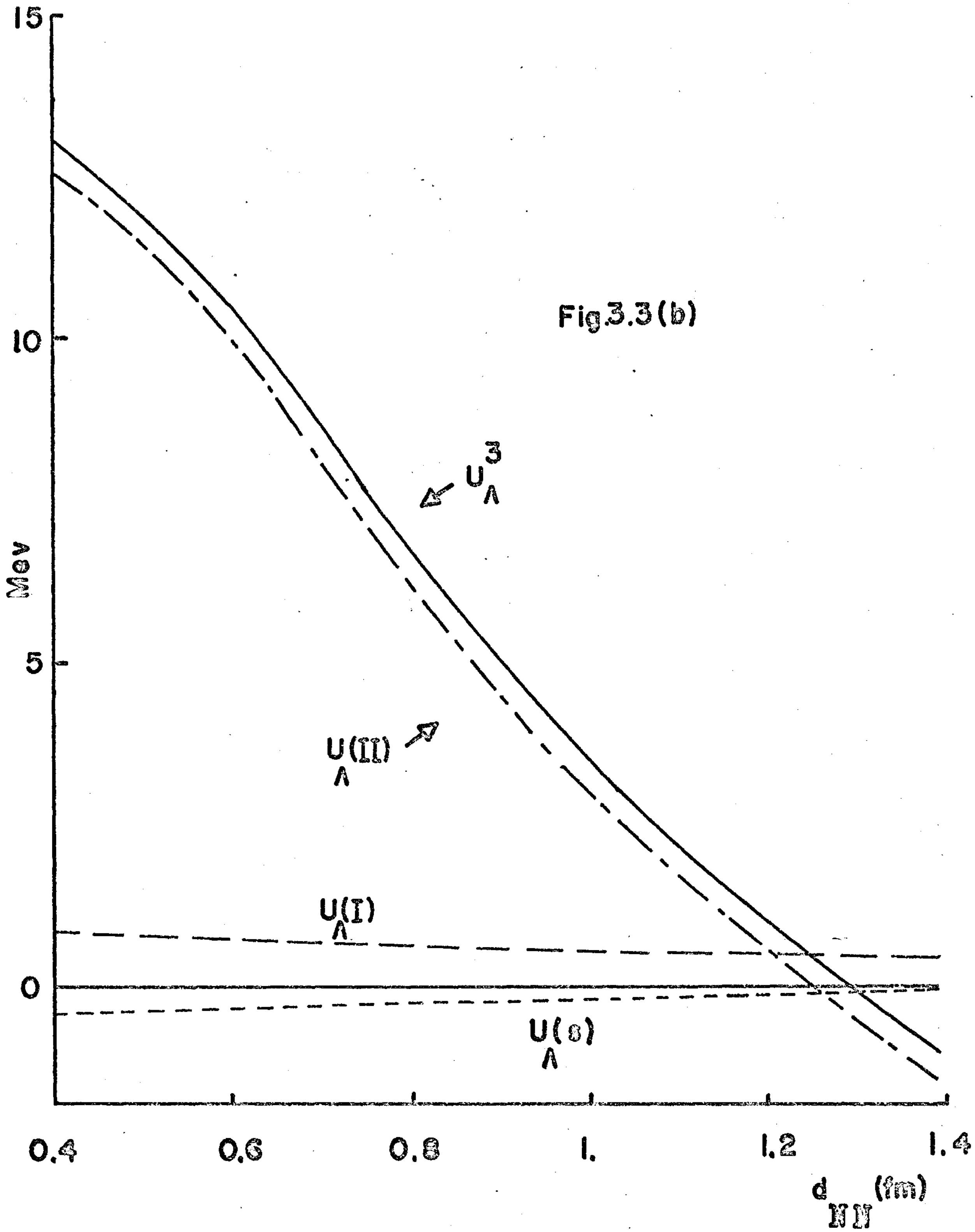

15
10
5
0
Mev
0.4
0.6
0.8
1.
1.2
1.4
d_{NN} (fm)
U_Λ^3
$U_\Lambda(II)$
$U_\Lambda(I)$
$U_\Lambda(s)$

Fig.3.3(b)

CHAPTER 4

EVALUATION OF THE INTERMEDIATE RANGE THREE-PION EXCHANGE ΛNN FORCE

As previously explained in chapter 2 the Λ-N and ΛNN potentials can be worked out in the one-or two-channel formalism. Since only the bound state is considered it is sufficiently accurate to use the one-channel formalism. The diagrams which contribute to the three-pion exchange ΛNN force are shown in Fig. 4.1c and 4.1d. Diagrams 4.1c and 4.1d, equally important, are as yet too complicated to be evaluated exactly. Therefore the contribution of diagram 4.1d is approximated by the sum of the contributions of diagrams 4.1a and 4.1b. The force arising from diagrams 4.1a and 4.1b is derived in the static approximation. It was stated by Uehara (1960) that the potential due to Fig. 4.1c has the asymptotic tail of the OPEP. This statement was incorrect and therefore this potential is written down and the correct estimate of the range in the asymptotic region is given.

First the contributions arising from diagrams 4.1a and 4.1b are considered. The contribution from the diagram of Fig. 4.2 has to be subtracted once since it is included in both 4.1a and 4.1b. The S matrix element is given by :

$$S = S_{1a} + S_{1b} - S_{2} + \tilde{S}_{1a} + \tilde{S}_{1b} - \tilde{S}_{2} \qquad \ldots(4.1)$$

where the suffix refers to the diagram number and the tilda indicates that nucleons 1 and 2 have been interchanged. Following Miyazawa (1956,1957) the S matrix element corresponding to the diagram 4.1a can be written

$(c = \hbar = 1)$:

$$S_{1a} = -\frac{2f_N^2}{(2\pi)^{10}\mu^2}\sum_{\alpha,\beta,\gamma}\int d^4p\, d^4q\, d^4k\, \delta(k_0)\, \frac{\langle\beta, q\,|\,S_\Lambda\,|\,\alpha, p\rangle\, \tau^1_\beta\, \boldsymbol{\sigma}^1\cdot\mathbf{q}}{(p_0^2-\omega_p^2)\,(q_0^2-\omega_q^2)\,(k_0^2-\omega_k^2)}$$

$$\frac{\langle\gamma, -k\,|\,S_1\,|\,\alpha, -p\rangle\, \tau^2_\gamma\, \boldsymbol{\sigma}^2\cdot\mathbf{k}}{k_0 - p_0 + i\epsilon}\, \exp\left[i\,(\mathbf{q}\cdot\mathbf{r}_1 - \mathbf{k}\cdot\mathbf{r}_2)\right] \qquad \ldots(4.2)$$

The subscripts α, β and γ refer to the index of the third component of the isotopic spin, f_N is the pseudo-vector πNN coupling constant ($f_N^2 = 0.08$), p, q and k (which have fourth components p_o, q_o and k_o) are the four momenta of the exchanged pions whose energies have been labelled ω_p, ω_q and ω_k respectively. The vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of the two nucleons and μ is the pion mass. The matrix elements for the π-N and π-Λ scattering parts are given by (Nogami and Bloore, 1964) :

$$\langle\gamma, -k\,|\,S_1\,|\,\alpha, -p\rangle = 2\pi i\,\delta(p_0 - k_0)\,\left[A_N(-p_0)\,\tau^1_\alpha \tau^1_\gamma\, \boldsymbol{\sigma}^1\cdot\mathbf{p}\, \boldsymbol{\sigma}^1\cdot\mathbf{k} + B_N(-p_0)\right.$$
$$\left(\tau^1_\gamma \tau^1_\alpha\, \boldsymbol{\sigma}^1\cdot\mathbf{p}\, \boldsymbol{\sigma}^1\cdot\mathbf{k} + \tau^1_\alpha \tau^1_\gamma\, \boldsymbol{\sigma}^1\cdot\mathbf{k}\, \boldsymbol{\sigma}^1\cdot\mathbf{p}\right) + \left. C_N(-p_0)\,\tau^1_\gamma \tau^1_\alpha\, \boldsymbol{\sigma}^1\cdot\mathbf{k}\, \boldsymbol{\sigma}^1\cdot\mathbf{p}\right]$$
$$\times \exp\left[-i\,(\mathbf{p} - \mathbf{k})\cdot\mathbf{r}_1\right]\; v_p\, v_k\,, \qquad \ldots(4.3)$$

$$\langle\beta, q\,|\,S_\Lambda\,|\,\alpha, p\rangle = 2\pi i\,\delta(p_0 - q_0)\,\left(A_\Lambda(p_0)\,\boldsymbol{\sigma}^\Lambda\cdot\mathbf{p}\, \boldsymbol{\sigma}^\Lambda\cdot\mathbf{q} + C_\Lambda(p_0)\,\boldsymbol{\sigma}^\Lambda\cdot\mathbf{q}\, \boldsymbol{\sigma}^\Lambda\cdot\mathbf{p}\right)\delta_{\alpha\beta}$$

$$\times \exp\left[i\,(\underset{\sim}{p} - \underset{\sim}{q}) \cdot \underset{\sim}{r}_\Lambda\right] v_p\, v_q \;, \qquad \ldots(4.4)$$

where $\underset{\sim}{r}_\Lambda$ is the coordinate of Λ. The function v_p is a cutoff factor which is chosen to be $\exp\,(-\,p^2/2p_m^2)$ where p_m is the momentum corresponding to the nucleon mass; it is introduced for the convenience of computation (Chew and Low, 1956; Nogami and Bloore 1964). The functions A_N, B_N, C_N, A_Λ and C_Λ are given in terms of the π-N and π-Λ scattering cross sections by the dispersion relations in the static approximation :

$$A_N(p_o) = C_N(-\,p_o) = \frac{4\pi\,(f_N/\mu)^2}{p_o - i\epsilon} + \frac{1}{4\pi}\int_0^\infty \frac{dk}{\omega_k}\,\frac{\sigma_{33}(k)}{\omega_k - p_o - i\epsilon} +$$

$$\frac{1}{36\pi}\int_0^\infty \frac{dk}{\omega_k}\,\frac{4\sigma_{11}(k) + 4\sigma_{13}(k) + \sigma_{33}(k)}{\omega_k + p_o - i\epsilon}\;, \qquad \ldots(4.5a)$$

$$B_N(p_o) = B_N(-\,p_o) = \frac{1}{12\pi}\int_0^\infty \frac{dk}{\omega_k}\,\frac{\sigma_{33}(k) + 2\sigma_{13}(k)}{\omega_k - p_o - i\epsilon} +$$

$$\frac{1}{12\pi}\int_0^\infty \frac{dk}{\omega_k}\,\frac{\sigma_{33}(k) + 2\sigma_{13}(k)}{\omega_k + p_o - i\epsilon}\;, \qquad \ldots(4.5b)$$

$$A_\Lambda(p_o) = C_\Lambda(-\,p_o) = \frac{4\pi\,(f_\Lambda/\mu)^2}{\Delta + p_o - i\epsilon} + \frac{1}{2\pi}\int_0^\infty \frac{dk}{\omega_k}\,\frac{\sigma_3(k)}{\omega_k - p_o - i\epsilon}$$

$$+ \frac{1}{6\pi} \int_0^{\infty} \frac{dk}{\omega_k} \frac{2\sigma_1(k) + \sigma_3(k)}{\omega_k + p_o - i\epsilon} . \qquad \ldots(4.6)$$

Here f_Λ is the renormalized π-Λ-Σ coupling constant, Δ is the mass difference between Σ and Λ, $\sigma_{2I,\,2J}$ is the total cross section of the p-wave π-N scattering in the state (I, J) and σ_{2J} is the total cross section of the p wave π-Λ scattering in the state with angular momentum J ($\frac{1}{2}$ or $\frac{3}{2}$). If expressions (4.3) and (4.4) are substituted in eq. (4.2) S_{1a} is obtained in the form $S_{1a} = 2\pi i\, \delta(0)\, V_a$. The quantity V_a which is interpreted as the ΛNN potential arising from diagram 4.1a is given by :

$$V_a = \frac{i\, 4\pi\, f_N^2}{(2\pi)^{10}\, \mu^2} \int_{-\infty}^{+\infty} dp_o \int d^3p\, d^3q\, d^3k\, V_p^2\, V_q\, V_k \frac{P_a \exp\left[i(\underset{\sim}{q} - \underset{\sim}{p}).\underset{\sim}{x} + \underset{\sim}{k}.\underset{\sim}{z}\right]}{(p_o^2 - \omega_p^2)(p_o^2 - \omega_q^2)\,\omega_k\,(p_o^2 - i\epsilon)} ,$$

$$\ldots(4.7)$$

where

$$P_a = \{\Lambda\} \sum_{\alpha,\,\beta,\,\gamma} \delta_{\alpha\beta}\, \tau_\beta^1\, \underset{\sim}{\sigma}^1 . \underset{\sim}{q} < \gamma, - k \,|\, T_1(p_o) \,|\, \alpha, - p > \tau_\gamma^2\, \underset{\sim}{\sigma}^2 . \underset{\sim}{k} ,$$

$$\ldots(4.8)$$

with eq. (4.3)

$$< \gamma, - k \,|\, S_1 \,|\, \alpha, - p > = 2\pi i\, \delta(p_o - k_o) \exp\left[- i(\underset{\sim}{p} - \underset{\sim}{k}).r_1\right] V_p V_k$$

$$\times < \gamma, - k \,|\, T_1(p_o) \,|\, \alpha, - p > , \qquad \ldots(4.9)$$

and from eq. (4.4)

$$\{\Lambda\} = A_\Lambda(p_o)\, \underset{\sim}{\sigma}^\Lambda\cdot\underset{\sim}{p}\;\; \underset{\sim}{\sigma}^\Lambda\cdot\underset{\sim}{q} + C_\Lambda(p_o)\, \underset{\sim}{\sigma}^\Lambda\cdot\underset{\sim}{q}\,\underset{\sim}{\sigma}^\Lambda\cdot\underset{\sim}{p}. \qquad \ldots(4.10)$$

Then,

$$P_a = P'_a + P''_a\,, \qquad \ldots(4.11)$$

with

$$P'_a = \underset{\sim}{\tau}^1\cdot\underset{\sim}{\tau}^2\;\{\Lambda\}\; P'_N\, \underset{\sim}{\sigma}^1\cdot\underset{\sim}{q}\;\; \underset{\sim}{\sigma}^1\cdot\underset{\sim}{p}\;\; \underset{\sim}{\sigma}^1\cdot\underset{\sim}{k}\,\underset{\sim}{\sigma}^2\cdot\underset{\sim}{k},$$

$$P'_N = A_N - 4B_N + 3C_N\,, \qquad \ldots(4.12)$$

$$P''_a = 2\,\underset{\sim}{\tau}^1\cdot\underset{\sim}{\tau}^2\;\{\Lambda\}\;\; P''_N\, \underset{\sim}{\sigma}^1\cdot\underset{\sim}{q}\,\underset{\sim}{\sigma}^2\cdot\underset{\sim}{k}\;\underset{\sim}{p}\cdot\underset{\sim}{k}\,,$$

$$P''_N = -A_N + 3B_N\,. \qquad \ldots(4.13)$$

Similar expressions are obtained for the potential V_b arising from diagram 4.1b. The potential V_2 ($S_2 = -2\pi i\,\delta(0)\,V_2$) could be obtained by replacing $\langle \gamma, -k\,|\,T_1(p_o)|\,\alpha, -p\rangle$ by $(4\pi/\mu^2)\, f_N^2\, \tau_\gamma^1\, \tau_\alpha^1\, \dfrac{\underset{\sim}{\sigma}^1\cdot\underset{\sim}{k}\,\underset{\sim}{\sigma}^1\cdot\underset{\sim}{p}}{p_o - i\epsilon}$ in the expression for V_a (4.7). However, it is simpler to notice that diagram 4.2 results only in terms proportional to f_N^4. Therefore $V_a + V_b - V_2$ can be replaced by $V = V_a + V_b$ with terms proportional to f_N^4 divided by two.

The part of V_a corresponding to P'_a, V'_a, can be written using eqs. (4.7) and (4.12),

$$V'_a = U\,(x)\;\; V\,(z)\,, \qquad \ldots(4.14)$$

with

$$V(z) = \frac{4\pi\, \overset{1}{\boldsymbol{\tau}} \cdot \overset{2}{\boldsymbol{\tau}}\, f_N}{(2\pi)^3\, \mu^2} \int \frac{d^3k\;\; \overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{k}\, \overset{2}{\boldsymbol{\sigma}} \cdot \boldsymbol{k}\, \exp(i\boldsymbol{k} \cdot \boldsymbol{z})}{\omega_k^2}$$

$$= \frac{4\pi\, \overset{1}{\boldsymbol{\tau}} \cdot \overset{2}{\boldsymbol{\tau}}\, f_N^2}{(2\pi)^3\, \mu^2}\;\; \overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_z\;\; \overset{2}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_z \int \frac{d^3k\;\; \exp(i\boldsymbol{k} \cdot \boldsymbol{z})}{\omega_k^2}\,.$$

$V(z)$ is simply the OPEP for the N-N potential :

$$V(z) = \frac{\overset{1}{\boldsymbol{\tau}} \cdot \overset{2}{\boldsymbol{\tau}}}{3}\;\; f_N^2\, (\overset{1}{\boldsymbol{\sigma}} \cdot \overset{2}{\boldsymbol{\sigma}} + T(z)\; S_{12}(z))\; Y(z) \qquad \ldots(4.15)$$

where $S_{12}(z)$, $T(z)$ and $Y(z)$ have been defined in chapter 2 (eqs. (2.11) and (2.12)).

Then,

$$U(x) = \frac{i}{(2\pi)^7} \int_{-\infty}^{+\infty} dp_o \int d^3p\; d^3q \left\{\Lambda\right\} \frac{P'_N\, \overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{q}\, \overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{p}\;\; \exp\left[i(\boldsymbol{q} - \boldsymbol{p}) \cdot \boldsymbol{x}\right]}{(p_o^2 - \omega_p^2)\; (p_o^2 - \omega_q^2)\; (p_o - i\epsilon)} \qquad \ldots(4.16)$$

That is,

$$U(x) = (\, \overset{\Lambda}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_{r'}\;\; \overset{\Lambda}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_r\; \overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_r\; \overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_{r'}\;\; I(A_\Lambda P'_N) + \overset{\Lambda}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_r\;\; \overset{\Lambda}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_{r'}\, \overset{1}{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}_r\;\; \overset{1}{\boldsymbol{\sigma}} \cdot \nabla_{r'}$$

$$\times\, I(C_\Lambda P'_N))_{r=r'=x} \qquad \ldots(4.17)$$

with

$$I(f(p_o), r, r') = \frac{i}{(2\pi)^7} \int_{-\infty}^{+\infty} dp_o \int d^3p\, d^3q\, \frac{f(p_o)\exp\left[i(\underset{\sim}{q}\cdot\underset{\sim}{r} - \underset{\sim}{p}\cdot\underset{\sim}{r}')\right]}{(p_o^2-\omega_p^2)(p_o^2-\omega_q^2)(p_o - i\epsilon)} \qquad \ldots(4.18)$$

After reduction of the differential operators, the potential (4.17) can be expressed as a sum of central (spin- independent and spin-dependent terms) and tensor parts :

$$U(x) = V_o(x) + V_s(x)\, \underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\sigma}^{1} + V_t(x)\, S_{\Lambda 1}(x) \qquad \ldots(4.19)$$

where

$$V_o(x) = \left[\left(\frac{2}{x^2} + \frac{\partial^2}{\partial r\, \partial r'}\right)\frac{\partial^2}{\partial r\, \partial r'}\; I\left[(A_\Lambda + C_\Lambda)\, P'_N\right]\right]_{r=r'=x}, \qquad \ldots(4.20a)$$

$$V_s(x) = \left[\frac{2}{3x}\left(\frac{1}{x} + \frac{\partial}{\partial r} + \frac{\partial}{\partial r'}\right)\frac{\partial^2}{\partial r\, \partial r'}\; I\left[(A_\Lambda - C_\Lambda)\, P'_N\right]\right]_{r=r'=x}, \qquad \ldots(4.20b)$$

and

$$V_T(x) = \left[\frac{1}{3x}\left(\frac{2}{x} - \frac{\partial}{\partial r} - \frac{\partial}{\partial r'}\right)\frac{\partial^2}{\partial r\,\partial r'}\; I\; \left[(A_\Lambda - C_\Lambda)\, P'_N\right]\right]_{r = r' = x} \quad . \qquad \ldots(4.20c)$$

$U(x)$ is the analytical expression of the TPE potential for Λ-N (Nogami and Bloore, 1964) and therefore it can be seen that the V'_a part of the potential corresponding to diagram 4.1a, is the product of the OPEP (N-N) and the dimensionless analytical expression for the TPEP (Λ-N).

Similarly the part corresponding to P'_b can be written

$$V'_b = V(z)\, U(x) \qquad \ldots(4.21)$$

Thus $V' = V'_a + V'_b = \left\{U(x),\, V(z)\right\}_+$, where $\left\{\ ,\ \right\}_+$ indicates the anticommutator and

$$V' = \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2\; \frac{f_N^2}{3} \left\{ \left(V_o(x) + V_s(x)\, \underset{\sim}{\sigma}^\Lambda \cdot \underset{\sim}{\sigma}^1 + V_T(x)\, S_{\Lambda 1}(x)\right), \left(\underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\sigma}^2 + T(z)\, S_{12}(z)\right)\right\}_+ Y(z). \qquad \ldots(4.22)$$

That is,

$$V' = \frac{2}{3}\, \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2\, \mu\, f_N^2 \left[V_o(x)\, \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\sigma}^2 + V_S(x)\, \underset{\sim}{\sigma}^\Lambda \cdot \underset{\sim}{\sigma}^2 + V_T(x)\,(1 - T(z))\, S_{\Lambda 2}(x) + (V_o(x) + V_S(x) - V_T(x))\, T(z)\, S_{\Lambda 2}(z) + V_T(x)\, T(z)\, \Sigma_{\Lambda 2}(x,z)\right] Y(z) \, , \qquad \ldots(4.23)$$

where

$$\Sigma_{\Lambda 2}(x, z) = 9\,\underset{\sim}{x}\cdot\underset{\sim}{z}\;\frac{\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{x}\;\;\underset{\sim}{\sigma}^{2}\cdot\underset{\sim}{z}}{x^{2}\;z^{2}} - \underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\sigma}^{2} \quad \ldots(4.24)$$

The part of V_a corresponding to P''_a, V''_a, can be written using eqs. (4.7) and (4.13)

$$V''_a = 2\,\tau^{1}\cdot\tau^{2}\;\frac{f_N^{2}}{\mu}\Big(\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\nabla}_{r'}\;\;\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\nabla}_{r}\;\;\underset{\sim}{\sigma}^{1}\cdot\underset{\sim}{\nabla}_{r}\;\underset{\sim}{\sigma}^{2}\cdot\underset{\sim}{\nabla}_{z}\;\;\underset{\sim}{\nabla}_{r'}\cdot\underset{\sim}{\nabla}_{z}\;\;I(A_{\Lambda}\;P''_N)$$

$$+\;\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\nabla}_{r}\;\;\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\nabla}_{r'}\;\;\underset{\sim}{\sigma}^{1}\cdot\underset{\sim}{\nabla}_{r}\;\;\underset{\sim}{\sigma}^{2}\cdot\underset{\sim}{\nabla}_{z}\;\underset{\sim}{\nabla}_{r'}\cdot\underset{\sim}{\nabla}_{z}\;\;I(C_{\Lambda}\;P''_N)\Big)\,Y(z)\;\Big|_{r = r' = x} \quad .$$

$$\ldots(4.25a)$$

Similarly

$$V''_b = 2\,\underset{\sim}{\tau}^{1}\cdot\underset{\sim}{\tau}^{2}\;\frac{f_N^{2}}{\mu}\Big(\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\nabla}_{r'}\;\;\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\nabla}_{r}\;\;\underset{\sim}{\sigma}^{1}\cdot\underset{\sim}{\nabla}_{r'}\;\;\underset{\sim}{\sigma}^{2}\cdot\underset{\sim}{\nabla}_{z}\;\underset{\sim}{\nabla}_{r}\cdot\underset{\sim}{\nabla}_{z}\;I(A_{\Lambda}\;P''_N)$$

$$+\;\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\nabla}_{r}\;\;\underset{\sim}{\sigma}^{\Lambda}\cdot\underset{\sim}{\nabla}_{r'}\;\;\underset{\sim}{\sigma}^{1}\cdot\underset{\sim}{\nabla}_{r'}\;\;\underset{\sim}{\sigma}^{2}\cdot\underset{\sim}{\nabla}_{z}\;\;\underset{\sim}{\nabla}_{r}\cdot\underset{\sim}{\nabla}_{z}\;\;I(C_{\Lambda}\;P''_N)\Big)\,Y(z)\;\Big|_{r = r' = x} \quad .$$

$$\ldots(4.25b)$$

Because $I(f(p_o), r, r')$ is a symmetric function of r and r' eq.(4.18), r and r' can be exchanged in eq. (4.25a) and then adding eq. (4.25b) gives

$$V'' = V''_a + V''_b = 4\;\underset{\sim}{\tau}^{1}\cdot\underset{\sim}{\tau}^{2}\;\frac{f_N^{2}}{\mu}\Big(\underset{\sim}{\nabla}_{r}\cdot\underset{\sim}{\nabla}_{r'}\;\;\underset{\sim}{\nabla}_{r}\cdot\underset{\sim}{\nabla}_{z}\;\underset{\sim}{\sigma}^{1}\cdot\underset{\sim}{\nabla}_{r'}\;\;\underset{\sim}{\sigma}^{2}\cdot\underset{\sim}{\nabla}_{z}$$

$$I\left[(A_{\Lambda} + C_{\Lambda})\;P''_N\right]\Big)\,Y(z),\quad r = r' = x \quad \ldots(4.26)$$

that is

$$V'' = \frac{4}{9}\,\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2\,\mu\, f_N^2 \left\{ \left[2\left(\frac{1}{x^2} + \frac{\partial^2}{2\,\partial r'\,\partial r}\right)\boldsymbol{\sigma}^1\cdot\boldsymbol{\sigma}^2 + \frac{3}{x^2}\,T(z)\,S_{12}(z) \right.\right.$$

$$\left. + 3\left(\frac{1}{\mu z} + \frac{1}{(\mu z)^2}\right)\left(\frac{1}{x^2} - \frac{\partial^2}{\partial r\,\partial r'}\right) S_{12}(x) - T(z)\left(\frac{1}{x^2} - \frac{\partial^2}{\partial r\,\partial r'}\right)\Sigma_{12}(x,z)\right]$$

$$\left.\frac{\partial^2}{\partial r\,\partial r'}\; I\;\left[(A_\Lambda + C_\Lambda)\,P''_N\right]\right\}_{r = r' = x} Y(z)\;. \qquad \ldots(4.27)$$

In the following sections these potentials will be considered in the cases where the nucleons and Λ are in the s-shell.

Therefore, with $i,\ j = 1,\ 2, \Lambda$

$$\langle \boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2\;\boldsymbol{\sigma}^i\cdot\mathbf{x}\;\boldsymbol{\sigma}^j\cdot\mathbf{z}\rangle = \frac{1}{3}\,\mathbf{x}\cdot\mathbf{z}\,\langle\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2\;\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^j\rangle\,, \qquad \ldots(4.28a)$$

$$\langle \boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2\;\Sigma_{ij}(x,z)\rangle = (3\cos^2\theta_{xz} - 1)\;\langle\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2\;\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^j\rangle\,, \qquad \ldots(4.28b)$$

$$\langle \boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2\;S_{ij}(x)\rangle = 0\,, \qquad \ldots(4.28c)$$

where $\cos\theta_{xz} = \mathbf{x}\cdot\mathbf{z}/xy$ as shown in Fig. 4.3. Now $V = V' + V''$ can be written as :

$$V(x,z) = \Big[\ S(x) + (3\cos^2\theta_{xz} - 1)\, T(z)\, W(x)\Big]\ Y(z) \qquad \ldots(4.29)$$

with

$$S(x) = -\frac{4}{3}\mu f_N^2 \Bigg[\langle \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2\ \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2 \rangle \left(\frac{1}{x^2} + \frac{\partial^2}{2\,\partial r\,\partial r'}\right) \frac{\partial^2}{\partial r\,\partial r'}\, Z(r, r')$$

$$+ \langle \boldsymbol{\sigma}^\Lambda \cdot \boldsymbol{\sigma}^2\ \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2 \rangle \left(\frac{1}{x^2} + \frac{2}{x}\frac{\partial}{\partial r}\right) \frac{\partial^2}{\partial r\,\partial r'}\ D(r,r')\Bigg]_{r = r' = x},$$

$$\ldots(4.30)$$

$$W(x) = -\frac{4}{3}\mu f_N^2 \Bigg[\langle \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2\ \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2 \rangle \left(-\frac{1}{x^2} + \frac{\partial^2}{\partial r\,\partial r'}\right) \frac{\partial^2}{\partial r\,\partial r'}\ X(r, r')$$

$$+ \langle \boldsymbol{\sigma}^\Lambda \cdot \boldsymbol{\sigma}^2\ \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2 \rangle \left(\frac{1}{x^2} - \frac{1}{x}\frac{\partial}{\partial r}\right) \frac{\partial^2}{\partial r\,\partial r'}\ D(r,r')\Bigg]_{r = r' = x},$$

$$\ldots(4.31)$$

where

$$Z(r,r') = I\left[(A_\Lambda + C_\Lambda)\,(3P'_N + 2P''_N)\right] = I\left[(A_\Lambda + C_\Lambda)\,(A_N - 6B_N + 9C_N)\right],$$

$$\ldots(4.32a)$$

$$D(r,r') = I\left[(A_\Lambda - C_\Lambda)\,P'_N\right] = I\left[(A_\Lambda - C_\Lambda)\,(A_N - 4B_N + 3C_N)\right]$$

$$\ldots(4.32b)$$

and

$$X(r, r') = I\left[(A_\Lambda + C_\Lambda)\, P''_N\right] = I\left[(A_\Lambda + C_\Lambda)\,(-A_N + 3B_N)\right] \quad . \qquad \ldots(4.32c)$$

In order to evaluate these expressions from eqs. (4.5), (4.6) and (4.18), it was assumed that the π-N and π-Λ scattering were dominated by the N^* (1238) and the $Y_1^*(1385)$ resonances respectively, that is σ_{11}, σ_{13} and σ_1 were set equal to zero in expressions (4.5) and (4.6). Also the integrals in (4.5) and (4.6) were evaluated replacing $\sigma(k)$ by $12\pi^2 g^2 k_r\ \delta(\omega_k - \omega_r)$ with $g^2 = \Gamma(k_r)/2k_r^3$, $\Gamma(k_r)$ being the width at the resonance energy (Fubini 1956). The values $g_N^2 = 0.057$, $\omega_N = 1.27\ fm^{-1}$ for $\sigma_{33}(k)$ and $g_\Lambda^2 = 0.047$ with $\omega_\Lambda = 1.24\ fm^{-1}$ for $\sigma_3(k)$ were used. The diagram shown in Fig. 4.4, which is included in diagrams 4.1a and 4.1b does not contribute to the three-body force. Therefore its contribution, which is proportional to $1/(p_o + i\epsilon)$ is subtracted by suppressing the term $-4\pi f_N^2/(p_o + i\epsilon)$ in $C_N(p_o)$ (4.5a). Now the expressions (4.32) can be calculated, and their explicit forms are given in the appendix 2.

Then the three-pion exchange potential corresponding to the S matrix (4.1) is :

$$P(\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{z}) = V(\underset{\sim}{x}, \underset{\sim}{z}) + \tilde{V}(\underset{\sim}{y}, \underset{\sim}{z}) \quad .$$

It can be seen that

$$P(\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{z}) = P_c(x, y, z) + P_T(\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{z}) \qquad \ldots(4.33)$$

with

$$P_c(x, y, z) = (S(x) + \tilde{S}(y))\, Y(z) \qquad \ldots(4.34)$$

and

$$P_T(\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{z}) = \left[(3\cos^2\theta_{xz} - 1)\, W(x) + (3\cos^2\theta_{yz} - 1)\, W(y)\right] T(z)\, Y(z) \qquad \ldots(4.35)$$

In the above equations the tilda indicates that superscript 1 and 2 in equations (4.31) are interchanged.

The contribution arising from diagram 4.1c is now considered.

Following Miyazawa (1956, 1957) the S matrix corresponding to this diagram can be written

$$S_c = \left(\frac{i}{(2\pi)^4}\right)^3 \sum_{\alpha,\beta,\gamma} \int d^4p\, d^4q\, d^4k \frac{\langle\beta, q \mid S_\Lambda \mid \alpha, p\rangle \langle\alpha, p \mid S_1 \mid \gamma, k\rangle}{(p_o^2 - \omega_p^2)(q_o^2 - \omega_q^2)}$$

$$\times \frac{\langle\beta, -q \mid S_2 \mid \gamma, -k\rangle}{(k_o^2 - \omega_k^2)} \qquad \ldots(4.36)$$

The notation is the same as that used in eq. (4.2). S_c is obtained in the form $S_c = -2\pi i\,\delta(0)\,V_c$, which is interpreted as the potential arising from diagram 4.1c, can be written

$$V_c = \frac{i}{(2\pi)^{10}} \sum_{\alpha,\beta,\gamma} \int_{-\infty}^{+\infty} dp_o \int d^3p\, d^3q\, d^3k \,\langle\beta, q \mid T_\Lambda(p_o) \mid \alpha, p\rangle\langle\alpha, p \mid T_1(p_o) \mid \gamma, k\rangle$$

$$\times \frac{\langle\beta, -q \mid T_2(p_o) \mid \gamma, -k\rangle \exp(-i\underset{\sim}{q}\cdot\underset{\sim}{y} + i\underset{\sim}{p}\cdot\underset{\sim}{x} - i\underset{\sim}{k}\cdot\underset{\sim}{z})}{(p_o^2 - \omega_p^2)(p_o^2 - \omega_q^2)(p_o^2 - \omega_k^2)} \,. \qquad \ldots(4.37)$$

As in Uehara (1960), only the Born terms are considered; that is

$$V_c = \frac{1}{6}\left[6 - (\underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2)^2\right] f_N^2 f_\Lambda^2 \left[(\underset{\sim}{\sigma}^\Lambda \cdot \underset{\sim}{\nabla}_x\, \underset{\sim}{\sigma}^\Lambda \cdot \underset{\sim}{\nabla}_y\, \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\nabla}_z\, \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\nabla}_x\, \underset{\sim}{\sigma}^2 \cdot \underset{\sim}{\nabla}_y\, \underset{\sim}{\sigma}^2 \cdot \underset{\sim}{\nabla}_z\right.$$

$$+ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_y \; \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_x \; \boldsymbol{\sigma}^1\cdot\boldsymbol{\nabla}_x \; \boldsymbol{\sigma}^1\cdot\boldsymbol{\nabla}_z \; \boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_z \; \boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_y \,) \, I_{+\Delta,\,0,\,0}(x,\, y,\, z)$$

$$- (\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_y \; \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_x \; \boldsymbol{\sigma}^1\cdot\boldsymbol{\nabla}_z \; \boldsymbol{\sigma}^1\cdot\boldsymbol{\nabla}_x \; \boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_y \; \boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_z + \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_x \; \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_y$$

$$\boldsymbol{\sigma}^1\cdot\boldsymbol{\nabla}_x \; \boldsymbol{\sigma}^1\cdot\boldsymbol{\nabla}_z \; \boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_z \; \boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_y \,) \, (I_{-\Delta,\,0,\,0}(x,\, y,\, z) - Y_{\Delta,\,0,\,0}(x,y,z))]$$

...(4.38)

with

$$I_{\alpha,\beta,\gamma}(x,y,z) = \frac{3}{\pi} \, \frac{1}{x\,y\,z} \int_0^{\infty} p\,dp \, \frac{\sin\left[p(x + y + z)\right]}{\omega_p\,(\omega_p + \alpha)\,(\omega_p + \beta)\,(\omega_p + \gamma)} \qquad \text{...(4.39a)}$$

and

$$Y_{\alpha,\beta,\gamma}(x,y,z) = \frac{\exp\left[-\left(\mu^2 - \alpha^2\right)^{1/2} (x + y + z)\right]}{(\alpha + \beta)\,(\alpha + \gamma)\;\; x\,y\,z} \qquad \text{...(4.39b)}$$

If the mass difference Δ is neglected in (4.38) and if the repetition term corresponding to $Y_{\Delta \to 0,0,0}(x,y,z)$ is subtracted, V_c can be written:

$$V_c = -\frac{4}{\mu^6} \left[6 - (\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2)^2\right] f_N^2 \, f_\Lambda^2 \, (\boldsymbol{\nabla}_z\cdot\boldsymbol{\nabla}_y \; \boldsymbol{\sigma}^\Lambda\cdot\boldsymbol{\nabla}_y \wedge \boldsymbol{\nabla}_x \; \boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_x \wedge \boldsymbol{\nabla}_z$$

$$+ \underset{\sim}{\nabla}_x \cdot \underset{\sim}{\nabla}_z \quad \underset{\sim}{\sigma}^{\wedge} \cdot \underset{\sim}{\nabla}_y \wedge \underset{\sim}{\nabla}_x \quad \underset{\sim}{\sigma}^{1} \cdot \underset{\sim}{\nabla}_z \wedge \underset{\sim}{\nabla}_y \,) \, I_{0,0,0}(x, y, z) \quad \ldots(4.40)$$

with

$$I_{0,0,0} = \frac{3}{4\mu} \frac{x + y + z}{x\,y\,z} \exp \left[- \mu \,(x + y + z)\right] \qquad \ldots(4.41)$$

Therefore the asymptotic form of V_c is proportional to $I_{0,0,0}$. If the equilateral triangle $x = y = z = r$ is considered, the asymptotic form is proportional to $\exp(-3\mu r)/\mu r^2$ which has the range expected for the exchange of three-pions. There is no term with the asymptotic tail of the OPEP. In Uehara (1960) the function $I_{\alpha,\beta,\gamma}(x,y,z)$ was miscalculated. The poles of the equation (4.37) have to be treated carefully. After the p_o integration, V_c (4.37) is proportional to J, where

$$J = \iiint_0^\infty dp\,dq\,dk \quad p \sin px \; q \sin qy \; k \sin kz \left(\frac{1}{\omega_p (\omega_q^2 - \omega_p^2)(\omega_k^2 - \omega_p^2)} \right.$$

$$\left. \frac{1}{(\omega_p + \alpha)(\omega_p + \beta)(\omega_p + \gamma)} \quad + \text{cyclic permutation of } p, q \text{ and } k \right).$$

The integral J was evaluated by assigning small, unequal imaginary masses to the mesons; that is $\omega_p \to (p^2 + \mu^2 + i\epsilon)^{1/2}$ etc. This leads to eq.† (4.39a). If poles are left on the real axis incorrect asymptotic behavior results.

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† This result agrees with that of a similar integral calculated by S.D. Drell and K. Huang, Phys. Rev. 91 (1953), 1527.

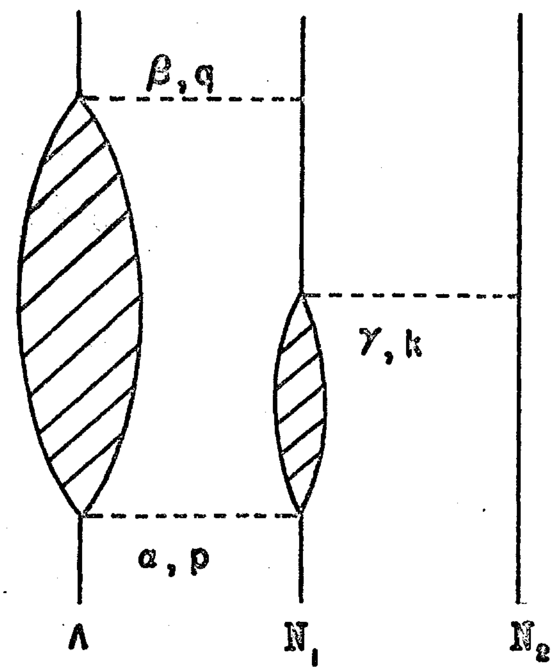


Fig.4.1(a)

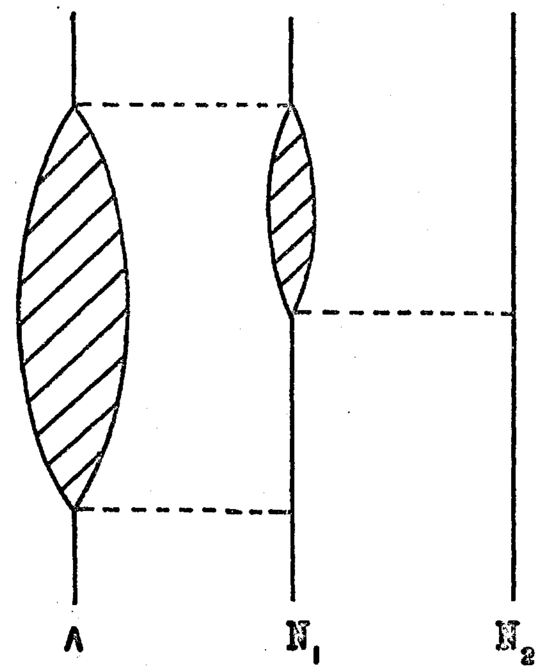


Fig.4.1(b)

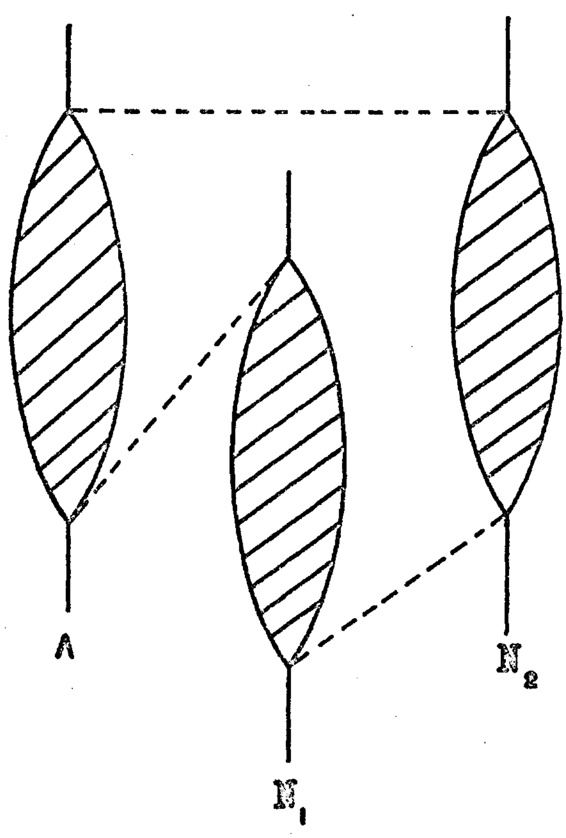


Fig.4.1(c)

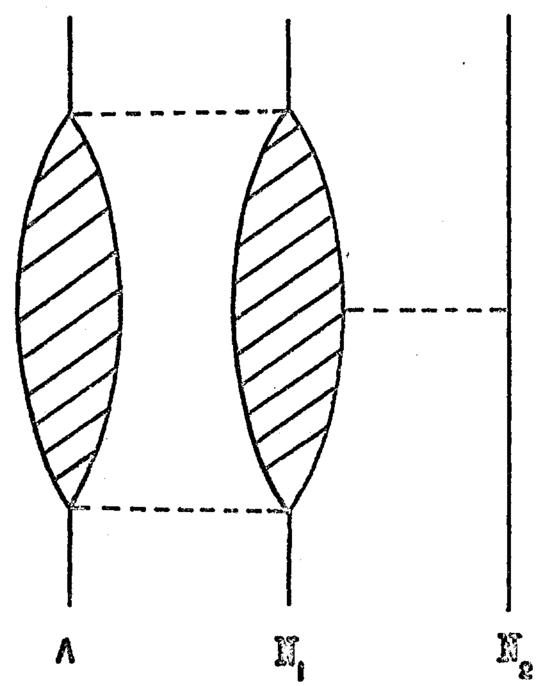


Fig.4.1(d)

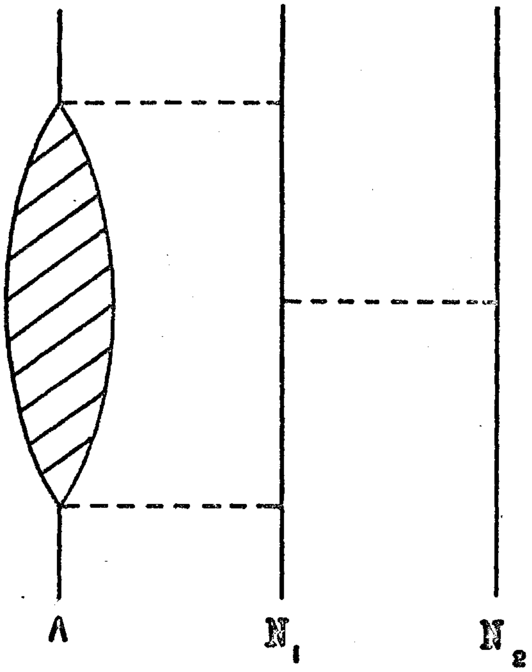


Fig.4.2

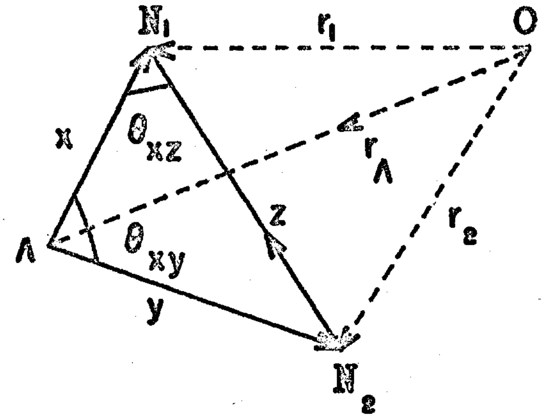
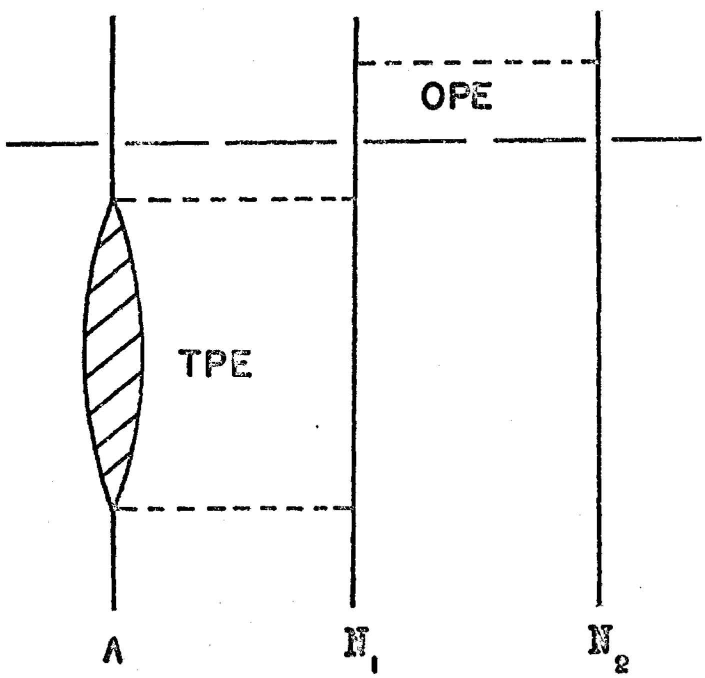


Fig.4.3

Fig. 4.4



CHAPTER 5

EFFECTS OF THE THREE-PION-EXCHANGE ΛNN FORCE

5.1 Effect in ${}^{3}_{\Lambda}H$

In a calculation similar to that of the TPE ΛNN force which has been done in § 3.1, the effects of the potentials P_c(4.34) and P_T(4.35) on B_Λ in ${}^{3}_{\Lambda}H$ are estimated by perturbation theory.

First the dependence of the three-pion exchange potential P(4.33) on the Λ-N distance, x, is compared with the x-dependence of the TPE ΛNN force, W_p. For ${}^{3}_{\Lambda}H$ it has been shown that W_p has the form (2.16 and 3.4) :

$$W_p = C_p \left[1 + (3 \cos^2 \theta_{xy} - 1) \; T(x) \; T(y) \right] Y(x) \; Y(y) \qquad \ldots(5.1)$$

with C_p = 1.43 Mev and $\cos\theta_{xy} = (\underset{\sim}{x} \cdot \underset{\sim}{y})/xy$ as shown on Fig. 2.1 . The functions Y and T have been defined in chapter 2,eqs. (2.12). The x-dependences of the central part and of the tensor part of W_p are denoted by C_2 and T_2 respectively,where from eq. (5.1) :

$$C_2 = C_p \; Y(x), \qquad \ldots(5.2)$$

$$T_2 = C_p \; Y(x) \; T(x). \qquad \ldots(5.3)$$

Similarly the x-dependences of P_c(4.34) and P_T(4.35) respectively can be written :

$$C_3 = S(x), \qquad \ldots(5.4)$$

$$T_3 = W(x) \qquad \ldots(5.5)$$

where S(x) and W(x) are given by the expressions (4.30) and (4.31) with the corresponding expectation values of the τ's and σ's replaced by their values for ${}^{3}_{\Lambda}H$:

$$\langle \underset{\sim}{\sigma}^{1} \cdot \underset{\sim}{\sigma}^{2} \; \underset{\sim}{\tau}^{1} \cdot \underset{\sim}{\tau}^{2} \rangle = -3, \qquad \ldots(5.6)$$

$$\langle \underset{\sim}{\sigma}^{i} \cdot \underset{\sim}{\sigma}^{\Lambda} \; \underset{\sim}{\tau}^{1} \cdot \underset{\sim}{\tau}^{2} \rangle = 6 \quad i = 1,2. \qquad \ldots(5.7)$$

The x-dependences of the central parts and tensor parts are plotted in Fig. 5.1 and Fig.5.2 respectively.

It can be seen that C_3 depends strongly on x and is larger than C_2 for $x \lesssim 1.2$ fm. The curve T_3 is also sensitive to x, but is of comparable strength to T_2, which becomes larger than T_3 only for $x \gtrsim .7$ fm. Therefore for $d_{\Lambda N} \lesssim 1.$ fm the central part P_c is expected to give a negative contribution to B_Λ(repulsive) larger than that of the TPE ΛNN force. The contribution of the tensor part, P_T , will depend on the average of the angular dependence '' $3 \cos^2 \theta_{xz} - 1$ ''.

The effect of the three-pion exchange potential P(4.35) on B_Λ in ${}^{3}_{\Lambda}H$ will now be considered. The notation is the same as that used in § 3.1. The expectation values of $P_c(x,y,z)$ (4.34) and $P_T(\underset{\sim}{x},\underset{\sim}{y},\underset{\sim}{z})$ (4.35) with respect to Y(3.1) are denoted by $\langle P_c \rangle$ and $\langle P_T \rangle$ respectively. Because of the complete symmetry of all the expressions in x and y, $\langle P_c \rangle$ and $\langle P_T \rangle$ can be written :

$$\langle P_c \rangle = 2\, I \left[S(x)\, Y(z) \right], \qquad \ldots(5.8)$$

$$\langle P_T \rangle = 2\, I \left[(3 \cos^2 \theta_{xz} - 1)\, W(x)\, T(z)\, Y(z) \right], \qquad \ldots(5.9)$$

where $I[\ldots]$ has been defined by eq. (3.9). The integration domain has also been explained in § 3.1.

The results are shown in table 5.1 for five different values of the cutoff $d_{\Lambda N}$ and three different values of the hard core radius D together with the results obtained for the central part $\langle W_p(I) \rangle$ and tensor part $\langle W_p(II) \rangle$ of the TPE ΛNN force(§ 3.1). It can be seen that for $d_{\Lambda N} \lesssim 1$ fm the central part $\langle P_c \rangle$ is always positive (repulsive) and larger than $\langle W_p(I) \rangle$, that the tensor part $\langle P_T \rangle$ is always negative (attractive) and larger than $\langle W_p(II) \rangle$ and that $\langle P_T \rangle$ is predominant. For $d_{\Lambda N} \lesssim 1$ fm the total effect $\langle P_c + P_T + W_p \rangle$ is always negative (attractive), for instance for $d_{\Lambda N} = 1$ fm and D = 0.4 fm, $\langle W_p \rangle = 0.164$ Mev and $\langle W_p + P_c + P_T \rangle = -0.132$ Mev. As in § 3.1 (eqs. 3.10 and 3.11), the relative importance of the ΛNN force P will be given by the ratio :

$$R = \frac{\langle P_c + P_T \rangle}{2\langle V_{\Lambda N} \rangle} = \begin{cases} 8.5\% \\ 1.5\% \end{cases} \quad \text{for } D = d_{\Lambda N} = \begin{cases} 0.4 \text{ fm} \\ 0.6 \text{ fm} \end{cases} . \qquad \ldots(5.10)$$

This small ratio justifies the use of the perturbation calculation of the effect of the ΛNN force. For the case of the TPE ΛNN force the corresponding ratios were found to be 2.6% and 1.3% respectively. Thus although the previous TPE results are modified, the overall effect is still small and can probably be taken into account by a suitable modification of the Λ-N force.

5.2 Effect in ${}^5_\Lambda$He

The Λ-α potential in ${}^5_\Lambda$He due to three-pion-exchange potential P, $P(r_\Lambda)$ is estimated in a way simalar to that used for the TPE ΛNN force (§ 3.2). First as in the previous section, the x-dependence of P(4.33) in ${}^5_\Lambda$He is compared with that of the TPE ΛNN force . The x-dependence of P_c(4.34) and P_T(4.35) respectively can be written :

$$C_3' = S'(x), \qquad \ldots(5.11)$$

$$T_3' = W'(x), \qquad \ldots(5.12)$$

where S'(x) and W'(x) are given by the expressions (4.30) and (4.31) with the expectation values of the τ's and σ's replaced by their values for ${}^{5}_{\Lambda}He$:

$$< \underset{\sim}{\sigma}^{1} \cdot \underset{\sim}{\sigma}^{2} \underset{\sim}{\tau}^{1} \cdot \underset{\sim}{\tau}^{2} > = -3, \qquad \ldots(5.13a)$$

$$< \underset{\sim}{\sigma}^{i} \cdot \underset{\sim}{\sigma}^{\Lambda} \underset{\sim}{\tau}^{i} \cdot \underset{\sim}{\tau}^{\Lambda} > = 0 \quad i = 1,2 . \qquad \ldots(5.13b)$$

Eq.(5.13b) follows from the spin saturation of the nucleons. Because of the difference in the expectations values of the τ's and σ's for ${}^{3}_{\Lambda}H$ and ${}^{5}_{\Lambda}He$, C_3' and T_3' differ from C_3 and T_3, respectively. The curves C_3' and T_3' are plotted in Fig. 5.1 and Fig. 5.2 . It can be seen that C_3' depends strongly on x, is larger than C_2 for $x \lesssim 1.2$ fm but smaller than C_2 for $x \gtrsim 1.6$ fm. The tensor part T_3' is not very sensitive to x, becomes negative for $x > 0.4$ fm and is always smaller in absolute value than T_2 and C_3' . Therefore the Λ-α potential due to C_3' will be positive (as for C_2) and larger than that due to T_3'.

The average one-body field, $P(r_\Lambda)$, that the Λ experiences in ${}^{5}_{\Lambda}He$ will now be estimated. As in (§ 3.2) $P(r_\Lambda)$ can be written :

$$P(r_\Lambda) = 6 \int d^3r_1 \, d^3r_2 \, \rho(r_1) \rho(r_2) \, P(\underset{\sim}{r}_1 - \underset{\sim}{r}_\Lambda, \underset{\sim}{r}_2 - \underset{\sim}{r}_\Lambda, \underset{\sim}{r}_1 - \underset{\sim}{r}_2), \qquad \ldots(5.14)$$

where the factor 6 comes from the six possible ΛNN bonds and $\rho(r_i)$ is the normalized density distribution for the nucleons which has been defined by eq. (3.15). The vectors $(\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{z})$ have been replaced by $(\underset{\sim}{r}_1 - \underset{\sim}{r}, \underset{\sim}{r}_2 - \underset{\sim}{r}, \underset{\sim}{r}_1 - \underset{\sim}{r}_2)$ (see Fig. 4.3). The potential $P(\underset{\sim}{r}_1 - \underset{\sim}{r}_\Lambda, \underset{\sim}{r}_2 - \underset{\sim}{r}_\Lambda, \underset{\sim}{r}_1 - \underset{\sim}{r}_2)$ is obtained from eq. (4.32) using eqs. (5.13a) and (5.13b). In this calculation the α was assumed to be undistorted by the presence of the Λ. The potential $P(r_\Lambda)$ can be written :

$$P(r_\Lambda) = P_c(r_\Lambda) + P_T(r_\Lambda) \qquad \ldots(5.15)$$

where

$$P_c(r_\Lambda) = 12 \int d^3x \, d^3z \, \rho(|\underset{\sim}{x} + \underset{\sim}{r}_\Lambda|) \rho(|\underset{\sim}{x} + \underset{\sim}{r}_\Lambda - \underset{\sim}{z}|) \, S'(x) \, Y(z) \qquad \ldots(5.16)$$

and

$$P_T(r_\Lambda) = 12 \int d^3x\, d^3z\, \rho(|\underset{\sim}{x} + \underset{\sim}{r}_\Lambda|)\, \rho(|\underset{\sim}{x} + \underset{\sim}{r}_\Lambda - \underset{\sim}{z}|)\, (3\cos^2\theta_{xz} - 1)\, W'(x)\, T(z)\, Y(z) \quad \ldots(5.17)$$

where $\cos\theta_{xz} = \underset{\sim}{x}\cdot\underset{\sim}{z}/xz$. The additional factor of two is due to the symmetry of the expressions (4.34) and (4.35) in x and y.

Equation (5.17) can be reduced to a four-dimensional integration which is done numerically. Similarly the integral (5.18) can be split into four-dimensional and five-dimensional integral, both of which are done numerically. Details about the reduction of the integrals are given in appendix 3. Since the short range part is not known, the spatial integration on the N-N distance z is done with z varying from a cutoff distance d_{NN} to infinity. As it will be seen, results are quite insensitive to d_{NN}. In fact the integrals (5.16) and (5.17) are even convergent for z starting from zero.

The results for five different values of r_Λ, two different values of the cutoff $d_{\Lambda N}$ and two different values of d_{NN} are shown in table 5.2. It can be seen that the results are sensitive to $d_{\Lambda N}$, but not to d_{NN}, and that the potential due to the central part is repulsive and larger than that due to the tensor part which is mostly attractive and small (for $r_\Lambda \approx 2$ fm $P_T(r_\Lambda)$ is repulsive). The potentials $P_c(r_\Lambda)$ and $P_T(r_\Lambda)$ are plotted on Fig. 5.3 for $d_{\Lambda N} = 1$ fm and $d_{NN} = 0.6$ fm and compared with the Λ-α potential , $U_p^3(\mathrm{I})$ (central) and $U_p^3(\mathrm{II})$ (tensor), of the TPE ΛNN force. It can be seen that $P(r_\Lambda)$ is smaller than $U_p^3(r_\Lambda)$ ($= U_p^3(\mathrm{I}) + U_p^3(\mathrm{II})$) and then reduces the overbinding of the Λ in ${}^5_\Lambda$He by an amount smaller than the reduction obtained using $U_p^3(r_\Lambda)$. Therefore in the case of ${}^5_\Lambda$He, for $d_{\Lambda N} \approx 1$ fm, the three-pion-exchange ΛNN force only slightly modifies the results obtained using the TPE ΛNN force.

5.3 Effect in Nuclear Matter

As in (§ 3.3) the binding energy P_Λ of a Λ particle in nuclear matter due to P_Λ is calculated by first order perturbation theory in a way similar to that formulated by Bodmer and Sampanthar (1962). Because two nucleons are involved there is a direct term and an exchange term. The direct term vanishes identically because of the spin isospin saturation. As in the case of ${}^{5}_{\Lambda}He$

$$\langle \underset{\sim}{\sigma}^{i} \cdot \underset{\sim}{\sigma}^{\Lambda} \quad \underset{\sim}{\tau}^{i} \cdot \underset{\sim}{\tau}^{\Lambda} \rangle = 0 .$$

Therefore the comparison between the x-dependence of the three-pion-exchange potential and the two-pion one is essentially the same as that discussed in the previous section. The only difference is a common multiplicative factor arising from the fact that

$$\langle \underset{\sim}{\sigma}^{1} \cdot \underset{\sim}{\sigma}^{2} \quad \underset{\sim}{\tau}^{1} \cdot \underset{\sim}{\tau}^{2} \rangle = 36 . \qquad \ldots(5.18)$$

Therefore the contribution of the central part P_c(4.34) to the binding energy, $P_{\Lambda c}$, is again expected to be predominant.

P_Λ can be written

$$P_\Lambda = P_{\Lambda c} + P_{\Lambda T} \qquad \ldots(5.19)$$

where $P_{\Lambda T}$ is the contribution from the tensor part P_T(4.35).

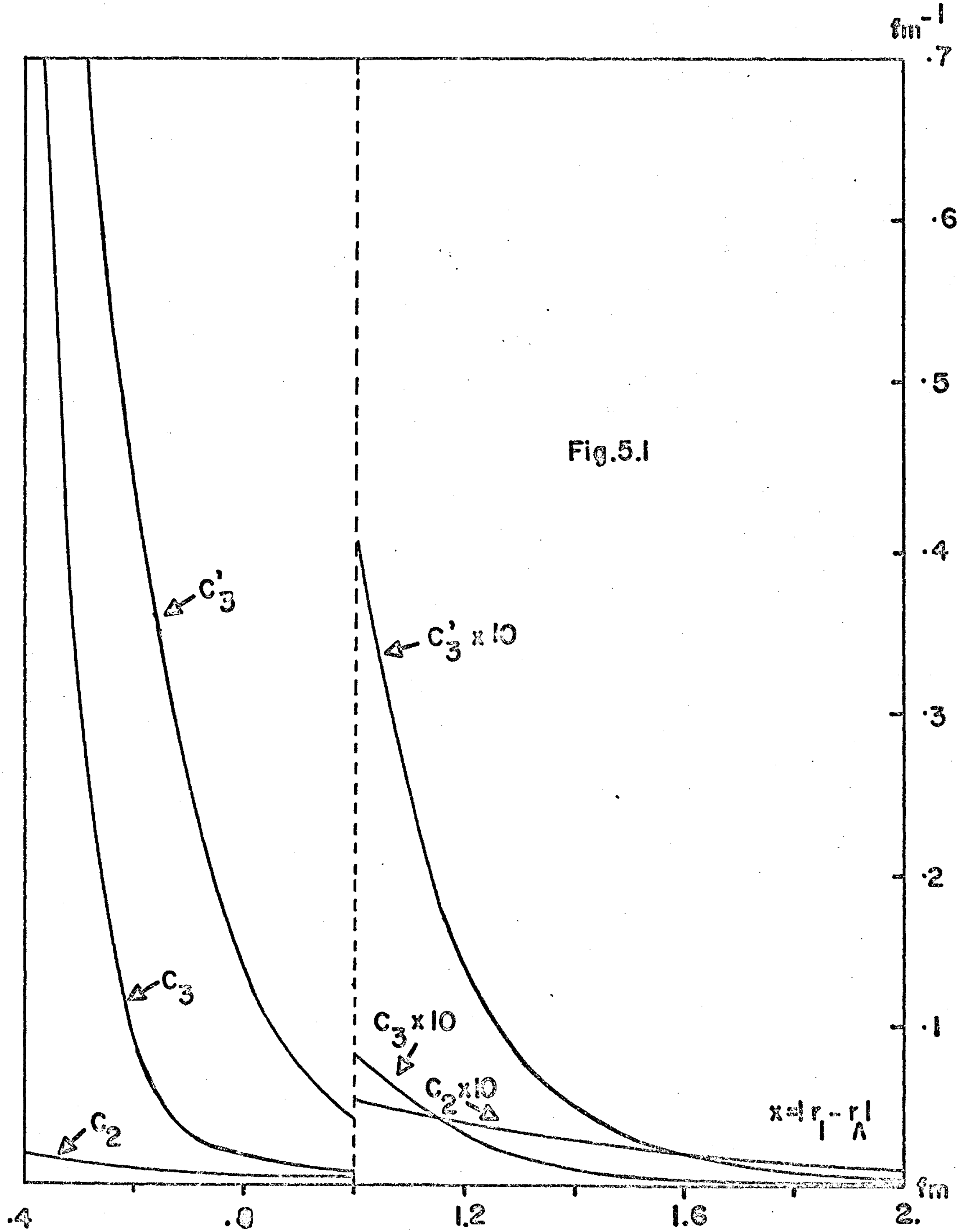
A straight forward calculation yields

$$P_{\Lambda c} = \frac{3}{4} \rho^2 \int D^2(k_f | \underset{\sim}{x} - \underset{\sim}{y} |) \, S'(x) \, Y(| \underset{\sim}{x} - \underset{\sim}{y} |) \, d^3x \, d^3y , \qquad \ldots(5.20)$$

$$P_{\Lambda T} = \frac{3}{4} \rho^2 \int D^2(k_f |\underset{\sim}{x} - \underset{\sim}{y}|)\, W'(x)(3\cos^2\theta_{xz} - 1)Y(|\underset{\sim}{x} - \underset{\sim}{y}|)T(|\underset{\sim}{x} - \underset{\sim}{y}|)d^3x\, dy^3 \quad \ldots(5.21)$$

ρ, D, k_f have been defined in § 3.3 and $\cos\theta_{xz} = \underset{\sim}{x}.\underset{\sim}{z}/xz$. As in the case of the TPE ΛNN force a step function,(eq. 3.3), is used as a nucleon-nucleon correlation function.

The integrals (5.20) and (5.21) are then evaluated numerically. The results are shown in table 5.3 for two different values of $d_{\Lambda N}$ and four different values of d_{NN}, together with the results obtained for the central part U_Λ (I) and tensor part U_Λ (II) of the TPE ΛNN force (§ 3.3). It can be seen that the effect is repulsive and large, that $P_{\Lambda c}$ is larger than $P_{\Lambda T}$, U_Λ(I) and U_Λ (II) and that for $d_{\Lambda N} \approx 1$ fm $P_{\Lambda T}$ is smaller than U_Λ (II). The main qualitative feature of the effect is that it is found to be repulsive, for instance with a reasonable value for d_{NN} of about 1 fm and that for $d_{\Lambda N} = 1$ fm, P gives a repulsive contribution of 7.43 Mev. For the same values of d_{NN} and $d_{\Lambda N}$, the TPE ΛNN force was found to give a repulsive contribution of 3.78 Mev. For $d_{\Lambda N} \approx 1$ fm, the three-pion-exchange three-body ΛNN force reduces therefore the binding energy of a Λ in nuclear matter by an amount greater than that obtained with the TPE ΛNN force. However the large values of the results obtained in table 5.3 may indicate that the use of a first order perturbation theory is not sufficient, and then their magnitude should not be taken too seriously. Nevertheless the calculation shows that the effect of the three-pion-exchange ΛNN force in nuclear matter can be very important.



fm⁻¹
.7
.6
.5
.4
.3
.2
.1
C'_3
C_3
C_2
$C'_3 \times 10$
$C_3 \times 10$
$C_2 \times 10$
$x = |r_1 - r_\Lambda|$
fm
.4
.8
1.2
1.6
2.

Fig.5.1

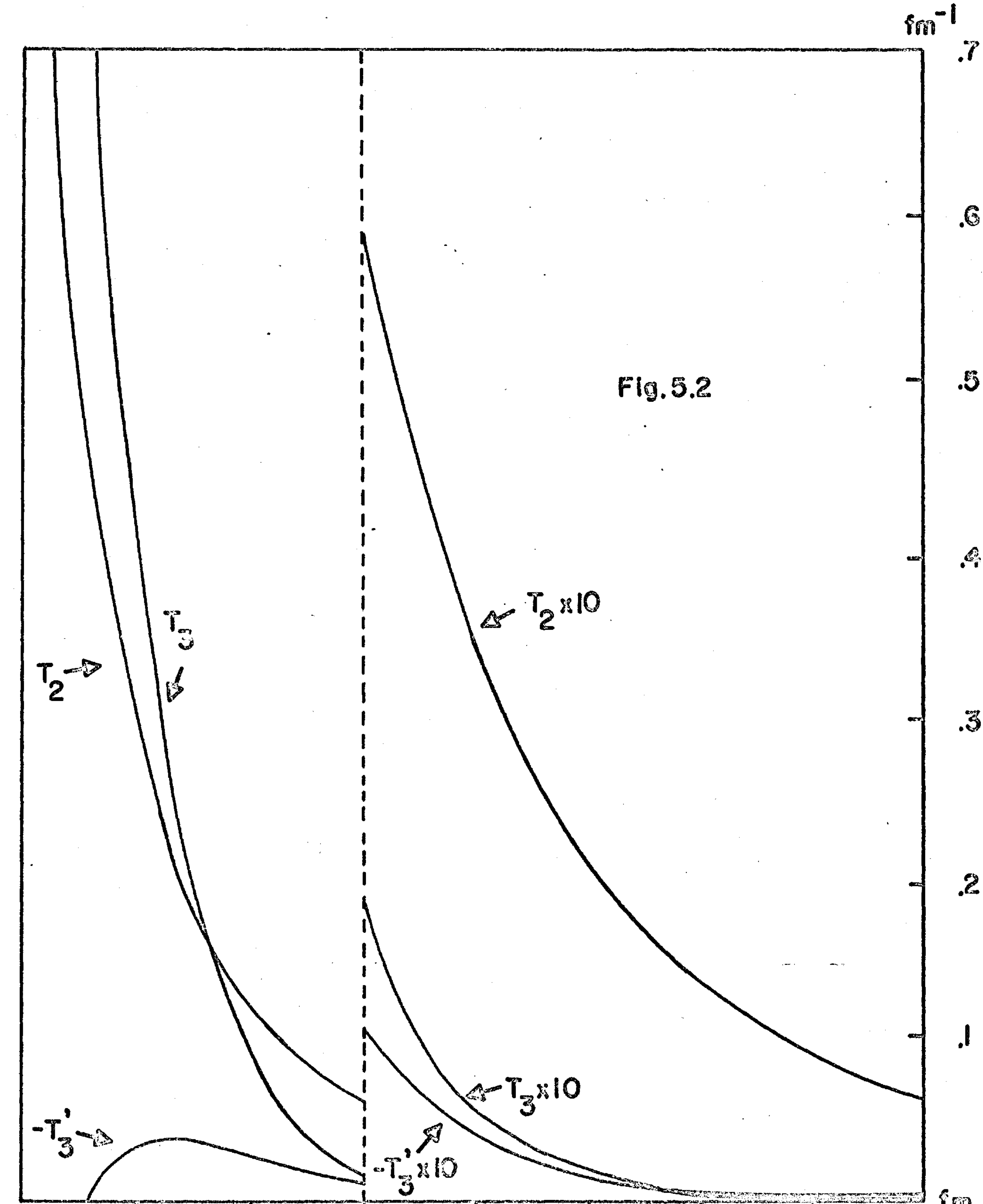


Fig. 5.2

TABLE 5.1

EXPECTATION VALUES IN ${}^{3}_{\Lambda}H$ OF THE THREE-PION-EXCHANGE ΛNN POTENTIAL IN MEV FOR DIFFERENT VALUES OF CUTOFF DISTANCE $d_{\Lambda N}$ AND HARD-CORE RADIUS D. THE RESULTS OBTAINED WITH THE TPE ΛNN FORCE (§ 3.1) ARE SHOWN IN PARENTHESIS.

D(fm)	$d_{\Lambda N}$(fm)	0.2	0.4	0.6	1.	1.4
0.2	$\langle P_c \rangle$	2.297	1.432	0.454	0.096	0.019
	$(\langle W_p(I) \rangle)$	(0.091)	(0.084)	(0.064)	(0.030)	(0.011)
0.4	$\langle P_c \rangle$		0.603	0.384	0.093	0.018
	$(\langle W_p(I) \rangle)$		(0.074)	(0.068)	(0.034)	(0.012)
0.6	$\langle P_c \rangle$			0.368	0.112	0.019
	$(\langle W_p(I) \rangle)$			(0.084)	(0.044)	(0.015)
0.2	$\langle P_T \rangle$	-8.438	-8.416	-4.839	-0.569	-0.078
	$(\langle W_p(II) \rangle)$	(-0.706)	(0.231)	(0.354)	(0.297)	(0.118)
0.4	$\langle P_T \rangle$		-2.659	-2.186	-0.390	-0.057
	$(\langle W_p(II) \rangle)$		(-0.700)	(-0.432)	(0.130)	(0.090)
0.6	$\langle P_T \rangle$			-1.435	-0.389	-0.056
	$(\langle W_p(II) \rangle)$			(-1.010)	(-0.075)	(0.074)

TABLE 5.2

THE Λ- α POTENTIAL IN ${}^{5}_{\Lambda}He$ DUE TO THE THREE-PION-EXCHANGE POTENTIAL FOR DIFFERENT VALUES OF r_{Λ}.

THE NOTATION IS EXPLAINED IN THE TEXT.

$d_{\Lambda N}$ (fm)		0.6					1.				
r_{Λ} (fm)		0.	1.	2.	3.	4.	0.	1.	2.	3.	4.
$P_c(r_{\Lambda})$	d_{NN} = 0.4 fm	35.48	19.71	3.05	0.115	0.066	4.36	3.30	1.07	0.093	0.007
	d_{NN} = 0.6 fm	33.10	19.01	3.33	0.131	0.069	4.52	3.21	1.10	0.101	0.007
$P_T(r_{\Lambda})$	d_{NN} = 0.4 fm	-22.57	- 0.395	0.279	-0.061	-0.005	-6.19	-0.809	0.274	-0.041	-0.005
	d_{NN} = 0.6 fm	-22.10	- 0.436	0.286	-0.061	-0.005	-6.11	-0.807	0.274	-0.041	-0.005

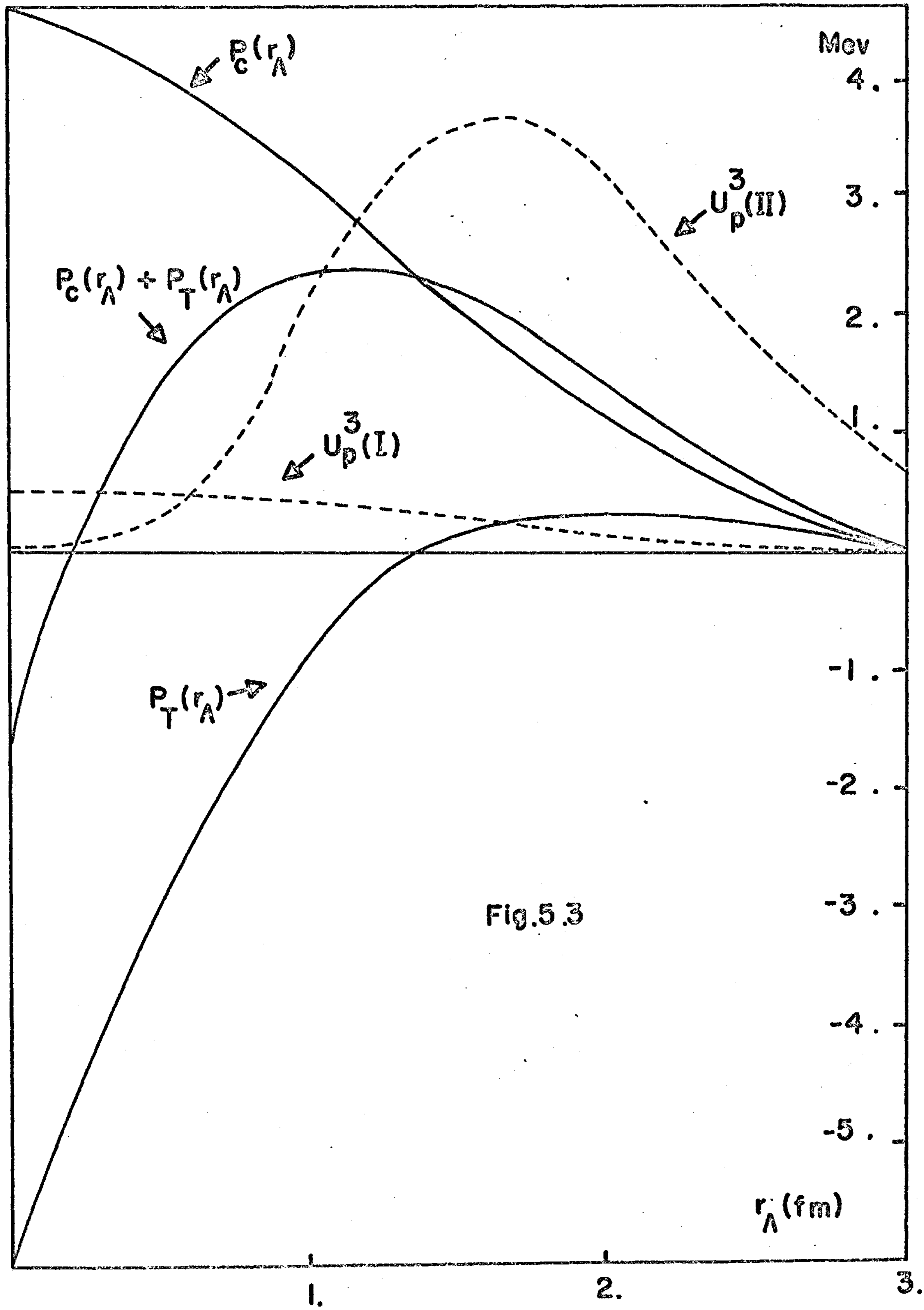

Mev
4.
3.
2.
1.
-1.
-2.
-3.
-4.
-5.
$P_c(r_\Lambda)$
$U_p^3(II)$
$P_c(r_\Lambda) + P_T(r_\Lambda)$
$U_p^3(I)$
$P_T(r_\Lambda)$
r_Λ(fm)
1.
2.
3.

Fig.5.3

TABLE 5.3

EFFECT OF THE THREE-PION-EXCHANGE ΛNN POTENTIALS IN NUCLEAR MATTER. THE RESULTS OBTAINED WITH THE TPE ΛNN FORCE (§ 3.3) ARE SHOWN IN PARENTHESIS. THE NOTATION IS EXPLAINED IN THE TEXT.

$d_{\Lambda N}$(fm)	0.6			1.			
d_{NN}(fm)	0.4	0.6	1.	0.4	0.6	1.	1.4
$P_{\Lambda c}$(Mev) (U_Λ(I))	41.11 (1.36)	35.81 (1.30)	23.66 (1.06)	9.71 (0.84)	8.40 (0.81)	5.52 (0.67)	3.02 (0.47)
$P_{\Lambda T}$(Mev) (U_Λ(II))	13.75 (13.95)	8.52 (4.69)	2.06 (-6.85)	6.38 (12.53)	4.47 (9.95)	1.91 (3.11)	0.60 (-1.35)
$P_{\Lambda c} + P_{\Lambda T}$ (U_Λ(I) + U_Λ(II))	54.86 (15.31)	44.33 (5.99)	25.72 (-5.79)	16.09 (13.37)	12.87 (10.76)	7.43 (3.78)	3.62 (-0.88)

CHAPTER 6

DERIVATION AND EFFECTS OF THE LONG RANGE TPE THREE-NUCLEON FORCE.

6.1 Derivation of the three-nucleon (3N) force

The long-range part of the three-body force arises mainly due to the TPE among three nucleons. The TPE 3N potential has been derived by Fujita and Miyazawa (1957) (FM) in the static approximation using a technique from dispersion theory (See also : Smith and Sharp, 1960; Fujita et al, 1962; Coury and Frank, 1963; Quang Ho-Kim, 1966). The TPE 3N potential consists of three terms,

$$F = F(1) + F(2) + F(3) \qquad \ldots(6.1)$$

where F(3) is due to the process depicted in Fig. 6.1.

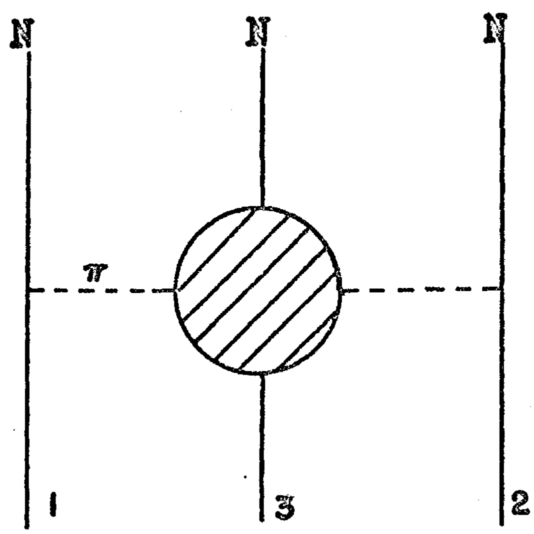


Fig.6.1

The other two terms are obtained by cyclic permutations of 1, 2 and 3. FM's expression for F(i) is further divided into two parts :

$$F(i) = F_p(i) + F_s(i)$$

where $F_p(i)$ and $F_s(i)$ are due respectively to the p- and s-wave scatterings of the virtual pion from the i-th nucleon. In order to obtain a rough idea of the effect of the TPE 3N force, F, in triton, it was assumed by FM that triton consists of an equilateral triangle with side 1.3 fm. The 3N potential energy, F, for this configuration was found to be -0.22 Mev (attractive). The contribution of $W_s(i)$ was found to be negligible.

The pion-pion interaction was not considered by FM. However, in addition to the well established I = J = 1 resonance, or the ρ-meson, there has accumulated in the last few years considerable, although not quite conclusive, evidence for an I = J = 0 resonance. This is the so-called σ-meson, with mass of about 410 Mev (Rosenfeld et al, 1967). The 3N force, which arises from the pion-nucleon interaction via the σ-meson as shown in Fig. 6.2, was examined by Harrington (1966).

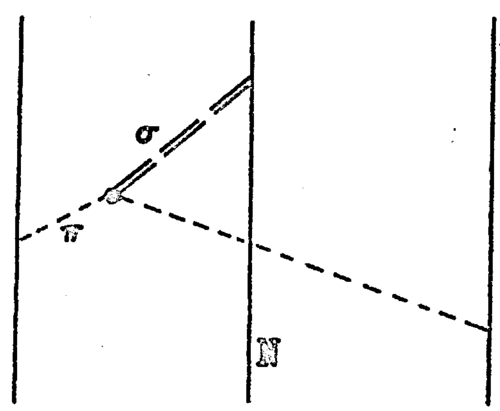


Fig.6.2

The effects of this 3N force in triton and in nuclear matter were then estimated. The effects turned out to be repulsive and quite substantial, especially in nuclear matter. As Harrington himself was aware of, however, it is dangerous to consider the diagram of Fig.6.2 alone. This is clear if the s-wave π-N scattering at low energy is considered. If only the diagram (a) of Fig. 6.3 is considered, where the pion interacts with the nucleon via the σ-meson, it would yield a very large scattering length in drastic disagreement with experiment. There must be other ''direct interactions'', here represented by the diagram (b) in Fig 6.3, which cancel the contribution from (a) so that the experimentally observed extremely small scattering length is reproduced.

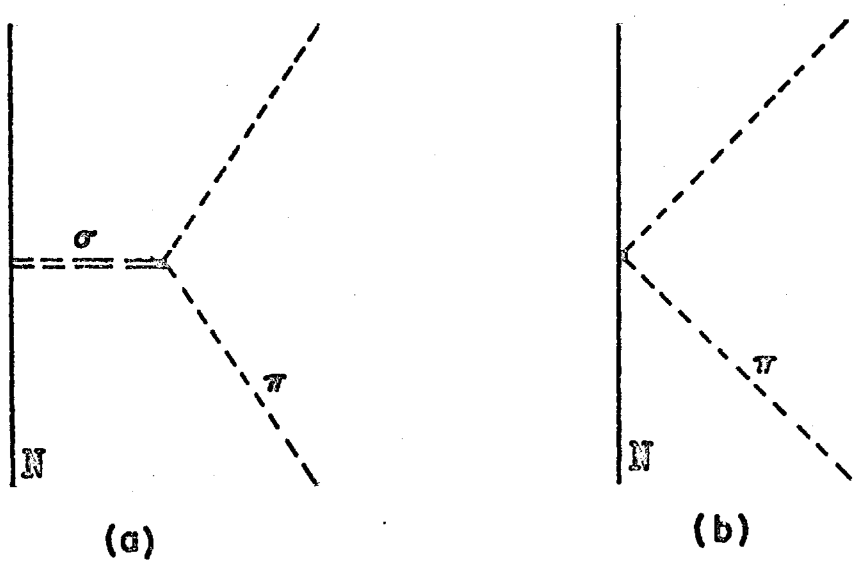


Fig.6.3

Including the effect of the σ-meson, F(i) can be written :

$$F(i) = F_p(i) + F_s(i) + F_\sigma(i) \qquad \ldots(6.3)$$

in place of (6.2). Here $F_\sigma(i)$ and $F_s(i)$ arises through the diagram (a) and (b) of Fig. 6.3 respectively. The potential $F_\sigma(i)$ is the one discussed by Harrington. The potential $F_s(i)$ has the same form as, but stronger than, that in (6.2). The potentials $F_\sigma(i)$ and $F_s(i)$ have opposite signs and, in fact, it will be seen that they almost completely cancel each other in the Fermi-gas model of nuclear matter. Strictly speaking, the diagram (a) in Fig. 6.3 contributes also to the p-wave π-N scattering, hence $F_p(i)$ has to be reajusted too. However, this is a very small effect, and the same $F_p(i)$, as was given by FM, is used here. According to FM, $F_p(i)$ and $F_s(i)$ are given by

$$F_p(3) = -(C_{pN}/8\mu^4)\ [5(\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^3)(\boldsymbol{\tau}^3\cdot\boldsymbol{\tau}^2) + 3(\boldsymbol{\tau}^2\cdot\boldsymbol{\tau}^3)(\boldsymbol{\tau}^3\cdot\boldsymbol{\tau}^1)]$$

$$\times\ (\boldsymbol{\sigma}^1\cdot\boldsymbol{\nabla}_x)(\boldsymbol{\sigma}^3\cdot\boldsymbol{\nabla}_y)(\boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_y)\ Y(x)\ Y(y) + (1 \rightleftarrows 2,\ x \rightleftarrows y), \qquad \ldots(6.4)$$

$$F_s(3) = C_{sN}\ \mu^{-2}\ (\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2)(\boldsymbol{\sigma}^1\cdot\boldsymbol{\nabla}_x)\ (\boldsymbol{\sigma}^2\cdot\boldsymbol{\nabla}_y)\ Y(x)\ Y(y) \qquad \ldots(6.5)$$

with $\quad \mathbf{x} = \mathbf{r}_2 - \mathbf{r}_3\ ,\ \mathbf{y} = \mathbf{r}_1 - \mathbf{r}_3, \qquad \ldots(6.6)$

$$Y(x) = \exp(-\mu x)/(\mu x). \qquad \ldots(6.7)$$

Here $\mathbf{r}_i$ is the coordinate of the i-th nucleon, and μ is the pion mass ($c = \hbar = 1$).

The coefficient C_{pN} is given by[†]

$$C_{pN} = \frac{f_N^2}{9\pi^2} \int_0^\infty \frac{\sigma_{33}(p)}{\omega_p^2} \, dp = 0.46 \text{ Mev} \qquad \ldots(6.8)$$

where $f_N^2 = 0.08$ is the π - N coupling constant and σ_{33} is the total cross section for p-wave π - N scattering in the I = J = 3/2 state. If the σ-meson is not considered, C_{sN} is related to the s-wave π - N scattering lengths, a_1 and a_3, by

$$C_{sN} = -(f_N)^2 (a_1 + 2a_3)/3 \; . \qquad \ldots(6.9)$$

This relation has to be modified if the contribution of the σ-meson is included.

For the σ-meson an effective interaction density for the σ-N and σ-π is assumed to be :

$$H = (4\pi)^{1/2} \left(g\bar{\Psi}\Psi\phi_\sigma + \frac{1}{2} h \; \underset{\sim}{\phi}\cdot\underset{\sim}{\phi}\phi_\sigma \right) \qquad \ldots(6.10)$$

where Ψ, $\underset{\sim}{\phi}$, and ϕ_σ are the nucleon, pion and σ-meson fields, respectively, and g and h are dimensionless coupling constants. Then as was

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[†] For the TPE ΛNN force, the corresponding strength factor is $C_{p\Lambda} \approx 1.43$ Mev (§ 2.1). For the TPE ΛNN force contributions come from the Σ and Y_1^* intermediate state, whereas for the 3N force only the N intermediate state contributes. The difference is thus mainly due to the Σ-contribution. However, there is only one ΛNN-bond in ${}^3_\Lambda H$, compared with 3N-bonds in ${}^3H$. Moreover, the average distance between particles in ${}^3H$ is shorter than in ${}^3_\Lambda H$ Thus it is expected that the effect of the 3N force in ${}^3H$ is more than that of the TPE ΛNN force in ${}^3_\Lambda H$.

shown by Harrington, $F_\sigma(3)$ may be written as

$$F_\sigma(3) = \frac{\lambda m^2}{4\pi^4 \mu^4} \; \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2 \;\; \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\nabla}_1 \;\; \underset{\sim}{\sigma}^2 \cdot \underset{\sim}{\nabla}_2$$

$$\times \int d^3 r \, \frac{\exp(-\mu s_1 - \mu s_2 - m s_3)}{s_1 s_2 s_3} \qquad \ldots(6.11)$$

where $s_i = |\underset{\sim}{r}_i - \underset{\sim}{r}|$, $i = 1, 2, 3$, m is the mass of the σ- meson, and

$$\lambda = f_N^2 g h \mu^3/m^2 \qquad \ldots(6.12)$$

The choice of C_{sN} and λ will be discussed in the next section.

6.2 <u>Effects of the TPE 3N force in triton</u>

Here it is assumed that the triton is in the s-state of complete spatial symmetry. Then the spin-isospin averages of $F_p(i)$ (6.4) and $F_s(i)$ (6.5) become :

$$F_p(3) = C_{pN} (3 \cos^2 \theta_{xy} - 1) \, M(x) \, M(y), \qquad \ldots(6.13)$$

$$F_p(3) = - C_{sN} \cos \theta_{xy} \, G(x) \, G(y) \qquad \ldots(6.14)$$

with

$$M(x) = \left(1 + \frac{3}{\mu x} + \frac{3}{(\mu x)^2}\right) Y(x), \; G(x) = \left(1 + \frac{1}{\mu x}\right) Y(x), \qquad \ldots(6.15)$$

$$\cos \theta_{xy} = (\underset{\sim}{x} \cdot \underset{\sim}{y})/(xy) \qquad \ldots(6.16)$$

Other terms with i = 1 or 2 can be obtained from the above formulae by cyclic permutations of 1, 2 and 3.

As was shown by Harrington, F_σ (3) (6.11) for the triton may be written as

$$F_\sigma(3) = -\frac{\lambda m^2}{4\pi^4 \mu^4} \int \frac{d^3 q_1 \, d^3 q_2 \;\; \mathbf{q}_1 \cdot \mathbf{q}_2 \;\; \exp\left[i(\mathbf{q}_1 \cdot \mathbf{x} + \mathbf{q}_2 \cdot \mathbf{y})\right]}{(q_1^2 + \mu^2)(q_2^2 + \mu^2) \;\; \left[(\mathbf{q}_1 + \mathbf{q}_2)^2 + m^2\right]} . \qquad \ldots(6.17)$$

Now, for the s-wave π-N scattering, the sum of the contributions of the diagrams (a) and (b) of Fig. 6.3 should give the observed (isospin-even part of) scattering length. This is achieved if C_s in (6.9) is replaced by $C_{sN} - \lambda$, namely

$$C_{sN} - \lambda = -\frac{1}{3} (\mu f_N)^2 (a_1 + 2 a_3) \qquad \ldots(6.18)$$

This can be seen in the following way. First, it is noted that the s-wave π-N scattering length does not depend on the shape of the source of the interaction, hence it is independent of m, provided that λ is kept constant. Therefore the limit $m \to \infty$ can be taken without affecting the scattering length. In this limit F_σ (3) becomes

$$F_\sigma(3) \to \lambda \mu^{-2} \, \nabla_x \cdot \nabla_y \, Y(x) \, Y(y) = \cos\theta_{xy} \, G(x) \, G(y) \qquad \ldots(6.19)$$

which is the same form as (6.14). In fact the σ - line in Fig. 6.2 or in diagram (c) of Fig. 6.3 shrinks to a point in this limit, and becomes indistinguishable from the ''direct interaction'' diagram (b) of Fig. 6.3.

Thus (6.18) replaces (6.9) in the limit $m \to \infty$. However, since the two sides of (6.18) are independent of m, this relation should hold for any value of m. Experimental values for the scattering lengths are $(a_1 + 2a_3) = (-0.035 \pm 0.012)/\mu$ (Samarayanake and Woolcock 1965). Substituting $a_1 + 2a_3 = -0.035/\mu$ into (6.18) it can be seen that :

$$C_{sN} - \lambda = 0.13 \text{ Mev} \qquad \ldots(6.20)$$

In (6.17) the long-range part of the potential is determined mainly by the part of the integrand with small momenta q_1 and q_2. Since m ($\sim$ 410 Mev) is much larger than μ, the propagator of the σ-meson may be approximated by

$$\left\{ (q_1 + q_2)^2 + m^2 \right\}^{-1} \to m^2 (q_1^2 + m^2)^{-1} (q_2^2 + m^2)^{-1} . \qquad \ldots(6.21)$$

Then

$$F_\sigma(3) = \lambda' \cos\theta_{xy} \left\{G(x) - G'(x)\right\} \left\{G(y) - G'(y)\right\} , \qquad \ldots(6.22)$$

where

$$\lambda' = \lambda m^4/(m^2 - \mu^2)^2 , \qquad \ldots(6.23)$$

and

$$G'(x) = \left(\frac{m}{\mu} + \frac{1}{\mu x}\right) \frac{e^{-\mu x}}{\mu x} . \qquad \ldots(6.24)$$

For the parameters g, h and m, Harrington assumed

$$g^2 = 10, \quad h^2 = 6, \quad g\,h > 0, \quad m = 4\mu , \qquad \ldots(6.25a)$$

which give

$$C_s = 5.48 \text{ Mev}, \quad \lambda = 5.35 \text{ Mev} \qquad \ldots(6.25b)$$

Here g and m were taken from an analysis of the N-N scattering in terms of the one-boson-exchange model (Bryan and Scott 1964). The π-σ coupling constant h was determined assuming the width of the σ-meson to be $\Gamma = \frac{1}{2}\mu$. The sign g h > 0 was suggested from the analysis of the two-pion contribution to the π-N scattering. A more recent analysis of the N-N scattering gives much smaller g^2(2.3 to 2.8) and slightly smaller m (420 to 470 Mev) (Arndt et al, 1966). Therefore the following set is also considered here :

$$g^2 = 2.5, \; h^2 = 6, \; g\,h > 0, \; m = 3\mu \; . \qquad \ldots(6.26a)$$

In this case, it can be seen that :

$$C_{sN} = 4.88 \text{ Mev}, \quad \lambda = 4.75 \text{ Mev} \qquad \ldots(6.26b)$$

Now the expectation values of $F_p(3)$, $F_s(3)$ and $F_\sigma(3)$ in the triton are estimated by perturbation theory. Since the short range part of the three nucleon force is not known, the three-body potentials $F_p(3)$, $F_s(3)$ and $F_\sigma(3)$ are taken to be zero for N-N distances less than a cutoff distance d. The unperturbed wavefunction is taken to be the wavefunction given by Ohmura (1959). Its spatial part is given by

$$\Psi(x,y,z) = N^{-\frac{1}{2}} \; f(x)\, f(y)\, f(z) \qquad \ldots(6.27)$$

where x,y and z are distances between the three nucleons and

$$\begin{aligned} f(x) &= 0 && \text{for } x < D \\ &= \exp\left[-\alpha(x - D)\right] - \exp\left[-\beta(x - D)\right] && \text{for } x > D. \end{aligned} \qquad \ldots(6.28)$$

Here D is the hard - core radius of the N-N force . The factor N normalizes Ψ to

$$\int \Psi(x,y,z)^2 \; xyz \; dx \; dy \; dz = 1 \qquad \ldots(6.29)$$

where it is understood that x, y, z satisfy the triangular relations $x + y \geqslant z$ etc... The values of the variational parameters α, β and the normalization factor N are listed in table 6.1 for the hard-core radii D = 0.2 , 0.4 and 0.6 fm. The expectation values of various parts of the 3N potential are listed in table 6.2 , where the notation is :

$$\Delta E_p = 3 \langle F_p(3) \rangle \equiv \int F_p(3) \; \Psi(x,y,z)^2 \; xyz \; dx \; dy \; dz \qquad \ldots(6.30)$$

$$\Delta E_s = 3 \langle F_s(3) \rangle , \quad \Delta E = 3 \langle F(3) \rangle , \quad \Delta E = \Delta E_p + \Delta E_s + \Delta E$$

D (fm)	α (fm^{-1})	β (fm^{-1})	N (fm^6)
0.2	0.462	5.03	1.084
0.4	0.457	4.09	1.676
0.6	0.450	4.20	2.718

TABLE 6.1

THE PARAMETERS OF THE TRITON WAVE FUNCTION †

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† This is taken from table 2 of the work of Ohmura (1959), for the case of exponential N-N potentials and r_{os} = 2.7 fm.

TABLE 6.2

THE EXPECTATION VALUE IN MEV OF THE 3N POTENTIAL IN THE TRITON AS DEFINED BY EQ.(6.30), FOR DIFFERENT HARD-CORE RADIUS D, CUTOFF DISTANCE d AND σ-MESON MASS m .

D(fm)	0.2			0.4			0.6	
d(fm)	0.2	0.6	1.0	0.4	0.6	1.	0.6	1.0
ΔE_p	-1.87	-0.14	0.27	-0.76	-0.60	0.01	-0.41	-0.14
$m = 4\mu$								
ΔE_s	-3.28	-2.85	-1.37	-1.53	-1.52	-1.06	-0.91	-0.80
ΔE_σ	2.21	2.03	1.18	1.27	1.26	0.94	0.83	0.74
ΔE	-2.94	-0.96	0.08	-1.02	-0.86	-0.11	-0.49	-0.20
$m = 3\mu$								
ΔE_s	-2.92	-2.54	-1.22	-1.30	-1.35	-0.94	-0.81	-0.71
ΔE_σ	1.44	1.34	0.84	0.90	0.89	0.69	0.71	0.56
ΔE	-3.35	-1.34	-0.11	-1.22	-1.06	-0.24	-0.51	-0.29

ΔE_p does not involve m. For ΔE_s and ΔE_σ, two cases are considered, namely $m = 4\mu$ and $m = 3\mu$. ΔE_s and ΔE_σ are both quite large but they tend to cancel each other. $\Delta E_s + \Delta E_\sigma$ is negative (attractive) and is more appreciable for smaller mass m. In the limit $m \to \infty$, as it has been discussed before, F_s and F_σ becomes indistinguishable. The value of $\Delta E_s + \Delta E_\sigma$ in this limit, is obtained by multiplying ΔE_s for $m = 4\mu$ by

$0.13/5.48 = 0.024$: For example if $D = d = 0.4$ fm, $\Delta E_s + \Delta E_\sigma$ becomes $- 0.04$ Mev, which is much smaller than ΔE_p. Results are all sensitive to d as well as to D. Presumably the most reasonable hard-core radius will be $D = 0.4$ fm. ΔE_p is most sensitive to d and can be much more appreciable than FM's simple estimate $- 0.22$ Mev. The total contribution to the binding energy can easily be as large as or even larger than 1 Mev (attractive).

The expectation value of F_p in triton was calculated by Pask (1967) who used a wave function for ^3H obtained from detailed variational calculations done by Davies (1967). The result is of the same sign and same order of magnitude as the one obtained here (- 1.38 Mev, attractive).

6.3 Effect in nuclear matter

The effect of F_σ and F_s, in nuclear matter is first considered. The contribution of F_σ has been estimated by Harrington(1966) who calculated the effective two-body potential obtained by averaging the 3N potential F_σ over the coordinate of the third particle. This potential was found to be repulsive and strong enough to dominate the OPEP at distances less than 2 fm. However, it will be shown here, that F_s yields a similar effective two-body potential which largely cancels the effect of F_σ, leaving a weakly attractive potential.

The effective two-body potential due to F_σ (3) (6.11) is given by

$$V_\sigma \ (12) = \rho \int d^3 r_3 \ F_\sigma \ (3)$$

$$= -\rho \lambda \ \mu^{-2} \ (\underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2)(\underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\nabla}_1) \ \underset{\sim}{\sigma}^2 \cdot \underset{\sim}{\nabla}_2 \ \times$$

$$\int d^3 r \, Y(s_1) \, Y(s_2), \qquad \ldots(6.31)$$

where ρ is the density of nucleons in nuclear matter. On the other hand, the effective two-body force due to $F_s(3)$ (6.5) is :

$$V_s(12) = \rho \, C_{sN} \, \mu^{-2} \, \underset{\sim}{\tau}^1 \cdot \underset{\sim}{\tau}^2 \; \underset{\sim}{\sigma}^1 \cdot \underset{\sim}{\nabla}_1 \; \underset{\sim}{\sigma}^2 \cdot \underset{\sim}{\nabla}_2 \int d^3 r_3 \, Y(x) \, Y(y)$$

$$\ldots(6.32)$$

The integrals in (6.31) and (6.32) are identical. Since $\lambda \approx C_{sN}$(6.20), V_σ (12) and V_s(12) almost completely cancel each other.

The expectation value of F_p in nuclear matter,using the Fermi gas model will now be estimated.The calculation is similar to that of§3.3.There is a direct term which vanishes identically because of spin isospin saturation. There are two exchange terms and the binding energy due to F_p in nuclear matter can be written :

$$U = U_1 + U_2 \qquad \ldots(6.33)$$

The single exchange term U_1 is

$$U_1 = \frac{A}{4} C_{pN} \, \rho^2 \int D^2(k_f z) \left[1 + (3\cos^2\theta_{xy} - 1) \; T(x) \; T(y)\right]$$

$$\times \, Y(x) \, Y(y) \; d^3x \, d^3y, \qquad \ldots(6.34)$$

and the double exchange term U_2 is :

$$U_2 = -\frac{A}{4} C_{pN} \rho^2 \int D(k_f x)\, D(k_f y)\, D(k_f z)$$

$$\left[1 + \frac{1}{4}(3 \cos^2 \theta_{xy} - 1)\, T(x)\, T(y)\right] \; Y(x)\, Y(y)\, d^3x\, d^3y. \qquad \ldots(6.35)$$

The function D(r) has been defined in § 3.3. The binding energies U_1 and U_2 consist of a central and a tensor term. The central terms $U_1(I)$ and $U_2(I)$ are given by the first terms of the integrals (6.34) and (6.35) respectively. and the tensor terms $U_1(II)$ and $U_2(II)$ by the second terms of the same integrals. The integral in U_1 is the same as the integral in $U_\Lambda(I) + U_\Lambda(II)$ (3.30 and 3.31) and

$$\frac{U_1(I)}{U_\Lambda(I)} = \frac{U_1(II)}{U_\Lambda(II)} = \frac{2}{3} \frac{C_{pN}}{C_{p\Lambda}} \times A = 0.2145 \times A \qquad \ldots(6.36)$$

As in the case of the TPE ΛNN force a step function is used as a N-N correlation function

$$\begin{aligned} \theta(|\underset{\sim}{x} - \underset{\sim}{y}| - c) &= 0 \qquad \text{for } |\underset{\sim}{x} - \underset{\sim}{y}| < c, \\ &= 1 \qquad \text{for } |\underset{\sim}{x} - \underset{\sim}{y}| > c. \end{aligned} \qquad \ldots(6.37)$$

The energies $U_1(I)$ and $U_1(II)$ are then obtained by multiplying $U_\Lambda(I)$ and $U_\Lambda(II)$ by the factor defined in (6.36). The integral (6.35) is evaluated numerically after putting in the N-N correlation function $\theta(|\underset{\sim}{x} - \underset{\sim}{y}| - c)$ in the integrand. The results are shown in table 6.3 for two different values of d and three different values of c.

TABLE 6.3

EFFECT OF THE TPE 3N POTENTIALS IN NUCLEAR MATTER. THE NOTATION IS EXPLAINED IN THE TEXT. d IS THE CUTOFF FOR THE 3N FORCE, WHILE C IS THE CUTOFF FOR A STEP-FUNCTION TYPE N-N CORRELATION.

d	0.6			1.0		
c	0.4	0.6	1.0	0.6	1.0	1.4
$U_1(I)/A$	0.292	0.278	0.227	0.173	0143	0.099
$U_1(II)/A$	2.990	1.060	-1.469	2.130	0.667	-0.290
$U_2(I)/A$	-0.164	-0.164	-0.137	-0.060	-0.060	-0.045
$U_2(II)/A$	0.437	0.437	0.718	0.159	0.159	0.215
U/A	3.558	1.559	-0.660	2.405	0.910	-0.119

For the values of d and c considered here, $U_1(I)/A$ and $U_2(II)/A$ are small and positive (repulsive), $U_2(I)/A$ is small and negative (attractive) and $U_1(I)/A$ is dominant and either positive (repulsive) or negative(attractive). For d = 0.6 fm, c = 0.4 fm the overall effect U/A is 3.558 Mev (repulsive) and for d = 0.6 fm, and c = 1 fm U/A = - 0.660 Mev (attractive). In summary the effect is small and sensitive to the N-N correlation function.

The contribution of the three-nucleon force, in nuclear matter, has been estimated by several authors (FM; Smith and Sharp, 1960). The results obtained by FM are :

$$U_1 = -0.3 \text{ Mev} \times A \qquad \text{(attractive)}$$

$$U_2 = +0.1 \text{ Mev} \times A \qquad \text{(repulsive)}$$

No details of the calculation are given. In the calculation of Smith and Sharp an angular average of the tensor part is taken and it is found that:

$$1 \text{ Mev} \times A \leqslant U_1 + U_2 \leqslant 3 \text{ Mev} \times A \qquad \text{(repulsive)}$$

These results cannot be directly compared to those of the present work since they are obtained with a different N-N correlation function.

As was mentioned in chapter 2, the contribution of the pion-pion interaction via the σ-meson has not been considered in the calculation of the TPE ΛNN force. However from the results of the present chapter it can be expected that this force will give an appreciable effect in hypertriton as F_σ does in triton. In the case of $^5_\Lambda$He and nuclear matter, since the contribution of the p-wave part (π-Λ interaction) of the TPE ΛNN force is dominant over that of the s-wave part, the effect of the σ-meson is expected to be small.

CHAPTER 7

DISCUSSION AND CONCLUSION

Three-body and many-body forces are predicted from the meson theory of nuclear forces. In the present work the long and intermediate range parts of the ΛNN force due to the two-and three-pion-exchange have been derived using the static approximation. Their effects on the binding energies of ${}^{3}_{\Lambda}H$, ${}^{5}_{\Lambda}He$ and nuclear matter have been estimated. Only the tail of these potentials was considered, since at short distances heavier meson exchanges may become important. The ΛNN force was arbitrarly taken to be zero for Λ-N distances less than a cutoff distance $d_{\Lambda N}$. The contributions of the two-pion-exchange part of the three-nucleon (3 N) force to the binding energy of the triton and nuclear matter have also been considered. Here again the force was taken to be zero for N-N distances less than a cutoff distance d. It was found that the effects of the ΛNN force can be important and also that the contribution of the 3N force is appreciable, although these results depend on the cutoff distances quite sensitively.

In deriving the TPE ΛNN force, W, the π-Λ interaction was considered to be dominated by the p- and s-wave. Then W can be written : $W = W_p + W_s$, where W_p and W_s arise from the p- and s-wave π-Λ interaction, respectively. As in the case of the s-wave π-N interaction, a ''suppression factor'' of the s-wave π-Λ interaction was here introduced for W_s. For a reasonable ''suppression factor'', W_s was found to be unimportant. W_p consist of central and tensor terms. The tensor term depends on the angle θ_{xy} as shown in Fig. 2.2. This term was found to be dominant.

In $^{3}_{\Lambda}H$, the contribution of W to the binding energy of a Λ, B_Λ, can be repulsive or attractive, depending on the cutoff $d_{\Lambda N}$ and the wavefunction used. A reasonable value for the Λ-N hard-core radius D may be taken to be 0.4 fm, and the cutoff radius for the ΛNN force about 1 fm. In this case W give a repulsive contribution of about 0.16 Mev. Although this is substantial in view of the small value of B_Λ in $^{3}_{\Lambda}H$, this could easily be accommodated for, by slightly changing the two-body Λ-N force.

In $^{5}_{\Lambda}He$, the overall effect is much larger, because there are six ΛNN bonds and also the average Λ-N distance is smaller than that of $^{3}_{\Lambda}H$. Here the contribution to B_Λ from the ΛNN forces is always repulsive, and is about 2 Mev for a cutoff of 1 fm. However, this repulsive effect is problably overestimated since no N-N correlation in the α-particle has been considered. This repulsive contribution of the ΛNN force substantially reduces the overbinding of the Λ in $^{5}_{\Lambda}He$ as was obtained by using the two-body forces alone (Bhaduri et al, 1967).

In nuclear matter, the contribution, $U^{(3)}_{\Lambda}$, of W to B_Λ was estimated by using perturbation theory. The effect is extremely sensitive to $d_{\Lambda N}$ as well as the N-N correlation function. For instance, with a cutoff value d_{NN} = 0.7 fm, for a step function type N-N correlation, $U^{(3)}_{\Lambda}$ = 1.8 Mev for $d_{\Lambda N}$ = 0.6 fm and is about 8.7 Mev for $d_{\Lambda N}$ = 1.0 fm see (Fig. 3.3). However, with a slightly larger value of d_{NN}, $U^{(3)}_{\Lambda}$ may well turn negative for the case where $d_{\Lambda N}$ is 0.6 fm. For $d_{\Lambda N}$ = d_{NN} = 1 fm, a repulsion of 3.535 Mev is obtained. If only the very long-range part of W ($d_{\Lambda N} > 1$) is considered, the effect is definitely repulsive and can be as large as 10 Mev.

In the short-range region, processes other than TPE will become important, so that the above results will be modified. The next lowest order contribution to the ΛNN force, after TPE, arises from three-pion-exchange. The three-pion-exchange ΛNN force has been derived. Its effects on light hypernuclei and nuclear matter have been compared with those of W_p, in order to see for what value of the cutoff $d_{\Lambda N}$, the TPE ΛNN force, W_p, is dominated by the three-pion-exchange ΛNN force.

The three-pion-exchange ΛNN force arises from diagrams 4.1c and 4.1d which are as yet too complicated to be evaluated exactly. The force arising from diagram 4.1d was approximated by the sum of the forces arising from diagrams 4.1a and 4.1b. This ΛNN force, P, derived in the static approximation, consists of a central and a tensor term, $P = P_c + P_T$, where the tensor term, P_T, depends on the angle θ_{xz} as shown in Fig. 4.3.The effect of the potential due to diagram 4.1(c) was not examined,because of its complexity.

In ${}^3_\Lambda H$, P depends strongly on the Λ-N distance and the contribution to B_Λ of P_T is larger than that of P_c. For $d_{\Lambda N} \lesssim 1$ fm, P_c always gives a positive contribution (repulsive), which is larger than that of the central part of W_p. P_T always gives a negative contribution (attractive) which is larger in absolute value than that of the tensor term of W_p. For $d_{\Lambda N} \lesssim 1$ fm the contribution to B_Λ of P is larger than that of W_p. For a reasonable value of 0.4 fm for the Λ-N hard-core radius, and $d_{\Lambda N} = 1$ fm, P gives an attractive contribution of 0.3 Mev. The contribution of W_p was previously shown to be repulsive (0.16 Mev), therefore the total contribution is attractive (0.14 Mev). In fact for $d_{\Lambda N} \lesssim 1$ fm, the overall effect is found to be always attractive.

The $\Lambda - \alpha$ potential in ${}^{5}_{\Lambda}He$ due to P, which is denoted by $P(r_\Lambda)$, was evaluated. The contribution $P_c(r_\Lambda)$ (central part) is always repulsive and larger in absolute value than the contribution $P_T(r_\Lambda)$ (tensor part) which is attractive (except for $r_\Lambda \approx 2$ fm). The overall contribution of P is thus repulsive (except for $d_{\Lambda N} = 1$ fm and $r_\Lambda \approx 0$ fm) and it further reduces the overbinding of ${}^{5}_{\Lambda}He$. Results are sensitive to $d_{\Lambda N}$ but not to d_{NN}. For $d_{\Lambda N} \approx 1$ fm, $P(r_\Lambda)$ is smaller than $U_p^3(r_\Lambda)$, the $\Lambda - \alpha$ potential of W_p, and therefore only slightly modifies the contribution of that force.

The contribution of P to the binding energy of Λ in nuclear matter depends strongly on the distance $d_{\Lambda N}$, but its dependence on d_{NN} is not as pronouced. The results in nuclear matter are more sensitive to d_{NN} than those of ${}^{5}_{\Lambda}He$ (see table 5.3 and 5.2, respectively). This can be explained by the fact that the nuclear matter expressions (5.20) and (5.21) have a stronger dependence on NN distance than the corresponding ${}^{5}_{\Lambda}He$ expressions (5.16) and (5.17). It can be also seen that in nuclear matter the tensor part contribution $P_{\Lambda T}$ depends less strongly on d_{NN} than the corresponding contribution $U_\Lambda(II)$ of the TPE ΛNN force (table 5.3). This is due to the difference in angular dependence of the two forces : P_T depends on the angle θ_{xz} (sensitive to the Λ-N distance) whereas the tensor part of W_p depends on the angle θ_{xy} (sensitive to the NN distance) (see Fig. 4.3) . The contribution of the central part P_c is found to be repulsive and larger than that of the tensor part P_T which is also repulsive. This total repulsive contribution is larger than that of W_p. For instance, with a cutoff for a step function type NN correlation of 1 fm and for

$d_{\Lambda N} = 1$ fm, B_Λ due to P is - 7.43 Mev and B_Λ due to W_p was found to be - 3.78 Mev. As in the case of W_p, P may have been overestimated because the coupling constant f_Λ^2 was chosen equal to $f_N^2(f_\Lambda^2 = f_N^2 = 0.08)$. Various estimated of f_Λ^2 have been done which indicated that f_Λ^2 is slightly smaller than f_N^2 (§ 2.1) . However, P does indicate the order of magnitude of the intermediate range part of the ΛNN force.

In summary, the contribution of the long-range ΛNN force, W, and the intermediate range ΛNN force, P, to the binding energy of a Λ-particle in s-shell hypernuclei, has been shown to be important. In ${}^3_\Lambda$H, P was found to be attractive; for $d_{\Lambda N} \approx 1$ fm the contribution of P is greater than that of W_p, although the overall effect $(P + W_p)$ is still relatively small and is attractive. The force P further reduces the binding energy of ${}^5_\Lambda$He; for $d_{\Lambda N} \approx 1$ fm P only slightly modifies the results obtained using W_p alone. In nuclear matter the contribution of P is repulsive and greater than that of W_p; even for $d_{\Lambda N} \approx 1$ fm the overall effect $(P + W_p)$ is quite large and is repulsive. Although there are ambiguities due to the unknown short-range part of the ΛNN force, it can be seen that the effects of the three-body ΛNN force cannot be ignored . Also, the '' effective'' Λ-N force extracted from binding energy data, assuming only a two-body force, can be significantly different from the ''free'' Λ-N force observed in two-body scattering.

The contributions of the long range part of the 3N force arising from two-pion-exchange to the binding energies of the triton and nuclear matter have been estimated. The effect of the pion-pion interaction was taken into account by the consideration of the contribution, E_σ , of the (virtual) π-N scattering via the σ -meson (the σ -meson is a controversial

$I = J = 0$ dipion resonance). Then the 3N potential can be written, $F = F_p + F_s + F_\sigma$, where F_p and F_s are due respectively to the p- and s-wave π-N scattering. The s-wave ''direct'' π-N interaction was formulated so that, together with the π-N interaction via the σ-meson it reproduces the observed π-N scattering length. It was found that the potentials F_s and F_σ have opposite signs and that they tend to cancel each other.

The contribution of F to the binding energy of the triton has been estimated by perturbation theory. The triton wavefunction is taken from a variational calculation for a hard-core two-nucleon potential (hard-core radius D). The results were found to be sensitive to the cutoff of the 3N force, d, as well as to D. The contribution of F_σ and F_s are large but their sum is relatively small and negative(attractive). The effect due to F_p is found to be mainly attractive. At $D = 0.4$ fm and $d \approx 0.4$ fm, the binding due to F is around 1 Mev (attractive). A perturbation calculation was done to estimate the effect of F_p in nuclear matter. This contribution is of the order of a few Mev, between - 0.7 Mev (attractive) and 4.0 Mev (repulsive), depending on the value of d and on the nucleon-nucleon correlation function. As in the case of the ΛNN force, the three-pion-exchange 3N force will probably modify the above results.

Since, in both cases (ΛNN and 3N forces), the short range part of the force is unknown it is not possible to draw any definite conclusion. Nevertheless the results of the present work clearly show that the three-body force can play an important role in nuclear structure problems.

APPENDIX 1

DETAILS OF THE CALCULATION OF THE EFFECT OF THE TPE ΛNN FORCE ON $^5_\Lambda He$

In this appendix some calculational details on $^5_\Lambda He$ (§ 3.2) are given, in particular the angular integration in (3.20) and (3.21). The z-axis is taken along $\underset{\sim}{r}_\Lambda$ and polar coordinates for $\underset{\sim}{x}$ and $\underset{\sim}{y}$ are introduced as follows

$$\underset{\sim}{x} = (x, \theta_x, \varphi_x), \quad \underset{\sim}{y} = (y, \theta_y, \varphi_y) \qquad \ldots(A1.1)$$

Then the φ-integration of $\cos^2\theta_{xy}$ becomes

$$\int_0^{2\pi} d\varphi_x \int_0^{2\pi} d\varphi_y \cos^2\theta_{xy}$$

$$= \int_0^{2\pi} d\varphi_x \int_0^{2\pi} d\varphi_y \left\{\cos\theta_x \cos\theta_y + \sin\theta_x \sin\theta_y \cos(\varphi_x - \varphi_y)\right\}^2$$

$$= \int_0^{2\pi} d\varphi_x \int_0^{2\pi} d\varphi_y (\cos^2\theta_x \cos^2\theta_y + \frac{1}{2}\sin^2\theta_x \sin^2\theta_y). \qquad \ldots(A1.2)$$

Thus the angular integral of the tensor part of W_p (2.16) becomes

$$\int d^3x \int d^3y\, M(x)\, M(y)\, (3\cos^2\theta_{xy} - 1)$$

$$= \int d^3x \int d^3y\, M(x)\, M(y) \left\{3\cos^2\theta_x \cos^2\theta_y + \frac{3}{2}\sin^2\theta_x \sin^2\theta_y - 1\right\}$$

$$= \frac{1}{2}\int d^3x \int d^3y\, M(x)\, M(y)\, (3\cos^2\theta_x - 1)(3\cos^2\theta_y - 1)$$

$$= \frac{1}{2} \left\{ \int d^3x \, F(x) \, (3 \cos^2 \theta_x - 1) \right\}^2 , \qquad \ldots (A1.3)$$

which explains (3.20).

The angular integral of $\bar{W}_s$ is simpler. Since the second term in

$$\cos \theta_{xy} = \cos \theta_x \cos \theta_y + \sin \theta_x \sin \theta_x \sin \theta_y \cos (\varphi_x - \varphi_y)$$

vanishes after φ-integration, it can be see that

$$\int d^3x \int d^3y \, M(x) \, M(y) \cos \theta_{xy} = \left\{ \int d^3x \, M(x) \cos \theta_x \right\}^2 \qquad \ldots (A1.4)$$

which gives (3.21).

The radial integrations in (3.19) - (3.21) can be done as follows.

For (3.19) it can be seen that :

$$\int d^3x \, \rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x}|) \, Y(x) = \frac{2\pi \, \rho(r)}{\beta^2 \, \mu r} \int_{d_{\Lambda N}}^{\infty} dx \, e^{-\beta^2 x^2 - \mu x} \cosh(2\beta^2 rx)$$

$$= \frac{e^{-(\mu/2\beta)^2}}{2\mu r} \left\{ e^{-\mu r} \left[1 - \mathrm{erf}\left(\beta(d_{\Lambda N} - r) + \frac{\mu}{2\beta}\right) \right] - e^{+\mu r} \left[1 - \mathrm{erf}\left(\beta(d_{\Lambda N} + r) + \frac{\mu}{2\beta}\right) \right] \right\} ,$$

$$\ldots (A1.5)$$

where

$$\mathrm{erf}(x) = (2/\pi^{1/2}) \int_0^x \exp(-t^2)\,dt \qquad \ldots(\mathrm{A1.6})$$

The integral (A1.5) is not very sensitive to the cutoff $d_{\Lambda N}$.

For (3.20) it can be seen that :

$$\int d^3x \; \rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x}|)\,(3\cos^2\theta_x - 1)\,T(x)\,Y(x)$$

$$= \frac{4\pi\,\rho(r)}{\beta^2\,\mu r} \int dx\, e^{-\beta^2 x^2 - \mu x}\, T(x) \left\{\left(1 + \frac{3}{z^2}\right)\sinh z - \frac{3\cosh z}{z}\right\}, \qquad \ldots(\mathrm{A1.7})$$

where $\qquad z = 2\beta^2 rx \qquad \ldots(\mathrm{A1.8})$

For small r, $\{\ \}$ in (A1.7) may be expanded as $\{\ \} = z^3/15 + \ldots$. Therefore, the integral (A1.7) vanishes at r = 0, as it should be.

Finally, for (3.21) it can be seen that

$$\int d^3x \;\rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x}|)\cos\theta_x\,(1+\mu x)\,Y(x)/(\mu x)$$

$$= \frac{2\pi\,\rho(r)}{\beta^2\,\mu r}\int_{d_{\Lambda N}}^{\infty} dx\, e^{-\beta^2 x^2}(1+\mu x)\,Y(x)\left\{\cosh z - \frac{\sinh z}{z}\right\} \qquad \ldots(\mathrm{A1.9})$$

where z is defined by (A1.8). For small r, $\{\ \}$ in (A1.9) may be expanded as $\{\ \} = z^2/3 + \ldots$. Therefore (A1.9) vanishes at $r = 0$. The integrals (A1.7) and (A1.9) have been evaluated numerically.

APPENDIX 2

EXPRESSIONS NEEDED IN THE CALCULATION OF THE THREE-PION-EXCHANGE ΛNN FORCE

The explicit form of the expressions (4.32) are given by†

$$Z(r,r') = -\frac{(4\pi)^2}{3}\Big[3 f_N^2 f_\Lambda^2 \Big(F_{\Delta 0}(r,r') + \frac{1}{\Delta} G_{\Delta 0}(r,r')\Big) +$$

$$6 f_N^2 g_\Lambda^2 \Big(F_{\omega_\Lambda 0}(r,r') + \frac{1}{\omega_\Lambda} G_{\omega_\Lambda 0}(r,r')\Big) + 32 g_N^2 f_\Lambda^2 \Big(F_{\omega_N \Delta}(r,r') + \frac{1}{\Delta} G_{\omega_N \Delta}(r,r')\Big) +$$

$$64 g_N^2 g_\Lambda^2 \Big(F_{\omega_N \omega_\Lambda}(r,r') + \frac{1}{\omega_\Lambda} G_{\omega_N \omega_\Lambda}(r,r')\Big)\Big] \qquad \ldots(A2.1),$$

$$D(r,r') = \frac{(4\pi)^2}{3}\Big[3 f_N^2 f_\Lambda^2 \frac{G_{\Delta 0}(r,r')}{\Delta} - 3 f_N^2 g_\Lambda^2 \frac{G_{\omega_\Lambda 0}(r,r')}{\omega} +$$

$$8 g_N^2 f_\Lambda^2 \frac{G_{\omega_N \Delta}(r,r')}{\Delta} - 8 g_N^2 g_\Lambda^2 \frac{G_{\omega_N \omega}(r,r')}{\omega}\Big] \qquad \ldots(A2.2)$$

and

$$X(r,r') = \frac{(4\pi)^2}{3}\Big[3 f_N^2 f_\Lambda^2 \Big(F_{\Delta 0}(r,r') + \frac{1}{\Delta} G_{\Delta 0}(r,r')\Big) + 6 f_N^2 g_\Lambda^2 \Big(F_{\omega 0}(r,r') +$$

$$+ \frac{1}{\omega_\Lambda} G_{\omega_\Lambda 0}(r,r')) - 4\, g_N^2 f_\Lambda^2 \left(F_{\omega_N \Delta}(r,r') + \frac{1}{\Delta} G_{\omega_N \Delta}(r,r')\right) -$$

$$8\, g_N^2 g_\Lambda^2 \left(F_{\omega_N \omega_\Lambda}(r,r') + \frac{1}{\omega_\Lambda} G_{\omega_N \omega_\Lambda}(r,r')\right)\Big] \qquad \ldots(A2.3)$$

with

$$F_{\alpha\beta}(r,r') = \frac{1}{2(2\pi)^3} \frac{1}{r\, r'} \int_0^\infty dk \, \frac{k \sin k(r + r')}{\omega^2 (\omega + \alpha)(\omega + \beta)} \qquad \ldots(A2.4)$$

and

$$G_{\alpha\beta}(r,r') = \frac{1}{2(2\pi)^3} \frac{1}{(\alpha + \beta)} \frac{1}{r\, r'} \int_0^\infty dk \, \frac{k \sin k(r + r')}{(\omega + \alpha)(\omega + \beta)} \left(1 + \frac{\alpha + \beta}{\omega}\right)$$

$$\ldots(A2.5)$$

† As pointed out previously terms proportional to f_N^2 are divided by a factor of two.

APPENDIX 3

CALCULATIONAL DETAILS ON THE EFFECT OF THE THREE-PION-EXCHANGE ΛNN FORCE ON $^{5}_{\Lambda}He$

The reduction of the integrals (5.16) and (5.17) can be done as follows. The $\underset{\sim}{z}$ integration is first considered. Take $\underset{\sim}{x}$ as the Z-axis and the plane defined by $\underset{\sim}{x}$, $\underset{\sim}{r}_\Lambda$ as the Z-X plane. In this frame, polar coordinates for $\underset{\sim}{z}$ are defined by

$$\underset{\sim}{z} = (z, \theta_{xz}, \varphi_z) \qquad \ldots(A3.1)$$

Next the $\underset{\sim}{x}$ integration is considered. $\underset{\sim}{r}_\Lambda$ is chosen as new Z-axis. Polar coordinates for $\underset{\sim}{x}$ are :

$$\underset{\sim}{x} = (x, \theta_{\Lambda x}, \varphi_x) \qquad \ldots(A3.2)$$

Consequently the product $\rho(r_1)\ \rho(r_2)$ can be written :

$$\rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x}|)\ \rho(|\underset{\sim}{r}_\Lambda + \underset{\sim}{x} - \underset{\sim}{z}|) = \left(\frac{\beta^2}{\pi}\right)^3 e^{-2\beta^2(r_\Lambda^2 + x^2)} \times$$

$$e^{-4\beta^2 x\, r_\Lambda \cos\theta_{\Lambda x}}\ e^{-\beta^2 z^2}\ e^{-2\beta^2 b z} \qquad \ldots(A3.3)$$

where

$$b = -r_\Lambda(\sin\theta_{xz}\sin\theta_{\Lambda x}\cos\varphi_z + \cos\theta_{xz}\cos\theta_{\Lambda x}) - x\cos\theta_{xz}\ .$$

$$\ldots(A3.4)$$

Then the integrals (5.16) and (5.17) become

$$P_c(r_\Lambda) = 48\pi \left(\frac{\beta^2}{\pi}\right)^3 e^{-2\beta^2 r_\Lambda^2} \int_{d_{\Lambda N}}^{\infty} dx\, x^2\, S'(x) \int_{-1}^{+1} dw\, e^{-4\beta^2 x r_\Lambda w}$$

$$\int_0^{\pi} d\varphi_z \int_{-1}^{+1} dv\; I_{cz} \,, \qquad \ldots(A3.5)$$

$$P_T(r_\Lambda) = 48\pi \left(\frac{\beta}{\pi}\right)^3 e^{-2\beta^2 r_\Lambda^2} \int_{d_{\Lambda N}}^{\infty} dx\, x^2\, W'(x) \int_{-1}^{+1} dw\, e^{-4\beta^2 x r_\Lambda w}$$

$$\int_0^{\pi} d\varphi \int_{-1}^{+1} dv\, (3v^2 - 1)\, I_{Tz} \,. \qquad \ldots(A3.6)$$

Here $v = \cos\theta_{xz}$, $w = \cos\theta_{\Lambda x}$ and (with unit $\mu = 1$)

$$I_{cz} = \int_{d_{NN}}^{\infty} dz\, z\, e^{-\beta^2 z^2 + (2\beta^2 b - 1)z} = \frac{e^{-\beta^2 d_{NN}^2}}{4\beta^2}\, e^{(2\beta^2 b - 1)\, d_{NN}}$$

$$\times \left[2 + \sqrt{\pi}\left(2\beta b - \frac{1}{\beta}\right)\right] \,, \qquad \ldots(A3.7)$$

$$I_{Tz} = I_1 + I_2 \,, \qquad \ldots(A3.8)$$

$$I_1 = \int_{d_{NN}}^{\infty} dz\,(z+3)\, e^{-\beta^2 z^2 + (2\beta^2 b - 1) z}$$

$$= \frac{e^{-\beta^2 d_{NN}^2}}{4\beta^2}\, e^{(2\beta^2 b - 1) d_{NN}} \left[2 + \sqrt{\pi}\left(2\beta b - \frac{1}{\beta} + 6\beta\right)\right], \quad \ldots(A3.9)$$

$$I_2 = 3 \int_{d_{NN}}^{\infty} \frac{dz}{z}\, e^{-\beta^2 z^2 + (2\beta^2 b - 1) z} \qquad \ldots(A3.10)$$

Thus the integral (5.16) is reduced to a four-dimensional integration (A3.5 with A3.7) which is done numerically. Similarly the integral (5.17) is split into a four-dimensional (A3.6 with A3.9) and five-dimensional (A3.6 with A3.10) integral, both of which are done numerically.

APPENDIX 4

THE VALUES OF NUMERICAL CONSTANTS USED THROUGHOUT THE COMPUTATION ARE GATHERED HERE

The mass of the pion is taken to be

$$\mu = 137.28 \text{ Mev} = 0.6939 \text{ fm}^{-1}$$

The units $\hbar = c = 1$ are used.

Values of other constants are :

$f_N^2 = 0.08$	πNN	coupling constant
$f_\Lambda^2 = 0.08$	$\pi \Sigma \Lambda$	coupling constant
$\Delta = 0.389 \text{ fm}^{-1}$	$\Sigma - \Lambda$	mass difference
$g_\Lambda^2 = 0.047$	$\pi Y_1^* \Lambda$	coupling constant
$\omega_\Lambda = 1.24 \text{ fm}^{-1}$	Y_1^*	resonance energy
$g_N^2 = 0.057$	$\pi N^* N$	coupling constant
$\omega_N = 1.27 \text{ fm}^{-1}$	N^*	resonance energy
$p_m = 6\mu = 4.286 \text{ fm}^{-1}$		cutoff momentum
$\beta = 0.85056 \text{ fm}^{-1}$		coefficient used for the normalized density distribution for the nucleons in ${}^5_\Lambda He$
$k_f = 1.36 \text{ fm}^{-1}$		Fermi momentum
$\rho = 0.170 \text{ fm}^{-3}$		density of nuclear matter

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