ANGULAR CORRELATIONS IN Mg^{24}

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SCOPE AND CONTENTS: A method is described by which spins of nuclear energy levels might be determined, which is a combination of coincidence experiments with Sodium Iodide detectors and angular distribution measurements with high resolution solid state detectors. The method is applied to the 8.87 MeV level in Mg^{24}.

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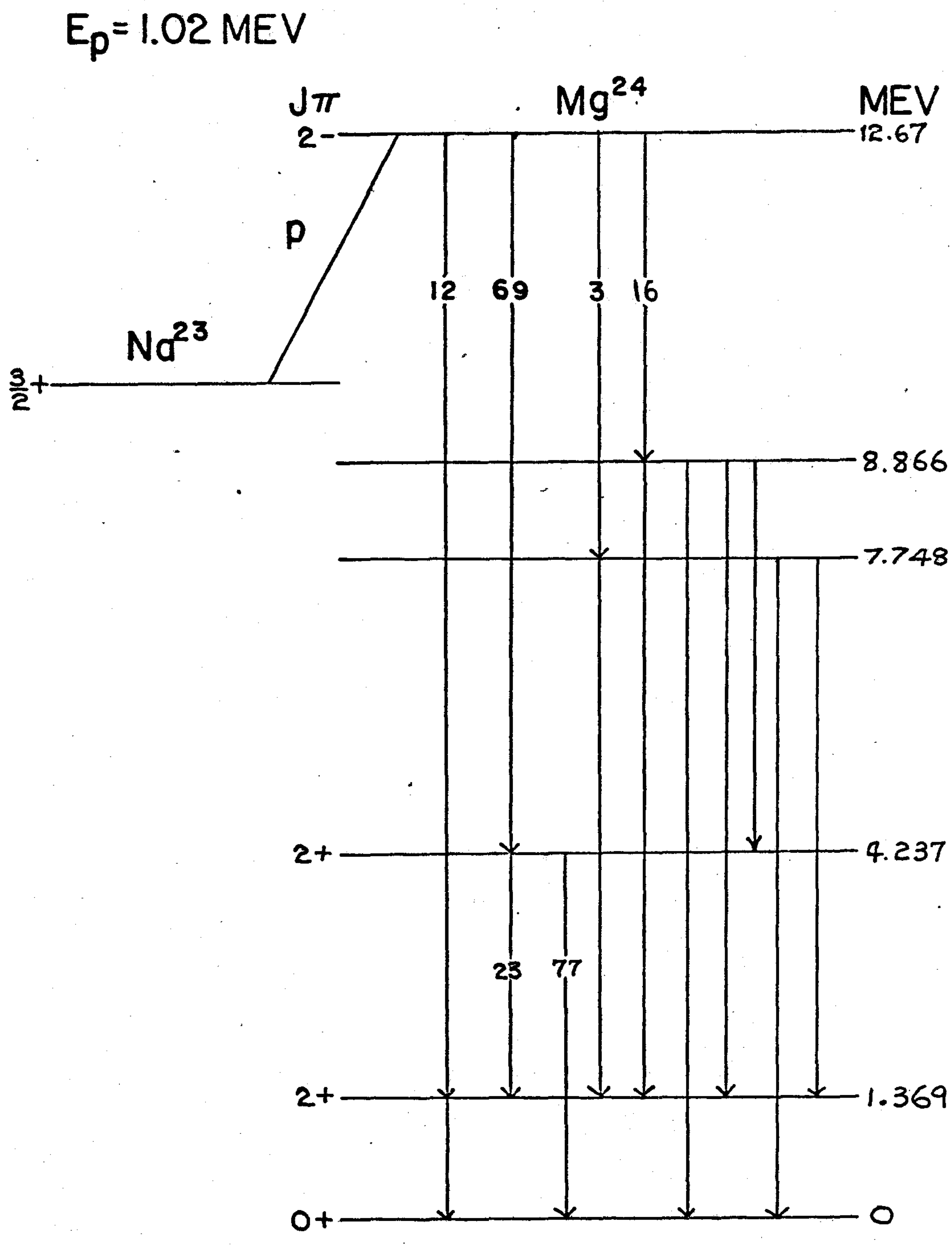
CHAPTER I

INTRODUCTION

The object of the experiments described here was to try to make spin assignments to certain levels in the Mg^{24} nucleus using angular correlation methods.

The work began with a re-examination of the levels of the nucleus making use of the superior resolving power of solid state detectors. This was achieved by observing the gamma ray spectrum in the $Na^{23}(p, \gamma)Mg^{24}$ reaction. During the course of this work, the resonance corresponding to a proton energy of 1,020 KeV was studied. This work is reported by K. J. Cassell (1968). It was shown that one mode of decay took place through the 8.87 MeV and 1.37 MeV levels. The gamma rays occurring in this cascade had previously been interpreted as indicating that the decay was from the resonance to the 5.23 MeV level to the 1.37 MeV level. This was due to the inferior resolving power of the Sodium Iodide detectors used previously, the order of the two gamma rays being inverted as a result.

These conclusions suggested a method by which a spin assignment might be made to the 8.87 MeV level. Any



E_p= 1.02 MEV
Jπ
Mg^{24}
MEV
2-
12.67
p
12
69
3
16
Na^{23}
$\frac{3}{2}$+
8.866
7.748
2+
4.237
23
77
2+
1.369
0+
0

FIG. 1

angular correlation measurement made directly on this level would necessitate the use of solid state detectors to give the required resolution. However, the relatively low efficiency of these counters would make a gamma-gamma coincidence measurement, for instance, impractical. On the other hand, an angular distribution measurement does not, in general, yield enough measured quantities for a spin assignment to be made uniquely. The procedure adopted was to determine details of the resonance state by making a gamma-gamma coincidence measurement on the decay to the ground state via the 4.24 MeV level, as a first step. The gamma rays involved here are sufficiently well resolved by Sodium Iodide to make such a measurement feasible. In the process it was hoped that the assignment of 2^- to the resonance by Prosser et al (1956) might be confirmed.

As a second step, the information gained above, would be used, together with that from an angular distribution measurement on the Res $\rightarrow$ 8.87 gamma ray using the Ge(Li) detector, and would hopefully yield a spin assignment for the 8.87 MeV level. The reasons for this procedure will be discussed in the following chapters.

CHAPTER II

THEORETICAL

Suppose that a quantum mechanical system is in a pure state $|n\rangle$. Then it may be expanded as a sum of eigenstates of suitable set of operators. For a state which is a function of angular momentum, the eigenstates $|a\alpha\rangle$ of a, where α is the magnetic substate value and has values -a, -a + 1, ... a, are chosen

$$|n\rangle = \sum_{a\alpha} |a\alpha\rangle\langle a\alpha|n\rangle$$

The expectation value of an operator Q in the state $|n\rangle$ is given by:

$$\langle Q\rangle = \langle n|Q|n\rangle = \sum_{\substack{aa' \\ \alpha\alpha'}} \langle n|a'\alpha'\rangle\langle a\alpha|n\rangle\langle a'\alpha'|Q|a\alpha\rangle$$

An ensemble of nuclei formed in a nuclear reaction cannot be specified completely in a quantum mechanical sense and is in a mixed state. It must therefore be described by an incoherent sum of pure states with weights g_n. The expectation value of Q is then given by:

$$\langle Q\rangle = \sum_{n} g_n\langle n|Q|n\rangle = \sum_{\substack{naa' \\ \alpha\alpha'}} g_n\langle n|a'\alpha'\rangle\langle a\alpha|n\rangle\langle a'\alpha'|Q|a\alpha\rangle$$

The density matrix elements are defined by:

$$\langle a\alpha|\rho|a'\alpha'\rangle = \sum_n g_n \langle n|a'\alpha'\rangle\langle a\alpha|n\rangle$$

and

$$\langle Q\rangle = \sum_{\substack{aa' \\ \alpha\alpha'}} \langle a\alpha|\rho|a'\alpha'\rangle\langle a'\alpha'|Q|a\alpha\rangle = \mathrm{Tr}(\rho Q)$$

This density matrix, defined by Fano (1953) represents an averaging over the whole ensemble analogous to that which takes place in classical statistical mechanics.

The theory of angular correlations is conveniently expressed in terms of this density matrix which describes the bound states of the decaying nucleus and by the efficiency matrix ε, defined by Coester and Jauch (1953) which represents a similar averaging for the emitted radiations. This latter may also be made to contain information about the efficiencies and positions of the detection equipment.

The correlation function or probability of a count being recorded in a counter or system of counters is then $\mathrm{Tr}(\rho\varepsilon)$.

Consider a bound state, spin a, which is formed as the final one in a nuclear reaction and which decays through states b and c with the emission of gamma rays. It is specified completely by its density matrix and no details of the reaction which formed it are necessary. Since no attempt is made to determine information about the intermediate states of the reaction, the treatment becomes independent of them.

The density matrix in its most general form has more elements than can be determined experimentally and in practice, certain restrictions are imposed which drastically reduce this number.

An alternative description is in terms of statistical tensors whose parameters $\rho_{k\kappa}(aa')$ are related to the density matrix by:

$$\rho_{k\kappa}(aa') = \sum_{\alpha\alpha'} (-)^{a' - \alpha'} (a\alpha\ a' - \alpha' | k\kappa) \langle a\alpha | \rho | a'\alpha' \rangle$$

where $(a\alpha a' - \alpha' | k\kappa)$ is a Clebsh Gordan coefficient. The correlation function is built up in terms of the statistical tensors because of their properties under rotation although it is convenient to revert to the density matrix formalism later for purposes of analysis.

In what follows, the states of the nucleus are considered to have sharp spin, i.e. $a = a'$.

If the state a is formed by a reaction in which unpolarised incident particles are used and any outgoing radiation is unobserved, then the state is symmetric about the incident particle direction which consequently is chosen as the Z-axis in the representation in which the theory is developed. In such a case, application of the rotation operator to the statistical tensor shows that $\kappa = 0$ and therefore $\alpha = \alpha'$ which makes the density matrix diagonal. If, further, it is required that the state has symmetry under reflection in a plane perpendicular to the beam direction, or definite parity, then the density matrix is symmetrical between positive and negative magnetic substate values. As a further consequence of the above, k is restricted to even values. Then, only a + 1 or a + 1/2 parameters, depending on whether a is integral or half integral respectively, are required to specify the state. Nuclei prepared to have the above properties are called aligned. The procedure which consists of preparing such a nucleus and performing a gamma-gamma coincidence measurement on it is called Method I by Litherland and Ferguson (1961).

An efficiency tensor may be defined by analogy with the statistical tensor and the correlation function for an aligned state is then

$$W = \sum_k \rho_{ko}(aa)\varepsilon_{ko}(aa)$$

This correlation function is built up by starting at the lowest state of the cascade and working upwards coupling the angular momenta involved to form the statistical tensor of the first gamma emitting state. Since a trace is invariant under unitary transformations, the efficiency tensors may then be transformed from an angular momentum representation into a co-ordinate representation and rotation operators applied to transform the efficiency tensors in the Z-direction into those in the direction of the detection equipment. It is assumed that the transitions of the cascade are accompanied by emission of gamma rays of multipolarities 2^{L_1} and $2^{L_1'}$ in the first and 2^{L_2} and $2^{L_2'}$ in the second.

The form of the correlation function is then given by Litherland and Ferguson as:

$$W(\theta_1, \theta_2, \phi) = \Sigma 2^n \rho_{ko}(aa)\ \delta_1^{p_1} \delta_2^{p_2} (-)^{f_2}\ \hat{L}_1 \hat{L}_1' \hat{L}_2 \hat{L}_2'$$

$$\times\ (L_1 1 L_1' - 1|k_1 o)(L_2 1 L_2' - 1|k_2 o)(k_1 - \kappa k_2 \kappa|ko)$$

$$\times \begin{Bmatrix} b & L_1 & a \\ b & L_1' & a \\ k_2 & k_1 & k \end{Bmatrix} W(bL_2bL_2'ck_2)\ Q_{k_1} Q_{k_2} X^{\kappa}_{k_1 k_2}(\theta_1, \theta_2, \phi)$$

$$\ldots\ldots\ldots\ldots(1)$$

The summation extends over $kk_1k_2\kappa L_1 L_1' L_2 L_2'$ with $L_1 \le L_1'$, $L_2 \le L_2'$, $\kappa \ge 0$ the duplication being accounted for by the factor 2^n.

$$n = n_1 + n_2 + n_3 \text{ where } n_{1(2)} = 1 \text{ if } L_{1(2)} \ne L'_{1(2)}$$
$$= 0 \text{ otherwise}$$
$$n_3 = 1 \text{ if } \kappa > 0$$
$$= 0 \text{ otherwise}$$

The statistical tensor is related to the density matrix elements by

$$\rho_{ko}(aa) = \sum_{\alpha} (-1)^{a-\alpha} (a\alpha a - \alpha|ko)\langle a\alpha|\rho|a\alpha\rangle$$

The δ_1 and δ_2 are the mixing ratios for the two gamma rays.

$$\delta_1 = \frac{\langle b||L_1 + 1||a\rangle}{\langle b||L_1||a\rangle}$$

The reduced matrix elements represent the amplitudes of the interfering multipolarities. p_1 is 0, 1 or 2 according as the pair (L_1, L_1') is (1, 1), (1, 2) or (2, 2).

The phase factor f_2 is given by:

$$f_2 = c - b + L_1' - L_2 + L_2' - k_2 + |\kappa|$$

The circumflex, e.g. $\hat{L}_1$ indicates the function $(2L_1 + 1)^{1/2}$. The notation for the Clebsh-Gordan, Racah, and 9-j coefficients is conventional.

The angular function $X^{\kappa}_{k_1k_2}(\theta_1, \theta_2, \phi)$ contains the rotation matrix elements which transform the efficiency tensors from the Z-direction to the actual counter directions.

$$X^{\kappa}_{k_1k_2}(\theta_1, \theta_2, \phi) = \hat{k}_1\hat{k}_2 \left[\frac{(k_1 - |\kappa|)!(k_2 - |\kappa|)!}{(k_1 + |\kappa|)!(k_2 + |\kappa|)!}\right]^{1/2}$$

$$\times P^{|\kappa|}_{k_1}(\cos \theta_1)P^{|\kappa|}_{k_2}(\cos \theta_2)\cos \kappa\phi$$

θ_1 is the angle between the beam and the axis of the counter detecting γ_1, θ_2 is the same for γ_2, ϕ is the azimuthal angle between the two counters, and the $P^{\kappa}_{k}(\cos \theta)$ are associated Legendre Functions.

In a real situation the gamma rays are not detected by point counters but by ones with finite size so that the gamma rays are detected in a cone, about the specified angle of the counter axis, which subtends a finite solid angle at the source. This is taken account of by Q_{k_1} and Q_{k_2} which are the attenuation coefficients for the counters which detect γ_1 and γ_2 respectively. For axially symmetric counters they are defined by

$$Q_k = \frac{J_k}{J_o}$$

$$J_k = \int_0^{\pi} \varepsilon(\xi)P_k(\cos \xi) \sin \xi \, d\xi$$

where $\varepsilon(\xi)$ is the efficiency of the counter for a gamma ray propagating at an angle ξ to the axis of the counter. The Q's have been tabulated by Rutledge (1959), Gove and Rutledge (1958) and Rose (1953).

The limits on $k_1 k_2 \kappa k$ are as follows:

$(L_1' - L_1) \le k_1 \le L_1' + L_1$ and is even.

$(L_2' - L_2) \le k_2 \le$ minimum $(2b, L_2 + L_2')$ and is even.

$0 \le \kappa \le$ minimum (k_1, k_2) and is even and odd.

$|k_1 - k_2| \le k \le$ minimum $(2a, k_1 + k_2)$ and is even.

Analysis is usually performed in terms of the so called population parameters. Conventions for them vary but in what follows they are defined by

$$p(\alpha) = \langle a\alpha|\rho|a\alpha\rangle + \langle a-\alpha|\rho|a-\alpha\rangle$$

They may be regarded as the relative probability of population of a magnetic substate and they must be positive. Details of the reaction may impose further conditions on them.

As a special case of formula (1) above, there is the angular distribution of a single gamma ray for a decay from a state c to a state d.

$$W(\theta) = \sum_{\gamma k p} (-)^{f_6} p(\gamma)\delta^{p}(c\gamma c-\gamma|ko)\ Z_1\ (LcL'\ c|d\ k)$$
$$\times\ Q_k P_k(\cos\theta) \qquad \ldots\ldots\ldots\ldots(2)$$

where $f_6 = d + \gamma + L + L' + k/2$, $\gamma \ge 0$, a factor 2 being introduced for $L \neq L'$. The coefficient Z_1 is defined and tabulated by Sharp et al (1954). The remaining terms have the same significance as in equation (1).

CHAPTER III

GENERAL CONSIDERATIONS

The equation (2) may be written in the form

$$W(\theta) = A_o + A_2P_2(\cos\theta) + A_4P_4(\cos\theta) + \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where the A's are functions of the spins, the population parameters and the mixing ratios. If no quadrupole radiation occurs in the transition $A_4 = 0$. Also, if the spin of the upper state is less than 2, only the first two terms will occur. Further restrictions might be imposed by the formation of the upper state. The results of an angular distribution measurement are usually analysed by least squares fitting and the coefficient A_o used for normalisation. Thus in the most favourable case, the experiment will yield two measured quantities, A_2/A_o and A_4/A_o. In general, this is insufficient to assign values to the parameters of which the A's are functions, far less to make a spin assignment.

In the formula (1), however, in a favourable case where there is quadrupole radiation, there are 19 terms in the expansion corresponding to the triplets κk_1k_2 which can occur. If the positions of the counters are chosen such that the angular functions $X^{\kappa}_{k_1k_2}(\theta_1\theta_2\phi)$ are linearly independent then there are in effect this many measured quantities from which to deduce the required parameters. With this degree of overdetermination, it is expected that spin assignments may be made uniquely.

Some use has been made of this general approach. To test its feasibility, Ferguson et al (1967), used an array of seven counters obtaining 42 correlations simultaneously. The counters were mounted on an icosahedral framework which had the disadvantage that the angular functions were not completely independent. The same group is currently working with a piece of apparatus called a Lotus which remedies this situation.

A less general approach has been used by Batchelor et al (1960), Litherland et al (1961), Broude and Gove (1963) and Glaudemans and Endt (1962). This approach consists of fixing two of the angles (θ_1, θ_2, ϕ) and varying the third. In this work, two counters were used, one of which was fixed at 90° to the beam direction and the other allowed to take up various settings in the horizontal plane. This yields two correlations for each setting corresponding to γ_2 being detected in the fixed counter and γ_1 being detected in the variable counter and the reverse situation. These two special cases or "geometries", as they are called, are designated geometries I and II by Litherland and Ferguson.

For these special cases the equation (1) reduces to the following:

$$W(\theta) = \Sigma\alpha^{\kappa}_{rk_1k_2}\ Q_{k_1}Q_{k_2}p(m)\delta_1^{p_1}\delta_2^{p_2}\ C^{\kappa}_{k_1k_2}P_r(\cos\ \theta) \qquad \text{...........(4)}$$

where θ is the angle between the beam direction and that of the moving detector.

The summation extends over $rk_1k_2\kappa L_1L_1'L_2L_2'$ and m.

The coefficients $C^{\kappa}_{k_1k_2}$ are given for useful cases by Smith (1962) and the $\alpha^{\kappa}_{rk_1k_2}$ are tabulated by Ferguson and Rutledge (1957).

The results for each geometry may now be expressed in the form of equation (3) and four measured quantities may be obtained. This number, in general, provides a small overdetermination, which, in previous work by Glaudemans and Endt, proved to be sufficient.

The procedure adopted in this work was to perform an experiment of this type on the cascade Res → 4.24 → 0 to determine the population parameters of the resonance with their errors and to insert these into the angular distribution formula (2). This leaves only the multipole mixing ratio δ to determine when analysing the results of the angular distribution obtained from the Res → 8.87 transition.

CHAPTER IV

EXPERIMENTAL

In these experiments the proton beam was produced by the High Voltage Engineering Corporation KN Van De Graaff at the Princess Margaret Hospital in Toronto. The beam energy was determined by a 25° deflecting magnet whose field was measured by a nuclear magnetic resonance device.

The target was prepared by vacuum evaporation of Sodium Chloride onto a 0.01 inch thick tantalum backing. It was set at an angle of 45° to the beam direction and contained in a 2.4 inch diameter brass target chamber symmetric about a vertical axis through the beam spot.

In the coincidence experiments two 5 in. diameter X 6 in. NaI(Tl) detectors were mounted on moveable platforms which could be rotated in the horizontal plane about a vertical axis through the beam spot. Both were equipped with lead shielding and set with their front faces 6.25 inches from the beam spot. One of these detectors was kept fixed at an angle of 90° to the beam direction and the other set at one of five angles to the beam in each run. These angles were 90°, 70°, 50°, 30°, 8°, in this order, the latter being chosen because the lead shielding prohibited smaller angles from being obtained at this distance.

A block diagram of the electronics is shown in Fig. 2. This setup allowed the coincidence intensities in both geometries to be measured simultaneously.

The pulses from each photomultiplier were fed into a preamplifier contained in the detector assembly and then into a main amplifier. The double delay line shaped pulses from each amplifier were then fed into two timing single channel analysers (S.C.A.), one with a narrow gate on the primary (8.43 MeV) gamma ray and the other with a wide gate which included the whole spectrum with the exception of a region of intense 0.51 MeV annihilation radiation at the low energy end. The output from the wide gate on one detector and the narrow gate on the other were then fed to a coincidence unit which gave an output signal when pulses from the two detectors had crossover points within a resolving time of 50 nanosec ($2\tau = 100$ nanosec). Initial tests indicated that the random coincidence count rate was very small compared with the true count rate for resolving times up to the maximum available on the coincidence unit ($2\tau = 100$ nanosec). Further tests showed that the yield was constant for $2\tau > 40$ nanosec. Thus the maximum value was used in these experiments to make sure that all the true coincidences were recorded. The signal from the coincidence unit opened

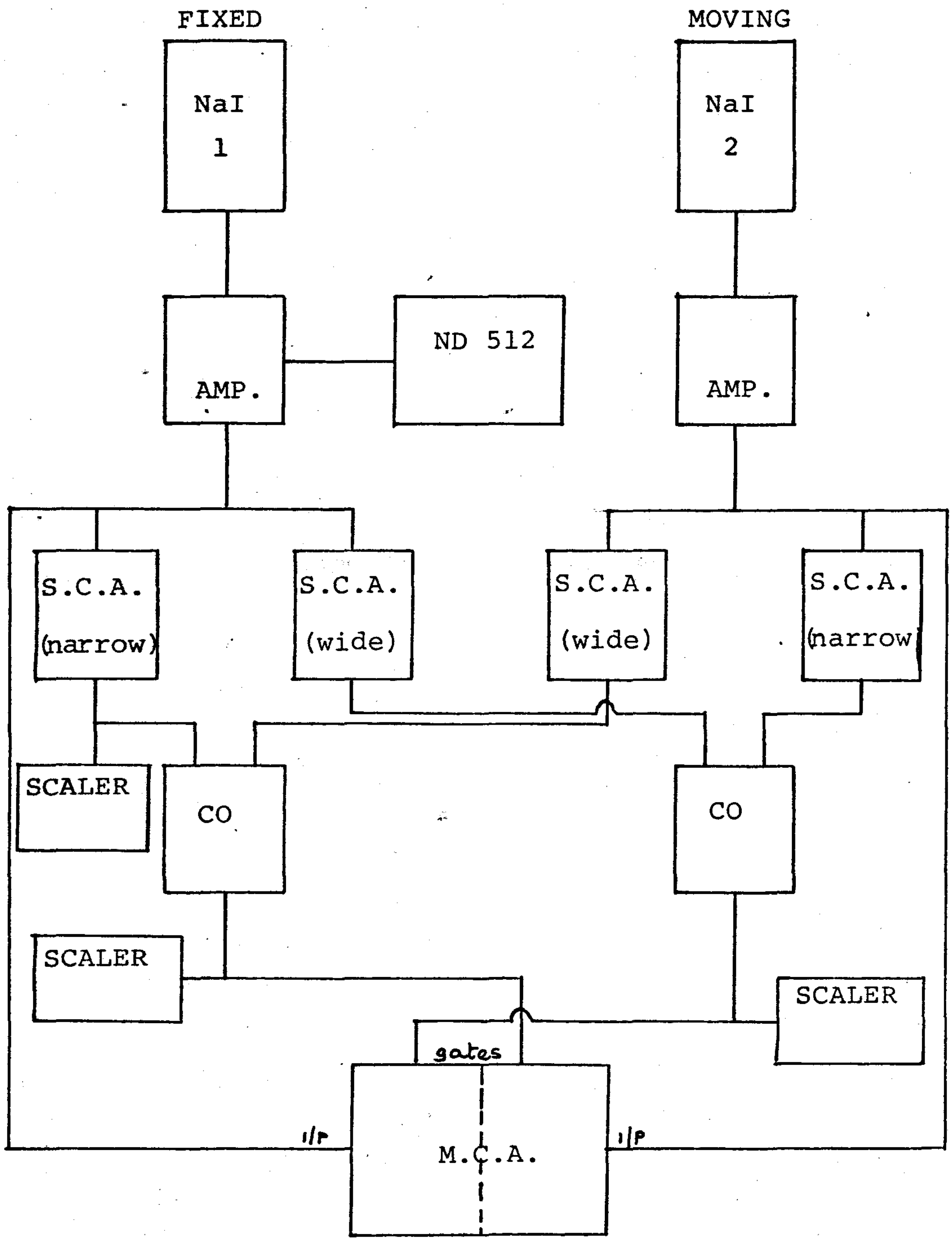

FIXED
MOVING
NaI
1
NaI
2
AMP.
ND 512
AMP.
S.C.A.
(narrow)
S.C.A.
(wide)
S.C.A.
(wide)
S.C.A.
(narrow)
SCALER
CO
CO
SCALER
SCALER
gates
i/p
M.C.A.
i/p

FIG.2

a gate allowing pulses from the wide gated detector, in coincidence with those falling within the narrow window on the other detector, to be analysed by the multichannel analyser (M.C.A).

The M.C.A. used was a Victoreen SCIPP 6400 whose memory may be divided into two independent parts of 3200 channels each, the two halves having separate analogue to digital converters (A.D.C.).

Thus the gamma rays detected by detector 1 in coincidence with primary gamma rays in detector 2 (Geometry I) were stored in the first half of the memory and those corresponding to the reverse situation (Geometry II) in the second half.

The experiment was monitored using a Nuclear Data 512 channel M.C.A. and three scalers. The analyser displayed the spectrum from the fixed detector. Two of the scalers were set on the output of the coincidence units and the third on the narrow window of the fixed detector.

For each setting of the moving detector the procedure consisted of four steps.

(a) Ungated spectra were stored in the first 400 channels of each memory unit of the M.C.A.

(b) The coincidence spectra were stored in the second 400 channels.

(c) A 300 nanosec delay was added to both narrow gates and the resulting spectra stored in the third 400 channels.

(d) A further ungated spectrum was stored in the fourth 400 channels.

The purpose of steps (a) to (d) was to estimate changes in gain, since no gain stabilisation device was used, so that any such changes could be corrected for in the analysis. Step (c) allowed a measure of random coincidences to be obtained. They proved to be negligible ($<\frac{1}{2}$ %).

The run at 90° was repeated at the end of the five runs to check consistency. There was agreement within statistics with the first 90° run.

As a further check, coincidence and non coincidence spectra at a proton energy just below 1.02 MeV were done corresponding, in energy, to the 1.01 MeV resonance which has a significant α width. The experimental energy resolution was not good enough to resolve these resonances and the intention was to estimate the possible contribution of the lower resonance to the coincidence intensity when running on the 1.02 MeV resonance. The yield observed in the run at lower energy could be accounted for by the "tail" of the 1.02 resonance and it was concluded that the contribution of the lower resonance was negligible.

The angular distribution measurements were performed using an R.C.A. Ge(Li) detector set at angles of 0°, 35°, 55°, 75°, 90° to the beam direction (K. J. Cassell). The experiment was monitored using a NaI detector fixed at an at an angle of 90° to the beam. The spectra thus obtained were stored in 3200 channels of the M.C.A.

CHAPTER V

ANALYSIS

The analysis of the experimental data is achieved in the first instance by least squares fitting to the theoretical expression

$$W(\theta) = \Sigma\alpha^{\kappa}_{rk_1k_2}Q_{k_1}Q_{k_2}p(m)\delta^{p}C^{\kappa}_{k_1k_2}P_r(\cos\theta)$$

(see Chapter II).

Only one mixing ratio occurs here since in the cascade studied, which is a $J \rightarrow 2 \rightarrow 0$, only a pure quadrupole radiation occurs in the second transition.

The theoretical expression above is a function of the unknown discrete parameter J, and of several continuous parameters, the mixing ratio δ, and the population parameters p(m). Evidently the expression is non-linear in the mixing ratio δ. In the computer programme written for an IBM 7040 the necessity for an iterative procedure is circumvented by allowing δ to take on certain values specified by the user and performing linear least squares fits to the population parameters for each proposed value of J.

The coefficients $\alpha^{\kappa}_{rk_1k_2}$ and $C^{\kappa}_{k_1k_2}$ are taken from the tabulations mentioned earlier and fed into the computer on punched cards.

The quantity δ has values in the range $-\infty \leq \delta \leq \infty$. In the programme δ is set equal to tan ε where ε may have the values $-90^{o} \leq \varepsilon \leq 90^{o}$. An initial and final value of ε and an amount by which ε is to be incremented in this range are specified by the user. The fit is achieved by minimising the quantity χ^2 defined by

$$\chi^2 = \sum_i (W_i(\theta) - N_i)^2 w_i$$

the summation being over the data points, N_i is the experimental coincidence intensity and w_i is the weight assigned to the data point i, taken here to be the reciprocal of the square of the experimental error. The theoretical estimate of χ^2 is the difference between the number of data points and the number of fitted parameters, that is, the number of degrees of freedom of the system.

Once a value of δ has been fixed, the quantities $\frac{\delta\chi^2}{\delta p(m)}$ for each m may be evaluated and set equal to zero giving rise to a set of linear equations in the p(m) which are then solved.

In order to represent a physical situation, the population parameters must be greater than or equal to zero. This condition is imposed by the programme by setting to zero any population parameter which comes out negative in the initial fit and then refitting the others. Having done this, the programme asks if $\frac{\delta\chi^2}{\delta p(m)}$ for the population parameter(s) equal to zero is non-negative for, if not, a better solution could be found by giving p(m) a positive variation. If this condition is fulfilled the best physical solution has been found. If not, p(m) is again allowed to vary. If this procedure does not reach a satisfactory conclusion after a given number of tries, the programme abandons the fitting attempt and proceeds to the next δ. This procedure is due to P. J. M. Smulders and A. J. Ferguson (private communication). It is illustrated by the flow chart in Fig. 3.

The output of the programme is then, for each value of δ, the population parameters which are normalised to add to unity and their errors and the value of χ^2. For each trial value of the spin J the value of χ^2 is plotted as a function of the mixing ratio δ and the plots inspected for minima in χ^2. Fits involving χ^2 values with a probability of less than 0.1 % were rejected. This value was obtained from χ^2 tables.

In practice, certain other restrictions may be placed on the p(m). In the current work, protons of spin ½ are captured by a target nucleus of spin 3/2. Since the beam direction is the Z-axis, the incident particles have zero projection of orbital angular momentum and so magnetic substates with $m > 2$ cannot be populated. Thus $p(m) = 0$ for $m > 2$, irrespective of the spin of the compound state.

An additional parameter which was fitted was the normalisation constant between the coincidence intensities in geometries I and II. Due to the symmetry of the experimental arrangement and the fact that measurements in

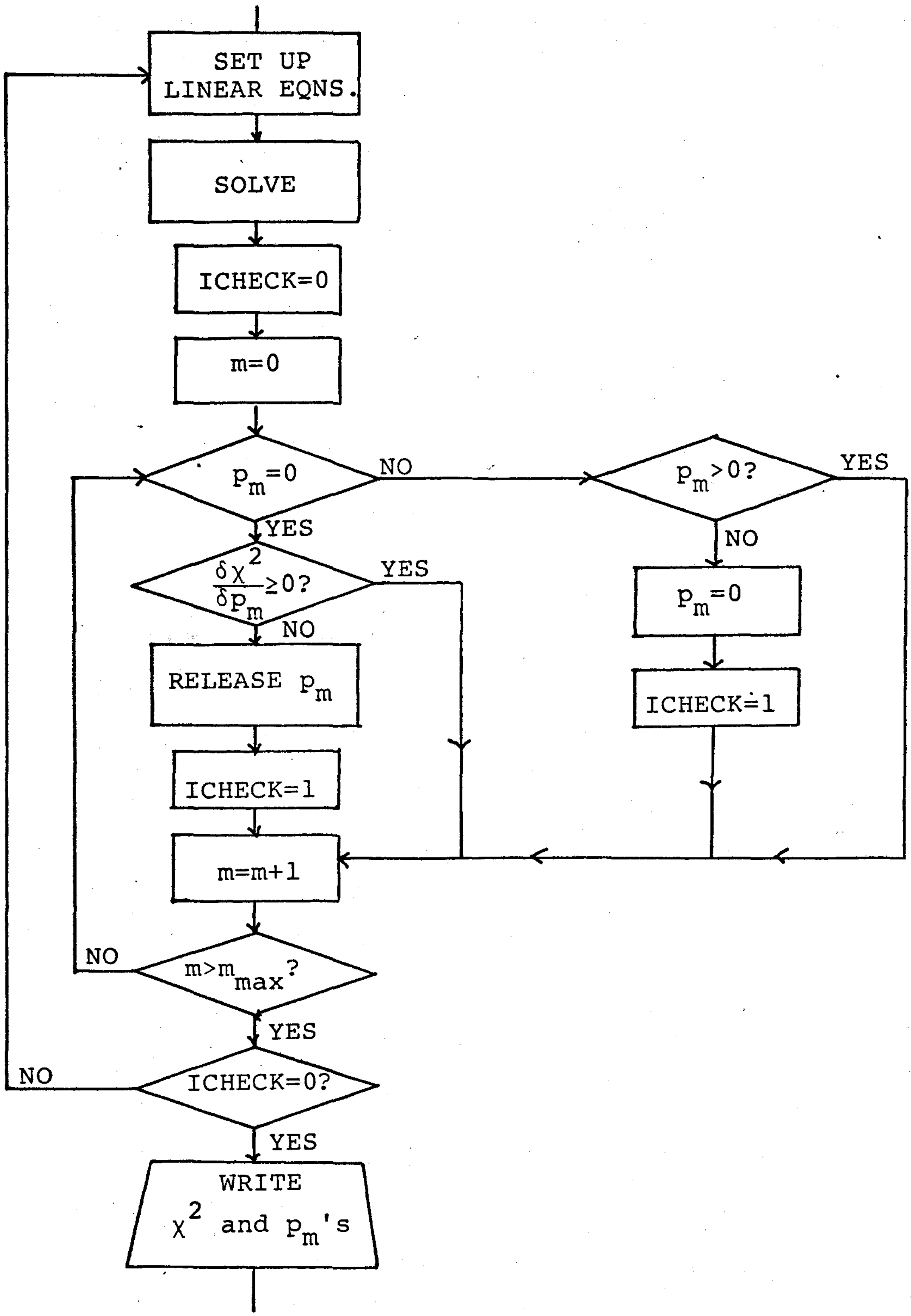

SET UP LINEAR EQNS.
SOLVE
ICHECK=0
m=0
p_m=0
NO
p_m>0?
YES
YES
$\frac{\delta\chi^2}{\delta p_m}\geq 0$?
YES
NO
NO
RELEASE p_m
p_m=0
ICHECK=1
ICHECK=1
m=m+1
NO
m>m_{max}?
YES
NO
ICHECK=0?
YES
WRITE χ^2 and p_m's

FIG.3

both geometries are performed simultaneously this is expected to be close to one, deviation being due to such factors as, for example, differences in gate widths on the γ - spectra.

The relative normalisation was varied by the programme over a limited range about the expected value to find that which gave the best fit.

The intensities in each geometry were also fitted independently by a Legendre Polynomial expansion of the form

$$W(\theta) = A_o + A_2P_2(\cos\,\theta) + A_4P_4(\cos\,\theta)$$

and the fits obtained compared to the simultaneous fit obtained from the procedure above. The normalisation constant can also be found from this procedure by making the fitted intensities at 90^o equal. The normalisations obtained by the two different methods showed good agreement (1.060 and 1.065).

The intensities obtained in the angular distribution measurements were also fitted to the above expression to obtain values of A_2/A_o and A_4/A_o and their errors. The angular distribution predicted by the population parameters and the mixing ratio for the primary gamma ray was then compared to the experimental distribution as a further check.

Finally, the values of A_2/A_o and A_4/A_o for the angular distribution of the gamma ray in the Res → 8.87 transition were calculated, using the population parameters obtained above, for the complete range of δ using equation (2). This was done for each trial value of the spin of the 8.87 MeV level. The results were then plotted and compared to the values of A_2/A_o and A_4/A_o obtained in the fit to this angular distribution.

CHAPTER VI

RESULTS

The direct and coincidence spectra obtained with both counters at 90^o are shown in Fig. 4.

A plot of χ^2 as a function of $\tan^{-1}\delta$ for each of the spins 1, 2 and 3 is shown in Fig. 5. The dotted parts of the curves indicate regions where negative population parameters were found and set equal to zero.

The curve for J = 2 shows a significantly deeper minimum than those for J = 1 and J = 3 which confirms the spin assignment of 2 to the resonance. Fig. 6 shows the variation of χ^2 with relative normalisation between geometries I and II for J = 2 and δ = 0.

The J = 2 curve in Fig. 5 has two minima corresponding to δ = 0 and δ = .158, the values of χ^2 being 4.97 and 5.19 respectively for an expected value of 5. In extracting a set of population parameters from the data neither possibility may be excluded. The ambiguity was resolved by referring to the angular distribution of the primary gamma ray. The values of A_2/A_o and A_4/A_o predicted using the mixing ratios and population parameters corresponding to each of the minima and those obtained by least squares

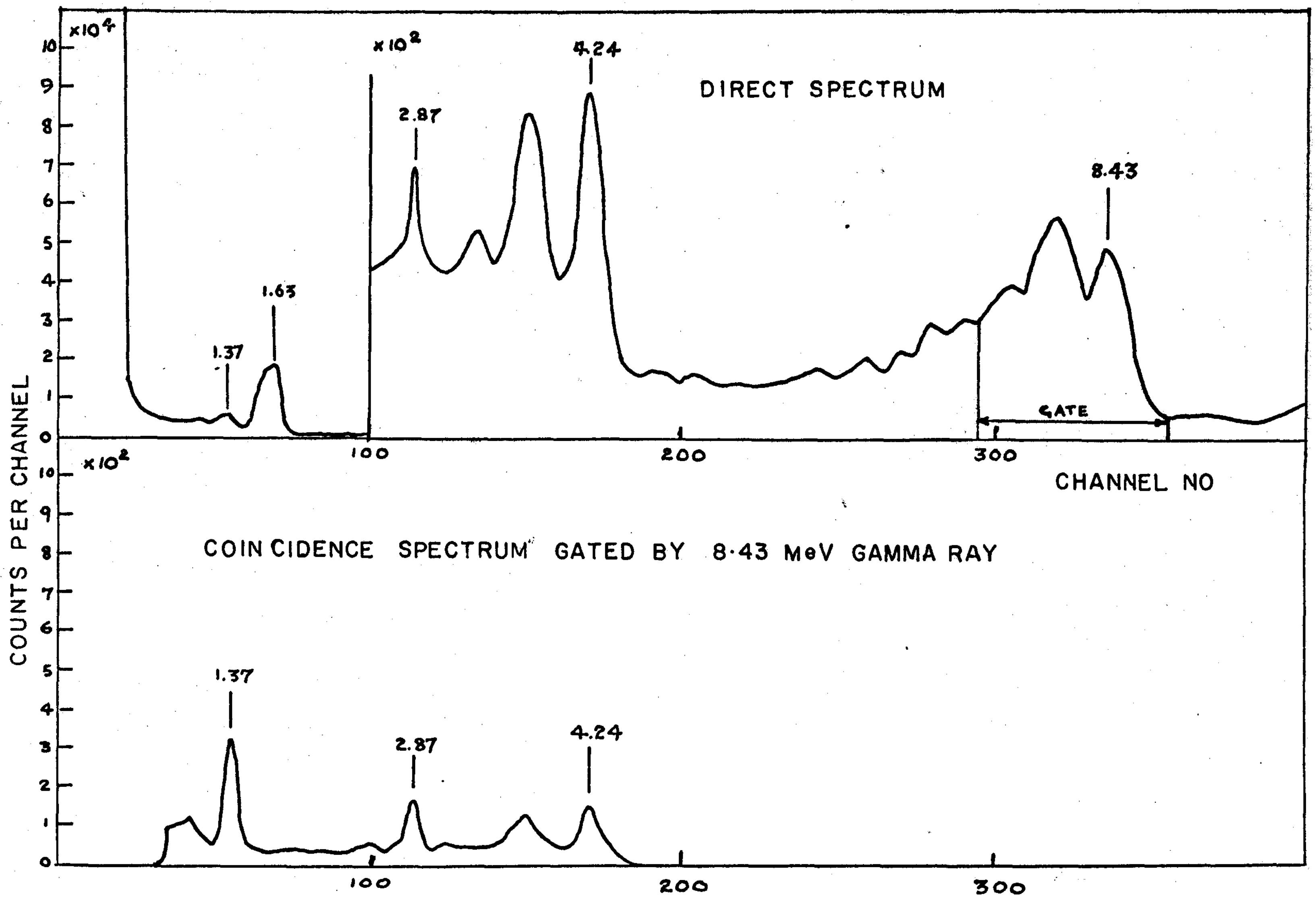
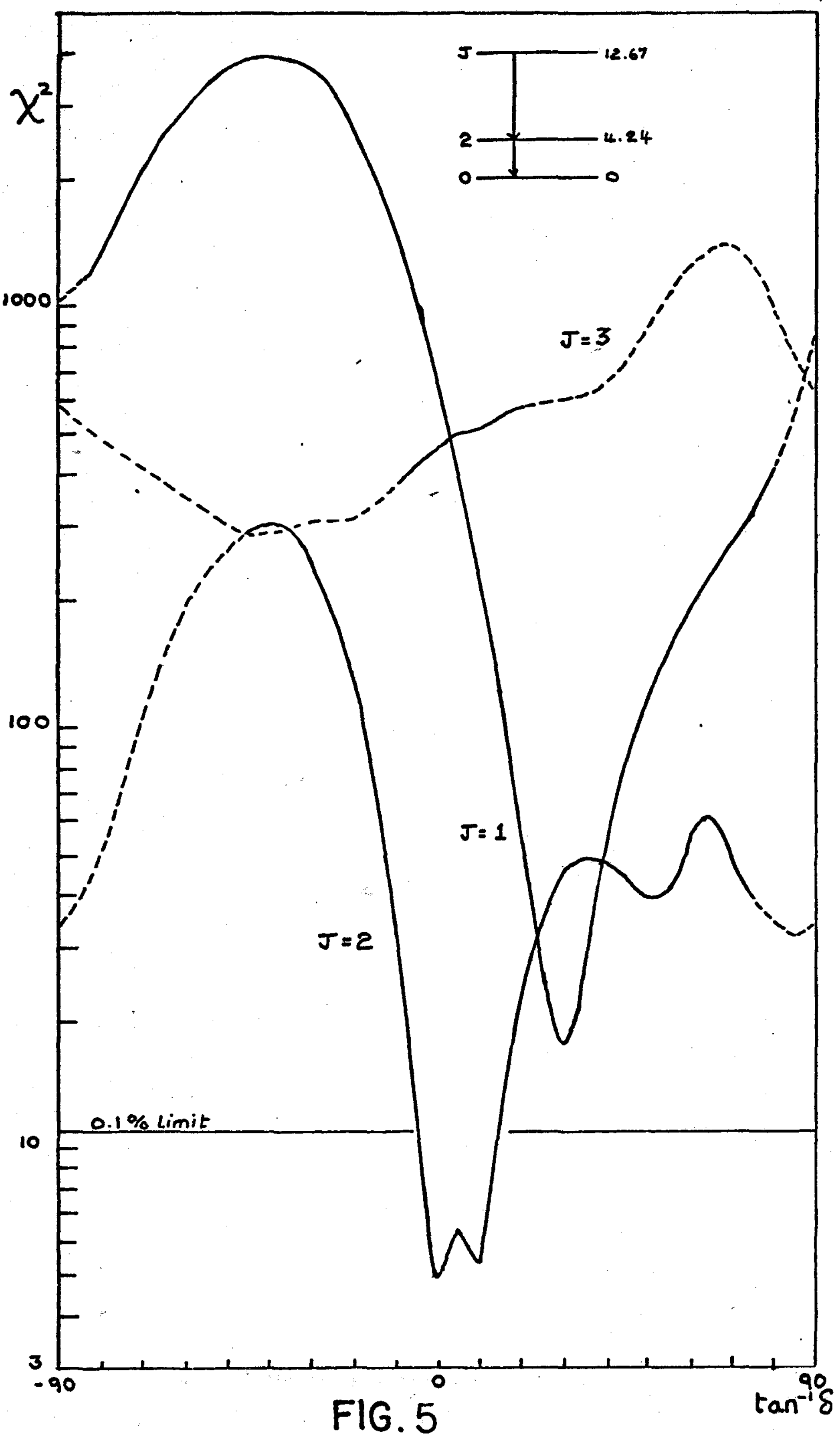
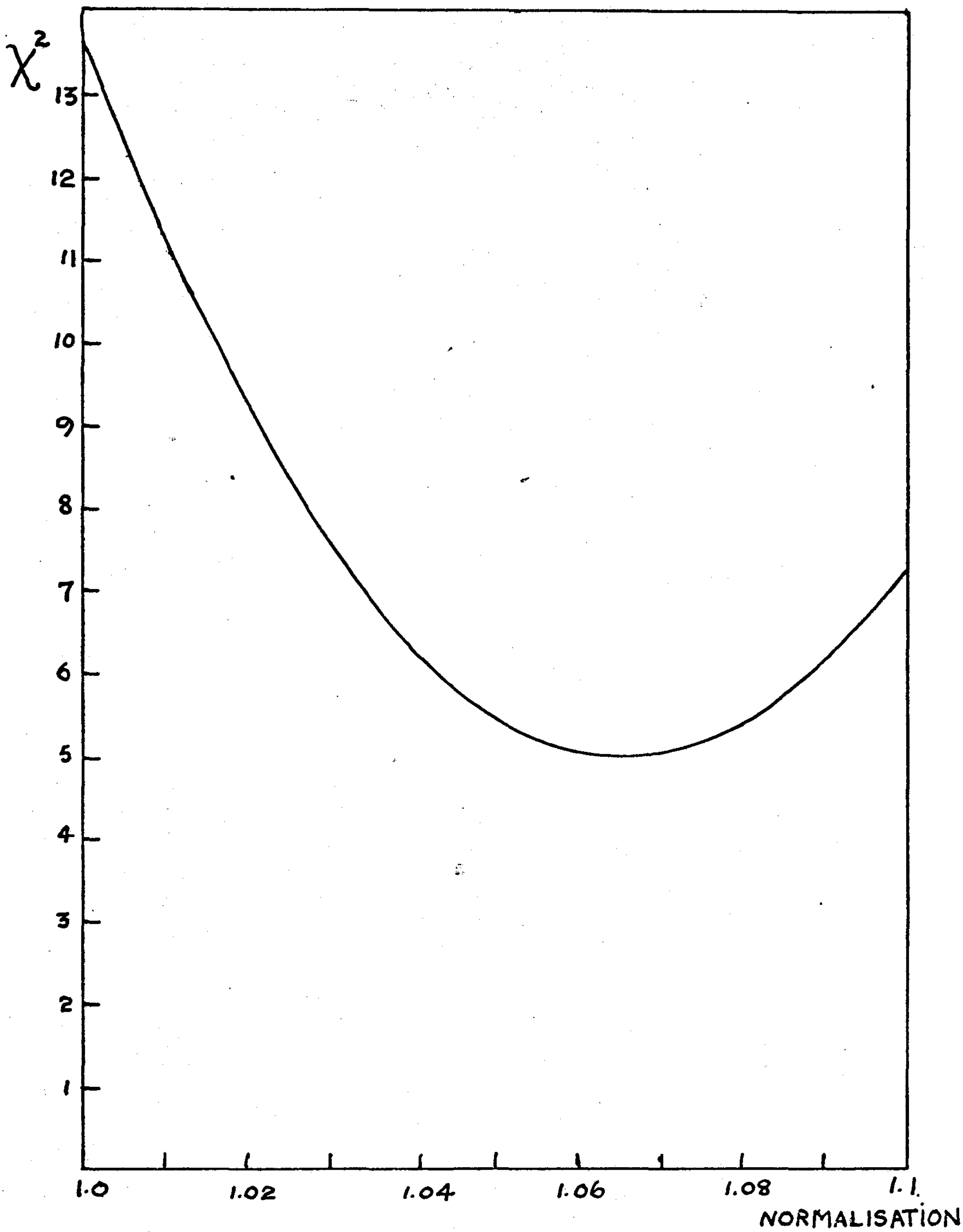

x10^4
x10^2
4.24
DIRECT SPECTRUM
2.87
8.43
1.63
1.37
GATE
100
200
300
CHANNEL NO
COUNTS PER CHANNEL
x10^2
COINCIDENCE SPECTRUM GATED BY 8·43 MeV GAMMA RAY
1.37
2.87
4.24
100
200
300

FIG.4



J
12.67
2
4.24
0
0
χ^2
1000
100
10
3
0.1% Limit
J=3
J=1
J=2
-90
0
90
$\tan^{-1}\delta$

FIG. 5



χ^2
13
12
11
10
9
8
7
6
5
4
3
2
1
1.0
1.02
1.04
1.06
1.08
1.1
NORMALISATION

FIG.6

fitting to the experimental angular distribution are shown on the following page. The experimental values of A_2/A_o and A_4/A_o given, have had the effect of the attenuation coefficients Q_2 and Q_4 removed from them (see Appendix).

The angular distribution clearly favours the minimum at $\delta = 0$. It is supposed that a simultaneous analysis of the angular distribution and the coincidence experiment would have yielded a single minimum. This indicates the desirability of the maximum possible overdetermination in these experiments.

The experimental errors on the data points in the angular distribution have clearly been underestimated. Fig. 7(III) shows the data points and the fitted curve. The errors shown include only the statistical errors. Further errors are expected to arise in the estimation of the background contribution to the peaks which were considered. This background was extremely large in each of the angular distribution measurements and in the worst case, that of the Res → 7.75 gamma ray, amounted to about 90 % of all the counts in the region of the peak. The value of χ^2 obtained in the fit to the Res → 4.24 angular distribution was four times as large as the expected value. If the convention is adopted of multiplying all errors by this number, the errors are such that the conclusions reached above are not altered.

δ	p(o)	p(1)	p(2)	χ^2	Predicted		Experimental	
					A_2/A_o	A_4/A_o	A_2/A_o	A_4/A_o
0	.125±.015	.322±.026	.553±.027	4.97	-.133±.017	0	-0.150±.028	0.061±0.034
.158	.177±.017	.434±.033	.389±.039	5.19	.001±.012	0	-0.150±.028	0.061±0.034

Table I showing the parameters associated with the two minima for J = 2 in Fig. 5 and the predicted and experimental coefficients for the angular distribution of the Res → 4.24 gamma ray.

Fig. 7(I) and (II) show the experimental coincidence intensities and the fitted curves for geometries I and II respectively, corresponding to the minimum at $\delta = 0$ and $J = 2$.

8.87 MeV Level

Fig. 8 shows the experimental data points and the fitted curve for the angular distribution of the Res → 8.87 gamma ray. The values obtained were:

$$A_2/A_o = .156 \pm .070$$

$$A_4/A_o = .032 \pm .094$$

Again these errors are regarded as underestimates, the value of χ^2 being twice the expected value.

Hird et al (1964) have shown that the 8.87 MeV level has unnatural parity $(-)^{J+1}$. Spins of greater than four for the lower state require octupole radiation or higher multipolarities and are excluded. This restricts possible spin assignments to 0^-, 1^+, 2^-, 3^+ or 4^-. The possibilities 0^- and 4^- require that the radiation be pure E2 and E2/M3 respectively.

Assuming that they are both pure E2, the predicted angular distribution coefficients are:

0^- $\quad A_2/A_o = -0.38$

4^- $\quad A_2/A_o = -0.11$

A_4/A_o is very small in both cases. Comparison with the experimental values excludes 0^-.

For both the E2 transitions, the single particle

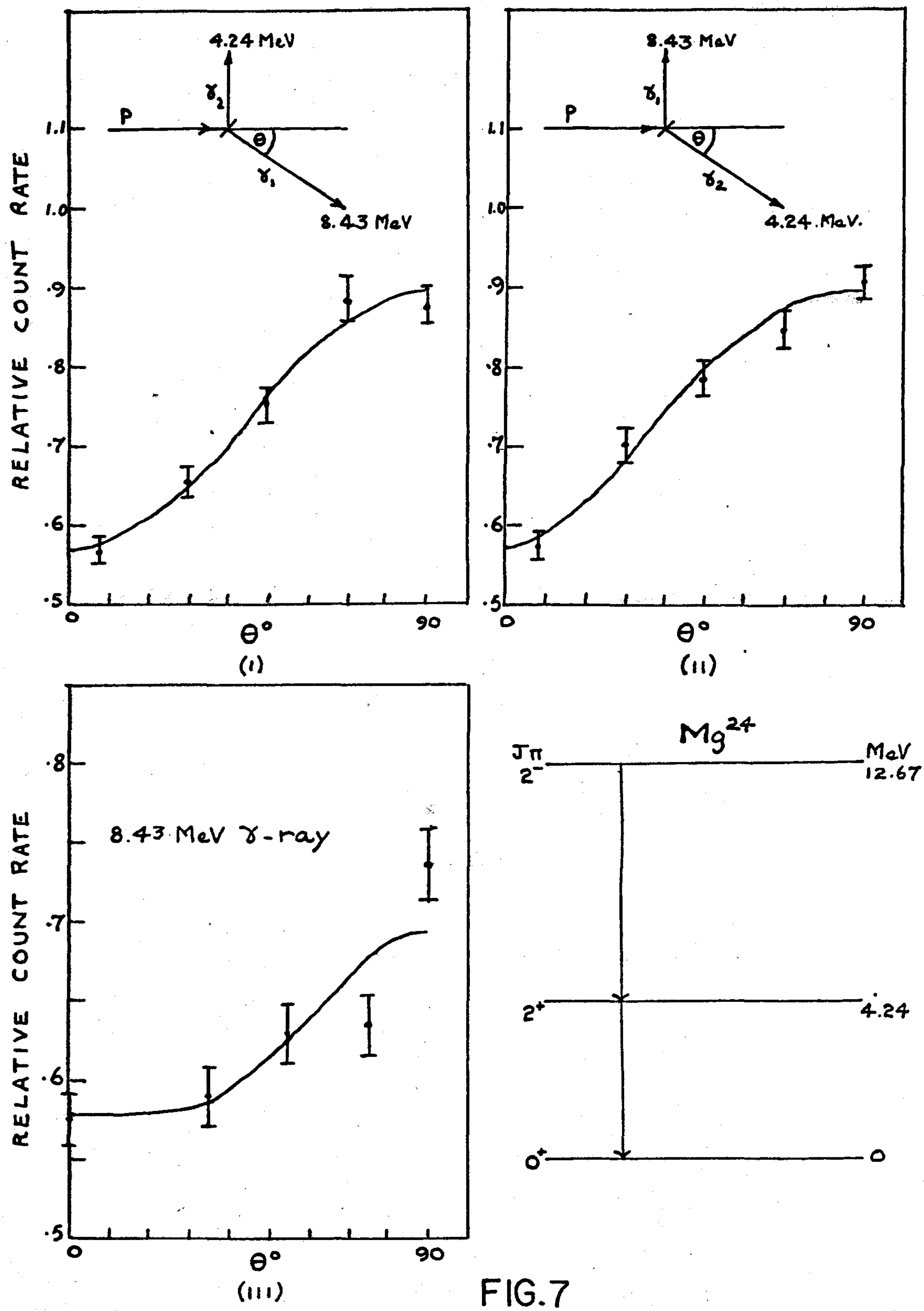

4.24 MeV
P
θ
8.43 MeV
8.43 MeV
4.24 MeV.
RELATIVE COUNT RATE
θ°
(I)
(II)
(III)
8.43 MeV γ-ray
Mg^{24}
Jπ
MeV
2⁻
12.67
2⁺
4.24
0⁺
0

FIG.7

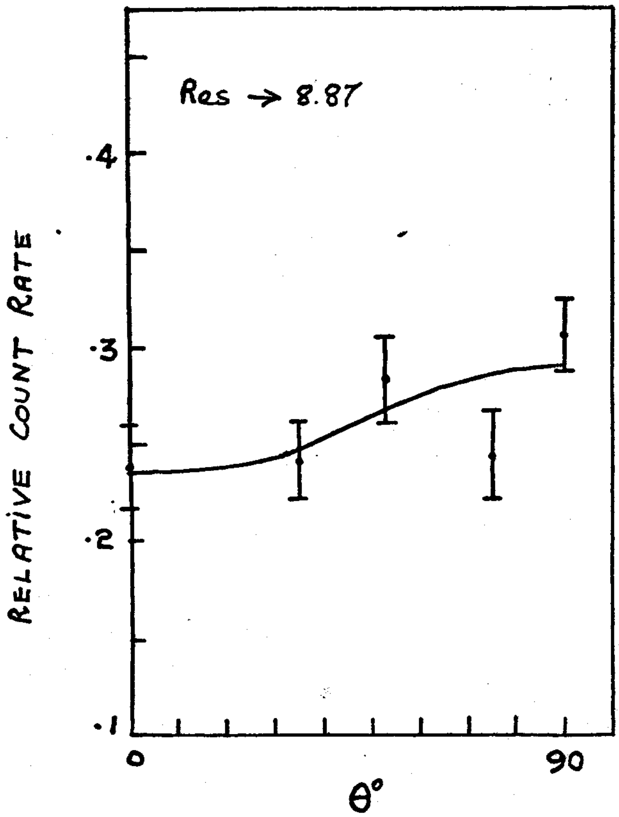


FIG.8

estimate of the radiative width is 3×10^{-3}eV (1 Weisskopf unit) whereas the observed width (K. J. Cassell) is 0.63eV. Thus if the transition is E2 it requires a radiative width of approximately 200 Weisskopf Units. Since in 98 observed E2 transitions in nuclei with $A < 40$ (Skorka, private communication) no radiative widths are observed to exceed 50 Weisskopf Units, the radiation seems unlikely to be pure E2. Thus a spin assignment of 4^- is excluded.

This restricts possible spin assignments to 1^+, 2^- and 3^+. If the spin is 1^+ or 3^+, the radiation is an E1/M2 mixture. The single particle estimate of the radiative width for an M2 transition in this case is 6.6×10^{-5}eV. Since M2 widths have never been observed to exceed their single particle estimates, δ cannot be significantly different from zero.

The predicted values of A_2/A_o as a function of δ for spin 1, 2 and 3 are shown plotted in Fig. 9. The predicted values of A_4/A_o are never greater than .001 for these population parameters. The error bars on the curve indicate the uncertainties in A_2/A_o due to the uncertainties in the population parameters. The experimental value of A_2/A_o is shown plotted at $\delta = 0$. This value does not compare favourably to those for spins 1 and 3. Spin and parity 2^- are therefore assigned to the 8.87 MeV level.

Summary

The method described here has been successful in

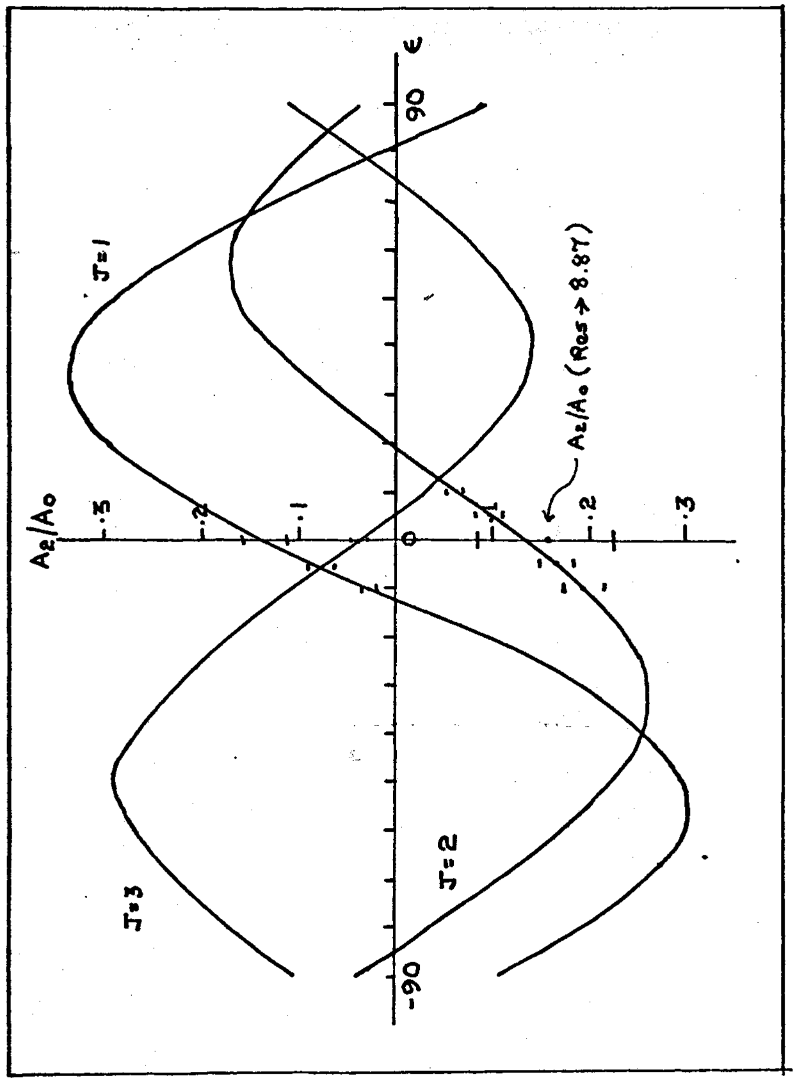


FIG. 9

this favourable case. Without any knowledge of the parities of the resonance and the 8.87 MeV level it would not have been possible to unambiguously assign a spin. Even if the 8.87 MeV level were a natural parity state, all three spins 1, 2 and 3 would have remained as possibilities. It is possible that in another application, the ratio A_4/A_o in the angular distribution would be significant providing additional information upon which to base a spin assignment. The future success of this method will depend very critically on the specific case under examination.

Spin and parity 2^- are assigned to the 8.87 MeV level in Mg^{24}.

APPENDIX

Finite Solid Angle Corrections for the Ge(Li) Detector

An approximate formula for finding the attenuation constants Q_2 and Q_4 is given by Smith (1962). A right cylindrical counter whose axis passes through the source is assumed.

If the face of the detector is located at a distance ℓ from the source, the length of the detector is D and its radius r the attenuation factor Q_κ is given by:

$$Q_\kappa = \frac{P_{\kappa-1}(\cos\alpha) - \cos\alpha P_\kappa(\cos\alpha)}{(\kappa+1)(1-\cos\alpha)}$$

$$\tan\alpha = \frac{r}{\ell + D/2}$$

The actual counter used is not cylindrical. Its cross-section may best be described as a trapezium with rounded corners. Its diameter varies between a maximum of 3.5 cm. and a minimum of 3 cm. Its length is 6 cm.

For the purposes of calculation, the detector was assumed to be a cylinder of diameter 3.25 cm. The resulting attenuation factors were

$$Q_2 = .943 \qquad Q_4 = .840$$

These values were intended only as estimates. It was expected that the uncertainties in the Q's would be insignificant compared to the large experimental errors.

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