PARITY MEASUREMENTS ON ^{26}Mg AND ^{28}Si

PARITY MEASUREMENTS ON ^{26}Mg AND ^{28}Si

by

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SCOPE AND CONTENTS:

An experiment is described whereby information can be gained concerning the parity of a state simply by the observation of that state. The result , based on the principles of parity conservation and time reversal invariance, is applied to measure parities of excited states in ^{26}Mg and ^{28}Si.

ACKNOWLEDGEMENTS

I would like to thank Professor John Kuehner for his help and understanding. I would also like to thank Dr. Alan Baxter, as well as the many people who have helped me in one way or another during the course of this work.

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CHAPTER I

INTRODUCTION

"'old men ought to be explorers' (T. S. Eliot).
Some have to be - because the frontiers of the
familiar are closed to them. But few succeed in
opening new lands" - Dag Hammarskjöld.

A great deal of effort has been expended to obtain information that can lead to a further understanding of the nucleus. The task has been made easier during the past decade with the development of electrostatic accelerators, high resolution detectors, sophisticated data collection techniques, and high speed computers.

The flexibility of electrostatic accelerators is especially evident when considering the light nuclei. These nuclei have not too many protons and hence a smaller Coulomb barrier for penetration by other nuclei. Among the better possibilities in this case is a study of resonance structure leading to information about states in the compound nucleus, as well as allowing states in the residual nucleus to be populated by several different reactions. Angular momentum information can be gathered from angular distributions of the reaction decay products, while other information relating to the nuclear matrix elements can be obtained from cross-sections. The result of approximately two decades of work on light nuclei has been a great deal of information on the properties of nuclear levels. A review of the available literature on light nuclei, and particularly of that pertaining to the two nuclei referred to here is a very long and detailed job. Fortunately this has been done, the latest compilation being in 1967 by P. M. Endt and C. Van der Leun$^{1)}$. They have tabulated work done on the nuclei from Z = 11 to Z = 21.

As one goes up in Z value, the number of possible reactions to a final nucleus decreases. Heavy ions become less practical, except for Coulomb excitation studies, because of the Coulomb barrier between two initial particles.

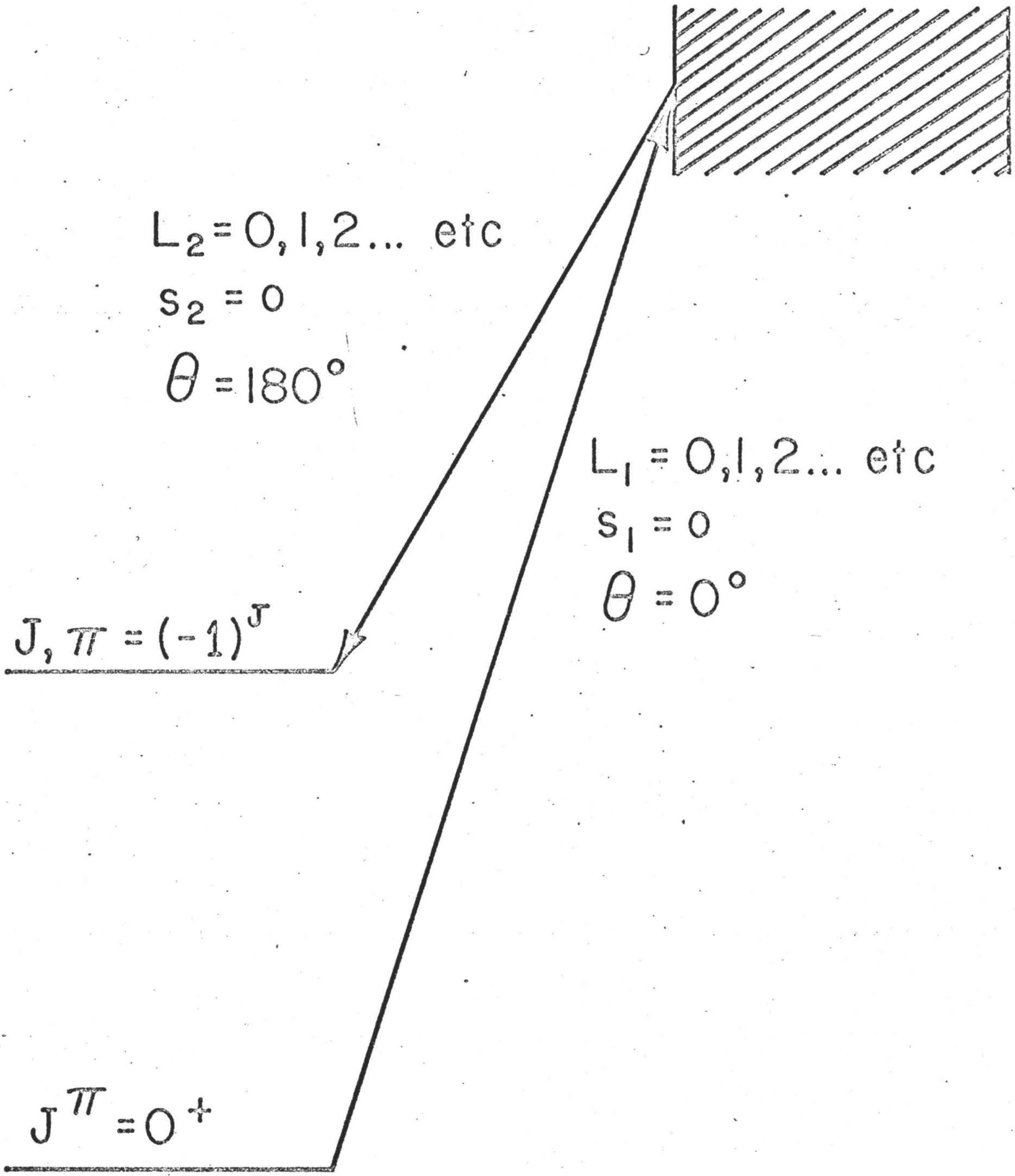
For the heavy elements,information is collected from stripping and pick-off reactions (which are also used for light nuclei). However over the years the most important source of information has been the work in β and γ-spectroscopy. One can learn something of angular momentum and other properties of the various states from γ-γ angular correlation studies. The physics in these measurements can be extracted from the transition rates and γ-ray multipolarities. In this region of the periodic table, one can make use of the internal conversion process which is measurable for the heavier nuclei, but quite unpractical to study for light nuclei. There are many effects that can be used to study the nucleus, and those mentioned here are only the more common ones. The net result however is that there is much more information concerning light nuclei than there is for the heavier ones.

From the theorists' point of view as well, light nuclei are more amenable to study since there are fewer particles involved. For example it is possible to attempt a shell model calculation for an s-d shell nucleus with only a minimum number of assumptions to limit the size of the problem.

Actually the goal of Nuclear Physics is to understand the nuclear force and how it affects Nuclear properties. This is where theory and experiment must interact. The experimentalist supplies information, including excitation energies, transition probabilities, and most important, spins and parities. The theorist then attempts to account for these properties within a framework which is consistent with the knowledge of the Nuclear force.

As was mentioned above, there has been much work done on light nuclei. Quite a few of the methods for making spin and parity determinations have been outlined in a recent symposium on the subject2).

The method to be outlined in this thesis (observation of (α,α') at 180° for a spin zero target) is quite a power-



YIELD AT 180° $\sim (L_2 J 0 0 / L_1 0)^2$

ful tool and can be used to complement the information obtained with stripping experiments. Generally, a stripping experiment leads to a definite parity but often to several possible spin values. When combined with results from (α,α') at 180°, the number of possibilities is reduced, often to a unique value. The principle of this technique will now be outlined.

It was pointed out by Litherland3) in 1961 that in a reaction where both initial particles and one final particle had zero intrinsic spin and positive parity, it followed that the emitted particle could be observed at 0° or 180° only if the residual nucleus left in a state of angular momentum J had parity $(-1)^J$ (see Figure 1). This result is definite and does not depend on the reaction mechanism.

The proof of this statement is contained in the appendix in terms of the initial and final angular momentum wave functions. The validity of this statement depends on time reversal invariance and parity considerations only.

The limitation hereby placed on the final energy state is the basis of this thesis and was used to make parity measurements on ^{26}Mg and ^{28}Si by inelastic scattering of alpha particles at 180°.

The results prove useful in helping to clear up uncertainties in previous assignments to some states in these nuclei. One can definitely eliminate the possibility of unnatural parity if a particular state is observed at 180°. If it is not observed, nothing definite can be stated, for this may be due to reasons other than parity being $(-1)^{J+1}$ i.e. statistical fluctuations or a small reaction cross-section for the state in question.

CHAPTER II

EXPERIMENTAL METHODS

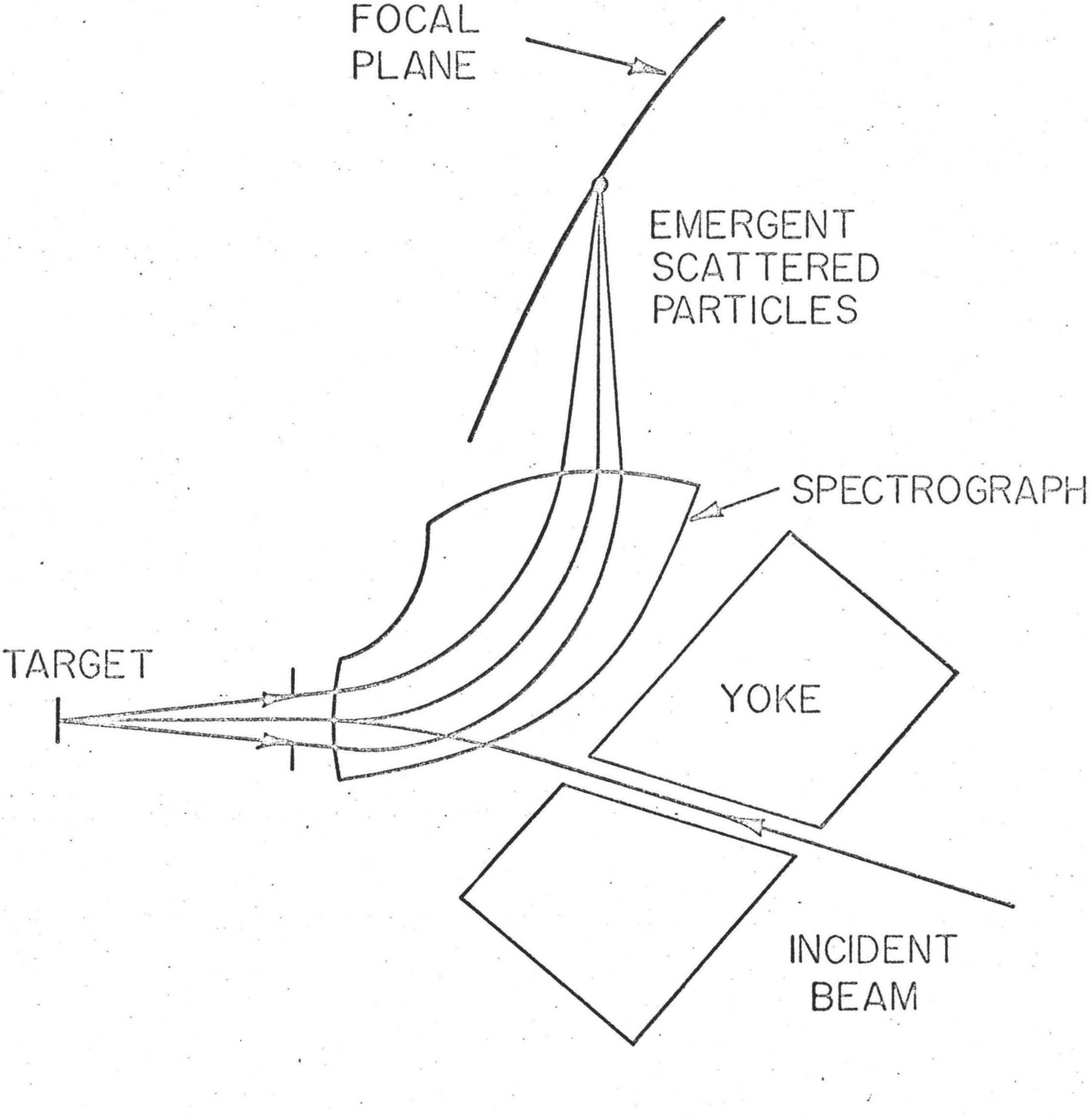
> "The best laid plans of mice and men gang aft
> aglee" Robt. Burns

A) FACILITIES

The experiment was carried out using a High Voltage Engineering Corporation, Model MP. Tandem Accelerator at Chalk River Nuclear Laboratories, producing a beam of alpha particles of approximately 19.5 to 22.5 MeV. Energy stabilization of the beam was acquired by deflecting it through 70° in an analyzing magnet, utilizing a slit system and a corona current control. A nuclear magnetic resonance system was used to measure the magnetic field.

The emitted spectrum of particles was analyzed in a magnetic spectrograph of the type described by Browne and Buechner4). In order to observe scattering at 180°, the beam must first pass through the back of the spectrograph so that it can pass through the entrance slit for the particles being analyzed in the field (as shown in Fig. 2).

The scattered alpha particles were then momentum analyzed in the spectrograph and detected by means of photographic


FOCAL PLANE
EMERGENT SCATTERED PARTICLES
SPECTROGRAPH
TARGET
YOKE
INCIDENT BEAM

plates coated with nuclear emulsion, and placed in the focal plane. The tracks were counted and plotted as the number of tracks per 1/4 mm. strip along the plate against plate position.

There was a solid state counter used as well during the ^{26}Mg experiments. It was set at a forward angle for the run at 19.93 MeV, and a backward angle (159°) for the 20.31 MeV run. The results obtained from this counter were useful for checking the progress of the experiment during the run.

After both runs on ^{26}Mg were completed, a yield curve at the backward angle was measured. The beam energy was raised in steps of .05 MeV over the range of energy used. A particle spectrum was measured after a short run at each beam energy. The intensities of several peaks (namely scattering to the ground state, 1.809 MeV and 3.941 MeV excited states in ^{26}Mg) were measured and compared. From this, an estimate of Γ, the compound nucleus coherence width was made. The purpose of this was to ensure that the increment in beam energy on the long runs was sufficient to provide results that were statistically independent.

B) ANALYSIS

i) Spectrograph data

Although from knowing the NMR frequency one can determine the magnetic field strength, one cannot accurately determine the energy because of the variable geometry in the system. Not knowing the beam energy with precision makes the problem of data analysis a little more difficult to approach analytically.

Another approach is simply to try to build a self-consistent calibration on the basis of trial and error, which is the procedure used here. First, one speculates on the energy of a few peaks and from this tries to extrapolate the energies of other peaks in a manner which is consistent with the information that is already known about the residual nucleus under study. In this way one can identify almost every peak in the spectrum (excluding background).

The method is checked out for reliability by comparing the shape of the calibration curve with a calculated curve and secondly by making use of the positions of the background peaks from carbon and oxygen target contaminants while using straightforward reaction kinematics over the range of energies detected, and requiring internal consistency.

ii) Counter Spectra

The spectrum measured during the low energy run on ^{26}Mg (which was the same angle of observation as for the yield curve) was used to establish a calibration. Several low lying states were observed and identified. This information was then used in order to find the ground state, 1.809 MeV first excited state, and 3.941 MeV state in each of the yield curve spectra. (These states are far enough from any other state that they can easily be resolved.) The area under each of these peaks for all yield curve spectra was tabulated and plotted as a function of the nominal beam energy. The energy interval between the maxima in the yield was measured. This number was then used to estimate the compound nucleus "coherence width". The calculation willbe described later on where results are discussed.

The reason for doing this, as has already been mentioned, was in order to establish whether or not the energy increment in the spectrograph runs was large enough to give independent spectra.

CHAPTER III

YIELD CURVE CONSIDERATION

> "Lies, ... damn lies ... and statistics"
> Winston Churchill.

The beam of alpha particles used in these experiments had lab energy in the region of 20 MeV. The emitted particles were observed at 180° where, for this back angle, most of the reaction goes by compound nucleus formation and decay. The reaction Q-value for compound nucleus formation is approximately +10 MeV. This leads to a compound nucleus excitation of about 30 MeV. At this excitation, the level spacing is much smaller than the widths of the individual levels because many inelastic channels for compound nuclear decay become available. This means that there will be interference between the different compound levels.

The observed fluctuations in cross-sections for various excited states is attributed to the effects of interference in the compound nucleus. This is based on two premises: the first is that the compound nuclear cross-section is much larger than the direct reaction cross-section; the second is that the contribution from the interfering resonant states is coherent. The resultant cross-section is not the sum of many cross-sections thereby giving rise to a relatively constant average cross-section; but rather it is the result of a coherent sum of amplitudes of the compound states. The only incoherent contributions come from the sum over the final substates. In the event that only a single final substate is involved, as in the present experiment, the situation is analagous to a random walk with the most probable amplitude being zero.

There is an energy "width" in the compound nucleus, outside of which one no longer observes the same levels interacting. This energy separation is associated with the average widths of the interfering levels and is known as the "coherence width". A detailed discussion of the quantitative aspects of this interference process has been given by Ericson$^{5)}$.

The "coherence width" is of interest in this experiment because it is important to ensure that the spectra being examined are independent with respect to one another. In order to assure this, the increment in beam energy between different runs should be greater than the coherence width. In this way one maximizes the information from the experiment.

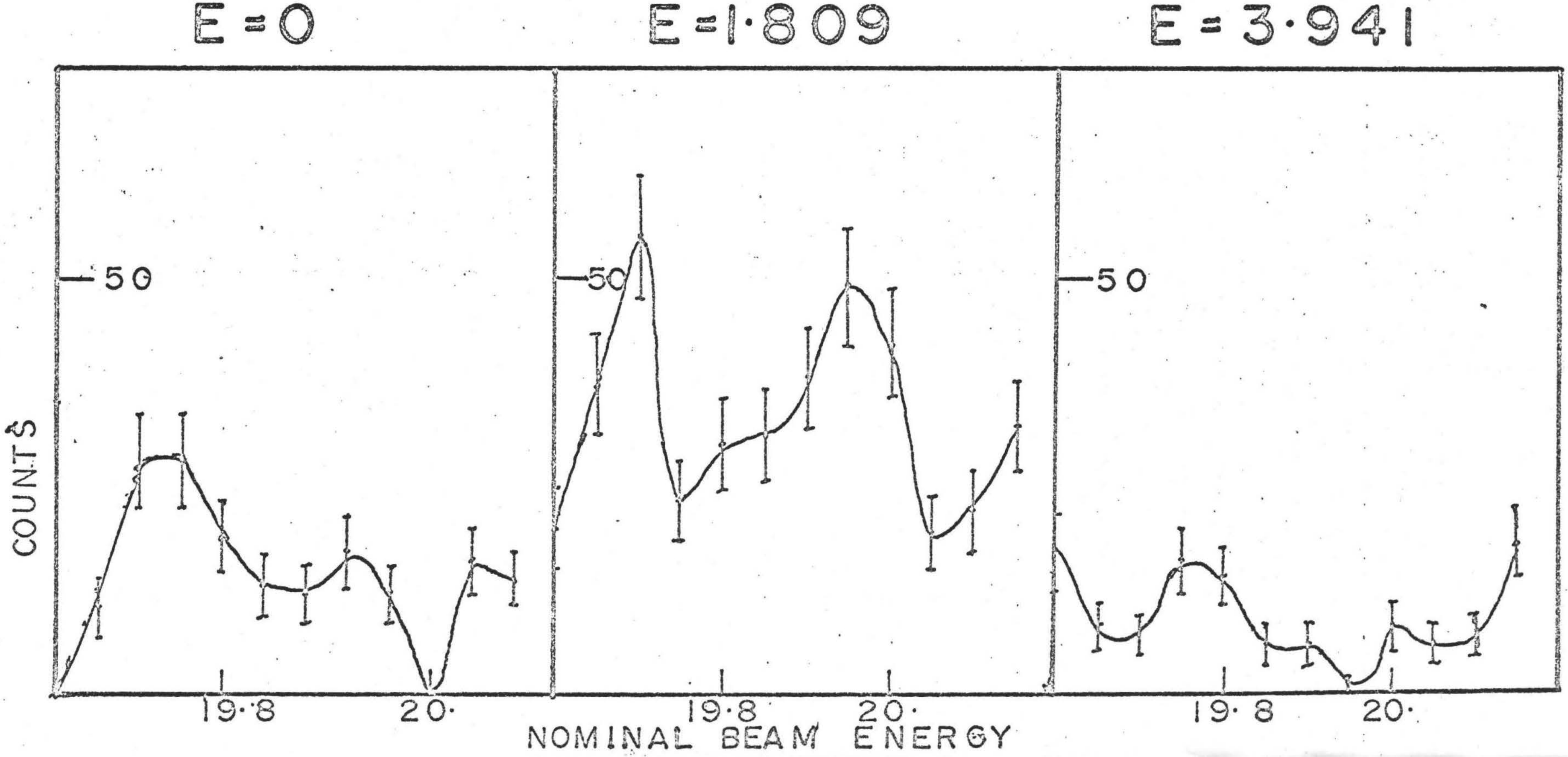
All that is required (in the light of the compound nuclear situation) is for a level to appear during one run for natural purity to be affirmed. On the other hand, the non-observation of a level does not require unnatural parity. However the non-observation of a level for a large number of beam energies separated by more than the coherence width tends to make the argument in favour of unnatural parity stronger.

The energy increment necessary to ensure independent results was measured from the yield curve spectra taken of the alpha particles emitted at 159° in the lab system.

The measurement of the coherence energy can be made according to a method described by Ericson$^{5)}$ by making use of an autocorrelation function. An alternate method, and the one used in this work, involves analyzing the data in terms of the observed peaks in the yield curve. This method is outlined by Brink and Stephen$^{6)}$.

These authors describe and give results from which one can obtain the coherence width provided that the widths of all the compound nuclear states in the region studied are approximately equal. One can measure the average width of the states directly from the yield curve by observing the average number of maxima per unit energy interval in the yield curve. The formula derived by the authors is

$$\Gamma = \frac{0.5}{K_1} \qquad (1)$$



E = 0

E = 1·809

E = 3·941

where Γ is the coherence width, and K_1 is the number of maxima per unit energy in the cross-section. This formula is valid for the case of only one coherent channel.

Since the lab angle of observation for the yield curve is 159°, the reaction is feeding $m = \pm 1$ substates as well as $m = 0$. The case for at least two coherent channels should be considered. There is then a further result from the work of Brink and Stephen,

$$K_n = b_n K_1 \tag{2}$$

where n is the number of coherent channels; $b_1=1$, $b_2=0.78$, $b_3=0.75$ with $b_\infty=0.707$.

The results from this experiment are in figure 3. Despite the fact that there are only approximately two maxima in the whole region of the yield curve and that the results have rather large errors, one can nevertheless estimate the size of Γ. It is in the vicinity of 100 keV, which is smaller than the 400 keV energy increment between the two runs on ^{26}Mg. It is noted that one expects a similar coherence energy for the $^{28}Si(\alpha,\alpha')^{28}Si$ reaction .

CHAPTER IV

RESULTS

> "The great tragedy of science - the slaying of a beautiful hypothesis by an ugly fact"
>
> Thomas Henry Huxley

As was pointed out earlier, if a level appears at 180°, then it has natural parity. If it does not appear one of two things can be true: 1) it is a state of unnatural parity 2) it is a state of natural parity but has a small cross-section. In this thesis emphasis is placed on positive results, and speculation on states which do not appear is omitted.

A visual picture of the results is given in the accompanying figures. The numbers which appear over the peaks in the spectra are the same as those in the first column of the tables. These numbers label the excitation energy according to the level ordering of the excited states given in the tables of Endt and Van der Leun$^{1)}$. The excitation energies (taken from Endt and Van der Leun) of the states observed in (α,α') are given in the second column, as well as their previously known spins and parities$^{1)}$ which are in the third column. Only states which are observed are shown in the tables, thus, only natural parity states are tabulated.

Brackets around a J^π assignment indicate that the assignment is as yet uncertain. A summary of the results is given in Table I for ^{26}Mg and Table II for ^{28}Si. An "x" indicates that the level appeared at the beam energy under which it appears.

A few remarks will now be made concerning states for which there is a discrepancy with previously reported results. It should be pointed out here that from the peak widths, the target thicknesses were found to be approximately 15 keV for the magnesium target and about three times this for the silicon target.

1) Levels in ^{26}Mg

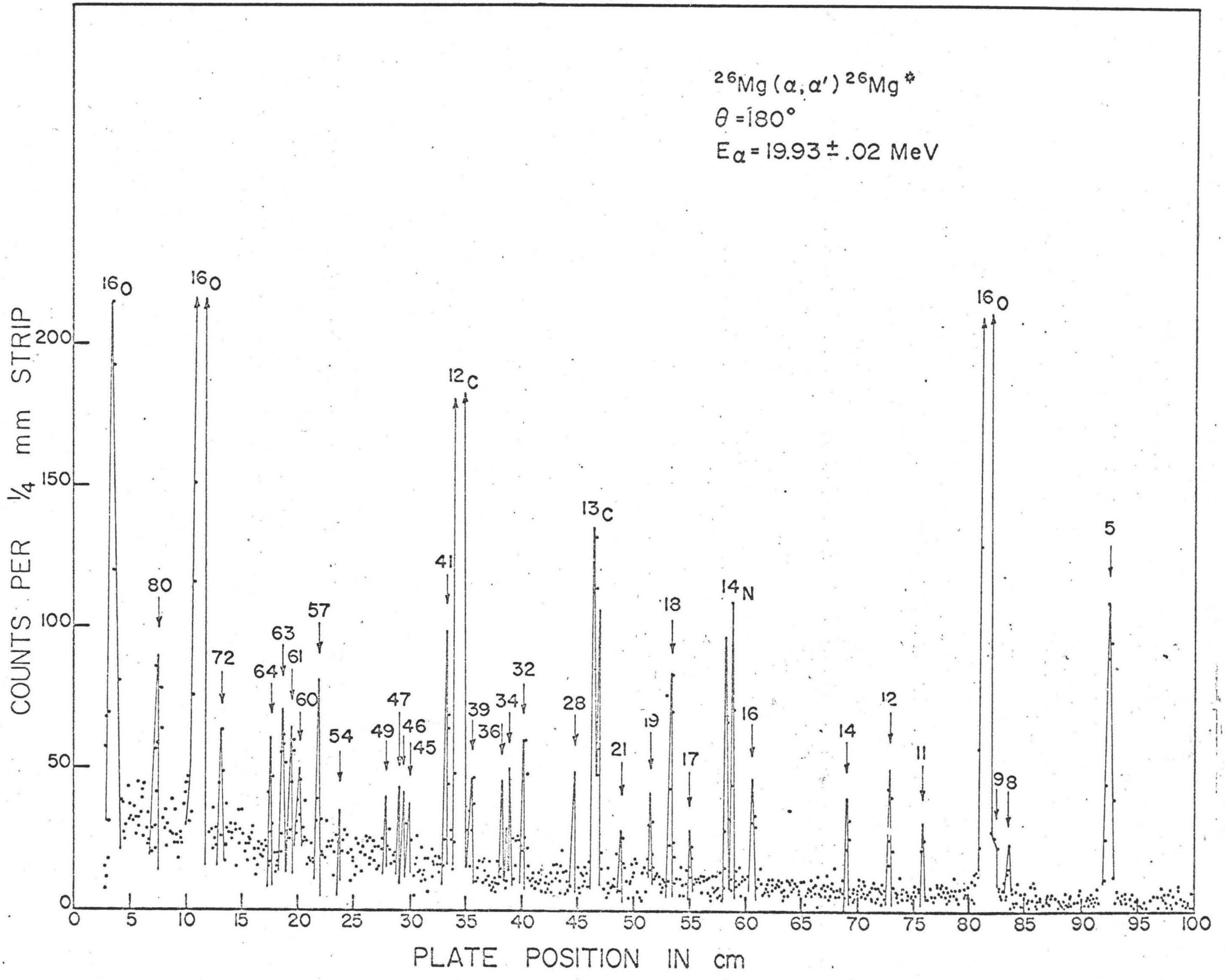
The states from ^{26}Mg which appear in this reaction range in excitation energy from approximately 4.3 to 10.2 MeV (see Figs. 4,5). There is, for several of these states, a discrepancy with previous assignments. However, in all cases, those assignments which disagree are not definite.

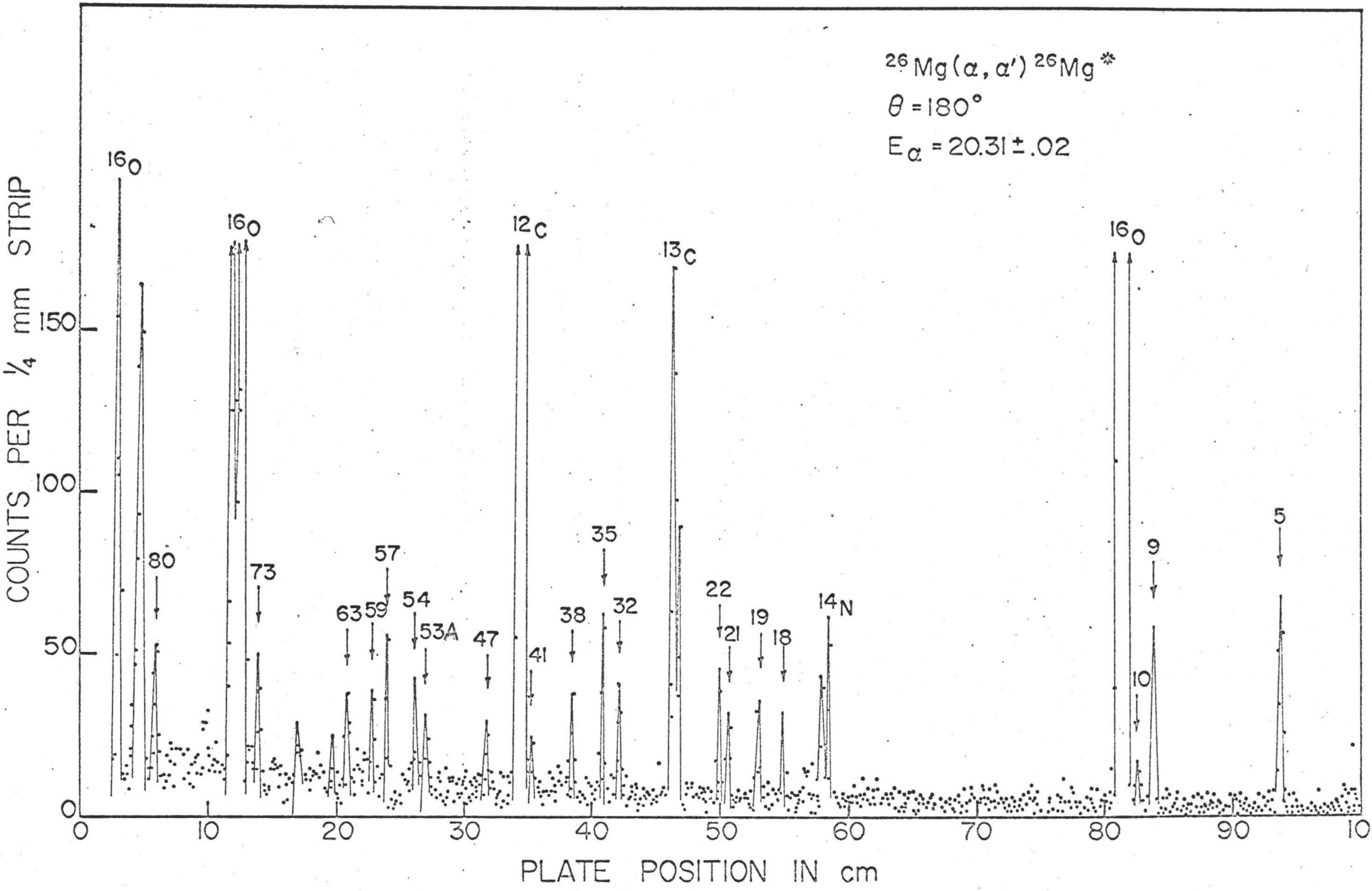
The states in particular are the levels at 4.896, 5.29, 5.71 and 6.62 MeV excitation. These levels are fed by ^{26}Mg(d,p)^{26}Mg with ℓ=2 stripping patterns$^{7)}$. However, because of the relatively large channel spin, the levels in ^{26}Mg can have any spin between 0 and 5, and positive parity. Further information was taken from ^{24}Mg(t,p)^{26}Mg$^{7)}$, which is expected to feed only natural parity levels in double stripping. Since none of these above mentioned states were observed,

TABLE I

$^{26}\text{Mg}(\alpha,\alpha')^{26}\text{Mg}^*$

Level No.	Ex	Jπ	20.31 MeV	19.93 MeV
5	4.313			
6	4.331	(4+)	x	x
7	4.350	2+		
8	4.83	2+		x
9	4.89	(2,3)+	x	x
10	4.97	0+	x	
11	5.29	(1)+		x
12	5.485	4+		x
14	5.710	(3)+		x
16	6.253	0+		x
17	6.616	(3+)		x
18	6.737	2+	x	x
19	6.879	3-	x	x
21	7.056	1-	x	x
22	7.095	2+	x	
28	7.358			x
32	7.668		x	x
34	7.714			x
35	7.761		x	
36	7.808			x
38	7.940		x	
39	8.020			x
41	8.175		x	x
45	8.388			x
46	8.451			x
47	8.494		x	x
49	8.565			x
53A	8.81		x	x
54	8.889		x	x
57	9.031,9.045		x	x
59	9.101		x	
60	9.157			x
61	9.225			x
63	9.294		x	x
64	9.366			x
72	9.76			x
73	9.814		x	
80	10.213,10.218		x	x


$^{26}Mg(\alpha,\alpha')^{26}Mg^*$
θ = 180°
E_α = 19.93 ± .02 MeV
^{16}O
^{16}O
^{12}C
^{13}C
^{14}N
^{16}O
80
72
64
63
61
60
57
54
49
47
46
45
41
39
36
34
32
28
21
19
18
17
16
14
12
11
9
8
5
COUNTS PER 1/4 mm STRIP
0
50
100
150
200
PLATE POSITION IN cm
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100


$^{26}Mg(\alpha,\alpha')^{26}Mg^*$
$\theta = 180°$
$E_\alpha = 20.31 \pm .02$
COUNTS PER 1/4 mm STRIP
PLATE POSITION IN cm
0
50
100
150
0
10
20
30
40
50
60
70
80
90
100
^{16}O
^{16}O
^{12}C
^{13}C
^{16}O
^{14}N
80
73
63
59
57
54
53A
47
41
38
35
32
22
21
19
18
10
9
5

it had been supposed they had unnatural parity. However, this conclusion is incorrect since all four of these are observed in (α,α') at 180° and hence they must have natural parity.

The explanation for this difference may lie in an incomplete understanding of the reaction mechanism for the double stripping experiment. The result from the present work is positive as well as being independent of any assumptions regarding reaction mechanism.

One can then say that these states have natural parity, and must also have positive parity (from the (d,p) results. There is one exception to this, in that the reaction to the 6.616 MeV level is not too well described by an $\ell=2$ transfer in $^{25}\mathrm{Mg}(d,p)^{26}\mathrm{Mg}$, so that all one can say about this state is that it has natural parity.

Further information on these states is available from $^{27}\mathrm{Al}(t,\alpha)^{26}\mathrm{Mg}$ 8) by applying the intensity rule, $(2J+1)\theta^2$, where θ^2 is the reduced width on the alpha particle transitions to these states. These results, of course, are not expected to be rigorous since the $(2J+1)\theta^2$ intensity rule supposes that the reduced widths of various levels are extremely uniform, a dubious assumption at best. These results are not going to be quoted here because it does not add any insight to the problem in light of this new parity information.

The spectra also show some new levels. First, there is a contribution from the previously known triplet near 4.33 MeV which is labelled 5 in both figures 3 and 4. Nothing can be said about it because it is unresolved. There is a previously unreported level at 8.81 MeV, labelled 53A. The peak numbered 80 is either the 10.118 or 10.213 MeV level; the uncertainty in this determination is the unreliability of the calibration curve in this region.

2) Levels in ^{28}Si

The results are plotted in figures 6 through 9 and are summarized in Table II.

One interesting point in these results is the state labelled 8 at 7.38 MeV excitation. It is reported in Endt and Van der Leun to be a 1^+ state as a result of the $^{27}\mathrm{Al}(p,\gamma)^{28}\mathrm{Si}$ reaction studied by Endt and Heylingers9). The branching ratios for the de-excitations to the 2^+ first excited state and the 0^+ ground state were found to be approximately 50-50. This was later confirmed by Carlson10). These results would leave open the possibilities of $1^\pm$, 2^+ for the J^π combination of this state. The same level however was not seen in the $^{16}\mathrm{O}(^{16}\mathrm{O},\alpha)^{28}\mathrm{Si}$ reaction at 180° by Alexander et al.11). It was assumed that this state did not have natural parity, so an assignment of 1^+ was made.

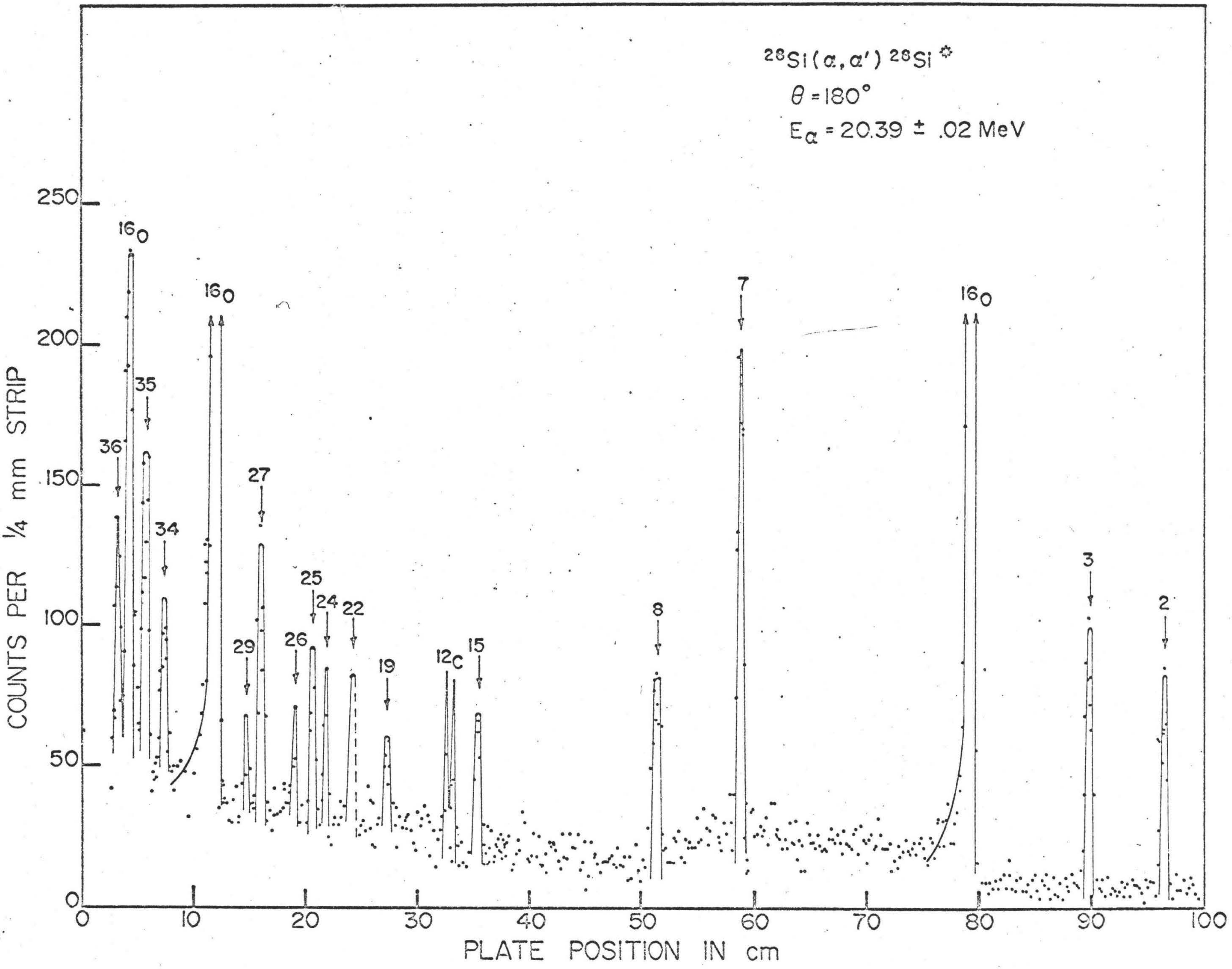
The 7.38 MeV state however is produced quite strongly by the (α,α') at 180°. The two experiments are based on the same principle, however $^{16}O(^{16}O,\alpha)^{28}Si$ produced a negative result. This then illustrates the danger of attaching a positive conclusion to a negative result with this experiment. One is now left with possible J^{π} combinations 1^-, 2^+ for the 7.38 MeV state in ^{28}Si.

There also appear several previously unreported levels which have been labelled 12A, 35, 35A which are approximately 8.45, 11.05, 11.19 MeV respectively. Despite the fact that the target thicknesses are above 35 keV and the statistics are poor, it is unlikely that these states could be identified with previously known levels in ^{28}Si.

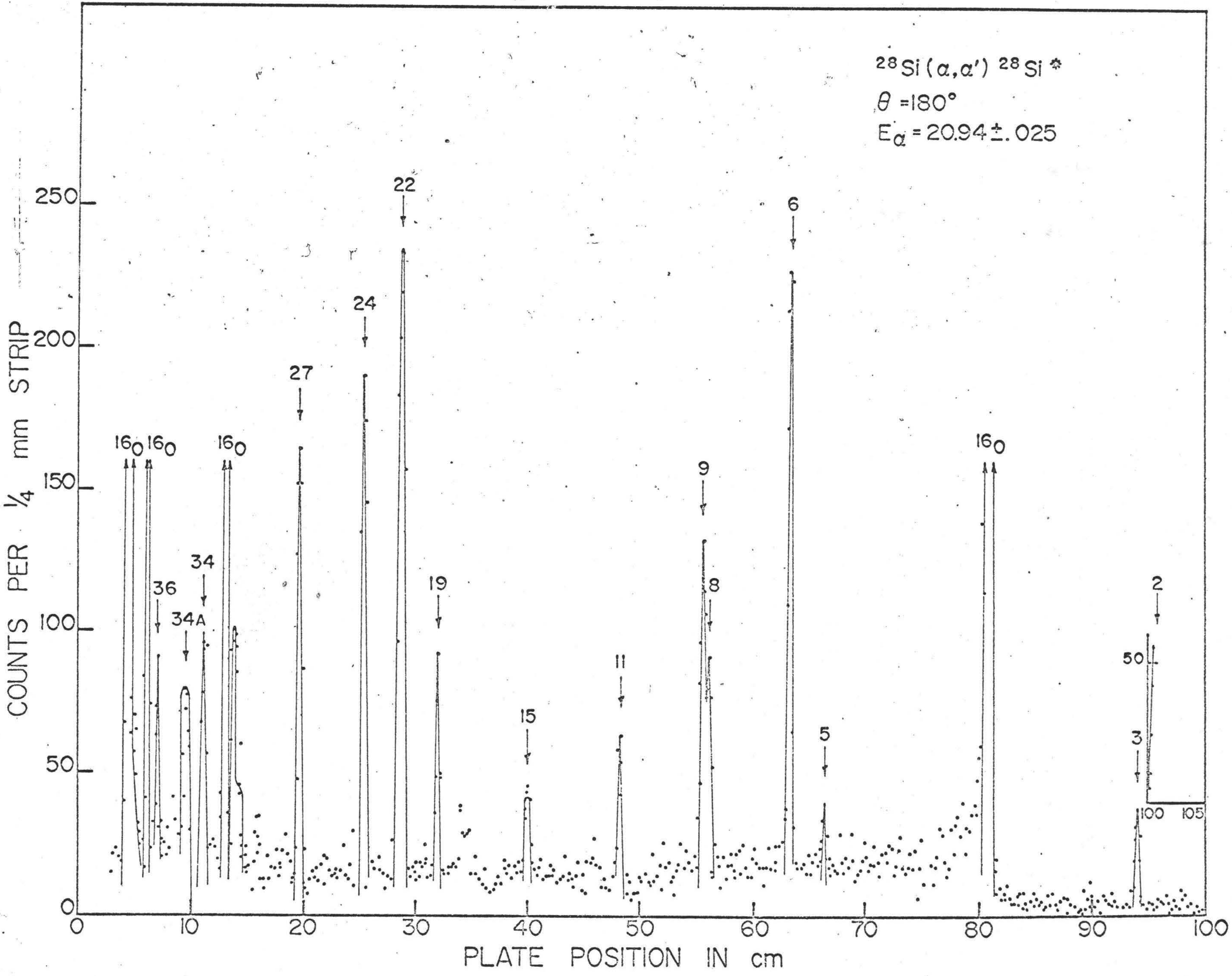
TABLE II

$^{28}Si\ (\alpha,\alpha')^{28}Si$.

LEVEL NO.	Ex	J^{π}	20.39 MeV	20.94 MeV	21.98 MeV	22.43 MeV
2	4.614	4+	x	x		
3	4.975	0+	x	x	x	x
5	6.690	0+	x			x
6	6.878, 6.887	3^-,4+	x	x	x	x
8	7.382	1+	x	x	x	x
9	7.415	2+	x	x		
11	7.932	2+		x	x	
12	8.260	1-			x	x
12A	8.45				x	
15	8.543	(6+)	x	x	x	x
17	8.902	1-			x	
19	9.167		x	x		
22	9.410		x	x		x
24	9.700		x	x	x	x
25	9.762		x		x	
26	9.932		x		x	
27	10.180		x	x	x	x
29	10.308		x			
34	10.909	π = +, T = 1	x	x		
35	11.089 (11.05)		x	x		
35A	11.19				x	
36	11.295	1-	x	x	x	
38	11.514	2+			x	
40	11.656	2+			x	
45	10.020				x	
47	12.180	1-			x	



$^{28}Si(\alpha,\alpha')^{28}Si^*$
θ = 180°
E_α = 20.39 ± .02 MeV
^{16}O
^{16}O
^{16}O
36
35
34
27
29
26
25
24
22
19
^{12}C
15
8
7
3
2
COUNTS PER 1/4 mm STRIP
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100
PLATE POSITION IN cm


$^{28}Si(\alpha,\alpha')^{28}Si^*$
$\theta = 180°$
$E_\alpha = 20.94 \pm .025$
COUNTS PER 1/4 mm STRIP
PLATE POSITION IN cm
0
50
100
150
200
250
0
10
20
30
40
50
60
70
80
90
100
^{16}O
^{16}O
^{16}O
^{16}O
36
34A
34
27
24
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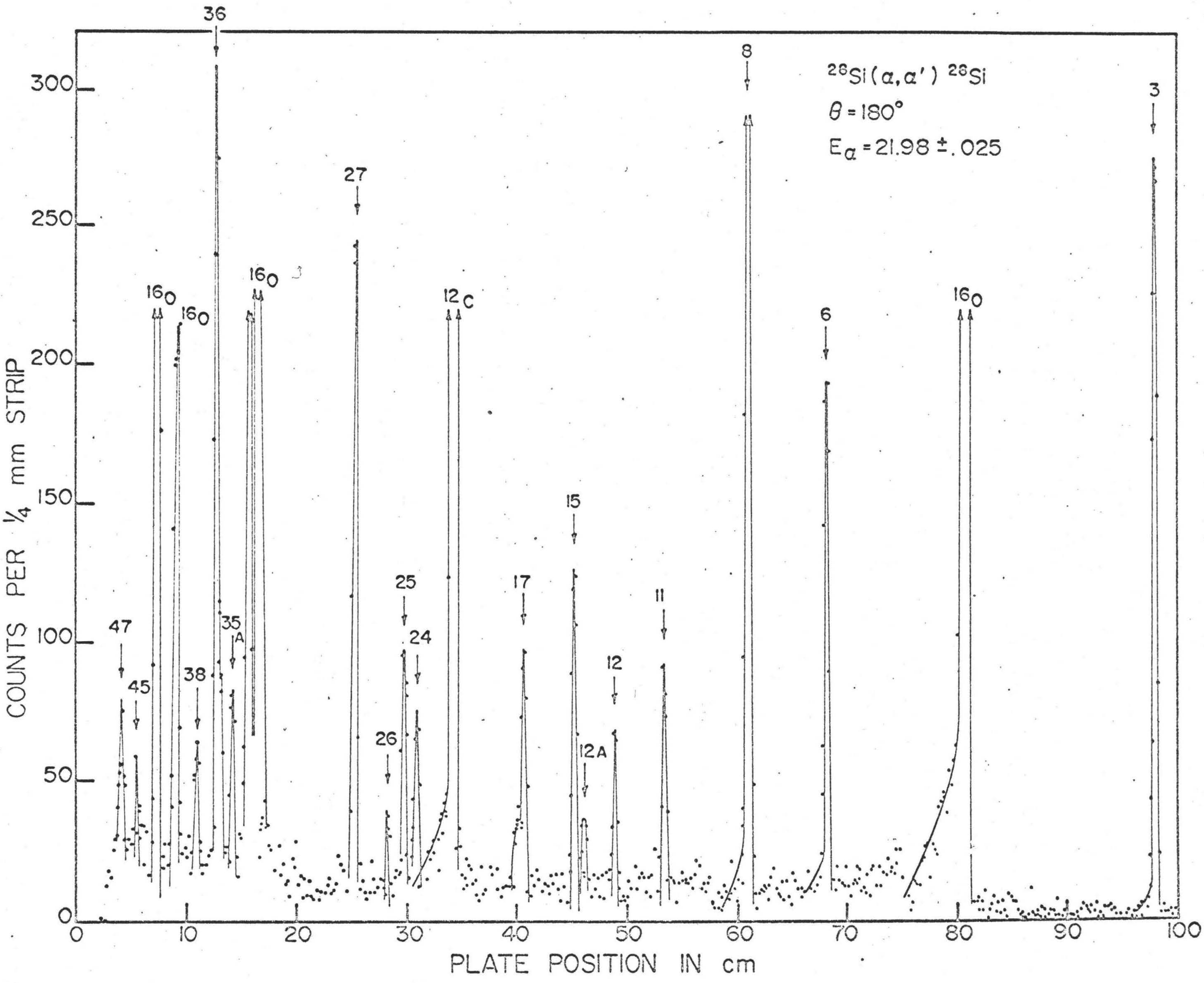
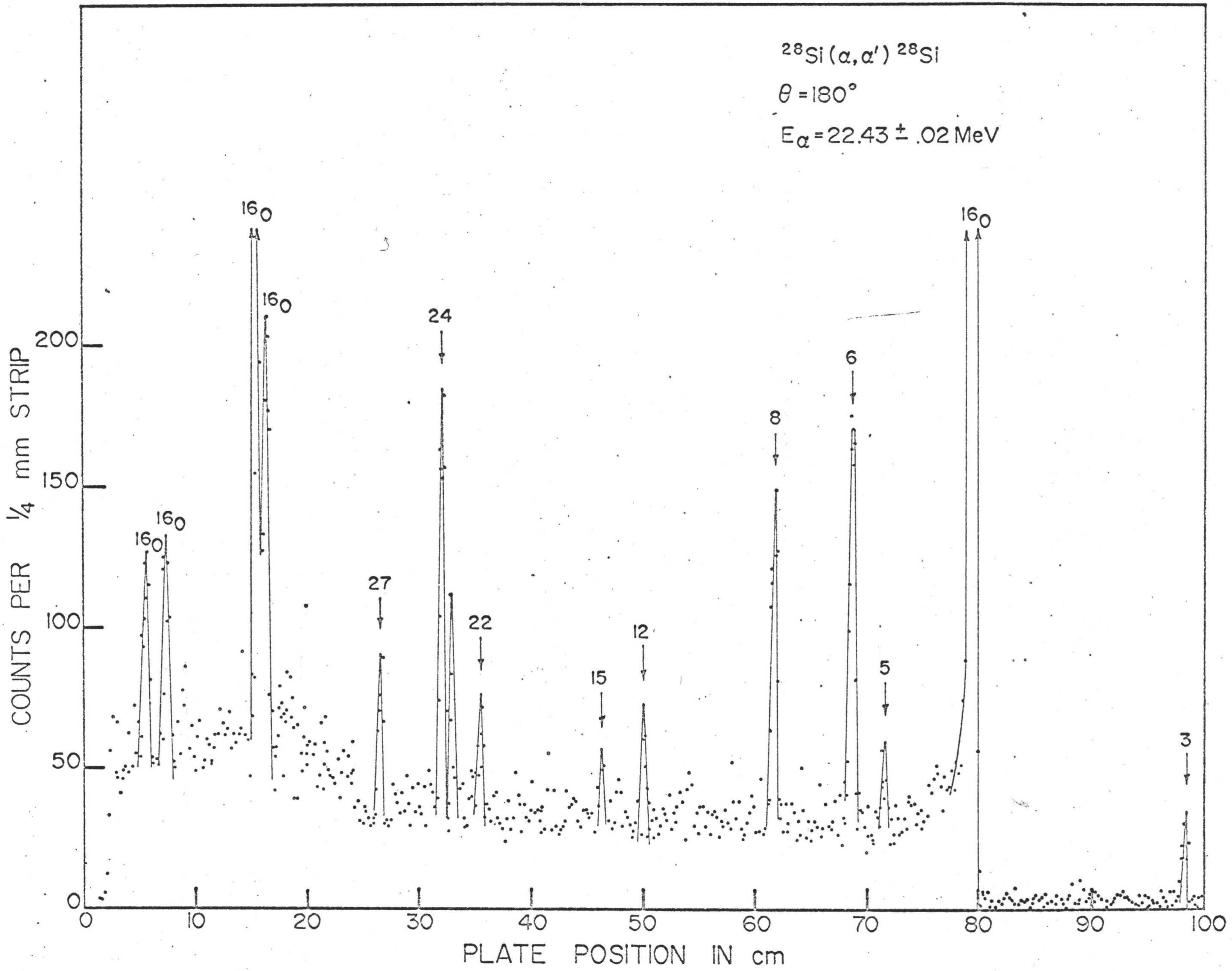
19
15
11
9
8
6
5
3
2
50
100
105


^{28}Si(α,α')^{28}Si
θ = 180°
E_α = 21.98 ± .025
COUNTS PER 1/4 mm STRIP
PLATE POSITION IN cm
0
50
100
150
200
250
300
0
10
20
30
40
50
60
70
80
90
100
47
45
^{16}O
^{16}O
38
36
35A
^{16}O
27
26
25
24
^{12}C
17
15
12A
12
11
8
6
^{16}O
3



$^{28}Si(\alpha,\alpha')^{28}Si$
θ = 180°
E_α = 22.43 ± .02 MeV
COUNTS PER 1/4 mm STRIP
PLATE POSITION IN cm
0
50
100
150
200
0
10
20
30
40
50
60
70
80
90
100
^{16}O
^{16}O
^{16}O
^{16}O
^{16}O
27
24
22
15
12
8
6
5
3

CHAPTER V

CONCLUSIONS

at the ending of this road,
a candle in a shrine:
its puniest flame persists
shaken by the sea
e.e. cummings

The results obtained from this experiment are quite certain since there are no ad hoc assumptions made about the reaction process. The only assumptions required are parity conservation in nuclear reactions and time reversal invariance.

Nothing has been stated about levels which do not appear here. The absence of certain levels may easily be consistent with the supposition that the parity is unnatural; however there may also be other factors influencing their cross-sections for this reaction and the beam energies used.

This experiment can yet be improved to yield better results. For example, there were many difficulties involved in reducing background, which of course affected statistics. As techniques improve, more accurate measurements will be made from this sort of experiment.

APPENDIX

PARITY CONSIDERATIONS

It has been pointed out that for the case where both initial particles and one final particle have zero intrinsic spin and positive parity, and if the emitted particle is scattered at 180°, then if the residual nucleus is left in an angular momentum state J, the parity of that state must be $(-1)^J$. It will now be shown how this statement holds true rigorously, and is independent of the reaction mechanism.

It is desired to describe the final angular momentum wave function in terms of the initial state. The dynamics are described by the relation

$$|JM\ell_2 m_2> \;=\; \sum_{b,\beta} <JM\ell_2 m_2|V|b\beta>|b\beta>.$$

J, ℓ_2, b are the angular momenta of the state in the residual nucleus, orbital angular momentum carried off by the emitted or scattered particle, and spin of the intermediate nucleus respectively. Since in this case the target has spin and parity 0+, the spin of the intermediate state is described by the angular momentum of the incoming projectile.

This experiment is looking at 180° scattering, so all angular momenta must have zero projection along the quantization axis taken as the beam direction. The above equation may now be written as

$$|J0\ell_2 0\rangle = \sum_{\ell_1} \langle J0\ell_2 0|V|\ell_1 0\rangle |\ell_1 0\rangle \qquad (1)$$

where V is the interacting potential which acts also on the other parts of the wave function which do not describe angular momentum ie. it causes the transition.

The Wigner-Eckhart theorem will now be used to simplify the above equation. It is based first on the assumption that the radiating system is independent of its surroundings so that the matrix elements $\langle J0\ell_2 0|V|\ell_1 0\rangle$ are independent of the orientation of the coordinate axes. It states that the matrix elements can be written in the form

$$\langle J0\ell_2 0|V|\ell_1 0\rangle = (J0\ell_2 0|\ell_1 0)\langle J|\ell_2||\ell_1\rangle \qquad (2)$$

where $\langle J|\ell_2||\ell_1\rangle$ is the reduced matrix element.

The final states may now be written

$$|J0\rangle|\ell_2 0\rangle = \sum_{\ell_1} \langle J|\ell_2||\ell_1\rangle(J0\ell_2 0|\ell_1 0)|\ell_1 0\rangle \qquad (3)$$

This is the form that is useful to work with for each combination of J and ℓ_2. For each state in the residual nucleus of spin J, there are several possible values of ℓ_2 however considering only one of these ie. not summing over ℓ_2 will not affect the generality.

The only allowed magnetic substates are M=0 for all states. Now consider the time reversal operator acting on both sides of (3) remembering that its property is

$$K|JM> = (-1)^{J+M}|J-M>.$$

where K is the time reversal operator described by Edmonds[12)]. This then gives

$$(-)^{J+\ell_2}|J0>|\ell_2 0> = \sum_{\ell} <J|\ell_2||\ell_1>(J0\ell_2 0|\ell_1 0)(-)^{\ell_1}|\ell_1 0> \qquad (4)$$

A time reversal, however, is the same as a change of parity followed by rotating the coordinate system through 180° about the quantization axis. This means that if these two operations were performed on (3), then the result should be identical to (4).

A parity operation on (3) gives

$$(-)^{\ell_2}[P|J0>]|\ell_2 0>= \sum_{\ell_1} <J|\ell_2||\ell_1>(J0\ell_2 0|\ell_1 0)(-1)^{\ell_1}|\ell_1 0> \qquad (5)$$

since the orbital angular momentum wave functions necessarily have parity $(-)^{\ell}$. A rotation of 180° about the quantization axis will not change any of the above phases because only M=0 substates are permitted.

The right hand side of both (4) and (5) are identical, but in order for the left hand sides to be equal there must be a relationship

$$P|J0\rangle = (-)^J|J0\rangle. \qquad (6)$$

If this relationship does not hold then neither does the equality needed for that state to be formed. Therefore if the final state is to be seen, it must have natural parity.

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