

PRESSURE DROP AND HEAT TRANSFER

SIMULATION OF TWO-PHASE PRESSURE DROPS IN HEATED CHANNELS
AND HEAT TRANSFER IN A HEATED FUEL ROD

By

ALBERT MIRZA KHACHADOUR, B.Eng.

A Thesis

Submitted to the School of Graduate Studies
in Partial Fulfilment of the Requirements

for the Degree

Master of Engineering

(Part B)

McMaster University

February, 1979

MASTER OF ENGINEERING (1979)
(Engineering Physics)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Simulation of Two-Phase Pressure Drops in Heated Channels
 and Heat Transfer in a Heated Fuel Rod

AUTHOR: Albert Mirza Khachadour, B.Eng. (Liverpool University, U.K.)

SUPERVISOR: G.I. Hadaller

NUMBER OF PAGES: vi, 27

ACKNOWLEDGEMENT

I would like to express my sincere thanks to F. Stern, Head of Atomic Power Division at WCL for his constant interest and guidance throughout this work. Thanks are also due to G.I. Hadaller for his direct supervision and useful suggestions. Lastly, I would like to thank WCL for providing me this opportunity and experience.

TABLE OF CONTENTS

	Page No.
1. Object	1
1.1 Pressure drop calculations	1
1.2 Heat Transfer	3
1.3 LOBI pump tests	4
2. Conclusions	6
References	7
Appendix A	12
Appendix B	21

LIST OF FIGURES

	Page No.
FIGURE 1: Experimental set-up	8
FIGURE 2: Typical flow rate vs. total pressure drop	9
FIGURE 3: Subcooling effect on pressure drop	10
FIGURE 4: Cross-sectional view of the fuel rod	11

OBJECT

Work was carried out in the following fields; pressure drop calculations in heated channels, heat transfer in a fuel rod and observation of a LOBI (Loss of Boiling Investigation) test on a pump.

1.1 Pressure Drop Calculations

Earlier last year, a group of workers at WCL carried out over fifty tests to study the premature heat transfer crisis in heated channels covering a range of pressure values, subcooling temperatures, heat fluxes, hydraulic diameters and flowrates. All these tests were carried out in horizontal, electrically-heated channels, with either an orifice or tubing to provide the outlet feeder resistance.

The prime objective of these tests was to determine the adequacy of using a pump with flat head characteristics and a single channel to simulate a multi-parallel channel system. If successful, this would allow single channel results to be used to predict multi-channel dynamics. A schematic diagram of the experimental set-up is shown in Fig. (1).

Therefore, a series of pressure drop calculations was done to see how theoretical values are compared to the experimentally measured readings. Also, graphs of single and two-phase flow were plotted for each test. Fig. (2) shows a typical curve of a test in single and two-phase flow conditions, where the total pressure drop is plotted versus flow rate. Test data with the same conditions but having one of the parameters different, were also plotted on a single graph to

see the effect of changes of that particular parameter. Fig. (3) shows a comparison of this type.

It has been found for the single phase pressure drops, that calculations based on either the imperial friction factor formula, namely,

$$f = 0.0014 + \frac{0.125}{\text{Re}^{0.32}}$$

or

$$f = \frac{0.0791}{\text{Re}^{0.25}}$$

gave lower values than the experimental readings. Therefore, some modifications had to be done to the friction factor to give consistent results with the experimental data. For example, in the channel region, the first term in the first formula was increased to 0.0016, and it was found that calculations based on this modified friction factor were within 10% of the measured data in all tests. Also, it was checked that friction factors calculated in this way would lie within reasonable values on the Moody diagram (a graph of friction factor versus Reynolds number) for all conditions.

For pre-channel and post-channel conditions, a different friction factor formula had to be used in order to take into account all friction losses due to bends, reducers, tubing or orifice meter and turbine flowmeter.

For the two-phase conditions, three different correlations were used to estimate the pressure drops. These are Martinelli-Nelson, Thom and HTFS (AECL) correlations. It has been found that the first

correlation gives higher values for the pressure drops compared to the measured readings, whereas Thom's correlation gives lower values than the measured ones. The last correlation gives values between the measured and Martinelli-Nelson correlation. Possible cause for discrepancies between measured and calculated values is that both Martinelli-Nelson's and Thom's correlations were developed for mass flow rates different from the mass flow rates used in these tests. Also, HTFS correlation is valid for hydraulic diameters greater than the one used in the tests. A sample of calculation using these correlations is given in Appendix A.

1.2 Heat Transfer in a Fuel Rod

Heat transfer calculations in an electrically heated fuel rod were carried out. The purpose of these calculations was to design and estimate the power requirement of the heater for the most critical situations; that is, when the material around the heater begins to melt. The design will be used in future in tests related to critical heat flux in reactors.

To calculate overall heat transfer, some temperature had to be assumed on the outside surface of the fuel rod. (In this case it was assumed to be 300°C). Also, a practical diameter for the heater was assumed, (0.183 inch). A schematic cross-sectional view of the rod arrangement is shown in Fig. 4.

Heat transfer due to conduction, convection and radiation was calculated to estimate the power of the heater and to obtain the maximum temperature which the rod elements near the centre can stand

without melting. Therefore, it was required to contact some companies to get some information on materials suitable for the above design. The materials investigated were, zirconia, alumina, thoria and brellia. Complete information on the last two materials was not available, so the choice was between zirconia and alumina. Although zirconia can stand higher temperatures than alumina (about 4000°C for zirconia and 3500°C for alumina), the cost was much higher. Therefore, the choice fell on alumina and all the calculations were based on this material. A sample calculation is given in Appendix B.

It was found that the effect of radiation heat transfer is not very significant and calculations can be simplified greatly by neglecting the radiation heat effect in this type of heat transfer problem.

1.3 LOBI Pump Tests

These are two-phase flow test which are called LOBI (Loss of Boiling Investigations) pump tests.

Several tests on the characteristics of a pump (supplied by EURATOM) and its behaviour under various flow conditions in the two-phase flow region were carried out at WCL laboratories. The purpose of these tests was to simulate a rupture condition in a pipe and observe the consequences. These tests are related to safety conditions in a nuclear reactor.

Some of these tests were run for two or three days continuously, and it was required to attend these tests and control or record some of the test parameters. Three heaters which were enclosed in vertical isolated pipings, provided the heat source, and the power on these

heaters was controlled manually and varied as required. Various temperatures, such as in the pump bearing and pressures at various points along the flow line were obtained, and the speed of the pump was controlled manually and adjusted as required. Void fractions were measured using two gamma-densitometers situated at the inlet and at the outlet of the pump. All experimental data were recorded by a computer when steady-state conditions were obtained.

CONCLUSIONS

Theoretical pressure drop calculations based on the imperial friction factor formula will give lower values than the experimental readings. Therefore some modifications needed on friction factor value to match the two cases. Heat transfer calculations in a heated fuel rod have shown that the contribution of radiation heat is very small and therefore it can be neglected in this case and in similar cases.

REFERENCES

- (1) Collier, J.G. Convective Boiling and Condensation, McGraw Hill (1972).
- (2) Wallis, G.B. One Dimensional Two-Phase Flow, McGraw Hill (1969).
- (3) Thom, J.R.S. "Prediction of pressure drop during forced circulation boiling of water", Int. J. Heat and Mass Transfer, V 7, 709-724 (1964).
- (4) A Compendium of Selected Two-Phase Heat Transfer and Fluid Flow Correlations, AECL, CRNL-1437 (1976).
- (5) Flow of Fluids Through Valves, Fittings, and Pipe, CRANE, Tech. paper No. 410-C (1976).
- (6) Kneith, F. Principles of Heat Transfer, Int. Text Book Co. (1958).
- (7) Holman, J.P. Heat Transfer, McGraw Hill (1976).

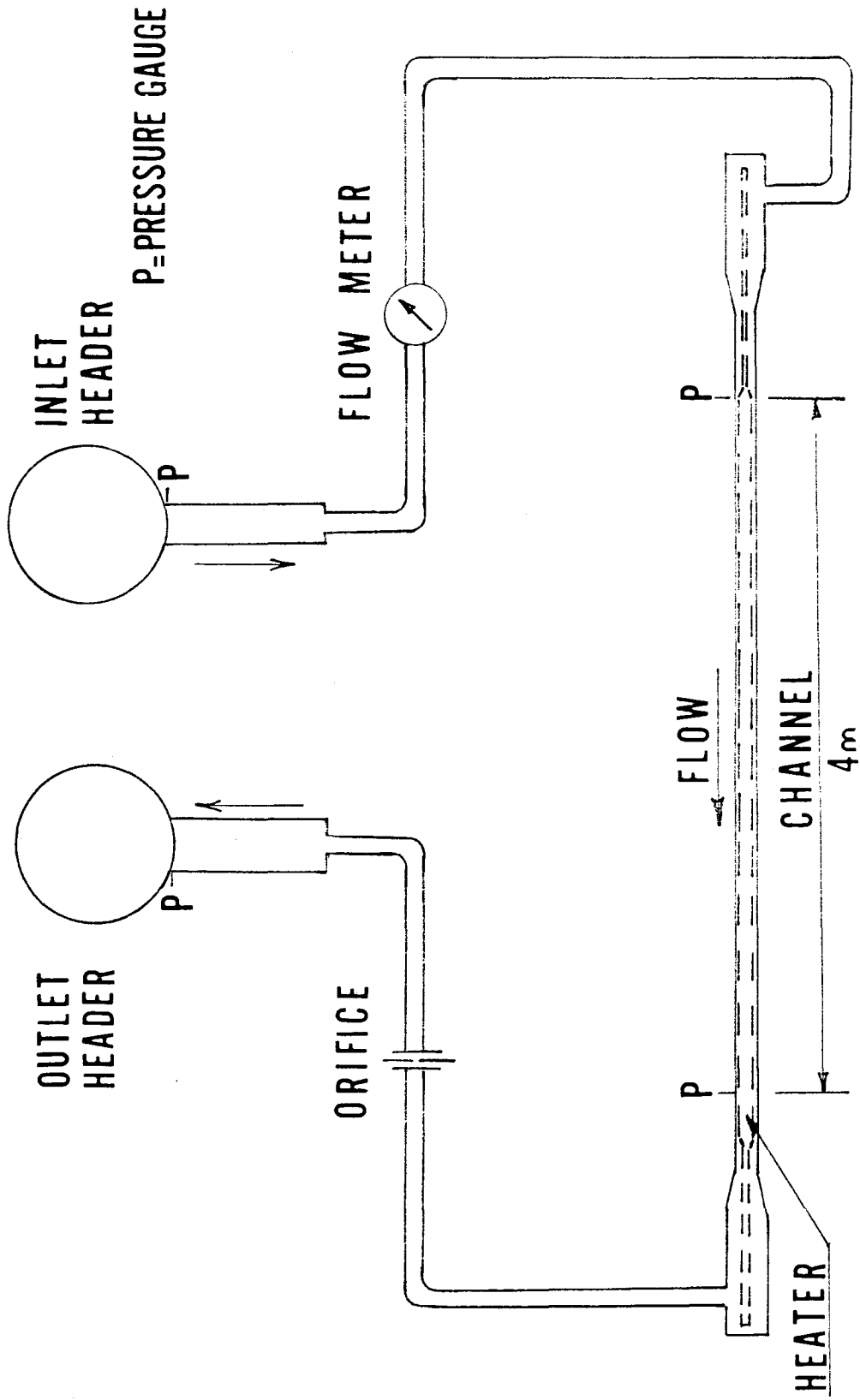


FIG.1. EXPERIMENTAL SET-UP

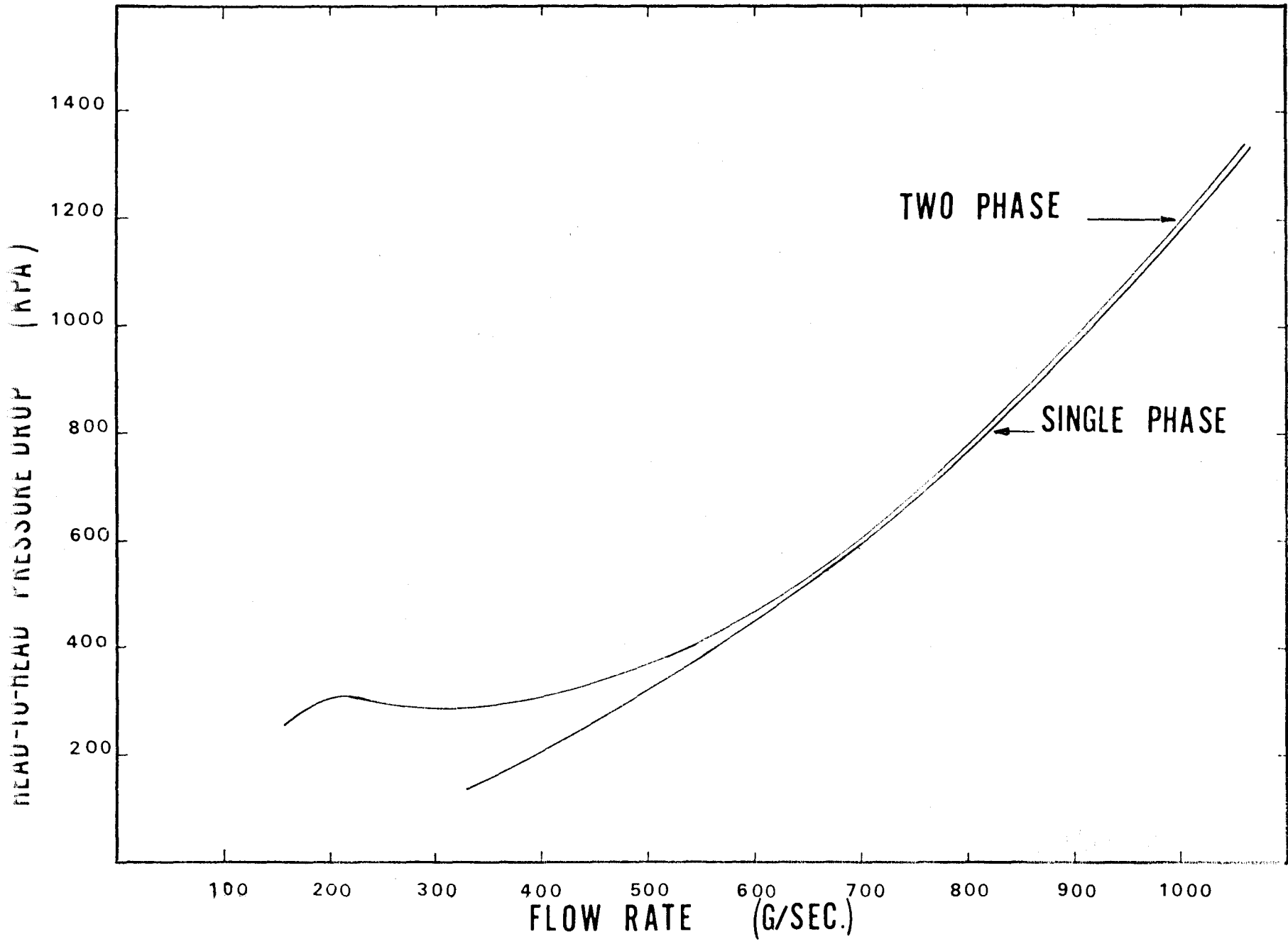


FIG. 2. TYPICAL FLOW RATE VS. TOTAL PRESSURE DROP

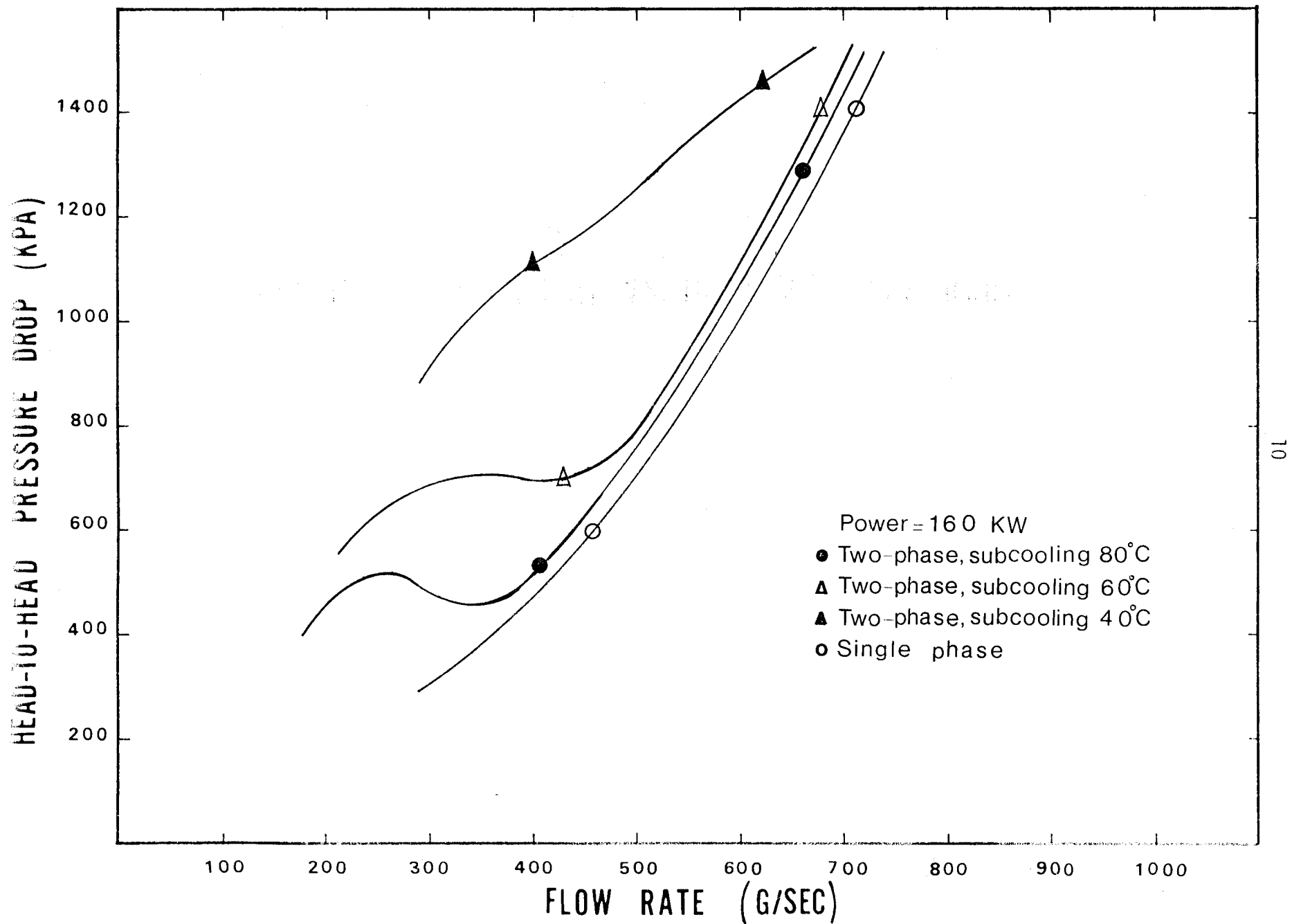


FIG.3. SUBCOOLING EFFECT ON PRESSURE DROP

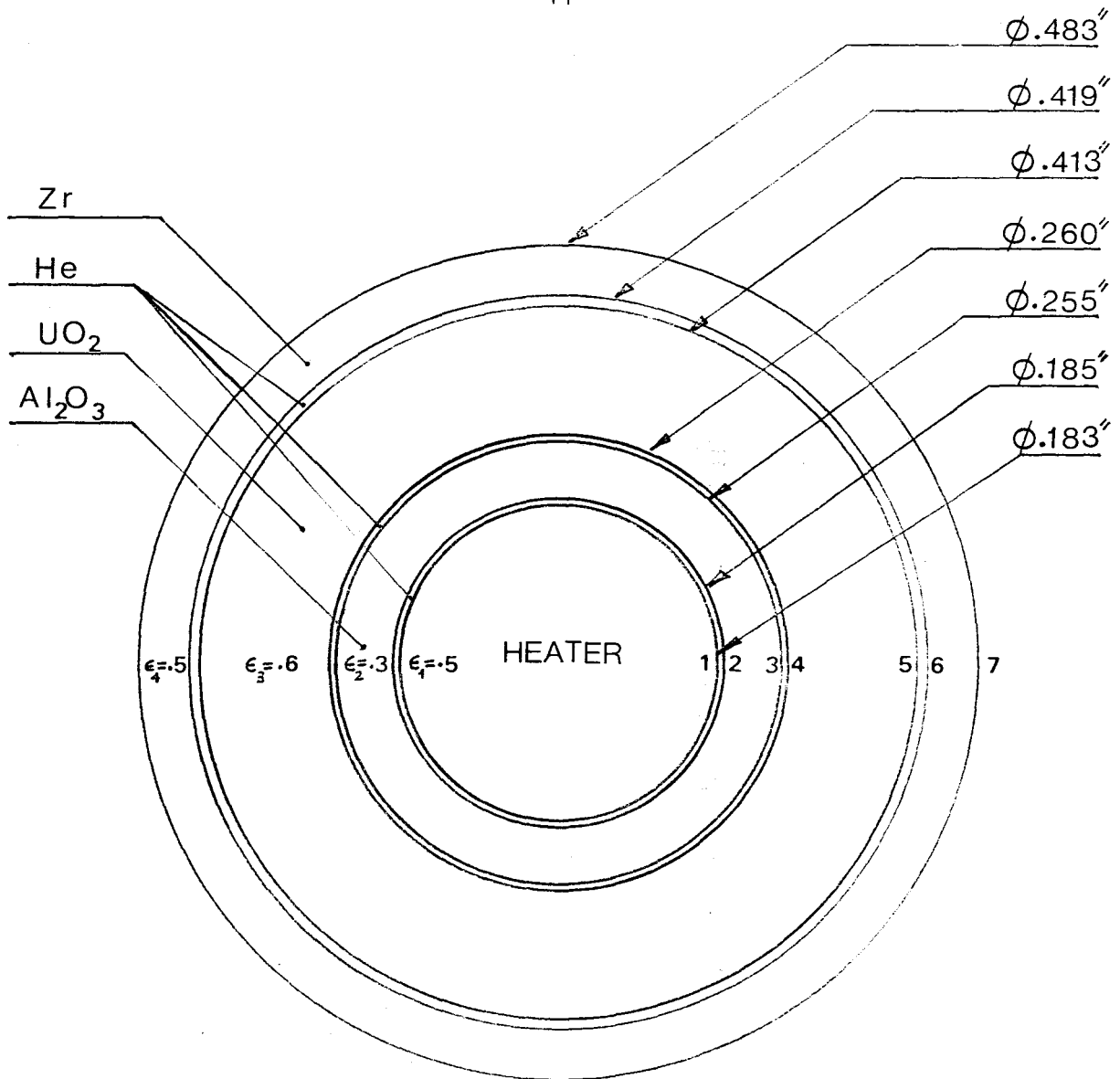


FIG. 4. CROSS-SECTIONAL VIEW OF THE FUEL ROD

APPENDIX A

The following are sample calculations of two-phase pressure drops in the heated channel.

Test data:

Exit quality, $x = 24.3\%$

Coolant flow rate, $Q = 0.224 \text{ kg/sec}$

Coolant inlet temperature = 210°C

Coolant inlet pressure, $P_{in} = 6273.7 \text{ KPa}$

Hydraulic diameter of the channel, $D_h = 3.87 \times 10^{-3} \text{ m}$

Channel length = 4 m

Power, $q = 160 \text{ KW}$

Measured pressure drop across the channel $\approx 131 \text{ KPa}$

Mass flow rate, $-G = \frac{Q}{x\text{-sec area}} = 1.76 \times 10^3 \text{ kg/m}^2 \text{ sec}$

Solution:

To find the friction factor, the following formula was used:

$$f = 0.0014 + \frac{0.125}{\text{Re}^{.32}}$$

From steam tables:

$$\mu_{210^\circ\text{C}} = 127.9 \times 10^{-6} \text{ kg/m sec}$$

$$V_{\&210^\circ\text{C}} = 1.1726 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$\begin{aligned} \text{Re} &= \frac{GD_h}{\mu} \\ &= \frac{1.76 \times 10^3 \times 3.87 \times 10^{-3}}{127.9 \times 10^{-6}} \\ &= 53254 \end{aligned}$$

$$\begin{aligned} \therefore f &= 0.0014 + \frac{0.125}{53254 \cdot 32} \\ &= 0.00524 \end{aligned}$$

Assuming the pressure drop in the channel is single phase only,
then:

$$\begin{aligned} \frac{dp}{dz} &= \frac{2fG^2V_l}{D_h} \\ &= \frac{2 \times 0.00524 \times (1.76 \times 10^3)^2 \times 1.1726 \times 10^{-3}}{3.87 \times 10^{-3}} \\ &= 9.84 \text{ KPa} \end{aligned}$$

For uniform pressure drop, ΔP along the channel length, we get:

$$\begin{aligned} \Delta P &= 9.84 \times 4 = 39.36 \text{ KPa} \\ \therefore \text{Outlet pressure, } P_{\text{out}} &= 6273.7 - 39.36 \\ &= 6234.34 \text{ KPa} \end{aligned}$$

From steam tables,

$$\begin{aligned} \text{Enthalpy at the outlet, } h_{\text{out}} &= 1226.7 \text{ KJ/kg} \\ \text{Enthalpy at the inlet (at } 210^\circ\text{C), } h_{\text{in}} &= 897.7 \text{ KJ/kg} \\ \therefore \text{Maximum heat removed} &= Q(h_{\text{out}} - h_{\text{in}}) \\ &= 0.224 (1226.7 - 897.7) \end{aligned}$$

$$q_{\max} = 73.7 \text{ KW}$$

∴ Some boiling is required to remove the input power of 160 KW.

Assuming a uniform heat distribution, then if Z is the single phase length, then:

$$Z = \frac{73.7}{160} \times 4 = 1.84 \text{ m}$$

∴ The two-phase length, $L = 4 - 1.84 = 2.16 \text{ m}$

∴ Single phase pressure drop, ΔP_{SP} is given by:

$$\begin{aligned} \Delta P_{SP} &= 1.84 \times 9.84 \\ &= 18.1 \text{ KPa} \end{aligned}$$

The pressure at the inlet of the two-phase region, $P_{TP_{in}}$ is therefore:

$$\begin{aligned} P_{TP_{in}} &= 6273.7 - 18.1 \\ &= 6255.6 \text{ KPa} \end{aligned}$$

Properties at this pressure are, (from steam tables):

$$\mu_l = 98.144 \times 10^{-6} \text{ Kg/msec}$$

$$\mu_g = 18.66 \times 10^{-6} \text{ Kg/msec}$$

$$V_l = 1.3268 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$V_g = 31.111 \times 10^{-3} \text{ m}^3/\text{kg}$$

The followings are the three correlations used to calculate theoretical pressure drop across the channel.

1 - HTFS (AECL) correlation [4]

$$\text{Re}_g = \frac{G \times D_h}{\mu_g}$$

$$= \frac{1.76 \times 10^3 \times 0.243 \times 3.87 \times 10^{-3}}{18.66 \times 10^{-6}}$$

$$= 8.8699 \times 10^4$$

$$\text{Re}_l = \frac{G(1-x)D_h}{\mu_l}$$

$$= \frac{1.76 \times 10^3 (1-0.243) \times 3.87 \times 10^{-3}}{98.144 \times 10^{-6}}$$

$$= 5.2536 \times 10^4$$

Now $f = 0.0014 + \frac{0.125}{\text{Re}^{.32}}$

$$f_g = 0.0014 + \frac{0.125}{\text{Re}_g^{.32}}$$

$$= 0.00466$$

$$f_l = 0.0014 + \frac{0.125}{\text{Re}_l^{.32}}$$

$$= 0.00526$$

$$\left[\frac{dp}{dz} \right]_g = \frac{2f_g G^2 x^2 v_g}{D_h}$$

$$= \frac{2 \times 0.00466 (1.76 \times 10^3)^2 \times 0.243^2 \times 31.111 \times 10^{-3}}{3.87 \times 10^{-3}}$$

$$= 1.3704 \times 10^4 \text{ Pa/m}$$

$$\left[\frac{dp}{dz} \right]_l = \frac{2f_l G^2 (1-x)^2 v_l}{D_h}$$

$$\left[\frac{dp}{dz} \right]_{\ell} = \frac{2 \times 0.00526 \times (1.76 \times 10^3)(1 - 0.243)^2 \times 1.3268 \times 10^{-3}}{3.87 \times 10^{-3}}$$

$$= 6.402 \times 10^3 \text{ Pa/m}$$

$$X = \left[\left(\frac{dp}{dz} \right)_{\ell} / \left(\frac{dp}{dz} \right)_g \right]^{1/2}$$

$$= \left(\frac{6.402 \times 10^3}{1.3704 \times 10^4} \right)^{1/2}$$

$$= 0.6835$$

$$f_1(G) = 28 - 0.3 \sqrt{G}$$

$$= 28 - 0.3 \sqrt{1.76 \times 10^3}$$

$$= 15.414$$

$$\Lambda = \frac{V_{\ell}}{V_g} \left[\frac{\mu_{\ell}}{\mu_g} \right]^{0.2}$$

$$= \frac{1.3268 \times 10^{-3}}{31.111 \times 10^{-3}} \left[\frac{98.144 \times 10^{-6}}{18.66 \times 10^{-6}} \right]^{0.2}$$

$$= 0.0594$$

$$T_1 = \exp \left[- \frac{(\log \Lambda + 2.5)^2}{2.4 - 10^{-4} G} \right]$$

$$= \exp \left[- \frac{(\log 0.0594 + 2.5)^2}{2.4 - 10^{-4} \times 1.76 \times 10^3} \right]$$

$$= 0.482$$

$$C = -2 + f_1(G)T_1$$

$$= 2 + 15.414 \times 0.482 = 5.43$$

$$\begin{aligned}
 \phi_{\ell}^2 &= 1 + \frac{c}{x} + \frac{1}{x^2} \\
 &= 1 + \frac{5.43}{.6835} + \frac{1}{.6835^2} \\
 &= 1 + 7.944 + 2.141 \\
 &= 11.085
 \end{aligned}$$

$$\begin{aligned}
 Re_{\ell_0} &= \frac{GD}{\mu_{\ell}} \\
 &= \frac{1.76 \times 10^3 \times 3.87 \times 10^{-3}}{98.144 \times 10^{-3}} \\
 &= 6.94 \times 10^4
 \end{aligned}$$

$$\begin{aligned}
 f_{\ell_0} &= 0.0014 + \frac{0.125}{(6.94 \times 10^4)^{.32}} \\
 &= 0.00493
 \end{aligned}$$

$$\begin{aligned}
 T_2 &= (1 - x)^2 \frac{f_{\ell}}{f_{\ell_0}} \\
 &= 0.757^2 \times \frac{0.00526}{0.00493} \\
 &= 0.611
 \end{aligned}$$

$$\begin{aligned}
 \phi_{\ell_0}^2 &= \phi_{\ell}^2 T_2 \\
 &= 11.085 \times 0.611 \\
 &= 6.773
 \end{aligned}$$

$$\begin{aligned}
 -\frac{dp}{dz} &= \frac{2f_{\ell_0} G^2 V_{\ell} \phi_{\ell_0}^2}{D_h} \\
 &= \frac{2 \times 0.00493 \times (1.76 \times 10^3)^2 \times 1.3268 \times 10^{-3} \times 6.773}{3.87 \times 10^{-3}}
 \end{aligned}$$

$$-\frac{dp}{dz} = 70.92 \text{ KPa/m}$$

$$\begin{aligned} \therefore (\Delta P_{TP})_{\text{friction}} &= 70.92 \times 2.16 \\ &= 153.19 \text{ KPa} \end{aligned}$$

$$\begin{aligned} \Delta P_{\text{TOTAL}} &= \Delta P_{\text{SP}} + (\Delta P_{TP})_f + (\Delta P_{TP})_{\text{acc}} \\ &= 18.1 + 153.19 + 11.92 \end{aligned}$$

(using pressure drop due to acceleration, $(\Delta P_{TP})_{\text{acc}}$ from Martinelli-Nelson correlation, see later)

$$\therefore \Delta P_{\text{TOTAL}} = 183.21 \text{ KPa}$$

2. Martinelli-Nelson Correlation

The two-phase pressure drop due to friction and acceleration in a horizontal pipe is given by:

$$\Delta P_{TP} = 2f G^2 v_\ell \frac{L}{D_h} [\phi_0^2] + G^2 v_\ell [r_2]$$

where $[\phi_0^2]$ = two-phase frictional multiplier = $\frac{1}{x} \int_0^x \phi_{fo}^2 dx$

$[r_2]$ = acceleration pressure drop multiplier for boiling flow of water and steam.

From Figs. 2.5 and 2.7, ref. [1], for inlet pressure of 6255.6 KPa we get:

$$[\phi_0^2] = 5.9 \text{ and } [r_2] = 2.9$$

Using average friction factor,

$$\begin{aligned} f_{\text{av}} &= \frac{0.00526 + 0.00466}{2} \\ &= 0.00496 \end{aligned}$$

$$\begin{aligned}
\therefore \Delta P_{TP} &= 2 \times 0.00496 \times (1.76 \times 10^3)^2 \times (1.3268 \times 10^{-3}) \times \frac{2.16}{3.87 \times 10^{-3}} \\
&\quad \times 5.9 + (1.76 \times 10^3)^2 \times (1.3268 \times 10^{-3}) \times 2.9 \\
&= 134 \times 10^3 + 11.92 \times 10^3 \text{ Pa} \\
&= 145.92 \text{ KPa}
\end{aligned}$$

$$\begin{aligned}
\therefore \Delta P_{TOTAL} &= \Delta P_{SP} + \Delta P_{TP} \\
&= 18.1 + 145.92 \\
&= 163.02 \text{ KPa}
\end{aligned}$$

3. Thom's Correlation

The two-phase pressure drop due to friction and momentum in a horizontal pipe using this correlation is the same as Martinelli-Nelson correlation, except that the values of the multipliers are different.

$$\text{i.e. } \Delta P_{TP} = 2fG^2 v_{\ell} \frac{L}{D_h} [\phi_0^2] + G^2 v_{\ell} [r_2]$$

From Figs. 7 and Fig. 9, ref. [3]

$$[\phi_0^2] = [r_3] = 3.5$$

$$[r_2] = 3.7$$

$$\begin{aligned}
\therefore \Delta P_{TP} &= 2 \times 0.00496 \times (1.76 \times 10^3)^2 \times (1.3268 \times 10^{-3}) \times \frac{2.16}{3.87 \times 10^{-3}} \\
&\quad \times 3.5 + (1.76 \times 10^3)^2 (1.3268 \times 10^{-3}) \times 3.7 \\
&= 79.5 \times 10^3 + 15.21 \times 10^3 \text{ Pa} \\
&= 94.71 \text{ KPa}
\end{aligned}$$

$$\begin{aligned}\therefore \Delta P_{\text{TOTAL}} &= \Delta P_{\text{SP}} + \Delta P_{\text{TP}} \\ &= 18.1 + 94.7 \\ &= 112.8 \text{ KPa}\end{aligned}$$

APPENDIX B

Heat Transfer in a Fuel Rod

Referring to Fig. 4, consider first heat transfer due to conduction and convection only.

Assuptions:

- (a) Surface temperature of the fuel rod is maintained at 300°C
- (b) Maximum temperature at the centre is 2900°K

1. Inner Sheath Temperature, T_6

$$T_6 = T_s + \frac{q}{2\pi K_c} \ln \frac{r_s}{r_6}$$

where 's' refers to surface

q = power input in KW/m

K_c = thermal conductivity of Zr
= 0.01276 KW/m°K

$$\therefore T_6 = 573 + q \frac{\ln \frac{0.483}{0.419}}{2\pi \times 0.01276}$$

$$\therefore T_6 = 573 + 1.773q \text{-----*}$$

2. Outer Pellet Temperature, T_5

$$T_5 = T_6 + q \frac{1}{2\pi r_6 h_{\text{gap}}}$$

where h_{gap} is thermal convective heat transfer coefficient

$$h_{\text{gap}} = \frac{\text{Nu} \times K_f}{D}$$

where Nu = Nusselt number

$$= 5$$

K_f = thermal conductivity

$$K_f = 0.251 \times 10^{-3} \text{ KW/m}^\circ\text{K (at } 700^\circ\text{K-assumed)}$$

D = gap spacing in meters

$$\therefore h_{\text{gap}} = \frac{5 \times 0.251 \times 10^{-3}}{\frac{0.003}{39.37}} = 16.47 \text{ KW/m}^2\text{ }^\circ\text{K}$$

$$\therefore T_5 = T_6 + q \frac{1}{2\pi \left(\frac{0.419}{2} \times 25.4 \times 10^{-3} \right) \times 16.47}$$

$$= (573 + 1.773q) + 1.8q$$

$$= 573 + 3.57 q \text{ -----*}$$

3. Inner Pellet Temperature, T_4

$$T_4 = T_5 + q \frac{1}{2\pi K_p} \ln \frac{r_5}{r_4}$$

$$(K_p = 3.116 \times 10^{-3} \text{ KW/m}^\circ\text{K at } 1350^\circ\text{K-assumed)}$$

$$\therefore T_4 = T_6 + q \frac{\ln \frac{.413}{.260}}{2\pi \times 3.116 \times 10^{-3}}$$

$$= (573 + 3.57q) + 23.64q$$

$$= 573 + 27.2q \text{ -----*}$$

4. Outer Insulation Temperature, T_3

$$T_3 = T_4 + q \frac{1}{2\pi r_4 h'_{\text{gap}}}$$

$$h'_{\text{gap}} = \frac{\text{Nu} \times K'_f}{D'}$$

$$K'_f = 0.429 \times 10^{-3} \text{ KW/m}^\circ\text{K (at 1250}^\circ\text{K - assumed)}$$

$$\begin{aligned} \therefore h'_{\text{gap}} &= \frac{5 \times 0.429 \times 10^{-3}}{\frac{0.0025}{39.37}} \\ &= 33.78 \text{ KW/m}^2\text{K} \end{aligned}$$

$$\begin{aligned} \therefore T_3 &= T_4 + \frac{q}{2\pi \left(\frac{0.26}{2} \times 25.4 \times 10^{-3}\right) \times 33.78} \\ &= (573 + 27.2 q) + 1.429 q \\ &= 573 + 28.63 q \text{ -----*} \end{aligned}$$

5. Inner Insulation Temperature, T_2

$$\begin{aligned} T_2 &= T_3 + \frac{q}{2\pi K} \ln \frac{r_3}{r_2} \\ &= T_3 + \frac{q}{2\pi \times 0.005} \ln \frac{.255}{.185} \\ &= (573 + 28.63 q) + 10.22 q \\ &= 573 + 38.85 q \text{ -----*} \end{aligned}$$

6. Outer Coil Surface, T_1

$$T_1 = T_2 + \frac{q}{2\pi r_2} h''_{\text{gap}}$$

$$h''_{\text{gap}} = \frac{\text{Nu} \times K''_f}{D''}$$

$$K''_f = 0.620 \times 10^{-3} \text{ KW/m}^\circ\text{K, (at 2000}^\circ\text{K-assumed)}$$

$$\therefore h''_{\text{gap}} = \frac{5 \times 0.620 \times 10^{-3}}{\frac{.001}{39.37}}$$

$$= 122.0 \text{ KW/m}^2 \text{ } ^\circ\text{K}$$

$$\begin{aligned} \therefore T_1 &= T_2 + q \frac{1}{2\pi \left(\frac{.185}{2} \times 25.4 \times 10^{-3}\right) \times 122} \\ &= (573 + 38.85 q) + 0.561 q \\ &= 573 + 39.41 q \text{ -----*} \end{aligned}$$

For maximum temperature of $T_1 = 2900^\circ\text{K}$ we get

$$2900 = 573 + 39.41 q$$

$$\therefore q = \frac{2327}{39.41}$$

$$= 59 \text{ KW/m}$$

If the insulating material used around the heater is the alumina, with a melting point approximately 2000°C , let us check the temperature of the alumina, T_2 using above results:

We have,
$$T_1 = T_2 + \frac{q}{2\pi r_2 h''_{\text{gap}}}$$

$$2900 = T_2 + \frac{59}{2\pi \left(\frac{.185}{2} \times 25.4 \times 10^{-3}\right) 122}$$

$$= T_2 + 59 \times 0.561$$

$$\therefore T_2 = 2900 - 35$$

$$= 2865^\circ\text{K}$$

This is obviously much higher than the M.P. of alumina. Therefore using the M.P. value (2273°K) we get:

$$T_1 = 2273 + 0.561 \times 59$$

$$= 2306^\circ\text{K max. temperature at centre.}$$

The new value of q will therefore be:

$$2306 = 573 + 39.41q \text{ (using equation in (6))}$$

$$= q = \frac{1733}{39.41} = 44 \text{ kW/m}$$

To check this value to see whether it is agreeable with the heat equation, we have:

$$\begin{aligned} T_{\max} &= 2273 + 0.561 \times 44 \\ &= 2298^{\circ}\text{K} \end{aligned}$$

which is very close. Therefore we will assume that $T_{\max} = T_1 = 2300^{\circ}\text{K}$.

Now in the following lines, heat transfer due to radiation will be included in the above equations.

Heat radiated between two concentric cylinders is given by:

$$q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1 \right)} \quad [7]$$

where,

q_{12} = rate of heat transfer, Watts

σ = Stefan-Boltzmann constant

$$= 5.669 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

A = surface area, m^2

ϵ = emissivity

T = temperature, $^{\circ}\text{K}$

$$\therefore q_{12} = K_{12} (T_1^4 - T_2^4)$$

where
$$K_{12} = \frac{\sigma A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1 \right)}$$

Using values shown in Fig. 4, we get:

$$K_{12} = \frac{5.669 \times 10^{-8} \times \pi \times 0.184 \times 25.4 \times 10^{-3} L}{\frac{1}{0.5} + \frac{0.184}{.2575} \left(\frac{1}{0.3} - 1 \right)}$$

where L is the length of the fuel rod.

$$= K_{12} = 22.7 \times 10^{-11} L$$

Similarly,

$$K_{34} = 31.096 \times 10^{-11} L$$

and

$$K_{56} = 86.430 \times 10^{-11} L$$

$$\text{Radiated heat } q_{12} = K_{12} (T_1^4 - T_2^4)$$

$$= 22.7 \times 10^{-11} L (2300^4 - 2273^4)$$

$$= 0.29 L \text{ KW}$$

$$= 0.29 \text{ KW/m}$$

$$q_3 = q + q_{12}$$

$$= 44 + 0.29$$

$$= 44.29 \text{ KW/m}$$

$$T_3 = 573 + 28.63 \times 44.29$$

$$= 1841^\circ\text{K}$$

$$T_4 = 573 + 27.2 \times 44.29$$

$$T_4 = 1778^\circ\text{K}$$

$$q_{34} = K_{34} (T_3^4 - T_4^4)$$

$$= 0.47 \text{ KW/m}$$

$$q_5 = q_3 + q_{34}$$

$$= 44.76 \text{ KW/m}$$

$$T_5 = 573 + 3.57 \times 44.76$$

$$= 732.8^\circ\text{K}$$

$$\begin{aligned}T_6 &= 573 + 1.773 \times 44.76 \\ &= 652.4^\circ\text{K}\end{aligned}$$

$$\begin{aligned}q_{56} &= K_{56} (T_5^4 - T_6^4) \\ &= 0.09 \text{ KW/m}\end{aligned}$$

$$\begin{aligned}q_{\text{TOTAL}} &= q_7 = q_5 + q_{56} \\ &= 44.85 \text{ KW/m}\end{aligned}$$

∴ Radiation effect $\approx 0.85 \text{ KW/m}$