OPTIMUM DESIGN OF HIGH-RISE
OFFICE BUILDINGS

OPTIMUM DESIGN OF HIGH-RISE OFFICE BUILDINGS

by

JAN JANOSIK, Dipl. Ing.

A Project Report
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree
Master of Engineering

McMaster University
Hamilton, Ontario

1972

MASTER OF ENGINEERING (1972)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Optimum Design of High-Rise Office Buildings

AUTHOR: Jan Janosik, Dipl. Ing. (Bratislava)

SUPERVISORS: Dr. P. R. Barnard, Dr. A. E. Heidebrecht, Dr. R. G. Drysdale

NUMBER OF PAGES: vi, 150

SCOPE AND CONTENTS:

This project report includes classification of a design of high-rise buildings. Then the survey of recent optimisation techniques is made. The optimum design of one-way flab, beam and girder slab is presented. Also optimum design of flat plate is presented as both problems are part of overall optimum design of high-rise buildings.

The project report also describes the development of the computer programs for optimum design of one-way slab and flat plate. The behaviour of cost function in vicinity of optimum solution is presented and comparison of conventional versus optimized design method is made.

ACKNOWLEDGEMENTS

The author would like to express his appreciation to Dr. P. R. Barnard, Dr. A. E. Heidebrecht and Dr. R. G. Drysdale for their guidance and advice during the course of this project report.

The author also takes this opportunity to thank the following:

1. McMaster University for financial support during this project; and
2. The National Research Council of Canada for financial assistance.

And, finally, the author wishes to express his gratitude to his friends at McMaster University for their encouragement and help during the course.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

Office buildings are investment-type projects, and therefore, the efficiency and economy of the structural and architectural solutions are the two most important factors affecting the final development. To achieve maximum efficiency and economy of any system, an optimization technique is utilized. Optimization, in the design of a load bearing structure, is to find means to make all of the various structural elements work to their maximum capacity and in such manner as to produce a structure that best fulfills its functions and is most economical.

The fundamental concepts of optimization with mathematical formulations are described in recent textbooks [(1), (2), (3), (4), (5), (6), (7), and (18)]*. The optimization techniques in Civil Engineering are applied to minimum cost design as well as weight minimization. Rubinstein and Karagozian(25) have, used a linear programming technique to obtain the minimum weight design of a multi-storey steel frame, using safety against collapse as the design criteria. The assumption that the final cost is reduced in accordance with the reduction in structural weight is

---

* Numbers in bracket refer to number in Reference list.

usually followed in most intuitive designs. The minimum cost design of reinforced girders using the technique of Lagrange multiplier has been published by Hill[10] . Hill also used the technique of steepest descent (or the path of steepest slope) in the automated optimum design of a practical one way solid slab, beam and girder in reinforced concrete building[26]. Templeman[8] introduced the application of geometric programming in Civil Engineering. However, this technique is not likely to succeed in large projects, because of the large number of interrelationships which must be specified. Bond[9] developed a computer program for the design of reinforced concrete structures supported on columns. However the limitation on the number of structural members and the uniformity of the structure along the height of the building make it inconvenient for the design of high-rise buildings.

Office-buildings are grouped in urban centres where the cost of land is very expensive, therefore, it is the purpose of this study to undertake the optimum structural design of the tower type high-rise, office building.

The structural design of tall buildings consists of the design for gravity loads and lateral loads. Conventionally, tall buildings are braced against wind load by providing concrete shear walls and/or a shear core. If the capacity of the core is not sufficient, then the designer has to turn to the exterior framing to achieve additional lateral stiffness. Recently, structural engineers conceived the idea of designing tall buildings so that they behave like hollow-tube cantilevers in resisting wind forces. The exterior walls are designed either as Vierendeel beams or as trusses with diagonal bracings, while the core is designed for gravity loads only[29]. More recently, a third approach has been developed by the consulting engineering firm of Severud, Sturm, Coulin and Baudel[30]. This approach draws upon the moment capacity of the core for lateral resistance and adds to it the axial resistance of the exterior columns by tieing the exterior columns to the core at sufficiently stiffened mechanical floors.

In this study, the latter approach is employed. The efficiency of this approach, in comparison to the previous two approaches for the design of the tower-shaped high-rise office building, is demonstrated on the model given in Reference (30).

This project report consists of six parts. In Chapter II, the evaluation system of the building is defined and the basic coordination model for an optimally designed tall building is given. Chapter III contains a survey of recent automatic optimization methods which have found or should find useful application in the optimum design of load bearing structural members using the computer. The optimum design of a one-way solid slab, beam and girder floor-framing system as part of the coordination model is described in Chapter IV. The flat plate form of construction is commonly used as floor-framing in modern office buildings.

Its optimum design is given in Chapter V, followed by conclusions and recommendations in Chapter VI. Appendices A and B contain computer programs for the optimum designs of the one-way solid slab, beam and girder, system and of the flat plate system respectively.

CHAPTER II

DESIGN APPROACH, THE DEFINITION OF THE EVALUATION SYSTEM AND THE BASIC COORDINATION MODEL

2.1 Introduction

The approach to optimization was summarized by Beveridge and Schechter(1). This approach, as adjusted to suit the present problem, is shown in Figure (2.1) in the form of a flow diagram. In this chapter, the problem is identified, the evaluation system is defined and construction of the basic coordination model for optimal design of a tall building is shown.

The object is the minimization of the cost of the reinforced concrete structure as based on the unit prices of construction materials. The three basic construction materials under consideration are: concrete, reinforcing steel and formwork.

The present chapter consists of the following sections: Section 2.2 introduces the different types of office buildings and describes the approach to the design of tall buildings which is being adopted in this study. The definition of the evaluation system is given in Section 2.3, which also contains a discussion on the decisions a designer has to make concerning the type of floor-framing and vertical supporting members. Section 2.4 presents a description of the basic coordination model for the optimal design of high-rise buildings. The determination of the unit prices of construction materials pertaining to this study is given in Section 2.5, followed by a summary of this chapter in Section 2.6.

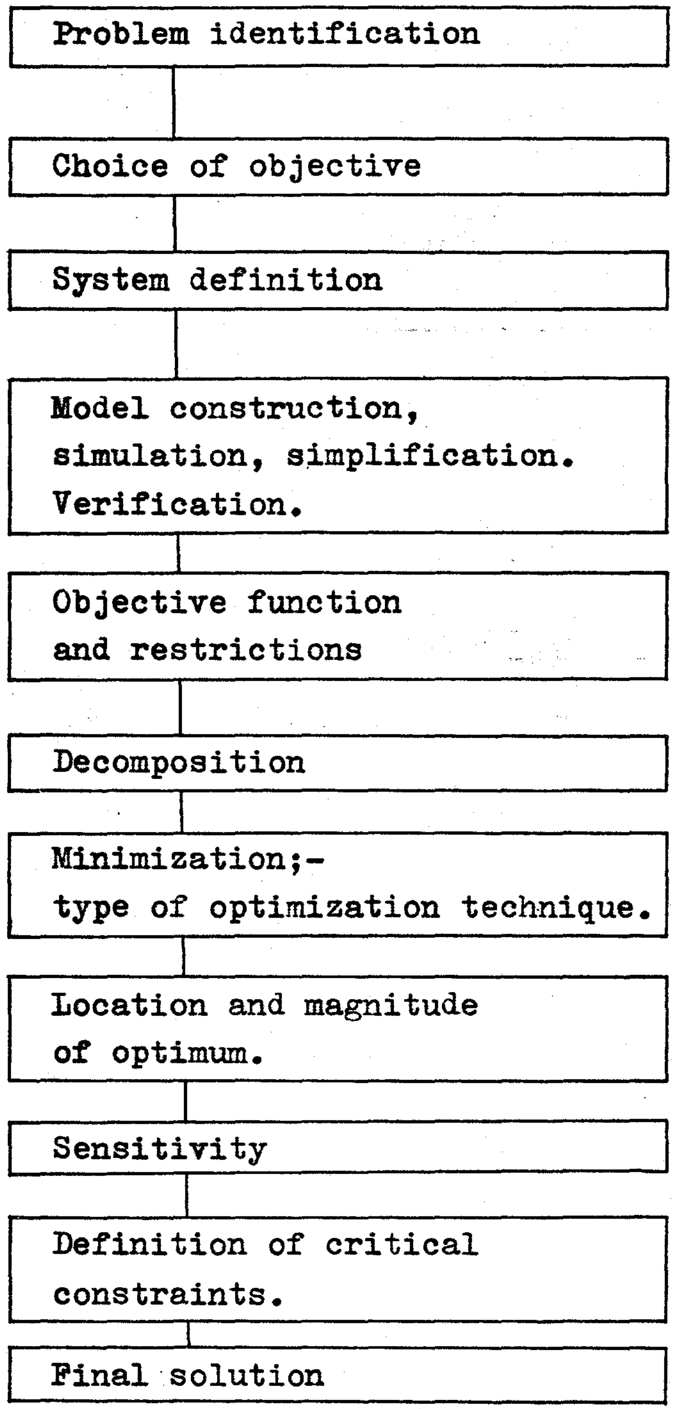


Fig. 2.1 Flow - diagram for optimization.

2.2 Design Approach

Usually, the space utilization in tall buildings varies over the height of the structure. Most of the recent office towers have a utilization distribution similar to that shown in Figure (2.2). This distribution induces substantially different live loads at different flows. The disposition on the plan depends on many factors such as the location and the shape of the building site, owner's demends, and the architect's solution. However, the building is generally planned in basic modules with mobile partitions to achieve multi-purpose usage of space. The elevators, staircases, service ducts, smoke shafts and other services are grouped together and enclosed in a continuous shaft for the full height of the structure. The layout of this core depends on the various user demands such as the number of persons per unit floor area and the number of storeys. An example core layout is suggested in Figure 2.3. The core can be placed near the centre of the plan (known as an inner core) as shown in Figure 2.4 or on an axis of symmetry of the plan at the circumference of the tower (known as an exterior core) as shown in Figure 2.5. The position of the core in the plan disposition of a building has the significant role as for the design assumption on the structure resisting the lateral loads. In this study, only tall buildings with inner cores are considered.

In tall buildings, there is always the consideration that the lateral movement caused by wind loads might be large enough

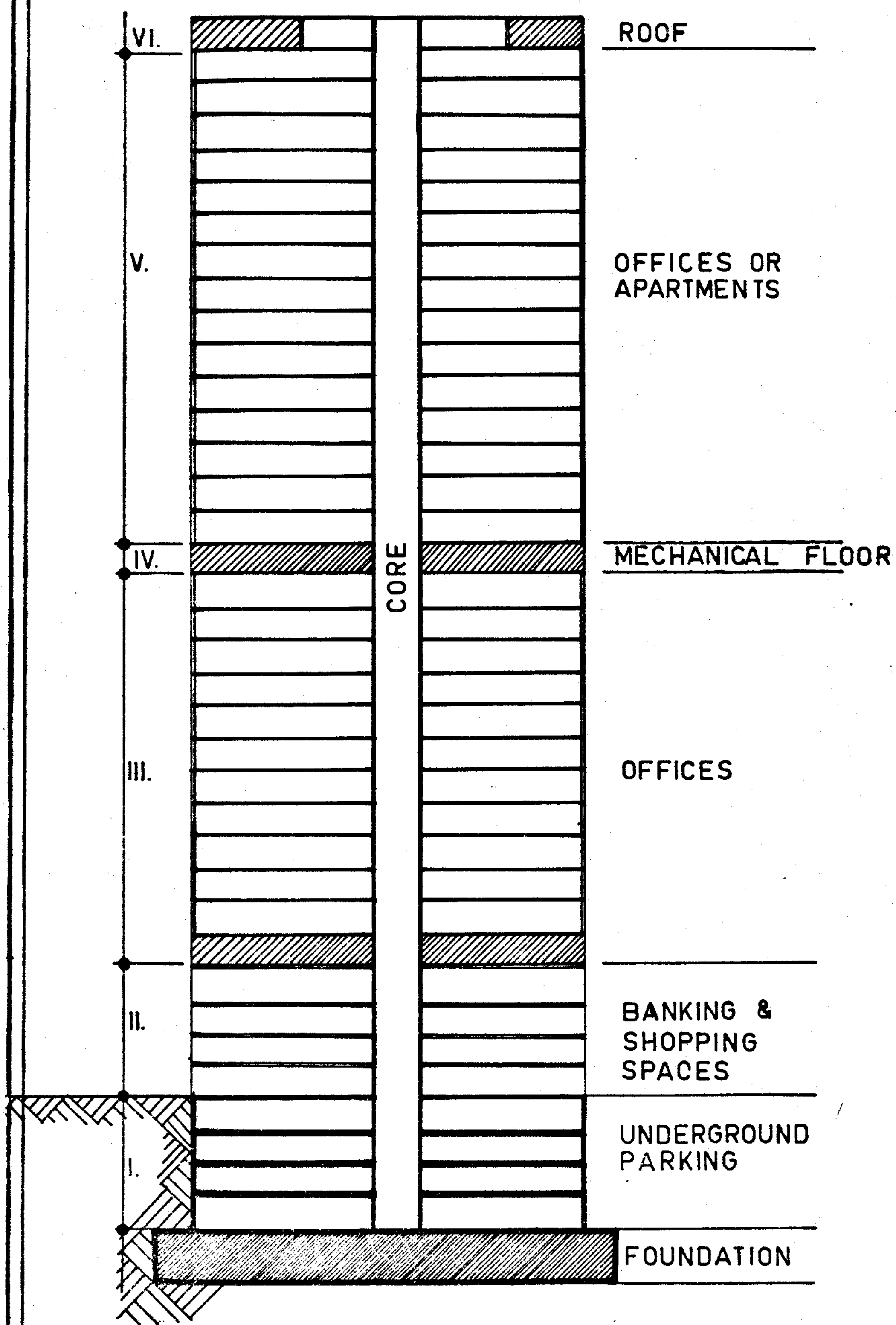


FIG. 2.2 ELEVATION OF A TALL BUILDING

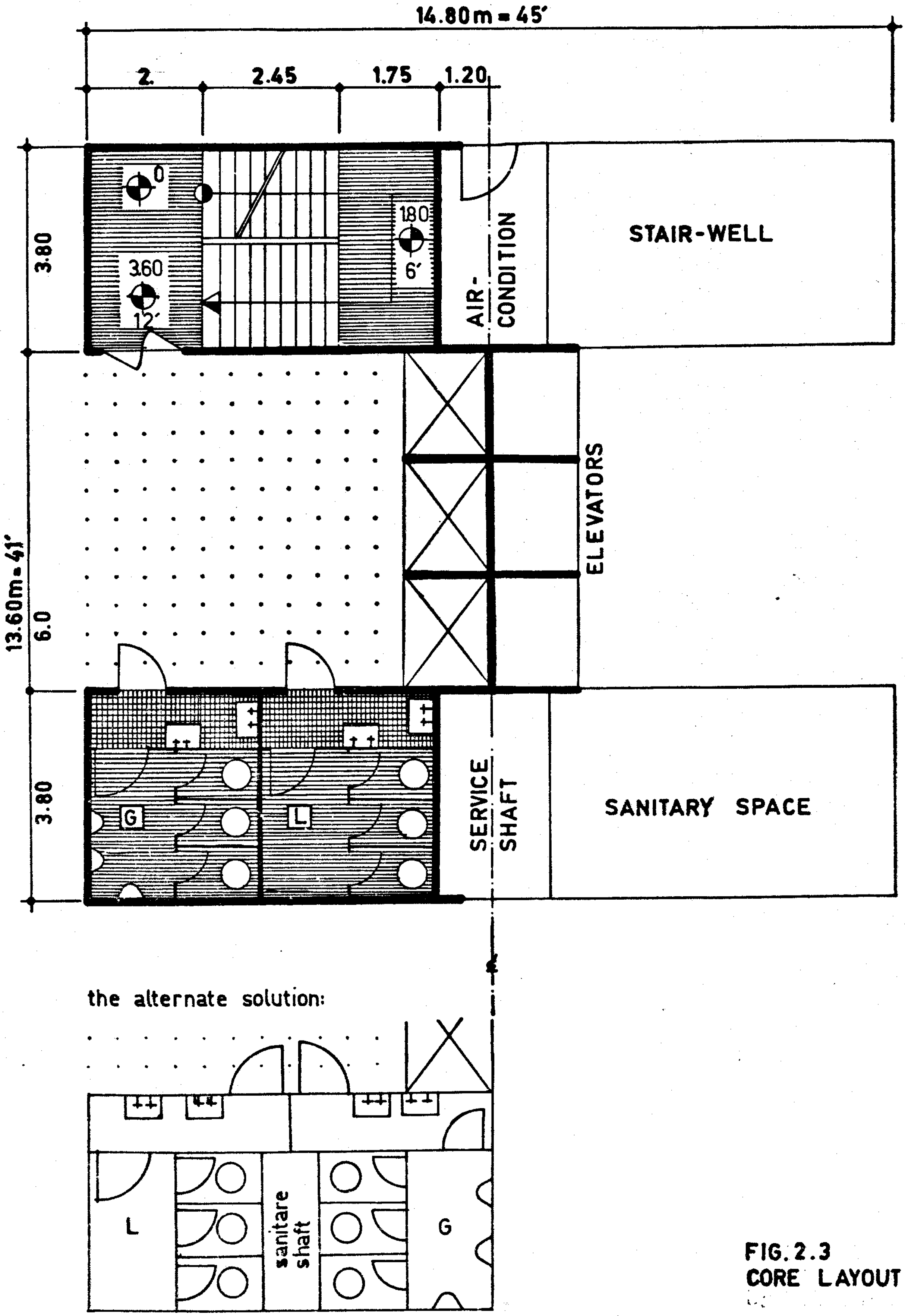


FIG. 2.3
CORE LAYOUT

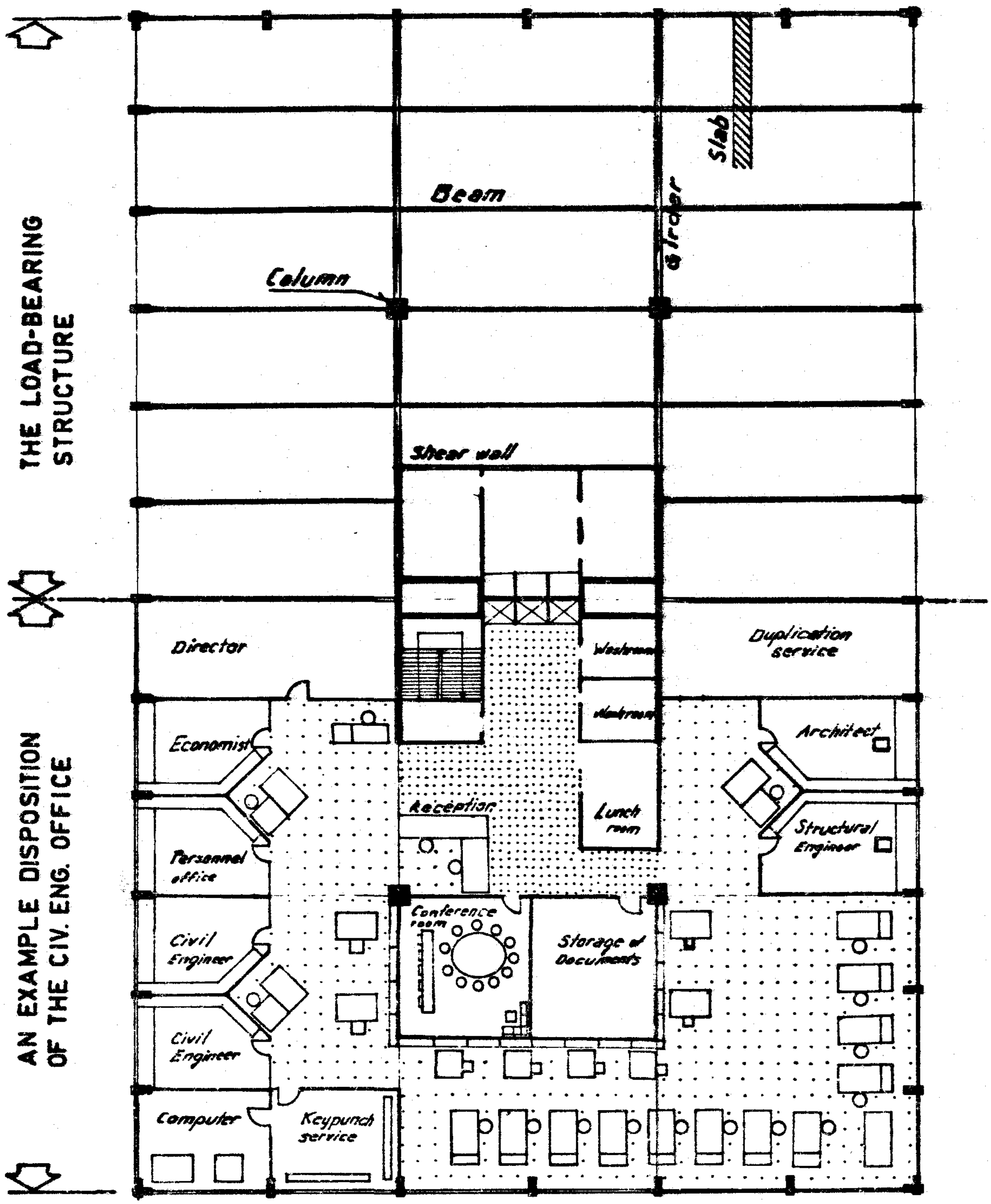


FIG.2.4 THE PLAN OF A OFFICE BUILDING WITH THE INNER-CORE DISPOSITION.

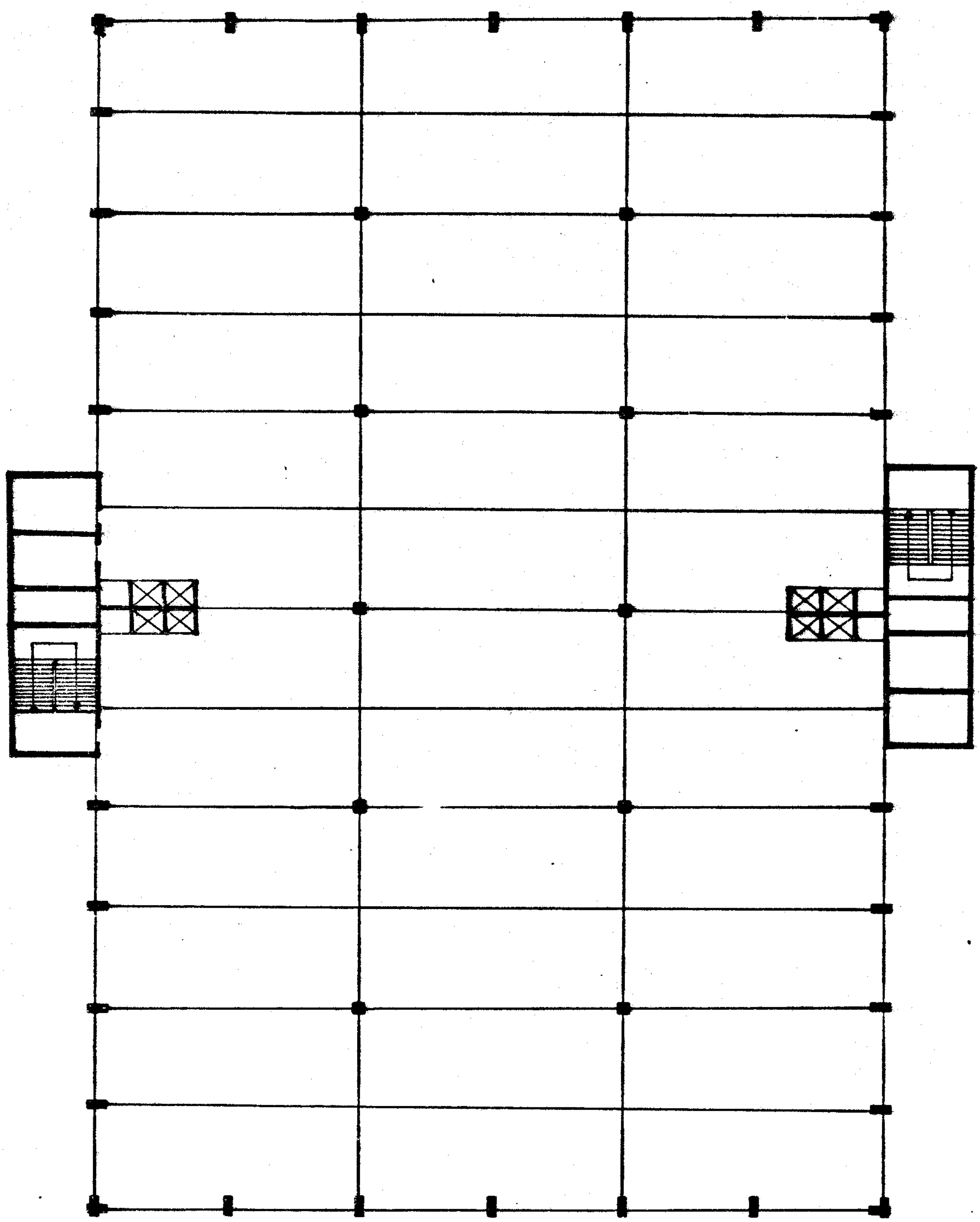
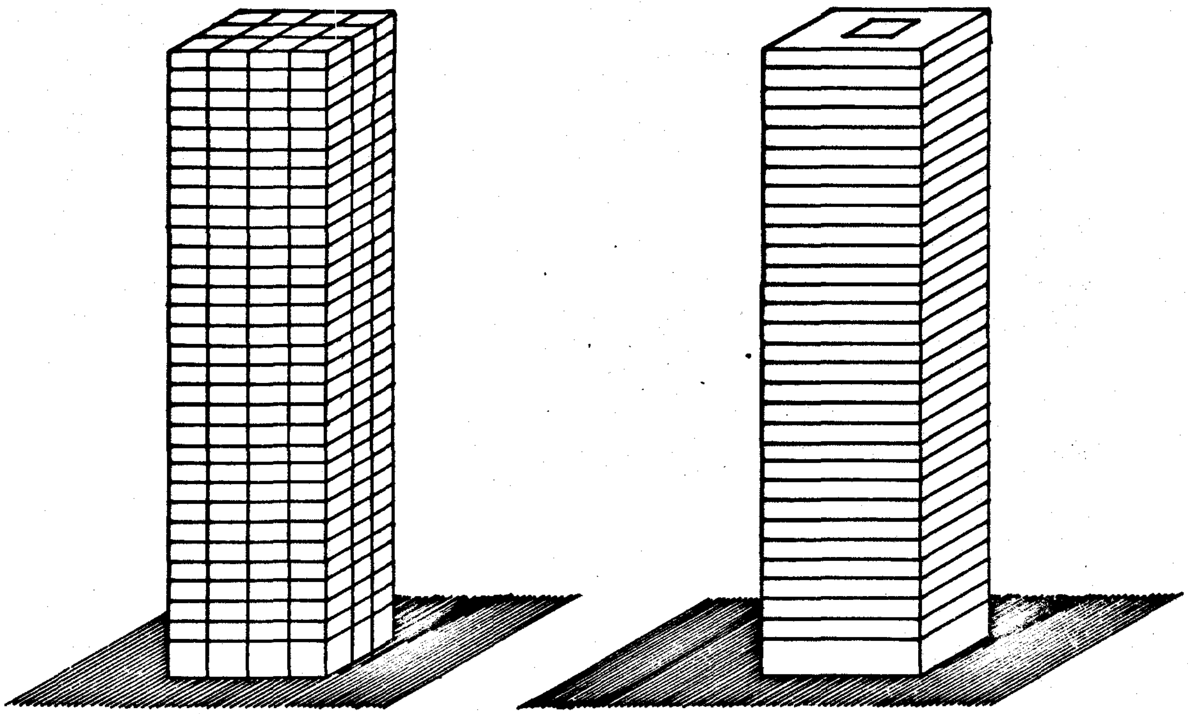


FIG. 2.5 STRUCTURAL PLAN OF A OFFICE BUILDING WITH THE EXTERIOR CORE.

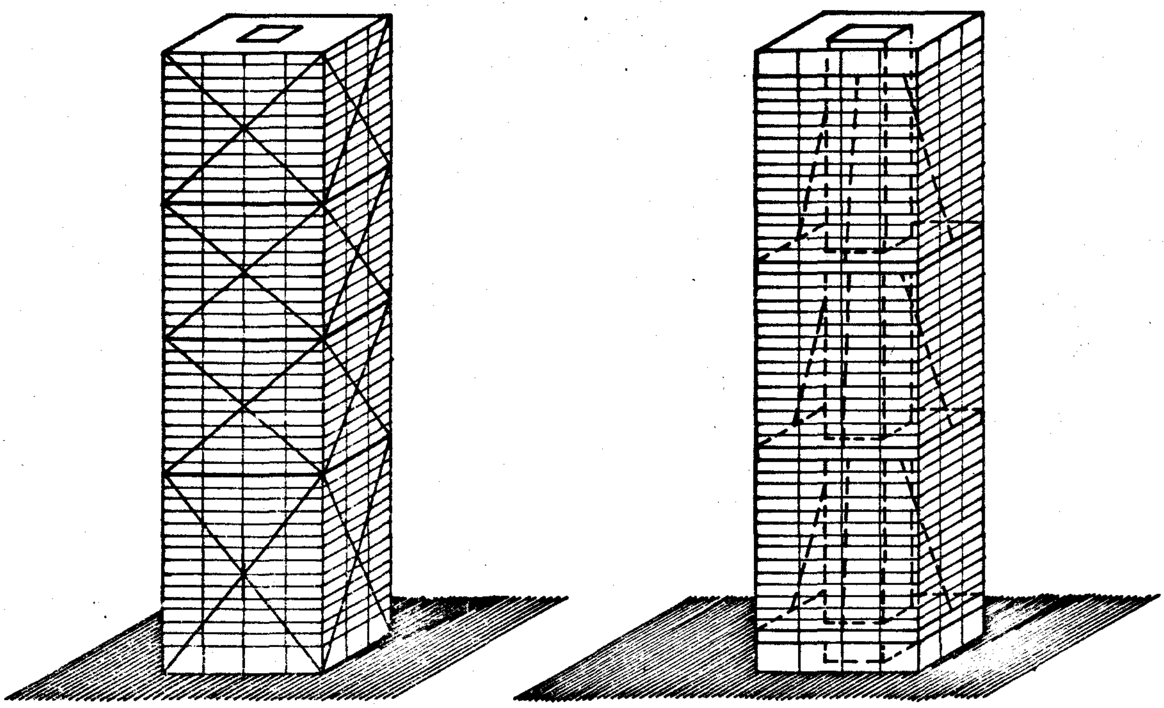
to cause cracking and unpleasant psychological reactions among the building occupants, without actually endangering the stability of the structure. To compensate for this, engineers design the columns and girders as rigid frames (wind bents), Figure (2.6a). A recent approach has designers stiffening the exterior wall framing so that the structure as a whole acts like a cantilever tube[29]. Walls are designed either as Vierendeel beams or as trusses (Figures 2.6, 2.7a). In this case, the core takes only gravity loading. The consulting engineering firm of Severud, Sturm, Coulin and Bandel [30] has developed a further appraoch which incorporates the moment capacity of the core and the axial resistance of the exterior columns by joining the columns to the core with diagonals at the mechanical floors (Figure 2.7b). The 51 storey steel I.D.S. Centre Tower in Minneapolis is a result of such a design concept.

The design of tall buildings in reinforced concrete by this approach can be conceived as follows. By providing concrete shear walls, the core is designed as a moment resisting element. Furthermore, the exterior columns are connected to the core at the mechanical floors (where diagonals will not interfere with the space usage), by means of diagonal trussing. The connections at the columns are effectively hinges. The reasons for this is to have the core engage the exterior columns but not induce any bending moment in them when the structure is laterally loaded by wind. The core by itself would deform as a cantilever; but due



a. Rigid frame — b. Vierendeel tube

FIG 2.6 STRUCTURES USED IN OFFICE BUILDINGS TO RESIST THE LATERAL LOAD



a. Trussed tube — b. Interior truss

FIG. 2.7 RECENT METHODS OF STIFFENING OF A TALL BUILDING AGAINST LATERAL MOVEMENT to the restraint at the mechanical floors, the moment capacity is considerably reduced at these levels, as well as along the whole height of the structure. The behaviour of this "primary structure" is best illustrated in Figure 2.13. The surrounding "secondary structure" is designed for gravity loads only. This design concept, as adjusted to suit the basic optimization model, is discussed in Section 2.4.

2.3 Definition of the project evaluation system

A system must be determined for the evaluation of any design project. The break-down of the evaluation system, in general, is illustrated in Figure 2.8. In descending order of priority with respect to decision-making, the system is subdivided into subsystems, assemblies, components and parts.

The load bearing structure forms a subsystem of the building design evaluation system, as shown in Figure 2.9. The determination of the assemblies under this subsystem is dictated by the structural design philosophy as described in Section 2.2. In this study, there are three assemblies. The primary structure is composed of the core, exterior columns, and the mechanical floors with diagonal bracing. It carries both gravity and lateral loads. The floor framing and the interior columns (which may be placed in different positions between mechanical floors as demanded by the space usage of the building) make up the secondary structure which carries the gravity load only. To

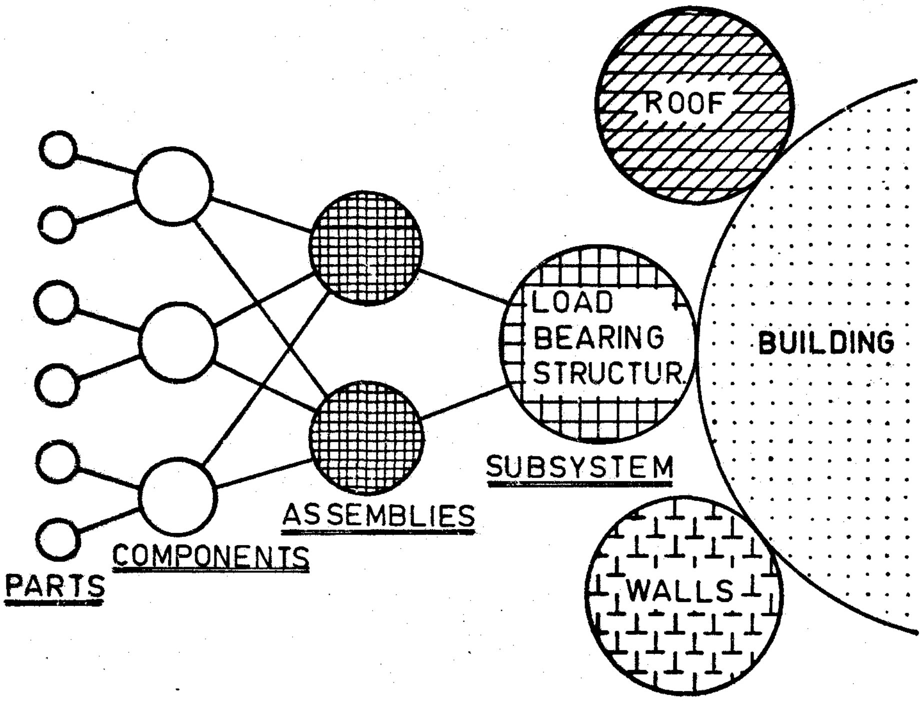


FIG.2.8 SYSTEM DEFINITION IN GENERAL

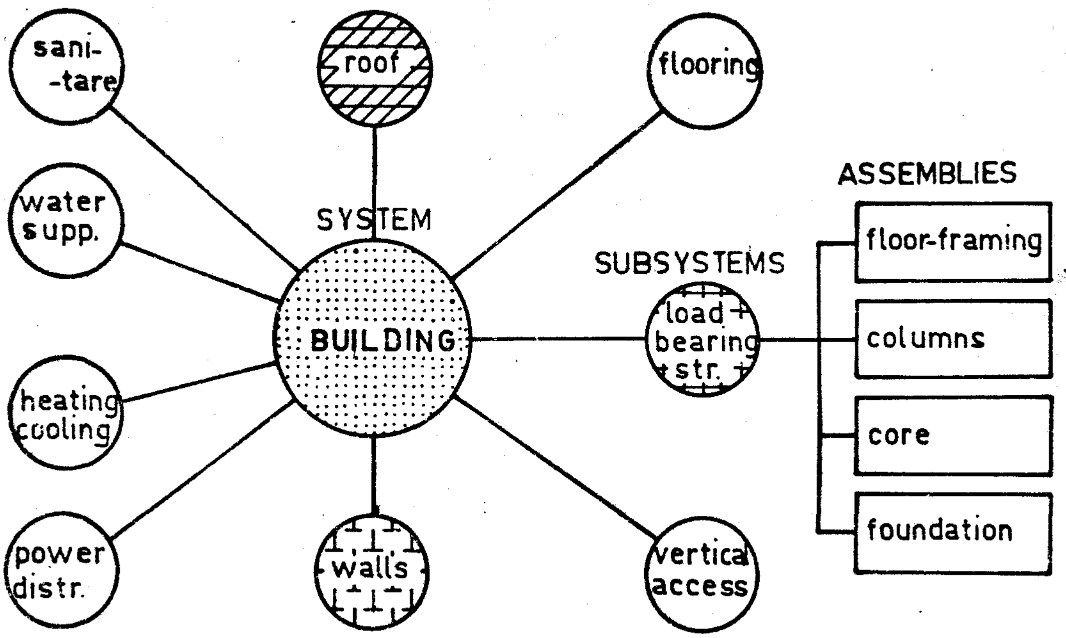


FIG.2.9 SYSTEM DEFINITION FOR BUILDING DESIGN AND EVALUATION

transmit the loads to the ground, there is the third assembly, the foundation. The above assemblies and the further breakdown of the load-bearing structure into components and parts are detailed in Figure 2.10. This figure also indicates the complexity of decision making in terms of the designer's choice of components and parts. For example for the study undertaken in this report, the arrows show the sequence of decision making.

The definition of the components and parts as outlined in Figure 2.10 can be found in publications on reinforced concrete. Recommended are References (11), (13), (14) and (17, part two). As the present study deals with the optimization of floor framing structures, further attention is directed to the description of such structures as follows.

A floor slab panel can be designed so that the load is carried in one or two directions. The type of support conditions and/or the dimensions generally control the way in which the load is carried. As shown in Figure 2.11. The one-way floor slab panels is supported on beams which, in turn, transmit the load to the girders. When the beams and girders are built integrally with the slab, they assume the shape of a tee-section. The floor slab panel is referred to as a joist floor when the spacing between beams is sufficiently small (20" to 30"). Removable steel pans may be used for this type of construction or the space between joists may be filled with hollow tiles. A typical plan of a concrete joist floor panel is shown in Figure 2.12.

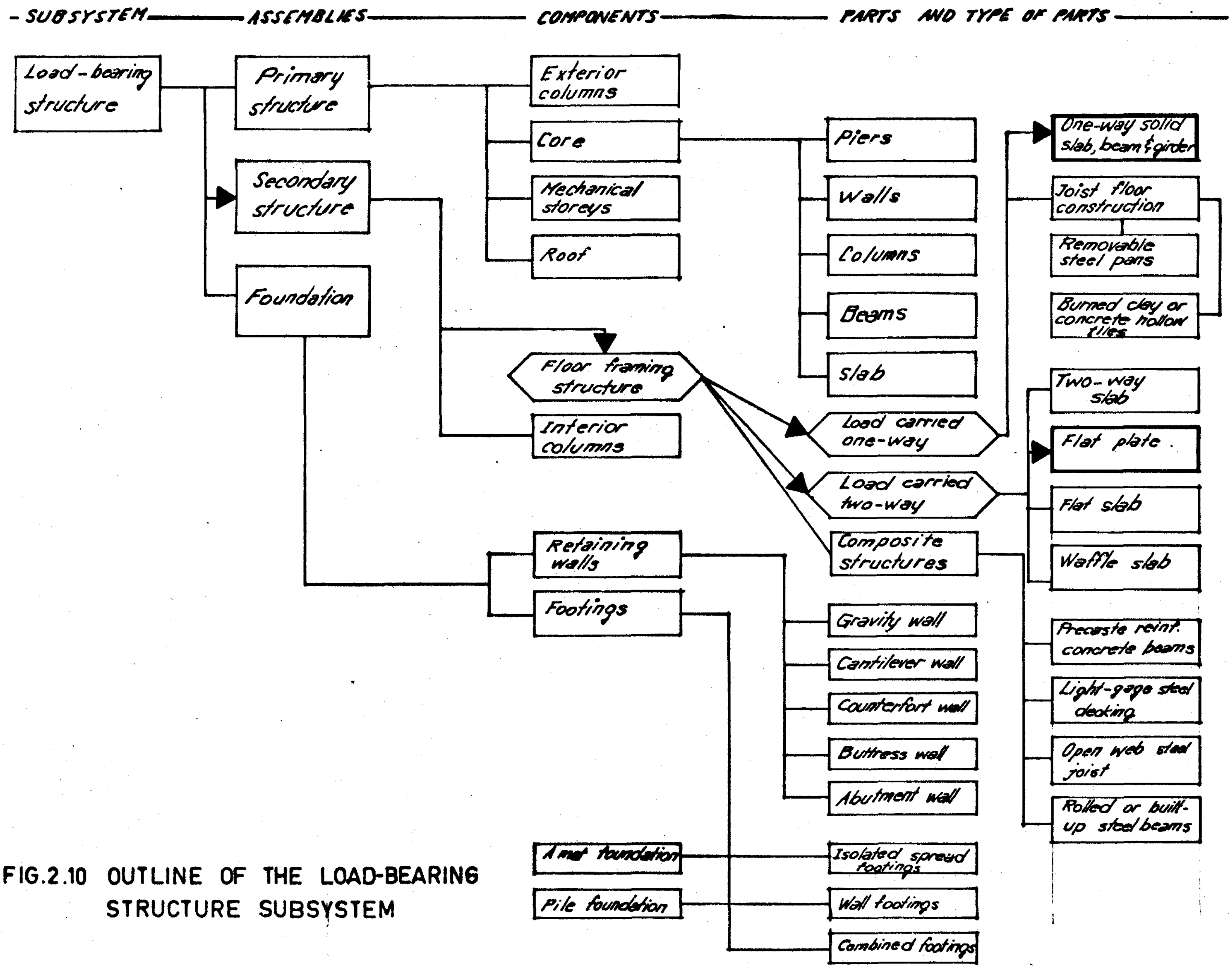


FIG.2.10 OUTLINE OF THE LOAD-BEARING STRUCTURE SUBSYSTEM

When the ratio of the long span to the short span is less than about two the bending in the long direction contributes a significant portion of the two way action whereas for larger ratios the load carrying action is basically one-way. This applies to cases when the floor slab panel is supported along its edges on the beams. Slabs which are supported at their corners by columns (the flat plate) are restricted by structural behaviour to a nearly square plan. In this latter case, the punching effect induced by the columns can be reduced by means of drop panels and/or column capitals. For heavier loads, the slab may consist of two-way ribs, known as the waffle slab.

Composite construction has a cast-in-place concrete slab placed upon and interconnected to a prefabricated beam so that the combined beam and slab acts together as a unit. The types of prefabricated beams are outlined in Figure 2.10. The interconnection is obtained by means of mechanical shear connectors such as 1. channel connectors, 2. stud connectors, (short pieces of straight or L-shaped round bars,) 3. spiral connectors, 4. Reinforcement ties and 5. shear keys plus vertical ties in prestressed concrete.

It is worth noting that in recent years a marked increase in the use of precast slabs has occured. These are generally either conventionally reinforced or prestressed placed on a monolithic beam and girder frame.

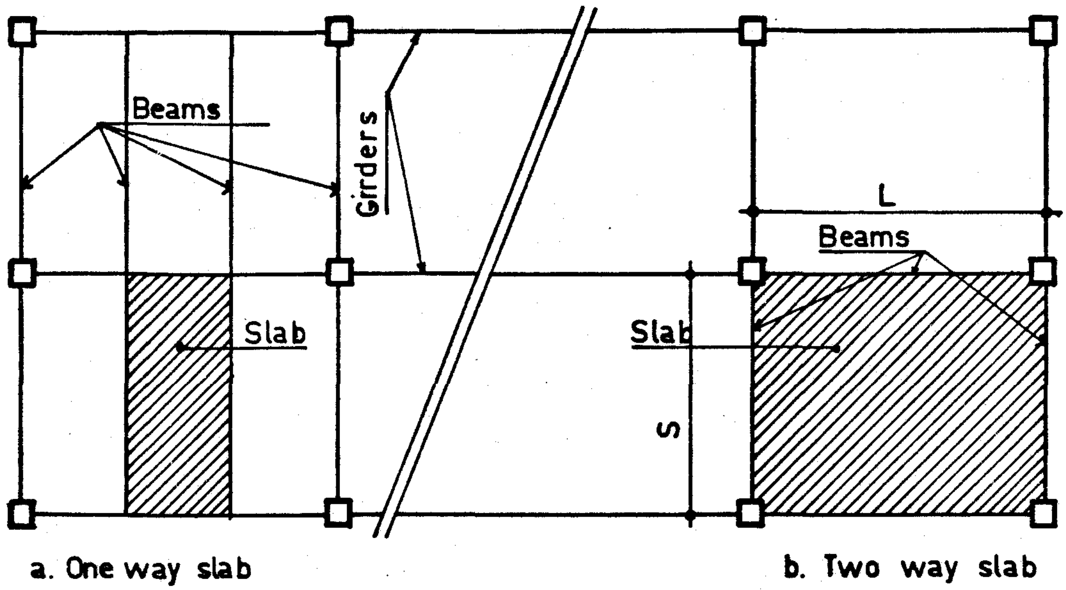


FIG.2.11 ONE WAY VERSUS TWO WAY REINFORCED SLAB

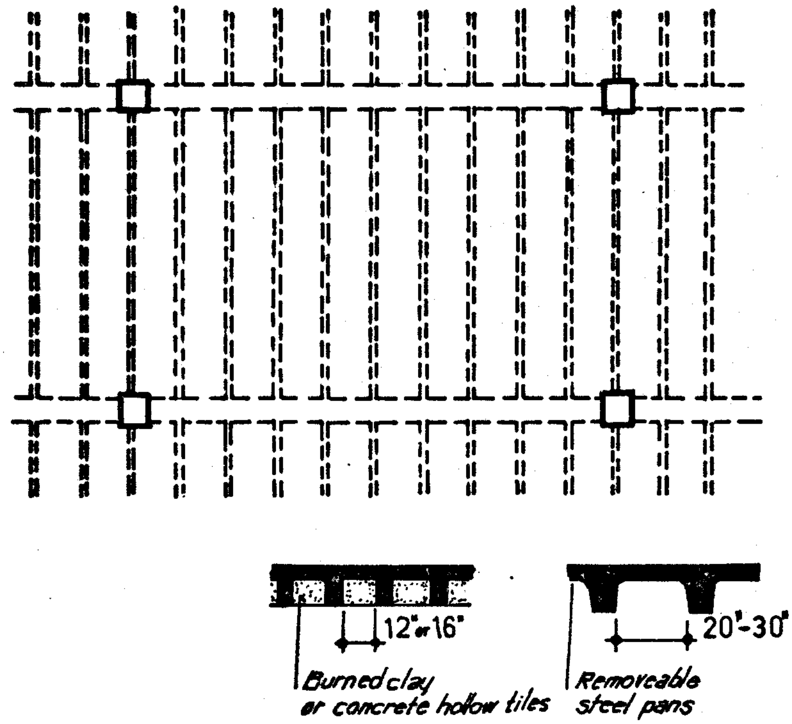


FIG.2.12 CONCRETE JOIST FLOOR CONSTRUCTION

2.4 Coordination model

The design of a tall building requires an organized approach. Due to the large number of requirements to be satisfied and decisions to be made, it is very unlikely that the optimum design of a tall building as a whole can be achieved in one giant step without first breaking it down into smaller methodical procedures. To this end, the coordination model which is shown diagrammatically in Figure 2.13 was developed.

The systematic approach to be used in association with the coordination model is outlined in Figure 2.14. By the cost comparisons, the designer, aided by the use of a small computer, is able to find the most economical design for the floor-framing component. In this study, computer programs for the optimum design of two floor-framing types were developed and are presented. The optimizing procedure is repeated for different levels (from mechanical floor to mechanical floor as shown in Figure 2.2) of the structure where there are significant differences in the live load. However, the decision to use different types of floor framing at various levels must also take into account the differences in construction technology and in addition the cost factors such as loss of efficiency interupting the repetitive nature of the work.

Next is the optimum design of the interior columns, based on the reactions from the floor-framing components. Change of structural floor framing and/or the change of columns disposition in the plane would induce the columns redesign.

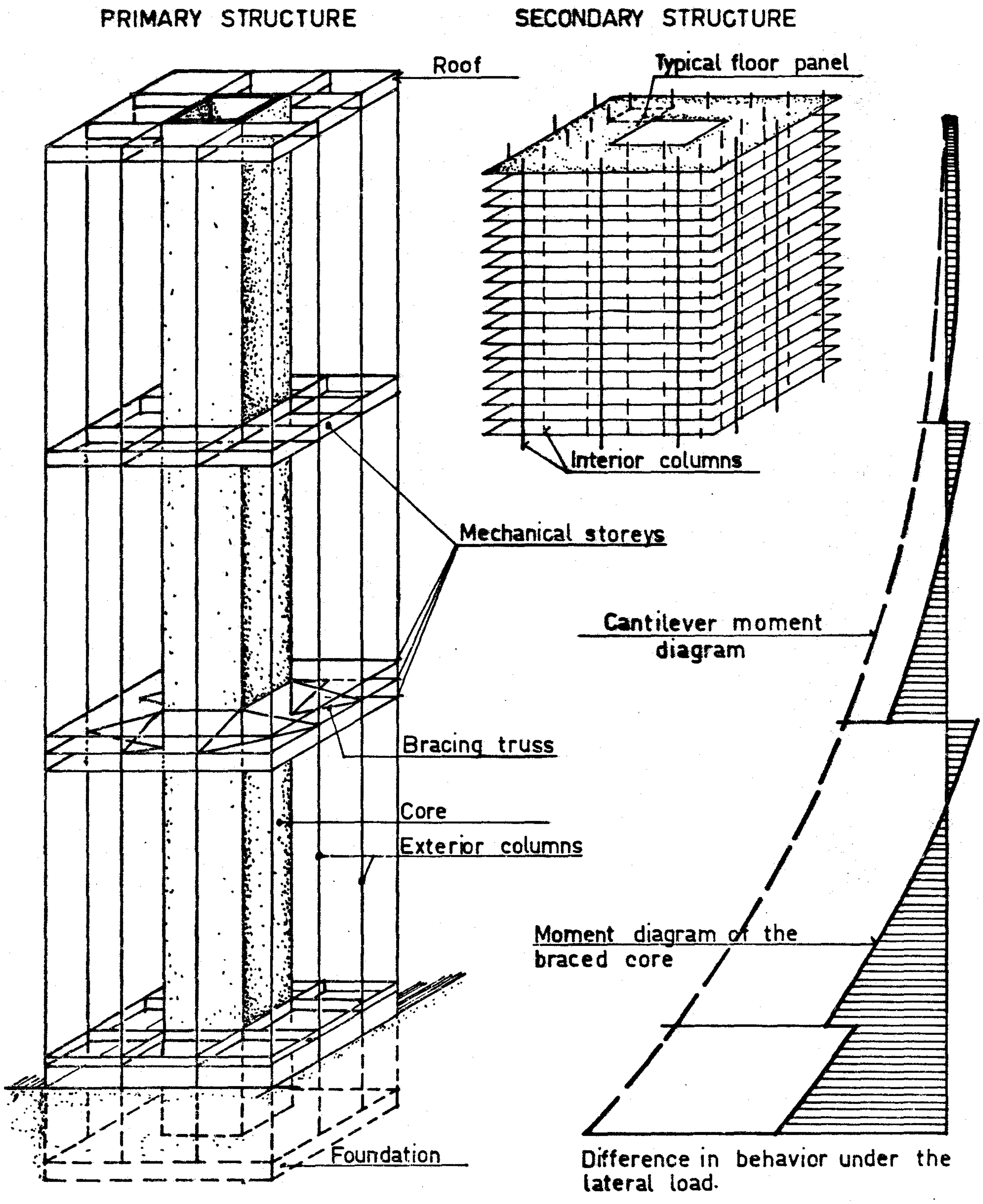


FIG.2.13 CO-ORDINATION MODEL OF INTERIOR TRUSS TYPE OF BUILDING

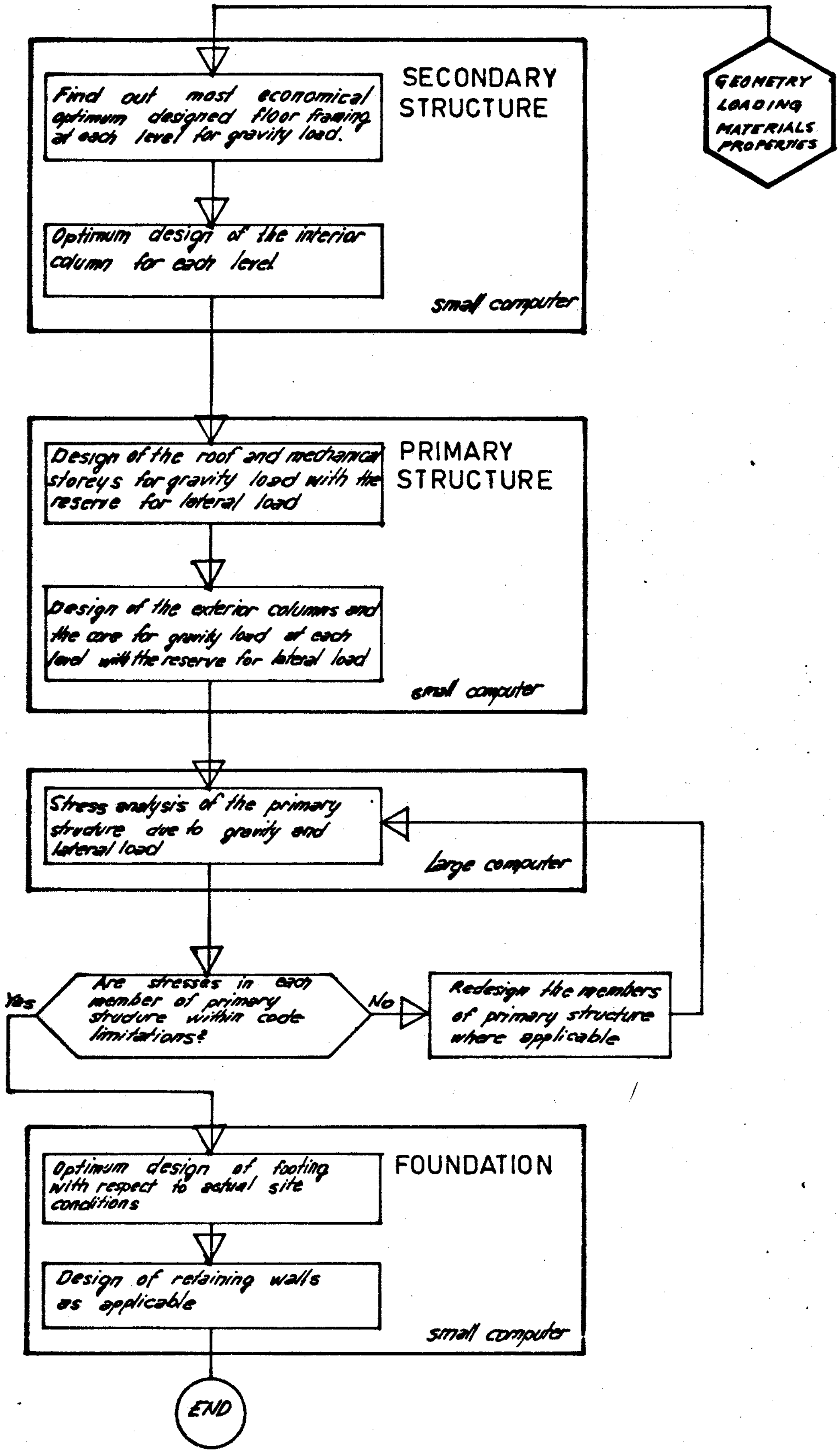


FIG. 2.14 CHART OF CO-ORDINATION MODEL FOR INTERIOR TRUSS TYPE OF BUILDING STIFFENING

This includes the design of the secondary structure. Note that the whole process is to be repeated for different plan dispositions.

The primary structure is designed as follows. The roof is designed for gravity load with reserve for lateral load. The same applies to mechanical floors which may also accommodate the load reactions from the interior columns, if applicable. The elements of the core are designed for gravity loads only throughout the height of the structure. The primary structure, as a whole, must be checked for its lateral load resisting capacity. The STRESS program which requires the facilities of large computers such as the CDC 6400 or IBM 360, is available for the analysis of the primary structure. If the stresses in the members of the primary structure are not satisfactory, it is necessary to redesign the members and to repeat the analysis. If the stresses and overall deflection are within specified limitations, then the design of the foundation takes place.

At the end of this procedure, the designer has accumulated all the necessary data for the drawings and the cost of the load bearing structure.

2.5 Unit Prices

For the optimalization process unit prices of material (constructed) must be established. The optimization in this study uses the following prices which where established through commercial contacts in 1970.

Concrete:

1. Price of ready-mixed concrete (vary according to desired strengths).

2. Finishing and curing the reasonable value = $4

3. Placing the reasonable value = $4

Subtotal price = sum of 1,2,3

25% of subtotal price to include compensation, supervison, overhead, and profit = 0.25 subtotal price

Total unit price, UPC = 1.25 subtotal price

Table 1. The values of UPC used in this thesis work

Strength of concrete	Price of ready-mixed concrete *	Total unit Price, UPC
3,000 psi	$ 17.50	$ 31.80 1cu.yd.
3,500 psi	$ 18.10	$ 32.60
3,750 psi	$ 18.50	$ 33.10
4,000 psi	$ 18.90	$ 33.60
4,500 psi	$ 19.60	$ 34.50
5,000 psi	$ 20.40	$ 35.50
6,000 psi	$ 21.40	$ 36.80

* The values obtained from Premier ready-mixed concrete company in Hamilton and should be used as a guide only.

These prices (UPC) depend on :
a) distance to site
b) required volume of concrete
c) time

Steel:

For the Hard Grade steel (FY = 50,000 psi) the price is determined as follows:

1.	Base price		= \$0.0575/lb
2.	Grade price		= \$0.000/lb
3.	Size extra		= \$0.0100/lb
a.	Price		= \$0.0675/lb
b.	Cageing and Placing	reasonable value	= \$0.05/lb
c.	Bending	reasonable value	= \$0.01/lb
Subtotal Price			= Sum of a, b, c.

+ 25% for compensation, supervision, overhead and profit

Total unit price, UPS = 1.25 Subtotal price

Table 2. The values of UPS used in this thesis work.

Yield Strength	* Base Price	* Grade Extra	* Size Extra **	Price	Total Unit Price, UPS
40,000	\$0.0585/lb	\$0./lb	0.01	\$0.0685	\$0.1605/lb
50,000	0.0575	0.	0.01	\$0.0675	\$0.1592/lb
60,000	0.0575	0.005	0.01	\$0.0725	\$0.1665/lb
75,000	0.0575	0.0125	0.01	\$0.0800	\$0.175/lb

Notes:

* Those values are obtained from the Steel Company of Canada and should be used as a guide only.

** The value is taken for bar #4. The size extras vary.

Formwork:

The unit costs of formwork were obtained from Handbook[32]. Accordingly, the cost of formwork for slabs was taken as $0.68 per sq.ft. of contact surface per 4 uses. It was $0.90 per square ft. for beams and girders. This includes the cost of materials, accessories, fabrication, erect and strip, cleaning and moving. The above mentioned items were increased 25 percent to include compensation, supervision, overhead and profit. Therefore the total prices used are:

Unit price of formwork for slab, UPFS = $0.75/sq.ft, and

Unit price of formwork for beam and girder, UPFB = $1.125/sq.ft.

The unit prices are input data in the developed computer programs. For future application current local values should be used once they have been established.

2.6 Summary

The design procedure as suited to the optimal design of high-rise office buildings has been outlined in this Chapter. The design assumption is made that the secondary structure does not resist lateral load; however, the primary and secondary structures act simultaneously to support the gravity load. Furthermore, the floor-framing is taken to consist of typical floor slab panels as shown in Figure 2.10.

CHAPTER III

OPTIMIZATION METHODS FOR COMPUTER-AIDED DESIGN

3.1 Introduction

This chapter surveys recent automatic optimization methods which either have found or should find useful application in the optimal design of load-bearing structural members. Detailed consideration is given to direct search methods as a solution of problems by sequential unconstrained minimisation. The consideration is given to the inequality constraints by means of transformation or penalities to unconstrained artificial objective function. Several one-dimensional and multi-dimensional strategies are discussed below.

Section 3.2 introduces fundamental concepts and definitions. Section 3.3 describes one-dimensional unconstrained optimization strategies, followed by a description of the multidimensional direct search method of Hooke and Jeeves [18]. Section 3.4 deals with the differential calculus method, followed by geometric programming in section 3.5. Finally section 3.6 deals with dynamic programming.

Inevitably, the material presented in this chapter tends to reflect the author's current problem - the optimum design of one way solid slab, beam, girder floor framing.

3.2 Fundamental Concepts and Definitions

The problem is to minimize U where

$$U = U(X) \tag{3.1}$$

and where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \tag{3.2}$$

U is called the objective function and the vector X represents a set of independent variables (such as the dimensions of a cross section). Minimizing a function is the same as maximizing the negative of the function, so there is no loss of generality.

In general, there will be constraints that must be satisfied either during optimization or by the optimum solution. Each variable might be constrained explicitly by an upper and lower bound as follows:

$$x_{min\ i} \leq x_i \leq x_{max\ i} \qquad i = 1, 2, \ldots, n \tag{3.3}$$

where $x_{min\ i}$ and $x_{max\ i}$ are lower and upper bounds, respectively. Furthermore, the problem could be constrained by a set of k implicit functions

$$\phi_k(X) > 0 \qquad k = 1, 2, \ldots, p \tag{3.4}$$

Any vector X which satisfies the constraints is termed feasible. Fig. (3.1) shows a two-dimensional contour sketch which illustrates some features encountered in optimization problems.

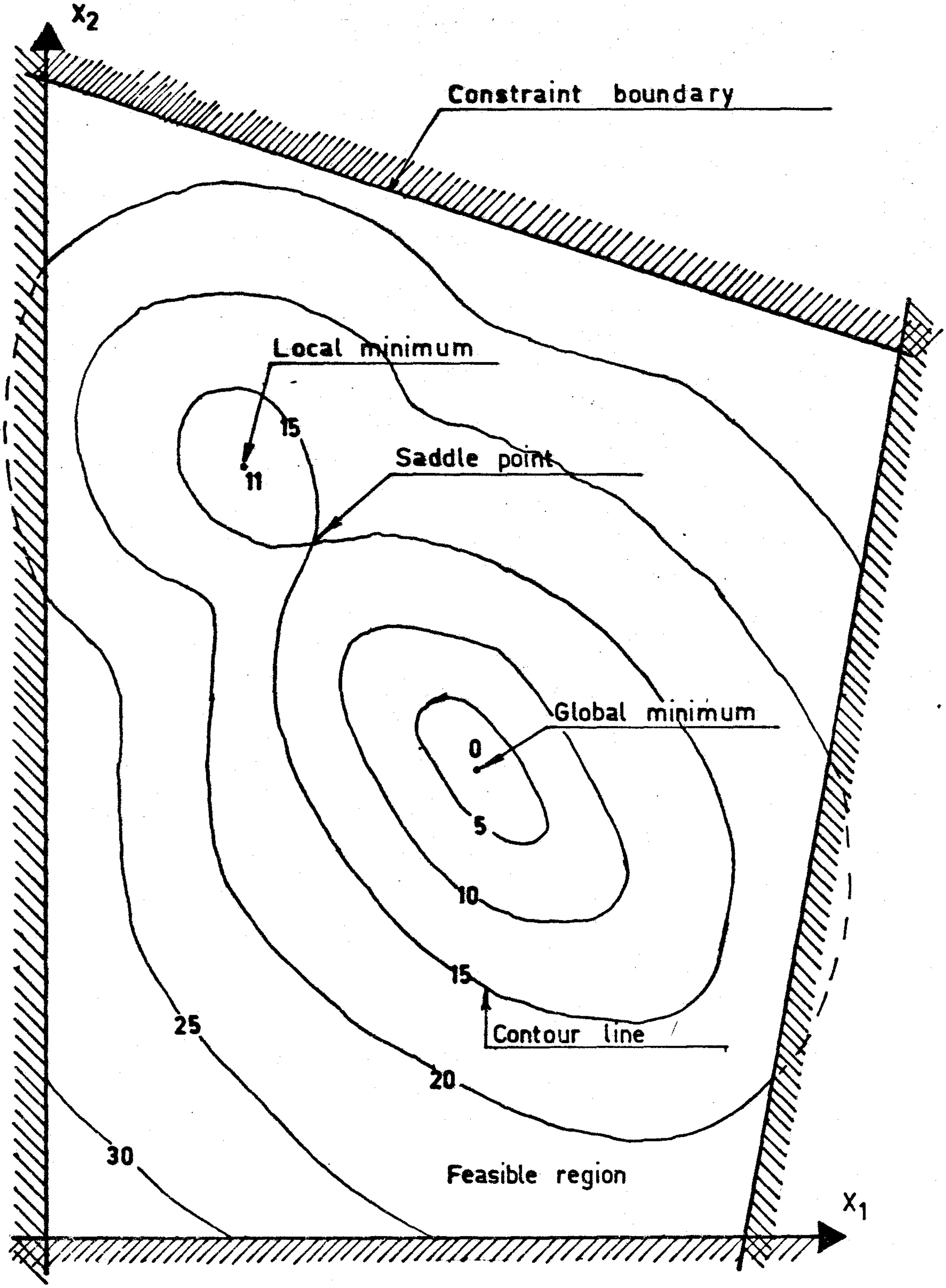


FIG.3.1 COUNTOR SKETCH ILLUSTRATING SOME FEATURES ENCOUNTERED IN OPTIMATIZATION PROBLEMS

A hypercontour, described by the relation

$$U(X) = U_{const.} \tag{3.5}$$

is the multidimensional generalization of a contour. The feasible region in Fig. (3.1) is determined by fixed upper and lower bounds on X. The feasible region is seen to contain one global minimum, and one saddle point. A minimum may be located by a point $\bar{X}$ on the response (artificial) hypersurface generated by U(X) such that

$$0 = U(\bar{X}) < U(X) \tag{3.6}$$

for any X in the immediate feasible neighbourhood of X. (The discussion must restrict itself to consideration of local minima because methods which guarantee convergence to a global minimum are not available). A more formal definition of a minimum follows.

The first three terms of the multidimensional Taylor series are given by

$$U(X + \Delta X) = U(X) + \nabla U^T \Delta X + 1/2 \Delta X^T H \Delta X + \ldots \tag{3.7}$$

where

$$\Delta X = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_k \end{bmatrix} \tag{3.8}$$

represents the variable increments,

$$\nabla U = \begin{bmatrix} \frac{\partial U}{\partial x_1} \\ \frac{\partial U}{\partial x_2} \\ \vdots \\ \frac{\partial U}{\partial x_k} \end{bmatrix} \tag{3.9}$$

is the gradient vector containing the first partial derivatives

and

$$H = \begin{bmatrix} \frac{\partial^2 U}{\partial x_1^2} & \frac{\partial^2 U}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 U}{\partial x_1 \partial x_k} \\ \frac{\partial^2 U}{\partial x_2 \partial x_1} & \cdots & & \cdots \\ \vdots & & & \\ \frac{\partial^2 U}{\partial x_k \partial x_1} & \cdots & & \frac{\partial^2 U}{\partial x_k^2} \end{bmatrix} \tag{3.10}$$

is the matrix of second partial derivatives, the Hessian matrix. Assuming the first and second derivatives exist, a point is a minimum if the gradient vector is zero and the Hessian matrix is positive definite at that point.

3.3 Direct Search Methods

Direct search methods are based on a sequential examination of trial solutions which by simple comparison give an indication for further searching procedure.

If a choice of controlled variables is made and the constraints are satisfied, the objective function is calculated. Another choice is then made and if this gives an improvement, this second choice is to be preferred to the first.

In general, the direct search methods do not give a rapid rate of ultimate convergence and hence are inefficient for finding a minimum with high precision. However, the problems related to reinforced concrete design do not require precise location of the minimum because the resulting independent variables, e.g. dimensions, often require rounding-off to the nearest higher integer value in order to be applicable in practical situations.

3.3.1 Unconstrained search along a line

The simplest optimization problem is that of finding a local minimum of a function when the values of the independent variable lie on a fixed line. There are three types of approaches to this problem:

a. Golden section algorithm

b. Fibonacci search

c. Repeated interpolation method

a) Golden Section Algorithm

Let U(x) have only one stationary value (minimum) in the interval of (a,b).

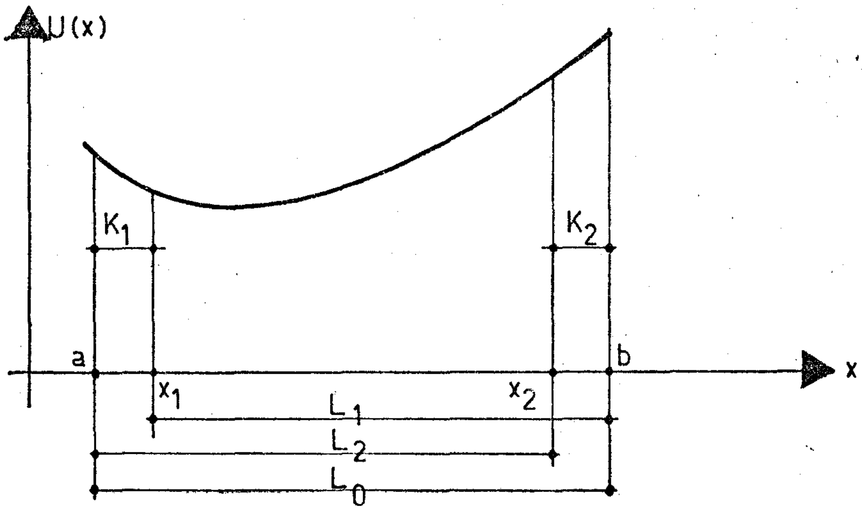


FIG.3.2 THE COURSE OF AN ASSUMED FUNCTION

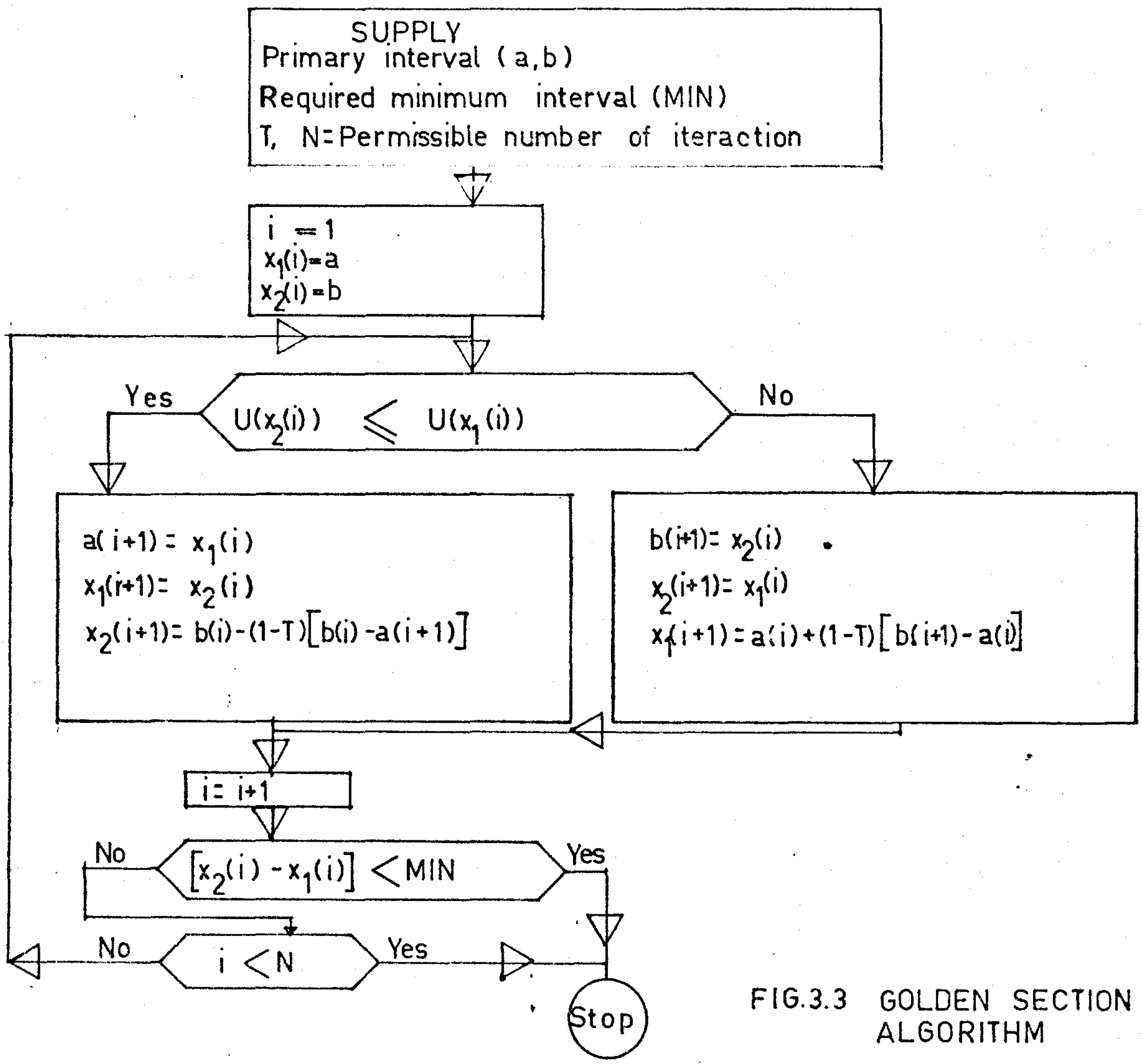


FIG.3.3 GOLDEN SECTION ALGORITHM

Say, the function U(x) was evaluated at the interior point x_1. Then the minimum of U(x) can lie to either side of x_1. It is necessary to evaluate function U(x) at an other interior point x_2. When $\underline{a} < x_1 < x_2 < b$; then if $U(x_1) < U(x_2)$, the minimum lies in interval of (a,x_2), otherwise in (x_1,b).

It is clear that one of the points used in reducing the size of the current interval must always lie in the interior of the reduced interval. Thus only one function evaluation is required for each subsequent step. By repeated evaluation of U(x) the minimum can be located to any required precision.

To establish the size of the increment T let the current interval be(a_i,b_i), and let the points at which the function is evaluated be $x_{1(i)}$ and $x_{2(i)}$, where $x_{1(i)} < x_{2(i)}$ in such a manner that:

$$\frac{L_2}{L_0} = \frac{L_1}{L_o} = T \tag{3.11}$$

and

$$T^2 - T - 1 = 0 \tag{3.15}$$

Therefore,

$$T = \frac{\sqrt{5} - 1}{2} = 0.618 \tag{3.16}$$

Then, the algorithm has the form shown in Figure 3.3.

b) Fibonacci Search

The Fibonacci search seeks an algorithm for finding the optimum such that it gives the largest ratio of the initial to the final interval for a fixed number of function evaluations. For more information refer to Kowalik and Osborne[2].

c) Repeated Interpolation Method

The Repeated Interpolation Method suits problems that require determination of the minimum with high precision and it is therefore not useful for the design of reinforced concrete structures. A full explanation of this method is presented in Reference 2.

3.3.2 Method of Hooke and Jeeves

The direct search method for more than one independent variable was developed by Hooke and Jeeves (18). This method changes one parameter at a time starting from the initial point. Once the full series of perturbations has been completed, it takes a step along the direction joining the last and the initial point. The portions of this method which are required to understand the application of this method to this research are reproduced below.

The developed algorithm solves two problems:

a. The direction in which to move from a given design point to a probably better design point;

b. How far to move in the direction chosen.

The first kind of move explores the local behaviour of the objective function. Introducing a starting point X, we prescribe step lengths dx_i in each direction, i=1,2,...,N.

The algorithm is performed as follows:

1. Set i=1 and compute U = U(X) where $X = (x_1, x_2, \ldots, x_N)$

2. Compute the new trial point

$$X = (x_1, x_2 \ldots, (x_i + dx_i), \ldots, x_N)$$

3. If U(X) < U, then set U = U(X) and accept this trial point as a starting point and repeat from step 2.

4. If U(X) ≧ U, then set $X = (x1, x_2, \ldots, (x_i - 2dx_i), \ldots . x_N)$ and check if U(X) < U. In case of success, the new trial point is retained. Set U = U(X) and repeat from step 2 with i=i+1. If again U(X) > U, then the move is rejected so that x_i remains unchanged and repeat from step 2 with i=i+1.

Figure 3.4 shows the flow-chart for this method. Several alternative techniques are based on this logic. At McMaster University the optimization group under Professor J.N. Siddall has transformed the ideas of optimization techniques into computer aided optimization subroutines. Of the nine available in 1971, it was found that subroutine SEEK 1 (references (19) and (24)) gives the best solution for the computation model described in Chapter 2.

SEEK 1 uses the direct search technique. The flow-diagram and program are given in Reference (24). This subroutine deals with the unconstrained artificial objective function and the constraints are incorporated in such a way that every invalid constraint is enlarged 10^{20} times and added to the artificial function. For more details refer to Ref.(19) and Ref.(24).

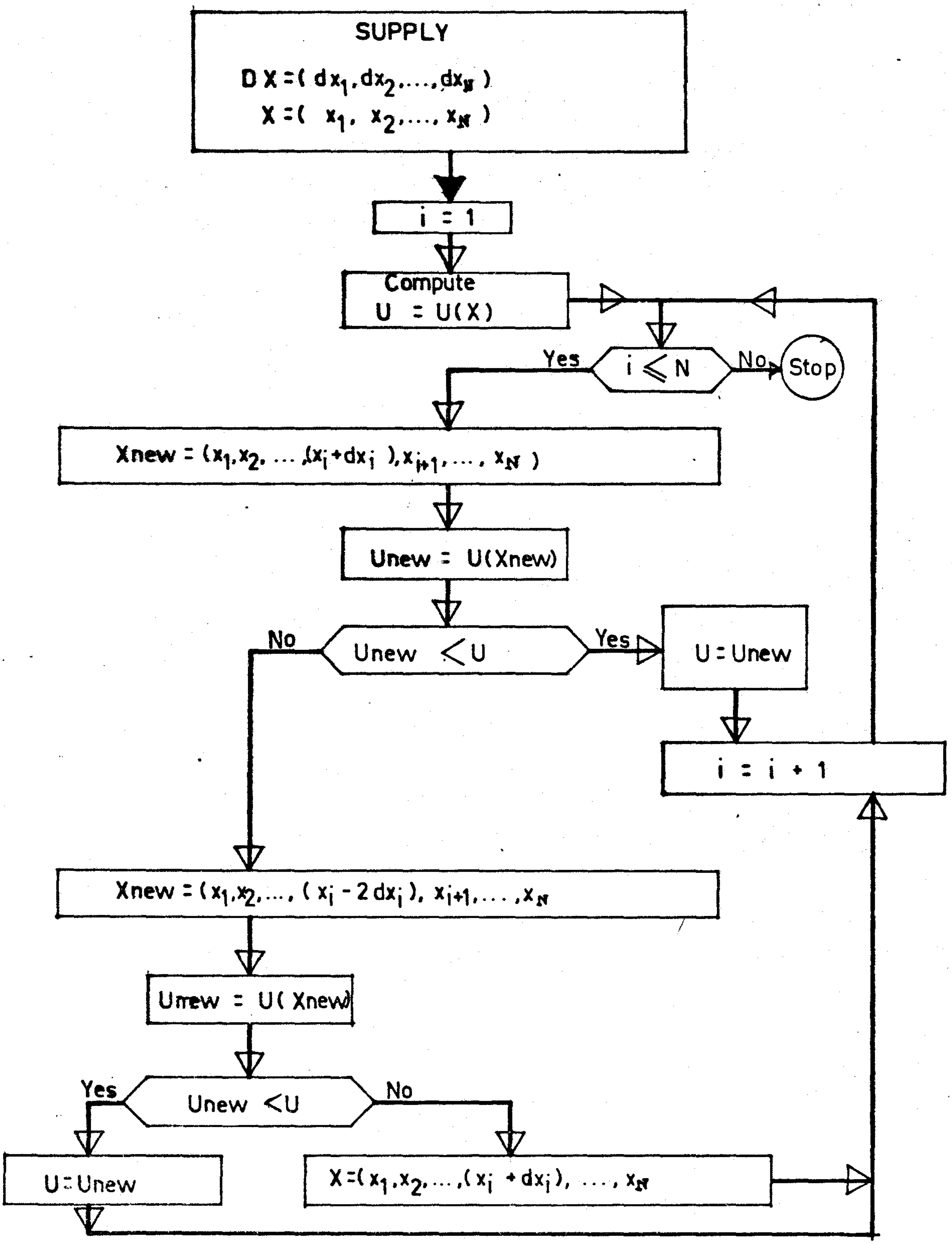


FIG. 34 FLOW-CHART FOR HOOK-JEEVES METHOD

3.4 Differential Calculus

This is a somewhat more formidable problem than simply differentiating and setting the derivate equal to zero. However, there are cases where the physical nature of the function assures us that it is continuous and differentiable and has only a single minimum in the interval of interest.

In the case of more than one variable, the task of finding a maximum is even more formidable. The first derivatives have to be made zero and the definiteness of the Hessian matrix (3.10) of second derivatives established at each solution; then the boundaries of each region of differentiability must be searched and compared with internal extrema.

Let us consider the objective function U(x). The problem is to find minimum of U(x) for which the values of the components $\underline{x}$ are restricted by side conditions or constraints. The classic methods of the calculus give the necessary condition:

$$\frac{dU}{dx_1} = \frac{dU}{dx_2} = \ldots\ldots = \frac{dU}{dx_N} = 0. \qquad (3.17)$$

which must be satisfied by a stationary value. The problem of finding a stationary value of a function U(x) is equivalent to that of solving a system of nonlinear equations:

$$U_i'(x) = \frac{dU}{dx_i} = 0. \qquad i=1,2,\ldots,N \qquad (3.18)$$

Then the solution of these equations also minimizes the function:

$$U(X) = \sum_{i=1}^{N} (U_i(X))^2 \qquad (3.19)$$

Let U(X) be minimized subject to the equality constraints:

$$g_i(x) = 0. \qquad i = 1,2,\ldots,M \qquad (3.20)$$

The classic Lagrange multiplier technique can be used to write down necessary conditions which a stationary point must satisfy.

Thus the classic constrained problem can be approached by:

a. solving the system of equations:

$$\left.\begin{aligned} &\frac{dU}{dx_i} + \sum_{j=1}^{M} L_j \frac{dg_j}{dx_i} = 0. \qquad && i=1,2,\ldots,M \\ & && j=1,2,\ldots,M \\ &g_i(x)=0. && \end{aligned}\right\} \quad (3.21)$$

or

b. optimizing the function:

$$H(x,L) = \sum_{i=1}^{N} \left(\frac{dF}{dx_i} + \sum_{j=1}^{M} L_j \frac{dg_i}{dx_i}\right) + \sum_{j=1}^{M} g_j^2 \qquad (3.22)$$

If function U(X) is to be minimized subject to the inequality constraints:

$$g_i(x) \leqslant 0. \qquad i=1,2,\ldots,M$$

and

$$x_i \geq 0. \qquad i=1,2,\ldots N \qquad (3.23)$$

then, it is called mathematical programming. To transform the above to the form of Equation (3.22) it is necessary to have a mechanism that indicates those constraints which are currently held as equalities. This is what makes the Lagrange multiplier approach unattractive in general. However, this appraoch has been used for optimization of reinforced concrete beam design[10].

3.5 Geometric programming

Geometric programming is a very recently developed technique the first reference to it being by its originator Zener (refer to reference 8) in 1961. The use of geometric programming in civil engineering has been minimal due partly to the fact that civil engineers were not greatly involved in the early development of this technique. The explanation of the mathematical basis of the method followed the description given by Templeman[8]. For easier reference to the original paper, the author uses the symbol from Reference (8).

The objective function, U, is a polynomial function as defined by Equations (3.1) and (3.2), and it's general form could be also written as follows:

$$
\begin{aligned}
U_m = & \pm C_{m1}\, x_1^{Em11}\, x_2^{Em12} \;\ldots\ldots\; x_N^{Em1N} \\
& \pm C_{m2}\, x_1^{Em21}\, x_2^{Em22} \;\ldots\ldots\; x_N^{Em2N} \\
& \quad \vdots \\
& + C_{mT}\, x_1^{EmT1}\, x_2^{EmT2} \;\ldots\ldots\; x_N^{EmTN}
\end{aligned}
\tag{3.25}
$$

The subscript, $m = 0,1,2...M$, distinguishes the several Equations of the form of Equation (3.25). The subscript N refers to the number of independent variables x_n of the vector X, where $n = 1,2,3,...N$. The number of terms in the Equation is subscripted with T ($t = 1,2,3,...,T$). The C_{mt} terms are constants. The signs of the powers E_{mtn} in Equation (3.25) are unrestricted.

Equation (3.25) may be expressed in the form:

$$U_m = \sum_{t=1}^{Tm} C_{mt} \prod_{n=1}^{N} x_n^{Emtn} \qquad (3.26)$$

The objective function (3.26) is to be minimize subject to M constraints. These constraints should have the same form in Equation (3.26), so that

$$Z_m = \sum_{t=1}^{Tm} C_{mt} \prod_{n=1}^{N} x_n^{Emtn} \leq 1 \qquad (3.27)$$

where:

$m = 1,2,\ldots\ldots M$

This is known as the primal problem. Geometric programming is a method of transforming the primal problem to a dual problem, the solution of the primal problem. The proof that a solution of the dual problem is exactly equivalent to a solution of the primal problem can be found elsewhere[7].

The dual of the primal objective function, $U_o(x_n)$, is given the symbol $D(\omega)$. This dual function D is defined by

$$D = \prod_{m=o}^{N} \prod_{t=1}^{Tm} \left(\frac{Cmt\ \omega mo}{\omega mt}\right)^{\omega mt} \qquad (3.28)$$

The dual problem relates to the primal problem such that at the minimum ${}^{min}U_o$.

$${}^{min}U_o({}^{min}x_n) \equiv {}^{max}D({}^{max}\omega) \qquad (3.29)$$

The problem of minimizing U_o subject to several inequlity constraints is now replaced by the dual problem of maximizing D subject to several constraining relationships between the dual variables ω. These are as follows:

a) There is one normality condition

$$\sum_{t=1}^{To} \omega ot = 1 \qquad (3.30a)$$

b) There are N orthogonality conditions, one for each x_n

$$\sum_{m=o}^{M} \sum_{t=1}^{Tm} E_{mtn}\, \omega_{mt} = 0 \qquad (3.30b)$$

c) There are M inequality constraints

$$\sum_{t=1}^{Tm} \omega_{mt} \equiv \omega_{mo} \geqq 0 \qquad (3.30c)$$

with m = 1,2,...,M.

d) There are T non-negativity conditions

$$\omega_{mt} > 0 \text{ for all } m,t$$

with formally understood $\omega_{oo} = 1$ (3.30d)

Now having maximized Equation (3.28), and having found the values of the dual variables ω from equations (3.30), the value of the optimum primal objective function ${}^{min}U_o$ is found from Equation (3.29) and the values of the corresponding optimum primal variables ${}^{min}x_n$ are given by

$$C_{ot} \prod_{n=1}^{N} x_n^{Eotn} = \omega_{ot}\ {}^{min}U_o \qquad (3.31a)$$

for t=1,2,...To

and

$$C_{mt} \prod_{n=1}^{N} x_n^{Emtn} = \frac{\omega_{mt}}{\omega_{mo}} \qquad (3.31b)$$

for t=1,2,...Tm, m = 1,2,...,M.

References (7) and (8) give further details of this method and also demonstrate it's use in several examples.

3.6 Dynamic Programming

Dynamic programming applies primarily to a situation in which many decisions have to be made to minimize the overall performance of a system, but the system is one in which distinct stages may be recognized where decisions at the later stages do not affect the performance of the earlier ones. It works best when the number of decisions at any stage is not too large and above all when the effect of the decisions can be represented by only a few variables.

The recommended references are (3), (4) and (5).

3.6.1 Discrete Deterministic Dynamic Programming

One kind of dynamic programming is the discrete deterministic dynamic programming. A general discrete deterministic decision process is shown in Fig. 3.5 and also the detail of the decision stage is shown in Fig. 3.6.

The following definitions of terms for dynamic programming are given:

Decision: The decision variables are those quantities that can be controlled or chosen at any stage of design (say, dimensions of the beam b,d). If there are m such variables $(x_{n1}, x_{n2}, \ldots\ldots, x_{nm})$, they may be regarded as forming a decision vector X_n.

The set of all decision vectors $X_1, X_2 \ldots\ldots X_n$ is called the operating policy or more briefly, just the policy.

State: The variables describing the geometry and an adequate set

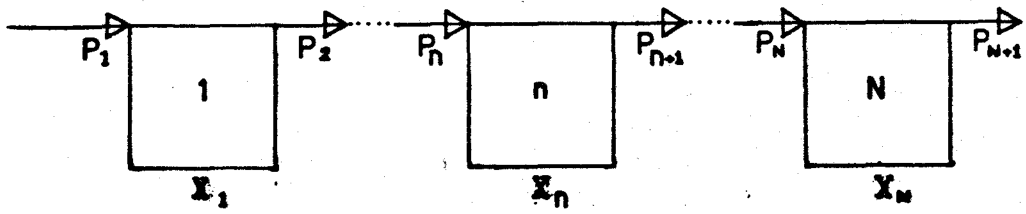


FIG. 3.5 THE DISCRETE DETERMINISTIC DECISION PROCESS

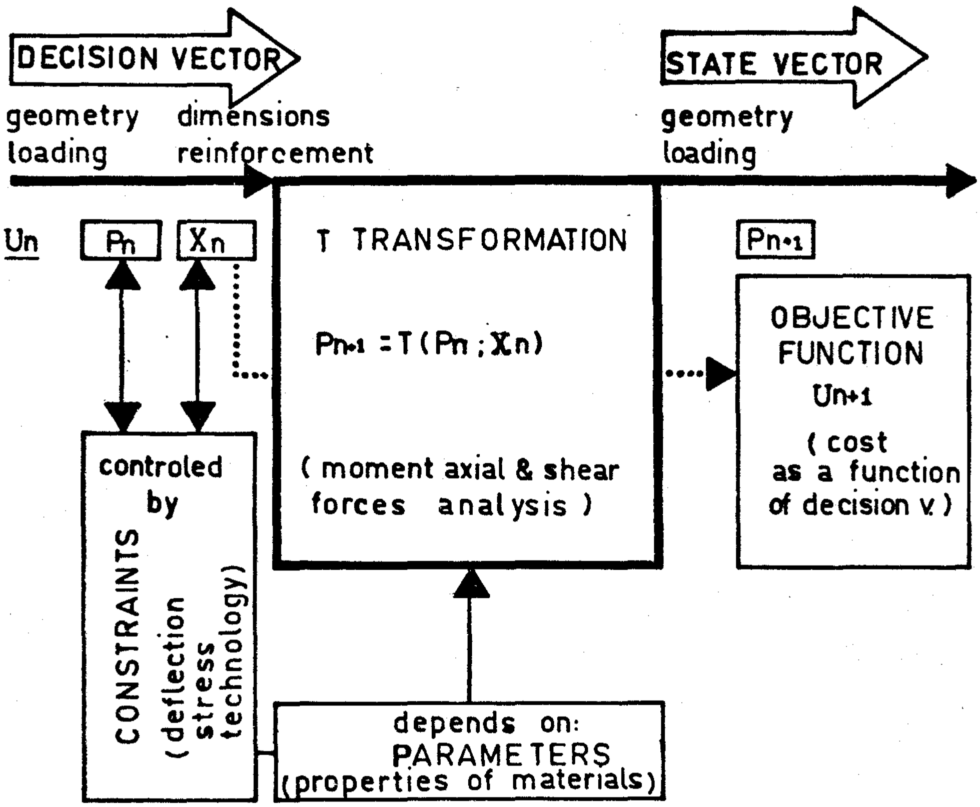


FIG. 3.6 THE DETAIL OF A DECISION STAGE

of loadings form a state vecotr or state vectors.

Transformation: The transformation may be expressed symbolically by writing:

$$P_{n+1} = T(P_n, X_n) \tag{3.25}$$

meaning that given P_n & X_n, it is possible to calculate P_{n+1}. This may represent a subroutine for a computer where many formulas are evaluated and put together.

If the design problem is not as well understood as one may wish, T may represent the judgement of the man who has had the best experience in the experimental work.

Parameters: The fixed constants of the system are its parameters. In general, it is a good practice to reduce the number of parameters by forming dimensionless variables.

Constraints: In general, both state and decision variables may be satisfied by some constraints either in the form of equations or inequilities. The constraints on the two sets of variables are inter-dependent.

In many situations the restrictions fall separately on the state and decision variables and often take the form of upper and lower bounds

$$Xmin \leq Xn \leq Xmax \tag{3.26}$$

A choice of decision variables that satisfies the constraints is called a d m i s s i b l e and we similarly speak of an admissible policy.

Objective function: The criterion by which various admissible policies can be judged is called an objective function.

We may refer to an objective function as being the sum of the profits from each stage.

The admissible policy Q_n that minimizes the objective function is called the <u>o p t i m a l p o l i c y</u>.

The process begins by considering only one stage (for example a slab) in which the initial state p_1 is to be transformed to p_2 by the choice of decision variables X_1 which minimizes the objective function

$$U_1 \ (p_1;\ X_1) \tag{3.27}$$

Let:

$$U_1 \ (p_1) = \underset{X_1}{\mathrm{Min}}(U_1(p_1;X_1)) \tag{3.28}$$

be the minimum price for one stage (slab).
It is obtained by the optimal choice of x_1 which depends on p_1. This choice may be denoted by $X_1^o,(p_1)$.

When there are two stages, the objective function is:

$$U_1 \ (p_1;X_1) + U_2(p_2;X_2) \tag{3.29}$$

The basis for minimizing the first term is known from previous study. Hence x_1 should be equal to $X_1^o(p_2)$ and the resulting minimum is $U\ (p_1)$. The p_2 in the second term is given bythe transformation

$$P_2 = T_1(p_1;X_1^o(p_1)) \tag{3.30}$$

Now if various admissible choices are made of X_2, it should be possible to arrive at the minimum of the sum of the two terms that is, at the minimum of the objective function for a two stage process.

Denoting this minimum by $U_2(p_2)$, this is symbolized by the equation:

$$U_2(p_2) = \underset{X_2}{\text{Min}} \left(U_1(p_1;X_1^o(p_1)) + U_2(p_2)\right). \qquad (3.31)$$

The optimal choice of X_2, however, will be a function of p_3.

Thus for n- stage policy the minimum of the objective function $\sum_1^n U_n$ is known and this minimum value may be denoted by $U_n(p_n)$ which is given by

$$U_n(p_n) = \text{Min}\left(\sum_1^n U_{n-1}(p_{n-1};X_{n-1}^o(p_{n-1})) + U_n(p_n)\right) \quad (3.32)$$

The steps to be taken in setting up and solving the optimization problem by dynamic programming are as follows:

1. Check that the problem "flows". If there are any feedback loops, a more careful analysis is needed.

2. Set up the problem by listing the elements:
   - (a) State
   - (b) Parameters
   - (c) Decision
   - (d) Constraints
   - (e) Transformation
   - (f) Objective function

3. Check to see that the state variables are sufficient and that the minimum of the objective function is a function of the variables p(n).

Define:

$$U_n(p_n) = \text{Min}\left({}_1^nU_{n-1}(p_{n-1};X_{n-1}^0(p_{n-1})) + U_n(p_n)\right) \qquad (3.32)$$

4. Check to see that the decision variables are the most suitable so that U_{n-1} is in the most readily calculable form.

5. Solve the above equation starting with n=1,2,...,N.

6. Present the results.

3.7 Summary

The following summary of the decisions regarding the application of the various optimization techniques were made as a result of trail solutions of the problems related to this study.

The method of direct search is employed in this study. The differential calculus approach is not attempted. Geometric programming requires the special formulation of the objective function and the constraints. This technique was used in the simplified design of one-way slabs. For more complex problems, this technique is not chosen because of previously mentioned difficulties. Also the dynamic programming is not selected; however, this technique may be added to the presented program for selecting the bar sizes to suit the already determined optimum cross sectional area of reinforcement (refer to Ref.(10)).

SEEK 1, (refer to Ref.(24)) is chosen as the most suitable subroutine from the package of optimization subroutines named OPTIPAC. The criterion for suitability is determined by:

a) The lowest computation time,

b) The accuracy of determining the minimum value of the objective function.

CHAPTER IV

OPTIMUM DESIGN OF ONE-WAY SOLID SLAB, BEAM AND GIRDER FLOOR

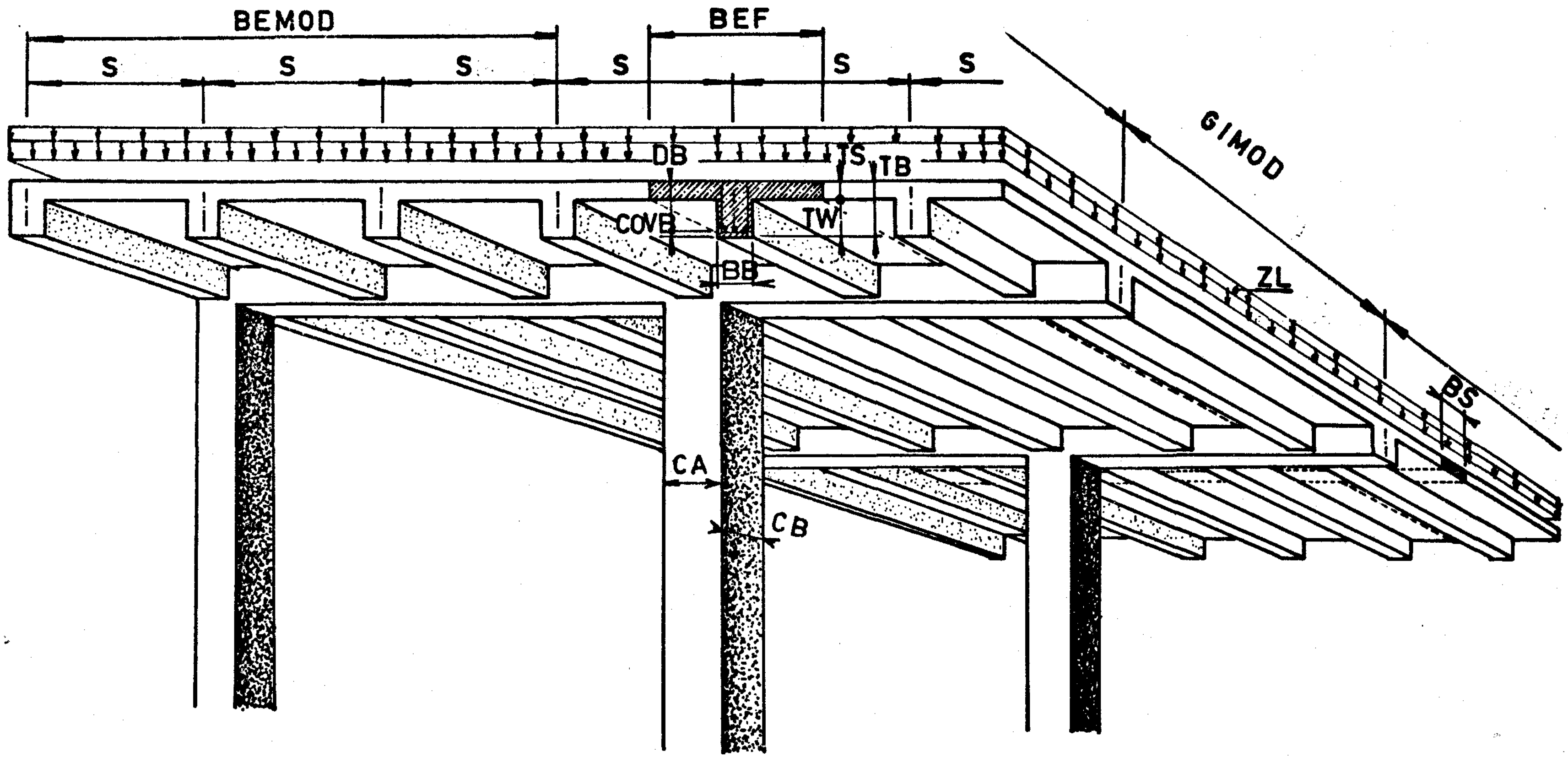
4.1 Introduction

The one-way solid slab, supported by beams that frame into girders and columns, is a conventional type floor. This type of floor is generally restricted to use in short spans (up to 15ft.) because of excessive dead weight. The optimum design of typical one-way solid slab, beam, girder floor panels is a part of the design of tall buildings as indicated in Figures 2.10 and 2.13.

This Chapter consists of the following sections: In section 4.2, the model panel is defined and the problem is stated. The objective function (cost function) as related to the model panel is determined in section 4.3. Section 4.4 describes the design constraints. The optimum solution is discussed in section 4.5. The sensitivity or the behavior of objective functions around the optimum solution is described in section 4.6. Section 4.7 shows the application of the developed computer program and a comparison between the conventional design and the optimal is made. The summary of this chapter follows in section 4.8.

4.2 Model panel

As stated in Chapter II, the floor is assumed to consist of typical panels. The one-way solid slab, beam, girder floor and its typical panel are shown in Figures 4.1 and 4.2 respectively. This panel serves as a model for the optimization.



GIMOD : DISTANCE BETWEEN AXIS OF TWO GIRDERS ADJACENT TO THE INVESTIGATED BAY (FT),
BEMOD : DISTANCE BETWEEN AXIS OF COLUMNS TAKEN PERPENDICULARLY TO GIMOD(FT)
CA : THICKNESS OF COLUMN SUPPORTING THE GIRDER TAKEN IN DIRECTION BEMOD(IN)
CB : THICKNESS OF COLUMN IN DIRECTION GIMOD(IN)
HS : HEIGHT OF STOREY(FT)

Fig. 4.1 Typical one-way solid slab, beam, girder floor framing.

The following parameters are specified:

a. Superimposed load, ZL, - which is the sum of all gravity loads except the dead load of the floor framing itself. This includes loads due to partitions, flooring and ceiling finishes, and live loads.

b. The distances BEMOD and GIMOD as defined on Fig. 4.2.

c. The strength of concrete, FC, (28 days after pouring) and the unit price of concrete, UPC, as defined in Section 2.5.

d. Yield stress of the reinforcing steel, FY, and the unit price of reinforcing steel, UPS, as defined in Section 2.5.

e. Unit price of formwork; UPFS, for slab, UPFB for beam.

The problem is to find the most economical solution for:

a. Layout of the beams (NS stands for number of spans, S, as defined in Fig. 4.2).

b. Size of: 1. Slab,
   2. Beam,
   3. Girder.

c. Amount of reinforcement steel area for
   1. Slab,
   2. Beam,
   3. Girder

d. Diameter of stirrups for beam, and the number of stirrups for one beam.

e. Diameter of stirrups for girder, and the number of stirrups for one girder.

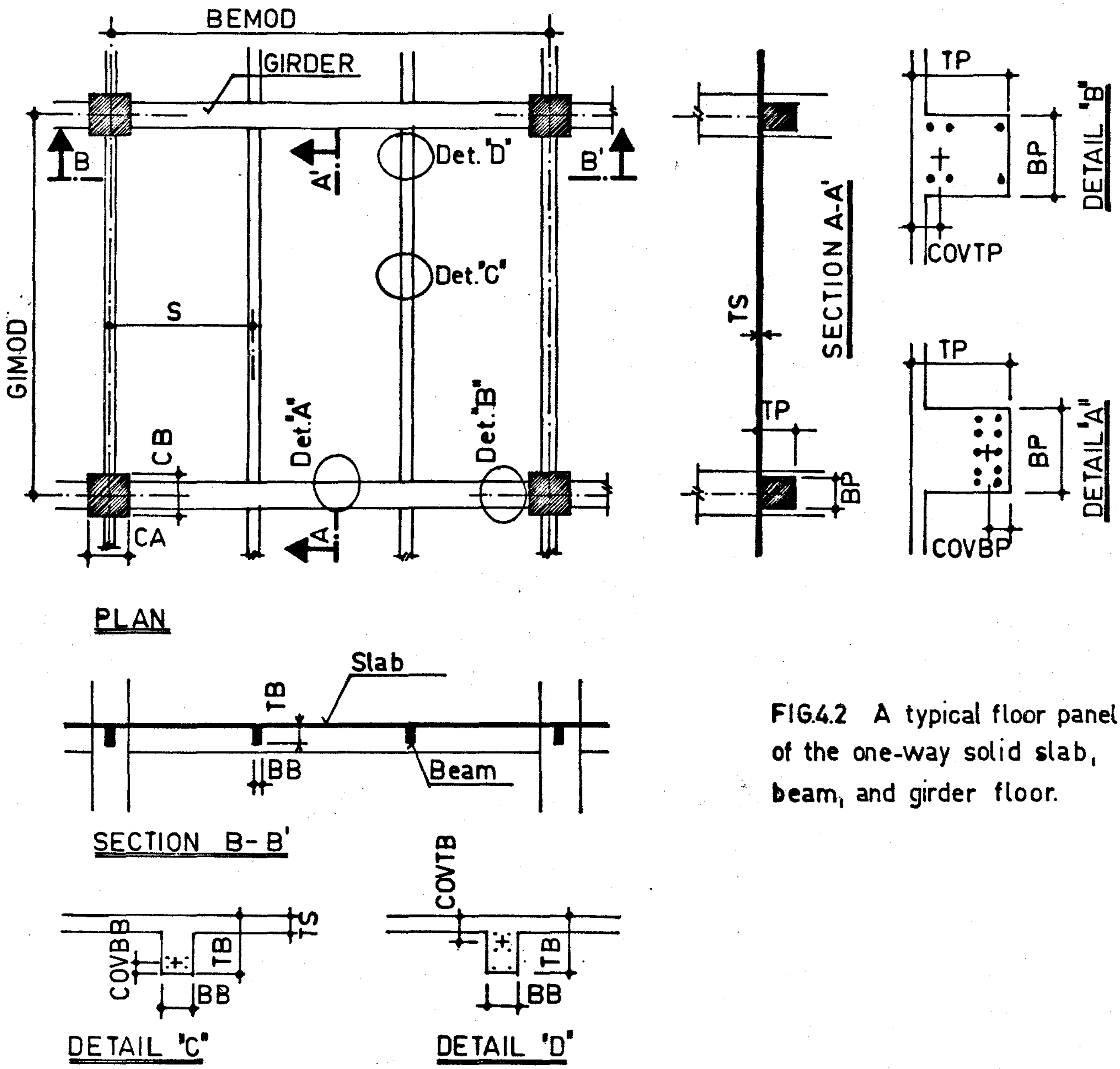


FIG.4.2 A typical floor panel of the one-way solid slab, beam, and girder floor.

VARIABLES:

X1 : NS, Number of spans
X2 : TS, Thickness of slab (in.)

BEAM:
X3 : TB, Overall depth (in.)
X4 : BB, Width of the web (in.)
X5 : Diameter (size) of the stirrups 2,...,8
X6 : Number of stirrups for one beam

GIRDER:
X7 : TP, Overall depth (in.)
X8 : BP Wrdth of the web (in.)
X9 : Diameter of the stirrups (size) 2,...,8
X10 : Number of stirrups for one girder

X(11) : Area of posit. reinforcement at midspan of slab (in^2)
X(12) : —"— negat. —"— support —"—
X(13) : Area of posit. reinforcement at midspan of beam (in^2)
X(14) : —"— negat. —"— support —"—
X(15) : Area of posit. reinforcement at midspan of girder (in^2)
X(16) : —"— negat. —"— support —"—

4.3 Objective function

The cost of a typical floor panel is the sum of costs of the slab, beams and the girder.

4.3.1 Slab:

The cost of a slab is expressed as follows

US = UPC + (AMCONS) + UPS* (AMSTS) + UPFS*(AMFOS) (4.1)

where:

US = Total price of slab ($)

UPC = Unit price of concrete } as defined in Section 2.5

UPS = Unit price of reinforcing steel } as defined in Section 2.5

UPFS = Unit price of slab formwork

AMCONS = Volume of poured concrete (ft^3)

AMSTS = Weight of reinforcing steel (lb)

AMFOS = Area of contact surface of formwork (ft^2)

The volume of concrete is given by:

AMCONS = TS* BEMOD* GIMOD (4.2)

where the symbols, as explained in Fig. 4.3, have the following meanings:

BS = Constant width of investigated strip = 1 ft.

TS = Thickness of the slab (ft)

TU = The distance from extreme compression fibre to centroid of tension reinforcement.

COV = Covering (0.75in) plus half diameter of reinforcement bar.

S = Span length (ft)

NS = Number of spans

BEMOD = Modulus of the beams (ft)

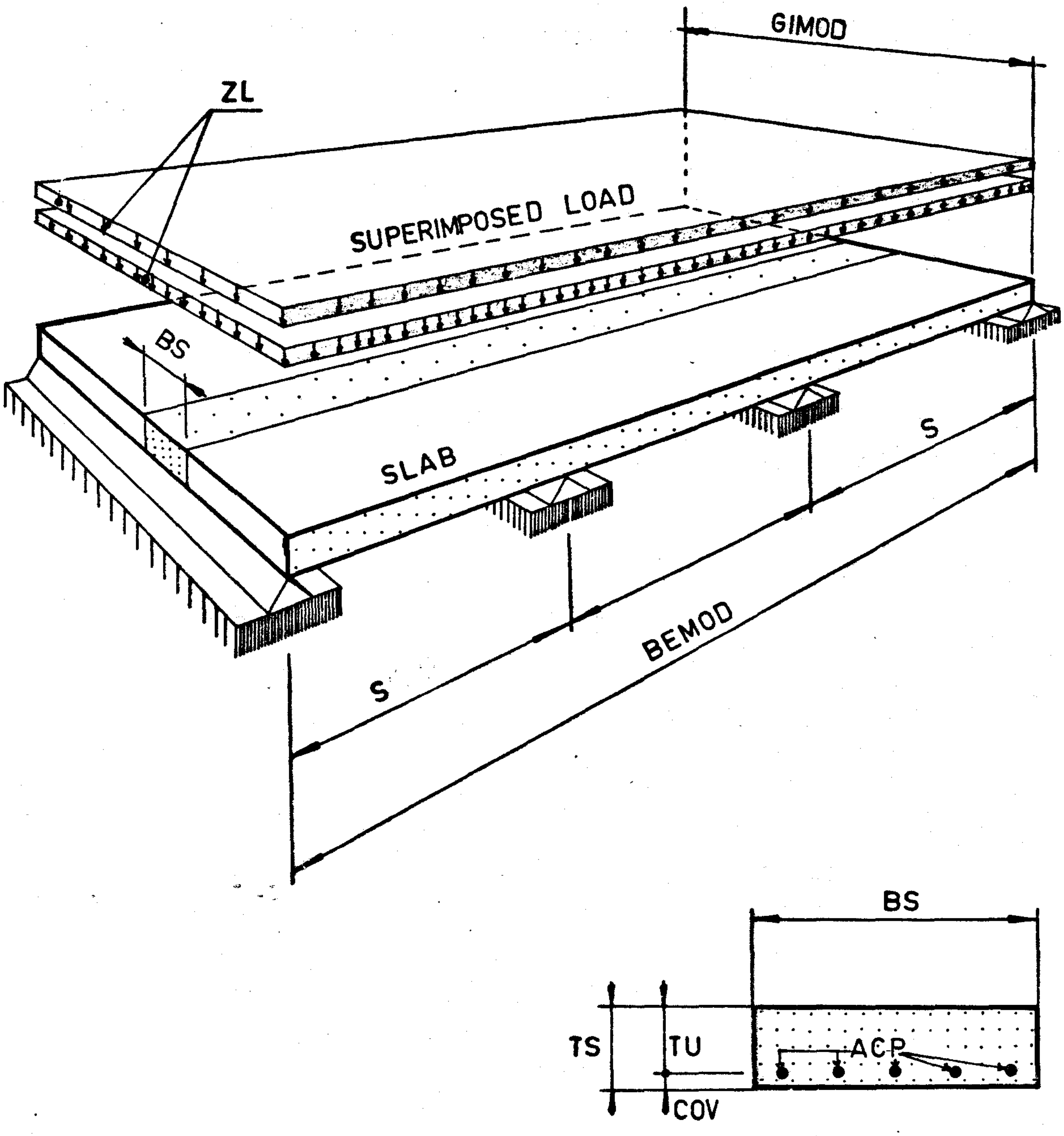


FIG.4.3 SYMBOLS USED IN SLAB DESIGN

GIMOD = Modulus of the girders (ft)

ZL = Superimposed load (lb/ft of the span)

The calculation of the weight of the positive reinforcing bars, ASp, (bottom of cross-section at midspan) follows the acting length assumptions for interior span as explained in Fig. 4.4 and as a result of the further assumptions in Fig. 4.5, it becomes:

$$ASp = X_{11} * W * (TS + BB + 0.8535*S) * GIMOD \qquad (4.3)$$

Where W stands for specific weight of steel (490 lb/ft^3)

The calculation of the weight of the negative reinforcing bars, ASn, is based on the assumptions in Fig. 4.6.

$$ASn = X_{12} * W * (TS + 0.418 *S) * GIMOD \qquad (4.4)$$

Thus,

$$AMST = ASp + ASn = W* GIMOD* [X_{11}*(TS + BB+0.8535*S) + X_{12}*(TS + 0.418*S)] \qquad (4.5)$$

The calculation of the area of contact surface of formwork, AMFOS, is expressed as:

$$AMFOS = (S - BB)* NS * GIMOD \qquad (4.6)$$

Substituting AMCONS, AMST and AMFOS from Equations (4.2), (4.5) and (4.6) respectively into Equation (4.1), results in:

$$US = GIMOD\{UPC* (TS*BEMOD) + UPS*W[X_{11}*(TS + BB + 0.8535*S) + X_{12}*(TS + 0.418)] + UPFS*(S - BB)* NS\} \qquad (4.7)$$

4.3.2 Beam

The beam is assumed to be built monolithically with the slab, so that the cross-section takes the shape of a tee. Therefore for the computation of the objective function the beam is represented by projection of the T-beam (Fig. 4.1) below the

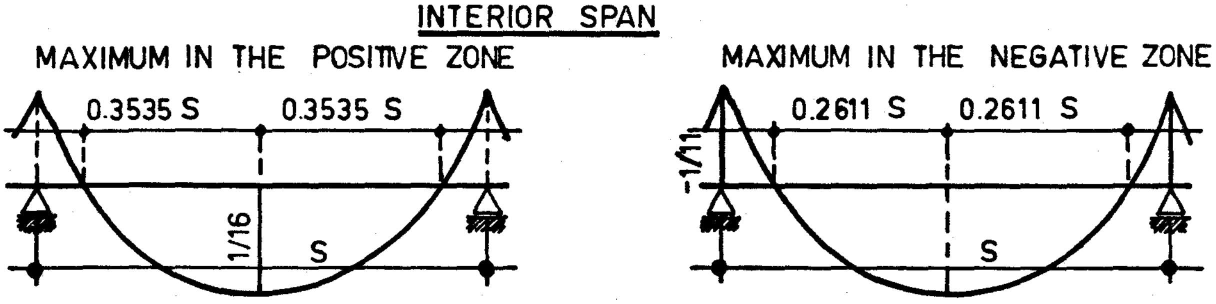


FIG. 4.4 ACI MOMENT COEFFICIENTS (ARTICLE 904 OF 1963 ACI CODE)

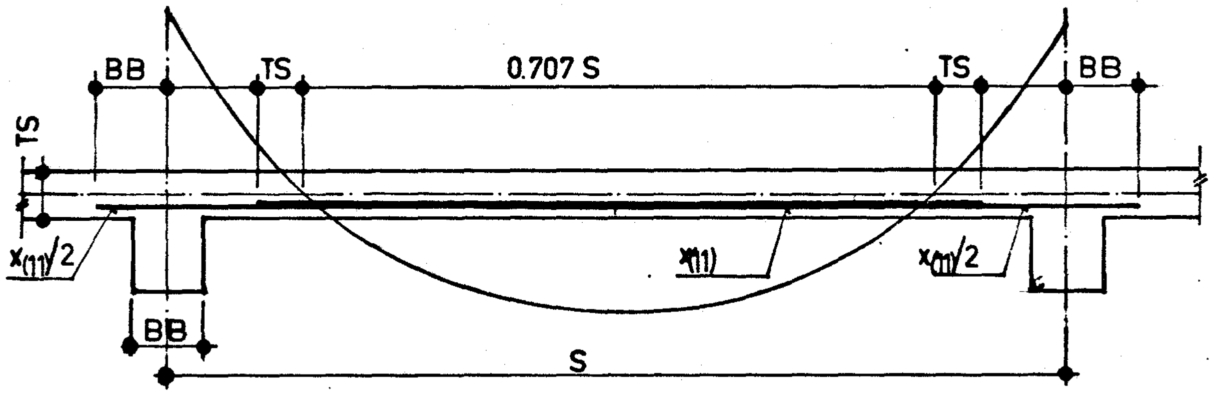


FIG.4.5 THE POSITIVE STEEL ARRANGEMENT FOR SLAB

FIG.4.6 THE NEGATIVE STEEL ARRANGEMENT FOR SLAB

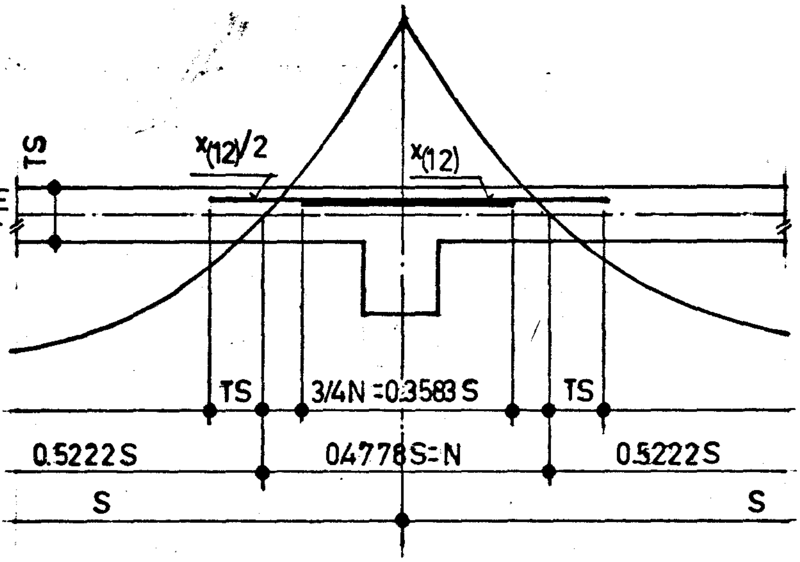
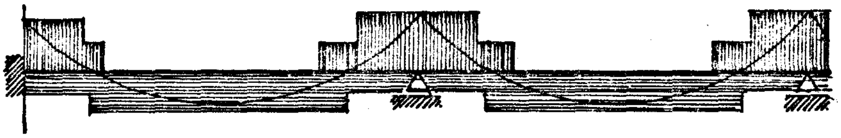


FIG.4.7 THE REINFORCING ENVELOPE OF MOMENT DIAGRAM.



slab.

The cost of the single beam, UB, is given by,

$$UB = UPC*(AMCB) + UPS*(AMSTEN + AMSH) + UPFB*(AMFB) \quad (4.8)$$

where:

AMCB = Volume of poured concrete (ft^3)

AMSTEN = Weight of main reinforcement (lb)

AMSH = Weight of shear reinforcement (lb)

AMFB = Area of surface of formwork (ft^2)

The volume of poured concrete, AMCB, is expressed by,

$$AMCB = TW * BB * (GIMOD - BP) \quad (4.9)$$

where:

$$TW = \text{Clear depth of the beam} = TB - TS \quad (4.10)$$

BP = Width of the girder

The calculation of the weight of main reinforcement, AMSTEN, is based on the same assumptions as for the weight of reinforced steel for the slab, AMSTS.

Therefore, for the beam, Equation (4.3) takes the form:

$$ASp = X_{13} * W * (TB + BP + 0.8535* GIMOD) \quad (4.11)$$

$$ASn = X_{14} * W * (TB + 0.418* GIMOD) \quad (4.12)$$

Hence, the total weight of main reinforcement, AMSTEN, is the sum of Equations (4.11) and (4.12):

$$AMSTEN = W*[X_{13}*(TB + BP + 0.8535*GIMOD) + X_{14}* (TB + 0.418* GIMOD)] \quad (4.13)$$

The weight of shear reinforcement, AMSH, is given by,

$$AMSH = W*(2TB + BB - 4COVBB - 2COVTB)* A * X_6 \quad (4.14)$$

where: $A = 3.1415(X_5)^2/256.$ = Sectional area of one stirrup for beam, (4.15)

X_5 = Bar size stirrups for the beam - 2,3,..., 8

X_6 = Number of stirrups for the beam,

COVBB, COVTB = As explained in Figure 4.2.

The area of contact surface of formwork, AMFB, is

$$AMFB = (2TW + BB)^* (GIMOD - BP) \quad (4.16)$$

Substituting AMCB, AMSTEN, AMSH and AMFB from Equations (4.9), (4.13) (4.14) and (4.16) respectively into Equation (4.8), the cost of a single beam, UB, is calculated as follows:

$$UB = UPC^*[TW^*BB^*(GIMOD - BP)]+UPS^*W^*[X_{13}^*(TB^*BP + 0.8535^*GIMOD) + X_{14}^*(TB + 0.418^*GIMOD) + (2TB+BB-4COVBB-2COVTB)^*A^*X_6] + UPFB^*[(2TW + BB)^* (GIMOD-BP)] \quad (4.17)$$

It is noted, that in the invesitgation of the model shown in Fig. 4.2, the number of beams is equal to number of spans, NS rather than the number of spans plus one.

4.3.3 Girder

The girder is assumed to be built monolithically with the slab and with the beam as shown in Fig. 4.1. The calculation of the objective function for the girder takes into account only the projection of the girder bellow the slab.

The cost of a single girder, UG, is expressed by

$$UG = UPC(AMCG) + UPS(AMSTG+AMSHG) + UPFB(AMFG) \quad (4.18)$$

where:

AMCG = Volume of poured concrete into the girder (ft^3)

AMSTG = Weight of main reinforcement (lb)

AMSHG = Weight of shear reinforcement (lb) and

AMFG = Area of formwork (ft^2)

The volume of concrete, AMCG, is given by,

AMCB = TWP * BP * BEMOD (4.19)

where

TWP = Clear depth of the girder = TP-TS

TP = Overall depth of the girder. (4.20)

The calculation of the weight, AMSTG, of the main reinforcement also follows the assumptions shown in Figures 4.5 and 4.6. For the girder, Equation (4.3) takes the following form:

ASp = X_{15}*W*(TP + BP + 0.8535 * BEMOD) (4.21)

and Equation (4.4) assumes the form:

AS_n = X_{16}*W*(TP + 0.418 * BEMOD) (4.22)

Hence, the total weight of main reinforcement, AMSTG, is given by,

AMSTG = W*[X_{15}*(TP+BP+ 0.8535 * BEMOD) + X_{16}*(TP+0.418*BEMOD) (4.23)

The weight of the stirrups for the girder, AMSHG, is calculated from following equation:

AMSHG = W*(2TP+ BP - 4COVBP - 2COVTP)* AG * X_{10} (4.24)

where:

AG = $3.1415(X_9)^2/256$ = Cross-sectional area of one stirrup, (4.25)

X_9 = Bar size of stirrup for girder, (in1/8th of an inch; say X_9=2 read 2/8).

X_{10} = Number of stirrups within the distance of BEMOD.

COVBP, COVTP = As explained in Fig. 4.2.

The area of contact surface of formwork, AMFG, from the geometry shown in Figures 4.1 and 4.2, is

$$AMFG = 2\{[(BEMOD-TP) * (TWP + 0.5BP)]-(BB*TW*NS)\} \quad (4.26)$$

Substituting AMCB, AMSTG, AMSHG and AMFG from Equations (4.19), (4.23), (4.24), and (4.26) respectively into Equation (4.18), the cost of the girder, UG, is computed as follows:

$$UG = UPC*[TWP*BP*BEMOD] + UPS*W*[X_{15}*(TP+BP+0.8535\ BEMOD)$$
$$+ X_{16}*(TP+0.418\ BEMOD) + (2TP \pm BP - 4COVBP - 2COVTP)*AG*X_{10}]$$
$$+ UPFB*2\{[(BEMOD-TP)* (TWP + 0.5BP)] - (BB*TW*NS)\} \quad (4.27)$$

Finally, the objective function, U, for model shown in Fig. 4.2 takes the form,

$$U = US + NS*(UB) + UG \quad (4.28)$$

where

US, UB and UG are to be substituted from Equations (4.7), (4.17) and (4.27) respectively.

4.4 Design constraints

All the three elements - Slab, Beam and Girder, of the floor-framing model shown in Figures 4.1 and 4.2, are flexural members. Moment, shear and deflection constraints are applied to each of above mentioned elements. The National Building Code of Canada[17] requires the actual reinforcing ratio (P) to be less than or equal to 75% of the reinforcing ratio which produces the balanced condition (PB) (refer also to Reference (11)). P should not be less than the minimum required for shrinkage and creep (refer to (17)). This gives the additional constraints to be applied to all three elements. There are also constraints of practical nature such as the minimum number of bars, min. depth, maximum spacing of stirrups which are applicable to particular elements.

4.4.1 Moment constraint

The general form of the moment constraint is expressed as follows,

$$UM - M \geq 0 \qquad (4.29)$$

where:

UM = Ultimate moment capacity of the cross-section of the particular element at the critical point.

M = Actual moment at the critical point caused by external forces.

The critical points are the sections at the midspan of the member and at the face of support. The actual moments, M, for the slab and beam are computed on the basis of moment coefficients as suggested by Reference (17). For the girder, the actual moments are computed as follows. It was observed that for equally spaced beams on the girder, the maximum possible negative moment (at the face of supporting column) occurs for a two-span girder with live load on both spans. However, the maximum possible positive moment for equally spaced beams occurs in a three-span girder with live load on the middle span and just dead load (self-weight) on the outer spans. The moment distribution method is employed in subroutines TWOSP, and THREEN where the names stand for a two-span girder, and a three-span girder respecrively. The ultimate moment capacity, UM, is computed by subroutines SIMPLY, DOUBLE, and TBEAM. All three subroutines follow the logic described in Reference (11).

Subroutine SIMPLY computes the ultimate moment capacity of the rectangular cross-section with tension reinforcement only. Such a section occurs at the midspan of the slab.

Subroutine DOUBLE calculates the ultimate moment capacity of the rectangular cross-section with both compression and tension reinforcement steel. The section, at the support, of all three elements i.e. slab, beam and girder is assumed to be doubly reinforced with the compression reinforcement equal to half of the tension reinforcement at the midspan of the element. The details of the arrangement of reinforcing steel at the support, and the reinforcing envelope of the moment diagram are contained in Figures 4.6 and 4.7 respectively. Examples 3.8,1 and 3.8.2 from Reference (11) were used to test subroutine DOUBLE.

Subroutine TBEAM compute the ultimate moment capacity of the T-section which relates to the midspan section of the beam and the girder. Here, the effective width of the flange for the beam is given by the least of the following.

$$\left.\begin{array}{l} BEF \leq 0.25*GIMOD \\ BEF \leq BB + 24\ TS \\ BEF \leq S \end{array}\right) \qquad (4.30)$$

The effective width of the flange, BEF, for the girder is taken to be two times of the width of the girder, BP, (refer to Reference (11)).

The subroutine TBEAM also controls the value of (P-PF) by setting to zero the ultimate moment capacity of the section if this value exceeds 0.75 PB. The area of reinforcement required to develope the compressive strength of the overhanging flanges is A_{sf}(17). Thus it is defined by

$$PF = \frac{A_{st}}{b'd} \qquad (4.31)$$

where:

b' = Width of web in T-section

d = Effective depth (refer to Reference (17)).

By means of example 8.5.1 from Reference (11), subroutine TBEAM was successfully tested and verified.

4.4.2 Shear constraint

The ultimate shear strength of a beam is expressed as the sum of two components:

$$V_u = V_{uc} + V_{us} \tag{4.32}$$

in which $V_u = V_u/bd$; V_{uc} = ultimate shear strength of the concrete when there is no web reinforcement; and V_{us} = ultimate shear strength contributed by web reinforcement.

The slab is defined as a beam with no web reinforcement. Thus, the ultimate shear produced by external loading must be less than that carried by the concrete alone. Hence,

$$\frac{Vu}{12TS} < 2\phi\sqrt{FC} \tag{4.33}$$

where

Vu = Maximum shear force at the distance TS from face of supporting beam,

ϕ = Undercapacity factor = 0.85 for diagonal tension[17],

b = Taken 12 inches.

For the beam and girder, vertical stirrups are assumed to carry the shear stress, V_{us}. The control spacing, s, is the least of the following:

$s = d/2$ if $v_u \leq 6\emptyset\sqrt{FC}$

$s = d/4$ if $v_u > 6\emptyset\sqrt{FC}$

$s = \emptyset Av\ FY/b\ v_{ws}$

$s = Av/0.0015\ b$

where:

Av = Two times the sectional area of a stirrup,

$\emptyset$ = As defined in Equation (4.33),

b = width of the web

d = distance from extreme compression fibre to centroid of tension reinforcement

The above formulas are taken from National Building Code (1965). The control spacing is calculated in the subroutine WEBRTG. Further, it is assumed that the spacing of the stirrups does not follow the shear stress diagram, but is kept constant throughout the span.

The subroutine WEBRTG was tested and found to be operative by means of example 4.10.1 from Wang and Salmon (11). Reference (11) contains a detailed explaination of the basis for the derivation of the above design formulas and procedures.

4.4.3 Deflection constraint

Any of the previously mentioned flexural members (slab, beam, and girder) should not deflect more than the allowable value of 1/360 of the span as prescribed by the National Building Code(17).

The midspan deflection is computed by the conjugate-beam method and the derivation of the general expression can be found in Reference (11). It is given by (Equation 14.2.4 in Reference (11)),

$$y_m = \frac{5L^2}{48EI} \left[Ms - 0.1(M_a + M_b)\right] \qquad (4.35)$$

where:

y_m = Total midspan deflection,

L = Span length

E = Modulus of elasticity for concrete computed as $E = w^{1.5}\ 33\sqrt{FC'}$

for w = 145pcf $E = 57{,}500\sqrt{FC'}$

I = Moment of inertia of section of the number,

M_s = Midspan moment

M_a and M_b = Moment at the left and right support of the span respectively.

The moment of inertia, I, can be obtained as follows

a) when $P*FY \leq 500$, use gross section

b) when $P*FY > 500$, use the transformed cracked section. (4.37)

where P = Reinforcement ratio of the section at midspan.

$M_a = 0$ and $M_b = 0.1*SU*L^2$ are assumed in Equation (4.35), whereas the moment at the midspan is given by,

$$M_s = 0.125*SU*L^2 - 0.5\ M_b \qquad (4.38)$$

where SU^{Δ} = Uniformly distributed service load as per Reference (17).

Δ Foot note: The value, SU, was redefined from ultimate to service load during the correction. There is no significant change in results, however, those effected are marked.

The deflection, y_m, computed from Equation (4.35) is taken to be the immediate live load deflection. The additional deflection due to dead load and deflection due to creep and shrinkage of the concrete is obtained as follows:

$$y_{ad} = \left(\frac{ZD+SU}{SU}\right) y_m * \text{creep factor} \qquad (4.39)$$

where ZD = Dead load, as per Reference (17). Creep factor as per Reference (17) = 2.0

Thus, the total deflection (sum of $y_m + y_{ad}$) of the slab, beam and the girder should not exceed the allowable. This is expressed mathematically by

$$y_m + y_{ad} \leq \frac{1}{360} L \qquad (4.40)$$

Reference (17) states that deflection must be checked when the net reinforcement ratio p, $p-p'$, or p_w-p_f in any section of a flexural member exceeds 0.18 FC/FY.

4.4.4 Geometric constraints

Constraints such as the minimum thickness of the slab are itemized as geometric constraints which can be found in the List of Constraints in subsection 4.4.5.

If, for some reason, a variable x_n is to be additionally constrained to a particular value, V, then setting the program at "adjusting run" (IRUN=2) and setting $RMAX_n$, $RMIN_n$ and $XSTRT_n$ equal to V, the program will keep the value of x_n constant and equal to V during the optimization process. $RMAX_n$ and $RMIN_n$ are the estimated upper and lower bounds of the variable x_n

respectively. $XSTRT_n$ is the starting value of the variable x_n.

It is noted that, in the program, the constraints are "scaled". That is, all the values of constraints are in the range between 0.0 and 1.0

4.4.5 List of the constraints (*)

1.	Ultimate moment capacity of the slab section at the midspan.	
2.	Minimum reinforcement ratio	of the slab section at the midspan
3.	Maximum reinforcement ratio	of the slab section at the midspan
4.	Ultimate moment capacity	of the slab section at the support
5.	Minimum r.r.	of the slab section at the support
6.	Maximum r.r.	of the slab section at the support
7.	Shear capacity	of the slab section at the support
8.	Deflection at the midspan of the slab	
9.	Ultimate moment capaicty	of the beam section at the midspan
10.	Minimum r.r.	of the beam section at the midspan
11.	Maximum r.r.	of the beam section at the midspan
12.	Ultimate moment capacity	of the beam section at the support
13.	Minimum r.r.	of the beam section at the support
14.	Maximum r.r.	of the beam section at the support
15.	Number of stirrups	for the beam
16.	Minimum diameter of stirrups	for the beam

(*) The numbering of constraints corresponds with the numbers in Figures (4.11, to 4.16)

17. Deflection at the midspan of the beam

18.	Ultimate moment capacity	of the girder section at the midspan of the girder
19.	Minimum r.r.	
20.	Maximum r.r.	
21.	Ultimate moment capacity	of the girder section at the support
22.	Minimum r.r.	
23.	Maximum r.r.	
24.	Mumber of stirrups	for the girder
25.	Minimum diameter of stirrups	

26. Min. thickness of the slab = 3"$^{\Delta}$

27. Min. number of spans to be 2.

28. Min. width of the beam = 3"$^{\Delta}$

29. Min. depth of the girder = 9"

30. Depth of a beam must not be larger than the depth of girder.

31. Min. 2-#4 bars of sectional area = 0.4in^2 for the beam at midspan.

32. Min. 2-#4 bars of sectional area = 0.4in^2 for the beam at support.

33. Min. 2-#4 bars of sectional area = 0.4in^2 for the girder at midspan.

34. Min. 2-#4 bars of sectional area = 0.4in^2 for the girder at support.

35. Min. 1-#2 per foot of slab at the midspan

36. Min. 1-#2 per foot of slab at the support

Foot note: Δ NBC(17) limites to 4 inches ASI limites to 3.5 inches.

4.5 Program Employment and Solution Analysis

Using the direct search technique, a computer program was developed for the optimum design of the one-way solid slab, beam and girder floor. Appendix A gives the program in its entirity in FORTRAN IV language. As an application of the program (with the aid of a CDC Model 6400 computer), the panel model shown in Figure 4.2 was optimized with respect to the following input parameter data:

BEMOD = 20 ft.	CA = 14 inches
GIMOD = 20 ft	CB = 14 inches
ZL = 100 psf	HS = 12 ft

Concrete Strength , FC = 3,00psi

Yield stress of Reinforcement,FY = 40,000 psi

For comparison purposes, these input data correspond to those in a design example in Reference (13).

An input variable of interest is IRUN which is to indicate whether the user specifies the starting values of the variables to be optimized ($XSTRT_i$, i = 1,2...,16) and their respective estimated upper and lower bounds ($RMAX_i$ and $RMIN_i$, i = 1,2..,16). If such is the case, IRUN is set equal to 2; then detailing variables such as COV, etc. (as defined in Section 4.2) must be included in the input data. Otherwise, IRUN is set equal to 1, where all the above mentioned data are assigned automatically within the computer program.

The program can be used for studying the effects of a range of concrete strengths (FC) as well as a range of yield strengths of steel (FY) by controlling the input value of JAK.

With JAK=1, only a single value of concrete strength (MB=1) with a single value of steel strangth (MS=1) is investigated. With JAK=2, all conbinations of FC(1,2,...,MB) and FY(1,2,..,MS) are processed, where MB and MS represent the number of strength values provided for concrete and steel respectively.

Thus, setting both IRUN and JAK equal to 1, the computer assigns upper and lower bounds of the variables and their respective starting values are listed in the first, second and third columns of Table 4.1. Using the unit prices from section 2.5, subroutine SEEK 1(Reference 24), minimized the objective function Eq.(4.28) subject to the constraints of section 4.4. The final values of the variables after the optimization process are listed in the fourth column of Table 4.1. However, it is to be noted that the final values as listed in the sixth column are not realistic. The number of spans (X_1) should not involve fractional quantities and from a practical point of view, the diameter of stirrups (x_5 and x_9) should be taken to the closest eighth of an inch. Hence, the last column in Table 4.6 is obtained by setting IRUN=2. $RMAX_i$, $RMIN_i$ and $XSTRT_i$ are taken as per the first second and third columns respectively, except for variables, x_1, x_5 and x_9 where they are constrained to the closest integer values of 2, 2 and 4 respectively. The detailing parameters are as given in Table 4.1.

Figure 4.8 shows the optimum solutions for different cases of live load. It is seen from the x_1 solutions that to accomodate the increasing live loads, there is practically no need to increase the number of beams except for exceedingly large values such as the 300 psf. While the number of beams is only mildly

Variables Column 1	Representation 2	RMAX 3	RMIN 4	XSTRT 5	Optim. Solution 6
x1	Number of spans	5	1	2.1	2.001
x2	Thickness of slab (in)	7.017	3.03	4.	3.18
x3	Depth of one beam (in)	30.	15.	23.	20.77
x4	Width of one beam (in)	20.	10.	9.07	6.84
x5	Diameter of stirrups for one beam (1/8 of in)	8.	2.	2.06	2.65
x6	Number of stirrups for one beam	50.	10.	31.37	27.41
x7	Depth of girder (in)	30.	15.	23.65	23.46
x8	Width of girder (in)	20.	10.	15.46	14.84
x9	Diameter of stirrups for girder (1/8 ")	8.	2.	3.08	3.08
x10	Number of stirrups for girder	50	10.	28.8	28.8
x11	Area of positive steel slab	1.44	0.072	0.252	0.252
x12	Area of negative steel slab	1.44	0.072	0.453	0.453
x13	Area of positive steel beam	3.34	0.83	1.022	1.146
x14	Area of negative steel beam	3.34	0.41	1.523	1.74
x15	Area of positive steel girder	4.02	0.804	4.278	3.04
x16	Area of negative steel girder	4.059	0.804	1.587	1.526
U	Cost of the panel $				903.34

PARAMETER:

FC = 3,000 psi BEMOD = 20 ft ZL = 100 psf CA = 14 in COV = 1.25 in COVBB = 2 in

FY = 40,000 psi GIMOD = 20 ft HS = 12 ft CB = 14 in COVTB = 3 in COVBP = 3 in

COVTP = 3 in

Table 4.1. Optimum Solution of One-way Slab, Beam and Girder Floor.

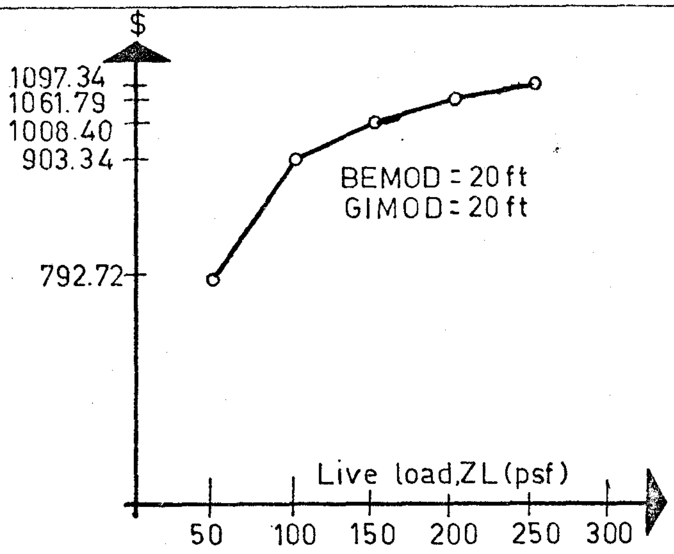
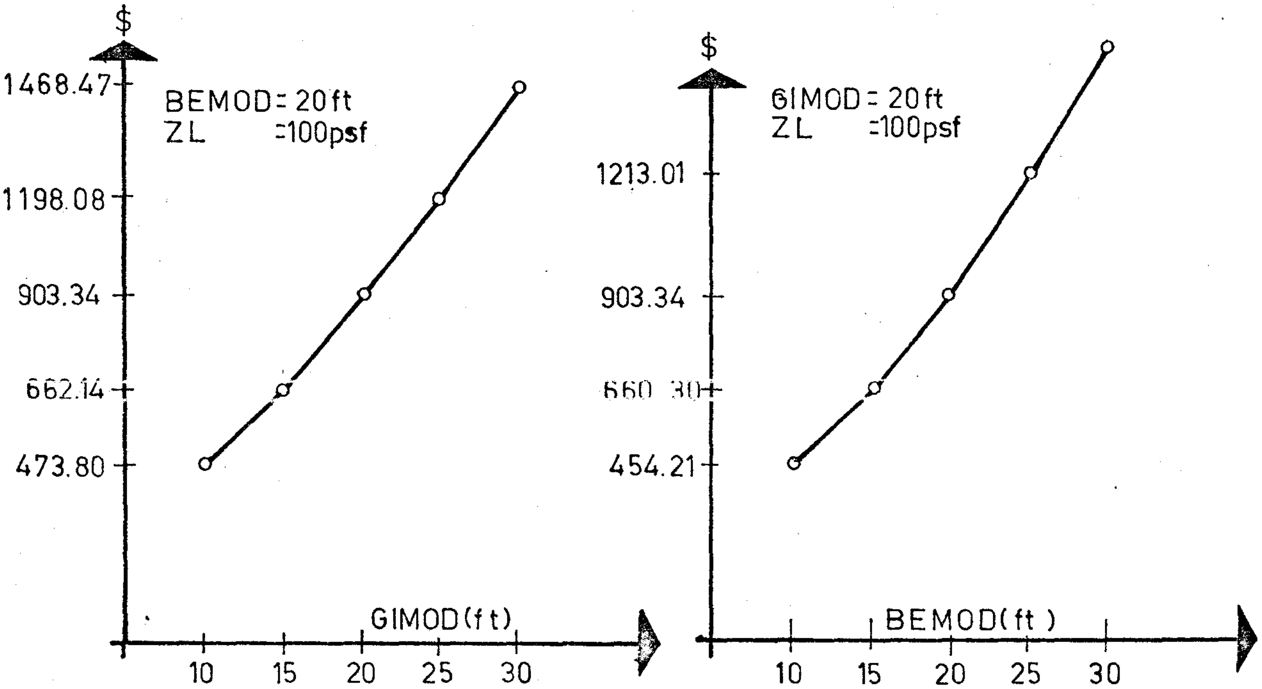


FIG.4.8 Relationship between the cost and various live load, ZL.



a, GIMOD variation

b, BEMOD variation

FIG. 4.9 Relationship between the cost and different sizes of a panel

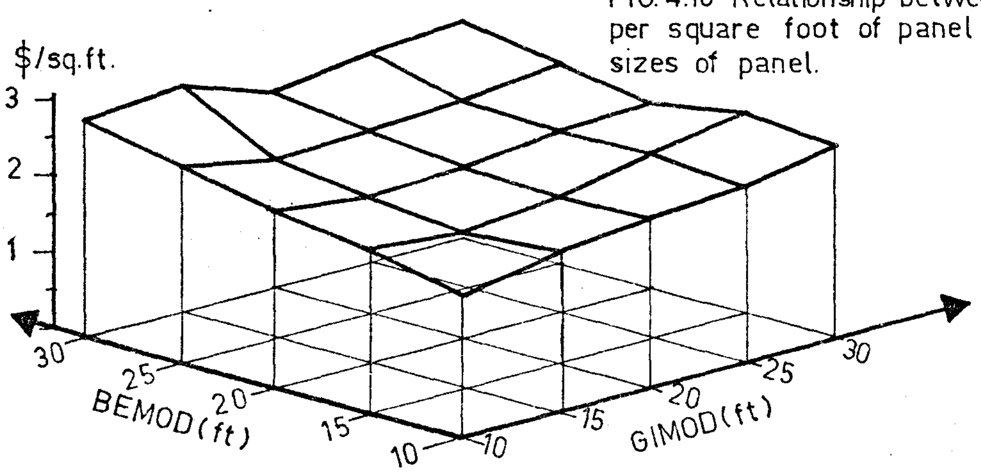


FIG. 4.10 Relationship between the cost per square foot of panel and various sizes of panel.

affected by the increase in live loads, the objective cost function scores an increase of almost 34% for an increase of ZL from 50 psf to 200 psf. This relationship is shown in Figure 4.8.

Figure 4.9a shows the variation of the optimum cost of panel with the distance GIMOD while BEMOD is kept constant at 20 ft. The increase in the cost with increasing GIMOD is to be expected. A similar variation for the distance BEMOD is shown in Figure 4.9b.

Figure 4.10 shows the relationship between the cost per square foot of a panel and various sizes of panels. The study of optimization through varying the panel size is a problem which would require placing a value on column free space. This aspect is beyond the scope of this work.

Table 4.2 contains a study of the optimum cost of the panel as effected by different combinations of concrete strengths and yield strength of reinforcing steel. The results are plotted in three-dimensional form for better visual impact. It is seen that over the ranges of concrete strengths and yield strengths considered, the variation in the cost is more prominant in the FY direction. In other words, better economy is obtained using a higher yield strength of steel rather than increasing the strength of concrete. Furthermore, the structure is most economical with a combination of the highest possible yield strength of steel and the lowest possible strength of concrete.

FY \ FC (psi)	3,000	3,500	4,000	4,500
psi	$	$	$	$
40,000	903.34	905.28	918.31	925.83
50,000	908.01	914.23	925.34	926.70
60,000	802.37	795.41	800.33	799.01
75,000	760.01	768.91	771.64	773.41

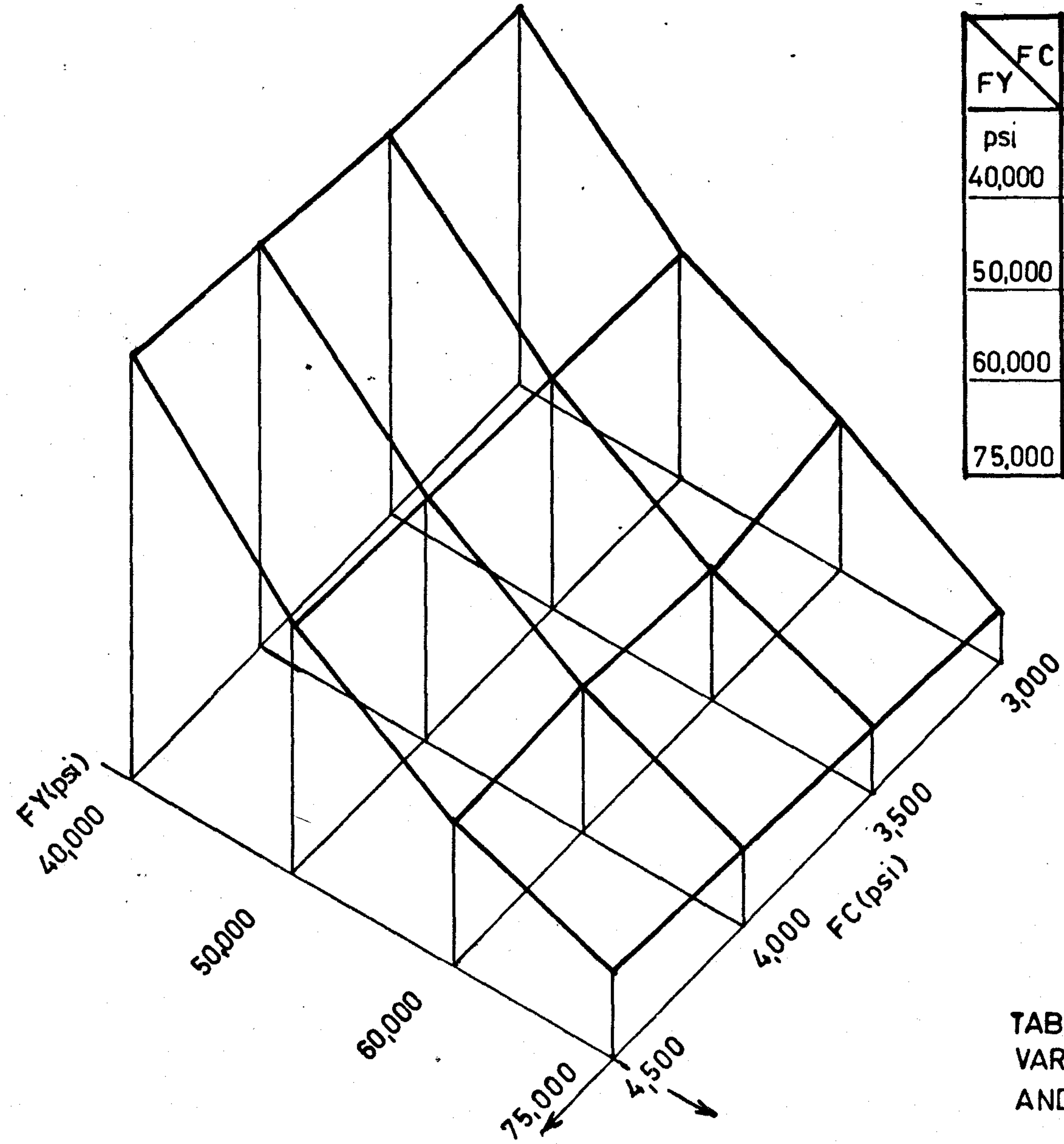
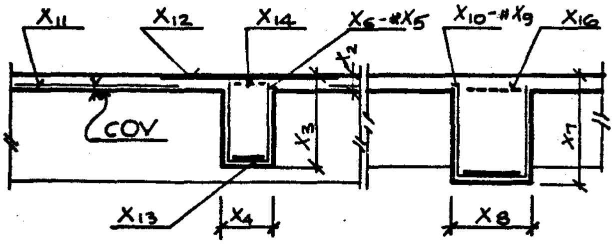
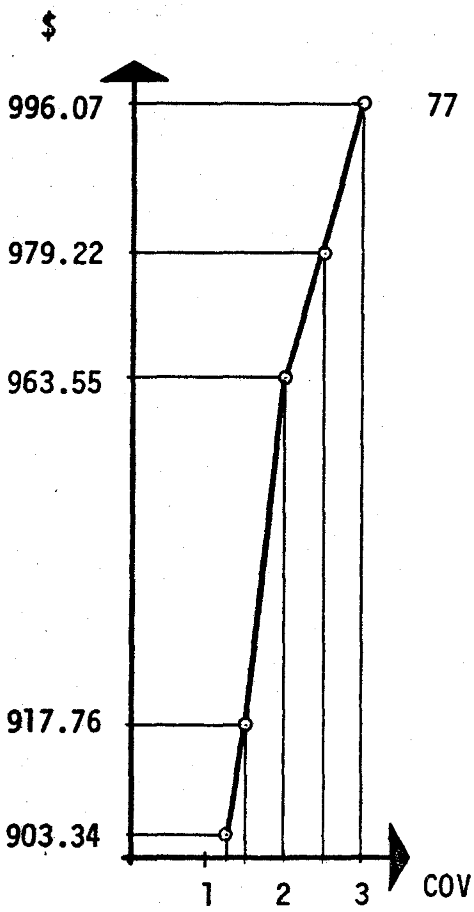


TABLE 4.2 COST ANALYSIS OF VARIOUS STRENGTH OF CONCRETE, FC, AND YIELD STRESS OF STEEL, FY.

It should be noted that these conclusions are relative due to the change in unit prices with time and locale. The user may arrive at different conclusions using different unit prices. However this phenomena has been recognized by designers and is one of the reasons that use of high strength steel has become quite common.

The National Building Code of Canada (17) specifies concrete covering protection for reinforcement as in Article 5.2.8. During the author's field inspections it was confirmed that those values of covering protection are reasonable well adhered to. Further, it was found that the water-cement ratio, diameter of reinforcing steel and curing of the concrete are the three main factors affecting the value of the concrete covering protection against corrosion. The reinforcing steel is wired into a "cage", however, even then the steel position moves during pouring and vibrating the concrete. Table 4.3 illustrates the effect of displacement of steel. Increasing cover, COV, as expected call for an increased thickness of slab and obviously such model is less economical. However, the thicker slab causes lower girder thickness -x_7, due to the larger compression zone in the section. This resulting greater moment arm for flexure T section capacity only occured where the natural axis lies below the slab (which is usually the case). The other four, COVTB, COVBB, COVTP and COVTB, do not cause any dramatic change in cost. For example a change of COVTB from 2 inches to 4" yields an increase of $6.93 only 0.61% increase in the cost of the model.





ZL = 100 psf

BEMOD = 20 ft

GIMOD = 20 ft

FC = 3,000 psi

FY = 40,000 psi

	COV = 1.25"	COV = 1.5"	COV = 2"	COV = 2.5"	COV = 3"
x_1	2.001	2.001	2.001	2.001	2.001
x_2	3.185	3.53	4.0	4.59	5.18
x_3	20.77	21.23	22.53	22.62	22.44
x_4	6.84	6.84	8.95	8.08	7.59
x_5	2.65	2.65	2.06	2.65	2.65
x_6	27.41	27.41	31.37	23.45	27.41
x_7	23.45	23.65	23.55	23.65	23.27
x_8	14.84	14.65	15.46	15.09	15.46
x_9	3.08	3.08	3.08	3.08	3.08
x_{10}	28.8	28.8	28.8	28.8	28.80
x_{11}	0.252	0.244	0.252	0.252	0.252
x_{12}	0.453	0.453	0.504	0.546	0.572
x_{13}	1.146	1.146	1.115	1.146	1.193
x_{14}	1.740	1.740	1.668	1.722	1.795
x_{15}	3.044	3.044	3.183	3.123	3.143
x_{16}	1.526	1.526	1.607	1.566	1.584
U	903.34	917.76	963.55	979.22	996.07

Table 4.3. Study of variation of COV as defined in Section 4.2.

4.6 Sensitivity

In optimization terminology, sensitivity refers to the behaviour of an objective function in the vicinity of the optimum. A sensitivity study of the objective function (cost), as defined in Section 4.3, was done and is graphically presented in Figures 4.11 to 4.16.

The computer library program PLOT3D was employed for plotting the objective function (Equation 4.28). However, the PLOT3D can handle only 20 constraints at a time. Therefore the program was split into two programs. The first program contained constraints 1 to 17 plus constraints number 26, 27 and 30. This gives a total of 20 critical constraints on the design of the slab and the beams. The second program contained constraints 18 to 36 with the exclusion of constraints 26, 27 and 30. This gives a total of 16 constraints relating mainly to the girder.

Since sixteen independent variables are involved, and the objective function may be plotted against any two variables at one time, the remaining variables had to be kept constant. Obviously the constant values of the remaining variables were taken to be those at the optimum solution as specified in Table 4.1, column 6.

The cost function is generally a sloped plane when two variables are plotted at the same time (refer to Figures 4.11 to 4.16). The cost function is multi-planar for this problem which deals with a multi-variable function. These multi-planes are inter-dependent. The minimum cost of the panel is obtained from the set of variables of lowest value which satisfy the constraints in the presented problem (refer to Figures 4.11 to 4.16).

A single variable may be constrained to a specified constant value, for instance the minimum stirrup size xg should not be less than 2 (refer to

Figure 4.14) which is the smallest bar size available. Other constraints may be functions of two or more variables. Furthermore, an inequality constraint may exist where both sides of inequality constraint are functions of one or more variables. An example is the moment constraint where both dead load and ultimate moment capacity of a section depend on the variable dimensions of a section. The latter type of constraint creates the boundaries within which the solution is found (Fig. 4.12). It was recognized that the solution is sensitive to starting values. This observation led next to the examination of starting values and upper and lower boundaries of each variable since the step size is determined as a fraction of the specified range between the upper boundary and the lower boundary. On the basis of trial-error using different combinations of starting values, it has been found that the starting values proportioned from the geometry of the panel as defined in Appendix A for automatic set up of starting values gives satisfactory results. For nearly square shaped panels the optimization works out to a true minimum. The proof of this is that any other combination of values of variables than those claimed to be a true minimum would give higher cost. For very unusual shapes the author advises checking the first solution obtained against others obtained using different sets of starting values. However, it has been found that even for example a 5 ft by 30 ft panel, the solution would be within 4% of the true minimum. Realizing that for practical purposes one would have to round-off all variables one by one, and adjust or optimize the remaining unconstrained variables, the solution obtained may differ by as much as 8% depending on the geometry. Hence not arriving at the true minimum on the trial is not critical. The true minimum is a more academic consideration.

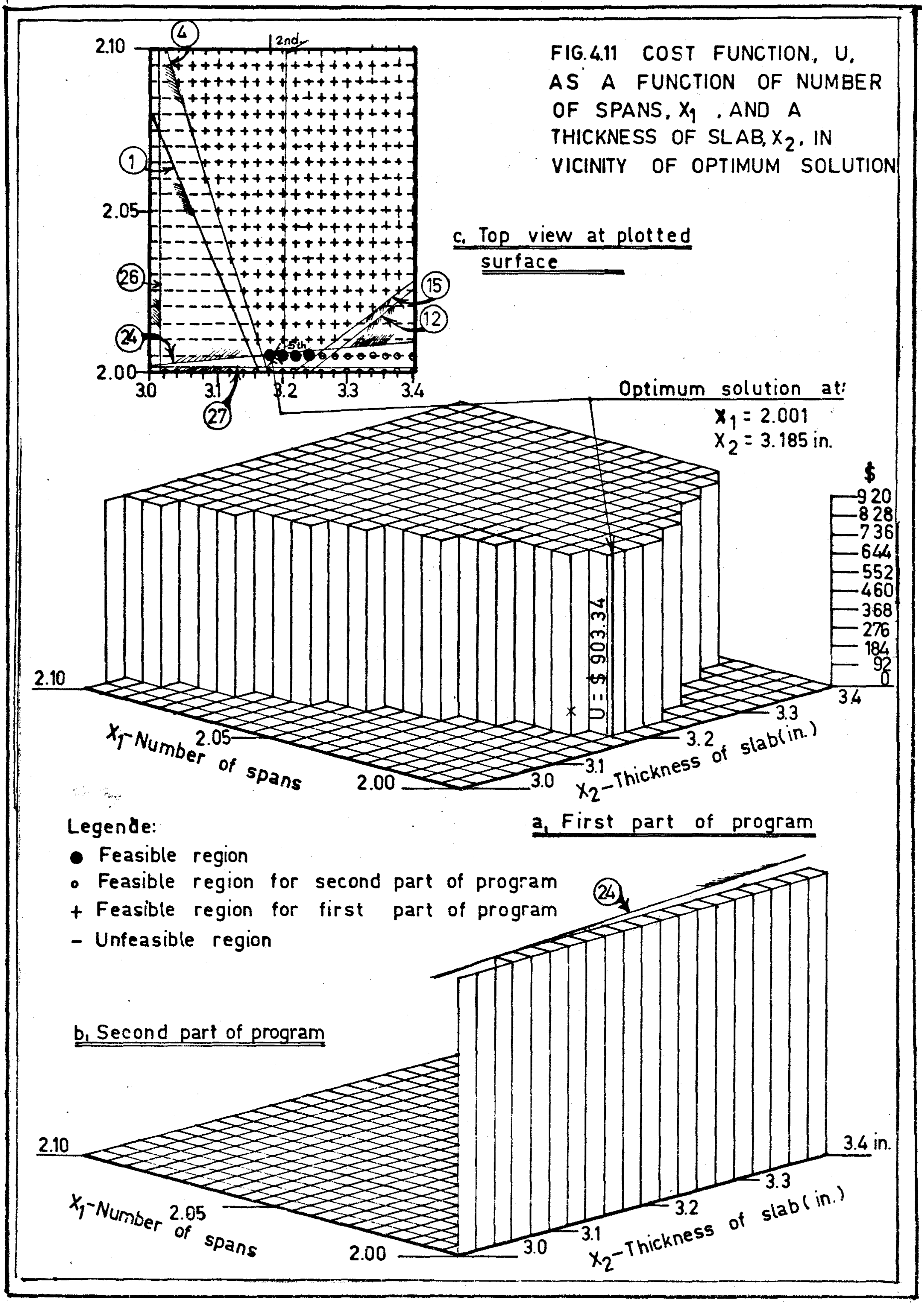
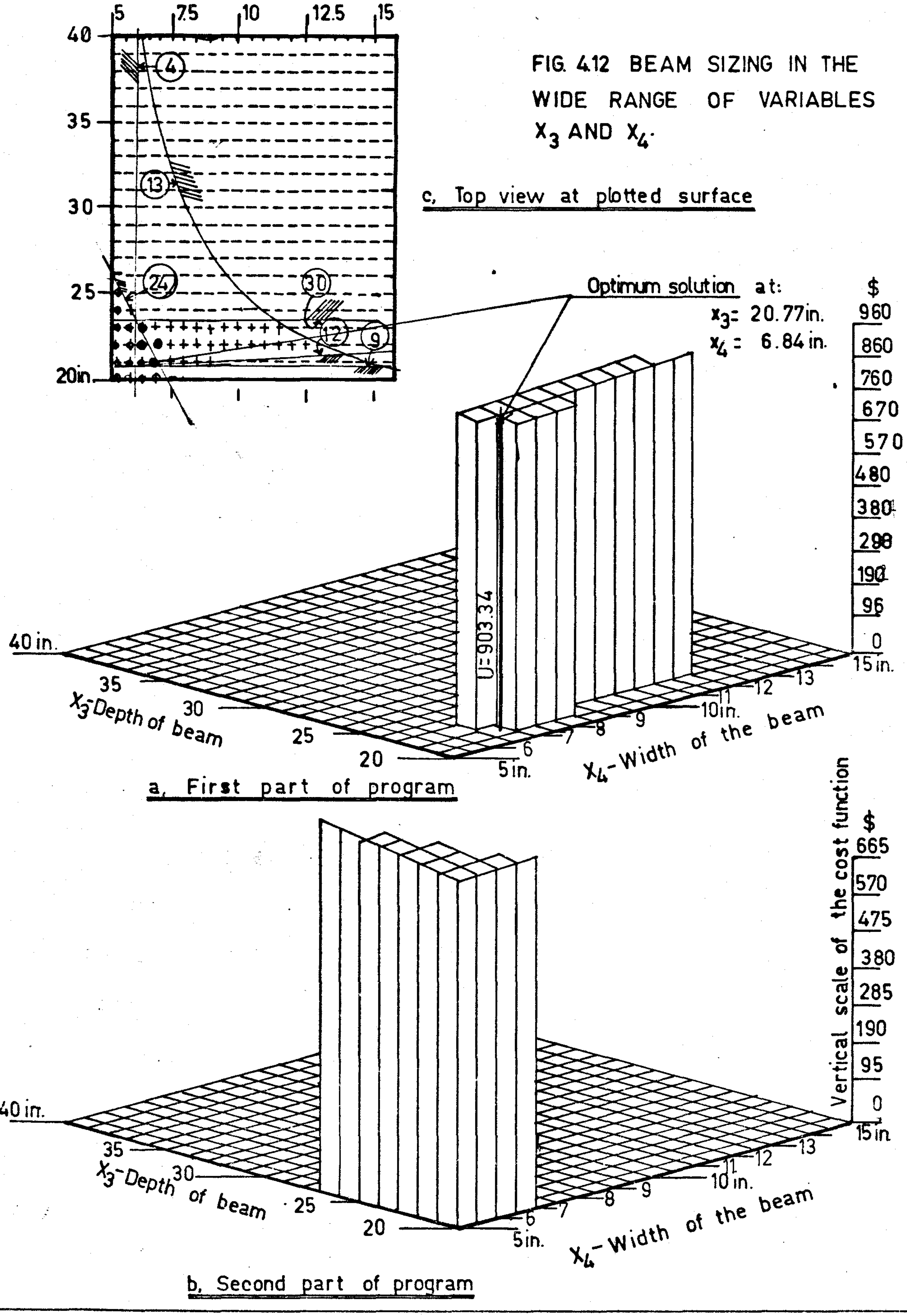


FIG. 4.11 COST FUNCTION, U, AS A FUNCTION OF NUMBER OF SPANS, X_1, AND A THICKNESS OF SLAB, X_2, IN VICINITY OF OPTIMUM SOLUTION

FIG. 4.12 BEAM SIZING IN THE WIDE RANGE OF VARIABLES X_3 AND X_4.



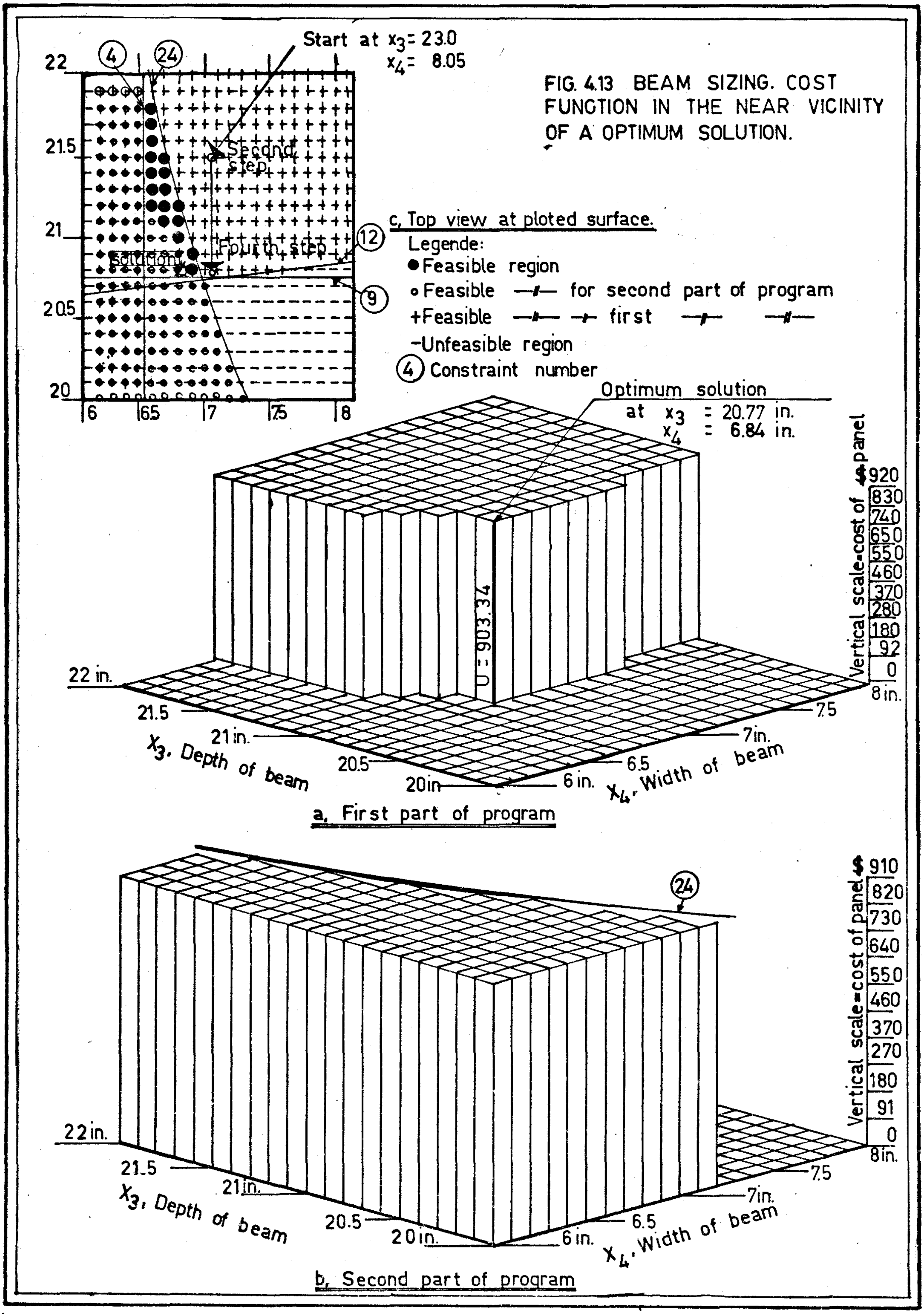


FIG. 4.13 BEAM SIZING. COST FUNCTION IN THE NEAR VICINITY OF A OPTIMUM SOLUTION.

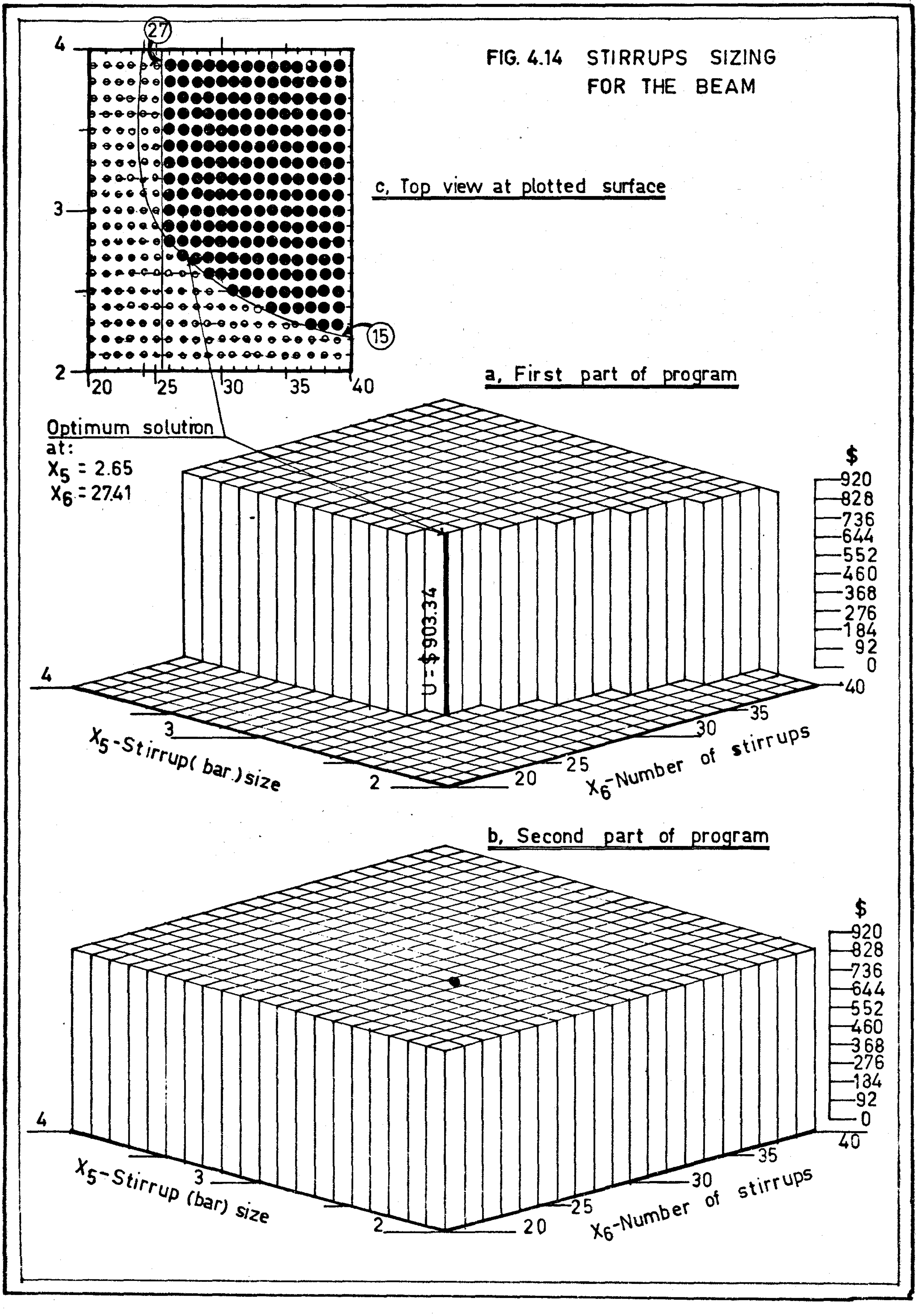


FIG. 4.14 STIRRUPS SIZING FOR THE BEAM

FIG. 4.15 COST AS A FUNCTION OF A THICKNESS OF SLAB, X_2, AND AN AREA OF POSITIVE STEEL IN SLAB, X_{11}

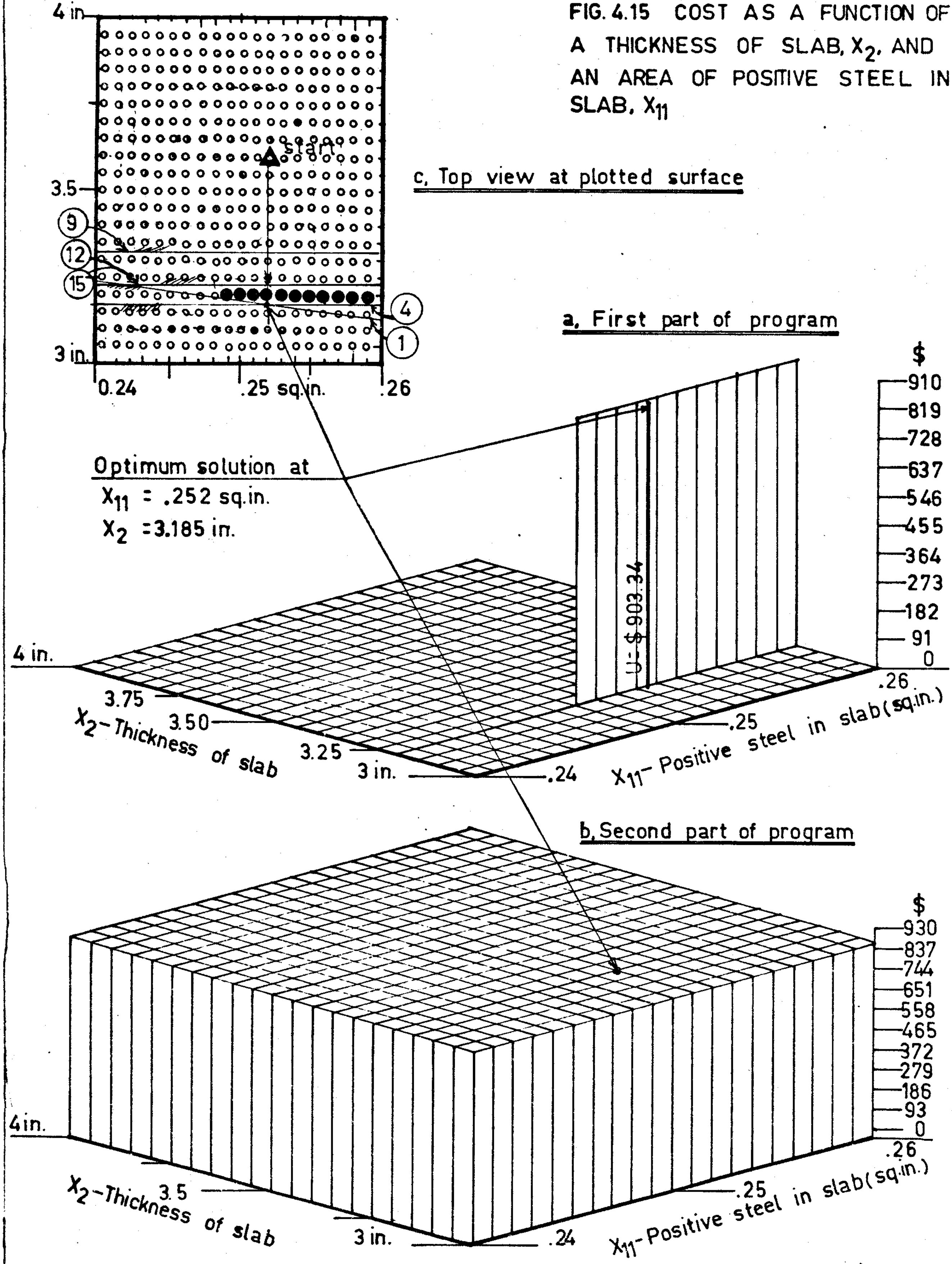
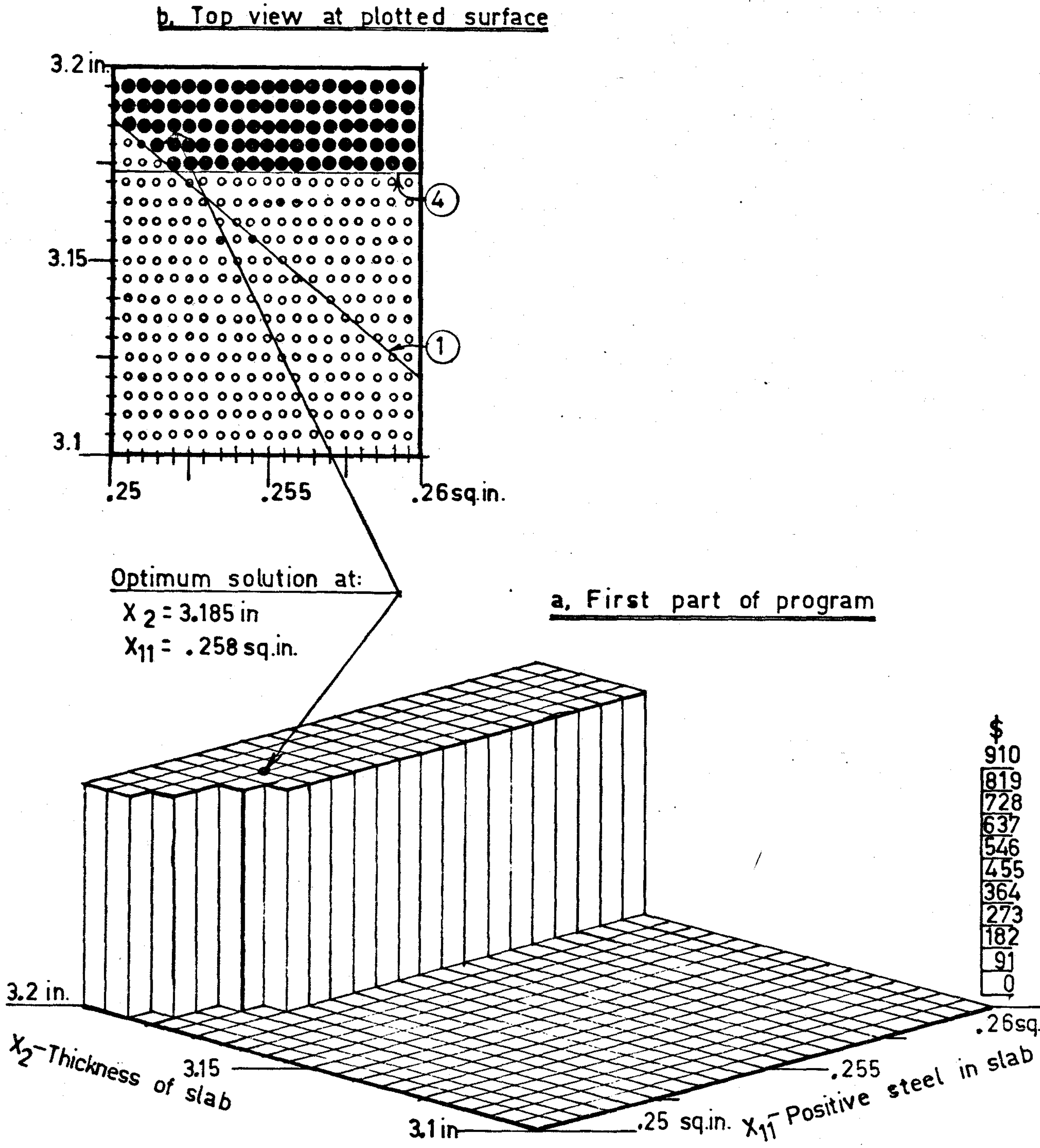


FIG. 4.16 COST AS A FUNCTION OF X_2 AND X_{11} IN NEAR VICINITY OF OPTIMUM SOLUTION.



Note: For second part of program see Figure 4.15 b.

Figure 4.11 shows the cost function in the vicinity of the optimum solution. In this case the cost function is the function of two variables x_1 (number of spans) and x_2 (thickness of the slab). When the regions of both parts are joined together (because in reality there is one program which had to be split into two) it is seen that only a narrow strip remains feasible for the search. However the number of spans should be constrained to an integer number, say a two, which is the minimum permissible in this program. Then the minimum U would be found at thickness $x_1 = 3.1806$.

Figure 4.12 shows the beam sizing. The wide variation of depth and width was taken into account in order to examine the variation of the cost as a function of these two variables. However the remaining variables were fixed at the position shown at "C", which is the top view of the plotted surface. Because of fixing the variables, only the small portion of cost function which satisfies the constraints is plotted. Therefore, the variation is not remarkable.

A closer view in the vicinity of the optimum cost function as a function of the depth of the beam x_3 and the width of the beam, x_4, is shown on Figure 4.13.

Figure 4.14 shows the cost function as a function of stirrup diameter, x_5, and number of stirrups, x_6, in a beam. It is seen that the solution lies on constraint number 15 at the lowest value of the cost function, so that any "artificial" constraining would inevitably induce a higher cost than that minimum.

In Figures 4.15 and 4.16 the cost function is shown as a function of the thickness of the slab, x_2, and the area of positive steel in the slab, x_{11}. In the first part of the program, Figure 4.15(a) shows the cost function has been bound by constraints so that for the variables considered only a single line represents the feasible region. The second part of the program (Figure 4.15(b)) does not control the feasible region. Figure 4.16 is drawn in such a way that the interval of the variables has been shortened in the vicinity of the optimum solution. This indicates the position of the feasible region and the position of the minimum cost. From this figure it is obvious that the true optimum has been found.

4.7 Appearance

The purpose of this section is to investigate the possible benefits of employing this optimization program in design. Here it must be emphasized that the multiplier effect produced by many floors in high-rise buildings is especially important and therefore seemingly small unit savings can be magnified into significant total savings.

A comparison is made between the optimum design aided by the computer and an example design of one-way solid slab, beam, girder floor taken from Reference 13. It should be noted that Reference 13 was published in 1954 when the Working Stress Design method was used and that the present method used here is Ultimate Strength Design; therefore no direct cost comparison can be made. To make it compatible the dimensions were taken as per example and the steel was adjusted to satisfy Ultimate Strength Design so in this way the example was thought to be representative of a non-computerized design. The saving shown in Table 4.4 of $69.94 which makes 7.2% cannot be due to optimization only. For practical reasons the number of spans, x_1, the bar number for beam stirrups, x_5, and the bar number for girder stirrups, x_9, should be integers. These three variables were constrained to different values and the results were tabulated in Table 4.5 for general interest.

Variables 1		Representation 2	Opt. Sol. 3	Example 4	Modified Example 5
Design Variables	x1	Number of spans	2.001	2	2
	x2	Thickness of slab	3.18	4	4
	x3	Depth of beam	20.77	15	15
	x4	Width of beam	6.84	15	15
	x5	Diameter of stirrups for beam	2.65	2	2
	x6	Number of stirrups for beam	27.41	14	31.2
	x7	Depth of girder	23.46	19	19
	x8	Width of girder	14.84	15	15
	x9	Diameter of stirrups for girder	3.08	2	2
	x10	Number of stirrups for girder	28.80	14	35.3
	x11	Area of positive steel slab	0.252	0.155	1.131
	x12	Area of negative steel slab	0.453	0.200	0.182
	x13	Area of positive steel beam	1.146	2.26	0.206
	x14	Area of negative steel beam	1.740	0.88	0.123
	x15	Area of positive steel girder	3.040	4.12	3.83
	x16	Area of negative steel girder	1.526	1.58	1.56
	U	Cost of the panel $	903.34	973.28	951.89

PARAMETERS:

FC = 3,000 psi	BEMOD = 20 ft	ZL = 100 psf	CA = 14 in	COV = 1.25 in	COVBB = 2 in
FY = 40,000 psi	GIMOD = 20 ft	HS = 10 ft	CB = 14 in	COVTB = 3 in	COVBP = 3 in
					COVTP = 3 in

Table 4.4. Optimum Solution Versus Example's Solution.

Variables Column No. 1		Representation 2	RMAX 3	RMIN 4	XSTRT 5	1st RUN 6	X(1)=2 7	X(1)=3 8	X(9)=4 X(5)=2 X(1)=2 9
Design Variables	x1	Number of spans	5	1	2.1	2.001	2	3	2
	x2	Thickness of slab	7.017	3.03	4.	3.18	3.1806	3.03	3.18
	x3	Depth of beam	30.	15.	23.	20.77	20.77	17.43	23.00
	x4	Width of beam	20.	10.	9.07	6.84	6.84	6.66	6.78
	x5	Diameter of stirrups for beam	8.	2.	2.06	2.65	2.65	2.06	2
	x6	Number of stirrups for beam	50.	10.	31.37	27.41	27.41	31.37	39.29
	x7	Depth of girder	30.	15.	23.65	23.46	23.46	24.94	23.18
	x8	Width of girder	20.	10.	15.46	14.84	14.84	13.97	15.09
	x9	Diameter of stirrups for girder	8.	2.	3.08	3.08	3.08	3.08	4
	x10	Number of stirrups for girder	50.	10.	28.8	28.8	28.8	28.8	23.85
	x11	Area of positive steel slab	1.44	0.072	0.252	0.252	0.253	0.125	0.252
	x12	Area of negative steel slab	1.44	0.072	0.453	0.453	0.453	0.182	0.453
	x13	Area of positive steel beam	3.339	0.83	1.022	1.146	1.147	0.976	1.022
	x14	Area of negative steel beam	3.34	0.41	1.523	1.74	1.74	1.505	1.52
	x15	Area of positive steel girder	4.02	0.804	4.278	3.04	3.044	3.08	3.06
	x16	Area of negative steel girder	4.059	0.804	1.587	1.526	1.526	1.546	1.52
	U	Cost of the panel $				903.34	903.42	923.33	920.05

PARAMETERS:

FC = 3,000 psi BEMOD = 20 ft ZL = 100 psf CA = 14 in COV = 1.25 in COVBB = 2 in

FY = 40,000 psi GIMOD = 20 ft HS = 12 ft CB = 14 in COVTB = 3 in COVBP = 3 in

COVTP = 3 in

Table 4.5. Solutions with Various Variables Constrained.

The cost of finding a solution must realistically be less than the costs incurred in engineering offices for the same job without a computer. When using a computer, time is proportional to cost. Therefore, this study seeks to arrive at a satisfactory compromise between the accuracy of the solution and the time required to obtain the results.

The program runs for 20 seconds in octal, which is 12 seconds or 1/300 of an hour. At McMaster Computer Centre one hour of computation costs $500. Hence one run costs $1.70. The additional cost for a detailer is estimated as follows: 1) the wage is $12,000 per year, which yields $33/day; 2) the time consumed for detailing (estimated) is 4 hours, which makes additional cost of $16.50 plus 2 hours for punching of $8.25. Therefore:

Total cost = 1.70 + 24.75 = $26.45

Total time from start to finish is reasonably estimated to be 6 hours.

The time taken for an experienced designer for the design of such a project as shown in Figure 2.1 is estimated to be one day. This costs $33.00 and requires a total time of 8 hours. Therefore the saving on the cost is approximately (6.55/33) x 100 = 19.8%, and the saving on time is about 25%.

The above values are, of course, very flexible due to the estimation. They are presented as a guide only. However, it should be noted that by using the computer for the design the errors in calculation should be almost eliminated. The remarkable advantage of optimization is that it automatically tries several dozens of solutions before arriving at the final solution.

4.8. Conclusion of Chapter IV

The question could arise as to whether the one-way slab, beam and girder floor is feasible or even applicable for office buildings. This type of floor could find application for underground garages, lobbies and mechanical floors where heavy concentrated loads can be expected and where spans of about 15 feet are acceptable.

An important feature of this program is its easy adaption to form a part of a master program which could be developed to automatically search for the most economical type of flooring. This master program would contain similar sub-programs for the different parts contained in the chart of the coordination model shown in Figure 2.14.

The future development of the program developed here may be divided into two directions. Firstly, to facilitate easy use, the organization of the input-output data could be tailored to the needs and practices of specified users such as architects and practicing engineers. Secondly, a minor modification in the input-output formats must be made if the program is to be used in the above suggested master program package.

In Section 4.6, Sensitivity, it has been recognized that the problem presented is to find a set of variables of lowest possible values which would satisfy the constraints rather than search for the minimum value of the objective function in the feasible region. Therefore it may be expected in the future that the whole problem will be somewhat redefined for the new search which is not known at the present.

CHAPTER V. FLAT PLATE

5.1 Introduction

The flat plate is one of today's most commonly used floors in office and apartment buildings. Flat plate floors offer many advantages due to not having beams project below the slab:

(1) Since beams are eliminated and reinforcing is of an extremely simple pattern, the speed of construction is increased. Work is even more simplified using welded wire mesh which also assures more exact placing of reinforcement, excellent bond to concrete, and good details around openings in the slab.

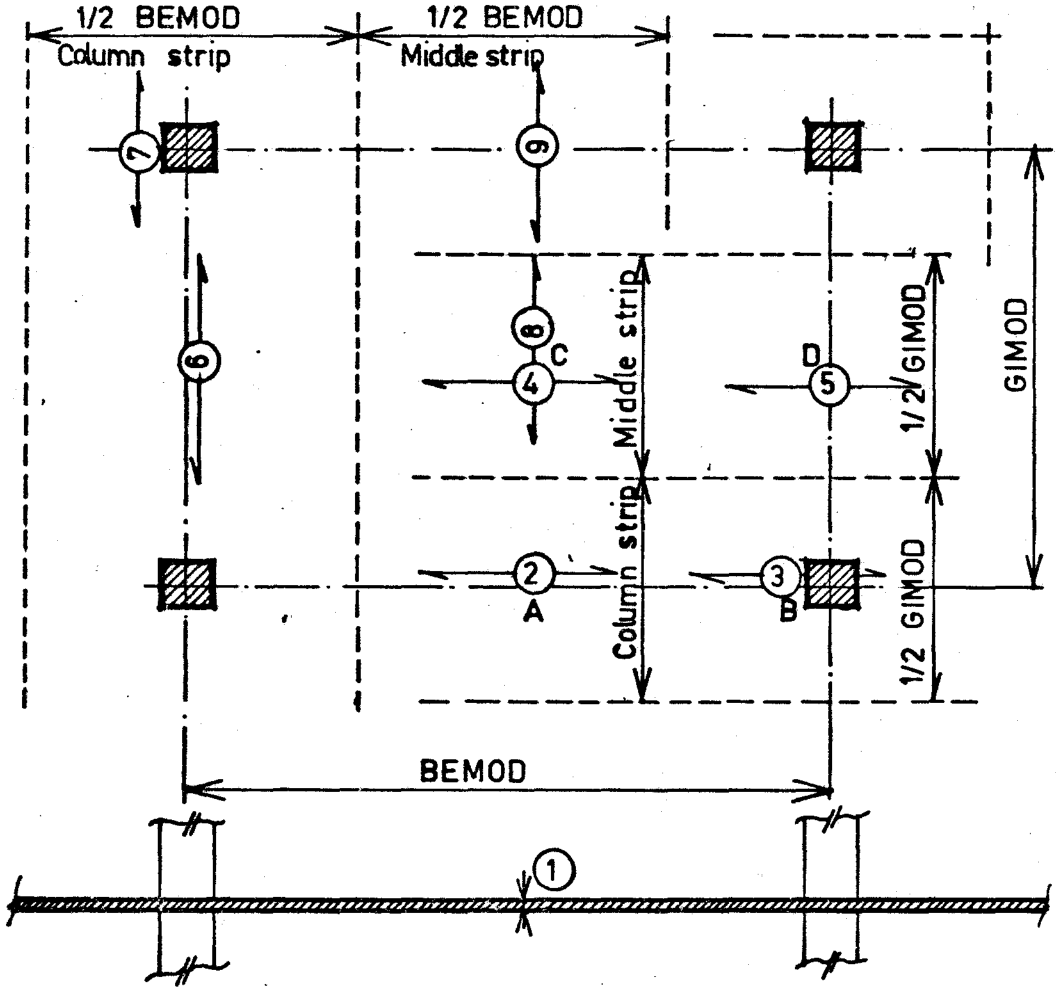
(2) A flat unbroken ceiling without any furring or additional hung ceiling produces additional savings.

A reduction in floor-to-floor distance lowers the overall building height. As a result this cuts down wind loads bringing about savings in the cost of the primary structure and the foundations. Also there are corresponding savings in mechanical and architectural details.

The designer satisfies the structural requirements for the plate thickness and the amount of steel at four critical points in both directions. The eight critical sections are indicated in Figure 5.1. Several alternatives must be tried in order to obtain a financially more effective plate than the initial design.

The aims of optimized design are:

(1) The systematic search for the design which will satisfy the structural criteria as well as achieving the financially most effective flat plate structure.



D e s i g n   v a r i a b l e s:

X1 = Overall thickness of the flat plate

X2 = Area of positive steel for column strip at A
X3 = —//— negative —//— —//— at B
X4 = —//— positive —//— for middle strip at C
X5 = —//— negative —//— —//— at D
(X2–X5: in the direction of BEMOD)

X6 = —//— positive —//— for column strip at A
X7 = —//— negative —//— —//— at B
X8 = —//— positive —//— for middle strip at C
X9 = —//— negative —//— —//— at D
(X6–X9: in the direction of GIMOD)

Note: The symbol BEMOD is assigned to the longer distance between columns.

FIG. 5.1 TYPICAL FLAT PLATE PANEL

(2) To save the designer's time spent doing repetitive calculation of several alternatives.

Therefore, the optimized design of flat plate floors is discussed in this Chapter.

This chapter consists of the following sections. Section 5.2 shows the computation model of the flat plate. Section 5.3 deals with the cost determination as the objective function. The inequality constraints are discussed in Section 5.4 and the solution for the typical panel is shown in Section 5.5, followed by the conclusion of this chapter in Section 5.6.

5.2 Computation Model

The flat plate floor is assumed to consist of typical plates as shown in Figure 5.1. A typical flat plate floor is assumed here to be of uniform thickness including the corner and end bays. As a result the designer has only to specify different steel reinforcement for the plates which differ from the typical plate.

The thickness of the plate and the reinforcement steel areas at the critical points (refer to Figure 5.1) are assumed as variables in the search for a feasible minimum of the objective function where the cost is the function of these variables.

5.3 Objective Function

The cost of the flat plate as based on the unit prices of materials defined in Section 2.5 is the object of minimization. A typical flat plate panel is shown on Figure 5.1. Also the independent variables $(x_1, x_2, \ldots, x_9)$ of the design vector X are specified on Figure 5.1.

The cost of the typical flat plate panel is formulated as the sum of the cost of concrete (CON), the cost of reinforcing steel (STEEL), and the cost of formwork (FORM).

$$U = CON + STEEL + FORM \tag{5.1}$$

The cost of concrete, CON, is expressed as follows:

$$CON = BEMOD*GIMOD*x_1*UPC \tag{5.2}$$

where BEMOD and GIMOD are panel dimensions as defined in Figure 5.1, and

UPC = unit price of concrete as defined in Section 2.5.

The cost of reinforcing steel, STEEL, is calculated as:

$$STEEL^* = 0.5*W*UPS[BEMOD(x_2 + x_3 + x_4 + x_5) + GIMOD(x_6 + x_7 + x_8 + x_9)] \tag{5.3}$$

where W = specific weight of the steel (490 lbs/ft^3), and

UPS = unit price of reinforcing steel as defined in Section 2.5.

The cost of formwork, FORM, is given by:

$$FORM = UPFS[(BEMOD*GIMOD) - (CA*CB)] \tag{5.4}$$

where UPFS = unit price of formwork as specified in Section 2.5;

CA = width of the column taken in direction BEMOD; and

CB = depth of the column taken in direction GIMOD.

5.4 Design Constraints

In general, the design of a flat plate is subjected to four basic constraints. They are:

(1) Moment constraint. The ultimate moment capacity of the section must not be less than the moment caused by external forces.

---

*The assumption that all the bars extend over the full span is slightly conservative and some steel could be saved by wing cut-offs.

(2) Shear constraint. The shear capacity of the section where the shear stress caused by external forces is maximum must be greater than the actual shear stress.

(3) Deformation constraint. Deformations of the structure must be within the allowable limits.

(4) Other constraints. There are other practical limitations on the vector of design variables.

5.4.1 Moment Constraint

A moment constraint has a general form:

$$ULM - BM > 0 \tag{5.5}$$

where BM = moment at the particular critical section due to external loading, and

ULM = ultimate moment capacity of the investigated section, where the section of flat plate is assumed to be reinforced with tension steel only.

Steel in the compression zone of flat plates is generally very close to the neutral axis and because of this as well as the negligible influence of compression steel in underreinforced sections, it is not taken into account.

The design is done by elastic analysis. The moments, BM, are found by means of the moment coefficients[17] as per Figure 4.4. For this purpose the structure is divided into number of bents "B" in the long direction and "G" in the short direction as defined in Figure 5.2. The critical moments are calculated across the full width of bents "B" and "G" assuming the entire width of the bent as a beam. In the office building the ratio of live load and dead load is such that for design purposes the total moment for maximum

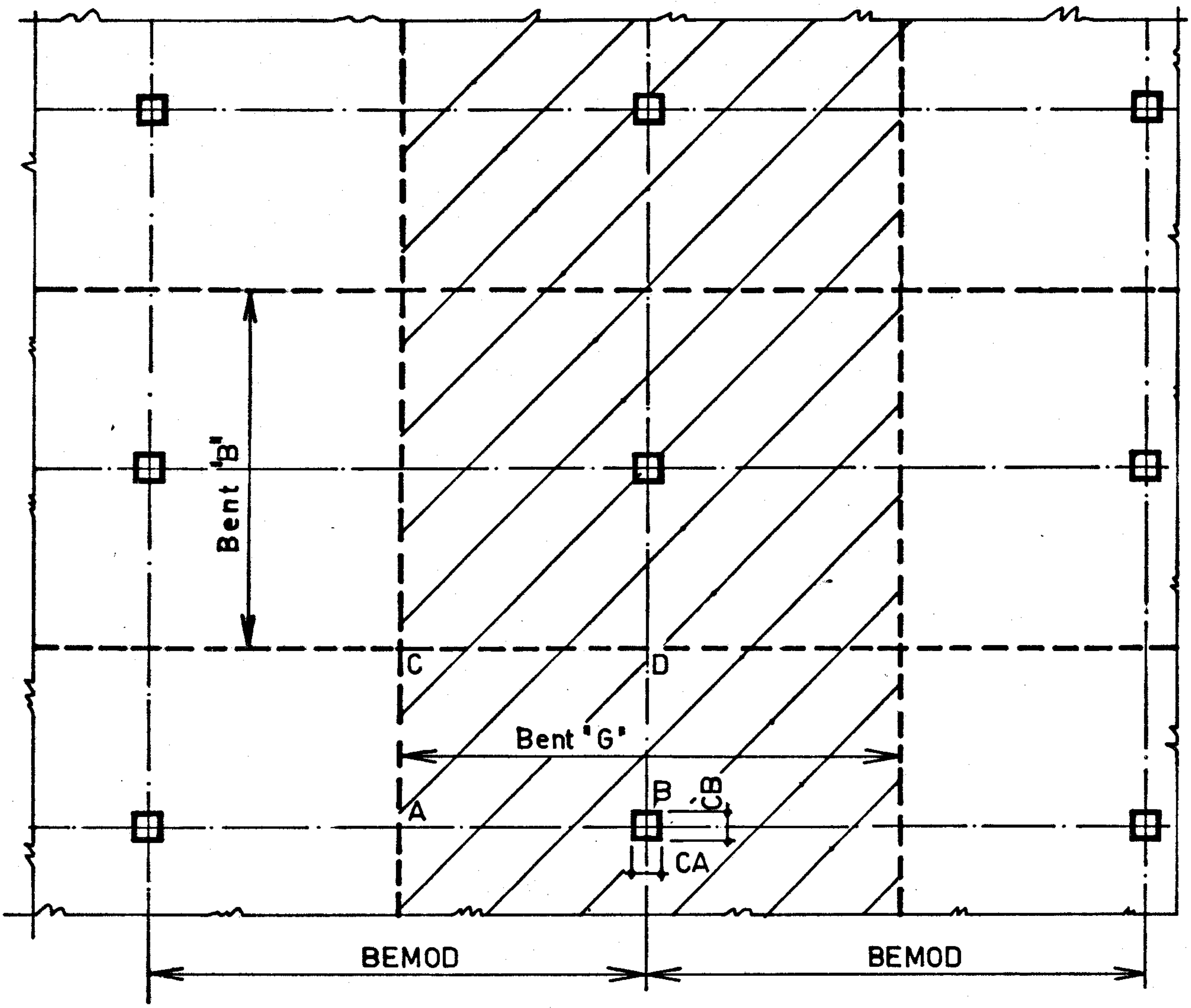


FIG. 5.2 Bents in the long and short directions as defined in National Building Code of Canada(17)

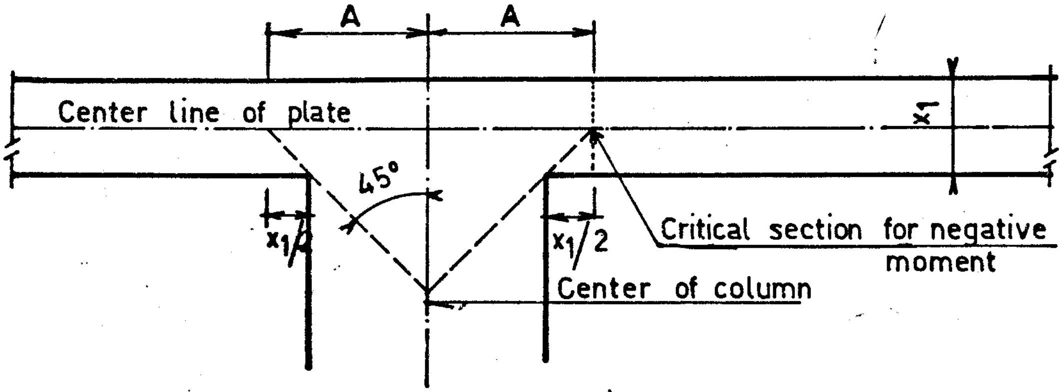


FIG. 5.3 Critical section for negative bending

bending is assumed to occur under full live load[17]. The negative moment in both column and middle strip is levelled-off at the distance A as defined in Figure 5.3 and this critical negative moment is considered constant over the support.

Next, the numerical sum of positive and negative bending moments in the direction of either side of the panel is checked against the moment calculated by moment formula (106) in the National Building Code[17]. However the value of the total moment was not reduced to agree with formula 106 because the generality of the program precluded satisfying the requirements of Section 4.5.7.5.[17] which would permit this. The moments calculated for the bents are then distributed to the column and the middle strips according to Table 4.5.7.B. of the National Building Code[17]. In such a way the obtained moments are substituted for BM in the Inequality (5.5).

The ultimate moment capacity of the investigated section, ULM, is calculated in subroutine SIMPLE, and also substituted for ULM in Inequality (5.5).

5.4.2 Shear Constraint

The situation in shear around a typical interior column in a flat plate is analogous to that in a single spread footing. The idea is that the area enclosed between the two parallel pairs of centerlines of panels is equivalent to the area of the footing. The NBC of Canada [Reference 17, Article 4.5.7.3.(3)(b)] has chosen to use Moe's[31] recent work for cases of two-way plate action as shown in Figure 5.5.

For a flat plate in which bending action is primarily in one direction (GIMOD/BEMOD $\geq$ 0.5), the procedure used for beams should be applied. The critical section for shear-compression failure and loaded area for the

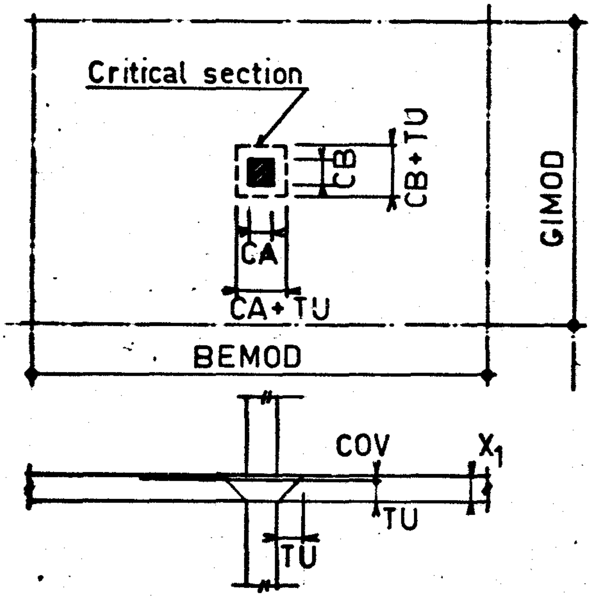
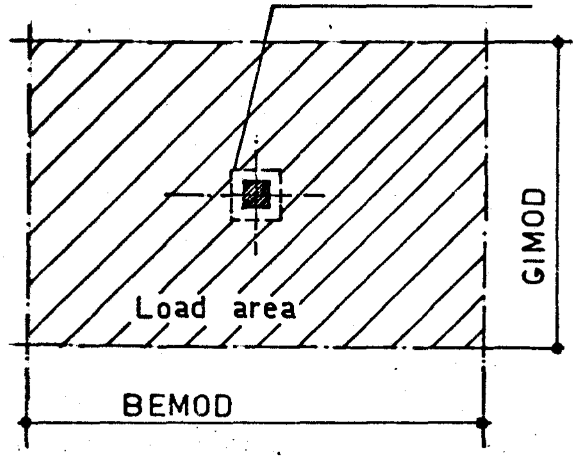
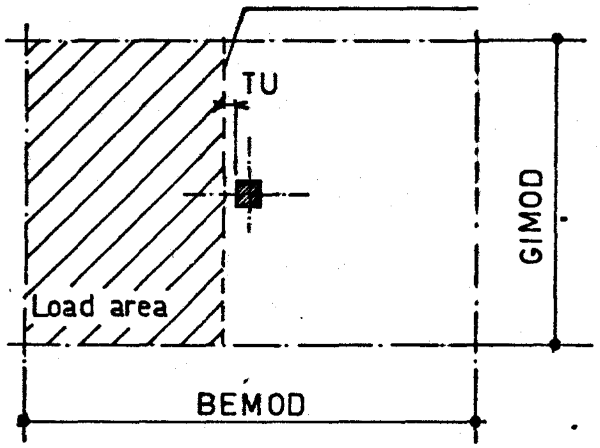


FIG. 5.5 CRITICAL SECTION FOR DIAGONAL TENSION FAILURE

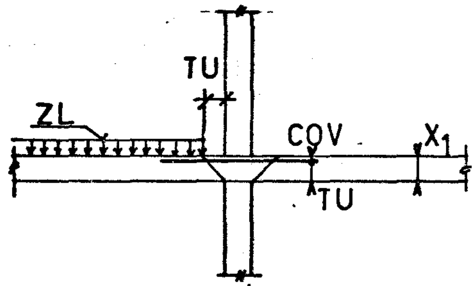


a, Two-way action

$$\frac{GIMOD}{BEMOD} \geqslant 0.5$$



$$\frac{GIMOD}{BEMOD} < 0.5$$



b, One-way action

FIG. 5.6 CRITICAL SECTIONS AND LOADED AREAS FOR FLATE PLATE investigated sections is shown on Figure 5.6.

For <u>two-way diagonal tension action</u>, the shear force (Figures 5.5 and 5.6a) is

$$V = (ZL + ZD)\{BEMOD*GIMOD - [4TU^2 + 2TU(CA + CB) + CA*CB]\} \qquad (5.6)$$

where ZL = live load (lb/ft^2), and

ZD = dead load = 12.5 x_1 (lb/ft^2), assuming weight of normal concrete equal to 150 lb/ft^3,

and the shear stress is

$$v = \frac{V}{TU*2(CA + CB + 2TU)} \qquad (5.7)$$

Assuming a section without shear reinforcement the allowable shear stress, as defined in Reference 11, is

$$\text{Allowable } v = 4\phi\sqrt{FC} \qquad (5.8)$$

$$\phi = 0.85$$

For one-way diagonal tension action, the shear force according to Figure 5.5b is

$$V = (ZL + ZD)[0.5(BEMOD - CA) - TU] \qquad (5.9)$$

and the shear stress is

$$v = \frac{V}{TU*GIMOD} \qquad (5.10)$$

The allowable shear stress is defined in Reference 17, Article 4.5.4B.13(5) as

$$\text{Allowable } v = 2\phi\sqrt{FC} \qquad (5.11)$$

Thus, shear constraint takes the following form:

$$\text{Allowable } v - v > 0 \qquad (5.12)$$

where Allowable v and v are taken from Equations (5.8) and (5.7) respectively for GIMOD/BEMOD $\geqslant$ 0.5. Where GIMOD/BEMOD 0.5 Equations (5.11 and (5.10) respectively are similarly used.

5.4.3 Deformation Constraint

The stiffness of a flat plate depends mainly on its thickness. Therefore the National Building Code[17] controls the deflection through the slab thickness as per Table 4.5.7.A of Reference 17. The designer may choose a plate of less thickness but then an analysis for deflection is required. Computed deflections increased by 1/3 should satisfy the deflection limit as per Section 4.5.4.9. of the National Building Code[17]. The test of reinforced concrete flat slabs done by Hatcher and Sozen[34] show that the deflection of the slab is a critical factor. Hatcher's test also shows that the edge panel is most critically deflected.

The flat plate deformation behaviour was approximated as a diagonal beam, as a plate using the theory of Bares[28] and as a grid. A realistic but still conservative value of centre panel deflection was obtained assuming the diagonal beam action.

In calculation of deflection which occurs immediately upon application of service (working) load the modulus of elasticity is specified as follows:

$$E_c = w_c^{1.5}\ 33\ \sqrt{FC} \qquad (5.13)$$

Taking w_c = 145 pcf for normal weight concrete, the Equation (5.13) becomes

$$E_c = 57{,}500\ \sqrt{FC} \qquad (5.14)$$

The moment of inertia, I, is based on the gross cross section.

For the deflection analysis the conjugate-beam method[11] is used where the midspan deflection, y_m, is expressed as

$$y_m = \frac{5}{48}\frac{L^2}{E_c I}[M_s - 0.1(M_a + M_b)] \qquad (5.15)$$

where M_s = positive moment at the middle of the span,

M_a and M_b = negative moments at the lefthand support a, and the righthand support b, respectively,

E_c = modulus of elasticity,

I = moment of inertia, and

L = diagonal clear-span defined as follows:

$$L = \left[(BEMOD^2) + (GIMOD)^2\right] - \left[(CA)^2 + (CB)^2\right] \qquad (5.16)$$

The additional long-term deflections are obtained by multiplying the immediate deflection caused by the sustained part of the load (assumed 60% of live load) by a creep factor of 2. The allowable limit for the sum of the immediate deflection due to live load, ZL, and the additional deflection due to shrinkage and creep under all sustained loads is defined in Reference 17 as not to exceed L/360.

5.4.4 Other Constraints

Several other provisions of the Building Code(17) must be satisfied:

(1) The area of reinforcement at any section must be greater than the minimum required for shrinkage and temperature(17) given by

$$A_s \geqslant 0.002\, A_c \qquad (5.17)$$

where A_c = gross concrete area.

(2) The minimum slab thickness must be greater than the values shown below:

fy (psi)	without drop panels
40,000	L/36 or 5 in.
50,000	L/33 or 5 in.
60,000	L/30 or 5 in.

(3) Another type of constraint which is not included in the program because bar sizes are not chosen, refers to the arrangement of slab reinforcement where:

(a) the spacing of the bars at any section shall not exceed two times the slab thickness x_1 nor 18 inches; and

(b) at least 25% of required negative reinforcement in column strip shall cross the periphery located at a distance x_1 from the column.

5.5 Solution

The computation model as shown in Figure 5.1 was designed with the aid of a direct optimization technique using the following input parameters:

FC = 3,000 psi	and	IPS = 1	
FY = 40,000 psi		JAK = 1	
UPC = $31.80/yd^3		MB = 1	
UPS = $0.1605/lb		MS = 1	
UPFS = $0.75/ft^2		IRUN = 1	
BEMOD = 20 ft		CA = 16 inches	
GIMOD = 20 ft		CB = 16 inches	
ZL = 50 psf		HS = 10 ft	
COV = 2 inches			

Since no column analysis is performed here, the above column size was chosen to safely support the axial bonding of 40 flat plates.

IRUN was specified to be 1, therefore the values of RMAX(I), RMIN(I), and XSTRT(I) were set automatically by the program and they are as listed in Table 5.1.

RMAX(I) - estimated upper bound on range of variables x(I) where

I = number of variables = 1,...,9 (refer to Figure 5.1);

RMIN(I) - estimated lower bound on range of variables x(I); and

XSTRT(I) - starting values of x(I).

In this example the solution has been found after fifteen successful iterations. Values of all the variables and the cost of the flat plate panel for the solution are listed in Table 5.1. The most critical constraint in this example is the shear around the column (the punching effect). As a practical dimension the thickness of the slab, x_1, was then constrained to 8.5 inches. Naturally, the cost of this flat plate panel is higher than the optimum solution. The results are listed in Table 5.1.

Effect of Changes in the Parameters

Two parameters are investigated, the loading and the panel size. These two parameters have the primary influence on the cost. Various live loads, ZL, were applied and the solutions are tabulated in Table 5.2. When live load, ZL, is plotted against cost function, U, it may be observed that the mutual dependence is not linear and the curve is similar to that in Figure 4.8. A much steeper curve is obtained when live load is plotted against the thickness of the slab. Therefore, there is no linear proportion between the thickness of slab and the cost. It should be noted that the column size was not increased with increased load.

The solutions for various sizes of panels are listed in Table 5.3. It is seen that for large size panel (say 30 ft by 20 ft) any increase in size rapidly increases the cost per square foot.

Symbol	Representation	RMAX(I)	RMIN	XSTRT	Solution Found	Constr. x1=8.5 in.
x1	Thickness of the slab	12.	5.	6.0	8.156	8.5
x2	Area of positive steel at A in BEMOD direction	0.72	0.144	0.42	0.265	0.248
x3	Area of negative steel at B in BEMOD direction	0.72	0.144	0.71	0.339	0.313
x4	Area of positive steel at C in BEMOD direction	0.72	0.144	0.338	0.196	0.205
x5	Area of negative steel at D in BEMOD direction	0.72	0.144	0.34	0.198	0.207
x6	Area of positive steel at A in GIMOD direction	0.72	0.144	0.42	0.199	0.208
x7	Area of negative steel at B in GIMOD direction	0.72	0.144	0.71	0.339	0.313
x8	Area of positive steel at C in GIMOD direction	0.72	0.144	0.338	0.196	0.205
x9	Area of negative steel at D in GIMOD direction	0.72	0.144	0.34	0.198	0.207
U	Cost of the flat plate panel				629.47	642.79

PARAMETERS: FC = 3,000 psi
FY = 40,000 psi
UPC = $31.8/cu yd
UPS = $ 0.1605/lb
UPFS = $0.75/sq.ft

BEMOD = 20 ft
GIMOD = 20 ft
ZL = 50 psf
COV = 2 in
CA = 16 in
CB = 16 in
HS = 10 ft

IRUN = 1, IRUN = 2

Table 5.1. Optimum Solution for Flat Plate Panel.

	ZL = 50 psf	ZL = 100 psf	ZL = 150 psf	ZL = 200 psf	ZL = 300 psf
x1	8.15	10.51	12.44	14.21	17.00
x2	0.265	0.292	0.314	0.340	0.411
x3	0.339	0.370	0.401	0.423	0.463
x4	0.196	0.254	0.302	0.342	0.408
x5	0.198	0.256	0.300	0.340	0.410
x6	0.199	0.252	0.300	0.340	0.411
x7	0.339	0.370	0.401	0.423	0.463
x8	0.196	0.254	0.302	0.342	0.408
x9	0.198	0.256	0.300	0.340	0.410
U	$ 629.47	724.07	801.70	869.15	984.69

Table 5.2. Various Live Load, ZL, on the Panel of Basic Parameters 20 x 20 ft.

Variable	BEMOD = 15 GIMOD = 15	20 ft 15 ft	20 ft 20 ft	25 ft 20 ft	30 ft 20 ft
x1	5.21	6.41	8.15	10.05	12.03
x2	0.222	0.232	0.265	0.287	0.309
x3	0.284	0.297	0.339	0.366	0.392
x4	0.127	0.154	0.196	0.245	0.289
x5	0.126	0.155	0.198	0.243	0.291
x6	0.126	0.155	0.199	0.242	0.290
x7	0.284	0.316	0.339	0.376	0.416
x8	0.127	0.156	0.196	0.241	0.290
x9	0.155	0.171	0.198	0.244	0.289
U $	288.40	420.21	629.47	880.91	1175.09
$	1.28/S.F.	1.40/S.F.	1.57/S.F.	1.76/S.F.	1.96/S.F.

Table 5.3. Various Size of the Panel (Live Load, ZL = 50 psf).

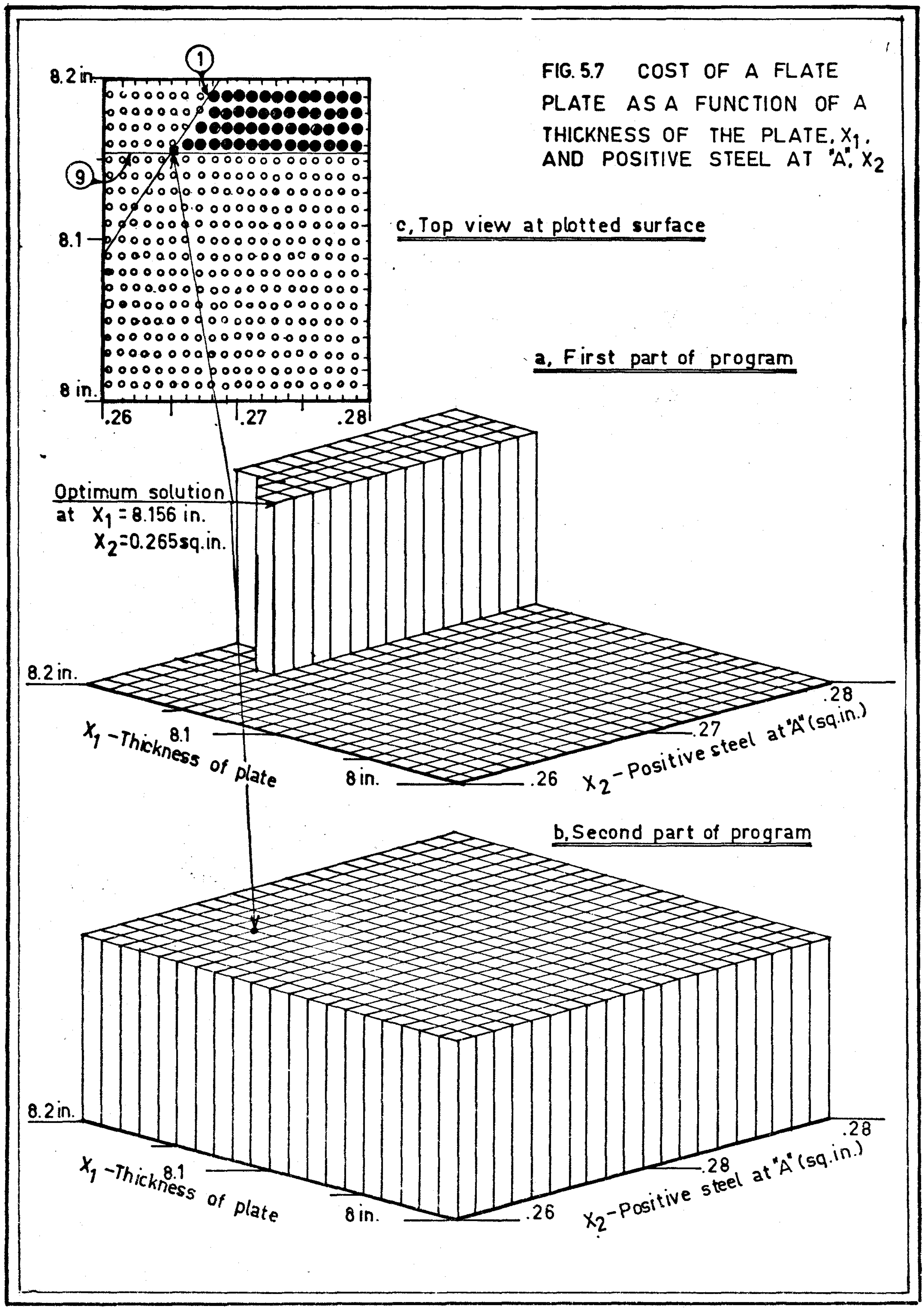


FIG. 5.7 COST OF A FLATE PLATE AS A FUNCTION OF A THICKNESS OF THE PLATE, X_1, AND POSITIVE STEEL AT "A", X_2

5.7 Sensitivity

The behaviour of an objective function (cost) in the vicinity of the optimum solution has been examined and is presented in Figure 5.7. Similarly as for the one-way slab, beam and girder floor the computer library program PLOT3D was used for plotting the objective function. As before the program had to be split into two programs because the flat plate optimum design program contains 29 constraints (which are detailed in Appendix B) and PLOT3D can handle only 20 constraints at a time. The first 13 constraints were included in the first program and the remaining 16 constraints were included in the second program.

In Figure 5.7 the objective function is plotted against variables x_1, the overall thickness of the flat plate, and x_2, the positive steel area in BEMOD direction at "A" (refer to Figure 5.1). The variables x_1 and x_2 were chosen for presentation because, firstly, the cost function changes most dramatically with variation of these two variables, and, secondly, that the most critical constraint, constraint 9 (the shear capacity of the plate against punching) depends on the x_1 only since shear reinforcement is not present. The remaining variables were assigned constant values as found for the optimum solution shown in Table 5.1.

The optimum solution illustrated in Figure 5.7 is found on the boundaries of the shear constraint and the ultimate moment capacity of a section at "A" in the BEMOD direction. The latter constraint is immediately followed by the constraint which requires the minimum reinforcement in the section. This means that the moment capacity around the optimum was achieved by providing only slightly more than minimum reinforcement. From the Figure 5.7 the conclusion was drawn that the shear constraint governs the plate thickness for the model with the parameters given on page 103. However some designers have suggested use of shear head reinforcement in order to increase the shear capacity of a plate against punching. To the author's knowledge this has not gained wide acceptance in North America and will probably require further study to fully document the effectiveness of shear head reinforcement. When the behaviour of a plate with shear head reinforcement is understood, then this may be introduced as a new variable x_{10} which would represent the area of shear head reinforcement. Obviously, it will intorudce an increase in number of constraints.

LIST OF CONSTRAINTS FOR FLAT PLATE

1.	Ultimate moment capacity of the section at:	A	in BEMOD direction
2.		B	
3.		C	
4.		D	
5.		A	in GIMOD direction
6.		B	
7.		C	
8.		D	

9. Shear capacity against punching.

10. Shear capacity in one-way action.

11. Immediate deflection due to live load.

12. Total deflection.

13. Minimum thickness of plate to be 5 inches.

14.	Minimum area of reinforcing steel required for shrinkage and temperature at:	A	in BEMOD direction
15.		B	
16.		C	
17.		D	
18.		A	in GIMOD direction
19.		B	
20.		C	
21.		D	

22.	Minimum one bar number 3 per one foot at:	A	in BEMOD direction
23.		B	
24.		C	
25.		D	
26.		A	in GIMOD direction
27.		B	
28.		C	
29.		D	

CHAPTER VI. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The purpose of this work is to provide help to the designer with a means for making correct decisions towards achieving the most economical solution. The design of a building creates a complex of a decision-making process, and, therefore, an evaluation system is needed for evaluation of the decisions. One of the possible evaluation systems is shown in Chapter II. The author tried to indicate the complexity of decision making even after cast in place reinforced concrete was chosen as the structural material for the sub-system of the load-bearing structure. The optimization of the whole sub-system is beyond the scope of this work, and therefore the sub-system has been broken down into assemblies, components and parts.

Computer programs were developed for the optimum design of one-way slab, beam and girder floors and for the optimal design of flat plates. According to the chart of coordination (Figure 2.4), both programs coupled with at least four more programs which could be developed in the future would make a master program package the purpose of which would be to select the most economical floor framing design.

The benefits of using such a master program package were illustrated by the presentation of two possible solutions. Designs were made for a one-way, beam and slab floor framing, and for a flat plate using the same parameters such as the distance between columns in both directions, strengths of used materials, etc., in both cases. For given unit prices as defined in Section 2.5, the flat plate design for a 20 ft by 20 ft panel with 50 psf live load is more economical than the corresponding one-way, beam and girder floor framing design (refer to Figure 4.8 and Table 5.2).

At the present time both programs work separately. However, they are valuable in this form as was discussed in Section 4.8. The minimum cost of a project before any detailed design has been made is obviously important for feasibility studies, tendering, etc.

The versatility of the programs presented herein gives the architect an opportunity to quickly determine sufficient dimensions. For the engineer the steel areas are calculated and his engineering judgement is applied for detailing. Also the estimator has ready access to the price of floor framing structure. For all of them time is saved which would otherwise have been used in making trial calculations without necessarily ever finding the least costly design.

6.2 Recommendations

The future development of programs presented may involve specified organization of input-output data for specific users such as an architect or a practicing engineer. Further, a minor modification will be required when employing these programs in a master program package. The changes will be with regard to the way the master program prepares the output so that such parameters as loads on column will be directly usable for the master program for the optimum design of the interior columns.

REFERENCES

1. Beveridge and Schechter, "Optimization: Theory and Practice", McGraw-Hill Book Company, New York, 1970.

2. Kowalik and Osborne, "Methods for Unconstrained Optimization Problems", American Elsevier Publishing Company, Inc., New York, 1968.

3. G. Hadley, "Nonlinear and Dynamic Programming", Addison-Wesley Publishing Company, Mass., 1964.

4. R. Bellman, "Dynamic Programming", Princeton: Princeton University Press, 1957.

5. Bellman and Dreyfus, "Applied Dynamic Programming", Princeton: Princeton University Press, 1957.

6. Rutherford, Aris, "Discrete Dynamic Programming", Blaisdel Publishing Company, 1964.

7. Duffin, Peterson, Zener, "Geometric Programming", John Wiley and Sons, Inc., New York, 1967.

8. A. B. Templeman, "Structural design for minimum cost using the method of geometric programming", Proceedings, Institution of Civil Engineers, Paper 7295, June, 1969.

9. D. Bond, "A computer program for studying the design of reinforced concrete structures supported on columns", Proceedings, Institution of Civil Engineers, Paper 7179, June, 1969.

10. Louis A. Hill, Jr., "Automated optimum cost design of building girders", Paper SP 16-8, Computer Application in Concrete Design and Technology, ACI Publication, March 1967.

11. Chu-kiz Wang and Charles Salmon, "Reinforced Concrete Design", International Textbook Co., Scaraton, Pennsylvania, 1969.

12. P. A. Stone, "Building Design Evaluation", E. & F Spon Ltd., 11 New Fetter Lane, London EC4, 1967.

13. Portland Cement Association, "Reinforced Concrete Floor Systems", 33 West Grand Avenue, Chicago.

14. Collective of authors, "Structural Steel Design", The Ronald Press Company, New York, 1964.

15. Pier L. Nervi, "Structures", F.W. Dodge Corporation, New York, 1956.

16. Heinz Rasch, "Some roofs of modern architecture", Translation Arts, New York, 1967.

17. National Research Council, Ottawa, Canada, "National Building Code of Canada 1970".

18. R. Hook and T. A. Jeeves, "Direct Search Solution of Numerical and Statistical Problems", Journal: ASSOC. Comput. Mach., 8 (1961), 212.

19. James N. Siddall, "OPTISEP - Designer's optimization subroutines", McMaster University, 1970.

20. Canadian Institute of Steel Construction, "Handbook of Steel Construction", 1815 Yonge Street, Toronto 7, 1969.

21. Coull G. R. Smith, "Tall Buildings", Symposium Publication Division, Pergamon Press Ltd., Oxford, 1967.

22. Bull and Sved, "Moment Distribution in Theory and Practice", Pergamon Press Ltd., The McMillan Co., 60 Fifth Avenue, New York 11, N.Y., 1964.

23. Novak, Horejsi, a kol., "Staticke Tabulky pro Stavebni Praxi", SNTL - Praha, 1968.

24. James N. Siddall, "OPTIPAC - The designer's optimization problem solver. Volume I, II, III", McMaster University, April, 1970.

25. Rubinstein, Karagozian, "Building design using Linear Programming", Journal of the Structural Division, Proceedings of ASCE, December, 1966.

26. Louis A. Hill, "Automated Optimum Cost Building Design", Journal of the Structural Division, Proceedings at ASCE, December, 1966.

27. C. K. Wang, "Statically Indeterminate Structures", McGraw-Hill Book Company, 1953.

28. Richard Bares, "Berechungs tafeln für Platten und Wandscheiben (Tables for the Analysis of Plates, Slab and Diaphragms), Bauverlag GmbH., Wiesbaden, 1969.

29. Kahn, Iyengar, Colago, "Computer design of 100-Storey John Hancock Center", Journal of the Structural Division, Proceedings of ASCE, December, 1966.

30. Sererud, Sturm, Conlin, "Optimizing structural design in very tall buildings", Architectural Record, August, 1970.

31. J. Moe, "Shearing Strength of Reinforced Concrete Slabs and Footings Under Concentrated Loads", Bulletin No. D47, Portland Cement Association, Research and Development Laboratories, April, 1961.

32. R. S. Godfrey, "Building Construction Cost Data 1970", Robert Snow Means Company, Inc., P.O. Box G, Duxbury, Mass., 02332, 1970.

33. Wire Reinforcement Institute, Inc., "Welded Wire Fabric - General Information Manual", Manual No. GF-21, A.I.A. File No. 4-E-2, Wire Reinforcement Institute, Inc., 5034 Wisconsin Avenue, N.W., Washington, D.C. 20016, May, 1967.

34. Hatcher, Sozen, "Test of a Reinforced Concrete Flat Slab", Journal of the Structural Division, Proceedings of the ASCE, June, 1969.

APPENDIX A

```
HRDZ,CM60000,T30.                                                   JANO.
RUN(S)
SETINDF.
REDUCE.
LGO.
'        6400 END OF RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     THIS IS THE PROGRAM FOR AN AUTOMATIC OPTIMATIZED   DESIGN OF
C     FOR TWO  PANELS
C     ***************************************************************
C
C
C
C     MC MASTER UNIVERSITY, DEPARTMENT OF CIVIL ENGINEERING
C     MECHANICS, HAMILTON , ONTARIO, CANADA.
C     PROGRAM DEVELOPER = JAN JANOSIK,  AUGUST,1972
C
C
C     ONE-WAY SLAB,BEAM,GIRDER FLOOR PANEL.  THE PANEL IS ASSUMED
C     AS A TYPICAL ONE WITH CONTINUITY IN BOTH DIRECTIONS  ATLEAST
C     FOR TWO  PANELS.
C
C     INPUT                                                            A
C     ***************************************************************  A
C                                                                      A
C     STRENGTH - PRICE                                                 A
C     ----------------                                                 A
C     IPS          1-PRINT-OUT OF THE INPUT DATA OF STRENGTH-PRICE     A
C                  2- NO PRINTOUT OF THE INPUT DATA OF STRENGTH-PRICE  A
C     JAK          1- FOR SINGLE FC,UPC,                               A
C                  2- FOR SET OF COUPLES FC,UPC                        A
C     MB           RANGE OF STRENGTHS OF CONCRETE                      A
C     MS           RANGE OF YIELD STENGTHS OF STEEL                    A
C     FC(1,....,MB) PARTICULAR COMPRESSION STRENGTH OF CONCRETE 28 DAYS A
C                  AFTER POURING                         (LB/IN**2)    A
C     UPC(1,.,MB)  THE UNIT PRICE OF READY MIXED CONCRETE   ($/Y**3)   A
C     FY(1,..,MS)  PARTICULAR YIELD STRENGTH OF REINFORCING STEEL      A
C                                                        (LB/IN**2)    A
C     UPS(1,.,MS)  UNIT PRICE OF STEEL                   ($/LB)        A
C                                                                      A
C     UPFS         UNIT PRICE OF FORMWORK FOR SLAB       ($/FT**2)     A
C     UPFB         UNIT PRICE OF FORMWORK FOR BEAM       ($/FT**2)     A
C                                                                      A
C     ZL           SUPERIMPOSED LOAD                     (LB/FT**2)    A
C                                                                      A
C     GEOMETRY                                                         A
C     --------------                                                   A
C     GIMOD        DISTANCE BETWEEN AXIS OF TWO GIRDERS ADJACENT TO THE A
C                  INVESTIGATED BAY                      (FT)          A
C     BEMOD        DISTANCE BETWEEN AXIS OF COLUMNS TAKEN PERPENDICULARY A
C                  TO ,GIMOD,                            (FT)          A
C     CA           THICKNESS OF COLUMN SUPPORTING THE GIRDER TAKEN     A
C                  IN DIRECTION ,BEMOD,                  (IN)          A
C     CB           THICKNESS OF COLUMN IN DIRECTION GIMOD  (IN)        A
C     HS           HEIGTH OF STOREY                      (FT)          A
C                                                                      A
C     DETAILING                                                        A
C     ------------                                                     A
```

```
C       IRUN            1- FOR THE FIRST RUN- THE VALUES OF COV,...,COVBP      A
C                       ARE SET INTERNALLY                                     A
C                       2- FOR THE ADJUSTING RUNS                              A
C       COV             0.75IN+HALF DIAMETER OF POSIT. STEEL IN SLAB (IN)      A
C       COVTB           DISTANCE BETWEEN THE CENTROID OF TENSION STEEL AT      A
C                       THE MIDSTAN OF BEAM  AND THE TESION FACE OF THE BEAM   A
C                                                               (IN)           A
C       COVBB           DISTANCE BETWEEN THE CETROID OF COMPRESSION STEEL AT   A
C                       THE SUPPORT AND THE COMPR. FACE OF THE BEAM (IN)       A
C       COVTP           DISTANCE BETWEEN THE CENTROID OF TENSION STEEL AT      A
C                       THE MIDSPAN OF THE GIRDER AND THE TENSION FACE OF      A
C                       THE GIRDER.                                 (IN)       A
C       COVBP           DISTANCE BETWEEN THE CENTROID OF COMPRESSION STEEL     A
C                       AT THE SUPPORT AND THE COMPRESSION FACE OF THE         A
C                       GIRDER.                                  (IN)          A
C                                                                              A
C                                                                              A
C       INPUT DATA PREPARATION.                                                A
C       ****************************************************************      A
C                                                                              A
C       SEQUENCE OF CARDS  VARIABLES                               FORMAT      A
C       +-----------------+---------------------------------------+---------  A
C       1                  ANY ALPHANUMERIC TITLE                  13A6        A
C       2                  IPS,JAK,MB,MS,IRUN                      5I5         A
C       3...               FC(I),I=1,MB                            4F20.5      A
C       4...               FY(J),J=1,MS                            4F20.5      A
C       5...               UPC(I),I=1,MB                           4F20.5      A
C       6...               UPS(J),J=1,MS                           4F20.5      A
C       7                  UPFS,UPFB                               2F20.5      A
C       8                  GIMOD,BEMOD,ZL                          3F20.5      A
C       9                  CA,CB,HS                                3F20.5      A
C       IF IRUN=2      -ADD THE FOLLOWING CARDS                                A
C       10...              RMAX(I),I=1,16                          5F16.8      A
C       11...              RMIN(I),I=1,16                          5F16.8      A
C       12...              XSTRT(I),I=1,16                         5F16.8      A
C       13                 COV,COVTB,COVBB,COVTP,COVBP             5F16.8      A
C                                                                              A
C       NOTE = IF SOME VARIABLE(J) IS DESIRED TO BE CERTAIN UNCHANGED          A
C       VALUE  - SET RMAX(J), RMIN(J), XSTRT(J)  EQUAL TO THIS VALUE.          A
C                                                                              A
C                                                                              A
C       OUTPUT                                                                 A
C       *****************                                                      A
C                                                                              A
C       X1       = NS, NUBER OF SPANS                                          A
C       X2       = TS, THICKNESS OF SLAB(IN)                                   A
C                                                                              A
C       X3       = TB, OVERALL DEPTH (IN)                          ...         A
C       X4       = BB, WIDTH OF THE WEB (IN)                         .         A
C       X5       =DIAMETER OF STIRRUPS 2...,8                        ...BEAM   A
C       X6       = NUMBER OF STIRRUPS FOR ONE BEAM                 ...         A
C                                                                              A
C       X7       = TP, OVERALL DEPTH (IN)                          ...         A
C       X8       = BP, WIDTH OF THE WEB (IN)                         .         A
C       X9       = DIAMETER OF THE STIRRUPS 2,...,8                  ..GIRDER  A
C       X10      = NUMBER OF STIRRUPS FOR ONE GIRDER               ...         A
C                                                                              A
```

```
C       X(11)   = AREA OF REINFORCEMENT AT MIDSPAN OF SLAB(IN**2)-POSIT.(BT   A
C       X(12)   = AREA OF REINFORCEMENT AT SUPPORT OF SLAB(IN**2) -NEGAT(TO   A
C       X(13)   = AREA OF REINFORCEMENT AT MIDSPAN OF BEAM(IN**2)-POSIT.(BT   A
C       X(14)   = AREA OF REINFORCEMENT AT SUPPORT OF BEAM(IN**2) -NEGAT(TO   A
C       X(15)   = AREA OF R.F. AT MIDSPAN OF GIRDER(IN**2) -POSIT.(TOP)       A
C       X(16)   = AREA OF R.F. AT SUPPORT OF GIRDER(IN**2) -NEG.(TOP)         A
C                                                                             A
C       THE PROGRAM BEGINS                                                    A
C       ==================                                                    A
C                                                                             A
        COMMON /BLOK1/ IPS,JAK/BLOK2/FC,FY/BLOK3/UPC,UPS,UPFS,UPFB            A
        COMMON /BLOK4/ GIMOD,BEMOD,COV,COVTB,COVBB,COVTP,COVBP,ZL             A
        COMMON /BLOK5/ CA,CB,HS                                               A
        DIMENSION WORK1(16), WORK2(16), WORK3(16), WORK4(16)                  A
        DIMENSION X(16), PHI(148), PSI(1), XSTRT(16), RMAX(16), RMIN(16)      A
        DIMENSION IX(10), DIF(10), XB(16)                                     A
        DIMENSION PEVB(20), PEVS(20), CENB(20), CENS(20)                      A
        DIMENSION TITLE(13)                                                   A
C                                                                             A
C                                                                             A
        DATA N,IPRINT,IDATA,NCONS,F,MAXM,G,NEQUS,NSHOT,NTEST/16,1,1,36,0.0    A
       101,3000,0.01,0,1,50/
        READ (5,35) (TITLE(I),I=1,13)                                         A
        WRITE (6,35) (TITLE(I),I=1,13)                                        A
C                                                                             A
        READ (5,40) IPS,JAK,MB,MS,IRUN                                        A
C                                                                             A
        READ (5,45) (PEVB(J),J=1,MB)                                          A
        READ (5,45) (PEVS(JJ),JJ=1,MS)                                        A
        READ (5,45) (CENB(J),J=1,MB)                                          A
        READ (5,45) (CENS(JJ),JJ=1,MS)                                        A
        IF (IPS.EQ.1) GO TO 5                                                 A
        GO TO 10                                                              A
5       WRITE (6,50)                                                          A
        WRITE (6,45) (PEVB(J),J=1,MB)                                         A
        WRITE (6,55)                                                          A
        WRITE (6,45) (CENB(J),J=1,MB)                                         A
        WRITE (6,60)                                                          A
        WRITE (6,45) (PEVS(JJ),JJ=1,MS)                                       A
        WRITE (6,65)                                                          A
        WRITE (6,45) (CENS(JJ),JJ=1,MS)                                       A
10      READ (5,70) UPFS,UPFB                                                 A
C                                                                             A
        READ (5,75) GIMOD,BEMOD,ZL                                            A
        READ (5,75) CA,CB,HS                                                  A
        IF (IRUN.EQ.2) GO TO 15                                               A
        RMAX(1)=5.                                                            A
        RMIN(1)=1.                                                            A
        XSTRT(1)=2.                                                           A
        RMAX(2)=BEMOD/2.85                                                    A
        RMIN(2)=BEMOD/6.6                                                     A
        XSTRT(2)=BEMOD/(XSTRT(1))*0.33                                        A
        RMAX(3)=GIMOD*1.5                                                     A
        RMIN(3)=GIMOD*0.75                                                    A
        XSTRT(3)=GIMOD*1.25                                                   A
        RMAX(4)=GIMOD                                                         A
        RMIN(4)=GIMOD/2.                                                      A
```

```
      XSTRT(4)=RMIN(4)                                                  A
      RMAX(5)=8.                                                        A
      RMIN(5)=2.                                                        A
      XSTRT(5)=2.                                                       A
      RMAX(6)=GIMOD*2.5                                                 A
      RMIN(6)=GIMOD/2.                                                  A
      XSTRT(6)=GIMOD*1.75                                               A
      RMAX(7)=BEMOD*1.5                                                 A
      RMIN(7)=BEMOD*0.75                                                A
      XSTRT(7)=BEMOD*1.25                                               A
      RMAX(8)=BEMOD                                                     A
      RMIN(8)=BEMOD/2.                                                  A
      XSTRT(8)=BEMOD*0.9                                                A
      RMAX(9)=8.                                                        A
      RMIN(9)=2.                                                        A
      XSTRT(9)=2.                                                       A
      RMAX(10)=BEMOD*2.5                                                A
      RMIN(10)=BEMOD /2.                                                A
      XSTRT(10)=BEMOD*1.1                                               A
      PLSL=12.*XSTRT(2)                                                 A
      RMAX(11)=0.03*PLSL                                                A
      RMIN(11)=0.0015*PLSL                                              A
      XSTRT(11)=0.00666*PLSL                                            A
      RMAX(12)=0.03*PLSL                                                A
      RMIN(12)=0.0015*PLSL                                              A
      XSTRT(12)=0.0123*PLSL                                             A
      PLOB=XSTRT(3)*XSTRT(4)                                            A
      RMAX(13)=0.016*PLOB                                               A
      RMIN(13)=0.0040*PLOB                                              A
      XSTRT(13)=0.0041*PLOB                                             A
      RMAX(14)=0.016*PLOB                                               A
      RMIN(14)=0.002*PLOB                                               A
      XSTRT(14)=0.0064*PLOB                                             A
      PLOP=XSTRT(7)*XSTRT(8)                                            A
      RMAX(15)=0.011*PLOP                                               A
      RMIN(15)=0.0022*PLOP                                              A
      XSTRT(15)=0.0084*PLOP                                             A
      RMAX(16)=0.0111*PLOP                                              A
      RMIN(16)=0.0022*PLOP                                              A
      XSTRT(16)=0.0042*PLOP                                             A
      DATA COV,COVTB,COVBB,COVTP,COVBP/1.25,3.,2.,3.,3./                A
      GO TO 20                                                          A
C                                                                       A
C     SUPPLIED VALUES OF EXPECTED BOUNDARIES MAX.,MIN.,AND              A
C     STARTING VALUES RESPECTIVELY                                      A
15    READ (5,80) (RMAX(I),I=1,16)                                      A
      READ (5,80) (RMIN(I),I=1,16)                                      A
      READ (5,80) (XSTRT(I),I=1,16)                                     A
      READ (5,85) COV,COVTB,COVBB,COVTP,COVBP                           A
C                                                                       A
20    DO 30 I=1,MB                                                      A
      FC=PEVB(I)                                                        A
      UPC=CENB(I)/27.                                                   A
      DO 25 J=1,MS                                                      A
      FY=PEVS(J)                                                        A
      UPS=CENS(J)                                                       A
      CALL SEEK1 (N,RMAX,RMIN,NCONS,NEQUS,F,G,XSTRT,NSHOT,NTEST,MAXM,IPR  A
```

```
     1INT,IDATA,X,U,PHI,PSI,WORK1,WORK2,WORK3,WORK4)                    A
      CALL ANSWER (U,X,PHI,PSI,N,NCONS,NEQUS)                           A
25    CONTINUE                                                          A
30    CONTINUE                                                          A
      STOP                                                              A
C                                                                       A
35    FORMAT (13A6)                                                     A
40    FORMAT (5I5)                                                      A
45    FORMAT (4F20.5)                                                   A
50    FORMAT (3X,*COMPRESSION STRENGTH OF CONCRETE FC(PSI)*)            A
55    FORMAT (3X,*UNIT PRICE OF CONCRETE UPC($/CU.YD.)*)                A
60    FORMAT (3X,*YIELD STRENGTH OF R.F. STEEL FY(PSI)*)                A
65    FORMAT (3X,*UNIT PRICE OF STEEL*)                                 A
70    FORMAT (2F20.5)                                                   A
75    FORMAT (3F20.5)                                                   A
80    FORMAT (5F16.8)                                                   A
85    FORMAT (5F16.8)                                                   A
      END                                                               A
```

CD TOT 0248

```
      SUBROUTINE UREAL (X,U)                                              B
C                                                                         B
C     ****************************************************************   B
C     PURPOSE OF THIS SUBROUTINE IS TO CALCULATE THE VALUE                B
C     OF THE OBJECTIVE FUNCTION                                           B
C     ****************************************************************   B
C                                                                         B
      COMMON /BLOK3/ UPC,UPS,UPFS,UPFB                                    B
      COMMON /BLOK4/ GIMOD,BEMOD,COV,COVTB,COVBB,COVTP,COVBP,ZL           B
      DIMENSION X(1)                                                      B
C     SPECIFIC WEIGTH OF THE STEEL                                        B
      W=490.                                                              B
C                                                                         B
C     THE CONVERTED AUXILIARY VALUES                                      B
      NS = X(1)
      X111 = NS
      S=BEMOD/X111                                                        B
      TS=X(2)/12.                                                         B
      TB=X(3)/12.                                                         B
      TP=X(7)/12.                                                         B
      TW=(X(3)-X(2))/12.
      TWP=(X(7)-X(2))/12.                                                 B
      BB=X(4)/12.                                                         B
      BP=X(8)/12.                                                         B
      COBB=COVBB/12.                                                      B
      COTB=COVTB/12.                                                      B
      COTP=COVTP/12.                                                      B
      COBP=COVBP/12.                                                      B
      X11=X(11)/144.                                                      B
      X12=X(12)/144.                                                      B
      X13=X(13)/144.                                                      B
      X14=X(14)/144.                                                      B
      X15=X(15)/144.                                                      B
      X16=X(16)/144.                                                      B
      AB=3.1415*X(5)**2/(256.*144.)                                       B
      AG=3.1415*X(9)**2/(256.*144.)                                       B
C                                                                         B
C     THE COST OF THE SLAB  US                                            B
      US=GIMOD*(UPC*TS*BEMOD+UPS*W*(X11*(TS+BB+0.8535*S)+X12*(TS+0.418*S  B
     1))+UPFS*(S-BB)*X111)                                                B
C                                                                         B
C     THE COST OF THE BEAM,  UB                                           B
      UB=UPC*(TW*BB*(GIMOD-BP))+UPS*W*(X13*(TB+BP+0.8535*GIMOD)+X14*(TB+  B
     10.418*GIMOD)+(2.*TB+BB-4.*COBB-2.*COTB)*AB*X(6))+UPFB*((2.*TW+BB)*  B
     2(GIMOD-BP))                                                         B
C                                                                         B
C     THE COST OF GIRDER,    UG                                           B
      UG=UPC*TWP*BP*BEMOD+UPS*W*(X15*(TP+BP+0.8535*BEMOD)+X16*(TP+0.418*  B
     1BEMOD)+(2.*TP+BP-4.*COBP-2.*COTP)*AG*X(10))+UPFB*2.*(((BEMOD-TP)*(  B
     2TP+0.5*BP))-(BB*TW*X111))                                           B
C                                                                         B
C                                                                         B
C     THE OBJECTIVE FUNCTION                                              B
C     ----------------------                                              B
C                                                                         B
      U=US+X(1)*UB+UG                                                     B
      RETURN                                                              B
      END                                                                 B
```

```
      SUBROUTINE CONST (X,NCONS,PHI)
C
C     ******************************************************************
C     PURPOSE OF THIS SUBROUTINE IS TO CALCULATE THE VALUES OF
C     THE INEQUALITY CONSTRAINTS
C     AT THE FEASIBLE POINT PHI.GE.0.
C     ******************************************************************
C
      DIMENSION X(1), PHI(1)
      COMMON /BLOK2/ FC,FY
      COMMON /BLOK4/ GIMOD,BEMOD,COV,COVTB,COVBB,COVTP,COVBP,ZL
      COMMON /BLOK5/ CA,CB,HS
C
C     THE AUXILIARY VALUES
C
      F1=0.85*FC
      SQ=SQRT(FC)
      NS = X(1)
      X111 = NS
      S=BEMOD/X111
      Z=S-X(4)/12.
      BB=X(4)
      BE=2.*X(8)
      BS=12.
      BB=X(4)
      TU=X(2)-COV
      TUU=12.*TU
      DB=X(3)-COVBB
      DT=X(3)-COVTB
      TW=X(3)-X(2)
      DP=X(7)-COVTP
      DPP=X(7)-COVBP
      TWP=X(7)-X(2)
      ZL1=S*ZL/12.
      GIM12=GIMOD*12.
C
C     THE MINIMUM AND MAXIMUM REINFORCEMENT RATIO PMIN, PMAX RESP.
      PMIN=200./FY
      FCC=FC-4000.
      IF (FCC) 5,5,10
5     EK=0.85
      GO TO 15
10    EK=0.85-0.05*(FCC/1000.)
15    PMAX=(F1*EK*87000./(FY*(87000.+FY)))*0.75
      X2=X(2)
      X8=X(8)
      X9=X(9)
      X11=X(11)
      X12=X(12)
      X13=X(13)
      X14=X(14)
      X15=X(15)
      X16=X(16)
      X111=X(11)/2.
      X133=X(13)/2.
      X155=X(15)/2.
C
C
C
```

```
C       THE SLAB                                                              C
C       -----------------------------------                                   C
C                                                                             C
        USLAB=18.75*X(2)+1.8*ZL                                               C
        ZA=USLAB*Z**2                                                         C
C                                                                             C
C       POSITIVE DESIGN MOMENT                                                C
        BMSLAB=ZA/16.                                                         C
C                                                                             C
C       NEGATIVE DESIGN MOMENT                                                C
        BMSL=ZA/11.                                                           C
C                                                                             C
C       THE SECTION AT THE MIDSPAN                                            C
        PS1=X(12)/TUU                                                         C
        PSDASH=X111/TUU                                                       C
        PM1=X(11)/TUU                                                         C
        CALL SIMPLE (BS,TU,X11,EK,USMSL,F1)                                   C
        PHI(1)=(USMSL-BMSLAB*12.)*1.E-05                                      C
        PHI(2)=PS1-PMIN                                                       C
        PHI(3)=PMAX-PS1                                                       C
C                                                                             C
C       THE SECTION AT THE SUPPORT                                            C
        CALL DOUBLE (F1,PMAX,TU,COV,PS1,PSDASH,X12,X111,BS,UBMS,EK)           C
        PHI(4)=(UBMS-BMSL*12.)*1.E-05                                         C
        PHI(5)=PM1-PMIN                                                       C
        PHI(6)=PMAX-PM1                                                       C
C                                                                             C
C       SHEAR CONSTRAINT                                                      C
        V=0.6*USLAB*Z                                                         C
        PHI(7)=1.7*SQ-V/TUU                                                   C
C                                                                             C
C       THE DEFLECTION CONSTRAINT                                             C
        IF (PM1*FY-500.) 20,20,30                                             C
20      IF (PS1*FY-500.) 25,25,30                                             C
25      SI=X(2)**3*BS/12.                                                     C
        GO TO 40                                                              C
30      AS=ABS(8.*X(11))                                                      C
        EXXX=AS+2.*TU                                                         C
        IF (EXXX) 25,25,35                                                    C
C       THE LOCATION OF THE NEUTRAL  AXIS                                     C
35      EX=AS+SQRT(AS*EXXX)                                                   C
C                                                                             C
C       THE MOMENT OF INERTIA FOR THE CRACKED TRANSFORM. SECTION              C
        SI=0.333*EX**3*BS+AS*(TU-EX)**2                                       C
40      UDEAD=12.50*X(2)                                                      C
        BPOD=0.1*(ZL+UDEAD)*Z**2                                              C
        BMS=0.125*(ZL+UDEAD)*Z**2-0.5*BPOD                                    C
C                                                                             C
C       THE MODULUS OF ELASTICITY FOR NORMAL WEIGTH CONCRETE(145PSF)          C
        EC=575500.*SQ                                                         C
        YM=5.*Z**2*(BMS-0.1*BPOD)/(48.*EC*SI*12.**3)                          C
        DLIM=ZL*YM/(ZL+UDEAD)                                                 C
C                                                                             C
C       ASSUME THAT 60 PER CENT OF LIVE LOAD IS SUSTAINED                     C
        SUSTDL=(0.6*ZL+UDEAD)*2.*YM/(ZL+UDEAD)                                C
        ALLDL=S*12./360.                                                      C
        PHI(8)=ALLDL-SUSTDL-DLIM                                              C
```

```
C
C
C      THE    T-BEAM
C      ===================
C
C
C      THE EFFECTIVE WIDTH DETERMINATION
C      ---------------------------------
C
       BEF1=X(4)+24.*X(2)
       BEF2=0.25*GIM12
       BEF3=S*12.
C
       IF (BEF1.LT.BEF2) GO TO 50
       IF (BEF2.LT.BEF3) GO TO 45
       BEF=BEF3
       GO TO 60
45     BEF=BEF2
       GO TO 60
50     IF (BEF1.LT.BEF3) GO TO 55
       BEF=BEF3
       GO TO 60
55     BEF=BEF1
C
C
C      ULTIMATE LOAD PER FOOT OF SPAN OF THE BEAM
60     UBEAM=(X(3)*X(4)+X(2)*(S*12.-X(4)))*1.56+S*1.8*ZL
       ZB=(GIMOD-X(8)/12.)**2*UBEAM
C
C      POSITIVE DESIGN MOMENT
       BMP=ZB/16.
C
C
C      NEGATIVE DESIGN MOMENT
       BMBE=ZB/11.
C
C
C      THE SECTION AT THE MIDSPAN OF BEAM
       PM2=X(13)/(X(4)*DT)
       PS2=X14/(X(4)*DT)
       PS2DAS=X13/(X(4)*DT)
       CALL TBEAM (BEF,BB,DB,X2,COVBB,X13,EK,UBM,F1,PMAX)
       PHI(9)=(UBM-BMP*12.)*1.E-06
       PHI(10)=PS2-PMIN
       PHI(11)=PMAX-PS2
C
C      THE SECTION ABOVE THE SUPPORT OF THE BEAM
       CALL DOUBLE (F1,PMAX,DT,COVTB,PS2DAS,PS2,X14,X133,BB,UBMB,EK)
       PHI(12)=(UBMB-BMBE*12.)*1.E-06
       PHI(13)=PM2-PMIN
       PHI(14)=PMAX-PM2
C
C      THE WEB REINFORCEMENT
C      ---------------------------------
C
C      THE ULTIMATE REACTION
       VR=UBEAM*0.5058*(GIMOD-X(8)/12.)
```

```
      DIA=X(5)
      CALL WEBRTG (BB,DB,VR,DIA,SQ,SB)
      PHI(15) =(X(6) - GIM12/SB)*0.01
      PHI(16)=(X(5)-1.99999)*0.1
C
C     THE DEFLECTION CONSTRAINT
C     ------------------------------
C
      IF (PM2*FY-500.) 65,65,70
C
C     EX1=DISTANCE FROM THE BOTTOM OF THE BEAM TO THE CENTRE OF GRAVITY
65    EX1=(X(2)*BEF*(X(3)-0.5*X(2))+0.5*X(4)*TW**2)/(X(2)*BEF+TW*X(4))
C
C     SI1=MOMENT OF INERTIA OF GROSS-SECTION
      SI1=BEF*X(2)*(X(2)**2/12.+(X(3)-.5*X(2)-EX1)**2)+X(4)*TW*(TW**2/12
     1.+(EX1-0.5*TW)**2)
      GO TO 85
70    Q=PM2*FC/FY
C
      IF (1.180*Q*DB-X(2)) 75,75,80
C
C
C     VZ=POSITION OF NEUTRAL AXIS FROM THE TOP OF THE BEAM
C
C     EX3=POSITION OF THE CENTRE OF GRAVITY MEASURED FROM THE BOTTOM
C     WHEN N.A. IS IN THE FLANGE
C
75    VZ=X(13)*FY/(F1*BEF*EK)
      EX3=(BEF*VZ*(X(3)-0.5*VZ)+50.*X(13))/(BEF*VZ+8.*X(13))
C
C     THE MOMENT OF INERTIA
      SI1=BEF*VZ*(X(3)-EX3)**2+8.*X(13)*(EX3-COVBB)**2
      GO TO 85
C
C     VY=POSITION OF NEUTRAL AXIS FROM THE TOP OF THR BEAM
80    ASF=F1*X(2)*(BEF-X(4))/FY
      VY=(X(13)-ASF)*FY/(F1*X(4))
C
C     EX4=THE CENTRE OF GRAVITY OF THE CRACKED SECTION LOCATED FROM
C     THE BITTOM OF THE BEAM
      EX4=(X(2)*BEF*(X(3)-X(2)/2.)+(VY-X(2))*X(4)*(TW-0.5*(VY-X(2)))+50.
     1*X(13))/(X(2)*BEF+(VY-X(2))*X(4)+8.*X(13))
C
C     THE MOMENT OF INERTIA OF CRACKED SECTION
      SI1=BEF*X(2)*(X(2)**2/12.+(X(3)-X(2)/2.-EX4)**2)+(VY-X(2))*X(4)*(X
     1(3)-EX4-0.5*(VY-X(2)))**2+8.*X(13)*(EX4-COVBB)
C
C     IMMEDIATE LIVE LOAD DEFLECTION
85    DLL=5.*ZL1*GIM12**4/(EC*SI1*384.)
C
C     ADDITIONAL LONG TIME DEFLECTION DUE TO CREEP AND SHRINKAGE
C     CREEP FACTOR = 2.
      ADS=UBEAM/ZL*DLL*2.
C     HALF-AN INCH IS TOLERABLE DEFLECTION
      PHI(17)=GIM12/360.-DLL-ADS+0.5
C
C
```

```
C     THE GIRDER
C     ============================
C
      A1=TWP*X(8)
      A2=X(2)*2.*X(8)
      A3=A1+A2
C
C     THE CENTROID OF THE GROSS-AREA OF THE T-SECTION
      YDASH=(X(8)*TWP**2/2.-2.*X(8)*X(2)**2/2.)/A3
C
C     THE MODULUS OF INERTIA OF THE GROSS-AREA
      SI=0.33*X(8)*(2.*X(2)**3+TWP**3)-A3*YDASH**2
C
C     THE STIFFNESS OF THE GIRDER GKR
      GKR=SI/BEMOD
C
C     THE STIFFNESS OF THE COLUMN CKL
      CKL=CA**3*CB/(12.*HS)
      CCC=GKR/CKL
C
C     THE DISTRIBUTION FACTORS
      DAB=-CCC/(CCC+2.)
      DBA=-CCC/(2.*CCC+2.)
C
C     THE UNIFORMLY DISTRIBUTED SELFWEIGTH OF GIRDEF
      UP=1.56*X(8)*X(7)
C
C     THE CONCENTRATED DEAD LOAD
      CD=GIMOD*1.56*((X(3)*X(4)+X(2)*(S*12.-X(4))))
C
C     THE CONCENTRATED LIVE LOAD
      CL=GIMOD*1.8*ZL*S
C
C     THE FIXED END MOMENT DUE TO DEAD AND LIVE LOAD
      FMDL=(CD+CL)*BEMOD*(X111**2-1.)/(12.*X111)+BEMOD**2*UP/12.
C
C     FIXED END MOMENT DUE TO DEAD LOAD ONLY
      FMD=CD*BEMOD*(X111**2-1.)/(12.*X111)+BEMOD**2*UP/12.
      CALL TWOSP (FMDL,FMD,DAB,DBA,BMNT)
      CALL THREEN (FMDL,FMD,DAB,DBA,CD,CL,UP,BMNP,BMNB,BMPP,X)
C
C
C     THE SECTION AT THE MIDSPAN OF THE GIRDER
      PM3=X(15)/(DP*2.*X(8))
      CALL TBEAM (BE,X8,DP,X2,COVBP,X15,EK,UMMP,F1,PMAX)
      PHI(18)=(UMMP-BMPP)*1.E-07
      PHI(19)=PM3-PMIN
      PHI(20)=PMAX-PM3
C
C     THE SECTION AT THE SUPPORT OF THE GIRDER
      PG=X16/(X(8)*DP)
      PGDASH=X155/(X(8)*DP)
      CALL DOUBLE (F1,PMAX,DP,COVTP,PG,PGDASH,X155,X15,X8,UBMG,EK)
      PHI(21)=(UBMG-BMNT    )*1.E-07
      PHI(22)=PG-PMIN
      PHI(23)=PMAX-PG
C
```

```
C      THE WEB REINFORCEMENT                                              C
C      ----------------------------                                       C
C      THE ULTIMATE LOAD PER FT OF LENGTH OF GIRDER                       C
       ULT=2.34*(X(2)*GIMOD+(X111-1.)*(X(3)-X(2))*(GIMOD-X(8)/12.)*X(4)/B C
      1EMOD+X(8)*(X(7)-X(2)))+1.8*ZL*GIMOD                                C
C      THE ULTIMATE REACTION                                              C
       VP=0.5056*BEMOD*ULT                                                C
       CALL WEBRTG (X8,DP,VP,X9,SQ,SG)                                    C
       PHI(24)=(X(10)-BEMOD*12./SG)*0.01                                  C
C                                                                         C
C      DEFLECTION CONSTRAINTS                                             C
       PHI(25)=0.18*FC/FY-PM3                                             C
C                                                                         C
C                                                                         C
C      CONSTRAINTS ON GEOMETRY                                            C
C      ------------------------                                           C
C                                                                         C
C      THE MIN. THICKNESS OF THE SLAB=3.0 INCHES                          C
       PHI(26)=(X(2)-3.)/10.                                              C
C                                                                         C
C      MINIMUM NUMBER OF SPANS TO BE 2.
       PHI(27)=(X(1)-2.)*1.E-08
C                                                                         C
C      THE MIN. WIDTH OF THE BEAM BB= 3. INCHES                           C
       PHI(28)=(X(4)-3.)*1.E-02                                           C
C                                                                         C
C      THE MIN. DEPTH OF THE GIRDER TP=9. INCHES                          C
       PHI(29)=(X(7)-9.)*1.E-02                                           C
       PHI(30) = X(7)-X(3)
C                                                                         C
C      THE MIN. 2 BARS NO.4 FOR THE BEAM AND THE GIRDER =0.4 IN**2        C
       PHI(31)=(X(13)-0.4)*1.E-01                                         C
       PHI(32)=(X(14)-0.4)*1.E-01                                         C
       PHI(33)=(X(15)-0.4)*1.E-01                                         C
       PHI(34)=(X(16)-0.4)*1.E-01                                         C
C                                                                         C
C      MINIMUM  1-NO.2 PER FOOT OF SLAB                                   C
       PHI(35)=X(11)-0.05                                                 C
       PHI(36)=X(12)-0.05                                                 C
       RETURN                                                             C
       END                                                                C
```

CD TOT   0328

```
      SUBROUTINE TWOSP (FMDL,FMD,DAB,DBA,BMNT)                          D
C     DISTRIBUTION FACTORS                                              D
      DBC=DBA                                                           D
      DCB=DAB                                                           D
C     FIXED END MOMENTS                                                 D
      FMA=FMDL                                                          D
      FMBA=-FMDL                                                        D
      FMBC=FMDL                                                         D
      FMC=-FMDL                                                         D
C     1. BALANCE                                                        D
      PBAB=FMA*DAB                                                      D
      PBBA=(FMBA+FMBC)*DBA                                              D
      PBBC=(FMBA+FMBC)*DBC                                              D
      PBCB=FMC*DCB                                                      D
C                                                                       D
      CAB=0.5*PBBA                                                      D
      CBA=0.5*PBAB                                                      D
      CBC=0.5*PBCB                                                      D
      CCB=0.5*PBBC                                                      D
C                                                                       D
C     2. BALANCE                                                        D
      BAB=CAB*DAB                                                       D
      BBA=(CBA+CBC)*DBA                                                 D
      BBC=(CBA+CBC)*DBC                                                 D
      BCB=CCB*DCB                                                       D
C                                                                       D
C     TOTAL MOMENT                                                      D
      BMNT=FMBA+PBBA+CBA+BBA                                            D
      BMA=FMA+PBAB+CAB+BAB                                              D
      BMBC=FMBC+PBBC+CBC+BBC                                            D
      BMC=FMC+PBCB+CCB+BCB                                              D
      RETURN                                                            D
      END                                                               D
```

CD TOT   0033

```
      SUBROUTINE THREEN (FMDL,FMD,DAB,DBA,CD,CL,UP,BMNP,BMNB,BMPP,X)         E
      DIMENSION X(1)                                                         E
      COMMON /BLOK4/ GIMOD,BEMOD,COV,COVTB,COVBB,COVTP,COVBP,ZL              E
C                                                                            E
C     DISTRIBUTION FACTORS                                                   E
      DDC=DAB                                                                E
      DBC=DBA                                                                E
      DCB=DBA                                                                E
      DCD=DBA                                                                E
C                                                                            E
C     FIXED END MOMENTS                                                      E
      I=1                                                                    E
      FMA=FMDL                                                               E
      FMBA=-FMDL                                                             E
      FMBC=FMDL                                                              E
      FMCB=-FMDL                                                             E
      FMCD=FMD                                                               E
      FMDC=-FMD                                                              E
      GO TO 10                                                               E
C     FEM FOR POSITIVE POSITION                                              E
5     I=2                                                                    E
      FMA=FMD                                                                E
      FMBA=-FMD                                                              E
      FMBC=FMDL                                                              E
      FMCB=-FMDL                                                             E
      FMCD=FMD                                                               E
      FMDC=-FMD                                                              E
C                                                                            E
C     1. BALANCE                                                             E
10    PBAB=DAB*FMA                                                           E
      PBBA=DBA*(FMBA+FMBC)                                                   E
      PBBC=DBC*(FMBA+FMBC)                                                   E
      PBCB=DCB*(FMCB+FMCD)                                                   E
      PBCD=DCD*(FMCB+FMCD)                                                   E
      PBDC=FMDC*DDC                                                          E
C     1. CARRY-OVER                                                          E
      PCAB=0.5*PBBA                                                          E
      PCBA=0.5*PBAB                                                          E
      PCBC=0.5*PBCB                                                          E
      PCCB=0.5*PBBC                                                          E
      PCCD=0.5*PBDC                                                          E
      PCDC=0.5*PBCD                                                          E
C                                                                            E
C     2. BALANCE                                                             E
      DBAB=DAB*PCAB                                                          E
      DBBA=DBA*(PCBA+PCBC)                                                   E
      DBBC=DBC*(PCBA+PCBC)                                                   E
      DBCB=DCB*(PCCB+PCCD)                                                   E
      DBCD=DCD*(PCCB+PCCD)                                                   E
      DBDC=DDC*PCDC                                                          E
C                                                                            E
C     2. CARRY-OVER                                                          E
      DCAB=0.5*DBBA                                                          E
      DCBA=0.5*DBAB                                                          E
      DCBC=0.5*DBCB                                                          E
      DCCB=0.5*DBBC                                                          E
      DCCD=0.5*DBDC                                                          E
      DCDC=0.5*DBCD                                                          E
C                                                                            E
```

```
C       3. BALANCE                                                        E
        TBDC=DDC*DCDC                                                     E
        TBCD=DCD*(DCCD+DCCB)                                              E
        TBCB=DCB*(DCCD+DCCB)                                              E
        TBBC=DBC*(DCBC+DCBA)                                              E
        TBBA=DBA*(DCBC+DCBA)                                              E
        TBAB=DAB*DCAB                                                     E
        BMA=FMA+PBAB+PCAB+DCAB+DBAB+TBAB                                  E
        BMBA=FMBA+PBBA+PCBA+DBBA+DCBA+TBBA                                E
        BMBC=FMBC+PBBC+PCBC+DBBC+DCBC+TBBC                                E
        BMCB=FMCB+PBCB+PCCB+DBCB+DCCB+TBCB                                E
        BMCD=FMCD+PBCD+PCCD+DBCD+DCCD+TBCD                                E
        BMDC=FMDC+PBDC+PCDC+DBDC+DCDC+TBDC                                E
        IF (I.EQ.1) GO TO 15                                              E
        IF (I.EQ.2) GO TO 20                                              E
15      BMNP=ABS(BMBA)                                                    E
        GO TO 5                                                           E
20      BMNB=BMBC                                                         E
C                                                                         E
C       THE BENDING MOMENT AT THE MIDSPAN OF THE SIMPLY SUPP. SPAN        E
C       -----------------------------------------                         E
        NS=X(1)                                                           E
        H=NS                                                              E
        DO 40 JJ = 1,10,2
        IF (JJ-NS)40,25,40
40      CONTINUE
        BMSC=H*(CD+CL)*BEMOD/4.                                           E
        GO TO 30                                                          E
25      BMSC=(H**2-1.)*BEMOD*(CD+CL)/(8.*H)                               E
30      BMSU=UP*BEMOD**2/8.                                               E
        BMS=BMSC+BMSU                                                     E
        BMPP=BMS-BMNB                                                     E
        RETURN                                                            E
        END                                                               E
```

CD TOT 0093

```
      SUBROUTINE TBEAM (BEF,B,TB,TS,COVBT,AS,EK,USM,F1,PMAX)               G
C                                                                           G
C     *****************************************************************    G
C     PURPOSE OF THIS SUBROUTINE IS TO DETERMINE  THE USABLE ULTIMATE      G
C     MOMENT CAPACITY OF THE T-SECTION.                                     G
C     *****************************************************************    G
C                                                                           G
      COMMON /BLOK2/ FC,FY                                                  G
C                                                                           G
C     FIND AS, SO THAT N.A. IS ON THE BOTTOM OF FLANGE                      G
      CC=F1*BEF*TS                                                          G
      T=AS*FY                                                               G
C                                                                           G
      IF (T-CC) 20,20,5                                                     G
C                                                                           G
C     TREAT BY THE TWO-COUPLE METHOD                                        G
5     A1=TS*(BEF-B)                                                         G
      A2=T/F1-A1                                                            G
      C1=F1*A2                                                              G
      C2=F1*A1                                                              G
      AS1=C1/FY                                                             G
      PW=AS1/(B*(TB-COVBT))                                                 G
      P=AS/(B*(TB-COVBT))                                                   G
      IF (PMAX-(P-PW)) 15,15,10                                             G
10    USM=(C1*(TB-COVBT-0.5*A2/B)+C2*(TB-COVBT-0.5*TS))*0.9                 G
      RETURN                                                                G
15    USM=0.1                                                               G
      RETURN                                                                G
C                                                                           G
C                                                                           G
C     SECTION IS TREATED AS A RECTANGULAR SECTION                           G
20    A=T/(F1*BEF)                                                          G
      USM=0.9*T*(TB-COVBT-0.5*A)                                            G
      RETURN                                                                G
      END                                                                   G
```

CD TOT   0035

```
      SUBROUTINE WEBRTG (B,D,VU,DIA,SQ,SP)                                 H
C                                                                          H
C     ****************************************************************     H
C     PURPOSE OF THIS SUBROUTINE IS TO DETERMINE THE MAXIMUM SPACING       H
C     OF THE VERTICAL -U- STIRRUPS  BY THE ULT.-STRENGTH DESIGN            H
C     THEORY FOR RECTANGULAR SECTION.                                      H
C     ****************************************************************     H
C                                                                          H
      COMMON /BLOK2/ FC,FY                                                 H
      AV=6.283*DIA**2/256.                                                 H
C                                                                          H
C     SHEAR STRESS AT A DISTANCE D FROM THE FACE OF SUPPORT                H
      VS=VU/(B*D)                                                          H
C                                                                          H
C     SHEAR STRESS CARRIED BY CONCRETE ALONE                               H
      VUC=1.7*SQ                                                           H
C                                                                          H
C     LIMITATION                                                           H
      VUC1=5.1*SQ                                                          H
      IF (VS-VUC1) 5,5,10                                                  H
5     SMAX=D/2.                                                            H
      GO TO 15                                                             H
10    SMAX=D/4.                                                            H
C                                                                          H
C     THE MAXIMUM SPACING FOR A MIN. REQUIRED AV.                          H
15    SMA=AV/(0.0015*B)                                                    H
C                                                                          H
C     ---------------------------                                          H
C                                                                          H
C     SHAER STRESS CARIED BY WEB REINFORC.                                 H
      VUS=VS-VUC                                                           H
      IF (VUS) 25,25,20                                                    H
C                                                                          H
C     MAX.SPACING FOR ACTUAL STRESS                                        H
20    SM=0.85*AV*FY/(B*VUS)                                                H
      GO TO 30                                                             H
C                                                                          H
C     THE WEB R. IS NOT REQUIRED - MIN. SPACING IS DETERNINED FROM PRACT   H
25    SM=14.                                                               H
      SMAX=1000.                                                           H
      SMA=100.                                                             H
30    IF (SMAX.LT.SMA) GO TO 40                                            H
      IF (SMA.LT.SM) GO TO 35                                              H
      SP=SM                                                                H
      GO TO 50                                                             H
35    SP=SMA                                                               H
      GO TO 50                                                             H
40    IF (SMAX.LT.SM) GO TO 45                                             H
      SP=SM                                                                H
      GO TO 50                                                             H
45    SP=SMAX                                                              H
50    RETURN                                                               H
      END                                                                  H
```

```
      SUBROUTINE SIMPLE (B,D,AS,EK,USB,F1)                                  I
C                                                                           I
C     ***************************************************************      I
C     PURPOSE OF THIS SUBROUTINE IS TO DETERMINE THE USABLE ULTIMATE        I
C     MOMENT OF THE RECTANGULAR SECTION WITH TENSION REINFORCEMENT ONLY.    I
C     ***************************************************************      I
C                                                                           I
      COMMON /BLOK2/ FC,FY                                                  I
C                                                                           I
C     BALANCED CONDITION                                                    I
C     ------------------------------                                        I
C                                                                           I
C                                                                           I
C     THE POSITION OF N.A.=EX                                               I
      EX=3.*D/(3.+FY/29000.)                                                I
C                                                                           I
C     THE FORCE DEVELOPED BY CONCRETE                                       I
      CB=F1*EK*B*EX                                                         I
      ASB=CB/FY                                                             I
C                                                                           I
C     PB= BALANCED RATIO OF REINFORCEMENT                                   I
      PB=(ASB/(B*D))*0.75                                                   I
C                                                                           I
C                                                                           I
C     ACTUAL CONDITION                                                      I
C     ------------------------------                                        I
C                                                                           I
C     P= AN ACTUAL RATIO OF REINFORCEMENT                                   I
      P=AS/(B*D)                                                            I
      IF (PB-P) 10,10,5                                                     I
5     T=AS*FY                                                               I
      EX1=T/(F1*B*EK)                                                       I
      USB=0.9*T*(D-EX1*EK/2.)                                               I
      RETURN                                                                I
10    USB=0.1                                                               I
      RETURN                                                                I
      END                                                                   I
```

CD TOT 0037

```
      SUBROUTINE SEEK1 (N,RMAX,RMIN,NCONS,NEQUS,F,G,XSTRT,NSHOT,NTEST,MA     G
     1XM,IPRINT,IDATA,X,U,PHI,PSI,WORK1,WORK2,WORK3,WORK4)                   G
      DIMENSION RMAX(1), RMIN(1), XSTRT(1), X(1), PHI(1), PSI(1), WORK1(     G
     11), WORK2(1), WORK3(1), WORK4(1)                                       G
      COMMON KO,NNDEX                                                        G
      IF (IDATA.NE.1) GO TO 1                                                G
      WRITE (6,13) IPRINT                                                    G
      WRITE (6,14) IDATA                                                     G
      WRITE (6,12) N                                                         G
      WRITE (6,15) NCONS                                                     G
      WRITE (6,16) F                                                         G
      WRITE (6,17) MAXM                                                      G
      WRITE (6,18) G                                                         G
      WRITE (6,24) NEQUS                                                     G
      WRITE (6,19) NSHOT                                                     G
      WRITE (6,20) NTEST                                                     G
      WRITE (6,21) (RMAX(I),I=1,N)                                           G
      WRITE (6,22) (RMIN(I),I=1,N)                                           G
      WRITE (6,23) (XSTRT(I),I=1,N)                                          G
1     WRITE (6,9)                                                            G
C     ZERO WORKING ARRAYS                                                    G
      DO 2 I=1,N                                                             G
      X(I)=0.0                                                               G
      WORK1(I)=0.0                                                           G
      WORK2(I)=0.0                                                           G
      WORK3(I)=0.0                                                           G
      WORK4(I)=0.0                                                           G
2     CONTINUE                                                               G
C                                                                            G
C     SEARCH IS USED BY BOTH SEEK1 AND SEEK3                                 G
C                                                                            G
      KO=0                                                                   G
      NNDEX=1                                                                G
      INDEX=1                                                                G
C     INDEX=0 INDICATES TO SEARCH THAT IT IS BEING USED BY FEASBL            G
      KOUNT=0                                                                G
      R=1.                                                                   G
3     CALL SEARCH (X,U,N,XSTRT,RMAX,RMIN,PHI,PSI,NCONS,NEQUS,MAXM,NVIOL,     G
     1F,G,IPRINT,INDEX,R,WORK1,WORK2,WORK3,WORK4)                            G
      CALL SHOT (U,X,N,KK,PHI,PSI,NCONS,NEQUS,RMAX,RMIN,F,NTEST,NSHOT,WO     G
     1RK1,WORK2,WORK3)                                                       G
C     CHECK TO SEE WHETHER SUBR.SHOT HAS FOUND AN IMPROVED POINT             G
      IF (KK.EQ.1) GO TO 4                                                   G
      IF (KO.EQ.0) RETURN                                                    G
C     KO  CANNOT BE RESET IN SUBR.SHOT, THEREFORE IF KO=1 AT THIS STAGE      G
C     THEN SUBR.SEARCH FAILED AND SHOT FOUND NO IMPROVEMENT                  G
      WRITE (6,8)                                                            G
      GO TO 7                                                                G
4     IF (IPRINT.GT.0) WRITE (6,11) U,(X(I),I=1,N)                           G
      KOUNT=KOUNT+1                                                          G
      IF (KOUNT.LE.NSHOT) GO TO 5                                            G
      WRITE (6,10) NSHOT                                                     G
      KO=1                                                                   G
      GO TO 7                                                                G
C     REDEFINE STARTING POINT FOR SEARCH                                     G
5     DO 6 I=1,N                                                             G
      XSTRT(I)=X(I)                                                          G
6     CONTINUE                                                               G
      GO TO 3                                                                G
```

```
C     PRINT OUT LAST ITERATION RESULTS(KO=1)                                    G
7     CALL ANSWER (U,X,PHI,PSI,N,NCONS,NEQUS)                                   G
      CALL EXIT                                                                 G
C                                                                               G
8     FORMAT (1H-,71HDIRECT SEARCH HAS HUNG UP AND SHOTGUN SEARCH CANNOT        G
     1 FIND A BETTER POINT/41HTRY A DIFFERENT STARTING POINT AT LEVEL=1/        G
     2)                                                                         G
9     FORMAT (1H1,10X,38HDIRECT SEARCH OPTIMIZATION USING SEEK1//)              G
10    FORMAT (1H-,48HSHOTGUN SEARCH FOUND AN IMPROVEMENT BUT  NSHOT =,I6        G
     1,18H HAS BEEN EXCEEDED)                                                   G
11    FORMAT (1H-,7H.SHOT. ,5E16.8/(24X,4E16.8))                                G
12    FORMAT (61H0NUMBER OF INDEPENDENT VARIABLES . . . . . . . . . . .         G
     1  N   =,I6)                                                               G
13    FORMAT (61H0INTERMEDIATE OUTPUT EVERY IPRINT(TH) CYCLE. . . .   IPR       G
     1INT   =,I6)                                                               G
14    FORMAT (61H0INPUT DATA IS PRINTED OUT FOR IDATA=1 ONLY. . . .    ID       G
     1ATA   =,I6)                                                               G
15    FORMAT (61H0NUMBER OF INEQUALITY (.GE.) CONSTRAINTS . . . . . .   NC      G
     1ONS   =,I6)                                                               G
16    FORMAT (61H0FRACTION OF RANGE USED AS STEP SIZE . . . . . . . .           G
     1  F   =,E19.8)                                                            G
17    FORMAT (61H0MAXIMUM NUMBER OF MOVES PERMITTED . . . . . . . . .     M     G
     1AXM   =,I6)                                                               G
18    FORMAT (61H0STEP SIZE FRACTION USED AS CONVERGENCE CRITERION.             G
     1  G   =,E19.8)                                                            G
19    FORMAT (61H0NUMBER OF SHOTGUN SEARCHES PERMITTED. . . . . . . .   NS      G
     1HOT   =,I6)                                                               G
20    FORMAT (61H0NUMBER OF TEST POINTS IN SHOTGUN SEARCH . . . . . .   NT      G
     1EST   =,I6)                                                               G
21    FORMAT (61H0ESTIMATED UPPER BOUND ON RANGE OF X(I). . . . . . . RMAX      G
     1(I)   =,//(5E16.8))                                                       G
22    FORMAT (61H0ESTIMATED LOWER BOUND ON RANGE OF X(I). . . . . . . RMIN      G
     1(I)   =,//(5E16.8))                                                       G
23    FORMAT (61H-STARTING VALUES OF X(I) . . . . . . . . . . . . . .XSTRT      G
     1(I)   =,//(5E16.8))                                                       G
24    FORMAT (61H0NUMBER OF EQUALITY CONSTRAINTS. . . . . . . . . . .   NE      G
     1QUS   =,I6)                                                               G
      END                                                                       G
```

CD TOT   0097

```
      SUBROUTINE SEARCH(X,U,N,XSTRT,RMAX,RMIN,PHI,PSI,NCONS,NEQUS,MAXM,N
     1VIOL,F,G,IPRINT,INDEX,R,XO,XB,DXXX,TXXX)
      DIMENSION X(1),XSTRT(1),RMAX(1),RMIN(1),PHI(1),PSI(1),XO(1),XB(1),
     1DXXX(1),TXXX(1)
      COMMON KO,NNDEX
C
C     DIRECT  SEARCH  PORTION  OF  SEEK1  AND  SEEK3
C
C     THIS IS THE DIRECT SEARCH ALGORITHM OF HOOKE AND JEEVES
C     SEARCH IS USED BE SEEK1 AND SEEK3
C     NNDEX=1  MEANS SEARCH HAS BEEN CALLED BY SEEK1
C     NNDEX=2  MEANS SEARCH HAS BEEN CALLED BY SEEK3
      NVIOL1=1
      KKK=0
      M1 = 0
   20 K1=1
      K2=N
   30 DO 40   I=K1,K2
      DXXX(I)=0.
      TXXX(I)=0.
      XO(I)=0.
   40 XB(I)=0.
      DO 60  I=K1,K2
   60 X(I) = XSTRT(I)
C  SET  FIRST  BASE  POINT
      DO 70 I=K1,K2
   70 XO(I) =X(I)
C     GENERATE  DELX(I)  AND  TEST(I)
      DO 80 I=K1,K2
      DXXX(I) = F*(RMAX(I)-RMIN(I))
   80 TXXX(I)=DXXX(I)*G
      NCALL=1
  100 CONTINUE
      GO TO (101,102)NNDEX
  101 CALL OPTIMF1(X,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
      GO TO 110
  102 CALL OPTIMF2(X,UART,PHI,PSI,NCONS,NEQUS,NVIOL,R)
  110 IF(NCALL.NE.1)GOTO   120
      UARTO = UART
  120 CONTINUE
      IF(NVIOL.EQ.0)NVIOL1=0
      IF(NNDEX.EQ.1) GO TO 130
C     INDEX=0 INDICATES TO SEARCH THAT IT IS BEING USED BY FEASBL
      IF(INDEX.EQ.1) GO TO 130
C     IF SEARCH IS BEING USED MERELY TO OBTAIN A FEASIBLE STARTING POINT
C     THEN RETURN AS SOON AS SOLUTION GOES FEASIBLE
      IF(NVIOL1.EQ.0)GO TO   385
  130 GO TO (170, 200,  210,  355) NCALL
  170 CONTINUE
C  MAKE  SEARCH
  180 NFAIL=0
      DO 240 I=K1,K2
      X(I)=X(I)+DXXX(I)
      NCALL=2
      GO TO 100
  200 CONTINUE
      IF(UART.LT.UARTO)  GOTO  230
      X(I)=X(I) - 2.0*DXXX(I)
      NCALL=3
```

```
      GO TO 100
  210 CONTINUE
      IF(UART.LT.UARTO)  GOTO  230
      NFAIL = NFAIL + 1
      X(I)=X(I)+DXXX(I)
      GOTO  240
  230 UARTO = UART
  240 CONTINUE
  250 IF(NFAIL.EQ.N)GOTO 260
      GOTO  315
  260 DO 280 I=K1,K2
      IF(DXXX(I).GT.TXXX(I)) GO TO  290
  280 CONTINUE
      GO TO 385
  290 DO 310  I=K1,K2
  310 DXXX(I)=DXXX(I)/2.
      GOTO 180
C     ESTABLISH  NEW  BASE  POINT
  315 DO 320  I=K1,K2
  320 XB(I) = X(I)
      M1 = M1 + 1
      IF (NNDEX.EQ.1) GO TO 330
      GO TO  340
  330 KKK=KKK+1
      IF(KKK.NE.IPRINT) GO TO 340
      CALL UREAL(X,ULOW)
      WRITE (6,2)  M1,ULOW , (X(I), I=1,N)
      KKK=0
  340 CONTINUE
      IF(M1.GT.MAXM) GO TO 385
C     MAKE A PATTERN MOVE
      DO 350  I=K1,K2
  350 X(I) = X(I) + (X(I) - XO(I))
      NCALL=4
      GO TO 100
  355 CONTINUE
      IF(UART.LT.UARTO)  GOTO 370
      DO 360 I=K1,K2
      XO(I) = XB(I)
  360 X(I) = XB(I)
      GOTO 180
  370 DO 380  I=K1,K2
  380 XO(I) = XB(I)
      UARTO = UART
      GOTO 180
  385 CALL UREAL(X,U)
      GO TO(103,104)NNDEX
  103 CALL OPTIMF1(X,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
      GO TO 105
  104 CALL OPTIMF2(X,UART,PHI,PSI,NCONS,NEQUS,NVIOL,R)
  105 IF(NVIOL.EQ.0)GOTO387
      IF(M1.GT.MAXM)WRITE(6,4)MAXM
      KO=1
  387 RETURN
    2 FORMAT(1H0,I4,3X,5E16.8/(24X,4E16.8))
    4 FORMAT(1H0,60HNO FEASIBLE SOLUTION AFTER ALLOWABLE NUMBER OF MOVES
     1, MAXM =,I6/)
      END
```

```
      SUBROUTINE SHOT(U,X,N,KK,PHI,PSI,NCONS,NEQUS,RMAX,RMIN,F,NTEST,NSH
     1OT,RR,XX,RF)
      DIMENSION PHI(1),PSI(1),RMAX(1),RMIN(1),X(1),RR(1),XX(1),RF(1)
      COMMON KO,NNDEX
C     U=OPTIMUM DETERMINED BY DIRECT SEARCH.  IT IS CHANGED TO IMPROVED
C     VALUE IF SUCH A VALUE IS OBTAINED
C     XX= TRIAL VALUES OF X(I) FROM SHOTGUN SEARCH
C     RF= FRACTION OF RANGE USED IN SHOTGUN SEARCH
C     KK= INDICATOR TO SHOW IF U RETURNED IS AN IMPROVEMENT
C     INITIALIZE RANDOM NUMBER GENERATOR
      CALL FRANDN(RR,N,1)
      UMIN=U
      KK=0
C     THIS SHOTGUN SEARCH IS INTENDED TO GET THE SOLUTION OFF A FENCE
C     RATHER THAN TO INCH IT TOWARDS THE OPTIMUM. THEREFORE LARGE STEPS,
C     EQUAL 10. TIMES THE INITIAL STEP SIZE IN SEARCH ARE TRIED.
      DO 1 I=1,N
    1 RF(I)=10.*F*ABS(RMAX(I)-RMIN(I))
      DO 4 J=1,NTEST
      CALL FRANDN(RR,N,0)
      DO 2 I=1,N
    2 XX(I)=(X(I)-RF(I))+RR(I)*2.0*RF(I)
      CALL OPTIMF1(XX,UTEST,PHI,PSI,NCONS,NEQUS,NVIOL)
      IF(NVIOL.NE.0)GOTO4
      IF(UTEST.GE.UMIN)GOTO4
      UMIN=UTEST
      U=UTEST
      DO 3 I=1,N
    3 X(I)=XX(I)
      KK=1
    4 CONTINUE
      RETURN
      END
      SUBROUTINE OPTIMF1(X,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
      DIMENSION X(1),PHI(1),PSI(1)
C     VERY MINOR VIOLATIONS OF INEQUALITY CONSTRAINTS SHOULD NOT MAKE
C     THE ENTIRE SOLUTION INFEASIBLE. THEREFORE TEST FOR PHI(I).GE.ZERO
C     WHERE ZERO=-1.0E-10
      ZERO=-1.0E-10
      NVIOL=0
      SUM1=0.0
      SUM2=0.0
      CALL UREAL(X,U)
C
C     SEEK1  PENALTY  FUNCTIONS -
C
C     A  ROUTINE TO CALCULATE A VALUE FOR AN ARTIFICIAL OBJECTIVE
C     FUNCTION OF THE FORM
C        UART=UREAL+SUM(ABS(PHI(I)))*10.E20+SUM(ABS(PSI(I)))*10.E20
C     WHERE
C     PSI(I) AND PHI(I) IN THE ABOVE EXPRESSION ARE THE VALUES OF THE
C     CORESPONDING EQUALITY AND INEQUALITY CONSTRAINTS THAT HAVE BEEN
C     VIOLATED
      IF(NCONS.EQ.0)GOTO2
      CALL CONST(X,NCONS,PHI)
      DO 1 I=1,NCONS
      IF(PHI(I).GE.ZERO)GOTO1
      SUM1=SUM1 + ABS(PHI(I))*10.0E+20
      NVIOL=NVIOL + 1
```

```
  1 CONTINUE
  2 IF(NEQUS.EQ.0)GOTO115
    CALL EQUAL(X,PSI,NEQUS)
    DO 3 I=1,NEQUS
  3 SUM2=SUM2 + ABS(PSI(I))*10.0E+20
115 UART=U+SUM1+SUM2
    RETURN
    END
```

CD TOT 0067

```
      SUBROUTINE FRANDN(A,N,M)
      DIMENSION A(1)
C     B  IS A MACHINE-DEPENDENT CONSTANT AND  B=2.0**(I/2+1)+3.0
C     WHERE  I = NUMBER OF BITS IN AN INTEGER WORD (I=47 FOR CDC6400)
      B=262147.0
      X=M
      X=X/0.8719467
   20 IF(X.NE.0.0)Y=AMOD(ABS(X),3.18967)
      DO 10 K=1,N
      DO 11 J=1,2
   11 Y=AMOD(B*Y,1.0)
      A(K)=Y
C     AVOID Y=0. AND Y=1. TO PREVENT DIVIDING INTO ZERO
   10 IF(Y.EQ.0.0.OR.Y.EQ.1.0)Y=0.182818285
      RETURN
      END
```

CD TOT   0016

```
      SUBROUTINE ANSWER(U,X,PHI,PSI,N,NCONS,NEQUS)
      DIMENSION X(1),PHI(1),PSI(1)
      COMMON KO,NNDEX
C     THIS SUBROUTINE IS USED MERELY TO OUTPUT THE FINAL SOLUTION IN A
C     STANDARD FORM. IF AN OPTIMUM IS NOT REACHED(KO=1)THEN THE RESULTS
C     AT THE LAST ITERATION MAY BE PRINTED OUT.
      CALL UREAL(X,U)
      IF(KO.EQ.0)GOTO1
      WRITE(6,18)
      WRITE(6,19)U
      GOTO2
    1 WRITE(6,20)
      WRITE(6,21)U
    2 WRITE(6,22)(I,X(I),I=1,N)
      IF(NCONS.EQ.0)GOTO3
      CALL CONST(X,NCONS,PHI)
      WRITE(6,23)
      WRITE(6,24)(I,PHI(I),I=1,NCONS)
    3 IF(NEQUS.EQ.0)GOTO30
      CALL EQUAL(X,PSI,NEQUS)
      WRITE(6,25)
      WRITE(6,26)(I,PSI(I),I=1,NEQUS)
   18 FORMAT(1H-,16X,25HRESULTS AT LAST ITERATION,/)
   19 FORMAT(29X,3HU =,E16.8//)
   20 FORMAT(1H1,21X,22HOPTIMUM SOLUTION FOUND,/)
   21 FORMAT(20X,12HMINIMUM  U =,E16.8//)
   22 FORMAT(25X,2HX(,I2,3H) =,E16.8)
   23 FORMAT(1H-,22HINEQUALITY CONSTRAINTS)
   24 FORMAT(23X,4HPHI(,I2,3H) =,E16.8)
   25 FORMAT(1H-,22H  EQUALITY CONSTRAINTS)
   26 FORMAT(23X,4HPSI(,I2,3H) =,E16.8)
   30 RETURN
      END
```

CD TOT   0033

APPENDIX B

```
HRDZ,CM60000,T30.                                                    JANO.
RUN(S)
SETINDF.
REDUCE.
LGO.
'        6400 END OF RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     MC MASTER UNIVERSITY, CIVIL ENGINEERING, HAMILTON, ONT., CANADA.
C
C     D E S I G N   O F   F L A T   P L A T E
C     ----------------------------------------
C
C     PROGRAM DEVELOPER- JAN JANOSIK
C
C     SCOPE
C     --------
C      THIS PROGRAM WAS DEVELOPED DURING 1970-1971 PERIOD AS THE PART
C     OF THE AUTHOR,S  THESIS WORK WITH INTENTION TO HELP THE DESIGNER T
C     TO  DESIGN MOST ECONOMICAL STRUCTURE.   THE CONSTRAINTS FOR
C     THE DESIGN ARE IN ACCORDANCE WITH NATIONAL BUILDING CODE OF
C     CANADA 1970.
C
C     INPUT                                                             A
C     ******************************************************************A
C                                                                       A
C     STRENGTH - PRICE                                                  A
C     ----------------                                                  A
C     IPS            1-PRINT-OUT OF THE INPUT DATA OF STRENGTH-PRICE    A
C                    2- NO PRINTOUT OF THE INPUT DATA OF STRENGTH-PRICE A
C     JAK            1- FOR SINGLE FC,UPC,                              A
C                    2- FOR SET OF COUPLES FC,UPC                       A
C     MB             RANGE OF STRENGTHS OF CONCRETE                     A
C     MS             RANGE OF YIELD STENGTHS OF STEEL                   A
C     FC(1,...,MB) PARTICULAR COMPRESSION STRENGTH OF CONCRETE 28 DAYS  A
C                    AFTER POURING                         (LB/IN**2)   A
C     UPC(1,.,MB)    THE UNIT PRICE OF READY MIXED CONCRETE    ($/Y**3) A
C     FY(1,..,MS)    PARTICULAR YIELD STRENGTH OF REINFORCING STEEL     A
C                                                          (LB/IN**2)   A
C     UPS(1,.,MS)    UNIT PRICE OF STEEL                   ($/LB)       A
C                                                                       A
C     UPFS           UNIT PRICE OF FORMWORK FOR SLAB       ($/FT**2)    A
C                                                                       A
C     ZL             SUPERIMPOSED LOAD                     (LB/FT**2)   A
C                                                                       A
C     GEOMETRY                                                          A
C     -------------                                                     A
C                                                                       A
C                              BEMOD (FT)                               A
C                +---------------------------------------+              A
C               *-*                                     *-*             A
C                                                           +           A
C               * *                                     * * G           A
C                                                           I           A
C                              C                        D   M           A
C                                                           O           A
C               * *                                     * * D           A
C                              A                        B   +           A
C               * *                                     * *             A
C                                                                       A
```

```
C       CA              THICKNESS OF COLUMN SUPPORTING THE GIRDER TAKEN        A
C                       IN DIRECTION ,BEMOD,                           (IN)    A
C       CB              THICKNESS OF COLUMN IN DIRECTION GIMOD         (IN)    A
C       HS              HEIGTH OF STOREY                               (FT)    A
C                                                                              A
C       DETAILING                                                              A
C       ------------                                                           A
C       IRUN            1- FOR THE FIRST RUN- THE VALUES OF COV,...,COVBP      A
C                       ARE SET INTERNALLY                                     A
C                       2- FOR THE ADJUSTING RUNS                              A
C       COV             0.75IN+HALF DIAMETER OF POSIT. STEEL IN SLAB (IN)      A
C                                                                              A
C                                                                              A
C       INPUT DATA PREPARATION.                                                A
C       ************************************************************          A
C                                                                              A
C       SEQUENCE OF CARDS  VARIABLES                              FORMAT       A
C       +-----------------+---------------------------------------+---------- A
C       1                 ANY ALPHANUMERIC TITLE                  13A6         A
C       2                 IPS,JAK,MB,MS,IRUN                      5I5          A
C       3...              FC(I),I=1,MB                            4F20.5       A
C       4...              FY(J),J=1,MS                            4F20.5       A
C       5...              UPC(I),I=1,MB                           4F20.5       A
C       6...              UPS(J),J=1,MS                           4F20.5       A
C       7                 UPFS                                     F20.5       A
C       8                 GIMOD,BEMOD,ZL                          3F20.5       A
C       9                 CA,CB,HS                                3F20.5       A
C       IF IRUN=2     -ADD THE FOLLOWING CARDS                                 A
C       10...             RMAX(I),I=1,16                          5F16.8       A
C       11...             RMIN(I),I=1,16                          5F16.8       A
C       12...             XSTRT(I),I=1,16                         5F16.8       A
C       13                COV                                      F16.8       A
C                                                                              A
C       NOTE = IF SOME VARIABLE(J) IS DESIRED TO BE CERTAIN UNCHANGED          A
C       VALUE  - SET RMAX(J), RMIN(J), XSTRT(J)  EQUAL TO THIS VALUE.          A
C                                                                              A
C                                                                              A
C       OUTPUT                                                                 A
C       *****************                                                      A
C                                                                              A
C       X1      = THICKNESS OF THE SLAB (IN)                                   A
C       X2      = AREA OF POSITIVE STEEL(IN**2) AT,A,                          A
C                                              ..COLUMN STRIP   -..B           A
C       X3      = AREA OF NEGATIVE STEEL(IN**2).AT,B,                  .E      A
C                                                                      .M      A
C       X4      = AREA OF POSITIVE STEEL(IN**2) AT,C,                  .O      A
C                                              ..COLUMN STRIP -B-..D           A
C       X5      = AREA OF NEGATIVE STEEL(IN**2).AT,D,                          A
C                                                                              A
C       X6      = AREA OF POSITIVE STEEL(IN**2) AT,A,                          A
C                                              ..COLUMN STRIP -C-..G           A
C       X7      = AREA OF NEGATIVE STEEL(IN**2).AT,B,                  .I      A
C                                                                      .M      A
C       X8      = AREA OF POSITIVE STEEL(IN**2) AT,C,                  .O      A
C                                              ..COLUMN STRIP -D-..D           A
C       X9      = AREA OF NEGATIVE STEEL(IN**2).AT,D,                          A
C       THE PROGRAM BEGINS                                                     A
```

```
C     THE PROGRAM BEGINS                                                A
C     ==================                                                A
C                                                                       A
      COMMON /BLOK1/ IPS,JAK/BLOK2/FC,FY/BLOK3/UPC,UPS,UPFS             A
      COMMON /BLOK4/ GIMOD,BEMOD,COV,ZL                                 A
      COMMON /BLOK5/ CA,CB,HS                                           A
      DIMENSION WORK1(16), WORK2(16), WORK3(16), WORK4(16)              A
      DIMENSION X(9), PHI(29), PSI(1), XSTRT(9), RMAX(9), RMIN(9)       A
      DIMENSION PEVB(20), PEVS(20), CENB(20), CENS(20)                  A
      DIMENSION TITLE(13)                                               A
C                                                                       A
C                                                                       A
      READ (5,7) (TITLE(I),I=1,13)                                      A
      DATA N,IPRINT,IDATA,NCONS,F,MAXM,G,NEQUS,NSHOT,NTEST/ 9,1,1,29,0.0A
     11,300,0.10,0,1,50/
      WRITE (6,7) (TITLE(I),I=1,13)                                     A
C                                                                       A
      READ (5,8) IPS,JAK,MB,MS,IRUN                                     A
C                                                                       A
      READ (5,9) (PEVB(J),J=1,MB)                                       A
      READ (5,9) (PEVS(JJ),JJ=1,MS)                                     A
      READ (5,9) (CENB(J),J=1,MB)                                       A
      READ (5,9) (CENS(JJ),JJ=1,MS)                                     A
      IF (IPS.EQ.1) GO TO 1                                             A
      GO TO 2                                                           A
1     WRITE (6,10)                                                      A
      WRITE (6,9) (PEVB(J),J=1,MB)                                      A
      WRITE (6,11)                                                      A
      WRITE (6,9) (CENB(J),J=1,MB)                                      A
      WRITE (6,12)                                                      A
      WRITE (6,9) (PEVS(JJ),JJ=1,MS)                                    A
      WRITE (6,13)                                                      A
      WRITE (6,9) (CENS(JJ),JJ=1,MS)                                    A
2     READ (5,14) UPFS                                                  A
C                                                                       A
      READ (5,15) GIMOD,BEMOD,ZL                                        A
      READ (5,15) CA,CB,HS                                              A
      IF (IRUN.EQ.2) GO TO 3                                            A
C                                                                       A
      XSTRT(1) = BEMOD *0.3
      XSTRT(2)=BEMOD*0.021                                              A
      XSTRT(3)=BEMOD*0.0355                                             A
      XSTRT(4)=BEMOD*0.0169                                             A
      XSTRT(5)=BEMOD*0.017                                              A
      XSTRT(6)=GIMOD*0.021                                              A
      XSTRT(7)=GIMOD*0.0355                                             A
      XSTRT(8)=GIMOD*0.0169                                             A
      XSTRT(9)=GIMOD*0.017                                              A
C                                                                       A
      AC = XSTRT(1)*12.
C     ASMAX  =  MAXIMUM STEEL IS TO BE 10 PERCENT OF  GROSS AREA OF CONCRET
      ASMAX = 0.1*AC
      RMAX(1)=(BEMOD+GIMOD)*0.3                                         A
      DO 30 M=2,9
      RMAX(M) = ASMAX
30    CONTINUE
C                                                                       A
      ASMIN = .002*AC
      RMIN(1)=(BEMOD+GIMOD)*0.125                                       A
```

```
      DO 20 L=2,9
      RMIN(L) = ASMIN
20    CONTINUE
      DATA COV/2.25/                                                    A
      GO TO 4                                                           A
C                                                                       A
C     SUPPLIED VALUES OF EXPECTED BOUNDARIES MAX.,MIN.,AND              A
C     STARTING VALUES RESPECTIVELY                                      A
3     READ (5,16) (RMAX(I),I=1,9)                                       A
      READ (5,16) (RMIN(I),I=1,9)                                       A
      READ (5,16) (XSTRT(I),I=1,9)                                      A
      READ (5,17) COV                                                   A
C                                                                       A
4     DO 6 I=1,MB                                                       A
      FC=PEVB(I)                                                        A
      UPC=CENB(I)/27.                                                   A
      DO 5 J=1,MS                                                       A
      FY=PEVS(J)                                                        A
      UPS=CENS(J)                                                       A
      CALL SEEK1 (N,RMAX,RMIN,NCONS,NEQUS,F,G,XSTRT,NSHOT,NTEST,MAXM,IPR A
     1INT,IDATA,X,U,PHI,PSI,WORK1,WORK2,WORK3,WORK4)                    A
      CALL ANSWER (U,X,PHI,PSI,N,NCONS,NEQUS)                           A
5     CONTINUE                                                          A
6     CONTINUE                                                          A
      STOP                                                              A
C                                                                       A
C                                                                       A
7     FORMAT (13A6)                                                     A
8     FORMAT (5I5)                                                      A
9     FORMAT (4F20.5)                                                   A
10    FORMAT (3X,*COMPRESSION STRENGTH OF CONCRETE FC(PSI)*)            A
11    FORMAT (3X,*UNIT PRICE OF CONCRETE UPC($/CU.YD.)*)                A
12    FORMAT (3X,*YIELD STRENGTH OF R.F. STEEL FY(PSI)*)                A
13    FORMAT (3X,*UNIT PRICE OF STEEL*)                                 A
14    FORMAT (1F20.5)                                                   A
15    FORMAT (3F20.5)                                                   A
16    FORMAT (5F16.8)                                                   A
17    FORMAT (F16.8)                                                    A
      END                                                               A
```

```
      SUBROUTINE UREAL (X,U)
C
C     ******************************************************************
C     PURPOSE OF THIS SUBROUTINE IS TO CALCULATE OBJECTIVE FUNCTION=
C     = U = PRICE OF R. F. FLAT PLATE SUPPORTED ON COLUMNS ONLY
C     ******************************************************************
C
      DIMENSION X(1)
      COMMON /BLOK1/ IPS,JAK/BLOK2/FC,FY/BLOK3/UPC,UPS,UPFS
      COMMON /BLOK5/ CA,CB,HS
      COMMON /BLOK4/ GIMOD,BEMOD,COV,ZL
      W=490.
C
C     AUXILIARY INVERTION OF UNITS
      X1=X(1)/12.
      X2=X(2)/144.
      X3=X(3)/144.
      X4=X(4)/144.
      X5=X(5)/144.
      X6=X(6)/144.
      X7=X(7)/144.
      X8=X(8)/144.
      X9=X(9)/144.
C
C     CON = PRICE OF CONCRETE
      CON=UPC*BEMOD*GIMOD*X1
C
C     STEEL = PRICE OF REINFORCING STEEL
      STEEL=UPS*W*0.5*(BEMOD*(X2+X3+X4+X5)+GIMOD*(X6+X7+X8+X9))
C
C     FORM = PRICE OF FORMWORK
      FORM=UPFS*(BEMOD*GIMOD-(CA*CB/144.))
C
C     THE OBJECTIVE FUNCTION ,U,
C     --------------------------
      U=CON+STEEL+FORM
      RETURN
      END
```

```
      SUBROUTINE CONST (X,NCONS,PHI)
C
C     ******************************************************************
C     PURPOSE OF THIS SUBROUTINE IS TO CALCULATE VALUES OF
C     THE INEQUALITY CONSTRAINTS.
C     AT THE FEASIBLE POINT PHI.GE.0.
C     ******************************************************************
C
      DIMENSION X(1), PHI(1)
      COMMON /BLOK2/ FC,FY
      COMMON /BLOK4/ GIMOD,BEMOD,COV,ZL
      COMMON /BLOK5/ CA,CB,HS
C     AUXILIARY VALUES
C     ------------------------------
      BEM=12.*BEMOD
      GIM=12.*GIMOD
      SQ=SQRT(FC)
      TU=X(1)-COV
      ZD=12.5*X(1)
      WU=1.5*ZD+1.8*ZL
      B=12.
      EC=57500.*SQ
      X2=X(2)
      X3=X(3)
      X4=X(4)
      X5=X(5)
      X6=X(6)
      X7=X(7)
      X8=X(8)
      X9=X(9)
      FCC=FC-4000.
      IF (FCC) 1,1,2
1     EK=0.85
      GO TO 3
2     EK=0.85-0.05*(FCC/1000.)
3     F1=0.85*FC
C
C      REDUCED SPANS
      BEMO=(BEM-CA)/12.
      GIMO=(GIM-CB)/12.
C
C     MOMENT CONSTRAINTS
C     ==============================
C
C     BENT -B- LONG DIRECTION
C     -------------------------
C
      WUB = WU*GIMOD/2.
      QB=WUB      *BEMO**2
C     NEGATIVE MOMENT IN BENT *B* AT*B*
      BMBNEG=QB/11.
C     POSITIVE MOMENT IN BENT*B* AT*A*
      BMBPOS =(QB/16.)*12.
C     CRITICAL SECTION(FT)
      AB=(CB+X(1))/24.
C
C     LEVELED NEGATIVE MOMENT AT -B-AB-
      RMB =(ABS(BMBNEG)-AB**2*WUB/2.)*12.
C
```

```
C     PERCENTAGE DISTRIBUTION OF BENT MOMEMTS TO STRIP MOMENTS
C
      AB = .60*RMB
      BB= .76*RMB
      OR = .40*BMBPOS
      DB= .24*BMBPOS
C
C     BENT -G- SHORT DIRECTION
C
      WUG = WU*BEMOD/2.
      QG=WUG      *GIMO**2
C     NEGATIVE MOMENT IN BENT*G* AT*B* IN LB-FT.
      BMGNEG=QG/11.
C     POSITIVE MOMENT IN BENT*G* AT*B* INLB-IN.
      BMGPOS=(QG/16.)*12.
C
C     CRITICAL SECTION (FT)
      AG=(CA+X(1))/24.
C
C     LEVELED NEGATIVE MOMENT
      RMG=(ABS(BMGNEG)-AG**2*WUG/2.)*12.
C
C     PERCENTAGE DISTRIBUTION , ACTING IN BEMOD DIRECTION
C
      AG=.24*RMG
      BG=.76*RMG
      CG=.40*BMGPOS
      DG=.60*BMGPOS
C
C     COMPUTE F,MO
      CR=SQRT(4.*CA*CB/3.1415)
      FG=1.15-CR/GIM
      IF (FG.LT.1.) FG=1.
      FB=1.15-CR/BEM
      IF (FB.LT.1.) FB=1.
C
      OMB= 0.1*WU*BEMOD**2*FB*(1.-2.*CR/(3.*BEM))**2
      OMG= 0.1*WU*GIMOD**2*FG*(1.-2.*CR/(3.*GIM))**2
      SUMB =RMB+BMBPOS
      IF (SUMB .LT.OMB) GO TO 4
      GO TO 5
4     BB=.76*(OMB-BMBPOS)
      AB=.60*(OMB-BMBPOS)
5     SUMG =RMG+BMGPOS
      IF(SUMG.LT.OMG) GO TO 6
      GO TO 7
6     BG =.76*(OMG-BMGPOS)
      AG=.600*(OMG-BMGPOS)
C
C     COMPUTE ULTIMATE MOMENT CAPACITY OF THE SECTIONS
7     CALL SIMPLE (B,TU,X2,FK,USM2,F1)
      CALL SIMPLE (B,TU,X3,FK,USM3,F1)
      CALL SIMPLE (B,TU,X4,FK,USM4,F1)
      CALL SIMPLE (B,TU,X5,FK,USM5,F1)
      CALL SIMPLE (B,TU,X6,FK,USM6,F1)
      CALL SIMPLE (B,TU,X7,FK,USM7,F1)
      CALL SIMPLE (B,TU,X8,FK,USM8,F1)
```

```
      CALL SIMPLE (B,TU,X9,FK,USM9,F1)
C     BE, ANDGI= UNIT STRIP   IN THIS CASE 1 FT. WIDE STRIP
      BE= BEMOD*.5
      GI=GIMOD*.5
      PHI(1)=(USM2-AB/BE)*1.E-05
      PHI(2)=(USM3-BB/BE)*1.E-05
      PHI(3)=(USM4-QR/BE)*1.E-05
      PHI(4)=(USM5-DB/BE)*1.E-05
      PHI(5)=(USM6-AG/GI)*1.E-05
      PHI(6)=(USM7-BG/GI)*1.E-05
      PHI(7)=(USM8-CG/GI)*1.E-05
      PHI(8)=(USM9-DG/GI)*1.E-05
C
C     ===============================
C     SHEAR
C     ===============================
C
      FI=.85
C     TWO-WAY ACTION REACTION(LB)
      VT=((BEMOD*GIMOD)-((CA+TU)*(CB+TU))/144.)*WU
C
C     TWO-WAY ACTION SHEAR STRESS(LB/IN**2)
      VST=VT/(TU*(4.*TU+2.*(CA+CB)))
C
C     ALLOWABLE SHEAR STRESS DUE TO TWO WAY ACTION
      AST=4.*SQ*FI
C
C
C     ONE WAY ACTION REACTION (LB)
      VO=(WU*(0.5*(BEM-CA-2.*TU)*GIM))/144.
C
C     ONE WAY ASTION SHEAR STRESS
      VSO=VO/(TU*GIM)
C
C     ALLOWABLE SHEAR STRESS DUO TO ONE WAY ACTION
      ASO=2.*SQ*FI
C     SHEAR CONSTRAINTS
C
      PHI(9)=(AST-VST)*1.E-03
      PHI(10)=(ASO-VSO)*1.E-03
C
C
C     ===============================
C     DEFLECTION
C     ===============================
C
C     MINIMUM SLAB THICKNESS
C     TABLE 7 OF NATIONAL BUILDING CODE OF CANADA
      IF (FY-50000.) 8,9,10
8     TS=BEM/36.
      GO TO 11
9     TS=BEM/33.
      GO TO 11
10    TS=BEM/30.
11    IF(X(1)-TS)15,16,16
16    PHI(11)=1.
      PHI(12)=1.
```

```
      PHI(13)=X(1)-5.
      GO TO 17
C     DIAGONAL CLEAR-SPAN
15    DL=SQRT(BEM**2+GIM**2)-SQRT(CA**2+CB**2)
      DLS=DL**2
      GL=ZL*DLS/12.
C
C     SUPPORT MOMENT, BMS
      BMS=GL/11.
C     MIDSPAN MOMENT, BMM
      BMM=GL/16.
C     MOMENT OF INERTIA, SI
      SI=X(1)**3
C
C     CRITICAL MIDSPAN DEFLECTION DUE TO LIVE LOAD-IMMIDIATE
      YM=5.*DLS*(BMM-0.1*BMS)/(48.*SI*EC)
C
C
C     ADDITIONAL DEFL. DUE TO SHRINKAGE AND CREEP( ASSUMING60 PERCENT OF
C     ZL TO BE SUSTAINED AND CREEP FACTOR = 2)
      YAD = 2.*YM*((ZD+.6*ZL)/(ZD+ZL))
C
C     THE SUBTOTAL DEFLECTION
      YSUB=YM +YAD
C
C     TOTAL DEFLECTION
      YT=1.3*YSUB
C
C     ALLOWABLE DEFLECTION
      YALL=DL/360.
C
C     CONSTRAINT
      PHI(11)=YALL-YT +2.
      PHI(12)=YALL-YM
      PHI(13)=X(1)-5.
C
C
C     =====================
C     OTHERS
C     =====================
C
C
C     MIN. AREA OF  REINFORCING STEEL REQUIRED FOR SHRINKAGE AND TEMP.
17    ASM=0.002*12.*X(1)
      PHI(14)=X(2)-ASM
      PHI(15)=X(3)-ASM
      PHI(16)=X(4)-ASM
      PHI(17)=X(5)-ASM
      PHI(18)=X(6)-ASM
      PHI(19)=X(7)-ASM
      PHI(20)=X(8)-ASM
      PHI(21)=X(9)-ASM
C
C     MIN. 1 BAR NO.3 PER FT.=ASB=0.11IN**2
      ASB=0.11
      PHI(22)=X(2)-ASB
      PHI(23)=X(3)-ASB
```

```
PHI(24)=X(4)-ASB
PHI(25)=X(5)-ASB
PHI(26)=X(6)-ASB
PHI(27)=X(7)-ASB
PHI(28)=X(8)-ASB
PHI(29)=X(9)-ASB
RETURN
END
```

```
      SUBROUTINE DOUBLE (F1,PMAX,D,DDASH,P,PDASH,AS,ASDASH,B,UBM,EK)         F
      COMMON /BLOK2/ FC,FY                                                   F
C                                                                            F
C     ****************************************************************      F
C                                                                            F
C     PURPOSE OF THIS SUBROUTINE IS TO DETERMINE THE USABLE ULTIMATE         F
C     MOMENT, UBM, IN THE ACTUAL CONDITION OF DOUBLE REINFORCED              F
C     RECTANGULAR SECTION.                                                   F
C                                                                            F
C     ****************************************************************      F
C     ESPT = 29,000,000 *0.003                                               F
      ESTP=87000.                                                            F
      PROZ=P-PDASH                                                           F
      IF (PMAX-PROZ) 20,5,5                                                  F
5     PROZD=F1*EK*DDASH/(FY*D)*(87000./(87000.+FY))                          F
      IF (PROZD-PROZ) 15,15,10                                               F
10    A=(AS-ASDASH)*FY/(F1*B)                                                F
      UBM=0.9*((AS-ASDASH)*FY*(D-A/2.)+ASDASH*FY*(D-DDASH))                  F
      RETURN                                                                 F
15    COE1=((ESTP-F1)*ASDASH-FY*AS)/(F1*B*EK)                                F
      COE2=ESTP*ASDASH*DDASH/(F1*B*EK)                                       F
      EX=-COE1/2.+SQRT(COE1**2/4.+COE2)                                      F
      CC=F1*B*EX*EK                                                          F
      CS=ESTP*ASDASH*(1.-DDASH/EX-F1/ESTP)                                   F
      UBM=0.9*(CC*(D-EX*EK/2.)+CS*(D-DDASH))                                 F
      RETURN                                                                 F
20    UBM=0.1                                                                F
      RETURN                                                                 F
      END                                                                    F
```