THE CHARGE DEPENDENT AND CHARGE SYMMETRY BREAKING EFFECTS OF THE γ-π-EXCHANGE PROCESS IN N-N AND λ-N INTERACTIONS

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SCOPE AND CONTENT:

Experimental and theoretical investigations on charge dependence and charge asymetry in N-N and λ-N interactions are reviewed. The γ-π-exchange N-N and λ-N potentials are derived and calculated in the static limit. The γ-π-exchange charge dependent component of the N-N potential is found to be larger than the charge dependent part of OPEP due to pion mass difference up to 3 fm and is of opposite sign, making the explanation of the splitting between p-n and p-p scattering lengths more difficult than was previously thought. Except at very short distances, this potential is of the order of only a few percent of the OPEP. In the light of this result, it is concluded that the very large charge-dependent correction at large distances required by Noyes' phenomenological analysis on the splitting of r_{pn} and r_{pp}

is inexplicable as an electromagnetic correction. The γ-π exchange charge asymmetric components of both N-N and λ-N interactions are found to be much weaker than the charge dependent one for the N-N case.

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PART I

EFFECTS ON N-N INTERACTIONS

CHAPTER 1

INTRODUCTION - LOW ENERGY N-N SCATTERING PARAMETERS

It is generally believed that the strong interaction is charge independent, and the observed charge dependence should be accounted for as due to electromagnetic corrections. A crucial test of charge symmetry (CS), as well as charge independence (CI) of the nuclear interaction is provided by measurements of the low-energy nucleon-nucleon scattering parameters in the ^1S-state, the scattering length a and effective range r. The CS hypothesis of the strong interaction states that, in the absence of electromagnetic interaction, the proton-proton interaction is exactly the same as its charge-symmetry counterpart, the neutron-neutron interaction. The CI hypothesis extends this equality to include the proton-neutron interaction as well.

The p-n scattering parameters can be measured directly.† Taking Engelke et al.'s (1963) recent results into account, Noyes (1963) has found that

$$a_{pn} = (-23.678 \pm 0.028) \text{ fm}, \quad r_{pn} = 2.51 \pm 0.11 \pm 0.043 \text{ fm}. \quad \ldots(1.1)$$

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†In an approximation where the shape parameters are taken to be zero, the four parameters a_s, a_t, r_s and r_t, the singlet and triplet scattering lengths and effective ranges are obtained from the three experimental quantities,

The second uncertainty in r_{pn} is due to the maximum possible deviation from shape-independence approximation.† Breit et al. (1965) analyzed the systematic errors that might occur in the experimental data, especially that of the coherent neutron-hydrogen scattering length. While confirming the Noyes result as the most probable values for the small statistical uncertainties used, they suspect

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the coherent neutron-hydrogen scattering length a_{nH}, total n-p scattering cross section σ_o and the binding energy of deutron ϵ_d, through the four relations:

$$a_{nH} = \frac{1}{2}(a_s + 3a_t)$$

$$\sigma_o = \pi(a_s^2 + 3a_t^2)$$

$$r_t = 2(1 - \frac{1}{a_t}k_o)/k_o \text{ with } \hbar^2 k_o^2 = -2 M_n M_p \epsilon_d \Big/ (M_n + M_p)$$

$$\sigma_o = \pi \left\{ \left[k^2 + \left(\frac{-1}{a_s} + \frac{1}{2} r_s k^2\right)^2\right]^{-1} + 3\left[k^2 + \left(-\frac{1}{a_t} + \frac{1}{2} r_t k^2\right)^2\right]^{-1} \right\}$$

where M_n and M_p are neutron and proton masses and k the centre of mass momentum of the neutron.

†As was pointed out by Noyes (1963), k^4 terms are expected to appear also in the expansion of the quantity $k \cot \delta$. Also, in order to determine the singlet parameters from experiment, we have to know the triplet parameter as well. The strong tensor force couples $^3S_1$ and $^3D_1$ states and the centrifugal shielding of the p-waves are not complete. The $^3S_1 - ^3D_1$ coupling parameter ϵ^1 and the p-wave phase shifts make contributions of order k^4 to the total cross section, hence these higher angular momentum states contribute to deviation from shape independence also.

that the latter is being underestimated, for besides deviation from the effective range approximation, uncertainties also arise from dynamic effects of molecular electrons and effects of molecular binding on the n-p experiments. It should be remembered that Engelke et al.'s data plays a decisive role in narrowing the uncertainty of r_{pn}. In fact, if one does not include their results, one would obtain the higher value $r_{pn} = (2.64 \pm 0.12)$ fm. (Noyes, 1963)

The p-p scattering experiment can be done with great accuracy. In order to obtain quantities which can be compared with those of p-n and n-n cases, however, one has to eliminate the effect of the coulomb force, e^2/r.† Let us denote the directly measured scattering parameters by a'_{pp} and r'_{pp}, in distinction from the so-called non-coulomb scattering parameters, a_{pp} and r_{pp}. The latter would describe the p-p scattering in the absence of the coulomb force. Gursky and Heller's (1964) experiment yielded the values

$$a'_{pp} = (-7.815 \pm 0.008) \text{ fm}, \quad r'_{pp} = (2.795 \pm 0.025) \text{ fm}. \quad \ldots(1.2)$$

The non-coulomb scattering parameters a_{pp} and r_{pp} have been determined for several current phenomenological

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†Because of the long range coulomb force, the so-called effective range function is not simply $k \cot \delta$, but contains in addition a function $h(\eta)$ where $\eta = e^2/h\upsilon$, υ being the laboratory velocity of the incident proton.

nuclear potentials by requiring that these potentials, together with the coulomb interaction between point protons, reproduce the observed p-p scattering data. In this way Heller, Signell and Yoder (1964) found†

$$a_{pp} = -(16.6 \sim 16.9) \text{ fm} \qquad \ldots(1.3a)$$

The difference between r_{pp} and r'_{pp} has been shown to be of the order of 1% only. (Breit, 1965; Noyes, 1965) Hence we may assume

$$r_{pp} = r'_{pp} \qquad \ldots(1.3b)$$

The n-n scattering length has been deduced from the analysis of the final state interaction in the reaction $\pi^- + d \rightarrow 2n + \gamma$. (Haddock et al., 1965) This is an exceptionally convenient reaction since all three of the final particles can be detected and the n-γ interactions are weak so that the momentum spectra is determined entirely by the final state n-n interaction. (McVoy, 1961; Bander, 1964) Haddock et al. (1965) gave the result

$$a_{nn} = -(16.4 \pm 1.9) \text{ fm} \qquad \ldots(1.4a)$$

The error in (1.4a) is experimental. Bander (1964) examined the various approximations employed in the analysis, and concluded that the theoretical error should be only 1 fm. More recently, a_{nn} has also been determined

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†Heller et al. assumed charge symmetry. In their paper, a'_{pp} and a_{pp} are denoted by a_{pp} and a_{nn} respectively.

from the reaction $d+H^3 \rightarrow 2n+He^3$ to be (Baumgartner et al., 1966)

$$a_{nn} = -(16.1 \pm 1.0) \text{ fm.} \qquad \ldots(1.4b)$$

The most recent analysis of the reaction $n+d \rightarrow 2n+p$ (Bar-Avraham et al., 1967) gives

$$a_{nn} = (-14 \pm 3) \text{ fm.} \qquad \ldots(1.4c)$$

The theoretical uncertainty for (1.4b) and (1.4c) would presumably be larger than that of (1.4a), because three strongly interacting particles are involved in the final state.

The effective range r_{nn} is unknown as yet.

Taken at face value, the scattering lengths a_{pp} and a_{nn} are consistent with CS. On the other hand, recent studies by Okamoto (1967) of the binding energies of the mirror nuclei He^3 and H^3 indicate the possible presence of a small charge symmetry breaking (CSB) component in the nuclear force. Moreover, although the substantial part of the rather differently measured rms radii of the charge distributions in He^3 and H^3 has been explained with a charge symmetric nuclear force, there may be some remaining discrepancy which is due to the CSB nuclear force. (Baumgartner et al., 1966)

Deviation from CI is obvious from (1.1) and (1.3);

$$\Delta a = a_{pn} - a_{pp} \approx -7 \text{ fm}, \quad \Delta r = r_{pn} - r_{pp} \approx -0.03 \text{ fm.} \qquad \ldots(1.5)$$

Although the splitting between a's may look very large, it can result from a difference of only a few percent between the p-p and p-n potentials. This is because the ^1S scattering length is extremely sensitive to the small change in the potential. The effective range is much less sensitive to the change in the potential. Thus, as will be discussed later, the rather small splitting between r's implies considerable difference between the p-p and p-n potentials.

Now the problem before us is to explain the observed charge dependence as due to electromagnetic corrections, starting from exactly charge-independent strong interaction. Considerable effort has been made to estimate various charge dependent effects, which can be classified into two types: direct and indirect effects. The direct effects include those of the modification of the coulomb force due to the vacuum polarization, the p-n mass difference, the magnetic interaction, and the finite size of the nucleon. These are characterized by not involving exchange of mesons between nucleons. The indirect effects include any process in which electromagnetic interaction modifies the nuclear force. Here having been considered so far are the mass splitting between charged and neutral mesons, coupling constant splitting, and meson mixing.

It should be noted that, except for the effect of meson-mass splitting which breaks CI but not CS, all the other effects break CS as well as CI.

The direct effects are well understood, and found to be too weak to explain the observed charge dependence (1.5). The indirect effects are fairly well understood, except for the coupling constant splitting. They are large enough to yield the splitting of a's. However, a serious difficulty arises with the splitting of r's. Noyes (1965) examined phenomenologically the difference between the p-p (V_{pp}) and p-n (V_{pn}) potentials which is required by the splitting $r_{pp}-r_{pn}$. He showed that V_{pp} should be more than 30% more attractive than V_{pn} in the range of 2$\sim$4 fm, and correspondingly less attractive for the same amount at shorter distances. This is very surprising because electromagnetic correction is generally of the order of only a few percent of the strong interaction. In fact, none of the effects considered so far can produce Noyes phenomenological potential.

There is, however, one mechanism which has not been considered. This is the photon-pion (γ-π) exchange process. It gives rise to a potential of one-pion range. We thought it might be quite strong, just as the two-pion-exchange potential (TPEP) is much stronger than the one-pion-exchange potential (OPEP) even at the distance around a pion compton wave length. The purpose of this part of the thesis is to investigate the potential due to the γ-π exchange process.

In Chapter 2, we summarize the previous theoretical work and recapitulate Noyes' phenomenological analysis. The γ-π potential is derived in Chapter 3. The numerical results are given in Chapter 4. It turns out that the strength of the γ-π exchange charge dependent potential is a few percent of that of the OPEP in the range of 1∿2 pion compton wave length, and it decreases (increases) more rapidly than the OPEP with increasing (decreasing) distance. Therefore, the γ-π exchange potential cannot produce such a strong long ranged correction as required by Noyes' phenomenological analysis. It becomes very strong at short distances so that if we take it literally, its effect on the scattering length becomes enormous. Of course, our potential at short distances, say $\lesssim 1$ fm, is not reliable. Since the scattering length is very sensitive to the potential at short distances, where all the theories become unreliable, we think it is rather futile to explain the splitting of a's with our present theoretical techniques.

The result is summarized and conclusion given in Chapter 4.

Throughout the whole calculation, we use the units $\hbar=c=\mu=1$.

CHAPTER 2

REVIEW OF PREVIOUS THEORETICAL WORK

2.1 Introduction

Set against a background of increasing evidences for charge independence, both in nuclear physics and in particle physics, the charge dependence of the ^1S_o state scattering parameter, as set out in (1.5), has long been the object of intensive study. This chapter gives a brief account of previous efforts (up to June 1967) to explain this deviation from the strict symmetry as due to electromagnetic processes. As has been remarked, the direct processes are well understood. The corrections due to them are, however, only weak. The indirect processes are less well studied, and, with a complete theory of strong interactions lacking, can at best be semi-phenomenological.

2.2 Vacuum Polarization

The evidence for vacuum polarization is overwhelming in atomic physics. It gives rise to a measurable contribution to the Lamb shift in hydrogen, in x-ray fine structure separations and in level splittings in mu-mesic atoms. In p-p scattering, the clever analysis of Foldy and Erikson (1955) favours the existence of vacuum polarization effect.

According to the Dirac theory of positron, an electromagnetic field induces in vacuum a charge and current distribution with the creation and annihilation of electron-positron pairs. Because of this polarization of the vacuum, interaction between two point charges is not merely a coulomb force e^2/r, but in addition modified by a vacuum polarization potential. To first order of e^2, Uehling (1935) gave this potential as

$$V_{vp}(r) = \frac{2\alpha e^2}{2\pi r} \int_1^\infty e^{-2kr\,x} \left(1 + \frac{1}{2x^2}\right) \frac{(x^2-1)^{\frac{1}{2}}}{x^2}\, dx, \quad k = {}^{m_e c}/\hbar \qquad \ldots(2.1)$$

Wichmann and Knoll (1956) considered the vacuum polarization potential for a strong coulomb field and found that the first order term agrees with the Uehling potential while effect of higher-than-first-order terms is small.[†]

[†] Wichmann and Knoll solve the Dirac equation with the coulomb field explicitly to obtain the charge density. They then consider the quantity which is the Laplace transform of this density times r^2.

The Uehling potential has a range of the compton wave length of the electron. For $2kr \rightarrow o$, it diverges as lnr/r while for 2kr>>1, it decays exponentially. At 1 fm, the Uehling potential is 0.4 kev, less than 1/2% of the coulomb potential.

Erikson, Foldy and Rarita (1956) showed that the correction due to vacuum polarization is relatively much more important in the ^3P states than the ^1S state, because of the long range of the Uehling potential and that the ^3P p-p nucleon force is much weaker than that of the ^1S state. They found that approximately one half of the ^3P phase shift observed between 1-5 Mev is due to the vacuum polarization potential. De Wit and Durand (1958) demonstrated that the Uehling potential produces scattering in many higher-than-S orbital angular momentum states, even at low photon energies. Hence, to deduce the correct nuclear S-wave phase shifts, the complete scattering amplitude of the Uehling potential should be taken into account , along with that of the coulomb potential. Thus, Heller (1960) solved the Schrödinger equation first with the coulomb plus vacuum polarization potential, and used this "electric" wave function as the assymtoptic form in the analysis of the nuclear p-p scattering. The work of Gursky and Heller (1964) follows exactly the same procedure. On calculating the nuclear parameters a_{pp} and r_{pp} from the experimental quantities

a'_{pp} and r'_{pp}, one should subtract the electric potential, instead of the coulomb potential only, from the phenomenological potential used to fit the experimental data, so as to account for the vacuum polarization effect properly. Heller, Signal and Yoder (1964) estimated that this will remove 0.2 fm in the difference Δa.

2.3 Nucleon Mass Difference

The mass difference of 1.3 Mev between neutron and proton is believed to be of electromagnetic origin. Downs and Nogami (1967) estimated its effect on low energy N-N scattering parameters through the Schrödinger equations, using the charge independent Hamada-Johnston potential. They found that the mass difference leads to $a_{nn}<a_{np}<a_{pp}$, with the difference $a_{pp}-a_{nn}$ being only about 0.5 fm.

2.4 Electromagnetic Structure of Nucleons

By electromagnetic structure is meant the modification at the vertex, from the case of a point Dirac particle, where a nucleon interacts with a virtual photon. Such a modification reflects the fact that nucleons are strongly coupled to mesons and does not react with photons as a point source. The most general form of the electromagnetic current associated with the vertex shown in Fig. 2.1, compatible with requirements of gauge invariance and Lorentz invariance, is given as (Drell and

Zachariasan, 1961)

$$\langle p'|j_\mu(o)|p\rangle = \frac{1}{\sqrt{4E_{p'}E_p}} \tilde{w}(p')\{F_1(q^2)\gamma_\mu + i\ F_2(q^2)\sigma_{\mu\nu}q_\nu\}w(p)$$

...(2.2)

The form factors F_1 and F_2 are real functions of q^2, the square of the momentum of the virtual photon. While F_1 and F_2 are related to the charge and magnetic moment distributions of the nucleon, the so-called spacial charge and magnetic moment density can be defined as linear combinations of their Fourier transforms only in the barycentric system, where the fourth component of q vanishes. In this system, however, the nucleon is not at rest. Hence the meaning of these densities is not clear. In the extreme non-relativistic limit, where the nucleon is infinitely heavy, the barycentric and the rest system of the nucleon coincide. Thus, in this limit, we may think of the Fourier transform of the form factors as physical density distribution functions.

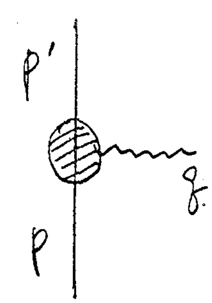

Fig. 2.1

There exist excellent measurements for values of the F's through electron-nucleon scattering experiments. Their analytic forms are determined mainly with the aids of models where vector mesons, viz. ρ and ω, are given the role of furnishing intermediate currents tying the photon to the nucleon.

The most natural method to deal with the electromagnetic interaction between nucleons with structures would be that of Riazudin (1958) who constructed the electromagnetic potential as the Fourier transform of the scattering amplitude due to the diagram for the exchange of a single photon. The effect of higher order corrections is generally believed to be small. Riazudin's calculation showed that the effect of non-coulomb electromagnetic interaction on low energy scattering parameters is small, even for a potential of the Yukawa type without a hard core. Heller et al. (1964) estimated that this can account for 0.8 fm of Δa.

Schneider and Thaler (1965) defined the electromagnetic potential as

$$V(r) = \frac{1}{(2\pi)^3} \int e^{iqr} \, V(q) \, d^3q$$

where ...(2.3)

$$V(q) = f_1(q) \; f_2(q) \; \mathcal{V}(q)$$

f_1 and f_2 are the form factors and $\mathcal{V}(q)$ the fourier transform of interaction for point sources. As remarked before, this treatment is legitimate in the extreme non-relativistic limit and equivalent to the Riazudin prescription. All the electric and magnetic potentials thus obtained are repulsive for both p-p and n-n cases. For p-n, the magnetic potential is attractive while the much weaker electric potential is repulsive. The strongest of all six potentials, the p-p electric potential, is about +1.4 Mev at 1.0 fm. To see the effect of these potentials on a and r, Schneider and Thaler assume a charge independent nucleon potential V_N. The parameters in V_N are adjusted to fit the p-p data. The p-n parameters then obtained are

$$a_{pn} = -16.64 \text{ fm} \qquad r_{pn} = 2.79 \text{ fm} \qquad \ldots(2.4)$$

Comparing with (1.1), we see that this result is in obvious disagreement with experiment.

2.5 Effect of Pion Mass Difference in OPEP

The mass difference between charged and neutral pions, generally believed to be of electrodynamic origin, induces a difference between the p-p and p-n interactions, since in the former case only neutral pions can be exchanged while in the latter case both charged and neutral pions may take part. Unfortunately the meson-theoretic treatment of nuclear forces is not a complete theory. The two pion exchange potential (TPEP) cannot be calculated with quantitative reliability in an unambiguous way. However, the one pion exchange potential can be calculated without ambiguity, except for the so called strong structure of the nucleon which is to be described by the pionic form factor, which is at present not well understood. Both pseudoscalar and pseudovector couplings give identical results for the scattering amplitude for the one pion exchange diagram. The singlet S state potentials are

$$V_{pp} = V_{nn} = \frac{f\mu_o}{2M}^2 \frac{e^{-\mu_o r}}{r}$$

$$V_{pn} = \frac{f}{2M}^2 \frac{1}{3r} (2\mu_\pm^2 e^{-\mu_\pm r} + \mu_o^2 e^{-\mu_o r}), \qquad ...(2.5)$$

giving a difference

$$\delta V = V_{pn} - V_{pp} = \frac{2}{3} \frac{f}{2M}^2 (\mu_o^2 - \mu_\pm^2) \left(\frac{1}{r} - \frac{1}{2\mu}\right) e^{-\mu r} \qquad ...(2.6)$$

Here the pionic form factor of the nucleon has not been considered. Heller et al. (1964) found that this difference, added to the p-p potential to obtain correction for the p-n case, accounts for 2 fm of the difference Δa. Schneider and Thaler (1965) showed that the mass difference in OPEP brings a_{pn} to between -18.5 - -20.14 fm, depending on the other parameters in the charge independent part of the nuclear potential, V_N. Henley and Morrison (1966) obtained a correction of Δa of 4 fm (a_{pn} = -19.9 fm) due to pion mass difference in OPEP using a boundary condition model. Roughly speaking, this effect can account for 1/3 of Δa.

2.6 Effect of Pion Mass Difference in TPEP

In contrast to the OPEP, the TPEP cannot be defined unambiguously. The ambiguity is indeed inherent in the concept of the potential, for the instantaneous nature of an interaction through a potential is at variance with the retarded nature of the interaction mediated by the meson field. This results in the ambiguity in treating the so-called repetition diagram of the TPEP. There are two typical prescriptions; perturbation theory and Brueckner-Watson's method. As is well known, Brueckner-Watson's method does not give the right result for the neutral scalar meson case where the exact answer is known. Although this may not necessarily be a fatal objection against Brueckner-Watson's method, we tend to think that the perturbation theory is more justified than Brueckner-Watson's method. The TPEP in perturbation theory was first derived by Taketani et al., hence referred to as the TMO potential. They assumed the ps-pv coupling for the pion-nucleon interaction. Later Miyazawa gave a prescription how to take account of the pion-nucleon re-scattering corrections in the static approximation. A relativistic derivation has been also developed but we will not go into the details of it. (Cottingham and Vinh Mau, 1963)

Sugie (1954) in his investigation of pion-mass difference effect employed essentially the Brueckner-Watson potential, using a ps-pv interaction Hamiltonian.

$$H = {}^{g}/_{\mu}\bar{\psi}\ (\sigma\cdot\nabla)\ \tau_{\alpha}\psi\,\phi_{\alpha} \qquad \ldots(2.7)$$

For ^1S state, the central part of the charge dependent potential is found to be

$$\delta V = \tau_3^{(1)}\tau_3^{(2)}\Delta\mu \quad G^2\left(\frac{\mu}{2M}\right)^2\left(\frac{2}{x}-1\right)e^{-x} + \left(\frac{G^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^4\frac{8}{\pi}\,[k_o(2x)$$

$$+\left(1+\frac{11}{4x^2}\right)k_1(2x) \qquad \ldots(2.8)$$

where $x=\mu r$, $\Delta\mu/\mu = [\mu_{\pm}-\mu_o]/\mu = \frac{1}{30}$ and $G^2/4\pi = 16$. Employing a wave function of the form

$$u = 1 + x/\alpha + Ae^{-\beta x} \quad \text{for } x \geqslant 0.54 \text{ fm}$$

$$u = o \quad \text{for } x \leqslant 0.54 \text{ fm},$$

he obtained the correction Δa as about 6 fm.†

Lin (1964) took exactly the same Hamiltonian density (2.7) from the Brueckner-Watson theory with completely suppressed pair terms. The coupling constants and hard core radius were treated as free parameters to be adjusted to experimental results. With proper care

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†The correction in Sugie's paper is given as $b/a_{pp} - b/a_{pn} = 0.40$, with $b = \frac{\hbar^2}{me^2}$.

about the pion mass difference taken, the p-n potential is more attractive than the pp case, while for the triplet state, the relation is reversed. The difference in the potentials is overall weak. At a separation distance of 1.4 fm, the ^1S state difference is less than 2.5 Mev.

Henley and Morrison (1966) employed the phenomenological potential derived by Feshbach and Loman (1961, 1964). This is essentially a perturbation potential but a pair suppression parameter λ and a "ladder" parameter ξ are introduced. That $\lambda<1$ is not a perturbative result. For $\xi=1$ one has the TMO potential, for $\xi=o$, the Brueckner-Watson one. In order to fit the p-p phase shifts up to 345 Mev, they employ slightly different coupling constants for OPEP and TPEP. These, together with λ, ξ and the pion mass μ are treated as parameters. To take into account the short range interaction, they employ a boundary condition at r_o by adjusting the logarithmic derivative $B = \frac{r}{u}\frac{du}{dr}\Big|_{r_o}$, where u(r) is the radial wave function. In order to see the effect of pion-mass difference, Henley and Morrison use the empirical pion mass μ_o = 135.01 Mev and $\mu_\pm$ = 139.59 Mev. The boundary condition and coupling constants are left charge independent. With such pion-mass-difference-corrected TPEP + OPEP, they obtained a_{np} = -20.80 fm, i.e., a total correction of 5 fm in Δa.

Heller *et al.* (1964) reported that it is possible to obtain the correct Δa by letting the effective mass in the TMO potential be 1.3 Mev larger in the p-p than the p-n case.

To sum up, one can only say that while pion mass difference does cause a correction in Δa in the right direction, it is quantitatively insufficient to explain the entire discrepancy.

2.7 Pion-Nucleon Coupling Constants

Charge independence would require the coupling constants between various pions and nucleons to be identical, in the absence of electromagnetic interaction. In the presence of electromagnetic interaction, there is a splitting between these coupling constants and they should be determined by quantum electrodynamics.

Sugie (1954) pointed out that the scattering length is much more sensitive to changes in the coupling constants than to the pion mass difference, since a decrease in the mass, say, would increase the range of the potential but reduces its strength at the same time, thus causing two mutually cancelling effects.

By treating the core radius and the coupling constants as free parameters, Lin (1964) was able to produce the right Δa required by experiment. With a core radius of 0.3 fm and $f_{\pi_\pm NN}/f_{\pi_0 NN} = 0.980/1.0140$, he obtained

$a_{nn} = -22.182$ F. Again, the result is extremely sensitive to ratio of the coupling constants. Heller et al. (1964) attained the right Δa by allowing $f_{\pi_{\pm}NN}$ to be larger than $f_{\pi_{o}NN}$ by 3.5%. It is to be noted that such a splitting is of the opposite sign to that of Lin's. In Henley and Morrison's (1966) analysis of the pion-mass difference and ρ mass difference effect, they find that in conjunction with these, a splitting of the coupling constants can bring out the right Δa. The splittings required are, for pseudoscalar coupling, $g_{\pm}/g_{o} = 0.9832$ and for pseudovector coupling $f_{\pm}/f_{o} = 1.017$, so that in the two different cases, the sign of splitting is opposite.

In view of the above, it is only fair to say that the analysis on coupling constants is, if anything, inconclusive.

2.8 Particle Mixing

In the classification of particles, we start for the sake of convenience with the highest symmetry and assume that each individual particle is an eigenstate of a set of operators, $\Sigma = (o_1, o_2, \ldots)$, invariant under the strong interaction. Because of other weaker interactions, the symmetry of the strong interaction is broken in actuality. Hence the physical particles are not really pure eigenstates of Σ but may be admixtures of them. In particular, the different eigenstates of the isospin operator are mixed by the electromagnetic interaction. Thus, the physical $\tilde{\pi}^o$ and $\tilde{\eta}$ are mixtures of the π^o and η, respectively the isovector and isoscalar members of the pseudoscalar octet. In the SU(3) model, the fact that the electromagnetic interaction conserves the so called U-spin enables one to write down a formula from which the extent of isospin mixing can be determined in terms of the mass splitting among isospin multiplets. Similarly, the physical $\tilde{\rho}^o$, $\tilde{\phi}$, and $\tilde{\omega}$ are mixtures of ρ^o, ϕ and ω, respectively the isovector member, the isoscalar member of the vector meson octet and that of the isoscalar unitary singlet. Here the medium strong interaction is responsible for the mixing between members of different unitary supermultiplets. Such a mixing of particles will result in the N-N potential terms proportional to the operator $(\tau_3^{(1)} + \tau_3^{(2)})$, where $\tau_3^{(i)}$ is the third component of the isospin operator of the ith nucleon. These terms violate charge symmetry. Downs and Nogami (1967) constructed the charge symmetry breaking (CSB) potentials due to mixing of the above two sets of particles, and investigated their corrections to the scattering lengths $a_{pp,nn}$ and effective ranges $r_{pp,nn}$, as given by the Hamada-Johnston potential (HJ) and the C. W. Wong potential y_{pp2}. Such phenomenological potentials contain CSB effects implicitly. Their results, displayed in Tables 2.1 and 2.2, depend on the F-D mixing parameter α_{ps} of the pseudoscalar mesons and the vector coupling constant $g_{NN\omega}$.

TABLE 2.1

Scattering Parameter Shifts for

Pseudoscalar-Meson-Exchange CSB p-p Potential

(Downs and Nogami, 1967)

	HJ		Y_{pp2}	
α_{ps}	Δa_{pp}†	Δr_{pp}	Δa_{pp}	Δr_{pp}
1/2	-0.01	-0.02	-0.15	-0.02
1	0.01	0.02	0.13	0.02

TABLE 2.2

Scattering Parameter Shifts for

Vector-Meson-Exchange CSB p-p Potential

(Downs and Nogami, 1967)

	HJ		Y_{pp2}	
$g_{NN\omega}$	Δa_{pp}	Δr_{pp}	Δa_{pp}	Δr_{pp}
6	0.63	0.02	0.75	0.02
-5	-0.78	-0.02	-0.94	-0.02

†All values of Δa and Δr in units of fm.

2.9 Difficulty with Effective Ranges and Noyes Phenomenological Analysis

It is clear from the above that while it is possible to give an account of the difference Δa -- mainly with the splitting of pion masses and coupling constants -- such efforts are far from being conclusive. Except for the direct effects, the effects of the other processes considered seem to depend on the juggling of a few parameters. The coupling constant splittings are especially unreliable, while a small disparity in them would induce large effects on the behaviour at short distances, as was pointed out by Noyes (1965). On the other hand, none of these effects explain the large difference in the effective ranges. In fact, the effective ranges are hardly intensively investigated. This, of course, is partly because r_{nn} is not available. Henley (1966) estimated the effect of pion mass splitting on the effective ranges, using the same boundary condition model in his other work. (Henley and Morrison, 1966) The correction obtained is $r_{nn} - r_{pn} = (0.12 \pm 0.03)$ fm.

Noyes analyzed phenomenologically the amount of charge dependence required in the N-N potentials to produce such a large discrepancy between r_{nn} and r_{pn}. He found that the updated HJ potential[†] which gives -23.68 fm

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[†]This potential is first mentioned in the work of Heller et al. (1964). As far as we know, it is not published anywhere.

for a_{pn} has a value of r_{pn} falling between 2.72 - 2.73 fm, the model dependent uncertainty being 0.02 fm. To obtain an r_{pn} of 2.68 fm would entail increasing the attraction of the p-n potential in the region between 2 to 4 fm by 5% and decreasing the attraction in the inner region to the same amount, as compared to the p-p potential. For r_{pn} = 2.4468 fm, the value given by the experiment of Engelke et al., corresponding changes of 30% are required.

None of the processes considered above can produce such a large correction to the potential at the long range part.

CHAPTER 3

DERIVATION OF THE PHOTON-PION EXCHANGE POTENTIAL

3.1 Introduction

We will derive the potential due to the photon-pion ($\gamma\pi$) exchange process, employing the techniques which have been developed for the calculation of the two-pion exchange potential. The γ-π exchange process is depicted in Fig. 3.1(a), where the shaded parts stand for all the possible processes. The sum over virtual interactions represented by this shaded part is identical to that appearing in the diagram for photon-pion production. Here, of course, is the difference that the virtual photon and pions exchanged are not on the mass shell and the light-cone, i.e., $p_o^2 \neq p^2 + \mu^2$ and $k_o{}^2 \neq k^2$. Especially, diagram (a) includes diagram (b) but since we are interchanging two different particles, we have not counted the process twice, as would be the case for the exchange of two pions. We ignore the nucleon recoil and treat the pion-nucleon interaction in the static approximation.

We are aware of the fact that the static limit of the two-pion-exchange potential is mathematically not well defined, and essentially the same difficulty arises with the γ-π exchange potential ($\gamma\pi$EP), namely, if it is written in the form

$$V(x) = \int_{\mu^2}^{\infty} dm^2 \rho(m^2) \exp(-mx/x) \qquad \ldots(3.1)$$

the inverse mass expansion of the spectral function $\rho(m^2)$ will not converge at the lower mass end ($m \to \mu$). The relativistic effect is therefore important in the asymtoptic region ($x \to \infty$), where the static limit would appear to be most justified. For the TPEP, however, it is known that the static approximation does provide us with a reasonable numerical approximation. This can be seen by comparing the λN and $\lambda\lambda$ TPEP derived in the approximation and those obtained relativistically. (Rimpault and Vinh Mau, 1966; Nogami et al., 1964; Deloff and Wrezecinko, 1964) Our calculation will be meaningful except for the extremely large distance where V(x) is small, and for very short distances.

In the absence of electromagnetic field, the Hamiltonian of the pion-nucleon system is given by

$$H_o = H_\mu + H_{\mu N}$$

$$H_\mu = \frac{1}{2} \sum_{\alpha=1}^{3} \int d^3x \left\{ \dot{\phi}_\alpha^2 + (\Delta\phi_\alpha)^2 + \mu^2\phi_\alpha^2 \right\}$$

$$= \sum_{\alpha=1}^{3} \int d^3p \; w_p \; a_{\alpha p}^{+} \; a_{\alpha p}$$

$$H_{\mu N} = (\sqrt{4\pi}\ fo/\mu) \int d^3x\ \rho(x)\tau_\alpha(\sigma\cdot\nabla)\phi_\alpha(x)$$

$$= i(\sqrt{4\pi}\ fo/\mu)(2\ \pi)^{-3/2}\int d^3p\ U_p(\sigma\cdot p)(2w_p)^{-1/2}$$

$$(a_{\alpha p} - a^+_{\alpha p})$$

Here, ϕ_α -- pion field with isospin index α

w_p -- pion energy $(p^2 + \mu^2)^{1/2}$

$a_{\alpha p}$ -- α-pion annihilation operator with momentum p

$\rho(x)$ -- source function of nucleus in real space, this represents a cut-off

U_p -- Fourier transform of $\rho(x)$

fo -- the unrenormalized coupling constant

μ -- pion mass.

Now let us introduce the electromagnetic field. The interaction between the pion nucleon system and the electromagnetic field is obtained through the prescription of making the replacement $\nabla\rightarrow\nabla-ie\bar{A}$ in (3.2). We work in the radiation (or coulomb or transverse) gauge.

$$\nabla\cdot\bar{A} = 0 \qquad \ldots(3.3)$$

Thus, no longitudinal component of the vector potential $\bar{A}$ enters into the calculation. The scalar potential is not an independent dynamical variable, but is determined by the instantaneous charge distribution. It is nothing but the familiar coulomb potential between charge distributions. In the reference frame of this gauge, the propagator of the electromagnetic field is given by

$$D_F^{tr}(x'-x)_{\mu\nu} = g_{\mu\nu}D_F(x'-x) - \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot(x'-x)}}{(k^2+i\varepsilon)}$$

$$\frac{k^2\eta_\nu\eta_\mu - (k\cdot\eta)(k_\nu\eta_\mu + \eta_\nu k_\mu) + k_\nu k_\mu}{(k\cdot\eta)^2 - k^2} \quad (3.4)$$

where $D_F(x'-x) = -\lim_{m^2\to o} \Delta_F(x'-x,m)$, the spin o boson propagator, $\eta^\mu = (1,0,0,0)$ and k is the four dimensional wave vector. In the calculation of quantities of physical interest, D_F^{tr} is always to appear between conserved currents. The term in (3.4) proportional to k_ν or k_μ will vanish because of current conservation. The term proportional to $\eta_\nu\,\eta_\mu$ gives the coulomb potential. Thus the effective part of the propagator is $g_{\mu\nu}D_F(x-x')$.

The interaction Hamiltonian is given by

$$H_1 = -\int d^3x\, \underset{\sim}{j}(x)\cdot\underset{\sim}{A}(x) \quad \ldots(3.5)$$

where the current of the pion-nucleon system is given by

$$\underset{\sim}{j} = \underset{\sim}{j}_N + \underset{\sim}{j}_\mu + \underset{\sim}{j}_{int}$$

$$\underset{\sim}{j}_N = \frac{e}{2m}\frac{1+\tau_3}{2}(\underset{\sim}{\sigma}\times\underset{\sim}{\nabla})$$

$$\underset{\sim}{j}_\mu = -e\,(\phi_1\underset{\sim}{\nabla}\phi_2 - \phi_2\underset{\sim}{\nabla}\phi_1) \quad \ldots(3.6)$$

$$\underset{\sim}{j}_{int} = \frac{efo}{\mu}(\tau_1\phi_2 - \tau_2\phi_1)\underset{\sim}{\sigma}$$

j_N is the nuclear magnetic moment current density, j_μ is the meson current and j_{int} is the interaction current because of the derivative coupling employed. The term of the order of e^2 has been omitted here.

However, we shall decompose the current $\underset{\sim}{j}$ in a different way, following Chew and Low (1956).

$$\underset{\sim}{j} = \underset{\sim}{j}_v + \underset{\sim}{j}_s + \underset{\sim}{j}_\pi \qquad \ldots(3.7)$$

where $\langle \Psi_o | \underset{\sim}{j}_\pi | \Psi_o \rangle = o$

$$\ldots(3.8)$$

and $[a_p, \underset{\sim}{j}_v] = [a_p, \underset{\sim}{j}_s] = o$

Here, Ψ_o stands for the one-nucleon state, an eigen-state of $H_o = H_\mu + H_N$. The subscripts v and s refer to the isospin character of the respective current densities; v stands for isovector and s for isoscalar.

3.2 The Scattering Matrix

The N-N scattering amplitude corresponding to the Feynman diagram in Fig. 3(a) can be written down with the help of a prescription given by Miyazawa (1956).

$$S= -(2\pi)^{-8} \iint d^4p d^4k \sum_{\alpha,\lambda} \frac{\langle \alpha p | s^{(1)} | \lambda k \rangle \langle \alpha -p | s^{(2)} | \lambda -k \rangle}{(k_o{}^2 - k^2)(k_o{}^2 - w_p{}^2)} \qquad (3.9)$$

$w_p{}^2 = p^2 + \mu^2, \quad \alpha = 1, 2, 3, \quad \lambda = 1, 2$

In the static model, we neglect the nucleon recoil effect. Hence, there is no index or variable pertaining to the nucleons in the state labels. $\langle \alpha p | s^{(i)} | \lambda k \rangle$ approaches

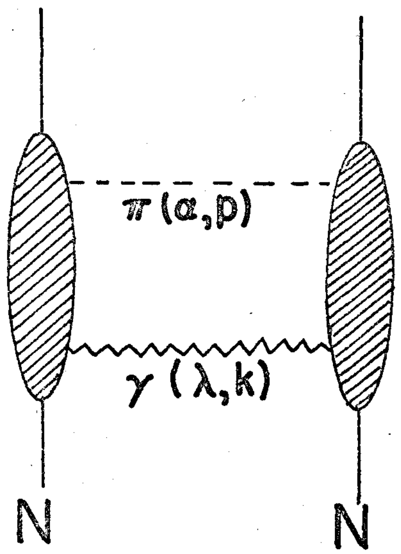


Fig. 3·1(a)

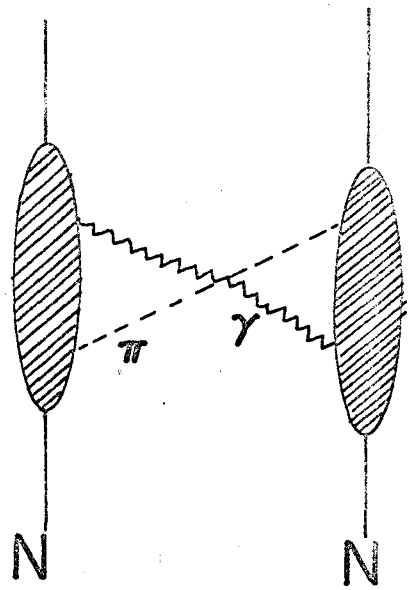


Fig. 3·1(b)

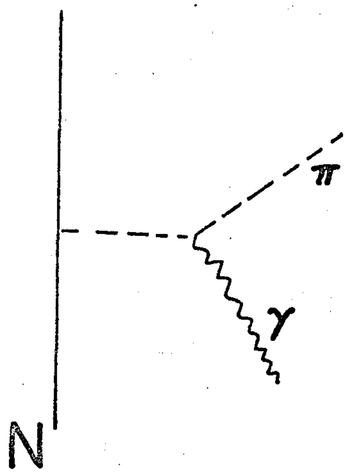


Fig.3·2

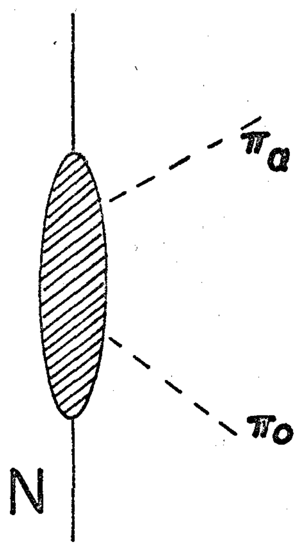


Fig. 3·3

the photo-pion production amplitude when the four momenta approach values on the mass shell and the light cone. It is then the amplitude for an incoming photon of momentum k, polarization λ, and an outgoing pion of momentum p and isospin label α. The matrix element $\langle \alpha p | s^{(i)} | \lambda k \rangle$ is related to the physical photoproduction amplitude through dispersion relations. Because of the virtuality of the pion and photon, we shall include in $\langle \alpha p | s^{(i)} | \lambda k \rangle$ the pionic and electromagnetic form factor at the vertex where the nucleon-pion and photon-nucleon interaction takes place. In their current analytic form, these form factors modify in a systematic way the function on which numerical calculations are to be carried out. We relegate a detailed discussion of this to Appendeces 2 and 3.

$s^{(i)}$ is proportional to the nucleon current density (3.7). We further separate from $j_\pi^{(i)}$ the part which is due to the so-called interaction current,

$$\underset{\sim}{j}_\pi^{(i)} = \underset{\sim}{j}_{int}^{(i)} + \underset{\sim}{j}_R^{(i)} \qquad \ldots(3.9)$$

Furthermore, we write $\underset{\sim}{j}_I = \underset{\sim}{j}_V + \underset{\sim}{j}_{int}$ ...(3.10)

The term $\underset{\sim}{j}_I^{(1)} \times \underset{\sim}{j}_I^{(2)}$ in (3.7) gives rise to charge independence breaking contribution while $\left(\underset{\sim}{j}_I^{(1)} \times \underset{\sim}{j}_s^{(2)} + \underset{\sim}{j}_s^{(1)} \times \underset{\sim}{j}_I^{(2)}\right)$ contains terms breaking charge symmetry. The terms containing $j_R^{(i)}$ belong to diagrams with vertex shown in Fig. 3.2. They are absorbed into the pion mass and coupling constant renormalization, so we shall not include them in the following calculation.

Corresponding to the three parts of nuclear current of interest to us, we divide $s^{(i)}$ into three parts, respectively pertaining to $\underset{\sim}{j}_v^{(i)}$, $\underset{\sim}{j}_{int}^{(i)}$ and $\underset{\sim}{j}_s^{(i)}$:

$$s^{(i)} = s_v^{(i)} + s_{int}^{(i)} + s_s^{(i)} \qquad \ldots(3.11)$$

Chew and Low (1956) show that $\underset{\sim}{j}_v$ effectively generates neutral mesons of momentum $\underset{\sim}{k} \times \underset{\sim}{\varepsilon}$ which are then scattered by the nucleon, $\underset{\sim}{\varepsilon}$ being the polarization vector of the photon. Hence, $s_v^{(i)}$ is proportional to the T-matrix element corresponding to the diagram in Fig. 3.3.

$$s_v^{(i)} = \frac{e}{f}\mu \, \frac{g_p - g_n}{4M} \, (\alpha p|s|3 \; \underset{\sim}{k} \times \underset{\sim}{\varepsilon}) \qquad \ldots(3.12)$$

f -- the renormalized unrationalized coupling constant

g_p, g_n -- the proton and neutron magnetic moment in units of nuclear magneton

M -- nucleon mass

Miyazawa (1956) wrote down the explicit form of the π-N scattering matrix-element.

$$(jp|s|iq) = 2\pi i \delta(q_o - p_o) [A(q_o) \tau_i \tau_j (\sigma q)(\sigma p)$$
$$+ B(q_o)(\tau_i \tau_j (\sigma p)(\sigma q) + \tau_j \tau_i (\sigma q)(\sigma p)) + C(q_o) \tau_j \tau_i (\sigma p)(\sigma q)]$$
$$\times e^{i(\underset{\sim}{q} - \underset{\sim}{p}) \cdot \underset{\sim}{x}} \qquad \ldots(3.13)$$

$\underset{\sim}{x}$ is the coordinate of the nucleon.

The functions $A(q_o)$, $B(q_o)$ and $C(q_o)$ are given in terms of the π-N scattering cross section, through dispersion relations.

$$A(t) = \frac{4\pi f^2}{\mu^2}\frac{1}{t-i\epsilon} + \frac{1}{4\pi}\int_o^\infty \frac{d_p}{w_p}\frac{\sigma_{33}(p)}{w_p-t-i\epsilon} + \frac{1}{36\pi}\int_o^\infty \frac{d_p}{w_p}\frac{(4\sigma_{11}+4\sigma_{13}+\sigma_{33})}{w_p+t-i\epsilon}$$

$$B(t) = \frac{1}{12}\int_o^\infty \frac{d_p}{w_p}\frac{\sigma_{33}+2\sigma_{13}}{w_p-t-i\epsilon} + \frac{1}{12\pi}\int_o^\infty \frac{d_p}{w_p}\frac{\sigma_{33}+2\sigma_{13}}{w_p+t-i\epsilon} \qquad \ldots(3.14)$$

$$C(t) = -\frac{4\pi f^2}{\mu^2}\frac{1}{t+i\epsilon} + \frac{1}{36\pi}\int_o^\infty \frac{d_p}{w_p}\frac{4\sigma_{11}+\sigma_{13}+\sigma_{33}}{w_p-t-i\epsilon} + \frac{1}{4\pi}\int_o^\infty \frac{d_p}{w_p}\frac{\sigma_{33}}{w_p+t-i\epsilon}$$

$\sigma_{2I,2J}$ is the partial cross section for π-N scattering in the state (I,J), I and J being the isospin and real spin.

It is well known that the π-N scattering is dominated by the 3-3 resonance and that the s-wave pion photo-production amplitude is small. $\underset{\sim}{j}_v$ produces p-wave pion, since magnetic dipole radiation has parity +1 and the pion an intrisic parity -1. Furthermore, it is the only part of the current which can give an intermediate state of the same quantum numbers as that of the 3-3 resonance. Hence, we expect $s_v^{(i)}$ to dominate over other contributions to $s^{(i)}$. In the above, this part has been treated exactly except for the static approximation. The other parts of $s^{(i)}$ we shall only approximate by the first Born term. This has been found to be a good approximation.

The current $j_{int}^{(i)}$ is responsible for the electric dipole photo-production of s-wave pion. As was noted above, photo-production amplitude for s-wave pion is described rather well by the first Born term. Hence, we only evaluate $s_{int}^{(i)}$ by a perturbation method. The first Born term gives

$$s_{int}^{(i)} = \frac{4\pi ef}{\mu} (\tau_2\delta_{\alpha 1}-\tau_1\delta_{\alpha 2})\sigma_\lambda e^{i(k-p)\cdot x} \qquad \ldots(3.15)$$

The current j_s is also magnetic and therefore produces p-pions, but it has nothing to do with the 3-3 resonances. Hence we also assume that its contribution is small and are ~~be~~ satisfied with a first Born approximation to $s_s^{(i)}$. Thus we have

$$s_s^{(i)} = 2\pi i\ \delta(p_o-k_o)\ \frac{e\mu}{f}\ \frac{g_n+g_p}{4M} \times\{A_s(k_o)(\sigma\times k)_\lambda(\sigma p)$$

$$+ C_s(k_o)(\sigma p)(\sigma\times k)_\lambda\}\times\tau_\alpha e^{i(k-p)\cdot x}$$

$$\ldots(3.16)$$

The functions A_s and C_s, in the zero meson approximation, are given as

$$A(t) = \frac{4\pi}{\mu^2}\frac{f^2}{t-i\epsilon}\ , \qquad C_s(t) = \frac{4\pi}{\mu^2}\frac{-f^2}{t+i\epsilon} \qquad \ldots(3.17)$$

Let us define

$$G = \frac{e}{f}\mu\ \frac{g_p-g_n}{4M}\ , \quad F = \frac{4\pi ef}{\mu} \quad \text{and} \quad G' = \frac{e\mu}{f}\ \frac{g_n+g_p}{4M}\ .$$

Substitute (3.12), (3.13), (3.15) and (3.16) into (3.11), we obtain

$$s^{(i)} = 2\pi i\delta(p_o-k_o)[G\{A\tau_3\tau_\alpha(\sigma\times k)_\lambda(\sigma p)+B\left(\tau_3\tau_\alpha(\sigma p)(\sigma\times k)_\lambda + \tau_\alpha\tau_3(\sigma\times k)_\lambda(\sigma p)\right) + C\tau_\alpha\tau_3(\sigma p)(\sigma\times k)_\lambda\} + F(\tau_2\delta_{\alpha 1}-\tau_1\delta_{\alpha 2})\sigma_\lambda + G'\{A_s(\sigma\times k)_\lambda(\sigma p) + C_s(\sigma p)(\sigma\times k)_\lambda\}]e^{i(\underset{\sim}{k}-\underset{\sim}{p})\cdot\underset{\sim}{x}} \quad \ldots(3.18)$$

$\alpha = 1, 2, 3, \quad \lambda = 1, 2$

The potential is defined by requiring it to give the same S-matrix element when inserted into the Schrödinger equation. Thus we define

$$S = -2\pi i\delta(o)V(x) \quad \ldots(3.19)$$

We further separate V(x) into three parts corresponding to the three products of the current components: $\underset{\sim}{j}_I^{(1)} \times \underset{\sim}{j}_I^{(2)}$, $\underset{\sim}{j}_s^{(1)} \times \underset{\sim}{j}_I^{(2)} + \underset{\sim}{j}_I^{(1)} \times \underset{\sim}{j}_s^{(2)}$ and $\underset{\sim}{j}_s^{(1)} \times \underset{\sim}{j}_s^{(2)}$.

$$V(x) = V_I(x) + V_{II}(x) + V_{III}(x) \quad \ldots(3.20)$$

Comparing (3.19) and (3.9), we have

$$V_\xi(x) = \frac{i}{(2\pi)^7}\int_{-o}^{-\infty} dt \iint d^3p\,d^3k\,\frac{e^{i(\underset{\sim}{k}-\underset{\sim}{p})\cdot\underset{\sim}{x}}}{(t^2-k^2)(t^2-w_p^2)} \times \mathcal{K}_\xi \quad \ldots(3.21)$$

$$\mathcal{K}_{I} = s_I^{(1)} \times s_I^{(2)}$$

$$\mathcal{K}_{II} = s_I \times s_s^{(2)} + s_s^{(1)} \times s_I^{(2)} \qquad \ldots(3.22)$$

$$\mathcal{K}_{III} = s_s^{(1)} \times s_s^{(2)}$$

The potentials thus defined in (3.21) are static, with no energy dependence and no free parameter.

We could also divide V(x) into parts according to their symmetries in isospin space. Thus, we define

$$V(x) = V_{CI}(x) + V_{CD}(x)\ T_3 + V_{CSB}(x)(\tau_3^{(1)} + \tau_3^{(2)}).$$

$$T_3 = \tau_3^{(1)} \tau_3^{(2)} \qquad \ldots(3.20a)$$

$V_{CSB}(x)$ is identical to $V_{II}(x)$. $V_I(x)$ gives contribution to $V_{CD}(x)$ while both $V_I(x)$ and $V_{III}(x)$ contribute to $V_{CI}(x)$.

3.3 The Charge Dependent (CD) Potential

The potential $V_I(x)$ curtains a charge dependent part, which is the coefficient of the operator $T_3 = \tau_3^{(1)} \tau_3^{(2)}$ in the isospin space.

$$V_I(x) = \frac{i}{(2\pi)^7} \int_{-\infty}^{\infty} dt \iint d^3p\, d^3k\, \frac{e^{i(\underset{\sim}{k} - \underset{\sim}{p}) \cdot \underset{\sim}{x}}}{(t^2 - k^2)(t^2 - w_p^2)} \mathcal{K}_{I}$$

$$\ldots(3.21a)$$

Substituting (3.12) and (3.15) into (3.22) and note that $A(k_o) = C(-k_o)$ and $B(k_o) = B(-k_o)$, we have,

$$\mathcal{K}_I = s_I^{(1)} \times s_I^{(2)} = G^2\{I\} + GF\{II\} + F^2\{III\}$$

$$= \sum_{\alpha,\lambda} \{G[A(k_o)\tau_3^1\tau_\alpha^1(\sigma^1\times k)_\lambda(\sigma^1 p) + B(k_o)(\tau_3^1\tau_\alpha^1(\sigma^1 p)(\sigma^1\times k)$$

$$+ \tau_\alpha^1\tau_3^1(\sigma^1\times k)_\lambda(\sigma^1 p))$$

$$+ C(k_o)\tau_\alpha^1\tau_3^1(\sigma^1 p)(\sigma^1\times k)_\lambda] + F(\tau_2^1\delta_{\alpha 1} - \tau_1^1\delta_{\alpha 2})\sigma_\lambda^1\}$$

$$\times \{G[C(k_o)\tau_3^2\tau_\alpha^2(\sigma^2\times k)_\lambda(\sigma^2 p) + B(k_o)(\tau_3^2\tau_\alpha^2(\sigma^2 p)(\sigma^2\times k)_\lambda$$

$$+ \tau_\alpha^2\tau_3^2(\sigma^2\times k)_\lambda(\sigma^2 p))$$

$$+ A(k_o)\tau_\alpha^2\tau_3^2(\sigma^2 p)(\sigma^2\times k)_\lambda] + F(\tau_2^2\delta_{\alpha 1} - \tau_1^2\delta_{\alpha 2})\sigma_\lambda^2\} \qquad \ldots(3.23)$$

Write $T = \sum_\alpha \tau_\alpha^1\tau_\alpha^2$, $T_3 = \tau_3^1\tau_3^2$

$$\{I\} = \sum_{\lambda=1}^{3} \{(\sigma^1 k)_\lambda(\sigma^1 p)(\sigma^2\times k)_\lambda(\sigma^2 p)[AC+BB-2AB)TT_3+4AB]$$

$$+ (\sigma^1\times k)_\lambda(\sigma^1 p)(\sigma^2 p)(\sigma^2\times k)_\lambda[(AB+BA-AA-BB)TT_3+2(AA+BB)]$$

$$+ (\sigma^1 p)(\sigma^1\times k)_\lambda(\sigma^2\times k)_\lambda(\sigma^2 p)[(2BC-BB-CC)TT_3+2(BB+CC)]$$

$$+ (\sigma^1 p)(\sigma^1\times k)_\lambda(\sigma^2 p)(\sigma^2\times k)\ [(BB+CA-2AB)TT_3+4AB]\}$$

$$\ldots(3.24)$$

We have here extended the sum of the index λ to run through $\lambda=1,2$ and 3 since $\epsilon_3 = \underset{\sim}{k}/|\underset{\sim}{k}|$ and therefore $(\sigma\times k)_3 = \sigma\times\underset{\sim}{k}\cdot\epsilon_3 = o$.

Note also that because of the form of the functions A, B and C, we have, under the integration sign, AA=CC and AB=BC.

Simplifying, we get

$$\{I\} = (p\times k)^2[TT_3 2(AC-AA)+4(A+B)^2]$$

$$+\Big[(\sigma^1\sigma^2)(pk)^2+ p^2(\sigma^1 k)(\sigma^2 k)-(pk)[(\sigma^1 p)(\sigma^2 k) - (\sigma^1 k)(\sigma^2 p)]\Big][TT_3 2(4AB -AC-AA-2BB)+4(A-B)^2] \quad ...(3.25)$$

Similarly,

$$\{II\}= \sum_{\alpha,\lambda}\Big\{A(k_o)\tau_3^1\tau_\alpha^1(\sigma^1\times k)_\lambda(\sigma^1 p)+B(k_o)\Big(\tau_3^1\tau_\alpha^1(\sigma^1 p)(\sigma^1\times k) + \tau_\alpha^1\tau_3^1(\sigma^1\times k)_\lambda(\sigma^1 p)\Big)$$

$$+ C(k_o)\tau_\alpha^1\tau_3^1(\sigma^1 p)(\sigma^1\times k)_\lambda\Big\}\Big\{\tau_2^2\delta_{\alpha 1}-\tau_1^2\delta_{\alpha 2})\sigma_\lambda^2$$

$$+ (\tau_2^1\delta_{\alpha 1}-\tau_1^1\delta_{\alpha 2})\sigma_\lambda^1[C(k_o)\tau_3^2\tau_\alpha^2(\sigma^2\times k)_\lambda(\sigma^2 p) +B(k_o)\Big(\tau_3^2\tau_\alpha^2(\sigma^2 p)(\sigma^2\times k)_\lambda$$

$$+ \tau_\alpha^2\tau_3^2(\sigma^2\times k)_\lambda(\sigma^2 p)\Big) + A(k_o)\tau_\alpha^2\tau_3^2(\sigma^2 p)(\sigma^2\times k)_\lambda\Big\}$$

$$= 2(A-B)(T-T_3)[(\sigma^1 k)(\sigma^2 p)+(\sigma^1 p)(\sigma^2 k)-2(\sigma^1\sigma^2)(pk)] \quad ...(3.26)$$

For the term {III}, we cannot simply extend the sum of λ to include the case $\lambda=3$.

$$\{III\} = \sum_{\alpha=1}^{3}\sum_{\lambda=1}^{2}(\tau_2^1\delta_{\alpha 1}-\tau_1^1\delta_{\alpha 2})(\tau_2^2\delta_{\alpha 1}-\tau_1^2\delta_{\alpha 2})(\sigma_\lambda^1\sigma_\lambda^2)$$

$$= (T - T_3)\left(\sigma^1\sigma^2 - \frac{(\sigma^1 k)(\sigma^2 k)}{k^2}\right) \quad ...(3.27)$$

Substituting (3.27), (3.26) and (3.25) into (3.23) we have

$$\mathcal{K}_I = G^2\Big\{(p\times k)^2[TT_3 2(AC-AA)+4(A+B)^2]$$
$$+ \Big[(\sigma^1\sigma^2)(pk)^2 + p^2(\sigma^1 k)(\sigma^2 k)-(pk)[(\sigma^1 p)(\sigma^2 k)+(\sigma^1 k)(\sigma^2 p)]\Big]$$
$$\Big[TT_3 2[4AB-AC-AA-2BB]+4(A-B)^2\Big]\Big\}$$
$$+ GF\{2(A-B)(T-T_3)[(\sigma^1 k)(\sigma^2 p)+(\sigma^1 p)(\sigma^2 k)-2(\sigma^1\sigma^2)^2(pk)]\}$$
$$+ F^2 \;(T-T_3)\left(\left(\sigma^1\sigma^2\right)- \frac{(\sigma^1 k)(\sigma^2 k)}{k^2}\right) \qquad \ldots(3.28)$$

To avoid complicated angular integrations, we employ the standard trick of making substitutions:

$$e^{i(\underset{\sim}{k}-\underset{\sim}{p})\cdot x} \longrightarrow e^{i(\underset{\sim}{k}\cdot\underset{\sim}{y}-\underset{\sim}{p}\cdot\underset{\sim}{z})}$$
$$\underset{\sim}{p} \longrightarrow i\underset{\sim}{\nabla}_z \qquad \ldots(3.29)$$
$$\underset{\sim}{k} \longrightarrow i\underset{\sim}{\nabla}_y$$

Furthermore, since the integrands are spherically symmetric, all angular differentiations vanish. Thus the above substitution may be replaced by

$$\underset{\sim}{p} \longrightarrow i\,\frac{\underset{\sim}{z}}{|\underset{\sim}{z}|}\,\frac{\partial}{\partial z}$$
$$\ldots(3.30)$$
$$\underset{\sim}{k} \longrightarrow i\,\frac{\underset{\sim}{y}}{|\underset{\sim}{y}|}\,\frac{\partial}{\partial y}$$

After making the above substitutions for the operators p and k in (3.28), we obtain

$$\mathcal{K}_I = 2G^2\left\{ \frac{2}{x}\left(\frac{1}{x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\frac{\partial^2}{\partial y\partial z}\left[2(A+B)^2 - \frac{1}{2}(A-C)^2 TT_3\right]\right.$$

$$+\left\{\frac{2}{3}(\sigma'\sigma^2)\left[\frac{3}{x^2} + \frac{1}{x}\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right) + \frac{\partial^2}{\partial y\partial z}\right] + \frac{1}{3}S_{12}\right.$$

$$\left.2\left(\frac{\partial}{\partial y} - \frac{\partial}{\partial z} - \frac{\partial^2}{\partial y\partial z}\right)\right\}\frac{\partial^2}{\partial y\partial z}$$

$$\left.\times\left[\left(\frac{1}{2}(A-C)^2 - 2(A-B)^2\right)TT_3 + 2(A-B)^2\right]\right\}$$

$$+ 2GF\left\{\frac{4}{3}(\sigma^1\sigma^2) - \frac{2}{3}S_{12}\right\}\frac{\partial^2}{\partial y\partial z}(T-T_3)(A-B)$$

$$+ F^2(T-T_3)\,\frac{1}{3}\,(2\sigma^1\sigma^2 - S_{12})\left(\frac{1}{xk^2}\frac{\partial}{\partial y} + 1\right) \qquad \ldots(3.31)$$

$$S_{12} = \frac{3(\sigma^1 x)(\sigma^2 x)}{x^2} - \left(\sigma^1\sigma^2\right)$$

We then simplify the functions A, B and C given in (3.14). First retain only the first Born terms and terms containing σ_{33}, ignoring the rest, which is equivalent to assuming that the 3-3 channel is the predominant one. Further, we assume that σ_{33} is approximated by the contribution from the resonance and make the substitution

$$\sigma_{33} \Rightarrow 12\pi^2 \frac{g^2}{\mu^2} p_r \delta(w-w_r)$$

$$g^2 = \Gamma(p_r)/2p_r^3 \qquad \ldots(3.32)$$

$$p_r = \text{Resonance momentum},\ w_r^2 = p_r^2 + \mu^2$$

$\Gamma(p_r)$ = Resonance width

Thus $\int \frac{dp}{w_p}\frac{\sigma(p)}{t-w_p}$ goes over to $12\pi^2\frac{g^2}{\mu^2}\int dw_p \cdot \frac{\delta(w_p-w)}{t-w_p} = \frac{12\pi^2 g^2}{\mu^2}\frac{1}{t-w}$

and (3.14) becomes

$$A(t) = \frac{4\pi}{\mu^2}\left(\frac{f^2}{t+o} - \frac{4}{3}\frac{g^2}{t-w_r} + \frac{1}{12}\frac{g^2}{t+w_r}\right)$$

$$B(t) = \frac{4\pi}{\mu^2}\left(-\frac{1}{4}\frac{g^2}{t-w_r} + \frac{1}{4}\frac{g^2}{t+w_r}\right) \qquad \ldots(3.33)$$

$$C(t) = \frac{4\pi}{\mu^2}\left(-\frac{f^2}{t-o} - \frac{1}{12}\frac{g^2}{t-w_r} + \frac{3}{4}\frac{g^2}{t-w_r}\right)$$

Substituting (3.33) into (3.31) and then into (3.21a), we find that there are five kinds of integrals to be dealt with:

$$I^+(o,\mu;\alpha,\beta) = \frac{i}{(2\pi)^7}\int_{-\infty}^{+\infty} dt \iint d^3p d^3k \frac{e^{i(\underset{\sim}{k}\cdot\underset{\sim}{y}-\underset{\sim}{p}\cdot\underset{\sim}{z})}}{(t^2-k^2)(t^2-w_p{}^2)(t-\alpha)(t-\beta)}$$

$$I^-(o,\mu;\alpha,\beta) = \frac{i}{(2\pi)^7}\int_{-\infty}^{+\infty} dt \iint d^3p d^3k \frac{e^{i(\underset{\sim}{k}\cdot\underset{\sim}{y}-\underset{\sim}{p}\cdot\underset{\sim}{z})}}{(t^2-k^2)(t^2-w_p{}^2)(t-\alpha)(t+\beta)}$$

$$I(o,\mu;\alpha) = \frac{i}{(2\pi)^7}\int_{-\infty}^{+\infty} dt \iint d^3p d^3k \frac{e^{i(\underset{\sim}{k}\cdot\underset{\sim}{y}-\underset{\sim}{p}\cdot\underset{\sim}{z})}}{(t^2-k^2)(t^2-w_p{}^2)(t-\alpha)}$$

$$I(o,\mu;1) = \frac{i}{(2\pi)^7}\int_{-\infty}^{+\infty} dt \iint d^3p d^3k \frac{e^{i(\underset{\sim}{k}\cdot\underset{\sim}{y}-\underset{\sim}{p}\cdot\underset{\sim}{z})}}{(t^2-k^2)(t^2-w_p{}^2)}$$

$$\Delta(\mu) = \frac{i}{(2\pi)^7}\frac{\partial}{\partial y}\int_{-\infty}^{+\infty} dt \iint d^3p d^3k \frac{e^{i(\underset{\sim}{k}\cdot\underset{\sim}{y}-\underset{\sim}{p}\cdot\underset{\sim}{z})}}{k^2(t^2-k^2)(t^2-w_p{}^2)}$$

$$\ldots(3.34)$$

After performing the t integration and the angular integrations, we obtain:

$$I^{+}(o,\mu;\alpha,\beta) = -F_{\alpha\beta}(y,z)$$

$$I^{-}(o,\mu;\alpha,\beta) = F_{\alpha\beta}(y,z) + 2G_{\alpha\beta}(y,z) \qquad \ldots(3.35)$$

$$I(o,\mu;\alpha) = F_{\alpha}(y,z)$$

$$I(o,\mu;1) = -F(y,z)$$

The functions $F_{\alpha\beta}$, F_{α}, F, $G_{\alpha\beta}$ and Δ are defined as

$$F_{\alpha\beta}(y,z) = \frac{1}{2(2\pi)^3 yz}\left\{\int_0^\infty \frac{p\cos w_p y \sin pz dp}{w_p(w_p+\alpha)(w_p+\beta)} + \int_\mu^\infty \frac{p\cos\sqrt{p^2-\mu^2}z \sin py dp}{p(p+\alpha)(p+\beta)} + \int_0^\mu \frac{pe^{-\sqrt{\mu^2-p^2}\,z}\sin py dp}{p(p+\alpha)(p+\beta)}\right\}$$

$$G_{\alpha\beta}(y,z) = \frac{1}{2(2\pi)^3 yz(\alpha+\beta)}\left\{\int_0^\infty \frac{p\cos w_p y \sin pz dp}{(w_p+\alpha)(w_p+\beta)} + \int_\mu^\infty \frac{p\cos\sqrt{p^2-\mu^2}\,z \sin py dp}{(p+\alpha)(p+\beta)} + \int_0^\mu \frac{pe^{-\sqrt{\mu^2-p^2}\,z}\sin pz dp}{(p+\alpha)(p+\beta)}\right\}$$

$$F_{\alpha}(y,z) = \frac{1}{2(2\pi)^3 yz}\left\{\int_0^\infty \frac{p\cos w_p y \sin pz dp}{w_p(w_p+\alpha)} + \int_\mu^\infty \frac{p\cos\sqrt{p^2-\mu^2}\,z \sin py dp}{p(p+\alpha)} + \int_0^\mu \frac{pe^{-\sqrt{\mu^2-p^2}\,z}\sin pz dp}{p(p+\alpha)}\right\}$$

$$F(y,z) = \frac{1}{2(2\pi)^3 yz} \left\{ \int_o^\infty \frac{p \cos w_p y \sin pzdp}{w_p} \right.$$

$$\left. + \int_\mu^\infty \cos\sqrt{p^2-\mu^2} \sin pzdp + \int_o^\mu e^{-\sqrt{\mu^2-p^2}\, z} \sin pydp \right\}$$

$$\Delta(\mu) = \frac{1}{2(2\pi)^3} \left\{ \int_o^\infty \frac{p \sin px}{x^2 w_p^2} \left(\frac{\cos w_p x - 2}{x\, w_p} + \sin w_p x \right) dp \right.$$

$$+ \int_o^\mu dp \frac{e^{-\sqrt{\mu^2-p^2}\, x}}{x^2 p} \left(\frac{\sin px}{xp} - \cos px \right)$$

$$\left. + \int_\mu^\infty \frac{\cos\sqrt{p^2-\mu^2}}{x^2 p} \left(\frac{\sin px}{xp} - \cos px \right) dp \right\}$$

...(3.36)

The charge dependent potential $V_{CD}(x)$ is the coefficient of T_3 in $V_I(x)$. We separate it into three parts:

$$V_{CD}(x) = V_{CD}^C(x) + V_{CS}^S(\sigma^1\sigma^2) + V_{CD}^T S_{12} \qquad ...(3.37)$$

Substituting (3.31), (3.33) and (3.35) into (3.21a), we obtain

$$V_{CD}^C(x) = -2G^2 \left(\frac{4\pi}{\mu^2}\right)^2 \left[\frac{1}{x} \left(\frac{1}{x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \frac{\partial^2}{\partial y \partial z} \right] \left\{ - \frac{16}{3} f^2 g^2 G_{ow} + \frac{16 g^4}{9} G_{ww} \right\}$$

$$V_{CD}^S(x) = 8G^2 \left(\frac{4\pi}{\mu^2}\right)^2 \left[\frac{1}{x^2} + \frac{1}{3x} \left(\frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) + \frac{1}{3} \frac{\partial^2}{\partial y \partial z} \right] \frac{\partial^2}{\partial y \partial z}\, x$$

$$\left\{ f^4 F_{oo} + \frac{2}{3} f^2 g^2 (F_{ow} + G_{ow}) + \frac{g^4}{9} (F_{ww} + G_{ww}) \right\}$$

$$+ \frac{8}{3} GF\left(\frac{4\pi}{\mu^2}\right) \frac{\partial^2}{\partial y \partial z}\left(f^2 F_o + \frac{g^2}{3} F_w\right) - F^2 \frac{2}{3} (\Delta(\mu) - F(y,z))$$

$$V_{CD}^{T}(x) = \frac{4G^2}{3} \left(\frac{4\pi}{\mu^2}\right)^2 \left[\frac{1}{x} \left(2\frac{\partial}{\partial y} - \frac{\partial}{\partial z}\right) - \frac{\partial^2}{\partial y \partial z}\right] \frac{\partial^2}{\partial y \partial z}$$

$$\left\{ f^4 F_{oo} + \frac{2}{3} f^2 g^2 \ (F_{ow} + G_{ow}) \right.$$

$$\left. + \frac{g^4}{9} (F_{ww} + G_{ww}) \right\} - \frac{4}{3} FG \frac{4\pi}{\mu^2} \frac{\partial^2}{\partial y \partial z} \left(f^2 F_o + \frac{g^2}{3} F_w\right)$$

$$+ \frac{F^2}{3} \ (\Delta(\mu) - F(y,z)) \qquad \ldots(3.38)$$

In obtaining (3.38) we have thrown away terms like G_{oo} which are divergent and obviously come from the "repetition term" where the intermediate state has identical energy as the initial state.

The separate terms in the potentials defined above can be evaluated numerically.

To include the effect of form factors, all that is involved is to modify the functions defined in (3.36). The detail of this operation will be given in the Appendix dealing with form factors.

3.4 The Charge Independent Potential

Let us now turn to the charge independent part of the γ-π exchange potential, $V_{CI}(x)$. As can be seen from the expression (3.31) of $\mathcal{K}_I$, $V_I(x)$, which belongs to the term $\underset{\sim}{j}_I^{(1)} \times \underset{\sim}{j}_I^{(2)}$, contains a charge independent part. $V_{III}(x)$, belonging to the term $\underset{\sim}{j}_s^{(1)} \times \underset{\sim}{j}_s^{(2)}$, is charge independent also. Substituting (3.16) into (3.22), we have

$$\mathcal{K}_{III} = \sum_{\alpha,\lambda} G'^2 [A_s(k_o)(\sigma^1 \times k)_\lambda (\sigma^1 p) + C_s(\sigma^1 p)(\sigma^1 \times k)_\lambda]$$

$$[C_s(k_o)(\sigma^2 \times k)_\lambda (\sigma^2 p) + A_s(k_o)(\sigma^2 p)(\sigma^2 \times k)_\lambda] \tau_\alpha^{(1)} \tau_\alpha^{(2)}$$

$$= G'^2 T \{(p \times k)^2 (A_s + A_c)^2 + [(\sigma^1 \sigma^2)(pk)^2 + p^2 (\sigma^1 k)(\sigma^2 k)$$

$$+ (pk)[(\sigma^1 k)(\sigma^2 p) + (\sigma^1 p)(\sigma^2 k)] \times (A_s - C_s)^2\}$$

...(3.39)

As in the case of $V_{CD}(x)$, we separate $V_{CI}(x)$ into three parts:

$$V_{CI}(x) = V_{CI}^C(x) + V_{CI}^S(x) \; (\sigma^1 \sigma^2) + V_{CI}^T(x) S_{12}. \qquad \text{...(3.40)}$$

Summing up (3.39) and the charge independent part in (3.31), we have

$$V_{CI}^C(x) = -4G^2 \left(\frac{4\pi}{\mu^2}\right)^2 \left[\frac{2}{x}\left(\frac{1}{x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\frac{\partial^2}{\partial y \partial z}\right] \left\{ f^4 F_{oo} + \frac{4}{3} g^4 \left(\frac{4}{3} F_{ww} + G_{ww}\right)\right.$$

$$\left. + 4 f^2 g^2 \left(\frac{2}{3} F_{ow} + G_{ow}\right)\right\}$$

$$V_{CI}^S(x) = -8G^2 \left(\frac{4\pi}{\mu^2}\right)^2 \left[\frac{1}{x^2} + \frac{1}{3x}\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) + \frac{\partial^2}{\partial y \partial z}\right] \frac{\partial^2}{\partial y \partial z}$$

$$\left\{ f^4 \left(1 + \frac{G'^2}{G^2}\right) F_{oo} \right.$$

$$+ g^4\left(\frac{1}{9}F_{ww} - \frac{1}{3}G_{ww}\right) + 2f^2g^2\left(\frac{1}{3}F_{ow} + G_{ow}\right)\Big]$$

$$- \frac{8}{3}\, GF\, \frac{\partial^2}{\partial y \partial z}\, \frac{4\pi}{\mu^2}\left(f^2F_o + \frac{g^2}{3}F_w\right) + F^2\,\frac{2}{3}\,(\Delta(\mu) - F(x,x))$$

$$V^T_{CI}(x) = -\frac{4}{3}\, G^2\left(\frac{4\pi}{\mu^2}\right)^2\left[\frac{1}{x^2}\left(2\frac{\partial}{\partial y} - \frac{\partial}{\partial z}\right)^2 - \frac{\partial^2}{\partial y\partial z}\right]\frac{\partial^2}{\partial y \partial z}$$

$$\Big\{ f^4\left(1 + \frac{G'^2}{G^2}\right)F_{oo}$$

$$+ g^4\left(\frac{1}{9}F_{ww} - \frac{1}{3}G_{ww}\right) + 2f^2g^2\left(\frac{1}{3}F_{ow} + G_{ow}\right)\Big\}$$

$$+ \frac{4}{3}\, GF\, \frac{\partial^2}{\partial y \partial z}\, \frac{4\pi}{\mu^2}\left(f^2F_o + \frac{g^2}{3}F_w\right) - F^2\,\frac{1}{3}\,(\Delta(\mu) - F(x,x))$$

...(3.41)

3.5 The Charge Symmetry Breaking Potential

The charge symmetry breaking (CSB) potential consists of terms which are the coefficients of the operator $(\tau_3^{(1)} + \tau_3^{(2)})$. Thus, from their definitions (3.20) and (3.20a), we have $V_{II}(x) = V_{CSB}(x)$. It can easily be shown that the term $(\underset{\sim}{j}^{(1)}_{int} \times \underset{\sim}{j}^{(2)}_{s} + \underset{\sim}{j}^{(1)}_{s} \times \underset{\sim}{j}^{(2)}_{int})$ vanishes identically. Hence, we are left in $\mathcal{K}_{II}$ only the term $(\underset{\sim}{j}^{(1)}_{v} \times \underset{\sim}{j}^{(2)}_{s} + \underset{\sim}{j}^{(1)}_{s} \times \underset{\sim}{j}^{(2)}_{v})$. Substituting (3.12), (3.13) and (3.16) into (3.22), we obtain,

$$\mathcal{K}_{II} = \sum_{\alpha,\lambda} \tau_3^1\, (\tau_\alpha^1\tau_\alpha^2)GG'\{C_s(A+B)(\sigma^1\times k)_\lambda(\sigma^2 p)(\sigma^2\times k)_\lambda(\sigma^2 p)$$

$$+ C_s(B+C)(\sigma^1\times k)_\lambda(\sigma^1 p)(\sigma^2 p)(\sigma^2\times k)_\lambda + A_s(B+C)(\sigma^1 p)(\sigma^1\, k)_\lambda$$

$$(\sigma^2 p)(\sigma^2\times k)_\lambda$$

$$+ A_s(A+B)(\sigma^1 p)(\sigma^1 \times k)_\lambda (\sigma^2 \times k)_\lambda (\sigma^2 p)\} + (1 \rightleftarrows 2)$$

...(3.42)

Here $(1 \rightleftarrows 2)$ stands for similar terms with index 1 and 2 interchanged. As in the previous cases, we divide $V_{CSB}(x)$ into three parts.

$$V_{CSB}(x) = V^C_{CSB}(x) + V^S_{CSB}(\sigma^1 \sigma^2) + V^T_{CSB}\, S_{12}$$

...(3.43)

After some simplification and performing the substitution (3.30), we have

$$V^C_{CSB}(x) = -\frac{8}{x}\, GG' \left(\frac{4\pi}{\mu^2}\right)^2 f^2 \left(\frac{1}{x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) \frac{\partial^2}{\partial y \partial z}\left[f^2 F_{oo} + \frac{4g^2}{3}(F_{ow} + G_{ow})\right]$$

$$V^S_{CSB}(x) = -\frac{16}{3}\, GG' \left(\frac{4\pi}{\mu^2}\right)^2 f^2 g^2 \left[\frac{1}{x^2} + \frac{1}{3x}\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) + \frac{1}{3}\frac{\partial^2}{\partial y \partial z}\right] \frac{\partial^2}{\partial y \partial z}[G_{ow}]$$

$$V^T_{CSB}(x) = -\frac{8}{9}\, GG' \left(\frac{4\pi}{\mu^2}\right)^2 f^2 g^2 \left[\frac{1}{x}\left(2\frac{\partial}{\partial y} - \frac{\partial}{\partial z}\right) - \frac{\partial^2}{\partial y \partial z}\right] \frac{\partial^2}{\partial y \partial z}[G_{ow}]$$

...(3.44)

In obtaining (3.44), we have again thrown away a term involving the function G_{oo} in $V^C_{CSB}(x)$, for the same reason mentioned before after (3.38).

CHAPTER 4

RESULTS AND CONCLUSION

Table 4.1 lists the $\gamma-\pi$ exchange charge dependent potential for the ^1S-state, $\Delta V_{\gamma\pi}$. This is the difference between the potentials in the p-n and p-p systems, $\Delta V = V_{pn} - V_{pp}$. It is plotted in Fig. 4.1. The charge dependent part of the OPEP due to pion mass difference, ΔV_{md}, and one-tenth of OPEP itself are also shown for the sake of comparison. $\Delta V_{\gamma\pi}$ is larger than 10% of the OPEP at 1 fm. For distances r<1 fm, it becomes very strong, reaching 20% of the OPEP at 0.85 fm. Between 1-3 fm, it is comparable, but on the whole stronger than ΔV_{md}. Thus, the $\gamma-\pi$ exchange process is presumably the most important long range effect. The sign of $\Delta V_{\gamma\pi}$ is opposite to that of ΔV_{md} before it changes sign at $2\lambda\!\!\!^{-}_{\pi}$. The sum $\Delta V_{\gamma\pi} + \Delta V_{md}$ is therefore positive everywhere, so that the net effect tends to make the p-n potential less negative than the p-p potential. This is at variance with their relation as indicated by the sign of the splitting Δa. The explanation of Δa would be much more difficult than was previous thought, for ΔV_{md} alone had contributed to one-third of Δa in the right direction, and now this effect is cancelled by that of $\Delta V_{\gamma\pi}$. Presumably, pion-mass difference effect in TPEP, which becomes also very strong at short distances, and splitting of the coupling constants would still be able to produce the correct value of Δa, but one doubts how meaningful such a fit would be, because of its sensitivity to the short range potential. It is to be remarked again that $\Delta V_{\gamma\pi}$ becomes very strong also, and with a wrong sign, at distances within 0.8 fm. We can expect, however, that contribution from form factors will suppress $\Delta V_{\gamma\pi}$ at short distances and reduce its intensity.

The very large charge dependent correction at very large distances required by Noyes' phenomenological analysis of the splitting Δr remains a mystery. $\Delta V_{\gamma\pi}$ is nowhere more than a few percent of the OPEP between 2-4 fm. Indeed, there is no other conceivable electromagnetic effect that can produce Noyes' correction.

The $\gamma-\pi$ exchange charge independent potential is listed in Table 4.2. Similar to $\Delta V_{\gamma\pi}$, it is of the order of a few percent of the OPEP between 1-3 fm. Hence we see that the OPEP can be considered as a reliable tail of the NN potential.

The charge symmetry breaking component of the $\gamma-\pi$ exchange potential, V_{CSB}, is listed in Table 4.3 and shown in Fig. 4.2. Beyond $1\lambda_\pi$, it is on the whole much weaker than the CSB potential due to $\pi^{o}-\eta$ mixing. Because V_{CSB} increases rapidly with decrease in distance, it can overcome the positive part of the $\pi^{o}-\eta$ mixing potential and make the n-n interaction everywhere stronger than that of p-p. This is in agreement with Okomoto and Lucas' (1967) analysis of the electromagnetic difference between the binding energies of He^3 and H^3. There is, however, still some qualification to this conclusion, since our potential is not as reliable at short distances. Also, suppression is again expected at short distances because of contribution from form factors.

TABLE 4.1

^{1}S-STATE CHARGE DEPENDENT

$\gamma-\pi$ EXCHANGE NN POTENTIAL

$$\Delta V = V_{pn} - V_{pp}$$

$X(\lambda_\pi)$	$\Delta V(\mu_\pi)$	ΔV(Mev)
0.3	0.270130	37.6561
0.4	0.098580	13.7420
0.5	0.036794	5.1290
0.6	0.014880	2.0742
0.7	0.006890	0.9604
0.8	0.003562	0.4965
0.9	0.002064	0.2877
1.0	0.001168	0.1628
1.1	0.000714	0.0995
1.2	0.000450	0.0627
1.3	0.000312	0.0434
1.4	0.000218	0.0303
1.5	0.000138	0.0192
1.6	0.000088	0.0122
1.7	0.000062	0.0086
1.8	0.000048	0.0066
1.9	0.000018	0.0025
2.0	0.000012	0.0016

TABLE 4.2

[1]S-STATE CHARGE INDEPENDENT γ-π EXCHANGE NN POTENTIAL

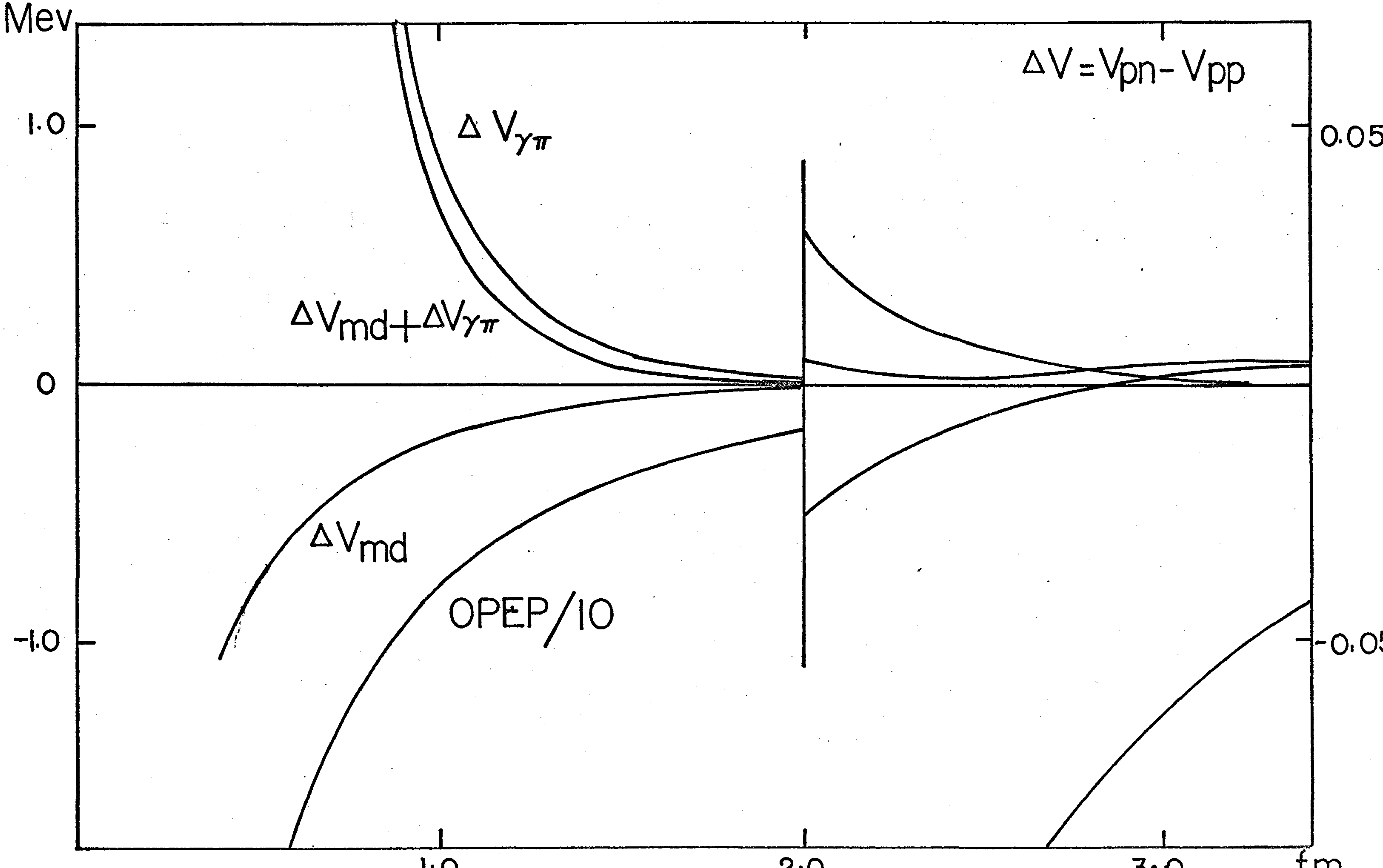
$X(\lambda_\pi)$	$V_{CI}(\mu_\pi)$	V_{CI}(Mev)
0.3	0.508958	70.9487
0.4	0.172740	24.0799
0.5	0.055195	7.6941
0.6	0.019848	2.7668
0.7	0.008329	1.1610
0.8	0.003945	0.5499
0.9	0.002022	0.2818
1.0	0.001135	0.1582
1.1	0.000672	0.0936
1.2	0.000410	0.0571
1.3	0.000286	0.0398
1.4	0.000173	0.0241
1.5	0.000111	0.0154
1.6	0.000077	0.0107
1.7	0.000051	0.0071
1.8	0.000039	0.0054
1.9	0.000020	0.0027
2.0	0.000016	0.0022

TABLE 4.3

^{1}S-STATE CHARGE SYMMETRY BREAKING $\gamma-\pi$ POTENTIAL

$$\Delta V = V_{nn} - V_{pp}$$

$X(\chi_\pi)$	$\Delta V(\mu_\pi)$	ΔV(Mev)
0.3	-0.074152	-9.9698
0.4	-0.028436	-3.9639
0.5	-0.009360	-1.3047
0.6	-0.003344	-0.4661
0.7	-0.001368	-0.1906
0.8	-0.000628	-0.0875
0.9	-0.000320	-0.0446
1.0	-0.000168	-0.0234
1.1	-0.000092	-0.0128
1.2	-0.000056	-0.0078
1.3	-0.000036	-0.0050
1.4	-0.000024	-0.0033
1.5	-0.000016	-0.0022
1.6	-0.000012	-0.0016
1.7	-0.000008	-0.0011
1.8	-0.000004	-0.0005



Mev
1.0
0
-1.0
$\Delta V_{\gamma\pi}$
$\Delta V_{md} + \Delta V_{\gamma\pi}$
ΔV_{md}
OPEP/10
$\Delta V = V_{pn} - V_{pp}$
0.05
-0.05
1·0
2·0
3·0
fm

Fig. 4·1

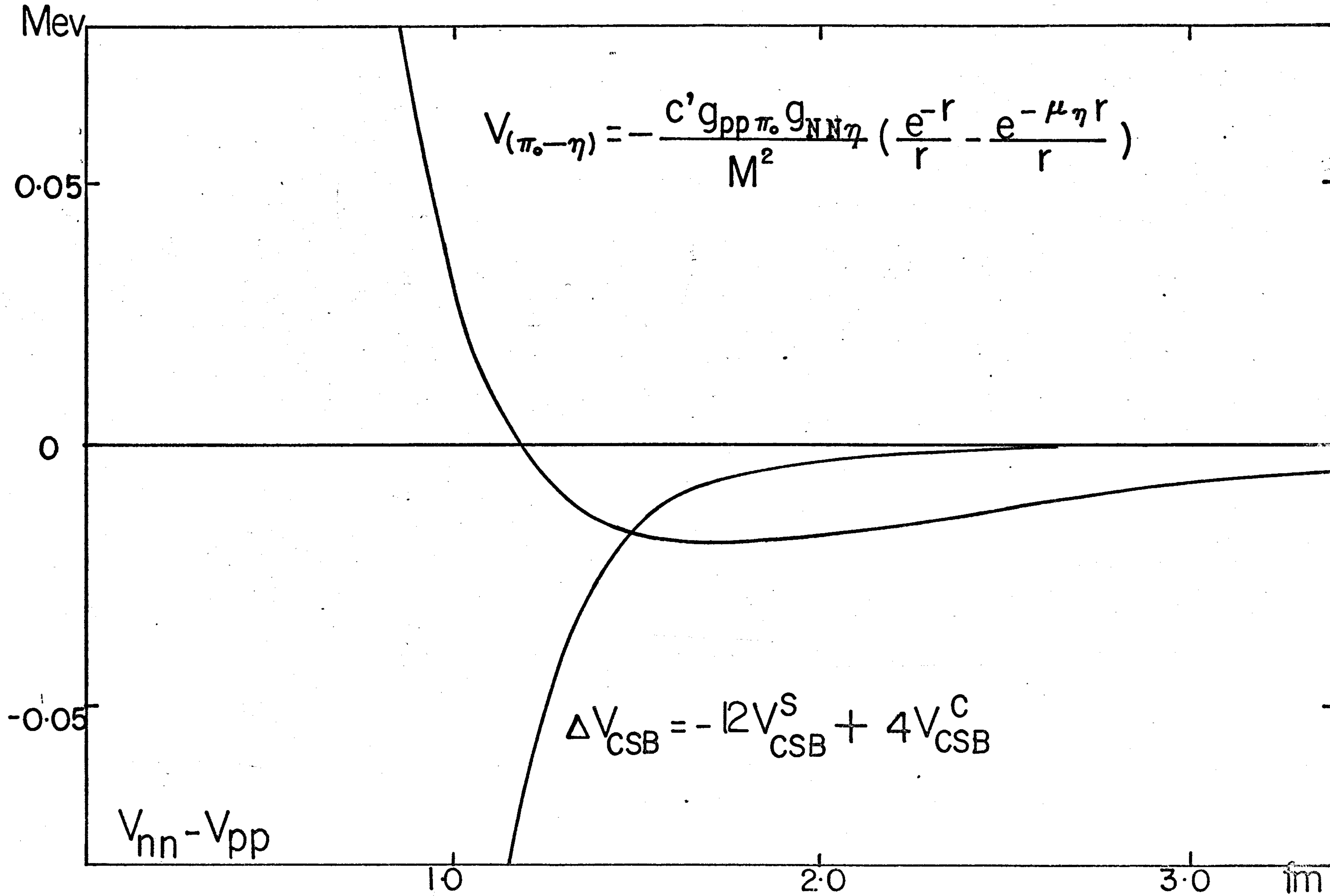

Mev
0·05
0
-0·05
$V_{(\pi_0-\eta)} = -\frac{c' g_{pp\pi_0} g_{NN\eta}}{M^2}\left(\frac{e^{-r}}{r} - \frac{e^{-\mu_\eta r}}{r}\right)$
$\Delta V_{CSB} = -12V^{S}_{CSB} + 4V^{C}_{CSB}$
$V_{nn} - V_{pp}$
1·0
2·0
3·0
fm

Fig. 4·2

PART II

EFFECTS ON λ-N INTERACTION

CHAPTER 5

CHARGE SYMMETRY VIOLATION IN λ-N INTERACTION

The most important piece of experimental evidence for the existence of a charge symmetry breaking component in the λ-N interaction is the difference in binding energies of λ-particles in the mirror hypernuclei ${}_{\lambda}He^{4}$ and ${}_{\lambda}H^{4}$. Precise determination of this difference has been difficult because the quantity sought is smaller than the typical measurement error of B_{λ} in the decay of either ${}_{\lambda}He^{4}$ or ${}_{\lambda}H^{4}$. Also, there were discrepancies in the results obtained from considerations of various decay modes. Raymund (1964) claimed that

$$\Delta B_{\lambda} = B_{\lambda}({}_{\lambda}He^{4}) - B_{\lambda}({}_{\lambda}H^{4}) = (0.30 \pm 0.14)\text{Mev} \qquad \ldots(5.1)$$

More recent work by Meyeur and collaborators (1966) indicated that $\Delta B_{\lambda} = (+0.12 \pm 0.17)$Mev, which is considered to be consistent with zero.

If the λ-N interaction is charge symmetric, the coulomb force will produce a negative ΔB_{λ}. This arises through two effects. The presence of a λ-particle, which interacts attractively with the nucleons, tends to compress the three-nucleon core. In the case of He^{3}, this compression results in a rise in the coulomb energy, i.e., the mass of the He^3-core in ${}_\lambda He^4$ is effectively larger than that of a free He^3 nucleus. The binding energy B_λ is calculated from the relation

$$B_\lambda = M_\lambda + M_z^{A-1} - \Sigma m_i - Q \qquad \ldots(5.2)$$

where M_λ is the mass of the hypernucleon, M_z^{A-1} the mass of the core, Σm_i the sum of individual component particles and Q the observed energy released. Thus, an increase in the effective mass of M_z^{A-1} will lower B_λ. Dalitz and Downs (1958) estimated that the core compression in ${}_\lambda He^4$ is about 10%. This will cause a rise of 0.08 Mev in the coulomb energy. Also, because of the coulomb force, the r.m.s. radius of He^3 is larger than that of H^3. As the λ-N interaction is of a short range character, a larger volume distribution of the nucleons means less chance for the λ-particles to feel them and hence gives rise to a more loosely bound system. Dalitz and Downs (1958) estimated that for a λ-N potential of the Gaussian form, dB_λ/dR, the gradient of binding energy with respect to core radius, is $-2.5\ B_\lambda$/fm. The difference between the radii $R(He^3)$ and $R(H^3)$ would contribute -0.35 Mev to ΔB_λ.

If we take the result given by Raymund seriously, charge assymmetry component in λ-N force would have to account for 0.75 Mev in ΔB_λ. There is however some dispute over this amount. Gal (1966) showed that a strong repulsive λNN three body force would reduce greatly the amount of core compression. He used, however, a three body force much stronger than the one derived meson-theoretically. Also, the value $^{dB}\lambda/dR$ given by Dalitz and Downs cannot be regarded as final, because of the particularly simple form of the potential and wave function employed. If we take the experimental value of ΔB_λ as the one given by Meyeur _et al_., then the effective ΔB_λ as due to charge symmetry breaking force in the λ-N interaction would only be 0.45 Mev.

Besides coulomb interaction, three electromagnetic effects have been considered, which give rise to charge-assymmetry components in the λ-N force. They are, the $\lambda-\Sigma^o$ and $\pi^o-\eta$ mixing, the $\Sigma^+-\Sigma^-$ mass difference and the $\omega-\phi-\rho^o$ mixing.

The hypothesis of charge symmetry eliminates the coupling $\lambda\lambda\pi^o$, since the π^o-state changes sign under the charge symmetry operator. Thus, without particle mixing, the λ-N interaction with the longest range is given by the exchange of two pions. Mixing of particles $\lambda-\Sigma^o$ and $\eta-\pi^o$, however, makes the coupling $\tilde{\lambda}\tilde{\lambda}\tilde{\pi}^o$ between the physical particles possible. In the SU(3) model, the amount of mixing can be calculated, making use of the U-spin invariant property of the electromagnetic mass operator. The coupling constant can then be derived. (Dalitz and Von Hippel, 1964)

$$g_{\lambda\lambda\pi} = -2 \frac{(\Sigma^{o}|\delta M|\lambda)}{M(\Sigma)-M(\lambda)} + \frac{(\pi^{o}|\delta m^{2}|\eta)}{m^{2}(\eta)-m^{2}(\pi)} \quad g_{\Sigma\lambda\pi} \qquad \ldots(5.3)$$

Here δM is the electromagnetic mass operator of the baryons and δm^2 the mass square operator of mesons. $g_{\Sigma\lambda\pi}$ is the coupling constant of the interaction $\Sigma \rightarrow \lambda + \pi$. The other symbols appearing in (5.3) are self explaining; $|\lambda)$ is the isospin eigenstate λ and $M(\lambda)$ its mass, etc. We have also

$$g_{\Sigma\lambda\pi} = \frac{2}{\sqrt{3}} g_{NN\pi} \alpha \qquad \ldots(5.4)$$

where α is the F-D mixing parameter. With this knowledge of the coupling constant $g_{\lambda\lambda\pi^o}$, one can derive the one-π^o-exchange λ-N potential. Because of (5.3), this potential is weak, but because of its long range, as compared to the TPEP, it might be quite important. Dalitz and Von Hippel (1964) estimated that the π^o-exchange potential gives rise to $+0.21 \pm 0.05$ Mev in ΔB_{λ}.

In a similar manner, one can discuss the λ-N potential given rise by the exchange of the physical vector mesons $\tilde{\omega}$, $\tilde{\phi}$ and $\tilde{\rho}^{o}$. The mixing of ω and ϕ is, however, not electromagnetic. It is a consequency of the so called medium-strong interaction. Downs (1965) estimated the effect of these potentials on ΔB_{λ}. He showed that it is possible to produce a ΔB_{λ} of 0.70 Mev with the combination of all the above mentioned one-boson-exchange potentials. The result, however, relies on the F-D mixing parameters and the coupling constant $g_{\lambda\lambda\omega}$, the value of which is not precisely determined. Furthermore, the one-boson-exchange potentials are highly singular for short distances and it is doubtful whether it is legitimate to take it literally.

The difference in the mass of the intermediate state for the TPEP of λ-p and λ-n cases may account for 0.05 Mev in ΔB_{λ}. (Dalitz and Von Hippel, 1964)

CHAPTER 6

DERIVATION OF THE

CHARGE SYMMETRY BREAKING γ-π EXCHANGE λ-N POTENTIAL

Although it is still under dispute to what extent charge symmetry in λ-N interaction is broken, it is generally agreed that charge symmetry violation is manifested in that ΔB_λ never becomes negative. It is thus of interest to examine possible processes that can give rise to a charge assymmetric interaction. Besides the processes already mentioned in Chapter 5, the γ-π exchange process also gives rise to a charge symmetry breaking λ-N interaction. We think this can be an important effect as it is important in the N-N case.

The diagram to be considered is shown in Fig. 6.1. We use the notations of Chapter 3 and write the scattering matrix as

$$S = -(2\pi)^8 \iint d^4p d^4k \sum_\lambda \frac{\langle 3p|s^{(\lambda)}|\lambda k\rangle\langle 3\text{-}p|s^{(N)}|\lambda\text{-}k\rangle}{(k_o{}^2-k^2)(k_o{}^2-w_p{}^2)} \quad \ldots(6.1)$$

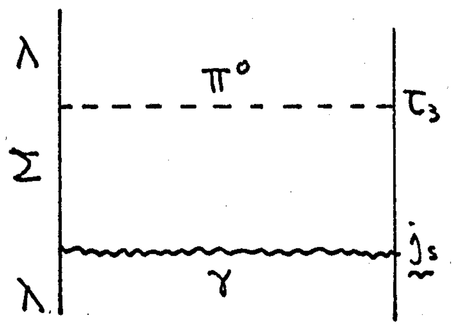


Fig. 6.1

Since the λ-particle is an isoscalar, only π^o can participate. Also, since we are interested in the charge asymmetry contribution, and the $NN\pi^o$ vertex involves the nucleon isospin operator τ_3, we need only consider the isoscalar current $\underset{\sim}{j}_s$ at the $NN\gamma$ vertex. The isovector current $\underset{\sim}{j}_v$ would in this case give rise to a charge independent contribution. We obtain the matrix element $\langle 3\text{-}p|s^{(N)}|\lambda k\rangle$ from (3.16), which gives us the perturbation result. The other matrix element in (6.1) is given by

$$\langle 3p|s^{(\lambda)}|\lambda k\rangle = 2\pi i\delta(p_o-k_o)\,\frac{e\mu}{f_\lambda}\,\frac{\mu(\Sigma\lambda)}{4M}\Big(A(k_o)(\sigma_\lambda\times k)_\lambda(\sigma_\lambda\cdot p) + C_s(k_o)(\sigma_\lambda\cdot p)(\sigma_\lambda\times k)\Big)\times e^{i(\overline{p}-\overline{k})\cdot\overline{x}_\lambda} \quad \text{...(6.2)}$$

Here f_λ is the renormalized $\Sigma\lambda\pi$ coupling constant, and $\mu(\Sigma\lambda)$ is the transition magnetic moment for the decay $\Sigma\rightarrow\lambda+\pi$, in units of nuclear nagneton. The functions A_λ and C_λ are given by static dispersion relations. (Nogami and Bloore, 1964)

$$A_\lambda(t) = \frac{4\pi f_\lambda^{\,2}}{\Delta+t+i\epsilon} + \frac{1}{2\pi}\int_0^\infty \frac{dp\ \sigma_3(p)}{w_p(w_p-t-i\epsilon)} + \frac{1}{6\pi}\int_0^\infty \frac{dp}{w_p}\ \frac{2\sigma_1(p)+\sigma_3(p)}{w_p+t+i\epsilon}$$

$$C_\lambda(t) = \frac{4\pi f_\lambda^{\,2}}{\Delta-t-i\epsilon} + \frac{1}{6\pi}\int_0^\infty \frac{dp}{w_p}\ \frac{2\sigma_1(p)+\sigma_3(p)}{w_p-t-i\epsilon}$$

$$+ \frac{1}{2\pi}\int_0^\infty \frac{dp}{w_p}\ \frac{\sigma_3(p)}{w_p+t-i\epsilon} \qquad \ldots(6.3)$$

Δ is the mass difference between the Σ- and λ- particles and σ_{2J} is the total cross section of the π-λ scattering in the state with angular momentum J. Again, we make use of the technique outlined in (3.32) and retain only σ_3 because of its predominant resonance. With the substitution

$$\begin{aligned} \sigma_3 &\Rightarrow 12\pi^2 \frac{g_\lambda^{\,2}}{\mu^2} p_\lambda \delta(w-w_\lambda) \\ g_\lambda^{\,2} &= \Gamma_\lambda(p_\lambda)/2p_\lambda^{\,3} \\ p_\lambda &= \text{Resonance momentum, } w_\lambda^2 = p_\lambda^{\,2} + \mu^2 \\ \Gamma &= \text{Resonance width} \end{aligned} \qquad \ldots(6.4)$$

(6.3) becomes

$$\begin{aligned} A_\lambda(t) &= \frac{4\pi}{\mu^2}\left\{ \frac{f_\lambda^{\,2}}{t+\Delta} + \frac{3}{2} - \frac{g_\lambda^{\,2}}{t-w} + \frac{1}{2}\,\frac{g_\lambda^{\,2}}{t+w_\lambda} \right\} \\ C_\lambda(t) &= \frac{4\pi}{\mu^2}\left\{ \frac{f_\lambda^{\,2}}{t-\Delta} + \frac{1}{2}\,\frac{g_\lambda^{\,2}}{t-w_\lambda} + \frac{3}{2} - \frac{g_\lambda^{\,2}}{t+w_\lambda} \right\} \end{aligned} \qquad \ldots(6.5)$$

Substituting (6.5), (6.3), (6.2) and (3.16) into (6.1) and employing the same technique outlined in (3.29), we can finally write the charge symmetry breaking potential as

$$V^{\lambda}_{CSB} = V^{C}_{\lambda} + V^{S}_{\lambda} \left(\sigma_N \cdot \sigma_{\lambda}\right) + V^{T}_{\lambda} S_{N\lambda} \qquad \ldots(6.6)$$

with

$$V^{C}_{\lambda} = -\frac{8}{x} G'G_{\lambda} f^2 \left(\frac{4\pi}{\mu^2}\right)^2 \left(\frac{1}{x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) \frac{\partial^2}{\partial y \partial z} [f^2 (F_{o\Delta} + G_{o\Delta}) + 2g_{\lambda}{}^2 (F_{ow_{\lambda}} + G_{ow_{\lambda}})]$$

$$V^{S}_{\lambda} = 8 G'G_{\lambda} f^2 \left(\frac{4\pi}{\mu^2}\right)^2 \left(\frac{1}{x^2} + \frac{1}{3x}\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) + \frac{1}{3}\frac{\partial^2}{\partial y \partial z}\right) \frac{\partial^2}{\partial y \partial z} [f^2_{\lambda} G_{o\Delta} - g_{\lambda}{}^2 G_{ow_{\lambda}}]$$

$$V^{T}_{\lambda} = \frac{4}{3} G'G_{\lambda} f^2 \left(\frac{4\pi}{\mu^2}\right)^2 \left(\frac{1}{x}\left(2\frac{\partial}{\partial y} - \frac{\partial}{\partial z}\right) - \frac{\partial^2}{\partial y \partial z}\right) \frac{\partial^2}{\partial y \partial z} [f^2_{\lambda} G_{o\Delta} - g_{\lambda}{}^2 G_{ow_{\lambda}}]$$

$$G_{\lambda} = \frac{e}{f_{\lambda}} \frac{\mu\mu(\Sigma_{\lambda})}{2M} \qquad \ldots(6.7)$$

For the value of f_{λ} and $\mu(\Sigma_{\lambda})$, we employ the SU(3) results,

$$f_{\lambda}/f = \frac{2}{\sqrt{3}} \alpha \qquad \ldots(6.8)$$

$$\mu(\Sigma_{\lambda}) = -\frac{\sqrt{3}}{2} \mu_n \qquad \ldots(6.9)$$

α is the so-called F-D mixing ratio and μ_n the magnetic moment of neutron in units of nuclear magneton. The value of α is not well determined, but likely to be above 0.6. We take it to be 0.75. Furthermore, we extend the relation (6.9) to write the magnetic form factor of the $\gamma\Sigma\lambda$ vertex as

$$G_{\Sigma\lambda}(t) = -\frac{\sqrt{3}}{2} G^n_M(t) \qquad \ldots(6.10)$$

For numerical calculation, we take the values

$$g^2 = 0.047$$

$$w_p = 1.79 \qquad \ldots(6.11)$$

$$\mu = \hbar = c = 1$$

CHAPTER 7

RESULT AND CONCLUSION

The charge assymetric component of the γ-π exchange λ-N potential is tabulated in Tables 7.1 and 7.2. The spin-dependent and spin-independent components are of the same signs. On the whole they are very weak compared to the π^o-exchange potential due to λ-Σ mixing in the physical $\tilde{\lambda}$. Within $1\lambda_\pi$, the γ-π exchange potential increases rapidly with decreasing distance. This is because its dependence on r^{-1} is of a higher power, due to the additional exchange of a photon. The quantity that enters into the calculation of ΔB_λ as defined in Chapter 5 is the combination $V_{\Delta B_\lambda} = (6V_\lambda^S + 2V_\lambda^C)$. This is plotted in Fig. 7.1 along with the π^o exchange potential, $V_\lambda^{\pi^o}$. $V_{\Delta B_\lambda}$ is very weak compared to $V_\lambda^{\pi^o}$, and is of the opposite sign. It was found that $V_\lambda^{\pi^o}$ alone was not enough to explain ΔB_λ. Our result will make its explanation slightly more difficult.

TABLE 7.1

^{1}S-STATE CHARGE SYMMETRY BREAKING

$\gamma-\pi$ EXCHANGE λN POTENTIAL (SPIN DEPENDENT)

$$\Delta V = V_{\lambda n} - V_{\lambda p}$$

$X(\lambda_\pi)$	$\Delta V(\mu_\pi)$	ΔV(Mev)
0.3	0.021366	2.9784
0.4	0.011748	1.6376
0.5	0.004170	0.5812
0.6	0.001362	0.1898
0.7	0.000498	0.0694
0.8	0.000204	0.0284
0.9	0.000090	0.0125
1.0	0.000042	0.0058
1.1	0.000018	0.0025
1.2	0.000006	0.0008

TABLE 7.2

^{1}S-STATE CHARGE SYMMETRY BREAKING

γ-π EXCHANGE λN POTENTIAL (SPIN INDEPENDENT)

$$\Delta V = V_{\lambda n} - V_{\lambda p}$$

$X(\lambda_\pi)$	$\Delta V(\mu_\pi)$	ΔV(Mev)
0.3	-0.079362	-11.063
0.4	-0.026354	- 3.6737
0.5	-0.007788	- 1.0856
0.6	-0.002600	- 0.3624
0.7	-0.001012	- 0.1421
0.8	-0.000444	- 0.0618
0.9	-0.000214	- 0.0298
1.0	-0.000110	- 0.0153
1.1	-0.000060	- 0.0083
1.2	-0.000034	- 0.0047
1.3	-0.000020	- 0.0027
1.4	-0.000012	- 0.0017
1.5	-0.000008	- 0.0011
1.6	-0.000006	- 0.0008
1.7	-0.000004	- 0.0005
1.8	-0.000002	- 0.0002

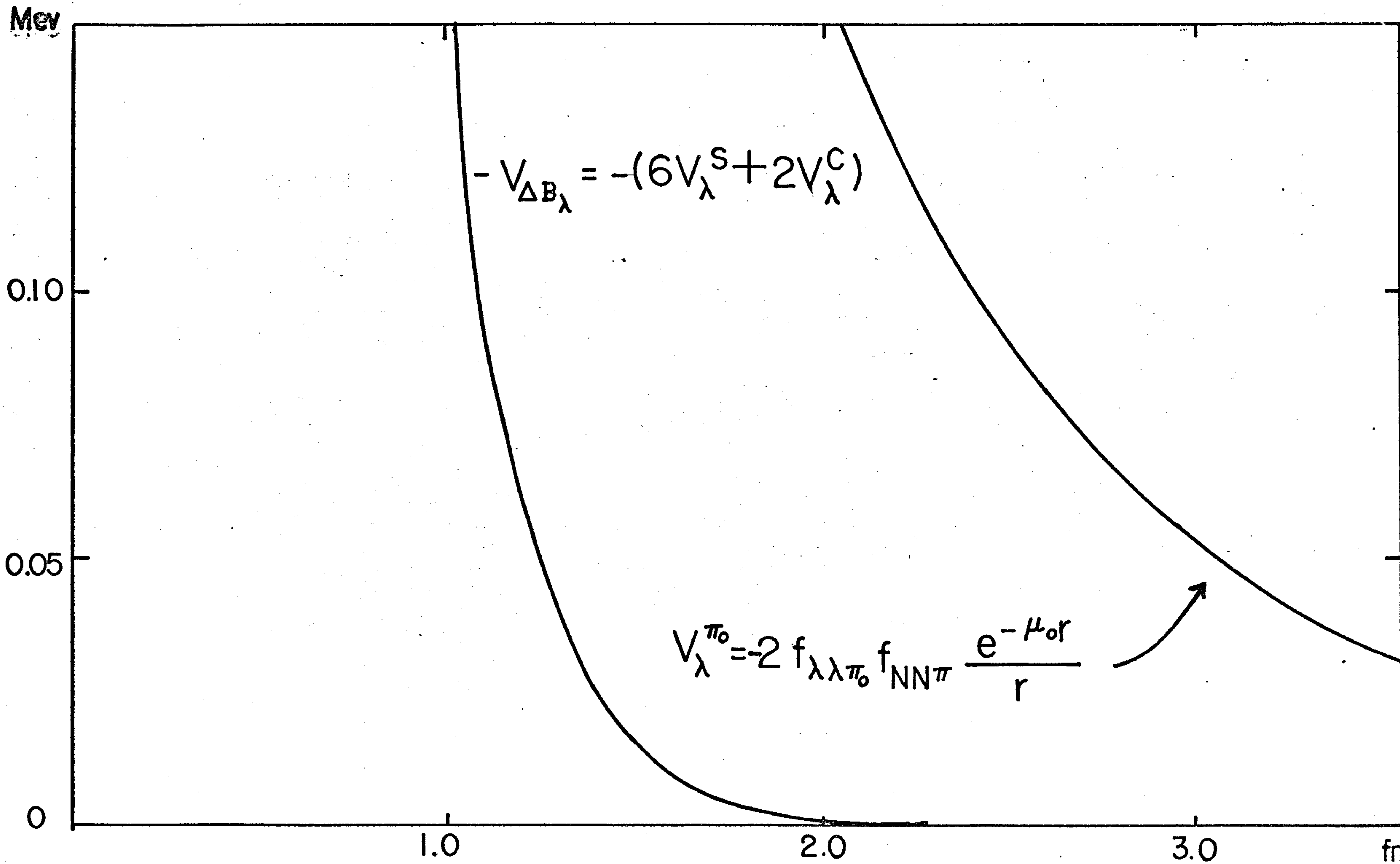

Mev
0.10
0.05
0
1.0
2.0
3.0
fm
$-V_{\Delta B_\lambda} = -(6V_\lambda^S + 2V_\lambda^C)$
$V_\lambda^{\pi_0} = -2 f_{\lambda\lambda\pi_0} f_{NN\pi} \frac{e^{-\mu_0 r}}{r}$

Fig. 7·1

APPENDIX 1

USEFUL RELATIONS

In simplifying the expressions for $\mathcal{K}_I$, $\mathcal{K}_{II}$ and $\mathcal{K}_{III}$, we found the following relations useful:

$$(\sigma\times k)_\lambda(\sigma p) = (p\times k)_\lambda + i\sigma_\lambda(kp) - ip_\lambda(\sigma k)$$
$$(\sigma p)(\sigma\times k)_\lambda = (p\times k)_\lambda - i\sigma_\lambda(kp) + ip_\lambda(\sigma k) \qquad \ldots(A1.1)$$

Making use of the above, we also have

$$\sum_{\lambda=1}^{3}(\sigma^1\times k)_\lambda(\sigma^1 p)(\sigma^2\times k)_\lambda(\sigma^2 p) = (p\times k)^2 - (\sigma^1\sigma^2)(pk)^2$$
$$- p^2(\sigma^1 k)(\sigma^2 k) + (pk)[(\sigma^1 p)(\sigma^2 k) + (\sigma^1 k)(\sigma^2 p)]$$
$$+ i(\sigma^1+\sigma^2)\cdot(p\times k)(pk)$$

$$\sum_{\lambda=1}^{3}(\sigma^1\times k)_\lambda(\sigma^1 p)(\sigma^2 p)(\sigma^2\times k)_\lambda = (p\times k)^2 + (\sigma^1\sigma^2)(pk)^2$$
$$+ p^2(\sigma^1 k)(\sigma^2 k) - (pk)[(\sigma^1 p)(\sigma^2 k) + (\sigma^1 k)(\sigma^2 p)]$$
$$+ i(\sigma^1-\sigma^2)\cdot(p\times k)(pk)$$

$$\sum_{\lambda=1}^{3}(\sigma^1 p)(\sigma^1\times k)(\sigma^2 p)(\sigma^2\times k)_\lambda = (p\times k)^2 - (\sigma^1\sigma^2)(pk)^2$$
$$- p^2(\sigma^1 k)(\sigma^2 k) + (pk)[(\sigma^1 p)(\sigma^2 k) + (\sigma^1 k)(\sigma^2 p)]$$
$$= i(\sigma^1+\sigma^2)\cdot(p\times k)(pk)$$

$$\sum_{\lambda=1}^{3}(\sigma^1 p)(\sigma^2\times k)_\lambda(\sigma^2\times k)_\lambda(\sigma^2 p) = (p\times k)^2+(\sigma^1\sigma^2)(kp)^2$$
$$+ p^2(\sigma^1 k)(\sigma^2 k)-(kp)[(\sigma^1 p)(\sigma^2 k)+(\sigma^1 k)(\sigma^2 p)$$
$$- i(\sigma^1-\sigma^2)\cdot(p\times k)(pk) \qquad \ldots(A1.2)$$

It is to be noted that the terms linear in the spins cancel out in the expressions for the $\mathcal{K}$'s, as it should be the case. We list the expressions for the other operators appearing in (A1.2) under the transformation (3.30).

$$p^2k^2 = \left(\frac{4}{x^2}+\frac{2}{x}\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)+\frac{\partial^2}{\partial y\partial z}\right)\frac{\partial^2}{\partial y\partial z} \qquad \ldots(A1.3)$$

$$(pk)^2 = \left(\frac{2}{x^2}+\frac{\partial^2}{\partial y\partial z}\right)\frac{\partial^2}{\partial y\partial z} \qquad \ldots(A1.4)$$

$$(p\times k)^2 = p^2k^2-(pk)^2 = \left(\frac{2}{x^2}+\frac{2}{x}\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\right)\frac{\partial^2}{\partial y\partial z} \qquad \ldots(A1.5)$$

$$(\sigma^1 p)(\sigma^2 p) = \frac{(\sigma^1 z)(\sigma^2 z)}{z^2}\frac{\partial}{\partial z} - \frac{(\sigma^1\sigma^2)}{z}\frac{\partial}{\partial z}$$
$$- \frac{(\sigma^1 z)(\sigma^2 z)}{z^2}\frac{\partial^2}{\partial z^2}$$
$$= \frac{1}{3}s_{12}\left(\frac{1}{x}\frac{\partial}{\partial z}-\frac{\partial^2}{\partial z^2}\right)-\frac{(\sigma^1\sigma^2)}{3}\left(\frac{2}{x}\frac{\partial}{\partial z}+\frac{\partial^2}{\partial z^2}\right) \qquad \ldots(A1.6)$$

$$(\sigma^1 p)(\sigma^2 k) = (\sigma^1 k)(\sigma^2 p)$$
$$= -\frac{(\sigma^1 y)(\sigma^2 z)}{yz}\frac{\partial^2}{\partial y\partial z} = -\frac{1}{3}(s_{12}+\sigma^1\sigma^2)\frac{\partial^2}{\partial y\partial z} \qquad \ldots(A1.7)$$

Making use of the above, we have

$$(\sigma^1\sigma^2)(pk)^2+p^2(\sigma^1k)(\sigma^2k)-(pk)[(\sigma^1p)(\sigma^2k)+(\sigma^1k)(\sigma^2p)]$$

$$= (\sigma^1\sigma^2)\left(\frac{2}{x^2}+\frac{2}{3x}\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)+\frac{2}{3}\frac{\partial^2}{\partial y\partial z}\right)\frac{\partial^2}{\partial y\partial z}$$

$$+\frac{1}{3}S_{12}\ \frac{1}{x}\left(2\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right)\frac{\partial^2}{\partial y\partial z}\ \frac{\partial^2}{\partial y\partial z} \quad \text{...(A1.8)}$$

$$\sigma^1\sigma^2-\frac{(\sigma^1k)(\sigma^2k)}{k^2}=\frac{1}{3}(2\sigma^1\sigma^2-S_{12})\left(\frac{1}{xk^2}\frac{\partial}{\partial y}+1\right) \quad \text{...(A1.9)}$$

The following combinations of the functions A, B and C, as given in (3.33) are also used in obtaining the expressions for the potentials (3.38), (3.41) and (3.44).

$$(A-B) = \frac{4\pi}{\mu^2}\left(\frac{f^2}{t+o}-\frac{1}{2}\ \frac{g^2}{t-w_r}-\frac{1}{6}\frac{g^2}{t+w_r}\right) \quad \text{...(A1.10)}$$

$$(A-C) = \frac{4\pi}{\mu^2}\left(\frac{f^2}{t+o}+\frac{f^2}{t-o}-\frac{2}{3}\frac{g^2}{t-w_r}-\frac{2}{3}\frac{g^2}{t-w_r}\right) \quad \text{...(A1.11)}$$

$$(A+B) = \frac{4\pi}{\mu^2}\left(\frac{f^2}{t+o}-\frac{g^2}{t-w_r}+\frac{1}{3}\frac{g^2}{t+w_r}\right) \quad \text{...(A1.12)}$$

Denote $$I[R(t)] = \frac{i}{(2\pi)^7}\int_{-\infty}^{+\infty}dt\iint d^3p\,d^3k\ R(t) \quad \text{...(A1.13)}$$

In what follows, we write w for w_r for simplicity.

$$I[(A-B)^2] = -\left(\frac{4\pi}{\mu^2}\right)^2\left\{f^4F_{oo}+f^2g^2\left(\frac{2}{3}F_{ow}+2G_{ow}\right)\right.$$

$$\left.+g^4\left(\frac{1}{9}F_{ww}-\frac{1}{3}G_{ww}\right)\right\} \quad \text{...(A1.14)}$$

$$I[(A-C)^2] = \left(\frac{4\pi}{\mu}\right)^2 \left(\frac{-16f^2g^2}{3} G_{ow} + \frac{16g^4}{9} G_{ww}\right) \quad \text{...(A1.15)}$$

In deriving (A1.15), we have dropped the term G_{oo}.

$$I[4(A-B)^2-(A-C)^2] = -4\left(\frac{4\pi}{\mu^2}\right)^2 \Big\{ f^4F_{oo} + \frac{2}{3} f^2g^2 \ (F_{ow} \pm G_{ow})$$

$$+ \frac{g^4}{9} (F_{ww} + G_{ww})\Big\} \quad \text{...(A1.16)}$$

$$I[(A+B)^2] = -\left(\frac{4\pi}{\mu^2}\right)^2 \Big\{ f^4F_{oo} + \frac{4}{3} g^4 \left(\frac{4}{3} F_{ww} + G_{ww}\right)$$

$$+ 4 f^2g^2 \left(\frac{2}{3} F_{ow} + G_{ow}\right)\Big\} \quad \text{...(A1.17)}$$

$$I[(A_s-C_s)^2] = -\left(\frac{4\pi}{\mu^2}\right)^2 4 f^4F_{oo} \quad \text{...(A1.18)}$$

$$I[A_s(A+2B+C)] = 2\left(\frac{4\pi}{\mu^2}\right)^2 f^2 \Big\{ f^2F_{oo} + \frac{4g^2}{3} (F_{ow} + G_{ow})\Big\}$$

$$\text{...(A1.19)}$$

$$I[A_s(A-C)] = -\left(\frac{4\pi}{\mu^2}\right)^2 f^2 \frac{4}{3} g^2G_{ow} \quad \text{...(A1.20)}$$

$$I[A_s(A_\lambda+C_\lambda)] = -2\left(\frac{4\pi}{\mu^2}\right)^2 f^2 \Big\{ f_\lambda^2 \ (F_{o\Delta}+G_{o\Delta}) + 2g_\lambda^{\ 2}(F_{ow}+G_{ow\lambda})\Big\}$$

$$\text{...(A1.21)}$$

$$I[A_s(A_\lambda-C_\lambda)] = 2\left(\frac{4\pi}{\mu^2}\right)^2 f^2 \ (f_\lambda^2 G_{o\Delta} - g_\lambda^{\ 2}G_{ow\lambda}) \quad \text{...(A1.22)}$$

$$I[(A-B)] = -\frac{4\pi}{\mu^2} \left(f^2F_o + \frac{g^2}{3} F_w \right) \quad \text{...(A1.23)}$$

APPENDIX 2

NUCLEON FORM FACTORS

i. Electromagnetic Form Factors

Electromagnetic form factors of the nucleon describes its charge and magnetization distribution. In a crude way we may picture this as the result of the strong coupling between nucleon and mesons, the manifestation of the fact that the bare Dirac particle is being dressed by a meson cloud. The most general form of the nucleon-photon vertex consistent with Lorentz invariance and gauge invariance can be written as

$$\langle p_f | j_\mu(o) | p_i \rangle = e \langle \bar{u}_{p_f} | F_1(q^2) r_\mu + i \frac{\kappa}{2\mu} \sigma_{\mu\nu} q_\nu F_2(q^2) | u_{p_i} \rangle \qquad \text{...(A2.1)}$$

It describes the vertex where a virtual photon with momentum square $q^2 = (p_f - p_i)^2$ is absorbed by a nucleon satisfying the free Dirac equation. p_f and p_i are final and initial momentum respectively. κ is the anomalous magnetic moment of the nucleon in unit of nuclear magneton. F_1 and F_2 are referred to as Dirac and Pauli form factors respectively. They are renormalized so that $F_1(o) = F_2(o) = 1$.

For the sake of more convenient physical interpretation, the following linear combination of F_1 and F_2 are defined

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4\,2} \kappa F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + \kappa F_2(q^2) \qquad \ldots(A2.2)$$

In the Breit frame where $q_o=o$, these two functions represent the Fourier transform of the nuclear charge and magnetic moment distributions. For vanishing q^2, we have, according to the above normalizations of F_1 and F_2,

$$G_{Ep}(o) = 1.0 \qquad G_{Mp} = g_p$$

$$G_{En}(o) = 0 \qquad G_{Mn}(o) = g_n \qquad \ldots(A2.3)$$

It is also convenient to define the isoscalar and isovector form factors:

$$G_E^V = \frac{1}{2}(G_{Ep}-G_{En})$$

$$G_E^S = \frac{1}{2}(G_{Ep}+G_{En})$$

$$G_M^V = \frac{1}{2}(G_{Mp}-G_{Mn}) \qquad \ldots(A2.4)$$

$$G_M^S = \frac{1}{2}(G_{Mp}+G_{Mn})$$

These form factors are determined experimentally by electron-nucleon scattering. The analysis of electron-nucleon scattering employs the Rosenbluth formula which assumes the exchange of a single photon and validity of the first Born approximation. Deviation from the Rosenbluth formula is not observed for value of q^2 up to 50 fm^{-2}. Contribution from two photon exchange can be tested through comparison of electron-proton and positron-proton scattering and measurement of polarization of the recoil proton. The theoretical prediction of two photon exchange contribution is small. Experimentally, it is found to be small except perhaps for very high value of q^2.

The current interpretation of nucleon e.m. form factor relates them to the vector mesons through dispersion relations. Assuming that $G_v(q^2)$ and $G_s(q^2)$ are analytic in the complex z-plane with cuts on the real axis and that it has the suitable assymtoptic property as $z \to \infty$, we can write

$$G(q^2) = \frac{1}{\pi} \int_{\omega}^{\infty} \frac{I_m G(q^{2\prime})}{q'^2 - q^2} \, dq'^2 \qquad \text{...(A2.5)}$$

If we further assume that $I_m G(q^2)$ is approximated by contributions arising from poles related to a few vector bosons, we get the Clementel-Villi formula

$$G(q^2) = \sum_n \frac{r_n}{1 - q^2/M_n^2} \qquad \text{...(A2.6)}$$

Here r_n is a measure of the coupling of the boson with mass M_n to the photon.

Attempts have been made to fit $G_V(q^2)$ and $G_s(q^2)$ with Clementel-Villi equations of a few terms. The isovector meson ρ and isoscalar mesons ϕ and ω are employed. It is found that the ρ meson mass has to be considerably smaller than the experimental value. A four pole fit preserving the ρ-mass introduces an additional vector meson at 875Mev, which is not discovered. Thus, the analytic form of $G_V(q^2)$ and $G_s(q^2)$ are ambiguous.

For our calculation, we have taken a two pole fit given by de Vries et al. (1962).

$$G_E^S\ (q^2) = \left(-0.17 + \frac{1.17}{1-q^2/23} \right) \frac{e}{2}$$

$$G_M^S\ (q^2) = -0.06 \left(1.5 - \frac{0.5}{1-q^2/23} \right) \frac{e}{2m}$$

$$G_E^V\ (q^2) = \left(0.08 + \frac{0.92}{1-q^2/18} \right) \frac{e}{2}$$

$$G_M^M\ (q^2) = 1.85 \left(-0.18 + \frac{1.15}{1-q^2/18} \right) \frac{e}{2m}$$

...(A2.7)

ii. The Pionic Form Factor

The Pionic form factor describes the so-called "strong structure" of the nuclear just as the e.m. form factor describes its e.m. structure. The concept of pionic form factor is first applied to the analysis of pion production in inelastic N-N scattering. Here the peripheral model, which assumes the exchange of a single pion and ignores the final state interaction between the two nucleons, is rather successful at low momentum transfer but fails at higher energies. This leads to the conjecture that the peripheral model might be preserved if one adds into the formalism the multiplicative function of q^2

$$G(q^2) = K^2(q^2)\ K'(q^2) \qquad \ldots(A2.8)$$

$K(q^2)$ stands for the form factor of the pion-nucleon vertex and $K'(q^2)$ the ratio between the complete and perturbative pion propagator. Thus, a quantitative determination of $G(q^2)$ can be carried out, and the consistence of $G(q^2)$ at different energies indicates that the peripheral model is reliable.

In contrast to the electromagnetic case, separate determination of K and K' is impossible. In electromagnetic interaction, the perturbation expansion in orders of the coupling constant $(e^2/4\pi)$ converges rapidly enough. Thus, the difference between the complete propagator of the photon and its perturbative propagator can be neglected as corrections of higher order. In strong interactions, this is no longer true, since the perturbation expansion is not reliable.

There is yet no theoretical determination as to the form of function $G(q^2)$. Ferrari and Selleri (1963) measured $G(q^2)$ from reactions $p+p\rightarrow p+n+\pi^+$ and $p+p\rightarrow p+p+\pi^o$. They give a phenomenological fit in the form of a Clementel-Villi formula:

$$G(q^2) = \frac{1-A}{1+(q^2+\mu^2)/\eta} + A \qquad \text{...(A2.9)}$$

$$A = 0.28, \qquad \eta = 4.73\ \mu^2$$

They claimed that other types of expressions are found to give bad agreement for any choice of the parameters.

APPENDIX 3

INCORPORATING NUCLEON FORM FACTORS

In this Appendix we deal with the nucleon form factors. Incorporating form factors in their current analytic form into our calculation introduces additional integrals into the expressions for the γ-π potentials. They are very similar to the functions encountered in Chapters 3 and 6. The numerical evaluations of these new integrals are, however, not as easy. Their contribution to the potentials are small, especially at distances beyond one pion compton wave length. We shall therefore not evaluate them, but satisfied with surveying their effects from the special case of V_{λ}^{C}, where the integration can be carried out analytically, we first deal with the electromagnetic form factors and then the pionic form factor. In the electromagnetic case, we only consider the magnetic form factors, since major contribution to the γ-π potential comes from the magnetic currents.

We have three kinds of terms which are the products of two currents, viz., $j_V^{(1)} \times j_V^{(2)}$, $j_S^{(1)} \times j_S^{(2)}$ and

$(j_v^{(1)} \times j_s^{(2)} + j_s^{(1)} \times j_v^{(2)})$. The first two terms have identical modifications form the incorporation of the e.m. form facotr, except for some numerical constants. We shall consider them together and the last term separately.

i. $j_v^{(1)} \times j_v^{(2)}$ and $j_s^{(1)} \times j_s^{(2)}$

In this section F(t) stands for either the iso-scalar or isovector magnetic form factor, $F(t) = a + \frac{b}{m^2+k^2-t^2}$.

Let us define formally the operator

$$J = \frac{i}{(2\pi)^7}\int_{-\infty}^{+\infty} dt \iint d^3p d^3k \qquad \ldots (A3.1)$$

so that

$$I = \frac{i}{(2\pi)^7}\int_{-\infty}^{+\infty} dt \int d^3p d^3k \; R(t)(t^2-k^2)^{-1}(t^2-w_p{}^2)^{-1}$$

$$= J[R(t)(t^2-k^2)^{-1}(t^2-w_p{}^2)^{-1}] \qquad \ldots (A3.2)$$

R(t) is any function of (t).

We further define

$$I_m = J\left[k(t)(m^2+k^2-t^2)^{-1}(t^2-w_p{}^2)^{-1}\right] \qquad \ldots (A3.3)$$

$$MI = J\left[R(t)(m^2+k^2-t^2)^{-1}(t^2-k^2)^{-1}(t^2-w_p{}^2)^{-1}\right] \qquad \ldots (A3.4)$$

$$M^2 I = J\left[R(t)(m^2+k^2-t^2)^{-2}(t^2-k^2)^{-1}(t^2-w_p{}^2)^{-1}\right] \qquad \ldots (A3.5)$$

Then

$$F^2(t) I = a^2 I + 2abMI + b^2 M^2 I \qquad \ldots (A3.6)$$

But it can be shown that

$$MI = \frac{1}{m^2}\,[I+I_m] \qquad \text{...(A3.7)}$$

$$M^2I = \frac{-\partial}{\partial m^2}\,MI = \frac{+1}{m^4}\,[I+I_m] - \frac{1}{m^2}\frac{\partial}{\partial m^2}\,I_m \qquad \text{...(A3.8)}$$

Substituting (A3.7) and (A3.8) into (A3.6), we have

$$F^2(t)I = I + (1-a^2)I_m - m^2(1-a^2)\,\frac{\partial}{\partial m^2}\,I_m \qquad \text{...(A3.9)}$$

Thus it is clear that the inclusion of magnetic form factors is equivalent to making the substitutions:

$$F_{\alpha\beta}(y,z) \longrightarrow F_{\alpha\beta}(y,z) - (1-a^2)H_{\alpha\beta}(y,z) + m^2(1-a)^2\,\frac{\partial}{\partial m^2}\,H_{\alpha\beta}$$

$$G_{\alpha\beta}(y,z) \longrightarrow G_{\alpha\beta}(y,z) - (1-a^2)E_{\alpha\beta}(y,z) + m^2(1-a)^2\,\frac{\partial}{\partial m^2}\,E_{\alpha\beta}$$

...(A3.10)

In obtaining (A3.10), we have made use of the normalization relation $(a + {}^{b}/_{m}2)=1$. The functions $H_{\alpha\beta}$, $E_{\alpha\beta}$, $\frac{\partial}{\partial m^2}H_{\alpha\beta}$ and $\frac{\partial}{\partial m^2}E_{\alpha\beta}$ are defined in the following:

$$H_{\alpha\beta}(y,z) = \frac{1}{2(2\pi)^3 yz}\left\{\int_0^\infty \frac{\cos(p^2+r^2)^{\frac{1}{2}}z\ \sin py\ pdp}{\delta(\delta+\alpha)(\delta+\beta)} + \int_0^r \frac{e^{-\sqrt{r^2-p^2}\,y}\sin pz\ pdp}{w_p(w_p+\alpha)(w_p+\beta)} + \int_r^\infty \frac{\cos\sqrt{p^2-r^2}\,y\ \sin pz\ pdp}{w_p(w_p+\alpha)(w_p+\beta)}\right\}.$$

$$E_{\alpha\beta}(y,z) = \frac{1}{2(2\pi)^3 yz(\alpha+\beta)} \left\{ \int_0^\infty \frac{\cos(p^2+r^2)^{\frac{1}{2}} z \ \sin py \ pdp}{(\delta+\alpha)(\delta+\beta)} \right.$$

$$\left. + \int_0^r \frac{e^{-\sqrt{r^2-p^2}y} \sin pz \ pdp}{(w_p+\alpha)(w_p+\beta)} + \int_r^\infty \frac{\cos\sqrt{p^2-r^2}y \ \sin pz \ pdp}{(w_p+\alpha)(w_p+\beta)} \right\}$$

$$\frac{\partial}{\partial m^2} H_{\alpha\beta}(y,z) = \frac{1}{2(2\pi)^3 yz} \left\{ -\frac{1}{2} \int_0^\infty \frac{pdp}{\delta(\delta+\alpha)(\delta+\beta)} \left\{ \frac{z \sin\sqrt{\delta^2-\mu^2}z \ \sin py}{\sqrt{\delta^2-\mu^2}} \right.\right.$$

$$\left. + \frac{\cos\sqrt{\delta^2-\mu^2}z \ \sin py}{\delta} \left(\frac{1}{\delta} + \frac{1}{\delta+\alpha} + \frac{1}{\delta+\beta} \right) \right\}$$

$$+ \frac{y}{2} \int_r^\infty \frac{p \sin\sqrt{p^2-r^2}y \ \sin pz \ pdp}{\sqrt{p^2-r^2} \ w_p(w_p+\alpha)(w_p+\beta)}$$

$$\left. - \frac{y}{2} \int_0^r \frac{pe^{-\sqrt{r^2-p^2} y} \sin pz \ dp}{\sqrt{r^2-p^2} \ w_p(w_p+\alpha)(w_p+\beta)} \right\}$$

$$\frac{\partial}{\partial m^2} E_{\alpha\beta}(y,z) = \frac{1}{2(2\pi)^3 yz(\alpha+\beta)} \left\{ -\frac{1}{2} \int_0^\infty \frac{pdp}{(\delta+\alpha)(\delta+\beta)} \right.$$

$$\left\{ \frac{z \sin\sqrt{\delta^2-\mu^2}z \ \sin py}{\sqrt{\delta^2-\mu^2}} + \frac{\cos\sqrt{\delta^2-\mu^2}z \ \sin py}{\delta} \left(\frac{1}{\delta+\alpha} + \frac{1}{\delta+\beta} \right) \right\}$$

$$\left. + \frac{y}{2} \int_r^\infty \frac{p \sin\sqrt{p^2-r^2}y \ \sin pz \ dp}{\sqrt{p^2-r^2} \ (w_p+\alpha)(w_p+\beta)} - \frac{y}{2} \int_0^r \frac{pe^{-\sqrt{r^2-p^2}y} \sin pz \ dp}{\sqrt{r^2-p^2} \ (w_p+\alpha)(w_p+\beta)} \right\}$$

$$\delta = \sqrt{p^2+m^2} \ , \quad r = \sqrt{m^2-\mu^2} \qquad \text{...(A3.11)}$$

To incorporate the pionic form factor into the potentials, we have to evaluate the expression

$F^2(t)F_\pi(t)I \quad F_\pi(t) = A + \frac{B}{\eta+\mu^2+p^2-t^2}$

$$I_\eta = J[R(t)(\eta+p^2+\mu^2-t^2)^{-1}(t^2-k^2)^{-1}]$$

$$I_{m\eta} = J\left[R(t)(\eta+p^2+\mu^2-t^2)^{-1}(m^2+k^2-t^2)^{-1}\right]$$

$$NI = J\left[R(t)(\eta+p^2+\mu^2-t^2)^{-1}(t^2-k^2)^{-1}(t^2-w_p^2)^{-1}\right]$$

$$MNI = J\left[R(t)(\eta+p^2+\mu^2-t^2)^{-1}(m^2+k^2-t^2)^{-1}(t^2-k^2)^{-1}(t^2-w_p^2)^{-1}\right]$$

$$M^2NI = J\left[R(t)(\eta+p^2+\mu^2-t^2)^{-1}(m^2+k^2-t^2)^{-2}(t^2-k^2)(t^2-w_p^2)^{-1}\right]$$

...(A3.12)

Then

$$F(t)^2F_\pi(t)I = (a^2A+2abM+b^2AM^2+a^2BN+2abBMN+b^2BM^2N)\cdot I$$

...(A3.13)

It can easily be shown that

$$NI = \frac{1}{\eta}[I+I_\eta] \qquad \text{...(A3.14)}$$

$$MNI = \frac{1}{m^2\eta}[I+I_m+I_\eta+I_{m\eta}] \qquad \text{...(A3.15)}$$

$$M^2NI = \frac{+1}{m^4\eta}[I+I_m+I_\eta+I_{m\eta}] - \frac{1}{m^2\eta}\frac{\partial}{\partial m^2}I_m + \frac{\partial}{\partial m^2}I_{m\eta}$$

...(A3.16)

Substituting (A3.7), (A3.8), (A3.14), (A3.15) and (A3.16) into (A3.13), we have

$$F^2(t)F_\pi(t)I = I+(1-a^2)I_m+(1-A)I_\eta+(1-A)(1-a)I_{m\eta}-m^2(1-a)^2 \frac{\partial}{\partial m^2} I_m \; -m^2(1-a)^2(1-a) \frac{\partial}{\partial m^2} I_{m\eta}$$

...(A3.17)

Corresponding to (A3.17), the substitutionson $F_{\alpha\beta}(y,z)$ and $G_{\alpha\beta}(y,z)$ are:

$$F_{\alpha\beta}(y,z) \longrightarrow F_{\alpha\beta}-(1-a^2)H_{\alpha\beta}-(1-A)L_{\alpha\beta}+(1-A)(1-a^2)O_{\alpha\beta} + m^2(1-a^2)\frac{\partial}{\partial m^2}H_{\alpha\beta}-m^2(1-a)^2(1-A)\frac{\partial}{\partial m^2}O_{\alpha\beta}$$

$$G_{\alpha\beta}(y,z) \longrightarrow G_{\alpha\beta}-(1-a^2)E_{\alpha\beta}-(1-A)D_{\alpha\beta}+(1-A)(1-a^2)C_{\alpha\beta} + m^2(1-a)^2\frac{\partial}{\partial m^2}E_{\alpha\beta}-m^2(1-a)^2(1-A)\frac{\partial}{\partial m^2}C_{\alpha\beta}$$

...(A3.18)

The functions $L_{\alpha\beta}$, $O_{\alpha\beta}$, $\frac{\partial}{\partial m^2}O_{\alpha\beta}$, $D_{\alpha\beta}$, $C_{\alpha\beta}$ and $\frac{\partial}{\partial m^2}C_{\alpha\beta}$ are defined as follows:

$$L_{\alpha\beta} = \frac{1}{2(2\pi)^3 yz}\left\{\int_0^\infty \frac{\cos\theta y \sin pz \, pdp}{\theta(\theta+\alpha)(\theta+\beta)} + \int_0^\rho \frac{e^{-\sqrt{\rho^2-p^2}z}\sin py \, dp}{(p+\alpha)(p+\beta)} + \int_\rho^\infty \frac{\cos\sqrt{p^2-\rho^2}z \sin py \, dp}{(p+\alpha)(p+\beta)}\right\}$$

$$O_{\alpha\beta} = \frac{1}{2(2\pi)^3 yz}\left\{\int_0^\infty \frac{\cos\sqrt{p^2+\chi^2}z \sin py \, pdp}{\delta(\delta+\alpha)(\delta+\beta)} + \int_0^\chi \frac{e^{-\sqrt{\chi^2-p^2}y}\sin pz \, pdp}{\theta(\theta+\alpha)(\theta+\beta)} + \int_\chi^\infty \frac{\cos\sqrt{p^2-\chi^2}y \sin pz \, pdp}{\theta(\theta+\alpha)(\theta+\beta)}\right\}$$

$$D_{\alpha\beta}=\frac{1}{2(2\pi)^3yz(\alpha+\beta)}\left\{\int_0^\infty\frac{\cos\theta y\ \sin pz\ pdp}{(\theta+\alpha)(\theta+\beta)}+\int_0^\rho\frac{e^{-\sqrt{\rho^2-p^2}z}\sin py\ pdp}{(p+\alpha)(p+\beta)}\right.$$

$$\left.+\int_\rho^\infty\frac{\cos\sqrt{p^2-\rho^2}z\ \sin py\ pdp}{(p+\alpha)(p+\beta)}\right\}.$$

$$C_{\alpha\beta}=\frac{1}{2(2\pi)^3yz(\alpha+\beta)}\left\{\int_0^\infty\frac{\cos\sqrt{p^2+\chi^2}z\ \sin py\ pdp}{(\delta+\alpha)(\delta+\beta)}\right.$$

$$\left.+\int_0^\chi\frac{e^{-\sqrt{\chi^2-p^2}y}\sin pz\ pdp}{(\theta+\alpha)(\theta+\beta)}+\int_\chi^\infty\frac{\cos\sqrt{p^2-\chi^2}y\ \sin pz\ pdp}{(\theta+\alpha)(\theta+\beta)}\right\}.$$

$$\frac{d}{dm^2}\,O_{\alpha\beta}=\frac{1}{2(2\pi)^3yz}\left\{-\frac{1}{2}\int_0^\infty\frac{pdp}{\delta(\delta+\alpha)(\delta+\beta)}\left[\frac{z\ \sin\sqrt{p^2+\chi^2}z\ \sin py}{\sqrt{p^2+\chi^2}}\right.\right.$$

$$\left.+\frac{\cos\sqrt{p^2+\chi^2}z\ \sin py}{\delta}\left(\frac{1}{\delta}+\frac{1}{\delta+\alpha}+\frac{1}{\delta+\beta}\right)\right]$$

$$+\frac{y}{2}\int_\chi^\infty\frac{p\ \sin\sqrt{p^2-\chi^2}y\ \sin pz\ dp}{\sqrt{p^2-\chi^2}\ \theta(\theta+\alpha)(\theta+\beta)}$$

$$\left.-\frac{y}{2}\int_0^\chi\frac{pe^{-\sqrt{\chi^2-p^2}y}\sin pz\ dp}{\sqrt{\chi^2-p^2}\ \theta(\theta+\alpha)(\theta+\beta)}\right\}$$

$$\frac{d}{dm^2}\,C_{\alpha\beta}=\frac{1}{2(2\pi)^3yz(\alpha+\beta)}\left\{-\frac{1}{2}\int_0^\infty\frac{pdp}{(\delta+\alpha)(\delta+\beta)}\left[\frac{z\ \sin\sqrt{p^2+\chi^2}z\ \sin py}{\sqrt{p^2+\chi^2}}\right.\right.$$

$$\left.+\frac{\cos\sqrt{p^2+\chi^2}z\ \sin py}{\delta}\left(\frac{1}{\delta+\alpha}+\frac{1}{\delta+\beta}\right)\right]$$

$$+ \frac{y}{2} \int_{\chi}^{\infty} \frac{p \sin\sqrt{p^2-\chi^2}y \sin pz \, dp}{\sqrt{p^2+\chi^2} \, (\theta+\alpha)(\theta+\beta)}$$

$$\left. - \frac{y}{2} \int_{0}^{\infty} \frac{p e^{-\sqrt{\chi^2-p^2}y} \sin pz \, dp}{\sqrt{\chi^2-p^2} \, (\theta+\alpha)(\theta+\beta)} \right\} \qquad \ldots (A3.19)$$

Here $\rho = \sqrt{\mu^2+\eta}$, $\chi = \sqrt{r^2+\eta}$ and $\theta = \sqrt{p^2+\rho^2}$.

For the fraction $F_\alpha(y,z)$, the magnetic form factor appears only once, i.e., $I \to F_V(t) F_\pi(t) I$. Correspondingly,

$F_\alpha(y,z) \longrightarrow F_\alpha(y,z) - (1-A)L_\alpha - (1-a)H_\alpha + (1-a)(1-A)O_\alpha$.

Here

$$H_\alpha(y,z) = \frac{1}{2(2\pi)^3 yz} \left\{ \int_0^\infty \frac{\cos(p^2+r^2)^{\frac{1}{2}}z \sin py \, pdp}{\delta(\delta+\alpha)} \right.$$

$$\left. + \int_0^r \frac{e^{-\sqrt{r^2-p^2}y} \sin pz \, pdp}{w_p(w_p+\alpha)} + \int_r^\infty \frac{\cos\sqrt{p^2-r^2}y \sin pz \, pdp}{w_p(w_p+\alpha)} \right\}$$

$$L_\alpha(y,z) = \frac{1}{2(2\pi)^3 yz} \left\{ \int_0^\infty \frac{\cos\theta y \sin pz \, pdp}{\theta(\theta+\alpha)} \right.$$

$$\left. + \int_0^\rho \frac{e^{-\sqrt{\rho^2-p^2}z} \sin py \, dp}{(p+\alpha)} + \int_\rho^\infty \frac{\cos\sqrt{p^2-\rho^2}z \sin py \, dp}{(p+\alpha)} \right\}$$

$$O_\alpha(y,z) = \frac{1}{2(2\pi)^3 yz} \left\{ \int_0^\infty \frac{\cos\sqrt{p^2+\chi^2}z \sin py \, pdp}{\delta(\delta+\alpha)} \right.$$

$$\left. + \int_0^\chi \frac{e^{-\sqrt{\chi^2-p^2}y} \sin pz \, pdp}{\theta(\theta+\alpha)} + \int_\chi^\infty \frac{\cos\sqrt{p^2-\chi^2}y \sin pz \, pdp}{\theta(\theta+\alpha)} \right\}$$

$$\ldots (A3.19a)$$

ii. $j_v^{(1)} \times j_s^{(2)} + j_s^{(1)} \times j_v^{(2)}$

For these terms, including the magnetic form factors, means replacing I by the expression $F_v(t)F_s(t)I$. Making use of the notations of the previous section, we can write

$$F_v(t)F_s(t)I = (aa' + ab'M + a'bM + bb'MM')I \qquad \text{...(A3.20)}$$

It can be shown that

$$MM'I = \frac{1}{m^2-m'^2} \quad (M'I-MI) \qquad \text{...(A3.21)}$$

The substitutions to be made on the function $F_{\alpha\beta}(y,z)$ and $G_{\alpha\beta}(y,z)$ are

$$F_{\alpha\beta}(y,z) \rightarrow F_{\alpha\beta}(y,z) - (1-a)\,\frac{m'^2-a'm^2}{m'^2-m^2}\,H^m_{\alpha\beta}(y,z)$$

$$-(1-a')\,\frac{am'^2-m^2}{m'^2-m^2}\,H^{m'}_{\alpha\beta}(y,z)$$

$$G_{\alpha\beta}(y,z) \rightarrow G_{\alpha\beta}(y,z) - (1-a)\,\frac{m'^2-a'm^2}{m'^2-m^2}\,E^m_{\alpha\beta}(y,z)$$

$$-(1-a')\,\frac{am'^2-m^2}{m'^2-m^2}\,E^{m'}_{\alpha\beta}(y,z)$$

Here, $H^m_{\alpha\beta}$, $H^{m'}_{\alpha\beta}$, $E^m_{\alpha\beta}$ and $E^{m'}_{\alpha\beta}$ are the same functions $H_{\alpha\beta}$ and $E_{\alpha\beta}$ defined in (A3.11). The superscript m and m' are added to indicate whether the isoscalar or isovector constants are used. This notation will apply also for functions appearing below.

To include the pionic form factor of the nucleons ultimately, the expression $F_\pi(t)F_S(t)F_V(t)I$ should replace I. The corresponding substitutions then become

$$F_{\alpha\beta}(y,z)\rightarrow F_{\alpha\beta}-(1-a)\frac{m'^2-a'm^2}{m'^2-m^2}H^m_{\alpha\beta}-(1-a)\frac{am'^2-m^2}{m'^2-m^2}H^{m'}_{\alpha\beta}$$

$$-(1-A)L_{\alpha\beta}+(1-a)(1-A)\frac{m'^2-a'm^2}{m'^2-m^2}O^m_{\alpha\beta}$$

$$+(1-A)(1-a)\frac{am'^2-m^2}{m'^2-m^2}O^{m'}_{\alpha\beta}$$

$$G_{\alpha\beta}(y,z)\rightarrow G_{\alpha\beta}-(1-a)\frac{m'^2-a'm^2}{m'^2-m^2}E^m_{\alpha\beta}-(1-a)\frac{am'^2-m^2}{m'^2-m^2}E^{m'}_{\alpha\beta}$$

$$-(1-A)D_{\alpha\beta}+(1-a)(1-A)\frac{m'^2-a'm^2}{m'^2-m^2}C^m_{\alpha\beta}$$

$$+(1-A)(1-a)\frac{am'^2-m^2}{m'^2-m^2}C^{m'}_{\alpha\beta}$$

...(A3.23)

The functions defined in (3.36), (A3.11), (A3.19) and (A3.19a) with their derivatives, up to fourth order w.r.t. y and z, enter into the expression of the potentials. Most of these integrals are, however, mathematically undefined. On the other hand, we know that integrals like

$$Q(x) = \int\frac{k^n \sin kx\, dx}{k^2+a^2} \qquad ...(A3.24)$$

can be written as the sum of δ-function-like terms and a nondivergent term. (The functions we are dealing with are not exactly of the form (A3.24), but very similar to it, and should have similar behaviours.) Therefore, if we are interested in cases where x is quite far from the origin, Q(x) does define a finite potential. The usual trick is to multiply the integrand with a gaussian e^{-p^2/λ^2} so that it becomes absolutely convergent, keeping always in mind that the limit $\lambda\to\infty$ is to be taken, although for practical purpose a finite λ has to be used. The crutch is that if we introduce this factor at the very beginning of the derivation as a restriction on the pion momentum, the expression of the integrals are not exactly correct except for $\lambda=\infty$. After the angular integrations, we are left with a two dimensional integral over p and k. The integral where k is integrated first and p left to be integrated numerically will not be affected whether the factor e^{-p^2/λ^2} is added at the beginning or at the end. The integral where p has to be integrated first will deviate from the expression given if the factor is added before doing this last-but-one integral. Our expression will be "more exact" as λ becomes larger. Thus we are furnished with a practical method for evaluating the integrals, provided the value of the integral with the cut-off factor included becomes insensitive to the choice of λ, beyond a relatively small positive value λ_m. A large λ_m would make the numerical integration prohibitive, since one then has to integrate up to very large values of k to attain the desired accuracy. It turns out that for functions appearing in Chapters 3 and 6, a relatively small λ_m (~ 6.0) exists for distances beyond 0.5 $\lambdabar_\pi$, while for the new integrals introduced with form factors, λ_m has to be very large (~ 25.0) even beyond 1.5 $\lambdabar_\pi$. There are ways to get around this point. For instance, one can separate out the δ-function like terms first to reduce the singularity in the defining integrals, or one may write down the exact expression, with the factor e^{-p^2/λ^2} included, for all the last-but-one integrations.

We shall not do this, for we believe the effect of form factor is small. It is also restricted to a short range as must be the case, since it involves the exchange of heavy vector bosons. The special case of V_λ^C, where all the integrals can be performed analytically, proves that our guess is correct. At the same time, this provides a check for the accuracy of our numerical evaluation of the original integrals. We now turn to this special case.

The first equation in (6.7) can be written slightly differently as

$$V_\lambda^C = 4G_\lambda G' \frac{1}{x}\left(\frac{1}{x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\frac{\partial^2}{\partial y \partial z} [f_\lambda^2 (I^+(o,\mu;\ o,\Delta) - I^-(o,\mu;\ o,\Delta)$$

$$+ 2g_\lambda{}^2 (I^+(o,\mu;\ o,w_\lambda) - I^-(o,\mu;\ o,w_\lambda))]$$

$$= 4G_\lambda G'\ f^2 \left(\frac{4\pi}{\mu^2}\right)^2 \frac{1}{x}\left(\frac{1}{x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\frac{\partial^2}{\partial y \partial z}$$

$$\times[f_\lambda{}^2 u_\Delta(o,\mu) + 2g_\lambda{}^2 u_{w\lambda}(o,\mu)]$$

...(A3.25)

$$u_\alpha(o,\mu) = I^+(o,\mu;\ o\alpha) - I^-(o,\mu;\ o\alpha)$$

$$= \frac{-1}{(2\pi)^6} \iint d^3p\, d^3k\ \frac{e^{i(k\cdot y - p\cdot z)}}{\alpha k^2 w_p{}^2}$$

$$= \frac{-1}{16\pi^2} \frac{1}{\alpha yz} e^{-\mu z}$$ ...(A3.26)

Following derivations outlined previously in this Appendix, it is easy to prove that incorporation of form factors would require the following substitution in (A3.25)

$$u_\alpha(o,\mu) \longrightarrow u_\alpha(o,\mu) + Au_\alpha(m,\mu) + Bu_\alpha(m',\mu)$$ ...(A3.27)

where A and B are appropriate constants and the new function $u_\alpha(m,\mu)$ is defined as

$$u_\alpha(m,\mu) = \frac{-1}{(2\pi)^6} \iint d^3p\, d^3k\ \frac{e^{i(k\cdot y - p\cdot z)}}{\alpha(k^2+m^2) w_p{}^2}$$

$$= \frac{1}{16\pi^2} \frac{1}{\alpha yz} e^{-my}\ e^{-\mu z}$$ ...(A3.28)

The value of V_λ^C evaluated from (A3.25) with (A3.27) is given in Table A3.1. Comparison with Table 7.2 shows that our evaluation of the original integrals is sufficiently accurate for value of x beyond 0.6 λ_π. The contribution from form factors is comparable to but smaller than that of the original integrals inside 0.8 λ_π

and falls off rapidly beyond this range. It is to be noted that for very short distances, the effect of form factors is to suppress the high singularity of our potential, as is to be expected, since the form factors effectively make corrections to the assumption of a point particle.

TABLE A3.1

V_λ^C FROM ANALYTIC EXPRESSION

$X(\lambda_\pi)$	V_λ^C (μ_π) Contribution Without f.f.	V_λ^C (μ_π) Contribution From f.f.
0.3	-0.171404	0.173376
0.4	-0.030162	0.029722
0.5	-0.007794	0.006704
0.6	-0.002566	0.002174
0.7	-0.000998	0.000740
0.8	-0.000438	0.000276
0.9	-0.000210	0.000110
1.0	-0.000108	0.000046
1.1	-0.000060	0.000020
1.2	-0.000034	0.000010
1.3	-0.000020	0.000004
1.4	-0.000012	0.000002
1.5	-0.000008	
1.6	-0.000006	
1.7	-0.000004	
1.8	-0.000002	

APPENDIX 4

We gather here the values of numerical constants used throughout the computation.

The mass of pion, μ_π is taken to be 139.4 Mev = $(1.415 \text{ fm})^{-1}$. We have set $\mu_\pi = \hbar = c = 1$. Values of other constants are:

$f^2 = 0.08$	Pion-nucleon coupling constant
$g_p = 2.793$	Proton magnetic moment in units of nuclear magneton
$g_n = -1.913$	~~Nucleon~~ Neutron magnetic moment
$M = 6.73$	Nucleon mass
$w_r = 1.73$	N^* resonance energy
$w_\lambda = 1.79$	Y^* resonance energy
$f_\lambda^2 = 0.06$	$\Sigma\lambda\pi$ coupling constant
$g^2 = 0.057$	$NN^*\pi$ coupling constant
$g_\lambda^2 = 0.047$	$\lambda Y^*\pi$ coupling constant
$e^2 = 1/137$	Fine structure constant
$\alpha = 0.75$	F-D mixing ratio
$\Delta = 0.552$	λ-Σ mass difference
$f^2_{\widetilde{\lambda\lambda\pi}^0} = 0.0485\ f_\lambda^2$	$\widetilde{\lambda\lambda\pi}^0$ coupling constant
$c' = 0.1$	π^0-η mixing constant

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