# AN APPLICATION OF THE NEAREST NEIGHBOUR TECHNIQUE:

# PATTERNS OF URBAN PLACES

# IN

# SOUTHERN SASKATCHEWAN

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SCOPE AND CONTENTS:

The patterns of certain groups of urban places, selected on the basis of population size and areal location, in southern Saskatchewan are classified by the use of the nearest neighbour technique. Through a study of the variations within the overall pattern, which are revealed by differences in the derived pattern statistic, a partial contribution is made to the understanding of the distributive process that underlies the observed settlement pattern. Explanations for the variations in the nature of the spatial arrangement of the various groups of places are suggested through the use of multivariate analysis, and by reference to theoretical and empirical works in the field of Central Place Theory.

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# CHAPTER I

## INTRODUCTION

The description of settlement patterns in geographical writing has relied, to a large extent, upon the subjective appreciation of the distinction between such qualitative statements as nucleated, dispersed, and regular arrangements of farms or towns within an area. However, the accurate description of pattern as a step in the scientific progression of, firstly, empirical observation and, secondly, conceptual generalization would seem to require the adoption of a rigorous system of definitions and a method of classification that are independent of an observer's judgement. An example of the differences in results and of the need to adopt an unambiguous set of definitions is provided by a comparison of the qualitative approach of Brush (1953), in a study of the hierarchy of central places in south western Wisconsin, and the later quantitative work of Dacey (1962) which was based on Brush's findings with respect to the rank of towns in the area.

Within the field of urban geography a theory already exists concerning the number, size, and the spatial arrangement of towns. Christaller's work (1933) on the Central Places of Southern Germany has provided the theoretical base for much of the recent research in urban geography. Many of these later works have been primarily interested in, or have mentioned, one aspect of the pattern of towns: that is the distance between towns, but few have been concerned with an appreciation of the pattern *per se*.

It is a contention of the author that any reformulation of, or addition to, that part of central place theory concerned with the pattern of towns will not be achieved solely by studying the distances between towns as suggested by Thomas (1961). Such a reformulation must be based upon the empirical observation of the nature of the patterns exhibited by different urban systems. It is suggested that this can be achieved by the utilization of an index which places the emphasis on spatial arrangement rather than upon distance.

The work of Brush and Bracey (1955) is an example of an attempt to compare patterns of settlement through the use of actual distances between towns. Brush and Bracey followed their individual studies of central places, in south-west Wisconsin and south-west England, respectively, by a comparison of the results obtained. One comparison concerned the distances between towns of the same hierarchical rank (although of different population sizes) by use of mean distances. Their conclusion, that the spatial arrangement of places is similar in both areas, can be faulted, because it is mainly derived from the similarity of the mean distances; the same mean distance can be exhibited in two different patterns, if the density of towns in the two patterns is different. King's comparative study (1962) of the patterns of urban places in different parts of the United States illustrates the value of utilizing a method which implies spatial arrangement rather than distance.

The terms 'pattern' and 'distribution' are normally used interchangeably in geographical writing, however, in the context of pattern analysis a strict distinction has to be made. Pattern is defined in this thesis as the nature of a phenomenon's arrangement over a two-dimensional surface. The distribution of a phenomenon is viewed as a dynamic process which is determined by the operation of certain distributive factors; pattern conveniently summarizes the result of the operation of these distributive forces at a specific moment in time.

The objectives of this thesis are: firstly, to apply the nearest neighbour technique of pattern analysis to the problem of describing the patterns of urban places in Saskatchewan; secondly, by consideration of the variations which exist within the observed patterns, it is hoped to make a contribution to the understanding of the distributive process. A discussion of limitations, which exist in the implementation of the technique, will be offered where this is felt to be appropriate.

The statistical universe of this thesis is the 481 incorporated places in Saskatchewan which are listed in the 1961 Census of Canada. Saskatchewan was chosen as the study area primarily because it is felt that it is the only Canadian province that has a reasonable uniformity of topography and that the major influences on the arrangement of settlements would be the distributive forces attributable to human occupancy of the area. The study area is defined as the area of the Census Divisions one through seventeen, as these contain the area of continuous settlement in the province. These impressions concerning Saskatchewan were gained from documentary sources, for example, the Report of the Royal Commission on Agriculture and Rural Life (1957), rather than from field observation.

The nearest neighbour method of analysis is applied to two aspects of the overall pattern of urban places: firstly, the overall pattern is viewed as containing subsidiary patterns of places, which are grouped on the basis of population size: secondly, the province is subdivided into small areas and the pattern of urban places in each area is analysed. The former aspect is studied by utilizing Central Place Theory to provide null hypotheses. The relevance of the statements in the theory concerning the spatial arrangement of urban places can be accepted, or rejected, for Saskatchewan, on the basis of the analysis, by use of standard statistical techniques. The variations within sets of related results and deviances from the theoretical norm of a uniform arrangement may be explainable by reference to the relevant theoretical and empirical work in this field. In considering the spatial arrangement of groups of settlements subdivided by area, variations in the patterns between areas may reflect responses to distributive forces that also exhibit areal variation. Using multivariate analysis, linkages are suggested between the variation of the pattern of urban places and other spatial variables that may be part of the distributive process.

The use of the term 'urban place' was preferred in this thesis to that of 'central place' as it suggests form rather than function, and it also allows for the inclusion into the statistical universe of non-central places which are, however, an integral part of the settlement pattern. It was felt that the application of the term 'central place' to a settlement solely on the basis of it having incorporated status was unwarranted without individual research into the nature of that place, the functions it performs, and the area and population it serves.

# CHAPTER II

# METHODOLOGY OF PATTERN ANALYSIS

The classification of patterns raises serious problems in definition because of the lack of clear cut boundaries between one type of pattern and another. As Bunge (1962, p. 70) has noted 'pattern is an élusive property' and its classification has relied on the utilization of a continuous index, in the form of a dimensionless parameter, derived from the observation of the spatial arrangement of the phenomena under consideration. Therefore, before proceeding with a review of the two major quantitative methods of pattern analysis, it is proposed to define the two most different forms of pattern, uniform and clustered, and, because the two methods of analysis are centred on the mathematical properties of a pattern that results from a stohastic process, a random pattern.

## The Three Definable Patterns

Because of the use of a continuous index to classify a point pattern, it is only possible to define the extremities of such an index, that is, clustered and uniform patterns, and, because of its use in the statistical theory of pattern analysis, a random pattern. In the formal definition of these patterns, and in the statistical theory that underlies the various indices, it is assumed that a pattern is comprised of an infinite number of points situated on an unbounded surface. The correspondence of these assumptions to those that underlie central place theory is immediate.

Random Pattern

A random pattern of points is defined by Clark and Evans (1954, p. 446) as a set of points such that:

> 'any point has had the same chance of occuring on any given sub-area as any other point; that any sub-area of specified size has had the same chance of receiving a point as any other sub-area of that size, and that the placement of each point has not been influenced by that of any other point.'

Although the validity of such a pattern in the formulation of an index of pattern is not questioned, the usefulness of such a definition, when applied to the spatial arrangement of a specific geographic phenomenon, is not clear. It may be that the concept of a random pattern is useful within the range of a continuous index solely as a reference point from which to measure tendencies towards clustering or uniformity.

Uniform Pattern

A uniform or regular pattern is defined as one in which each point is at the maximum possible distance from every other point. Such a pattern results when the total surface containing the points is divided equally between the points and each point is central to its particular share of the total surface. The most suitable geometric form for this mean area is a hexagon if total coverage of the surface is to be obtained (Haggett, 1965, p. 48-50). The centres of these hexagons form a triangular lattice. Therefore, the definition of a uniform pattern may be extended in the following manner: the points contained in a uniform pattern are arranged such that each point is a vertex of six equilateral triangles.

Clustered Pattern

Logically a pattern should only be classified as clustered when all the points in that pattern occupy a single locus. However, it could be argued that in such a situation, as the points have no separate locii, there is no pattern because the component points are indistinguishable. In order to avoid this problem, and those introduced by Dacey and Tung (1962) concerning single- and multiple-clump patterns, an operational definition is put forward.[1] A clustered pattern is defined in terms of a distributive process that is biased towards particular locations in the spatial plane resulting in the close proximity of a number of points, when close proximity is understood as allowing the determination of a unique locus for each point. Close proximity may also be visualised as being a finite distance between two points. Dacey (1962) required towns to be one mile apart before they could be considered to occupy uniquely determinable locii. In the analysis of the pattern of urban places in Saskatchewan it was possible to assign unique locii to urban places one half of a mile apart.

## Quadrat Methods of Pattern Analysis

The development of methodologies for the analysis of patterns has relied strongly on the work of plant ecologists and biometricians. In ecology three distinct approaches -- assessment of abundance, quadrat methods, and distance measurement techniques -- have been made in the quantification of pattern analysis and description (Kershaw, 1964). The

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[1]It is precisely because of these problems that a diagramatic representation of a clustered pattern cannot be offered; although it is possible to depict both random and uniform patterns (Haggett, 1965, p. 89). It is felt that the problem of depicting the many different forms a clustered pattern may assume is overcome, to some extent, by defining the nature of such a pattern in terms of constraints within the distributive process.

use of quadrats resulted in the first appropriate method of pattern classification. The use of this approach by geographers has been very limited: Dacey (1964-B) and Getis (1964) have utilised it as an appendage to studies applying the more recent nearest-neighbour methods in order to obtain corroboration of the results of their primary analysis. Goodall (1952) and Kershaw (1964) have extensively reviewed the development and application of quadrat methods. This section is, therefore, concerned primarily with a description of the nature of quadrat analysis and its application to the study of punctiform patterns.

Methodology

Gleason (1920) was the first ecologist to show that the terms of a Poisson expansion,

$$\frac{d^x c^{-d}}{x!} \qquad (1)$$

(where d is the average number of points per cell and x is the number of points expected per cell in a random distribution) could be utilised as an assessment of the nature of a pattern. The method basically consists of comparing the observed number of quadrats containing a given number of points against the expected number of quadrats in a random pattern that contain the same number of points (i.e., as obtained from the Poisson series), by means of a Chi-square test of goodness-of-fit. Comparing the distribution curves of the 'observed' and 'expected' data results in an assessment of the spatial arrangement of the phenomena and the probability of it being produced by a random process.

In a Poisson series the variance is equal to the mean, thus a continuous index of pattern can be obtained by computing the ratio of the variance and the mean of the observed data. A random pattern has a value equal to 1, a regular pattern has a value equal to 0, and a clustered pattern greater than 1.

Data is obtained for the above two methods, either by placing quadrats of the same size at random within the area under study, or, by taking a complete census of the point population by means of a regular grid laid down across the area. A study by Matui (vide Dacey, 1964-B) of the distribution of rural settlements in the Tonami Plain, Japan, utilised the complete coverage method. It would seem that this is probably the most useful method to adopt in quadrat analysis because it possesses advantages for testing the data by other methods.

Criticisms of quadrat analysis have been levelled by Curtis and McIntosh (1950) and Skellam (1952) on the grounds that the frequency data is influenced by the size of quadrat. Getis (1964) noted the related problem of trying to decide on an appropriate cell size for geographic work. Evans (1952) has shown that the variance: mean ratio may give a widely different estimate of non-randomness from the Chi-square test. These apparent disadvantages have been utilised by ecologists to improve the quadrat method of pattern analysis. Data for different sizes of quadrats may be obtained by successively combining quadrats for both the Chi-square test and the variance: mean ratio. By repeating the analysis at the different scales of quadrats it is possible to assess changes in the nature of complex spatial patterns through use of graphs of the relationship of the variance, or variance: mean ratio against the size of quadrats. The main purpose of this, in ecology, is to obtain an estimate of the size of area within which clustering of a species occurs. Although quadrat methods are an acceptable approach to pattern classification it is felt that the necessity to perform the calculations for a number of quadrat scales makes it an inefficient method and also one open to errors of interpretation.

## Nearest Neighbour Methods of Pattern Analysis

Nearest neighbour methods were originally developed by ecologists disatisfied with quadrat methods because of the disadvantages mentioned above and because of the related problems of assessing the degree of departure from random expectation, and the significance of differences in patterns of two or more phenomena, or strata of a single phenomenon. Geographers have utilised this technique much more readily than the quadrat methods; and Dacey has made a number of contributions to the mathematical theory underlying the statistics in order that they might be more applicable to geographical problems (1963, 1964-A, 1966-A).

### Methodology

Nearest neighbour methods describe spatial patterns by evaluating the distances between nearest and other near neighbours in a punctiform pattern. As in the quadrat approach to pattern analysis, the statistical theory that underlies the method is derived from consideration of the attributes of a pattern that would result from a stochastic process. Clark and Evans (1954) showed that the average minimum distances separating two points in a random pattern having the same number of points and area as the pattern under consideration is given by:

$$\bar{r}_e = \frac{0.5}{\sqrt{d}} \qquad (2)$$

where, d is the mean density of N points per unit area over the total area (A) under consideration. By comparing the observed average distance,

$$\bar{r}_o = \frac{\Sigma r}{N} \qquad (3)$$

when r is the distance between a point and its nearest neighbour (measured in the same units as A), with the expected average minimum distance a dimensionless index R is obtained which classifys the observed pattern:

$$R = \frac{\bar{r}_o}{\bar{r}_e} \qquad (4)$$

Values of the pattern statistic R occur within a range,

$$0 \leq R \leq 2.1491$$

A clustered pattern is denoted by an R value of 0, a random pattern by 1 and a uniform pattern by 2.1491.

The pattern index R has a number of advantages which result in it being a much more efficient analytical tool than the quadrat method. The index has a limited range of possible values and, thus, it is possible to make meaningful interpretations of R. In any given pattern the average observed distance between nearest neighbours is R times as great as the distance which would be expected in a random pattern. Thus, a value of R equal to 0.5 would indicate that nearest neighbours are, on the average, half as far apart in reality as the expected distance under conditions of randomness. It is also possible to evaluate $\bar{r}_u$, the distance between points in a uniform pattern of the same density as the observed pattern

$$\bar{r}_u = \frac{1.0750}{\sqrt{d}} \qquad (5)$$

The reliability of R can be evaluated by assessment of the significance of the departure of $\bar{r}_o$ from $\bar{r}_e$; letting $z$ equal the standard variate of the normal curve

$$z = \frac{\bar{r}_o - \bar{r}_e}{\sigma_{\bar{r}_e}} \qquad (6)$$

when, $$\sigma_{\bar{r}_e} = \frac{0.26136}{\sqrt{Nd}} \qquad (7)$$

Values of $z$ equal to 1.96 and 2.58 represent, respectively, the 95 per cent level of confidence and the 99 per cent level of confidence. By manipulating formula 6 confidence intervals around R equal to 1 can be obtained

$$1 - z.\sigma_{\bar{r}_e} \leq 1 \leq 1 + z.\sigma_{\bar{r}_e} \qquad (8)$$

Substituting for $\sigma_{\bar{r}_e}$ and $z$ the required confidence interval around R equal to 1 can be found. The signifance of the difference between two patterns can be found from the data used to obtain R by use of Snedecor's F-test or Student's t-test. (Clark and Evans, 1954, p. 452)

The relationships of nearest neighbours have been examined by Clark and Evans (1955) who found, from statistical theory and empirical observation, that a random pattern contains a high percentage of reflexive pairs, i.e., two points serving as one another's nearest neighbour. In a random pattern 62 per cent of the population is theoretically reflexive. In a synthetic random pattern of 1,000 points the proportion was found to be 60 per cent. Their analysis of the synthetic random pattern was extended and the proportion of points serving as nearest neighbours to 0, 1, 2, 3, and 4 or more, points was found (Table 2:1).

Because of the prevalence of reflexivity in a random pattern these findings have resulted in the development of the nearest neighbour method in order to increase the power of the analysis. The two major developments have been the order method and the regional, or sector, method (Dacey and Tung, 1962). Figure 2:1 illustrates these two techniques. The order method consists of measuring the distance to the first, second, ....nth neighbour and comparing the averages of each order to the expected value of a random pattern. The formulae for $\bar{r}_e$ for each order were derived by Thompson (1956); the constants are tabulated in Table 2:2 for the first four orders of nearest neighbours. The regional method consists of dividing the area around each point into a number of equal sectors (Figure 2:1(B)) and finding the nearest neighbour in each sector. For each point the distances to the nearest neighbour are ordered and used to derive the sector means. The same mathematical procedures as the simple method are utilised: the constants for $\bar{r}_e$ in each sector have been tabulated by Dacey (1962).

The regional method has a number of disadvantages when compared to the single-sector approach of the order method. The main disadvantage is that concerned with the placement of the sectors; the assumptions underlying the derivation of $\bar{r}_e$ and $\bar{r}_e$ preclude the existence of empty sectors. A second disadvantage results from the procedural definition

TABLE 2:1

INTERNAL CHARACTERISTICS OF A SYNTHETIC RANDOM PATTERN*

| | | |
|---|---|---|
| Number of Points | | 1000 |
| Proportion of Points which have reflexive relations | | .602 |
| Proportion of Points serving as Nearest Neighbour to: . . . | 0 points | .297 |
| | 1 " | .453 |
| | 2 " | .225 |
| | 3 " | .025 |
| | 4 " | 0.0 |

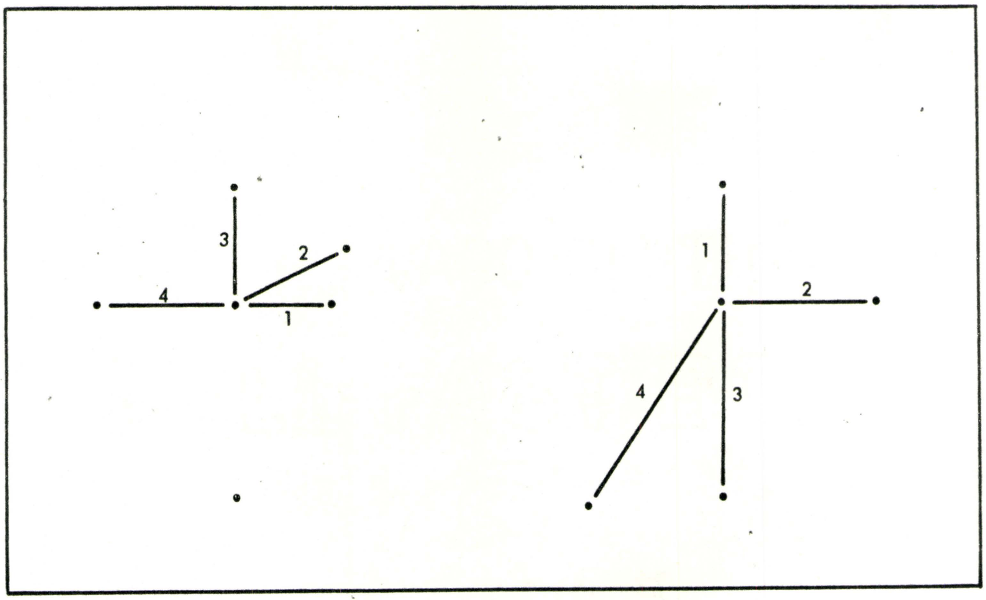
*Source: Clark and Evans (1955, p. 397)

TABLE 2:2

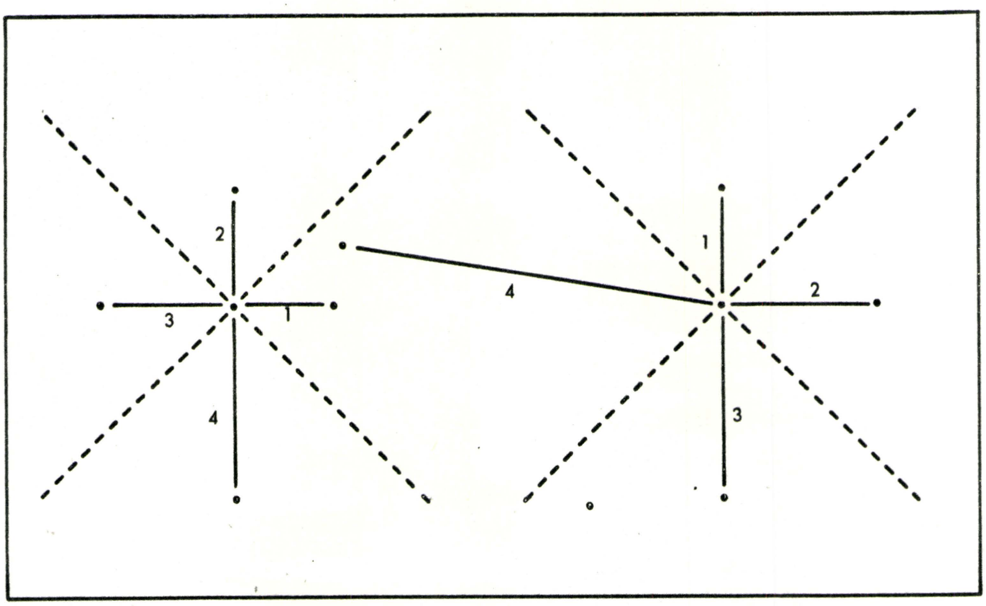
CONSTANTS FOR EXPECTED AVERAGE DISTANCE AND THE STANDARD DEVIATION FOR NEAREST NEIGHBOURS IN A RANDOM PATTERN (ORDER METHOD)

| Parameter | Order of Nearest Neighbour 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $E(\bar{r}_e)$ | $0.5(\sqrt{d})^{-1}$ | $0.75(\sqrt{d})^{-1}$ | $0.93(\sqrt{d})^{-1}$ | $1.0937(\sqrt{d})^{-1}$ |
| $\bar{r}_e$ | $0.2614(\sqrt{Nd})^{-1}$ | $0.2723(\sqrt{Nd})^{-1}$ | $0.2757(\sqrt{Nd})^{-1}$ | $0.2774(\sqrt{Nd})^{-1}$ |

FIG 2:1 EXTENSIONS OF THE NEAREST NEIGHBOUR METHOD




A) ORDER METHOD




B) SECTOR METHOD

of the sectors; a sector of a certain order need not have the same orientation from each measuring point as is shown in Figure 2:1(B). It is possible for the nearest neighbour in a sector to be farther from the measuring point than other points. Because of these disadvantages it is difficult to express the results meaningfully in terms of spatial arrangement. The single sector approach of the order method obviates certain of these problems in that the first four measurements represent the distance to the four nearest points and the problems of empty sectors and different orientations of sectors are avoided.

Nearest neighbour methods utilise statistics based on the assumptions of an infinite number of points and an unbounded surface in order to derive $\bar{r}_e$, however, empirical observation of a pattern is normally concerned with a defined area. The existence of a boundary affects the derivation of R in certain known ways. Firstly, it limits the known number of points and, therefore, affects the calculation of the density of points; secondly, the distance from a point to its nearest neighbour within the bounded area may not be the shortest distance in reality because a third point, the actual nearest neighbour may be across the boundary. Therefore $\bar{r}_o$ is magnified to some extent. Three methods of accounting for the problems raised by the existence of a boundary can be formulated.

The occurence of points outside the study area may be ignored and only the points inside the designated area used to calculate the density and the average observed distance. Secondly, measurements to points outside the area can be made but not from external to internal points. Getis (1964) and King (1962) utilised this approach in their studies. Dacey (1962) suggested a third approach: measurements of the distance to nearest neighbour are only used if they are smaller than the distance from the measuring point to its nearest boundary. The second and third methods have certain drawbacks, in that R is calculated, respectively, for an area greater than, or smaller than the study area. Although the second method may be the best indicator of R, in that the boundary effect does not intrude into the calculations, if variations in the pattern of towns are to be related to the variation of other phenomena, for which the data is collected on the basis of the defined study area, the variables are not areally compatible. This disadvantage is much more important when study areas are adjacent rather than separated.

In the analysis of the pattern of urban places in Saskatchewan the distance to first nearest neighbour is calculated because the number of towns, in certain of the groupings made, is often small and to go beyond first nearest neighbours would result in values of R that are not statistically different from random, although their actual value might well suggest otherwise. The first solution to the boundary problem is used in all sections of the analysis, that is, the distance to nearest neighbour of all points within the study area is used in the calculation of the observed average distance.

## Related Techniques

The first quantitative attempts to understand the nature of a phenomenon's arrangement over a two-dimensional surface were concerned with the isolation of certain of the parameters previously mentioned and discussing them in relationship to the phenomenon's environment (Goodall, 1952). The work of some European geographers (Bernard, 1931;

Demangeon, 1933; Colas, 1945) closely followed the work of ecologists prior to 1925, with discussions of the density of the lowest orders of rural settlement, the average area per settlement (i.e., the reciprocal of the density), and the use of certain arbitrary distances to indicate the nature of the pattern of such settlements. Zierhoffer (1934), for example, regarded dwellings as dispersed if they were separated by a distance of 150-200 metres, whilst Debouverie (1943) regarded 100 metres between farms as a critical index of a dispersed settlement pattern. Examples of certain of the above methods, and others, are to be found in Monkhouse and Wilkinson (1952, pp. 308-313). Criticism of this type of approach has been made by a geographer (Houston, 1953, pp. 81-85) and on ecologist (Goodall, 1952) on the basis of its generality and insensitivity to changes in the pattern. Specific criticism can be levelled at the abstraction of density, in that the same density can result from widely different patterns if the number of points and the containing area remain constant.

The measurement of minimum distances between towns is not unique to pattern analysis by means of the nearest neighbour method. Lösch (1954, p. 392) measured the distances between towns for different areas of the United States in an attempt to show that there are similarities from one area to another. Further studies of the spacing of settlements have been made (Thomas, 1961, 1962; King, 1961) which have increased our knowledge of the relationships which exist between the members of a system of urban places.

Dacey (1965) has shown that it is possible to fit a gamma distribution to the frequency distributions published by Lösch. The results are inconclusive with regard to the form of the pattern in the two areas to which Dacey applied his method, with the exception that: 'the differences between midwestern United States and England is conspicuous', (1965, p. 7). It would appear, on the basis of Dacey's work, that the use of minimum distances is of little value in the classification and description of a pattern.

The major distinction between studies of spacing and those concerned with the analysis of patterns is with respect to their objectives. The purpose of Lösch's work and of the other spacing studies has been to increase our understanding of the internal attributes of a pattern. Pattern analysis by the nearest neighbour method, although it utilises similar data to that which is required for a study of spacing, is concerned with the spatial arrangement of settlements. This objective is achieved through the introduction of the concept of a random pattern, of the same density as the pattern to be analysed, against which the observed pattern may be measured.

# CHAPTER III

## POPULATION SIZE AND THE PATTERNS OF URBAN PLACES

The pattern of urban places in an area can be regarded as a single pattern with each place being considered the same, or it may be viewed as containing several sub-patterns, each being related to a set of towns differentiated from the remainder by some factor. Population size is one characteristic that differentiates urban places from one another. The urban structure of an area is composed of towns of various sizes and an analysis of the overall pattern should take into account the effect that this variation may induce. In doing so the results may be related to facets of urban geography other than that concerned with the nature of the settlement pattern. The objective of this chapter is to analyse the patterns of urban places of different sizes and to relate the results to the overall pattern of urban places in Saskatchewan and to relevant central place studies.

### Population Size as a Continuum

In order to analyse the patterns of urban places of varying population size it was found necessary to treat population as a continuous variable. It was viewed as doubtful that any satisfactory division of the towns could be made on the basis of the functions they contain, or the services performed, because of the lack of the relevant data. Also, any division based on apparent breaks in a rank-size diagram would have been questionable, as no rigid definition of 'central place' could be applied to the urban places selected, nor could it be certain that all central places in Saskatchewan had been included. Therefore, in order to obtain an appreciation of the nature of the pattern of urban places of different sizes, arbitrary class boundaries were selected within the continuum. Once selected, these figures were kept constant as an aid to the comparison of different sets of results. Graphical, as well as tabular, methods have been utilised to present the pattern statistics obtained, so that the selection of arbitrary class boundaries should not unduly influence the discussion and evaluation of the results.

The use of population size as a continuous variable can be supported by reference to the work of Thomas (1962) on the spacing of towns and to various works which have shown that central places are differentiated along a continuum of population size. Berry and Barnum (1962) have shown that a number of relationships exist between population size and various indices traditionally used to measure the 'centrality' and importance of a place. Beckmann (1958) has shown that it is possible to produce the continuous distribution of the rank-size rule from the stepped distribution that follows from Christaller's fixed K assumption, by the simple addition of a random variable. Berry and Garrison (1958) independently arrived at the same conclusion.

Certain, unknown, errors may result from this operational procedure. It is felt that any such errors will be small and only affect conclusions made about certain parts of the continuum. The number of places that do not possess any central place functions, i.e., point-bound places, but are incorporated, are probably both small in number and in population size. Whilst central places that are excluded, because they do not possess corporate status, are also most likely to be small in terms of population. Further errors may result from the inclusion of a town in a particular sub-set of places because of its size, although the functions it performs would have placed it in another group, if a division on the basis of functional importance had been made for this work. Although this last source of error may be the most important, in terms of the analysis, without the relevant data it is impossible to assess its effect on the results.

## Hypothesis I

### Formulation

Central Place Theory, as stated by Christaller and Lösch, has as its basis a clear statement of the nature of the overall pattern of central places in an area. The pattern of central places in the theoretical model is uniform, i.e., a punctiform pattern, with the points situated at the apexes of a mesh of equilateral triangles (Christaller, 1966, p.63). Figure 3:1(A) shows such a pattern, which remained inviolate throughout Christaller's formulation of various principles concerning the weighting of the points and the shape of their market areas.

In the marketing principle the points receive different values, to form a hierarchical system, such that each point is the centre of a hexagon formed by the six triangles that surround it, and the triangles having at their other apexes places of a lower rank (Figure 3:1(B)). At each level of the hierarchy all places of that rank and higher ranks form secondary meshes of equilateral triangles. Thus, a feature of the marketing principle is that at any level of the hierarchy the distances

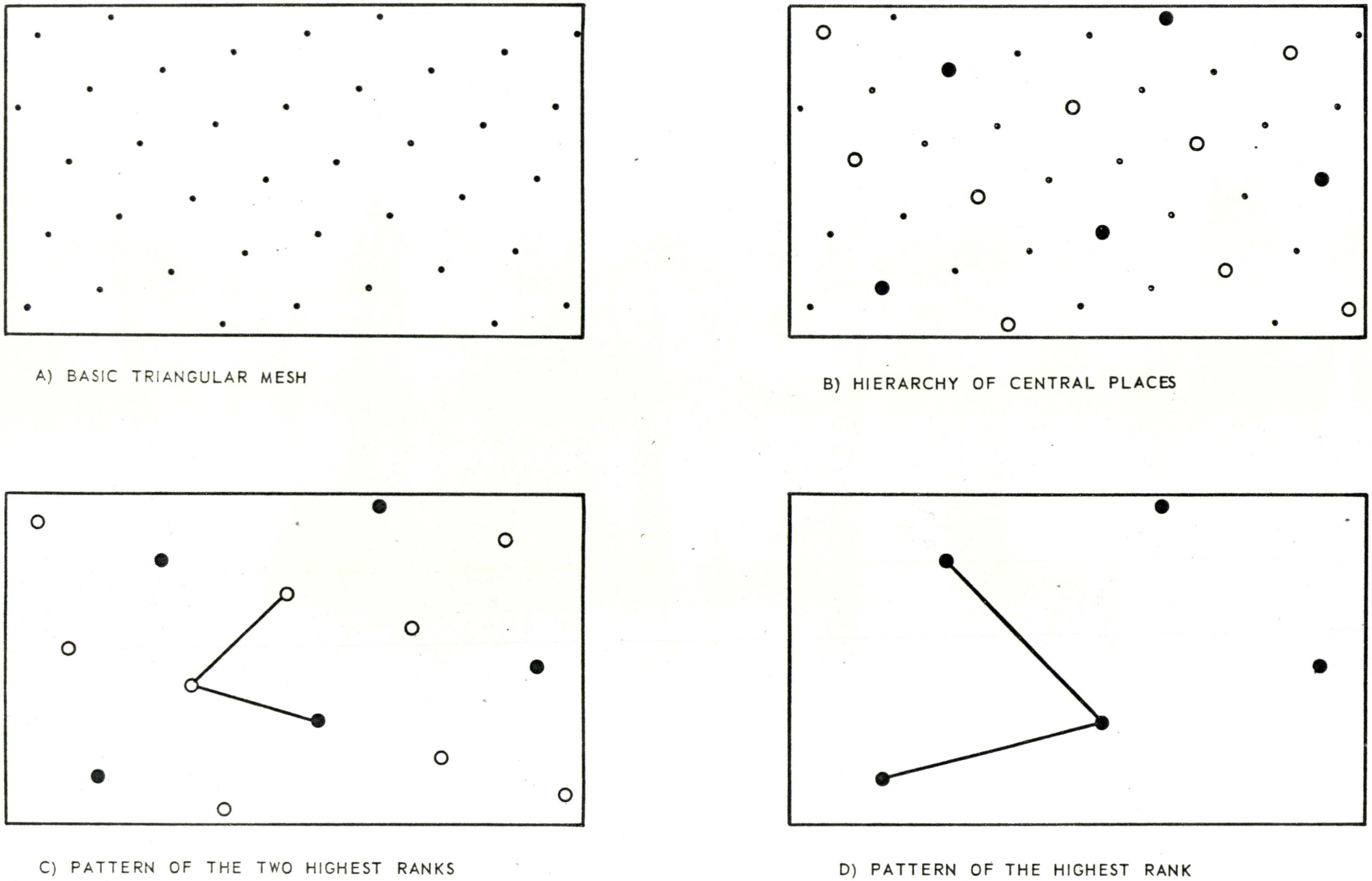



FIG 3:1 THE PATTERN OF TOWNS IN THE CENTRAL PLACE MODEL

between towns of a rank equal to, or greater than, that level are equal, and the pattern of such places is uniform. This principle is illustrated in Figures 3:1(C) and (D), where low rank towns are removed from the basic mesh in (B).

If this feature of the arrangement of central places is viewed in the light of Berry and Barnum's findings (1962) on the relationship of function and population, as mentioned above, it can be hypothesized that the pattern of urban places above a given size is uniform. Because of limitations in data handling it was decided to limit the analysis of pattern to first nearest neighbours. Therefore, Hypothesis I is stated as: the pattern of urban places, with respect to first nearest neighbours, above a certain size is uniform.

Test of Hypothesis I

In order to verify the hypothesis, the pattern statistic, R, was calculated for the urban places of Saskatchewan nine times. At each stage of the analysis places with a population below an arbitrary point in the continuum of population size were removed from consideration and R was obtained for first nearest neighbours above that size. The nearest neighbour of any measuring point may have a larger, or smaller, population than that place, the criterion for its selection being that it is the nearest place that has a population equal to, or greater than, the arbitrary population size for that particular set of calculations.

The results of the analysis are shown in Table 3:1 and Figure 3:2. If the pattern of urban places was regular R would have a value of 2.1491; at no stage in the analysis is such a value obtained, therefore, Hypothesis I

TABLE 3:1

RESULTS OF PATTERN ANALYSIS FOR HYPOTHESIS I

| Pop. of Towns > | No. of Towns | Density[1] per Square Mile | Av. Expe.[2] Distance (Miles) | Av. Obs. Distance (Miles) | Pattern[3] Statistic R | Standard Variate of Normal Curve |
|---|---|---|---|---|---|---|
| 1 | 481 | .003927 | 8.0 | 8.7 | 1.088 | 3.70 * |
| 75 | 452 | .003691 | 8.2 | 9.1 | 1.106 | 4.31 * |
| 100 | 421 | .003943 | 8.5 | 9.6 | 1.121 | 4.75 * |
| 250 | 254 | .002074 | 11.0 | 12.8 | 1.168 | 5.11 * |
| 500 | 113 | .000922 | 16.5 | 19.5 | 1.187 | 3.80 * |
| 750 | 67 | .000547 | 21.4 | 26.6 | 1.243 | 3.80 * |
| 1000 | 48 | .000391 | 25.3 | 31.4 | 1.244 | 3.24 * |
| 5000 | 10 | .000081 | 55.3 | 66.6 | 1.204 | 1.23 |
| 10000 | 6 | .000048 | 71.4 | 77.2 | 0.997 | 0.01 |

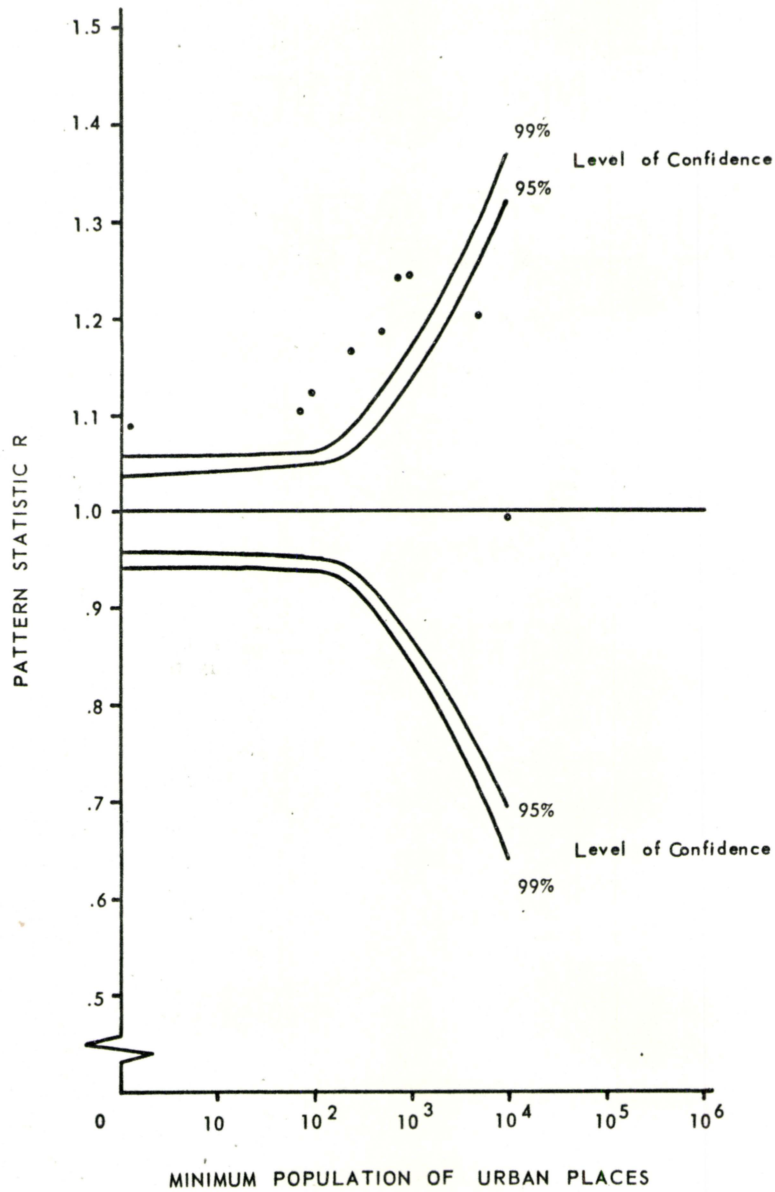
1. d = No. of Towns/Area of Saskatchewan (122466) square miles

2. $\bar{r}_e = .500 \times \sqrt{d}$

3. $Re = \bar{r}_{obs} / \bar{r}_e$

* Significantly different from a Random Pattern at the 99% level of confidence.

FIG 3:2 THE PATTERN OF URBAN PLACES OF A CERTAIN SIZE OR GREATER



is rejected on the basis of the analysis of distances to first nearest neighbour.

The most regular pattern that is shown to exist is that of towns with populations greater than 1,000 (R equal to 1.244). Almost as significant is the value of R (1.234) for towns greater than 750. Figure 3:2 shows that the pattern of urban places in Saskatchewan increases towqrds regularity as the smaller places are removed from the analysis of pattern. However, the value of R rapidly declines when it is obtained for towns greater than 5,000 and 10,000. This is due mainly to the mathematics of the calculation of R, i.e., the average expected distance is based on the assumption of an infinite number of measurements, whilst the average observed distance results from 10 and 6 measurements, respectively. In both of these stages the value of R is inside both the 95% and 99% confidence intervals placed around the respective random pattern with the same density of points. A further hypothesis which might be put forward on the basis of the results is that, if the number of places in these sub-sets were greater, necessitating a larger study area, the value of R which would be obtained would indicate greater uniformity of the pattern.

## Implications of the Results of Hypothesis I

Two reasons can be suggested to explain the increase in the regularity of the pattern of urban places as the smaller settlements are excluded. Firstly, there may be at least two different distributive processes affecting the overall pattern. The overall pattern, that is, the pattern of all urban places used in this analysis, may be affected by more local considerations than the subsidiary pattern of those towns over 1,000. A comparison of Figures 3:3 and 3:4 shows that the overall pattern is dominated by a linear network, which can be closely correlated with the major road and rail networks; however, the pattern of places with a population greater than 1,000 appears to be much less influenced by any linear control.

The control of railways on the location of settlements has been noted by others working on the settlement pattern of Saskatchewan (Royal Commission, 1957, v. 12, p. 23). The construction of the trans-continental railways in Canada resulted in a linear transportation pattern having an east-west orientation within Saskatchewan. At intervals along these routes and the later, predominantly north-south branch lines, stations were situated in order to store and tranship grain from road to rail haulage. Such sites became the nuclei for other functions. Two of the criteria used by the Royal Commission (1957, v. 12, pp. 30-31) to classify the lowest order of service centre were the occurence of a grain elevator and rail facilities. There are also records of pre-rail settlements migrating towards a railway line that passed near, but not through, the settlement. Extensions to the rail network and the improvement of road transportation did not considerably alter the basic settlement pattern, mainly because the major roads tended to be built parallel to the main rail routes. The subsequent growth of certain centres (Regina, Saskatoon, and Moose Jaw), selected by the railway companies as major focal points of their systems, was continued with the development of transportation methods, because such places were also the centres of the road pattern. If the linear pattern was to be altered it would require a much denser rural road network than that which is in existence

FIG 3:3 URBAN PLACES IN SASKATCHEWAN

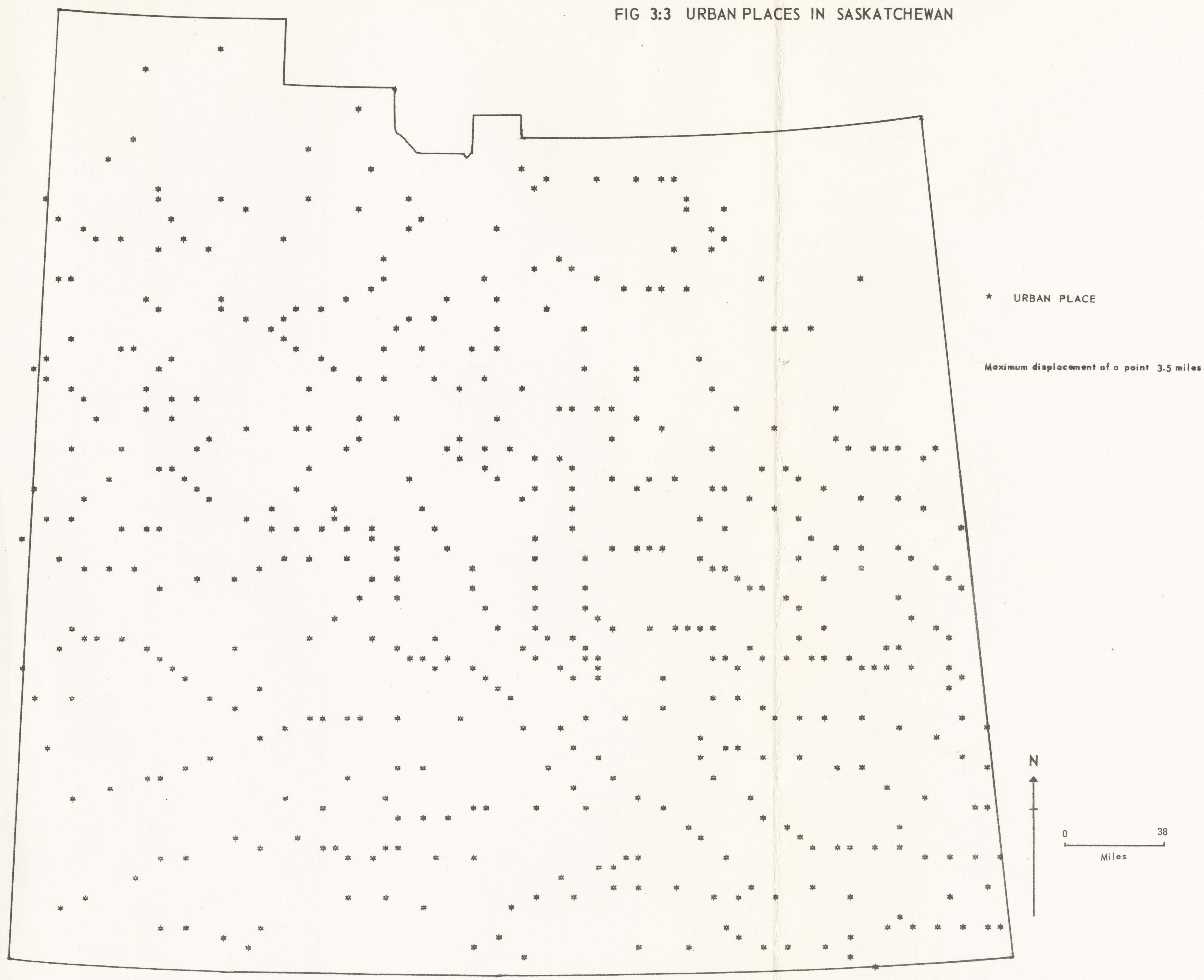
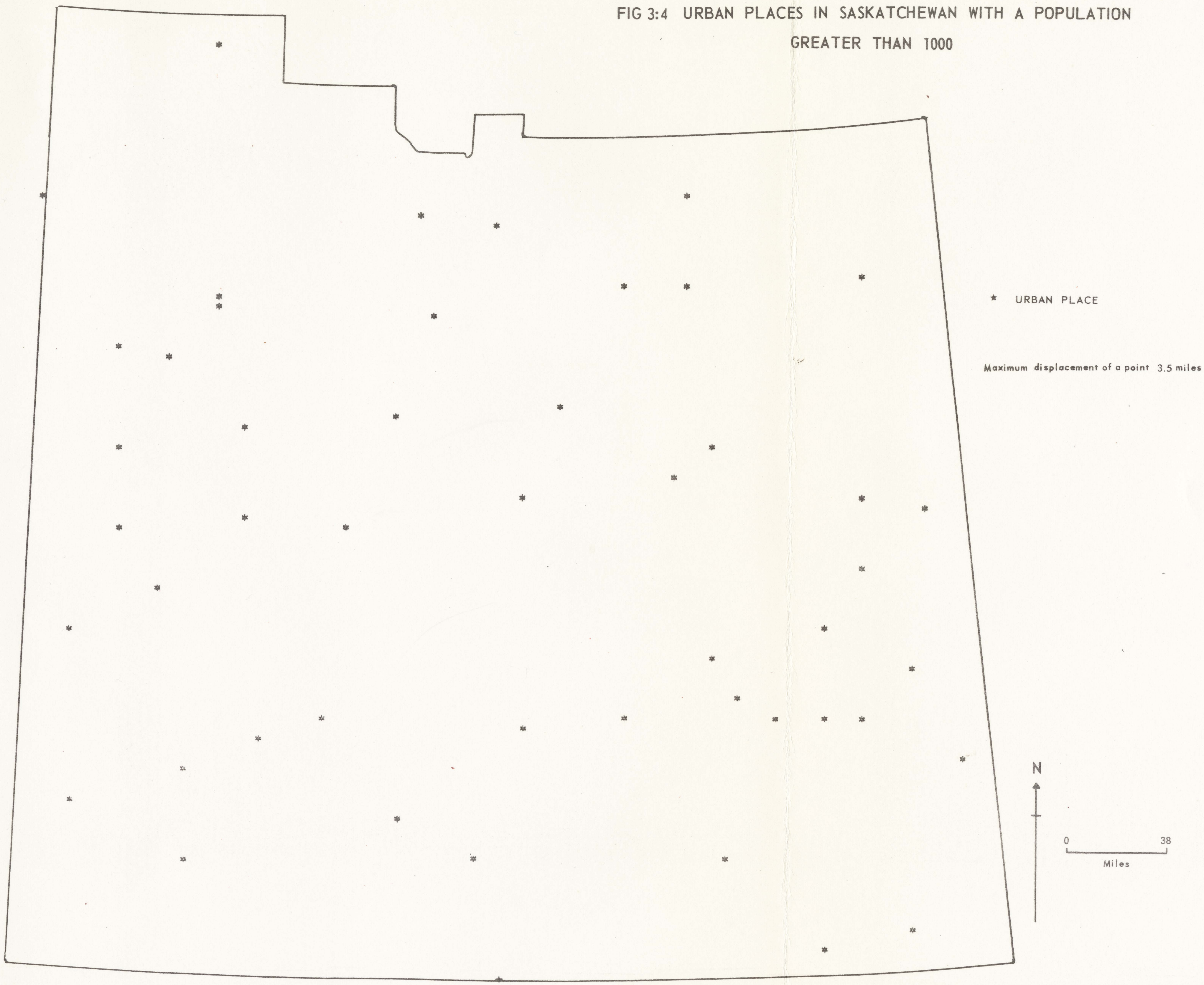


FIG 3:4 URBAN PLACES IN SASKATCHEWAN WITH A POPULATION GREATER THAN 1000



at the present in order that the marketing principle might come into effect and alter the status of presently non-central place settlements (Royal Commission, 1957, v. 12, pp. 69-70).

The close spacing of settlements along the lines of transportation, with large areas of intervening territory without settlements, results in a low value for the average observed distance (vide King, 1962, p. 6). Thus R is comparatively lower than if the towns are uniformly distributed over space. It is, therefore, suggested that the hypothesised uniform pattern of all settlements is disturbed by major transportation routes after the manner suggested by Christaller in his traffic principle (1966, pp. 72-77, 111-117). In the traffic principle the basic mesh of equilateral triangles is not disturbed by the introduction of through transport. However, if long haul transportation is introduced into a region either before, or shortly after, the original settlement of thatarea the weightings of the points forming the mesh, in terms of central place functions and, thus, population are affected. There is a strong tendency for places near to the transportation line to develop as central places rather than places not so situated. That is, instead of the uniformly distributed resource, population, which governs the location and functions of a central place under the conditions of the marketing principle, transportation, a highly localized resource, becomes the prime control of the form of the central place pattern. Harris and Ullman (1945) in their discussion of the support of cities modified the basic central place pattern, derived from the marketing principle, to take into account the effect of transportation as a controlling factor.

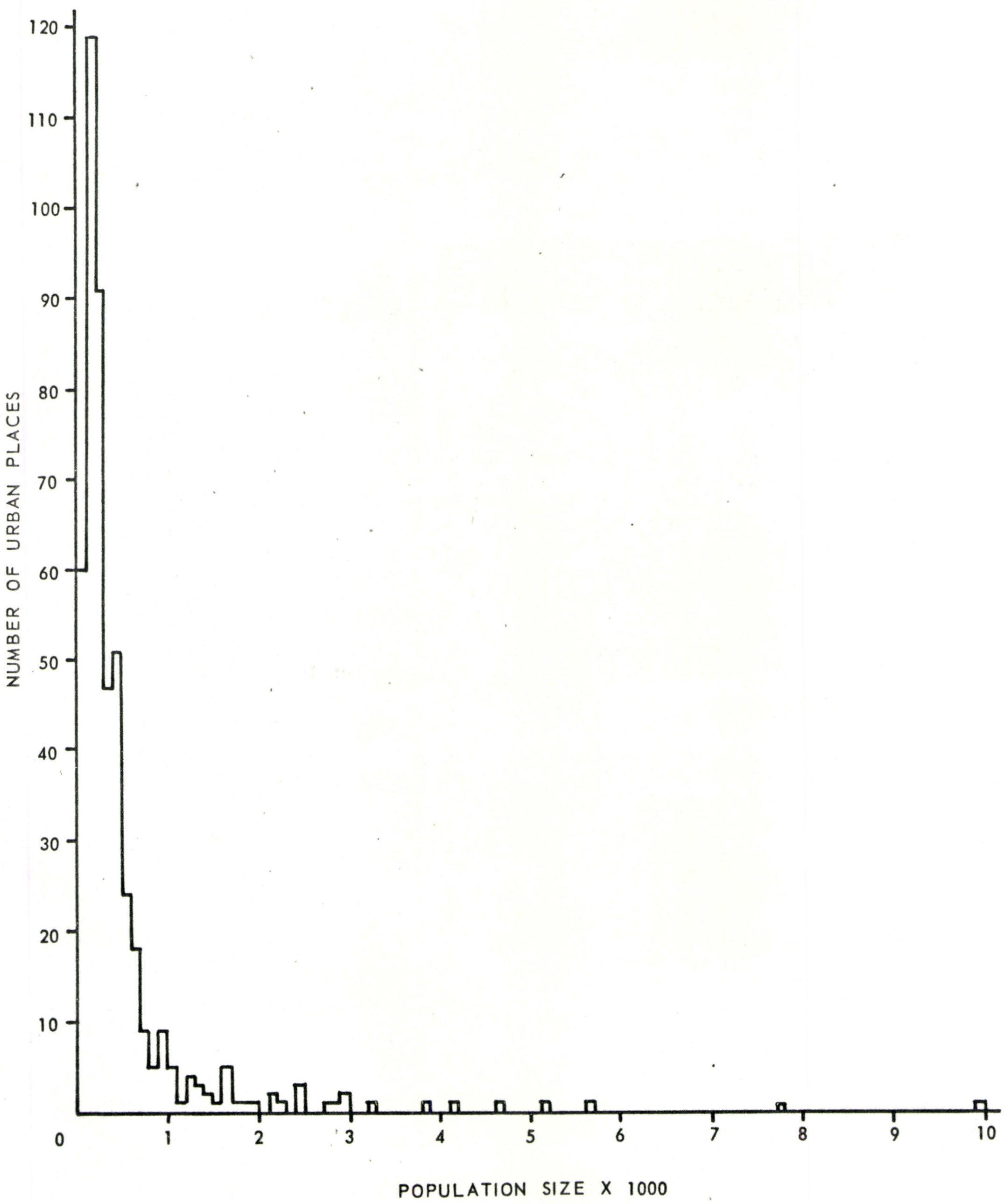
One corollary of the operation of the traffic principle on the pattern of towns is that the number of central places required to service a region is increased over the number required if they are distributed according to the marketing principle. The Royal Commission (1957, v. 12, p. 9) noted that Saskatchewan had a mugh higher proportion of service centres to total population than any other Canadian province; this may well be due, if Christaller's traffic principle is applicable to the distributive process in Saskatchewan, to the linear control of the railways over the settlement pattern.

The higher values of R for towns over 750 and 1,000 suggest that at these levels the influence of railway development may be less. By this it is meant that at this level, although the settlements are located on railways and owe part of their development to rail transportation, they also have developed, in terms of population size, according to the marketing principle. Christaller (1966, p. 76) does not discount the possibility of actual patterns being due to the operation of two, or more, of his principles:

> ' Both principles traffic and marketing are theoretically correct ...... either the traffic principle has such a weight that it outweighs the marketing principle, ......, or the marketing principle is the stronger one, or finally, the most favourable system is obtained through a combination of both principles, i.e., through a compromise.'

The distribution graph of town sizes (Figure 3:5) shows that the curve is highly skewed to the left. However, it is not the J-curve which would be logically expected for such a graph. According to central place theory and empirical observations of the frequency of town sizes, the J-curve should fall, concave upwards, from a large number of small

FIG 3:5 FREQUENCY DISTRIBUTION OF TOWN SIZES




Six Towns with a population greater than 10,000 not included.

places to a few places of large size. In Saskatchewan the distribution curve increases from 60 urban places with a population size of less than 100, to 119 places with populations between 100 and 199, from this point the curve descends in the manner that theory and empirical evidence would suggest is normal. Thus a second reason for the lower values of R, when the population size criterion for the inclusion of towns into the analysis is low, may be due to the ommission of some small urban places because they are not incorporated and, therefore, not included in the census. The inclusion of more small urban places into the analysis, given that the population data were available, would decrease the average expected distance between points in a random pattern because of the increase in unit density. The value of R would only be increased if the average observed distance was not decreased; thus, the location of extra settlements would be required to be in those areas between major transportation routes, rather than on such routes, if a more regular pattern was to be obtained.

## Hypothesis II

### Formulation

The results of the analysis of pattern under the conditions of Hypothesis I show that the value of R for the total pattern is low (1.088) and that this is only slightly more uniform than random, although the difference is statistically significant. A second hypothesis is now derived to provide information concerning the suggestions that there may be different distributive processes affecting the subsidiary patterns of various subsets of urban places and that the low value of R represents a mean value for the whole population.

If a sample of urban places was chosen at random from the 481 that constitute the statistical universe, the average distance from each of the places in the sample to all points, including the other sample members, and R' would differ from the values 8.7 miles and 1.088 only by chance, i.e., sampling error. If this procedure were repeated several times, the values of the average observed distance and R' would form a narrow band around the respective values for the total population, any differences not being statistically significant. In order to verify or disprove the suggestion that different distributive processes affect different strata of the population size continuum a null hypothesis is set up on the basis that stratified sampling of the continuum would produce the same result as simple random sampling. Hypothesis II is stated as: the patterns of stratified samples in relation to the overall pattern are not statistically different. This follows from Central Place Theory in that central places, whatever their size, are situated on the apexes of equilateral triangles (Figure 3:1(B)) and thus, the distance from any place to its nearest neighbour is a constant. However, if the results of the pattern analysis are statistically different it can be assumed that this represents the operation of different distributive processes for different population size classes of urban places.

Test of Hypothesis II

In order to stratify the population size continuum the same arbitrary class boundaries that were utilised in the test of Hypothesis I were chosen. At each stage of the analysis the value of the average expected distance is constant (7.98 miles) because the number of possible nearest neighbours of any size is 480 (i.e., 481 urban places minus the town from which the measurement is made.) The standard deviation of the average expected distance is 0.1901524. Thus, it is possible to compare directly the pattern statistics obtained. Because of the constant value of the average expected distance there is a one-to-one relationship between the average observed distance and the pattern statistic R for each sample.

On the basis of the results shown in Table 3:2 and Figure 3:6 and an analysis of variance (Clark and Evans, 1954, p. 452), which showed that there are significant differences in the results, the null hypothesis is rejected. Therefore, it is accepted that the pattern of various groups with relation to the overall pattern varies with the size of the towns forming the groups.

In Figures 3:6 and 3:7 the pattern statistic and the average observed distance to first nearest neighbour is plotted against the average size of the towns in each sample. It is seen that as the average population size of towns increases so the value of R increases, and because of the one-to-one relationship between distance and R, average distance alters in a similar manner. A linear regression of average distance on average population size takes the form

$$\text{Average Distance} = 4.420 + 1.614.\ \text{Log}_{10}(\text{Av. Pop. Size})$$

This equation explains 92.17% of the variation in the results as plotted in Figure 3:7.

TABLE 3:2

RESULTS OF PATTERN ANALYSIS FOR HYPOTHESIS II

| Pop. Range of Towns | Average Pop. | No. of Towns | Av. Obs. Distance (Miles) | Pattern Statistic R | Standard Variate of Normal Curve |
|---|---|---|---|---|---|
| 1 - 75 | 45.28 | 29 | 6.2 | 0.777 | - 9.33 * |
| 75 - 100 | 88.74 | 31 | 7.5 | 0.947 | - 2.19 + |
| 100 - 250 | 169.51 | 167 | 8.5 | 1.074 | 3.13 * |
| 250 - 500 | 360.57 | 141 | 8.8 | 1.113 | 4.76 * |
| 500 - 750 | 598.96 | 46 | 8.8 | 1.104 | 4.38 * |
| 750 - 1000 | 873.47 | 19 | 9.6 | 1.204 | 8.56 * |
| 1000 - 5000 | 1962.34 | 38 | 9.9 | 1.246 | 10.36 * |
| 5000 - 10000 | 7158.75 | 4 | 10.3 | 1.296 | 12.45 * |
| 10000 - 112141 | 48076.17 | 6 | 11.7 | 1.469 | 19.68 * |

TABLE 3:3

CUMULATIVE RESULTS

| Max. Pop. of Towns | No. of Towns | Av. Obs. Distance (Miles) | Pattern Statistic R | Standard Variate of Normal Curve |
|---|---|---|---|---|
| 75 | 29 | 6.2 | 0.777 | -9.33 * |
| 100 | 60 | 6.9 | 0.865 | -5.64 * |
| 250 | 227 | 8.1 | 1.019 | 0.80 |
| 500 | 368 | 8.4 | 1.055 | 2.32 + |
| 750 | 414 | 8.5 | 1.060 | 2.55 + |
| 1000 | 433 | 8.5 | 1.067 | 2.81 * |
| 5000 | 471 | 8.6 | 1.082 | 3.42 * |
| 10000 | 475 | 8.6 | 1.083 | 3.50 * |
| 112141 | 481 | 8.7 | 1.088 | 3.70 * |

* Significantly different from a Random Pattern at the 99% level of confidence.

+ Significantly different from a Random Pattern at the 95% level of confidence.

Density of Towns in Saskatchewan 0.0039276

Average Expected Distance Between Towns 8.0 Miles

FIG 3:6 THE PATTERN OF STRATIFIED SAMPLES WITH RESPECT TO ALL URBAN PLACES

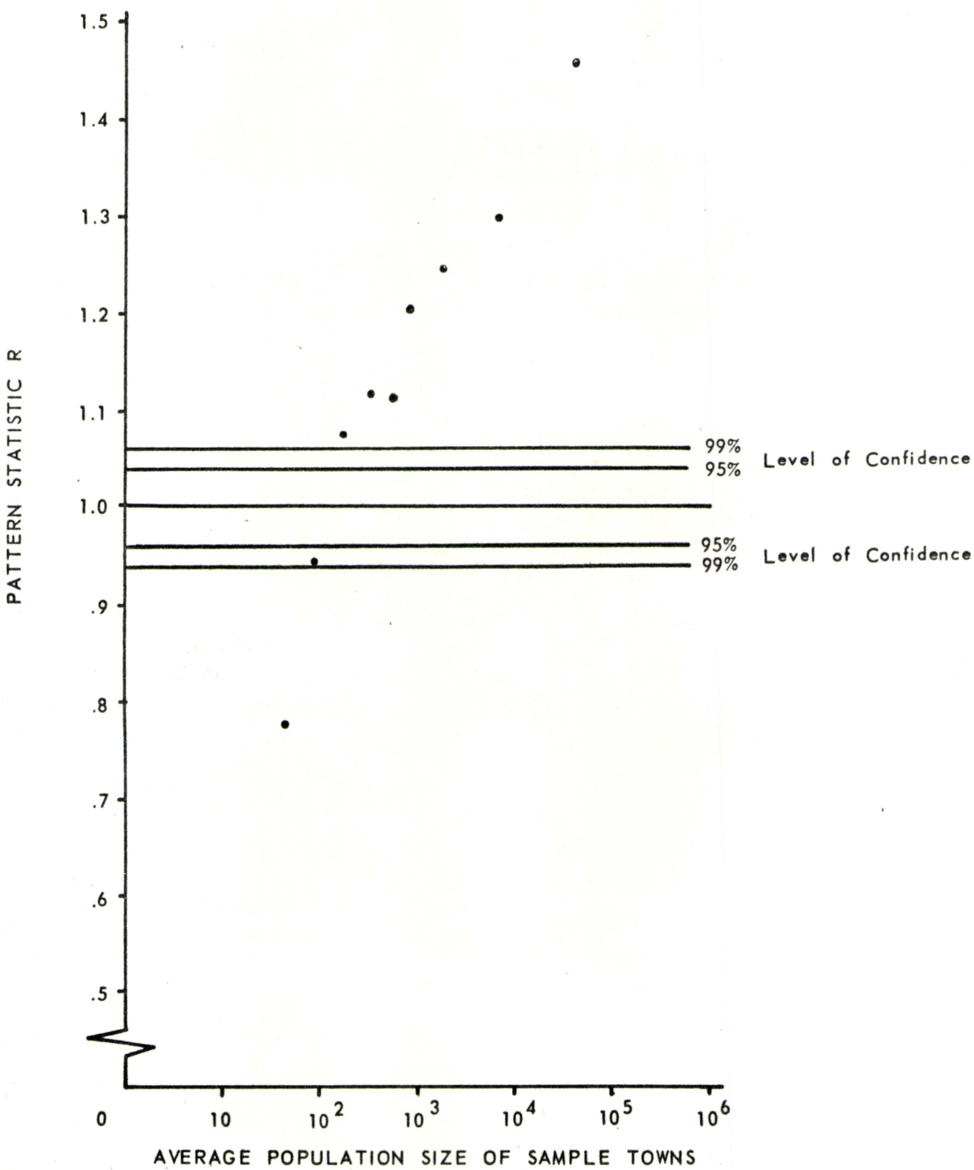


FIG 3:7 AVERAGE DISTANCE TO NEAREST NEIGHBOUR FOR STRATIFIED SAMPLES

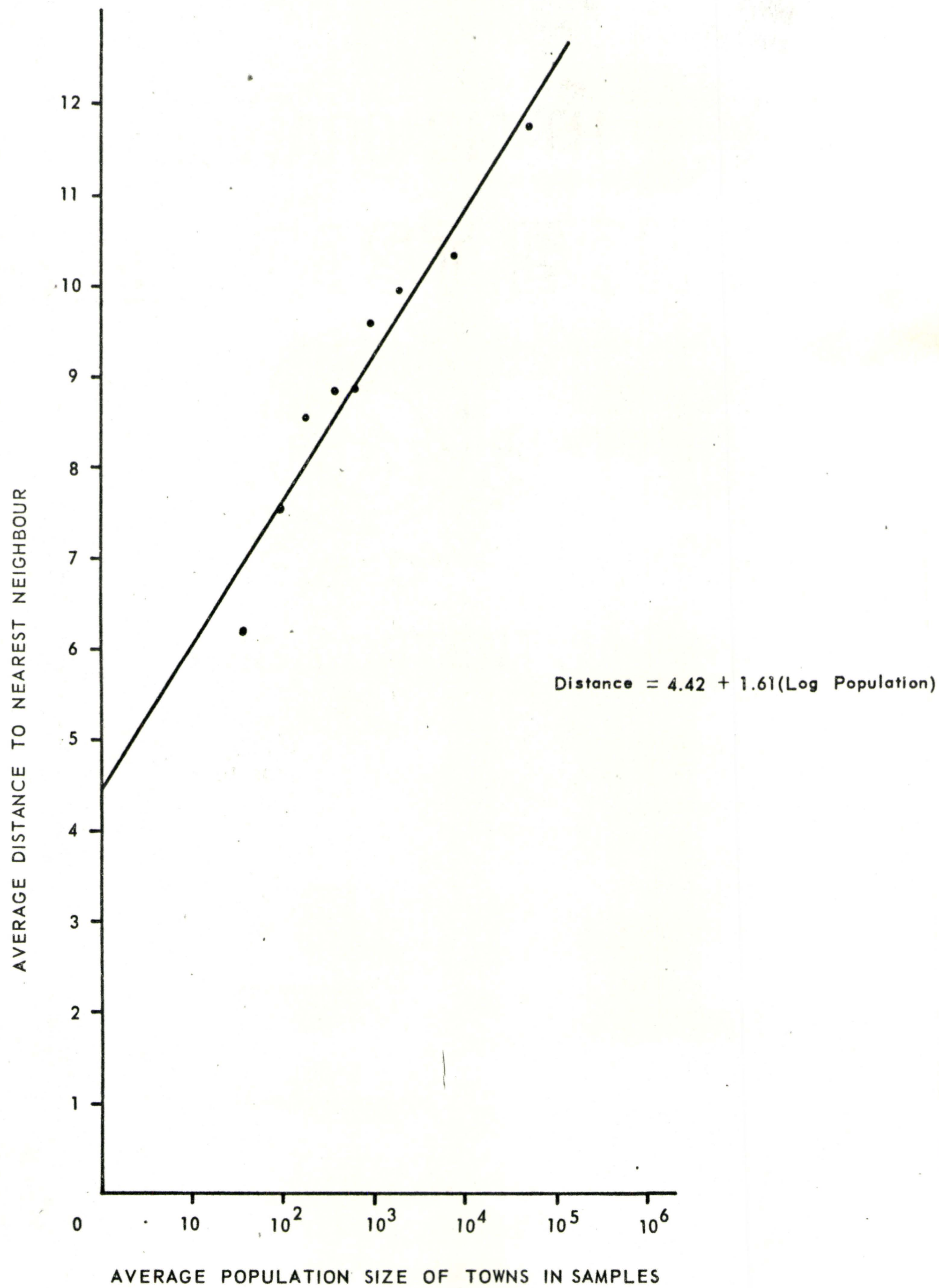
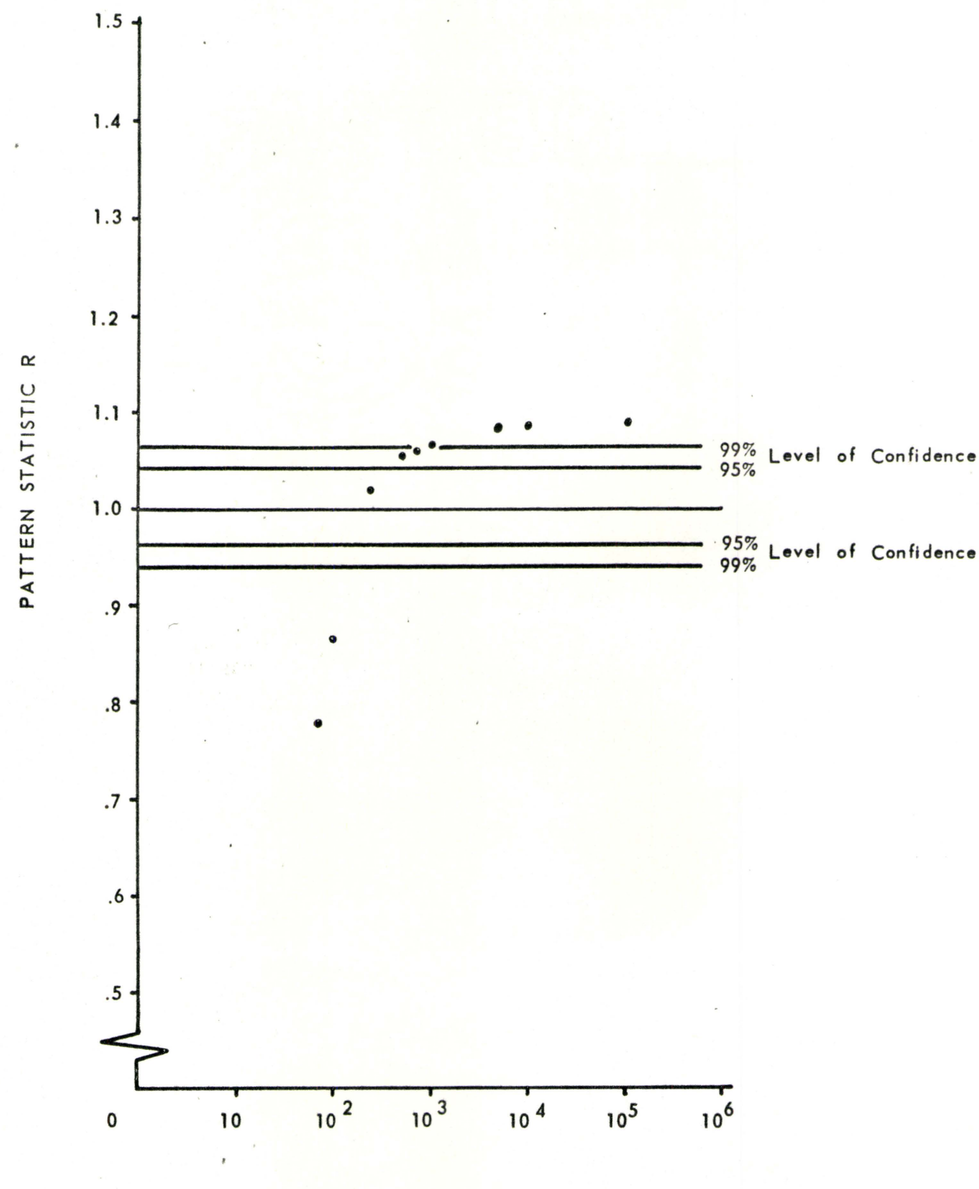


FIG 3:8 THE PATTERN OF URBAN PLACES WITH LESS THAN A CERTAIN POPULATION WITH RESPECT TO ALL URBAN PLACES



A second set of results are shown in Table 3:3 and Figure 3:8. The sum of the observed distances for each sample are cumulatively summed and R calculated for towns under a certain size. It can be seen that the results are curvilinear, as the addition of the smaller number of distances from the higher population size samples does not radically alter the cumulative average of the smaller places. The final line of Table 3:3 is the same as the first in Table 3:1 because both represent the overall pattern of urban places in Saskatchewan (that is, all 481 towns in the sample). The value of R for the overall pattern is the same in both Tables, thus providing a useful check on the results.

Implications of the Results of Hypothesis II

The results shown in Table 3:2 allow a comparison to be made between the relative locations of urban places of various sizes within the overall pattern and the construction of a simple model of the possible arrangement of urban places with respect to one another. It is also possible to describe some facets of the process by which the present pattern of urban places in Saskatchewan has been formed.

The pattern of urban places with respect to the overall pattern tends to uniformity as the average population size of the samples increases; small places are more clustered than the overall pattern, whilst large places are more uniform. Also it can be assumed that the nearest neighbours of small places must be other small places, rather than large places, because the nearest neighbours of large places are farther away, on average, than those of small places; however, the nearest neighbour of a large place may be of any size. This phenomenon of a greater distance separating a large centre from its immediate neighbours than the distances between the places surrounding such a centre has also been noted by other workers in this field (Kolb, 1923; Brush, 1953; Bracey, 1956).

The results and the above comments are in direct contradiction to the central place model of the pattern of urban places as stated by Christaller. Olsson and Persson (1964) have suggested that the central place model might well be improved by inclusion of the concept of retail gravitation as stated by Reilly (1931). To do so would require the relaxation of the assumption that the competitive influence of urban places of unequal population size, offering the same good or service, is equal.

The law of retail gravitation states that the point of competitive equilibrium between two places is, in miles from town B,

$$\frac{\text{Miles between A and B}}{1 + \sqrt{\dfrac{\text{population of A}}{\text{population of B}}}}$$

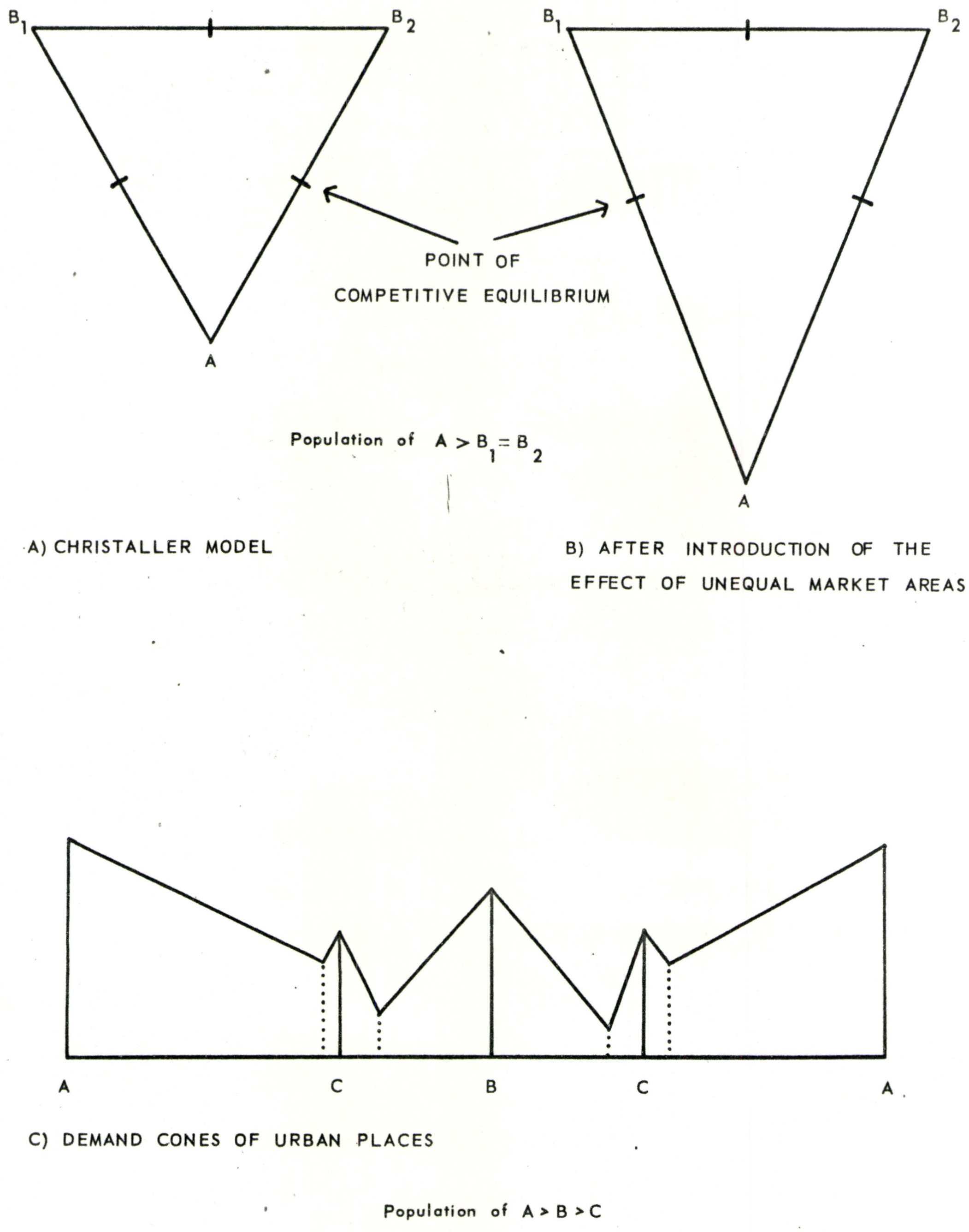
Thus, given that the density of population is uniform in all cases, the trade area of a large centre will contain a greater area than that of a small centre, for the supply of the same good or service. Examples of the existence of such differences in the size of trade areas for the same good have been mapped by Berry (1967, pp. 10-20).

Given certain assumptions (that there is the minimum population required in the trade area of a place for a given good to be offered at that place, and that service centres are located at the centre of approximately circular trade areas) it can be shown that the distance between a large place and its nearest neighbour (of smaller size) is greater than the distance between two small places (Figure 3:9(B)). In the system shown the trade area of the large place, demarcated by the point of competitive equilibrium, will contain more than the required threshold population. In order for the trade area of the smaller place to contain the minimum and for the place to be at the centre of its trade area, it is necessary to displace the smaller centre from its theoretical location in the Central Place Model. Thus, the equilateral triangle of Christaller's model (Figure 3:9(A)) is replaced by an isoceles triangle (Figure 3:9(B)) if the two smaller places are assumed to be of equal size. The small places are closer to their nearest neighbour than the average distance, whilst the large place is farther from other places than the average.

The above explanation of the results obtained from the testing of Hypothesis II is relevant to the static situation considered by this thesis. However, the factors which influence the development of such a pattern require further explication in order that some conclusions concerning the distributive process might be made.

Rushton, Golledge and Clark (1967, p. 392) have shown that a large proportion of the rural population, in the sample space which they studied, does not necessarily make its largest grocery purchase in the nearest town which offers such a function, but has a tendency (fifty-two percent) to patronise the nearest centre with a population greater than 1,200. A possible explanation of this phenomenon is to be found in the work of Baumol and Ide (1956) who approached the concept

FIG 3:9 POSSIBLE EXPLANATION OF THE OBSERVED RESULTS




After B. J. L. BERRY, 1967 p. 85.

of trade area size through the medium of probability theory. They argued that large centres have greater trade areas for low order goods than smaller places because of the higher incidence of multi-purpose shopping trips to large towns with the greater probability of fulfilling all purchase requirements. Thus, the effective population supporting a trade centre, or a given function, can be reduced or enlarged by the spatial purchasing habits of the population itself. Figure 3:9(C) shows Berry's interpretation of the demand cones of centres of different sizes utilizing Baumol and Ide's ideas (Berry, 1967, p. 85).

The development of the disparity in the areal extent of trade areas for the same good or service in Saskatchewan is also dependent on a number of other factors. Hodge (1965) has shown that the competitive influence of certain towns in Saskatchewan has increased through time and that this has affected the pattern of service centres. It has been shown (Royal Commission, 1957, v. 12, pp. 123-127) that the increase in the trade areas of large centres is associated with the reduction of the number of small urban places that serve as service centres. This effect is partially due to a great increase in the mobility of the rural population and the reduction of the gap between farm and town -- a marked urbanizing of rural habits and values. The farm population has an increased propensity to require goods and services available only in large centres. The ability to purchase higher order goods has also increased due to higher per capita income in the rural areas, associated with the reduction of the rural population and the increase in agricultural profitability. The resultant increase in the movement to large centres has resulted in such centres having an expanded trade area for all goods and services. Hodge (1965, p. 98) has summarized the areas which are most affected by this process.

> 'There is clearly a zone of attrition of small centres up to ten miles around large centres and the indications are that attrition may soon extend farther out. ... It would seem that the spatial integrity of small centres beyond fifteen miles is more secure.'

The effect of the expansion of the trade areas of large towns on smaller places is a decline of the areal extent and, therefore, rural population of their trade areas. Eventually the population of the trade area is reduced below that which can support the offering of a given good or service from the small place which then becomes an uneconomic location from which to supply that good or service.

Figure 3:9(C) shows that the trade areas, denoted by the demand cones, of small centres are asymmetrical, having their largest extent in the direction away from the largest centres. This is an indication of the ongoing process of trade area expansion and contraction. Curry (1962), on the other hand, has suggested that one reason for the observed clustering of small places and the asymmetry of their trade areas in the vicinity of larger places is the 'desire' to protect their hinterlands by collectively repulsing the influence of larger centres. However, this may be an artificial impression due to the thinning out of small urban places in the vicinity of a large centre because of the loss of functions as the areal influence of the large centre increases. At a distance greater than the range of influence of the large centre the pattern of the small centres is not affected by this process and, thus, it has the appearance of being more clustered than the overall pattern.

Hart and Salisbury (1965) found that the increase in population of small places was inversely proportional to their distance from cities of 25,000 or greater, for the nine state area of the Great Plains in the United States. Although their results are based on the observation of a similar phenomenon to that which Hodge studied in Saskatchewan, it would appear that there is some discrepancy between the findings of the two studies. One solution of the difference can be found in the work of Hassinger (1957), who studied part of the Great Plains in southern Minnesota. He divided incorporated urban places into two categories: small towns with less than 2,000 inhabitants, and large towns having a population greater than 2,000. The second category was further sub-divided into towns over 5,000 and between 2,000 and 5,000. It was found that small places had a slower rate of population growth the nearer they were to towns in the 2,000 to 5,000 range, but that this relationship was not apparent with respect to the distance from towns over 5,000. Hassinger speculates (1957, p. 134) that the reasons for this difference lies in the nature of the trade patterns of the larger towns. He reasons that towns with between 2,000 and 5,000 inhabitants rival their smaller neighbours with respect to the services which they offer and, therefore, tend to be 'more destructively competitive' than towns over 5,000 which dominate, rather than rival, the small places near them which take on the characteristics of suburbs. Thus, it may be that Hodge's conclusions with respect to Saskatchewan were based on the observation of the competitive situation, whilst Hart and Salisbury, because of the use of the distance from a city of 25,000 inhavitants, may be studying the suburban stage of the development of inter-relationships between urban places.

Summary

The analysis of the results obtained from testing two hypotheses concerning the relationship of the patterns of urban places to population size has shown: firstly, that the overall pattern of settlements in Saskatchewan exhibits a high degree of linearity. The control of transportation routes over settlement location has been suggested as the prime cause for the observed linearity. Secondly, it would appear that the marketing principle increases in importance when discussing the distribution of functions the larger the population size of the group of towns considered. Thirdly, in the analysis of the relationship of different population size groups to the overall pattern, an explanation of the results requires consideration of the rôle of retail competition between centres. It is shown that the Central Place Model requires reformulating to accommodate the ideas expressed in the law of retail gravitation. It was further suggested that the pattern of urban places undergoes change through time and that this is due to changes in consumer patronage of centres affecting both the size of trade areas and the economic viability of urban functions.

# CHAPTER IV

## AREAL VARIATION AND THE PATTERN OF URBAN PLACES

The preceding chapter of this thesis was concerned with the evaluation of the pattern of urban places either in total or for groups of towns selected on the basis of population. One assumption of the analysis was that the overall pattern, or the patterns of the various groups, did not exhibit extreme areal variation and that R, therefore, is applicable as a measure of a single pattern, rather than being a mean value representing the summation of two, or more, patterns of different characteristics in separate areas of Saskatchewan. The first objective of this chapter is to describe statistically and cartographically the pattern of urban places for small areas in Saskatchewan in order to evaluate the validity of the above assumption. The second objective is to determine how the size of the collecting area affects the results obtained by use of the nearest neighbour method of pattern analysis.

### Methodology

In order to produce maps of the spatial characteristics of the pattern of urban places it was necessary to obtain a set of values of R for the surface of Saskatchewan. A network of grid points located at regular intervals across the province were used as control points for the assessment of R and for the plotting of isopleth maps. R was observed for each grid point by centering a circle of given radius on the point and calculating the distances to first nearest neighbour for all towns within the circle. Thus, it was possible to find the average observed distance between points.

The major difficulty encountered in the calculation of R was that of obtaining an accurate estimate of the area in which the points may occur, so that the average expected distance between points in a random pattern of the same density could be calculated. In the case of many grid points the use of a circle does not present any difficulty in the calculation of its area. However, many grid points are situated close to the Saskatchewan boundary, and a circle centered on such a point will contain areas outside of Saskatchewan, for which no information was available concerning the location of urban places.[1] The number of grid points to which the foregoing applies increases as the radius of the circle utilised increases. A method was devised that allowed machine computation of the area of the circle that falls within Saskatchewan. A rectangle was determined (Figure 4:1) which occupied the greatest areal extent of Saskatchewan. The grid points were located on or within this rectangle which forms the study area for this chapter. The rectangle occupies 85.33 per cent of the seventeen census divisions

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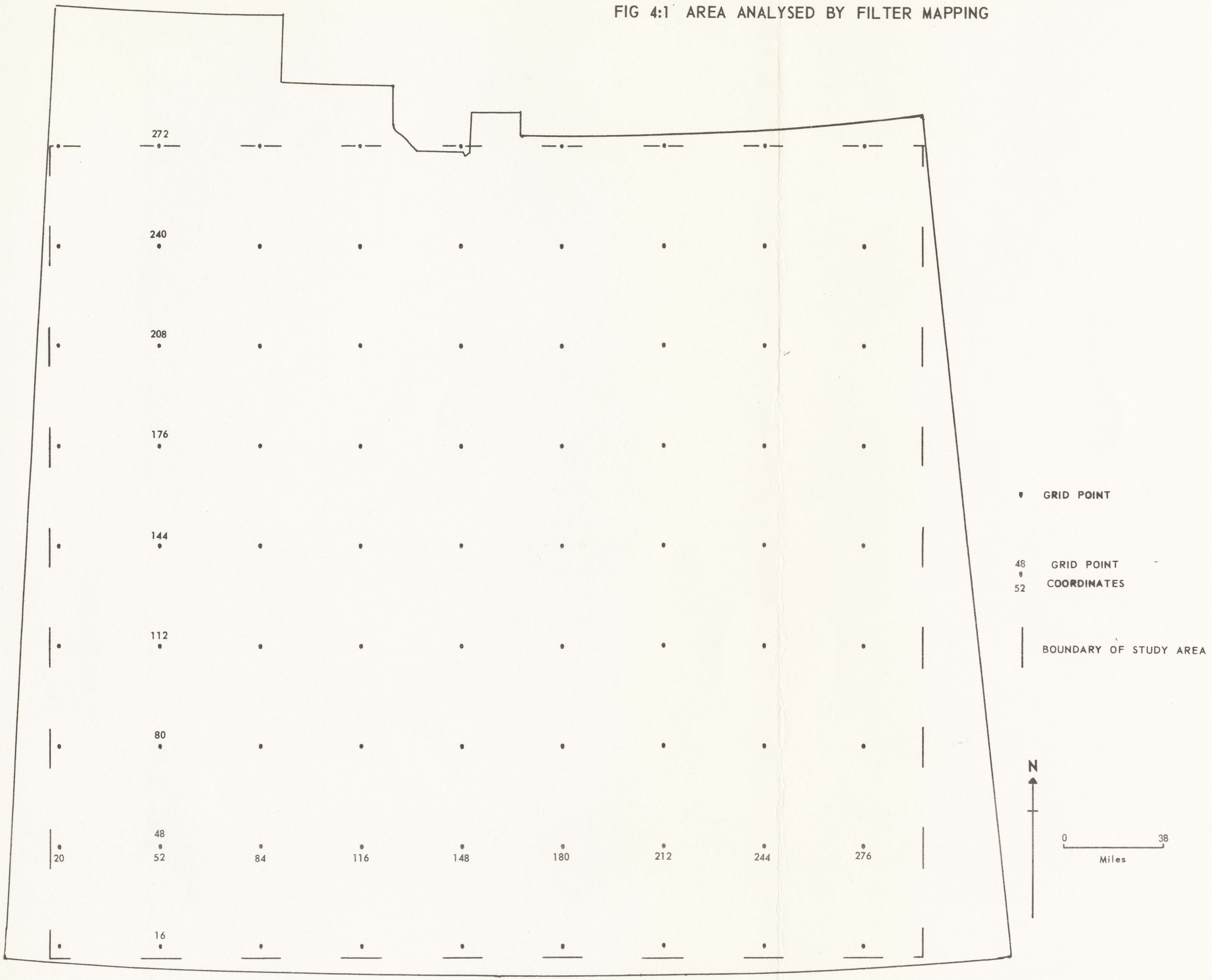
[1]Locational coordinates for urban places in the adjoining provinces of Alberta and Manitoba were not determined because there were no counterparts to the map used as the basis of the calculations for Saskatchewan. Whilst it may have been possible to determine locational coordinates for urban places in the three provinces from other maps, it was felt that this was not a worthwhile exercise because: population data are only available for incorporated centres and differences do exist between provinces in the criteria used to approve the incorporation of a centre.

FIG 4:1 AREA ANALYSED BY FILTER MAPPING



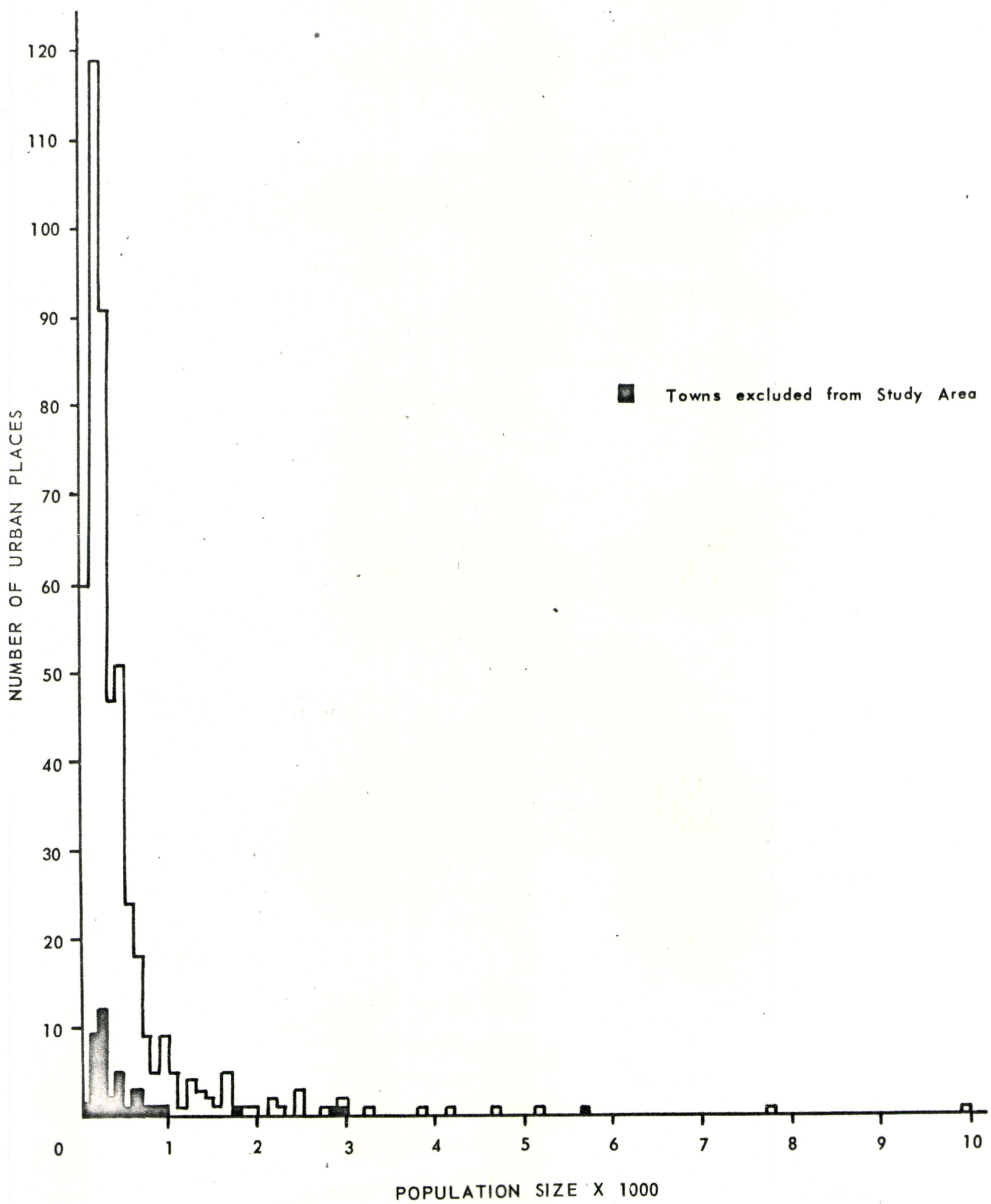
used previously and contains 441 of the 481 towns. The places outside the rectangle have a similar frequency distribution to the 481 towns (Figure 4:2), therefore, their exclusion should not seriously affect any conclusions drawn from the results.

In order to calculate the area of a circle that falls partly within the rectangle, the distance from the grid point to each side of the rectangle was found. If the distances in two adjacent directions, e.g., north and west, are greater than the radius of the circle, the area in this quarter is equal to one quarter of the total area of the circle. If one, or two adjacent directions are nearer to their respective boundaries than the radius of the circle, the area in this quarter is equal to the rectangle contained. The resultant areas of the four quarters are then summed to give the area of the circle that lies within the rectangle. The calculation of the average expected distance and the ratio R from this point follows the method outlined in Chapter II.

## Procedure

The ring and centre method used in this chapter is derived from the filter mapping method described by Haggett (1965, pp. 269-270). Filter mapping normally consists of finding the average value of the data, collected for small areas, within a circular or rectangular filter; the average value for the units within the filter being assigned to the point on which the filter is centered. The purpose of filter mapping is to separate broad-regional trends from local, unsystematic variations in the surface. Cassetti (1966) has discussed the effect of the aggrega-

FIG 4:2 FREQUENCY DISTRIBUTION OF PLACES EXCLUDED FROM STUDY AREA



tion of data for small units in terms of 'spatial harmonics' and has shown that the result of this process is to reduce that variation which has a lower amplitude than the size of the filter used. The centre points of the filters are arranged so that the filters overlap one another in a manner similar to the finding of 'running-means' in meteorological data (Panofsky and Brier, 1965, pp. 147-149), except that in filter mapping areal rather than time series data are analysed.

The ring and centre method used here foregoes the collection of data for small units because of the need to calculate the pattern statistic for a sizeable area. Increased generalization is obtained by increasing the size of the filter for which the pattern statistic is individually calculated rather than finding the mean value of an increased number of subsidiary units. Three different sizes of filter were used in the analysis of the areal variation of the pattern of urban places, that is, circles of 50, 60, and 70 miles radius. The upper limit was set by the manageability of the data required to perform the computations. The lower limit was set by an operational problem; if a radius of less than 50 miles had been used a number of circles, especially the part circles near the boundary, would have contained very few urban places or none at all.

The occurence of circles with few urban places did create problems when plotting isopleths of the pattern statistic. The grid point 276/272 (Figure 4:1) was not used in interpolating the isopleths in Figures 4:3 and 4:4 because it was felt that the values of R for this point would result in serious distortion of the finished maps if they were utilised. It was decided to accept for plotting the isopleths values of R derived from three or more measurements of distance to nearest neighbour. Values of R to two decimal places were used in the interpolation of the isopleths which have an interval of 0.1 units of R.

Linear Trend Surfaces

The surfaces which result from the above procedure are capable of being compared visually and qualitatively. However, in order to provide a more precise form of comparison a linear trend surface was obtained by fitting a least squares regression plane to the data for each scale of filter. The grid point coordinates were used as independent variables which are correlated with R, but not with each other (because they are orthagonal). The mathematical expression of the linear surface takes the form

$$R = A + B_1(\text{east}) + B_2(\text{north})$$

The linear least squares surface generalises as a tilted plane the orientation and the dip of a phenomenon. Normal statistical parameters can be used to make statements about the surface, such as, the degree of correlation, the standard error of the estimate and the amount of the original variation explained by the linear surface (Chorley and Haggett, 1965). Haggett (1961) has performed sequential testing of the residuals from the regional trend surface in order to increase the amount of variation that can be explained by consideration of sub-regional trends. However, this expansion of the procedure requires a larger number of control points than that used in the present analysis.

Areal Variation of the Pattern of Urban Places

Figures 4:3, 4:4, and 4:5 are, respectively, the isopleth maps of the 50, 60, and 70 mile radius filters. The results are tabulated in Appendix A. In this section it is proposed to describe the surface of each map.

The form of the surface of the 50 mile filter (Figure 4:3) is divided by a broken north-south ridge of R values greater than 1.2 which broadens out in the south east. In all four corners of the study area except the south east the pattern of urban places becomes more clustered than random, the dip in values being most strongly developed in the north east. The western half of the map is characterised by a depression having values between 1.0 and 1.1 which dips to more clustered than random values in the north and south. The central portion of this depression approximates to a col between a block of values rising to 1.4 on the western boundary and the central ridge of high values of the pattern statistic.

The isopleths drawn from the 60 mile radius filter show a slightly different result. Although high values still predominate in the south-east corner and in the north-centre, the surface might be more graphically described as saucer shaped: a central depression (R between 1.0 and 1.1) surrounded by areas of higher values. Lower values of R in the north west and south west are still present at this scale although of much smaller areal extent. A strongly developed 'valley' of low values trends north eastwards from the centre of the map breaking through the belt of higher values that surround the central depression. The occurrence of this band of low values explains the change in the direction of the trends

FIG 4:3 AREAL VARIATION OF THE PATTERN OF URBAN PLACES
FIFTY MILE RADIUS FILTER

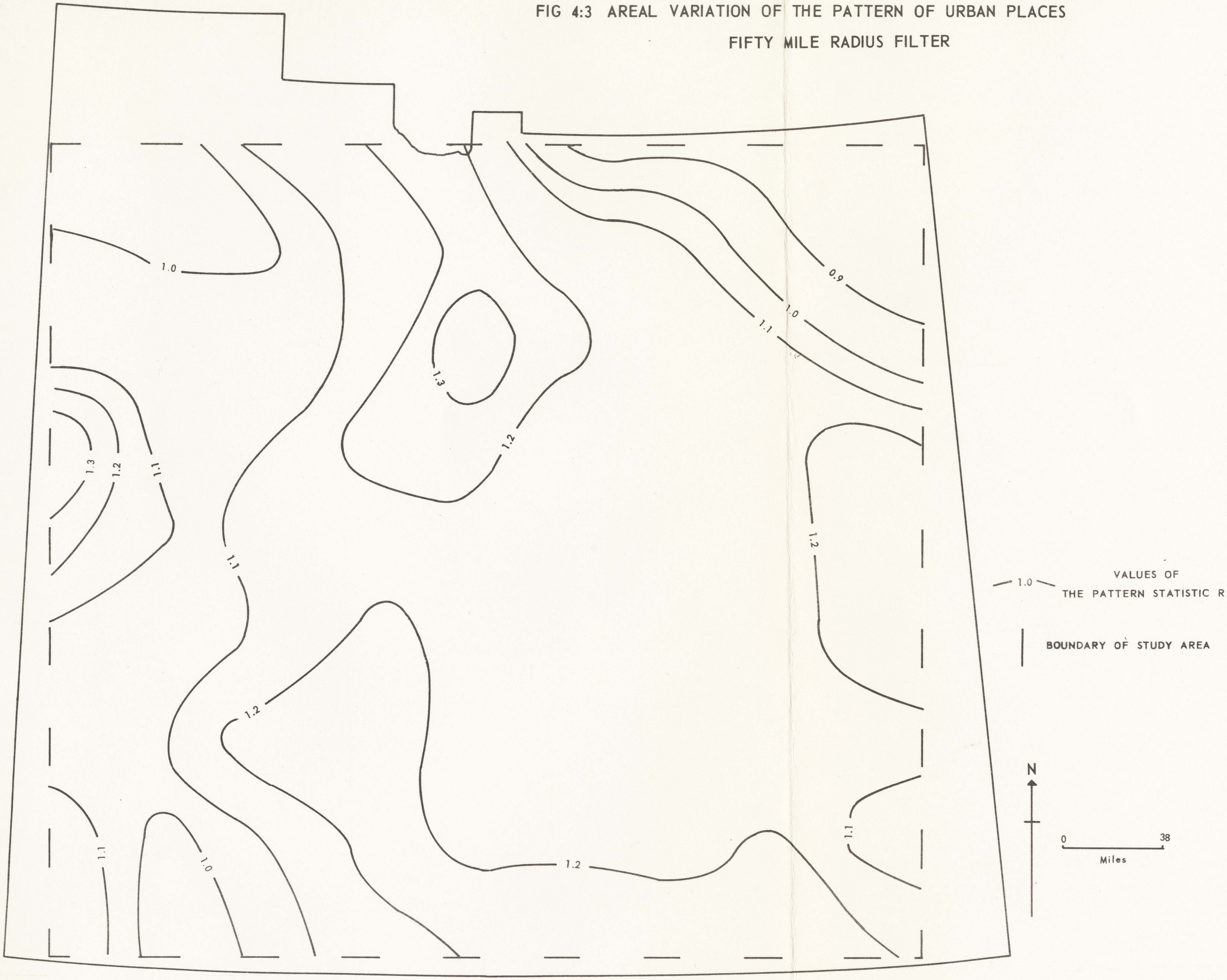
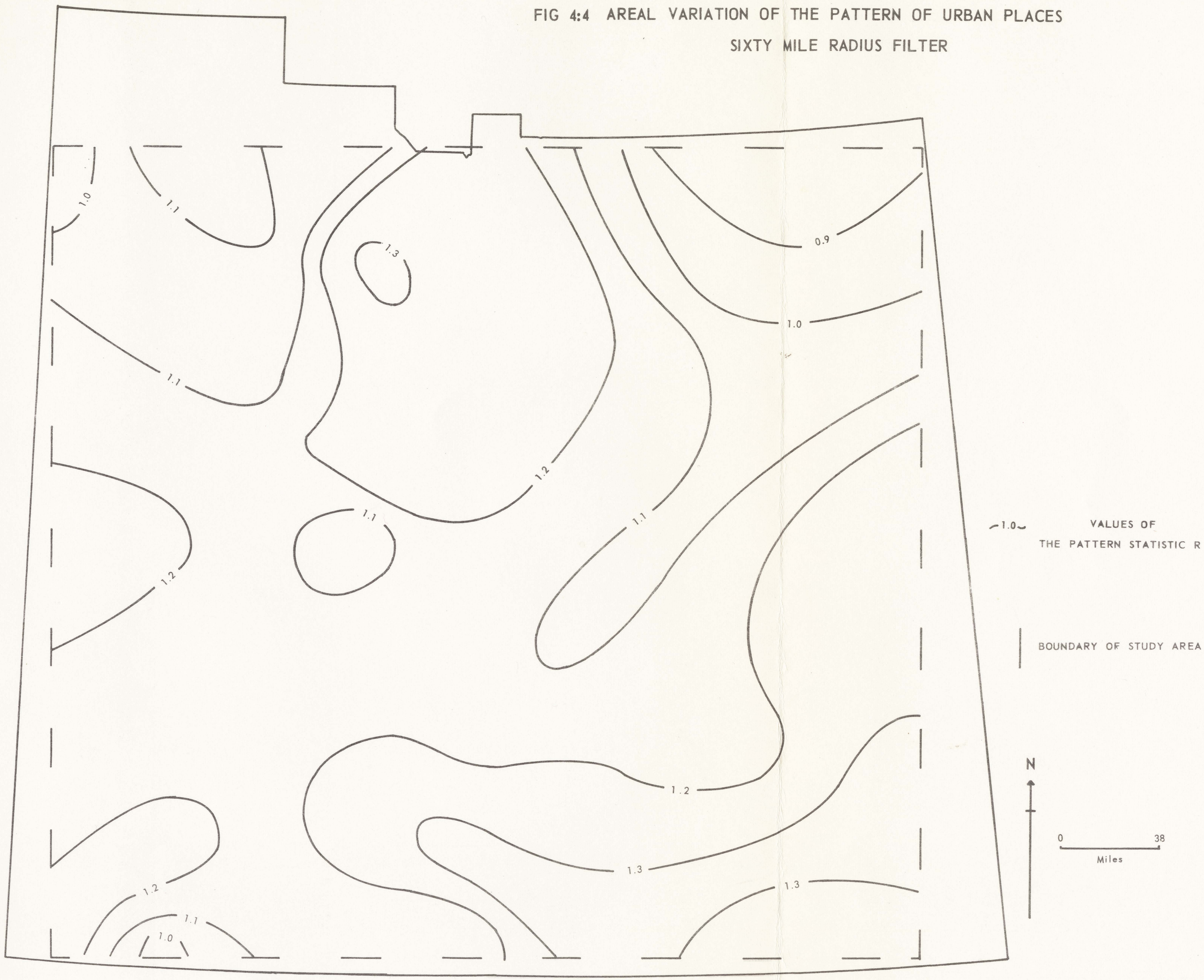


FIG 4:4 AREAL VARIATION OF THE PATTERN OF URBAN PLACES
SIXTY MILE RADIUS FILTER

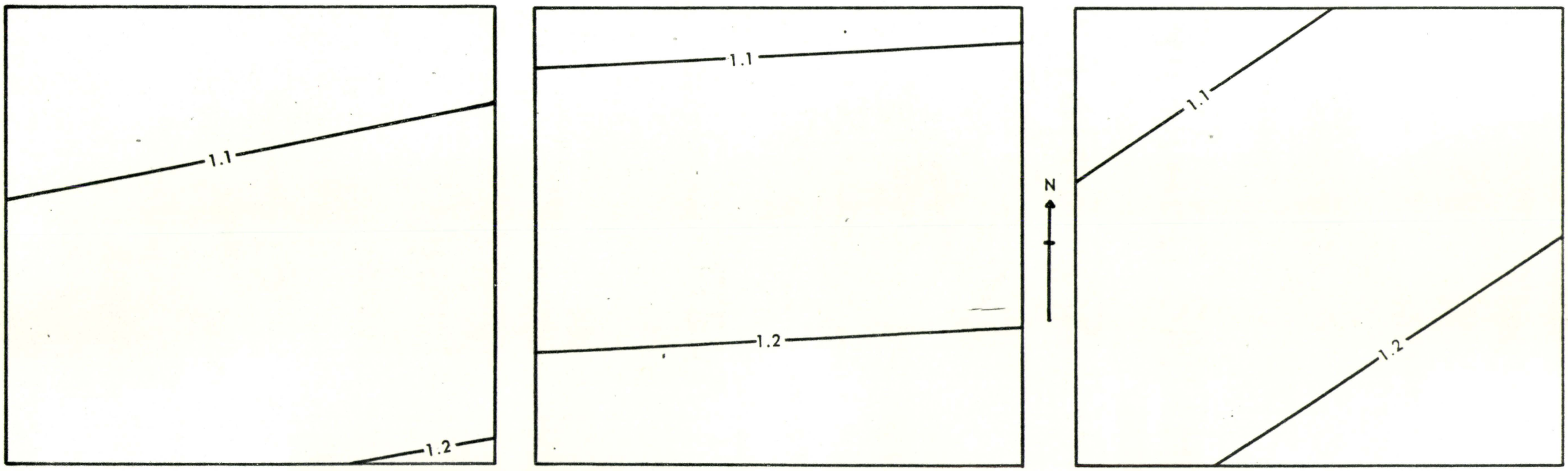


derived from the linear surfaces (Figure 4:6) for the 50 mile and 60 mile radius filters. The large number of low values in the north east of the study area in the 60 mile radius filter 'pulls' the trend surface round so that the direction of dip is approximately south to north.

The features of the isopleth map drawn from the 70 mile radius filter (Figure 4:5) have a similar appearance to the two preceding maps. Low values predominate in the southwestern and the two northern corners; the central ridge is still present, although the area above R equal to 1.3 is much smaller in the south-east than in the 60 mile radius filter map. The linear surface has a similar appearance to the 50 mile radius surface, that is, it dips from south-east to north-west. A feature of Figure 4:5 is the large areal extent of values of R between 1.1 and 1.2 which suggests that at this scale the pattern of urban places in Saskatchewan is approximately the same from one area to another.

A comparison of the isopleth maps and the map of all urban places in Saskatchewan (Figure 3:3) shows that the lower values of R, that is, the more clustered than random patterns, occur in those areas of the province where towns are sparse and tend to be located along the same major road or railway. The higher values occur in those areas where towns are more numerous and where roads and railways are more highly developed, suggesting that in the latter areas the towns are located either in accordance with the triangular mesh of the central place model or, that, because the transport surface is more uniform, the 'attraction' of a single transport route is of little importance for the location of settlements.

FIG 4:6 LINEAR TREND SURFACES OF THE PATTERN OF URBAN PLACES




50 MILE RADIUS FILTER

60 MILE RADIUS FILTER

70 MILE RADIUS FILTER

R = 1.186 + .000099 (EAST) − .000533 (NORTH)

R = 1.24 + .00003 (EAST) − .000607 (NORTH)

R = 1.175 + .0003 (EAST) − .000462 (NORTH)

Scale circa 96 miles : 1 inch

Rectangle as denoted in Figure 4:1

Affect of Variation in the Size of the Filter

The filter method used above enables the effect of altering the size of the area for which R is calculated to be studied. Firstly, a comparison can be made between the results at each size of filter, utilising only those measurements of R that are based on a complete circle. However, there are few complete circles; only twenty-five on the 70 mile radius filter map. Figure 4:7 shows the difference between the values of R obtained with a 70 mile radius filter and with a 50 mile radius filter for the aforementioned twenty-five points. It can be seen that the difference is often only significant in the second decimal place, which suggests that the alteration of the size of filter has little importance when assessing the nature of the pattern.

Secondly, the effect of altering the size of the filter area can also be described by comparing the highest and lowest values of R for each size of filter.

TABLE 4:1

VARIATION OF THE PATTERN STATISTIC 'R' FOR DIFFERENT SIZES OF FILTER

| Filter Radius (miles) | Highest Value of R | Lowest Value of R | Range | Percentage of variation explained by Linear Surface |
|---|---|---|---|---|
| 50* | 1.4297 | 0.6962 | .7335 | 12.98 |
| 60* | 1.3650 | 0.8095 | .5555 | 20.76 |
| 70 | 1.3517 | 0.9389 | .4128 | 24.81 |

*80 grid points.

The high values decrease and the low values of R increase towards a value of R equal to 1.0 as the size of filter increases. This is in accordance with the properties of filter mapping described previously: as the size of the filter increases local variations are smoothed out and regional tendencies predominate.

The percentage of the variation in the values of R that is explained by the regression planes (Figure 4:6) also increases as the size of filter is increased. This is due to the suppression of local variation in the pattern of towns. Thus, the value of R equal to 1.088, which was obtained in the analysis of the pattern of urban places for the total area of Saskatchewan, represents the suppression of all variation within the pattern of urban places.

The validity of the use of a single value for the whole province can be supported by reference to the significance of the values obtained in the filter mapping exercise (Table 4:2). It can be seen that as the filter size is increased the percentages of the values of R which are significantly different from a random pattern of the same density of points increases at both the 95 per cent level of confidence and at the 99 per cent level.

TABLE 4:2

PERCENTAGES OF VALUES OF 'R' THAT ARE SIGNIFICANTLY DIFFERENT FROM RANDOM

| Filter Radius (miles) | Level of Confidence | Significantly different from random | No. of Points |
|---|---|---|---|
| 50 | 95% | 30% | 80 |
| | 99% | 15% | |
| 60 | 95% | 52% | 80 |
| | 99% | 32% | |
| 70 | 95% | 59% | 81 |
| | 99% | 39% | |

Whilst one might expect some of the values of R obtained to signify a random pattern, 70 per cent of the values would appear to be a very high proportion. The explanation of the high number of patterns that are not significantly different from a random pattern is similar to that offered in Chapter III when the results of the test of Hypothesis I were discussed. A small circle or part circle often contains very few towns, therefore, the number of measurements made is also small, and the confidence interval around the value of R equal to 1.0 is correspondingly wider than in those cases where, because of a greater density of towns or because a complete circle is used to define a group of urban places (therefore, including more towns), a greater number of measurements are made. One example of the width that can be attained by the confidence interval is that surrounding the lowest value of the pattern statistic obtained in the 50 mile radius filter calculations. The value of R is equal to 0.6962 which is only significantly different from a random pattern at the 80 per cent level of confidence. The use of a value of

R equal to 1.088 for the whole of Saskatchewan would seem to be justified as it is different from a random pattern at a highly significant level.

In the plotting of the isopleths all the values obtained were accepted without alteration, except the two which were discounted as previously explained. However, if all the patterns that were not significantly different from random had been given a value of 1.0 and these values mapped, the appearance of the maps would be very different. The 50 mile radius filter map would probably exhibit less variation than the 70 mile radius filter map. It would appear that the use of the pattern statistic in describing the pattern of towns has two disadvantages which have to be balenced against one another. The larger the area for which the parameter is calculated the more general is the result, whilst the smaller the area, the higher the probability that the pattern is not significantly different from random because of the small number of measurements that can be made from the few places which are included.

Summary

The three isopleth maps which have resulted from the analysis of the pattern of small groups of settlements show that areal variation does exist within the overall pattern. However, on the basis of the examination of the effect of altering the size of the filter, it is concluded that a single value for the overall pattern is valid because of the existence of a number of patterns that are not statistically different from a random pattern. The above comment does not detract from the value of the maps: a review of the pertinent literature (for example:

Monkhouse and Wilkinson, 1952, pp. 308-313) would suggest that these may be the first maps to objectively measure the areal variation of the pattern of urban places. However, in view of the problems discussed in various sections of this chapter it would appear that a map of this nature should be concerned with, firstly, a much larger area than Saskatchewan in order that data for a large number of full circles can be utilised; and, secondly, the radius of the circles (filters) should be at least seventy miles, if not greater, so that the number of measurements is sufficient to narrow the range of values that may be considered to represent a random pattern.

# CHAPTER V

## MULTIVARIATE ANALYSIS OF THE PATTERN OF URBAN PLACES

One of the stated aims of this thesis is to obtain some knowledge of the distributive process that underlies the observed pattern of urban places in order that differences between the patterns of groups of places may be explained. In the preceding chapter areal variation was shown to exist in the settlement pattern of Saskatchewan. The purpose of this chapter is to expand upon the cartographic and statistical description of this source of variation by relating it to the spatial variation of other criteria that may be part of the distributive process.

In order to investigate the nature of the relationships between the settlement pattern and various social and economic factors the techniques of regression analysis were used. It would have been desirable to have utilised the values of R which were obtained in the previous chapter but the data for the independent variables are not available on the same basis. It was decided to use data collected on the basis of Census Divisions because these are the smallest units for which all the data are readily available.

### The Dependent Variable

The dependent variable for the multiple regression analysis is the value of the pattern statistic R for each of the seventeen Census

Divisions. R was derived for first nearest neighbours from the average observed distance between all places in a Census Division. The urban places were regarded as having equal importance and no measurements were made across a Census Division Boundary. Thus, the value of R is derived for separate groups of towns. The values have a range between 0.8538 to 1.3391, and exhibit a similar areal variation to that described in the previous chapter.

## The Independent Variables

The selection of the independent variables that are used in the regression analysis was based on the hypothesized relevance of such variables to the distributive process. The amount of variation in the dependent variable (i.e., the pattern statistic R) that is unexplained by the regression equation is indicative of the importance of the variables selected and of those excluded from the analysis.

Firstly, it is hypothesized that the extent to which the pattern of urban places in a Census Division tends towards uniformity should reflect the proportion of the total area that has been settled. The average observed distance between first nearest neighbours is determined by the location of the urban places, which may be presumed to be in the settled area. However, the average expected distance is dependent upon the density of urban places over the total surface. If the settled area is less than the total area, the average expected distance will be relatively larger and, therefore, the resultant value of R will tend to be lower than if the total area was settled. It is, therefore, postulated that the greater the proportion of the total area that is settled, the more uniform will be the pattern of settlement. For the purposes of this investigation the settled area is taken as the area that is in farmland.

Secondly, it is hypothesized that the uniformity of the pattern of urban places is dependent upon the percentage of the total area under cropland. Central Place Theory was developed from the empirical observation of an area that is characterized by a mixed farming economy. Major differences between the theoretical and actual system of towns in southern Germany occur in those areas that are not favourable to such a farm economy, (Christaller, 1966, pp. 170-197. The comparative work of King (1962) substantiates this hypothesis: he found that the most regular patterns of urban settlement in the United States occurred in the Great Plains in association with an intensive grain-feed-livestock economy. Also, because of the poor road transport at the time of settlement formation, and of the need to move large amounts of grain at the present time to transhipment points, an arable economy requires a regular arrangement of transhipment centres in those areas where grain is the major crop. Such a system would not be as important in a pastoral economy.

As an adjunct to the previous hypothesis it is postulated that there is a positive relationship between the percentage of farmland under crops and the pattern of urban places.

Fourthly, the pattern of urban places is hypothesized as being influenced by the purchasing power of the rural population; the greater the purchasing power the more uniform should be the pattern of the places that service the area. Assuming a uniform distribution of the rural population across the province, in poorer areas the urban places are likely to be located in positions which favour the provision of services to a larger area than if the purchasing power was high, because the population can support fewer centres. The geographic centre of a market area may not prove to be the most favourable position for a service centre in an area of low purchasing power, whilst a location on a major transport artery, which facilitates the movement of people, might well be more advantageous. Thus, if this consideration affects a number of towns the visual impression is one of towns arranged along lines, which results in a low value of the pattern statistic. For the purposes of the regression analysis the capital value of farms per square mile was used as an index of rural purchasing power.

Fifthly, it is hypothesized that the greater the density of the rural farm population the more uniform will be the pattern of urban places required to service the area. If the density of the population is low the threshold population required for the offering of a good, or service, may not be reached within the range of the good if the towns are arranged uniformly. Such a situation results in the demise of certain centres and the expansion of those most favourably located for the servicing of a large area. Similar factors to those mentioned in the previous hypothesis might well be important in such a situation.

Sixthly, the extent to which the pattern of urban places trends towards uniformity may well reflect the percentage of the total population that is classified as 'rural farm'. If the value of this criterion is low because of the effect of employment in industries other than agriculture, the pattern of places may be determined by factors other than that of the efficient provision of goods and services to a rural population. It has been noted that the lower the proportion of agricultural population to the total population of an area the more clustered are the urban places due to the economies derived from proximity in a manufacturing economy (Lösch, 1954, p. 438).

In Chapter III it was postulated that the value of R was low for the overall pattern because of the control of railway routes in the period of settlement formation. It is hypothesized that the greater the density of rail routes the more uniform will be the pattern of urban places. As the density of rail routes increases the uniform transport surface assumption of the Central Place Model becomes much more feasible and, thus, one effect of rail transport as a controlling factor of settlement location may be disregarded.

In order to meet the assumption of regression analysis that the variables are normally distributed, certain of the variables required transformation as detailed in Table 5:1.

TABLE 5:1

VARIABLES USED IN MULTIVARIATE ANALYSIS

| Variable | Transformation | Subscript |
|---|---|---|
| Percent of Census Division in Farmland | Log | x(1) |
| Percent of Census Division under Crops | | x(2) |
| Percent of Farmland under Crops | Log | x(3) |
| Capital value of Farms p.s. mile | | x(4) |
| Density of Rural Farm Population p.s. mile | Log | x(5) |
| Percent of Total Population Rural Farm | | x(6) |
| Miles of railways p.s. mile | Log | x(7) |
| Pattern Statistic R | | R |

TABLE 5:2

VALUES OF VARIABLES BY CENSUS DIVISIONS AND AGRICULTURAL REGIONS

| Census Division | | R | x(1) | x(2) | x(3) | x(4) | x(5) | x(6) | x(7) |
|---|---|---|---|---|---|---|---|---|---|
| Prarie | 1 | 1.2421 | 89.72 | 33.31 | 37.12 | 22202. | 2.55 | 39.09 | .08714 |
| | 2 | 1.1689 | 96.00 | 34.53 | 36.56 | 22682. | 2.07 | 41.04 | .08061 |
| | 3 | 1.1933 | 92.60 | 33.68 | 36.30 | 21915. | 1.90 | 51.49 | .04394 |
| | 4 | 1.0547 | 94.77 | 20.79 | 21.94 | 15561. | 1.22 | 51.73 | .03417 |
| | 6 | 1.1329 | 92.70 | 40.85 | 44.07 | 34774. | 3.13 | 13.79 | .08973 |
| | 7 | 1.0994 | 92.14 | 33.85 | 36.74 | 24284. | 2.00 | 24.46 | .06371 |
| | 8 | 1.1499 | 95.81 | 33.30 | 34.76 | 26110. | 1.58 | 35.57 | .04987 |
| | 11 | 1.2687 | 93.61 | 40.87 | 43.66 | 28828. | 2.52 | 11.98 | .07492 |
| | 12 | 1.3049 | 89.14 | 37.14 | 41.67 | 25572. | 2.27 | 48.09 | .08425 |
| | 13 | 1.1701 | 92.44 | 37.11 | 40.14 | 24660. | 2.31 | 47.98 | .08177 |
| Park | 5 | 1.3391 | 92.60 | 30.21 | 32.62 | 26960. | 3.80 | 48.30 | .08069 |
| | 9 | 1.3055 | 88.97 | 31.71 | 35.64 | 28677. | 4.64 | 46.55 | .07265 |
| | 10 | 1.1748 | 93.46 | 35.81 | 38.31 | 30005. | 3.29 | 56.20 | .04320 |
| | 14 | 0.8538 | 39.38 | 16.57 | 42.07 | 13872. | 1.85 | 45.61 | .02451 |
| | 15 | 1.2596 | 77.10 | 36.25 | 47.02 | 32448. | 4.12 | 40.35 | .04871 |
| | 16 | 1.1769 | 62.13 | 25.57 | 34.71 | 16258. | 2.40 | 44.88 | .02928 |
| | 17 | 1.0547 | 60.60 | 18.11 | 29.89 | 15345. | 1.86 | 44.71 | .02632 |

## Test of the Initial Hypotheses

In order to test the validity of the hypotheses each independent variable is separately related to the pattern statistic without reference to the effect of the other variables. Five of the seven independent variables are significantly related to the pattern statistic at the 95 per cent level of confidence (Table 5:3). The two variables that are rejected by the correlation analysis are the percentage of farmland under crops, x(3), and the percentage of the total population which is classified as rural farm. The significant variables each separately account for at least 37 per cent of the variation in the pattern statistic, the most important variable being the density of railways (which accounts for 47 per cent of the variation), supporting previous statements concerning the importance of railways in the location of settlements in Saskatchewan.

From the matrix of cross correlations (Table 5:4) it can be seen that a number of the independent variables have a high degree of relationship with one another. If these cross-correlations are taken into account 72.29 per cent of the variation in the pattern statistic is explained by the regression equation:

$$R_c = -.40 + .75\, x(1) + .007\, x(2) - .000014\, x(4) + .663\, x(5)$$

The standard error of the estimated value of R using this equation is 0.0618. The density of railways, x(7), does not enter this equation because it is highly correlated with all the variables that are included.

## Expansion of the Analysis

The correlation analysis revealed that the variables associated with agriculture are significantly related with the pattern of urban

TABLE 5:3

RELATIONSHIP OF PATTERN STATISTIC 'R' TO THE INDEPENDENT VARIABLES

| Independent Variable | Simple Correlation Coefficient | Coefficient of Determination |
|---|---|---|
| x(1) | .6506* | .4231 |
| x(2) | .6533* | .4268 |
| x(3) | .2014 | .0405 |
| x(4) | .6125* | .3751 |
| x(5) | .6229* | .3880 |
| x(6) | -.0247 | .0006 |
| x(7) | .6742* | .4545 |

*Significant at the 95 per cent level of confidence.

TABLE 5:4

MATRIX OF CORRELATION COEFFICIENTS

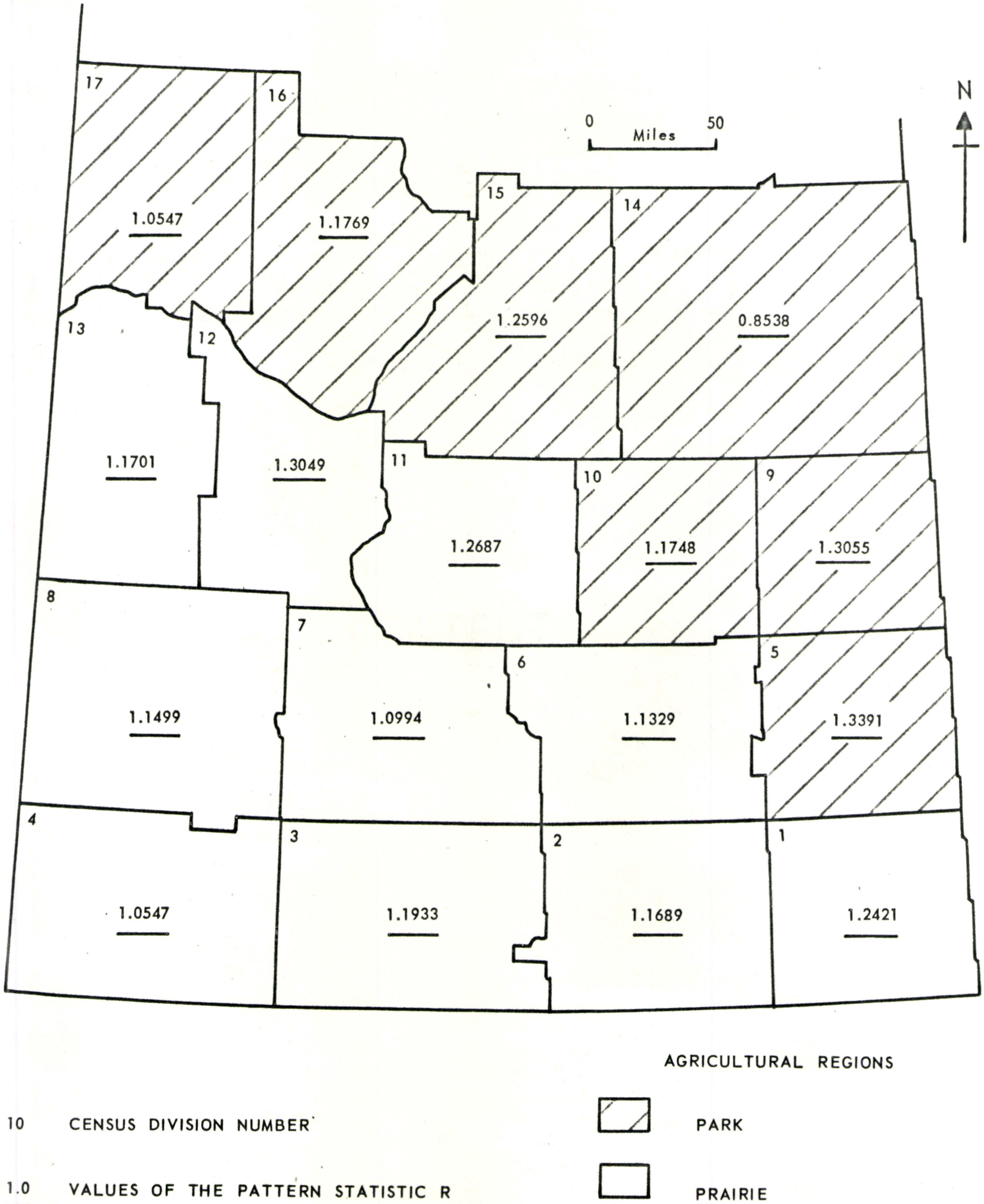
| | x(1) | x(2) | x(3) | x(4) | x(5) | x(6) | x(7) | R |
|---|---|---|---|---|---|---|---|---|
| x(1) | ..... | .722 | -.098 | .593 | .135 | -.166 | .717 | .650 |
| x(2) | | ..... | .605 | .858 | .444 | -.455 | .807 | .653 |
| x(3) | | | ..... | .577 | .538 | -.412 | .372 | .201 |
| x(4) | | | | ..... | .694 | -.415 | .682 | .612 |
| x(5) | | | | | ..... | -.063 | .427 | .623 |
| x(6) | | | | | | ..... | -.382 | -.025 |
| x(7) | | | | | | | ..... | .674 |
| R | | | | | | | | ..... |

places. In order to attempt an improvement of the level of prediction and the percentage of variation explained by the regression equation it was decided to divide the Census Divisions into groups based on the predominant type of agriculture. King (1961) found in a study of spacing that grouping the towns by agricultural regions revealed more information concerning the importance of certain of the independent variables which he utilised. One purpose of sub-dividing the original data into separate groups is to emphasize the variation in the rôle of variables under different conditions.

The Saskatchewan Royal Commission on Agriculture and Rural Life (1957, v. 12, pp. 20-22) divided the Census Divisions into two broad agricultural regions (Fig. 5:1). The ten Census Divisions in the south and west, the Prarie Region, are characterized by ranching in the extreme south-west, where rainfall and large areas of poor soil predominate, and, elsewhere by straight cereal production, mainly wheat. The seven Census Divisions that form the Park region have a mixed farming economy based on the production of the coarser grains and livestock.

In the Prairie region the population density is generally lower than the Saskatchewan average especially in the cattle raising areas, whilstit is higher in the Park region. The Royal Commission noted that the type of transactions differ markedly in the two areas and that this is reflected in the importance and nature of the service centres. In the Prairie region agricultural transactions predominate, whilst personal and household transactions are of greater importance in the Park region. The service centres in the Park region tend to be small and undifferentiated as to functional importance, whilst in the Prairie region there

FIG 5:1 CENSUS DIVISIONS



is a tendency for greater variability to exist in both size and functions of the centers. It is postulated that the hypotheses concerning the independent variables will be more applicable in the Park region, given the nature of the population distribution and the similarity of the settlements. In the Prairie region the degree of uniformity of the settlement pattern is probably associated with the amount of the total area under crops because of the requirements of a grain exporting economy as mentioned previously.

Table 5:5 details the correlation coefficients and the coefficients of determination derived from simple correlation analyses for each independent variable against the appropriate dependent variable for each agricultural region. As hypothesized the independent variables are, on the average, more highly correlated with the dependent variable in the Park region than in the Prairie Census Divisions. In the Park region the variables which are significantly related to the pattern statistic are those which are significant at the provincial level. However, the percentage of the variation which each of these variables explains, without reference to the effect of the other variables, has increased to at least 58 per cent. The most significant variable is the proportion of the total area in farmland, x(1). That is, the more uniform settlement patterns are associated with those areas where farming is widespread. The density of rural farm population, x(5), and the density of railways, x(7) each separately account for more than 75 per cent of the variation in the dependent variable, if they are correlated without regard to the effect of the other variables. However, in the Prairie region the only variable that is significantly correlated with the variation in the pattern of urban places is the percentage of farmland under crops, x(3), which is not a significant variable at the provincial level or in the Park region. This most probably reflects the demand of the grain cultivating areas for the regular location of collection and transhipment points for their produce.

TABLE 5:5

MULTIVARIATE ANALYSIS FOR AGRICULTURAL REGIONS

| | PARK | | | PRAIRIE | | |
|---|---|---|---|---|---|---|
| Variable | Corr. Coeff. | Coeff. of Det. | 'b' value | Corr. Coeff. | Coeff. of Det. | 'b' value |
| x(1) | .9104* | .8288 | 1.29 | -.5660 | .3203 | |
| x(2) | .7968* | .6348 | | .5877 | .3455 | |
| x(3) | -.1380 | .0190 | | .6435* | .4140 | .572 |
| x(4) | .7642* | .5840 | | .2812 | .0790 | |
| x(5) | .8752* | .7659 | | .5320 | .2830 | |
| x(6) | .0590 | .0034 | .0215 | -.0359 | .0012 | |
| x(7) | .8688* | .7548 | | .5549 | .3079 | |
| Pure Constant | = | -.288 | | | = | .283 |
| Multiple Corr. Coeff. | = | .9698* | | | = | .6435* |
| Variation explained | = | 94.06% | | | = | 41.41% |
| Standard Error of the Estimate | = | .049 | | | = | .062 |

*Significant at the 95 per cent level of Confidence

Table 5:5 also gives the 'b' values for the variables that are included in the regression equation for each agricultural region. In the Park region the major variable is the percentage of the total area in farmland. Although the simple correlation coefficient between the dependent variable and the percentage of the total population that is classified as rural farm is not significant, this variable is included in the regression equation, emphasizing the complexity of the inter-relationships between the variables used in this analysis. The only variable included in the regression equation for the Prairie region is the percentage of farmland in crops.

Approximately 94 per cent of the variation of the dependent variable in the Park region is explained by the regression equation whilst in the Prairie region 41 per cent of the variation is explained. Although both these equations reach a significant level of explanation, it would seem that to increase the ability to explain variations in the pattern of urban places it would be necessary to include further variables into the analysis.

Summary

The results of the multivariate analyses reveal the complexity of the distributive process for the location of urban places and also they allow statements to be made concerning certain of the reasons for areal variation in the pattern of urban places. The variation of phenomena concerned with the type and profitability of agriculture, in association with concomitant variation in rural farm population densities, would appear to be of prime importance in determining the nature of the pattern of urban settlement. The density of railway routes is also highly correlated with the pattern of settlement. The reason for its exclusion from the regression equations is the high correlation between this variable and those which are included. This is to be expected as the majority of railways were constructed to remove agricultural produce from those areas where grain and other cash crops were important. Settlements developed at the storage and transhipment points along these routes because they were the focus of economic activity for the surrounding area.

The level of explanation achieved by the analysis was high, although the inclusion of further variables might well be advantageous especially to increase the predictive ability of the regression equation in the Prairie region. One possible limitation of the analysis is that the data for all the variables utilised represent present day conditions, although it may seem more applicable to investigate the distributive process during the major period of settlement formation. However, the pattern of urban places has been modified through time by changes in technology and by the demands of the rural population for goods and services. Therefore, it would seem valid to relate the present pattern of urban places to other variables in order to discuss the relationships that exist between them at the moment as an indication of the distributive process in the past and present.

## CHAPTER VI

## CONCLUSIONS

Through the use of the nearest neighbour method of pattern analysis, it has been possible to give a readily understood value to the complex spatial patterns of many groups of places in Saskatchewan. By utilizing the pattern statistic calculated for a number of areas within the province, it has proved possible to construct isopleth maps which reveal the spatial variation within the overall pattern of settlement. The depiction of this source of variation has not previously been achieved in this manner by qualitative or other quantitative methods.

Several inadequacies in the nearest neighbour technique were revealed by the investigation. However, it is felt that these do not detract from its usefulness in the study. An observed pattern should not be classified solely on the basis of the derived pattern statistic - certain additional information is required. The calculation of the standard variate of the normal curve allows the assessment of the significance of the departure from randomness of the observed pattern. Also, the variance of the observed distances to the nearest neighbour is of value, because it indicates the spread of the values around the mean distance.

In Chapters III and IV, it was shown, by calculating the standard variate of the normal curve, that certain values of the pattern statistic do not represent a pattern that is significantly different from a random pattern, although the values of R that were obtained would, nominally, suggest otherwise. In a number of cases the pattern that was measured may well have been random. In other instances, because of a small number of measurements made, the pattern statistic was a poor indication of the nature of the pattern, because of the wide range of values of R which, statistically, must be regarded as representing a random pattern. In Chapter IV, it was shown that the range of these values was dependent upon the size of the area used to determine the group of towns for which the pattern statistic was calculated: the larger the area, or the greater the number of places included in the sample, the smaller was the range of values which indicate a random pattern.

To resolve this problem, in addition to the use of the standard variate of the normal curve, alternative and more powerful methods of assessing the departure from randomness would seem to be required. One such method would be to utilise the Poisson distribution to generate the frequency distribution of expected distances between nearest neighbours in a random pattern of the same density. This set of figures could then be compared to the frequency distribution of the observed distances by means of the Chi-square test of goodness of fit.

The use of the nearest neighbour method of pattern analysis has enabled the majority of observed patterns to be classified into two groups: more regular than random, or more clustered than random. The values of R also allowed the degree of departure from randomness to be assessed. In discussing the variations between different patterns, the pattern statistic proved to be a useful method of comparison; however, the reasons for variations between patterns were not revealed by the pattern statistic. The possible causes of variation were discussed by inspection of the pattern, and, in Chapter III, by reference to the relevant literature. In Chapter V, the observed areal variation was discussed by the utilization of multivariate analysis to suggest linkages between the nature of the pattern and possible factors within the distributive process.

The overall pattern of the 481 places in Saskatchewan is significantly different from a random pattern, though the degree of departure from randomness (towards uniformity) is slight. It is suggested that the traffic principle of Central Place Theory is applicable to a partial explanation of the overall settlement pattern in Saskatchewan, because of the influence of railway routes on the location of a great many urban places. The railway network induced a pronounced visual impression of linearity in the settlement pattern because of the establishment of marketing points at regular intervals along these lines of communication. However, the patterns of various sub-groups, determined on the basis of population size or location within the province, exhibit a wide range of values around a random pattern.

The pattern of those places greater than a certain arbitrary size increases towards uniformity as the smaller places are excluded from the determination of the pattern index. It is suggested by the author that the observed increase in the regularity of the pattern of places may possibly be due to the increasing importance of the marketing principle of Central Place Theory in the location of the larger places. Similarly, the pattern of urban places, which fall into certain arbitrary size classes, with respect to the overall pattern, increases towards uniformity the larger the average population-size of the group of towns.

The most clustered group of places in this analysis were those with a population size of less than 75 people, whilst the most regular group, with respect to the overall pattern, were the six towns with a population greater than 10,000. It would appear that one reason for this observed relationship may be the result of the differential abilities of centres of different sizes to be spatially dominant.

The isopleth maps, constructed from the multiple calculation of the pattern statistic for small areas within the province, indicate that there is areal variation in the overall pattern. Utilizing values of R calculated for the urban places in each of the seventeen Census Divisions, it was shown that the areal variation within the overall pattern can be related to the variation of other spatial variables. The most important factor in determining the nature of the pattern was the variation in the density of railways. Railways were constructed for two main purposes within the province: firstly, certain of the routes form part of a trans-Canada system and, secondly, local branch lines were built to service potential agricultural land. As stated previously, the railway network preceded the development of the majority of the towns, whose location was influenced by the prior location of the communication network. Because a large proportion of the railway mileage in the province was constructed to serve areas which had agricultural potential, a certain amount of the variation in the pattern of urban places can also be linked to the variations within the agricultural economy that has developed.

During the investigation several inadequacies were found in the methodology which may have affected the results to some extent. Throughout the study the overall pattern is regarded as being composed of 481 places, that is, those listed in the Census, although there are other places within the Province for which there is no information in the Census. If these places had been included, the nature of the pattern may well have been different. The rejection of the hypotheses derived from Central Place Theory in Chapter III may have been due to the use of arbitrary divisions of the population size continuum. Further work in this field most probably would require an initial investigation into the functional characteristics of the urban places under consideration. This would enable: firstly, the patterns of groups of towns selected on the basis of functional similarity to be studied and, secondly, it would provide additional information for the explanation of any differences between the patterns of various groups. The replacement of population size by a functional classification would thus improve our understanding of the relationships between towns.

The method of analysis could well be extended to consider the pattern to more than the first nearest neighbour in order to improve our knowledge concerning the geometry of a system of towns. The investigation of the extent to which pairs of towns are each other's nearest neighbours would not only further the description of the spatial structure, but it may also provide information concerning the functional dependence and/or the division of functions between places.

This study was essentially static in its approach to the analysis of the patterns considered, in that the data were applicable to a specific moment in time, that is, 1961. The stability through time of the relationships shown by the curves derived from the investigations concerned with the population-size continuum is one topic that merits further attention The complexity of the distributive process requires further investigation and this also should include recognition of time-dependent elements.

APPENDIX A

TABLES OF "R" VALUES FOR THE

ISOPLETH MAPS IN CHAPTER IV

# RESULTS FROM 50 MILE RADIUS FILTER

| COORDINATES EAST | NORTH | R | AREA | NO OF TOWNS | Z |
|---|---|---|---|---|---|
| 20.0 | 16.0 | 1.2589 | 2284.72 | 4 | 0.9905 |
| 20.0 | 48.0 | 1.1902 | 4296.88 | 8 | 1.0293 |
| 20.0 | 80.0 | 1.0351 | 4167.00 | 11 | 0.2230 |
| 20.0 | 112.0 | 1.0587 | 4167.00 | 15 | 0.4351 |
| 20.0 | 144.0 | 1.2231 | 4167.00 | 20 | 1.9088 |
| 20.0 | 176.0 | 1.4297 | 4167.00 | 24 | 4.0274 |
| 20.0 | 208.0 | 1.0142 | 4167.00 | 21 | 0.1247 |
| 20.0 | 240.0 | 1.0235 | 4095.66 | 19 | 0.1963 |
| 20.0 | 272.0 | 0.9095 | 2083.50 | 10 | -0.5476 |
| 52.0 | 16.0 | 0.9053 | 4352.17 | 10 | -0.5730 |
| 52.0 | 48.0 | 0.9555 | 7838.89 | 17 | -0.3511 |
| 52.0 | 80.0 | 1.1077 | 8007.00 | 21 | 0.9438 |
| 52.0 | 112.0 | 1.0154 | 8007.00 | 27 | 0.1526 |
| 52.0 | 144.0 | 1.1043 | 8007.00 | 35 | 1.1807 |
| 52.0 | 176.0 | 1.0335 | 8007.00 | 33 | 0.3685 |
| 52.0 | 208.0 | 1.0658 | 8007.00 | 36 | 0.7557 |
| 52.0 | 240.0 | 0.9824 | 7490.22 | 27 | -0.1746 |
| 52.0 | 272.0 | 0.9207 | 4003.50 | 14 | -0.5674 |
| 84.0 | 16.0 | 1.0392 | 4311.00 | 13 | 0.2702 |
| 84.0 | 48.0 | 1.0940 | 8151.00 | 25 | 0.8990 |
| 84.0 | 80.0 | 1.2618 | 7854.00 | 26 | 2.5537 |
| 84.0 | 112.0 | 1.1062 | 7854.00 | 26 | 1.0363 |
| 84.0 | 144.0 | 1.1597 | 7854.00 | 36 | 1.8334 |
| 84.0 | 176.0 | 1.0952 | 7854.00 | 44 | 1.2077 |
| 84.0 | 208.0 | 1.0811 | 7854.00 | 34 | 0.9044 |
| 84.0 | 240.0 | 0.9699 | 7767.00 | 25 | -0.2879 |
| 84.0 | 272.0 | 1.1540 | 3927.00 | 11 | 0.9774 |
| 116.0 | 16.0 | 1.1770 | 4311.00 | 14 | 1.2670 |
| 116.0 | 48.0 | 1.2243 | 8151.00 | 25 | 2.1455 |
| 116.0 | 80.0 | 1.2402 | 7854.00 | 33 | 2.6402 |
| 116.0 | 112.0 | 1.2330 | 7854.00 | 36 | 2.6749 |
| 116.0 | 144.0 | 1.1159 | 7854.00 | 37 | 1.3486 |
| 116.0 | 176.0 | 1.2427 | 7854.00 | 44 | 3.0801 |
| 116.0 | 208.0 | 1.1129 | 7854.00 | 35 | 1.2773 |
| 116.0 | 240.0 | 1.1466 | 7767.00 | 29 | 1.5099 |
| 116.0 | 272.0 | 1.2036 | 3927.00 | 10 | 1.2318 |
| 148.0 | 16.0 | 1.2054 | 4311.00 | 15 | 1.5222 |
| 148.0 | 48.0 | 1.1278 | 8151.00 | 27 | 1.2707 |
| 148.0 | 80.0 | 1.1793 | 7854.00 | 36 | 2.0578 |
| 148.0 | 112.0 | 1.1785 | 7854.00 | 42 | 2.2131 |
| 148.0 | 144.0 | 1.1695 | 7854.00 | 46 | 2.1995 |
| 148.0 | 176.0 | 1.2500 | 7854.00 | 41 | 3.0628 |
| 148.0 | 208.0 | 1.3867 | 7854.00 | 39 | 4.6199 |
| 148.0 | 240.0 | 1.2231 | 7767.00 | 30 | 2.3378 |
| 148.0 | 272.0 | 1.1884 | 3927.00 | 11 | 1.1951 |
| 180.0 | 16.0 | 1.2637 | 4311.00 | 17 | 2.0800 |
| 180.0 | 48.0 | 1.1714 | 8151.00 | 31 | 1.8253 |
| 180.0 | 80.0 | 1.1351 | 7854.00 | 37 | 1.5716 |
| 180.0 | 112.0 | 1.1919 | 7854.00 | 47 | 2.5164 |
| 180.0 | 144.0 | 1.1001 | 7854.00 | 45 | 1.2852 |
| 180.0 | 176.0 | 1.1236 | 7854.00 | 39 | 1.4766 |

50 MILE FILTER CONTINUED/

| COORDINATES EAST | NORTH | R | AREA | NO OF TOWNS | Z |
|---|---|---|---|---|---|
| 180.0 | 208.0 | 1.2307 | 7854.00 | 37 | 2.6846 |
| 180.0 | 240.0 | 1.1737 | 7767.00 | 24 | 1.6278 |
| 180.0 | 272.0 | 0.8984 | 3927.00 | 11 | -0.6445 |
| 212.0 | 16.0 | 1.2359 | 4311.00 | 22 | 2.1164 |
| 212.0 | 48.0 | 1.1528 | 8151.00 | 35 | 1.7292 |
| 212.0 | 80.0 | 1.1375 | 7854.00 | 45 | 1.7649 |
| 212.0 | 112.0 | 1.0746 | 7854.00 | 51 | 1.0189 |
| 212.0 | 144.0 | 1.1384 | 7854.00 | 48 | 1.8349 |
| 212.0 | 176.0 | 1.1878 | 7854.00 | 38 | 2.2144 |
| 212.0 | 208.0 | 1.1342 | 7854.00 | 30 | 1.4059 |
| 212.0 | 240.0 | 1.1253 | 7767.00 | 26 | 1.2220 |
| 212.0 | 272.0 | 0.8547 | 3927.00 | 13 | -1.0022 |
| 244.0 | 16.0 | 1.3359 | 4311.00 | 21 | 2.9446 |
| 244.0 | 48.0 | 1.2191 | 8151.00 | 39 | 2.6175 |
| 244.0 | 80.0 | 1.1275 | 7854.00 | 40 | 1.5421 |
| 244.0 | 112.0 | 1.1579 | 7854.00 | 49 | 2.1144 |
| 244.0 | 144.0 | 1.1454 | 7854.00 | 48 | 1.9274 |
| 244.0 | 176.0 | 1.1609 | 7854.00 | 36 | 1.8466 |
| 244.0 | 208.0 | 1.1165 | 7854.00 | 22 | 1.0452 |
| 244.0 | 240.0 | 0.9090 | 7767.00 | 18 | -0.7382 |
| 244.0 | 272.0 | 0.8928 | 3927.00 | 11 | -0.6799 |
| 276.0 | 16.0 | 1.2387 | 3576.88 | 22 | 2.1415 |
| 276.0 | 48.0 | 1.0826 | 6510.64 | 27 | 0.8211 |
| 276.0 | 80.0 | 1.1273 | 6567.00 | 33 | 1.3992 |
| 276.0 | 112.0 | 1.2402 | 6567.00 | 39 | 2.8697 |
| 276.0 | 144.0 | 1.2613 | 6567.00 | 42 | 3.2397 |
| 276.0 | 176.0 | 1.2699 | 6567.00 | 27 | 2.6827 |
| 276.0 | 208.0 | 0.9208 | 6567.00 | 14 | -0.5669 |
| 276.0 | 240.0 | 0.6962 | 6217.26 | 5 | -1.2995 |
| 276.0 | 272.0 | 0.0000 | 3283.50 | 1 | 0.0000 |

RESULTS FROM 60 MILE RADIUS FILTER

| COORDINATES EAST | NORTH | R | AREA | NO OF TOWNS | Z |
|---|---|---|---|---|---|
| 20.0 | 16.0 | 1.3055 | 3093.84 | 7 | 1.5461 |
| 20.0 | 48.0 | 1.1842 | 4757.84 | 14 | 1.3187 |
| 20.0 | 80.0 | 1.1502 | 5854.88 | 19 | 1.2525 |
| 20.0 | 112.0 | 1.2069 | 5854.88 | 22 | 1.8562 |
| 20.0 | 144.0 | 1.2479 | 5854.88 | 28 | 2.5094 |
| 20.0 | 176.0 | 1.1917 | 5854.88 | 34 | 2.1384 |
| 20.0 | 208.0 | 1.1644 | 5854.88 | 30 | 1.7225 |
| 20.0 | 240.0 | 1.0290 | 4591.44 | 23 | 0.2657 |
| 20.0 | 272.0 | 0.9452 | 2927.44 | 13 | -0.3783 |
| 52.0 | 16.0 | 0.9832 | 4796.24 | 11 | -0.1066 |
| 52.0 | 48.0 | 1.2446 | 7484.24 | 22 | 2.1950 |
| 52.0 | 80.0 | 1.1677 | 9054.88 | 29 | 1.7273 |
| 52.0 | 112.0 | 1.1331 | 9054.88 | 33 | 1.4631 |
| 52.0 | 144.0 | 1.2417 | 9054.88 | 46 | 3.1358 |
| 52.0 | 176.0 | 1.1619 | 9054.88 | 48 | 2.1458 |
| 52.0 | 208.0 | 1.0821 | 9054.88 | 47 | 1.0766 |
| 52.0 | 240.0 | 1.0565 | 7215.44 | 35 | 0.6393 |
| 52.0 | 272.0 | 1.1717 | 4527.44 | 17 | 1.3540 |
| 84.0 | 16.0 | 1.1390 | 5974.88 | 19 | 1.1588 |
| 84.0 | 48.0 | 1.1881 | 9174.88 | 33 | 2.0676 |
| 84.0 | 80.0 | 1.1176 | 11309.76 | 35 | 1.3305 |
| 84.0 | 112.0 | 1.1656 | 11309.76 | 41 | 2.0290 |
| 84.0 | 144.0 | 1.1186 | 11309.76 | 54 | 1.6677 |
| 84.0 | 176.0 | 1.1339 | 11309.76 | 58 | 1.9506 |
| 84.0 | 208.0 | 1.0582 | 11309.76 | 55 | 0.8261 |
| 84.0 | 240.0 | 1.1051 | 8854.88 | 34 | 1.1723 |
| 84.0 | 272.0 | 1.1039 | 5654.88 | 15 | 0.7702 |
| 116.0 | 16.0 | 1.1514 | 5974.88 | 21 | 1.3269 |
| 116.0 | 48.0 | 1.2280 | 9174.88 | 35 | 2.5806 |
| 116.0 | 80.0 | 1.2197 | 11309.76 | 47 | 2.8812 |
| 116.0 | 112.0 | 1.1592 | 11309.76 | 53 | 2.2176 |
| 116.0 | 144.0 | 1.0691 | 11309.76 | 51 | 0.9435 |
| 116.0 | 176.0 | 1.2946 | 11309.76 | 65 | 4.5442 |
| 116.0 | 208.0 | 1.2064 | 11309.76 | 51 | 2.8197 |
| 116.0 | 240.0 | 1.3070 | 8854.88 | 39 | 3.6681 |
| 116.0 | 272.0 | 1.0522 | 5654.88 | 13 | 0.3601 |
| 148.0 | 16.0 | 1.0932 | 5974.88 | 22 | 0.8362 |
| 148.0 | 48.0 | 1.3650 | 9174.88 | 36 | 4.1900 |
| 148.0 | 80.0 | 1.1677 | 11309.76 | 51 | 2.2905 |
| 148.0 | 112.0 | 1.1693 | 11309.76 | 62 | 2.5508 |
| 148.0 | 144.0 | 1.1809 | 11309.76 | 67 | 2.8334 |
| 148.0 | 176.0 | 1.2645 | 11309.76 | 62 | 3.9847 |
| 148.0 | 208.0 | 1.2869 | 11309.76 | 53 | 3.9958 |
| 148.0 | 240.0 | 1.2794 | 8854.88 | 37 | 3.2518 |
| 148.0 | 272.0 | 1.2859 | 5654.88 | 18 | 2.3208 |
| 180.0 | 16.0 | 1.3205 | 5974.88 | 21 | 2.8098 |
| 180.0 | 48.0 | 1.2841 | 9174.88 | 39 | 3.3940 |
| 180.0 | 80.0 | 1.1820 | 11309.76 | 56 | 2.6062 |
| 180.0 | 112.0 | 1.0811 | 11309.76 | 58 | 1.1820 |
| 180.0 | 144.0 | 1.1239 | 11309.76 | 72 | 2.0114 |
| 180.0 | 176.0 | 1.2060 | 11309.76 | 57 | 2.9751 |
| 180.0 | 208.0 | 1.2586 | 11309.76 | 53 | 3.6023 |

60 MILE FILTER CONTINUED/

| COORDINATES EAST | NORTH | R | AREA | NO OF TOWNS | Z |
|---|---|---|---|---|---|
| 180.0 | 240.0 | 1.2172 | 8854.88 | 39 | 2.5946 |
| 180.0 | 272.0 | 1.1617 | 5654.88 | 20 | 1.3835 |
| 212.0 | 16.0 | 1.3113 | 5974.88 | 28 | 3.1511 |
| 212.0 | 48.0 | 1.2921 | 9174.88 | 45 | 3.7486 |
| 212.0 | 80.0 | 1.1334 | 11309.76 | 64 | 2.0413 |
| 212.0 | 112.0 | 1.1477 | 11309.76 | 71 | 2.3815 |
| 212.0 | 144.0 | 1.0864 | 11309.76 | 71 | 1.3924 |
| 212.0 | 176.0 | 1.1282 | 11309.76 | 51 | 1.7510 |
| 212.0 | 208.0 | 1.1516 | 11309.76 | 45 | 1.9453 |
| 212.0 | 240.0 | 1.0172 | 8854.88 | 31 | 0.1834 |
| 212.0 | 272.0 | 0.8747 | 5654.88 | 21 | -1.0988 |
| 244.0 | 16.0 | 1.2760 | 5974.88 | 31 | 2.9402 |
| 244.0 | 48.0 | 1.3238 | 9174.88 | 52 | 4.4669 |
| 244.0 | 80.0 | 1.1559 | 11309.76 | 59 | 2.2915 |
| 244.0 | 112.0 | 1.2328 | 11309.76 | 66 | 3.6184 |
| 244.0 | 144.0 | 1.1870 | 11309.76 | 62 | 2.8164 |
| 244.0 | 176.0 | 1.0970 | 11309.76 | 51 | 1.3252 |
| 244.0 | 208.0 | 1.0440 | 11309.76 | 36 | 0.5055 |
| 244.0 | 240.0 | 0.9105 | 8854.88 | 22 | -0.8035 |
| 244.0 | 272.0 | 0.8095 | 5654.88 | 12 | -1.2623 |
| 276.0 | 16.0 | 1.2330 | 4157.84 | 25 | 2.2290 |
| 276.0 | 48.0 | 1.3384 | 6461.84 | 40 | 4.0947 |
| 276.0 | 80.0 | 1.3156 | 7854.88 | 47 | 4.1386 |
| 276.0 | 112.0 | 1.2316 | 7854.88 | 50 | 3.1325 |
| 276.0 | 144.0 | 1.2729 | 7854.88 | 50 | 3.6921 |
| 276.0 | 176.0 | 1.2089 | 7854.88 | 34 | 2.3299 |
| 276.0 | 208.0 | 1.0163 | 7854.88 | 21 | 0.1428 |
| 276.0 | 240.0 | 0.9594 | 6231.44 | 10 | -0.2455 |
| 276.0 | 272.0 | 1.8180 | 3927.44 | 2 | 2.2130 |

RESULTS FROM 70 MILE RADIUS FILTER

| COORDINATES EAST | NORTH | R | AREA | NO OF TOWNS | Z |
|---|---|---|---|---|---|
| 20.0 | 16.0 | 1.0670 | 4158.19 | 10 | 0.4051 |
| 20.0 | 48.0 | 1.1347 | 6088.86 | 15 | 0.9978 |
| 20.0 | 80.0 | 1.0398 | 7930.25 | 24 | 0.3732 |
| 20.0 | 112.0 | 1.2156 | 7930.25 | 31 | 2.2962 |
| 20.0 | 144.0 | 1.1778 | 7930.25 | 36 | 2.0403 |
| 20.0 | 176.0 | 1.1943 | 7930.25 | 41 | 2.3797 |
| 20.0 | 208.0 | 1.0141 | 7930.25 | 40 | 0.1710 |
| 20.0 | 240.0 | 1.0628 | 5895.79 | 31 | 0.6689 |
| 20.0 | 272.0 | 1.1124 | 3965.13 | 16 | 0.8598 |
| 52.0 | 16.0 | 1.0791 | 6127.26 | 17 | 0.6238 |
| 52.0 | 48.0 | 1.0951 | 11384.65 | 30 | 0.9961 |
| 52.0 | 80.0 | 1.1589 | 13408.92 | 37 | 1.8492 |
| 52.0 | 112.0 | 1.1413 | 13408.92 | 48 | 1.8732 |
| 52.0 | 144.0 | 1.1734 | 13408.92 | 57 | 2.5046 |
| 52.0 | 176.0 | 1.0461 | 13408.92 | 60 | 0.6831 |
| 52.0 | 208.0 | 1.0184 | 13408.92 | 63 | 0.2794 |
| 52.0 | 240.0 | 0.9802 | 10959.18 | 44 | -0.2506 |
| 52.0 | 272.0 | 0.9423 | 6704.46 | 22 | -0.5178 |
| 84.0 | 16.0 | 1.0817 | 8234.52 | 23 | 0.7498 |
| 84.0 | 48.0 | 1.1379 | 13610.52 | 43 | 1.7299 |
| 84.0 | 80.0 | 1.1479 | 15393.84 | 49 | 1.9803 |
| 84.0 | 112.0 | 1.0824 | 15393.84 | 63 | 1.2518 |
| 84.0 | 144.0 | 1.1490 | 15393.84 | 72 | 2.4193 |
| 84.0 | 176.0 | 1.0822 | 15393.84 | 75 | 1.3626 |
| 84.0 | 208.0 | 1.1264 | 15393.84 | 71 | 2.0372 |
| 84.0 | 240.0 | 1.0904 | 13072.92 | 54 | 1.2708 |
| 84.0 | 272.0 | 1.0967 | 7696.92 | 28 | 0.9790 |
| 116.0 | 16.0 | 1.2177 | 8234.52 | 29 | 2.2424 |
| 116.0 | 48.0 | 1.1535 | 13610.52 | 41 | 1.8804 |
| 116.0 | 80.0 | 1.1763 | 15393.84 | 57 | 2.5465 |
| 116.0 | 112.0 | 1.2129 | 15393.84 | 69 | 3.3825 |
| 116.0 | 144.0 | 1.1694 | 15393.84 | 73 | 2.7695 |
| 116.0 | 176.0 | 1.1635 | 15393.84 | 82 | 2.8332 |
| 116.0 | 208.0 | 1.2192 | 15393.84 | 68 | 3.4581 |
| 116.0 | 240.0 | 1.2126 | 13072.92 | 49 | 2.8475 |
| 116.0 | 272.0 | 1.2715 | 7696.92 | 22 | 2.4366 |
| 148.0 | 16.0 | 1.1725 | 8070.25 | 29 | 1.7775 |
| 148.0 | 48.0 | 1.2233 | 11803.59 | 51 | 3.0507 |
| 148.0 | 80.0 | 1.1876 | 15393.84 | 73 | 3.0667 |
| 148.0 | 112.0 | 1.1495 | 15393.84 | 79 | 2.5417 |
| 148.0 | 144.0 | 1.2016 | 15393.84 | 88 | 3.6171 |
| 148.0 | 176.0 | 1.2288 | 15393.84 | 84 | 4.0110 |
| 148.0 | 208.0 | 1.2004 | 15393.84 | 73 | 3.2752 |
| 148.0 | 240.0 | 1.3517 | 11430.25 | 49 | 4.7098 |
| 148.0 | 272.0 | 1.2101 | 7696.92 | 26 | 2.0491 |
| 180.0 | 16.0 | 1.2551 | 8070.25 | 32 | 2.7603 |
| 180.0 | 48.0 | 1.3183 | 11803.59 | 54 | 4.4743 |
| 180.0 | 80.0 | 1.1782 | 15393.84 | 78 | 3.0104 |
| 180.0 | 112.0 | 1.1339 | 15393.84 | 82 | 2.3202 |
| 180.0 | 144.0 | 1.1503 | 15393.84 | 93 | 2.7730 |
| 180.0 | 176.0 | 1.2162 | 15393.84 | 73 | 3.5338 |
| 180.0 | 208.0 | 1.1164 | 15393.84 | 63 | 1.7678 |

70 MILE FILTER CONTINUED/

| COORDINATES EAST | NORTH | R | AREA | NO OF TOWNS | Z |
|---|---|---|---|---|---|
| 180.0 | 240.0 | 1.1924 | 11430.25 | 53 | 2.6795 |
| 180.0 | 272.0 | 1.2246 | 7696.92 | 31 | 2.3919 |
| 212.0 | 16.0 | 1.2654 | 8070.25 | 37 | 3.0882 |
| 212.0 | 48.0 | 1.3359 | 11803.59 | 59 | 4.9363 |
| 212.0 | 80.0 | 1.1321 | 15393.84 | 81 | 2.2751 |
| 212.0 | 112.0 | 1.1960 | 15393.84 | 95 | 3.6538 |
| 212.0 | 144.0 | 1.1769 | 15393.84 | 95 | 3.2994 |
| 212.0 | 176.0 | 1.1299 | 15393.84 | 76 | 2.1671 |
| 212.0 | 208.0 | 1.1262 | 15393.84 | 62 | 1.9005 |
| 212.0 | 240.0 | 1.0856 | 11430.25 | 44 | 1.0856 |
| 212.0 | 272.0 | 0.9401 | 7696.92 | 24 | -0.5611 |
| 244.0 | 16.0 | 1.2522 | 7357.93 | 39 | 3.0127 |
| 244.0 | 48.0 | 1.3223 | 10952.59 | 61 | 4.8153 |
| 244.0 | 80.0 | 1.2176 | 13996.92 | 82 | 3.7688 |
| 244.0 | 112.0 | 1.1735 | 13996.92 | 85 | 3.0595 |
| 244.0 | 144.0 | 1.2377 | 13996.92 | 86 | 4.2179 |
| 244.0 | 176.0 | 1.1077 | 13996.92 | 62 | 1.6229 |
| 244.0 | 208.0 | 1.1142 | 13996.92 | 52 | 1.5753 |
| 244.0 | 240.0 | 1.0615 | 10593.13 | 30 | 0.6448 |
| 244.0 | 272.0 | 0.9389 | 6998.46 | 18 | -0.4960 |
| 276.0 | 16.0 | 1.2989 | 5388.86 | 29 | 3.0792 |
| 276.0 | 48.0 | 1.2758 | 7959.53 | 45 | 3.5392 |
| 276.0 | 80.0 | 1.2475 | 10263.59 | 62 | 3.7278 |
| 276.0 | 112.0 | 1.2597 | 10263.59 | 61 | 3.8810 |
| 276.0 | 144.0 | 1.2592 | 10263.59 | 67 | 4.0588 |
| 276.0 | 176.0 | 1.2207 | 10263.59 | 46 | 2.8638 |
| 276.0 | 208.0 | 1.0059 | 10263.59 | 27 | 0.0584 |
| 276.0 | 240.0 | 0.9873 | 7702.46 | 12 | -0.0841 |
| 276.0 | 272.0 | 1.0022 | 5131.79 | 7 | 0.0114 |

APPENDIX B

PROGRAMS USED IN THE STUDY

```
C       PATTERNS OF GROUPS ABOVE CERTAIN SIZES
C
        DIMENSION ANO(481),EA(481), DIST(481),HPOP(7), N(7)
        WRITE(6,13)
   13 FORMAT(1H0,    10X,     95HMIN.POP   TOWN   DENSITY    SUMDIS      SD
     1IS2   MEAN DIST   DIST EXP   PATTERN     SDREXPE   NORM DEV    )
C
C N = NO OF POINTS
        READ(5,10)(HPOP(J),N(J),J = 1,7)
   10 FORMAT(F10.0,I5)
        DATA AREA/122466./
        READ(5,15)EA(J),ANO(J), J = 1,481)
   15 FORMAT(20X,2F10.1)
C
        DO 70 L = 1,7
        TOWN = N(L)
        NN = N(L)
        HHPOP = HPOP(L)
        SUMDIS = 0.0
        SDIS2 =0.0
C
        DO 50 K = 1, NN
        DIST(K)  =  9999.
        DO 40 J = 1, NN
        IF(J.EQ.K) GO TO 40
C NOTE SQUARED DISTANCES USED TO COMPARE
        DIS = (ANO(K)-ANO(J))**2 +(EA(K)-EA(J))**2
        IF(DIS.LT.DIST(K)) DIST(K)=DIS
   40 CONTINUE
        DIST(K) = DIST(K)*1.44
        SDIS2 = SDIS2 + DIST(K)
        DIST(K) = DIST(K) **.5
        SUMDIS = SUMDIS+ DIST(K)
   50 CONTINUE
C CALCULATE THE CLARK AND EVANS PARAMETERS AND TABULATE THEM
        DENS = TOWN/AREA
        REXPE = .5/(DENS**.5)
        SDREXP = 0.26136/(TOWN*DENS)**.5
        ROBS = SUMDIS/TOWN
        PAT = ROBS/REXPE
        C = (ROBS-REXPE)/SDREXP
        WRITE(6,61)HHPOP,TOWN,DENS,SUMDIS,SDIS2,ROBS,REXPE,PAT,SDREXP,C
   61 FORMAT(1H ,10X, F10.4,F5.0,F10.8,4F10.1,F10.3,F10.7,F10.5)
   70 CONTINUE
        STOP
        END
```

```
C     PATTERN OF STRATIFIED SAMPLES  (AND BELOW POINTS IN CONTINUUM)
C
      DIMENSION  EAS(481), ANO(481), N(9), IPOP(9),DIST(481)
      DENS = 481./122466.
      REXPE = .5/(DENS**.5)
      SDREXP = 0.26136/((481.*DENS)**.5)
      WRITE(6,16) DENS,REXPE,SDREXP
   16 FORMAT(1H1, 16HDENSITY OF TOWNS,F10.7, 1X, 9HREXPECTED, F10.2,
     11X, 11HSDREXPECTED, F10.7)
      WRITE(6,10)
   10 FORMAT( 1H0, 15X,38HPATTERN OF TOWNS BETWEEN CERTAIN SIZES,18X, 36
     1HPATTERN OF TOWNS BELOW CERTAIN SIZES)
      WRITE(6,11)
   11 FORMAT(1H0, 114HMIN POP MAX POP NUMBER MEAN DIST   PATTERN   Z
     1 SUMDIS       SDIS2     LIMIT MEAN DIST    PATTERN    Z       SUMDIS    )
C READ IN NUMBER OF PLACES BELOW EACH CUTOFF POINT IN GROUPS
      READ(5,14) (N(L),IPOP(L), L = 1,9)
   14 FORMAT(2I10)
      READ(5,15)(EAS(K),ANO(K), K =1,481)
   15 FORMAT(20X,2F10.1)
      SUM = 0.
      IPIP = 1
      L = 1
   18 NN = 1
      M = N(1)
      GO TO 19
   17 NN = NN + N(L)
      L = L + 1
      M = NN + N(L) - 1
   19 SUMDIS = 0.
      SDIS2 = 0.
      DO 21 I = NN,M
      DIST(I) = 9999.
      DO 20 K = 1,481
      IF(I.EQ.K) GO TO 20
      SOUTH = ANO(I)- ANO(K)
      EAST = EAS(I)- EAS(K)
      DIS = SOUTH*SOUTH + EAST*EAST
      IF(DIS.LT.DIST(I)) DIST(I) = DIS
   20 CONTINUE
      DIST(I) = DIST(I)*1.44
      SDIS2 = SDIS2 + DIST(I)
      DIST(I) =  DIST(I)**.5
      SUMDIS = SUMDIS + DIST(I)
   21 CONTINUE
      ROBS = SUMDIS/FLOAT(N(L))
      R = ROBS/REXPE
      C = (ROBS-REXPE)/SDREXP
      SUM = SUM + SUMDIS
      ROBSO = SUM/FLOAT(M)
      R2 = ROBSO/ REXPE
      C2 = (ROBSO-REXPE)/SDREXP
      WRITE(6,22) IPIP,    IPOP(L),N(L),ROBS,R,C,SUMDIS,SDIS2,IPOP(L),
     1ROBSO,R2,C2,SUM
   22 FORMAT(1H0, I7,I8,I7,2F10.4,F7.4,2F10.3,I8,2F10.4, F7.4,F10.3)
      IF(L.EQ.9) STOP
      IPIP = IPOP(L)
      GO TO 17
      END
```

```
C                  PATTERN IN CENSUS DIVISIONS
C
      DIMENSION EA(481), ANO(481), BOUND(481), TIT(12), DIST(481)
      READ(5,1)(TIT(J),J=1,12)
    1 FORMAT(12A6)
      WRITE(6,2)(TIT(J),J=1,12)
    2 FORMAT(1H1, 10X, 12A6)
      WRITE(6,36)
   36 FORMAT(1H0, 120H TOWN   DENSITY   SDREXPE     REXPE     SDIS2
     1 ROBS   PATTERN  NORM DEV  BOUNDARY  NO.      ROBS   PATTERN  NORM
     2 DEV      )
C USE A COUNTER FOR CENSUS DIVISIONS
      CD = 0.
  100 READ(5,3) N, AREA
    3 FORMAT( I5,F10.0)
      CD = CD + 1.
      READ(5,10) (EA(J),ANO(J),BOUND(J),J =1,N)
   10 FORMAT(20X, 2F10.0,10X,F10.0)
      TOWN    = N
      DENS    = TOWN/AREA
      REXPE = .5/(DENS**.5)
      SDREXP = 0.26136/((TOWN*DENS)**.5)
      SUMDIS = 0.0
      SUMD2 =0.0
      DO 20 K = 1,N
      DIST(K) = 999.
      DO 25 J = 1,N
      IF(J.EQ.K) GO TO 25
      EAST = ABS(EA(K)-EA(J))
      ORTH = ABS(ANO(K)-ANO(J))
      DIS = EAST*EAST + ORTH*ORTH
      IF(DIS.LT.DIST(K))DIST(K) = DIS
   25 CONTINUE
      DIST(K) = DIST(K)*1.44
      SUMD2 = SUMD2 + DIST(K)
      DIST(K) = DIST(K)**.5
      SUMDIS = SUMDIS + DIST(K)
   20 CONTINUE
      ROBS = SUMDIS/TOWN
      PAT = ROBS/REXPE
      C = (ROBS-REXPE)/SDREXP
      RBOUN = 0.0
      ANOS = 0.
      DO 30 K = 1,N
      IF(DIST(K).GT.BOUND(K)) GO TO 30
      RBOUN = RBOUN + DIST(K)
      ANOS = ANOS + 1.
   30 CONTINUE
      RO = RBOUN/ANOS
      PATB = RO/REXPE
      CB = ( RO -REXPE)/SDREXP
      WRITE(6,35)    N,DENS,SDREXP, REXPE, SUMD2,ROBS,PAT,C,ANOS,RO,PATB
     1,CB
   35 FORMAT(1H0, I5,F10.7,4F10.1,F10.4,F10.3, 10X,F5.0,F10.1,2F10.4)
      IF(CD.EQ.17.) STOP
      GO TO 100
      END
```

```
C     PRODUCTION OF ISOPLETH MAPS OF PATTERN STATISTIC  (PART A)
C   PRODUCTION OF GRID POINTS
      DIMENSION GEAS(144), GNO(144)
C 20EAST AND 16NORTH = LOWER LEFT HAND GRID POINT
      L = 0
      DO 1 K = 1,12
      KK = K-1
      EAST = 20 + (32 * KK)
      IF(EAST.GT.298.) GO TO 3
      DO 1 J = 1,12
      JJ = J - 1
      NORTH = 16 +(32*JJ)
      IF(NORTH.GT.272) GO TO 1
      L = L + 1
      GNO(L) = NORTH
      GEAS(L) = EAST
    1 CONTINUE
    3 N = L
      WRITE(7,2)  (GEAS(L), GNO(L), L = 1,N)
    2 FORMAT(8F10.1)
      STOP
      END
```

```
C       PRODUCTION OF ISOPLETH MAPS OF PATTERN STATISTIC  (PART B)
C        PRODUCTION OF DISTANCE RANGES FOR PART C
        DIMENSION GEAS(81), GNO(81), EAS(481), ANO(481), POP(481),DIST(481
       1), EA(481), AN(481), POPU(481)
C READ GRID POINTS
        READ(5,9) IF,IL
     9  FORMAT(2I3)
        READ(5,10) (GEAS(L), GNO(L), L = 1,81)
    10  FORMAT(8F10.1)
C READ TOWN COORDINATES AND POPULATION SIZES
        READ(5,11) (EAS(K), ANO(K), POP(K), K = 1,481)
    11  FORMAT( 20X,3F10.1)
C STORE THOSE TOWNS ETC. WITHIN STUDY AREA AND LESS THAN 70 MILES
C FROM THE GRID POINT IN QUESTION
        DO 20 L = IF,IL
        M = 0
        DO 21 K = 1, 481
        IF(EAS(K).LT.18.0.OR.EAS(K).GT.298.) GO TO 21
        IF(ANO(K).LT.12.8.OR.ANO(K).GT.272.) GO TO 21
        EAST = GEAS(L) - EAS(K)
        SOUTH = GNO(L) - ANO(K)
        DIS = SOUTH* SOUTH + EAST * EAST
        IF( DIS.GT.3402.77) GO TO 21
        M = M + 1
        DIST(M) = (DIS*1.44)**.5
        EA(M) = EAS(K)
        AN(M) = ANO(K)
        POPU(M) = POP(K)
    21  CONTINUE
        WRITE(7,25) GEAS(L), GNO(L), M
    25  FORMAT(2F10.1, I5)
        WRITE(7,23) (DIST(K), EA(K), AN(K), POPU(K), K = 1,M)
    23  FORMAT(4F10.1)
    20  CONTINUE
        STOP
        END
```

```
C      PRODUCTION OF ISOPLETH MAPS OF PATTERN STATISTIC  (PART C)
C MULTIPLE CALCULATION OF PATTERN STATISTIC
      DIMENSION GEAS(81), GNO(81), EAS(481), REAS(481), ANO(481),
     1RNO(481), DIST(481), SAREA(4), AREA(81), ROBS(81), DENS(81),
     2REXPE(81), SDREXP(81), C(81), POP(481), ADIST(481), PAT(81)
C
      READ(5,10) RAD
   10 FORMAT(F10.2)
      RANGE = RAD/1.2
      QUAD = (3.1416*(RAD**2))/4.
C
      DO 20 L = 1,81
      READ(5,11)GEAS(L), GNO(L), M
   11 FORMAT(2F10.1, I5)
C
C     CALCULATION OF AREA OF CIRCLE THAT MAY BE TRUNCATED BY STUDY AREA
      AREA(L) = 0.0
      DO 12 I = 1,4
      SAREA(I) = 0.0
   12 CONTINUE
      ORTH = 272.0 - GNO(L)
      WEST = GEAS(L) - 18.0
      EAST = 298.0 - GEAS(L)
      SOUT = GNO(L)- 12.8
      IF(ORTH.GE.RANGE.AND.WEST.GE.RANGE) SAREA(1) = QUAD
      IF(ORTH.GE.RANGE.AND.EAST.GE.RANGE) SAREA(2) = QUAD
      IF(SOUT.GE.RANGE.AND.WEST.GE.RANGE) SAREA(3) = QUAD
      IF(SOUT.GE.RANGE.AND.EAST.GE.RANGE) SAREA(4) = QUAD
      IF(ORTH.GE.RANGE) ORTH = RANGE
      IF(WEST.GE.RANGE) WEST = RANGE
      IF(EAST.GE.RANGE) EAST = RANGE
      IF(SOUT.GE.RANGE) SOUT = RANGE
      IF(SAREA(1).LE.0.) SAREA(1) =(ORTH*WEST)*1.44
      IF(SAREA(2).LE.0.) SAREA(2) =(ORTH*EAST)*1.44
      IF(SAREA(3).LE.0.) SAREA(3) =(SOUT*WEST)*1.44
      IF(SAREA(4).LE.0.) SAREA(4) =(SOUT*EAST)*1.44
C
      DO 13 I = 1,4
      AREA(L) = AREA(L) + SAREA(I)
   13 CONTINUE
C
      READ(5,14) (DIST(K),EAS(K),ANO(K),POP(K), K = 1,M)
   14 FORMAT(4F10.1)
      MM = 0
      DO 15 K = 1,M
      IF(DIST(K).GT.RAD) GO TO 15
      MM = MM + 1
      REAS(MM) = EAS(K)
      RNO(MM) = ANO(K)
   15 CONTINUE
      DENS(L) = (FLOAT(MM)-1.)/AREA(L)
      REXPE(L) = .5/(DENS(L)**.5)
      SDREXP(L) = 0.26136/((FLOAT(MM)*DENS(L))**.5)
C
      SUM = 0.0
      DO 17 K = 1,MM
      ADIST(K) = 9999.
      DO 16 J = 1,MM
```

```
      IF(J.EQ.K) GO TO 16
      EASY = REAS(K)-REAS(J)
      SOUTY = RNO(K)-RNO(J)
      DIS = (EASY*EASY) + (SOUTY*SOUTY)
      IF(DIS.LT.ADIST(K)) ADIST(K) = DIS
   16 CONTINUE
      SUM = SUM + ((ADIST(K)*1.44)**.5)
   17 CONTINUE
      ROBS(L) = SUM/FLOAT(MM)
      PAT(L) = ROBS(L)/REXPE(L)
      C(L) = (ROBS(L) - REXPE(L))/SDREXP(L)
      WRITE(6,18) GEAS(L), GNO(L), MM, AREA(L), DENS(L), REXPE(L),
     1SDREXP(L), ROBS(L), PAT(L), C(L)
   18 FORMAT(1H , 2F10.1,I5,F10.2, F10.5, F10.2, F10.6, F10.2, 2F10.4)
      WRITE(7,19)  GEAS(L), GNO(L), PAT(L)
   19 FORMAT(3F10.5)
      WRITE(7,119) GEAS(L), GNO(L), PAT(L), AREA(L),MM, C(L)
  119 FORMAT(10X,2F10.1,F10.4,F10.2,I5,F10.4)
   20 CONTINUE
      STOP
      END
```

```
C DISTRIBUTION OF TOWNS BELOW 10000 BY 100,S
      DIMENSION  POP(500), PO(101), X(101)
      READ(5,10)(POP(K),K=1,481)
   10 FORMAT(40X,F10.1)
      DO 20 J =1,481
      IF(POP(J).GT.10000.) GO TO 20
      IF(POP(J).LT.100.) GO TO 21
      L = (POP(J)/100.) +1.
      PO(L) = PO(L) + 1.
      GO TO 20
   21 PO(1) = PO(1) +1.
   20 CONTINUE
      WRITE(6,12)(L,PO(L), L= 1, 101)
   12 FORMAT(1H , I5, F10.1)
      DO 50 L= 1,101
      X(L) = L
   50 CONTINUE
      CALL PLOT3(X,PO,101,150.,0.,120.,0.,50,125,25)
      STOP
      END
```

```
$IBFTC SASKAT
      SUBROUTINE SASKAT(FE,FN,IFIRST,ILAST,IJ)
C
C SUBROUTINE USED TO MAP LOCATIONS IN SASKATCHEWAN
C
C FE IS THE EASTING OF THE POINTS IN SASKATCHEWAN TO BE PLOTTED
C FN IS THE NORTHING
C IFIRST IS THE SUBSCRIPT OF THE FIRST POINT IN THE ARRAY TO BE PLOTTED
C  THIS IS NORMALLY 1
C ILAST IS THE SUBSCRIPT OF THE LAST POINT IN THE ARRAY TO BE PLOTTED
C AST(1),PLUS,2), DOT(3), CIRC(4),
C
      DIMENSION FE(500), FN(500), OUT(105), IE(500), IN(500), IGE(7),
     1IGN(7),FIG(4)
      DATA IGE, IGN/ 11,15,84,91,50,52,42,9,104,93,9,7,91,96/
      DATA FIG/1H* , 1H+, 1H., 1HO/
      DATA PLUS, BLAN/1H+, 1H /
      FAG = FIG(IJ)
      WRITE(6,9)
    9 FORMAT(1H1)
      DO 10 L = IFIRST,ILAST
      IN(L) = (FN(L)/3.2) + 5.
      IE(L) = (FE(L)/4.0) + 10.
   10 CONTINUE
      DO 15 J = 1,115
      DO 11 K = 1,105
      OUT(K) = BLAN
   11 CONTINUE
      DO 12 N = 1,7
      IF(IGE(N).NE.J) GO TO 12
      K = IGN(N)
      OUT(K) = PLUS
   12 CONTINUE
      DO 13 L = IFIRST,ILAST
      IF(J.NE.IE(L)) GO TO 13
      K = IN(L)
      OUT(K) = FAG
   13 CONTINUE
      WRITE(6,14)(OUT(K), K = 1,105)
   14 FORMAT(1H , 105A1)
   15 CONTINUE
      RETURN
      END
```

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