A STATISTICAL ANALYSIS OF HYDROCYCLONE PARAMETERS

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SCOPE AND CONTENTS:

The separation of a mixture of glass spheres in water using 2 inch hydrocyclones was studied. Three operating parameters were investigated: feed concentration, volume split and feed flow rate. In addition, three design parameters were cone angle, inlet diameter, and vortex finder length. The performance criterion parameters were the efficiency with which the solids were separated from the liquid, and the energy required per unit mass flowing through the hydrocyclone.

First the experimental data were analyzed by three different statistical methods and the results compared in an attempt to determine which statistical method was most suitable for this two criteria system. The three methods were principal component analysis, canonical correlation analysis and multiple regression analysis. The theory behind these methods is briefly

outlined. Our conclusion is that using all three methods give much more insight than could be obtained from any individual method.

Second, an analysis of the above eight hydrocyclone parameters of hydrocyclones with cylindrical sections indicated that for the range of parameters covered in this work, feed flow rate and inlet diameter influenced the energy loss most; volume split influenced the separation efficiency the most. Energy loss and separation efficiency are quite independent; this means that it is possible to design and run a hydrocyclone with high separation efficiency and low energy loss. The dilute concentrations used in this work indicate that a hydrocyclone of conventional design can be used in waste water treatment. When the parameters were correlated, a power model gave more consistent interpretation than a linear model.

Third, the effect of the three operating parameters on hydrocyclones with three different body shapes suggested that the most efficient cyclone was one with a straight cone and no cylindrical section. The body shape dictated which parameters would significantly affect performance.

TABLE OF CONTENTS

PART I

PART II

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

A = p_1 - component or p-component column vector

a = element of vector A

B = p_2 - component column vecotr

D = cyclone diameter

D_p = particle size

$\langle D_G \rangle$ = geometric mean diameter

D_1 = inlet diameter

D_2 = overflow diameter

D_3 = underflow diameter

G = pressure loss factor

g = gravitational acceleration

H = height of cylindrical section

L = overall vertical length of cone

L_2 = vortex finder length

N = sample size

P_1 = feed stream pressure

P_2 = overflow stream pressure

P_3 = underflow stream pressure

p = number of variates

p_1 = number of criterion variates

p_2 = number of predictor variates

R	=	covariance matrix
U, V	=	canonical variates
V_1	=	feed stream velocity
V_2	=	overflow stream velocity
V_3	=	underflow stream velocity
W_1	=	feed mass flow rate
W_2	=	overflow mass flow rate
W_3	=	underflow mass flow rate
w	=	wt. fraction of solid in liquid
X	=	independent variables
Y	=	dependent variable
$\hat{Y}$	=	estimated value of dependent variable
Z	=	random sample matrix
β	=	the coefficient of least square equation
$\hat{\beta}$	=	estimated value of the coefficient of least square equation
ε	=	random error
Θ	=	angle of rotation; cone angle
η	=	separation efficiency
λ	=	Lagrangian multiplier; eigenvalue
ζ	=	Lagrangian multiplier
μ	=	micron
σ_G	=	geometric standard deviation
$\Delta\xi$	=	energy loss

PART I

1. INTRODUCTION

Multiple regression analysis is a technique commonly used to correlate a single dependent variate in terms of a set of independent variates. Since the regression analysis is restricted to single response study, the application is quite limited. More generally, the study of the correlations between a set of predictor (independent) variates and a set of criterion (dependent) variates (multiple responses study) is called canonical correlation analysis (Hotelling, 1936). This may be considered as multivariate case of a simple correlation. The regression analysis may be considered as a special case of it. The basic approach in the canonical correlation technique is to determine those linear functions of the predictor variates and of the criterion variates which produce the maximum correlation between these two sets. The canonical correlation analysis was generalized by Roy (1957). It should be emphasized that canonical correlation analysis is different from canonical reduction used by Box (1954). (The form resulting from canonical reduction is called canonical form, which is a standard process of axes transformation from the origin to the center of the system in coordiate geometry).

From the regression analysis we understand that multiple correlation demands that one response be dependent upon some or all of the remaining variates. Similarly, for a canonical correlation analysis the responses must be collected into two or more sets. All these choices depend upon the nature of the responses and other information external to the mere value of their correlations. Therefore, the dependence structure will in turn depend upon those choices. Furthermore, if the analyses are repeated for different choices of the dependent or constant variates, the successive findings will hardly be independent or contain mutually exclusive bits of information about the structure.

A new class of technique will be required for picking apart the dependence structure in an attempt to identify those hidden factors which have generated the dependence or variation in the responses. That is, the observable variates are represented as functions of a smaller number of latent factor variates. Principal component analysis is one technique among this field. It explains observed relations among numerous variates in term of simpler relations. The simplification consists of producing a smaller number of hypothetical variates (called "principal components" (Hotelling, 1933) or "principal factors" (Harman, 1967)). Therefore, this method gives a short representation of a random sample from a population of multivariate measurements; the method searches for the basic underlying influences. Principal component analysis isolates and develops hypothetical constructs out of observed phenomena. In statistical practice, the method of principal component analysis is used to find the linear combinations with large variances. The methodology originated with Pearson (1901) as a means of fitting planes by orthogonal least squares, but was proposed by Hotelling (1933, 1936a).

In this part we consider first a brief description of these three methods; then these methods are applied to a practical problem to illustrate the different types of information that are produced.

2. REVIEW OF THEORIES

2.1 Canonical Correlation Analysis

2.1.1 General Statement

Suppose we have a sample from a p-dimensional space. In the canonical correlation analysis (Hotelling, 1936) the search is for a linear function of the first p_1-variates and a linear function of the last p_2-variates $(p_1+p_2=p)$, such that these two linear functions have the highest possible correlation coefficient. Under the assumption of normality, if the canonical correlation is zero, these two sets are completely independent, and it is useless to predict the dependent variates by means of the independent variates. If the canonical correlation is unity, this means that the dependent variates would be predicted perfectly by means of the independent variates based on the particular linear functions. For the special case in which the number of dependent variates is equal to one, the problem becomes simply one of multiple regression.

The assumption is that the observed variates are linear functions of the canonical variates. Furthermore we assume that the observed variates are normally distributed in order to make a statistical inference on the dependence between the two sets and to derive the probability distribution of the canonical correlation coefficients.

A response surface which is an homogeneous expression of a quadratic form is reduced to a linear combination of only squares, the cross-product terms being eliminated. A form of this type is said to be a canonical form. For instance, the canonical reduction mentioned by Box (1954) is algebraically equivalent to the canonical analysis of Hotelling's method, but their underlying physical meanings are completely different.

2.1.2. Theory

Suppose Z_{ij} $(i=1,2,...,p,\ j=1,2,..,N, N>p)$ is a random sample of size N from a p - dimensional distribution. Suppose further that Z_{ij} has a covariance matrix, R, which is known to be a positive definite, real, symmetric matrix. Without loss of generality we may suppose that Z_i has zero mean.

We partition Z into two subvectors of p_1 and p_2 components $(p=p_1+p_2)$ respectively,

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \tag{1}$$

For convenience we assume $p_1 \leq p_2$. The covariance matrix is partitioned into matrices as follows:

$$R = \left[\begin{array}{c|c} R_{11} & R_{12} \\ \hline R_{21} & R_{22} \end{array}\right] \tag{2}$$

Where R_{11} is the correlation matrix for Z_1, R_{22} for Z_2, and $R_{12} = R'_{21}$ is the correlation matrix between Z_1 and Z_2. The canonical variates U and V are defined by the arbitrary linear combinations:

$$U = A'Z_1 \quad , \quad V = B'Z_2 \tag{3}$$

We require A and B to be such that U and V have unit variance, that is

$$1 = E\ [U^2] = A'R_{11}A \tag{4}$$

$$1 = E\ [V^2] = B'R_{22}B \tag{5}$$

For this condition, we can obtain an expression for the expected value of UV

$$E\ [UV] = A'R_{12}B \tag{6}$$

The problem is to find A and B to maximize Equation (6) subject to the constraints of Equations (4) and (5). Lagrangian multipliers, $\frac{1}{2}\lambda$ and $\frac{1}{2}\zeta$, are introduced to describe the constraints of Equations (4) and (5). The composite equation is

$$\psi = A'R_{12}B - \frac{1}{2}\lambda(AR_{11}A-1) - \frac{1}{2}\zeta(B'R_{22}B-1) \tag{7}$$

We differentiate ψ with respect to the variables A and B and equate to zero. The results are:

$$R_{12}B - \lambda R_{11}A = 0 \tag{8}$$

$$R'_{12}A - \zeta R_{22}B = 0 \qquad (9)$$

From Equations (8) and (9) we note that the Lagrangian multipliers are equal, that is,

$$\lambda = \zeta = A'R_{12}B \qquad (10)$$

Rearrangement of Equations (8) and (9) and use of Equation (10) gives

$$\lambda R_{11}A + R_{12}B = 0 \qquad (11)$$

$$R_{21}A - \lambda R_{22}B = 0 \qquad (12)$$

Solving for A we obtain

$$(R_{11}^{-1} R_{12} R_{22}^{-1} R_{21} - \lambda^2 I) A = 0 \qquad (13)$$

Where I is the identity matrix. The solution involves finding eigenvalues, λ^2, of the equation

$$| R_{11}^{-1} R_{12} R_{22}^{-1} R_{21} - \lambda^2 I | = 0 \qquad (14)$$

From Equation (10) we see that $\lambda = A'R_{12}B$ is the correlation between U and V. The λ's were called the canonical correlation coefficient by Hotelling (1936). Values of A_i in equation (3) are the eigenvectors associated with λ_i^2. Solving for B from equation (12), we have B_i for a particular λ_i is given by

$$B_i = R_{22}^{-1} R_{21} A_i / \lambda_i \qquad (\lambda_i \neq 0) \qquad (15)$$

The terms A_i and B_i are normalized, so that we have

$$A_i' A_i = 1 \text{ and } B_i' B_i = 1 \qquad (16)$$

Then U_i and V_i are normalized linear functions of Z_1 and Z_2, respectively, with maximum correlation.

Next we want to find other linear functions of Z_1 and Z_2, respectively, such that each of these two linear functions has the maximum correlation and is uncorrelated with the first linear functions. This procedure is continued with two linear equations being generated at each step. At the r-th step we have obtained linear combinations $U_1 = A_1'Z_1$, $V_1 = B_1'Z_2, \ldots,$ $U_r = A_r'Z_1, V_r = B_r'Z_2$, with corresponding correlations, $\lambda_1, \ldots, \lambda_r$. These values are called canonical correlation coefficients (canonical roots).

Following a similar line of argument to that used above, we can obtain expressions for the maximum correlation. The resulting equations for the i^{th} equations turn out to be Equations (11) and (12). The details are given by Anderson (1958) p. 290 ff. Therefore, any λ_i from the p_1 roots satisfies the conditions of Equations (4) and (5) for $i = 1, 2, 3 \ldots . r$.

A criterion which is useful in detecting the simultaneous departure of several roots λ_i^2 from zero was suggested by Bartlett (1939) for testing of significance.

$$\chi^2 = -[N - \frac{1}{2}(p_1 + p_2 + 1)] \ \ell n \ \Lambda \tag{17}$$

Where $\Lambda = \prod_{i=1+\gamma}^{p_1} (1 - \lambda_i^2)$ follows approximately a chi-square distribution with $(p_1 - \gamma)(p_2 - r)$ degrees of freedom. The assumption is that Z_1 and Z_2 follow a multivariate normal distribution with zero means.

A FORTRAN IV program was written (Lee, 1967) for this analysis.

2.2 Multiple Regression Analysis

If a function which is linear in the independent variables is used, the mathematical model can be written in matrix form simply as

$$Y = X\beta + \varepsilon \tag{18}$$

Where X is the N x p matrix of independent variables and β is the p x 1 vector of unknown parameters to be estimated. Let Y be the N x 1 vector of observations.

Elements, ε_i, of $\varepsilon(nx1)$ are uncorrelated, unobservable random variables and are normally distributed with zero mean and variance of σ^2. If the matrix X'X is nonsingular, least squares estimates of β, called $\hat{\beta}$, are readily obtained from the equation

$$\hat{\beta} = [X'X]^{-1}X'Y \tag{19}$$

It is then possible to construct a response surface of the predicted value, $\hat{Y}$, in terms of the independent variables X :

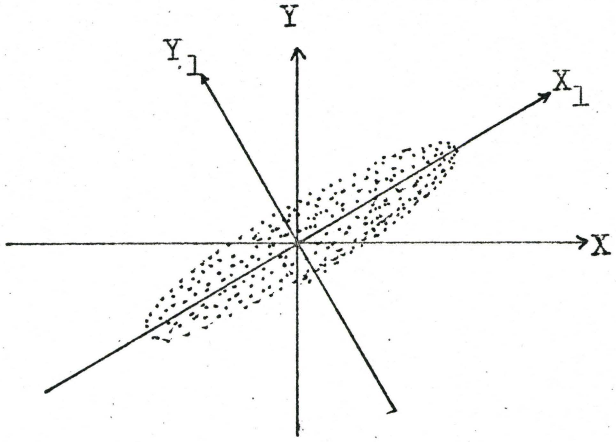
$$\hat{Y} = X\hat{\beta} \tag{20}$$

This surface, in the form of contours (loci of constant $\hat{Y}$ values), can be studied visually to gain an appreciation of the relationship between the variables X and the response Y. In this analysis, we assume no error resides in X, and have considered only one response, Y.

2.3 Principal Component Analysis

2.3.1 General Statement

Consider a random sample of size N where two random variables X, and Y have been measured in each individual in the sample. This sample may be represented geometrically as a sample cluster of N points in a 2-dimensional Euclidean space.



If the X-Y coordinate system is rotated rigidly into a new position such that the X_1 coincides with the long axis of the sample cluster, whereas Y_1 is perpendicular to X_1, then X_1 and Y_1 are two nearly uncorrelated random variables.

The X_1 axis accounts for the most variation (variance) of the sample cluster, whereas the contribution to the sample variation from Y_1 is nearly zero. The new axis, X_1, is called the principal axis or principal component (Hotelling, 1933), or principal factor (Harman, 1967, p. 135). This bivariate case can be generalized into the multivariate case. This rotation, or more precisely, this orthogonal linear transformation, is the underlying concept in multivariate statistical analysis. In this example, X_1 and Y_1 are linear combinations of X and Y, such as

$$(X_1 \quad Y_1) = (X \quad Y) \begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix} \tag{21}$$

where $\begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix}' = I$

I is the identity matrix.

Principal components are linear combinations of these measurements or random variables which have special properties in terms of variances. Most of the variations from observation to observation may reside in fewer linear combinations than the number of variates the experimenter started with; then he can confine his study to fewer quantities, because the other linear combinations vary so little that one cannot detect variations from observation to observation.

2.3.2. <u>Theory</u>

Suppose (Z_{ij}, i=1,2,...,p, j=1,2,...,N) is a sample of size N>p from a p-dimensional distribution whose covariance matrix, R, is positive definite. In developing the ideas, we do not need to assume that Z is normally distributed, but the normality assumption is needed to derive the sampling theory.

Let A be a p-component column vector such that A'A=1. The variance of linear combination U=A'Z is

$$E[A'Z]^2 = E[A'ZZ'A] = A'RA \qquad (22)$$

To determine the normalized linear combination $A'Z$ that has the maximum variance, we must find a vector A, such that Equation (22) is maximized subject to the constraint $A'A=1$.
Introduce the Lagrangian multiplier λ and Equation (22) becomes

$$0 = A'RA - \lambda (A'A-1) \qquad (23)$$

Partial derivatives respect with A are set equalzero to give:

$$2RA - 2\lambda A = 0 \qquad (24)$$

In terms of the identity matrix I this becomes

$$(R - \lambda I) A = 0 \qquad (25)$$

The nontrivial solution to Equation (25) is

$$|R - \lambda I| = 0 \qquad (26)$$

Equation (26) is a polynomial in λ of degree p. Multiplication of Equation (25) by A' yields

$$A'R A = \lambda A'A = \lambda \qquad (27)$$

This means that if A satisfies Equation (25), then the variance of $A'Z$ is λ (Combining Equation (7) with Equation (22)). Hence for the maximum variance we should select the largest root λ_1. Let A_1 be a normalized solution of $(R-\lambda_1 I)$ $A = 0$ then $U_1=A_1' Z$ is a normalized linear combination with maximum variance. We can refer to this as the first principal component.

A similar analysis can be made for the second principal component U_2; in fact, this can be continued up to p steps with p principal components, $U_i = A_i Z$, coresponding to p eigenvalues $\lambda_i (i = 1,2,3 \ldots p)$. Anderson (1958) p. 274 ff discusses this in more detail.

Let us summarize this process as: for the p-component random vector Z with E[Z]=0 and E[ZZ']=R there exists an orthogonal linear transformation

$$U = A'Z \tag{28}$$

such that the covariance matrix of U is $E[UU'] = \Lambda$ where

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ 0 & & & \lambda_p \end{bmatrix} \tag{29}$$

Where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p \geq 0$ are the roots of Equation (26). The component $U_i = A_i' Z$ has maximum variance of all normalized linear combinations uncorrelated with $U_1, \ldots, U_{i-1}$.

2.3.3. Interpretations

What an experimenter is looking for is a physical meaning for each principal component. Physical meanings can be obtained by an examination of the elements of matrix A on each random variable. It is easier to interpret the physical meaning through rotation of matrix A like in factor analysis (Harman, 1967). One rotation method, varimax method introduced by Kaiser (1958), is used in the present study. The varimax method attempts to produce numerous zero coefficients in each principal component rather than in each random variable and also attempts to maximize the differences between coefficients for each principal component.

Two principal components are rotated at a time. After the rotation of all possible pairs of principal components, the varimax criterion, V, is evaluated as follows

$$V = N \sum_{i=1}^{p} \sum_{j=1}^{p} (a_{ji}/h_j)^4 - \sum_{i=1}^{p} (\sum_{j=1}^{p} a_{ji}^2 / h_j^2)^2 \qquad (30)$$

Where a_{ji} are elements of matrix A.

h_j is defined as

$$h_j^2 = a_{j1}^2 + a_{j2}^2 + \cdots + a_{jp}^2 \qquad (31)$$

The rotation process is continued until the difference of two successive varimax criteria is less than a predetermined value.

Principal component, may be obtained from either a correlation matrix or a covariance matrix. If random variables have been measured in non-comparable units, the correlation matrix is recommended (Anderson, 1963), because the correlation coefficients may be regarded as constituting a standardized sample covariance matrix. Conversely if the random variables are reasonably comparable, the covariance form has a greater statistical appeal (Anderson, 1963). Principal components do not necessarily need to have any intrinsic physical meaning.

One problem is to determine the number of statistical significant principal components. A number of large-sample distributional properties of component coefficients and characteristic roots permits the construction of tests of hypotheses and confidence intervals for the population component structure. The tests are fully discussed by Anderson (1963). No tests are used in the present study.

A FORTRAN IV program was written (Lee, 1967a) for this analysis according to the theory described in this chapter. Also varimax rotation of principal components is included.

3. AN EXAMPLE OF APPLICATION

3.1 Introduction

A hydrocyclone is a device used to separate solid/liquid mixture. The usual criteria used to evaluate a hydrocyclone are the degree of separation that is attained, given by efficiency η or a critical separation diameter of the particle, $(D_p)_{50}$; and the amount of energy required to perform the separation, Δp. Often, the better the separation, the more energy is required. What we want is excellent separation for a minimum of energy. Workers in this field have tended to generate expressions for either of these single criterion parameters in terms of design and operating parameters.

The example used in the present work demonstrates how the three statistical techniques can be applied to this hydrocyclone problem, illustrates the type of information that is produced from these three methods and discusses the implications of the answers.

Based on a survey of the literature, conveniently condensed in the text by Bradley (1965), three design and three operating parameters were selected as having the most important effect on the two performance criteria. Some 52 runs (samples were observed for the separation of glass beads (geometric mean diameter, $\langle D_G \rangle = 37$ microns, standard deviation of log-normal drop size distribution,

σ_g, = 1.24) from water; the experiments were arranged on a three level, incomplete block design for a linear correlation model (Box (1960)). Details are given in Part II. The parameters selected were: Operating parameters: feed concentration, ratio of fluid flowing out the overflow to fluid out the underflow (volume split) and the volumetric feed flowrate. Design parameters: cone angle, inlet diameter of feed line, vortex finder length. Performance criterion parameters: separation efficiency, η, and the energy loss per unit of mass flowing through the hydrocyclone.

The aim of this study was first, understanding the underlying factors influencing the hydrocyclone performance, second, to find out an overall concept of the influences of six predictor variates (three operating parameters and three design parameters) on two criterion variates (two performance criterion parameters).

3.2. Statistical Analyses and Discussion

The principal component analysis, canonical correlation analysis and multiple regression analysis were conducted on a set of experiment data (sample size = 52, number of variates = 8). We will discuss the results (by each analysis method) individually. Principal component analysis tries to understand a group of observations, one normally searches first for the underlying principles. Hence we discuss principle component analysis first.

3.2.1. From Principal Component Analysis

The principal component analysis was conducted on correlation matrix and eight eigenvectors were rotated (Table 1). We expect that some of these factors will have a low total eigenvalue and can be discarded. Otherwise if we start with eight variables and end up with eight factors we have accomplished very little in learning about the underlying factors. Of the eight factors, the last two in the table were discarded because the eigenvalue was too small. Of the remaining factors, factors 4, 5, and 6 have almost zero factor loadings by all of the variates. Hence, these were discarded. The whole analysis was repeated with only the first three factors being retained. The results are shown in Table 1.

Thus we have three factors. Consider the physical significance of these factors by noting the magnitude of the factor loadings in the table (the sign indicates the direction in which the variates interact). For the first factor the variates with the largest factor loading are the inlet diameter, the cone angle, the flow rate and the energy loss. Since we are interested in the criterion of energy loss, we could interpret this first factor as being indicative of the energy loss. The inlet diameter and vortex finder length are negative correlated with energy loss. On the contrary, the flow rate, cone angle, and efficiency are positively

TABLE 1. RESULTS OF PRINCIPAL COMPONENT ANALYSIS FOR LINEAR MODEL (FOR PART I STUDY)

Variates \ Factor	1	2	3	4	5	6	7	8
Inlet diameter	1.00	-0.04	-0.05	0.01	0.00	-0.00	-0.12	-0.14
Vortex finder length	0.80	0.60	-0.01	0.00	0.00	-0.00	0.23	0.04
Cone angle	-0.99	0.14	-0.07	0.01	-0.00	0.00	0.02	-0.29
Flow rate	-0.99	0.10	-0.02	0.00	0.00	-0.00	0.15	-0.01
Energy loss	-1.00	0.02	0.00	-0.00	0.00	-0.00	0.13	0.07
Volume split	0.04	1.00	0.06	-0.01	-0.00	0.00	0.21	-0.01
Efficiency	-0.24	0.97	-0.03	0.01	0.00	-0.00	0.79	0.01
Concentration	0.00	0.02	1.00	-0.00	-0.00	0.00	0.01	0.00
Eigenvalue	2.08	1.55	1.00	1.00	1.00	1.00	0.26	0.10

Variate \ Factor	1	2	3
Inlet diameter	1.00	-0.03	-0.02
Vortex finder length	0.76	0.65	-0.01
Cone angle	-0.99	0.14	-0.04
Flow rate	-1.00	0.09	-0.01
Energy loss	-1.00	0.01	-0.00
Volume split	0.03	1.00	0.01
Efficiency	-0.25	0.97	-0.03
Concentration	0.01	-0.01	1.00
Eigenvalue	2.08	1.55	1.00

correlated with energy loss. The contribution of volume split and concentration to this factor is negligible. To a certain degree this grouping makes sense in that we would expect a high correlation between inlet diameter and feed velocity with energy loss. The effect of cone angle and vortex finder length are surprising in that this dependence has not been noted before. The interaction between the criteria, energy loss and separation efficiency, is not strong in this factor.

The second factor has high factor loadings for the efficiency, the volume split and to a limited extent the vortex finder length. Physically this is interpreted as being the separation efficiency factor because we would like to relate the efficiency criterion variate to one factor. The factor loadings imply there is a high correlation between efficiency and volume split. Burrill (1967) found similar results for his study of a hydrocyclone is separate liquid-liquid systems. The suggestion that an increase in vortex finder length increases the efficiency could be justified by arguments concerning the existence of a short circuit flow of feed directly to the overflow without undergoing separation. Nevertheless, this relatively strong dependence is surprising considering the evidence published literature to date. The contribution of the other variates to this factor is negligible.

The small effect of concentration found in this analysis coincides with the findings of Burrill (1967).

The authors interprets the third factor as a unique factor that suggests the independence of the concentration effect. It is unique because the factor loadings of all but one variate are very small.

The above three mutually independent (orthogonal) factors describe the main underlying influences of the hydrocyclone performance. The first two factors are called common factors which account for the most variance of the variates. The third factor is called unique factor which accounts for the remaining variance of that variate. The identification of hidden relationships is the main value of principal component analysis. It supplies a single means of reducing the number of variates to be treated in more extensive studies.

3.2.2. From Canonical Correlation Analysis

Results from canonical correlation analysis are listed in Table 2. The influence of the six predictor variates (three design parameters and three operating parameters) on two criterion variates (two performance criterion parameters) are shown. The two canonical variates describe the overall concept from two points of view. High values of the canonical roots (canonical correlation coefficients) indicate that the criterion variates are predicted well by means of the predictor variates based on the particular linear functions. Furthermore, the two canonical variates are statistically significant at 1% level. The first canonical variate has a high canonical root, and it is predominated by the energy loss. It shows the relationship that these predictor variates can increase the efficiency and the energy loss or decrease the efficiency and the energy loss simultaneously. The second canonical variate has a canonical root that is high yet lower than that of the first canonical variate. This variate illustrates how these predictor variates can increase the efficiency and decrease the energy loss simultaneously or vice versa. This canonical variate is predominated by efficiency.

The descending order of importance of the predictor variates in the first canonical variate is: inlet diameter, flow rate, volume split, cone angle. The influence of concentration and vortex finder length is negligible. Thus, if we decrease the

TABLE 2. RESULT OF CANONICAL CORRELATION ANALYSIS FOR LINEAR MODEL (FOR PART I STUDY)

Canonical variate	1	2
Efficiency	0.46	0.84
Energy Loss	0.89	-0.54
Concentration	-0.03	-0.04
Volume split	0.37	0.93
Flow Rate	0.50	-0.17
Cone Angle	0.18	-0.05
Inlet Diameter	-0.76	0.32
Vortex Finder Length	-0.02	0.05
Canonical Root	0.906	0.710
Significant level	0.01	0.01

inlet diameter and increase the flow rate, volume split, and cone angle, both the efficiency and the energy loss will increase. Also, we realize that the change will be greater for energy loss than for efficiency.

In the second canonical variate, the relative importance of the predictor variates is arranged in descending order: volume split, inlet diameter, feed rate. Also we realize that the influence on efficiency is larger than that on the energy loss. Thus, if we increase the volume split and the inlet diameter and decrease the flow rate, we may achieve our purpose of simultaneously increasing the efficiency and decreasing the energy loss. Since the ideal condition of a hydrocyclone performance is with minimum energy loss and maximum separation efficiency. The result from the canonical correlation analysis is very useful.

In this problem, since we have two criterion variates, we obtain two canonical roots, and the two canonical variates are mutually orthogonal. Usually the second canonical variate describes the phenomena that the first canonical variate does not fully described. To understand a system the conclusions from both canonical variates should be considered.

Since canonical correlation analysis deals with relationship between two sets of variates, it is extremely powerful in predicting multiple criteion variates based on multiple predictor variates. This technique is also useful in reducing two large sets of variates to a few linear combinations of a set of canonical variates.

Sometimes if we are only interested in finding the relative importance of these predictor variates influencing on the energy loss or efficiency separately, the regression analysis-single response study is useful. We discuss regresssion analysis in the next section.

3.2.3. Regression Analysis

The results from regression analysis are listed in Table 3. Two correlations are presented, one for each criterion variate. Since the experimental design was a three level, second order incomplete block design (Box 1960), a second degree, graduating polynomial equation was used for the correlation. The correlations are given in Table 3 in coded form. The relative importance of each predictor variate can be judged by the relative absolute magnitude of the coefficients. The missing terms in the equations were statistically insignificant as judged by an F test at the 1% significance level. From the energy loss regression equation the most important variate is inlet diameter. The second important variate is flow rate and then the cone angle. The sign of each coefficient shows the relative direction of influence of that variate on energy loss.

The efficiency regression equation indicates that the descending order of importance of the variates is: volume split, inlet diameter, flow rate. The contribution of concentration and cone angle is very small.

TABLE 3. RESULTS OF MULTIPLE REGRESSION ANALYSIS (FOR PART I STUDY)

$$\text{Energy Loss} = 14.8177 + 12.8173\ X_3 + 4.5787\ X_4 - 20.2068\ X_5 + 13.2879\ X_5^2$$

Multiple correlation coefficient = 0.94

$$\text{Efficiency} = 63.7047 - 0.57301\ X_1 + 10.5822\ X_2 + 3.6747\ X_3 + 0.93214\ X_4 - 9.9617\ X_5 + 1.4133\ X_1^2 - 6.5178\ X_2^2 - 2.4750\ X_3^2 - 0.8773\ X_4^2 - 4.8656\ X_5^2$$

Multiple correlation coefficient = 0.99

X_1 = concentration, X_2 = volume split, X_3 flow rate, X_4 = cone angle, X_5 = inlet diameter, X_6 = vortex finder length.

Since the regression analysis is a curve-fitting technique, interpretation of the physical meaning from the resulting equation such as the polynomials reported in Table 3 can be misleading. In this sense, prinicpal component analysis is the technique recommended to indicate the correlations among the variates in the objective system. Furthermore, regression analysis deals with the study of a correlation between only a single dependent variate and multiple independent variates. This is probably the greatest limitation to the application of regression analysis. When systems have multiple criterion variates, like the case studied in the paper, regression analysis gives a regression for each criterion variate. We can only understand the effect of the predictor variates influencing each criterion variate separately; we do not have an overall feel for how predictor variates influence the criterion variates simultaneously. Therefore, the usefulness of regression analysis is quite limited as compared with canonical correlation analysis.

3.2.4. The Comparison of the Results from Three Methods

Because the underlying philosophy and the presentation of results of the three statistical methods are different, the results are difficult to compare quantitatively. Nevertheless, the qualitative trends suggested from the different methods should be comparable. Let us compare the results in terms of the parameters.

Concentration

All three methods indicate that concentration has a negligible effect on the energy loss and on the separation efficiencies. Perhaps the strongest evidence is that the principal component analysis suggests that concentration is a unique factor.

Volume Split

All three methods agree that the volume split is the most significant parameter in influencing the efficiency and that an increase in volume split increases the separation efficiency. The first factor from the principal component analysis suggests that the volume split is of no importance in influencing the energy loss and so does the result from the energy loss regression equation, whereas the canonical variate dominated by the energy loss suggests that volume split is third in importance with a positive effect. This apparent contradiction merely emphasizes the difference in the type of result each method produces. The criterion variates in the canonical variate are the combination of efficiency and energy loss, even though the energy loss tends to dominate. Since the volume split predominates the efficiency effect, any correlation technique that combines the criterion variates should show a dependence upon volume split that bears some relation to the amount the efficiency is accounted for in the correlation.

Feed Flowrate

Principal component analysis suggests that this and the inlet diameter are the major parameters affecting the energy loss. The canonical variate dominated by energy loss indicates that the feed flowrate is the second most important parameter. The result of regression analysis also agrees with this. Furthermore all three statistical techniques suggest that an increase in flowrate increases the energy loss. Considering the effect of flowrate on separation efficiency, it is not a very important parameter. The first factor resulted from principal component analysis shows the insignificance of flowrate in influencing the efficiency. Also, the flowrate ranks third in importance in both separation efficiency regression equation and the canonical variate dominated by efficiency. An increase of flowrate increases the efficiency. The negative correlation between the flowrate and the efficiency in the second canonical variate is due to the direct relation between the flowrate and the energy loss criterion.

Cone angle

All three methods suggest that cone angle has a very small effect on the efficiency. Concerning its effect on the energy loss, cone angle ranks third in both energy loss regression equation and the first factor resulted from principal component analysis, and fourth in the canonical variate dominated by the energy loss. Since the correlation between the cone angle and the energy loss is near unity, perhaps, the principal component analysis over emphasizes the role of cone angle.

Inlet diameter

All three methods indicate it is the most important parameter influencing the energy loss; an increase of inlet diameter decreases the energy loss directly. Both regression analysis and the canonical variate dominated by the efficiency also suggest it is quite an important parameter in influencing the separation efficiency, whereas the result from principal component analysis does not suggest this. The inlet diameter influences the efficiency in the opposite direction.

Vortex finder length

Both the canonical correlation analysis and regression analysis suggest it to be insignificant in influencing the energy loss and the separation efficiency, whereas vortex finder length ranks the fourth in the first factor in influencing the energy loss and the second in the second factor in influencing the separation efficiency. This is the most significant difference in results between the findings from principal component analysis to the findings from canonical correlation analysis and multiple regression analysis.

To sum up, we found some discrepancy among the result of three methods, but generally, the findings from different methods are quite coincident.

4. CONCLUSIONS

1. The combination of these three statistical techniques provides a very powerful means of interpreting data. The use of the three is not a means of quantitatively checking one statistical technique against the other. Rather this combination supplies three different ways of looking at the same data. Nevertheless the qualitative trends in the data should be the same for all three statistical methods.

2. The principal component analysis indicates the main underlying influences. It aims to explain observed relations among numerous variates in terms of simpler relations; the logical nature is to isolate the underlying influences into several independent main factors. However, from this paper we note that the results may not qualitatively agree with the results from the canonical correlation analysis or the regression analysis. In the particular example, the dilemma exists as to the effect of the vortex finder length. Great care needed to be taken if the principal component analysis alone is used. It further suggests the form of the correlation assumed is very important. As we have mentioned, the mathematical model used in this work is a linear one. This is also

the assumption and limitation of a linear correlation form. If the real system should be expressed as a nonlinear correlation among the variates, then a linear model expression will not give a reliable description of the real system. One direction of recent work is the development of factor analysis in a nonlinear model (McDonald, 1965, 1967).

3. The canonical correlation analysis presents correlations between a set of predictor (independent) variates and a set of criterion (dependent) variates that describes most of the total covariance between the two sets of variates. The main advantage of this method is that it indicates the way of influence from a set of predictor variates to a set of criterion variates, hence it is extremely useful especially in analysing a system with multiple criterion variates. Also in exploratory studies, if the two sets are very large, the investigator may apply canonical correlation analysis and then study only the few linear combinations that are most highly correlated.

4. The multiple regression analysis presents a correlation between a single criterion variate and a set of predictor variates that the linear function of the predictor variates describes most of the total variance. Since the regression analysis is restricted to single dependent variate study, its application is quite limited comparing with canonical correlation analysis.

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PART II

1. INTRODUCTION

A hydrocyclone is a simple device that separates solid particles from liquid using centrifugal force. In a sense, it is like a centrifuge. Two factors of key importance are the efficiency of the separation and the amount of energy required in carrying out the separation. Few attempts have been made to relate the effect of cyclone parameters to separation efficiency and energy loss simultaneously. Indeed, few researchers have used statistics to aid in the interpretation of the data or in determining the significance of the operating and design parameters on even one of these factors.

The hydrocyclone field has been reviewed by Bradley (1965). From a review of this reference and other literature in the hydrocyclone field, the problems chosen to be studied in this work are:

1. To find out the possibility of separating very dilute concentrations (ppm) of solid in liquid with cyclone and to see, for this separation problem, if a configuration radically different from that used for more concentrated separation is needed.
2. To use and evaluate statistical methods to determine significant design and operating parameters.

In Part I, three important statistical techniques were introduced. These will be used in this work to

a. identify the underlying factors influencing hydrocyclone performance.

b. study of the correlation between a set of predictor variates (hydrocyclone design and operating parameters) and a set of criterion variates (performance criterion parameters) -multiple responses study.

c. to find out the relative importance among the design and operating parameters in influencing the performance, and to compare the performance among different cyclone designs.

For convenience, the experimental work and results discussion are divided into two parts: A and B.

PART A:

An analysis of eight hydrocyclone parameters (three design parameters, three operating parameters and two performance criterion parameters) on the data obtained from a set of the hydrocyclones with a cylindrical section and fixed body shape.

The three operating parameters are:

feed solid concentration	(X_1)
volume split	(X_2)
feed flow rate	(X_3)

The three design parameters are:

cone angle (X_4)

inlet diameter (X_5)

vortex finder length (X_6)

The two performance criteria

energy loss per unit mass of feed $\Delta\xi$/mass

separation efficiency η

Details of these parameters will be discussed in the next chapter. The term "cyclone" and "hydrocyclone" will be used interchangably, with the understanding that the cyclone feed has liquid as the continuous phase.

PART B:

In this part three hydrocyclones with different body shape are used:

1. hydrocyclone with a cylindrical section (called hydrocyclone a)
2. hydrocyclone without a cylindrical section (called hydrocyclone b)
3. hydrocyclone with a curved wall (called hydrocyclone c)

Thus the work for Part B is to analyze five hydrocyclone parameters (three operating parameters, and two performance criterion parameters) on the data obtained from the experiments using hydrocyclone a , b and c.

2. EXPERIMENTAL APPROACH

This chapter describes the design of experiments, the design of the hydrocyclones, the operating and performance criterion parameters, the equipment, and the experimental procedure.

2.1 Design of Experiments

The design of experiments for this work are three levels, second order, incomplete block design (Box 1960). This was selected because there was no much theoretical background in hydrocyclone field. An empirical model, say a second order, graduating function, fits the desirable level of simplicity. The design is shown in Table 4.

The three coded levels for these variables are:

$$-1,\ 0,\ +1$$

The symbol ($\pm 1, \pm 1, \ldots \pm 1$) means that all combinations of plus and minus levels are to be run.

TABLE 4. THE DESIGN OF EXPERIMENTS

Design Pattern Used in Part	Number of Variables	Design Matrix	Number of Points (Runs)
B	3	$\begin{matrix} X_1 & X_2 & X_3 \end{matrix}$ $\begin{pmatrix} \pm 1 & \pm 1 & 0 \\ \pm 1 & 0 & \pm 1 \\ 0 & \pm 1 & \pm 1 \\ 0 & 0 & 0 \end{pmatrix}$	12 4 N = 16
A	6	$\begin{matrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \end{matrix}$ $\begin{pmatrix} \pm 1 & \pm 1 & 0 & \pm 1 & 0 & 0 \\ 0 & \pm 1 & \pm 1 & 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 & \pm 1 & 0 & \pm 1 \\ \pm 1 & 0 & 0 & \pm 1 & \pm 1 & 0 \\ 0 & \pm 1 & 0 & 0 & \pm 1 & \pm 1 \\ \pm 1 & 0 & \pm 1 & 0 & 0 & \pm 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	48 4 N = 52

2.2. Hydrocyclone Design, Operating and Performance Criterion Parameters

2.2.1 Hydrocyclone Design Parameters

The hydrocyclone design parameters determine its shape and dimensions. These are given in Table 5.

TABLE 5. HYDROCYCLONE DESIGN PARAMETERS

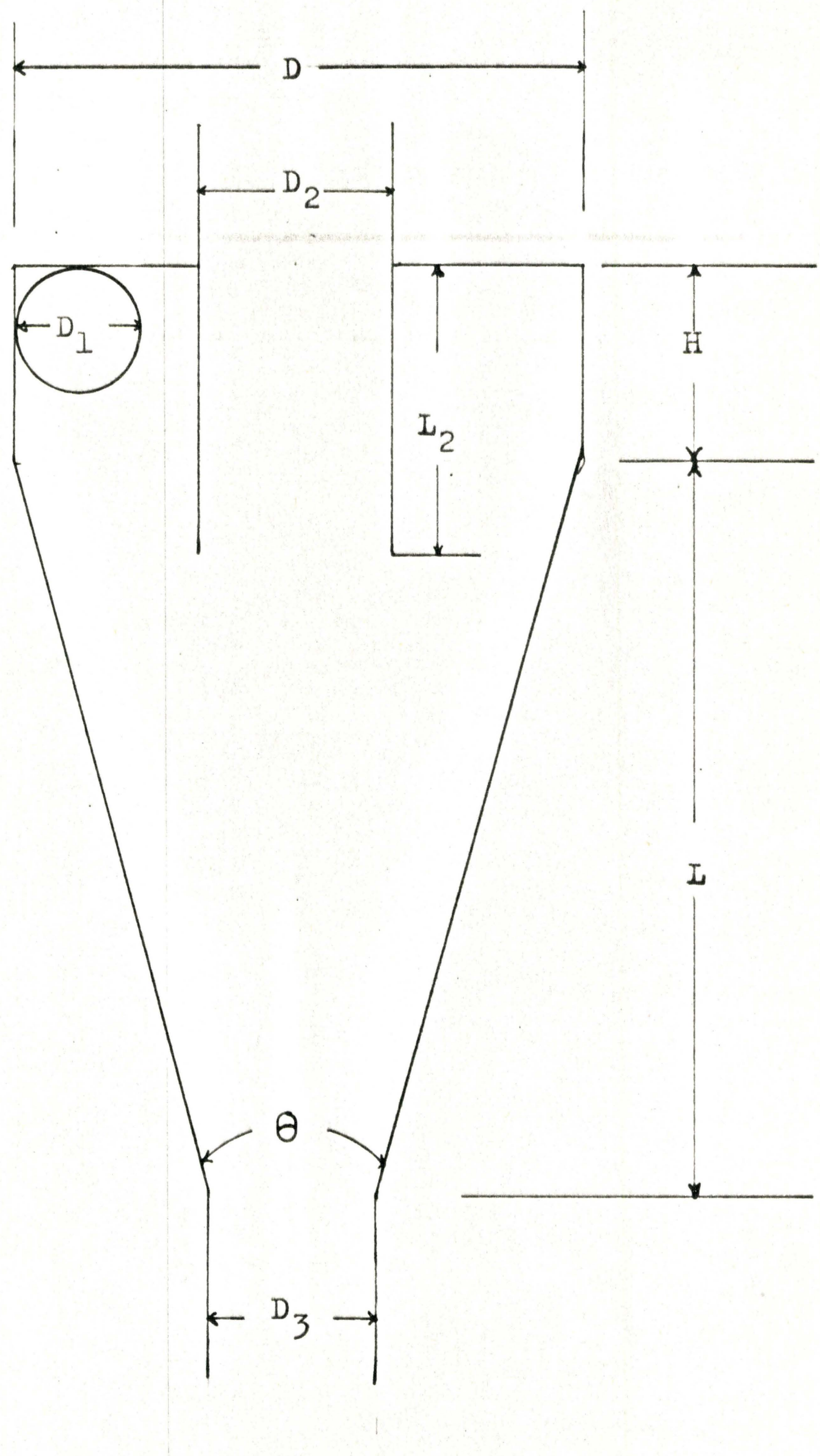
Cyclone diameter	D
Inlet diameter	D_1
Overflow diameter	D_2
Underflow diameter	D_3
Included cone angle	Θ
Overall vertical length of cone	L
Height of cylinder	H
Vortex finder length	L_2
Presence of valves on outlets	Yes or No
Round feed cross section	Yes or No
Body shape	Conical or curved
Presence of internals	Yes or No
Presence of side taps	Yes or No

The cyclone dimension is shown in Figure 1.

(a) Design of hydrocyclone for Part A of the study.

In Part A, three hydrocyclones with the same type of body shape but with different cone angles were used. These hydrocyclones were made of glass with interchangeable ground glass inlet pieces and vortex finders,

FIGURE 1. CYCLONE DIMENSIONS



as shown in Figure 2.

The range of design parameters used in this study is shown in Table 6. Table 7 lists the remaining design parameters.

TABLE 6. THE DESIGN PARAMETERS STUDIED IN THIS WORK

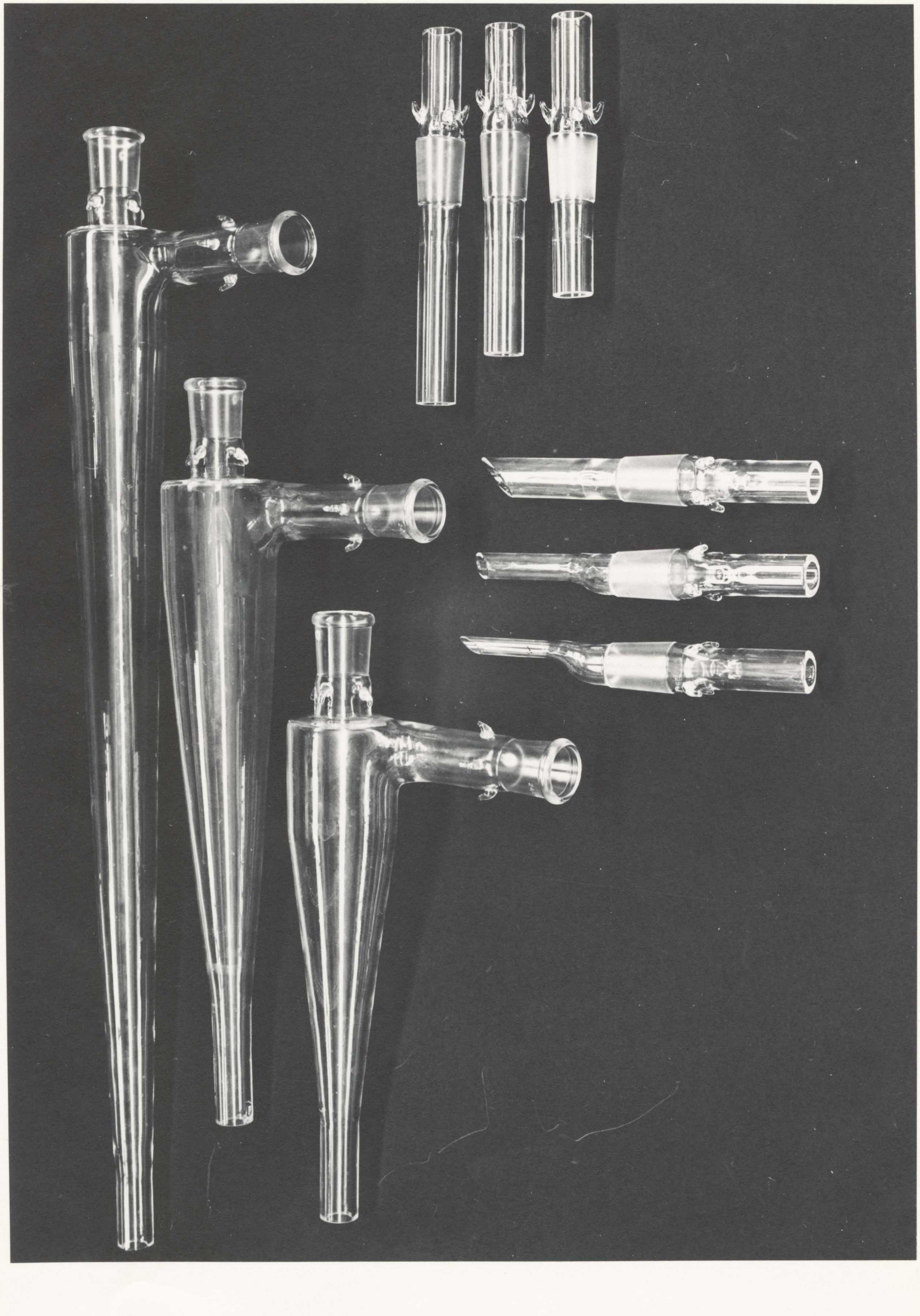
Parameters	Range(three level studied)		
	-1,	0,	+1
inlet diameter (D_1), inches (I,D.)	1/4	3/8	1/2
vortex finder length (L_2), inches	1	2	3
cone angle (Θ), degree	5	10	15

TABLE 7. THE FIXED DESIGN PARAMETERS

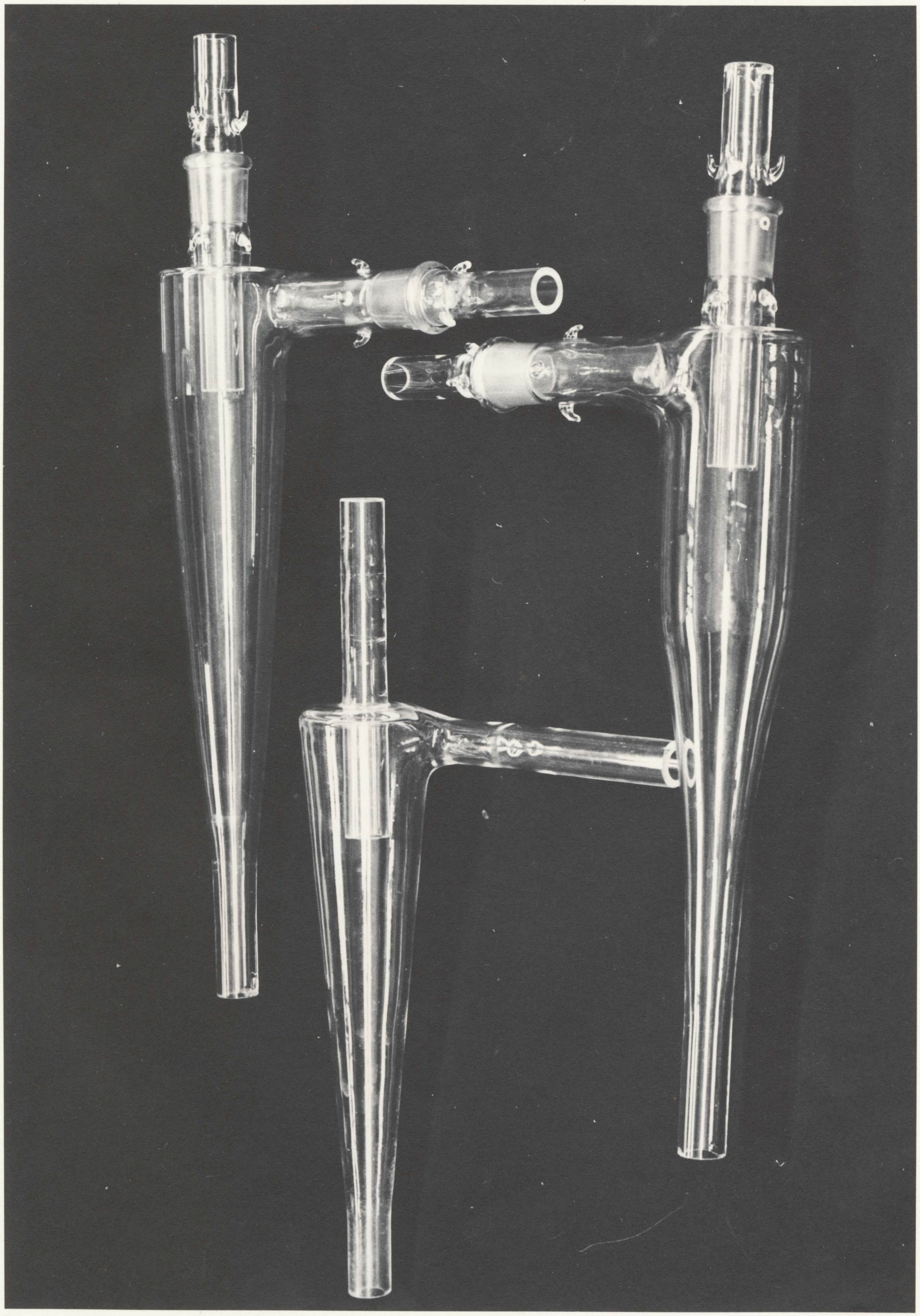
Hydrocyclone No.	1	2	3
D , inches (I.D.)		2	
D_2, inches (I.D.)		1/2	
D_3, inches (I.D.)		1/2	
L , inches	17.2	8.6	5.7
H , inches		2	
Presence of valves on outlet		Yes	
Round feed cross section		Yes	

The presence of valves was desirable since the cyclones were to be used for different operating conditions.

43-A



43-B



(b) Design of hydrocyclone for Part B.

In Part B, three hydrocyclones with different body shape but with the same "cone angle" were used.

These three hydrocyclones were:

1. hydrocyclone with a cylindrical section, Figure 3-a
2. hydrocyclone without a cylindrical section, Figure 3-b
3. hydrocyclone with a curved cone wall, Figure 3-c

In this part of the work, only the influence of the operating parameters on performance was studied.

The designs of the three cyclones are given in Table 8.

TABLE 8. THE DESIGNS OF HYDROCYCLONE a, b, and c.

Hydrocyclone No.	a	b	c
D, inches (I.D.)		2	
D_1, inches (I.D.)		1/2	
D_2, inches (I.D.)		1/2	
D_3, inches (I.D.)		1/2	
Θ, degree	10	10	not constant
L, inches		8.6	
H, inches	2	NO	2
L_2, inches		2	
Presence of valves on outlets		YES	
Round feed cross section		YES	
Comment	with a cylindrical section	without a cylindrical section	with curved wall

Usually cyclone are constructed with a cylindrical section and a conical section, or a conical section only. In this work, a novel cyclone shape with curved cone wall (Figure 3-c) was designed to see what the results would be.

2.2.2. Hydrocyclone Operating Parameters

The operating parameters of any hydrocyclone for a solid/liquid feed are:

feed physical properties : density, viscosity, pH

particle properties: density, behaviour under shear,

aggomeration tendency, size, size distribution

concentration of solid in feed

feed flow rate

volume split (overflow rate/underflow rate)

feed pressure (P_1), and back pressure (P_2 and P_3)

temperature

In this work the solid/liquid system was chosen to be a mixture of solid glass spheres and distilled water. The size distribution of the sphere was log-normal. The geometric mean diameter, $\langle D_G \rangle$ was 37 microns; the geometric standard deviation, σ_G, was 1.24. The particles were spherical. The density of the glass spheres were found to be 2.5 g/cc. The feed was controlled at room temperature ($25^o \pm 2^o C$). The operating parameters that were chosen for study in both Part A and Part B are given in Table 9. The operation was such that no air core existed under any of the operating conditions.

TABLE 9. THE OPERATING PARAMETERS STUDIED IN THIS WORK

Parameter	Range (three level studied)		
	-1	0	+1
feed solid wt. concentration (ppm)	97	514	941
Volume split (overflow/underflow)	1/1	2/1	3/1
Volume flow rate (US Gal/Min)	4	6	8

2.2.3. Performance Criteria Used in This Work

In this work, two hydrocyclone performance criteria were used. The energy loss per unit mass of feed is given by the expression:

$$\Delta\xi/\text{mass} = \frac{\left(\frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2g_c}\right)W_2 + \left(\frac{P_1 - P_3}{\rho} + \frac{V_1^2 - V_3^2}{2g_c}\right)W_3}{W_1} \qquad (32)$$

Other expressions have been reported in the literature to describe the pressure loss factor (Bradley 1965), (Mitzmager and Mizrahi 1964).

$$G = \frac{\Delta P}{\frac{1}{2}\rho V_1^2} \qquad (V_1 : \text{feed velocity})$$

However, for the present work these definitions could not be applied because the pressures at the exit of both underflow and overflow valves were not equal.

The separation efficiency can be considered from a wide variety of viewpoints. Some workers define a cut off diameter, $(D_p)_{50}$, that represented the diameter of a particle that had 50% chance to appear in the overflow and 50% chance to appear in the underflow. Other approaches to defining the separation involve relating the concentrations and liquid volumes of the exit streams to the inlet conditions. Tengbergen and Rietema (1961) discuss the difficulty of selecting a meaningful measure of separation efficiency. The definition chosen for the present study is

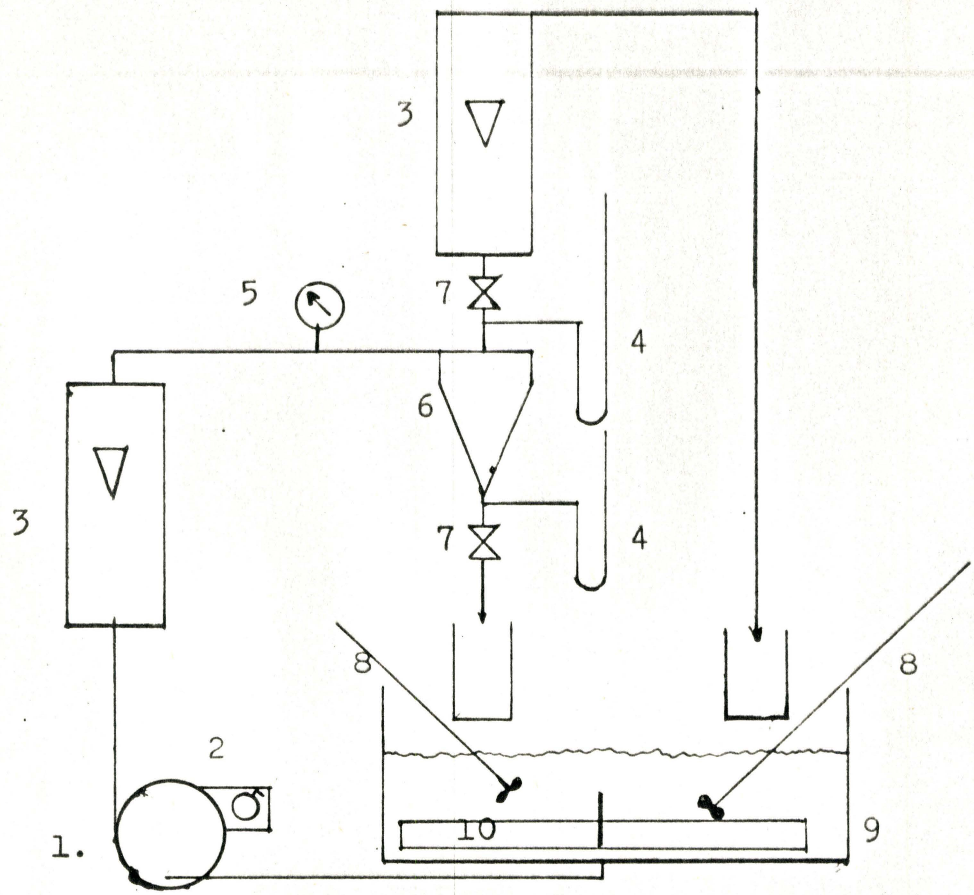
$$\eta = \left| \frac{W_2}{W_1} \left(\frac{w_2 - w_1}{1 - w_1} \right) + \frac{W_3}{W_1} \left(\frac{w_1 - w_3}{w_1} \right) \right| \qquad (33)$$

This expression includes the effects of both fluid and solid on the ideal separation which is to have all fluid out the overflow and all solid out the underflow.

2.3 Equipment

The flow diagram for the equipment is shown in Figure 4.

FIGURE 4. OVERALL FLOW SHEET



1. PUMP
2. MOTOR
3. ROTAMETERS
4. MANOMETERS
5. PRESSURE GAUGE
6. HYDROCYCLONE
7. VALVES
8. AGITATORS
9. RESERVOIR
10. VORTEX BREAKER

2.4 Experiment Procedures

For the work in Part A eight hydrocyclone parameters were studied (three design parameters, three operating parameters, and two performance criterion parameters) with 52 runs being done. For the work in Part B, five hydrocyclone parameters (three operating parameters and two performance criterion parameters) were studied, with 16 runs to be done for each of three hydrocyclone designs. For the three hydrocyclones with different body designs, a total of 48 runs (16x3) were to be done for the work in Part B. The operation procedures for both Part A and Part B were the same, with the only difference being that for the work in Part A, besides three operating parameters, three design parameters were considered. In Part B, only three operating parameters were considered (the design parameters were fixed).

2.4.1 The Preparation of Feed

To keep the feed concentration at each concentration level constant, percisely weighed glass spheres were put in a known volume of distilled water. The temperature of the water and the room was maintained at $25^{o} \pm 2^{o}C$. The feed was agitated violently and assumed to be completely mixed. For each run a pair of samples of the overflow and underflow streams were taken directly by placing beakers under both streams, simultaneously. Feed concentrations were calculated from the concentrations of overflow and underflow streams and their respective feed rate and also checked by taking sample directly from the feed.

The feed concentration was constant (±3%) at each level.

2.4.2 The Operation Procedures

Mixtures of glass spheres and water were taken from the reservoir (9) and pumped by (1) through a rotameter (3) to the hydrocyclone (6). The overflow stream passed through another rotameter so that we could easily measure the volume split. Effluent samples were taken directly by placing beakers under both outlet streams, simultaneously. The feed flow rate was controlled by the variable speed motor drive (2). The installation of valves on the outlet lines of the cyclone permitted the volume split to be varied easily. A pressure gauge (5) measured the feed pressure (P_1). Two manometers (4) measured the back pressures (P_2 and P_3) of the overflow and underflow streams. The rotameters were calibrated for the solid/liquid system used in this work, and the solid particles had a negligible influence on the rotameters as compared with pure water.

2.4.3 Analysis of Samples

For each run the feed pressure, two outlet pressures, feed flow rate, overflow rate and design parameters were recorded. For each run two samples were taken, one from the overflow stream and one from the underflow stream simultaneously. The weight percent of solid in the sample was then measured by filtration. The energy loss per unit mass and the separation efficiency were calculated from

Equations (32) and (33) respectively. A mass balance on the solids over the hydrocyclone system indicated a negligible error.

3. ANALYSIS OF THE RESULTS AND DISCUSSION

3.1 Introduction

For convenience, the analysis of the results and discussion are divided into Part A and B. The theories, characteristics and discussion of usefulness of principal component analysis, canonical correlation analysis, and regression analysis have been fully described in Part I of this thesis.

It should be noted that:

1. The relative signs of the loadings or the coefficients show the relative direction of mutual influence.
2. The relative absolute magnitudes of the loadings or the coefficients show the importance of mutual influence.
3. The missing terms in the regression equations are statistically insignificant as judged by an F test at the 0.01 significant level.

3.2 Analysis of the Results and Discussion for the Work of Part A

The work for Part A is to analyze the operating and design parameters on hydrocyclone performance from the experiment by using a set of three hydrocyclones with cylindrical section. The experimental data (sample size = 52, number of variates = 8) are shown in Appendix A, Table A-1.

The example discussed in Chapter 3 of Part I was taken from this Part A's work. The results from three analysis methods were listed in Tables 1, 2, and 3 in Part I. From that work a study of the results of the three statistical methods indicated some discrepancies in the importance of vortex finder length. This was possibly because the parameters of the hydrocyclone have certain correlation other than a linear one; that is, a linear combination of hydrocyclone parameters is an unsatisfactory way to express the relations among the parameters. Perhaps the parameters correlate as products of the parameters; each parameter with its own exponent. Principal component analysis and canonical correlation analysis were applied again, but for this time a log transformation on raw data (by taking the common logarithm of raw data value) was the first step for each analysis. The results are discussed in the following sections.

3.2.1 Principal Component Analysis on Log Transformed Data.

The principal component analysis was conducted on the correlation matrix, and the results of the rotation of the eight eigenvectors is given in Table 10. The eigenvalue of last two eigenvectors are very small and can be neglected. The analysis was repeated and the first six eigenvectors which are common and unique factors were orthogonally rotated. The result of the second run is listed in Table 10.

TABLE 10. RESULTS OF PRINCIPAL COMPONENT ANALYSIS ON LOG TRANSFORMED DATA FOR THE WORK OF PART A

Variate \ Factor	1	2	3	4	5	6	7	8
Concentration	-0.00	-0.02	-0.01	0.00	-0.01	1.00	-0.02	0.00
Volume split	0.03	1.00	-0.00	0.02	0.01	0.01	-0.03	-0.00
Flow rate	-0.06	0.02	0.01	-1.00	0.02	-0.00	0.03	0.01
Cone angle	-0.04	-0.00	-0.00	-0.01	-1.00	0.01	0.01	-0.00
Inlet diameter	1.00	0.02	-0.01	-0.06	-0.04	-0.01	-0.03	-0.05
Vortex finder length	0.00	-0.01	1.00	-0.00	0.00	-0.01	-0.00	0.00
Energy loss	-0.79	0.07	-0.03	-0.55	-0.20	-0.01	0.09	-0.12
Efficiency	-0.20	0.76	-0.01	-0.12	-0.02	-0.07	0.61	-0.00
Eigenvalue	2.23	1.54	1.03	0.99	0.98	0.97	0.21	0.13

Variate \ Factor	1	2	3	4	5	6
Concentration	-0.01	-0.04	-0.01	0.00	-0.01	1.00
Volume split	0.08	1.00	0.00	0.05	0.01	0.03
Flow rate	-0.06	0.04	0.01	-1.00	0.02	-0.00
Cone angle	-0.05	0.00	-0.00	-0.01	-1.00	0.01
Inlet diameter	1.00	-0.04	-0.01	-0.06	-0.04	-0.01
Vortex finder length	0.00	-0.01	1.00	-0.00	0.00	-0.01
Energy loss	-0.80	0.12	-0.03	-0.55	-0.21	-0.01
Efficiency	-0.22	0.96	-0.02	-0.14	-0.02	-0.08
Eigenvalue	2.23	1.54	1.03	0.99	0.98	0.97

The first factor is interpreted as the inlet diameter controlling factor. The energy loss is directly correlated with the inlet diameter. Increase of inlet diameter decreases the energy loss. Also, an increase of inlet diameter decreases efficiency, but the influence is not so direct. The contribution of other variates to this factor is negligible.

The second factor is considered as a volume split controlling factor. The efficiency is positively directly correlated with the volume split. Although the influence of volume split on energy loss is not significant, they are correlated in the same direction.

The third factor is considered a unique factor which describes the independence of vortex finder length effect.

The fourth factor loads highly on the flow rate, moderately on the energy loss and lowly on the efficiency. This implies that the increase of feed rate increases the energy loss and efficiency, but the influence on energy loss is much more significant than on efficiency.

The fifth factor is related to the influence by cone angle. It shows that increase of cone angle increases the energy loss. The influence on other variates is negligible.

The last factor is also considered as an unique factor and describes the independence of concentration effect. The result from this analysis reveals the role of each factor, and the interaction of the six design and operating parameters is negligible. The factors which influence the energy loss are the first, fourth, and fifth factors. The second factor is the only factor which controls the efficiency. The first factor has only a little influence on efficiency. Thus, the underlying influences on the energy loss and efficinecy may be considered separately. This implies that the control of energy loss and efficiency may be approached individually, since the mutual correlation of these two performance criteria is insignificant. In particular, the vortex finder length and the concentration are unique factors.

3.2.2 From Canonical Correlation Analysis

The result of canonical correlation analysis is listed in Table 11.

Both of the two canonical variates have a high canonical root, which indicate that the criterion variates are predicted well by means of the predictor variates based on the particular linear combinations. Furthermore, the two canonical variates are statistically significant at 1% level.

The first canonical variate shows that these predictor variates can decrease the energy loss with negligible efficiency variation. The relative importance of the predictor variates is arranged in descending order; inlet diameter, flow rate, cone angle. The contribution of the other variates is insignificant. Thus, an increase in inlet diameter, and a decrease of feed rate and of the cone angle decrease the energy loss with negligible

TABLE 11. RESULTS OF CANONICAL CORRELATION ANALYSIS ON LOG TRANSFORMED DATA FOR THE WORK OF PART A

Canonical Variate	1	2
Efficiency	-0.06	0.96
Energy Loss	1.00	-0.27
Concentration	0.01	-0.12
Volume split	0.01	0.99
Flow rate	0.61	0.00
Cone angle	0.25	-0.05
Inlet diameter	-0.75	-0.02
Vortex finder length	-0.04	-0.01
Canonical roots	0.986	0.769
Significance level	0.01	0.01

efficiency increase.

The second canonical variate is a complement of the first and usually describes the phenomena that the first does not fully described. From the second canonical variate we find that volume split is the only controlling parameter in influencing the efficiency; an increase of volume split increases the efficiency directly, and this canonical variate is efficiency predominant.

From the conclusion of both canonical variates we find the controlling parameters for energy loss and efficiency are different; the correlation between efficiency and energy loss is insignificant. This coincides to the findings from principal component analysis. We could increase the efficiency and decrease the energy loss simultaneously by increasing volume split and inlet diameter and by decreasing the feed rate and the cone angle.

3.2.3 The Comparison of Results from Principal Component Analysis and Canonical Correlation Analysis on Log Transformed Data

Although the results from these two methods are difficult to compare quantitatively, the qualitative trends suggested from the two different methods should be comparable. The comparison was studied in terms of the parameters as we did in Section 3.2.4 of Part I.

Concentration

Both of the methods suggest that concentration has a negligible effect on the energy loss and on the separation efficiency. Principal component analysis shows that concentration is a unique factor - the independence of concentration effect.

Volume split

Both of the methods indicate that it is the most important parameter in influencing the efficiency; an increase of the volume split increases the separation efficiency. Also, both methods agree that the volume split has a negligible effect on the energy loss.

Feed flowrate

Principal component analysis suggests that feed flowrate ranks second in affecting the energy loss. Feed flowrate ranks second in the canonical variate dominated by the energy loss. Furthermore, both methods indicate that an increase in flowrate increases the energy loss. Concerning the influence of feed flowrate on the separation efficiency, both methods agree again that it has a negligible effect.

Cone angle

Both methods coincide in predicting the role of cone angle. Principal component analysis suggests that cone angle has negligible influence in the separation efficiency and has very small positive effect (rank the third) on the energy loss.

Cone angle also stands at the third place in the canonical variate dominated by the energy loss, and shows a negligible effect in the canonical variate dominated by the separation efficiency.

Inlet diameter

Both principal component analysis and canonical correlation analysis indicate that inlet diameter is the most important parameter in affecting the energy loss; the direction of influence is in opposite direction. Concerning its effect on the efficiency, principal component analysis shows that it has only a very small effect on the efficiency (the first factor) whereas the canonical variate dominated by the efficiency shows a negligible effect of the inlet diameter. This apparent contradiction merely emphasizes the difference in the type of result each method produced. The criterion variates in the canonical variate are the combination of efficiency and energy loss. Since the inlet diameter predominates the energy loss effect, any correlation technique that combines the criterion variates should show a dependence upon inlet diameter that bears some relation to the amount the energy loss is accounted for in the canonical variate, even though the efficiency tends to dominate.

Vortex finder length

The results from both methods agree again that vortex finder length has a negligible effect on the energy loss and the separation efficiency.

To sum up, the results from both methods coincide very well. The products of the parameters correlation model appear to be better than a linear combination model (comparing with Section 3.2.4 in Part I). Therefore we might assume that the hydrocyclone parameters should correlate as products of the parameters; each parameter with its own exponent.

3.3 Analysis of The Results and Discussion for The Work of Part B

3.3.1 Introduction

In Part A the effect of operating and design parameters on hydrocyclone performance for a set of hydrocyclones with a cylindrical section was studied. In this part we want to study especially the influence of three operating parameters on the performance of hydrocyclones with three different body shapes. Besides body shape, the other design parameters were kept the same. Principal component analysis, canonical correlation analysis, and regression analysis are applied to each of the three sets of experiments of using hydrocyclones a, b, and c respectively.

The experimental data for each hydrocyclone are listed in Appendix A, Table A-2, A-3 and A-4 respectively. Since we are especially interested in comparing the performance among the three body shape hydrocyclones, the regression analysis is presented first for this purpose. Then we discuss the results from the other two methods for finding different information for each body shape hydrocyclone.

3.3.2 Analysis of Results from Multiple Regression Analysis for Hydrocyclone a, b, and c

Hydrocyclone a

The result from regression analysis is listed in Table 12 in coded form.

TABLE 12. RESULT OF REGRESSION ANALYSIS FOR HYDROCYCLONE a

$$\text{Efficiency} = 41.4406+1.964\ X_1 + 11.6137\ X_2 + 3.1575\ X_3 + 3.4187\ X_1^2 - 3.5762\ X_2^2$$

(multiple correlation coefficient = 0.98)

$$\text{Energy loss} = 5.9863+0.4861\ X_2 + 4.0967\ X_3 + 0.9462\ X_3^2$$

(multiple correlation coefficient = 0.99)

The separation efficiency regression equation shows that volume split is the most important parameter among operating parameters in influencing efficiency. The effect of concentration and flow rate is small comparing with volume split. The energy loss regression equation shows that the flow rate is the most important parameter among operating parameters in influencing energy loss. The influence from volume split is very small.

Hydrocyclone b

The result from regression analysis is listed in Table 13 in coded form.

TABLE 13. RESULT OF REGRESSION ANALYSIS FOR HYDROCYCLONE b

$$\text{Efficiency} = 66.2562 + 12.4712\ X_2 + 0.6175\ X_3 - 4.1675\ X_2^2 - 0.4550\ X_3^2$$

(multiple correlation coefficient = 0.96)

$$\text{Energy loss} = 12.8069 + 0.5485\ X_2 + 8.1911\ X_3 - 1.0063\ X_2^2 + 1.0027\ X_3^2$$

(multiple correlation coefficient = 0.98)

The efficiency regression equation shows that the volume split is still the most important parameter among operating parameters in influencing the efficiency. The influence from the flow rate is very small comparing with volume split. The energy loss regression equation shows that the flow rate is the most important parameter among operating parameters in influencing the energy loss. The influence from volume split is very small comparing with the flow rate.

Hydrocyclone c

The result from regression analysis is listed in Table 14 in coded form.

TABLE 14. RESULT FROM REGRESSION ANALYSIS FOR HYDROCYCLONE c

$$\text{Efficiency} = 30.20 + 11.035\ X_2 + 11.2412\ X_3 + 7.7587\ X_3^2$$

(multiple correlation coefficient = 0.89)

$$\text{Energy loss} = 6.293 - 1.97874\ X_1 + 4.9933\ X_3$$

(multiple correlation coefficient = 0.92)

The efficiency regression equation shows that the flow rate and volume split are two importance operating parameters in influencing the efficiency. The increase of these two operating variates increases the efficiency. The energy loss regression equation shows that the flowrate is the most important operating variate. The effect of concentration on energy loss ranks second in importance.

3.3.3 The Comparison of Performance among Hydrocyclone a, b, and c

The understanding of relative importance of operating parameters in influencing performance has been described in each section; we are not repeating here, but somebody might ask "what kind of hydrocyclone body shape has the best performance?". To answer this question we should apply the Analysis of Covariance (Fisher, 1954) (Davies, 1961) to compare between groups of results and find the differences, but unfortunately, significant differences between regression slopes are found and therefore Analysis of Covariance is invalid in this case. However, qualitatively, we still might compare the performance among three hydrocyclones. Concerning the separation efficiency, hydrocyclone b has the best separation. Hydrocyclone a stands in the second place. Hydrocyclone c has the worst separation. Possibly, the poor separation of the curved wall hydrocyclone is due to re-entrainment of the particles by the upward axial current in the lower part of the hydrocyclone. Concerning the energy loss hydrocyclone b has the highest energy loss. The difference of energy loss between hydrocyclone a and hydrocyclone c is small and the energy loss of both of them is less than hydrocyclone b. Therefore, the choice of hydrocyclone body shape should depend on the real situation that which criterion is more concerned.

3.3.4 The Analysis and Comparison of Results from Principal Component Analysis and Canonical Correlation Analysis for Hydrocyclone a, b, and c

Realizing that principal component and canonical correlation analyses are not designed to compare the three alternative designs based on performance, nevertheless let us use these statistical methods to see if radical changes in the significance of the operating parameters occur among different body designs.

Hydrocyclone a

The results from principal component analysis and canonical correlation analysis on log transformed data for hydrocyclone a are listed in Table 15 and 16 respectively.

The results from both principal component analysis and canonical correlation analysis show that first, concentration has a negligible effect on the energy loss and the separation efficiency, second, feed flowrate is the most important operating parameter in influencing the energy loss, and third, the volume split is the most important operating parameter in influencing the efficiency. The correlation between the energy loss and the efficiency is insignificant.

TABLE 15. RESULTS OF PRINCIPAL COMPONENT ANALYSIS FOR HYDROCYCLONE a (log tansformation on raw data)

Factor / Variate	1	2	3
Concentration	1.00	-0.03	0.01
Volume split	-0.03	-1.00	-0.02
Flow rate	1.00	-0.03	0.01
Energy loss	0.99	-0.15	-0.02
Efficiency	0.23	-0.97	0.06
Eigenvalue	2.34	1.60	1.00

TABLE 16. RESULTS OF CANONICAL CORRELATION ANALYSIS FOR HYDROCYCLONE a (log transformation on raw data)

Canonical variate	1	2
Efficiency	-0.13	0.97
Energy loss	0.99	-0.25
Concentration	-0.03	0.09
Volume split	0.00	1.00
Flow rate	1.00	0.00
Canonical root	0.998	0.968
Significant level	0.01	0.01

Hydrocyclone b

The results from principal component analysis and canonical correlation analysis on log transformed data for hydrocyclone b are listed in Table 17 and 18 respectively.

TABLE 17. RESULTS OF PRINCIPAL COMPONENT ANALYSIS FOR HYDROCYCLONE b (log transformation on raw data)

Factor / Variate	1	2	3
Concentration	0.01	0.00	1.00
Volume split	-0.00	-1.00	0.00
Flow rate	-1.00	0.01	0.01
Energy loss	-1.00	-0.07	-0.03
Efficiency	-0.06	-1.00	-0.01
Eigenvalue	2.11	1.87	1.00

TABLE 18. RESULTS OF CANONICAL CORRELATION ANALYSIS FOR HYDROCYCLONE b (log transformation on raw data)

Canonical variate	1	2
Efficiency	-0.53	0.88
Energy loss	0.85	0.48
Concentration	-0.03	-0.03
Volume split	-0.50	0.87
Flow rate	0.86	0.50
Canonical root	0.999	0.993
Significant level	0.01	0.01

For hydrocyclone b, the roles of feed flowrate and volume split are same as we discussed for hydrocyclone a. Principal component analysis also suggests that a correlation between the energy loss and the efficiency is negligible. On the contrary, canonical correlation analysis suggests a dependency between the two criterion parameters. The cause of this apparent contradiction is not known.

Hydrocyclone c

The results from principal component analysis and canonical correlation analysis on log transformed data for hydrocyclone c are listed in Tables 19 and 20 respectively.

TABLE 19. RESULTS OF PRINCIPAL COMPONENT ANALYSIS FOR HYDROCYCLONE c(log transformation on raw data)

Variate \ Factor	1	2	3
Concentration	0.08	-0.01	1.00
Volume split	0.07	-1.00	-0.02
Flow rate	-1.00	-0.06	0.07
Energy loss	-0.96	-0.13	-0.24
Efficiency	-0.56	-0.83	0.06
Eigenvalue	2.50	1.30	1.00

TABLE 20. RESULTS OF CANONICAL CORRELATION ANALYSIS FOR HYDROCYCLONE c (log transformation on raw data)

Canonical variate	1	2
Efficiency	0.19	0.83
Energy loss	0.98	-0.55
Concentration	-0.27	0.34
Volume split	0.20	0.93
Flow rate	0.94	-0.10
Canonical root	0.992	0.735
Significant level	0.01	0.01

Principal component analysis shows that first, the feed flowrate is still the most important operating parameter in influencing the energy loss, but the effect on the efficiency from flow rate can not be neglected. This is also the main difference between hydrocyclone c and hydrocyclone b. Secondly, the volume split is still the most important parameter in affecting the separation efficiency. Third, it is interesting to note that concentration has an opposite effect on the energy loss. Also in both canonical variates, the concentration appears to have some small effect on the criterion variates. However, the effect from the dilute concentration on the performance is questionable. It is also possible that we might attribute this phenomena to the unknwon flow pattern in a curved wall hydrocyclone.

4. CONCLUSIONS

1. The combination of three statistical techniques provides a very powerful means of interpreting data. Since the underlying philosophy of each method is different, and each method has its different function and emphasis, this combination supplies three different ways of looking at the same data. The qualitative trends in the data should be the same for all three statistical methods. However, in this work some discrepancies did occur. For example, the contradiction exists as to the correlation between the energy loss and the separation efficiency for hydrocyclone b in Part B. In general, the results from three methods agree each other.

2. Concerning the correlation model for the hydrocyclone parameters, the parameters correlate as products of the parameters; each parameter with its own exponent appears to be better than a simple linear combination one. The search for the real correlation model in nonlinear form is needed.

3. For the range of parameters covered in this work, feed flowrate and inlet diameter influenced the energy loss the most; volume split influenced the separation efficiency the most.

4. The most efficient cyclone was a straight conical hydrocyclone with no cylindrical section; this shape however required the largest amount of energy.

5. The body shape affects which parameters significantly affect performance.

6. The correlation between energy loss and separation are not very significant, especially for a cyclone that does not have a cylindrical section.

7. This problem provided a meaningful and useful comparison of three statistical techniques and demonstrated the importance to a chemical engineer of becoming very familiar with the statistical tools available.

8. The ppm concentrations common to waste water studies can be separated in a hydrocyclone. No design shape that is drastically different from common practice is indicated.

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APPENDICES

APPENDIX A

EXPERIMENTAL DATA

x_1 = concentration of feed, ppm
x_2 = volume split(overflow rate/underflow rate)
x_3 = feed flow rate, US GAL/min
x_4 = cone angle, degree
x_5 = inlet diameter, inch
x_6 = vortex finder length, inch
η = separation efficiency×100
$\Delta\xi/mass$ = energy loss per unit mass, (ft)(lb force)/(lb mass)

TABLE A-1: EXPERIMENTAL DATA FOR THE WORK OF PART I & PART A(of PART II)

UNCODED LEVELS						CODED LEVELS						η	$\Delta\xi/mass$
x_1	x_2	x_3	x_4	x_5	x_6	x_1	x_2	x_3	x_4	x_5	x_6		
514.	3.	8.	10.	0.500	2.	0.	1.	1.	0.	1.	0.	53.93	11.71
514.	1.	8.	10.	0.500	2.	0.	-1.	1.	0.	1.	0.	30.27	10.03
514.	1.	4.	10.	0.500	2.	0.	-1.	-1.	0.	1.	0.	21.57	2.38
514.	3.	4.	10.	0.500	2.	0.	1.	-1.	0.	1.	0.	44.84	3.07
514.	1.	8.	10.	0.250	2.	0.	-1.	1.	0.	-1.	0.	49.39	31.59
514.	3.	8.	10.	0.250	2.	0.	1.	1.	0.	-1.	0.	74.24	35.57
514.	1.	4.	10.	0.250	2.	0.	-1.	-1.	0.	-1.	0.	48.32	20.25
514.	3.	4.	10.	0.250	2.	0.	1.	-1.	0.	-1.	0.	73.16	21.06
514.	2.	6.	10.	0.375	2.	0.	0.	0.	0.	0.	0.	63.73	14.56
514.	2.	6.	10.	0.375	2.	0.	0.	0.	0.	0.	0.	63.69	14.67
514.	2.	6.	10.	0.375	2.	0.	0.	0.	0.	0.	0.	63.87	14.70
514.	2.	6.	10.	0.375	2.	0.	0.	0.	0.	0.	0.	63.56	14.75
514.	1.	6.	10.	0.500	1.	0.	-1.	0.	0.	1.	-1.	47.64	9.67
514.	3.	6.	10.	0.500	1.	0.	1.	0.	0.	1.	-1.	72.49	9.24
514.	3.	6.	10.	0.500	3.	0.	1.	0.	0.	1.	1.	52.24	6.21
514.	1.	6.	10.	0.500	3.	0.	-1.	0.	0.	1.	1.	30.85	5.22
514.	3.	6.	10.	0.250	3.	0.	1.	0.	0.	-1.	1.	72.83	45.19
514.	1.	6.	10.	0.250	3.	0.	-1.	0.	0.	-1.	1.	47.39	43.75
514.	1.	6.	10.	0.250	1.	0.	-1.	0.	0.	-1.	-1.	45.76	49.22
514.	3.	6.	10.	0.250	1.	0.	1.	0.	0.	-1.	-1.	69.39	49.77
514.	2.	4.	15.	0.375	1.	0.	0.	-1.	1.	0.	-1.	50.33	7.15
514.	2.	8.	15.	0.375	1.	0.	0.	1.	1.	0.	-1.	61.30	34.36
514.	2.	8.	15.	0.375	3.	0.	0.	1.	1.	0.	1.	64.98	32.48
514.	2.	4.	15.	0.375	3.	0.	0.	-1.	1.	0.	1.	58.88	6.97
514.	2.	8.	5.	0.375	3.	0.	0.	1.	-1.	0.	1.	62.24	17.14
514.	2.	4.	5.	0.375	3.	0.	0.	-1.	-1.	0.	1.	58.53	3.64
514.	2.	4.	5.	0.375	1.	0.	0.	-1.	-1.	0.	-1.	54.39	3.85
514.	2.	8.	5.	0.375	1.	0.	0.	1.	-1.	0.	-1.	57.61	17.31

TABLE A-1 (continued)

97.	2.	4.	10.	0.375	1.	-1.	0.	-1.	0.	0.	-1.	56.71	5.36
97.	2.	8.	10.	0.375	1.	-1.	0.	1.	0.	0.	-1.	63.42	28.24
97.	2.	4.	10.	0.375	3.	-1.	0.	-1.	0.	0.	1.	62.66	5.75
97.	2.	8.	10.	0.375	3.	-1.	0.	1.	0.	0.	1.	66.24	27.69
97.	3.	6.	15.	0.375	2.	-1.	1.	0.	1.	0.	0.	73.09	16.93
97.	1.	6.	15.	0.375	2.	-1.	-1.	0.	1.	0.	0.	47.81	15.59
97.	2.	6.	15.	0.500	2.	-1.	0.	0.	1.	1.	0.	66.14	13.26
97.	2.	6.	15.	0.250	2.	-1.	0.	0.	1.	-1.	0.	60.90	53.31
97.	2.	6.	5.	0.250	2.	-1.	0.	0.	-1.	-1.	0.	49.81	30.93
97.	2.	6.	5.	0.500	2.	-1.	0.	0.	-1.	1.	0.	62.20	5.71
97.	1.	6.	5.	0.375	2.	-1.	-1.	0.	-1.	0.	0.	46.56	8.22
97.	3.	6.	5.	0.375	2.	-1.	1.	0.	-1.	0.	0.	70.30	9.81
941.	2.	4.	10.	0.375	1.	1.	0.	-1.	0.	0.	-1.	56.44	5.83
941.	2.	8.	10.	0.375	1.	1.	0.	1.	0.	0.	-1.	63.41	26.72
941.	2.	4.	10.	0.375	3.	1.	0.	-1.	0.	0.	1.	61.44	5.82
941.	2.	8.	10.	0.375	3.	1.	0.	1.	0.	0.	1.	65.77	26.21
941.	3.	6.	15.	0.375	2.	1.	1.	0.	1.	0.	0.	68.58	17.49
941.	1.	6.	15.	0.375	2.	1.	-1.	0.	1.	0.	0.	44.06	15.78
941.	2.	6.	15.	0.500	2.	1.	0.	0.	1.	1.	0.	65.88	13.10
941.	2.	6.	15.	0.250	2.	1.	0.	0.	1.	-1.	0.	55.93	51.10
941.	3.	6.	5.	0.375	2.	1.	1.	0.	-1.	0.	0.	69.76	9.25
941.	1.	6.	5.	0.375	2.	1.	-1.	0.	-1.	0.	0.	45.66	8.13
941.	2.	6.	5.	0.500	2.	1.	0.	0.	-1.	1.	0.	61.90	5.56
941.	2.	6.	5.	0.250	2.	1.	0.	0.	-1.	-1.	0.	52.35	39.02

TABLE A-2: EXPERIMENTAL DATA FOR HYDROCYCLONE a

UNCODED LEVELS			CODED LEVELS			η	$\Delta \xi$/mass
x_1	x_2	x_3	x_1	x_2	x_3		
514.	3.	8.	0.	1.	1.	53.93	11.71
514.	1.	8.	0.	-1.	1.	30.27	10.03
514.	1.	4.	0.	-1.	-1.	21.57	2.38
514.	3.	4.	0.	1.	-1.	44.84	3.07
514.	2.	6.	0.	0.	0.	41.75	5.94
514.	2.	6.	0.	0.	0.	41.92	5.91
514.	2.	6.	0.	0.	0.	41.20	5.84
514.	2.	6.	0.	0.	0.	41.74	5.96
97.	2.	4.	-1.	0.	-1.	39.84	2.98
97.	1.	6.	-1.	-1.	0.	27.81	5.64
97.	3.	6.	-1.	1.	0.	50.12	6.52
97.	2.	8.	-1.	0.	1.	46.66	11.33
941.	2.	4.	1.	0.	-1.	45.72	2.91
941.	3.	6.	1.	1.	0.	55.86	6.35
941.	1.	6.	1.	-1.	0.	32.19	5.71
941.	2.	8.	1.	0.	1.	46.37	11.06

TABLE A-3: EXPERIMENTAL DATA FOR HYDROCYCLONE b

UNCODED LEVELS			CODED LEVELS				
x_1	x_2	x_3	x_1	x_2	x_3	η	$\Delta\xi$/mass
514.	3.	8.	0.	1.	1.	74.74	21.13
514.	1.	8.	0.	-1.	1.	49.78	18.77
514.	1.	4.	0.	-1.	-1.	48.57	4.72
514.	3.	4.	0.	1.	-1.	73.56	5.15
514.	2.	6.	0.	0.	0.	66.27	12.51
514.	2.	6.	0.	0.	0.	66.29	12.29
514.	2.	6.	0.	0.	0.	66.25	12.47
514.	2.	6.	0.	0.	0.	66.33	12.42
97.	2.	4.	-1.	0.	-1.	65.14	5.39
97.	1.	6.	-1.	-1.	0.	49.73	12.08
97.	3.	6.	-1.	1.	0.	74.66	13.03
97.	2.	8.	-1.	0.	1.	66.63	22.23
941.	2.	4.	1.	0.	-1.	65.13	5.19
941.	3.	6.	1.	1.	0.	74.37	12.09
941.	1.	6.	1.	-1.	0.	49.48	11.43
941.	2.	8.	1.	0.	1.	66.19	22.86

TABLE A-4: EXPERIMENTAL DATA FOR HYDROCYCLONE c

UNCODED LEVELS			CODED LEVELS				
x_1	x_2	x_3	x_1	x_2	x_3	η	$\Delta\xi$/mass
514.	3.	4.	0.	1.	-1.	34.77	2.96
514.	1.	4.	0.	-1.	-1.	13.23	2.38
514.	1.	8.	0.	-1.	1.	48.51	9.61
514.	3.	8.	0.	1.	1.	72.99	11.68
514.	2.	6.	0.	0.	0.	30.64	6.02
514.	2.	6.	0.	0.	0.	30.48	5.93
514.	2.	6.	0.	0.	0.	30.33	5.90
514.	2.	6.	0.	0.	0.	30.15	5.86
97.	2.	4.	-1.	0.	-1.	30.29	2.93
97.	1.	6.	-1.	-1.	0.	19.44	8.97
97.	3.	6.	-1.	1.	0.	40.88	10.39
97.	2.	8.	-1.	0.	1.	37.84	18.54
941.	2.	4.	1.	0.	-1.	28.58	2.44
941.	3.	6.	1.	1.	0.	40.25	6.30
941.	1.	6.	1.	-1.	0.	19.43	5.43
941.	2.	8.	1.	0.	1.	37.46	10.84

APPENDIX B

FEED SIZE DISTRIBUTION

a. Introduction

Many calculations may be performed on the data collected in this work. For example in testing the applicability of the $(D_p)_{50}$ criterion, the feed size distribution must be known. The purpose of this appendix will be to give complete measurements on the glass beads used in the feed so that the data will be as complete as possible.

b. Particle Size Distribution

(i) Theory and Measurements

There are two general size distributions laws in nature: the normal distribution, and the log-normal distribution. To test whether our size distribution follows either law, we will plot measurements of bead size versus cumulative percent on normal-probability and log-noraml probability paper and see which is better described by a straight line.

To measure the bead size, a small sample of the beads was placed on a glass slide and photographed through a microscope. A graticule was also photographed under the microscope so that the total magnification could be determined.

The negatives were printed on very thin photographic paper and the bead size measured with a Zeiss particle size analyzer. This instrument compares the image size of the bead on the paper with a circular beam of light generated by the instrument. A recorder with 48 size intervals registers the drop counted under a given size interval.

(ii) Results

Three different bead samples were photographed and roughly 330 beads were sized and counted per photograph. A total of 999 beads were measured.

This size-number information was plotted on log-normal probability paper (Figure A1) and found to yield a good straight over most of its length.

The following data result:

Geometric mean diameter	37 μ
Geometric standard deviation	1.24

About 68% of the beads lay within the size range $37 \pm 8\mu$, and about 95% of the beads lay within $37 \pm 13\mu$.

The photographs also indicated that the beads were spherical in shape.

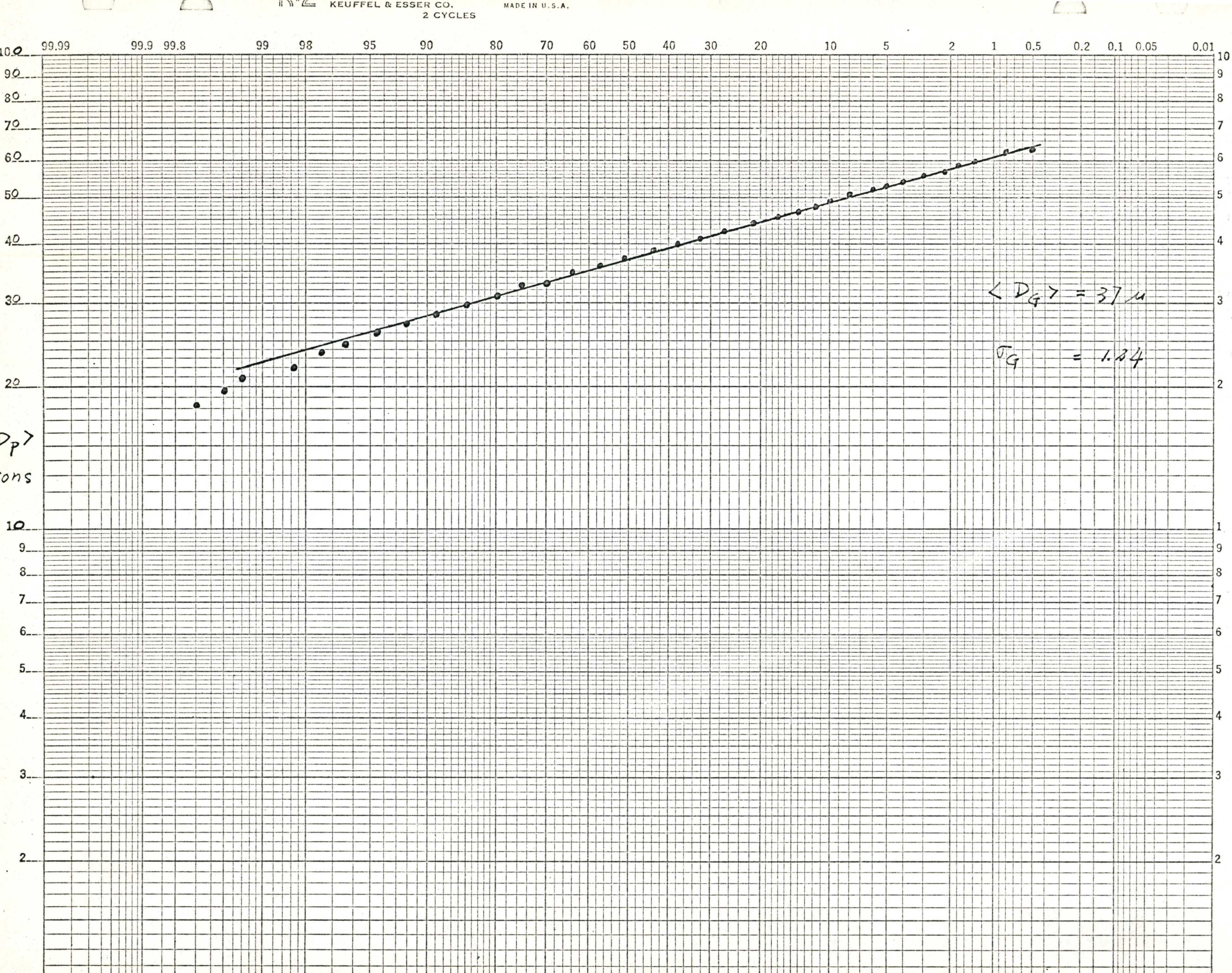

FIGURE A1. PARTICLE SIZE DISTRIBUTION

APPENDIX C

EQUIPMENT SPECIFICATIONS AND SUPPLIERS

EQUIPMENT	SPECIFICATIONS	SUPPLIER
Feed Pump	Frame 1L3, Type CDQ, Tubular Moyno pump	The Robbins & Myers Co., Brantford, Ont. Canada.
Reeves variable speed motodrive	Size 100	Reliance Electric & Engineering Co., Columbus, Indiana, U.S.A.
Rotameters	0-10 USGPM 316 Stainless Steel and Teflon	Fischer-Porter 1110 A Wilson Ave. Downsview, Ont.
1" & 5/8" O.D. Tubings	Type 304 Stainless Steel	Atlas Alloy Metal Sales, 215 Lakeshore Rd. Toronto, Ont.
Swagelok Fittings	Type 316 Stainless Steel	Niagara Valve 102 Parkdale Ave. N. Hamilton, Ont.
Pressure Gage	0-60 p.s.i. 316 Stainless Steel	Thomson-Gordon Ltd. 200 Queen St. N. Hamilton, Ont.
Glass Cyclone	See Chapter II (of Part II) for details	Glass Blower McMaster University Hamilton, Ont.
Reservoir	316 Stainless Steel,	Machine Shop Eng. Bldg. McMaster University, Hamilton, Ont.

Glass Beads	Sample #908 $\rho = 2.5$ g/c.c.	3M Company, St. Paul, Minnesota, U.S.A.
Filter	Triacetate Metrical Filter, GA-1.	Gelman Instrument Company, Ann Arbor, Michigan, U.S.A.
Mayon Flexible Tubing	P.V.C.	Warehouse Plastic Sales 571 Gerrard Street Toronto 8, Ont.
Tubing compressor clamp	3/4 x 1 nickel plated brass	Fisher Scientific Limited, Toronto.