A STUDY OF LEVELS POPULATED IN 166 Er by THE (3 He,d) REACTION

A STUDY OF LEVELS POPULATED

IN ¹⁶⁶Er BY THE (³He,d) REACTION

BY

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The reaction 165 Ho (3 He,d) 166 Er was studied using the University of Rochester Tandem Van de Graaff accelerator and an Enge-type splitpole magnetic spectrograph. The observed cross sections for the ground state rotational band and the gamma vibrational band were consistent with predictions based on the unified nuclear model. The population of the 1574 keV level was consistent with that expected for the K^{π}=4⁻, 7/2-[523] + 1/2+[411] assignment obtained from decay scheme studies. The K^{π}=2⁻ octupole vibrational band based on the 1460 keV state was not populated as strongly as expected, indicating that the admixture of the 7/2-[523] - 3/2+[411] two proton state into this vibration is only about one-fifth as large as predicted by Soloviev et al.

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CHAPTER I

INTRODUCTION

Most investigations of the nuclear structure of heavy deformed nuclei have been done by beta and gamma decay studies. Such studies often involve measurement of gamma-ray transition probabilities, branching ratios, internal conversion coefficients, gamma-gamma and beta-gamma coincidences. Spins and parities of many states below 2 MeV excitation and of some higher states have thus been established. However, many states still remain unexplained.

In spite of the fact that beta-gamma techniques are well established methods of obtaining nuclear structure information, some limitations to their usefulness exist. Firstly, some levels are not populated or are populated very weakly in the decay scheme studies because of the selection rules of beta and gamma decay processes. These selection rules are stronger than those of the nuclear reactions. Most of the beta decays in heavy nuclei are allowed or first forbidden, which means that a maximum of one or two units of angular momentum are transferred. In a typical (³He,d) reaction, such as studied in the present work, observable cross sections are obtained when five or six units of angular momentum are transferred. Also in a beta decay study, the Q-value is small enough so that only a limited range of excitation energy can be populated. In cases where the Q-value is greater the number of de-exciting gamma-rays is very large and an analysis becomes complicated.

It is now technically possible to carry out reaction studies on heavy nuclei with sufficient precision to obtain results complementary to those provided by the decay scheme studies. The word "complementary" should be stressed, because some states can be populated by the beta decay and not by the reactions and vice versa.

The Coulomb barrier has provided one of the difficulties encountered in carrying out reaction studies in heavy nuclei, because the older accelerators which were capable of energies sufficient to overcome the Coulomb barrier did not have well defined beam energies. As a result, the closely spaced levels commonly found in heavy nuclei could not be resolved. The development of the Tandem Van de Graaff accelerator has almost removed this difficulty. The model MP(so-called "Emperor" Tandem) accelerators which are installed at several institutions accelerate Helium-ions up to 30 MeV with an energy spread of \pm 3 KeV.

Another technical problem has been the design of a detector system with high resolution to analyze the reaction products. At present, the newest magnetic spectrographs offer the best resolving power for this type of study.

Up to now the levels in 166 Er have been investigated by means of decay scheme studies using sources of 166 Ho $(J^{\pi}=7^{-})^{(1,2)}$, 166m Ho $(J^{\pi}=0^{-})^{(2)}$ and 166 Tm $(J^{\pi}=2^{+})^{(2,3)}$, and spins and parities of many states have been established. In addition, Coulomb excitation ${}^{(4)}$, (d,d'), ${}^{(5)}$ and $(d,t)^{(6)}$ reaction studies have been used to study the levels in this nucleus. So far, in spite of the large number of data available, there is still considerable uncertainty concerning the interpretation of some of the levels.

In the present work, the $({}^{3}\text{He},d)$ reaction has been used to provide further information in hopes of clarifying some of the uncertainties. It is to be noted that the (d,t) reaction transfers a neutron and thus can be used to populate two-neutron states in ${}^{166}\text{Er}$ whereas the (${}^{3}\text{He},d$) reaction transfers a proton and thus can populate two-proton states. In this respect these two single particle transfer processes are complementary.

The level structure of this nucleus is usually explained within the framework of the unified nuclear model^(7,8), which has been quite successful in this mass region. On the basis of this model the differential cross section for a single nucleon transfer reaction leading to a member of a rotational band can be calculated using the theory of Satchler⁽⁹⁾, a single particle cross section obtained from a DWBA ⁽¹⁰⁾ (Distorted Wave Born Approximation) calculation and the Nilsson wave functions.⁽⁸⁾

Soloviev ⁽¹¹⁾, Bés ⁽¹²⁾ and co-workers have made extensive microscopic calculations for the vibrational states in deformed nuclei. Whereas the vibrational states have been commonly described as superpositions of large numbers of single particle components each with a small amplitude, the calculations have shown that certain of the amplitudes can sometimes be fairly large. In an even-even nucleus the vibrational states consist mainly of superpositions of two quasi-particle states. It is not uncommon to find amplitudes as large as 0.7 or 0.8. In particular the calculations have shown that the $K^{\pi}=2^{-}$ octupole vibration in ¹⁶⁶Er should contain a large two-neutron component and a large twoproton component. The first of these has been verified experimentally by the use of the (d,t) reaction (6). One of the main reasons for carrying out the present experiment was to look for the predicted two-proton component.

CHAPTER 2

THEORETICAL CONSIDERATIONS

2.1 Unified Nuclear Model

2.1.1. Introduction

The Nuclear shell model has successfully explained the magic numbers, ground state spins, magnetic moments, etc., but it could not explain the quadrupole moments which were observed in many nuclei. In the strongly deformed region, these are $10 \sim 100$ times larger than the values calculated on the basis of the single particle model. It could also not explain the strong enhancement of E2 transitions which are a conspicuous feature of many nuclei (13).

To account for the large quadrupole moment which shows that nucleus is no longer spherical, N. Bohr and J.A. Wheeler (14) introduced the liquid drop model. This model also has shortcomings, however. It predicts too few levels and these are too widely spread.

A. Bohr and B. Mottelson ⁽⁷⁾ combined the nuclear shell model and the liquid drop model into a single "unified model". They studied a system consisting of a few nucleons moving in a field provided by a nuclear drop.

They postulated the following two properties;

1) The existence of non-spherical equilibrium shape.

2) The separability of the nuclear wave functions.

The property (1) comes from the experimental evidence of the large quadrupole moment which indicates the deformation of the nucleus. The second property is the so-called adiabatic assumption which implies that

the rotation of the nucleus is very slow compared with the rotation of individual particles in their orbits, so that one can treat the nuclear rotation adiabatically.

One writes the nuclear Hamiltonian

 $H = H_{vib} + H_{rot} + H_{int} + H_{coup}$

Where H_{vib} and H_{rot} are the energy operators of vibrational motion and rotational motion, respectively. H_{int} is the internal motion, independent of the rotation of nucleus. H_{coup} represents any coupling interaction. Therefore, the total wave function is of the form of

$\Psi \sim \psi_{\rm wib} D_{\rm X}$

The function ψ_{vib} describes the nuclear shape and its state of vibration. The function D describes the orientation of the shape, and the deformed shell model wave function χ describes the nucleon's motion in the deformed potential well.

Since the potential is no longer spherical, the quantum numbers j and ℓ of each nucleon are no longer constants of motion. However, the axial symmetry of the lowest states of the deformed nucleus is preserved and therefore, Ω_j , the projection of j on the symmetry axis remains a good quantum number for each orbit.

The rotational motions of the deformed nucleus are well described in a number of references (15) and therefore only a few remarks are made here. As shown in Fig. 1, the total angular momentum \hat{I} is a vector sum of a rotational operator \hat{R} and an internal angular momentum operator \hat{J} ;

$$\vec{1} = \vec{R} + \vec{J}$$
$$\vec{J} = \Sigma \vec{j}_i$$

J is not a good quantum number, but its projection on the symmetry axis Z' is a good quantum number. One writes this quantum number $\Omega(=\Sigma\Omega_j)$.

FIGURE 1

Angular momentum diagram for a deformed nucleus. Z and Z' stand for the space fixed axis and the symmetry axis, respectively.



•

The projections of \vec{I} on the symmetry axis and on the space fixed axis are K and M, and they are also good quantum numbers.

When one takes into account the interaction between the rotational and vibrational motions, one obtains the following expression for the total energy;

 $E_{tot} = \epsilon_{IK} + A[I(I+1) - K(K+1)] + B[I^2(I+1)^2 - K^2(K+1)^2].$ (1) Here A and B are constants within a band and are usually determined by the experiment.

In recent years, many calculations of the microscopic structure of the vibrational motions have been carried out by Soloviev (11), Bés(12) and co-workers. The properties of these states were studied under the assumption of the superconducting pairing correlations (16) plus quadrupole-quadrupole, octupole-octupole, and other interactions which are responsible for the collective properties of nuclei.

Therefore, the interaction between nucleons in the nucleus is described by a Hamiltonian of the form;

 $H = H_{av} + H_{pair} + H_{coll}$

 H_{av} describes the average nuclear field. The interaction H_{pair} leads to the superconducting pair correlations which are described by a superposition of various two-quasiparticle states in the case of an even-even nucleus. The solutions for the problem are derived by means of second quantization, that is, two-quasiparticle states are obtained by a canonical transformation of Bogolyubov operating on the single particle Nilsson states. It is assumed that G in $H_{pair} = -G \sum_{ss'} A^+_{s} A^+_{s} A_{s'} A_{s'+}$ is constant. A^+ and A are a creation and annihilation operator, respectively, for the Fermi particle, while A_{s-} stands for the time reversal operator of A_{s+} and vice versa. Energies and wave functions for the vibrational states of strongly deformed even-even nuclei were calculated by solving a secular equation derived by a variational principle. As discussed in the Introduction in Chapter I, the results of these calculations show that some of the two-quasiparticle components making up the vibrational motion have rather large admixtures. Some of these components have been measured by means of single particle transfer reactions and good qualitative agreement between experiment and theory has been obtained in many cases (17).

2.1.2. Nilsson Model

The Nilsson model ⁽⁸⁾ has proven to be very successful in predicting many aspects of nuclear structure. The model assumes a single particle Hamilton representing the interaction of one nucleon with the nuclear field provided by the rest of nucleons;

$$H = \frac{\vec{p}^2}{2m} + V(\vec{r},\beta) + C\vec{z}\cdot\vec{s} + D\vec{z}^2$$

where C, D and β are adjustable parameters. A further substitution is made;

$$\kappa = -\frac{C}{2h\omega_{0}}$$
$$\mu = \frac{2D}{C}$$

The values of κ and μ have been fixed so as to make the isotropic levels (zero deformation) coincide with those of the shell model as closely as possible.

The wave function χ in the body fixed axis is expanded in terms of the shell model wave functions which are the eigenfunctions of angular momentum.

2.2. Theoretical Cross Sections

2.2.1. Spectroscopic Factor

The stripping reactions were originally discussed by Butler who used the plane wave Born approximation. As the name implies, this approximation treated the incoming beam and the reaction products as plane waves. These reactions have been shown to be very useful for studying nuclear structure. Both theory and experiment showed unique angular distribution patterns for the various *L*-values and in many cases one could determine the parity and the orbital angular momentum of states in the final nucleus. It was in the spirit of this theory to neglect the excitation of the target nucleus before capture and of the final nucleus after capture. In general the plane wave theory was successful in predicting the position of the first maximum in the angular distributions for reactions on light nuclei. However, it did not yield accurate values for the absolute cross sections or the correct shapes for the angular distributions.

The plane wave theory has now been replaced by the more sophisticated distorted wave theory which takes into account effects of the Coulomb and nuclear forces which "distort" the incoming and outgoing "plane waves".

The reactions in this work are considered to be direct reactions, in which a nucleon is transferred from the projectile to the target nucleus and the remaining nucleons in the projectile continue on their way. It is assumed that there is no formation of a compound nucleus.

French and Raz ⁽¹⁸⁾ have given the following cross sections for such "stripping" reactions;

$$\frac{d \sigma(\theta)}{d\Omega} = \frac{2I_{f}+1}{2I_{i}+1} \sum_{\ell} S_{\ell} \phi_{\ell}(\theta)$$
(2)

 I_i and I_f are the total angular momenta of the initial and final nucleus, respectively. θ is the angle between the incident beam and the observed reaction products. $\phi_{\ell}(\theta)$ is the single particle reaction cross section for angular momentum transfer ℓ at an angle of θ . This quantity can be obtained by means of the DWBA calculations ⁽¹⁰⁾ as discussed in the next section. The spectroscopic factor S_{ℓ} is determined by the nuclear structure only, and is thus independent of angle θ , Q-value, etc. If one uses the Nilsson wave functions, the value of S_{ℓ} for a transition to a rotational state in a deformed nucleus is;

$S_{\mathfrak{L}} = \Sigma \Theta 2$ $\mathbf{j} \mathbf{j} \mathcal{L}$			
$\theta_{j\ell} = g \sqrt{\frac{2I+1}{2If+1}} < $	I _i j <u>+</u> K∓∆K I _f	$K_{f} > C_{j\ell} < \psi_{i}$	^ψ f >

The $C_{j\ell}$ values ⁽¹⁹⁾ are characteristic of the orbital into which the proton is stripped and are related to the Nilsson coefficients $A_{\ell\Lambda}$ by means of the Clebsch-Gordon transformation;

$$C_{j\ell} = \sum_{\Lambda} A_{\ell\Lambda} < \ell \quad 1/2 \quad \Lambda \quad \Sigma | j \quad \Omega >$$

where the quantities Λ, Σ , K and Ω are the same as those used by Nilsson. K_i and K_f are the spin components parallel to the symmetry axis in the initial and the final nucleus, respectively. ΔK is the K-value of the orbital into which the nucleon is stripped. $\langle \psi_i | \psi_f \rangle$ is the overlap integral for the initial and the final states of core and is expected to be nearly unity if the nuclear deformation does not vary greatly. The factor g is equal to $\sqrt{2}$ if $K_i = 0$ or $K_f = 0$, and is otherwise equal to unity.

The pairing effects⁽¹⁶⁾, which are a common feature in heavy nuclei, were taken into account simply by multiplying the cross section, eg. (2) by an emptiness factor U^2 (or a fullness factor V^2), so that one has;

$$\frac{d \sigma(\theta)}{d\Omega} = \frac{2I_{f}+1}{2I_{f}+1} \sum_{\ell} S_{\ell} \phi_{\ell}(\theta) \{ o_{\ell} \}$$
(3)

A simple explanation for this is that the cross section for putting a particle into a given state is proportional to the probability of the state being empty. However, V^2 should be used when the final state is the ground state of an even-even nucleus, since the probability of finding the unpeired proton in the target ground state orbital v is unity, and the cross section for putting a proton in this state is given in eg. (2). Following this process there is a condition of two protons in the state v which, however, is not the ground state of the even nucleus, since the latter has only a probability $V^2(v)$ of state v being full. Hence, the cross section eg.(2) should be multiplied by the factor $V^2(v)$, which is the overlap between these two cases.

2.2.2. DWBA Theory

The DWBA theory of calculating direct nuclear reaction cross sections has become widely used during the last several years. In this approximation, the transition amplitude is calculated as a first-order matrix element between initial wave functions and final wave functions. That is, for the A(a,b)B reaction, the transition amplitude is;

 $T_{ba} = (\psi_{B}\psi_{b}\chi_{b} (-) (\vec{k}_{b}, \vec{r}_{b}), V\psi_{A}\psi_{a}\chi_{a} (+) (\vec{k}_{a}, \vec{r}_{a}))$ (4)

 ψ_{B} , ψ_{b} , ψ_{A} and ψ_{a} are the internal wave functions for the noninteracting, separated particles B, b, A and a. The interaction V is the interaction whose off-diagonal matrix elements are responsible for the transition. Here \vec{r}_{a} is the displacement of a from A, and \vec{r}_{b} is the displacement of b from B. The functions χ_{a} and χ_{b} are the "distorted waves". They are elastic scattering wave functions which describe the relative motion of the pair a,A (asymptotically, with relative momentum K_{a}) before the collision, or of the pair b, B (with K_{b}) after the collision. Asymptotically the χ have the form of plane wave and scattered wave; in the absence of a Coulomb field when r becomes large, the χ approaches the right hand side of the following equation;

$$\chi^{(+)}(\vec{k},\vec{r}) \rightarrow \exp(i\vec{k}\cdot\vec{r}) + f(\theta) \frac{\exp(iKr)}{r}$$

The superscript (+) or (-) denotes the usual outgoing or ingoing wave boundary conditions. It is convenient to first isolate in eg.(4) the matrix element of the interaction, taken between the initial and the final states;

< Bb | V |Aa > =
$$f\psi_{B}^{*}\psi_{b}^{*}V\psi_{A}\psi_{a}d\xi$$
 (5)

 ξ represents all the co-ordinates independent of \vec{r}_a and \vec{r}_b . Generally, the calculations of eg. (5) are fairly easy. In terms of this matrix element, the calculation of the transition amplitude is completed in the form,

$$T_{ba} = \int dr_a \int dr_b \chi_b (-)^* (\vec{k}_b, \vec{r}_b) \cdot Bb | V| Aa > \chi_a^{(+)} (\vec{k}_a, \vec{r}_a)$$
(6)

Several approximations other than the first order perturbation theory were made in the calculation of eg. (6). Certain interactions, which depend on the type of the reaction, are neglected. For example, in the (3 He,d) reaction, the interaction between the deuteron and the target nucleus, and some exchange terms are neglected. In addition to these, the DWBA calculations consider only direct processes and compound processes are neglected.

Uncertainties in the calculated DWBA cross sections depend mainly upon the following factors;

- Insufficient knowledge of Helium-3 potential and deuteron potential.
- 2. Radius R_o where proton is captured.
- 3. If one uses cut-off, "how", "where" and "how much".

CHAPTER 3

EXPERIMENTAL PROCEDURE AND RESULTS

Targets of 165 Ho were bombarded with beams of 28 MeV 3 He-ions obtained from the Model MP Tandem Van de Graaff accelerator at the University of Rochester. The scattered particles were analyzed with an Enge split-pole broad range magnetic spectrograph using Kodak NTB nuclear-emulsions 50µ thick to record the deuteron groups. An Aluminum foil 0.83 mm thick was placed in front of the emulsions to stop triton groups which had the same magnetic rigidity as the deuterons being observed. The energies of protons, deuterons, Helium-3 and alphaparticles resulting from the 3 He-induced reactions with the 165 Ho targets were low enough , so that these particles could not interfere with the deuteron groups being studied. At small angles, however, some peaks in the spectra were obscured by deuterons from $({}^{3}\text{He},d)$ reactions on the 13 C in the carbon backing. Two 165 Ho targets with thickness of approximately $100\mu q/cm^2$ evaporated on carbon backings were purchased from Fodor Accelerator Targets of Pittsburg, Pennsylvania. Other ¹⁶⁵Ho targets were made in this laboratory by vacuum-evaporating ¹⁶⁵Ho from a crucible on carbon foils which were 10 to 30 μ g/cm² thick.

The desired beam of particles obtained from the Tandem Van de Graaff was deflected 90 degrees by the analyzing magnet which selected the particles with the required energy. The beam then passed to the magnetic spectrograph. Magnetic quadrupole lenses were located at the various points along the beam line to focus the beam to a small spot.

The properties of the Enge-type split-pole magnetic spectrograph (20) used in this study are similar to those of the Elbek-type (21) magnetic spectrograph. The advantage of these spectrographs over the previously designed magnets is that they have completely eliminated second order aberrations over the full momentum range. The usable momentum range is P_{max}/P_{min} and is 2.5 for the Elbek magnet and 2.8 for the Enge magnet. A cross sectional view of the Enge-type magnetic spectrograph is shown in Fig. 2.

A monitor counter located in the spectrograph target chamber was used to count the ³He-ions which had been elastically scattered through an angle of 90 degrees. The monitor counter consisted of a 1 mm thick NaI scintillator and a photomultiplier. Pulses from the monitor were amplified and sent to a multichannel pulse height analyzer and a singlechannel pulse height analyzer simultaneously. A typical block diagram is shown in Fig. 3. Pulses corresponding to the elastic scattering events were selected by the single-channel pulse height analyzer and sent to a scaler. The scaler readings were used to normalize the intensity of the deuteron peaks recorded on nuclear emulsions. The multichannel pulse height analyzer was used to observe the whole spectrum from the monitor counter while the experiment was in progress and to check whether the spectrum appeared to be reasonable.

The camera assembly included a six sided rotatable drum so that six sets of emulsions could be loaded at the same time. The photographic plates were scanned by counting the tracks in strips, 0.25 mm wide, using microscopes.

FIGURE 2

Enge-type split-pole magnetic spectrograph and the target chamber. Two pole pieces are shown by dashed curves.



FIGURE 3

Block diagram for the counting circuitry.



The conventional Faraday-Cup and the current integrator were used to obtain the absolute value of the integrated current or the charge, that is, the total number of 3 He-ions which passed through the target. This information was used to estimate the target thickness.

Slits were used at the entrance to the magnetic field region of the spectrograph to limit the entrance aperture and define the solid angle.

The reaction cross sections were determined by comparing the intensities of the observed deuteron groups with that of the elastically scattered 3 He peak in the monitor counter. The ratio of the solid angles for the spectrograph and the monitor counter was determined from the geometrical dimensions of apertures. The cross section for the groups thus observed is;

 $\left(\frac{d \sigma}{d\Omega}\right)_{\text{spectrograph}} = \left(\frac{d \sigma}{d\Omega}\right)_{\text{monitor}} \cdot \frac{d\Omega \text{ monitor}}{d\Omega \text{ spectrograph}} \cdot \frac{N \text{ spectrograph}}{N \text{ monitor}}$

 $d \, \Omega$ and N refer to the solid angle and the number of counts observed, respectively.

In order to determine $\left(\frac{d \sigma}{d\Omega}\right)_{elastic}$, it was assumed that the cross section for small angle scattering approaches the Rutherford cross section, since for the small angle scatterings the projectile does not come close to the nucleus. By measuring the angular distributions of elastically scattered ³He and equating the small angle values to the Rutherford cross sections, the elastic scattering cross sections at large angles could be determined. The angular distributions were thus normalized to the absolute values. This procedure was not carried out for the ¹⁶⁵Ho target, but was interpolated from similar data for ¹⁷⁴Yb and ¹⁸²W targets. A typical angular distribution showing the elastic scattering for 174 Yb is presented in Fig.4. For the 165 Ho experiments, the elastic scattering cross section at θ = 90 degrees was taken to be ≈ 6.0 % of the Rutherford cross section or $_{\%}7.8$ mb/sr.

Energy calibration of the spectrograph was carried out using 6.0498 MeV and 8.7864 MeV alpha-particles from an active deposit of ThB

placed in the target position. The α -particle momentum is proportional to the magnetic rigidity B_{ρ} , where ρ is the radius of the trajectory of the α -particles in a magnetic field of strength B. By selecting a set of values for the magnetic field, the α -particles of certain energy take on a set of values for ρ and hence a set of peaks at various positions along the focal plane are obtained. In this way, one can establish a mathematical relation between ρ and the distance along the focal plane, D.

The deuteron spectra were measured at angles of 15, 22, 29.5, 45, 53, 60, 65 and 70 degrees and the observed spectrum at the angle of 53 degrees is shown in Fig. 5. The interpretations shown for some of the levels are described in Chapter 4. The excitation energies and absolute cross sections for many of the states populated in the present reaction are presented in Table 1. As the ground state was not populated very strongly, the excitation energies were measured relative to that of the 265 keV state whose position was well known in the decay scheme studies.

The Q-value for the present reaction is estimated from the mass tables (22) to be 1902 keV with an uncertainty of 34 keV. The experimental Q-value obtained in our reaction study is 1880 keV and the error is \pm 50 keV. The error is large since the energy of the incident beam has not been measured precisely.

FIGURE 4

Elastic scattering of ³He-ions on ¹⁷⁴Yb. Circles indicate the experimental values of $(\frac{d \sigma}{d\Omega})_{elastic} / (\frac{d \sigma}{d\Omega})_{Rutherford}$.



FIGURE 5

Deuteron spectrum from the $^{165}\mathrm{Ho}~(^{3}\mathrm{He},\mathrm{d})$ $^{166}\mathrm{Er}$ reaction at the angle of 53 degrees.



TABLE 1

Energy Present (keV)	Energy Previo (keV)	ous	κ,Ι ^π	Mode	15 ° #	22°	29.5°	45 ° #	53 °	60°	65°	70°
1 80 265 547 788 859 957 1573 1658 1686 1785 1825 1916 1986 2054 2137 2221 2604	0 81 265 545 787 860 957 1574 1662 1688 1785 1825	a) a) a) a) a) b) b) b)	$\begin{array}{c} 0, 0^{+} \\ 0, 2^{+} \\ 0, 4^{+} \\ 0, 6^{+} \\ 2, 2^{+} \\ 2, 3^{+} \\ 2, 4^{+} \\ 4, 4^{-} \\ (4), 5^{-} \\ (4), 6^{-} \\ (4), 6^{-} \\ (4), 6^{-} \end{array}$	A A B B C * C * C * C *	23.5 6.6 10.7 2.6 5.3 5.2 36.7 43.2 33.3 19.5	21.9 6.0 9.9 1.22 3.3 11.6 45.2 51.8 49.8 30.0 72.5	0.16 1.05 3.8 2.1 0.26 26.8 8.2 11.2 2.9 6.5 18.8 50.5 52.0 49.8 35.6 139	$\begin{array}{c} 0.18\\ 1.34\\ 4.4\\ 2.1\\ 0.30\\ 0.48\\ 7.8\\ 5.7\\ 7.2\\ 1.6\\ 3.5\\ 11.5\\ 28.8\\ 33.8\\ 32.4\\ 20.1\\ 75.5 \end{array}$	$\begin{array}{c} 0.20\\ 0.86\\ 3.4\\ 1.7\\ 0.33\\ 0.29\\ 13.6\\ 4.7\\ 6.6\\ 1.07\\ 3.0\\ 9.3\\ 24.8\\ 25.2\\ 23.3\\ 14.4\\ 45.4 \end{array}$	$\begin{array}{c} 0.05\\ 0.73\\ 2.3\\ 1.22\\ 0.20\\ 0.20\\ 0.11\\ 9.9\\ 3.1\\ 4.6\\ 1.07\\ 2.5\\ 6.4\\ 17.7\\ 20.4\\ 19.1\\ 12.5\\ 35.9 \end{array}$	$\begin{array}{c} 0.18\\ 0.40\\ 2.1\\ 1.15\\ 0.09\\ 0.14\\ 0.08\\ 6.8\\ 3.1\\ 3.9\\ 0.70\\ 1.7\\ 4.6\\ 13.8\\ 12.9\\ 15.0\\ 8.2\\ 26.1 \end{array}$	0.45 1.8 0.72 0.10 0.10 0.16 5.7 2.6 1.8 0.83 1.6 3.6 10.8 10.9 11.7 8.1 23.8

THE EXCITATION ENERGIES AND THE OBSERVED CROSS SECTIONS (ub/sr)

- a) Reference (3)
- b) Reference (1)
- * See discussion in 4.5
- A Ground State Rotational Band
- B Gamma Vibrational Band
- C $K^{\kappa} = 4^{-}$ Rotational Band

Experimental data at 15° and 45° were divided by a factor of 1.7 in order to make the angular distributions smoothly varying functions of angles at 15° and 45°. This normalization is necessary partly because these data were taken on a different day using different apertures for the monitor and spectrograph. Other experimental data obtained with the same geometrical conditions at the same times indicate that a correction of 1.25 should be applied.

CHAPTER 4

INTERPRETATIONS OF THE SPECTRA

4.1. Introduction

For some (3 He,d) reactions on heavy even-even nuclei (23), the angular distributions have been found to give some indications of the &-value, but with 28 MeV 3 He-particles the patterns are not nearly so unique as for many studies on light nuclei. As mentioned in Chapter 2, the cross sections to each state of 166 Er can have contributions from several different &-values. Even if the calculation predicts that some levels have predominantly one &-value, it is quite difficult to determine the &-value definitely, because of inevitable errors in the cross section and the similarity of the angular distributions for various &-values. Detailed discussions on the observed cross sections for several rotational bands are made in the following sections.

DWBA calculations for the reaction studied in this work have not been carried out. However, the calculations were available for Samarium and Wolfram targets. An interpolation of these calculations has been performed to obtain estimates of theoretical cross sections for Holmium. The justification and procedure for this interpolation is discussed in an Appendix.

4.2. Ground State Rotational Band

The ground state configuration of the odd-Z nucleus 165 Ho has been confirmed to have proton in the 7/2 - [523] Nilsson orbital⁽²²⁾. The five lowest levels of 166 Er constitute a rotational band with K=0,

7/2-[523] - 7/2-[523] in agreement with Bohr and Mottelson's unified nuclear model. A member of this band can thus be populated with the (³He,d) reaction by transferring one proton into the 7/2-[523] orbital. For this Nilsson state, the calculated C_{jk}^2 coefficient is largest for j = 11/2 (k=5). The C_{jk}^2 coefficients for j = 7/2(k=3) and j = 9/2 (k=5) are 30 ~40 times smaller than for j = 11/2. The single particle cross section $\phi_{k=5}$ is fairly small, so it is expected that this band will be populated rather weakly, though substitution of these values to eg. (2) show that the 4⁺ and the 6⁺ states are expected to have the largest cross sections.

As shown in Fig. 5, the cross sections for this band are small, especially to the ground state and the fourth excited state (8^+) , so that the statistical errors are large. If some unwanted particles mix in the peak, they change the cross sections appreciably. At the angles of 15 and 22 degrees almost all the peaks of this band were obscured by the strong intensity of deuteron groups from Carbon and other impurities.

The angular distributions of this band are shown in Fig. 6. Good agreement is not obtained for the ground state transition because of poor statistics. However, good agreement for the 265 keV state (4^+) and the 545 keV state (6^+) are obtained. The 81 keV state (2^+) shows a fairly good agreement.

It should be mentioned that the theoretical values were multiplied by a factor of 0.9 to fit the experimental data. In other words, the fullness factor, V^2 , for the 7/2-[523] Nilsson orbital in the ground state band in ¹⁶⁶Er is 0.9. The value 0.9 is somewhat larger than normally expected for this orbital, because a state near the

FIGURE 6

Angular distributions of the ground state rotational band. The theoretical curves are all multiplied by a factor of 0.9.



GROUND STATE BAND

Fermi level would be expected to have V^2 equal to about one-half. However, the probable error in deduced value of V^2 is large because of poor statistics and uncertainties in the theoretical cross sections with which experimental values were compared.

4.3. Gamma Vibrational Band

The rotational band based on the gamma vibrational state at 787 keV has been studied by other groups (1,2,3), and has been found to be in good agreement with theoretical predictions by Bohr and Mottelson's unified nuclear model.

Microscopic calculations of the structure of this band show no large quasi-particle component which could be populated in the $({}^{3}He,d)$ reaction.

Indeed, in the spectrum of the (³He,d) reaction, the cross sections to the members of the gamma vibrational band are small. As we have no assurance that the observed cross sections are, in fact, due to a direct reaction process, no further discussion concerning this band will be made.

4.4. Octupole Vibrational Band

Žylicz <u>et al</u> ⁽³⁾ have suggested that levels observed in the decay scheme study at 1460, 1516 and 1784 keV form the 2⁻, 3⁻ and 6⁻ members of the $K^{\pi}=2^{-}$ octupole band. The octupole nature of this band was confirmed by Tjøm and Elbek ⁽⁵⁾ who performed the inelastic deuteron scattering experiment and observed a large B(E3) value connecting the 3⁻ member of this band to the ground state.

The microscopic calculations of Soloviev (11) predict that the rotational band based on the $K^{\pi}=2^{-}$ vibration has two relatively large two-quasiparticle components; the two-neutron configuration 7/2+[633] - 3/2-[521] with an admixture of $\frac{1}{2}58\%$, and the two-proton configuration 7/2-[523] - 3/2+[411] with an admixture of $\frac{1}{2}36\%$. The two-neutron component was confirmed in the (d,t) reaction ⁽⁶⁾ with an admixture of $\frac{1}{2}50\%$ in agreement with prediction.

One of the major reasons for undertaking the present experiment was to search for the two-proton configuration 7/2-[523] - 3/2+[411]. This component should contribute to cross sections of the octupole states in the $({}^{3}He,d)$ reaction with an intensity of ${}^{3}36\%$ of those indicated by eq. (3). However, the experimental data show no strong population for the 1460 keV level, the band member, which is expected to have the largest cross section. There are not enough counts to make meaningful measurements of peak positions and intensities. However, an attempt can be made to set an upper limit on the observed cross section for the 1460 keV level. The results are shown in Table 2 where the entries in column 3 are the numbers of tracks observed in an energy interval corresponding to a peak width at 1460 keV. The cross sections corresponding to these numbers are shown in column 4. The cross sections which would be expected for this state can be obtained from eq. (3) using a value of $U^2 \approx 15\%$. This estimate of U^2 is based on an analysis such as that shown in Fig. 23 of reference 24, assuming that A $_{lpha}$ 800 keV $^{(25)}$ and that the 3/2+[411] state (22) in the ¹⁶⁵Ho target is found at the excitation energy 360 keV.

As discussed above, this result should also be multiplied by a factor of 0.36, because the two-proton component being populated is expected to make up this fraction of the states. The predicted cross sections obtained in this manner were shown in column 2 of Table 2.

TABLE 2

<u>(µb/sr)</u>

Angles (degree)	Expected Cross Sections	A Number of Counts Observed	Corresponding Maximum Cross Sections	Ratios
15	1.00	obscured		
22	0.84	obscured		
29.5	1.16	obscured		
45	0.69	7	0.14	0.20
53	0.43	7	0.12	0.28
60	0.32	not observed	0	0
65	0.26	5	0.059	0.23
70	0.21	5	0.044	0.21

Ratios of experimental values to theoretical estimates are given in the last column. Their average is $_{3V}$ 0.18 and indicates that the two-proton configuration 7/2-[523] - 3/2+[411] has an admixture $\gtrsim 7\%$ in the K^T=2⁻ octupole vibration; not $_{3V}36\%$ as the microscopic calculations of Soloviev predict.

4.5. $K^{\pi}=4^{-}$ Rotational Band

In an even-even nucleus the level density above the first single particle state increases rapidly as the excitation energy, so that experimental studies on these states become complicated. Very little knowledge about the physical nature of these states has been obtained. With more sophisticated methods, people have made attempts to investigate the nature of these high excited states.

Indeed in ¹⁶⁶Er, above the level at 1574 keV, many states are populated in the present reaction studies. Before discussing in detail some of the levels populated in the (³He,d) reaction, it is important to review the investigations done by other groups. The results of these attempts are listed in Table 3.

Some of the levels listed in Table 3 have been discussed in connection with the $K^{\pi}=2^{-}$ octupole band. $\dot{Z}ylicz \underline{et} \underline{al}^{(3)}$ suggested that the 6⁻ state at 1784 keV belonged to the octupole band and from the I(I+1) rule they predicted 4⁻ and 5⁻ states should be located at 1589 and 1679 keV. Two 5⁻ states have previously been observed at 1662 and 1688 keV. $\dot{Z}ylicz \underline{et} \underline{al}$ suggested that the 1688 keV level did not belong to the octupole band, but to a $K^{\pi}=4^{-}$ band based on a 4⁻ level at 1574 keV. As discussed in 4.4, in the inelastic

TABLI	Ξ3
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(keV)	
•		

Żylicz <u>et al</u>		Reich and Cline		Burson <u>et</u> <u>al</u>		Tjøm and Elbek		(d,t)	Present Work
Excitation Energy	Ι ^π	Excitation Energy	Ι ^π	Excitation Energy	I ^π	Excitation Energy	Iπ	Excite. Energy	Excite. Energy
1574	(4)-			1572	4-			1571	157 3
1590	(4 ⁻)							1597	
		i		1663	ו ⁺	1662			
		1662	(5)	1667	5			1667	1658
1687	(5)	1688	(5 ⁻)	1693	(5)	1698		1693	1686
				1704	(3 ⁺ ,4 ⁺)				
						1719	3		
								1722	
						1759			
					(\cdot)			1761	
1787	(6 ⁻)	1784	6	1787	$6^{(+)}$			1787	1785
		1825	(6 ⁻)	1828	(5,6)			1826	1825
				1831	ר י				

I and π in parentheses indicate tentative assignment.

deuteron scattering experiment the strongly excited level at 1516 keV was confirmed to be the spin 3 member of the octupole band. However, the proposed 5⁻ state around 1679 keV was not observed and possible 5⁻ states of the octupole band were found at 1662 keV and 1698 keV.

Żylicz <u>et al</u> have assigned the $K^{\pi=4}$ band with 4⁻, 5⁻ and 6⁻ members at 1574, 1688 and 1825 keV to be the two-proton configuration. However, in view of the uncertainties which are still present, it can not be clearly said which 5⁻ or 6⁻ states belong to the octupole band and which to the $K^{\pi}=4^-$ band. In principle the stripping and the pickup reactions should be useful in resolving this ambiguity. For instance, the rotational band based on the above two-proton configuration should not be populated in the (d,t) reaction ⁽⁶⁾ which transfers a neutron. The 1574 keV was indeed populated only very weakly in the (d,t) reaction. However, weak peaks were found in the (d,t) spectrum at or near the energies expected for both the 5⁻ and 6⁻ states belonged to the octupole band from that reaction.

One might expect that the $({}^{3}\text{He},d)$ reaction would solve this problem. As seen in last section, the 2⁻ member of the octupole band was essentially not populated in this reaction. Since the 2⁻ member was expected to have the largest cross section, it is logical to assume that any cross sections for higher spin states in this band would be negligible. As the 4⁻, 5⁻ and 6⁻ members of the band based on the K^{π} =4⁻ two-proton state should have appreciable cross sections in the $({}^{3}\text{He},d)$ reaction, one might hope to isolate them from the corresponding members of the octupole band. However, two peaks with approximately

the same intensity appear in the (3 He,d) spectrum with the energies roughly corresponding to the two previously known 5⁻ states. Similarly two peaks with comparable intensity are found with energies at or near the two 6⁻ states. This does not mean, for example, that both 5⁻ states have been populated. The density of levels is such that some other state (or states) could be near one of the 5⁻ states. The energy resolution and precision in the present experiment is poor enough so that any state within 15 keV of a given energy could not be discerned as being distinct. In fact, there is a known 1⁺ level at 1663 keV and 1⁻ state at 1831 keV. The net result is that one still can not decide which 5⁻ and 6⁻ levels belong to the octupole band and which to the K^{π}=4⁻ band.

The angular distributions of the observed levels at 1573, 1658, 1686, 1785 and 1825 keV are shown in Fig. 7, where the relative theoretical cross sections calculated using the proposed two-proton configuration are shown. As discussed above, the two possible 5⁻ groups have nearly same magnitude, so that both levels are eligible as a member of the K^{π}=4⁻ rotational band. On the other hand, the 1825 keV state seems to be the preferred choice for the 6⁻ member of the band, but the existence of the level at 1831 keV (1⁻) weakens this preference, becuase it is impossible to separate these levels in the present reaction. Again it is not possible on the basis of the angular distribution to decide which of the observed peaks are the members of the K^{π}=4⁻ rotational band.

FIGURE 7

Angular distributions of the $K^{\pi}=4^{-}$ band. Theoretical curves are multiplied by a factor of 0.43. This gives a good fit to the data. Black circles show the level at 1573 keV. Crosses and white circles show the possible 5⁻ state at 1658 keV and 1686 keV, respectively. Triangles and squares indicate the levels at 1785 keV and 1825 keV, respectively.



CHAPTER 5

CONCLUDING REMARKS

The ¹⁶⁵Ho (³He,d) ¹⁶⁶Er reactions have been performed and the results were compared with predictions based on the unified model of nuclear structure. The observed cross sections for the ground state rotational band and the gamma vibrational band were consistent with theory.

The admixture of the two-proton configuration 7/2-[523] -3/2+[411] in the $K^{\pi}=2^{-}$ octupole band should have been observable if it had been approximately 36% as predicted by Soloviev. However, in this study, the 1460 keV state was populated very weakly and an upper limit of approximately 7% was placed on this admixture. It is not possible to decide which of two previously known 5⁻ states belong to the $K^{\pi}=4^{-}$ rotational band based on the 1574 keV state, since both states have similar cross sections. For the same reason it is still not known which of the two previously known 6⁻ states belong to the $K^{\pi}=4^{-}$ rotational band.

At about 2600 keV excitation, a very intense peak appears. Its cross section at ϕ = 53 degrees is $\sqrt{245} \ \mu b/sr$, a value so large that it could be obtained theoretically only by $\ell = 0$, 1, 2 or 3 transitions. The angular distribution favors an $\ell = 0$ or $\ell = 2$ transition.

It should be pointed out that the DWBA calculations used were interpolated from calculations carried out for nearby nuclei. This interpretation is justified by the fact that the structure of the two sets of calculations was quite similar. It was an unfortunate feature of the angular distributions that the cross sections dropped off with

increasing angle at about the same slope for all values. Thus it was difficult to determine *L*-values from the angular distributions.

The ambiguity of level structures at the excitation energy about 1500 keV to 1800 keV could be removed by carrying out more sophisticated experiments.

APPENDIX

No DWBA calculations of ϕ_{ℓ} for the present 165 Ho (3 He,d) 166 Er reaction have been done, but the results of the DWBA calculations for the 152 Sm (3 He,d) 153 Eu and the 182 W (3 He,d) 183 Re reactions were available.

In the present work an interpolation between the two sets of available data were carried out in order to obtain the shapes of the angular distributions. These cross sections were then normalized to fit existing experimental data for well known transitions from targets of Sm and Yb. The justification for the above interpolation is based on the fact that the shapes of the predicted angular distributions for Sm and W are quite similar - corresponding maxima and minima for the two curves occur at the same angles. A typical example for $\ell = 2$ is shown in Fig. 8. The similarity between the curves results because ϕ_{ℓ} is expected to be a slowly varying function of atomic number ⁽⁹⁾ due to the geometrical aspects of the reaction. Since the atomic number of Ho is close to the average of Sm and W, the interpolation was carried out by taking the mean values for the Sm and W curves at each angle.

This procedure was carried out for each ℓ - value, which means that the cross sections obtained have the mean ℓ -dependence predicted by DWBA for the Sm and W cases. This is reasonable because relative intensities for various ℓ -values can be predicted quite well with DWBA. This is because relative intensities are also smoothly varying functions of ℓ -values.

Since the DWBA calculations have not always been successful in

FIGURE 8

Angular distributions calculated by DWBA for Sm and W and the interpolated curve. Value shown up is $\ell = 2$.



46.

predicting the absolute cross sections, the angular distributions obtained in the above manner have been normalized using experimental (3 He,d) data for some well known &=2 transitions.

The experimental values used for normalization were cross sections in the 152 Sm (3 He,d) 153 Eu reaction leading to the 172 keV state in 153 Eu (23) and the 172 Yb (3 He,d) 173 Lu reaction leading to the 357 keV state in 173 Lu (26). In each case the data at θ =30 degrees were used. These states correspond to the 5/2 3/2+[411] and the 5/2 5/2+[402] configurations. Both of these states have large values of C_{j}^{2} , therefore, large cross sections.

Only the case of the state in Eu will be discussed in detail. For an even-even target nucleus, eg. (3) simplifies to

$$\frac{d\sigma}{d\Omega} = 2C_{j\ell}^2 \phi_{\ell} U^2$$

because of the fact that $J_i = 0$ and $J_f = j$. Thus one has

$$\frac{d\sigma}{d\Omega} = 2C_{j\ell}^{2}\phi_{\ell}U^{2} = 230 \ \mu b/sr \ (observed)$$

$$C_{j\ell}^{2} = 0.79 \ (19)$$

$$U^{2} \approx 0.85 \ (estimated in a similar manner as the octupole state)$$

Hence,

$$\phi_{n-2} = 179 \ \mu b/sr$$

In a similar manner, one has

$$\phi_{\ell=2} = 91 \ \mu b/sr$$
 for the Lu case.

The Q-value dependence of ϕ_{ℓ} for the 152 Sm (3 He,d) 153 Eu reactions obtained from the DWBA calculations is shown in Fig. 9, where ϕ_{ℓ} is normalized to unity at Q=-1.000 MeV.

Assuming that the Q-value dependence does not vary rapidly as mass number, Fig. 9 was used to normalize other data. The ϕ_{ℓ} values for the ^{182}W (3 He,d) 183 Re were calculated at Q=-1.210MeV, so one has to multiply them by 0.96 to normalize to Q=-1.000 MeV. The Q-values to the states at 172 keV in 153 Eu and at 357 keV in 173 Lu are 0.238 MeV and -0.842 MeV respectively. Therefore, they must be multiplied by 1.30 and 1.03 to be normalized to Q=-1.000 MeV.

The last column of a Table 4 shows the cross sections obtained at 30 degrees both from experiment and theory after a Q-value correction. These values are plotted againstatomic number in Fig. 10. At Z=67 (atomic number of ¹⁶⁵Ho) the experimental points show $\phi_{l=2}$ is \approx 130 µb/sr which corresponds nearly to the midpoint of the two theoretical predictions. Thus it is seen that the normalization to the experimental results is almost identical with that which would have been obtained assuming the absolute value of DWBA calculations.

The angular distributions thus interpolated for the present reaction are shown in Fig. 11.

FIGURE 9

Q-value dependence of the DWBA cross sections. Three cross marks indicate Q-value at which theoretical calculations have been done for the 152 Sm (3 He,d) 153 Eu reactions.



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TABLE 4

					(<u>d σ</u>)
Reaction	Excitation Energy (MeV)	Q-value (MeV)	Q-value Factor	(<u>d σ</u>) μb/sr	µb/sr Q= -1.0MeV
¹⁵² Sm (³ He,d) ¹⁵³ Eu	0.172	0.238	1.30	179	232
¹⁵² Sm (³ He,d) ¹⁵³ Eu	DWBA	-1.000	1.00	230	230
¹⁷² Yb (³ He,d) ¹⁷³ Lu	0.357	-0.842	1.03	91	94
¹⁸² W (³ He,d) ¹⁸³ Re	DWBA	-1.210	0.96	84	80

FIGURE 10

Interpolation of the cross section $\phi_{l=2}$ for the present study at θ = 30 degrees. Cross marks are experimental points for Sm and Yb while circles show theoretical predictions. Triangles are interpolated points.



FIGURE 11

Interpolated $\boldsymbol{\varphi}_{\boldsymbol{\ell}}$ for various $\boldsymbol{\ell}\text{-values}$.



BIBLIOGRAPHY

- 1. Reich, C.W. and Cline, J.E., Phys. Rev. <u>137</u> (1965) B1424.
- Burson, S.B., Goudsmit, P.F.A., and Konijin, J. Phys. Rev. <u>158</u>(1967) 1161.
- Zylicz, J., Jørgensen, M.H., Nielsen, O.B. and Skilbreid, O.
 Nucl. phys. 81 (1966) 88.
- Yoshizawa, Y., Elbek, B. Herskind, B. and Olesen, M.C. Nucl. Phys.
 <u>73</u> (1965) 273.
- 5. Tjøm, P.O. and Elbek, B. Nucl. Phys. A107 (1968) 385.
- 6. Burke, D.G., Nelson, D.E. and Reich, C.W. To be published.
- Bohr, A. and Mottelson, B. Mat. Rys. Medd. Dan. Vid. Selsk.
 30 (1955) No. 1.
- 8. Nilsson, S.G. Mat. Fys. Medd. Dan. Vid. Selsk. 29 (1955) No. 16.
- 9. Satchler, G.R. Ann. Phys. 3 (1958) 275.
- Bassel, R.H., Drisko, R.M. and Satchler, G.R. Oak Ridge National Laboratory Report No. 3240 Unpublished.
- Zheleznova, K.M., Korneichuk, A.A., Soloviev, V.G., Vogel, P. and Jungclaussen, G. Dubna Preprint D-2157 (1965).
 - Soloviev, V.G., Vogel, P. and Jungclaussen Izv. Akad. Nauk SSSR (Ser. fiz.) 31 (1967) 518.
- Bés, D.R., Federman, P., Maqueda, E. and Zuker, A. Nucl. Phys.
 65 (1965) 1.

Bés, D.R. and Cho Yi-Chung Nucl. Phys. <u>86</u> (1966) 581.

 See, for example, Elliot, J.P. Lecture notes of Collective Motion in Nuclei (1958) Unpublished.

- 14. Bohr, N. and Wheeler, J.A. Phys. Rev. <u>56</u> (1939) 426.
- See, for example, Preston, M.A. <u>Physics of the Nucleus</u>.Addison-Wesley Publishing Company, Inc. (1963).
- 16. Belyaev, S.T. Mat. Fys. Medd. Dan. Vid. Selsk. 31 (1959) No. 11
- Sheline, R.K. Preprint of paper form International Symposium of Nuclear Structure.

Burke, D.G. and Elbek, B. Mat. Fys. Medd. Dan. Vid. Selsk. <u>36</u> (1967) no.6.

- 18. French, J.B. and Raz, B.J. Phys. Rev. 104 (1956) 1411
- 19. Chi, B.E. Nuc. Phys. <u>83</u> (1966) 97.
- 20. Spencer, J. and Enge, H.A. Nucl. Instr. and Meth. 49 (1967) 181
- 21. Borggren, J., Elbek, B. and Nielsen, L.P. Nucl. Instr. and Meth. <u>24</u> (1963) 1.
- 22. Nuclear Data Sheets, Editor Way, K. National Academy of Science, Washington.
- 23. Ungrin, J. and Ming, T.L. Private communication.
- 24. Burke, D.G., Zeideman, B., Elbek, B., Herskind, B.and Olesen, M.

Mat. Fys. Medd. Dan. Vid. Selsk. 35 (1966) No.2.

- 25. Bés, D.R. and Szymański, Z. Nucl. Phys. <u>28</u> (1961)[•] 42.
- 26. Presently being analyzed at McMaster University.