THE MEASUREMENT OF DIFFUSIVITY AND TURBULENCE IN

FULLY DEVELOPED PIPE FLOW

THE MEASUREMENT OF DIFFUSIVITY AND TURBULENCE

IN

FULLY DEVELOPED PIPE FLOW

BY

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SCOPE AND CONTENTS:

An experimental study of turbulent air flow in a pipe is reported in this paper.

A determination was made of the mean velocity distribution and longitudinal mean turbulent velocity distribution, both in the turbulent core and boundary layer for four different Reynolds numbers from 7300 to 58300.

A traversing mechanism was designed in order to measure the turbulence correlations between two points. The variation of the macro scale length, one of the fundamental quantities in recent statistical turbulence theory, across the pipe diameter, was calculated for Reynolds number equal to 58300, by integrating the correlation curves. The turbulent momentum diffusivity at the center of a pipe was calculated from the correlation study and the dimension-

(ii)

less value was found to be 0.111.

Ethylene gas was injected into the center of the pipe, and in order to investigate the turbulent mass diffusivities, the concentration distribution curves of ethylene were measured at different test positions downstream from the injection point, for the same series of Reynolds numbers used in the turbulence measurement. A numerical method for calculating the diffusivities was developed. The values of diffusivity obtained in these experiments show that the assumptions which were used by most of the authors, that $\boldsymbol{\alpha}$ (turbulent mass diffusivity/turbulent momentum diffusivity) has a value between 1.0 to 1.6, is correct.

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1. INTRODUCTION

The transport phenomenon, such as heat, mass or momentum, in a pipe flow is always an important problem for the engineers. At present, most of the authors who deal with this problem solve it by assuming some eddy diffusivity at the wall region and applying Reynolds analogy $/1.7^*$ to the turbulent core. Reynolds analogy states that the eddy viscosity for momentum, the eddy conductivity for heat and eddy diffusivity for mass are same. Since Reynolds, in 1874, first pointed out the analogy phenomenon, there have been many authors who devoted themselves to investigate this problem and found that Reynolds analogy was only correct for a Prandtl number equal to 1.0. For air they found that the ratio between eddy viscosity and eddy diffusivity had a value in the range 1.0 to 1.6. 27

Actually, most of the engineering problems require only the overall picture of transfer processes, such as described by Nusselt number for heat transfer or the Sherwood number for mass transfer, the modified Reynolds Analogy do provide a fairly good solution, which will be demonstrated later in the section of Literature Survey. However, the analogy approach gives no detailed information concerning

^{*} Figures in parentheses <a>[7] refer to the reference at the end of the paper.

the transport processes.

In general, two different types of flow may be observed in pipe flow. At low velocity, the path of every particle of fluid is well defined. This flow is called viscous or laminar flow. In laminar flow the transfer of mass, heat or momentum results only from the motion of the molecules of the fluid. Theoretically, therefore, knowing the fluid properties, it is possible to obtain analytical solutions relating transfer rates to the flow condition. However, the turbulent flow is the dominated phenomenon encountered in most of the industrial processes. A secondary transport mechanism in addition to the molecular motion is involved in turbulent flow: that of mixing and interchange between macroscopic fluid entities, or eddies. The presence of the eddies provides the main obstacle which prevents a complete theoretical solution to the problem. Recently, the statistical theory of turbulence $_3_7$, provides further insight which may lead to a complete solution but there is still a long way to go.

Unlike the molecular transport mechanism, the turbulent transport mechanism is not a property of the fluid at all,—it will not exist if the flow is laminar or the fluid is still— notwithstanding, the problem given to the author was to measure this quantity in a pipe with fully developed turbulent air flow. Since the mechanism of the turbulent transport of mass and heat is the same $\sum 3_7$, and

for the reason of convenience, only the eddy diffusivity of mass was measured.

In order to understand the structure of turbulence in pipe flow according to statistical theory, the velocity profile, turbulence level and the lateral double correlation coefficient (the relationship of the fluctuation velocity at two adjacent positions in the flow field) have to be measured at several Reynolds numbers. From this information, the turbulent momentum diffusivity at the center of the pipe, which still seems an unsolved problem, may be calculated.

The entire experiment was carried out in an acrylic plastic pipe of 5.5 inches inside diameter. Air was used as the flow medium. The velocity profiles and turbulence levels were measured by using the hot wire anemometer. The mass transfer effects were measured using ethylene gas injected by a 0.25 inches inside diameter tube which was positioned at the center of the main pipe and at an axial location approximately 52 diameters downstream from the entrance. The radial concentration distribution of ethylene gas was measured at 8 successive test position, each 1 foot apart, downstream from the injector. The concentration gradients in both the axial and radial direction enable one to calculate the mass diffusivity as a function of radial position and Reynolds numbers.

2. LITERATURE SURVEY

Reynolds $\begin{bmatrix} -1 \end{bmatrix}$ was the first to introduce the analogy between mass, heat and momentum transfer. The word "analogy" means to relate any one rate process to the other two. However, since fluid flow has been studied in much more detail than either mass or heat transfer, it is usual to develop relations between heat transfer and fluid friction, or between mass transfer and fluid friction. Following is a brief description of the analogy which was developed by previous authors $\begin{bmatrix} -4 \\ -5 \\ -6 \\ -7 \end{bmatrix}$

Most authors, in pursuing the analogy, write the shear stress, mass transfer and heat transfer fluxes as follows:

$$\frac{\gamma g_{c}}{\rho} = -(\nu + \epsilon_{v}) \frac{du}{dy} \qquad (1)^{*}$$

$$N_{A} = (D + \epsilon_{m}) \frac{dC}{dy}$$
(2)

$$\frac{\mathbf{q}}{\mathbf{c}_{\mathbf{p}}\boldsymbol{\beta}} = \left(\frac{\mathbf{k}}{\mathbf{c}_{\mathbf{p}}\boldsymbol{\beta}} + \boldsymbol{\epsilon}_{\mathbf{h}}\right) \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}\mathbf{y}}$$
(3)

Von Kármán [4,7] used equation (1) in his analysis. Martinelli [8,7] derived the third equation. While the second equation follows automatically since the heat transfer and mass transfer are similar.

A further sumplification was made by Rannie (-5, 7)

^{*} All symbols used in equation are shown and defined in the Nomenclature.

in which he used the dimensionless velocity u^+ and dimensionless distance y^+ , which are defined as follows,

$$u^{+} = u / \sqrt{\tau_{w} g_{c} / f} = u / u_{*} = \frac{u}{u_{o}} \sqrt{2 / f}$$

 $y^+ = y u_* / v$

 u_{\star}^2

where

$$= \frac{1}{2} f' u_0^2 = \frac{\tau_w g_c}{\rho}$$

here u_0 is equal to the average velocity while f' is the friction factor.

Again in a pipe the shear stress τ may relate to τ_w by

$$\tau g_{c} = \tau_{w}g_{c}\frac{r}{R} = \tau_{w}g_{c}\frac{R-y}{R} = \frac{1}{2}f'u_{o}^{2}\frac{r}{R}$$

By substituting the above relations into equations (1) to (3), after some rearrangement, equation (1) becomes

$$\frac{du^{+}}{dy^{+}} = \frac{r}{R} \left(\frac{1}{1 + \epsilon_{v}/\nu} \right)$$
(4)

equation (2) has the form

$$\frac{u_{*}}{N_{A}} \frac{dC}{dy^{+}} = \frac{1}{(D/\nu) + (\epsilon_{m}/\nu)} = \frac{1}{(1/Sc) + (\epsilon_{m}/\nu)}$$
(5)

while equation (3) can be written

$$\frac{c_{p} p^{u_{*}}}{q} \frac{dT}{dy^{+}} = \frac{1}{(k/\mu c_{p}) + (\epsilon_{h}/\nu)} = \frac{1}{(1/Pr) + (\epsilon_{h}/\nu)}$$
(6)

Since equation (5) and (6) are similar, only equation (5) will be used in continuing analysis.

To solve equation (4), (5) and (6), several assumptions are usually made.

1. In the wall region the shear stress τ , mass transfer flux N_A, and heat transfer flux q are assumed to be constant and equal to the value at the wall.

2. In the fully developed turbulent region the variation of shear stress, mass transfer, and heat transfer is such that τ/N_A and τ/q are constant.

3. In the case of mass transfer the interfacial velocity (at right angles to the surface) is assumed to be negligible.

4. The eddy diffusivities of momentum ϵ_v , mass ϵ_m and heat ϵ_h are equal in turbulent core.

Using assumption (1), equation (4) was integrated, giving, cv^+

$$u_{o}^{+} = \int_{0}^{y} \frac{1}{1 + \epsilon_{v}/\nu} + \int_{y_{1}^{+}}^{y_{o}} \frac{r}{R} \frac{dy^{+}}{\epsilon_{v}/\nu}$$
(7)

where y_1^+ is at a position within which assumption (1) remains valid and beyond which the molecular contribution to transfer is negligible and y_0^+ is the distance from wall at which the local mean velocity equal to the average bulk velocity.

Now by assumption (2)

$$\frac{\boldsymbol{\tau}'}{N_{A}} = \text{constant} = \frac{\boldsymbol{\tau}_{W}}{(N_{A})_{W}}$$

$$\frac{N_{A}}{(N_{A})_{W}} = \frac{\gamma}{\gamma_{W}} = \frac{r}{R}$$

Integrating equation (5) gives,

$$\frac{u_{*}}{(N_{A})_{w}} (C_{w} - C_{o}) = \int_{0}^{y_{1}^{+}} \frac{dy^{+}}{(\epsilon_{m}/\nu) + (1/Sc)} + \int_{y_{1}^{+}}^{y_{o}^{+}} \frac{r}{R} \frac{dy^{+}}{\epsilon_{m}/\nu}$$
(8)

7

where the value of y_1^+ is the same as before, C_0 is the value corresponding to y_0^+ .

Now if assumption (4) holds, then equation (7) may be subtracted from equation (8) to provide the expression

$$\frac{u_{*}}{(N_{A})_{w}} (C_{w}-C_{o}) = u_{o}^{+} + \int_{0}^{y_{1}} \frac{1}{(\epsilon_{m}/\nu) + (1/Sc)} - \frac{1}{(\epsilon_{v}/\nu) + 1} dy^{+} (9)$$

Since

$$u_{0}^{+} = \frac{u_{0}}{u_{0}} \sqrt{2/f'} = \sqrt{2/f'}$$

and

$$= u_0 \sqrt{f'/2}$$

The above equation reduce to

u.,

$$\frac{(C_{w}-C_{o})u_{o}}{(N_{A})_{w}} = \frac{2}{f}, + \sqrt{2/f} \int_{0}^{y_{1}^{+}} \left[\frac{1}{(\epsilon_{m}/\nu) + (1/S_{c})} - \frac{1}{(\epsilon_{v}/\nu) + 1}\right] dy^{+} \quad (10)$$

If the mass transfer coefficient (K), is defined as

$$K = (N_A)_w / (C_w - C_o)$$

then left hand side of equation (10) becomes

$$\frac{u_{o}}{K} = \frac{u_{o}}{(2KR/D)(D/2R)} = \frac{(2Ru_{o}/\nu)(\nu/D)}{Sh} = \frac{Re Sc}{Sh} = 1/St_{r}$$

Hence equation (10) can be written as,

$$\frac{1}{\mathrm{St}_{\mathrm{m}}} = \frac{2}{\mathrm{f}} + \sqrt{2/\mathrm{f}} \int_{0}^{\mathrm{y}_{1}} \left[\frac{1}{(\epsilon_{\mathrm{m}}/\nu) + (1/\mathrm{Sc})} - \frac{1}{(\epsilon_{\mathrm{v}}/\nu) + 1} \right] \mathrm{dy}^{+} \quad (11)$$

Equation (11) can be evaluated if the relation between ϵ_m and y⁺ is known. Actually equation (11) is the most general form of many of the analogies. From equation (11), the familar Reynolds analogy follows for Sc = 1.0. Prandtl's analogy $\sum_{0}^{-6} 2$ allows for varying Sc (or Pr) but leaves $\epsilon_v = 0$ within $y^+ < y_1^+$. He chose the value of y_1^+ empirically as 8.7 and with f' = 0.08 Re⁻¹ obtained

$$\frac{1}{\mathrm{St}_{\mathrm{m}}} = 25 \ \mathrm{Re}^{-\frac{1}{4}} \left[1 + 1.74 \ \mathrm{Re}^{-1/8} \ (\ \mathrm{Sc} - 1 \) \right]$$

Murphree $\left[7_{7} \right]$ in 1932 was the first to modify equation (10) by assuming,

$$\boldsymbol{\epsilon}_{\mathrm{m}} = (\boldsymbol{\epsilon}_{\mathrm{m}})_{1} (\boldsymbol{y}/\boldsymbol{y}_{1})^{3}$$
(12)

where $(\epsilon_m)_1$ is the main stream value, and y_1 is the value of y at which the velocity given by a parabolic velocitydeficiency equation $\sqrt{7}$ for the core agrees with that calculated for the film condition by substituting equation (12) into equation (1).

Von Kármán $[4_7, in 1939, extended the range of integration for y⁺ equal to 30 with the following assumptions for <math>e_y$,

 $\epsilon_v / \nu = 0$ $0 < y^+ < 5$ $\epsilon_v / \nu = \frac{y^+}{5} - 1$ $5 < y^+ < 30$

and integrated equation (11), the results of which gave good agreement with experimental data in the range of Prandtl numbers from 0.73 to 40.

> Lin, Moulton and Putnam $[-9_7]$ in 1953 assumed $e/v = (y^+/14.5)^3$ $0 < y_+ < 5$

$$\epsilon / v = (y^{+}/5) - 0.949$$
 $5 < y^{+} < 33$

and substituted these relation in equation (11) and integrated. Their results were shown to agree with a variety of experimental mass transfer data, including both gas systems (Sc = 0.5 to 1.7) and liquid systems for which Sc ranges from 325 to 3200. The agreement with heat transfer data is also excellent over the range of Pr from 0.5 to 10.

In 1956, Rannie $[5_7]$ proposed the first assumption of eddy diffusivities near the wall, which did not lead to a discontinuity with that of the turbulent core. His assumption was,

 $\epsilon/\nu = \sinh^2(y^+/14.53)$ $y^+ \leq 27.5$ The velocity distribution, being integrated according to equation (1) may be written,

 $u^+ = 14.53 \tanh(y^+/14.53)$ $y^+ \leq 27.5$ (13) The numerical values in equation (13) were chosen to join the logarithmic velocity profile,

 $u^+ = 5.5 + 2.5 \ln y^+$ $y^+ \ge 27.5$ so that at the position $y^+ = 27.5$, the velocity and velocity gradient were continuous at the junction. Comparison of his results with experimental data gave good agreement for the Prandtl number range between 1.0 and 100.

Recently, Wasan et al $\sum 10_7$ derived theoretically an expression for eddy diffusivity from the statistical theory of turbulence, in the following form:

$$\frac{\epsilon}{\nu} = \frac{4.16 \times 10^{-4} (y^+)^3 - 15.15 \times 10^{-6} (y^+)^4}{1 - 4.16 \times 10^{-4} (y^+)^3 + 15.15 \times 10^{-6} (y^+)^4} \quad 0 \le y^+ \le 20 \quad (14)$$

Numerical integration of equation (11) by using the relations of equation (14) provided a set of tables [11_7. Good agreement of these results with experimental data was observed for Prandtl numbers greater than 0.7.

Martinelli [5_7 without reference to Reynolds analogy, used the following generalized velocity profiles,

> $u^{+} = y^{+}$ $0 < y^{+} < 5$ $u^{+} = -3.05 + 5.0 \ln y^{+}$ $5 < y^{+} < 30$ $u^{+} = 5.5 + 2.5 \ln y^{+}$ $y^{+} > 30$

He calculated ϵ_v from above relation with the aid of equation (1) and assuming $\epsilon_v/\epsilon_h = 1.0$, carried out the integration of equation (3) all the way from the wall region to turbulent core. His results were in fair agreement with various experimental data on the heat transfer for a mercury system for which the Pr is about 0.024 but required an adjustment to handle higher Prandtl number fluids.

One of the few published papers concerned with diffusivity measurements is that by F. Page et al $/12_7$, in which they measured the temperature field between two parallel plates which were kept at constant but different wall temperatures. Equation (3) was then employed to calculate values of ϵ_h . When a comparison between ϵ_v and ϵ_h was made, it showed that the ratio of ϵ_h/ϵ_v not only varied with the position between the plates but also varied with

the flow Reynolds numbers; with $\epsilon_{\rm h}/\epsilon_{\rm v}$ almost equal to 1.0 at the highest Reynolds number (Re = 53450), while at a Re equal to 9190, the value of $\epsilon_{\rm h}/\epsilon_{\rm v}$ was found to be approximately 1.3.

Sleicher [13]7 apparently measured some diffusivities in a pipe flow with air as the working fluid and solved the following equation using an analogue computer.

$$u \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{\nu}{Pr} + \epsilon_{h} \right) \frac{\partial T}{\partial r}$$

In his published material, however, there was no mention of his experimental results for $\epsilon_{\rm h}.$

3. EXPERIMENTAL APPARATUS

The apparatus used in this experiment will be discussed in the following sections.

3.1 The Flow System

The experimental test section consisted of a long straight clear acrylic plastic pipe of 5.5 inches inside diameter and approximately 32 feet in length. The exit plane of the pipe was connected to an air box (3 feet X 3 feet X 2 feet) which was in turn connected to an exhauster type fan. The fan was driven by a 3 hp d.c. motor, the electrical supply of which was adjustable by mean of a rheostat, hence controlling the air velocity in the pipe. In order to avoid contamination of the room air by the gas (C_2H_A) , ethylene), which was injected into the pipe center when the measurement of concentration distribution was being made, the exit of the fan was connected to an adjacent room (equipped with an exhaust fan) by an exhaust pipe about 20 feet long and 6 inches in diameter. The complete flow system is shown in Fig-I.

Since, as in most of the references [14_7, fully developed turbulent flow occurs at a distance of at least 20 to 40 diameters of the pipe from the entrance, the first test position was fixed at a distance of 24 feet (equivalent

to 52 diameters) from the entrance. The experimental test positions, which were 1 foot apart, are shown in Fig-I. The individual test position was designed to allow the traversing mechanism to be easily installed or dismounted as shown in Fig-II.

3.2 Traversing Mechanism

The traversing mechanism was designed so that it could be used for four different purposes. Firstly, it was made to hold the pitot-static probe so that measurements could be made of the velocity distribution across the tube. Secondly, it was designed to hold the hot wire probe which was used to check the velocity distribution particularly in the sublayer near the wall and also to measure the turbulence level at each position. Thirdly, it enabled one to measure the correlation of turbulence since its design permitted the positioning of two hot wires at any position across the pipe and also permitted movement relative to The fourth purpose of this traverse mechanism each other. was to hold the sample collector, essentially a reverse pitot tube, so that the concentration of ethylene gas at each point across the pipe and at each test position could Early experimental results showed that the velobe taken. city and concentration profiles were quite symmetric about

The split plug and hold down nut which held the probe at the test position were made to fit as closely as possible to avoid leakage into the pipe. the axis of the pipe and all reported measurements were made with traversing mechanism installed vertically.

As shown in Fig-III the traversing mechanism consists of a main semi-circular body of 3 inches width and with an inside diameter of 6 inches (the outside diameter of the main pipe). A 12 inches diameter hole bored at the top of the main body was necessary in order to fit the protruding portion of the test position plug shown in Fig-II. The traversing mechanism was fixed to the pipe by the other half of the circular body, which has been omitted in Fig-III for clarity, through four screws, two on each side of the main body. The main traversing plate was attached to the body through four guiding rods which were fixed onto the main body. The traversing distance of the main plate was measured by marks, which were cut into one of the guide rods, 0.275 inches apart (equivalent to 1 tenth of the pipe inside radius). For the purpose of measuring the laminar sublayer velocity profile and turbulent correlations, a second traversing plate driven by a micrometer, was rigged above the main traversing plate by two short guide rods. The micrometer head provided a maximum traversing distance of 1.0 inches. The hot wire probe support was made from two concentric brass tubes with 1/4 inches and 1/8 inches outside diameters respectively. The larger tube passed through a center hole in the main traversing plate and was fixed in position with respect to the plate by a set screw.

The smaller tube was attached to the secondary micro-traversing plate and moved axially through the larger tube. Hence the whole traversing portion could be moved in the vertical direction by moving the main traversing plate, while only the micro-traversing plate could be moved by turning the micrometer head. Fig-III shows the traversing michanism complete with two hot wire probes, which were used to measure the turbulence correlation coefficient. It should be noted that one of the probe supports has been bent through a small angle so that the two hot wires can be effectively brought together. The lower one was bent upward since it also served to contact with the pipe wall during the measurement of sublayer velocity while the upper probe was removed. A very simple traversing mechanism employing a threaded shaft (20 threads per inch) was used for other traversing requirements.

3.3 Instruments for Velocity and Turbulence Measurements and Correlation Studies.

All the instruments used in measuring these quantities were available from commercial sources, therefore only a brief description of their function is given in the following section.

The duct velocity head was measured by a L-type pitot-static tube, and was recorded either on inclined manometer^{*} for higher velocity ($\Delta h > 0.1$ inches) or on the

Inclined Manometer (0-2 inches range of water) Dwyer MFG.
Co. Michigan city. Ind.

micro-manometer^{*} for lower velocity. The pitot-static tube also served the purpose of calibrating the hot wire probes⁺, which were used to measure the velocity profiles especially at lower velocity and to detect the turbulence level and also used in the double correlation coefficient measurements. The output from the hot wires was measured on a DISA Constant Temperature Anemometer^{**}. The DISA Random Signal Indicator and Correlator⁺⁺ was used to register the signal of the fluctuation velocity from the probes and to measure the double correlation coefficient.

3.4 <u>Injection System and Equipment for Concentration</u> <u>Measurements</u>

Since the injector was placed along the center-line of the pipe, its effect on the air flow had to be minimized and care was taken in its design to reduce this effect as much as possible. To meet this requirement, as shown in Fig-IV, the injector was made from a thin wall tube, 9 inches long and 1/4 inches inside diameter. Near the upstream end of the injector, two 2.625 inches long and 1/8 inches outside diameter tubes were soldered perpendicular to the main 'tube. The downstream end of the injector was positioned by attaching three small diameter steel wires, which were soldered to the 1/4 inches tube near the head

^{*} Merian, Micromanometer, Model 34 F B2.

⁺ DISA Miniature Hot-wire Probe, Model 55A25.

^{**} DISA Constant Temperature Anemometer, Model 55A01.

⁺⁺ DISA Random Signal Indicator and Correlator, Model 55A06.

of the injector at 120° intervals. These wires were passed through 1/64 inches holes drilled at the corresponding position in the acrylic tube. Mounting plugs were provided in the plastic tube which accepted the 1/8 inches brass tubes fastened to the upstream end of the injector. The proper position of the injector was obtained by adjusting the three wires and tighting or loosening these retaining plugs. The gas path was through both the 1/8 inches brass tubes to the injector.

The volume flow rate of the ethylene gas was controlled by a needle valve and measured by a rotameter . A vacuum pump was used to obtain the gas sample from the main stream by mean of the sample collector, which was positioned by the traversing mechanism at the desired radius inside the acrylic plastic tube. A large tank was installed between the pump and the sample collector, the sample air, therefore delivered to the container from the tank, was actually an average value over a period of say 3 minutes. From the container the sample gas was extracted by a 5 ml syringe.

The sample mixture of gas and air was analyzed by a Fisher Gas Partitioner⁺ using a Honeywell self-balancing potentiometer to recorded the output. The analysis

- Brooks Rotameter
- +

Fisher Gas Partitioner, Model 25 M, Fisher. Honeywell Recorder, Model SY 153 X 16-(VAHI)-II-III-** 157-D, Minneapolis, Honeywell Regulator Co.

of the output after calibration allowed one to calculate the percentage of C_2H_4 in the gas-air mixture.

4. EXPERIMENTAL PROCEDURE AND RESULTS

4.1 Calibration of the Maximum Volocity in the Pipe

The relation between the maximum velocity of air in the pipe and the fan speed (rpm) was required with as much precision as possible. The result is given in Fig-V. The pitot-static probe was placed at the center of the pipe, the pressure difference between static and dynamic pressure was measured using either the inclined manometer for higher velocity (say when $\triangle h > 0.1$ inches of water) or the micromanometer, if $\Delta h < 0.1$ inches of water. The accuracy of the micromanometer was no more than one thousandth of an inch This meant that a measured velocity which corresof water. ponded to a pressure drop of approximately 0.01 inches of water gave an error greater than 10%, therefore the lower velocity used in this experiment. (approximately 3.7 ft/sec) was adopted from the extrapolated value of Fig-V. This extrapolated value was checked using the hot wire anemometer, the linear calibration curve of which shown in section 4.2. The linear relation between rpm and center line velocity was therefore assumed to hold at the lower velocity.

4.2 Calibration of Hot Wire Anemometer

Experience has shown that the calibration curve

for hot wires will not be the same even with the same wire at different temperatures, hence each hot wire was calibrated individually before using it. The wires were calibrated in situ using a pitot-static tube and fan speed calibration curve. For very low velocity readings, the calibration curve was extrapolated using the linear relationship obtained with the hot wire and matching the curves at a higher velocity where the accuracy of the pitot-static tube was acceptable. A typical calibration curve is shown in Fig-VI, and also shows a linear relation between the square root of velocities and the square of the d.c. voltage from the anemometer.

4.3 The Velocity Distribution in the Turbulent Core

The measured velocity distribution is shown in Fig-VII, and Laufer's data of Re = $50000 \ \ 15 \ 7$ is also given on the same figure by way of comparison. These experiments were performed at four different Reynolds numbers, 7300, 19200, 38600 and 58300. The Reynolds numbers are based on the mass flows, which were in turn calculated from the experimental data. The agreement is fairly good for the two highest Reynolds numbers, comparing the present results with those of Laufer, as it should be since the Reynolds numbers are very close to those given by Laufer who incidently based Reynolds number on the center-line mean velo-

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city. The profiles for the lower Reynolds numbers shown a trend which is in agreement with that given by Schlichting (-14_7) .

the first of the

At the two highest Reynolds numbers the velocity profiles were measured both by pitot-static probe and hot wire anemometer, while, for the velocity profile under 10 ft/sec only the hot wire anemometer was used. At higher velocities, the resulting profiles from both instruments were indistinguishable.

4.4 The Velocity Distribution Near the Wall

The velocity distribution near the wall was measured using the hot wire anemometer combined with the microtraversing mechanism. The results of these measurements are presented in a non-dimensional form $(u^+ vs. y^+)$ in Fig-VIII^{*}, where the Universal velocity distribution is also given. The values obtained from the measurement show a higher velocity profile in the turbulent core and good agreement in the transition region for all Reynolds numbers but the values in the laminar sublayer region shown considerable scatter, possibly due to both the inaccuracy in position measurement and the actual velocity measurement.

^{*} The dimensionless value of y^+ is a function of both Re and the distance from the wall. For example, at r/R = 0.95, y^+ is 11.1 for Re = 7300, y^+ is 27.2 for Re = 19200, y^+ is 50.2 for Re = 38600 and y^+ is 72.3 for Re = 58300.

4.5 Turbulence Level in the Turbulent Core

A DISA Random Signal Indicator and Correlator was used to detect the rms values which were produced by the hot wire probe. The hot wire probe was mounted on the lower part of the probe support as shown in Fig-III, with the wire parallel to the pipe wall.

Fig-IX gives the turbulence level measurements made in the core plotted in dimensionless form u'/u_* against the dimensionless radius. The same four Reynolds numbers were used in this experiment. Laufer's data are shown for comparison. It would appear that if the turbulence is expressed in the dimensionless form u'/u_* , it is independent of Reynolds numbers.

4.6 Turbulence Level Near the Wall

No direct method was available to position the probe exactly at the wall except by the feel of the micrometer head and by eye. And this positioning error is apparent in all sublayer measurements. However, repeated readings produced reasonable uniformity in the measurement of the wall position.

Fig-X shows the results of turbulence measurement taken near the wall at the same Reynolds numbers, as indicated earlier. Once again Laufer's data at Re = 50000 are shown for comparison. It appears that for high Reynolds numbers (greater than 19200) the pattern is quite consistent and in general agreement with the work of Laufer. The wall turbulence at the lowest Reynolds number shows a somewhat different pattern than the others. It seems that there is a transition of the wall turbulence pattern occurring somewhere in the Reynolds number range from 7300 and 19200. However there are no experimental data to confirm this statement.

From the curves in Fig-X, it can be seen that the boundary sublayer becomes thinner with increasing Reynolds number.

4.7 The Measurement of Double Correlation Coefficient

As a first step to understand the modern statistical theory of turbulence, the double lateral correlation coefficient was measured by using two hot wire probes whose assembly is shown in Fig-III.

The double lateral correlation coefficient in homogeneous turbulent flow is defined in reference 27 by the following expression.

$$g(x_2) = \frac{\Delta u(r) \Delta u(x_2 + r)}{u'(r) u'(x_2 + r)}$$
(15)

Here the averaging procedure is carried out with respect to time. The actual meaning of this correlation has been demonstrated in detail in reference 27. In brief this correlation measures the relation of the turbulence between two point in the flow and has the properties that g(0) = 1 and $g(\infty) = 0$. A fuller description of this function will be given later in Section 5.4.

In measuring the correlation coefficient, the main traversing plate was moved first to the appropriate location (in the vertical plane) in the pipe, and then the two probes were at first brought together by adjusting the micrometer. The relative distance between the two probes and the corresponding value on the correlator were the data which were needed for plotting Fig-XI. As shown in Fig-XI, the double correlations were measured at six different locations in the vertical direction for a Reynolds number of 58300.

4.8 The Macro Scale of Turbulence.

According to a recent statistical theory of turbulence $\begin{bmatrix} 2 \\ 7 \end{bmatrix}$, the macro scale is the measurement of the longest connection, or correlation distance, between the velocities at two points in the flow field. It has been defined as $f \infty$

$$\Lambda_{g}(\frac{\mathbf{r}}{R}) = \int_{0}^{\infty} g(\mathbf{x}_{2}) d\mathbf{x}_{2}$$
(16)

The macro scales at the six different positions in one plane of the duct can be obtained by numerical integrating the curves given in Fig-XI. The values so obtained are tabulated in the following table.

TABLE 1

Values of Macro Scale

r/R 0 .2 .4 .6 .8 .95 $\Lambda_{p}(r/R)$.378" .382" .382" .403" .370" .241"

4.9 Concentration Profile Measurement

a. General Description:

Ethylene gas was chosen as the additive for the work on the diffusivity since it has approximately the same molecular weight as air. It was felt that the mixing of ethylene with air would not involve any mechanism other than the molecular and / or turbulent diffusion.

The ethylene gas was injected into the main stream by the injector as shown in Fig-IV. The quantity of ethylene injected was controlled by a needle valve and was measured on a flowmeter (rotameter). The gas volumetric flow rate was set so as to have the velocity of injection equal to the maximum stream velocity. In so doing preliminary tests showed that at this injection rate, it gave the minimum distortion to the velocity profile of the main stream. The velocity profile was carefully checked at each section after the insertion of the injector running trial tests with air as the injected fluid. It was found that the effect of the injector was insignificant downstream of the second test position for all Reynolds numbers used during the test.

In order to take the gas sample from the test positions, a sample collector with a tip having a rectangular cross section of approximate dimensions $1/8" \ge 1/32"$, was mounted on the traversing mechanism. The end of the sample collector was connected to a large tank which had a volume

of approximate 20,000 cc and served as an averaging device. The vacuum pump was then connected in series with the tank and a glass sample container from which one could extract the sample using a syringe of the desired volume. The sample volume was always equal to 3.2 cc, a figure decided by the response of the recording apparatus. In order to make the most accurate use of the Honeywell recorder, the sample volume was so chosen that the total peak signal from the Fisher gas partitioner at the 2% sensitivity position would produce a full deflection on the recorder when its minimum range was set (0 to 5 millivolts full scale). In other words, at this injection volume, the partitioner gave a 5 mv signal at the 2% sensitivity position. By trial and error, it was found that the best sample volume was 3.2 cc. The peak height was found to vary slightly even when the injection value was the same. There are two different reasons for this phenomenon: i) a higher injection speed usually gives a higher peak. This is due to the principle of the gas partitioner. However, the areas under the peak curves are the same if the equal quantities and same mixture gas are injected. ii) When the injector's needle was inserted into the gas partitioner, the pressure (26.5 psia) of helium, the carrier gas, made it difficult to control the injection speed and sometimes let some helium into the

^{*} The detailed descriptions of the principles of the gas partitioner may be found in the instruction book of Fisher Gas Partitioner, Model 25 and 25M, Nos. 11-134 & 11-134-50.
gas partitioner, thus causing a lower peak height. However, it was found that the area under the curve was still the same. Hence, in order to determine the concentration of C_2H_4 the area under the curve must measured for each sample. This was done using a planimeter, but as we are interested only in the ratio of areas between the sample and 100% of C_2H_4 with the same injection volume, the absolute area was not necessary. Moreover, it was very difficult to measure the full area under the peak, when the concentration of ethylene was very small. An easier way to do this was to measure their half peak area as a basis for comparison. The results measured by this method were quite satisfactory.

Another reason to use the half peak area was that ethylene peak appeared immediately after the total peak on the recorder and before the total peak had entirely died This meant that there was an overlap between the away. total peak and the ethylene peak. The overlap was not very important for higher concentrations of ethylene, but it became most important when the concentration of ethylene was, say, below 10% of the total volume. Since most of the concentrations of ethylene measured in this experiment were below 1%, it can became a very critical problem. This is because the response on the recorder chart at the end of the total peak is much higher than that of the ethylene itself. For instance, the response of the total peak at the place when the ethylene should show its peak is usually

4 subdivisions on the chart if the sensitivity of gas partitioner is set on 100%, (i.e. the recording potentiometer may register 100% of signal which is produced by the gas partitioner.) while 0.02% of ethylene of 3.2 cc sample volume only gives one subdivision of response on the chart of the potentiometer at the same sensitivity on the gas partitioner. In order to remove this effect, many response curves for air alone at different injection conditions were measured, and making the assumption that the response of air and ethylene was additive, therefore the overlapped area, which was produced by the air, was subtracted before we measured the half peak area which was produced by ethylene gas. This method was found to be quite satisfactory, the accuracy of this measurement giving an absolute error within the range + 0.02% of ethylene. The term absolute error here meant that when a sample which contained 0.02% of ethylene was measured, the measured value may have a relative error as high as 100%.

All the experiments in connection with the measurement of concentrations were conducted at night for two reasons. Firstly, the contaminated air was exhausted to a fan in a room next door, thereby minimising the pollution effect on the experimental room. The opening of windows or doors was found to influence the ambient flow condition in the experimental apparatus and to eliminate this effect all the doors and windows were kept closed. Secondly, it is found

much easier to control the room temperature at night than at day time. During the whole experiment, the room temperature varied from about $74^{\circ}F$ to $78^{\circ}F$, but on a single run the ambient temperature could be controlled within the range $\pm 1^{\circ}F$ by adjusting a heater in the room. Within this temperature variation no corrections were made for the change of temperature, and all the properties of air were evaluated at a temperature of $76^{\circ}F$.

b. Procedure for Measuring the Concentrations.

In order to get a stable response, the Honeywell recording potentiometer was turned on at least 4 hours before the experiment. The Fisher gas partitioner was stable approximately an hour after it was turned on. The motor speed was adjusted according to the curve of Fig-I for the test at hand and it was found that it required generally more than 2 hours to achieve stability according to the speed required. The instability was due to the change of the resistance of the motor rheostat as a result of changes in room temperature. For example, it was found that the motor speed changed from one to five percent of the original value for a 1°F change in room temperature. A hand held tachometer was used to check the motor speed at least once every half hour.

Helium was obtained from a gas cylinder linked to the gas partitioner by a 1/4 inches outside diameter copper tube. A drying tube was used to remove all water vapour in the helium. The flow of helium was regulated by a 0 to 100 psia regulator, and its flow rate was calibrated by a flowmeter. It was found that a regulator setting of 26.5 psia gave the desired helium flow rate of about 100 cc/min. Two steps of control were necessary in order to obtain the steady-state flow rate of the ethylene gas. A 0 to 200 psia regulator was used to take the ethylene gas from the gas cylinder to the injector using copper tubing of 1/4 inches outside diameter. A needle valve, which actually controlled the flow rate, was installed between the gas cylinder and a rotameter, which was used to measure the flow rate into the injector. The calibration curves for these rotameters were obtained from the company.

Injection rates varied as the Reynolds number were changed. These are shown below in Table 2.

	Flow Conditions and Injection Bates					
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Re No.	Max. Vel. (ft/sec)	Mean Vel. (ft/sec)	Inj. Vel. (ft/sec)	Inj. Rate (cu-ft/sec)		
7300	3.66	2.63	4.5	1.54 x 10 ⁻³		
19200	8.94	6.96	8.94	3.06×10^{-3}		
38600	17.40	13.99	17.40	6.06 X 10 ⁻³		
58300	26.50	21.10	26.50	9.06 x 10 ⁻³		

TABLE 2

The concentration distribution of ethylene for

different Reynolds number as a function of test section length are present in Fig-XII through Fig-XVI. Fig-XII and XIII show the ethylene distribution for Re = 7300 at test position 1 through test position 8. The Reynolds. number was obtained by integrating the velocity distribution measured using the hot wire anemometer. The first test position was at a distance 18 inches downstream from the injector, but it should be noted that the sample collector had a length of 7 inches, so that the actual distance of the first test position according to the tip of the collector was 11 inches downstream from the injector. Each test position was 1 foot apart, that is, the eighth test position was 95 inches from the injector. The solid lines are consiered to best fit the experimental data and at the same time give the proper boundary conditions. From Fig-XIV to XVI, the results show the concentration distribution of C_2H_4 only for the last 5 or 6 test positions. This is because the cencentration distribution curves for ethylene before that test position gave no values to calculate the diffusivities as far as our experiment concerned; i.e. the ethylene gas had not diffused to the wall.

4.10 Calculation of the Diffusivities

Fig-XVII shows the diffusivities which were calculated from the previous concentration distribution curves. Before calculation were made, any irregularities were eli-

minated from the concentration curves, since it was the slope of the curves which was important for the diffusivity calculation. The \pm 0.02% absolute error range of the gas partitioner gave a band of 0.04% width rather than a line as shown in these figures. As a result, the diffusivity distribution shows a rather large deviation especially at the center-line and near the wall. This is because, in these regions, the gradient of the concentration is very small and an accurate reading of the value is impossible with the limitations of the present apparatus. A small deviation in the readings taken in this region will give a large error in the gradient, and this will show later in the data analysis.

For these reasons, the individual value of diffusivities at each position shows considerable spread, hence only the average value of the diffusivities for different Reynolds number is given in Fig-XVII. The averages were assembled by taking the values at each different test position maintaining the same vertical position in the pipe (r/R). The points in Fig-XVII are the average values for each Reynolds number and the diffusivities have been plotted in the dimensionless form, ϵ_m/u_*R . The ordinate in Fig-XVII shows ϵ/u_*R for both the momentum and the mass diffusivities. According to the theoretical derivation of momentum, if the turbulent diffusivities of momentum are expressed in a dimensionless form they should be independent of Rey-

nolds number [14_7. Therefore the assumption has also been made that the dimensionless mass diffusivities are also independent of Reynolds number. The averaging process taken over all the Reynolds numbers, gives the mean value curve which is shown in Fig-XVII. The turbulent momentum diffusivity after Nikuradse (from reference 14) is also given for comparison.

A comparison of the experimental values of diffusivity with those obtained by F. Page et al <u>[12]</u> using two parallel plates, is given in Fig-XVIII. Although the same geometry of flow system does not exist, similar tendencies are shown and the results are of the same order of magnitude.

5. DATA ANALYSIS AND SAMPLE CALCULATION

In the following sections the equations used in the calculation of the results from the experimental data are outlined and sample calculations are also given as necessary. The meaning of the symbols used in this section are defined under Nomenclature.

5.1 The Calculation of Velocities from Measured Pressure Differences.

Bernoulli's equation its usual form was used to calculate the velocity from the differential pressure, i.e.,

$$p_1 + \frac{1}{2} \frac{f_a}{g_c} u_1^2 = p_2 + \frac{1}{2} \frac{f_a}{g_c} u_2^2 = p_0$$

$$u = \sqrt{2(p_0 - p)} g_0 / f_a$$

for air

and

$$f_a = P_{atm}/RT_a$$

 $p_o - p_1 = f_w \Delta h/12$

finally one has

$$u = 15.89 \sqrt{T_a \Delta h/P_{atm}}$$
(17)

5.2 Calibration of the Hot Wire Probe.

King's relation has been used to correlate the d.c. voltage vs. velocity in the form

$$E^2 = A \sqrt{u} + B \tag{18}$$

Accordingly the relation between E^2 and \sqrt{u} should be linear. Fig-VI is a typical correlation curve for a particular hot wire probe from which one can obtain the constants,

A = 5.3417 & B = 27.25King's equation then becomes,

$$E^2 = 5.3417 \sqrt{u} + 27.25$$
(19)

5.3 Turbulence Calculations

The formula for calculating turbulence from experimental can be derived as following.

By applying the following boundary conditions to equation (18),

$$E = E_0$$
 when $u = 0$

one has

$$E^2 = A \sqrt{u} + E_0^2$$
 (20)

then differentiating both sides one obtains,

$$2 \text{ EdE} = \frac{1}{2} \text{ A } \frac{du}{\sqrt{u}}$$

Evaluating A from equation (20) and writing ΔE and Δu instead dE and du in above equation, one obtains the expression.

$$\frac{\Delta u}{u} = \frac{4E}{E^2 - E_0^2} \Delta E$$
 (21)

However, since only the rms value of ΔE can be measured from the DISA hot wire anemometer, the fluctuation velocity is also a mean value and equation (21) becomes,

$$\frac{u}{u} = 100 E_{rms} \frac{4E}{E^2 - E_0^2}$$

Equation (22) expresses the percentage of turbulence.

A sample calculation follows.

For a Re = 58300, at y = 0.045", the following measurements were taken using the hot wire anemometer.

E = 6.87 volts and $E_{rms} = 150$ mv. Accordingly, equation (19) give,

u = 13.94 ft/sec

while from equation (22)

u'/u = 20.7%

and therefore

u' = 2.88 ft/sec

The average velocity in the pipe was obtained by integrating the local velocities over the cross section and then dividing by the cross sectional area. i.e.,

$$\overline{u} = \int_0^n u r \, dr / \int_0^n r \, dr \qquad (23)$$

The numerator was integrated by numerical techniques using experimental values of velocity distribution as shown in Fig-VII and combining it with the measured mean centerline velocity given in Table 2. The average velocity was found equal to 21.2 ft/sec. A Reynolds number was then calculated using the expression,

Re = $\overline{u} 2R/\nu$ (24) Since 2R = 5.5/12 ft and $\nu = 0.166 \times 10^{-3} \text{ ft}^2/\text{sec}$ for air

(22)

at 76° F, the numerical value for Reynolds number becomes Re = 58300

The experimental value differed from that value originally assumed using a 1/7 power law (Re = 60000) by only 2.8% and was therefore quite satisfactory.

From the Moody diagram $/ 16_7$, the corresponding friction factor f for Re = 58300 is given as 0.0198 for a smooth surface. The friction velocity was obtained using the expression,

$$u_{*} = \sqrt{f/8} \ \overline{u} = 1.05 \ ft/sec^{\ddagger}$$

The other properties were calculated as follows: Dimensionless velocity based on maximum velocity,

 $u/u_{max} = 13.94/26.5 = 0.526$

where $u_{max} = 26.5$ for Re = 58300.

Dimensionless turbulent velocity,

 $u'/u_{*} = 2.74$

Universal dimensionless distance from the wall

$$y^+ = yu_*/1 = 23.7$$

The universal dimensionless velocity

$$u^+ = u/u_* = 13.28$$

Dimensionless radius

r/R = 0.9936

All the experimental data calculated as above are shown in Fig-VII to Fig-X.

The frictional velocity may also be defined as $u_{\star} = \sqrt{f'/2} \overline{u}$ Fig-VII shows the velocity profile which is plotted using u/u_{max} vs. r/R.

The information given in Fig-VII has been replotted in Fig-VIII in the Universal form with distance (y^+) plotted as a function of the dimensionless velocity (u^+) . The Universal Distribution Law is also presented for comparison.

Fig-IX and Fig-X show the dimensionless turbulence level measured across the pipe with particular emphasis being placed on the region near the wall.

5.4 Double Lateral Correlation Coefficient Measurement.

As a first step to understand the modern statistical theory of turbulence, the double lateral correlation was measured using two hot wire probes whose assembly is shown in Fig-III. The double lateral correlation has been defined earlier in the report by equation (15) in section 4.7 (pp. 22).

From the previous section, it has been shown that

∆u∝∆E

and u'∝E_{rms}

If two hot wires are placed paralled to each other in the flow medium, one at r and the other displaced $(r+x_2)$, and their sensitivities are different the output signals can be written,

 $\Delta u(r) = A \Delta E(r)$ u'(r) = A E_{rms}(r) and

$$\Delta u(x_{2}+r) = B \Delta E(x_{2}+r)$$

$$u'(x_{2}+r) = B E_{rms}(x_{2}+r)$$
(26)

Therefore equation (15) may be written as

$$g(x_{2}) = \frac{AB}{AB} \frac{\overline{\Delta E(r) \Delta E(x_{2}+r)}}{\overline{AB E_{rms}(r) E_{rms}(x_{2}+r)}}$$

$$= \frac{\overline{\Delta E(r) \Delta E(x_{2}+r)}}{\overline{E_{rms}(r) E_{rms}(x_{2}+r)}}$$
(27)

The time average should be taken after the multiplication. For convenience the relation given in equation (27) is not measured directly but rather the following ratio (K_1) ,

$$K_{1} = \int \frac{\left[\Delta E(r) + \Delta E(x_{2}+r)\right]^{2}}{\left[\Delta E(r) - \Delta E(x_{2}+r)\right]^{2}}$$
(28)

The relation between $g(x_2)$ and the above expression can be shown by expanding the terms in both the numerator and denomerator on right hand side of equation (28) as follows:

$$K_{1} = \int \frac{\overline{\Delta E(r)^{2} + 2\Delta E(r)\Delta E(x_{2}+r) + \Delta E(x_{2}+r)^{2}}}{\overline{\Delta E(r)^{2} - 2\Delta E(r)\Delta E(x_{2}+r) + \Delta E(x_{2}+r)^{2}}}$$
$$= \int \frac{\frac{1 + 2\Delta E(r)\Delta E(x_{2}+r)}{1 - 2\Delta E(r)\Delta E(x_{2}+r)/[\Delta E(r)^{2} + \Delta E(x_{2}+r)^{2}]}}{1 - 2\Delta E(r)\Delta E(x_{2}+r)/[\Delta E(r)^{2} + \Delta E(x_{2}+r)^{2}]}$$
(29)

Finally one has

$$K_{1} = \sqrt{\frac{1 + g(x_{2})}{1 - g(x_{2})}}$$

Here the assumption has been made that $\overline{\Delta E(r)^2} = \overline{\Delta E(x_2+r)^2}$

= $E_{rms}(r)^2$, which is true of course when $x_2 = 0$. It can be demonstrated however that this assumption satisfactory for small values of x_2 . For example, if there is a relatively large difference between $\overline{\Delta E(r)^2}$ and $\overline{\Delta E(x_2+r)^2}$, use is made of the relationship,

$$\overline{\Delta E(r)^2} + \overline{\Delta E(x_2 + r)^2} = 2 E_{rms}(r) E_{rms}(x_2 + r)$$

Now, if $g(x_2) = 0$ when $x_2 = 1.0$ inches, Fig-IX shows that the maximum turbulence difference between two points, 1.0 inches apart, is around 40%. The percentage deviation $\overline{\Delta E(r)^2} + \overline{\Delta E(x_2+r)^2}$ and 2 $E_{rms}(r) E_{rms}(x_2+r)$ is only 5.4%, so that the assumption is quite satisfactory. Equation (29) can be rewritten to show

$$g(\mathbf{x}_2) = \frac{K_1^2 - 1}{K_1^2 + 1}$$
(30)

From these relationships a clearer idea of $g(x_2)$ may be obtained. If $x_2 = 0$, $\Delta E(r) = \Delta E(x_2+r)$ both in magnitude and phase, then $K_1 = \infty$, thus $g(x_2) = 1$, as $K_1 \rightarrow \infty$ as a limit. As $x_2 \rightarrow \infty$ then $\Delta E(r) + \Delta E(x_2+r) = \Delta E(r) - \Delta E(x_2+r)$, since the turbulence is random, both the signal of $\Delta E(r)$ and $\Delta E(x_2+r)$ are gathered in an arbitrary direction and each has an arbitrary phase angle. From the probability that at this condition, $\Delta E(r) + \Delta E(x_2+r)$ does not differ much from $\Delta E(r) - \Delta E(x_2+r)$. It gives $K_1 = 1$ and the result is that the $g(x_2) = 0$ as $x_2 \rightarrow \infty$. Both K_1 and $g(x_2)$ can be read directly from the DISA Random Signal Correlator. The results of $g(x_2)$ measured at r/R equal to 0, 0.2, 0.4, 0.6, 0.8 and 0.95 are given in Fig-XI.

The Macro or Integral scale of turbulence is another significant scale for the statistical theory of turbulence. It has been defined by equation (16). Values of these turbulence parameters were obtained by integrating the curves shown in Fig-XI numerically and are shown in Table 1 (pp. 23). The usefulness of this scale of turbulence will be discussion in the following section.

5.5 Prediction of the Value of the Momentum Diffusivity at The Center of the Pipe.

As described in the Appendix I, the relation between diffusivity and the macro scale of turbulence may be correlated by the expression,

$$\boldsymbol{\epsilon}_{\mathbf{v}} \stackrel{\boldsymbol{\omega}}{=} \mathbf{v} \cdot \boldsymbol{\Lambda}_{\mathbf{g}} \tag{31}$$

In this experiment, v' the lateral turbulence component has not been measured, however from Fig-IX, the longitudinal fluctuation velocity in the turbulent core is independent of Reynolds number if the fluctuation velocity is expressed in dimensionless form by dividing by the frictional velocity, u_* . We may expect that it is also true for v', if it is also expressed in the form of v'/ u_* .

Rewriting equation (31) and dividing both sides by u_R , one has

$$\frac{\epsilon_{\rm v}}{u_{\rm *}R} = \frac{\rm v}{u_{\rm *}R} \Lambda_{\rm g}$$

41

(32)

If Λ_g is independent of Reynolds number, the dimensionless group ϵ_v/u_*R is also independent of Reynolds number. Although only one Reynolds number was used for experimentally measuring $g(x_2)$ and Λ_g , and since by definition

$$g(x_2) = \frac{\Delta u(r) \Delta u(x_2 + r)}{u'(r) u'(x_2 + r)}$$
(15)

it follows that:

$$g(x_2) = \frac{\frac{\Delta u(r) \Delta u(x_2+r)}{u_*}}{\frac{u^*(r) u^*(x_2+r)}{u_*}}$$
(33)

Now if $\Delta u/u_*$ and u'/u_* are independent of Reynolds number in the turbulent core, it follows that both $g(x_2)$ and Λ_g should also be independent of Reynolds number. Although v' has not been measured during the experiment, the value of ϵ_v/u_*R at the pipe center-line can be estimated by assuming isotropic turbulent flow along the centerline. This assumption is quite valid since Laufer \sum_{15}^{7} in a similar experiment did get very nearly the same value of u' and v' at the center of the pipe. By taking $u'/u_* = v'/u_* = 0.81$ from Fig-IX and substituting the value of $\Lambda_g(0)$, measured on the center line of the pipe, one then obtains

 $\frac{\epsilon_{\rm v}}{u_{\rm *}R} = 0.81 \ {\rm X} \ 0.378/2.75 = 0.111$

This is a rather high value in comparison with the mass diffusivities calculated in this experiment by means of concentration profiles. However it is really a very interesting result, since most of the theoretical results show $\epsilon_v/u_*R = 0$ at the center of pipe. This is undoubtedly due to the fact that most theoretical analyses assume velocity profiles which do not match properly on the center line to give a velocity gradient, with respect to lateral distance, equal to zero. For example, rearranging equation (1) in the form

$$\epsilon_{\mathbf{v}} = \frac{\tau g_{\mathbf{c}}}{\mathcal{F}_{\mathbf{a}} \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\mathbf{v}}}$$
(34)

and applying to the center line position both τ and du/dy are equal to zero. To find the value of ϵ_v resolves itself into the problem of finding the indeterminate form (0/0) of equation (34) in a mathematical sense. Nikuradse $\sum_{i=1}^{n} \frac{14}{7}$ has obtained $\epsilon_v/u_*R = 0.012$, but the present result is much higher than this value.

Since equation (29) has been derived using the assumption that the turbulent flow field is isotropic, this is only true along the center line of the pipe and although the values of the macro scale have been calculated at six positions in the radial direction, only the momentum diffusivity at the center line has been calculated (for the want of a proper value of v').

5.6 <u>Calculation of the Mass Diffusivities from the</u> Concentration Distributions. In order to calculate the diffusivities, a numerical technique has been derived, starting from the accepted equation in the form

$$u \frac{\partial C}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} (\epsilon_m + D) r \frac{\partial C}{\partial r}$$
(35)

For the turbulent region, the molecular diffusivity, D, can be neglected in comparison with the large value of $\epsilon_{\rm m}$. Another assumption which is related to equation (35) is that the concentration distribution is axially symmetric.

In order to integrate numerically, the radius R has been divided into 10 intervals with the assumption that parameters within the individual $(r_{i+1} - r_i)$ intervals were constant. Equation (35) can be recast in the following form to show the integration over R.

$$\epsilon_{m} r \frac{\partial C}{\partial r} \Big|_{r=r_{i}}^{r=r_{i+1}} = \int_{r_{i}}^{r_{i+1}} r u \frac{\partial C}{\partial x} dr \qquad (36)$$

The final form used in the calculation becomes,

$$r_{i+1}(\epsilon_{m})_{i+1} \frac{\partial C}{\partial r} \Big| r_{i+1} = \frac{1}{2} u_{i} \frac{\partial C}{\partial x} \Big| r_{i} (r_{i+1}^{2} - r_{i}^{2}) + r_{i} (\epsilon_{m})_{i} \frac{\partial C}{\partial r} \Big| r_{i}$$
(37)

where $\partial C/\partial r$ and $\partial C/\partial x$ may be expressed in the numerical form

$$\frac{\partial C}{\partial r}\Big|_{r_{i+1}} = \frac{1}{2} \left(\frac{C_{j,i+1} - C_{j,i}}{r_{i+1} - r_{i}} + \frac{C_{j+1,i+1} - C_{j+1,i}}{r_{i+1} - r_{i}} \right) \quad (38)$$

and

$$\frac{\partial C}{\partial x}\Big|_{r_{i}} = \frac{C_{j+1,i} - C_{j,i}}{x_{j+1} - x_{j}}$$
(39)

where the subscript j denotes the jth test position and

the subscript i denotes the ith radius interval.

The intervals in radial direction have been chosen as $r_0 = 0$, $r_1/R = 0.05$, $r_2/R = 0.15$ and $r_{10}/R = .95$. But one must remember that the value of $r_1 - r_0$ in equation (38) have also been taken as 0.275 inches.

The $C_{j,i}$'s were obtained from concentration distribution curves as shown from Fig-XIII to Fig-XVI. The value of $C_{j,i}$ except when i = 0 is actually the middle value between r_i and r_{i+1} at jth test position.

 u_i , the velocity, which appears in equation (37), is obtained from the experimental results and is also an average value between r_i and r_{i+1} .

The physical meaning of equation (37) may be demonstrated much more clearly by the balance diagram of the concentration flow of an elemental volume as shown in Appendix II.

To evaluate equation (37) one has to start from i = 0, and since $(\partial C/\partial r)_{r=0} = 0$, the equation becomes,

$$\mathbf{r}_{1}(\boldsymbol{\epsilon}_{m})_{1} \frac{\partial C}{\partial \mathbf{r}} \Big|_{\mathbf{r}_{1}} = \frac{1}{2} \mathbf{u}_{0} \frac{C_{j+1,0} - C_{j,0}}{\mathbf{x}_{j+1} - \mathbf{x}_{j}} (\mathbf{r}_{1}^{2} - 0)$$
(40)

By dividing the right hand side by $r_1(\partial C/\partial r)_{r_1}$, a value of $(\epsilon_m)_1$ is obtained.

In order to find $r_2(\epsilon_m)_2(\partial C/\partial r)_{r_2}$, the value of $r_1(\epsilon_m)_1(\partial C/\partial r)_{r_1}$ has to be used in equation (37) and so on. The $(\epsilon_m)_i$'s are obtained by dividing the right hand

side by $r_i(\partial C/\partial r)_{r_i}$, respectively. The dimensionless diffusivity is obtained by dividing $(\epsilon_m)_i$ by u_*R .

A sample calculation follows:

For Re = 19200, the following data were measured,

$$C_{4,0} = 1.24$$
 $C_{4,1} = 1.17$ $C_{4,2} = 1.00$
 $C_{5,0} = 0.96$ $C_{5,1} = 0.91$ $C_{5,2} = 0.80$

The above data were taken from Fig-XIV, the first subscript denoted the number of the test position downstream from the injector and the second subscript denoted the dimensionless radius. For example, the subscript showed that the data were measured at distances of 47 inches downstream from the injector. Hence $x_4 = 47$ " and x_5 , the next position is at a distance of 59 inches from the injector.

Other data required for the calculation include the mean pipe velocity $u_0 = 8.94$ ft/sec and $u_1 = 8.86$ ft/sec.

The following procedures were necessary in order to calculate the diffusivity.

From equation (40)

 $\left. r_{1}(\epsilon_{m})_{1\overline{\partial r}} \right|_{r_{1}} = \frac{1}{2} \times 8.94 \times \frac{0.96 - 1.24}{(59 - 47)/12} \times (.05 \times 2.75/12)^{2}$ = -0.000165 ft²/sec

and from equation (37) and substituting the above value, one has

$$\mathbf{r}_{2}(\epsilon_{\rm m})_{2\frac{\partial C}{\partial r}}\Big|_{r_{2}} = \frac{1}{2} \times 8.86 \times \frac{0.91 - 1.17}{(59 - 47)/12} \times (.15^{2} - .05^{2}) \times 2.75^{2}/12^{2}$$

- 0.000165 = -0.00139 ft²/sec

These are the values which have been plotted in

Fig-XIX^{*}. (47"-59" at r/R = 0.5 & 0.15 respectively)

The gradient of the concentration was obtained according to equation (38), which was the average value between test position 4 and 5, i.e.,

$$\frac{\partial C}{\partial r}\Big|_{r_1} = \frac{1}{2} \left(\frac{1.17 - 1.24}{2.75/12} + \frac{0.91 - 0.96}{2.75/12} \right) = -2.618$$

For convenience the gradient is taken with respect to the dimensionless radius rather than to the radius itself, as shown in Fig-XX^{*}, hence,

$$\frac{\partial C}{\partial (r/R)} \Big|_{r_1} = -2.618 \times 2.75/12 = -0.6$$

and

$$\frac{\partial C}{\partial (r/R)} \Big|_{r_2} = \frac{1}{2} (1.00 - 1.17 + 0.80 - 0.91) = -1.4$$

These two values are shown in Fig-XX at the test position 47"-59" intervals and r/R = 0.05 & 0.15 respectively.

The diffusivity may now be calculated as follows:

$$(\epsilon_{\rm m})_1 = -0.000165 / \left[\frac{r}{R} \frac{\partial C}{\partial (r/R)} | r_1 \right] = 0.0055 \ {\rm ft}^2 / {\rm sec}$$

while

$$(\epsilon_{\rm m})_2 = -0.00139 / \left[\frac{r}{R} \frac{\partial C}{\partial (r/R)} | r_2 \right] = 0.00662 \ {\rm ft}^2/{\rm sec}$$

The frictional velocity is needed in order to find the dimensionless diffusivity. The procedures required to calculate u_* were given in section 5.3 and it was found that $u_* = 0.41$ ft/sec for Re = 19200. Therefore the dimensionless diffusivity is calculated as

Absolute values were used during the plot of these Figures.

$$\frac{(\epsilon_m)_1}{u_*R} = 0.0055/(0.41 \times 2.75/12) = 0.0585$$

and

$$\frac{(\epsilon_{\rm m})_2}{u_{\rm w}R} = 0.00662/0.094 = 0.0705$$

these were the data which were plotted in Fig-XXI, (at test position 47"-59" and r/R = 0.05 & 0.15 respectively) If the assumption is made that the diffusivity is independent of the test position, the average values can be calculated and these are shown by the solid curve in Fig-XXI.

6. DISCUSSION:

This section concerns the analysis of error in the correlation of the experimental results and discusses some of the results.

6.1 Motor Speed.

While the experiment was performed, the motor speed was adjusted once every half hour using a tachometer, so that the maximum variation of the speed would be + 5 rpm. For example, at Reynolds number 58300, the maximum fan speed is 825 + 5 rpm, so that at 830 rpm, the maximum mean velocity in the center line of the pipe will be 26.7 ft/sec, and it will be 26.3 ft/sec at 820 rpm. Deviation of velocity is only + 0.75% and could be neglected while measuring the concentration distribution and turbulence level, as they have only an average value. It could not however be neglected when the velocity profile across the pipe was being measured. This error could be eliminated by adjusting the motor speed more frequently, say at an interval of 5 minutes, while the measurement of the velocity profiles was being made. In other words the errors induced by variation of motor speed can be neglected in comparison with other sources of error. As shown in Fig-VII, the measured velocity profile is quite consistence and satisfac-

tory compared with the data obtained by other authors.

6.2 Errors from Non-uniformity of Pipe Diameter and Traversing Mechanism.

A careful measurement of the pipe diameter along the test positions showed a variation in diameter from 5.505 inches to 5.481 inches. Because the traverse mechanism was designed for a uniform pipe cross section of 5.5 inches inside diameter, all the scales were marked on the guide rod of the traversing mechanism by thin grooves, each 0.275 inches apart, i.e., one tenth of the pipe radius, and are not adjustable for a different diameter test position. This was not however a critical problem in our experiment.

As far as the measurement of the boundary layer was concerned, only one hot wire was necessary. The hot wire was mounted on the lower part of the adapter, as shown in Fig-III. This adapter could be moved over a 1 inch distance along the radius direction by a micrometer head which was attached to the traversing mechanism. As noted earlier the adapter was slightly bent upward which allowed the hot wire to be brought into contact with the pipe wall. During the measurement, the hot wire was brought into contact with the wall by turning the micrometer backward, while the contact condition was decided by the feel of mecrometer and by a few thousandth of an inch error could be expected. eye: From Fig-VIII, at a Reynolds number of 7300, for example,

it would appear that the measured distance was 10 thousandth of an inch larger than it should have been if the standard Universal Velocity Distribution Law was correct. The measured distances seem correct for both Re = 19200 and 38600. At the highest Reynolds number (58300), the measured distance was 5 thousandth of an inch smaller than it should have been. If the above corrections were made, all the points would fall on the theoretical universal velocity distribution curves. But in order to maintain the true picture of this experiment, no attempt was made to correct the experimental data in this manner.

No correction were made in the hot wire readings taken near the wall although Laufer $_15_7$ indicated that because of the turbulent fluctuation in this area, a correction could be applied to obtain the proper value of u. The measured values of u are probably less than the actual value in the immediate wall area.

During the concentration measurements, the importance of the exact location of the traversing mechanism was dependent on the test positions and the concentration distributions. As shown in Fig-XII and Fig-XIII, the concentration gradient was steeper at the first few test positions than at the later ones. As we were interested only in the concentration distributions at the last 5 or 6 test positions, where the distribution curves were rather flat, a few hundredth of an inch of deviation does not lead to

any serious error and so can also be neglected.

6.3 The Errors Involved in the Measurement of the Turbulence level.

The turbulence level was measured using the hot wire apparatus and its rms value was given on the DISA Random Signal Indicator. As the average value of rms was taken by averaging over a certain time interval, which seemed to be too short to give a stable value of rms, the indicating needle of the rms meter fluctuated slowly over a rather wide band when the rms value fell below 80 mv. Since most of the rms values in the turbulent core for all the Reynolds numbers investigated during this experiment had values below 80 mv, it was necessary to observe the meter over a certain time interval, and the readings were taken as an average of the rms value over a 5 minutes period. From the results of turbulence level shown in Fig-IX, it would seem that this procedure was quite satisfactory, since most of the data for different Reynolds numbers obtained in this experiment, fell on the same curve within + 5%, if the turbulent level was expressed in dimensionless form u'/u_{\star} . Laufer's data taken at a Re = 50000 has also been plotted in the same figure. The agreement is good up to a r/Rvalue of 0.5. However there is some discrepancy in the region of the turbulent core (for values of r/R > 0.7). The present results show some scatter in this region, although in general the results are quite consistent when plotted in this non-dimensional form.

Fig-X gives the turbulence level near the wall and as before, Laufer's data of Re = 50000 has also been given. The present data, except for Re = 7300, are a little higher in value than that of Laufer. As the rms value in this region is quite stable, the error, if any, must come from the d.c. voltage, which denoted the mean local velocity. Actually, during the experiment the d.c. voltage was observed to vary over a wide range in the vicinity of the wall, primarily, it is suspected, because of the high turbulence level in this region. As before an average reading time of 5 minutes was used with a resultant error of + 5%. According to the discussion presented in a former section, the distance measured from the wall for Re = 58300 was 5 thousandth of an inch less than it should be, but this error is not significant in this plot because of the scale used for the abscissa. The pattern of wall turbulence at Re = 7300 is somewhat different from the others. While the velocity profile and mass diffusivity measured in this experiment are fully turbulent for this Reynolds number, there appear to be a transition occuring in the u'/u_{\star} as a function of Reynolds numbers. More experimental data are needed in order to confirm this result.

Fig-XI shows the correlation of turbulence between two points as a function of the distance from the center

line of the pipe. Although the response of the meter for this correlation is very slow, it is also very stable, and the points given in Fig-XI show the least deviation of any of the experimental results.

The macro scale was obtained according to equation (16), and has been tabulated in Table 1 (pp. 23) for six values of r/R. Because of the limitations of using equation (32), only the value at the center line was used to calculate the momentum diffusivity at the pipe center. However, the macro scales may provide further general information on the diffusion in pipe flow. From Table 1, it can be seen that the maximum value of the macro scale occurs at r/R = 0.6 and the general trend of these results is consistent, except at the center, with the experimental values of the mass diffusivity which are shown in Fig-XVII. Since it will be shown later that the data on the mass diffusivity at the center region (see Fig-XVII) are subject to a great deal of uncertainty and since the macro scales are quite stable, one might be tempted to extrapolate the curve of mass diffusivity in Fig-XVII, according to the tendency of the macro scale, from r/R = 0.5 to the center region. It might in fact provide a somewhat more accurate value of diffusivity at the center of the pipe than the value obtained from equation (32) directly. Using this analysis, the mass diffusivity at the center of pipe could have a value around 0.07 (in a dimensionless form). From Fig-XVIII, the results

of diffusivities from Page et al 2^{-12}_{7} apparently show much more reasonable values than that of the present experimental work in the center region, although the authors admit that there is great uncertainty about the value on the center line.

6.4 The Errors in Concentration Measurement and their Effect on the Calculation of Diffusivities.

In this experiment, the measurements of velocity profiles and turbulence were quite straightforward but the measurement of concentration was much more difficult and the calculation of the turbulent mass diffusivities from the measured concentration distribution was open to considerable error. In the present apparatus, there were two major factors which caused the inaccuracies in the concentration distribution and its measurement.

Firstly, although the injection system was carefully designed, in order to reduce the disturbance in the downstream flow as much as possible, preliminary experiments showed that the injector was slightly off center giving a higher concentration distribution in the lower part of the pipe than in the upper part. Although the injection tip as shown in Fig-IV may be adjusted slightly upward to compensate, it is difficult to ensure at all times that the flow with the additive gas is completely symmetrical, since the traversing mechanism and support allows one to probe in the vertical plane only.

During measurement the injector was adjusted first

by trial and error to obtain the most symmetric distribution at the fourth test position, and then the experiment was It was also found that the best position of the repeated. injector was different for each Reynolds number, so that the injector had to be readjusted with a change in Reynolds number. From the results of this adjustment, it was found that the concentration distribution was very sensitive to a change in injection angle. For example, a one degree change, upward, in the injector angle gave a highly distorted concentration profile as far downstream as the eighth test position. A series of micrometer adjustments is needed if more accurate positioning is required. During the experiment some, but not all, concentrations were measured on both side of the pipe in order to get a more accurate distribution. This procedure involved a considerable increase in experimental time, since each concentration measurement took at least 6 minutes, and there were a total of 264 points taken for 4 different Reynolds numbers as shown in Fig-XII to Fig-XVI. The curves in those figures, therefore, were smoothed more or less according to the data obtained on both sides of the center line. Some of the experimental points given in the figures show duplication, however not all of these duplicate readings have been given since they tended to confuse the picture.

The additional complication was in the limitations of the gas partitioner. These limitations have been dis-

cussed in section 4.9 and are omitted here, except to point out once again that the sensitivity of the gas partitioner will give an error of \pm 0.02% absolute of the ethylene concentration. This was another very serious error as far as the calculation of the diffusivities is concerned.

Besides these two major deficiencies in the experimental apparatus, there was another possible error involved in the concentration measurement at the highest Reynolds number. At this Reynolds number, (Re = 58300), the injection volume rate was quite large (0.011 ft^3 /sec) and as the expansion of the C_2H_4 from the high bottle pressure to the lower line pressure absorbed a large quantity of heat from the surroundings, the regulator had a tendency to freeze. thus causing some instability in the pressure control. Under these conditions, the proper injecting volume had to be continually adjusted using the needle valve. This procedure was difficult to follow during the experimental runs simply because of the other details which required attention. These included (1) setting the traversing mechanism to the proper position; (2) adjusting the motor speed once every half hour; (3) extracting the sample from the container and injecting it into the gas partitioner; (4) manipulating the chart control speed of the Honeywell recorder and at the same time selecting the proper sensitivity knob on gas partitioner (at proper time). In addition, the room temperature had also to be adjusted and of course the traverse me-

chanism had to be changed from one test position of another as the experiment proceeded.

Consider now Fig-XII to Fig-XVI, which show the concentration distribution curves for four different Reynolds number at the different test positions. These curves which were obtained by experiment and combined with the compensation techniques mentioned in the previous section, appear to be quite satisfactory as far as the ethylene distribution was concerned. In addition, as there was no sink or source of ethylene in the flow system, save for the injection section, the mean concentration profile of C_2H_4 should obey the conservation law, which states that the flow rate of ethylene must remain unchanged downstream from the injection position. The integrated flow rate at each section for different Reynolds number was calculated numerically and is tabulated in Table 3. The smoothed data from Fig-XII to XIV were used in these calculations. The injection volume flow rates used in the same table for comparison purposes were obtained from the calibration curves for the flow meter supplied by Brooks Instrument Company Inc.

The measured volume flow rate was obtained using the following equation,

Volume flow rate = $2\pi \int_0^R u \ C \ r \ dr$ where both u and C were obtained from the experiment. Table 3 shows that the accuracy of our concentra-

TABLE 3

1		· · · · · · · · · · · · · · · · · · ·			
Reynolds No.		7300	19200	38600	58300
Injection volume rate		.001535	.00306	.00606	.00906
Integrated Volume Rate at each test position	1	.001337			
	2	.001378			
	3	.001359	.003248		
	4	.001365	.003179	.006516	.01085
	5	.001371	.003277	.006326	.01055
	6	.001377	.003277	.006292	.01088
	7	.001377	.003302	.006290	.01149
	8	.001383	.003287	.006339	.01120
Average value		.001368	.003262	.006352	.01099
The Max. percent deviation from Average value		-2.29%	-2.54%	-0.98%	-4.00%
		+1.04%	+1.25%	+2.57%	+4.50%

Comparison of the Volume Rate of C_2H_4 (Between Injected and Measured Values)

Note: All the volume rates in above table have the dimension of ft3/sec and can be converted to lb_m /sec, knowing the density of ethylene gas.

tion measurements may deviate by $\pm 2.6\%^*$. Also the average integrated value at the highest flow rate is 21% higher than that recorded on the flow meter. Since the flow meters

Except for the highest Reynolds number where difficulty was experienced with maintaining a constant flow rate.

had overlapping calibration curves, it was possible to check one flow meter against the other over a limited range. These tests showed inconsistencies in the calibration curves for two different tubes. For example the calibration curve for the tube used for the highest Reynolds number gave a lower value of volume flow rate than that of the tube used for the middle Reynolds number range. The inconsistency could not be explained and the individual tubes could not be checked without the aid of a primary standard. It should be pointed out that the deviation of the volume flow rate between the one measured and the one recorded on the flow meter had the effect of slightly changing the injection velocity and did not in fact produce a serious error in the calculation of the mass diffusivity. As long as the injecting velocity did not change the main flow condition too much, the calculation of mass diffusivity needed only the relative concentration distribution of the ethylene gas. The absolute value of the volume rate was not necessary.

The conclusion can be drawn that the concentration curves are reliable within the percent error of $\pm 2.6\%$. The large deviation for Re = 58300 was due to the freezing effect on the regulation system caused by the expansion of the C₂H_A in large quantity.

The results of diffusivities calculated from concentration curves are not as near to an optimum value as the concentration curves themselves. From equation (37)

the diffusivities calculated from the concentration curves are subject to two inherent errors by the numerical integration. For example, the calculation of the RHS (right hand side) of equation (37) at the ith radius is actually the accumulated values of previous (i-1)th radius intervals. That is, if the ith value of the RHS of equation (37) is to be calculated, the value of (i-1)th value is needed, but the (i-1)th value contains the (i-2)th value and so on. Hence when i=10, the value of the last interval, it will be affected by all the concentration values at the j and j+1 test positions which are involved in the calculation of the RHS of equation (37). Therefore the errors in the results of the equation may either be cancelled by individual readings or may be accumulated. However the integrating procedures involved in calculating the last interval of the equation (37) consisted of 10 additions and 10 subtractions. If we assume the error of the individual reading to be + 3%, (a value selected according to Table 3) therefore, the probable error for the last term of above integration will be 13.4%. This estimation was made under the assumption that the readings involved in the subtraction and addition were independent, however the data from the concentration distribution curves are somewhat dependent on each other after the smoothing procedures. Fig-XIX shows the integrated values of RHS of equation (37) for Re = 19200 at the last six test positions. The probable error estimated before

seems acceptable. It will also be observed that in order to calculate the diffusivities according to equation (37). the RHS should be divided by $r_i \frac{\partial C}{\partial r} | r_i$ which appeared on the LHS (left hand side). To obtain an accurate value of the gradient with respect to the radius, 2C/2r, for this numerical integration is difficult. As a first attempt, the least squares method for correlating the experimental points was tried, but it failed, since neither the polynomial series of 8th power, nor the Gauss distribution fitted the data. The problem was solved by using equation (38) to find the average values of the concentration gradient. However the error using this finite difference technique cannot be estimated very well. From Fig-XX, the curves of $\frac{\partial C}{\partial (r/R)}$ vs. r/R, show that this error may be as high as 100% at r/R = 0.95. On the other hand from Fig-XXI, the diffusivities obtained by equation (37), show that most of the points fall within + 20% of the average diffusivity curve. This percentage error appears to be the same for the calculations carried out at the other Reynolds numbers. Therefore an estimation that the total percentage error is of the order of + 20% is not unreasonable However it seems that there is not much to be gained by duplicating Fig-XXI for the complete Reynolds number range. The average values between positions for different Reynolds numbers are given as shown in Fig-XVII. They were assumed to be independent of Reynolds number as discussed in the previous section. All the points

.62
on Fig-XVII show a random tendency within a certain range and the scatter of these points must be deduced from the errors discussed before. Taking the mean value over the four different Reynolds numbers, again, provides the solid curve as shown in Fig-XVII. 90% of the experimental points fit the mean curve within $\pm 10\%$.

As a comparison, the eddy momentum diffusivity distribution 2^{-14}_{-7} is also given in Fig-XVII and it can be seen that the mass diffusivity values are slightly greater than the values of the momentum diffusivities at the same radius ratio. Because of the magnitude of the errors involved in the calculation of ϵ_m the calculation of α has little meaning. Based on the average values, the ratio between mass diffusivities and momentum diffusivities (α) can be seen to lie between 1.1 and 1.8 with an average value of 1.3. Most texts quoted values between 1.0 and 1.6.

In the calculation of diffusivities, the concentrations have not only been considered axisymmetric but the molecular diffusivity has been neglected. In order to give some idea how good this assumptions is the following table gives some turbulent diffusivities, the maximum value and the value near the center line, for four different Reynolds numbers in the turbulent core. These values have been obtained from Fig-XVII by multiplying the ordinate by u_*R .

TABLE 4

Reynolds number	7300	19200	38600	58300
$\epsilon_{\rm m}$ at $r/R = .05$.00215	.0063	.0107	.0118
Max. ϵ_{m}	.00411	.0078	.01436	.02363

Some Values of the Turbulent Mass Diffusivities in the Turbulent Core

Note: The diffusivities are expressed in absolute units of ft^2/sec .

The molecular diffusivity for ethylene gas in air is approximately 0.00018 $ft^2/sec [17_7]$, and therefore even at the lowest Reynolds number it will give an error of less than 10% if we neglect this effect in equation (35).

Another reason why the last 5 or 6 test positions were chosen for the diffusivity calculations was that at those positions the ethylene concentration was usually below 2% of the air by volume and therefore the binary effect for molecular diffusivity could be ignored as well $\sqrt{2}$.

6.5 Some Comment on the Apparatus

Most of the apparatus used in this experiment performed satisfactorily although in the case of the gas partitioner it was asked to perform well outside of its accuracy limits. Although the gas partitioner may measure 2 parts per 10 thousandth of ethylene gas from air, it was still not accurate enough to produce the proper concentration gradient with respect to radius. As stated in a previous section 4.9, the problem concerning the injection of the sample gas by syringe can be eliminated if a two way sample valve had been installed on the gas partitioner. Unfortunately during early tests it was found that there was a small leakage within the sample valve and in order to be on the safe side the syringe injector had to be used. The installation of a proper sampling valve, in my opinion, will extend the range of accuracy up to at least 1 part per 10 thousandth of ethylene gas from the air. Moreover, at the end of this experiment it was found that two columns which served to separate the mixture of the gases inside the gas partitioner were place in wrong order. If the positions of these columns had been reversed, then the peaks of the signal appearing on the potentiometer would have been in the following order: the total peak followed by the oxygen, then nitrogen. these having been released by the first column. The last appearances would have been ethylene and the carbon dioxide which are separated in the second column. This may be compared with the present installation in which the ethylene came out right after the total peak, a condition which necessitated a great deal of unnecessary work in the elimination of the effects of the total peak from the ethylene peak.

The position of the $C_{2}H_{4}$ injector was very important in order to obtain a symmetric concentration profile. A more precise adjustment on the injector is a necessary addition to the experiment. The adjustment of the symmetrical concentration should be performed along both the vertical and lateral direction in the pipe. With the limitations of the present equipment only the concentration profile in the vertical direction has been established.

As far as the flow meters are concerned, the calibration curves are not consistent. However the exact value of volume flow rate of the ethylene gas is not very important although it was considered wise to keep the gas velocity equal to the air velocity at the point of injection. The important point was to keep the volume flow rate constant for a particular Reynolds number and series of tests.

In order to measure accurately the distance of the measuring device from the wall an ocular micrometer is needed as Laufer $/ 15_7$ used in his measurements.

The air box used in this experiment is marginal in size. The difference in height between the inlet and exit plane of the fan seem to affect the velocity profile in the pipe at the last test position which was only 6 inches from the air box. This was not a serious defect but could have been avoided by a more symmetrical installation or possibly a larger air box.

During the experiment, too many procedures were

involved simultaneously. Two people working together would have simplified the test procedure and produced more nearly ideal test conditions and results.

7. CONCLUSIONS

7.1 The velocity profiles measured in this experiment are in good agreement with the data from other authors. In general the present data fits the Universal Velocity Distribution Law very well in the sublayer and transition region, although there is some scattering in the sublayer due mainly to the inaccuracy in measuring the wall distance. The comparison of the experimental results with existing theory is adequate in the turbulent core.

7.2 The turbulence level in the core if expressed in the dimensionless form, u'/u_* , is independent of the Reynolds number, at least in the Reynolds number range between 7300 to 58300, although the results differ slightly from Laufer's data.

7.3 The turbulent momentum diffusivity on the center line of the pipe has been calculated using the macro scale of turbulence and has a value equal to 0.111 (in dimensionless form). This value does not agree with the accepted results. Referring to discussion presented in section 6.3 (pp.51), the dimensionless momentum diffusivity on the center line of pipe should have a value of approximately

^{0.07.} It is generally believed that the momentum diffusivity is always slightly lower than the mass and heat diffusivity, for air.

7.4 The concentration distribution curves for ethylene are bell-shaped across the pipe cross section. But it is not a Gaussian distribution, since the turbulent diffusivities are not a constant across the radius direction. The bell curves become flatter as the distance from the injection point is increased, and since there is no source or sink, the total volume rate is conserved.

7.5 The dimensionless mass diffusivity (ϵ_m/u_*R) is independent of Reynolds number. The dependence of α (the ratio of turbulent mass diffusivity to turbulent momentum diffusivity), on Reynolds number as found by Page et al $/12_7$ between two parallel plates, no longer exists in pipe flow. This is due to the fact that the dimensionless turbulent momentum diffusivity is also independent of Reynolds number, as shown theoretically (or experimentally) by most of the aothors $/14_7$.

7.6 The value of \bigotimes for air lies between 1.1 and 1.8 with an average value of 1.3. This is in general agreement with the presently accepted range of 1.0 to 1.6 $\sum 2_7$ which is used in analogies between momentum, mass and heat transfer. The conclusion is that, if it is the overall picture of heat or mass transfer, that is the Nusselt or Sherwood number respectively, in which one is interested, the exact values of turbulent diffusivity in the turbulent core is not necessary, because the analogy methods do provide a good solution to this kind of problem, and because the analogy approach does not need the values of turbulent diffusivity in the core as shown in the previous section of literature survey. However, the diffusivity value is important in order to calculate the temperature distribution or mass concentration along the axial direction and across the pipe.

In the present experiment, although only for air, 7.7 which has a Prandtl number of 0.7, it seems worthwhile to restate the conclusions given by Sherwood 2_2 , as a final conclusion to this present paper. Sherwood pointed out at the conclusion of an analogy treatment which he reviewed, that the best results for large values of Pr (or Sc) are obtained by workers who assumed a certain amount of turbulence in the sublayer region below $y^+ = 5$. The best results for very low values of Pr (or Sc) on the other hand are obtained by assuming \measuredangle to vary with Pr (or Sc). In this range it is more important to know the manner in which ϵ_m (or ϵ_h) and \propto vary with Pr (or Sc) and y⁺ at a considerable distance from wall. When the Prandtl number (or Sch-

midt number) has a value of approximately one, both assumptions as stated are good, since the experimental and theoretical results are in good agreement in this range.

8. NOMENCLATURE

Arabic Description Units Symbols lb_m -mole/ft³ Concentration С per cent by Vol. (dimensionless) Or Concentration of $C_2 H_A$ Molecular velocity ft/sec С Btu/lb_m-⁰R Specific heat °p ft²/sec Molecular Diffusivity D E d.c. volts volts ΔE Instantaneous change of mv d.c. voltage Erms Root mean square value of $\triangle E$ mv f Frictional factor defined by dimensionless $u_{\star} = \sqrt{f/8} \overline{u}$ f! Frictional factor defined by dimensionless $u_x = \sqrt{f'/2} \overline{u}$ 32.2 ft/sec^2 Acceleration due to gravity g $32.2 \frac{1 \text{bm-ft}}{1 \text{bf-sec}^2}$ Conversion factor gc $g(x_2)$ Double correlation Coefficient dimensionless defined by equation (15) Δh Differential pressure inches of water Btu/hr-ft²-°R h Heat transfer coefficient Btu/hr-ft-^oR k Thermal conductivity

К	Mass transfer coefficint	ft/sec
ĸı	Defined by equation (28)	dimensionless
l	Mean free path of molecules	ft
N	Collision number	dimensionless
NA	Mass transfer flux	lb _m -mole/ft ² -sec
р	Pressure	lb _f /ft ²
P_{atm}	Atmospheric pressure	inches of Hg
P	Probability	dimensionless
q	Heat flux	Btu/hr-ft ²
r	Radius	ft
R	Radius of pipe	ft
$R_{L}(s)$	Lagrangian correlation coefficient	dimensionless
8	Time	sec
t	Time	sec
T _a	Room temperature	°R
u	Local mean velocity	ft/sec
∆u	Instantaneous longitudinal fluctuation velocity	ft/sec
ū	Bulk mean velocity	ft/sec
u'	Root mean square value of Δu	ft/se c
u _*	Frictional velocity	ft/sec
u ⁺	Dimensionless velocity, u/u_{\star}	dimensionless
v(t)	Lateral fluctuation velocity at time t	ft/sec
v	Root mean square value of lateral fluctuation velocity	ft/sec

73[.]

x	Axial direction	ft
x ₂	Relative distances between two hot wire probes	inches
y _i	Small displacement of turbulent lump at ith collision	ft
у	Distances measured from the wall	ft
y .+	Dimensionless distances from the wall, yu_*/v .	dimensionless
y ²	Mean square value of the dis- placement of a turbulent lump after N collisions	ft ²

Greek Symbols	Description	Units	
Ø	Ratio between turbulent mass (or heat) diffusivity to turbulent momentum diffusivity	dimensionless	
E	Turbulent Diffusivity	ft ² /sec	
$\boldsymbol{\Lambda}_{\mathrm{L}}$	Lagrangian integral scale	inches	
$\Lambda_{\rm g}$	Lateral integral scale	inches	
M	Dynamic viscosity	lb _m /sec-ft	
V	Kinematic viscosity	ft ² /sec	
₿ _a	Density of air	lb _m /ft ³	
\mathcal{S}_{w}	Density of water	lb _m /ft ³	
ዮ	Frictional stress or shear stress	lb _f /ft ²	

. 74 Dimensionless parameters

Descriptions

Nusselt	number	=	$\frac{2 h_c^R}{k}$

Prandtl number = $\frac{\mu c_p}{k}$

 \Pr

Nu

Sc

Sh

Re

Schmidt number = $\frac{1}{D}$ Sherwood number= $\frac{2 \text{ K R}}{D}$ Reynolds number= $\frac{2 \overline{u} R}{\sqrt{2}}$

 $\operatorname{St}_{\mathbf{h}}$

Stanton number for heat transfer

$$St_{h} = \frac{Nu}{Pr Re} = \frac{h_{c}}{f_{a}c_{p}u}$$

 st_m

Stanton number for mass transfer

 $St_m = \frac{Nu}{Re Sc} = \frac{K}{\overline{u}}$

Letter Subscripts

Descriptions

h Relates to heat
i Number of interval in radius direction
j Number of test position
m Relates to mass
v Relates to momentum
w Relates to wall

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10. ILLUSTRATIONS

APPENDIX

C







Fig-III TRAVERSING MECHANISM



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APPENDIX I

Derivation of the Relation between Turbulent Diffusivity and Lagrangian Integral (Macro) scale.

The derivation of the relation between momentum diffusivity and the Lagrangian integral scale can be found in Ref. 2 (pp.42 to 49). However for the completeness of this paper, a brief description will be given below.

The one dimension molecular diffusion equation is generally written as follows,

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$
 I-1

The simplest solution of this equation is ,

$$C(x,t) = \frac{\text{const.}}{\sqrt{t}} \exp(-x^2/4Dt) \qquad I-2$$

Equation (I-2) gives the value of the concentration C at x after the lapse of a time t since the molecules start from the plane x = 0.

The Kinetic Theory of Gases $[18_7]$ may also be used to solve the one dimensional diffusion problem with the proper assumptions. These include the assumption of random molecular motion and also that the mean free paths of the molecules are the same, constant and parallel to the direction x. It can be shown that the probability P that a molecule has advanced a distance x after N free paths is equal to,

$$P(x,N) = \sqrt{\frac{2}{\pi N}} \exp(-x^2/2N \ell^2)$$
 I-3

where \mathcal{L} is the mean free path of the molecule. Since it can be shown that

$$N = ct/\ell$$

where c is the molecular velocity. Equation (I-3) becomes,

$$P(x,t) = \sqrt{\frac{2\ell}{\pi ct}} \exp(-x^2/2c\ell t)$$
 I-4

Furthermore, if one supposes that the original concentration at x = 0 when t = 0 is C_0 , then C(x,t) in equation (I-2) denotes only the fraction of C_0 which advances a distance x at time t. Therefore the ratio of C(x,t) to C_0 has the same mean as the probability as shown in equation (I-4), i.e.,

$$P(x,t) = C(x,t)/C_0$$

Then it can be seen that equation (I-4) has the same form as equation (I-2) with the possible difference of a constant. Hence one may write,

$$D = N l^2 / 2t = \frac{1}{2} c l$$
 I-5

Consider next the case of successive displacements Δy_i of a turbulent lump with the same assumption as made previously for molecular motion. Since the assumption has been made that each displacement is independent of its history, therefore after N collisions of the lump, the mean square value of y reads

$$\overline{y^{2}} = \overline{\left(\sum_{i=1}^{N} \Delta y_{i}\right)^{2}} = \sum_{i=1}^{N} \sum_{i=1}^{N} \Delta y_{i} \Delta y_{i} = N \overline{\Delta y^{2}}$$
 I-6

I - 7

The average value of the product terms vanishes in the above equation since the independence of each displacement has been assumed. Therefore one may difine the turbulent diffusivity, ϵ , according to equation (I-5)

$$\epsilon = \overline{y^2}/2t$$

Now let v(t) be the turbulent velocity in the y direction. The distance y(t) - y(0) traveled by the lump in this direction after time t is,

$$y(t) = y(0) + \int_0^t v(t^*) dt^*$$
 I-8

If the diffusion start from y(0) = 0, when $t = t_0$, then

$$y(t_{o}+t) = \int_{0}^{t} dt' v(t_{o}+t')$$

Therefore the mean square value $\overline{y^2}(t)$, where the averaging has been carried out with respect to a large number of particles with different starting time t_0 , will be

$$\overline{y^2}(t) = \frac{1}{T} \int_0^T y^2(t_0 + t) dt_0$$

$$= \frac{1}{T} \int_0^T dt_0 \int_0^t dt' \int_0^t dt'' v(t_0 + t') v(t_0 + t'')$$

$$= \int_0^t dt' \int_0^t dt'' \overline{v(t') v(t'')}$$

where

$$\overline{v(t') v(t'')} = \frac{1}{T} \int_0^T dt_0 v(t_0 + t') v(t_0 + t'')$$

The double integration is carried out over the area ABCD

as shown on right figure, and may split into two part,

$$\int_{0}^{t} \int_{0}^{t} dt' dt'' = t$$

$$= \int_{0}^{t} dt'' \int_{0}^{t''} dt' + \int_{0}^{t} dt' \int_{0}^{t''} dt'' = t$$
I-9

The integration of the first term of RHS of above equation is carried out over the triangular ADC and the second term of RHS is integrated over the area ABC. Since $\overline{v(t')} \ v(t'')$ is symmetrical with respect to t' and t'', i.e. the interchange of t' and t'' will not affect the integration results in equation (I-9) and therefore, one can write,

$$\int_0^t dt'' \int_0^{t''} dt' = \int_0^t dt' \int_0^{t'} dt''$$

Hence,

÷

$$\frac{1}{y^{2}} = 2 \int_{0}^{t} dt' \int_{0}^{t'} dt'' \overline{v(t') v(t'')}$$
 I-10

Now let t''-t' = s and equation (I-10) may be transformed into,

$$\overline{y^{2}(t)} = 2 \int_{0}^{t} dt' \int_{-t'}^{0} v(t') v(s+t') ds$$

= 2 $\int_{0}^{t} dt' \int_{0}^{t'} v(t') v(t'-s) ds$
= 2 $v'^{2} \int_{0}^{t} dt' \int_{0}^{t'} R_{L}(s) ds$ I-11

t'!

D

where

$$R_{L}(s) = \frac{v(t') v(t'-s)}{v'^{2}}$$
I-12

Equation (I-12) defines the Lagrangian correlation coefficient. Equation (I-11) may be written somewhat differently by carrying out a partial integration as follows,

$$\overline{y^{2}}(t) = 2 v'^{2} \left\{ \left| t' \int_{0}^{t'} R_{L}(s) ds \right|_{0}^{t} - \int_{0}^{t} t' R_{L}(t') dt' \right\}$$
$$= 2 v'^{2} \left\{ t \int_{0}^{t} R_{L}(s) ds - \int_{0}^{t} s R_{L}(s) ds \right\}$$
$$= 2 v'^{2} \int_{0}^{t} (t-s) R_{L}(s) ds \qquad I-13$$

Since $R_L(s)$ has properties similar to $g(x_2)$, which was measured experimentally, it is to be expected that $R_L(s) \rightarrow 0$ as s increases. Therefore one may assume $R_L(s) = 0$ when $s = t_1$. Equation (I-13) gives,

$$\overline{y^2}(t) = 2 v'^2 \left\{ t \int_0^{t_1} R_L(s) ds - \int_0^{t_1} s R_L(s) ds \right\}$$

For $t >> t_1$, the second term on the right hand side will become very small with respect to the first term so it may be neglected.

If one defines the Lagrangian macro scale as,

$$\Lambda_{\rm L} = v' \int_0^\infty R_{\rm L}(s) ds$$

Then it can be seen that

$$\epsilon_{v} = \frac{\overline{y^{2}(t)}}{2t} = \frac{2v't\Lambda_{L}}{2t} = v'\Lambda_{L}$$

Empirical relationships have been found which show that Λ_L is roughly of the same magnitude as Λ_g , accordingly, the momentum diffusivity at the center of the pipe can be calculated by

$$\epsilon_{v} \cong v \cdot \Lambda_{g}$$

where Λ_g , the macro or integral scale of turbulence, is an experimentally measured quantity.

The above proof while lacking rigor provides a relationship between the diffusivity and the scale of turbulence. A more complete analysis is given in Reference 2.

APPENDIX II

Shell]	Balance	Techr	nique	Apr	lied	to	a (Cylind	lriçal	
Volume	Element	; for	the	Calc	ulat	Lon	of	Mass	Diffu-	-
sivity.	<u>.</u>									

The meaning of equation (37) in the text may be demonstrated much more clearly by the balance diagram of the mass flow in an elemental annular volume as shown in the figure.



The diffusion effects in the volume element $r = r_i$ to r_{i+1} and a longitudinal distance x_j to x_{j+1} can be written as follows.

The mass flow into the volume is

$$u_i C_{j,i} \pi (r_{i+1}^2 - r_i^2) - \frac{\partial C}{\partial r} \Big|_{r_i} (\epsilon_m)_i 2\pi r_i (x_{j+1} - x_j)$$

while mass flow out of the volume is

$$u_i c_{j+1,i} \pi (r_{i+1}^2 - r_i^2) - \frac{\partial c}{\partial r} \Big|_{r_{i+1}} (\epsilon_m)_{i+1} 2\pi r_{i+1} (x_{j+1} - x_j)$$

.

Since there is a steady-state flow therefore,

The mass flow in = The mass flow out. Hence one has

$$\pi \mathbf{r}_{i+1} (\epsilon_m)_{i+1} \frac{\partial C}{\partial r} |_{r_{i+1}}$$
$$= u_i \frac{C_{j+1,i} - C_{j,i}}{x_{j+1} - x_j} \frac{\pi}{2} (r_{i+1}^2 - r_i^2) + \pi r_i (\epsilon_m)_i \frac{\partial C}{\partial r} |_{r_i}$$

It is exactly equation (37) with the cancellation of π .