Three Essays in Corporate Investment and Financing

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Abstract

This thesis explores the effects of three important factors on a firm's investment and financing decisions, using contingent claim structural model. The first essay investigates how implementation lag impacts investment timing for a levered firm. The main finding is that implementation lag can potentially have a substantial effect on a company's investment trigger. A crucial determinant of the lag-investment relationship is the fraction of investment cost that has to be incurred upfront. If this fraction is small, investment trigger is a decreasing function of implementation lag and the effect can be economically significant. If this fraction is large, investment trigger can be either increasing or decreasing in lag, but the magnitude of the effect is not large.

The second essay investigates how future uncertain growth opportunity impacts a firm's investment timing decision and optimal leverage ratio. The firm has an option to expand profits after the first investment. However, the exercise of the growth option depends not only on the underlying profit flow but also on the uncertain arrival of the growth opportunity. The model illustrates that such uncertainty can significantly impact the initial investment timing for unlevered firm in a non-monotonic way. For levered firm, the future growth uncertainty, along with debt overhang problem, can shape the firm's financing decision at initial investment.

The third essay shows how risk-compensating performance-sensitive debt can be used to mitigate the "overinvestment" agency problem. We show that properly designed performance-sensitive debt can add significant value relative to fixed-coupon debt, and identify the risk-compensation level that maximizes shareholder wealth. The optimal risk-compensation level is found to be smaller than that required to eliminate overinvestment; thus, it is optimal for shareholders to incur some agency cost of overinvestment.

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1. Chapter One

Thesis Introduction

The firm's investment and financing decisions are among the most important topics in corporate finance. These decisions can be impacted by many factors. The first essay investigates the impact of time-to-build on a levered firm's investment decision. Time-to-build is very common in most industrial firms since capital investment unavoidably requires time to construct. And different industries require different time to build. For example, a nuclear utility firm may need more than 10 years to finish the construction while a real estate firm only need 2 years to build an apartment, thus the capital budgeting decision is closely related to the construction duration. Since most firms issue debt to fund investment we have to consider leverage ratio which affects firm value. Therefore the investment behaviours of levered firms will be different from that of unlevered firms. The impact of time-to-build for unlevered firms have been studied by (Bar-Ilan and Strange 1996, Sarkar and Zhang 2013) . Our treatment of the time-to-build is similar to Margsiri et al. (2008) because it could make the model more tractable and lead to semi-closed form solutions. To implement the construction duration, the firm is assumed to have two stages investment and the construction is not finished until the two investments are implemented. The firm have to raise both equity and debt to fund initial investment and choose the optimal timing and coupon level. However, there is no profit flow occurred until the second stage investment, which also requires the input of sunk cost funded by both debt and equity. Moreover, due to the existence of debt, equity holders may default before the second stage investment. We show that the cost distribution, namely, which fraction of total cost in each stage, has a significant impact on the investment timing of both unlevered and levered firms. For unlevered firm, this paper shows that when the first-stage investment fraction is large, the optimal investment trigger is an increasing function of implementation lag; otherwise, the trigger is a U-shaped function of lag. However, when the first-stage investment fraction is small, the optimal investment trigger is strictly decreasing for all realistic lags. For optimally levered firm, when the initial investment fraction is small, the investment trigger is a decreasing function of implementation lag, and the effect can be economically significant. When the initial investment fraction is large, the investment trigger is less sensitive to implementation lag, and can be either increasing or decreasing. Thus we conclude that time-to-build can have positive impact on the optimally levered firm.

The second essay studies the impact of uncertain growth opportunities on a firm's investment and financing decisions. The reasons for uncertainty of a firm’s growth opportunity are multiple. For example, some firms need technology innovation to generate new products; the success of new technology is uncertain and unknown to managers (R&D, new drug development, etc). Some firms may have trouble tackling with uncertain policies, selection of factory location and upstream suppliers when they decide to expand either domestically or internationally. Thus it will be quite useful for a firm to consider such future uncertainties when it prepares to make the initial investment decision. Despite the important effect of future growth uncertainty on the current investment decision, only a few papers have examined such a phenomenon. We model the uncertainty by assuming it arrives with Poisson process, similar to Li and Mauer (2013). However, Li and Mauer assume that the expansion option must be exercised immediately if at all, while in our paper the firm can time its exercise of the growth opportunity. Suppose a firm has made the first investment and wishes to expand current revenue. However, such an expansion cannot be realized without more advanced technology and some investment cost. The irreversible sunk cost implies that the firm has to wait until the price reaches a higher level to exercise the growth option. However, the new technology will arrive with uncertainty. It may have always been accessible for the time being, or need some time to develop and we don’t know when it can be implementable. For unlevered firm, we show that the growth uncertainty has non-linear effect on the initial investment timing. That is, the investment trigger is a decreasing function of arrival rate of growth opportunity when the rate is relatively small and an increasing function of arrival rate when the rate is relatively large. The reason can be attributable to the relative weight of cost and benefit of growth opportunity. When the arrival rate of growth is small, the benefit (earnings effect) of growth dominates the cost and vice versa as the arrival rate is large. The uncertain arrival provides a channel through which the two above-mentioned forces can interact to impact initial investment timing. We also extend the model to levered firms and find that the introduction of debt financing doesn't alter the investment-growth uncertainty relationship. We show that the optimal leverage also presents a U shaped curve with arrival rate of growth opportunity. For levered firm, the decision of how much debt to issue at the beginning is crucially dependent on the trade-off among tax shield benefit, pre-arrival default chance as well as debt overhang problem. The result for levered firm has valuable empirical implication since most empirical papers which test the leverage-growth options relation.

The third essay provides an investment-based explanation for firms to use Performance-Sensitive-Debt (PSD). PSD is an innovation in the corporate debt market that is very popular today, particularly in bank loans and Telecom corporate bonds (Asquith, et al., 2005, Koziol and Lawrenz, 2010, Manso, et al., 2010, Mjos, et al., 2012, Myklebust, 2012). The novel feature of this debt is that the coupon payment varies with the firm's performance, typically increasing when firm performance deteriorates, in order to compensate debt holder for the additional default risk (Manso, et al., 2010). This risk-compensation provision is the unique feature of performance-sensitive debt. On the other hand, the over-investment problem (Mauer and Sarkar 2005) occurs because the equity holders have limited liability and wish to accelerate investment henceforth transfer the premature risk to debt holders. We found that the optimally designed PSD (by optimizing the risk compensation factor of PSD) could mitigate the over-investment agency conflicts and optimize equity value under the second-best strategy, namely, the firm choose optimal investment timing through maximizing equity holders' value rather than total firm value. The reason is that as risk compensation factor increases both first- and second- best investment triggers will rise while the later rise faster thus overinvestment problem is decreased.

We use different sets of base-case parameter values as benchmark cases for each essay. The reason is that we would like to follow the parameter values used by the related literature on each sub-topic, to facilitate our comparison with the previous results. To make sure that the results are robust and not dependent on the exact parameter values, we conduct extensive comparative statics by repeating the computations with a wide range of parameter values. The results do not change in any of the cases.

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2. Chapter Two

Investment Policy with Time-to-build

2.1. Introduction

Real-option models of corporate investment generally assume that, when the investment decision is taken, the project is completed instantaneously and starts delivering cash flows immediately (Dixit and Pindyck, 1994, Mauer and Ott, 2000, McDonald and Siegel, 1986). However, it is well known that most capital projects involve significant time to completion before they start generating cash flows (Agliardi and Koussis, 2013, Koeva, 2000, Bar-Ilan and Strange, 1996). This time lag is known in the literature as "implementation lag" or "time-to-build." In recent research, some attempts have been made to study the effect of implementation lag on the investment decision (Alvarez and Keppo, 2002, Bar-Ilan and Strange, 1996, Sarkar and Zhang, 2013), but they are limited to all-equity (unlevered) firms.

Most firms use some leverage, which affects firm value via tax shields and bankruptcy costs; thus, leverage affects the project value and thereby changes the attractiveness of the project. Clearly, then, leverage should affect the investment policy. Implementation lag plays a role in the investment policy because it impacts the leverage ratio, which, as mentioned above, affects the investment policy. Thus the effect of implementation lag could be different for levered firms than for unlevered firms. This motivates our paper, the main objective of which is to determine the effect of implementation lag on a levered firm's investment decision. This issue has not yet been addressed in the literature, to our knowledge.[1]

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[1] Agliardi and Koussis (2013) determine the optimal capital structure with time-to-build, but do not consider optimal investment policy (assuming instead that investment is made at time $t = 0$). Egami (2009) and Tsyplakov (2008) examine the firm's decision to expand its existing operations rather than the initial investment decision. Since the implementation lag is more important for an initial investment decision than for an expansion decision (Sarkar and Zhang, 2013), our model focuses on the initial investment, unlike Egami (2009) and Tsyplakov (2008).

We examine the effect of time-to-build on a levered firm's investment timing. Time-to-build is incorporated in the manner of Margsiri et al. (2008), which allows us to develop a tractable model with quasi-analytical solutions. Specifically, the project is assumed to be implemented in two stages, with an initial (or first-stage) investment and a final (second-stage) investment. The project starts generating cash flows only after the second-stage investment. The implementation lag is then simply the time elapsed between the first-stage and the second-stage investments.

This paper contributes to the literature by establishing the effect of the ubiquitous implementation lag on corporate investment decisions. The main results are as follows. The time-distribution of investment cost plays a crucial role in determining the effect of implementation lag on the investment trigger. For an unlevered firm, when the investment is front-loaded (i.e., the first-stage investment fraction is large), the optimal investment trigger is an increasing function of implementation lag; otherwise, the trigger is a U-shaped function of lag. However, when the first-stage investment fraction is small, the optimal investment trigger is strictly decreasing for all realistic lags (below 12 years). These are new results, since the role of the time-distribution of investment cost has not been examined in the literature; earlier papers ignore this issue by assuming the entire investment cost is incurred at one point in time (either at the end or at the beginning).

For a levered firm, the relationship is more complicated: implementation lag could potentially have a significant effect on the investment trigger, but the exact effect depends on the level of debt used. We briefly examine the case of exogenously-specified debt level, but the main focus of our paper is on the optimally-levered firm, for which we obtain the following results. When the initial investment fraction is small, optimal investment trigger is a decreasing function of implementation lag, and the effect can be economically significant. When the investment cost is front-loaded, the investment trigger is not very sensitive to implementation lag, and can be either increasing (when growth rate and tax rate are low, and interest rate is high) or decreasing (all other cases). Optimally levering a firm causes the implementation lag to have a more favorable impact on investment relative to using no leverage. Thus, if for an unlevered firm the investment trigger is a decreasing (increasing) function of implementation lag, then for an optimally-levered firm it will be a more decreasing (less increasing or even decreasing) function of lag. Overall, for an optimally-levered firm, implementation lag has a positive effect or a minor negative effect on investment; this is very different from an unlevered firm, particularly for front-loaded investment projects.

Although our paper's main focus is investment policy, we also take a brief look at financing policy and find that the optimal leverage ratio is an increasing function of implementation lag, consistent with Agliardi and Koussis (2013).

The main practical implication of our paper is that, for optimally-levered firms, implementation lag will have a positive effect on investment, except when the initial investment fraction is large, interest rate is high, and tax rate and growth rate are low, in which cases it might have a minor negative effect.

The rest of the paper is organized as follows. Section 2.2 develops the model, describes the implementation lag in detail, and evaluates the investment decision for an unlevered firm. Section 2.3 examines the more important case of a levered firm. Section 2.4 presents the results, and Section 2.5 concludes.

2.2. The Model

As in traditional real-option models (Mauer and Sarkar, 2005, Roques and Savva, 2009), we assume that the firm has an investment opportunity which costs \$I to implement, and it can choose the time of investment. Prior to the investment, the firm consists of just the investment option. Unlike the above models, however, there is an implementation lag or time-to-build, because of which the project starts generating earnings or cash flows not immediately but only after a lag (the implementation lag). Implementation lag is modeled as in Margsiri, et al. (2008) and discussed in Section 2.2.1 below.

After the implementation lag, the project generates a continuous cash flow stream of \$$x_t$ per unit time, which is assumed to follow the usual lognormal process:

$$dx = \mu x dt + \sigma x dz \tag{1}$$

where μ is the expected growth rate and σ is the volatility of the earnings process, both assumed constant, and dz is an increment to a standard Brownian Motion Process. The firm's earnings (after interest, if any) are taxed at a constant rate of τ, and all cash flows are discounted at a constant discount rate of r. Shareholders receive all residual cash flows after interest and taxes.

2.2.1. Implementation Lag

The existing literature on implementation lag generally treats the lag as a fixed length of time that is known in advance, e.g., Alvarez and Keppo (2002), Bar-Ilan and Strange (1996), Sarkar and

Zhang (2013). Our paper uses a different approach (described below) following Margsiri et al. (2008), where implementation lag is denoted by a parameter β.[2]

If the firm wants to implement the project, it must invest in two stages, with some elapsed time between the two stages, before it can realize any benefits from the project. Thus, investment takes place in two stages – in the first stage, the firm invests a fraction θ of the total investment cost (or $θI) and receives a fraction θ of the total set of assets of the project, where $0 \leq \theta \leq 1$. The first-stage investment allows the firm to proceed to the second stage. In the second stage, the firm pays the remainder of the investment cost, or $(1–θ)I, and receives the remaining fraction (1–θ) of the assets. The project starts generating cash flows only at the second stage, hence there are no cash inflows between the first and the second stage.

To represent the implementation lag, we specify that if the first-stage investment takes place at a certain level of x (say, when x rises to x_f), then the second-stage investment must take place (and cash flows will start) when x rises to $x = \beta x_f$, where $\beta > 1$. Thus, some time has to elapse between the first stage and the second stage, i.e., the time required for x to increase from x_f to βx_f. This elapsed time is the implementation lag.

Since x is stochastic, the implementation lag is a random variable, with an expected value of:[3]

$$E(L) = \frac{\ln \beta}{\mu - 0.5\sigma^2}$$

(2)

[2] This approach has the advantage of tractability. As Margsiri et al. (2008) confirm (footnote 8, p. 643), the main results are not affected if a fixed lag is used.
[3] See Margsiri et al. (2008), Section 2.4.

An increase in β implies a longer expected lag. The distribution of implementation lag is independent of the first-stage investment threshold x_f, hence the expected lag is unaffected by x_f.

Note that in our representation, the implementation lag is stochastic, as opposed to the known (i.e., with no uncertainty) implementation lag in earlier papers such as Alvarez and Keppo (2002), Bar-Ilan and Strange (1996) and Sarkar and Zhang (2013).

2.2.2. Time-distribution of Investment Cost (θ)

In all the existing papers, the entire investment cost is incurred either at the beginning (Bar-Ilan and Strange, 1996) or at the end (Alvarez and Keppo, 2002, and Sarkar and Zhang, 2013). In contrast, our model makes the more general assumption that the firm incurs investment cost of θI at the beginning and $(1-\theta)I$ at the end. As shown in Sections 2.2.3 and 2.4, the parameter θ plays an important role in determining how implementation lag affects the investment trigger. A higher θ means that a larger fraction of the total investment cost has to be incurred upfront. Then the effect of a higher θ is to increase the effective investment cost (in present value terms), which should result in a higher investment trigger. This is confirmed by the numerical results of Section 2.4.2.

Further, the response of the investment trigger to a longer time-to-build will also depend on the parameter θ. For small θ, the firm is paying most of the investment cost at the second stage, hence a longer gap between the two stages will make the project relatively more attractive, everything else remaining the same. Therefore, for small θ, a longer time-to-build will result in earlier investment (lower investment trigger). Conversely, for large θ, a longer time-to-build is more likely to lead to delayed investment (higher trigger). These relationships are confirmed by the numerical results of Section 2.4.

There is only one paper, to our knowledge, that examines empirically the time-distribution of a project's investment cost. Krainer (1968) examines a number of investment projects in the automobile industry, and finds that investment costs are incurred in different time patterns for different projects, even within the same industry. In his Table 2, he lists, for 25 investment projects, the proportions of total project time elapsed when 25%, 50% and 75% of the investment cost was incurred. Although this does not give us an explicit value of θ, we can conclude whether θ is small, intermediate or large from his data. For instance, for project 24, 50% of the total investment cost is spent in 33% of the project's implementation time, indicating that most of the investment cost is incurred upfront (i.e., θ is large). For project 25, 50% of the cost is spent in 70% of the time, indicating most of the cost is incurred later in the project (i.e., θ is small). Finally, for project 9, 50% of the cost is spent in 50% of the time, indicating the cost is evenly distributed (i.e., θ has an intermediate value). Thus, the parameter value θ in our model can take on a wide range of values, depending on the project specifications.

2.2.3. The Deep-Pocket Firm without Access to Debt Financing

First we look at the all-equity firm, i.e., a deep-pocket firm without access to debt financing. Suppose it makes the first-stage investment (at a cost of θI) when x rises to x_i, and the second-stage investment (at a further cost of $(1-\theta)I$) when x rises further to βx_i. Let the firm value be $V_0(x)$ before investment, $V_1(x)$ after the first-stage but before second-stage investment, and $V_2(x)$ after the second-stage investment. Then, as shown in Appendix A, the three value functions are:

$$V_2(x) = \frac{(1-\tau)x}{r-\mu} \tag{3}$$

$$V_1(x) = B x^{\gamma_1} \tag{4}$$

$$V_0(x) = A x^{\gamma_1} \tag{5}$$

where A and B are constants to be determined by boundary conditions (and are given by equations (12) and (13) below), and γ_1 and γ_2 are the positive and negative solutions, respectively, of the quadratic equation: $0.5\sigma^2\gamma(\gamma-1)+\mu\gamma-r=0$, and are given by

$$\gamma_1 = 0.5 - \mu/\sigma^2 + \sqrt{2r/\sigma^2 + \left(0.5-\mu/\sigma^2\right)^2} \tag{6}$$

$$\gamma_2 = 0.5 - \mu/\sigma^2 - \sqrt{2r/\sigma^2 + \left(0.5-\mu/\sigma^2\right)^2} \tag{7}$$

(Note that $\gamma_1 > 1$ and $\gamma_2 < 0$.)

Boundary Conditions

There are two boundaries (corresponding to the two investment triggers): $x = x_i$ and $x = \beta x_i$. The boundary conditions are:

$$V_1(\beta x_i) = V_2(\beta x_i) - (1-\theta)I \tag{8}$$

$$V_0(x_i) = V_1(x_i) - \theta I \tag{9}$$

$$\left.\frac{dV_0(x)}{dx}\right|_{x=x_i} = \frac{dV_1(x_i)}{dx_i} \tag{10}$$

Conditions (8) and (9) are value-matching conditions to ensure continuity of the valuation functions, and condition (10) is a smooth-pasting condition to ensure optimality of the investment decision. The three boundary conditions (8) – (10) are solved for the three unknowns A, B and x_i, to get:

$$x_i = \frac{1 - \theta + \theta\beta^{\gamma_1}}{\beta} \frac{(r - \mu)I}{(1 - \tau)(1 - 1/\gamma_1)} \tag{11}$$

$$A = \frac{(1 - \tau)}{(r - \mu)} \frac{(\beta x_i)^{1-\gamma_1}}{\gamma_1} \tag{12}$$

$$B = \left[\frac{(1 - \tau)}{(r - \mu)} \beta x_i - (1 - \theta)I\right] (\beta x_i)^{-\gamma_1} \tag{13}$$

The investment trigger x_i in equation (11) represents the optimal investment policy.

From equation (11), it is clear that the effect of implementation lag on investment trigger (i.e., $dx_i/d\beta$) depends crucially on the time-distribution of investment cost, θ. Differentiating x_i in equation (11) with respect to β, we get, after some simplification:

(i) for $\theta \geq 1/\gamma_1$, $dx_i/d\beta > 0$, hence the investment trigger is an increasing function of lag;

(ii) for $\theta < 1/\gamma_1$, $dx_i/d\beta$ is initially negative and subsequently positive, hence the investment trigger is a

U-shaped function of lag, where x_i is minimized at $\beta^* = [(1/\theta - 1)/(\gamma_1 - 1)]^{1/\gamma_1}$.

(Note that in (ii), $\beta^* \to \infty$ when $\theta \to 0$, implying that for very small θ it is downward-sloping.)

In the earlier papers (Alvarez and Keppo, 2002, Bar-Ilan and Strange, 1996, Sarkar and Zhang, 2013), the role of the parameter θ could not be examined because the entire investment cost was assumed to be incurred at one particular point in time (either beginning or end of project). However, as shown above, θ is a crucial factor in determining the relationship between investment trigger and implementation lag (also confirmed by numerical results in Section 2.4).

2.3. The Levered Firm

Here we look at a firm that has access to debt financing. Prior to the first-stage investment, the firm consists of just the option to invest. It makes the first-stage investment (at a cost of θI) when x rises to x^*, and the second-stage investment (at a cost of $(1-\theta)I$) when x rises further to βx^*. As in Agliardi and Koussis (2013), the firm can finance both investments stages with a mix of debt and equity. We assume the first-stage coupon level is c_1 and the second-stage coupon is c_2; that is, debt with coupon of c_1 is issued when $x = x^*$, and additional equal-seniority debt with coupon (c_2-c_1) is issued when $x = \beta x^*$. The company's debt and equity are valued in a backward manner, starting with the post-second-stage results.

2.3.1. Second-stage Valuation

After the second-stage investment (when the company is earning \$x per unit time), the company will declare bankruptcy if x falls sufficiently (Leland, 1994, Goldstein et al., 2001, Hackbarth and Mauer, 2012, etc); let this bankruptcy trigger be x_{b2}. When bankruptcy is declared, bondholders acquire the assets of the firm (after incurring fractional bankruptcy cost α, where $0 \leq \alpha \leq 1$) while shareholders exit with zero payoff. This is in accordance with the APR (absolute priority rule), and implies that the surviving (reorganized) firm will be owned by the erstwhile bondholders; moreover, the reorganized firm will be unlevered.

Let the debt and equity values after the second-stage investment be $D_2(x)$ and $E_2(x)$ respectively. As shown in Appendix A, these are given by:

$$D_2(x) = c_2 / r + H_1 x^{\gamma_2} \tag{14}$$

$$E_2(x) = (1-\tau)[x/(r-\mu) - c_2/r] + H_2 x^{\gamma_2} \tag{15}$$

where H_1 and H_2 are constants. Note that $D_2(x)$ represents the value of the total debt (including the old debt issued at the first stage).

The bankruptcy trigger gives us a boundary condition for equity value:

$$E_2(x_{b2}) = 0 \tag{16}$$

and another for debt value:

$$D_2(x_{b2}) = (1-\alpha)V_2(x_{b2}) = (1-\alpha)(1-\tau)x_{b2}/(r-\mu) \tag{17}$$

Also, for the bankruptcy trigger x_{b2} to be optimal, it must satisfy the smooth-pasting condition:

$$\left.\frac{dE_2(x)}{dx}\right|_{x=x_{b2}} = 0 \tag{18}$$

The three boundary conditions (16) – (18) give us the three constants H_1, H_2 and x_{b2}:

$$x_{b2} = (1-\mu/r)c_2/(1-1/\gamma_2) \tag{19}$$

$$H_1 = [(1-\alpha)V_2(x_{b2}) - c_2/r](x_{b2})^{-\gamma_2} \tag{20}$$

$$H_2 = -\frac{(1-\tau)}{(r-\mu)} \frac{(x_{b2})^{1-\gamma_2}}{\gamma_2} \tag{21}$$

Recall that the second-stage investment is made when x = βx^*. At this point, the company must also decide on the level of debt or coupon c_2. We assume this is done optimally, i.e., so as to maximize the total firm value, as in Leland (1994), Mauer and Sarkar (2005), etc. Thus, c_2 is chosen to maximize $\{D_2(\beta x^*, c_2) + E_2(\beta x^*, c_2)\}$. After some algebra, this gives the optimal coupon level:

$$c_2^* = \frac{(1-1/\gamma_2)x^*}{(1-\mu/r)h} \tag{22}$$

where $h = [1-\gamma_2(1-\alpha+\alpha/\tau)]^{-\frac{1}{\gamma_2}}$.

2.3.2. First-stage Valuation

The first-stage valuation is a little more complicated because of the possibility of bankruptcy prior to the second stage. Suppose that, after completion of the first stage but before x rises to βx^* (i.e., between first stage and second stage), x falls so far (say, to x_{b1}) that the company decides to declare bankruptcy. When this happens, shareholders will walk away with nothing (in accordance with the APR), as in the second-stage bankruptcy. However, the bondholders will acquire a partly-completed project that generates no cash flows yet. Because of the nature of the project, it must be completed when x rises to βx^*, as discussed in Section 2.2.1 (note that the expected lag depends on the project, specifically the parameter β, and not on the firm's ownership structure).[4] Therefore, once the first-stage investment is made at $x = x^*$, the second-stage investment will be made at $x = \beta x^*$, irrespective of the firm's ownership situation.

After the first-stage investment but before the second-stage, let the debt and equity values of the levered firm be $D_1(x)$ and $E_1(x)$ respectively. These are given by:

$$D_1(x) = N_1 x^{\gamma_1} + N_2 x^{\gamma_2} + c_1 / r \tag{23}$$

and

$$E_1(x) = N_3 x^{\gamma_1} + N_4 x^{\gamma_2} - (1-\tau) c_1 / r \tag{24}$$

where N_1, N_2, N_3 and N_4 are constants to be determined from the boundary conditions.

Boundary Conditions for Equity Value

At the bankruptcy boundary ($x = x_{b1}$), shareholders will exit with zero payoff, which gives the value-matching condition:

$$E_1(x_{b1}) = 0 \tag{25}$$

The other boundary is the trigger βx^*, at which the second-stage investment is implemented, giving the boundary condition:

$$E_1(\beta x^*) = E_2(\beta x^*) - (1-\theta) I + (1 - c_1 / c_2) D_2(\beta x^*) \tag{26}$$

Boundary condition (26) requires some explanation. At the second-stage investment, the firm issues some additional debt (of equal seniority) so that the total coupon obligation increases from c_1 to c_2. That is, the additional coupon obligation is $(c_2 - c_1)$. Since it is of equal priority, old

[4] We assume that, at bankruptcy, bondholders take over the firm, and are unable to get rid of the unfinished project. This can be justified by liquidation costs such as cleaning up the soil and repairing environmental damage. These costs are substantial in many industries with long implementation lags, e.g., mining and nuclear industries.

bondholders will now hold a fraction (c_1/c_2) of the total debt and new bondholders will hold fraction ($1–c_1/c_2$). Thus, the old debt will be worth (c_1/c_2) $D_2(\beta x^*)$ and new debt will be worth ($1–c_1/c_2$) $D_2(\beta x^*)$. Since there is no informational asymmetry, the amount raised by the firm from the new debt issue will be ($1–c_1/c_2$) $D_2(\beta x^*)$. The total investment at this stage is $(1–\theta)I$, hence the amount contributed by shareholders will be the difference, or $\{(1–\theta)I – (1–c_1/c_2)\, D_2(\beta x^*)\}$. Then the shareholders payoff at the second stage will be $E_2(\beta x^*)$ less their contribution, which gives boundary condition (26). Solving equations (25) and (26), we obtain the constants N_3 and N_4 as functions of the trigger x^*. These are given in Appendix 2.C.

For ease of interpretation, we can also write the equity value as follows (after some rearrangement):

$$E_1\left(x \middle| x^*\right) = p_{ib}\left[E_2\left(\beta x^*\right) + D_n\left(\beta x^*\right) - (1-\theta)I\right] - (1-\tau)\frac{c_1}{r}\left(1 - p_{ib} - p_{bi}\right) \tag{27}$$

where

$$D_n\left(\beta x^*\right) = D_2\left(\beta x^*\right) - D_1\left(\beta x^*\right) \qquad p_{ib} = \frac{x_{b1}^{\gamma_2} x^{\gamma_1} - x_{b1}^{\gamma_1} x^{\gamma_2}}{x_{b1}^{\gamma_2}\left(\beta x^*\right)^{\gamma_1} - x_{b1}^{\gamma_1}\left(\beta x^*\right)^{\gamma_2}} \quad p_{bi} = \frac{\left(\beta x^*\right)^{\gamma_1} x^{\gamma_2} - \left(\beta x^*\right)^{\gamma_2} x^{\gamma_1}}{x_{b1}^{\gamma_2}\left(\beta x^*\right)^{\gamma_1} - x_{b1}^{\gamma_1}\left(\beta x^*\right)^{\gamma_2}}$$

Here, p_{ib} is the present value of one dollar to be received at the first passage time of the stochastic shock x to βx^*, conditional on the firm not defaulting (i.e., x not falling to x_{b1}) by then. Similarly, p_{bi} is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_{b1}, conditional on x not having risen to βx^* by then. The above formula for equity value can be interpreted as follows. The first term is the value of the existing (or seasoned) equity at the second-stage investment point (conditional on no default prior to second stage) multiplied by the corresponding probability. The second term is the value of the tax-adjusted coupon stream given no first-stage default and no second-stage investment, multiplied by the corresponding probability.

(Note that there is no corresponding bankruptcy-condition term here, since the payoff to shareholders at bankruptcy is zero).

Finally, there is a smooth-pasting condition at the bankruptcy boundary x_{b1}, which ensures the bankruptcy trigger x_{b1} is optimal. Since the payoff to shareholders at bankruptcy is zero, the smooth-pasting condition is:

$$E_1'(x_{b1}) = 0 \tag{28}$$

Equation (28) can be solved numerically for the optimal bankruptcy trigger x_{b1}.

<u>Boundary Conditions for Debt Value</u>

When the company declares bankruptcy (at $x = x_{b1}$), bondholders acquire the company which now has two characteristics (i) it is unlevered, and (ii) the second-stage investment must occur at $x = \beta x^*$, as explained at the beginning of this section. As shown in Appendix B, the value of such a company is given by $M(x_{b1})^{\gamma_1}$, where

$$M = \left[((1-\tau)/(r-\mu))\beta x^* - (1-\theta)I\right]\left(\beta x^*\right)^{-\gamma_1} \tag{29}$$

This gives us the bankruptcy boundary condition for debt value:

$$D_1(x_{b1}) = (1-\alpha)M(x_{b1})^{\gamma_1} \tag{30}$$

The other boundary condition for debt is:

$$D_1(\beta x^*) = \frac{c_1}{c_2} D_2(\beta x^*) \tag{31}$$

This is derived in the same way as the boundary condition for equity (equation (26), discussed above). Equations (30) and (31) give N_1 and N_2 (as functions of x^*), which are given in Appendix C.

The debt value can also be written as follows, for easier interpretation:

$$D_1\left(x \middle| x^*\right) = p_{ib} D_1\left(\beta x^*\right) + p_{bi} D_1\left(x_{b1}\right) + \frac{c_1}{r}\left(1 - p_{ib} - p_{bi}\right) \tag{32}$$

The first term in this expression is the seasoned debt value at second-stage investment (given default has not occurred by then), the second term is the debt value at default (given second-stage investment has not occurred by then), and the third term is the value of the coupon stream in the absence of both second-stage investment and default. Each term is adjusted for the corresponding probability.

Optimal First-Stage Coupon

If the firm chooses the debt level optimally when making the first-stage investment, then the coupon c_1 will maximize the total firm value at that time, or $\left\{D_1\left(x^*, c_1\right) + E_1\left(x^*, c_1\right)\right\}$. This will have to be done numerically, as there is no analytical expression for the optimal coupon level at this stage (unlike the second stage). Thus the optimal first-stage coupon is given by:

$$c_1^* = \underset{c_1}{Arg\,max} \left\{D_1\left(x^*, c_1\right) + E_1\left(x^*, c_1\right)\right\} \tag{33}$$

2.3.3. The Investment Decision

The investment decision is represented by the first-stage investment trigger x^* (the second-stage trigger does not represent a decision, since it follows automatically from the first-stage trigger). As stated in the beginning of Section 2.2, prior to the first-stage investment the firm consists of just the option to invest. It can be shown that this option value is given by the function $\Omega(x) = Fx^{\gamma_1}$, where F is a constant to be determined by the boundary condition. If the first-stage investment is made when $x = x^*$, the value-matching condition is:

$$\Omega(x^*) = F\left(x^*\right)^{\gamma_1} = E_1\left(x^*\right) + D_1\left(x^*\right) - \theta I \tag{34}$$

For the investment trigger x^* to be optimal, it must satisfy the smooth-pasting condition:

$$\left.\frac{d\Omega(x)}{dx}\right|_{x=x^*} = \frac{\partial E_1\left(x^*\right)}{\partial x^*} + \frac{\partial D_1\left(x^*\right)}{\partial x^*} \tag{35}$$

which simplifies to

$$\gamma_1 F\left(x^*\right)^{\gamma_1} = \frac{\partial(N_1 + N_3)}{\partial x^*}\left(x^*\right)^{\gamma_1} + \gamma_1(N_1 + N_3)\left(x^*\right)^{\gamma_1} + \frac{\partial(N_2 + N_4)}{\partial x^*}\left(x^*\right)^{\gamma_2} + \gamma_2(N_2 + N_4)\left(x^*\right)^{\gamma_2}$$

This smooth-pasting condition is too complex to implement because N_1, N_2, N_3 and N_4 are all complicated functions of x^*. We therefore use the following direct approach to identify the optimal investment trigger. The optimal trigger x^* is the one that maximizes ex-ante equity value, which, prior to investment, is just the option value $\Omega(x)$. Thus, the objective is to maximize $\Omega(x) = Fx^{\gamma_1}$ for all x. Since $\gamma_1 > 1$, this is equivalent to maximizing the parameter F, which (from equation (34), after some simplification) is:

$$F = N_1 + N_3 + (N_2 + N_4)\left(x^*\right)^{\gamma_2 - \gamma_1} + \left(\tau c_1 / r - \theta I\right)\left(x^*\right)^{-\gamma_1} \tag{36}$$

Therefore, the optimal first-stage investment trigger x^* is the one that maximizes F in equation (36) (numerically, since no analytical solution is available). This gives our desired solution.

2.4. Results

This section presents results obtained by solving the models of Sections 2.2 (unlevered firm) and 2.3 (levered firm). The results are derived numerically, since the model is too complicated to allow analytical solutions. For numerical results, we need to specify values of the input parameters. We start with a set of reasonable "base-case" parameter values, and then repeat the computations with a wide range of parameter values, in order to ensure robustness of the results.

2.4.1. Base-case Parameter Values

Our choice of base-case parameter values is guided by existing papers in the real-option literature.[5] For the discount rate, we choose r = 7%, as in Grenadier and Weiss (1997), Lambrecht and Perraudin (2003), Mauer and Ott (2000), and Tsyplakov (2008). For the earnings growth rate, we take a simple average of Grenadier and Weiss (1997) and Roques and Savva (2009), who use a growth rate of 5% and 3% respectively, so we set μ = 4%. For earnings volatility, we choose σ = 10%, as in Chu (2011), Lambrecht and Perraudin (2003), Titman and Tsyplakov (2007), and Tsyplakov (2008). For the corporate tax rate, we use the statutory rate of τ = 35%, as in Mauer and Ott (2000), Pawlina (2010), Titman and Tsyplakov (2007), and Tsyplakov (2008). For bankruptcy cost, we choose α = 25%, as in Hackbarth and Mauer (2012). Finally, for the

[5] Since there is no comparable paper in the existing literature (that studies the problem addressed in this paper), we look at various real-option and contingent-claim papers to guide our choice of base-case parameter values.

investment cost, we use I = \$20, but this is just a normalization, with no loss of generality. All computations are done for three levels of θ, $\theta = 0.3$, 0.5 and 0.7, as well as for different lengths of expected time-to-build from 0 to 12 years (that is, the parameter β is chosen so as to give expected lag of 0, 1 year, etc., all the way to 12 years).

2.4.2. Firm with No Access to Debt Financing

With the base-case parameter values specified above, we compute the optimal investment trigger x_i as in Section 2.2.3, for various lengths of implementation lag (or various values of β)[6], and various levels of initial investment fraction θ. To illustrate, let us look at the case of $\theta = 0.5$ and expected implementation lag = 6 years. With the base-case parameter values, equation (2) gives $\beta = 1.2337$ for this expected lag. With this β, the optimal investment trigger comes to $x_i = 2.3441$. Thus, the investment rule is: the first-stage investment should be done when x rises to 2.3441, and the second-stage investment when x rises further to $\beta x_i = 2.8919$. When the computations are repeated with expected lag of zero ($\beta = 1$) and 12 years ($\beta = 1.5220$), the investment trigger x_i comes to 2.4036 and 2.3512 respectively. Thus, x_i is a U-shaped function of implementation lag. Figure 2.1 illustrates the results for various levels of θ.

<u>Figure 2.1 about here</u>

[6] The parameter β is related to expected lag as specified in equation (2).

In Figure 2.1, the first point worth noting is that the investment trigger is an increasing function of the initial investment fraction θ. This is not surprising, since a higher θ implies a higher effective investment cost, as discussed in Section 2.2.2.

The second point of interest is the shape of the relationship. If $\theta > (\leq)\ 1/\gamma_1$, x_i should be an increasing (U-shaped) function of lag, from Section 2.3. With the base-case parameter values, we get $\gamma_1 = 1.6235$ or $1/\gamma_1 = 0.616$; hence x_i should be an increasing (U-shaped) function for $\theta >$ $(\leq)$ 0.616, and a decreasing function for $\theta \to 0$. This is consistent with the results displayed in Figure 1. For $\theta = 0.75$ and 1, x_i is an increasing function of lag; for $\theta = 0.25$ and 0.5, x_i is a U-shaped function of lag, and for $\theta = 0$, x_i is a decreasing function of lag. Note that, for $\theta = 0.5$, x_i starts rising when lag is 8 years, and for $\theta = 0.25$, x_i starts rising when lag is 28 years; for the latter we do not observe the upward-sloping part of the curve since Figure 2.1 does not include lags beyond 12 years (because such long lags are not observed in practice, see Koeva, 2000).

From Figure 2.1, it is clear that the effect of implementation lag on the investment trigger can potentially be economically significant. The above computations were repeated with a wide range of parameter values, and the results were qualitatively very similar. This gives:

Result 1. *For an unlevered firm, the effect of implementation lag on investment trigger depends on the first-stage investment fraction θ. For large θ, investment trigger is an increasing function of lag; otherwise, it is a U-shaped function. However, for small θ, it is a strictly decreasing function of lag for all realistic lags (below 12 years).*

The economic trade-off that leads to the above results for an unlevered firm is discussed below. A longer implementation lag will have two effects on the firm's cash flows. One, the cash outflow (the remaining investment cost), $(1-\theta)I$, will be delayed, which is a positive effect. Two, the cash inflow stream (from operations) will be delayed, which is a negative effect. The value of this cash inflow stream is given by $(1-\tau)x_i\beta/(r-\mu)$, since the cash flow stream starts when $x = x_i\beta$ (second-stage investment). Thus, there are two opposing effects, and the net effect cannot be unambiguously stated. If the overall effect on cash flows is positive, then the firm will invest earlier, or x_i will be smaller, as lag is increased.

In general, for short lag the second effect will be smaller (because β is small), hence the overall effect will be positive and x_i is more likely to be a decreasing function of lag. For longer lag (larger β) the second effect will be larger, hence the overall effect is more likely to be negative and x_i an increasing function of lag. This explains the general U-shaped relation between lag and investment trigger. However, when θ is large enough (i.e., $(1-\theta)$ is small enough) the first effect will also be very small, and the overall effect will be negative. Thus, for large enough θ, x_i will be an increasing function of lag. Similarly, for small enough θ, x_i will be a decreasing function of lag.

2.4.3. Levered Firm (Exogenously Specified Debt Level)

This section focuses on the effect of implementation lag on investment timing for a levered firm. To illustrate this effect, we present numerical solutions to the model of Section 2.3 for an exogenously-specified first-stage debt level (c_1).[7] The base-case parameter values of Section 2.4.1

[7] The second-stage coupon is assumed to be set optimally, as discussed in Section 3.1.

are used, as well as two different debt levels – high (c_1 = 3) and low (c_1 = 1.5).[8] Figure 2.2 displays the results with θ = 0.3, 0.5 and 0.7. The investment trigger for an unlevered firm (from Section 2.4.2) is also shown as a benchmark.

Figures 2.2(a) – 2.2(c) about here

Debt financing has two effects: tax benefits and bankruptcy costs. Because of the tax benefits of debt, the project is more valuable for a levered firm, which should result in earlier investment (or lower x^*) relative to an unlevered firm. On the other hand, the bankruptcy costs resulting from debt make the project less valuable with debt financing, and this will delay investment (raise x^*) relative to an unlevered firm. If the tax benefit dominates, debt financing will lead to earlier investment. If bankruptcy costs dominate (which is likely when debt level is high), debt financing will result in delayed investment.

In Figure 2.2, we note that for low debt level (c_1 = 1.5) the optimal investment trigger is lower than the unlevered trigger in all cases. However, a high debt level (c_1 = 3) results in a higher optimal investment trigger (delaying investment); in some cases (low θ and long time-to-build) the levered trigger is even higher than the unlevered trigger. These findings are consistent with the discussion in the previous paragraph.

Low Debt Level (c_1 = 1.5)

[8] For the base-case parameter values, along with θ = 0.5 and expected lag = 6 years, we get an optimal debt level of c_1 = 1.9434 (these results are discussed in Section 4.4). We therefore used c_1 = 1.5 and 3 to represent the low debt level and the high debt level respectively.

For all values of θ, we note that the investment trigger x^* is a decreasing function of implementation lag. Therefore a longer implementation lag encourages investment, even when it discourages investment for an unlevered firm (e.g., for $\theta = 0.7$). The intuition behind this result is given below.

From Result 1, we know how lag affects the investment trigger for an unlevered firm. When the firm is levered, the additional effect will be as follows. Suppose the implementation lag is lengthened, keeping everything else unchanged. This will result in a reduced equity value, since equity holders will have to make the interest payments to bondholders for a longer period of time without earning any cash flows. Thus, debt value will rise relative to equity value, leading to a higher leverage ratio in the financing package. Now, when the leverage ratio of the financing package is higher, the investment trigger will be lower (as shown by Lyandres and Zhdanov, 2010, Mauer and Sarkar, 2005). Thus, a longer implementation lag will result in a higher leverage ratio (this can be confirmed by computing the stage-one leverage ratio for different values of the lag), which will lead to a lower investment trigger. For low debt levels, this "low debt level" effect dominates the "unlevered" effect of Section 4.2 in all cases, hence the investment trigger x^* is a decreasing function of implementation lag.

There are two points worth noting in the above results with low debt level – implementation lag can have an economically significant effect on investment trigger, and the effect can be significantly different from that for an unlevered firm (particularly for large θ).

High Debt Level ($c_1 = 3$)

For high c_1, implementation lag has a two-part effect on investment trigger x^*. In all cases, x^* is initially a sharply-increasing function of implementation lag. As the lag is increased further, x^* becomes much less sensitive to changes in lag; for $\theta = 0.3$, it is slightly decreasing, and for $\theta = 0.5$

and 0.7, it is slightly increasing, in lag. Thus, for small θ, x^* is initially a strongly-increasing function, and subsequently a slightly decreasing function, of lag. For large θ, x^* is initially a strongly-increasing function, and subsequently a slightly increasing function, of lag.

This result can be explained as follows. In addition to the two effects mentioned above (the "unlevered" effect of Section 2.4.2 and the "low debt level" effect discussed above), there is another effect at play here, resulting from the high debt level. Because of the high debt level, any increase in the implementation lag is undesirable, since it requires shareholders to make the (large) interest payments to bondholders (before receiving any earnings) for a longer period of time. This makes the investment less attractive, everything else remaining the same, thus it results in a higher investment trigger. We call this the "high debt level" effect. The overall effect will be a combination of the "unlevered" effect, "low debt" effect and "high debt" effect. For short implementation lag, the "high debt" effect seems to dominate, and x^* is an increasing function of lag as a result. For longer implementation lag, the overall effect is quite weak, and x^* is only slightly affected by implementation lag.

There are again two points worth noting about the results with high debt level – implementation lag can have an economically significant effect on investment trigger (particularly for short lags), and the effect can be significantly different from that for an unlevered firm (particularly for small θ).

Repeating the computations for both high and low debt levels with a wide range of parameter values, we get very similar results. The effect of time-to-build on investment trigger for a levered firm, when the debt level is exogenously specified, is summarized in Result 2 below.

Result 2. *With an exogenously-specified debt level, the effect of implementation lag on investment trigger (i) can be economically significant, and (ii) can be significantly different from that for an unlevered firm. The exact effect depends on the debt level: for low debt level, the investment trigger is a decreasing function of implementation lag; for high debt level, the investment trigger is initially a sharply-increasing function of implementation lag and subsequently a slightly decreasing (increasing) function of lag for small (large)* θ.

Since the effect of implementation lag depends on the debt level, an immediate question of interest is: what will be the level of debt when the firm makes the investment? On one hand, it is well known that many firms take a long time to reach the optimal capital structure, presumably because of adjustment costs; thus the actual capital structure at the time of investment might be quite different from the optimal capital structure (Leary and Roberts, 2005, Li and Mauer, 2012). Therefore, the case of non-optimally levered firm is interesting in its own right. On the other hand, it is commonly assumed that the firm is using the optimal level of debt when making the investment decision (Leland, 1994, Mauer and Ott, 2000). Therefore, we look at the investment decision of an optimally-levered firm in the next section.

2.4.4. Optimally Levered Firm

As discussed in Section 2.4.3, leverage has two effects – a positive effect (tax shield) that makes the project more attractive, and a negative effect (bankruptcy cost) that makes the project less attractive. These effects vary in magnitude depending on the amount of debt used. When the firm chooses the optimal amount of debt, it maximizes firm value, and thus makes the project as attractive as possible. With a more attractive project, the firm will be more willing to invest, hence the investment trigger will be lower. Therefore, when the leverage ratio is chosen optimally, the investment trigger should be lower. This is indeed what we find later in this section.

Figure 2.3 shows the effect of implementation lag on investment trigger for an optimally-levered firm (optimal debt level being identified as in Section 2.3.2), along with that for an unlevered firm. The base-case parameter values are used, as well as three first-stage investment fractions, $\theta = 0.3$, 0.5 and 0.7.

Figures 2.3(a) – 2.3(c) about here

From Figure 2.3 it is clear that, for the optimally-levered firm, the effect of implementation lag on investment trigger x^* depends on the initial investment fraction θ. For $\theta = 0.3$, x^* is a decreasing function of implementation lag, similar to an unlevered firm. For $\theta = 0.5$, x^* is again a decreasing function of lag, but this is different from an unlevered firm (where x^* is a slightly U-shaped function of lag). Finally, for $\theta = 0.7$ the investment trigger is quite insensitive to implementation lag, also different from the case of unlevered firm (where x^* is a strongly increasing function of lag). To summarize the results for optimally-levered firm with the base-case parameter values: (i) for small θ the effect of lag on investment trigger is economically significant but this effect is not significantly different from unlevered firm, (ii) for intermediate values of θ the effect of lag is economically significant as well as significantly different from unlevered firm, and (iii) for large θ the effect of lag is not economically significant but is significantly different from unlevered firm.

In the above results, the following points are worth noting. First, the optimally-levered firm always has a lower investment trigger than the unlevered firm, thus it will make the investment earlier. The optimal use of leverage will therefore have a positive effect on investment, consistent with the discussion at the beginning of this section. Second, for an optimally-levered firm, the investment trigger is a decreasing function of lag for small and intermediate θ, but can be either increasing or decreasing in lag (although with small magnitude) for large θ; thus, for an optimally-levered firm, implementation lag has a positive or a minor negative effect on investment. Third, the optimally-levered firm's investment response to changes in implementation lag is generally different from that of an unlevered firm. Fourth, using the optimal amount of leverage makes the lag-trigger curve more downward-sloping (or less upward-sloping) than for an unlevered firm, except for small θ (in which case the difference is negligible).[9] Thus, implementation lag is more likely to have a positive effect (or less likely to have a negative effect) on investment for an optimally-levered firm than for an unlevered firm. Any negative effect of implementation lag on investment (that might show up for an unlevered firm, e.g., when θ is large) can be mitigated by the proper use of financial leverage.

Repeating the computations with a wide range of parameter values, we get results that are very similar to the above. We now state the main result of the paper:

Result 3.

(a) For an optimally-levered firm,

[9] For an intuitive explanation of this result, we need to know how lag affects optimal leverage ratio, which is discussed in Section 2.4.6. Therefore, this explanation is deferred to Section 2.4.6.

i. *the effect of the implementation lag on the investment trigger depends on the initial investment fraction θ,*

ii. *for large θ, lag has a minor effect on investment trigger (increasing or decreasing); for small and intermediate θ, investment trigger is a decreasing function of lag,*

iii. *the effect of lag on investment trigger can be significantly different from an unlevered firm;*

(b) The result of optimally-levering the firm, relative to using no leverage, is generally to make implementation lag more favorable (or less unfavorable) for investment; that is, if the investment trigger is a decreasing (an increasing) function of lag for the unlevered firm, it will be a more decreasing (less increasing, or even decreasing) function for the optimally-levered firm.

Result 3 is important because it implies that, for an optimally-levered firm, an implementation lag does not discourage investment as it might do in the case of an unlevered firm (Alvarez and Keppo, 2002, Sarkar and Zhang, 2013). In fact, if the implementation lag were to have a non-negligible effect on investment trigger, it would likely be a positive effect (i.e., would lower the investment trigger). Also importantly, the effect of implementation lag on investment for a levered firm (which has not been examined to date in the literature) can be very different from that of an unlevered firm (which has been examined). In fact, it is possible that implementation lag will have a negative effect on investment for an unlevered firm, but will have a positive effect if the same firm is optimally-levered.

2.4.5. Comparative Static Results

The above results for an optimally-levered firm were repeated with different parameter values, in order to establish the robustness of the results, and to identify the comparative static relationships. First, we look at the effect of tax rate (τ) on the lag-trigger relationship, illustrated in Figure 2. 4(a)–(c). When τ is higher, there are two effects: (i) "earnings effect:" higher tax rate reduces the after-tax earnings, making the project less attractive, which should delay investment or raise the investment trigger; and (ii) "tax shield effect:" higher tax rate makes the tax shield (from debt) more valuable, which makes the project more attractive and should accelerate investment or lower the investment trigger. For longer implementation lag, the cash flows will start after greater delay, hence the "earnings effect" will be less important. For longer implementation lag, therefore, we expect the "tax shield effect" to dominate, i.e., a higher tax rate should lead to a lower investment trigger. For short implementation lag, it will be just the opposite, hence the "earnings effect" should dominate, and a higher tax rate should lead to a higher investment trigger.

This is exactly what we find in Figure 2.4(a)–(c). For short implementation lag, investment trigger is higher when the tax rate is higher; however, this effect shrinks as the lag is lengthened. In fact, when the implementation lag is long enough, a higher tax rate may actually lead to a lower investment trigger. The tax rate has a significant effect on the shape of the lag-trigger relationship:

(i) For short lag, a higher τ increases the downward slope of the investment trigger curve; that is, a longer implementation lag is more favorable for investment when the tax rate is higher;

(ii) For long lag, a higher τ reduces the upward slope of the investment trigger curve, thereby making longer implementation lag less unfavorable for investment.[10]

[10] For small θ, this starts only when the implementation lag is very long. Since our figures show lag up to 12 years, there is no upward-sloping region in Figure 2.4(a) for small θ.

Thus, the effect of longer implementation lag on investment is more favorable (or less unfavorable) when the tax rate is higher. Also, in Figure 2.4(c) we see an instance of the lag having a negative effect on investment (albeit a small effect and only when θ is large) when τ is sufficiently small.

<u>Figures 2.4 – 2.7 about here</u>

The comparative static results for earnings growth rate (μ) are shown in Figure 2.5(a)–(c). A higher μ will make the project more attractive, resulting in earlier investment or a lower investment trigger. This is indeed what we find, since x^* is a decreasing function of μ in all cases. Also, this effect is greater for a longer implementation lag because of the compounding effect of earnings growth rate. Thus, a higher μ will have the following effect: (i) if trigger is a decreasing function of lag, a higher μ will increase the downward slope, and thus make it more favorable for investment; (ii) if trigger is an increasing function of lag, it will reduce the upward slope, and thus make it less unfavorable for investment. Therefore, for $\theta = 0.3$ and 0.5, the investment trigger is a more strongly decreasing function of implementation lag when μ is higher. For $\theta = 0.7$, the investment trigger becomes less upward-sloping as μ is increased, and can even become downward-sloping for large enough μ (in contrast with the unlevered firm, where the trigger is always an increasing function of lag).

Clearly, implementation lag is more likely to have a positive effect on investment when the earnings growth rate is higher. Another instance of implementation lag having a negative effect on investment (also small in magnitude and for large θ only) is when μ is sufficiently small.

The comparative static results for interest rate (r) are shown in Figure 2.6(a)–(c). A higher interest rate will reduce the present value of the project, hence the project will become less attractive. This will result in delayed investment or higher investment trigger. Thus, a higher interest rate will raise the investment trigger. Moreover, this effect will be stronger when the lag is longer, because the discounting effect is stronger over a longer time period. This is what we observe in all cases in Figure 2.6(a)–(c).

Further, the effect of higher interest rate should be more important when more of the investment cost is incurred upfront (i.e., θ is larger). This is consistent with Figure 2.6(c), which shows that, for $\theta = 0.7$, the gap between the lines increases with lag; for r = 6% the trigger is a decreasing function of lag, while for r = 7% it is an increasing function. Thus, when θ is large, the interest rate can play a significant role in determining the shape of the lag-trigger curve.

To summarize the effect of interest rate on the lag-trigger relationship, for $\theta = 0.3$ and 0.5 the investment trigger is a decreasing function of implementation lag for all interest rates, while for $\theta = 0.7$ it is a decreasing (an increasing) function of lag for low (high) interest rate. Once again, another instance of implementation lag having a negative effect on investment is when r is sufficiently large (small effect and only for large θ).

Finally, Figure 2.7(a)–(c) shows the trigger-lag relationship for different volatilities σ. A higher volatility raises the investment trigger in all cases, which is a standard result from option theory. However, unlike in the earlier cases of growth rate and discount rate, the effect of volatility does not vary with lag. This is because the impact of volatility on option value is independent of the implementation lag. Although volatility impacts the investment trigger as discussed above, it has no effect on the *shape* of the trigger-lag curve. Thus, volatility plays an important role in determining the investment trigger, but has no role in determining the lag-trigger relationship.

We do not discuss the comparative static results for bankruptcy cost, because it has no effect on the shape of the trigger-lag curve.

To summarize the results in this section, the investment trigger becomes a more negative function of implementation lag (or implementation lag has a more positive effect on investment) when the tax rate is higher, earnings growth rate is higher, and interest rate is lower, while earnings volatility and bankruptcy cost have no significant effect on the trigger-lag relationship. In certain cases (large θ and one or more of the following: low earnings growth rate, low tax rate, and high interest rate), the investment trigger is an increasing function of lag (or lag has a negative effect on investment). However, in such cases the magnitude of the effect is small. In all other cases, investment trigger is a decreasing function of lag (or lag has a positive effect on investment). Thus, for optimally-levered firms, implementation lag generally has a positive effect on investment; in those cases when it has a negative effect, the magnitude is small. Moreover, the lag-investment relationship for an optimally-levered firm can be very different from an unlevered firm. The comparative static results are briefly summarized in Table 2.1.

<u>Table 2.1 about here</u>

2.4.6. Optimal Coupon Level and Leverage Ratio

Figure 2.8(a)–(b) shows the optimal first-stage coupon level (c_1^*) and optimal first-stage leverage ratio, given by $D_1(x^*)/[D_1(x^*)+ E_1(x^*)]$, as a function of the expected implementation lag, for the three cases $\theta = 0.3$, 0.5 and 0.7. We note that c_1^* is a decreasing function of lag for $\theta = 0.3$, virtually independent of lag for $\theta = 0.5$, and an increasing function of lag for $\theta = 0.7$.

With a longer implementation lag, the company will have to make the out-of-pocket coupon payments to bondholders for a longer period of time (out-of-pocket, since there will be no cash inflows before the second stage). This creates an incentive for the firm to reduce the coupon amount. Thus, the coupon c_1^* should be decreasing in implementation lag. On the other hand, a longer lag delays the arrival of cash inflows from the project, thereby reducing project value; the firm will try to offset this effect by increasing the coupon, so as to take advantage of the larger tax shield resulting from the higher coupon level. This implies c_1^* should be an increasing function of implementation lag. Clearly, there are two effects of implementation lag on optimal coupon level, and they act in opposite directions. Thus, the overall effect is ambiguous, and would depend on which effect dominates.

When θ is large, the company pays more upfront, making the coupon payments over the implementation lag period relatively less important, hence the former effect becomes less important. Therefore, for large θ, the second effect dominates, and the coupon (c_1^*) is an increasing function of lag, as we noted in Figure 2.8(a). For small θ, it is just the opposite, hence c_1^* is a decreasing function of lag.

<u>Figure 2.8 about here</u>

When considering the optimal leverage ratio, however, we must keep in mind that longer implementation lag results in a lower equity value because the payoff to equity holders starts later, although they have to keep making coupon payments to bondholders. The lower equity value results in a higher leverage ratio. This effect will be stronger for a longer implementation lag. Therefore, the optimal first-stage leverage ratio is an increasing function of implementation lag. This result is consistent with Agliardi and Koussis (2013).

As discussed in Section 2.2.2, a higher θ will increase the effective investment cost. Also, as shown in Sarkar (2011), the optimal leverage ratio is an increasing function of the investment cost. Therefore, it follows that a higher θ will result in a higher optimal leverage ratio. This is indeed what we find in Figure 2.6(b). This result is supported empirically by Gaud, et al. (2004) who find that more attractive projects (or lower-cost projects) tend to be financed by more equity and less debt (alternatively, higher investment cost results in higher leverage ratio).

Recall from Section 2.4.4 that the effect of implementation lag on investment trigger for an optimally-levered firm is more favorable for investment than for an unlevered firm (except for small θ, in which case they are similar). This result can be explained by Figure 2.8. For small θ, we note from Figure 2.8(a) that the optimal debt level (c_1^*) is a decreasing function of implementation lag. This means a longer lag will lead to a less valuable tax shield, thereby reducing the attractiveness of the project and raising the investment trigger. On the other hand, Figure 8(b) shows that a longer lag leads to a higher leverage ratio; as discussed in Section 2.4.3, greater use of debt in the financing package leads to a lower investment trigger. Thus, when optimal leverage is introduced, there are two opposing effects of a longer lag on investment trigger. These effects largely offset one another; thus, for small θ, the effect of lag on investment trigger for an optimally-levered firm is similar to that for an unlevered firm, as observed in Figure 2.3(a).

For large θ, we note from Figure 2.8(a) that c_1^* is an increasing function of implementation lag; this means a longer lag will result in a more valuable tax shield, increasing the project's attractiveness. At the same time, from Figure 2.8(b) we note that a longer lag leads to a higher leverage ratio and therefore (as in the above paragraph) to a lower investment trigger. In this case, the two effects reinforce each other. Thus the overall effect is that a longer lag leads to a lower investment trigger or earlier investment, relative to an unlevered firm. Moreover, the longer the lag, the stronger is the effect, hence the gap between optimally-levered and unlevered widens as the lag is increased. Therefore, when θ is large, implementation lag has a more positive (less negative) effect on investment, for optimally-levered firm relative to unlevered firm. This is exactly what we observe in Figure 2.3(c).

2.5. Conclusion

This is the first paper, to our knowledge, that examines the effect of implementation lag on a levered firm's investment decision, which is an important issue because most firms are levered and most capital projects have non-trivial implementation lags.

The main result is that implementation lag can have a substantial impact on a levered firm's investment trigger, and this effect can be significantly different from that of an unlevered firm. For a levered (but not optimally levered) firm the effect depends on the level of debt used. For an optimally-levered firm, the investment trigger can be increasing or decreasing in lag when the initial investment fraction is large (but the magnitude of the effect is small); otherwise it is a decreasing function of lag. A major conclusion from the numerical results is that, if the firm uses leverage optimally, the implementation lag will generally have either a positive effect or an insignificant effect on investment (unlike for an unlevered firm). In a sense, this is good news for investment because most projects have some implementation lag.

Our results are relevant for firms considering investing in a project with a significant implementation lag, e.g., electricity generation, bulk chemicals, real estate (Koeva, 2000, Bar-Ilan and Strange, 1996). However, our results will have different implications for different projects, depending mainly on the time-distribution of the investment cost. For those projects where most of the cost is incurred at the beginning, the investment trigger for an optimally-levered firm is quite insensitive to implementation lag, hence the company might as well ignore the complexities of implementation lag when making their investment decisions. In other cases, the investment trigger will be a decreasing function of lag. Therefore, for such projects, a longer implementation lag will actually have a positive effect on investment.

The empirical implications of our model relate to how leverage ratio and investment trigger are impacted by implementation lag. Regarding the leverage ratio, the model implies that it is an increasing function of lag (consistent with Agliardi and Koussis, 2013). While this is theoretically easy to test (since leverage figures are easily available and lag can be determined from historical data, see Koeva, 2000), there is a practical difficulty – leverage ratio is for the whole firm while the implementation lag applies to an individual project. Therefore, any empirical test will have to be restricted to single-project firms. Regarding the effect of lag on the investment trigger, testing the implications is more challenging since the investment trigger is not directly measurable. However, a few papers have used proxies for the investment trigger in real estate development and in resource industries. Bulan et al. (2009) use a hazard model of time-to-development as a proxy for real-estate development trigger, and Moel and Tufano (2002) use a probit model of operational state of a mine (i.e., open/closed) as a proxy for mine opening/closing triggers. The empirical implications of our model may be tested along these lines.

In terms of policy relevance, governments around the world wish to encourage investment in capital projects, hence anything that affects investment (or investment triggers) is of interest to policymakers. The major policy implication of our model is that implementation lag generally has a positive effect on investment. However, if the initial investment fraction is large, along with low growth rate, low tax rate and/or high interest rate, implementation lag will have a negative effect, which the government might want to counter. It might, for instance, try to reduce the time required to get the necessary clearances for project completion, or to lighten the regulatory requirements associated with the project. These steps would reduce the implementation lag, and thereby encourage investment. This is consistent with the general argument that lightened regulatory requirements can encourage investment. On the other hand, in many scenarios (small initial investment fraction, high tax rate, high growth rate, etc), our model's implication is just the opposite, since implementation lag has a positive impact on investment. Thus we have the surprising and counter-intuitive result that, in these scenarios, reduced regulatory burden might actually discourage investment, contradictory to received wisdom.

Finally, this model makes some simplifying assumptions for tractability, e.g., debt is issued only when investment costs are incurred. There will be other factors, not considered here, that might impact the optimal leverage ratio and investment policy under implementation lag. However, our focus was on the implementation lag and its effects, and we feel that the important factors relating to this have been taken into account; hence the model's results should be valid even in a more comprehensive model.

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Appendix 2.A

Derivations of valuation results for unlevered firm

Post-Completion: After completion of project (i.e., after the second stage), the cash flow stream is x_t per unit time to perpetuity. The value of the project, $V_2(x)$, is the expected present value of the after-tax cash flows from the project:

$$V_2(x) = E\left[\int_0^{\infty} (1-\tau)x_t\, e^{-rt}\, dt \,\middle|\, x_0 = x\right] \tag{A1}$$

Since x_t follows a lognormal process (Geometric Brownian Motion process), it is given by:

$$x_t = x_0\, e^{(\mu - 0.5\sigma^2)t + \sigma z_t}$$

Thus,

$$V_2(x) = (1-\tau)x \int_0^{\infty} E\left[e^{\sigma z_t - 0.5\sigma^2 t}\right] e^{(\mu - r)t}\, dt \tag{A2}$$

From the martingale property, $E_0\left[e^{\sigma z_t - 0.5\sigma^2 t}\right] = e^{\sigma z_0 - 0.5\sigma^2 (0)} = e^0 = 1$. Then equation (A2) simplifies to

$$V_2(x) = (1-\tau)x \int_0^{\infty} e^{(\mu - r)t}\, dt = \frac{(1-\tau)x}{(r-\mu)},$$

which is equation (3) in the paper.

Pre-completion: Before completion, there are no cash flows, just the expectation of future cash flows. Prior to completion, x is the (implied) cash flow that would have occurred if the project had started generating cash flows. The project value at this stage will be a function of x, say $V_1(x)$.

After a small time interval dt, the project value will be $\{V_1(x)+dV_1(x)\}$. The expected present value of this, discounted at a rate r, will be $\{V_1(x)+E[dV_1(x)]\}e^{-rdt}$. Using the approximation $e^{-rdt} = (1–rdt)$, this comes to $V_1+E(dV_1)–rV_1dt$ (since $dV_1\, dt= 0$). This expected present value must be the current project value V_1. That is, $V_1 = V_1+E(dV_1)–rV_1dt$, or

$$E(dV_1) - rV_1dt = 0 \tag{A3}$$

From Ito's lemma, $dV_1 = V_1'(x)\, dx + 0.5V_1''(x)\, (dx)^2$. Using equation (1) to substitute for dx, we get

$$dV_1 = V_1'(x)\, (\mu xdt + \sigma xdz) + 0.5\, V_1''(x)\, \sigma^2x^2\, dt \tag{A4}$$

Taking expectations, we get

$$E(dV_1) = [\mu x\, V_1'(x) + 0.5\, \sigma^2x^2\, V_1''(x)]\, dt \tag{A5}$$

Combining equations (A3) and (A5), we get the basic differential equation:

$$0.5\sigma^2x^2V_1''(x) + \mu xV_1'(x) - rV_1(x) = 0 \tag{A6}$$

The solution to the basic differential equation (A6) is:

$$V_1(x) = Bx^{\gamma_1} + B_2x^{\gamma_2},$$

where B and B_2 are constants to be determined by the boundary conditions, and γ_1 and γ_2 are solutions of the quadratic equation: $0.5\sigma^2\gamma(\gamma-1) + \mu\gamma - r = 0$, and are given by equations (6) and (7) of the paper.

If x falls to zero, the project will be worthless (since x = 0 is an absorbing boundary), which implies $B_2 = 0$. Thus, we can write the project value as: $V_1(x) = Bx^{\gamma_1}$, which is equation (4) of the paper. In a similar manner, we can derive equation (5) of the paper (which has the same form, but a different constant A since the boundary conditions are different).

Derivations of results for levered firm

Post-Completion:

Now the cash inflow from operations is \$x per unit time, of which \$$c_2$ goes to bondholders, and the residual after-tax cash flow of $(1-\tau)$ $(x-c_2)$ goes to shareholders, as in Mauer and Ott (2000), Mauer and Sarkar (2005), Pawlina (2010), etc.

Debt: The basic differential equation (A6) must be augmented by the cash flow to debt holders (here, it is c_2 per unit time). If the debt value is $D_2(x)$, the appropriate differential equation will be:

$$0.5\sigma^2 x^2 D_2''(x) + \mu x D_2'(x) - rD_2(x) + c_2 = 0, \qquad \text{(A7)}$$

with the particular solution c_2/r. Thus, the complete solution for debt value is:

$$D_2(x) = H_{11} x^{\gamma_1} + H_1 x^{\gamma_2} + c_2 / r \qquad \text{(A8)}$$

When x is very high ($x \to \infty$), the debt becomes riskless, hence its value will be the risk-free value or c_2/r, which implies that $H_{11} = 0$ in equation (A8). This gives equation (14) of the paper.

Equity: The procedure is very similar for equity value $E_2(x)$, except that the cash flow to equity holders is $(1-\tau)(x-c_2)$. Then the differential equation for equity value is:

$$0.5\sigma^2 x^2 E_2''(x) + \mu x E_2'(x) - rE_2(x) + (1-\tau)(x - c_2) = 0, \qquad \text{(A9)}$$

with the particular solution $(1-\tau)[x/(r-\mu)-c_2/r]$. Thus, the complete solution for equity value is:

$$E_2(x) = (1-\tau)[x/(r-\mu) - c_2/r] + H_{22}x^{\gamma_1} + H_2 x^{\gamma_2} \tag{A10}$$

When x is very high ($x \to \infty$), the debt becomes riskless, hence equity value will be just the project value, $(1-\tau)x/(r-\mu)$, plus the value of the tax shield, $\tau c_2/r$, less the value of the risk-free debt, c_2/r. Substituting into equation (10), this implies $H_{22} = 0$, giving equation (15) of the paper.

The pre-completion debt and equity values $D_1(x)$ and $E_1(x)$ are derived in a similar manner.

Appendix 2.B

Firm Value in First-stage Bankruptcy

The levered firm makes the first-stage investment when $x = x^*$; suppose x subsequently falls to x_{b1}, when the firm declares bankruptcy and is taken over by bondholders. We want to value the assets of the firm at the point of bankruptcy. At this point, the firm is owned by the erstwhile bondholders, has no leverage, and has no tangible assets; all it has is the second-stage investment, which will be made when x rises to βx^*.

Let the value be given by $V_b(x)$. Then it can be shown that $V_b(x) = Mx^{\gamma_1}$, where M is a constant. Since the firm will make the second-stage investment at $x = \beta x^*$, we have the boundary condition:

$$V_b(\beta x^*) = V_2(\beta x^*) - (1-\theta)I\,, \tag{A11}$$

where $V_2(\beta x^*) = (1-\tau)\beta x^*/(r-\mu)$. This gives:

$$M = \left[\frac{(1-\tau)\beta x^*}{(r-\mu)} - (1-\theta)I\right]\left(\beta x^*\right)^{-\gamma_1} \tag{A12}$$

Appendix 2.C

Expressions for Constants

Solving equations (25), (26), (30) and (31) in Section 2.3.2, we get the constants:

$$N_1(x^*) = \frac{(\beta x^*)^{\gamma_2} D_1(x_{b1}) - x_{b1}{}^{\gamma_2} D_1(\beta x^*) - \frac{c_1}{r}\left[(\beta x^*)^{\gamma_2} - x_{b1}{}^{\gamma_2}\right]}{x_{b1}{}^{\gamma_1}(\beta x^*)^{\gamma_2} - x_{b1}{}^{\gamma_2}(\beta x^*)^{\gamma_1}} \tag{A13}$$

$$N_2(x^*) = \frac{(\beta x^*)^{\gamma_1} D_1(x_{b1}) - x_{b1}{}^{\gamma_1} D_1(\beta x^*) - \frac{c_1}{r}\left[(\beta x^*)^{\gamma_1} - x_{b1}{}^{\gamma_1}\right]}{x_{b1}{}^{\gamma_2}(\beta x^*)^{\gamma_1} - x_{b1}{}^{\gamma_1}(\beta x^*)^{\gamma_2}} \tag{A14}$$

$$N_3(x^*) = \frac{x_{b1}{}^{\gamma_2}\left[E_2(\beta x^*) + D_n(\beta x^*) - (1-\theta)I\right] - (1-\tau)\frac{c_1}{r}\left[(\beta x^*)^{\gamma_2} - x_{b1}{}^{\gamma_2}\right]}{x_{b1}{}^{\gamma_2}(\beta x^*)^{\gamma_1} - x_{b1}{}^{\gamma_1}(\beta x^*)^{\gamma_2}} \tag{A15}$$

$$N_4(x^*) = \frac{(1-\tau)\frac{c_1}{r}\left[(\beta x^*)^{\gamma_1} - x_{b1}{}^{\gamma_1}\right] - x_{b1}{}^{\gamma_1}\left[E_2(\beta x^*) + D_n(\beta x^*) - (1-\theta)I\right]}{x_{b1}{}^{\gamma_2}(\beta x^*)^{\gamma_1} - x_{b1}{}^{\gamma_1}(\beta x^*)^{\gamma_2}} \tag{A16}$$

Figure 2.1. Optimal investment trigger for an unlevered firm (x_i) as a function of expected implementation lag, for different values of first-stage investment fraction θ. The base-case parameter values are used: r = 7%, μ = 4%, σ = 10%, τ = 35% and I = 20.

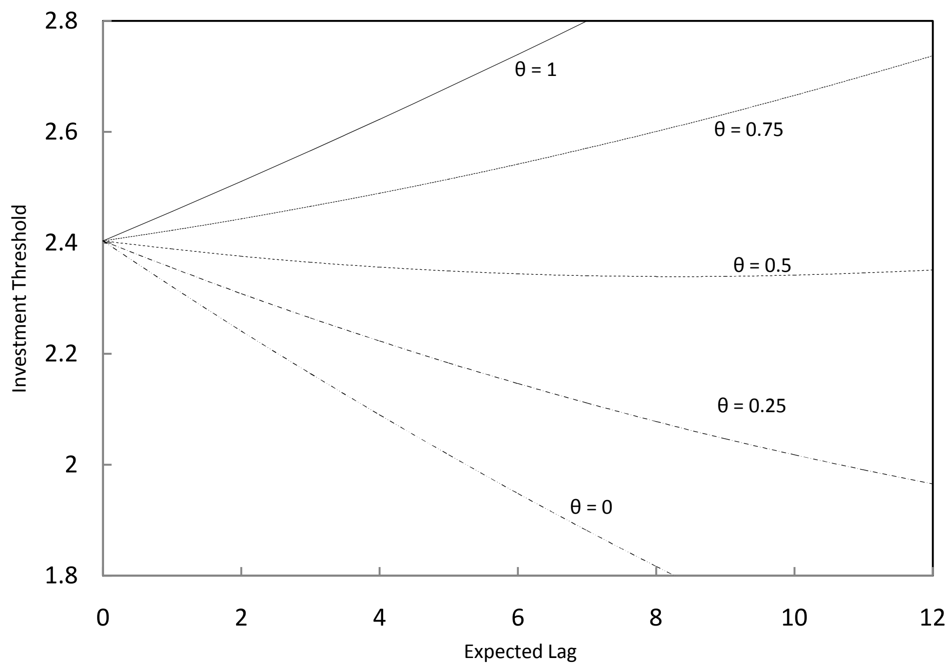
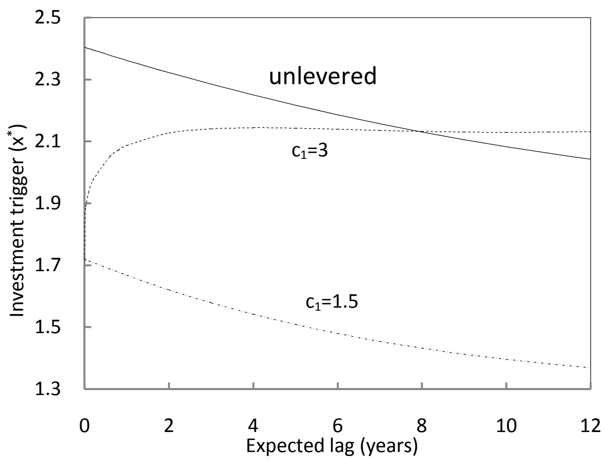
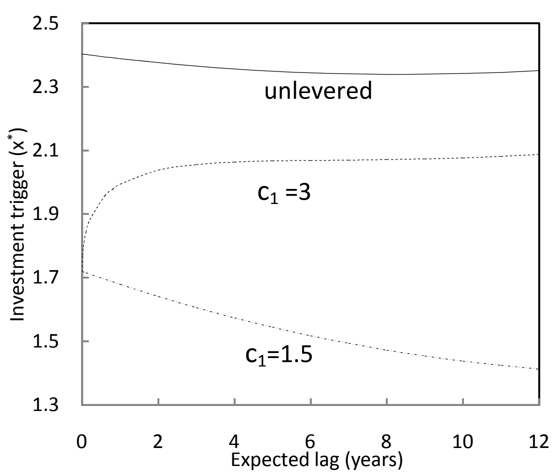


Figure 2.2 (a)–(c). Optimal investment trigger for a levered firm (x^*) as a function of expected implementation lag, for two different exogenously-specified debt levels, c_1 = 1.5 and 3, along with the unlevered investment trigger. The base-case parameter values are used: r = 7%, μ = 4%, σ = 10%, τ = 35%, α = 25% and I = 20. Parts (a), (b) and (c) show results for θ = 0.3, 0.5 and 0.7 respectively.

(a) θ = 0.3



(b) θ = 0.5



(c) θ = 0.7

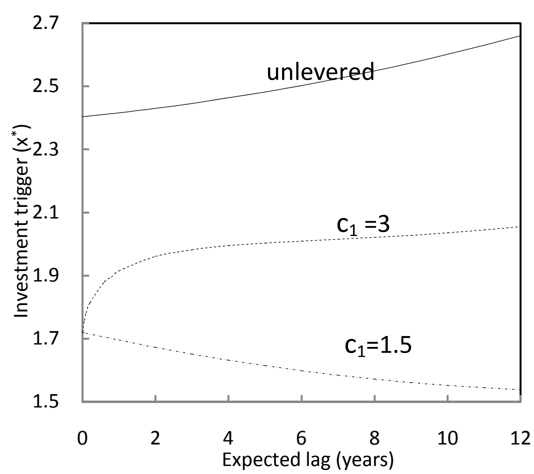
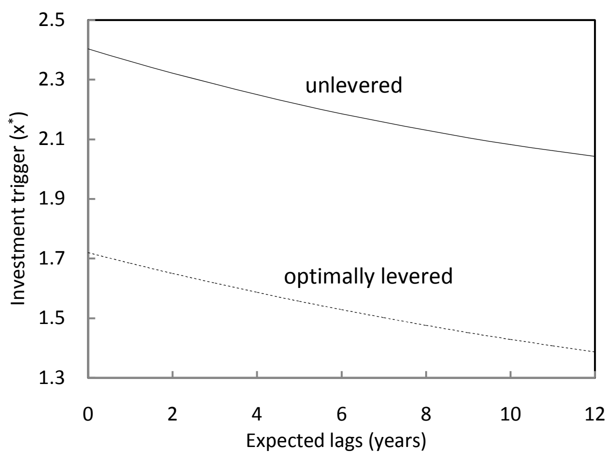
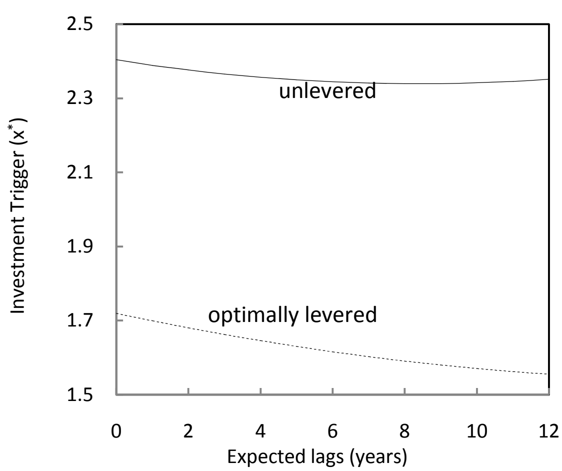


Figure 2.3(a)–(c). Optimal investment trigger (x^*) for *optimally-levered* and unlevered firm, as a function of expected implementation lag. The base-case parameter values are used: r = 7%, μ = 4%, σ = 10%, τ = 35%, α = 25% and I = 20. Parts (a), (b) and (c) show results for θ = 0.3, 0.5 and 0.7 respectively.

(a) θ = 0.3



(b) θ = 0.5



(c) θ = 0.7

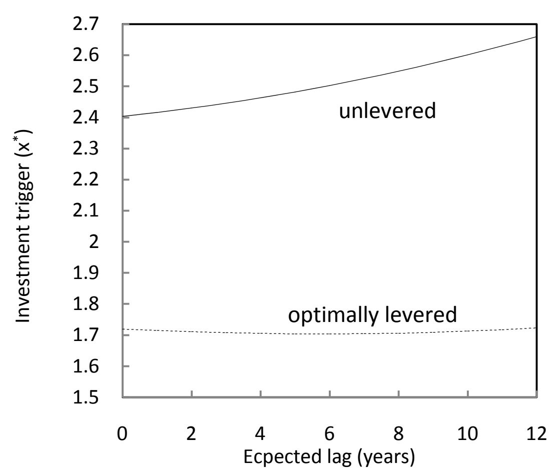
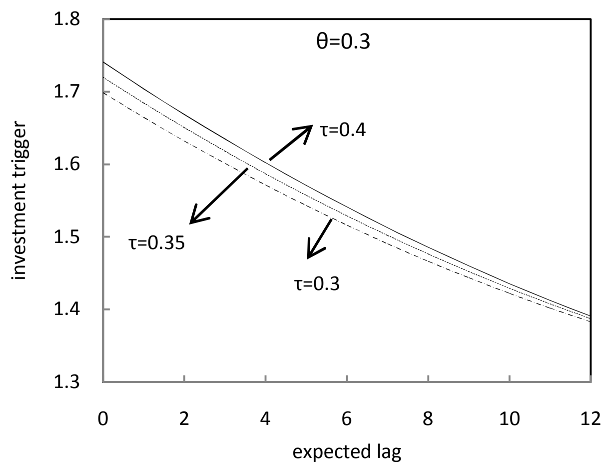
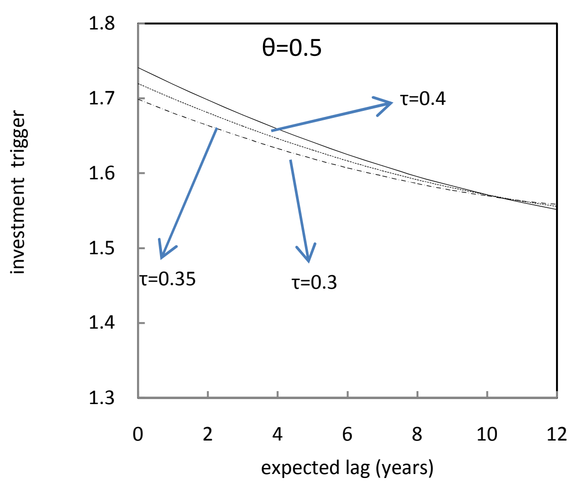


Figure 2.4(a)-(c). Comparative static results for tax rate (τ). Optimal investment trigger (x^*) for *optimally-levered* firm, as a function of expected implementation lag, for three different tax rates, τ = 30%, 35% and 40%. The base-case parameter values are used: r = 7%, μ = 4%, σ = 10%, α = 25% and I = 20. Parts (a), (b) and (c) show results for θ = 0.3, 0.5 and 0.7 respectively.

(a)



(b)



(c)

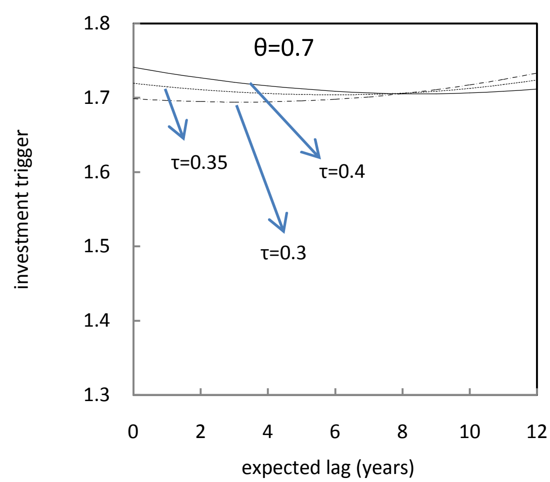
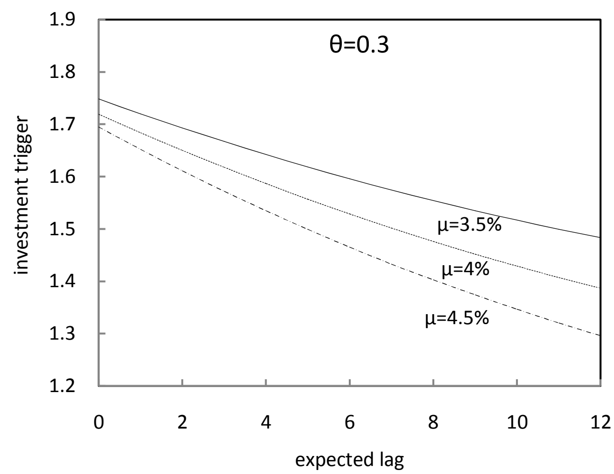
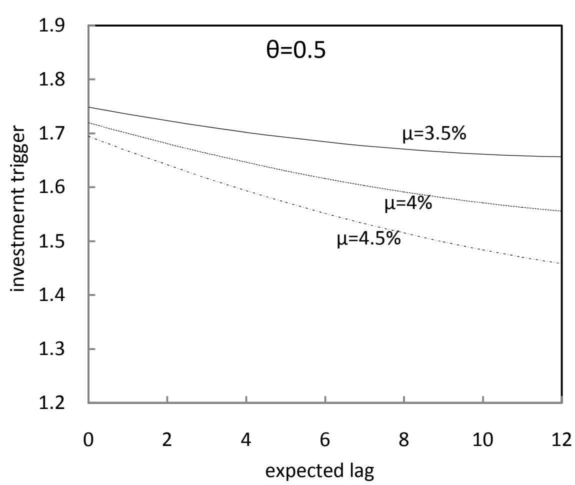


Figure 2.5 (a)-(c). Comparative static results for earnings growth rate (μ). Optimal investment trigger (x^*) for *optimally-levered* firm, as a function of expected implementation lag, for three different earning growth rates, μ = 3.5%, 4% and 4.5%. The base-case parameter values are used: r = 7%, σ = 10%, τ = 35% and I = 20. Parts (a), (b) and (c) show results for θ = 0.3, 0.5 and 0.7 respectively.

(a)



(b)



(c)

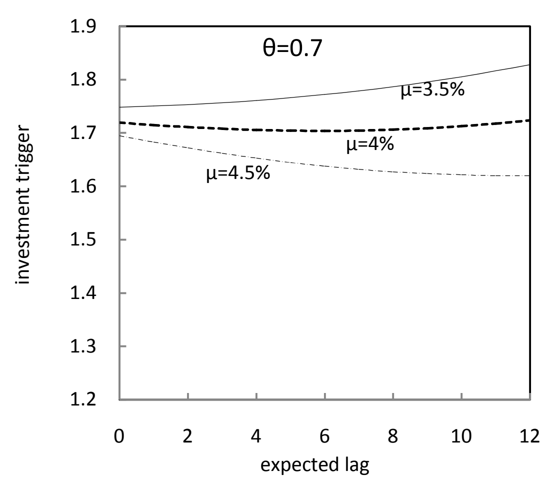
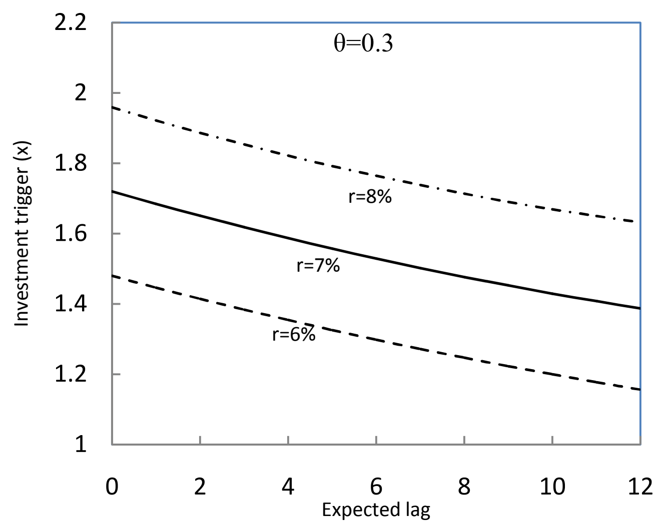
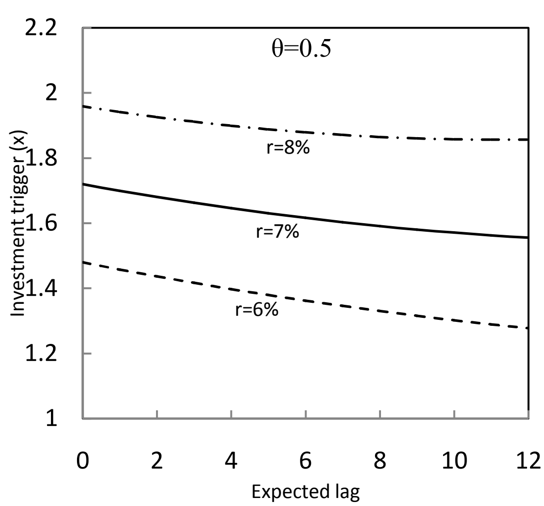


Figure 2.6(a)-(c). Comparative static results for interest rate (r). Optimal investment trigger (x^*) for *optimally-levered* firm, as a function of expected implementation lag, for three different earning growth rates, r = 6%, 7% and 8%. The base-case parameter values are used: σ = 10%, μ = 4%, τ = 35% and I = 20. Parts (a), (b) and (c) show results for θ = 0.3, 0.5 and 0.7 respectively.

(a)



(b)



(c)

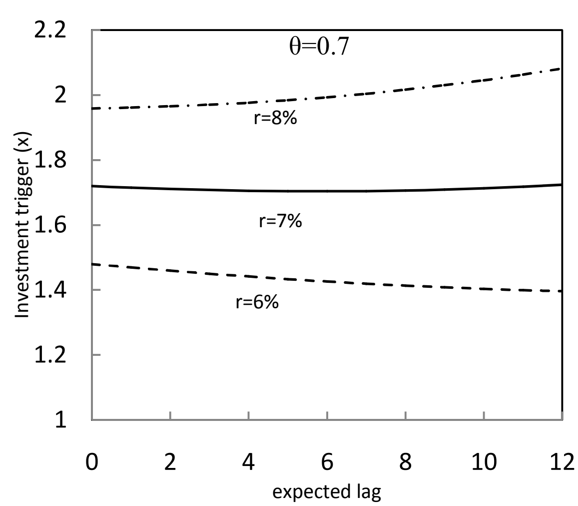
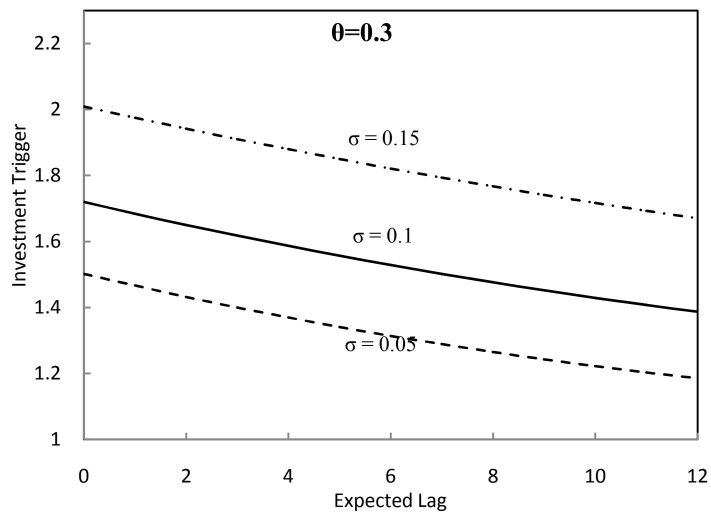
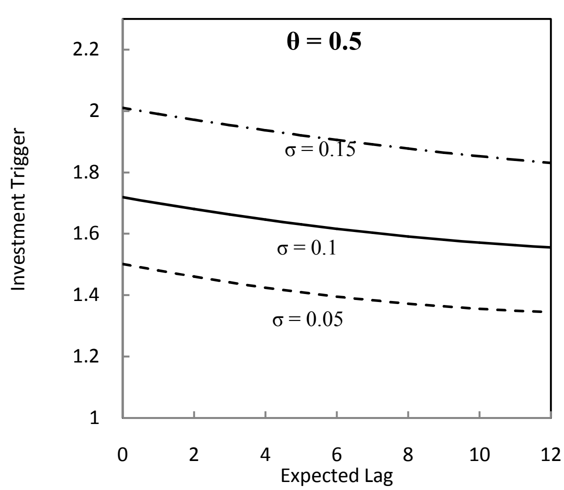


Figure 2.7(a)-(c). Comparative static results for volatility (σ). Optimal investment trigger (x^*) for *optimally-levered* firm, as a function of expected implementation lag, for three different idiosyncratic volatilities, σ = 15%, 10% and 5%. The base-case parameter values are used: r = 7%, μ = 4%, τ = 35% and I = 20. Parts (a), (b) and (c) show results for θ = 0.3, 0.5 and 0.7 respectively.

(a)



(b) **(c)**



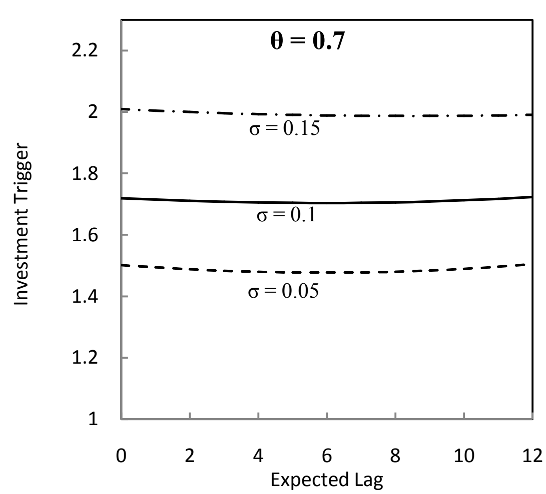
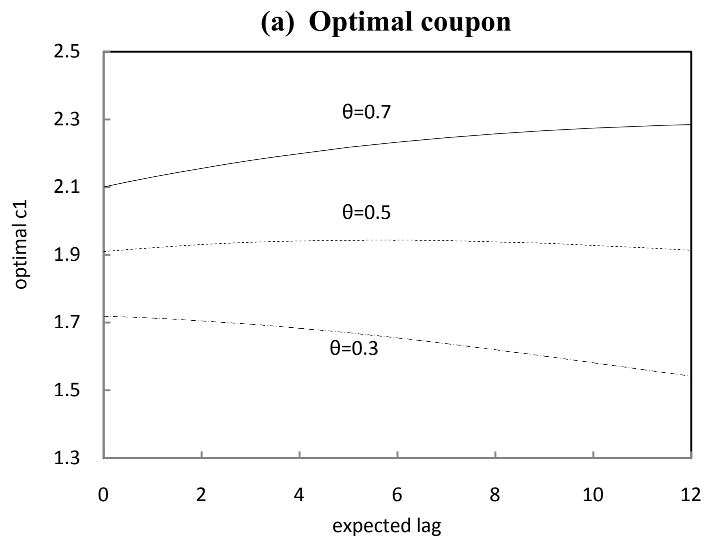


Figure 2.8(a)-(b). First-stage optimal coupon level and corresponding (optimal) leverage ratio, as functions of expected implementation lag. The base-case parameter values are used: r = 7%, μ = 4%, σ = 10%, τ = 35%, α = 25% and I = 20.



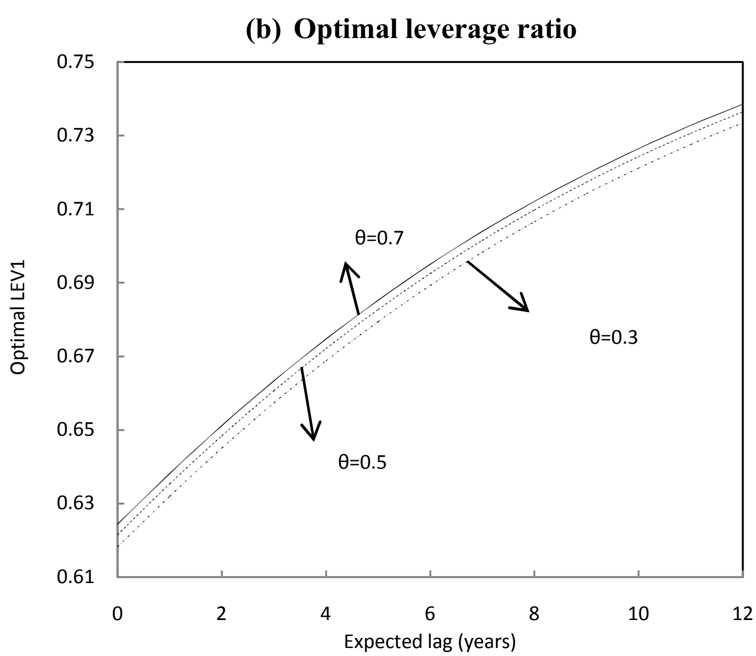


Table 2.1. Effect of various inputs on the investment trigger–lag curve

Effect of	
1. Tax rate τ →	A higher τ makes the curve more downward-sloping or less upward-sloping
2. Growth rate μ →	A higher μ makes the curve more downward-sloping or less upward-sloping
3. Interest rate r →	A higher r makes the curve less downward-sloping or more upward-sloping
4. Volatility σ →	Shape of curve is the same for all σ

Chapter 3

A Framework of Investment with Growth Opportunities

3.1. Introduction

The firm's investment/financing decisions are among the most important topics in corporate finance. These decisions can be impacted by many factors such as agency conflict (Mauer and Sarkar (2005)), asset tangibility (Campello and Hackbarth (2012)), debt issuance constraint (Shibata and Nishihara (2012)), liquidity constraint (Bolton, Wang and Yang (2013), Guthrie and Boyle (2003)), macroeconomic conditions (Morellec, Miao, Hackbarth), and market competition (Mason and Weeds (2011), Grenadier (2002)).

In this paper, we consider a new channel, uncertain arrival of growth opportunities, through which the investment and financing decisions are distorted. The reasons for uncertainty of a firm's growth opportunity are multiple. For example, some firms need technology innovation to generate new products; the success of new technology is uncertain and unknown to managers (R&D, new drug development, etc). Some firms may have trouble dealing with uncertain policies, selection of factory location and upstream supplier when they decide to expand either domestically or internationally. Thus it will be quite useful for a firm to consider such future uncertainties when it prepares to make the initial investment decision.

Despite the important effect of future growth uncertainty on the current investment decision, only a few papers have examined such a phenomenon. While our treatment of growth uncertainty is similar to Anderson and Carverhill (2012), there are some differences in modeling the uncertainty. They assume the growth is fully stochastic, thus the value of each contingent claim is the probability weighted average of the value assuming the growth does occur and does not occur. We model the uncertainty by assuming it arrives with Poisson process, similar to Li and Mauer (2013). However, Li and Mauer assume that the expansion option must be exercised immediately if at all, while in our paper the firm can time its exercise of the growth opportunity.

Our model assumes there are two investment options, namely, initial investment and growth opportunity. The firm has no assets before initial investment. After initial investment, the firm obtains one unit of assets-in-place which generates continuous profit flow and a growth option claim to the second investment, which will further enhance the profit level. The firm can fund the initial investment by issuing a mix of debt and equity. Only equity (or retained earnings) will be used to finance the second investment. All the investment and bankruptcy thresholds will be endogenously derived.

Suppose a firm has made the first investment and wishes to expand current earnings. However, such an expansion cannot be realized without more advanced technology and some investment cost. The irreversible sunk cost implies the firm has to wait until the price reaches a higher level to exercise the growth option. However, the new technology will arrive with uncertainty. It may have always been accessible for the time being, or need some time to develop and we don't know when it can be implementable. Consequently, if the technology exists before the optimal expansion trigger, then the firm can expand at this trigger, in line with traditional real option model. If the opportunity has not arrived, then it will not expand until the technology is available. To our knowledge, this paper is the first to relate future expansion opportunity to current investment decision, through arrival uncertainty. We document that the magnitude of the impact of future uncertainty (embodied in the Poisson process) on firm's initial investment can be economically significant. The concise symbolization by Poisson parameter not only makes our model more tractable, but also can capture a wide range of realities as described before. Thus our model provides a useful guideline for the firm's investment decisions.

Our paper generates several new results. Firstly, without arrival uncertainty, we illustrate that the expansion option will not impact the initial investment for an all-equity (unlevered) firm, consistent with Kort, Murto and Pawlina (2010) and Sundaresan, Wang and Yang (2013). However, for a levered firm, the initial investment can be impacted, the magnitude of which depends on the interaction between growth size and agency problem of debt overhang.

Secondly, if there is uncertainty of arrival, the results are very different. The mechanism through which the initial investment timing is impacted lies in the additional option terms caused by uncertainty of arrival of growth opportunity. Our analysis shows that for unlevered firm when the possibility of technology adoption is very low, the firm would like to accelerate the investment as the possibility increases. On the other hand, when the arrival possibility of growth option is relatively high, the firm will delay investment. Hence there is a non-monotonic relationship between initial investment timing and exogenous uncertainty, in contrast with conventional wisdom that the initial investment is independent of the second investment decision when the investment is in sequence. Moreover, our results demonstrate that the impact of arrival uncertainty will be dampened as the expansion scale becomes small. Alternatively put, the firm's initial investment timing will not be significantly impacted by future uncertainty growth opportunity if the size of the growth is not attractive.

We also extend the unlevered model to debt financing to examine the investment timing and optimal capital structure simultaneously. The firm uses equity (i.e., retained earnings or internal financing) to fund the modernization, similar to the benchmark case. This assumption is made for two reasons. Firstly, Li and Mauer (2013) have found that there is an interaction impact of debt issuance timing and uncertain growth arrival, and we wish to isolate such impact since we only focus on how growth uncertainty affects initial investment and financing decision. The introduction of second debt issuance would complicate our model because of additional interaction impact of first and second debt financing. Secondly, in reality debt issuance is time-consuming and firms normally issue equity or use retained earnings to finance the expansion. Our results illustrate that the U-shaped relation of initial investment timing and arrival rate is robust to the introduction of debt financing. On the other hand, the optimal financial leverage presents non-monotonic relation with the arrival rate. Apparently, the decision of how much debt to issue at the beginning is crucially dependent on the trade-off among tax shield benefit of debt issuance, pre-arrival default chance as well as debt overhang problem (especially when the growth size is large). Our result for levered firm has valuable empirical implications since most empirical papers which test the underinvestment problem by regressing growth option and financial leverage neglect the extent to which the growth option can be finally realized, and our model predicts the proxy variable for the different levels of realization of growth opportunity can reshape the regression result.

Related Literature: Sundareson, Wang and Yang (2013) model a firm's investment and financing decisions with multiple growth options. However, they assume the growth options can be exercised optimally without uncertainty. They find that firm chooses conservative leverage to mitigate the debt overhang problem. We, instead, only assume one growth option since we focus on how the future growth uncertainty impacts the firm's initial investment and financing decisions. Kort *et al* (2010) also model sequential investments but they focus on the trade-off between value of flexibility and economic scale instead of growth uncertainty. Moreover, they do not consider the firm's financing decision. Our paper is also related to Murto (2007). However we have several distinct features. Although he models technology arrival uncertainty as Poisson process and its impact on the investment timing, he focuses the interaction between technology uncertainty and idiosyncratic volatility while we assume the technology uncertainty only occurs at expansion stage. Also we examine how such expansion uncertainty impacts the financing decision.

The remainder of the paper is organized as follows: section 3.2 introduces and describes the model, section 3.3 presents results along with discussion, and section 3.4 concludes.

3.2. Model Set up

We use a standard real-option model (e.g., Sundaresan and Wang, 2007) but with the addition of a jump process for the modernization opportunity (similar to Li and Mauer, 2013). The jump process is used to represent the uncertain arrival of the modernization opportunity. Table 1 gives a brief list of the different variables used in the model.

Consider a start-up firm with no asset-in-place but with an investment opportunity, which costs $\$I_1$ to implement. The investment opportunity, once implemented, will generate a cash flow (or earnings) stream of x_t per unit time. The earnings stream follows a lognormal (Geometric Brownian Motion) process:

$$dx_t = \mu x_t dt + \sigma x_t dz_t \tag{1}$$

where μ and σ are positive constants, and $\left(z_t\right)_{t\geq 0}$ is a standard Brownian Motion Process. Time is continuous and varies over $[0, \infty)$. Uncertainty is represented by the filtered probability space space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ over which all stochastic processes are defined. The firm is subjected to a constant corporate tax rate of τ $(0< \tau <1)$. We also assume that agents are risk-neutral, such that all future cash flows are discounted at the risk-free interest rate r. The initial cash flow level x_0 is low enough such that the firm will not make the investment immediately. The investment will be financed by a mix of debt and equity.

3.2.1. The Modernization Opportunity

After the investment has been made, a new technology might arrive that allows the firm to modernize its operation, and thereby increase the existing cash flow by a factor of δ (δ >1). To avail itself of this modernization opportunity, the firm has to pay fixed cost I_2. We assume that the cost of the upgrade/modernization is large enough so that the initial investment is always made first, and the firm does not leapfrog to the modern technology without going through the initial investment. Thus, the investment and modernization stages always happen in sequence.

Thus there is an optimal profit level that triggers the expansion (we use *expansion*, *growth* and *modernization* interchangeably in the following). However, the time when the new opportunity arrives is not necessarily the exact same time that it is optimal to modernize. Therefore, modernization doesn't necessarily take place at the optimal time because the opportunity might not have arrived.

The arrival of the modernization opportunity is modeled as a Poisson process with mean intensity λ. This means the expected wait time for arrival of opportunity is $E(\mathrm{T}) = \lambda^{-1}$. Thus, if λ = 0.1 then the mean waiting length is 10 years. The assumption of Poisson arrival has been used widely to capture uncertainties, in Murto (2007), Farzin *et al* (1998), Berk *et al* (2004). The arrival intensity is independent on the cash flow generated by assets-in-place, hence the uncertainty is exogenous to the firm. Note such a simplification only represents arrival rate of uncertainty but not the quality or direction. Also, we assume that there is only one modernization opportunity, rather than multiple such opportunities in real life. This is done for tractability (as in

Li and Mauer, 2013). We also assume that the modernization is financed entirely by equity (or retained earnings), as in Mauer and Ott (2000).

3.2.2 The Unlevered Firm

We begin the analysis with an all-equity or unlevered firm. Before proceeding to the general case, we look at two special cases: $\lambda = 0$ (when there is no modernization opportunity at all) and $\lambda = \infty$ (when the firm has the modernization opportunity with certainty, as soon as it makes the initial investment). These special cases not only give some sense to our final results, but also their result will be used in our following steps. Since these special cases have been covered in earlier real-option models, we present just the results and not their derivations.

3.2.2.1. No Modernization Opportunity (λ= 0)

When there is no modernization opportunity, the investment threshold x_{ui} is:

$$x_{ui} = \frac{\gamma_1}{\gamma_1 - 1}\frac{r-\mu}{1-\tau}I_1 \tag{2}$$

where $\gamma_1(\gamma_2)$ is the positive (negative) root of $\frac{1}{2}\sigma^2\gamma(\gamma-1)+\mu\gamma - r = 0$

3.2.2.2. Certain Modernization Opportunity ($\lambda = \infty$)

In this case, when making the initial investment the firm knows that there is a modernization opportunity that can be exploited when the conditions are right. Thus, the investment and modernization will be done in sequence. After the initial investment, what the firm has is assets-in-place generating earnings, and an option to modernize. Let the modernization trigger be x_{ue}.

Also, suppose the firm value is U(x) after modernization and V(x) after initial investment but before modernization. Then, the post-modernization firm value is given by:

$$U(x) = (1-\tau)\delta x/(r-\mu) \tag{3}$$

This is just the expected present value of the after-tax cash flows from the modernized project.

The pre-modernization firm value is given by:

$$V(x) = A_1 x^{\gamma_1} + (1-\tau)x/(r-\mu) \tag{4}$$

where $$A_1 = \left[\frac{1-\tau}{r-\mu}(\delta-1)x_{ue} - I_2\right]x_{ue}^{-\gamma_1} \quad \text{and} \quad x_{ue} = \frac{\gamma_1}{\gamma_1 - 1}\frac{r-\mu}{1-\tau}\frac{I_2}{\delta-1}.$$

The initial investment trigger is

$$x_{ui} = \frac{\gamma_1}{\gamma_1 - 1}\frac{r-\mu}{1-\tau}I_1 \tag{5}$$

It can be seen that the investment trigger x_{ui} is the same as in the special case $\lambda = 0$. To ensure that the investments are sequential, we require $I_1 < I_2 / (\delta - 1)$. If this condition is violated, then the initial investment and modernization would occur simultaneously. We now state:

Proposition **1** (i) The investment trigger x_{ui} is same for the polar cases $\lambda = 0$ and $\lambda = \infty$; (ii) For the case $\lambda = \infty$, the investment trigger is independent of the characteristics of the modernization opportunity (δ, I_2).

This result is not surprising because the modernization characteristics have been taken into account when determining the modernization trigger x_{ue}. Since the firm can choose the modernization trigger optimally, the impact of the modernization characteristics will be reflected in the choice of x_{ue} and will therefore not affect the investment trigger x_{ui}. The modernization opportunity is completely taken into account in setting the modernization trigger, hence the investment trigger will be independent of the modernization opportunity. Since the modernization opportunity has no effect on investment, the investment trigger x_{ui} is the same as when there is no modernization opportunity ($\lambda = 0$). In the next section we will see that Proposition 1 would not hold if the firm's ability to optimize the modernization timing was undermined by any jump risk or uncertainty regarding the timing of arrival of the modernization opportunity, it might be the case that the opportunity arrives only after x exceeds the optimal modernization trigger. In this case, the firm is clearly unable to follow the optimal modernization timing policy because of its ability to optimize the modernization decision is undermined by the random arrival of the opportunity, which might not arrive exactly when it becomes optimal to modernize. Thus, when $0 < \lambda < \infty$, the firm will be unable to fully optimize its modernization timing; as a result, the modernization opportunity (δ, I_2) will affect the initial investment decision.

3.2.2.3. General Case ($0 < \lambda < \infty$)

Now we are in a position to examine the intermediate case $\lambda \in (0, \infty)$. That is, the firm's modernization is contingent on the arrival of the modernization opportunity, which follows a jump process. Therefore, there are now two conditions that must be satisfied before the second investment takes place: (i) the modernization opportunity must have arrived, and (ii) the state variable x_t must have risen to the level of the modernization trigger.

Suppose the modernization opportunity arrives at time $\tilde{T}$. If the state variable at that time is not high enough to trigger modernization (i.e., $x_{\tilde{T}} < x_{ue}$), the firm will not modernize as soon as the opportunity arrives. On the other hand, if $x_{\tilde{T}} > x_{ue}$, the firm will modernize immediately on arrival of the opportunity. Therefore, the modernization decision is driven not just by some critical price trigger but also by the exogenous arrival uncertainty. To facilitate our analysis, we partition the process into two separate valuation regions: time before opportunity arrival, $T < \tilde{T}$ and time after opportunity arrival, $T > \tilde{T}$. We describe them backwardly begin with the second region after arrival.

After the arrival of the modernization opportunity:

- If $x < x_{\text{ue}}$, the firm will not modernize immediately; let the firm value be W(x)
- If $x \geq x_{\text{ue}}$, the firm will modernize immediately; let the firm value be Z(x)

In the former case, the firm value must satisfy the differential equation:

$$rW = \frac{1}{2}\sigma^2 x^2 W_{xx} + \mu x W x + (1-\tau)x + \lambda(V - W) \tag{6}$$

The left hand side of equation denotes the required return for holding the assets per unit of time, and the right hand side represents the realized return (expected change in asset value). This equation is very similar to the differential equation in a standard real-option model, except the last term which captures the impact of jump process, being the product of instantaneous probability of opportunity arrival and the corresponding value change. The general solution takes form:

$$W(x) = B_1 x^{\beta_1} + A_1 x^{\gamma_1} + (1-\tau)x/(r-\mu) \tag{7}$$

This expression is equivalent to an option term plus $V(x)$. The parameter $\beta_1(\beta_2)$ is the positive (negative) root of $\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-(r+\lambda)=0$

In the latter case ($x > x_{ue}$), the firm value must satisfy the differential equation:

$$rZ = \frac{1}{2}\sigma^2 x^2 Z_{xx} + \mu x Zx + (1-\tau)x + \lambda(U - I_2 - Z) \quad (8)$$

The last term on the right hand side represents the discrete change in firm value resulting from immediate modernization. The general solution is:

$$Z(x) = B_2 x^{\beta_2} + \frac{1-\tau}{r-\mu}\frac{\mu - r - \delta\lambda}{\mu - r - \lambda}x - \frac{\lambda I_2}{r+\lambda} \quad (9)$$

This expression is equivalent to an option term plus the post-modernization asset value minus the investment cost, both of which are adjusted by λ. To solve for B_1 and B_2 we need the following two conditions:

$$W(x_{ue}) = Z(x_{ue}) \quad (10)$$

$$\left.\frac{\partial W(x)}{\partial x}\right|_{x=x_{ue}} = \left.\frac{\partial Z(x)}{\partial x}\right|_{x=x_{ue}} \quad (11)$$

The first expression is value matching condition which requires that the two firm values in each region be equal at switching point x_{ue}. The second is smooth-pasting condition: since x is random diffused across this boundary, the value function cannot change abruptly, thus they must be continuously differentiable. Then we obtain

$$B_2 = \frac{(\beta_1 - \gamma_1) A_1 x_{ue}^{\gamma_1} - (\beta_1 - 1) \frac{1-\tau}{r-\mu} \frac{\delta - 1}{r - \mu + \lambda} \lambda x_{ue} + \beta_1 \frac{\lambda I_2}{r + \lambda}}{(\beta_1 - \beta_2) x_{ue}^{\beta_2}} \tag{12}$$

$$B_1 = \frac{(\beta_2 - \gamma_1) A_1 x_{ue}^{\gamma_1} - (\beta_2 - 1) \frac{1-\tau}{r-\mu} \frac{\delta - 1}{r - \mu + \lambda} \lambda x_{ue} + \beta_2 \frac{\lambda I_2}{r + \lambda}}{(\beta_1 - \beta_2) x_{ue}^{\beta_1}} \tag{13}$$

The two option terms are key features in our model. Now we are in a position to solve for the initial investment trigger. Denote firm value as *F*(x) given the investment option has not been exercised; according to standard protocol, it is similar to an American call option:

$$F(x) = Hx^{\gamma_1} \tag{14}$$

The boundary conditions are:

$$F(x_{ui}) = W(x_{ui}) - I_1 \tag{15}$$

$$\left.\frac{\partial F(x)}{\partial x}\right|_{x = x_{ui}} = \left.\frac{\partial W(x)}{\partial x}\right|_{x = x_{ui}} \tag{16}$$

from which x_{ui} can be solved by following nonlinear equation

$$(\beta_1 - \gamma_1) B_1 x_{ui}^{\beta_1} - (\gamma_1 - 1)(1 - \tau) x_{ui} / (r - \mu) + \gamma_1 I_1 = 0 \text{ for } x_{ui} < x_{ue} \tag{17}$$

Since no closed form solution exists, equation (17) has to be solved numerically. However, as Proposition 2 below states, we can prove that a unique optimal investment trigger x_{ui} (or a unique solution to equation (17)) does exist.

***Proposition* 2** *There exists a unique solution* x_{ui} *to equation* (17).

We can also state the following properties of the optimal investment trigger (Propositions 3 and 4 below).

***Proposition* 3.** *There exists some* λ^* *such that* $\partial x_{ui} / \partial \lambda < 0$ *when* $\lambda < \lambda^*$ *and* $\partial x_{ui} / \partial \lambda > 0$ *when* $\lambda > \lambda^*$. *Thus,* x_{ui} *is a U-shaped function of* λ.

***Proposition* 4.** *For a given* λ*, the optimal investment trigger* x_{ui} *is a decreasing function of* δ *and an increasing function of* I_2.

All the proofs are in Appendix.

By what mechanism should the arrival speed λ affect the optimal investment trigger x_{ui}? On one hand, a higher λ (faster arrival) means that the increased earnings from modernization are more likely to start earlier; since this is a positive effect, a higher λ will result in earlier investment or a lower trigger x_{ui}. On the other hand, a higher λ also means that the modernization cost (I_2) is more likely to be incurred earlier, which has a negative effect and results in a higher x_{ui}. Thus, the overall effect of λ on investment trigger is likely to be non-monotonic, depending on which effect dominates, the "earlier earnings" effect or the "earlier cost" effect. When λ is large, the arrival is close to imminent; in such cases, the cost effect will dominate, since the one-time investment cost I_2 is incurred immediately while the benefits of higher earnings stream will come over a period of time. Therefore, for large λ, any further increase in λ will result in delayed investment or higher investment trigger x_{ui}. For small λ, the cost effect is smaller, hence the earnings effect dominates, resulting in accelerated investment; thus, x_{ui} is a decreasing function of λ when λ is small.

In addition, there is a third effect that comes into play when λ is small. For small λ, it is likely that the modernization opportunity will arrive only after x reaches the optimal modernization trigger x_{ue}, in which case it will not be possible for the firm to optimize its modernization decision. A higher λ makes this sub-optimality less likely (i.e., makes it less likely that the modernization opportunity will arrive after the modernization trigger has been reached), hence the firm is more likely to be able to optimize its modernization decision, hence it will be more willing to make the initial investment. Thus, an increase in λ should result in a lower investment trigger x_{ui}. Note that this factor is not important when λ is large, because in that case the opportunity is likely to arrive before the trigger is reached. Therefore, for small λ, there is an additional reason for x_{ui} being a decreasing function of λ. This gives the *U*-shaped curve discussed in Proposition 3. Numerical results in section **3.3.2** discuss this further.

Proposition 4 is consistent with economic intuition. Higher (lower) δ or lower (higher) I_2 are more (less) attractive for the project thus accelerate (delay) the initial investment given there is an arrival uncertainty. However, this result does not hold when the arrival uncertainty disappears.

3.2.3 Levered Firm

Now we examine the case when the initial investment is financed with both debt and equity. When issuing debt, the firm is assumed to choose the optimal amount of debt, trading off the tax benefits of debt versus the bankruptcy cost associated with debt. Similar to discussion in unlevered case, we first examine the two special cases of no modernization ($\lambda = 0$) and modernization with certainty ($\lambda = \infty$).

3.2.3.1. No Modernization Opportunity (λ = 0)

The firm issues debt and equity upon investment, after which equity holders receive the earnings stream and serve debt obligation by paying the coupon. When x falls to a sufficiently low level (say x_{b0}), the firm declares bankruptcy. At bankruptcy, the APR (absolute priority rule) is followed, so the shareholders exit with zero payoff, and bondholders take over the assets of the firm after incurring fractional bankruptcy cost of α ($0 \leq \alpha \leq 1$). The solutions are straightforward and can be obtained in closed form. We present the results here; interested readers can refer to existing papers for derivations (e.g., Shibata and Nishihara (2010)).

The investment and default triggers are

$$x_i = \frac{\gamma_1}{\gamma_1 - 1}\frac{r-\mu}{1-\tau}\frac{I_1}{\varphi} \text{ and } x_{b_0} = \frac{\gamma_2}{\gamma_2 - 1}\frac{r-\mu}{r}c^*$$

The optimal coupon and leverage are

$$c^* = \frac{x_i}{h}\frac{\gamma_2 - 1}{\gamma_2}\frac{r}{r-\mu} \text{ and } LEV^* = \frac{1}{\varphi h}\frac{1-\omega}{1-\tau}\frac{\gamma_2 - 1}{\gamma_2}$$

where $\varphi = 1 + \tau / h/(1-\tau)$ and $h = [1 - \gamma_2(1 - \alpha + \alpha/\tau)]^{-1/\gamma_2}$.

3.2.3.2. Certain Modernization Opportunity (λ = ∞)

The firm issues debt once at the initial investment, the optimal amount of debt reflecting the tradeoff between bankruptcy cost and tax benefit, growth opportunity and value loss caused by agency problem of debt overhang. Let us define x_{b3} and x_{b2} as the default trigger after and before modernization respectively. After modernization, the market value of equity and debt can be written as:

$$E_3(x) = (1-\tau)\left[\delta x/(r-\mu) - c/r - \left(\delta x_{b_3}/(r-\mu) - c/r\right)\left(x/x_{b_3}\right)^{\gamma_2}\right] \tag{18}$$

$$D_3(x) = c/r + \left[(1-\alpha)U\left(x_{b_3}\right) - c/r\right]\left(x/x_{b_3}\right)^{\gamma_2} \tag{19}$$

and

$$x_{b_3} = \frac{\gamma_2}{\gamma_2 - 1}\frac{r-\mu}{r}\frac{c}{\delta}$$

Before expansion, the market value of equity is given by:

$$E_2(x) = M_1 x^{\gamma_1} + M_2 x^{\gamma_2} + (1-\tau)\left[x/(r-\mu) - c/r\right] \tag{20}$$

Where M_1 and M_2 are constants to be solved using following conditions:

$$E_2(x_e) = E_3(x_e) - I_2 \tag{21}$$

$$E_2\left(x_{b_2}\right) = 0 \tag{22}$$

Those are value matching conditions. The first one indicates that the market value of old equity should be exactly the post-expansion equity value minus investment cost at expansion trigger. The second one states that the equity holders receive a zero payoff at bankruptcy. We use the two conditions to solve for M_1 and M_2, substituting them into equity value to obtain:

$$E_2(x) = (1-\tau)\left[\frac{x}{r-\mu} - \frac{c}{r} - p_{bi}\left(\frac{x_{b_2}}{r-\mu} - \frac{c}{r}\right)\right] + p_{ib}\left[E_3(x_e) - (1-\tau)\left(\frac{x_e}{r-\mu} - \frac{c}{r}\right) - I_2\right] \tag{23}$$

where $p_{ib} = \dfrac{x_{b_2}^{\gamma_2} x^{\gamma_1} - x_{b_2}^{\gamma_1} x^{\gamma_2}}{x_{b_2}^{\gamma_2} x_e^{\gamma_1} - x_{b_2}^{\gamma_1} x_e^{\gamma_2}}$ and $p_{bi} = \dfrac{x_e^{\gamma_1} x^{\gamma_2} - x_e^{\gamma_2} x^{\gamma_1}}{x_{b_2}^{\gamma_2} x_e^{\gamma_1} - x_{b_2}^{\gamma_1} x_e^{\gamma_2}}$

Note that

(i) p_{ib} is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_e, given that x has not fallen to x_{b_2} by then, and

(ii) p_{bi} is the present value of one dollar to be received at the first passage time of the stochastic shock x to x_{b_2}, given that x has not risen to x_e by then.

The expression (23) can be interpreted as follows: The first term represents the pre-expansion profit flow deducting coupon payment and bankruptcy value adjusted by corresponding probability, multiplied by the corresponding probability. The second term captures the incremental value of equity by exercising the growth option after deducting the expansion cost at x_e, adjusted by the corresponding probability.

Also, both expansion and default decisions are made optimally, giving two smooth pasting conditions:

$$\left.\frac{\partial E_2(x)}{\partial x}\right|_{x=x_e} = \left.\frac{\partial E_3(x)}{\partial x}\right|_{x=x_e} \quad \text{and} \quad \left.\frac{\partial E_2(x)}{\partial x}\right|_{x=x_{b_2}} = 0 \tag{24}$$

The debt value $D_2(x)$ is given by:

$$D_2(x) = N_1 x^{\gamma_1} + N_2 x^{\gamma_2} + c/r \tag{25}$$

in which N_1 and N_2 are constants to be solved using following conditions

$$D_2(x_e) = D_3(x_e) \tag{26}$$

$$D_2(x_{b_2}) = (1-\alpha)(1-\tau)x_{b_2} / (r-\mu) \tag{27}$$

The first condition is the value matching which requires that the two debt values are equal at modernization, since the modernization is financed entirely by equity. The second condition states that, at bankruptcy, the creditors take over the firm after incurring bankruptcy cost which accounts for a fraction α of the firm's assets. Note that we assume the creditors will not exercise growth option or re-lever the firm, following Mauer and Hackbarth (2012). Then we obtain:

$$D_2(x) = \frac{c}{r}\left[1 - p_{bi} - p_{ib}\left(\frac{x_e}{x_{b_3}}\right)^{\gamma_2}\right] + (1-\alpha)U(x_{b_3})p_{ib}\left(\frac{x_e}{x_{b_3}}\right)^{\gamma_2} + (1-\alpha)V(x_{b_2})p_{bi} \quad (28)$$

To interpret (28), the first term in this expression is the value of the coupon stream in the absence of both pre-investment default and expansion, the second term captures the residual claim value at post-expansion default, and the third term represents residual claim value at pre-expansion default. All items are adjusted by the corresponding probabilities.

The Investment Decision

At initial investment the firm does not have asset-in-place and debt, thus its value boils down to American option, similar to unlevered case:

$$G(x) = Jx^{\beta_1} \quad (29)$$

which is subject to value matching and smooth pasting conditions

$$G(x_i) = E_2(x_i) + D_2(x_i) - I_1 \quad (30)$$

$$\left.\frac{\partial G(x)}{\partial x}\right|_{x=x_i} = \left.\frac{\partial E_2(x)}{\partial x}\right|_{x=x_i} + \left.\frac{\partial D_2(x)}{\partial x}\right|_{x=x_i} \quad (31)$$

Also, the firm needs to optimize capital structure upon investment i.e. to maximize total firm value for any arbitrary x

$$c^* = \arg\max_c \{E_2(c,x) + D_2(c,x)\}$$

Combining it with previous equation, we can identify the solution (c^* , x_i). Since it is too complicated to obtain analytical solution we have to solve it numerically.

3.2.3.3. The General Case [$\lambda \in (0, \infty)$]

The existence of arrival uncertainty will impact initial investment trigger, default trigger and optimal capital structure. We discuss them backwardly.

1. *After growth opportunity arrival*

If the price level is not high enough when the opportunity arrives it will not be optimal to expand, hence the firm will wait until the price increases to the optimal expansion trigger x_e. Also, after arrival of the opportunity but before expansion, if the price becomes sufficiently low, the firm will default. All values and thresholds in this region are the same as the special case $\lambda = \infty$.

2. *Before growth opportunity arrival*

Similar to the above, the price level can be higher or lower than x_e. However, if the price falls to the default trigger, the firm will default before the arrival of growth opportunity. We make an assumption that the debt holders take over the control and still can utilize the growth options. After initial investment, we define the bankruptcy trigger before opportunity arrives as x_b,. Before proceeding to that, we need to outline the region's boundary which requires following information:

$$x_{b_3} < x_{b_2} < x_b < x_{b_0} < x_i < x_e$$

x_i is yet to be computed, but it obviously should be lower than expansion threshold x_e. Also the investment threshold must be higher than the default trigger, thus $x_i > x_b$. Now we clarify the relation among x_{b2}, x_{b0} and x_b. Note that x_{b0} is the default trigger given the firm only has one investment option, hence it clearly should be higher than x_{b2} which is for the firm that has second investment option. However, when there is possible arrival of growth, x_b should be lower than x_{b0} since the firm would like to delay bankruptcy anticipating future growth opportunity, meanwhile, x_b should be higher than x_{b2}. Since if x_{b2} is larger than x_b, then the firm will default immediately upon the arrival of growth opportunity. Thus the firm loses the chance of expansion. Thus $x_b < x_{b0}$. Now we can proceed to the valuation for each region

In the region $x_b < x < x_e$, the firm will never default neither exercise the growth option if it arrives in the next instant, the equity value is governed by

$$rE = \frac{1}{2}\sigma^2 x^2 E_{xx} + \mu x Ex + (1-\tau)(x-c) + \lambda(E_2 - E) \tag{32}$$

where E_2 is defined in Section 1. The last term on the right-hand side represents impact of jump on the equity value, which is the product of probability of jump and change of equity. The general solution is

$$E(x) = M_3 x^{\beta_1} + M_4 x^{\beta_2} + M_1 x^{\gamma_1} + M_2 x^{\gamma_2} + (1-\tau)[x/(r-\mu) - c/r] \tag{33}$$

The debt value is governed by equation

$$rD = \frac{1}{2}\sigma^2 x^2 D_{xx} + \mu x D_x + c + \lambda(D_2 - D) \tag{34}$$

The last term on the right-hand side represents impact of jump on the debt value, which is the product of probability of jump and change in debt value. The general solution is

$$D(x) = N_3 x^{\beta_1} + N_4 x^{\beta_2} + N_1 x^{\gamma_1} + N_2 x^{\gamma_2} + c/r \tag{35}$$

It is clear that both equity and debt values in this region are actually E_2 and D_2 plus two options term to reflect possible default before growth opportunity arrival and option to transit to next region.

In the region $x > x_e$, the equity holders will immediately engage in expansion when the growth opportunity arrives. The equity value is governed by

$$rE_1 = \frac{1}{2}\sigma^2 x^2 E_{1xx} + \mu x E_{1x} + (1-\tau)(x-c) + \lambda(E_3 - I_2 - E_1) \tag{36}$$

The general solution is

$$E_1(x) = M_5 x^{\beta_2} + (1-\tau)\left[\frac{x}{r-\mu}\frac{\mu - r - \delta\lambda}{\mu - r - \lambda} - \frac{c}{r} - \left(\frac{\delta x_{b_3}}{r-\mu} - \frac{c}{r}\right)\left(\frac{x}{x_{b_3}}\right)^{\gamma_2}\right] - \frac{\lambda I_2}{r+\lambda} \tag{37}$$

The debt value is governed by equation

$$rD_1 = \frac{1}{2}\sigma^2 x^2 D_{1xx} + \mu x D_{1x} + c + \lambda(D_3 - D_1) \tag{38}$$

The general solution is given by

$$D_1(x) = N_5 x^{\beta_2} + \left[(1-\alpha)U(x_{b_3}) - \frac{c}{r}\right]\left(\frac{x}{x_{b_3}}\right)^{\gamma_2} + \frac{c}{r} \tag{39}$$

It is clear that equity value is no more than an option term plus post-expansion value adjusted by λ. The option term represents claims to transit to previous region. The debt value has a similar format except the second and third term is not impacted by λ since the coupon is continuously served. To solve for M_3, M_4, N_3, N_4, M_5 and N_5, we need following conditions

$$E(x_b) = 0 \tag{40}$$

$$E(x_e) = E_1(x_e) \tag{41}$$

$$D(x_e) = D_1(x_e) \tag{42}$$

$$D(x_b) = (1-\alpha)(1-\tau)x_b / (r-\mu) \tag{43}$$

They are value-matching conditions. The first states that the equity holders obtain zero when the firm goes bankrupt. The second states that equity values (note here x_e is not "real" expansion trigger, but the switching threshold) in two regions are equal at x_e. Eq (42) specifies that the two debt values are equal at the switching point x_e. Eq (43) states that the bondholders are left with asset less default cost. Also, we have

$$\left.\frac{\partial E(x)}{\partial x}\right|_{x=x_b} = 0 \tag{44}$$

$$\left.\frac{\partial E(x)}{\partial x}\right|_{x=x_e} = \left.\frac{\partial E_1(x)}{\partial x}\right|_{x=x_e} \tag{45}$$

$$\left.\frac{\partial D(x)}{\partial x}\right|_{x=x_e} = \left.\frac{\partial D_1(x)}{\partial x}\right|_{x=x_e} \tag{46}$$

Those are smooth-pasting conditions. Eq (44) ensures the equity holders maximize their value at default. The other two ensure both equity and debt value in two regions diffuse continuously across switching trigger without discontinuous changes.

The initial investment threshold is identified using value matching and smooth pasting conditions similar to the prior section. After obtaining solutions to all coefficients we substitute and reorganize them to obtain following equation for x_i

$$\begin{aligned}\gamma_1\left(I_1 - \tau c / r\right) = {} & \left(\gamma_1 - \beta_1\right)\left(M_3 + N_3\right)x^{\beta_1} + \left(\gamma_1 - \beta_2\right)\left(M_4 + N_4\right)x^{\beta_2} \\ & + \left(\gamma_1 - \gamma_2\right)\left(M_2 + N_2\right)x^{\gamma_2} + \left(\gamma_1 - 1\right)\left(1 - \tau\right)x / \left(r - \mu\right)\end{aligned} \tag{47}$$

Meanwhile the optimal capital structure is obtained through

$$c^* = \arg\max_c \left\{E_2\left(c, x \middle| \lambda\right) + D_2\left(c, x \middle| \lambda\right)\right\}$$

There is no closed form solution to those systems of nonlinear equations (39-46) and we have to use numerical solutions.

3.3. Numerical Results

Although some special cases of our model can be solved analytically, there are no closed-form solutions for the general case ($0 < \lambda < \infty$). We therefore use numerical solutions to characterize the quantitative effects of the jump risk (of arrival) on investment and financing decisions. In order to solve the model numerically, we need to specify values of the input parameters. We start with a set of reasonable "base case" parameter values taken from well-established papers in the existing real-option/contingent-claim literature, and repeat the computations with a wide range of parameter values to ensure robustness of the results. Since there is no single paper which studies the exact problem studied in this paper, we use a number of papers to choose our base-case parameter values.

3.3.1. Base-case Parameter Values

For the base case, we adopt following parameter values: the risk-free rate is r = 7% (as in Tsyplakov, 2008, Mauer and Ott, 2000), the expected growth rate is μ = 1% (as in Hackbarth & Mauer, 2012, Huang and Li, 2013, Morellec and Zhdanov, 2008, Shibata and Nishihara, 2012, Lyandres and Zhdanov, 2010). For the volatility of the earnings stream, we use σ = 35% (as in Schwartz and Moon, 2000, and in between Li, 2011 (40%) and Hackbarth and Campello, 2012 (30%)). The corporate tax rate is τ = 15% (Leland, 1994, Morellec and Zhdanov, 2008, Shibata and Nishihara, 2012), and the bankruptcy cost is α = 50% (as in Leland, 1994, Chu, 2009, Huang and Li, 2013, Mauer and Ott, 2000, Childs, Mauer and Ott, 2005). The growth multiple at modernization is $\delta = 2$, the initial investment cost is $I_1 = 15$, and the modernization cost is $I_2 = 20$.

3.3.2. Unlevered Firm

3.3.2.1 Base Case

Figure 3.1 shows the investment threshold (x_{ui}) as a function of the arrival rate (λ) for the base-case parameter values. As expected from Proposition 2, for the two polar cases ($\lambda = 0$ and $\lambda = \infty$) the optimal investment triggers are same: $x_{ui} = 2.9283$. As λ is increased from 0, we find that x_{ui} initially falls and subsequently rises. It falls from 2.9283 to 2.6028 as λ increases from 0 to 0.08, and then starts rising; thus, for the base case, we have $\lambda^* = 0.08$. Consistent with Proposition 2, there is a *U*-shaped relationship between investment trigger and speed of arrival. Note also that for $\lambda > \lambda^*$, x_{ui} is increasing at a decreasing rate, converging asymptotically to x_{ui} ($\lambda = \infty$) = 2.9283. Incidentally, the U-shaped relationship is obtained in all cases (for all parameter values) examined, as discussed below.

<u>Figure 3.1 about here</u>

3.3.2.2 Two special cases

Although we have discussed the reason in Section 3.2.2.3, to further formalize our intuition, we consider two special cases: it's known with certainty that: (1) the modernization opportunity will arrive before the profit flow hits the optimal level, that is $x(\widetilde{T}) < x_{ue}$, and (2) the modernization opportunity will arrive after the profit flow hits the optimal level that is $x(\widetilde{T}) > x_{ue}$. We present the solutions to them in Appendix 3.B.

It is worth noting that both solutions are same as in special cases $\lambda = 0$ or $\lambda = \infty$. In the first case, the reason is straightforward. Since the firm can always expand optimally with certainty, the initial investment trigger will not be impacted. For the second case, if we take a closer look at the solution $\widetilde{x}_{ue}(\lambda)$ in Appendix B, we find that the uncertainty has been incorporated in the optimal expansion threshold, thus the initial investment threshold is still a constant independent of the growth property.

To summarize, when the arrival timing is unknown all through the life after initial investment, the growth quality δ and I_2 play an important role in the initial investment timing. But if the firm knows some specific information of the arrival timing, it can handle the arrival uncertainty, growth opportunity will not impact initial investment trigger.

3.3.2.3 Comparative statics

Figures 3.2(a) and 3.2(b) show the results for different modernization characteristics (I_2, δ). First, we note that in all cases the relationship is U-shaped as mentioned above. As expected from Proposition 4, the investment trigger x_{ui} is an increasing function of the modernization cost I_2. Also worth noting is the fact that x_{ui} starts rising at lower levels of λ when I_2 is higher (that is, λ^* is a decreasing function of I_2) since the second effect mentioned in the discussion following Proposition 4 (i.e., the "earlier cost effect") is more important. For instance, when I_2 is increased from 16 to 26, we find that λ^* falls from 0.1 to 0.07. For different δ the results are also similar. In all cases, x_{ui} has a U-shaped relationship with λ. Also, as expected, x_{ui} is a decreasing function of δ. Finally, x_{ui} starts rising at higher levels of λ when δ is higher (that is, λ^* is an increasing function of δ), because the first effect (the "earnings effect") is more important; henceforth x_{ui} will be falling for a longer stretch and will start rising later. For instance, when δ is increased from 1.7 to 2.3, we find that λ^* rises from 0.07 to 0.1.

<u>Figure 3.2 about here</u>

Figure 3.2(c) and (d) depict the impact of risk-free rate r and idiosyncratic volatility σ. For a given λ, when the discount rate r is higher, the present value of the benefits of modernization will be smaller, hence the investment trigger will be higher, since the investment is less attractive. Further, for r = 8%, r = 7% and r = 6% the inflection point λ^* is 9%, 8% and 7% respectively. This indicates that r has an impact on the relative dominance of "earning effect" and "cost effect". The channel through which the impact of r functions can be explained mathematically in equation (6), (7), (11) and (12). However, due to the complex form we present a heuristic explanation: when the arrival speed λ is relatively low, the higher the interest rate r the more weakly I_2 affects investment option compared to revenue (it is partially reflected in last two items from equation (8)) hence the cost effect will become less important; as a result, the investment trigger will start rising later. Thus, λ^* will be higher when r is larger. This is also what we observe in the numerical results. For volatility, higher σ means a larger value to holding the option, hence a smaller value of exercising option to invest. Therefore the investment threshold increases with σ, which is standard result in the literature. Similar to r, when σ becomes higher λ^* becomes larger since the revenue effect is impacted more by volatility than the cost effect. For example, for σ = 50%, σ = 35% and σ = 20% the lowest point λ^* is 9%, 8% and 6%.

3.3.3. Levered Firm

3.3.3.1 Base Case

3.3.3.1.1 Optimal Financial Leverage

The optimal financial leverage is calculated from $D(x_i, c^*) / (D(x_i, c^*) + E(x_i, c^*))$. From Fig 3.3 we note that the optimal leverage is U shaped. It decreases sharpely from 34.15% to 30.44% as λ increases from 0 to 0.3 and then increases slowly to 30.68% when $\lambda = 1$. For comparison we also plot the dotted line representing the optimal leverage when $\lambda = \infty$ (which is 30.89%). The U shaped leverage curve is driven by several reasons. On one hand, there will be a debt overhang problem due to the equity value maximization strategy at expansion; however, this impact is not important when λ is low. Our unreported result shows that if the firm follows first-best policy to set optimal expansion trigger, the optimal leverage is also U shaped and the degree of magnitude changes trivially. The reason is twofold. Firstly, Sundareson, Wang and Yang (2014) have shown that when the initial investment and financing is endogenized, the agency cost of debt overhang at expansion stage will decrease significantly. Secondly, in our case, although the firm can set second best or first best optimal expansion trigger, the arrival of growth opportunity is essentially independent of it. Thus the debt overhang problem, even if it exists, largely depends on the availability of modernization. If the opportunity arrives with low possibility or after the optimal expansion trigger, the debt overhang impact is nearly negligible. Especially in Fig 3.3, when λ is low, the future debt overhang problem can be neglected. Thus the agency conflicts contribute trivially to the result. (Note: this paragraph's discussion relies on base parameter of $\delta = 2$, in which debt overhand is not prominent, in later discussion we will reveal that when $\delta > 2$, the effect of debt overhang becomes larger).

The decreasing leverage when λ is relatively low is due to the firm raising low debt to decrease bankruptcy probability, thus enhancing the chance that growth opportunity arrives, rather than avoid debt overhang. As we discussed in unlevered case, "earnings effect" dominates at low λ region. Thus the lower chance of bankruptcy indicates the higher possibility that the firm can reap the modernization benefit and increases the equity value. However, when λ is relatively high, the "cost effect" and "debt overhang" looms and the value of equity is reduced. Thus the optimal leverage increases. Notably, the slope of the curve in the "high λ" region is much flatter than in "low λ" region, reflecting those effects function more softly than that of "earning effect". We can indirectly decompose these two effects further, as discussed in section 3.3.3.2

<u>Figure 3.3 about here</u>

3.3.3.1.2. Optimal investment threshold

Fig. 3.4 shows that the optimal investment threshold decreases from 2.8476 to 2.5509 as λ increases from 0 to 0.08 then it increases to 2.7905 when λ =1. The dotted line shows that x_i = 2.8986 for $\lambda = \infty$. Thus we can see that the shape of investment threshold with respect to arrival uncertainty, which is similar to unlevered firm. However, there are a few things requiring discussion.

<u>Figure 3.4 about here</u>

Firstly, comparing Fig.3.1 and 3.4, the investment threshold for levered firm is lower than the unlevered case for all range of λ. It is anticipated since the existence of debt makes the investment attractive, which lowers the investment threshold compared to unlevered case (see Mauer and Sarkar (2005)). Secondly, the investment trigger without growth opportunity, x_i (λ = 0), is lower than that with certain growth $x_i(\lambda = \infty)$. As we have discussed, without debt financing they are equal. From this result, we can see even without modernization uncertainty, the financing can distort investment decision with growth option. The reason is straightforward: the growth opportunity caused the debt overhang problem which makes the expansion less attractive thus the initial investment is delayed, although the magnitude is extremely small (since debt overhang effect is not so apparent). We need to clarify one important thing here: it is not the growth option that impacts the investment trigger but the agency conflict caused by wealth transfer occurring upon the growth option being exercised. Recall that in unlevered case, the guaranteed modernization has no impact on the investment decision because the impact brought by δ and I_2 can be accommodated by choosing an appropriate expansion trigger. However, the debt overhang can't be eliminated completely by choosing optimal expansion trigger, thus the initial investment timing will be negatively affected.

3.3.3.2 Comparative Statics

Figure 3.5 and 3.6 plots optimal investment threshold and leverage for a range of parameter values. In Figure 3.5 it is noteworthy that all curves are U-shaped and all the minimum point λ^* are almost same as unlevered case shown in Fig 3.2. Thus we can conclude that the investment decisions are mainly driven by the tradeoff between "earnings effect" and "cost effect" while debt overhang effect, compared to the aformentioned effects, is negligible in shaping firm's initial investment decision.

<u>Figure 3.5 about here</u>

We focus on Figure 3.6 to explain leverage behavior. We can see that all leverage curves are U shaped. We firstly look at panel (a) for the effect of δ. It can be observed that the larger the growth size, the more strongly U-shaped is the curve. For example, when $\delta = 1.7$, the U shape becomes fairly weak. Moreover, the slope before around $\lambda = 0.2$ is much steeper than after. The reason is that earnings effect leads to increased equity value as λ increases when it is relatively low. After that, the probability of arrival becomes fairly large, thus earnings effect decreases.

Compared with other two cases, we can note that the debt overhang effect is a little more important than "cost effect" to impact optimal leverage. For cost effect, all three cases should be similar (with same I_2), thus the only reason for more severe U shape is debt overhang. For $\delta = 1.7$, since growth size is low, debt overhang is negligible and we can see cost effect does harm to equity value while for $\delta = 2.3$, since the growth size is very large, debt overhang effect dominates. We will continue the discussion of the impact of δ in the following section, from another angle to highlights the "synergic effect" between δ and λ.

<u>Figure 3.6 about here</u>

Panel (b) consolidates our intuition. We can find that given certain λ the higher the modernization cost the lower the optimal leverage. As for low I_2 (such as $I_2 = 16$) the U shape is stronger than other curves. As documented by Mauer and Ott (2000), the higher the expansion cost the smaller the agency cost since the high expansion cost will both delay first-and second-best investment decision, rendering their gap narrower. Thus we can see that when I_2 is fairly large ($I_2 = 26$) the debt overhang effect will be very small hence the "cost effect" dominates thus the increasing trend is almost flat. On the other hand, when I_2 is small, the debt overhang effect becomes more important thus the increasing trend is clearer.

Panels (c) and (d) report the impact of interest rate r and volatility σ, respectively, on optimal leverage. It can be seen that the higher the r the higher the optimal leverage and the higher the σ the lower the optimal leverge. The reason for the former is ambiguous since it is attributed to the interaction among growth options, debt and equity value. Since equity can be viewed as a call option thus higher volatility leads to higher equity value which lowers optimal leverage

3.3.3.3 Effect of growth size δ

So far we have discussed the impact of arrival uncertainty λ on investment timing x_i and optimal leverage. The comparative static results show that the growth characteristics (δ, I_2) are heavily involved in the firm's decision through arrival uncertainty for both unlevred and levered firm. Now we examine their impacts further. For simplicity, we only focus on δ since I_2 functions in a similar manner, but in a reverse direction.

From Fig 3.7 we can find that when $\lambda \to \infty$, the investment threshold increases from 2.8476 to 2.9768 as the growth size increases from 1 to 2.45. Firstly, it is note worthy that the investment is still in sequence (our result shows that the optimal expansion timing is $x_e = 3.029 > x_i$ at $\delta =$ 2.45, also note that as our numerical result shown when $\delta > 2.45$ the investments become simualtaneous, that is, $x_e \leq x_i$) even the growth size is larger than 2.33, a critical value above which the investment will be implemented simultaneuously for unlevered firm given our base parameters. The reason is that the existence of debt overhang delay the optimal expansion timing. And such an effect is more severe when the growth size becomes larger since equity holders have to give more value upside to creditors. Secondly, debt issuance accelerates the initial investment. Thus, in case of levered firm, even if the growth size is large enough to increasing return to scale (in our base case, $\delta = 2.45 > 2.33$), the investment is still in sequence.

<u>Figure 3.7 about here</u>

Contrary to the case when $\lambda = \infty$, when there is growth uncertainty such that $\lambda = 1$, the investment trigger decreases from 2.8476 to 2.56. This result is similar as described by Proposition 4 for unlevered firm. This is because the impact of debt overhang on initial investment becomes far less important. It is noteworthy that when the growth size is relatively small, the effect of arrival uncertainty is negligble. For example when δ is 1.5, the initial investment trigger is 2.8499, very close to 2.8544 at $\delta = 1$. Following the same logic, when the arrival rate λ is as low as 0.1, we can find that the investment trigger decreases sharpely from 2.8476 to 2.56 as δ increases from 1 to 2.45.

Now let's take a look at the optimal financial leverage. From Figure 3.8 it can be observed that when $\delta = 1$, all cases converge to 34.15%. when there is no uncertainty of growth opportunity ($\lambda = \infty$) the optimal leverage decreases first to 30.85% at $\delta = 1.9$ and then increases to 31.44% at $\delta = 2.45$. When the uncertainty level is $\lambda = 1$ the shape is simialr to $\lambda = \infty$ albeit it decreases to 30.67% at $\delta = 2.1$ and then increases to 30.85%. When $\lambda = 0.1$, the slope is always decreasing to 30.04%. For $\lambda = \infty$, on one hand, more debt brings tax shield benefit thus firm has incentive to issue debt and increases the optimal leverage. On the other hand, more debt causes agenct cost upon expansion which disincentivize issuing too much debt. The two forces lead to the U shaped curve. Typically, when the growth size is relatively small the growth opportunity contributes to enhance equity value while debt overhang effect is negligible thus as δ increases the equity value increases relative to debt value and the optimal financial leverage decreases. This result is similarly documented by Hackbarth & Mauer (2012). When growth size is relatively large (in our case $\delta \geq 2.1$), debt overhang undermines the pre-expansion equity value and this effect is larger than the increased value by growth option, therefore the optimal leverage increases in this region.

<u>Figure 3.8 about here</u>

When $\lambda = 1$, we can observe that the curve is still U-shaped but some points need further discussion. Compared to $\lambda = \infty$, it can be easily found that the optimal leverage are almost same in the range of $1< \delta < 1.7$. The reason is that although there has been some uncertainty, $\lambda = 1$ means the opportunity will be expected arrive in one year, thus the expansion is likely (at least for the base parameters) and will still enhance the equity value. For $\delta \geq 1.7$, we can observe that the curve is a bit lower than that of $\lambda = \infty$ because the debt overhang impact is less important due to uncertain arrival. Thus it can be summarized that the agency cost of debt overhang is more sensitive to the arrival uncertainty compared to the equity value enhancement caused by growth opportunity.

Further, when $\lambda = 0.1$, the curve is downward-sloping. It is interesting that at low δ region the optimal leverage for $\lambda = 0.1$ is higher than other two cases and is lower than other two cases at high δ region. For example, the leverage for $\lambda = 0.1$ is higher than $\lambda = \infty$ when $\delta < 1.94$ and lower than the latter when $\delta > 1.94$. This result supports our previous explanation: when $\delta < 1.94$ the growth option increases the equity value hence the optimal leverage decreases as δ increases. However, since the arrival probability is relatively low (it is expected to arrive in 10 years) the contribution of growth option is discounted, thus the equity value is less increased than $\lambda = \infty$ or 1, and the optimal leverage is higher than them. When $\delta > 1.94$, as δ increases the growth option continues to enhance the equity value while debt overhang is of second order importance. Those two reasons lead to the result. To summarize, the synergic effect of degree of uncertain arrival of growth opportunity, λ, and growth size, δ, plays an important role in balancing the equity enhancement from growth option and potential value loss from debt overhang. The lower the probabillity of arrival, the lower the impact of both two effects. However, as the growth size increases, the first effect dominates the second effect.

3.4. Concluding Remarks

This paper develops a continuous-time model of the firm's sequential investments when the future growth opportunity may arrive with uncertainty. The existing literature neglects the relationship between first-stage investment and future expansion with arrival uncertainty. For unlevered firms, our results show that the future growth uncertainty has economically significant impact on the initial investment decision. This impact is non-monotonic, depending how large the growth multiple will be and how much it costs to expand. Although we don't illustrate that such an uncertainty is a necessary mechanism through which the initial investment is impacted by future growth opportunity, it provides a valuable economic guidance on the capital budgeting decision, especially for those projects with potential large scale expansion when there is uncertainty about the availability of the expansion.

When the firm issues both debt and equity to finance the first-stage investment, we show that the financing decision can be greatly impacted by the interaction between arrival possibility and growth size. The relationship between optimal capital structure and level of arrival probability also presents a non-monotonic shape. When the arrival probability of future expansion is small, the equity holders would like to lower the amount of debt as the probability increases, since it will lower the probability of default, so as to take advantage of possible expansion. On the contrary, when the arrival probability is high, the debt overhang effect becomes important, and equity holders' value will be reduced; the larger the growth size the more severe the debt overhang problem. As a consequence, both growth size and arrival uncertainty will impact the firm's financing decision.

Our findings also shed light on the empirical testing of agency cost of underinvestment; there has been debate on the regression between growth option and financial leverage to predict the debt overhang problem (summarized by Chen and Zhao (2006)). However, as our model implies, the inverse (or even non-monotonic) relation between growth option and market leverage is not a secure signal of debt overhang since the extent to which the growth opportunity can be realized also plays an important role. Thus, testing can be refined by using some subset of companies. Also, it raises another question: how do we choose the proxy variable for the degree of uncertainty of growth opportunity? Hoppe (2002) has a brief survey on the empirical testing of technology adoptions. He pointed that most papers use probit/logit and hazard rate models to describe some specific technology modification. And there are other factors linked to the technology adoption such as firms' scale (e.g. small vs. large firms).

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Appendix 3.A

Proof to Proposition 1 Before proceeding to the formal proof, we first state two lemmas.

***Lemma* 1** The positive root $1 < \gamma_1 < r/\mu$. This is easily derived, hence it is not presented here.

***Proposition* 2** The constant $B_1 < 0$.

It is obvious that denominator of B_1 is always positive because $\beta_1 > \beta_2$. Thus the sign of B_1 is the same as that of the numerator. Let us define the numerator as $\Psi \equiv \Psi(\lambda)$; after substituting for A_1 and x_{ue} we obtain

$$\Psi(\lambda) = \left(\frac{\beta_2 - \gamma_1}{\gamma_1 - 1} + \frac{\lambda\gamma_1}{\gamma_1 - 1} \frac{1-\beta_2}{r - \mu + \lambda} + \frac{\lambda\beta_2}{r+\lambda} \right) I_2 \tag{A1}$$

Let's define the items in the bracket as $\varphi \equiv \varphi(\lambda)$ we can further reorganize it to

$$\varphi(\lambda) = \frac{r(\beta_2 - \gamma_1)(r - \mu + \lambda) + \gamma_1\lambda\mu(1-\beta_2)}{(\gamma_1 - 1)(r - \mu + \lambda)(r+\lambda)} \tag{A2}$$

Note the denominator is positive so we only need to prove the numerator is negative. To do it, let us define the numerator as $\kappa(\lambda)$, then we have

$$\begin{aligned} \kappa(\lambda) &= r(\beta_2 - \gamma_1)(r - \mu + \lambda) + \gamma_1\lambda\mu(1-\beta_2) \\ &< r(\beta_2 - \gamma_1)(r - \mu + \lambda) + \gamma_1\lambda\mu(\gamma_1 - \beta_2) \\ &= (\beta_2 - \gamma_1)[r(r - \mu + \lambda) - \gamma_1\lambda\mu] \end{aligned} \tag{A3}$$

Now define the item in the second bracket as

$$\theta(\lambda) = r(r - \mu + \lambda) - \gamma_1\lambda\mu \tag{A4}$$

It can be easily verified that $\partial\theta/\partial\lambda = r - \gamma_1\mu > 0$ according to Proposition 1. Also, we have $\theta(0) = r(r-\mu) > 0$, thus we immediately have $\theta(\lambda) > 0$ for $\lambda \geq 0$. Thus we have $\kappa(\lambda) < 0$ since the first item is always negative. Hence B_1 is negative.

Actually we can easily observe that as $\lambda \to 0$, $B_1 \to -A_1$ when $\lambda \to \infty$, $B_1 x^{\beta_1} \to 0$, because

$\left(B_1 x^{\beta_1}\right)_{\lambda\to\infty} = \frac{\Psi_{\lambda\to\infty}}{(\beta_1-\beta_2)\left(\frac{x_{ue}}{x}\right)^{\beta_1}} = \frac{\beta_2}{\beta_1-\beta_2}\left(\frac{x_{ue}}{x}\right)^{-\beta_1} \to \frac{1}{2} \times 0 = 0$ given $x < x_{ue}$. Note we are not sure the value of B_1 as $\lambda\to\infty$ since it depends on x_{ue}

Now we are able to prove the *Proposition 1*

i. To prove the existence we firstly rewrite

$$\Gamma(x) = (\beta_1 - \gamma_1)B_1 x^{\beta_1} - (\gamma_1 - 1)\frac{1-\tau}{r-\mu}x + \gamma_1 I_1 \tag{A5}$$

note $\Gamma(0) = \gamma_1 I_1 > 0$ and

$$\Gamma(x_{ue}) = \frac{\beta_1-\gamma_1}{\beta_1-\beta_2}\Psi(\lambda) - (\gamma_1 - 1)\frac{1-\tau}{r-\mu}x_{ue} + \gamma_1 I_1 = \frac{\beta_1-\gamma_1}{\beta_1-\beta_2}\Psi(\lambda) - \gamma_1\left(\frac{I_2}{\delta-1} - I_1\right) \tag{A6}$$

Given the investment is sequential thus the second term is positive and $\Gamma(x_{ue})$ is negative, since the function is continuous and differentiable in entire region thus there exist at least one solution within $x \in [0, x_{ue}]$

ii. To prove the uniqueness, we have

$$\frac{d\Gamma}{dx} = \beta_1(\beta_1 - \gamma_1)B_1 x^{\beta_1-1} - (\gamma_1 - 1)\frac{1-\tau}{r-\mu} \tag{A7}$$

Combined with $\beta_1 > \gamma_1$ thus we immediately have the first item is negative since we have proven $B_1 < 0$ hence the first derivative is negative. Also, we have the second derivative

$$\frac{d^2\Gamma}{dx^2} = \beta_1(\beta_1 - 1)(\beta_1 - \gamma_1)B_1 x^{\beta_1 - 2} < 0 \tag{A8}$$

Thus the solution is unique.

Proof to proposition 2. To prove it we take first derivative to λ on both sides of equation; after some rearrangement we have

$$\frac{\partial x_{ui}}{\partial \lambda} = \frac{\frac{\partial \beta_1}{\partial \lambda} B_1 x_{ui}{}^{\beta_1} + (\beta_1 - \gamma_1) x_{ui}{}^{\beta_1}\left(\frac{\partial B_1}{\partial \lambda} + B_1 \frac{\partial \beta_1}{\partial \lambda} \ln \beta_1\right)}{(\gamma_1 - 1)\frac{1-\tau}{r-\mu} - (\beta_1 - \gamma_1) B_1 \beta_1 x_{ui}{}^{\beta_1 - 1}} \tag{A9}$$

It can be verified that $\left.\frac{\partial x_{ui}}{\partial \lambda}\right|_{\lambda=0} = -\frac{\partial \gamma_1}{\partial \lambda}\frac{I_2}{(\gamma_1 - 1)^2}\frac{r-\mu}{1-\tau} < 0$. Suppose that $\frac{\partial x_{ui}}{\partial \lambda} \le 0$ for all $\lambda \in (0, +\infty)$ then $x_{ui}\big|_{\lambda \to \infty}$ will be smaller than $x_{ui}\big|_{\lambda=0}$ which violates our analytic result thus there is at least one λ^* above which the slope will be positive. However, it is too complex to prove the uniqueness and we have to numerically prove it until sufficiently large λ.

Proof to proposition 3. To prove it let's take derivative to δ on both sides of equation, which leads to

$$(\beta_1 - \gamma_1)\left(\frac{\partial B_1}{\partial \delta} x_{ui}^{\beta_1} + B_1 \beta_1 x_{ui}^{\beta_1 - 1}\frac{\partial x_{ui}}{\partial \delta}\right) - (\gamma_1 - 1)\frac{1-\tau}{r-\mu}\frac{\partial x_{ui}}{\partial \delta} = 0 \tag{A10}$$

We reorganize it into $$\frac{\partial x_{ui}}{\partial \delta} = \frac{(\beta_1 - \gamma_1)\frac{\partial B_1}{\partial \delta} x_{ui}^{\beta_1}}{-(\beta_1 - \gamma_1) B_1 \beta_1 x_{ui}^{\beta_1 - 1} + (\gamma_1 - 1)\frac{1-\tau}{r-\mu}} \tag{A11}$$

Note $\frac{\partial B_1}{\partial \delta} < 0$ since $\frac{\partial B_1}{\partial \delta} = \frac{\partial B_1}{\partial x_{ue}} \frac{dx_{ue}}{d\delta}$ and the first term of RHS is positive (because $B_1 < 0$) the second term is negative, thus we have $\frac{\partial x_{ui}}{\partial \delta} > 0$

Similarly for I_2 We reorganize it into

$$\frac{\partial x_{ui}}{\partial I_2} = \frac{(\beta_1 - \gamma_1)\frac{\partial B_1}{\partial I_2} x_{ui}^{\beta_1}}{-(\beta_1 - \gamma_1)B_1\beta_1 x_{ui}^{\beta_1 - 1} + (\gamma_1 - 1)\frac{1-\tau}{r-\mu}} \tag{A12}$$

Note $B_1(I_2) = \frac{\Psi I_2}{(\beta_1 - \beta_2)x_{ue}^{\beta_1}}$ hence $\frac{\partial B_1(I_2)}{\partial I_2} = \frac{\Psi}{\beta_1 - \beta_2}\frac{1-\beta_1}{x_{ue}^{\beta_1}} > 0$ thus we have $\frac{\partial x_{ui}}{\partial I_2} > 0$

Appendix 3.B

If the firm knows the opportunity will surely arrive before the profit flow hits x_{ue}, the firm can still optimally exercise the expansion option; thus the post-expansion firm value is same as eq (3). The pre-expansion firm value can be similarly expressed by eq (6). However, it is apparent that the coefficient B_1 becomes zero. Thus the initial investment trigger is same as (1) or (4).

If the firm knows the opportunity will surely arrives after the profit flow hits $\widetilde{x}_{ue}$ the firm value at $x > \widetilde{x}_{ue}$ can be similarly expressed by eq (8) and note this $\widetilde{x}_{ue}$ is different from

$$\overline{V}(x) = \frac{1-\tau}{r-\mu}\frac{r-\mu+\delta\lambda}{r-\mu+\lambda}x - \frac{\lambda I_2}{r+\lambda} \tag{B1}$$

Then the firm value at $x < \widetilde{x}_{ue}$ is $\underline{V}(x) = L_1 x^{\gamma_1} + \frac{1-\tau}{r-\mu}x$

Then we need to solve L_1 and $\tilde{x}_{ue}$ the value matching and smooth pasting conditions are

$$\underline{V}(\tilde{x}_{ue}) = \overline{V}(\tilde{x}_{ue}) \tag{B2}$$

$$\left.\frac{\partial \underline{V}(x)}{\partial x}\right|_{x=\tilde{x}_{ue}} = \left.\frac{\partial \overline{V}(x)}{\partial x}\right|_{x=\tilde{x}_{ue}} \tag{B3}$$

Then we obtain

$$\tilde{x}_{ue}(\lambda) = \frac{\gamma_1}{\gamma_1 - 1}\frac{r-\mu}{1-\tau}\frac{r-\mu+\lambda}{r+\lambda}\frac{I_2}{\delta - 1} \tag{B4}$$

$$L_1 = \left[\frac{1-\tau}{r-\mu}\frac{\lambda(\delta-1)x_{ue}}{r-\mu+\lambda} - \frac{\lambda I_2}{r+\lambda}\right]x_{ue}^{-\gamma_1} \tag{B5}$$

Define the initial investment option value as $F(x) = L_0 x^{\gamma_1}$ we obtain the initial investment trigger

$$x_{ui} = \frac{\gamma_1}{\gamma_1 - 1}\frac{r-\mu}{1-\tau}I_1 \tag{B6}$$

Table 3.1

Variables summary	
I_2: expansion investment cost	I_1: initial investment cost
x_{ue}: optimal expansion trigger for unlevered firm	λ: arrival rate of growth opportunity
x_e: optimal expansion trigger for levered firm	c: c coupon payment
x_{b3}: default trigger after adopting new technology	δ: growth option payoff factor
x_{b2}: default trigger of a firm doesn't invest when new tech arrives	η: proportional bankruptcy cost
x_b: default trigger of a firm loses change to adopt new tech	r: risk-free interest rate
τ: corporate tax rate	σ: idiosyncratic volatility of profit flow
x_i: optimal initial investment trigger for levered firm	μ: expected growth rate of profit flow
V: unlevered firm value before expansion but after opportunity arrival	x_{ui}: optimal initial investment trigger for unlevered firm
U unlevered firm value after expansion	W: unlevered firm value before expansion and opportunity arrival
	Z: unlevered firm value after expansion and before opportunity arrival

Fig 3.1

It plots the initial investment trigger x_{ui} for an unlevered firm against arrival rate of growth opportunity λ, the black dotted line corresponds to the special case that no uncertainty of arrival. The parameters in this model are set: The risk-free interest is 7% per year, the mean growth rate μ and standard deviation σ of profit flow are 1% and 35%, the cost for initial investment is I_1=15 and I_2=20 respectively.

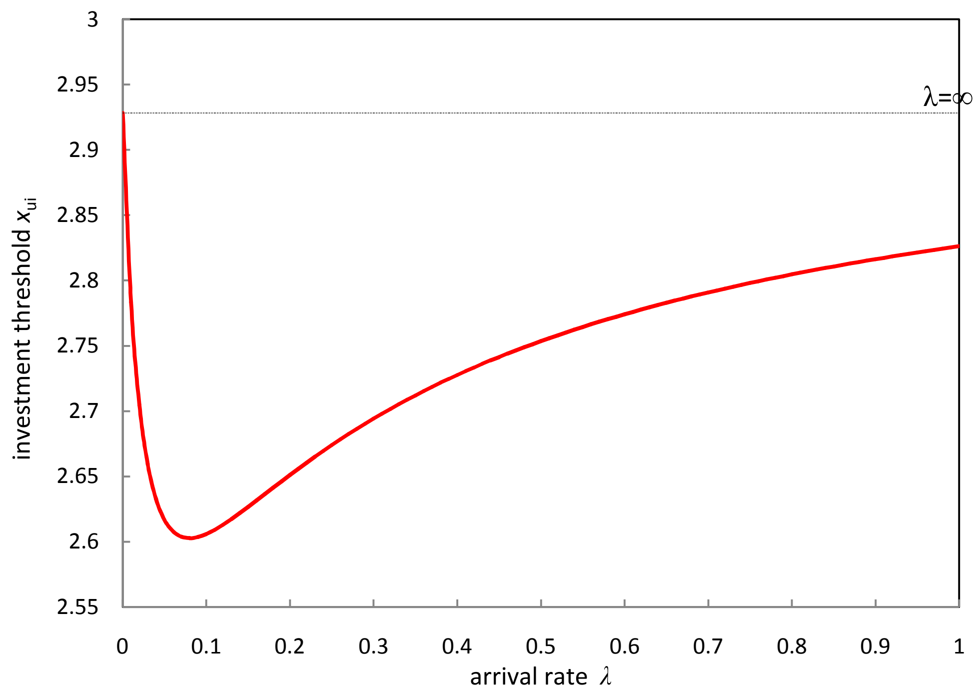


Fig 3.2 Comparative static for unlevered firm investment threshold

Figure (a)-(b) plot the initial investment trigger x_{ui} for an unlevered firm against arrival rate of growth opportunity, panel (a) and (b) for impact of modernization property of size δ and cost I_2 panel (c) is for risk free rate r panel (d) is for idiosyncratic volatility σ . In all panels black line represents base case. The parameters in this model are set: The risk-free interest is 7% per year, the mean growth rate μ and standard deviation σ of profit flow are 1% and 35%, the cost for initial investment is I_1=15 and I_2=20 respectively.

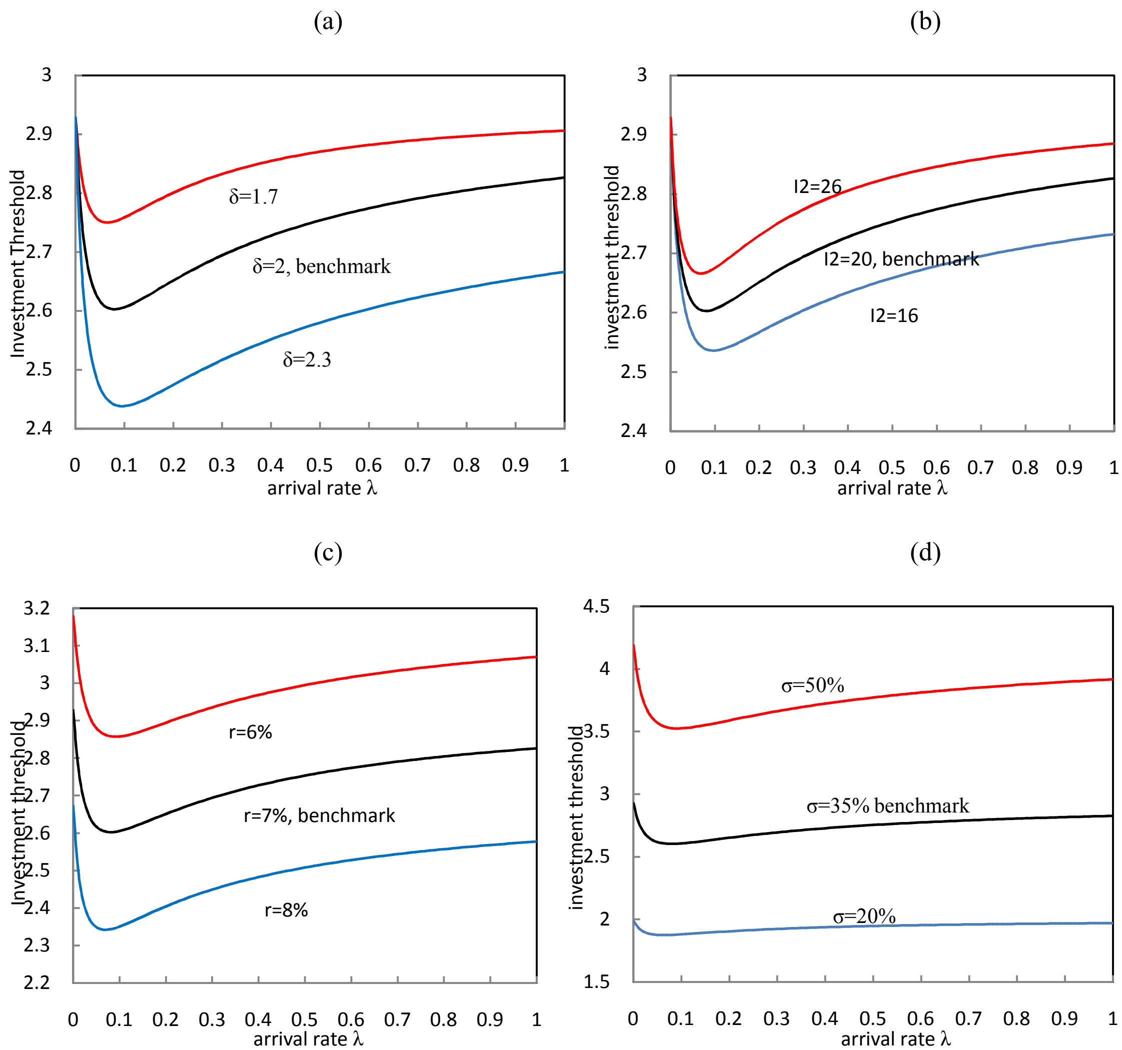


Fig 3.3

This figure plots the optimal financial leverage against arrival rate of growth opportunity given the coupon is optimally chosen , the black dotted line corresponds to the special case that no uncertainty of arrival: The parameters in this model are set: The risk-free interest is 7% per year, the mean growth rate μ and standard deviation σ of profit flow are 1% and 35%, respectively.

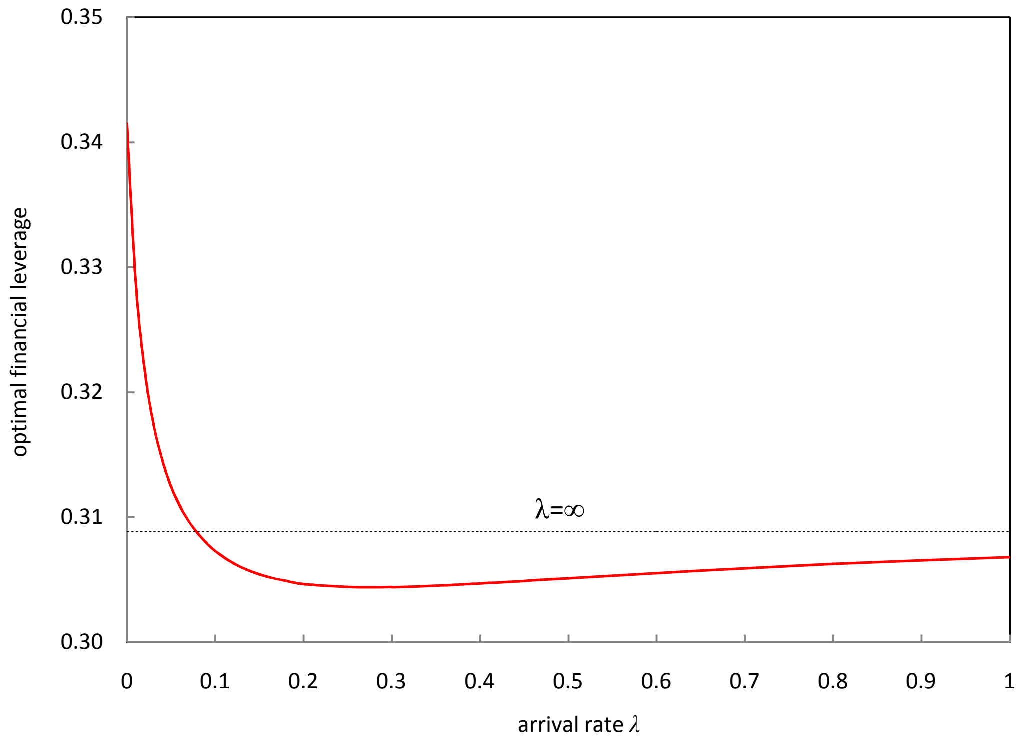


Fig 3.4

This figure plots the initial investment trigger x_i for an optimally levered firm against arrival rate of growth opportunity, the black dotted line corresponds to the special case that no uncertainty of arrival: The parameters in this model are set: The risk-free interest is 7% per year, the mean growth rate μ and standard deviation σ of profit flow are 1% and 35%, respectively.

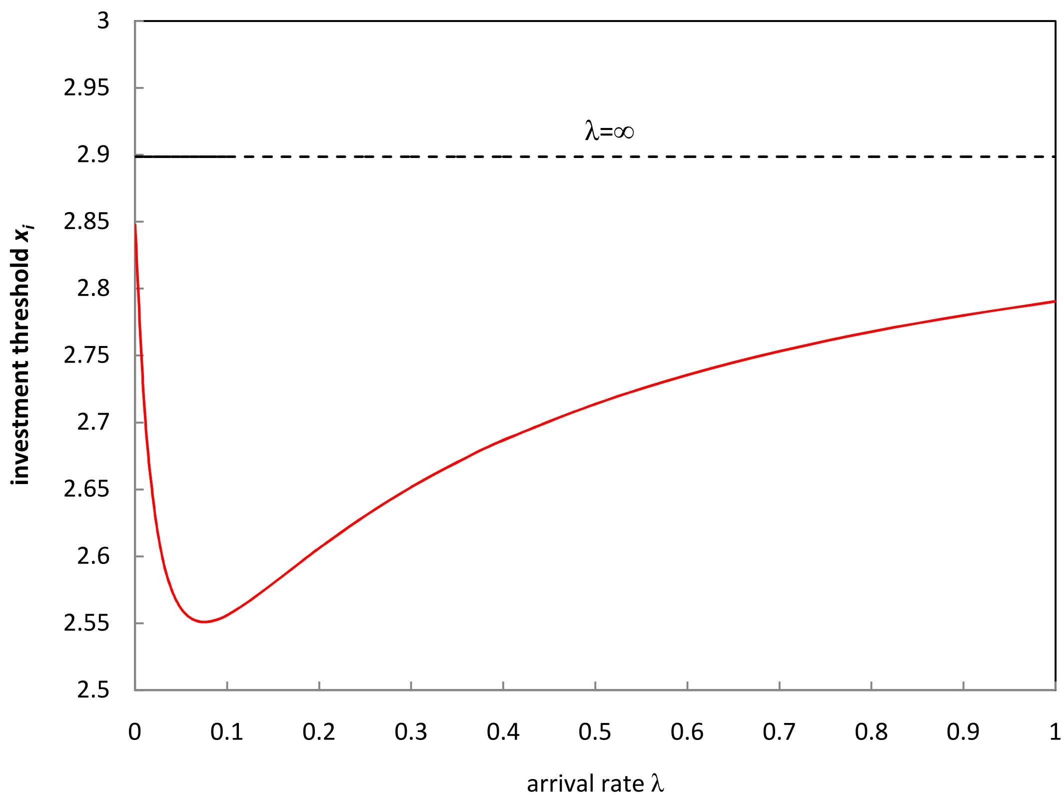


Fig 3.5 Comparative statics for investment trigger for levered firm

Figure (a)-(b) plot the initial investment trigger x_i for a levered firm against arrival rate of growth opportunity, panel (a) and (b) for impact of modernization property of size δ and cost I_2 panel (c) is for risk free rate r panel (d) is for idiosyncratic volatility σ . In all panels black line represents base case. The parameters in this model are set: The risk-free interest is 7% per year, the mean growth rate μ and standard deviation σ of profit flow are 1% and 35%, the cost for initial investment is I_1=15 and I_2=20 respectively.

(a) (b)

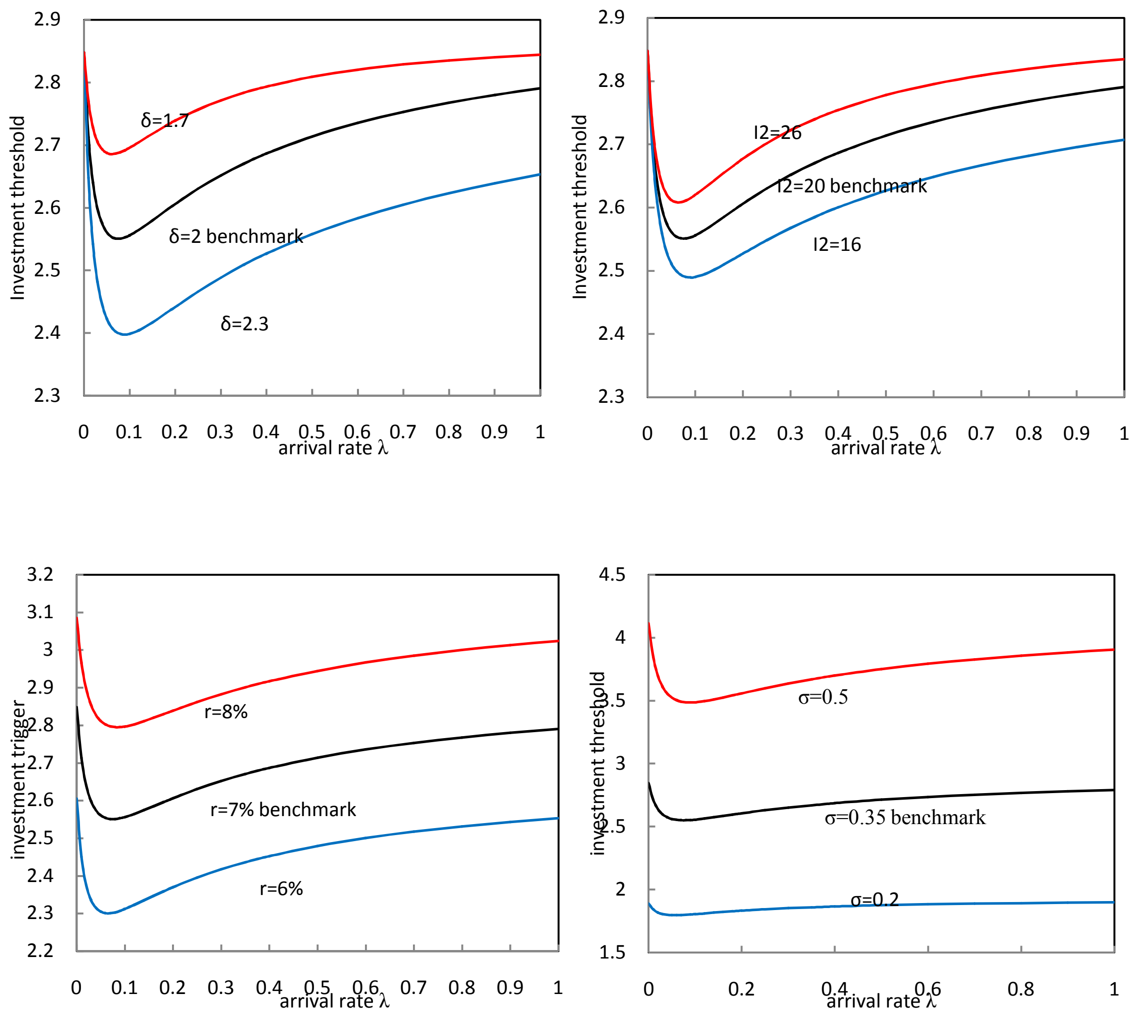


Fig 3.6 Comparative statics for optimal leverage for levered firm

Figure (a)-(b) plot the optimal leverage for a levered firm against arrival rate of growth opportunity, panel (a) and (b) for impact of modernization property of size δ and cost I_2 panel (c) is for risk free rate r panel (d) is for idiosyncratic volatility σ . In all panels black line represents base case. The parameters in this model are set: The risk-free interest is 7% per year, the mean growth rate μ and standard deviation σ of profit flow are 1% and 35%, the cost for initial investment is I_1=15 and I_2=20 respectively.

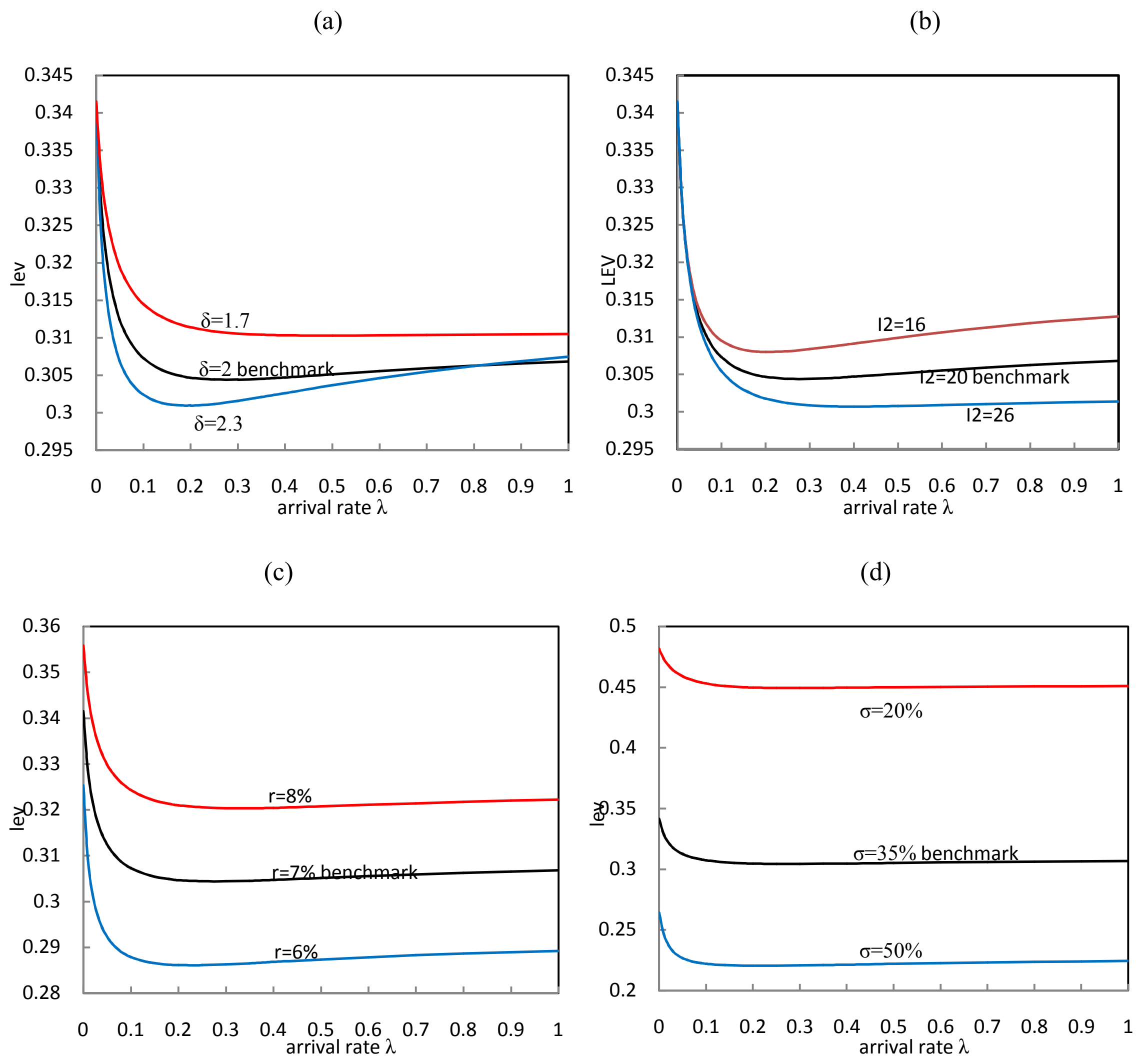


Fig 3.7

This figure plots the optimal financial leverage for a levered firm against growth size δ, given certain arrival rate λ: The parameters in this model are set: the risk-free interest is 7% per year, the mean growth rate μ and standard deviation σ of profit flow are 1% and 35%, respectively.

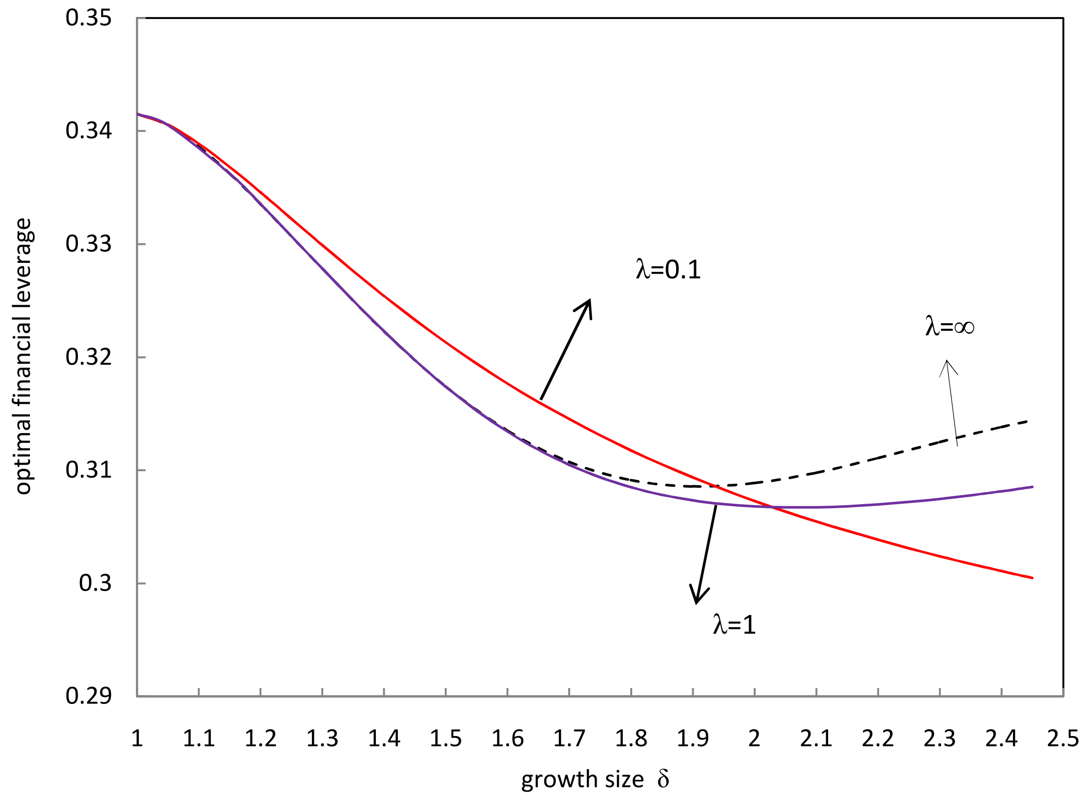
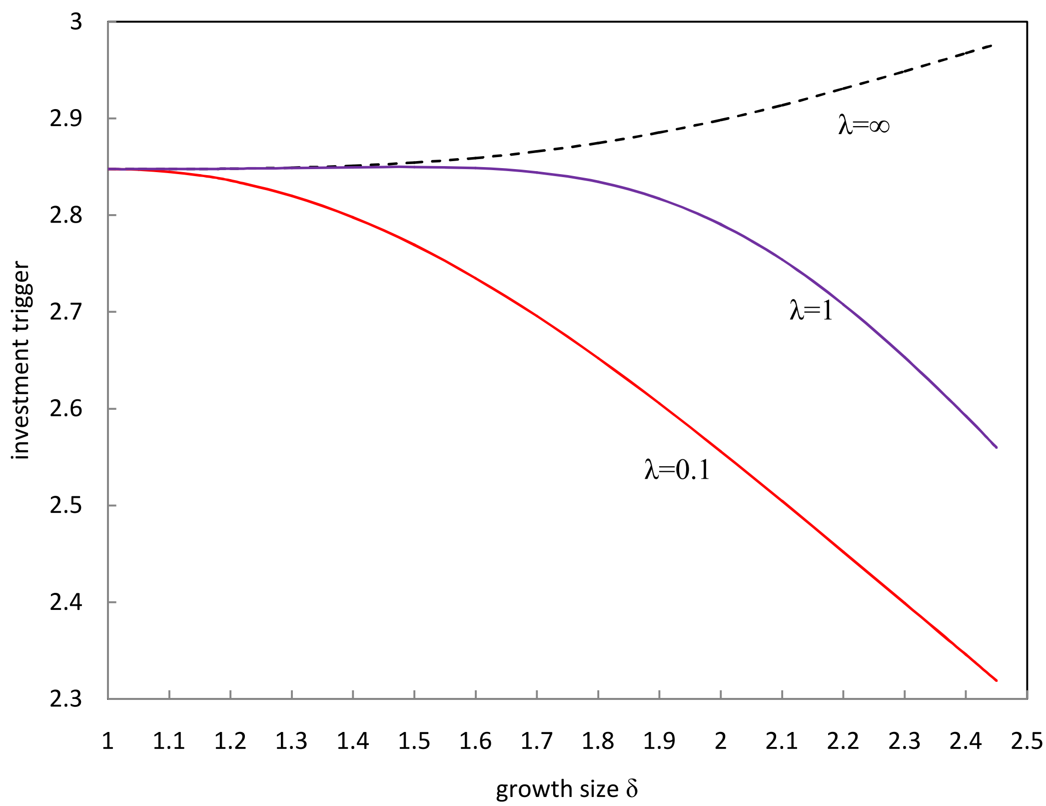


Fig 3.8

This figure plots the investment threshold for an optimally levered firm against growth size δ, given certain arrival rate λ. The parameters in this model are set: the risk-free interest is 7% per year, the mean growth rate μ and standard deviation σ of profit flow are 1% and 35%, respectively.



Chapter 4

The "Overinvestment" Agency Problem and Performance-Sensitive Debt

4.1. Introduction

The "overinvestment" agency problem, discussed by Mauer and Sarkar (2005), is essentially that of equity-value-maximizing shareholders investing too early relative to the firm-value-maximizing strategy.[11] In corporate finance parlance, the second-best (equity-value-maximizing) investment trigger is lower than the first-best (firm-value-maximizing) trigger. The resulting sub-optimal investment policy leads to a loss in firm value, commonly viewed as the agency cost of overinvestment. This agency cost can be economically significant; in Mauer and Sarkar (2005), for instance, the agency cost is 9.4% of firm value with base-case parameter values.

Agency problems can often be mitigated by appropriate design of corporate debt; for instance, convertible debt mitigates risk-shifting (Green, 1984) and short-maturity debt or renegotiable debt mitigates underinvestment (Myers, 1977). This paper shows how *Performance-Sensitive Debt* or PSD (where the coupon payment varies with firm performance)[12] can be used to mitigate the overinvestment agency problem. Using a contingent-claim model similar to Mauer and Sarkar (2005), we show that it is possible to eliminate entirely the overinvestment problem, by using PSD with the correct degree of risk-compensation. However, this PSD is not necessarily optimal, in the sense that it does not maximize shareholder wealth. The wealth-maximizing PSD

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[11] Overinvestment is equivalent to the well-known risk-shifting or asset substitution agency problem (Jensen and Meckling, 1976), as explained by Hirth and Uhrig-Homburg (2010) and illustrated by Mauer and Sarkar (2005, Section 3.2).

[12] PSD is an innovation in the corporate debt market that is very popular today, particularly in bank loans and Telecom corporate bonds (Asquith, et al., 2005, Koziol and Lawrenz, 2010, Manso, et al., 2010, Mjos, et al., 2012, Myklebust, 2012). For example, Asquith et al. (2005) observe that about 54 % of bank loan contracts by dollar volume have performance pricing provisions. Manso et al. (2008) document that 40% of the loans have performance pricing provisions. In this type of debt, the coupon payment varies with the firm's performance, typically increasing when firm performance deteriorates, in order to compensate debt holder for the additional default risk (Manso, et al., 2010). This risk-compensation provision is the unique feature of performance-sensitive debt.

design generally requires a lower level of risk-compensation than that required for eliminating overinvestment.

With traditional fixed-coupon debt, overinvestment arises from a conflict of interest between shareholders and creditors regarding investment timing. Because shareholders have limited liability, they can transfer to creditors the risk of premature investment while preserving for themselves the upside potential of the project; thus they tend to invest too soon (Mauer and Sarkar, 2005). With risk-compensating PSD, both first-best and second-best investment triggers will rise with the level of risk-compensation, but the latter will rise faster. Hence the degree of overinvestment shrinks as the degree of risk-compensation is increased, and at some point the first-best and second-best investment triggers are identical, eliminating the overinvestment problem. Thus, PSD mitigates the overinvestment agency problem, and can even eliminate it if properly designed.

Further, as the level of PSD risk-compensation is increased, the ex-ante equity value initially rises and subsequently falls, indicating there is an optimal level of risk-compensation. The optimal PSD is found to be less risk-compensating than the agency-problem-eliminating PSD of the previous paragraph. This implies that it is optimal for shareholders to incur some agency cost of overinvestment. Comparative static analysis indicates that the optimal risk-compensation is increasing in earnings growth rate and corporate tax rate, and decreasing in interest rate and bankruptcy cost. Our results help identify conditions under which PSD offers significant improvement in shareholder wealth, relative to traditional fixed-coupon debt. If the PSD is chosen optimally, it can increase shareholder wealth significantly relative to fixed-coupon debt, amounting to over 5% in the base case. Thus, there can be an economically significant benefit to shareholders from using PSD instead of fixed-coupon debt.

The main contributions of this paper are as follows: (i) it shows how to design performance-sensitive debt to eliminate the overinvestment agency problem or to maximize shareholder wealth; (ii) it provides an additional rationale for the existence of performance-sensitive debt,[13] and (iii) it identifies conditions under which the benefits of PSD are significantly larger than with fixed-coupon debt.

The remainder of the paper is organized as follows. Section 4.2 describes the model and identifies the firm's investment and financing decisions, as well as the first-best policy. Section 4.3 presents and discusses the results of the paper, and Section 4.4 concludes.

4.2. The Model

Similar to Mauer and Sarkar (2005), we assume that the terms of the debt issue (amount, coupon, etc) are prearranged between shareholders and creditors, as in a loan commitment or a revolving line of credit. The timing of the investment is at the discretion of the shareholders, but lenders set the terms of the debt issue anticipating that shareholders will rationally choose an investment policy that maximizes equity value rather than total firm value. The main difference between Mauer and Sarkar (2005) and this paper is that we consider performance-sensitive debt rather than fixed-coupon debt, and are therefore able to design the debt contract so as to eliminate the overinvestment agency problem.

With performance-sensitive debt, the interest rate depends on the borrowing firm's performance. In stochastic real-option models of PSD (Koziol and Lawrenz, 2010, Manso, et al., 2010, Myklebust, 2012), the state variable is generally the asset value or the earnings stream; in our model, it is the earnings level (see below). In such models, firm performance can be captured

[13] Some papers examine the rationale for issuing performance-sensitive debt. Manso, et al. (2010) show that issuing PSD can be explained by asymmetric information and signaling, and Koziol and Lawrenz (2010) show it can mitigate the asset substitution agency problem, in contrast to Bhanot and Mello (2006).

by the state variable; thus, in our model, the firm's performance is indicated by the level of earnings. In risk-compensating PSD, when earnings fall, the coupon payment must rise.[14] The increase in coupon usually occurs in discrete steps (Manso, et al., 2010), but for model tractability, it is common to assume that the coupon is a linear continuous function of the performance measure (Manso, et al., 2010, Myklebust, 2012, etc). We use the same assumption of linear PSD, i.e., $c(x) = c_0 - c_1x$, where x = earnings level, and a larger c_1 implies greater risk-compensation or performance-sensitivity.

4.2.1. The Investment Project

A firm has an option to invest in a production facility at any time by paying a fixed investment cost \$I. Once the investment is made, the project generates a cash flow stream of x_t per unit time, which follows the usual lognormal diffusion process (Goldstein, et al., 2001, Sundaresan and Wang, 2007):

$$dx/x = \mu dt + \sigma dZ \tag{1}$$

where μ is the expected growth rate and σ the volatility of the earnings process, and dZ is the increment of a standard Wiener process.[15] Future cash flows are discounted at a constant discount rate of r (we assume $r > \mu$ to preclude infinite values). The corporate tax rate τ is constant.

The investment is financed with a combination of equity and performance-sensitive debt. As in Kim and Maksimovic (1990), Mauer and Sarkar (2005), Sarkar (2011), etc., we assume that, prior exercising the investment option, the firm negotiates a contract under which creditors will lend the firm \$K in return for a continuous coupon payment of $c(x) = \{c_0 - c_1x\}$ in perpetuity or until the firm defaults and declares bankruptcy.

[14] The vast majority of PSD is the risk-compensating type (Manso, et al., 2010).
[15] Here our model differs slightly from Mauer and Sarkar (2005), where (i) the state variable is output price, which follows a diffusion process; and (ii) there is a constant (per-unit) operating cost. Hence the cash flow stream in Mauer and Sarkar (2005) is not exactly a lognormal process. However, this is a minor difference and will make no qualitative difference in the results. The main reason for using a lognormal cash flow process is tractability (as in Goldstein, et al., 2001, Sundaresan and Wang, 2007, etc).

This kind of arrangement is similar to a loan commitment or revolving line of credit from a bank, as discussed by Mauer and Sarkar (2005). Loan commitments allow the firm to borrow in future at terms specified in advance, and are becoming increasingly important in financing new investments. To quote Mauer and Sarkar (2005, p. 1408): "In recent years, this type of commitment-based lending has eclipsed other forms of corporate financing." Bradley and Roberts (2003) show that since 1994, the dollar amount of private corporate debt – approximately 80% of which are loan commitments – has been two or three times larger than the amount of public debt issues. Clearly, this type of corporate borrowing is an important component of corporate finance, and particularly relevant for private and bank debt. Since performance-sensitive provisions are more common for bank and private debt than public debt, this type of pre-arranged financing is also particularly relevant when studying PSD.

When $x < c(x)$, the firm is not making enough money to cover interest payments, and shareholders must make out-of-pocket payments to keep the company alive. But if x falls far enough (say, to x_b), then shareholders will not find it worth keeping alive, and the company will declare bankruptcy. Thus, x_b is the bankruptcy trigger. At bankruptcy, the debt holders will take over the assets of the company after incurring fractional bankruptcy cost of α ($1 \geq \alpha \geq 0$), and shareholders will exit with zero payoff. Also, since the firm becomes unlevered at bankruptcy, all tax shields will be lost (as in Leland, 1994, Mauer and Sarkar, 2005).

The agency problem of overinvestment arises from the fact that the timing of investment is chosen by shareholders, and cannot be contracted in advance. We assume that lenders have rational expectations and fully anticipate that shareholders may choose an exercise policy that harms the value of their fixed coupon claim. Thus, creditors will require that their commitment of $K at investment be fair relative to the promised coupon and investment exercise strategy adopted by shareholders.

4.2.2. Valuation of Equity and Debt

With performance-sensitive debt, the coupon obligation is given by:

$$c(x) = c_0 - c_1 x \tag{2}$$

where c_0, $c_1 > 0$. When $c_1 = 0$, the PSD becomes a traditional fixed-coupon bond; a higher c_1 implies greater risk-compensation. Debt holders then receive a cash flow stream of $(c_0 – c_1 x)$ per unit time, and shareholders receive a flow of $\{(1–\tau)(x–c_0+c_1 x)\}$ per unit time. It can be shown that debt and equity values are given by:[16]

$$D(x) = c_0 / r - c_1 x / (r - \mu) + Z_1 x^{\gamma_2} \tag{3}$$

$$E(x) = (1 - \tau)\left[x(1 + c_1)/(r - \mu) - c_0 / r\right] + Z_2 x^{\gamma_2} \tag{4}$$

where Z_1 and Z_2 are constants to be determined by the boundary conditions, and γ_1 and γ_2 are the positive and negative roots, respectively, of the quadratic equation:

$$0.5\sigma^2\gamma(\gamma - 1) + \mu\gamma - r = 0 \tag{5}$$

and are given by:

$$\gamma_1 = 0.5 - \mu / \sigma^2 + \sqrt{2r / \sigma^2 + \left(0.5 - \mu / \sigma^2\right)^2} \tag{6}$$

$$\gamma_2 = 0.5 - \mu / \sigma^2 - \sqrt{2r / \sigma^2 + \left(0.5 - \mu / \sigma^2\right)^2} \tag{7}$$

When the firm declares bankruptcy (at $x = x_b$), shareholders receive zero payoff. Therefore, the value-matching and smooth-pasting boundary conditions (see Leland, 1994) are as follows:

Value-matching: $$E(x_b) = 0 \tag{8}$$

Smooth-pasting: $$E'(x_b) = 0 \tag{9}$$

[16] All derivations are available from authors on request.

Also, at bankruptcy the payoff to debt holders is $(1-\alpha)(1-\tau)x_b/(r-\mu)$, giving the boundary condition:

$$D(x_b) = (1-\alpha)(1-\tau)x_b/(r-\mu) \tag{10}$$

Solving the three boundary conditions (8), (9) and (10), we get the three unknowns:

$$x_b = \frac{c_0(1-\mu/r)}{(1+c_1)(1-1/\gamma_2)} \tag{11}$$

$$Z_1 = (c_0/r)\left[\frac{c_1 + (1-\alpha)(1-\tau)}{(1+c_1)(1-1/\gamma_2)} - 1\right](x_b)^{-\gamma_2} \tag{12}$$

$$Z_2 = -(1-\tau)(1+c_1)(x_b)^{1-\gamma_2}/[\gamma_2(r-\mu)] \tag{13}$$

The market leverage ratio is then given by $D(x)/[D(x)+E(x)]$.

4.2.3. The Financing Decision

For a given risk-compensation level c_1, the company chooses the debt level c_0 optimally, i.e., so as to maximize the total firm value $V(x) = \{E(x) + D(x)\}$, as in Leland (1994), Sundaresan and Wang (2007), etc. Differentiating $V(x)$ with respect to c_0 and setting the derivative $dV(x)/dc_0 = 0$, we get the optimal debt level, after some simplification:

$$c_0^*(x) = \frac{x(1+c_1)(1-1/\gamma_2)}{(1-\mu/r)}\left[1 - \frac{\gamma_2[1-(1-\alpha)(1-\tau)]}{\tau(1+c_1)}\right]^{1/\gamma_2} \tag{14}$$

This gives the optimal capital structure of the firm, for a given level of risk-compensation. Because lenders are rational, the loan amount \$K will be equal to the value of the debt when it is issued. Since the investment will be made at the second-best investment trigger ($x = x^{SB}$), this means K = $D(x^{SB})$, where D(.) is given by equation (3) and the coupon is $c_0^*(x^{SB})$ from equation (14).

4.2.4. The Investment Decision

As in Mauer and Sarkar (2005), the investment decision is equivalent to exercising the (American) option to invest in the project. The value of this option is the same as pre-investment firm value (or pre-investment equity value, since there is no debt prior to investment). Suppose the option value is F(x). Then it is easily shown that:

$$F(x) = \Omega_{SB} x^{\gamma_1}, \tag{15}$$

where Ω_{SB} is a constant to be determined by boundary conditions.

Thus, the ex-ante or pre-investment shareholder wealth is given by F(x), as in Sundaresan and Wang (2007). When the investment option is exercised, shareholders pay the investment cost (less the amount raised from lenders) and receive the equity value of the firm. Since the shareholders control the investment decision, they will choose the investment trigger to maximize equity value (i.e., the second-best investment trigger); let this trigger be x^{SB}. Then there are two boundary conditions:

Value-matching: $$F(x^{SB}) = \Omega_{SB}\left(x^{SB}\right)^{\gamma_1} = E(x^{SB}) - (I - K) \tag{16}$$

Smooth-pasting: $$F'(x^{SB}) = \Omega_{SB}\gamma_1\left(x^{SB}\right)^{\gamma_1 - 1} = E'(x^{SB}) \tag{17}$$

As mentioned above, rational creditors will not agree to lend \$K unless it is a fair price for the debt. Creditors cannot force the company to choose a particular investment policy; hence, they will value the debt (and thereby determine K) under the assumption that the investment policy is one that maximizes equity value rather than firm value. Therefore, K in equation (16) must equal debt value at the second-best investment trigger, or

$$K = D(x^{SB}) = \frac{c_0}{r} - \frac{c_1 x^{SB}}{r-\mu} + Z_1\left(x^{SB}\right)^{\gamma_2} \qquad (18)$$

From the boundary conditions, we get:

$$\Omega_{SB} = \frac{E(x^{SB}) + D(x^{SB}) - I}{\left(x^{SB}\right)^{\gamma_1}} \qquad (19)$$

and an equation that can be solved for x^{SB}:

$$[Z_1 + Z_2(1-\gamma_2/\gamma_1)](x^{SB})^{\gamma_2} + x^{SB}[(1-\tau)(1-1/\gamma_1)(1+c_1) - c_1]/(r-\mu) + \tau c_0/r - I = 0$$

(20)

In equation (20), c_0 is chosen optimally, i.e., $c_0 = c_0(x^{SB})$ from equation (14). Since equation (20) has no analytical solution, it has to be solved numerically for x^{SB}. The second-best investment trigger x^{SB} describes the investment policy that will be actually followed by the company.

4.2.5. The First-best (Firm-value-maximizing) Investment Policy/Trigger

As a benchmark, we also identify the first-best investment trigger x^{FB} and the resulting firm value Ω_{FB}, as in Mauer and Sarkar (2005). Here, the total firm value is maximized rather than equity value, giving the following boundary conditions:

Value-matching: $$F(x^{FB}) = \Omega_{FB}\left(x^{FB}\right)^{\gamma_1} = D(x^{FB}) + E(x^{FB}) - I \qquad (21)$$

Smooth-pasting: $$F'(x^{FB}) = \Omega_{FB}\gamma_1\left(x^{FB}\right)^{\gamma_1 - 1} = D'(x^{FB}) + E'(x^{FB}) \qquad (22)$$

The boundary conditions (21) and (22) give:

$$\Omega_{FB} = \frac{E(x^{FB}) + D(x^{FB}) - I}{\left(x^{FB}\right)^{\gamma_1}} \qquad (23)$$

as well as an equation that can be solved for x^{FB}:

$$x^{FB}(1-\tau-\tau c_1)(1-1/\gamma_1)/(r-\mu) + (Z_1 + Z_2)(1-\gamma_2/\gamma_1)(x^{FB})^{\gamma_2} + \tau c_0/r - I = 0$$

(24)

where $c_0 = c_0(x^{FB})$ from equation (14). Since equation (24) has no analytical solution, it has to be solved numerically for x^{FB}. We now state our first result.

Result 1. *Shareholders' optimal investment and financing decisions are given by equations (20) and (14) respectively, while the first-best investment and financing decisions are given by equations (24) and (14) respectively.*

4.3. Model Results

4.3.1. Base-case Parameter Values

Since equations (20) and (24) have no analytical solutions, we illustrate the results of the model numerically. We use the same "base-case" input parameter values as Mauer and Sarkar (2005). Thus, the discount rate (or interest rate) is $r = 5\%$, earnings growth rate $\mu = 3\%$,[17] earnings volatility is $\sigma = 25\%$, tax rate $\tau = 30\%$, bankruptcy cost $\alpha = 35\%$, and investment cost $I = 20$.[18] We also repeat the computations with a wide range of parameter values to ensure robustness of the results.

4.3.2. First-Best versus Second-Best Investment

Traditional Fixed-Coupon Debt ($c_1 = 0$)

If we set the risk-compensation parameter $c_1 = 0$, PSD becomes traditional fixed-coupon debt. With the above base-case parameter values and $c_1 = 0$, the firm's (second-best) decision is as

[17] Mauer and Sarkar (2005) use $r = 5\%$ and convenience yield $\delta = 2\%$. Since the convenience yield is given by $\delta = r-\mu$, the implied growth rate is $\mu = 3\%$.

[18] Mauer and Sarkar (2005) use $I = 5$ and operating cost of \$0.75 per unit time. Since our model has no operating cost, we capitalized the operating cost of Mauer and Sarkar (2005) to $0.75/0.05 = 15$, and added this to their investment cost of 5, to give $I = 20$ in our model.

follows: investment trigger $x^{SB} = 1.1906$, debt level $c_0 = 2.0766$, with resulting firm value $\Omega_{SB} = 22.8407$.

For comparison, the ideal (first-best) solution is as follows: $x^{FB} = 2.2096$, debt level $c_0 = 3.8538$, with resulting firm value $\Omega_{FB} = 25.3291$. Since $x^{SB} < x^{FB}$, following the second-best investment policy results in earlier investment (or overinvestment) relative to the firm-value-maximizing policy. Rational lenders will anticipate that the manager, acting on behalf of shareholders, will choose the investment policy that maximizes equity value, at the expense of total firm value. They will therefore incorporate this behavior in the pricing of the debt (i.e., reduce the proceeds from the debt issue). As a result, shareholders will end up bearing the resulting "agency cost." As in Mauer and Sarkar (2005), the agency cost is the reduction in firm value resulting from the second-best investment policy, which in this case is $(\Omega_{FB}/\Omega_{SB} - 1) = 10.9\%$. Thus, with traditional fixed-coupon debt, the agency cost of overinvestment is 10.9% with the base-case parameter values, similar to Mauer and Sarkar (2005).

Performance-sensitive Debt ($c_1 > 0$)

Next we repeat the computations for performance-sensitive debt, with increasing levels of risk-compensation (c_1). By varying c_1, we are able to examine the effect of risk-compensation on the overinvestment problem.

With $c_1 = 0.2$, we get the following output: $x^{SB} = 1.6891$, $c_0 = 3.9016$, $\Omega_{SB} = 23.9805$; and $x^{FB} = 2.2718$, $c_0 = 5.2476$, $\Omega_{FB} = 24.4413$, giving an agency cost of 1.92%. We note that both first-best and second-best triggers are higher because of the risk-compensation feature, but the second-best trigger has increased by a larger margin, hence the gap between the two triggers has narrowed. Not surprisingly, the agency cost of overinvestment is substantially smaller than with fixed-coupon debt.

Figure 4.1 about here

Figure 4.1 shows the first-best and second-best investment triggers as functions of the risk-compensation level c_1. It can be noted that both investment triggers are increasing functions of c_1. However, while the first-best trigger rises very slowly, the second-best trigger rises rapidly, as c_1 is increased. In fact, for large enough risk-compensation, the problem of overinvestment is even reversed. For instance, with $c_1 = 0.4$, the results are as follows: $x^{SB} = 2.7376$, $c_0 = 7.9815$, $\Omega_{SB} = 23.5608$; and $x^{FB} = 2.3296$, $c_0 = 6.7920$, $\Omega_{FB} = 23.6647$, and agency cost = 0.44%. In this case, because of the large risk-compensation, the second-best investment trigger exceeds the first-best trigger, and the overinvestment problem has been turned into an "underinvestment" problem.

The behavior of the two investment triggers can be explained as follows. With $c_1 = 0$ (fixed-coupon debt), we have $x^{SB} < x^{FB}$, as explained in Mauer and Sarkar (2005). As c_1 is increased, both triggers will rise because risk-compensation causes earlier bankruptcy (Manso, et al., 2010, Koziol and Lawrenz, 2010); since bankruptcy is costly, this increases the expected deadweight bankruptcy cost. This leads to delayed investment in both cases (first-best and second-best), hence both x^{SB} and x^{FB} rise with c_1. However, there is an additional factor that affects only x^{SB}: with higher c_1, shareholders have to make larger payments to debt holders (as compensation for risk) when x falls; but this is exactly when they can least afford it. This makes them even less willing to invest. Therefore, as c_1 is increased, x^{SB} rises faster than x^{FB}. At some point (say c_1^{FS}) the two triggers will be identical, when overinvestment is nullified by the risk-compensating PSD. We state our second result.

Result 2. *A firm that maximizes equity value will overinvest with traditional fixed-coupon debt. With performance-sensitive debt, the degree of overinvestment will decline with level of risk-compensation; for large enough risk-compensation, the firm will underinvest.*

Risk-Compensation Level that Eliminates Agency Cost (c_1^{FS})

In Figure 4.1, it can be noted that the two curves do intersect. At the point of intersection, there will be neither overinvestment nor underinvestment. Therefore, it is possible to choose the degree of risk-compensation so that the equity-maximizing strategy is identical to the firm-value-maximizing strategy, and thus eliminate the agency problem of overinvestment. In Figure 1, this overinvestment-eliminating c_1 is seen to be between 0.3 and 0.4.

To compute the agency-cost-eliminating level of risk-compensation (say c_1^{FS}), we can set $x^{FB} = x^{SB} = x^{inv}$ (say) in equations (20) and (24), and solve them simultaneously for c_1 and x^{inv}. Using this procedure, we get $c_1^{FS} = 0.3389$, with the other outputs being: debt level $c_0 = 6.3053$, firm value $\Omega = 23.8914$, and $x^{inv} = x^{FB} = x^{SB} = 2.3124$ (and of course, zero agency cost). This leads to our next result.

Result 3. *It is possible to ensure a first-best or firm-value-maximizing investment policy by setting the degree of risk-compensation (c_1) such that equations (20) and (24) are satisfied when $x^{FB} = x^{SB}$.*

Comparative Static Analysis of c_1^{FS}

Table 4.1 shows how c_1^{FS} varies with the different parameters. We note that it is insensitive to all the parameters except the tax rate τ and bankruptcy cost α. As τ is increased, c_1^{FS} also rises significantly. This is because a higher tax rate makes shareholders more willing to invest early, in order to take advantage of the larger tax shield. Hence there is a larger gap between x^{FB} and x^{SB} (i.e., greater degree of overinvestment) when the tax rate is higher, because of which a higher level of risk-compensation is required to bring the two triggers together. On the other hand, c_1^{FS} is a significantly decreasing function of α. As α is increased, shareholders are less willing to make the investment, hence the degree of overinvestment declines. Therefore, a smaller level of risk-compensation is required to eliminate overinvestment.

Table 4.1 about here

4.3.3. Optimal PSD Design

We have shown above that setting $c_1 = c_1^{FS}$ will eliminate the overinvestment agency problem. However, this is not optimal for shareholders if it does not maximize shareholder wealth. Since the shareholders' objective is to maximize their own wealth, the optimal PSD design should maximize the ex-ante, pre-investment equity/firm value, which is given by equation (15): $F(x) = \Omega_{SB} x^{\gamma_1}$. But maximizing $\Omega_{SB} x^{\gamma_1}$ for all x is equivalent to maximizing Ω_{SB}. We therefore call Ω_{SB} the (normalized) pre-investment equity value. The optimal risk-compensation level, then, will be the one that maximizes Ω_{SB}, anticipating the firm's investment and financing decisions, i.e.

$$c_1^* = \operatorname*{Arg\,max}_{c_1} \; \Omega_{SB}(c_1) \tag{25}$$

There being no analytical solution to equation (25), it is solved numerically.

Figure 4.2 shows Ω_{SB} and Ω_{FB} as functions of c_1. The equity value under the first-best investment policy, Ω^{FB}, is a decreasing function of c_1. This is not surprising, since the only effect of increasing c_1 on total firm value is negative (higher expected bankruptcy cost). However, for the second-best investment policy, as c_1 is increased, Ω_{SB} first rises and then falls.

Figure 4.2 about here

When c_1 is increased, the investment trigger will increase, as discussed in Section 4.3.2. At the higher earnings level, the firm will take on more debt (higher c_0); this increases the tax benefit to shareholders, hence a higher c_1 has a positive effect on shareholder wealth. However, a higher c_1 also increases the probability of bankruptcy and thereby increases expected bankruptcy cost; thus, a higher c_1 also has a negative effect on shareholder wealth. Initially (for low c_1) the additional tax benefit dominates the additional expected bankruptcy cost resulting from higher c_1, hence the net effect is that Ω_{SB} increases with c_1. For large c_1, however, the bankruptcy cost effect dominates, and Ω_{SB} decreases with c_1. Thus, Ω_{SB} has an inverted-U shaped relationship with c_1, implying a unique optimal c_1 that maximizes Ω_{SB}. In Figure 4.2, the optimal c_1 is seen to be between 0.2 and 0.3.

Numerical Results

Solving equation (25) numerically with the base-case parameter values, we get an optimal risk-compensation level of $c_1^* = 0.2559$. The other outputs from the model are: $c_0 = 4.7006$, leverage ratio = 62.41%, $\Omega_{SB} = 24.0435$, $\Omega_{FB} = 24.2140$, $x^{FB} = 2.2884$, $x^{SB} = 1.8988$, and agency cost = 0.71%.

Comparing c_1^* with c_1^{FS} (which was 0.3389 for the base case), we see that the optimal risk-compensation level is significantly smaller than that which ensures a first-best investment policy. The implication is that it is optimal for shareholders to overinvest (and of course to incur the resulting agency cost). Even if the resulting agency cost is not very large (0.71%), the degree of overinvestment is quite large, with x^{FB} being about 20% higher than x^{SB}.

Further, the inequality $c_1^* < c_1^{FS}$ was found to be valid for all parameter value combinations examined (see the section on comparative statics below). Despite a large number of computations, we could not find a single instance when this inequality was violated. We therefore conclude that it is generally optimal to invest early relative to the firm-value-maximizing or first-best investment policy. In other words, it is generally optimal to overinvest relative to first-best, and to incur the resulting agency cost. Shareholders are willing to incur this agency cost because the tax benefit from the earlier investment (net of bankruptcy cost) is large enough to make it worthwhile. Increasing the risk-compensation c_1 might reduce the agency cost of overinvestment, but it would also reduce the net benefit (tax shield minus bankruptcy cost) because bankruptcy cost would rise with c_1, and this would leave the shareholders worse off. Therefore, when designing performance-sensitive debt, the objective should be to maximize Ω_{SB}, not minimize or eliminate agency cost (Bhanot and Mello, 2006, Koziol and Lawrenz, 2010).

Next, comparing $\Omega_{SB}(c_1^*)$ with $\Omega_{SB}(0)$, i.e., with the traditional fixed-coupon debt, we find that $\Omega_{SB}(c_1^*)$ is 5.27% higher than $\Omega_{SB}(0)$. Clearly, an economically significant value is added to shareholder wealth by using performance-sensitive debt. This provides a substantive motivation for companies to use performance-sensitive debt rather than fixed-coupon debt. As we will see in the comparative static section below, this increase in wealth varies substantially across parameter values. We can identify situations where performance-sensitive debt is particularly attractive vis-à-vis fixed-coupon debt.

Finally, comparing $\Omega_{SB}(c_1^*)$ with $\Omega_{SB}(c_1^{FS})$, we find that the difference is not large, with the former exceeding the latter by only 0.64%. Thus, the difference in equity value between optimal and first-best risk-compensation levels is not large. Nevertheless, shareholders are better off with the former.

The above computations were repeated for a wide range of parameter values, and the results discussed in the next section.

Our results indicate that a properly-designed PSD can increase shareholder wealth significantly. Thus, there is a strong case for using PSD instead of fixed-coupon debt when companies use loan-commitment-type borrowing (which is becoming increasingly popular). In the comparative statics section, we examine the conditions under which the value added by PSD (relative to fixed-coupon debt) is more likely to be economically significant. We now state our next result.

Result 4. *(a) Optimally-designed PSD increases shareholder wealth (and this wealth increase can be economically significant) when firms use loan-commitment-type borrowing; (b) The optimal PSD design results in some overinvestment relative to the first-best investment strategy.*

It is also worth noting that shareholders would be better off, ex-ante, if the first-best investment policy could always be followed (as is clear from Figure 4.2). However, ex-post (i.e., after debt arrangements have been made), it is always optimal for shareholders to follow the second-best investment policy; hence there is a time-inconsistency problem that makes it impossible for the first-best policy to be followed when shareholders make the investment decision. The actual investment will therefore be the second-best one. However, as shown above, using PSD instead of the traditional fixed-coupon debt can mitigate this problem significantly; with the base-case parameter values, the optimally designed PSD will increase shareholder wealth by 5.27%.

Comparative Static Analysis of c_1^*

Table 4.2 shows c_1^* for a range of parameter values, along with c_1^{FS} for comparison. The last column shows $\%\Delta\Omega$, which is the percentage increase in shareholder wealth resulting from use of performance-sensitive debt instead of traditional fixed-coupon debt, or $[\Omega_{SB}(c_1^*)/\Omega_{SB}(0) - 1]$.

Note that in every single case, we have $c_1^* < c_1^{FS}$. This indicates that the performance-sensitive debt must be less risk-compensating than required to ensure first-best investment. It also implies that the optimal PSD design will result in some overinvestment.

First, c_1^* is found to be a decreasing function of interest rate r. This is because the tax benefit (present value of future stream of tax shields) declines when r is higher. Therefore, in the trade-off between tax benefit and bankruptcy cost, the optimal c_1 is reached earlier, and c_1^* is smaller as a result.

Table 4.2 about here

Next, c_1^* is an increasing function of earnings growth rate μ. With higher earnings growth, the possibility of bankruptcy becomes more remote hence expected bankruptcy cost declines. In the trade-off between tax benefits and bankruptcy cost, the optimal c_1 is reached later, resulting in a larger c_1^*.

Also, c_1^* is found to be an increasing function of tax rate τ. With a higher tax rate, the tax benefit is higher, hence the effect will be just the opposite of a higher interest rate, resulting in a higher c_1^*. Finally, c_1^* is a decreasing function of bankruptcy cost α. With higher bankruptcy cost, the optimal c_1 is reached earlier, resulting in a lower c_1^*. The other parameters (earnings volatility, investment cost) do not have a material effect on c_1^*.

To summarize, c_1^* is large (more risk-compensating PSD) when μ and τ are large, and small when r and α are large. Since PSD with small c_1^* resembles fixed-coupon debt, this means PSD is more attractive for high earnings growth and high tax rate, and low interest rate and low bankruptcy cost.

We also note a significant dispersion in the values of $\%\Delta\Omega$. When $\%\Delta\Omega$ is small, not much value is added by using performance-sensitive debt, but when it is large PSD becomes much more attractive. From Table 4.2, PSD is more attractive when growth rate and tax rate are higher, and when interest rate and bankruptcy cost are low. Volatility has virtually no effect on $\%\Delta\Omega$.

4.4. Conclusion

With traditional fixed-coupon corporate debt, it has been shown (Mauer and Sarkar, 2005) that shareholders have an incentive to invest too early (overinvest); alternatively, the actual (equity-value-maximizing or second-best) investment trigger is below the socially optimal (firm-value-maximizing or first-best) investment trigger.

This paper shows that performance-sensitive debt can be used to mitigate this overinvestment problem by appropriately choosing the level of risk-compensation in the performance-sensitive debt. In fact, for a certain risk-compensation level, the overinvestment problem is eliminated entirely. However, this risk-compensation level does not necessarily maximize shareholder wealth.

We also identify the optimal or shareholder-wealth-maximizing level of risk-compensation. This is found, in all cases, to be smaller than that required to eliminate overinvestment. Thus, it is optimal for shareholders to bear some agency cost of early investment, because of the tax benefits resulting from early investment. We identify situations where performance-sensitive debt adds significantly to shareholder wealth (relative to fixed-coupon debt), e.g., high μ and τ, and low r and α. Therefore, firms in high-growth industries will benefit more from PSD financing than the traditional fixed-coupon debt.

While we have made the simplifying assumption of linear PSD (following Manso, et al., 2010, Myklebust, 2012), the main results of the model (that proper PSD design can eliminate overinvestment and also significantly increase shareholder wealth relative to fixed-coupon debt) should be unaltered even with a more realistic modeling of performance-sensitive debt (although it would add substantial complexity to the model).

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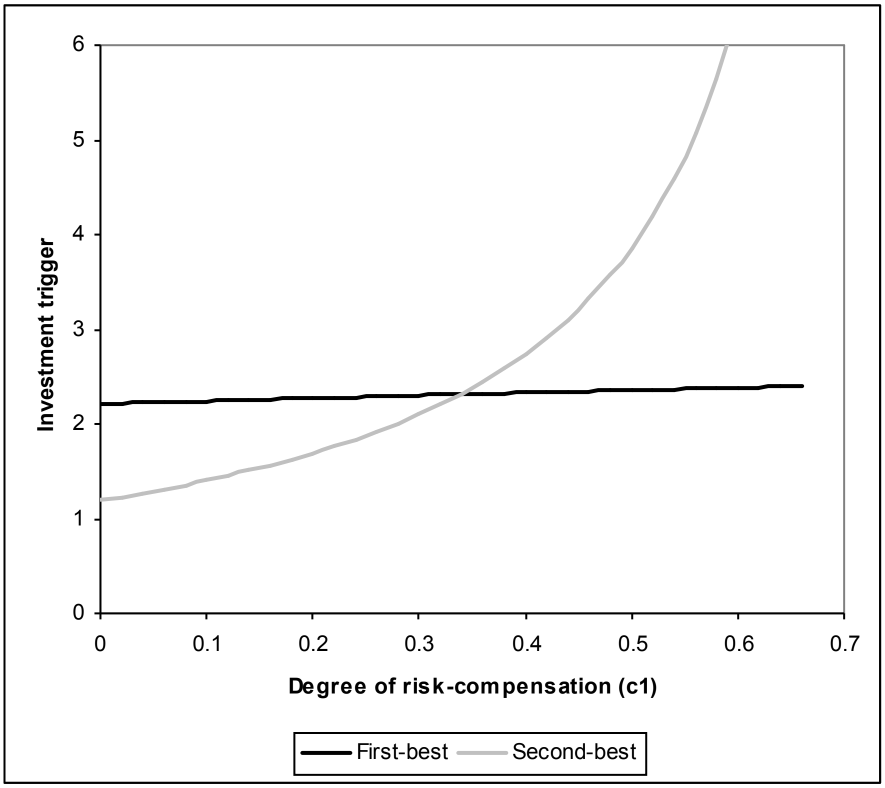

Figure 4.1. First-best and second-best investment triggers (x^{FB} and x^{SB} respectively) as functions of risk-compensation level c_1. Base-case parameter values: $r = 5\%$, $\mu = 3\%$, $\sigma = 25\%$, $\tau = 30\%$, $\alpha = 35\%$, and $I = 20$. The two curves intersect (i.e., overinvestment is eliminated) for c_1 between 0.3 and 0.4.

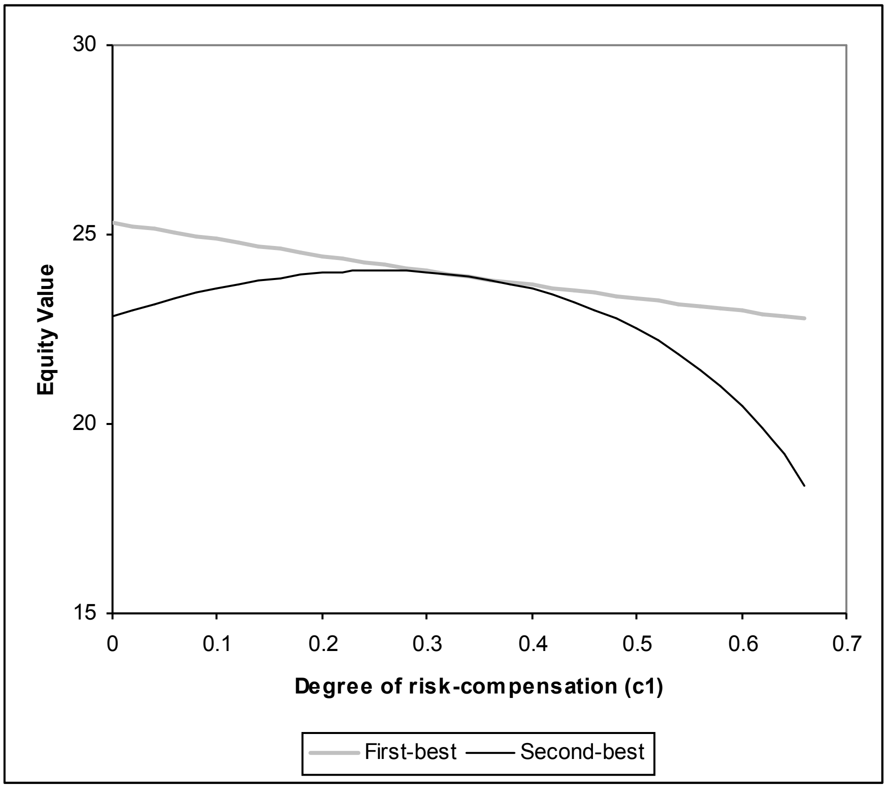

Figure 4.2. Pre-investment equity/firm values (Ω_{FB} and Ω_{SB}, for first-best and second-best, respectively) as functions of risk-compensation level c_1. Base-case parameter values: $r = 5\%$, $\mu = 3\%$, $\sigma = 25\%$, $\tau = 30\%$, $\alpha = 35\%$, and $I = 20$. Second-best equity value is maximized for c_1 between 0.2 and 0.3.

Table 4.1. Effect of Parameter Values on c_1^{FS}

Shows the effect of various parameters on the risk-compensation that equates first-best and second-best investment triggers (c_1^{FS}). Base-case parameter values: μ = 3%, r = 5%, σ = 25%, α = 35%, τ = 30%, and I = 20.

r	c_1^{FS}	c_0	$x^{FB}(x^{SB})$
3.5%	0.3393	14.0750	1.7832
4%	0.3391	8.7369	1.9636
5%	0.3389	6.3053	2.3124
6%	0.3389	5.7018	2.6487
7%	0.3389	4.5441	2.9757

μ	c_1^{FS}	c_0	$x^{FB}(x^{SB})$
1%	0.3395	3.7882	2.6473
2%	0.3391	4.5925	2.4713
3%	0.3389	6.3053	2.3124
4%	0.3389	10.8994	2.1712
4.5%	0.3389	21.7113	2.1073

σ	c_1^{FS}	c_0	$x^{FB}(x^{SB})$
20%	0.3394	5.0259	1.9204
22.5%	0.3389	5.6124	2.1092
25%	0.3389	6.3053	2.3124
27.5%	0.3392	7.1147	2.5303
30%	0.3398	8.0522	2.7632

τ	c_1^{FS}	c_0	$x^{FB}(x^{SB})$
20%	0.1838	4.1193	2.1231
25%	0.2565	5.1437	2.2131
30%	0.3389	6.3053	2.3124
35%	0.4328	7.6622	2.4236
40%	0.5405	9.2906	2.5502

α	c_1^{FS}	c_0	$x^{FB}(x^{SB})$
25%	0.4170	7.2760	2.2942
30%	0.3740	6.7476	2.3037
35%	0.3389	6.3053	2.3124
40%	0.3098	5.9286	2.3204
45%	0.2853	4.9388	2.3280

I	c_1^{FS}	c_0	$x^{FB}(x^{SB})$
10	0.3389	3.1527	1.1562
15	0.3389	4.7290	1.7343
20	0.3389	6.3053	2.3124
25	0.3389	7.8816	2.8905
30	0.3389	8.2824	3.4686

Table 4.2. Effect of Parameter Values on c_1^*

Shows the effect of various parameters on the *optimal* risk-compensation level c_1^*. Base-case parameter values: μ = 3%, r = 5%, σ = 25%, α = 35%, τ = 30%, and I = 20.

	c_1^*	c_1^{FS}	c_0	Leverage	x^{FB}	x^{SB}	Ω_{SB}	%$\Delta\Omega$
Base case:	0.2559	0.3389	4.7006	62.41%	2.2884	1.8988	24.0435	5.27%

r	c_1^*	c_1^{FS}	c_0	Leverage	x^{FB}	x^{SB}	Ω_{SB}	%$\Delta\Omega$
3.5%	0.3136	0.3393	11.1006	60.17%	1.7773	1.4496	136.7800	12.40%
4%	0.2917	0.3391	6.7448	60.99%	1.9518	1.6027	60.8745	8.60%
5%	0.2559	0.3389	4.7006	62.41%	2.2884	1.8988	24.0435	5.27%
6%	0.2276	0.3389	4.1387	63.61%	2.6125	2.1865	12.6484	3.69%
7%	0.2041	0.3389	3.9427	64.65%	2.9272	2.4669	7.4913	2.76%

μ	c_1^*	c_1^{FS}	c_0	Leverage	x^{FB}	x^{SB}	Ω_{SB}	%$\Delta\Omega$
1%	0.1439	0.3395	2.3644	61.54%	2.5774	2.1289	6.3080	1.23%
2%	0.2029	0.3391	3.1577	61.89%	2.4275	2.0080	11.8340	2.75%
3%	0.2559	0.3389	4.7006	62.41%	2.2884	1.8988	24.0435	5.27%
4%	0.3014	0.3389	9.2589	63.08%	2.1029	1.8013	62.8134	10.00%
4.5%	0.3212	0.3389	18.3438	63.47%	1.9021	1.7573	141.7652	14.66%

σ	c_1^*	c_1^{FS}	c_0	Leverage	x^{FB}	x^{SB}	Ω_{SB}	%$\Delta\Omega$
20%	0.2545	0.3394	3.8684	66.30%	1.9021	1.6190	23.7612	5.05%
22.5%	0.2545	0.3389	4.2456	64.20%	2.087	1.7537	23.8626	5.13%
25%	0.2559	0.3389	4.7006	62.41%	2.2884	1.8988	24.0435	5.27%
27.5%	0.2582	0.3392	5.2387	60.86%	2.5037	2.0545	24.2796	5.45%
30%	0.2612	0.3398	5.8699	59.53%	2.7342	2.2214	24.5529	5.65%

Table 4.2 (continued). Effect of Parameter Values on c_1^*

Shows the effect of various parameters on the *optimal* risk-compensation level c_1^*. Base-case parameter values: μ = 3%, r = 5%, σ = 25%, α = 35%, τ = 30%, and I = 20.

τ	c_1^*	c_1^{FS}	c_0	Leverage	x^{FB}	x^{SB}	Ω_{SB}	%$\Delta\Omega$
20%	0.1366	0.1838	3.3841	53.31%	2.1129	1.8598	26.7424	2.17%
25%	0.1931	0.2565	4.0316	58.28%	2.1970	1.8787	25.3909	3.59%
30%	0.2559	0.3389	4.7006	62.41%	2.2884	1.8988	24.0435	5.27%
35%	0.3256	0.4328	5.4107	65.95%	2.3889	1.9202	22.6876	7.16%
40%	0.4033	0.5405	6.1829	69.08%	2.5010	1.9433	21.3135	9.26%

α	c_1^*	c_1^{FS}	c_0	Leverage	x^{FB}	x^{SB}	Ω_{SB}	%$\Delta\Omega$
25%	0.3278	0.4170	5.4811	66.32%	2.2726	1.9021	24.2739	7.11%
30%	0.2883	0.3740	5.0568	64.28%	2.2808	1.9006	24.1535	6.10%
35%	0.2559	0.3389	4.7006	62.41%	2.2884	1.8988	24.0435	5.27%
40%	0.2290	0.3098	4.3989	60.68%	2.2953	1.8975	23.9424	4.56%
45%	0.2061	0.2853	4.1363	59.08%	2.3017	1.8955	23.8492	3.97%

I	c_1^*	c_1^{FS}	c_0	Leverage	x^{FB}	x^{SB}	Ω_{SB}	%$\Delta\Omega$
10	0.2559	0.3389	2.3503	62.41%	1.1442	0.9494	29.2962	5.27%
15	0.2559	0.3389	3.5254	62.41%	1.7163	1.4241	26.0984	5.27%
20	0.2559	0.3389	4.7006	62.41%	2.2884	1.8988	24.0435	5.27%
25	0.2559	0.3389	5.8757	62.41%	2.8605	2.3735	22.5616	5.27%
30	0.2559	0.3389	7.0508	62.41%	3.4325	2.8482	21.4190	5.27%

Chapter 5

Conclusion

This thesis contributes to the literature in the interface of real options and corporate finance. The first essay examines the impact of time-to-build on a levered firm's investment decision. We show that the relation between investment trigger and implementation lag depends largely on the construction cost distribution。 For an optimally-levered firm, the investment trigger can be increasing or decreasing in lag when the initial investment fraction is large albeit the magnitude is relative small; otherwise it is a decreasing function of lag. The result implicates that, if the firm uses leverage optimally, the implementation lag will generally have either a positive effect or an insignificant effect on investment (unlike for an unlevered firm). In this regard, this is good news for investment because most projects have some implementation lag.

The second essay develops a continuous-time model of the firm’s sequential investments when the future growth opportunity may arrive with uncertainty. The existing literature which studies firm's investment decisions neglects the arrival uncertainty of future growth opportunities. Our results show that for unlevered firms, the future growth uncertainty has an economically significant impact on the initial investment decision. This impact is non-monotonic, depending on how profitable and costly the growth opportunity will be. When the firm issues both debt and equity to finance the first-stage investment, we show that the financing decision can be greatly affected by the interaction between the arrival possibility and growth size. The relationship between optimal capital structure and level of arrival uncertainty also presents a non-monotonic shape. My findings shed light on the empirical testing of agency cost of underinvestment; there has been debate on the regression between growth option and financial leverage to predict the debt overhang problem. However, as our model implies, the inverse relation between growth option and market leverage is not a secure signal of debt overhang since the degree to which the growth opportunity can be realized also plays an important role. Thus demands are required for a refinement of testing some subset of companies.

The third essay shows that Performance-Sensitive-Debt (PSD) can dampen or even totally eliminate the over-investment agency problem between equity- and debt- holders, which happens due to the fact that traditional fixed coupon bond normally induce equity holders to investment earlier (e.g. lower investment trigger) . When the risk compensation factor is high enough, the agency conflict can be eliminated. However, such risk compensation level is not necessarily the same one as to maximize post-investment equity value. We show further that the optimal or shareholder-wealth-maximizing level of risk-compensation is smaller than that required to eliminate overinvestment for all range of parameters. Thus, our results indicate that it is optimal for shareholders to bear some agency cost of early investment to reap the tax benefits resulting from early investment.